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## CHAPTER 2

### Differentiation

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# CHAPTER 2

## Differentiation

### Section 2.1 The Derivative and the Slope of a Graph

#### Skills Warm Up

1.  $P(3, 1), Q(3, 6)$   
 $\frac{6-1}{3-3}$

$m = \frac{6-1}{3-3}$ ;  $m$  is undefined.

2.  $P(2, 2), Q = (-5, 2)$

$m = \frac{2-2}{-2-2} = 0$

$-5-2$   
 $-2 = 0(x-2)$   
 $y = 2$

$P(1, 5), Q(4, -1)$

$m = \frac{-1-5}{4-1} = \frac{-6}{3} = -2$

$4-13$   
 $-5 = -2(x-1)$   
 $y-5 = -2x+2$

$y = -2x+7$

$P(3, 5), Q(-1, -7)$

$m = \frac{-7-5}{-1-3} = \frac{-12}{-4} = 3$

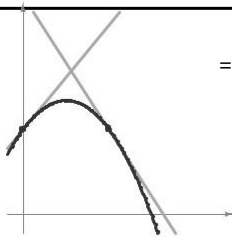
$y-5 = 3(x-3)$

$y = 3x-4$

5.  $\lim_{x \rightarrow 0} \frac{2x^2 - x + (x)^2}{x} = \lim_{x \rightarrow 0} x(2x + x)$

$\lim_{x \rightarrow 0} \frac{2x^2 - x + x^2}{x} = \lim_{x \rightarrow 0} \frac{3x^2 - x}{x}$

$= 2x$



1.

6.  $\lim_{x \rightarrow 0} \frac{3x^2 - x + 3x(x)^2 + (x)^3}{x}$

$\lim_{x \rightarrow 0} \frac{3x^2 - x + 3x^3 + x^3}{x}$

$= \lim_{x \rightarrow 0} \frac{3x^2 - x + 4x^3}{x}$

$\frac{-0}{-0} = \frac{3x^2}{x}$

7.  $\lim_{x \rightarrow 0} \frac{1}{x^2} = \frac{1}{x^2}$

8.  $\lim_{x \rightarrow 0} \frac{(x+x)^2 - x^2}{x} = \frac{x^2 + 2x + x^2 - x^2}{x}$

$= \lim_{x \rightarrow 0} \frac{2x + x^2}{x}$

$= \lim_{x \rightarrow 0} \frac{2x + x^2}{x}$

$= \lim_{x \rightarrow 0} \frac{x(2+x)}{x}$

$2x$

$f(x) = 3x$

Domain:  $(-\infty, \infty)$

10.  $f(x) = \frac{1}{-1}$

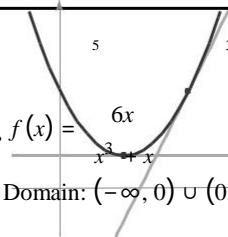
Domain:  $(-\infty, 1) \cup (1, \infty)$

11.  $f(x) = \frac{1}{x^3} - 2x^2 + \frac{1}{x-1}$

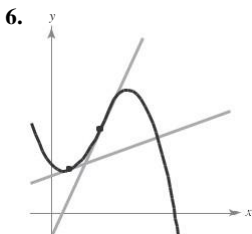
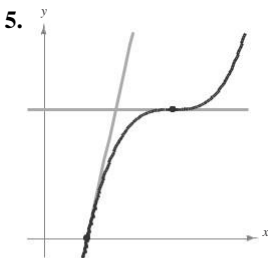
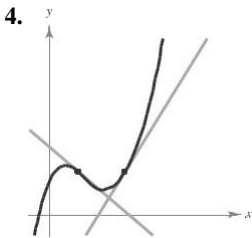
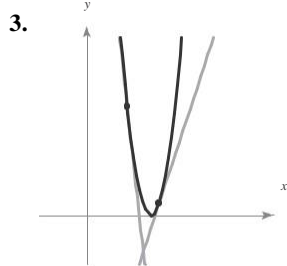
12.  $f(x) = \frac{6x}{x^3 + x}$

Domain:  $(-\infty, 0) \cup (0, \infty)$

2.







7. The slope is  $m = 1$ .

8. The slope is  $m = \frac{4}{3}$ .

9. The slope is  $m = 0$ .

10. The slope is  $m = \frac{1}{4}$ .

11. The slope is  $m = -\frac{1}{3}$ .

12. The slope is  $m = -3$ .

13. 2009:  $m \approx 118$

2011:  $m \approx 375$

The slope is the rate of change in millions of dollars per year of revenue for the years 2009 and 2011 for Under Armour.

2010:  $m \approx 500$

2012:  $m \approx 500$

The slope is the rate of change in millions of dollars per year of sales for the years 2010 and 2012 for Fossil.

15.  $t = 3$ :  $m \approx 8$

$= 7$ :  $m \approx 1$

$t = 10$ :  $m \approx -10$

The slope is the rate of change of the average temperature in degrees Fahrenheit per month in Bland, Virginia, for March, July, and October.

16. (a) At  $t_1$ ,  $f'(t_1) > g'(t_1)$ , so the runner given by  $f$  is running faster.

(b) At  $t_2$ ,  $g'(t_2) > f'(t_2)$ , so the runner given by  $g$  is running faster. The runner given by  $f$  has traveled farther.

(c) At  $t_3$ , the runners are at the same location, but the runner given by  $g$  is running faster.

(d) The runner given by  $g$  will finish first because that runner finishes the distance at a lesser value of  $t$ .

17.  $f(x) = -1$  at  $(0, -1)$

$$m_{\text{sec}} = \frac{f(0+x) - f(0)}{x} = \frac{-1 - (-1)}{x} = \frac{0}{x} = 0$$

$$m = \lim_{x \rightarrow 0} m_{\text{sec}} = \lim_{x \rightarrow 0} 0 = 0$$

18.  $f(x) = 6$  at  $(-2, 6)$

$$m_{\text{sec}} = \frac{f(-2+x) - f(-2)}{x} = \frac{6 - 6}{x} = \frac{0}{x} = 0$$

$$m = \lim_{x \rightarrow 0} m_{\text{sec}} = \lim_{x \rightarrow 0} 0 = 0$$

19.  $f(x) = 13 - 4x$  at  $(3, 1)$

$$m_{\text{sec}} = \frac{f(3 + \Delta x) - f(3)}{\Delta x}$$

$$= \frac{13 - 4(3 + \Delta x) - 1}{\Delta x}$$

$$= \frac{13 - 12 - 4\Delta x - 1}{\Delta x}$$

$$= \frac{-4\Delta x}{\Delta x}$$

$$= -4$$

$m = \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} -4 = -4$

$f(x) = 6x + 3$  at  $(1, 9)$

$$m_{\text{sec}} = \frac{f(1 + \Delta x) - f(1)}{\Delta x}$$

$$= \frac{6(1 + \Delta x) + 3 - 9}{\Delta x}$$

$$= \frac{6 + 6\Delta x + 3 - 9}{\Delta x}$$

$$= \frac{6\Delta x}{\Delta x} = 6$$

$m = \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} 6 = 6$

$f(x) = 2x^2 - 3$  at  $(2, 5)$

$$m_{\text{sec}} = \frac{f(2 + \Delta x) - f(2)}{\Delta x}$$

$$= \frac{2(2 + \Delta x)^2 - 3 - 5}{\Delta x}$$

$$= \frac{2(4 + 4\Delta x + \Delta x^2) - 8}{\Delta x}$$

$$= \frac{8 + 8\Delta x + 2\Delta x^2 - 8}{\Delta x}$$

$$= \frac{8\Delta x + 2\Delta x^2}{\Delta x}$$

$$= 8 + 2\Delta x$$

$f(x) = 11 - x^2$  at  $(3, 2)$

$$m_{\text{sec}} = \frac{f(3 + \Delta x) - f(3)}{\Delta x}$$

$$= \frac{11 - (3 + \Delta x)^2 - 2}{\Delta x}$$

$$= \frac{11 - 9 - 6\Delta x - \Delta x^2 - 2}{\Delta x}$$

$$= \frac{-6\Delta x - \Delta x^2}{\Delta x}$$

$$= -6 - \Delta x$$

$m = \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} (-6 - \Delta x) = -6$

$f(x) = x^3 - 4x$  at  $(-1, 3)$

$$m_{\text{sec}} = \frac{f(-1 + \Delta x) - f(-1)}{\Delta x}$$

$$= \frac{(-1 + \Delta x)^3 - 4(-1 + \Delta x) - (-1) + 4}{\Delta x}$$

$$= \frac{-1 + 3\Delta x - 3(\Delta x)^2 + (\Delta x)^3 + 4 - 4\Delta x + 4}{\Delta x}$$

$$= \frac{-\Delta x - 3(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$

$$= -1 - 3\Delta x + (\Delta x)^2$$

$m = \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} (-1 - 3\Delta x + (\Delta x)^2) = -1$

$f(x) = 7x - x^3$  at  $(-3, 6)$

$$m_{\text{sec}} = \frac{f(-3 + \Delta x) - f(-3)}{\Delta x}$$

$$= \frac{7(-3 + \Delta x) - (-3 + \Delta x)^3 - 7(-3) - (-3)^3}{\Delta x}$$

=

$x$

$$m = \lim_{x \rightarrow 0} \frac{m}{\sec} = \lim_{x \rightarrow 0} \frac{24 + x}{( \quad )} = 8$$

$$\frac{-21 + 7x - \left( -27 + 27x - 9(x)^2 + (x)^3 \right) - 6}{x}$$

$$\frac{-20x + 9x^2 - x^3}{x}$$

$$= \frac{-20 + 9x - (x)^2}{x}$$

$$m = \lim_{x \rightarrow 0} \frac{m}{\sec} = \lim_{x \rightarrow 0} \left( -20 + 9x - (x)^2 \right) = -20$$



$$= \frac{1}{\sqrt{9} + 3}$$
$$\frac{1}{6}$$



$$g(s) = \frac{1}{3}s + 2$$

$$g'(s) = \lim_{\Delta s \rightarrow 0} \frac{g(s + \Delta s) - g(s)}{\Delta s}$$

$$g(s) = \frac{1}{3}(s + \Delta s) + 2 = \frac{1}{3}s + \frac{1}{3}\Delta s + 2$$

$$= \lim_{\Delta s \rightarrow 0} \frac{\frac{1}{3}s + \frac{1}{3}\Delta s + 2 - (\frac{1}{3}s + 2)}{\Delta s}$$

$$= \lim_{\Delta s \rightarrow 0} \frac{\frac{1}{3}\Delta s}{\Delta s} = \lim_{\Delta s \rightarrow 0} \frac{1}{3}$$

$$= \lim_{\Delta s \rightarrow 0} \frac{1}{3}$$

$$= \frac{1}{3}$$

$$\frac{1}{3}$$

$$f(x) = 4x^2 - 5x$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{4(x + \Delta x)^2 - 5(x + \Delta x) - (4x^2 - 5x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{4(x^2 + 2x\Delta x + (\Delta x)^2) - 5x - 5\Delta x - 4x^2 + 5x}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{4x^2 + 8x\Delta x + 4(\Delta x)^2 - 5x - 5\Delta x - 4x^2 + 5x}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{4x^2 + 8x\Delta x + 4(\Delta x)^2 - 5x - 5\Delta x - 4x^2 + 5x}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{4x^2 + 8x\Delta x + 4(\Delta x)^2 - 5x - 5\Delta x - 4x^2 + 5x}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} (4x + 4\Delta x - 5) = 4x - 5$$

$$f(x) = 2x^2 + 7x$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{2(x + \Delta x)^2 + 7(x + \Delta x) - (2x^2 + 7x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{2(x^2 + 2x\Delta x + (\Delta x)^2) + 7x + 7\Delta x - 2x^2 - 7x}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 + 7x + 7\Delta x - 2x^2 - 7x}{\Delta x}$$

$$h(t) = 6 - \frac{1}{2}t$$

$$h'(t) = \lim_{\Delta t \rightarrow 0} \frac{h(t + \Delta t) - h(t)}{\Delta t}$$

$$= \lim_{\Delta t \rightarrow 0} \frac{6 - \frac{1}{2}(t + \Delta t) - (6 - \frac{1}{2}t)}{\Delta t}$$

$$= \lim_{\Delta t \rightarrow 0} \frac{6 - \frac{1}{2}t - \frac{1}{2}\Delta t - 6 + \frac{1}{2}t}{\Delta t}$$

$$= \lim_{\Delta t \rightarrow 0} \frac{-\frac{1}{2}\Delta t}{\Delta t}$$

$$= \lim_{\Delta t \rightarrow 0} -\frac{1}{2}$$

$$\frac{1}{2}$$

$$= -\frac{1}{2}$$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{2x^2 + x + \frac{7}{2}}{x} \\ &= \lim_{x \rightarrow 0} \left( 2x + x + \frac{7}{2} \right) = 4x + 7 \end{aligned}$$

$$35. h(t) = \sqrt{t-3}$$

$$\begin{aligned} h'(t) &= \lim_{t \rightarrow 0} \frac{h(t+t) - h(t)}{t} \\ &= \lim_{t \rightarrow 0} \frac{\sqrt{t+t-3} - \sqrt{t-3}}{t} \\ &= \lim_{t \rightarrow 0} \frac{\sqrt{t+t-3} - \sqrt{t-3}}{t} \cdot \frac{\sqrt{t+t-3} + \sqrt{t-3}}{\sqrt{t+t-3} + \sqrt{t-3}} \\ &= \lim_{t \rightarrow 0} \frac{t+t-3 - (t-3)}{t(\sqrt{t+t-3} + \sqrt{t-3})} \\ &= \lim_{t \rightarrow 0} \frac{t}{t(\sqrt{t+t-3} + \sqrt{t-3})} \\ &= \lim_{t \rightarrow 0} \frac{1}{\sqrt{t+t-3} + \sqrt{t-3}} \\ &= \frac{1}{2\sqrt{t-3}} \\ &= \frac{\sqrt{t-3}}{2t-3} \end{aligned}$$

$$36. f(x) = \sqrt{x+2}$$

$$\begin{aligned} f'(x) &= \lim_{x \rightarrow 0} \frac{f(x+x) - f(x)}{x} \\ &= \lim_{x \rightarrow 0} \frac{\sqrt{x+x+2} - \sqrt{x+2}}{x} \\ &= \lim_{x \rightarrow 0} \frac{\sqrt{x+x+2} - \sqrt{x+2}}{x} \cdot \frac{\sqrt{x+x+2} + \sqrt{x+2}}{\sqrt{x+x+2} + \sqrt{x+2}} \\ &= \lim_{x \rightarrow 0} \frac{x+x+2 - (x+2)}{x(\sqrt{x+x+2} + \sqrt{x+2})} \\ &= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+x+2} + \sqrt{x+2})} \\ &= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+x+2} + \sqrt{x+2}} \\ &= \frac{1}{2\sqrt{x+2}} \end{aligned}$$

37.  $f(t) = t^3 - 12t$

$$\begin{aligned}
 f'(t) &= \lim_{t \rightarrow 0} \frac{f(t+h) - f(t)}{h} \\
 &= \lim_{t \rightarrow 0} \frac{(t+h)^3 - 12(t+h) - (t^3 - 12t)}{h} \\
 &= \lim_{t \rightarrow 0} \frac{t^3 + 3t^2h + 3t(h)^2 + (h)^3 - 12t - 12h - t^3 + 12t}{h} \\
 &= \lim_{t \rightarrow 0} \frac{3t^2h + 3t(h)^2 + (h)^3 - 12h}{h} \\
 &= \lim_{t \rightarrow 0} \frac{h(3t^2 + 3th + (h)^2 - 12)}{h} \\
 &= \lim_{t \rightarrow 0} (3t^2 + 3th + (h)^2 - 12)
 \end{aligned}$$

38.  $f(t) = t^3 + t^2$

$$\begin{aligned}
 f'(t) &= \lim_{t \rightarrow 0} \frac{f(t+h) - f(t)}{h} \\
 &= \lim_{t \rightarrow 0} \frac{(t+h)^3 + (t+h)^2 - (t^3 + t^2)}{h} \\
 &= \lim_{t \rightarrow 0} \frac{t^3 + 3t^2h + 3t(h)^2 + (h)^3 + t^2 + 2th + (h)^2 - t^3 - t^2}{h} \\
 &= \lim_{t \rightarrow 0} \frac{3t^2h + 3t(h)^2 + (h)^3 + 2th + (h)^2}{h} \\
 &= \lim_{t \rightarrow 0} \frac{h(3t^2 + 3th + (h)^2 + 2t + h)}{h} \\
 &= \lim_{t \rightarrow 0} (3t^2 + 3th + (h)^2 + 2t + h)
 \end{aligned}$$

39.  $f(x) = \frac{1}{x+2}$

$$\begin{aligned}
 f'(x) &= \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{x \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x+2}}{h} \\
 &= \lim_{x \rightarrow 0} \frac{\frac{1}{x+h} \cdot \frac{x+2}{x+2} - \frac{1}{x+2} \cdot \frac{x+h}{x+h}}{h} \\
 &= \lim_{x \rightarrow 0} \frac{\frac{x+2 - (x+h)}{(x+h)(x+2)}}{h} \\
 &= \lim_{x \rightarrow 0} \frac{-h}{x(x+h)(x+2)} \\
 &= \lim_{x \rightarrow 0} \frac{-1}{(x+h)(x+2)} \\
 &= \frac{-1}{(x+2)^2}
 \end{aligned}$$

40.  $g(s) = \frac{1}{s-4}$

$$g'(s) = \lim_{\Delta s \rightarrow 0} \frac{g(s+\Delta s) - g(s)}{\Delta s}$$

$$= \lim_{\Delta s \rightarrow 0} \frac{\frac{1}{s+\Delta s-4} - \frac{1}{s-4}}{\Delta s}$$

$$= \lim_{\Delta s \rightarrow 0} \frac{s - (s + \Delta s - 4)}{(s + \Delta s - 4)(s - 4)\Delta s}$$

$$= \lim_{\Delta s \rightarrow 0} \frac{-\Delta s}{(s + \Delta s - 4)(s - 4)\Delta s}$$

$$= \lim_{\Delta s \rightarrow 0} \frac{-1}{(s + \Delta s - 4)(s - 4)}$$

$$= \lim_{\Delta s \rightarrow 0} \frac{-1}{(s + \Delta s - 4)(s - 4)}$$

$$= \lim_{\Delta s \rightarrow 0} \frac{-1}{(s + \Delta s - 4)(s - 4)}$$

$$= \lim_{\Delta s \rightarrow 0} \frac{-1}{(s + \Delta s - 4)(s - 4)}$$

$$= \lim_{\Delta s \rightarrow 0} \frac{-1}{(s + \Delta s - 4)(s - 4)}$$

$$= -\frac{1}{(s-4)^2}$$

42.  $f(x) = -\frac{1}{8}x^2$  at  $(-4, -2)$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{8}(x+\Delta x)^2 - (-\frac{1}{8}x^2)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{8}(x^2 + 2x\Delta x + (\Delta x)^2) + \frac{1}{8}x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{8}x^2 - \frac{1}{4}x\Delta x - \frac{1}{8}(\Delta x)^2 + \frac{1}{8}x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{4}x\Delta x - \frac{1}{8}(\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{4}x - \frac{1}{8}\Delta x}{1}$$

$$= \lim_{\Delta x \rightarrow 0} (-\frac{1}{4}x - \frac{1}{8}\Delta x)$$

$$= -\frac{1}{4}x$$

$$m = f'(-4) = -\frac{1}{4}(-4) = 1$$

$$-(-2) = 1x - (-$$

41.  $f(x) = \frac{1}{2}x^2$  at  $(2, 2)$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{2}(x+\Delta x)^2 - \frac{1}{2}x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{2}(x^2 + 2x\Delta x + (\Delta x)^2) - \frac{1}{2}x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{2}x^2 + x\Delta x + \frac{1}{2}(\Delta x)^2 - \frac{1}{2}x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x\Delta x + \frac{1}{2}(\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x + \frac{1}{2}\Delta x}{1}$$

$$= \lim_{\Delta x \rightarrow 0} (x + \frac{1}{2}\Delta x)$$

$$= x + \frac{1}{2}\Delta x$$

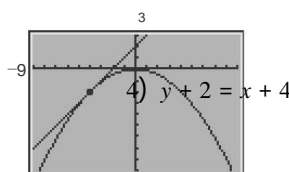
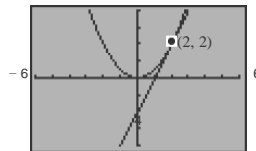
$$= \lim_{\Delta x \rightarrow 0} (x + \frac{1}{2}\Delta x)$$

$$= x$$

$$= f'(2) = 2$$

$$-2 = 2(x-2)$$

$$y = 2x - 2$$



$(-4, -2)$

$$y = x + 2$$

Chapter 2 Differentiation

43.  $f(x) = (x - 1)^2$  at  $(-2, 9)$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{(x + \Delta x - 1)^2 - (x - 1)^2}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - 2x + 1 - x^2 + 2x - 1}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{2x\Delta x + \Delta x^2 - 2\Delta x}{\Delta x}$$

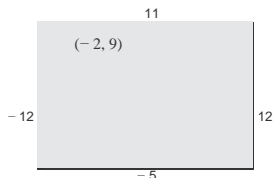
$$= \lim_{x \rightarrow 0} \frac{\Delta x(2x + \Delta x - 2)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} (2x + \Delta x - 2)$$

$$= 2x - 2$$

$m = f'(-2) = 2(-2) - 2 = -6$

$y - 9 = -6x - (-2)$   
 $y = -6x - 3$



44.  $f(x) = 2x^2 - 5$  at  $(-1, -3)$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{2(x + \Delta x)^2 - 5 - (2x^2 - 5)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 - 5 - 2x^2 + 5}{\Delta x}$$

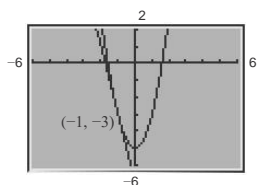
$$= \lim_{x \rightarrow 0} \frac{4x\Delta x + 2(\Delta x)^2}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{\Delta x(4x + 2\Delta x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} (4x + 2\Delta x)$$

$$= 4x$$

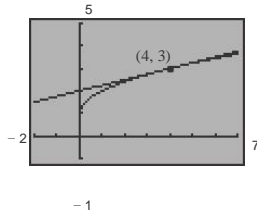
$m = f'(-1) = 4(-1) = -4$   
 $y - (-3) = -4x - (-1)$   
 $+ 3 = -4x - 4$   
 $y = -4x - 7$



$$f(x) = x\sqrt{x+1} \text{ at } (4, 3)$$

$$\begin{aligned} f'(x) &= \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{x \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\ &= \lim_{x \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}} \\ &= \lim_{x \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\ &= \lim_{x \rightarrow 0} \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\ &= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+1} + \sqrt{x+1}} \\ &= \frac{1}{2\sqrt{x+1}} \end{aligned}$$

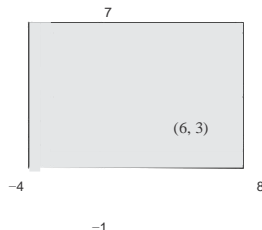
$$\begin{aligned} m = f'(4) &= \frac{1}{2\sqrt{4+1}} = \frac{1}{10} \\ -3 &= \frac{1}{10}(x-4) \\ y - 3 &= \frac{1}{10}x - \frac{4}{10} \\ y &= \frac{1}{10}x + \frac{26}{10} \end{aligned}$$



$$f(x) = x\sqrt{x+3} \text{ at } (6, 3)$$

$$\begin{aligned} f'(x) &= \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{x \rightarrow 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} \\ &= \lim_{x \rightarrow 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} \cdot \frac{\sqrt{x+h+3} + \sqrt{x+3}}{\sqrt{x+h+3} + \sqrt{x+3}} \\ &= \lim_{x \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h+3} + \sqrt{x+3})} \\ &= \lim_{x \rightarrow 0} \frac{h}{h(\sqrt{x+h+3} + \sqrt{x+3})} \\ &= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+3} + \sqrt{x+3}} \\ &= \frac{1}{2\sqrt{x+3}} \end{aligned}$$

$$\begin{aligned} m = f'(6) &= \frac{1}{2\sqrt{6+3}} = \frac{1}{10} \\ y - 3 &= \frac{1}{10}(x-6) \\ y - 3 &= \frac{1}{10}x - \frac{6}{10} \\ y &= \frac{1}{10}x + \frac{24}{10} \end{aligned}$$





Chapter 2 Differentiation

47.  $f(x) = \frac{1}{5x}$  at  $-\frac{1}{5}, -1$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{5(x+\Delta x)} - \frac{1}{5x}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{x - (x + \Delta x)}{5x(x+\Delta x)}}{x}$$

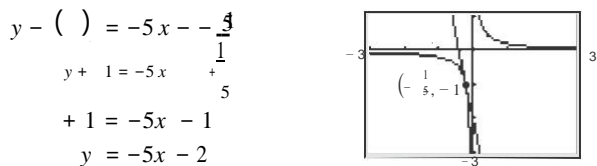
$$= \lim_{x \rightarrow 0} \frac{x - (x + \Delta x)}{5x(x+\Delta x)} \cdot \frac{1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{-\Delta x}{5x \cdot x \cdot (x + \Delta x)}$$

$$= \lim_{x \rightarrow 0} \frac{-1}{5x(x + \Delta x)}$$

$$= -\frac{1}{5x^2}$$

$$m = f'(-\frac{1}{5}) = -\frac{1}{5(-\frac{1}{5})^2} = -\frac{1}{5 \cdot \frac{1}{25}} = -5$$



48.  $f(x) = \frac{1}{x-3}$  at  $(2, -1)$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{x+\Delta x - 3} - \frac{1}{x-3}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{x+\Delta x - 3} \cdot \frac{x-3}{x-3} - \frac{1}{x-3} \cdot \frac{x+\Delta x - 3}{x+\Delta x - 3}}{x}$$

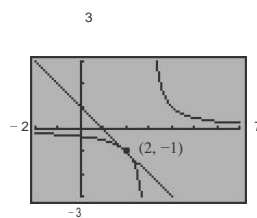
$$= \lim_{x \rightarrow 0} \frac{\frac{x-3 - (x+\Delta x - 3)}{(x+\Delta x - 3)(x-3)}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{-\Delta x}{(x+\Delta x - 3)(x-3)x}$$

$$= \lim_{x \rightarrow 0} \frac{-1}{(x+\Delta x - 3)(x-3)} = -\frac{1}{(x-3)^2}$$

$$m = f'(2) = -\frac{1}{(2-3)^2} = -1$$

$y - (-1) = -1(x - 2)$   
 $y + 1 = -x + 2$   
 $y = -x + 1$



49.  $f(x) = -\frac{1}{4}x^2$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-\frac{1}{4}(x+\Delta x)^2 - (-\frac{1}{4}x^2)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-\frac{1}{4}x^2 - \frac{1}{2}x\Delta x - \frac{1}{4}\Delta x^2 + \frac{1}{4}x^2}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-\frac{1}{2}x\Delta x - \frac{1}{4}\Delta x^2}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-\frac{1}{2}x - \frac{1}{4}\Delta x}{1}$$

$$= \lim_{x \rightarrow 0} -\frac{1}{2}x - \frac{1}{4}$$

$$= -\frac{1}{2}(0) - \frac{1}{4} = -\frac{1}{4}$$

1

Since the slope of the given line is  $-1$ ,  
 $-\frac{1}{2}x = -1$

$$x = 2 \text{ and } f(2) = -\frac{1}{4}$$

At the point  $(2, -\frac{1}{4})$ , the tangent line parallel to

$$x + y = 0 \text{ is } y - (-\frac{1}{4}) = -1(x - 2)$$

$$y = -x + \frac{7}{4}$$

51.  $f(x) = -\frac{1}{3}x^3$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-\frac{1}{3}(x+\Delta x)^3 - (-\frac{1}{3}x^3)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-\frac{1}{3}(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3) + \frac{1}{3}x^3}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-\frac{1}{3}x^3 - x^2\Delta x - x(\Delta x)^2 - \frac{1}{3}(\Delta x)^3 + \frac{1}{3}x^3}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-x^2\Delta x - x(\Delta x)^2 - \frac{1}{3}(\Delta x)^3}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \left( -x^2 - x\Delta x - \frac{1}{3}(\Delta x)^2 \right) = -x^2$$

Since the slope of the given line is  $-9$ ,

$$x^2 = -9$$

50.  $f(x) = x^2 - 7$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{(x+\Delta x)^2 - 7 - (x^2 - 7)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 7 - x^2 + 7}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{x \rightarrow 0} (2x + \Delta x) = 2x$$

Since the slope of the given line is  $-2$ ,

$$2x = -2$$

$$x = -1 \text{ and } f(-1) = -6$$

At the point  $(-1, -6)$ , the tangent line parallel to

$$2x + y = 0 \text{ is}$$

$$-6 = -2x - 1$$

$$y = -2x - 8$$

$$x^2 = 9$$

$$x = \pm 3 \text{ and } f(3) = -9 \text{ and } f(-3) = 9.$$

$$-(-9) = -9(x - 3)$$

$$y = -9x + 18.$$

$$-9 = -9(x - (-3))$$

$$y = -9x - 18.$$

Chapter 2 Differentiation

$$f(x) = x^3 + 2$$

$$\begin{aligned} f'(x) &= \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{x \rightarrow 0} \frac{(x+h)^3 + 2 - (x^3 + 2)}{h} \\ &= \lim_{x \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 2 - x^3 - 2}{h} \\ &= \lim_{x \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ &= \lim_{x \rightarrow 0} (3x^2 + 3xh + h^2) \\ &= 3x^2 \end{aligned}$$

The slope of the given line is

$$3x - y - 4 = 0$$

$$= 3x - 4$$

$$m = 3.$$

$$\begin{aligned} 3x^2 \\ x^2 \end{aligned}$$

$$= 3$$

$$= 1$$

$$= \pm 1$$

$$= 1 \text{ and } f(1) = 3$$

$$= -1 \text{ and } f(-1) = 1$$

$$-3 = 3(x - 1)$$

$$y - 3 = 3x - 3$$

$$= 3x.$$

$$-1 = 3(x - (-1))$$

$$-1 = 3(x + 1)$$

$$y - 1 = 3x + 3$$

$$= 3x + 4.$$

53.  $y$  is differentiable for all  $x \neq -3$ .

At  $(-3, 0)$ , the graph has a node.

54.  $y$  is differentiable for all  $x \neq \pm 3$ .

At  $(\pm 3, 0)$ , the graph has a cusp.

55.  $y$  is differentiable for all  $x \neq -2^{\frac{1}{2}}$ .

At  $(-2^{\frac{1}{2}}, 0)$ , the graph has a vertical tangent line.

56.  $y$  is differentiable for all  $x > 1$ .

The derivative does not exist at endpoints.

57.  $y$  is differentiable for all  $x \neq \pm 2$ .

The function is not defined at  $x = \pm 2$ .

58.  $y$  is differentiable for all  $x \neq 0$ .

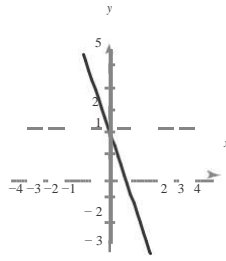
The function is discontinuous at  $x = 0$ .

59. Since  $f'(x) = -3$  for all  $x$ ,  $f$  is a line of the form

$$f(x) = -3x + b.$$

$f(0) = 2$ , so  $2 = (-3)(0) + b$ , or  $b = 2$ .

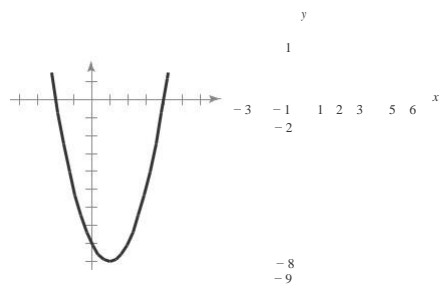
Thus,  $f(x) = -3x + 2$ .



60. Sample answer: Since  $f(-2) = f(4) = 0$ ,  $(x^2 + 2x - 4) = 0$ .

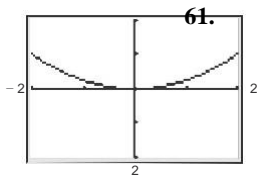
A function with these zeros is  $f(x) = x^2 - 2x - 8$ .

$$\begin{aligned} \text{Then } f'(x) &= \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{x \rightarrow 0} \frac{(x+h)^2 - 2(x+h) - 8 - (x^2 - 2x - 8)}{h} \\ &= \lim_{x \rightarrow 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - 8 - x^2 + 2x + 8}{h} \\ &= \lim_{x \rightarrow 0} \frac{2xh + h^2 - 2h}{h} \\ &= \lim_{x \rightarrow 0} (2x + h - 2) \\ &= 2x - 2. \end{aligned}$$



So  $f'(1) = 2(1) - 2 = 0$ . Sketching  $f(x)$  shows that

$f'(x) < 0$  for  $x < 1$  and  $f'(x) > 0$  for  $x > 1$ .

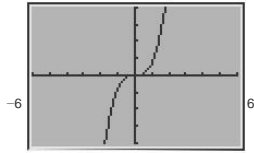


	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$f(x)$	1	0.5625	0.25	0.0625	0	0.0625	0.25	0.5625	1
$f'(x)$	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1

Analytically, the slope of  $f(x) = \frac{1}{4}x^2$  is

$$\begin{aligned} m &= \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{x \rightarrow 0} \frac{\frac{1}{4}(x+h)^2 - \frac{1}{4}x^2}{h} \\ &= \lim_{x \rightarrow 0} \frac{\frac{1}{4}x^2 + \frac{1}{2}xh + \frac{1}{4}h^2 - \frac{1}{4}x^2}{h} \\ &= \lim_{x \rightarrow 0} \frac{\frac{1}{2}xh + \frac{1}{4}h^2}{h} \\ &= \lim_{x \rightarrow 0} \left( \frac{1}{2}x + \frac{1}{4}h \right) \\ &= \frac{1}{2}x. \end{aligned}$$

Chapter 2 Differentiation

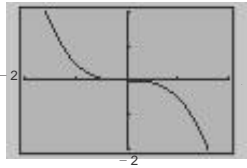


-4

$x$	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	-6	-2.53	-0.75	-0.1	0	0.1	0.75	2.53	6
$f'(x)$	9	5.0625	2.25	0.5625	0	0.5625	2.25	5.0625	9

Analytically, the slope of  $f(x)$  is

$$\begin{aligned}
 m &= \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\
 &= \lim_{x \rightarrow 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x} \\
 &= \lim_{x \rightarrow 0} \frac{(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3) - x^3}{\Delta x} \\
 &= \lim_{x \rightarrow 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x} \\
 &= \lim_{x \rightarrow 0} (3x^2 + 3x\Delta x + (\Delta x)^2) \\
 &= 3x^2
 \end{aligned}$$



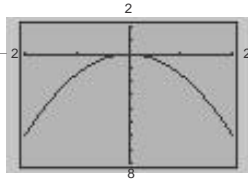
$x$	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	4	1.6875	0.5	0.0625	0	-0.0625	-0.5	-1.6875	-4
$f'(x)$	-6	-3.375	-1.5	-0.5	0	-0.375	-1.5	-3.375	-6

Analytically, the slope of  $f(x)$  is

$$\begin{aligned}
 m &= \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\
 &= \lim_{x \rightarrow 0} \frac{-(x+\Delta x)^3 + 3(x+\Delta x)^2 - 4(x+\Delta x) + 4 - (-x^3 + 3x^2 - 4x + 4)}{\Delta x} \\
 &= \lim_{x \rightarrow 0} \frac{-x^3 - 3x^2\Delta x - 3x(\Delta x)^2 - (\Delta x)^3 + 3x^2 + 6x\Delta x + 3(\Delta x)^2 - 4\Delta x}{\Delta x} \\
 &= \lim_{x \rightarrow 0} (-3x^2 + 3x\Delta x + (\Delta x)^2 - 4) \\
 &= -3x^2 - 4
 \end{aligned}$$

$$-2x^2.$$

64.

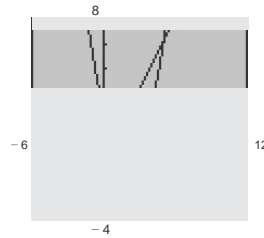


$x$	-2	-1	0	1	2
$f(x)$	-6	-3.375	0	-3.375	-6
$f'(x)$	6	4.5	0	-4.5	-6

Analytically, the slope of  $f(x) = -\frac{3}{2}x^2$  is

$$\begin{aligned}
 m &= \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{x \rightarrow 0} \frac{-\frac{3}{2}(x+h)^2 - (-\frac{3}{2}x^2)}{h} \\
 &= \lim_{x \rightarrow 0} \frac{-\frac{3}{2}x^2 - 3xh - \frac{3}{2}h^2 + \frac{3}{2}x^2}{h} \\
 &= \lim_{x \rightarrow 0} \frac{-3xh - \frac{3}{2}h^2}{h} \\
 &= \lim_{x \rightarrow 0} (-3x - \frac{3}{2}h) \\
 &= -3x
 \end{aligned}$$

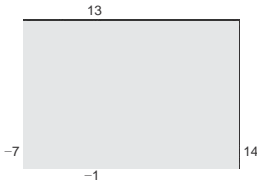
$$\begin{aligned}
 65. f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 4(x+h) - (x^2 - 4x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 4h}{h} \\
 &= \lim_{h \rightarrow 0} (2x + h - 4) \\
 &= 2x - 4
 \end{aligned}$$



The  $x$ -intercept of the derivative indicates a point of horizontal tangency for  $f$ .

66.

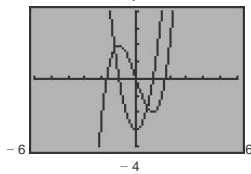
$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= \lim_{x \rightarrow 0} \frac{2 + 6(x+h) - x - (2 + 6x - x^2)}{h} \\
 &= \lim_{x \rightarrow 0} \frac{6x + 6h - x^2}{h} \\
 &= \lim_{x \rightarrow 0} (6 - 2x - x) = 6 - 2x
 \end{aligned}$$



The  $x$ -intercept of the derivative indicates a point of horizontal tangency for  $f$ .

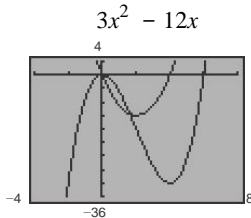


$$\begin{aligned}
 67. f'(x) &= \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{x \rightarrow 0} \frac{(x+h)^3 - 3x^2 + x - (x^3 - 3x^2 - 3x)}{h} \\
 &= \lim_{x \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 3x^2 + x - x^3 + 3x^2 + 3x}{h} \\
 &= \lim_{x \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3 + 3x}{h} \\
 &= \lim_{x \rightarrow 0} (3x^2 + 3xh + h^2 + 3) \\
 &= 3x^2 + 3
 \end{aligned}$$



The  $x$ -intercepts of the derivative indicate points of horizontal tangency for  $f$ .

$$\begin{aligned}
 68. f'(x) &= \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{x \rightarrow 0} \frac{(x+h)^3 - 6x^2 + x - (x^3 - 6x^2)}{h} \\
 &= \lim_{x \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 6x^2 + x - x^3 + 6x^2}{h} \\
 &= \lim_{x \rightarrow 0} (3x^2 + 3xh + h^2 + x) \\
 &= 3x^2 + x
 \end{aligned}$$



The  $x$ -intercepts of the derivative indicate points of horizontal tangency for  $f$ .

69. True. The slope of the graph is given by  $f'(x) = 2x$ ,

which is different for each different  $x$  value.

70. False.  $f(x) = |x|$  is continuous, but not differentiable at  $x = 0$ .

True. See page 122.

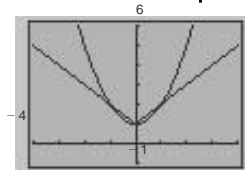
72. True. See page 115.

73. The graph of

2

$f(x) = x + 1$  is smooth at  $(0, 1)$ , but the

graph of  $g(x) = |x + 1|$  has a node at  $(0, 1)$ . The function  $g$  is not differentiable at  $(0, 1)$ .





$$6x^3 =$$

$$3x^3$$

$$4. f(x) = 1 - x^6$$

$$8. \frac{2}{t}$$

$x^6$

$$f'(x) = -6x^{-7} = -\frac{6}{x^7}$$

$$g(t) = \frac{3}{2t}$$

\*

$$9. f(x) = 4x$$

$$5. h(x) = 3x$$

$f'$

$$h(x) = 9x^2$$

Chapter 2 Differentiation

$$g(x) = 3^x = \frac{1}{3}$$

$$x g'(x) = \frac{1}{3}$$

$$y = 8 - x^3$$

$$y' = -3x^2$$

$$y = t^2 - 6$$

$$y' = 2t$$

$$f(x) = 4x^2 - 3x$$

$$f'(x) = 8x - 3$$

$$g(x) = 3x^2 + 5x^3$$

$$g'(x) = 6x + 15x^2 = 15x^2 + 6x$$

$$f(t) = -3t^2 + 2t -$$

$$4f'(t) = -6t + 2$$

$$y = 7x^3 - 9x^2 + 8$$

$$y' = 21x^2 - 18x$$

$$s(t) = 4t^4 - 2t + t + 3$$

$$s'(t) = 16t^3 - 4t + 1$$

$$y = 2x^3 - x^2 + 3x - 1$$

$$y' = 6x^2 - 2x + 3$$

$$g(x) = x^{2/3}$$

$$g'(x) = \frac{2}{3} x^{-1/3} = \frac{2}{3x^{1/3}}$$

$$20. h(x) = x^{5/2}$$

$$h'(x) = \frac{5}{2} x^{3/2}$$

$$21. y = 4t^{4/3}$$

$$y' = 4 \left( \frac{4}{3} \right) t^{1/3} = \frac{16}{3} t^{1/3}$$

$$22. f(x) = 10x^{5/6}$$

$$f'(x) = \frac{5}{6} \cdot 10 x^{-1/6} = \frac{50}{6} x^{-1/6} = \frac{25}{3} x^{-1/6} = \frac{25}{3\sqrt[6]{x}}$$

$$f'(x) = \frac{5}{6} \cdot 10 x^{-1/6} = \frac{50}{6} x^{-1/6} = \frac{25}{3} x^{-1/6} = \frac{25}{3\sqrt[6]{x}}$$

$$y = 4x^{-2} + 2x^2$$

$$y' = -8x^{-3} + 4x = -\frac{8}{x^3} + 4x$$

$$s(t) = 8t^{-4} + t$$

$$s'(t) = 8(-4t^{-5}) + 1 = -\frac{32}{t^5} + 1$$

Function	Rewrite	Differentiate	Simplify
25. $y = \frac{2}{7x^4}$	$y = \frac{2}{7} x^{-4}$	$y' = -\frac{8}{7} x^{-5}$	$y' = -\frac{8}{7x^5}$
26. $y = \frac{2}{3x^2}$	$y = \frac{2}{3} x^{-2}$	$y' = -\frac{4}{3} x^{-3}$	$y' = -\frac{4}{3x^3}$
27. $y = \frac{1}{(4x)^3}$	$y = \frac{1}{64} x^{-3}$	$y' = -\frac{3}{64} x^{-4}$	$y' = -\frac{3}{64x^4}$
28. $y = \frac{\pi}{x^6}$	$y = \frac{\pi}{64} x^{-6}$	$y' = -\frac{6\pi}{64}$	$y' = -\frac{3\pi}{32x^7}$
29. $y = \frac{4}{(2x)^{-5}}$	$y = 128x^5$	$y' = 128(5)x^4$	$y' = 640x^4$
30. $y = \frac{4x}{x^{-3}}$	$y = 4x^4$	$y' = 4(4)x^3$	$y' = 16x^3$
31. $y = 6\sqrt{x}$	$y = 6x^{1/2}$	$y' = 6\left(\frac{1}{2}\right)x^{-1/2}$	$y' = \frac{3}{\sqrt{x}}$
32. $y = \frac{3\sqrt{x}}{4}$	$y = \frac{3}{4} x^{1/2}$	$y' = \frac{3}{4} \cdot \frac{1}{2} x^{-1/2}$	$y' = \frac{3}{8\sqrt{x}}$

$$y' = 8x^3$$

Function	Rewrite	Differentiate	Simplify
33. $y = \frac{1}{7\sqrt[4]{x}}$	$y = \frac{1}{7}x^{-1/4}$	$y' = \frac{1}{7} \cdot \frac{1}{4}x^{-5/4}$	$y' = -\frac{1}{28\sqrt[4]{x^5}}$
34. $y = \frac{3}{2\sqrt{x^3}}$	$y = \frac{3}{2}x^{-3/2}$	$y' = \frac{3}{2} \cdot \frac{3}{2}x^{-5/2}$	$y' = -\frac{9}{4\sqrt{x^5}}$
35. $y = \sqrt[5]{8x}$	$y = (8x)^{1/5} = 8^{1/5}x^{1/5}$	$y' = 8^{1/5} \cdot \frac{1}{5}x^{-4/5}$	$y' = \frac{8^{1/5}}{5\sqrt[5]{x^4}} = \frac{2\sqrt[5]{6}}{5\sqrt[5]{x^4}}$
36. $y = \frac{\sqrt[3]{x^2}}{6x}$	$y = \frac{x^{2/3}}{6x} = \frac{1}{6}x^{-1/3}$	$y' = \frac{1}{6} \cdot \frac{1}{3}x^{-4/3}$	$y' = \frac{1}{18\sqrt[3]{x^4}}$

At the point (1, 1),  $y' = \frac{3}{-2}(1)^{1/2} = -\frac{3}{2} = m$ .

$$y = x^{-1}$$

$$y' = x^{-2} = -\frac{1}{x^2}$$

At the point  $(\frac{3}{4}, \frac{16}{9})$ ,  $y' = -\frac{1}{(\frac{3}{4})^2} = -\frac{16}{9} = m$ .

$$f(t) = t^{-4}$$

$$f'(t) = -4t^{-5} = -\frac{4}{t^5}$$

At the point

$$\left(\frac{1}{2}, 16\right), f' = -\frac{4}{(\frac{1}{2})^5} = -128 = m.$$

$$f(x) = x^{-1/3}$$

$$f'(x) = -\frac{1}{3}x^{-4/3} = -\frac{1}{3x^{4/3}}$$

At the point 8,  $f'(8) = -\frac{1}{3(8)^{4/3}} = -\frac{1}{48} = m$ .

$$f(x) = 2x^3 + 8x^2 - x - 4$$

$$f'(x) = 6x^2 + 16x - 1$$

At the point

$$(-1, 3), f'(-1) = 6(-1)^2 + 16(-1) - 1 = -11 = m.$$

$$f(x) = x^4 - 2x^3 + 5x^2 - 7x$$

$$f'(x) = 4x^3 - 6x^2 + 10x - 7$$

43.  $f(x) = -\frac{1}{2}x + x^2 = -\frac{1}{2}x + x^2$

$$f'(x) = -\frac{1}{2} + 2x$$

44.  $f(x) = 3(5-x)^2 = 75 - 30x + 3x^2$

$$f'(x) = -30 + 6x$$

$$f'(5) = -30 + (6)(5) = 0$$

(a)  $y = -2x^4 + 5x^2 - 3$

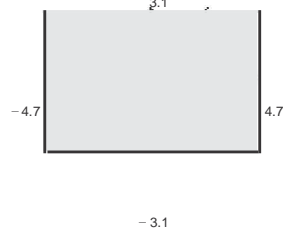
$$y' = -8x^3 + 10x$$

$$m = y'(1) = -8 + 10 = 2$$

The equation of the tangent line is  $y - 0 = 2(x - 1)$

$$= 2x - 2.$$

(b) and (c)



(a)  $y = x^3 + x + 4$

$$y' = 3x^2 + 1$$

$$= y'(-2) = 3(-2)^2 + 1 = 13$$

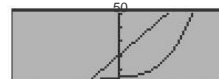
The equation of the tangent line is

$$-(-6) = 13x - (-2)$$

$$y + 6 = 13x + 26$$

$$y = 13x + 20.$$

(b) and (c)



At the point

$$\begin{aligned} & (-1, 15), f'(-1) \\ & = 4(-1)^3 - 6(-1)^2 + 10(-1) - 7 \\ & = -4 - 6 - 10 - 7 = -27 = m. \end{aligned}$$

- 5

5

- 50



Chapter 2 Differentiation

$$47. (a) f(x) = 3\sqrt{x} + 5x = 3x^{1/2} + 5x^{1/1}$$

$$f'(x) = 3 \cdot \frac{1}{2} x^{-1/2} + 5 \cdot 1 x^{-1/1} = \frac{3}{2\sqrt{x}} + 5$$

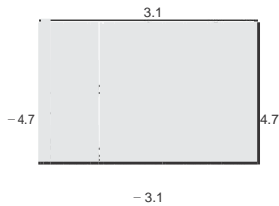
$$m = f'(1) = \frac{3}{2} + 5 = \frac{13}{2}$$

The equation of the tangent line is

$$y - 8 = \frac{13}{2}(x - 1)$$

$$y = \frac{13}{2}x + \frac{3}{2}$$

(b) and (c)



$$48. (a) f(x) = \frac{1}{x^2} - x = x^{-2} - x$$

$$f'(x) = -2x^{-3} - 1 = -\frac{2}{x^3} - 1$$

$$m = f'(-1) = -\frac{2}{(-1)^3} - 1 = 2 - 1 = 1$$

The equation of the tangent line is

$$y - 1 = 1(x + 1)$$

$$y = x + 2$$

(b) and (c)



$$(a) y = 3x^2 - \frac{2}{x}$$

$$y = 3x^3 - 6$$

$$y' = 9x^2$$

$$y' = 9(2)^2 = 36$$

The equation of the tangent line is

$$(a) y = (2x + 1)^2$$

$$= 4x^2 + 4x + 1$$

$$y' = 8x + 4$$

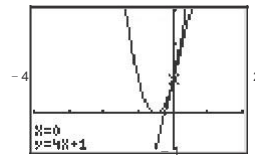
$$y' = 8(0) + 4 = 4$$

The equation of the tangent line is

$$y - 1 = 4(x - 0)$$

$$y = 4x + 1$$

(b) and (c)



$$51. f(x) = x^2 - 4x^{-1} - 3x^{-2}$$

$$f'(x) = 2x + 4x^{-2} + 6x^{-3} = 2x + \frac{4}{x^2} + \frac{6}{x^3}$$

$$52. f(x) = 6x^2 - 5x^{-2} + 7x^{-3}$$

$$f'(x) = 12x + 10x^{-3} - 21x^{-4} = 12x + \frac{10}{x^3} - \frac{21}{x^4}$$

$$f'(1) = 12(1) + \frac{10}{1^3} - \frac{21}{1^4} = 12 + 10 - 21 = 1$$

$$f(x) = x^2 - 2x - \frac{2}{x^4} = x^2 - 2x - 2x^{-4}$$

$$f'(x) = 2x - 2 + 8x^{-5} = 2x - 2 + \frac{8}{x^5}$$

$$f'(1) = 2(1) - 2 + \frac{8}{1^5} = 0 + 8 = 8$$

$$54. f(x) = x^2 + 4x + \frac{1}{x} = x^2 + 4x + x^{-1}$$

$$f'(x) = 2x + 4 - x^{-2} = 2x + 4 - \frac{1}{x^2}$$

$$f(x) = x^{4/5} + x$$

$$f'(x) = \frac{4}{5}x^{-1/5} + 1 = \frac{4}{5x^{1/5}} + 1$$

$$f(x) = x^{1/3} - 1$$

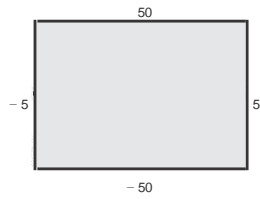
$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

$$57. f(x) = x(x^2 + 1) = x^3 + x$$

$$-18 = 36(x - 2)$$

$$y = 36x - 54.$$

(b) and (c)



$$f(x) = 3x^2 + 1$$

58.  $f(x) = (x^2 + 2x)(x + 1) = x^3 + 3x^2 + 2x$

$$f'(x) = 3x^2 + 6x + 2$$

$$59. f(x) = \frac{2x^3 - 4x^2 + 3}{x^2} = 2x - 4 + 3x^{-2}$$

$$f'(x) = 2 - 6x^{-3} = 2 - \frac{6}{x^3} = \frac{2x^3 - 6}{x^3}$$

$$60. f(x) = \frac{2x^2 - 3x + 1}{x} = 2x - 3 + x^{-1}$$

$$f'(x) = 2 - x^{-2} = 2 - \frac{1}{x^2} = \frac{2x^2 - 1}{x^2}$$

$$61. f(x) = \frac{4x^3 - 3x^2 + 2x + 5}{x^3} = 4x - 3 + 2x^{-1} + 5x^{-2}$$

$$f'(x) = 4 - 2x^{-2} - 10x^{-3} = 4 - \frac{2}{x^2} - \frac{10}{x^3} = \frac{4x^3 - 2x - 10}{x^3}$$

$$62. f(x) = \frac{-6x^4 + 3x^2 - 2x + 1}{x} = -6x^3 + 3x - 2 + x^{-1}$$

$$f'(x) = -12x^2 + 3 - x^{-2} = -12x^2 + 3 - \frac{1}{x^2}$$

$$63. y = x^4 - 2x + 3$$

$$y' = 4x^3 - 2 = 4x(x^2 - 1) = 0 \text{ when } x = 0, \pm 1$$

$$\text{If } x = \pm 1, \text{ then } y = (\pm 1)^4 - 2(\pm 1) + 3 = 2.$$

The function has horizontal tangent lines at the points

$(0, 3)$ ,  $(1, 2)$ , and  $(-1, 2)$ .

$$y = x^3 + 3x^2$$

$$y' = 3x^2 + 6x = 3x(x + 2) = 0 \text{ when } x = 0, -2.$$

The function has horizontal tangent lines at the points  $(0, 0)$  and  $(-2, 4)$ .

$$y = \frac{1}{2}x^2 + 5x$$

$$y' = x + 5 = 0 \text{ when } x = -5.$$

The function has a horizontal tangent line at the point  $(-5, -\frac{25}{2})$ .

$$y = x^2 + 2x$$

$$y' = 2x + 2 = 0 \text{ when } x = -1.$$

The function has a horizontal tangent line at the point  $(-1, -1)$ .

$$67. y = x^2 + 3$$

$$y' = 2x$$

$$\text{Set } y' = 4.$$

$$2x = 4$$

$$x = 2$$

$$\text{If } x = 2, y = (2)^2 + 3 = 7 \rightarrow (2, 7).$$

The graph of  $y = x^2 + 3$  has a tangent line with slope  $m = 4$  at the point  $(2, 7)$ .

$$68. y = x^2 + 2x$$

$$y' = 2x + 2$$

$$\text{Set } y' = 10.$$

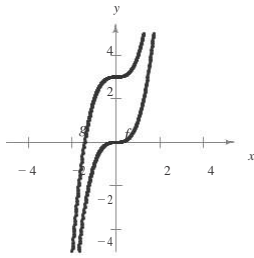
$$2x + 2 = 10$$

$$x = 4$$

$$\text{If } x = 4, y = (4)^2 + 2(4) = 24 \rightarrow (4, 24).$$

The graph of  $y = x^2 + 2x$  has a tangent line with slope  $m = 10$  at the point  $(4, 24)$ .

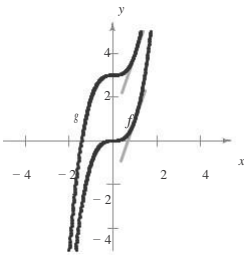
69. (a)



(b)  $f'(x) = g'(x) = 3x^2$   
 $f'(1) = g'(1) = 3$

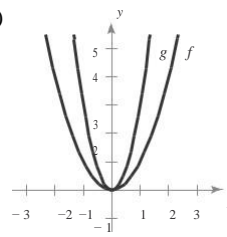
(c) Tangent line to  $f$  at  $x = 1$ :  
 $f(1) = 1$   
 $-1 = 3(x - 1)$   
 $y = 3x - 2$

Tangent line to  $g$  at  $x = 1$ :  
 $g(1) = 4$   
 $y - 4 = 3(x - 1)$   
 $= 3x + 1$



(d)  $f'$  and  $g'$  are the same.

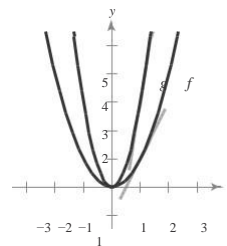
70. (a)



(b)  $f'(x) = 2x$   
 $f'(1) = 2$   
 $g'(x) = 6x$   
 $g'(1) = 6$

(c) Tangent line to  $f$  at  $x = 1$ :  
 $f(1) = 1$   
 $y - 1 = 2(x - 1)$   
 $y = 2x - 1$

Tangent line to  $g$  at  $x = 1$ :  
 $g(1) = 3$   
 $-3 = 6(x - 1)$   
 $y = 6x - 3$



(d)  $g'$  is 3 times  $f'$ .

71. If  $g(x) = f(x) + 6$ , then  $g'(x) = f'(x)$  because the derivative of a constant is 0,  $g'(x) = f'(x)$ .

If  $g(x) = 2f(x)$ , then  $g'(x) = 2f'(x)$  because of the Constant Multiple Rule.

If  $g(x) = -5f(x)$ , then  $g'(x) = -5f'(x)$  because of the Constant Multiple Rule.

74. If  $g(x) = 3f(x) - 1$ , then  $g'(x) = 3f'(x)$  because of the Constant Multiple Rule and the derivative of a constant is 0.

(a)  $R = -4.1685t^3 + 175.037t^2 - 1950.88t + 7265.3$   
 $R' = -12.5055t^2 + 350.074t - 1950.88$

2009:  $R'(9) = -12.5055(9)^2 + 350.074(9) - 1950.88 \approx \$186.8$  million per year

2011:  $R'(11) = -12.5055(11)^2 + 350.074(11) - 1950.88 \approx \$386.8$  million per year

These results are close to the estimates in Exercise 13 in Section 2.1.

The slope of the graph at time  $t$  is the rate at which sales are increasing in millions of dollars per year.

$$(a) R = -2.67538t^4 + 94.0568t^3 - 1155.203t^2 + 6002.42t - 9794.2$$

$$R' = -10.70152t^3 + 282.1704t^2 - 2310.406t + 6002.42$$

$$2010: R'(10) = -10.70152(10)^3 + 282.1704(10)^2 - 2310.406(10) + 6002 \approx \$413.88 \text{ million per year}$$

$$2012: R'(12) = -10.70152(12)^3 + 282.1704(12)^2 - 2310.406(12) + 6002 \approx \$417.86 \text{ million per year}$$

These results are close to the estimates in Exercise 14 in Section 2.1.

The slope of the graph at time  $t$  is the rate at which sales are increasing in millions of dollars per year.

77. (a) More men and women seem to suffer from migraines between 30 and 40 years old. More females than

males suffer from migraines. Fewer people whose income is greater than or equal to \$30,000 suffer from migraines than people whose income is less than \$10,000.

The derivatives are positive up to approximately 37 years old and negative after about 37 years of age. The percent of adults suffering from migraines increases up to about 37 years old, then decreases. The units of the derivative are percent of adults suffering from migraines per year.

- (a) The attendance rate for football games,  $g'(t)$ , is greater at game 1.

- (b) The attendance rate for basketball games,  $f'(t)$ ,

is greater than the rate for football games,  $g'(t)$ , at game 3.

- (c) The attendance rate for basketball games,  $f'(t)$ ,

is greater than the rate for football games,  $g'(t)$ , at game 4. In addition, the attendance rate for football games is decreasing at game 4.

At game 5, the attendance rate for football continues to increase, while the attendance rate for basketball continues to decrease.

$$C = 7.75x + 500$$

$$C' = 7.75, \text{ which equals the variable cost.}$$

$$C = 150x + 7000$$

$$P = R - C$$

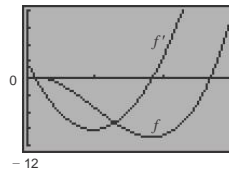
$$P = 500x - (150x + 7000)$$

$$P = 350x - 7000$$

$$P' = 350, \text{ which equals the profit on each dinner sold.}$$

81.  $f(x) = 4.1x^3 - 12x^2 + 2.5x$

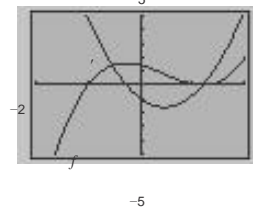
$$f'(x) = 12.3x^2 - 24x + 2.5$$



$f$  has horizontal tangents at  $(0.110, 0.135)$  and  $(1.841, -10.486)$ .

82.  $f(x) = x^3 - 1.4x^2 - 0.96x + 1.44$

$$f'(x) = 3x^2 - 2.8x - 0.96$$



$f$  has horizontal tangents at  $(1.2, 0)$  and  $(-0.267, 1.577)$ .

False. Let  $f(x) = x$  and  $g(x) = x + 1$ . Then

$$f'(x) = g'(x) = 1, \text{ but } f(x) \neq g(x).$$

84. True.  $c$  is a constant.

### Section 2.3 Rates of Change: Velocity and Marginals

#### Skills Warm Up

$$1. \frac{-63 - (-105)}{21 - 7} = \frac{42}{14} = 3$$

$$\frac{-43 - 35}{-78}$$

$$2. 6 - (-7) = 13$$

$$= -6$$

$$\frac{24 - 33}{-9}$$

$$3. 9 - 6 = 3 = -3$$

$$4. \frac{40 - 16}{18 - 8} = \frac{24}{10} = \frac{12}{5}$$

$$5. y = 4x^2 - 2x + 7$$

$$y' = 8x - 2$$

$$6. s = -2t^3 + 8t^2 - 7t$$

$$s' = -6t^2 + 16t - 7$$

$$7. s = -16t^2 + 24t + 30$$

$$s' = -32t + 24$$

$$8. y = -16x^2 + 54x + 70$$

$$y' = -32x + 54$$

$$9. A = 10^1 (-2r^3 + 3r^2 + 5r)$$

$$A' = \frac{1}{10} (-6r^2 + 6r + 5)$$

$$A' = -\frac{3}{5}r^2 + \frac{3}{5}r + \frac{1}{2}$$

$$10. y = 9^1 (6x^3 - 18x^2 + 63x - 15)$$

$$y' = \frac{1}{9} (18x^2 - 36x + 63)$$

$$y' = 2x^2 - 4x + 7$$

$$11. y = 12x - \frac{x^2}{5000}$$

$$y' = 12 - \frac{2x}{5000}$$

$$y' = 12 - \frac{x}{2500}$$

$$12. y = 138 + 74x - \frac{x^3}{10,000}$$

$$y' = 74 - \frac{3x^2}{10,000}$$

$$1. (a) \frac{1980-1986: \frac{120 - 63}{6 - 0} = \$9.5 \text{ billion/yr}}{\quad \quad \quad} \quad \quad \quad (e) \frac{2004-2010: \frac{408 - 305}{30 - 24} \approx \$17.2 \text{ billion/yr}}{\quad \quad \quad}$$

$$(b) 1986-1992: \frac{165 - 120}{12 - 6} = \$7.5 \text{ billion/yr}$$

$$(f) 1980-2012: \frac{453 - 63}{32 - 0} \approx \$12.2 \text{ billion/yr}$$

$$(c) 1992-1998: \frac{226 - 165}{18 - 12} \approx \$10.2 \text{ billion/yr}$$

$$(g) 1990-2012: \frac{453 - 152}{32 - 10} \approx \$13.7 \text{ billion/yr}$$

$$(d) 1998-2004: \frac{305 - 226}{24 - 18} \approx \$13.2 \text{ billion/yr}$$

$$(h) 2000-2012: \frac{453 - 269}{32 - 20} \approx \$15.3 \text{ billion/yr}$$

2. (a) Imports:

$$\frac{495 - 245}{10 - 0} = \dots$$

1980–1990:  $\frac{495 - 245}{10 - 0} = \$25$  billion yr  
Exports:

$$1980\text{--}1990: \frac{394 - 226}{10 - 0} = \$16.8 \text{ billion yr}$$

Imports:

$$\frac{1218 - 495}{1990 - 2000} \approx \$72.3 \text{ billion yr}$$

1990–2000:  $\frac{1218 - 495}{1990 - 2000} \approx \$72.3$  billion yr

Exports:

$$1990\text{--}2000: \frac{782 - 394}{10} = \$38.8 \text{ billion yr}$$

Imports:

$$\frac{1560 - 1218}{2000 - 2010} = \$38.0 \text{ billion yr}$$

2000–2010:  $\frac{1560 - 1218}{2000 - 2010} = \$38.0$  billion yr

(f) Exports:

$$1056 - 782$$

2000–2010:  $\frac{1056 - 782}{10} = \$30.4$  billion yr

Imports:

$$\frac{2268 - 245}{1980 - 2013} \approx \$61.3 \text{ billion yr}$$

1980–2013:  $\frac{2268 - 245}{1980 - 2013} \approx \$61.3$  billion yr

Exports:

$$\frac{1580 - 226}{1980 - 2013} \approx \$41.0 \text{ billion yr}$$

1980–2013:  $\frac{1580 - 226}{1980 - 2013} \approx \$41.0$  billion yr

$$f(t) = 3t + 5; [1, 2]$$

Average rate of change:

$$\frac{y}{x} = \frac{f(2) - f(1)}{2 - 1} = \frac{11 - 8}{1} = 3$$

$$f'(t) = 3$$

Instantaneous rates of change:  $f'(1) = 3, f'(2) = 3$

$$h(x) = 7 - 2x; [1, 3]$$

Average rate of change:

$$\frac{h(3) - h(1)}{3 - 1} = \frac{-5 - (-1)}{2} = -2$$

$$h'(x) = -2$$

$$6. f(x) = -x^2 - 6x - 5; [-3, 1]$$

Average rate of change:

$$\frac{f(1) - f(-3)}{1 - (-3)} = \frac{-12 - 4}{4} = -4$$

$$f'(x) = -2x - 6$$

Instantaneous rates of change:  $f'(-3) = 0, f'(1) = -8$

$$f(x) = 3x^{4/3}; [1, 8]$$

Average rate of change:  $\frac{f(8) - f(1)}{8 - 1} = \frac{48 - 3}{7} = 6.43$

$$f'(x) = 4x^{1/3}$$

Instantaneous rates of change:  $f'(1) = 4, f'(8) = 8$

$$f(x) = x^{3/2}; [1, 4]$$

Average rate of change:

$$\frac{f(4) - f(1)}{4 - 1} = \frac{8 - 1}{3} = \frac{7}{3}$$

$$f'(x) = \frac{3}{2}x^{1/2}$$

Instantaneous rates of change:  $f'(1) = \frac{3}{2}, f'(4) = 3$

$$f(x) = \frac{1}{x}; [1, 5]$$

Average rate of change:

$$\frac{f(5) - f(1)}{5 - 1} = \frac{\frac{1}{5} - 1}{4} = -\frac{4}{5}$$

$$f'(x) = -\frac{1}{x^2}$$

Instantaneous rates of change:

$$f'(1) = -1, f'(5) = -\frac{1}{25}$$

$$10. f(x) = \frac{1}{\sqrt{x}}; [1, 9]$$

$$h'(t) = -2$$

Instantaneous rates of change:  $h(1) = -2, h(3) = -2$

$$h(x) = x^2 - 4x + 2; [-2, 2]$$

Average rate of change:

$$\frac{h}{x} = \frac{h(2) - h(-2)}{2 - (-2)} = \frac{-2 - 14}{4} = -4$$

$$h'(x) = 2x - 4$$

Instantaneous rates of change:  $h'(-2) = -8, h'(2) = 0$

Average rate of change:

$$\frac{f}{x} = \frac{f(9) - f(1)}{9 - 1} = \frac{-\frac{1}{8} - \frac{1}{4}}{8} = -\frac{\frac{1}{8} + \frac{2}{8}}{8} = -\frac{\frac{3}{8}}{8} = -\frac{3}{64}$$

$$f'(x) = \frac{1}{2x^{3/2}}$$

Instantaneous rates of change:

$$f'(1) = -\frac{1}{2}, f'(9) = \frac{1}{54}$$



$$f(t) = t^4 - 2t^2; [-2, -1]$$

Average rate of change:  $-1-8$

$$\frac{f(-1) - f(-2)}{-1 - (-2)} = \frac{-1 - 8}{1} = -9$$

$$f'(t) = 4t^3 - 4t$$

Instantaneous rates of change:

$$f'(-2) = -24, f'(-1) = 0$$

$$g(x) = x^3 - 1; [-1, 1]$$

Average rate of change:

$$\frac{g(1) - g(-1)}{1 - (-1)} = \frac{0 - 2}{2} = -1$$

$$g'(x) = 3x^2$$

Instantaneous rates of change:

$$g'(-1) = 3, g'(1) = 3$$

$$( ) \quad ( )$$

13. (a)  $\approx \frac{0 - 1400}{3} \approx -467$

The number of visitors to the park is decreasing at an average rate of 467 people per month from

September to December.

Answers will vary. Sample answer: [4, 11]

Both the instantaneous rate of change at  $t = 8$  and the

average rate of change on [4, 11] are about zero.

14. (a)  $\frac{M}{t} = \frac{800 - 200}{3 - 1} = \frac{600}{2} = 300 \text{ mg/hr}$

Answers will vary. Sample answer: [2, 5]

Both the instantaneous rate of change at  $t = 4$  and the average rate of change on [2, 5] is about zero.

$$s = -16t^2 + 30t + 250$$

Instantaneous:  $v(t) = s'(t) = -32t + 30$

(a) Average: [0, 1]:

$$\frac{s(1) - s(0)}{1 - 0} = \frac{264 - 250}{1} = 14 \text{ ft/sec}$$

$$v(0) = s'(0) = 30 \text{ ft/sec}$$

$$v(1) = s'(1) = -2 \text{ ft/sec}$$

(c) Average: [2, 3]:

$$\frac{s(3) - s(2)}{3 - 2} = \frac{196 - 246}{1} = -50 \text{ ft/sec}$$

$$v(2) = s'(2) = -34 \text{ ft/sec}$$

$$v(3) = s'(3) = -66 \text{ ft/sec}$$

(d) Average: [3, 4]:

$$\frac{s(4) - s(3)}{4 - 3} = \frac{114 - 196}{1} = -82 \text{ ft/sec}$$

$$v(3) = s'(3) = -66 \text{ ft/sec}$$

$$v(4) = s'(4) = -98 \text{ ft/sec}$$

16. (a)  $H'(v) = 3310 \frac{1}{v^2} - 1 = 33 \frac{5}{2} \sqrt{v} - 1$

Rate of change of heat loss with respect to wind speed.

(b)  $H'(2) = 33 \frac{5}{\sqrt{2}} - 1$   
 $\approx 83.673 \frac{\text{kcal} \cdot \text{m} \cdot \text{hr}}{\text{m}^3 \cdot \text{sec}}$

$$= 83.673 \frac{\text{kcal} \cdot \text{sec}}{\text{m}^3 \cdot \text{hr}} = 83.673 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{1}{\text{hr}}$$

$$= 0.023 \frac{\text{kcal}}{\text{m}^3}$$

$$= 40.790 \frac{\text{kcal} \cdot \text{m} \cdot \text{hr}}{\text{m}^3 \cdot \text{sec}}$$

$$= 40.790 \frac{\text{kcal} \cdot \text{hr}}{\text{m}^3 \cdot \text{sec}} = 40.790 \frac{\text{kcal} \cdot \text{hr}}{\text{m}^3 \cdot 3600 \text{ sec}} = 0.11 \frac{\text{kcal}}{\text{m}^3}$$

17.  $s = -16t^2 + 555$

(a) Average velocity =  $\frac{s(3) - s(2)}{3 - 2}$

$$\frac{411 - 491}{1} = -80 \text{ ft/sec}$$

(b)  $v = s'(t) = -32t, v(2) = -64 \text{ ft/sec},$

(b) Average:  $\frac{[ \quad ]}{( \quad )}$

$$\frac{s(2) - s(1)}{2 - 1} = \frac{-24 - 24}{1} = -48 \text{ ft/sec}$$

$$v(1) = s'(1) = -2 \text{ ft/sec}$$

$$v(2) = s'(2) = -34 \text{ ft/sec}$$

$$v(3) = \frac{-96}{2} \text{ ft/sec}$$

$$s = -16t^2 + 555 = 0 \quad 16t^2 = 555$$

$$t^2 = \frac{555}{16}$$

$$\approx 5.89 \text{ seconds}$$

$$v(5.89) \approx -188.5 \text{ ft/sec}$$

(a)  $s(t) = -16t^2 - 18t + 210$   
 $v(t) = s'(t) = -32t - 18$

(b)  $\frac{1}{2} \left[ \frac{s_2 - s_1}{t_2 - t_1} \right] = \frac{1}{2} \left[ \frac{-66}{1} \right] = -33 \text{ ft/sec}$

$v(1) = -50 \text{ ft/sec}$   
 $v(2) = -82 \text{ ft/sec}$

Set  $s(t) = 0$ .

$-16t^2 - 18t + 210 = 0$

$t = - \frac{(-18) \pm \sqrt{(-18)^2 - 4(-16)(210)}}{2(-16)} = \frac{18 \pm \sqrt{13,764}}{-32} \approx 3.10 \text{ sec}$

(e)  $v(3.10) = -117.2 \text{ ft/sec}$

19.  $C = 205,000 + 9800x$

$\frac{dC}{dx} = 9800$

20.  $C = 150,000 + 7x^3$

$\frac{dC}{dx} = 21x^2$

$C = 55,000 + 470x - 0.25x^2, 0 \leq x \leq 940$

$\frac{dC}{dx} = 470 - 0.5x$

$\frac{dC}{dx}$

$C = 100(9 + 3\sqrt{x})$

$\frac{dC}{dx}$

$\frac{dC}{dx} = \frac{100}{2} \cdot \frac{1}{\sqrt{x}} = \frac{50}{\sqrt{x}}$

$R = 50x - 0.5x^2$

$\frac{dR}{dx} = 50 - x$

$\frac{dR}{dx}$

$R = 30x - x^2$

$\frac{dR}{dx} = 30 - 2x$

$R = -6x^3 + 8x^2 + 200x$

$\frac{dR}{dx}$

$\frac{dR}{dx} = -18x^2 + 16x + 200$

$R = 2x(900 + 32x - x^2)$

$R = 1800x + 64x^2 - 2x^3$   
 $R'(x) = 1800 + 128x - 6x^2$

$R'(14) = \$2416$

$R(15) - R(14) = 2(15)900 + 32(15) - 15^3 - 2(14)900 + 32(14) - 14^3$   
 $34,650 - 32,256 = \$2394$

The answers are close.

26.  $R = 5020x - x^3$

$\frac{dR}{dx} = 5020 - 3x^2 = 1000 - 75\sqrt{x}$

27.  $P = -2x^2 + 72x - 145$

$\frac{dP}{dx} = -4x + 72$

$P = -0.25x^2 + 2000x - 1,250,000$

$\frac{dP}{dx}$

$\frac{dP}{dx} = -0.5x + 2000$

$P = 0.0013x^3 + 12x$

$\frac{dP}{dx}$

$\frac{dP}{dx} = 0.0039x^2 + 12$

$P = -0.5x^3 + 30x^2 - 164.25x - 1000$

$\frac{dP}{dx} = -1.5x^2 + 60x - 164.25$

$C = 3.6\sqrt{x} + 500$

$C'(x) = 1.8/\sqrt{x}$

$C'(9) = \$0.60 \text{ per unit.}$

$C(10) - C(9) \approx \$0.584$

The answers are close.

$$P = -0.04x^2 + 25x - 1500$$

$$\frac{dP}{dx} = -0.08x + 25 =$$

$$P'(x) \quad P'(150) = \$13$$

(b)  $\frac{P}{x} = \frac{P(151) - P(150)}{151 - 150} = \frac{1362.96 - 1500}{1} = -137.04 = -\$137.04$   
 The results are close.

$$P = 36,000 + 2048 \sqrt[3]{x} - \frac{1}{8}x^2, \quad 150 \leq x \leq 275$$

$$\frac{dP}{dx} = 2048 \cdot \frac{1}{3} x^{-2/3} - \frac{1}{4} x^{-1} = \frac{2048}{3} x^{-2/3} - \frac{1}{4x}$$

- (a) When  $x = 150$ ,  $\frac{dP}{dx} \approx \$83.61$ .      (b) When  $x = 175$ ,  $\frac{dP}{dx} \approx \$77.41$ .      (c) When  $x = 200$ ,  $\frac{dP}{dx} \approx \$72.41$ .
- (d) When  $x = 225$ ,  $\frac{dP}{dx} \approx \$68.27$ .      (e) When  $x = 250$ ,  $\frac{dP}{dx} \approx \$64.76$ .      (f) When  $x = 275$ ,  $\frac{dP}{dx} \approx \$61.75$ .

$$P = 1.73t^2 + 190.6t + 16,994$$

$$P(0) = 16,994 \text{ thousand people}$$

$$P(3) = 17,581.37 \text{ thousand people}$$

$$P(6) = 18,199.88 \text{ thousand people}$$

$$P(9) = 18,849.53 \text{ thousand people}$$

$$P(12) = 19,530.32 \text{ thousand people}$$

$$P(15) = 20,242.25 \text{ thousand people}$$

$$P(18) = 20,985.32 \text{ thousand people}$$

$$P(21) = 21,759.53 \text{ thousand people}$$

The population is increasing from 1990 to 2011.

$$\frac{dP}{dt} = P'(t) = 3.46t + 190.6$$

$\frac{dP}{dt}$  represents the population growth rate.

$$P'(0) = 190.6 \text{ thousand people per year}$$

$$P'(3) = 200.98 \text{ thousand people per year}$$

$$P'(6) = 211.36 \text{ thousand people per year}$$

$$P'(9) = 221.74 \text{ thousand people per year}$$

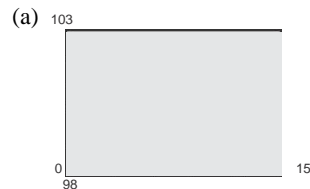
$$P'(12) = 232.12 \text{ thousand people per year}$$

$$P'(15) = 242.5 \text{ thousand people per year}$$

$$P'(18) = 252.88 \text{ thousand people per year}$$

$$P'(21) = 263.26 \text{ thousand people per year}$$

The rate of growth is increasing.



- (a) For  $t < 4$ , the slopes are positive, and the fever is increasing. For  $t > 4$ , the slopes are negative, and the fever is decreasing.

$$T(0) = 100.4^\circ\text{F}$$

$$T(4) = 101^\circ\text{F}$$

$$T(8) = 100.4^\circ\text{F}$$

$$T(12) = 98.6^\circ\text{F}$$

$\frac{dT}{dt} = -0.075t + 0.3$ ; the rate of change of temperature with respect to time

$$T'(0) = 0.3^\circ\text{F per hour}$$

$$T'(4) = 0^\circ\text{F per hour}$$

$$T'(8) = -0.3^\circ\text{F per hour}$$

$$T'(12) = -0.6^\circ\text{F per hour}$$

For  $0 \leq t < 4$ , the rate of change of the temperature is positive; therefore, the temperature is increasing.

For  $4 < t \leq 12$ , the rate of change of the temperature is decreasing; therefore, the temperature is decreasing back to a normal temperature of  $98.6^\circ\text{F}$ .

37. (a)  $TR = -10Q^2 + 160Q$

(b)  $TR = MR = -20Q + 160$

$Q$	0	2	4	6	8	10
Model	160	120	80	40	0	-40
Table	-	130	90	50	10	-30

38. (a)  $R = xp = x(5 - 0.001x) = 5x - 0.001x^2$

(b)  $P = R - C = (5x - 0.001x^2) - (35 + 1.5x)$   
 $= -0.001x^2 + 3.5x - 35$

$\frac{dR}{dx} = 5 - 0.002x$   
 $\frac{dP}{dx} = 3.5 - 0.002x$

$x$	600	1200	1800	2400	3000
$dR/dx$	3.8	2.6	1.4	0.2	-1.0
$dP/dx$	2.3	1.1	-0.1	-1.3	-2.5
$P$	1705	2725	3025	2605	1465

$(36,000, 30), (32,000, 35)$

Slope =  $\frac{35-30}{32,000 - 36,000} = -\frac{5}{4000} = -\frac{1}{800}$

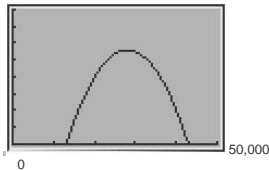
$-30 = -\frac{1}{800}(x - 36,000)$

$= -\frac{1}{800}x + 75$  (demand function)

(a)  $P = R - C = xp - C = x - \frac{1}{800}x + 75$

$= \frac{1}{800}x^2 + 70x - 700,000$

(b) 400,000



At  $x = 18,000$ ,  $P$  has a positive slope.

At  $x = 28,000$ ,  $P$  has a 0 slope.

At  $x = 36,000$ ,  $P$  has a negative slope.

$P'(x) = -\frac{1}{400}x + 70$

$P'(18,000) = \$25$  per ticket

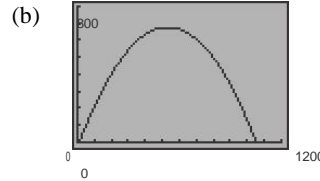
$P'(28,000) = \$0$  per ticket

$P'(36,000) = -\$20$  per ticket

39. (a)  $\left(\frac{1.50 - 1.75}{500 - 400}\right) (x - 400)$

Slope =  $\frac{500 - 400}{p - 1.75} = -0.0025$   
 $p = -0.0025x + 2.75$

$P = R - C = xp - c$   
 $= x(-0.0025x + 2.75) - (0.1x + 25)$   
 $= -0.0025x^2 + 2.65x - 25$



At  $x = 300$ ,  $P$  has a positive slope.

At  $x = 530$ ,  $P$  has a 0 slope.

At  $x = 700$ ,  $P$  has a negative slope.

$P'(x) = -0.005x + 2.65$

$P'(300) = \$1.15$  per unit

$P'(530) = \$0$  per unit

$P'(700) = -\$0.85$  per unit

Chapter 2 Differentiation

41. (a)  $C(x) = \frac{15,000 \text{ mi} \cdot 1 \text{ gal} \cdot 2.60 \text{ dollars}}{\text{yr} \cdot x \text{ mi} \cdot 1 \text{ gal}}$

$$C(x) = \frac{39,000 \text{ dollars}}{x \text{ yr}}$$

(b)  $\frac{dC}{dx} = -\frac{39,000 \text{ dollars}}{x^2 \text{ mpg}}$

The marginal cost is the change of savings for a 1-mile per gallon increase in fuel efficiency.

(c)

$x$	10	15	20	25	30	35	40
$C$	3900	2600	1950	1560	1300	1114.29	975
$dC/dx$	-390	-173.33	-97.5	-62.4	-43.33	-31.84	-24.38

The driver who gets 15 miles per gallon would benefit more than the driver who gets 35 miles per gallon. The value of  $dC/dx$  is a greater savings for  $x = 15$  than for  $x = 35$ .

(a)  $f'(2.959)$  is the rate of change of the number of gallons of gasoline sold when the price is \$2.959/gallon.

In general, it should be negative. Demand tends to decrease as price increases. Answers will vary.

43. (a) Average rate of change from 2000 to 2013:  $\frac{p}{t} = \frac{16,576.66 - 10,786.85}{13 - 0} \approx \$445.37/\text{yr}$

(b) Average rate of change from 2003 to 2007:  $\frac{p}{t} = \frac{13,264.82 - 10,453.92}{7 - 3} \approx \$702.73/\text{yr}$

So, the instantaneous rate of change for 2005 is  $p'(5) \approx \$702.73/\text{yr}$ .

(c) Average rate of change from 2004 to 2006:  $\frac{p}{t} = \frac{12,463.15 - 10,783.01}{6 - 4} \approx \$840.07/\text{yr}$

So, the instantaneous rate of change for 2005 is  $p'(5) \approx \$840.07/\text{yr}$ .

(d) The average rate of change from 2004 to 2006 is a better estimate because the data is closer to the years in question.

44. Answers will vary. *Sample answer:*

The rate of growth in the lag phase is relatively slow when compared with the rapid growth in the acceleration phase.

The population grows slower in the deceleration phase, and there is no growth at equilibrium. These changes could be explained by food supply or seasonal growth.

## Section 2.4 The Product and Quotient Rules

### Skills Warm Up

$$(x^2 + 1)(2) + (2x + 7)(2x) = 2x^2 + 2 + 4x^2 + 14x$$

$$6x^2 + 14x + 2$$

$$2(3x^2 + 7x + 1)$$

2.  $(2x - x^3)(8x) + (4x^2)(2 - 3x^2) = 16x^2 - 8x^4 + 8x^2 - 12x^4$

$$24x^2 - 20x^4$$

$$4x^2(6 - 5x^2)$$

$$x(4)(x^2 + 2)^3(2x) + (x^2 + 4)(1) = 8x^2(x^2 + 2)^3(x^2 + 4)$$

## Skills Warm Up—continued—

$$x_2(2)(2x+1)(2) + (2x+1)^4(2x) = 4x_2(2x+1) + 2x(2x+1)^4$$

$$= 2x(2x+1)2x + (2x+1)^3$$

$$5. \frac{(2x+7)(5) - (5x+6)(2)}{(2x+7)^2} = \frac{10x+35-10x-12}{(2x+7)^2}$$

$$= \frac{23}{(2x+7)^2}$$

$$6. \frac{(x^2-4)(2x+1) - (x^2-4)(2x)}{(x^2-4)^2} = \frac{2x^3+x_2-8x-4-2x^3-2x}{(x^2-4)^2}$$

$$= \frac{-x^2-8x-4}{(x^2-4)^2}$$

$$7. \frac{(x^2+1)(2) - (2x+1)(2x)}{(x^2+1)^2} = \frac{2x^2+2-4x^2-2x}{(x^2+1)^2}$$

$$= \frac{-2x^2-2x+2}{(x^2+1)^2}$$

$$8. \frac{(1-x^4)(4x-1) - (4x-1)(-4x^3)}{(1-x^4)^2} = \frac{4-4x+16x-4x}{(1-x^4)^2}$$

$$= \frac{12x^4-4x^3+4}{(1-x^4)^2}$$

$$= \frac{4(3x^4-x^3+1)}{(1-x^4)^2}$$

$$9. (x^{-1}+x)(2) + (2x-3)(-x^{-2}+1) = 2x^{-1}+2x + (-2x^{-1}+2x+3x^{-2}-3)$$

$$= 4x+3x^{-2}-3$$

$$= 4x + \frac{3}{x^2} - 3$$

$$= \frac{4x^3-3x^2+3}{x^2}$$

$$10. \frac{(1-x)(1-x-4) - (1-x)(-1)}{(1-x)^2} = \frac{-1-2x^{-1}+x^{-2}-x^2}{(1-x)^2}$$

$$= \frac{x^2-2x+4}{x^2-2x+1}$$

$$= \frac{x^2-2x+4}{(x-1)^2}$$

**Skills Warm Up —continued—**

11.  $f(x) = 3x^2 - x + 4$   
 $f'(x) = 6x - 1$

$f'(2) = 6(2) - 1$   
 $= 12 - 1$   
 $= 11$

12.  $f(x) = -x^3 + x^2 + 8x$   
 $f'(x) = -3x^2 + 2x + 8$   
 $f'(2) = (-3(2)^2 + 2(2) + 8)$   
 $= -34 + 4 + 8$   
 $= 0$

2

4

1.  $f(x) = (2x - 3)(1 - 5x)$   
 $f'(x) = (2x - 3)(-5) + (1 - 5x)(2)$   
 $= -10x + 15 + 2 - 10x$   
 $= -20x + 17$

2.  $g(x) = (4x^2 - 7)(3x + 1)$   
 $g'(x) = (4x^2 - 7)(3) + (3x + 1)(8)$   
 $= 12x^2 - 21 + 12x + 8$   
 $= 12x^2 + 12x - 13$

3.  $f(x) = (6x - x^2)(4 + 3x)$   
 $f'(x) = (6x - x^2)(3) + (4 + 3x)(6 - 2x)$   
 $= 18x - 3x^2 + 24 - 8x + 18x - 6x^2$   
 $= -9x^2 + 28x + 24$

4.  $f(x) = (5x - x^3)(2x + 9)$   
 $f'(x) = (5x - x^3)(2) + (2x + 9)(5 - 3x^2)$   
 $= 10x - 2x^3 + 10 - 6x^3 + 45 - 27x^2$   
 $= -8x^3 - 27x^2 + 55x + 55$

13.  $f(x) = \frac{2}{7x} = \frac{2}{7}x^{-1}$   
 $f'(x) = -\frac{2}{7}x^{-2} = -\frac{2}{7x^2}$   
 $f'(2) = -\frac{2}{7(2)^2} = -\frac{2}{28} = -\frac{1}{14}$

14.  $f(x) = x^2 - \frac{1}{2}$   
 $f'(x) = 2x + \frac{1}{x^3}$   
 $f'(2) = 2(2) + \frac{1}{2^3}$   
 $= 4 + \frac{1}{8}$   
 $= 4 + \frac{1}{4}$   
 $= \frac{17}{4}$

5.  $f(x) = x(x^2 + 3)$   
 $f'(x) = x(2x) + (x^2 + 3)(1)$   
 $= 2x^2 + x^2 + 3$   
 $= 3x^2 + 3$

6.  $f(x) = x^3(3x - 1)$   
 $f'(x) = 3x^2(3x - 1) + x^3(3)$   
 $= 9x^3 - 3x^2 + 3x^3$   
 $= 12x^3 - 3x^2$

7.  $h(x) = -3(x^2 + 7)(x^2 + 7)$   
 $h'(x) = -3(2x)(x^2 + 7) - 3(x^2 + 7)(2x)$   
 $= -6x(x^2 + 7) - 6x(x^2 + 7)$   
 $= -12x(x^2 + 7)$   
 $= -12x^3 - 84x$

8.  $f(x) = (3 - x^2)(4 - x)$   
 $f'(x) = (3 - x^2)(-1) + (4 - x)(-2x)$   
 $= -3 + x^2 - 8x + 2x^2$   
 $= 3x^2 - 8x - 3$

$f'(x) = 3x^2 - 8x - 3$       5(-1)



Chapter 2 Differentiation

$$= -24x^{-3} + 8x^{-2} - 4x^{-2} + 5$$

$$= -\frac{24}{x^3} - \frac{4}{x^2} + 5$$

$$g(x) = (x^2 - 4x + 3)(x - 2)$$

$$g'(x) = (x^2 - 4x + 3)(1) + (x - 2)(2x - 4)$$

$$x^2 - 4x + 3 + 2x^2 - 4x - 4x + 8$$

$$3x^2 - 12x + 11$$

$$g(x) = (x^2 - 2x + 1)(x^3 - 1)$$

$$g'(x) = (x^2 - 2x + 1)(3x^2) + (x^3 - 1)(2x - 2)$$

$$3x^4 - 6x^3 + 3x^2 + 2x^4 - 2x^3 - 2x + 2$$

$$5x^4 - 8x^3 + 3x^2 - 2x + 2$$

11.  $h(x) = \frac{x}{x-5}$

$$h'(x) = \frac{(x-5)(1) - x(1)}{(x-5)^2} = \frac{x-5-x}{(x-5)^2} = \frac{-x}{(x-5)^2}$$

12.  $h(x) = \frac{x+3}{x^2+3x-1}$

$$h' = \frac{(x^2+3x-1)(1) - (x+3)(2x-2)}{(x^2+3x-1)^2}$$

$$= \frac{2x^2+6x-x^2-6x+2}{(x^2+3x-1)^2}$$

$$= \frac{x^2+2}{(x^2+3x-1)^2}$$

13.  $f(t) = \frac{2t-3}{3t+1}$

$$f'(t) = \frac{(3t+1)(2) - (2t-3)(3)}{(3t+1)^2}$$

$$= \frac{6t+2-6t+9}{(3t+1)^2} = \frac{11}{(3t+1)^2}$$
  
 $f(x) = \frac{7x+3}{4x-9}$ 

$$f'(x) = \frac{(4x-9)(7) - (7x+3)(4)}{(4x-9)^2}$$

$$= \frac{28x-63-28x-12}{(4x-9)^2} = \frac{-75}{(4x-9)^2}$$

$$f(t) = \frac{t^2-8}{t^2-8t-12}$$

$$f'(t) = \frac{(t^2-8)(2t) - (t^2-8t-12)(2t)}{(t^2-8t-12)^2}$$

$$= \frac{2t^3-16t - (2t^3-16t-24t)}{(t^2-8t-12)^2}$$

$$= \frac{2t^3-16t - 2t^3+16t+24t}{(t^2-8t-12)^2} = \frac{24t}{(t^2-8t-12)^2}$$
  
 $g(x) = \frac{4x-5}{x^2-1}$ 

$$g'(x) = \frac{(x^2-1)(4) - (4x-5)(2x)}{(x^2-1)^2}$$

$$= \frac{4x^2-4-8x^2+10x}{(x^2-1)^2}$$

$$= \frac{-4x^2+10x-4}{(x^2-1)^2}$$

$f(x) = \frac{x^2+6x+5}{2x-1}$ 

$$f'(x) = \frac{(2x-1)(2x+6) - (x^2+6x+5)(2)}{(2x-1)^2}$$

$$= \frac{4x^2+12x-2x-6-2x^2-12x-10}{(2x-1)^2}$$

$$= \frac{2x^2-2x-16}{(2x-1)^2}$$

$f(x) = \frac{4x^2-x+2}{3-4x}$ 

$$f'(x) = \frac{(3-4x)(-4) - (4x^2-x+2)(-4)}{(3-4x)^2}$$

$$= \frac{-16x+16+16x^2-4x+8}{(3-4x)^2}$$

$$= \frac{16x^2-20x+24}{(3-4x)^2}$$

19.  $f(x) = \frac{6 + 2x^{-1}}{3x - 1}$

$f'(x) = \frac{(3x - 1) \cdot \frac{-2x^{-2}}{x^2} - (6 + 2x^{-1}) \cdot (3)}{(3x - 1)^2}$

$$\frac{-6x^{-1} + 2x^{-2} - 18 - 6x^{-1}}{(3x - 1)^2}$$

$$\frac{2x^{-2} - 12x^{-1} - 18}{(3x - 1)^2}$$

$$\frac{\frac{2}{x^2} - \frac{12}{x} - 18}{(3x - 1)^2} = \frac{2 - 12x - 18x^2}{x^2(3x - 1)^2}$$

$f(x) = \frac{5 - x - 2}{+ 2}$

$f'(x) = \frac{(x + 2) \cdot 2x^{-3} - (5 - x - 2) \cdot 1}{(x + 2)^2}$

$$\frac{2x^{-2} + 4x^{-3} - 5 + x^{-2}}{(x + 2)^2}$$

$$\frac{4x^{-3} + 3x^{-2} - 5}{(x + 2)^2}$$

$$\frac{-\frac{5}{x^3} - \frac{5}{x^2}}{(x + 2)^2}$$

$$\frac{4 + 2x - 5x^3}{x^3(x + 2)^2}$$

Function	Rewrite	Differentiate	Simplify
$f(x) = x^3 + \frac{6x}{3}$	$f(x) = \frac{1}{3}x^3 + 2x$	$f'(x) = x^2 + 2$	$f'(x) = x^2 + 2$
$f(x) = x^3 + \frac{2x}{2}$	$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2$	$f'(x) = \frac{3}{3}x^2 + \frac{2}{2}x$	$f'(x) = x^2 + \frac{2}{2}x$
23. $y = \frac{7x^2}{5}$	$y = \frac{7}{5}x^2$	$y' = \frac{7}{5} \cdot 2x$	$y' = \frac{14}{5}x$
24. $y = \frac{2x}{9}$	$y = \frac{2}{9}x^4$	$y' = \frac{2}{9} \cdot 4x^3$	$y' = \frac{8}{9}x^3$
25. $y = 3x^3$	$y = \frac{1}{3}x^{-3}$	$y' = -7x^{-4}$	$y' = -\frac{7}{4}x^{-4}$
26. $y = -\frac{4}{5x^2}$	$y = -\frac{4}{5}x^{-2}$	$y' = -\frac{8}{5}x^{-3}$	$y' = -\frac{8}{5x^3}$
27. $y = \frac{4x^2 - 3x}{8\sqrt{x}}$	$y = \frac{1}{8}x^{3/2} - \frac{3}{8}x^{1/2}, x \neq 0$	$y' = \frac{3}{4}x^{1/2} - \frac{3}{16}x^{-1/2}$	$y' = \frac{3\sqrt{x}}{4} - \frac{3}{16\sqrt{x}}$
28. $y = \frac{3x + 2x}{6\sqrt[3]{x}}$	$y = \frac{5}{6}x^{2/3} + \frac{2}{3}x^{1/3}, x \neq 0$	$y' = \frac{5}{9}x^{-1/3} + \frac{2}{9}x^{-2/3}$	$y' = \frac{5\sqrt[3]{x}}{9} + \frac{2}{9\sqrt[3]{x}}$
29. $y = \frac{x^2 - 4x + 3}{2x - 1}$	$y = \frac{1}{2}x - 3, x \neq 1$	$y' = \frac{1}{2}, x \neq 1$	$y' = \frac{1}{2}, x \neq 1$
30. $y = \frac{x^2 - 4}{4(x + 2)}$	$y = \frac{1}{4}(x - 2), x \neq -2$	$y' = \frac{1}{4}(1), x \neq -2$	$y' = \frac{1}{4}, x \neq -2$
31. $f'(x) = (x^3 - 3x)(4x + 3) + (3x^2 - 3)(2x^2 + 3x + 5)$			

$$= 4x^4 + 3x^3 - 12x^2 - 9x + 6x^4 + 9x^3 + 9x^2 - 9x - 15$$
$$10x^4 + 12x^3 - 3x^2 - 18x - 15$$

Product Rule and Simple Power Rule

$$32. h'(t) = (t^5 - 1)(8t - 7) + (5t^4)(4t^2 - 7t - 3)$$

$$= 8t^6 - 7t^5 - 8t + 7 + 20t^6 - 35t^5 - 15t^4$$

$$28t^6 - 42t^5 - 15t^4 - 8t + 7$$

Product Rule and Simple Power Rule

$$h(t) = \frac{1}{3}(6t - 4)$$

-3

$$h'(t) = \frac{1}{3}(6) = 2$$

Constant Multiple and Simple Power Rules

$$f(x) = \frac{1}{2}(3x - 8)$$

$$f'(x) = \frac{1}{2}(3) = \frac{3}{2}$$

Constant Multiple and Simple Power Rules

$$35. f(x) = \frac{(x^2 - 1)(3x^2 + 3)}{(x^3 + 3x + 2)(2x)}$$

$$f(x) = \frac{(x^2 - 1)(3x^2 + 3)}{(x^3 + 3x + 2)(2x)}$$

$$= \frac{3x^4 - 3 - 2x^4 - 6x^2 - 4x}{(x^2 - 1)^2}$$

$$= \frac{x^4 - 6x^2 - 4x - 3}{(x^2 - 1)^2}$$

Quotient Rule and Simple Power Rule

$$f(x) = \frac{2x^3 - 4x^2 - 9}{x^3 - 5}$$

$$f'(x) = \frac{(x^3 - 5)(3x^2 - 8x) - (2x^3 - 4x^2 - 9)(3x^2)}{(x^3 - 5)^2}$$

$$\frac{3x^5 - 8x^4 - 15x^2 + 40x - 6x^5 + 12x^4 - 27x^2}{(x^3 - 5)^2}$$

$$\frac{-3x^5 + 4x^4 - 42x^2 + 40x}{(x^3 - 5)^2}$$

Quotient Rule and Simple Power Rule

$$37. f(x) = \frac{20}{x^2 - x - x + 4} = \frac{20}{x^2 - 2x + 4} = \frac{20}{(x - 1)^2 + 3} = x - 5, x \neq -4$$

$$f'(x) = 1$$

Simple Power Rule

$$38. h(t) = \frac{3t^2 + 22t + 7}{t + 7} = \frac{3t + 1}{t + 7} = 3t + 1, t \neq -7$$

$$h(t) = 3t + 1, t \neq -7$$

Simple Power Rule

$$39. g(t) = 2t^3 - 1 = 2t^3 - 12t^3 - 1$$

$$g(t) = \frac{(2t^3 - 1)(6t^2 + 2t - 1)}{3t^2 - 12t^3 - 1}$$

$$12t^2(2t^3 - 1)$$

Product Rule and Simple Power Rule

$$f(x) = (4x^3 - 2x - 3)^2 = (4x^3 - 2x - 3)(4x^3 - 2x - 3)$$

$$f'(x) = (4x^3 - 2x - 3)(12x^2 - 2) + (4x^3 - 2x - 3)(12x^2 - 2)$$

$$48x^5 - 24x^3 - 36x^2 - 8x^3 + 4x + 6 + 48x^5 - 24x^3 - 36x^2 - 8x^3 + 4x + 6$$

$$96x^5 - 48x^3 - 72x^2 - 16x^3 + 8x + 12$$

Product Rule and Simple Power Rule

Chapter 2 Differentiation

$$41. g(s) = \frac{s^2 - 2s + 5}{\sqrt{s}} = \frac{s^2 - 2s + 5}{s^{1/2}}$$

$$g(s) = \frac{2s^{3/2} - 2s^{1/2} - \frac{1}{2}s^{-1/2} + \frac{5}{2}s^{-1/2}}{s}$$

$$= \frac{3s^2 - 2s - 5}{2s^3}$$

Quotient Rule and Simple Power Rule

$$42. f(x) = \frac{x^3 - 5x^2 - 6x}{\sqrt{x}} = \frac{x^3 - 5x^2 - 6x}{x^{1/2}}$$

$$f(x) = 2x^2 - 2x - 3x^{-1/2}$$

$$= \frac{5x^2}{2} - \frac{15x}{2} - \frac{3}{2x^{1/2}}$$

Constant Multiple and Simple Power Rules

$$43. f(x) = \frac{(x-2)(3x+1)}{(4x+2)^2} = \frac{3x^2 - 5x - 2}{(4x+2)^2}$$

$$f'(x) = \frac{6x - 5}{(4x+2)^2} - \frac{2(4x+2)(6x-5)}{(4x+2)^4}$$

$$= \frac{12x^2 + 12x - 2}{4(2x+1)^2} - \frac{2(6x^2 - 5x - 2)}{(2x+1)^2}$$

$$= \frac{6x^2 - 5x - 2}{(2x+1)^2}$$

Quotient Rule and Simple Power Rule

$$44. f(x) = \frac{(x+1)(2x-7)}{2x+1} = \frac{2x^2 - 5x - 7}{2x+1}$$

$$f'(x) = \frac{(2x+1)(4x-5) - (2x^2 - 5x - 7)(2)}{(2x+1)^2}$$

$$f(x) = (x+4)(2x+9)(x-3)$$

$$= (2x^2 + 17x + 36)(x-3)$$

$$f'(x) = (2x^2 + 17x + 36)(1) + (x-3)(4x+17)$$

$$= (2x^2 + 17x + 36) + (4x^2 + 5x - 51)$$

$$= 6x^2 + 22x - 15$$

Product Rule and Simple Power Rule

$$f(x) = (3x^3 + 4x)(x-5)(x+1)$$

$$f'(x) = (3x^3 + 4x)(2x-4) + (x^2 - 4x - 5)(9x^2 + 4)$$

$$= (6x^4 - 12x^3 + 8x^2 - 16x) + (9x^3 - 36x^2 - 41x - 20)$$

$$= 15x^4 - 48x^3 - 33x^2 - 32x - 20$$

Product Rule and Simple Power Rule

$$f(x) = (5x+2)(x^2+x+5)$$

$$f'(x) = 5x+2(2x+1) + x^2+x+5(5)$$

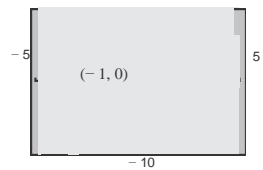
$$= 10x^2 + 9x + 2 + 5x^2 + 5x$$

$$= 15x^2 + 14x + 2$$

$$m = f'(-1) = 3$$

$$y - 0 = 3(x - (-1))$$

$$y = 3x + 3$$



$$f(x) = (x^2 - 1)(x^3 - 3x)$$

$$f'(x) = (x^2 - 1)(3x^2 - 3) + (x^2 - 3x)(2x)$$

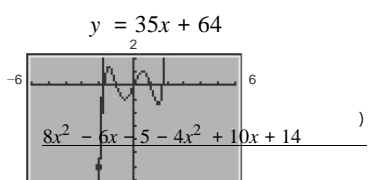
$$= 3x^4 - 6x^2 + 3 + 2x^4 - 6x^2$$

$$= 5x^4 - 12x^2 + 3$$

$$= f'(-2) = 5(-2)^4 - 12(-2)^2 + 3 = 35$$

$$-(-6) = 35(x - (-2))$$

$$y + 6 = 35(x + 2)$$



Chapter 2 Differentiation

$$\left( \frac{2}{x} + \frac{1}{2} \right)_{(-2, -6)}$$
$$\frac{4x^2 + 4x + 9}{2}$$
$$\left( \frac{1}{2} \right)$$

Quotient Rule and Simple Power Rule

49.  $f(x) = x^3(x^2 - 4)$

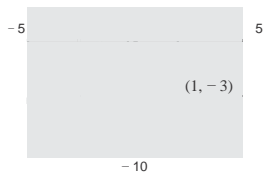
$f'(x) = x^2(2x) + x^2 - 4(3x^2)$

$= 5x^4 - 12x^2$

$m = f'(1) = -7$

$y - (-3) = -7(x - 1)$

$y = -7x + 4$

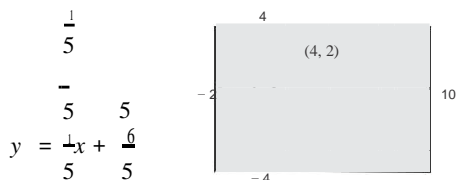


$f(x) = \frac{3x - 2}{x + 1}$

$f'(x) = \frac{(3)(1) - (3x - 2)(1)}{(x + 1)^2} = \frac{3 - 3x + 2}{(x + 1)^2} = \frac{5 - 3x}{(x + 1)^2}$

$f'(4) = \frac{5 - 3(4)}{(4 + 1)^2} = \frac{5 - 12}{25} = -\frac{7}{25}$

$f'(4) = -\frac{7}{25}$



$y = \frac{1}{5}x + \frac{6}{5}$

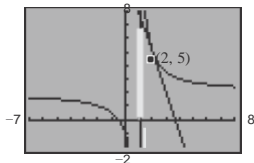
52.  $f'(x) = (x-1)^2 - (2x+1) = -3$

$f'(2) = -3$

$f'(2) = -3$

$-5 = -3(x - 2)$

$= -3x + 11$



50.  $f(x) = \sqrt{x}(x - 3) = x^{1/2}(x - 3)$

$f'(x) = x^{1/2}(1) + (x - 3)(-1/2)x^{-1/2}$

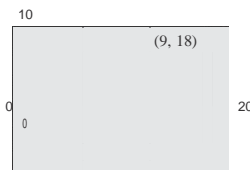
$= \frac{1}{2}x^{1/2} - \frac{3 - x}{2x^{1/2}}$

$= \frac{3 - 3}{2x^{1/2}} - \frac{-x}{2x^{1/2}}$

$= \frac{0}{2\sqrt{x}} + \frac{x}{2\sqrt{x}} = \frac{x}{2\sqrt{x}} = \frac{\sqrt{x}}{2}$

$y - 18 = 4(x - 9)$

$y = 4x - 18$



53.  $f(x) = \frac{(3x - 2)(6x + 5)}{2x - 3} = \frac{18x^2 + 3x - 10}{2x - 3}$

$f'(x) = \frac{(2)(18x^2 + 3x - 10) - (18x^2 + 3x - 10)(2)}{(2x - 3)^2}$

$f'(x) = \frac{36x^2 + 6x - 20 - 36x^2 - 6x + 20}{(2x - 3)^2} = \frac{0}{(2x - 3)^2} = 0$

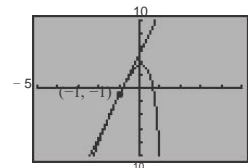
$\frac{72x^2 - 102x - 9 - 36x^2}{(2x - 3)^2} = \frac{36x^2 - 108x + 11}{(2x - 3)^2}$

$= f'(-1) = \frac{36(-1)^2 - 108(-1) + 11}{(2(-1) - 3)^2} = \frac{31}{25}$

$(2(-1) - 3)^2 = 25$

$y - (-1) = \frac{31}{5}(x - (-1))$

$y = \frac{31}{5}x + \frac{26}{5}$





54.  $f(x) = \frac{x^3 + 3x^2 + 2x}{x - 4}$

$$f'(x) = \frac{(x-4)(3x^2 + 6x + 2) - (x^3 + 3x^2 + 2x)(1)}{(x-4)^2}$$

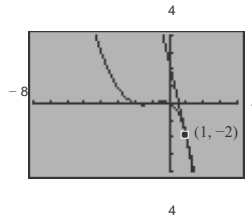
$$= \frac{3x^3 - 12x^2 + 6x^2 - 24x + 2x - 8 - x^3 - 3x^2 - 2x}{(x-4)^2}$$

$$= \frac{2x^3 - 9x^2 - 24x - 8}{(x-4)^2}$$

$m = f'(1) = \frac{2(1)^3 - 9(1)^2 - 24(1) - 8}{(1-4)^2} = -13$

$y - (-2) = -13(x - 1)$

$$y = -13x + 11$$



55.  $f(x) = \frac{(x-1)(2x)}{x-1} = 2x$

$$f'(x) = 2$$

$f'(x) = 0$  when  $x^2 - 2x = x(x - 2) = 0$ , which implies that  $x = 0$  or  $x = 2$ . Thus, the horizontal tangent lines occur at  $(0, 0)$  and  $(2, 4)$ .

$$f(x) = \frac{x^2 + 1}{x^2 - 2x}$$

56.  $f'(x) = \frac{x^2 + 1}{x^2 - 2x}$

$f'(x) = 0$  when  $2x = 0$ , which implies that  $x = 0$ .

Thus, the horizontal tangent line occurs

at  $(0, 0)$ .

57.  $f(x) = \frac{(x+1)(x-4)}{x^3 - x}$

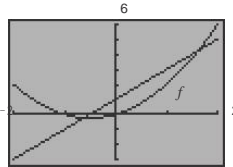
$$f'(x) = \frac{(x^3 - x)(x - 4) - (x + 1)(x - 4)^2}{(x^3 - x)^2}$$

$f'(x) = 0$  when  $x^6 + 4x^3 = x^3(x^3 + 4) = 0$ , which implies that  $x = 0$  or  $x = \sqrt[3]{-4}$ . Thus, the horizontal

tangent lines occur at  $(0, 0)$  and  $(\sqrt[3]{-4}, -2.117)$ .

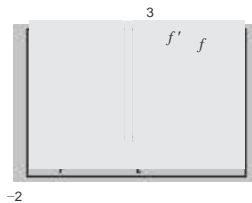
59.  $f(x) = x(x + 1) = x^2 + x$

$f'(x) = 2x + 1$



60.  $f(x) = x^2(x + 1) = x^3 + x^2$

$f(x) = 3x^2 + 2x = x(3x + 2)$



61.  $f(x) = x(x + 1)(x - 1)$

$f'(x) = 3x^2 - x$

Chapter 2 Differentiation

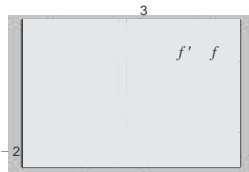
$$58. f'(x) = \frac{2x(x^2 + 3)(x^2 - 1)}{(x^2 + 1)^2}$$

$f'(x) = 0$  when  $2x(x^2 + 3)(x^2 - 1) = 0$ , which implies that  $x = 0$  or  $x = \pm 1$ . Thus, the horizontal tangent lines occur at  $(0, 3)$ ,  $(1, 2)$ , and  $(-1, 2)$ .



$$62. f(x) = x^2(x+1)(x-1) = x^4 - x^2$$

$$f'(x) = 4x^3 - 2x$$



3 p

63.  $x = 2751 - 5p + 1$

$$\frac{dx}{dp} = \frac{d}{dp}(2751 - 5p + 1) = -5$$

$$dp = -\frac{dx}{5}$$

When  $p = 4$ ,  $\frac{dx}{dp} = -5 \approx -1.87$  units

per dollar.

$$\frac{dx}{dp} = \frac{(p+1)(2) - (2p)(1)}{(p+1)^2}$$

64.  $dp = 0 - 1 -$

$$= -1 - \frac{2}{(p+1)^2}$$

$$= \frac{p+1-2}{(p+1)^2}$$

$$= \frac{-1}{(p+1)^2} \approx -0.24$$

$$= \frac{-p^2 - 2p - 3}{(p+1)^2}$$

$$\frac{-9-6-3}{(4+1)^2}$$

When  $p = 3$ ,  $\frac{dx}{dp} = -1.13$  units

per dollar.

$$\frac{dx}{dt} = \frac{200 - 4t^2}{(t^2 + 1)^2}$$

65.  $P' = 500$

$$\frac{dP}{dt} = \frac{500 - 184}{(50 + t^2)^2}$$

When  $t = 2$ ,  $P' = 500 - \frac{184}{(50+4)^2} \approx 31.55$  bacteria/hour.

(54)<sup>2</sup>  
level of oxygen in the pond is changing at that particular time.

68.  $T = 10 \frac{4t^2 + 16t + 75}{t^2 + 4t + 10}$

$$T = 10 \frac{4t^2 + 16t + 75}{t^2 + 4t + 10}$$

Initial temperature:  $T(0) = 75^\circ\text{F}$

$$T'(t) = 10 \frac{4t^2 + 16t + 75}{(t^2 + 4t + 10)^2}$$

$$dP = \frac{d}{dt} \left( \frac{50(t+2)}{t+1750} \right)$$

$$= \frac{50(1) - (t+2)(1)}{(t+1750)^2}$$

66.  $dt = \frac{d}{dt} \left( \frac{50(t+2)}{t+1750} \right)$

$$= \frac{50(1) - (t+2)(1)}{(t+1750)^2}$$

$$= \frac{2500(t+2) - 1748}{(t+1750)^2}$$

(a) When  $t = 1$ ,  $\frac{dP}{dt} = \frac{-874}{225} \approx -3.88$  percent/day.

(b) When  $t = 10$ ,  $\frac{dP}{dt} = \frac{-874}{3600}$

$$= -\frac{437}{1800}$$

$$\frac{1800}{3600} = 0.5$$

67.  $P = t^2 + 1$

$$P' = \frac{d}{dt} (t^2 + 1) = 2t$$

(a)  $P'(0.5) = 1$

(b)  $P'(2) = 4$

(c)  $P'(8) = 16$

Each rate in parts (a), (b), and (c) is the rate at which the

$$T'(1) \approx -9.33^\circ\text{F/hr}$$

$$T'(3) \approx -3.64^\circ\text{F hr/}$$

$$T'(5) \approx -1.62^\circ\text{F /hr}$$

$$T'(10) \approx -0.37^\circ\text{F hr/}$$

$$= (t^2 + 4t + 10)^2$$

Each rate in parts (a), (b), (c), and (d) is the rate at which the temperature of the food in the refrigerator is changing at that particular time.

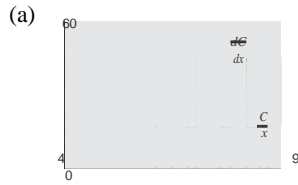
Chapter 2 Differentiation  
 $C = x^3 - 15x^2 + 87x - 73, 4 \leq x \leq 9$

$$\frac{dC}{dx}$$

Marginal cost:  $dx = 3x^2 - 30x + 87$

$$\frac{C}{x} = x^2 - 15x + 87 - \frac{73}{x}$$

Average cost:  $x = x^2 - 15x + 87 - \frac{73}{x}$



(b) Point of intersection:

$$3x^2 - 30x + 87 = x^2 - 15x + 87 - \frac{73}{x}$$

$$2x^2 - 15x + \frac{73}{x} = 0$$

$$2x^3 - 15x^2 + 73 = 0$$

$$x \approx 6.683$$

When  $x = 6.683, \frac{C}{x} = \frac{dC}{dx} \approx 20.50$ .

Thus, the point of intersection is (6.683, 20.50).  
 At this point average cost is at a minimum.

(a) As time passes, the percent of people aware of the product approaches approximately 95%.

As time passes, the rate of change of the percent of people aware of the product approaches zero.

71.  $C = 100 - \frac{200}{x^2} + \frac{x}{x+30}, x \geq 1$

$$C' = 100 - 2(200x^{-3}) + \left( \frac{x}{x+30} \right)' = \frac{x}{(x+30)^2} - \frac{30}{x^3}$$

$$100 - \frac{400}{x^3} + \frac{30}{(x+30)^2}$$

(a)  $C'(10) = 100 - \frac{400}{10^3} + \frac{30}{40^2} = -38.125$

$$C'(15) \approx -10.37$$

$$C'(20) \approx -3.8$$

Increasing the order size reduces the cost per item.  
 An order size of 2000 should be chosen since the cost per item is the smallest at  $x = 20$ .

(a)  $P = ax^2 + bx + c$

When  $x = 10, P = 50: 50 = 100a + 10b + c$ .

When  $x = 12, P = 60: 60 = 144a + 12b + c$ .

When  $x = 14, P = 65: 65 = 196a + 14b + c$ .

$$= -\frac{5}{8}, b = \frac{75}{4}, \text{ and } c = -75.$$



(c) Marginal profit:  $P' = -\frac{5}{4}x + \frac{75}{4} = 0 \Rightarrow x = 15$

This is the maximum point on the graph of  $P$ , so selling 15 units will maximize the profit.

$$f(x) = 2g(x) + h(x)f$$

$$f'(x) = 2g'(x) + h'(x)f$$

$$f'(2) = 2(-2) + 4 = 0$$

$$f(x) = 3 - g(x)$$

$$f'(x) = -g'(x)$$

$$f'(2) = -(-2) = 2$$

$$f(x) = g(x)h(x)$$

$$f(2) = g(2)h(2) + h(2)g'(2)$$

$$( ) ( ) ( ) ( )$$

$$= 3 \cdot 4 + (-1) \cdot 2$$

$$= 14$$

76.  $f(x) = \frac{g(x)}{h(x)}$

$$h(x)g'(x) - g(x)h'(x)$$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{h(x)^2}$$

$$f'(2) = \frac{(-1)(-2) - (3)(4)}{(-1)^2} = -10$$

Answers will vary.

### Chapter 2 Quiz Yourself

$$f(x) = 5x + 3$$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{5(x+\Delta x) + 3 - (5x + 3)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{5x + 5\Delta x + 3 - 5x - 3}{x}$$

$$= \lim_{x \rightarrow 0} \frac{5\Delta x}{x}$$

$$\lim_{\Delta x \rightarrow 0} 5 = 5$$

At  $(-2, -7)$ :  $m = 5$

$$f(x) = 3x - x^2$$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{3(x+\Delta x) - (x+\Delta x)^2 - (3x - x^2)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{3x + 3\Delta x - (x^2 + 2x\Delta x + (\Delta x)^2) - (3x - x^2)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{3x + 3\Delta x - x^2 - 2x\Delta x - (\Delta x)^2 - 3x + x^2}{x}$$

$$= \lim_{x \rightarrow 0} \frac{3\Delta x - 2x\Delta x - (\Delta x)^2}{x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(3 - 2x - \Delta x)}{x}$$

At  $(4, -4)$ :  $m = 3 - 2(4) = 3 - 8 = -5$

4.  $f(x) = 12$

$$f'(x) = 0$$

5.  $f(x) = 19x + 9$

$$f'(x) = 19$$

6.  $f(x) = x^4 - 3x^3 - 5x^2 + 8$

$$f'(x) = 4x^3 - 9x^2 - 10x$$

$$f(x) = 12x^{1/4}$$

$$f'(x) = 3x^{-3/4} = \frac{3}{x^{3/4}}$$

2.  $f(x) = \sqrt{x+3}$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{x+\Delta x+3} - \sqrt{x+3}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{x(\sqrt{x+\Delta x+3} + \sqrt{x+3})}{x(\sqrt{x+\Delta x+3} + \sqrt{x+3})}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+\Delta x+3} + \sqrt{x+3}}$$

$$= \frac{1}{2\sqrt{x+3}}$$

At  $(1, 2)$ :  $m = \frac{1}{2(1+3)} = \frac{1}{8}$

8.  $f(x) = 4x^{-2}$

$$f'(x) = -8x^{-3} = -\frac{8}{x^3}$$

9.  $f(x) = 10x^{-1/5} + x^{-3}$

$$f'(x) = -2x^{-6/5} - 3x^{-4} = -\frac{2}{x^{6/5}} - \frac{3}{x^4}$$

10.

$$3x + 2$$

$$f(x) = \frac{2x+3}{3x}; [-5, -2]$$

$$f'(x) = \frac{(3x+2)(2) - (2x+3)(3)}{(3x+2)^2}$$

$$= \frac{6x+4-6x-9}{2}$$

$$\frac{(3x+2)^2}{5}$$

$$= -\frac{5}{(3x+2)^2}$$

11.  $f(x) = (x^2 + 1)(-2x + 4)$

$$f'(x) = (x^2 + 1)(-2) + (-2x + 4)(2x) - 6x^2 + 8x - 2$$

$$f(x) = (x^2 + 3x + 4)(5x - 2)$$

$$f'(x) = (x^2 + 3x + 4)(5) + (5x - 2)(2x + 3) 5x^2 + 15x + 20 + 10x^2 + 11x - 6$$

$$15x^2 + 26x + 14$$

$$\frac{x}{x^2 + 3}$$

$$f'(x) = \frac{x}{x^2 + 3}$$

$$f'(x) = \frac{(x^2 + 3)(4) - 4x(2x)}{(x^2 + 3)^2}$$

$$\frac{4x^2 + 12 - 8x^2}{(x^2 + 3)^2}$$

$$\frac{-4x^2 + 12}{(x^2 + 3)^2}$$

$$\frac{4(x^2 - 3)}{(x^2 + 3)^2}$$

$$(x^2 + 3)^2 + 3$$

$$f(x) = x^2 - 3x + 1; [0, 3]$$

Average rate of change:  $\frac{1-1}{3-0} = 0$

$$\frac{f(3) - f(0)}{3 - 0} = \frac{0 - 1}{3} = -\frac{1}{3}$$

$$x = 3 - 0 = 3$$

Average rate of change:

$$\frac{1-1}{3} = 0$$

$$\frac{f(2) - f(5)}{2 - 5}$$

$$= \frac{6 - 15}{-2 - (-5)} = \frac{-9}{3} = -3$$

$$f'(x) = -\frac{1}{x^2}$$

$$3x$$

Instantaneous rates of change:

$$f(-2) = -12, f(-5) = -75$$

17.  $f(x) = \sqrt[3]{x}$

$$x = 27, 8$$

Average rate of change:

$$\frac{f(27) - f(8)}{27 - 8} = \frac{3 - 2}{19} = \frac{1}{19}$$

$$f(x) = \sqrt[3]{x} = x^{1/3}$$

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

Instantaneous rates of change:  $f'(8) = \frac{1}{6}$

$$f'(27) = \frac{1}{12}$$

$$\frac{1}{27}$$

$$P = -0.0125x^2$$

$\frac{dP}{dx}$

$$dx = -0.025x + 16$$

$$\text{When } x = 175, \frac{dP}{dx} = -\$11.625$$

$$P(176) - P(175) = 1828.8 - 1817.1875 = \$11.6125$$

The results are approximately equal.

$$f(x) = 5x^2 + 6x - 1$$

$$f'(x) = 10x + 6$$

$$f'(x) = 2x - 3$$

Instantaneous rates of change:  $f'(0) = -3$ ,  $f'(3) = 3$

$$f(x) = 2x^3 + x^3 - x + 4; [-1, 1]$$

Average rate of change:

$$\frac{y}{x} = \frac{f(x) - f(-1)}{x - (-1)} =$$

$$f'(x) = 10x + 6$$

At  $(-1, -2)$ ,  $m = -4$ .

$$+ 2 = -4(x + 1) y$$
$$= -4x - 6$$

2

-5

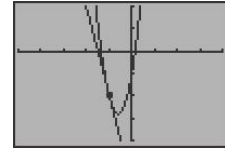
$$x \quad 1 - -1 \quad 4 ( )$$



$$= \frac{6-4}{2} = 1 \quad (-1, -2)$$

$$f'(x) = 6x^2 + 2x - 1$$

Instantaneous rates of change:  $f'(-1) = 3$ ,  $f'(1) = 7$



20.  $f(x) = \frac{8}{\sqrt{x^3}} = 8x^{-3/2}$

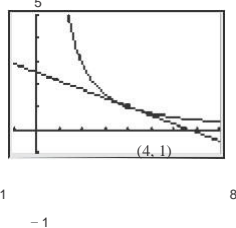
$f'(x) = -12x^{-5/2} = -\frac{12}{x^2\sqrt{x}}$

$m = f'(4) = -\frac{12}{(4)^2\sqrt{4}} = -\frac{3}{8}$

$y - 1 = -\frac{3}{8}(x - 4)$

$y - 1 = -\frac{3}{8}x + \frac{3}{2}$

$y = -\frac{3}{8}x + \frac{5}{2}$



$f(x) = \frac{5x + 4}{-3x}$

$(2 - 3x)(5) - (5x + 4)(-3) = 10 - 15x + 15x + 12$

$f'(x) = \frac{(2 - 3x)^2}{22} = \frac{(2 - 3x)^2}{22}$

$m = f'(1) = \frac{(2 - 3(1))^2}{22} = \frac{1}{22}$

$-(-9) = 22(x - 1)$

$y + 9 = 22x - 22$

$y = 22x - 31$

21.  $f(x) = x^2 + 1(4x - 3)$

$f'(x) = 2x + 4 = 2x + 4$

$4x^2 + 4 + 8x^2 - 6x$

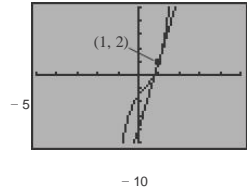
$12x^2 - 6x + 4$

$m = f'(1) = 12(1) - 6 + 4 = 10$

$6(1)$

$-2 = 10(x - 1)$

$= 10x - 10$



$= 22$

$S = -0.01722t^3 + 0.7333t^2 - 7.657t + 45.47, 7 \leq t \leq 13$

$\frac{dS}{dt} = S'(t) = -0.051666t^2 + 1.4666t - 7.657$

2008:  $S'(8) \approx \$0.77/\text{yr}$

2011:  $S'(11) \approx \$2.22/\text{yr}$

2012:  $S'(12) \approx \$2.50/\text{yr}$

## Section 2.5 The Chain Rule

### Skills Warm Up

1.  $\frac{d}{dx}(1 - 5x)^2 = 2(1 - 5x)(-5) = -10(1 - 5x)$

2.  $\frac{d}{dx}\sqrt[4]{2x - 1} = \frac{1}{4}(2x - 1)^{-3/4} \cdot 2 = \frac{1}{2}(2x - 1)^{-3/4}$

3.  $\frac{d}{dx}\frac{1}{\sqrt{4x^2 + 1}} = -\frac{1}{2}(4x^2 + 1)^{-3/2} \cdot 8x = -\frac{4x}{(4x^2 + 1)^{3/2}}$

5.  $\frac{d}{dx}\sqrt[3]{1 - 2x} = \frac{1}{3}(1 - 2x)^{-2/3} \cdot (-2) = -\frac{2}{3(1 - 2x)^{2/3}}$

$\frac{d}{dx}\sqrt{3 - 7x} = \frac{1}{2}(3 - 7x)^{-1/2} \cdot (-7) = -\frac{7}{2\sqrt{3 - 7x}}$

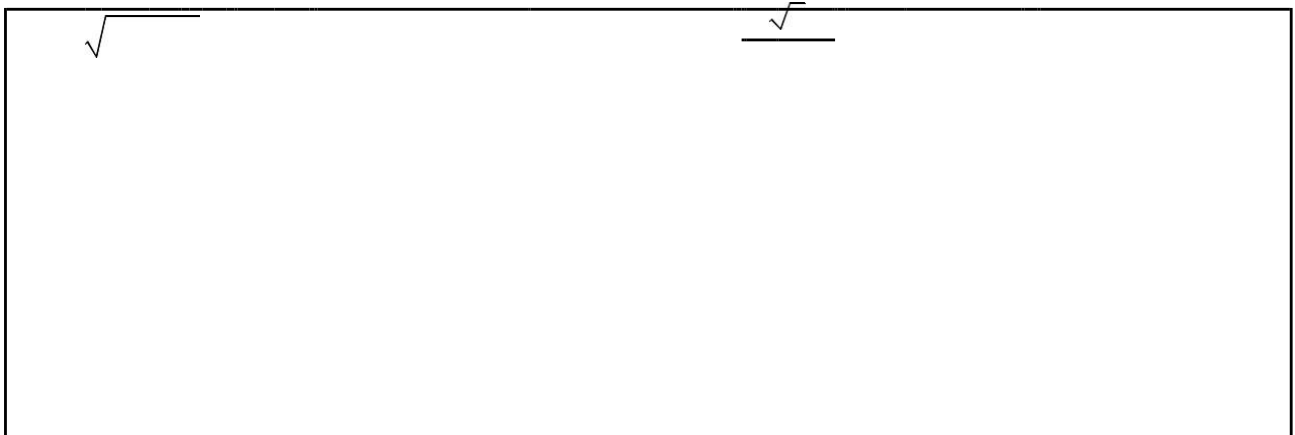
6.  $\frac{d}{dx}(2x)^{3 - 7x} = (2x)^{3 - 7x} \left[ \frac{1}{2x} + (3 - 7x)(-\ln 2) \right]$

7.  $3x^3 - 6x^2 + 5x - 10 = 3x^2(x - 2) + 5(x - 2)$

4.  $\frac{1}{\sqrt{2x^3+9}}$   $\frac{1}{\sqrt{2x^3+9}}$

$\frac{-(2x^3+9)^{-1/2}}{2}$

$\frac{1}{\sqrt{2x^3+9}}$  ( )



**Skills Warm Up—continued—**

$$8. \sqrt{5x} \cdot x - x - \sqrt{5x} + 1 = x^2 \sqrt{5x} - 1 - 1 \sqrt{5x} - 1$$

$$= (x^2 - 1) \sqrt{5x} - 1$$

$$9. 4(x+1)^2 - x(x+1) = (x+1)^2 \cdot 4 - x(x+1)$$

$$= 4x^2 + 8x + 4 - x^2 - x$$

$$10. -x^5 + 6x^3 + 7x^2 - 42 = -x^3(x^2 - 6) + 7(x^2 - 6)$$

$$= (-x^3 + 7)(x^2 - 6)$$

$$= -(x^3 - 7)(x^2 - 6)$$

$y = f(g(x))$        $u = g(x)$

1.  $y = (6x - 5)^4$        $u = 6x - 5$        $y = u^4$

2.  $y = x^2 - 2x + 3$        $u = x^2 - 2x + 3$        $y = u^3$

3.  $y = 5\sqrt{x-2}$        $u = 5x - 2$        $y = \sqrt{u}$

4.  $y = \sqrt[3]{9-x^2}$        $u = 9-x^2$        $y = u^{-1/3}$

5.  $y = 3x + 1$        $u = 3x + 1$        $y = u^{-1/2}$

6.  $y = (x^2 - 3)^{-1/2}$        $u = x^2 - 3$        $y = u^{-1/2}$

$$y = (4x + 7)^2$$

$$y' = 2(4x + 7)^1 (4)$$

$$y' = 8(4x + 7)$$

$$32x + 56$$

$$y = (3x^2 - 2)^3$$

$$y' = 3(3x^2 - 2)^2 (6x)$$

$$y' = 18x(3x^2 - 2)^2$$

$$y = 3\sqrt{x^2} = (3 - x^2)^{1/2}$$

$$y' = \frac{1}{2}(3 - x^2)^{-1/2} (-2x)$$

$$y' = -\frac{x}{\sqrt{3 - x^2}}$$

$$y' = -x(3 - x)^{-1/2} = -(3 - x^2)^{-1/2}$$

$$y' = -\frac{x}{\sqrt{3 - x^2}}$$

$$y = (5x^4 - 2x)^{2/3}$$

$$y' = \frac{2}{3}(5x^4 - 2x)^{-1/3} (20x^3 - 2)$$

$$y' = \frac{2}{3} 5x^4 - 2x^{-1/3} 10x^3 - 1$$

$$y' = \frac{2}{3} (5x^4 - 2x)^{-1/3} (10x^3 - 1)$$

$$= \frac{2(10x^3 - 1)}{3(5x^4 - 2x)^{1/3}}$$

$$y = \frac{3x^2 - 2x + 1}{\sqrt{4x^2 - 2x}}$$

$$y = (x^3 + 2x^2)^{-1}$$

$$y' = (-1)(x^3 + 2x^2)^{-2}$$

$$y' = -\frac{3x^2 + 4x}{(x^3 + 2x^2)^2}$$

13.  $f(x) = \frac{1-x^3}{2} = 2(1-x^3)^{-1}$ ; (c) General Power Rule

14.  $f(x) = \frac{7}{1-x^3} = 7(1-x^3)^{-3}$ ; (c) General Power Rule

$$f(x) = 3\sqrt[3]{8^2}$$
; (b) Constant Rule

$f(x) = 3\sqrt{x^2} = x^{2/3}$ ; (a) Simple Power Rule

17.  $f(x) = \frac{x^3 + 4x^2 - 6}{x^3 + 2x - 5}$ ; (d) Quotient Rule

18.  $f(x) = \frac{x^{1/2}}{x^3 + 2x - 5}$ ; (d) Quotient Rule

10.  $y = 4^4 6x + 5 = 4 6x + 5 14$

$$y' = 3(2x - 7)^2 (2) = 6(2x - 7)^2$$

$$y' = 4 \frac{1}{4} 6(x + 5)^{-5/4} (6)$$

$$y' = 6(6x + 5)^{-5/4} = \frac{6}{(6x + 5)^{5/4}}$$

$$y = (3 - 5x)^4$$

$$y' = 4(3 - 5x)^3 (-5) = -20(3 - 5x)^3$$

$$h'(x) = 2(6x - x^3)(6 - 3x^2) = 6x(6 - x^2)(2 - x^2)$$

$$f(x) = 2x^3 - 6x^4$$

$$f'(x) = \frac{d}{dx}(2x^3 - 6x^4) = 6x^2 - 24x^3$$

$$f'(x) = 6x^2 - 24x^3$$

$$f'(x) = 6x^2 - 24x^3$$

$$f(t) = \sqrt{t+1} = (t+1)^{1/2}$$

$$f'(t) = \frac{1}{2}(t+1)^{-1/2} = \frac{1}{2\sqrt{t+1}}$$

$$g(x) = \sqrt{5-3x} = (5-3x)^{1/2}$$

$$g'(x) = \frac{1}{2}(5-3x)^{-1/2}(-3) = -\frac{3}{2\sqrt{5-3x}}$$

$$s(t) = \sqrt{2t^2 + 5t + 2} = (2t^2 + 5t + 2)^{1/2}$$

$$s'(t) = \frac{1}{2}(2t^2 + 5t + 2)^{-1/2}(4t + 5) = \frac{4t + 5}{2\sqrt{2t^2 + 5t + 2}}$$

$$y = 9\sqrt[3]{4x^2 + 3} = 9(4x^2 + 3)^{1/3}$$

$$y' = 9 \cdot \frac{1}{3}(4x^2 + 3)^{-2/3} \cdot 8x = \frac{24x}{(4x^2 + 3)^{2/3}}$$

$$y' = 24x(4x^2 + 3)^{-2/3}$$

$$y' = \frac{24x}{(4x^2 + 3)^{2/3}}$$

$$f(x) = 2(2-9x)^{-3}$$

$$f'(x) = 2(-3)(2-9x)^{-4}(-9) = \frac{54}{(2-9x)^4}$$

$$28. g(x) = \frac{3}{7x^2 + 6x} = 3(7x^2 + 6x)^{-1}$$

$$g'(x) = 3(-1)(7x^2 + 6x)^{-2}(14x + 6) = -\frac{3(14x + 6)}{(7x^2 + 6x)^2}$$

$$g'(x) = -\frac{3(14x + 6)}{(7x^2 + 6x)^2}$$

$$29. f(x) = \frac{1}{\sqrt{x^2 + 11}} = (x^2 + 11)^{-1/2}$$

$$f'(x) = -\frac{1}{2}(x^2 + 11)^{-3/2}(2x) = -\frac{x}{(x^2 + 11)^{3/2}}$$

$$f'(x) = -\frac{x}{(x^2 + 11)^{3/2}}$$

$$f'(x) = -\frac{x}{(x^2 + 11)^{3/2}} = -\frac{x}{\sqrt{(x^2 + 11)^3}}$$

$$30. y = (4-x^3)^{-4/3}$$

$$y' = -\frac{4}{3}(4-x^3)^{-7/3}(-3x^2) = \frac{4x^2}{(4-x^3)^{7/3}}$$

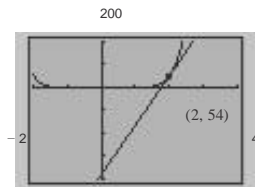
$$31. f'(x) = 2(3-x^2) - 1^2(2x) = 12x - 2x = 10x$$

$$f'(2) = 10(2) = 20$$

$$f(2) = 54$$

$$y - 54 = 20(x - 2)$$

$$y = 20x - 38$$



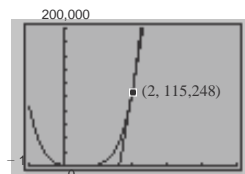
$$32. f'(x) = 12(9x - 4)^3 (9) = 108(9x - 4)^3$$

$$f'(2) = 12(14)^3 (9) = 296,352$$

$$f(2) = 3(14)^4 = 115,248$$

$$-115,248 = 296,352(x - 2)$$

$$y = 296,352x - 477,456$$



$$33. f(x) = \sqrt{4x^2 - 7} = (4x^2 - 7)^{1/2}$$

$$g'(x) = -15 \cdot 7x^2 + 6x - 6 \cdot 14x + 6$$

$$= -\frac{15(14x + 6)}{(7x^2 + 6x)^6}$$

$$= -\frac{15(14x + 6)}{(7x^2 + 6x)^6}$$

$$f(x) = \frac{1}{(4x^2 - 7)^{-1/2}} (8x) = \frac{4x}{\sqrt{4x^2 - 7}}$$

$$2$$

$$4x^2 - 7$$

$$\frac{8}{3}$$

$$f'(2) = \frac{8}{3}$$

$$y = \frac{8}{3}x - \frac{7}{3}$$

$$f(2) = 3$$

$$3 \quad 3$$

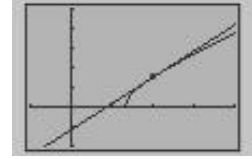
$$y - 3 = \frac{8}{3}(x - 2)$$

10

(  
2  
·  
3  
)

$\frac{1}{4}$

4





Chapter 2 Differentiation

34.  $f(x) = x\sqrt{x^2 + 5} = x(x^2 + 5)^{1/2}$

$f'(x) = x^{1/2}x^2 + 5^{-1/2}2x + x^2 + 5^{1/2}$

$f'(2) = \frac{13}{3}$

$f(2) = 6$

$y - 6 = \frac{13}{3}(x - 2)$

$y = \frac{13}{3}x - \frac{8}{3}$



35.  $f(x) = \sqrt{x^2 - 2x + 1} = x^2 - 2x + 1^{1/2}$

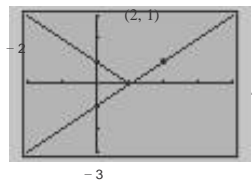
$f'(x) = 2(x^2 - 2x + 1)^{-1/2}(2x - 2)$

$f'(2) = 1$

$f(2) = 1$

$-1 = 1(x - 2)$

$y = x - 1$



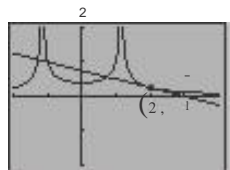
36.  $f(x) = -2(4 - 3x^2)^{-2/3}$

$f'(2) = \frac{8}{-32} = -\frac{1}{4}$

$f(2) = (-8)^{-2/3} = \frac{1}{4}$

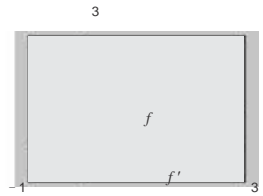
$y - \frac{1}{4} = -\frac{1}{4}(x - 2)$

$y = -\frac{1}{4}x + \frac{1}{2}$



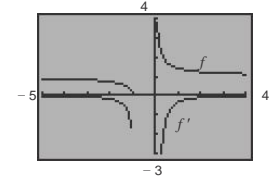
38.  $f'(x) = \frac{2}{\sqrt{x}\sqrt{x+1}}$

$f'$  is never 0.



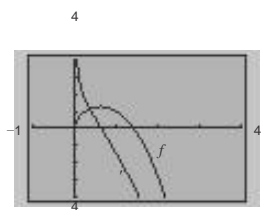
39.  $f(x) = -\sqrt{x+1}$

$f'$  is never 0.



40.  $f'(x) = \frac{2 - 5x^2}{2\sqrt{x}}$

has a horizontal tangent when  $f' = 0$ .



$y = 4 - x^2$

41.  $y' = -2x$

$y = 4 - x^2$

$y' = -2x$

$y = (4 - x^2)^2$

General Power Rule

42.  $s(t) = t^2 + 3t - 1$

$s'(t) = 2t + 3$

$s'(2) = 7$

$s(2) = 7$

General Power Rule

43.  $y = \frac{5t}{(t+8)^2}$

$y' = \frac{(t+8)^2(5) - (5t)(2)(t+8)(1)}{(t+8)^4}$

$y' = -\frac{1}{4}(x-2)$

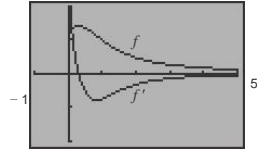
Chapter 2 Differentiation

$$y' = \frac{5(t+8)(t+8) - 2t}{(t+8)^4}$$

$$= -\frac{1}{4}x + 4\frac{3}{2}$$

$$f'(x) = \frac{1 - 3x^2 - 4x^{\frac{3}{2}}}{2x\sqrt{x^2+1}}$$

$f$  has a horizontal tangent when  $f' = 0$ .



$$y = \frac{(t+8)^4}{5t+8-t+8} = \frac{5t-8}{(t+8)^3}$$

Quotient Rule and Chain Rule

$$f(x) = 3x(x^3 - 4)^{-2}$$

$$f'(x) = 3x(-2)(x^3 - 4)^{-3} (3x^2) + (x^3 - 4)^{-2} (-18x^3)$$

$$= -3(x^3 - 4)^{-3} 6x^3 - (x^3 - 4)^{-2} 18x^3$$

$$= \frac{3(5x^3 + 4)}{(x^3 - 4)^3}$$

Product Rule and Chain Rule

45.  $f(x) = (2x - 1)(9 - 3x^2)$

$$f'(x) = (2x - 1)(-6x) + (9 - 3x^2)(2)$$

$$= -12x^2 + 6x + 18 - 6x^2$$

$$= 18 + 6x - 18x^2 - 6(3x^2 - 2x - 3)$$

Product Rule and Simple Power Rule

$$y = (7x + 4)(x^3 - 2x^2)$$

$$y' = (7x + 4)(3x^2 - 4x) + (x^3 - 2x^2)(7)$$

$$= 21x^3 - 16x^2 - 16x + 7x^3 - 14x^2$$

$$= 28x^3 - 30x^2 - 16x$$

Product Rule and Simple Power Rule

47.  $y = \frac{1}{\sqrt{x+2}} = (x+2)^{-1/2}$

$$y' = -\frac{1}{2}(x+2)^{-3/2} = -\frac{1}{2(x+2)^{3/2}}$$

General Power Rule

48.  $g(x) = \sqrt[3]{x^3 - 1} = (x^3 - 1)^{1/3}$

$$g'(x) = \frac{1}{3}(x^3 - 1)^{-2/3} (3x^2) = \frac{x^2}{(x^3 - 1)^{2/3}}$$

General Power Rule

49.  $f(x) = x(3x - 9)^3$

$$f'(x) = x(3)(3x - 9)^2 (3) + (3x - 9)^3 (1)$$

$$(3x - 9)^2 9x + (3x - 9)^3$$

$$f(x) = x^3(x - 4)^2$$

$$= x^3(x^2 - 8x + 16)$$

$$= x^5 - 8x^4 + 16x^3$$

$$f'(x) = 5x^4 - 32x^3 + 48x^2$$

$$= x^2(5x^2 - 32x + 48)$$

$$= x^2(5x - 12)(x - 4)$$

Simple Power Rule

$$y = x\sqrt{x+3} = x(2x+3)^{1/2}$$

$$y' = x \cdot \frac{1}{2}(2x+3)^{-1/2} (2) + (2x+3)^{1/2} (2)$$

$$= (2x+3)^{-1/2} x + (2x+3)$$

$$= \frac{3(x+1)}{\sqrt{2x+3}}$$

Product and General Power Rule

$$y = 2t\sqrt{t+6} = 2t(t+6)^{1/2}$$

$$y' = 2t \cdot \frac{1}{2}(t+6)^{-1/2} (1) + (t+6)^{1/2} (2)$$

$$= t(t+6)^{-1/2} + 2(t+6)^{1/2}$$

$$= \frac{t}{\sqrt{t+6}} + 2\sqrt{t+6}$$

$$= \frac{t + 2(t+6)}{\sqrt{t+6}} = \frac{3t+12}{\sqrt{t+6}} = \frac{3(t+4)}{\sqrt{t+6}}$$

Product and General Power Rule

53.  $y = t^2\sqrt{t-2} = t^2(t-2)^{1/2}$

$$y' = t^2 \cdot \frac{1}{2}(t-2)^{-1/2} (1) + 2t(t-2)^{1/2} (2)$$

$$= \frac{t^2}{2\sqrt{t-2}} + 4t\sqrt{t-2}$$

$$= \frac{t^2 + 4t(t-2)}{2\sqrt{t-2}}$$

$$= \frac{t(5t-8)}{2\sqrt{t-2}}$$

Product and General Power Rule

$$9(x - 3)^2 (12x - 9)$$

$$27(x - 3)^2 (4x - 3)$$

Product and General Power Rule

Chapter 2 Differentiation

$$y = x\sqrt{x-2}^2 = x^{1/2}(x-2)^2$$

$$y' = x^{1/2} \cdot 2(x-2)^1 (1) + (x-2)^2 \cdot \frac{1}{2} x^{-1/2}$$

$$= 2\sqrt{x}(x-2) + \frac{(x-2)^2}{2\sqrt{x}}$$

$$= \frac{4x(x-2) + (x-2)^2}{2\sqrt{x}}$$

$$= \frac{(x-2)4x + (x-2)^2}{2\sqrt{x}}$$

$$\frac{(x-2)(5x-2)}{\sqrt{x}}$$

Product and General Power Rule

55.  $y = \frac{6-5x}{x^2-1}$

$$y' = \frac{(6-5x)'(x^2-1) - (6-5x)(x^2-1)'}{(x^2-1)^2}$$

$$y' = \frac{2(6-5x)(5x^2-12x+5)}{(x^2-1)^3}$$

$$\frac{2(6-5x)(5x^2-12x+5)}{(x^2-1)^3}$$

Quotient and General Power Rule

$$y = \frac{4x^2-5}{2-x}$$

$$-x$$

$$y' = \frac{(4x^2-5)'(2-x) - (4x^2-5)(2-x)'}{(2-x)^2}$$

$$\frac{4x^2-5}{(2-x)^2}$$

$$= \frac{3(4x^2-5)(-4x^2+16x-5)}{(2-x)^3}$$

$$= \frac{3(4x^2-5)(-4x^2+16x-5)}{(2-x)^3}$$

$$= \frac{3(4x^2-5)(-4x^2+16x-5)}{(2-x)^3}$$

$$f(x) = (3x^3 + 4x)^{1/5}$$

$$f'(x) = \frac{1}{5} (3x^3 + 4x)^{-4/5} (9x^2 + 4)$$

$$m = f'(2) = \frac{1}{4}$$

$$y-2 = \frac{1}{4}(x-2)$$

$$y = \frac{1}{4}x + 1$$



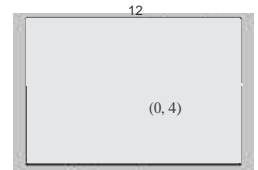
59.  $f(t) = \frac{36}{(3-t)^2} = 36(3-t)^{-2}$

$$f'(t) = -72(3-t)^{-3} (-1) = \frac{72}{(3-t)^3}$$

$$f'(0) = \frac{8}{27}$$

$$y-4 = \frac{8}{27}(t-0)$$

$$y = \frac{8}{27}t + 4$$



60.  $s(x) = \frac{1}{\sqrt{x^2-3x+4}}$

$$s'(x) = \frac{1}{2} (x^2-3x+4)^{-3/2} (2x-3)$$

$$= \frac{3-2x}{2(x^2-3x+4)^{3/2}}$$

$$= \frac{3-2x}{2(x^2-3x+4)^{3/2}}$$

$$s'(3) = \frac{3-6}{2(4)^{3/2}} = -\frac{3}{16}$$

$$= -\frac{3}{16}$$

$$\frac{1}{3} (x-3)$$

Q tie d General Power Rule

$$y = (x^3 - 2x^2 - x + 1)^2$$

Chapter 2 Differentiation

$$y' = 2(x^3 - 2x^2 - x + 1)(3x^2 - 4x - 1)$$

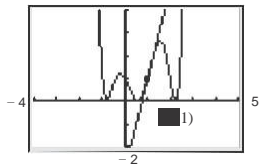
4

$$= y'(1) = 2(1^3 - 2(1)^2 - (1) + 1)(3(1)^2 - 4(1) - 1)$$

$$= 2(-1)(-2) = 4$$

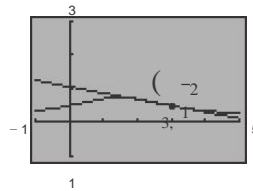
$$-1 = 4(x - 1)$$

$$y = 4x - 3$$



$$y - 2 = -16$$

$$= -\frac{1}{16}x + \frac{17}{16}$$



61.  $f(t) = (t^2 - 9)(t + 2) = (t^2 - 9)(t + 2)^{1.2}$

63.  $f(x) = \frac{x+1}{2x-3} = \frac{x+1}{(2x-3)^{1.2}}$

$$f'(t) = (t^2 - 9) \cdot 2(t + 2)^{-1.2} + (t^2 - 9)' \cdot (t + 2)^{1.2}$$

$$= \frac{1}{2}(t^2 - 9)(t + 2)^{-1.2} + 2t(t + 2)^{1.2}$$

2

$$= (t + 2)^{-1.2} \frac{t^2 - 9 + 2t(t + 2)}{2}$$

$$= \frac{t^2 - 9 + 2t^2 + 4t}{2(t + 2)^{1.2}}$$

$$= \frac{2t^2 + 4t - 9}{2(t + 2)^{1.2}}$$

$$y - (-8) = -6t - (-1)$$

$$y - (-8) = -6t - (-1)$$

$$y = -6t - 14$$

$$\frac{2x}{2x}$$

62.  $y = (1-x)^{-1.2} = (1-x)^{-1.2}$

$$y' = -\frac{1-x}{(1-x)^{2.2}} \cdot (-1) = \frac{1-x}{(1-x)^{2.2}}$$

1/2

$$= -\frac{(1-x)^{-1.2} + x \cdot (-1.2)(1-x)^{-2.2}}{(1-x)^2}$$

1-x

$$= -\frac{(1-x)^{-1.2} + x \cdot (-1.2)(1-x)^{-2.2}}{(1-x)^2}$$

1-x

$$= -\frac{(1-x)^{-1.2} + x \cdot (-1.2)(1-x)^{-2.2}}{(1-x)^2}$$

$$(1-x)^{-1.2}(2-x)$$

$$= \frac{x+1}{2x-3} = \frac{x+1}{(2x-3)^{1.2}}$$

$$f'(x) = \frac{2(x-3)^{1.2} \cdot 1 - (x+1) \cdot 1.2(2x-3)^{0.2}}{(2x-3)^{2.4}}$$

$$= \frac{2(x-3)^{1.2} - 1.2(x+1)(2x-3)^{0.2}}{(2x-3)^{2.4}}$$

$$= \frac{2(x-3)^{1.2} - 1.2(x+1)(2x-3)^{0.2}}{(2x-3)^{2.4}}$$

$$= 2$$

$$\frac{x-4}{1-3}$$

$$f'(2) = 1 = -2$$

$$y - 3 = -2(x - 2)$$

$$y = -2x + 7$$

$$y = \frac{x}{25+x^2} = x(25+x^2)^{-1.2}$$

64.

$$y' = \frac{1 \cdot (25+x^2)^{-1.2} - x \cdot (-1.2)(25+x^2)^{-2.2} \cdot 2x}{(25+x^2)^{2.4}}$$

$$y' = \frac{(25+x^2)^{-1.2} + 2.4x^2(25+x^2)^{-2.2}}{(25+x^2)^{2.4}}$$

$$= \frac{25}{(25+x^2)^{3.2}} + \frac{2.4x^2}{(25+x^2)^{2.2}}$$

$$= \frac{25}{(25+x^2)^{3.2}} + \frac{2.4x^2}{(25+x^2)^{2.2}}$$

$$(y)' = \frac{1}{5}$$

$$y - 0 = \frac{1}{5}(x - 0)$$

$$y = \frac{1}{5}x$$

65.  $f(x) = x^2 + 4 = (x^2 + 4)^{1.3}$

$$f'(x) = 2x + 4^{-2.3} \cdot 2.3x$$

$$= - \frac{\quad}{1-x}$$

$$\frac{2-x}{\quad}$$

$$= - \frac{1-x}{32}$$

$$= \frac{x-2}{3x^2}$$

$$(1-x)^{3/2}$$

$$\left( \frac{-3-2}{5} \right) = \frac{-5}{5} = -1$$

$$y'(-3) = \frac{-5}{5} = -1$$

$$f'(x) = 2x$$

$$3(x+4)^{2/3}$$

$$2x = 0$$

$$\text{Set } f'(x) = 0 \Rightarrow x = -4$$

$$2x = 0$$

$$x = 0 \rightarrow y = f(0) = 4$$

Horizontal tangent at:  $(0, 4)$



$$y - 3 = - \frac{5}{8} (x - (-3))$$

6

$$y - 3 = - \frac{5}{8} x - \frac{15}{8}$$

$$y = - \frac{5}{8} x + \frac{9}{8}$$

(-3, 3)

-7

1

-1

66.  $f(x) = \sqrt{5x^2 + x - 3} = (5x^2 + x - 3)^{1/2}$

$$f'(x) = \frac{1}{2} (5x^2 + x - 3)^{-1/2} (10x + 1)$$

$$f'(x) = \frac{10x + 1}{2\sqrt{5x^2 + x - 3}}$$

Set  $f'(x) = \frac{10x + 1}{2\sqrt{5x^2 + x - 3}} = 0$

$$10x + 1 = 0$$

$$x = -\frac{1}{10} \rightarrow y = f\left(-\frac{1}{10}\right) = \sqrt{\frac{61}{20}}$$

Because  $\sqrt{\frac{61}{20}}$  is not a real number, there is no point of horizontal tangency.

67.  $f(x) = \sqrt{2x-1} = (2x-1)^{1/2}$

$$f'(x) = \frac{1}{2} (2x-1)^{-1/2} (2) = \frac{1}{\sqrt{2x-1}}$$

$$f(x) = (2x-1)^{1/2}$$

$$f'(x) = \frac{2x-1 - x(2x-1)}{(2x-1)^{3/2}}$$

$$f'(x) = \frac{2x-1 - 2x^2 + x}{(2x-1)^{3/2}}$$

$$f'(x) = \frac{x-1}{(2x-1)^{3/2}}$$

Set  $f'(x) = \frac{x-1}{(2x-1)^{3/2}} = 0$

$$x - 1 = 0$$

$$x = 1 \rightarrow y = f(1) = \frac{1}{\sqrt{1}} = 1$$

Horizontal tangent at: (1, 1)

68.  $f(x) = \frac{5x}{\sqrt{3x-2}} = 5x(3x-2)^{-1/2}$

$$f'(x) = (3x-2)^{1/2} (5) - 5x \cdot \frac{1}{2} (3x-2)^{-3/2} (3)$$

$$f'(x) = \frac{5(3x-2)^{3/2} - \frac{15}{2}x(3x-2)^{1/2}}{(3x-2)^2}$$

$$f'(x) = \frac{5(3x-2)^{1/2} \left[ (3x-2) - \frac{3}{2}x \right]}{(2x+1)^2}$$

$$f'(x) = \frac{5(3x-2)^{-1/2} (3-x)}{(2x-1)^2}$$

$$f'(x) = \frac{5(3x-4)}{2(3x-2)^{3/2}}$$

Set  $f'(x) = \frac{5(3x-4)}{2(3x-2)^{3/2}} = 0$

$$5(3x-4) = 0$$

$$3x - 4 = 0$$

$$x = \frac{4}{3} \rightarrow y = f\left(\frac{4}{3}\right) = \frac{20}{3\sqrt{2}}$$

Horizontal tangent at:  $\left(\frac{4}{3}, \frac{20}{3\sqrt{2}}\right)$

69.  $A' = 1000 \left(\frac{60}{100}\right)^{12} + \frac{1}{12} = 50001 + \frac{1}{12}$

(a)  $A'(0.08) = 501 + \frac{0.08}{12}$

≈ \$74.00 per percentage point

(b)  $A'(0.10) = 501 + \frac{0.10}{12}$

\$81.59 per percentage point

(c)  $A'(0.12) = 501 + \frac{0.12}{12}$

\$89.94 per percentage point

$$70. N = 4001 - 3t^2 + 2t^{-2}$$

$$\frac{dN}{dt} = N'(t) = 400(-3)(-2)(t^{-2} + 2)^{-3}(2t)$$

$$= \frac{4800t}{t^2 + 2^3}$$

$$N'(0) = 0 \text{ bacteria/day}$$

$$N'(1) \approx 177.8 \text{ bacteria/day}$$

$$N'(2) \approx 44.4 \text{ bacteria/day}$$

$$N'(3) \approx 10.8 \text{ bacteria/day}$$

$$N'(4) \approx 3.3 \text{ bacteria/day}$$

(f) The rate of change of the population is decreasing as time passes.

$$71. V = \frac{k}{\sqrt{t+1}}$$

When  $t = 0$ ,  $V = 10,000$ .

$$10,000 = \frac{k}{\sqrt{0+1}} \quad k = 10,000$$

$$V = \frac{10,000}{\sqrt{t+1}}$$

$$\frac{dV}{dt} = -5000(t+1)^{-3/2} \quad (1) = -\frac{5000}{(t+1)^{3/2}}$$

When  $t = 1$ ,

$$\frac{dV}{dt} = -\frac{5000}{(2)^{3/2}} = -\frac{2500}{\sqrt{2}} \approx -\$1767.77 \text{ per year.}$$

When  $t = 3$ ,  $\frac{dV}{dt} = -\frac{5000}{(4)^{3/2}} = -\$625.00 \text{ per year.}$

72. (a) From the graph, the tangent line at  $t = 4$  is steeper than the tangent line at  $t = 1$ . So, the rate of change after 4 hours is greater.  
 (b) The cost function is a composite function of  $x$  units, which is a function of the number of hours, which is not a linear function.

73. (a)  $r = (0.3017t^4 - 9.657t^3 + 97.35t^2 - 266.8t - 242)^{1/2}$

$$\frac{dr}{dt} = r'(t) = \frac{1}{2} (0.3017t^4 - 9.657t^3 + 97.35t^2 - 266.8t - 242)^{-1/2} \cdot (1.2068t^3 - 28.971t^2 + 194.7t - 266.8)$$

$$= \frac{1.2068t^3 - 28.971t^2 + 194.7t - 266.8}{\sqrt{0.3017t^4 - 9.657t^3 + 97.35t^2 - 266.8t - 242}}$$

Chain Rule



The rate of change appears to be the greatest when  $t = 8$  or 2008.

The rate of change appears to be the least when  $t \approx 9.60$ , or 2009, and when  $t \approx 12.57$ , or 2012.

## Section 2.6 Higher-Order Derivatives

### Skills Warm Up

1.  $-16t^2 + 292 = 0$

$$-16t^2 = -292$$

$$t^2 = \frac{73}{4}$$

$$t = \pm \frac{\sqrt{73}}{2}$$

2.  $-16t^2 + 88t = 0$

$$-8t(2t - 11) = 0$$

$$-8t = 0 \rightarrow t = 0$$

$$2t - 11 = 0 \rightarrow t = \frac{11}{2}$$

**Skills Warm Up —continued—**

$$-16t^2 + 128t + 320 = 0$$

$$-16(t^2 - 8t - 20) = 0$$

$$-16(t - 10)(t + 2) = 0$$

$$t - 10 = 0 \rightarrow t = 10$$

$$t + 2 = 0 \rightarrow t = -2$$

$$-16t^2 + 9t + 1440 = 0$$

$$= -9 \pm \sqrt{9^2 - 4(-16)(1440)}$$

$$t = \frac{-9 \pm \sqrt{81 - 92160}}{-32}$$

$$\frac{-9 \pm \sqrt{92169}}{-32}$$

$$\frac{9 \pm 302.44}{-32}$$

$$\approx -9.21 \text{ and } t \approx 9.77$$

$$y = x^2(2x + 7)$$

$$\frac{dy}{dx} = x^2(2) + 2x(2x + 7)$$

$$2x^2 + 4x^2 + 14x$$

$$6x^2 + 14x$$

$$y = (x^2 + 3x)(2x^2 - 5)$$

$$\frac{dy}{dx} = (2x)(2x^2 - 5) + (x^2 + 3x)(4x)$$

$$dx$$

$$= 4x^3 + 12x^2 + 4x^3 - 10x + 6x^2 - 15$$

$$= 8x^3 + 18x^2 - 10x - 15$$

$$7. y = \frac{x^2}{2x + 7}$$

$$\frac{dy}{dx} = \frac{(2x + 7)(2x) - (x^2)(2)}{(2x + 7)^2}$$

$$= \frac{4x^2 + 14x - 2x^2}{(2x + 7)^2}$$

$$= \frac{2x^2 + 14x}{(2x + 7)^2}$$

$$\frac{2-x(x+7)}{(2x+7)^2}$$

$$y = \frac{x^2 + 3x}{2x^2 - 5}$$

$$\frac{dy}{dx} = \frac{(2x^2 - 5)(2x + 3) - (x^2 + 3x)(4x)}{(2x^2 - 5)^2}$$

$$dx$$

$$\frac{4x^3 + 6x^2 - 10x - 15 - 4x^3 - 12x^2}{(2x^2 - 5)^2}$$

$$\frac{-6x^2 - 10x - 15}{(2x^2 - 5)^2}$$

9. ( )

$$f(x) = x^2 - 4$$

Domain:  $(-\infty, \infty)$

Range:  $[-4, \infty)$

10.  $f(x) = \sqrt{x-7}$

Domain:  $[7, \infty)$

Range:  $[0, \infty)$

1.  $f(x) = 9 - 2x$

$$f'(x) = -2$$

$$f''(x) = 0$$

2.  $f(x) = 4x + 15$

$$f''(x) = 0$$

3.  $f(x) = x^2 + 7x - 4$

$$f'(x) = 2x + 7$$

$$f''(x) = 2$$

4.  $f(x) = 3x^2 + 4x + 6$

$$f'(x) = 6x + 4$$

$$f''(x) = 6$$

5.  $g(t) = \frac{1}{3}t^3 - 4t^2 + 2t$

$$g'(t) = t^2 - 8t + 2$$

$$g''(t) = 2t - 8$$

6.  $j(x) = -\frac{5}{4}x^4 + 3x^2 - 6x$

$$j'(x) = -5x^3 + 6x - 6$$

$$j''(x) = -15x^2 + 6$$

$$j'''(x) = -30x$$

$$7. f(t) = \frac{2}{3} = 2t^{-3}$$

$$f'(t) = -6t^{-4}$$

$$f''(t) = 24t^{-5} = \frac{24}{t^5}$$

$$g(t) = \frac{5}{6} = \frac{5}{6}t^{-1}$$

$$g'(t) = -\frac{5}{6}t^{-2}$$

$$g''(t) = \frac{50}{3}t^{-3} = \frac{50}{3t^3}$$

$$f(x) = 3(2 - x^2)^3$$

$$f'(x) = 9(2 - x^2)^2(-2x) = -18x(2 - x^2)^2(-18)$$

$$= 18(2 - x^2)4x^2 - (2 - x^2)$$

$$= 182 - x^2 - 5x^2 - 2$$

$$10. y = 4(x^2 + 5x)^3$$

$$y' = 4(3)(x^2 + 5x)^2(2x + 5)$$

$$(24x + 60)(x^4 + 10x^3 + 25x^2)$$

$$24x^5 + 300x^4 + 1200x^3 + 1500x^2$$

$$y'' = 120x^4 + 1200x^3 + 3600x^2 + 3000x$$

$$f(x) = \frac{x \pm 1}{-1}$$

$$f'(x) = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2}$$

$$= -\frac{2}{(x-1)^2} = -2(x-1)^{-2}$$

$$(x-1)^2 \quad ( )$$

$$f''(x) = 4(x-1)^{-3} = \frac{4}{(x-1)^3}$$

$$( ) \quad ( ) \quad ( ) \quad x-1_3$$

$$g(x) = \frac{1-4x}{-3}$$

$$g'(x) = \frac{(1-4x)(-1) - (-4)(1)}{(x-3)^2}$$

$$f(x) = x^5 - 3x^4$$

$$f'(x) = 5x^4 - 12x^3$$

$$f''(x) = 20x^3 - 36x^2$$

$$f'''(x) = 60x^2 - 72x$$

$$f(x) = x^4 - 2x^3$$

$$f'(x) = 4x^3 - 6x^2$$

$$f''(x) = 12x^2 - 12x \quad ( )$$

$$f'''(x) = 24x - 12 = 12(2x - 1)$$

$$f(x) = 5x(x+4)^3$$

$$5x(x^3 + 12x^2 + 48x + 64)$$

$$5x^4 + 60x^3 + 240x^2 + 320x$$

$$f'(x) = 20x^3 + 180x^2 + 480x + 320$$

$$f''(x) = 60x^2 + 360x + 480$$

$$f'(x) = 120x + 360$$

$$f(x) = (x^3 - 6)^4$$

$$f'(x) = 4(x^3 - 6)^3(3x^2)$$

$$= 12x^{11} - 216x^8 + 1296x^5 - 2592x^2$$

$$f''(x) = 132x^{10} - 1728x^7 + 6480x^4 - 5184x$$

$$f'''(x) = 1320x^9 - 12,096x^6 + 25,920x^3 - 5184$$

$$17. f(x) = \frac{3}{8x^4} = \frac{3}{8}x^{-4}$$

$$f'(x) = -\frac{3}{2}x^{-5} = -\frac{3}{2x^5}$$

$$f''(x) = \frac{15}{2}x^{-6} = \frac{15}{2x^6}$$

$$f'''(x) = -\frac{45}{2}x^{-7} = -\frac{45}{2x^7}$$

$$f(x) = -\frac{2}{25}x^{-5}$$

$$f'(x) = \frac{2}{25}x^{-6} = \frac{2}{25x^6}$$

$$\frac{-4x + 12 - 1 + 4x^2}{(x-3)}$$

$$= -\frac{11}{(x-3)^2} = 11x - 3 \quad ( )$$

$$g''x = -22x - 3^{-3} \quad 1 = -\frac{22}{x-3} \quad ( )$$

$$f'(x) = 5x^{-6}$$

$$f(x) = -5x^{-7}$$

$$\frac{84}{-8} = \frac{84}{-8}$$

$$f'''(x) = 5x^{-8} = 5x^8$$

19.  $g(t) = 5t^4 + 10t^2 + 3$

$$g'(t) = 20t^3 + 20t$$

$$g''(2) = 60(4) + 20 = 260$$

$$f(x) = 9 - x^2 f'(x)$$

$$= -2x$$

$$f''(x) = -2$$

$$f''(\sqrt{5}) = -2$$

21.  $f(x) = \sqrt{4-x} = (4-x)^{1/2}$

$$f'(x) = \frac{1}{2} (4-x)^{-1/2}$$

$$f''(x) = -\frac{1}{4} (4-x)^{-3/2}$$

$$f'''(x) = \frac{3}{8} (4-x)^{-5/2}$$

$$f'''(-5) = \frac{3}{8} (9)^{5/2} = \frac{3}{8} \cdot 243 = \frac{729}{8}$$

$$f(t) = 2t + 3\sqrt{(2t+3)^{1/2}}$$

$$f''(t) = -\frac{1}{2} (2t+3)^{-3/2} (2) = -(2t+3)^{-3/2}$$

$$f'''(t) = \frac{3}{4} (2t+3)^{-5/2} (2) = \frac{3}{2} (2t+3)^{-5/2}$$

$$f'''(2) = \frac{3}{2} (7)^{-5/2} = \frac{3}{2 \cdot 7^2 \sqrt{7}} = \frac{3}{98\sqrt{7}}$$

23.  $f(x) = (x^3 - 2x)^3 = x^9 - 6x^7 + 12x^5 - 8x^3$

$$f'(x) = 9x^8 - 42x^6 + 60x^4 - 24x^2$$

$$f''(x) = 72x^7 - 252x^5 + 240x^3 - 48x$$

$$f'(1) = 12$$

24.  $g(x) = (x^2 + 3x)^4 = x^8 + 12x^7 + 54x^6 + 108x^5 + 81x^4$

''

26.  $f(x) = 20x^3 - 36x^2$

$$f'(x) = 60x^2 - 72x = 12x(5x - 6)$$

$$f''(x) = 4x^{-4}$$

()

$$f^{(4)}(x) = -16x^{-5}$$

$$f^{(5)}(x) = 80x^{-6} = \frac{80}{x^6}$$

28.  $f(x) = 4(x-2)^{1/2}$

$$f'(x) = \frac{1}{\sqrt{2}} (x-2)^{-1/2}$$

$$f''(x) = -\frac{1}{4\sqrt{2}} (x-2)^{-3/2}$$

$$f'''(x) = \frac{3}{16\sqrt{2}} (x-2)^{-5/2}$$

$$f^{(5)}(x) = \frac{3}{2} (x-2)^{-5/2} = \frac{3}{2(x-2)^{5/2}}$$

$$f^{(5)}(x) = 2(x^2 + 1)(2x) = 4x^3 + 4x$$

$$f^{(6)}(x) = 12x^2 + 4$$

30.  $f'''(x) = 4x + 7$

$$f^{(4)}(x) = 4$$

$$f^{(5)}(x) = 0$$

31.  $f(x) = 3x^2 - 18x + 27$

$$f'(x) = 6x - 18$$

$$0 = 6x - 18$$

$$x = 3$$

$$f(x) = (x+2)(x-2)(x+3)(x-3)$$

$$= (x^2 - 4)(x^2 - 9)$$

$$= x^4 - 13x^2 + 36$$

$$f'(x) = 4x^3 - 26x$$

$$f''(x) = 12x^2 - 26 = 26$$



Chapter 2 Differentiation

$$g'(x) = 8x^7 + 84x^6 + 324x^5 + 540x^4 + 324x^3$$
$$g''(x) = 56x^6 + 504x^5 + 1620x^4 + 2160x^3 + 972x^2$$
$$g''(-1) = -16$$

$$f'(x) = 2x^2 f$$
$$f''(x) = 4x$$

$$f(x) = 0 \quad 12x$$

$$x = \pm \sqrt{\frac{13}{6}} = \pm \frac{\sqrt{78}}{6}$$

33.

$$f(x) = x\sqrt{x^2 - 1} = x(x^2 - 1)^{1/2}$$

$$f'(x) = x \cdot \frac{1}{2}(x^2 - 1)^{-1/2}(2x) + (x^2 - 1)^{1/2} = \frac{x^2}{(x^2 - 1)^{1/2}} + (x^2 - 1)^{1/2}$$

$$f''(x) = \frac{2x \cdot (x^2 - 1)^{1/2} - x^2 \cdot \frac{1}{2}(x^2 - 1)^{-1/2}(2x)}{(x^2 - 1)^2} + \frac{1}{2}(x^2 - 1)^{-1/2}(2x)$$

$$f''(x) = \frac{2x^3 - 3x}{(x^2 - 1)^{3/2}} + \frac{x}{(x^2 - 1)^{3/2}} = \frac{2x^3 - 3x + x}{(x^2 - 1)^{3/2}} = \frac{2x^3 - 2x}{(x^2 - 1)^{3/2}} = \frac{2x(x^2 - 1)}{(x^2 - 1)^{3/2}} = \frac{2x}{(x^2 - 1)^{1/2}}$$

$$f''(x) = 0 \implies \frac{2x}{(x^2 - 1)^{1/2}} = 0 \implies 2x = 0 \implies x = 0$$

$$x = \pm \sqrt{\frac{3}{2}} = \pm \frac{\sqrt{6}}{2}$$

$x = 0$  is not in the domain of  $f$ .

34.  $f(x) = \frac{x + 3(1-x)}{(x^2 + 3)^{-2}}$

$$f'(x) = \frac{(1-3x) \cdot (-2)(x^2 + 3)^{-3}(2x) + (x^2 + 3)^{-2}(1-3x)'}{(x^2 + 3)^{-4}}$$

$$f'(x) = \frac{-2x(2-x) + (x^2 + 3)^{-2}(1-3x)'}{(x^2 + 3)^{-4}}$$

$$f''(x) = (3 - x^2) - 2(x^2 + 3)^{-3}(2x) + (x^2 + 3)^{-2}(-2x)$$

$$= -2x(x^2 + 3)^{-3}(2 - x^2) + (x^2 + 3)^{-2}(-2x)$$

$$= \frac{-2x(2 - x^2)}{(x^2 + 3)^3} - \frac{2x}{(x^2 + 3)^2}$$

$$= \frac{-2x(x^2 - 9)}{(x^2 + 3)^3}$$

$$f''(x) = 0 \implies \frac{-2x(x^2 - 9)}{(x^2 + 3)^3} = 0$$

$$x = 0, \pm 3$$

(a)  $s(t) = -16t^2 + 144t$   
 $v(t) = s'(t) = -32t + 144$   
 $a(t) = v'(t) = s''(t) = -32$

$$s(3) = 288 \text{ ft}$$

$$v(3) = 48 \text{ ft/sec}$$

$$a(3) = -32 \text{ ft/sec}^2$$

$$v(t) = 0$$

$$32t + 144 = 0$$

$$32t = -144$$

$$t = -4.5 \text{ sec}$$

$$s(4.5) = 324 \text{ ft}$$

$$s(t) = 0$$

$$-16t^2 + 144t = 0$$

-

1

6

t

(

t

-

9

)

=

0

$$t = 0 \text{ sec} \quad t = 9 \text{ sec}$$

$$v(9) = -32(9) + 144 = -144 \text{ ft/sec}$$

This is the same speed as the initial velocity.

$$(a) s(t) =$$

$$-16t^2 +$$

$$1250 v(t)$$

$$) = s'(t) = -32t a(t)$$

$$= v'(t) = -32$$

$$(b) s(t) = 0 \text{ when } 16t^2 = 1250, \text{ or}$$

$$t =$$

$$78.125 \approx$$

$$8.8 \text{ sec. (c)}$$

$$v(8.8) \approx$$

$$-282.8 \text{ ft/sec}$$



$$\frac{d^2s}{dt^2} = \frac{(t+10)(90) - (90t)(1)}{(t+10)^2} = \frac{-900}{(t+10)^2}$$

37.  $\frac{ds}{dt} = \frac{1}{(t+10)^2} = (t+10)^{-2}$

$t$	0	10	20	30	40	50	60
$\frac{ds}{dt}$	0	45	60	67.5	72	75	77.14
$\frac{d^2s}{dt^2}$	9	2.25	1	0.56	0.36	0.25	0.18

As time increases, the acceleration decreases. After 1 minute, the automobile is traveling at about 77.14 feet per second.

$$s(t) = -8.25t^2 + 66t$$

$$v(t) = s'(t) = -16.50t + 66$$

$$a(t) = s''(t) = -16.50$$

$t$	0	1	2	3	4	5
$s(t)$	0	57.75	99	123.75	132	123.75
$v(t)$	66	49.50	33	16.50	0	-16.50
$a(t)$	-16.50	-16.50	-16.50	-16.50	-16.50	-16.50

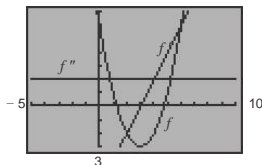
It takes 4 seconds for the car to stop, at which time it has traveled 132 feet.

39.  $f(x) = x^2 - 6x + 6$

$$f'(x) = 2x - 6$$

$$f''(x) = 2$$

(a)

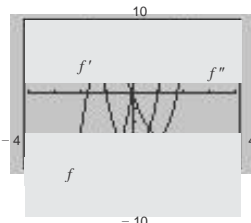


The degree decreased by 1 for each successive derivative.

(c)  $f(x) = 3x^2 - 9x$

$$f'(x) = 6x - 9$$

$$f''(x) = 6$$



The degree decreases by 1 for each successive derivative.

Graph A is the position function. Graph B is the velocity function. Graph C is the acceleration function.

Explanations will vary. Sample explanation:

The position function appears to be a third-degree function, while the velocity is a second-degree function, and the acceleration is a linear function.

41. (a)  $y(t) = -21.944t^3 + 701.75t^2 - 6969.4t + 27,164$

(b)  $y'(t) = -65.832t^2 + 1403.5t - 6969.4$

$$y'(t) = -131.664t + 1403.5$$

(c) Over the interval  $8 \leq t \leq 13$ ,  $y'(t) > 0$ ; therefore,

$y$  is increasing over  $8 \leq t \leq 13$ , or from 2008 to 20013.

(d)

$$-131.664t + 1403.5 = 0$$

$$-131.664t = -1403.5$$

$$t \approx 10.66 \text{ or } 2010$$

Let  $y = xf(x)$ .

Then,  $y' = xf'(x) + f(x)$

$$y'' = xf''(x) + f'(x) + f'(x)$$

$$xf''(x) + 2f'(x)$$

$$y''' = xf'''(x) + f''(x) + 2f''(x)$$

$$xf'''(x) + 3f''(x)$$

In general  $y_n^{(n)} = xf^{(n)}(x) + nf^{(n-1)}(x)$ .

True. If  $y = (x+1)(x+2)(x+3)(x+4)$ , then  $y$  is a fourth-degree polynomial function and its fifth derivative

$\frac{d^5y}{dx^5}$  equals 0.

44. True. The second derivative represents the rate of change of the first derivative, the same way that the first derivative represents the rate of change of the function.

45. Answers will vary.

### Section 2.7 Implicit Differentiation

#### Skills Warm Up

$$\begin{aligned}
 1. \quad & x - x = 2 \\
 & x^2 - y = 2x \\
 & -y = 2x - x^2 \\
 & y = x^2 - 2x
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{4}{x-3} = \frac{1}{y} \\
 & 4y = x - 3 \\
 & y = \frac{x-3}{4}
 \end{aligned}$$

$$\begin{aligned}
 & xy - x + 6y = 6 \\
 & xy + 6y = 6 + x \\
 & y(x + 6) = 6 + x
 \end{aligned}$$

$$\begin{aligned}
 y &= \frac{6+x}{x+6} \\
 -2 & \quad \quad \quad x+6 \\
 y &= 1, x \neq -6
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & 7 + 4y = 3x^2 + x^2y \\
 & 4y - x^2y = 3x^2 - 7 \\
 & y(4 - x^2) = 3x^2 - 7 \\
 & y = \frac{3x^2 - 7}{4 - x^2}, x \neq \pm 2
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & x = \pm\sqrt{6 - y^2} \\
 & x^2 = 6 - y^2 \\
 & x^2 - 6 = -y^2 \\
 & 6 - x^2 = y^2 \\
 & \pm\sqrt{6 - x^2} = y
 \end{aligned}$$

$$\begin{aligned}
 & \frac{3x}{2} = \frac{-4}{3(1^2)} = \frac{-4}{3} = 3 \\
 & \frac{3x}{2} = \frac{-4}{3} \\
 & 3x = \frac{-8}{3} \\
 & x = \frac{-8}{9}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \frac{x^2 - 2}{1 - y}, (0, -3)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{0^2 - 2}{1 - (-3)} = \frac{-2}{4} = -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & \frac{7x}{4y^2 + 13y + 3} = -\frac{1}{7}, -2
 \end{aligned}$$

$$\begin{aligned}
 & \frac{7(-2)}{4(-2)^2 + 13(-2) + 3} = \frac{-14}{16 - 26 + 3} = \frac{-14}{-7} = 2
 \end{aligned}$$

$$4 - 2 + 13 - 2 + 3$$

$$\begin{aligned}
 5. \quad & x^2 + y^2 = 5 \\
 & y^2 = 5 - x^2 \\
 & y = \pm\sqrt{5 - x^2}
 \end{aligned}$$

$$\begin{aligned}
 1. \quad & x^3y = 6 \\
 & x^3 \frac{dy}{dx} + 3x^2y = 0 \\
 & x^3 \frac{dy}{dx} = -3x^2y \\
 & \frac{dy}{dx} = \frac{-3x^2y}{x^3} = \frac{-3y}{x}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & 3x^2 - y = 8x \\
 & 6x - \frac{dy}{dx} = 8 \\
 & -\frac{dy}{dx} = 8 - 6x \\
 & \frac{dy}{dx} = 6x - 8
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & y^2 = 1 - x^2 \\
 & 2y \frac{dy}{dx} = -2x \\
 & \frac{dy}{dx} = -\frac{x}{y}
 \end{aligned}$$

$$y^3 = 5x^3 + 8x$$

$$3y^2 \frac{dy}{dx} = 15x^2 + 8$$

$$\frac{dy}{dx} = \frac{5x^2 + \frac{8}{3}}{y^2}$$

$$y^4 - y^2 + 7y - 6x = 9$$

$$4y^3 \frac{dy}{dx} - 2y \frac{dy}{dx} + 7 \frac{dy}{dx} - 6 = 0$$

$$\frac{dy}{dx}$$

$$(4y^3 - 2y + 7) \frac{dy}{dx} = 6$$

$$\frac{dy}{dx} = \frac{6}{4y^3 - 2y + 7}$$

6.  $4y^3 + 5y^2 - y - 3x^3 = 8x$

$$12y^2 \frac{dy}{dx} + 10y \frac{dy}{dx} - \frac{dy}{dx} - 9x^2 = 8$$

$$\frac{dy}{dx} (12y^2 + 10y - 1) = 8 + 9x^2$$

$$\frac{dy}{dx} = \frac{8 + 9x^2}{12y^2 + 10y - 1}$$

$$\frac{dy}{dx} = \frac{8 + 9x^2}{12y^2 + 10y - 1}$$

7.  $xy^2 + 4xy = 10$

$$y^2 + 2xy \frac{dy}{dx} + 4y + 4x \frac{dy}{dx} = 0$$

$$(2xy + 4x) \frac{dy}{dx} = -y^2 - 4y$$

$$\frac{dy}{dx} = -\frac{y^2 + 4y}{2xy + 4x}$$

$$\frac{dy}{dx} = -\frac{y^2 + 4y}{2xy + 4x}$$

8.  $2xy^3 - x^2y = 2$

$$2y^3 + 6xy^2 \frac{dy}{dx} - 2xy - x^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (6xy^2 - x^2) = 2xy - 2y^3$$

$$\frac{dy}{dx} = \frac{2xy - 2y^3}{6xy^2 - x^2}$$

$$\frac{dy}{dx} = \frac{2xy - 2y^3}{6xy^2 - x^2}$$

$$\frac{x + y}{-5y} = 1$$

$$x + y = -5y$$

$$2x + y = x - 5y$$

$$6y = -x$$

10.  $xy = y^2$

$$y - x = 1$$

$$xy - y^2 = y - x$$

$$y(x - y) = -(x - y)$$

$$= -1$$

$$\frac{dy}{dx} = 0$$

11.  $\frac{2y}{y^2 + 3} = 4x$

$$2y = 4x(y^2 + 3)$$

$$2y = 4xy^2 + 12x$$

$$2 \frac{dy}{dx} = 8xy \frac{dy}{dx} + 4y^2 + 12$$

$$\frac{dy}{dx} (2 - 8xy) = 4y^2 + 12$$

$$\frac{dy}{dx} = \frac{4y^2 + 12}{2 - 8xy}$$

$$\frac{dy}{dx} = \frac{4y^2 + 12}{2 - 8xy}$$

$$\frac{dy}{dx} = \frac{4y^2 + 12}{2 - 8xy}$$

12.  $\frac{4y^2}{y^2 - 9} = x^2$

$$\left( \frac{y^2 - 9}{y^2 - 9} \right) 8y \frac{dy}{dx} = 4y^2 \frac{dy}{dx} =$$

$$\frac{8y \frac{dy}{dx} (y^2 - 9 - y^2)}{(y^2 - 9)^2} = x$$

$$\frac{8y \frac{dy}{dx} (y^2 - 9 - y^2)}{(y^2 - 9)^2} = x$$

$$(y^2 - 9)$$

$$\frac{-72y \frac{dy}{dx}}{(y^2 - 9)^2} = x$$

$$\frac{-72y \frac{dy}{dx}}{(y^2 - 9)^2} = x$$

$$\frac{dy}{dx} = \frac{2x(y-9)^2}{72y}$$

$$\frac{dy}{dx} = \frac{2x(y-9)^2}{72y}$$

$$\frac{dy}{dx} = \frac{-x(y^2-9)}{36y}$$

$$x^2 + y^2 = 16$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{1}{6}x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\text{At } (0, 4), \frac{dy}{dx} = -\frac{0}{4} = 0.$$

$$x^2 - y^2 = 25$$

$$\frac{dy}{dx}$$

$$2x - 2y \frac{dy}{dx} = 0$$

$$-2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\frac{dy}{dx} = \frac{x}{y}$$

At (5, 0),  $\frac{dy}{dx}$  is undefined.

15.  $y + xy = 4$

$$\frac{dy}{dx} + x \frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx}(1+x) = -y$$

$$\frac{dy}{dx} = -\frac{y}{x+1}$$

At (-5, -1),  $\frac{dy}{dx} = -\frac{1}{4}$

$$\frac{dy}{dx}$$

$$xy - 3y^2 = 2$$

$$x \frac{dy}{dx} + y - 6y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x - 6y) = -y$$

$$\frac{dy}{dx} = -\frac{y}{x-6y}$$

$$\frac{dy}{dx} = \frac{2}{2-6}$$

At (7, 2),  $\frac{dy}{dx} = -\frac{2}{7-12} = \frac{2}{5}$

$$x^2 - xy + y^2 = 4$$

$$2x - x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0$$

$$2x - x \frac{dy}{dx} - y + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y - x) = y - 2x$$

$$\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y-2x}{2y-x}$$

$\frac{-1-3-2}{2} = -\frac{6}{2} = -3$ ;  $-3$  is

18.  $x^2y + y^3x = -6$

$$x^2 \frac{dy}{dx} + 2xy + y^3 + 3y^2 \frac{dy}{dx}x = 0$$

$$\frac{dy}{dx}$$

$$\frac{dy}{dx}$$

$$\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^3$$

$$\frac{dy}{dx} = \frac{-2xy - y^3}{x^2 + 2xy}$$

$$\frac{dy}{dx} = \frac{-y(2x + y^2)}{x(x + 3y)}$$

$$\frac{dy}{dx} = \frac{-1(2(-1) + (-1)^2)}{-1(-1 + 3(-1))} = \frac{-1(-2+1)}{-1(-1-3)} = \frac{-1(-1)}{-1(-4)} = \frac{1}{4}$$

At (2, -1),  $\frac{dy}{dx} = \frac{1}{4}$

$$xy - x = y$$

$$\frac{dy}{dx}x + y - 1 = \frac{dy}{dx}$$

$$\frac{dy}{dx}(x-1) = 1-y$$

$$\frac{dy}{dx}$$

$$\frac{dy}{dx}(x-1) = 1-y$$

$$\frac{dy}{dx} = \frac{1-y}{x-1} = -\frac{y-1}{x-1}$$

$$\frac{dy}{dx} = -\frac{3-1}{2-1} = -\frac{2}{1} = -2$$

At (3, 2),  $\frac{dy}{dx} = -\frac{2}{1} = -2$

$$2, 3, \frac{dy}{dx}$$

$$2-1 = 1$$

20.  $\frac{3}{x} + \frac{3}{y} = 6xy$

$$3x^{-2} + 3y^{-2} = 6xy$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 6x \frac{dy}{dx} + 6y$$

$$(3y^2 - 6x) \frac{dy}{dx} = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

$$\frac{dy}{dx} = \frac{3(2y - x^2)}{3(y^2 - 2x)}$$

$$\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$$

At (4, 8),  $\frac{dy}{dx} = \frac{16-16}{64-8} = 0$



$$\text{At } (-2, -1), \frac{dy}{dx} = 0$$

undefined.

$$3x^3 + 5y^3 = 9$$

$$x^3 + y^3 = 9$$

$$\frac{1}{2}x^{-1/2} + 2\frac{1}{y^{-1/2}} \frac{dy}{dx} = 0$$

$$x^{-1/2} + y^{-1/2} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-1/2}}{y^{-1/2}} = -\sqrt{\frac{y}{x}}$$

$$\text{At } (16, 25), \frac{dy}{dx} = -\frac{5}{4}$$

$$x^{2/3} + y^{2/3} = 5$$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \frac{dy}{dx} = 0$$

$$\frac{dx}{dy} = \frac{-x^{-1/3}}{y^{-1/3}} = -\frac{y^{1/3}}{x^{1/3}} = -\sqrt[3]{\frac{y}{x}}$$

At (8, 1),  $\frac{dy}{dx} = -\frac{1}{2}$

23.

$$\sqrt{xy} = x - 2y$$

$$\sqrt{x}\sqrt{y} = x - 2y$$

$$\frac{1}{2}\sqrt{x}^{-1/2}y^{-1/2} \frac{dy}{dx} = 1 - 2 \frac{dy}{dx}$$

$$\frac{\sqrt{x} dy}{2y dx} + \frac{2dy}{dx} = 1 - \frac{\sqrt{y}}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{\frac{\sqrt{y}}{2\sqrt{x}}}{\frac{\sqrt{x}}{2y} + 2} = \frac{\sqrt{y}\sqrt{y}}{\sqrt{x} + 4xy}$$

$$\frac{2(x - 2y) - y}{4x - 2y} = \frac{2x - 5y}{4x - 2y}$$

At (4, 1),  $\frac{dy}{dx} = \frac{1}{2}$

$$(x + y)^3 = x^3 + y^3$$

$$3(x + y)^2 \left(1 + \frac{dy}{dx}\right) = 3x^2 + 3y^2 \frac{dy}{dx}$$

$$3(x + y)^2 + 3(x + y)^2 \frac{dy}{dx} = 3x^2 + 3y^2 \frac{dy}{dx}$$

$$(x + y)^2 \frac{dy}{dx} - y^2 \frac{dy}{dx} = x^2 - (x + y)^2$$

$$\frac{dy}{dx} (x + y)^2 - y^2 = x^2 - x^2 - 2xy + y^2$$

At (-1, 1),  $\frac{dy}{dx} = -1$

25.  $y^2(x^2 + y^2) = 2x^2$

$$y^2 2x + 2y \frac{dy}{dx} + (x^2 + y^2) 2y \frac{dy}{dx} = 4x$$

$$2xy^2 + 2y^3 \frac{dy}{dx} + 2x^2y \frac{dy}{dx} + 2y^3 \frac{dy}{dx} = 4x$$

$$\frac{dy}{dx} (4y^3 + 2x^2y) = 4x - 2xy^2$$

$$\frac{dy}{dx} = \frac{2x(2 - y^2)}{y(2y^2 + x^2)}$$

$$\frac{dy}{dx} = \frac{x(2 - y^2)}{y(2y^2 + x^2)}$$

$$\frac{dy}{dx} = \frac{1}{y(2y^2 + x^2)}$$

At (1, 1),  $\frac{dy}{dx} = \frac{1}{3}$

26.  $(x^2 + y^2)^2 = 8x^2y$

$$2(x^2 + y^2) \left(2x \frac{dx}{dx} + 2y \frac{dy}{dx}\right) = 8x^2 \frac{dy}{dx} + y(16x)$$

$$4x^3 + 4x^2y \frac{dy}{dx} + 4xy^2 + 4y^3 \frac{dy}{dx} = 8x^2 \frac{dy}{dx} + 16xy$$

$$\frac{dy}{dx} (4x^2y + 4y^3 - 8x^2) = 16xy - 4x^3 - 4xy^2$$

$$\frac{dy}{dx} = \frac{4(4xy - x^3 - xy^2)}{4(xy + y^3 - 2x^2)}$$

At (2, 2),  $\frac{dy}{dx} = 0$

$$3x^2 - 2y + 5 = 0$$

$$\frac{dy}{dx} = 3x$$

$$\frac{dy}{dx} = 3x$$

At (1, 4),  $\frac{dy}{dx} = 3$

$$\frac{dy}{dx} = \frac{-(2xy + y)}{x^2 + 2xy} = -\frac{y(2x + y)}{x(x + 2y)}$$

$$4x^2 + 2y - 1 = 0$$

$$\frac{d}{dx} (4x^2 + 2y - 1) = 0$$

$$\frac{d}{dx} (4x^2) + \frac{d}{dx} (2y) - \frac{d}{dx} (1) = 0$$

$$8x + 2 \frac{dy}{dx} - 0 = 0$$

$$\frac{dy}{dx} (-1) = -4(-1) = 4$$

29.  $x^2 + y^2 = 4$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

At  $(\sqrt{3}, 1)$ ,  $\frac{dy}{dx} = -\frac{\sqrt{3}}{1} = -\sqrt{3}$ .

30.  $4x^2 + 9y^2 = 36$

$$8x + 18y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x}{9y}$$

At  $(\sqrt{5}, \frac{2\sqrt{5}}{3})$ ,  $\frac{dy}{dx} = -\frac{4\sqrt{5}}{9 \cdot \frac{2\sqrt{5}}{3}} = -\frac{\sqrt{5}}{3}$ .

31.  $x^2 - y^3 = 0$

$$2x - 3y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2x}{3y^2}$$

At  $(-1, 1)$ ,  $\frac{dy}{dx} = -\frac{2}{3}$ .

32.  $(4-x)y^2 = x^3$

$$y^2 = \frac{x^3}{4-x}$$

$$2y \frac{dy}{dx} = \frac{(4-x) \cdot 3x^2 - x^3(-1)}{(4-x)^2}$$

$$\frac{dy}{dx} = \frac{12x^2 - 3x^3 + x^3}{(4-x)^2}$$

$$2y \frac{dy}{dx} = \frac{12x^2 - 2x^3}{(4-x)^2}$$

$$\frac{dy}{dx} = -\frac{2x^2(x-6)}{2y(4-x)^2}$$

$$\frac{dy}{dx} = -\frac{x(x-6)}{y(4-x)^2}$$

At  $(2, 7)$ ,  $\frac{dy}{dx} = 2$ .

33. Implicitly:  $1 - 2y \frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{1}{2y}$$

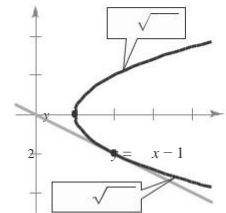
Explicitly:  $y = \pm \sqrt{x-1}$

$$\frac{dy}{dx} = \pm \frac{1}{2\sqrt{x-1}}$$

$$\frac{dy}{dx} = \pm \frac{1}{2(x-1)^{1/2}}$$

$$= \pm \frac{1}{2} (x-1)^{-1/2}$$

$$= 2 \left( \pm \frac{1}{2} (x-1)^{-1/2} \right)$$



At  $(2, -1)$ ,  $\frac{dy}{dx} = -\frac{1}{2}$ .

34. Implicitly:  $8y \frac{dy}{dx} - 2x = 0$

$$\frac{dy}{dx} = \frac{x}{4y}$$

Explicitly:  $y = \pm \frac{1}{2} \sqrt{x^2 + 7}$

$$\frac{dy}{dx} = \pm \frac{1}{4} (x^2 + 7)^{-1/2} (2x)$$

$$= \pm \frac{x}{2\sqrt{x^2 + 7}}$$

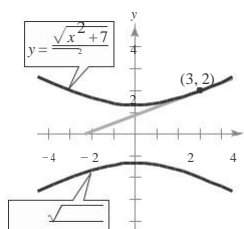
$$= \frac{2x^2 + 7}{4\sqrt{x^2 + 7}}$$

$$= \frac{4 \pm 1}{4} \sqrt{x^2 + 7}$$

$$= \frac{x}{2}$$

$$= 4y$$

At  $(3, 2)$ ,  $\frac{dy}{dx} = \frac{3}{8}$ .



$$y = -\frac{x^2 + 7}{2}$$

$$x^2 + y^2 = 100$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

At (8, 6):

$$m = -\frac{8}{6} = -\frac{4}{3}$$

$$-6 = -\frac{4}{3}(x - 8)$$

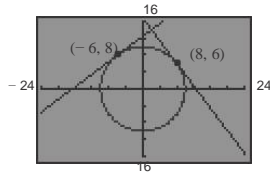
$$= -\frac{4}{3}x + \frac{32}{3}$$

50 At (-6, 8):

$$m = \frac{3}{4}$$

$$-8 = \frac{3}{4}(x + 6)$$

$$= \frac{3}{4}x + \frac{18}{4}$$



$$x^2 + y^2 = 9$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

At (0, 3):

$$m = 0$$

$$-3 = 0(x - 0)$$

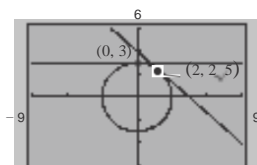
$$y = 3$$

At (2, √5):

$$m = -\frac{2}{\sqrt{5}}$$

$$y - \sqrt{5} = -\frac{2}{\sqrt{5}}(x - 2)$$

$$= -\frac{2\sqrt{5}}{5}x + \frac{4\sqrt{5}}{5}$$



$$y^2 = 5x^3$$

$$2y \frac{dy}{dx} = 15x^2$$

$$\frac{dy}{dx} = \frac{15x^2}{2y}$$

At (1, √5):

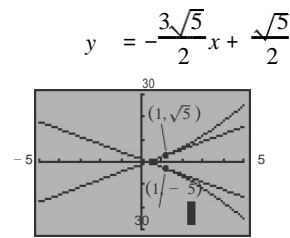
$$m = \frac{15}{2\sqrt{5}} = \frac{3\sqrt{5}}{2}$$

$$y - \sqrt{5} = \frac{3\sqrt{5}}{2}(x - 1)$$

At (1, -√5):

$$m = \frac{-15}{2\sqrt{5}} = -\frac{3\sqrt{5}}{2}$$

$$y + \sqrt{5} = -\frac{3\sqrt{5}}{2}(x - 1)$$



$$4xy + x^2 = 5$$

$$4x \frac{dy}{dx} + 4y + 2x = 0$$

$$\frac{dy}{dx} = -\frac{4y + 2x}{4x} = -\frac{2y + x}{2x}$$

At (1, 1):

$$m = -\frac{3}{2}$$

$$-1 = -\frac{3}{2}(x - 1)$$

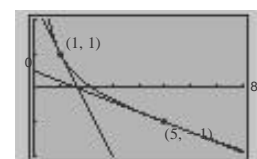
$$= -\frac{3}{2}x + \frac{3}{2}$$

$$+ \frac{3}{2} \text{ At } (5, -1):$$

$$m = -\frac{1}{10}$$

$$y + 1 = -\frac{1}{10}(x - 5)$$

$$y = -\frac{1}{10}x + \frac{1}{2}$$





$$x^3 + y^3 = 8$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 0$$

$$3y^2 \frac{dy}{dx} = -3x^2$$

$$\frac{dy}{dx} = -\frac{x^2}{y^2}$$

At (0, 2):

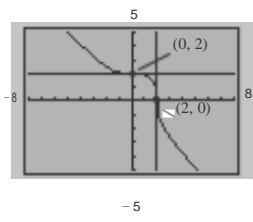
$$m = \frac{dy}{dx} = 0$$

$$-2 = 0(x - 0) + 2$$

At (2, 0):

$$\frac{dy}{dx} \text{ is undefined.}$$

The tangent line is  $x = 2$ .



40.  $x^2y - 8 = -4y$   
 $x^2y + 4y = 8$   
 $y(x^2 + 4) = 8$

$$y = \frac{8}{x^2 + 4} = 8(x^2 + 4)^{-1}$$

$$\frac{dy}{dx} = 8(-1)(x^2 + 4)^{-2}(2x)$$

$$\frac{dy}{dx} = \frac{-16x}{(x^2 + 4)^2}$$

At (-2, 1):

$$\frac{dy}{dx} = \frac{-16(-2)}{(-2)^2 + 4} = \frac{32}{8} = 4$$

$$m = \frac{dy}{dx} = -\frac{16(-2)}{((-2)^2 + 4)^2} = \frac{32}{64} = \frac{1}{2}$$

$$-1 = \frac{1}{2}(x - (-2))$$

$$\frac{1}{2}$$

$$= \frac{1}{2}x +$$

$$\frac{1}{2}$$

At (6, 5):

$$m = \frac{dy}{dx} = -\frac{16(6)}{(6^2 + 4)^2} = -\frac{96}{1600} = -\frac{3}{50}$$

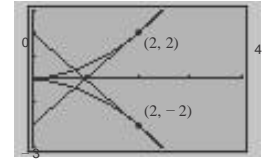
$$y^{-1} = -\frac{1}{x^2 + 4}$$

41.  $y^2 = \frac{x^3}{4-x}$   
 $\frac{dy}{dx} = \frac{3x^2(4-x) - x^3(-1)}{(4-x)^2}$

$$2y \frac{dy}{dx} = \frac{12x^2(4-x) + x^3}{(4-x)^2}$$

$$\frac{dy}{dx} = \frac{2x^2(6-x)}{(4-x)^2}$$

$$\frac{dy}{dx} = \frac{2x^2(6-x)}{y^4 - x^2}$$



At (2, 2):

$$m = 2$$

$$-2 = 2(x - 2) + 2$$

$$= 2x - 2$$

At (2, -2):

$$= -2$$

$$+ 2 = -2(x - 2)$$

$$y = -2x + 2$$

$$x + y^3 = 6xy^3 - 1y^3 - 6xy^3 = -1 - x$$

$$y^3(1 - 6x) = -(1 + x)$$

$$y^3$$

$$x + \frac{1}{y^3}$$

$$=$$

$$\frac{6x - 1}{(6x - 1)^2}$$

$$\frac{6x - 11}{(6x - 1)^2}$$

$$\frac{dy}{dx} = \frac{6x - 11}{(6x - 1)^2}$$

$$\frac{dy}{dx} = \frac{6x - 11 - 6x - 6}{(6x - 1)^2}$$

$$3y^2 \frac{dy}{dx} = \frac{6x - 11 - 6x - 6}{(6x - 1)^2}$$

$$\frac{dy}{dx} = \frac{7}{(6x - 1)^2}$$

$$\frac{dy}{dx} = -3y^2(6x - 1)^2$$

At (-1, 0):

$$= \frac{dy}{dx} \text{ is undefined. The tangent line is } x = -1.$$

At (0, -1):

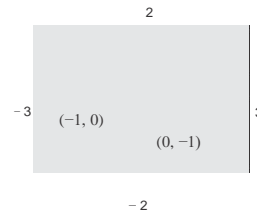
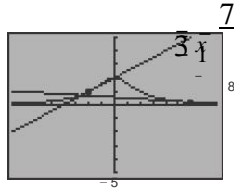
$$\frac{dy}{dx} = 7$$

$$m = \frac{dy}{dx} = -\frac{3}{50}$$



(6, 1/5)

$$y = -\frac{3}{50}x + \frac{14}{25}$$



$$43. p = 0.00001x^3 + 0.1x, x \geq 0$$

$$0.00001x^3 + 0.1x = \frac{2}{p}$$

$$0.00003x^2 \frac{dx}{dp} + 0.1 \frac{dx}{dp} = -\frac{2}{p^2}$$

$$\frac{dx}{dp} = -\frac{2}{p^2(0.00003x^2 + 0.1)}$$

$$44. p = \frac{4}{0.000001x^2 + 0.05x + 1}, x \geq 0$$

$$0.000001x^2 + 0.05x + 1 = \frac{4}{p}$$

$$0.000002x \frac{dx}{dp} + 0.05 \frac{dx}{dp} = -\frac{4}{p^2}$$

$$(0.000002x + 0.05) \frac{dx}{dp} = -\frac{4}{p^2}$$

$$45. p = \sqrt{\frac{200-x}{2x}}, 0 < x \leq 200$$

$$2xp^2 = 200 - x$$

$$2x(2p) + p^2 2 \frac{dx}{dp} = -\frac{dx}{dp}$$

$$(2p^2 + 1) \frac{dx}{dp} = -4xp$$

$$\frac{dx}{dp} = -\frac{4xp}{2p^2 + 1}$$

$$49. (a) y^2 - 35,892.5 = -27.0021t^3 + 888.789t^2 - 9753.25t$$

$$y^2 = -27.0021t^3 + 888.789t^2 - 9753.25t + 35,892.5$$

$$y = \pm \sqrt{-27.0021t^3 + 888.789t^2 - 9753.25t + 35,892.5}$$



The numbers of cases of Chickenpox decreases from 2008 to 2012.

(b) It appears that the number of reported cases was decreasing at the greatest rate during 2008,  $t = 8$ .

$$46. p = \sqrt{\frac{500-x}{2x}}, 0 < x \leq 500$$

$$2xp^2 = 500 - x$$

$$2x(2p) + p^2 2 \frac{dx}{dp} = -\frac{dx}{dp}$$

$$\frac{dx}{dp} (2p^2 + 1) = -4xp$$

$$\frac{dx}{dp} = -\frac{4xp}{2p^2 + 1}$$

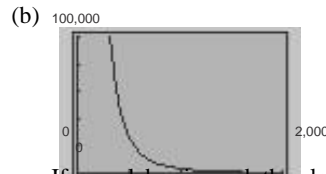
$$47. (a) 100x^{0.75}y^{0.25} = 135,540$$

$$100x^{0.75} \cdot 0.25 y^{-0.75} \frac{dy}{dx} + y^{0.25} (75x^{-0.25}) = 0$$

$$\frac{25x^{0.75}}{y^{0.75}} \cdot \frac{dy}{dx} = -\frac{75y^{0.25}}{x^{0.25}}$$

$$\frac{dy}{dx} = -\frac{3y}{x}$$

When  $x = 1500$  and  $y = 1000$ ,  $\frac{dy}{dx} = -2$ .



If more labor is used, then less capital is available.  
If more capital is used, then less labor is available.

(a) As price increases, the demand decreases.

For  $x > 0$ , the rate of change of demand,  $x$ , with respect to the price,  $p$ , is always decreasing; that is,

for  $x > 0$ ,  $\frac{dx}{dp}$  is never increasing.

$$(c) y^2 - 35,892.5 = -27.0021t^3 + 888.789t^2 - 9753.25t$$

$$2y \frac{dy}{dt} = -81.0063t^2 + 1777.578t - 9753.25$$

$$y' = \frac{dy}{dt} = \frac{-81.0063t^2 + 1777.578t - 9753.25}{2y}$$

t	8	9	10	11	12
y	30.40	20.51	15.39	14.51	13.40
y'	-11.79	-7.71	-2.54	-0.06	-3.26

The table of values for  $y'$  agrees with the answer in part (b) when the greatest value of  $y'$  is  $-11.79$  thousand cases per year.

### Section 2.8 Related Rates

#### Skills Warm Up

$$A = \pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

$$SA = 6s^2$$

$$V = s^3$$

$$V = \frac{1}{3}\pi r^2 h$$

$$A = \frac{1}{2}bh$$

$$x^2 + y^2 = 9$$

$$\frac{d}{dx}x^2 + y^2 = \frac{d}{dx}9$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$= \frac{-x}{y}$$

$$3xy - x^2 = 6$$

$$\frac{d}{dx}3xy - x^2 = \frac{d}{dx}[6]$$

$$3y + 3x \frac{dy}{dx} - 2x = 0$$

$$\frac{dy}{dx} = 2x - 3y$$

$$\frac{dy}{dx} = \frac{2x - 3y}{3x}$$

$$x^2 + 2y + xy = 12$$

$$\frac{d}{dx}x^2 + 2y + xy = \frac{d}{dx}12 \quad (12)$$

$$2x + 2 \frac{dy}{dx} + y + x \frac{dy}{dx} = 0$$

$$2 \frac{dy}{dx} + x \frac{dy}{dx} = -y - 2x$$

$$\frac{dy}{dx}(2 + x) = -y - 2x$$

$$\frac{dy}{dx} = \frac{-y - 2x}{2 + x}$$

$$\frac{dy}{dx} = \frac{-y - 2x}{2 + x}$$

$$x + xy^2 - y^2 = xy$$

$$\frac{d}{dx}x + xy^2 - y^2 = \frac{d}{dx}[xy]$$

$$1 + y^2 + 2xy \frac{dy}{dx} - 2y \frac{dy}{dx} = y + x \frac{dy}{dx}$$

$$2xy \frac{dy}{dx} - 2y \frac{dy}{dx} - x \frac{dy}{dx} = y - y^2 - 1$$

$$\frac{dy}{dx}(2xy - 2y - x) = y - y^2 - 1$$

$$\frac{dy}{dx} = \frac{y - y^2 - 1}{2xy - 2y - x}$$

$$1. y = \sqrt{x}, \quad \frac{dy}{dt} = 2x^{-1/2} \frac{dx}{dt} = \frac{1}{2\sqrt{x}} \frac{dx}{dt} = 2\sqrt{x} \frac{dy}{dt}$$

(a) When  $x = 4$  and  $\frac{dx}{dt} = 3$ ,  $\frac{dy}{dt} = 2\sqrt{4}(3) = 4$ .

(b) When  $x = 25$  and  $\frac{dx}{dt} = 2$ ,  $\frac{dy}{dt} = 2\sqrt{25}(2) = 20$ .

$$2. y = 3x^2 - 5x, \quad \frac{dy}{dt} = 6x \frac{dx}{dt} - 5 \frac{dx}{dt}, \quad \frac{dy}{dt} = (6x - 5) \frac{dx}{dt}, \quad \frac{dy}{dt} = \frac{dx}{dt} (6x - 5)$$

(a) When  $x = 3$  and  $\frac{dx}{dt} = 2$ ,  $\frac{dy}{dt} = (6(3) - 5)(2) = 26$ .

(b) When  $x = 2$  and  $\frac{dx}{dt} = 4$ ,  $\frac{dy}{dt} = (6(2) - 5)(4) = 20$ .

$$3. xy = 4, \quad x \frac{dy}{dt} + y \frac{dx}{dt} = 0, \quad \frac{dy}{dt} = -\frac{y}{x} \frac{dx}{dt}, \quad \frac{dy}{dt} = -\frac{4}{x^2} \frac{dx}{dt}$$

(a) When  $x = 8$ ,  $y = \frac{1}{2}$ , and  $\frac{dx}{dt} = 10$ ,  $\frac{dy}{dt} = -\frac{1/2}{8}(10) = -\frac{5}{8}$ .

(b) When  $x = 1$ ,  $y = 4$ , and  $\frac{dy}{dt} = -6$ ,  $\frac{dx}{dt} = -\frac{1}{4}(-6) = \frac{3}{2}$ .

$$4. x^2 + y^2 = 25, \quad 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0, \quad \frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}, \quad \frac{dy}{dt} = -\frac{y}{x} \frac{dx}{dt}$$

(a) When  $x = 3$ ,  $y = 4$ , and  $\frac{dx}{dt} = 8$ ,  $\frac{dy}{dt} = -\frac{3}{4}(8) = -6$ .

(b) When  $x = 4$ ,  $y = 3$ , and  $\frac{dx}{dt} = -2$ ,  $\frac{dy}{dt} = -\frac{3}{4}(-2) = \frac{3}{2}$ .

$$5. A = \pi r^2, \quad \frac{dA}{dt} = 3, \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 6\pi r$$

When  $r = 6$ ,  $\frac{dA}{dt} = 2\pi(6)(3) = 36\pi \text{ in.}^2/\text{min}$ .

When  $r = 24$ ,  $\frac{dA}{dt} = 2\pi(24)(3) = 144\pi \text{ in.}^2/\text{min}$ .

$$6. V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dt} = 3, \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 12\pi r^2$$

When  $r = 9$ ,  $\frac{dV}{dt} = 12\pi(9)^2 = 972\pi \text{ in.}^3/\text{min}$ .

When  $r = 16$ ,

$$8. V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

If  $\frac{dr}{dt}$  is constant,  $\frac{dV}{dt}$  is not constant since

it is proportional to the square of  $r$ .

$$9. V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dt} = 10, \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi r^2} \frac{dV}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi(10)^2} = \frac{1}{400\pi}$$

$$\frac{dV}{dt} = 12\pi(16)^2 = 3072\pi \text{ in.}^3 \text{ min.} \quad (\text{a}) \quad \text{When } r = 1, \quad \frac{dV}{dt} = 4\pi(1)^2 \quad ( )$$

2π ft min.

-

$$A = \pi r^2, \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$(\text{b}) \quad \text{When } r = 2, \quad \frac{dr}{dt} = \frac{1}{4\pi(2)^2}(10) = \frac{5}{8\pi} \text{ ft/min.}$$

If  $\frac{dr}{dt}$  is constant, then  $\frac{dA}{dt}$  is not constant;  $\frac{dA}{dt}$  is proportional to  $r$ .

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 (3r) = \pi r^3$$

$$\frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt} = 6\pi r^2$$

When  $r = 6$ ,  $\frac{dV}{dt} = 6\pi (6)^2 = 216\pi \text{ in.}^3/\text{min.}$

When  $r = 24$ ,  $\frac{dV}{dt} = 6\pi (24)^2 = 3456\pi \text{ in.}^3/\text{min.}$

11. (a)  $\frac{dC}{dt} = 0.75 \frac{dx}{dt} = 0.75(150)$   
112.5 dollars per week

$$\frac{dR}{dt} = 250 \frac{dx}{dt} - 5 \frac{1}{x} \frac{dx}{dt}$$

$$250(150) - 5(1000)(150)$$

7500 dollars per week

$P = R - C$

$$\frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt} = 7500 - 112.5$$

7387.5 dollars per week

$$\frac{dC}{dt} = \frac{dx}{dt}$$

(a)  $\frac{dC}{dt} = 1.05$   $\frac{dx}{dt} = 1.05(250) = 262.5$  dollars/week

(b)  $\frac{dR}{dt} = 500 - \frac{2x}{25} \frac{dx}{dt} = 500 - \frac{2(5000)}{25} (250)$

$$\frac{dR}{dt} = 500 - \frac{25}{25} \frac{dx}{dt} = 250,000 \text{ dollars/week}$$

$P = R - C$   
 $\frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt} = 25,000 - 262.5$

$$\frac{dP}{dt} = 24,737.5 \text{ dollars/week}$$

13.  $R = 1200x - x^2$ ,  $\frac{dR}{dt} = 1200 \frac{dx}{dt} - 2x \frac{dx}{dt}$

$$\frac{dR}{dt} = (1200 - 2x) \frac{dx}{dt}$$

When  $\frac{dx}{dt} = 23$  units/day and  $x = 300$  units,  $\frac{dR}{dt}$

$$\frac{dR}{dt} = 1200 - 2(300)(23) = \$13,800 \text{ per day.}$$

When  $\frac{dx}{dt} = 23$  units/day and  $x = 450$  units,  $\frac{dR}{dt}$

14.  $R = 510x - 0.3x^2$ ,  $\frac{dR}{dt} = 510 \frac{dx}{dt} - 0.6x \frac{dx}{dt}$

$$\frac{dR}{dt} = (510 - 0.6x) \frac{dx}{dt}$$

(a) When  $\frac{dx}{dt} = 9$  units/day and  $x = 400$  units,  $\frac{dR}{dt}$

$$\frac{dR}{dt} = 510 - 0.6(400)(9) = \$2430 \text{ per day.}$$

When  $\frac{dx}{dt} = 9$  units/day and  $x = 600$  units,  $\frac{dR}{dt}$

$$\frac{dR}{dt} = 510 - 0.6(600)(9) = \$1350 \text{ per day.}$$

$V = x^3$ ,  $\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

(a) When  $x = 2$ ,  $\frac{dV}{dt} = 3(2)^2 (6) = 72 \text{ cm}^3/\text{sec.}$

(b) When  $x = 10$ ,  $\frac{dV}{dt} = 3(10)^2 (6) = 1800 \text{ cm}^3/\text{sec.}$

$A = 6x^2$ ,  $\frac{dA}{dt} = 12x \frac{dx}{dt}$

$$\frac{dA}{dt}$$

When  $x = 2$ ,  $\frac{dA}{dt} = 12(2)(6) = 144 \text{ cm}^2/\text{sec.}$

When  $x = 10$ ,  $\frac{dA}{dt} = 12(10)(6) = 720 \text{ cm}^2/\text{sec.}$

Let  $x$  be the distance from the boat to the dock and  $y$  be the length of the rope.

$$12^2 + x^2 = y^2$$

$$\frac{dy}{dt} = -4$$

$$\frac{dx}{dt} = 2y \frac{dy}{dt}$$

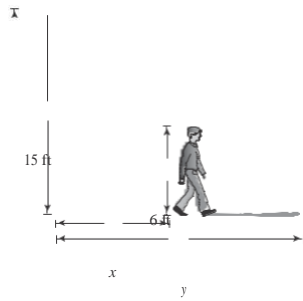
$$\frac{dx}{dt} = \frac{y}{x} \frac{dy}{dt}$$

When  $y = 13$ ,  $x = 5$  and  $\frac{dy}{dt}$

$$\frac{dx}{dt} = \frac{13}{5}(-4) = -10.4 \text{ ft/sec.}$$

As  $x \rightarrow 0$ ,  $\frac{dx}{dt}$  increases.

$$\frac{dR}{dt} = 1200 - 2(450)(23) = \$6900 \text{ per day.}$$



(a)  $15 = \frac{y}{x} \cdot 15 \implies y - 15x = 6y$

$6y = 15x$

$9y = 15x$

$y = \frac{5}{3}x$

Find  $\frac{dy}{dt}$  if  $\frac{dx}{dt} = 5$  ft/sec when  $x = 10$  ft.

$\frac{dy}{dt} = \frac{5}{3} \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{5}{3} \cdot 25$

$\frac{dy}{dt} = 3(5) = 15$  ft/sec

Find  $\frac{d}{dt}(y - x)$  if  $\frac{dx}{dt} = 5$  ft/sec and

$\frac{dy}{dt} = \frac{25}{3}$

$\frac{dy}{dt} = 15$  ft/sec when  $x = 10$  ft.

$\frac{d}{dt}(y - x) = \frac{dy}{dt} - \frac{dx}{dt}$

$= \frac{25}{3} - 5 = \frac{10}{3}$  ft/sec

$x^2 + 6^2 = s^2$

$2x \frac{dx}{dt} = 2s \frac{ds}{dt}$

$2x \frac{dx}{dt} = 2s \frac{ds}{dt}$

$\frac{dx}{dt} = \frac{s}{x} \frac{ds}{dt}$

$\frac{ds}{dt}$

When  $s = 10$ ,  $x = 8$  and  $\frac{dx}{dt} = 240$ :

$\frac{dx}{dt} = \frac{10}{8}(-240) = 300$  mi/hr.

$s^2 = 90^2 + x^2, x = 26, \frac{dx}{dt} = -30$

$2s \frac{ds}{dt} = 2x \frac{dx}{dt}$

$\frac{ds}{dt} = \frac{x}{s} \frac{dx}{dt}$

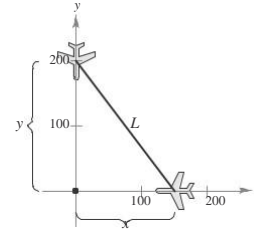
When  $\frac{dx}{dt} = \frac{26}{26}(-30) = -30$

21. (a)  $L^2 = x^2 + y^2, \frac{dx}{dt} = -450, \frac{dy}{dt} = -600$ , and

$\frac{dL}{dt} = \frac{x(\frac{dx}{dt}) + y(\frac{dy}{dt})}{L}$

When  $x = 150$  and  $y = 200$ ,  $L = 250$  and

$\frac{dL}{dt} = \frac{150(-450) + 200(-600)}{250} = -750$  mph.



$t = \frac{250}{750} = \frac{1}{3}$  hr = 20 min

$S = 2250 + 50x + 0.35x^2$

$\frac{dS}{dt} = 50 \frac{dx}{dt} + 0.70x \frac{dx}{dt}$

$\frac{dS}{dt} = 50(125) + 0.70(1500)(125)$

\$137,500 per week

23.  $V = \pi r^2 h, h = 0.08, V = 0.08\pi r^2, \frac{dV}{dt} = 0.16\pi r \frac{dr}{dt}$

When  $r = 150$  and  $\frac{dr}{dt} = \frac{1}{2}$ ,

$\frac{dV}{dt} = 0.16\pi (150) \frac{1}{2} = 12\pi = 37.70$  ft<sup>3</sup>/min.

$P = R - C$

$xP - C$

$x(50 - 0.01x) - (4000 + 40x - 0.02x^2)$   
 $= 50x - 0.01x^2 - 4000 - 40x + 0.02x^2$   
 $0.01x^2 + 10x - 4000$

$\frac{dP}{dt} = 0.02x \frac{dx}{dt} + 10 \frac{dx}{dt}$

When  $x = 800$  and  $\frac{dx}{dt} = 25$ ,

$\frac{dP}{dt}$



$$\begin{aligned}(-30) &\approx -8.33 \text{ ft sec.} \frac{dt}{dt} = 0.02(800)(25) \\ &+ (10)(25) = \$650 \\ &\text{week.}\end{aligned}$$

$$P = R - C = xp - C = x(6000 - 25x) - (2400x + 5200)$$

$$-25x^2 + 3600x - 5200$$

$$\frac{dP}{dx} = -50x + 3600$$

$$\frac{dx}{dP} = \frac{1}{3600 - 50x}$$

When  $x = 44$  and  $dP = 5600$ ,  $\frac{dx}{dP} = \frac{1}{3600 - 50(44)} (5600) = 4$  units per week.

26. (a) For supply, if  $\frac{dx}{dt}$  is negative, then  $\frac{dp}{dt}$  is negative. For demand, if  $\frac{dx}{dt}$  is negative, then  $\frac{dp}{dt}$  is positive.
- (b) For supply, if  $\frac{dp}{dt}$  is positive, then  $\frac{dx}{dt}$  is positive. For demand, if  $\frac{dp}{dt}$  is positive, then  $\frac{dx}{dt}$  is negative.

### Review Exercises for Chapter 2

$$\text{Slope} \approx \frac{-4}{2} = -2$$

$$\text{Slope} \approx \frac{4}{2} = 2$$

$$\text{Slope} \approx 0$$

$$\text{Slope} \approx \frac{-2}{42} = -\frac{1}{21}$$

Answers will vary. Sample answer:

- = 8; slope  $\approx$  \$225 million/yr; Revenue was increasing by about \$225 million per year in 2008.
- = 10; slope  $\approx$  \$350 million/yr; Revenue was increasing by about \$350 million per year in 2010.

Answers will vary. Sample answer:

- = 10; slope  $\approx$  -20 thousand/year; The number of farms was decreasing by about 20 thousand per year in 2010.
- = 12; slope  $\approx$  -10 thousand/year; The number of

farms was decreasing by about 10 thousand per year in 2012.

Answers will vary. Sample answer:

- = 1:  $m \approx$  65 hundred thousand visitors/month; The number of visitors to the national park is increasing at about 65,000,000/per month in January.
- = 8:  $m \approx$  0 visitors/month; The number of visitors to the national park is neither increasing nor decreasing in August.
- = 12:  $m \approx$  -1000 hundred thousand/month; The number of visitors to the national park is decreasing at about 1,000,000,000 visitors per month in December.

8. (a) At  $t_1$ , the slope of  $g(t)$  is greater than the slope of  $f(t)$ , so the rafter whose progress is given by  $g(t)$

( ) is traveling faster.

- (b) At  $t_2$ , the slope of  $f(t)$  is greater than the slope of  $g(t)$ , so the rafter whose progress is given by  $f(t)$  is traveling faster.

- (c) At  $t_3$ , the slope of  $f(t)$  is greater than the slope of  $g(t)$ , so the rafter whose progress is given by  $f(t)$  is traveling faster.

- (d) The rafter whose progress is given by  $f(t)$  finishes first. The value of  $t$  where  $f(t) = 9$  is smaller than the value of  $t$  where  $g(t) = 9$ .

9.  $f(x) = -3x - 5; (-2, 1)$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-3(x+\Delta x) - 5 - (-3x - 5)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-3\Delta x}{\Delta x} = -3$$

$$f'(-2) = -3$$

10.  $f(x) = 7x + 3; (-1, -4)$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{7(x+\Delta x) + 3 - (7x + 3)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{7\Delta x}{\Delta x} = 7$$

$$f'(-1) = 7$$

11.  $f(x) = x^2 + 9; (3, 18)$

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{x \rightarrow 0} \frac{(x+\Delta x)^2 + 9 - (x^2 + 9)}{\Delta x} \\ &= \lim_{x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 9 - x^2 - 9}{\Delta x} \\ &= \lim_{x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} \\ &= \lim_{x \rightarrow 0} \Delta x(2x + \Delta x) \\ &= \lim_{x \rightarrow 0} (2x + \Delta x) = 2x \\ f'(3) &= 2(3) = 6 \end{aligned}$$

$f(x) = x^2 - 7x; (1, -6)$

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{x \rightarrow 0} \frac{(x+\Delta x)^2 - 7(x+\Delta x) - (x^2 - 7x)}{\Delta x} \\ &= \lim_{x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 7x - 7\Delta x - x^2 + 7x}{\Delta x} \\ &= \lim_{x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2 - 7\Delta x}{\Delta x} \\ &= \lim_{x \rightarrow 0} (2x + \Delta x - 7) = 2x - 7 \\ f'(1) &= 2(1) - 7 = -5 \end{aligned}$$

$f(x) = x + \sqrt{x+9}; (-5, 2)$

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{x \rightarrow 0} \frac{\sqrt{x+\Delta x+9} - \sqrt{x+9}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x+9} + \sqrt{x+9}}{\sqrt{x+\Delta x+9} + \sqrt{x+9}} \\ &= \lim_{x \rightarrow 0} \frac{(x+\Delta x+9) - (x+9)}{\Delta x(\sqrt{x+\Delta x+9} + \sqrt{x+9})} \\ &= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+\Delta x+9} + \sqrt{x+9}} = \frac{1}{2\sqrt{x+9}} \\ f'(-5) &= \frac{1}{4} \end{aligned}$$

$f(x) = x - 1; (10, 3)$

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{x \rightarrow 0} \frac{\sqrt{x+\Delta x-1} - \sqrt{x-1}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x-1} + \sqrt{x-1}}{\sqrt{x+\Delta x-1} + \sqrt{x-1}} \\ &= \lim_{x \rightarrow 0} \frac{(x+\Delta x-1) - (x-1)}{\Delta x(\sqrt{x+\Delta x-1} + \sqrt{x-1})} \\ &= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+\Delta x-1} + \sqrt{x-1}} = \frac{1}{2\sqrt{x-1}} \\ f'(10) &= \frac{1}{6} \end{aligned}$$

15.  $f(x) = \frac{1}{x-5}; (6,1)$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{x+\Delta x-5} - \frac{1}{x-5}}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{x-5 - (x+\Delta x-5)}{(x+\Delta x-5)(x-5)}}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-\Delta x}{\Delta x(x+\Delta x-5)(x-5)}$$

$$= \lim_{x \rightarrow 0} \frac{-1}{(x+\Delta x-5)(x-5)}$$

$$= \frac{-1}{(6-5)(6-5)} = -\frac{1}{1}$$

$f(6) = -1$

16.  $f(x) = x + 6$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{(x+\Delta x) + 6 - (x+6)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{\Delta x}{\Delta x}$$

$$= \lim_{x \rightarrow 0} 1 = 1$$

$f(-3) = -\frac{1}{(-2+6)^2} = -\frac{1}{16}$

19.  $f(x) = -\frac{1}{2}x^2 + 2x$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-\frac{1}{2}(x+\Delta x)^2 + 2(x+\Delta x) - (-\frac{1}{2}x^2 + 2x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-\frac{1}{2}(x^2 + 2x\Delta x + \Delta x^2) + 2x + 2\Delta x + \frac{1}{2}x^2 - 2x}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-x\Delta x - \frac{1}{2}\Delta x^2 + 2\Delta x}{\Delta x}$$

$$= \lim_{x \rightarrow 0} (-x - \frac{1}{2}\Delta x + 2)$$

$$= -0 - \frac{1}{2}(0) + 2 = 2$$

$f(x) = 9x + 1$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{9(x+\Delta x) + 1 - (9x+1)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{9x + 9\Delta x + 1 - 9x - 1}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{9\Delta x}{\Delta x}$$

$$= \lim_{x \rightarrow 0} 9 = 9$$

$f(x) = 1 - 4x$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{1 - 4(x+\Delta x) - (1 - 4x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{1 - 4x - 4\Delta x - 1 + 4x}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{-4\Delta x}{\Delta x}$$

$$= \lim_{x \rightarrow 0} -4 = -4$$

$$\lim_{x \rightarrow 0} \left( -x - \frac{1}{2}x + 2 \right) = -x + 2$$

20.  $f(x) = 3x^2 - \frac{1}{4}x$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{3(x+\Delta x)^2 - \frac{1}{4}(x+\Delta x) - 3x^2 - \frac{1}{4}x}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{3x^2 + 6x(\Delta x) + 3(\Delta x)^2 - \frac{1}{4}x - \frac{1}{4}\Delta x - 3x^2 - \frac{1}{4}x}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{6x(\Delta x) + 3(\Delta x)^2 - \frac{1}{4}\Delta x}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \left( \frac{6x + 3\Delta x - \frac{1}{4}}{\Delta x} \right) \Delta x$$

$$= \lim_{x \rightarrow 0} (6x + 3\Delta x - \frac{1}{4}) = 6x - \frac{1}{4}$$

21.  $f(x) = \sqrt{x-5}$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{x+\Delta x-5} - \sqrt{x-5}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x-5} + \sqrt{x-5}}{\sqrt{x+\Delta x-5} + \sqrt{x-5}}$$

$$= \lim_{x \rightarrow 0} \frac{(x+\Delta x-5) - (x-5)}{\Delta x (\sqrt{x+\Delta x-5} + \sqrt{x-5})}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+\Delta x-5} + \sqrt{x-5}} = \frac{1}{2\sqrt{x-5}}$$

22.  $f(x) = \sqrt{x} + 3$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{x+\Delta x} + 3 - (\sqrt{x} + 3)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$= \lim_{x \rightarrow 0} \frac{(x+\Delta x) - x}{\Delta x (\sqrt{x+\Delta x} + \sqrt{x})}$$

23.  $f(x) = \frac{5}{x}$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{5}{x+\Delta x} - \frac{5}{x}}{\Delta x}$$

$$= \lim_{x \rightarrow 0} \frac{5x - 5(x+\Delta x)}{\Delta x x(x+\Delta x)}$$

$$= \lim_{x \rightarrow 0} \frac{-5\Delta x}{\Delta x x(x+\Delta x)}$$

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{x(\sqrt{x+x} + \sqrt{x})}{\sqrt{x+x} + x} \\
 &= \lim_{x \rightarrow 0} \frac{x(\sqrt{2x} + \sqrt{x})}{\sqrt{2x} + x} = \lim_{x \rightarrow 0} \frac{x(\sqrt{2} + 1)\sqrt{x}}{\sqrt{2x} + x} \\
 &= \lim_{x \rightarrow 0} \frac{x(\sqrt{2} + 1)\sqrt{x}}{\sqrt{x}(\sqrt{2} + x)} = \lim_{x \rightarrow 0} \frac{x(\sqrt{2} + 1)}{\sqrt{2} + x} = \frac{0(\sqrt{2} + 1)}{\sqrt{2} + 0} = 0
 \end{aligned}$$

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{-5x}{x(x+x)} \\
 &= \lim_{x \rightarrow 0} \frac{-5}{x(x+x)} = \lim_{x \rightarrow 0} \frac{-5}{x^2 + x} = -\frac{5}{0} = -\infty
 \end{aligned}$$

24.  $f(x) = \frac{1}{x+4}$

$\lim_{x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$$\begin{aligned} f(x) &= \frac{1}{x+4} \\ &= \lim_{x \rightarrow 0} \frac{\frac{1}{x+\Delta x+4} - \frac{1}{x+4}}{\Delta x} \\ &= \lim_{x \rightarrow 0} \frac{\frac{(x+4) - (x+\Delta x+4)}{(x+\Delta x+4)(x+4)}}{\Delta x} \\ &= \lim_{x \rightarrow 0} \frac{-\Delta x}{\Delta x(x+\Delta x+4)(x+4)} \\ &= \lim_{x \rightarrow 0} \frac{-1}{(x+4)(x+\Delta x+4)(x+4)} = \frac{-1}{(x+4)^2} \end{aligned}$$

y is not differentiable at  $x = -1$ . At  $(-1, 0)$ , the graph has a vertical tangent line.

y is not differentiable at  $x = 0$ . At  $(0, 3)$ , the graph has a node.

y is not differentiable at  $x = 0$ . The function is discontinuous at  $x = 0$ .

y is not differentiable at  $x = -1$ . At  $(-1, 0)$ , the graph

has a cusp.

$y = -6$   
 $y' = 0$

$f(x) = 5^x$

$f'(x) = 0$

$f(x) = x^7$

$f'(x) = 7x^6$

$h(x) = x^{\frac{1}{-4}}$

$h(x) = x^{-4}$

$f(x) = \frac{5}{4}x^3$

$f'(x) = \frac{15x^2}{4}$

$y = 3x^{2/3}$

$= 2x^{-1/3}$

$y' = \frac{2}{x^{4/3}}$

$g(x) = 2x^4 + 3x^2$

$g'(x) = 8x^3 + 6x$

$f(x) = 6x^2 - 4x$

$f'(x) = 12x - 4$

$y = x^2 + 6x - 7$

$y' = 2x + 6$

$y = 2x^4 - 3x^3 + x$

$y' = 8x^3 - 9x^2 + 1$

$f(x) = 2x^{-1/2}; (4, 1)$

$f'(x) = -x^{-3/2}$

$f'(4) = -4^{-3/2} = -0.125$

$y = 2^{\frac{3}{2}}x + 3; 2^{\frac{1}{2}}, 6$

$y = 2x^{-1} + 3$

$y' = -\frac{3}{2}x^{-2} = -\frac{3}{2x^2}$

$y' = \frac{1}{2}x^{-3} = \frac{1}{2x^3}$

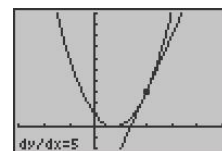
$y' = \frac{1}{2}x^{-2} = \frac{1}{2x^2}$

$y' = \frac{1}{2}x^{-2} = \frac{1}{2x^2}$

43.  $g(x) = x^3 - 4x^2 - 6x + 8; (-1, 9)$

$g'(x) = 3x^2 - 8x - 6$

$g'(-1) = 3(-1)^2 - 8(-1) - 6 = 5$



( )



$$h'(x) = -4x^{-5}$$

$$h'(x) = \frac{4}{5}x^{-4}$$

$$f(x) = 4x^2$$

$$f'(x) = 8x$$

$$g(t) = 8t^6$$

$$g'(t) = 48t^5$$

44.  $y = 2x^4 - 5x^3 + 6x^2 - x; 1, 2$

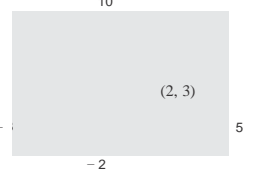
$$y' = 8x^3 - 15x^2 + 12x - 1$$

45.  $y'(1) = 8(1)^3 - 15(1)^2 + 12(1) - 1 = 4$

$$y'(2) = 5$$

$$y - 3 = 5(x - 2)$$

$$y = 5x - 7$$

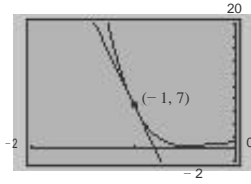


$$y' = 44x^3 - 10x$$

$$y'(-1) = -34$$

$$-7 = -34(x + 1)$$

$$y = -34x - 27$$



47.  $f(x) = \sqrt{x} - \frac{1}{x} = x^{1/2} - x^{-1}$

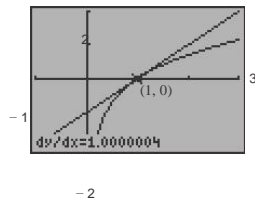
$$f'(x) = \frac{1}{2}x^{-1/2} + x^{-2}$$

$$= \frac{1}{2\sqrt{x}} + \frac{1}{x^2}$$

$$f'(1) = 1$$

$$-0 = 1(x - 1)$$

$$y = x - 1$$



48.  $f(x) = \sqrt[3]{x} - x = x^{1/3} - x$

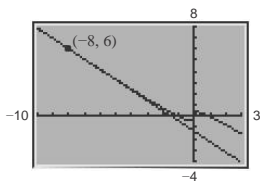
$$f'(x) = \frac{1}{3}x^{-2/3} - 1 = \frac{1}{3\sqrt[3]{x^2}} - 1$$

$$f'(-8) = \frac{1}{3\sqrt[3]{(-8)^2}} - 1 = \frac{1}{12} - 1 = -\frac{11}{12}$$

$$-6 = -\frac{11}{12}(x + 8)$$

$$y - 6 = -\frac{11}{12}x - \frac{22}{3}$$

$$= -\frac{11}{12}x - \frac{4}{3}$$



$$R = -0.5972t^3 + 51.187t^2 - 485.54t + 2199.0$$

$$\frac{dR}{dt} = R'(t) = -1.7916t^2 + 102.374t - 485.54$$

2008:  $R'(8) = m \approx 218.8$

2010:  $R'(10) = m \approx 359.0$

Results should be similar.  
 The slope shows the rate at which sales were increasing or decreasing in that particular year, or value of  $t$ .  
 In 2008, the revenue was increasing about

$$N = 0.2083t^4 - 7.954t^3 + 11.96t^2 - 706.5t + 3891$$

$$\frac{dN}{dt} = N'(t) = 0.8332t^3 - 23.862t^2 + 23.92t - 706.5$$

(a) 2010:  $N'(10) = m \approx -2020.33$

2012:  $N'(12) = m \approx -2415.85$

(b) Results should be similar.  
 (c) The slope shows the rate at which the number of farms was increasing or decreasing in that particular year, or value of  $t$ .

In 2010, the number of farms was decreasing about 2020.33 thousand per year, and in 2012, the number of farms was decreasing about 2415.85 thousand per year.

$$f(t) = 4t + 3; [-3, 1]$$

Average rate of change:  $\frac{f(1) - f(-3)}{1 - (-3)} = \frac{7 - (-9)}{4} = 4$

Instantaneous rate of change:  
 $f'(1) = 4$   
 $f'(-3) = 4$

52.  $f(x) = x^2 + 3x - 4; [0, 1]$

Average rate of change:  $\frac{f(1) - f(0)}{1 - 0} = \frac{0 - (-4)}{1} = 4$

Instantaneous rate of change:  
 $f'(x) = 2x + 3$   
 $f'(1) = 5$   
 $f'(0) = 3$

53.  $f(x) = x^{2/3}; [1, 8]$

Average rate of change:  $\frac{f(8) - f(1)}{8 - 1} = \frac{4 - 1}{7} = \frac{3}{7}$

Instantaneous rate of change:  
 $f'(x) = \frac{2}{3}x^{-1/3}$

\$218.8 million per year, and in 2010, revenue was increasing about \$359.0 million per year.

$$f'(8) = \frac{1}{3}$$
$$f'(2) = \frac{1}{2}$$
$$f(1) = 3$$

$$f(x) = x^3 - x^2 + 3; [-2, 2]$$

Average rate of change:  $\frac{f(2) - f(-2)}{2 - (-2)} = \frac{7 - (-9)}{4} = 4$

Instantaneous rate of change:

$$f'(-2) = 16$$

$$s(t) = -16t^2 - 30t + 600$$

(a) Average velocity =  $\frac{s(3) - s(1)}{3 - 1} = \frac{366 - 554}{2} = -94$  ft/sec

$$v(t) = s'(t) = -32t - 30$$

$$v(1) = -62 \text{ ft/sec}$$

$$v(3) = -126 \text{ ft/sec}$$

(c)  $s(t) = 0$

$$-16t^2 - 30t + 600 = 0$$

$$16t^2 + 30t - 600 = 0$$

$$t = \frac{-30 \pm \sqrt{30^2 - 4(16)(-600)}}{2(16)} = \frac{-30 \pm 39.300}{32}$$

$$\approx 5.26 \text{ sec}$$

$$v(t) = s'(5.26) = -32(5.26) - 30 = -198.32 \text{ ft/sec}$$

56. (a)  $s(t) = -16t^2 + 276$

(b) Average velocity =  $\frac{s(2) - s(0)}{2 - 0}$

$$= \frac{212 - 276}{2}$$

$$= -32 \text{ ft/sec}$$

(c)  $v(t) = -32t$

$$v(2) = -64 \text{ ft/sec}$$

$$v(3) = -96 \text{ ft/sec}$$

(d)  $s(t) = 0$

$$-16t^2 + 276 = 0$$

$$16t^2 = 276$$

$$t^2 = \frac{276}{16}$$

$$t \approx 4.15 \text{ sec}$$

(e)  $v(4.15) = -132.8 \text{ ft/sec}$

57.  $C = 2500 + 320x$

$$\frac{dC}{dx} = 320$$

58.  $C = 24,000 + 450x - x^2, 0 \leq x \leq 225$

$$\frac{dC}{dx} = 450 - 2x$$

59.  $C = 370 + 2.55\sqrt{x} = 370 + 2.25x^{1/2}$

$$\frac{dC}{dx} = \frac{1}{2}(2.55)(x^{-1/2}) = \frac{1.275}{\sqrt{x}}$$

60.  $C = 475 + 5.25x^{2/3}$

$$\frac{dC}{dx} = 5.25 \cdot \frac{2}{3} x^{-1/3} = \frac{3.5}{\sqrt[3]{x}}$$

61.  $R = 150x - 0.6x^2$

$$\frac{dR}{dx} = 150 - 1.2x$$

62.  $R = 150x - \frac{3}{4}x^2$

$$\frac{dR}{dx} = 150 - \frac{3}{2}x$$

63.  $R = -4x^3 + 2x^2 + 100x$

$$\frac{dR}{dx} = -12x^2 + 4x + 100$$

$$R = 4x + 10x^{1/2}$$

$$\frac{dR}{dx} = 4 + \frac{5}{x^{1/2}}$$

$$dx$$

$$P = -0.0002x^3 + 6x^2 - x - 2000$$

$$\frac{dP}{dx} = -0.0006x^2 + 12x - 1$$

$$P = -15x^3 + 4000x^2 - 120x - 144,000$$

$$\frac{dP}{dx} = -45x^2 + 8000x - 120$$

$$dx$$

$$P = -0.05x^2 + 20x - 1000$$

$$\frac{dP}{dx}$$

Find  $\frac{dP}{dx}$  when  $x = 100$ .

$$\frac{dP}{dx}$$

$$dx = -0.1x + 20 = P'(x)$$

$$\frac{dP}{dx}$$

When  $x = 100$ ,  $\frac{dP}{dx} = P'(100) = \$10$ .

(b) Find  $\frac{P}{x}$  for  $100 \leq x \leq 101$ .

$$\frac{P(101)}{P(100)}$$

$$= 509.95 - 500 =$$

$$\$9.95$$

Parts (a) and (b) differ by only \$0.05.

$$P = -0.021t^2 + 2.77t + 148.9$$

$$P(0) = 148.9$$

$$P(4) = 159.644$$

$$P(8) = 169.716$$

$$P(12) = 179.116$$

$$P(16) = 187.844$$

$$P(20) = 195.9$$

$$P(23) = 201.501$$

These values are the populations in millions for Brazil from 1990 to 2013.

$$\frac{dP}{dt} = -0.042t + 2.77 = P'(t)$$

$$P'(0) = 2.77$$

$$f(x) = x^3(5 - 3x^2) = 5x^3 - 3x^5$$

$$f'(x) = 15x^2 - 15x^4 = 15x^2(1 - x^2)$$

Simple Power Rule

$$70. f(x) = 4x^2 - 2x^2 - 5 = 8x^4 - 20x^2$$

$$f'(x) = 32x^3 - 40x = 8x(4x^2 - 5)$$

Simple Power Rule

$$71. y = (4x - 3)(x^3 - 2x^2)$$

$$y' = (4x - 3)(3x^2 - 4x) + 4x^3 - 2x^2$$

$$12x^3 - 25x^2 + 12x + 4x^3 - 8x^2$$

Product Rule and Simple Power Rule

$$72. s = 4 - \frac{1}{2}(t^2 - 3t) = (4 - \frac{1}{2}t^2)(t^2 - 3t)$$

$$s' = (4 - \frac{1}{2}t^2)(2t - 3) + t(t^2 - 3t)$$

$$8t - 12 - 2t^{-1} + 3t^{-2} + 2t^{-1} - 6t^{-2}$$

$$8t - 12 - 3t^{-2}$$

Product Rule and Simple Power Rule

$$73. g(x) = \frac{x}{x+3}$$

$$g'(x) = \frac{(x+3)(1) - x(1)}{(x+3)^2}$$

$$g'(x) = \frac{3}{(x+3)^2}$$

Quotient Rule and Simple Power Rule

$$f(x) = \frac{2 - 5x}{3x + 1}$$

$$f'(x) = \frac{(3x+1)(-5) - (2-5x)(3)}{(3x+1)^2}$$

$$f'(x) = \frac{-15x - 5 - 6 + 15x}{(3x+1)^2}$$

$$\frac{-11}{2}$$

$$P'(4) = 2.602$$

$$P'(8) = 2.434$$

$$P'(12) = 2.266$$

$$P'(16) = 2.098$$

$$P'(20) = 1.93$$

$$P'(23) = 1.804$$

These are the rates at which the population of Brazil is changing in millions per year from 1990 to 2013.

$$f'(x) = -(3x + 1)$$

Quotient Rule and Simple Power Rule

$$f(x) = \frac{6x - 5}{x^2 + 1}$$

$$f'(x) = \frac{(6)(x^2 + 1) - (6x - 5)(2x)}{(x^2 + 1)^2}$$

$$= \frac{6x^2 + 6 - 12x^2 + 10x}{(x^2 + 1)^2}$$

$$= \frac{-6x^2 + 10x + 6}{(x^2 + 1)^2}$$

Quotient Rule and Simple Power Rule

$$\frac{x^2 + x - 1}{x^2 + 1}$$

76.  $f(x) = \frac{x^2 - 1}{x^2 + x - 1}$

$$f'(x) = \frac{(2x)(x^2 + x - 1) - (x^2 - 1)(2x + 1)}{(x^2 + x - 1)^2}$$

$$= \frac{2x^3 + 2x^2 - 2x - 1 - 2x^3 - 2x^2 + 2x}{(x^2 + x - 1)^2}$$

$$= \frac{-x^2 - 1}{(x^2 + x - 1)^2}$$

Quotient Rule and Simple Power Rule

$$f(x) = (5x^2 + 2)^3$$

$$f'(x) = 3(5x^2 + 2)^2 (10x)$$

$$= 30x(5x^2 + 2)^2$$

General Power Rule

78.  $f(x) = \sqrt{x^2 - 1} = (x^2 - 1)^{1/2}$

$$f'(x) = \frac{1}{2}(x^2 - 1)^{-1/2} (2x)$$

$$= \frac{x}{\sqrt{x^2 - 1}}$$

General Power Rule

79.  $h(x) = \frac{2}{\sqrt{x+1}} = 2(x+1)^{-1/2}$

$$h'(x) = 2(-1/2)(x+1)^{-3/2} = -\frac{1}{\sqrt{x+1}}$$

80.  $g(x) = \frac{6}{(3x^2 - 5x)^4} = 6(3x^2 - 5x)^{-4}$

$$g'(x) = 6(-4)(3x^2 - 5x)^{-5}(6x - 5)$$

$$= -\frac{24(6x - 5)}{(3x^2 - 5x)^5}$$

General Power Rule

81.  $g(x) = x \sqrt{x^2 + 1} = x(x^2 + 1)^{1/2}$

$$g'(x) = x \cdot \frac{1}{2}(x^2 + 1)^{-1/2}(2x) + (1)(x^2 + 1)^{1/2}$$

$$= x^2(x^2 + 1)^{-1/2} + (x^2 + 1)^{1/2}$$

$$= \frac{x^2 + x^2 + 1}{\sqrt{x^2 + 1}} = \frac{2x^2 + 1}{\sqrt{x^2 + 1}}$$

Product and General Power Rule

82.  $g(t) = (1 - t)^3$

$$g'(t) = \frac{1-t}{(1-t)^6} \cdot \frac{1-t-3(1-t)^2}{(1-t)^6}$$

$$= \frac{1-t-3+6t-3}{(1-t)^6} = \frac{1-t-6+6t-3}{(1-t)^6} = \frac{-8+5t}{(1-t)^6}$$

Quotient Rule and General Power Rule

$$f(x) = x(1 - 4x^2)^2$$

$$f'(x) = x(2)(1 - 4x^2)(-8x) + (1 - 4x^2)^2$$

$$= -16x^2(1 - 4x^2) + (1 - 4x^2)^2$$

$$= (1 - 4x^2)(-16x^2 + 1 - 4x^2)$$

$$= (1 - 4x^2)(1 - 20x^2)$$

$$1 - 4x^2 - 20x^2$$

Product and General Power Rule

$$h'(x) = 2 - \frac{1}{2} (x+1)^{-3/2}$$

1

$$\frac{1}{(x+1)^{3/2}}$$

General Power Rule

$$f(x) = x^2 + \frac{1}{x} = (x^2 + x^{-1})^5$$

$$\begin{aligned} f'(x) &= 5(x^2 + x^{-1})^4(2x - x^{-2}) \\ &= 5x^2 \frac{1}{x^2} (2x^2 - x^{-2}) \\ &= 5(2x^2 - x^{-2}) \end{aligned}$$



## General Power Rule

$$h(x) = x^2(2x + 3)^3 = x^6(2x + 3)^3$$

$$h'(x) = x^6 \cdot 3(2x + 3)^2(2) + 6x^5(2x + 3)^3$$

$$6x^5(2x + 3)^2 \cdot 2 + (2x + 3)$$

$$18x^5(2x + 3)^2(x + 1)$$

Product and General Power Rule

86.  $f(x) = (x - 2)(x + 4)^2$

$$f'(x) = 2(x - 2)(x + 4)(x - 2) + (x + 4)$$

$$2(x - 2)(x + 4)(2x + 2)$$

$$4(x - 2)(x + 4)(x + 1)$$

Product and General Power Rule

89.  $h(t) = \frac{\sqrt{3t+1}}{1-3t} = \frac{(3t+1)^{1/2}}{1-3t}$

$$h'(t) = \frac{(1-3t)^{-2} \cdot \frac{1}{2}(3t+1)^{-1/2} \cdot 3 - (3t+1)^{-1/2} \cdot (-3)}{(1-3t)^3}$$

$$= \frac{(3t+1)^{-1/2} \cdot 1 - 3t \cdot 3/2 + 3t+1 \cdot 6}{(1-3t)^3}$$

$$= \frac{3(9t+5)}{2\sqrt{3t+1}(1-3t)^3}$$

Quotient and General Power Rule

$g(x) = \frac{3x+1}{x^2+1} = \frac{(3x+1)^2}{(x^2+1)^2}$

$$\frac{(3x+1)^2}{(x^2+1)^2} = \frac{(3x+1)^2 \cdot 2(x^2+1) \cdot (-2x) - (x^2+1)^2 \cdot 6x}{(x^2+1)^4}$$

$$g'(x) = \frac{6(x^2+1)^2(3x+1) - 4x(3x+1)^2(x^2+1)}{(x^2+1)^4}$$

$$= \frac{6(x^2+1)^2(3x+1) - 4x(3x+1)^2(x^2+1)}{(x^2+1)^4}$$

$$= \frac{2(3x+1)(x^2+1)3(x^2+1) - 2x(3x+1)^2(x^2+1)}{(x^2+1)^4}$$

$$\frac{2(3x+1)(x^2+1)(-3x^2-2x+3)}{(x^2+1)^4}$$

$$\frac{2(3x+1)(-3x^2-2x+3)}{(x^2+1)^3}$$

Quotient and General Power Rule.

$$f(x) = x^2(x-1)^5$$

$$f'(x) = 5x^2(x-1)^4 + 2x(x-1)^5$$

$$x(x-1)^4(5x+2(x-1))$$

$$x(x-1)^4(7x-2)$$

Product and General Power

Rule 88.  $f(s) = s^3(s^2-1)^{5/2}$

$$f'(s) = s^3 \cdot \frac{5}{2}(s^2-1)^{3/2} \cdot 2s + 3s^2(s^2-1)^{5/2}$$

$$= s^3 \cdot 5s^2(s^2-1)^{3/2} + 3s^2(s^2-1)^{5/2}$$

$$= s^2(s^2-1)^{3/2}(8s^2-3)$$

Product and General Power Rule

91.  $T = \frac{1300}{t^2 + 2t + 25} = 1300(t^2 + 2t + 25)^{-1}$

$T'(t) = -1300(t^2 + 2t + 25)^{-2}(2t + 2)$

$= -\frac{2600(t + 1)}{(t^2 + 2t + 25)^2}$

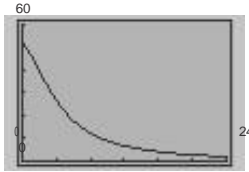
$T'(1) = -\frac{325}{49} \approx -6.63^\circ\text{F/hr}$

$T'(3) = -\frac{13}{2} \approx -6.5^\circ\text{F/hr}$

$T'(5) = -\frac{13}{3} \approx -4.33^\circ\text{F/hr}$

$T'(10) = -\frac{1144}{841} \approx -1.36^\circ\text{F/hr}$

(b)



The rate of decrease is approaching zero.

When  $L = 12$ ,

$V = \frac{1}{16}L(D - 4)^2 = \frac{1}{16}(12)(D - 4)^2 = \frac{3}{4}(D - 4)^2$

$\frac{dV}{dD} = \frac{3}{2}(D - 4)$

When  $D = 8$ ,  $\frac{dV}{dD} = \frac{3}{2}(8 - 4) = 6$  board ft in.

When  $D = 16$ ,

$\frac{dV}{dD} = \frac{3}{2}(16 - 4) = 18$  board ft in.

(c) When  $D = 24$ ,

$\frac{dV}{dD} = \frac{3}{2}(24 - 4) = 30$  board ft in.

(d) When  $D = 36$ ,

$\frac{dV}{dD} = \frac{3}{2}(36 - 4) = 48$  board ft in.

95.  $f(x) = -\frac{3}{x^4} = -3x^{-4}$

$f'(x) = 12x^{-5}$

$f''(x) = -60x^{-6}$

$f'''(x) = 360x^{-7} = \frac{360}{x^7}$

96.  $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f''(x) = -\frac{1}{4}x^{-3/2}$

$f'''(x) = \frac{3}{8}x^{-5/2}$

$f^{(4)}(x) = -\frac{15}{8}x^{-7/2}$

97.  $f(x) = -16x^{5/2}$

$f'(x) = 20x^{3/2}$

$f''(x) = 30x^{1/2}$

$f^{(3)}(x) = \frac{15}{x^{1/2}}$

$f^{(4)}(x) = -\frac{15}{2x^{3/2}}$

98.  $f(x) = 9x^3 = 9x^{3/3}$

$f'(x) = 3x^2$

$f''(x) = -2x^{-5/3}$

$f^{(3)}(x) = \frac{10}{x^{8/3}} = \frac{10}{x^{2+2/3}}$

99.  $f(x) = x^2 + \frac{3}{x} = x^2 + 3x^{-1}$

$f'(x) = 2x - 3x^{-2}$

$f''(x) = 2 + 6x^{-3} = 2 + \frac{6}{x^3}$

100.  $f'''(x) = 20x^4 - \frac{2}{x^3} = 20x^4 - 2x^{-3}$

$f^{(4)}(x) = 80x^3 + 6x^{-4}$

$f(x) = 3x^2 + 7x + 1$

$$f'(x) = 6x + 7$$

$$f''(x) = 6$$

$$f^{(5)}(x) = 240x^2 - 24x^{-5} = 240x^2 - \frac{24}{x^5}$$

94.  $f(x) = 5x^4 - 6x^2 + 2x$   
"  $(20x^3 - 12x + 2)$   
 $f'''(x) = 60x^2 - 12 = 12(5x^2 - 1)$

(a)  $s(t) = -16t^2 + 5t + 30$   
 $v(t) = s'(t) = -32t + 5$   
 $a(t) = v'(t) = s''(t) = -32$

(b)  $s(t) = 0 = -16t^2 + 5t + 30$   
 Using the Quadratic Formula,  $t \approx 1.534$  seconds.

$v(t) = s'(t) = -32t + 5$   
 $v(1.534) \approx -44.09$  ft/sec  
 $a(t) = v'(t) = -32$  ft/sec<sup>2</sup>

102.  $s(t) = \frac{1}{t^2 + 2t + 1} = (t + 1)^{-2}$

$v(t) = s'(t) = -2(t + 1)^{-3} = -\frac{2}{(t + 1)^3}$

$a(t) = v'(t) = \frac{6}{(t + 1)^4}$

$x^2 + 3xy + y^3 = 10$

$2x + 3x \frac{dy}{dx} + 3y + 3y^2 \frac{dy}{dx} = 0$   
 $\frac{dy}{dx} (3x + 3y^2) = -2x - 3y$

$\frac{dy}{dx} = \frac{-2x - 3y}{3x + 3y^2} = -\frac{2x + 3y}{3(x + y^2)}$

$x^2 + 9xy + y^2 = 0$

$2x + 9y + 9x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$

$(9x + 2y) \frac{dy}{dx} = -2x - 9y$

$\frac{dy}{dx} = \frac{-2x - 9y}{9x + 2y} = -\frac{2x + 9y}{9x + 2y}$

$y^2 - x^2 + 8x - 9y - 1 = 0$

$\frac{dy}{dx} (2y - x) - 2x + 8 - 9 = 0$

$(2y - x) \frac{dy}{dx} = 2x - 8$

$\frac{dy}{dx} = \frac{2x - 8}{2y - x}$

$y^2 = x - y$   
 $2y \frac{dy}{dx} = 1 - \frac{dy}{dx}$

$2y \frac{dy}{dx} + \frac{dy}{dx} = 1$

$(2y + 1) \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y + 1}$

At (2, 1),  $\frac{dy}{dx} = \frac{1}{3}$

$\frac{dy}{dx} = \frac{1}{3}$

$-1 = \frac{1}{3}(x - 2)$

$y = \frac{1}{3}x + \frac{1}{3}$

$2x^{1/3} + 3y^{1/2} = 10$

$\frac{2}{3}x^{-2/3} + \frac{3}{2}y^{-1/2} \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{4y^{1/2}}{9x^{2/3}}$

At (8, 4),  $\frac{dy}{dx} = -\frac{2}{9}$

$-4 = -\frac{2}{9}(x - 8)$

$\frac{dy}{dx} = -\frac{2}{9}$

$y^2 - 2x = xy$

$2y \frac{dy}{dx} - 2 = x \frac{dy}{dx} + y$

$\frac{dy}{dx} = \frac{y + 2}{2y - x}$

At (1, 2),  $\frac{dy}{dx} = \frac{4}{3}$

$\frac{dy}{dx} = \frac{4}{3}$

$-2 = \frac{4}{3}(x - 1)$

$y^2 + x^2 - 6y - 2x - 5 = 0$

$$y = \frac{4}{3}x + \frac{2}{3}$$

$$dx \frac{dy}{dx} + 2x - 6 dx \frac{dy}{dx} - 2 = 0$$

$$\frac{dy}{dx} (2y - 6) = 2 - 2x$$

$$\frac{dy}{dx} = \frac{2 - 2x}{2y - 6} = \frac{1 - x}{y - 3}$$

$$dx \quad 2y - 6 \quad y - 3$$

110.  $y^3 - 2x^2y + 3xy^2 = -1$   
 $3y^2 \frac{dy}{dx} - 2x^2 \frac{dy}{dx} - 4xy + 6xy \frac{dy}{dx} + 3y^2 = 0$   
 $\frac{dy}{dx}(3y^2 - 2x^2 + 6xy) = 4xy - 3y^2$   
 $\frac{dy}{dx} = \frac{4xy - 3y^2}{3y^2 - 2x^2 + 6xy}$

At (0, -1),  $\frac{dy}{dx} = -1$ .

$+1 = -1(x - 0)y = -x - 1$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

(a) Find  $\frac{dA}{dt}$  when  $r = 3$  in. and  $\frac{dr}{dt} = 2$  in. min.

$\frac{dA}{dt} = 2\pi(3)(2) = 12\pi \text{ in.}^2/\text{min}$   
 $\frac{dA}{dt} = 37.7 \text{ in.}^2/\text{min}$

$P = 375x - 1.5x^2$

$\frac{dP}{dt} = 375 - 3.0x \frac{dx}{dt}$

$\frac{dP}{dt} = (375 - 3.0x) \frac{dx}{dt}$

$\frac{dP}{dt} = 375 - 3.0(50)(2) = \$450 \text{ day}$

$\frac{dP}{dt} = 375 - 3.0(100)(2) = \$150 \text{ day}$

Let  $b$  be the horizontal distance of the water and  $h$  be the depth of the water at the deep end.

Then  $b = 8h$  for  $0 \leq h \leq 5$ .

$V = \frac{1}{2}bh(20) = 10bh = 10(8h)h = 80h^2$

$\frac{dV}{dt} = 160h \frac{dh}{dt}$

$\frac{dV}{dt} = 160h \frac{dh}{dt} = \frac{1}{160h} \frac{dV}{dt} = \frac{1}{160(10)} = \frac{1}{1600}$

When  $h = 4$ ,  $\frac{dh}{dt} = \frac{1}{16(4)} = \frac{1}{64} \text{ ft min.}$

$P = R - C$

$xP - C$

$x(211 - 0.002x) - (30x + 1,500,000)$

$181x - 0.002x^2 - 1,500,000$

$\frac{dP}{dx}$

$\frac{dP}{dx} = 181 - 0.004x$

$\frac{dP}{dx} = 181 - 0.004(1600)(15) = \$2619 \text{ week}$

**Chapter 2 Test Yourself**

1.  $f(x) = x^2 + 3; (3, 12)$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 + 3 - (x^2 + 3)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + \Delta x^2 + 3 - x^2 - 3}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + \Delta x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 2x + \Delta x \\ &= 2x \end{aligned}$$

At  $(3, 12): m = 2 \cdot 3 = 6$

$(y - 12) = 6(x - 3)$

$f(x) = x - \sqrt{x}; (4, 0)$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x) - \sqrt{x + \Delta x} - (x - \sqrt{x})}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x - (\sqrt{x + \Delta x} - \sqrt{x})}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x - \frac{\Delta x}{\sqrt{x + \Delta x} + \sqrt{x}}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x \left(1 - \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}\right)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1 - \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}}{1} \\ &= \frac{1 - \frac{1}{\sqrt{x} + \sqrt{x}}}{1} \\ &= \frac{1 - \frac{1}{2\sqrt{x}}}{1} \\ &= \frac{2\sqrt{x} - 1}{2\sqrt{x}} \end{aligned}$$

At  $(4, 0): m = 2 \cdot \frac{1}{\sqrt{4}} = \frac{1}{2}$

$f(t) = t^3 + 2t$

$f'(t) = 3t^2 + 2$

$f'(x) = 8x - 8$

$f(x) = x^{3/2} + 6x^{1/2}$

$f'(x) = \frac{3}{2}x^{1/2} + 3x^{-1/2} = \frac{3\sqrt{x}}{2} + \frac{3}{\sqrt{x}}$

$f(x) = (x + 3)(x^2 + 2x)$

$f(x) = x^3 + 5x^2 + 6x$

$f'(x) = 3x^2 + 10x + 6$

(Or use the Product Rule.)

8.  $f(x) = x(\sqrt{x} + x) = 5x^{1/2} + x^3$

$f'(x) = \frac{5}{2}x^{-1/2} + 3x^2 = \frac{5}{2\sqrt{x}} + 3x^2$

9.  $f(x) = (3x^2 + 4)^2$   
 $f'(x) = 2(3x^2 + 4)(6x)$

$36x^3 + 48x$

10.  $f(x) = \sqrt{1 - 2x} = (1 - 2x)^{1/2}$

$f'(x) = \frac{1}{2}(1 - 2x)^{-1/2} = \frac{1}{2\sqrt{1 - 2x}}$

$(y - 2) = \frac{1}{2\sqrt{1 - 2x}}(x - 1)$

$f(x) = \frac{(5x - 1)^3}{x}$

$f'(x) =$

$\frac{(3)(5x - 1)^2(5) - (5x - 1)^3}{x^2}$

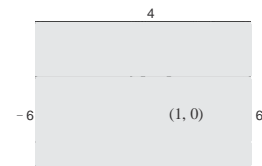
$\frac{15(5x - 1)^2 - (5x - 1)^3}{x^2}$

$= \frac{(5x - 1)^2(10x + 1)}{x^2}$

12.  $f(x) = x - \frac{1}{x}$

$f'(1) = 1 + \frac{1}{1^2} = 2$

$y - 0 = 2(x - 1)$   
 $y = 2x - 2$





$$f(x) = 2x^2 + 3x = 2x^2 + 3x$$

$$6. f(x) = 5x^2 - \frac{3}{x} = 5x^2 - 3x^{-3}$$

$$f'(x) = 10 + 9x^{-4} = 10x + \frac{9}{x^4}$$

$$S = -2.1083t^3 + 70.811t^2 - 777.05t + 2893.6$$

$\frac{S}{t}$  for  $10 \leq t \leq 12$

$$\frac{S(12) - S(10)}{12 - 10} = \frac{122.6416}{2} = 61.3208$$

\$13.3708 billion/yr

$$S'(t) = -6.3249t^2 + 141.622t - 777.05$$

$$2010: S'(10) = \$6.68 \text{ billion/yr}$$

$$2012: S'(12) = \$11.6284 \text{ billion/yr}$$

The annual sales of CVS Caremark from 2010 to 2012 increased by an average of about \$13.37 billion per year, and the instantaneous rates of change for 2010 and 2012 are \$6.68 billion per year and \$11.63 billion per year, respectively.

$$P = 1700 - 0.016x, C = 715,000 + 240x$$

Profit = Revenue - Cost

$$\begin{aligned} \text{Revenue: } R &= xp \\ &= x(1700 - 0.016x) \\ &= 1700x - 0.016x^2 \end{aligned}$$

$$P = R - C$$

$$P = (1700x - 0.016x^2) - (715,000 + 240x)$$

$$P = -0.016x^2 + 1460x - 715,000$$

$\frac{dP}{dx}$

$$dx = -0.032x + 1460 = P'(x)$$

$$P'(700) = \$1437.60$$

$$f(x) = 2x^2 + 3x + 1$$

$$f'(x) = 4x + 3$$

$$f''(x) = 4$$

$$f'''(x) = 0$$

$$f(x) = 3\sqrt{x} = (3-x)^{1/2}$$

$$f'(x) = \frac{1}{2}(3-x)^{-1/2}(-1) = -\frac{1}{2}(3-x)^{-1/2}$$

$$f''(x) = -\frac{1}{2} - \frac{1}{2}(3-x)^{-3/2}(-1) = -\frac{1}{4}(3-x)^{-3/2}$$

$$\begin{aligned} f'''(x) &= -\frac{1}{4} - \frac{3}{2}(3-x)^{-5/2}(-1) \\ &= -\frac{3}{8}(3-x)^{-5/2} \end{aligned}$$

$$= -\frac{3}{8(3-x)^{5/2}}$$

$$f(x) = \frac{2x+1}{2x-1}$$

$$f'(x) = \frac{(2x-1)(2) - (2x+1)(2)}{(2x-1)^2}$$

$$= \frac{-4(2x-1)}{(2x-1)^2} = -4(2x-1)^{-2}$$

$$f''(x) = 8(2x-1)^{-3}(2) = 16(2x-1)^{-3}$$

$$f'''(x) = -48(2x-1)^{-4}(2) = -96(2x-1)^{-4}$$

18.  $s(t) = -16t^2 + 30t + 75$

$$v(t) = s'(t) = -32t + 30$$

$$a(t) = v'(t) = -32$$

At  $t = 2$ :  $s(2) = 71$  ft

$$v(2) = -34 \text{ ft/sec}$$

$$a(2) = -32 \text{ ft/sec}^2$$

$$x + xy = 6$$

$$x \frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = -\frac{y}{x-1}$$

$$\frac{y+1}{x}$$

$$dx = -$$

$$y^2 + 2x - 2y + 1 = 0$$

$$2y \frac{dy}{dx} + 2 - 2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y-2) = -2$$

$$\frac{dy}{dx} = -\frac{1}{y-1}$$

21.  $4x^2 - 3y^2 + x^3y = 5$

$$8x - 6y \frac{dy}{dx} + x^3 \frac{dy}{dx} + 3x^2y = 0$$

$$6y \frac{dy}{dx} + x^3 \frac{dy}{dx} = -8x - 3x^2y$$

$$(x^3 - 6y) \frac{dy}{dx} = -(8x + 3x^2y)$$

$$\frac{dy}{dx} = -\frac{8x + 3x^2y}{x^3 - 6y}$$

$$\frac{dy}{dx} = \frac{8x + 3x^2y}{6y - x^3} = \frac{x(8 + 3xy)}{6y - x^3}$$

*Chapter 2 Differentiation*

$$V = \pi r^2 h = 20\pi r^3$$

$$\frac{dV}{dt} = 60\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 60\pi (0.5)^2 (0.25) = 3.75\pi \text{ cm}^3 / \text{min}$$

$$\frac{dV}{dt} = 60\pi (1)^2 (0.25) = 15\pi \text{ cm}^3 / \text{min}$$