CALCULUS

SECOND EDITION

William Briggs
University of Colorado at Denver

Lyle Cochran
Whitworth University

Bernard Gillett
University of Colorado at Boulder

with the assistance of

Eric Schulz
Walla Walla Community College

PEARSON

Boston Columbus Indianapolis New York San Francisco Upper Saddle River Amsterdam Cape Town Dubai London Madrid Milan Munich Paris Montreal Toronto Delhi Mexico City São Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo

Reproduced by Pearson from electronic files supplied by the author.

Publishing as Pearson, 75 Arlington Street, Boston, MA 02116.

ISBN-13: 978-0-321-95485-5
ISBN-10: 0-321-95485-8

123456OPM17161514

www.pearsonhighered.com

PEARSON

Contents

3 Derivatives **109**

4 Applications of the Derivative **213**

Chapter 1

Functions

1.1 Review of Functions

1.1.1 A function is a rule which assigns each domain element to a unique range element. The independent variable is associated with the domain, while the dependent variable is associated with the range.

1.1.2 The independent variable belongs to the domain, while the dependent variable belongs to the range.

1.1.3 The vertical line test is used to determine whether a given graph represents a function. (Specifically, it tests whether the variable associated with the vertical axis is a function of the variable associated with the horizontal axis.) If every vertical line which intersects the graph does so in exactly one point, then the given graph represents a function. If any vertical line $x = a$ intersects the curve in more than one point, then there is more than one range value for the domain value $x = a$, so the given curve does not represent a function.

1.1.4 $f(2) = \frac{1}{2^3+1} = \frac{1}{9}$. $f(y^2) = \frac{1}{(y^2)^3+1} = \frac{1}{y^6+1}$.

1.1.5 Item i. is true while item ii. isn't necessarily true. In the definition of function, item i. is stipulated. However, item ii. need not be true – for example, the function $f(x) = x^2$ has two different domain values associated with the one range value 4, because $f(2) = f(-2) = 4$.

1.1.6

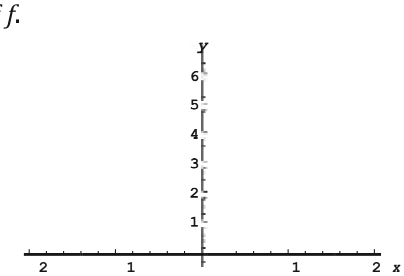
$$(f \circ g)(x) = f(g(x)) = f(x^3 - 2) = \sqrt{x^3 - 2}$$
$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = x^{3/2} - 2.$$
$$(f \circ f)(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}.$$
$$(g \circ g)(x) = g(g(x)) = g(x^3 - 2) = (x^3 - 2)^3 - 2 = x^9 - 6x^6 + 12x^3 - 10$$

1.1.7 $f(g(2)) = f(-2) = f(2) = 2$. The fact that $f(-2) = f(2)$ follows from the fact that f is an even function. $g(f(-2)) = g(f(2)) = g(2) = -2$.

1.1.8 The domain of $f \circ g$ is the subset of the domain of g whose range is in the domain of f. Thus, we need to look for elements x in the domain of g so that $g(x)$ is in the domain of f.

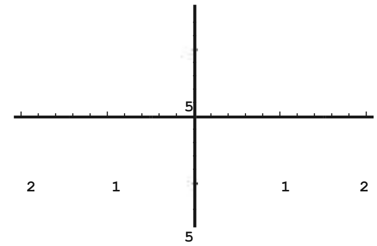
When f is an even function, we have $f(-x) = f(x)$

1.1.9 for all x in the domain of f, which ensures that the graph of the function is symmetric about the y-axis.



y

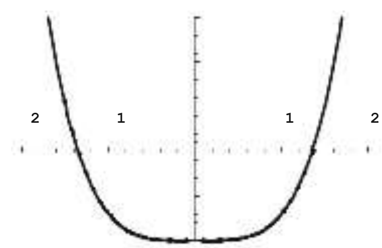
1.1.10 When f is an odd function, we have $f(-x) = -f(x)$ for all x in the domain of f, which ensures that the graph of the function is symmetric about the origin.



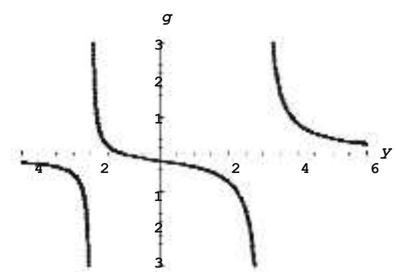
1.1.11 Graph A does not represent a function, while graph B does. Note that graph A fails the vertical line test, while graph B passes it.

1.1.12 Graph A does not represent a function, while graph B does. Note that graph A fails the vertical line test, while graph B passes it.

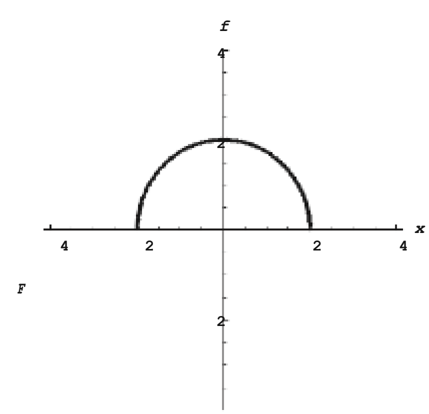
1.1.13 The domain of this function is the set of a real numbers. The range is $[-10, \infty)$.



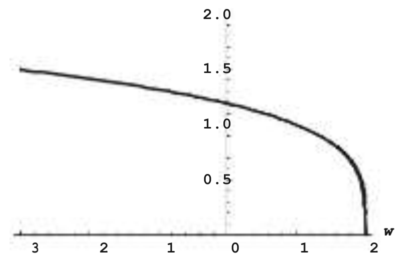
1.1.14 The domain of this function is $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$. The range is the set of all real numbers.

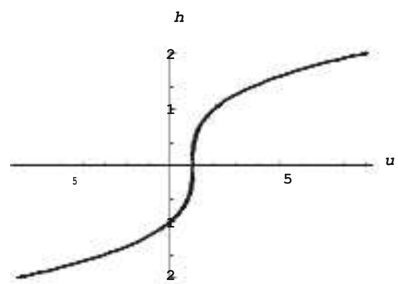


1.1.15 The domain of this function is $[-2,2]$. The range is $[0,2]$.



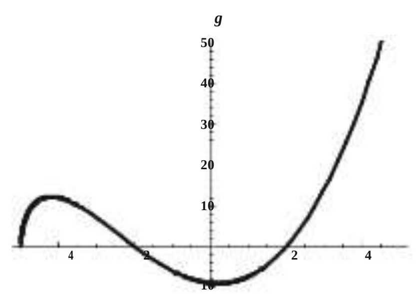
1.1.16 The domain of this function is (−∞,2]. The range is [0,∞).



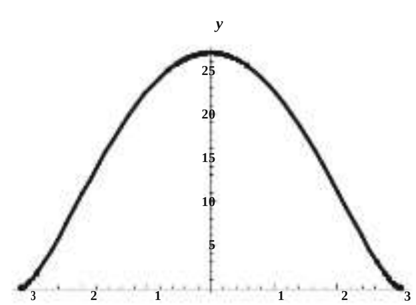


1.1.17 The domain and the range for this function are both the set of all real numbers.

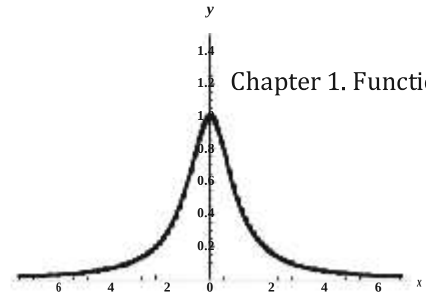
1.1.18 The domain of this function is [−5,∞). The range is approximately [−9.03,∞).



1.1.19 The domain of this function is [−3,3]. The range is [0,27].



1.1.35



1.1.20 The domain of this function is (−∞,∞)]. The range is (0,1].

1.1.21 The independent variable *t* is elapsed time and the dependent variable *d* is distance above the ground. The domain in context is [0,8]

1.1.22 The independent variable *t* is elapsed time and the dependent variable *d* is distance above the water. The domain in context is [0,2]

1.1.23 The independent variable *h* is the height of the water in the tank and the dependent variable *V* is the volume of water in the tank. The domain in context is [0,50]

1.1.24 The independent variable *r* is the radius of the balloon and the dependent variable *V* is the volume of the balloon. The domain in context is [0, $\sqrt[3]{3/(4\pi)}$]

$f(10) = 96$ **1.1.26** $f(p^2) = (p^2)^2 - 4 = p^4 - 4$

$g(1/z) = (1/z)^3 = \frac{1}{z^3}$ **1.1.28** $F(y^4) = \frac{1}{y^4-3}$

$F(g(y)) = F(y^3) = \frac{1}{y^3-3}$ **1.1.30** $f(g(w)) = f(w^3) = (w^3)^2 - 4 = w^6 - 4$

$g(f(u)) = g(u^2-4) = (u^2-4)^3$

$\frac{f(2+h)-f(2)}{h} = \frac{(2+h)^2-4-0}{h} = \frac{4+4h+h^2-4}{h} = \frac{4h+h^2}{h} = 4+h$

$F(F(x)) = F\left(\frac{1}{x-3}\right) = \frac{1}{\frac{1}{x-3}-3} = \frac{1}{\frac{1}{x-3}-\frac{3(x-3)}{x-3}} = \frac{1}{\frac{10-3x}{x-3}} = \frac{x-3}{10-3x}$

$g(F(f(x))) = g(F(x^2-4)) = g\left(\frac{1}{x^2-4-3}\right) = \left(\frac{1}{x^2-7}\right)^3$

$f(\sqrt{x+4}) = (\sqrt{x+4})^2 - 4 = x+4-4 = x.$

$F((3x+1)/x) = \frac{1}{\frac{3x+1}{x}-3} = \frac{1}{\frac{3x+1-3x}{x}} = \frac{x}{3x+1-3x} = x.$

1.1.25

1.1.27

1.1.29

1.1.31

1.1.32

1.1.33

1.1.34

1.1.35

1.1.36

1.1.37 $g(x) = x^3 - 5$ and $f(x) = x^{10}$. The domain of h is the set of all real numbers.

1.1.38 $g(x) = x^6 + x^2 + 1$ and $f(x) = \frac{2}{x^2}$. The domain of h is the set of all real numbers.

1.1.39 $g(x) = x^4 + 2$ and $f(x) = \sqrt{x}$. The domain of h is the set of all real numbers.

1.1.40 $g(x) = x^3 - 1$ and $f(x) = \frac{1}{\sqrt{x}}$. The domain of h is the set of all real numbers for which $x^3 - 1 > 0$, which corresponds to the set $(1,\infty)$.

1.1.41 $(f \circ g)(x) = f(g(x)) = f(x^2 - 4) = |x^2 - 4|$. The domain of this function is the set of all real numbers.

1.1.42 $(g \circ f)(x) = g(f(x)) = g(|x|) = |x|^2 - 4 = x^2 - 4$. The domain of this function is the set of all real numbers.

1.1.43 $(f \circ G)(x) = f(G(x)) = f\left(\frac{1}{x-2}\right) = \left|\frac{1}{x-2}\right|$. The domain of this function is the set of all real numbers except for the number 2.

1.1.44 $(f \circ g \circ G)(x) = f(g(G(x))) = f\left(g\left(\frac{1}{x-2}\right)\right) = f\left(\left(\frac{1}{x-2}\right)^2 - 4\right) = \left|\left(\frac{1}{x-2}\right)^2 - 4\right|$. The domain of this function is the set of all real numbers except for the number 2.

1.1.45 $(G \circ g \circ f)(x) = G(g(f(x))) = G(g(|x|)) = G(x^2 - 4) = \frac{1}{x^2-4-2} = \frac{1}{x^2-6}$. The domain of this function is the set of all real numbers except for the numbers $\pm\sqrt{6}$.

1.1.46 $(F \circ g \circ g)(x) = F(g(g(x))) = F(g(x^2 - 4)) = F((x^2 - 4)^2 - 4) = \sqrt{(x^2 - 4)^2 - 4} = \sqrt{x^4 - 8x^2 + 12}$. The domain of this function consists of the numbers x so that $x^4 - 8x^2 + 12 \geq 0$. Because $x^4 - 8x^2 + 12 = (x^2 - 6) \cdot (x^2 - 2)$, we see that this expression is zero for $x = \pm\sqrt{6}$ and $x = \pm\sqrt{2}$. By looking between these points, we see that the expression is greater than or equal to zero for the set $(-\infty, -\sqrt{6}] \cup [-\sqrt{2}, \sqrt{2}] \cup [\sqrt{6}, \infty)$.

1.1.47 $(g \circ g)(x) = g(g(x)) = g(x^2 - 4) = (x^2 - 4)^2 - 4 = x^4 - 8x^2 + 16 - 4 = x^4 - 8x^2 + 12$. The domain is the set of all real numbers.

1.1.48 $(G \circ G)(x) = G(G(x)) = G(1/(x-2)) = \frac{1}{\frac{1}{x-2} - 2} = \frac{1}{\frac{1-2(x-2)}{x-2}} = \frac{x-2}{1-2x+4} = \frac{x-2}{5-2x}$. Then $G \circ G$ is defined except where the denominator vanishes, so its domain is the set of all real numbers except for $x = \frac{5}{2}$.

1.1.49 Because $(x^2 + 3) - 3 = x^2$, we may choose $f(x) = x - 3$.

1.1.50 Because the reciprocal of $x^2 + 3$ is $\frac{1}{x^2+3}$, we may choose $f(x) = \frac{1}{x}$.

1.1.51

1.1.66

a. $(f \circ g)(2) = f(g(2)) = f(2) = 4.$

b. $g(f(2)) = g(4) = 1.$

c. $f(g(4)) = f(1) = 3.$

d. $g(f(5)) = g(6) = 3.$

e. $f(f(8)) = f(8) = 8.$

f. $g(f(g(5))) = g(f(2)) = g(4) = 1.$

1.1.56

a. $h(g(0)) = h(0) = -1.$

b. $g(f(4)) = g(-1) = -1.$

c. $h(h(0)) = h(-1) = 0.$

d. $g(h(f(4))) = g(h(-1)) = g(0) = 0.$

e. $f(f(f(1))) = f(f(0)) = f(1) = 0.$

f. $h(h(h(0))) = h(h(-1)) = h(0) = -1.$

Because $(x^2 + 3)^2 = x^4 + 6x^2 + 9$, we may choose $f(x) = x^2$.

1.1.52 Because $(x^2 + 3)^2 = x^4 + 6x^2 + 9$, and the given expression is 11 more than this, we may choose $f(x) = x^2 + 11$.

1.1.53 Because $(x^2)^2 + 3 = x^4 + 3$, this expression results from squaring x^2 and adding 3 to it. Thus we may choose $f(x) = x^2$.

1.1.54 Because $x^{2/3} + 3 = (\sqrt[3]{x})^2 + 3$, we may choose $f(x) = \sqrt[3]{x}$.

g. $f(h(g(2))) = f(h(3)) = f(0) = 1.$

h. $g(f(h(4))) = g(f(4)) = g(-1) = -1.$

i. $g(g(g(1))) = g(g(2)) = g(3) = 4.$

j. $f(f(h(3))) = f(f(0)) = f(1) = 0.$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-x^2}{h} = \frac{(x^2+2hx+h^2)-x^2}{h} = \frac{h(2x+h)}{h} = 2x+h$$

$$\frac{f(x+h)-f(x)}{h} = \frac{4(x+h)-3-(4x-3)}{h} = \frac{4x+4h-3-4x+3}{h} = \frac{4h}{h} = 4.$$

1.1.57.

1.1.58

1.1.59 $\frac{f(x+h)-f(x)}{h} = \frac{\frac{2}{x+h}-\frac{2}{x}}{h} = \frac{\frac{2x-2(x+h)}{x(x+h)}}{h} = \frac{2x-2x-2h}{hx(x+h)} = -\frac{2h}{hx(x+h)} = -\frac{2}{x(x+h)}.$

1.1.60 $\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2-3(x+h)+1-(2x^2-3x+1)}{h} = \frac{2x^2+4xh+2h^2-3x-3h+1-2x^2+3x-1}{h} = \frac{4xh+2h^2-3h}{h} = \frac{h(4x+2h-3)}{h} = 4x+2h-3.$

1.1.61 $\frac{f(x+h)-f(x)}{h} = \frac{\frac{x+h}{x+h+1}-\frac{x}{x+1}}{h} = \frac{\frac{(x+h)(x+1)-x(x+h+1)}{(x+1)(x+h+1)}}{h} = \frac{x^2+x+hx+h-x^2-xh-x}{h(x+1)(x+h+1)} = \frac{h}{h(x+1)(x+h+1)} = \frac{1}{(x+1)(x+h+1)}$

1.1.62 $\frac{f(x)-f(a)}{x-a} = \frac{x^4-a^4}{x-a} = \frac{(x^2-a^2)(x^2+a^2)}{x-a} = \frac{(x-a)(x+a)(x^2+a^2)}{x-a} = (x+a)(x^2+a^2).$

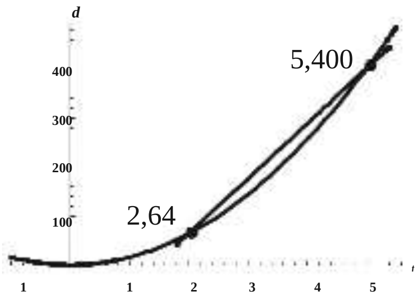
1.1.63 $\frac{f(x)-f(a)}{x-a} = \frac{x^3-2x-(a^3-2a)}{x-a} = \frac{(x^3-a^3)-2(x-a)}{x-a} = \frac{(x-a)(x^2+ax+a^2)-2(x-a)}{x-a} = \frac{(x-a)(x^2+ax+a^2-2)}{x-a} = x^2+ax+a^2-2.$

1.1.64 $\frac{f(x)-f(a)}{x-a} = \frac{4-4x-x^2-(4-4a-a^2)}{x-a} = \frac{-4(x-a)-(x^2-a^2)}{x-a} = \frac{-4(x-a)-(x-a)(x+a)}{x-a} = \frac{(x-a)(-4-(x+a))}{x-a} = -4-x-a.$

1.1.65 $\frac{f(x)-f(a)}{x-a} = \frac{\frac{-4}{x^2}-\frac{-4}{a^2}}{x-a} = \frac{\frac{-4a^2+4x^2}{a^2x^2}}{x-a} = \frac{4(x^2-a^2)}{(x-a)a^2x^2} = \frac{4(x-a)(x+a)}{(x-a)a^2x^2} = \frac{4(x+a)}{a^2x^2}.$

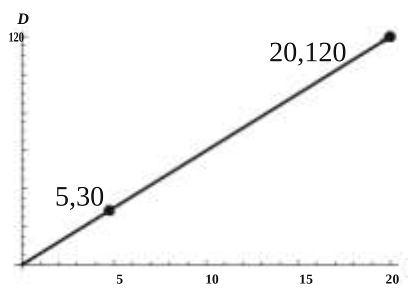
1.1.66 $\frac{f(x)-f(a)}{x-a} = \frac{\frac{1}{x}-x^2-(\frac{1}{a}-a^2)}{x-a} = \frac{\frac{1}{x}-\frac{1}{a}}{x-a} - \frac{x^2-a^2}{x-a} = \frac{\frac{a-x}{ax}}{x-a} - \frac{(x-a)(x+a)}{x-a} = -\frac{1}{ax} - (x+a).$

1.1.67

a. 

b. The slope of the secant line is given by $\frac{400-64}{5-2} = \frac{336}{3}$ = 112 feet per second. The object falls at an average rate of 112 feet per second over the interval $2 \le t \le 5$.

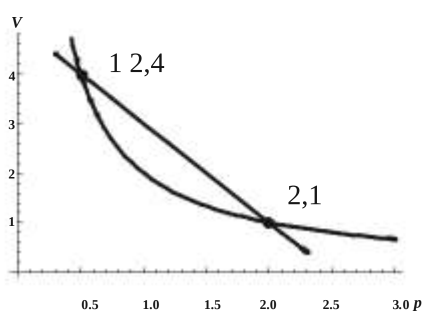
1.1.68

a. 

b. The slope of the secant line is given by $\frac{120-30}{20-5} = \frac{90}{15}$ = 6 degrees per second. The second hand moves at an average rate of 6 degrees per second over the interval $5 \le t \le 20$.

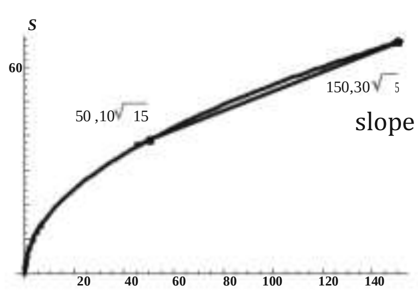
100 80 60 40 20

1.1.69

a. 

b. The slope of the secant line is given by $\frac{1-4}{2-(1/2)} = -\frac{3}{3/2} = -2$ cubic cm per atmosphere. The volume decreases at an average rate of 2 cubic cm per atmosphere over the interval $0.5 \le p \le 2$.

1.1.70

a. 

b. The slope of the secant line is given by $\frac{30\sqrt{5}-10\sqrt{15}}{150-50} \approx .2835$ mph per foot. The speed of the car changes with an average rate of about .2835 mph per foot over the interval $50 \le l \le 150$.

50 40 30 20 10

1.1.71 This function is symmetric about the y-axis, because $f(-x) = (-x)^4+5(-x)^2-12 = x^4+5x^2-12 = f(x)$.

1.1.72 This function is symmetric about the origin, because $f(-x) = 3(-x)^5 + 2(-x)^3 - (-x) = -3x^5 - 2x^3 + x = -(3x^5 + 2x^3 - x) = f(x)$.

1.1.73 This function has none of the indicated symmetries. For example, note that $f(-2) = -26$, while $f(2) = 22$, so f is not symmetric about either the origin or about the y-axis, and is not symmetric about the x-axis because it is a function.

1.1.74 This function is symmetric about the y-axis. Note that $f(-x) = 2|-x| = 2|x| = f(x)$.

1.1.75 This curve (which is not a function) is symmetric about the x-axis, the y-axis, and the origin. Note that replacing either x by $-x$ or y by $-y$ (or both) yields the same equation. This is due to the fact that $(-x)^{2/3} = ((-x)^2)^{1/3} = (x^2)^{1/3} = x^{2/3}$, and a similar fact holds for the term involving y.

1.1.76 This function is symmetric about the origin. Writing the function as $y = f(x) = x^{3/5}$, we see that $f(-x) = (-x)^{3/5} = -(x)^{3/5} = -f(x)$.

1.1.77 This function is symmetric about the origin. Note that $f(-x) = (-x)|(-x)| = -x|x| = -f(x)$.

1.1.78 This curve (which is not a function) is symmetric about the x-axis, the y-axis, and the origin. Note that replacing either x by $-x$ or y by $-y$ (or both) yields the same equation. This is due to the fact that $|-x| = |x|$ and $|-y| = |y|$.

1.1.79 Function A is symmetric about the y-axis, so is even. Function B is symmetric about the origin, so is odd. Function C is also symmetric about the y-axis, so is even.

1.1.80 Function A is symmetric about the y-axis, so is even. Function B is symmetric about the origin, so is odd. Function C is also symmetric about the origin, so is odd.

1.1.81

a. True. A real number z corresponds to the domain element $z/2 + 19$, because $f(z/2 + 19) = 2(z/2 + 19) - 38 = z + 38 - 38 = z$.

b. False. The definition of function does not require that each range element comes from a unique domainelement, rather that each domain element is paired with a unique range element.

c. True. $f(1/x) = \frac{1}{1/x} = x$, and $\frac{1}{f(x)} = \frac{1}{1/x} = x$.

d. False. For example, suppose that f is the straight line through the origin with slope 1, so that $f(x) = x$. Then $f(f(x)) = f(x) = x$, while $(f(x))^2 = x^2$.

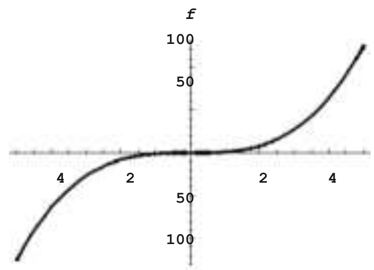
e. False. For example, let $f(x) = x+2$ and $g(x) = 2x-1$. Then $f(g(x)) = f(2x-1) = 2x-1+2 = 2x+1$, while $g(f(x)) = g(x + 2) = 2(x + 2) - 1 = 2x + 3$.

f. True. This is the definition of $f \circ g$.

g. True. If f is even, then $f(-z) = f(z)$ for all z, so this is true in particular for $z = ax$. So if $g(x) = cf(ax)$, then $g(-x) = cf(-ax) = cf(ax) = g(x)$, so g is even.

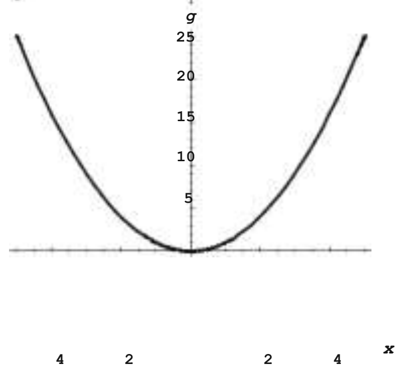
h. False. For example, $f(x) = x$ is an odd function, but $h(x) = x + 1$ isn't, because $h(2) = 3$, while $h(-2) = -1$ which isn't $-h(2)$.

i. True. If $f(-x) = -f(x) = f(x)$, then in particular $-f(x) = f(x)$, so $0 = 2f(x)$, so $f(x) = 0$ for all x.

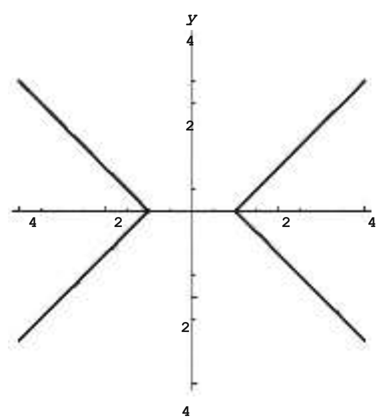


1.1.82 If n is odd, then $n = 2k + 1$ for some integer k, and $(x)^n = (x)^{2k+1} = x(x)^{2k}$, which is less than 0 when $x < 0$ and greater than 0 when $x > 0$. For any number P (positive or negative) the number $\sqrt[n]{P}$ is a real number when n is odd, and $f(\sqrt[n]{P}) = P$. So the range of f in this case is the set of all real numbers.

If n is even, then $n = 2k$ for some integer k, and $x^n = (x^2)^k$. Thus $g(-x) = g(x) = (x^2)^k \geq 0$ for all x. Also, for any nonnegative number M, we have $g(\sqrt[n]{M}) = M$, so the range of g in this case is the set of all nonnegative numbers.



1.1.83 We will make heavy use of the fact that $|x|$ is x if $x > 0$, and is $-x$ if $x < 0$. In the first quadrant where x and y are both positive, this equation becomes $x - y = 1$ which is a straight line with slope 1 and y-intercept -1. In the second quadrant where x is negative and y is positive, this equation becomes $-x - y = 1$, which is a straight line with slope -1 and y-intercept -1. In the third quadrant where both x and y are negative, we obtain the equation $-x - (-y) = 1$, or $y = x + 1$, and in the fourth quadrant, we obtain $x + y = 1$. Graphing these lines and restricting them to the appropriate quadrants yields the following curve:



1.1.84

a. No. For example $f(x) = x^2 + 3$ is an even function, but $f(0)$ is not 0.

b. Yes. because $f(-x) = -f(x)$, and because $-0 = 0$, we must have $f(-0) = f(0) = -f(0)$, so $f(0) = -f(0)$, and the only number which is its own additive inverse is 0, so $f(0) = 0$.

1.1.85 Because the composition of f with itself has first degree, f has first degree as well, so let $f(x) = ax+b$. Then $(f \circ f)(x) = f(ax + b) = a(ax + b) + b = a^2x + (ab + b)$. Equating coefficients, we see that $a^2 = 9$ and $ab + b = -8$. If $a = 3$, we get that $b = -2$, while if $a = -3$ we have $b = 4$. So the two possible answers are $f(x) = 3x - 2$ and $f(x) = -3x + 4$.

1.1.86 Since the square of a linear function is a quadratic, we let $f(x) = ax+b$. Then $f(x)^2 = a^2x^2+2abx+b^2$. Equating coefficients yields that $a = \pm 3$ and $b = \pm 2$. However, a quick check shows that the middle term is correct only when one of these is positive and one is negative. So the two possible such functions f are $f(x) = 3x - 2$ and $f(x) = -3x + 2$.

1.1.87 Let $f(x) = ax^2 + bx + c$. Then $(f \circ f)(x) = f(ax^2 + bx + c) = a(ax^2 + bx + c)^2 + b(ax^2 + bx + c) + c$. Expanding this expression yields $a^3x^4 + 2a^2bx^3 + 2a^2cx^2 + ab^2x^2 + 2abcx + ac^2 + abx^2 + b^2x + bc + c$, which simplifies to $a^3x^4 + 2a^2bx^3 + (2a^2c + ab^2 + ab)x^2 + (2abc + b^2)x + (ac^2 + bc + c)$. Equating coefficients yields $a^3 = 1$, so $a = 1$. Then $2a^2b = 0$, so $b = 0$. It then follows that $c = -6$, so the original function was $f(x) = x^2 - 6$.

1.1.88 Because the square of a quadratic is a quartic, we let $f(x) = ax^2 + bx + c$. Then the square of f is $c^2 + 2bcx + b^2x^2 + 2acx^2 + 2abx^3 + a^2x^4$. By equating coefficients, we see that $a^2 = 1$ and so $a = \pm 1$. Because the coefficient on x^3 must be 0, we have that $b = 0$. And the constant term reveals that $c = \pm 6$. A quick check shows that the only possible solutions are thus $f(x) = x^2 - 6$ and $f(x) = -x^2 + 6$.

1.1.89 $\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{\sqrt{x+h}+\sqrt{x}}$.

$\frac{f(x)-f(a)}{x-a} = \frac{\sqrt{x}-\sqrt{a}}{x-a} = \frac{\sqrt{x}-\sqrt{a}}{x-a} \cdot \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}} = \frac{x-a}{(x-a)(\sqrt{x}+\sqrt{a})} = \frac{1}{\sqrt{x}+\sqrt{a}}$.

1.1.90 $\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{1-2(x+h)}-\sqrt{1-2x}}{h} = \frac{\sqrt{1-2(x+h)}-\sqrt{1-2x}}{h} \cdot \frac{\sqrt{1-2(x+h)}+\sqrt{1-2x}}{\sqrt{1-2(x+h)}+\sqrt{1-2x}} =$
$\frac{1-2(x+h)-(1-2x)}{h(\sqrt{1-2(x+h)}+\sqrt{1-2x})} = -\frac{2}{\sqrt{1-2(x+h)}+\sqrt{1-2x}}$.

$\frac{f(x)-f(a)}{x-a} = \frac{\sqrt{1-2x}-\sqrt{1-2a}}{x-a} = \frac{\sqrt{1-2x}-\sqrt{1-2a}}{x-a} \cdot \frac{\sqrt{1-2x}+\sqrt{1-2a}}{\sqrt{1-2x}+\sqrt{1-2a}} = \frac{(1-2x)-(1-2a)}{(x-a)(\sqrt{1-2x}+\sqrt{1-2a})} =$
$\frac{(-2)(x-a)}{(x-a)(\sqrt{1-2x}+\sqrt{1-2a})} = -\frac{2}{(\sqrt{1-2x}+\sqrt{1-2a})}$.

1.1.91 $\frac{f(x+h)-f(x)}{h} = \frac{\frac{-3}{\sqrt{x+h}}-\frac{-3}{\sqrt{x}}}{h} = \frac{-3(\sqrt{x}-\sqrt{x+h})}{h\sqrt{x}\sqrt{x+h}} = \frac{-3(\sqrt{x}-\sqrt{x+h})}{h\sqrt{x}\sqrt{x+h}} \cdot \frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}} =$
$\frac{-3(x-(x+h))}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})} = \frac{3}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$.

$\frac{f(x)-f(a)}{x-a} = \frac{\frac{-3}{\sqrt{x}}-\frac{-3}{\sqrt{a}}}{x-a} = \frac{-3\left(\frac{\sqrt{a}-\sqrt{x}}{\sqrt{a}\sqrt{x}}\right)}{x-a} = \frac{(-3)(\sqrt{a}-\sqrt{x})}{(x-a)\sqrt{a}\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}} = \frac{(3)(x-a)}{(x-a)(\sqrt{a}\sqrt{x})(\sqrt{a}+\sqrt{x})} = \frac{3}{\sqrt{ax}(\sqrt{a}+\sqrt{x})}$

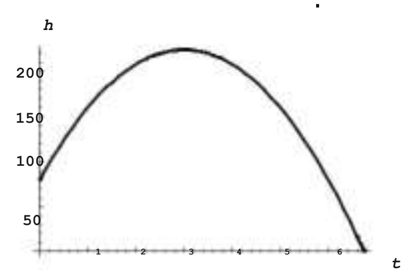
1.1.92 $\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h} = \frac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h} \cdot \frac{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}} =$
$\frac{(x+h)^2+1-(x^2+1)}{h(\sqrt{(x+h)^2+1}+\sqrt{x^2+1})} = \frac{x^2+2hx+h^2-x^2}{h(\sqrt{(x+h)^2+1}+\sqrt{x^2+1})} = \frac{2x+h}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}$.

$\frac{f(x)-f(a)}{x-a} = \frac{\sqrt{x^2+1}-\sqrt{a^2+1}}{x-a} = \frac{\sqrt{x^2+1}-\sqrt{a^2+1}}{x-a} \cdot \frac{\sqrt{x^2+1}+\sqrt{a^2+1}}{\sqrt{x^2+1}+\sqrt{a^2+1}} = \frac{x^2+1-(a^2+1)}{(x-a)(\sqrt{x^2+1}+\sqrt{a^2+1})} =$
$\frac{(x-a)(x+a)}{(x-a)(\sqrt{x^2+1}+\sqrt{a^2+1})} = \frac{x+a}{\sqrt{x^2+1}+\sqrt{a^2+1}}$.

1.1.93

a. The formula for the height of the rocket is valid from t = 0 until the rocket hits the ground, which is the positive solution to $-16t^2 + 96t + 80 = 0$, which the quadratic formula reveals is $t = 3 + \sqrt{14}$. Thus, the domain is $[0, 3+\sqrt{14}]$.

b.



The maximum appears to occur at t = 3. The height at that time would be 224.

1.1.94

a. $d(0) = (10 - (2.2)\cdot 0)^2 = 100$.

b. The tank is first empty when $d(t) = 0$, which is when $10 - (2.2)t = 0$, or $t = 50/11$.

c. An appropriate domain would $[0, 50/11]$.

1.1.95 This would not necessarily have either kind of symmetry. For example, $f(x) = x^2$ is an even function and $g(x) = x^3$ is odd, but the sum of these two is neither even nor odd.

1.1.96 This would be an odd function, so it would be symmetric about the origin. Suppose f is even and g is odd. Then $(f \cdot g)(-x) = f(-x)g(-x) = f(x)\cdot(-g(x)) = -(f \cdot g)(x)$.

1.1.97 This would be an odd function, so it would be symmetric about the origin. Suppose f is even and g is odd. Then $\frac{f}{g}(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f}{g}(x)$.

1.1.98 This would be an even function, so it would be symmetric about the y-axis. Suppose f is even and g is odd. Then $f(g(-x)) = f(-g(x)) = f(g(x))$.

1.1.99 This would be an even function, so it would be symmetric about the y-axis. Suppose f is even and g is even. Then $f(g(-x)) = f(g(x))$, because $g(-x) = g(x)$.

1.1.100 This would be an odd function, so it would be symmetric about the origin. Suppose f is odd and g is odd. Then $f(g(-x)) = f(-g(x)) = -f(g(x))$.

1.1.101 This would be an even function, so it would be symmetric about the y-axis. Suppose f is even and g is odd. Then $g(f(-x)) = g(f(x))$, because $f(-x) = f(x)$.

1.1.102

a. $f(g(-1)) = f(-g(1)) = f(3) = 3$

b. $g(f(-4)) = g(f(4)) = g(-4) = -g(4) = 2$

c. $f(g(-3)) = f(-g(3)) = f(4) = -4$

d. $f(g(-2)) = f(-g(2)) = f(1) = 2$

e. $g(g(-1)) = g(-g(1)) = g(3) = -4$

f. $f(g(0) - 1) = f(-1) = f(1) = 2$

g. $f(g(g(-2))) = f(g(-g(2))) = f(g(1)) = f(-3) = 3$

h. $g(f(f(-4))) = g(f(-4)) = g(-4) = 2$

i. $g(g(g(-1))) = g(g(-g(1))) = g(g(3)) = g(-4) = 2$

1.1.103

a. $f(g(-2)) = f(-g(2)) = f(-2) = 4$ c.

$f(g(-4)) = f(-g(4)) = f(-1) = 3$ e.

$g(g(-7)) = g(-g(7)) = g(-4) = -1$

b. $g(f(-2)) = g(f(2)) = g(4) = 1$

d. $g(f(5) - 8) = g(-2) = -g(2) = -2$

f. $f(1 - f(8)) = f(-7) = 7$

1.2 Representing Functions

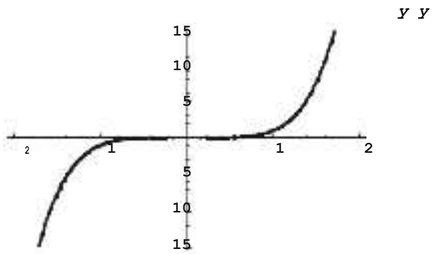
1.2.1 Functions can be defined and represented by a formula, through a graph, via a table, and by using words.

1.2.2 The domain of every polynomial is the set of all real numbers.

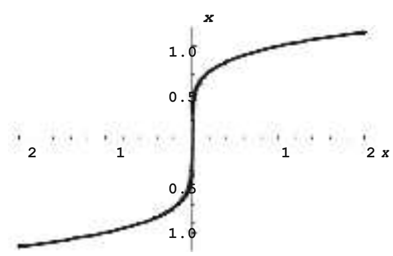
1.2.3 The domain of a rational function $\frac{p(x)}{q(x)}$ is the set of all real numbers for which $q(x) \neq 0$.

1.2.4 A piecewise linear function is one which is linear over intervals in the domain.

1.2.5



1.2.6



1.2.7 Compared to the graph of $f(x)$, the graph of $f(x + 2)$ will be shifted 2 units to the left.

1.2.8 Compared to the graph of $f(x)$, the graph of $-3f(x)$ will be scaled vertically by a factor of 3 and flipped about the x axis.

1.2.9 Compared to the graph of $f(x)$, the graph of $f(3x)$ will be scaled horizontally by a factor of 3.

1.2.10 To produce the graph of $y = 4(x + 3)^2 + 6$ from the graph of x^2, one must

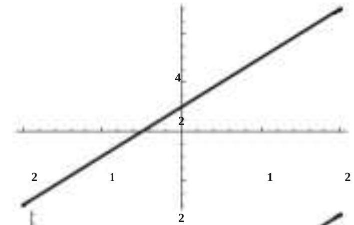
1. shift the graph horizontally by 3 units to left
2. scale the graph vertically by a factor of 4
3. shift the graph vertically up 6 units.

1.2.11 The slope of the line shown is 3. The y-intercept is $b = -1$. Thus the function is given by $f(x) = (-2/3)x - 1$.

1.2.12 The slope of the line shown is 5. The y-intercept is $b = 5$. Thus the function is given by $f(x) = (-4/5)x + 5$. **1.2.13**

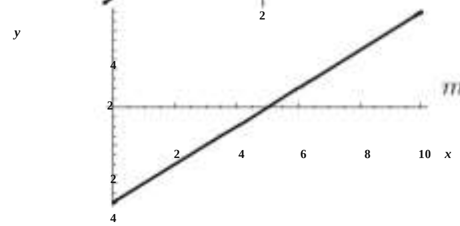
y

The slope is given by = 2, so the equation of the line is $y - 3 = 2(x - -1)$, which can be written as $y = 2x - 2 + 3$, or $y = 2x + 1$.



1.2.14

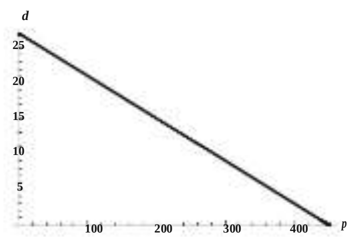
The slope is given by $\frac{0-(-3)}{5-2} = 1$, so the equation of the line is $y - 0 = 1(x - 5)$, or $y = x - 5$.



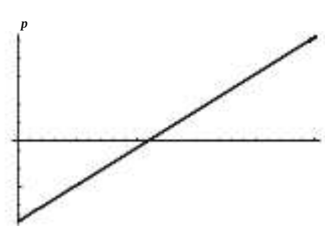
$m = \frac{15-12}{200-250} = -\frac{3}{50}$

1.2.15

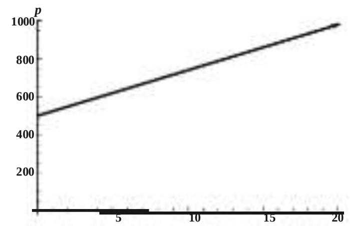
Using price as the independent variable p and the average number of units sold per day as the dependent variable d, we have the ordered pairs (250,12) and (200,15). The slope of the line determined by these points is. Thus the demand function has the form $d(p) = (-3/50)p + b$ for some constant b. Using the point (200, 15), we find that $15 = (-3/50) \cdot 200 + b$, so $b = 27$. Thus the demand function is $d = (-3/50)p + 27$. While the domain of this linear function is the set of all real numbers, the formula is only likely to be valid for some subset of the interval (0,450), because outside of that interval either $p \le 0$ or $d \le 0$.



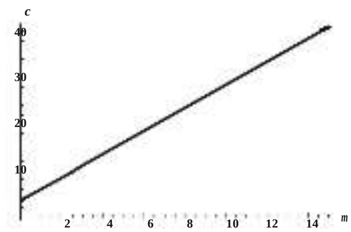
1.2.16 The profit is given by $p = f(n) = 8n - 175$. The break-even point is when $p = 0$, which occurs when $n = 175/8 = 21.875$, so they need to sell at least 22 tickets to not have a negative profit.



1.2.17 The slope is given by the rate of growth, which is 24. When $t = 0$ (years past 2015), the population is 500, so the point (0,500) satisfies our linear function. Thus the population is given by $p(t) = 24t + 500$. In 2030, we have $t = 15$, so the population will be approximately $p(15) = 360 + 500 = 860$.



1.2.18 The cost per mile is the slope of the desired line, and the intercept is the fixed cost of 3.5. Thus, the cost per mile is given by $c(m) = 2.5m + 3.5$. When $m = 9$, we have $c(9) = (2.5)(9) + 3.5 = 22.5 + 3.5 = 26$ dollars.



1.2.19 For $x < 0$, the graph is a line with slope 1 and y- intercept 3, while for $x > 0$, it is a line with slope $-1/2$ and y-intercept 3. Note that both of these lines contain the point (0,3). The function shown can thus be written

$$f(x) = \begin{cases} x + 3 & \text{if } x < 0; \\ -\frac{1}{2}x + 3 & \text{if } x \geq 0. \end{cases}$$

1.2.20 For $x < 3$, the graph is a line with slope 1 and y- intercept 1, while for $x > 3$, it is a line with slope $-1/3$. The portion to the right thus is represented by $y = (-1/3)x + b$, but because it contains the point (6,1), we must have $1 = (-1/3)(6) + b$ so $b = 3$. The function shown can thus be written

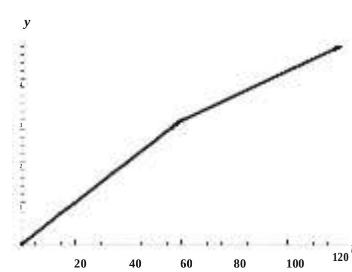
$$f(x) = \begin{cases} x + 1 & \text{if } x < 3; \\ -\frac{1}{3}x + 3 & \text{if } x \geq 3. \end{cases}$$

Note that at $x = 3$ the value of the function is 2, as indicated by our formula.

1.2.21

The cost is given by

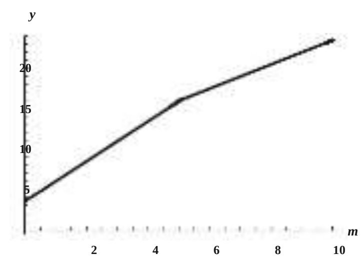
$$c(t) = \begin{cases} 0.05t & \text{for } 0 \leq t \leq 60 \\ 1.2 + 0.03t & \text{for } 60 < t \leq 120 \end{cases}.$$



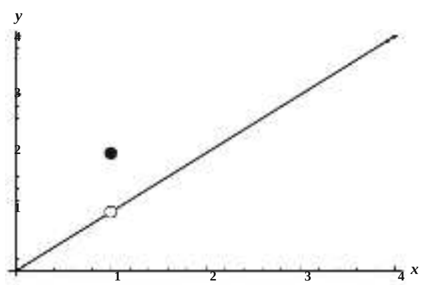
1.2.22

The cost is given by

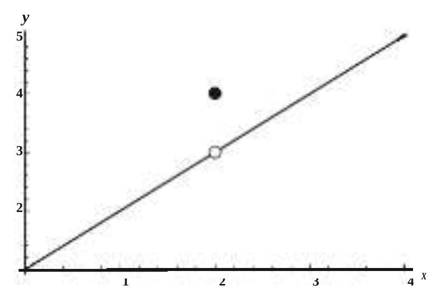
$$c(m) = \begin{cases} 3.5 + 2.5m & \text{for } 0 \le m \le 5 \\ 8.5 + 1.5m & \text{for } m > 5 \end{cases}$$



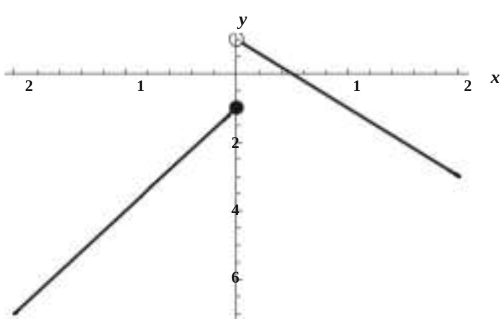
1.2.23



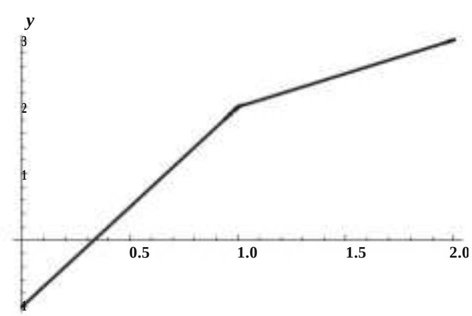
1.2.24



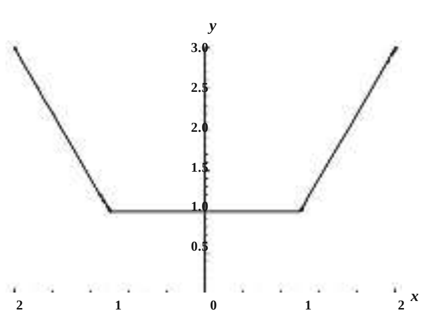
1.2.25



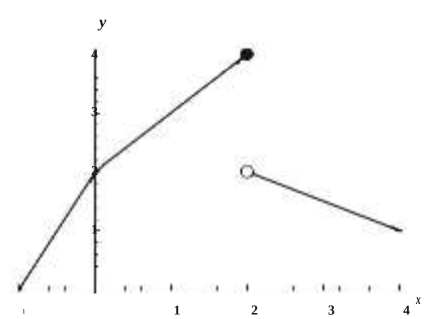
1.2.26



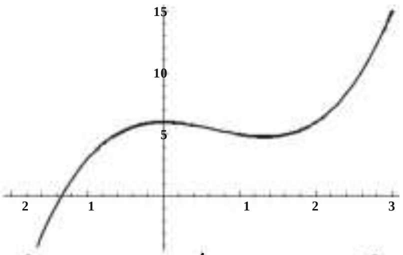
1.2.27



1.2.28



1.2.29



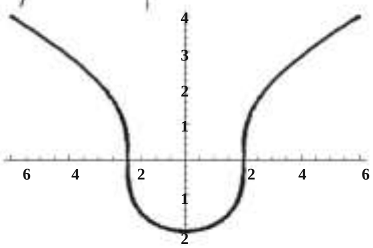
a.

b. The function is a polynomial, so its domain is the setof all real numbers.

c. It has one peak near its y-intercept of (0,6) and one valley between $x = 1$ and $x = 2$. Its x-intercept is x − near $x = 4/3$.

1.2.30

y



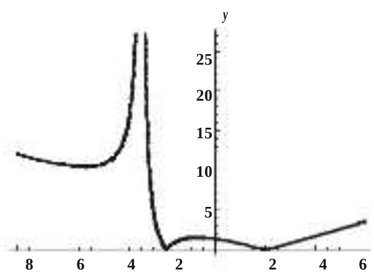
a.

b. The function's domain is the set of all real numbers.

c. It has a valley at the y-intercept of $(0,-2)$, and is very steep at $x = -2$ and $x = 2$ which are the x-intercepts.

x It is symmetric about the y-axis.

1.2.31

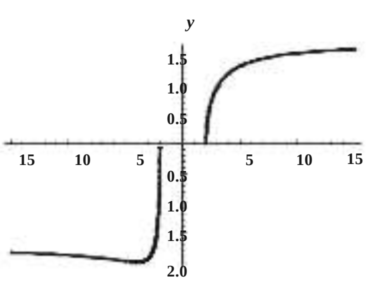


a.

b. The domain of the function is the set of all real num-bers except −3.

c. There is a valley near $x = -5.2$ and a peak near $x = -0.8$. The x-intercepts are at −2 and 2, where the curve does not appear to be smooth. There is a vertical asymptote at $x = -3$. The function is never *x* below the x-axis. The y-intercept is (0,4/3).

1.2.32



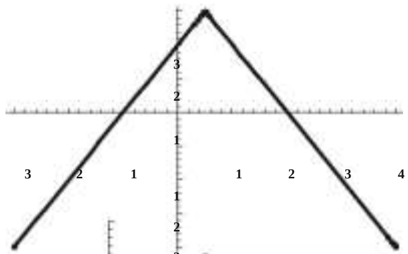
a.

b. The domain of the function is $(-\infty,-2] \cup [2,\infty)$

x c. x-intercepts are at −2 and 2. Because 0 isn't in the domain, there is no y-intercept. The function has a valley at $x = -4$.

1.2.33

y

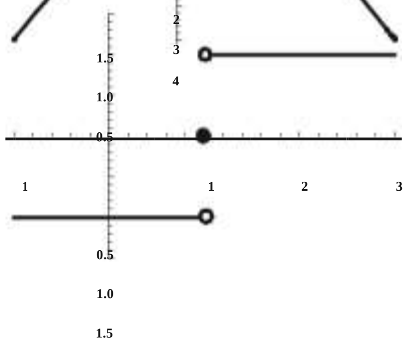


a.

b. The domain of the function is $(-\infty,\infty)$

c. The function has a maximum of 3 at $x = 1/2$, and a y-intercept of 2.

1.2.34



a.

b. The domain of the function is $(-\infty,\infty)$

c. The function contains a jump at $x = 1$. The maximum value of the function is 1 and the minimum value is -1.

1.2.35 The slope of this line is constantly 2, so the slope function is $s(x) = 2$.

1.2.36 The function can be written as $x = \begin{cases} -x & \text{if } x \le 0 \\ x & \text{if } x > 0 \end{cases}$.

The slope function is $s(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$.

1.2.37 The slope function is given by $s(x) = \begin{cases} 1 & \text{if } x < 0; \\ -1/2 & \text{if } x > 0. \end{cases}$

1.2.38 The slope function is given by $s(x) = \begin{cases} 1 & \text{if } x < 3; \\ -1/3 & \text{if } x > 3. \end{cases}$

1.2.39

a. Because the area under consideration is that of a rectangle with base 2 and height 6, $A(2) = 12$.

b. Because the area under consideration is that of a rectangle with base 6 and height 6, $A(6) = 36$.

c. Because the area under consideration is that of a rectangle with base x and height 6, $A(x) = 6x$.

1.2.40

a. Because the area under consideration is that of a triangle with base 2 and height 1, $A(2) = 1$.

b. Because the area under consideration is that of a triangle with base 6 and height 3, the $A(6) = 9$.

c. Because $A(x)$ represents the area of a triangle with base x and height $(1/2)x$, the formula for $A(x)$ is $\frac{1}{2} \cdot x \cdot \frac{x}{2} = \frac{x^2}{4}$.

1.2.41

a. Because the area under consideration is that of a trapezoid with base 2 and heights 8 and 4, we have $A(2) = 2 \cdot \frac{8+4}{2} = 12.$

b. Note that $A(3)$ represents the area of a trapezoid with base 3 and heights 8 and 2, so $A(3) = 3 \cdot \frac{8+2}{2} = 15.$ So (3)), and $A(6)-A(3)$ represents the area of a triangle with base 3 and height 2. Thus

$$A(6) = 15 + (A(6) - A$$
$$A(6) = 15 + 6 = 21.$$

c. For x between 0 and 3, $A(x)$ represents the area of a trapezoid with base x, and heights 8 and $8 - 2x$. Thus the area is $x \cdot \frac{8+8-2x}{2} = 8x - x^2$. For $x > 3$, $A(x) = A(3)+A(x)-A(3) = 15+2(x-3) = 2x+9$. Thus

$$A(x) = \begin{cases} 8x - x^2 & \text{if } 0 \le x \le 3; \\ 2x + 9 & \text{if } x > 3. \end{cases}$$

1.2.42

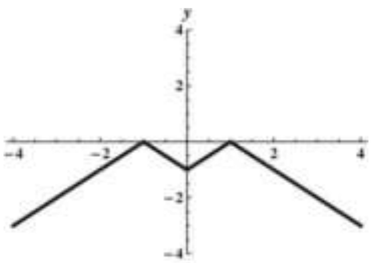
a. Because the area under consideration is that of trapezoid with base 2 and heights 3 and 1, we have $A(2) = 2 \cdot \frac{3+1}{2} = 4.$
b. Note that $A(6) = A(2)+(A(6)-A(2)$, and that $A(6)-A(2)$ represents a trapezoid with base $6-2 = 4$ and heights 1 and 5. The area is thus $4 + \left(4 \cdot \frac{1+5}{2}\right) = 4 + 12 = 16.$
c. For x between 0 and 2, $A(x)$ represents the area of a trapezoid with base x, and heights 3 and $3 - x$.

Thus the area is $x \cdot \frac{3+3-x}{2} = 3x - \frac{x^2}{2}$. For $x > 2$, $A(x) = A(2)+A(x)-A(2) = 4+(A(x)-A(2))$. Note that $A(x) - A(2)$ represents the area of a trapezoid with base $x - 2$ and heights 1 and $x - 1$. Thus $A(x) = 4 + (x-2) \cdot \frac{1+x-1}{2} = 4 + (x-2)\left(\frac{x}{2}\right) = \frac{x^2}{2} - x + 4$. Thus

$$A(x) = \begin{cases} 3x - \frac{x^2}{2} & \text{if } 0 \le x \le 2; \\ \frac{x^2}{2} - x + 4 & \text{if } x > 2. \end{cases}$$

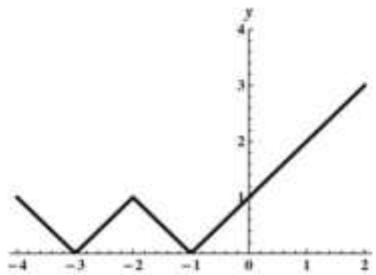
1.2.43 $f(x) = |x-2|+3$, because the graph of f is obtained from that of $|x|$ by shifting 2 units to the right and 3 units up.

$g(x) = -|x + 2| - 1$, because the graph of g is obtained from the graph of $|x|$ by shifting 2 units to the left, then reflecting about the x-axis, and then shifting 1 unit down. **1.2.44**

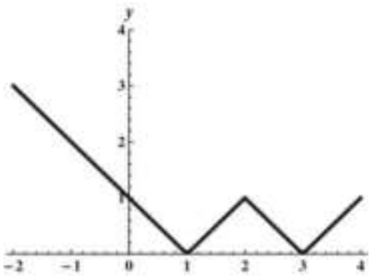


x

a.b.

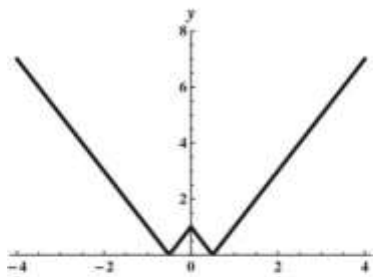


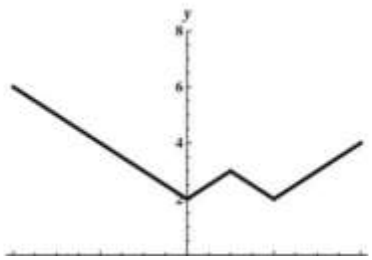
x



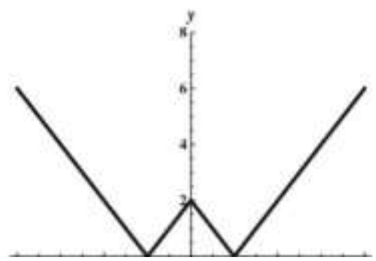
xx

c.d.





xx

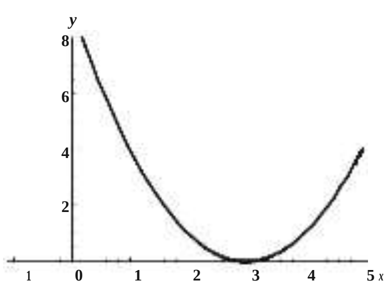


e. 4
2
0
2
4

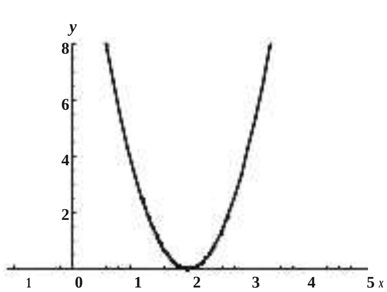
f. 4 2 0 2 4

1.2.45

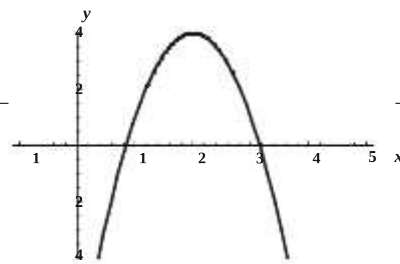
a. Shift 3 units to the right.



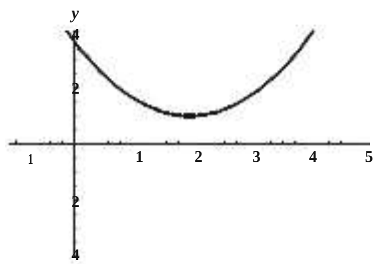
b. Horizontal scaling by a factor of 2, then shift 2 units to the right.



c. Shift to the right 2 units, vertical scaling by a factor of 3 and flip, shift up 4 units.

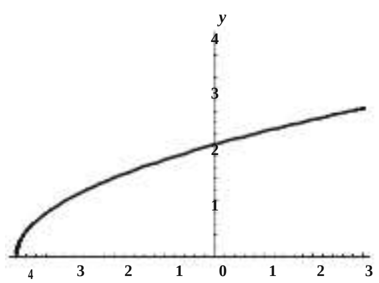


d. Horizontal scaling by a factor of $\frac{1}{3}$, horizontal shift right 2 units, vertical scaling by a factor of 6, vertical shift up 1 unit.

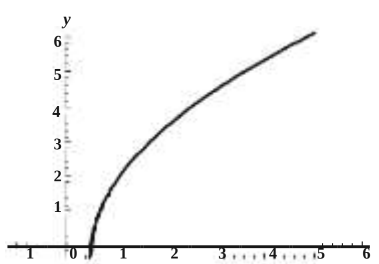


1.2.46

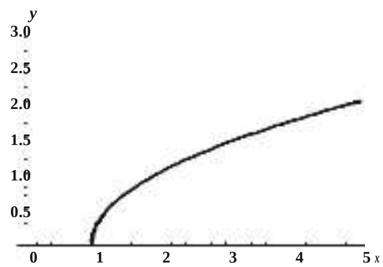
a. Shift 4 units to the left.



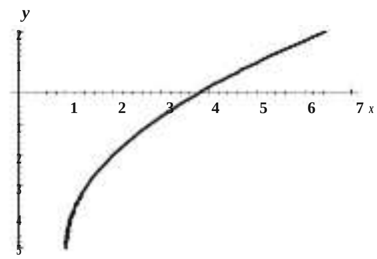
b. Horizontal scaling by a factor of 2, shift $\frac{1}{2}$ unit to the right, vertical scaling by a factor of 2.



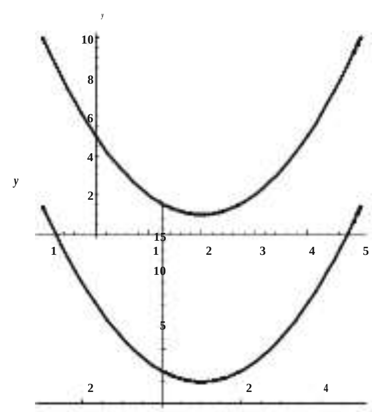
c. Shift 1 unit to the right.



d. Shift 1 unit to the right, vertical scaling by a factor of 3, vertical shift down 5 units.

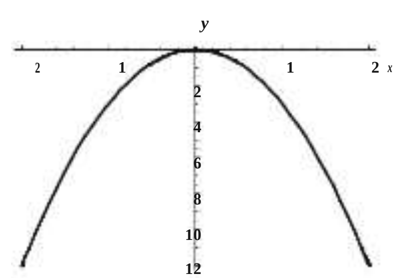


1.2.47 The graph is obtained by shifting the graph of x^2 two units to the right and one unit up.



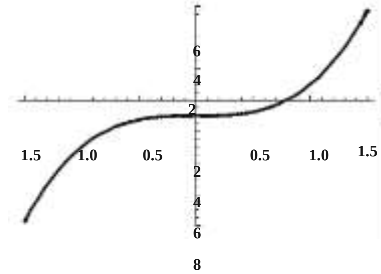
1.2.48 Write $x^2 - 2x + 3$ as $(x^2 - 2x + 1) + 2 = (x-1)^2 + 2$. The graph is obtained by shifting the graph of x^2 one unit to the right and two units up.

1.2.49 This function is $-3f(x)$ where $f(x) = x^2$. Vertically scale the graph of f by a factor of 3 and then flip.

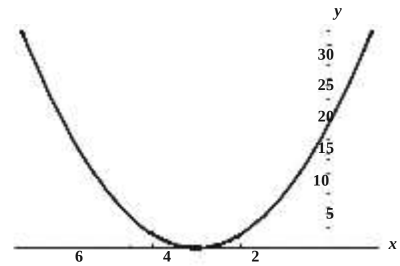


y

1.2.50 This function is $2f(x) - 1$ where $f(x) = x^3$. Vertically scale the graph of f by a factor of 2 and then vertically shift down 1 unit.

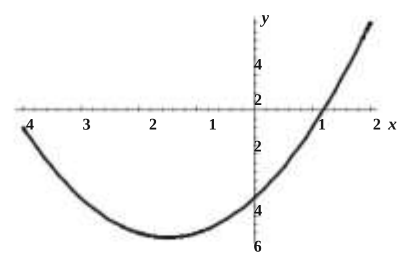


1.2.51 This function is $2f(x + 3)$ where $f(x) = x^2$. Vertically scale the graph of f by a factor of 2 and then shift left 3 units.

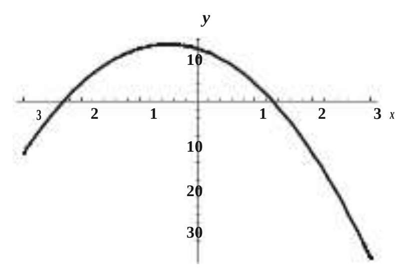


1.2.52. By completing the square, we have that $p(x) = (x^2 + 3x + (9/4)) - (29/4) = (x + (3/2))^2 - (29/4)$. So it is $f(x + (3/2)) - (29/4)$ where $f(x) = x^2$.

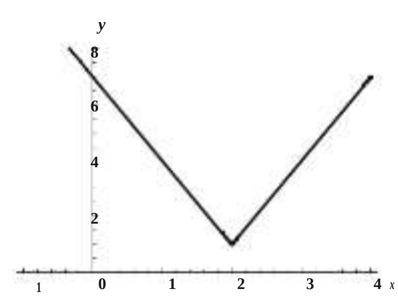
Shift the graph of f 3/2 units to the left and then down 29/4 units.



1.2.53 By completing the square, we have that $h(x) = -4(x^2 + x - 3) = -4\left(x^2 + x + \frac{1}{4} - \frac{1}{4} - 3\right) = -4(x + (1/2))^2 + 13$. So it is $-4f(x + (1/2)) + 13$ where $f(x) = x^2$. Vertically scale the graph of f by a factor of 4, then reflect about the x-axis, then shift left 1/2 unit, and then up 13 units.



1.2.54 Because $|3x-6|+1 = 3|x-2|+1$, this is $3f(x-2)+1$ where $f(x) = |x|$. Shift the graph of f 2 units to the right, vertically scale by a factor of 3, and then shift 1 unit up.



1.2.55

a. True. A polynomial $p(x)$ can be written as the ratio of polynomials $\frac{p(x)}{1}$, so it is a rational function. However, a rational function like $\frac{1}{x}$ is not a polynomial.

b. False. For example, if $f(x) = 2x$, then $(f \circ f)(x) = f(f(x)) = f(2x) = 4x$ is linear, not quadratic.

c. True. In fact, if f is degree m and g is degree n, then the degree of the composition of f and g is $m \cdot n$, regardless of the order they are composed.

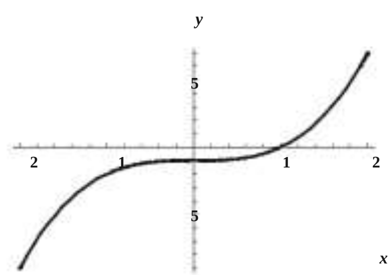
d. False. The graph would be shifted two units to the left.

1.2.56 The points of intersection are found by solving $x^2 + 2 = x + 4$. This yields the quadratic equation $x^2 - x - 2 = 0$ or $(x - 2)(x + 1) = 0$. So the x-values of the points of intersection are 2 and −1. The actual points of intersection are (2,6) and (−1,3).

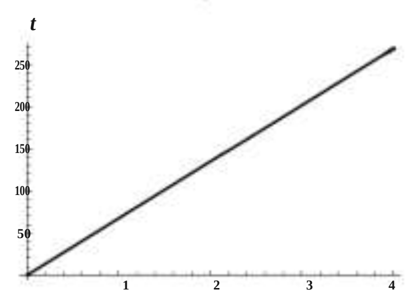
1.2.57 The points of intersection are found by solving $x^2 = -x^2 + 8x$. This yields the quadratic equation $2x^2 - 8x = 0$ or $(2x)(x-4) = 0$. So the x-values of the points of intersection are 0 and 4. The actual points of intersection are (0,0) and (4,16).

1.2.58 $y = x + 1$, because the y value is always 1 more than the x value.

1.2.59 $y = \sqrt{x} - 1$, because the y value is always 1 less than the square root of the x value.



1.2.60 $y = x^3 - 1$. The domain is $(-\infty,\infty)$.

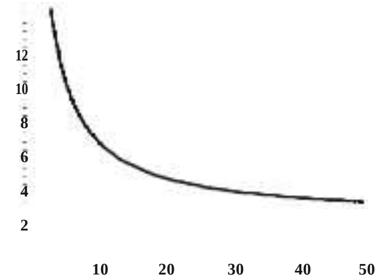


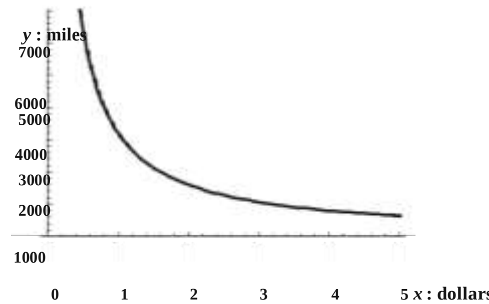
1.2.61 The car moving north has gone $30t$ miles after t hours and the car moving east has gone $60t$ miles. Using the Pythagorean theorem, we have $s(t) = \sqrt{(30t)^2 + (60t)^2} = \sqrt{900t^2 + 3600t^2} = \sqrt{4500t^2} = 30\sqrt{5}t$ miles. The context domain could be [0,4].

y

$y = \frac{50}{x}$. Theoretically the domain is $(0,\infty)$, but

1.2.62 the world record for the “hour ride” is just short of 50 miles.





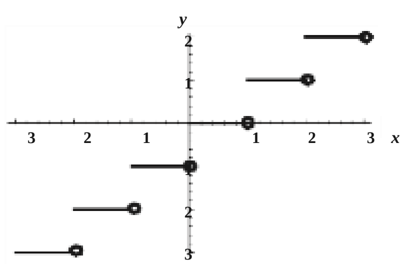
$y = \frac{3200}{x}$. Note that x 32 dollars per gallon miles per gallon $\cdot\, y$ miles would represent the numbers of dollars, so this

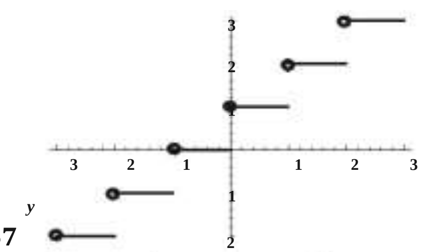
1.2.63 must be 100. So we have $\frac{xy}{32} = 100$, or $y = \frac{3200}{x}$.

We certainly have $x > 0$, and a reasonable upper bound to imagine for x is \$5 (let’s hope), so the context domain is $(0,5]$.

1.2.64

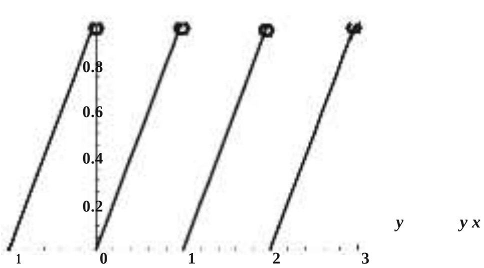
1.2.65

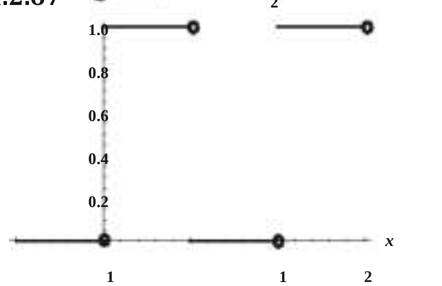




1.2.66

1.2.67

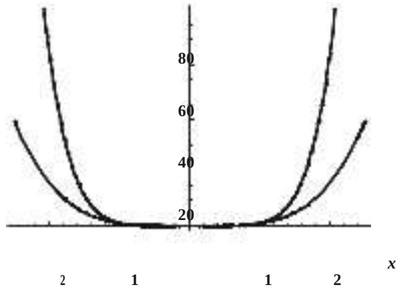


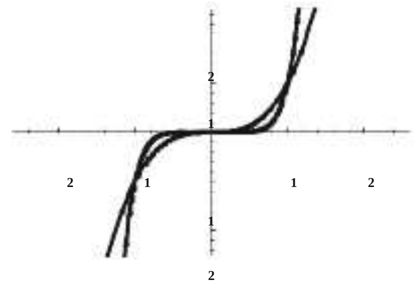


1.2.68

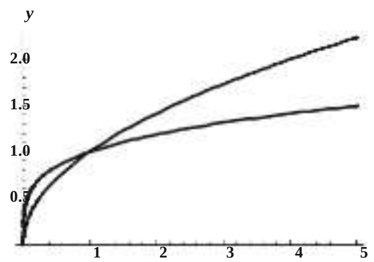
1.2.69

y y





1.2.70



1.2.71

a. The zeros of *f* are the points where the graph crosses the *x*-axis, so these are points *A*, *D*, *F*, and *I.*

b. The only high point, or peak, of *f* occurs at point *E*, because it appears that the graph has larger and larger *y* values as *x* increases past point *I* and decreases past point *A*.

c. The only low points, or valleys, of *f* are at points *B* and *H*, again assuming that the graph of *f* continues its apparent behavior for larger values of *x*.

d. Past point *H*, the graph is rising, and is rising faster and faster as *x* increases. It is also rising between points *B* and *E*, but not as quickly as it is past point *H*. So the marked point at which it is rising most rapidly is *I*.

e. Before point *B*, the graph is falling, and falls more and more rapidly as *x* becomes more and more negative. It is also falling between points *E* and *H*, but not as rapidly as it is before point *B*. So the marked point at which it is falling most rapidly is *A*.

1.2.72

a. The zeros of g appear to be at $x = 0$, $x = 1$, $x = 1.6$, and $x \approx 3.15$.

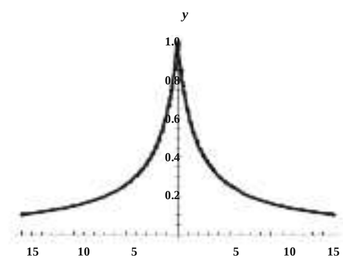
b. The two peaks of g appear to be at $x \approx 0.5$ and $x \approx 2.6$, with corresponding points $\approx (0.5, 0.4)$ and $\approx (2.6, 3.4)$.

c. The only valley of g is at $\approx (1.3, -0.2)$.

d. Moving right from $x \approx 1.3$, the graph is rising more and more rapidly until about $x = 2$, at which point it starts rising less rapidly (because, by $x \approx 2.6$, it is not rising at all). So the coordinates of the point at which it is rising most rapidly are approximately $(2.1, g(2)) \approx (2.1, 2)$. Note that while the curve is also rising between $x = 0$ and $x \approx 0.5$, it is not rising as rapidly as it is near $x = 2$.

e. To the right of $x \approx 2.6$, the curve is falling, and falling more and more rapidly as x increases. So the point at which it is falling most rapidly in the interval [0,3] is at $x = 3$, which has the approximate coordinates (3,1.4). Note that while the curve is also falling between $x \approx 0.5$ and $x \approx 1.3$, it is not falling as rapidly as it is near $x = 3$.

1.2.73

a. 

b. This appears to have a maximum when $\theta = 0$. Our vision is sharpest when we look straight ahead.

c. For $|\theta| \leq .19^\circ$. We have an extremely narrow range where our eyesight is sharp.

1.2.74

a. $f(.75) = \frac{.75^2}{1-2(.75)(.25)} = .9$. There is a 90% chance that the server will win from deuce if they win 75% of their service points.

b. $f(.25) = \frac{.25^2}{1-2(.25)(.75)} = .1$. There is a 10% chance that the server will win from deuce if they win 25% of their service points.

1.2.75

a. Using the points (1986,1875) and (2000,6471) we see that the slope is about 328.3. At $t = 0$, the value of p is 1875. Therefore a line which reasonably approximates the data is $p(t) = 328.3t + 1875$.

b. Using this line, we have that $p(9) = 4830$.

1.2.76

a. We know that the points (32,0) and (212,100) are on our line. The slope of our line is thus $\frac{100-0}{212-32} = \frac{100}{180} = \frac{5}{9}$. The function $f(F)$ thus has the form $C = (5/9)F + b$, and using the point (32,0) we see that $0 = (5/9)32 + b$, so $b = -(160/9)$. Thus $C = (5/9)F - (160/9)$

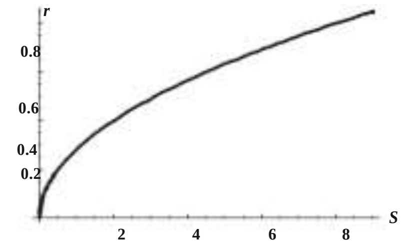
b. Solving the system of equations $C = (5/9)F - (160/9)$ and $C = F$, we have that $F = (5/9)F - (160/9)$, so $(4/9)F = -160/9$, so $F = -40$ when $C = -40$.

1.2.77

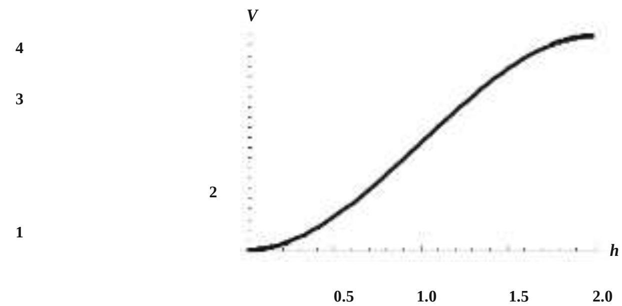
a. Because you are paying \$350 per month, the amount paid after m months is $y = 350m + 1200$.

b. After 4 years (48 months) you have paid $350 \cdot 48 + 1200 = 18000$ dollars. If you then buy the car for \$10,000, you will have paid a total of \$28,000 for the car instead of \$25,000. So you should buy the car instead of leasing it.

1.2.78 Because $S = 4\pi r^2$, we have that $r^2 = \frac{S}{4\pi}$, so $|r| = \frac{\sqrt{S}}{2\sqrt{\pi}}$, but because r is positive, we can write $r = \frac{\sqrt{S}}{2\sqrt{\pi}}$.

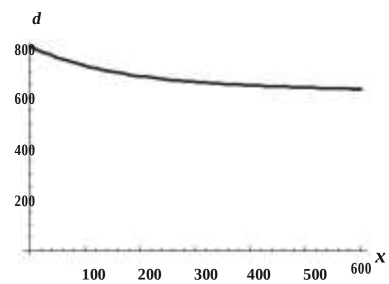


1.2.79 The function makes sense for $0 \le h \le 2$.

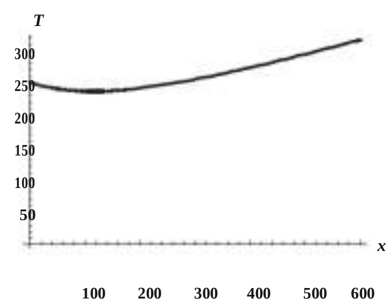


1.2.80

a. Note that the island, the point P on shore, and the point down shore x units from P form a right triangle. By the Pythagorean theorem, the length of the hypotenuse is $\sqrt{40000 + x^2}$. So Kelly must row this distance and then jog $600 - x$ meters to get home. So her total distance $d(x) = \sqrt{40000 + x^2} + (600 - x)$.



b. Because distance is rate times time, we have that time is distance divided by rate. Thus $T(x) = \frac{\sqrt{40000+x^2}}{2} + \frac{600-x}{4}$.



c. By inspection, it looks as though she should head to a point about 115 meters down shore from P. This would lead to a time of about 236.6 seconds.