Solution Manual for Brief Applied Calculus 7th Edition Berresford by Berresford Rockett ISBN 1305085329 9781305085329

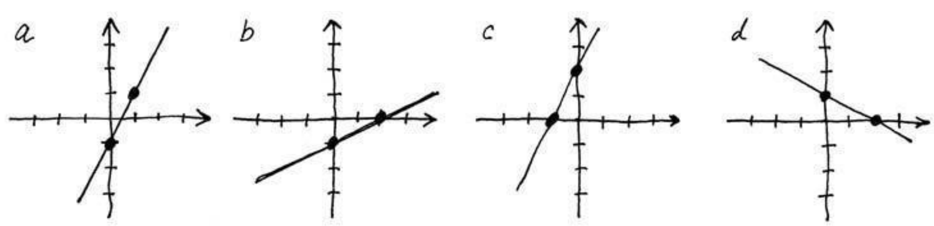
DIAGNOSTIC TEST

Are you ready to study calculus?

Algebra is the language in which we express the ideas of calculus. Therefore, to understand calculus and express its ideas with precision, you need to know some algebra.

If you are comfortable with the algebra covered in the following problems, you are ready to begin your study of calculus. If not, turn to the Algebra Appendix beginning on page A.xxx and review the Complete Solutions to these problems, and continue reading the other parts of the Appendix that cover anything that you do not know.

Problems	Answers
1. True or False? $\frac{1}{2} < 3$	False
2. Express $\{x \mid 4 < x \quad 5\}$ in interval notation.	4, 5] (
3. What is the slope of the line through the points (6, 7) and (9, 8)?	5
4. On the line $y = 3x + 4$, what value of y corresponds to $x = 2$?	6
5. Which sketch shows the graph of the line $y = 2x \quad 1$?	a



True or False? $x^{-2} = y_2$ — yx √p | True

7. Find the zeros of the function $f(x) = 9x^2 \quad 6x \quad 1$ — $x = \pm 1$ 2 3 p

8. Expand and simplify $x(8 \quad x) \quad (3x + 7)$. — 7 ^{2} + 5x x

9. What is the domain of $f(x) = \frac{x^2 \quad 3x + 2}{x^3 + x^2 \quad 6x}$? — } , x= 26 3, x= 06 x=6 | x{

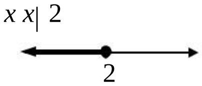
10. For $f(x) = x^2 \quad 5x$, find the di↵erence quotient $\frac{f(x+h) \quad f(x)}{h}$ — 5 + 2hx

Chapter 1: Functions

EXERCISES 1.1

x 0| x 6

x x| 2


2

a. Since x = 3 and m = 5, then y, the change in y, is

y = 3 • m = 3 • 5 = 15

Since x = –2 and m = 5, then y, the change in y, is

y = –2 • m = –2 • 5 = –10

For (2, 3) and (4, –1), the slope is

$\frac{1\ 3}{4\ 2}\quad\frac{4}{2}\ 2$

For (–4, 0) and (2, 2), the slope is

$\frac{2\ 0}{2\ (4)}\quad\frac{2}{2\ 4}\quad\frac{2}{6}\quad\frac{1}{3}$

For (0, –1) and (4, –1), the slope is

$\frac{1(1)}{4\ 0}\quad\frac{1\ 1}{4}\quad\frac{0}{4}\ 0$

For (2, –1) and (2, 5), the slope is

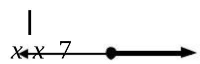
$\frac{5\ (1)}{2\ 2}\quad\frac{5\ 1}{0}$ undefined

Since $y = 3x - 4$ is in slope-intercept form, m = 3 and the y-intercept is (0, –4). Using the slope m = 3, we see that the point 1 unit to the right and 3 units up is also on the line.

x 3 x 5


–3 5

4. x x 7


7

a. Since x = 5 and m = –2, then y, the change in y, is

y = 5 • m = 5 • (–2) = –10

Since x = –4 and m = –2, then y, the change in y, is

y = –4 • m = –4 • (–2) = 8

For (3, –1) and (5, 7), the slope is

$\frac{7\ (1)}{5\ 3}\quad\frac{7\ 1}{2}\quad\frac{8}{2}\ 4$

For (–1, 4) and (5, 1), the slope is

$\frac{1\ 4}{5\ (1)}\quad\frac{3}{5\ 1}\quad\frac{3}{6}\quad\frac{1}{2}$

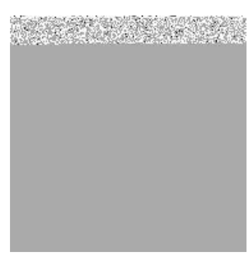
12. For 2, $\frac{1}{2}$ and 5, $\frac{1}{2}$, the slope is

$\frac{\frac{1}{2}\ \frac{1}{2}}{5\ (2)}\quad\frac{0}{5\ 2}\quad\frac{0}{7}\quad 0$

For (6, –4) and (6, –3), the slope is

$\frac{3(4)}{6\ 6}\quad\frac{3\ 4}{0}$ undefined

Since $y = 2x$ is in slope-intercept form, m = 2 and the y-intercept is (0, 0). Using m = 2, we see that the point 1 to the right and 2 units up is also on the line.

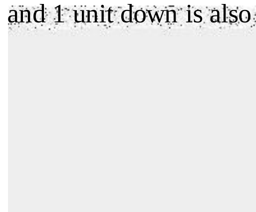


17. Since $y = \frac{1}{2}x$ is in slope-intercept form,

$m = \frac{1}{2}$ and the y-intercept is (0, 0). Using $m = \frac{1}{2}$, we see that the point 2 units to the right and 1 unit down is also on the line.

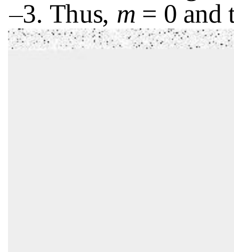
18. Since $y = \frac{1}{3}x + 2$ is in slope-intercept form,

$m = \frac{1}{3}$ and the y-intercept is (0, 2). Using $m = \frac{1}{3}$, we see that the point 3 units to the right and 1 unit down is also on the line.



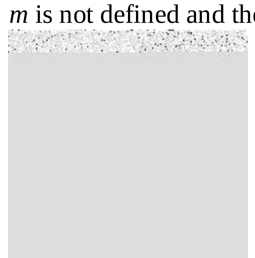
The equation $y = 4$ is the equation of the hori-zontal line through all points with y-coordinate 4. Thus, $m = 0$ and the y-intercept is (0, 4).

The equation $y = –3$ is the equation of the hori-zontal line through all points with y-coordinate –3. Thus, $m = 0$ and the y-intercept is (0, –3).



The equation $x = 4$ is the equation of the vertical line through all points with x-coordinate 4. Thus, m is not defined and there is no y-intercept.

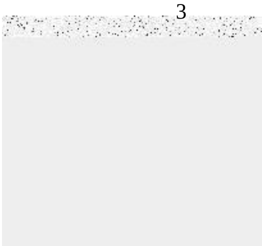
The equation $x = –3$ is the equation of the vertical line through all points with x-coordinate –3. Thus, m is not defined and there is no y-intercept.



23. First, solve for y:

$$2x \quad 3y \quad 12$$
$$3y \quad 2x \quad 12$$
$$y \quad \frac{2}{3}x \quad 4$$

Therefore, $m = \frac{2}{3}$ and the y-intercept is (0, –4).



24. First, solve for y:

$$3x \quad 2y \quad 18$$
$$2y \quad 3x \quad 18$$
$$y \quad \frac{3}{2}x \quad 9$$

Therefore, $m = \frac{3}{2}$ and the y-intercept is (0, 9).

First, solve for y:

$y \;\; 0$

x

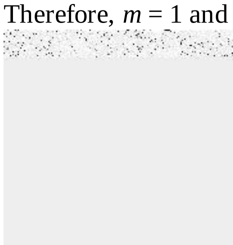
Therefore, m = –1 and the y-intercept is (0, 0).

First, solve for y:

$y\ 0\ y\ x$

$y \quad x$

Therefore, m = 1 and the y-intercept is (0, 0).



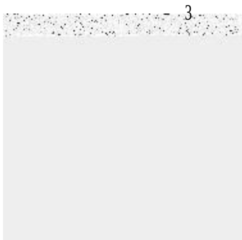
First, put the equation in slope-intercept form:

$x \;\; \underline{2}$

$y \;\; 3$

$y \;\; \frac{1}{3} \; x \;\; \frac{2}{3}$

Therefore, m = $\frac{1}{3}$ and the y-intercept is $\left(0, \frac{2}{3}\right)$.



First, solve for y:

$2\,y \;\; 4$

$2\,y \qquad x \;\; 4$

$y \quad - \quad \frac{1}{2}\,x \;\; 2$

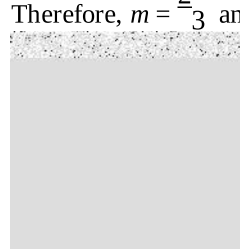
Therefore, m = $\frac{1}{2}$ and the y-intercept is (0, –2).

First, put the equation in slope-intercept form:

$y \;\; \frac{2}{3}\,x\ 3$

$y \;\; -\frac{2}{3}\,x \;\; 2$

Therefore, m = $\frac{2}{3}$ and the y-intercept is (0, –2).



First, solve for y:

$\_x \;\; \_y$

$2 \quad 3 \quad 1$

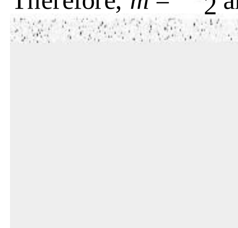
$\_y1$

$3 \quad 2 \; x \; 1 \; y$

$\underline{3}$

$2\,x\,3$

Therefore, m = $\frac{3}{2}$ and the y-intercept is (0, 3).



First, solve for y:

$\frac{2}{3}x$

$y \; 1$

$y \; \frac{2}{3} x \; 1$

$y \; \frac{2}{3}x \; 1$

2

Therefore, $m = \frac{2}{3}$ and the y-intercept is (0, –1).

First, solve for y:

$y \; 1$

$2 \quad 1$

$1 \; y \; 1 \; 2 \; x \; 1$

$y \; 2 \; 2$

x

Therefore, m = –1 and the y-intercept is (0, 0).

y = –2.25x + 3

$y \; 2 \; 5 \; x \; 1 \; y \; 2 \; 5 \; x \; 5$

$y \; 5 \; x \; 3$

y = –4

x = 1.5

$y \; \frac{2}{3} x \; 8$

$y \; 3 \; 1 \; x \; 4 \; y \; 3 \; x \; 4$

$y x \; 7$

$y \; \frac{3}{4}$

$x \; \frac{1}{2}$

First, find the slope.

$m \quad \frac{1 \; 3}{7 \; 5} \quad \frac{4}{2} \quad 2$

Then use the point-slope formula with this slope and the point (5, 3).

$y \; 3 \quad 2 \; x \quad 5$

$y \quad 3 \; 2 \; x \quad 10$

$y \quad 2 \; x \quad 13$

42. First, find the slope.

$m \quad \frac{0 \quad 1}{6 \; 3} \quad \frac{1}{3}$

Then use the point-slope formula with this slope and the point (6, 0).

$y \quad 0 \quad \frac{1}{3} x \; 6$

$y \quad \frac{1}{3} x \; 2$

43. First, find the slope.

$m \quad \frac{1 \quad 1}{5 \; 1} \quad \frac{1 \; 1}{4} \quad 0$

Then use the point-slope formula with this slope and the point (1,–1).

$y \quad 1 \quad 0 \; x \quad 1$

$1 \; 0 \; y$

1

First, find the slope.

$m \quad \frac{4 \; 0}{2 \; 2} \quad \frac{4}{0}$ undefined

Since the slope of the line is undefined, the line is a vertical line. Because the x-coordinates of the points are 2, the equation is x = 2.

a. First find the slope of the line $4y\ \ 3x\ \ 5$.

Write the equation in slope-intercept form.

$$y\ \ \frac{3}{4}x\ \frac{5}{4}$$

The slope of the parallel line is $m\ \ \frac{3}{4}$.

Next, use the point-slope form with the point (12, 2):

$$y\ \ y_1\ \ \ m\ x\ \ \ x_1$$
$$y\ \ 2\ \ \ \frac{3}{4}\ x\ \ 12$$

The slope of the line perpendicular to

$y\ \frac{3}{4}x\ \frac{5}{4}$ is $m\ \frac{4}{3}$.

Next, use the point-slope form with the point (12, 2):

$$y\ \ y_1\ \ m\ x\ \ \ x_1$$
$$y\ \ 2\ \ \frac{4}{3}\ x\ \ 12$$

46. a. First find the slope of the line $x\ \ 3y\ \ 7$.

Write the equation in slope-intercept form.

$$y\ \ \frac{1}{3}x\ \frac{7}{3}.$$

The slope of the parallel line is $m\ \ \frac{1}{3}$.

Next, use the point-slope form with the point (–6, 5):

$$y\ \ y_1\ \ \ m\ x\ \ \ x_1$$
$$y\ 5\ \frac{1}{3}\ x\ \ 6$$

b. The slope of the line perpendicular to

$y\ \ \frac{1}{3}x\ \frac{7}{3}$ is $m\ \ 3$.

Next, use the point-slope form with the point (–6, 5):

$$y\ \ y_1\ \ \ m\ x\ \ \ x_1$$
$$y\ 5\ \ \ 3\ x\ \ \ 6$$
$$y\ \ 3x\ \ \ 23$$

The y-intercept of the line is (0, 1), and y = –2 for x = 1. Thus, $m\ \frac{y}{x}\ \frac{2}{1}\ \ 2$. Now, use the slope-intercept form of the line:
y = –2x + 1.

The y-intercept is (0, –2), and y = 3 for x = 2. Thus, $m\ \frac{y}{x}\ \frac{3}{2}$. Now, use the slope-intercept form of the line: $y\ \ \frac{3}{2}x\ 2$

The y-intercept of the line is (0, –2), and y = 3 for x = 1. Thus, $m\ \frac{y}{x}\ \frac{3}{1}\ 3$. Now, use the slope-intercept form of the line: y = 3x – 2

The y-intercept is (0, 1), and y = –2 for x = 3. Thus, $m\ \frac{y}{x}\ \frac{2}{3}\ \ \frac{2}{3}$. Now, use the slope-intercept form of the line: $y\ \ \frac{2}{3}x\ 1$

51. First, consider the line through the points (0, 5) and (5, 0). The slope of this line is $m\ \frac{0\ 5}{5\ 0}\ \frac{5}{5}\ 1$. Since (0, 5) is the y-intercept of this line, use the slope-intercept form of the line: y = –1x + 5 or y = –x + 5.
Now consider the line through the points (5, 0) and (0, –5). The slope of this line is $m\ \frac{5\ 0}{0\ 5}\ \frac{5}{5}\ 1$. Since (0,–5) is the y-intercept of the line, use the slope-intercept form of the line: y = 1x – 5 or y = x – 5
Next, consider the line through the points (0, –5) and (–5, 0). The slope of this line is $m\ \frac{0\ 5}{5\ 0}\ \frac{5}{5}\ 1$. Since (0, –5) is the y-intercept, use the slope-intercept form of the line: y = –1x – 5 or y = –x – 5
Finally, consider the line through the points (–5, 0) and (0, 5). The slope of this line is $m\ \frac{5\ 0}{0\ 5}\ \frac{5}{5}\ 1$. Since (0, 5) is the y-intercept, use the slope-intercept form of the line: y = 1x + 5 or y = x + 5

52. The equation of the vertical line through (5, 0) is x = 5.
~~The equation of the vertical line through (–5, 0)~~ is x = –5.
The equation of the horizontal line through (0, 5) is y = 5.
The equation of the horizontal line through

53. If the point (x_1, y_1) is the y-intercept (0, b), then substituting into the point-slope form of the line gives

$$y\ \ y_1\ \ m\,(x\ \ x_1)$$
$$y\ \ b\ \ m\,(x\ \ 0)$$
$$y\ \ b\ \ mx$$
$$y\ \ mx\ \ b$$

(0, -5) is $y = -5$.

To find the x-intercept, substitute $y = 0$ into the equation and solve for x:

x y 1

x $\frac{}{a}$ $\frac{}{b}$ $\frac{0}{}$ 1

x $\frac{}{a}$ 1 b

x a Thus, $(a, 0)$ is the x-intercept.

To find the y-intercept, substitute $x = 0$ into the equation and solve for y:

x y 1

$\frac{0}{a}$ $\frac{}{b}$ y 1

y a $\frac{}{b}$ 1

$\frac{}{b}$

y b Thus, $(0, b)$ is the y-intercept.

a.

on [–5, 5] by [–5, 5]

Low demand: [0, 8);
average demand: [8, 20);
high demand: [20, 40);
critical demand: [40,)

a.

on [–5, 5] by [–5, 5]

b.

on [–5, 5] by [–5, 5]

b.

on [–5, 5] by [–5, 5]

A: [90, 100]; B: [80,90); C: [70, 80);
D: [60, 70); F: [0, 60)

a. The value of x corresponding to the year 2020 is x = 2020 – 1900 = 120. Substituting x = 120 into the equation for the regression line gives

0.356 x 257.44

y 0.356(120) 257.44 214.72 seconds

Since 3 minutes = 180 seconds, 214.72 = 3 minutes 34.72 seconds. Thus, the world record in the year 2020 will be 3 minutes 34.72 seconds.

To find the year when the record will be 3 minutes 30 seconds, first convert 3 minutes 30 seconds to seconds: 3 minutes 30 seconds = 3 minutes • $\frac{60\text{ sec}}{1\text{ min}}$ + 30 seconds = 210 seconds.

Now substitute y = 210 seconds into the equation for the regression line and solve for x.

y 0 .356 x 257.44

210 0 .356 x 257.44

0.356 x 257.44210

0.356 x 47.44

x $\frac{47.44}{0.356}$ 133.26

Since x represents the number of years after 1900, the year corresponding to this value of x is 1900 + 133.26 = 2033.26 2033. The world record will be 3 minutes 30 seconds in 2033.

60. For x = 720:

y 0 .356x 257.44

0.356 720257.44

256.32 257.44 1.12 seconds

For x = 722:

y 0.356x 257.44

0.356 722 257.44

257.744 257.44 0.408 second

~~These are both unreasonable times for running 1 mile.~~

a. To find the linear equation, first find the slope of the line containing these points.

$m \quad \frac{146 \quad 70}{3 \quad 1} \quad \frac{76}{2} \; 38$

Next, use the point-slope form with the point (1, 70):

$y \quad y_1 \quad m\, x \quad x_1$

$70 \quad 38\, x \quad 1\, y$

$38\, x \quad 32$

Sales are increasing by 38 million units per year.

The sales at the end of 2020 is

y = 38(10) + 32 = 412 million units.

a. First, find the slope of the line containing the points.

$m \quad \frac{42.8 \quad 38.6}{4 \quad 1} \quad \frac{4.2}{3} \; 1.4$

Next, use the point-slope form with the point (1, 38.6):

$m\, x\, x_1\, y$

$38.6 \; 1.4\, x \; 1\, y \; 1.4\, x$

37.2

PCPI increases by about $1400 (or $1.4 thousand) each year.

The value of x corresponding to 2020 is = 2020 – 2008 = 12. Substitute 12 into the equation:

yy = 1.4(12) + 37.2 = $54 thousand

or $54,000

a. First, find the slope of the line containing the points.

$m \quad \frac{212 \quad 32}{100 \quad 0} \quad \frac{180}{100} \quad \frac{9}{5}$

Next, use the point-slope form with the point (0, 32):

$y \quad y_1 \quad m\, x \quad x_1$

$y \quad 32 \quad \frac{9}{5}\, x \quad 0$

$y \quad \frac{9}{5}\, x \quad 32$

b. Substitute 20 into the equation.

$y \quad \frac{9}{5}\, x \quad 32$

$y \quad \frac{9}{5}(20) \quad 32 \quad 36 \quad 32 \quad 68$ F

a. First, find the slope of the line containing the points.

$m \quad \frac{89.8 \quad 74.8}{4 \quad 0} \quad \frac{15}{4} \; 3.75$

Next, use the point-slope form with the point (0, 74.8):

$m\, x \quad x_1$

$y \quad 74.8 \quad 3.75\, x \quad 0\, y$

$3.75\, x \quad 74.8$

Since 2021 is 12 years after 2009, substitute 11 into the equation.

yy

$y \quad 3.75\, x \quad 74.8$

y 3.75(12) 74.8 119.8 thousand dollars or $119,800

65. a. Price = $50,000; useful lifetime = 20 years; scrap value = $6000

$V \quad 50{,}000 \quad \frac{50{,}000 \quad 6000}{20} \quad t \quad 0 \quad t \quad 20$

$50{,}000 \; 2200\, t \quad 0 \quad t \quad 20$

Substitute t = 5 into the equation. V 50,000 2200t

50,000 2200 5

50,000 11,000 $39, 000

on [0, 20] by [0, 50,000]

a. Price = $800,000; useful lifetime = 20 yrs; scrap value = $60,000

$V \quad 800{,}000 \quad \frac{800{,}000 \quad 60{,}000}{20} \quad 0 \quad t \quad 20$

$800{,}000 \quad 37{,}000t \quad 0 \quad t \quad 20$

Substitute t = 10 into the equation. V 800,000 37,000 t

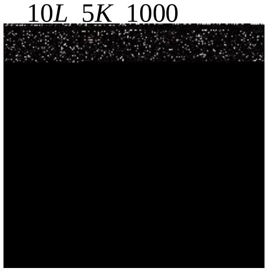
800,000 37,000 10

800,000 370, 000 $430, 000

on [0, 20] by [0, 800,000]

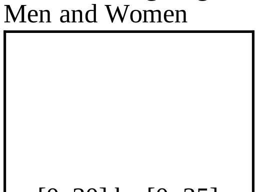
67. **a.** Substitute $w = 10$, $r = 5$, $C = 1000$ into the equation.

10*L* 5*K* 1000



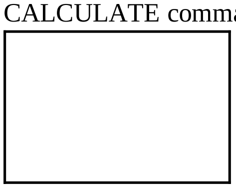
Substitute each pair into the equation. For (100, 0), 10 100 5 0 1000 For (75, 50), 10 75 5 50 1000 For (20, 160), 10 20 5 160 1000 For (0, 200), 10 0 5 200 1000 Every pair gives 1000.

a. Median Marriage Age for Men and Women

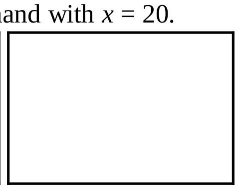


on [0, 30] by [0, 35]

he *x*-value corresponding to the year 2020 is $x = 2020 - 2000 = 20$. The following screens are a result of the CALCULATE command with $x = 20$.



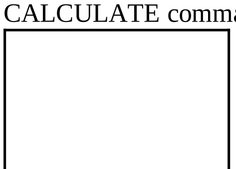
Median Age at Marriage for Men in 2020



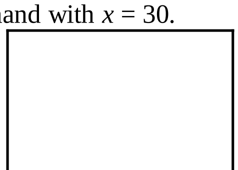
Median Age at Marriage for Women in 2020.

So, the median marriage age for men in 2020 will be 30.3 years and for women it will be 27.8 years.

The *x*-value corresponding to the year 2030 is $x = 2030 - 2000 = 30$. The following screens are a result of the CALCULATE command with $x = 30$.



Median Age at Marriage for Men in 2030

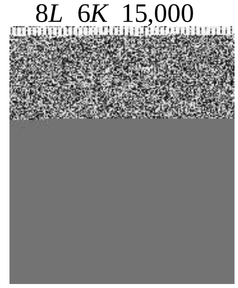


Median Age at Marriage for Women in 2030.

So, the median marriage age for men in 2030 will be 32.1 years and for women it will be 29.2 years.

68. **a.** Substitute $w = 8$, $r = 6$, $C = 15{,}000$ into the equation.

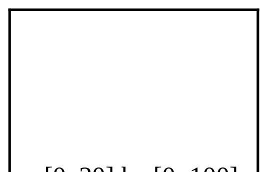
8*L* 6*K* 15,000



Substitute each pair into the equation. For (1875, 0), 8 1875 6 0 15, 000 For (1200, 900), 8 1200 6 900 15,000 For (600, 1700), 8 600 6 1700 15,000 For (0, 2500), 8 0 6 2500 15, 000

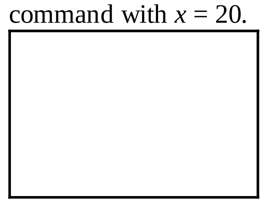
Every pair gives 15,000.

a. Women's Annual Earnings as a Percent of Men's



on [0, 30] by [0, 100]

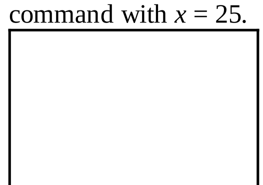
The *x*-value corresponding to the year 2020 is $x = 2020 - 2000 = 20$. The following screen is a result of the CALCULATE command with $x = 20$.



Women's Annual Earnings as a Percent of Men's

So, in the year 2020 women's wages will be about 84.2% of men's wages.

The *x*-value corresponding to the year 2025 is $x = 2025 - 2000 = 25$. The following screen is a result of the CALCULATE command with $x = 25$.



Women's Annual Earnings as a Percent of Men's

So in the year 2025 women's wages will be about 86% of men's wages.

a.

on [0, 100] by [0, 50]

To find the probability that a person with a family income of $40,000 is a smoker, substitute 40 into the equation

y 0 .31x 40

y 0 .31(40) 40 27.6 or 28%.

The probability that a person with a family income of $70,000 is a smoker is y 0.31(70) 40 18.3 or 18%.

a. To find the reported "happiness" of a person with an income of $25,000, substitute 25 into the equation

0.065 x 0.613

0.065(25) 0.613 1.0.

The reported "happiness" of a person with an income of $35,000 is

0.065(35) 0.613 1.7.

The reported "happiness" of a person with an income of $45,000 is

0.065(45) 0.613 2.3.

73. **a.**

y

0.094x 1.582

Cigarette consumption is declining by about 94 cigarettes (from 0.094 thousand, so about 5 packs) per person per year.

y 0 .094 13 1.582 0.36 thousand

(360 cigarettes)

a.

2.13x 65.35

The male life expectancy is increasing by 2.13 years per decade, which is 0.213 years (or about 2.6 months per year).

y 2.13(6.5) 65.35 79.2 years

a.

0.864x 75.46

Future longevity decreases by 0.864 (or about 10.44 months) per year.

y 0.864 25 75.46 53.9 years

It would not make sense to use the regression line to predict future longevity a age 90 because the line predicts –2.3 years of life remaining.

74. **a.**

y 5.8x 24.5

Each year the usage increases by about 5.8 percentage points.

y 5.8 11 24.5 88.3%

a.

1.41x 73.97

The female life expectancy is increasing by 1.41 years per decade, which is 0.141 years (or about 1.7 months per year).

y 1.41 6.5 73.97 83.1 years

a.

8.5x 53

Seat belt use increases by 8.5% each 5 years (or about 1.7% per year).

y 8.5 5.4 53 98.9%

It would not make sense to use the regressio line to predict seat belt use in 2025 (x = 7) because the line predicts 112.5%.

False: Infinity is not a number.

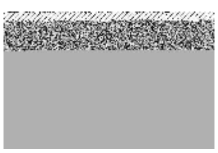
$m \; \frac{y_2 \;\; y_1}{x_2 \;\; x_1}$ for any two points (x_1, y_1) and (x_1, y_1) on the line *or* the slope is the amount that the line rises when x increases by 1.

False: The slope of a vertical line is undefined.

True: The slope is $\frac{a}{b}$ and the y-intercept is $\frac{c}{b}$.

False: It should be $\frac{y_2 \;\; y_1}{x_2 \;\; x_1}$.

Drawing a picture of a right triangle.



$$x^2 \;\; 4^2 \;\; 5^2$$
$$x^2 \; 16 \; 25 \; x^2$$
$$9 \; x \; 3$$

The slope is $m \;\; \frac{4}{3}$ or $\frac{4}{3}$ if the ladder slopes downward.

To find the x-intercept, substitute $y = 0$ into the equation and solve for x:

$$y \;\; mx \;\; b$$
$$0 \;\; mx \;\; b$$
$$mx \;\; b$$
$$x \;\; \frac{b}{m}$$

If $m \neq 0$, then a single x-intercept exists. So $a \; \frac{b}{m}$. Thus, the x-intercept is $\frac{b}{m}, 0$.

True: All negative numbers must be less than zero, and all positive numbers are more than zero. Therefore, all negative numbers are less than all positive numbers.

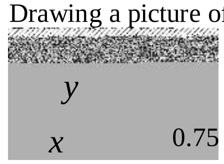
"Slope" is the answer to the first blank. The second blank would be describing it as negative, because the slope of a line slanting downward as you go to the right is a "fall" over "run".

False: The slope of a vertical line is undefined, so a vertical line does not have a slope.

True: $x = c$ will always be a vertical line because the x values do not change.

False. A vertical line has no slope, so there is no m for $y \; mx \; b$.

Drawing a picture of a right triangle.



$$x^2 \;\; y^2 \;\; 5^2$$
$$0.75 \;\; y \;\; 0.75x$$
$$x^2 \;\; (0.75x)^2 \;\; 5^2$$
$$x^2 \;\; 0.5625x^2 \;\; 25$$
$$1.5625x^2 \;\; 25$$
$$x^2 \;\; 16$$
$$x \;\; 4$$
$$y \;\; 0.75(4) \;\; 3$$

The upper end is 3 feet high.

i. To obtain the slope-intercept form of a line, solve the equation for y:

$$ax \;\; by \;\; c$$
$$by \;\; ax \;\; c$$
$$y \;\; \frac{a}{b}x \;\; \frac{c}{b}$$

Substitute 0 for b and solve for x:

$$ax \;\; by \;\; c$$
$$ax \;\; 0\,y \;\; c$$
$$ax \;\; c$$
$$x \;\; \frac{c}{a}$$

93. Consider $R > 1$ and $0 < x < K$

$x < K$ means that $K - x > 0$ and $0 < \frac{x}{K} < 1$.

Since $K - x > 0$, then

$$K - x + Rx > Rx$$

$$K + (R-1)x > Rx$$

$$K\left[1 + \frac{R-1}{K}x\right] > Rx$$

Therefore, $K > \dfrac{Rx}{1 + \frac{R-1}{K}x} = y$

Additionally, since $0 < \frac{x}{K} < 1$,

$$1 < 1 + (R-1)\frac{x}{K} < 1 + (R-1)$$

So, $y = \dfrac{Rx}{1 + \frac{R-1}{K}x} > \dfrac{Rx}{1 + (R-1)} = \dfrac{Rx}{R} = x$

We have $x < y < K$.

94. $x > K$ means that $K - x < 0$ and $\frac{x}{K} > 1$.

Since $K - x < 0$, then

$$K - x + Rx < Rx$$

$$K + (R-1)x < Rx$$

$$K\left[1 + \frac{R-1}{K}x\right] < Rx$$

Therefore, $K < \dfrac{Rx}{1 + \frac{R-1}{K}x} = y$

Additionally, since $\frac{x}{K} > 1$,

$$1 + (R-1)\frac{x}{K} > 1 + (R-1)$$

So, $y = \dfrac{Rx}{1 + \frac{R-1}{K}x} < \dfrac{Rx}{1 + (R-1)} = \dfrac{Rx}{R} = x$

We have $K < y < x$.

EXERCISES 1.2

1. $2^2 \cdot 2^2 \cdot 2^2 = 2^{2+2+2} = 2^{3\cdot 2} = 2^6 = 64$

2. $(5^2 \cdot 4)^2 = (5^2 \cdot 2^2)^2 = (10^2)^2 = 10^4 = 10{,}000$

3. $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

4. $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

5. $\left(\frac{1}{2}\right)^{-3} = (2^{-1})^{-3} = 2^3 = 8$

6. $\left(\frac{1}{3}\right)^{-2} = (3^{-1})^{-2} = 3^2 = 9$

7. $\left(\frac{5}{8}\right)^{-1} = \frac{8}{5}$

8. $\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$

9. $4^{-2} \cdot 2^{-1} = (2^2)^{-2} \cdot 2^{-1} = 2^{-4} \cdot 2^{-1} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

10. $3^{-2} \cdot 9^{-1} = 3^{-2}(3^2)^{-1} = 3^{-2} \cdot 3^{-2} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

11. $\left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

12. $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$

13. 1 / 3 2 ... 1 3 ... 3 ... 2 2 ... 3 2 ... 3 2 ... 2 ... 1 1 1 1 ... 9 8 1

14. 1 / 3 2 ... 1 2 ... 3 ... 2 2 ... 3 2 ... 2 2 ... 1 ... 9 4 5

15. 2 2 1 ... 3 2 1 ... 3 ... 2 2 ... 2 2 ... 4 ... 2 2 ... 3 3 ... 9 5

16. 2 2 1 ... 5 2 1 ... 5 2 ... 2 ... 2 ... 4 ... 2 2 ... 5 5 ... 25

17. $25^{1/2} = \sqrt{25} = 5$

18. $36^{1/2} = \sqrt{36} = 6$

19. $25^{3/2}$ $\sqrt{25}$ ^{3} 5^3 125

20. $16^{3/2}$ $\sqrt{16}^3$ ^{3} 64

21. $16^{3/4}$ [redacted] ^{3} 2^3 8

22. $27^{2/3}$ ^{3} 27 ^{2} [redacted] 3^2 9

23. $8^{2/3}$ [redacted] 2^2 2 $\sqrt{4}$

24. $27^{2/3}$ [redacted] 3^2 2 $\sqrt{9}$

25. $8^{5/3}$ [redacted] 2^5 5 $\sqrt{32}$

26. $27^{5/3}$ [redacted] 3^5 5 $\sqrt{243}$

27. $\left(\frac{25}{36}\right)^{3/2} = \left(\sqrt{\frac{25}{36}}\right)^3 = \left(\frac{5}{6}\right)^3 = \frac{5^3}{6^3} = \frac{125}{216}$

28. $\left(\frac{16}{25}\right)^{3/2} = \left(\sqrt{\frac{16}{25}}\right)^3 = \left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3} = \frac{64}{125}$

29. $\left(\frac{27}{125}\right)^{2/3} = \left(\sqrt[3]{\frac{27}{125}}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$

30. $\left(\frac{125}{8}\right)^{2/3} = \left(\sqrt[3]{\frac{125}{8}}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

31. $\left(\frac{1}{32}\right)^{2/5} = \left(\sqrt[5]{\frac{1}{32}}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

32. $\left(\frac{1}{32}\right)^{3/5} = \left(\sqrt[5]{\frac{1}{32}}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

33. $4^{-1/2} = \frac{1}{4^{1/2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

34. $9^{-1/2} = \frac{1}{9^{1/2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

35. $4^{-3/2} = \frac{1}{4^{3/2}} = \frac{1}{\left(\sqrt{4}\right)^3} = \frac{1}{2^3} = \frac{1}{8}$

36. $9^{-3/2} = \frac{1}{9^{3/2}} = \frac{1}{\left(\sqrt{9}\right)^3} = \frac{1}{3^3} = \frac{1}{27}$

37. $8^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{\left(\sqrt[3]{8}\right)^2} = \frac{1}{2^2} = \frac{1}{4}$

38. $16^{-3/4} = \frac{1}{16^{3/4}} =$ [redacted] $= \frac{1}{2^3} = \frac{1}{8}$

39. $8^{-1/3} = \frac{1}{8^{1/3}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

40. $27^{-1/3} = \frac{1}{27^{1/3}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

41. $(-8)^{-2/3} = \frac{1}{(-8)^{2/3}} = \frac{1}{\left(\sqrt[3]{-8}\right)^2} = \frac{1}{(-2)^2} = \frac{1}{4}$

42. $(-27)^{-2/3} = \frac{1}{(-27)^{2/3}} = \frac{1}{\left(\sqrt[3]{-27}\right)^2} = \frac{1}{(-3)^2} = \frac{1}{9}$

43. $\left(\frac{25}{16}\right)^{-1/2} = \left(\frac{16}{25}\right)^{1/2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

44. $\left(\frac{16}{9}\right)^{-1/2} = \left(\frac{9}{16}\right)^{1/2} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

45. $\left(\frac{25}{16}\right)^{-3/2} = \left(\frac{16}{25}\right)^{3/2} = \left(\sqrt{\frac{16}{25}}\right)^3 = \left(\frac{4}{5}\right)^3 = \frac{64}{125}$

46. $\left(\frac{16}{9}\right)^{-3/2} = \left(\frac{9}{16}\right)^{3/2} = \left(\sqrt{\frac{9}{16}}\right)^3 = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$

$\left(\frac{1}{27}\right)^{-5/3} = 27^{5/3} = \left(\sqrt[3]{27}\right)^5 = 3^5 = 243$

$\left(\frac{1}{8}\right)^{-5/3} = 8^{5/3} = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$

$7^{0.39} \approx 2.14$

$5^{0.47} \approx 2.13$

$8^{2.7} \approx 274.37$

$5^{3.9} \approx 532.09$

$0.1^{0.1}$

$0.1 \quad 0.977$

$\left(1 - \frac{1}{1000}\right)^{1000} \approx 2.717$

$\left(1 + \frac{1}{1000}\right)^{1000} \approx 2.720$

$\left(1 + 10^{-6}\right)^{10^6} \approx 2.718$

$\frac{4}{x^5} = 4x^{-5}$

$\frac{6}{2x^3} = \frac{3}{x^3} = 3x^{-3}$

59. $\frac{4}{\sqrt[3]{8x^4}} = \frac{4}{2x^{4/3}} = 2x^{-4/3}$

$\frac{6}{\sqrt{x^3}} = \frac{6}{x^{3/2}} = 6x^{-3/2}$

24 . 24 .

60. $4x \quad 2x^{3/2} \; 3x$

61. $2 \quad \sqrt{x}^{\,3} \quad 8x^{3/2} \; 3x^{3/2}$

$\sqrt{\frac{9}{x^4}} \; \frac{3}{x^2} \quad 3x^2$

$\frac{5}{x^2} \; \frac{5}{x^2} \qquad 2\;1/2 \qquad 3/2$

$\sqrt{x} \quad x^{1/2} \qquad 5\,x \qquad 5x$

62. $\frac{18}{3 \quad 2} \; \frac{18}{2/3} \; 2x^{2/3}$

$3 \quad$ [redacted] $\quad 9x$

$\sqrt[3]{\frac{8}{x^6}} \; \frac{2}{x^2} \; 2x^2$

$\frac{3}{x}\sqrt{x} \; \frac{x}{3x} \quad \frac{1/2}{x} \quad 3\,x^{1/2\;1} \; 3x^{1/2}$

67. $\frac{12}{3\sqrt{x^2}} \quad \frac{12x}{3x^{2}} \quad \frac{12}{3} x^{2/3\;2} \quad 4x^{4/3}$

68. $\frac{10\sqrt{x}}{2\,\text{[redacted]}} \quad \frac{10\,x^{1/2}}{2x^{1/3}} \quad \frac{10}{2} x^{1/2\;1/3} \quad 5x^{1/6}$

69. $\frac{\sqrt{36\,x}}{2\,x} \quad \frac{6\sqrt{x}}{2\,x} \quad \frac{6\,x^{1/2}}{2\,x} \quad \frac{6}{2} x^{1/2\;1} \quad 3x^{1/2}$

70. $\frac{\sqrt[3]{8\,x^2}}{4\,x} \quad \frac{2^3\,x^2}{4\,x} \quad \frac{2x^{2/3}}{4\,x} \quad \frac{2}{4} x^{2/3\;1} \; \frac{1}{2} x^{1/3}$

$x^3 x^{2\ 2}\ x^{5\ 2}\ x^{10}$

$x^4\ x^{3\ 2}\ x^{7\ 2}\ x^{14}$

73. $z^2\ zz\ ^2\ z^{2\ 3}\ z^2\ z^{3\ 2}\ z^3\ z^2\ z^6\ z^3$

3

$z^9\ z^{27}$

74. $zz^3\ z^2\ z^{2\ 2}\ zz^4\ z^{2\ 2\ 2}\ zz^8\ z^2\ 2$

2

$z^{11}\ z^{22}$

75. $x^{2\ 2\ 2}\ x^{4\ 2}\ x^8$

76. $x^3\ _3\ ^3\ x^{9\ 3}\ x^{27}$

77. $(3x^2 y^5 z)^3\ 3^3 x^{2\ 3} y^{5\ 3} z^3\ 27x^6\ ^{15} z^3$

y

5

$(ww^2)^3\ (w^3)^3\ w^9\ w$

79. $w^3 w\ w^4\ w^4$

78. $(2x^4 yz^6)^4\ 2^4 x^{4\ 4} y^4 z^{6\ 4}\ 16x^{16} y^4 z^{24}$

3 2 4 2 8

$(ww)\ (w)\ w\ w^3$

80. $w^3\ w^2\ w^5\ w^5$

4 2 2 8

81. $5xy\ \ 25x\ y\ \ y^5$

$25x^3 y^3\ 25x^3 y^3\ x$

$9xy^3 z^2\ _2\ 81x^2 y^6 z^2\ _2\ _2\ _2\ 27y^4$

$3\ xyz 3x\ y\ z$

$2u^2 vw^{3\ 2}\ 4u^4 v^2 w^6\ u^2 v$

$^2 w^2\ 4uw^{2\ 2}\ 4u^2 w^4$

$4x^3 y^2\ \ 6\ 2$

$16x\ y\ 2x^4$

$^2 y^3\ 8x^2 y^3\ ^y 8x$

$5x^2 y^3 z\ \ 25x^4 y^6 z^2$

2 2

$5x^2 y^4\ 5\ xyz^2\ 5x^2 y^2 z$

$u^3 vw^{2\ 2}\ u\ v^6 w^2\ u^4 v^2 w^2\ 2$

$u^2 w^2\ 9u^4 w^2\ 9\ 9$

Average body thickness
0.4(hip-to-shoulder length)$^{3/}$

$^2\ 0.4(16)^{3/2}\ 0.4\ 16^3$

$\sqrt{\ }$

25.6 ft

Average body thickness
0.4(hip-to-shoulder length)$^{3/}$

$^2\ 0.4(14)^{3/2}\ 0.4\ 14^3$

$\sqrt{\ }$

21.0 ft

0.6

$C\ x\ C$

$4^{0.6}\ C\ 2.3C$

To quadruple the capacity costs about 2.3 times as much.

$C\ x^{0.6} C$

$3^{0.6}\ C\ 1.9C$

To triple the capacity costs about 1.9 times as much

a. Given the unemployment rate of 2 percent, the inflation rate is

1.394

9.638 2 0.900

2.77 percent.

Given the unemployment rate of 5 percent, the inflation rate is

a. Given the unemployment rate of 3 percent, the inflation rate is

1.54

45.4 3 1

$9.638\ 5^{1.394}\ 0.900$

0.12 percent.

$\text{Heart rate } 250 \text{ weight}^{1/4} \; 250$

$16^{1/4}$

125 beats per minute

7.36 percent.

Given the unemployment rate of 8 percent, the inflation rate is

$45.4 \; 8^{1.54} \; 1$

0.85 percent.

$\text{Heart rate } 250 \text{ weight}^{1/4} \; 250$

$625^{1/4}$

50 beats per minute

(Time to build the 50th Boeing 707)
$150(50)^{0.322}$
42.6 thousand work-hours

It took approximately 42,600 work-hours to build the 50th Boeing 707.

Increase in energy $32^{B\ A}$
$32^{7.8\ 6.7}$
$32^{1.1}$ 45

The 1906 San Francisco earthquake had about 45 times more energy released than the 1994 Northridge earthquake.

K 3000 $225^{1/2}$ 200

$S\ \frac{60}{11}\ x^{0.5}$
$\frac{60}{11}\ 3281^{0.5}$ 312 mph

103.

on [0, 100] by [0,4]
x 18.2. Therefore, the land area must be increased by a factor of more than 18 to double the number of species.

y $9.4x^{0.37}$
$9.4(150)^{0.37}$ 60 miles per hour
The speed of a car that left 150-foot skid marks was 60 miles per hour.

a.

$79.9x^{0.138}$ (rounded)
For year 2020, $x = 10$.
$79.9\ 10^{0.138}$ $110 billion

a.

$41.6x^{0.0388}$ (rounded)
For year 2020, $x = 11$.
$41.6\ 11^{0.0388}$ $45.7 million

(Time to build the 250th Boeing 707)
$150(250)^{0.322}$
25.3 thousand work-hours

It took approximately 25,300 work-hours to build the 250th Boeing 707.

Increase in energy $32^{B\ A}$
$32^{9.0\ 7.7}$
$32^{1.3}$ 91

The 2011 Japan earthquake had about 91 times more energy released than the 2011 India earthquake.

K 4000 $125^{2/3}$ 160

$S\ \frac{60}{11}\ x^{0.5}$
$\frac{60}{11}\ 1650^{0.5}$ 222 mph

104.

on [0, 100] by [0,4]
x 99. Therefore, the land area must be increased by almost 100 times to triple the number of species.

y $9.4(350)^{0.37}$ 82 miles per hour
The speed of a car that left 350-foot skid marks was 82 miles per hour.

a.

$607x^{0.0866}$ (rounded)
For year 2020, $x = 12$
$607\ 12^{0.0866}$ $753

Ā Ā □ Ā □

.

□ Ā □ Ā □
$6.6x^{0.176}$ (rounded)

For year 2020, $x = 12$.
□ Ā □ Ā □
$6.6\ 12^{0.176}$ $41.2 billion

3, since $\sqrt{9}$ means the principal square foot.

(To get 3 you would have to write $\sqrt{9}$.)

False: $\frac{2}{2}^{6}_{6} \frac{64}{4}$ 16 , while $2^{6/2}$ 2^{3} 8 . (The

correct statement is $\frac{x^m}{x^n} x^{m-n}$.)

$x^{1/2} \quad \sqrt{x}$, so x must be nonnegative for the expression to be defined.

$x^{-1} \frac{1}{x}$, so all values of x except 0, because

you cannot divide by 0.

False: $2^2\ 2^3$ 4 8 32 , while $2^{2\ 3}$ 2^6 64 . (The

correct statement is $x^m\ x^n\ x^{m\ n}$.

False: $2^{3\ 2}$ 8^2 64 , while 2^{3^2} 2^9 512 . (The

correct statement is $x^{m\,n}$ $x^{m\,n}$.

$x^{1/3}$ [redacted] x, so all values of x. For example,
$8^{1/3}$ 2 and $8^{1/3}$ 2 .

If the exponent $\frac{m}{n}$ is not fully reduced, it will indicate an even root of a negative number, which is not defined in the real number set.

EXERCISES 1.3

1.	Yes	**2.**	No
5.	No	**6.**	Yes

3.	No	**4.**	Yes
7.	No	**8.**	Yes

Domain = $\{x \mid x \le 0 \text{ or } x \ge 1\}$ Range = $\{y \mid y \ge -1\}$

Domain = $\{x \mid x \le -1 \text{ or } x \ge 0\}$
Range = $\{y \mid y \le 1\}$

11. **a.** $f(x)\ \sqrt{x\ 1}$
$f(10)\ \sqrt{10\ 1}\ \sqrt{9}\ 3$

b. Domain = $\{x \mid x \ge 1\}$ since $f(x)\ \sqrt{x\ 1}$ is defined for all values of $x \ge 1$.

Range = $\{y \mid y \ge 0\}$

12. **a.** $f(x)\ \sqrt{x\ 4}$
$f(40)\ \sqrt{40\ 4}\ \sqrt{36}\ 6$

b. Domain = $\{x \mid x \ge 4\}$ since $f(x)\ \sqrt{x\ 4}$ is defined for all values of $x \ge 4$.

Range = $\{y \mid y \ge 0\}$

13. **a.** $h(z)\ \frac{1}{z\ 4}$

$h(5)\ \frac{1}{5\ 4}\ 1$

Domain = $\{z \mid z \ne -4\}$ since $h(z)\ \frac{1}{z\ 4}$ is defined for all values of z except $z = -4$.
Range = $\{y \mid y \ne 0\}$

14. **a.** $h(z)\ \frac{1}{z\ 7}$

$h(8)\ \frac{1}{8\ 7}\ 1$

Domain = $\{z \mid z \ne -7\}$ since $h(z)\ \frac{1}{z\ 7}$ is defined for all values of z except $z = -7$.
Range = $\{y \mid y \ne 0\}$

a. $h(x)\ x^{1/4}$
$h(81)\ 81^{1/4}$ [redacted] 3

Domain = $\{x \mid x \ge 0\}$ since $h(x)\ x^{1/4}$ is defined only for nonnegative values of x.
Range = $\{y \mid y \ge 0\}$

a. $h(x)\ x^{1/6}$
$h(81)\ 64^{1/4}$ [redacted] 2

Domain = $\{x \mid x \ge 0\}$ since $h\ x\ x^{1\ 6}$ is defined for nonnegative values of x.
Range = $\{y \mid y \ge 0\}$

a. $f(x)\ x^{2/3}$

$(8)(8)^{2/3}$ [redacted] $(2)^2$ 4

a. $f(x)\ x^{4/5}$

$(32)\ (32)^{4/5}$ [redacted] 32^4 $(2)^4$ 16

Ā Ā Ā

omain =

Ā Ā Ā

ange = {y | $y \geq 0$}

Domain = Ā Ā

Range = {y | $y \geq 0$}

Ā Ā

19. **a.** $f(x)=\sqrt{4-x^2}$

$f(0)=\sqrt{4-0^2}=\sqrt{4}=2$

b. $f(x)=\sqrt{4-x^2}$ is defined for values of x such that $4-x^2\ge 0$. *Thus,*

$4-x^2\ge 0$

$x^2\le 4$

$x^2\le 4$

$-2\le x\le 2$

Domain = $\{x \mid -2\le x\le 2\}$

Range = $\{y \mid 0\le y\le 2\}$

a. $f(x)=\frac{1}{\sqrt{x}}$

$f(4)=\frac{1}{\sqrt{4}}=\frac{1}{2}$

Domain = $\{x \mid x>0\}$ since $f(x)=\frac{1}{\sqrt{x}}$ is defined only for positive values of x.

Range = $\{y \mid y>0\}$

21. **a.** $f(x)=\sqrt{-x}$

$f(-25)=\sqrt{-(-25)}=\sqrt{25}=5$

b. $f(x)=\sqrt{-x}$ is defined only for values of x such that $-x\ge 0$. Thus $x\le 0$.

Domain = $\{x \mid x\le 0\}$

c. Range = $\{y \mid y\ge 0\}$

22. **a.** $f(x)=-\sqrt{-x}$

$f(-100)=-\sqrt{-(-100)}=-\sqrt{100}=-10$

b. $f(x)=-\sqrt{-x}$ is defined only for values of x such that $-x\ge 0$. Thus $x\le 0$.

Domain = $\{x \mid x\le 0\}$

c. Range = $\{y \mid y\le 0\}$

23.

24.

25.

26.

27.

28.

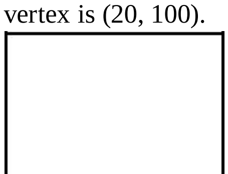
29.

a. $x \frac{b}{2a} \frac{40}{2 \cdot 1} = \frac{40}{2} = 20$

To find the y-coordinate, evaluate f at $x = 20$.

$20 \; 20^2 \; 40 \; 20 \; 500 \; 100$ The vertex is (20, 100).

b.



on [15, 25] by [100, 120]

30.

$x \; \frac{b}{} \; \frac{40}{} \; 40 \; 20$

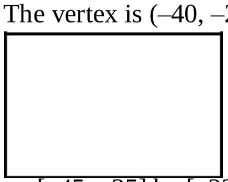
a. $x \frac{b}{2a} \frac{(80)}{2(1)} \frac{80}{2} 40$

To find the y-coordinate, evaluate f at $x = -40$.

$(40) \; (40)^2 \; 80(40) \; 1800 \; 200$

The vertex is (–40, –200).

b.



on [–45, –35] by [–220, –200]

32. **a.** $2a \quad 2 \; 1 \quad \overline{2}$

To find the y-coordinate, evaluate f at $x = -20$.

$f \; 20 \; 20^2 \; 40 \; 20 \; 500 \; 100$ The vertex is (–20, 100).

b.

on [–25, –15] by [100, 120]

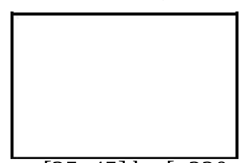
34. **a.** $x \; \frac{b}{2a} \; \frac{80}{2(1)} \; \frac{80}{2} \; 40$

To find the y-coordinate, evaluate f at $x = 40$.

$(40) \; (40)^2 \; 80(40) \; 1800 \; 200$

The vertex is (40, –200).

b.



on [35, 45] by [–220, –200]

35. $x^2 \; 6x \; 7 \; 0$

$x \quad 7 \; x \quad 1 \quad 0$

Equals 0 Equals 0 at
x 7 at x 1 x
7, x 1

36. $x^2 \; x \; 20 \; 0$

$x \quad 5 \; x \quad 4 \quad 0$

Equals 0 Equals 0 at
x 5 at x 4
x 5, x 4

37. $2 \; x^2 \; 2x \; 15$

$x \; 2x \; 15 \; 0$

$x \; 5 \; x \; 3 \; 0$

Equals 0 Equals 0
at x 5 at x 3

x 5, x 3

38. $2 \; x^2 \; 3x \; 54$

$x \; 3x \; 54 \; 0$

$x \; 9 \; x \; 6 \; 0$

Equals 0 Equals 0
at x 9 at x 6

x 9, x 6

$2x^2 \quad 40 \; 18x \; 2$

$x^2 \quad 18x \; 40 \; 0$

$x^2 \quad 9x \quad 20 \quad 0$

$x \quad 4 \quad x \quad 5 \quad 0$

Equals 0 at x 4   Equals 0 at x 5

$x \quad 4, \quad x \quad 5$

$x^2 \quad 50x \;\; 0$

$x^2 \quad 10x \;\; 0$

$x \;\; 10 \quad 0$

Equals 0 at x 0   Equals 0 at x 10

$x \;\; 0, \qquad x \; 10$

$2x^2 \quad 50 \;\; 0$

$x^2 \quad 25 \quad 0$

$x \quad 5 \; x \quad 5 \quad 0$

Equals 0 at x 5   Equals 0 at x 5

$x \quad 5, \quad x \quad 5$

$4x^2 \quad 24x \;\; 40$

$4 \; 4x^2 \quad 24x \;\; 36$

$0 \; x^2 \quad 6x \;\; 9 \;\; 0 \; ($

$x \;\; 3)^2 \quad 0$

Equals 0 at x 3

$4x^2 \quad 12x \;\; 8 \;\; 4$

$x^2 \quad 12x \; 8 \; 0$

$x^2 \quad 3x \quad 2 \quad 0$

$x \quad 2 \quad x \quad 1 \quad 0$

Equals 0 at x 2   Equals 0 at x 1

$x \quad 2, \quad x \quad 1$

$2x^2 \quad 12x \;\; 20 \;\; 0$

$x^2 \quad 6x \quad 10 \quad 0$

Use the quadratic formula with $a = 1$, $b = –6$, and $c = 10$.

$$x \quad \frac{(\;6) \quad \sqrt{(\;6)^2 \quad 4(1)(10)}}{2(1)}$$

$$\frac{6 \; \sqrt{36 \quad 40}}{2}$$

$$\frac{6 \; \sqrt{4}}{2} \quad \text{Undefined}$$

$2x^2 \quad 12x \quad 20 \quad 0$ has no real solutions.

$3x^2 \quad 12 \;\; 0$

$x^2 \quad 4 \;\; 0$

$3x^2 \quad 18 \; 15x \; 3$

$x^2 \quad 15x \; 18 \; 0$

$x^2 \quad 5x \quad 6 \quad 0$

$x \quad 3 \quad x \quad 2 \quad 0$

Equals 0 at x 3   Equals 0 at x 2

$x \quad 3, \quad x \quad 2$

$3x^2 \quad 36x \;\; 0$

$x^2 \quad 12x \quad 0$

$x \;\; 12 \quad 0$

Equals 0 at x 0   Equals 0 at x 12

$x \;\; 0, \qquad x \; 12$

$3x^2 \quad 27 \;\; 0$

$x^2 \quad 9 \quad 0$

$x \quad 3 \; x \quad 3 \quad 0$

Equals 0 at x 3   Equals 0 at x 3

$x \quad 3, \quad x \quad 3$

$3x^2 \quad 6x \;\; 9 \;\; 6 \; 3x^2$

$6x \;\; 3 \;\; 0 \; x^2$

$2x \; 1 \;\; 0 \; (x \; 1)^2$

0

Equals 0 at x 1

$3x^2 \quad 6x \quad 24$

$3x^2 \quad 6x \;\; 24 \;\; 0$

$x^2 \quad 2x \quad 8 \quad 0$

$x \quad 4 \quad x \quad 2 \quad 0$

Equals 0 at x 4   Equals 0 at x 2

$x \quad 4, \quad x \quad 2$

$2x^2 \quad 8x \;\; 10 \;\; 0$

$x^2 \quad 4x \quad 5 \quad 0$

Use the quadratic formula with $a = 1$, $b = –4$, and $c = 5$.

$$x \quad \frac{(\;4) \quad \sqrt{(\;4)^2 \quad 4(1)(5)}}{2(1)}$$

$$\frac{4 \; \sqrt{16 \quad 20}}{2}$$

$$\frac{4 \; \sqrt{4}}{2} \quad \text{Undefined}$$

$2x^2 \quad 8x \quad 10 \quad 0$ has no real solutions.

$5x^2 \quad 20 \;\; 0$

$x^2 \quad 4 \;\; 0$

x^2 4 0

4

x $\sqrt{4}$ Undefined

2

$3x$ 12 0 has no real solutions.

4

x $\sqrt{4}$ Undefined

$5x^2$ 20 0 has no real solutions.

53.

on [–5, 6] by [–22, 6]
$x = –4$, $x = 5$

54.

on [–6, 4] by [–20, 6]
$x = –5$, $x = 3$

55.

on [–1, 9] by [–10, 40]
$x = 4$, $x = 5$

56.

on [0, 5] by [–3, 15]
$x = 2$, $x = 3$

57.

on [–7, 1] by [–2, 16]
$x = –3$

58.

on [–2, 3] by [–2, 18]
$x = 1$

59.

on [–5, 3] by [–5, 30]
No real solutions

60.

on [–5, 3] by [–5, 30]
No real solutions

61.

on [–4, 3] by [–9, 15]

$x = –2.64$, $x = 1.14$

62.

on [–4, 3] by [–10, 10]

$x = –2.57$, $x = 0.91$

63.

on [–10, 10] by [–10, 10]

Their slopes are all 2, but they have different *y*-intercepts.
The line 2 units below the line of the equation $y = 2x – 6$ must have *y*-intercept –8. Thus, the equation of this line is $y = 2x – 8$.

Let x = the number of board feet of wood. Then $C(x) = 4x + 20$

64.

on [–10, 10] by [–10, 10]

The lines have the same *y*-intercept, but their slopes are different.

$y \quad \frac{1}{2} x \quad 4$

Let x = the number of bicycles. Then $C(x) = 55x + 900$

Let x = the number of hours of overtime. Then
$P(x) = 15x + 500$

a. $p\ d\ 0.45d\ 15\ p\ 6\ 0.45$
6 15

17.7 pounds per square inch

$p\ d \quad 0.45d \quad 15$

$p\ 35,000 \quad 0.45\ 35,000 \quad 15$

15,765 pounds per square inch

$D\ v \quad 0.055v^2 \quad 1.1v$
2

$D\ 40 \quad 0.055\ 40 \quad 1.1\ 40 \quad 132\ \text{ft}$

a. $N(t)\ 200\ 50t^2$

2

$N(2) \quad 200 \quad 50(2)$
400 cells

b. $N(t)\ 200\ 50t^2$
$N(10)\ 200\ 50(10)^2$
5200 cells

75. $v(x) \quad \frac{60}{11}\sqrt{x}$

$v(1776) \quad \frac{60}{11}\sqrt{1776} \quad 230$ mph

77.

on [0, 5] by [0, 50]
The object hits the ground in about 2.92 seconds

a. To find the break-even points, set $C(x)$ equal to $R(x)$ and solve the resulting equation.

$C(x) \quad R(x)$

$180x\ 16,000 \quad 2x^2 \quad 660x$

$2x^2 \quad 480x\ 16,000 \quad 0$

Use the quadratic formula with a = 2, b = –480 and c = 16,000.

$x \quad \frac{480 \quad \sqrt{(480)^2 \quad 4(2)(16,000)}}{2(2)}$

$x \quad \frac{480 \quad \sqrt{102,400}}{4} \quad \frac{480 \quad 320}{4}$

$x \quad \frac{800}{4}$ or $\frac{160}{4}$

$x \quad 200$ or 40

The company will break even when it

Let x = the total week's sales. Then
$P(x) = 0.02x + 300$

70. $B\ h \quad 1.8h \quad 212$

$98.6 \quad 1.8h \quad 212$
$1.8h \quad 113.4$
$h \quad 63$ thousand feet above sea level

72. $D(v)\ 0.55v^2 \quad 1.1v$

$D(60)\ 0.55(60)^2 \quad 1.1(60) \quad 264$ ft

74. **a.** $T(h)\ 0.5\sqrt{h}$

$T(4)\ 0.5\sqrt{4} \quad 1$ second
$T(8)\ 0.5\sqrt{8} \quad 1.4$ seconds

$T(h)\ 0.5\sqrt{h} \qquad T(h)\ 0.5\sqrt{h}$
$2\ 0.5\sqrt{h} \qquad 3\ 0.5\sqrt{h}$
$4\ \sqrt{h} \qquad 6\ \sqrt{h}$
$h\ 16$ ft $\qquad h\ 36$ ft

$s(d)\ 3.86\sqrt{d}$

$s(15,000) \quad 3.86\sqrt{15,000} \quad 473$ mph

78.

on [0, 5] by [0, 50]
The object hits the ground in about 2.6 seconds.

To find the number of devices that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.

$P(x) \quad (2x^2 \quad 660x) \quad (180x\ 16,000)$

$2x^2 \quad 480x\ 16,000$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

$x \quad \frac{480}{2\ 2} \quad \frac{480}{4} \quad 120$

Thus, profit is maximized when 120 devices are produced per week. The maximum profit is found by evaluating $P(120)$.

$P(120) \quad 2(120)^2 \quad 480(120)\ 16,000$
$12,800

makes either 40 devices or 200 devices.	Therefore, the maximum profit is $12,800.

a. To find the break-even points, set $C(x)$ equal to $R(x)$ and solve the resulting equation.

$$C(x) \quad R(x)$$
$$420x \quad 72,000 \quad 3x^2 \quad 1800x$$

$$3x^2 \quad 1380x \quad 72,000 \quad 0$$

Use the quadratic formula with $a = 3$, $b = –1380$ and $c = 72,000$.

$$x \quad \frac{1380 \quad \sqrt{(1380)^2 \quad 4(3)(72,000)}}{2(3)}$$

$$x \quad \frac{1380 \quad \sqrt{1,040,400}}{6} \quad \frac{1380 \quad 1020}{6}$$

$$x \quad \frac{2400}{6} \text{ or } \frac{360}{6}$$

The store will break even when it sells either 60 bicycles or 400 bicycles.

a. To find the break-even points, set $C(x)$ equal to $R(x)$ and solve the resulting

equation.

$$C(x) \quad R(x)$$
$$100x \quad 3200 \quad 2x^2 \quad 300x$$
$$2x^2 \quad 200x \quad 3200 \quad 0$$

$b = –200$ and $c = 3200$.

$$x \quad \frac{200 \quad \sqrt{(1020)^2 \quad 4(2)(3200)}}{2(2)}$$

$$x \quad \frac{200 \quad \sqrt{14,400}}{4} \quad \frac{200 \quad 120}{4}$$

$$x \quad \frac{320}{4} \text{ or } \frac{80}{4}$$

$$x \quad 80 \text{ or } 20$$

The store will break even when it sells either 20 exercise machines or 80 exercise machines.

Since this is a parabola that opens downward, the monthly price that maximizes visits is found at the vertex.

$$x \quad \frac{0.56}{2(0.004)} \quad \$70$$

To find the number of bicycles that maximizes profit, first find the profit function, $P(x) = R(x) – C(x)$.

$$P(x) \quad (3x^2 \quad 1800x) \quad (420x \quad 72,000)$$
$$3x^2 \quad 1380x \quad 72,000$$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

$$x \quad \frac{1380}{2 \quad 3} \quad \frac{1380}{6} \quad 230$$

Thus, profit is maximized when 230 bicycles are sold per month. The maximum profit is found by evaluating $P(230)$.

$$P(230) \quad 3(230)^2 \quad 1380(230) \quad 72,000$$
$$\$86,700$$

Therefore, the maximum profit is \$86,700.

To find the number of exercise machines that maximizes profit, first find the profit function, $P(x) = R(x) – C(x)$.

$$P(x) \quad (2x^2 \quad 300x) \quad (100x \quad 3200)$$
$$2x^2 \quad 200x \quad 3200$$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

$$x \quad 2 \quad 2 \quad 4 \quad 50$$

Thus, profit is maximized when 50 exercise machines are sold per day. The maximum profit is found by evaluating $P(50)$.

$$P\,50 \quad 2\,50^2 \quad 200\,50 \quad 3200$$
$$\$1800$$

Therefore, the maximum profit is \$1800.

$$w \quad a \quad v \quad b \quad c$$

$$b \quad \frac{c}{w \quad a}$$
$$\frac{c}{w \quad a} \quad b$$

84. **a.** $f\ 1 \quad 0.077 \quad 1 \ ^{2} 0.057 \quad 1 \ \ 1$

$0.077 \quad 0.057\ 1$

0.866

So a 65-year-old person has an 86.6% chance of living another decade.

$f\ 2\ 0\ .077\ 2^{2}\ 0.057\ 2\ 1\ 0.308$

$0.114\ 1\ 0.578$

So a 65-year-old person has a 57.8% chance of living two more decades.

$f\ 3\ 0\ .077\ 3^{2}\ 0.057\ 3\ 1\ 0.693$

$0.171\ 1\ 0.136$

So a 65-year-old person has a 13.6% chance of living three more decades.

a.

On [0, 20] by [0, 300].

2015 – 1995 = 20

$0.9x^{2} \quad 3.9x \ \ 12.4$

$0.9(20)^{2}\ 3.9(20)\ 12.4\ y$

294.4

So the global wind power generating capacity in the year 2015 is about 294 thousand megawatts.

2020 – 1995 = 25

$0.9x^{2} \quad 3.9x \ \ 12.4$

$0.9(25)^{2}\ 3.9(25)\ 12.4\ y$

477.4

So the global wind power generating capacity in the year 2020 is about 477 thousand megawatts.

a. The upper limit is

$(\ x\ ) \ \ 0.7(220 \ \ x\ ) \ \ 154 \ \ 0.7x$

The lower limit is

$f\ (\ x\ ) \quad 0.5(220 \ \ x\ ) \quad 110 \quad 0.5x$

85. **a.**

On [10, 16] by [0, 100].

$y \ \ 0.831\ x^{2} \quad 18.1\ x \ \ 137.3$

2

$0.831(12) \quad 18.1(12)\ 137.3\ y$

39.764

The probability that a high school graduate smoker will quit is 40%.

$y \ \ 0.831\ x^{2} \quad 18.1\ x \ \ 137.3$

2

$0.831(16) \quad 18.1(16)\ 137.3\ y$

60.436

The probability that a college graduate smoker will quit is 60%.

87. **a.** $(100 \ \ x\ )\ x \ \ 100x \ \ x^{2} \text{ or } \ x^{2} \quad 100$

$f\ (x\ ) \quad 100x \quad x^{2} \text{ or } f\ (\ x\ ) \quad x^{2} \quad 100x$

Since this function represents a parabola opening downward (because a = –1), it is maximized at its vertex, which is found using the vertex formula, $x \ \frac{b}{2a}$, with a 1 and b 100 .

$x \quad \frac{100}{2} \_\_50$

She should charge \$50 to maximize her revenue.

The lower cardio limit for a 20-year old is
$g(20)\ 110\ 0.5(20)\ 100$ bpm

The upper cardio limit for a 20-year old is
$f\ (20) \quad 154 \quad 0.7(20) \quad 140$ bpm

The lower cardio limit for a 60-year old is
$g(60) \quad 110 \quad 0.5(60) \quad 80$ bpm

The upper cardio limit for a 60-year old is
$f\ (60) \quad 154 \quad 0.7(60) \quad 112$ bpm

a.

$0.434x^2 - 3.26x + 11.6$

b. For year 2016, $x = 8.6$.

$0.434(8.6)^2 - 3.26(8.6) + 11.6 \approx 15.7\%$

a.

$0.153x^2 - 0.312x + 0.335$

b. For year 2020, $x = 7$.

$0.153(7)^2 - 0.312(7) + 0.335 \approx \$10$

For year 2030, $x = 8$.

$0.153(8)^2 - 0.312(8) + 0.335 \approx \$12.60$

A function can have more than one x-intercept. Many parabolas cross the x-axis twice. A function cannot have more than one y-intercept because that would violate the vertical line test.

Because the function is linear and 5 is halfway between 4 and 6, $f(5) = 9$ (halfway between 7 and 11).

95. m is blargs per prendle and $\frac{y}{x}$, so x is in prendles and y is in blargs.

No, that would violate the vertical line test. Note: A *parabola* is a geometric shape and so *may* open sideways, but a quadratic function,

being a *function*, must pass the vertical line test.

$f(4) = 9$, (since the two given values show that x increasing by 1 means y increases by 2.).

The units of $f(x)$ is widgets and the units of x are blivets, so the units of the slope would be widgets per blivet.

If a is negative, then it will have a vertex that is its highest value. If a is positive, then the equation will have a vertex that is its lowest value.

Either by the symmetry of parabolas, or, better, by taking the average of the two x-intercepts: the part of the quadratic formula will cancel out,

leaving just $\frac{-b}{2a}$.

EXERCISES 1.4

Domain = $\{x \mid x < -4 \text{ or } x > 0\}$
Range = $\{y \mid y < -2 \text{ or } y > 0\}$

Domain = $\{x \mid x \le 0 \text{ or } x > 3\}$
Range = $\{y \mid y \le -2 \text{ or } y > 2\}$

3. **a.** $f(x) = \frac{1}{x+4}$

$f(-3) = \frac{1}{-3+4} = 1$

Domain = $\{x \mid x \neq -4\}$
Range = $\{y \mid y \neq 0\}$

4. **a.** $f(x) = \frac{1}{(x-1)^2}$

$f(-1) = \frac{1}{(-1-1)^2} = \frac{1}{4}$

Domain = $\{x \mid x \neq 1\}$
Range = $\{y \mid y > 0\}$

5. **a.** $f(x) = \frac{x^2}{x-1}$

$f(-1) = \frac{(-1)^2}{-1-1} = -\frac{1}{2}$

Domain = $\{x \mid x \neq 1\}$
Range = $\{y \mid y \le 0 \text{ or } y \ge 4\}$

6. **a.** $f(x) = \frac{x^2}{x+2}$

$f(-2) = \frac{2^2}{2+2} = 1$

Domain = $\{x \mid x \neq -2\}$
Range = $\{y \mid y \le -8 \text{ or } y \ge 0\}$

7. **a.** $f(x) = \frac{12}{x(x+4)}$

$f(-2) = \frac{12}{-2(-2+4)} = -1$

Domain = {x | $x \neq 0$, $x \neq -4$}

Range = {y | $y \leq -3$ or $y > 0$}

a. $g(x) = |x - 2|$

$g(5) = |5 - 2| = |3| = 3$

Domain =

Range = {y | $y \geq 0$}

$x^5 2x^4 3x^3 0 \; x^3 x^2 2x 3 0$

$x^3 \; x \; 3 \; x \; 1 \; 0$

Equals 0 Equals 0 Equals 0

at $x = 0$ at x 3 at x 1

x 0, x 3,and x 1

$5x^3 20x 0 \; x^2 4 05x$

$5x(x\,2)(x\;2)\;0$

Equals 0 Equals 0 Equals 0

at $x = 0$ at x 2 at x 2

x 0, x 2,and x 2

$2x^3 18x 12x^2 \; 2x^3 12x^2 18x$

0

$2x\,x^2 \; 6x \; 9 \; 0$

$2x(x\,3)^3 \; 0$

Equals 0 Equals 0

at $x = 0$ at x 3

x 0 and x 3

$6x^5 \; 30x^4$

$6x^5 \; 30x^4 \; 0 \; x \; 5$

$06x^4$

Equals 0 Equals 0

at $x = 0$ at x 5

x 0 and x 5

$3x^{5/2} \; 6x^{3/2} \; 9x^{1/2}$

$3x^{5/2} \; 6x^{3/2} 9x^{1/2} \; 0$

$3x^{1/2} \; x^2 \; 2x \; 3 \; 0$

$3x^{1/2}(x \; 3)(x \; 1) \; 0$

Equals 0

Equ

als 0

Equ

als 0

at x

0

at x

3

at x

8. **a.** $f(x) = \frac{16}{x(x+4)}$

$f(-4) = \frac{16}{-4(-4+4)} \quad \frac{1}{2}$

x

0,

x

3 and x

1

Valid solutions are $x = 0$ and $x = 3$.


1

Domain = $\{x \mid x \neq 0, x \neq 4\}$
Range = $\{y \mid y \leq -4 \text{ or } y > 0\}$

a. $g\ x \quad x \quad 2$ | |

$g \quad 5 \quad |5| \quad 2 \quad 5 \quad 2 \quad 7$

Domain =
Range = $\{y \mid y \geq 2\}$

$x^6 \ x^5 \ 6x^4 \ 0 \ x^4 \ x^2 \ x \ 6 \ 0$
4

$x \quad x \quad 3 \quad x \quad 2 \quad 0$

Equals 0 at $x = 0$ | Equals 0 at x 3 | Equals 0 at x 2

$x \quad 0, x \quad 3$, and $x \quad 2$

$2x^5 \quad 50x^3 \quad 0 \quad x^3 \quad 2 \quad 25 \ 0 \ 2x^3$

$2x \quad (x\ 5)(x \quad 5) \quad 0$

Equals 0 at $x = 0$ | Equals 0 at x 5 | Equals 0 at x 5

$x \quad 0, x \quad 5$, and $x \quad 5$

$3x^3_3 \ 12_2 \ x^2_2 \ 12x^3_2 \quad 4 \quad 3x \quad 12x$

$12x \quad 0 \ 3x \quad x \quad 4x \ 4 \ 0$

$3x^2 \ (x\ 2)^2 \quad 0$

Equals 0 at $x = 0$ | Equals 0 at x 2

$x \ 0$ and $x \quad 2$

18. $5x^4 \quad 20x^3$

$5x^4 \quad 20x^3 \quad 0$
3

$5x \quad x \ 4 \quad 0$

Equals 0 at $x = 0$ | Equals 0 at x 4

$x \quad 0$ and $x \quad 4$

$2x^{7/2} \quad 8x^{5/2} \quad 24x^{3/2} \quad 2x^{7/2} \quad 8x^{5}$

$/2 \quad 3/2$

$24x \ 2 \quad 0 \quad 3/2$

$x \quad 4x \ 12 \quad 0\ 2x$

$2x^{3/2} \quad (x\ 6)(x \quad 2) \quad 0$

Equals 0 at x 0 | Equals 0 at x 6 | Equals 0 at x 2

$x \ 0$, $\quad x \quad 6 \quad$ and $x \ 2$

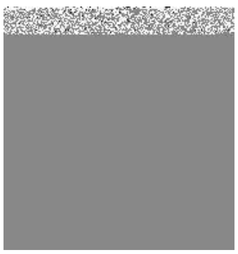
Valid solutions are $x = 0$ and $x = 2$.

21. on [0, 4] by [–5,25]
$x = 0$ and $x = 1$

22. on [0, 6] by [–300, 25]
$x = 0$ and $x = 6$

23. on [–3, 3] by [–25, 10]
x –1.79, $x = 0$, and x 2.79

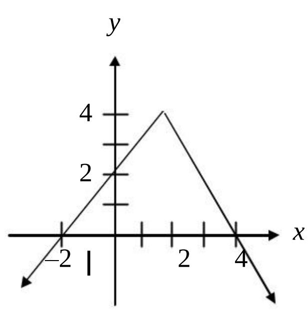
24. on [–4,2] by [–25, 10]
x –3.45, $x = 0$, and x 1.45

25. 

26. 

27. 

28. 

29. 

30. 

31. on [–2, 10] by [–5, 5]

32. on [–2, 10] by [–5, 5]

33. Polynomial

34. Piecewise linear function

35. Piecewise linear function

36. Polynomial

37. Polynomial

38. Piecewise linear function

39. Rational function

40. Polynomial

Piecewise linear function
Polynomial
None of these

Rational function
None of these
Polynomial

47. **a.** $y = 4$

$y = 1$

c.

on [–3, 3] by [0, 5]

(0, 1) because $a^0 = 1$ for any constant $a \neq 0$.

48.

on [–1, 1] by [0, 1]

The parabola is inside and the semicircle is outside.

49. **a.** $f(g(x)) = (g(x))^5 = (7x+1)^5$

$g(f(x)) = 7f(x) + 1$

$= 7x^5 + 1$

50. **a.** $f(g(x)) = (g(x))^8 = (2x+5)^8$

$g(f(x)) = 2f(x) + 5 = 2x^8 + 5$

52. **a.** $f(g(x)) = \sqrt{g(x)} = \sqrt[3]{x} + 1$

b. $g(f(x)) = \sqrt[3]{f(x)} + 1$

$= \sqrt[3]{x^3} + 1 = x + 1$

51. **a.** $f(g(x)) = \frac{1}{g(x)} = \frac{1}{\frac{1}{x^2} - 1}$

b. $g(f(x)) = \frac{1}{(f(x))^2} - 1 = \frac{1}{x^2} - 1$

54. **a.** $f(g(x)) = \sqrt{g(x)} = \sqrt{x^2 - 1}$

b. $g(f(x)) = (f(x))^2 - 1 = x - 1$

53. **a.** $f(g(x)) = (g(x))^3 - (g(x))^2$

$= (\sqrt[3]{x} + 1)^3 - (\sqrt[3]{x} + 1)^2$

b. $g(f(x)) = \sqrt[3]{f(x)} + 1 = \sqrt[3]{x^3 - x^2} + 1$

56. **a.** $f(g(x)) = \frac{g(x)^4 + 1}{g(x)^4 - 1}$

b. $g(f(x)) = \sqrt[3]{f(x)}$

$= \sqrt[3]{\frac{x^4 + 1}{x^4 - 1}}$

55. **a.** $f(g(x)) = \frac{g(x)^3 + 1}{x^2} = \frac{x^3 + 1}{x^2}$

b. $g(f(x)) = (f(x))^2 = \left(\frac{x^3 + 1}{x^3 - 1}\right)^2$

58. **a.** $f(g(x)) = g(x) - 1$

[redacted] 1

$x - 1 - 1$
x

57. **a.** $f(g(x)) = 2g(x) - 6$

$= 2\left(\frac{x}{2} + 3\right) - 6$

$= x + 6 - 6$

b. $g(f(x)) = \sqrt[3]{f(x) + 1}$

$= \sqrt[3]{x^3 - 1 + 1}$

$= \sqrt[3]{x^3}$

$= x$

b. $g\ f\ x$ $\frac{f(x)}{}$. 3

$\frac{2x\ 6}{}$ ^{2} $_3$

$x\ 2^3\ 3$

x

$f\ x\ h\ \ 5\,x\ h^2\ \ 5\,x^2\ \ 2hx\ \ h^2$

$5\,x^2\ \ 10\,hx\ \ 5h^2$

$f\ x\ h\ \ 2\,x\ h^2\ \ 5\,x\ h\ \ 1$

$2\,x^2\ \ 4\,xh\ \ 2\,h^2\ \ 5\,x\ \ 5h\ \ 1$

$f(x)\ \ 5x^2$

$\dfrac{f(x\ h)\ \ f(x)}{hh}\ \ 5(x\ h)^2\ \ 5x^2$

$\dfrac{5(x^2\ \ 2\,xh\ \ h^2)\ 5x^2}{h}$

$\dfrac{5\,x^2\ \ 10\,xh\ 5h^2\ \ 5x^2\ \ h}{}$

$\dfrac{10\,xh\ \ 5h^2}{h}$

$\dfrac{h\,(10\,x\ \ 5\,h)}{h}$

$10x\ \ 5h$ or $5(2x\ \ h)$

65. $f(x)\ \ 2x^2\ \ 5x\ \ 1$

$\dfrac{f(x\ h)\ f(x)}{h}\ \ \dfrac{2(x\ h)^2\ \ 5(x\ h)\ \ 1\ \ (2x^2\ \ 5x\ \ 1)}{h}$

$\dfrac{2(x^2\ \ 2xh\ \ h^2)\ \ 5(x\ h)\ \ 1\ \ (2x^2\ \ 5x\ \ 1)}{h}$

$\dfrac{2x^2\ \ 4xh\ \ 2h^2\ \ 5x\ \ 5h\ \ 1\ \ 2x^2\ \ 5x\ \ 1}{h}$

$\dfrac{4xh\ \ 2h^2\ \ 5h}{h}$

$\dfrac{h\,(4x\ 2h\ 5)}{h}$

$4x\ \ 2h\ \ 5$

$f(x)\ \ 7x^2\ \ 3x\ \ 2$

$\dfrac{f(x\ h)\ f(x)}{hh}\ 7(x\ h)^2\ \ 3(x\ h)\ \ 2\ \ (7x^2\ \ 3x\ \ 2)$

$\dfrac{7(x^2\ \ 2xh\ \ h^2)\ \ 3(x\ h)\ \ 2\ \ (7x^2\ \ 3x\ \ 2)}{h}$

$7x^2\ 14xh\ \ 7h^2\ 3x\ \ 3h\ \ 2\ \ 7x^2\ 3x\ \ 2\ h$

$\dfrac{14xh\ \ 7h^2\ \ 3h}{h}$

$\dfrac{h\,(14x\ 7h\ 3)}{h}$

$14x\ \ 7h\ \ 3$

$f\,x\,h\ 3\,x\,h^2\ 3\,x^2\ 2hx\,h^2\ 3\,x^2\ 6\,hx\ 3h^2$

$f\ x\ h\ \ 3\,x\ h^2\ \ 5\,x\ h\ \ 2$

$3\,x^2\ \ 6hx\ \ 3h^2\ \ 5\,x\ \ 5h\ \ 2$

$f(x)\ \ 3x^2$

$\dfrac{f(x\ h)\ \ f(x)}{hh}\ \ 3(x\ h)^2\ \ 3x^2$

$\dfrac{3(x^2\ \ 2xh\ \ h^2)\ 3x^2}{h}$

$3x^2\ 6xh\ 3h^2\ 3x^2\ h$

$\dfrac{6xh\ \ 3h^2}{h}$

$h\,(6x\ \ 3h)$

$6x\ \ 3h$ or $3(2x\ \ h)$

66. $f(x)\ \ 3x^2\ \ 5x\ \ 2$

$\dfrac{f(x\ h)\ \ f(x)}{h}$

$\dfrac{3(x\ h)^2\ \ 5(x\ h)\ \ 2\ \ (3\,x^2\ \ 5\,x\ \ 2)}{h}$

$3(x^2\ \ 2\,xh\ \ h^2)\ \ 5(x\ h)\ \ 2\ \ (3\,x^2\ \ 5\,x\ \ 2)\ h$

$\dfrac{3\,x^2\ 6\,xh\ \ 3h^2\ 5\,x\ \ 5\,h\ \ 2\ \ 3\,x^2\ 5\,x\ \ 2}{h}$

$\dfrac{6\,xh\ \ 3h^2\ \ 5h}{h}$

$\dfrac{h\,(6\,x\ \ 3h\ \ 5)}{h}$

$6\,x\ \ 3h\ \ 5$

$f(x)\ 4x^2\ 5x\ 3\ f(x\ h)\ f\ (x)$

h

$\dfrac{4(x\ h)^2\ \ 5(x\ h)\ \ 3\ \ (4\,x^2\ \ 5\,x\ \ 3)}{h}$

$4(x^2\ \ 2\,xh\ \ h^2)\ \ 5(x\ h)\ \ 3\ \ (4\,x^2\ \ 5\,x\ \ 3)\ h$

$\dfrac{4\,x^2\ \ 8\,xh\ \ 4\,h^2\ \ 5\,x\ \ 5\,h\ \ 3\ \ 4\,x^2\ \ 5\,x\ \ 3}{h}$

$\dfrac{8\,xh\ \ 4\,h^2\ \ 5h}{h}$

$\dfrac{h\,(8\,x\ \ 4\,h\ \ 5)}{h}$

$8\,x\ \ 4\,h\ \ 5$

69. $f(x) = x^3$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^3 - x^3}{h}$$
$$= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$
$$= \frac{3x^2h + 3xh^2 + h^3}{h}$$
$$= \frac{h(3x^2 + 3xh + h^2)}{h}$$
$$= 3x^2 + 3xh + h^2$$

70. $f(x) = x^4$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^4 - x^4}{h}$$
$$= \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$
$$= \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$
$$= \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h}$$
$$= 4x^3 + 6x^2h + 4xh^2 + h^3$$

71. $f(x) = \frac{2}{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$$
$$= \frac{\frac{2}{x+h} - \frac{2}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$
$$= \frac{2x - 2(x+h)}{hx(x+h)}$$
$$= \frac{2x - 2x - 2h}{hx(x+h)}$$
$$= \frac{-2h}{hx(x+h)}$$
$$= \frac{-2}{x(x+h)}$$

72. $f(x) = \frac{3}{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{3}{x+h} - \frac{3}{x}}{h}$$
$$= \frac{\frac{3}{x+h} - \frac{3}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$
$$= \frac{3x - 3(x+h)}{hx(x+h)}$$
$$= \frac{3x - 3x - 3h}{hx(x+h)}$$
$$= \frac{-3h}{hx(x+h)}$$
$$= \frac{-3}{x(x+h)}$$

73. $f(x) = \frac{1}{x^2}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$
$$= \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \cdot \frac{x^2(x+h)^2}{x^2(x+h)^2}$$
$$= \frac{x^2 - (x+h)^2}{hx^2(x+h)^2}$$
$$= \frac{x^2 - (x^2 + 2xh + h^2)}{hx^2(x+h)^2}$$
$$= \frac{-2xh - h^2}{hx^2(x+h)^2}$$
$$= \frac{h(-2x - h)}{hx^2(x+h)^2}$$
$$= -\frac{2x+h}{x^2(x+h)^2} \text{ or } -\frac{2x+h}{x^2(x^2 + 2xh + h^2)}$$
$$\text{or } -\frac{2x+h}{x^4 + 2x^3h + x^2h^2}$$

74. $f(x) = \sqrt{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

75. **a.** 2.70481

b. 2.71815

c. 2.71828

d. Yes, 2.71828

76.

x	$\left(1 + \frac{1}{x}\right)^x$
100	2.70481
10,000	2.71815
1,000,000	2.71828
10,000,000	2.71828

Yes, 2.71828

The graph of $y\ x\ 3^{3}\ 6$ is the same shape as the graph of $y\ x^{3}$ but it is shifted left 3 units and up 6 units.

Check:

on [–10, 10] by [–10, 10]

The graph of $y\ x\ 4^{2}\ 8$ is the same shape as the graph of $y\ x^{2}$ but it is shifted right 4 units and up 8 units.

Check:

on [–10, 10] by [–10, 10]

$P\ x\ \ 522\ 1 .0053^{x}$

$P\ 50\ \ 522\ 1.0053^{50}$ 680 million people in 1750

$P\ 100\ \ 522\ 1.0053^{100}$

886 million people in 1800

a. For x = 3000, use $f(x) = 0.10x$. $f\ x$ 0.10 x

3000 0.10 3000 \$300 **b.**

For x = 5000,

use $f(x) = 0.10x$.

$x\ \ 0.10x$

5000 0.10(5000) \$500 **c.**

For x = 10,000,

use $f(x) = 500 + 0.30(x – 5000)$.

$f\ x\quad 500\quad 0.30\ x\quad 5000$

$f\ 10,000\quad 500\quad 0.30\ 10,000\quad 5000$

500 0.30 5000

\$2000

d.

a. For x = 3000, use $f(x) = 0.15x$. $f\ x$ 0.15 x

f 3000 *0.15 3000 \$450*

For x = 6000,

use $f(x) = 0.15x$.

$x\ \ 0.15x$

6000 0.15(6000) \$900 **c.**

For x = 10,000,

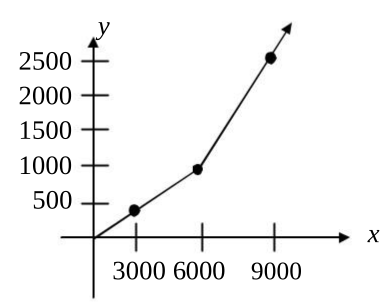
use $f(x) = 900 + 0.40(x – 6000)$.

$f\ x\quad 900\quad 0.40\ x\quad 6000$

$f\ 10,000\quad 900\quad 0.40\ 10,000\quad 6000$

900 0.40 4000

\$2500



83. a. For $x \le \frac{2}{3}$, use $f(x) = 10.5x$.

$f(x) = 10.5x$

$f\left(\frac{2}{3}\right) = 10.5\left(\frac{2}{3}\right)$

$= 7$ years

For $x \le \frac{4}{3}$, use $f(x) = 10.5x$.

$f(x) = 10.5x$

$f\left(\frac{4}{3}\right) = 10.5\left(\frac{4}{3}\right)$

$= 14$ years

For $x = 4$, use $f(x) = 21 + 4(x - 2)$.

$f(x) = 21 + 4(x - 2)$

$f(4) = 21 + 4(4 - 2)$

$= 21 + 4(2)$

$= 29$ years

For $x = 10$, use $f(x) = 21 + 4(x - 2)$.

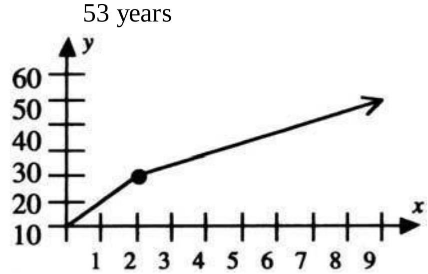
$f(x) = 21 + 4(x - 2)$

$f(10) = 21 + 4(10 - 2)$

$= 21 + 4(8)$

$= 53$ years

b.



Substitute $K = 24L^{-1}$ into $3L + 8K = 48$.

$3L + 8(24L^{-1}) = 48$

$3L + \frac{192}{L} = 48$

$3L^2 + 192 = 48L$

$3L^2 - 48L + 192 = 0$

$3(L^2 - 16L + 64) = 0$

$3(L - 8)^2 = 0$

So, $L = 8$.

And $K = 24(8)^{-1} = \frac{24}{8} = 3$

The intersection point is (8, 3).

First find the composition $R(v(t))$.

$R(v(t)) = 2(v(t))^{0.3}$

$= 2(60 + 3t)^{0.3}$

Then find $R(v(10))$.

84. a. For $x \le \frac{2}{3}$, use $f(x) = 15x$.

$f(x) = 15x$

$f\left(\frac{2}{3}\right) = 15\left(\frac{2}{3}\right)$

$= 10$ years

For $x = \frac{4}{3}$, use $f(x) = 15 + 9(x - 1)$.

$f(x) = 15 + 9(x - 1)$

$f\left(\frac{4}{3}\right) = 15 + 9\left(\frac{4}{3} - 1\right)$

$= 18$ years

For $x = 4$, use $f(x) = 15 + 9(x - 1)$.

$f(x) = 15 + 9(x - 1)$

$f(4) = 15 + 9(4 - 1) = 15 + 9(3)$

$= 32$ years

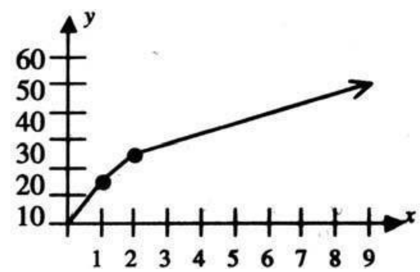
For $x = 10$, use $f(x) = 15 + 9(x - 1)$.

$f(x) = 15 + 9(x - 1)$

$f(10) = 15 + 9(10 - 1) = 15 + 9(8)$

$= 56$ years

b.



Substitute $K = 180L^{-1}$ into $5L + 4K = 120$.

$5L + 4(180L^{-1}) = 120$

$5L + \frac{720}{L} = 120$

$5L^2 + 720 = 120L$

$5L^2 - 120L + 720 = 0$

$5(L^2 - 24L + 144) = 0$

$5(L - 12)^2 = 0$

So, $L = 12$.

And $K = 180(12)^{-1} = \frac{180}{12} = 15$

The intersection point is (12, 15).

We must find the composition $R(p(t))$.

$R(p(t)) = 3(p(t))^{0.25}$

$= 3(55 + 4t)^{0.25}$

$R\ v\ 10 \quad 2\ 60 \quad 3(10)^{0.3} \quad 2\ 90^{0.3}$

$\$7.714$ million

$R(5)\ \ 3[55 \quad 4(5)]^{.0.25}$

$3(75)^{0.25} \quad \$8.8$ million

a. $f(x)$ 4^{10} 1,048,576 cells 1 million cells

b. $f(15)$ 4^{15} 1,073, 741,824 cells No, the

mouse will not survive beyond day 15.

91. **a.**

y 9 0.547^x (rounded)

b. For year 2020, $x = 6$.

y

9 0.547^6 0.24 weeks

or less than 2 days

One will have “missing points” at the excluded x-values.

A slope of 1 is a tax of 100%. That means, all dollars taxed are paid as the tax.

f $f(x)f(x)$ a

$(x\ a)\ a\ x$

$2a$

$f(x$ 10) is shifted to the left by 10 units.

False: $f(x\ h)\ (x\ h)^2\ x^2\ 2xh\ h^2$, not $x^2\ h^2$.

a.

on [–5, 5] by [–5, 5]

Note that each line segment in this graph includes its left end-point, but excludes its right endpoint. So it should be drawn like ●——○.

Domain = ; range = the set of integers

105. **a.**

$f\ g\ x\ \ a\ g\ \ x\ \ b\ a\ cx\ d\ b$

$acx\ ad\ b$

b. Yes

The value 2020 – 2012 = 8 corresponds to the year 2020. Substitute 8 for x.

$f(8)$ $226(1.11)^8$ \$521 billion

There will be about \$521 billion of e-commerce in the year 2020.

92. **a.**

y 19.04 1.096^x (rounded)

b. For year 2020, $x = 5$.

y 19.04 1.096^5 30.1 million

$x^2\ 1$ $\sqrt{\ }$ is not a polynomial because the exponent is not a non-negative integer.

$f\ f(x)\ a\ f(x)\ a\ (ax)$

$a^2\ x$

$f(x)$ 10 is translated up by 10 units.

$f(x$ 10) 10 is shifted up 10 units and left 10 units.

True: $f(x\ h)\ m(x\ h)\ b\ mx\ b\ mh$

$f(x)\ mh$

a.

on [–10, 10] by [–10, 10]

Domain = ; range = the set of even integers.

106. **a.**

$f\ g\ x\ \ g\ x\ \ {}_2x^2\ {}^2\ \ x^4$

Yes, because the composition of two polynomials involves raising integral powers to integral powers.

REVIEW EXERCISES AND CHAPTER TEST FOR CHAPTER 1

$\{x \mid 2 < x \le 5\}$

$\{x \mid -2 \le x < 0\}$

$\{x \mid x \ge 100\}$

$\{x \mid x \le 6\}$

Hurricane: [74,); storm: [55, 74);
gale: [38, 55); small craft warning: [21,38)

a. (0,)
(– ,0)
[0,)
(– ,0]

$y\ 3\ 2\ x\ 1\ y\ 3\ 2\ x\ 2$
$y\ \ 2\ x\ \ 5$

$y\ 6\ 3\ x\ 1\ y\ 6\ 3x\ 3$

$y\ \ 3x\ \ 3$

Since the vertical line passes through the point with x-coordinate 2, the equation of the line is $x = 2$.

Since the horizontal line passes through the point with y-coordinate 3, the equation of the line is
$y = 3$.

First, calculate the slope from the two points.

$m\ \frac{3\ 3}{21}\ \frac{6}{3}\ 2$

Now use the point-slope formula with this slope and the point (–1, 3).

$3\ 2\ x1$

$y\ 3\ \ 2\ x\ \ 2\ y$
$2\ x\ \ 1$

First find the slope of the line $x\ 2\ y\ 8$. Write the equation in slope-intercept form.

$\frac{1}{2}\ x\ \ 4.$

The slope of the perpendicular line is $m\ 2$.
Next, use the point-slope form with the point (6, –1):

$y\ y_1\ \ m\ x\ \ x_1$
$y\ \ 1\ \ 2\ x\ \ 6$
$y\ \ 2\ x\ \ 13$

Since the y-intercept is (0, –1), $b = -1$. To find the slope, use the slope formula with the points (0, –1) and (1, 1).

$m\ 1\ \frac{1}{1\ 0}\ \ 2$

Thus the equation of the line is $y = 2x - 1$.

14. Since the y-intercept is (0, 1), $b = 1$. To find the slope, use the slope formula with the points (0, 1) and (2, 0).

$m\ \frac{0\ 1}{2}\ 0\ \frac{1}{2}$

The equation of the line is $y\ \frac{1}{2}\ x\ \ 1$

a. Use the straight-line depreciation formula with price = 25,000, useful lifetime = 8, and scrap value = 1000.

$\text{Value}\ \text{price}\ \frac{\text{price} - \text{scrap value}}{\text{useful lifetime}}\ t$

$25,000\ \frac{25,000\ \ 1000}{8}\ t$

$25,000\ \frac{24,000}{8}\ t$

$25,000\ 3000t$

Value after 4 years 25, 000 3000 4 25, 000 12,000 \$13, 000

a. Use the straight-line depreciation formula with price = 78,000, useful lifetime = 15, and scrap value = 3000.

$\text{Value}\ \ \text{price}\ \ \frac{\text{price} - \text{scrap value}}{\text{useful lifetime}}\ t$

$78,000\ \frac{78,000\ \ 3000}{15}\ t$

$78,000\ \frac{75,000}{15}$

$t\ 78,000\ 5000t$

Value after 8 years 78, 000 5000 8 78, 000 40, 000 \$38, 000

a.

5.04 x 2.45

c. For the year 2020, $x = 12$

5.04 12 2.45 58.03

or about 58 thousand

The number is increasing by about 5000 3D screens per year.

$1\frac{6}{}2 \quad 1\frac{6}{2}2 \quad 36$

12/

20. 64   64 8

/   /   3 3

22. 81   3 4   1 3 4 $\sqrt[4]{\frac{1}{81}}$ 1

81   3   27

2

1

$4^{\frac{}{3}} 3 \; 4^{\frac{3}{}}$

$1000^{1/3}$ [redacted] 10

23. 100 3/2 $\frac{1}{100}$ / 3 2 $\frac{1}{\sqrt{100}}$ 3 $\frac{1}{10}$ 3 $\frac{1}{1000}$

3

24. $\left(\frac{8}{27}\right)^{2/3}$ $\left(\frac{27}{8}\right)^{2/3}$ $\sqrt{\sqrt[3]{\frac{27}{8}}}$

$\frac{3}{2}2$ $\frac{9}{4}$

25. $\left(\frac{9}{16}\right)^{3/2}$ $\left(\frac{16}{9}\right)^{3/2}$ $\sqrt{\frac{16}{9}}$ $\frac{4}{3}$ 3 $\frac{64}{27}$

26. 13.97

112.32

28. **a.** $y \;\; 0.86x^{0.47}$

$y \;\; 0.86(4000)^{0.47} \;\; 42.4$

The weight for the top cold-blooded meat-eating animals in Hawaii is 42.4 lbs.

$y \;\; 0.86x^{0.47}$

$y \;\; 0.86(9,400,000)^{0.47} \;\; 1628.8$

The weight for the top cold-blooded meat-eating animals in North America is 1628.8 lbs.

29. **a.** $y \;\; 1.7x^{0.52}$

$1.7(4000)^{0.52} \;\; 126.9$

The weight for the top warm-blooded plant-eating animals in Hawaii is 126.9 lbs.

$y \;\; 1.7x^{0.52}$

$1.7(9,400,000)^{0.52} \;\; 7185.8$

The weight for the top warm-blooded plant-eating animals in Hawaii is 7185.8 lbs.

a.

$4.55x^{0.643}$ (rounded)

For year 2020, $x = 12$.

$4.55 \; 12^{0.643}$ \$22.5 billion

is defined only for all values of $x \geq 7$.

Range = $\{y \mid y \geq 0\}$

31. **a.** $f\,11 \;\; \sqrt{11 \; 7} \;\; \sqrt{4} \;\; 2$

b. Domain = $\{x \mid x \geq 7\}$ because $\sqrt{x \; 7}$

32. **a.** $g\ 1 \quad \frac{1}{1\ \ 3^{2}} \quad \frac{1}{\ }$

b. Domain = $\{t \mid t \neq -3\}$

c. Range = $\{y \mid y \neq 0\}$

33. **a.** $h(16) = 16^{-3/4} = \frac{1}{16^{3/4}} = \frac{1}{\left(\sqrt[4]{16}\right)^3} = \frac{1}{2^3} = \frac{1}{8}$

Domain = {w | $w > 0$} because the fourth root is defined only for nonnegative numbers and division by 0 is not defined.
Range = {y | $y > 0$}

34. **a.** $w(8) = 8^{-4/3} = \frac{1}{8^{4/3}} = \frac{1}{\left(\sqrt[3]{8}\right)^4} = \frac{1}{2^4} = \frac{1}{16}$

Domain = {z | z ≠ 0} because division by 0 is not defined.

Range = {y | $y > 0$}

Yes

No

37.

38.

39.

40.

41. **a.** $3x^2 + 9x = 0$

$3x(x + 3) = 0$

Equals 0 at $x = 0$; Equals 0 at $x = -3$

$x = 0$ and $x = -3$

Use the quadratic formula with $a = 3$, $b = 9$, and $c = 0$

$$\frac{-9 \pm \sqrt{9^2 - 4 \cdot 3 \cdot 0}}{2 \cdot 3} = \frac{-9 \pm \sqrt{81}}{6} = \frac{-9 \pm 9}{6} = 0, -3$$

$x = 0$ and $x = -3$

42. **a.** $2x^2 - 8x - 10 = 0$

$2(x^2 - 4x - 5) = 0$

$2(x - 5)(x + 1) = 0$

Equals 0 at $x = 5$; Equals 0 at $x = -1$

$x = 5$ and $x = -1$

Use the quadratic formula with $a = 2$, $b = -8$, and $c = -10$

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-10)}}{2(2)} = \frac{8 \pm \sqrt{64 + 80}}{4} = \frac{8 \pm 12}{4} = 5, -1$$

$x = 5$ and $x = -1$

43. **a.** $3x^2\ 3x\ 5\ \ 11$

$3x^2\ 3x\ 6\ \ 0$

$3\ x^2\ \ x\ 2\ \ 0$

$x\ 2\ x\ 1\ \ 0$

Equals 0 at x 2 Equals 0 at x 1

x 2 and x 1

b. $3\ 3^2\ 436$ / 2 3 $\quad$ 3 972 / 6 $\quad$ 3 81 / 6 $\quad$ 3 9 / 6

2,1

x = –2 and x = 1

44. **a.** $4x^2\ \ 2\ 2$

$4x^2\ \ 4$

x^2

x 1

x1

x = 1 and x = –1

b. 0 $\sqrt{0^2\ 44\ 4}$ / 2 4 $\quad$ 64 / 8 $\quad$ 8 / 8 $\quad$ 1

x = 1 and x = –1

a. Use the vertex formula with a = 1 and b = –10.

$x\ \ \frac{b}{2a}\ \ \frac{10}{2\ 1}\ \ \frac{10}{2}\ \ 5$

To find y, evaluate f(5).

$5\ \ (5)^2\ \ 10\ 5\ \ 25\ \ 50$

The vertex is (5, –50).

b.

on [–5, 15] by [–50, 50]

a. Use the vertex formula with a = 1 and b = 14.

$x\ \ \frac{b}{2a}\ \ \frac{14}{2\ 1}\ \ 7$

To find y, evaluate f(–7).

$f\ 7\ 7^2\ \ 14\ 7\ 15\ 64$ The vertex is (–7, –64)

b.

on [–20, 10] by [–65, 65]

Let x = number of miles per day. $C(x) = 0.12x + 45$

Let x = the altitude in feet.

$T\ x\ \ 70\ \frac{x}{300}$

Use the interest formula with P = 10,000 and r = 0.08.

$I(t) = 10{,}000(0.08)t = 800t$

Let t = the number of years after 2010.

$C\ t\ \ 0.45t\ \ 20.3\ 25$

$0.45t\ \ 20.3$

t 10.4 years after 2010; in the year 2020

a. To find the break even points, solve the equation $C(x) = R(x)$ for x.

$C\ x \qquad R\ x$

$80\ x\ \ 1950\ \ \ 2\ x^2\ \ \ 240\ x$

$2\ x^2\ \ 160\ x\ \ 1950\ \ 0$

$x^2\ \ 80\ x\ \ 975\ \ 0$

$x\ \ 65\ x\ \ 15\ \ 0$

Equals 0 at x 65 Equals 0 at x 15

x 65 and x 15

The store breaks even at 15 receivers and at 65 receivers.

a. To find the break even points, solve the equation $C(x) = R(x)$ for x.

$C\ x \qquad R\ x$

$220x\ \ 202{,}500\ \ \ 3\ x^2\ \ 2020x$

$3x^2\ \ 1800x\ \ 202{,}500\ \ 0$

$x^2\ \ 600x\ \ 67{,}500\ \ 0$

$x\ \ 450\ \ x\ \ 150\ \ 0$

Equals 0 at x 450 Equals 0 at x 150

x 450 and x 150

The outlet breaks even at 150 units and 450 units.

b. To find the number of receivers that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.

$P(x) \quad 2x^2 \quad 240x \quad 80x \quad 1950$

$2x^2 \quad 160x \quad 1950$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

$x \quad \frac{b}{2a} \quad \frac{160}{2 \quad 2} \quad \frac{160}{4} \quad 40$

Thus, profit is maximized when 40 receivers are installed per week. The maximum profit is found by evaluating $P(40)$.

$P\ 40 \quad 2\ 40^2 \quad 160\ 40 \quad 1950$

$1250

Therefore, the maximum profit is $1250.

a.

$1.675x^2 \quad 0.435x \quad 21.625$ (rounded)

54. a. $f\ 1 \quad \frac{3}{1 \quad 1 \quad 2} \quad \frac{3}{3}\ 1$

Domain = $\{x \mid x \neq 0, x \neq 2\}$

Range = $\{y \mid y > 0 \text{ or } y \leq -3\}$

$g\ 4 \quad 4\ 2\ 2\ 2\ 2\ 0$

56. a. | |

Domain =

Range = $\{y \mid y \geq 0\}$

$5x^4 \quad 10x^3 \quad 15x^2$

$5x^4 \quad 10x^3 \quad 15x^2 \quad 0$

$5x^2 \quad x^2 \quad 2x \quad 3 \quad 0$

$x^2 \quad x \quad 3 \quad x \quad 1 \quad 0$

Equals 0 at x 0 Equals 0 at x 3 Equals 0 at x 1

x 0, x 3, and x 1

$2x^{5/2} \quad 8x^{3/2} \quad 10x^{1/2}$

$2x^{5/2} \quad 8x^{3/2} \quad 10x^{1/2} \quad 0$

$2x^{1/2}(x^2 \quad 4x \quad 5) \quad 0$

$x^{1/2}(x \quad 5)(x \quad 1) \quad 0$

Equals 0 Equals 0 Equals 0

at x 0 at x 5

at x 1

x 0, x 5

and x 1

Only $x = 0$ and $x = 5$ are solutions.

8 8 4 32 2

Domain = $\{x \mid x \neq 0, x \neq -4\}$

Range = $\{y \mid y > 0 \text{ or } y \leq -4\}$

52. **b.** To find the number of units that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.

$P\ x \quad 3x^2 \ 2020x \quad 220x \quad 202,500$

$3x^2 \quad 1800x \ 202,500$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

$x \quad \frac{b}{2a} \quad \frac{1800}{2 \cdot 3} \quad \frac{1800}{6} \quad 300$

Thus, profit is maximized when 300 units ar installed per month. The maximum profit is found by evaluating $P(300)$.

$300\ 3\ 300^2\ 1800\ 300\ 202{,}500$

$67,500

Therefore, the maximum profit is $67,500.

For year 2020, $x = 12$.

$1.675\ 12^2\ 0.435\ 12\ 21.6$

$268 billion

---

55. **a.** $f\ 8 \quad \frac{16}{} \quad \frac{16}{} \quad \frac{1}{}$

57. **a.** $g\ 5 \quad 5 \quad |5| \quad 5\ 5 \quad 10$

Domain =

Range = $\{y \mid y \geq 0\}$

$4x^5\ 8x^4\ 32x^3\ 4x^5\ 8x^4\ 32x^3$
0

$4x^3\ x^2\ 2x\ 8\ 0$

$^3\ x\ 4\ x\ 2\ 0$

Equals 0 Equals 0 Equals 0 at
x 0 at x 4 at x 2 x
0, x 4, and x 2

$3x^{5/2}\ 3x^{3/2}\ 18x^{1/2}\ 3x^{5/2}\ 3x$

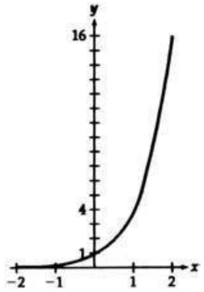
$^{3/2}\ 18x^{1/2}\ 0$

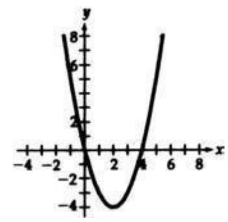
$3x^{1/2}(x^2\ x\ 6)\ 0$

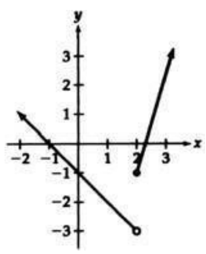
$x^{1/2}(x\ 3)(x\ 2)\ 0$

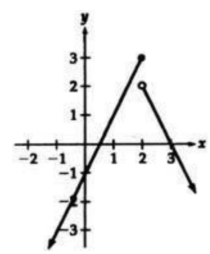
Equals 0 Equals 0 Equals 0
at x 0 at x ~~3~~ at x 2
x 0, ~~x 3~~ and x 2

Only $x = 0$ and $x = 2$ are solutions.

62. 

63. 

64. 

65. 

66. **a.** $(f \circ g)(x) = f(g(x)) = (g(x))^2 + 1 = \left(\frac{1}{x}\right)^2 + 1 = \frac{1}{x^2} + 1$

b. $(g \circ f)(x) = g(f(x)) = \frac{1}{f(x)} = \frac{1}{x^2 + 1}$

67. **a.** $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{5x - 4}$

b. $(g \circ f)(x) = g(f(x)) = 5f(x) - 4 = 5\sqrt{x} - 4$

68. **a.** $(f \circ g)(x) = \frac{g(x) + 1}{g(x) - 1} = \frac{x^3 + 1}{x^3 - 1}$

b. $(g \circ f)(x) = (f(x))^3 = \left(\frac{x + 1}{x - 1}\right)^3$

69. **a.** $(f \circ g)(x) = |g(x)| = |x - 2|$

b. $(g \circ f)(x) = f(x) - 2 = |x| - 2$

$f(x) = 2x^2 - 3x + 1$

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$$

$$= \frac{4xh + 2h^2 - 3h}{h}$$

$$= \frac{h(4x + 2h - 3)}{h}$$

$$= 4x + 2h - 3$$

$f(x) = \frac{5}{x}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{5}{x+h} - \frac{5}{x}}{h}$$

$$= \frac{\frac{5}{x+h} - \frac{5}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \frac{5x - 5(x+h)}{hx(x+h)}$$

$$= \frac{5x - 5x - 5h}{hx(x+h)}$$

$$= \frac{-5h}{hx(x+h)}$$

$$= \frac{-5}{x(x+h)}$$

The advertising budget A as a function of t is the composition of $A(p)$ and $p(t)$.

0.15

$A\ p\ t\quad 2\quad p\quad t\qquad 2\ 18\ 2t\quad 0.15$

$A\ 4\ 2\quad 18\ 2\quad 4\ {}_{0.15}\quad 2\ 26^{0.15}$

$3.26 million

73. a. $x^4\quad 2x^3\quad 3x^2\quad 0$

$x^2(x^2\quad 2x\ \ 3)\ \ 0$

$x^2(x\ \ 3)(x\ \ 1)\ \ 0$

Equals 0 Equals 0 Equals 0
at x 0 at x 3 at x 1
x 0, x 3 and x 1

b.

on [–5, 5] by [–5, 5]

74. a. $x^3\quad 2x^2\quad 3x\ \ 0$

$x\,x^2\quad 2x\ \ 3\quad 0$

$x\,x\ \ 3\ \ x\ \ 1\quad 0$

Equals 0 Equals 0 Equals 0
at x 0 at x 3 at x 1
x 0, x 3, and x 1

b.

on [–5, 5] by [–5, 5]

a.

$6.52\ 0.761^x$

b. For year 2020, $x = 4$.

$6.52\ 0.761^4\quad 2.2$

2.2 crimes per 100,000

Chapter 2: Derivatives and Their Uses

EXERCISES 2.1

1.

x	$5x-7$
1.9	2.500
1.99	2.950
1.999	2.995

x	$5x-7$
2.1	3.500
2.01	3.050
2.001	3.005

$\lim_{x\to 2^-}(5x-7) = 3$

$\lim_{x\to 2^+}(5x-7) = 3$

c. $\lim_{x\to 2}(5x-7) = 3$

2.

x	$2x+1$
3.9	8.800
3.99	8.980
3.999	8.998

x	$2x+1$
4.100	9.200
4.010	9.020
4.001	9.002

$\lim_{x\to 4^-}(2x+1) = 9$

$\lim_{x\to 4^+}(2x+1) = 9$

c. $\lim_{x\to 4}(2x+1) = 9$

3.

x	$\frac{x^3-1}{x-1}$
0.900	2.710
0.990	2.970
0.999	2.997

x	$\frac{x^3-1}{x-1}$
1.1	3.310
1.01	3.030
1.001	3.003

$\lim_{x\to 1^-}\frac{x^3-1}{x-1} = 3$

$\lim_{x\to 1^+}\frac{x^3-1}{x-1} = 3$

c. $\lim_{x\to 1}\frac{x^3-1}{x-1} = 3$

4.

x	$\frac{x^4-1}{x-1}$
0.9	3.439
0.99	3.940
0.999	3.994

x	$\frac{x^4-1}{x-1}$
1.1	4.641
1.01	4.060
1.001	4.006

$\lim_{x\to 1^-}\frac{x^4-1}{x-1} = 4$

$\lim_{x\to 1^+}\frac{x^4-1}{x-1} = 4$

c. $\lim_{x\to 1}\frac{x^4-1}{x-1} = 4$

5.

x	$(1+2x)^{1/x}$
–0.1	9.313
–0.01	7.540
–0.001	7.404

x	$(1+2x)^{1/x}$
0.1	6.192
0.01	7.245
0.001	7.374

$\lim_{x\to 0}(1+2x)^{1/x} \approx 7.4$

6.

x	$(1+x)^{1/x}$
–0.1	0.386
–0.01	0.370
–0.001	0.368

x	$(1+x)^{1/x}$
0.1	0.349
0.01	0.366
0.001	0.368

$\lim_{x\to 0}(1-x)^{1/x} \approx 0.368$

7.

x	$\frac{\frac{1}{x}-\frac{1}{2}}{x-2}$
1.9	–0.263
1.99	–0.251
1.999	–0.250

x	$\frac{\frac{1}{x}-\frac{1}{2}}{x-2}$
2.1	-0.238
2.01	-0.249
2.001	-0.250

$\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2} = -0.25$

8.

x	$\frac{\sqrt{x}-1}{x-1}$
0.9	0.513
0.99	0.501
0.999	0.500

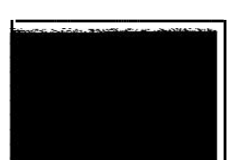
x	$\frac{\sqrt{x}-1}{x-1}$
1.1	0.488
1.01	0.499
1.001	0.500

$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1} = 0.5$

9.

on [0, 2] by [0, 5]

10.



on [0, 3] by [0, 10]

$$\lim_{x \to 1} \frac{\frac{1}{x} - 1}{1 - x} = 1$$

$$\lim_{x \to 1.5} \frac{2x^2 - 4.5}{x - 1.5} = 6$$

11. [redacted]

$\lim_{x \to 4} \frac{x^{1.5} - 4x^{0.5}}{x^{1.5} - 2x} = 2$

12. [redacted]

$\lim_{x \to 1} \frac{x - 1}{\sqrt{x} - x} = -2$

13. $\lim_{x \to 3} (4x^2 - 10x + 2) = 4(3)^2 - 10(3) + 2 = 8$

14. $\lim_{x \to 7} \frac{x^2 - x}{2x - 7} = \frac{(7)^2 - 7}{2(7) - 7} = 6$

15. $\lim_{x \to 5} \frac{3x^2 - 5x}{7x - 10} = \frac{3(5)^2 - 5(5)}{7(5) - 10} = 2$

16. $\lim_{t \to 3} \sqrt[3]{t^2 + t - 4} = \sqrt[3]{(3)^2 + 3 - 4} = 2$

17. $\lim_{x \to 3} \sqrt{2} = \sqrt{2}$ because the limit of a constant is just the constant.

18. $\lim_{q \to 9} \frac{8 + 2\sqrt{q}}{8 - 2\sqrt{q}} = \frac{8 + 2\sqrt{9}}{8 - 2\sqrt{9}} = \frac{14}{2} = 7$

19. $\lim_{t \to 25} \left[(t + 5)t^{-1/2}\right] = (25 + 5)(25)^{-1/2} = 6$

20. $\lim_{s \to 4} (s^{3/2} - 3s^{1/2}) = 4^{3/2} - 3(4)^{1/2} = 2$

21. $\lim_{h \to 0} (5x^3 + 2x^2h - xh^2) = 5x^3 + 2x^2 \cdot 0 - x(0)^2 = 5x^3$

22. $\lim_{h \to 0} (2x^2 + 4xh + h^2) = 2x^2 + 4x \cdot 0 + (0)^2 = 2x^2$

23. $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x - 2)(x + 2)}{x - 2} = \lim_{x \to 2} (x + 2) = 2 + 2 = 4$

24. $\lim_{x \to 1} \frac{x - 1}{x^2 + x - 2} = \lim_{x \to 1} \frac{x - 1}{(x - 1)(x + 2)} = \lim_{x \to 1} \frac{1}{x + 2} = \frac{1}{1 + 2} = \frac{1}{3}$

25. $\lim_{x \to 3} \frac{x - 3}{x^2 - 8x + 15} = \lim_{x \to 3} \frac{x - 3}{(x - 3)(x - 5)} = \lim_{x \to 3} \frac{1}{x - 5} = \frac{1}{3 - 5} = -\frac{1}{2}$

26. $\lim_{x \to 4} \frac{x^2 - 9x + 20}{x - 4} = \lim_{x \to 4} \frac{(x - 5)(x - 4)}{x - 4} = \lim_{x \to 4} (x - 5) = 4 - 5 = -1$

27. $\lim_{x \to -1} \frac{3x^3 - 3x^2 - 6x}{x^2 + x} = \lim_{x \to -1} \frac{3x(x^2 - x - 2)}{x(x + 1)} = \lim_{x \to -1} \frac{3x(x - 2)(x + 1)}{x(x + 1)} = \lim_{x \to -1} 3(x - 2) = 3(-1 - 2) = -9$

28. $\lim_{x \to 0} \frac{x^2 - x}{x^2 + x} = \lim_{x \to 0} \frac{x(x - 1)}{x(x + 1)} = \lim_{x \to 0} \frac{x - 1}{x + 1} = -1$

29. $\lim_{h \to 0} \frac{2xh - 3h^2}{h} = \lim_{h \to 0} (2x - 3h) = 2x - 3(0) = 2x$

30. $\lim_{h \to 0} \frac{5x^4h - 9xh^2}{h} = \lim_{h \to 0} (5x^4 - 9xh)$

$5x \quad 9x(0) \;\; 5x$

31. $\lim_{h \to 0} \frac{4x^2h \;\; xh^2 \;\; h^3}{h} \quad \lim_{h \to 0} (4x^2 \quad xh \;\; h^2)$

$4x^2 \quad x(0) \quad (0)^2 \quad 4x^2$

32. $\lim_{h \to 0} \frac{x^2h \;\; xh^2 \;\; h^3}{h} \quad \lim_{h \to 0} (x^2 \;\; xh \;\; h^2)$

$x^2 \quad x(0) \quad (0)^2 \quad x^2$

33. **a.** $\lim_{x\ 2} f(x)\ 1$

b. $\lim_{x\ 2} f(x)\ 3$

c. $\lim_{x\ 2} f(x)$ does not exist.

34. **a.** $\lim_{x\ 2} f(x)\ 1$

b. $\lim_{x\ 2} f(x)\ 2$

c. $\lim_{x\ 2} f(x)$ does not exist.

35. **a.** $\lim_{x\ 2} f(x)\quad 1$

b. $\lim_{x\ 2} f(x)\quad 1$

c. $\lim_{x\ 2} f(x)\quad 1$

36. **a.** $\lim_{x\ 2} f(x)\ 3$

b. $\lim_{x\ 2} f(x)\ 3$

c. $\lim_{x\ 2} f(x)\ 3$

37. **a.** $\lim_{x\ 4} f(x)\quad \lim_{x\ 4}(3\ x)$

$3(4)\ 1$

b. $\lim_{x\ 4} f(x)\ \lim_{x\ 4}(10\ 2x)$

$10\ 2(4)\ 2$

c. $\lim_{x\ 4} f(x)$ does not exist.

38. **a.** $\lim_{x\ 4} f(x)\quad \lim_{x\ 4}(5\ x)$

$5\ 4\ 1$

b. $\lim_{x\ 4} f(x)\quad \lim_{x\ 4}(2x\ 5)$

$2(4)\ 5\ 3$

c. $\lim_{x\ 4} f(x)$ does not exist.

39. **a.** $\lim_{x\ 4} f(x)\quad \lim_{x\ 4}(2\ x)$

$2\ 4\quad 2$

b. $\lim_{x\ 4} f(x)\quad \lim_{x\ 4}(x\ 6)$

$4\ 6\quad 2$

c. $\lim_{x\ 4} f(x)\quad 2$

40. **a.** $\lim_{x\ 4} f(x)\quad \lim_{x\ 4}(2\ x)$

$2\ 4\quad 2$

b. $\lim_{x\ 4} f(x)\quad \lim_{x\ 4}(2x\ 10)$

$2(4)\ 102$

c. $\lim_{x\ 4} f(x)\quad 2$

41. **a.** $\lim_{x\ 0} f(x)\quad \lim_{x\ 0}(\ x)$

$0\quad 0$

b. $\lim_{x\ 0} f(x)\ \lim_{x\ 0}(x)$

0

c. $\lim_{x\ 0} f(x)\ 0$

42. **a.** $\lim_{x\ 0} f(x)\ \lim_{x\ 0}[\ (\ x)]$

$(0)\ 0$

b. $\lim_{x\ 0} f(x)\ \lim_{x\ 0}(x)$

$0\quad 0$

c. $\lim_{x\ 0} f(x)\ 0$

43. **a.**

x	$\frac{\lvert x \rvert}{x}$
–0.1	–1
–0.01	–1
–0.001	–1

$\lim_{x\ 0} f(x)\quad 1$

b.

x	$\frac{\lvert x \rvert}{x}$
0.1	1
0.01	1
0.001	1

$\lim_{x\ 0} f(x)\quad 1$

c. $\lim_{x\ 0} f(x)$ does not exist.

44. **a.**

x	$\frac{\lvert x \rvert}{x}$
–0.1	1
–0.01	1
–0.001	1

$\lim_{x\ 0} f(x)\ 1$

b.

x	$\frac{\lvert x \rvert}{x}$
0.1	–1
0.01	–1
0.001	–1

$\lim_{x\ 0} f(x)\quad 1$

c. $\lim_{x\ 0} f(x)$ does not exist.

45. $\lim_{x} f(x)\quad 0;\qquad \lim_{x\ 3} f(x)$ and $\lim_{x\ 3} f(x),$

so $\lim_{x\ 3} f(x)$ does not exist and $\lim_{x} f(x)\quad 0.$

46. $\lim_{x} f(x)\quad 0;\qquad \lim_{x\ 3} f(x)$ *and* $\lim_{x\ 3} f(x)\quad ,$

so $\lim_{x\ 3} f(x)$ and $\lim_{x} f(x)\quad 0.$

47. $\lim_{x} f(x)$ 0; $\lim_{x\ 0} f(x)$ and $\lim_{x\ 0} f(x)$,

so $\lim_{x\ 0} f(x)$and $\lim_{x} f(x)$ 0.

48. $\lim_{x} f(x)$ 0; $\lim_{x\ 0} f(x)$ and $\lim_{x\ 0} f(x)$,

so $\lim_{x\ 0} f(x)$ does not exist and $\lim_{x} f(x)$ 0.

49. $\lim_{x} f(x)$ 2; $\lim_{x\ 1} f(x)$ and $\lim_{x\ 1} f(x)$,

so $\lim_{x\ 1} f(x)$ does not exist and $\lim_{x} f(x)$ 2.

50. $\lim_{x} f(x)$ 1; $\lim_{x\ 3} f(x)$*and* $\lim_{x\ 3} f(x)$,

so $\lim_{x\ 3} f(x)$ does not exist and $\lim_{x} f(x)$ 1.

51. $\lim_{x} f(x)$ 1; $\lim_{x\ 2} f(x)$ and $\lim_{x\ 2} f(x)$,

so $\lim_{x\ 2} f(x)$and $\lim_{x} f(x)$ 1.

52. $\lim_{x} f(x)$ 2; $\lim_{x\ 1} f(x)$ and $\lim_{x\ 1} f(x)$,

so $\lim_{x\ 1} f(x)$and $\lim_{x} f(x)$ 2.

Continuous

Discontinuous at c because $f(c)$ is not defined.

55. Discontinuous at c because $\lim_{x\ c} f(x)$ $f(c)$.

56. Discontinuous at c because $\lim_{x\ c} f(x)$ does not exist.

Discontinuous at c because $f(c)$ is not defined.

58. Continuous

59. Discontinuous at c because $\lim_{x\ c} f(x)$ does not exist.

60. Discontinuous at c because $\lim_{x\ c} f(x)$ $f(c)$.

a. 

b. $\lim_{x\ 3} f(x)$ 3; $\lim_{x\ 3} f(x)$ *3*

c. Continuous

a. 

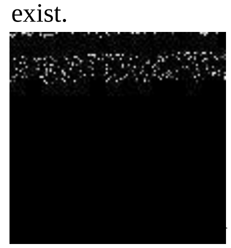
b. $\lim_{x\ 3}$ $\lim_{x\ 3} f(x)$ 1

c. Discontinuous because $\lim_{x\ 3} f(x)$ does not exist.

63. a. 

b. $\lim_{x\ 3} f(x)$ 3; $\lim_{x\ 3} f(x)$ *4*

c. Discontinuous because $\lim_{x\ 3} f(x)$ does not exist.

64. a. 

b. $\lim_{x\ 3}$ $\lim_{x\ 3} f(x)$ 2

c. Continuous

---

65. Continuous

66. Continuous

67. Discontinuous at $x = 1$

68. Discontinuous at $x = –7$ and $x = 2$

69. $f(x) = \frac{2}{5x^3 - 5x}$ is discontinuous at values of x for which the denominator is zero. Thus, consider

$$5x^3 - 5x = 0$$
$$5x(x^2 - 1) = 0$$

$5x$ equals zero at $x = 0$ and $x^2 - 1$ equals zero at $x = \pm 1$.

Thus, the function is discontinuous at $x = 0$, $x = -1$, and $x = 1$.

70. $f(x) = \frac{x - 2}{x^4 - 3x^3 - 4x^2}$ is discontinuous at values of x for which the denominator is zero. Thus, consider

$$x^4 - 3x^3 - 4x^2 = 0$$
$$x^2(x^2 - 3x - 4) = 0$$
$$x^2(x - 4)(x + 1) = 0$$

x^2 equals zero at $x = 0$, $x - 4$ equals zero at $x = 4$, and $x + 1$ equals zero at $x = -1$. Thus, the function is discontinuous at $x = 0$, $x = 4$, and $x = -1$.

From Exercise 37, we know $\lim_{x \to 4} f(x)$ does not exist. Therefore, the function is discontinuous at $x = 4$.

From Exercise 38, we know $\lim_{x \to 4} f(x)$ does not exist. Therefore, the function is discontinuous at $x = 4$.

From Exercise 39, we know $\lim_{x \to 4} f(x) = 2 = f(4)$. Therefore, the function is continuous.

From Exercise 40, we know $\lim_{x \to 4} f(x) = 2 = f(4)$. Therefore, the function is continuous.

From Exercise 41, we know $\lim_{x \to 0} f(x) = 0 = f(0)$. Therefore, the function is continuous.

76. From Exercise 43, we know $\lim_{x \to 0} f(x)$ does not exist. Therefore the function is discontinuous at $x = 0$.

77. From the graph, we can see that $\lim_{x \to 6} f(x)$ does not exist because the left and right limits do not agree. $f(x)$ is discontinuous at $x = 6$.

78. From the graph, we can see that $\lim_{x \to 7} f(x)$ does not exist because the left and right limits do not agree. $f(x)$ is discontinuous at $x = 7$.

79. Since the function $\frac{(x-1)(x+2)}{x-1}$ is not defined at $x = 1$ and the function $x + 2$ equals 3 at $x = 1$, the functions are not equal.

80. $\lim_{v \to c^-} \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \frac{c^2}{c^2}} = 0$

81.

x	$\left(1 + \frac{x}{10}\right)^{1/x}$
–0.1	1.11
–0.01	1.11
–0.001	1.11
–0.0001	1.11

x	$\left(1 + \frac{x}{10}\right)^{1/x}$
0.1	1.10
0.01	1.11
0.001	1.11
0.0001	1.11

$\lim_{x \to 0} \left(1 + \frac{x}{10}\right)^{1/x} \approx \$1.11$

82.

x	$\left(1 + \frac{x}{20}\right)^{1/x}$
–0.1	1.05
–0.01	1.05
–0.001	1.05
–0.0001	1.05

x	$\left(1 + \frac{x}{20}\right)^{1/x}$
0.1	1.05
0.01	1.05
0.001	1.05
0.0001	1.05

$\lim_{x \to 0} \left(1 + \frac{x}{20}\right)^{1/x} \approx \$1.05$

83. $\lim_{x \to 0} \frac{100}{1 + .001x^2} = \frac{100}{1 + .001(0)^2} = \frac{100}{1} = 100$

The left and right limits at 1 ounce, 2 ounces, and at 3 ounces do not agree. This function is discontinuous at 1 ounce, 2 ounces and at 3 ounces.

As *x* approaches *c*, the function is approaching $\lim_{x \to c} f(x)$ even if the value of the function at *c* is different, so the limit is where the function is "going".

In a continuous function, when *x* equals *c*, the function equals $\lim_{x \to c} f(x)$. This is not true for a discontinuous function.

As x approaches c, the function is approaching $\lim_{x \to c} f(x)$ even if the value of the function at c is different, so the limit is where the function is "going".

False: The value of the function at 2 has nothing to do with the limit as x approaches 2.

False: Both one-sided limits would have to exist and agree to guarantee that the limit exists.

False: On the left side of the limit exists and equals 2 (as we saw in Example 4), but on the right side of the denominator of the fraction is zero. Therefore one side of the equation is defined and the other is not.

True: The third requirement for continuity at 2 is that the limit and the value at 2 must agree, so if one is 7 the other must be 7.

In a continuous function, when x equals c, the function equals $\lim_{x \to c} f(x)$. This is not true for a discontinuous function.

False: There could be a "hole" or "jump" at 2.

90. True: If $\lim_{x \to 2} f(x) = 7$, then $\lim_{x \to 2^-} f(x) = 7$ and $\lim_{x \to 2^+} f(x) = 7$.

True: A function must be defined at $x = c$ to be continuous at $x = c$.

True: If a function is continuous at every x-value, then its graph has no jumps or breaks. The jumps or breaks would make it discontinuous.

95. $\lim_{x \to 0^-} f(x)$ does not exist; $\lim_{x \to 0^+} f(x)$ does not exist; $\lim_{x \to 0} f(x)$ does not exist

EXERCISES 2.2

The slope is positive at P_1. The slope is negative at P_2. The slope is zero at P_3.

The slope is zero at P_1. The slope is positive at P_2. The slope is negative at P_3.

The slope is positive at P_1. The slope is negative at P_2. The slope is zero at P_3.

The slope is negative at P_1. The slope is zero at P_2. The slope is negative at P_3.

The tangent line at P_1 contains the points (0, 2) and (1, 5). The slope of this line is

$$\frac{5-2}{1-0} = 3.$$

The slope of the curve at P_1 is 3.

The tangent line at P_2 contains the points (3, 5) and (5, 4). The slope of this line is

$$m = \frac{4-5}{5-3} = -\frac{1}{2}$$

[redacted] at P_2 is $-\frac{1}{2}$.

Your [redacted]raph should look roughly like the [redacted]llowing:

The tangent line at P_1 contains the points (1, 4) and (4, 3). The slope of this line is

$$m = \frac{3-4}{4-1} = -\frac{1}{3}.$$

The slope of the curve at P_1 is $-\frac{1}{3}$.

The tangent line at P_2 contains the points (5, 3) and (6, 5). The slope of this line is

$$\frac{5-3}{6-5} = 2$$

[redacted] P_2 is 2.

Y[redacted]k roughly like the

a. $\frac{f(3)-f(1)}{2} = \frac{12-2}{2} = 5$

$\frac{f(2)-f(1)}{1} = \frac{6-2}{1} = 4$

$\frac{f(1.5)-f(1)}{.5} = \frac{3.75-2}{.5} = 3.5$

$\frac{f(1.1)-f(1)}{.1} = \frac{2.31-2}{.1} = 3.1$

$\frac{f(1.01)-f(1)}{.01} = \frac{2.0301-2}{.01} = 3.01$

f. Answers seem to be approaching 3.

a. $\frac{f(3)-f(1)}{2} = \frac{23-7}{2} = 8$

$\frac{f(2)-f(1)}{1} = \frac{13-7}{1} = 6$

$\frac{f(1.5)-f(1)}{.5} = \frac{9.5-7}{.5} = 5$

$\frac{f(1.1)-f(1)}{.1} = \frac{7.42-7}{.1} = 4.2$

$\frac{f(1.01)-f(1)}{.01} = \frac{7.0402-7}{.01} = 4.02$

f. Answers seem to be approaching 4.

11. a. $\frac{f(4)-f(2)}{2} = \frac{34-8}{2} = 13$

$\frac{f(3)-f(2)}{1} = \frac{19-8}{1} = 11$

$\frac{f(2.5)-f(2)}{.5} = \frac{13-8}{.5} = 10$

$\frac{f(2.1)-f(2)}{.1} = \frac{8.92-8}{.1} = 9.2$

$\frac{f(2.01)-f(1)}{.01} = \frac{8.0902-8}{.01} = 9.02$

f. Answers seen to be approaching 9.

12. a. $\frac{f(4)-f(2)}{2} = \frac{23-7}{2} = 8$

$\frac{f(3)-f(2)}{1} = \frac{14-7}{1} = 7$

$\frac{f(2.5)-f(2)}{.5} = \frac{10.25-7}{.5} = 6.5$

$\frac{f(2.1)-f(2)}{.1} = \frac{7.61-7}{.1} = 6.1$

$\frac{f(2.01)-f(1)}{.01} = \frac{7.0601-7}{.01} = 6.01$

f. Answers seem to be approaching 6.

13. a. $\frac{f(5)-f(3)}{2} = \frac{26-16}{2} = 5$

$\frac{f(4)-f(3)}{1} = \frac{21-16}{1} = 5$

$\frac{f(3.5)-f(3)}{.5} = \frac{18.5-16}{.5} = 5$

$\frac{f(3.1)-f(3)}{.1} = \frac{16.5-16}{.1} = 5$

$\frac{f(3.01)-f(3)}{.01} = \frac{16.05-16}{.01} = 5$

f. Answers seem to be approaching 5.

14. a. $\frac{f(5)-f(3)}{2} = \frac{33-19}{2} = 7$

$\frac{f(4)-f(3)}{1} = \frac{26-19}{1} = 7$

$\frac{f(3.5)-f(3)}{.5} = \frac{22.5-19}{.5} = 7$

$\frac{f(3.1)-f(3)}{.1} = \frac{19.7-19}{.1} = 7$

$\frac{f(3.01)-f(3)}{.01} = \frac{19.07-19}{.01} = 7$

f. Answers seem to be approaching 7.

15. a. $\frac{f(6)-f(4)}{2} = \frac{2.4495-2}{2} = 0.2247$

$\frac{f(5)-f(4)}{1} = \frac{2.236-2}{1} = 0.2361$

$\frac{f(4.5)-f(4)}{.5} = \frac{2.121-2}{.5} = 0.2426$

$\frac{f(4.1)-f(4)}{.1} = \frac{2.025-2}{.1} = 0.2485$

$\frac{f(4.01)-f(4)}{.01} = \frac{2.0025-2}{.01} = 0.2498$

Answers seem to approach 0.25.

16. a. $\frac{f(6)-f(4)}{2} = \frac{.6667-1}{2} = 0.1667$

$\frac{f(5)-f(4)}{1} = \frac{.8-1}{1} = 0.2000$

$\frac{f(4.5)-f(4)}{.5} = \frac{.889-1}{.5} = 0.2222$

$\frac{f(4.1)-f(4)}{.1} = \frac{.976-1}{.1} = 0.2439$

$\frac{f(4.01)-f(4)}{.01} = \frac{.9975-1}{.01} = 0.2494$

Answers seem to approach –0.25.

17. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2 + h}{h}$

$= \lim_{h \to 0} (2x + h + 1) = 2x + 1$

Evaluating at $x = 1$ gives $2(1) + 1 = 3$, which matches the answer from Exercise 9.

18. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) + 1 - (x^2 + 2x + 1)}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h + 1 - x^2 - 2x - 1}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h}$

$= \lim_{h \to 0} (2x + h + 2) = 2x + 2$

Evaluating at $x = 2$ gives $2(2) + 2 = 6$, which matches the answer from Exercise 12.

19. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{5(x+h) + 1 - (5x + 1)}{h}$

$= \lim_{h \to 0} \frac{5x + 5h + 1 - 5x - 1}{h}$

$= \lim_{h \to 0} \frac{5h}{h}$

$= \lim_{h \to 0} 5 = 5$

Evaluating at $x=3$ gives 5, which matches the answer from Exercise 13.

20. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{4}{(x+h)} - \frac{4}{x}}{h}$

$= \lim_{h \to 0} \frac{1}{h}\left(\frac{4}{x+h} - \frac{4}{x}\right)$

$= \lim_{h \to 0} \frac{1}{h}\left(\frac{4x - 4(x+h)}{(x+h)x}\right)$

$= \lim_{h \to 0} \frac{4x - 4x - 4h}{h(x^2 + xh)}$

$= \lim_{h \to 0} \frac{-4}{x^2 + xh} = -\frac{4}{x^2}$

Evaluating at $x=4$ gives $-\frac{4}{4^2} = -.25$, which matches the answer from Exercise 16.

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{2(x+h)^2 + (x+h) + 2 - (2x^2 + x + 2)}{h}$

$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 + x + h + 2 - 2x^2 - x - 2}{h}$

$= \lim_{h \to 0} \frac{4xh + 2h^2 + h}{h}$

$= \lim_{h \to 0} (4x + 2h + 1) = 4x + 1$

The slope of the tangent line is at $x = 2$ is $4(2) + 1 = 9$, which matches the answer from Exercise 11.

22. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2(x+h)^2 - 5 - (2x^2 - 5)}{h}$

$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 5 - 2x^2 + 5}{h}$

$= \lim_{h \to 0} \frac{4xh + 2h^2}{h}$

$= \lim_{h \to 0} (4x + 2h) = 4x$

The slope of the tangent line at $x = 1$ is $4(1) = 4$, which matches the answer from Exercise 10.

23. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

$= \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$= \lim_{h\to 0} \frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x+0}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$

The slope of the tangent line at $x = 4$ is $\frac{1}{2\sqrt{4}} = 0.25$, which matches the answer from Exercise 15.

24. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{7(x+h)-2-(7x-2)}{h}$

$= \lim_{h\to 0} \frac{7x+7h-2-7x+2}{h}$

$= \lim_{h\to 0} \frac{7h}{h}$

$= \lim_{h\to 0} 7 = 7$

The slope of the tangent line at $x = 3$ is 7, which matches the answer from Exercise 14.

25. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2-3(x+h)+5-(x^2-3x+5)}{h}$

$= \lim_{h\to 0} \frac{x^2+2xh+h^2-3x-3h+5-x^2+3x-5}{h}$

$= \lim_{h\to 0}(2x+h-3)$

$= 2x-3$

26. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2+5(x+h)-1-(2x^2+5x-1)}{h}$

$= \lim_{h\to 0} \frac{2x^2+4xh+2h^2+5x+5h-1-2x^2-5x+1}{h}$

$= \lim_{h\to 0}(4x+2h+5)$

$= 4x+5$

27. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{1-(x+h)^2-(1-x^2)}{h}$

$= \lim_{h\to 0} \frac{1-x^2-2xh-h^2-1+x^2}{h}$

$= \lim_{h\to 0}(-2x-h)$

$= -2x$

28. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{2}(x+h)^2-1-\left(\frac{1}{2}x^2-1\right)}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{2}x^2+xh+\frac{1}{2}h^2-1-\frac{1}{2}x^2+1}{h}$

29. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{9(x+h)-2-(9x-2)}{h}$

$= \lim_{h\to 0} \frac{9x+9h-2-9x+2}{h} = 9$

30. $f'(x) = \lim_{h\to 0} \frac{\frac{1}{2}(x+h)-\frac{1}{2}x}{h}$

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

31. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{(x+h) - x}{h}$

32. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{3(x+h)-5-(3x-5)}{h}$

$= \lim_{h\to 0} \frac{3x+3h-5-3x+5}{h} = \lim_{h\to 0} \frac{3h}{h}$

$$\lim_{h \to 0} \frac{2}{h} \cdot \frac{2}{} \lim_{h \to 0} \frac{2}{h}$$

$$\lim_{h \to 0} \frac{h}{2h} = \frac{1}{2}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{0.01(x+h) + 0.05 - (0.01x + 0.05)}{h}$$

$$\frac{0.01x + 0.01h + 0.05 - 0.01x - 0.05}{h}$$

$$\frac{0.01h}{h} = 0.01$$

33. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{4-4}{h} = 0$

34. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\quad}{h} = 0$

35. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{a(x+h)^2+b(x+h)+c-(ax^2+bx+c)}{h}$

$= \lim_{h\to 0}\frac{ax^2+2axh+ah^2+bx+bh+c-ax^2-bx-c}{h}$

$= \lim_{h\to 0}(2ax+ah+b) = 2ax+b$

Use $(x-a)^2 = x^2-2ax+a^2$.

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-2a(x+h)+a^2-(x^2-2ax+a^2)}{h}$

$= \lim_{h\to 0}\frac{x^2+2xh+h^2-2ax-2ah+a^2-x^2+2ax-a^2}{h}$

$= \lim_{h\to 0}(2x+h-2a)$

$= 2x-2a$

37. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{(x+h)^5-x^5}{h}$

$= \lim_{h\to 0}\frac{x^5+5x^4h+10x^3h^2+5xh^4+h^5-x^5}{h}$

$= \lim_{h\to 0} 5x^4+10x^3h+5xh^3+h^4 = 5x^4$

38. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{(x+h)^4-x^4}{h}$

$= \lim_{h\to 0}\frac{x^4+4x^3h+6x^2h^2+4xh^3+h^4-x^4}{h}$

$= \lim_{h\to 0} 4x^3+6x^2h+4xh^2+h^3 = 4x^3$

39. $f(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{\frac{2}{x+h}-\frac{2}{x}}{h}$

$= \lim_{h\to 0}\frac{\frac{2x}{x(x+h)}-\frac{2(x+h)}{x(x+h)}}{h}$

$= \lim_{h\to 0}\frac{2x-2x-2h}{x(x+h)}\cdot\frac{1}{h}$

$= \lim_{h\to 0}\frac{-2h}{x(x+h)}\cdot\frac{1}{h}$

$= \lim_{h\to 0}\frac{-2}{x(x+h)} = -\frac{2}{x^2}$

40. $f(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

$= \lim_{h\to 0}\frac{\frac{x^2}{x^2(x+h)^2}-\frac{(x+h)^2}{x^2(x+h)^2}}{h}$

$= \lim_{h\to 0}\frac{x^2-x^2-2xh-h^2}{x^2(x+h)^2}\cdot\frac{1}{h}$

$= \lim_{h\to 0}\frac{-2xh-h^2}{x^2(x+h)^2}\cdot\frac{1}{h}$

$= \lim_{h\to 0}\frac{-2x-h}{x^2(x+h)^2} = -\frac{2x}{x^4} = -\frac{2}{x^3}$

41. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

$= \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$= \lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h\to 0}\frac{h}{h(\sqrt{x+h}+\sqrt{x})}$

$$\lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

42. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h} = \lim_{h\to 0}\frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h} = \lim_{h\to 0}\frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}}{h}$

$= \lim_{h\to 0}\frac{x-(x+h)}{\sqrt{x}\sqrt{x+h}\,(\sqrt{x}+\sqrt{x+h})\,h} = \lim_{h\to 0}\frac{-h}{\sqrt{x}\sqrt{x+h}\,(\sqrt{x}+\sqrt{x+h})}\cdot\frac{1}{h}$

$= \lim_{h\to 0}\frac{-1}{\sqrt{x}\sqrt{x+h}\,(\sqrt{x}+\sqrt{x+h})} = \frac{-1}{\sqrt{x}\sqrt{x}\,(\sqrt{x}+\sqrt{x})} = \frac{-1}{x\,2\sqrt{x}}$

$= -\frac{1}{2x\sqrt{x}}$

43. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^3+(x+h)^2-(x^3+x^2)}{h}$

$= \lim_{h\to 0}\frac{(x^3+3x^2h+3xh^2+h^3)+(x^2+2xh+h^2)-x^3-x^2}{h}$

$= \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3+2xh+h^2}{h} = \lim_{h\to 0}(3x^2+3xh+h^2+2x+h)$

$= 3x^2+2x$

44. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{2(x+h)}-\frac{1}{2x}}{h} = \lim_{h\to 0}\frac{\frac{1}{2x+2h}-\frac{1}{2x}}{h} = \lim_{h\to 0}\frac{\frac{2x-(2x+2h)}{2x(2x+2h)}}{h} = \lim_{h\to 0}\frac{\frac{-2h}{2x(2x+2h)}}{h}$

$= \lim_{h\to 0}\frac{-h}{x(2x+2h)}\cdot\frac{1}{h} = \lim_{h\to 0}\frac{-1}{x(2x+2h)} = -\frac{1}{2x^2}$

45. **a.** The slope of the tangent line at $x = 2$ is $f'(2) = 2(2) - 3 = 1$. To find the point of the curve at $x = 2$, we calculate $y = f(2) = 2^2 - 3(2) + 5 = 3$. Using the point-slope form with the point (2, 3), we have

$y - 3 = 1(x - 2)$

$y = 3 + x - 2$

$y = x + 1$

b. on viewing window [–10, 10] by [–10, 10]

46. **a.** The slope of the tangent line at $x = 2$ is $f'(2) = 4(2) - 5 = 3$. To find the point of the curve at $x = 2$, we calculate $y = f(2) = 2(2)^2 - 5(2) + 1 = -1$. Using the point-slope form with the point (2, –1), we have

$y - (-1) = 3(x - 2)$

$y + 1 = 3x - 6$

$y = 3x - 7$

b. on viewing window [–10, 10] by [–10, 10]

47. a. 

b. 

48. a. 

b. 

49. a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h) - 4 - (3x - 4)}{h} = \lim_{h \to 0} \frac{3x + 3h - 4 - 3x + 4}{h} = \lim_{h \to 0} 3 = 3$

b. The graph of $f(x) = 3x - 4$ is a straight line with slope 3.

50. a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2(x+h) - 9 - (2x - 9)}{h} = \lim_{h \to 0} \frac{2x + 2h - 9 - 2x + 9}{h} = \lim_{h \to 0} 2 = 2$

b. The graph of $f(x) = 2x - 9$ is a straight line with slope 2.

51. a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{5 - 5}{h} = \lim_{h \to 0} 0 = 0$

b. The graph of $f(x) = 5$ is a straight line with slope 0.

52. a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{12 - 12}{h} = \lim_{h \to 0} 0 = 0$

b. The graph of $f(x) = 12$ is a straight line with slope 0.

53. a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{m(x+h) + b - (mx + b)}{h} = \lim_{h \to 0} \frac{mx + mh + b - mx - b}{h} = \lim_{h \to 0} \frac{mh}{h}$
$= \lim_{h \to 0} m = m$

b. The graph of $f(x) = mx + b$ is a straight line with slope m.

a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{b - b}{h} = \lim_{h \to 0} 0 = 0$

b. The graph of $f(x) = b$ is a straight line with slope 0.

a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 8(x+h) + 110 - (x^2 - 8x + 110)}{h}$
$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - 8x - 8h + 110 - x^2 + 8x - 110}{h}$
$= \lim_{h \to 0} (2x + h - 8) = 2x - 8$

$f'(2) = 2(2) - 8 = -4$. The temperature is decreasing at a rate of 4 degrees per minute after 2 minutes.

$f'(5) = 2(5) - 8 = 2$. The temperature is increasing at a rate of 2 degrees per minute after 5 minutes.

56. **a.** $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{3(x+h)^2-12(x+h)+200-(3x^2-12x+200)}{h}$

$=\lim\limits_{h\to 0}\dfrac{3x^2+6hx+3h^2-12x-12h+200-3x^2+12x-200}{h}$

$=\lim\limits_{h\to 0}(6x+3h-12)=6x-12$

b. $f'(1)=6(1)-12=-6$. The population is decreasing by 6 people per week.

c. $f'(5)=6(5)-12=18$. The population is increasing by 18 people per week.

57. **a.** $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{2(x+h)^2-(x+h)-(2x^2-x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{2x^2+4hx+2h^2-x-h-2x^2+x}{h}=\lim\limits_{h\to 0}(4x+2h-1)=4x-1$

b. $f'(5)=4(5)-1=19$. When 5 words have been memorized, the memorization time is increasing at a rate of 19 seconds per word.

58. **a.** $S'(x)=\lim\limits_{h\to 0}\dfrac{S(x+h)-S(x)}{h}=\lim\limits_{h\to 0}\dfrac{-(x+h)^2+10(x+h)-(-x^2+10x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{-x^2-2hx-h^2+10x+10h+x^2-10x}{h}$

$=\lim\limits_{h\to 0}(-2x-h+10)=-2x+10$

$S'(3)=-2(3)+10=4$. The number of cars sold on the third day of the advertising campaign is increasing at a rate of 4 cars per day.

$S'(6)=-2(6)+10=-2$. The number of cars sold on the sixth day of the advertising campaign is decreasing at a rate of 2 cars per day.

59. **a.** $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{\frac{1}{2}(x+h)^2-3.7(x+h)+12-\left(\frac{1}{2}x^2-3.7x+12\right)}{h}$

$=\lim\limits_{h\to 0}\dfrac{\frac{1}{2}x^2+xh+\frac{1}{2}h^2-3.7x-3.7h+12-\frac{1}{2}x^2+3.7x-12}{h}$

$=\lim\limits_{h\to 0}\left(x+\frac{1}{2}h-3.7\right)=x-3.7$

b. $f'(1)=1-3.7=-2.7$. In 1940, the percentage of immigrants was decreasing by 2.7 percentage points per decade (which is about 0.27 of a percentage point decrease per year).

$f'(8)=8-3.7=4.3$. Increasing by 4.3 percentage points per decade (so about 0.43 of a percentage point per year).

60. **a.** $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{6(x+h)^2-16(x+h)+109-(6x^2-16x+109)}{h}$

$=\lim\limits_{h\to 0}\dfrac{6x^2+12xh+6h^2-16x-16h+109-6x^2+16x-109}{h}$

$=\lim\limits_{h\to 0}\dfrac{12xh+6h^2-16h}{h}=\lim\limits_{h\to 0}(12x+6h-16)$

$=12x-16$

b. $f'(0)=12(0)-16=-16$. In 2010, net income was decreasing at the rate of \$16 million per year.

c. $f'(2)=12(2)-16=8$. In 2012, net income was increasing at the rate of \$8 million per year.

61. The average rate of change requires two *x*-values and is the change in the function - values divided by the change in the *x*-values. The instantaneous rate of change is at a single *x*-value, and is found from the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Substitution $h = 0$ would make the denominator zero, and we can't divide by zero. That's why we need to do some algebra on the difference quotient, to cancel out the terms that are zero so that afterwards we can evaluate by direct substitution.

The units of *x* are blargs and the units of *f* are prendles because the derivative $f'(x)$ is equivalent to the slope of $f(x)$, which is the change in *f* over the change in *x*.

The patient's health is deteriorating during the first day (temperature is rising above normal). The patient's health is improving during the second day (temperature is falling).

A secant line crosses the function twice while a tangent line crosses only once. To find the slope of a secant line, use the formula for average rate of change. For the slope of a tangent line, use the formula for instantaneous rate of change.

The units of the derivative $f'(x)$ is in widgets per blivet because the derivative is equivalent to the slope $f(x)$ which is the change in *f* over the change in *x*.

The population is decreasing because the negative derivative implies a negative slope.

The temperature at 6 a.m. is the lowest temperature throughout the first half of the day because the temperature falls until 6 a.m. and rises after 6 a.m.

EXERCISES 2.3

$f'(x) = \frac{d}{dx}(x^4) = 4x^{4-1} = 4x^3$

$f'(x) = \frac{d}{dx}(x^5) = 5x^{5-1} = 5x^4$

$f'(x) = \frac{d}{dx}(x^{500}) = 500x^{500-1} = 500x^{499}$

$f'(x) = \frac{d}{dx}(x^{1000}) = 1000x^{1000-1} = 1000x^{999}$

$f'(x) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2}$

$f'(x) = \frac{d}{dx}(x^{1/3}) = \frac{1}{3}x^{1/3-1} = \frac{1}{3}x^{-2/3}$

$g'(x) = \frac{d}{dx}\frac{1}{4}x^4 = \frac{1}{2}4x^{4-1} = 2x^3$

$g'(x) = \frac{d}{dx}\frac{1}{3}x^9 = \frac{1}{3}9x^{9-1} = 3x^8$

$g'(w) = \frac{d}{dw}(6w^{1/3}) = 6\cdot\frac{1}{3}w^{1/3-1} = 2w^{-2/3}$

$g'(w) = \frac{d}{dw}(12w^{1/2}) = 12\cdot\frac{1}{2}w^{1/2-1} = 6w^{-1/2}$

$h'(x) = \frac{d}{dx}(3x^{-2}) = 3(-2)x^{-2-1} = -6x^{-3}$

$h'(x) = \frac{d}{dx}(4x^{-3}) = 4(-3)x^{-3-1} = -12x^{-4}$

$f'(x) = \frac{d}{dx}(4x^2 - 3x + 2) = 4\cdot 2x^{2-1} - 3\cdot 1x^{1-1} + 0 = 8x - 3$

$f'(x) = \frac{d}{dx}(3x^2 - 5x + 4) = 3\cdot 2x^{2-1} - 5\cdot 1x^{1-1} + 0 = 6x - 5$

15. $f'(x) = \frac{d}{dx}\frac{1}{x^{1/2}} = \frac{d}{dx}x^{-1/2} = -\frac{1}{2}x^{-3/2} = -\frac{1}{2x^{3/2}}$.

16. $f'(x) = \frac{d}{dx}\frac{1}{x^{2/3}} = \frac{d}{dx}x^{-2/3} = -\frac{2}{3}x^{-5/3} = -\frac{2}{3x^{5/3}}$.

2 $2x^{3/2}$ 3 $3x^{5/3}$

17. $f'(x)\ \frac{d}{dx}\ \frac{6}{3\sqrt{x}}\quad \frac{d}{dx}\ 6x^{1/3}$

$6\ \frac{1}{3}x^{4/3}\ 2$

$3x^{4/3}$

$f'(r)\ \frac{d}{dr}\ r^2\ (2r)\ 2r$

$\frac{d}{dx}\ \frac{1}{6}x^3\ \frac{1}{2}x^2\quad x\ 1$

$f'(x)$

$\frac{1}{6}(3x^2)\ \frac{1}{2}(2x)\ 1$

$\frac{1}{2}x^2\quad x\ 1$

$g(x)\ \frac{d}{dx}\ (x^{1/2}\quad x^1)\ \frac{1}{2}x^{1/2\,1}\quad (1)x^{1\,1}\ \frac{1}{2}$

$x^{1/2}\ \ x^2$

$h'(x)\ \frac{d}{dx}\ 6x^{2/3}\ 12x^{1/3}\ 6\ \frac{3}{2}x^{2/3\,1}\ 12\ \frac{1}{3}x^{1/3\,1}$

$4x^{1/3}\ 4x^{4/3}$

$h'(x)\ \frac{d}{dx}\ 8x^{3/2}\ 8x^{1/4}\ 8\ \frac{3}{2}x^{3/2\,1}\ 8\ \frac{1}{4}x^{1/4\,1}$

$12x^{1/2}\ 2x^{5/4}$

$f'(x)\ \frac{d}{dx}\ 10x^{1/2}\ \ \frac{9}{5}x^{5/3}\ \ 17\quad 10\ \frac{1}{2}x^{1/2\,1}\quad \frac{9}{5}\ \frac{5}{3}x^{5/3\,1}\quad 0$

$5x^{3/2}\ 3x^{2/3}$

$f'(x)\ \frac{d}{dx}\ \frac{9}{2}x^{2/3}\quad 16x^{5/2}\quad 14\quad \frac{9}{2}\ \frac{2}{3}x^{2/3\,1}\quad 16\ \frac{5}{2}x^{5/2\,1}\quad 0$

$3x^{5/3}\quad 40x^{3/2}$

$f'(x)\quad \frac{d}{dx}\ \frac{2x^3}{x}\quad \frac{d}{dx}\ (x^2\ x)\ 1\ 2x$

$f'(x)\ \frac{d}{dx}\ x^2\ (x\ 1)\ \frac{d}{dx}\ (x^3\quad x^2)\ 3x^2\quad 2x$

$f(x)\ \frac{d}{dx}\ (x^5)\ 5x^4;\ f(2)\ 5(2)^4\quad 80$

18. $f'(x)\ \frac{d}{dx}\ \frac{4}{\sqrt{x}}\quad \frac{d}{dx}\ 4x^{1/2}$

$4\ \frac{1}{2}x^{3/2}$

$\frac{2}{x^{3/2}}$

$f'(r)\ \frac{d}{dr}\ \frac{4}{3}r^3\quad \frac{4}{3}(3r^2)\ \ 4r^2$

$f'(x)\ \frac{d}{dx}\ \frac{1}{24}x^4\quad \frac{1}{6}x^3\quad \frac{1}{2}x^2\quad x\ 1$

$\frac{1}{24}(4x^3)\ \frac{1}{6}(3x^2)\ \frac{1}{2}(2x)\ 1$

$\frac{1}{6}x^3\ \frac{1}{2}x^2\quad x\ 1$

$g(x)\ \frac{d}{dx}\ (x^{1/3}\quad x^1)\ \frac{1}{3}x^{1/3\,1}\ (1)x^{1\,1}\ \frac{1}{3}x^{2/3}\ 2$

$\frac{d}{dx}\ (x^4)\quad 4x^3$

$f(x)\ \frac{d}{dx}\ (x^{2/3})\ 4x^{1/3}\ ;\ f(3)\ 4(3)^{1/3}\ 108\ \frac{2}{}\ \frac{1}{}\ {}^{4/3}$

$f(x)\ \frac{d}{dx}\ (6x^{2/3}\ \ 48x^{1/3})\ 6\ \ 3\ x\ \ 48\ \ 3\ x$

$4x^{1/3}\ \ 16x^{4/3}$

$f(8)\ \ 4(8)^{1/3}\ \ 16(8)^{4/3}\ \ \frac{4}{\sqrt[3]{8}}\ \ \frac{16}{\sqrt[3]{8^4}}\ \ \frac{4}{2}\ \ \frac{16}{\sqrt[3]{8^3}\ \sqrt[3]{8}}\ \ 2\ \ \frac{16}{8\ 2}\ \ 3$

$f'(x) \; \frac{d}{dx} \; 12x^{2/3} \; 48x^{1/3} \; 12\frac{2}{3}x^{-1/3} \; 48\frac{1}{3}x^{1/3}$

$4x^{-1/3} \; 16x^{1/3}$

$f'(8) \; 4(8)^{-1/3} \; 16(8)^{1/3} \; \frac{4}{\sqrt[3]{8}} \; 16\sqrt[3]{8} \; \frac{4}{2} \; 16 \cdot 2 \; 2 \; 32 \; 34$

35. $\frac{df}{dx} \; \frac{d}{dx}(x^3) \; 3x^2$; $\left.\frac{df}{dx}\right|_{x\,3} \; 3(3)^2 \; 27$ **6.** $\frac{df}{dx} \; \frac{d}{dx}(x^4) \; 4x^3$; $\left.\frac{df}{dx}\right|_{x\,2} \; 4(2)^3 \; 32$

$\frac{df}{dx} \; \frac{d}{dx} \; 16x^{1/2} \; 8x^{-1/2} \; 16\frac{1}{2}x^{-1/2} \; 8\frac{1}{2}x^{-3/2} \; 8x^{-1/2}$

$4x^{-3/2}$

$\left.\frac{df}{dx}\right|_{x\,4} \; 8(4)^{-1/2} \; 4(4)^{-3/2} \; \frac{8}{\sqrt{4}} \; \frac{4}{\sqrt{4}^3} \; \frac{8}{2} \; \frac{4}{8} \; \frac{7}{2}$

$\frac{df}{dx} \; \frac{d}{dx} \; 54x^{1/2} \; 12x^{-1/2} \; 54\frac{1}{2}x^{-1/2} \; 12\frac{1}{2}x^{-3/2} \; 27x^{-1/2}$

$6x^{-3/2}$

$\left.\frac{df}{dx}\right|_{x\,9} \; 27(9)^{-1/2} \; 6(9)^{-3/2} \; \frac{27}{\sqrt{9}} \; \frac{6}{\sqrt{9}^3} \; \frac{27}{3} \; \frac{6}{27} \; 9 \; \frac{2}{9}$

39. a. $f'(x) \; 2x^1 \; 2 \; 0 \; 2x \; 2$

$f'(3) \; 2(3) \; 2 \; 4$

The slope of the tangent line is 4.

$y \; 5 \; 4(x \; 3)$

$y \; 5 \; 4x \; 12$

$y \; 4x \; 7$

The equation for the tangent line is $y \; 4x \; 7$.

b.

on [–1, 6] by [–10, 20]

40. a. $f'(x) \; 2x^1 \; 4 \; 0 \; 2x \; 4$

$f'(1) \; 2(1) \; 4 \; 2$

The slope of the tangent line is –2.

$y \; 3 \; 2(x \; 1)$

$y \; 3 \; 2x \; 2$

$y \; 2x \; 5$

The equation for the tangent line is $y \; 2x \; 5$.

b.

on [–1, 5] by [–2, 10]

a. $'(x) \; 3x^2 \; 3(2)x^1 \; 2 \; 0 \; 3x^2 \; 6x \; 2$

$'(2) \; 3(2)^2 \; 6(2) \; 2 \; 2$

The slope of the tangent line is 2.

$y \; 2 \; 2(x \; 2)$

$y \; 2 \; 2x \; 4$

$2x \; 6$

The equation for the tangent line is $2x \; 6$.

b.

on [–1, 4] by [–7, 5]

a. $'(x) \; 3(2)x^1 \; 3x^2 \; 5x \; 3x^2 \; f$

$'(1) \; 6(1) \; 3(1)^2 \; 3$

The slope of the tangent line is 3.

$2 \; 3(x \; 1) \; y$

$2 \; 3x \; 3$

$y \; 3x \; 1$

The equation for the tangent line is $3x \; 1$.

b.

on [–1, 3] by [–2, 5]

For y_1 5 and viewing rectangle [–10, 10] by [–10, 10], your graph should look roughly like the following:

For y_1 $3x$ 4 and viewing rectangle [–10, 10] by [–10, 10], your graph should look roughly like the following:

y' 9.638 1.394 $x^{2.394}$ 13.435372$x^{2.394}$
$y'(2)$ 13.435372 $2^{2.394}$ 2.6
$y'(5)$ 13.435372 $5^{2.394}$ 0.29

Interpretation: Near full employment, inflation greatly decreases with small increases in unemployment but with higher unemployment, inflation hardly decreases with increases in unemployment.

y' 45.4 1 .54 $x^{2.54}$ 69.916$x^{2.54}$
$y'(3)$ 69.916 $3^{2.54}$ 4.3
$y'(8)$ 69.916 $8^{2.54}$ 0.36

Interpretation: Near full employment, inflation greatly decreases with small increases in unemployment but with higher unemployment, inflation hardly decreases with increases in unemployment.

47. **a.** $C'(x)\ \frac{16}{\sqrt[3]{x}}$

$C'(8)\ \frac{16}{\sqrt[3]{8}}\ \frac{16}{2}\ 8$

When 8 items are purchased, the cost of the last item is about \$8.

$C'(64)\ \frac{16}{\sqrt[3]{64}}\ \frac{16}{4}\ 4$

When 8 items are purchased, the cost of the last item is about \$4.

a. $C'(x)\ 140x^{1/6}$

$C'(1)$ [redacted] 140

When 1 license is purchased, the cost is about \$140.

$C'(64)$ [redacted] $\frac{140}{2}$ 70

When 64 licenses are purchased, the cost is about \$70.

$C(64)\ C(63)\ \ 24(64)^{2/3}\ \ 24(63)^{2/3}\ \ 4.01$

The answer is close to \$4.

$C(64)\ C(63)\ \ 168(64)^{5/6}\ \ 168(63)^{5/6}\ \ 70.09$

The answer is close to \$70.

a. The rate of change of the population in x years is the derivative of the population function

$P'(x)\quad x^2\quad 5x\quad 3$

To find the rate of change of the population 20 years from now, evaluate $P(x)$ for $x = 2$.

$P'(2)\quad 2^2\quad 5(2)\quad 3\quad 3$

In 2030, this population group will be increasing by 3 million per decade.
To find the rate of change of the population 0 years from now, evaluate $P(x)$ for $x = 0$.

$P'(0)\quad 0^2\quad 5(0)\quad 3\quad 3$

In 2010, this population group will be decreasing by 3 million per decade.

a. The rate of change of the number of newly infected people is the derivative of the function $f(t)\ 13t^2\ t^3$

$(t)\ 2\ 13t\ 3t^2\quad 26t\ 3t^2$

The rate of change on day 5 is found by evaluating $f(t)$ for $t = 5$.

$f'(5)\quad 26(5)\quad 3(5)^2\quad 55$

The number of newly infected people on day 5 is increasing by about 55 people per day.
The rate of change on day 10 is found by evaluating $f(t)$ for $t = 10$.

$f(10)\quad 26(10)\quad 3(10)^2\quad 40$

The number of newly infected people on day 10 is decreasing by about 40 people per day.

The rate of change of the pool of potential customers is the derivative of the function

$N(x)\quad 400,000\quad \frac{200,000}{}\ .$

$N(x)\quad \frac{d}{dx}(400,000\ \ 200,000\ x^{1})$

$N(x)\quad 0\quad (1)200,000x^{2}\quad \frac{200,000}{x^2}$

To find the rate of change of the pool of potential customers when the ad has run for 5 days, evaluate $N(x)$ for $x = 5$.

$N(5)\quad \frac{200,000}{5^2}\quad 8000$

The pool of potential customers is increasing by about 8000 people per additional day.

54. $A(t)\quad 0.01t^2\quad 1\quad t\quad 5$

The instantaneous rate of change of the cross-sectional area t hours after administration of nitroglycerin is given by

$A(t)\quad 2(0.01)t^{2\ 1}\quad 0.02t$

$A(4)\quad 0.02(4)\quad 0.08$

After 4 hours the cross-sectional area is increasing by about 0.08 cm^2 per hour.

55. The rate of change of 3D movies is the derivative of the function $f(x)\quad \frac{1}{2}x^2\ 11x\ \ 8$.

$f'(x)\ x\ \ 11$

The rate of change of number of 3D movies in 10 years is found by evaluating $f(x)$ at $x = 10$.

$'(10)\ \ 10\ \ 11\ \ 21$

In 2020, the number of 3D movies will be increasing by 21 per year.

56. **a.** The rate of change of the function is the derivative of the function:

$f'(x) \quad \frac{1}{2}x^{3/2} \quad \frac{1}{2\sqrt{x^3}}$.

Find the rate of change for $x = 4$.

$f'(4) \quad \frac{1}{2\sqrt{4^3}} \quad \frac{1}{16}$

In 2005, the number of fatalities is decreasing by $\frac{1}{16}$ per hundred million vehicle miles traveled every five years.

Find the rate of change for $x = 4$.

$f'(9) \quad \frac{1}{2\sqrt{9^3}} \quad \frac{1}{54}$

In 2030, the number of fatalities is decreasing by $\frac{1}{54}$ per hundred million vehicle miles traveled every five years.

The instantaneous rate of change of the number of phrases students can memorize is the derivative of the function $p(t)\ 24\sqrt{t}$.

$p'(t)\ \frac{d}{dt}(24\,t^{1/2})$

$\frac{1}{2}(24t^{1/2})\ 12t^{1/2}$

$p'(4)\ 12(4)^{1/2} \quad \frac{12}{\sqrt{}} \quad 6$

The number of phrases students can memorize after 4 hours is increasing by 6.

a. The instantaneous rate of change of the amount of dissolved oxygen x miles downstream is the derivative of the function $D(x)\ 0.2x^2\ 2x\ 10$.

$D(x)\ 2(0.2)x\ 2\ 0\ 0.4x\ 2\ D(1)$

$0.4(1)\ 2\ 1.6$

The amount of dissolved oxygen 1 mile downstream is decreasing by about 1.6 mpl per mile.

b. $D'(10) \quad 0.4(10) \quad 2 \quad 2$

The amount of dissolved oxygen 10 miles downstream is increasing by about 2 mpl per mile.

a. $U(x) \quad 100\sqrt{x} \quad 100\,x^{1/2}$

$MU(x) \quad U(x) \quad \frac{1}{2}(100)x^{1/2\ 1} \quad 50\,x^{1/2}$

b. $MU(1) \quad U(1) \quad 50(1)^{1/2} \quad 50$

The marginal utility of the first dollar is 50.

c. $MU(1{,}000{,}000) \quad U(1{,}000{,}000) \quad 50(10^6)^{1/2} \quad 50(10)^3 \quad \frac{50}{1000} \quad 0.05$

The marginal utility of the millionth dollar is 0.05.

a. $U(x)\ 12\sqrt[3]{}\ 12\,x^{1/3}$

$MU(x) \quad U'(x) \quad \frac{1}{3}(12\,x^{2/3})\ 4\,x^{2/3}$

$MU(1)\ U(1)\ 4(1)^{2/3} \quad 4$

The marginal utility of the first dollar is 4.

c. $MU(1{,}000{,}000)\ U(1{,}000{,}000)\ 4(1{,}000{,}000)^{2/3} \quad \frac{4}{\sqrt[3]{(1{,}000{,}000)}^{2}} \quad \frac{4}{100^2}\ 0.0004$

The marginal utility of the millionth dollar is 0.0004.

a. $f(12)\quad 0.831(12)^2\quad 18.1(12)\quad 137.3\quad 39.764$

A smoker who is a high school graduate has a 39.8% chance of quitting.

(x) 0.831(2)x 18.1 1.662 x 18.1 $f(12)$

1.662(12) 18.1 1.844

When a smoker has a high school diploma, the chance of quitting is increasing at the rate of 1.8% per year of education.

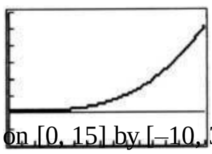
b. $f(16)\quad 0.831(16)^2\quad 18.1(16)\quad 137.3\quad 60.436$

A smoker who is a college graduate has a 60.4% chance of quitting.

$f(16)\quad 1.662(16)\quad 18.1\quad 8.492$

When a smoker has a college degree, the chance of quitting is increasing at the rate of 8.5% per year of education.

a.



on [0, 15] by [–10, 30]

b. $N(10)\quad 0.00437(10)^{3.2}\quad 6.9$

A group with 10 years of exposure to asbestos will have 7 cases of lung cancer.

$N(t)\quad 0.00437(3.2)t^{3.2\ 1}\quad 0.013984t^{2.2}$

$N(10)\quad 0.013984(10)^{2.2}\quad 2.2$

When a group has 10 years of exposure to asbestos, the number of cases of lung cancer is increasing at the rate of 2.2 cases per year.

63. **a.** $f(5)\quad 650(5)^2\quad 3000(5)\quad 12,000$

$43, 250

$f'(x)$ 1300 x 3000

$f'(5)$ 1300(5) 3000 $9500

In 2020-2021, private college tuition will be increasing by $9500 every 5 years.

$1900

64. **a.** $f(5)\quad 400(5)^2\quad 500(5)\quad 2700$

$15,200

$f'(x)$ 800 x 500

$f'(5)$ 800(5) 500 $4500

In 2020-2021, public college tuition will be increasing by $4500 every 5 years.

$900

$f(x)$ 2 will have a graph that is a horizontal line at height 2, and the slope (the derivative) of a horizontal line is zero. A function that stays constant ill have a rate of change of zero, so its derivative (instantaneous rate of change) will be zero.

$f(x)$ 3 x 5 is a line with a slope (derivative) of A function that has a slope of 3 will have a rate of change of 3, so its derivative (instantaneous rate of change) will be three.

If f has a particular rate of change, then the rate of change of 2 $f(x)$ will be twice as large, and the rate of change of $c f(x)$ will be $c f'(x)$, which is just the constant multiple rule.

If f and g have particular rates of change, then the rate of change of $f(x)$ $g(x)$ would be the rate of change of $f(x)$ plus the rate of change of $g(x)$. For example, if $f(x)$ 2x and $g(x)$ 3x, then the rate of change of 2 x 3 x 5x is the same as the rate of change of 2x plus the rate of change of 3x. The derivative of $f(x)$ $g(x)$ will be $f'(x)$ $g'(x)$, which is just the sum rate.

Since –f slopes down by the same amount that f slopes up, the slope of –f should be the negative of the slope of f. The constant multiple rule with c 1 also says that the slope of –f will be the negative of the slope of f.

The slopes of the f and f 10 will be the same because f 10 is just f raised 10 units. Using the sum rule with $g(x)$ 10, also says that the slope will just be f because $g'(x)$ 0.

Evaluating first would give a constant and the derivative of a constant is zero, so evaluating and the differentiating would always give zero, regardless of the function and number. This supports the idea that we should always differentiate and then evaluate to obtain anything meaningful.

To find a function that is positive but does not have a positive slope at a particular x-value, we need to find an equation with a negative slope. y 5x will work at x 1 .

Each additional year of education increases life expectancy by 1.7 years.

The probability of an accident increases by 13% as the speed exceeds the limit.

75. **a.** If $D(c)$ 0, then $f(c)$ $L(c)$ 0 so $f(c)$ $L(c)$.

Since the line $L(x)$ must pass through the point $(c, L(c))$, part (a) shows that this point is $(c, f(c))$. Then the point-slope form $y\ y_1\ m(x\ x_1)$ with $x_1\ c$ and $y_1\ f(c)$ becomes $y\ f(c)$ $m(x\ c)$

or y $m(x\ c)$ $f(c)$, giving the line $L(x)\ m(x\ c)$ $f(c)$

$D(x)\ f(x)\ L(x)\ f(x)\ (m(x\ c)\ f(c))\ f(x)\ m(x\ c)\ f(c)$

Differentiating the last expression for $D(x)$ above gives $D(x)\ f(x)\ m$

which at $x\ c$ becomes $D(c)\ f(c)$ m, and setting this equal to zero gives $m\ f(c)$.

e. $L(x)\ f(c)(x\ c)\ f(c)$ so $y\ L(x)$ is the same as $y\ f(c)\ f(c)(x\ c)$, which is the point-slope form of the equation of the tangent line to the curve $y\ f(x)$ at $x\ c$.

The tangent line has zero error at the point and errors near the point that differ as little as possible from zero. So the tangent line is the best linear approximation to the function since it has the smallest possible differences between a line and the curve at and near the point.

EXERCISES 2.4

a. Using the product rule:

$$\frac{d}{dx}(x^4\ x^6)\quad 4x^3\ x^6\quad x^4(6x^5)\quad 4x^9\quad 6x^9\quad 10x^9$$

Using the power rule:

$$\frac{d}{dx}(x^4\ x^6)\ \frac{d}{dx}(x^{10})\ 10x^9$$

a. Using the product rule:

$$\frac{d}{dx}(x^7\ x^2)\quad 7x^6\ x^2\quad x^7\ 2x\quad 7x^8\ 2x^8\quad 9x^8$$

Using the power rule:

$$\frac{d}{dx}(x^7\ x^2)\ \frac{d}{dx}(x^9)\ 9x^8$$

a. Using the product rule:

$$\frac{d}{dx}[x^4(x^5\quad 1)]\quad 4x^3(x^5\quad 1)\quad x^4(5x^4)\quad 4x^8\ 4x^3\quad 5x^8\quad 9x^8\quad 4x^3$$

Using the power rule:

$$\frac{d}{dx}[x^4(x^5\quad 1)]\ \frac{d}{dx}(x^9\quad x^4)\ 9x^8\quad 4x^3$$

a. Using the product rule:

$$\frac{d}{dx}[x^5(x^4 \quad 1)] \quad 5x^4(x^4 \quad 1) \quad x^5(4x^3) \quad 5x^8 \quad 5x^4 \; 4x^8 \quad 9x^8 \quad 5x^4$$

Using the power rule:

$$\frac{d}{dx}[x^5(x^4 \quad 1)] \quad \frac{d}{dx}(x^9 \quad x^5) \; 9x^8 \quad 5x^4$$

$f(x)\; 2x(x^3 \quad 1)\; x^2(3x^2)\; 2x^4 \quad 2x\; 3x^4 \quad 5x^4 \quad 2x$

2 2 3 4 2 4 4 2

$f(x)\; 3x \quad (x \quad 1)\; x \quad (2x)\; 3x \quad 3x \quad 2x \quad 5x \quad 3x$

2 2 2 2

$f(x)\; 1(5x \quad 1)\; x(10x)\; 5x \quad 1\; 10x \quad 15x \quad 1$

4 3 4 4 4

$f(x)\; 2(x \quad 1)\; 2x(4x \quad)\; 2x \quad 2\; 8x \quad 10x \quad 2$

9. $f'(x)\; \frac{1}{2}x^{1/2}\, 6x\; 2 \quad x^{1/2}(6)\; 3x^{1/2} \quad x^{1/2}\; 6x^{1/2} \quad 9x^{1/2}\; x^{1/2} \quad 9\sqrt{x} \quad \frac{1}{\sqrt{x}}$

10. $f'(x)\; 6\; \frac{1}{3}x^{2/3} \quad 2x\; 1 \quad 6x^{1/3}(2)\; 4x^{1/3} \quad 2x^{2/3} \quad 12x^{1/3} \quad 16x^{1/3} \quad 2x^{2/3} \quad 16\sqrt[3]{x} \quad \frac{2}{\sqrt[3]{x^2}}$

$f(x)\; (2x)(x^2 \quad 1)\; (x^2 \quad 1)(2x)\; 2x^3 \quad 2x\; 2x^3 \quad 2x\; 4x^3$

$f(x)\; 3x^2(x^3 \quad 1)\; (x^3 \quad 1)(3x^2)\; 3x^5 \quad 3x^2 \quad 3x^5 \quad 3x^2 \quad 6x^5$

$f(x)\; (2x \quad 1)(3x \quad 1)\; (x^2 \quad x)(3)\; 6x^2 \quad 5x \quad 1\; 3x^2 \quad 3x\; 9x^2 \quad 8x \quad 1$

$f(x)\; (2x \quad 2)(2x \quad 1)\; (x^2 \quad 2x)(2)\; 4x^2 \quad 6x\; 2\; 2x^2 \quad 4x\; 6x^2 \quad 10x \quad 2$

$f'(x)\; 2x\,x^2 \quad 3x\; 1 \quad x^2(2x \quad 3)\; 2x^3 \quad 6x^2 \quad 2x\; 2x^3 \quad 3x^2 \quad 4x^3 \quad 9x^2 \quad 2x$

$f'(x)\; 3x^2\; x^2 \quad 4x\; 3 \quad x^3(2x \quad 4)\; 3x^4 \quad 12x^3 \quad 9x^2 \quad 2x^4 \quad 4x^3 \quad 5x^4 \quad 16x^3 \quad 9x^2$

$f'(x)\; (4x)(1 \quad x)\; (2x^2 \quad 1)(1)\; 4x\; 4x^2 \quad 2x^2 \quad 1 \quad 6x^2 \quad 4x\; 1$

$f'(x)\; (2x)(1 \quad x^2)\; (2x \quad 1)(\; 2x)\; 2\; 2x^2 \quad 4x^2 \quad 2x\; 6x^2 \quad 2x\; 2$

$f'(x)\; \frac{1}{2}x^{1/2} \quad x^{1/2} \quad 1 \quad x^{1/2} \quad 1\; \frac{1}{2}x^{1/2} \quad \frac{1}{2} \quad \frac{1}{2}x^{1/2} \quad \frac{1}{2} \quad \frac{1}{2}x^{1/2} \quad 1$

$f'(x)\; \frac{1}{2}x^{1/2} \quad x^{1/2} \quad 2 \quad x^{1/2} \quad 2\; \frac{1}{2}x^{1/2} \quad \frac{1}{2}\; x^{1/2} \quad \frac{1}{2}\; x^{1/2} \quad 1$

$f'(t)\; 8t^{1/3}(3t^{2/3} \quad 1)\; 6t^{4/3}(2t^{1/3})\; 24t\; 8t^{1/3} \quad 12t\; 36t\; 8t^{1/3}$

$f(t)\; 6t^{1/2}\; (2t^3 \quad 1/2 \quad 1)\; 4t^{3\;3/2} \quad (t^{4\;1/2} \quad 2$

$) \; 12t \;\; 6t \qquad 1/2 \quad 4t \quad 1/2$

$16t$

$2 \qquad 6t$

$f(z) \;\; (4z \quad 2z)(z \quad z) \;\; (z \quad z \quad 1)(3z \quad 1)$

$6 \quad 4 \quad 2 \quad 6 \quad 4 \quad 2 \quad 4 \quad 2$

$4z \quad 2z \quad 2z \quad 3z \quad 3z \quad 3z \quad z \quad z \quad 1$

$7z^{6} \quad 1$

24. $f(z)\ \frac{1}{4} z^{3/4}\ \frac{1}{2} z^{1/2}\ (z^{1/4}\ z^{1/2})\ (z^{1/4}\ z^{1/2})\ \frac{1}{4} z^{3/4}\ \frac{1}{2} z^{1/2}$

$\frac{1}{4} z^{1/2}\ \frac{1}{4} z^{1/4}\ \frac{1}{2} z^{1/4}\ \frac{1}{2}\ \frac{1}{4} z^{1/2}\ \frac{1}{2} z^{1/4}\ \frac{1}{4} z^{1/4}\ \frac{1}{2}$

$\frac{1}{4} z^{1/2}\ \frac{1}{4} z^{1/2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2} z^{1/2}\ 1$

$f'(x)\ 2\ 4\ \frac{1}{2} z^{1/2}\ 2 z^{1/2}\ 1\ 2\ z\ 4 z^{1/2}\ 1\ 2\ \frac{1}{2} z^{1/2}\ 4 z^{1/2}\ 2$

$4\ 2 z^{1/2}\ 2 z^{1/2}\ 4 z^{1/2}$

$6 z^{1/2}\ z^{1/2}\ 10$

$6\ \sqrt{z}\ \frac{1}{\sqrt{z}}\ 10$

$f'(x)\ 1\ 6\ \frac{1}{2} z^{1/2}\ z\ 2 z^{1/2}\ 1\ z\ 6 z^{1/2}\ 1\ 2\ \frac{1}{2} z^{1/2}\ z\ 3 z^{1/2}\ 2$

$z^{1/2}\ 6\ 1\ 3 z^{1/2}\ z\ z^{1/2}\ 6 z^{1/2}\ 6$

$2 z\ 6 z^{1/2}\ 3 z^{1/2}\ 11$

$2 z\ 6 \sqrt{z}\ \frac{3}{\sqrt{z}}\ 11$

a. Using the quotient rule:

$\frac{d}{dx}\frac{x^8}{x^2}\ \frac{x^2(8x^7)\ 2x(x^8)}{(x^2)^2}\ \frac{8x^9\ 2x^9}{x^4}\ 6x^5$

Using the power rule:

$\frac{d}{dx}\frac{x^8}{x^2}\ \frac{d}{dx}(x^6)\ 6x^5$

a. Using the quotient rule:

$\frac{d}{dx}\frac{x^9}{x^3}\ \frac{x^3(9x^8)\ 3x^2(x^9)}{(x^3)^2}\ \frac{9x^{11}\ 3x^{11}}{x^6}\ 6x^5$

Using the power rule:

$\frac{d}{dx}\frac{x^9}{x^3}\ \frac{d}{dx}(x^6)\ 6x^5$

a. Using the quotient rule:

$\frac{d}{dx}\frac{1}{x^3}\ \frac{x^3(0)\ 3x^2(1)}{(x^3)^2}\ \frac{3x^2}{x^6}\ \frac{3}{x^4}$

Using the power rule:

$\frac{d}{dx}\frac{1}{x^3}\ \frac{d}{dx}(x^{-3})\ 3x^{-4}\ \frac{3}{x^4}$

a. Using the quotient rule:

$\frac{d}{dx}\frac{1}{x^4}\ \frac{x^4(0)\ 4x^3(1)}{(x^4)^2}\ \frac{4x^3}{x^8}\ \frac{4}{x^5}$

Using the power rule:

$$\frac{d}{dx}\left(\frac{1}{x^4}\right) = \frac{d}{dx}\left(x^{-4}\right) = -4x^{-5} = -\frac{4}{x^5}$$

31. $$f'(x) = \frac{x^3(4x^3) - 3x^2(x^4 - 1)}{(x^3)^2} = \frac{4x^6 - 3x^6 + 3x^2}{x^6} = \frac{x^6 + 3x^2}{x^6} = 1 + \frac{3}{x^4}$$

32. $f(x)$ $\frac{x^2(5x^4) 2x(x^5 1)}{(x^2)^2}$ $\frac{5x^6 2x^6 2x}{x^4}$ $\frac{3x^6 2x}{x^4}$ $3x^2$ $\frac{2}{x^3}$

33. $f(x)$ $\frac{(x 1)(1) (1)(x 1)}{(x 1)^2}$ $\frac{x 1 x 1}{(x 1)^2}$ $\frac{2}{(x 1)^2}$

34. $f(x)$ $\frac{(x 1)(1) (1)(x 1)}{(x 1)^2}$ $\frac{x 1 x 1}{(x 1)^2}$ $\frac{2}{(x 1)^2}$

35. $f'(x)$ $\frac{(2 x)(3) (1)(3x 1)}{(2 x)^2}$ $\frac{6 3x 3x 1}{(2 x)^2}$ $\frac{5}{(2 x)^2}$

$f'(x)$ $\frac{(2x^2 1)(1) (4x)(x 1)}{2x^2 1^2}$ $\frac{2x^2 1 4x^2 4x}{2x^2 1^2}$ $\frac{2x^2 4x 1}{2x^2 1^2}$

37. $f(t)$ $\frac{(t^2 1)(2t) (2t)(t^2 1)}{(t^2 1)^2}$ $\frac{2t^3 2t 2t^3 2t}{(t^2 1)^2}$ $\frac{4t}{(t^2 1)^2}$

38. $f(t)$ $\frac{(t^2 1)(2t) (2t)(t^2 1)}{(t^2 1)^2}$ $\frac{2t^3 2t 2t^3 2t}{(t^2 1)^2}$ $\frac{4t}{(t^2 1)^2}$

39. $f(s)$ $\frac{(s 1)(3s^2) (1)(s^3 1)}{(s 1)^2}$ $\frac{3s^3 3s^2 s^3 1}{(s 1)^2}$ $\frac{2s^3 3s^2 1}{(s 1)^2}$

40. $f(s)$ $\frac{(s 1)(3s^2) (1)(s^3 1)}{(s 1)^2}$ $\frac{3s^3 3s^2 s^3 1}{(s 1)^2}$ $\frac{2s^3 3s^2 1}{(s 1)^2}$

$f'(x)$ $\frac{(x 1)(2x 2) (1)(x^2 2x 3)}{(x 1)^2}$ $\frac{2x^2 2x^2 2x 3}{(x 1)^2}$ $\frac{x^2 2x 5}{(x 1)^2}$

$f'(x)$ $\frac{(x 1)(2x 3) (1)(x^2 3x 1)}{(x 1)^2}$ $\frac{2x^2 x 3 x^2 3x 1}{(x 1)^2}$ $\frac{x^2 2x 2}{(x 1)^2}$

43. $f(x)$ $\frac{(x^2 1)(4x^3 2x) (2x)(x^4 x^2 1)}{(x^2 1)^2}$ $\frac{4x^3(x^2 1) 2x(x^2 1) 2x(x^4) 2x(x^2 1)}{(x^2 1)^2}$

$\frac{4x^5 4x^3 2x}{(x^2 1)^2}$ $\frac{2x^5 4x^3}{(x^2 1)^2}$

$f(x)$ $\frac{x^5 x^3 x}{x^3}$ $\frac{x(x^4 x^2 1)}{x^2}$ $x^4 x^2 1$

$xx(x 1)x 1$

Thus, $f(x)$ $\frac{2x^5 4x^3}{2}$.

$f'(t)$ $(t$ $(x$ $t^2 3)(2t 2) (t$

$1)^2$

$^2\ \underline{2}\ t\ \underline{1)(2}\ t^2\ t\ 3)^2$ - -

$\underline{1)}\ (t$

$$\frac{2t^3 \quad 4t^2 \quad 4t \;\; 6 \;\; 2t^3 \quad 5t^2 \quad 1}{(t^2 \quad t \;\; 3)^2} \qquad \frac{t^2 \quad 4t \;\; 5}{(t^2 \quad t \;\; 3)^2}$$

$f'(t) = \frac{(t^2 - t - 2)(4t - 1) - (2t^2 - t - 5)(2t - 1)}{(t^2 - t - 2)^2} = \frac{4t^3 - 3t^2 - 7t + 2 - 4t^3 - 11t + 53t^2 - 18t + 3}{(t^2 - t - 2)^2} \quad (t^2 - t - 2)^2$

47. Rewrite 1 Differentiate Rewrite

$y\ 3x \quad dy \quad 2 \quad dy \quad 3$

$dx \quad 3x \quad dx \quad x^2$

48. Rewrite Differentiate Rewrite

$1 \quad 2 \quad dy \quad 1 \quad \frac{dy}{dx} \quad \frac{x}{2}$

$y\ 4x \quad dx \quad 2^x$

49. Rewrite Differentiate Rewrite

$3 \quad 4 \quad \frac{dy}{dx} \quad \frac{3}{2} x^3 \quad dy \quad \frac{3x^3}{2}$

$y = \frac{3}{8} x \quad dx$

50. Rewrite Differentiate Rewrite

$\frac{3}{2} x^2 \quad \frac{dy}{dx} \quad 3 \quad \frac{dy}{dx} \quad \frac{3}{x^3}$

$y \quad dx\ 3x$

51. Rewrite Differentiate Rewrite

$\frac{1}{2} \quad 5 \quad dy \quad 2 \quad 5 \quad \frac{dy}{dx} \quad \frac{2x}{3}\ 5$

2

x

$y\ 3x \quad 3 \quad dx \quad 3x \quad 3 \quad dx \quad 3$

52. Rewrite Differentiate Rewrite

$y\ 4x^{1/2} \quad dy \quad 3/ \quad \frac{dy}{dx} \quad \frac{2}{\sqrt{x^3}}$

$dx\ 2x$

53. $\frac{d}{dx}\left(x^3 - 2\right)\frac{x^2}{x}\ 1 \quad \frac{d}{dx}(x \quad 2) \quad \frac{2}{x}\ 1 \ (x \quad 2) \quad \frac{d}{dx}\ \frac{2}{x}\ 1$

$dx \quad x \quad 1 \quad dx \quad x \quad 1 dx \quad x \quad 1$

2
$3x$
x

$x^2\ 1\ (x \quad 3 \quad (x \quad 1)(2x) \quad (1)(x^2 \quad 1) \quad 2) \quad 1 \quad (x\ 1)^2$

2

$x\ 1 \quad 3x^2 \quad 1\ (x^3 \quad 2) \quad \frac{x^2\ 2x\ 1}{(x\ 1)^2}$

54. $\frac{d}{dx}\ 5 \ (x \quad 1) \quad x^3\ 2 \quad d\ 5\ (x \quad 1) \quad x^3\ 2 \quad 5\ (x \quad 1) \quad d\ 3 \quad 2$

4

$x\ 1 \quad 5x \quad x^3\ 2 \quad (x \quad 5 \quad 1) \quad \frac{(x\ 1)(3x^2) \ (1)(x^3 \quad 2)}{(x\ 1)^2}$

$x\ 1 \quad 4\ 5x \quad x^3\ 2 \quad (x \quad 5 \quad 1) \quad \frac{3x^3\ 3x^2\ x^3\ 2}{(x\ 1)^2}$

$x\ 1 \quad 4\ 5x \quad x^3\ 2 \quad (x \quad 5 \quad 1) \quad \frac{2x^3\ 3x^2\ 2}{(x\ 1)^2}$

55. $\frac{d}{dx}\ \frac{(x^2 \quad 3)(x^3 \quad 1)}{x^2 \quad 2} \quad \frac{(x^2 \quad 2)\ \frac{d}{dx}\ (x^2\ 3)(x^3 \quad 1) \quad (x^2 \quad 3)(x^3 \quad 1)\ \frac{d}{dx}(x^2 \quad 2)}{(x^2 \quad 2)^2}$

$\frac{(x^2 \quad 2)\ \frac{d}{dx}(x^2 \quad 3)(x^3 \quad 1) \quad (x^2 \quad 3)\frac{d}{dx}(x^3 \quad 1) \quad 2x(x^2 \quad 3)(x^3 \quad 1)}{(x^2 \quad 2)^2}$

$(x^2 \quad 2)(2x)(x^3 \quad 1) \quad (x^2 \quad 3)(3x^2) \quad 2x(x^2 \quad 3)(x^3 \quad 1)$

$$(x^2\ \ 2)^2$$

$$\frac{(x^2\ \ 2)(2x^4\ \ 2x\ \ 3x^4\ \ 9x^2)\ \ (2x)(x^5\ \ 3x^3\ \ x^2\ \ 3)}{(x^2\ \ 2)^2}$$

$$\frac{2x^6\ \ 2x^3\ \ 3x^6\ \ 9x^4\ \ 4x^4\ \ 4x\ \ 6x^4\ \ 18x^2\ \ 2x^6\ \ 6x^4\ \ 2x^3\ \ 6x}{(x^2\ \ 2)^2}$$

$$\frac{3x^6\ \ 13x^4\ \ 18x^2\ \ 2x}{(x^2\ \ 2)^2}$$

56. $$\frac{d}{dx}\frac{(x^3-2)(x^2-2)}{x^3-1} = \frac{(x^3-1)\frac{d}{dx}(x^3-2)(x^2-2) - \frac{d}{dx}(x^3-1)\,(x^3-2)(x^2-2)}{(x^3-1)^2}$$

$$= \frac{(x^3-1)\left[(3x^2)(x^2-2)+(x^3-2)(2x)\right] - 3x^2(x^3-2)(x^2-2)}{(x^3-1)^2}$$

$$= \frac{(x^3-1)(3x^2)(x^2-2) + (x^3-1)(x^3-2)(2x) - 3x^2(x^3-2)(x^2-2)}{(x^3-1)^2}$$

$$= \frac{3x^2(x^2-2)\left[(x^3-1)-(x^3-2)\right] + (x^3-1)(x^3-2)(2x)}{(x^3-1)^2}$$

$$= \frac{3x^4 - 6x^2 + 2x^7 - 6x^4 + 4x}{(x^3-1)^2}$$

$$= \frac{2x^7 - 3x^4 - 6x^2 + 4x}{(x^3-1)^2}$$

57. $$\frac{d}{dx}\frac{\sqrt{x}-1}{\sqrt{x}+1} = \frac{d}{dx}\frac{x^{1/2}-1}{x^{1/2}+1} = \frac{(x^{1/2}+1)\left(\frac{1}{2}x^{-1/2}\right) - \frac{1}{2}x^{-1/2}(x^{1/2}-1)}{(x^{1/2}+1)^2}$$

$$= \frac{\frac{1}{2} + \frac{1}{2}x^{-1/2} - \frac{1}{2} + \frac{1}{2}x^{-1/2}}{(x^{1/2}+1)^2} = \frac{x^{-1/2}}{(x^{1/2}+1)^2} = \frac{1}{\sqrt{x}(\sqrt{x}+1)^2}$$

58. $$\frac{d}{dx}\frac{\sqrt{x}+1}{\sqrt{x}-1} = \frac{d}{dx}\frac{x^{1/2}+1}{x^{1/2}-1} = \frac{(x^{1/2}-1)\left(\frac{1}{2}x^{-1/2}\right) - \frac{1}{2}x^{-1/2}(x^{1/2}+1)}{(x^{1/2}-1)^2}$$

$$= \frac{\frac{1}{2} - \frac{1}{2}x^{-1/2} - \frac{1}{2} - \frac{1}{2}x^{-1/2}}{(x^{1/2}-1)^2} = \frac{-x^{-1/2}}{(x^{1/2}-1)^2} = -\frac{1}{\sqrt{x}(\sqrt{x}-1)^2}$$

59. $$\frac{d}{dx}\frac{R(x)}{x} = \frac{xR'(x) - 1\cdot R(x)}{x^2} = \frac{xR'(x) - R(x)}{x^2}$$

60. $$\frac{d}{dx}\frac{P(x)}{x} = \frac{xP'(x) - 1\cdot P(x)}{x^2} = \frac{xP'(x) - P(x)}{x^2}$$

a. The instantaneous rate of change of cost with respect to purity is the derivative of the cost function $C(x) = \frac{100}{100-x}$ on $50 \le x < 100$.

$$C'(x) = \frac{(100-x)\,0 - (-1)(100)}{(100-x)^2} = \frac{100}{(100-x)^2} \quad \text{on } 50 \le x < 100$$

To find the rate of change for a purity of 95%, evaluate $C'(x)$ at $x = 95$.

$$C'(95) = \frac{100}{(100-95)^2} = \frac{100}{5^2} = 4$$

The cost is increasing by 4 cents per additional percent of purity.

To find the rate of change for a purity of 98%, evaluate $C'(x)$ at $x = 98$.

$$C'(98) = \frac{100}{(100-98)^2} = \frac{100}{2^2} = 25$$

The cost is increasing by 25 cents per additional percent of purity.

a. $AC(x) = \frac{C(x)}{x} = \frac{6x + 50}{x}$

b. The marginal average cost function $MAC(x)$ is the derivative of the average cost function

$AC(x) = \frac{C(x)}{x} = \frac{6x + 50}{x}$.

$MAC(x) = \frac{d}{dx}\left(\frac{6x + 50}{x}\right) = \frac{x(6) - 1(6x + 50)}{x^2}$

$= \frac{6x - 6x - 50}{x^2} = -\frac{50}{x^2}$

$MAC(25) = -\frac{50}{25^2} = -0.08$

The average cost is decreasing at the rate of 8 cents per additional truck.

a.

on [50, 100] by [0, 20]

b. Rate of change of cost is 4 for $x = 95$; rate of change of cost is 25 for $x = 98$.

a.

on [0, 400] by [0,50]

Rate of change is –0.03 for $x = 200$.

a. $AC(x) = \frac{C(x)}{x} = \frac{8x + 45}{x}$

The marginal average cost function $MAC(x)$ is the derivative of the average cost function

$AC(x) = \frac{C(x)}{x} = \frac{8x + 45}{x}$, or $8 + \frac{45}{x}$.

$MAC(x) = \frac{d}{dx}\left(\frac{8x + 45}{x}\right) = \frac{x(8) - 1(8x + 45)}{x^2} = \frac{8x - 8x - 45}{x^2} = -\frac{45}{x^2}$

$MAC(30) = -\frac{45}{30^2} = -\frac{1}{20} = -0.05$

The average cost is decreasing at the rate of 5 cents per additional clock.

To find the rate of change of the number of bottles sold, find $N'(p)$.

$N'(p) = \frac{(p + 7) \cdot 0 - (1)(2250)}{(p + 7)^2} = -\frac{2250}{(p + 7)^2}$

When $p = 8$, $N'(8) = -\frac{2250}{(8 + 7)^2} = -\frac{2250}{225} = -10$.

At $8 per bottle, the number of bottles of whiskey sold will decrease by about 10 bottles for each $1 increase in price.

To find the rate of change of temperature, find $T'(x)$.

$T'(x) = 3x^2(4 - x^2) + x^3(-2x)$

$= 12x^2 - 3x^4 - 2x^4 = 12x^2 - 5x^4$

For $x = 1$, $T'(1) = 12(1)^2 - 5(1)^4 = 12 - 5 = 7$.

After 1 hour, the person’s temperature is increasing by 7 degrees per hour.

To find the rate of change of the sales, find $S(x)$.

$$S(x) \quad 2x(8 \quad x^3) \quad x^2 \; (\; 3x^2)$$
$$16x \;\; 2x^4 \quad 3x^4 \quad 16x \;\; 5x^4$$
$$4$$

For $x = 1$, $S(1) \quad 16(1) \quad 5(1) \quad 16 \quad 5 \quad 11$.

After 1 month, sales are increasing by 11 thousand per month.

a.

on [0, 2] by [90, 110]
The rate of change at $x = 1$ is 7.
The maximum temperature is about 104.5 degrees.

a.

on [0, 2] by [0, 12]
The rate of change at $x = 1$ is 11.
The maximum sales are about 10.4 thousand.

a. $y_3(8) \; \frac{73.1 \; 8^2 \;\; 1270 \; 8 \;\; 16,280}{2.3 \; 8 \;\; 314} \; 65.468$

The per capita national debt in 2020 would be $65,468.

$$y_3(13) \; \frac{73.1 \; 13^2 \qquad 1270 \; 13 \;\; 16,280}{2.3 \; 13 \quad 314} \; 59.425$$

The per capita national debt in 2025 would be $59,425.

$y_3(8) \quad 0.151$

In 2020 the per capita national debt will be shrinking by $151 per year.

$y_3(13) \quad 2.231$

In 2025 the per capita national debt will be shrinking by $2231 per year.

72. a. $g'(x) \; \frac{(x^2 \;\; 110x \;\; 3500)(\; 30x \;\; 1125) \;\; (2x \;\; 110)(\; 15x^2 \quad 1125x)}{(x^2 \quad 110x \;\; 3500)^2}$

$$\frac{525x^2 \;\; 105,000x \;\; 3,937,500}{(x^2 \quad 110x \quad 3500)^2}$$

$$g'(40) \; \frac{525(40)^2 \;\; 105,000(40) \; 3,937,500}{(40)^2 \;\; 110(40) \; 3500^2}$$

$$1.1785714 \; \frac{\text{mi/gallon}}{\text{mi/hour}}$$

At 40 mph, your gas mileage increases by 1.1785714 for each additional mile per hour.

$$g'(50) \; \frac{525(50)^2 \quad 105,000(50) \quad 3,937,500}{2}$$

$$0 \; \frac{\text{mi/gallon}}{\text{mi/hour}}$$

At 50 mph, your gas mileage will not change for each additional mile per hour.

$$g'(60) \frac{525(60)^2 \quad 105,000(60) \quad 3,937,500}{(60)^2 \quad 110(60) \quad 3500^2}$$

$$1.89 \frac{\text{mi/gallon}}{\text{mi/hour}}$$

At 60 mph, your gas mileage decreases by 1.89 for each additional mile per hour.

The positive sign of $g'(40)$ tells you that gas mileage increases with speed when driving at 40 mph. The negative sign of $g'(60)$ tells you that gas mileage decreases with speed when driving at 60 mph. The fact that $g'(50)$ is zero tells you that gas mileage neither increases nor decreases with speed when driving at 50 mph. This means it is the most economical speed.

$I\ x \quad 0.45(x \ 1.7)(x^2 \quad 12.5x \ 43)$

$I\ x\ 0.45\ 1(x^2\ 12.5x\ 43)\ (x\ 1.7)(2x\ 12.5)\ I\ 5\ 0.45\ (5)^2$

$12.5(5)\ 43\ (5\ 1.7)(2(5)\ 12.5)$

1.2375

$I \quad 6 \quad 0.45\ (6)^2 \quad 12.5(6) \quad 43 \quad (6 \quad 1.7)(2(6) \quad 12.5)$

0.8325

Interest rates were declining by about 1.24 percent per year in 2010 and rising by 0.83 percent per year in 2011.

$I\ x \quad 0.106(x^2 \quad 6.85x \quad 14)(x^2 \quad 14.5x \quad 56)$

$I\ x \quad 0.106 \quad (2x \quad 6.85)(x^2 \quad 14.5x \quad 56) \quad (x^2 \quad 6.85x \quad 14)(2x \quad 14.5)$

$I \quad 3 \quad 0.106\ (2(3) \quad 6.85)((3)^2 \quad 14.5(3) \quad 56) \quad ((3)^2 \quad 6.85(3) \quad 14)(2(3) \quad 14.5)$

4.1446

$I \quad 5 \quad 0.106\ (2(5) \quad 6.85)((5)^2 \quad 14.5(5) \quad 56) \quad ((5)^2 \quad 6.85(5) \quad 14)(2(5) \quad 14.5)$

0.5724

Interest rates were declining by about 4.14 percent per year in 2008 and rising by 0.57 percent per year in 2010.

75. $E\ x \quad \frac{261x}{x \ 8.84} \quad 647$

$$E\ x \quad \frac{(x \ 8.84)\ 261 \quad 1\ (261x)}{x \ 8.84^2} \quad \frac{2307.24}{x \ 8.84^2}$$

$$E\ 4 \frac{2307.24}{4 \ 8.84^2} \quad 13.99$$

$$E\ 8 \frac{2307.24}{8 \ 8.84^2} \quad 8.14$$

Median weekly earnings were rising by \$13.99 per year in 2009 and by \$8.14 per year in 2013.

76. $C\ x \frac{75(44 \ 5x)}{x^2 \quad 18.3x \quad 85} \ 115x \quad 1605$

$$75 \frac{44 \quad 5x}{x^2 \quad 18.3x \quad 85} \quad 115x \quad 1605$$

$$C\ x\ 75 \frac{(x^2 \quad 18.3x \quad 85)(\ 5) \quad (2x \quad 18.3)(44 \quad 5x)}{(x^2 \quad 18.3x \quad 85)^2} \ 115$$

$$75 \frac{5x^2 \quad 88x \ 380.2}{(x^2 \ 18.3x \ 85)^2} \ 115\ (x$$

$C'(7) \approx 75\frac{5x^2-88x+380.2}{(x^2-18.3x+85)^2} \approx 115 \quad 134.82$

$C'(9) \approx 75\frac{5x^2-88x+380.2}{(x^2-18.3x+85)^2} \approx 115 \quad 186.78$

$C'(11) \approx 75\frac{5x^2-88x+380.2}{(x^2-18.3x+85)^2} \approx 115 \quad 173.34$

Outstanding credit was rising by about $135 million per year in 2007, falling by $187 million per year in 2009, and rising by $173 million per year in 2011.

77. a. $\frac{d}{dx}\frac{1}{x^2} = \frac{d}{dx}(x^{-2}) = -2x^{-3} = -\frac{2}{x^3}$

$\left.\frac{d}{dx}\right|_{x=0} = -\frac{2}{0^3}$ Undefined

Answers will vary.

78. a. $\frac{d}{dx}\frac{1}{x} = \frac{d}{dx}(x^{-1}) = -x^{-2} = -\frac{1}{x^2}$

$\left.\frac{d}{dx}\right|_{x=0} = -\frac{1}{0^2}$ Undefined

Answers will vary.

False: the product rule gives the correct right-hand side.

True: $\frac{d}{dx}\left(x \cdot f\right) = \frac{d}{dx}(x) \cdot f + x \cdot f' = f + x \cdot f'$

$\frac{d}{dx}\left(\frac{f \cdot g}{f \cdot g}\right) \cdot \frac{f \cdot g' + f' \cdot g}{?}$ … $g \cdot f' + f \cdot g'$

The right-hand side multiplies out to $g \cdot f' + f \cdot g'$ which agrees with the product rule.

False: This would be the same as saying that the derivative (instantaneous rate of change) of a product is a product of the derivatives. The product rule gives the correct way of finding the derivative of a product.

$\frac{d}{dx}(f \cdot g \cdot h) = \frac{d}{dx}[f \cdot (g \cdot h)] = \frac{df}{dx}(g \cdot h) + f\frac{d}{dx}(g \cdot h)$

$= \frac{df}{dx}(g \cdot h) + f\left(\frac{dg}{dx}h + g\frac{dh}{dx}\right)$

$= \frac{df}{dx}g\,h + f\frac{dg}{dx}h + f\,g\frac{dh}{dx}$

False: the quotient rule gives the correct right-hand side.

82. True: $\frac{d}{dx}\left(\frac{f}{x}\right) = \frac{x \cdot f' - \frac{d}{dx}x \cdot f}{(x)^2} = \frac{x \cdot f' - f}{x^2}$

84. $\frac{d}{dx}\frac{f}{g} = \frac{f}{g}\left(\frac{f'}{f} - \frac{g'}{g}\right)$

$= \frac{f'}{g} - \frac{f\,g'}{g\,g}$

$= \frac{g\,f' - f\,g'}{g^2}$

The right-hand side multiplies out to $\frac{g\,f' - f\,g'}{g^2}$ which agrees with the quotient rule.

False: This would be the same as saying that the derivative (instantaneous rate of change) of a quotient is a quotient of the derivatives. The quotient rule gives the correct way of finding the derivative of a quotient.

a. $Q(x) = \frac{f(x)}{g(x)}$

$g(x)\,Q(x) = g(x)\frac{f(x)}{g(x)}$

$g(x)\,Q(x) = f(x)$

$Q'(x)\,g(x) + Q(x)\,g'(x) = f'(x)$

$Q'(x)\,g(x) = f'(x) - Q(x)\,g'(x)$

$Q'(x) = \frac{f'(x) - Q(x)\,g'(x)}{g(x)}$

e. $Q'(x) = \frac{f'(x) - \frac{f(x)}{g(x)}\,g'(x)}{g(x)} \cdot \frac{1}{g(x)}\frac{[g(x)\,f'(x) - f(x)\,g'(x)]}{g(x)} = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}$

$\frac{d}{dx}[f(x)]^2 = \frac{d}{dx}[f(x)\,f(x)]$

$= \frac{d}{dx}f(x)\,f(x) + f(x)\frac{d}{dx}f(x)$

$= f'(x)\,f(x) + f(x)\,f'(x) = 2f(x)\,f'(x)$

90. Rewrite $\frac{1}{f(x)} = [f(x)]^{-1}$ and find $\frac{d}{dx}\frac{1}{f(x)}$.

$\frac{d}{dx}\frac{1}{f(x)} = \frac{f(x)\cdot 0 - f'(x)\cdot 1}{[f(x)]^2} = -\frac{f'(x)}{[f(x)]^2}$

91. $\frac{dy}{dx} = \frac{Rx}{1+\frac{R-1}{K}x} = \frac{\left(1+\frac{R-1}{K}x\right)R - Rx\frac{R-1}{K}}{\left(1+\frac{R-1}{K}x\right)^2} = \frac{R + Rx\frac{R-1}{K} - Rx\frac{R-1}{K}}{\left(1+\frac{R-1}{K}x\right)^2} = \frac{R}{\left(1+\frac{R-1}{K}x\right)^2}$

To show $y' > 0$, show that both the numerator and denominator are greater than zero.

$R > 1 > 0$ *numerator is greater than zero.*

$R > 1 \Rightarrow R - 1 > 0.$

$R-1>0; K>0 \Rightarrow \frac{R-1}{K} > 0$

$\frac{R-1}{K} > 0,\ x > 0 \Rightarrow 1 + \frac{R-1}{K}x > 0 \Rightarrow$ denominator is greater than zero.

$y' > 0.$

This means the density of the offspring is always increasing faster than the density of the parents.

92. $w > 5$ and $3.5 > 0$, $(w-1.5)^2 > 0$ and $w - 1.5 > 0 \Rightarrow R'(w) = \frac{w-5}{w-1.5} > 0.$

A task that expends w kcal/min of work for $w > 5$ requires more than w minutes of rest.

EXERCISES 2.5

1. **a.** $f'(x) = 4x^3 + 3(2)x^2 + 2(3)x + 5 + 0$
$= 4x^3 + 6x^2 + 6x + 5$

b. $f''(x) = 3(4)x^2 + 2(6)x + 6 + 0 = 12x^2 + 12x + 6$

c. $f'''(x) = 2(12)x + 12 + 0 = 24x + 12$

d. $f^{(4)}(x) = 24$

2. **a.** $f'(x) = 4x^3 - 9x^2 + 4x - 8$

b. $f''(x) = 12x^2 - 18x + 4$

c. $f'''(x) = 24x - 18$

d. $f^{(4)}(x) = 24$

3. **a.** $f(x)\ 1\ x\ \ \frac{1}{2}x^2\ \ \frac{1}{6}x^3\ \ \frac{1}{24}x^4$

b. $f(x)\ \ 1\ x\ \ \frac{1}{2}x^2\ \ \frac{1}{6}x^3$

c. $f^{(4)}(x)\ 1\ x\ \ \frac{1}{2}x^2$

d. $f^{(4)}(x)\ 1\ x$

5. $f(x)\ \sqrt{x^5}\ \ x^{5/2}$

a. $f'(x)\ \frac{5}{2}x^{3/2}$

b. $f''(x)\ \frac{3}{2}\frac{5}{2}x^{1/2}\ \ \frac{15}{4}\sqrt{x}$

c. $f'''(x)\ \frac{d}{dx}\frac{15}{4}x^{1/2}\ \ \frac{1}{2}\frac{15}{4}x^{-1/2}$

$\frac{15}{8}x^{-1/2}\ \ \frac{15}{8\sqrt{x}}$

d. $f^{(4)}(x)\ \frac{d}{dx}\frac{15}{8}x^{-1/2}\ \ \frac{1}{2}\frac{15}{8}x^{-3/2}$

$\frac{15}{16}x^{-3/2}$

7. $f(x)\ \frac{x\ 1}{x}\ 1\ \frac{1}{x}$

a. $f'(x)\ \frac{1}{x^2}$

$\frac{d}{dx}$

$\frac{2}{x^3}$

$f''(3)\ \frac{2}{3^3}\ \frac{2}{27}$

$f(x)\ \frac{x\ 1}{2x}\ \frac{1}{2}\ \frac{1}{2}\frac{1}{x}$

$f'(x)\ \frac{1}{2x^2}$

$f''(x)\ \frac{d}{dx}\frac{1}{2x^2}\ \ \frac{d}{dx}\frac{1}{2}x^{-2}\ \ \frac{1}{x^3}$

$f''(3)\ \frac{1}{3^3}\ \frac{1}{27}$

$\frac{d}{dx}f''(x)\ 2x$

4. **a.** $f(x)\ 1\ x\ \ \frac{1}{2}x^2\ \ \frac{1}{6}x^3$

$f(x)\ 1\ x\ \ \frac{1}{2}x^2$

$f(x)\ 1\ x$

$f^{(4)}(x)\ 1$

$f(x)\ \ x^3\sqrt{x}\ \ x^{3/2}$

a. $f(x)\ \frac{3}{2}x^{1/2}\ \ \frac{3}{2}\sqrt{x}$

$f(x)\ \frac{3}{4}x^{-1/2}\ \ \frac{3}{4\sqrt{x}}$

$f(x)\ \frac{3}{8}x^{-3/2}$

d. $f^{(4)}(x)\ \frac{9}{16}x^{-5/2}$

$f(x)\ \frac{x\ 2}{x}\ 1\ \frac{2}{x}$

$f'(x)\ \frac{2}{x^2}$

$f''(x)\ \frac{d}{dx}\frac{2}{x^2}\ \ \frac{d}{dx}2x^{-2}\ \ \frac{4}{x^3}$

$f''(3)\ \frac{4}{3^3}\ \frac{4}{27}$

$f(x)\ \frac{x\ 2}{4x}\ \frac{1}{4}\ \frac{1}{2x}$

$f'(x)\ \frac{1}{2x^2}$

$f''(x)\ \frac{d}{dx}\frac{1}{2x^2}\ \ \frac{d}{dx}\frac{1}{2}x^{-2}\ \ \frac{1}{x^3}$

$f''(3)\ \frac{1}{3^3}\ \frac{1}{27}$

$f(x)\ \frac{1}{12x}$

$$f(x) = \frac{1}{6x^{2}}$$

$$f'(x) = -2\left(\frac{1}{6}\right)x^{-3} = -\frac{1}{3x^{3}}$$

$$f''(x) = \frac{d}{dx}\left(-\frac{1}{3}x^{-3}\right) = x^{-4} = \frac{1}{x^{4}}$$

$$f''(3) = 3^{-4} = \frac{1}{81}$$

$$f'(x) = -\frac{1}{4x^{4}}$$

$$f''(x) = \frac{d}{dx}\left(-\frac{1}{4}x^{-4}\right) = x^{-5} = \frac{1}{x^{5}}$$

$$f''(3) = 3^{-5} = \frac{1}{243}$$

$f(x) = x^2 2x^2 3x^4 x^2 6$ $f'(x) = 4x^3$ $2x$

$f''(x) = 12x^2 \; 2$

$f(x) = x^2 1x^2 2x^4 x^2 2$ $f'(x) = 4x^3$ $2x$

$f''(x) = 12x^2 \; 2$

15. $f(x) = \frac{27}{\sqrt[3]{x}} = 27x^{1/3}$

$f'(x) = \frac{1}{3} 27x^{4/3} \; 9x^{4/3}$

$f''(x) = \frac{4}{3} 9x^{7/3} \quad 12x^{7/3}$

16. $f(x) = \frac{32}{\sqrt[4]{x}} = 32x^{1/4}$

$f'(x) = \frac{1}{4} 32x^{5/4} \quad 8x^{5/4}$

$f''(x) = \frac{5}{4} 8x^{9/4} \quad 10x^{9/4}$

$f(x) = \frac{x}{x \; 1}$

$f'(x) = \frac{(x\;1)\frac{d}{dx}(x) \; (x)\frac{d}{dx}(x\;1)}{(x\;1)^2} = \frac{(x\;1)(1) \; (1)(x)}{(x\;1)^2} = \frac{1}{(x\;1)^2} = \frac{1}{x^2 \; 2x \; 1}$

$f''(x) = \frac{(x^2 \; 2x \; 1)\frac{d}{dx}(1) \; (1)\frac{d}{dx}(x^2 \; 2x \; 1)}{[(x\;1)^2]^2} = \frac{0 \; (2x \; 2)(1)}{(x\;1)^4} = \frac{2x\;2}{(x\;1)^4} = \frac{2(x\;1)}{(x\;1)^4} = \frac{2}{(x\;1)^3}$

18. $f(x) = \frac{x}{x\;2}$

$f'(x) = \frac{(x\;2)(1) \; (1)(x)}{(x\;2)^2} = \frac{x\;2\;x}{(x\;2)^2} = \frac{2}{(x\;2)^2} = \frac{2}{x^2\;4x\;4}$

$f''(x) = \frac{(x^2\;4x\;4)\,0 \; (2x\;4)(2)}{[(x\;2)^2]^2} = \frac{4x\;8}{(x^2\;4x\;4)^2} = \frac{4(x\;2)}{(x\;2)^4} = \frac{4}{(x\;2)^3}$

$\frac{d}{dr}\left(\frac{r^2}{2}\right) = 2r$

$\frac{d^2}{dr^2}\left(\frac{r^2}{2}\right) = \frac{d}{dr}(2r) = 2$

$\frac{d}{dr}\left(\frac{4}{3} r^3\right) = 4r^2$

$\frac{d^2}{dr^2}\left(\frac{4}{3} r^3\right) = \frac{d}{dr}(4r^2) = 8r$

$\frac{d^3}{dr^3}\left(\frac{4}{3} r^3\right) = \frac{d}{dr}(8r) = 8$

21. $\frac{d}{dx} x^{10} = 10x^9$

$\frac{d^2}{dx^2} x^{10} = \frac{d}{dx}(10x^9) = 90x^8$

$\left.\frac{d^2}{dx^2} x^{10}\right|_{x=1} = 90(1)^8 = 90$

22. $\frac{d}{dx} x^{11} = 11x^{10}$

$\frac{d^2}{dx^2} x^{11} = \frac{d}{dx}(11x^{10}) = 110x^9$

$\left.\frac{d^2}{dx^2} x^{11}\right|_{x=1} = 110(1)^9 = 110$

23. From Exercise 21, we know

$$\frac{d^2}{dx^2} x^{10} = 90x^8$$

$$\frac{d^3}{dx^3} x^{10} = \frac{d}{dx}(90x^8) = 720x^7$$

Thus, $\left.\frac{d^3}{dx^3} x^{10}\right|_{x=-1} = 720(-1)^7 = -720$

24. From Exercise 22, we know

$$\frac{d^2}{dx^2} x^{11} = 110x^9$$

$$\frac{d^3}{dx^3} x^{11} = \frac{d}{dx}(110x^9) = 990x^8$$

Thus, $\left.\frac{d^3}{dx^3} x^{11}\right|_{x=-1} = 990(-1)^8 = 990$

25. $\frac{d}{dx}\sqrt{x^3} = \frac{d}{dx}x^{3/2} = \frac{3}{2}x^{1/2}$

$\frac{d^2}{dx^2}\sqrt{x^3} = \frac{d}{dx}\frac{3}{2}x^{1/2} = \frac{3}{4}x^{-1/2}$

$\left.\frac{d^2}{dx^2}\sqrt{x^3}\right|_{x=1/16} = \frac{3}{4}\cdot\frac{1}{\sqrt{1/16}} = \frac{3}{4}\sqrt{16} = 3$

26. $\frac{d}{dx}\sqrt[3]{x^4} = \frac{d}{dx}x^{4/3} = \frac{4}{3}x^{1/3}$

$\frac{d^2}{dx^2}\sqrt[3]{x^4} = \frac{d}{dx}\frac{4}{3}x^{1/3} = \frac{4}{9}x^{-2/3}$

$\left.\frac{d^2}{dx^2}\sqrt[3]{x^4}\right|_{x=1/27} = \frac{4}{9}\cdot\frac{1}{(1/27)^{2/3}} = \frac{4}{9}(27)^{2/3} = 4$

$$\frac{d}{dx}[(x^2+x+1)(x^3-1)] = (2x+1)(x^3-1) + (x^2+x+1)(3x^2)$$
$$= 2x^4 + x^3 - 2x - 1 + 3x^4 + 3x^3 + 3x^2 = 5x^4 + 4x^3 + 3x^2 - 2x - 1$$

$$\frac{d^2}{dx^2}[(x^2+x+1)(x^3-1)] = \frac{d}{dx}(5x^4+4x^3+3x^2-2x-1)$$
$$= 20x^3 + 12x^2 + 6x - 2$$

$$\frac{d}{dx}[(x^3-x+1)(x^3-1)] = (3x^2-1)(x^3-1) + (x^3-x+1)(3x^2)$$
$$= 3x^5 - 3x^2 - x^3 + 1 + 3x^5 - 3x^3 + 3x^2 = 6x^5 - 4x^3 + 1$$

$$\frac{d^2}{dx^2}[(x^3-x+1)(x^3-1)] = \frac{d}{dx}(6x^5-4x^3+1) = 30x^4 - 12x^2$$

29. $\frac{d}{dx}\frac{x}{x^2+1} = \frac{(x^2+1)\frac{d}{dx}x - x\frac{d}{dx}(x^2+1)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = \frac{1-x^2}{x^4+2x^2+1}$

$$\frac{d^2}{dx^2}\frac{x}{x^2+1} = \frac{d}{dx}\frac{1-x^2}{x^4+2x^2+1} = \frac{(x^4+2x^2+1)\frac{d}{dx}(1-x^2) - (1-x^2)\frac{d}{dx}(x^4+2x^2+1)}{[(x^2+1)^2]^2}$$
$$= \frac{(x^4+2x^2+1)(-2x) - (1-x^2)(4x^3+4x)}{(x^2+1)^4} = \frac{(x^2+1)[(x^2+1)(-2x) - (1-x^2)(4x)]}{(x^2+1)^4}$$
$$= \frac{-2x^3 - 2x - 4x + 4x^3}{(x^2+1)^3} = \frac{2x(x^2-3)}{(x^2+1)^3}$$

30. $\frac{d}{dx}\frac{x}{x^2-1} = \frac{(x^2-1)(1) - (2x)x}{(x^2-1)^2} = \frac{x^2-2x^2-1}{(x^2-1)^2} = \frac{-x^2-1}{(x^2-1)^2} = \frac{-x^2-1}{x^4-2x^2+1}$

$$\frac{d^2}{dx^2}\frac{x}{x^2-1} = \frac{(x^2-1)^2(-2x) - (4x^3-4x)(-x^2-1)}{(x^2-1)^4}$$
$$= \frac{-2x^5+4x^3-2x-(-4x^5+4x^3-4x^3+4x)}{(x^2-1)^4}$$
$$= \frac{-2x^5+4x^3-2x+4x^5-4x^3+4x^3-4x}{(x^2-1)^4} = \frac{2x^5+4x^3-6x}{(x^2-1)^4}$$

31. $\frac{d}{dx}\frac{2x+1}{2x-1} = \frac{(2x-1)2 - 2(2x+1)}{(2x-1)^2} = \frac{4x-2-4x-2}{(2x-1)^2} = \frac{-4}{\ldots}$

$$dx \quad 2x\,1 \qquad 2\,(2x\ \ 1)^2 \qquad 4x^2\ \ 4x\ \ 1 \quad 4x^2 \qquad 4x\ \ 1$$

$$\frac{d^2}{dx^2}\ \frac{2x\,1}{2x\,1} \quad \frac{(4x^2\ \ 4x\,1)\,0\ \ (8x\ \ 4)(4)}{(4x^2\ \ 4x\ \ 1)^2} \quad \frac{32x\ \ 16}{[(2x\ \ 1)^2]^2} \quad \frac{32x\,16}{(2x\ \ 1)^4}$$

$$\frac{16(2x\ \ 1)}{(2x\,1)^4} \quad \frac{16}{(2x\ \ 1)^3}.$$

32. $\frac{d}{dx^2}\ \frac{3x\ 1}{3x\ 1}$ $\frac{(3x\ 1)(3)\ 3(3x\ 1)}{(3x\ 1)^2}$ $\frac{9x\ 3\ 9x\ 3}{9x^2\ 6x\ 1}$ $\frac{6}{9x^2\ 6x\ 1}$

$\frac{d}{dx^2}\ \frac{3x\ 1}{3x\ 1}$ $\frac{(9x^2\ 6x\ 1)\ 0\ 6\ (18x\ 6)}{(9x^2\ 6x\ 1)^2}$ $\frac{36(3x\ 1)}{[(3x\ 1)^2]^2}$ $\frac{36}{(3x\ 1)^3}$

To find velocity, we differentiate the distance function $s(t)\ 18t^2\ \ 2t^3$.

$v(t)\ \ s(t)\ \ 36t\ \ 6t^2$

For $t = 3$, $v(3)\ \ s(3)\ \ 36(3)\ \ 6(3)^2\ \ 108\ \ 54\ \ 54$ miles per hour

For $t = 7$, $v(7)\ \ s(7)\ \ 36(7)\ \ 6(7)^2\ \ 252\ \ 294$ 42 miles per hour

To find the acceleration, we differentiate the velocity function $s(t)\ 36t\ 6t^2$. $a(t)\ s(t)$ $36\ 12t$.

For $t = 1$, $a(1)\ \ s(1)\ \ 36\ \ 12(1)\ \ 24$ mi / hr^2

To find velocity, we differentiate the distance function $s(t)\ 24t^2\ 2t^3$.

$v(t)\ s(t)\ 48t\ 6t^2$

For $t = 4$, $v(4)\ \ s(4)\ \ 48(4)\ \ 6(4)^2\ \ 96$ miles per hour

For $t = 10$, $v(10)\ \ s(10)\ \ 48(10)\ \ 6(10)^2$ 120 miles per hour

To find the acceleration, we differentiate the velocity function $s(t)\ 48t\ 6t^2$. $a(t)\ s(t)$ $48\ 12t$.

For $t = 1$, $a(1)\ \ s(1)\ \ 48\ \ 12(1)\ \ 36$ mi / hr^2

35. $v(t)\ \ h(t)\ \ 3t^2\ \ 2(0.5)t\ \ 3t^2\ \ t$

$v(10)\ \ 3(10^2)\ \ \ 10\ \ 310$ feet per second

$a(t) = v(t) = \frac{d}{dt}(3t^2\ \ t)\ \ 6t\ \ 1$

$a(10)\ \ 6(10)\ \ 1\ \ 61$ ft / sec^2

36. $v(t)\ \ s(t)\ \ 60\ \frac{(t\ 3)\ 0\ \ (1)(100)}{(t\ 3)^2}$

$60\ \frac{100}{(t\ 3)^2}$

$v(2)\ \ 60\ \frac{100}{(2\ \ 3)^2}\ \ 60\ \frac{100}{25}$ = 56 miles per hour

37. a. To find when the steel ball will reach the ground, we need to determine what value of t produces $s(t)\ \ 0$. Thus, set $s(t)\ \ 0$ and solve the equation.

$0\ \ s(t)\ \ 2717\ \ 16t^2$

$0\ \ 2717\ \ 16t^2$

$16t^2\ \ 2717$

$t^2\ \ 169.8125$

$t\ \ 13.0312$ sec

The steel ball will reach the ground after about 13.03 seconds.

b. $v(t)\ \ s'(t)\ \ 32t$

$v(13.03)\ \ s'(13.03)\ \ 417$ feet per second

c. $a\ t\ \ v(t)\ \ s(t)$

32 feet per second per second

38. a. To find how long it will take to reach the ground, we need to determine what value of t produces $s(t)\ \ 0$. Thus, set $s(t)\ \ 0$ and solve the equation.

$s(t)\ \ 1451\ \ 16t^2\ \ \ 0$

$t^2\ \ \frac{1451}{16}$

$t\ \ 9.52$ sec

It will take about 9.52 seconds to reach the ground.

b. $v(t)\ \ s'(t)\ \ 32t$

$v(9.52)\ \ 32(9.52)\ \ 305$ feet per second

c. $a\ t\ \ v(t)\ \ s(t)$

32 feet per second per second

a. $v(t)\ s(t)\ \ 32t\ \ 1280$

When $s(t)$ is a maximum, $s(t)\ 0$.
$32t\ 1280\ 0$

$32t\ \ 1280$
40 seconds

At $t\ \ 40$,

$s(40)\ \ 16(40)^{2}\ \ 1280(40)$

25, 600 51, 200
25, 600 feet

$D'(t)\ \frac{4}{3}\ 9t^{1/3}\ \ 12t^{1/3}$
$1/3$

$D'(8)\ \ 12(8)^{1/3}\ \ 12(2)\ \ 24$

The national debt is increasing by 24 billion dollars per year after 8 years.

$D(t)\ \frac{d}{dt}\ 12\ t^{1/3}\ \ \frac{1}{3}\ 12\ t^{2/3}\ \ 4t^{2/3}$

$D(8)\ \ 4(8)^{2/3}\ \ 4\frac{1}{8}^{2/3}\ 4\frac{1}{4}\ \ 1$

The rate of growth of the national debt is increasing by 1 billion dollars per year each year after 8 years.

$L(10)\ \ 9.3$

By 2100, sea levels may have risen by 93 cm (about 3 feet).

$L'(t)\ 0.06\,x^{4}\ \ 0.14\,x\ \ 8$
$L'(10)\ \ 12.6$

In 2100 sea levels will be rising by about 12.6 centimeters per decade, or about 1.26 cm (about half an inch) per year.

$L(x)\ \ 0.12\,x\ \ 0.14$
$L(10)\ \ 1.06$

The rise in the sea level will be speeding up (by about 1 cm per decade per decade).

40. $T(2)\ \ 98\ \ \frac{8}{\sqrt{2}}\ 103.7$

The temperature of a patient 2 hours after taking the medicine is 103.7 degrees.

$T(t)\ \ 98\ \ 8t^{1/2}$
$3/2\ \ \frac{4}{}$

$T(t)\ \ 4t\ \ t^{3/2}$

$T(2)\ \ \frac{4}{2^{3/2}}\ \ 1.4$

Two hours after taking the medicine, the patient's temperature is decreasing at the rate of 1.4 degrees per hour.

$T(t)\ 6t^{5/2}\ \ \frac{6}{t^{5/2}}$

$T(2)\ \frac{6}{2^{5/2}}\ \ 1.1$

After 2 hours, the rate of decrease of the patient's temperature is increasing by about 1.1 degree per hour each hour.

$T(10)\ \ 6.3$

In 2100 global temperatures will have risen by about 6.3°F.

$'(t)\ 0.35t^{0.4}$

$T'(10)\ 0.88$

In 2100 global temperatures will be rising by 0.88 degrees per decade, or about a tenth of a degree per year.

$(t)\ 0.14t^{0.6}$
$T(10)\ 0.035$

The temperature increases will be speeding up (by about 0.035 degrees per decade per decade).

$P(x)\ \ 1.581x^{0.7}\ \ 0.70376\,x^{0.52}$

$P(x)\ 1.1067x^{1.7}\ 0.3660\,x^{0.48}\ P(3)$
4.87

The profit 3 years from now will be \$4.87 million.

$P(3)\ 0.51$

The profit will be decreasing by about \$0.51 million per year 3 years from now.

$P(3)\ \ \ 0.39$

In 3 years, the rate of decline of profit will be accelerating by about \$0.39 million per year each year.

a. For $x = 15$, approximately 21.589 ; for $x = 30$, approximately 17.597 .

on [0,50] by [0,40]

Each 1-mph increase in wind speed lowers the wind-chill index. As wind speed increases, the rate with which the wind-chill index decreases slows.
For $x = 15$, $y' = -0.363$. For a wind speed of 15 mph, each additional mile per hour decreases the wind-chill index by about 0.363. For $x = 30$, $y' = -0.203$. For a wind speed of 30 mph, each additional mile per hour decreases the wind-chill index by about 0.203˚.

a.

on [1, 20] by [0, 800]
The second derivative is $f''(x)\ 0.2184x^2\ 3.156x\ 2.6$, which equals zero when x is approximately equal to 13.57. The AIDS epidemic began to slow in 1993.

a. The first derivative is negative because the temperature is dropping.
The second derivative is negative because the temperature is dropping increasingly rapidly making the change in temperature speed up in a negative direction.

a. The first derivative is positive because the economy is growing.
The second derivative is negative because the growth is slowing.

a. The first derivative is negative, because the stock market is declining.
The second derivative is positive because the change in the stock market is slowing down in a negative direction.

a. The first derivative is positive because the population is growing.
The second derivative is positive because the change in population is speeding up.

51. True: For example $\frac{d^4}{dx^4}\ 4x^3\ 0$.

a. $f'(1)$ will be positive because the altitude is increasing. $f''(1)$ will be positive because acceleration is positive.
$f'(59)$ will be negative because the altitude is decreasing.
$f''(59)$ will be negative because acceleration is negative.

a. iii (showing a stop, then a slower velocity)
i (showing a stop, then a negative velocity)
ii (showing stops and starts and then a higher velocity)

a. ii (showing positive and increasing slope)
iii (showing negative but increasing slope)
i (showing positive but decreasing slope)

55. $\frac{d^{100}}{dx^{100}}(x^{99}\ 4x^{98}\ 3x^{50}\ 6)\ 0$

$\frac{d}{dx}(x^{1})\ x^{2}$

$\frac{d^2}{dx^2}(x^{1})\ 2x^{3}\ \frac{2!}{x^3}$

$\frac{d^3}{dx^3}(x^{1})\ 6x^{4}\ \frac{3!}{x^4}$

$\frac{d^4}{dx^4}(x^{1})\ 24x^{5}\ \frac{4!}{x^5}$

$\frac{d^n}{dx^n}(x^{1})\ (1)^n\ \frac{n!}{x^{n\ 1}}$

$\frac{d^2}{dx^2}(f \cdot g) = \frac{d}{dx}\left(\frac{df}{dx} g + f \frac{dg}{dx}\right)$ or $\frac{d}{dx}(f' g + f g')$

$= \frac{d}{dx}\frac{df}{dx} \cdot g + \frac{df}{dx}\frac{dg}{dx} + \frac{df}{dx}\frac{dg}{dx} + f \frac{d}{dx}\frac{dg}{dx}$

$\frac{d^2}{dx^2}(f \cdot g) = \frac{d^2 f}{dx^2} g + 2\frac{df}{dx}\frac{dg}{dx} + f\frac{d^2 g}{dx^2}$ or $f'' g + f' g' + f' g' + f g''$

$= f'' g + 2 f' g' + f g''$

58. From Exercise 31, we know that $\frac{d^2}{dx^2}(f \cdot g) = f'' g + 2 f' g' + f g''$.

$\frac{d^3}{dx^3}(f \cdot g) = \frac{d}{dx}\frac{d^2}{dx^2}(f \cdot g) = \frac{d}{dx}(f'' g + 2 f' g' + f g'') = f''' g + f'' g' + 2 f'' g' + 2 f' g'' + f' g'' + f g'''$

$= f''' g + 3 f'' g' + 3 f' g'' + f g'''$

EXERCISES 2.6

1. $f(g(x)) = \sqrt{x^2 - 3x + 1}$

The outside function is $\sqrt{x}$ and the inside function is $x^2 - 3x + 1$.

Thus, we take $f(x) = \sqrt{x}$, $g(x) = x^2 - 3x + 1$

$f(g(x)) = (5x^2 - x + 2)^4$

The outside function is x^4 and the inside function is $5x^2 - x + 2$.

Thus, we take $f(x) = x^4$, $g(x) = 5x^2 - x + 2$

$f(g(x)) = (x^2 - x)^3$

The outside function is x^3 and the inside function is $x^2 - x$.

Thus, we take $f(x) = x^3$, $g(x) = x^2 - x$

$f(g(x)) = \frac{1}{x^2 - x}$

The outside function is $\frac{1}{x}$ and the inside function is $x^2 - x$.

Thus, we take $f(x) = \frac{1}{x}$, $g(x) = x^2 - x$

$f(g(x)) = \frac{x^3 + 1}{x^3 - 1}$

$f(x) = \frac{x + 1}{x - 1}$

$g(x) = x^3$

$f(g(x)) = \left(\frac{x + 1}{x - 1}\right)^4$ $f(x) = x^4$

$g(x) = \frac{x + 1}{x - 1}$

6. $f(g(x)) = \frac{\sqrt{x} + 1}{\sqrt{x} - 1}$

$f(x) = \frac{x + 1}{x - 1}$

$g(x) = \sqrt{x}$

8. $f(g(x)) = \sqrt{\frac{x + 1}{x - 1}}$

$f(x) = \sqrt{x}$

$g(x) = \frac{x + 1}{x - 1}$

9. $f(g(x)) = \sqrt{x^2 - 9} + 5$

$f(x) = \sqrt{x} + 5$

$g(x) = x^2 - 9$

10. $f(g(x)) = \sqrt[3]{x^3 - 8} + 5$ [redacted]

$g(x)\ \ x\ \ 9$

$f(x)\ \ (x^{2}\ \ 1)^{3}$

$f(x)\ \ 3(x^{2}\ \ 1)^{2}\ \ (2x)\ \ 6x(x^{2}\ \ 1)^{2}$

$f(x)\ \ x\ \ 5$

$g(x)\ \ x^{3}\ \ 8$

$f(x)\ \ (x^{3}\ \ 1)^{4}$

$f(x)\ \ 4(x^{3}\ \ 1)^{3}(3x^{2})\ \ 12x^{2}(x^{3}\ \ 1)^{3}$

$g(x)\ \ 2x^2\ \ 7x\ \ 3^4$

$g'(x)\ \ 4\ 2x^2\ \ 7x\ \ \ 3^3\ (4x\ \ 7)$

$g(x)\ \ 3x^3\ \ x^2\ \ 1^5$

$g'(x)\ 5\ 3x^3\ \ x^2\ \ 1^4\ (9x^2\ \ 2x)$

$h(z)\ \ (3z^2\ \ 5z\ \ 2)^4$

$h(z)\ \ 4(3z^2\ \ 5z\ \ 2)^3\ \ (6z\ \ 5)$

$h(z)\ \ (5z^2\ \ 3z\ \ 1)^3$

$h(z)\ \ 3(5z^2\ \ 3z\ \ 1)^2\ (10z\ \ 3)$

$f(x)\ x^4\sqrt{5x\ 1}\ x^4\ 5x\ 1^{1/2}\ f'(x)$

$\frac{1}{2}\ x^4\ 5x\ 1^{1/2}\ 4x^3\ 5$

$f(x)\ x^6\ 3\sqrt{x\ 1}\ x^6\ 3x\ 1^{1/2}\ f'(x)\ \frac{1}{2}$

$x^6\ 3x\ 1^{1/2}\ 6x^5\ 3$

$w(z)\ \ \sqrt[3]{9z\ \ 1}\ \ (9z\ \ 1)^{1/3}$

$w(z)\ \ \frac{1}{3}(9z\ \ 1)^{2/3}\ \ (9)\ \ 3(9z\ \ 1)^{2/3}$

$w(z)\ \ \sqrt[5]{10z\ \ 4}\ \ (10z\ \ 4)^{1/5}$

$w(z)\ \ \frac{1}{5}(10z\ \ 4)^{4/5}\ \ (10)\ \ 2(10z\ \ 4)^{4/5}$

$y\ \ (4\ \ x^2)^4$

$y\ \ 4(4\ \ x^2)^3\ (2x)\ \ 8x(4\ \ x^2)^3$

$y\ \ (1\ \ x)^{50}$

$y\ \ 50(1\ \ x)^{49}\ (1)\ \ 50(1\ \ x)^{49}$

23. $y\ \ \frac{1}{4}\ [(w^3\ \ 1)]^{1\ 4}\ (w^3\ \ 1)^4$

$y\ \ \frac{1}{3}\ 4(w^3\ \ 1)^5\ (3w^2)\ \ 12w^2\ (w^3\ \ 1)^5$

24. $y\ \ \frac{1}{5}\ [(w^4\ \ 1)]^{1\ 5}\ (w^4\ \ 1)^5$

$y\ \ \frac{1}{4}\ 5(w^4\ \ 1)^6\ (4w^3)\ \ 20w^3\ (w^4\ \ 1)^6$

$y\ x^4\ \ (1\ \ x)^4$

$y\ \ 4x^3\ \ 4(1\ \ x)^3\ \ (1)\ \ 4x^3\ \ 4(1\ \ x)^3$

$f(x)\ \ (x^2\ \ 4)^3\ \ (x^2\ \ 4)^2$

$(x)\ 3(x^2\ 4)^2\ (2x)\ 2(x^2\ 4)(2x)\ 6x(x^2\ 4)^2\ \ 4x(x^2\ \ 4)$

$\sqrt{3x^2\ 5x\ 1}$

$'(x)\ \frac{1}{2}\ 3x^2\ \ 5x\ 1\ \ (6x\ 5)$

28. $f(x)\ \ \frac{1}{\sqrt{2x^2\ \ 7x\ \ 1}}\ 2x^2\ \ 7x\ \ 1^{1/2}$

$'(x)\ \frac{1}{2}\ 2x^2\ \ 7x\ 1^{3/2}\ (4x\ 7)$

29. $f(x)\ \frac{1}{2}\sqrt[3]{9x\ \ 1^2}\ \ \frac{1}{2}\ 9x\ \ 1^{2/3}\ 9x\ \ 1^{2/3}$

30. $f(x)\ \ \frac{1}{\sqrt[3]{3x\ 1^2}}\ 3x\ \ 1^{2/3}$

$'(x)\ \ \frac{2}{3}\ 3x\ \ 1^{5/3}\ (3)\ \ 2\ 3x\ \ 1^{5/3}$

31. $f(x)\ \ \frac{1}{\sqrt[3]{2x^2\ \ 3x\ 1^2}}\ 2x^2\ \ 3x\ \ 1^{2/3}$

$'(x)\ \frac{2}{3}\ 2x^2\ \ 3x\ 1^{5/3}\ (4x\ 3)$

32. $f(x)\ \ \frac{1}{\sqrt[3]{x^2\ \ x\ 9^2}}\ x^2\ \ x\ \ 9^{2/3}$

$2\ \ \ 5/3$

'(x)2 3 x __ x 9 (2 x 1)

$$f(x)\ [(x^2\ 1)^3\ x]^3$$

$$(x)\ 3[(x^2\ 1)^3\ x]^2\ [3(x^2\ 1)^2\ (2x)\ 1]\ 3[(x^2\ 1)^3\ x]^2\ [6x(x^2\ 1)^2\ 1]$$

$$f(x)\ 3x^2\ (2x\ 1)^5$$

$$(x)\ 6x(2x\ 1)^5\ 3x^2\ [5(2x\ 1)^4\ (2)]$$

$$6x(2x\ 1)^5\ 30x^2\ (2x\ 1)^4$$

$$f(x)\ [(x^3\ 1)^2\ x]^4$$

$$(x)\ 4[(x^3\ 1)^2\ x]^3\ [2(x^3\ 1)(3x^2)\ 1]\ 4[(x^3\ 1)^2\ x]^3\ [6x^2\ (x^3\ 1)\ 1]$$

$$f(x)\ 2x(x^3\ 1)^4$$

$$(x)\ 2(x^3\ 1)^4\ 2x[4(x^3\ 1)^3\ (3x^2)]\ 2(x^3\ 1)^4\ 24x^3\ (x^3\ 1)^3$$

$f(x)$ (2x 1)^3 (2 x 1)^4

2 4 3 3

(x) 3(2 x 1) (2)(2x 1) (2 x 1) [4(2 x 1) (2)] 6(2 x

2 4 3 3

1) (2 x 1) 8(2 x 1) (2 x 1)

3 4

$f(x)$ (2x 1) (2 x 1)

2 4 3 3

(x) 3(2 x 1) (2)(2 x 1) (2 x 1) [4(2 x 1) (2)] 6(2 x

2 4 3 3

1) (2 x 1) 8(2 x 1) (2 x 1)

$g(z)$ 2 z 3 z^2 z 1^4

$g'(z)$ 2 3 z^2 z 1^4 2 z 4 3 z^2 z 1^3 6 z 1

$g(z)$ z^2 2 z^3 z 5^4

$g'(z)$ 2 z 2 z^2 z 5 4 2 ^{2} 4 2 z^2 z 5 3 6 z ^{2}

3 1

2 z 2 z^2 z 5 14 z^3 3 z 5

41. $f(x)$ $\frac{x\ 1}{x\ 1}$ 3 2

$f'(x)$ 3 x1 (x 1)(1) (1)(x 1) 2

x1 (x 1)

x1 2 x 1 x 1 x 1 2 2

3 x1 (x 1) 2 3 x 1 (x 1) 2

6 (x 1)^2 4

(x1)

42. x 1 5

$f(x)$ x 1

$f'(x)$ 5 x1 4(x 1)(1) (1)(x 1)

x1 (x 1)

2 (x 1)^4

5 x1 4 x 1 x 1 2 10

x1 (x 1) (x 1) 6

43. $f(x)$ x $\sqrt{1x^2}$ x 2 x 21/ 2

'(x) 2 x 1 x^2 1/ 2 x^2 1 1 x^2 1/ 2 (2 x)

2

2 x 1 x^2 1/ 2 x^3 1 x^2 1/ 2

44. $f(x)$ x $\sqrt{x^2 1}$ x 2 x^2 1/ 2

'(x) 2 x x^2 1 1/ 2 x^2 1 x^2 1 1/ 2 (2 x)

2

2 x x^2 1 1/ 2 x^3 x^2 1 1/ 2

45. $f(x)$ $\sqrt{1 \sqrt{x}}$ 1/ 2 1/ 2

'(x) 1 2 1 x 1/ 2 1/ 2 1 2 x 1/

2 1 4 x 1/ 2 1 x 1/ 2 1/ 2

$f(x)$ 3 1 $\sqrt[3]{x}$ 1 x 1/ 3 1/ 3

1/ 3 2 / 3 2 / 3

'(x) 1 3 1 x 1 3 x

1 9 x 2 / 3 1 x 1/ 3 2 / 3

$f(x)$ 2 x 1^4

1 2 1 1^4 1

The point is (–1, 1).

(x) 4 2 x 1^3 2 8 2 x 1^3 f(1)

8 2 1 1^3 8

The tangent line has slope –8 at (–1, 1).

1 8 x 1 y 8 x

$f(x)$ x $\sqrt{3 x^2}$ 2 3 1/2

f 1 $\sqrt{1^2 3}$ 2

The point is (1, 2).

$f(x)$ $\frac{1}{2}$ x^2 3 1/2 2x $\frac{2x}{2\sqrt{x^2 3}}$

f(1) $\frac{2\ 1}{\sqrt{\ }}$ $\frac{1}{\ }$

7

$2 \quad 1^2 \quad 3 \qquad 2$

The tangent line has slope $\frac{1}{2}$ at (1, 2).

$y \quad 2 \quad \frac{1}{2} \quad x \_\_ 1$

2

$f(x)\ \ x\ 1^{2}\ \ x^{2}$

$21^{2}2^{2}1$

The point is (2, 1).

$(x)\ 2\ \ x\ 1^{2}\ \ x\ 2x\ 1\ 1\ \ 1$

$2\ \ x\ 1^{2}\ \ x\ \ \ 2x\ \ \ 3$

$(2)221^{2}\ 22\ 232$

The tangent line has slope –2 at (2, 1).

$1\ 2x\ 2y\ 2x$

5

$f(x)\ \ 2x\ 3^{3}\ \ 3x\ 2^{2}\ \ 4x\ 5$

$2223^{3}322^{2}425$

2

The point is (–2, 2).

$(x)\ 3\ 2x\ 3^{2}\ \ 2\ 2\ 3x\ 2\ 3\ 4\ 6\ 2x\ 3^{2}$

$6\ 3x\ 2\ 4$

2

$(2)6223\ \ 63224$

14

The tangent line has slope –14 at (–2, 2).

$2\ \ 14x\ \ 2y$

$14x\ \ 26$

51. a. $\frac{d}{dx}[(x^{2}\ \ 1)^{2}]\ \ 2(x^{2}\ \ 1)(2x)\ \ 4x^{3}\ \ 4x$

$\frac{d}{dx}[(x^{2}\ \ 1)^{2}]\ \ \frac{d}{dx}(x^{4}\ \ 2x^{2}\ \ 1)\ \ 4x^{3}\ \ 4x$

52. a. $\frac{d}{dx}\frac{1}{x^{2}}\ \ \frac{x^{2}\ 0\ (2x)(1)}{(x^{2})^{2}}\ \ \frac{2x}{x^{4}}\ \ \frac{2}{x^{3}}$

b. $\frac{d}{dx}\frac{1}{x^{2}}\ \ \frac{d}{dx}[(x^{2})^{1}]\ \ 1(x^{2})^{2}(2x)$

$\frac{1}{dx}\ \ 3\ \ 2x^{3}\ \ 2x\ \ 3$

53. a. $\frac{d}{dx}\frac{1}{3x\ 1}\ \ \frac{(3x\ 1)0\ \ 3(1)}{(3x\ 1)^{2}}\ \ \frac{3}{(3x\ \ 1)^{2}}$

b. $\frac{d}{dx}[(3x\ \ 1)^{1}]\ \ (3x\ \ 1)^{2}(3)$

$\frac{3}{(3x\ \ 1)^{2}}$

c. $x^{2}\ \ \frac{d}{dx}(x^{2})\ f$

$\frac{d}{dx}f(g(h(x)))\ \ \ (h(x))h(x)$

$(g(h(x)))g$

$f(x)\ \ (x^{2}\ \ 1)^{10}$

$(x)\ \ 10(x^{2}\ \ 1)^{9}(2x)\ \ 20x(x^{2}\ \ 1)^{9}$

$(x)\ 20(x^{2}\ \ 1)^{9}\ \ 20x[9(x^{2}\ \ 1)^{8}\ \ (2x)]\ 20(x^{2}$

$1)^{9}\ \ 40x\ \ [9(x^{2}\ \ 1)^{8}\ ]\ 20(x^{2}\ \ 1)^{9}$

$360x^{2}\ (x^{2}\ \ 1)^{8}$

$f(x)\ \ (x^{3}\ \ 1)^{5}$

$(x)\ \ 5(x^{3}\ \ 1)^{4}(3x^{2})\ \ 15x^{2}(x^{3}\ \ 1)^{4}$

$(x)\ 30x(x^{3}\ \ 1)^{4}\ \ 15x^{2}\ \ [4(x^{3}\ \ 1)^{3}\ (3x^{2})]\ 30$

$x(x^{3}\ \ 1)^{4}\ \ 180x^{4}\ \ (x^{3}\ \ 1)^{3}$

The marginal cost function is the derivative of the function $C(x)\ 4\sqrt{x^{2}\ 900}\ (4x^{2}\ 900)^{1/2}$.

$MC(x)\ \frac{1}{}\ \ 4x^{2}\ 900^{1/2}\ 8x$

58.

$4^{2}\ \ x\ 4x^{2}\ \ 900^{1/2}$

$2\ \ \ 1/2$

$MC20\ \ 420\ 420^{2}\ \ 900^{1/2}$

$80\ 4\ 400\ \ 900^{1/2}$

$80\ 2500^{1/2}\ \ \frac{80}{50}\ \ 1.60$

on [0, 30] by [–10, 70]

x 27

$S(e)$ 0.22(e 4)$^{2.1}$ S(
e) 0.462(e 4)$^{1.1}$
S(12) 0.462(12 4)$^{1.1}$ 9.75

At a level of 12 units, a person's social status increases by about 9.75 units per additional year of education.

$S(i)$ 17.5(i 1)$^{0.53}$
0.47
$S(i)$ 9.275(i 1)
S(25) 9.275(25 1)$^{0.47}$ 2.08

At an income of $25,000 social status increases by about 2.08 units per additional $1000 of income.

$V(r)$ 1000(1 0.01r)^{5}
4
(r) 5000(1 0.01r) (0.01)
50(1 0.01r)^{4}

V(6) 50[1 0.01(6)]^{4} 63.12
At a rate of 6%, the value increases by about $63.12 for each additional percentage point of interest.

63. $R(x)$ 4x $\sqrt{11\ 0.5x}$ 4x(11 0.5x)$^{1/2}$

$R'(x)$ 4x $\frac{1}{2}$(11 0.5x)$^{1/2}$ (0.5) 4(11 0.5x)$^{1/2}$
$\frac{x}{\sqrt{11\ 0.5x}}$ 4 $\sqrt{11\ 0.5x}$

The sensitivity to a dose of 50 mg is

$R'(50)$ $\frac{x}{\sqrt{11\ 0.5(50)}}$ 4 $\sqrt{11\ 0.5(50)}$
$\frac{50}{\sqrt{11\ 25}}$ 4 11 25 $\frac{50}{\sqrt{36}}$ 4 $\sqrt{36}$
$\frac{50}{6}$ 24 $\frac{25}{3}$ $\frac{72}{3}$ $\frac{97}{3}$ 32$\frac{1}{3}$

64.

on [1, 5] by [0, 3]
x 3.6 years

65.

on [0, 140] by [0, 50]
x 26 mg

$R(x)$ 0.25(1 x)^{4}
$R(x)$ 4(0.25)(1 x)^{4} (1 x)^{3}
R(0) (1 0)^{3} 1
R(1) (1 1)^{3} 8

$P(t)$ 0.02(12 2 t)$^{3/2}$ 1 $P(t)$
0.03(12 2t)$^{1/2}$ (2)
1/2
0.06(12 2 t)
P(2) 0.06[12 2(2)]$^{1/2}$ 0.24

$T(p)$ 36(p 1)$^{1/3}$
$T(p)$ $\frac{1}{3}$ (36)(p 1)$^{4/3}$ 12(p 1)$^{4/3}$ T
(7) 12(7 1)$^{4/3}$ $\frac{3}{4}$
The time is decreasing by about $\frac{3}{4}$ minute for each additional practice session.

b.

on [0, 20] by [60, 62]

$\frac{1.8}{x}$ 0.05184
x 34.7, so about 35 years

slope = 0.05184

$h(40) \approx 8.2 - (0.01(40) - 2.8)^2 \approx 2.44$

$h'(x) = -2(0.01t - 2.8)(0.01) = -0.0002t + 0.056$ $h'(40) \approx 0.048$

At a temperature of 40 degrees, happiness approximately 2.4, and each additional degree of temperature would raise happiness by about 0.05.

False: There should not be a prime of the first g on the right-hand side.

The power rule has only x as the inner function, but the generalized power rule can have any inner function.

76. True: $\frac{d}{dx} f\left(\frac{x}{2}\right) = f'\left(\frac{x}{2}\right) \frac{d}{dx}\frac{x}{2} = f'\left(\frac{x}{2}\right)\frac{1}{2} = \frac{f'\left(\frac{x}{2}\right)}{2}$

True: $\frac{d}{dx} f(x+5) = f'(x+5)(1) = f'(x+5)$

No: since instantaneous rates of change are derivatives, this would be saying that $\frac{d}{dx}[f(x)]^3 = [f'(x)]^3$, where $f(x)$ is the length of a side. The chain rule gives the correct derivative of $[f(x)]^3$.

$\frac{d}{dx} E(g(x)) = E'(g(x))\, g'(x)$

Since $E'(x) = E(x)$, $E'(g(x)) = E(g(x))$.

Thus, $\frac{d}{dx} E(g(x)) = E(g(x))\, g'(x)$.

False: There should not be a prime of the first g on the right-hand side.

The generalized power rule is a special case of the chain rule, when the outer function is just a power of the variable.

True: $\frac{d}{dx} f(5x) = \frac{d}{dx} 5x \cdot f'(5x) = 5f'(5x)$

77. False: The outer function, $\sqrt{x}$ was not differentiated. The correct right-hand side is $\frac{1}{2}[g(x)]^{-1/2} g'(x)$.

No: Since instantaneous rates of change are derivatives, this would be saying that $\frac{d}{dx}[f(x)]^2 = [f'(x)]^2$, where $f(x)$ is the length of a side. The chain rule gives the correct derivative of $[f(x)]^2$.

$\frac{d}{dx} L(g(x)) = L'(g(x))g'(x)$

But since $L'(x) = \frac{1}{x}$, $L'(g(x)) = \frac{1}{g(x)}$.

Thus, $\frac{d}{dx} L(g(x)) = \frac{g'(x)}{g(x)}$.

Let $G(x) = g(h(x))$, then the chain rule gives $G'(x) = g'(h(x))\, h'(x)$. We use the chain rule to find $\frac{d}{dx} f(G(x))$ and substitute the expressions for $G(x)$ and $G'(x)$.

$$\frac{d}{dx} f(G(x)) = f'(G(x))\, G'(x) = f'(g(h(x)))\, g'(h(x))\, h'(x)$$

a. Use $\frac{d}{dx} f(g(h(x))) f(g(h(x)) g(h(x)) h(x)$

to find $\frac{d}{dx}((x^2)^3)^4$.

$$\frac{d}{dx}((x^2)^3)^4 \quad 4((x^2)^3)^3 \quad 3(x^2)^2 \quad 2x$$

$$4x^{18} \quad 3x^4 \quad 2x$$

$$24x^{23}$$

$$((x^2)^3)^4 \quad (x^6)^4 \quad x^{24}$$

$$\frac{d}{dx} x^{24} \quad 24x^{23}$$

which is the same result as in part (a)

85. Solve $f(x\ h)\ \ f(x)\ \ F(h)\,h$ for $F(h)$ to find $F(h)\ \ \frac{f(x\ h)\ \ f(x)}{h}$ and use the continuity of F at 0 to obtain

$$F(0)\ \ \lim_{h\ 0} F(h)\ \ \lim_{h\ 0} \frac{f(x\ h)\ \ f(x)}{h},$$

which is the definition of the derivative.

a. Apply Caratheodory's definition since g is differentiable.
Apply Caratheodory's definition since f is differentiable.
In the first step add and subtract $g(x)$.
In the second step, use the associative property to group $g(x\ h)\ \ g(x)$.
In the third step, use Caratheodory's definition as given in part (b), where "h" is $g(x\ h)\ g(x)$. In the last step, use Caratheodory's definition as given in part (a).
The derivative of $f(g(x))$,

$$\frac{d}{dx} f(g(x))$$

$$\lim_{h\ 0} \frac{F(g(x\ h)\ \ g(x))\, G(h)\, h}{h}$$

$\lim_{h\ 0} F(g(x\ h)\ \ g(x))\, G(h)$ Remove a factor of h

$F(g(x\ 0)\, g(x))\, G(0)$ Apply the limit

$F(0)\, G(0)$ Simplify $g(x)\ \ g(x)\ \ 0$

$f(g(x))\ \ g(x)$ Substitute $F(0)\ \ f\ \ (g(x))$ and $G(0)\ \ g(x)$

EXERCISES 2.7

1. The derivative does not exist at the corner points x 2, 0, 2.

2. The derivative does not exist at the discontinuous points x 3, 0, 3.

3. The derivative does not exist at the discontinuous points x 3, 3.

4. The derivative does not exist at the vertical tangents at x 4, 2, 2, 4.

5. For positive h, $\lim_{h\ 0} \frac{f(x\ h)\ f(x)}{h} \lim_{h\ 0} \frac{|2x\ 2h|\ |2x|}{h}$. For $x = 0$, this becomes $\lim_{h\ 0} \frac{|0\ 2h|\ |0|}{h}\ \ \frac{|2h|}{h}\ \ 2$ because h is positive. For h negative,

$\lim_{h\ 0} \frac{f(x\ h)\ f(x)}{h} \lim_{h\ 0} \frac{|2x\ 2h|\ |2x|}{h}$. Since $x = 0$, we get

$\lim_{h\ 0} \frac{|0\ 2h|\ |0|}{h}\ \ \frac{|2h|}{h}\ \ 2$. Thus, the derivative does not exist.

6. For positive h and $x = 0$, $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{|0+3h|-|0|}{h} = \frac{3h}{h} = 3$.

For negative h and $x = 0$, $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{|0+3h|-|0|}{h} = \frac{3h}{h} = 3$.

Thus, the derivative does not exist.

7. For $x = 0$, $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{(0+h)^{2/5}-0}{h} = \lim_{h \to 0} \frac{h^{2/5}}{h} = \lim_{h \to 0} \frac{1}{h^{3/5}}$ which does not exist. Thus, the derivative does not exist at $x = 0$.

8. For $x = 0$, $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{(0+h)^{4/5}-0}{h} = \lim_{h \to 0} \frac{h^{4/5}}{h} = \lim_{h \to 0} \frac{1}{h^{1/5}}$ which does not exist.

Thus, the derivative does not exist at $x = 0$.

If you get a numerical answer, it is wrong because the function is undefined at $x = 0$. Thus, the derivative at $x = 0$ does not exist.

a. For $x = 0$,

$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0}$ [redacted] $/h$

$= \lim_{h \to 0}$ [redacted] $/h$

b.

h	$\frac{\sqrt[5]{h}}{h}$
0.1	6.3
0.001	251.2
0.00001	10,000

No, the limit does not exist. No, the derivative does not exist at $x = 0$.

d.

On [–1, 1] by [–1, 1]

At a corner point, a proposed tangent line can tip back and forth and so there is no well-defined slope.

At a discontinuity, the values of the function take a sudden jump, and so a (steady) rate of change cannot be defined.

True: For a function to be differentiable, it cannot jump or break.

If you get a numerical answer, it is wrong because the function is undefined at $x = 0$. Thus, the derivative at $x = 0$ does not exist.

a. For $x = 0$,

$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0}$ [redacted] $/h$

$= \lim_{h \to 0}$ [redacted] $/h$

b.

h	$\frac{\sqrt[3]{h}}{h}$
0.1	4.64
0.0001	464.16
0.0000001	46,415.89

No, the limit does not exist. No, the derivative does not exist at $x = 0$.

d.

on [–1, 1] by [–1, 1]

A vertical tangent has an undefined slope, or derivative.

False: A function can have a corner or vertical tangent and still be continuous.

False: A function could have a vertical tangent.

REVIEW EXERCISES AND CHAPTER TEST FOR CHAPTER 2

1.

x	$4x + 2$
1.9	9.6
1.99	9.96
1.999	9.996

x	$4x + 2$
2.1	10.4
2.01	10.04
2.001	10.004

a. $\lim_{x\ 2} 4x\ \ 2\ \ 10$

b. $\lim_{x\ 2} 4x\ \ 2\ \ 10$

c. $\lim_{x\ 2} 4x\ \ 2\ \ 10$

2.

x	$\frac{\sqrt{x\ 1}\ 1}{x}$
–0.1	0.513
–0.01	0.501
–0.0001	0.500

x	$\frac{\sqrt{x\ 1}\ 1}{x}$
0.1	0.488
0.01	0.499
0.001	0.500

a. $\lim_{x\ 0} \frac{\sqrt{x\ 1}\ 1}{x}\ 0.5$

b. $\lim_{x\ 0} \frac{\sqrt{x\ 1}\ 1}{x}\ 0.5$

c. $\lim_{x\ 0} \frac{\sqrt{x\ 1}\ 1}{x}\ 0.5$

3. **a.** $\lim_{x\ 5} f(x)\ \ \lim_{x\ 5}(2x\ \ 7)$
$2(5)\ \ 7)\ \ 3$

b. $\lim_{x\ 5} f(x)\ \ \lim_{x\ 5}(3\ \ x)$
$3\ \ 5\ \ \ 2$

c. $\lim_{x\ 5} f(x)$ does not exist.

4. **a.** $\lim_{x\ 5} f(x)\ \ \lim_{x\ 5}(4\ \ x)$
$4\ \ 5\ \ \ 1$

b. $\lim_{x\ 5} f(x)\ \ \lim_{x\ 5}(2x\ \ 11)$
$2(5)\ \ 11\ \ \ 1$

c. $\lim_{x\ 5} f(x)\ \ \ 1$

5. $\lim_{x\ 4} \sqrt{x^2\ \ \ \ x\ \ 5}\ \sqrt{4^2\ 4\ 5}\ \ \ \sqrt{16\ \ 9}$
$\sqrt{25}\ \ \ 5$

6. $\lim_{x\ 0}$

$\frac{1}{}\ \ \ 1/2\frac{1}{}\ \ \ \ 1/2$
$_s\lim_{16}\ \ 2\ s\ \ s\ \ \ \frac{2}{2}(16)\ \ 16\_\ \ \ \_\_\_\_8\ \ 4$

8. $\lim_{r\ 8} \frac{r}{r^2\ \ 30\sqrt[3]{r}}\ \ \frac{8}{8^2\ \ 30\sqrt[3]{8}}\ \ \frac{8}{64\ \ 30(2)}\ \ 2$

9. $\lim_{x\ 1} \frac{x^2\ x}{x^2\ \ 1}\ \ \lim_{x\ 1} \frac{x(x\ \ 1)}{(x\ 1)(x\ 1)}$
$\lim_{x\ 1} \frac{x}{x\ \ 1}\ \ \ \frac{1}{2}$

$\lim_{1} \frac{3x^3\ \ 3x}{2x^2\ \ 2x}\ \ \lim_{1} \frac{3x(x\ \ 1)}{2x(x\ 1)}$
$\lim_{1} \frac{3x(x\ 1)(x\ 1)}{2x(x\ 1)}$
$\lim_{x\ \ 1} \frac{3(x\ \ 1)}{2}\ \ \ \frac{3(1\ 1)}{2}\ \ 3$

11. $\lim_{h\ 0} \frac{2x^2h\ \ xh^2}{h}\ \ \lim_{h\ 0} \frac{xh(2x\ \ h)}{h}$
$\lim_{h\ 0} x(2x\ \ h)$
$x(2x\ \ 0)\ \ 2x^2$

12. $\lim_{h\ 0} \frac{6xh^2\ \ \ x^2h}{h}\ \ \lim_{h\ 0} \frac{xh(6h\ \ x)}{h}$
$\lim_{h\ 0} x(6h\ \ x)$
$x(0\ \ x)\ \ \ x^2$

13. $\lim_{x} f(x)\ \ 0;\ \ \lim_{x\ \ 2} f(x)$ and $\lim_{x\ 2} f(x)\ \ ,$
so $\lim_{x\ 2} f(x)$ and $\lim_{x} f(x)\ \ \ 0.$

14. $\lim_{x} f(x)\ \ 3;\ \ \lim_{x\ 2} f(x)$ and $\lim_{x\ 2} f(x)\ \ ,$
so $\lim_{x\ 2} f(x)$ does not exist and $\lim_{x} f(x)\ \ 3.$

15. Continuous

16. Continuous

---

17. Discontinuous at $x = –1$

18. Continuous

The function is discontinuous at values of x for which the denominator is zero. Thus, we consider

$x^2 + x = 0$ and solve. $x^2 + x = 0$

$x(x + 1) = 0$

$x = 0, -1$

Discontinuous at $x = 0, -1$

Discontinuous at $x = -3, 3$

From Exercise 3, we know $\lim_{x \to 5} f(x)$ does not exist. Therefore, the function is discontinuous at $x = 5$.

22. From Exercise 4, we know $\lim_{x \to 5} f(x) = 1 = f(5)$. Therefore, the function is continuous.

23. $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h} = \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1}{h} = \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h} = \lim_{h \to 0} (4x + 2h - 3) = 4x - 3$$

24. $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 - 2(x+h) + 3 - (3x^2 - 2x + 3)}{h} = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 2x - 2h + 3 - 3x^2 + 2x - 3}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2 - 2h}{h} = \lim_{h \to 0} (6x + 3h - 2) = 6x - 2$$

25. $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h} = \lim_{h \to 0} \frac{\frac{3x - 3(x+h)}{x(x+h)}}{h} = \lim_{h \to 0} \frac{3x - 3x - 3h}{x(x+h)h} = \lim_{h \to 0} \frac{-3h}{x(x+h)h} = \lim_{h \to 0} \frac{-3}{x(x+h)} = -\frac{3}{x^2}$$

26. $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{4\sqrt{x+h} - 4\sqrt{x}}{h} = \lim_{h \to 0} \frac{4(\sqrt{x+h} - \sqrt{x})}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \lim_{h \to 0} 4\frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} 4\frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{4}{\sqrt{x+h} + \sqrt{x}} = \frac{4}{2\sqrt{x}} = \frac{2}{\sqrt{x}}$$

27. $f(x) = 6\sqrt[3]{x^5} - \frac{4}{\sqrt{x}} + 1 = 6x^{5/3} - 4x^{-1/2} + 1$

$f'(x) = \frac{5}{3}6x^{2/3} - \left(-\frac{1}{2}\right)4x^{-3/2} + 0$

$= 10x^{2/3} + 2x^{-3/2}$

$f(x) = 4\sqrt{x^5} - \frac{6}{\sqrt[3]{x}} + 1 = 4x^{5/2} - 6x^{-1/3} + 1$

$f'(x) = \frac{5}{2}4x^{3/2} - \left(-\frac{1}{3}\right)6x^{-4/3} + 0$

$= 10x^{3/2} + 2x^{-4/3}$

29. $f(x) = \frac{1}{x^2} = x^{-2}$

$f'(x) = -2x^{-3}$

30. $f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -x^{-2}$

$$f'\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^{-3} = 2(2)^{3} = 16$$

$$f'\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^{-2} = (3)^{2} = 9$$

31. $f(x) = 12\sqrt[3]{x} = 12x^{1/3}$

$$f'(x) = \frac{1}{3}\cdot 12x^{-2/3} = 4x^{-2/3}$$

$$f'(8) = 4(8)^{-2/3} = 4\left(\frac{1}{8}\right)^{2/3} = 4\cdot\frac{1}{8^{2/3}}$$

$$= 4\cdot\frac{1}{\sqrt[3]{8^2}} = 4\cdot\frac{1}{\sqrt[3]{64}} = 1$$

32. $f(x) = 6\sqrt[3]{x} = 6x^{1/3}$

$$f'(x) = \frac{1}{3}\cdot 6x^{-2/3} = 2x^{-2/3}$$

$$f'(-8) = 2(-8)^{-2/3} = 2\left(-\frac{1}{8}\right)^{2/3} = 2\sqrt[3]{\frac{1}{(-8)^2}} = \frac{1}{2}$$

$y \frac{1}{x} x^2 x^1 x^2 \frac{1}{1}^2$ 1 2y1

1

The point is (1, 2).

y 1 x^2 2 x $\frac{1}{x^2}$ 2x
1

$y(1)$ ___ 2 1 1

1 2

The tangent line has slope 1 at (1, 2).
2 1 x 1 y x
1

a. $C'(x)$ 20$x^{1/6}$ C

$'(1)$ $\frac{20}{\sqrt[6]{1}}$ 20

Costs are increasing by about $20 per additional license.

$C'(64)$ [redacted] $\frac{20}{2}$ 10

Costs are increasing by about $10 per additional license.

$f(x)$ 150 $x^{0.322}$

1.322 1.322

(x) 0.322(150 x) 48.3x f
1.322

(10) 48.3(10) 2.3

After 10 planes, construction time is decreasing by about 2300 hours for each additional plane built.

2

a. A r A' 2
r

As the radius increases, the area grows by “a circumference.”

37. **a.** V $\frac{4}{3}$ r^3

A 3 $\frac{4}{3}$ r^2 4 r^2

As the radius increases, the volume grows by “a surface area.”

$f(x)$ 2 x(5 x^3 3)
3 2
(x) 2(5x 3) 2 x(15x)

2(5x^3 3) 30 x^3 40 x^3 6

39. $f(x)$ x^2 (3x 3 1)

$f(x)$ 2 x(3 x 3 1) x^2 (9 x^2)

2 x(3 x^3 1) 9x^4 15x^4 2 x

40. $f(x)$ (x^2 5)(x^2 5)

$f(x)$ 2 x(x^2 5) (x^2 5)(2 x)

2 x^3 10 x 2 x^3 10 x 4 x^3

$f(x)$ (x^2 3)(x^2 3)
2 2
(x) 2 x(x 3) (x 3)(2 x)
2 x^3 6x 2 x^3 6 x 4x^3

y (x^4 x^2 1)(x^5 x^3 x)
3 5 3 4 2 4 2
y (4 x 2 x)(x x x) (x x 1)(5 x 3x 1)
8 6 4 6 4 2 8 6 4 6 4 2 4 2
4 x 4x 4x 2x 2 x 2 x 5 x 3x x 5x 3 x x 5 x 3x 1
9 x^8 5 x^4 1
5 3 4 2

y (x x x)(x x 1)
4 2 4 2 5 3 3
y (5 x 3x 1)(x x 1) (x x x)(4x 2 x)
8 6 4 6 4 2 4 2 8 6 4 6 4
5x 5x 5 x 5 x 2 3x 2 x 3x 1 4 x 3x 3 x
8 6 4 2 8 6 4

$2x \quad 2x$

$x \quad x \quad 1 \quad 4x \quad 4x \; 2x \quad 2x^2$

$2x^2 \; 9x^8 \quad 4x \; 2x$

$5x^4$

1

44. $y \dfrac{x\ 1}{x\ 1}$

$y' \dfrac{(x\ 1)(1)\ (1)(x\ 1)}{(x\ 1)^2} \quad \dfrac{x\ 1\ x\ 1}{(x\ 1)^2} \quad \dfrac{2}{(x\ 1)^2}$

45. $y \dfrac{x\ 1}{x\ 1}$

$y' \dfrac{(x\ 1)(1)\ (1)(x\ 1)}{(x\ 1)^2} \quad \dfrac{x\ 1\ x\ 1}{(x\ 1)^2} \quad \dfrac{2}{(x\ 1)^2}$

46. $y = \frac{x^5 1}{x^5 1}$

$y = \frac{(x^5 1)(5x^4) (5x^4)(x^5 1)}{(x^5 1)^2}$

$\frac{5x^9 \; 5x^4 \; 5x^9 \; 5x^4}{(x^5 1)^2} \quad \frac{10x^4}{(x^5 1)^2}$

48. a. $f(x) \; \frac{2x\;1}{x}$

$f(x) \; \frac{x(2) \; (1)(2x\;1)}{x^2}$

$\frac{2x \; 2x \; 1}{x^2} \quad \frac{1}{x^2}$

b. $f(x) \; (2x \; 1)(x^{1})$

$(x) \; 2(x^{1}) \; (2x \; 1)(\;1)x^{2}$

$\frac{2}{x} \; \frac{2x\;1}{x^2} \; \frac{2x}{x^2} \; \frac{2x\;1}{x^2} \; \frac{1}{x^2}$

Dividing $2x + 1$ by x, we get

$f(x) \; 2 \; \frac{1}{x} \; 2 \; x^{1}$

$f(x) \; 0 \; (1)x^{2} \; \frac{1}{x^2}$

$S(x) \; \frac{2250}{x \; 9} \; 2250(x \; 9)^{1}$

$22502250(xS(9)2x) \; \frac{2250}{(x\;9)^2}$

$S(6) \; \frac{2250}{(6\;9)^2} \; 10$

At \$6 per flash drive, the number of drives sold is decreasing by 10 per dollar increase in price.

52. a. $AC(x) \; \frac{C(x)}{x} \; \frac{7.5x \; 50}{x} \; 7.5 \; \frac{50}{x}$

$y \; \frac{x^6 \; 1}{x^6 \; 1}$

$y \; \frac{(x^6 \; 1)(6x^5) \; (6x^5)(x^6 \; 1)}{(x^6 \; 1)^2}$

$\frac{6x^{11} \; 6x^5 \; 6x^{11} \; 6x^5}{(x^6 \; 1)^2} \quad \frac{12x^5}{(x^6 \; 1)^2}$

$y \; \frac{3}{x} \; \frac{4}{1x \; 1}$

$\frac{1^3 \; 1 \; 1}{1^4 \; 1}$

The point is (–1, –1).

$y \; \frac{x^4 \; 2 \; 4x^3 \; x^3}{x^4}$

$\frac{6x^3 \; 2 \; 4x \; 3x^2}{x^4}$

$y(1) \; \frac{6 \; 4 \; 3 \; 3 \; 2}{1^4 \; 1^2} \; \frac{4\;3}{4} \; \frac{1}{2}$

The tangent line has slope $\frac{1}{2}$ at (–1, –1).

$1 \; \frac{1}{2}x \; 1 \; y$

$\frac{1}{2}x \; \frac{3}{2}$

a. To find the average profit function, divide $P(x) \; 6x \; 200$ by x.

$AP(x) \; \frac{6x \; 200}{x}$

To find the marginal average profit, take the derivative of $AP(x)$.

$MAP(x) \; \frac{x(6) \; (1)(6x \; 200)}{x^2} \; \frac{200}{x^2}$

$MAP(10) \; \frac{200}{(10)^2} \; \frac{200}{100} \; 2$

Average profit is increasing by about \$2 for each additional unit.

$f(x) \; 12\sqrt{x^3} \; 9\sqrt[3]{x} \; 12x^{3/2} \; 9x^{1/3}$

$(x) \; \frac{3}{2}\,12x^{1/2} \; \frac{1}{3}\,9x^{2/3} \; 18x^{1/2} \; 3x^{2/3}$

$\frac{1}{\;} \; 1/2 \; \frac{2}{\;} \; 5/3$

$(x) \quad 2 \; 18x \qquad 3 \; 3x$

$9x^{1/2} \quad 2x^{5/3}$

b. $MAC(x) \quad \frac{d}{dx} 7.5 \quad \frac{50}{x} \quad \frac{50}{x^2}$

c. $MAC(50) \quad -\frac{50}{50^2} \quad \frac{1}{50}. \quad 0.02$

Average cost is decreasing by about $0.02 per additional mouse.

$f(x)$ 18 ^{3} x^2 4 $x\sqrt{^3}$ 18 $x\sqrt{^{2/3}}$ $4x^{3/2}$

'(x) 12 $x^{1/3}$ $6x^{1/2}$

$f(x)$ $4x^{4/3}$ $3x^{1/2}$

55. $f(x)$ $\frac{1}{3x_2}$ $\frac{1}{3}$ 2

$f'(x)$ 2 x

$\frac{1}{3x}$ 3 $\frac{2}{3}$ x^3

$f(x)$ 3 $\frac{2}{3x}$ ^{4} $2x^4$

56. $f(x)$ 3 $2x$ 1 1 3

$2x$ x^3

$f(x)$ $\frac{3}{2}$ 4 x

$f(x)$ $6x$ 5

57. $f(x)$ 2 $2x^3$

$f'(x)$ 3 2 x^4 6 x^4 $\frac{6}{x^4}$

$f(x)$ 4 6 x^5 $24x^5$ $\frac{24}{x^5}$

$f(1)$ $\frac{24}{(1)^5}$ 24

58. $f(x)$ $\frac{3}{x^4}$ 3 x^4

$f(x)$ 12 x^5

$f(x)$ 60 x^6 $\frac{60}{x^6}$

$f(1)$ $\frac{60}{(1)^6}$ 60

59. $\frac{d}{dx}$ x^6 $6x^5$

$\frac{d}{dx}$ 2 x^6 $\frac{d}{dx}$ $6x^5$ 30 x^4

$\frac{d^2}{dx^2}$ x^6 $\Big|_{x\,2}$ 30(2)^4 480

60. $\frac{d}{dx}$ x^2 2 x^3

$\frac{d^2}{dx^2}$ x^2 $\frac{d}{dx}$ (2 x^3) 6 x^4 $\frac{6}{x^4}$

$\frac{d^2}{dx^2}$ x^2 $\Big|_{x\,2}$ $\frac{6}{(2)^4}$ $\frac{6}{16}$ $\frac{3}{8}$

61. $\frac{d}{dx}$ x^5 $\frac{d}{dx}$ $x^{5/2}$ $\frac{5}{2}$ $x^{3/2}$

$\frac{d^2}{dx^2}$ $\sqrt{x^5}$ $\frac{d}{dx}$ $\frac{5}{2}$ $x^{3/2}$ $\frac{15}{4}$ $x^{1/2}$

$\frac{d^2}{dx^2}$ $\sqrt{x^5}$ $\Big|_{x\,16}$ $\frac{15}{4}$ (16)$^{1/2}$ 15

62. $\frac{d}{dx}$ $\sqrt{x^7}$ $\frac{d}{dx}$ $x^{7/2}$ $\frac{7}{2}$ $x^{5/2}$

$\frac{d^2}{dx^2}$ $\sqrt{x^7}$ $\frac{d}{dx}$ $\frac{7}{2}$ $x^{5/2}$ $\frac{35}{4}$ $x^{3/2}$

$\frac{d^2}{dx^2}$ $\sqrt{x^7}$ $\Big|_{x\,4}$ $\frac{35}{4}$ (4)$^{3/2}$ 70

$P(t)$ $0.25t^3$ $3t^2$ $5t$ 200

$P(t)$ 3(0.25t^2) 2(3t) 1(5) 0 $0.75t^2$ 6t 5

$P(t)$ 2(0.75t) 6(1) 0 1.5t 6

$P(10)$ 0.25(10)^3 3(10)^2 5(10) 200 250 300 50 200 200

In 10 years, the population will be 200,000.

$P'(10) \quad 0.75(10) \quad 6(10) \quad 5 \quad 75 \quad 60 \quad 5 \quad 20$

In 10 years, the population will be increasing by about 20,000 per year.

$P''(10) \quad 1.5(10) \quad 6 \quad 9$

In 10 years, the rate of growth of the increase will be 9000 per year each year.

$s(t)\ 8t^{5/2}$

$v(t)\quad s(t)\quad \frac{5}{2}(8t^{3/2})\ 20t^{3/2}$

$v(25)\quad 20(25)^{3/2}\quad 2500$ ft / sec

a. When the height is a maximum, the velocity is zero. Thus, to find the maximum height, set $v(t)$ 0 and solve. First, we find $v(t)\ s'(t)$.

$s(t)\quad 16t^2\quad 148t\quad 5$

$v(t)\quad s'(t)\quad 32t\quad 148$

Now set $v(t)$ 0 and solve.

$v(t)\quad 32t\quad 148\quad 0$

$32t\quad 148$

4.625

To find the height when $t = 4.625$, evaluate $s(4.625)$.

$s(4.625)\quad 16(4.625)^2\quad 148(4.625)\quad 5$

347.25 feet

b.

$h(z)\ (4z^2\ \ 3z\ \ 1)^3$

$h(z)\quad 3(4z^2\quad 3z\quad 1)^2\ (8z\ 3)$

$h(z)\ (3z^2\ \ 5z\ \ 1)^4$

$h(z)\quad 4(3z^2\quad 5z\quad 1)^3\ (6z\ 5)$

$g(x)\ (100\ x)^5$

$(x)\ 5(100\ x)^4\ (1)\quad 5(100\ x)^4$

$g(x)\ (1000\ x)^4$

$g(x)\quad 4(1000\ x)^3\ (1)\quad 4(1000x)^3$

$f(x)\ \sqrt{x^2\ 2x}\ \ x^2\ 2x^{1/2}\quad f'(x)\ \frac{1}{2}\ x^2\ 2x^{1/2}\ (2x\ 1)$

$f(x)\ \sqrt{x^2\ 5x\ 1}\ \ x^2\ 5x\ 1^{1/2}\quad f'(x)\ \frac{1}{2}\ x^2\ 5x\ 1^{1/2}\ (2x\ 5)$

$w(z)\ \sqrt[3]{6z\ 1}\ \ (6z\ 1)^{1/3}$

$w(z)\quad \frac{1}{3}(6z\ \ 1)^{2/3}\ (6)\ \ 2(6z\ \ 1)^{2/3}$

$w(z)\ \sqrt[3]{3z\ 1}\ \ (3z\ 1)^{1/3}$

$w(z)\quad \frac{1}{3}(3z\ \ 1)^{2/3}(3)\ \ (3z\ \ 1)^{2/3}$

74. $h(x)\quad \frac{1}{\sqrt[5]{(5x\ 1)^2}}\ \ 5x\ \ 1^{2/5}$

$h'(x)\quad \frac{2}{5}\ 5x\ \ 1^{7/5}\quad (5)\ 2(5x\ \ 1)^{7/5}$

75. $h(x)\quad \frac{1}{\sqrt[5]{(10x\ 1)^3}}\quad 10x\ 1^{3/5}$

$h'(x)\quad \frac{3}{5}\ 10x\ \ 1^{8/5}\quad (10)\ 6(10x\ \ 1)^{8/5}$

$g(x)\ x^2\ (2x\ \ 1)^4$

$g(x)\quad 2x(2x\ \ 1)^4\quad x^2\ [4(2x\ \ 1)^3\ (2)]$

$2x(2x\quad 1)^4\quad 8x^2\ (2x\ \ 1)^3$

$g(x)\ 5x(x^3\ \ 2)^4$

$g(x)\quad 5(x^3\quad 2)^4\quad 5x[4(x^3\ \ 2)^3\ (3x^2)]$

$5(x \quad 2) \quad 60\,x \quad (x \quad 2)^3$

78. $y\ x^{3}\ \sqrt[3]{x^3\ \ 1}\ \ x^{3}\ (x^{3}\ \ 1)^{1/3}$

$y\ \ 3x^{2}\ (x^{3}\ \ 1)^{1/3}\ \ x^{3}\ \tfrac{1}{3}\ (x^{3}\ \ 1)^{2/3}\ (3x^{2})$

$3x^{2}\ (x^{3}\ \ 1)^{1/3}\ \ x^{5}\ (x^{3}\ \ 1)^{2/3}$

79. $y\ x^{4}\ \sqrt{x^{2}\ \ 1}\ \ x^{4}\ (x^{2}\ \ 1)^{1/2}$

$y\ \ 4x^{3}\ (x^{2}\ \ 1)^{1/2}\ \ x^{4}\ \tfrac{1}{2}\ (x^{2}\ \ 1)^{1/2}\ (2x)$

$4x^{3}\ (x^{2}\ \ 1)^{1/2}\ \ x^{5}\ (x^{2}\ \ 1)^{1/2}$

80. $f(x)\ \ [(2x^{2}\ \ 1)^{4}\ \ x^{4}]^{3}$

$f(x)\ \ 3[(2x^{2}\ \ 1)^{4}\ \ x^{4}]^{2}[4(2x^{2}\ \ 1)^{3}(4x)\ \ 4x^{3}]$

$3[(2x^{2}\ \ 1)^{4}\ \ x^{4}]^{2}[16x(2x^{2}\ \ 1)^{3}\ \ 4x^{3}]$

81. $f(x)\ \ [(3x^{2}\ \ 1)^{3}\ \ x^{3}]^{2}$

$f(x)\ \ 2[(3x^{2}\ \ 1)^{3}\ \ x^{3}][3(3x^{2}\ \ 1)^{2}(6x)\ \ 3x^{2}]$

$2[(3x^{2}\ \ 1)^{3}\ \ x^{3}][18x(3x^{2}\ \ 1)^{2}\ \ 3x^{2}]$

82. $f(x)\ \sqrt{(x^{2}\ \ 1)^{4}\ \ x^{4}}\ \ [(x^{2}\ \ 1)^{4}\ \ x^{4}]^{1/2}$

$f(x)\ \tfrac{1}{2}[(x^{2}\ \ 1)^{4}\ \ x^{4}]^{1/2}\ [4(x^{2}\ \ 1)^{3}(2x)\ \ 4x^{3}]$

$\tfrac{1}{2}[(x^{2}\ \ 1)^{4}\ \ x^{4}]^{1/2}\ [8x(x^{2}\ \ 1)^{3}\ 4x^{3}]$

83. $f(x)\ \sqrt{(x^{3}\ \ 1)^{2}\ \ x^{2}}\ \ [(x^{3}\ \ 1)^{2}\ \ x^{2}]^{1/2}$

$f(x)\ \tfrac{1}{2}[(x^{3}\ \ 1)^{2}\ \ x^{2}]^{1/2}[2(x^{3}\ \ 1)(3x^{2})\ \ 2x]$

$\tfrac{1}{2}[(x^{3}\ \ 1)^{2}\ \ x^{2}]^{1/2}[6x^{2}\ (x^{3}\ \ 1)\ \ 2x]$

$[(x^{3}\ \ 1)^{2}\ \ x^{2}]^{1/2}\ [3x^{2}\ (x^{3}\ \ 1)\ \ x]$

$f(x)\ \ (3x\ \ 1)^{4}\ (4x\ \ 1)^{3}$

$(x)\ 4\ (3x\ 1)^{3}\ \ (3)(4x\ 1)^{3}\ \ (3x\ 1)^{4}\ \ (3)(4x\ 1)^{2}\ \ (4)\ 12\ (3x$

$1)^{3}\ \ (4x\ 1)^{3}\ \ 12(3x\ 1)^{4}\ \ (4x\ 1)^{2}$

$12\ (3x\ 1)^{3}\ \ (4x\ 1)^{2}\ \ [(4x\ 1)\ (3x\ 1)]\ 12$

$(3x\ 1)^{3}\ \ (4x\ 1)^{2}\ \ (7x\ 2)$

$f(x)\ \ (x^{2}\ \ 1)^{3}\ (x^{2}\ \ 1)^{4}$

$(x)\ 3(x^{2}\ \ 1)^{2}\ \ (2x)(x^{2}\ \ 1)^{4}\ \ (x^{2}\ \ 1)^{3}\ \ [4(x^{2}\ \ 1)^{3}\ \ (2x)]\ 6x(x^{2}$

$1)^{2}\ (x^{2}\ 1)^{4}\ 8x(x^{2}\ 1)^{3}\ (x^{2}\ 1)^{3}$

86. $f(x)\ \left(\frac{x\ \ 5}{x}\right)^{4}$

$f(x)\ \ 4\left(\frac{x\ \ 5}{x}\right)^{3}\ \frac{x(1)\ \ (1)(x\ \ 5)}{x^{2}}$

$4\left(\frac{x\ \ 5}{x}\right)^{3}\ \frac{x\ \ x\ \ 5}{x^{2}}\ \ \frac{20\ (x\ \ 5)^{3}}{x^{2}x^{3}}$

$\frac{20(x\ \ 5)^{3}}{x^{5}}$

$f(x)\ \left(\frac{x\ \ 4}{x}\right)^{5}$

$(x)\ \ 5\left(\frac{x\ \ 4}{x}\right)^{4}\ \frac{x(1)\ \ (1)(x\ \ 4)}{x^{2}}$

$5\left(\frac{x\ \ 4}{x}\right)^{4}\ \frac{4}{x^{2}}\ \ \frac{20}{x^{2}}\left(\frac{x\ \ 4}{x}\right)^{4}$

$\frac{20(x\ \ 4)^{4}}{x^{6}}$

$y\ \sqrt[3]{x^{2}\ \ 4}\ \ (x^{2}\ \ 4)^{1/3}$

$y\ 2\ \sqrt[3]{2^{2}\ \ 4}\ \ 2$

$h(w)\ \ (2w^{2}\ \ 4)^{5}$

$h(w)\ \ 5(2w^{2}\ \ 4)^{4}\ (4w)$

$2\ \ 3\ \ 2\ \ 4$

The point is (2, 2).

$$y'(x) = \frac{1}{3}\left(x^2+4\right)^{-2/3}(2x) = \frac{2x}{3\left(x^2+4\right)^{2/3}}$$

$$y'(2) = \frac{2 \cdot 2}{3\left(2^2+4\right)^{2/3}} = \frac{1}{3}$$

The tangent line has slope $\frac{1}{3}$ at (2, 2).

$$y - 2 = \frac{1}{3}(x - 2)$$

$$h'(w) = 20(2w^2-4)^3(4w)(4w) + 5(2w^2-4)^4(4)$$

$$= 20(16w^2)(2w^2-4)^3 + 20(2w^2-4)^4 = 320w^2(2w^2-4)^3 + 20(2w^2-4)^4$$

$h(w)\ (3w^2\ 1)^4$

$(w)\ 4(3w^2\ 1)^3\ (6w)$

$h\ (w)\ \ 12(3w^2\ \ 1)^2\ (6\ w)(6\ w)\ 4(3w^2\ \ 1)^3\ (6)$

$432w^2\ (3w^2\ \ 1)^2\ \ 24(3w^2\ \ 1)^3$

$g(z)\ z^3\ (z\ 1)^3$

$(z)\ 3z^2\ (z\ 1)^3\ \ z^3\ [3(z\ 1)^2\ ]\ 3z^2\ (z\ 1)^3\ \ 3z^3\ (z\ 1)^2\ \ g\ (z$

$)\ 6\ z(z\ 1)^3\ \ 3z^2\ [3(z\ 1)^2\ ]\ 9\ z^2\ (z\ 1)^2\ \ 3z^3\ [2(z\ 1)]$

$6\ z(z\ 1)^3\ \ 18z^2\ (z\ 1)^2\ \ 6\ z^3\ (z\ 1)$

$g(z)\ z^4\ (z\ 1)^4$

$(z)\ 4\ z^3\ (z\ 1)^4\ \ 4\ z^4\ (z\ 1)^3$

$g\ (z)\ \ 12\ z^2\ (z\ 1)^4\ \ 4\ z^3\ [4(z\ 1)^3\ ]\ 16\ z^3\ (z\ 1)^3\ \ 4\ z^4\ [3(z\ 1)^2\ ]$

$12\ z^2\ (z\ 1)^4\ \ 32\ z^3\ (z\ 1)^3\ \ 12z^4\ (z\ 1)^2$

93. a. $\frac{d}{dx}(x^3\ 1)^2\ 2(x^3\ 1)(3\ x^2)\ 6\ x^2\ (x^3\ 1)\ 6\ x^5\ \ 6\ x^2$

$\frac{d}{dx}(x^3\ 1)^2\ \frac{d}{dx}(x^6\ \ 2\ x^3\ \ 1)\ 6\ x^5\ \ 6\ x^2$

94. a. $\frac{d}{dx}\ \frac{1}{x^3\ 1}\ \ \frac{(x^3\ 1)(0)\ (3\ x^2)(1)}{(x^3\ 1)^2}\ \ \frac{3\ x^2}{(x^3\ 1)^2}$

b. $\frac{d}{dx}\ \frac{1}{x^3\ 1}\ \ \frac{d}{dx}(x^3\ 1)^1\ \ 1(x^3\ 1)^2\ (3x^2)\ \ \frac{3x^2}{(x^3\ 1)^2}$

$P(x)\ \sqrt{x^3\ \ 3\ x\ \ 34}\ \ (x^3\ \ 3x\ \ 34)^{1/2}$

$P\ (x)\ \ \frac{1}{2}\ (x^3\ 3\ x\ \ 34)^{1/2}\ (3\ x^2\ \ 3)\ \ \frac{3x^2\ \ 3}{2\sqrt{x^3\ \ 3x\ \ 34}}$

$P\ (5)\ \ \frac{3(5)^2\ \ 3}{2\sqrt{(5)^3\ \ 3(5)\ \ 34}}\ \ \frac{72}{2\sqrt{144}}\ \ \frac{72}{24}\ 3$

When 5 tons is produced, profit is increasing at about $3000 for each additional ton.

$V(r)\ \ 500(1\ \ 0.01r)^3$

$V\ (r)\ 1500(1\ 0.01r)^2\ (0.01)\ 15(1\ 0.01r)^2\ \ V\ (8)$

$15[1\ 0.01(8)]^2\ \ 17.50$

For 8 percent interest, the value increases by about $17.50 for each additional percent interest.

97. on [0, 10] by [0, 30]

98. on [0, 20] by [–2, 10]

a. $P(5)\ \ P(4) 12\ \ 9.27\ \ 2.73$ $\qquad x\ \ 16$

$P(6)\ \ P(5) 15.23\ \ 12\ \ \ 3.23$

Both values are near 3.

$x\ \ 7.6$

$R(x)\ 0.25(0.01x\ \ 1)^4$

$R(x)\ \ 0.25[4(0.01x\ \ 1)^3\ (0.01)]$

$0.01(0.01x\ \ 1)^3$

$R(100)\ \ 0.01[0.01(100)\ \ 1)]^3\ \ 0.08$

$N(x)\ 1000\ 100\sqrt{\ x}\ 1000\ 100\ x^{1/2}$

$N'(x)\ \ 500(100\ \ x)^{1/2}\ (\ 1)\ \ \frac{500}{\sqrt{100\ \ x}}$

$N'(96)\ \ \frac{500}{\sqrt{100\ \ 96}}\ 250$

At age 96, the number of survivors is decreasing by 250 people per year.

The derivative does not exist at corner points 3 and x 3 and at the discontinuous point 1 .

The derivative does not exist at the corner point 2 and at the discontinuous point x 2 .

The derivative does not exist at the corner point x 3.5 and the discontinuous point x 0 .

The derivative does not exist at the corner points x 0 and x 3 .

For positive h,

$\lim_{h\ 0} \frac{f(x\ \ h)\ f(x)}{h}\ \lim_{h\ 0} \frac{|5\ x\ \ 5h|\ \ |5\ x|}{h}\ \lim_{h\ 0} \frac{|5(0)\ 5\ h|\ \ |5\ 0|}{h}$ for x 0

$\lim_{h\ 0} \frac{5\ h}{h}\ 5$

For negative h,

$\lim_{h\ 0} \frac{f(x\ \ h)\ f(x)}{h}\ \lim_{h\ 0} \frac{|5\ x\ \ 5h|\ \ |5\ x|}{h}\ \lim_{h\ 0} \frac{|5(0)\ \ 5h|\ \ |5(0)|}{h}$ for x 0

$\lim_{h\ 0} \frac{5h}{h}\ 5$

Thus, the limit does not exist, and so the derivative does not exist.

106. $\lim_{h\ 0} \frac{f(x\ \ h)\ f(x)}{h}\ \lim_{h\ 0} \frac{(x\ \ h)^{3/5}\ \ x^{3/5}}{h}\ \lim_{h\ 0} \frac{h^{3/5}}{h}$ for x 0

$\lim_{h\ 0} \frac{1}{h^{2/5}}$

which does not exist. Thus, the derivative does not exist.