Chapter 2 Differentiation

Chapter Comments

The material presented in Chapter 2 forms the basis for the remainder of calculus. Much of it needs to be memorized, beginning with the definition of a derivative of a function found on page 103. Students need to have a thorough understanding of the tangent line problem and they need to be able to find an equation of a tangent line. Frequently, students will use the function $f'(x)$ as the slope of the tangent line. They need to understand that $f'(x)$ is the formula for the slope and the actual value of the slope can be found by substituting into $f'(x)$ the appropriate value for x On pages 105–106 of Section 2.1, you will find a discussion of situations where the derivative fails to exist. These examples (or similar ones) should be discussed in class.

As you teach this chapter, vary your notations for the derivative. One time write y'; another time write $dy\ dx$ or $f'(x)$. Terminology is also important. Instead of saying "find the derivative," sometimes say, "differentiate." This would be an appropriate time, also, to talk a little about Leibnitz and Newton and the discovery of calculus.

Sections 2.2, 2.3, and 2.4 present a number of rules for differentiation. Have your students memorize the Product Rule and the Quotient Rule (Theorems 2.7 and 2.8) in words rather than symbols. Students tend to be lazy when it comes to trigonometry and therefore, you need to impress upon them that the formulas for the derivatives of the six trigonometric functions need to be memorized also. You will probably not have enough time in class to prove every one of these differentiation rules, so choose several to do in class and perhaps assign a few of the other proofs as homework.

The Chain Rule, in Section 2.4, will require two days of your class time. Students need a lot of practice with this and the algebra involved in these problems. Many students can find the derivative of $f(x) = x^2\sqrt{1 - x^2}$ without much trouble, but simplifying the answer is often difficult for them. Insist that they learn to factor and write the answer without negative exponents. Strive to get the answer in the form given in the back of the book. This will help them later on when the derivative is set equal to zero.

Implicit differentiation is often difficult for students. Have students think of as a function of x and therefore y^3 is $[f(x)]^3$. This way they can relate implicit differentiation to the Chain Rule studied in the previous section.

Try to get your students to see that related rates, discussed in Section 2.6, are another use of the Chain Rule.

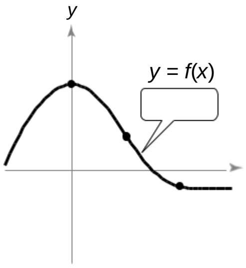
Section 2.1 The Derivative and the Tangent Line Problem

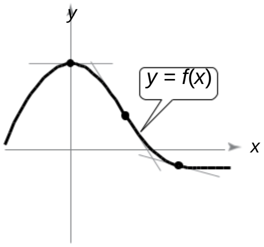
Section Comments

2.1 **The Derivative and the Tangent Line Problem**—Find the slope of the tangent line to a curve at a point. Use the limit definition to find the derivative of a function. Understand the relationship between differentiability and continuity.

Teaching Tips

Ask students what they think "the line tangent to a curve" means. Draw a curve with tangent lines to show a visual picture of tangent lines. For example:





When talking about the tangent line problem, use the suggested example of finding the equation of the tangent line to the parabola $y = x^2$ at the point $(1, 1)$.

Compute an approximation of the slope m by choosing a nearby point $Q(x, x^2)$ on the parabola and computing the slope m_{PQ} of the secant line PQ.

After going over Examples 1–3, return to Example 2 where $f(x) = x^2 + 1$ and note that $f'(x) = 2x$. How can we find the equation of the line tangent to f and parallel to $4x - y = 0$? Because the slope of the line is 4,

$$2x = 4$$

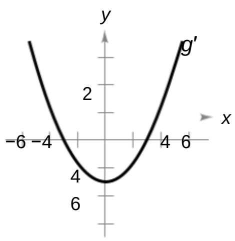
$$x = 2.$$

So, at the point $(2, 5)$, the tangent line is parallel to $4x - y = 0$. The equation of the tangent line is $y - 5 = 4(x - 2)$ or $y = 4x - 3$.

Be sure to find the derivatives of various types of functions to show students the different types of techniques for finding derivatives. Some suggested problems are $f(x) = 4x^3 - 3x^2$, $g(x) = 2 (x - 1)$, and $h(x) = 2x + 5$.

How Do You See It? Exercise

Page 108, Exercise 64 The figure shows the graph of g'.



(a) $g'(0) =$

(b) $g'(3) =$

(c) What can you conclude about the graph of g knowing that $g'(1) = -\frac{8}{3}$?

(d) What can you conclude about the graph of g knowing that $g'(-4) = \frac{7}{3}$?

(e) Is $g(6) - g(4)$ positive or negative? Explain.

(f) Is it possible to find $g(2)$ from the graph? Explain.

Solution

(a) $g'(0) = -3$

(b) $g'(3) = 0$

(c) Because $g'(1) = -\frac{8}{3}$, g is decreasing (falling) at $x = 1$.

(d) Because $g'(-4) = \frac{7}{3}$, g is increasing (rising) at $x = -4$.

(e) Because $g'(4)$ and $g'(6)$ are both positive, $g(6)$ is greater than $g(4)$ and $g(6) - g(4) > 0$.

(f) No, it is not possible. All you can say is that g is decreasing (falling) at $x = 2$.

Suggested Homework Assignment

Pages 107–109: 1, 3, 7, 11, 21–27 odd, 37, 43–47 odd, 53, 57, 61, 77, 87, 93, and 95.

Section 2.2 Basic Differentiation Rules and Rates of Change

Section Comments

2.2 **Basic Differentiation Rules and Rates of Change**—Find the derivative of a function using the Constant Rule. Find the derivative of a function using the Power Rule. Find the derivative of a function using the Constant Multiple Rule. Find the derivative of a function using the Sum and Difference Rules. Find the derivatives of the sine function and of the cosine function. Use derivatives to find rates of change.

Teaching Tips

Start by showing proofs of the Constant Rule and the Power Rule. Students who are mathematics majors need to start seeing proofs early on in their college careers as they will be taking Functions of a Real Variable at some point.

Go over an example in class like $f(x) = \frac{5x^2 + x}{x}$. Show students that before differentiating they can rewrite the function as $f(x) = 5x + 1$. Then they can differentiate to obtain $f'(x) = 5$. Use this example to emphasize the prudence of examining the function first before differentiating. Rewriting the function in a simpler, equivalent form can expedite the differentiating process.

Give mixed examples of finding derivatives. Some suggested examples are:

$f(x) = 3x^6 - x^{2/3} + 3\sin x$ and $g(x) = \frac{4}{x^3} + \frac{2}{(3x)^2} - 3\cos x + 7x + \pi^3$.

This will test students' understanding of the various differentiation rules of this section.

How Do You See It? Exercise

Page 119, Exercise 76 Use the graph of f to answer each question. To print an enlarged copy of the graph, go to *MathGraphs.com*.

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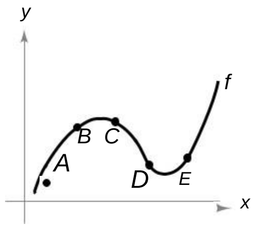


(a) Between which two consecutive points is the average rate of change of the function greatest?

(b) Is the average rate of change of the function between *A* and *B* greater than or less than the instantaneous rate of change at *B*?

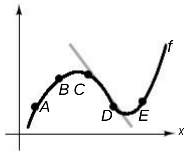
(c) Sketch a tangent line to the graph between *C* and *D* such that the slope of the tangent line is the same as the average rate of change of the function between *C* and *D*.

Solution

(a) The slope appears to be steepest between *A* and *B*.

(b) The average rate of change between *A* and *B* is **greater** than the instantaneous rate of change at *B*.

(c)



Suggested Homework Assignment

Pages 118–120: 1, 3, 5, 7–29 odd, 35, 39–53 odd, 55, 59, 65, 75, 85–89 odd, 91, 95, and 97.

Section 2.3 Product and Quotient Rules and Higher-Order Derivatives

Section Comments

2.3 Product and Quotient Rules and Higher-Order Derivatives—Find the derivative of a function using the Product Rule. Find the derivative of a function using the Quotient Rule. Find the derivative of a trigonometric function. Find a higher-order derivative of a function.

Teaching Tips

Some students have difficulty simplifying polynomial and rational expressions. Students should review these concepts by studying Appendices A.2–A.4 and A.7 in *Precalculus*, 10th edition, by Larson.

When teaching the Product and Quotient Rules, give proofs of each rule so that students can see where the rules come from. This will provide mathematics majors a tool for writing proofs, as each proof requires subtracting and adding the same quantity to achieve the desired results. For the Project Rule, emphasize that there are many ways to write the solution. Remind students that there must be one derivative in each term of the solution. Also, the Product Rule can be extended to more that just the product of two functions. Simplification is up to the discretion of the instructor. Examples such as $f(x) = (2x^2 - 3x)(5x^3 + 6)$ can be done with or without the Product Rule. Show the class both ways.

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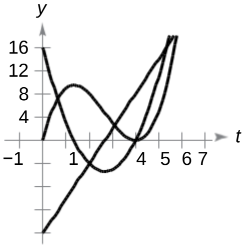
After the Quotient Rule has been proved to the class, give students the memorization tool of LO d HI – HI d LO. This will give students a way to memorize what goes in the numerator of the Quotient Rule.

Some examples to use are $f(x) = \frac{2x-1}{x^2+7x}$ and $g(x) = \frac{4-(1/x)}{3-x^2}$. Save $f(x)$ for the next section as this will be a good example for the Chain Rule. $g(x)$ is a good example for first finding the least common denominator.

How Do You See It? Exercise

Page 132, Exercise 120 The figure shows the graphs of the position, velocity, and acceleration functions of a particle.

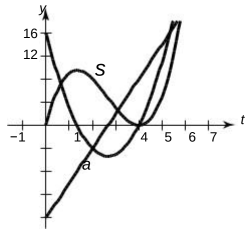


(a) Copy the graphs of the functions shown. Identify each graph. Explain your reasoning. To print an enlarged copy of the graph, go to *MathGraphs.com*.

(b) On your sketch, identify when the particle speeds up and when it slows down. Explain your reasoning.

Solution

(a)

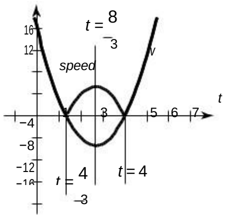


s position function

v velocity function

a acceleration function

(b) The speed of the particle is the absolute value of its velocity. So, the particle's speed is slowing down on the intervals $(0, \frac{4}{3})$, and $(\frac{8}{3}, 4)$ and it speeds up on the intervals $(\frac{4}{3}, \frac{8}{3})$ and $(4, 6)$.



Suggested Homework Assignment

Pages 129–132: 1, 3, 9, 13, 19, 23, 29–55 odd, 59, 61, 63, 75, 77, 91–107 odd, 111, 113, 117, and 131–135 odd.

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Section 2.4 The Chain Rule

Section Comments

2.4 **The Chain Rule**—Find the derivative of a composite function using the Chain Rule. Find the derivative of a function using the General Power Rule. Simplify the derivative of a function using algebra. Find the derivative of a trigonometric function using the Chain Rule.

Teaching Tips

Begin this section by asking students to consider finding the derivative of $F(x) = \sqrt{x^2 + 1}$. F is a composite function. Letting $y = f(u) = u$ and $u = g(x) = x^2 + 1$, then $y = F(x) = f(g(x))$ or $F = f \circ g$. When stating the Chain Rule, be sure to state it using function notation and using Leibniz notation as students will see both forms when studying other courses with other texts. Following the definition, be sure to prove the Chain Rule as done on page 134.

Be sure to give examples that involve all rules discussed so far. Some examples include: $f(x) = (\sin(6x))^4$, $g(x) = \dfrac{(3 + \sin(2x))^2}{\sqrt[3]{x + 3}}$, and $h(x) = \sqrt{x - \dfrac{2}{x}} \cdot [8x + \cos(x^2 + 1)]^3$.

You can use Exercise 98 on page 141 to review the following concepts:

- Product Rule
- Chain Rule
- Quotient Rule
- General Power Rule

Students need to understand these rules because they are the foundation of the study of differentiation.

Use the solution to show students how to solve each problem. As you apply each rule, give the definition of the rule verbally. Note that part (b) is not possible because we are not given $g'(3)$.

Solution

(a) $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + g'(x)h(x)$

$f'(5) = (-3)(-2) + (6)(3) = 24$

(b) $f(x) = g(h(x))$

$f'(x) = g'(h(x))h'(x)$

$f'(5) = g'(3)(-2) = -2g'(3)$

Not possible. You need $g'(3)$ to find $f'(5)$.

(c) $f(x) = \dfrac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$$

$$f'(x) = \frac{(3)(6) - (-3)(-2)}{(3)^2} = \frac{12}{9} = \frac{4}{3}$$

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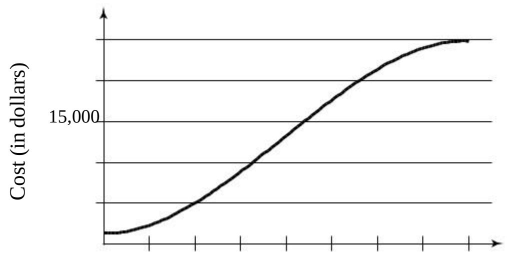
(d) $f(x) = [g(x)]^3$

$f'(x) = 3[g(x)]^2 g'(x)$

$f'(5) = 3(-3)^2(6) = 162$

How Do You See It? Exercise

Page 142, Exercise 106 The cost C (in dollars) of producing x units of a product is $C = 60x + 1350$. For one week, management determined that the number of units produced x at the end of t hours can be modeled by $x = -1.6t^3 + 19t^2 - 0.5t - 1$. The graph shows the cost C in terms of the time t.



(a) Using the graph, which is greater, the rate of change of the cost after 1 hour or the rate of change of the cost after 4 hours?

(b) Explain why the cost function is not increasing at a constant rate during the eight-hour shift.

Solution

(a) According to the graph, $C'(4) > C'(1)$.

(b) Answers will vary.

Suggested Homework Assignment

Pages 140–143: 1–53 odd, 63, 67, 75, 81, 83, 91, 97, 121, and 123.

Section 2.5 Implicit Differentiation

Section Comments

2.5 **Implicit Differentiation**—Distinguish between functions written in implicit form and explicit form. Use implicit differentiation to find the derivative of a function.

Teaching Tips

Material learned in this section will be vital for students to have for related rates. Be sure to ask students to find $\frac{dy}{dx}$ when $x = c$.

You can use the exercise below to review the following concepts:

- Finding derivatives when the variables agree and when they disagree
- Using implicit differentiation to find the derivative of a function

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Determine if the statement is true. If it is false, explain why and correct it. For each statement, assume is a function of x

(a) $\frac{d}{dx}\cos(x^2) = -2x\sin(x^2)$

(b) $\frac{d}{dy}\cos(y^2) = 2y\sin(y^2)$

(c) $\frac{d}{dx}\cos(y^2) = -2y\sin(y^2)$

Implicit differentiation is often difficult for students, so as you review this concept remind students to think of y as a function of x. Part (a) is true, and part (b) can be corrected as shown below. Part (c) requires implicit differentiation. Note that the result can also be written as $-2y\sin(y^2)\frac{dy}{dx}$.

Solution

(a) True

(b) False. $\frac{d}{dy}\cos(y^2) = -2y\sin(y^2)$.

(c) False. $\frac{d}{dx}\cos(y^2) = -2yy'\sin(y^2)$.

A good way to teach students how to understand the differentiation of a mix of variables in part (c) is to let g y. Then $g' = y'$. So,

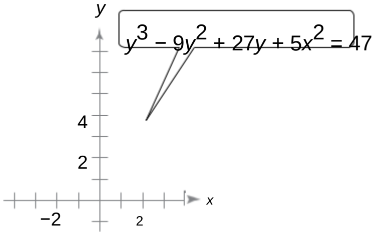
$$\frac{d}{dx}\cos(y^2) = \frac{d}{dx}\cos(g^2)$$

$$-\sin(g^2)\cdot 2gg'$$

$$-\sin(y^2)\cdot 2yy'$$

How Do You See It? Exercise

Page 151, Exercise 70 Use the graph to answer the questions.



(a) Which is greater, the slope of the tangent line at $x = -3$ or the slope of the tangent line at $= -1$?

(b) Estimate the point(s) where the graph has a vertical tangent line.

(c) Estimate the point(s) where the graph has a horizontal tangent line.

Solution

(a) The slope is greater at $x = -3$.

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(b) The graph has vertical tangent lines at about $(-2, 3)$ and $(2, 3)$.

(c) The graph has a horizontal tangent line at about $(0, 6)$.

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Suggested Homework Assignment

Pages 149–150: 1–17 odd, 25–35 odd, 53, and 61.

Section 2.6 Related Rates

Section Comments

2.6 **Related Rates**—Find a related rate. Use related rates to solve real-life problems.

Teaching Tips

Begin this lesson with a quick review of implicit differentiation with an implicit function in terms of *x* and differentiated with respect to time. Follow this with an example similar to Example 1 on page 152, outlining the step-by-step procedure at the top of page 153 along with the guidelines at the bottom of page 153. Be sure to tell students, that for every related rate problem, to write down the given information, the equation needed, and the unknown quantity. A suggested problem to work out with the students is as follows:

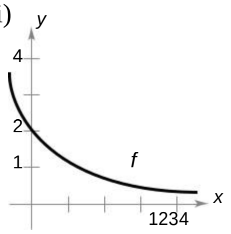
A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 foot per second, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall?

Be sure to go over a related rate problem similar to Example 5 on page 155 so that students are exposed to working with related rate problems involving trigonometric functions.

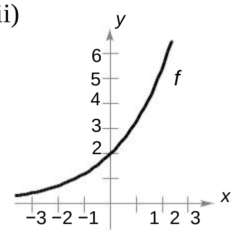
How Do You See It? Exercise

Page 159, Exercise 34 Using the graph of *f*, (a) determine whether *dy dt* is positive or negative given that *dx dt* is negative, and (b) determine whether *dx dt* is positive or negative given that *dy dt* is positive. Explain.

(i)



(ii)



Solution

(a) $\frac{dx}{dt}$ negative ⇒ $\frac{dy}{dt}$ positive

$\frac{dy}{dt}$ positive ⇒ $\frac{dx}{dt}$ negative

(ii) (a) $\frac{dx}{dt}$ negative ⇒ $\frac{dy}{dt}$ negative

$\frac{dy}{dt}$ positive ⇒ $\frac{dx}{dt}$ positive

Suggested Homework Assignment

Pages 157–160: 1, 7, 11, 13, 15, 17, 21, 25, 29, and 41.

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Chapter 2 Project

Timing a Handoff

You are a competitive bicyclist. During a race, you bike at a constant velocity of k meters per second. A chase car waits for you at the ten-mile mark of a course. When you cross the ten-mile mark, the car immediately accelerates to catch you. The position function of the chase car is given by the equation $s(t) = \frac{15}{4}t^2 - \frac{5}{12}t^3$, for $0 \le t \le 6$, where t is the time in seconds and s is the distance traveled in meters. When the car catches you, you and the car are traveling at the same velocity, and the driver hands you a cup of water while you continue to bike at k meters per second.

Exercises

1. Write an equation that represents your position s (in meters) at time t (in seconds).

2. Use your answer to Exercise 1 and the given information to write an equation that represents the velocity k at which the chase car catches you in terms of t.

3. Find the velocity function of the car.

4. Use your answers to Exercises 2 and 3 to find how many seconds it takes the chase car to catch you.

5. What is your velocity when the car catches you?

6. Use a graphing utility to graph the chase car's position function and your position function in the same viewing window.

7. Find the point of intersection of the two graphs in Exercise 6. What does this point represent in the context of the problem?

8. Describe the graphs in Exercise 6 at the point of intersection. Why is this important for a successful handoff?

9. Suppose you bike at a constant velocity of 9 meters per second and the chase car's position function is unchanged.

(a) Use a graphing utility to graph the chase car's position function and your position function in the same viewing window.

(b) In this scenario, how many times will the chase car be in the same position as you after the 10-mile mark?

(c) In this scenario, would the driver of the car be able to successfully handoff a cup of water to you? Explain.

10. Suppose you bike at a constant velocity of 8 meters per second and the chase car's position function is unchanged.

(a) Use a graphing utility to graph the chase car's position function and your position function in the same viewing window.

(b) In this scenario, how many times will the chase car be in the same position as you after the ten-mile mark?

(c) In this scenario, why might it be difficult for the driver of the chase car to successfully handoff a cup of water to you? Explain.

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CHAPTER 2
Differentiation

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Section 2.1 The Derivative and the Tangent Line Problem

The problem of finding the tangent line at a point P is essentially finding the slope of the tangent line at point P. To do so for a function f, if f is defined on an open interval containing c, and if the limit

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = m$$

exists, then the line passing through the point $P(c, f(c))$ with slope m is the tangent line to the graph of f at the point P.

Some alternative notations for $f'(x)$ are

$$\frac{dy}{dx},\ y',\ \frac{d}{dx}\left[f(x)\right], \text{ and } D_x[y].$$

The limit used to define the slope of a tangent line is also used to define differentiation. The key is to rewrite the difference quotient so that Δx does not occur as a factor of the denominator.

If a function f is differentiable at a point $x = c$, then f is continuous at $x = c$. The converse is not true. That is, a function could be continuous at a point, but not differentiable there. For example, the function $y = |x|$ is continuous at $x = 0$, but is not differentiable there.

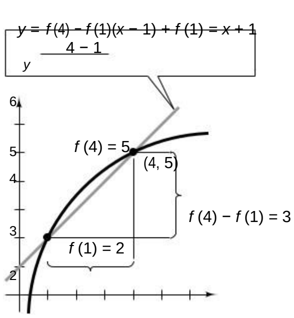
5. At (x_1, y_1), slope $= 0$.

At (x_2, y_2), slope $= \frac{5}{2}$.

6. At (x_1, y_1), slope $= \frac{2}{3}$.

At (x_2, y_2), slope $= -\frac{2}{5}$.

7. (a)–(c)



8. (a) $\dfrac{f(4) - f(1)}{4 - 1} = \dfrac{5 - 2}{3} = 1$

$\dfrac{f(4) - f(3)}{4 - 3} \approx \dfrac{5 - 4.75}{1} = 0.25$

So, $\dfrac{f(4) - f(1)}{4 - 1} > \dfrac{f(4) - f(3)}{4 - 3}$.

The slope of the tangent line at $(1, 2)$ equals $f'(1)$. This slope is steeper than the slope of the line through $(1, 2)$ and $(4, 5)$. So, $\dfrac{f(4) - f(1)}{4 - 1} < f'(1)$.

$f(x) = 3 - 5x$ is a line. Slope $= -5$

$g(x) = \frac{3}{2}x + 1$ is a line. Slope $= \frac{3}{2}$

11. Slope at $(2, 5) = \displaystyle\lim_{\Delta x \to 0} \frac{f(2 + \Delta x) - f(2)}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} \frac{2(2 + \Delta x)^2 - 3 - \left[2(2)^2 - 3\right]}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} \frac{2\left[4 + 4\Delta x + (\Delta x)^2\right] - 3 - (5)}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} \frac{8\Delta x + 2(\Delta x)^2}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} (8 + 2\Delta x) = 8$

12. Slope at $(3, -4) = \displaystyle\lim_{\Delta x \to 0} \frac{f(3 + \Delta x) - f(3)}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} \frac{5 - (3 + \Delta x)^2 - (-4)}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} \frac{5 - 9 - 6\Delta x - (\Delta x)^2 + 4}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} \frac{-6\Delta x - (\Delta x)^2}{\Delta x}$

1 (1, 2)

4 − 1 = 3

1 2 3 4 5 6 x

(d) $y = \frac{f(4) - f(1)}{4 - 1}(x - 1) + f(1)$

$= \frac{3}{3}(x - 1) + 2$

$= 1(x - 1) + 2$

$= x + 1$

$= \lim_{\Delta x \to 0}(-6 - \Delta x) = -6$

13. Slope at $(0, 0) = \lim_{t \to 0} \frac{f(0 + t) - f(0)}{t}$

$= \lim_{t \to 0} \frac{3t - t^2 - 0}{t}$

$= \lim_{t \to 0}(3 - t) = 3$

14. Slope at $(1, 5) = \lim_{t \to 0} \frac{h(1 + t) - h(1)}{t}$

$= \lim_{t \to 0} \frac{(1 + t)^2 + 4(1 + t) - 5}{t}$

$= \lim_{t \to 0} \frac{1 + 2 t + t^2 + 4 + 4 t - 5}{t}$

$= \lim_{t \to 0} \frac{6 t + (t)^2}{t}$

$= \lim_{t \to 0} (6 + t) = 6$

15. $f(x) = 7$

$f'(x) = \lim_{x \to 0} \frac{f(x + x) - f(x)}{x}$

$= \lim_{x \to 0} \frac{7 - 7}{x}$

$= \lim_{x \to 0} 0 = 0$

16. $g(x) = -3$

$g'(x) = \lim_{x \to 0} \frac{g(x + x) - g(x)}{x}$

$= \lim_{x \to 0} \frac{-3 - (-3)}{x}$

$= \lim_{x \to 0} \frac{0}{x} = 0$

17. $f(x) = -5x$

$f'(x) = \lim_{x \to 0} \frac{f(x + x) - f(x)}{x}$

$= \lim_{x \to 0} \frac{-5(x + x) - (-5x)}{x}$

$= \lim_{x \to 0} \frac{-5x - 5x + 5x}{x}$

$= \lim_{x \to 0} \frac{-5 x}{x}$

$= \lim_{x \to 0} (-5) = -5$

18. $f(x) = 7x - 3$

$f'(x) = \lim_{x \to 0} \frac{f(x + x) - f(x)}{x}$

$= \lim_{x \to 0} \frac{7(x + x) - 3 - (7x - 3)}{x}$

$= \lim_{x \to 0} \frac{7x + 7 x - 3 - 7x + 3}{x}$

$= \lim_{x \to 0} \frac{7(x)}{x}$

$= \lim_{x \to 0} 7 = 7$

19. $h(s) = 3 + \frac{2}{3}s$

$h'(s) = \lim_{s \to 0} \frac{h(s + s) - h(s)}{s}$

$= \lim_{s \to 0} \frac{3 + \frac{2}{3}(s + s) - \left(3 + \frac{2}{3}s\right)}{s}$

$= \lim_{s \to 0} \frac{3 + \frac{2}{3}s + \frac{2}{3} s - 3 - \frac{2}{3}s}{s}$

$= \lim_{s \to 0} \frac{\frac{2}{3} s}{s} = \frac{2}{3}$

20. $f(x) = 5 - \frac{2}{3}x$

$f'(x) = \lim_{x \to 0} \frac{f(x + x) - f(x)}{x}$

$= \lim_{x \to 0} \frac{5 - \frac{2}{3}(x + x) - \left(5 - \frac{2}{3}x\right)}{x}$

$= \lim_{x \to 0} \frac{5 - \frac{2}{3}x - \frac{2}{3} x - 5 + \frac{2}{3}x}{x}$

$= \lim_{x \to 0} \frac{-\frac{2}{3}(x)}{x}$

$$= \lim_{x \to 0} \left(\frac{2}{3} \right) = \frac{2}{3}$$

$$f(x) = x^2 + x - 3$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 + (x + \Delta x) - 3 - (x^2 + x - 3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 + x + \Delta x - 3 - x^2 - x + 3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2 + \Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x + 1) = 2x + 1$$

$f(x) = x^2 - 5$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - 5 - (x^2 - 5)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 - 5 - x^2 + 5}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

$f(x) = x^3 - 12x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 - 12(x + \Delta x)\right] - \left[x^3 - 12x\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12x - 12\Delta x - x^3 + 12x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\Delta x + (\Delta x)^2 - 12\right) = 3x^2 - 12$$

24. $g(t) = t^3 + 4t$

$$g'(t) = \lim_{\Delta t \to 0} \frac{g(t + \Delta t) - g(t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\left[(t + \Delta t)^3 + 4(t + \Delta t)\right] - \left[t^3 + 4t\right]}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{t^3 + 3t^3\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + 4t + 4\Delta t - t^3 - 4t}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + 4\Delta t}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \left(3t^2 + 3t\Delta t + (\Delta t)^2 + 4\right)$$

$$= 3t^2 + 4$$

25. $f(x) = \dfrac{1}{x-1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x+\Delta x-1} - \dfrac{1}{x-1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x-1) - (x+\Delta x-1)}{\Delta x(x+\Delta x-1)(x-1)}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x(x+\Delta x-1)(x-1)}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x-1)(x-1)}$$

$$= -\frac{1}{(x-1)^2}$$

26. $f(x) = \dfrac{1}{x^2}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{(x+\Delta x)^2} - \dfrac{1}{x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 - (x+\Delta x)^2}{\Delta x(x+\Delta x)^2 x^2}$$

$$= \lim_{\Delta x \to 0} \frac{-2x\Delta x - (\Delta x)^2}{\Delta x(x+\Delta x)^2 x^2}$$

$$= \lim_{\Delta x \to 0} \frac{-2x - \Delta x}{(x+\Delta x)^2 x^2}$$

$$= \frac{-2x}{x^4}$$

$$= -\frac{2}{x^3}$$

27. $f(x) = \sqrt{x+4}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x+4} - \sqrt{x+4}}{\Delta x} \cdot \left(\frac{\sqrt{x+\Delta x+4} + \sqrt{x+4}}{\sqrt{x+\Delta x+4} + \sqrt{x+4}}\right)$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x+4) - (x+4)}{\Delta x\left[\sqrt{x+\Delta x+4} + \sqrt{x+4}\right]}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x+4} + \sqrt{x+4}} = \frac{1}{\sqrt{x+4} + \sqrt{x+4}} = \frac{1}{2\sqrt{x+4}}$$

28. $h(s) = -2\sqrt{s}$

$$h'(s) = \lim_{\Delta s \to 0} \frac{h(s+\Delta s) - h(s)}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{-2\sqrt{s+\Delta s} - (-2\sqrt{s})}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{-2\left(\sqrt{s+\Delta s} - \sqrt{s}\right)}{\Delta s} \cdot \frac{\sqrt{s+\Delta s} + \sqrt{s}}{\sqrt{s+\Delta s} + \sqrt{s}}$$

$$= \lim_{\Delta s \to 0} \frac{-2(s+\Delta s - s)}{\Delta s\left(\sqrt{s+\Delta s} + \sqrt{s}\right)}$$

$$= \lim_{s \to 0} \frac{-2}{\sqrt{s + \quad s} + \sqrt{s}}$$

$$= -\frac{2}{2\sqrt{s}} = -\frac{1}{\sqrt{s}}$$

29. (a) $f(x) = x^2 + 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 + 3\right] - \left(x^2 + 3\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 3 - x^2 - 3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

At $(-1, 4)$, the slope of the tangent line is $m = 2(-1) = -2$.

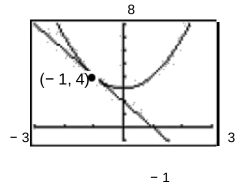
The equation of the tangent line is

$y - 4 = -2(x + 1)$

$y - 4 = -2x - 2$

$y = -2x + 2$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = -2$ at $(-1, 4)$.

30. (a) $f(x) = x^2 + 2x - 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 + 2(x + \Delta x) - 1\right] - \left[x^2 + 2x - 1\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[x^2 + 2x\Delta x + (\Delta x)^2 + 2x + 2\Delta x - 1\right] - \left[x^2 + 2x - 1\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 + 2\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x + 2) = 2x + 2$$

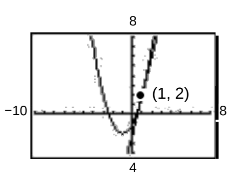
At $(1, 2)$, the slope of the tangent line is $m = 2(1) + 2 = 4$.

The equation of the tangent line is

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$= 4x - 2.$



Graphing utility confirms $\frac{dy}{dx} = 4$ at $(1, 2)$.

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31. (a) $f(x) = x^3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(3x^2 + 3x(\Delta x) + (\Delta x)^2\right)}{\Delta x}$$

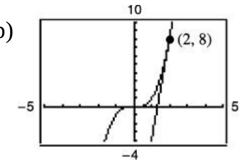
$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\Delta x + (\Delta x)^2\right) = 3x^2$$

At $(2, 8)$, the slope of the tangent is $m = 3(2)^2 = 12$.

The equation of the tangent line is

$$y - 8 = 12(x - 2)$$
$$y - 8 = 12x - 24$$
$$y = 12x - 16.$$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = 12$ at $(2, 8)$.

32. (a) $f(x) = x^3 + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x+\Delta x)^3 + 1\right] - \left(x^3 + 1\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 + 1 - x^3 - 1}{\Delta x}$$

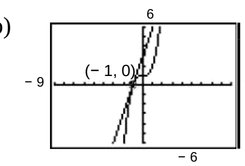
$$= \lim_{\Delta x \to 0} \left[3x^2 + 3x\Delta x + (\Delta x)^2\right] = 3x^2$$

At $(-1, 0)$, the slope of the tangent line is $m = 3(-1)^2 = 3$.

The equation of the tangent line is

$$y - 0 = 3(x + 1)$$
$$y = 3x + 3.$$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = 3$ at $(-1, 0)$.

33. (a) $f(x) = \sqrt{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

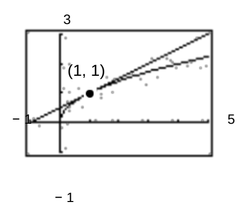
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x) - x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

At $(1, 1)$, the slope of the tangent line is $m = \frac{1}{2\sqrt{1}} = \frac{1}{2}$.

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = \frac{1}{2}$ at $(1, 1)$.

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2 1 2

The equation of the tangent line is

$$-1 = \frac{1}{2}(x-1)$$

$$y - 1 = \frac{1}{2}x - \frac{1}{2}$$

$$y = \frac{1}{2}x + \frac{1}{2}.$$

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34. (a) $f(x) = \sqrt{x-1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x-1} - \sqrt{x-1}}{\Delta x} \cdot \frac{\left(\sqrt{x+\Delta x-1} + \sqrt{x-1}\right)}{\left(\sqrt{x+\Delta x-1} + \sqrt{x-1}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x-1) - (x-1)}{\Delta x\left(\sqrt{x+\Delta x-1} + \sqrt{x-1}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x-1} + \sqrt{x-1}} = \frac{1}{2\sqrt{x-1}}$$

At $(5, 2)$, the slope of the tangent line is $m = \frac{1}{2\sqrt{5-1}} = \frac{1}{4}$.

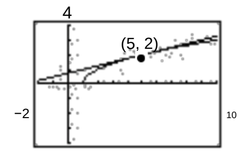
The equation of the tangent line is

$$y - 2 = \frac{1}{4}(x-5)$$

$$y - 2 = \frac{1}{4}x - \frac{5}{4}$$

$$y = \frac{1}{4}x + \frac{3}{4}.$$

(b)



Graphing utility confirms $\frac{dy}{dx} = \frac{1}{4}$ at $(5, 2)$.

35. (a) $f(x) = x + \frac{4}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left(x+\Delta x\right) + \frac{4}{x+\Delta x} - \left(x + \frac{4}{x}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x(x+\Delta x)(x+\Delta x) + 4x - x^2(x+\Delta x) - 4(x+\Delta x)}{x(\Delta x)(x+\Delta x)}$$

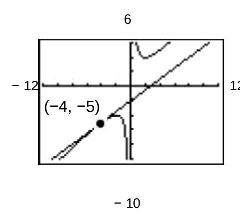
$$= \lim_{\Delta x \to 0} \frac{x^3 + 2x^2(\Delta x) + x(\Delta x)^2 - x^3 - x^2(\Delta x) - 4(\Delta x)}{x(\Delta x)(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2(\Delta x) + x(\Delta x)^2 - 4(\Delta x)}{x(\Delta x)(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + x(\Delta x) - 4}{x(x+\Delta x)}.$$

2

(b)



Graphing utility confirms $\frac{dy}{dx} = \frac{3}{4}$ at $(-4, -5)$.

$$= \frac{x^2 - 4}{x^2} = 1 - \frac{4}{x^2}$$

At $(-4, -5)$, the slope of the tangent line is $m = 1 - \frac{4}{(-4)^2} = \frac{3}{4}$.
The equation of the tangent line is

$$y + 5 = \frac{3}{4}(x + 4)$$

$$y + 5 = \frac{3}{4}x + 3$$

$$y = \frac{3}{4}x - 2.$$

36. (a) $f(x) = x - \frac{1}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left(x + \Delta x - \frac{1}{x+\Delta x}\right) - \left(x - \frac{1}{x}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)(x+\Delta x)x - x - x^2(x+\Delta x) + (x+\Delta x)}{x(x+\Delta x)\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 2x^2(\Delta x) + x(\Delta x)^2 - x - x^3 - x^2(\Delta x) + x + \Delta x}{x(x+\Delta x)\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x^2(\Delta x) + x(\Delta x)^2 - x^2(\Delta x) + \Delta x}{x(x+\Delta x)\Delta x}$$

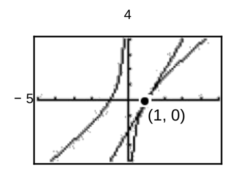
$$= \lim_{\Delta x \to 0} \frac{2x^2 + x(\Delta x) - x^2 + 1}{(x+\Delta x)x}$$

$$= \frac{x^2+1}{x^2} = 1 + \frac{1}{x^2}$$

At $(1, 0)$, the slope of the tangent line is $m = f'(1) = 2$. The equation of the tangent line is

$$y - 0 = 2(x - 1)$$
$$y = 2x - 2.$$

(b)



$\frac{dy}{dx} = 2$ at $(1, 0)$ confirms.

37. Using the limit definition of a derivative, $f'(x) = -\frac{1}{2}x$.

Because the slope of the given line is −1, you have

$$-\frac{1}{2}x = -1$$
$$x = 2.$$

At the point $(2, -1)$, the tangent line is parallel to $x + y = 0$. The equation of this line is

$$y - (-1) = -1(x - 2)$$
$$y = -x + 1.$$

38. Using the limit definition of derivative, $f'(x) = 4x$.

Because the slope of the given line is –4, you have

$$4x = -4$$
$$x = -1.$$

At the point $(-1, 2)$ the tangent line is parallel to $4x + y + 3 = 0$. The equation of this line is

$$y - 2 = -4(x + 1)$$
$$y = -4x - 2.$$

39. From Exercise 31 we know that $f'(x) = 3x^2$.

Because the slope of the given line is 3, you have

$$3x^2 = 3$$
$$x = \pm 1.$$

Therefore, at the points $(1, 1)$ and $(-1, -1)$ the tangent lines are parallel to $3x - y + 1 = 0$.

These lines have equations

$$y - 1 = 3(x - 1) \quad \text{and} \quad y + 1 = 3(x + 1)$$
$$y = 3x - 2 \qquad\qquad y = 3x + 2.$$

40. Using the limit definition of derivative, $f'(x) = 3x^2$.

Because the slope of the given line is 3, you have

$$3x^2 = 3$$
$$x^2 = 1 \Rightarrow x = \pm 1.$$

Therefore, at the points $(1, 3)$ and $(-1, 1)$ the tangent lines are parallel to $3x - y - 4 = 0$. These lines have equations

$$y - 3 = 3(x - 1) \quad \text{and} \quad y - 1 = 3(x + 1)$$
$$y = 3x \qquad\qquad y = 3x + 4.$$

Using the limit definition of derivative,

$$f'(x) = \frac{-1}{2x\sqrt{x}}.$$

Because the slope of the given line is $-\frac{1}{2}$, you have

$$\frac{-1}{2x\sqrt{x}} = -\frac{1}{2}$$

$$x = 1.$$

Therefore, at the point $(1, 1)$ the tangent line is parallel to $x + 2y - 6 = 0$. The equation of this line is

$$y - 1 = -\frac{1}{2}(x - 1)$$

$$y = -\frac{1}{2}x + \frac{3}{2}.$$

Using the limit definition of derivative,

$$f'(x) = \frac{-1}{2(x-1)^{3/2}}.$$

Because the slope of the given line is $-\frac{1}{2}$, you have

$$\frac{-1}{2(x-1)^{3/2}} = -\frac{1}{2}$$

$$1 = (x - 1)^{3/2}$$

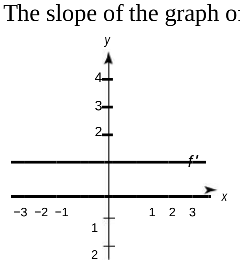
$$1 = x - 1 \Rightarrow x = 2.$$

At the point $(2, 1)$, the tangent line is parallel to $x + 2y + 7 = 0$. The equation of the tangent line is

$$y - 1 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 2.$$

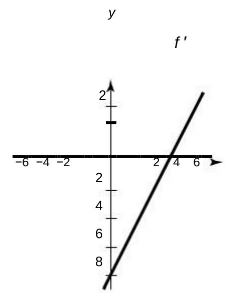
The slope of the graph of f is 1 for all x-values.



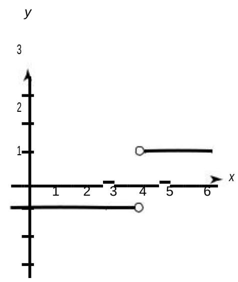
The slope of the graph of f is 0 for all x-values.

y

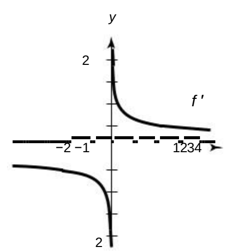
The slope of the graph of f is negative for $x < 4$, positive for $x > 4$, and 0 at $x = 4$.



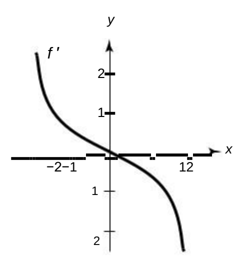
The slope of the graph of f is –1 for $x < 4$, 1 for $x > 4$, and undefined at $x = 4$.

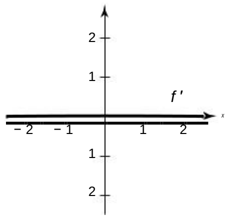


The slope of the graph of f is negative for $x < 0$ and positive for $x > 0$. The slope is undefined at $x = 0$.



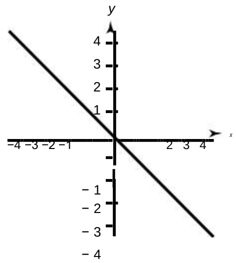
48. The slope is positive for $-2 < x < 0$ and negative for $0 < x < 2$. The slope is undefined at $x = \pm 2$, and 0 at $x = 0$.




f ′
−2
−1
1
2

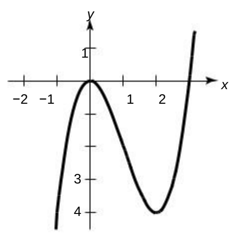
Answers will vary.

Sample answer: $y = -x$



The derivative of $y = -x$ is $y' = -1$. So, the derivative is always negative.

50. Answers will vary. *Sample answer*: $y = x^3 - 3x^2$



Note that $y' = 3x^2 - 6x = 3x(x - 2)$.

So, $y' = 0$ at $x = 0$ and $x = 2$.

51. No. For example, the domain of $f(x) = \sqrt{x}$ is $x \geq 0$, whereas the domain of $f'(x) = \frac{1}{2\sqrt{x}}$ is $x > 0$.

No. For example, $f(x) = x^3$ is symmetric with respect to the origin, but its derivative, $f'(x) = 3x^2$, is symmetric with respect to the y-axis.

$g(4) = 5$ because the tangent line passes through $(4, 5)$.

$$g'(4) = \frac{5 - 0}{4 - 7} = -\frac{5}{3}$$

$h(-1) = 4$ because the tangent line passes through $(-1, 4)$.

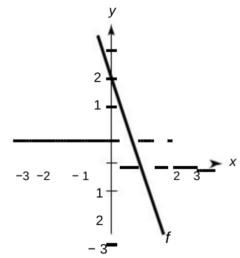
$$h'(-1) = \frac{6 - 4}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

$f(x) = 5 - 3x$ and $c = 1$

$f(x) = x^3$ and $c = -2$

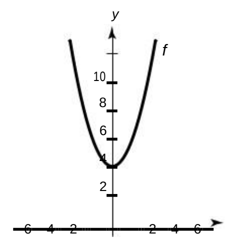
59. $f(0) = 2$ and $f'(x) = -3, -\infty < x < \infty$

$f(x) = -3x + 2$



60. $f(0) = 4, f'(0) = 0; f'(x) < 0$ for $x < 0, f'(x) > 0$ for $x > 0$

Answers will vary: *Sample answer:* $f(x) = x^2 + 4$



Let (x_0, y_0) be a point of tangency on the graph of f. By the limit definition for the derivative, $f'(x) = 4 - 2x$. The slope of the line through $(2, 5)$ and (x_0, y_0) equals the derivative of f at x_0:

$$\frac{5 - y_0}{2 - x_0} = 4 - 2x_0$$

$$5 - y_0 = (2 - x_0)(4 - 2x_0)$$

$$5 - (4x_0 - x_0^2) = 8 - 8x_0 + 2x_0^2$$

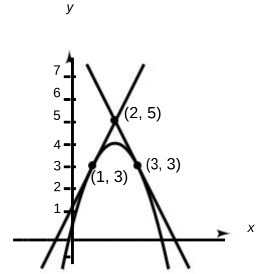
$$0 = x_0^2 - 4x_0 + 3$$

$$0 = (x_0 - 1)(x_0 - 3) \Rightarrow x_0 = 1, 3$$

Therefore, the points of tangency are $(1, 3)$ and $(3, 3)$, and the corresponding slopes are 2 and –2. The equations of the tangent lines are:

$y - 5 = 2(x - 2)$ $\qquad$ $y - 5 = -2(x - 2)$

$y = 2x + 1$ $\qquad$ $y = -2x + 9$



−2 1 2 3 6

$f(x) = -x^2$ and $c = 6$

$f(x) = 2\sqrt{x}$ and $c = 9$

Let (x_0, y_0) be a point of tangency on the graph of f.

By the limit definition for the derivative, $f'(x) = 2x$.

The slope of the line through $(1, -3)$ and (x_0, y_0) equals the derivative of f at x_0:

$$\frac{-3 - y_0}{1 - x_0} = 2x_0$$

$$-3 - y_0 = (1 - x_0)2x_0$$

$$-3 - x_0^2 = 2x_0 - 2x_0^2$$

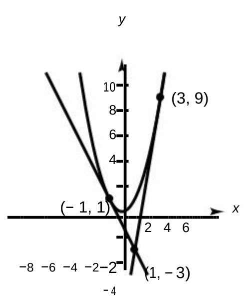
$$x_0^2 - 2x_0 - 3 = 0$$

$$(x_0 - 3)(x_0 + 1) = 0 \Rightarrow x_0 = 3, -1$$

Therefore, the points of tangency are $(3, 9)$ and $(-1, 1)$, and the corresponding slopes are 6 and –2. The equations of the tangent lines are:

$y + 3 = 6(x - 1)$ $\quad$ $y + 3 = -2(x - 1)$

$y = 6x - 9$ $\quad$ $y = -2x - 1$



63. (a) $f(x) = x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

At $x = -1$, $f'(-1) = -2$ and the tangent line is

$y - 1 = -2(x + 1)$ or $y = -2x - 1$.

At $x = 0$, $f'(0) = 0$ and the tangent line is $y = 0$.

At $x = 1$, $f'(1) = 2$ and the tangent line is $y = 2x - 1$.

(b) $$g'(x) = \lim_{\Delta x \to 0} \frac{g(x + \Delta x) - g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(3x^2 + 3x(\Delta x) + (\Delta x)^2\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x(\Delta x) + (\Delta x)^2\right) = 3x^2$$

At $x = -1$, $g'(-1) = 3$ and the tangent line is

$y + 1 = 3(x + 1)$ or $y = 3x + 2$.

At $x = 0$, $g'(0) = 0$ and the tangent line is $y = 0$.

At $x = 1$, $g'(1) = 3$ and the tangent line is

$y - 1 = 3(x - 1)$ or $y = 3x - 2$.

For this function, the slopes of the tangent lines are sometimes the same.

(a) $g'(0) = -3$

$g'(3) = 0$

(c) Because $g'(1) = -\frac{8}{3}$, g is decreasing (falling) at $x = 1$.

Because $g'(-4) = \frac{7}{3}$, g is increasing (rising) at $x = -4$.

Because $g'(4)$ and $g'(6)$ are both positive, $g(6)$ is greater than $g(4)$, and $g(6) - g(4) > 0$.

No, it is not possible. All you can say is that g is decreasing (falling) at $x = 2$.

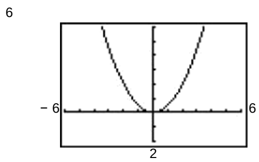
= 2 and the tangent line is

− 3 3

3

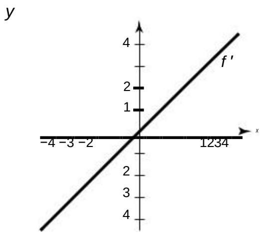
For this function, the slopes of the tangent lines are always distinct for different values of x.

$f(x) = \frac{1}{2}x^2$



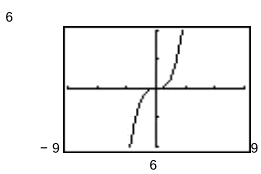
$f'(0) = 0, f'(1/2) = 1/2, f'(1) = 1, f'(2) = 2$

By symmetry: $f'(-1/2) = -1/2, f'(-1) = -1, f'(-2) = -2$



(d) $f'(x) = \lim_{x \to 0} \frac{f(x + x) - f(x)}{x} = \lim_{x \to 0} \frac{\frac{1}{2}(x + x)^2 - \frac{1}{2}x^2}{x} = \lim_{x \to 0} \frac{\frac{1}{2}\left(x^2 + 2x(x) + (x)^2\right) - \frac{1}{2}x^2}{x} = \lim_{x \to 0}\left(x + \frac{x}{2}\right) = x$

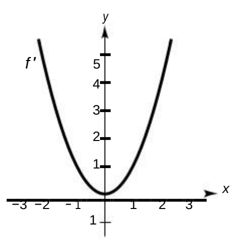
$f(x) = \frac{1}{3}x^3$



$f'(0) = 0, f'(1/2) = 1/4, f'(1) = 1, f'(2) = 4, f'(3) = 9$

(b) By symmetry: $f'(-1/2) = 1/4, f'(-1) = 1, f'(-2) = 4, f'(-3) = 9$

(c)



(d) $f'(x) = \lim_{x \to 0} \frac{f(x + x) - f(x)}{x}$

$= \lim_{x \to 0} \frac{\frac{1}{3}(x + x)^3 - \frac{1}{3}x^3}{x}$

$= \lim_{x \to 0} \frac{\frac{1}{3}\left(x^3 + 3x^2(x) + 3x(x)^2 + (x)^3\right) - \frac{1}{3}x^3}{x}$

$= \lim_{x \to 0}\left[x^2 + x(x) + \frac{1}{3}(x)^2\right] = x^2$

67. $f(2) = 2(4-2) = 4$, $f(2.1) = 2.1(4-2.1) = 3.99$

$$f'(2) \approx \frac{3.99-4}{2.1-2} = -0.1 \quad [\text{Exact: } f'(2) = 0]$$

$$f(2) = \frac{1}{4}(2^3) = 2, \quad f(2.1) = 2.31525$$

$$f'(2) \approx \frac{2.31525-2}{2.1-2} = 3.1525 \quad [\text{Exact: } f'(2) = 3]$$

69. $f(x) = x^3 + 2x^2 + 1$, $c = -2$

$$f'(-2) = \lim_{x \to -2} \frac{f(x) - f(-2)}{x+2}$$

$$= \lim_{x \to -2} \frac{(x^3 + 2x^2 + 1) - 1}{x+2}$$

$$= \lim_{x \to -2} \frac{x^2(x+2)}{x+2} = \lim_{x \to -2} x^2 = 4$$

$g(x) = x^2 - x$, $c = 1$

$$g'(1) = \lim_{x \to 1} \frac{g(x) - g(1)}{x-1}$$

$$= \lim_{x \to 1} \frac{x^2 - x - 0}{x-1}$$

$$= \lim_{x \to 1} \frac{x(x-1)}{x-1}$$

$$= \lim_{x \to 1} x = 1$$

71. $g(x) = \sqrt{|x|}$, $c = 0$

$$g'(0) = \lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x \to 0} \frac{\sqrt{|x|}}{x}.$$ Does not exist.

As $x \to 0^-$, $\frac{\sqrt{|x|}}{x} = \frac{-1}{\sqrt{|x|}} \to -\infty$.

As $x \to 0^+$, $\frac{\sqrt{|x|}}{x} = \frac{1}{\sqrt{x}} \to \infty$.

Therefore $g(x)$ is not differentiable at $x = 0$.

$f(x) = (x-6)^{2/3}$, $c = 6$

$$f'(6) = \lim_{x \to 6} \frac{f(x) - f(6)}{x - 6}$$

$$= \lim_{x \to 6} \frac{(x-6)^{2/3} - 0}{x-6} = \lim_{x \to 6} \frac{1}{(x-6)^{1/3}}.$$

Does not exist.

Therefore $f(x)$ is not differentiable at $x = 6$.

$g(x) = (x+3)^{1/3}$, $c = -3$

$$g'(-3) = \lim_{x \to -3} \frac{g(x) - g(-3)}{x - (-3)}$$

$$= \lim_{x \to -3} \frac{(x+3)^{1/3} - 0}{x+3} = \lim_{x \to -3} \frac{1}{(x+3)^{2/3}}.$$

Does not exist.

Therefore $g(x)$ is not differentiable at $x = -3$.

$h(x) = |x+7|$, $c = -7$

$$h'(-7) = \lim_{x \to -7} \frac{h(x) - h(-7)}{x - (-7)}$$

$$= \lim_{x \to -7} \frac{|x+7| - 0}{x+7} = \lim_{x \to -7} \frac{|x+7|}{x+7}$$

Does not exist.

Therefore $h(x)$ is not differentiable at $x = -7$.

$f(x) = |x-6|$, $c = 6$

$$f'(6) = \lim_{x \to 6} \frac{f(x) - f(6)}{x - 6}$$

$$= \lim_{x \to 6} \frac{|x-6| - 0}{x-6} = \lim_{x \to 6} \frac{|x-6|}{x-6}$$

Does not exist.

Therefore $f(x)$ is not differentiable at $x = 6$.

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$f(x) = \frac{3}{x}, c = 4$

$$f'(4) = \lim_{x\to4} \frac{f(x) - f(4)}{x - 4}$$

$$= \lim_{x\to4} \frac{\frac{3}{x} - \frac{3}{4}}{x - 4}$$

$$\lim_{x\to4} \frac{12 - 3x}{4x(x - 4)}$$

$$\lim_{x\to4} \frac{-3(x - 4)}{4x(x - 4)}$$

$$= \lim_{x\to4} -\frac{3}{4x} = -\frac{3}{16}$$

$f(x)$ is differentiable everywhere except at $x = -4$. (Sharp turn in the graph)

$f(x)$ is differentiable everywhere except at $x = \pm2$. (Discontinuities)

$f(x)$ is differentiable on the interval $(-1, \infty)$. (At $= -1$ the tangent line is vertical.)

$f(x)$ is differentiable everywhere except at $x = 0$. (Discontinuity)

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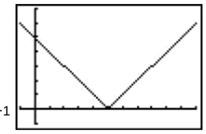
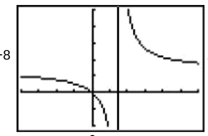
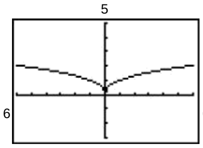
81. $f(x) = |x - 5|$ is differentiable everywhere except at $x = 5$. There is a sharp corner at $x = 5$.



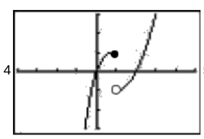
82. $f(x) = \frac{4x}{x-3}$ is differentiable everywhere except at $x = 3$. f is not defined at $x = 3$. (Vertical asymptote)



$f(x) = x^{2/5}$ is differentiable for all $x \neq 0$. There is a sharp corner at $x = 0$.



84. f is differentiable for all $x \neq 1$. f is not continuous at $x = 1$.



$f(x) = |x - 1|$

The derivative from the left is

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^-} \frac{|x-1| - 0}{x - 1} = -1.$$

The derivative from the right is

$$\lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{|x-1| - 0}{x - 1} = 1.$$

The one-sided limits are not equal. Therefore, f is not differentiable at $x = 1$.

86. $f(x) = \sqrt{1 - x^2}$

The derivative from the left does not exist because

87. $f(x) = \begin{cases} (x-1)^3, & x \leq 1 \\ (x-1)^2, & x > 1 \end{cases}$

The derivative from the left is

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^-} \frac{(x-1)^3 - 0}{x - 1} = \lim_{x\to 1^-} (x-1)^2 = 0.$$

The derivative from the right is

$$\lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{(x-1)^2 - 0}{x - 1} = \lim_{x\to 1^+} (x-1) = 0.$$

The one-sided limits are equal. Therefore, f is differentiable at $x = 1$. $(f'(1) = 0)$

$f(x) = (1 - x)^{2/3}$

The derivative from the left does not exist.

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^-} \frac{(1-x)^{2/3} - 0}{x - 1} = \lim_{x\to 1^-} \frac{-1}{(1-x)^{1/3}} = -\infty$$

Similarly, the derivative from the right does not exist because the limit is ∞.

Therefore, f is not differentiable at $x = 1$.

89. Note that f is continuous at $x = 2$.

$f(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ 4x - 3, & x > 2 \end{cases}$

The derivative from the left is

$$\lim_{x\to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^-} \frac{(x^2 + 1) - 5}{x - 2} = \lim_{x\to 2^-} (x + 2) = 4.$$

The derivative from the right is

$$\lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^+} \frac{(4x - 3) - 5}{x - 2} = \lim_{x\to 2^+} 4 = 4.$$

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$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^-} \frac{\sqrt{1 - x^2} - 0}{x - 1}$$

$$= \lim_{x\to 1^-} \frac{\sqrt{1 - x^2}}{x - 1} \cdot \frac{\sqrt{1 - x^2}}{\sqrt{1 - x^2}}$$

$$= \lim_{x\to 1^-} -\frac{1 + x}{\sqrt{1 - x^2}} = -\infty.$$

(Vertical tangent)

The limit from the right does not exist since f is undefined for $x > 1$. Therefore, f is not differentiable at $x = 1$.

The one-sided limits are equal. Therefore, f is differentiable at $x = 2$. $(f'(2) = 4)$

90. $f(x) = \begin{cases} \frac{1}{2}x + 2, & x < 2 \\ \sqrt{2x}, & x \geq 2 \end{cases}$

is not differentiable at $x = 2$ because it is not continuous at $x = 2$.

$$\lim_{x\to 2^-} f(x) = \frac{1}{2}(2) + 2 = 3$$

$$\lim_{x\to 2^+} f(x) = \sqrt{2(2)} = 2$$

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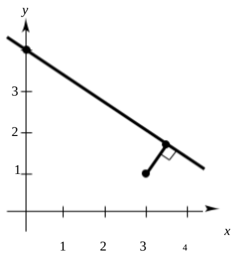
91. (a) The distance from $(3, 1)$ to the line $mx - y + 4 = 0$ is $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}} = \frac{|m(3) - 1(1) + 4|}{\sqrt{m^2+1}} = \frac{|3m+3|}{\sqrt{m^2+1}}$.



(b)

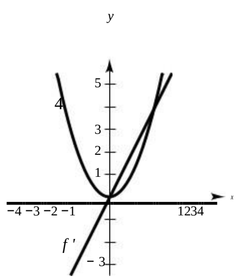
5

−4 4

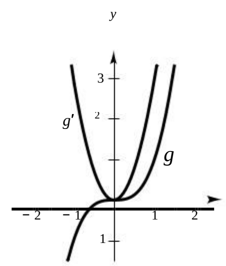
1

The function d is not differentiable at $m = -1$. This corresponds to the line $y = -x + 4$, which passes through the point $(3, 1)$.

(a) $f(x) = x^2$ and $f'(x) = 2x$



(b) $g(x) = x^3$ and $g'(x) = 3x^2$



The derivative is a polynomial of degree 1 less than the original function. If $h(x) = x^n$, then $h'(x) = nx^{n-1}$.

If $f(x) = x^4$, then

$$f'(x) = \lim_{x \to 0} \frac{f(x + x) - f(x)}{x}$$

$$= \lim_{x \to 0} \frac{(x + x)^4 - x^4}{x}$$

$$= \lim_{x \to 0} \frac{x^4 + 4x^3(x) + 6x^2(x)^2 + 4x(x)^3 + (x)^4 - x^4}{x}$$

$$= \lim_{x \to 0} \frac{x\left(4x^3 + 6x^2(x) + 4x(x)^2 + (x)^3\right)}{x} = \lim_{x \to 0}\left(4x^3 + 6x^2(x) + 4x(x)^2 + (x)^3\right) = 4x^3.$$

So, if $f(x) = x^4$, then $f'(x) = 4x^3$ which is consistent with the conjecture. However, this is not a proof because you must verify the conjecture for all integer values of n, $n \geq 2$.

93. False. The slope is $\lim_{x \to 0} \frac{f(2 + x) - f(2)}{x}$.

False. $y = |x - 2|$ is continuous at $x = 2$, but is not differentiable at $x = 2$. (Sharp turn in the graph)

False. If the derivative from the left of a point does not equal the derivative from the right of a point, then the derivative does not exist at that point. For example, if $f(x) = |x|$, then the derivative from the left at $x = 0$ is –1 and the derivative from the right at $x = 0$ is 1. At

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$x = 0$, the derivative does not

exist. True. See Theorem 2.1.

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97. $f(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Using the Squeeze Theorem, you have $-|x| \leq x \sin(1/x) \leq |x|$, $x \neq 0$. So, $\lim_{x \to 0} x \sin(1/x) = 0 = f(0)$ and f is continuous at $x = 0$. Using the alternative form of the derivative, you have

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{x \sin(1/x) - 0}{x - 0} = \lim_{x \to 0}\left(\sin \frac{1}{x}\right).$$

Because this limit does not exist ($\sin(1/x)$ oscillates between –1 and 1), the function is not differentiable at $x = 0$.

$g(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Using the Squeeze Theorem again, you have $-x^2 \leq x^2 \sin(1/x) \leq x^2$, $x \neq 0$. So, $\lim_{x \to 0} x^2 \sin(1/x) = 0 = g(0)$ and g is continuous at $x = 0$. Using the alternative form of the derivative again, you have

$$\lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x \to 0} \frac{x^2 \sin(1/x) - 0}{x - 0} = \lim_{x \to 0} x \sin\frac{1}{x} = 0.$$

Therefore, g is differentiable at $x = 0$, $g'(0) = 0$.

98.

3

−3 3

1

As you zoom in, the graph of $y_1 = x^2 + 1$ appears to be locally the graph of a horizontal line, whereas the graph $y_2 = |x| + 1$ always has a sharp corner at $(0, 1)$. y_2 is not differentiable at $(0, 1)$.

Section 2.2 Basic Differentiation Rules and Rates of Change

The derivative of a constant function is 0.

$$\frac{d}{dx}[c] = 0$$

To find the derivative of $f(x) = cx^n$, multiply n by c, and reduce the power of x by 1.

$$f'(x) = ncx^{n-1}$$

The derivative of the sine function, $f(x) = \sin x$, is the cosine function, $f'(x) = \cos x$.

The derivative of the cosine function, $g(x) = \cos x$, is the negative of the sine function, $g'(x) = -\sin x$.

The average velocity of an object is the change in distance divided by the change in time. The velocity is the instantaneous change in velocity. It is the derivative of the position function.

(a) $y = x^{1/2}$

$y' = \frac{1}{2} x^{-1/2}$

$y'(1) = \frac{1}{2}$

$y = x^3$

$y' = 3x^2$

$y'(1) = 3$

(a) $y = x^{-1/2}$

$y' = -\frac{1}{2} x^{-3/2}$

$y = x^{-1}$

$y' = -x^{-2}$

$y'(1) = -1$

$y = 12$
$y' = 0$

$f(x) = -9$
$f'(x) = 0$

$y = x^7$
$y' = 7x^6$

$y = x^{12}$
$y' = 12x^{11}$

$y = \frac{1}{x^5} = x^{-5}$

$y' = -5x^{-6} = -\frac{5}{x^6}$

$y = \frac{3}{x^7} = 3x^{-7}$

$y' = 3(-7x^{-8}) = -\frac{21}{x^8}$

13. $f(x) = \sqrt[9]{x} = x^{1/9}$
$f'(x) = \frac{1}{9}x^{-8/9} = \frac{1}{9x^{8/9}}$

14. $y = \sqrt[4]{x} = x^{1/4}$
$y' = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}}$

15. $f(x) = x + 11$
$f'(x) = 1$

$g(x) = 6x + 3$
$g'(x) = 6$

$f(t) = -3t^2 + 2t - 6$
$f'(t) = -6t + 2$

$y = t^2 - 3t + 1$
$y' = 2t - 3$

$g(x) = x^2 + 4x^3$
$g'(x) = 2x + 12x^2$

$y = 4x - 3x^3$
$y' = 4 - 9x^2$

$s(t) = t^3 + 5t^2 - 3t + 8$

$s'(t) = 3t^2 + 10t - 3$

$y = 2x^3 + 6x^2 - 1$
$y' = 6x^2 + 12x$

$y = \frac{\pi}{2} \sin\theta$

$y' = \frac{\pi}{2} \cos\theta$

$g(t) = \pi \cos t$
$g'(t) = -\pi \sin t$

$y = x^2 - \frac{1}{2}\cos x$

$y' = 2x + \frac{1}{2}\sin x$

$y = 7x^4 + 2\sin x$

$y' = 7(4x^3) + 2\cos x = 28x^3 + 2\cos x$

Function	*Rewrite*	*Differentiate*	*Simplify*
27. $y = \frac{2}{7x^4}$	$y = \frac{2}{7}x^{-4}$	$y' = -\frac{8}{7}x^{-5}$	$y' = -\frac{8}{7x^5}$
28. $y = \frac{8}{5x^{-5}}$	$y = \frac{8}{5}x^5$	$y' = \frac{8}{5}(5x^4)$	$y' = 8x^4$
29. $y = \frac{6}{(5x)^3}$	$y = \frac{6}{125}x^{-3}$	$y' = -\frac{18}{125}x^{-4}$	$y' = -\frac{18}{125x^4}$

125

30. $y = \dfrac{3}{(2x)^{-2}}$ $\quad y = 12x^2 \quad y' = 12(2x) \quad y' = 24x$

31. $f(x) = \frac{8}{x^2} = 8x^{-2}, (2, 2)$

$f'(x) = -16x^{-3} = -\frac{16}{x^3}$

$f'(2) = -2$

32. $f(t) = 2 - \frac{4}{t} = 2 - 4t^{-1}, (4, 1)$

$f'(t) = 4t^{-2} = \frac{4}{t^2}$

$f'(4) = \frac{1}{4}$

33. $f(x) = -\frac{1}{2} + \frac{7}{5}x^3, \left(0, -\frac{1}{2}\right)$

$f'(x) = \frac{21}{5}x^2$

$f'(0) = 0$

$y = 2x^4 - 3, (1, -1)$

$y' = 8x^3$

$y'(1) = 8$

$y = (4x + 1)^2, (0, 1)$

$= 16x^2 + 8x + 1$

$y' = 32x + 8$

$y'(0) = 32(0) + 8 = 8$

$f(x) = 2(x - 4)^2, (2, 8)$

$= 2x^2 - 16x + 32$

$f'(x) = 4x - 16$

$f'(2) = 8 - 16 = -8$

37. $f(\theta) = 4\sin\theta - \theta, (0, 0)$

$f'(\theta) = 4\cos\theta - 1$

$f'(0) = 4(1) - 1 = 3$

$g(t) = -2\cos t + 5, (\pi, 7)$

$g'(t) = 2\sin t$

$g'(\pi) = 0$

$g(t) = t^2 - \frac{4}{t^3} = t^2 - 4t^{-3}$

$g'(t) = 2t + 12t^{-4} = 2t + \frac{12}{t^4}$

42. $f(x) = 8x + \frac{3}{x^2} = 8x + 3x^{-2}$

$f'(x) = 8 - 6x^{-3} = 8 - \frac{6}{x^3}$

$f(x) = \frac{x^3 - 3x^2 + 4}{x^2} = x - 3 + 4x^{-2}$

$f'(x) = 1 - \frac{8}{x^3} = \frac{x^3 - 8}{x^3}$

$h(x) = \frac{4x^3 + 2x + 5}{x} = 4x^2 + 2 + 5x^{-1}$

$h'(x) = 8x - 5x^{-2} = 8x - \frac{5}{x^2}$

45. $g(t) = \frac{3t^2 + 4t - 8}{t^{3/2}} = 3t^{1/2} + 4t^{-1/2} - 8t^{-3/2}$

$g'(t) = \frac{3}{2}t^{-1/2} - 2t^{-3/2} + 12t^{-5/2}$

$= \frac{3t^2 - 4t + 24}{2t^{5/2}}$

$h(s) = \frac{s^5 + 2s + 6}{s^{1/3}} = s^{14/3} + 2s^{2/3} + 6s^{-1/3}$

$h'(s) = \frac{14}{3}s^{11/3} + \frac{4}{3}s^{-1/3} - 2s^{-4/3}$

$= \frac{14s^5 + 4s - 6}{3s^{4/3}}$

$y = x(x^2 + 1) = x^3 + x$

$y' = 3x^2 + 1$

$y = x^2(2x^2 - 3x) = 2x^4 - 3x^3$

$y' = 8x^3 - 9x^2 = x^2(8x - 9)$

49. $f(x) = \sqrt{x} - 6\sqrt[3]{x} = x^{1/2} - 6x^{1/3}$

$f'(x) = \frac{1}{2}x^{-1/2} - 2x^{-2/3} = \frac{1}{2\sqrt{x}} - \frac{2}{x^{2/3}}$

$$f(x) = x^2 + 5 - 3x^{-2}$$

$$f'(x) = 2x + 6x^{-3} = 2x + \frac{6}{x^3}$$

$$f(t) = t^{2/3} - t^{1/3} + 4$$

$$f'(t) = \frac{2}{3}t^{-1/3} - \frac{1}{3}t^{-2/3} = \frac{2}{3t^{1/3}} - \frac{1}{3t^{2/3}}$$

$$f(x) = x^3 - 2x + 3x^{-3}$$

$$f'(x) = 3x^2 - 2 - 9x^{-4} = 3x^2 - 2 - \frac{9}{x^4}$$

51. $f(x) = 6\sqrt{x} + 5\cos x = 6x^{1/2} + 5\cos x$

$$f'(x) = 3x^{-1/2} - 5\sin x = \frac{3}{\sqrt{x}} - 5\sin x$$

52. $f(x) = \frac{2}{\sqrt[3]{x}} + 3\cos x = 2x^{-1/3} + 3\cos x$

$f'(x) = -\frac{2}{3}x^{-4/3} - 3\sin x = -\frac{2}{3x^{4/3}} - 3\sin x$

53. $y = \frac{1}{(3x)^{-2}} - 5\cos x = (3x)^2 - 5\cos x = 9x^2 - 5\cos x$

$y' = 18x + 5\sin x$

54. $y = \frac{3}{(2x)^3} + 2\sin x = \frac{3}{8}x^{-3} + 2\sin x$

$y' = -\frac{9}{8}x^{-4} + 2\cos x$

$= -\frac{9}{8x^4} + 2\cos x$

(a) $f(x) = -2x^4 + 5x^2 - 3$

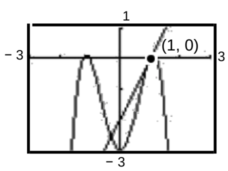
$f'(x) = -8x^3 + 10x$

At $(1, 0)$: $f'(1) = -8(1)^3 + 10(1) = 2$

Tangent line: $y - 0 = 2(x - 1)$

$= 2x - 2$

(b) and (c)



(a) $y = x^3 - 3x$
$y' = 3x^2 - 3$

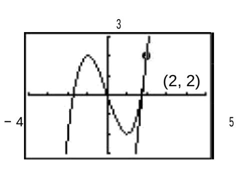
At $(2, 2)$: $y' = 3(2)^2 - 3 = 9$

Tangent line: $y - 2 = 9(x - 2)$

$y = 9x - 16$

$9x - y - 16 = 0$

(b) and (c)



57. (a) $f(x) = \frac{2}{\sqrt[4]{x^3}} = 2x^{-3/4}$

$f'(x) = -\frac{3}{2}x^{-7/4} = -\frac{3}{2x^{7/4}}$

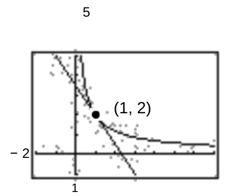
At $(1, 2)$: $f'(1) = -\frac{3}{2}$

Tangent line: $y - 2 = -\frac{3}{2}(x - 1)$

$y = -\frac{3}{2}x + \frac{7}{2}$

$3x + 2y - 7 = 0$

(b) and (c)



(a) $y = (x - 2)(x^2 + 3x) = x^3 + x^2 - 6x$
$y' = 3x^2 + 2x - 6$

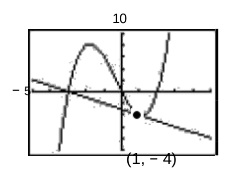
At $(1, -4)$: $y' = 3(1)^2 + 2(1) - 6 = -1$

Tangent line: $y - (-4) = -1(x - 1)$

$y = -x - 3$

$x + y + 3 = 0$

(b) and (c)



$y = x^4 - 2x^2 + 3$

$y' = 4x^3 - 4x$

$= 4x(x^2 - 1)$

$= 4x(x - 1)(x + 1)$

$y' = 0 \Rightarrow x = 0, \pm 1$

Horizontal tangents: $(0, 3), (1, 2), (-1, 2)$

$y = x^3 + x$

$y' = 3x^2 + 1 > 0$ for all x.

Therefore, there are no horizontal tangents.

$$y = \frac{1}{x^2} = x^{-2}$$

$$y' = -2x^{-3} = -\frac{2}{x^3}$$ cannot equal zero.

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$y = x^2 + 9$

$y' = 2x = 0 \Rightarrow x = 0$

At $x = 0, y = 1$.

Horizontal tangent: $(0, 9)$

$y = x + \sin x,\ 0 \le x < 2\pi$

$y' = 1 + \cos x = 0$

$\cos x = -1 \Rightarrow x = \pi$

At $x = \pi : y = \pi$

Horizontal tangent: (π, π)

$y = \sqrt{3}x + 2\cos x,\ 0 \le x < 2\pi$

$y' = \sqrt{3} - 2\sin x = 0$

$\sin x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

At $x = \frac{\pi}{3} : y = \frac{\sqrt{3}\pi + 3}{3}$

At $x = \frac{2\pi}{3} : y = \frac{2\sqrt{3}\pi - 3\sqrt{3}}{3}$

Horizontal tangents: $\left(\frac{\pi}{3}, \frac{\sqrt{3}\pi + 3}{3}\right), \left(\frac{2\pi}{3}, \frac{2\sqrt{3}\pi - 3\sqrt{3}}{3}\right)$

65. $f(x) = k - x^2,\ y = -6x + 1$

$f'(x) = -2x$ and slope of tangent line is $m = -6$.

$f'(x) = -6$

$-2x = -6$

$= 3$

$= -6(3) + 1 = -17$

$-17 = k - 3^2$

$-8 = k$

66. $f(x) = kx^2,\ y = -2x + 3$

$f'(x) = 2kx$ and slope of tangent line is $m = -2$.

$f'(x) = -2$

$2kx = -2$

$= -\frac{1}{k}$

$y = -2\left(-\frac{1}{k}\right) + 3 = \frac{2}{k} + 3$

$\frac{2}{k} + 3 = k\left(-\frac{1}{k}\right)^2$

$\frac{2}{k} + 3 = \frac{1}{k}$

$\frac{1}{k} = -3$

$k = -\frac{1}{3}$

67. $f(x) = \frac{k}{x},\ y = -\frac{3}{4}x + 3$

$f'(x) = -\frac{k}{x^2}$ and slope of tangent line is $m = -\frac{3}{4}$.

$f'(x) = -\frac{3}{4}$

$-\frac{k}{x^2} = -\frac{3}{4}$

$x^2 = \frac{4k}{3}$

$x = \sqrt{\frac{4k}{3}}$

$y = -\frac{3}{4}x + 3 = -\frac{3}{4}\sqrt{\frac{4k}{3}} + 3$

$-\frac{3}{4}\sqrt{\frac{4k}{3}} + 3 = k\sqrt{\frac{3}{4k}}$

$k = 3$

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$f(x) = k\sqrt{x},\ y = x + 4$

$f'(x) = \frac{k}{2\sqrt{x}}$ and slope of tangent line is $m = 1$.

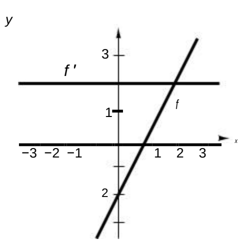
$$f'(x) = 1$$
$$\frac{k}{2\sqrt{x}} = 1$$
$$\frac{k}{2} = \sqrt{x}$$
$$\frac{k^2}{4} = x$$
$$y = x + 4 = \frac{k^2}{4} + 4$$
$$\frac{k^2}{4} + 4 = k \cdot \frac{k}{2}$$
$$\frac{k^2}{4} - \frac{k^2}{2} = -4$$
$$-\frac{k^2}{4} = -4$$
$$k^2 = 16$$
$$k = 4$$

$g(x) = f(x) + 6 \Rightarrow g'(x) = f'(x)$

$g(x) = 2f(x) \Rightarrow g'(x) = 2f'(x)$

$g(x) = -5f(x) \Rightarrow g'(x) = -5f'(x)$

$g(x) = 3f(x) - 1 \Rightarrow g'(x) = 3f'(x)$

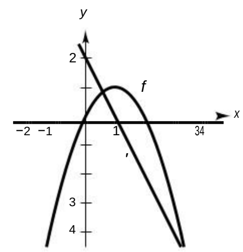


If f is linear then its derivative is a constant function.

$f(x) = ax + b$

$f'(x) = a$

74.

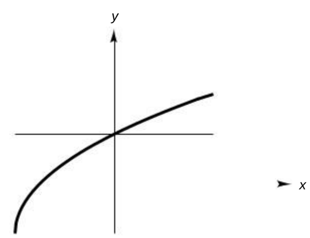


If f is quadratic, then its derivative is a linear function.

$f(x) = ax^2 + bx + c$

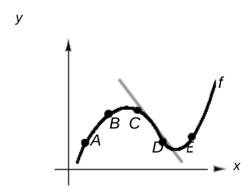
$f'(x) = 2ax + b$

The graph of a function f such that $f' > 0$ for all x and the rate of change of the function is decreasing (i.e., as x increases, f' decreases) would, in general, look like the graph below.



(a) The slope appears to be steepest between A and B.

The average rate of change between A and B is **greater** than the instantaneous rate of change at B.



Chapter 2 Differentiation

Let (x_1, y_1) and (x_2, y_2) be the points of tangency on $y = x^2$ and $y = -x^2 + 6x - 5$, respectively.

The derivatives of these functions are:

$y' = 2x \Rightarrow m = 2x_1$ and $y' = -2x + 6 \Rightarrow m = -2x_2 + 6$

$m = 2x_1 = -2x_2 + 6$

$x_1 = -x_2 + 3$

Because $y_1 = x_1^2$ and $y_2 = -x_2^2 + 6x_2 - 5$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-x_2^2 + 6x_2 - 5) - (x_1^2)}{x_2 - x_1} = -2x_2 + 6$$

$$\frac{(-x_2^2 + 6x_2 - 5) - (-x_2 + 3)^2}{x_2 - (-x_2 + 3)} = -2x_2 + 6$$

$$(-x_2^2 + 6x_2 - 5) - (x_2^2 - 6x_2 + 9) = (-2x_2 + 6)(2x_2 - 3)$$

$$-2x_2^2 + 12x_2 - 14 = -4x_2^2 + 18x_2 - 18$$

$$2x_2^2 - 6x_2 + 4 = 0$$

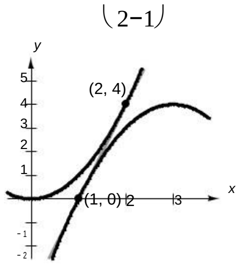
$$2(x_2 - 2)(x_2 - 1) = 0$$

$$x_2 = 1 \text{ or } 2$$

$x_2 = 1 \Rightarrow y_2 = 0, x_1 = 2$ and $y_1 = 4$

So, the tangent line through $(1, 0)$ and $(2, 4)$ is

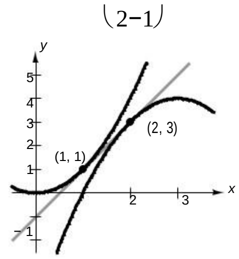
$$y - 0 = \left(\frac{4 - 0}{2 - 1}\right)(x - 1) \Rightarrow y = 4x - 4.$$



$x_2 = 2 \Rightarrow y_2 = 3, x_1 = 1$ and $y_1 = 1$

So, the tangent line through $(2, 3)$ and $(1, 1)$ is

$$y - 1 = \left(\frac{3 - 1}{2 - 1}\right)(x - 1) \Rightarrow y = 2x - 1.$$



78. m_1 is the slope of the line tangent to $y = x$. m_2 is the slope of the line tangent to $y = 1/x$. Because

$$y = x \Rightarrow y' = 1 \Rightarrow m_1 = 1 \text{ and } y = \frac{1}{x} \Rightarrow y' = -\frac{1}{x^2} \Rightarrow m_2 = -\frac{1}{x^2}.$$

The points of intersection of $y = x$ and $y = 1/x$ are

$$x = \frac{1}{x} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1.$$

At $x = \pm 1$, $m_2 = -1$. Because $m_2 = -1/m_1$, these tangent lines are perpendicular at the points of intersection.

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79. $f(x) = 3x + \sin x + 2$

$f'(x) = 3 + \cos x$

Because $|\cos x| \le 1$, $f'(x) \ne 0$ for all x and f does not have a horizontal tangent line.

80. $f(x) = x^5 + 3x^3 + 5x$

$f'(x) = 5x^4 + 9x^2 + 5$

Because $5x^4 + 9x^2 \ge 0$, $f'(x) \ge 5$. So, f does not have a tangent line with a slope of 3.

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81. $f(x) = \sqrt{x},\ (-4, 0)$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = \frac{0 - y}{-4 - x}$

$4 + x = 2\sqrt{x}\,y$

$4 + x = 2\sqrt{x}\sqrt{x}$

$4 + x = 2x$

$x = 4,\ y = 2$

The point $(4, 2)$ is on the graph of f.

Tangent line: $y - 2 = \frac{0 - 2}{-4 - 4}(x - 4)$

$4y - 8 = x - 4$

$0 = x - 4y + 4$

$f(x) = \frac{2}{x},\ (5, 0)$

$f'(x) = -\frac{2}{x^2}$

$-\frac{2}{x^2} = \frac{0 - y}{5 - x}$

$-10 + 2x = -x^2 y$

$-10 + 2x = -x^2\left(\frac{2}{x}\right)$

$-10 + 2x = -2x$

$4x = 10$

$x = \frac{5}{2},\ y = \frac{4}{5}$

The point $\left(\frac{5}{2}, \frac{4}{5}\right)$ is on the graph of f. The slope of the tangent line is $f'\left(\frac{5}{2}\right) = -\frac{8}{25}$.

Tangent line: $y - \frac{4}{5} = -\frac{8}{25}\left(x - \frac{5}{2}\right)$

$25y - 20 = -8x + 20$

$8x + 25y - 40 = 0$

(a) One possible secant is between $(3.9, 7.7019)$ and $(4, 8)$:

$$y - 8 = \frac{8 - 7.7019}{4 - 3.9}(x - 4)$$

$y - 8 = 2.981(x - 4)$

$y = S(x) = 2.981x - 3.924$

20

(4, 8)

−2 12

2

(b) $f'(x) = \frac{3}{2}x^{1/2} \Rightarrow f'(4) = \frac{3}{2}(2) = 3$

$T(x) = 3(x - 4) + 8 = 3x - 4$

The slope (and equation) of the secant line approaches that of the tangent line at $(4, 8)$ as you choose points closer and closer to $(4, 8)$.

(c) As you move further away from $(4, 8)$, the accuracy of the approximation T gets worse.

20

f T

−2 12

−2

(d)

Δx	–3	–2	–1	–0.5	–0.1	0	0.1	0.5	1	2	3
$f(4 + \Delta x)$	1	2.828	5.196	6.548	7.702	8	8.302	9.546	11.180	14.697	18.520
$T(4 + \Delta x)$											

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–1 2
5
6.5
7.7
8
8.3
9.5
11
14
17

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(a) Nearby point: (1.0073138, 1.0221024)

Secant line: $y - 1 = \frac{1.0221024 - 1}{1.0073138 - 1}(x - 1)$

$y = 3.022(x - 1) + 1$

(Answers will vary.)

2 (1, 1) −3 3 −2

(b) $f'(x) = 3x^2$

$T(x) = 3(x - 1) + 1 = 3x - 2$

(c) The accuracy worsens as you move away from (1, 1).

2 (1, 1) −3 3 f T −2

(d)

Δx	–3	–2	–1	–0.5	–0.1	0	0.1	0.5	1	2	3
$f(x)$	–8	–1	0	0.125	0.729	1	1.331	3.375	8	27	64
$T(x)$	–8	–5	–2	–0.5	0.7	1	1.3	2.5	4	7	10

The accuracy decreases more rapidly than in Exercise 85 because $y = x^3$ is less "linear" than $y = x^{3/2}$.

85. False. Let $f(x) = x$ and $g(x) = x + 1$. Then

$f'(x) = g'(x) = x$, but $f(x) \neq g(x)$.

True. If $y = x^{a+2} + bx$, then

$\frac{dy}{dx} = (a + 2)x^{(a+2)-1} + b = (a + 2)x^{a+1} + b.$

False. If $y = \pi^2$, then $dy/dx = 0$. (π^2 is a constant.)

88. True. If $f(x) = -g(x) + b$, then

$f'(x) = -g'(x) + 0 = -g'(x)$.

False. If $f(x) = 0$, then $f'(x) = 0$ by the Constant Rule.

90. False. If $f(x) = \frac{1}{x^n} = x^{-n}$, then

$f'(x) = -nx^{-n-1} = \frac{-n}{x^{n+1}}.$

92. $f(t) = t^2 - 7$, $[3, 3.1]$

$f'(t) = 2t$

Instantaneous rate of change:

At $(3, 2)$: $f'(3) = 6$

At $(3.1, 2.61)$: $f'(3.1) = 6.2$

Average rate of change:

$\frac{f(3.1) - f(3)}{3.1 - 3} = \frac{2.61 - 2}{0.1} = 6.1$

93. $f(x) = -\frac{1}{x}$, $[1, 2]$

$f'(x) = \frac{1}{x^2}$

Instantaneous rate of change:

$(1, -1) \Rightarrow f'(1) = 1$

$\left(2, -\frac{1}{2}\right) \Rightarrow f'(2) = \frac{1}{}$

91. $f(t) = 3t + 5, \quad [1, 2]$

$f'(t) = 3$. So, $f'(1) = f'(2) = 3$.

Instantaneous rate of change is the constant 3.

Average rate of change:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{11 - 8}{1} = 3$$

$\left(\quad 2\right) \qquad 4$

Average rate of change:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{-1/2 - (-1)}{2 - 1} = \frac{1}{2}$$

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94. $f(x) = \sin x, \left[0, \frac{\pi}{6}\right]$

$f'(x) = \cos x$

Instantaneous rate of change:

$(0, 0) \Rightarrow f'(0) = 1$

$\left(\frac{\pi}{6}, \frac{1}{2}\right) \Rightarrow f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \approx 0.866$

Average rate of change:

$$\frac{f(\pi/6) - f(0)}{(\pi/6) - 0} = \frac{(1/2) - 0}{(\pi/6) - 0} = \frac{3}{\pi} \approx 0.955$$

(a) $s(t) = -16t^2 + 1362$ $v(t) = -32t$

$\frac{s(2) - s(1)}{2 - 1} = 1298 - 1346 = -48$ ft/sec

$v(t) = s'(t) = -32t$

When $t = 1$: $v(1) = -32$ ft/sec

When $t = 2$: $v(2) = -64$ ft/sec

(d) $-16t^2 + 1362 = 0$

$t^2 = \frac{1362}{16} \Rightarrow t = \frac{\sqrt{1362}}{4} \approx 9.226$ sec

(e) $v\left(\frac{\sqrt{1362}}{4}\right) = -32\left(\frac{\sqrt{1362}}{4}\right)$

$-8\sqrt{1362} \approx -295.242$ ft/sec

$s(t) = -16t^2 - 22t + 220$ $v(t) = -32t - 22$

$v(3) = -118$ ft/sec

$s(t) = -16t^2 - 22t + 220$

112 (height after falling 108 ft)

$(t = 2)$

$v(2) = -32(2) - 22$

-86 ft/sec

$s(t) = -4.9t^2 + v_0t + s_0$

$-4.9t^2 + 120t$

$v(t) = -9.8t + 120$

98. (a) $s(t) = -4.9t^2 + v_0t + s_0 = -4.9t^2 + 214$

$s'(t) = v(t) = -9.8t$

Average velocity $= \frac{s(5) - s(2)}{5 - 2}$

$\frac{91.5 - 194.4}{3}$

-34.3 m/sec

$s'(2) = -9.8(2) = -19.6$ m/sec

$s'(5) = -9.8(5) = 49.0$ m/sec

$s(t) = -4.9t^2 + 214 = 0$

$4.9t^2 = 214$

$t^2 = \frac{214}{4.9}$

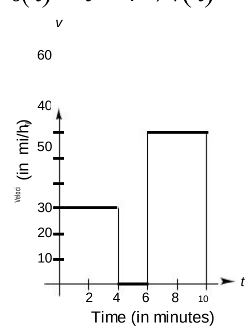
$t \approx 6.61$ sec

$v(6.61) = -9.8(6.61) \approx -64.8$ m/sec

From $(0, 0)$ to $(4, 2)$, $s(t) = \frac{1}{2}t \Rightarrow v(t) = \frac{1}{2}$ mi/min. $v(t) = \frac{1}{2}(60) = 30$ mi/h for $0 < t < 4$

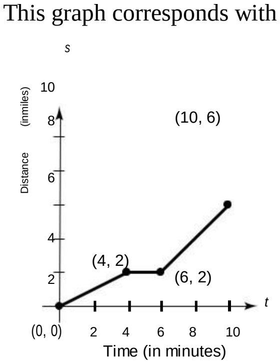
Similarly, $v(t) = 0$ for $4 < t < 6$. Finally, from $(6, 2)$ to $(10, 6)$,

$s(t) = t - 4 \Rightarrow v(t) = 1$ mi/min. $= 60$ mi/h.



(The velocity has been converted to miles per hour.)

This graph corresponds with Exercise 101.



$v(5) = -9.8(5) + 120 = 71$ m/sec

$v(10) = -9.8(10) + 120 = 22$ m/sec

$V = s^3, \frac{dV}{ds} = 3s^2$

When $s = 6$ cm, $\frac{dV}{ds} = 108$ cm^3 per cm change in s.

$A = s^2, \dfrac{dA}{ds} = 2s$

When $s = 6$ m, $\dfrac{dA}{ds} = 12 \text{ m}^2$ per m change in s.

(a) Using a graphing utility,

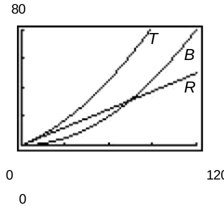
$R(v) = 0.417v - 0.02.$

(b) Using a graphing utility,

$B(v) = 0.0056v^2 + 0.001v + 0.04.$

(c) $T(v) = R(v) + B(v) = 0.0056v^2 + 0.418v + 0.02$

(d)



$\dfrac{dT}{dv} = 0.0112v + 0.418$

For $v = 40$, $T'(40) \approx 0.866$

For $v = 80$, $T'(80) \approx 1.314$

For $v = 100$, $T'(100) \approx 1.538$

(f) For increasing speeds, the total stopping distance increases.

$C = (\text{gallons of fuel used})(\text{cost per gallon})$

$= \left(\dfrac{15{,}000}{x}\right)(3.48) = \dfrac{52{,}200}{x}$

$\dfrac{dC}{dx} = -\dfrac{52{,}200}{x^2}$

x	10	15	20	25	30	35	40
C	5220	3480	2610	2088	1740	1491.4	1305
dC/dx	–522	–232	–130.5	–83.52	–58	–42.61	–32.63

The driver who gets 15 miles per gallon would benefit more. The rate of change at $x = 15$ is larger in absolute value than that at $x = 35$.

105. $s(t) = -\frac{1}{2}at^2 + c$ and $s'(t) = -at$

Average velocity: $\dfrac{s(t_0 + t) - s(t_0 - t)}{(t_0 + t) - (t_0 - t)}$

$= \dfrac{\left[-\frac{1}{2}a(t_0 + t)^2 + c\right] - \left[-\frac{1}{2}a(t_0 - t)^2 + c\right]}{2t}$

$= \dfrac{-\frac{1}{2}a\left(t_0^2 + 2t_0 t + t^2\right) + \frac{1}{2}a\left(t_0^2 - 2t_0 t + t^2\right)}{2t}$

$= \dfrac{-2at_0 t}{2t} = -at_0 = s'(t_0)$ instantaneous velocity at $t = t_0$

$C = \dfrac{1{,}008{,}000}{Q} + 6.3Q$

$\dfrac{dC}{dQ} = -\dfrac{1{,}008{,}000}{Q^2} + 6.3$

$C(351) - C(350) \approx 5083.095 - 5085 \approx -\$1.91$

When $Q = 350$, $\dfrac{dC}{dQ} \approx -\$1.93.$

$y = ax^2 + bx + c$

Because the parabola passes through $(0, 1)$ and $(1, 0)$, you have:

$(0, 1): 1 = a(0)^2 + b(0) + c \Rightarrow c = 1$

$(1, 0): 0 = a(1)^2 + b(1) + 1 \Rightarrow b = -a - 1$

So, $y = ax^2 + (-a - 1)x + 1$. From the tangent line $y = x - 1$, you know that the derivative is 1 at the point $(1, 0)$.

$y' = 2ax + (-a - 1)$

$1 = 2a(1) + (-a - 1)$

$1 = a - 1$

$a = 2$

$b = -a - 1 = -3$

Therefore, $y = 2x^2 - 3x + 1$.

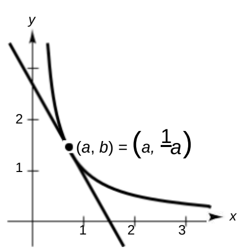
$y = \frac{1}{x}, x > 0$

$y' = -\frac{1}{x^2}$

At (a, b), the equation of the tangent line is $y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$ or $y = -\frac{x}{a^2} + \frac{2}{a}$.

The x-intercept is $(2a, 0)$. The y-intercept is $\left(0, \frac{2}{a}\right)$.

The area of the triangle is $A = \frac{1}{2}bh = \frac{1}{2}(2a)\left(\frac{2}{a}\right) = 2$.



$y = x^3 - 9x$

$y' = 3x^2 - 9$

Tangent lines through $(1, -9)$:

$y + 9 = (3x^2 - 9)(x - 1)$

$(x^3 - 9x) + 9 = 3x^3 - 3x^2 - 9x + 9$

$0 = 2x^3 - 3x^2 = x^2(2x - 3)$

$x = 0$ or $x = \frac{3}{2}$

The points of tangency are $(0, 0)$ and $\left(\frac{3}{2}, -\frac{81}{8}\right)$. At $(0, 0)$, the slope is $y'(0) = -9$. At $\left(\frac{3}{2}, -\frac{81}{8}\right)$, the slope is $y'\left(\frac{3}{2}\right) = -\frac{9}{4}$.

$y - 0 = -9(x - 0)$ and $y + \frac{81}{8} = -\frac{9}{4}\left(x - \frac{3}{2}\right)$

$y = -9x$ $\quad$ $y = -\frac{9}{4}x - \frac{27}{4}$

$9x + y = 0$ $\quad$ $9x + 4y + 27 = 0$

$y = x^2 \quad y' = 2x$

Tangent lines through $(0, a)$:

$$y - a = 2x(x - 0)$$
$$x^2 - a = 2x^2$$
$$-a = x^2$$
$$\pm\sqrt{-a} = x$$

The points of tangency are $\left(\pm\sqrt{-a}, -a\right)$. At $\left(\sqrt{-a}, -a\right)$, the slope is $y'\left(\sqrt{-a}\right) = 2\sqrt{-a}$.

At $\left(-\sqrt{-a}, -a\right)$, the slope is $y'\left(-\sqrt{-a}\right) = -2\sqrt{-a}$.

Tangent lines: $y + a = 2\sqrt{-a}\left(x - \sqrt{-a}\right)$ and $y + a = -2\sqrt{-a}\left(x + \sqrt{-a}\right)$

$$y = 2\sqrt{-a}x + a \qquad y = -2\sqrt{-a}x + a$$

Restriction: a must be negative.

(b) Tangent lines through $(a, 0)$:

$$y - 0 = 2x(x - a)$$
$$x^2 = 2x^2 - 2ax$$
$$0 = x^2 - 2ax = x(x - 2a)$$

The points of tangency are $(0, 0)$ and $\left(2a, 4a^2\right)$. At $(0, 0)$, the slope is $y'(0) = 0$. At $\left(2a, 4a^2\right)$, the slope is $y'(2a) = 4a$.

Tangent lines: $y - 0 = 0(x - 0)$ and $y - 4a^2 = 4a(x - 2a)$

$$y = 0 \qquad y = 4ax - 4a^2$$

Restriction: None, a can be any real number.

111. $f(x) = \begin{cases} ax^3, & x \le 2 \\ x^2 + b, & x > 2 \end{cases}$

f must be continuous at $x = 2$ to be differentiable at $x = 2$.

$$\left.\begin{array}{l} \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} ax^3 = 8a \\ \lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} \left(x^2 + b\right) = 4 + b \end{array}\right\} \begin{array}{l} 8a = 4 + b \\ 8a - 4 = b \end{array}$$

$$f'(x) = \begin{cases} 3ax^2, & x < 2 \\ 2x, & x > 2 \end{cases}$$

For f to be differentiable at $x = 2$, the left derivative must equal the right derivative.

$$3a(2)^2 = 2(2)$$
$$12a = 4$$
$$a = \frac{1}{3}$$
$$b = 8a - 4 = -\frac{4}{3}$$

112. $f(x) = \begin{cases} \cos x, & x < 0 \\ ax + b, & x \geq 0 \end{cases}$

$f(0) = b = \cos 0 = 1 \Rightarrow b = 1$

$f'(x) = \begin{cases} -\sin x, & x < 0 \\ a, & x > 0 \end{cases}$

So, $a = 0$.

Answer: $a = 0, b = 1$

$f_1(x) = |\sin x|$ is differentiable for all $x \neq n\pi$, n an integer.

$f_2(x) = \sin|x|$ is differentiable for all $x \neq 0$.

You can verify this by graphing f_1 and f_2 and observing the locations of the sharp turns.

Let $f(x) = \cos x$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cos x(\cos \Delta x - 1)}{\Delta x} - \lim_{\Delta x \to 0} \sin x \left(\frac{\sin \Delta x}{\Delta x}\right)$$

$$= 0 - \sin x(1) = -\sin x$$

You are given $f : R \to R$ satisfying

$$(*) \quad f'(x) = \frac{f(x + n) - f(x)}{n}$$ for all real numbers x and all positive integers n. You claim that $f(x) = mx + b$, $m, b \in R$.

For this case, $f'(x) = \frac{[m(x + n) + b] - [mx + b]}{n} = m$.

Furthermore, these are the only solutions:

Note first that $f'(x + 1) = \frac{f(x + 2) - f(x + 1)}{1}$, and $f'(x) = f(x + 1) - f(x)$. From (*) you have

$$2f'(x) = f(x + 2) - f(x)$$
$$= [f(x + 2) - f(x + 1)] + [f(x + 1) - f(x)]$$
$$= f'(x + 1) + f'(x).$$

Thus, $f'(x) = f'(x + 1)$.

Let $g(x) = f(x + 1) - f(x)$.

Let $m = g(0) = f(1) - f(0)$.

Let $b = f(0)$. Then

$$g'(x) = f'(x + 1) - f'(x) = 0$$

$$g(x) = \text{constant} = g(0) = m$$

$$f'(x) = f(x + 1) - f(x) = g(x) = m$$

$$\Rightarrow f(x) = mx + b.$$

Section 2.3 Product and Quotient Rules and Higher-Order Derivatives

To find the derivative of the product of two differentiable functions f and g, multiply the first function f by the derivative of the second function g, and then add the second function g times the derivative of the first function f.

To find the derivative of the quotient of two differentiable functions f and g, where $g(x) \neq 0$, multiply the denominator by the derivative of the numerator minus the numerator times the derivative of the denominator, all of which is divided by the square of the denominator.

$d \qquad d$

$g(x) = (2x - 3)(1 - 5x)$

$g'(x) = (2x - 3)(-5) + (1 - 5x)(2)$

$= -10x + 15 + 2 - 10x$

$= -20x + 17$

$y = (3x - 4)(x^3 + 5)$

$y' = (3x - 4)(3x^2) + (x^3 + 5)(3)$

$= 9x^3 - 12x^2 + 3x^3 + 15$

$= 12x^3 - 12x^2 + 15$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

Higher-order derivatives are successive derivatives of a function.

$$h(t) = \sqrt{t}\left(1 - t^2\right) = t^{1/2}\left(1 - t^2\right)$$

$$h'(t) = t^{1/2}(-2t) + \left(1 - t^2\right)\frac{1}{2}t^{-1/2}$$

$$= -2t^{3/2} + \frac{1}{2t^{1/2}} - \frac{1}{2}t^{3/2}$$

$$= -\frac{5}{2}t^{3/2} + \frac{1}{2t^{1/2}}$$

$$= \frac{1 - 5t^2}{2t^{1/2}} = \frac{1 - 5t^2}{2\sqrt{t}}$$

8. $g(s) = \sqrt{s}\left(s^2 + 8\right) = s^{1/2}\left(s^2 + 8\right)$

$g'(s) = s^{1/2}(2s) + \left(s^2 + 8\right)\frac{1}{2}s^{-1/2}$

$= 2s^{3/2} + \frac{1}{2}s^{3/2} + 4s^{-1/2}$

$= \frac{5}{2}s^{3/2} + \frac{4}{s^{1/2}}$

$= \frac{5s^2 + 8}{2\sqrt{s}}$

9. $f(x) = x^3 \cos x$

$f'(x) = x^3(-\sin x) + \cos x\left(3x^2\right)$

$= 3x^2 \cos x - x^3 \sin x$

$= x^2(3 \cos x - x \sin x)$

10. $g(x) = \sqrt{x} \sin x$

$g'(x) = \sqrt{x} \cos x + \sin x\left(\frac{1}{2\sqrt{x}}\right)$

$= \sqrt{x} \cos x + \frac{1}{2\sqrt{x}} \sin x$

11. $f(x) = \frac{x}{x - 5}$

$f'(x) = \frac{(x - 5)(1) - x(1)}{(x - 5)^2} = \frac{x - 5 - x}{(x - 5)^2} = -\frac{5}{(x - 5)^2}$

$g(t) = \frac{3t^2 - 1}{2t + 5}$

$g'(t) = \frac{(2t + 5)(6t) - \left(3t^2 - 1\right)(2)}{(2t + 5)^2}$

$= \frac{12t^2 + 30t - 6t^2 + 2}{(2t + 5)^2}$

$= \frac{6t^2 + 30t + 2}{(2t + 5)^2}$

$f(x) = \left(x^3 + 4x\right)\left(3x^2 + 2x - 5\right)$

$f'(x) = \left(x^3 + 4x\right)(6x + 2) + \left(3x^2 + 2x - 5\right)\left(3x^2 + 4\right)$

13. $h(x) = \frac{\sqrt{x}}{x^3 + 1} = \frac{x^{1/2}}{x^3 + 1}$

$h'(x) = \frac{\left(x^3 + 1\right)\frac{1}{2}x^{-1/2} - x^{1/2}\left(3x^2\right)}{\left(x^3 + 1\right)^2}$

$= \frac{x^3 + 1 - 6x^3}{2x^{1/2}\left(x^3 + 1\right)^2}$

$= \frac{1 - 5x^3}{2\sqrt{x}\left(x^3 + 1\right)^2}$

14. $f(x) = \frac{x^2}{2\sqrt{x} + 1}$

$f'(x) = \frac{\left(2\sqrt{x} + 1\right)(2x) - x^2\left(x^{-1/2}\right)}{\left(2\sqrt{x} + 1\right)^2}$

$= \frac{4x^{3/2} + 2x - x^{3/2}}{\left(2\sqrt{x} + 1\right)^2}$

$= \frac{3x^{3/2} + 2x}{\left(2\sqrt{x} + 1\right)^2}$

$= \frac{x\left(3\sqrt{x} + 2\right)}{\left(2\sqrt{x} + 1\right)^2}$

15. $g(x) = \frac{\sin x}{x^2}$

$g'(x) = \frac{x^2(\cos x) - \sin x(2x)}{\left(x^2\right)^2} = \frac{x \cos x - 2 \sin x}{x^3}$

$f(t) = \frac{\cos t}{t^3}$

$f'(t) = \frac{t^3(-\sin t) - \cos t\left(3t^2\right)}{\left(t^3\right)^2} = -\frac{t \sin t + 3 \cos t}{t^4}$

$= 6x^4 + 24x^2 + 2x^3 + 8x + 9x^4 + 6x^3 - 15x^2 + 12x^2 \ldots$

$= 15x^4 + 8x^3 + \ldots$

$21x^2 + 16x - 20$

$f'(0) = -20$

$(t^3)^2$ t^4

$8x - 20$

$f(x) = (2x^2 - 3x)(9x + 4)$

$$f'(x) = (2x^2 - 3x)(9) + (9x + 4)(4x - 3)$$
$$= 18x^2 - 27x + 36x^2 + 16x - 27x - 12$$
$$= 54x^2 - 38x - 12$$

$$f'(-1) = 54(-1)^2 - 38(-1) - 12 = 80$$

$f(x) = \frac{x^2 - 4}{x - 3}$

$$f'(x) = \frac{(x-3)(2x) - (x^2 - 4)(1)}{(x-3)^2}$$
$$= \frac{2x^2 - 6x - x^2 + 4}{(x-3)^2}$$
$$= \frac{x^2 - 6x + 4}{(x-3)^2}$$
$$f'(1) = \frac{1 - 6 + 4}{(1-3)^2} = -\frac{1}{4}$$

$f(x) = \frac{x - 4}{x + 4}$

$$f'(x) = \frac{(x+4)(1) - (x-4)(1)}{(x+4)^2}$$
$$= \frac{x + 4 - x + 4}{(x+4)^2}$$
$$= \frac{8}{(x+4)^2}$$
$$f'(3) = \frac{8}{(3+4)^2} = \frac{8}{49}$$

$f(x) = x \cos x$

$$f'(x) = x(-\sin x) + (\cos x)(1) = \cos x - x \sin x$$
$$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{8}(4 - \pi)$$

$f(x) = \frac{\sin x}{x}$

$$f'(x) = \frac{(x)(\cos x) - (\sin x)(1)}{x^2} = \frac{x \cos x - \sin x}{x^2}$$
$$f'\left(\frac{\pi}{6}\right) = \frac{(\pi/6)\left(\sqrt{3}/2\right) - (1/2)}{\pi^2/36} = \frac{3\sqrt{3}\pi - 18}{\pi^2}$$
$$= \frac{3\left(\sqrt{3}\pi - 6\right)}{\pi^2}$$

	Function	Rewrite	Differentiate	Simplify
23.	$y = \frac{x^3 + 6x}{3}$	$y = \frac{1}{3}x^3 + 2x$	$y' = \frac{1}{3}(3x^2) + 2$	$y' = x^2 + 2$
24.	$y = \frac{5x^2 - 3}{4}$	$y = \frac{5}{4}x^2 - \frac{3}{4}$	$y' = \frac{10}{4}x$	$y' = \frac{5x}{2}$
25.	$y = \frac{6}{7x^2}$	$y = \frac{6}{7}x^{-2}$	$y' = -\frac{12}{7}x^{-3}$	$y' = -\frac{12}{7x^3}$
26.	$y = \frac{10}{3x^3}$	$y = \frac{10}{3}x^{-3}$	$y' = -\frac{30}{3}x^{-4}$	$y' = -\frac{10}{x^4}$

27. $y = \dfrac{4x^{3/2}}{x}$	$y = 4x^{1/2},\ x > 0$	$y' = 2x^{-1/2}$	$y' = \dfrac{2}{\sqrt{x}},\ x > 0$
28. $y = \dfrac{2x}{x^{1/3}}$	$y = 2x^{2/3}$	$y' = \dfrac{4}{3}x^{-1/3}$	$y' = \dfrac{4}{3x^{1/3}}$

29. $f(x) = \dfrac{4 - 3x - x^2}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(-3 - 2x) - (4 - 3x - x^2)(2x)}{(x^2 - 1)^2}$$

$$= \frac{-3x^2 + 3 - 2x^3 + 2x - 8x + 6x^2 + 2x^3}{(x^2 - 1)^2}$$

$$= \frac{3x^2 - 6x + 3}{(x^2 - 1)^2}$$

$$= \frac{3(x - 1)^2}{(x - 1)^2(x + 1)^2} = \frac{3}{(x + 1)^2},\ x \neq 1$$

30. $f(x) = \dfrac{x^2 + 5x + 6}{x^2 - 4}$

$$f'(x) = \frac{(x^2 - 4)(2x + 5) - (x^2 + 5x + 6)(2x)}{(x^2 - 4)^2}$$

$$= \frac{2x^3 + 5x^2 - 8x - 20 - 2x^3 - 10x^2 - 12x}{(x^2 - 4)^2}$$

$$= \frac{-5x^2 - 20x - 20}{(x^2 - 4)^2}$$

$$= \frac{-5(x^2 + 4x + 4)}{(x - 2)^2(x + 2)^2} = \frac{-5(x + 2)^2}{(x - 2)^2(x + 2)^2}$$

$$= -\frac{5}{(x - 2)^2},\ x \neq 2, -2$$

Alternate solution:

$$f(x) = \frac{x^2 + 5x + 6}{x^2 - 4}$$

31. $f(x) = x\left(1 - \dfrac{4}{x + 3}\right) = x - \dfrac{4x}{x + 3}$

$$f'(x) = 1 - \frac{(x + 3)4 - 4x(1)}{(x + 3)^2}$$

$$= \frac{(x^2 + 6x + 9) - 12}{(x + 3)^2}$$

$$= \frac{x^2 + 6x - 3}{(x + 3)^2}$$

$f(x) = x^4\left[1 - \dfrac{2}{x + 1}\right] = x^4\left[\dfrac{x - 1}{x + 1}\right]$

$$f'(x) = x^4\left[\frac{(x + 1) - (x - 1)}{(x + 1)^2}\right] + \left[\frac{x - 1}{x + 1}\right](4x^3)$$

$$= x^4\left[\frac{2}{(x + 1)^2}\right] + \left[\frac{x - 1}{x + 1}\right]4x^3$$

$$= \frac{2x^3(2x^2 + x - 2)}{(x + 1)^2}$$

33. $f(x) = \dfrac{3x - 1}{\sqrt{x}} = 3x^{1/2} - x^{-1/2}$

$$f'(x) = \frac{3}{2}x^{-1/2} + \frac{1}{2}x^{-3/2} = \frac{3x + 1}{2x^{3/2}}$$

$$f(x) = \frac{3x - 1}{\sqrt{x}} = \frac{3x - 1}{x^{1/2}}$$

$$f'(x) = \frac{x^{1/2}(3) - (3x - 1)\left(\frac{1}{2}\right)x^{-1/2}}{x}$$

$$= \frac{\frac{1}{2}x^{-1/2}(3x + 1)}{x}$$

$$= \frac{3x + 1}{2x^{3/2}}$$

$$f(x) = \frac{\ldots}{x^2 - 4}$$

$$= \frac{(x+3)(x+2)}{(x+2)(x-2)}$$

$$= \frac{x+3}{x-2}, \quad x \neq -2$$

$$f'(x) = \frac{(x-2)(1) - (x+3)(1)}{(x-2)^2}$$

$$= -\frac{5}{(x-2)^2}$$

$$f(x) = \sqrt[3]{x}\left(\sqrt{x} + 3\right) = x^{1/3}\left(x^{1/2} + 3\right)$$

$$f'(x) = x^{1/3}\left(\frac{1}{2}x^{-1/2}\right) + \left(x^{1/2} + 3\right)\left(\frac{1}{3}x^{-2/3}\right)$$

$$= \frac{5}{6}x^{-1/6} + x^{-2/3}$$

$$= \frac{5}{6x^{1/6}} + \frac{1}{x^{2/3}}$$

Alternate solution:

$$f(x) = \sqrt[3]{x}\left(\sqrt{x} + 3\right) = x^{5/6} + 3x^{1/3}$$

$$f'(x) = \frac{5}{6}x^{-1/6} + x^{-2/3} = \frac{5}{6x^{1/6}} + \frac{1}{x^{2/3}}$$

(

$\sqrt{}$ $\sqrt{}$ / /

/ /

/ /

6 $6x^{1\,6}$ $x^{2\,3}$

35. $f(x) = \dfrac{2 - \frac{1}{x}}{x - 3} = \dfrac{2x - 1}{x(x-3)} = \dfrac{2x - 1}{x^2 - 3x}$

$$f'(x) = \frac{(x^2 - 3x)2 - (2x - 1)(2x - 3)}{(x^2 - 3x)^2}$$

$$= \frac{2x^2 - 6x - 4x^2 + 8x - 3}{(x^2 - 3x)^2}$$

$$= \frac{-2x^2 + 2x - 3}{(x^2 - 3x)^2} = -\frac{2x^2 - 2x + 3}{x^2(x-3)^2}$$

36. $h(x) = \dfrac{\frac{1}{x^2} + 5x}{x + 1} = \dfrac{1 + 5x^3}{x^3 + x^2}$

$$h'(x) = \frac{(x^3 + x^2)(15x^2) - (1 + 5x^3)(3x^2 + 2x)}{(x^3 + x^2)^2}$$

$$= \frac{15x^5 + 15x^4 - 3x^2 - 2x - 15x^5 - 10x^4}{x^4(x+1)^2}$$

$$= \frac{5x^4 - 3x^2 - 2x}{x^4(x+1)^2}$$

$$= \frac{5x^3 - 3x - 2}{x^3(x+1)^2}$$

37. $g(s) = s^3\left(5 - \dfrac{s}{s+2}\right) = 5s^3 - \dfrac{s^4}{s+2}$

$$g'(s) = 15s^2 - \frac{(s+2)4s^3 - s^4(1)}{(s+2)^2}$$

$$= 15s^2 - \frac{3s^4 + 8s^3}{(s+2)^2}$$

$$= \frac{15s^2(s+2)^2 - (3s^4 + 8s^3)}{(s+2)^2}$$

$$= \frac{15s^2(s^2 + 4s + 4) - 3s^4 - 8s^3}{(s+2)^2}$$

$$= \frac{12s^4 + 52s^3 + 60s^2}{(s+2)^2}$$

$$= \frac{4s^2(3s^2 + 13s + 15)}{(s+2)^2}$$

38. $g(x) = x^2\left(\dfrac{2}{x} - \dfrac{1}{x+1}\right) = 2x - \dfrac{x^2}{x+1}$

$$g'(x) = 2 - \frac{(x+1)2x - x^2(1)}{(x+1)^2} = \frac{2(x^2 + 2x + 1) - x^2 - 2x}{(x+1)^2} = \frac{x^2 + 2x + 2}{(x+1)^2}$$

$$f(x) = (2x^3 + 5x)(x - 3)(x + 2)$$

$$f'(x) = (6x^2 + 5)(x - 3)(x + 2) + (2x^3 + 5x)(1)(x + 2) + (2x^3 + 5x)(x - 3)(1)$$

$$(6x^2 + 5)(x^2 - x - 6) + (2x^3 + 5x)(x + 2) + (2x^3 + 5x)(x - 3)$$

$$= (6x^4 + 5x^2 - 6x^3 - 5x - 36x^2 - 30) + (2x^4 + 4x^3 + 5x^2 + 10x) + (2x^4 + 5x^2 - 6x^3 - 15x)$$

$$10x^4 - 8x^3 - 21x^2 - 10x - 30$$

Note: You could simplify first: $f(x) = (2x^3 + 5x)(x^2 - x - 6)$

40. $f(x) = (x^3 - x)(x^2 + 2)(x^2 + x - 1)$

$$f'(x) = (3x^2 - 1)(x^2 + 2)(x^2 + x - 1) + (x^3 - x)(2x)(x^2 + x - 1) + (x^3 - x)(x^2 + 2)(2x + 1)$$

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$$= \left(3x^4 + 5x^2 - 2\right)\left(x^2 + x - 1\right) + \left(2x^4 - 2x^2\right)\left(x^2 + x - 1\right) + \left(x^5 + x^3 - 2x\right)(2x + 1)$$

$$\left(3x^6 + 5x^4 - 2x^2 + 3x^5 + 5x^3 - 2x - 3x^4 - 5x^2 + 2\right)$$

$$\left(2x^6 - 2x^4 + 2x^5 - 2x^3 - 2x^4 + 2x^2\right)$$

$$\left(2x^6 + 2x^4 - 4x^2 + x^5 + x^3 - 2x\right)$$

$$7x^6 + 6x^5 + 4x^3 - 9x^2 - 4x + 2$$

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41. $f(t) = t^2 \sin t$

$f'(t) = t^2 \cos t + 2t \sin t = t(t \cos t + 2 \sin t)$

42. $f(\theta) = (\theta + 1)\cos\theta$

$f'(\theta) = (\theta + 1)(-\sin\theta) + (\cos\theta)(1)$

$= \cos\theta - (\theta + 1)\sin\theta$

43. $f(t) = \dfrac{\cos t}{t}$

$f'(t) = \dfrac{-t \sin t - \cos t}{t^2} = -\dfrac{t \sin t + \cos t}{t^2}$

$f(x) = \dfrac{\sin x}{x^3}$

$f'(x) = \dfrac{x^3 \cos x - \sin x(3x^2)}{(x^3)^2} = \dfrac{x \cos x - 3 \sin x}{x^4}$

45. $f(x) = -x + \tan x$

$f'(x) = -1 + \sec^2 x = \tan^2 x$

$y = x + \cot x$

$y' = 1 - \csc^2 x = -\cot^2 x$

47. $g(t) = \sqrt[4]{t} + 6 \csc t = t^{1/4} + 6 \csc t$

$g'(t) = \dfrac{1}{4} t^{-3/4} - 6 \csc t \cot t$

$= \dfrac{1}{4t^{3/4}} - 6 \csc t \cot t$

$h(x) = \dfrac{1}{x} - 12 \sec x = x^{-1} - 12 \sec x$

$h'(x) = -x^{-2} - 12 \sec x \tan x$

$= \dfrac{-1}{x^2} - 12 \sec x \tan x$

49. $y = \dfrac{3(1 - \sin x)}{2 \cos x} = \dfrac{3 - 3 \sin x}{2 \cos x}$

$y = \dfrac{\sec x}{x}$

$y' = \dfrac{x \sec x \tan x - \sec x}{x^2}$

$= \dfrac{\sec x(x \tan x - 1)}{x^2}$

51. $y = -\csc x - \sin x$

$y' = \csc x \cot x - \cos x$

$= \dfrac{\cos x}{\sin^2 x} - \cos x$

$= \cos x(\csc^2 x - 1)$

$= \cos x \cot^2 x$

$y = x \sin x + \cos x$

$y' = x \cos x + \sin x - \sin x = x \cos x$

$f(x) = x^2 \tan x$

$f'(x) = x^2 \sec^2 x + 2x \tan x = x(x \sec^2 x + 2 \tan x)$

$f(x) = \sin x \cos x$

$f'(x) = \sin x(-\sin x) + \cos x(\cos x) = \cos 2x$

$y = 2x \sin x + x^2 \cos x$

$y' = 2x \cos x + 2 \sin x + x^2(-\sin x) + 2x \cos x$

$= 4x \cos x + (2 - x^2)\sin x$

$h(\theta) = 5\theta \sec\theta + \theta \tan\theta$

$h'(\theta) = 5\theta \sec\theta \tan\theta + 5 \sec\theta + \theta \sec^2\theta + \tan\theta$

$g(x) = \left(\dfrac{x + 1}{x + 2}\right)(2x - 5)$

$g'(x) = \left(\dfrac{x + 1}{x + 2}\right)(2) + (2x - 5)\left[\dfrac{(x + 2)(1) - (x + 1)(1)}{(x + 2)^2}\right]$

$= \dfrac{2x^2 + 8x - 1}{(x + 2)^2}$

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$$y' = \frac{(-3\cos x)(2\cos x) - (3 - 3\sin x)(-2\sin x)}{(2\cos x)^2}$$

$$= \frac{-6\cos^2 x + 6\sin x - 6\sin^2 x}{4\cos^2 x}$$

$$= \frac{3}{2}\left(-1 + \tan x \sec x - \tan^2 x\right)$$

$$= \frac{3}{2}\sec x\left(\tan x - \sec x\right)$$

$(x + 2)$

(Form of answer may vary.)

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58. $f(x) = \dfrac{\cos x}{1 - \sin x}$

$$f'(x) = \frac{(1 - \sin x)(-\sin x) - (\cos x)(-\cos x)}{(1 - \sin x)^2}$$

$$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1 - \sin x)^2}$$

$$= \frac{1 - \sin x}{(1 - \sin x)^2}$$

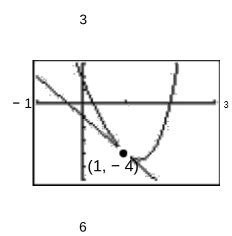
$$= \frac{1}{1 - \sin x}$$

(Form of answer may vary.)

$$y = \frac{1 + \csc x}{1 - \csc x}$$

$$y' = \frac{(1 - \csc x)(-\csc x \cot x) - (1 + \csc x)(\csc x \cot x)}{(1 - \csc x)^2} \quad \frac{-2 \csc x \cot x}{(1 - \csc x)^2}$$

$$y'\left(\frac{\pi}{6}\right) = \frac{-2(2)\sqrt{3}}{(1 - 2)^2} = -4\sqrt{3}$$



Graphing utility confirms $\dfrac{dy}{dx} = -3$ at $(1, -4)$.

60. $f(x) = \tan x \cot x = 1$

$f'(x) = 0$

$f'(1) = 0$

61. $h(t) = \dfrac{\sec t}{t}$

$$h'(t) = \frac{t(\sec t \tan t) - (\sec t)(1)}{t^2} = \frac{\sec t\,(t \tan t - 1)}{t^2}$$

$$h'(\pi) = \frac{\sec \pi\,(\pi \tan \pi - 1)}{\pi^2} = \frac{1}{\pi^2}$$

(a) $f(x) = (x - 2)(x^2 + 4)$, $(1, -5)$

$$f'(x) = (x - 2)(2x) + (x^2 + 4)(1)$$

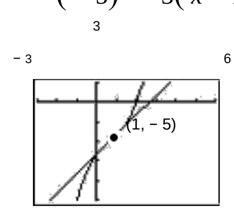
$$2x^2 - 4x + x^2 + 4$$

$$3x^2 - 4x + 4$$

$f'(1) = -3$; Slope at $(1, -5)$

Tangent line:

$-(-5) = 3(x - 1) \Rightarrow y = 3x - 8$



$f(x) = \sin x(\sin x + \cos x)$

$$f'(x) = \sin x(\cos x - \sin x) + (\sin x + \cos x)\cos x$$

$$\sin x \cos x - \sin^2 x + \sin x \cos x + \cos^2 x$$

$$\sin 2x + \cos 2x$$

$$f'\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1$$

63. (a) $f(x) = (x^3 + 4x - 1)(x - 2)$, $(1, -4)$

$$f'(x) = (x^3 + 4x - 1)(1) + (x - 2)(3x^2 + 4)$$

$\dfrac{dy}{dx}$ = 3 +

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$4x - 1 + 3x^3 - 6x^2 + 4x - 8$

$= 4x^3 - 6x^2 + 8x - 9$

$f'(1) = -3$; Slope at $(1, -4)$

Tangent line: $y + 4 = -3(x - 1) \Rightarrow y = -3x - 1$

Graphing utility confirms $dx = 3$ at $(1, -5)$.

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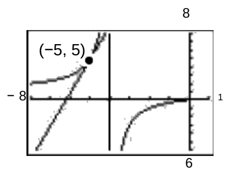
65. (a) $f(x) = \dfrac{x}{x+4}$, $(-5, 5)$

$$f'(x) = \frac{(x+4)(1) - x(1)}{(x+4)^2} = \frac{4}{(x+4)^2}$$

$$f'(-5) = \frac{4}{(-5+4)^2} = 4; \text{ Slope at } (-5, 5)$$

Tangent line: $y - 5 = 4(x+5) \Rightarrow y = 4x + 25$

(b)



Graphing utility confirms $\dfrac{dy}{dx} = 4$ at $(-5, 5)$.

66. (a) $f(x) = \dfrac{x+3}{x-3}$, $(4, 7)$

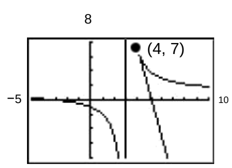
$$f'(x) = \frac{(x-3)(1) - (x+3)(1)}{(x-3)^2} = -\frac{6}{(x-3)^2}$$

$$f'(4) = \frac{-6}{1} = -6; \quad \text{Slope at } (4, 7)$$

Tangent line:

$y - 7 = -6(x-4) \Rightarrow y = -6x + 31$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = -6$ at $(4, 7)$.

67. (a) $f(x) = \tan x$, $\left(\dfrac{\pi}{4}, 1\right)$

$f'(x) = \sec^2 x$

$\left(\pi\right)$ $\left(\pi\right)$

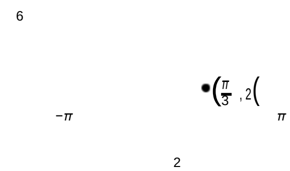
68. (a) $f(x) = \sec x$, $\left(\dfrac{\pi}{3}, 2\right)$

$$f'\left(\frac{\pi}{3}\right) = 2\sqrt{3}; \text{ Slope at } \left(\frac{\pi}{3}, 2\right)$$

Tangent line:

$$y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

$$\sqrt{3}x - 3y + 6 - 2\sqrt{3}\pi = 0$$



Graphing utility confirms $\dfrac{dy}{dx} = 2\sqrt{3}$ at $\left(\dfrac{\pi}{3}, 2\right)$.

69. $f(x) = \dfrac{8}{x^2+4}$; $(2, 1)$

$$f'(x) = \frac{(x^2+4)(0) - 8(2x)}{(x^2+4)^2} = \frac{-16x}{(x^2+4)^2}$$

$$f'(2) = \frac{-16(2)}{(4+4)^2} = -\frac{1}{2}$$

$$y - 1 = -\frac{1}{2}(x-2)$$

$$y = -\frac{1}{2}x + 2$$

$$2y + x - 4 = 0$$

$f(x) = \dfrac{27}{x^2+9}$; $\left(-3, \dfrac{3}{2}\right)$

$$f'(x) = \frac{(x^2+9)(0) - 27(2x)}{(x^2+9)^2} = \frac{-54x}{(x^2+9)^2}$$

f ; Slope at

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$(x^2$

$+ 9)^2 = (x^2 + 9)^2$

$\left(4\right) \qquad \left(4\right)$

$f'(-3) = \dfrac{-54(-3)}{(9+9)^2} = \dfrac{1}{2}$

Tangent line: $y - 1 = 2\left(x - \dfrac{\pi}{4}\right)$

$y - \dfrac{3}{2} = \dfrac{1}{2}(x + 3)$

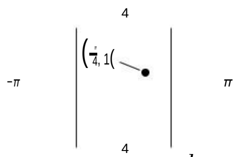
$y - 1 = 2x - \dfrac{\pi}{2}$

$= \dfrac{1}{2}x + 3$

$4x - 2y - \pi + 2 = 0$

$2y - x - 6 = 0$

(b)



Graphing utility confirms $\dfrac{dy}{dx} = 2$ at $\left(\dfrac{\pi}{4}, 1\right)$.

71. $f(x) = \frac{16x}{x^2 + 16}$; $\left(-2, -\frac{8}{5}\right)$

$$f'(x) = \frac{(x^2 + 16)(16) - 16x(2x)}{(x^2 + 16)^2} = \frac{256 - 16x^2}{(x^2 + 16)^2}$$

$$f'(-2) = \frac{256 - 16(4)}{20^2} = \frac{12}{25}$$

$$y + \frac{8}{5} = \frac{12}{25}(x + 2)$$

$$y = \frac{12}{25}x - \frac{16}{25}$$

$25y - 12x + 16 = 0$

72. $f(x) = \frac{4x}{x^2 + 6}$; $\left(2, \frac{4}{5}\right)$

$$f'(x) = \frac{(x^2 + 6)4 - 4x(2x)}{(x^2 + 6)^2} = \frac{24 - 4x^2}{(x^2 + 6)^2}$$

$$f'(2) = \frac{24 - 16}{10^2} = \frac{2}{25}$$

$$y - \frac{4}{5} = \frac{2}{25}(x - 2)$$

$$y = \frac{2}{25}x + \frac{16}{25}$$

$25y - 2x - 16 = 0$

$$f(x) = \frac{2x - 1}{x^2} = 2x^{-1} - x^{-2}$$

$$f'(x) = -2x^{-2} + 2x^{-3} = \frac{2(-x + 1)}{x^3}$$

$f'(x) = 0$ when $x = 1$, and $f(1) = 1$.

Horizontal tangent at $(1, 1)$.

$$f(x) = \frac{x^2}{x^2 + 1}$$

$$f'(x) = \frac{(x^2 + 1)2x - x^2(2x)}{}$$

75. $f(x) = \frac{x^2}{x - 1}$

$$f'(x) = \frac{(x - 1)(2x) - x^2(1)}{(x - 1)^2} = \frac{x^2 - 2x}{(x - 1)^2} = \frac{x(x - 2)}{(x - 1)^2}$$

$f'(x) = 0$ when $x = 0$ or $x = 2$.

Horizontal tangents are at $(0, 0)$ and $(2, 4)$.

$$f(x) = \frac{x - 4}{x^2 - 7}$$

$$f'(x) = \frac{(x^2 - 7)(1) - (x - 4)(2x)}{(x^2 - 7)^2} = \frac{x^2 - 7 - 2x^2 + 8x}{(x^2 - 7)^2} = \frac{-x^2 + 8x - 7}{(x^2 - 7)^2} = -\frac{(x - 7)(x - 1)}{(x^2 - 7)^2}$$

$f'(x) = 0$ for $x = 1, 7$; $f(1) = \frac{1}{2}$, $f(7) = \frac{1}{14}$

f has horizontal tangents at $\left(1, \frac{1}{2}\right)$ and $\left(7, \frac{1}{14}\right)$.

$$f(x) = \frac{x + 1}{x - 1}$$

$$f'(x) = \frac{(x - 1) - (x + 1)}{(x - 1)^2} = \frac{-2}{(x - 1)^2}$$

$2y + x = 6 \Rightarrow y = -\frac{1}{2}x + 3$; Slope: $-\frac{1}{2}$

$$\frac{-2}{(x - 1)^2} = -\frac{1}{2}$$

$$(x - 1)^2 = 4$$

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$- 2 \cdot \cdot \frac{-x}{(x^2 + 1)^2}$

$$= \frac{2x}{(x^2 + 1)^2}$$

$f'(x) = 0$ when $x = 0$.

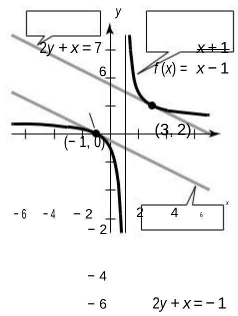
Horizontal tangent is at $(0, 0)$.

$x - 1 = \pm 2$

$x = -1, 3;\ f(-1) = 0,\ f(3) = 2$

$$y - 0 = -\frac{1}{2}(x + 1) \Rightarrow y = -\frac{1}{2}x - \frac{1}{2}$$

$$y - 2 = -\frac{1}{2}(x - 3) \Rightarrow y = -\frac{1}{2}x + \frac{7}{2}$$



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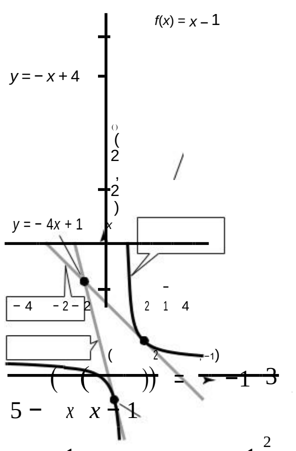
78. $f(x) = \dfrac{x}{x-1}$

$f'(x) = \dfrac{(x-1) - x}{(x-1)^2} = \dfrac{-1}{(x-1)^2}$

Let $(x, y) = (x, x/(x-1))$ be a point of tangency on the graph of f.



$\dfrac{5 - (x/(x-1))}{-1 - x} = \dfrac{-1}{(x-1)^2}$

$\dfrac{4x - 5}{(x-1)(x+1)} = \dfrac{-1}{(x-1)^2}$

$(4x - 5)(x - 1) = x + 1$

$4x^2 - 10x + 4 = 0$

$(x - 2)(2x - 1) = 0 \Rightarrow x = \dfrac{1}{2}, 2$

$f\left(\dfrac{1}{2}\right) = -1, f(2) = 2; f'\left(\dfrac{1}{2}\right) = -4, f'(2) = -1$

Two tangent lines:

$y + 1 = -4\left(x - \dfrac{1}{2}\right) \Rightarrow y = -4x + 1$

$y - 2 = -1(x - 2) \Rightarrow y = -x + 4$

79. $f'(x) = \dfrac{(x+2)3 - 3x(1)}{(x+2)^2} = \dfrac{6}{(x+2)^2}$

$g'(x) = \dfrac{(x+2)5 - (5x+4)(1)}{(x+2)^2} = \dfrac{6}{(x+2)^2}$

$g(x) = \dfrac{5x+4}{x+2} = \dfrac{3x}{x+2} + \dfrac{2x+4}{x+2} = f(x) + 2$

f and g differ by a constant.

80. $f'(x) = \dfrac{x(\cos x - 3\sin x + 3x) - (\sin x - 3\cos x + 3x^2 ...)(1)}{x^2} = \dfrac{x\cos x - \sin x}{x^2}$

$g'(x) = \dfrac{x(\cos x + 2) - (\sin x + 2x)(1)}{x^2} = \dfrac{x\cos x - \sin x}{x^2}$

$g(x) = \dfrac{\sin x + 2x}{x} = \dfrac{\sin x - 3x + 5x}{x} = f(x) + 5$

f and g differ by a constant.

81. (a) $p'(x) = f'(x)g(x) + f(x)g'(x)$

$p'(1) = f'(1)g(1) + f(1)g'(1) = 1(4) + 6\left(-\dfrac{1}{2}\right) = 1$

(b) $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$q'(4) = \dfrac{(1)(-1) - 7(0)}{1^2} = -1$

82. (a) $p'(x) = f'(x)g(x) + f(x)g'(x)$

$p'(4) = \dfrac{1}{2}(8) + 1(0) = 4$

(b) $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$q'(7) = \dfrac{4(2) - 4(-1)}{4^2} = \dfrac{12}{16} = \dfrac{3}{4}$

83. Area $= A(t) = (6t + 5)\sqrt{t} = 6t^{3/2} + 5t^{1/2}$

$A'(t) = 9t^{1/2} + \dfrac{5}{2}t^{-1/2} = \dfrac{18t + 5}{2\sqrt{t}}$ cm^2/sec

84. $V = \pi r^2 h = \pi(t + 2)\left(\dfrac{1}{2}\sqrt{t}\right) = \dfrac{1}{2}\left(t^{3/2} + 2t^{1/2}\right)\pi$

$V'(t) = \ldots t^{-1/2} \ldots 6$

$$g'(x) = \frac{x + 25 - 5x + 41}{(x+2)^2} = \frac{\ldots}{(x+2)^2}$$

$$g(x) = \frac{5x+4}{(x+2)} = \frac{3x}{(x+2)} + \frac{2x+4}{(x+2)} = f(x) + 2$$

f and *g* differ by a constant.

85. $C = 100\left(\frac{200}{x^2} + \frac{x}{x+30}\right),\ 1 \le x$

$$\frac{dC}{dx} = 100\left(-\frac{400}{x^3} + \frac{30}{(x+30)^2}\right)$$

(a) When

(b) When

$= \frac{3t+2}{\ldots} \pi \text{ in.}^3/\text{sec}$

$x = 10$: $\frac{dC}{dx} = -\$38.15$ thousand/100 components

$x = 15$: $\frac{dC}{dx} = -\$10.37$ thousand/100 components

(c) When $x = 20$: $\frac{dC}{dx} = -\$3.80$ thousand/100 components

As the order size increases, the cost per item decreases.

86. $P(t) = 500\left[1 + \frac{4t}{50 + t^2}\right]$

$$P'(t) = 500\left[\frac{(50 + t^2)(4) - (4t)(2t)}{(50 + t^2)^2}\right] = 500\left[\frac{200 - 4t^2}{(50 + t^2)^2}\right] = 2000\left[\frac{50 - t^2}{(50 + t^2)^2}\right]$$

$P'(2) \approx 31.55$ bacteria/h

87. (a) $\sec x = \frac{1}{\cos x}$

$$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{(\cos x)(0) - (1)(-\sin x)}{(\cos x)^2} = \frac{\sin x}{\cos x \cos x} = \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x} = \sec x \tan x$$

(b) $\csc x = \frac{1}{\sin x}$

$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{(\sin x)(0) - (1)(\cos x)}{(\sin x)^2} = -\frac{\cos x}{\sin x \sin x} = -\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x} = -\csc x \cot x$$

$\cot x = \frac{\cos x}{\sin x}$

$$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\sin x(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2} = -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

$f(x) = \sec x$

$g(x) = \csc x, \quad [0, 2\pi)$

$f'(x) = g'(x)$

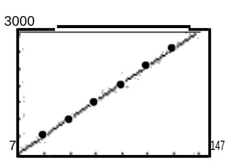
$$\sec x \tan x = -\csc x \cot x \Rightarrow \frac{\sec x \tan x}{\csc x \cot x} = -1 \Rightarrow \frac{\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}}{\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}} = -1 \Rightarrow \frac{\sin^3 x}{\cos^3 x} = -1 \Rightarrow \tan^3 x = -1 \Rightarrow \tan x = -1$$

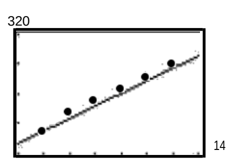
$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

89. (a) $h(t) = 101.7t + 1593$

$p(t) = 2.1t + 287$

(b)





(c) $A = \frac{101.7t + 1593}{2.1t + 287}$

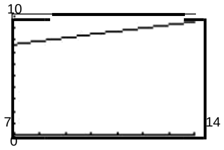
(d) $A'(t) \approx \frac{25{,}842.6}{4.41t^2 + 1205.4t + 82{,}369}$

$A'(t)$ represents the rate of change of the average health care expenditures per person for the given year t.

90. (a) $\sin\theta = \frac{r}{r + h}$

$r + h = r\csc\theta$

$h = r\csc\theta - r = r(\csc\theta - 1)$



A represents the average health care expenditures per person (in thousands of dollars).

(b) $h'\theta = r - \csc\theta \cdot \cot\theta$

$h'(30°) = h'\left(\frac{\pi}{6}\right)$

$= -4000\left(2 \cdot \sqrt{3}\right) = -8000\sqrt{3}$ mi / rad

$f(x) = x^2 + 7x - 4$

$f'(x) = 2x + 7$

$f''(x) = 2$

$f(x) = 4x^5 - 2x^3 + 5x^2$

$f'(x) = 20x^4 - 6x^2 + 10x$

$f''(x) = 80x^3 - 12x + 10$

$f(x) = 4x^{3/2}$

$f'(x) = 6x^{1/2}$

$f''(x) = 3x^{-1/2} = \dfrac{3}{\sqrt{x}}$

$f(x) = x^2 + 3x^{-3}$

$f'(x) = 2x - 9x^{-4}$

$f''(x) = 2 + 36x^{-5} = 2 + \dfrac{36}{x^5}$

95. $f(x) = \dfrac{x}{x-1}$

$f'(x) = \dfrac{(x-1)(1) - x(1)}{(x-1)^2} = \dfrac{-1}{(x-1)^2}$

$f''(x) = \dfrac{2}{(x-1)^3}$

96. $f(x) = \dfrac{x^2 + 3x}{x-4}$

$f'(x) = \dfrac{(x-4)(2x+3) - (x^2+3x)(1)}{(x-4)^2}$

$= \dfrac{2x^2 - 5x - 12 - x^2 - 3x}{(x-4)^2} = \dfrac{x^2 - 8x - 12}{x^2 - 8x + 16}$

$f''(x) = \dfrac{(x-4)^2(2x-8) - (x^2-8x-12)(2x-8)}{(x-4)^4}$

$= \dfrac{(x-4)\left[(x-4)(2x-8) - 2(x^2-8x-12)\right]}{(x-4)^4}$

$x - 4 \quad 2x - 8 - 2x^2 - 8x - 12$

$f(x) = x \sin x$

$f'(x) = x \cos x + \sin x$

$f''(x) = x(-\sin x) + \cos x + \cos x$

$= -x \sin x + 2 \cos x$

$f(x) = x \cos x$

$f'(x) = \cos x - x \sin x$

$f''(x) = -\sin x - (\sin x + x \cos x) = -x \cos x - 2 \sin x$

$f(x) = \csc x$

$f'(x) = -\csc x \cot x$

$f''(x) = -\csc x(-\csc^2 x) - \cot x(-\csc x \cot x)$

$= \csc^3 x + \cot^2 x \csc x$

$f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$

$= \sec x(\sec^2 x + \tan^2 x)$

101. $f'(x) = x^3 - x^{2/5}$

$f''(x) = 3x^2 - \dfrac{2}{5}x^{-3/5}$

$f'''(x) = 6x + \dfrac{6}{25}x^{-8/5} = 6x + \dfrac{6}{25x^{8/5}}$

102. $f^{(3)}(x) = \sqrt[5]{x^4} = x^{4/5}$

$f^{(4)}(x) = \dfrac{4}{5}x^{-1/5} = \dfrac{4}{5x^{1/5}}$

$f''(x) = -\sin x$

$f^{(3)}(x) = -\cos x$

$f^{(4)}(x) = \sin x$

$f^{(5)}(x) = \cos x$

$f^{(6)}(x) = -\sin x$

$f^{(7)}(x) = -\cos x$

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$$= \frac{\cdot}{(x-4)^3}$$

$$= \frac{2x^2 - 16x + 32 - 2x^2 + 16x + 24}{(x-4)^3}$$

$$= \frac{56}{(x-4)^3}$$

x f $\sin x$

$f^{(8)}(x)$

$f^{(4)}(t)$ f $t\cos t$

$\cos t - t\sin t$

$f^{(5)}(t)$

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$$f(x) = 2g(x) + h(x)$$
$$f'(x) = 2g'(x) + h'(x)$$
$$f'(2) = 2g'(2) + h'(2)$$
$$= 2(-2) + 4$$
$$= 0$$

$$f(x) = 4 - h(x)$$
$$f'(x) = -h'(x)$$
$$f'(2) = -h'(2) = -4$$

$$f(x) = \frac{g(x)}{h(x)}$$
$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{\left[h(x)\right]^2}$$
$$f'(2) = \frac{h(2)g'(2) - g(2)h'(2)}{\left[h(2)\right]^2}$$
$$= \frac{(-1)(-2) - (3)(4)}{(-1)^2}$$
$$= -10$$

$$f(x) = g(x)h(x)$$
$$f'(x) = g(x)h'(x) + h(x)g'(x)$$
$$f'(2) = g(2)h'(2) + h(2)g'(2)$$
$$= (3)(4) + (-1)(-2)$$
$$= 14$$

109. Polynomials of degree $n - 1$ (or lower) satisfy $f^{(n)}(x) = 0$. The derivative of a polynomial of the 0th degree (a constant) is 0.

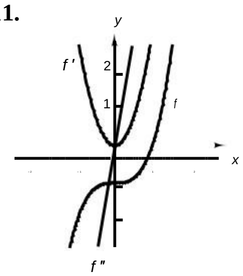
To differentiate a piecewise function, separate the function into its pieces, and differentiate each piece.

If $f(x) = x|x|$, then on $(-\infty, 0)$ you have $f(x) = -x^2$, $f'(x) = -2x$, and $f''(x) = -2$.

On $(0, \infty)$ you have $f(x) = x^2$, $f'(x) = 2x$, and $f''(x) = 2$.

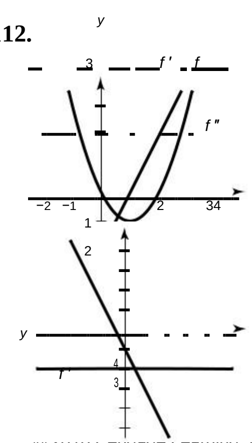
Notice that $f(0) = 0$, $f'(0) = 0$, but $f''(0)$ does not exist (the left-hand limit is -2, whereas the right-hand limit is 2).

111.



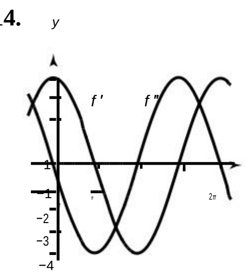
It appears that f is cubic, so f' would be quadratic and f'' would be linear.

112.

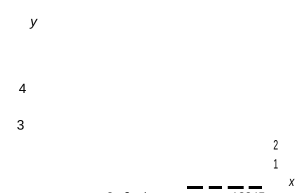


It appears that f is quadratic so f' would be linear and f'' would be constant.

114.



The graph of a differentiable function f such that $f(2) = 0$, $f' < 0$ for $-\infty < x < 2$, and $f' > 0$ for $2 < x < \infty$ would, in general, look like the graph below.

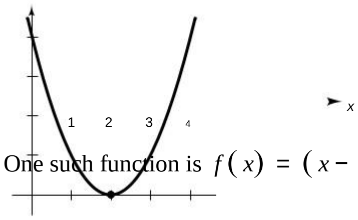


2

1

f''

− 3
4
5



One such function is $f(x) = (x - 2)^2$.

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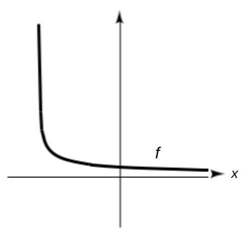
The graph of a differentiable function f such that $f > 0$ and $f' < 0$ for all real numbers x would, in general, look like the graph below.



$v(t) = 36 - t^2, \; 0 \le t \le 6$

$a(t) = v'(t) = -2t$

$v(3) = 27$ m/sec

$a(3) = -6$ m/sec^2

The speed of the object is decreasing.

$v(t) = \dfrac{100t}{2t + 15}$

$$a(t) = v'(t) = \frac{(2t + 15)100 - 100t(2)}{(2t + 15)^2} = \frac{1500}{(2t + 15)^2}$$

(a) $a(5) = \dfrac{1500}{[2(5) + 15]^2} = 2.4$ ft/sec^2

(b) $a(10) = \dfrac{1500}{[2(10) + 15]^2} \approx 1.2$ ft/sec^2

(c) $a(20) = \dfrac{1500}{[2(20) + 15]^2} \approx 0.5$ ft/sec^2

119. $s(t) = -8.25t^2 + 66t$

$v(t) = s'(t) = -16.50t + 66$

$a(t) = v'(t) = -16.50$

t(sec)	0	1	2	3	4
$s(t)$(ft)	0	57.75	99	123.75	132
$v(t) = s'(t)$ (ft / sec)	66	49.5	33	16.5	0
$a(t) = v'(t)$ (ft / sec^2)	–16.5	–16.5	–16.5	–16.5	–16.5

Average velocity on:

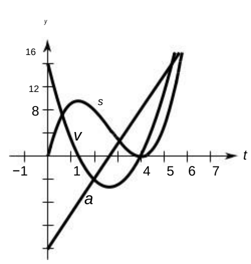
$[0, 1]$ is $\dfrac{57.75 - 0}{1 - 0} = 57.75$

$[1, 2]$ is $\dfrac{99 - 57.75}{2 - 1} = 41.25$

$[2, 3]$ is $\dfrac{123.75 - 99}{3 - 2} = 24.75$

$[3, 4]$ is $\dfrac{132 - 123.75}{4 - 3} = 8.25$

120. (a)

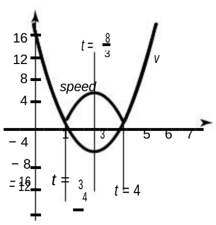


s position function

v velocity function

a acceleration function

The speed of the particle is the absolute value of its velocity. So, the particle's speed is slowing down on the intervals $(0, 4/3)$ and $(8/3, 4)$ and it speeds up on the intervals $(4/3, 8/3)$ and $(4, 6)$.



121. $f(x) = x^n$

$$f^{(n)}(x) = n(n-1)(n-2)\cdots(2)(1) = n!$$

Note: $n! = n(n-1)\cdots 3\cdot 2\cdot 1$ (read "n factorial")

122. $f(x) = \dfrac{1}{x}$

$$f^{(n)}(x) = \frac{(-1)^n n(n-1)(n-2)\cdots(2)(1)}{x^{n+1}} = \frac{(-1)^n n!}{x^{n+1}}$$

123. $f(x) = g(x)h(x)$

$$f'(x) = g(x)h'(x) + h(x)g'(x)$$

$$f''(x) = g(x)h''(x) + g'(x)h'(x) + h(x)g''(x) + h'(x)g'(x)$$
$$= g(x)h''(x) + 2g'(x)h'(x) + h(x)g''(x)$$

$$f'''(x) = g(x)h'''(x) + g'(x)h''(x) + 2g'(x)h''(x) + 2g''(x)h'(x) + h(x)g'''(x) + h'(x)g''(x)$$
$$= g(x)h'''(x) + 3g'(x)h''(x) + 3g''(x)h'(x) + g'''(x)h(x)$$

$$f^{(4)}(x) = g(x)h^{(4)}(x) + g'(x)h'''(x) + 3g'(x)h'''(x) + 3g''(x)h''(x) + 3g''(x)h''(x) + 3g'''(x)h'(x)$$
$$+ g'''(x)h'(x) + g^{(4)}(x)h(x)$$
$$= g(x)h^{(4)}(x) + 4g'(x)h'''(x) + 6g''(x)h''(x) + 4g'''(x)h'(x) + g^{(4)}(x)h(x)$$

$$f^{(n)}(x) = g(x)h^{(n)}(x) + \frac{n(n-1)(n-2)\cdots(2)(1)}{1\left[(n-1)(n-2)\cdots(2)(1)\right]}g'(x)h^{(n-1)}(x) + \frac{n(n-1)(n-2)\cdots(2)(1)}{(2)(1)\left[(n-2)(n-3)\cdots(2)(1)\right]}g''(x)h^{(n-2)}(x)$$
$$+ \frac{n(n-1)(n-2)\cdots(2)(1)}{(3)(2)(1)\left[(n-3)(n-4)\cdots(2)(1)\right]}g'''(x)h^{(n-3)}(x) + \cdots$$
$$+ \frac{n(n-1)(n-2)\cdots(2)(1)}{\left[(n-1)(n-2)\cdots(2)(1)\right](1)}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)$$

$$= g(x)h^{(n)}(x) + \frac{n!}{1!(n-1)!}g'(x)h^{(n-1)}(x) + \frac{n!}{2!(n-2)!}g''(x)h^{(n-2)}(x) + \cdots$$
$$+ \frac{n!}{(n-1)!1!}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)$$

Note: $n! = n(n-1)\cdots 3\cdot 2\cdot 1$ (read "n factorial")

124. $\left[xf(x)\right]' = xf'(x) + f(x)$

$$\left[xf(x)\right]'' = xf''(x) + f'(x) + f'(x) = xf''(x) + 2f'(x)$$

$$\left[xf(x)\right]''' = xf'''(x) + f''(x) + 2f''(x) = xf'''(x) + 3f''(x)$$

In general, $\left[xf(x)\right]^{(n)} = xf^{(n)}(x) + nf^{(n-1)}(x)$.

125. $f(x) = x^n \sin x$

$$f'(x) = x^n \cos x + nx^{n-1}\sin x$$

When $n = 1$: $f'(x) = x\cos x + \sin x$

When $n = 2$: $f'(x) = x^2 \cos x + 2 \sin x$

When $n = 3$: $f'(x) = x^3 \cos x + 3x^2 \sin x$

When $n = 4$: $f'(x) = x^4 \cos x + 4x^3 \sin x$

For general n, $f'(x) = x^n \cos x + nx^{n-1} \sin x$.

126. $f(x) = \dfrac{\cos x}{x^n} = x^{-n}\cos x$

$f'(x) = -x^{-n}\sin x - nx^{-n-1}\cos x$

$= -x^{-n-1}(x\sin x + n\cos x)$

$= -\dfrac{x\sin x + n\cos x}{x^{n+1}}$

When $n = 1$: $f'(x) = -\dfrac{x\sin x + \cos x}{x^2}$

When $n = 2$: $f'(x) = -\dfrac{x\sin x + 2\cos x}{x^3}$

When $n = 3$: $f'(x) = -\dfrac{x\sin x + 3\cos x}{x^4}$

When $n = 4$: $f'(x) = -\dfrac{x\sin x + 4\cos x}{x^5}$

For general n, $f'(x) = -\dfrac{x\sin x + n\cos x}{x^{n+1}}$.

127. $y = \dfrac{1}{x}$, $y' = -\dfrac{1}{x^2}$, $y'' = \dfrac{2}{x^3}$

$x^3y'' + 2x^2y' = x^3\left[\dfrac{2}{x^3}\right] + 2x^2\left[-\dfrac{1}{x^2}\right] = 2 - 2 = 0$

128. $y = 2x^3 - 6x + 10$

$y' = 6x^2 - 6$

$y'' = 12x$

$y''' = 12$

$-y''' - xy'' - 2y' = -12 - x(12x) - 2(6x^2 - 6) = -24x^2$

129. $y = 2\sin x + 3$

$y' = 2\cos x$

$y'' = -2\sin x$

$y'' + y = -2\sin x + (2\sin x + 3) = 3$

130. $y = 3\cos x + \sin x$

$y' = -3\sin x + \cos x$

$y'' = -3\cos x - \sin x$

$y'' + y = (-3\cos x - \sin x) + (3\cos x + \sin x) = 0$

131. False. If $y = f(x)g(x)$, then

$\dfrac{dy}{dx} = f(x)g'(x) + g(x)f'(x)$.

132. True. y is a fourth-degree polynomial.

$\dfrac{d^ny}{dx^n} = 0$ when $n > 4$.

133. True

$h'(c) = f(c)g'(c) + g(c)f'(c)$

$= f(c)(0) + g(c)(0)$

$= 0$

134. True

135. True

136. True

137. $\dfrac{d}{dx}\left[f(x)g(x)h(x)\right] = \dfrac{d}{dx}\left[\left(f(x)g(x)\right)h(x)\right]$

$= \dfrac{d}{dx}\left[f(x)g(x)\right]h(x) + f(x)g(x)h'(x)$

$= \left[f(x)g'(x) + g(x)f'(x)\right]h(x) + f(x)g(x)h'(x)$

$= f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$

138. (a) $(fg' - f'g)' = fg'' + f'g' - f'g' - f''g$

$= fg'' - f''g$ True

(b) $(fg)'' = (fg' + f'g)'$

$= fg'' + f'g' + f'g' + f''g$

$= fg'' + 2f'g' + f''g$

$\neq fg'' + f''g$ False

Section 2.4 The Chain Rule

To find the derivative of the composition of two differentiable functions, take the derivative of the outer function and keep the inner function the same. Then multiply this by the derivative of the inner function.

$$\left[f(g(x))\right]' = f'(g(x))\,g'(x)$$

The (Simple) Power Rule is $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$.

The General Power Rule uses the Chain Rule:

$$\frac{d}{dx}\left(u^n\right) = nu^{n-1}\frac{du}{dx}.$$

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
3. $y = (6x - 5)^4$	$u = 6x - 5$	$y = u^4$
4. $y = \sqrt[3]{4x + 3}$	$u = 4x + 3$	$y = u^{1/3}$
5. $y = \dfrac{1}{3x + 5}$	$u = 3x + 5$	$y = u^{-1}$
6. $y = \dfrac{2}{\sqrt{x^2 + 10}}$	$u = x^2 + 10$	$y = \dfrac{2}{\sqrt{u}}$
7. $y = \csc^3 x$	$u = \csc x$	$y = u^3$
8. $y = \sin \dfrac{5x}{2}$	$u = \dfrac{5x}{2}$	$y = \sin u$

9. $y = (2x - 7)^3$

$$y' = 3(2x - 7)^2(2) = 6(2x - 7)^2$$

$y = 5(2 - x^3)^4$

$$y' = 5(4)(2 - x^3)^3(-3x^2) = -60x^2(2 - x^3)^3 = 60x^2(x^3 - 2)^3$$

$g(x) = 3(4 - 9x)^{5/6}$

$$g'(x) = 3\left(\frac{5}{6}\right)(4 - 9x)^{-1/6}(-9) = \frac{-45}{2(4 - 9x)^{1/6}} = \frac{-45}{2\sqrt[6]{4 - 9x}}$$

$f(t) = (9t + 2)^{2/3}$

$$f'(t) = \frac{2}{3}(9t + 2)^{-1/3}(9) = \frac{6}{\sqrt[3]{9t + 2}}$$

$y = \sqrt[3]{6x^2 + 1} = (6x^2 + 1)^{1/3}$

$$y' = \frac{1}{3}(6x^2 + 1)^{-2/3}(12x) = \frac{4x}{(6x^2 + 1)^{2/3}} = \frac{4x}{\sqrt[3]{(6x^2 + 1)^2}}$$

16. $y = 2\sqrt[4]{9 - x^2} = 2(9 - x^2)^{1/4}$

$$y' = 2\left(\frac{1}{4}\right)(9 - x^2)^{-3/4}(-2x) = \frac{-x}{(9 - x^2)^{3/4}} = \frac{-x}{\sqrt[4]{(9 - x^2)^3}}$$

$y = (x - 2)^{-1}$

$$y' = -1(x - 2)^{-2}(1) = \frac{-1}{(x - 2)^2}$$

18. $s(t) = \dfrac{1}{4 - 5t - t^2} = (4 - 5t - t^2)^{-1}$

$$s'(t) = -(4 - 5t - t^2)^{-2}(-5 - 2t) = \frac{5 + 2t}{(4 - 5t - t^2)^2} = \frac{2t + 5}{(t^2 + 5t - 4)^2}$$

13. $h(s) = -2\sqrt{5s^2 + 3} = -2\left(5s^2 + 3\right)^{1/2}$

$$h'(s) = -2\left(\frac{1}{2}\right)\left(5s^2 + 3\right)^{-1/2}(10s)$$

$$= \frac{-10s}{\left(5s^2 + 3\right)^{1/2}} = -\frac{10s}{\sqrt{5s^2 + 3}}$$

$g(x) = \sqrt{4 - 3x^2} = \left(4 - 3x^2\right)^{1/2}$

$$g'(x) = \frac{1}{2}\left(4 - 3x^2\right)^{-1/2}(-6x) = -\frac{3x}{\sqrt{4 - 3x^2}}$$

19. $g(s) = \dfrac{6}{\left(s^3 - 2\right)^3} = 6\left(s^3 - 2\right)^{-3}$

$$g'(s) = 6(-3)\left(s^3 - 2\right)^{-4}\left(3s^2\right)$$

$$= -\frac{54s^2}{\left(s^3 - 2\right)^4}$$

20. $y = -\dfrac{3}{(t-2)^4} = -3(t-2)^{-4}$

$y' = 12(t-2)^{-5} = \dfrac{12}{(t-2)^5}$

21. $y = \dfrac{1}{\sqrt{3x+5}} = (3x+5)^{-1/2}$

$y' = -\dfrac{1}{2}(3x+5)^{-3/2}(3)$

$= \dfrac{-3}{2(3x+5)^{3/2}}$

$= -\dfrac{3}{2\sqrt{(3x+5)^3}}$

22. $g(t) = \dfrac{1}{\sqrt{t^2-2}} = (t^2-2)^{-1/2}$

$g'(t) = -\dfrac{1}{2}(t^2-2)^{-3/2}(2t)$

$= \dfrac{-t}{(t^2-2)^{3/2}}$

$= -\dfrac{t}{\sqrt{(t^2-2)^3}}$

$f(x) = x^2(x-2)^7$

$f'(x) = 2x(x-2)^7 + 7(x-2)^6 x^2$

$= x(x-2)^6\left[2(x-2) + 7x\right]$

$= x(x-2)^6(9x-4)$

$f(x) = x(2x-5)^3$

$y = x^2\sqrt{16-x^2} = x^2(16-x^2)^{1/2}$

$y' = 2x(16-x^2)^{1/2} + \dfrac{1}{2}(16-x^2)^{-1/2}(-2x)x^2$

$= \dfrac{x}{(16-x^2)^{1/2}}\left[2(16-x^2) - x^2\right]$

$= \dfrac{x(32-3x^2)}{\sqrt{16-x^2}}$

27. $y = \dfrac{x}{\sqrt{x^2+1}} = \dfrac{x}{(x^2+1)^{1/2}}$

$y' = \dfrac{(x^2+1)^{1/2}(1) - x\left(\frac{1}{2}\right)(x^2+1)^{-1/2}(2x)}{\left[(x^2+1)^{1/2}\right]^2}$

$= \dfrac{(x^2+1)^{1/2} - x^2(x^2+1)^{-1/2}}{x^2+1}$

$= \dfrac{(x^2+1)^{-1/2}\left[x^2+1-x^2\right]}{x^2+1}$

$= \dfrac{1}{(x^2+1)^{3/2}} = \dfrac{1}{\sqrt{(x^2+1)^3}}$

28. $y = \dfrac{x}{\sqrt{x^4+4}}$

$y' = \dfrac{(x^4+4)^{1/2}(1) - x\left(\frac{1}{2}\right)(x^4+4)^{-1/2}(4x^3)}{x^4+4}$

$= \dfrac{x^4+4-2x^4}{(x^4+4)^{3/2}} = \dfrac{4-x^4}{(x^4+4)^{3/2}} = \dfrac{4-x^4}{\sqrt{(x^4+4)^3}}$

$$f'(x) = x(3)(2x-5)^2(2) + (2x-5)^3(1)$$
$$= (2x-5)^2\left[6x + (2x-5)\right]$$
$$= (2x-5)^2(8x-5)$$

25. $y = x\sqrt{1-x^2} = x\left(1-x^2\right)^{1/2}$

$$y' = x\left[\frac{1}{2}\left(1-x^2\right)^{-1/2}(-2x)\right] + \left(1-x^2\right)^{1/2}(1)$$
$$= -x^2\left(1-x^2\right)^{-1/2} + \left(1-x^2\right)^{1/2}$$
$$= \left(1-x^2\right)^{-1/2}\left[-x^2 + \left(1-x^2\right)\right]$$
$$= \frac{1-2x^2}{\sqrt{1-x^2}}$$

$$g(x) = \left(\frac{x+5}{x^2+2}\right)^2$$

$$g'(x) = 2\left(\frac{x+5}{x^2+2}\right)\left[\frac{\left(x^2+2\right) - (x+5)(2x)}{\left(x^2+2\right)^2}\right]$$
$$= \frac{2(x+5)\left(2-10x-x^2\right)}{\left(x^2+2\right)^3}$$
$$= \frac{-2(x+5)\left(x^2+10x-2\right)}{\left(x^2+2\right)^3}$$

30. $h(t) = \left(\dfrac{t^2}{t^3+2}\right)^2$

$$h'(t) = 2\left(\frac{t^2}{t^3+2}\right)\left(\frac{(t^3+2)(2t) - t^2(3t^2)}{(t^3+2)^2}\right)$$

$$= \frac{2t^2(4t - t^4)}{(t^3+2)^3} = \frac{2t^3(4 - t^3)}{(t^3+2)^3}$$

31. $s(t) = \left(\dfrac{1+t}{t+3}\right)^4$

$$s'(t) = 4\left(\frac{1+t}{t+3}\right)^3\left[\frac{(t+3)(1) - (1+t)(1)}{(t+3)^2}\right]$$

$$= \frac{4(1+t)^3(2)}{(t+3)^5}$$

$$= \frac{8(t+1)^3}{(t+3)^5}$$

32. $g(x) = \left(\dfrac{3x^2-2}{2x+3}\right)^{-2} = \left(\dfrac{2x+3}{3x^2-2}\right)^2$

$$g'(x) = 2\left(\frac{2x+3}{3x^2-2}\right)\left[\frac{(3x^2-2)(2) - (2x+3)(6x)}{(3x^2-2)^2}\right]$$

$$= \frac{2(2x+3)(-6x^2-18x-4)}{(3x^2-2)^3}$$

$$= \frac{-4(2x+3)(3x^2+9x+2)}{(3x^2-2)^3}$$

$$f(x) = \left((x^2+3)^5 + x\right)^2$$

$$f'(x) = 2\left((x^2+3)^5 + x\right)\left(5(x^2+3)^4(2x) + 1\right)$$

$$= 2\left[10x(x^2+3)^9 + (x^2+3)^5 + 10x^2(x^2+3)^4 + x\right] = 20x(x^2+3)^9 + 2(x^2+3)^5 + 20x^2(x^2+3)^4 + 2x$$

$$g(x) = \left(2 + (x^2+1)^4\right)^3$$

$$g'(x) = 3\left(2 + (x^2+1)^4\right)^2\left(4(x^2+1)^3(2x)\right) = 24x(x^2+1)^3\left(2 + (x^2+1)^4\right)^2$$

$y = \cos 4x$

$\dfrac{dy}{dx} = -4\sin 4x$

$y = \sin \pi x$

$\dfrac{dy}{dx} = \pi \cos \pi x$

$g(x) = 5\tan 3x$

$g'(x) = 15\sec^2 3x$

$h(x) = \sec 6x$

$y = \csc(1-2x)^2$

$$y' = -\csc(1-2x)^2\cot(1-2x)^2\left[2(1-2x)(-2)\right]$$

$$= 4(1-2x)\csc(1-2x)^2\cot(1-2x)^2$$

$h(x) = \sin 2x\cos 2x$

$$h'(x) = \sin 2x(-2\sin 2x) + \cos 2x(2\cos 2x)$$

$$= 2\cos^2 2x - 2\sin^2 2x$$

$$= 2\cos 4x$$

Alternate solution: $h(x) = \frac{1}{2}\sin 4x$

$$h'(x) = \sec 6x \tan 6x\,(6)$$
$$= 6 \sec 6x \tan 6x$$

$$y = \sin(\pi x)^2 = \sin(\pi^2 x^2)$$

$$y' = \cos(\pi^2 x^2)\left[2\pi^2 x\right] = 2\pi^2 x \cos(\pi^2 x^2)$$
$$= 2\pi^2 x \cos(\pi x)^2$$

$$h'(x) = \frac{1}{2}\cos 4x\,(4) = 2\cos 4x$$

$$g(\theta) = \sec\left(\tfrac{1}{2}\theta\right)\tan\left(\tfrac{1}{2}\theta\right)$$

$$g'(\theta) = \sec\left(\tfrac{1}{2}\theta\right)\left[\sec^2\left(\tfrac{1}{2}\theta\right)\left(\tfrac{1}{2}\right)\right] + \tan\left(\tfrac{1}{2}\theta\right)\left[\sec\left(\tfrac{1}{2}\theta\right)\tan\left(\tfrac{1}{2}\theta\right)\left(\tfrac{1}{2}\right)\right]$$
$$= \frac{1}{2}\sec\left(\tfrac{1}{2}\theta\right)\left[\sec^2\left(\tfrac{1}{2}\theta\right) + \tan^2\left(\tfrac{1}{2}\theta\right)\right]$$

$f(x) = \dfrac{\cot x}{\sin x} = \dfrac{\cos x}{\sin^2 x}$

$f'(x) = \dfrac{\sin^2 x(-\sin x) - \cos x(2 \sin x \cos x)}{\sin^4 x}$

$= \dfrac{-\sin^2 x - 2\cos^2 x}{\sin^3 x} = \dfrac{-1 - \cos^2 x}{\sin^3 x}$

$g(v) = \dfrac{\cos v}{\csc v} = \cos v \cdot \sin v$

$g'(v) = \cos v(\cos v) + \sin v(-\sin v)$

$\cos^2 v - \sin^2 v = \cos 2v$

$y = 4 \sec^2 x$

$y' = 8 \sec x \cdot \sec x \tan x = 8 \sec^2 x \tan x$

$g(t) = 5 \cos^2 \pi t = 5(\cos \pi t)^2$

$g'(t) = 10 \cos \pi t(-\sin \pi t)(\pi)$

$-10\pi (\sin \pi t)(\cos \pi t)$

$-5\pi \sin 2\pi t$

$f(\theta) = \frac{1}{4} \sin^2 2\theta = \frac{1}{4}(\sin 2\theta)^2$

$f'(\theta) = 2\left(\frac{1}{4}\right)(\sin 2\theta)(\cos 2\theta)(2)$

$\sin 2\theta \cos 2\theta = \frac{1}{2} \sin 4\theta$

$h(t) = 2 \cot^2(\pi t + 2)$

$h'(t) = 4 \cot(\pi t + 2)\left[-\csc^2(\pi t + 2)(\pi)\right]$

$-4\pi \cot(\pi t + 2) \csc^2(\pi t + 2)$

$f(t) = 3 \sec(\pi t - 1)^2$

$f'(t) = 3 \sec(\pi t - 1)^2 \tan(\pi t - 1)^2 (2)(\pi t - 1)(\pi)$

$6\pi(\pi t - 1) \sec(\pi t - 1)^2 \tan(\pi t - 1)^2$

$y = 5 \cos(\pi x)^2$

$y' = -5 \sin(\pi x)^2 (2)(\pi x)(\pi)$

$-10\pi^2 x \sin(\pi x)^2$

$y = \sin(3x^2 + \cos x)$

$y' = \cos(3x^2 + \cos x)(6x - \sin x)$

$y = \cos(5x + \csc x)$

$y' = -\sin(5x + \csc x)(5 - \csc x \cot x)$

$y = \sin\sqrt{\cot 3\pi x} = \sin(\cot 3\pi x)^{1/2}$

$y' = \cos(\cot 3\pi x)^{1/2}\left[\frac{1}{2}(\cot 3\pi x)^{-1/2}(-\csc^2 3\pi x)(3\pi)\right]$

$-\dfrac{3\pi \cos\left(\sqrt{\cot 3\pi x}\right) \csc^2(3\pi x)}{2\sqrt{\cot 3\pi x}}$

54. $y = \cos\sqrt{\sin(\tan \pi x)}$

$y' = -\sin\sqrt{\sin(\tan \pi x)} \cdot \frac{1}{2}(\sin(\tan \pi x))^{-1/2} \cos(\tan \pi x) \sec^2 \pi x(\pi) = \dfrac{-\pi \sin\sqrt{\sin(\tan \pi x)} \cos(\tan \pi x) \sec^2 \pi x}{2\sqrt{\sin(\tan \pi x)}}$

$y = \dfrac{\sqrt{x} + 1}{x^2 + 1}$

$y' = \dfrac{1 - 3x^2 - 4x^{3/2}}{2\sqrt{x}(x^2 + 1)^2}$

The zero of y' corresponds to the point on the graph of y where the tangent line is horizontal.

2

y

−1 5

$y = \sqrt{\dfrac{2x}{x + 1}}$

7

y

$y' = \dfrac{1}{\sqrt{2x}(x + 1)^{3/2}}$

y'

−6 6

y' has no zeros.

1

$y = \sqrt{\dfrac{x + 1}{x}}$

4

$y' = -\dfrac{\sqrt{(x + 1)/x}}{2x(x + 1)}$

y

−5 4

2 y' has no zeros. − 2

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58. $g(x) = \sqrt{x-1} + \sqrt{x+1}$

$g'(x) = \dfrac{1}{2\sqrt{x-1}} + \dfrac{1}{2\sqrt{x+1}}$

g' has no zeros.

6

g

$'$

−2 10

2

$y = \dfrac{\cos \pi x + 1}{x}$

$\dfrac{dy}{dx} = \dfrac{-\pi x \sin \pi x - \cos \pi x - 1}{x^2}$

$= -\dfrac{\pi x \sin \pi x + \cos \pi x + 1}{x^2}$

The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.

3

y

−5 5

$'$

3

$y = x^2 \tan \dfrac{1}{x}$

$\dfrac{dy}{dx} = 2x \tan \dfrac{1}{x} - \sec^2 \dfrac{1}{x}$

The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.

6

y

−4 5

$'$

6

$y = \sin 3x$ $y' = 3 \cos 3x$

$y'(0) = 3$

3 cycles in $[0, 2\pi]$

$y = \sin \dfrac{x}{2}$

$y' = \dfrac{1}{2} \cos \dfrac{x}{2}$

$\dfrac{1}{2}$ cycle in $[0, 2\pi]$

$y = \sqrt{x^2 + 8x} = (x^2 + 8x)^{1/2}$, $(1, 3)$

$y' = \dfrac{1}{2}(x^2 + 8x)^{-1/2}(2x + 8) = \dfrac{2(x+4)}{2\sqrt{x^2 + 8x}} = \dfrac{x + 4}{\sqrt{x^2 + 8x}}$

$y'(1) = \dfrac{1 + 4}{\sqrt{1^2 + 8(1)}} = \dfrac{5}{\sqrt{9}} = \dfrac{5}{3}$

64. $y = (3x^3 + 4x)^{1/5}$, $(2, 2)$

$y' = \dfrac{1}{5}(3x^3 + 4x)^{-4/5}(9x^2 + 4)$

$= \dfrac{9x^2 + 4}{5(3x^3 + 4x)^{4/5}}$

$y'(2) = \dfrac{1}{2}$

65. $f(x) = 5(x^3 - 2)^{-1}$, $\left(-2, -\dfrac{1}{2}\right)$

$f'(x) = -5(x^3 - 2)^{-2}(3x^2) = \dfrac{-15x^2}{(x^3 - 2)^2}$

66. $f(x) = \dfrac{1}{(x^2 - 3x)^2} = (x^2 - 3x)^{-2}$, $\left(4, \dfrac{1}{16}\right)$

$f'(x) = -2(x^2 - 3x)^{-3}(2x - 3) = \dfrac{-2(2x - 3)}{(x^2 - 3x)^3}$

$f'(4) = -\dfrac{5}{32}$

67. $y = \dfrac{4}{(x + 2)^2} = 4(x + 2)^{-2}$, $(0, 1)$

$y' = -8(x + 2)^{-3} = \dfrac{-8}{(x + 2)^3}$

$y'(0) = -\dfrac{8}{8} = -1$

8

$$f'(-2) = -\frac{60}{100} = -\frac{3}{5}$$

$y = \dfrac{4}{(x^2 - 2x)^3} = 4(x^2 - 2x)^{-3}, \ (1, -4)$

68. $y' = -12(x^2 - 2x)^{-4}(2x - 2)$

$y'(1) = -12(-1)^{-4}(0) = 0$

69. $y = 26 - \sec^3 4x, \quad (0, 25)$

$y' = -3\sec^2 4x \sec 4x \tan 4x \, 4$

$= -12\sec^3 4x \tan 4x$

$y'(0) = 0$

70. $y = \dfrac{1}{x} + \sqrt{\cos x} = x^{-1} + (\cos x)^{1/2}, \ \left(\dfrac{\pi}{2}, \dfrac{2}{\pi}\right)$

$y' = -x^{-2} + \dfrac{1}{2}(\cos x)^{-1/2}(-\sin x) = -\dfrac{1}{x^2} - \dfrac{\sin x}{2\sqrt{\cos x}}$

$y'(\pi/2)$ is undefined.

71. (a) $f(x) = (2x^2 - 7)^{1/2}, \ (4, 5)$

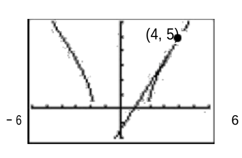
$f'(x) = \dfrac{1}{2}(2x^2 - 7)^{-1/2}(4x) = \dfrac{2x}{\sqrt{2x^2 - 7}}$

$f'(4) = \dfrac{8}{5}$

Tangent line:

$y - 5 = \dfrac{8}{5}(x - 4) \Rightarrow 8x - 5y - 7 = 0$

(b)



72. (a) $f(x) = \dfrac{1}{3}x\sqrt{x^2 + 5} = \dfrac{1}{3}x(x^2 + 5)^{1/2}, \ (2, 2)$

73. (a) $y = (4x^3 + 3)^2, \quad (-1, 1)$

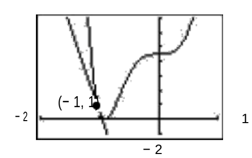
$y' = 2(4x^3 + 3)(12x^2) = 24x^2(4x^3 + 3)$

$y'(-1) = -24$

Tangent line:

$y - 1 = -24(x + 1) \Rightarrow 24x + y + 23 = 0$

(b)



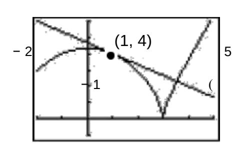
74. (a) $f(x) = (9 - x^2)^{2/3}, \quad (1, 4)$

$f'(x) = \dfrac{2}{3}(9 - x^2)^{-1/3}(-2x) = \dfrac{-4x}{3(9 - x^2)^{1/3}}$

$f'(1) = \dfrac{-4}{3(8)^{1/3}} = -\dfrac{2}{3}$

Tangent line:

$y - 4 = -\dfrac{2}{3}(x - 1) \Rightarrow 2x + 3y - 14 = 0$



75. (a) $f(x) = \sin 8x, \quad (\pi, 0)$

$f'(x) = 8\cos 8x$

$f'(\pi) = 8$

Tangent line: $y = 8(x - \pi) = 8x - 8\pi$

(b) 2

$(\pi, 0)$

$$f'(x) = \frac{1}{3}x\left[\frac{1}{2}\left(x^2+5\right)^{-1/2}(2x)\right] + \frac{1}{3}\left(x^2+5\right)^{1/2}$$

$$= \frac{x^2}{2\sqrt{x^2+5}} + \frac{1}{3}\sqrt{x^2+5}$$

$$f'(2) = \frac{4}{3(3)} + \frac{1}{3}(3) = \frac{13}{9}$$

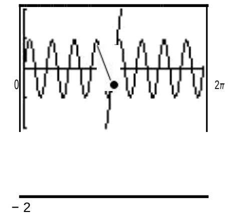
Tangent line:

$$y - 2 = \frac{13}{9}(x - 2) \Rightarrow 13x - 9y - 8 = 0$$

6

−9 9

6



76. (a) $y = \cos 3x, \left(\frac{\pi}{4}, -\frac{\sqrt{2}}{2}\right)$

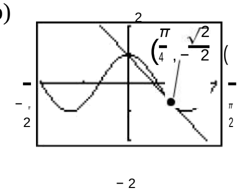
$y' = -3\sin 3x$

$y'\left(\frac{\pi}{4}\right) = -3\sin\left(\frac{3\pi}{4}\right) = \frac{-3\sqrt{2}}{2}$

Tangent line: $y + \frac{\sqrt{2}}{2} = \frac{-3\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$y = \frac{-3\sqrt{2}}{2}x + \frac{3\sqrt{2}\pi}{8} - \frac{\sqrt{2}}{2}$

(b)



77. (a) $f(x) = \tan^2 x, \left(\frac{\pi}{4}, 1\right)$

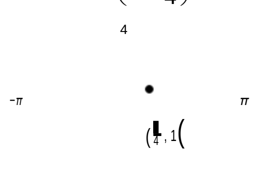
$f'(x) = 2\tan x\sec^2 x$

$f'\left(\frac{\pi}{4}\right) = 2(1)(2) = 4$

Tangent line:

$y - 1 = 4\left(x - \frac{\pi}{4}\right) \Rightarrow 4x - y + (1 - \pi) = 0$

(b)



78. (a) $y = 2\tan^3 x, \left(\frac{\pi}{4}, 2\right)$

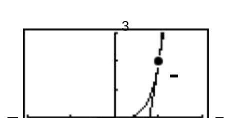
$y' = 6\tan^2 x \cdot \sec^2 x$

$y'\left(\frac{\pi}{4}\right) = 6(1)(2) = 12$

Tangent line:

$y - 2 = 12\left(x - \frac{\pi}{4}\right) \Rightarrow 12x - y + (2 - 3\pi) = 0$

(b)



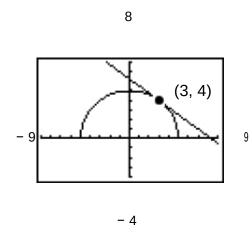
79. $f(x) = \sqrt{25 - x^2} = \left(25 - x^2\right)^{1/2}, \quad (3, 4)$

$f'(x) = \frac{1}{2}\left(25 - x^2\right)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}$

$f'(3) = -\frac{3}{4}$

Tangent line:

$y - 4 = -\frac{3}{4}(x - 3) \Rightarrow 3x + 4y - 25 = 0$

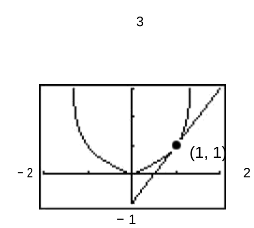


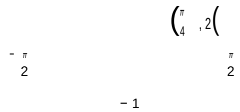
80. $f(x) = \frac{|x|}{\sqrt{2 - x^2}} = |x|\left(2 - x^2\right)^{-1/2}, \quad (1, 1)$

$f'(x) = \frac{2}{\left(2 - x^2\right)^{3/2}}$ for $x > 0$

$f'(1) = 2$

Tangent line: $y - 1 = 2(x - 1) \Rightarrow 2x - y - 1 = 0$




(π/4, 2)
−π/2
π/2
−1

81. $f(x) = 2\cos x + \sin 2x, \quad 0 < x < 2\pi$

$f'(x) = -2\sin x + 2\cos 2x$

$= -2\sin x + 2 - 4\sin^2 x = 0$

$2\sin^2 x + \sin x - 1 = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x = -1 \Rightarrow x = \frac{3\pi}{2}$

$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

Horizontal tangents at $x = \frac{\pi}{6}, \frac{3\pi}{2}, \frac{5\pi}{6}$

Horizontal tangent at the points $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right), \left(\frac{3\pi}{2}, 0\right)$, and $\left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{2}\right)$

82. $f(x) = \frac{-4x}{\sqrt{2x-1}}$

$f'(x) = \frac{(2x-1)^{1/2}(-4) + 4x\left(\frac{1}{2}\right)(2x-1)^{-1/2}(2)}{2x-1}$

$= \frac{(2x-1)(-4) + 4x}{(2x-1)^{3/2}}$

$= \frac{4-4x}{(2x-1)^{3/2}}$

$f'(x) = 0 = \frac{4-4x}{(2x-1)^{3/2}} \Rightarrow x = 1$

Horizontal tangent at $(1, -4)$

$f(x) = 5(2-7x)^4$

$f'(x) = 20(2-7x)^3(-7) = -140(2-7x)^3$

$f''(x) = -420(2-7x)^2(-7) = 2940(2-7x)^2$

$f(x) = 6(x^3+4)^3$

$f'(x) = 18(x^3+4)^2(3x^2) = 54x^2(x^3+4)^2$

$f''(x) = 54x^2(2)(x^3+4)(3x^2) + 108x(x^3+4)^2$

$= 108x(x^3+4)\left[3x^3 + x^3 + 4\right]$

$= 432x(x^3+4)(x^3+1)$

$f(x) = \sec^2 \pi x$

85. $f(x) = \frac{1}{11x-6} = (11x-6)^{-1}$

$f'(x) = -(11x-6)^{-2}(11)$

$f''(x) = -22(11x-6)^{-3}(11)$

$= 242(11x-6)^{-3}$

$= \frac{242}{(11x-6)^3}$

86. $f(x) = \frac{8}{(x-2)^2} = 8(x-2)^{-2}$

$f'(x) = -16(x-2)^{-3}$

$f''(x) = 48(x-2)^{-4} = \frac{48}{(x-2)^4}$

87. $f(x) = \sin x^2$

$f'(x) = 2x\cos x^2$

$f''(x) = 2x\left[2x\left(-\sin x^2\right)\right] + 2\cos x^2$

$= 2\cos x^2 - 2x^2 \sin x^2$

$f'(x) = 2\sec \pi x(\pi \sec \pi x \tan \pi x)$

$2\pi \sec^2 \pi x \tan \pi x$

$f''(x) = 2\pi \sec^2 \pi x(\sec^2 \pi x)(\pi) + 2\pi \tan\pi x(2\pi \sec^2 \pi x \tan \pi x)$

$2\pi^2 \sec^4 \pi x + 4\pi^2 \sec^2 \pi x \tan^2 \pi x$

$2\pi^2 \sec^2 \pi x(\sec^2 \pi x + 2 \tan^2 \pi x)$

$2\pi^2 \sec^2 \pi x(3 \sec^2 \pi x - 2)$

89. $h(x) = \frac{1}{9}(3x+1)^3, \left(1, \frac{64}{9}\right)$

$h'(x) = \frac{1}{9}3(3x+1)^2(3) = (3x+1)^2$

$h''(x) = 2(3x+1)(3) = 18x + 6$

$h''(1) = 24$

90. $f(x) = \frac{1}{\sqrt{x+4}} = (x+4)^{-1/2}, \left(0, \frac{1}{2}\right)$

$f'(x) = -\frac{1}{2}(x+4)^{-3/2}$

$f''(x) = \frac{3}{4}(x+4)^{-5/2} = \frac{3}{4(x+4)^{5/2}}$

$f''(0) = \frac{3}{128}$

91. $f(x) = \cos x^2, (0, 1)$

$f'(x) = -\sin(x^2)(2x) = -2x\sin(x^2)$

$f''(x) = -2x\cos(x^2)(2x) - 2\sin(x^2)$

$= -4x^2\cos(x^2) - 2\sin(x^2)$

$f''(0) = 0$

92. $g(t) = \tan 2t, \left(\frac{\pi}{6}, \sqrt{3}\right)$

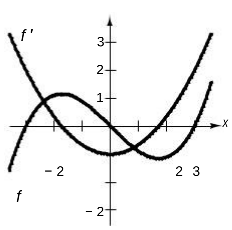
$g'(t) = 2\sec^2(2t)$

$g''(t) = 4\sec(2t)\cdot\sec(2t)\tan(2t)(2)$

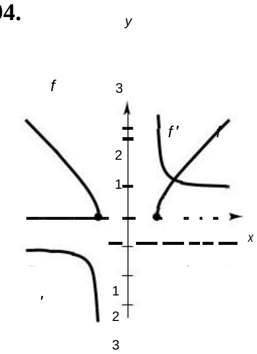
$= 8\sec^2(2t)\tan(2t)$

$g''\left(\frac{\pi}{6}\right) = 32\sqrt{3}$

93.



94.



f is decreasing on $(-\infty, -1)$ so f' must be negative there.

f is increasing on $(1, \infty)$ so f' must be positive there.

95. (a) $g(x) = f(3x)$

$g'(x) = f'(3x)(3) \Rightarrow g'(x) = 3f'(3x)$

The rate of change of g is three times as fast as the rate of change of f.

(b) $g(x) = f(x^2)$

$g'(x) = f'(x^2)(2x) \Rightarrow g'(x) = 2xf'(x^2)$

The rate of change of g is $2x$ times as fast as the rate of change of f.

96. $r(x) = \frac{2x-5}{(3x+1)^2}$

If $h(x) = \frac{f(x)}{g(x)}$, then write $h(x) = f(x)(g(x))^{-1}$ and use the Product Rule.

$r(x) = (2x-5)(3x+1)^{-2}$

$r'(x) = (2x-5)(-2)(3x+1)^{-3}(3) + (3x+1)^{-2}(2)$

$= \frac{-6(2x-5) + 2(3x+1)}{(3x+1)^3}$

$= \frac{-6x+32}{(3x+1)^3}$

The zeros of f' correspond to the points where the graph of f has horizontal tangents.

$$r'(x) = \frac{(3x+1)^2(2) - (2x-5)(2)(3x+1)(3)}{(3x+1)^4}$$

$$= \frac{(3x+1)(2) - 6(2x-5)}{(3x+1)^3}$$

$$= \frac{-6x+32}{(3x+1)^3}$$

(d) Answers will vary.

97. (a) $g(x) = f(x) - 2 \Rightarrow g'(x) = f'(x)$

$h(x) = 2f(x) \Rightarrow h'(x) = 2f'(x)$

(c) $r(x) = f(-3x) \Rightarrow r'(x) = f'(-3x)(-3) = -3f'(-3x)$

So, you need to know $f'(-3x)$.

$r'(0) = -3f'(0) = (-3)\left(-\frac{1}{3}\right) = 1$

$r'(-1) = -3f'(3) = -3(-4) = 12$

(d) $s(x) = f(x+2) \Rightarrow s'(x) = f'(x+2)$

So, you need to know $f'(x+2)$.

x $f'(x)$	-2	$\frac{2}{3}$ -1	0 $-\frac{1}{3}$	1	2	3
$g'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$h'(x)$	4	$\frac{4}{3}$	$-\frac{2}{3}$	-1	-2	-4
$r'(x)$		12	1			
$s'(x)$	$-\frac{1}{3}$					

-1 -2 -4

$s'(-2) = f'(0) = -\frac{1}{3}$, etc.

(a) $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + g'(x)h(x)$

$f'(5) = (-3)(-2) + (6)(3) = 24$

$f(x) = g(h(x))$

$f'(x) = g'(h(x))h'(x)$

$f'(5) = g'(3)(-2) = -2g'(3)$

Not possible, you need $g'(3)$ to find $f'(5)$.

$f(x) = \frac{g(x)}{h(x)}$

$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$

$f'(5) = \frac{(3)(6) - (-3)(-2)}{(3)^2} = \frac{12}{9} = \frac{4}{3}$

$f(x) = [g(x)]^3$

$f'(x) = 3[g(x)]^2 g'(x)$

$f'(5) = 3(-3)^2(6) = 162$

99. (a) $h(x) = f(g(x)),\ g(1) = 4,\ g'(1) = -\frac{1}{2},\ f'(4) = -1$

$h'(x) = f'(g(x))g'(x)$

$h'(1) = f'(g(1))g'(1) = f'(4)g'(1) = (-1)\left(-\frac{1}{2}\right) = \frac{1}{2}$

$s(x) = g(f(x)),\ f(5) = 6,\ f'(5) = -1,\ g'(6)$ does not exist. $s'(x) = g'(f(x))f'(x)$

$s'(5) = g'(f(5))f'(5) = g'(6)(-1)$

$s'(5)$ does not exist because g is not differentiable at 6.

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(a) $h(x) = f(g(x))$

$h'(x) = f'(g(x))g'(x)$

$h'(3) = f'(g(3))g'(3) \quad f'(5)(1) = \frac{1}{2}$

$s(x) = g(f(x))$

$s'(x) = g'(f(x))f'(x)$

$s'(9) = g'(f(9))f'(9) \quad g'(8)(2) = (-1)(2) = -2$

(a) $F = 132{,}400(331 - v)^{-1}$

$F' = (-1)(132{,}400)(331 - v)^{-2}(-1) = \frac{132{,}400}{(331 - v)^2}$

When $v = 30$, $F' \approx 1.461$.

(b) $F = 132{,}400(331 + v)^{-1}$

$F' = (-1)(132{,}400)(331 + v)^{-2}(1) = \frac{-132{,}400}{(331 + v)^2}$

When $v = 30$, $F' \approx -1.016$.

102. $y = \frac{1}{3}\cos 12t - \frac{1}{4}\sin 12t$

$v = y' = \frac{1}{3}[-12\sin 12t] - \frac{1}{4}[12\cos 12t]$

$-4\sin 12t - 3\cos 12t$

When $t = \pi/8$, $y = 0.25$ ft and $v = 4$ ft/sec.

$\theta = 0.2\cos 8t$

The maximum angular displacement is $\theta = 0.2$ (because $-1 \le \cos 8t \le 1$).

$\frac{d\theta}{dt} = 0.2[-8\sin 8t] = -1.6\sin 8t$

When $t = 3$, $d\theta/dt = -1.6\sin 24 \approx 1.4489$ rad/sec.

$y = A\cos \omega t$

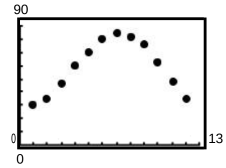
Amplitude: $A = \frac{3.5}{2} = 1.75$

$y = 1.75\cos \omega t$

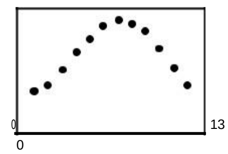
Period: 10 $\Rightarrow \omega = \frac{2\pi}{10} = \frac{\pi}{5}$

$= 1.75\cos\frac{\pi t}{5}$

105. (a) $T(t) = 27.3\sin(0.49t - 1.90) + 57.1$

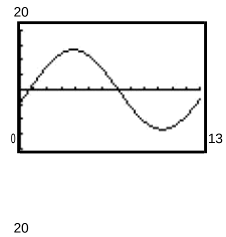


(b)



The model is a good fit.

$T'(t) = 13.377\cos(0.49t - 1.90)$



The temperature changes most rapidly around spring (March–May) and fall (Oct.–Nov.). The temperature changes most slowly around winter (Dec.–Feb.) and summer (Jun.–Aug.). Yes. Explanations will vary.

(a) According to the graph $C'(4) > C'(1)$.

Answers will vary.

107. $N = 400\left[1 - \frac{3}{(t^2 + 2)^2}\right] = 400 - 1200(t^2 + 2)^{-2}$

$N'(t) = 2400(t^2 + 2)^{-3}(2t) = \frac{4800t}{(t^2 + 2)^3}$

(a) $N'(0) = 0$ bacteria/day

(b) $N'(1) = \frac{4800(1)}{(1 + 2)^3} = \frac{4800}{27} \approx 177.8$ bacteria/day

(c) $N'(2) = \frac{4800(2)}{(4 + 2)^3} = \frac{9600}{216} \approx 44.4$ bacteria/day

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(b) $v = y' = 1.75\left[-\frac{\pi}{5}\sin\frac{\pi t}{5}\right] = -0.35\pi \sin\frac{\pi t}{5}$

(d) $N'(3) = \frac{4800(3)}{(9+2)^3} = \frac{14{,}400}{1331} \approx 10.8$ bacteria/day

(e) $N'(4) = \frac{4800(4)}{(16+2)^3} = \frac{19{,}200}{5832} \approx 3.3$ bacteria/day

(f) The rate of change of the population is decreasing as $t \to \infty$.

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108. (a) $V = \dfrac{k}{\sqrt{t+1}}$

$V(0) = 10{,}000 = \dfrac{k}{\sqrt{0+1}} = k$

$V = \dfrac{10{,}000}{\sqrt{t+1}} = 10{,}000(t+1)^{-1/2}$

(b) $\dfrac{dV}{dt} = 10{,}000\left(-\dfrac{1}{2}\right)(t+1)^{-3/2} = \dfrac{-5000}{(t+1)^{3/2}}$

$V'(1) = \dfrac{-5000}{2^{3/2}} \approx -1767.77$ dollars/year

(c) $V'(3) = \dfrac{-5000}{4^{3/2}} = \dfrac{-5000}{8} = -625$ dollars/year

$f(x) = \sin \beta x$

(a) $f'(x) = \beta \cos \beta x$

$f''(x) = -\beta^2 \sin \beta x$

$f'''(x) = -\beta^3 \cos \beta x$

$f^{(4)} = \beta^4 \sin \beta x$

$f''(x) + \beta^2 f(x) = -\beta^2 \sin \beta x + \beta^2(\sin \beta x) = 0$

(c) $f^{(2k)}(x) = (-1)^k \beta^{2k} \sin \beta x$

$f^{(2k-1)}(x) = (-1)^{k+1} \beta^{2k-1} \cos \beta x$

110. (a) Yes, if $f(x+p) = f(x)$ for all x, then

$f'(x+p) = f'(x)$, which shows that f is periodic as well.

Yes, if $g(x) = f(2x)$, then $g'(x) = 2f'(2x)$. Because f' is periodic, so is g'.

(a) $r'(x) = f'(g(x))g'(x)$

$r'(1) = f'(g(1))g'(1)$

Note that $g(1) = 4$ and $f'(4) = \dfrac{5-0}{6-2} = \dfrac{5}{4}$.

Also, $g'(1) = 0$. So, $r'(1) = 0$.

112. (a) $g(x) = \sin^2 x + \cos^2 x = 1 \Rightarrow g'(x) = 0$

$g'(x) = 2 \sin x \cos x + 2 \cos x(-\sin x) = 0$

$\tan^2 x + 1 = \sec^2 x$

$g(x) + 1 = f(x)$

Taking derivatives of both sides, $g'(x) = f'(x)$.
Equivalently,

$f'(x) = 2 \sec x \cdot \sec x \tan x = 2\sec^2 x \tan x$ and

$g'(x) = 2 \tan x \cdot \sec^2 x = 2\sec^2 x \tan x$, which are the same.

113. (a) If $f(-x) = -f(x)$, then

$\dfrac{d}{dx}\left[f(-x)\right] = \dfrac{d}{dx}\left[-f(x)\right]$

$f'(-x)(-1) = -f'(x)$

$f'(-x) = f'(x)$.

So, $f'(x)$ is even.

(b) If $f(-x) = f(x)$, then

$\dfrac{d}{dx}\left[f(-x)\right] = \dfrac{d}{dx}\left[f(x)\right]$

$f'(-x)(-1) = f'(x)$

$f'(-x) = -f'(x)$.

So, f' is odd.

114. $|u| = \sqrt{u^2}$

$\dfrac{d}{dx}\left[|u|\right] = \dfrac{d}{dx}\left[\sqrt{u^2}\right] = \dfrac{1}{2}(u^2)^{-1/2}(2uu')$

$= \dfrac{uu'}{\sqrt{u^2}} = u'\dfrac{u}{|u|}, \ u \neq 0$

$g(x) = |3x - 5|$

$g'(x) = 3\left(\dfrac{3x-5}{|3x-5|}\right), \ x \neq \dfrac{5}{3}$

$f(x) = |x^2 - 9|$

$s'(x) = g'(f(x))f'(x)$

$s'(4) = g'(f(4))f'(4)$

Note that $f(4) = \frac{5}{2}$, $g'\left(\frac{5}{2}\right) = \frac{6-4}{6-2} = \frac{1}{2}$ and

$f'(4) = \frac{5}{4}$. So, $s'(4) = \frac{1}{2}\left(\frac{5}{4}\right) = \frac{5}{8}$.

$$f'(x) = 2x\left(\frac{x^2 - 9}{|x^2 - 9|}\right),\ x \neq \pm 3$$

117. $h(x) = |x|\cos x$

$$h'(x) = -|x|\sin x + \frac{x}{|x|}\cos x,\ x \neq 0$$

$f(x) = |\sin x|$

$$f'(x) = \cos x\left(\frac{\sin x}{|\sin x|}\right),\ x \neq k\pi$$

119. (a)

$f(x) = \tan x \qquad f(\pi/4) = 1$

$f'(x) = \sec^2 x \qquad f'(\pi/4) = 2$

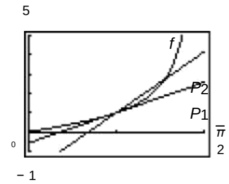
$f''(x) = 2\sec^2 x \tan x \qquad f''(\pi/4) = 4$

$P_1(x) = 2(x - \pi/4) + 1$

$P_2(x) = \frac{1}{2}(4)(x - \pi/4)^2 + 2(x - \pi/4) + 1$

$= 2(x - \pi/4)^2 + 2(x - \pi/4) + 1$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = \pi/4$.

120. (a)

$f(x) = \sec x \qquad f(\pi/6) = \frac{2}{\sqrt{3}}$

$f'(x) = \sec x \tan x \qquad f'(\pi/6) = \frac{2}{3}$

$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x) \qquad f''(\pi/6) = \frac{10\sqrt{3}}{9}$

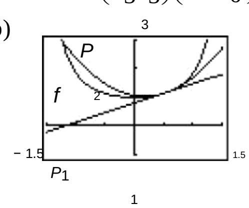
$= \sec^3 x + \sec x \tan^2 x$

$P_1(x) = \frac{2}{3}\left(x - \frac{\pi}{6}\right) + \frac{2}{\sqrt{3}}$

$P_2(x) = \frac{1}{2} \cdot \left(\frac{10}{3\sqrt{3}}\right)\left(x - \frac{\pi}{6}\right)^2 + \frac{2}{3}\left(x - \frac{\pi}{6}\right) + \frac{2}{\sqrt{3}}$

$= \left(\frac{5}{3\sqrt{3}}\right)\left(x - \frac{\pi}{6}\right)^2 + \frac{2}{3}\left(x - \frac{\pi}{6}\right) + \frac{2}{\sqrt{3}}$

(b)



P_2 is a better approximation than P_1.

The accuracy worsens as you move away from $x = \pi/6$.

121. True

122. False. $f'(x) = -b \sin x$ and $f'(0) = 0$

123. True

124. True

125.

$f(x) = a_1 \sin x + a_2 \sin 2x + \cdots + a_n \sin nx$

$f'(x) = a_1 \cos x + 2a_2 \cos 2x + \cdots + na_n \cos nx$

$f'(0) = a_1 + 2a_2 + \cdots + na_n$

$|a_1 + 2a_2 + \cdots + na_n| = |f'(0)| = \lim_{x\to 0}\left|\frac{f(x) - f(0)}{x - 0}\right| = \lim_{x\to 0}\left|\frac{f(x)}{\sin x} \cdot \frac{\sin x}{x}\right|$

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$$\frac{d}{dx}\left[\frac{P_n(x)}{\left(x^k-1\right)^{n+1}}\right] = \frac{\left(x^k-1\right)^{n+1}P_n'(x) - (n+1)kx^{k-1}\left(x^k-1\right)^{n}P_n(x)}{\left(x^k-1\right)^{2n+2}} = \frac{\left(x^k-1\right)P_n'(x) - (n+1)kx^{k-1}P_n(x)}{\left(x^k-1\right)^{n+2}}.$$

126.

$$P_n(x) = \left(x^k-1\right)^{n+1}\frac{d^n}{dx^n}\left[\frac{1}{x^k-1}\right] \Rightarrow$$

$$P_{n+1}(x) = \left(x^k-1\right)^{n+2}\frac{d}{dx}\left[\frac{d^n}{dx^n}\left[\frac{1}{x^k-1}\right]\right] = \left(x^k-1\right)P_n'(x) - (n+1)kx^{k-1}P_n(x)$$

$$P_{n+1}(1) = -(n+1)kP_n(1)$$

For $n = 1$, $\frac{d}{dx}\left[\frac{1}{x^k-1}\right] = \frac{-kx^{k-1}}{\left(x^k-1\right)^2} = \frac{P_1(x)}{\left(x^k-1\right)^{2}} \Rightarrow P_1(1) = -k$. Also, $P_0(1) = 1$.

You now use mathematical induction to verify that $P_n(1) = (-k)^n n!$ for $n \geq 0$. Assume true for n. Then

$$P_{n+1}(1) = -(n+1)kP_n(1) = -(n+1)k(-k)^n n! = (-k)^{n+1}(n+1)!.$$

Section 2.5 Implicit Differentiation

Answers will vary. *Sample answer:* In the explicit form of a function, the variable is explicitly written as a function of x. In an implicit equation, the function is only implied by an equation. An example of an implicit function is $x^2 + xy = 5$. In explicit form it would be $y = \left(5 - x^2\right)/x$.

Answers will vary. *Sample answer:* Given an implicit equation, first differentiate both sides with respect to x. Collect all terms involving y' on the left, and all other terms to the right. Factor out y' on the left side. Finally, divide both sides by the left-hand factor that does not contain y'.

You use implicit differentiation to find the derivative y' when it is difficult to express y explicitly as a function of x.

If y is an implicit function of x, then to compute y', you differentiate the equation with respect to x. For example, if $xy^2 = 1$, then $y^2 + 2xyy' = 0$. Here, the derivative of y^2 is $2yy'$.

$x^2 + y^2 = 9$
$2x + 2yy' = 0$
$y' = -\frac{x}{y}$

$x^5 + y^5 = 16$
$5x^4 + 5y^4y' = 0$
$5y^4y' = -5x^4$
$y' = -\frac{x^4}{y^4}$

$2x^3 + 3y^3 = 64$
$6x^2 + 9y^2y' = 0$
$9y^2y' = -6x^2$
$y' = \frac{-6x^2}{9y^2} = -\frac{2x^2}{3y^2}$

$x^3 - xy + y^2 = 7$
$3x^2 - xy' - y + 2yy' = 0$
$(2y - x)y' = y - 3x^2$
$y' = \frac{y - 3x^2}{2y - x}$

$x^2y + y^2x = -2$
$x^2y' + 2xy + y^2 + 2yxy' = 0$

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$x^2 - y^2 = 25$ $2x -$
$2yy' = 0$

$$y' = \frac{x}{y}$$

$x^2 + 2xy) y' = -(y^2 + 2xy)$

$$y' = \frac{-y(y + 2x)}{x(x + 2y)}$$

$x^3 y^3 - y - x = 0$ $3x^3 y^2 y' + 3x$
$^2 y^3 - y' - 1 = 0$

$$(3x^3 y^2 - 1) y' = 1 - 3x^2 y^3$$

$$y' = \frac{1 - 3x^2 y^3}{3x^3 y^2 - 1}$$

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12. $\sqrt{xy} = x^2y + 1$

$\frac{1}{2}(xy)^{-1/2}(xy' + y) = 2xy + x^2y'$

$\frac{x}{2\sqrt{xy}}y' + \frac{y}{2\sqrt{xy}} = 2xy + x^2y'$

$\left(\frac{x}{2\sqrt{xy}} - x^2\right)y' = 2xy - \frac{y}{2\sqrt{xy}}$

$y' = \frac{2xy - \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} - x^2}$

$y' = \frac{4xy\sqrt{xy} - y}{x - 2x^2\sqrt{xy}}$

14. $x^4y - 8xy + 3xy^2 = 9$

$x^4y' + 4x^3y - 8xy' - 8y + 6xyy' + 3y^2 = 0$

$(x^4 - 8x + 6xy)y' = 8y - 4x^3y - 3y^2$

$y' = \frac{8y - 4x^3y - 3y^2}{x^4 - 8x + 6xy}$

$\sin x + 2\cos 2y = 1$

$\cos x - 4(\sin 2y)y' = 0$

$y' = \frac{\cos x}{4\sin 2y}$

16. $(\sin \pi x + \cos \pi y)^2 = 2$

$2(\sin \pi x + \cos \pi y)\left[\pi\cos \pi x - \pi(\sin \pi y)y'\right] = 0$

$\pi\cos \pi x - \pi(\sin \pi y)y' = 0$

$y' = \frac{\cos \pi x}{\sin \pi y}$

$\csc x = x(1 + \tan y)$

$\csc x \cot x = (1 + \tan y) + x(\sec^2 y)y'$

$y' = -\frac{\csc x \cot x + 1 + \tan y}{x\sec^2 y}$

$\cot y = x - y$

$(-\csc^2 y)y' = 1 - y'$

$y' = \frac{1}{1 - \csc^2 y} = \frac{1}{-\cot^2 y} = -\tan^2 y$

$x^3 - 3x^2y + 2xy^2 = 12$

$3x^2 - 3x^2y' - 6xy + 4xyy' + 2y^2 = 0$

$(4xy - 3x^2)y' = 6xy - 3x^2 - 2y^2$

$y' = \frac{6xy - 3x^2 - 2y^2}{4xy - 3x^2}$

$x = \sec\frac{1}{y}$

$1 = -\frac{y'}{y^2}\sec\frac{1}{y}\tan\frac{1}{y}$

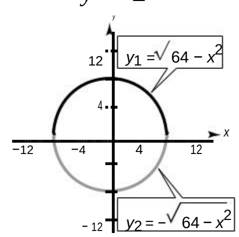
$y' = \frac{-y^2}{\sec(1/y)\tan(1/y)} = -y^2\cos\left(\frac{1}{y}\right)\cot\left(\frac{1}{y}\right)$

(a) $x^2 + y^2 = 64$

$y^2 = 64 - x^2$

$y = \pm\sqrt{64 - x^2}$

(b)



(c) Explicitly:

$\frac{dy}{dx} = \pm\frac{1}{2}(64 - x^2)^{-1/2}(-2x) = \frac{x}{\sqrt{64 - x^2}}$

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$$= \frac{-x}{\pm\sqrt{64 - x^2}} = -\frac{x}{y}$$

Implicitly: $2x + 2yy' = 0 \quad y' = -\frac{x}{y}$

$y = \sin xy$

$$y' = [xy' + y]\cos(xy)$$

$$y' - x\cos(xy)y' = y\cos(xy)$$

$$y' = \frac{y\cos(xy)}{1 - x\cos(xy)}$$

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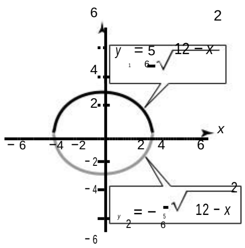
(a) $25x^2 + 36y^2 = 300$

$$36y^2 = 300 - 25x^2 = 25(12 - x^2)$$

$$y^2 = \frac{25}{36}(12 - x^2)$$

$$y = \pm\frac{5}{6}\sqrt{12 - x^2}$$

(b)



(c) Explicitly:

$$\frac{dy}{dx} = \pm\frac{5}{6}\left(\frac{1}{2}\right)(12 - x^2)^{-1/2}(-2x)$$

$$= \frac{\mp 5x}{6\sqrt{12 - x^2}}$$

$$= -\frac{25x}{36y}$$

(d) Implicitly: $50x + 72y \cdot y' = 0$

$$y' = \frac{-50x}{72y} = -\frac{25x}{36y}$$

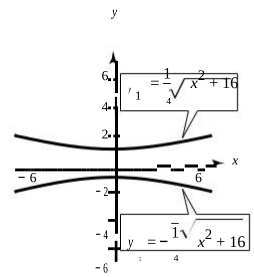
(a) $16y^2 - x^2 = 16$

$$16y^2 = x^2 + 16$$

$$y^2 = \frac{x^2}{16} + 1 = \frac{x^2 + 16}{16}$$

$$y = \frac{\pm\sqrt{x^2 + 16}}{4}$$

(b)



(c) Explicitly:

$$\frac{dy}{dx} = \frac{\pm\frac{1}{2}(x^2 + 16)^{-1/2}(2x)}{4}$$

$$= \frac{\pm x}{4\sqrt{x^2 + 16}} = \frac{\pm x}{4(\pm 4y)} = \frac{x}{16y}$$

Implicitly: $16y^2 - x^2 = 16$

$$32yy' - 2x = 0$$

$$32yy' = 2x$$

$$y' = \frac{2x}{32y} = \frac{x}{16y}$$

24. (a)

$$x^2 + y^2 - 4x + 6y + 9 = 0$$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -9 + 4 + 9$$

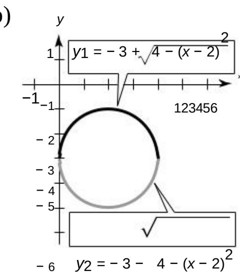
$$(x - 2)^2 + (y + 3)^2 = 4$$

$$(y + 3)^2 = 4 - (x - 2)^2$$

$$y + 3 = \pm\sqrt{4 - (x - 2)^2}$$

$$y = -3 \pm \sqrt{4 - (x - 2)^2}$$

(b)



(c) Explicitly:

$$\frac{dy}{dx} = \pm\frac{1}{2}\left[4 - (x - 2)^2\right]^{-1/2}\left[-2(x - 2)\right]$$

$$= \mp\frac{x - 2}{\sqrt{4 - (x - 2)^2}}$$

(d) Implicitly:

$$2x + 2yy' - 4 + 6y' = 0$$

$$2yy' + 6y' = -2x + 4$$

$$y'(2y + 6) = -2(x - 2)$$

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$$= -\frac{x-2}{y+3}$$

$$y' = \frac{-2(x-2)}{2(y+3)} = -\frac{x-2}{y+3}$$

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$xy = 6$

$xy' + y(1) = 0$

$xy' = -y$

$y' = -\frac{y}{x}$

At $(-6, -1)$: $y' = -\frac{1}{6}$

$3x^3 y = 6 \quad x^3 y = 2$

$3x^2 y + x^3 y' = 0$

$x^3 y' = -3x^2 y$

$y' = \frac{-3x^2 y}{x^3} = -\frac{3y}{x}$

At $(1, 2)$: $y' = \frac{-3(2)}{1} = -6$

$y^2 = \frac{x^2 - 49}{x^2 + 49}$

$2yy' = \frac{(x^2 + 49)(2x) - (x^2 - 49)(2x)}{(x^2 + 49)^2}$

$= \frac{196x}{(x^2 + 49)^2}$

$y' = \frac{98x}{y(x^2 + 49)^2}$

At $(7, 0)$: y' is undefined.

$4y^3 = \frac{x^2 - 36}{x^3 + 36}$

$= \frac{(x^3 + 36)(2x) - (x^2 - 36)(3x^2)}{(x^3 + 36)^2}$

$y' = \frac{72x + 108x^2 - x^4}{12y^2(x^3 + 36)^2}$

At $(6, 0)$: y' is undefined (division by 0).

29. $(x + y)^3 = x^3 + y^3$

$x^3 + 3x^2 y + 3xy^2 + y^3 = x^3 + y^3$

$3x^2 y + 3xy^2 = 0$

$x^2 y + xy^2 = 0$

$x^2 y' + 2xy + 2xyy' + y^2 = 0$

$(x^2 + 2xy)y' = -(y^2 + 2xy)$

$y' = -\frac{y(y + 2x)}{x(x + 2y)}$

$x^3 + y^3 = 6xy - 1$

$3x^2 + 3y^2 y' = 6xy' + 6y$

$(3y^2 - 6x)y' = 6y - 3x^2$

$y' = \frac{6y - 3x^2}{3y^2 - 6x}$

At $(2, 3)$: $y' = \frac{18 - 12}{27 - 12} = \frac{6}{15} = \frac{2}{5}$

$\tan(x + y) = x$

$(1 + y')\sec^2(x + y) = 1$

$y' = \frac{1 - \sec^2(x + y)}{\sec^2(x + y)}$

$= \frac{-\tan^2(x + y)}{\tan^2(x + y) + 1}$

$= -\sin^2(x + y)$

$= -\frac{x^2}{x^2 + 1}$

At $(0, 0)$: $y' = 0$

$x \cos y = 1$

$x[-y' \sin y] + \cos y = 0$

$y' = \frac{\cos y}{x \sin y}$

$= \frac{1}{x} \cot y$

$= \frac{\cot y}{x}$

At $\left(2, \frac{\pi}{3}\right)$: $y' = \frac{1}{2\sqrt{3}}$

33. $(x^2 + 4)y = 8$

$(x^2 + 4)y' + y(2x) = 0$

$y' = \frac{-2xy}{x^2 + 4}$

$= \frac{-2x\left[8/(x^2 + 4)\right]}{x^2 + 4}$

$= \frac{-16x}{(x^2 + 4)^2}$

At $(2, 1)$: $y' = \frac{-32}{64} = -\frac{1}{2}$

At $(-1, 1): y' = -1$

(Or, you could just solve for y: $y = x + 4$)

34. $(4-x)y^2 = x^3$

$(4-x)(2yy') + y^2(-1) = 3x^2$

$$y' = \frac{3x^2 + y^2}{2y(4-x)}$$

At $(2, 2)$: $y' = 2$

35. $(x^2 + y^2)^2 = 4x^2y$

$2(x^2 + y^2)(2x + 2yy') = 4x^2y' + y(8x)$

$4x^3 + 4x^2yy' + 4xy^2 + 4y^3y' = 4x^2y' + 8xy$

$4x^2yy' + 4y^3y' - 4x^2y' = 8xy - 4x^3 - 4xy^2$

$4y'(x^2y + y^3 - x^2) = 4(2xy - x^3 - xy^2)$

$$y' = \frac{2xy - x^3 - xy^2}{x^2y + y^3 - x^2}$$

At $(1, 1)$: $y' = 0$

$x^3 + y^3 - 6xy = 0$

$3x^2 + 3y^2y' - 6xy' - 6y = 0$

$y'(3y^2 - 6x) = 6y - 3x^2$

$$y' = \frac{6y - 3x^2}{3y^2 - 6x} = \frac{2y - x^2}{y^2 - 2x}$$

At $\left(\frac{4}{3}, \frac{8}{3}\right)$: $y' = \dfrac{(16/3) - (16/9)}{(64/9) - (8/3)} = \dfrac{32}{40} = \dfrac{4}{5}$

37. $(y-3)^2 = 4(x-5)$, $(6, 1)$

$2(y-3)y' = 4$

$$y' = \frac{2}{y-3}$$

At $(6, 1)$: $y' = \dfrac{2}{1-3} = -1$

Tangent line: $y - 1 = -1(x - 6)$

$y = -x + 7$

39. $x^2y^2 - 9x^2 - 4y^2 = 0$, $(-4, 2\sqrt{3})$

$x^2 2yy' + 2xy^2 - 18x - 8yy' = 0$

$$y' = \frac{18x - 2xy^2}{2x^2y - 8y}$$

At $(-4, 2\sqrt{3})$: $y' = \dfrac{18(-4) - 2(-4)(12)}{2(16)(2\sqrt{3}) - 16\sqrt{3}} = \dfrac{24}{48\sqrt{3}} = \dfrac{1}{2\sqrt{3}} = \dfrac{\sqrt{3}}{6}$

Tangent line: $y - 2\sqrt{3} = \dfrac{\sqrt{3}}{6}(x + 4)$

$y = \dfrac{\sqrt{3}}{6}x + \dfrac{8}{3}\sqrt{3}$

40. $x^{2/3} + y^{2/3} = 5$, $(8, 1)$

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$

$$y' = \frac{-x^{-1/3}}{y^{-1/3}} = -\left(\frac{y}{x}\right)^{1/3}$$

At $(8, 1)$: $y' = -\dfrac{1}{2}$

Tangent line: $y - 1 = -\dfrac{1}{2}(x - 8)$

$y = -\dfrac{1}{2}x + 5$

41. $3(x^2 + y^2)^2 = 100(x^2 - y^2)$, $(4, 2)$

$6(x^2 + y^2)(2x + 2yy') = 100(2x - 2yy')$

At $(4, 2)$: $6(16 + 4)(8 + 4y') = 100(8 - 4y')$

$960 + 480y' = 800 - 400y'$

$880y' = -160$

$y' = -\dfrac{2}{11}$

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38. $(x+2)^2 + (y-3)^2 = 37, \quad (4, 4)$

$$2(x+2) + 2(y-3)y' = 0$$

$$(y-3)y' = -(x+2)$$

$$y' = -\frac{(x+2)}{y-3}$$

At $(4, 4)$: $y' = -\frac{6}{1} = -6$

Tangent line: $y - 4 = -6(x-4)$

$$y = -6x + 28$$

Tangent line: $y - 2 = -\frac{2}{11}(x-4)$

$$11y + 2x - 30 = 0$$

$$y = -\frac{2}{11}x + \frac{30}{11}$$

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42. $y^2(x^2 + y^2) = 2x^2, \quad (1, 1)$

$$y^2x^2 + y^4 = 2x^2$$
$$2yy'x^2 + 2xy^2 + 4y^3y' = 4x$$

At $(1, 1)$:

$$2y' + 2 + 4y' = 4$$
$$6y' = 2$$
$$y' = \frac{1}{3}$$

Tangent line: $y - 1 = \frac{1}{3}(x - 1)$

$$= \frac{1}{3}x + \frac{2}{3}$$

Answers will vary. *Sample answers*:

$xy = 2 \Rightarrow y = \frac{2}{x}$

$yx^2 + x = 2 \Rightarrow y = \frac{2 - x}{x^2}$

$x^2 + y^2 = 4$

$xy + y^2 = 2$

The equation $x^2 + y^2 + 2 = 1$ implies $x^2 + y^2 = -1$, which has no real solutions.

45. (a) $\frac{x^2}{2} + \frac{y^2}{8} = 1, \quad (1, 2)$

$$x + \frac{yy'}{4} = 0$$
$$y' = -\frac{4x}{y}$$

At $(1, 2)$: $y' = -2$

Tangent line: $y - 2 = -2(x - 1)$

$$= -2x + 4$$

(b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} + \frac{2yy'}{b^2} = 0 \Rightarrow y' = \frac{-b^2x}{a^2y}$

$y - y_0 = \frac{-b^2x_0}{a^2y_0}(x - x_0)$, Tangent line at (x_0, y_0)

$$\frac{y_0y}{b^2} - \frac{y_0^2}{b^2} = -\frac{x_0x}{a^2} + \frac{x_0^2}{a^2}$$

46. (a) $\frac{x^2}{6} - \frac{y^2}{8} = 1, \quad (3, -2)$

$$\frac{x}{3} - \frac{y}{4}y' = 0$$
$$\frac{y}{4}y' = \frac{x}{3}$$
$$y' = \frac{4x}{3y}$$

At $(3, -2)$: $y' = \frac{4(3)}{3(-2)} = -2$

Tangent line: $y + 2 = -2(x - 3)$

$$= -2x + 4$$

(b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} - \frac{2yy'}{b^2} = 0 \Rightarrow y' = \frac{xb^2}{ya^2}$

$y - y_0 = \frac{x_0b^2}{y_0a^2}(x - x_0)$, Tangent line at (x_0, y_0)

$$\frac{yy_0}{b^2} - \frac{y_0^2}{b^2} = \frac{x_0x}{a^2} - \frac{x_0^2}{a^2}$$

Because $\frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1$, you have $\frac{x_0x}{a^2} - \frac{yy_0}{b^2} = 1$.

Note: From part (a),

$\frac{3x}{6} - \frac{(-2)y}{8} = 1 \Rightarrow \frac{1}{2}x + \frac{y}{4} = 1 \Rightarrow y = -2x + 4$, Tangent line.

$\tan y = x$

$y' \sec^2 y = 1$

$$y' = \frac{1}{\sec^2 y} = \cos^2 y, \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

$$\sec^2 y = 1 + \tan^2 y = 1 + x^2$$

$$y' = \frac{1}{1 + x^2}$$

$\cos y = x$

$-\sin y \cdot y' = 1$

$$y' = \frac{-1}{\sin y}, \quad 0 < y < \pi$$

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$b^2 \quad b^2 \quad a^2 \quad a^2$

Because $\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1$, you have $\frac{y_0 y}{b^2} + \frac{x_0 x}{a^2} = 1$.

Note: From part (a),

$$\frac{1(x)}{2} + \frac{2(y)}{8} = 1 \Rightarrow \frac{1}{4}y = -\frac{1}{2}x + 1 \Rightarrow y = -2x + 4,$$

Tangent line.

$\sin y$

$$\sin^2 y + \cos^2 y = 1$$

$$\sin^2 y = 1 - \cos^2 y$$

$$\sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - x^2}$$

$$y' = \frac{-1}{\sqrt{1 - x^2}}, \quad -1 < x < 1$$

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49. $x^2 + y^2 = 4$

$2x + 2yy' = 0$

$y' = \dfrac{-x}{y}$

$y'' = \dfrac{y(-1) + xy'}{y^2}$

$= \dfrac{-y + x(-x/y)}{y^2}$

$= \dfrac{-y^2 - x^2}{y^3}$

$= -\dfrac{4}{y^3}$

50. $x^2y - 4x = 5$

$x^2y' + 2xy - 4 = 0$

$y' = \dfrac{4 - 2xy}{x^2}$

$x^2y'' + 2xy' + 2xy' + 2y = 0$

$x^2y'' + 4x\left[\dfrac{4 - 2xy}{x^2}\right] + 2y = 0$

$x^4y'' + 4x(4 - 2xy) + 2x^2y = 0$

$x^4y'' + 16x - 8x^2y + 2x^2y = 0$

$x^4y'' = 6x^2y - 16x$

$y'' = \dfrac{6xy - 16}{x^3}$

51. $x^2y - 2 = 5x + y$

$2xy + x^2y' = 5 + y'$

$(x^2 - 1)y' = 5 - 2xy$

$y' = \dfrac{5 - 2xy}{x^2 - 1}$

$2xy' + (x^2 - 1)y'' = -2y - 2xy'$

$(x^2 - 1)y'' = -2y - 4xy' = -2y - 4x\left(\dfrac{5 - 2xy}{x^2 - 1}\right)$

$y'' = \dfrac{-2y}{x^2 - 1} - \dfrac{4x(5 - 2xy)}{(x^2 - 1)^2}$

$= \dfrac{-2y(x^2 - 1) - 20x + 8x^2y}{(x^2 - 1)^2} = \dfrac{6x^2y - 20x + 2y}{(x^2 - 1)^2}$

$xy - 1 = 2x + y^2$

$xy' + y = 2 + 2yy'$

$xy' - 2yy' = 2 - y$

$(x - 2y)y' = 2 - y$

$y' = \dfrac{2 - y}{x - 2y}$

$xy'' + y' + y' = 2yy'' + 2(y')^2$

$xy'' - 2yy'' = 2(y')^2 - 2y'$

$(x - 2y)y'' = 2(y')^2 - 2y' = 2\left(\dfrac{2 - y}{x - 2y}\right)^2 - 2\left(\dfrac{2 - y}{x - 2y}\right)$

[illegible]

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$2\,2\ -y\lceil 2\ -y\ -\ x-2y\rceil$

$(x-2y)^3 \qquad\qquad (x-2y)^3$

$= \dfrac{2(4-2x+2y-2y+xy-y^2)}{(\quad)\ (x-2y)^3} = \dfrac{2(y^2-xy+2x-4)}{(2y-x)^3}$

$2\ \ -5$

$= (2y-x)^3 \quad = (x-2y)^3$

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$$7xy + \sin x = 2$$

$$7xy' + 7y + \cos x = 0$$

$$y' = \frac{-7y - \cos x}{7x}$$

$$7xy'' + 7y' + 7y' - \sin x = 0$$

$$7xy'' = \sin x - 14y' = \sin x - 14\left(\frac{-7y - \cos x}{7x}\right)$$

$$7xy'' = \sin x + \frac{14y + 2\cos x}{x}$$

$$y'' = \frac{\sin x}{7x} + \frac{14y + 2\cos x}{7x^2}$$

$$y'' = \frac{x\sin x + 14y + 2\cos x}{7x^2}$$

$$3xy - 4\cos x = -6 \qquad 3xy' + 3y + 4\sin x = 0$$

$$y' = \frac{-4\sin x - 3y}{3x}$$

$$3xy'' + 3y' + 3y' + 4\cos x = 0$$

$$3xy'' = -6y' - 4\cos x = -6\left(\frac{-4\sin x - 3y}{3x}\right) - 4\cos x$$

$$= \frac{8\sin x + 6y - 4x\cos x}{x}$$

$$y'' = \frac{8\sin x + 6y - 4x\cos x}{3x^2}$$

55. $\sqrt{x} + \sqrt{y} = 5$

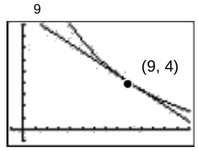
$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$$

$$y' = \frac{-\sqrt{y}}{\sqrt{x}}$$

At $(9, 4)$: $y' = -\frac{2}{3}$

Tangent line: $y - 4 = -\frac{2}{3}(x - 9)$

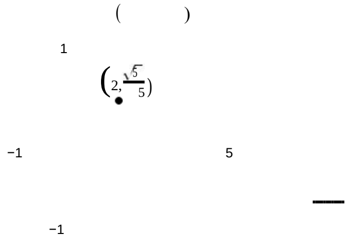
$$2x + 3y - 30 = 0$$



56. $y^2 = \frac{x - 1}{x^2 + 1}$

$$2yy' = \frac{(x^2 + 1)(1) - (x - 1)(2x)}{(x^2 + 1)^2} = \frac{x^2 + 1 - 2x^2 + 2x}{(x^2 + 1)^2}$$

$$y' = \frac{1 + 2x - x^2}{2y(x^2 + 1)^2}$$



At $\left(2, \frac{\sqrt{5}}{5}\right)$: $y' = \dfrac{1 + 4 - 4}{2\left(\frac{\sqrt{5}}{5}\right)(4 + 1)^2} = \dfrac{1}{10\sqrt{5}} = \dfrac{\sqrt{5}}{50}$

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Tangent line: $y - \sqrt{5} = \frac{1}{10\sqrt{5}}(x - 2)$

$$10\sqrt{5}y - 10 = x - 2$$

$$x - 10\sqrt{5}y + 8 = 0$$

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$x^2 + y^2 = 25$ $2x + 2yy' = 0$

$$y' = -\frac{x}{y}$$

At $(4, 3)$:

Tangent line:

$y - 3 = -\frac{4}{3}(x - 4) \Rightarrow 4x + 3y - 25 = 0$ Normal line: $y - 3 = \frac{3}{4}(x - 4) \Rightarrow 3x - 4y = 0$

6

• (4, 3)

− 9 9

6

At $(-3, 4)$:

Tangent line:

$y - 4 = \frac{3}{4}(x + 3) \Rightarrow 3x - 4y + 25 = 0$ Normal line: $y - 4 = -\frac{4}{3}(x + 3) \Rightarrow 4x + 3y = 0$

6

(− 3, 4) •

−9 9

6

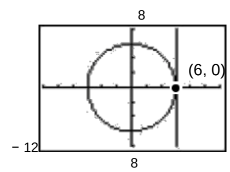
$x^2 + y^2 = 36$ $2x + 2yy' = 0$

$$y' = -\frac{x}{y}$$

At $(6, 0)$; slope is undefined.

Tangent line: $x = 6$

Normal line: $y = 0$



At $(5, \sqrt{11})$, slope is $\frac{-5}{\sqrt{11}}$.

Tangent line: $y - \sqrt{11} = \frac{-5}{\sqrt{11}}(x - 5)$

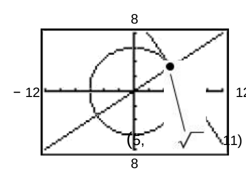
$$\sqrt{11}y - 11 = -5x + 25$$

$$5x + \sqrt{11}y - 36 = 0$$

Normal line: $y - \sqrt{11} = \frac{\sqrt{11}}{5}(x - 5)$

$$5y - 5\sqrt{11} = \sqrt{11}x - 5\sqrt{11}$$

$$5y - \sqrt{11}x = 0$$



$x^2 + y^2 = r^2$ $2x + 2yy' = 0$

$$y' = -\frac{x}{y} = \text{slope of tangent line}$$

$$\frac{y}{x} = \text{slope of normal line}$$

Let (x_0, y_0) be a point on the circle. If $x_0 = 0$, then the tangent line is horizontal, the normal line is vertical and, hence, passes through the origin. If $x_0 \neq 0$, then the equation of the normal line is

$$y - y_0 = \frac{y_0}{x_0}(x - x_0)$$

$$y = \frac{y_0}{x_0}x$$

which passes through the origin.

$y^2 = 4x$

$2yy' = 4$

$y' = \frac{2}{y} = 1$ at $(1, 2)$

Equation of normal line at $(1, 2)$ is $y - 2 = -1(x - 1)$, $y = 3 - x$. The centers of the circles must be on the normal line and at a distance of 4 units from $(1, 2)$. Therefore,

$$(x - 1)^2 + \left[(3 - x) - 2\right]^2 = 16$$

$$2(x - 1)^2 = 16$$

$$x = 1 \pm 2\sqrt{2}.$$

Centers of the circles: $(1 + 2\sqrt{2}, 2 - 2\sqrt{2})$ and $(1 - 2\sqrt{2}, 2 + 2\sqrt{2})$

Equations: $(x - 1 - 2\sqrt{2})^2 + (y - 2 + 2\sqrt{2})^2 = 16$

$$(x - 1 + 2\sqrt{2})^2 + (y - 2 - 2\sqrt{2})^2 = 16$$

$4x^2 + y^2 - 8x + 4y + 4 = 0$

$8x + 2yy' - 8 + 4y' = 0$

$$y' = \frac{8 - 8x}{2y + 4} = \frac{4 - 4x}{y + 2}$$

Horizontal tangents occur when $x = 1$:

$4(1)^2 + y^2 - 8(1) + 4y + 4 = 0$

$$y^2 + 4y = y(y + 4) = 0 \Rightarrow y = 0, -4$$

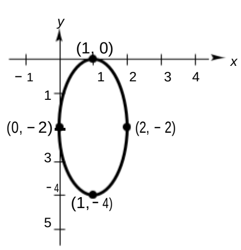
Horizontal tangents: $(1, 0)$, $(1, -4)$

Vertical tangents occur when $y = -2$:

$4x^2 + (-2)^2 - 8x + 4(-2) + 4 = 0$

$$4x^2 - 8x = 4x(x - 2) = 0 \Rightarrow x = 0, 2$$

Vertical tangents: $(0, -2)$, $(2, -2)$



$25x^2 + 16y^2 + 200x - 160y + 400 = 0$

$50x + 32yy' + 200 - 160y' = 0$

$$y' = \frac{200 \pm 50x}{160 - 32y}$$

Horizontal tangents occur when $x = -4$:

$25(16) + 16y^2 + 200(-4) - 160y + 400 = 0$

$$y(y - 10) = 0 \Rightarrow y = 0, 10$$

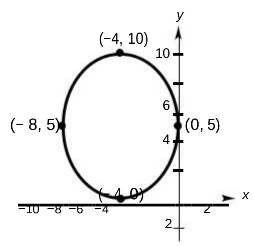
Horizontal tangents: $(-4, 0)$, $(-4, 10)$

Vertical tangents occur when $y = 5$:

$25x^2 + 400 + 200x - 800 + 400 = 0$

$$25x(x + 8) = 0 \Rightarrow x = 0, -8$$

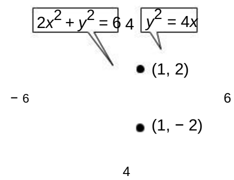
Vertical tangents: $(0, 5)$, $(-8, 5)$



63. Find the points of intersection by letting $y^2 = 4x$ in the equation $2x^2 + y^2 = 6$.

$2x^2 + 4x = 6$ and $(x + 3)(x - 1) = 0$

The curves intersect at $(1, \pm 2)$.



Ellipse:	*Parabola*:
$4x + 2yy' = 0$	$2yy' = 4$
$y' = -\frac{2x}{y}$	$y' = \frac{2}{y}$

At $(1, 2)$, the slopes are:

$y' = -1$	$y' = 1$

At $(1, -2)$, the slopes are:

$y' = 1$	$y' = -1$

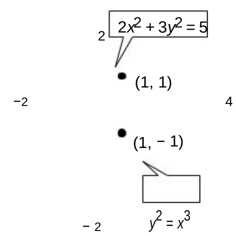
Tangents are perpendicular.

Find the points of intersection by letting $y^2 = x^3$ in the equation $2x^2 + 3y^2 = 5$.

$2x^2 + 3x^3 = 5$ and $3x^3 + 2x^2 - 5 = 0$

Intersect when $x = 1$.

Points of intersection: $(1, \pm 1)$



$y^2 = x^3$:	$2x^2 + 3y^2 = 5$:
$2yy' = 3x^2$	$4x + 6yy' = 0$
$y' = \frac{3x^2}{2y}$	$y' = -\frac{2x}{3y}$

At $(1, 1)$, the slopes are:

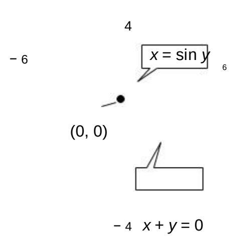
$y' = \frac{3}{2}$	$y' = -\frac{2}{3}$

At $(1, -1)$, the slopes are:

$y' = -\frac{3}{2}$	$y' = \frac{2}{3}$

Tangents are perpendicular.

$y = -x$ and $x = \sin y$ Point of intersection: $(0, 0)$



$y = -x$:	$x = \sin y$:
$y' = -1$	$1 = y' \cos y$
	$y' = \sec y$

At $(0, 0)$, the slopes are:

$y' = -1$	$y' = 1$

Tangents are perpendicular.

Rewriting each equation and differentiating:

$$x^3 = 3(y - 1) \qquad x(3y - 29) = 3$$

$$y = \frac{x^3}{3} + 1 \qquad y = \frac{1}{3}\left(\frac{3}{x} + 29\right)$$

$$y' = x^2 \qquad y' = -\frac{1}{x^2}.$$

$x(3y - 29) = 3$

$x^3 = 3y - 3$

15

−15

12

3

For each value of x, the derivatives are negative reciprocals of each other.
So, the tangent lines are orthogonal at both points of intersection.

67.

$$xy = C \qquad x^2 - y^2 = K$$

$$xy' + y = 0 \qquad 2x - 2yy' = 0$$

$$y' = -\frac{y}{x} \qquad y' = \frac{x}{y}$$

At any point of intersection (x, y) the product of the slopes is $(-y/x)(x/y) = -1$. The curves are orthogonal.

2

−3

3

$C = 1$

$= -1$

2

2

$C = 4$

−3

3

$K = 2$

2

68.

$$x^2 + y^2 = C^2 \qquad y = Kx$$

$$2x + 2yy' = 0 \qquad y' = K$$

$$y' = -\frac{x}{y}$$

At the point of intersection (x, y), the product of the slopes is $(-x/y)(K) = (-x/Kx)(K) = -1$. The curves *are orthogonal*.

2

$K = 1$

−3

3

$C = 2$

2

2

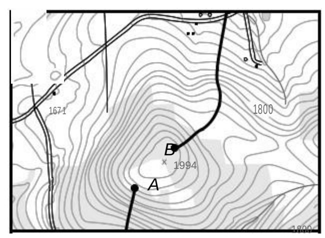
$K = -1$

$C = 1$

−3

3

2



Use starting point B.

(a) The slope is greater at $x = -3$.

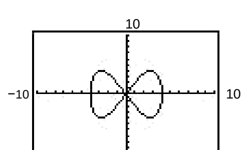
The graph has vertical tangent lines at about $(-2, 3)$ and $(2, 3)$.

The graph has a horizontal tangent line at about $(0, 6)$.

(a) $x^4 = 4(4x^2 - y^2)$

$$4y^2 = 16x^2 - x^4$$

$$y^2 = 4x^2 - \frac{1}{4}x^4$$



$$y = \pm\sqrt{4x^2 - \frac{1}{4}x^4}$$

− 10

$$y = 3 \Rightarrow 9 = 4x^2 - \frac{1}{4}x^4$$

$$36 = 16x^2 - x^4$$

$$x^4 - 16x^2 + 36 = 0$$

$$x^2 = \frac{16 \pm \sqrt{256 - 144}}{2} = 8 \pm \sqrt{28}$$

Note that $x^2 = 8 \pm \sqrt{28} = 8 \pm 2\sqrt{7} = \left(1 \pm \sqrt{7}\right)^2$. So, there are four values of x:

$-1 - \sqrt{7},\ 1 - \sqrt{7},\ -1 + \sqrt{7},\ 1 + \sqrt{7}$

To find the slope, $2yy' = 8x - x^3 \Rightarrow y' = \frac{x\left(8 - x^2\right)}{2(3)}$.

For $x = -1 - \sqrt{7}$, $y' = \frac{1}{3}\left(\sqrt{7} + 7\right)$, and the line is

$$y_1 = \frac{1}{3}\left(\sqrt{7} + 7\right)\left(x + 1 + \sqrt{7}\right) + 3 = \frac{1}{3}\left[\left(\sqrt{7} + 7\right)x + 8\sqrt{7} + 23\right].$$

For $x = 1 - \sqrt{7}$, $y' = \frac{1}{3}\left(\sqrt{7} - 7\right)$, and the line is

$$y_2 = \frac{1}{3}\left(\sqrt{7} - 7\right)\left(x - 1 + \sqrt{7}\right) + 3 = \frac{1}{3}\left[\left(\sqrt{7} - 7\right)x + 23 - 8\sqrt{7}\right].$$

For $x = -1 + \sqrt{7}$, $y' = -\frac{1}{3}\left(\sqrt{7} - 7\right)$, and the line is

$$y_3 = -\frac{1}{3}\left(\sqrt{7} - 7\right)\left(x + 1 - \sqrt{7}\right) + 3 = -\frac{1}{3}\left[\left(\sqrt{7} - 7\right)x - \left(23 - 8\sqrt{7}\right)\right].$$

For $x = 1 + \sqrt{7}$, $y' = -\frac{1}{3}\left(\sqrt{7} + 7\right)$, and the line is

$$y_4 = -\frac{1}{3}\left(\sqrt{7} + 7\right)\left(x - 1 - \sqrt{7}\right) + 3 = -\frac{1}{3}\left[\left(\sqrt{7} + 7\right)x - \left(8\sqrt{7} + 23\right)\right].$$

(c) Equating y_3 and y_4:

$$-\frac{1}{3}\left(\sqrt{7} - 7\right)\left(x + 1 - \sqrt{7}\right) + 3 = -\frac{1}{3}\left(\sqrt{7} + 7\right)\left(x - 1 - \sqrt{7}\right) + 3$$

$$\left(\sqrt{7} - 7\right)\left(x + 1 - \sqrt{7}\right) = \left(\sqrt{7} + 7\right)\left(x - 1 - \sqrt{7}\right)$$

$$\sqrt{7}x + \sqrt{7} - 7 - 7x - 7 + 7\sqrt{7} = \sqrt{7}x - \sqrt{7} - 7 + 7x - 7 - 7\sqrt{7}$$

$$\sqrt{7} = 14x$$

$$x = \frac{8\sqrt{7}}{7}$$

If $x = \frac{8\sqrt{7}}{7}$, then $y = 5$ and the lines intersect at $\left(\frac{8\sqrt{7}}{7}, 5\right)$.

7 $\left(7 \right)$

72. $\sqrt{x} + \sqrt{y} = \sqrt{c}$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

Tangent line at (x_0, y_0): $y - y_0 = -\frac{\sqrt{y_0}}{\sqrt{x_0}}(x - x_0)$

x-intercept: $\left(x_0 + \sqrt{x_0}\sqrt{y_0}, 0\right)$

y-intercept: $\left(0, y_0 + \sqrt{x_0}\sqrt{y_0}\right)$

Sum of intercepts:

$$\left(x_0 + \sqrt{x_0}\sqrt{y_0}\right) + \left(y_0 + \sqrt{x_0}\sqrt{y_0}\right) = x_0 + 2\sqrt{x_0}\sqrt{y_0} + y_0 = \left(\sqrt{x_0} + \sqrt{y_0}\right)^2 = \left(\sqrt{c}\right)^2 = c$$

73. $y = x^{p/q}$; p, q integers and $q > 0$

$$y^q = x^p$$

$$qy^{q-1}y' = px^{p-1}$$

$$y' = \frac{px^{p-1}}{qy^{q-1}} = \frac{px^{p-1}}{q} \cdot \frac{y}{y^q}$$

$$= \frac{p}{q} \cdot \frac{x^{p-1}}{x^p}x^{p/q} = \frac{p}{q}x^{p/q-1}$$

So, if $y = x^n$, $n = p/q$, then $y' = nx^{n-1}$.

74. $x^2 + y^2 = 100$, slope $= \frac{3}{4}$

$$2x + 2yy' = 0$$

$$y' = -\frac{x}{y} = \frac{3}{4} \Rightarrow y = -\frac{4}{3}x$$

$$x^2 + \left(\frac{16}{9}x^2\right) = 100$$

$$\frac{25}{9}x^2 = 100$$

$$x = \pm 6$$

Points: $(6, -8)$ and $(-6, 8)$

75. $\frac{x^2}{4} + \frac{y^2}{9} = 1$, $(4, 0)$

$$\frac{2x}{4} + \frac{2yy'}{9} = 0$$

$$y' = \frac{-9x}{4y}$$

$$\frac{-9x}{4y} = \frac{y - 0}{x - 4}$$

$$-9x(x - 4) = 4y^2$$

But, $9x^2 + 4y^2 = 36 \Rightarrow 4y^2 = 36 - 9x^2$. So, $-9x^2 + 36x = 4y^2 = 36 - 9x^2 \Rightarrow x = 1$.

Points on ellipse: $\left(1, \pm\frac{3}{2}\sqrt{3}\right)$

At $\left(1, \frac{3}{2}\sqrt{3}\right)$: $y' = \frac{-9x}{4y} = \frac{-9}{4\left[(3/2)\sqrt{3}\right]} = -\frac{\sqrt{3}}{2}$

At $\left(1, -\frac{3}{2}\sqrt{3}\right)$: $y' = \frac{\sqrt{3}}{2}$

(2) 2

Tangent lines: $y = -\frac{\sqrt{3}}{2}(x - 4) = -\frac{\sqrt{3}}{2}x + 2\sqrt{3}$

$y = \frac{\sqrt{3}}{2}(x - 4) = \frac{\sqrt{3}}{2}x - 2\sqrt{3}$

$x = y^2$

$1 = 2yy'$

$y' = \frac{1}{2y}$, slope of tangent line

Consider the slope of the normal line joining $(x_0, 0)$ and $(x, y) = (y^2, y)$ on the parabola.

$$-2y = \frac{y - 0}{y^2 - x_0}$$

$$y^2 - x_0 = -\frac{1}{2}$$

$$y^2 = x_0 - \frac{1}{2}$$

$-\frac{1}{4}$, then $y^2 = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$, which is impossible. So, the only normal line is the x-axis ($y = 0$).

(b) If $x_0 = \frac{1}{2}$, then $y^2 = 0 \Rightarrow y = 0$. Same as part (a).

If $x_0 = 1$, then $y^2 = \frac{1}{2} = x$ and there are three normal lines.

The x-axis, the line joining $(x_0, 0)$ and $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$, and the line joining $(x_0, 0)$ and $\left(\frac{1}{2}, -\frac{1}{\sqrt{2}}\right)$

If two normals are perpendicular, then their slopes are –1 and 1. So,

$$-2y = -1 = \frac{y - 0}{y^2 - x_0} \Rightarrow y = \frac{1}{2}$$

and

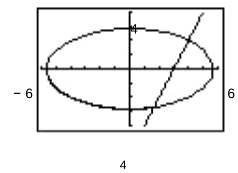
$$\frac{1/2}{(1/4) - x_0} = -1 \Rightarrow \frac{1}{4} - x_0 = -\frac{1}{2} \Rightarrow x_0 = \frac{3}{4}.$$

The perpendicular normal lines are $y = -x + \frac{3}{4}$ and $y = x - \frac{3}{4}$.

77. (a) $\frac{x^2}{32} + \frac{y^2}{8} = 1$

$$\frac{2x}{32} + \frac{2yy'}{8} = 0 \Rightarrow y' = \frac{-x}{4y}$$

(b)



At $(4, 2)$: $y' = \frac{-4}{4(2)} = -\frac{1}{2}$

Slope of normal line is 2.

$$y - 2 = 2(x - 4)$$
$$= 2x - 6$$

$$\frac{x^2}{32} + \frac{(2x - 6)^2}{8} = 1$$

$$x^2 + 4(4x^2 - 24x + 36) = 32$$

$$17x^2 - 96x + 112 = 0$$

$$(17x - 28)(x - 4) = 0 \Rightarrow x = 4, \frac{28}{17}$$

Second point: $\left(\frac{28}{17}, -\frac{46}{17}\right)$

Section 2.6 Related Rates

A related-rate equation is an equation that relates the rates of change of various quantities.

Answers will vary. See page 153.

$y = \sqrt{x}$

$$\frac{dy}{dt} = \left(\frac{1}{2\sqrt{x}}\right)\frac{dx}{dt}$$

$$\frac{dx}{dt} = 2\sqrt{x}\,\frac{dy}{dt}$$

When $x = 4$ and $dx/dt = 3$:

$$= \frac{1}{2\sqrt{4}}(3) = \frac{3}{4}$$

$\frac{dy}{dt}$

When $x = 25$ and $dy/dt = 2$: $\frac{dx}{dt}$

$= 2\sqrt{25}(2) = 20$

$y = 3x^2 - 5x$

$\frac{dy}{dt} = (6x - 5)\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{1}{6x - 5}\frac{dy}{dt}$

(a) When $x = 3$ and $\frac{dx}{dt} = 2$:

$\frac{dy}{dt} = \left[6(3) - 5\right]2 = 26$

(b) When $x = 2$ and $\frac{dy}{dt} = 4$:

$\frac{dx}{dt} = \frac{1}{6(2) - 5}(4) = \frac{4}{7}$

$xy = 4$

$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$

$\frac{dy}{dt} = \left(-\frac{y}{x}\right)\frac{dx}{dt}$

$\frac{dx}{dt} = \left(-\frac{x}{y}\right)\frac{dy}{dt}$

(a) When $x = 8$, $y = 1/2$, and $dx/dt = 10$:

$\frac{dy}{dt} = -\frac{1/2}{8}(10) = -\frac{5}{8}$

When $x = 1$, $y = 4$, and $dy/dt = -6$:

$\frac{dx}{dt} = -\frac{1}{4}(-6) = \frac{3}{2}$

$x^2 + y^2 = 25$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \left(-\frac{x}{y}\right)\frac{dx}{dt}$

$\frac{dx}{dt} = \left(-\frac{y}{x}\right)\frac{dy}{dt}$

$y = 2x^2 + 1$

$\frac{dx}{dt} = 2$

$\frac{dy}{dt} = 4x\frac{dx}{dt}$

(a) When $x = -1$:

$\frac{dy}{dt} = 4(-1)(2) = -8$ cm/sec

(b) When $x = 0$:

$\frac{dy}{dt} = 4(0)(2) = 0$ cm/sec

(c) When $x = 1$:

$\frac{dy}{dt} = 4(1)(2) = 8$ cm/sec

8. $y = \frac{1}{1 + x^2}$, $\frac{dx}{dt} = 6$

$\frac{dy}{dt} = \left(\frac{-2x}{(1 + x^2)^2}\right) \cdot \frac{dx}{dt}$

$= \frac{-2x}{(1 + x^2)^2}(6) = \frac{-12x}{(1 + x^2)^2}$

(a) When $x = -2$:

$\frac{dy}{dt} = \frac{-12(-2)}{\left[1 + (-2)^2\right]^2} = \frac{24}{25}$ in./sec

(b) When $x = 0$:

$\frac{dy}{dt} = \frac{-12(0)}{(1 + 0^2)^2} = 0$ in./sec

(c) When $x = 2$:

$\frac{dy}{dt} = \frac{-12(2)}{(1 + 2^2)^2} = -\frac{24}{25}$ in./sec

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dt ⎝ *x* ⎠ *dt*

(a) When $x = 3$, $y = 4$, and $dx/dt = 8$:

$$\frac{dy}{dt} = -\frac{3}{4}(8) = -6$$

(b) When $x = 4$, $y = 3$, and $dy/dt = -2$:

$$\frac{dx}{dt} = -\frac{3}{4}(-2) = \frac{3}{2}$$

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$y = \tan x, \frac{dx}{dt} = 3$

$\frac{dy}{dt} = \sec^2 x \cdot \frac{dx}{dt} = \sec^2 x(3) = 3\sec^2 x$

(a) When $x = -\frac{\pi}{3}$:

$\frac{dy}{dt} = 3\sec^2\left(-\frac{\pi}{3}\right) = 3(2)^2 = 12$ ft/sec

(b) When $x = -\frac{\pi}{4}$:

$\frac{dy}{dt} = 3\sec^2\left(-\frac{\pi}{4}\right) = 3\left(\sqrt{2}\right)^2 = 6$ ft/sec

(c) When $x = 0$:

$\frac{dy}{dt} = 3\sec^2(0) = 3$ ft/sec

$y = \cos x, \quad \frac{dx}{dt} = 4$

$\frac{dy}{dt} = -\sin x \cdot \frac{dx}{dt} = -\sin x(4) = -4\sin x$

(a) When $x = \frac{\pi}{6}$:

$\frac{dy}{dt} = -4\sin\left(\frac{\pi}{6}\right) = -4\left(\frac{1}{2}\right) = -2$ cm/sec

(b) When $x = \frac{\pi}{4}$:

$\frac{dy}{dt} = -4\sin\left(\frac{\pi}{4}\right) = -4\left(\frac{\sqrt{2}}{2}\right) = -2\sqrt{2}$ cm/sec

(c) When $x = \frac{\pi}{3}$:

$\frac{dy}{dt} = -4\sin\left(\frac{\pi}{3}\right) = -4\left(\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$ cm/sec

$A = \pi r^2$

$\frac{dr}{dt} = 4$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

When $r = 37$,

$V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = 3$

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

(a) When $r = 9$,

$\frac{dV}{dt} = 4\pi(9)^2(3) = 972\pi$ in.^{3}/min.

When $r = 36$,

$\frac{dV}{dt} = 4\pi(36)^2(3) = 15{,}552\pi$ in.^{3}/min.

If dr/dt is constant, dV/dt is proportional to r^2.

$V = \frac{4}{3}\pi r^3, \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$\frac{dV}{dt} = 800$

$\frac{dr}{dt} = \frac{dV/dt}{4\pi r^2}$

At $r = 30$, $\frac{dr}{dt} = \frac{800}{4\pi(30)^2} = \frac{2}{9\pi}$ cm/min.

At $r = 85$, $\frac{dr}{dt} = \frac{800}{4\pi(85)^2} = \frac{8}{289\pi}$ cm/min.

$\frac{dr}{dt}$ depends on r^2, not r.

$V = x^3$

$\frac{dx}{dt} = 6$

$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$

(a) When $x = 2$,

$\frac{dV}{dt} = 3(2)^2(6) = 72$ cm^3/sec.

(b) When $x = 10$,

$\frac{dV}{dt} = 3(10)^2(6) = 1800$ cm^3/sec.

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$\frac{dA}{dt} = 2\pi($ $s = 6x^2$ /

$37)(4) = 296$ $\frac{dx}{dt} =$ 6

π cm^2 min.

$A = \frac{s^2\sqrt{3}}{4}$

$\frac{ds}{dt} = 13$

$\frac{dA}{dt} = \frac{\sqrt{3}}{4}(2s)\frac{ds}{dt} = \frac{\sqrt{3}s}{2}\frac{ds}{dt}$

When $s = 41$, $\frac{dA}{dt} = \frac{\sqrt{3}}{2}(41)(13) = \frac{533\sqrt{3}}{2}$ ft^2/hr.

$\frac{ds}{dt} = 12x\frac{dx}{dt}$

(a) When $x = 2$,

$\frac{ds}{dt} = 12(2)(6) = 144$ cm^2/sec.

(b) When $x = 10$,

$\frac{ds}{dt} = 12(10)(6) = 720$ cm^2/sec.

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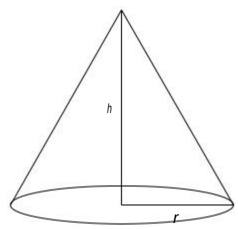
17. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{9}{4}h^2\right)h = \frac{3}{4}\pi h^3$ [because $2r = 3h$]

$\frac{dV}{dt} = 10$

$\frac{dV}{dt} = \frac{9\pi}{4}h^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{4(dV/dt)}{9\pi h^2}$

When $h = 15$,

$\frac{dh}{dt} = \frac{4(10)}{9\pi(15)^2} = \frac{8}{405\pi}$ ft/min.



$V = \pi r^2 h$

$\frac{dV}{dt} = 150$

$h = 10r \Rightarrow r = \frac{h}{10}$

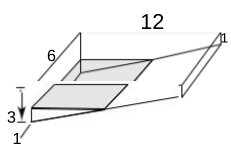
$V = \pi\left(\frac{h}{10}\right)^2 h = \frac{\pi}{100}h^3$

$\frac{dV}{dt} = \frac{3\pi}{100}h^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{100}{3\pi h^2}\frac{dV}{dt}$

When $h = 35$, $\frac{dh}{dt} = \frac{100}{3\pi(35)^2}(150) = \frac{200}{49\pi}$ in./sec.

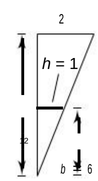
19.



(a) Total volume of pool $= \frac{1}{2}(2)(12)(6) + (1)(6)(12) = 144 \text{ m}^3$

Volume of 1 m of water $= \frac{1}{2}(1)(6)(6) = 18 \text{ m}^3$ (see similar triangle diagram)

% pool filled $= \frac{18}{144} = 12.5\%$



(b) Because for $0 \le h \le 2$, $b = 6h$, you have

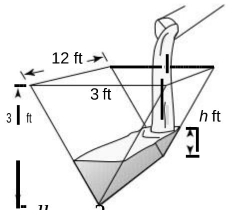
$V = \frac{1}{2}bh(6) = 3bh = 3(6h)h = 18h^2$

$\frac{dV}{dt} = 36h\frac{dh}{dt} = \frac{1}{4} \Rightarrow \frac{dh}{dt} = \frac{1}{144h} = \frac{1}{144(1)} = \frac{1}{144}$ m/min.

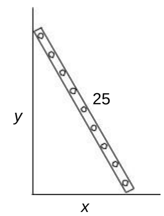
20. $V = \frac{1}{2}bh(12) = 6bh = 6h^2$ (since $b = h$)

(a) $\frac{dV}{dt} = 12h\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{12h}\frac{dV}{dt}$

When $h = 1$ and $\frac{dV}{dt} = 2$, $\frac{dh}{dt} = \frac{1}{12(1)}(2) = \frac{1}{6}$ ft/min.



(b) If $\frac{dh}{dt} = \frac{3}{8}$ in./min $= \frac{1}{32}$ ft/min and $h = 2$ ft, then $\frac{dV}{dt} = 12(2)\left(\frac{1}{32}\right) = \frac{3}{4}$ ft^3/min.



$$x^2 + y^2 = 25^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-x}{y}\cdot\frac{dx}{dt} = \frac{-2x}{y} \quad \text{because } \frac{dx}{dt} = 2.$$

(a) When $x = 7$, $y = \sqrt{576} = 24$, $\dfrac{dy}{dt} = \dfrac{-2(7)}{24} = -\dfrac{7}{12}$ ft/sec.

When $x = 15$, $y = \sqrt{400} = 20$, $\dfrac{dy}{dt} = \dfrac{-2(15)}{20} = -\dfrac{3}{2}$ ft/sec.

When $x = 24$, $y = 7$, $\dfrac{dy}{dt} = \dfrac{-2(24)}{7} = -\dfrac{48}{7}$ ft/sec.

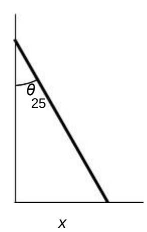
$$A = \frac{1}{2}xy$$

$$\frac{dA}{dt} = \frac{1}{2}\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)$$

From part (a) you have $x = 7$, $y = 24$, $\dfrac{dx}{dt} = 2$, and $\dfrac{dy}{dt} = -\dfrac{7}{12}$. So,

$$\frac{dA}{dt} = \frac{1}{2}\left[7\left(-\frac{7}{12}\right) + 24(2)\right] = \frac{527}{24}\ \text{ft}^2\text{/sec.}$$

$$\tan\theta = \frac{x}{y}$$



$$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{y}\cdot\frac{dx}{dt} - \frac{x}{y^2}\cdot\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \cos^2\theta\left[\frac{1}{y}\cdot\frac{dx}{dt} - \frac{x}{y^2}\cdot\frac{dy}{dt}\right]$$

Using $x = 7$, $y = 24$, $\dfrac{dx}{dt} = 2$, $\dfrac{dy}{dt} = -\dfrac{7}{12}$ and $\cos\theta = \dfrac{24}{25}$, you have

$$\frac{d\theta}{dt} = \left(\frac{24}{25}\right)^2\left[\frac{1}{24}(2) - \frac{7}{(24)^2}\left(-\frac{7}{12}\right)\right] = \frac{1}{12}\ \text{rad/sec.}$$

$$x^2 + y^2 = 25 \qquad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{y}{x}\cdot\frac{dy}{dt} = -\frac{0.15y}{x}\quad\left(\text{because } \frac{dy}{dt} = 0.15\right)$$

When $x = 2.5$, $y = \sqrt{18.75}$, $\dfrac{dx}{dt} = -\dfrac{\sqrt{18.75}}{2.5}(0.15) \approx -0.26$ m/sec.

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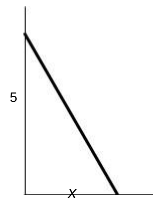

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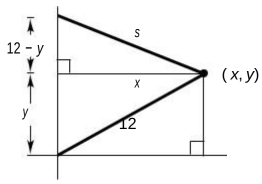
23. When $y = 6$, $x = \sqrt{12^2 - 6^2} = 6\sqrt{3}$, and $s = \sqrt{x^2 + (12 - y)^2} = \sqrt{108 + 36} = 12$.



$$x^2 + (12 - y)^2 = s^2$$

$$2x\frac{dx}{dt} + 2(12 - y)(-1)\frac{dy}{dt} = 2s\frac{ds}{dt}$$

$$x\frac{dx}{dt} + (y - 12)\frac{dy}{dt} = s\frac{ds}{dt}$$

Also, $x^2 + y^2 = 12^2$.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

So, $x\frac{dx}{dt} + (y - 12)\left(\frac{-x}{y}\frac{dx}{dt}\right) = s\frac{ds}{dt}$.

$$\frac{dx}{dt}\left[x - x + \frac{12x}{y}\right] = s\frac{ds}{dt} \Rightarrow \frac{dx}{dt} = \frac{sy}{12x}\cdot\frac{ds}{dt} = \frac{(12)(6)}{(12)(6\sqrt{3})}(-0.2) = \frac{-1}{5\sqrt{3}} = \frac{-\sqrt{3}}{15} \text{ m/sec (horizontal)}$$

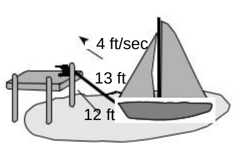
$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt} = \frac{-6\sqrt{3}}{6}\cdot\left(\frac{-\sqrt{3}}{15}\right) = \frac{1}{5} \text{ m/sec (vertical)}$$

Let L be the length of the rope.

(a)
$$L^2 = 144 + x^2$$

$$2L\frac{dL}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{L}{x}\cdot\frac{dL}{dt} = -\frac{4L}{x} \quad \left(\text{since } \frac{dL}{dt} = -4 \text{ ft/sec}\right)$$



When $L = 13$:

$$x = \sqrt{L^2 - 144} = \sqrt{169 - 144} = 5$$

$$\frac{dx}{dt} = -\frac{4(13)}{5} = -\frac{52}{5} = -10.4 \text{ ft/sec}$$

Speed of the boat increases as it approaches the dock.

(b) If $\frac{dx}{dt} = -4$, and $L = 13$:

$$\frac{dL}{dt} = \frac{x}{L}\frac{dx}{dt} = \frac{5}{13}(-4) = \frac{-20}{13} \text{ ft/sec}$$

$$\frac{dL}{dt} = \frac{x}{L}\frac{dx}{dt} = \frac{\sqrt{L^2 - 144}}{L}(-4)$$

$$\lim_{L \to 12^+}\frac{dL}{dt} = \lim_{L \to 12^+}\frac{-4}{L}\sqrt{L^2 - 144} = 0$$

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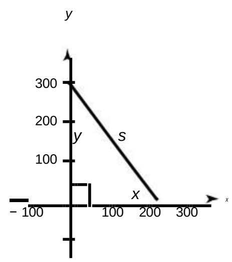
(a) $s^2 = x^2 + y^2$

$\frac{dx}{dt} = -450$

$\frac{dy}{dt} = -600$

$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{ds}{dt} = \frac{x(dx/dt) + y(dy/dt)}{s}$



When $x = 225$ and $y = 300$, $s = 375$ and

$\frac{ds}{dt} = \frac{225(-450) + 300(-600)}{375} = -750$ mi/h.

(b) $t = \frac{375}{750} = \frac{1}{2}$ h = 30 min

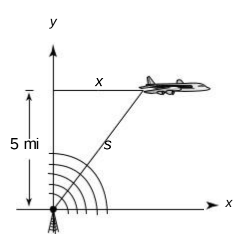
26. $x^2 + y^2 = s^2$

$2x\frac{dx}{dt} + 0 = 2s\frac{ds}{dt}$ $\left(\text{because } \frac{dy}{dt} = 0\right)$

$\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$

When $s = 10$, $x = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}$,

$\frac{dx}{dt} = \frac{10}{5\sqrt{3}}(240) = \frac{480}{\sqrt{3}} = 160\sqrt{3} \approx 277.13$ mi/h.



$s^2 = 90^2 + x^2$

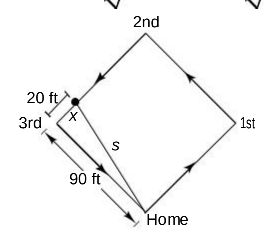
$x = 20$

$\frac{dx}{dt} = -25$

$2s\frac{ds}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{ds}{dt} = \frac{x}{s}\cdot\frac{dx}{dt}$

When $x = 20$, $s = \sqrt{90^2 + 20^2} = 10\sqrt{85}$,

$\frac{ds}{dt} = \frac{20}{10\sqrt{85}}(-25) = \frac{-50}{\sqrt{85}} \approx -5.42$ ft/sec.



28. $s^2 = 90^2 + x^2$

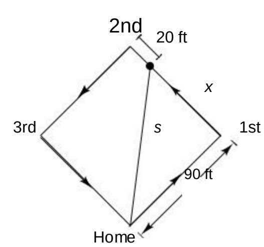
$x = 90 - 20 = 70$

$\frac{dx}{dt} = 25$

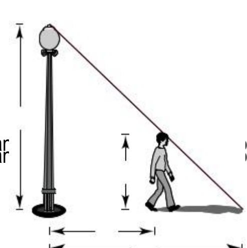
$\frac{ds}{dt} = \frac{x}{s}\cdot\frac{dx}{dt}$

When $x = 70$, $s = \sqrt{90^2 + 70^2} = 10\sqrt{130}$,

$\frac{ds}{dt} = \frac{70}{10\sqrt{130}}(25) = \frac{175}{\sqrt{130}} \approx 15.35$ ft/sec.



29. (a) $\frac{15}{6} = \frac{y}{y - x} \Rightarrow 15y - 15x = 6y$



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$$y = \frac{5}{3}x$$

$$\frac{dx}{dt} = 5$$

$$\frac{dy}{dt} = \frac{5}{3} \cdot \frac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3} \text{ ft/sec}$$

15

6

x

y

(b) $$\frac{d(y-x)}{dt} = \frac{dy}{dt} - \frac{dx}{dt} = \frac{25}{3} - 5 = \frac{10}{3} \text{ ft/sec}$$

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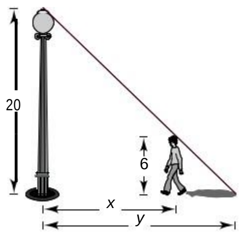
30. (a)

$$\frac{20}{6} = \frac{y}{y-x}$$
$$20y - 20x = 6y$$
$$14y = 20x$$
$$y = \frac{10}{7}x$$
$$\frac{dx}{dt} = -5$$
$$\frac{dy}{dt} = \frac{10}{7}\frac{dx}{dt} = \frac{10}{7}(-5) = \frac{-50}{7} \text{ ft/sec}$$



(b)
$$\frac{d(y-x)}{dt} = \frac{dy}{dt} - \frac{dx}{dt}$$
$$= \frac{-50}{7} - (-5)$$
$$= \frac{-50}{7} + \frac{35}{7} = \frac{-15}{7} \text{ ft/sec}$$

$$x(t) = \frac{1}{2}\sin\frac{\pi t}{6},\ x^2 + y^2 = 1$$

Period: $\dfrac{2\pi}{\pi/6} = 12$ seconds

(b) When $x = \frac{1}{2}$, $y = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$ m.

Lowest point: $\left(0, \frac{\sqrt{3}}{2}\right)$

(c) When $x = \frac{1}{4}$,

$$y = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4} \text{ and } t = 1:$$

$$\frac{dx}{dt} = \frac{1}{2}\left(\frac{\pi}{6}\right)\cos\frac{\pi t}{6} = \frac{\pi}{12}\cos\frac{\pi t}{6}$$

$$x^2 + y^2 = 1$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

So, $\dfrac{dy}{dt} = -\dfrac{1/4}{\sqrt{15}/4}\cdot\dfrac{\pi}{12}\cos\left(\dfrac{\pi}{6}\right)$

$$= \frac{-\pi}{\sqrt{15}}\left(\frac{1}{12}\right)\frac{\sqrt{3}}{2} = \frac{-\pi}{24}\frac{1}{\sqrt{5}} = \frac{-\sqrt{5}\pi}{120}.$$

Speed $= \left|\dfrac{-\sqrt{5}\pi}{120}\right| = \dfrac{\sqrt{5}\pi}{120}$ m/sec

$$x(t) = \frac{3}{5}\sin\pi t,\ x^2 + y^2 = 1$$

Period: $\dfrac{2\pi}{\pi} = 2$ seconds

(c) When $x = \frac{3}{10}$, $y = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4}$ and

$$\frac{3}{10} = \frac{3}{5}\sin\pi t \Rightarrow \sin\pi t = \frac{1}{2} \Rightarrow t = \frac{1}{6}$$

$$\frac{dx}{dt} = \frac{3}{5}\pi\cos\pi t$$

$$x^2 + y^2 = 1$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

So, $\dfrac{dy}{dt} = \dfrac{-3/10}{\sqrt{15}/4}\cdot\dfrac{3}{5}\pi\cos\left(\dfrac{\pi}{6}\right)$

$$= \frac{-9\pi}{25\sqrt{5}} = \frac{-9\sqrt{5}\pi}{125}.$$

Speed $= \left|\dfrac{-9\sqrt{5}\pi}{125}\right| \approx 0.5058$ m/sec

33. Because the evaporation rate is proportional to the surface area, $dV/dt = k(4\pi r^2)$. However, because $V = (4/3)\pi r^3$, you have $\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$. Therefore, $k(4\pi r^2) = 4\pi r^2\dfrac{dr}{dt} \Rightarrow k = \dfrac{dr}{dt}$.

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dt ·

34. (i) (a) $\frac{dx}{dt}$ negative $\Rightarrow \frac{dy}{dt}$ positive

(b) $\frac{dy}{dt}$. positive $\Rightarrow \frac{dx}{dt}$ negative

(ii) (a) $\frac{dx}{dt}$ negative $\Rightarrow \frac{dy}{dt}$ negative

(b) $\frac{dy}{dt}$. positive $\Rightarrow \frac{dx}{dt}$ positive

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35. (a) $\frac{dy}{dt} = 3\left(\frac{dx}{dt}\right)$ means that y changes three times as fast as x changes.

y changes slowly when $x \approx 0$ or $x \approx L$. y changes more rapidly when x is near the middle of the interval.

No. $V = s^3$, $\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

If $\frac{ds}{dt}$ is constant, then $\frac{dV}{dt}$ is $3s^2$ times that constant.

37. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$\frac{dR_1}{dt} = 1$

$\frac{dR_2}{dt} = 1.5$

$$\frac{1}{R^2} \cdot \frac{dR}{dt} = \frac{1}{R_1^2} \cdot \frac{dR_1}{dt} + \frac{1}{R_2^2} \cdot \frac{dR_2}{dt}$$

When $R_1 = 50$ and $R_2 = 75$:

$R = 30$

$$\frac{dR}{dt} = (30)^2\left[\frac{1}{(50)^2}(1) + \frac{1}{(75)^2}(1.5)\right] = 0.6 \text{ ohm/sec}$$

38. $V = IR$

$$\frac{dV}{dt} = I\frac{dR}{dt} + R\frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{1}{R}\frac{dV}{dt} - \frac{I}{R}\frac{dR}{dt}$$

When $V = 12, R = 4$, $\frac{dV}{dt} = 3$, and

$\frac{dR}{dt} = 2$, $I = \frac{V}{R} = \frac{12}{4} = 3$ and

$$\frac{dI}{dt} = \frac{1}{4}(3) - \frac{3}{4}(2) = -\frac{3}{4} \text{ amperes/sec.}$$

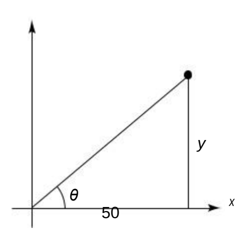
$\sin 18° = \frac{x}{y}$

$\tan\theta = 50$

$\frac{dy}{dt} = 4$ m/sec

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{50}\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{50}\cos^2\theta \cdot \frac{dy}{dt}$$



When $y = 50$, $\theta = \frac{\pi}{4}$, and $\cos\theta = \frac{\sqrt{2}}{2}$. So,

$$\frac{d\theta}{dt} = \frac{1}{50}\left(\frac{\sqrt{2}}{2}\right)^2(4) = \frac{1}{25} \text{ rad/sec.}$$

41. $\sin\theta = \frac{10}{x}$

$\frac{dx}{dt} = -1$ ft/sec

$$\cos\theta\left(\frac{d\theta}{dt}\right) = \frac{-10}{x^2}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{-10}{x^2}\frac{dx}{dt}(\sec\theta)$$

$$= \frac{-10}{25^2}(-1)\frac{25}{\sqrt{25^2 - 10^2}}$$

$$= \frac{10}{25}\frac{1}{5\sqrt{21}} = \frac{2}{25\sqrt{21}}$$

$$= \frac{2\sqrt{21}}{525} \approx 0.017 \text{ rad/sec}$$

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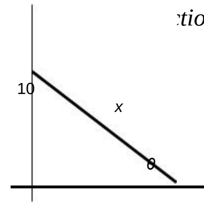
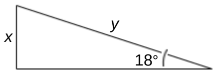


$$0 = -\frac{x}{y^2}\cdot\frac{dy}{dt} + \frac{1}{y}\cdot\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{x}{y}\cdot\frac{dy}{dt} = (\sin 18^\circ)(275) \approx 84.9797 \text{ mi/hr}$$



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$\tan\theta = \frac{y}{x}, y = 5$

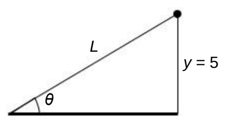
$\frac{dx}{dt} = -600$ mi/h

$(\sec^2\theta)\frac{d\theta}{dt} = -\frac{5}{x^2}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \cos^2\theta\left(-\frac{5}{x^2}\right)\frac{dx}{dt} = \frac{x^2}{L^2}\left(-\frac{5}{x^2}\right)\frac{dx}{dt}$

$= \left(-\frac{5^2}{L^2}\right)\left(\frac{1}{5}\right)\frac{dx}{dt}$

$= (-\sin^2\theta)\left(\frac{1}{5}\right)(-600) = 120\sin^2\theta$



(a) When $\theta = 30°$,

$\frac{d\theta}{dt} = \frac{120}{4} = 30$ rad/h $= \frac{1}{2}$ rad/min.

(b) When $\theta = 60°$,

$\frac{d\theta}{dt} = 120\left(\frac{3}{4}\right) = 90$ rad/h $= \frac{3}{2}$ rad/min.

(c) When $\theta = 75°$,

$\frac{d\theta}{dt} = 120\sin^2 75° \approx 111.96$ rad/h ≈ 1.87 rad/min.

$\tan\theta = \frac{x}{50}$

$\frac{d\theta}{dt} = 30(2\pi) = 60\pi$ rad/min $= \pi$ rad/sec

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{50}\left(\frac{dx}{dt}\right)$

$\frac{dx}{dt} = 50\sec^2\theta\left(\frac{d\theta}{dt}\right)$

Police

$\frac{d\theta}{dt} = (10 \text{ rev/sec})(2\pi \text{ rad/rev}) = 20\pi$ rad/sec

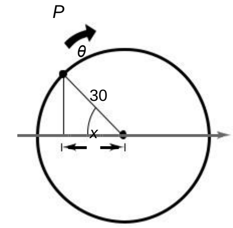
(a) $\cos\theta = \frac{x}{30}$

$-\sin\theta\frac{d\theta}{dt} = \frac{1}{30}\frac{dx}{dt}$

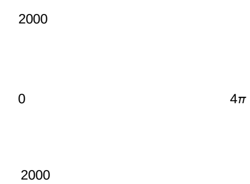
$\frac{dx}{dt} = -30\sin\theta\frac{d\theta}{dt}$

$= -30\sin\theta(20\pi)$

$= -600\pi\sin\theta$



(b)



$|\sin\theta| = 1 \Rightarrow \theta = \frac{\pi}{2} + n\pi$ (or $90° + n\cdot180°$).

(d) For $\theta = 30°$,

$\frac{dx}{dt} = -600\pi\sin(30°) = -600\pi\frac{1}{2} = -300\pi$ cm/sec.

For $\theta = 60°$,

$\frac{dx}{dt} = -600\pi\sin(60°)$

$= -600\pi\frac{\sqrt{3}}{2} = -300\sqrt{3}\pi$ cm/sec.

45. (a) $\sin\frac{\theta}{2} = \frac{(1/2)b}{s} \Rightarrow b = 2s\sin\frac{\theta}{2}$

$\cos\frac{\theta}{2} = \frac{h}{s} \Rightarrow h = s\cos\frac{\theta}{2}$

$A = \frac{1}{2}bh = \frac{1}{2}\left(2s\sin\frac{\theta}{2}\right)\left(s\cos\frac{\theta}{2}\right)$

$= \frac{s^2}{2}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = \frac{s^2}{2}\sin\theta$

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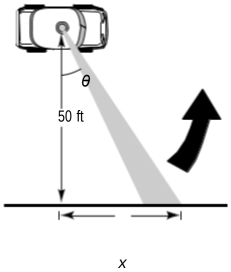


When $\theta = 30°$, $\frac{dx}{dt} = \frac{200\pi}{3}$ ft/sec.

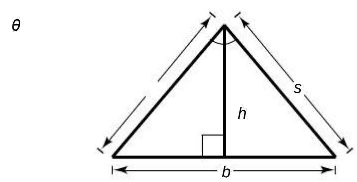
When $\theta = 60°$, $\frac{dx}{dt} = 200\pi$ ft/sec.

When $\theta = 70°$, $\frac{dx}{dt} \approx 427.43\pi$ ft/sec.

(b) $\frac{dA}{dt} = \frac{s}{2}\cos\theta\frac{d\theta}{dt}$ where $\frac{d\theta}{dt} = \frac{1}{2}$ rad/min.

When $\theta = \frac{\pi}{6}$, $\frac{dA}{dt} = \frac{s^2}{2}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}s^2}{8}$.

When $\theta = \frac{\pi}{3}$, $\frac{dA}{dt} = \frac{s^2}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{s^2}{8}$.



(c) If s and $\frac{d\theta}{dt}$ is constant, $\frac{dA}{dt}$ is proportional to $\cos\theta$.

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46. $\tan\theta = \frac{x}{50} \Rightarrow x = 50\tan\theta$

$$\frac{dx}{dt} = 50\sec^2\theta\,\frac{d\theta}{dt}$$

$$2 = 50\sec^2\theta\,\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{25}\cos^2\theta, \quad -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$$

47. (a) Using a graphing utility,

$$r(f) = 0.0096f^3 - 0.559f^2 + 10.54f - 61.5.$$

(b) $\frac{dr}{dt} = \frac{dr}{df}\frac{df}{dt} = \left(0.0288f^2 - 1.118f + 10.54\right)\frac{df}{dt}$

For $t = 9$, $f = 16.3$ from the table under the year 2009.

Also, $\frac{df}{dt} = 1.25$, so you have

$$\frac{dr}{dt} = \left(0.0288(16.3)^2 - 1.118(16.3) + 10.54\right)(1.25)$$

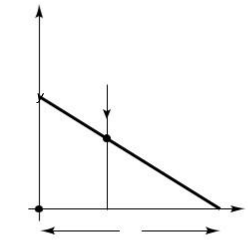
$$= -0.03941 \text{ million participants per year.}$$

48. $y(t) = -4.9t^2 + 20$

$$\frac{dy}{dt} = -9.8t$$

$$y(1) = -4.9 + 20 = 15.1$$

$$y'(1) = -9.8$$



By similar triangles: $\frac{20}{x} = \frac{y}{x-12}$

$$20x - 240 = xy$$

When $y = 15.1$: $20x - 240 = x(15.1)$

$$(20 - 15.1)x = 240$$

$$x = \frac{240}{4.9}$$

$$20x - 240 = xy$$

$$20\frac{dx}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{x}{20 - y}\frac{dy}{dt}$$

At $t = 1$, $\frac{dx}{dt} = \frac{240/4.9}{20 - 15.1}(-9.8) \approx -97.96$ m/sec.

49. $x^2 + y^2 = 25$; acceleration of the top of the ladder $= \frac{d^2y}{dt^2}$

First derivative: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

Second derivative: $x\frac{d^2x}{dt^2} + \frac{dx}{dt}\cdot\frac{dx}{dt} + y\frac{d^2y}{dt^2} + \frac{dy}{dt}\cdot\frac{dy}{dt} = 0$

$$\frac{d^2y}{dt^2} = \left(\frac{1}{y}\right)\left[-x\frac{d^2x}{dt^2} - \left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2\right]$$

When $x = 7$, $y = 24$, $\frac{dy}{dt} = -\frac{7}{12}$, and $\frac{dx}{dt} = 2$ (see Exercise 25). Because $\frac{dx}{dt}$ is constant, $\frac{d^2x}{dt^2} = 0$.

$$\frac{d^2y}{dt^2} = \frac{1}{24}\left[-7(0) - (2)^2 - \left(-\frac{7}{12}\right)^2\right] = \frac{1}{24}\left[-4 - \frac{49}{144}\right] = \frac{1}{24}\left[-\frac{625}{144}\right] \approx -0.1808 \text{ ft/sec}^2$$

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50. $L^2 = 144 + x^2$; acceleration of the boat $= \dfrac{d^2x}{dt^2}$

First derivative: $2L\dfrac{dL}{dt} = 2x\dfrac{dx}{dt}$

$$L\frac{dL}{dt} = x\frac{dx}{dt}$$

Second derivative: $L\dfrac{d^2L}{dt^2} + \dfrac{dL}{dt}\cdot\dfrac{dL}{dt} = x\dfrac{d^2x}{dt^2} + \dfrac{dx}{dt}\cdot\dfrac{dx}{dt}$

$$\frac{d^2x}{dt^2} = \left(\frac{1}{x}\right)\left[L\frac{d^2L}{dt^2} + \left(\frac{dL}{dt}\right)^2 - \left(\frac{dx}{dt}\right)^2\right]$$

When $L = 13$, $x = 5$, $\dfrac{dx}{dt} = -10.4$, and $\dfrac{dL}{dt} = -4$ (see Exercise 28). Because $\dfrac{dL}{dt}$ is constant, $\dfrac{d^2L}{dt^2} = 0$.

$$\frac{d^2x}{dt^2} = \frac{1}{5}\left[13(0) + (-4)^2 - (-10.4)^2\right]$$

$$= \frac{1}{5}[16 - 108.16] = \frac{1}{5}[-92.16] = -18.432 \text{ ft/sec}^2$$

Review Exercises for Chapter 2

1. $f(x) = 12$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{12 - 12}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{0}{\Delta x} = 0$$

2. $f(x) = 5x - 4$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[5(x + \Delta x) - 4] - (5x - 4)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{5x + 5\Delta x - 4 - 5x + 4}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{5\Delta x}{\Delta x} = 5$$

3. $f(x) = x^3 - 2x + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 - 2(x + \Delta x) + 1\right] - \left[x^3 - 2x + 1\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - 2x - 2(\Delta x) + 1 - x^3 + 2x - 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - 2(\Delta x)}{\Delta x}$$

$$= \lim \left[3x^2 + 3x(\Delta x) + (\Delta x)^2 - 2\right]$$

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$x \to 0 \lfloor \quad 1 \quad \rfloor$

$= 3x^2 - 2$

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$f(x) = \frac{6}{x}$

$f'(x) = \lim_{x \to 0} \frac{f(x + x) - f(x)}{x}$

$= \lim_{x \to 0} \frac{\frac{6}{x + x} - \frac{6}{x}}{x} = \lim_{x \to 0} \frac{6x - (6x + 6x)}{x(x + x)x} = \lim_{x \to 0} \frac{-6x}{x(x + x)x} = \lim_{x \to 0} \frac{-6}{(x + x)x} = \frac{-6}{x^2}$

$g(x) = 2x^2 - 3x, c = 2$

$g'(2) = \lim_{x \to 2} \frac{g(x) - g(2)}{x - 2}$

$\lim_{x \to 2} \frac{(2x^2 - 3x) - 2}{x - 2}$

$\lim_{x \to 2} \frac{(x - 2)(2x + 1)}{x - 2}$

$\lim_{x \to 2} (2x + 1) = 2(2) + 1 = 5$

$f(x) = \frac{1}{x + 4}, c = 3$

$f'(3) = \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$

$= \lim_{x \to 3} \frac{\frac{1}{x + 4} - \frac{1}{7}}{x - 3}$

$= \lim_{x \to 3} \frac{7 - x - 4}{(x - 3)(x + 4)7}$

$= \lim_{x \to 3} \frac{-1}{(x + 4)7} = -\frac{1}{49}$

f is differentiable for all $x \neq 3$.

f is differentiable for all $x \neq -1$.

$y = 25$

$y' = 0$

$f(t) = \frac{\pi}{6}$

$f'(t) = 0$

14. $f(x) = x^{1/2} - x^{-5/6}$

$f'(x) = \frac{1}{2}x^{-1/2} + \frac{5}{6}x^{-11/6}$

$g(t) = \frac{2}{3}t^{-2}$

$g'(t) = \frac{-4}{3}t^{-3} = -\frac{4}{3t^3}$

16. $h(x) = \frac{8}{5x^4} = \frac{8}{5}x^{-4}$

$h'(x) = -\frac{32}{5}x^{-5} = -\frac{32}{5x^5}$

17. $f(\theta) = 4\theta - 5 \sin \theta$

$f'(\theta) = 4 - 5 \cos \theta$

18. $g(\alpha) = 4 \cos \alpha + 6$

$g'(\alpha) = -4 \sin \alpha$

19. $f(\theta) = 3 \cos \theta - \frac{\sin \theta}{4}$

$f'(\theta) = -3 \sin \theta - \frac{\cos \theta}{4}$

20. $g(\alpha) = \frac{5 \sin \alpha}{3} - 2\alpha$

$g'(\alpha) = \frac{5 \cos \alpha}{3} - 2$

$f(x) = \frac{27}{x^3} = 27x^{-3}, (3, 1)$

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$f(x) = x^3 - 11x^2$ f

$'(x) = 3x^2 - 22x$

$g(s) = 3s^5 - 2s^4$

$g'(s) = 15s^4 - 8s^3$

13. $h(x) = 6\sqrt{x} + 3\sqrt[3]{x} = 6x^{1/2} + 3x^{1/3}$

$h'(x) = 3x^{-1/2} + x^{-2/3} = \dfrac{3}{\sqrt{x}} + \dfrac{1}{\sqrt[3]{x^2}}$

$f'(x) = 27(-3)x^{-4} = -\dfrac{}{x^4}$

$f'(3) = -\dfrac{81}{3^4} = -1$

22. $f(x) = 3x^2 - 4x, (1, -1)$

$f'(x) = 6x - 4$

$f'(1) = 6-4=2$

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23. $f(x) = 4x^5 + 3x - \sin x, (0, 0)$

$f'(x) = 20x^4 + 3 - \cos x$

$f'(0) = 3 - 1 = 2$

24. $f(x) = 5\cos x - 9x, (0, 5)$

$f'(x) = -5\sin x - 9$

$f'(0) = -5\sin 0 - 9 = -9$

25. $F = 200\sqrt{T}$

$F'(t) = \frac{100}{\sqrt{T}}$

When $T = 4$, $F'(4) = 50$ vibrations/sec/lb.

When $T = 9$, $F'(9) = 33\frac{1}{3}$ vibrations/sec/lb.

$S = 6x^2$

$\frac{dS}{dx} = 12x$

When $x = 4$, $\frac{dS}{dx} = 12(4) = 48$ in.^{2}/in.

$s(t) = -16t^2 + v_0 t + s_0$; $s_0 = 600$, $v_0 = -30$

$s(t) = -16t^2 - 30t + 600$

$s'(t) = v(t) = -32t - 30$

Average velocity $= \frac{s(3) - s(1)}{3 - 1}$

$= \frac{366 - 554}{2}$

$= -94$ ft/sec

$v(1) = -32(1) - 30 = -62$ ft/sec $v(3) = -32(3) - 30 = -126$ ft/sec

$s(t) = 0 = -16t^2 - 30t + 600$

Using a graphing utility or the Quadratic Formula, ≈ 5.258 seconds.

When ≈ 5.258, $v(t) \approx -32(5.258) - 30 \approx -198.3$ ft/sec.

$f(x) = (5x^2 + 8)(x^2 - 4x - 6)$

$f'(x) = (5x^2 + 8)(2x - 4) + (x^2 - 4x - 6)(10x)$

$10x^3 + 16x - 20x^2 - 32 + 10x^3 - 40x^2 - 60x$

$20x^3 - 60x^2 - 44x - 32$

$4(5x^3 - 15x^2 - 11x - 8)$

$g(x) = (2x^3 + 5x)(3x - 4)$

$g'(x) = (2x^3 + 5x)(3) + (3x - 4)(6x^2 + 5)$

$6x^3 + 15x + 18x^3 - 24x^2 + 15x - 20$

$24x^3 - 24x^2 + 30x - 20$

$f(x) = (9x - 1)\sin x$

$f'(x) = (9x - 1)\cos x + 9\sin x$

$9x\cos x - \cos x + 9\sin x$

$f(t) = 2t^5\cos t$

$f'(t) = 2t^5(-\sin t) + \cos t(10t^4)$

$-2t^5\sin t + 10t^4\cos t$

$f(x) = \frac{x^2 + x - 1}{x^2 - 1}$

$f'(x) = \frac{(x^2 - 1)(2x + 1) - (x^2 + x - 1)(2x)}{(x^2 - 1)^2}$

$= \frac{-(x^2 + 1)}{(x^2 - 1)^2}$

34. $f(x) = \frac{2x + 7}{x^2 + 4}$

$f'(x) = \frac{(x^2 + 4)(2) - (2x + 7)(2x)}{(x^2 + 4)^2}$

$= \frac{2x^2 + 8 - 4x^2 - 14x}{(x^2 + 4)^2}$

$= \frac{-2x^2 - 14x + 8}{(x^2 + 4)^2} = \frac{-2(x^2 + 7x - 4)}{(x^2 + 4)^2}$

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28. $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$

$-16t^2 + 450$

$v(t) = s'(t) = -32t$

$v(2) = -32(2) = -64$ ft/sec

$v(5) = -32(5) = -160$ ft/sec

$(x^2 + 4)^2$ $(x^2 + 4)^2$

x^4

$y = \frac{x^4}{\cos x}$

$y' = \frac{(\cos x)4x^3 - x^4(-\sin x)}{\cos^2 x}$

$\frac{4x^3 \cos x + x^4 \sin x}{\cos^2 x}$

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$y = \frac{\sin x}{x^4}$

$y' = \frac{(x^4)\cos x - (\sin x)(4x^3)}{(x^4)^2} = \frac{x\cos x - 4\sin x}{x^5}$

$y = 3x^2 \sec x$

$y' = 3x^2 \sec x \tan x + 6x \sec x$

$y = -x^2 \tan x$

$y' = -x^2 \sec^2 x - 2x \tan x$

$y = x\cos x - \sin x$

$y' = -x\sin x + \cos x - \cos x = -x\sin x$

$g(x) = x^4 \cot x + 3x\cos x$

$g'(x) = 4x^3 \cot x + x^4(-\csc^2 x) + 3\cos x - 3x\sin x$

$4x^3 \cot x - x^4 \csc^2 x + 3\cos x - 3x\sin x$

41. $f(x) = (x+2)(x^2+5), \ (-1, 6)$

$f'(x) = (x+2)(2x) + (x^2+5)(1)$

$2x^2 + 4x + x^2 + 5 = 3x^2 + 4x + 5$ $f'(-1) = 3 - 4 + 5 = 4$

Tangent line: $y - 6 = 4(x+1)$

$= 4x + 10$

42. $f(x) = (x-4)(x^2+6x-1), \ (0, 4)$

$f'(x) = (x-4)(2x+6) + (x^2+6x-1)(1)$

$= 2x^2 - 2x - 24 + x^2 + 6x - 1$

$= 3x^2 + 4x - 25$

$f'(0) = 0 + 0 - 25 = -25$

Tangent line: $y - 4 = -25(x-0)$

$= -25x + 4$

$f(x) = \frac{1+\cos x}{1-\cos x}, \ \left(\frac{\pi}{2}, 1\right)$

$f'(x) = \frac{(1-\cos x)(-\sin x) - (1+\cos x)(\sin x)}{(1-\cos x)^2}$

$= \frac{-2\sin x}{(1-\cos x)^2}$

$f'\left(\frac{\pi}{2}\right) = \frac{-2}{1} = -2$

Tangent line: $y - 1 = -2\left(x - \frac{\pi}{2}\right)$

$= -2x + 1 + \pi$

$g(t) = -8t^3 - 5t + 12$

$g'(t) = -24t^2 - 5$

$g''(t) = -48t$

$h(x) = 6x^{-2} + 7x^2$

$h'(x) = -12x^{-3} + 14x$

$h''(x) = 36x^{-4} + 14 = \frac{36}{x^4} + 14$

$f(x) = 15x^{5/2}$

$f'(x) = \frac{75}{2}x^{3/2}$

$f''(x) = \frac{225}{4}x^{1/2} = \frac{225}{4}\sqrt{x}$

48. $f(x) = 20\sqrt[5]{x} = 20x^{1/5}$

$f'(x) = 4x^{-4/5}$

$f''(x) = \frac{-16}{5}x^{-9/5} = -\frac{16}{5x^{9/5}}$

49. $f(\theta) = 3\tan\theta$

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43. $f(x) = \dfrac{x+1}{x-1}, \left(\dfrac{1}{2}, -3\right)$

$$f'(x) = \frac{(x-1) - (x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

$$f'\left(\frac{1}{2}\right) = \frac{-2}{(1/4)} = -8$$

Tangent line: $y + 3 = -8\left(x - \dfrac{1}{2}\right)$

$$y = -8x + 1$$

$$f'(\theta) = 3\sec^2\theta$$
$$f''(\theta) = 6\sec\theta(\sec\theta\tan\theta)$$
$$= 6\sec^2\theta\tan\theta$$

$$h(t) = 10\cos t - 15\sin t$$

$$h'(t) = -10\sin t - 15\cos t$$

$$h''(t) = -10\cos t + 15\sin t$$

$$g(x) = 4\cot x$$
$$g'(x) = -4\csc^2 x$$
$$g''(x) = -8\csc x(-\csc x\cot x)$$
$$= 8\csc^2 x\cot x$$

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$h(t) = -12\csc t$

$h'(t) = -12(-\csc t\cot t) = 12\csc t\cot t$

$h''(t) = 12\csc t(-\csc^2 t) + 12\cot t(-\csc t\cot t)$

$= -12(\csc^3 t + \csc t\cot^2 t)$

53. $v(t) = 20 - t^2,\ 0 \le t \le 6$

$a(t) = v'(t) = -2t$

$v(3) = 20 - 3^2 = 11$ m/sec

$a(3) = -2(3) = -6$ m/sec^2

54. $v(t) = \dfrac{90t}{4t + 10}$

$a(t) = \dfrac{(4t + 10)90 - 90t(4)}{(4t + 10)^2}$

$= \dfrac{900}{(4t + 10)^2} = \dfrac{225}{(2t + 5)^2}$

(a) $v(1) = \dfrac{90}{14} \approx 6.43$ ft/sec

$a(1) = \dfrac{225}{49} \approx 4.59$ ft/sec^2

(b) $v(5) = \dfrac{90(5)}{30} = 15$ ft/sec

$a(5) = \dfrac{225}{15^2} = 1$ ft/sec^2

$v(10) = \dfrac{90(10)}{50} = 18$ ft/sec

$a(10) = \dfrac{225}{25^2} = 0.36$ ft/sec^2

$y = (7x + 3)^4$

$y' = 4(7x + 3)^3(7) = 28(7x + 3)^3$

$y = (x^2 - 6)^3$

$y' = 3(x^2 - 6)^2(2x) = 6x(x^2 - 6)^2$

58. $f(x) = \dfrac{1}{(5x + 1)^2} = (5x + 1)^{-2}$

$f'(x) = -2(5x + 1)^{-3}(5) = -\dfrac{10}{(5x + 1)^3}$

$y = 5\cos(9x + 1)$

$y' = -5\sin(9x + 1)(9) = -45\sin(9x + 1)$

$y = -6\sin 3x^4$

$y' = -6\cos(3x^4)(12x^3) = -72x^3\cos 3x^4$

$y = \dfrac{x}{2} - \dfrac{\sin 2x}{4}$

$y' = \dfrac{1}{2} - \dfrac{1}{4}(\cos 2x)(2) = \dfrac{1}{2}(1 - \cos 2x) = \sin^2 x$

62. $y = \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5}$

$y' = \sec^6 x(\sec x\tan x) - \sec^4 x(\sec x\tan x)$

$= \sec^5 x\tan x(\sec^2 x - 1)$

$= \sec^5 x\tan^3 x$

$y = x(6x + 1)^5$

$y' = x5(6x + 1)^4(6) + (6x + 1)^5(1)$

$= 30x(6x + 1)^4 + (6x + 1)^5$

$= (6x + 1)^4(30x + 6x + 1)$

$= (6x + 1)^4(36x + 1)$

64. $f(s) = (s^2 - 1)^{5/2}(s^3 + 5)$

$f'(s) = (s^2 - 1)^{5/2}(3s^2) + (s^3 + 5)\left(\frac{5}{2}\right)(s^2 - 1)^{3/2}(2s)$

$= s(s^2 - 1)^{3/2}\left[3s(s^2 - 1) + 5(s^3 + 5)\right]$

$= s(s^2 - 1)^{3/2}(8s^3 - 3s + 25)$

65. $f(x) = \left(\dfrac{x}{\sqrt{}}\right)^3$

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57. $y = \frac{1}{(x^2+5)^3} = (x^2+5)^{-3}$

$$y' = -3(x^2+5)^{-4}(2x)$$

$$= -\frac{6x}{(x^2+5)^4}$$

$$f'(x) = 3\left(\frac{x}{\sqrt{x+5}}\right)^2 \left[\frac{(x+5)^{1/2}(1) - x\left(\frac{1}{2}\right)(x+5)^{-1/2}}{x+5}\right]$$

$$= \frac{3x^2}{x+5}\left[\frac{2(x+5) - x}{2(x+5)^{3/2}}\right]$$

$$= \frac{3x^2(x+10)}{2(x+5)^{5/2}}$$

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$$h(x) = \left(\frac{x+5}{x^2+3}\right)^2$$

$$h'(x) = 2\left(\frac{x+5}{x^2+3}\right)\left(\frac{(x^2+3)(1)-(x+5)(2x)}{(x^2+3)^2}\right)$$

$$= \frac{2(x+5)(-x^2-10x+3)}{(x^2+3)^3}$$

67. $f(x) = \sqrt{1-x^3}$, $(-2, 3)$

$$f'(x) = \frac{1}{2}(1-x^3)^{-1/2}(-3x^2) = \frac{-3x^2}{2\sqrt{1-x^3}}$$

$$f'(-2) = \frac{-12}{2(3)} = -2$$

68. $f(x) = \sqrt[3]{x^2-1}$, $(3, 2)$

$$f'(x) = \frac{1}{3}(x^2-1)^{-2/3}(2x) = \frac{2x}{3(x^2-1)^{2/3}}$$

$$f'(3) = \frac{2(3)}{3(4)} = \frac{1}{2}$$

69. $f(x) = \dfrac{x+8}{(3x+1)^{1/2}}$, $(0, 8)$

$$f'(x) = \frac{(3x+1)^{1/2}(1)-(x+8)\left(\frac{1}{2}\right)(3x+1)^{-1/2}(3)}{3x+1}$$

$$f'(0) = \frac{1-4(3)}{1} = -11$$

70. $f(x) = \dfrac{3x+1}{(4x-3)^3}$, $(1, 4)$

$$f'(x) = \frac{(4x-3)^3(3)-(3x+1)3(4x-3)^2(4)}{(4x-3)^6}$$

$$f'(1) = \frac{3-4(3)(4)}{1} = -45$$

$y = \frac{1}{2}\csc 2x$, $\left(\frac{\pi}{4}, \frac{1}{2}\right)$

$$y' = -\csc 2x \cot 2x$$

$$y'\left(\frac{\pi}{4}\right) = 0$$

72. $y = \csc 3x + \cot 3x$, $\left(\frac{\pi}{6}, 1\right)$

$$y' = -3\csc 3x\cot 3x - 3\csc^2 3x$$

$$y'\left(\frac{\pi}{6}\right) = 0-3 = -3$$

$y = (8x+5)^3$

$$y' = 3(8x+5)^2(8) = 24(8x+5)^2$$

$$y'' = 24(2)(8x+5)(8) = 384(8x+5)$$

74. $y = \dfrac{1}{5x+1} = (5x+1)^{-1}$

$$y' = -1(5x+1)^{-2}(5) = -5(5x+1)^{-2}$$

$$y'' = -5(-2)(5x+1)^{-3}(5) = \frac{50}{(5x+1)^3}$$

$f(x) = \cot x$

$$f'(x) = -\csc^2 x$$

$$f''(x) = -2\csc x(-\csc x \cdot \cot x)$$

$$= 2\csc^2 x \cot x$$

$$y = x\sin^2 x$$
$$y' = \sin^2 x + 2x\sin x\cos x$$
$$y'' = 2\sin x\cos x + 2\sin x\cos x + 2x\cos^2 x - 2x\sin^2 x$$
$$= 4\sin x\cos x + 2x\left(\cos^2 x - \sin^2 x\right)$$

77. $T = \frac{700}{t^2 + 4t + 10}$

$= 700(t^2 + 4t + 10)^{-1}$

$T' = \frac{-1400(t+2)}{(t^2 + 4t + 10)^2}$

When $t = 1$,

$T' = \frac{-1400(1+2)}{(1+4+10)^2} \approx -18.667$ deg/h.

When $t = 3$,

$T' = \frac{-1400(3+2)}{(9+12+10)^2} \approx -7.284$ deg/h.

(c) When $t = 5$,

$T' = \frac{-1400(5+2)}{(25+20+10)^2} \approx -3.240$ deg/h.

(d) When $t = 10$,

$T' = \frac{-1400(10+2)}{(100+40+10)^2} \approx -0.747$ deg/h.

78. $y = \frac{1}{4}\cos 8t - \frac{1}{4}\sin 8t$

$y' = \frac{1}{4}(-\sin 8t)8 - \frac{1}{4}(\cos 8t)8$

$= -2\sin 8t - 2\cos 8t$

At time $t = \frac{\pi}{4}$,

$y\left(\frac{\pi}{4}\right) = \frac{1}{4}\cos\left[8\left(\frac{\pi}{4}\right)\right] - \frac{1}{4}\sin\left[8\left(\frac{\pi}{4}\right)\right]$

$= \frac{1}{4}(1) = \frac{1}{4}$ ft.

$v(t) = y'\left(\frac{\pi}{4}\right) = -2\sin\left[8\left(\frac{\pi}{4}\right)\right] - 2\cos\left[8\left(\frac{\pi}{4}\right)\right]$

$-2(0) - 2(1) = -2$ ft/sec

$x^2 + y^2 = 64$

81. $x^3y - xy^3 = 4$

$x^3y' + 3x^2y - x3y^2y' - y^3 = 0$

$x^3y' - 3xy^2y' = y^3 - 3x^2y$

$y'(x^3 - 3xy^2) = y^3 - 3x^2y$

$y' = \frac{y^3 - 3x^2y}{x^3 - 3xy^2}$

$y' = \frac{y(y^2 - 3x^2)}{x(x^2 - 3y^2)}$

$\sqrt{xy} = x - 4y$

$\frac{\sqrt{x}}{2\sqrt{y}}y' + \frac{\sqrt{y}}{2\sqrt{x}} = 1 - 4y'$

$xy' + y = 2\sqrt{xy} - 8\sqrt{xy}\,y'$

$(x + 8\sqrt{xy})y' = 2\sqrt{xy} - y$

$y' = \frac{2\sqrt{xy} - y}{x + 8\sqrt{xy}}$

$= \frac{2(x - 4y) - y}{x + 8(x - 4y)}$

$= \frac{2x - 9y}{9x - 32y}$

$x\sin y = y\cos x$

$(x\cos y)y' + \sin y = -y\sin x + y'\cos x$

$y'(x\cos y - \cos x) = -y\sin x - \sin y$

$y' = \frac{y\sin x + \sin y}{\cos x - x\cos y}$

$\cos(x + y) = x$

$2x + 2yy' = 0$

$2yy' = -2x$

x

$-(1 + y')\sin(x + y) = 1$

$y' = -y$

$x^2 + 4xy - y^3 = 6$

$2x + 4xy' + 4y - 3y^2 y' = 0$

$(4x - 3y^2)y' = -2x - 4y$

$y' = \dfrac{2x + 4y}{3y^2 - 4x}$

$y' \sin(x + y) = 1 + \sin(x + y)$

$y' = -\dfrac{1 + \sin(x + y)}{\sin(x + y)} = -\csc(x + 1) - 1$

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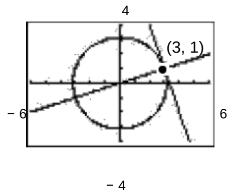
85. $x^2 + y^2 = 10$

$2x + 2yy' = 0$

$y' = \dfrac{-x}{y}$



At $(3, 1)$, $y' = -3$

Tangent line: $y - 1 = -3(x - 3) \Rightarrow 3x + y - 10 = 0$

Normal line: $y - 1 = \frac{1}{3}(x - 3) \Rightarrow x - 3y = 0$

$x^2 - y^2 = 20$

$2x - 2yy' = 0$

$y' = \dfrac{x}{y}$

At $(6, 4)$, $y' = \frac{3}{2}$

Tangent line: $y - 4 = \frac{3}{2}(x - 6)$

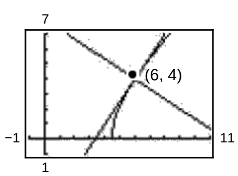
$y = \frac{3}{2}x - 5$

$2y - 3x + 10 = 0$

Normal line: $y - 4 = -\frac{2}{3}(x - 6)$

$= -\frac{2}{3}x + 8$

$3y + 2x - 24 = 0$



$y = \sqrt{x}$

$\dfrac{dy}{dt} = 2$ units/sec

$\dfrac{dy}{dt} = \dfrac{1}{2\sqrt{x}}\dfrac{dx}{dt} \Rightarrow \dfrac{dx}{dt} = 2\sqrt{x}\dfrac{dy}{dt} = 4\sqrt{x}$

(a) When $x = \frac{1}{2}$, $\frac{dx}{dt} = 2\sqrt{2}$ units/sec.

When $x = 1$, $\frac{dx}{dt} = 4$ units/sec.

When $x = 4$, $\frac{dx}{dt} = 8$ units/sec.

Surface area $= A = 6x^2$, $x =$ length of edge

$\dfrac{dx}{dt} = 8$

$\dfrac{dA}{dt} = 12x\dfrac{dx}{dt} = 12(6.5)(8) = 624\text{ cm}^2/\text{sec}$

$\tan\theta = x$

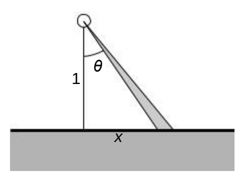
$\dfrac{d\theta}{dt} = 3(2\pi)$ rad/min

$\sec^2\theta\left(\dfrac{d\theta}{dt}\right) = \dfrac{dx}{dt}$

$\dfrac{dx}{dt} = (\tan^2\theta + 1)(6\pi) = 6\pi(x^2 + 1)$

When $x = \frac{1}{2}$,

$\dfrac{dx}{dt} = 6\pi\left(\dfrac{1}{4} + 1\right) = \dfrac{15\pi}{2}$ km/min $= 450\pi$ km/h.



$s(t) = s'(t$ $- 4.9t^2$

$) =$ $-9.8t$

$s =$ $= 60 - 4.9t^2$

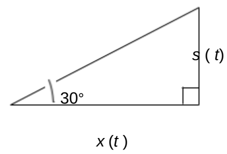
$4.9t^2 =$

$t = \frac{5}{\sqrt{4.9}}$

$\tan 30 = \frac{}{\sqrt{3}} = \frac{s(t)}{x(t)}$

$x(t) = \sqrt{3}s(t)$

$\frac{dx}{dt} = \sqrt{3}\frac{ds}{dt} = \sqrt{3}\left(-9.8\frac{5}{\sqrt{4.9}}\right) \approx -38.34$ m/sec



Problem Solving for Chapter 2

(a) $x^2 + (y - r)^2 = r^2$, Circle

$x^2 = y$, Parabola

Substituting:

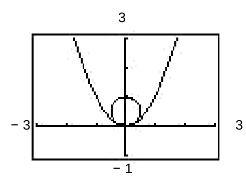
$y - r)^2 = r^2 - y$

$y^2 - 2ry + r^2 = r^2 - y$

$y^2 - 2ry + y = 0$

$y\, y - 2r + 1 = 0$

Because you want only one solution, let $1 - 2r = 0 \Rightarrow r = \frac{1}{2}$ Graph $y = x^2$ and $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$



(b) Let (x, y) be a point of tangency:

$x^2 + (y - b)^2 = 1 \Rightarrow 2x + 2(y - b)y' = 0 \Rightarrow y' = \frac{x}{b - y}$, Circle

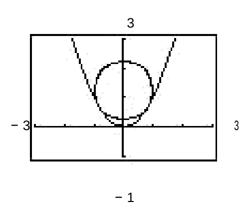
$= x^2 \Rightarrow y' = 2x$, Parabola

Equating:

$2x = \frac{x}{b - y}$

$2(b - y) = 1$

$b - y = \frac{1}{2} \Rightarrow b = y + \frac{1}{2}$



Also, $x^2 + (y - b)^2 = 1$ and $y = x^2$ imply:

$y + (y - b)^2 = 1 \Rightarrow y + \left[y - \left(y + \frac{1}{2}\right)\right]^2 = 1 \Rightarrow y + \frac{1}{4} = 1 \Rightarrow y = \frac{3}{4}$ and $b = \frac{5}{4}$

Center: $\left(0, \frac{5}{4}\right)$

Graph $y = x^2$ and $x^2 + \left(y - \frac{5}{4}\right)^2 = 1$.

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2. Let (a, a^2) and $(b, -b^2 + 2b - 5)$ be the points of tangency. For $y = x^2$, $y' = 2x$ and for $y = -x^2 + 2x - 5$, $y' = -2x + 2$. So, $2a = -2b + 2 \Rightarrow a + b = 1$, or $a = 1 - b$. Furthermore, the slope of the common tangent line is

$$\frac{a^2 - (-b^2 + 2b - 5)}{a - b} = \frac{(1 - b)^2 + b^2 - 2b + 5}{(1 - b) - b} = -2b + 2$$

$$\frac{1 - 2b + b^2 + b^2 - 2b + 5}{1 - 2b} = -2b + 2$$

$$2b^2 - 4b + 6 = 4b^2 - 6b + 2$$

$$2b^2 - 2b - 4 = 0$$

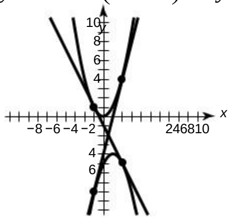
$$b^2 - b - 2 = 0$$

$$(b - 2)(b + 1) = 0$$

$$= 2, -1$$

For $b = 2$, $a = 1 - b = -1$ and the points of tangency are $(-1, 1)$ and $(2, -5)$. The tangent line has slope -2: $y - 1 = -2(x + 1) \Rightarrow y = -2x - 1$

For $b = -1$, $a = 1 - b = 2$ and the points of tangency are $(2, 4)$ and $(-1, -8)$. The tangent line has slope $y - 4 = 4(x - 2) \Rightarrow y = 4x - 4$



Let $p(x) = Ax^3 + Bx^2 + Cx + D$, $p'(x) = 3Ax^2 + 2Bx + C$.

At $(1, 1)$:

$A + B + C + D = 1$ Equation 1

$3A + 2B + C = 14$ Equation 2

At $(-1, -3)$:

$A + B - C + D = -3$ Equation 3

$3A + 2B + C = -2$ Equation 4

Adding Equations 1 and 3: $2B + 2D = -2$

Subtracting Equations 1 and 3: $2A + 2C = 4$ $D = \frac{1}{2}(-2 - 2B) = -5.$

Adding Equations 2 and 4: $6A + 2C = 12$

Subtracting Equations 2 and 4: $4B = 16$

So, $B = 4$ and $D = \frac{1}{2}(-2 - 2B) = -5$. Subtracting $2A + 2C = 4$ and $6A + 2C = 12$, you obtain $4A = 8 \Rightarrow A = 2$. Finally, $C = \frac{1}{2}(4 - 2A) = 0$. So, $p(x) = 2x^3 + 4x^2 - 5$.

$f(x) = a + b\cos cx$

$f'(x) = -bc\sin cx$

At $\left(\frac{\pi}{4}, \frac{3}{2}\right)$: $a + b\cos\left(\frac{c\pi}{4}\right) = \frac{3}{2}$ Equation 2

$-bc\sin\left(\frac{c\pi}{4}\right) = 1$ Equation 3

$(1 - b) + b\cos\left(\frac{c\pi}{4}\right) = \frac{3}{2} \Rightarrow -b + b\cos\frac{c\pi}{4} = \frac{1}{2}.$

From Equation 3, $b = \frac{-1}{c\sin(c\pi/4)}$. So:

$\frac{1}{c\sin(c\pi/4)} - \frac{1}{c\sin(c\pi/4)}\cos\left(\frac{c\pi}{4}\right) = \frac{1}{2}$

$1 - \cos\left(\frac{c\pi}{4}\right) = \frac{1}{2}c\sin\left(\frac{c\pi}{4}\right)$

Graphing the equation

$2\left[c\sin\left(\frac{c\pi}{4}\right) + \cos\left(\frac{c\pi}{4}\right)\right]$

you see that many values of c will work. One answer:

$c = 2, b = -\frac{1}{2}, a = \frac{3}{2} \Rightarrow f(x) = \frac{3}{2} - \frac{1}{2}\cos 2x$

(a) $y = x^2$, $y' = 2x$, Slope $= 4$ at $(2, 4)$

Tangent line: $y - 4 = 4(x - 2)$
$y = 4x - 4$

Slope of normal line: $-\frac{1}{4}$

Normal line: $y - 4 = -\frac{1}{4}(x - 2)$

$y = -\frac{1}{4}x + \frac{9}{2}$

$y = -\frac{1}{4}x + \frac{9}{2} = x^2$

$\Rightarrow 4x^2 + x - 18 = 0$

$\Rightarrow (4x + 9)(x - 2) = 0$

$x = 2, -\frac{9}{4}$

Second intersection point: $\left(-\frac{9}{4}, \frac{81}{16}\right)$

(c) Tangent line: $y = 0$
Normal line: $x = 0$

(d) Let be a point on the parabola

Tangent line at is

Normal line at is

To find points of intersection, solve:

x	-1.0	-0.1	-0.001	0	0.001	0.1	1.0
$\cos x$	0.5403	0.9950	≈ 1	1	≈ 1	0.9950	0.5403
$P_2(x)$	0.5	0.9950	≈ 1	1	≈ 1	0.9950	0.5

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6. (a) $f(x) = \cos x$ $\quad P_1(x) = a_0 + a_1 x$

$f(0) = 1$ $\quad P_1(0) = a_0 \Rightarrow a_0 = 1$

$f'(0) = 0$ $\quad P_1'(0) = a_1 \Rightarrow a_1 = 0$

$P_1(x) = 1$

(b)

$f(x) = \cos x$ $\quad P_2(x) = a_0 + a_1 x + a_2 x^2$

$f(0) = 1$ $\quad P_2(0) = a_0 \Rightarrow a_0 = 1$

$f'(0) = 0$ $\quad P_2'(0) = a_1 \Rightarrow a_1 = 0$

$f''(0) = -1$ $\quad P_2''(0) = 2a_2 \Rightarrow a_2 = -\frac{1}{2}$

$P_2(x) = 1 - \frac{1}{2}x^2$

$P_2(x)$ is a good approximation of $f(x) = \cos x$ when x is near 0.

16)

$(a, a^2), a \neq 0,$ $\quad y = x^2.$

(a, a^2) $\quad y = 2a(x - a) + a^2.$

(a, a^2) $\quad y = -(1/2a)(x - a) + a^2.$

The normal line intersects a second time at

$$x^2 = -\frac{1}{2a}(x - a) + a^2$$

$$x^2 + \frac{1}{2a}x = a^2 + \frac{1}{2}$$

$$x^2 + \frac{1}{2a}x + \frac{1}{16a^2} = a^2 + \frac{1}{2} + \frac{1}{16a^2}$$

$$\left(x + \frac{1}{4a}\right)^2 = \left(a + \frac{1}{4a}\right)^2$$

$$x + \frac{1}{4a} = \pm\left(a + \frac{1}{4a}\right)$$

$$x + \frac{1}{4a} = a + \frac{1}{4a} \Rightarrow x = a \quad \text{(Point of tangency)}$$

$$x + \frac{1}{4a} = -\left(a + \frac{1}{4a}\right) \Rightarrow x = -a - \frac{1}{2a} = -\frac{2a^2 + 1}{2a}$$

$$x = -\frac{2a^2 + 1}{2a}.$$

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(d) $f(x) = \sin x$ $\quad P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$f(0) = 0 \quad P_3(0) = a_0 \Rightarrow a_0 = 0$

$f'(0) = 1 \quad P_3'(0) = a_1 \Rightarrow a_1 = 1$

$f''(0) = 0 \quad P_3''(0) = 2a_2 \Rightarrow a_2 = 0$

$f'''(0) = -1 \quad P_3'''(0) = 6a_3 \Rightarrow a_3 = -\frac{1}{6}$

$P_3(x) = x - \frac{1}{6}x^3$

7. (a) $x^4 = a^2x^2 - a^2y^2$

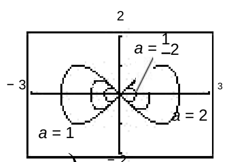
$a^2y^2 = a^2x^2 - x^4$

$y = \frac{\pm\sqrt{a^2x^2 - x^4}}{a}$

Graph: $y_1 = \frac{\sqrt{a^2x^2 - x^4}}{a}$

and $y_2 = -\frac{\sqrt{a^2x^2 - x^4}}{a}$.

(b)



$(\pm a, 0)$ are the x-intercepts,

along with $(0, 0)$.

(c) Differentiating implicitly:

$4x^3 = 2a^2x - 2a^2yy'$

$y' = \frac{2a^2x - 4x^3}{2a^2y} = \frac{x(a^2 - 2x^2)}{a^2y} = 0 \Rightarrow 2x^2 = a^2 \Rightarrow x = \frac{\pm a}{\sqrt{2}}$

$\left(\frac{a^2}{2}\right)^2 = a^2\left(\frac{a^2}{2}\right) - a^2y^2$

$\frac{a^4}{4} = \frac{a^4}{2} - a^2y^2$

$a^2y^2 = \frac{a^4}{4}$

$y^2 = \frac{a^2}{4}$

$y = \pm\frac{a}{2}$

Four points: $\left(\frac{a}{\sqrt{2}}, \frac{a}{2}\right), \left(\frac{a}{\sqrt{2}}, -\frac{a}{2}\right), \left(-\frac{a}{\sqrt{2}}, \frac{a}{2}\right), \left(-\frac{a}{\sqrt{2}}, -\frac{a}{2}\right)$

8. (a) $b^2y^2 = x^3(a - x);\ a, b > 0$

$y^2 = \frac{x^3(a - x)}{b^2}$

Graph $y_1 = \frac{\sqrt{x^3(a - x)}}{b}$ and $y_2 = -\frac{\sqrt{x^3(a - x)}}{b}$.

a determines the x-intercept on the right: $(a, 0)$. b affects the height.

Differentiating implicitly:

$2b^2yy' = 3x^2(a - x) - x^3 = 3ax^2 - 4x^3$

$y' = \frac{3ax^2 - 4x^3}{2b^2y} = 0$

$3ax^2 = 4x^3$

$3a = 4x$

$\frac{3a}{4} = x$

$b^2y^2 = \left(\frac{3a}{4}\right)^3\left(a - \frac{3a}{4}\right) = \frac{27a^3}{64}\left(\frac{1}{4}a\right)$

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$$\left(\frac{\quad}{4}\right)\left(a - \frac{\quad}{4}\right) = \frac{\quad}{64}\left(\frac{a}{4}\right)$$

$$y^2 = \frac{27a^4}{256b^2} \Rightarrow y = \pm\frac{3\sqrt{3}a^2}{16b}$$

Two points: $\left(\frac{3a}{4}, \frac{3\sqrt{3}a^2}{16b}\right), \left(\frac{3a}{4}, -\frac{3\sqrt{3}a^2}{16b}\right)$

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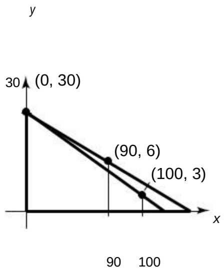
9. (a)



Not drawn to scale

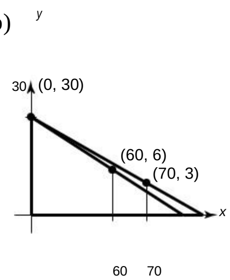
Line determined by $(0, 30)$ and $(90, 6)$:

$$y - 30 = \frac{30-6}{0-90}(x - 0) = \frac{24}{-90}x = -\frac{4}{15}x \Rightarrow y = -\frac{4}{15}x + 30$$

When $x = 100$: $y = -\frac{4}{15}(100) + 30 = \frac{10}{3} > 3$

As you can see from the figure, the shadow determined by the man extends beyond the shadow determined by the child.

(b)



Not drawn to scale

Line determined by $(0, 30)$ and $(60, 6)$:

$$y - 30 = \frac{30-6}{0-60}(x - 0) = -\frac{2}{5}x \Rightarrow y = -\frac{2}{5}x + 30$$

When $x = 70$: $y = -\frac{2}{5}(70) + 30 = 2 < 3$

As you can see from the figure, the shadow determined by the child extends beyond the shadow determined by the man.

(c) Need $(0, 30)$, $(d, 6)$, $(d + 10, 3)$ collinear.

$$\frac{30-6}{0-d} = \frac{6-3}{d-(d+10)} \Rightarrow \frac{24}{d} = \frac{3}{10} \Rightarrow d = 80 \text{ feet}$$

Let y be the distance from the base of the street light to the tip of the shadow. You know that $dx/dt = -5$.

For $x > 80$, the shadow is determined by the man.

$$\frac{y}{30} = \frac{y-x}{6} \Rightarrow y = \frac{5}{4}x \text{ and } \frac{dy}{dt} = \frac{5}{4}\frac{dx}{dt} = \frac{-25}{4}$$

For $x < 80$, the shadow is determined by the child.

$$\frac{y}{30} = \frac{y-x-10}{3} \Rightarrow y = \frac{10}{9}x + \frac{100}{9} \text{ and } \frac{dy}{dt} = \frac{10}{9}\frac{dx}{dt} = -\frac{50}{9}$$

Therefore:

$$\frac{dy}{dt} = \begin{cases} -\frac{25}{4}, & x > 80 \\ -\frac{50}{9}, & 0 < x < 80 \end{cases}$$

dy/dt is not continuous at $x = 80$.

ALTERNATE SOLUTION for parts (a) and (b):

(a) As before, the line determined by the man's shadow is

$$y_m = -\frac{4}{15}x + 30$$

The line determined by the child's shadow is obtained by finding the line through

$$y - 30 = \frac{30-3}{0-100}(x - 0)$$

$$\Rightarrow y_c = -\frac{27}{100}x + 30$$

100 100

By setting $y_m = y_c = 0$, you can determine how far the shadows extend:

$(0, 30)$ and $(100, 3)$:

Man: $y_m = 0 \Rightarrow \frac{4}{15}x = 30 \Rightarrow x = 112.5 = 112\frac{1}{2}$

Child: $y_c = 0 \Rightarrow -\frac{27}{100}x = 30 \Rightarrow x = 111.\overline{11} = 111\frac{1}{9}$

The man's shadow is $112\frac{1}{2} - 111\frac{1}{9} = 1\frac{7}{18}$ ft beyond the child's shadow.

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(b) As before, the line determined by the man's shadow is

$$y_m = -\frac{2}{5}x + 30$$

For the child's shadow,

$$y - 30 = \frac{30-3}{0-70}(x-0) \Rightarrow y_c = -\frac{27}{70}x + 30$$

Man: $y_m = 0 \Rightarrow \frac{2}{5}x = 30 \Rightarrow x = 75$

Child: $y_c = 0 \Rightarrow \frac{27}{70}x = 30 \Rightarrow x = \frac{700}{9} = 77\frac{7}{9}$

So the child's shadow is $77\frac{7}{9} - 75 = 2\frac{7}{9}$ ft beyond the man's shadow.

10. (a) $y = x^{1/3} \Rightarrow \frac{dy}{dt} = \frac{1}{3}x^{-2/3}\frac{dx}{dt}$

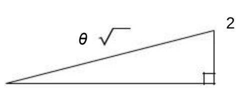
$$1 = \frac{1}{3}(8)^{-2/3}\frac{dx}{dt}$$

$$\frac{dx}{dt} = 12 \text{ cm/sec}$$

(b) $D = \sqrt{x^2+y^2} \Rightarrow \frac{dD}{dt} = \frac{1}{2}(x^2+y^2)^{-1/2}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{x(dx/dt) + y(dy/dt)}{\sqrt{x^2+y^2}}$

$$= \frac{8(12) + 2(1)}{\sqrt{64+4}} = \frac{98}{\sqrt{68}} = \frac{49}{17}\sqrt{17} \text{ cm/sec}$$

(c) $\tan\theta = \frac{y}{x} \Rightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{x(dy/dt) - y(dx/dt)}{x^2}$



From the triangle, $\sec\theta = \sqrt{68}/8$. So $\frac{d\theta}{dt} = \frac{1}{68/64}\left(\frac{8(1) - 2(12)}{64}\right) = \frac{-16}{68} = -\frac{4}{17}$ rad/sec.

(a) $v(t) = -\frac{27}{5}t + 27$ ft/sec

$a(t) = -\frac{27}{5}$ ft/sec^2

$v(t) = -\frac{27}{5}t + 27 = 0 \Rightarrow \frac{27}{5}t = 27 \Rightarrow t = 5$ seconds

$s(5) = -\frac{27}{10}(5)^2 + 27(5) + 6 = 73.5$ feet

The acceleration due to gravity on Earth is greater in magnitude than that on the moon.

12. $E'(x) = \lim\limits_{\Delta x\to 0}\frac{E(x+\Delta x) - E(x)}{\Delta x} = \lim\limits_{\Delta x\to 0}\frac{E(x)E(\Delta x) - E(x)}{\Delta x} = \lim\limits_{\Delta x\to 0} E(x)\left(\frac{E(\Delta x) - 1}{\Delta x}\right) = E(x)\lim\limits_{\Delta x\to 0}\frac{E(\Delta x) - 1}{\Delta x}$

But, $E'(0) = \lim_{x \to 0} \frac{E(x) - E(0)}{x} = \lim_{x \to 0} \frac{E(x) - 1}{x} = 1$. So, $E'(x) = E(x)E'(0) = E(x)$ exists for all x.

For example: $E(x) = e^x$.

13. $L'(x) = \lim_{\Delta x \to 0} \frac{L(x+\Delta x) - L(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{L(x) + L(\Delta x) - L(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{L(\Delta x)}{\Delta x}$.

Also, $L'(0) = \lim_{\Delta x \to 0} \frac{L(\Delta x) - L(0)}{\Delta x}$. But, $L(0) = 0$ because $L(0) = L(0+0) = L(0) + L(0) \Rightarrow L(0) = 0$.

So, $L'(x) = L'(0)$ for all x. The graph of L is a line through the origin of slope $L'(0)$.

14. (a)

z (degrees)	0.1	0.01	0.0001
$\frac{\sin z}{z}$	0.0174524	0.0174533	0.0174533

(b) $\lim_{z \to 0} \frac{\sin z}{z} \approx 0.0174533$

In fact, $\lim_{z \to 0} \frac{\sin z}{z} = \frac{\pi}{180}$.

(c) $\frac{d}{dz}(\sin z) = \lim_{\Delta z \to 0} \frac{\sin(z + \Delta z) - \sin z}{\Delta z}$

$= \lim_{\Delta z \to 0} \frac{\sin z \cdot \cos \Delta z + \sin \Delta z \cdot \cos z - \sin z}{\Delta z}$

$= \lim_{\Delta z \to 0} \left[\sin z \left(\frac{\cos \Delta z - 1}{\Delta z}\right)\right] + \lim_{\Delta z \to 0} \left[\cos z \left(\frac{\sin \Delta z}{\Delta z}\right)\right]$

$= (\sin z)(0) + (\cos z)\left(\frac{\pi}{180}\right) = \frac{\pi}{180} \cos z$

$S(90) = \sin\left(\frac{\pi}{180} 90\right) = \sin \frac{\pi}{2} = 1$

$C(180) = \cos\left(\frac{\pi}{180} 180\right) = -1$

$\frac{d}{dz} S(z) = \frac{d}{dz} \sin(cz) = c \cdot \cos(cz) = \frac{\pi}{180} C(z)$

The formulas for the derivatives are more complicated in degrees.

$j(t) = a'(t)$

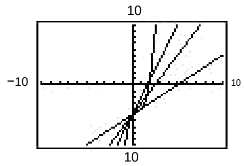
$j(t)$ is the rate of change of acceleration.

$s(t) = -8.25t^2 + 66t$
$v(t) = -16.5t + 66$
$a(t) = -16.5$
$a'(t) = j(t) = 0$

The acceleration is constant, so $j(t) = 0$.

a is position.
b is acceleration.
c is jerk.
d is velocity.

Chapter 2, Section 1, page 100



The line $y = x - 5$ appears to be tangent to the graph of f at the point $(0, -5)$ because it seems to intersect the graph at only that point.

Chapter 2, Section 2, page 110

$f(x) = x^1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 1$$
$$= 1$$

$f(x) = x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x)$$
$$= 2x$$

c. $f(x) = x^3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x\left[3x^2 + 3x\Delta x + (\Delta x)^2\right]}{\Delta x}.$$
$$= \lim_{\Delta x \to 0} \left[3x^2 + 3x\Delta x + (\Delta x)^2\right]$$
$$3x^2$$

$f(x) = x^4$

$f'(x) = \lim_{x \to 0} \frac{f(x + x) - f(x)}{x}$

$= \lim_{x \to 0} \frac{(x + x)^4 - x^4}{x}$

$= \lim_{x \to 0} \frac{x^4 + 4x^3 x + 6x^2 (x)^2 + 4x(x)^3 + (x)^4 - x^4}{x}$

$= \lim_{x \to 0} \frac{4x^3 x + 6x^2 (x)^2 + 4x(x)^3 + (x)^4}{x}$

$= \lim_{x \to 0} \frac{x\left[4x^3 + 6x^2 x + 4x(x)^2 + (x)^3\right]}{x}$

$= \lim_{x \to 0} \left[4x^3 + 6x^2 x + 4x x^2 + x^3\right]$

$4x^3$

$\frac{1}{2}$ $\sqrt{}$

e. $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \lim_{x \to 0} \frac{f(x + x) - f(x)}{x}$

$= \lim_{x \to 0} \frac{\sqrt{x + x} - \sqrt{x}}{x}$

$= \lim_{x \to 0} \left(\frac{\sqrt{x + x} - \sqrt{x}}{x}\right)\left(\frac{\sqrt{x + x} + \sqrt{x}}{\sqrt{x + x} + \sqrt{x}}\right)$

$= \lim_{x \to 0} \frac{(x + x) - x}{x\left(\sqrt{x + x} + \sqrt{x}\right)}$

$= \lim_{x \to 0} \frac{x}{x\left(\sqrt{x + x} + \sqrt{x}\right)}$

$= \lim_{x \to 0} \frac{1}{\sqrt{x + x} + \sqrt{x}}$

$= \frac{1}{2\sqrt{x}}$

$\frac{1}{2} x^{-1/2}$

f. $f(x) = x^{-1} = \frac{1}{x}$

$f'(x) = \lim_{x \to 0} \frac{f(x + x) - f(x)}{x}$

$= \lim_{x \to 0} \frac{\frac{1}{x + x} - \frac{1}{x}}{x}$

$= \lim_{x \to 0} \frac{\frac{x - (x + x)}{x(x + x)}}{x}$

$= \lim_{x \to 0} \frac{-x}{x(x)(x + x)}$

$= \lim_{x \to 0} \frac{-1}{x(x + x)}$

$= -\frac{1}{x^2}$

$= -x^{-2}$

The exponent of *f* becomes the coefficient of f' and the power of *x* decreases by 1.

$(x^n)' = n(x^{n-1})$

Chapter 2, Section 4, page 134

a. Using the Quotient Rule: $y' = \dfrac{(3x+1)(0) - 2(3)}{(3x+1)^2} = -\dfrac{6}{(3x+1)^2}$.

Using the Chain Rule: $y' = \left[2(3x+1)^{-1}\right]'$

$= 2(-1)(3x+1)^{-2}(3)$

$= -\dfrac{6}{(3x+1)^2}$

b. Using algebra before differentiating: $y' = \left[x^3 + 6x^2 + 12x + 8\right]' = 3x^2 + 12x + 12$

Using the Chain Rule: $y' = 3(x+2)^2(1) = 3x^2 + 12x + 12$

Using a trigonometric identity and the Product Rule:

$y = \sin 2x = 2\sin x\cos x$

$y' = 2\left[(\sin x)(-\sin x) + (\cos x)(\cos x)\right]$

$2\left[\cos^2 x - \sin^2 x\right]$

$2\cos 2x$

Using the Chain Rule: $y' = (\cos 2x)(2)$

$2\cos 2x$

In general, the Chain Rule is simpler.

Chapter 2, Section 6, page 152

$$V = \frac{\pi}{3}r^2h$$

$$\frac{dV}{dt} = \frac{\pi}{3}\left(r^2\frac{dh}{dt} + 2rh\frac{dr}{dt}\right)$$

$$\frac{\pi}{3}\left[(1\text{ ft})^2(-0.2\text{ ft/min}) + 2(1\text{ ft})(2\text{ ft})(-0.1\text{ ft/min})\right]$$

$$\frac{\pi}{3}\left(-0.2\text{ ft}^3/\text{min} - 0.4\text{ ft}^3/\text{min}\right)$$

$$\frac{\pi}{3}\left(-0.6\text{ ft}^3/\text{min}\right)$$

$$-\frac{\pi}{5}\text{ ft}^3/\text{min}$$

Given: $\dfrac{dh}{dt} = -0.2$ ft/min

$\dfrac{dr}{dt} = -0.1$ ft/min

$r = 1$ ft

$h = 2$ ft

The rate of

change in the volume does depend on the values of r and h because both variables are in the function $\frac{dV}{dt}$.

CHAPTER 11
Vectors and the Geometry of Space

Section 11.1 Vectors in the Plane

Answers will vary. *Sample answer*: A scalar is a real number such as 2. A vector is represented by a directed line segment. A vector has both magnitude and direction.

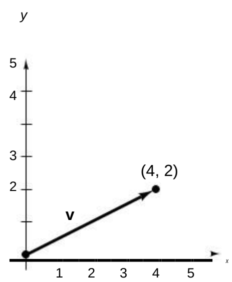
For example $\langle\sqrt{3},1\rangle$ has direction $\frac{\pi}{6}$ and a magnitude of 2.

2. Notice that $\mathbf{v} = \langle 6,-7\rangle \neq \langle 2-(-4), -1-6\rangle = QP$.

Hence, Q is the initial point and P is the terminal point.

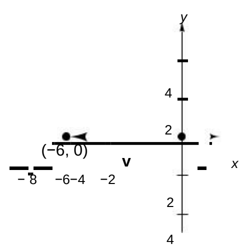
3. (a) $\mathbf{v} = \langle 5-1, 4-2\rangle = \langle 4, 2\rangle$

(b)



4. (a) $\mathbf{v} = \langle -4-2, -3-(-3)\rangle = \langle -6, 0\rangle$

(b)



$\mathbf{u} = \langle 5-3, 6-2\rangle = \langle 2, 4\rangle$

$= \langle 3-1, 8-4\rangle = \langle 2,4\rangle$

$= \mathbf{v}$

$\mathbf{u} = \langle 1-(-4), 8-0\rangle = \langle 5, 8\rangle$

$= \langle 7-2, 7-(-1)\rangle = \langle 5,8\rangle$

$= \mathbf{v}$

$\mathbf{u} = \langle 6-0, -2-3\rangle = \langle 6, -5\rangle$

$= \langle 9-3, 5-10\rangle = \langle 6,-5\rangle$

$= \mathbf{v}$

$\mathbf{u} = \langle 11-(-4), -4-(-1)\rangle = \langle 15, -3\rangle$

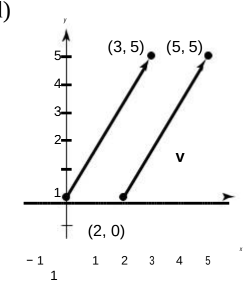
$= \langle 25-0, 10-13\rangle = \langle 15,-3\rangle$

$\mathbf{u} = \mathbf{v}$

9. (b) $\mathbf{v} = \langle 5-2, 5-0\rangle = \langle 3, 5\rangle$

$\mathbf{v} = 3\mathbf{i} + 5\mathbf{j}$

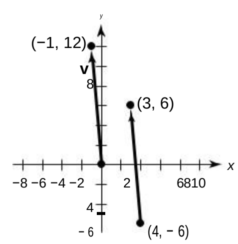
(a), (d)



(b) $\mathbf{v} = \langle 3-4, 6-(-6)\rangle = \langle -1, 12\rangle$

$\mathbf{v} = -\mathbf{i} + 12\mathbf{j}$

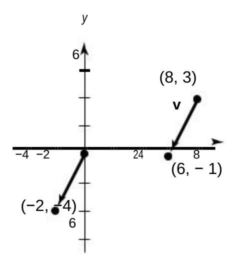
(a), (d)



(b) $\mathbf{v} = \langle 6-8, -1-3\rangle = \langle -2, -4\rangle$

$\mathbf{v} = -2\mathbf{i} - 4\mathbf{j}$

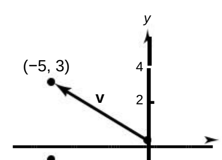
(a), (d)



12. (b) $\mathbf{v} = \langle -5-0, -1-(-4)\rangle = \langle -5, 3\rangle$

$\mathbf{v} = -5\mathbf{i} + 3\mathbf{j}$

(a) and (d).



−6 −4 −2 2

(−5, −1) −2

(0, −4)

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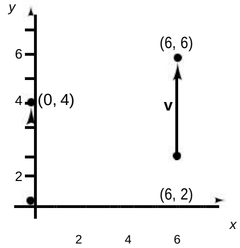
13. (b) $\mathbf{v} = \langle 6-6, 6-2 \rangle = \langle 0, 4 \rangle$

$\mathbf{v} = 4\mathbf{j}$

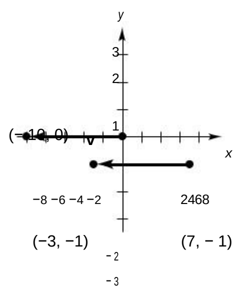
and (d).



14. (b) $\mathbf{v} = \langle -3-7, -1-(-1) \rangle = \langle -10, 0 \rangle$

$\mathbf{v} = -10\mathbf{i}$

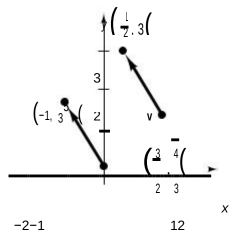
(a) and (d).



(b) $\mathbf{v} = \left\langle \frac{1}{2} - \frac{3}{2}, 3 - \frac{4}{3} \right\rangle = \left\langle -1, \frac{5}{3} \right\rangle$

$\mathbf{v} = -\mathbf{i} + \frac{5}{3}\mathbf{j}$

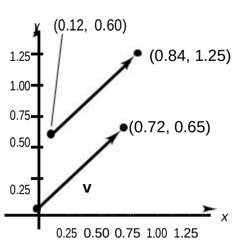
(a) and (d)



16. (b) $\mathbf{v} = \langle 0.84 - 0.12, 1.25 - 0.60 \rangle = \langle 0.72, 0.65 \rangle$

$\mathbf{v} = 0.72\mathbf{i} + 0.65\mathbf{j}$

(a) and (d).



17. $u_1 - 4 = -1$ $\quad u_1 = 3$

$u_2 - 2 = 3$ $\quad u_2 = 5$

$Q = (3, 5)$ Terminal point

18. $u_1 - 5 = 4$ $\quad u_1 = 9$

$u_2 - 3 = -9$ $\quad u_2 = -6$

$Q = (9, -6)$ Terminal point

19. $\mathbf{v} = 4i$

$\|\mathbf{v}\| = \sqrt{4^2} = 4$

20. $\mathbf{v} = -9j$

$\|\mathbf{v}\| = \sqrt{(-9)^2} = 9$

$\mathbf{v} = \langle 8, 15 \rangle$

$\|\mathbf{v}\| = \sqrt{8^2 + 15^2} = 17$

22. $\mathbf{v} = \langle -24, 7 \rangle$

$\|\mathbf{v}\| = \sqrt{(-24)^2 + 7^2} = 25$

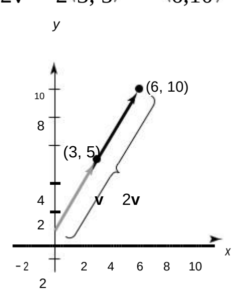
23. $\mathbf{v} = -\mathbf{i} - 5\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{(-1)^2 + (-5)^2} = \sqrt{26}$

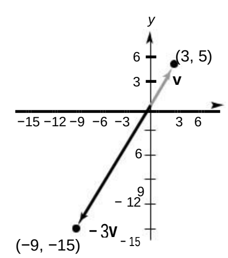
24. $\mathbf{v} = 3\mathbf{i} + 3\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$

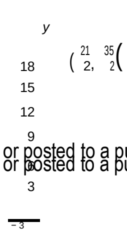
25. (a) $2\mathbf{v} = 2\langle 3, 5 \rangle = \langle 6, 10 \rangle$



$-3\mathbf{v} = \langle -9, -15 \rangle$



$\frac{7}{2}\mathbf{v} = \left\langle \frac{21}{2}, \frac{35}{2} \right\rangle$



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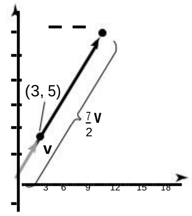

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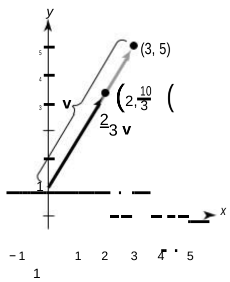
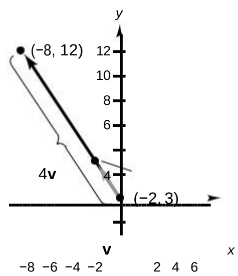
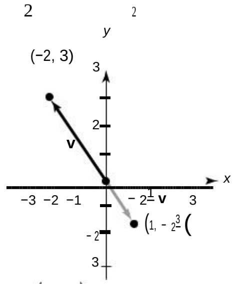
$\frac{2}{3}\mathbf{v} = \left\langle 2, \frac{10}{3} \right\rangle$



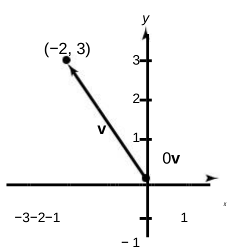
26. (a) $4\mathbf{v} = 4\langle -2, 3\rangle = \langle -8, 12\rangle$



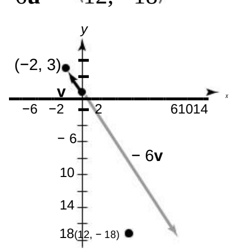
(b) $-\frac{1}{2}\mathbf{v} = \left\langle 1, -\frac{3}{2} \right\rangle$



$0\,\mathbf{v} = \langle 0, 0\rangle$



(d) $-6\mathbf{u} = \langle 12, -18\rangle$



$\mathbf{u} = \langle 4, 9\rangle$, $\mathbf{v} = \langle 2, -5\rangle$

$\frac{2}{3}\mathbf{u} = \frac{2}{3}\langle 4, 9\rangle = \left\langle \frac{8}{3}, 6 \right\rangle$

$3\mathbf{v} = 3\langle 2, -5\rangle = \langle 6, -15\rangle$

$\mathbf{v} - \mathbf{u} = \langle 2, -5\rangle - \langle 4, 9\rangle = \langle -2, -14\rangle$

$2\mathbf{u} + 5\mathbf{v} = 2\langle 4, 9\rangle + 5\langle 2, -5\rangle$

$= \langle 8, 18\rangle + \langle 10, -25\rangle$

$= \langle 18, -7\rangle$

$\mathbf{u} = \langle -3, -8\rangle$, $\mathbf{v} = \langle 8, 7\rangle$

(a) $\frac{2}{3}\mathbf{u} = \frac{2}{3}\langle -3, -8\rangle = \left\langle -2, -\frac{16}{3} \right\rangle$

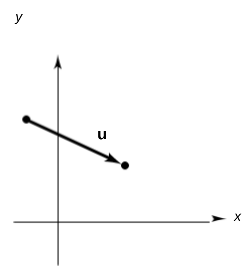
$3\mathbf{v} = 3\langle 8, 7\rangle = \langle 24, 21\rangle$

$\mathbf{v} - \mathbf{u} = \langle 8, 7\rangle - \langle -3, -8\rangle = \langle 11, 15\rangle$

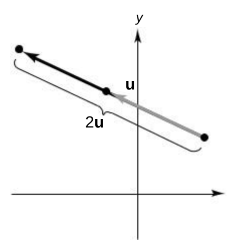
$2\mathbf{u} + 5\mathbf{v} = 2\langle -3, -8\rangle + 5\langle 8, 7\rangle$

$= \langle -6, -16\rangle + \langle 40, 35\rangle$

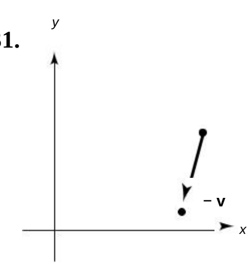
$= \langle 34, 19\rangle$



Twice as long as given vector **u.**



31.



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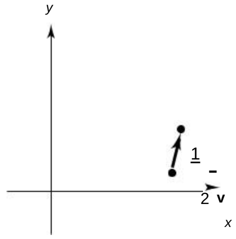
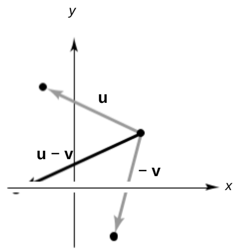
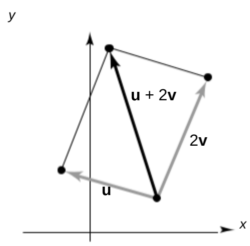
32.



33.





$\mathbf{v} = \langle 3, 12 \rangle$

$\|\mathbf{v}\| = \sqrt{3^2 + 12^2} = \sqrt{153}$

$\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{\langle 3, 12 \rangle}{\sqrt{153}} = \left\langle \dfrac{3}{\sqrt{153}}, \dfrac{12}{\sqrt{153}} \right\rangle$

$= \left\langle \dfrac{\sqrt{17}}{17}, \dfrac{4\sqrt{17}}{17} \right\rangle$ unit vector

$\mathbf{v} = \langle -5, 15 \rangle$

$\|\mathbf{v}\| = \sqrt{25 + 225} = \sqrt{250} = 5\sqrt{10}$

$\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{\langle -5, 15 \rangle}{5\sqrt{10}} = \left\langle -\dfrac{\sqrt{10}}{10}, \dfrac{3\sqrt{10}}{10} \right\rangle$ unit vector

$\mathbf{v} = \left\langle \dfrac{3}{2}, \dfrac{5}{2} \right\rangle$

$\|\mathbf{v}\| = \sqrt{\left(\dfrac{3}{2}\right)^2 + \left(\dfrac{5}{2}\right)^2} = \dfrac{\sqrt{34}}{2}$

$\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{\left\langle \frac{3}{2}, \frac{5}{2} \right\rangle}{\frac{\sqrt{34}}{2}} = \left\langle \dfrac{3}{\sqrt{34}}, \dfrac{5}{\sqrt{34}} \right\rangle$

$= \left\langle \dfrac{3\sqrt{34}}{34}, \dfrac{5\sqrt{34}}{34} \right\rangle$

38. $\mathbf{v} = \langle -6.2, 3.4 \rangle$

$\|\mathbf{v}\| = \sqrt{(-6.2)^2 + (3.4)^2} = \sqrt{50} = 5\sqrt{2}$

$\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{\langle -6.2, 3.4 \rangle}{5\sqrt{2}} = \left\langle -\dfrac{31\sqrt{2}}{50}, \dfrac{17\sqrt{2}}{50} \right\rangle$ unit vector

$\mathbf{u} = \langle 1, -1 \rangle$, $\mathbf{v} = \langle -1, 2 \rangle$

(a) $\|\mathbf{u}\| = \sqrt{1 + 1} = \sqrt{2}$

(b) $\|\mathbf{v}\| = \sqrt{1 + 4} = \sqrt{5}$

(c) $\mathbf{u} + \mathbf{v} = \langle 0, 1 \rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{0 + 1} = 1$

(d) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{\sqrt{2}} \langle 1, -1 \rangle$

$\left\| \dfrac{\mathbf{u}}{\|\mathbf{u}\|} \right\| = 1$

(e) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{\sqrt{5}} \langle -1, 2 \rangle$

$\left\| \dfrac{\mathbf{v}}{\|\mathbf{v}\|} \right\| = 1$

(f) $\dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \langle 0, 1 \rangle$

$\left\| \dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} \right\| = 1$

$\mathbf{u} = \langle 0, 1 \rangle$, $\mathbf{v} = \langle 3, -3 \rangle$

$\|\mathbf{u}\| = \sqrt{0 + 1} = 1$

(b) $\|\mathbf{v}\| = \sqrt{9 + 9} = 3\sqrt{2}$

(c) $\mathbf{u} + \mathbf{v} = \langle 3, -2 \rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{9 + 4} = \sqrt{13}$

$\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \langle 0, 1 \rangle$

$\left\| \dfrac{\mathbf{u}}{\|\mathbf{u}\|} \right\| = 1$

(e) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{3\sqrt{2}} \langle 3, -3 \rangle$

$\left\| \dfrac{\mathbf{v}}{\|\mathbf{v}\|} \right\| = 1$

(f) $\dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \dfrac{1}{\sqrt{13}} \langle 3, -2 \rangle$

$\left\| \dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} \right\| = 1$

u + **v**

unit vector

$\mathbf{u} = \left\langle 1, \frac{1}{2} \right\rangle, \mathbf{v} = \langle 2, 3 \rangle$

(a) $\|\mathbf{u}\| = \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}$

(b) $\|\mathbf{v}\| = \sqrt{4 + 9} = \sqrt{13}$

$\mathbf{u} + \mathbf{v} = \left\langle 3, \frac{7}{2} \right\rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{9 + \frac{49}{4}} = \frac{\sqrt{85}}{2}$

(d) $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{2}{\sqrt{5}}\left\langle 1, \frac{1}{2} \right\rangle$

$\left\| \frac{\mathbf{u}}{\|\mathbf{u}\|} \right\| = 1$

(e) $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\sqrt{13}}\langle 2, 3 \rangle$

$\left\| \frac{\mathbf{v}}{\|\mathbf{v}\|} \right\| = 1$

(f) $\frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \frac{2}{\sqrt{85}}\left\langle 3, \frac{7}{2} \right\rangle$

$\left\| \frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} \right\| = 1$

$\mathbf{u} = \langle 2, -4 \rangle, \mathbf{v} = \langle 5, 5 \rangle$

(a) $\|\mathbf{u}\| = \sqrt{4 + 16} = 2\sqrt{5}$

(b) $\|\mathbf{v}\| = \sqrt{25 + 25} = 5\sqrt{2}$

(c) $\mathbf{u} + \mathbf{v} = \langle 7, 1 \rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{49 + 1} = 5\sqrt{2}$

(d) $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{2\sqrt{5}}\langle 2, -4 \rangle$

$\left\| \frac{\mathbf{u}}{\|\mathbf{u}\|} \right\| = 1$

(e) $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{5\sqrt{2}}\langle 5, 5 \rangle$

$\left\| \frac{\mathbf{v}}{\|\mathbf{v}\|} \right\| = 1$

(f) $\frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \frac{1}{5\sqrt{2}}\langle 7, 1 \rangle$

$\left\| \frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} \right\| = 1$

43. $\mathbf{u} = \langle 2, 1 \rangle$

$\|\mathbf{u}\| = \sqrt{5} \approx 2.236$

$\mathbf{v} = \langle 5, 4 \rangle$

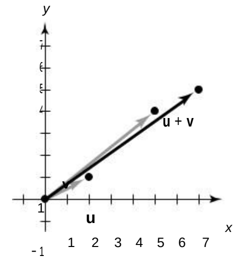
$\|\mathbf{v}\| = \sqrt{41} \approx 6.403$

$\mathbf{u} + \mathbf{v} = \langle 7, 5 \rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{74} \approx 8.602$

$\|\mathbf{u} + \mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$

$\sqrt{74} \le \sqrt{5} + \sqrt{41}$



44. $\mathbf{u} = \langle -3, 2 \rangle$

$\|\mathbf{u}\| = \sqrt{13} \approx 3.606$

$\mathbf{v} = \langle 1, -2 \rangle$

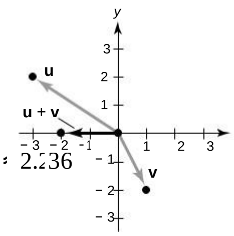
$\|\mathbf{v}\| = \sqrt{5} \approx 2.236$

$\mathbf{u} + \mathbf{v} = \langle -2, 0 \rangle$

$\|\mathbf{u} + \mathbf{v}\| = 2$

$\|\mathbf{u} + \mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$

$2 \le \sqrt{13} + \sqrt{5}$



45. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{3}\langle 0, 3 \rangle = \langle 0, 1 \rangle$

$6\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = 6\langle 0, 1 \rangle = \langle 0, 6 \rangle$

$\mathbf{v} = \langle 0, 6 \rangle$

46. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{2}}\langle 1, 1 \rangle$

$4\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = 2\sqrt{2}\langle 1, 1 \rangle$

$\mathbf{v} = \langle 2\sqrt{2}, 2\sqrt{2} \rangle$

47. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{5}}\langle -1, 2 \rangle = \left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

$5\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = 5\left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle = \langle -\sqrt{5}, 2\sqrt{5} \rangle$

$\mathbf{v} = \langle -\sqrt{5}, 2\sqrt{5} \rangle$

48. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{2\sqrt{3}}\langle \sqrt{3}, 3 \rangle$

$2\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = \frac{1}{\sqrt{3}}\langle \sqrt{3}, 3 \rangle$

$\mathbf{v} = \langle 1, \sqrt{3} \rangle$

49. $\mathbf{v} = 3\left[(\cos 0°)\mathbf{i} + (\sin 0°)\mathbf{j}\right] = 3\mathbf{i} = \langle 3, 0\rangle$

$\mathbf{v} = 5\left[(\cos 120°)\mathbf{i} + (\sin 120°)\mathbf{j}\right]$

$= -\frac{5}{2}\mathbf{i} + \frac{5\sqrt{3}}{2}\mathbf{j} = \left\langle -\frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle$

51. $\mathbf{v} = 2\left[(\cos 150°)\mathbf{i} + (\sin 150°)\mathbf{j}\right]$

$= -\sqrt{3}\mathbf{i} + \mathbf{j} = \left\langle -\sqrt{3}, 1 \right\rangle$

$\mathbf{v} = 4\left[(\cos 3.5°)\mathbf{i} + (\sin 3.5°)\mathbf{j}\right]$

$3.9925\mathbf{i} + 0.2442\mathbf{j}$

$= \langle 3.9925, 0.2442 \rangle$

$\mathbf{u} = (\cos 0°)\mathbf{i} + (\sin 0°)\mathbf{j} = \mathbf{i}$

$\mathbf{v} = 3(\cos 45°)\mathbf{i} + 3(\sin 45°)\mathbf{j} = \frac{3\sqrt{2}}{2}\mathbf{i} + \frac{3\sqrt{2}}{2}\mathbf{j}$

$\mathbf{u} + \mathbf{v} = \left(\frac{2 + 3\sqrt{2}}{2}\right)\mathbf{i} + \frac{3\sqrt{2}}{2}\mathbf{j} = \left\langle \frac{2 + 3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2} \right\rangle$

54. $\mathbf{u} = 4(\cos 0°)\mathbf{i} + 4(\sin 0°)\mathbf{j} = 4\mathbf{i}$

$\mathbf{v} = 2(\cos 30°)\mathbf{i} + 2(\sin 30°)\mathbf{j} = \mathbf{i} + \sqrt{3}\mathbf{j}$

$\mathbf{u} + \mathbf{v} = 5\mathbf{i} + \sqrt{3}\mathbf{j} = \langle 5, \sqrt{3} \rangle$

$\mathbf{u} = 2(\cos 4)\mathbf{i} + 2(\sin 4)\mathbf{j}$ $\mathbf{v} = (\cos 2)\mathbf{i} + (\sin 2)\mathbf{j}$

$\langle 2\cos 4 + \cos 2, 2\sin 4 + \sin 2 \rangle$

$\mathbf{u} = 5\left[\cos(-0.5)\right]\mathbf{i} + 5\left[\sin(-0.5)\right]\mathbf{j}$

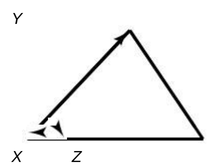
$= 5(\cos 0.5)\mathbf{i} - 5(\sin 0.5)\mathbf{j}$

$= 5(\cos 0.5)\mathbf{i} + 5(\sin 0.5)\mathbf{j}$

$+ \mathbf{v} = 10(\cos 0.5)\mathbf{i} = \langle 10\cos 0.5, 0 \rangle$

The forces act along the same direction. $\theta = 0°$.

The forces cancel out each other. $\theta = 180°$.



$XY + YZ + ZX = 0$.

Vectors that start and end at the same point have a magnitude of 0.

(a) True. **d** has the same magnitude as **a** but is in the opposite direction.

True. **c** and **s** have the same length and direction.

True. **a** and **u** are the adjacent sides of a parallelogram. So, the resultant vector, $\mathbf{a} + \mathbf{u}$, is the diagonal of the parallelogram, **c**.

False. The negative of a vector has the opposite direction of the original vector.

True. $\mathbf{a} + \mathbf{d} = \mathbf{a} + (-\mathbf{a}) = 0$

False. $\mathbf{u} - \mathbf{v} = \mathbf{u} - (-\mathbf{u}) = 2\mathbf{u}$

$-2(\mathbf{b} + \mathbf{t}) = -2(\mathbf{b} + \mathbf{b}) = -2(2\mathbf{b}) = -2\left[2(-\mathbf{u})\right] = 4\mathbf{u}$

61. $\mathbf{v} = \langle 4, 5 \rangle = a\langle 1, 2 \rangle + b\langle 1, -1 \rangle$

$4 = a + b$

$5 = 2a - b$

Adding the equations, $9 = 3a \Rightarrow a = 3$.

Then you have $b = 4 - a = 4 - 3 = 1$.

$a = 3, b = 1$

62. $\mathbf{v} = \langle -7, -2 \rangle = a\langle 1, 2 \rangle + b\langle 1, -1 \rangle$

$-7 = a + b$

$-2 = 2a - b$

Adding the equations, $-9 = 3a \Rightarrow a = -3$.

63. $\mathbf{v} = \langle -6, 0 \rangle = a\langle 1, 2 \rangle + b\langle 1, -1 \rangle$

$-6 = a + b$

$0 = 2a - b$

Adding the equations, $-6 = 3a \Rightarrow a = -2$.

Then you have $b = -6 - a = -6 - (-2) = -4$.

$a = -2, b = -4$

64. $\mathbf{v} = \langle 0, 6 \rangle = a\langle 1, 2 \rangle + b\langle 1, -1 \rangle$

$0 = a + b$

$6 = 2a - b$

Adding the equations, $6 = 3a \Rightarrow a = 2$.

Then you have $b = -7 - a = -7 - (-3) = -4$.

$a = -3, b = -4$

Then you have $b = -a = -2$.

$a = 2, b = -2$

65. $\mathbf{v} = \langle 1, -3\rangle = a\langle 1, 2\rangle + b\langle 1, -1\rangle$

$1 = a + b$

$-3 = 2a - b$

Adding the equations, $-2 = 3a \Rightarrow a = -\frac{2}{3}$.

Then you have $b = 1 - a = 1 - \left(-\frac{2}{3}\right) = \frac{5}{3}$.

$a = -\frac{2}{3}, b = \frac{5}{3}$

66. $\mathbf{v} = \langle -1, 8\rangle = a\langle 1, 2\rangle + b\langle 1, -1\rangle$

$-1 = a + b$

$8 = 2a - b$

Adding the equations, $7 = 3a \Rightarrow a = \frac{7}{3}$.

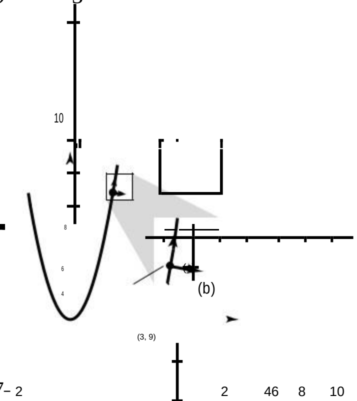
Then you have $b = -1 - a = -1 - \frac{7}{3} = -\frac{10}{3}$.

$a = \frac{7}{3}, b = -\frac{10}{3}$

67. $f(x) = x^2, f'(x) = 2x, f'(3) = 6$

(a) $m = 6$. Let $\mathbf{w} = \langle 1, 6\rangle$, $\|\mathbf{w}\| = \sqrt{37}$, then $\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{37}}\langle 1, 6\rangle$.

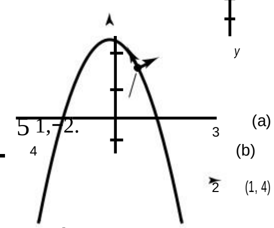
(b) $m = -\frac{1}{6}$. Let $\mathbf{w} = \langle -6, 1\rangle$, $\|\mathbf{w}\| = \sqrt{37}$, then $\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{37}}\langle -6, 1\rangle$.



68. $f(x) = -x^2 + 5, f'(x) = -2x, f'(1) = -2$

(a) $m = -2$. Let $\mathbf{w} = \langle 1, -2\rangle$, $\|\mathbf{w}\| = \sqrt{5}$, then $\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{5}}\langle 1, -2\rangle$.

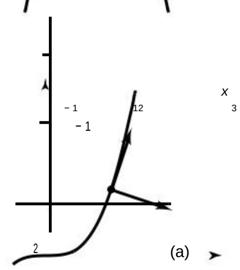
(b) $m = \frac{1}{2}$. Let $\mathbf{w} = \langle 2, 1\rangle$, $\|\mathbf{w}\| = \sqrt{5}$, then $\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{5}}\langle 2, 1\rangle$.



69. $f(x) = x^3, f'(x) = 3x^2 = 3$ at $x = 1$.

(a) $m = 3$. Let $\mathbf{w} = \langle 1, 3\rangle$, $\|\mathbf{w}\| = \sqrt{10}$, then $\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{10}}\langle 1, 3\rangle$.

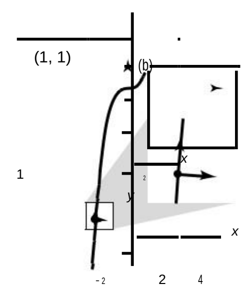
(b) $m = -\frac{1}{3}$. Let $\mathbf{w} = \langle 3, -1\rangle$, $\|\mathbf{w}\| = \sqrt{10}$, then $\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{10}}\langle 3, -1\rangle$.



70. $f(x) = x^3, f'(x) = 3x^2 = 12$ at $x = -2$.

(a) $m = 12$. Let $\mathbf{w} = \langle 1, 12\rangle$, $\|\mathbf{w}\| = \sqrt{145}$, then $\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{145}}\langle 1, 12\rangle$.

(b) $m = -\frac{1}{12}$. Let $\mathbf{w} = \langle 12, -1\rangle$, $\|\mathbf{w}\| = \sqrt{145}$, then $\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{145}}\langle 12, -1\rangle$.

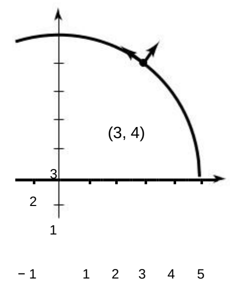


71. $f(x) = \sqrt{25 - x^2}$ $\quad f'(x) = \frac{-x}{\sqrt{25 - x^2}}$ $\quad \frac{3}{4}$

$= \sqrt{3}$ at $x = 3$. $\frac{1}{4}$

(a) $m = -\frac{3}{4}$. Let $\mathbf{w} = \langle -4, 3 \rangle$, $\|\mathbf{w}\| = 5$, then $\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{5}\langle -4, 3 \rangle$.

(b) $m = \frac{4}{3}$. Let $\mathbf{w} = \langle 3, 4 \rangle$, $\|\mathbf{w}\| = 5$, then $\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{5}\langle 3, 4 \rangle$.

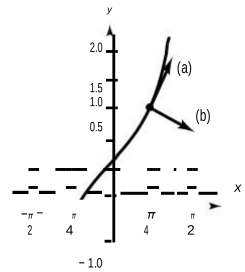


72. $f(x) = \tan x$

$f'(x) = \sec^2 x = 2$ at $x = \frac{\pi}{4}$

(a) $m = 2$. Let $\mathbf{w} = \langle 1, 2\rangle$, $\|\mathbf{w}\| = \sqrt{5}$, then $\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{5}}\langle 1, 2\rangle$.

(b) $m = -\frac{1}{2}$. Let $\mathbf{w} = \langle -2, 1\rangle$, $\|\mathbf{w}\| = \sqrt{5}$, then $\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{5}}\langle -2, 1\rangle$.



73. $\mathbf{u} = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$

$\mathbf{u} + \mathbf{v} = \sqrt{2}\mathbf{j}$

$\mathbf{v} = (\mathbf{u} + \mathbf{v}) - \mathbf{u} = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} = \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle$

74. $\mathbf{u} = 2\sqrt{3}\mathbf{i} + 2\mathbf{j}$

$\mathbf{u} + \mathbf{v} = -3\mathbf{i} + 3\sqrt{3}\mathbf{j}$

$\mathbf{v} = (\mathbf{u} + \mathbf{v}) - \mathbf{u} = \left(-3 - 2\sqrt{3}\right)\mathbf{i} + \left(3\sqrt{3} - 2\right)\mathbf{j}$

$= \left\langle -3 - 2\sqrt{3}, 3\sqrt{3} - 2\right\rangle$

75. $\mathbf{F}_1 + \mathbf{F}_2 = (500\cos 30°\mathbf{i} + 500\sin 30°\mathbf{j}) + (200\cos(-45°)\mathbf{i} + 200\sin(-45°)\mathbf{j}) = \left(250\sqrt{3} + 100\sqrt{2}\right)\mathbf{i} + \left(250 - 100\sqrt{2}\right)\mathbf{j}$

$\|\mathbf{F}_1 + \mathbf{F}_2\| = \sqrt{\left(250\sqrt{3} + 100\sqrt{2}\right)^2 + \left(250 - 100\sqrt{2}\right)^2} \approx 584.6$ lb

$\tan\theta = \frac{250 - 100\sqrt{2}}{250\sqrt{3} + 100\sqrt{2}} \Rightarrow \theta \approx 10.7°$

76. (a) $180(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) + 275\mathbf{i} \approx 430.88\mathbf{i} + 90\mathbf{j}$

Direction: $\alpha \approx \arctan\left(\frac{90}{430.88}\right) \approx 0.206\ (\approx 11.8°)$

Magnitude: $\sqrt{430.88^2 + 90^2} \approx 440.18$ newtons

(b) $M = \sqrt{(275 + 180\cos\theta)^2 + (180\sin\theta)^2}$

$\alpha = \arctan\left[\frac{180\sin\theta}{275 + 180\cos\theta}\right]$

(c)

θ	0°	30°	60°	90°	120°	150°	180°
M	455	440.2	396.9	328.7	241.9	149.3	95
α	0°	11.8°	23.1°	33.2°	40.1°	37.1°	0

(d) 500 M 0 180 50 α 0 180

(e) M decreases because the forces change from acting in the same direction to acting in the opposite direction as θ increases from 0° to 180°.

77. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (75\cos 30°\mathbf{i} + 75\sin 30°\mathbf{j}) + (100\cos 45°\mathbf{i} + 100\sin 45°\mathbf{j}) + (125\cos 120°\mathbf{i} + 125\sin 120°\mathbf{j})$

$= \left(\frac{75}{2}\sqrt{3} + 50\sqrt{2} - \frac{125}{2}\right)\mathbf{i} + \left(\frac{75}{2} + 50\sqrt{2} + \frac{125}{2}\sqrt{3}\right)\mathbf{j}$

$\|\mathbf{R}\| = \|\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\| \approx 228.5$ lb

$\theta_{\mathbf{R}} = \theta_{\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3} \approx 71.3°$

78. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 400[\cos(-30°)\mathbf{i} + \sin(-30°)\mathbf{j}] + 280[\cos 45°\mathbf{i} + \sin 45°\mathbf{j}] + 350[\cos 135°\mathbf{i} + \sin 135°\mathbf{j}]$

$= \left[200\sqrt{3} + 140\sqrt{2} - 175\sqrt{2}\right]\mathbf{i} + \left[-200 + 140\sqrt{2} + 175\sqrt{2}\right]\mathbf{j}$

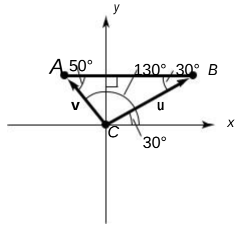
$\|\mathbf{R}\| = \sqrt{\left(200\sqrt{3} - 35\sqrt{2}\right)^2 + \left(-200 + 315\sqrt{2}\right)^2} \approx 385.2483$ newtons

$\theta_{\mathbf{R}} \left(\frac{-200 + 315\sqrt{2}}{\ldots}\right)$

$$= \arctan\left(\frac{}{200\sqrt{3} - 35\sqrt{2}}\right) \approx 0.6908 \approx 39.6°$$

79. $\mathbf{u} = \overrightarrow{CB} = \|\mathbf{u}\|(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$

$\mathbf{v} = \overrightarrow{CA} = \|\mathbf{v}\|(\cos 130°\mathbf{i} + \sin 130°\mathbf{j})$



Vertical components:
$\|\mathbf{u}\|\sin 30° + \|\mathbf{v}\|\sin 130° = 3000$

Horizontal components:
$\|\mathbf{u}\|\cos 30° + \|\mathbf{v}\|\cos 130° = 0$

Solving this system, you obtain

$\|\mathbf{u}\| \approx 1958.1$ pounds,

$\|\mathbf{v}\| \approx 2638.2$ pounds.

80. $\theta = \arctan\left(\frac{24}{20}\right) \approx 0.8761$ or $50.2°$

$\theta_2 = \arctan\left(\frac{24}{-10}\right) + \pi \approx 1.9656$ or $112.6°$

$= \|\mathbf{u}\|(\cos\theta_1\mathbf{i} + \sin\theta_1\mathbf{j})$

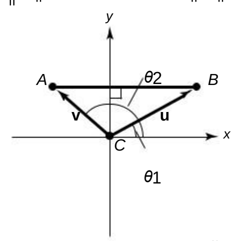
$= \|\mathbf{v}\|(\cos\theta_2\mathbf{i} + \sin\theta_2\mathbf{j})$

Vertical components:
$\|\mathbf{u}\|\sin\theta_1 + \|\mathbf{v}\|\sin\theta_2 = 5000$

Horizontal components:
$\|\mathbf{u}\|\cos\theta_1 + \|\mathbf{v}\|\cos\theta_2 = 0$

Solving this system, you obtain

$\|\mathbf{u}\| \approx 2169.4$ and $\|\mathbf{v}\| \approx 3611.2$.



81. Horizontal component $= \|\mathbf{v}\|\cos\theta$

$= 1200\cos 6° \approx 1193.43$ ft/sec

Vertical component $= \|\mathbf{v}\|\sin\theta$

$= 1200\sin 6° \approx 125.43$ ft/sec

To lift the weight vertically, the sum of the vertical components of **u** and **v** must be 100 and the sum of the horizontal components must be 0.

$= \|\mathbf{u}\|(\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$

$= \|\mathbf{v}\|(\cos 110°\mathbf{i} + \sin 110°\mathbf{j})$

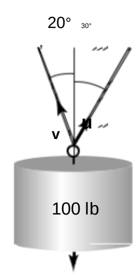
So, $\|\mathbf{u}\|\sin 60° + \|\mathbf{v}\|\sin 110° = 100$, or $\|\mathbf{u}\|\left(\frac{\sqrt{3}}{2}\right) + \|\mathbf{v}\|\sin 110° = 100$.

And $\|\mathbf{u}\|\cos 60° + \|\mathbf{v}\|\cos 110° = 0$ or $\|\mathbf{u}\|\left(\frac{1}{2}\right) + \|\mathbf{v}\|\cos 110° = 0$.

Multiplying the last equation by $(\sqrt{3})$ and adding to the first equation gives

$\|\mathbf{u}\|(\sin 110° - \sqrt{3}\cos 110°) = 100 \Rightarrow \|\mathbf{v}\| \approx 65.27$ pounds

Then, $\|\mathbf{u}\|\left(\frac{1}{2}\right) + 65.27\cos 110° = 0$ gives $\|\mathbf{u}\| \approx 44.65$ pounds



(a) The tension in each rope: $\|\mathbf{u}\| = 44.65$ lb,

$\|\mathbf{v}\| = 65.27$ lb

(b) Vertical components: $\|\mathbf{u}\|\sin 60° \approx 38.67$ lb,

$\|\mathbf{v}\|\sin 110° \approx 61.33$ lb

$\mathbf{u} = 900(\cos 148°\,\mathbf{i} + \sin 148°\,\mathbf{j})$

$\mathbf{v} = 100(\cos 45°\,\mathbf{i} + \sin 45°\,\mathbf{j})$

$$\mathbf{u} + \mathbf{v} = (900\cos 148° + 100\cos 45°)\,\mathbf{i} + (900\sin 148° + 100\sin 45°)\,\mathbf{j}$$
$$\approx -692.53\,\mathbf{i} + 547.64\,\mathbf{j}$$
$$\theta \approx \arctan\left(\frac{547.64}{-692.53}\right) \approx -38.34°;\ 38.34° \text{ North of West}$$
$$\|\mathbf{u} + \mathbf{v}\| \approx \sqrt{(-692.53)^2 + (547.64)^2} \approx 882.9 \text{ km/h}$$

$\mathbf{u} = 400\mathbf{i}$ (plane)

$\mathbf{v} = 50(\cos 135°\,\mathbf{i} + \sin 135°\,\mathbf{j}) = -25\sqrt{2}\,\mathbf{i} + 25\sqrt{2}\,\mathbf{j}$ (wind)

$$\mathbf{u} + \mathbf{v} = \left(400 - 25\sqrt{2}\right)\mathbf{i} + 25\sqrt{2}\,\mathbf{j} \approx 364.64\mathbf{i} + 35.36\mathbf{j}$$
$$\tan\theta = \frac{35.36}{364.64} \Rightarrow \theta \approx 5.54°$$

Direction North of East: ≈ N 84.46° E

Speed: ≈ 336.35 mi/h

85. False. Weight has direction.

86. True

87. True

88. True

89. True

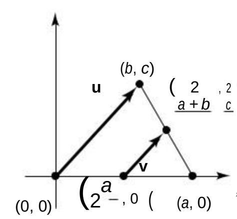
90. True

91. True

Let the triangle have vertices at $(0, 0)$, $(a, 0)$, and (b, c).

Let $\mathbf{u}$ be the vector joining $(0, 0)$ and (b, c), as indicated in the figure. Then $\mathbf{v}$, the vector joining the midpoints, is

$$\mathbf{v} = \left(\frac{a+b}{2} - \frac{a}{2}\right)\mathbf{i} + \frac{c}{2}\mathbf{j}$$
$$= \frac{b}{2}\mathbf{i} + \frac{c}{2}\mathbf{j}$$
$$= \frac{1}{2}(b\mathbf{i} + c\mathbf{j}) = \frac{1}{2}\mathbf{u}.$$



92. False

$a = b = 0$

93. False

$\|a\mathbf{i} + b\mathbf{j}\| = \sqrt{2}|a|$

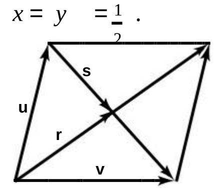
94. True

$$\|\mathbf{u}\| = \sqrt{\cos^2\theta + \sin^2\theta} = 1,$$
$$\|\mathbf{v}\| = \sqrt{\sin^2\theta + \cos^2\theta} = 1$$

97. Let $\mathbf{u}$ and $\mathbf{v}$ be the vectors that determine the parallelogram, as indicated in the figure. The two diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{v} - \mathbf{u}$. So, $\mathbf{r} = x(\mathbf{u} + \mathbf{v})$, $\mathbf{s} = 4(\mathbf{v} - \mathbf{u})$. But,

$$\mathbf{u} = \mathbf{r} - \mathbf{s}$$
$$= x(\mathbf{u} + \mathbf{v}) - y(\mathbf{v} - \mathbf{u}) = (x + y)\mathbf{u} + (x - y)\mathbf{v}.$$

So, $x + y = 1$ and $x - y = 0$. Solving you have $x = y = \frac{1}{2}$.



98. $\mathbf{w} = \|\mathbf{u}\|\mathbf{v} + \|\mathbf{v}\|\mathbf{u}$

$$= \|\mathbf{u}\|\left[\|\mathbf{v}\|(\cos\theta_{\mathbf{v}}\,\mathbf{i} + \sin\theta_{\mathbf{v}}\,\mathbf{j})\right] + \|\mathbf{v}\|\left[\|\mathbf{u}\|(\cos\theta_{\mathbf{u}}\,\mathbf{i} + \sin\theta_{\mathbf{u}}\,\mathbf{j})\right]$$

$$= \|\mathbf{u}\|\,\|\mathbf{v}\|\left[(\cos\theta_{\mathbf{u}} + \cos\theta_{\mathbf{v}})\mathbf{i} + (\sin\theta_{\mathbf{u}} + \sin\theta_{\mathbf{v}})\mathbf{j}\right]$$

$$= 2\|\mathbf{u}\|\,\|\mathbf{v}\|\left[\cos\left(\frac{\theta_{\mathbf{u}}+\theta_{\mathbf{v}}}{2}\right)\cos\left(\frac{\theta_{\mathbf{u}}-\theta_{\mathbf{v}}}{2}\right)\mathbf{i} + \sin\left(\frac{\theta_{\mathbf{u}}+\theta_{\mathbf{v}}}{2}\right)\cos\left(\frac{\theta_{\mathbf{u}}-\theta_{\mathbf{v}}}{2}\right)\mathbf{j}\right]$$

$$\tan\theta_{\mathbf{w}} = \frac{\sin\left(\frac{\theta_{\mathbf{u}}+\theta_{\mathbf{v}}}{2}\right)\cos\left(\frac{\theta_{\mathbf{u}}-\theta_{\mathbf{v}}}{2}\right)}{\cos\left(\frac{\theta_{\mathbf{u}}+\theta_{\mathbf{v}}}{2}\right)\cos\left(\frac{\theta_{\mathbf{u}}-\theta_{\mathbf{v}}}{2}\right)} = \tan\left(\frac{\theta_{\mathbf{u}}+\theta_{\mathbf{v}}}{2}\right)$$

So, $\theta_{\mathbf{w}} = (\theta_{\mathbf{u}} + \theta_{\mathbf{v}})/2$ and **w** bisects the angle between **u** and **v**.

The set is a circle of radius 5, centered at the origin.

$$\|\mathbf{u}\| = \|\langle x, y\rangle\| = \sqrt{x^2 + y^2} = 5 \Rightarrow x^2 + y^2 = 25$$

100. Let $x = v_0 t\cos\alpha$ and $y = v_0 t\sin\alpha - \frac{1}{2}gt^2$.

$$t = \frac{x}{v_0\cos\alpha} \Rightarrow y = v_0\sin\alpha\left(\frac{x}{v_0\cos\alpha}\right) - \frac{1}{2}g\left(\frac{x}{v_0\cos\alpha}\right)^2$$

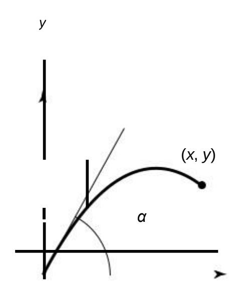
$$= x\tan\alpha - \frac{g}{2v_0^2}x^2\sec^2\alpha$$

$$= x\tan\alpha - \frac{gx^2}{2v_0^2}\left(1 + \tan^2\alpha\right)$$

$$= \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2} - \frac{gx^2}{2v_0^2}\tan^2\alpha + x\tan\alpha - \frac{v_0^2}{2g}$$

$$= \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2} - \frac{gx^2}{2v_0^2}\left[\tan^2\alpha - \frac{2v_0^2}{gx}\tan\alpha + \frac{v_0^4}{g^2x^2}\right]$$

$$= \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2} - \frac{gx^2}{2v_0^2}\left(\tan\alpha - \frac{v_0^2}{gx}\right)^2$$



If $y \le \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}$, then α can be chosen to hit the point (x, y). To hit $(0, y)$: Let $\alpha = 90°$. Then

$$y = v_0 t - \frac{1}{2}gt^2 = \frac{v_0^2}{2g} - \frac{v_0^2}{2g}\left(\frac{g}{v_0}t - 1\right)^2, \text{ and you need } y \le \frac{v_0^2}{2g}.$$

The set H is given by $0 \le x$, $0 < y$ and $y \le \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}$

Note: The parabola $y = \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}$ is called the "parabola of safety."

Section 11.2 Space Coordinates and Vectors in Space

x_0 is directed distance to yz-plane. y_0 is directed distance to xz-plane. z_0 is directed distance to xy-plane.

The y-coordinate of any point in the xz-plane is 0.

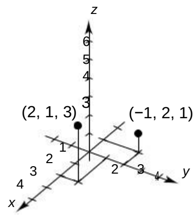
(a) $x = 4$ is a point on the number line.

$x = 4$ is a vertical line in the plane.

$x = 4$ is a plane in space.

The nonzero vectors **u** and **v** are parallel if there exists a scalar such that $\mathbf{u} = c\mathbf{v}$.

5.



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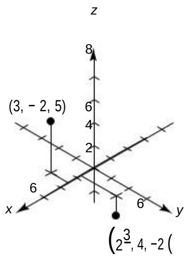

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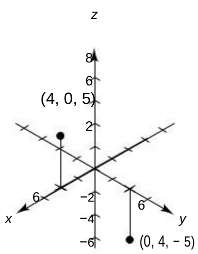
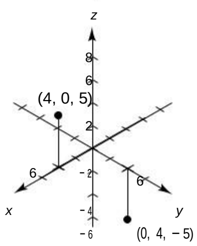


8.



$x = -3, y = 4, z = 5: (-3, 4, 5)$

$x = 7, y = -2, z = -1:$

$(7, -2, -1)$

$y = z = 0, x = 12: (12, 0, 0)$

$x = 0, y = 3, z = 2: (0, 3, 2)$

The point is 1 unit above the *xy*-plane.

The point is 6 units in front of the *xz*-plane.

The point is on the plane parallel to the *yz*-plane that passes through $x = -3$.

The point is 5 units below the *xy*-plane.

The point is to the left of the *xz*-plane.

The point more than 4 units away from the *yz*-plane.

The point is in front of the plane $x = 4$.

The point (x, y, z) is 3 units below the *xy*-plane, and below either quadrant I or III.

The point (x, y, z) is 4 units above the *xy*-plane, and above either quadrant II or IV.

The point could be above the *xy*-plane and so above quadrants II or IV, or below the *xy*-plane, and so below quadrants I or III.

The point could be above the *xy*-plane, and so above quadrants I and III, or below the *xy*-plane, and so below quadrants II or IV.

$$d = \sqrt{(8-4)^2 + (2-1)^2 + (6-5)^2}$$
$$\sqrt{16+1+1}$$
$$\sqrt{18} = 3\sqrt{2}$$

$$d = \sqrt{(-3-(-1))^2 + (5-1)^2 + (-3-1)^2}$$
$$\sqrt{4+16+16}$$
$$\sqrt{36} = 6$$

$$d = \sqrt{(3-0)^2 + (2-2)^2 + (8-4)^2}$$
$$\sqrt{9+0+16}$$
$$\sqrt{25} = 5$$

The point is on or between the planes $y = 3$ and $y = -3$.

28. $d = \sqrt{(-5-(-3))^2 + (8-7)^2 + (-4-1)^2}$

$= \sqrt{4+1+25}$

$= \sqrt{30}$

$A(0, 0, 4), B(2, 6, 7), C(6, 4, -8)$

$|AB| = \sqrt{2^2+6^2+3^2} = \sqrt{49} = 7$

$|AC| = \sqrt{6^2 + 4^2 + (-12)^2} = \sqrt{196} = 14$

$|BC| = \sqrt{4^2 + (-2)^2 + (-15)^2} = \sqrt{245} = 7\sqrt{5}$

$|BC|^2 = 245 = 49 + 196 = |AB|^2 + |AC|^2$

Right triangle

$A(3, 4, 1), B(0, 6, 2), C(3, 5, 6)$

$|AB| = \sqrt{9 + 4 + 1} = \sqrt{14}$

$|AC| = \sqrt{0 + 1 + 25} = \sqrt{26}$

$|BC| = \sqrt{9 + 1 + 16} = \sqrt{26}$

Because $|AC| = |BC|$, the triangle is isosceles.

$A(-1, 0, -2), B(-1, 5, 2), C(-3, -1, 1)$

$|AB| = \sqrt{0 + 25 + 16} = \sqrt{41}$

$|AC| = \sqrt{4 + 1 + 9} = \sqrt{14}$

$|BC| = \sqrt{4 + 36 + 1} = \sqrt{41}$

Because $|AB| = |BC|$, the triangle is isosceles.

$A(4, -1, -1), B(2, 0, -4), C(3, 5, -1)$

$|AB| = \sqrt{4 + 1 + 9} = \sqrt{14}$

$|AC| = \sqrt{1 + 36 + 0} = \sqrt{37}$

$|BC| = \sqrt{1 + 25 + 9} = \sqrt{35}$

Neither

33. $\left(\frac{4+8}{2}, \frac{0+8}{2}, \frac{-6+20}{2}\right) = (6, 4, 7)$

Center is midpoint of diameter:

$\left(\frac{2+1}{2}, \frac{1+3}{2}, \frac{3-1}{2}\right) = \left(\frac{3}{2}, 2, 1\right)$

Radius is distance from center to endpoint:

$d = \sqrt{\left(\frac{3}{2}\right)^2 \ldots} = \sqrt{\frac{1}{4} \ldots} = \frac{\sqrt{21}}{2}$

$\left(x - \frac{3}{2}\right)^2 + (y - 2)^2 + (z - 1)^2 = \frac{21}{4}$

40. Center is midpoint of diameter:

$\left(\frac{-2 - 4}{2}, \frac{4 + 0}{2}, \frac{-5 + 3}{2}\right) = (-3, 2, -1)$

Radius is distance from center to endpoint:

$d = \sqrt{(-4 - (-3))^2 + (0 - 2)^2 + (3 - (-1))^2} = \sqrt{1 + 4 + 16} = \sqrt{21}$

$(x + 3)^2 + (y - 2)^2 + (z + 1)^2 = 21$

41. Center: $(-7, 7, 6)$

Tangent to *xy*-plane

34. $\left(\frac{7-5}{2}, \frac{2-2}{2}, \frac{-2-3}{2}\right) = \left(1, 0, -\frac{1}{2}\right)$

35. $\left(\frac{3+1}{2}, \frac{4+8}{2}, \frac{6+0}{2}\right) = (2, 6, 3)$

36. $\left(\frac{5+(-2)}{2}, \frac{-9+3}{2}, \frac{7+3}{2}\right) = \left(\frac{3}{2}, -3, 5\right)$

37. Center: $(7, 1, -2)$

Radius: 1

$(x - 7)^2 + (y - 1)^2 + (z + 2)^2 = 1$

Center: $(-1, -5, 8)$

Radius: 5

$(x + 1)^2 + (y + 5)^2 + (z - 8)^2 = 25$

42. Center: $(-4, 0, 0)$

Tangent to *yz*-plane

Radius is z-coordinate, 6.

$$(x+7)^2+(y-7)^2+(z-6)^2=36$$

Radius is distance to yz-plane, 4.

$$(x+4)^2+y^2+z^2=16$$

43. $x^2+y^2+z^2-2x+6y+8z+1=0$

$$(x^2-2x+1)+(y^2+6y+9)+(z^2+8z+16)=-1+1+9+16$$

$$(x-1)^2+(y+3)^2+(z+4)^2=25$$

Center: $(1, -3, -4)$

Radius: 5

44. $x^2 + y^2 + z^2 + 9x - 2y + 10z + 19 = 0$

$$\left(x^2 + 9x + \frac{81}{4}\right) + \left(y^2 - 2y + 1\right) + \left(z^2 + 10z + 25\right) = -19 + \frac{81}{4} + 1 + 25$$

$$\left(x + \frac{9}{2}\right)^2 + (y - 1)^2 + (z + 5)^2 = \frac{109}{4}$$

Center: $\left(-\frac{9}{2}, 1, -5\right)$

Radius: $\frac{\sqrt{109}}{2}$

$9x^2 + 9y^2 + 9z^2 - 6x + 18y + 1 = 0$

$$x^2 + y^2 + z^2 - \frac{2}{3}x + 2y + \frac{1}{9} = 0$$

$$\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) + \left(y^2 + 2y + 1\right) + z^2 = -\frac{1}{9} + \frac{1}{9} + 1$$

$$\left(x - \frac{1}{3}\right)^2 + (y + 1)^2 + (z - 0)^2 = 1$$

Center: $\left(\frac{1}{3}, -1, 0\right)$

Radius: 1

$4x^2 + 4y^2 + 4z^2 - 24x - 4y + 8z - 23 = 0$

$$\left(x^2 - 6x + 9\right) + \left(y^2 - y + \frac{1}{4}\right) + \left(z^2 + 2z + 1\right) = \frac{23}{4} + 9 + \frac{1}{4} + 1$$

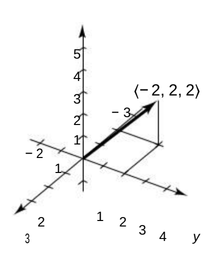
$$(x - 3)^2 + \left(y - \frac{1}{2}\right)^2 + (z + 1)^2 = 16$$

Center: $\left(3, \frac{1}{2}, -1\right)$

Radius: 4

(a) $\mathbf{v} = \langle 2 - 4, 4 - 2, 3 - 1\rangle = \langle -2, 2, 2\rangle$

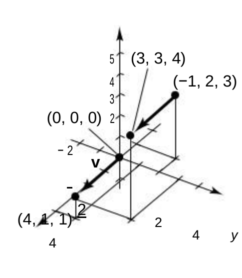
$\mathbf{v} = -2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$



(b) $\mathbf{v} = \langle 3 - (-1), 3 - 2, 4 - 3\rangle = \langle 4, 1, 1\rangle$

$\mathbf{v} = 4\mathbf{i} + \mathbf{j} + \mathbf{k}$

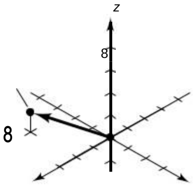
(a), (d)



48. (a) $\mathbf{v} = \langle 4 - 0, 0 - 5, 3 - 1\rangle = \langle 4, -5, 2\rangle$

(b) $\mathbf{v} = 4\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$

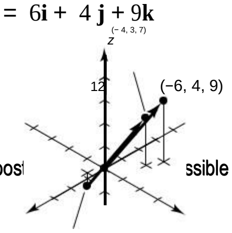
(c)

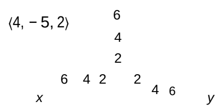


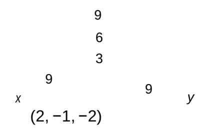
50. (b) $\mathbf{v} = \langle -4 - 2, 3 - (-1), 7 - (-2)\rangle = \langle -6, 4, 9\rangle$

(c) $\mathbf{v} = -6\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$

(a), (d)




⟨4, − 5, 2⟩
6
4
2
6 4 2
2 4 6
x
y


9
6
3
9
9
x
y
(2, −1, −2)

51. $\mathbf{v} = \langle 4-3, 1-2, 6-0\rangle = \langle 1, -1, 6\rangle$

$\|\mathbf{v}\| = \sqrt{1+1+36} = \sqrt{38}$

Unit vector: $\dfrac{\langle 1, -1, 6\rangle}{\sqrt{38}} = \left\langle \dfrac{1}{\sqrt{38}}, \dfrac{-1}{\sqrt{38}}, \dfrac{6}{\sqrt{38}}\right\rangle$

52. $\mathbf{v} = \langle 2-1, 4-(-2), -2-4\rangle = \langle 1, 6, -6\rangle$

$\|\mathbf{v}\| = \sqrt{1+36+36} = \sqrt{73}$

Unit vector: $\dfrac{\langle 1, 6, -6\rangle}{\sqrt{73}} = \left\langle \dfrac{1}{\sqrt{73}}, \dfrac{6}{\sqrt{73}}, \dfrac{-6}{\sqrt{73}}\right\rangle$

53. $\mathbf{v} = \langle 0-4, 5-2, 2-0\rangle = \langle -4, 3, 2\rangle$

$\|\mathbf{v}\| = \sqrt{(-4)^2+3^2+2^2} = \sqrt{16+9+4} = \sqrt{29}$

Unit vector:

$\dfrac{1}{\sqrt{29}}\langle -4, 3, 2\rangle = \left\langle -\dfrac{4}{\sqrt{29}}, \dfrac{3}{\sqrt{29}}, \dfrac{2}{\sqrt{29}}\right\rangle$

54. $\mathbf{v} = \langle 1-1, -2-(-2), -3-0\rangle = \langle 0, 0, -3\rangle$

$\|\mathbf{v}\| = \sqrt{0^2+0^2+(-3)^2} = 3$

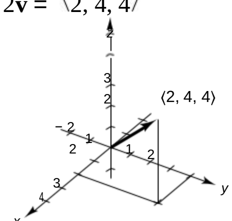
Unit vector:

$\frac{1}{3}\langle \quad \rangle \quad \langle \quad \rangle$

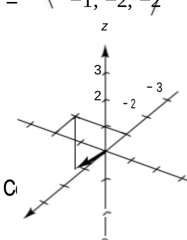
$(q_1, q_2, q_3) - (0, 6, 2) = (3, -5, 6)$ $Q = (3, 1, 8)$

$(q_1, q_2, q_3) - (0, 2, \frac{5}{2}) = (1, -\frac{2}{3}, \frac{1}{2})$ $Q = (1, -\frac{4}{3}, 3)$

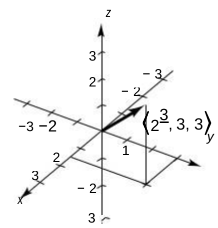
57. (a) $2\mathbf{v} = \langle 2, 4, 4\rangle$



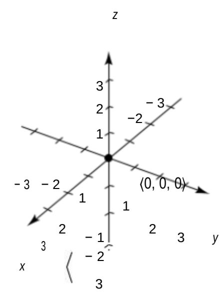
(b) $-\mathbf{v} = \langle -1, -2, -2\rangle$



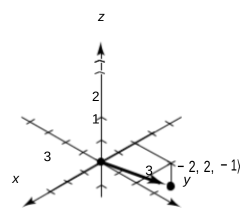
$\frac{3}{2}\mathbf{v} = \langle \frac{3}{2}, 3, 3\rangle$



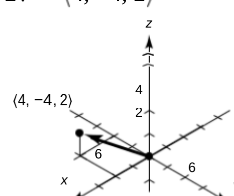
$0\mathbf{v} = \langle 0, 0, 0\rangle$



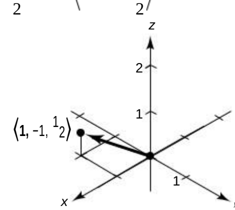
58. (a) $-\mathbf{v} = \langle -2, 2, -1\rangle$



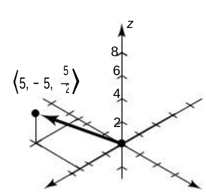
(b) $2\mathbf{v} = \langle 4, -4, 2\rangle$



(c) $\frac{1}{2}\mathbf{v} = \langle 1, -1, \frac{1}{2}\rangle$



(d) $\frac{5}{2}\mathbf{v} = \langle 5, -5, \frac{5}{2}\rangle$



− 3 −2
−1, −2, −2⟩
2
3
1 2 3 y
x
− 2
3

x 6 y

59. $\mathbf{z} = \mathbf{u} - \mathbf{v} + \mathbf{w}$

$= \langle 1,2,3\rangle - \langle 2,2,-1\rangle + \langle 4,0,-4\rangle$

$= \langle 3,0,0\rangle$

60. $\mathbf{z} = 5\mathbf{u} - 3\mathbf{v} - \frac{1}{2}\mathbf{w}$

$= \langle 5,10,15\rangle - \langle 6,6,-3\rangle - \langle 2,0,-2\rangle$

$= \langle -3,4,20\rangle$

62. $2\mathbf{u} + \mathbf{v} - \mathbf{w} + 3\mathbf{z} = 2\langle 1,2,3\rangle + \langle 2,2,-1\rangle - \langle 4,0,-4\rangle + 3\langle z_1,z_2,z_3\rangle = \langle 0,0,0\rangle$

$\langle 0,6,9\rangle + \langle 3z_1,3z_2,3z_3\rangle = \langle 0,0,0\rangle$

$0 + 3z_1 = 0 \Rightarrow z_1 = 0$

$6 + 3z_2 = 0 \Rightarrow z_2 = -2$

$9 + 3z_3 = 0 \Rightarrow z_3 = -3$

$= \langle 0,-2,-3\rangle$

(a) and (b) are parallel because

$\langle -6,-4,10\rangle = -2\langle 3,2,-5\rangle$ and

$\langle 2,\frac{4}{3},-\frac{10}{3}\rangle = \frac{2}{3}\langle 3,2,-5\rangle$.

(b) and (d) are parallel because

$-\mathbf{i} + \frac{4}{3}\mathbf{j} - \frac{3}{2}\mathbf{k} = -2\left(\frac{1}{2}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{3}{4}\mathbf{k}\right)$

and $\frac{3}{4}\mathbf{i} - \mathbf{j} + \frac{9}{8}\mathbf{k} = \frac{3}{2}\left(\frac{1}{2}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{3}{4}\mathbf{k}\right)$.

$\mathbf{z} = -3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

is parallel because $-6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k} = 2\mathbf{z}$.

$\mathbf{z} = \langle -7,-8,3\rangle$

is parallel because $(-z)\mathbf{z} = \langle 14,16,-6\rangle$.

$P(0,-2,-5), Q(3,4,4), R(2,2,1)$

$PQ = \langle 3,6,9\rangle$

$PR = \langle 2,4,6\rangle$

$\langle 3,6,9\rangle = \frac{3}{2}\langle 2,4,6\rangle$

So, *PQ* and *PR* are parallel, the points are collinear.

$P(4,-2,7), Q(-2,0,3), R(7,-3,9)$

$PQ = -6,2,-4$

$PR = 3,-1,2$

$\langle 3,-1,2\rangle = -\frac{1}{2}\langle -6,2,-4\rangle$

So, *PQ* and *PR* are parallel. The points are collinear.

61. $\frac{1}{3}\mathbf{z} - 3\mathbf{u} = \mathbf{w}$

$\frac{1}{3}\mathbf{z} = 3\mathbf{u} + \mathbf{w}$

$\mathbf{z} = 9\mathbf{u} + 3\mathbf{w}$

$= 9\langle 1,2,3\rangle + 3\langle 4,0,-4\rangle$

$= \langle 9,18,27\rangle + \langle 12,0,-12\rangle$

$= \langle 21,18,15\rangle$

$P(1,2,4), Q(2,5,0), R(0,1,5)$

$PQ = \langle 1,3,-4\rangle$

$PR = \langle -1,-1,1\rangle$

Because *PQ* and *PR* are not parallel, the points are not collinear.

$P(0,0,0), Q(1,3,-2), R(2,-6,4)$

$PQ = 1,3,-2$

$PR = 2,-6,4$

Because *PQ* and *PR* are not parallel, the points are not collinear.

$A(2,9,1), B(3,11,4), C(0,10,2), D(1,12,5)$

$AB = \langle 1,2,3\rangle$

$CD = 1,2,3$

$AC = -2,1,1$

$BD = (-2,1,1)$

Because *AB* = *CD* and *AC* = *BD*, the given points form the vertices of a parallelogram.

$A(1,1,3)\ B(9,-1,-2), C(11,2,-9), D(3,4,-4)$

$AB = 8,-2,-5$

$DC = 8,-2,-5$

$AD = \langle 2,3,-7\rangle$

$BC = \langle 2,3,-7\rangle$

Because *AB* = *DC* and *AD* = *BC*, the given points form the vertices of a parallelogram.

Section 11.2 Space Coordinates and Vectors in Space **1108**

$\|\mathbf{v}\| = \|\langle -1, 0, 1\rangle\|$

$\sqrt{(-1)^2 + 0^2 + 1^2}$

$= \sqrt{1+1} = \sqrt{2}$

$\|\mathbf{v}\| = \|\langle -5, -3, -4\rangle\|$

$\sqrt{(-5)^2 + (-3)^2 + (-4)^2}$

$\sqrt{25+9+16}$

$\sqrt{50}$

$5\sqrt{2}$

75. $\mathbf{v} = 3\mathbf{j} - 5\mathbf{k} = \langle 0, 3, -5\rangle$

$\|\mathbf{v}\| = \sqrt{0+9+25} = \sqrt{34}$

76. $\mathbf{v} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k} = \langle 2, 5, -1\rangle$

$\|\mathbf{v}\| = \sqrt{4+25+1} = \sqrt{30}$

77. $\mathbf{v} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k} = \langle 1, -2, -3\rangle$

$\|\mathbf{v}\| = \sqrt{1+4+9} = \sqrt{14}$

78. $\mathbf{v} = -4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} = \langle -4, 3, 7\rangle$

$\|\mathbf{v}\| = \sqrt{16+9+49} = \sqrt{74}$

79. $\mathbf{v} = \langle 2, -1, 2\rangle$

$\|\mathbf{v}\| = \sqrt{4+1+4} = 3$

$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{3}\langle 2, -1, 2\rangle$

(b) $-\frac{\mathbf{v}}{\|\mathbf{v}\|} = -\frac{1}{3}\langle 2, -1, 2\rangle$

$\mathbf{v} = \langle 6, 0, 8\rangle$

$\|\mathbf{v}\| = \sqrt{36 + 0 + 64} = 10$

$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{10}\langle 6, 0, 8\rangle$

$-\frac{\mathbf{v}}{\|\mathbf{v}\|} = -\frac{1}{10}\langle 6, 0, 8\rangle$

81. $\mathbf{v} = 4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

$\|\mathbf{v}\| = \sqrt{16 + 25 + 9} = \sqrt{50} = 5\sqrt{2}$

(a) $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{5\sqrt{2}}(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) = \frac{2\sqrt{2}}{5}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j} + \frac{3\sqrt{2}}{10}\mathbf{k}$

(b) $-\frac{\mathbf{v}}{\|\mathbf{v}\|} = -\frac{1}{5\sqrt{2}}(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) = -\frac{2\sqrt{2}}{5}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} - \frac{3\sqrt{2}}{10}\mathbf{k}$

82. $\mathbf{v} = 5\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

$\|\mathbf{v}\| = \sqrt{25+9+1} = \sqrt{35}$

(a) $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\sqrt{35}}(5\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = \frac{\sqrt{35}}{7}\mathbf{i} + \frac{3\sqrt{35}}{35}\mathbf{j} - \frac{\sqrt{35}}{35}\mathbf{k}$

(b) $-\frac{\mathbf{v}}{\|\mathbf{v}\|} = -\frac{1}{\sqrt{35}}(5\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = -\frac{\sqrt{35}}{7}\mathbf{i} - \frac{3\sqrt{35}}{35}\mathbf{j} + \frac{\sqrt{35}}{35}\mathbf{k}$

$\langle 0, 3, 3\rangle$

83. $\mathbf{v} = 10\frac{\mathbf{u}}{\|\mathbf{u}\|} = 10 \frac{\langle 0, 3, 3\rangle}{3\sqrt{2}} = 10\left\langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle = \langle 0, 5\sqrt{2}, 5\sqrt{2}\rangle$

85. $\mathbf{v} = \frac{3}{2}\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{3}{2}\frac{\langle 2, -2, 1\rangle}{3} = \frac{3}{2}\left\langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right\rangle = \left\langle 1, -1, \frac{1}{2}\right\rangle$

=100, 2 , 2 = 0, 2 , 2

u

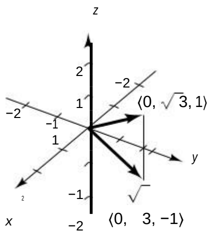
84. $\mathbf{v} = 3\frac{\mathbf{u}}{\|\mathbf{u}\|} = 3\frac{\langle 1, 1, 1\rangle}{\sqrt{3}}$

$= 3\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle = \left\langle \frac{3}{\sqrt{3}}, \frac{3}{\sqrt{3}}, \frac{3}{\sqrt{3}} \right\rangle$

86. $\mathbf{v} = 7\frac{\mathbf{u}}{\|\mathbf{u}\|} = 7\frac{\langle -4, 6, 2\rangle}{2\sqrt{14}} = \left\langle \frac{-14}{\sqrt{14}}, \frac{21}{\sqrt{14}}, \frac{7}{\sqrt{14}} \right\rangle$

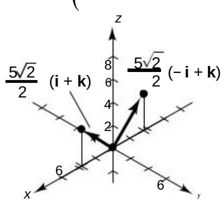
$\mathbf{v} = 2\left[\cos(\pm 30°)\mathbf{j} + \sin(\pm 30°)\mathbf{k}\right]$

$= \sqrt{3}\,\mathbf{j} \pm \mathbf{k} = \langle 0, \sqrt{3}, \pm 1\rangle$



88. $\mathbf{v} = 5(\cos 45°\mathbf{i} + \sin 45°\mathbf{k}) = \frac{5\sqrt{2}}{2}(\mathbf{i} + \mathbf{k})$ or

$\mathbf{v} = 5(\cos 135°\mathbf{i} + \sin 135°\mathbf{k}) = \frac{5\sqrt{2}}{2}(-\mathbf{i} + \mathbf{k})$



89. $\mathbf{v} = \langle -3, -6, 3\rangle$

$\frac{2}{3}\mathbf{v} = \langle -2, -4, 2\rangle$

$(4, 3, 0) + (-2, -4, 2) = (2, -1, 2)$

90. $\mathbf{v} = \langle 5, 6, -3\rangle$

$\frac{2}{3}\mathbf{v} = \left\langle \frac{10}{3}, 4, -2\right\rangle$

$(1, 2, 5) + \left(\frac{10}{3}, 4, -2\right) = \left(\frac{13}{3}, 6, 3\right)$

(x_1, y_1, z_1).

91. A sphere of radius 4 centered at

$\|\mathbf{v}\| = \|\langle x - x_1, y - y_1, z - z_1\rangle\|$

$= \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} = 4$

$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = 16$

92. $\|\mathbf{r} - \mathbf{r}_0\| = \sqrt{(x-1)^2 + (y-1)^2 + (z-1)^2} = 2$

$(x-1)^2 + (y-1)^2 + (z-1)^2 = 4$

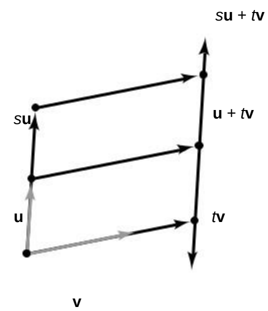
This is a sphere of radius 2 and center $(1, 1, 1)$.

$(x, y, z) = (4, 4, 8)$

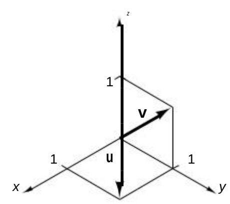
$\mathbf{v} = \langle 4, 4, 8\rangle - \langle 4, 0, 0\rangle$

$= \langle 4-4, 4-0, 8-0\rangle = \langle 0, 4, 8\rangle$

The terminal points of the vectors $t\mathbf{u}$, $\mathbf{u} + t\mathbf{v}$ and $s\mathbf{u} + t\mathbf{v}$ are collinear.



96. (a)



$\mathbf{w} = a\mathbf{u} + b\mathbf{v} = a\mathbf{i} + (a + b)\mathbf{j} + b\mathbf{k} = 0$

$= 0, a + b = 0, b = 0$

So, a and b are both zero.

$a\mathbf{i} + (a + b)\mathbf{j} + b\mathbf{k} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $a = 1$,

$a + b = 2, b = 1$

$\mathbf{w} = \mathbf{u} + \mathbf{v}$

$a\,\mathbf{i} + (a + b)\mathbf{j} + b\mathbf{k} = \mathbf{i} + 2\,\mathbf{j} + 3\mathbf{k}$ $a =$

$1, a + b = 2, b = 3$

Not possible

Let α be the angle between $\mathbf{v}$ and the coordinate axes.

$\mathbf{v} = (\cos\alpha)\mathbf{i} + (\cos\alpha)\mathbf{j} + (\cos\alpha)\mathbf{k}$

$\|\mathbf{v}\| = \sqrt{3}\cos\alpha = 1$

$\cos\alpha = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

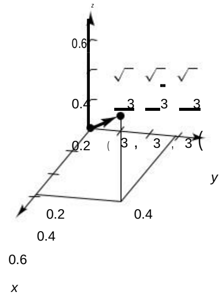
$\frac{\sqrt{3}}{3}$ $\frac{\sqrt{3}}{3}\langle\ \rangle$

The set of all points (x, y, z) such that $\|\mathbf{r}\| > 1$ represent outside the sphere of radius 1 centered at the origin.

(a) $(x, y, z) = (3, 3, 3)$

$$= \langle 3,3,3\rangle - \langle 3,0,0\rangle$$
$$\langle 3-3, 3-0, 3-0\rangle = \langle 0,3,3\rangle$$

$\mathbf{v} = 3(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 3\langle 1,1,1\rangle$



$550 = \|c(75\mathbf{i} - 50\mathbf{j} - 100\mathbf{k})\|$

$302{,}500 = 18{,}125c^2$

$c^2 = 16.689655$

$c \approx 4.085$

$\mathbf{F} \approx 4.085(75\mathbf{i} - 50\mathbf{j} - 100\mathbf{k})$

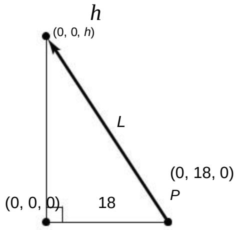
$\approx 306\mathbf{i} - 204\mathbf{j} - 409\mathbf{k}$

99. (a) The height of the right triangle is $h = \sqrt{L^2 - 18^2}$.

The vector $\overrightarrow{PQ}$ is given by $\overrightarrow{PQ} = \langle 0, -18, h\rangle$.

The tension vector $\mathbf{T}$ in each wire is $\mathbf{T} = c\langle 0, -18, h\rangle$ where $ch = \frac{24}{3} = 8$.

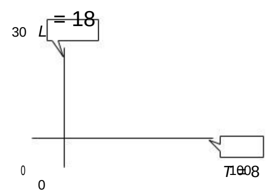
So, $\mathbf{T} = \frac{8}{h}\langle 0, -18, h\rangle$ and $T = \|\mathbf{T}\| = \frac{8}{h}\sqrt{18^2 + h^2} = \frac{8}{\sqrt{L^2 - 18^2}}\sqrt{18^2 + (L^2 - 18^2)} = \frac{8L}{\sqrt{L^2 - 18^2}}$, $L > 18$.



(b)

L	20	25	30	35	40	45	50
T	18.4	11.5	10	9.3	9.0	8.7	8.6

(c)



$x = 18$ is a vertical asymptote and $y = 8$ is a horizontal asymptote.

(d) $\lim_{L \to 18^+} \frac{8L}{\sqrt{L^2 - 18^2}} = \infty$

$\lim_{L \to \infty} \frac{8L}{\sqrt{L^2 - 18^2}} = \lim_{L \to \infty} \frac{8}{\sqrt{1 - (18/L)^2}} = 8$

(e) From the table, $T = 10$ implies $L = 30$ inches.

100. As in Exercise 99(c), $x = a$ will be a vertical asymptote. So, $\lim_{r_0 \to a^-} T = \infty$.

101. $\overrightarrow{AB} = \langle 0, 70, 115\rangle$, $\mathbf{F}_1 = C_1\langle 0, 70, 115\rangle$

$\overrightarrow{AC} = \langle -60, 0, 115\rangle$, $\mathbf{F}_2 = C_2\langle -60, 0, 115\rangle$

$\overrightarrow{AD} = \langle 45, -65, 115\rangle$, $\mathbf{F}_3 = C_3\langle 45, -65, 115\rangle$

$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \langle 0, 0, 500\rangle$

So:

$$-60C_2 + 45C_3 = 0$$
$$70C_1 - 65C_3 = 0$$
$$115(C_1 + C_2 + C_3) = 500$$

Solving this system yields $C_1 = \frac{104}{69}$, $C_2 = \frac{28}{23}$, and $C_3 = \frac{112}{69}$. So:

$\|\mathbf{F}_1\| \approx 202.919\text{N}$

$\|\mathbf{F}_2\| \approx 157.909\text{N}$

$\|\mathbf{F}_3\| \approx 226.521\text{N}$

Section 11.2 Space Coordinates and Vectors in Space 1109

Let A lie on the y-axis and the wall on the x-axis. Then $A = (0, 10, 0)$, $B = (8, 0, 6)$, $C = (-10, 0, 6)$ and $\overrightarrow{AB} = \langle 8, -10, 6\rangle$, $\overrightarrow{AC} = \langle -10, -10, 6\rangle$.

$\|\overrightarrow{AB}\| = 10\sqrt{2}$, $\|\overrightarrow{AC}\| = 2\sqrt{59}$

Thus, $\mathbf{F}_1 = 420\dfrac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|}$, $\mathbf{F}_2 = 650\dfrac{\overrightarrow{AC}}{\|\overrightarrow{AC}\|}$

$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 \approx \langle 237.6, -297.0, 178.2\rangle + \langle -423.1, -423.1, 253.9\rangle \approx \langle -185.5, -720.1, 432.1\rangle$

$\|\mathbf{F}\| \approx 860.0$ lb

$$\sqrt{x^2 + (y+1)^2 + (z-1)^2} = 2\sqrt{(x-1)^2 + (y-2)^2 + z^2}$$

$$x^2 + y^2 + z^2 + 2y - 2z + 2 = 4(x^2 + y^2 + z^2 - 2x - 4y + 5)$$

$$0 = 3x^2 + 3y^2 + 3z^2 - 8x - 18y + 2z + 18$$

$$-6 + \frac{16}{9} + 9 + \frac{1}{9} = \left(x^2 - \frac{8}{3}x + \frac{16}{9}\right) + (y^2 - 6y + 9) + \left(z^2 + \frac{2}{3}z + \frac{1}{9}\right)$$

$$\frac{44}{9} = \left(x - \frac{4}{3}\right)^2 + (y-3)^2 + \left(z + \frac{1}{3}\right)^2$$

Sphere; center: $\left(\frac{4}{3}, 3, -\frac{1}{3}\right)$, radius: $\dfrac{2\sqrt{11}}{3}$

Section 11.3 The Dot Product of Two Vectors

The vectors are orthogonal (perpendicular) if the dot product of the vectors is zero.

2. If $\arccos\dfrac{v_2}{\|\mathbf{v}\|} = 30°$, then $\cos 30° = \dfrac{v_2}{\|\mathbf{v}\|}$.

So, the angle between $\mathbf{v}$ and $\mathbf{j}$ is 30°.

$\mathbf{u} = \langle 3, 4\rangle$, $\mathbf{v} = \langle -1, 5\rangle$

$\mathbf{u}\cdot\mathbf{v} = 3(-1) + 4(5) = 17$

$\mathbf{u}\cdot\mathbf{u} = 3(3) + 4(4) = 25$

$\|\mathbf{v}\|^2 = (-1)^2 + 5^2 = 26$

$(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = 17\langle -1, 5\rangle = \langle -17, 85\rangle$

$\mathbf{u}\cdot(3\mathbf{v}) = 3(\mathbf{u}\cdot\mathbf{v}) = 3(17) = 51$

$\mathbf{u} = \langle 4, 10\rangle$, $\mathbf{v} = \langle -2, 3\rangle$

$\mathbf{u}\cdot\mathbf{v} = 4(-2) + 10(3) = 22$

$\mathbf{u}\cdot\mathbf{u} = 4(4) + 10(10) = 116$

$\mathbf{u} = \langle 6, -4\rangle$, $\mathbf{v} = \langle -3, 2\rangle$

$\mathbf{u}\cdot\mathbf{v} = 6(-3) + (-4)(2) = -26$

$\mathbf{u}\cdot\mathbf{u} = 6(6) + (-4)(-4) = 52$

$\|\mathbf{v}\|^2 = (-3)^2 + 2^2 = 13$

$(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = -26\langle -3, 2\rangle = \langle 78, -52\rangle$

$\mathbf{u}\cdot(3\mathbf{v}) = 3(\mathbf{u}\cdot\mathbf{v}) = 3(-26) = -78$

$\mathbf{u} = \langle -7, -1\rangle$, $\mathbf{v} = \langle -4, -1\rangle$

$\mathbf{u}\cdot\mathbf{v} = -7(-4) + -1(-1) = 29$

$\mathbf{u}\cdot\mathbf{u} = -7(-7) + -1(-1) = 50$

$\|\mathbf{v}\|^2 = (-4)^2 + (-1)^2 = 17$

$(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = 29\langle -4, -1\rangle = \langle -116, -29\rangle$

$\mathbf{u}\cdot(3\mathbf{u}) = 3(\mathbf{u}\cdot\mathbf{v}) = 3(29) = 87$

$\|\mathbf{v}\|^2 = (-2)^2 + 3^2 = 13$

$(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = 22\langle -2, 3\rangle = \langle -44, 66\rangle$

$\mathbf{u} \cdot (3\mathbf{v}) = 3(\mathbf{u} \cdot \mathbf{v}) = 3(22) = 66$

$\mathbf{u} = \langle 2, -3, 4\rangle$, $\mathbf{v} = \langle 0, 6, 5\rangle$

$\mathbf{u}\cdot\mathbf{v} = 2(0) + (-3)(6) + (4)(5) = 2$

$\mathbf{u}\cdot\mathbf{u} = 2(2) + (-3)(-3) + 4(4) = 29$

$\|\mathbf{v}\|^2 = 0^2 + 6^2 + 5^2 = 61$

$(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = 2\langle 0, 6, 5\rangle = \langle 0,12,10\rangle$

$\mathbf{u}\cdot(3\mathbf{v}) = 3(\mathbf{u}\cdot\mathbf{v}) = 3(2) = 6$

$\mathbf{u} = \langle -5, 0, 5\rangle$, $\mathbf{v} = \langle -1, 2, 1\rangle$

$\mathbf{u}\cdot\mathbf{v} = -5(-1) + 0(2) + 5(1) = 10$

$\mathbf{u}\cdot\mathbf{u} = (-5)(-5) + (0)(0) + 5(5) = 50$

$\|\mathbf{v}\|^2 = (-1)^2 + 2^2 + 1^2 = 6$

$(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = 10\langle -1, 2, 1\rangle = \langle -10, 20, 10\rangle$

$\mathbf{u}\cdot(3\mathbf{v}) = 3(\mathbf{u}\cdot\mathbf{v}) = 3(10) = 30$

$\mathbf{u} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} - \mathbf{k}$

$\mathbf{u}\cdot\mathbf{v} = 2(1) + (-1)(0) + 1(-1) = 1$

$\mathbf{u}\cdot\mathbf{u} = 2(2) + (-1)(-1) + (1)(1) = 6$

$\|\mathbf{v}\|^2 = 1^2 + (-1)^2 = 2$

$(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = \mathbf{v} = \mathbf{i} - \mathbf{k}$

$\mathbf{u}\cdot(3\mathbf{v}) = 3(\mathbf{u}\cdot\mathbf{v}) = 3(1) = 3$

$\mathbf{u} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\mathbf{v} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

$\mathbf{u}\cdot\mathbf{v} = 2(1) + 1(-3) + (-2)(2) = -5$

$\mathbf{u}\cdot\mathbf{u} = 2(2) + 1(1) + (-2)(-2) = 9$

$\|\mathbf{v}\|^2 = 1^2 + (-3)^2 + 2^2 = 14$

$(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = -5(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = -5\mathbf{i} + 15\mathbf{j} - 10\mathbf{k}$

$\mathbf{u}\cdot(3\mathbf{v}) = 3(\mathbf{u}\cdot\mathbf{v}) = 3(-5) = -15$

$\mathbf{u} = \langle 1,1\rangle$, $\mathbf{v} = \langle 2, -2\rangle$

$\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{0}{\sqrt{2}\sqrt{8}} = 0$

13. $\mathbf{u} = 3\mathbf{i} + \mathbf{j}$, $\mathbf{v} = -2\mathbf{i} + 4\mathbf{j}$

$\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{-2}{\sqrt{10}\sqrt{20}} = \dfrac{-1}{5\sqrt{2}}$

(a) $\theta = \arccos\left(-\dfrac{1}{5\sqrt{2}}\right) \approx 1.713$

(b) $\theta \approx 98.1°$

14. $\mathbf{u} = \cos\left(\dfrac{\pi}{6}\right)\mathbf{i} + \sin\left(\dfrac{\pi}{6}\right)\mathbf{j} = \dfrac{\sqrt{3}}{2}\mathbf{i} + \dfrac{1}{2}\mathbf{j}$

$\mathbf{v} = \cos\left(\dfrac{3\pi}{4}\right)\mathbf{i} + \sin\left(\dfrac{3\pi}{4}\right)\mathbf{j} = -\dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j}$

$\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$

$= \dfrac{\sqrt{3}}{2}\left(-\dfrac{\sqrt{2}}{2}\right) + \dfrac{1}{2}\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$

(a) $\theta = \arccos\left[\dfrac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)\right] = \dfrac{7\pi}{12}$

$\theta = 105°$

$\mathbf{u} = \langle 1,1,1\rangle$, $\mathbf{v} = \langle 2,1,-1\rangle$

$\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{2}{\sqrt{3}\sqrt{6}} = \dfrac{\sqrt{2}}{3}$

(a) $\theta = \arccos\dfrac{\sqrt{2}}{3} \approx 1.080$

$\theta \approx 61.9°$

$\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

$\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{3(2) + 2(-3) + 0}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = 0$

$\theta = \dfrac{\pi}{2}$

$\theta = 90°$

$\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$, $\mathbf{v} = -2\mathbf{j} + 3\mathbf{k}$

$\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{-8}{5\sqrt{13}} = \dfrac{-8\sqrt{13}}{65}$

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(a) $\theta = \arccos\left(-\frac{8\sqrt{13}}{65}\right) \approx 2.031$

(b) $\theta \approx 116.3°$

(a) $\theta = \frac{\pi}{2}$ (b) $\theta = 90°$

$\mathbf{u} = \langle 3, 1 \rangle$, $\mathbf{v} = \langle 2, -1 \rangle$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{5}{\sqrt{10}\sqrt{5}} = \frac{1}{\sqrt{2}}$$

(a) $\theta = \frac{\pi}{4}$ (b) $\theta = 45°$

18. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{9}{\sqrt{14}\sqrt{6}} = \frac{9}{2\sqrt{21}} = \frac{3\sqrt{21}}{14}$$

(a) $\theta = \arccos\left(\dfrac{3\sqrt{21}}{14}\right) \approx 0.190$

$\theta \approx 10.9°$

$$\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \cos\theta$$

$$\mathbf{u}\cdot\mathbf{v} = (8)(5)\cos\frac{\pi}{3} = 20$$

20. $\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \cos\theta$

$$\mathbf{u}\cdot\mathbf{v} = (40)(25)\cos\frac{5\pi}{6} = -500\sqrt{3}$$

$\mathbf{u} = \langle 4, 3\rangle$, $\mathbf{v} = \left\langle \frac{1}{2}, -\frac{2}{3}\right\rangle$

$\mathbf{u} \neq c\mathbf{v} \Rightarrow$ not parallel $\mathbf{u}\cdot\mathbf{v}$
$= 0 \Rightarrow$ orthogonal

22. $\mathbf{u} = -\frac{1}{3}(\mathbf{i} - 2\mathbf{j})$, $\mathbf{v} = 2\mathbf{i} - 4\mathbf{j}$

$= -\frac{1}{6}\mathbf{v} \Rightarrow$ parallel

$\mathbf{u} = \mathbf{j} + 6\mathbf{k}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$
$\neq c\mathbf{v} \Rightarrow$ not parallel

$\mathbf{u}\cdot\mathbf{v} = -8 \neq 0 \Rightarrow$ not orthogonal
Neither

$\mathbf{u} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ $\mathbf{u}$ $-$
$c\mathbf{v} \Rightarrow$ not parallel
$\mathbf{u}\cdot\mathbf{v} = 0 \Rightarrow$ orthogonal

$\mathbf{u} = \langle 2, -3, 1\rangle$, $\mathbf{v} = \langle -1, -1, -1\rangle$
$\mathbf{u} \neq c\mathbf{v} \Rightarrow$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 0 \Rightarrow$ orthogonal

$\mathbf{u} = \langle \cos\theta, \sin\theta, -1\rangle$,

Consider the vector $\langle -3, 0, 0\rangle$ joining $(0, 0, 0)$ and $(-3, 0, 0)$, and the vector $\langle 1, 2, 3\rangle$ joining $(0, 0, 0)$ and $(1,2,3)$: $\langle -3,0,0\rangle \cdot \langle 1,2,3\rangle = -3 < 0$

The triangle has an obtuse angle, so it is an obtuse triangle.

29. $A(2,0,1)$, $B(0,1,2)$, $C\left(-\frac{1}{2}, \frac{3}{2}, 0\right)$

$AB = \langle -2,1,1\rangle$ $\quad BA = \langle 2, -1, -1\rangle$

$AC = \left\langle -\frac{5}{2}, \frac{3}{2}, -1\right\rangle$ $\quad CA = \left\langle \frac{5}{2}, -\frac{3}{2}, 1\right\rangle$

$BC = \left\langle -\frac{1}{2}, \frac{1}{2}, -2\right\rangle$ $\quad CB = \left\langle \frac{1}{2}, -\frac{1}{2}, 2\right\rangle$

$AB\cdot AC = 5 + \frac{3}{2} - 1 > 0$

$BA\cdot BC = -1 - \frac{1}{2} + 2 > 0$

$CA\cdot CB = -\frac{5}{4} + -\frac{3}{4} + 2 > 0$

The triangle has three acute angles, so it is an acute triangle.

$A(2, -7, 3)$, $B(-1, 5, 8)$, $C(4, 6, -1)$

$AB = \langle -3, 12, 5\rangle$ $\quad BA = \langle 3, -12, -5\rangle$
$AC = \langle 2, 13, -4\rangle$ $\quad CA = \langle -2, -13, 4\rangle$
$BC = \langle 5, 1, -9\rangle$ $\quad CB = \langle -5, -1, 9\rangle$

$AB\cdot AC = -6 + 156 - 20 > 0$
$BA\cdot BC = 15 - 12 + 45 > 0$

$CA\cdot CB = 10 + 13 + 36 > 0$

The triangle has three acute angles, so it is an acute triangle.

31. $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\|\mathbf{u}\| = \sqrt{1+4+4} = 3$

$\cos\alpha = \frac{1}{3} \Rightarrow \alpha \approx 1.2310$ or $70.5°$
$\cos\beta = \frac{2}{3} \Rightarrow \beta \approx 0.8411$ or $48.2°$
$\cos\gamma = \frac{2}{3} \Rightarrow \gamma \approx 0.8411$ or $48.2°$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = 1$$

$= \langle \sin\theta, -\cos\theta, 0\rangle$ $\mathbf{u}$

32. $\mathbf{u} = 5\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\|\mathbf{u}\| = \sqrt{25+9+1} = \sqrt{35}$

$\neq c\mathbf{v} \Rightarrow$ not parallel $\mathbf{u} \cdot \mathbf{v}$

$= 0 \Rightarrow$ orthogonal

The vector $\langle 1, 2, 0 \rangle$ joining $(1, 2, 0)$ and $(0, 0, 0)$ is perpendicular to the vector $\langle -2, 1, 0 \rangle$ joining $(-2, 1, 0)$ and $(0, 0, 0)$: $\langle 1, 2, 0 \rangle \cdot \langle -2, 1, 0 \rangle = 0$

The triangle has a right angle, so it is a right triangle.

$\cos\alpha = \frac{5}{\sqrt{35}} \Rightarrow \alpha \approx 0.5639$ or $32.3°$

$\cos\beta = \frac{3}{\sqrt{35}} \Rightarrow \beta \approx 1.0390$ or $59.5°$

$\cos\gamma = \frac{-1}{\sqrt{35}} \Rightarrow \gamma \approx 1.7406$ or $99.7°$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{25}{35} + \frac{9}{35} + \frac{1}{35} = 1$

33. $\mathbf{u} = 7\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\|\mathbf{u}\| = \sqrt{49+1+1} = \sqrt{51}$

$\cos\alpha = \frac{7}{\sqrt{51}} \Rightarrow \alpha \approx 11.4°$

$\cos\beta = \frac{1}{\sqrt{51}} \Rightarrow \beta \approx 82.0°$

$\cos\gamma = -\frac{1}{\sqrt{51}} \Rightarrow \gamma \approx 98.0°$

34. $\mathbf{u} = -4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$, $\|\mathbf{u}\| = \sqrt{16+9+25} = \sqrt{50} = 5\sqrt{2}$

$\cos\alpha = \frac{-4}{5\sqrt{2}} \Rightarrow \alpha \approx 2.1721$ or $124.4°$

$\cos\beta = \frac{3}{5\sqrt{2}} \Rightarrow \beta \approx 1.1326$ or $64.9°$

$\cos\gamma = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \gamma \approx \frac{\pi}{4}$ or $45°$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{16}{50} + \frac{9}{50} + \frac{25}{50} = 1$

35. $\mathbf{u} = \langle 0, 6, -4 \rangle$, $\|\mathbf{u}\| = \sqrt{0+36+16} = \sqrt{52} = 2\sqrt{13}$

$\cos\alpha = 0 \Rightarrow \alpha = \frac{\pi}{2}$ or $90°$

$\cos\beta = \frac{3}{\sqrt{13}} \Rightarrow \beta \approx 0.5880$ or $33.7°$

$\cos\gamma = -\frac{2}{\sqrt{13}} \Rightarrow \gamma \approx 2.1588$ or $123.7°$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 0 + \frac{9}{13} + \frac{4}{13} = 1$

36. $\mathbf{u} = \langle -1, 5, 2 \rangle$, $\|\mathbf{u}\| = \sqrt{1+25+4} = \sqrt{30}$

$\cos\alpha = \frac{-1}{\sqrt{30}} \Rightarrow \alpha \approx 1.7544$ or $100.5°$

$\cos\beta = \frac{5}{\sqrt{30}} \Rightarrow \beta \approx 0.4205$ or $24.1°$

$\cos\gamma = \frac{2}{\sqrt{30}} \Rightarrow \gamma \approx 1.1970$ or $68.6°$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{1}{30} + \frac{25}{30} + \frac{4}{30} = 1$

$\mathbf{u} = \langle 6, 7 \rangle$, $\mathbf{v} = \langle 1, 4 \rangle$

$\mathbf{w}_1 = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{6(1)+7(4)}{1^2+4^2}\langle 1, 4\rangle = \frac{34}{17}\ldots$

$\mathbf{u} = \langle 9, 7 \rangle$, $\mathbf{v} = \langle 1, 3 \rangle$

$\mathbf{w}_1 = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{9(1)+7(3)}{1^2+3^2}\langle 1, 3\rangle = \frac{30}{10}\langle 1, 3\rangle = \langle 3, 9\rangle$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 9, 7\rangle - \langle 3, 9\rangle = \langle 6, -2\rangle$

$\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} = \langle 2, 3\rangle$, $\mathbf{v} = 5\mathbf{i} + \mathbf{j} = \langle 5, 1\rangle$

$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{2(5)+3(1)}{5^2+1}\langle 5, 1\rangle = \frac{13}{26}\langle 5, 1\rangle = \left\langle \frac{5}{2}, \frac{1}{2}\right\rangle$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 2, 3\rangle - \left\langle \frac{5}{2}, \frac{1}{2}\right\rangle = \left\langle -\frac{1}{2}, \frac{5}{2}\right\rangle$

$\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} = \langle 2, -3\rangle$, $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} = \langle 3, 2\rangle$ (a) $\mathbf{w}_1$

$\mathbf{w}_1 = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$

2(3) + (−3)(2)

3, 2

$3^2 + 2^2$

⟨3, 2⟩ = ⟨0, 0⟩

17 ⟨1, 4⟩ = ⟨2, 8⟩

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 2, -3 \rangle =$ proj

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 6, 7 \rangle - \langle 2, 8 \rangle = \langle 4, -1 \rangle =$ proj

$\mathbf{u} = \langle 0, 3, 3\rangle, \mathbf{v} = \langle -1, 1, 1\rangle$

$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{0(-1) + 3(1) + 3(1)}{1+1+1}\langle -1,1,1\rangle = \frac{6}{3}\langle -1,1,1\rangle = \langle -2, 2, 2\rangle$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 0, 3, 3\rangle - \langle -2, 2, 2\rangle = \langle 2, 1, 1\rangle$

$\mathbf{u} = \langle 8, 2, 0\rangle, \mathbf{v} = \langle 2, 1, -1\rangle$

$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{8(2) + 2(1) + 0(-1)}{2^2 + 1 + 1}\langle 2, 1, -1\rangle = \frac{18}{6}\langle 2, 1, -1\rangle = \langle 6, 3, -3\rangle$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 8, 2, 0\rangle - \langle 6, 3, -3\rangle = \langle 2, -1, 3\rangle$

43. $\mathbf{u} = -9\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}, \mathbf{v} = 4\mathbf{j} + 4\mathbf{k}$

(a) $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{-2(4) + (-4)(4)}{4^2 + 4^2}\right)\langle 0,4,4\rangle = -\frac{3}{4}\langle 0,4,4\rangle = \langle 0, -3, -3\rangle$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle -9, -2, -4\rangle - \langle 0, -3, -3\rangle = \langle -9, 1, -1\rangle$

44. $\mathbf{u} = 5\mathbf{i} - \mathbf{j} - \mathbf{k}, \mathbf{v} = -\mathbf{i} + 5\mathbf{j} + 8\mathbf{k}$

(a) $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{5(-1) + (-1)(5) + (-1)(8)}{(-1)^2 + 5^2 + 8^2}\right)\langle -1, 5, 8\rangle = \frac{-18}{90}\langle -1, 5, 8\rangle = \frac{-1}{5}\langle -1, 5, 8\rangle = \left\langle \frac{1}{5}, -1, -\frac{8}{5}\right\rangle$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 5, -1, -1\rangle - \left\langle \frac{1}{5}, -1, -\frac{8}{5}\right\rangle = \left\langle \frac{24}{5}, 0, \frac{3}{5}\right\rangle$

$\mathbf{u}$ is a vector and $\mathbf{v}\cdot\mathbf{w}$ is a scalar. You cannot add a vector and a scalar.

(a) Orthogonal, $\theta = \frac{\pi}{2}$

Acute, $0 < \theta < \frac{\pi}{2}$

Obtuse, $\frac{\pi}{2} < \theta < \pi$

46. $\left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \mathbf{u} \Rightarrow \mathbf{u} = c\mathbf{v} \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are parallel.

47. Yes, $\left\|\left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}\right\| = \left\|\left(\frac{\mathbf{v}\cdot\mathbf{u}}{\|\mathbf{u}\|^2}\right)\mathbf{u}\right\|$

$\mathbf{u} = \langle 3240, 1450, 2235\rangle$ $\mathbf{v} = \langle 2.25, 2.95, 2.65\rangle$

$$\frac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{v}\|^2} = \frac{|\mathbf{v}\cdot\mathbf{u}|}{\|\mathbf{u}\|^2}$$

$$\frac{1}{\|\mathbf{v}\|^2} = \frac{1}{\|\mathbf{u}\|^2}$$

$$\|\mathbf{u}\| = \|\mathbf{v}\|$$

$$\mathbf{u}\cdot\mathbf{v} = 3240(2.25) + 1450(2.95) + 2235(2.65)$$

$$\$17{,}490.25$$

This represents the total revenue the restaurant earned on its three products.

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$\mathbf{u} = \langle 3240, 1450, 2235\rangle$

$\mathbf{v} = \langle 2.25, 2.95, 2.65\rangle$

Decrease prices by 2%: $0.98\mathbf{v}$

New total revenue:

$0.98\langle 3240, 1450, 2235\rangle \cdot \langle 2.25, 2.95, 2.65\rangle = 0.98(17490.25)$

$= \$17{,}140.45$

51. Answers will vary. *Sample answer:*

$\mathbf{u} = -\frac{1}{4}\mathbf{i} + \frac{3}{2}\mathbf{j}$. Want $\mathbf{u}\cdot\mathbf{v} = 0$.

$\mathbf{v} = 12\mathbf{i} + 2\mathbf{j}$ and $-\mathbf{v} = -12\mathbf{i} - 2\mathbf{j}$ are orthogonal to $\mathbf{u}$.

52. Answers will vary. *Sample answer:*

$\mathbf{u} = 9\mathbf{i} - 4\mathbf{j}$. Want $\mathbf{u}\cdot\mathbf{v} = 0$.

$= 4\mathbf{i} + 9\mathbf{j}$ and $-\mathbf{v} = -4\mathbf{i} - 9\mathbf{j}$ are

orthogonal to $\mathbf{u}$.

Answers will vary. *Sample answer:*

$= \langle 3, 1, -2\rangle$. Want $\mathbf{u}\cdot\mathbf{v} = 0$.

$= \langle 0, 2, 1\rangle$ and $-\mathbf{v} = \langle 0, -2, -1\rangle$ are orthogonal to $\mathbf{u}$.

Answers will vary. *Sample answer:*

$= \langle 4, -3, 6\rangle$. Want $\mathbf{u}\cdot\mathbf{v} = 0$

$= \langle 0, 6, 3\rangle$ and $-\mathbf{v} = \langle 0, -6, -3\rangle$

are orthogonal to $\mathbf{u}$.

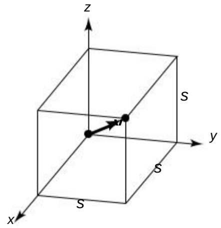
Let s = length of a side.

$= \langle s, s, s\rangle$

$\|\mathbf{v}\| = s\sqrt{3}$

$\cos\alpha = \cos\beta = \cos\gamma = \frac{s}{s\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\alpha = \beta = \gamma = \arccos\left(\frac{1}{\sqrt{3}}\right) \approx 54.7°$



56. $\mathbf{v}_1 = \langle s, s, s\rangle$

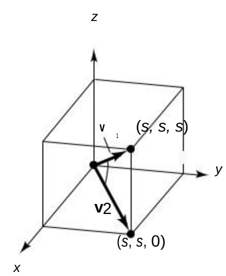
$\|\mathbf{v}_1\| = s\sqrt{3}$

$\mathbf{v}_2 = \langle s, s, 0\rangle$

$\|\mathbf{v}_2\| = s\sqrt{2}$

$\cos\theta = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{6}}{3}$

$= \arccos\frac{\sqrt{6}}{3} \approx 35.26°$



(a) Gravitational Force $\mathbf{F} = -48{,}000\mathbf{j}$

$= \cos 10°\mathbf{i} + \sin 10°\mathbf{j}$

$\mathbf{w}_1 = \frac{\mathbf{F}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = (\mathbf{F}\cdot\mathbf{v})\mathbf{v}$

$(-48{,}000)(\sin 10°)\mathbf{v}$

$\approx -8335.1(\cos 10°\mathbf{i} + \sin 10°\mathbf{j})$

$\|\mathbf{w}_1\| \approx 8335.1$ lb

$\mathbf{w}_2 = \mathbf{F} - \mathbf{w}_1$

$-48{,}000\mathbf{j} + 8335.1(\cos 10°\mathbf{i} + \sin 10°\mathbf{j})$

$8208.5\mathbf{i} - 46{,}552.6\mathbf{j}$

$\|\mathbf{w}_2\| \approx 47{,}270.8$ lb

(a) Gravitational Force $\mathbf{F} = -5400\mathbf{j}$

$= \cos 18°\mathbf{i} + \sin 18°\mathbf{j}$

$\mathbf{w}_1 = \frac{\mathbf{F}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = (\mathbf{F}\cdot\mathbf{v})\mathbf{v}$

$(-5400)(\sin 18°)\mathbf{v}$

$-1668.7(\cos 18°\mathbf{i} + \sin 18°\mathbf{j})$

$\|\mathbf{w}_1\| = 1668.7$ lb

(b) $\mathbf{w}_2 = \mathbf{F} - \mathbf{w}_1$

$= -5400\mathbf{i} + 1668.7(\cos 18°\mathbf{i} + \sin 18°\mathbf{i})$

$\approx 1587.0\mathbf{i} - 4884.3\mathbf{j}$

$\|\mathbf{w}_2\| \approx 5135.7$ lb

59. $\mathbf{F} = 85\left(\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right)$

$= 10\mathbf{i}$

$= \mathbf{F} \cdot \mathbf{v} = 425$ ft-lb

60. $W = \left\|\text{proj}_{PQ}\mathbf{F}\right\| \left\|PQ\right\|$

$= \cos 20° \left\|\mathbf{F}\right\| \left\|PQ\right\|$

$= (\cos 20°)(65)(50)$

≈ 3054.0 ft-lb

61. $\mathbf{F} = 1600(\cos 25°\,\mathbf{i} + \sin 25°\,\mathbf{j})$

$\mathbf{v} = 2000\mathbf{i}$

$= \mathbf{F} \cdot \mathbf{v} = 1600(2000)\cos 25°$

2,900,184.9 Newton meters (Joules)

2900.2 km-N

62. $W = \left\|\text{proj}_{PQ}\mathbf{F}\right\| \left\|PQ\right\|$

$= (\cos 60°)\left\|\mathbf{F}\right\| \left\|PQ\right\|$

$= \frac{1}{2}(400)(40)$

$= 8000$ Joules

63. False.

For example, let $\mathbf{u} = \langle 1, 1\rangle$, $\mathbf{v} = \langle 2, 3\rangle$ and $\mathbf{w} = \langle 1, 4\rangle$. Then $\mathbf{u} \cdot \mathbf{v} = 2 + 3 = 5$ and $\mathbf{u} \cdot \mathbf{w} = 1 + 4 = 5$.

64. True

$\mathbf{w} \cdot (\mathbf{u} + \mathbf{v}) = \mathbf{w} \cdot \mathbf{u} + \mathbf{w} \cdot \mathbf{v} = 0 + 0 = 0$ so, $\mathbf{w}$ and $\mathbf{u} + \mathbf{v}$ are orthogonal.

(a) The graphs $y_1 = x^2$ and $y_2 = x^{1/3}$ intersect at $(0, 0)$ and $(1,1)$.

(b) $y_1' = 2x$ and $y_2' = \frac{1}{3x^{2/3}}$.

At $(0, 0)$, $\pm\langle 1, 0\rangle$ is tangent to y_1 and $\pm\langle 0,1\rangle$ is tangent to y_2.

At $(1,1)$, $y_1' = 2$ and $y_2' = \frac{1}{3}$.

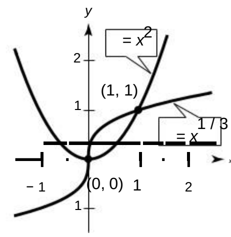
$\pm\frac{1}{\sqrt{5}}\langle 1, 2\rangle$ is tangent to y_1, $\pm\frac{1}{\sqrt{10}}\langle 3,1\rangle$ is tangent to y_2.

At $(0, 0)$, the vectors are perpendicular $(90°)$.

At $(1,1)$,

$$\cos\theta = \frac{\frac{1}{\sqrt{5}}\langle 1,2\rangle \cdot \frac{1}{\sqrt{10}}\langle 3,1\rangle}{(1)(1)} = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}$$

$= 45°$



(a) The graphs $y_1 = x^3$ and $y_2 = x^{1/3}$ intersect at $(-1, -1)$, $(0, 0)$ and $(1,1)$.

(b) $y_1' = 3x^2$ and $y_2' = \frac{1}{3x^{2/3}}$.

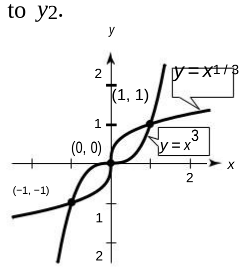
At $(0, 0)$, $\pm\langle 1, 0\rangle$ is tangent to y_1 and $\pm\langle 0,1\rangle$ is tangent to y_2.

At $(1,1)$, $y_1' = 3$ and $y_2' = \frac{1}{3}$.

$\pm\frac{1}{\sqrt{10}}\langle 1, 3\rangle$ is tangent to y_1, $\pm\frac{1}{\sqrt{10}}\langle 3,1\rangle$ is tangent to y_2.

At $(-1, -1)$, $y_1' = 3$ and $y_2' = \frac{1}{3}$.

$\pm\frac{1}{\sqrt{10}}\langle 1, 3\rangle$ is tangent to y_1, $\pm\frac{1}{\sqrt{10}}\langle 3,1\rangle$ is tangent to y_2.



At $(0, 0)$, the vectors are perpendicular $(90°)$.

At $(1,1)$,

$$\cos\theta = \frac{\frac{1}{\sqrt{10}}\langle 1,3\rangle \cdot \frac{1}{\sqrt{10}}\langle 3,1\rangle}{} = \frac{6}{} = \frac{3}{}$$

$\cos\theta = \quad 1\ 1 \quad = 10 \quad = 5$
00

≈ 0.9273 or $53.13°$

By symmetry, the angle is the same at $(-1, -1)$.

(a) The graphs of $y_1 = 1 - x^2$ and $y_2 = x^2 - 1$ intersect at $(1, 0)$ and $(-1, 0)$.

(b) $y_1' = -2x$ and $y_2' = 2x$.

At $(1, 0)$, $y_1' = -2$ and $y_2' = 2$. $\pm\frac{1}{\sqrt{5}}\langle 1, -2\rangle$ is tangent to y_1, $\pm\frac{1}{\sqrt{5}}\langle 1, 2\rangle$ is tangent to y_2.

At $(-1, 0)$, $y_1' = 2$ and $y_2' = -2$. $\pm\frac{1}{\sqrt{5}}\langle 1, 2\rangle$ is tangent to y_1, $\pm\frac{1}{\sqrt{5}}\langle 1, -2\rangle$ is tangent to y_2.

(c) At $(1, 0)$, $\cos\theta = \frac{1}{\sqrt{5}}\langle 1, -2\rangle \cdot \frac{-1}{\sqrt{5}}\langle 1, -2\rangle = \frac{3}{5}$.

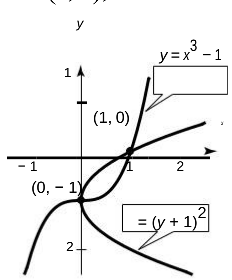
$\theta \approx 0.9273$ or $53.13°$

By symmetry, the angle is the same at $(-1, 0)$.

(a) To find the intersection points, rewrite the second equation as $y + 1 = x^3$. Substituting into the first equation

$(y + 1)^2 = x \Rightarrow x^6 = x \Rightarrow x = 0, 1.$

There are two points of intersection, $(0, -1)$ and $(1, 0)$, as indicated in the figure.



First equation:

$(y + 1)^2 = x \Rightarrow 2(y + 1)y' = 1 \Rightarrow y' = \frac{1}{2(y + 1)}$.

At $(1, 0)$, $y' = \frac{1}{2}$.

Second equation: $y = x^3 - 1 \Rightarrow y' = 3x^2$. At $(1, 0)$, $y' = 3$.

$\pm\frac{1}{\sqrt{5}}\langle 2, 1\rangle$ unit tangent vectors to first curve,

$\pm\frac{1}{\sqrt{10}}\langle 1, 3\rangle$ unit tangent vectors to second curve

At $(0, 1)$, the unit tangent vectors to the first curve are $\pm\langle 0, 1\rangle$, and the unit tangent vectors to the second curve are $\pm\langle 1, 0\rangle$.

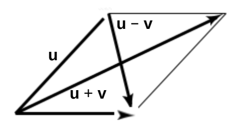
(c) At $(1, 0)$,

$$\cos\theta = \frac{1}{\sqrt{5}}\langle 2, 1\rangle \cdot \frac{1}{\sqrt{10}}\langle 1, 3\rangle = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}.$$

In a rhombus, $\|\mathbf{u}\| = \|\mathbf{v}\|$. The diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.

$$(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) = (\mathbf{u} + \mathbf{v}) \cdot \mathbf{u} - (\mathbf{u} + \mathbf{v}) \cdot \mathbf{v}$$
$$= \mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{u} - \mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{v}$$
$$= \|\mathbf{u}\|^2 - \|\mathbf{v}\|^2 = 0$$

So, the diagonals are orthogonal.



If $\mathbf{u}$ and $\mathbf{v}$ are the sides of the parallelogram, then the diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$, as indicated in the figure.

the parallelogram is a rectangle.

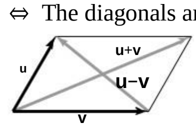
$\Leftrightarrow \mathbf{u} \cdot \mathbf{v} = 0$

$\Leftrightarrow 2\mathbf{u} \cdot \mathbf{v} = -2\mathbf{u} \cdot \mathbf{v}$

$\Leftrightarrow (\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} + \mathbf{v}) = (\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v})$

$\Leftrightarrow \|\mathbf{u} + \mathbf{v}\|^2 = \|\mathbf{u} - \mathbf{v}\|^2$

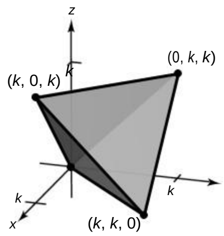
$\Leftrightarrow$ The diagonals are equal in length.



$\approx \frac{\pi}{4}$ or 45°

At $(0, -1)$ the vectors are perpendicular, $\theta = 90°$.

71. (a)



(b) Length of each edge: $\sqrt{k^2 + k^2 + 0^2} = k\sqrt{2}$

(c) $\cos\theta = \dfrac{k^2}{\left(k\sqrt{2}\right)\left(k\sqrt{2}\right)} = \dfrac{1}{2}$

$\theta = \arccos\left(\dfrac{1}{2}\right) = 60°$

(d) $r_1 = \langle k, k, 0\rangle - \left\langle \dfrac{k}{2}, \dfrac{k}{2}, \dfrac{k}{2}\right\rangle = \left\langle \dfrac{k}{2}, \dfrac{k}{2}, -\dfrac{k}{2}\right\rangle$

$r_2 = \langle 0, 0, 0\rangle - \left\langle \dfrac{k}{2}, \dfrac{k}{2}, \dfrac{k}{2}\right\rangle = \left\langle -\dfrac{k}{2}, -\dfrac{k}{2}, -\dfrac{k}{2}\right\rangle$

$\cos\theta = \dfrac{-\dfrac{k^2}{4}}{\left(\dfrac{k}{2}\right)^2 \cdot 3} = -\dfrac{1}{3}$

$= 109.5°$

$\mathbf{u} = \langle \cos\alpha, \sin\alpha, 0\rangle, \mathbf{v} = \langle \cos\beta, \sin\beta, 0\rangle$

The angle between **u** and **v** is $\alpha - \beta$. (Assuming that $> \beta$). Also,

$$\cos(\alpha - \beta) = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{\cos\alpha \cos\beta + \sin\alpha \sin\beta}{(1)(1)} = \cos\alpha \cos\beta + \sin\alpha \sin\beta.$$

73. $\|\mathbf{u} + \mathbf{v}\|^2 = (\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} + \mathbf{v})$

$(\mathbf{u} + \mathbf{v}) \cdot \mathbf{u} + (\mathbf{u} + \mathbf{v}) \cdot \mathbf{v}$

$\mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbf{v}$

$\|\mathbf{u}\|^2 + 2\mathbf{u} \cdot \mathbf{v} + \|\mathbf{v}\|^2$

$\|\mathbf{u}\|^2 + 2\|\mathbf{u}\|\|\mathbf{v}\| + \|\mathbf{v}\|^2 \le (\|\mathbf{u}\| + \|\mathbf{v}\|)^2$

So, $\|\mathbf{u} + \mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$.

Let $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u}$, as indicated in the figure. Because $\mathbf{w}_1$ is a scalar multiple of **v**, you can write

$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = c\mathbf{v} + \mathbf{w}_2.$

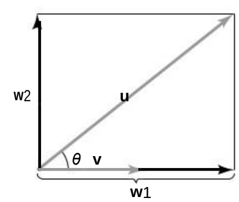
Taking the dot product of both sides with **v** produces

$\mathbf{u} \cdot \mathbf{v} = (c\mathbf{v} + \mathbf{w}_2) \cdot \mathbf{v} = c\mathbf{v} \cdot \mathbf{v} + \mathbf{w}_2 \cdot \mathbf{v}$

$c\|\mathbf{v}\|^2$, because $\mathbf{w}_2$ and **v** are orthogonol.

So, $\mathbf{u} \cdot \mathbf{v} = c\|\mathbf{v}\|^2 \Rightarrow c = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}$ and

$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = c\mathbf{v} = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}.$



$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\|\cos\theta$

$|\mathbf{u} \cdot \mathbf{v}| = \|\mathbf{u}\|\|\mathbf{v}\|\cos\theta|$

$\|\mathbf{u}\|\|\mathbf{v}\||\cos\theta|$

$\le \|\mathbf{u}\|\|\mathbf{v}\|$ because $|\cos\theta| \le 1$.

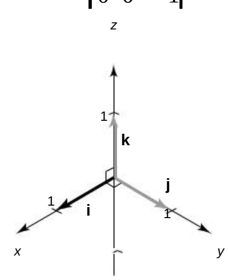
Section 11.4 The Cross Product of Two Vectors in Space

$\mathbf{u} \times \mathbf{v}$ is a vector that is perpendicular (orthogonal) to both **u** and **v**.

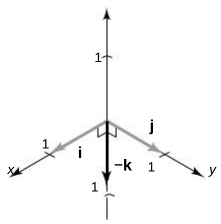
If **u** and **v** are the adjacent sides of a parallelogram, then $A = \|\mathbf{u} \times \mathbf{v}\|$.

3. $\mathbf{j} \times \mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = -\mathbf{k}$

4. $\mathbf{j} \times \mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \mathbf{i}$



z



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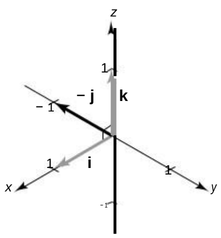
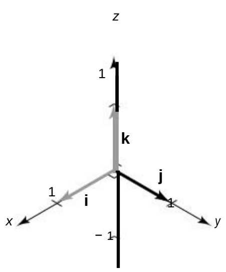
5. $\mathbf{i} \times \mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\mathbf{j}$



6. $\mathbf{k} \times \mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \mathbf{j}$



7. (a) $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 4 & 0 \\ 3 & 2 & 5 \end{vmatrix} = 20\mathbf{i} + 10\mathbf{j} - 16\mathbf{k}$

$\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = -20\mathbf{i} - 10\mathbf{j} + 16\mathbf{k}$

$\mathbf{v} \times \mathbf{v} = \mathbf{0}$

8. (a) $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 5 \\ 2 & 3 & -2 \end{vmatrix} = -15\mathbf{i} + 16\mathbf{j} + 9\mathbf{k}$

$\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = 15\mathbf{i} - 16\mathbf{j} - 9\mathbf{k}$

$\mathbf{v} \times \mathbf{v} = \mathbf{0}$

9. (a) $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 3 & 2 \\ 1 & -1 & 5 \end{vmatrix} = 17\mathbf{i} - 33\mathbf{j} - 10\mathbf{k}$

$\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = -17\mathbf{i} + 33\mathbf{j} + 10\mathbf{k}$

$\mathbf{v} \times \mathbf{v} = \mathbf{0}$

10. (a) $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -9 \\ 6 & -2 & -1 \end{vmatrix}$

$-19\mathbf{i} + 56\mathbf{j} + 2\mathbf{k}$

$\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = 19\mathbf{i} - 56\mathbf{j} - 2\mathbf{k}$

$\mathbf{v} \times \mathbf{v} = 0$

11. $\mathbf{u} = \langle 4, -1, 0 \rangle$, $\mathbf{v} = \langle -6, 3, 0 \rangle$

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 0 \\ -6 & 3 & 0 \end{vmatrix} = 6\mathbf{k} = \langle 0, 0, 6 \rangle$

$\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) = 4(0) + (-1)(0) + 0(6) = 0 \Rightarrow \mathbf{u} \perp \mathbf{u} \times \mathbf{v}$

$\mathbf{v} \cdot (\mathbf{u} \times \mathbf{v}) = -6(0) + 3(0) + 0(6) = 0 \Rightarrow \mathbf{v} \perp \mathbf{u} \times \mathbf{v}$

12. $\mathbf{u} = \langle -5, 2, 2 \rangle$, $\mathbf{v} = \langle 0, 1, 8 \rangle$

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 2 & 2 \\ 0 & 1 & 8 \end{vmatrix} = 14\mathbf{i} + 40\mathbf{j} - 5\mathbf{k} = \langle 14, 40, -5 \rangle$

$\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) = -5(14) + 2(40) + 2(-5) = 0 \Rightarrow \mathbf{u} \perp \mathbf{u} \times \mathbf{v}$

$$\mathbf{v} \cdot (\mathbf{u} \times \mathbf{v}) = (0)(14) + (14)(0) + (8)(-5) = 0 \Rightarrow \mathbf{v} \perp (\mathbf{u} \times \mathbf{v})$$

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13. $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k} = \langle -2, 3, -1 \rangle$$

$$\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) = 1(-2) + 1(3) + 1(-1) = 0 \Rightarrow \mathbf{u} \perp \mathbf{u} \times \mathbf{v}$$

$$\mathbf{v} \cdot (\mathbf{u} \times \mathbf{v}) = 2(-2) + 1(3) + (-1)(-1) = 0 \Rightarrow \mathbf{v} \perp \mathbf{u} \times \mathbf{v}$$

14. $\mathbf{u} = \mathbf{i} + 6\mathbf{j}$, $\mathbf{v} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 6 & 0 \\ -2 & 1 & 1 \end{vmatrix} = 6\mathbf{i} - \mathbf{j} + 13\mathbf{k}$$

$$\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) = 1(6) + 6(-1) = 0 \Rightarrow \mathbf{u} \perp (\mathbf{u} \times \mathbf{v})$$

$$\mathbf{v} \cdot (\mathbf{u} \times \mathbf{v}) = -2(6) + 1(-1) + 1(13) = 0 \Rightarrow \mathbf{v} \perp (\mathbf{u} \times \mathbf{v})$$

$\mathbf{u} = \langle 4, -3, 1 \rangle$ $\mathbf{v} = \langle 2, 5, 3 \rangle$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -3 & 1 \\ 2 & 5 & 3 \end{vmatrix} = -14\mathbf{i} - 10\mathbf{j} + 26\mathbf{k}$$

$$\frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{1}{\sqrt{972}} \langle -14, -10, 26 \rangle = \frac{1}{18\sqrt{3}} \langle -14, -10, 26 \rangle = \left\langle -\frac{7}{9\sqrt{3}}, -\frac{5}{9\sqrt{3}}, \frac{13}{9\sqrt{3}} \right\rangle$$

$\mathbf{u} = \langle -8, -6, 4 \rangle$ $\mathbf{v} = \langle 10, -12, -2 \rangle$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & -6 & 4 \\ 10 & -12 & -2 \end{vmatrix} = 60\mathbf{i} + 24\mathbf{j} + 156\mathbf{k}$$

$$\frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{1}{36\sqrt{22}} \langle 60, 24, 156 \rangle = \left\langle \frac{5}{3\sqrt{22}}, \frac{2}{3\sqrt{22}}, \frac{13}{3\sqrt{22}} \right\rangle$$

$\mathbf{u} = -3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ $\mathbf{v} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 2 & -5 \\ 1 & -1 & 4 \end{vmatrix} = 3\mathbf{i} + 7\mathbf{j} + \mathbf{k}$$

$$\frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{1}{\sqrt{59}} \langle 3, 7, 1 \rangle = \left\langle \frac{3}{\sqrt{59}}, \frac{7}{\sqrt{59}}, \frac{1}{\sqrt{59}} \right\rangle$$

$\mathbf{u} = 2\mathbf{k}$

$\mathbf{v} = 4\mathbf{i} + 6\mathbf{k}$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 4 & 0 & 6 \end{vmatrix} = 8\mathbf{j}$$

$$\frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{1}{8}(8\mathbf{j}) = \mathbf{j} = \langle 0, 1, 0 \rangle$$

$\mathbf{u} = \mathbf{j}$

$\mathbf{v} = \mathbf{j} + \mathbf{k}$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \mathbf{i}$$

$$= \|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{i}\| = 1$$

$\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ $\mathbf{v} = \mathbf{j} + \mathbf{k}$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -\mathbf{j} + \mathbf{k}$$

$$A = \|\mathbf{u} \times \mathbf{v}\| = \|-\mathbf{j} + \mathbf{k}\| = \sqrt{2}$$

$\mathbf{u} = \langle 3, 2, -1 \rangle$ $\mathbf{v} = \langle 1, 2, 3 \rangle$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -1 \\ 1 & 2 & 3 \end{vmatrix} = \langle 8, -10, 4 \rangle$$

$$A = \|\mathbf{u} \times \mathbf{v}\| = \|\langle 8, -10, 4 \rangle\| = \sqrt{180} = 6\sqrt{5}$$

$\mathbf{u} = \langle 2, -1, 0 \rangle$

$= \langle -1, 2, 0 \rangle$

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 0 \\ -1 & 2 & 0 \end{vmatrix} = \langle 0, 0, 3 \rangle$

$= \left\| \mathbf{u} \times \mathbf{v} \right\| = \left\| \langle 0, 0, 3 \rangle \right\| = 3$

$A(0, 3, 2), B(1, 5, 5), C(6, 9, 5), D(5, 7, 2)$

$AB = 1,2,3$

$DC = 1,2,3$

$BC = 5,4,0$

$AD = 5,4,0$

Because $AB = DC$ and $BC = AD$, the figure $ABCD$ is

parallelogram.

AB and AD are adjacent sides

$AB \times AD = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 5 & 4 & 0 \end{vmatrix} = \langle -12, 15, -6 \rangle$

$A = \left\| AB \times AD \right\| = \sqrt{144+225+36} = 9\sqrt{5}$

$A(2, -3, 1), B(6, 5, -1), C(7, 2, 2), D(3, -6, 4)$

$AB = \langle 4, 8, -2 \rangle$

$DC = \langle 4, 8, -2 \rangle$

$BC = \langle 1, -3, 3 \rangle$

$AD = \langle 1, -3, 3 \rangle$

Because $AB = DC$ and $BC = AD$, the figure $ABCD$ is

parallelogram.

AB and AD are adjacent sides

$AB \times AD = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 8 & -2 \\ 1 & -3 & 3 \end{vmatrix} = \langle 18, -14, -20 \rangle$

$A = \left\| AB \times AD \right\| = \sqrt{324 + 196 + 400} = 2\sqrt{230}$

$A(0, 0, 0), B(1, 0, 3), C(-3, 2, 0)$

$AB = \langle 1, 0, 3 \rangle, AC = \langle -3, 2, 0 \rangle$

$AB \times AC = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 3 \\ -3 & 2 & 0 \end{vmatrix} = \langle -6, -9, 2 \rangle$

$A = \frac{1}{2} \left\| AB \times AC \right\| = \frac{1}{2}\sqrt{36+81+4} = \frac{11}{2}$

$A(2, -3, 4), B(0, 1, 2), C(-1, 2, 0)$

$AB = \langle -2, 4, -2 \rangle, AC = \langle -3, 5, -4 \rangle$

$AB \times AC = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 4 & -2 \\ -3 & 5 & -4 \end{vmatrix} = -6\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$

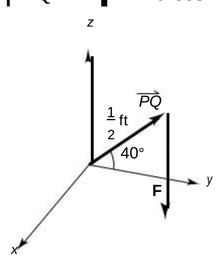
$A = \frac{1}{2} \left\| AB \times AC \right\| = \frac{1}{2}\sqrt{44} = \sqrt{11}$

27. $\mathbf{F} = -20\mathbf{k}$

$PQ = \frac{1}{2}(\cos 40°\, \mathbf{j} + \sin 40°\, \mathbf{k})$

$PQ \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \cos 40°/2 & \sin 40°/2 \\ 0 & 0 & -20 \end{vmatrix} = -10 \cos 40° \mathbf{i}$

$\left\| PQ \times \mathbf{F} \right\| = 10 \cos 40° \approx 7.66$ ft-lb



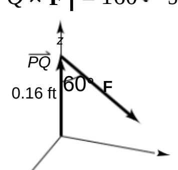
28. $\mathbf{F} = -2000(\cos 30°\, \mathbf{j} + \sin 30° \mathbf{k}) = -1000\sqrt{3}\, \mathbf{j} - 1000\mathbf{k}$

$PQ = 0.16\mathbf{k}$

$PQ \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.16 \\ 0 & -1000\sqrt{3} & -1000 \end{vmatrix}$

$= 160\sqrt{3}\mathbf{i}$

$\left\| PQ \times \mathbf{F} \right\| = 160\sqrt{3}$ ft-lb



y

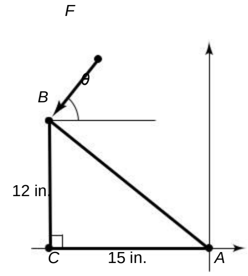
x

29. (a) $AC = 15 \text{ inches} = \frac{5}{4} \text{ feet}$

$BC = 12 \text{ inches} = 1 \text{ foot}$

$\overrightarrow{AB} = -\frac{5}{4}\mathbf{j} + \mathbf{k}$

$\mathbf{F} = -180(\cos\theta\,\mathbf{j} + \sin\theta\,\mathbf{k})$



(b) $\overrightarrow{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -\frac{5}{4} & 1 \\ 0 & -180\cos\theta & -180\sin\theta \end{vmatrix}$

$= (225\sin\theta + 180\cos\theta)\mathbf{i}$

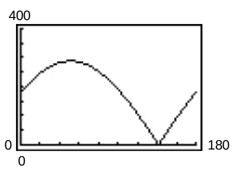
$\left\|\overrightarrow{AB} \times \mathbf{F}\right\| = \left|225\sin\theta + 180\cos\theta\right|$

(c) When $\theta = 30^\circ$, $\left\|\overrightarrow{AB} \times \mathbf{F}\right\| = 225\left(\frac{1}{2}\right) + 180\left(\frac{\sqrt{3}}{2}\right) \approx 268.38$

(d) If $T = \left|225\sin\theta + 180\cos\theta\right|$, $T = 0$ for $225\sin\theta = -180\cos\theta \Rightarrow \tan\theta = -\frac{4}{5} \Rightarrow \theta \approx 141.34^\circ$.

For $0 < \theta < 141.34$, $T'(\theta) = 225\cos\theta - 180\sin\theta = 0 \Rightarrow \tan\theta = \frac{5}{4} \Rightarrow \theta \approx 51.34^\circ$. $\overrightarrow{AB}$ and $\mathbf{F}$ are perpendicular.

(e)



From part (d), the zero is $\theta \approx 141.34^\circ$, when the vectors are parallel.

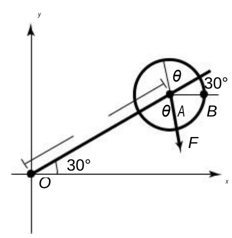
30. (a) Place the wrench in the *xy*-plane, as indicated in the figure.

The angle from $\overrightarrow{AB}$ to $\mathbf{F}$ is $30^\circ + 180^\circ + \theta = 210^\circ + \theta$

$\left\|\overrightarrow{OA}\right\| = 18 \text{ inches} = 1.5 \text{ feet}$

$\overrightarrow{OA} = 1.5\left[\cos(30^\circ)\,\mathbf{i} + \sin(30^\circ)\,\mathbf{j}\right] = \frac{3\sqrt{3}}{4}\mathbf{i} + \frac{3}{4}\mathbf{j}$

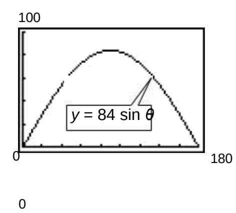
$\mathbf{F} = 56\left[\cos(210^\circ + \theta)\,\mathbf{i} + \sin(210^\circ + \theta)\,\mathbf{j}\right]$



$\overrightarrow{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{3\sqrt{3}}{4} & \frac{3}{4} & 0 \\ 56\cos(210^\circ + \theta) & 56\sin(210^\circ + \theta) & 0 \end{vmatrix}$

$= \left[42\sqrt{3}(\sin 210^\circ\cos\theta + \cos 210^\circ\sin\theta) - 42(\cos 210^\circ\cos\theta - \sin 210^\circ\sin\theta)\right]\mathbf{k}$

$= \left[42\sqrt{3}\left(-\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right) - 42\left(-\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta\right)\right]\mathbf{k} = (-84\sin\theta)\mathbf{k}$



$\left\|\overrightarrow{OA} \times \mathbf{F}\right\| = 84\sin\theta,\ 0 \le \theta \le 180^\circ$

(b) When $\theta = 45^\circ$, $\left\|\overrightarrow{OA} \times \mathbf{F}\right\| = 84\frac{\sqrt{2}}{2} = 42\sqrt{2} \approx 59.40$

Let $T = 84\sin\theta$

$\frac{dT}{d\theta} = 84\cos\theta = 0$ when $\theta = 90^\circ$.

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$d\theta$

This is reasonable. When $\theta = 90°$, the force is perpendicular to the wrench.

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31. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$

32. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1$

33. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 6$

34. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 2 \end{vmatrix} = 0$

35. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 2$

$V = \left|\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})\right| = 2$

36. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 1 & 3 & 1 \\ 0 & 6 & 6 \\ -4 & 0 & -4 \end{vmatrix} = -72$

$V = \left|\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})\right| = 72$

37. $\mathbf{u} = \langle 3, 0, 0\rangle$

$\mathbf{v} = \langle 0, 5, 1\rangle$

$\mathbf{w} = \langle 2, 0, 5\rangle$

$\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 3 & 0 & 0 \\ 0 & 5 & 1 \\ 2 & 0 & 5 \end{vmatrix} = 75$

$V = \left|\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})\right| = 75$

38. $\mathbf{u} = \langle 0, 4, 0\rangle$

$\mathbf{v} = \langle -3, 0, 0\rangle$

$\mathbf{w} = \langle -1, 1, 5\rangle$

$\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 0 & 4 & 0 \\ -3 & 0 & 0 \\ -1 & 1 & 5 \end{vmatrix} = -4(-15) = 60$

$V = \left|\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})\right| = 60$

39. (a) $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = (\mathbf{v}\times\mathbf{w})\cdot\mathbf{u}$ (b)

$= \mathbf{w}\cdot(\mathbf{u}\times\mathbf{v}) = (\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}$ (c)

$= \mathbf{v}\cdot(\mathbf{w}\times\mathbf{u}) = (\mathbf{u}\,x-\mathbf{w})\cdot\mathbf{v}$ (d)

$= \mathbf{v}\cdot(\mathbf{w}\times\mathbf{u}) = (\mathbf{w}\times\mathbf{u})\cdot\mathbf{v}$ (h)

(e) $\mathbf{u}\cdot(\mathbf{w}\times\mathbf{v}) = \mathbf{w}\cdot(\mathbf{v}\times\mathbf{u})$ (f) $= (-\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}$ (g)

So, $a = b = c = d = h$ and $e = f = g$

40. $\mathbf{u}\times\mathbf{v} = \mathbf{0} \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are parallel.

$\mathbf{u}\cdot\mathbf{v} = 0 \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are orthogonal.

So, **u** or **v** (or both) is the zero vector.

41. The cross product is orthogonal to the two vectors, so it is orthogonal to the *yz*-plane. It lies on the *x*-axis, since it is of the form $\langle k, 0, 0\rangle$.

42. Form the vectors for two sides of the triangle, and compute their cross product.

$\langle x_2 - x_1, y_2 - y_1, z_2 - z_1\rangle \times \langle x_3 - x_1, y_3 - y_1, z_3 - z_1\rangle$

43. False. If the vectors are ordered pairs, then the cross product does not exist.

44. False. The cross product is zero if the given vectors are parallel.

45. False. Let $\mathbf{u} = \langle 1, 0, 0\rangle$, $\mathbf{v} = \langle 1, 0, 0\rangle$, $\mathbf{w} = \langle -1, 0, 0\rangle$.

Then, $\mathbf{u}\times\mathbf{v} = \mathbf{u}\times\mathbf{w} = \mathbf{0}$, but $\mathbf{v} \neq \mathbf{w}$.

46. True

$\mathbf{u} = \langle u_1, u_2, u_3\rangle$, $\mathbf{v} = \langle v_1, v_2, v_3\rangle$, $\mathbf{w} = \langle w_1, w_2, w_3\rangle$

$\mathbf{u}\times(\mathbf{v}+\mathbf{w}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 + w_1 & v_2 + w_2 & v_3 + w_3 \end{vmatrix}$

$\lceil u(v + w) - u(v$ 2)⌋ ⌊1(3 3) 3(1 1)⌋ ⌊1(2 2) 2(1 1)⌋

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⌊23332 + w ⌉**i** − ⌈u v + w − u v + w⌉**j** + ⌈u v + w − u v + w⌉**k**

$(u_2 v_3 - u_3 v_2)\mathbf{i} - (u_1 v_3 - u_3 v_1)\mathbf{j} + (u_1 v_2 - u_2 v_1)\mathbf{k} + (u_2 w_3 - u_3 w_2)\mathbf{i} - (u_1 w_3 - u_3 w_1)\mathbf{j} + (u_1 w_2 - u_2 w_1)\mathbf{k}$

$(\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$

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48. $\mathbf{u} = \langle u_1, u_2, u_3\rangle$, $\mathbf{v} = \langle v_1, v_2, v_3\rangle$, c is a scalar:

$$(c\mathbf{u}) \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ cu_1 & cu_2 & cu_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= c\left[(u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}\right] = c\mathbf{u} \times \mathbf{v}$$

$\mathbf{u} = \langle u_1, u_2, u_3\rangle$

$$\mathbf{u} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = (u_2u_3 - u_3u_2)\mathbf{i} - (u_1u_3 - u_3u_1)\mathbf{j} + (u_1u_2 - u_2u_1)\mathbf{k} = \mathbf{0}$$

50. $$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{w} \cdot (\mathbf{u} \times \mathbf{v}) = \begin{vmatrix} w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$w_1(u_2v_3 - v_2u_3) - w_2(u_1v_3 - v_1u_3) + w_3(u_1v_2 - v_1u_2)$$
$$u_1(v_2w_3 - w_2v_3) - u_2(v_1w_3 - w_1v_3) + u_3(v_1w_2 - w_1v_2) = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$$

$\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = (u_2v_3 - u_3v_2)u_1 + (u_3v_1 - u_1v_3)u_2 + (u_1v_2 - u_2v_1)u_3 = \mathbf{0}$ $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = (u_2v_3 - u_3v_2)v_1 + (u_3v_1 - u_1v_3)v_2 + (u_1v_2 - u_2v_1)v_3 = \mathbf{0}$ So, $\mathbf{u} \times \mathbf{v} \perp \mathbf{u}$ and $\mathbf{u} \times \mathbf{v} \perp \mathbf{v}$.

If $\mathbf{u}$ and $\mathbf{v}$ are scalar multiples of each other, $\mathbf{u} = c\mathbf{v}$ for some scalar c. $\mathbf{u} \times \mathbf{v} = (c\mathbf{v}) \times \mathbf{v} = c(\mathbf{v} \times \mathbf{v}) = c(\mathbf{0}) = \mathbf{0}$

If $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then $\|\mathbf{u}\|\|\mathbf{v}\| \sin\theta = 0$. Assume $\mathbf{u} \neq \mathbf{0}$, $\mathbf{v} \neq \mathbf{0}$. So, $\sin\theta = 0$, $\theta = 0$, and $\mathbf{u}$ and $\mathbf{v}$ are parallel. So,

$\mathbf{u} = c\mathbf{v}$ for some scalar c.

53. $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\|\mathbf{v}\| \sin\theta$

If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal, $\theta = \pi/2$ and $\sin\theta = 1$. So, $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\|\mathbf{v}\|$.

54. $\mathbf{u} = \langle a_1, b_1, c_1\rangle$, $\mathbf{v} = \langle a_2, b_2, c_2\rangle$, $\mathbf{w} = \langle a_3, b_3, c_3\rangle$

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = (b_2c_3 - b_3c_2)\mathbf{i} - (a_2c_3 - a_3c_2)\mathbf{j} + (a_2b_3 - a_3b_2)\mathbf{k}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & b_1 & c_1 \\ (b_2c_3 - b_3c_2) & (a_3c_2 - a_2c_3) & (a_2b_3 - a_3b_2) \end{vmatrix}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \left[b_1(a_2b_3 - a_3b_2) - c_1(a_3c_2 - a_2c_3)\right]\mathbf{i} - \left[a_1(a_2b_3 - a_3b_2) - c_1(b_2c_3 - b_3c_2)\right]\mathbf{j} + \left[a_1(a_3c_2 - a_2c_3) - b_1(b_2c_3 - b_3c_2)\right]\mathbf{k}$$

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$$= \left[a_2(a_1a_3 + b_1b_3 + c_1c_3) - a_3(a_1a_2 + b_1b_2 + c_1c_2)\right]\mathbf{i} + \left[b_2(a_1a_3 + b_1b_3 + c_1c_3) - b_3(a_1a_2 + b_1b_2 + c_1c_2)\right]\mathbf{j} + \left[c_2(a_1a_3 + b_1b_3 + c_1c_3) - c_3(a_1a_2 + b_1b_2 + c_1c_2)\right]\mathbf{k}$$

$$= (a_1a_3 + b_1b_3 + c_1c_3)\langle a_2, b_2, c_2\rangle - (a_1a_2 + b_1b_2 + c_1c_2)\langle a_3, b_3, c_3\rangle =$$

$$(\mathbf{u}\cdot\mathbf{w})\mathbf{v} - (\mathbf{u}\cdot\mathbf{v})\mathbf{w}$$

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55. $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$, $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$, $\mathbf{w} = w_1\mathbf{i} + w_2\mathbf{j} + w_3\mathbf{k}$

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \langle v_2 w_3 - w_2 v_3, -(v_1 w_3 - w_1 v_3), v_1 w_2 - w_1 v_2 \rangle$$

$$\times (\mathbf{v} \times \mathbf{w}) = \langle u_1, u_2, u_3 \rangle \cdot \langle v_2 w_3 - w_2 v_3, -(v_1 w_3 - w_1 v_3), v_1 w_2 - w_1 v_2 \rangle$$

$$u_1 v_2 w_3 - u_1 v_3 w_2 - u_2 v_1 w_3 + u_2 v_3 w_1 + u_3 v_1 w_2 - u_3 v_2 w_1$$

$$\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \; w_1 \\ w_2 & w_3 & \end{vmatrix}$$

Section 11.5 Lines and Planes in Space

The parametric equations of a line L parallel to $= \langle a, b, c, \rangle$ and passing through the point $P(x_1, y_1, z_1)$ are $= x_1 + at, y = y_1 + bt, z = z_1 + ct$. The symmetric equations are

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}.$$

In the equation of the plane $2(x - 1) + 4(y - 3) - (z + 5) = 0$, $a = 2$, $b = 4$, and $c = -1$. Therefore, the normal vector is $\langle 2, 4, -1 \rangle$.

Answers will vary. Any plane that has a missing x-variable in its equation is parallel to the x-axis.
Sample answer: $3y - z = 5$

First choose a point Q in one plane. Then use Theorem 11.13:

$$D = \frac{|PQ \cdot n|}{\|\mathbf{n}\|}$$

where P is a point in the other plane and $\mathbf{n}$ is normal to that plane.

$x = -2 + t, y = 3t, z = 4 + t$

(a) $(0, 6, 6)$: For $x = 0 = -2 + t$, you have $t = 2$. Then $y = 3(2) = 6$ and $z = 4 + 2 = 6$. Yes, $(0, 6, 6)$ lies on the line.

$(2, 3, 5)$: For $x = 2 = -2 + t$, you have $t = 4$. Then $y = 3(4) = 12 \neq 3$. No, $(2, 3, 5)$ does not lie on the line.

(c) $(-4, -6, 2)$: For $x = -4 = -2 + t$, you have $= -2$.

Then $y = 3(-2) = -6$ and $z = 4 - 2 = 2$. Yes, $(-$

$$\frac{x - 3}{28} = \frac{y - 7}{} = z + 2$$

(a) $(7, 23, 0)$: Substituting, you have

$$\frac{7 - 3}{2} = \frac{23 - 7}{8} = 0 + 2$$

$$2 = 2 = 2$$

Yes, $(7, 23, 0)$ lies on the line.

(b) $(1, -1, -3)$: Substituting, you have

$$\frac{1 - 3}{2} = \frac{-1 - 7}{8} = -3 + 2$$

$$-1 = -1 = -1$$

Yes, $(1, -1, -3)$ lies on the line.

(c) $(-7, 47, -7)$: Substituting, you have

$$\frac{7 - 3}{28} = 47 - 7 = -7 + 2$$

$$5 \neq 5 \neq -5$$

No, $(-7, 47, -7)$ does not lie on the line.

Point: $(0, 0, 0)$

Direction vector: $\langle 3, 1, 5 \rangle$

Direction numbers: 3, 1, 5

Parametric: $x = 3t, y = t, z = 5t$

x z

$4, -6, 2)$ lies on the line.

Symmetric: $3 - = y = 5$

8. Point: $(0,0,0)$

Direction vector: $\mathbf{v} = \left\langle -2, \frac{5}{2}, 1 \right\rangle$

Direction numbers: $-4, 5, 2$

(a) Parametric: $x = -4t,\ y = 5t,\ z = 2t$

(b) Symmetric: $\frac{x}{-4} = \frac{y}{5} = \frac{z}{2}$

9. Point: $(-2, 0, 3)$

Direction vector: $\mathbf{v} = \langle 2, 4, -2 \rangle$

Direction numbers: $2, 4, -2$

(a) Parametric: $x = -2 + 2t,\ y = 4t,\ z = 3 - 2t$

(b) Symmetric: $\frac{x+2}{2} = \frac{y}{4} = \frac{z-3}{-2}$

Point: $(-3, 0, 2)$

Direction vector: $\mathbf{v} = \langle 0, 6, 3 \rangle$

Direction numbers: $0, 2, 1$

Parametric: $x = -3,\ y = 2t,\ z = 2 + t$

Symmetric: $\frac{y}{2} = z - 2,\ x = -3$

Point: $(1, 0, 1)$

Direction vector: $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

Direction numbers: $3, -2, 1$

Parametric: $x = 1 + 3t,\ y = -2t,\ z = 1 + t$

(b) Symmetric: $\frac{x-1}{3} = \frac{y}{-2} = \frac{z-1}{1}$

12. Point: $(-3, 5, 4)$

Directions numbers: $3, -2, 1$

(a) Parametric: $x = -3 + 3t,\ y = 5 - 2t,\ z = 4 + t$

(b) Symmetric: $\frac{x+3}{3} = \frac{y-5}{-2} = z - 4$

Points: $(5, -3, -2),\ \left(-\frac{2}{3}, \frac{2}{3}, 1\right)$

Direction vector: $\mathbf{v} = \frac{17}{3}\mathbf{i} - \frac{11}{3}\mathbf{j} - 3\mathbf{k}$

14. Points: $(0, 4, 3),\ (-1, 2, 5)$

Direction vector: $\langle 1, 2, -2 \rangle$

Direction numbers: $1, 2, -2$

(a) Parametric: $x = t,\ y = 4 + 2t,\ z = 3 - 2t$

(b) Symmetric: $x = \frac{y-4}{2} = \frac{z-3}{-2}$

15. Points: $(7, -2, 6),\ (-3, 0, 6)$

Direction vector: $\langle -10, 2, 0 \rangle$

Direction numbers: $-10, 2, 0$

(a) Parametric: $x = 7 - 10t,\ y = -2 + 2t,\ z = 6$

(b) Symmetric: Not possible because the direction number for z is 0. But, you could describe the line as $\frac{x-7}{10} = \frac{y+2}{-2},\ z = 6.$

16. Points: $(0, 0, 25),\ (10, 10, 0)$

Direction vector: $\langle 10, 10, -25 \rangle$

Direction numbers: $2, 2, -5$

(a) Parametric: $x = 2t,\ y = 2t,\ z = 25 - 5t$

$\frac{x}{2} = \frac{y}{2} = \frac{z-25}{-5}$

Point: $(2, 3, 4)$

Direction vector: $\mathbf{v} = \mathbf{k}$

Direction numbers: $0, 0, 1$

Parametric: $x = 2,\ y = 3,\ z = 4 + t$

Point: $(-4, 5, 2)$

Direction vector: $\mathbf{v} = \mathbf{j}$

Direction numbers: $0, 1, 0$

Parametric: $x = -4,\ y = 5 + t,\ z = 2$

Point: $(2, 3, 4)$

(a) Parametric:
$x = 5 + 17t,\ y = -3 - 11t,\ z = -2 - 9t$

Direction vector: $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

Direction numbers: 3, 2, −1

Parametric: $x = 2 + 3t$, $y = 3 + 2t$, $z = 4 - t$

(b) Symmetric: $\dfrac{x-5}{17} = \dfrac{y+3}{-11} = \dfrac{z+2}{-9}$

Point $(-4, 5, 2)$

Direction vector: $\mathbf{v} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

Direction numbers: −1, 2, 1

Parametric: $x = -4 - t$, $y = 5 + 2t$, $z = 2 + t$

Point: $(5, -3, -4)$

Direction vector: $\mathbf{v} = \langle 2, -1, 3\rangle$

Direction numbers: 2, −1, 3

Parametric: $x = 5 + 2t,\ y = -3 - t,\ z = -4 + 3t$

Point: $(-1, 4, -3)$

Direction vector: $\mathbf{v} = 5\mathbf{i} - \mathbf{j}$

Direction numbers: 5, −1, 0

Parametric: $x = -1 + 5t,\ y = 4 - t,\ z = -3$

Point: $(2,1,2)$

Direction vector: $\langle -1,1,1\rangle$

Direction numbers: −1, 1, 1

Parametric: $x = 2 - t,\ y = 1 + t,\ z = 2 + t$

Point: $(-6, 0, 8)$

Direction vector: $\langle -2, 2, 0\rangle$

Direction numbers: −2, 2, 0

Parametric: $x = -6 - 2t,\ y = 2t,\ z = 8$

Let $t = 0$: $P = (3, -1, -2)$ (other answers possible)

$= \langle -1, 2, 0\rangle$ (any nonzero multiple of $\mathbf{v}$ is correct)

26. Let $t = 0$: $P = (0, 5, 4)$ (other answers possible)

$= \langle 4, -1, 3\rangle$ (any nonzero multiple of $\mathbf{v}$ is correct)

Let each quantity equal 0:

$= (7, -6, -2)$ (other answers possible)

$= \langle 4, 2,1\rangle$ (any nonzero multiple of $\mathbf{v}$ is correct)

Let each quantity equal 0:

$= (-3, 0, 3)$ (other answers possible)

$= \langle 5, 8, 6\rangle$ (any nonzero multiple of $\mathbf{v}$ is correct)

29. $L_1 : \mathbf{v}_1 = \langle -3, 2, 4\rangle$ and $P = (6, -2, 5)$ on L_1

$L_2 : \mathbf{v}_2 = \langle 6, -4, -8\rangle$ and $P = (6, -2, 5)$ on L_2

The lines are identical.

L_1: $\mathbf{v}_1 = \langle 2, -1, 3\rangle$ and $P = (1, -1, 0)$ on L_1

$L_2 : \mathbf{v}_2 = \langle 2, -1, 3\rangle$ and P not on L_1

The lines are parallel.

31. $L_1 : \mathbf{v}_1 = \langle 4, -2, 3\rangle$ and $P = (8, -5, -9)$ on L_1

$L_2 : \mathbf{v}_2 = \langle -8, 4, -6\rangle$ and $P = (8, -5, -9)$ on L_2

The lines are identical.

$L_1 : \mathbf{v}_1 = \langle 4, 2, 4\rangle$ and $P = (1, 1, -3)$ on L_1

$L_2 : \mathbf{v}_2 = \langle 1, 0.5, 1\rangle$ and P not on

L_2 The lines are parallel.

At the point of intersection, the coordinates for one line equal the corresponding coordinates for the other line. So,

$4t + 2 = 2s + 2$, (ii) $3 = 2s + 3$, and

$-t + 1 = s + 1$.

From (ii), you find that $s = 0$ and consequently, from (iii), $t = 0$. Letting $s = t = 0$, you see that equation (i) is satisfied and so the two lines intersect. Substituting zero for s or for t, you obtain the point $(2, 3, 1)$.

$\mathbf{u} = 4\mathbf{i} - \mathbf{k}$ (First line)

$\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ Second line

$$\cos\theta = \frac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{8-1}{\sqrt{17}\sqrt{9}} = \frac{7}{3\sqrt{17}} = \frac{7\sqrt{17}}{51}$$

$\theta \approx 55.5°$

By equating like variables, you have

(i) $-3t + 1 = 3s + 1$, (ii) $4t + 1 = 2s + 4$, and

(iii) $2t + 4 = -s + 1$.

From (i) you have $s = -t$, and consequently from (ii), $= \frac{1}{2}$ and from (iii), $t = -3$. The lines do not intersect.

Writing the equations of the lines in parametric form you have

$x = 3t \qquad y = 2 - t \qquad z = -1 + t$

$x = 1 + 4s \qquad y = -2 + s \qquad z = -3 - 3s.$

For the coordinates to be equal, $3t = 1 + 4s$ and $2 - t = -2 + s$. Solving this system yields $t = \frac{17}{7}$ and $= \frac{11}{7}$. When using these values for s and t, the z coordinates are not equal. The lines do not intersect.

Writing the equations of the lines in parametric form you have

$x = 2 - 3t \quad y = 2 + 6t \quad z = 3 + t$

$x = 3 + 2s \quad y = -5 + s \quad z = -2 + 4s.$

By equating like variables, you have $2 - 3t = 3 + 2s$, $2 + 6t = -5 + s$, $3 + t = -2 + 4s$. So, $t = -1$, $s = 1$

and the point of intersection is $(5, -4, 2)$.

$\mathbf{u} = \langle -3, 6, 1 \rangle$ (First line)

$\mathbf{v} = \langle 2, 1, 4 \rangle$ Second line

$$\cos\theta = \frac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{4}{\sqrt{46}\sqrt{21}} = \frac{4}{\sqrt{966}} = \frac{2\sqrt{966}}{483}$$

$\approx 82.6°$

$x + 2y - 4z - 1 = 0$

(a) $(-7, 2, -1)$: $(-7) + 2(2) - 4(-1) - 1 = 0$ Point is in plane.

$(5, 2, 2)$: $5 + 2(2) - 4(2) - 1 = 0$

Point is in plane.

$(-6, 1, -1)$: $-6 + 2(1) - 4(-1) - 1 = -1 \neq 0$

Point is not in plane.

$2x + y + 3z - 6 = 0$

$(3, 6, -2)$: $2(3) + 6 + 3(-2) - 6 = 0$

Point is in plane.

$(-1, 5, -1)$: $2(-1) + 5 + 3(-1) - 6 = -6 \neq 0$

Point is not in plane.

$(2, 1, 0)$: $2(2) + 1 + 3(0) - 6 = -1 \neq 0$

Point is not in plane.

Point: $(1, 3, -7)$

Normal vector: $\mathbf{n} = \mathbf{j} = \langle 0, 1, 0 \rangle$

$0(x - 1) + 1(y - 3) + 0(z - (-7)) = 0$

$y - 3 = 0$

Point: $(0, -1, 4)$

Normal vector: $\mathbf{n} = \mathbf{k} = \langle 0, 0, 1 \rangle$

$0(x - 0) + 0(y + 1) + 1(z - 4) = 0$

$z - 4 = 0$

Point: $(3, 2, 2)$

Normal vector: $\mathbf{n} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

$2(x - 3) + 3(y - 2) - 1(z - 2) = 0$

$2x + 3y - z - 10 = 0$

Point: $(0, 0, 0)$

Normal vector: $\mathbf{n} = -3\mathbf{i} + 2\mathbf{k}$

$-3(x - 0) + 0(y - 0) + 2(z - 0) = 0$

$-3x + 2z = 0$

Point: $(-1, 4, 0)$

Normal vector: $\mathbf{v} = \langle 2, -1, -2 \rangle$

$2(x + 1) - 1(y - 4) - 2(z - 0) = 0$

$2x - y - 2z + 6 = 0$

Point: $(3, 2, 2)$

Normal vector: $\mathbf{v} = 4\mathbf{i} + \mathbf{j} - 3\mathbf{k}$

$4(x - 3) + (y - 2) - 3(z - 2) = 0$

$4x + y - 3z - 8 = 0$

Let $\mathbf{u}$ be the vector from $(0, 0, 0)$ to $(2, 0, 3)$: $\mathbf{u} = \langle 2, 0, 3 \rangle$

Let $\mathbf{u}$ be the vector from $(0, 0, 0)$ to $(-3, -1, 5)$: $\mathbf{v} = \langle -3, -1, 5 \rangle$

$$\text{Normal vectors: } \mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 3 \\ -3 & -1 & 5 \end{vmatrix} = \langle 3, -19, -2 \rangle$$

$3(x - 0) - 19(y - 0) - 2(z - 0) = 0$

$3x - 19y - 2z = 0$

Let $\mathbf{u}$ be the vector from $(3, -1, 2)$ to $(2, 1, 5)$: $\mathbf{u} = \langle -1, 2, 3 \rangle$

Let $\mathbf{u}$ be the vector from $(3, -1, 2)$ to $(1, -2, -2)$: $= \langle -2, -1, -4 \rangle$

Normal vector:

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ & & \\ & & \end{vmatrix} \langle$$

$-1 \quad 2 \quad 3 = \langle -5, -10, 5 \rangle = -5 \langle 1, 2, -1 \rangle$

$-2 \quad -1$

-4

$1(x - 3) + 2(y + 1) - (z - 2) =$

$0 \quad x + 2y - z + 1 = 0$

Let **u** be the vector from $(1, 2, 3)$

to $(3, 2, 1)$: $\mathbf{u} = 2\mathbf{i} - 2\mathbf{k}$

Let **v** be the vector from $(1, 2, 3)$ to

$(-1, -2, 2)$: $\mathbf{v} = -2\mathbf{i} - 4\mathbf{j} - \mathbf{k}$

Normal vector:

$$\left(\tfrac{1}{2}\mathbf{u}\right) \times (-\mathbf{v}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & -1 \\ 2 & 4 & 1 \end{vmatrix} = 4\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$$

$$4(x-1) - 3(y-2) + 4(z-3) = 0$$
$$4x - 3y + 4z - 10 = 0$$

$(1, 2, 3)$, Normal vector:

$= \mathbf{i}$, $1(x-1) = 0$, $x - 1 = 0$

$(1, 2, 3)$, Normal vector:

$= \mathbf{k}$, $1(z-3) = 0$, $z - 3 = 0$

The plane passes through the three points

$(\sqrt{3}, 0, 1), (0,0,0), (0,1,0)$,

The vector from $(0, 0, 0)$ to $(0, 1, 0)$: $\mathbf{u} = \mathbf{j}$

The vector from $(0, 0, 0)$ to $(\sqrt{3}, 0, 1)$: $\mathbf{v} = \sqrt{3}\mathbf{i} + \mathbf{k}$

$$\text{Normal vector: } \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ \sqrt{3} & 0 & 1 \end{vmatrix} = \mathbf{i} - \sqrt{3}\mathbf{k}$$

$x - \sqrt{3}z = 0$

The direction vectors for the lines are $\mathbf{u} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$,

$= -3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$.

$$\text{Normal vector: } \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ -3 & 4 & -1 \end{vmatrix} = -5(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

Point of intersection of the lines: $(-1, 5, 1)$

$x + 1) + (y - 5) + (z - 1) = 0$

$+ y + z - 5 = 0$

The direction of the line is $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$. Choose any point on the line, $[(0, 4, 0)$, for example$]$, and let **v** be the vector from $(0,4,0)$ to the given point $(2, 2, 1)$:

$\mathbf{v} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

$$\text{Normal vector: } \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = \mathbf{i} - 2\mathbf{k}$$

$x - 2) - 2(z - 1) = 0$

$- 2z = 0$

Let **v** be the vector from $(-1, 1, -1)$ to $(2, 2, 1)$:

$= 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

Let **n** be a vector normal to the plane

$2x - 3y + z = 3$: $\mathbf{n} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

Because **v** and **n** both lie in the plane *P*, the normal vector to *P* is

$$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 2 \\ 2 & -3 & 1 \end{vmatrix} = 7\mathbf{i} - \mathbf{j} - 11\mathbf{k}$$

$$7(x-2) + 1(y-2) - 11(z-1) = 0$$
$$7x + y - 11z - 5 = 0$$

Let **v** be the vector from $(3, 2, 1)$ to $(3, 1, -5)$:

$= -\mathbf{j} - 6\mathbf{k}$

Let **n** be the normal to the given plane:

$= 6\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$

Because **v** and **n** both lie in the plane *P*, the normal vector to *P* is:

$$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & -6 \\ 6 & 7 & 2 \end{vmatrix} = 40\mathbf{i} - 36\mathbf{j} + 6\mathbf{k}$$

$2(20\mathbf{i} - 18\mathbf{j} + 3\mathbf{k})$

$20x - 18y + 3z - 27 = 0$

Let $\mathbf{u} = \mathbf{i}$ and let **v** be the vector from $(1, -2, -1)$

to $(2, 5, 6)$: $\mathbf{v} = \mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$

Because **u** and **v** both lie in the plane P, the normal vector to P is:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 1 & 7 & 7 \end{vmatrix} = -7\mathbf{j} + 7\mathbf{k} = -7(\mathbf{j} - \mathbf{k})$$

$$\left[y - (-2)\right] - \left[z - (-1)\right] = 0$$

$$y - z + 1 = 0$$

Let **u** = **k** and let **v** be the vector from $(4, 2, 1)$ to $(-3, 5, 7)$: $\mathbf{v} = -7\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$

Because **u** and **v** both lie in the plane *P*, the normal vector to *P* is:

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -7 & 3 & 6 \end{vmatrix} = -3\mathbf{i} - 7\mathbf{j} = -(3\mathbf{i} + 7\mathbf{j})$$

$$3(x - 4) + 7(y - 2) = 0$$
$$3x + 7y - 26 = 0$$

Let (x, y, z) be equidistant from $(2, 2, 0)$ and $(0, 2, 2)$.

$$\sqrt{(x-2)^2 + (y-2)^2 + (z-0)^2} = \sqrt{(x-0)^2 + (y-2)^2 + (z-2)^2}$$
$$x^2 - 4x + 4 + y^2 - 4y + 4 + z^2 = x^2 + y^2 - 4y + 4 + z^2 - 4z + 4$$
$$4x + 8 = -4z + 8$$
$$-z = 0 \text{ Plane}$$

$$\sqrt{(x-1)^2 + (y-0)^2 + (z-2)^2} = \sqrt{(x-2)^2 + (y-0)^2 + (z-1)^2}$$

$$x^2 - 2x + 1 + y^2 + z^2 - 4z + 4 = x^2 - 4x + 4 + y^2 + z^2 - 2z + 1$$
$$-2x - 4z + 5 = -4x - 2z + 5$$
$$2x - 2z = 0$$
$$x - z = 0 \text{ Plane}$$

Let (x, y, z) be equidistant from $(-3, 1, 2)$ and $(6, -2, 4)$.

$$\sqrt{(x+3)^2 + (y-1)^2 + (z-2)^2} = \sqrt{(x-6)^2 + (y+2)^2 + (z-4)^2}$$
$$x^2 + 6x + 9 + y^2 - 2y + 1 + z^2 - 4z + 4 = x^2 - 12x + 36 + y^2 + 4y + 4 + z^2 - 8z + 16$$
$$6x - 2y - 4z + 14 = -12x + 4y - 8z + 56$$
$$18x - 6y + 4z - 42 = 0$$
$$9x - 3y + 2z - 21 = 0 \text{ Plane}$$

Let (x, y, z) be equidistant from $(-5, 1, -3)$ and $(2, -1, 6)$

$$\sqrt{(x+5)^2 + (y-1)^2 + (z+3)^2} = \sqrt{(x-2)^2 + (y+1)^2 + (z-6)^2}$$
$$x^2 + 10x + 25 + y^2 - 2y + 1 + z^2 + 6z + 9 = x^2 - 4x + 4 + y^2 + 2y + 1 + z^2 - 12z + 36$$
$$10x - 2y + 6z + 35 = -4x + 2y - 12z + 41$$
$$14x - 4y + 18z - 6 = 0$$
$$7x - 2y + 9z - 3 = 0 \text{ Plane}$$

61. First plane: $\mathbf{n}_1 = \langle -5, 2, -8\rangle$ and $P = (0, 3, 0)$ on plane

Second plane: $\mathbf{n}_2 = \langle 15, -6, 24\rangle = -3\mathbf{n}_1$ and *P* not on plane

Parallel planes

(Note: The equations are not equivalent.)

62. First plane: $\mathbf{n}_1 = \langle 2, -1, 3\rangle$ and $P = (4, 0, 0)$ on plane

Second plane: $\mathbf{n}_2 = \langle 8, -4, 12\rangle = 4\mathbf{n}_1$ and *P* not on plane.

Parallel planes

(Note: The equations are not equivalent.)

First plane: $\mathbf{n}_1 = \langle 3, -2, 5\rangle$ and $P = (0, 0, 2)$ on plane

Second plane: $\mathbf{n}_2 = \langle 75, -50, 125\rangle = 25\mathbf{n}_1$ and P on plane

Planes are identical.

(Note: The equations are equivalent.)

First plane: $\mathbf{n}_1 = \langle -1, 4, -1\rangle$ and $P = (-6, 0, 0)$ on plane

Second plane: $\mathbf{n}_2 = \left\langle -\frac{5}{2}, 10, -\frac{5}{2}\right\rangle = \frac{5}{2}\mathbf{n}_1$ and P on

plane Planes are identical.

(Note: The equations are equivalent.)

(a) $\mathbf{n}_1 = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{n}_2 = \mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{|-7|}{\sqrt{14}\sqrt{21}} = \frac{\sqrt{6}}{6}$$

$$\theta \approx 65.91°$$

The direction vector for the line is

$$\mathbf{n}_2 \times \mathbf{n}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 2 \\ 3 & 2 & -1 \end{vmatrix} = 7(\mathbf{j} + 2\mathbf{k}).$$

Find a point of intersection of the planes.

$$\begin{aligned} 6x + 4y - 2z &= 14 \\ x - 4y + 2z &= 0 \\ \hline 7x &= 14 \\ x &= 2 \end{aligned}$$

Substituting 2 for x in the second equation, you have $-4y + 2z = -2$ or $z = 2y - 1$. Letting $y = 1$, a

point of intersection is $(2,1,1)$.

$= 2, y = 1 + t, z = 1 + 2t$

66. (a) $\mathbf{n}_1 = \langle -2, 1, 1\rangle$ and $\mathbf{n}_2 = \langle 6, -3, 2\rangle$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{|-13|}{\sqrt{6}\sqrt{49}} = \frac{13\sqrt{6}}{42}$$

$$\theta \approx 40.70°$$

The direction vector for the line is

$$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ 6 & -3 & 2 \end{vmatrix} = 5(\mathbf{i} + 2\mathbf{j}).$$

Find a point of intersection of the planes.

$6x + 3y + 3z = 6$ $6x$

(a) $\mathbf{n}_1 = \langle 3, -1, 1\rangle$ and $\mathbf{n}_2 = \langle 4, 6, 3\rangle$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{|9|}{\sqrt{11}\sqrt{61}} = \frac{9\sqrt{671}}{671}$$

$$\theta \approx 69.67°$$

The direction vector for the line is

$$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 1 \\ 4 & 6 & 3 \end{vmatrix} = -9\mathbf{i} - 5\mathbf{j} + 22\mathbf{k}.$$

Find a point of intersection of the planes.

$$\begin{aligned} 18x - 6y + 6z &= 42 \\ 4x + 6y + 3z &= 2 \\ \hline 22x + 9z &= 44 \end{aligned}$$

Let $z = 0$, $22x = 44 \Rightarrow x = 2$ and $3(2) - y + 0 = 7 \Rightarrow y = -1$.

A point of intersection is $(2, -1, 0)$.

$= 2 - 9t, y = -1 - 5t, z = 22t$

(a) $\mathbf{n}_1 = 6\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{n}_2 = -\mathbf{i} + \mathbf{j} + 5\mathbf{k}$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{|-4|}{\sqrt{46}\sqrt{27}} = \frac{2\sqrt{138}}{207}$$

$$\approx 1.6845 \approx 96.52°$$

The direction vector for the line is

$$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -3 & 1 \\ -1 & 1 & 5 \end{vmatrix} = \langle -16, -31, 3\rangle.$$

$-3y + 2z = 4$ $5z =$

$10z = 2$

Substituting 2 for z in the first equation, you have $2x + y = 0$ or $y = 2x$. Letting $x = 0$, a point of intersection is $(0, 0, 2)$.

$= 5t$, $y = 10t$, $z = 2$ or $x = t$, $y = 2t$, $z = 2$

Find a point of intersection of the planes.

$$\begin{aligned} 6x - 3y + z = 5 &\Rightarrow \quad 6x - 3y + z = 5 \\ x + y + 5z = 5 &\Rightarrow \underline{-6x + 6y + 30z = 30} \\ &\qquad\qquad 3y + 31z = 35 \end{aligned}$$

Let $y = -9$, $z = 2 \Rightarrow x = -4 \Rightarrow (-4, -9, 2)$.

$x = -4 - 16t$, $y = -9 - 31t$, $z = 2 + 3t$

The normal vectors to the planes are

$\mathbf{n}_1 = \langle 5, -3, 1\rangle$, $\mathbf{n}_2 = \langle 1, 4, 7\rangle$, $\cos\theta = \dfrac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = 0.$

So, $\theta = \pi/2$ and the planes are orthogonal.

The normal vectors to the planes are
$\mathbf{n}_1 = \langle 3, 1, -4\rangle$, $\mathbf{n}_2 = \langle -9, -3, 12\rangle$.

Because $\mathbf{n}_2 = -3\mathbf{n}_1$, the planes are parallel, but not equal.

The normal vectors to the planes are
$\mathbf{n}_1 = \mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$, $\mathbf{n}_2 = 5\mathbf{i} + \mathbf{j} - \mathbf{k}$,

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{|5 - 3 - 6|}{\sqrt{46}\sqrt{27}} = \frac{4\sqrt{138}}{414} = \frac{2\sqrt{138}}{207}.$$

So, $\theta = \arccos\left(\dfrac{2\sqrt{138}}{207}\right) \approx 83.5°$.

The normal vectors to the planes are
$\mathbf{n}_1 = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{n}_2 = \mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$,

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{|3 - 8 - 2|}{\sqrt{14}\sqrt{21}} = \frac{7\sqrt{6}}{42} = \frac{\sqrt{6}}{6}.$$

So, $\theta = \arccos\left(\dfrac{\sqrt{6}}{6}\right) \approx 65.9°$.

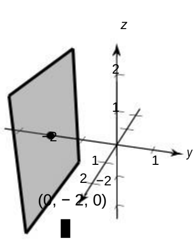
The normal vectors to the planes are $\mathbf{n}_1 = \langle 1, -5, -1\rangle$ and $\mathbf{n}_2 = \langle 5, -25, -5\rangle$. Because $\mathbf{n}_2 = 5\mathbf{n}_1$, the planes are parallel, but not equal.

The normal vectors to the planes are
$\mathbf{n}_1 = \langle 2, 0, -1\rangle$, $\mathbf{n}_2 = \langle 4, 1, 8\rangle$,

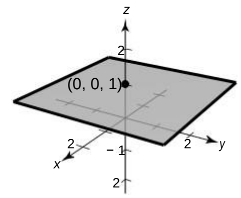
$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = 0$$

So, $\theta = \dfrac{\pi}{2}$ and the planes are orthogonal.

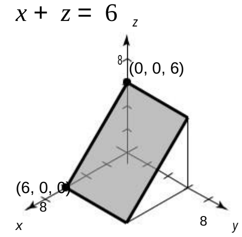
75. $y \le -2$



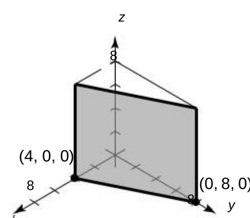
76. $z = 1$



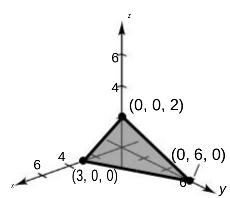
$x + z = 6$



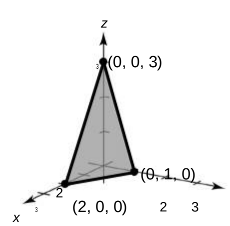
$2x + y = 8$



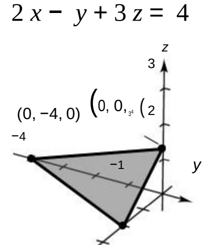
$4x + 2y + 6z = 12$



$3x + 6y + 2z = 6$

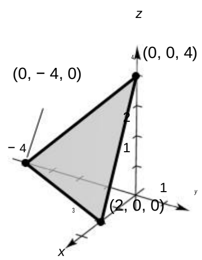


$2x - y + 3z = 4$



3 (2, 0, 0)
x

$2x - y + z = 4$



Writing the equation of the line in parametric form and substituting into the equation of the plane you have:

$= -7 + 2t,\ y = 4 + t,\ z = -1 + 5t$

$(-7 + 2t) + 3(4 + t) - (-1 + 5t) = 6\ 6 = 6$

The equation is valid for all t.

The line lies in the plane.

Writing the equation of the line in parametric form and substituting into the equation of the plane you have:

$= 1 + 4t,\ y = 2t,\ z = 3 + 6t$

$2(1 + 4t) + 3(2t) = -5,\ t = -\frac{1}{2}$

Substituting $t = -\frac{1}{2}$ into the parametric equations for the line you have the point of intersection $(-1, -1, 0)$. The line does not lie in the plane.

Writing the equation of the line in parametric form and

substituting into the equation of the plane you have:

$= 1 + 3t,\ y = -1 - 2t,\ z = 3 + t$

$2(1 + 3t) + 3(-1 - 2t) = 10,\ -1 = 10$, contradiction

So, the line does not intersect the plane.

Writing the equation of the line in parametric form and substituting into the equation of the plane you have:

$= 4 + 2t,\ y = -1 - 3t,\ z = -2 + 5t$

$5(4 + 2t) + 3(-1 - 3t) = 17,\ t = 0$

Substituting $t = 0$ into the parametric equations for the line you have the point of intersection $(4, -1, -2)$.

The line does not lie in the plane.

Point: $Q(0, 0, 0)$

Plane: $2x + 3y + z - 12 = 0$

Normal to plane: $\mathbf{n} = \langle 2, 3, 1\rangle$

Point in plane: $P(6, 0, 0)$

Vector $PQ = \langle -6, 0, 0\rangle$

Point: $Q(0, 0, 0)$

Plane: $5x + y - z - 9 = 0$

Normal to plane: $\mathbf{n} = \langle 5, 1, -1\rangle$

Point in plane: $P(0, 9, 0)$

Vector $PQ = \langle 0, -9, 0\rangle$

$D = \frac{|PQ \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-9|}{\sqrt{27}} = \sqrt{3}$

Point: $Q(2, 8, 4)$

Plane: $2x + y + z = 5$

Normal to plane: $\mathbf{n} = \langle 2, 1, 1\rangle$

Point in plane: $P\langle 0, 0, 5\rangle$

Vector: $PQ = \langle 2, 8, -1\rangle$

$D = \frac{|PQ \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{11}{\sqrt{6}} = \frac{11\sqrt{6}}{6}$

Point: $Q(1, 3, -1)$

Plane: $3x - 4y + 5z - 6 = 0$

Normal to plane: $\mathbf{n} = \langle 3, -4, 5\rangle$

Point in plane: $P(2, 0, 0)$

Vector PQ: $\langle -1, 3, -1\rangle$

$D = \frac{|PQ \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-20|}{\sqrt{50}} = 2\sqrt{2}$

The normal vectors to the planes are $\mathbf{n}_1 = \langle 1, -3, 4\rangle$ and $\mathbf{n}_2 = \langle 1, -3, 4\rangle$. Because $\mathbf{n}_1 = \mathbf{n}_2$, the planes are parallel. Choose a point in each plane.

$P(10, 0, 0)$ is a point in $x - 3y + 4z = 10$.

$Q(6, 0, 0)$ is a point in $x - 3y + 4z = 6$.

$PQ = \langle -4, 0, 0\rangle$, $D = \frac{|PQ \cdot \mathbf{n}_1|}{\|\mathbf{n}_1\|} = \frac{4}{\sqrt{26}} = \frac{2\sqrt{26}}{13}$

The normal vectors to the planes are $\mathbf{n}_1 = \langle 2, 7, 1\rangle$ and $\mathbf{n}_2 = \langle 2, 7, 1\rangle$. Because $\mathbf{n}_1 = \mathbf{n}_2$, the planes are parallel. Choose a point in each plane.

$P(0, 0, 13)$ is a point in $2x + 7y + z = 13$.

$Q(0, 0, 9)$ is a point in $2x + 7y + z = 9$.

$\overrightarrow{PQ} = \langle 0, 0, 4\rangle$

$$D = \frac{\left|\overrightarrow{PQ}\cdot \mathbf{n}_1\right|}{\|\mathbf{n}_1\|} = \frac{4}{\sqrt{54}} = \frac{2\sqrt{6}}{9}$$

$$D = \frac{\left|\overrightarrow{PQ}\cdot \mathbf{n}\right|}{\|\mathbf{n}\|} = \frac{|-12|}{\sqrt{14}} = \frac{6\sqrt{14}}{7}$$

$D = \dfrac{\left\| \overrightarrow{PQ} \times \mathbf{u} \right\|}{\|\mathbf{u}\|} = \dfrac{\sqrt{5}}{\sqrt{9}} = \dfrac{\sqrt{5}}{3}$

The normal vectors to the planes are $\mathbf{n}_1 = \langle -3, 6, 7 \rangle$ and $\mathbf{n}_2 = \langle 6, -12, -14 \rangle$. Because $\mathbf{n}_2 = -2\mathbf{n}_1$, the planes are parallel. Choose a point in each plane.

$P(0, -1, 1)$ is a point in $-3x + 6y + 7z = 1$.

$Q\left(\dfrac{25}{6}, 0, 0\right)$ is a point in $6x - 12y - 14z = 25$.

$\overrightarrow{PQ} = \left\langle \dfrac{25}{6}, 1, -1 \right\rangle$

$D = \dfrac{\left| \overrightarrow{PQ} \cdot \mathbf{n}_1 \right|}{\|\mathbf{n}_1\|} = \dfrac{\left| -27/2 \right|}{\sqrt{94}} = \dfrac{27}{2\sqrt{94}} = \dfrac{27\sqrt{94}}{188}$

The normal vectors to the planes are $\mathbf{n}_1 = \langle -1, 6, 2 \rangle$ and $\mathbf{n}_2 = \left\langle -\frac{1}{2}, 3, 1 \right\rangle$. Because $\mathbf{n}_1 = 2\mathbf{n}_2$, the planes are parallel. Choose a point in each plane.

$P(-3, 0, 0)$ is a point in $-x + 6y + 2z = 3$.

$Q(0, 0, 4)$ is a point in $-\frac{1}{2}x + 3y + z = 4$.

$\overrightarrow{PQ} = \langle 3, 0, 4 \rangle$

$D = \dfrac{\left| \overrightarrow{PQ} \cdot \mathbf{n}_1 \right|}{\|\mathbf{n}_1\|} = \dfrac{5}{\sqrt{41}} = \dfrac{5\sqrt{41}}{41}$

$\mathbf{u} = \langle 4, 0, -1 \rangle$ is the direction vector for the line.

$Q(1, 5, -2)$ is the given point, and $P(-2, 3, 1)$ is on the line.

$\overrightarrow{PQ} = \langle 3, 2, -3 \rangle$

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -3 \\ 4 & 0 & -1 \end{vmatrix} = \langle -2, -9, -8 \rangle$$

$D = \dfrac{\left\| \overrightarrow{PQ} \times \mathbf{u} \right\|}{\|\mathbf{u}\|} = \dfrac{\sqrt{149}}{\sqrt{17}} = \dfrac{\sqrt{2533}}{17}$

$\mathbf{u} = \langle 2, 1, 2 \rangle$ is the direction vector for the line.

$Q(1, -2, 4)$ is the given point, and $P(0, -3, 2)$ is a point on the line (let $t = 0$).

$\overrightarrow{PQ} = \langle 1, 1, 2 \rangle$

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 2 \\ 2 & 1 & 2 \end{vmatrix} = \langle 0, 2, -1 \rangle$$

$Q(-2, 1, 3)$ is the given point, and $P(1, 2, 0)$ is on the line (let $t = 0$ in the parametric equations for the line).

$\overrightarrow{PQ} = \langle -3, -1, 3 \rangle$

$\mathbf{u} = \langle -1, 1, -2 \rangle$ is the direction vector for the line.

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -1 & 3 \\ -1 & 1 & -2 \end{vmatrix} = \langle -1, -9, -4 \rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{1+81+16}}{\sqrt{1+1+4}} = \frac{\sqrt{98}}{\sqrt{6}} = \frac{7}{\sqrt{3}} = \frac{7\sqrt{3}}{3}$$

$\mathbf{u} = \langle 0, 3, 1 \rangle$ is the direction vector for the line.

$Q(4, -1, 5)$ is the given point, and $P(3, 1, 1)$ is on the line.

$\overrightarrow{PQ} = \langle 1, -2, 4 \rangle$

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 4 \\ 0 & 3 & 1 \end{vmatrix} = \langle -14, -1, 3 \rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{14^2 + 1 + 9}}{\sqrt{9 + 1}} = \sqrt{\frac{206}{10}} = \sqrt{\frac{103}{5}} = \frac{\sqrt{515}}{5}$$

The direction vector for L_1 is $\mathbf{v}_1 = \langle -1, 2, 1 \rangle$. The direction vector for L_2 is $\mathbf{v}_2 = \langle 3, -6, -3 \rangle$. Because $\mathbf{v}_2 = -3\mathbf{v}_1$, the lines are parallel.

Let $Q(2, 3, 4)$ to be a point on L_1 and $P(0, 1, 4)$ a point on L_2. $\overrightarrow{PQ} = \langle 2, 2, 0 \rangle$

$= \mathbf{v}_2$ is the direction vector for L_2.

$$\overrightarrow{PQ} \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 0 \\ 3 & -6 & -3 \end{vmatrix} = \langle -6, 6, -18 \rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{v}_2\|}{\|\mathbf{v}_2\|} = \frac{\sqrt{36+36+324}}{\sqrt{9+36+9}} = \sqrt{\frac{396}{54}} = \sqrt{\frac{22}{3}} = \frac{\sqrt{66}}{3}$$

The direction vector for L_1 is $\mathbf{v}_1 = \langle 6, 9, -12\rangle$.

The direction vector for L_2 is $\mathbf{v}_2 = \langle 4, 6, -8\rangle$.

Because $\mathbf{v}_1 = \frac{3}{2}\mathbf{v}_2$, the lines are parallel.

Let $Q(3, -2, 1)$ to be a point on L and $P(-1, 3, 0)$ a point on L_2. $\overrightarrow{PQ} = \langle 4, -5, 1\rangle$.

$\mathbf{u} = \mathbf{v}_2$ is the direction vector for L_2.

$$\overrightarrow{PQ} \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -5 & 1 \\ 4 & 6 & -8 \end{vmatrix} = \langle 34, 36, 44\rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{v}_2\|}{\|\mathbf{v}_2\|} = \frac{\sqrt{34^2 + 36^2 + 44^2}}{\sqrt{16+36+64}} = \frac{\sqrt{4388}}{\sqrt{116}} = \sqrt{\frac{1097}{29}} = \frac{\sqrt{31813}}{29}$$

Exactly one plane contains the point and line. Select two points on the line and observe that three noncolinear points determine a unique plane.

There are an infinite number of planes orthogonal to a given plane in space.

Yes, Consider two points on one line, and a third distinct point on another line. Three distinct points determine a unique plane.

(a) $ax + by + d = 0$ matches (iv). The plane is parallel to the z-axis.

$ax + d = 0$ matches (i). The plane is parallel to the yz-plane.

$cz + d = 0$ matches (ii). The plane is parallel to the xy-plane.

$ax + cz + d = 0$ matches (iii). The plane is parallel to the y-axis.

$z = 0.23x + 0.14y + 6.85$

(a)

Year	2009	2010	2011	2012	2013	2014
z(Approx)	18.93	19.46	20.31	21.10	21.58	22.62

The approximations are close to the actual values.

If x and y both increase, then so does z.

On one side you have the points $(0, 0, 0)$, $(6, 0, 0)$, and $(-1, -1, 8)$.

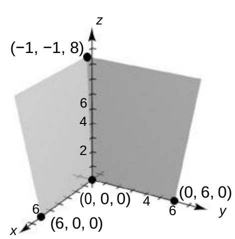
$$\mathbf{n}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ -1 & -1 & 8 \end{vmatrix} = -48\mathbf{j} - 6\mathbf{k}$$

On the adjacent side you have the points $(0, 0, 0)$, $(0, 6, 0)$, and $(-1, -1, 8)$.

$$\mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ -1 & -1 & 8 \end{vmatrix} = 48\mathbf{i} + 6\mathbf{k}$$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{36}{2340} = \frac{1}{65}$$

$$\theta = \arccos\frac{1}{65} \approx 89.1^\circ$$



$L_1: x_1 = 6 + t; y_1 = 8 - t, z_1 = 3 + t$

$L_2: x_2 = 1 + t, y_2 = 2 + t, z_2 = 2t$

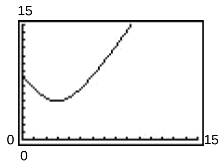
At $t = 0$, the first insect is at $P_1(6, 8, 3)$ and the second insect is at $P_2(1, 2, 0)$.

$$\text{Distance} = \sqrt{(6-1)^2 + (8-2)^2 + (3-0)^2} = \sqrt{70} \approx 8.37 \text{ inches}$$

(b) $\text{Distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} = \sqrt{5^2 + (6-2t)^2 + (3-t)^2} = \sqrt{5t^2 - 30t + 70}, 0 \le t \le 10$

The distance is never zero.

Using a graphing utility, the minimum distance is 5 inches when $t = 3$ minutes.



108. First find the distance D from the point $Q(-3, 2, 4)$ to the plane. Let $P(4, 0, 0)$ be on the plane.

$= \langle 2, 4, -3\rangle$ is the normal to the plane.

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|\langle -7, 2, 4\rangle \cdot \langle 2, 4, -3\rangle|}{\sqrt{4+16+9}} = \frac{|-14+8-12|}{\sqrt{29}} = \frac{18}{\sqrt{29}} = \frac{18\sqrt{29}}{29}$$

The equation of the sphere with center $(-3, 2, 4)$ and radius $18\sqrt{29}/29$ is $(x+3)^2 + (y-2)^2 + (z-4)^2 = \frac{324}{29}$.

The direction vector $\mathbf{v}$ of the line is the normal to the plane, $\mathbf{v} = \langle 3, -1, 4\rangle$.

The parametric equations of the line are $x = 5 + 3t$, $y = 4 - t$, $z = -3 + 4t$.

To find the point of intersection, solve for t in the following equation:

$$3(5 + 3t) - (4 - t) + 4(-3 + 4t) = 7$$
$$26t = 8$$
$$t = \frac{4}{13}$$

$$\left(5 + 3\left(\frac{4}{13}\right), 4 - \frac{4}{13}, -3 + 4\left(\frac{4}{13}\right)\right) = \left(\frac{77}{13}, \frac{48}{13}, -\frac{23}{13}\right)$$

The normal to the plane, $\mathbf{n} = \langle 2, -1, -3\rangle$ is perpendicular to the direction vector $\mathbf{v} = \langle 2, 4, 0\rangle$ of the line because $\langle 2, -1, -3\rangle \cdot \langle 2, 4, 0\rangle = 0$.

So, the plane is parallel to the line. To find the distance between them, let $Q(-2, -1, 4)$ be on the line and $P(2, 0, 0)$ on the plane. $\overrightarrow{PQ} = \langle -4, -1, 4\rangle$.

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|\langle -4, -1, 4\rangle \cdot \langle 2, -1, -3\rangle|}{\sqrt{4+1+9}} = \frac{19}{\sqrt{14}} = \frac{19\sqrt{14}}{14}$$

111. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -5 & 1 \\ -3 & 1 & 4 \end{vmatrix} = -21\mathbf{i} - 11\mathbf{j} - 13\mathbf{k}$

Direction numbers: 21, 11, 13

$= 21t$, $y = 1 + 11t$, $z = 4 + 13t$

The unknown line L is perpendicular to the normal vector $= \langle 1,1,1\rangle$ of the plane, and perpendicular to the direction vector $\mathbf{u} = \langle 1,1,-1\rangle$. So, the direction vector of L is

$$\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \langle -2, 2, 0\rangle.$$

The parametric equations for L are $x = 1 - 2t$, $y = 2t$, $z = 2$.

True

False. They may be skew lines. (See Section Project.)

True

116. False. For example, the lines $x = t, y = 0, z = 1$ and $x = 0, y = t, z = 1$ are both parallel to the plane $= 0$, but the lines are not parallel.

False. For example, planes $7x + y - 11z = 5$ and $5x + 2y - 4z = 1$ are both perpendicular to plane $2x - 3y + z = 3$, but are not parallel.

True

Section 11.6 Surfaces in Space

Quadric surfaces are the three-dimensional analogs of conic sections.

In the *xz*-plane, $z = x^2$ is a parabola. In three-space, $z = x^2$ is a cylinder.

The trace of a surface is the intersection of the surface with a plane. You find a trace by setting one variable equal to a constant, such as $x = 0$ or $z = 2$.

No. For example, $x^2 + y^2 + z^2 = 0$ is a single point and $x^2 + y^2 = 1$ is a right circular cylinder.

Ellipsoid Matches

graph (c)

Hyperboloid of two sheets

Matches graph (e)

Hyperboloid of one sheet

Matches graph (f)

Elliptic cone

Matches graph (b)

Elliptic paraboloid

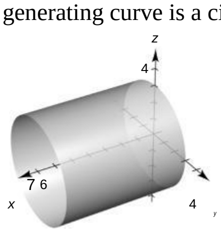
Matches graph (d)

Hyperbolic paraboloid

Matches graph (a)

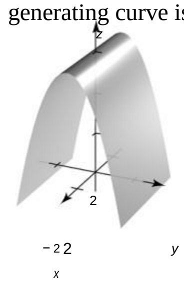
$y^2 + z^2 = 9$

The *x*-coordinate is missing so you have a right circular cylinder with rulings parallel to the *x*-axis. The generating curve is a circle.



$y^2 + z = 6$

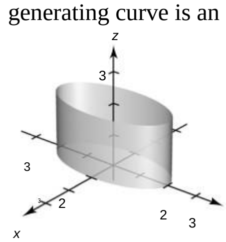
The *x*-coordinate is missing so you have a parabolic cylinder with the rulings parallel to the *x*-axis. The generating curve is a parabola.



$4x^2 + y^2 = 4$

$$\frac{x^2}{1} + \frac{y^2}{4} = 1$$

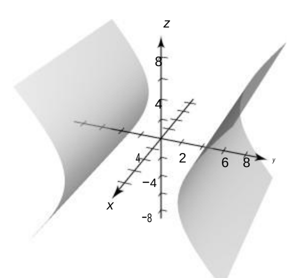
The *z*-coordinate is missing so you have an elliptic cylinder with rulings parallel to the *z*-axis. The generating curve is an ellipse.



$y^2 - z^2 = 25$

$$\frac{y^2}{25} - \frac{z^2}{25} = 1$$

The *x*-coordinate is missing so you have a hyperbolic cylinder with rulings parallel to the *x*-axis. The generating curve is a hyperbola.



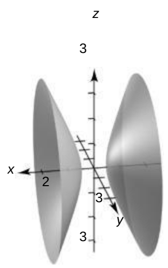
$4x^2 - y^2 - z^2 = 1$

Hyperboloid of two sheets

xy-trace: $4x^2 - y^2 = 1$ hyperbola

yz-trace: none

xz-trace: $4x^2 - z^2 = 1$ hyperbola

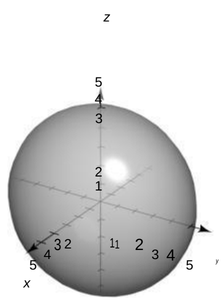


$\frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{25} = 1$

Ellipsoid

xy-trace: $\frac{x^2}{16} + \frac{y^2}{25} = 1$ ellipse

xz-trace: $\frac{x^2}{16} + \frac{z^2}{25} = 1$ ellipse



yz-trace: $y^2 + z^2 = 25$ circle

$16x^2 - y^2 + 16z^2 = 4$

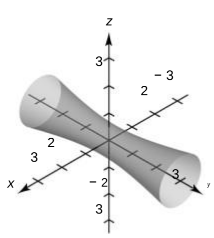
$4x^2 - \frac{y^2}{4} + 4z^2 = 1$

Hyperboloid of one sheet

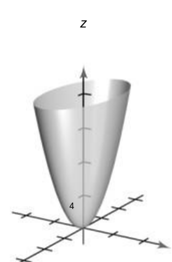
xy-trace: $4x^2 - \frac{y^2}{4} = 1$ hyperbola

xz-trace: $4x^2 + z^2 = 1$ circle

yz-trace: $-\frac{y^2}{4} + 4z^2 = 1$ hyperbola



18. $z = x^2 + 4y^2$



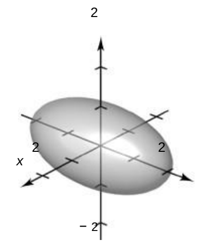
19. $\frac{x^2}{1} + \frac{y^2}{4} + \frac{z^2}{1} = 1$

Ellipsoid

xy-trace: $\frac{x^2}{1} + \frac{y^2}{4} = 1$ ellipse

xz-trace: $x^2 + z^2 = 1$ circle

yz-trace: $\frac{y^2}{4} + \frac{z^2}{1} = 1$ ellipse



20. $z^2 - x^2 - \frac{y^2}{4} = 1$

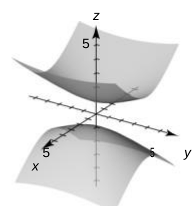
Hyperboloid of two sheets

xy-trace: none

xz-trace: $z^2 - x^2 = 1$ hyperbola

yz-trace: $z^2 - \frac{y^2}{4} = 1$ hyperbola

$z = \pm\sqrt{10}$: $\frac{x^2}{9} + \frac{y^2}{36} = 1$ ellipse



$z^2 = x^2 + \frac{y^2}{9}$

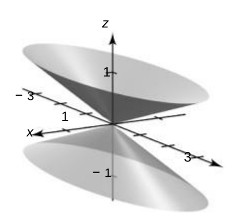
Elliptic cone

xy-trace: point $(0, 0, 0)$

xz-trace: $z = \pm x$

yz-trace: $z = \pm \overline{3}$

When $z = \pm 1$, $x^2 + \frac{y^2}{9} = 1$ ellipse

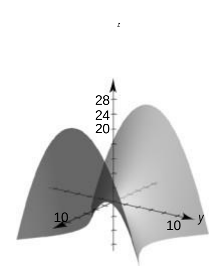


22. $3z = -y^2 + x^2$

Hyperbolic paraboloid

xy-trace: $y = \pm x$

xz-trace: $z = \frac{1}{3}x^2$



Elliptic paraboloid xy-trace: point $(0, 0, 0)$

xz-trace: $z = x^2$ parabola

yz-trace: $z = 4y^2$ parabola

1 3 2 12 y x

yz-trace: $z = -\frac{1}{3}y^2$

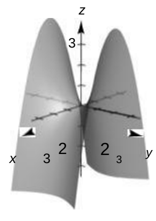
x

$x^2 - y^2 + z = 0$

Hyperbolic paraboloid

xy-trace: $y = \pm x$

xz-trace: $z = -x^2$



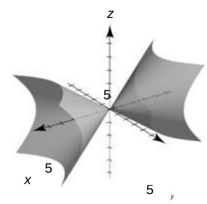
yz-trace: $z = y^2$

$y = \pm 1$: $z = 1 - x^2$

$x^2 = 2y^2 + 2z^2$

Elliptic Cone

xy-trace: $x = \pm \sqrt{2}y$



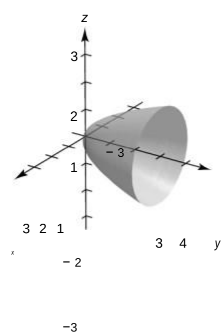
xz-trace: $x = \pm \sqrt{2}z$

yz-trace: point: $(0, 0, 0)$

$x^2 - y + z^2 = 0$

Elliptic paraboloid

xy-trace: $y = x^2$



xz-trace: $x^2 + z^2 =$

0, point $(0, 0, 0)$

yz-trace: $y = z^2$

$= 1$: $x^2 + z^2 = 1$

$-8x^2 + 18y^2 + 18z^2 = 2$

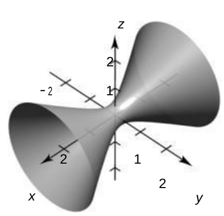
$9y^2 + 9z^2 - 4x^2 = 1$

Hyperboloid of one sheet

xy-trace: $9y^2 - 4x^2 = 1$ hyperbola

yz-trace: $9y^2 + 9z^2 = 1$ circle

xz-trace: $9z^2 - 4x^2 = 1$ hyperbola



27. These have to be two minus signs in order to have a hyperboloid of two sheets. The number of sheets is the same as the number of minus signs.

$z = x^2 + y^2$

You are viewing the paraboloid from the x-axis: $(20, 0, 0)$

You are viewing the paraboloid from above, but not on the z-axis: $(10, 10, 20)$

You are viewing the paraboloid from the z-axis: $(0, 0, 20)$

You are viewing the paraboloid from the y-axis: $(0, 20, 0)$

31. $x^2 + z^2 = [r(y)]^2$ and $z = r(y) = 5y$, so

$x^2 + z^2 = 25y^2$.

32. $x^2 + z^2 = [r(y)]^2$ and $z = r(y) = \pm 3\sqrt{y}$, so

$x^2 + z^2 = 9y$.

33. $x^2 + y^2 = [r(z)]^2$ and $y = r(z) = 2z^{1/3}$, so

$x^2 + y^2 = 4z^{2/3}$.

34. $x^2 + y^2 = [r(z)]^2$ and $x = r(z) = e^z$, so

$x^2 + y^2 = e^{2z}$.

35. $y^2 + z^2 = [r(x)]^2$ and $y = r(x) = \frac{2}{x}$, so

$y^2 + z^2 = \left(\frac{2}{x}\right)^2 \Rightarrow y^2 + z^2 = \frac{4}{x^2}$.

36. $y^2 + z^2 = [r(x)]^2$ and $z = r(x) = \frac{1}{2}\sqrt{4 - x^2}$, so

$y^2 + z^2 = \frac{1}{4}(4 - x^2) \Rightarrow x^2 + 4y^2 + 4z^2 = 4$.

37. $x^2 + y^2 - 2z = 0$

$x^2 + y^2 = \left(\sqrt{2z}\right)^2$

Equation of generating curve: $y = \sqrt{2z}$ or $x = \sqrt{2z}$

$x^2 + z^2 = \cos^2 y$

28. Yes. Every trace is an ellipse (or circle or point).

Equation of generating curve: $x = \cos y$ or $z = \cos y$

29. No. See the table on pages 800 and 801.

$$y^2 + z^2 = 5 - 8x^2 = \left(\sqrt{5 - 8x^2}\right)^2$$

Equation of generating curve: $y = \sqrt{5 - 8x^2}$ or $z = \sqrt{5 - 8x^2}$

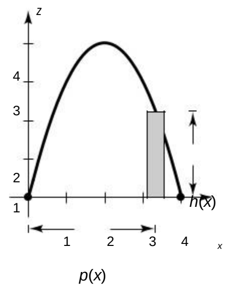
40. $6x^2 + 2y^2 + 2z^2 = 1$

$$y^2 + z^2 = \frac{1}{2} - 3x^2 = \left(\sqrt{\frac{1}{2} - 3x^2}\right)^2$$

Equation of generating curve: $y = \sqrt{\frac{1}{2} - 3x^2}$ or

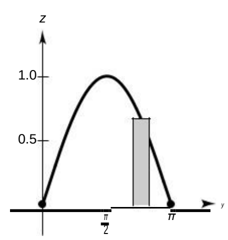
$z = \sqrt{\frac{1}{2} - 3x^2}$

41. $V = 2\pi\int_0^4 x\left(4x - x^2\right)dx = 2\pi\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^4 = \frac{218\pi}{3}$



$V = 2\pi\int_0^\pi y \sin y\, dy$

$= 2\pi\left[\sin y - y\cos y\right]_0^\pi = 2\pi^2$



$z = \frac{x^2}{2} + \frac{y^2}{4}$

(a) When $z = 2$ we have $2 = \frac{x^2}{2} + \frac{y^2}{4}$, or

$1 = \frac{x^2}{4} + \frac{y^2}{8}$

Major axis: $2\sqrt{8} = 4\sqrt{2}$

Minor axis: $2\sqrt{4} = 4$

$c^2 = a^2 - b^2, c^2 = 4, c = 2$

(b) When $z = 8$ we have $8 = \frac{x^2}{2} + \frac{y^2}{4}$, or

$1 = \frac{x^2}{16} + \frac{y^2}{32}$.

Major axis: $2\sqrt{32} = 8\sqrt{2}$

Minor axis: $2\sqrt{16} = 8$

$c^2 = 32 - 16 = 16, c = 4$

Foci: $(0, \pm 4, 8)$

44. $z = \frac{x^2}{2} + \frac{y^2}{4}$

(a) When $y = 4$ you have $z = \frac{x^2}{2} + 4$,

$4\left(\frac{1}{2}\right)(z - 4) = x^2$.

Focus: $\left(0, 4, \frac{9}{2}\right)$

(b) When $x = 2$ you have

$z = 2 + \frac{y^2}{4}$, $4(z - 2) = y^2$.

Focus: $(2, 0, 3)$

If (x, y, z) is on the surface, then

$(y + 2)^2 = x^2 + (y - 2)^2 + z^2$

$y^2 + 4y + 4 = x^2 + y^2 - 4y + 4 + z^2$

$x^2 + z^2 = 8y$

Elliptic paraboloid

Traces parallel to xz-plane are circles.

If (x, y, z) is on the surface, then

$z^2 = x^2 + y^2 + (z - 4)^2$

$z^2 = x^2 + y^2 + z^2 - 8z + 16$

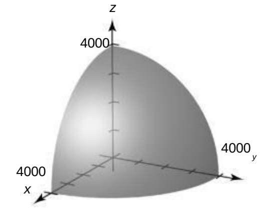
$8z = x^2 + y^2 + 16 \Rightarrow z = \frac{x^2}{8} + \frac{y^2}{8} + 2$

Elliptic paraboloid shifted up 2 units. Traces parallel to xy-plane are circles.

$x^2 \quad y^2 \quad z^2$

Foci: $(0, \pm 2, 2)$

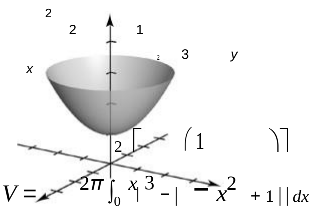
47. $\dfrac{x^2}{3963^2} + \dfrac{y^2}{3963^2} + \dfrac{z^2}{3950^2} = 1$



48. (a) $x^2 + y^2 = \left[r(z)\right]^2$

$$= \left[\sqrt{2(z-1)}\right]^2$$

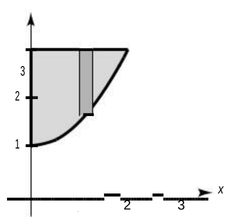
$$x^2 + y^2 - 2z + 2 = 0$$



(b) $V = 2\pi\int_0^2 x\left[3 - \left(\frac{1}{2}x^2 + 1\right)\right]dx$

$$= 2\pi\int_0^2 \left(2x - \frac{1}{2}x^3\right)dx$$

$$= 2\pi\left[x^2 - \frac{x^4}{8}\right]_0^2 = 4\pi \approx 12.6 \text{ cm}^3$$

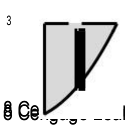


(c) $V = 2\pi\int_{1/2}^2 x\left[3 - \left(\frac{1}{2}x^2 + 1\right)\right]dx$

$$= 2\pi\int_{1/2}^2 \left(2x - \frac{1}{2}x^3\right)dx$$

$$= 2\pi\left[x^2 - \frac{x^4}{8}\right]_{1/2}^2$$

$$= 4\pi - \frac{31\pi}{64} = \frac{225\pi}{64} \approx 11.04 \text{ cm}^3$$



49. $\frac{y^2}{b^2} - \frac{x^2}{a^2} = z, \; z = bx + ay$

$$bx + ay = \frac{y^2}{b^2} - \frac{x^2}{a^2}$$

$$\frac{1}{a^2}\left(x^2 + a^2bx + \frac{a^4b^2}{4}\right) = \frac{1}{b^2}\left(y^2 - ab^2y + \frac{a^2b^4}{4}\right)$$

$$\frac{\left(x + \frac{a^2b}{2}\right)^2}{a^2} = \frac{\left(y - \frac{ab^2}{2}\right)^2}{b^2}$$

$$y = \pm\frac{b}{a}\left(x + \frac{a^2b}{2}\right) + \frac{ab^2}{2}$$

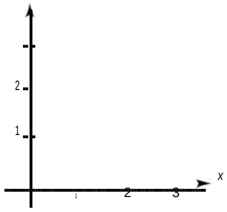
Letting $x = at$, you obtain the two intersecting lines $x = at$, $y = -bt$, $z = 0$ and $x = at$, $y = bt + ab^2$, $z = 2abt + a^2b^2$.

50. Equating twice the first equation with the second equation: $2x^2 + 6y^2 - 4z^2 + 4y - 8 = 2x^2 + 6y^2 - 4$

$$z^2 - 3x - 2\,4y - 8 = -3x - 2$$

$$3x + 4y = 6, \text{ a plane}$$

The Klein bottle *does not* have both an "inside" and an "outside." It is formed by inserting the small open end through the side of the bottle and making it contiguous with the top of the bottle.



Section 11.7 Cylindrical and Spherical Coordinates

The cylindrical coordinate system is an extension of the polar coordinate system. In this system, a point P in space is represented by an ordered triple (r, θ, z). (r, θ) is a polar representation of the projection of P in the xy-plane, and z is the directed distance from (r, θ) to P.

The point is 2 units from the origin, in the xz-plane, and makes an angle of 30° with the z-axis.

$(-7, 0, 5)$, cylindrical

$x = r\cos\theta = -7\cos 0 = -7$

$y = r\sin\theta = -7\sin 0 = 0$

$z = 5$

$(-7, 0, 5)$, rectangular

$(2, -\pi, -4)$, cylindrical

$x = r\cos\theta = 2\cos(-\pi) = -2$

$y = r\sin\theta = 2\sin(-\pi) = 0$

$= -4$

$(-2, 0, -4)$, rectangular

$\left(3, \frac{\pi}{4}, 1\right)$, cylindrical

$x = r\cos\theta = 3\cos\frac{\pi}{4} = \frac{3\sqrt{2}}{2}$

$y = r\sin\theta = 3\sin\frac{\pi}{4} = \frac{3\sqrt{2}}{2}$

$= 1$

$\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}, 1\right)$, rectangular

6. $\left(\ \frac{3\pi}{2}\ \right)$

$x = r\cos\theta = 6\cos\left(-\frac{3\pi}{2}\right) = 0$

$y = r\sin\theta = 6\sin\left(-\frac{3\pi}{2}\right) = 6$

$z = 2$

$(0, 6, 2)$, rectangular

$\left(4, \frac{7\pi}{6}, -3\right)$, cylindrical

$x = r\cos\theta = 4\cos\frac{7\pi}{6} = 4\left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$

$y = r\sin\theta = 4\sin\frac{7\pi}{6} = 4\left(-\frac{1}{2}\right) = -2$

$= -3$

$(-2\sqrt{3}, -2, -3)$, rectangular

$\left(-\frac{2}{3}, \frac{4\pi}{3}, 8\right)$, cylindrical

$(0, 5, 1)$, rectangular

$= \sqrt{(0)^2 + (5)^2} = 5$

$\tan\theta = \frac{5}{2} \Rightarrow \theta = \arctan\frac{5}{0} = \frac{\pi}{2}$

$= 1$

$\left(5, \frac{\pi}{2}, 1\right)$, cylindrical

$(6, 2\sqrt{3}, -1)$, rectangular

$r = \sqrt{6^2 + (2\sqrt{3})^2} = \sqrt{36 + 12} = \sqrt{48} = 4\sqrt{3}$

$\tan\theta = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3} \Rightarrow \theta = \arctan\frac{\sqrt{3}}{3} = \frac{\pi}{6}$

$= -1$

$\left(4\sqrt{3}, \frac{\pi}{6}, -1\right)$, cylindrical

11. $(2, -2, -4)$, rectangular

$r = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$

$\tan\theta = \frac{-2}{2} \Rightarrow \theta = \arctan(-1) = -\frac{\pi}{4}$

$= -4$

$\left(2\sqrt{2}, -\frac{\pi}{4}, -4\right)$, cylindrical

$(3, -3, 7)$, rectangular

$r = \sqrt{3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$

$\tan\theta = \frac{-3}{3} \Rightarrow \theta = \arctan(-1) = -\frac{\pi}{4}$

$z = 7$

$\left(3\sqrt{2}, -\frac{\pi}{4}, 7\right)$, cylindrical

13. $(1, \sqrt{3}, 4)$, rectangular

$$x = r\cos\theta = -\frac{2}{3}\cos\frac{4\pi}{3} = \left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right) = \frac{1}{3}$$

$$y = r\sin\theta = -\frac{2}{3}\sin\frac{4\pi}{3} = \left(-\frac{2}{3}\right)\left(-\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{3}$$

$$z = 8$$

$\left(\frac{1}{3}, \frac{\sqrt{3}}{3}, 8\right)$, rectangular

$$r = \sqrt{1 + \left(\sqrt{3}\right)^2} = 2$$

$$\tan\theta = \frac{\sqrt{3}}{1} \Rightarrow \theta = \arctan\sqrt{3} = \frac{\pi}{3}$$

$$z = 4$$

$\left(2, \frac{\pi}{3}, 4\right)$ cylindrical

$\left(2\sqrt{3}, -2, 6\right)$, rectangular

$r = \sqrt{12 + 4} = 4$

$\theta = \arctan\left(-\frac{1}{\sqrt{3}}\right) = \frac{5\pi}{6}$

$z = 6$

$\left(4, -\frac{\pi}{6}, 6\right)$, cylindrical

$z = 4$ is the equation in cylindrical coordinates. (plane)

$x = 9$, rectangular equation

$r\cos\theta = 9$

$r = 9\sec\theta$, cylindrical equation

17. $x^2 + y^2 - 2z^2 = 5$, rectangular equation

$r^2 - 2z^2 = 5$, cylindrical equation

18. $z = x^2 + y^2 - 11$, rectangular equation

$z = r^2 - 11$, cylindrical equation

$y = x^2$, rectangular equation

$r\sin\theta = (r\cos\theta)^2$

$\sin\theta = r\cos^2\theta$

$r = \sec\theta \cdot \tan\theta$, cylindrical equation

20. $x^2 + y^2 = 8x$, rectangular equation

$r^2 = 8r\cos\theta$

$r = 8\cos\theta$, cylindrical equation

$y^2 = 10 - z^2$, rectangular equation

$(r\sin\theta)^2 = 10 - z^2$

$r^2\sin^2\theta + z^2 = 10$, cylindrical equation

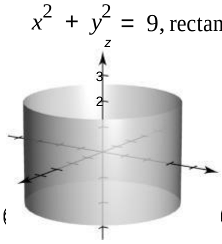
22. $x^2 + y^2 + z^2 - 3z = 0$, rectangular equation

$r^2 + z^2 - 3z = 0$, cylindrical equation

$r = 3$, cylindrical equation

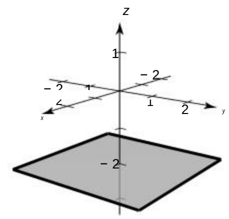
$\sqrt{x^2 + y^2} = 3$

$x^2 + y^2 = 9$, rectangular equation



$z = -2$, cylindrical equation

$z = -2$, rectangular equation



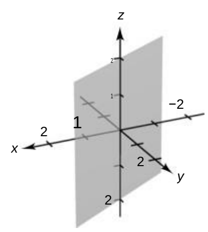
25. $\theta = \frac{\pi}{6}$, cylindrical equation

$\tan\frac{\pi}{6} = \frac{y}{x}$

$\frac{1}{\sqrt{3}} = \frac{y}{x}$

$x = \sqrt{3}y$

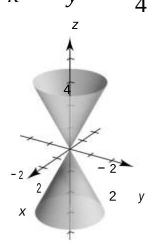
$x - \sqrt{3}y = 0$, rectangular equation



$r = \frac{z}{2}$, cylindrical equation

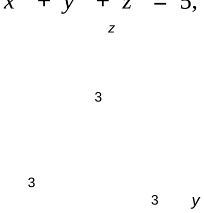
$\sqrt{x^2 + y^2} = \frac{z}{2}$

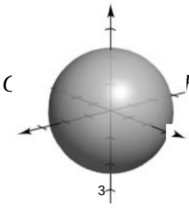
$x^2 + y^2 - \frac{z^2}{4} = 0$, rectangular equation



$r^2 + z^2 = 5$, cylindrical equation

$x^2 + y^2 + z^2 = 5$, rectangular equation





x 4 3 3 4 y

3

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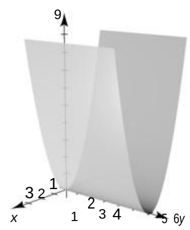
$z = r^2 \cos^2 \theta$, cylindrical equation

$= x^2$, rectangular equation



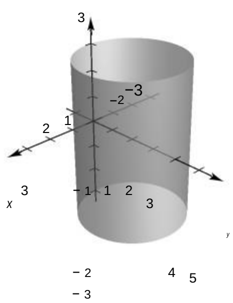
29. $r = 4 \sin \theta$, cylindrical equation

$r^2 = 4r \sin \theta$

$x^2 + y^2 = 4y$

$x^2 + y^2 - 4y + 4 = 4$

$x^2 + (y - 2)^2 = 4$, rectangular equation



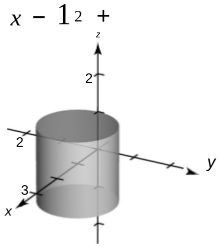
30. $r = 2 \cos \theta$, cylindrical equation

$r^2 = 2r \cos \theta$

$x^2 + y^2 = 2x$

$x^2 + y^2 - 2x = 0$

$x - 1^2 +$



$(4, 0, 0)$, rectangular

$(-4, 0, 0)$, rectangular

$\rho = \sqrt{(-4)^2 + 0^2 + 0^2} = 4$

$\tan \theta = \frac{y}{x} = 0 \Rightarrow \theta = 0$

$\varphi = \arccos \frac{z}{\rho} = \arccos 0 = \frac{\pi}{2}$

$\left(4, 0, \frac{\pi}{2}\right)$, spherical

33. $(-2, 2\sqrt{3}, 4)$, rectangular

$\rho = \sqrt{(-2)^2 + (2\sqrt{3})^2 + 4^2} = 4\sqrt{2}$

$\tan \theta = \frac{y}{x} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

$\theta = \frac{2\pi}{3}$.

$\varphi = \arccos \frac{z}{\rho} = \arccos \frac{1}{\sqrt{2}} = \frac{\pi}{4}$

$\left(4\sqrt{2}, \frac{2\pi}{3}, \frac{\pi}{4}\right)$, spherical

$(-5, -5, 2)$, rectangular

$\rho = \sqrt{(-5)^2 + (-5)^2 + (\sqrt{2})^2} = \sqrt{52} = 2\sqrt{13}$

$\tan \theta = \frac{y}{x} = \frac{-5}{-5} = 1 \Rightarrow \theta = \frac{\pi}{4}$

$\varphi = \arccos \frac{z}{\rho} = \arccos \frac{\sqrt{2}}{2\sqrt{13}} = \arccos \frac{\sqrt{26}}{26}$

$\rho = 4^2 + 0^2 + 0^2 = 4$

$\tan \theta = \frac{y}{x} = 0 \Rightarrow \theta = 0$

$= \arccos 0 = \frac{\pi}{2}$

ρ 2
13
26

$y^2 = 1$, rectangular equation

spherical

$\tan\theta = \frac{y}{x} = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

$\left(2\sqrt{13}, \frac{\pi}{4}, \arccos\frac{\sqrt{26}}{26}\right)$

$\varphi = \arccos\frac{z}{\rho} = \arccos\frac{\sqrt{3}}{2} = \frac{\pi}{6}$

$\left(4, \frac{\pi}{6}, \frac{\pi}{6}\right)$, spherical

35. $\left(\sqrt{3}, 1, 2\sqrt{3}\right)$, rectangular

$= \sqrt{3+1+12} = 4$

$\left(4, 0, \frac{\pi}{2}\right)$, spherical

$\rho = \sqrt{(-1)^2+2^2+1^2} = \sqrt{6}$

$\tan\theta = \frac{y}{x} = -2 \Rightarrow \theta = \arctan(-2) + \pi$

$= \arccos\frac{z}{\rho} = \arccos\frac{1}{\sqrt{6}}$

$\left(\sqrt{6}, \arctan(-2) + \pi, \arccos\frac{1}{\sqrt{6}}\right)$, spherical

$\left(4, \frac{\pi}{6}, \frac{\pi}{4}\right)$, spherical

$x = \rho \sin\varphi \cos\theta = 4 \sin\frac{\pi}{4} \cos\frac{\pi}{6} = \sqrt{6}$

$y = \rho \sin\varphi \sin\theta = 4 \sin\frac{\pi}{4} \sin\frac{\pi}{6} = \sqrt{2}$

$z = \rho \cos\varphi = 4 \cos\frac{\pi}{4} = 2\sqrt{2}$

$\left(\sqrt{6}, \sqrt{2}, 2\sqrt{2}\right)$, rectangular

$\left(6, \pi, \frac{\pi}{2}\right)$, spherical

$x = \rho \sin\varphi \cos\theta = 6 \sin\frac{\pi}{2} \cos\pi = -6$

$y = \rho \sin\varphi \sin\theta = 6 \sin\frac{\pi}{2} \sin\pi = 0$

$z = \rho \cos\varphi = 6 \cos\frac{\pi}{2} = 0$

$(-6, 0, 0)$, rectangular

$\left(12, -\frac{\pi}{4}, 0\right)$, spherical

$x = \rho \sin\varphi \cos\theta = 12 \sin 0 \cos\left(-\frac{\pi}{4}\right) = 0$

$y = \rho \sin\varphi \sin\theta = 12 \sin 0 \sin\left(-\frac{\pi}{4}\right) = 0$

$= \rho \cos\varphi = 12 \cos 0 = 12$

$(0, 0, 12)$, rectangular

$\left(9, \frac{\pi}{4}, \pi\right)$, spherical

$-4 \quad \pi\big)$, spherical

$x = \rho \sin\varphi \cos\theta = 5 \sin\frac{\pi}{12} \cos\frac{\pi}{4} \approx 0.915$

$y = \rho \sin\varphi \sin\theta = 5 \sin\frac{\pi}{12} \sin\frac{\pi}{4} \approx 0.915$

12

$(0.915, 0.915, 4.830)$, rectangular

$\left(7, \frac{3\pi}{4}, \frac{\pi}{9}\right)$, spherical

$x = \rho \sin\varphi \cos\theta = 7 \sin\frac{\pi}{9} \cos\frac{3\pi}{4} \approx -1.693$

$y = \rho \sin\varphi \sin\theta = 7 \sin\frac{\pi}{9} \sin\frac{3\pi}{4} \approx 1.693$

$= \rho \cos\varphi = 7 \cos\frac{\pi}{9} \approx 6.578$

$(-1.693, 1.693, 6.578)$, rectangular

$y = 2$, rectangular equation

$\rho \sin\varphi \sin\theta = 2$

$= 2 \csc\varphi \csc\theta$, spherical equation

$z = 6$, rectangular equation

$\rho \cos\varphi = 6$

$= 6 \sec\varphi$, spherical equation

45. $x^2 + y^2 + z^2 = 49$, rectangular equation

$^2 = 49$

$\rho = 7$, spherical equation

$x^2 + y^2 - 3z^2 = 0$, rectangular equation

$^2 + y^2 + z^2 = 4z^2$

$^2 = 4\rho^2 \cos^2\varphi$

$1 = 4 \cos^2\varphi$

$\cos\varphi = \frac{1}{2}$

$\varphi = \frac{\pi}{3}$, (cone) spherical equation

)

$x = \rho \sin\varphi \cos\theta = 9 \sin\pi \cos\frac{\pi}{4} = 0$

$y = \rho \sin\varphi \sin\theta = 9 \sin\pi \sin\frac{\pi}{4} = 0$

$= \rho \cos\varphi = 9 \cos\pi = -9$ (0,

0, −9), rectangular

$x^2 + y^2 = 16$, rectangular equation

$\rho^2 \sin^2 \varphi \sin^2 \theta + \rho^2 \sin^2 \varphi \cos^2 \theta = 16$

${}^2 \sin^2 \varphi \left(\sin^2 \theta + \cos^2 \theta \right) = 16$

${}^2 \sin^2 \varphi = 16$

$\sin \varphi = 4$

$= 4 \csc \varphi$, spherical equation

$x = 13$, rectangular equation

$\sin \varphi \cos \theta = 13$

$= 13 \csc \varphi \sec \theta$, spherical equation

49. $x^2 + y^2 = 2z^2$, rectangular equation

$\rho^2 \sin^2 \varphi \cos^2 \theta + \rho^2 \sin^2 \varphi \sin^2 \theta = 2\rho^2 \cos^2 \varphi$

$\rho^2 \sin^2 \varphi \left[\cos^2 \theta + \sin^2 \theta\right] = 2\rho^2 \cos^2 \varphi$

$\rho^2 \sin^2 \varphi = 2\rho^2 \cos^2 \theta$

$\dfrac{\sin^2 \varphi}{\cos^2 \varphi} = 2$

$\tan^2 \varphi = 2$

$\tan \varphi = \pm\sqrt{2}$, spherical equation

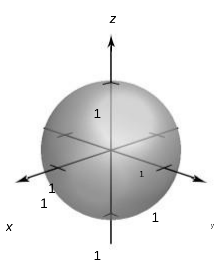
50. $x^2 + y^2 + z^2 - 9z = 0$, rectangular equation

${}^2 - 9\rho \cos \varphi = 0$

$= 9 \cos \varphi$, spherical equation

$\rho = 1$, spherical equation

$x^2 + y^2 + z^1 = 1$, rectangular equation



$\theta = \frac{3\pi}{4}$, spherical

equation $\tan \theta = \frac{y}{x}$

$-1 = \frac{y}{x}$

$x + y = 0$, rectangular equation

z

3

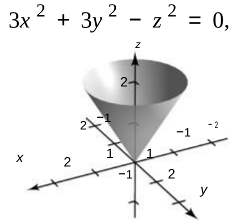
$\varphi = \frac{\pi}{6}$, spherical equation

$\cos \varphi = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$

$\dfrac{\sqrt{3}}{2} = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$

$\dfrac{3}{4} = \dfrac{z^2}{x^2 + y^2 + z^2}$

$3x^2 + 3y^2 - z^2 = 0, z \geq 0$, rectangular equation



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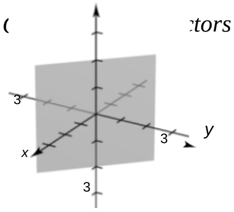

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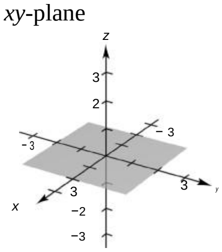
$\varphi = \frac{\pi}{2}$, spherical equation

$\cos\varphi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$

$0 = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$

$= 0$, rectangular equation

xy-plane

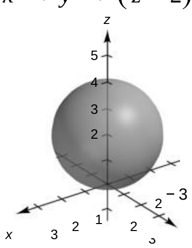


$\rho = 4\cos\varphi$, spherical equation

$\sqrt{x^2 + y^2 + z^2} = \frac{4z}{\sqrt{x^2 + y^2 + z^2}}$

$x^2 + y^2 + z^2 - 4z = 0$

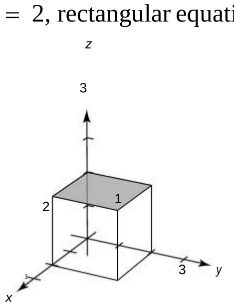
$x^2 + y^2 + (z - 2)^2 = 4,\ z \geq 0$, rectangular equation



$\rho = 2\sec\varphi$, spherical equation

$\cos\varphi = 2$

$= 2$, rectangular equation

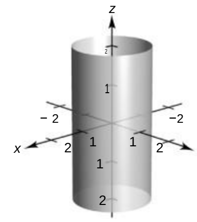


57. $\rho = \csc\varphi$, spherical equation

$\rho\sin\varphi = 1$

$\sqrt{\ ^2 + y^2} = 1$

$x^2 + y^2 = 1$, rectangular equation

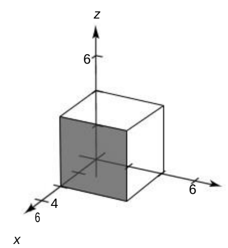


$\rho = 4\csc\varphi\sec\varphi$, spherical equation

$= \frac{4}{\sin\varphi\cos\theta}$

$\rho\sin\varphi\cos\theta = 4$

$= 4$, rectangular equation



$\left(4, \frac{\pi}{4}, 0\right)$, cylindrical

$\rho = \sqrt{4^2 + 0^2} = 4$

$= \frac{\pi}{4}$

$\varphi = \arccos\frac{z}{\sqrt{r^2 + z^2}} = \arccos 0 = \frac{\pi}{2}$

$\left(4, \frac{\pi}{4}, \frac{\pi}{2}\right)$, spherical

$\left(3, -\frac{\pi}{4}, 0\right)$, cylindrical

$\rho = \sqrt{3^2 + 0^2} = 3$

$= -\frac{\pi}{4}$

$\varphi = \arccos\frac{z}{\sqrt{r^2 + z^2}} = \arccos\frac{0}{9} = \frac{\pi}{2}$

$\left(\quad \pi \quad \pi \quad\right)$

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$\left(6, \frac{\pi}{2}, -6\right)$, cylindrical

$\rho = \sqrt{6^2 + (-6)^2} = \sqrt{72} = 6\sqrt{2}$

$\theta = \frac{\pi}{2}$

$\varphi = \arccos\frac{z}{\sqrt{r^2 + z^2}} = \arccos\left(\frac{-6}{6\sqrt{2}}\right) = \arccos\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4}$

$\left(6\sqrt{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right)$, spherical

62. $\left(-4, \frac{\pi}{3}, 4\right)$, cylindrical

$\rho = \sqrt{(-4)^2 + 4^2} = 4\sqrt{2}$

$\theta = \frac{\pi}{3}$

$\varphi = \arccos\frac{z}{\sqrt{r^2 + z^2}} = \arccos\frac{1}{\sqrt{2}} = \frac{\pi}{4}$

$\left(4\sqrt{2}, \frac{\pi}{3}, \frac{\pi}{4}\right)$, spherical

$(12, \pi, 5)$, cylindrical

$= \sqrt{12^2 + 5^2} = 13$

$\theta = \pi$

$\varphi = \arccos\frac{z}{\sqrt{r^2 + z^2}} = \arccos\frac{5}{13}$

$\left(13, \pi, \arccos\frac{5}{13}\right)$, spherical

$\left(4, \frac{\pi}{2}, 3\right)$, cylindrical

$\rho = \sqrt{4^2 + 3^2} = 5$

$= \frac{\pi}{2}$

$\varphi = \arccos\frac{z}{\sqrt{r^2 + z^2}} = \arccos\frac{3}{5}$

π $3)$

$\left(4, \frac{\pi}{18}, \frac{\pi}{2}\right)$, spherical

$= 4 \sin\frac{\pi}{2} = 4$

$= \frac{\pi}{18}$

$= 4 \cos\frac{\pi}{2} = 0$

$\left(4, \frac{\pi}{18}, 0\right)$, cylindrical

$\left(6, -\frac{\pi}{6}, \frac{\pi}{3}\right)$, spherical

$r = 6 \sin\frac{\pi}{3} = 3\sqrt{3}$

$= -\frac{\pi}{6}$

$z = 6 \cos\frac{\pi}{3} = 3$

$\left(3\sqrt{3}, -\frac{\pi}{6}, 3\right)$, cylindrical

68. $\left(5, -\frac{5\pi}{6}, \pi\right)$, spherical

$= 5 \sin \pi = 0$

$= -\frac{5\pi}{6}$

$= 5 \cos \pi = -5$

$\left(0, -\frac{5\pi}{6}, -5\right)$, cylindrical

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$\left(5, 2, \arccos 5\right)$, spherical

$\left(10, \frac{\pi}{6}, \frac{\pi}{2}\right)$, spherical

$r = 10 \sin \frac{\pi}{2} = 10$

$\theta = \frac{\pi}{6}$

$z = 10 \cos \frac{\pi}{2} = 0$

$\left(10, \frac{\pi}{6}, 0\right)$, cylindrical

$\left(8, \frac{7\pi}{6}, \frac{\pi}{6}\right)$, spherical

$r = 8 \sin \frac{\pi}{6} = 4$

$\theta = \frac{7\pi}{6}$

$z = 8 \cos \frac{\pi}{6} = \frac{8\sqrt{3}}{2}$

$\left(4, \frac{7\pi}{6}, 4\sqrt{3}\right)$, cylindrical

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70. $\left(7, \frac{\pi}{4}, \frac{3\pi}{4}\right)$, spherical

$r = 7\sin\frac{3\pi}{4} = \frac{7\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}$

$z = 7\cos\frac{3\pi}{4} = -\frac{7\sqrt{2}}{2}$

$\left(\frac{7\sqrt{2}}{2}, \frac{\pi}{4}, -\frac{7\sqrt{2}}{2}\right)$, cylindrical

71. $r = 5$

Cylinder

Matches graph (d)

72. $\theta = \frac{\pi}{4}$

Plane

Matches graph (e)

73. $\rho = 5$

Sphere

Matches graph (c)

74. $\varphi = \frac{\pi}{4}$

Cone

Matches graph (a)

75. $r^2 = z, x^2 + y^2 = z$

Paraboloid

Matches graph (f)

76. $\rho = 4\sec\varphi, z = \rho\cos\varphi = 4$

Plane

Matches graph (b)

77. $\theta = c$ is a half-plane because of the restriction $r \geq 0$.

78. (a) The surface is a cone. The equation is (i)

$$x^2 + y^2 = \frac{4}{9}z^2$$

In cylindrical coordinates, the equation is

(b) The surface is a hyperboloid of one sheet. The equation is (ii) $x^2 + y^2 - z^2 = 2.$

In cylindrical coordinates, the equation is

$$x^2 + y^2 - z^2 = 2$$
$$r^2 - z^2 = 2$$
$$r^2 = z^2 + 2.$$

79. $x^2 + y^2 + z^2 = 27$

(a) $r^2 + z^2 = 27$

(b) $\rho^2 = 27 \Rightarrow \rho = 3\sqrt{3}$

80. $4\left(x^2 + y^2\right) = z^2$

(a) $4r^2 = z^2 \Rightarrow 2r = z$

(b) $4\left(\rho^2\sin^2\varphi\cos^2\theta + \rho^2\sin^2\varphi\sin^2\theta\right) = \rho^2\cos^2\varphi$

$$4\sin^2\varphi = \cos^2\varphi,$$
$$\tan^2\varphi = \frac{1}{4},$$
$$\tan\varphi = \frac{1}{2} \Rightarrow \varphi = \arctan\frac{1}{2}$$

81. $x^2 + y^2 + z^2 - 2z = 0$

(a) $r^2 + z^2 - 2z = 0 \Rightarrow r^2 + (z - 1)^2 = 1$

(b) $\rho^2 - 2\rho\cos\varphi = 0$

$$\rho(\rho - 2\cos\varphi) = 0$$
$$\rho = 2\cos\varphi$$

82. $x^2 + y^2 = z$

(a) $r^2 = z$

(b) $\rho^2\sin^2\varphi = \rho\cos\varphi$

$$\rho\sin^2\varphi = \cos\varphi$$
$$\rho = \frac{\cos\varphi}{\sin^2\varphi}$$
$$\rho = \csc\varphi\cot\varphi$$

83. $x^2 + y^2 = 4y$

(a) $r^2 = 4r\sin\theta, r = 4\sin\theta$

(b) $\rho^2\sin^2\varphi = 4\rho\sin\varphi\sin\theta$

$$x^2 + y^2 = \frac{4}{9}z^2$$

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$$r^2 = \frac{4}{9}z^2$$

$$r = \frac{2}{3}z.$$

$$\rho \sin\varphi(\rho \sin\varphi - 4\sin\theta) = 0$$

$$\rho = \frac{4\sin\theta}{\sin\varphi}$$

$$\rho = 4\sin\theta \csc\varphi$$

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$x^2 + y^2 = 45$

(a) $r^2 = 45$ or $r = 3\sqrt{5}$

(b) $\rho^2 \sin^2\varphi \cos^2\theta + \rho^2 \sin^2\varphi \sin^2\theta = 45$

$\rho^2 \sin^2\varphi = 45$

$\rho = 3\sqrt{5} \csc\varphi$

$x^2 - y^2 = 9$

(a) $r^2 \cos^2\theta - r^2 \sin^2\theta = 9$

$$r^2 = \frac{9}{\cos^2\theta - \sin^2\theta}$$

(b) $\rho^2 \sin^2\varphi \cos^2\theta - \rho^2 \sin^2\varphi \sin^2\theta = 9$

$$\rho^2 \sin^2\varphi = \frac{9}{\cos^2\theta - \sin^2\theta}$$

$$\rho^2 = \frac{9 \csc^2\varphi}{\cos^2\theta - \sin^2\theta}$$

86. $y = 4$

(a) $r \sin\theta = 4 \Rightarrow r = 4 \csc\theta$

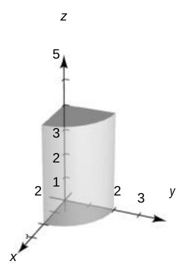
(b) $\rho \sin\varphi \sin\theta = 4,$

$\rho = 4 \csc\varphi \csc\theta$

87. $0 \le \theta \le \frac{\pi}{2}$

$0 \le r \le 2$

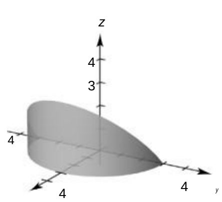
$0 \le z \le 4$



$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ 0

$\le r \le 3$

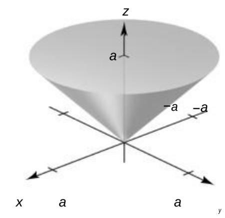
$0 \le z \le r\cos\theta$



$0 \le \theta \le 2\pi$

$0 \le r \le a$ r

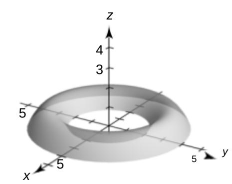
$\le z \le a$



$0 \le \theta \le 2\pi$

$2 \le r \le 4$

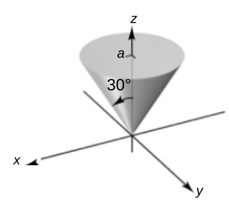
$z^2 \le -r^2 + 6r - 8$



$0 \le \theta \le 2\pi$

$0 \le \varphi \le \frac{\pi}{6}$

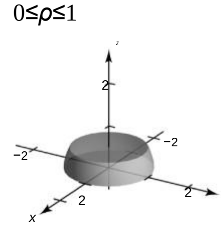
$0 \le \rho \le a \sec\varphi$



$0 \le \theta \le 2\pi$

$\frac{\pi}{4} \le \varphi \le \frac{\pi}{2}$

$0 \le \rho \le 1$



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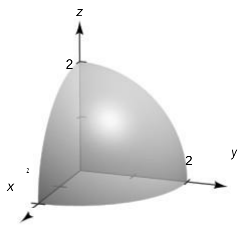
$0 \le \theta \le \frac{\pi}{2}$

$0 \le \varphi \le \frac{\pi}{2}$

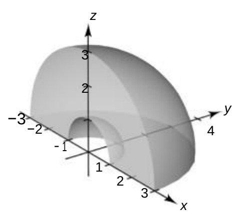
$0 \le \rho \le 2$



$0 \le \theta \le \pi$

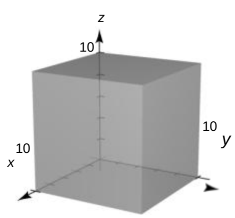
$0 \le \varphi \le \frac{\pi}{2}$

$1 \le \rho \le 3$



Rectangular

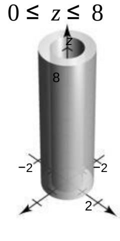
$0 \le x \le 10$ $0 \le y \le 10$ $0 \le z \le 10$



Cylindrical:

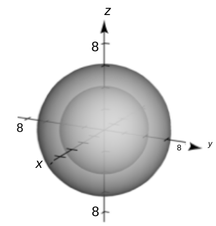
$0.75 \le r \le 1.25$

$0 \le z \le 8$



Spherical

$4 \delta \rho \delta 6$

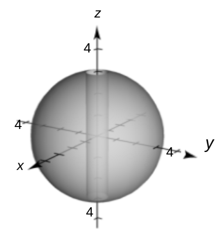


Cylindrical

$\frac{1}{2} \le r \le 3$

$0 \le \theta \le 2\pi$

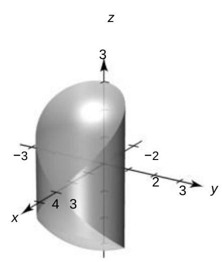
$-\sqrt{9 - r^2} \le z \le \sqrt{9 - r^2}$



Cylindrical coordinates:

$r^2 + z^2 \le 9,$

$r \le 3\cos\theta, 0 \le \theta \le \pi$

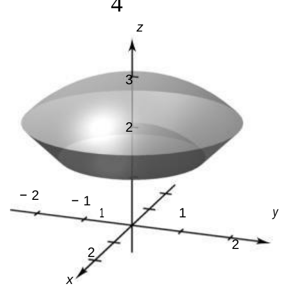


Spherical coordinates:

≥ 2

≤ 3

$0 \le \varphi \le \frac{\pi}{4}$



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False. $(r, \theta, z) = (0, 0, 1)$ and $(r, \theta, z) = (0, \pi, 1)$ represent the same point $(x, y, z) = (0, 0, 1)$.

True (except for the origin).

103. $z = \sin\theta,\ r = 1$

$z = \sin\theta = \frac{y}{r} = \frac{y}{1} = y$

The curve of intersection is the ellipse formed by the intersection of the plane $z = y$ and the cylinder $r = 1$.

104. $\rho = 2\sec\varphi \Rightarrow \rho\cos\varphi = 2 \Rightarrow z = 2$ plane

$= 4$ sphere

The intersection of the plane and the sphere is a circle.

Review Exercises for Chapter 11

$P = (1, 2),\ Q = (4, 1),\ R = (5, 4)$

(a) $\mathbf{u} = PQ = \langle 4 - 1, 1 - 2\rangle = \langle 3, -1\rangle$

$\mathbf{v} = PR = \langle 5 - 1, 4 - 2\rangle = \langle 4, 2\rangle$

(c) $\|\mathbf{u}\| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$ $\quad \|\mathbf{v}\| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$

$-3\mathbf{u} + \mathbf{v} = -3\langle 3, -1\rangle + \langle 4, 2\rangle = \langle -5, 5\rangle$

$P = (-2, -1),\ Q = (5, -1),\ R = (2, 4)$

(a) $\mathbf{u} = PQ = \langle 5 - (-2), -1 - (-1)\rangle = \langle 7, 0\rangle$

$\mathbf{v} = PR = \langle 2 - (-2), 4 - (-1)\rangle = \langle 4, 5\rangle$

$\mathbf{u} = 7\mathbf{i},\ \mathbf{v} = 4\mathbf{i} + 5\mathbf{j}$

(c) $\|\mathbf{u}\| = \sqrt{7^2 + 0^2} = \sqrt{49} = 7$ $\quad \|\mathbf{v}\| = \sqrt{4^2 + 5^2} = \sqrt{41}$

$-3\mathbf{u} + \mathbf{v} = -3\langle 7, 0\rangle + \langle 4, 5\rangle = \langle -17, 5\rangle$

$\mathbf{v} = \|\mathbf{v}\|(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$

$8(\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j})$

$= 8\left(\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right) = 4\mathbf{i} + 4\sqrt{3}\mathbf{j} = \langle 4, 4\sqrt{3}\rangle$

$\mathbf{v} = \|\mathbf{v}\|\cos\theta\,\mathbf{i} + \|\mathbf{v}\|\sin\theta\,\mathbf{j}$

$\frac{1}{2}\cos 225°\mathbf{i} + \frac{1}{2}\sin 225°\mathbf{j}$

$= -\frac{\sqrt{2}}{4}\mathbf{i} - \frac{\sqrt{2}}{4}\mathbf{j} = \left\langle -\frac{\sqrt{2}}{4}, -\frac{\sqrt{2}}{4}\right\rangle$

7. $d = \sqrt{(-2 - 1)^2 + (3 - 6)^2 + (5 - 3)^2}$

$= \sqrt{9 + 9 + 4} = \sqrt{22}$

8. $d = \sqrt{(4 - (-2))^2 + (-1 - 1)^2 + (-1 - (-5))^2}$

$= \sqrt{36 + 4 + 16} = \sqrt{56} = 2\sqrt{14}$

9. $(x - 3)^2 + (y + 2)^2 + (z - 6)^2 = 4^2$

$(x - 3)^2 + (y + 2)^2 + (z - 6)^2 = 16$

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5. $z = 0, y = 4, x = -5: (-5, 4, 0)$

$y = 3$ describes a plane parallel to the xz-plane and passing through $(0, 3, 0)$.

10. Center: $\left(\frac{0+4}{2}, \frac{0+6}{2}, \frac{4+0}{2}\right) = (2, 3, 2)$

Radius:

$\sqrt{(2-0)^2 + (3-0)^2 + (2-4)^2} = \sqrt{4+9+4} = \sqrt{17}$

$(x-2)^2 + (y-3)^2 + (z-2)^2 = 17$

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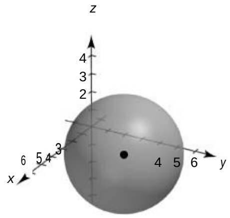
$\left(x^2 - 4x + 4\right) + \left(y^2 - 6y + 9\right) + z^2 = -4 + 4 + 9$

$x - 2)^2 + (y - 3)^3 + z^2 = 9$

Center: $(2, 3, 0)$

Radius: 3



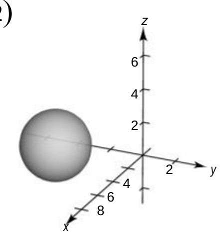
$\left(x^2 - 10x + 25\right) + \left(y^2 + 6y + 9\right)$

$\left(z^2 - 4z + 4\right) = -34 + 25 + 9 + 4$

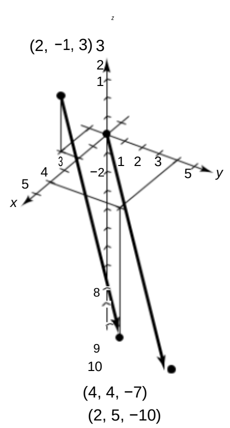
$x - 5)^2 + (y + 3)^2 + (z - 2)^2 = 4$

Center: $(5, -3, 2)$

Radius: 2



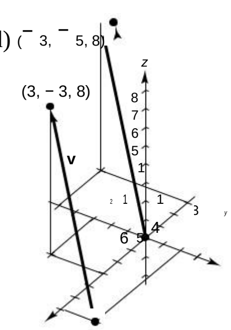
13. (a), (d)



$\mathbf{v} = \langle 4 - 2, 4 - (-1), -7 - 3\rangle = \langle 2, 5, -10\rangle$

$\mathbf{v} = 2\mathbf{i} + 5\mathbf{j} - 10\mathbf{k}$

(a), (d)



$z = -\mathbf{u} + 3\mathbf{v} + \frac{1}{2}\mathbf{w}$

$-\langle 5, -2, 3\rangle + 3\langle 0, 2, 1\rangle + \frac{1}{2}\langle -6, -6, 2\rangle$

$\langle -5, 2, -3\rangle + \langle 0, 6, 3\rangle + \langle -3, -3, 1$

$\langle -8, 5, 1\rangle$

$\mathbf{u} - \mathbf{v} + \mathbf{w} - 2\mathbf{z} = 0$

$= \frac{1}{2}(\mathbf{u} - \mathbf{v} + \mathbf{w})$

$\frac{1}{2}(\langle 5, -2, 3\rangle - \langle 0, 2, 1\rangle + \langle -6, -6, 2\rangle)$

$\frac{1}{2}\langle -1, -10, 4\rangle$

$\left\langle -\frac{1}{2}, -5, 2\right\rangle$

$\mathbf{v} = \langle -1 - 3, 6 - 4, 9 + 1\rangle = \langle -4, 2, 10\rangle$ $\mathbf{w} = \langle 5 - 3, 3 - 4, -6 + 1\rangle = \langle 2, -1, -5\rangle$ Because $-2\mathbf{w} = \mathbf{v}$, the points lie in a straight line.

$\mathbf{v} = \langle 8 - 5, -5 + 4, 5 - 7\rangle = \langle 3, -1, -2\rangle$ $\mathbf{w} = \langle 11 - 5, 6 + 4, 3 - 7\rangle = \langle 6, 10, -4\rangle$

Because **v** and **w** are not parallel, the points do not lie in a straight line.

19. Unit vector: $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \left\langle \frac{2, 3, 5}{\sqrt{38}}\right\rangle = \left\langle \frac{2}{\sqrt{38}}, \frac{3}{\sqrt{38}}, \frac{5}{\sqrt{38}}\right\rangle$

20. $8\frac{\langle 6, -3, 2\rangle}{\sqrt{49}} = \frac{8}{7}\langle 6, -3, 2\rangle = \left\langle \frac{48}{7}, -\frac{24}{7}, \frac{16}{7}\right\rangle$

$P = \langle 5, 0, 0\rangle, Q = \langle 4, 4, 0\rangle, R = \langle 2, 0, 6\rangle$

(a) $\mathbf{u} = PQ = \langle -1, 4, 0\rangle$

$\mathbf{v} = PR = \langle -3, 0, 6\rangle$

$\mathbf{u} \cdot \mathbf{v} = (-1)(-3) + 4(0) + 0(6) = 3$

$\mathbf{v} \cdot \mathbf{v} = 9 + 36 = 45$

$P = \langle 2, -1, 3\rangle, Q = \langle 0, 5, 1\rangle, R = \langle 5, 5, 0\rangle$

(a) $\mathbf{u} = PQ = \langle -2, 6, -2\rangle$

$\langle$

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(6, 2, 0)

$\mathbf{v} = \overrightarrow{PR} = \langle 3, 6, -3 \rangle$

$\mathbf{u} \cdot \mathbf{v} = (-2)(3) + (6)(6) + (-2)(-3) = 36$

$\mathbf{v} \cdot \mathbf{v} = 9 + 36 + 9 = 54$

$\mathbf{v} = \langle 3 - 6, -3 - 2, 8 - 0 \rangle = \langle -3, -5, 8 \rangle$

$\mathbf{v} = -3\mathbf{i} - 5\mathbf{j} + 8\mathbf{k}$

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23. $\mathbf{u} = 5\left(\cos\frac{3\pi}{4}\mathbf{i} + \sin\frac{3\pi}{4}\mathbf{j}\right) = \frac{5\sqrt{2}}{2}(-\mathbf{i} + \mathbf{j})$

$\mathbf{v} = 2\left(\cos\frac{2\pi}{3}\mathbf{i} + \sin\frac{2\pi}{3}\mathbf{j}\right) = -\mathbf{i} + \sqrt{3}\mathbf{j}$

$\mathbf{u}\cdot\mathbf{v} = \frac{5\sqrt{2}}{2}\left(1 + \sqrt{3}\right)$

$\|\mathbf{u}\| = \sqrt{\frac{25}{2} + \frac{25}{2}} = 5 \qquad \|\mathbf{v}\| = \sqrt{1+3} = 2$

$\cos\theta = \frac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{\left(5\sqrt{2}/2\right)\left(1+\sqrt{3}\right)}{5(2)} = \frac{\sqrt{2}+\sqrt{6}}{4}$

(a) $\theta = \arccos\frac{\sqrt{2}+\sqrt{6}}{4} = \frac{\pi}{12} \approx 0.262$

$\theta \approx 15°$

$\mathbf{u} = \langle 1, 0, -3\rangle$ $\mathbf{v} = \langle 2, -2, 1\rangle$

$\|\mathbf{u}\| = \sqrt{1+9} = \sqrt{10}$

$\|\mathbf{v}\| = \sqrt{4+4+1} = 3$

$\cos\theta = \frac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{1}{3\sqrt{10}}$

(a) $\theta = \arccos\left(\frac{1}{3\sqrt{10}}\right) \approx 1.465$

$\theta = 83.9°$

$\mathbf{u} = \langle 7, -2, 3\rangle$, $\mathbf{v} = \langle -1, 4, 5\rangle$

Because $\mathbf{u}\cdot\mathbf{v} = 0$, the vectors are orthogonal.

$\mathbf{u} = \langle -3, 0, 9\rangle = -3\langle 1, 0, -3\rangle = -3\mathbf{v}$

The vectors are parallel.

$\mathbf{u} = \langle 4, 2\rangle$, $\mathbf{v} = \langle 3, 4\rangle$

(a) $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$

$= \left(\frac{20}{}\right)$

28. $\mathbf{u} = \langle 1, -1, 1\rangle$, $\mathbf{v} = \langle 2, 0, 2\rangle$

(a) $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$

$= \frac{4}{8}\langle 2, 0, 2\rangle = \langle 1, 0, 1\rangle$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 1, -1, 1\rangle - \langle 1, 0, 1\rangle = \langle 0, -1, 0\rangle$

There are many correct answers.

For example: $\mathbf{v} = \pm\langle 6, -5, 0\rangle$.

30. $W = \mathbf{F}\cdot\overrightarrow{PQ} = \|\mathbf{F}\|\|\overrightarrow{PQ}\|\cos\theta = (75)(8)\cos 30°$

$= 300\sqrt{3}$ ft-lb

31. (a) $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 6 \\ 5 & 2 & 1 \end{vmatrix} = -9\mathbf{i} + 26\mathbf{j} - 7\mathbf{k}$

$\mathbf{v}\times\mathbf{v} = \mathbf{0}$

32. (a) $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 1 \\ 1 & -3 & 4 \end{vmatrix} = 11\mathbf{i} + \mathbf{j} - 2\mathbf{k}$

$\mathbf{v}\times\mathbf{u} = -(\mathbf{u}\times\mathbf{v}) = -11\mathbf{i} - \mathbf{j} + 2\mathbf{k}$

$\mathbf{v}\times\mathbf{v} = \mathbf{0}$

33. $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -10 & 8 \\ 4 & 6 & -8 \end{vmatrix} = 32\mathbf{i} + 48\mathbf{j} + 52\mathbf{k}$

$\|\mathbf{u}\times\mathbf{v}\| = \sqrt{6032} = 4\sqrt{377}$

Unit vector: $\frac{1}{\sqrt{377}}\langle 8, 12, 13\rangle$

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34. $\mathbf{u} = \langle 3, -1, 5 \rangle$, $\mathbf{v} = \langle 2, -4, 1 \rangle$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 5 \\ 2 & -4 & 1 \end{vmatrix} = 19\mathbf{i} + 7\mathbf{j} - 10\mathbf{k}$$

$$\|\mathbf{u} \times \mathbf{v}\| = \sqrt{19^2 + 7^2 + (-10)^2} = 5\sqrt{10}$$

$$\left(\frac{4}{25}\right)\langle 3, 4 \rangle$$

$$\frac{4}{5}\langle 3, 4 \rangle = \left\langle \frac{12}{5}, \frac{16}{5} \right\rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 4, 2 \rangle - \left\langle \frac{12}{5}, \frac{16}{5} \right\rangle = \left\langle \frac{8}{5}, -\frac{6}{5} \right\rangle$$

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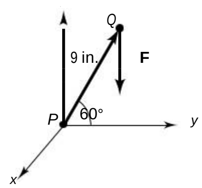
35. $\mathbf{F} = -40\mathbf{k}$ $\left(9 \text{ in.} = \frac{3}{4} \text{ ft}\right)$

$PQ = \frac{3}{4}\left(\cos 60° \mathbf{j} + \sin 60° \mathbf{k}\right)$ $\frac{3}{8}\mathbf{j} + \frac{3\sqrt{3}}{8}\mathbf{k}$

The moment of **F** about *P* is

$$M = PQ \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \frac{3}{8} & \frac{3\sqrt{3}}{8} \\ 0 & 0 & -40 \end{vmatrix} -15\mathbf{i}$$

Torque = 15 ft-lb



36. $V = \left|\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})\right| = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \end{vmatrix} = 2(5) = 10$

$\mathbf{v} = \langle 9 - 3, 11 - 0, 6 - 2\rangle = \langle 6, 11, 4\rangle$

Parametric equations:

$= 3 + 6t,\ y = 11t,\ z = 2 + 4t$

(b) Symmetric equations: $\frac{x-3}{6} = \frac{y}{11} = \frac{z-2}{4}$

$\mathbf{v} = \langle 8 + 1, 10 - 4, 5 - 3\rangle = \langle 9, 6, 2\rangle$

Parametric equations:

$= -1 + 9t,\ y = 4 + 6t,\ z = 3 + 2t$

(b) Symmetric equations: $\frac{x+1}{9} = \frac{y-4}{6} = \frac{z-3}{2}$

39. $P = (-6, -8, 2)$

$= \mathbf{j} = \langle 0, 1, 0\rangle$

$= -6,\ y = -8 + t,\ z = 2$

Direction numbers: 1, 1, 1, $\mathbf{v} = \langle 1, 1, 1\rangle$ $P(1, 2, 3)$

$= 1 + t,\ y = 2 + t,\ z = 3 + t$

$P = (-3, -4, 2),\ Q = (-3, 4, 1),\ R = (1, 1, -2)$

$PQ = \langle 0, 8, -1\rangle,\ PR = [4, 5, -4]$

$$\mathbf{n} = PQ \times PR = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -1 \\ 4 & 5 & -4 \end{vmatrix} = -27\mathbf{i} - 4\mathbf{j} - 32\mathbf{k}$$

$-27(x + 3) - 4(y + 4) - 32(z - 2) = 0$

$27x + 4y + 32z = -33$

42. $\mathbf{n} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$

$3(x + 2) - 1(y - 3) + 1(z - 1) = 0$

$3x - y + z + 8 = 0$

43. The two lines are parallel as they have the same direction numbers, −2, 1, 1. Therefore, a vector parallel to the plane is $\mathbf{v} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$. A point on the first line is $(1, 0, -1)$ and a point on the second line is $(-1, 1, 2)$. The vector $\mathbf{u} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ connecting these two points is

also parallel to the plane. Therefore, a normal to the plane is

$$\mathbf{v} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ 2 & -1 & -3 \end{vmatrix}$$

$= -2\mathbf{i} - 4\mathbf{j} = -2(\mathbf{i} + 2\mathbf{j}).$

Equation of the plane: $(x - 1) + 2y = 0$

$x + 2y = 1$

Let $\mathbf{v} = \langle 5 - 2, 1 + 2, 3 - 1\rangle = \langle 3, 3, 2\rangle$ be the direction vector for the line through the two points. Let $\mathbf{n} = \langle 2, 1, -1\rangle$ be the normal vector to the plane. Then

$$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & 2 \\ 2 & 1 & -1 \end{vmatrix} = \langle -5, 7, -3\rangle$$

is the normal to the unknown plane.

$-5(x - 5) + 7(y - 1) - 3(z - 3) =$

$0\ -5x + 7y - 3z + 27 = 0$

$Q(1, 0, 2)$ point

$2x - 3y + 6z = 6$

A point *P* on the plane is $(3, 0, 0)$.

$PQ = \langle -2, 0, 2\rangle$

$= 2\langle -3, 6\rangle$ normal to plane

$$D = \frac{\left|\overrightarrow{PQ} \cdot \mathbf{n}\right|}{\left\|\mathbf{n}\right\|} = \frac{8}{7}$$

$Q(3, -2, 4)$ point

$2x - 5y + z = 10$

A point P on the plane is $(5, 0, 0)$.

$\overrightarrow{PQ} = \langle -2, -2, 4 \rangle$

$\mathbf{n} = \langle 2, -5, 1 \rangle$ normal to plane

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{10}{\sqrt{30}} = \frac{\sqrt{30}}{3}$$

The normal vectors to the planes are the same, $\mathbf{n} = (5, -3, 1)$.

Choose a point in the first plane $P(0, 0, 2)$. Choose a point in the second plane, $Q(0, 0, -3)$.

$\overrightarrow{PQ} = \langle 0, 0, -5 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-5|}{\sqrt{35}} = \frac{5}{\sqrt{35}} = \frac{\sqrt{35}}{7}$$

$Q(-5, 1, 3)$ point

$\mathbf{u} = \langle 1, -2, -1 \rangle$ direction vector

$P(1, 3, 5)$ point on line

$\overrightarrow{PQ} = -6, -2, -2$

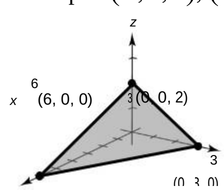
$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & -2 & -2 \\ 1 & -2 & -1 \end{vmatrix} = \langle -2, -8, 14 \rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{264}}{\sqrt{6}} = 2\sqrt{11}$$

$x + 2y + 3z = 6$

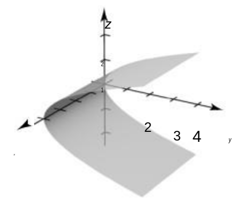
Plane

Intercepts: $(6, 0, 0)$, $(0, 3, 0)$, $(0, 0, 2)$,



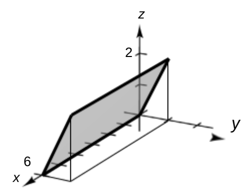
$y = z^2$

Because the x-coordinate is missing, you have a cylindrical surface with rulings parallel to the x-axis. The generating curve is a parabola in the yz-coordinate plane.



$y = \frac{1}{2}z$

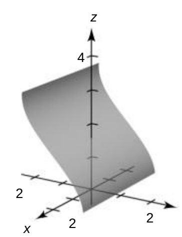
Plane with rulings parallel to the x-axis.



$y = \cos z$

Because the x-coordinate is missing, you have a

The generating curve is $y = \cos z$.



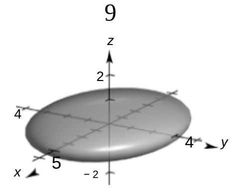
$$\frac{x^2}{16} + \frac{y^2}{9} + z^2 = 1$$

Ellipsoid

xy-trace: $\frac{x^2}{16} + \frac{y^2}{9} = 1$

xz-trace: $\frac{x^2}{16} + z^2 = 1$

yz-trace: $\frac{y^2}{9} + z^2 = 1$



$16x^2 + 16y^2 - 9z^2 = 0$

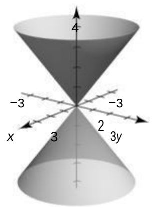
Cone

xy-trace: point $(0, 0, 0)$

xz-trace: $z = \pm \frac{4}{3}x$

yz-trace: $z = \pm \frac{4}{3}y$

$z = 4, x^2 + y^2 = 9$



$\frac{x^2}{16} - \frac{y^2}{9} + z^2 = -1$

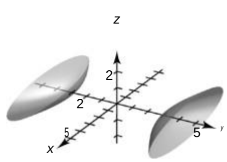
$\frac{y^2}{9} - \frac{x^2}{16} - z^2 = 1$

Hyperboloid of two sheets

xy-trace: $\frac{y^2}{9} - \frac{x^2}{16} = 1$

xz-trace: None

yz-trace: $\frac{y^2}{9} - z^2 = 1$



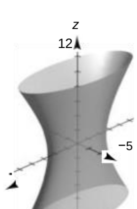
$\frac{x^2}{25} + \frac{y^2}{4} - \frac{z^2}{100} = 1$

Hyperboloid of one sheet

xy-trace: $\frac{x^2}{25} + \frac{y^2}{4} = 1$

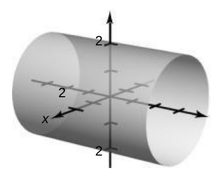
xz-trace: $\frac{x^2}{25} - \frac{z^2}{100} = 1$

yz-trace: $\frac{y^2}{4} - \frac{z^2}{100} = 1$



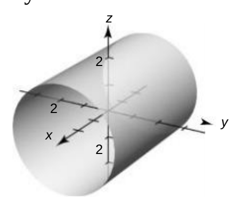
$x^2 + z^2 = 4.$

Cylinder of radius 2 about y-axis



$y^2 + z^2 = 16.$

Cylinder of radius 4 about x-axis



59. $z^2 = 2y$ revolved about y-axis

$z = \pm\sqrt{2y}$

$x^2 + z^2 = [r(y)]^2 = 2y$

$x^2 + z^2 = 2y$

60. $2x + 3z = 1$ revolved about the x-axis

$z = \frac{1 - 2x}{3}$

$y^2 + z^2 = [r(x)]^2 = \left(\frac{1 - 2x}{3}\right)^2$, Cone

61. $(-\sqrt{3}, 3, -5)$, rectangular

(a) $r = \sqrt{(-\sqrt{3})^2 + 3^2} = \sqrt{12} = 2\sqrt{3}$

$\tan\theta = \frac{3}{-\sqrt{3}} \Rightarrow \theta = -\frac{\pi}{3}$

$z = -5$

$\left(2\sqrt{3}, -\frac{\pi}{3}, -5\right)$, cylindrical

(b) $\rho = \sqrt{(-\sqrt{3})^2 + 3^2 + (-5)^2} = \sqrt{37}$

$\tan\theta = -\frac{3}{\sqrt{3}} \Rightarrow \theta = -\frac{\pi}{3}$

$\varphi = \arccos\frac{z}{\rho} = \arccos\left(\frac{-5}{\sqrt{37}}\right)$

$\left(\sqrt{37}, -\frac{\pi}{3}, \arccos\left(-\frac{5\sqrt{37}}{37}\right)\right)$, spherical

x 5 y

3 37

62. $(8, 8, 1)$, rectangular

(a) $r = \sqrt{8^2 + 8^2} = 8\sqrt{2}$

$\tan\theta = \frac{8}{8} = 1 \Rightarrow \theta = \frac{\pi}{4}$

$z = 1$

$\left(8\sqrt{2}, \frac{\pi}{4}, 1\right)$, cylindrical

(b) $\rho = \sqrt{8^2 + 8^2 + 1^2} = \sqrt{129}$

$\tan\theta = \frac{8}{8} = 1 \Rightarrow \theta = \frac{\pi}{4}$

$\varphi = \arccos\frac{z}{\rho} = \arccos\frac{1}{\sqrt{129}}$

$\left(\sqrt{129}, \frac{\pi}{4}, \arccos\frac{\sqrt{129}}{129}\right)$, spherical

63. $(5, \pi, 1)$, cylindrical

$x = r\cos\theta = 5\cos\pi = -5$

$y = r\sin\theta = 5\sin\pi = 0$

$= 1$

$(-5, 0, 1)$, rectangular

$\left(8, -\frac{\pi}{6}, \frac{\pi}{3}\right)$, spherical

$8\sin\frac{\pi}{3}\cos\left(-\frac{\pi}{6}\right) = 8\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = 6$

$y = \rho\sin\varphi\sin\theta = 8\sin\frac{\pi}{3}\sin\left(-\frac{\pi}{6}\right) = 8\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) = -2\sqrt{3}$

$= \rho\cos\varphi = 8\cos\frac{\pi}{3} = 4$

$(6, -2\sqrt{3}, 4)$, rectangular

64. $\left(-2, \frac{\pi}{3}, 3\right)$, cylindrical

$x = r\cos\theta = -2\cos\frac{\pi}{3} = -1$

$y = r\sin\theta = -2\sin\frac{\pi}{3} = -\sqrt{3}$

$z = 3$

$(-1, -\sqrt{3}, 3)$, rectangular

65. $\left(4, \pi, \frac{\pi}{4}\right)$, spherical

$x = \rho\sin\varphi\cos\theta = 4\sin\frac{\pi}{4}\cos\pi = -2\sqrt{2}$

$y = \rho\sin\varphi\sin\theta = 4\sin\frac{\pi}{4}\sin\pi = 0$

$z = \rho\cos\varphi = 4\cos\frac{\pi}{4} = 2\sqrt{2}$

$(-2\sqrt{2}, 0, 2\sqrt{2})$, rectangular

67. $x^2 - y^2 = 2z$

(a) Cylindrical:

$r^2\cos^2\theta - r^2\sin^2\theta = 2z \Rightarrow r^2\cos 2\theta = 2z$

(b) Spherical:

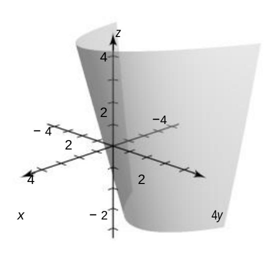
$\rho^2\sin^2\varphi\cos^2\theta - \rho^2\sin^2\varphi\sin^2\theta = 2\rho\cos\varphi$

$\rho\sin^2\varphi\cos 2\theta - 2\cos\varphi = 0$

$\rho = 2\sec 2\theta\cos\varphi\csc^2\varphi$

69. $z = r^2\sin^2\theta + 3r\cos\theta$, cylindrical equation

$z = y^2 + 3x$, rectangular equation



68. $x^2 + y^2 + z^2 = 16$

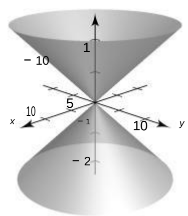
(a) Cylindrical: $r^2 + z^2 = 16$

(b) Spherical: $\rho = 4$

70. $r = -5z$, cylindrical equation

$$\sqrt{x^2 + y^2} = -5z$$

$$x^2 + y^2 - 25z^2 = 0, \text{ rectangular equation}$$



71. $\varphi = \frac{\pi}{4}$, spherical equation

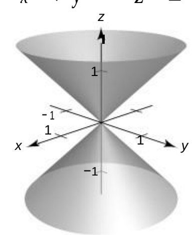
$$\varphi = \arccos \frac{z}{\sqrt{x^2 + y^2 + z^2}} = \frac{\pi}{4}$$

$$\frac{z}{\sqrt{x^2 + y^2 + z^2}} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$z^2 = \frac{1}{2}(x^2 + y^2 + z^2)$$

$$2z^2 = x^2 + y^2 + z^2$$

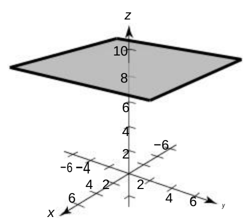
$$x^2 + y^2 - z^2 = 0, \text{ rectangular equation}$$



72. $\rho = 9 \sec\theta$, spherical equation

$$\rho\cos\theta = 9$$

$$z = 9, \text{ rectangular equation}$$



Problem Solving for Chapter 11

1. $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$, $\mathbf{b} \times (\mathbf{a} + \mathbf{b} + \mathbf{c}) = \mathbf{0}$

$$(\mathbf{b} \times \mathbf{a}) + (\mathbf{b} \times \mathbf{c}) = \mathbf{0}$$

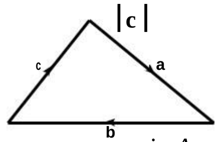
$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{b} \times \mathbf{c}\|$$

$$\|\mathbf{b} \times \mathbf{c}\| = \|\mathbf{b}\|\|\mathbf{c}\|\sin A$$

$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\|\sin C$$

Then,

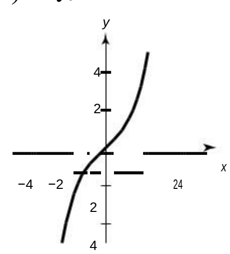
$$\frac{\sin A}{\|\mathbf{a}\|} = \frac{\|\mathbf{b} \times \mathbf{c}\|}{\|\mathbf{a}\|\|\mathbf{b}\|\|\mathbf{c}\|} = \frac{\|\mathbf{a} \times \mathbf{b}\|}{\|\mathbf{a}\|\|\mathbf{b}\|\|\mathbf{c}\|} = \frac{\sin C}{\|\mathbf{c}\|}.$$



The other case, $\frac{\sin A}{\|\mathbf{a}\|} = \frac{\sin B}{\|\mathbf{b}\|}$ is similar.

2. $f(x) = \int_0^x \sqrt{t^4 + 1}\, dt$

(a)



(b) $f'(x) = \sqrt{x^4 + 1}$

$f'(0) = 1 = \tan\theta$

$\theta = \frac{\pi}{4}$

$\mathbf{u} = \frac{\sqrt{2}}{2}(\mathbf{i} + \mathbf{j}) = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

(c) $\pm\left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$

(d) The line is $y = x : x = t, y = t$.

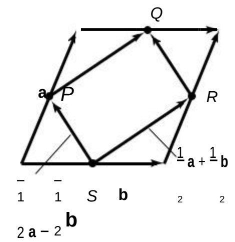
Label the figure as indicated.

From the figure, you see that

$$\overrightarrow{SP} = \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} = \overrightarrow{RQ} \text{ and } \overrightarrow{SR} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} = \overrightarrow{PQ}.$$

Because $\overrightarrow{SP} = \overrightarrow{RQ}$ and $\overrightarrow{SR} = \overrightarrow{PQ}$,

PSRQ is a parallelogram.



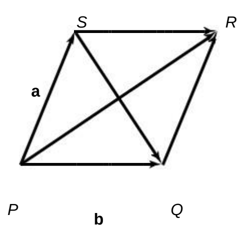
Label the figure as indicated.

$\overrightarrow{PR} = \mathbf{a} + \mathbf{b}$

$\overrightarrow{SQ} = \mathbf{b} - \mathbf{a}$

$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{b} - \mathbf{a}) = \|\mathbf{b}\|^2 - \|\mathbf{a}\|^2 = 0,$

because $\|\mathbf{a}\| = \|\mathbf{b}\|$ in a rhombus.

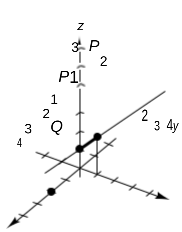


5. (a) $\mathbf{u} = \langle 0, 1, 1\rangle$ is the direction vector of the line determined by P_1 and P_2.

$$D = \frac{\|\overrightarrow{P_1Q} \times \mathbf{u}\|}{\|\mathbf{u}\|}$$

$$= \frac{\|\langle 2, 0, -1\rangle \times \langle 0, 1, 1\rangle\|}{\sqrt{2}}$$

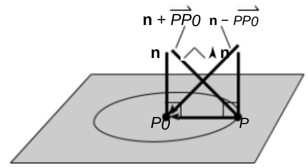
$$= \frac{\|\langle 1, -2, 2\rangle\|}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$



6. $(\mathbf{n} + \overrightarrow{PP_0}) \perp (\mathbf{n} - \overrightarrow{PP_0})$

Figure is a square.

So, $\|\overrightarrow{PP_0}\| = \|\mathbf{n}\|$ and the points P form a circle of radius $\|\mathbf{n}\|$ in the plane with center at P_0.



7. (a) $V = \pi\int_0^1 (\sqrt{z})^2\, dz = \left[\pi \frac{z^2}{2}\right]_0^1 = 2\pi$

Note: $\frac{1}{2}$(base)(altitude) $= \frac{1}{2}\pi(1)^2(1) = \frac{1}{2}\pi$

(b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = z$: (slice at $z = c$)

$$\frac{x^2}{(\sqrt{c}\,a)^2} + \frac{y^2}{(\sqrt{c}\,b)^2} = 1$$

At $z = c$, figure is ellipse of area

$$\pi(\sqrt{c}\,a)(\sqrt{c}\,b) = \pi abc.$$

$$V = \int_0^k \pi abc \cdot dc = \left[\frac{\pi abc^2}{2}\right]_0^k = \frac{\pi abk^2}{2}$$

(c) $V = \frac{1}{2}(\pi abk)k = \frac{1}{2}$ (area of base)(height)

8. (a) $V = 2\int_0^r \pi(r^2 - x^2)\,dx = 2\pi\left[r^2x - \frac{x^3}{3}\right]_0^r = \frac{4}{3}\pi r^3$

(b) At height $z = d > 0$,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{d^2}{c^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{d^2}{c^2} = \frac{c^2 - d^2}{c^2}$$

$$\frac{x^2}{\frac{a^2(c^2 - d^2)}{c^2}} + \frac{y^2}{\frac{b^2(c^2 - d^2)}{c^2}} = 1.$$

$c^2 \; c^2$

$(\quad 2 \quad 2 \quad 2)(\quad 2 \quad\quad 2)$

(b) The shortest distance to the line **segment** is

$$\|P_1Q\| = \|\langle 2,0,-1\rangle\| = \sqrt{5}.$$

$$\text{Area} = \pi\sqrt{\left(\frac{a}{c^2}\left(c^2-d^2\right)\right)\left(\frac{b}{c^2}\left(c^2-d^2\right)\right)} = \frac{\pi ab}{c^2}\left(c^2-d^2\right)$$

$$V = 2\int_0^c \frac{\pi ab}{c^2}\left(c^2-d^2\right)dd = \frac{2\pi ab}{c^2}\left[c^2 d - \frac{d^3}{3}\right]_0^c = \frac{4}{3}\pi abc$$

From Exercise 54, Section 11.4,

$$(\mathbf{u}\times\mathbf{v})\times(\mathbf{w}\times\mathbf{z}) = \left[(\mathbf{u}\times\mathbf{v})\cdot\mathbf{z}\right]\mathbf{w} - \left[(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}\right]\mathbf{z}.$$

10. $x = -t + 3,\ y = \frac{1}{2}t + 1,\ z = 2t - 1;\ Q = (4, 3, s)$

$\mathbf{u} = \langle -2, 1, 4\rangle$ direction vector for line $P =$

$(3, 1, -1)$ point on line

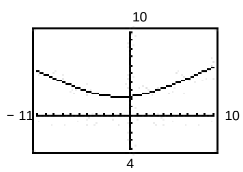
$$PQ = \langle 1, 2, s+1\rangle$$

$$PQ \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & s+1 \\ -2 & 1 & 4 \end{vmatrix}$$

$$= (7 - s)\mathbf{i} + (-6 - 2s)\mathbf{j} + 5\mathbf{k}$$

$$D = \frac{\|PQ \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{(7-s)^2 + (-6-2s)^2 + 25}}{\sqrt{21}}$$

(b)



The minimum is $D \approx 2.2361$ at $s = -1$.

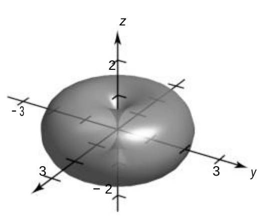
(c) Yes, there are slant asymptotes. Using $s = x$, you have

$$D(s) = \frac{1}{\sqrt{21}}\sqrt{5x^2 + 10x + 110} = \frac{\sqrt{5}}{\sqrt{21}}\sqrt{x^2 + 2x + 22} = \frac{\sqrt{5}}{\sqrt{21}}\sqrt{(x+1)^2 + 21} \to \pm\sqrt{\frac{5}{21}}(x+1)$$

$y = \pm\frac{\sqrt{105}}{21}(s+1)$ slant asymptotes.

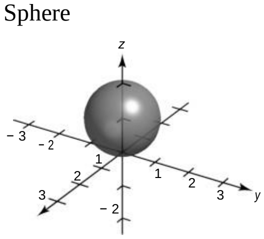
(a) $\rho = 2\sin\varphi$

Torus

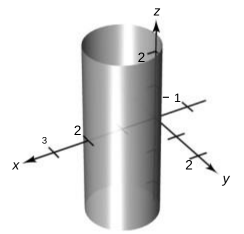


$\rho = 2\cos\varphi$

Sphere

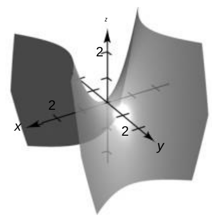


12. (a) $r = 2\cos\theta$ (b) Cylinder



$$z = r^2 \cos 2\theta$$
$$= r^2\left(\cos^2\theta - \sin^2\theta\right)$$
$$= (r\cos\theta)^2 - (r\sin\theta)^2$$
$$= x^2 - y^2$$

Hyperbolic paraboloid



(a) $\mathbf{u} = \|\mathbf{u}\|(\cos 0\,\mathbf{i} + \sin 0\,\mathbf{j}) = \|\mathbf{u}\|\mathbf{i}$

Downward force $\mathbf{w} = -\mathbf{j}$

$= \|\mathbf{T}\|(\cos(90° + \theta)\,\mathbf{i} + \sin(90° + \theta)\,\mathbf{j})$

$\|\mathbf{T}\|(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$

$\mathbf{0} = \mathbf{u} + \mathbf{w} + \mathbf{T} = \|\mathbf{u}\|\mathbf{i} - \mathbf{j} + \|\mathbf{T}\|(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$

$\|\mathbf{u}\| = \sin\theta\|\mathbf{T}\|$

$1 = \cos\theta\|\mathbf{T}\|$

If $\theta = 30°$, $\|\mathbf{u}\| = (1/2)\|\mathbf{T}\|$ and $1 = (\sqrt{3}/2)\|\mathbf{T}\| \Rightarrow \|\mathbf{T}\| = \dfrac{2}{\sqrt{3}} \approx 1.1547$ lb and $\|\mathbf{u}\| = \dfrac{1}{2}\left(\dfrac{2}{\sqrt{3}}\right) \approx 0.5774$ lb

(b) From part (a), $\|\mathbf{u}\| = \tan\theta$ and $\|\mathbf{T}\| = \sec\theta$.

Domain: $0 \le \theta \le 90°$

(c)

θ	0°	10°	20°	30°	40°	50°	60°
T	1	1.0154	1.0642	1.1547	1.3054	1.5557	2
$\|\mathbf{u}\|$	0	0.1763	0.3640	0.5774	0.8391	1.1918	1.7321

(d) 2.5

T

u

0 0 60

(e) Both are increasing functions.

(f) $\lim_{\theta\to\pi/2^-} T = \infty$ and $\lim_{\theta\to\pi/2^-} \|\mathbf{u}\| = \infty$.

Yes. As θ increases, both T and $\|\mathbf{u}\|$ increase.

(a) The tension T is the same in each tow line.

$$6000\,\mathbf{i} = T\left(\cos 20° + \cos(-20)\right)\mathbf{i} + T\left(\sin 20° + \sin(-20°)\right)\mathbf{j}$$
$$2T\cos 20°\mathbf{i}$$

$$\Rightarrow T = \frac{6000}{2\cos 20°} \approx 3192.5 \text{ lb}$$

(b) As in part (a), $6000\,\mathbf{i} = 2T\cos\theta$

$$\Rightarrow T = \frac{3000}{\cos\theta}$$

Domain: $0 < \theta < 90°$

(c)

θ	10°	20°	30°	40°	50°	60°
T	3046.3	3192.5	3464.1	3916.2	4667.2	6000.0

(d) 10,000

0
0 90

As θ increases, there is less force applied in the direction of motion.

Let $\theta = \alpha - \beta$, the angle between **u** and **v.** Then

$$\sin(\alpha - \beta) = \frac{\|\mathbf{u}\times\mathbf{v}\|}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{\|\mathbf{v}\times\mathbf{u}\|}{\|\mathbf{u}\|\|\mathbf{v}\|}.$$

For $\mathbf{u} = \langle\cos\alpha, \sin\alpha, 0\rangle$ and $\mathbf{v} = \langle\cos\beta, \sin\beta, 0\rangle$, $\|\mathbf{u}\| = \|\mathbf{v}\| = 1$ and

$$\mathbf{v}\times\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos\beta & \sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix} = \left(\sin\alpha\cos\beta - \cos\alpha\sin\beta\right)\mathbf{k}.$$

So, $\sin(\alpha - \beta) = \|\mathbf{v}\times\mathbf{u}\| = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

16. (a) Los Angeles: $(4000, -118.24°, 55.95°)$ Rio de Janeiro: $(4000, -43.23°, 112.90°)$

(b) Los Angeles: $x = 4000\sin(55.95°)\cos(-118.24°)$ Rio de Janeiro: $x = 4000\sin(112.90°)\cos(-43.23°)$

$y = 4000\sin(55.95°)\sin(-118.24°)$ $y = 4000\sin(112.90°)\sin(-43.23°)$

$z = 4000\cos 55.95°$ $z = 4000\cos 112.90°$

$(x, y, z) \approx (-1568.2, -2919.7, 2239.7)$ $(x, y, z) \approx (2684.7, -2523.8, -1556.5)$

(c) $$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{(-1568.2)(2684.7) + (-2919.7)(-2523.8) + (2239.7)(-1556.5)}{(4000)(4000)} \approx -0.02047$$

$\approx 91.17°$ or 1.59 radians

$s = r\theta = 4000(1.59) \approx 6360$ miles

(e) For Boston and Honolulu:

a. Boston: $(4000, -71.06°, 47.64°)$ Honolulu: $(4000, -157.86°, 68.69°)$

b. Boston: $x = 4000 \sin 47.64° \cos(-71.06°)$ Honolulu: $x = 4000 \sin 68.69° \cos(-157.86°)$

$y = 4000 \sin 47.64° \sin(-71.06°)$ $y = 4000 \sin 68.69° \sin(-157.86°)$

$z = 4000 \cos 47.64°$ $z = 4000 \cos 68.69°$

$(959.4, -2795.7, 2695.1)$ $(-3451.7, -1404.4, 1453.7)$

c. $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \dfrac{(959.4)(-3451.7) + (-2795.7)(-1404.4) + (2695.1)(1453.7)}{(4000)(4000)} \approx 0.28329$

$\theta \approx 73.54°$ or 1.28 radians d.

$s = r\theta = 4000(1.28) \approx 5120$ miles

17. From Theorem 11.13 and Theorem 11.7 (6) you have

$$D = \frac{|\overrightarrow{PQ}\cdot\mathbf{n}|}{\|\mathbf{n}\|}$$

$$= \frac{|\mathbf{w}\cdot(\mathbf{u}\times\mathbf{v})|}{\|\mathbf{u}\times\mathbf{v}\|} = \frac{|(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}|}{\|\mathbf{u}\times\mathbf{v}\|} = \frac{|\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})|}{\|\mathbf{u}\times\mathbf{v}\|}.$$

18. Assume one of a, b, c, is not zero, say a. Choose a point in the first plane such as $(-d_1/a, 0, 0)$. The distance between this point and the second plane is

$$D = \frac{|a(-d_1/a) + b(0) + c(0) + d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|-d_1 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}.$$

19. $x^2 + y^2 = 1$ cylinder

$z = 2y$ plane

Introduce a coordinate system in the plane $z = 2y$.

The new u-axis is the original x-axis.

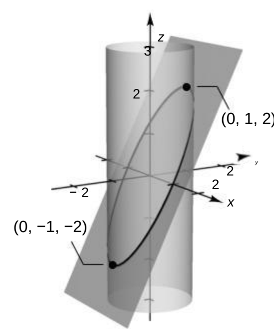
The new v-axis is the line $z = 2y$, $x = 0$.

Then the intersection of the cylinder and plane satisfies the equation of an ellipse:

$x^2 + y^2 = 1$

$x^2 + \left(\dfrac{z}{2}\right)^2 = 1$

$x^2 + \dfrac{z^2}{4} = 1$ ellipse

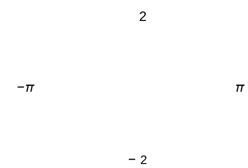


20. Essay

SECTION PROJECTS

Chapter 1, Section 5, page 94 Graphs and Limits of Trigonometric Functions

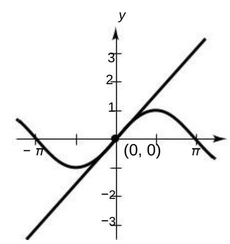
(a) On the graph of f, it appears that the y-coordinates of points lie as close to 1 as desired as long as you consider only those points with an x-coordinate near to but not equal to 0.



Use a table of values of x and $f(x)$ that includes several values of x near 0. Check to see if the corresponding values of $f(x)$ are close to 1. In this case, because f is an even function, only positive values of x are needed.

x	0.5	0.1	0.01	0.001
$f(x)$	0.9589	0.9983	1.0000	1.0000

(c) The slope of the sine function at the origin appears to be 1. (It is necessary to use radian measure and have the same unit of length on both axes.)



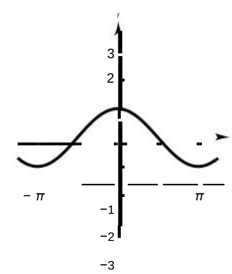
(d) In the notation of Section 1.1, $c = 0$ and $c + \quad x = x$. Thus, $m_{\sec} = \frac{\sin x - 0}{x - 0}$.

This formula has a value of 0.998334 if $x = 0.1$; $m_{\sec} = 0.999983$ if $x = 0.01$.

The exact slope of the tangent line to g at $(0, 0)$ is $\lim_{x \to 0} m_{\sec} = \lim_{x \to 0} \frac{\sin x}{x} = 1$.

(e) The slope of the tangent line to the cosine function at the point $(0, 1)$ is 0. The analytical proof is as follows:

$$\lim_{x \to 0} m_{\sec} = \lim_{x \to 0} \frac{\cos(0 + x) - 1}{x} = -\lim_{x \to 0} \frac{1 - \cos(x)}{x} = 0.$$



(f) The slope of the tangent line to the graph of the tangent function at $(0, 0)$ is:

$$\lim_{x \to \infty} m_{\sec} = \lim_{x \to 0} \frac{\tan(0 + x) - 0}{x} = \lim_{x \to 0} \frac{\sin(x)}{x} \cdot \frac{1}{\cos x} = 1 \cdot \frac{1}{1} = 1.$$

Chapter 2, Section 5, page 151 Optical Illusions

(a) $x^2 + y^2 = C^2$

$2x + 2yy' = 0$

$y' = -\frac{x}{y}$

at the point $(3, 4)$, $y' = -\frac{3}{4}$

(b) $xy = C$

$xy' + y = 0$

$y' = -\frac{y}{x}$

at the point $(1, 4)$, $y' = -4$

