Solution Manual for Calculus and Its Applications 11th Edition by Bittinger Ellenbogen Surgent ISBN 0321979397 9780321979391

Chapter 2
Applications of Differentiation

Exercise Set 2.1

1. $f\ x\ \ x^2\ \ 6\ x\ 3$

First, find the critical points.

$f\ '\ x\ \ 2x\ 6$

$f\ '\ x$ exists for all real numbers. We solve $f\ '\ x\ 0$

$2x\ 6\ 0$

$2x\ \ 6$

$x\ \ 3$

The only critical value is 3 . We use 3 to divide the real number line into two intervals,

A: , 3 and B:3, .

We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 4, $f\ '\ 4\ \ 2\ 4\ 6\ 2\ 0$

B: Test 0, $f\ '\ 0\ \ 2\ 0\ 6\ \ 6\ 0$

We see that $f\ x$ is decreasing on , 3 and

increasing on 3, , and the change from

decreasing to increasing indicates that a relative minimum occurs at $x\ 3$. We substitute into the original equation to find $f\ 3$:

$f\ 3\ 3^2\ \ 6\ 3\ 3\ 12$

x	$f\ x$

2. $f x x^2$ 4 x 5 f ' x 2x
4
f ' x exists for all real numbers. Solve f ' x
0

2x 4 0

2 x 4

x 2

The only critical value is 2 . We use 2 to divide the real number line into two intervals,
A: , 2 and B: 2, .

A: Test 3, f ' 3 2 3 4 2 0

B: Test 0, f ' 0 2 0 4 4 0

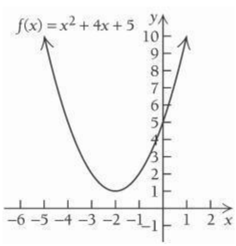
We see that $f x$ is decreasing on , 2

and increasing on 2, , there is a relative minimum at x 2 .

f 2 2^2 4 2 5 1

Thus, there is a relative minimum at 2,1 . We sketch the graph.

x	$f x$
5	10
4	5
3	2
2	1
1	2
0	5
1	10



Thus, there is a relative minimum at 3, 12 . We use the information obtained to sketch the graph. Other function values are listed below.

3. $f\ x\ \ 2\ 3x\ 2x^2$

$f\ '\ x\ \ 3\ 4x$

$f\ '\ x$ exists for all real numbers. We solve $f\ '\ x\ 0$

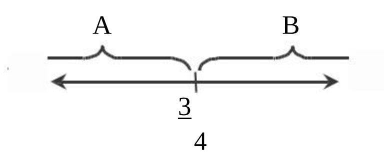
$3\ 4x\ 0$

$x \frac{3}{4}$

The solution is continued on the next page.

The only critical value is $\frac{3}{4}$. We use $\frac{3}{4}$ ~~to~~

divide the real number line into two intervals,
A: $, \frac{3}{4}$ and B: $\frac{3}{4}, .$



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 1, $f'\ 1\ 3\ 4\ 1\ 1\ 0$

B: Test 0, $f'\ 0\ \ 3\ 4\ 0\ \ 3\ 0$

We see that $f\ x$ is increasing on $, \frac{3}{4}$

and decreasing on $\frac{3}{4}$, , and the change

from increasing to decreasing indicates that a

relative maximum occurs at $x \ \frac{3}{4}$. We substitute into the original equation to find

$f\ \frac{3}{4}$:

$$f\ \frac{3}{4}\ \ 2\ 3\ \frac{3}{4}\ \ 2\ \ \frac{3}{4}^2\ \ -\frac{25}{8}$$

Thus, there is a relative maximum at $\frac{3}{4}, \frac{25}{8}$.

4. $f\ x \ \ 5\ x\ x^2$

$f'\ x\ \ 1\ 2x$

$f'\ x$ exists for all real numbers.

Solve $f'\ x\ 0$

$1\ 2x\ \ 0$

$2x\ 1$

$x\ \ \frac{1}{2}$

The only critical value is $\frac{1}{2}$. We use $\frac{1}{2}$ to divide the real number line into two intervals, $\frac{1}{2}\ \frac{1}{2}$

A: Test 1, $f'\ 1\ 1\ 2\ 1\ 1\ 0$

B: Test 0, $f'\ 0\ \ 1\ 2\ 0\ \ 1\ 0$

We see that $f\ x$ is increasing on $, \frac{1}{2}$

and decreasing on $\frac{1}{2}$, , which indicates

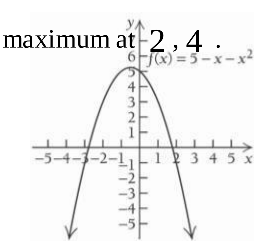
there is a relative maximum at $x\ \ \frac{1}{2}$.

$$f\ \frac{1}{2}\ \ 5\ \ \frac{1}{2}\ \ \frac{1}{2}^2\ \ \frac{21}{4}$$

$\frac{1}{2}\ \frac{21}{4}$

Thus, there is a relative maximum at $\frac{1}{2}, \frac{21}{4}$.

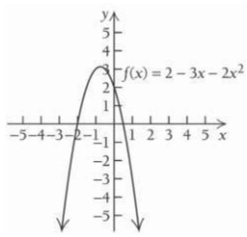
We sketch the graph.



4 8

We use the information obtained to sketch the graph. Other function values are listed below.

x	$f\ x$
3	7
2	0
1	3
4 3	$\frac{25}{8}$
0	2
1	3
2	12



x	$f\ x$
3	1
2	3
1	5
$\frac{1}{2}$	$\frac{21}{4}$
0	5
1	3
2	1

5. $F\ x\ 0.5x^2\ \ 2\ x\ 11$

First, find the critical points.

$F\ '\ x\ \ x\ 2$

$F\ '\ x$ exists for all real numbers. We solve:

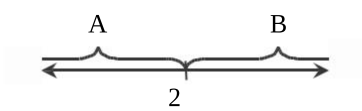
$F\ '\ x\ \ 0$

$x\ 2\ 0$

$x\ \ 2$

The solution is continued on the next page

The only critical value is 2 . We use 2 to divide the real number line into two intervals, A: , 2 and B:2, .



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 3, $F\ '\ 3\ 3\ 2\ \ 1\ 0$

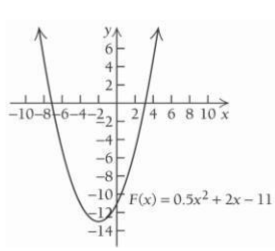
B: Test 0, $F'0\ 0\ 2\ 2\ 0$

We see that $F\ x$ is decreasing on , 2 and increasing on 2, , and the change from decreasing to increasing indicates that a relative minimum occurs at x 2 . We substitute into the original equation to find F 2 :

$F\ 2\ 0.5\ 2^2\ \ 2\ 2\ 11\ 13$

Thus, there is a relative minimum at 2, 13 . We use the information obtained to sketch the graph. Other function values are listed below.

x	$F\ x$
5	$1\frac{17}{2}$
4	25
3	$\frac{2}{13}$
2	13
1	$\frac{25}{2}$
0	11
1	17

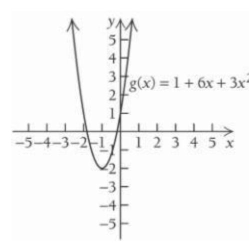


We see that $g\ '\ x$ is decreasing on , 1 and increasing on 1, , which indicates there is a relative minimum at x 1 .

$g\ 1\ 1\ 6\ 1\ 3\ 1^2\ \ 2$

Thus, there is a relative minimum at 1, 2 . We sketch the graph.

x	$g\ x$
4	25
3	10
2	1
1	2
0	1
1	10
2	25



7. $g\ x\ \ x^3\ \frac{1}{2}x^2\ \ 2\ x\ 5$

First, find the critical points.

$g\ '\ x\ \ \ 3x^2\ \ \ x\ \ 2$

$g\ '\ x$ exists for all real numbers. We solve

$g\ '\ x\ \ 0$

$3x^2\ \ x\ 2\ 0$

$3x\ 2\ x\ 1\ 0$

$3x\ 2\ 0$ or $x\ 1\ 0$

$x\ \ \frac{2}{3}$ or $x\ \ 1$

The critical values are 1 and $\frac{2}{3}$. We use them to divide the real number line into three intervals,

A: , 1, B: 1, $\frac{2}{3}$, and C: $\frac{2}{3}$, .

3 3

6. $g\ x\ 1\ 6x\ 3x^2$

$g\ '\ x\ \ 6\ 6x$

$g\ '\ x$ exists for all real numbers.

Solve $g\ '\ x\ 0$

$6\ 6\ x\ \ 0$

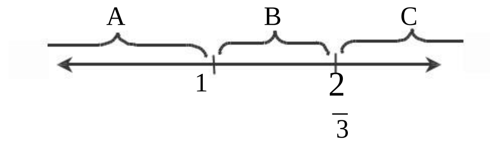
$x\ \ 1$

The only critical value is 1 . We use 1 to divide the real number line into two intervals,

A: , 1 and B: 1, :

A: Test 2, $g\ '\ 2\ 6\ 6\ 2\ \ 6\ \ 0$

B: Test 0, $g\ '\ 0\ 6\ 6\ 0\ 6\ 0$



We use a test value in each interval to determine the sign of the derivative in each interval.

The solution is continued on the next page.

A: Test 2,

$g'2\ 3\ 2^2\ 2\ 2\ 8\ 0$

B: Test 0,

$g'0\ 3\ 0^2\ 0\ 2\ 2\ 0$

C: Test 1,

$g'1\ 3\ 1^2\ 1\ 2\ 2\ 0$

We see that $g\ x$ is increasing on , 1 , decreasing on 1, $\frac{2}{3}$, and increasing on $\frac{2}{3}$, . So there is a relative maximum at x 1 and a relative minimum at x $\frac{2}{3}$.

We find g 1 :

$g\ 1\ 1^3\ \frac{1}{2}\ 2\ 1^2\ 2\ 1\ 5$

$1\ \frac{1}{2}\ 2\ 5\ \frac{13}{2}$

Then we find $g\ \frac{2}{3}$:

$g\ \frac{2}{3}\ \frac{2}{3}^3\ \frac{1}{2}\ \frac{2}{3}^2\ 2\ \frac{2}{3}\ 5$

$\frac{8}{27}\ \frac{2}{9}\ \frac{4}{3}\ 5\ \frac{113}{27}$

There is a relative maximum at $1, \frac{13}{2}$, and there is a relative minimum at $\frac{2}{3}, \frac{113}{27}$.

We use the information obtained to sketch the graph. Other function values are listed

$G'x$ exists for all real numbers. We solve

$G'x\ 0$

$3x^2\ 2x\ 1\ 0$

$3x\ 1x\ 1\ 0$

$3x\ 1\ 0$ or $x\ 1\ 0$

$3x\ 1$ or $x\ 1$

$x\ \frac{1}{3}$ or $x\ 1$

The critical values are $\frac{1}{3}$ and 1 . We use them to divide the real number line into three intervals,

A: , $\frac{1}{3}$, B: $\frac{1}{3}$,1 , and C: 1, .

A: Test 1,

$G'1\ 3\ 1^2\ 2\ 1\ 1\ 4\ 0$

B: Test 0,

$G'0\ 3\ 0^2\ 2\ 0\ 1\ 1\ 0$

C: Test 2,

$G'2\ 3\ 2^2\ 2\ 2\ 1\ 7\ 0$

We see that $G\ x$ is increasing on , $\frac{1}{3}$, decreasing on $\frac{1}{3}$,1 , and increasing on 1, . So there is a relative maximum at $x\ \frac{1}{3}$ and a relative minimum at x 1 .

$G\ \frac{1}{3}\ \frac{1}{3}^3\ \frac{1}{3}^2\ \frac{1}{3}\ \frac{59}{3}$

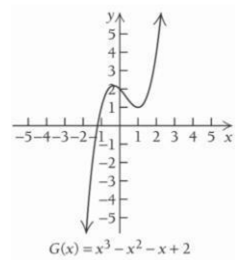
$G(1) = 1^3 - 1^2 - 1 + 2 = 1$

$\frac{2}{}\quad 27$

There is a relative maximum at $\left(-\frac{1}{3}, \frac{59}{27}\right)$, and there is a relative minimum at $(1, 1)$.

We use the information obtained to sketch the graph. Other function values are listed below.

x	$G\ x$
2	8
1	1
0	2
2	4
3	17



8. $G\,x \;\; x^3 \;\; x^2 \;\; x\,2$

$G\,'\,x \;\; 3x^2 \;\; 2\,x\,1$

9. $f\, x \;\; x^3 \;\; 3x^2$

First, find the critical points.

$f\,'\, x \;\; 3x^2 \;\; 6\, x$

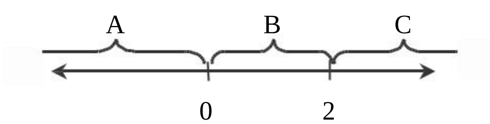
$f\,'\, x$ exists for all real numbers. We solve $f\,'\, x\; 0$

$3x^2 \;\; 6\, x \;\; 0$

$3x\, x\, 2 \;\; 0$

$x \;\; 0$ or $x \;\; 2$

The critical values are 0 and 2 . We use them to divide the real number line into three intervals, A: , 0, B: 0, 2, and C:2, .



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 1, $f\,'\, 1\; 3\; 1^2 \;\; 6\; 1\; 9 \;\; 0$

B: Test 1,$f\,'\, 1\; 31^2 \;\; 6\; 1\; 3\; 0$

C: Test 3, $f\,'\, 3\; 3\; 3^2 \;\; 6\; 3\; 9\; 0$

We see that $f\, x$ is increasing on , 0 ,

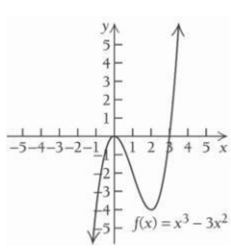
decreasing on 0, 2 , and increasing on 2, . So there is a relative maximum at x 0 and a relative minimum at x 2 .

We find $f\,0$:

$f\,0\; 0^3 \;\; 30^2 \;\; 0.$

$f\,2\; 2^3 \;\; 32^2 \;\; 4.$

There is a relative maximum at 0, 0 , and there is a relative minimum at 2, 4 . We use the information obtained to sketch the graph. Other function values are listed below.



10. $f\, x\; x^3\, 3x\; 6\; f$
$'\, x\; 3x^2\; 3$

$f\,'\, x$ exists for all real numbers. We solve $f\,'\, x\; 0$

$3x^2 \;\; 3\; 0$

$3x^2 \;\; 3$

$x^2 \;\; 1$

$x \;\; 1$

The critical values are 1 and 1 . We use them to divide the real number line into three intervals, A: , 1, B: 1,1, and C:1, .

A: Test 3, $f\,'\, 3\; 3\; 3^2 \;\; 3\; 24\; 0$

B: Test 0,$f\,'\, 0\; 3\; 0^2 \;\; 3\; 3\; 0$

C: Test 2,$f\,'\, 2\; 32^2 \;\; 3\; 9\; 0$

We see that $f\, x$ is increasing on , 1 ,

decreasing on 1,1 , and increasing on 1, .

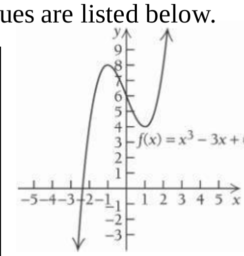
So there is a relative maximum at x 1 and a relative minimum at x 1 .

$f\, 1\; 1^3 \;\; 31\; 6\; 136\; 8$

$f\, 1\; 1^3 \;\; 316\; 136 \;\; 4$

There is a relative maximum at 1, 8 , and there is a relative minimum at 1, 4 . We use the information obtained to sketch the graph. Other function values are listed below.

x	$f\, x$
3	12
2	4
0	6
2	8
3	24



x	$f x$
2	20
1	4
1	2
3	0
4	16

11. $f x \; x^3 \; 3x$

First, find the critical points.

$f\,'\,x \;\; 3x^2 \;\; 3$

$f\,'\,x$ exists for all real numbers. We solve $f\,'\,x \; 0$

$3x^2 \;\; 3 \; 0$

$x^2 \;\; 1$

There are no real solutions to this equation. Therefore, the function does not have any critical values.

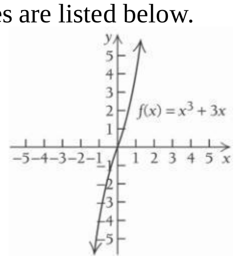
We test a point

$f\,'\,0 \; 30^2 \;\; 3 \; 3 \; 0$

We see that $f x$ is increasing on , , and

that there are no relative extrema. We use the information obtained to sketch the graph. Other function values are listed below.

x	$f x$
2	14
1	4
0	0
1	4
2	14



12. $f x \; 3x^2 \; 2 \; x^3$

$f\,'\,x \; 6x \; 6x^2$

$f\,'\,x$ exists for all real numbers. We solve $f\,'\,x \; 0$

$6x \; 6 \; x^2 \;\; 0$

$6x \; 1 \; x \;\; 0$

$6x \;\; 0$ or $x \; 1 \; 0$

$x \;\; 0$ or $x \;\; 1$

We know the critical values are 1 and 0 . We use them to divide the real number line into three intervals,

A: , 1, B: 1, 0, and C: 0, .

A: Test 2,

We see that $f x$ is increasing on , 1 ,

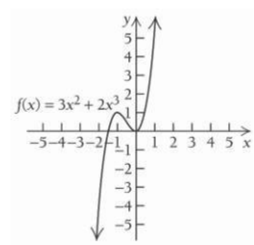
decreasing on 1, 0 , and increasing on 0, . So there is a relative maximum at x 1 and a relative minimum at x 0 .

$f \; 1 \; 31^2 \;\; 2 \; 1^3 \; 1$

$f \; 0 \; 30^2 \;\; 2 \; 0^3 \; 0$

There is a relative maximum at 1,1 , and there is a relative minimum at 0, 0 . We use the information obtained to sketch the graph. Other function values are listed below.

x	$f x$
3	27
2	4
$\frac{1}{2}$	1
2	28



13. $F x \; 1 \; x^3$

First, find the critical points.

$F\,'\,x \;\; 3x^2$

$F\,'\,x$ exists for all real numbers. We solve $F\,'\,x \; 0$

$3x^2 \;\; 0$

$x \;\; 0$

The only critical value is 0 . We use 0 to divide the real number line into two intervals, A: , 0, and B: 0, .

$f\,'\,2 \;\; 6 \; 2 \; 6 \; 2^2 \;\; 12 \; 0$
1

B: Test ,

We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 1, $F'\ 1\ 3\ 1^2\ \ 3\ 0$

B: Test 1, $F'1\ 31^2\ \ 3\ 0$

$f'\ \frac{1}{2}\ 6\ \frac{1}{2}\ 6\ \frac{1}{2}\ {}^2\ \frac{3}{2}\ 0$

C: Test 1,

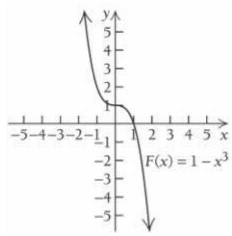
$f'\ 1\ 6\ 1\ 6\ 1^2\ \ 12\ 0$

We see that $F\ x$ is decreasing on , 0 and decreasing on 0, , so the function has no relative extema. We use the information obtained to sketch the graph on the next page.

Exercise Set 2.1

Using the information from the previous page and deteriming other function values are listed below, we sketch the graph.

x	$F\ x$
2	9
1	2
0	1
1	0
2	7



14. $g\ x\ \ 2\ x^3\ 16$

First, find the critical points.

$g\ '\ x\ \ 6\ x^2$

$g\ '\ x$ exists for all real numbers. We solve $g\ '\ x\ 0$

$6x^2\ \ 0$

$x\ \ 0$

The only critical value is 0 . We use 0 to

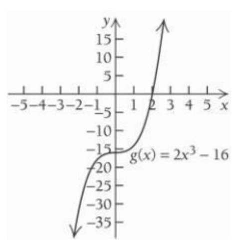
divide the real number line into two intervals, A: , 0, and B: 0, .

A: Test 1, $g\ '\ 1\ 6\ 1^2\ \ 6\ \ 0$

B: Test 1, $g\ '\ 1\ 6\ 1^2\ \ 6\ \ 0$

We see that $g\ x$ is increasing on , 0 and increasing on 0, , so the function has no relative extema. We use the information obtained to sketch the graph. Other function values are listed below.

x	$g\ x$
2	32
1	18
0	16
1	14
2	0
3	38



$G\ x\ \ x^3\ \ 6\ x^2\ \ 10$

15. First, find the critical points.

$G\ '\ x\ 3x^2\ 12\ x$

$G\ '\ x$ exists for all real numbers. We solve $G\ '\ x\ 0$

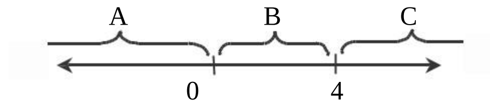
$x^2\ \ 4\ x\ \ 0$ Dividing by 3

$x\ x\ 4\ 0$

$x\ \ 0$ or $x\ 4\ 0$

$x\ \ 0$ or $x\ \ 4$

The critical values are 0 and 4 . We use them to divide the real number line into three intervals, A: , 0, B: 0, 4, and C: 4, .



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 1, $G\ '\ 1\ 3\ 1^2\ \ 12\ 1\ 15\ 0$

B: Test 1, $G\ '\ 1\ 3\ 1^2\ \ 12\ 1\ 9\ 0$

2

C: Test 5, $G'5\ 35\ \ \ 125\ 15\ 0$

We see that $G\ x$ is increasing on , 0 , decreasing on 0, 4 , and increasing on 4, . So there is a relative maximum at x 0 and a relative minimum at x 4 .

We find G 0 :

3 2

$G\ 0\ 0\ \ \ 6\ 0\ \ \ 10$

10

Then we find G 4 :

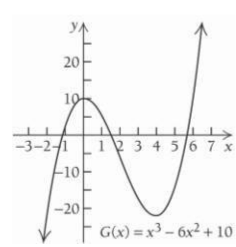
$G4\ \ \ 4^3 64^2 10$

64 96 10

22

There is a relative maximum at 0,10 , and there is a relative minimum at 4, 22 . We use the information obtained to sketch the graph. Other function values are listed below.

x	$G\ x$
2	22
1	3
1	5
2	6
3	17



16. $f\ x\ 12\ 9\ x\ 3x^2\ x^3$

$f\ '\ x\ 9\ 6\ x\ 3x^2$

$f\ '\ x$ exists for all real numbers. Solve

$f\ '\ x\ \ 0$

$9\ 6\ x\ 3x^2\ \ 0$

$x^2\ \ 2\ x\ 3\ 0$ Dividing by 3

$x\ 3x\ 1\ 0$

$x\ 3\ 0$ or $x\ 1\ 0$

$x\ \ 3$ or $x\ 1$

The critical values are 3 and 1 . We use them to divide the real number line into three intervals, A: , 3, B: 3,1, and C:1, .

We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 4,

$f\ '\ 4\ \ 9\ 6\ 4\ 3\ 4^2\ 15\ \ 0$

B: Test 0,

$f\ '\ 0\ 9\ 6\ 0\ 3\ 0^2\ 9\ 0$

C: Test 2,

$f\ '2\ 96232^2\ 15\ 0$

We see that $f\ x$ is decreasing on , 3 ,

increasing on 3,1 , and decreasing on 1, .

So there is a relative minimum at x 3 and a

relative maximum at x 1 .

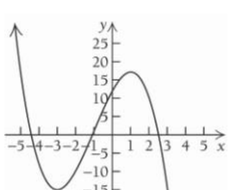
$f\ 3\ 12\ 9\ 3\ 33^2\ \ 3^3\ \ 15$

$f\ 1\ 12\ 9\ 1\ 31^2\ \ 1^3\ \ 17$

There is a relative minimum at 3, 15 , and there is a relative maximum at 1,17 . We use the information obtained to sketch the graph.

Other function values are listed below.

x	$f\ x$
5	17
4	8
2	10



17. $g\ x\ x^3\ x^4$

First, find the critical points.

$g\ '\ x\quad 3x^2\ \ 4\ x^3$

$g\ '\ x$ exists for all real numbers. We solve $g\ '\ x\ 0$

$3x^2\quad 4x^3\quad 0$

$x^2\ \ 3\ 4x\ \ 0$

$x^2\quad 0$ or $3\ 4\ x\ 0$

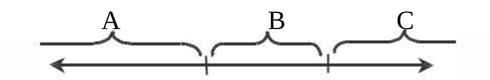
$x\quad 0$ or $4\ x\quad 3$

$x\quad 0$ or $x\quad \frac{3}{4}$

The critical values are 0 and $\frac{3}{4}$.

We use the critical values to divide the real number line into three intervals,

A: , 0, B: $0, \frac{3}{4}$, and C: $\frac{3}{4}$, .



$0 \qquad \frac{3}{4}$

We use a test value in each interval to determine

the sign of the derivative in each interval.

A: Test 1, $g\ '\ 1\ 3\ 1^2\qquad 4\ 1^3\qquad 7\ 0$

B: Test $\frac{1}{2}$, $g\ '\frac{1}{2}\quad 3\ \frac{1}{2}^2\ \ 4\ \frac{1}{2}^3$

$3\frac{1}{4}\ 4\frac{1}{8}\ \frac{1}{4}\quad 0$

C: Test 1, $g\ '\ 1\ 3\ 1^2\ \ 4\ 1^3\quad 1\ 0$

We see that $g\ x$ is increasing on , 0 and

$0, \frac{3}{4}$, and is decreasing on $\frac{3}{4}$,. So there

is no relative extrema at $x\ \ 0$ but there is a relative maximum at $x = \frac{3}{4}$.

We find $g\left(\frac{3}{4}\right)$:

$$g\left(\frac{3}{4}\right) = \left(\frac{3}{4}\right)^3 - \left(\frac{3}{4}\right)^4 = \frac{27}{64} - \frac{81}{256} = \frac{27}{256}$$

The solution is continued on the next page.

From the previous page, we determine there is

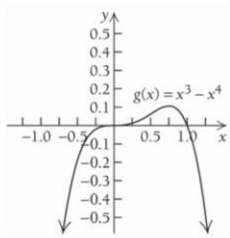
3 27

a relative maximum at , . We use the

4 256

information obtained to sketch the graph. Other function values are listed below.

x	$g\,x$
2	24
1	2
0	0
$\frac{1}{2}$	$\frac{1}{16}$
1	0
2	8



18. $f\,x\;x^4\;2\,x^3$

$f'\,x\;4\,x^3\;6\,x^2$

$f'\,x$ exists for all real numbers. Solve $f'\,x\;0$

$4x^3\;6\,x^2\;0$

$2\,x^2\,2\,x\,3\,0$

$x^2\;0$ or $2x\,3\,0$

$x\;0$ or $x\;\frac{3}{2}$

The critical values are 0 and $\frac{3}{2}$. We use them to divide the real number line into three intervals, A: , 0, B: 0, $\frac{3}{2}$, and C: $\frac{3}{2}$, .

A: Test 1, $f\;'1\;41^3\;61^2\;10\;0$

B: Test 1, $f'\,1\,4\,1^3\,6\,1^2\;2\;0$

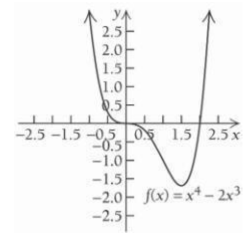
C: Test 2, $f'\,2\;4\,2^3\;6\,2^2\;8\;0$

Since $f\,x$ is decreasing on both , 0 and 0, $\frac{3}{2}$, and increasing on $\frac{3}{2}$, , there is no relative extrema at x 0 but there is a relative minimum at x $\frac{3}{2}$

$f\;\frac{3}{2}\;\;\frac{3}{2}^4\;2\;\frac{3}{2}^3\;\;\frac{27}{16}$

We use the information obtained to sketch the graph. Other function values are listed below.

x	$f\,x$
2	32
1	3
0	0
1	1
2	0
3	27



19. $f\,x\;\frac{1}{3}x^3\;2\,x^2\;4\,x\;1$

First, find the critical points.

$f'\,x\;x^2\;4\,x\,4$

$f'\,x$ exists for all real numbers. We solve $f'\,x\;0$

$x^2\;4\,x\,4\,0$

$x\,2^2\;0$

$x\;2$

The only critical value is 2 . We divide the real number line into two intervals,

A: , 2 and B: 2, .

2

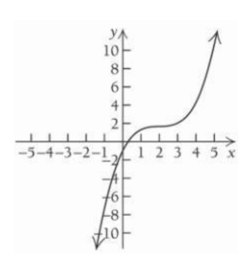
We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 0, $f'\,0\;0^2\;4\,0\,4\,4\,0$

B: Test 3, $f'\,3\;3^2\;4\,3\,4\,1\,0$

We see that $f\,x$ is increasing on both , 2 and 2, . Therefore, there are no relative extrema. We use the information obtained to sketch the graph. Other function values are listed below.

x	$f\,x$
3	40
2	$\frac{59}{3}$
1	$\frac{22}{3}$
0	1



There is a relative minimum at $\left(\frac{3}{2}, \frac{27}{16}\right)$.

20. $F\ x \quad \frac{1}{3} x^3\ 3x^2\ 9\ x\ 2$

$F'\ x\ x^2\ \ 6\ x\ 9$

$F'\ x$ exists for all real numbers.

Solve $F'\ x\ 0$

$x^2\ \ 6\ x\ 9\ 0$

$x^2\ \ 6\ x\ 9\ 0$

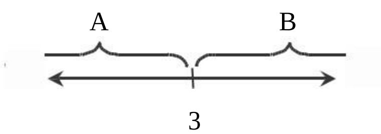
$x\ 3 \quad 0$

$x\ 3\ 0$

$x\ \ 3$

The only critical value is 3 . We divide the real number line into two intervals,

A: , 3 and B: 3, .



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 0, $F'\ 0\ \ 0^2\ \ 6\ 0\ 9\ \ 9\ 0$

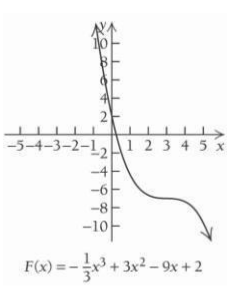
B: Test 4, $F'\ 4\ \ 4^2\ \ 6\ 4\ 9\ \ 1\ 0$

We see that $F\ x$ is decreasing on both ,3

and 3, . Therefore, there are no relative extrmea.

We use the information obtained to sketch the graph. Other function values are listed below.

x	$F\ x$
3	65
2	$\frac{104}{3}$
1	$\frac{43}{3}$
0	2
1	13
2	$3\frac{20}{3}$
3	7



21. $f\ x\ \ 3\ x^4\ \ 15x^2\ \ 12$

First, find the critical points.

$f'\ x\ 12\ x^3\ \ 30\ x$

$f'\ x$ exists for all real numbers. We solve $f'\ x\ 0$

$12\ x^3\ \ 30\ x\ \ 0$

$6\ x\ 2x^2\ \ 5\ 0$

$6\ x\ \ 0 \quad$ or $\quad 2\ x^2\ \ \ 5\ 0$

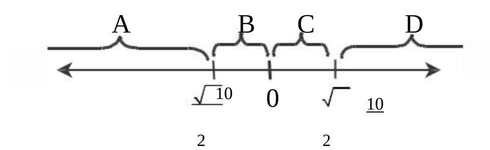
$x\ \ 0 \quad$ or $\quad x^2\ \frac{5}{2}$

$x\ \ 0 \quad$ or $\quad x\ \ \frac{\sqrt{10}}{2}$

The critical values are 0, [redacted] and $\frac{\sqrt{10}}{2}$. We use them to divide the real number line into four intervals,

A: , $\frac{\sqrt{10}}{2}$, B: $\frac{\sqrt{10}}{2}$, 0 ,

C: 0, $\frac{\sqrt{10}}{2}$, and D: $\frac{\sqrt{10}}{2}$, .



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 2,

$f'\ 2\ 12\ 2^3\ \ 30\ 2\ 36\ 0$

B: Test 1,

$f'\ 1\ 12\ 1^3\ \ 30\ 1\ 18\ 0$

C: Test 1,

$f'\ 1\ 12\ 1^3\ \ 30\ 1\ 18\ 0$

D: Test 2,

$f'\ 2\ 12\ 2^3\ \ 30\ 2\ 36\ 0$

The solution is continued on the next page.

From the previous page, we see that $f\ x$ is

$\sqrt{\ }$ 10

decreasing on , 2 , increasing on $\frac{\sqrt{10}}{}$, 0 , decreasing again on 0, $\sqrt{10}$,

2

2

and increasing again on $\frac{\sqrt{10}}{2}$, . Thus, there

is a relative minimum at x $\frac{\sqrt{10}}{2}$, a relative maximum at x 0 , and another relative minimum at x [redacted]

2

We find f [redacted] :

2

4 2

f $\frac{\sqrt{10}}{2}$ 3 $\frac{\sqrt{10}}{2}$ 15 $\frac{\sqrt{10}}{2}$ 12

$\frac{27}{4}$

Then we find $f\ 0$:

$f\ 0\ 30^4\ \ 150^2\ 12\ 12$

Then we find f $\frac{\sqrt{10}}{2}$:

f [redacted] 3 $\frac{10}{2}$ 4 15 [redacted] 2 12

2 2 2

$\frac{27}{}$

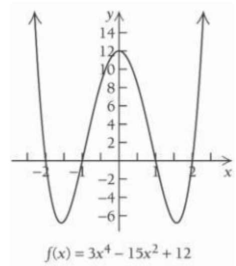
4

There are relative minima at $\frac{\sqrt{10}}{2}$, $\frac{27}{4}$ and

[redacted], $\frac{27}{}$.

2 4

x	$f\ x$
3	120
2	0
1	0
1	0
2	0
3	120


$f(x) = 3x^4 - 15x^2 + 12$

22. $g\ x\ 2x^4\ 20\ x^2\ 18\ g$ $'\ x\ 8x^3\ 40\ x$

$g\ '\ x$ exists for all real numbers. We solve $g\ '\ x\ 0$

$8x^3\ \ 40\ x\ \ 0$

2

$8x\ x$ 5 0

2

$8x$ 0 or x 5 0

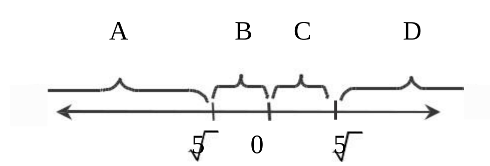
x 0 or ^2x 5

x 0 or x $\sqrt{5}$

The critical values are 0, $\sqrt{5}$ and 5 . We use them to divide the real number line into four intervals,

A: , $\sqrt{\ }$ 5, B: $\sqrt{\ }$5, 0,

C: 0, 5 , and D: 5 , .



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 3,

403 96 0

$g\ '\ 3\ 8\ 3^3$

B: Test 1,

There is a relative maximum at 0,12.

We use the information obtained above to sketch the graph. Other function values are listed at the top of the next column.

$g'(1) = 8(1)^3 - 40(1) = -32 < 0$

C: Test 1,

$g'(1) = 8(1)^3 - 40(1) = -32 < 0$

D: Test 3,

$g'(3) = 8(3)^3 - 40(3) = 96 > 0$

The solution is continued on the next page.

From the previoius page, we see that $g x$ is

decreasing on , 5 , increasing on

5, 0 , decreasing again on 0, 5 , and

increasing again on $\sqrt{5}$, . Thus, there is a

relative minimum at x $\sqrt{5}$, a relative maximum at x 0 , and another relative minimum at x $\sqrt{5}$.

g $\sqrt{}$5 2 5 ^{4}20 $\sqrt{}$5 ^{2}18 32

g 0 2 0^4 20 0^2 18 18

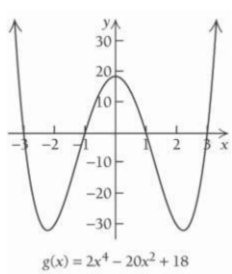
g $\sqrt{}$[redacted]2 $\sqrt{}$5 20 5 1832

There are relative minima at $\sqrt{5}$, 32 and

$\sqrt{}$[redacted]32 . There is a relative maximum at

0,18 We use the information obtained to sketch the graph. Other function values are listed below.

x	$g x$
4	210
3	0
1	0
1	0
3	0
4	210



23. $G x$ ^{3} x 2 x 2 ^{3} 1⁄

First, find the critical points.

$G ' x$ $\frac{1}{3}$ x 2 $^{2/3}$ 1

$\frac{1}{3\, x\, 2^{\,2/3}}$

$G ' x$ does not exist when x 2 . The equation

$G ' x$ 0 has no solution, therefore, the only

critical value is x 2 .

We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 3, G'3 1 1 0

332^3 ⁄ 3
1 2 1

B: Test 1,G '1 3 0

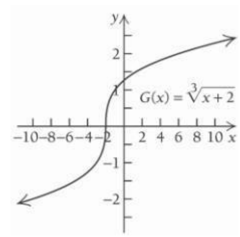
3 1 2 ^{3} 2⁄

We see that $G x$ is increasing on both , 2 and 2, . Thus, there are no

relative extrema for $G x$.

We use the information obtained to sketch the graph. Other function values are listed below.

x	$G x$
10	2
3	1
2	0
1	1
6	2



24. $F x$ ^{3} x 1 x 1 ^{3} 1⁄

$F ' x$ $\frac{1}{3}$ x 1$^{2/3}$ 1

$\frac{1}{3\, x\, 1^{\,2/3}}$

$F ' x$ does not exist when $3x$ 1 $^{3\,2}$ 0 , which means that $F ' x$ does not exist

when x 1 . The equation $F ' x$ 0 has no solution, therefore, the only critical value is x 1 .

We use 1 to divide the real number line into

two intervals,

A: ,1 and B: 1, .

We use a test value in each interval to determine

the sign of the derivative in each interval.

1 1

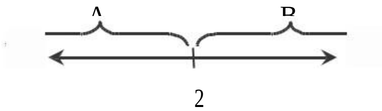
We use 2 to divide the real number line into two intervals, A: Test 0, *F* ' 0

301³ ₂ 3
0

B: Test 2, *F* ' 2

1 1
2
0
3 2 1 ₃ 3

A: , 2 and B: 2, :

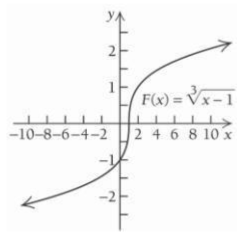


We see that $F x$ is increasing on both ,1

and 1, . Thus, there are no relative extrema for $F x$. We use the information obtained to sketch the graph at the top of the next page.

Using the information from the previous page, we sketch the graph. Other function values are listed.

x	$F(x)$
7	2
0	1
1	0
2	1
9	2



25. $f\ x\ 1\ x^{2/3}$

First, find the critical points.

$$f'\ x \quad \frac{2}{3}\ x^{\ 1/3}$$

$$\frac{2}{3\sqrt[3]{x}}$$

$f'\ x$ does not exist when $3\sqrt[3]{x}\ \ 0$, which means that $f'\ x$ does not exist when $x\ \ 0$. The equation $f'\ x\ \ 0$ has no solution, therefore, the only critical value is $x\ \ 0$.

We use 0 to divide the real number line into two intervals,

A: , 0 and B: 0, :

We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 1, $f'\ 1 \quad \frac{2}{3\sqrt[3]{1}} \quad \frac{2}{3} \quad 0$

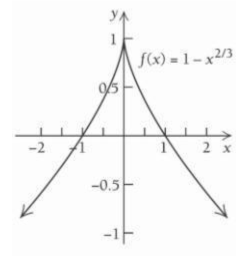
B: Test 1, $f'\ 1 \quad \frac{2}{3\sqrt[3]{1}} \quad \frac{2}{3} \quad 0$

We see that $f\ x$ is increasing on , 0 and decreasing on 0, . Thus, there is a relative maximum at $x\ 0$.

We find $f\ 0$:

$^{2/}$

x	$f\ x$
8	3
1	0
1	0
8	3



26. $f\ x\ x\ 3^{\ 3}\ \ 5^{\ 2/}$

$$f'\ x\ \frac{2}{3}\ x\ 3^{\ 1/3}$$

$$\frac{2}{3x\ \ 3^{\ 1/3}}$$

$f'\ x$ does not exist when $x\ 3$. The equation $f'\ x\ 0$ has no solution, therefore, the only critical value is $x\ \ 3$.

We use 3 to divide the real number line into two intervals, A: , 3 and B: 3, :

A: Test 4, $f'\ 4 \quad \frac{2}{3\ 4\ 3^{\ 1/3}} \quad \frac{2}{3} \quad 0$

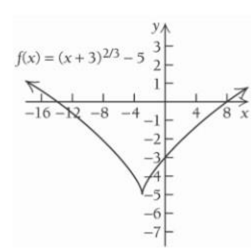
B: Test 2, $f'\ 2 \quad \frac{2}{3\ 2\ 3^{\ 1/3}} \quad \frac{2}{3} \quad 0$

We see that $f\ x$ is decreasing on , 3 and increasing on 3, . Thus, there is a relative minimum at $x\ 3$.

$f\ 3\ \ 3\ 3^{\ 3}\ \ 5\ 5^{\ 2/}$:

Therefore, there is a relative minimum at 3, 5 . We use the information obtained to sketch the graph. Other function values are listed below.

x	$f\ x$
11	1
4	4
2	4
5	1



$f\ 0\ 1\ 0^3\ \ 1$.

Therefore, there is a relative maximum at 0,1 . We use the information obtained to sketch the graph. Other function values are listed at the top of the next column.

27. $G\ x \quad \frac{8}{x^2\ \ 1} \quad 8\ x^2\ \ 1^{\ 1}$

First, find the critical points.

$G'\ x\ \ 8\ 1x^2\ \ 1^{\ 2}\ \ 2\ x$

$\frac{16\ x}{x^2\ \ 1^{\ 2}}$

$G'\ x$ exists for all real numbers. Setting the derivative equal to zero, we have:

$G'\ x\ \ 0$

$\frac{16x}{x^2\ \ 1^{\ 2}}\ \ 0$

$16x\ 0$

$x\ \ 0$

The only critical value is 0 .

We use 0 to divide the real number line into two intervals,

A: , 0 and B: 0, :

0

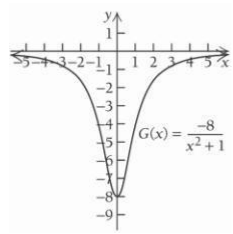
We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 1, $G'\ 1 \quad \frac{16\ 1}{1^2\ \ 1^{\ 2}} \quad \frac{16}{4}\ \ 4\ 0$

B: Test 1, $G'\ 1 \quad \frac{16\ 1}{1^2\ \ 1^{\ 2}} \quad \frac{16}{4}\ \ 4\ 0$

We see that $G\ x$ is decreasing on , 0 and

x	$G\ x$
3	$-\frac{4}{5}$
2	$-\frac{8}{5}$
1	-4
1	-4
2	$-\frac{8}{5}$
3	$-\frac{4}{5}$



28. $F\ x \quad \frac{5}{x^2\ \ 1} \quad 5\ x^2\ \ 1^{\ 1}$

$F'\ x\ \ 5\ 1x^2\ \ 1^{\ 2}\ \ 2x$

$\frac{10\ x}{x^2\ \ 1^{\ 2}}$

$F'\ x$ exists for all real numbers. We solve $F'\ x\ 0$

$\frac{10\ x}{x^2\ \ 1^{\ 2}}\ \ 0$

$x\ \ 0$

The only critical values is 0 .

We use 0 to divide the real number line into two intervals,

A: , 0 and B: 0, :

A: Test 1,

$F'1\ \frac{10\ 1}{1^2\ \ 1^{\ 2}}\ \frac{10}{4}\ \ \frac{5}{2}\ 0$

B: Test 1,

$\frac{10\ 1}{} \quad \frac{10}{} \quad \frac{5}{}$

2 4 2

increasing on $0, $. Thus, a relative minimum occurs at $x\ 0$.

We find $G\ 0$:

$$G\ 0 \quad \frac{8}{0^2 \quad 1} \quad 8$$

Thus, there is a relative minimum at $0, 8$.

We use the information obtained to sketch the graph. Other function values are listed at the top of the next column.

2

$F'\ 1 \qquad 2 \qquad 0$

1 1

We see that $F\ x$ is increasing on $, 0$ and decreasing on $0, $. Thus, a relative maximum occurs at $x\ 0$.

We find $F\ 0$:

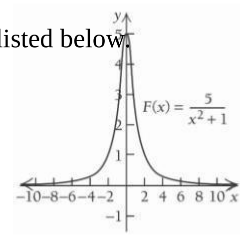
$$F\ 0 \quad \frac{5}{0^2 \quad 1} \quad 5$$

Thus, there is a relative maximum at $0, 5$.

The solution is continued on the next page.

We use the information obtained on the previous page to sketch the graph. Other function values are listed below.

x	$F(x)$
3	$\frac{1}{2}$
2	1
1	$\frac{5}{2}$
1	$\frac{5}{2}$
2	1
3	$\frac{1}{2}$



29. $g(x) = \frac{4x}{x^2+1}$

First, find the critical points.

$g'(x) = \frac{(x^2+1)(4) - 4x(2x)}{(x^2+1)^2}$ Quotient Rule

$= \frac{4x^2 + 4 - 8x^2}{(x^2+1)^2}$

$= \frac{4 - 4x^2}{(x^2+1)^2}$

$g'(x)$ exists for all real numbers. We solve $g'(x) = 0$

$\frac{4 - 4x^2}{(x^2+1)^2} = 0$

$4 - 4x^2 = 0$ Multiplying by $(x^2+1)^2$

$x^2 - 1 = 0$ Dividing by -4

A: Test -2, $g'(-2) = \frac{4 - 4(-2)^2}{((-2)^2+1)^2} = -\frac{12}{25} < 0$

B: Test 0, $g'(0) = \frac{4 - 4(0)^2}{((0)^2+1)^2} = 4 > 0$

C: Test 2, $g'(2) = \frac{4 - 4(2)^2}{((2)^2+1)^2} = -\frac{12}{25} < 0$

We see that $g(x)$ is decreasing on $(-\infty, -1)$, increasing on $(-1,1)$, and decreasing again on $(1, \infty)$. So there is a relative minimum at $x = -1$ and a relative maximum at $x = 1$. We find $g(-1)$:

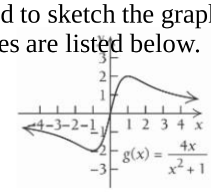
$g(-1) = \frac{4(-1)}{(-1)^2+1} = \frac{-4}{2} = -2$

Then we find $g(1)$:

$g(1) = \frac{4(1)}{(1)^2+1} = \frac{4}{2} = 2$

There is a relative minimum at $(-1, -2)$, and there is a relative maximum at $(1, 2)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$g(x)$
−3	$-\frac{6}{5}$
−2	$-\frac{8}{5}$
0	0
2	$\frac{8}{5}$
3	$\frac{6}{5}$

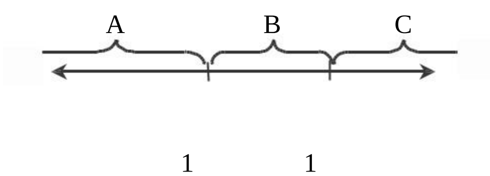


x 1

$x \quad 1$

The critical values are 1 and 1 . We use them to divide the real number line into three intervals,

A: , 1, B: 1,1, and C:1, .



We use a test value in each interval to determine the sign of the derivative in each interval.

30. $g\ x \quad \frac{x^2}{x^2 \quad 1}$

$g\ '\ x \quad \frac{x^2 \quad 1 \quad 2\ x \quad x^2 \quad 2\ x}{x^2 \quad 1^{\ 2}}$

$g\ '\ x \quad \frac{2\ x}{x^2 \quad 1^{\ 2}}$

The solution is continued on the next page.