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Chapter 2

Differentiation: Basic Concepts

2.1 The Derivative

If $f(x) = 4$, then $f(x + h) = 4$. The difference quotient (DQ) is

$$\frac{f(x+h)-f(x)}{h} = \frac{4-4}{h} = 0.$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = 0$$

The slope is $m = f'(0) = 0$.

$f(x) = 3$

The difference quotient is

$$\frac{f(x+h)-f(x)}{h} = \frac{3-(3)}{h} = 0$$

The slope of the line tangent to the graph of f at $x = 1$ is $f'(1) = 7$.

If $f(x) = 2x^2 - 3x - 5$, then

$f(x+h) = 2(x+h)^2 - 3(x+h) - 5$.

$h \quad 0h \quad h$

The difference quotient (DQ) is

$\frac{f(x h) f(x)}{h}$.

Then $f'(x) \lim_{h 0} 0 0$.

The slope of the line tangent to the graph

of f at $x = 1$ is $f'(1) 0$.

If $f(x) = 5x 3$, then
$f(x + h) = 5(x + h) 3$.

$\frac{[2(x h)^2 3(x h) 5] [2x^2 3x 5]}{h}$

$\frac{4xh 2(h)^2 3h}{h}$

$4x 2h 3$

$f'(x) \lim_{h 0} \frac{f(x h) f(x)}{h} 4x 3$

The difference quotient (DQ) is
$\frac{f(x h) f(x)}{} [5(x h) 3] [5x 3]$

h h

$\frac{5h\ h}{5}$

$f(x) \lim_{h 0} \frac{f(x\ h)\ f(x)}{h}\ 5$

The slope is $m\ \ f(2)\ 5$.

$f(x) = 2\ 7x$

The difference quotient is

$\frac{(x\ h)\ f(x)}{h}\ \frac{(2\ 7(x\ h))\ (2\ 7x)}{h}$

$\frac{2\ 7x\ 7h\ 2\ 7x}{h}$

$\frac{7h}{h}\ 7$

Then $f(x)\ \lim_{h 0}(7)\ 7$.

The slope is $m\ \ f(0)\ 3$.

6. $f(x)\ x^2\ \ 1$

The difference quotient is

$\frac{f(x\ h)\ f(x)}{h}\ \frac{((x\ h)^2\ \ 1)\ (x^2\ \ 1)}{h}$

$\frac{x^2\ \ 2hx\ h^2\ \ 1\ x^2\ \ 1}{h}$

$\frac{2hx\ h^2}{h}\ \ 2x\ h$

Then $f(x)\ \lim_{h 0}(2x\ h)\ \ 2x$.

The slope of the line tangent to the graph of f at $x\ \ 1$ is $f(1)\ 2$.

Chapter 2. Differentiation: Basic Concepts 89

7. If $f(x) = x^3 - 1$, then

$f(x+h) = (x+h)^3 - 1$

$= (x^2 + 2xh + h^2)(x+h) - 1$

$\frac{f(x+h) - f(x)}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 1 - (x^3 - 1)}{h}$

$= \frac{3x^2h + 3xh^2 + h^3}{h}$

$= \frac{h(3x^2 + 3xh + h^2)}{h}$

$= 3x^2 + 3xh + h^2$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} 3x^2 + 3xh + h^2$

$= 3x^2$

The slope is $m = f'(2) = 3(2)^2 = 12$.

$f(x) = x^3$

The difference quotient is

$\frac{f(x+h) - f(x)}{h} = \frac{((x+h)^3) - (x^3)}{h}$

$= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$

$= \frac{3x^2h + 3xh^2 + h^3}{h}$

$= 3x^2 + 3xh + h^2$

Then

2 2 2

$\frac{g(t+h) - g(t)}{h} = \frac{\frac{2}{t+h} - \frac{2}{t}}{h}$

$= \frac{\frac{2t - 2(t+h)}{t(t+h)}}{h} = \frac{2t - 2(t+h)}{h(t)(t+h)}$

$= \frac{-2}{t(t+h)}$

$g'(t) = \lim_{h \to 0} \frac{g(t+h) - g(t)}{h} = -\frac{2}{t^2}$

The slope is $m = g'(1) = -\frac{2}{1} = -2$.

10. $f(x) = \frac{1}{x^2}$

The difference quotient is

$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

$= \frac{\frac{1}{x^2 + 2hx + h^2} - \frac{1}{x^2}}{h}$

$= \frac{\frac{x^2 - (x^2 + 2hx + h^2)}{(x^2 + 2hx + h^2)x^2}}{h}$

$= \frac{-2hx - h^2}{h(x^2 + 2hx + h^2)x^2}$

$= \frac{-2x - h}{(x^2 + 2hx + h^2)x^2}$

Then $f'(x) = \lim_{h \to 0} \frac{-2x - h}{(x^2 + 2hx + h^2)x^2} = -\frac{2x}{x^4}$

$(x) \lim_{h \to 0} (3x \quad 3xh\ h \quad) 3x \quad .$

The slope of the line tangent to the graph

of f at $x = 1$ is $f(1)$ 3.

9. If $g(t) \ \frac{2}{t}$, then $g(t\ h) \ \frac{2}{t\ h}$.

The difference quotient (DQ) is

$\_\_.$

x^3

The slope of the line tangent to the graph of f at $x \quad 2$ is $f(2) \quad .\frac{1}{4}$

11. If $H(u) \quad \frac{1}{\sqrt{u}}$, then $H(u\ h) \quad \frac{1}{\sqrt{\ \ u\ h}}$.

Chapter 2. Differentiation: Basic Concepts 89

The difference quotient is

Chapter 2. Differentiation: Basic Concepts 89

(x h) f (x)

h

1 1

u √h u √u u √ √h h

√u √u h

[redacted] u h √ u √ u h √

h √[redacted] √ u h √ √ u u h

(u h)

h √[redacted] √ u h u √ √ u h

h

h √[redacted] √ u h u √ √ u h

1

√u √u h √u √u h

$H(u) \lim_{h \to 0} \frac{f(x h) f(x)}{h}$

lim 1

h0 √ √ √

u u h u u h

1

u u [redacted] √ √

√ √ √ √

1

u 2 √u

1

2u [redacted]

The slope is m H (4) 1 1 .

√

f (x) x √

Then $f(x) \lim_{h0} \frac{1}{\sqrt{x h} \sqrt{x}} \frac{1}{2\sqrt{x}}$

The slope of the line tangent to the graph of f at x = 9 is f (9) $\frac{1}{6}$.

If f(x) = 2, then f(x + h) = 2. The difference quotient (DQ) is

$\frac{f(x h) f(x)}{h} \frac{2 2}{h} 0.$

$f(x) \lim_{h0} \frac{f(x h) f(x)}{h} \lim_{h0} 0 0$

The slope of the tangent is zero for all values of x. Since f(13) = 2.

y 2 = 0(x 13), or y = 2.

For f (x) 3 ,

$f(x) \lim_{h \; 0} \frac{f(x h) f(x)}{h} \lim_{h \; 0} \frac{3 3}{h} 0$

for all x. So at the point c 4 , the slope of the tangent line is m f (4) 0 . The point (4, 3) is on the tangent line so by the point-slope formula the equation of the

tangent line is y 3 0[x (4)] or

3 .

If f(x) = 7 2x, then f(x + h) = 7 2(x + h).

The difference quotient is

(x h) f (x)

Chapter 2. Differentiation: Basic Concepts 89

2(4) 4 16

The difference quotient (DQ) is f (x h) f (x) [7 2(x h)] [7 2 x]

h

h

2

f x m f

(x h) f (x)

2

( ) li

Chapter 2. Differentiation: Basic Concepts

h

$x\ h\ x\ h$ [redacted]

$x\ h\quad x\quad x\ h\quad x$

$h\quad x$

$x\ h\ x$

[redacted] x

h

$h\ x\ h\quad x$

1

$x\ h\quad x$

$h0\qquad h$

The slope of the line is $m\ \ f(5)\ 2$.

Since $f(5) = 3$, $(5, 3)$ is a point on the curve and the equation of the tangent line is $y\ (3) = 2(x\ 5)$ or $y = 2x + 7$.

For $f(x) = 3x$,

$$f(x) \lim_{h0} \frac{f(x\ h)\ f(x)}{h}$$

$$\lim_{h0} \frac{3x\ 3h\ 3x}{h}$$

3

for all x. So at the point $c = 1$, the slope of

Chapter 2. Differentiation: Basic Concepts

the tangent line is $m = f'(1) = 3$. The point (1, 3) is on the tangent line so by the point-slope formula the equation of the tangent line is $y - 3 = 3(x - 1)$ or $y = 3x$.

17. If $f(x) = x^2$, then $f(x+h) = (x+h)^2$.

The difference quotient (DQ) is

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{2xh + h^2}{h} = 2x + h$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 2x$$

The slope of the line is $m = f'(1) = 2$.

Since $f(1) = 1$, (1, 1) is a point on the curve and the equation of the tangent line is $y - 1 = 2(x - 1)$ or $y = 2x - 1$.

For $f(x) = 2 - 3x^2$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(2 - 3(x+h)^2) - (2 - 3x^2)}{h} = \lim_{h \to 0} (-6x - 3h) = -6x$$

for all x. At the point $c = -1$, the slope of the tangent line is $m = f'(-1) = 6$. The point $(-1, -1)$ is on the tangent line so by the point-slope formula the equation of the tangent line is $y - (-1) = 6(x - (-1))$ or $y = 6x + 5$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = -\frac{2}{x^2}$$

The slope of the line is $m = f'(-1) = -2$. Since $f(-1) = -2$, $(-1, -2)$ is a point on the curve and the equation of the tangent line is $y + 2 = -2(x - (-1))$ or $y = -2x - 4$.

For $f(x) = \frac{3}{x^2}$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{3}{(x+h)^2} - \frac{3}{x^2}}{h} = \lim_{h \to 0} \frac{-6x - 3h}{(x+h)^2 x^2} = -\frac{6}{x^3}$$

At the point $c = \frac{1}{2}$, the slope of the tangent line is $m = f'\left(\frac{1}{2}\right) = -48$. The point $\left(\frac{1}{2}, 12\right)$ is on the tangent line so by the point-slope formula the equation of the tangent line is

$y - 12 = -48\left(x - \frac{1}{2}\right)$ or $y = -48x + 36$.

$f(x) = \frac{2}{x}$, then $\frac{2}{}$

Chapter 2. Differentiation: Basic Concepts

First we obtain the derivative of
$g(x) = \sqrt{x}$.

19. If x $f(xh)$. $x\ h$

The difference quotient is

The difference quotient (DQ) is

$$\frac{f(x\ h)\ f(x)}{h} \qquad \frac{\frac{2}{x\ h}\ \frac{2}{x}}{h}$$

$$\frac{x}{h}\ \frac{2}{x}\ \frac{2}{h}\ \frac{x(\ x\ h)}{x(\ x\ h)}$$

$$\frac{2\ x\ 2(\ x\ h)}{h(\ x)(\ x\ h)}$$

$$\frac{2}{x(\ x\ h)}$$

$$\frac{g(x+h)-g(x)}{h}$$
$$=\frac{\sqrt{x+h}-\sqrt{x}}{h}$$
$$=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$
$$=\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$
$$=\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$
$$=\frac{1}{\sqrt{x+h}+\sqrt{x}}$$
$$g'(x)=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

Then
$$\frac{d}{dx}k\cdot f(x)=k\cdot\frac{d}{dx}f(x),$$

Now since

$f(x)\ 2\ \frac{1}{2\sqrt{x}}\ \frac{1}{\sqrt{x}}$.

The slope is $m \quad f(4)\ \frac{1}{2}$, $f(4) = 4$, the equation of the tangent line is

$y\ 4\ \frac{1}{2}(x\ 4)$, or $\quad y\ \frac{1}{2}\ x\ 2.$

22. For $f(x)\ \frac{1}{\sqrt{x}}$,

$f(x) \lim_{h 0} \frac{f(x\ h)\ f(x)}{h}$

$\lim_{h 0} \frac{xh \ldots \sqrt{\ }}{h}$

$\lim_{h 0} \frac{\sqrt{x}\ x\ h\sqrt{\ }}{h\ \sqrt{2}\ x\ xh}$

$\lim \frac{x\ (x\ h)}{\ldots}$

So at the point $c = 1$, the slope of the tangent line is $f(1)\ \frac{1}{2}$. The point $(1,\ \frac{\ }{2}$ is on the tangent line so by the point-slope formula, the equation of the tangent line is $\quad y\ 1\ \frac{1}{2}(x\ 1)$ or $\quad y\ \frac{1}{2}\ x\frac{3}{2}$.

23. If $f(x)\ \frac{1}{x^3}$, then $f(x\ h)\ \frac{1}{(x\ h)^3}$.

The difference quotient (DQ) is

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h}\cdot\frac{x^3(x+h)^3}{x^3(x+h)^3}$$
$$=\frac{x^3-(x+h)^3}{hx^3(x+h)^3}$$
$$=\frac{x^3-(x^3+3x^2h+3xh^2+h^3)}{hx^3(x+h)^3}$$
$$=\frac{-3x^2h-3xh^2-h^3}{hx^3(x+h)^3}$$
$$=\frac{h(-3x^2-3xh-h^2)}{hx^3(x+h)^3}$$
$$=\frac{-3x^2-3xh-h^2}{x^3(x+h)^3}$$

$f(x) \lim_{h 0} \frac{f(x\ h)\ f(x)}{h}$

$\lim_{h 0} \frac{3x^2\ 3xh\ h^2}{x^3\ (x\ h)^3}$

$\frac{3x^2}{x^3\ (x)^3}$

$3\ \frac{\ }{\ }$.

$$x \sqrt{\ } \quad \sqrt{x\ h}$$

$_{h0}\ h$

1

lim

4

The slope is $m\ f(1) - \frac{3}{} \ 3.$

$h0 \sqrt{\ } 2$

$\sqrt{\ \ x \ \ xh} \sqrt{\ x} / \sqrt{\sqrt{x\ h}}$

4

(1)

$\frac{1}{\sqrt{x}^2 \ 2\sqrt{x}}$

$\frac{1}{\ }$

$x^{3/2}$

Further, $f(1) = 1$ so the equation of the line is y 1 = 3(x 1), or $y = 3x + 4$.

From Exercise 7 of this section

$f(x)\ 3x^2$. At the point c 1 , the slope of the tangent line is $m = f'(1) = 3$. The

point (1, 0) is on the tangent line so by the point-slope formula the equation of the

tangent line is $y - 0 = 3(x - 1)$ or $y = 3x - 3$.

If $y = f(x) = 3$, then $f(x + h) = 3$. The difference quotient (DQ) is

$$\frac{f(x+h) - f(x)}{h} = \frac{3 - 3}{h} = 0.$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 0$$

$\frac{dy}{dx} = 0$ when $x = 2$.

26. For $f(x) = 17$, $\frac{dy}{dx}$ at $x_0 = 14$ is

$$f'(14) = \lim_{h \to 0} \frac{f(14+h) - f(14)}{h}$$

$$= \lim_{h \to 0} \frac{17 - (17)}{h}$$

$$= \lim_{h \to 0} \frac{0}{h}$$

$$= \lim_{h \to 0} 0 = 0$$

If $y = f(x) = 3x + 5$, then $f(x + h) = 3(x + h) + 5 = 3x + 3h + 5$. The difference quotient (DQ)

is $\frac{f(x+h) - f(x)}{h} = \frac{3x + 3h + 5 - (3x + 5)}{h}$

$$= \frac{3h}{h} = 3$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 3 = 3$$

dy

29. If $y = f(x) = x(1 - x)$, or $f(x) = x - x^2$,

then $f(x + h) = (x + h) - (x + h)^2$.

The difference quotient (DQ) is $\frac{f(x+h) - f(x)}{h}$

$$= \frac{[(x+h) - (x+h)^2] - [x - x^2]}{h}$$

$$= \frac{h - 2xh - h^2}{h}$$

$$= 1 - 2x - h$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 1 - 2x$$

$\frac{dy}{dx} = -3$ when $x = 1$.

30. For $f(x) = x^2 - 2x$, $\frac{dy}{dx}$ at $x_0 = 1$ is

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{((1+h)^2 - 2(1+h)) - (1^2 - 2(1))}{h}$$

$$= \lim_{h \to 0} \frac{h^2}{h}$$

$$= 0$$

If $y = f(x) = x - \frac{1}{x}$, then

$$f(x+h) = x + h - \frac{1}{x+h}.$$

The difference quotient (DQ) is

$$\frac{f(x+h) - f(x)}{h} = \frac{x + h - \frac{1}{x+h} - \left(x - \frac{1}{x}\right)}{h}$$

$$= \frac{h - \frac{1}{x+h} + \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \frac{hx(x+h) - x + x + h}{}$$

Chapter 2. Differentiation: Basic Concepts 97

dx 3 when $x = -1$.

28. For $f(x)$ 6 $2x$, $\frac{dy}{dx}$ at x_0 3 is

$f(3)$ lim f (3 h) f (3)

$h0 \quad h$

$$\lim_{h0} \frac{(6\ 2(3\ h))\ (6\ 2(3))}{h}$$

$$\lim_{h0} \frac{2h}{h}$$

2

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} x^2 \quad hx \quad 1$$

$$= x^2 \quad 1$$

When $x = 1$, $\frac{dy}{dx} = (1)^2 \quad 1 \quad 2.$

32. For $f(x) = \frac{1}{2+x}$, $\frac{dy}{dx}$ at $x_0 = 3$ is

$$f'(3) = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{1}{2+(3+h)} - \frac{1}{2+(3)}}{h}$$

$$= \lim_{h\to 0}\frac{1}{5(5+h)}$$

$$= \frac{1}{25}$$

33. (a) If $f(x) = x^2$, then $f(2) = (2)^2 = 4$ and $f(1.9) = (1.9)^2 = 3.61$. The slope of the secant line joining the points (2, 4) and (1.9, 3.61) on the graph of f is

$$m_{\text{sec}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3.61 - 4}{1.9 - (2)} = 3.9.$$

34. (a)
$$m = \frac{f\left(\frac{1}{2}\right) - f(0)}{\frac{1}{2} - 0}$$

$$= \frac{2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^2 - (2(0) - 0^2)}{\frac{1}{2}}$$

$$= \frac{\frac{3}{4} - 0}{\frac{1}{2}}$$

$$= \frac{3}{2}$$

$$f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0}\frac{2h - h^2 - 0}{h}$$

$$= \lim_{h\to 0}(2 - h)$$

$$= 2$$

The answer is part (a) is a relatively good approximation to the slope of the tangent line.

35. (a) If $f(x) = x^3$, then $f(1) = 1$,

$f(1.1) = (1.1)^3 = 1.331.$

The slope of the secant line joining the points (1, 1) and (1.1, 1.331) on the graph of f is

$$\frac{y_2 - y_1}{} = \frac{1.331 - 1}{}$$

(b) If $f(x) = x^2$, then

$$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2.$$

The difference quotient (DQ) is

$$\frac{f(x+h)-f(x)}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{2xh + h^2}{h}$$

m_{sec} 3.31.

x_2 x_1 1.1 1

If $f(x) = x^3$, then f

$(x+h) = (x+h)^3$.

$\frac{h(2x\ h)}{h}$

$2x\ h$

Chapter 2. Differentiation: Basic Concepts 100

The difference quotient (DQ)

is

3 3

$(x h) f (x) \quad (x h) \quad x$

$f(x) \lim f (x h) f (x)$

$h0 \quad h$

$\lim 2 x h$

$h0$

x

The slope of the tangent line at the point (2, 4) on the graph of *f* is

$m_{tan} f(2) 2(2) 4.$

$h \quad 2 \quad h \quad 2 \quad 3$

$\frac{3x \quad h\, 3xh \quad h}{h}$

$3x^2 \quad 3xh\, h^2$

$f (x h) f (x)$

$f (x) \lim \quad 3x^2$

$h0 \quad h$

The slope is $m_{tan} \quad f(1) 3.$

Notice that this slope was approximated by the slope of the secant in part (a).

Chapter 2. Differentiation: Basic Concepts

Chapter 2. Differentiation: Basic Concepts 100

36. (a) $m = \dfrac{f\left(\frac{1}{2}\right) - f(1)}{\frac{1}{2} - (1)}$

$= \dfrac{\frac{1}{\frac{1}{2}+1} - \frac{1}{1+1}}{\frac{1}{2} - 1}$

$= \dfrac{\frac{1}{3} - \frac{1}{2}}{-\frac{1}{2}}$

$= \dfrac{1}{3}$

$f'(1) = \lim\limits_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$

$= \lim\limits_{h \to 0} \dfrac{\frac{1}{1+h+1} - \frac{1}{1+1}}{h}$

$= \lim\limits_{h \to 0} \dfrac{-1}{2(h+2)}$

$= -\dfrac{1}{4}$

The answer in part (a) is a relatively good approximation to the slope of the tangent line.

37. (a) If $f(x) = 3x^2 - x$, the average rate of change of f is $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$.

Since $f(0) = 0$ and

$f\left(\frac{1}{16}\right) = 3\left(\frac{1}{16}\right)^2 - \frac{1}{16} = -\frac{13}{256}$,

$\dfrac{f(x) - f(x)}{} \quad \dfrac{-\frac{13}{256} - 0}{}$

$\dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{3(x+h)^2 - (x+h) - (3x^2 - x)}{h}$

$= \dfrac{3x^2 + 6xh + 3h^2 - x - h - 3x^2 + x}{h}$

$= \dfrac{6xh + 3h^2 - h}{h}$

$= 6x + 3h - 1.$

$f'(x) = \lim\limits_{h \to 0} (6x + 3h - 1) = 6x - 1$

The instantaneous rate of change at $x = 0$ is $f'(0) = -1$. Notice that this rate is estimated by the average rate in part **(a)**.

38. (a) $f_{ave} = \dfrac{f\left(\frac{1}{2}\right) - f(0)}{\frac{1}{2} - 0}$

$= \dfrac{\frac{1}{2} - 2\left(\frac{1}{2}\right)^2 - (0 - 2(0))}{\frac{1}{2}}$

$= \dfrac{0 - 0}{\frac{1}{2}}$

$= 0$

(b) $f'(0) = \lim\limits_{h \to 0} \dfrac{f(0+h) - f(0)}{h}$

$= \lim\limits_{h \to 0} \dfrac{h(1 - 2h) - 0}{h}$

$= \lim\limits_{h \to 0} (1 - 2h)$

$= 1$

Chapter 2. Differentiation: Basic Concepts

Chapter 2. Differentiation: Basic Concepts 106

2 1 256
x x 1 0
2 1 16
13

16
0.8125.

The answer in part a is not a very good approximation to the average rate of change.

1

(a) If *s*(*t*) *t* 1 , the average rate of

s(*t*2) *s*(*t*1)

Chapter 2. Differentiation: Basic Concepts

Chapter 2. Differentiation: Basic Concepts 100

(b) If $f(x)\ 3x^2\ x,$ then

$(x\ h)\ 3(x\ h)^2\ (x\ h).$

The difference quotient (DQ) is

change of s is $t_2\ t_1$.

1 1

Since s 3 and

1

$s(0)$ $\frac{0^2\ 1}{0\ 1}$ 1, $\frac{3\ \frac{1}{2}^2\ 1}{\frac{1}{2}\ 0}$ 4.

Chapter 2. Differentiation: Basic Concepts

If $s(t) = \frac{t-1}{t+1}$, then

$$s(t+h) = \frac{(t+h)-1}{(t+h)+1}.$$

The difference quotient (DQ) is

$$\frac{s(t+h)-s(t)}{h} = \frac{\frac{t+h-1}{t+h+1} - \frac{t-1}{t+1}}{h}.$$

Multiplying numerator and denominator by $(t + h + 1)(t + 1)$.

$$\frac{(t+h-1)(t+1)-(t-1)(t+h+1)}{h(t+h+1)(t+1)}$$

$$= \frac{t^2+th-t+t+h-1-t^2-th-t+t+h+1}{h(t+h+1)(t+1)}$$

$$= \frac{2h}{h(t+h+1)(t+1)}$$

$$= \frac{2}{(t+h+1)(t+1)}$$

$$s'(t) = \lim_{h\to 0} \frac{2}{(t+h+1)(t+1)} = \frac{2}{(t+1)^2}$$

The instantaneous rate of change when $t = -\frac{1}{2}$ is

$$s'\left(-\frac{1}{2}\right) = \frac{2}{\left(-\frac{1}{2}+1\right)^2} = 8.$$

Notice that the estimate given by the average rate in part **(a)** differs significantly.

40. (a) $s_{ave} = \frac{s\left(\frac{1}{4}\right) - s(1)}{\frac{1}{4} - 1}$

$$= \frac{\sqrt{\frac{1}{4}} - \sqrt{1}}{-\frac{3}{4}} = \frac{1}{3}$$

$$s'(1) = \lim_{h\to 0} \frac{s(1+h)-s(1)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}$$

$$= \lim_{h\to 0} \frac{1+h-1}{h\left(\sqrt{1+h}+1\right)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1}$$

$$= \frac{1}{2}$$

The answer in part a is a relatively good approximation to the instantaneous rate of change.

(a) The average rate of temperature change between t_0 and $t_0 + h$ hours after midnight. The instantaneous rate of temperature change t_0 hours after midnight.

The average rate of change in blood alcohol level between t_0 and $t_0 + h$ hours after consumption. The instantaneous rate of change in blood alcohol level t_0 hours after consumption.

The average rate of change of the 30-year fixed mortgage rate between

Chapter 2. Differentiation: Basic Concepts

4 t_0 and t_0 h years after 2005. The instantaneous rate of change of 30-year fixed mortgage rate t_0 years after 2005.

(a) ... the averag

e rate of change of revenue when the production level changes from x_0 to x_0 h units.

Chapter 2. Differentiation: Basic Concepts

$\frac{2}{3}$

... the instantaneous rate of change of revenue when the production level is x_0 units.

... the average rate of change in the fuel level, in lb/ft, as the rocket travels between x_0 and x_0 h feet above the ground.

Chapter 2. Differentiation: Basic Concepts

... the instantaneous rate in fuel level when the rocket is x_0 feet above the ground.

... the average rate of change in volume of the growth as the drug dosage changes from x_0 to $x_0 h$ mg.

... the instantaneous rate in the growth's volume when x_0 mg of the drug have been injected.

$P(x) = 4{,}000(15\ x)(x\ 2)$

The difference quotient (DQ) is

$$\frac{P(x\ h)\ P(x)}{h}$$

$$\frac{[4{,}000(15\ (x\ h))((x\ h)\ 2)]}{h}$$

$$\frac{[4{,}000(15\ x)(x\ 2)]}{h}$$

$$\frac{4{,}000[(15\ x\ h)(x\ h\ 2)\ (15\ x)(x\ 2)]}{h}$$

$$\frac{4{,}000(17h\ 2xh\ h^2)}{h}$$

$4{,}000(17\ 2x\ h)$

$$P(x)\ \lim_{h\ 0} \frac{P(x\ h)\ P(x)}{h}$$

$4{,}000(17\ 2x)$

$P(x)\ 0$ when $4{,}000(17\ 2x) = 0$.

$x\ \frac{17}{2}\ 8.5$, or 850 units.

When $P(x)\ 0$, the line tangent to the graph of P is horizontal. Since the graph of P is a parabola which opens down, this horizontal tangent indicates a maximum profit.

(a) Profit = (number sold)(profit on each)
Profit on each
selling price cost to obtain
$P(p)\ (120\ p)(p\ 50)$

Since $q = 120\ p$, $p = 120\ q$. $P(q) = q[(120\ q)\ 50]$

The average rate as q increases from $q = 0$ to $q = 20$ is

$$\frac{P(20)\ P(0)}{20}\ \frac{[70(20)\ (20)^2]\ 0}{20}$$

\$50 per recorder

The rate the profit is changing at $q = 20$ is $P(20)$.

The difference quotient is

$$\frac{P(q\ h)\ P(q)}{h}$$

$$\frac{[70(q\ h)\ (q\ h)^2]\ [70q\ q^2]}{h}$$

$$\frac{70q\ 70h\ q^2\ 2qh\ h^2\ 70q\ q^2}{h}$$

$$\frac{70h\ 2qh\ h^2}{h}$$

$2q\ h$

$$P(q)\ \lim_{h\ 0} \frac{P(q h)\ P(q)}{h}\ 70\ 2q$$

$P(20)\ 70\ 2(20)$ \$30 per recorder.

Since $P(20)$ is positive, profit is increasing.

Chapter 2. Differentiation: Basic Concepts
Chapter 2. Differentiation: Basic Concepts 106

nge is

45.

$$C(x) = 0.04x^2 + 2.1x + 60$$

$$C(11) = 0.04(11)^2 + 2.1(11) + 60 = 87.94$$

$$C(10) = 0.04(10)^2 + 2.1(10) + 60 = 85$$

$$\frac{87.94 - 85}{11 - 10} = 2.94$$

or $2,940 per unit.

As x increases from 10 to 11, the average rate of cha

Chapter 2. Differentiation: Basic Concepts
Chapter 2. Differentiation: Basic Concepts 107

or $P(q) \; q(70 \; q) \; 70q \; q^{2}$.

(b) $C(x+h)$

$= 0.04(x+h)^2 + 2.1(x+h) + 60$

So, the difference quotient (DQ) is

$$\frac{C(x+h)-C(x)}{h}$$

$$= \frac{\left[0.04(x+h)^2 + 2.1(x+h) + 60\right.}{h}$$

$$\frac{\left.-\left(0.04x^2 + 2.1x + 60\right)\right]}{h}$$

$$= \frac{\left[0.04x^2 + 0.08xh + 0.04h^2 + 2.1x\right.}{h}$$

$$\frac{\left.+2.1h + 60 - 0.04x^2 - 2.1x - 60\right]}{h}$$

$$= \frac{0.08xh + 0.04h^2 + 2.1h}{h}$$

$= 0.08x + 0.04h + 2.1$

46. (a) Q_{ave} $\frac{Q(3{,}100) \;\; Q(3{,}025)}{3{,}100 \;\; 3{,}025}$

3,100 3,100 3 ,100 3,025

75

3,1001 √ 1 55

75

28.01

The average rate of change in output is about 28 units per worker-hour.

$C'(x) = \lim_{h\to 0}(0.08x + 0.04h + 2.1)$

$= 0.08x + 2.1$

$C'(10) = 0.08(10) + 2.1 = 2.90$

or $2,900 per unit

The average rate of change is close to this value and is an estimate of this instantaneous rate of change.

Since $C'(10)$ is positive, the cost will increase.

 Chapter 2. Differentiation: Basic Concepts 106

$$Q'(3,025) \quad \lim_{h0} \frac{Q(3,025\ \ h)\ \ Q(3,025)}{h}$$

$$\lim_{h0} \frac{3,100\ \ 3,025\,h\ 3,1\ 00\ 3,\ 025}{h}$$

$$3,100$$

$$\lim_{h0} \frac{\sqrt{3,025\,h}\ \ 55}{h} \cdot \frac{\sqrt{3,025\,h}\ 55}{\sqrt{3,025\,h}\ \ 55}$$

$$\lim_{h0} \frac{3,100(3,025\,h\ 3,025)}{h\ \sqrt{3,025\,h}\ 55}$$

$$\lim_{h0} \frac{3,100}{\sqrt{3,025\,h}\ \ 55}$$

$$\frac{3,100}{110}$$

$$28.2$$

The instantaneous rate of change is 28.2 units per worker-hour.

Writing Exercise Answers will vary.

(a) $E(x)\ x\ D(x)$

$x(35x^2\ 200)$

(b) $E_{ave}\ \ \frac{E(5)\ E(4)}{5\ 4}$

$$35(5)^2\ \ 200(5)\ (35(4)^2\ \ 200(4))$$

$$240$$

$$115$$

The average change in consumer expenditures is \$115 per unit.

$$E'(4)\ \lim_{h0} \frac{E(4\ h)\ E(4)}{h}$$

$$\lim_{h0} \frac{35(4\ h)^2\ \ 200(4\ h)\ (35(4)^2\ \ 200(4))}{h}$$

$$\lim_{h0} \frac{35h^2\ \ 80h}{h}$$

$$\lim_{h0} (35h\ 80)$$

$$80$$

The instantaneous rate of change is \$80 per unit when $x = 4$. The expenditure is decreasing when $x = 4$.

When $t = 30$, $\frac{dV}{}\ \frac{65\ 50\ 3}{}$.

dt 5030 4

In the “long run,” the rate at which *V* is changing with respect to time is getting smaller and smaller, decreasing to zero.

Answers will vary. Drawing a tangent line at each of the indicated points on the curve shows the population is growing at approximately 10/day after 20 days and 8/day after 36 days. The tangent line slope is steepest between 24 and 30 days at approximately 27 days.

51. When h = 1,000 meters, $\frac{dT}{dh}$ $\frac{6\ \ 0}{2,000\ \ 1,000}$ $\frac{6}{1,000}$ 0.006C/meter

When h = 2,000 meters, $\frac{dT}{dh}$ 0C/meter.

Since the line tangent to the graph at h = 2,000 is horizontal, its slope is zero.

52. $P(t) = -6t^2 + 12t + 151$

(a) The average rate of change is $\frac{P(t_2)-P(t_1)}{t_2-t_1} = \frac{P(2)-P(0)}{2-0}$.

Since $P(2) = -6(2)^2 + 12(2) + 151 = 151$

and $P(0) = -6(0)^2 + 12(0) + 151 = 151$,

$$\frac{P(2)-P(0)}{2-0} = \frac{151-151}{2} = 0.$$

The population’s average rate of change for 2010–2012 is zero.

To find the instantaneous rate, calculate P’(2).

$P(t+h) = -6(t+h)^2 + 12(t+h) + 151$ so the difference quotient (DQ) is

$$\begin{aligned} \text{DQ} &= \frac{P(t+h)-P(t)}{h} \\ &= \frac{-6(t+h)^2 + 12(t+h) + 151 - \left(-6t^2 + 12t + 151\right)}{h} \\ &= \frac{-6t^2 - 12ht - 6h^2 + 12t + 12h + 151 + 6t^2 - 12t - 151}{h} \\ &= \frac{-12ht - 6h^2 + 12h}{h} \\ &= -12t - 6h + 12 \end{aligned}$$

$$P'(x) = \lim_{h\to 0} \text{DQ} = \lim_{h\to 0}\left(-12t - 6h + 12\right) = -12t + 12$$

For 2012, t = 2, so the instantaneous rate of change is P’(2) = –12(2) + 12 = –12, or a decrease of 12,000 people/year.

$H(t)$ $4.4t$ $4.9t^2$

(a)

$$\begin{aligned} H(t+h) &= 4.4(t+h) - 4.9(t+h)^2 \\ &= 4.4t + 4.4h - 4.9(t^2 + 2th + h^2) \\ &= 4.4t + 4.4h - 4.9t^2 - 9.8th - 4.9h^2 \end{aligned}$$

The difference quotient (DQ) is

$$\begin{aligned} \frac{H(t+h)-H(t)}{h} &= \frac{4.4t + 4.4h - 4.9t^2 - 9.8th - 4.9h^2 - (4.4t - 4.9t^2)}{h} \\ &= \frac{4.4h - 9.8th - 4.9h^2}{h} \\ &= \frac{h(4.4 - 9.8t - 4.9h)}{h} \\ &= 4.4 - 9.8t - 4.9h \end{aligned}$$

$$\begin{aligned} H'(t) &= \lim_{h\to 0} \frac{H(t+h)-H(t)}{h} \\ &= \lim_{h\to 0} 4.4 - 9.8t - 4.9h \\ &= 4.4 - 9.8t \end{aligned}$$

After 1 second, *H* is changing at a rate of *H* (1) 4.4 9.8(1) 5.4 m/sec, where the negative represents that *H* is decreasing.

H (*t*) 0 when 4.4 9.8*t* = 0, or

t 0.449 seconds.

This represents the time when the height is not changing (neither increasing nor decreasing). That is, this represents the highest point in the jump.

When the flea lands, the height *H*(*t*) will be zero (as it was when *t* = 0).

$4.4t$ $4.9t^2$ 0

(4.4 4.9*t*)*t* 0

4.4 4.9*t* 0

t $\frac{44}{49}$ 0.898 seconds

At this time, the rate of change is H $\frac{44}{49}$ 4.4 9.8$\frac{44}{49}$ 4.4 m/sec.

Again, the negative represents that *H* is decreasing.

(a) If *P*(*t*) represents the blood pressure function then *P*(0.7) 80 , *P*(0.75) 77 , and *P*(0.8) 85 .

The average rate of change on [0.7,0.75] is approximately $\frac{77\ 80}{0.5}$ 6 mm/sec while on [0.75, 0.8] the average rate of change is about $\frac{85\ 77}{}$ 16 mm/sec. The rate of change is greater in magnitude in the period following the burst of blood.

Writing exerciseanswers will vary.

Chapter 2 Differentiation: Basic Concepts Chapter 2. Differentiation: Basic Concepts 110

$D(p) = 0.0009p^2 + 0.13p + 17.81$ The

average rate of change is

$\frac{D(p_2) - D(p_1)}{p_2 - p_1}$.

Since

$D(60) = 0.0009(60)^2 + 0.13(60) + 17.81 = 22.37$

and

$D(61) = 0.0009(61)^2 + 0.13(61) + 17.81 = 22.3911$,

$\frac{22.3911 - 22.37}{61 - 60} = 0.0211$ mm per mm of mercury

$D(p + h) = 0.0009(p + h)^2 + 0.13$

So, the difference quotient (DQ) is

$\frac{D(p+h) - D(p)}{h}$

$= \frac{[0.0009(p+h)^2 + 0.13(p+h) + 17.81 - (0.0009p^2 + 0.13p + 17.81)]}{h}$

$= \frac{[0.0009p^2 + 0.0018ph + 0.0009h^2 + 0.13p + 0.13h + 17.81 - 0.0009p^2 - 0.13p - 17.81]}{h}$

$= \frac{0.0018ph + 0.0009h^2 + 0.13h}{h}$

$= 0.0018p + 0.0009h + 0.13$

$D'(x) = \lim_{h \to 0} (0.0018p + 0.0009h + 0.13) = 0.0018p + 0.13$

The instantaneous rate of change when $p = 60$ is

$D'(60) = 0.0018(60) + 0.13 = 0.022$ mm per mm of mercury. Since $D'(60)$ is positive, the pressure is increasing when $p = 60$.

$0.0018p + 0.13 = 0$

$p = -72.22$ mm of mercury

At this pressure, the diameter is neither increasing nor decreasing.

(a) The rocket is $h(6) = -576 + 1200 = 624$ feet above ground.

The average velocity between 0 and 40 seconds is given by

$\frac{h(6) - h(0)}{6} = \frac{624}{6} = 104$ feet/second.

(c) $h'(0) = 200$ ft/sec and $h'(40) = -1080$ ft/sec. The negative sign in the second velocity indicates the rocket is falling.

57. $s(t) = 4\sqrt{t+1} - 4$

$= 4(t+1)^{1/2} - 4$

(a) $s(t+h) = 4[(t+h)+1]^{1/2} - 4$

So, the difference quotient (DQ) is

$\frac{4(t+h+1)^{1/2} - 4 - [4(t+1)^{1/2} - 4]}{h}$

$= \frac{4(t+h+1)^{1/2} - 4 - 4(t+1)^{1/2} + 4}{h}$

$= \frac{4(t+h+1)^{1/2} - 4(t+1)^{1/2}}{h}$

Multiplying the numerator and denominator by

$4(t+h+1)^{1/2} + 4(t+1)^{1/2}$

gives

$$\frac{16(t+h+1)-16(t+1)}{h\left[4(t+h+1)^{1/2}+4(t+1)^{1/2}\right]}$$

$$=\frac{16t+16h+16-16t-16}{4h\left[(t+h+1)^{1/2}+(t+1)^{1/2}\right]}$$

$$=\frac{16h}{4h\left[(t+h+1)^{1/2}+(t+1)^{1/2}\right]}$$

$$=\frac{4}{(t+h+1)^{1/2}+(t+1)^{1/2}}$$

$$s'(t)=\lim_{h\to 0}\frac{4}{(t+h+1)^{1/2}+(t+1)^{1/2}}$$

$$=\frac{4}{(t+1)^{1/2}+(t+1)^{1/2}}$$

$$=\frac{4}{2(t+1)^{1/2}}$$

$$v_{\text{ins}}(t)=\frac{2}{(t+1)^{1/2}}=\frac{2}{\sqrt{t+1}}$$

(b) $v_{\text{ins}}(0)=\frac{2}{(0+1)^{1/2}}=\frac{2}{\sqrt{1}}=2\text{ m/sec}\sqrt{2}$

(c) $s(3)=4\sqrt{3+1}-4=8-4=4\text{ m}$

$$v_{\text{ins}}(3)=\frac{2}{\sqrt{3+1}}=\frac{2}{2}=1\text{ m/sec}$$

(a) $f(x) \lim_{h 0} \frac{(3(x h) 2) (3x 2)}{h}$

$\lim_{h 0} \frac{3h}{h}$

The line tangent to a straight line at any point is the line itself.

(a) For $y f(x) x^2$,

$(x h)(x h)^2$.

The difference quotient (DQ) is

$\frac{f(x h) f(x)}{h}$ $\frac{(x h)^2 x^2}{h}$

$\frac{2xh h^2}{h}$

$2x h$

$\frac{dy}{dx}$ $f(x)$

$\lim_{h0} \frac{f(x h) f(x)}{h}$

x

For y $f(x) x^2$ 3,

$f(x h)(x h)^2$ 3.

The difference quotient (DQ) is $[(x h)^2$ 3] $(x^2$ 3) $2xh h^2$

h h

$2x h$

$\frac{dy}{dx}$ $f(x)$

$\lim_{h0} \frac{f(x h) f(x)}{h}$

$2x$

The graph of y x^2 3 is the graph of $y x^2$ shifted down 3 units. So the graphs are parallel and their tangent lines have the same slopes for any value of x. This accounts geometrically for the fact that their derivatives are identical.

(b) Since $y x^2$ 5 is the parabola

2

Chapter 2. Differentiation: Basic Concepts

Chapter 2. Differentiation: Basic Concepts 110

3

At x 1 ,y 3(1) 2 5 and

(1, 5) is a point on the tangent line.

Using the point-slope formula with

x shifted up 5 units and the constant appears to have no effect on the derivative, the derivative of the

2

function y x 5 is also 2x.

Chapter 2. Differentiation: Basic Concepts

m 3 gives y (5) 3(x (1)) or
3x 2 .

Chapter 2. Differentiation: Basic Concepts

60. (a) For $f(x)$ x^2 $3x$, the derivative is

$f(x)$

$$\lim_{h0} \frac{[(x\ h)^2\quad 3(x\ h)]\ (x^2\quad 3x)}{h}$$

$$\lim_{h0} \frac{x^2\quad 2hx\ h^2\quad 3x\ 3h\ x^2\quad 3x}{h}$$

$$\lim_{h0}(2x\ h\ 3)\ 2x\ 3$$

(b) For $g(x)\ x^2$, the derivative is

$$g(x) \lim_{h0} \frac{(x\ h)^2\quad x^2}{h}$$

$$\lim_{h0} \frac{x^2\quad 2hx\ h^2\quad x^2}{h}$$

$$\lim_{h0}(2\ x\ h)$$

x

While for $h(x)\ 3x$, the derivative is

$$h(x) \lim_{h0} \frac{3(x\ h)\ 3x}{h} \lim_{h0} \frac{3h}{h}\ 3$$

The derivative of the sum is the sum of the derivatives.

The derivative of $f(x)$ is the sum of the derivative of $g(x)$ and $h(x)$.

61. (a) For y $f(x)\ x^2$,

$(x\ h)\ (x\ h)^2$.

The difference quotient (DQ) is f

$$\frac{(x\ h)\ f(x)}{h} \quad \frac{(x\ h)^2\quad x^2}{\frac{2}{2}}$$

$2\ x\ h$

dy

Chapter 2. Differentiation: Basic Concepts
Chapter 2. Differentiation: Basic Concepts 111

dx $f(x)$

$\lim f(x\ h)\ f(x)$

$(x\,h)^3\;x^3 \qquad \dfrac{3x^2\,h\;3xh^2\;h^3}{}$

$\dfrac{}{h} \qquad \dfrac{}{h}\;3x^2\;\;3xh\,h^2$

$\dfrac{dy}{dx}\;f(x)$

$\lim_{h0} \dfrac{f(x\,h)\,f(x)}{h}$

$3x^2$

The pattern seems to be that the derivative of x raised to a power (x^n) power decreased by one (nx^{n1}). So, the derivative of the function $y\,x^4$ is $4x^3$ and the derivative of the function $y\,x^{27}$ is $27\,x^{26}$.

If $y\;mx\;b$ then

$\dfrac{dy}{dx}\;\lim_{h0} \dfrac{[m(x\,h)\,b]\,(mx\,b)}{h}$

$\lim_{h0} \dfrac{mx\;mh\;b\;mx\;b}{h}$

$\text{li}_{h0}\;\dfrac{mh}{h}$

$\lim_{h0} m$

m, a constant.

When $x < 0$, the difference quotient (DQ)

is $\dfrac{f(x\,h)\,f(x)}{h}\;\dfrac{(x\,h)\,(x)}{h}\;\dfrac{h}{h}\;1$

So, $f(x) \lim_{h0} 1\;1.$

When $x > 0$, the difference quotient (DQ)

is $f(x\,h)\,f(x)\;(x\,h)\;x\;1.$

h h

Chapter 2. Differentiation: Basic Concepts 112

$h0$ h
x

For y $f(x)$ x^3 ,
$(x\ h)$ $(x\ h)^3$.
The difference quotient (DQ) is

So, $f(x)\ \lim_{h0} 1$ 1.

Since there is a sharp corner at $x = 0$ (graph changes from $y = x$ to $y = x$), the

graph makes an abrupt change in direction at $x = 0$. So, f is not differentiable at $x = 0$.

Chapter 2. Differentiation: Basic Concepts

(a) Write any number x as $x\ c\ h$. If

the value of x is approaching c, then h is approaching 0 and vice versa. Thus

the indicated limit is the same as the

limit in the definition of the derivative. Less formally, note that if

$x\ c$ then $\frac{f(x)\ f(c)}{x\ c}$ is the slope of

a secant line. s x approaches c the slopes of the secant lines approach the slope of the tangent at c.

$\lim_{xc} [f(x)\ f(c)]$

$\lim_{xc} \frac{f(x)\ f(c)}{x\ c} (x\ c)$

$\lim_{xc} [f(x)\ f(c)]$

$\lim_{xc} \frac{f(x)\ f(c)}{x\ c} \lim_{xc} (x\ c)$

$(c)\ 0$

0

using part (a) for the first limit on the right.

Using the properties of limits and the result of part (b)

$0 \lim_{xc} [f(x)\ f(c)]$

$\lim_{xc} f(x) \lim_{xc} f(c)$

$\lim_{xc} f(x)\ f(c)$

so $\lim_{xc} f(x)\ f(c)$ meaning $f(x)$ is

continuous at $x\ c$.

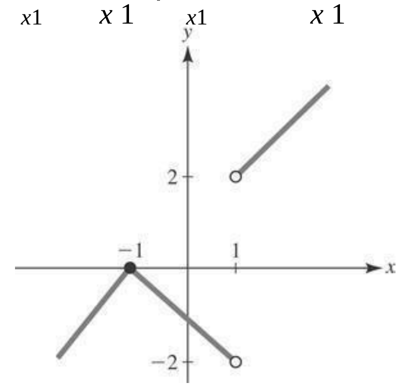
65. To show that $f(x) \frac{|x^2\ 1|}{x\ 1}$ is not

differentiable at $x = 1$,

press $y =$ and input (abs(x^2

$\lim_{x1} \frac{x^2\ 1}{x\ 1} \lim_{x1} \frac{(x\ 1)(x\ 1)}{x\ 1}\ 2$

$\lim_{x1} \frac{x^2\ 1}{x\ 1} \lim_{x1} \frac{(x\ 1)(x\ 1)}{x\ 1}\ 2$



Using the TRACE feature of a calculator

with the graph of $y\ 2\ x^3\ \ 0.8x^2\ \ 4$

shows a peak at x 0 and a valley at

0.2667 . Note the peaks and valleys are hard to see on the graph unless a small rectangle such as [0.3, 0.5] [3.8, 4.1] is used.

To find the slope of line tangent to the

graph of $f(x)\sqrt{x^2\ \ 2x}\ \ \sqrt{3x}$ at

$x = 3.85$, fill in the table below. The $x + h$ row can be filled in manually. For $f(x)$, press $y =$ and input

$\sqrt{x \wedge 2\ 2x}\ \ \sqrt{(3x)}$ for $y1$

Use window dimensions [1, 10]1 by [1, 10]1.
Use the value function under the calc

menu and enter $x = 3.85$ to find $f(x) = 4.37310$.

For $f(x + h)$, use the value function under the calc menu and enter $x = 3.83$ to find

Chapter 2. Differentiation: Basic Concepts

for $y1$ 1)) / (x 1)

$f(x + h) = 4.35192$. Repeat this process for

Chapter 2. Differentiation: Basic Concepts
Chapter 2. Differentiation: Basic Concepts 116

The abs is under the NUM menu in the math application.

Use window dimensions [4, 4]1 by [4, 4]1

Press Graph

We see that *f* is not defined at $x = 1$. There can be no point of tangency.

$x = 3.84, 3.849, 3.85, 3.851, 3.86,$ and 3.87.

The $\frac{f(x h) f(x)}{h}$ can be filled in manually given that the rest of the table is now complete. So, slope $f(3.85)$ 1.059.

Chapter 2. Differentiation: Basic Concepts

h			
$x + h$	0.02	0.01	0.001
	3.83	3.84	3.849
$f(x)$	4.37310	4.37310	4.37310
$f(x + h)$	4.35192	4.36251	4.37204
$\frac{f(x+h) - f(x)}{h}$	1.059	1.059	1.059

0	0.001	0.01	0.02
3.85	3.851	3.86	3.87
4.37310	4.37310	4.37310	4.37310
4.37310	4.37415	4.38368	4.39426
undefined	1.058	1.058	1.058

2.2 Techniques of Differentiation

Since the derivative of any constant is zero,

$y = 2$

$\frac{dy}{dx} = 0$

(Note: $y = 2$ is a horizontal line and all horizontal lines have a slope of zero, so $\frac{dy}{dx}$ must be zero.)

$y = 3$ $\frac{dy}{dx} = 0$

$y = 5x - 3$

$\frac{dy}{dx} = \frac{d}{dx}(5x) - \frac{d}{dx}(3)$

$\frac{dy}{dx} = 5 - 0 = 5$

4. $y = 2x + 7$

$\frac{dy}{dx} = 2(1) + 0 = 2$

$y = x^{7/3}$

$\frac{dy}{dx} = \frac{7}{3}x^{7/3-1} = \frac{7}{3}x^{4/3}$

$y = x^{3.7}$

$\frac{dy}{dx} = 3.7x^{3.7-1} = 3.7x^{2.7}$

$y = 4x^{1.2}$

$\frac{dy}{dx} = 4(1.2)x^{1.2-1} = 4.8x^{0.2}$

$y = r^2$

$\frac{dy}{dr} = 2r^{2-1} = 2r$

$y = \frac{4}{3}\pi r^3$

$\frac{dy}{dr} = \frac{4}{3}\pi(3r^2) = 4\pi r^2$

$y = \sqrt{2x} = \sqrt{2}\sqrt{x} = \sqrt{2}x^{1/2}$

$$\frac{dy}{dx} = \sqrt{2}\left(\frac{1}{2}x^{1/2-1}\right)$$

$$= \sqrt{2}\left(\frac{1}{2}x^{-1/2}\right)$$

$$= \sqrt{2}\cdot\frac{1}{2x^{1/2}} \text{ or } \frac{\sqrt{2}}{2\sqrt{x}}$$

12. $y = 2\sqrt[4]{x^3} = 2x^{3/4}$

$\frac{dy}{dx} = 2\left(\frac{3}{4}\right)x^{3/4-1} = \frac{3}{2}x^{-1/4} = \frac{3}{2\sqrt[4]{x}}$

13. $y = \frac{9}{\sqrt{t}} = 9t^{-1/2}$

Chapter 2. Differentiation: Basic Concepts

dx

5. *y x* 4

$\frac{dy}{dx}$ 9 1 *t* 1/21
2

9 1 *t* 3/ 2

2
9
or

2*t*3/ 2 √ 3

2 *t*

 Chapter 2. Differentiation: Basic Concepts 117

$\frac{dy}{dx}$ $4x^{41}$ $4x^5$ $\frac{4}{x^5}$

14. $y = \dfrac{3}{2t^2} = \dfrac{3}{2}t^{-2}$

$$\frac{dy}{dt} = \frac{3}{2}(-2t^{-3}) = -\frac{3}{t^3}$$

$y = x^2 + 2x + 3$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(2x) + \frac{d}{dx}(3)$$

$$\frac{dy}{dx} = 2x + 2$$

$y = 3x^5 - 4x^3 + 9x - 6$

$$\frac{dy}{dx} = 3(5x^4) - 4(3x^2) + 9(1) - 0$$

$$= 15x^4 - 12x^2 + 9$$

$y = x^9 - 5x^8 + x + 12$

$$\frac{dy}{dx} = \frac{d}{dx}(x^9) - \frac{d}{dx}(5x^8) + \frac{d}{dx}(x) + \frac{d}{dx}(12)$$

$$\frac{dy}{dx} = 9x^8 - 40x^7 + 1$$

$f(x) = \dfrac{1}{4}x^8 - \dfrac{1}{2}x^6 - x + 2$

$$f'(x) = 8\left(\frac{1}{4}\right)x^7 - 6\left(\frac{1}{2}\right)x^5 - 1 + 0 = 2x^7 - 3x^5 - 1$$

$f(x) = 0.02x^3 + 0.3x$

$$f'(x) = \frac{d}{dx}(0.02x^3) + \frac{d}{dx}(0.3x)$$

$$f'(x) = 0.02(3x^2) + 0.3 = 0.06x^2 + 0.3$$

3 0.3

20. $f(u)$ 0.07u 1.21u 3u 5.2

3 2

$f'(u)$ 4(0.07u) 3(1.21u) 3

3 2

0 0.28u 3.63u 3

21. $y=\frac{1}{t}+\frac{1}{t^2}-\frac{1}{\sqrt{t}}=t^{-1}+t^{-2}-t^{-1/2}$

$\frac{dy}{dt}$ $\frac{d}{dt}(t^{1})$ $\frac{d}{dt}(t^{2})$ $\frac{d}{dt}(t^{1/2})$

11 21 $\frac{1}{}$ 1/21

$1t$ $2t$ t 2

$1t^{2}$ $2t^{3}$ $\frac{1}{}$ $t^{3/2}$ 2

$\frac{1}{t^2}$ $\frac{2}{t^3}$ $\frac{1}{2t^{3/2}}$,

or $\frac{1}{t^2}$ $\frac{2}{t^3}$ $\frac{1}{2\sqrt{t^3}}$

22. y $\frac{3}{2x^2}$ $\frac{1}{2x^2}$ $\frac{2}{3x^3}$ $3x$ $2x$ $\frac{2}{3}$ $\frac{2}{3}$ $\frac{3}{3}$

$\frac{dy}{dx}$ $(1)(3x^{2})$ $(2)(2x^{3})$ (3) $\frac{2}{}x^{4}$

$\frac{2}{3x^3}$ $\frac{3}{4x^4}$ $\frac{4}{2x^2}$

$x^2\sqrt{x^3}$ $\frac{4}{\sqrt{x}}$ 1

23. $f(x)$ $\sqrt{x^3}$ $\sqrt[3]{x}\,x^{3/2}$ $x^{3/2}$,

$f(x)$ $\frac{d}{dx}(x^{3/2})$ $\frac{d}{dx}(x^{3/2})$

$\frac{3}{2}x^{3/2-1}$ $\frac{3}{2}x^{3/2-1}$

$\frac{3}{2}x^{1/2}$ $\frac{3}{2}x^{5/2}$ $2x$

$\frac{3}{2}x^{1/2}$ $\frac{3}{2x^{5/2}}$, or $\frac{3}{2}\sqrt{x}$ $\frac{3}{2x\sqrt{5}}$

24. $f(t)$ $2\sqrt{t^3}$ $\frac{4}{\sqrt{t}}$ $\sqrt{2}$

$2t^{3/2}$ $4t^{1/2}$ 2 — $\sqrt{\ }$

(t)

Chapter 2. Differentiation: Basic Concepts 119

3 (2t 3/ 21)

1 (4t 1/

–

21) 0 2 2

3t1/ 2 2t 3 / 2

$\sqrt{3}$ t 2

$\sqrt{t^3}$

Chapter 2. Differentiation: Basic Concepts

Chapter 2. Differentiation: Basic Concepts 126

x 2 2 3/ 2 1
x
25. y x . .
2
16 x 3x 3
1 x 2 2x 1x 3/ 2 1 x 2 1 x,
16 3 3

$$\frac{dy}{dx} = \frac{d}{dx}\left(-\frac{1}{16}x^2\right) + \frac{d}{dx}(2x^{-1}) - \frac{d}{dx}(x^{3/2}) + \frac{d}{dx}\left(\frac{1}{3}x^{-2}\right) + \frac{d}{dx}\left(\frac{1}{3}x\right)$$

$$= -\frac{1}{16}(2x) + 2(-1x^{-1-1}) - \frac{3}{2}x^{3/2-1} + \frac{1}{3}(-2x^{-2-1}) + \frac{1}{3}$$

$$= -\frac{1}{8}x - 2x^{-2} - \frac{3}{2}x^{1/2} - \frac{2}{3}x^{-3} + \frac{1}{3}$$

$$= -\frac{1}{8}x - \frac{2}{x^2} - \frac{3}{2}x^{1/2} - \frac{2}{3x^3} + \frac{1}{3},$$

$$\text{or } -\frac{1}{8}x - \frac{2}{x^2} - \frac{3}{2}\sqrt{x} - \frac{2}{3x^3} + \frac{1}{3}$$

75
y 7 x1.2 5x2.1 x1.2
x2.1
dy
1.21 2.11
dx 1.2(7 x) 2.1(5x)
2.2 1.1
8.4 x 10.5x
8.4
10.5x 1.1
x2.2

27. y x5 4x2
3
x
x 5 4 x 2
3 3
x x
x 2 4

29. y x 3 5x 2 3x 1
dy
2
3x 10x 3
dx

At x = 1, dy/dx 10. The equation of the tangent line at (1, 8) is
y + 8 = 10(x + 1), or y = 10x + 2.

5 3
Given y x 3x 5x 2 and the point

dy 4 2
(1, 5) , then dx 5x 9x 5 and

the slope of the tangent line at x 1 is
4 2
5(1) 9(1) 5 9 . The equation of

the tangent line is then
y (5) 9(x 1) or y 9 x 4 .

y 1 1 2 1 1/ 2
1x 2 x
√x
dy 1 1
2 2
dx x x3/ 2 x x3/ 2

At 4, 7 , dy 1 . The equation of
4 dx 16

7 1
the tangent line is y (x 4), or
4 16
1
16x 2.

3 2 √16 2
Given yx x and the point x
√
(4, 7) , then dy 3 x 2 x 32 and the
3
dx 2 x
slope of the tangent line at x 4 is
3 √

Chapter 2. Differentiation: Basic Concepts
Chapter 2. Differentiation: Basic Concepts 120

x

x^{2} $4x^{1}$

$2 \quad 1$

$\frac{dy}{dx} \; \frac{d}{dx}(x^{2}) \; \frac{d}{dx}(4x^{1})$

$2x \; 4(1x^{11})$

$4 \; 2(4) \; \frac{32 \; 11}{3}$. The equation of the tangent line is then

$y \; (7) \; \frac{11}{2}(x \; 4)$ or $y \; \frac{11}{2} x \; 15$.

Chapter 2. Differentiation: Basic Concepts *Chapter 2. Differentiation: Basic Concepts* 120

$2x\ 4x^2$

$2x$ $\quad \frac{4}{x^2}$

28. $y\ x^2\ (x^3\ 6x\ 7)\ x^5\ 6x^3\ 7x^2$

$\frac{dy}{dx}\ 5x^4\ 18x^2\ 14x$

33. $y\ (x^2\ x)(3\ 2x)\ 2x^3\ x^2\ 3x$

$\frac{dy}{dx}\ 6x^2\ 2x\ 3$

At $x = 1$, $\frac{dy}{dx}$ 1. The equation of the

Chapter 2. Differentiation: Basic Concepts

tangent line at (1, 2) is y 2 = 1(x + 1), or y = x + 3.

34. Given y 2x 4 $\sqrt{x}$ $\frac{3}{x}$ and the point (1, 4) , then $\frac{dy}{dx}$ 8x 3 $\frac{1}{2\sqrt{x}}$ $\frac{3}{x^2}$. and the slope of the tangent line at x 1 is

m 8(1)3 $\frac{1}{2(\sqrt{1})}$ $\frac{3}{1^2}$ $\frac{9}{2}$. The equation of the tangent line is then y 4 $\frac{9}{2}$ (x 1)

or y $\frac{9}{2}x$ $\frac{1}{2}$.

35. $f(x)$ 2x^3 $\frac{1}{x^2}$ 2x^3 x^{-2}

$f(x)$ 6x^2 $\frac{2}{x^3}$

At x = 1, f(1) 4. Further,

y = f(1) = 3. The equation of the tangent line at (1, 3) is y 3 = 4(x + 1), or

= 4x 1.

36. $f(x)$ x^4 3x^3 2x^2 x 6; x 2

(x) 4x^3 9x^2 4x

f(2) 16 24 8 6 6 so (2, 6) is a point on the tangent line. The slope is

f(2) 32 36 8 4 . The equation

of the tangent line is y (6) 4(x 2) or

f '(2) 48 $\frac{1}{44}$ $\frac{193}{}$. The equation of

the tangent line is y 66 $\frac{193}{4}$(x 4) or

y $\frac{193}{4}x$ 127 .

39. $f(x)$ $\frac{1}{3}x^3$ $\frac{1}{3}$ $8\sqrt{x}$ x^3 8 $x^{1/2}$

$f(x)$ x^2 $\frac{\sqrt{8}}{2}$ $x^{1/2}$ $\frac{\sqrt{8}}{2\sqrt{x}}$

At x = 2, f(2) 4 $\frac{\sqrt{8}}{2\sqrt{2}}$

4 $\frac{1}{2}\sqrt{\frac{8}{2}}$

4 $\frac{1}{2}$ 2

3.

Further, y f(2) $\frac{8}{3}$ 4 $\frac{4}{3}$. The equation of the tangent line at 2, $\frac{4}{3}$ is

y $\frac{4}{3}$ 3(x 2), or y 3x $\frac{22}{3}$.

40. $f(x)$ x(x 1) $x^{3/2}$ x; x 4

$f(x)$ $\frac{3}{2}\sqrt{x}$ 1

4x 14 .

Chapter 2. Differentiation: Basic Concepts

$f(x)\ x\ \frac{1}{2}\ x\ x\ \frac{2}{}\ 2\ x$

$(x)\ 1\ \frac{2}{x^3}$

At $x = 1$,

$f(4)\ 8\ 4\ 4$ so $(4, 4)$ is a point on the tangent line. The slope is $f(4)\ 3\ 1\ 2$. The equation of the tangent line is $y\ 4\ 2(x\ 4)$ or $2x\ 4$.

$f(x)\ 2x^4\ 3x\ 1$

3

 Chapter 2. Differentiation: Basic Concepts

$f(1)$ 3. Further, $f(x)$ $8x$ 3
$y = f(1) = 0$.

The equation of the tangent line at (1, 0) is $0 = 3(x$ 1), or $y = 3x$ 3.

38. $f(x)$ x 3 2 1 $\sqrt{x}$; x 4

$f(x)$ $3x$ $\frac{}{2\sqrt{x}}$

$f(4)$ 64 2 66 so (4, 66) is a point on the tangent line. The slope is

The rate of change of f at $x = 1$ is $f(1)$ 5.

42. $f(x)$ x 3 2 $3x$ 5; x 2

$f(x)$ $3x$ 3

Chapter 2: Differentiation: Basic Concepts

43. $f(x) = x\sqrt{x} - \frac{1}{x^2} \quad x^{1/2} \quad 2 \quad x\,x \quad x$

$f'(x) = 1 - \frac{1}{2x^{1/2}} \cdot \frac{2}{x^3}$

The rate of change of f at $x = 1$

is $f'(1) = \frac{3}{2}$

44. $f(x) = \sqrt{x} \;\; 5x; \quad x = 4$

$f'(x) = \frac{1}{2\sqrt{x}} \; 5$

$f'(4) = \frac{1}{2(2)} \; 5 \; \frac{21}{4}$

$f(x) = \frac{x - x\sqrt{x}}{\sqrt{x}} = \frac{x}{\sqrt{x}} - \frac{x\sqrt{x}}{\sqrt{x}} = \sqrt{x} - x$

$x^{1/2} \; 1$

$f'(x) = \frac{1}{2x^{1/2}}$

The rate of change of f at $x = 1$

is $f'(1) = \frac{1}{2}$

46. $f(x) = \frac{2}{x} \; x\sqrt{x}; \quad x = 1$

$f'(x) = \frac{2}{x^2} \; \frac{3}{2} \sqrt{x}$

$f'(1) = 2 \; \frac{3}{2} \; \frac{7}{2}$

47. $f(x) = 2x^3 \; 5x^2 \; 4$

$f'(x) = 6x^2 \; 10x$

The relative rate of change is

$\frac{f'(x)}{f(x)} = \frac{6x^2 \; 10x}{\;}$

Chapter 2. Differentiation: Basic Concepts

$f(x)$ 3 2 4 .

Chapter 2: Differentiation: Basic Concepts

$f(x) = x^{\frac{1}{2}} \; x\,x^{1}$; $f(1) = 1 + 1 = 2$ x

$f'(x) = 1 \; x^{2} \quad 1 \quad \frac{1}{x^{2}}$; $f'(1) = 1 \; 1 \; 0$

At $c = 1$, the relative rate of change is

$\frac{f'(1)}{f(1)} \quad \frac{0}{2} \quad 0$

$f(x) = x \; x\,x^{2}$

$x^{1/2} \; x^{2}$

$x^{3/2} \; x^{2}$

$f'(x) \quad \frac{3}{2} x^{1/2} \quad 2x \quad \frac{3}{2}\sqrt{x} \; 2x$

The relative rate of change is

$\frac{f'(x)}{f(x)} \quad \frac{\frac{3}{2}\sqrt{x} \; 2x}{\sqrt{x}\,x^{2} \; x^{2}} \quad \frac{3 \; x \; 4 \; x}{2\sqrt{x}\,x\,x^{2}}$.

When $x = 4$, $\frac{f'(x)}{f(x)} \quad \frac{3 \; 4 \; 4(4)}{2\sqrt{4}\;4\;4\;2} \quad \frac{11}{24}$

50. $f(x) \quad (4 \quad x)\,x^{1} \quad 4x^{1} \quad 1$;

$f(3) \quad \frac{4}{3} \quad \frac{1}{3}$

$f'(x) \; 4x^{2}$; $f'(3) \quad \frac{4}{9}$

At $c = 3$, the relative rate of change is

$\frac{f'(3)}{f(3)} \quad \frac{-\frac{4}{9}}{\frac{1}{3}} \quad \frac{4}{3}$.

(a) $A(t) \; 0.1t^{2} \quad 10t \; 20$

$A'(t) \; 0.2t \; 10$

In the year 2008, the rate of change is

$A'(4) \; 0.8 \; 10$ or $10,800 per year.

$f'(1) \; 6 \; 10$

Chapter 2. Differentiation: Basic Concepts

When $x = 1$, 4.

$A(4) = (0.1)(16) + 40 + 20 = 61.6$, so the percentage rate of change is

$\frac{(100)(10.8)}{61.6}$ 17.53%.

$f(1)$ 2 5 4

(a) Since $f(x)$ x^3 $6x^2$ $15x$ is the number of radios assembled x hours after 8:00 A.M., the rate of assembly after x hours is

$f(x)\ 3x^2\ 12x\ 15$ radios per hour.

The rate of assembly at 9:00 A.M.

$(x\ 1)$ is

$f(1)\ 3\ 12\ 15\ 24$ radios per hour.

At noon, $t = 4$.

$f'(4) = -3(4)^2 + 12(4) + 15 = 15$

and $f'(1) = 24$. So, Lupe is correct: the assembly rate is less at noon than

at 9 A.M.

(a) $T(x)\ 20x^2\ 40x\ 600$ dollars

The rate of change of property tax is $T(x)\ 40x\ 40$ dollars/year.
In the year 2008, $x = 0$,
$T(0)\ 40$ dollars/year.

In the year 2012, $x = 4$ and $T(4) = \$1{,}080$. In the year 2008, $x = 0$ and $T(0) = \$600$.
The change in property tax is $T(4)\ T(0) = \$480$.

54. $M(x)\ 2,300\ \frac{125}{x}\ \frac{517}{x^2}$

$M(x)\ \frac{125}{x^2}\ \frac{1034}{x^3}$

$M(9)\ \frac{125}{9^2}\ \frac{1034}{9^3}\ 0.125$. Sales are

decreasing at a rate of approximately 1/8 motorcycle per \$1,000 of advertising.

(a) Cost = cost driver + cost gasoline cost driver 20(# hrs)

$20\ \frac{250 \text{ mi}}{x}$

cost gasoline
4.0(# gals)

$4.0(250)\frac{1\ 1,200}{250\ \ x}\ x$

$\frac{4{,}800}{x}\ 4.0x$ dollars

So, the cost function is

$C(x)\ \frac{9,800}{}\ \ 4x.$

The rate of change of the cost is $C(x)$.

$C(x)\ 9{,}800x^{1}\ 4x$

$C(x)\ \frac{9,800}{x^2}\ 4$ dollars/mi per hr

When $x = 40$,
$C(40)\ 2.125$ dollars/mi per hr.
Since $C(40)$ is negative, the cost is decreasing.

(a) Since $C(t)\ 100t^2\ 400t\ 5,000$ is the circulation t years from now, the rate of change of the circulation in t years is
$C(t)\ 200t\ 400$ newspapers per year.

The rate of change of the circulation 5 years from now is

$C(5)\ 200(5)\ 400\ 1,400$ newspap

ers per year. The circulation is increasing.

The actual change in the circulation during the 6th year is

$\frac{5{,}000}{x}$

Chapter 2. Differentiation: Basic Concepts 126

$C(6)$ $C(5)$ 11, 000
9, 500
1, 500 newspapers.

(a) Since Gary's starting salary is

$45,000 and he gets a raise of

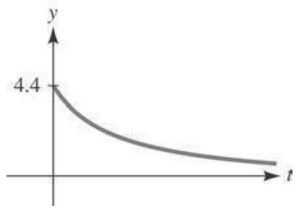
$2,000 per year,
his salary t years from now will be
$S(t) = 45{,}000 + 2{,}000t$ dollars.

The percentage rate of change of this salary t years from now is

Chapter 2. Differentiation: Basic Concepts
Chapter 2. Differentiation: Basic Concepts 128

$$100\left[\frac{S'(t)}{S(t)}\right] = 100\left(\frac{2{,}000}{45{,}000+2{,}000t}\right)$$

$$= \frac{200}{45+2t} \text{ percent per year}$$



The percentage rate of change after 1 year is

$$\frac{200}{47} \approx 4.26\%$$

In the long run, $\frac{200}{45+2t}$ approaches 0. That is, the percentage rate of

Gary's salary will approach 0 (even though Gary's salary will continue to increase at a constant rate.)

Let $G(t)$ be the GDP in billions of dollars where t is years and t 0 represents 1997. Since the GDP is growing at a constant

rate, $G(t)$ is a linear function passing through the points (0,125) and (8,155) . Then

$G(t)$ $\frac{155\,125}{8\,0}$ t 125 $\frac{15}{4}$ t 125 .

In 2012, t 15 and the model predicts a

GDP of $G(15)$ 181.25 billion dollars.

(a) $f(x) = 6x + 582$

The rate of change of SAT scores is

(x) 6.

The rate of change is constant, so the drop will not vary from year to year.

$N(x)$ 18x^2 500 commuters per

week. After 8 weeks this rate is

$N(8)$ 18(8^2) 500 1652 users per week.

The actual change in usage during the 8th week is

$N(8)$ $N(7)$ 15,072 13,558 1,

514 riders.

(a) $P(x)$ 2x 4$x^{3/2}$ 5, 000 is the

population x months from now. The rate of population growth is

$P(x)$ 2 4 $\frac{3x^{1/2}}{2}$

2 6$x^{1/2}$ people per month.

Nine months from now, the population will be changing at the rate

of $P(9)$ 2 6(9$^{1/2}$) 20 people per month.

The percentage rate at which the population will be changing 9 months from now is

$$100\,\frac{P(9)}{P(9)} \quad \frac{100(20)}{2(9)\ 4(9^{3/2})\ 5,000}$$

$$\frac{2,000}{5,126}$$

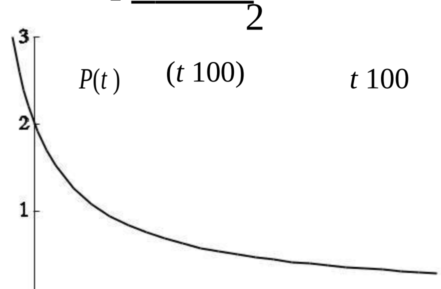
0.39%.

62. (a) $P(t)$ t^2 200t 10, 000 $(t\ 100)^2$

$P(t)$ 2t 200 2(t 100)

The percentage rate of change is

$$100\,\frac{P(t)}{P(t)}\ \frac{200(t\ 100)}{(t\ 100)^2}\ \frac{200}{t\ 100}.$$



Chapter 2. Differentiation: Basic Concepts *Chapter 2. Differentiation: Basic Concepts* 128

The rate of change is negative, so the scores are declining.

(a) Since $N(x)\ 6x^{3}\ \ 500x\ 8,000$ is

the number of people using rapid transit after *x* weeks, the rate at which system use is changing after *x* weeks is

Chapter 2. Differentiation: Basic Concepts Chapter 2. Differentiation: Basic Concepts 128

The percentage rate of changes approaches 0 since $\lim_{t \to \infty} \frac{200}{t+100} = 0$.

63. $N(t) = 10t^3 + 5t + \sqrt{t} = 10t^3 + 5t + t^{1/2}$

The rate of change of the infected population is

$N'(t) = 30t^2 + 5 + \frac{1}{2t^{1/2}}$ people/day.

On the 9th day, $N'(9) \approx 2,435$ people/day.

(a) $N(t) = 5,175 - t^3(t-8)$

$= 5,175 - t^4 + 8t^3$

$N'(3) = -4(3^3) + 8 \cdot 3(3^2) = 108$ people per week.

The percentage rate of change of N is given by

$$100\frac{N'(t)}{N(t)} = \frac{100(-4t^3 + 24t^2)}{5,175 - t^4 + 8t^3}.$$

A graph of this function shows that it never exceeds 25%.

Writing exercise–answers will vary.

(a) $T(t) = -68.07t^3 + 30.98t^2 + 12.52t + 37.1$

$T'(t) = -204.21t^2 + 61.96t + 12.52$

$T'(t)$ represents the rate at which the bird's temperature is changing after t days, measured in °C per day.

(b) $T'(0) = 12.52$°C/day since $T'(0)$ is positive, the bird's temperature is increasing.

$T'(0.713) \approx -47.12$°C/day

Since $T'(0.713)$ is negative, the bird's temperature is decreasing.

is $T(0.442) \approx 42.8$°C.

The bird's temperature starts at $T(0) = 37.1$°C, increases to $T(0.442) = 42.8$°C, and then begins to decrease.

(a) Using the graph, the x-value (tax rate) that appears to correspond to a y-value (percentage reduction) of 50 is 150, or a tax rate of 150 dollars per ton carbon.

Using the points (200, 60) and (300, 80), from the graph, the rate of change is approximately

$$\frac{dP}{dT} \approx \frac{80-60}{300-200} = \frac{20}{100} = 0.2\%$$

or increasing at approximately 0.2% per dollar. (Answers will vary depending on the choice of h.)

(c) Writing exercise – Answers will vary.

(a) $Q(t) = 0.05t^2 + 0.1t + 3.4$ PPM

$Q'(t) = 0.1t + 0.1$ PPM/year

The rate of change of Q at $t = 1$ is $Q'(1) = 0.2$ PPM/year.

$Q(1) = 3.55$ PPM, $Q(0) = 3.40$, and $Q(1) - Q(0) = 0.15$ PPM.

Chapter 2. Differentiation: Basic Concepts Chapter 2. Differentiation: Basic Concepts 128

$Q(2) = 0.2 + 0.2 + 3.4 =$
3.8, $Q(0) = 3.4$, and
$Q(2)$ $Q(0) = 0.4$ PPM.

2 2

4 π 4 N

temperature is decreasing.

Find t so that $T(t)$ 0.
2

68. P – T 1
πN
3 $3kT$ $9k$

dP $4\pi N$ 2 T^2 $4\pi N$

2
dt $9k$ $9kT$

Chapter 2. Differentiation: Basic Concepts
Chapter 2. Differentiation: Basic Concepts 130

Since g represents the acceleration due to gravity for

$$t \quad \frac{61.96 \qquad \sqrt{(61.96)^2 \quad 4(204.21)(12.52)}}{2(204.21)}$$

0.442 days

The bird's temperature when $t = 0.442$

the planet our spy is on, the formula for the rock's height is

$$H(t) = -\frac{1}{2}gt^2 + V_0 t + H_0$$

Since he throws the rock from ground

level, $H_0 = 0$. Also, since it returns to the ground after 5 seconds,

$$0 = -\frac{1}{2}g(5)^2 + V_0(5)$$
$$0 = -12.5g + 5V_0$$
$$V_0 = \frac{12.5g}{5} = 2.5g$$

The rock reaches its maximum height halfway through its trip, or when $t = 2.5$. So,

$$37.5 = -\frac{1}{2}g(2.5)^2 + V_0(2.5)$$
$$37.5 = -3.125g + 2.5V_0$$

Substituting $V_0 = 2.5g$

$$37.5 = -3.125g + 2.5(2.5g)$$
$$37.5 = -3.125g + 6.25g$$
$$37.5 = 3.125g$$
$$g = 12 \text{ ft/sec}^2$$

So, our spy is on Mars.

70. (a) $s(t)\ t^2\ \ 2t\ 6$ for $0\ t\ 2$

$v(t)\ 2t\ 2$
$a(t)\ 2$

The particle is stationary when $v(t)\ 2t\ 2\ 0$ which is at time $t\ 1$.

(a) $s(t)\ 3t^2\ \ 2t\ 5$ for $0\ t\ 1$

$v(t) = 6t + 2$ and $a(t) = 6$

$6t + 2 = 0$ at $t = 3$. The particle is not stationary between $t = 0$ and $t = 1$.

(a) $s(t)\ t^3\ \ 9t^2\ \ 15t\ \ 25$ for $0\ t\ 6$

$v(t)\ 3t^2\ \ 18t\ 15\ 3(t\ 1)(t\ 5)$

The particle is stationary when $v(t)\ 3(t\ 1)(t\ 5)\ 0$ which is at

73. (a) $s(t)\ t^4\ \ \frac{4t^3}{3}\ \ \frac{8t^2}{2}$ for $0\ t\ 4$

$v(t)\ 4t\ \ 12t\ \ \ 8$ and

To find all time in given interval when stationary,

$$4t^3 - 12t^2 + 8 = 0$$
$$4(t^3 - 3t^2 + 2) = 0$$
$$t^3 - 3t^2 + 2 = 0$$
$$(t-1)(t^2 - 2t - 2) = 0$$
$$t = 1 \text{ or } t = \frac{2 \pm \sqrt{4 - 4\cdot 1\cdot(-2)}}{2}$$

Since $0 \le t \le 4$, $t = 1$ or $t = 1 + \sqrt{3}$.

(a) Since the initial velocity is V_0 0 feet per second, the initial height is

0 144 feet and g 32 feet per second per second, the height of the stone at time t is

$$H(t)\ \ \frac{1}{2}gt^2\ \ V_0 t\ H_0$$

$16t^2\ \ 144.$

The stone hits the ground when

$H(t)\ 16t^2\ \ 144\ 0$, that is when

$t^2\ \ 9$ or after t 3 seconds.

The velocity at time t is given by $H(t)\ 32t$. When the stone hits the ground, its velocity is $H(3)\ 96$

feet per second.

(a) If after 2 seconds the ball passes you

on the way down, then $H(2)\ H_0$, where $H(t)\ 16t\ V\ t\ H$.

times
t 1 and t 5 .

Chapter 2. Differentiation: Basic Concepts

So, $16(2^{0}\ )\ (V_0)(2)\ H_0 \quad 0 \quad 0 \quad H_0$,

$64\ 2V_0\ 0$, or $V_0\ \ 32\ \frac{\text{ft}}{\text{sec}}$.

The height of the building is H_0 feet.
From part **(a)** you know that

$H(t)\ 16t^2\ \ 32t\ H_0$. Moreover,

 Chapter 2. Differentiation: Basic Concepts 131

$H(4) = 0$ since the ball hits the ground after 4 seconds. So,

$\frac{f(x+h)-f(x)}{h} = 4x^3 + 6x^2 h + 4xh^2 + h^3$

$16(4)^2 + 32(4) + H_0 = 0$, or

$H_0 = 128$ feet.

From parts **(a)** and **(b)** you know that

2

speed of the ball is

$H'(t) = 32t + 32 \frac{\text{ft}}{\text{sec}}$.

After 2 seconds, the speed will be

$H'(2) = -32$ feet per second, where

the minus sign indicates that the

direction of motion is down.

The speed at which the ball hits the

$\frac{\text{ft}}{\text{sec}}$

Let (x, y) be a point on the curve where the tangent line goes through (0, 0). Then the slope of the tangent line is equal to

$\frac{y-0}{x-0} = \frac{y}{x}$. The slope is also given by

$f'(x) = 2x + 4$. Thus $\frac{y}{x} = 2x + 4$ or

$y = 2x^2 + 4x$.

Since (x, y) is a point on the curve, we

must have $y = x^2 + 4x + 25$. Setting the

two expressions for y equal to each other

78. **(a)** If $f(x) = x^4$ then

$f(x+h) = (x+h)^4$

$= x^4 + 4x^3 h + 6x^2 h^2 + 4xh^3 + h^4$

$f(x+h) - f(x) = 4x^3 h + 6x^2 h^2 + 4xh^3 + h^4$

Chapter 2. Differentiation: Basic Concepts 133

gives

$x^2 \quad 4x \; 25 \; 2x^2 \quad 4x$

$x^2 \quad 25$

5

If $x \; 5$, then $y \; 70$, the slope is

14 and the tangent line is $y \; 14 \; x$.

If $x \; 5$, then $y \; 30$, the slope is 6 and

the tangent line is $y \; 6x$.

$f(x) \; ax^2 \quad bx \; c$

Since $f(0) = 0$, $c = 0$ and $f(x) \; ax^2 \quad bx$.

Since $f(5) = 0$, $0 = 25a + 5b$.

Further, since the slope of the tangent is 1

when $x = 2$, $f(2) \; 1$.

$f(x) \; 2ax \; b$

$1 \; 2a(2) \; b \; 4a \; b$

Now, solve the system: $0 = 25a + 5b$ and $1 = 4a + b$. Since $1 \; 4a = b$, using substitution

$25a \; 5(1 \; 4a)$

$25a \; 5 \; 20a$

$0 \; 5a \; 5$

or $a = 1$ and $b = 1 \; 4(1) = 5$.

So, $f(x) \; x^2 \quad 5x$.

If $f(x) = x^n$ then

$$(x+h)^n = x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \dots + nxh^{n-1} + h^n$$

$$f(x+h) = f(x) + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \dots + nxh^{n-1} + h^n$$

Chapter 2. Differentiation: Basic Concepts 134

and

$$\frac{f(x h) f(x)}{h} \quad nx^{n1} \quad \frac{n(n\ 1)}{2} x^{n2} h \ldots nx h^{n2} \quad h^{n1}$$

From part (b)

$$\frac{f(x h) f(x)}{h} \quad nx^{n1} \quad h \frac{n(n\ 1)}{2} x^{n2} \ldots h^{n2}$$

The first term on the right does not involve h while the second term approaches 0 as $h\ 0$.

Thus $\frac{d}{dx}[x^n] \lim_{h0} \frac{f(x h) f(x)}{h} \quad nx^{n1}$.

$(f\ g)(x)$

$$\lim_{h0} \frac{(f g)(x h)\ (f g)(x)}{h}$$

$$\lim_{h0} \frac{f(x h)\ g(x h)\ [f(x)\ g(x)]}{h}$$

$$\lim_{h0} \frac{f(x h) f(x)\ g(x h)\ g(x)}{h}$$

$$\lim_{h0} \frac{f(x h) f(x)}{h} \quad \lim_{h0} \frac{g(x h)\ g(x)}{h}$$

$(x)\ g(x)$

2.3 Product and Quotient Rules; Higher-Order Derivatives

$f(x) = (2x + 1)(3x\ 2)$,

$$f(x)\ (2x\ 1) \frac{1}{dx}(3x\ 2)$$

$$(3x\ 2) \frac{d}{}(2x\ 1)$$

$(3x\ 1)(3)\ (3x\ 2)(2)$

$12x\ 1$

$f(x)\ (x\ 5)(1\ 2x)$

$$f(x)\ (x\ 5) \frac{d}{dx}(1\ 2x)\ (1\ 2x) \frac{d}{dx}(x\ 5)$$

$2(x\ 5)\ 1(1\ 2x)$

$11\ 4x$

$y = 10(3u + 1)(1 5u)$,

$\frac{dy}{du}$ $\frac{d}{du}(3u\ 1)(1\ 5u)$

$10\ (3u\ 1)\ \frac{d}{du}\ 1\ 5u\ 1\ 5u\ \frac{d}{du}\ (3u\ 1)$

$10[(3u\ 1)(5)\ (1\ 5u)(3)]$

$300u\ 20$

$y\ \ 400(15\ x^2\ )(3x\ 2)$

$\frac{dy}{dx}\ 400\ \frac{d}{dx}(15\ x^2\ )(3x\ 2)$

$400\ (15\ x^2\ )\ \frac{d}{dx}(3x\ 2)\ (3x\ 2)\ \frac{d}{dx}(15\ x^2\ )$

$400\ (15\ x^2\ )(3)\ (3x\ 2)(2x)$

$400(9\ x^2\ \ 4\ x\ 45)$

5. $f(x)\ \frac{1}{3}\ (x^5\ 2x^3\ 1)\ \frac{d}{dx}\ x\ \frac{1}{x}$

$\frac{1}{3}\ x\ \frac{1}{x}\ \frac{d}{dx}(x^5\ \ 2x^3\ \ 1)$

$\frac{1}{3}(x^5\ \ 2x^3\ \ 1)\ 1\ \frac{1}{x^2}$

$x\ \frac{1}{x}\ (5x^4\ 6x^2)$

$\frac{2x^5\ 4x^3\ 4}{3}\ x\ \frac{1}{3}\ \frac{1}{3x^2}\ \frac{2}{3}\ \sqrt{}$

6. $f(x)\ \ 3(5x^3\ \ 2x\ 5)(\sqrt{x}\ \ 2x)$

$f'(x)\ 3\ (5x^3\ \ 2x\ 5)\ \frac{1}{2\sqrt{x}}\ 2\ \sqrt{x}\ \ 2x\ (15x^2\ \ 2)$

$\frac{105}{2}x^{5/2}\ \ 120x^3\ \ 9x^{1/2}\ 24x\ \frac{15}{2x^{1/2}}\ \ 30$

Chapter 2. Differentiation: Basic Concepts 136

$$y \; \frac{x\,1}{x\,2},$$

$$\frac{dy}{dx} \; \frac{(x\,2)\frac{d}{dx}(x\,1)\;(x\,1)\frac{d}{dx}(x\,2)}{(x\,2)^2}$$

$$\frac{(x\,2)(1)\;(x\,1)(1)}{(x\,2)^2} \; \frac{3}{(x\,2)^2}$$

$$y \; \frac{2x\,3}{5x\,4}$$

$$\frac{dy}{dx} \; \frac{(5x\,4)\frac{d}{dx}(2x\,3)\;(2x\,3)\frac{d}{dx}(5x\,4)}{(5x\,4)^2}$$

$$\frac{2(5x\,4)\;5(2x\,3)}{(5x\,4)^2} \; \frac{23}{(5x\,4)^2}$$

9. $f(t) \; \frac{t}{t^2\;2},$

$$f'(t) \; \frac{(t^2\;2)\frac{d}{dt}(t)\;t\frac{d}{dt}(t^2\;2)}{(t^2\;2)^2}$$

$$\frac{(t^2\;2)(1)\;(t)(2t)}{(t^2\;2)^2} \; \frac{2\;t^2}{(t^2\;2)^2}$$

$$f(x) \; \frac{1}{x\;2}$$

$$f'(x) \; \frac{(x\,2)(0)\;1(1)}{(x\,2)^2} \; \frac{1}{(x\,2)^2}$$

11. $y \; \frac{3}{x\,5},$

$$\frac{dy}{dx} \; \frac{(x\,5)\frac{d}{dx}(3)\;3\frac{d}{dx}(x\,5)}{(x\,5)^2}$$

$$\frac{(x\,5)(0)\;3(1)}{(x\,5)^2} \; \frac{3}{(x\,5)^2}$$

$y \; \frac{t^2}{} \; \frac{1}{} \; 1\,t^2$

$\frac{dy}{dt} \; \frac{(1\,t^2)(2t)\,(t^2\;1)(2t)}{(1\,t^2)^2} \; \frac{4t}{(1\,t^2)^2}$

13. $f(x) \; \frac{x^2\;3x\;2}{2x^2\;5x\;1}$

$f'(x) \; \frac{(2x^2\;5x\;1)\frac{d}{dx}(x^2\;3x\;2)\;\;(x^2\;3x\;2)\frac{d}{dx}(2x^2\;5x\;1)}{(2x^2\;5x\;1)^2}$

$\frac{(2x^2\;5x\;1)(2x\;3)}{(2x^2\;5x\;1)^2} \;\; \frac{(x^2\;3x\;2)(4x\;5)}{(2x^2\;5x\;1)^2}$

$\frac{11x^2\;10x\;7}{}$

14. $g(x) \; \frac{(x^2\;x\;1)(4\;x)}{2x\;1}$

$g'(x) \; \frac{(2x\;1)[1(x^2\;x\;1)\;(4\;x)(2x\;1)]\;(x^2\;x\;1)(4\;x)(2)}{(2x\;1)^2}$

$\frac{4x^3\;9x^2\;6x\;11}{(2x\;1)^2}$

15. $f(x)\;(2\;5x)^2\;(2\;5x)(2\;5x)$ $f'(x)\;(2\;5x)\frac{d}{dx}(2\;5x)\;\;(2\;5x)\frac{d}{dx}(2\;5x)$

$2(2\;5x)\frac{d}{dx}(2\;5x)$

$2(2\;5x)(5)$

$50x$

$10(2\;5x)$

16. $f(x)\ x \quad 1 \quad {}^{2}\ x\ 2 \quad 2 \quad \frac{1}{x^2} \quad x$

$f(x)\ 2x\ \frac{2}{x}$

17. $g(t)$ $\frac{t^2 - t}{\sqrt{\ }}$ $\frac{t^2\ t^{1/2}}{}$

$2t\ \ 5$ $\quad 2t\ 5$

$(2t\ 5)\ \frac{d}{dt}(t\ \ t^{1/2})$

$(t^2\ t^{1/2})\ \frac{d}{dt}(2t\ 5)$

$g(t)$ ———— 2

$(2t\ 5)$

$(2t\ 5)\ 2t\ \frac{1}{2}\ t^{1/2}\ (t^2\ \ t^{1/2})(2)$

2

$(2t\ 5)$

$2t^2\ \ 10t\ t^{1/2}\ \ 5\ \ 2t^{1/2}$

$2t^{1/2}$ 2

$(2t\ 5)$

$2t^{1/2}$

5/ 2 3/ 2

$4t\ \ \ 20t\ \ \ 2t\ 5$

1/ 2 2

$2t\ \ \ (2t\ \ 5)$

$\sqrt{5}\ \ \sqrt{3}$

4 [redacted] 20 t $2t\ 5$

2

2 $\sqrt{\ }$ [redacted] (2 [redacted] 5)

$h(x)$ $\frac{4x}{}$

2 2

1 x 1

2 2

$(x\ \ 1)(1)\ x(2x)\ \ \ (x\ \ 1)(1)\ (4\ x)(2\ x)$

2 2

$h(x)$ $(x\ \ \ 1)$

2 2

$(x\ \ 1)$

2 2

$x\ \ 1\ \ \ \ x\ \ 8x\ 1\ ($

2 2 2

$(x\ \ 1)\ \ \ \ x^2\ \ 1)$

$y\ (5x\ 1)(4\ 3x)$

$\frac{dy}{dx}\ \ 30\ x\ 17$

When $x = 0$, $y = 4$ and $\frac{dy}{dx}$ 17. The equation of the tangent line at (0, 4) is $y + 4 = 17(x\ 0)$, or $y = 17x\ 4$.

$y\ \ (x^2\ \ 3x\ 1)(2\ x)$

$(x^2\ \ 3x\ 1)(1)\ (2\ x\ 3)(2\ x)$

At $x_0\ 1$, $y\ (3)(1)\ 3$ and $y\ (3)(1)\ (5)(1)\ 2$. The equation of the tangent line is then

y 3 2(x 1) or y 2x 1 .

21. $y \;\; \frac{x}{2\,x\;3}$

$\frac{dy}{\;} \;\; \frac{3}{\;}$

$dx \ (2x\ 3)^2$

When $x = 1$, $y = 1$ and $\frac{dy}{dx}$ 3. The equation of the tangent line at (1, 1) is

$y + 1 = 3(x + 1)$, or $y = 3x + 2$.

Chapter 2. Differentiation: Basic Concepts 140

$y \frac{x\ 7}{5\ 2x}$

$\frac{(5\ 2x)(1)\ (x\ 7)(2)}{(5\ 2x)^2}$

At $x_0\ 0$, $y\ \frac{7}{5}$ and $y\ \frac{5\ 14}{5^2}\ \frac{19}{25}$.

The equation of the tangent line is then $y\ \frac{7}{5}\ \frac{19}{25}(x\ 0)$ or $y\ \frac{19}{25}x\ \frac{7}{5}$.

$y\ 3\sqrt{x}\ x(2\ x^2)$

$(3x^{1/2}\ x)(2\ x^2)$

$\frac{dy}{dx}\ 3x^2\ \frac{15}{2}x^{3/2}\ \frac{3}{2x^{1/2}}\ 2$

When $x = 1$, $y = 4$ and $\frac{dy}{dx}\ \frac{11}{2}$.

The equation of the tangent line at (1, 4) is $y\ 4\ \frac{11}{2}(x\ 1)$, or $y\ \frac{11}{2}x\ \frac{19}{2}$.

$f(x)\ (x\ 1)(x^2\ 8x\ 7)$

$f(x)\ 1\ (x^2\ 8x\ 7)\ (x\ 1)(2x\ 8)$

$3x^2\ 18x\ 15$

$3(x\ 1)(x\ 5)$

$f(x)\ 0$ when $x = 1$ and $x = 5$.

$f(1)\ (1\ 1)(1^2\ 8\ 1\ 7)\ 0$

$f(5)\ (5\ 1)(5^2\ 8\ 5\ 7)\ 32$

The tangent lines at (1, 0) and (5, 32) are horizontal.

$f(x)\ (x\ 1)(x^2\ x\ 2)$

$f(x)\ (x\ 1)(2x\ 1)\ (x^2\ x\ 2)(1)$

Since $f(x)$ represents the slope of the tangent line and the slope of a horizontal line is zero, need to solve

26. $f(x)\ \frac{x^2\ x\ 1}{x^2\ x\ 1}$

$(x)\ \frac{(2x\ 1)(x^2\ x\ 1)\ (x^2\ x\ 1)(2x\ 1)}{(x^2\ x\ 1)^2}$

$\frac{2x^2\ 4x}{}$

$0 \quad 3x^2 \quad 3 \quad 3(x \quad 1)(x \quad 1)$ or $x = 1, 1$.

When $x = 1$, $f(1) = 0$ and when $x = 1$, $f(1) = 4$. So, the tangent line is horizontal at the points (1, 0) and

$(x^2 \quad x \quad 1)^2$

$$\frac{2x(x \quad 2)}{(x^2 \quad x \quad 1)^2}$$

(1, 4).

$f(x) = 0$ when $x = 0$ and $x = 2$

2
0 0 1
2 1
$f(0)$ 0 0 1
2 2 1 5
$f(2)$
2
2 2 1 3

The tangent lines at (0, 1) and $2, \frac{5}{3}$ are horizontal.

$$f(x) = \frac{1}{x^2 - x + 1}$$

$$f'(x) = \frac{2 - 2x}{(x^2 - x + 1)^2}$$

Since $f'(x)$ represents the slope of the tangent line and the slope of a horizontal line is zero, need to solve

$$0 = \frac{x^2 - 2x}{(x^2 - x + 1)^2}$$

2

$0 = x^2 - 2x = x(x - 2)$ or $x = 0, 2$.
When $x = 0$, $f(0) = 1$ and when $x = 2$, $f(2) = \frac{1}{3}$. So, the tangent line is horizontal at the points (0, 1) and $2, \frac{1}{3}$.

$$y = (x^2 - 2)(x + \sqrt{x})$$

$$\frac{dy}{dx} = (x^2 - 2)\, 1 \frac{1}{2\sqrt{x}} \quad 2x\, x \quad x$$

At $x = 4$,

$$\frac{dy}{dx} = (18)\, 1 \frac{1}{4} \quad 8(6) = 70.5$$

$y = (x \quad 3)(5 \quad 2x \quad)$

31. $y = x - \frac{3}{2 - 4x}$

$$\frac{dy}{dx} = 1 - \frac{(2 - 4x)(0) - 3(-4)}{(2 - 4x)^2}$$

When $x = 0$, $\frac{dy}{dx} = 1 - \frac{12}{(2)^2} = 4$.

$y = x^2 - 3x + 5$

$2x - 3$

At $x = 0$, $y' = -3$ so the slope of the perpendicular line is $m = \frac{1}{3}$. The perpendicular line passes through the point (0, 5) and so has equation

$$y = \frac{1}{3}x + 5.$$

$$y = \frac{2}{x} - \sqrt{x} = 2x^{-1} - x^{1/2}$$

$$\frac{dy}{dx} = -\frac{2}{x^2} - \frac{1}{2x^{1/2}}$$

When $x = 1$, $\frac{dy}{dx} = -2 - \frac{1}{2} = -\frac{5}{2}$.

The slope of a line perpendicular to the tangent line at $x = 1$ is $\frac{2}{5}$.

The equation of the normal line at (1, 1) is

$y - 1 = \frac{2}{5}(x - 1)$, or $y = \frac{2}{5}x + \frac{3}{5}$.

$$\frac{dy}{dx} = 2$$

$(x$

Chapter 2. Differentiation: Basic Concepts

dx

5 5 5

2 3

3)(6x) (5 2 x)(2 x)

When x = 1,

$\frac{dy}{dx}$ (1 3)(6) (5 2)(2) 18.

34. y (x 3) 1 x √

1

y (x 3) 1 x √

2 x √

At x 1, y 2 so the slope of the

1