Chapter 2

EXPONENTIAL, LOGARITHMIC, AND TRIGONOMETRIC FUNCTIONS

2.1 Exponential Functions

1.

number of folds	1	2	3	4	5 ...	10 ...	50
layers of paper	2	4	8	16	32 ...	1024 ...	2^{50}

$2^{50} = 1.125899907 \times 10^{15}$

500 sheets are 2 inches high

$\frac{500}{2 \text{ in.}} = \frac{2^{50}}{x \text{ in.}}$

$x = \frac{2 \cdot 2^{50}}{500} = 4.503599627 \times 10^{12}$ in.

71,079,539.57 mi

The graph of $y = 3^x$ is the graph of an exponential function $y = a^x$ with $a > 1$.
This is graph **E**.

The graph of $y = 3^{-x}$ is the graph of $y = 3^x$ reflected across the y-axis.
This is graph **D**.

The graph of $y = \left(\frac{1}{3}\right)^{1-x}$ is the graph of $= \left(3^{-1}\right)^{1-x}$ or $y = 3^{x-1}$. This is the graph of $y = 3^x$ translated 1 unit to the right.
This is graph **C.**

The graph of $y = 3^{x+1}$ is the graph of $y = 3^x$ translated 1 unit to the left. This is graph **F**.

The graph of $y = 3(3)^x$ is the same as the graph of $y = 3^{x+1}$. This is the graph of y

$= 3^x$ translated 1 unit to the left. This is graph **F**.

The graph of $y = \left(\frac{1}{3}\right)^x$ is the graph of

$= (3^{-1})^x = 3^{-x}$. This is the graph of $y = 3^x$ reflected in the y-axis. This is graph **D**.

The graph of $y = 2 - 3^{-x}$ is the same as the

graph of $y = -3^{-x} + 2$. This is the graph of

$= 3^x$ reflected in the x-axis, reflected across the y-axis, and translated up 2 units. This is graph **A**.

The graph of $y = -2 + 3^{-x}$ is the same as the

graph of $y = 3^{-x} - 2$. This is the graph of

$= 3^x$ reflected in the y-axis and translated two units downward. This is graph **B**.

The graph of $y = 3^{x-1}$ is the graph of $y = 3^x$ translated 1 unit to the right. This is graph **C**.

Some of the functions are equivalent to each

other. $y = 3^{x-1} = \left(\frac{1}{3}\right)^{1-x}$ (graph C),

$= 3^{-x} = \left(\frac{1}{3}\right)^x$ (graph D), and

$3^{x+1} = 3(3)^x$ (graph F).

13.
$$2^x = 32$$
$$2^x = 2^5$$
$$x = 5$$

14.
$$4^x = 64$$
$$4^x = 4^3$$
$$x = 3$$

15.
$$3^x = \frac{1}{81}$$
$$3^x = \frac{1}{3^4}$$
$$3^x = 3^{-4}$$
$$x = -4$$

16.
$$e^x = \frac{1}{e^5}$$
$$e^x = e^{-5}$$
$$x = -5$$

17.
$$4^x = 8^{x+1}$$
$$(2^2)^x = (2^3)^{x+1}$$
$$2^{2x} = 2^{3x+3}$$
$$2x = 3x + 3$$
$$-x = 3$$
$$x = -3$$

18.
$$25^x = 125^{x+2}$$
$$(5^2)^x = (5^3)^{x+2}$$
$$5^{2x} = 5^{3x+6}$$
$$2x = 3x + 6$$
$$-6 = x$$

19.
$$16^{x+3} = 64^{2x-5}$$
$$(2^4)^{x+3} = (2^6)^{2x-5}$$
$$2^{4x+12} = 2^{12x-30}$$
$$4x + 12 = 12x - 30$$
$$42 = 8x$$
$$\frac{21}{4} = x$$

20.
$$(e^3)^{-2x} = e^{-x+5}$$
$$e^{-6x} = e^{-x+5}$$
$$-6x = -x + 5$$
$$-5x = 5$$
$$x = -1$$

$e^{-x} = (e^{4})^{x+3}$

$e^{-x} = e^{4x+12}$

$x = 4x +$

$12 - 5x = 12$

$= -\dfrac{12}{5}$

$2^{|x|} = 8$

$2^{|x|} = 2^{3}$

$|x| = 3$

$= 3$ or $x = -3$

$5^{-|x|} = \dfrac{1}{25}$

$5^{-|x|} = 5^{-2}$

$|x| = 2$

$x = 2$ or $x = -2$

24. $2^{x^2-4x} = \left(\dfrac{1}{16}\right)^{x-4}$

$2^{x^2-4x} = (2^{-4})^{x-4}$

$2^{x^2-4x} = 2^{-4x+16}$

$x^2 - 4x = -4x + 16$

$x^2 - 16 = 0$

$(x+4)(x-4) = 0$

$x = -4$ or $x = 4$

25. $5^{x^2+x} = 1$

$5^{x^2+x} = 5^{0}$

$x^2 + x = 0$

$x(x+1) = 0$

$x = 0$ or $x + 1 = 0$

$x = 0$ or $x = -1$

$8^{x^2} = 2^{x+}$

$4\,(2^{3})^{x^2} = 2^{x}$

$+4\; 2^{3x^2} = 2^{x}$

$+4\; 3x^2 = x + 4$

$3x^2 - x - 4 = 0$

$(3x-4)(x+1) = 0$

$x = \dfrac{4}{3}$ or $x = -1$

$27^{x} = 9^{x^2+x}$ $(3^{3})^{x} = (3^{2})^{x^2+x}$

$3^{3x} = 3^{2x^2+2x}$

$3x = 2x^2 + 2x$

$0 = 2x^2 - x$

$0 = x(2x-1)$

$x = 0$ or $2x - 1 = 0$

$x = 0$ or $x = \dfrac{1}{2}$

$e^{x^2+5x+6} = 1$

$e^{x^2+5x+6} = e^{0}$

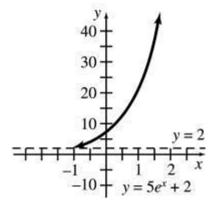
$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

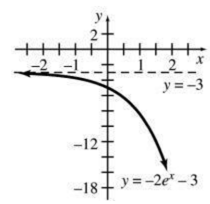
$x + 3 = 0$ or $x + 2 = 0$

$x = -3$ or $x = -2$

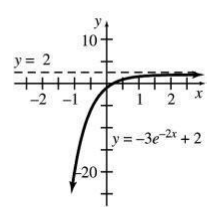
Graph of $y = 5e^{x} + 2$



Graph of $y = -2e^{x} - 3$



Graph of $y = -3e^{-2x} + 2$



Graph of $y = 4e^{-x/2} - 1$

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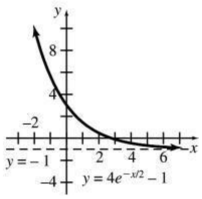
Section 2.1 EXPONENTIAL FUNCTIONS 91



33. Answers will vary.

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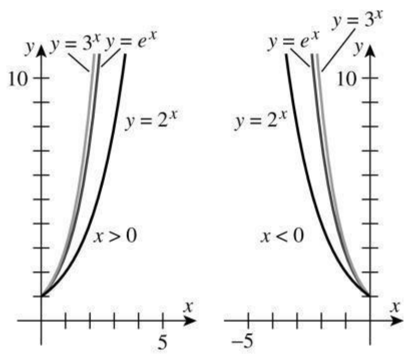
Section 2.1 EXPONENTIAL FUNCTIONS

4 and 6 cannot be easily written as powers of the same base, so the equation $4^x = 6$ cannot be solved using this approach.

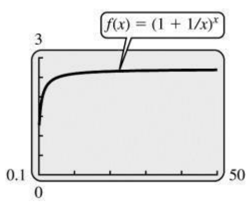
When $x > 0$, $3^x > e^x > 2^x$ because the functions are increasing. When $x < 0$,

$3^x < e^x < 2^x \Rightarrow \frac{1}{3}^{-x} < \frac{1}{3}^{-x} < \frac{1}{2}^{-x}$, and

the functions are decreasing. The graphs below illustrate this.



36.



e

$f(x)$ approaches $e \approx 2.71828$.

$A(t) = 3100e^{0.0166t}$

1970: $t = 10$

$A(10) = 3100e^{(0.0166)(10)}$
$3100e^{0.166}$
3659.78

The function gives a population of about 3660 million in 1970. This is very close to the actual population of about 3686 million.

2000: $t = 40$

$A(40) = 3100e^{0.0166(40)} = 3100e^{0.664}$
6021.90

The function gives a population of about 6022 million in 2000.

2015 : $t = 55$

$f(x) = 500 \cdot 2^{3t}$

After 1 hour:

$f(1) = 500 \cdot 2^{3(1)} = 500 \cdot 8 = 4000$ bacteria

initially:

$f(0) = 500 \cdot 2^{3(0)} = 500 \cdot 1 = 500$ bacteria

The bacteria double every $3t = 1$ hour, or every $\frac{1}{3}$ hour, or 20 minutes.

When does $f(t) = 32{,}000$?

$32{,}000 = 500 \cdot 2^{3t}$
$= 2^{3t}$
$6 = 2^{3t}$
$6 = 3t$
$= 2$

The number of bacteria will increase to 32,000 in 2 hours.

(a) Hispanic population: $h(t) =$ t

$A(55) = 3100e^{0.0166(55)} = 3100^{0.913}$
7724.54

From the function, we estimate that the world population in 2015 will be 7725 million.

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$h(5) = 37.79(1.021)^5 \approx 41.93$

The projected Hispanic population in 2005 is 41.93 million, whic

h is slightly less than the actual value of 42.69 million.

37.79(1.021)

Asian population:

$h(t) = 11.14(1.023)^t$

$h(5) = 11.14(1.023)^t \approx 12.48$

The projected Asian population in 2005 is 12.48 million, which is very close to the actual value of 12.69 million.

Annual Hispanic percent increase:

$1.021 - 1 = 0.021 = 2.1\%$

Annual Asian percent increase:

$1.023 - 1 = 0.023 = 2.3\%$

The Asian population is growing at a slightly faster rate.

Black population:

$b(t) = 0.5116t + 35.43$

$b(5) = 0.5116(5) + 35.43$

37.99

The projected Black population in 2005 is 37.99 million, which is extremely close to the actual value of 37.91 million.

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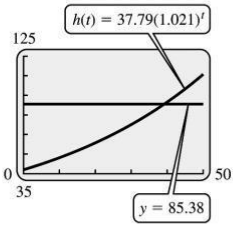
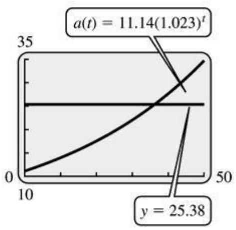
Section 2.1 EXPONENTIAL FUNCTIONS

Hispanic population:
Double the actual 2005 value is 2(42.69) = 85.38 million.



The doubling point is reached when t H 39, or in the year 2039.

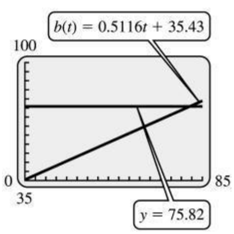
Asian population:
Double the actual 2005 value is 2(12.69) = 25.38 million.



The doubling point is reached when

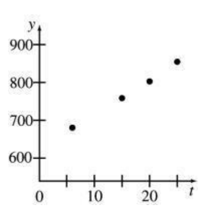
H 36, or in the year 2036. Black population:

Double the actual 2005 value is 2(37.91) = 75.82 million.



The doubling point is reached when t H 79, or in the year 2079.

40. (a)



The data appear to fit an exponential curve.

$f(t) = Ce^{kt}$

$f(6) = Ce^{k(6)}$ | $f(15) = Ce^{15k}$

$680.5 = Ce^{6k}$ | $758.6 = Ce^{15k}$

$C = \frac{680.5}{e^{6k}}$ | $C = \frac{758.6}{e^{15k}}$

The function passes through the points $(6, 680.5)$ and $(15, 758.6)$. Now solve for k.

$$\frac{680.5}{e^{6k}} = \frac{758.6}{e^{15k}}$$

$$\frac{e^{15k}}{e^{6k}} = \frac{758.6}{680.5}$$

$$e^{9k} = \frac{758.6}{680.5}$$

$$9k = \ln\frac{758.6}{680.5}$$

$$k = \frac{1}{9}\ln\frac{758.6}{680.5} \text{ H } 0.01207186$$

Substitute this value into $C = \frac{680.5}{e^{6k}}$ and solve for C.

$$C = \frac{680.5}{e^{6(0.01207186)}} \text{ H } 632.95$$

$$(t) = 632.95e^{0.01207t}$$

$t = 20$ corresponds to 2020 and $t = 25$ corresponds to 2025.

$$(t) = 632.95e^{0.01207t}$$

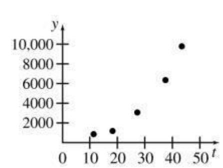
$$(20) = 632.95e^{0.01207(20)} \text{ H } 805.8$$

$$f(25) = 632.95e^{0.01207(25)} \text{ H } 855.9$$

The demand for physicians in 2020 will be about 805,800 and in 2020 will be about 855,900. These are very close to the values in the data.

The exponential regression function is $f(t) = 631.81e^{0.01224t}$. This is close to the function found in part (b).

41. (a)



The data appear to fit an exponential curve.

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The year 1963 corresponds to $t = 0$ and the year 2006 corresponds to $t = 43$.

$$f(t) = f_0 a^t$$

$$f(0) = f_0 a^0 \qquad f(43) = f_0 a^{43}$$

$$487 = f_0 \qquad 9789 = f_0 a^{43}$$

$$f_0 = \frac{9789}{a^{43}}$$

The function passes through the points (0, 487) and (43, 9789). Now solve for a.

$$487 = \frac{9789}{a^{43}} \Rightarrow a^{43} = \frac{9789}{487} \Rightarrow a = \left(\frac{9789}{487}\right)^{1/43} \Rightarrow a \approx 1.0723$$

Thus, $f(t) = 487(1.0723)^t$.

From part (b), we have $a \approx 1.0723$, so the number of breeding pairs is about 1.0723 times the number of breeding pairs in the previous year. Therefore, the average annual percentage increase is about 7.2%.

(d) Using a TI-84 Plus, the exponential regression is $f(t) = 398.8(1.0762)^t$.

42. (a) $t = 10$

$$= 100\exp\{9.8901 \exp - \exp(2.5420 - 0.2167(10))\}$$ 006

$t = 20$

$$= 100\exp\{9.8901 \exp - \exp(2.5420 - 0.2167(20))\}$$ 432,500

$t = 30$

$$= 100\exp\{9.8901 \exp - \exp(2.5420 - 0.2167(30))\}$$ 1,637,000

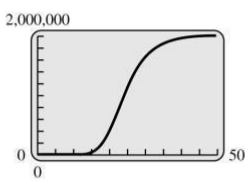
$t = 40$

$$= 100\exp\{9.8901 \exp - \exp(2.5420 - 0.2167(40))\}$$ 1,931,000

$t = 50$

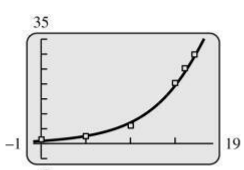
$$= 100\exp\{9.8901 \exp - \exp(2.5420 - 0.2167(50))\}$$ 1,969,000

(b)



The number of bacteria levels off at about 2,000,000.

(a)



$f(11) = 0.8454e^{0.2081(11)} \approx 8.341$ The risk increases about 6.8%

$f(6) = 0.8454e^{0.2081(6)} \approx 2.947$

The risk for a man with a score of 6 is about 2.9%.

$(14) = 0.8454e^{0.2081(14)} \approx 15.572$

The risk increases by about 12.6%.

$f(3) = 0.1210e^{0.2249(3)} \approx 0.2376$

The risk for a woman with a score of 3 is about 0.24%.

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The function fits the data closely.

$f(3) = 0.8454e^{0.2081(3)} \approx 1.578$

The risk for a man with a score of 3 is about 1.6%.

$f(12) = 0.1210e^{0.2249(12)} \approx 1.7983$

The risk increases by about 1.6%.

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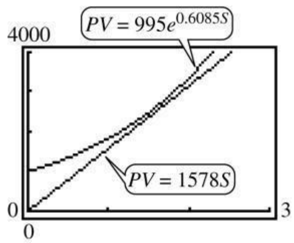
$f(3) = 0.1210e^{0.2249(6)} \approx 0.4665$

The risk for a woman with a score of 6 is about 0.47%.

$(15) = 0.1210e^{0.2249(15)} \approx 3.5308$

The risk increases by about 3.1%

44.



45. $C = \dfrac{D \times a}{V(a-b)}\left(e^{-bt} - e^{-at}\right)$

(a) At time $t = 0$,

$C = \dfrac{D \times a}{V(a-b)}\left(e^{-b0} - e^{-a0}\right)$

$= \dfrac{D \times a}{V(a-b)}\left(e^{0} - e^{0}\right)$

$= \dfrac{D \times a}{V(a-b)}(0) = 0$

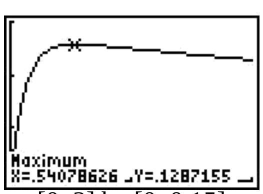
This makes sense because no cortisone has been administered yet.

As a large amount of time passes, the concentration is going to approach zero. The longer the drug is in the body, the lower the concentration.

$D = 500,\ a = 8.5,\ b = 0.09,\ V = 3700$

$C = \dfrac{500 \times 8.5}{3700(8.5 - 0.09)}\left(e^{-0.09t} - e^{-8.5t}\right)$

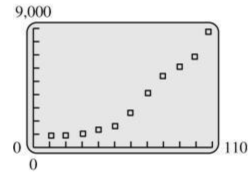
$\dfrac{4250}{31{,}117}\left(e^{-0.09t} - e^{-8.5t}\right)$



[0, 2] by [0, 0.15]

The maximum concentration occurs at about $t = 0.54$ hour.

46. (a)



The emissions appear to grow exponentially.

$f(x) = f_0 a^x$

$f_0 = 534$

Use the point (108, 8749) to find a.

$8749 = 534a^{108}$

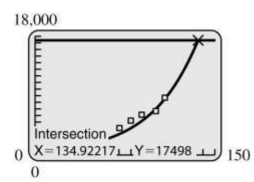
$a^{108} = \dfrac{6750}{534}$

$a = \sqrt[108]{\dfrac{8749}{534}} \approx 1.0262$

$f(x) = 534\,(1.0262)^x$

(c) From part (b), we have $a \approx 1.0262$, so the amount of carbon dioxide emissions is about 1.0262 times the amount in the previous year. Therefore, the average annual percentage increase is about 2.62%.

(d)



The doubling point is reached when $x \approx 135$. Thus, the first year in which emissions equal or exceed that threshold is 2035.

$Q(t) = 1000(5^{-0.3t})$

$Q(6) = 1000\,5^{-0.3(6)} \approx 55.189$

In 6 months, there will be about 55 grams.

$8 = 1000(5^{-0.3t})$

$\dfrac{1}{125} = 5^{-0.3t}$

$5^{-3} = 5^{-0.3t}$

$-3 = -0.3t$

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$10 = t$

It will take 10 months to reduce the substance to 8 grams.

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(a) When $x = 0$, $P = 1013$. When $x = 10{,}000$, $P = 265$.

First we fit $P = ae^{kx}$.

$$P_0 = 1013$$

$$P = 1013e^{kx}$$

$$265 = 1013e^{k(10{,}000)}$$

$$\frac{265}{1013} = e^{10{,}000k}$$

$$10{,}000k = \ln\frac{265}{1013}$$

$$k = \frac{\ln\left(\frac{265}{1013}\right)}{10{,}000} \approx -1.34\times10^{-4}$$

Therefore $P = 1013e^{(-1.34\times10^{-4})x}$.

We use the points (0, 1013) and (10,000, 265).

$$m = \frac{265 - 1013}{10{,}000 - 0} = -0.0748$$

$$b = 1013$$

Therefore $P = -0.0748x + 1013$.

Finally, we fit $P = \frac{1}{ax + b}$.

$$1013 = \frac{1}{a(0) + b}$$

$$b = \frac{1}{1013} \approx 9.87\times10^{-4}$$

$$P = \frac{1}{ax + \frac{1}{1013}}$$

$$265 = \frac{1}{10{,}000a + \frac{1}{1013}}$$

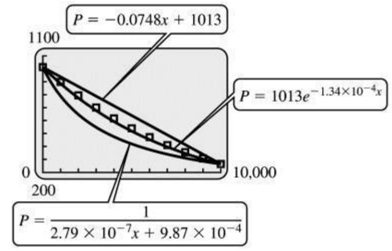
$$\frac{1}{265} = 10{,}000a + \frac{1}{1013}$$

$$10{,}000a = \frac{1}{265} - \frac{1}{1013}$$

$$a = \frac{\frac{1}{265} - \frac{1}{1013}}{10{,}000} \approx 2.79\times10^{-7}$$

Therefore,

(b)



$P = 1013e^{(-1.34\times10^{-4})x}$ is the best fit.

$$P(1500) = 1013e^{-1.34\times10^{-4}(1500)} \approx 829$$

$$P(11{,}000) = 1013e^{-1.34\times10^{-4}(11{,}000)} \approx 232$$

We predict that the pressure at 1500 meters will be 829 millibars, and at 11,000 meters will be 232 millibars.

Using exponential regression, we obtain $P = 1038(0.9998661)^x$ which differs slightly from the function found in part (b) which can be rewritten as $P = 1013(0.9998660)^x$.

(a) $y = mt + b$

$b = 0.275$

Use the points (0, 0.275) and (24, 1900) to find m.

$$m = \frac{1900 - 0.275}{24 - 0} = \frac{1899.725}{24} \approx 79.155$$

$$y = 79.155t + 0.275$$

$$y = at^2 + b$$

$$b = 0.275$$

Use the point (24, 1900) to find a.

$$1900 = a(24)^2 + 0.275$$

$$1899.925 = 576a$$

$$a = \frac{1899.725}{576} \approx 3.298$$

$$y = 3.298t^2 + 0.275$$

$$y = ab^t$$

$$a = 0.275$$

Use the point (24, 1900) to find b.

$$1900 = 0.275b^{24}$$

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$$P = \frac{1}{(2.79 \times 10^{-7})x + (9.87 \times 10^{-4})}.$$

$$b^{24} = \frac{1900}{0.275}$$

$$b = \sqrt[24]{\frac{1900}{275}} \approx 1.445$$

$$= 0.275(1.445)^t$$

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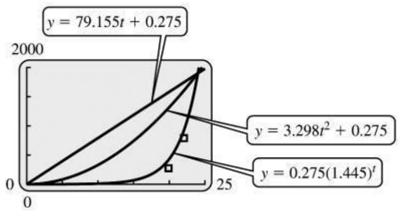
(b)



The function $y = 0.275(1.445)^t$ is the best fit.

$x = 30$ corresponds to 2015.

$= 0.275(1.445)^{30} = 17193.65098 \approx 17,200$

The regression function is

$= 0.1787(1.441)^t$.

This is close to the function in part (b).

(a) $C = mt + b$

Use the points (1, 24,322) and (11, 237,016) to find m.

$m = \dfrac{237,016 - 24,322}{11 - 1} = \dfrac{212,694}{10}$

$= 21,269.4$

Use the point (1, 24,322) to find b.

$= 21,269.4t + b$

$24,322 = 21,269.4(1) + b$

$b = 3052.6$

Thus, the linear function is

$C = 21,269.4t + 3052.6$

$C = at^2 + b$

Use the points (1, 24,322) and (11, 237,016) to find a and b.

$24,322 = a(1)^2 + b$	$237,016 = a(11)^2 + b$
$24,322 = a + b$	$237,016 = 121a + b$

Now solve the system

$121a + b = 237,016$

$\underline{\quad a + b = 24,322}$

$120a = 212,694$

$a = \dfrac{212,694}{120} = 1772.45$

Now solve for b.

$a + b = 24,322$

$= 24,322 - 1772.45 = 22,549.55$

Thus, the quadratic function is

$= 1772.45t^2 + 22,549.55$

$C = ab^t$

Use the points (1, 24,322) and (11, 237,016) to find a and b. $24,322 = ab^1 = ab$ $237,016 = ab^{11}$

Solve for b.

$\dfrac{237,016}{24,322} = \dfrac{ab^{11}}{ab} = b^{10}$

$b = \sqrt[10]{\dfrac{237,016}{24,322}} \approx 1.2557$

Now solve for a.

$ab = 24,322$

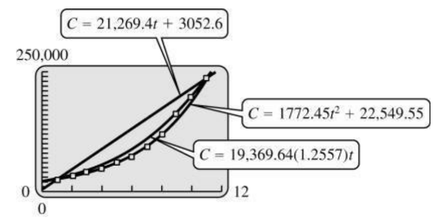
$a\sqrt[10]{\dfrac{237,016}{24,322}} = 24,322$

$a = \dfrac{24,322}{\sqrt[10]{\dfrac{237,016}{24,322}}} \approx 19,369.64$

Thus, the exponential function is

$= 19,369.64\,(1.2557)^t$

(b)



The exponential function

$= 19,369.64\,(1.2557)^t$ is the best fit.

Using a TI-84 Plus, the exponential regression is $f(t) = 19,259.86\,(1.2585)^t$.

This is very close to the function in part (b).

The year 2012 corresponds to $x = 12$.

Using the linear function, we have

$= 21,269.4t + 3052.6$

$= 21,269.4(12) + 3052.6 = 258,285.4$

Using the quadratic function, we have

$= 1772.45t^2 + 22,549.55$

$= 1772.45(12)^2 + 22,549.55 = 277,782.35$

Using the exponential function, we have

$= 19,369.64\,(1.2557)^t$

$= 19,369.64(1.2557)^{12} \approx 297,682$

Using the regression function, we have

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$t) = 19,259.86\,(1.2585)^t$

$(12) = 19,259.86(1.2585)^{12}$

$304,013$

Using the linear function, the total world wind energy capacity will be about 258,285 MW in 2012.

(*continued on next page*)

(*continued*)

Using the quadratic function, the total world wind energy capacity will be about 277,782 MW in 2012.
Using the exponential function, the total world wind energy capacity will be about 297,682 MW in 2012.

Using the regression function, the total

world wind energy capacity will be about 304,013 MW in 2012.
The quadratic function value of 277,782 is the value closest to the predicted value of 273,000.

51. $A = P\left(1 + \frac{r}{m}\right)^{tm}$, $P = 10{,}000$, $r = 0.04$, $t = 5$

annually, $m = 1$

$A = 10{,}000\left(1 + \frac{0.04}{1}\right)^{5(1)}$

$10{,}000(1.04)^5$
$\$12{,}166.53$
Interest = \$12,166.53 − \$10,000
\$2166.53

semiannually, $m = 2$

$A = 10{,}000\left(1 + \frac{0.04}{2}\right)^{5(2)}$

$10{,}000(1.02)^{10}$
$\$12{,}189.94$
Interest = \$12,189.94 − \$10,000
\$2189.94

quarterly, $m = 4$

$A = 10{,}000\left(1 + \frac{0.04}{4}\right)^{5(4)}$

$10{,}000(1.01)^{20}$
\$12, 201.90
Interest = \$12, 201.90 − \$10,000
= \$2201.90

(d) monthly, $m = 12$

$A = 10{,}000\left(1 + \frac{0.04}{12}\right)^{5(12)}$

$= 10{,}000(1.003\overline{)}^{60}$

= \$12, 209.97
Interest = \$12, 209.97 − \$10,000
= \$2209.97

52. $A = P\left(1 + \frac{r}{m}\right)^{tm}$, $P = 26{,}000$, $r = 0.06$, $t = 4$

annually, $m = 1$

$A = 26{,}000\left(1 + \frac{0.06}{1}\right)^{4(1)}$

$= 26{,}000(1.06) = \$32{,}824.40$

Interest = \$32,824.40 − \$26,000
= \$6824.40

(b) semiannually, $m = 2$

$A = 26{,}000\left(1 + \frac{0.06}{2}\right)^{4(2)}$

$26{,}000(1.03)^8$
\$32.936.02
Interest = \$32.936.02 − \$26,000
\$6936.02

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uarterly, $m = 4$

$A = 26{,}000\left(1 + \frac{0.06}{4}\right)^{4(4)}$

$26{,}000(1.015)^{16}$
\$32,993.62
Interest = \$32,993.62 −
\$26,000 = \$6993.62

(d) monthly, $m = 12$

$A = 26{,}000\left(1 + \frac{0.06}{12}\right)^{4(12)}$

$26{,}000(1.005)^{48}$
\$33,032.72
Interest = \$33,032.72 − \$26,000
\$7032.72

continuously, $t = 4$

$= 26{,}000e^{(0.06)(4)}$
= \$33,052.48
Interest = \$33,052.48 − \$26,000
\$7052.48

For 6% compounded annually for 2 years,
$= 18{,}000\left(1 + \frac{0.06}{2}\right)^2 =$
$18{,}000(1.06) = 20{,}224.80$

(e) continuously, $t = 5$

$(0.04)(5)$

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Section 2.1 EXPONENTIAL FUNCTIONS 98

For 5.9% compounded monthly for 2 years,

0

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= 10,000*e* = $12,

214.03 Interest = $12, 214.03 − $10,000
$2214.03

$$A = 18,000\left(1 + \frac{.059}{12}\right)^{12(2)}$$

$$= 18,000\left(\frac{12.059}{12}\right)^{24} = 20,248.54$$

The 5.9% investment is better. The additional interest is
$20, 248.54 − $20, 224.80 = $23.74.

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54. $A = P\left(1 + \frac{r}{m}\right)^{tm}$, $P = 5000$, $A = \$7500$, $t = 5$

(a) $m = 1$

$$7500 = 5000\left(1 + \frac{r}{1}\right)^{5(1)}$$

$$\frac{3}{2} = (1 + r)^5$$

$$\left(\frac{3}{2}\right)^{1/5} - 1 = r$$

$$0.084 \approx r$$

The interest rate is about 8.4%

$m = 4$

$$7500 = 5000\left(1 + \frac{r}{4}\right)^{5(4)}$$

$$\frac{3}{2} = \left(1 + \frac{r}{4}\right)^{20}$$

$$\left(\frac{3}{2}\right)^{1/20} - 1 = \frac{r}{4}$$

$$4\left[\left(\frac{3}{2}\right)^{1/20} - 1\right] \approx r$$

$$0.082 = r$$

The interest rate is about 8.2%.

55. $A = Pe^{rt}$

$r = 3\%$

$= 10e^{0.03(3)} \approx \$10.94$

$r = 4\%$

$= 10e^{0.04(3)} \approx \$11.27$

$r = 5\%$

$= 10e^{0.05(3)} \approx \$11.62$

56. $P = \$25{,}000$, $r = 5.5\%$

Use the formula for continuous compounding, $A = Pe^{rt}$.

$t = 1$

$= 25{,}000e^{0.055(1)}$

57. $1200 = 500\left(1 + \frac{r}{4}\right)^{(14)(4)}$

$$\frac{1200}{500} = \left(1 + \frac{r}{4}\right)^{56}$$

$$2.4 = \left(1 + \frac{r}{4}\right)^{56}$$

$$1 + \frac{r}{4} = (2.4)^{1/56}$$

$$4 + r = 4(2.4)^{1/56}$$

$$r = 4(2.4)^{1/56} - 4$$

$$r \approx 0.0630$$

The required interest rate is 6.30%.

58. (a) $30{,}000 = 10{,}500\left(1 + \frac{r}{4}\right)^{(4)(12)}$

$$\frac{300}{105} = \left(1 + \frac{r}{4}\right)^{48}$$

$$1 + \frac{r}{4} = \left(\frac{300}{105}\right)^{1/48}$$

$$4 + r = 4\left(\frac{300}{105}\right)^{1/48}$$

$$r = 4\left(\frac{300}{105}\right)^{1/48} - 4$$

$$\approx 0.0884$$

The required interest rate is 8.84%.

(b) $30{,}000 = 10{,}500\left(1 + \frac{r}{365}\right)^{12(365)}$

$$\frac{300}{105} = \left(1 + \frac{r}{365}\right)^{4380}$$

$$1 + \frac{r}{365} = \left(\frac{300}{105}\right)^{1/4380}$$

$$365 + r = 365\left(\frac{300}{105}\right)^{1/4380}$$

$$r = 365\left(\frac{300}{105}\right)^{1/4380} - 365$$

$26, 413.52

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$t = 5$

$= 25{,}000e^{0.055(5)}$
\$32,913.27

$t = 10$

$= 25{,}000e^{0.055(10)}$
\$43,331.33

0.0875
The required interest rate is 8.75%.

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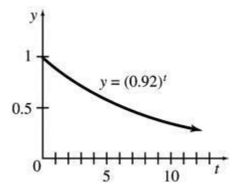
$y = (0.92)^t$

(a)

t	y
	$(0.92)^0 = 1$
	$(0.92)^1 = 0.9$
	$(0.92)^2$ H 0.8
	$(0.92)^3$ H 0.7
	$(0.92)^4$ H 0.7
	$(0.92)^5$ H 0.66
	$(0.92)^7$ H 0.5
	$(0.92)^6$ H 0.61
	$(0.92)^9$ H 0.47
	$(0.92)^8$ H 0.51
10	$(0.92)^{10}$ H 0.43

(b)



Let x = the cost of the house in 10 years.

Then, $0.43x = 165{,}000 \Rightarrow x$ H 383,721. In 10 years, the house will cost about

$384,000.

Let x = the cost of the book in 8 years.
Then, $0.51x = 50 \Rightarrow x$ H 98.
In 8 years, the textbook will cost

about $98.

60. $A = P\left(1+\frac{r}{m}\right)^{tm}$

$A = 1000\left(1+\frac{j}{2}\right)^{5(2)} = 1000\left(1+\frac{j}{2}\right)^{10}$

This represents the amount in Bank X on January 1, 2005.

$A = P\left(1+\frac{r}{m}\right)^{tm}$

$= 1000\left(1+\frac{j}{2}\right)^{10}\left(1+\frac{k}{4}\right)^{3(4)}$

$1000\left(1+\frac{j}{2}\right)^{10}\left(1+\frac{k}{4}\right)^{12}$

This represents the amount in Bank Y on

January 1, 2008, $1990.76.

$A = P\left(1+\frac{r}{m}\right)^{tm} = 1000\left(1+\frac{k}{4}\right)^{8(4)}$

$1000\left(1+\frac{k}{4}\right)^{32}$

This represents the amount he could have had from January 1, 2000, to January 1, 2008, at a rate of k per annum compounded quarterly, $2203.76.

So,

$1000\left(1+\frac{j}{2}\right)^{10}\left(1+\frac{k}{4}\right)^{12} = 1990.76$

and $1000\left(1+\frac{k}{4}\right)^{32} = 2203.76.$

$\left(1+\frac{k}{4}\right)^{32} = 2.20376$

$1+\frac{k}{4} = (2.20376)^{1/32}$

$1+\frac{k}{4} = 1.025$

$\frac{k}{4} = 0.025$

$k = 0.1$ or 10%

Substituting, we have

$1000\left(1+\frac{j}{2}\right)^{10}\left(1+\frac{0.1}{4}\right)^{12} = 1990.76$

$1000\left(1+\frac{j}{2}\right)^{10}(1.025)^{12} = 1990.76$

$\left(1+\frac{j}{2}\right)^{10} = 1.480$

$1+\frac{j}{2} = (1.480)^{1/10}$

$1+\frac{j}{2} = 1.04$

$\frac{j}{2} = 0.04$

$j = 0.08$ or 8%

The ratio $\frac{k}{j} = \frac{0.1}{0.08} = 1.25$, is choice **a**.

2.2 Logarithmic Functions

$5^3 = 125$

Since $a^y = x$ means $y = \log_a x$, the equation in logarithmic form is $\log_5 125 = 3$.

$7^2 = 49$

Since $a^y = x$ means $y = \log_a x$, the equation in logarithmic form is $\log_7 49 = 2$.

$3^4 = 81$

Since $a^y = x$ means $y = \log_a x$, the equation in logarithmic form is $\log_3 81 = 4$.

$2^7 = 128$

Since $a^y = x$ means $y = \log_a x$, the equation in logarithmic form is $\log_2 128 = 7$.

$3^{-2} = \frac{1}{9}$

Since $a^y = x$ means $y = \log_a x$, the equation in logarithmic form is $\log_3 \frac{1}{9} = -2$.

$\left(\frac{5}{4}\right)^{-2} = \frac{16}{25}$

Since $a^y = x$ means $y = \log_a x$, the equation in logarithmic form is $\log_{5/4} \frac{16}{25} = -2$.

$\log_2 32 = 5$

Since $y = \log_a x$ means $a^y = x$, the equation in exponential form is $2^5 = 32$.

$\log_3 81 = 4$

Since $y = \log_a x$ means $a^y = x$, the equation

$\ln \frac{1}{e} = -1$

equation in exponential form is $e^{-1} = \frac{1}{e}$.

$\log_2 \frac{1}{8} = -3$

Since $y = \log_a x$ means $a^y = x$, the equation in exponential form is $2^{-3} = \frac{1}{8}$.

$\log 0.001 = -3$

$\log_{10} 0.001 = -3$

$10^{-3} = 0.001$

When no base is written, $\log_{10} x$ is understood.

Let $\log_8 64 = x$. Then,

$8^x = 64 \Rightarrow 8^x = 8^2 \Rightarrow x = 2$

Thus, $\log_8 64 = 2$.

Let $\log_9 81 = x$. Then,

$9^x = 81 \Rightarrow 9^x = 9^2 \Rightarrow x = 2$

Thus $\log_9 81 = 2$.

$\log_4 64 = x \Rightarrow 4^x = 64 \Rightarrow 4^x = 4^3 \Rightarrow x = 3$

$\log_3 27 = x \Rightarrow 3^x = 27 \Rightarrow 3^x = 3^3 \Rightarrow x = 3$

$\log_2 \frac{1}{16} = x \Rightarrow 2^x = \frac{1}{16} \Rightarrow 2^x = 2^{-4} \Rightarrow x = -4$

$\log_3 \frac{1}{81} = x \Rightarrow 3^x = \frac{1}{81} \Rightarrow 3^x = 3^{-4} \Rightarrow x = -4$

19. $\log_2 \sqrt[3]{\frac{1}{4}} = x \Rightarrow 2^x = \left(\frac{1}{4}\right)^{1/3} \Rightarrow$

$2^x = \left(\frac{1}{2^2}\right)^{1/3} \Rightarrow 2^x = 2^{-2/3} \Rightarrow x = -\frac{2}{3}$

$\log_8 \sqrt[4]{\frac{1}{2}} = x$

$8^x = \sqrt[4]{\frac{1}{2}} = \left(\frac{1}{2}\right)^{1/4}$

$(2^3)^x = 2^{-1/4}$

$3x = -\frac{1}{4} \Rightarrow x = -\frac{1}{12}$

$\ln e = x$

$\log 100{,}000 = 5$

$\log_{10} 100{,}000 = 5$

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Section 2.2 LOGARITHMS

$10^5 = 100,000$

When no base is written, $\log_{10} x$ is understood.

Recall that ln y means $\log_e y$.

$e^x = e \Rightarrow x = 1$

$\ln e^3 = x$

Recall that ln y means $\log_e y$.

$e^x = e^3 \Rightarrow x = 3$

$\ln e^{5/3} = x \Rightarrow e^x = e^{5/3} \Rightarrow x = \frac{5}{3}$

$\ln \ln 1 = x \Rightarrow e^x = 1 \Rightarrow e^x = e^0 \Rightarrow x = 0$

The logarithm to the base 3 of 4 is written $\log_3 4$. The subscript denotes the base.

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Answers will vary.

$\log_5 (3k) = \log_5 3 + \log_5 k$

$\log_9 (4m) = \log_9 4 + \log_9 m$

$\log_3 \frac{3p}{5k} = \log_3 3p - \log_3 5k$

$= (\log_3 3 + \log_3 p) - (\log_3 5 + \log_3 k)$

$= 1 + \log_3 p - \log_3 5 - \log_3 k$

30. $\log_7 \frac{15p}{7y}$

$\log_7 15p - \log_7 7y$

$(\log_7 15 + \log_7 p) - (\log_7 7 + \log_7 y)$

$\log_7 15 + \log_7 p - \log_7 7 - \log_7 y$

$\log_7 15 + \log_7 p - 1 - \log_7 y$

31. $\ln \frac{3\sqrt{5}}{\sqrt[3]{6}} = \ln 3\sqrt{5} - \ln \sqrt[3]{6}$

$= \ln 3 \cdot 5^{1/2} - \ln 6^{1/3}$

$= \ln 3 + \ln 5^{1/2} - \ln 6^{1/3}$

$= \ln 3 + \frac{1}{2}\ln 5 - \frac{1}{3}\ln 6$

32. $\ln \frac{9\sqrt[3]{5}}{\sqrt[4]{3}} = \ln 9\sqrt[3]{5} - \ln \sqrt[4]{3}$

$= \ln 9 \cdot 5^{1/3} - \ln 3^{1/4}$

$= \ln 9 + \ln 5^{1/3} - \ln 3^{1/4}$

$= \ln 9 + \frac{1}{3}\ln 5 - \frac{1}{4}\ln 3$

33. $\log_b 32 = \log_b 2^5 = 5\log_b 2 = 5a$

34. $\log_b 18 = \log_b (2 \cdot 9) = \log_b (2 \cdot 3^2)$

$= \log_b 2 + \log_b 3^2$

$= \log_b 2 + 2\log_b 3$

$= a + 2c$

35. $\log_b 72b = \log_b 72 + \log_b b = \log_b 72 + 1$

$= \log_b \left(2^3 \cdot 3^2\right) + 1$

$= \log_b 2^3 + \log_b 3^2 + 1$

38. $\log_{12} 210 = \frac{\ln 210}{\ln 12} \approx 2.152$

39. $\log_{1.2} 0.95 = \frac{\ln 0.95}{\ln 1.2} \approx -0.281$

40. $\log_{2.8} 0.12 = \frac{\ln 0.12}{\ln 2.8} \approx -2.059$

41. $\log_x 36 = -2$

$x^{-2} = 36$

$(x^{-2})^{-1/2} = 36^{-1/2}$

$x = \frac{1}{6}$

42. $\log_9 27 = m$

$9^m = 27$

$(3^2)^m = 3^3$

$3^{2m} = 3^3$

$2m = 3$

$m = \frac{3}{2}$

43. $\log_8 16 = z$

$8^z = 16$

$(2^3)^z = 2^4$

$2^{3z} = 2^4$

$3z = 4$

$z = \frac{4}{3}$

44. $\log_y 8 = \frac{3}{4}$

$y^{3/4} = 8$

$(y^{3/4})^{4/3} = 8^{4/3}$

$y = (8^{1/3})^4$

$y = 2^4$

$y = 16$

45. $\log_r 5 = \frac{1}{2}$

$r^{1/2} = 5$

$(r^{1/2})^2 = 5^2$

$r = 25$

46. $\log_4 (5x + 1) = 2$

$4^2 = 5x + 1$

$16 = 5x + 1$

$5x = 15$

$x = 3$

47. $\log_5 (9x - 4) = 1$

$5^1 = 9x - 4$

$= 9x$

$= x$

$\log_4 x - \log_4 (x + 3) = -1$

$\log_4 \frac{x}{x+3} = -1$

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$= 3\log_b 2 + 2\log_b 3 + 1$
$= 3a + 2c + 1$

36. $\log_b (9b^2) = \log_b 9 + \log_b b^2$
$= \log_b 3^2 + \log_b b^2$
$= 2\log_b 3 + 2\log_b b$
$= 2c + 2(1) = 2c + 2$

37. $\log_5 30 = \dfrac{\ln 30}{\ln 5} \approx \dfrac{3.4012}{1.6094} \approx 2.113$

$4^{-1} = \dfrac{x}{x+3}$
$\dfrac{1}{4} = \dfrac{x}{x+3}$
$4x = x + 3$
$3x = 3 \Rightarrow x = 1$

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$$\log_9 m - \log_9 (m - 4) = -2$$
$$\log_9 \frac{m}{m-4} = -2$$
$$9^{-2} = \frac{m}{m-4}$$
$$\frac{1}{81} = \frac{m}{m-4}$$
$$m - 4 = 81m$$
$$-4 = 80m \Rightarrow -0.05 = m$$

This value is not possible since $\log_9(-0.05)$ does not exist. Thus, there is no solution to the original equation.

50. $\log(x + 5) + \log(x + 2) = 1$
$$\log[(x + 5)(x + 2)] = 1$$
$$(x + 5)(x + 2) = 10^1$$
$$x^2 + 7x + 10 = 10$$
$$x^2 + 7x = 0$$
$$x(x + 7) = 0$$
$$x = 0 \text{ or } x = -7$$

$x = -7$ is not a solution of the original equation because if $x = -7$, $x + 5$ and $x + 2$ would be negative, and log (–2) and log (–5) do not exist. Therefore, $x = 0$.

51. $\log_3(x - 2) + \log_3(x + 6) = 2$
$$\log_3[(x - 2)(x + 6)] = 2$$
$$(x - 2)(x + 6) = 3^2$$
$$x^2 + 4x - 12 = 9$$
$$x^2 + 4x - 21 = 0$$
$$(x + 7)(x - 3) = 0$$
$$x = -7 \text{ or } x = 3$$

$x = -7$ is not a solution of the original equation because if $x = -7$, $x + 6$ would be negative, and log (–1) does not exist. Therefore, $x = 3$.

$$\log_3(x^2 + 17) - \log_3(x + 5) = 1$$
$$\log_3 \frac{x^2+17}{x+5} = 1$$
$$3^1 = \frac{x^2 + 17}{x + 5}$$
$$3x + 15 = x^2 + 17$$
$$0 = x^2 - 3x + 2$$
$$0 = (x - 1)(x - 2)$$
$$x = 1 \text{ or } x = 2$$

$$\log_2(x^2 - 1) - \log_2(x + 1) = 2$$
$$\log_2 \frac{x^2 - 1}{x + 1} = 2$$
$$2^2 = \frac{x^2 - 1}{x + 1}$$
$$4 = \frac{x^2 - 1}{x + 1}$$
$$4x + 4 = x^2 - 1$$
$$x^2 - 4x - 5 = 0$$
$$(x - 5)(x + 1) = 0$$
$$x = 5 \text{ or } x = -1$$

$x = -1$ is not a solution of the original equation because if $x = -1$, $x + 1$ would not exist. Therefore, $x = 5$.

$$\ln(5x + 4) = 2$$
$$5x + 4 = e^2$$
$$5x = e^2 - 4$$
$$x = \frac{e^2 - 4}{5} \approx 0.6778$$

$$\ln x + \ln 3x = -1$$
$$\ln 3x^2 = -1$$
$$3x^2 = e^{-1}$$
$$x^2 = \frac{e^{-1}}{3}$$
$$x = \sqrt{\frac{e^{-1}}{3}} = \frac{1}{\sqrt{3e}} \approx 0.3502$$

$$\ln(x + 1) - \ln x = 1$$
$$\ln \frac{x + 1}{x} = 1$$
$$\frac{x + 1}{x} = e^1$$
$$x + 1 = ex$$
$$ex - x = 1$$
$$x(e - 1) = 1$$
$$x = \frac{1}{e - 1} \approx 0.5820$$

57. $2^x = 6$
$$\ln 2^x = \ln 6$$
$$x \ln 2 = \ln 6$$
$$x = \frac{\ln 6}{\ln 2} \approx 2.5850$$

58.

$$5^x = 12$$
$$x \log 5 = \log 12$$
$$x = \frac{\log 12}{} \approx 1.5440$$

$e^{k-1} = 6$

$\ln e^{k-1} = \ln 6$

$k - 1 = \frac{\ln 6}{\ln e}$

$k - 1 = \frac{\ln 6}{1}$

$k = 1 + \ln 6 \approx 2.7918$

60. $e^{2y} = 15$

$\ln e^{2y} = \ln 15$

$2y \ln e = \ln 15$

$2y(1) = \ln 15$

$y = \frac{\ln 15}{2} \approx 1.3540$

$3^{x+1} = 5^x$

$\ln 3^{x+1} = \ln 5^x$

$(x+1)\ln 3 = x \ln 5$

$x \ln 5 - x \ln 3 = \ln 3$

$x(\ln 5 - \ln 3) = \ln 3$

$\frac{\ln 3}{\ln(5/3)} \approx 2.1507$

$2^{x+1} = 6^{x-1}$

$\ln 2^{x+1} = \ln 6^{x-1}$

$(x+1)\ln 2 = (x-1)\ln 6$

$x \ln 2 + \ln 2 = x \ln 6 - \ln 6$

$x \ln 6 - x \ln 2 = \ln 2 + \ln 6$

$x(\ln 6 - \ln 2) = \ln 2 + \ln 6$

$x \ln 3 = \ln 12$

$x = \frac{\ln 12}{\ln 3} \approx 2.2619$

$5(0.10)^x = 4(0.12)^x$

$\ln[5(0.10)^x] = \ln[4(0.12)^x]$

$\ln 5 + x \ln 0.10 = \ln 4 + x \ln 0.12$

$x(\ln 0.12 - \ln 0.10) = \ln 5 - \ln 4$

$x = \frac{\ln 5 - \ln 4}{\ln 0.12 - \ln 0.10} = \frac{\ln 1.25}{\ln 1.2} \approx 1.2239$

64. $1.5(1.05)^x = 2(1.01)^x$

$\ln[1.5(1.05)^x] = \ln[2(1.01)^x]$

$\ln 1.5 + x \ln 1.05 = \ln 2 + x \ln 1.01$

$x(\ln 1.01 - \ln 1.05) = \ln 1.5 - \ln 2$

$x = \frac{\ln 0.75}{\ln(1.01/1.05)} \approx 7.4069$

For exercises 65−68, use the formula $a^x = e^{(\ln a)x}$.

$10^{x+1} = e^{(\ln 10)(x+1)}$

$10^{x^2} = e^{(\ln 10)x^2}$

$e^{3x} = (e^3)^x \approx 20.09^x$

$e^{-4x} = (e^{-4})^x \approx 0.0183^x$

$f(x) = \log(5 - x)$

$5 - x > 0 \Rightarrow -x > -5 \Rightarrow x < 5$

The domain of f is $x < 5$ or $(-\infty, 5)$.

$f(x) = \ln(x^2 - 9)$

Since the domain of $f(x) = \ln x$ is $(0, \infty)$, the domain of $f(x) = \ln(x^2 - 9)$ is the set of all real numbers x for which $x^2 - 9 > 0$. To solve this quadratic inequality, first solve the corresponding quadratic equation.

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x + 3 = 0$ or $x - 3 = 0$

$x = -3$ or $x = 3$

These two solutions determine three intervals on the number line: $(-\infty, -3)$, $(-3, 3)$, and $(3, \infty)$. If $x = -4$, $(-4+3)(-4-3) > 0$. If $x = 0$, $(0+3)(0-3) > 0$. ⁄
If $x = 4$, $(4+3)(4-3) > 0$.
The domain is $x < -3$ or $x > 3$, which is written in interval notation as $(-\infty, -3) \cup (3, \infty)$.

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$\log A - \log B = 0$

$\log \frac{A}{B} = 0$

$= 10^0 = 1$

$A - B = 0$

Thus, solving $\log A - \log B = 0$ is equivalent to solving $A - B = 0$

$(\log(x+2))^2 \neq 2\log(x+2)$

$(\log(x+2))^2 = (\log(x+2))(\log(x+2))$ $2\log(x+2) \neq 2(\log x + \log 2)$

$2\log(x+2) = \log(x+2)^2$

$\log 2 \neq 100$

$\log 2 \approx 0.30103$ because $10^{0.30103} = 2$

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Let $m = \log_a \frac{x}{y}$, $n = \log_a x$, and $p = \log_a y$.

Then $a^m = \frac{x}{y}$, $a^n = x$, and $a^p = y$.

Substituting gives $a^m = \frac{x}{y} = \frac{a^n}{a^p} = a^{n-p}$.

So $m = n - p$. Therefore,

$$\log_a \frac{x}{y} = \log_a x - \log_a y.$$

Let $m = \log_a x^r$ and $n = \log_a x$.

Then, $a^m = x^r$ and $a^n = x$. Substituting gives $a^m = x^r = (a^n)^r = a^{nr}$. Therefore, $m = nr$, or $\log_a x^r = r \log_a x$.

75. (a) $t = \dfrac{\ln 2}{\ln(1 + 0.03)} \approx 23.4$ years

(b) $t = \dfrac{\ln 2}{\ln(1 + 0.06)} \approx 11.9$ years

(c) $t = \dfrac{\ln 2}{\ln(1 + 0.08)} \approx 9.0$ years

76. (a) $t = \dfrac{\ln 2}{\ln(1 + 0.06)} \approx 12$

It will take about 12 years for the population to at least double.

(b) $t = \dfrac{\ln 3}{\ln(1 + 0.06)} \approx 19$

It will take about 19 years for the population to at least triple.

77. (a) $h(t) = 37.79(1.021)^t$

Double the 2005 population is $2(42.69) = 85.38$ million.

$$85.38 = 37.79(1.021)^t$$
$$\frac{85.38}{37.79} = (1.021)^t$$
$$\log_{1.021} \frac{85.38}{37.79} = t$$
$$t = \frac{\ln\left(\frac{85.38}{37.79}\right)}{\ln 1.021} \approx 39.22$$

$h(t) = 11.14(1.023)^t$

Double the 2005 population is $2(12.69) = 25.38$ million.

$$25.38 = 11.14(1.023)^t$$
$$\frac{25.38}{11.14} = (1.023)^t$$
$$\log_{1.023} \frac{25.38}{11.14} = t$$
$$t = \frac{\ln\left(\frac{25.38}{11.14}\right)}{\ln 1.023} \approx 36.21$$

The Asian population is estimated to double their 2005 population in 2036.

Double the 2006 population is $2(9789) = 19{,}578$.

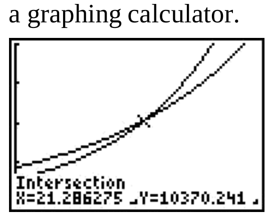
$$19{,}578 = 487(1.0723)^x$$
$$\frac{19{,}578}{487} = (1.0723)^x$$
$$\ln \frac{19{,}578}{487} = x \ln 1.0723$$
$$x = \frac{\ln \frac{19{,}578}{487}}{\ln 1.0723} \approx 52.92$$

The number of bald eagle breeding pairs will

Set the exponential growth functions $= 4500(1.04)^t$ and $y = 3000(1.06)^t$ equal to each other and solve for t.

$$4500(1.04)^t = 3000(1.06)^t$$
$$\ln\left(1.5(1.04)^t\right) = \ln(1.06)^t$$
$$\ln 1.5 + t \ln 1.04 = t \ln 1.06$$
$$\ln 1.5 = t(\ln 1.06 - \ln 1.04)$$
$$t = \frac{\ln 1.5}{\ln 1.06 - \ln 1.04} \approx 21.29$$

After about 21.3 years, the black squirrels will outnumber the gray squirrels. Verify this with a graphing calculator.



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The Hispanic population is estimated to double their 2005 population in 2039.

[0, 40] by [0, 20,000]

(a) New York: $f(t) = 18.2(1.001)^t$

Florida: $f(t) = 14.0(1.017)^t$

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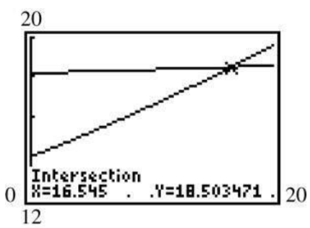
$$18.2(1.001)^t = 14.0(1.017)^t$$

$$\frac{18.2}{14.0} = \frac{(1.017)^t}{(1.001)^t} = \left(\frac{1.017}{1.001}\right)^t$$

$$\ln \frac{18.2}{14.0} = t \ln \left(\frac{1.017}{1.001}\right)$$

$$t = \frac{\ln \frac{18.2}{14.0}}{\ln \left(\frac{1.017}{1.001}\right)} \approx 16.54$$

If this trend continued, then the population of Florida would have exceeded that of New York about 16.5 years after 1994, or sometime in 2011.



New York: $f(t) = 19.5(1.004)^t$

Florida: $f(t) = 19.1(1.012)^t$

$$19.5(1.004)^t = 19.1(1.012)^t$$

$$\frac{(1.004)^t}{(1.012)^t} = \left(\frac{1.004}{1.012}\right)^t = \frac{19.1}{19.5}$$

$$t \ln \frac{1.004}{1.012} = \ln \frac{19.1}{19.5}$$

$$t = \frac{\ln \frac{19.1}{19.5}}{\ln \frac{1.004}{1.012}} \approx 2.61$$

If this trend continued, then the population of Florida would have exceeded that of New York about 2.6 years after 2011, or sometime in 2014.

It is less accurate to predict events far beyond the given data set.

(c) Yes, $\ln 2 \approx 0.693$.

82. $H = -[P_1 \ln P_1 + P_2 \ln P_2 + P_3 \ln P_3 + P_4 \ln P_4]$

$H = -[0.521 \ln 0.521 + 0.324 \ln 0.324 + 0.081 \ln 0.081 + 0.074 \ln 0.074]$

$H = 1.101$

83. (a) 3 species, $\frac{1}{3}$ each:

$$P_1 = P_2 = P_3 = \frac{1}{3}$$

$$= -(P_1 \ln P_1 + P_2 \ln P_2 + P_3 \ln P_3)$$

$$-3\left(\frac{1}{3} \ln \frac{1}{3}\right) = -\ln \frac{1}{3}$$

$$1.099$$

4 species, $\frac{1}{4}$ each:

$$P_1 = P_2 = P_3 = P_4 = \frac{1}{4}$$

$$H = -(P_1 \ln P_1 + P_2 \ln P_2 + P_3 \ln P_3 + P_4 \ln P_4)$$

$$-4\left(\frac{1}{4} \ln \frac{1}{4}\right) = -\ln \frac{1}{4}$$

$$1.386$$

Notice that

$$\ln \frac{1}{3} = \ln (3^{-1})^{-1} = \ln 3 \approx 1.099$$

and

$$\ln \frac{1}{4} = \ln (4^{-1})^{-1} = \ln 4 \approx 1.386$$

by Property **c** of logarithms, so the populations are at a maximum index of diversity.

(a) The total number of individuals in the community is 50 + 50, or 100.

Let $P_1 = \frac{50}{100} = 0.5$, $P_2 = 0.5$.

$$= -1[P_1 \ln P_1 + P_2 \ln P_2]$$

$$= -1[0.5 \ln 0.5 + 0.5 \ln 0.5]$$

$$0.693$$

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84. $mX + N$

$= m \log_b x + \log_b n$

lo

$\text{g}_b\ x^m + \log_b n$

$\log_b nx^m$

$\log_b y$

Y

Thus, $Y = mX + N$.

$$A = 4.688w^{0.8168 - 0.0154 \log w}$$

$$A = 4.688(4000)^{0.8168 - 0.0154 \log 4000}$$

2590 cm^2

$$A = 4.688(8000)^{0.8168 - 0.0154 \log 8000}$$

4211 cm^2

B □ ? □ □ □ □ A

or 2 species, the maximum diversity is ln 2.

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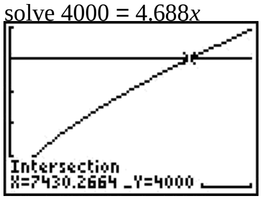
Section 2.2 LOGARITHMIC FUNCTIONS

Graph the equations

$Y_1 = 4.688x^{0.8168 - 0.0154 \log x}$ and

$Y_2 = 4000$, then find the intersection to

solve $4000 = 4.688x^{0.8168 - 0.0154 \log w}$.



[0, 10,000] by [0, 5000]

An infant with a surface area of 4000 square cm weighs about 7430 g.

If the number N is proportional to $m^{-0.6}$, where m is the mass, then $N = km^{-0.6}$, for some constant of proportionality k.

Taking the common log of both sides, we have

$$\log N = \log(km^{-0.6})$$

$$\log k + \log m^{-0.6}$$

$$\log k - 0.6 \log m.$$

This is a linear equation in log m. Its graph is a straight line with slope –0.6 and vertical intercept log k.

$$C(t) = C_0 e^{-kt}$$

When $t = 0$, $C(t) = 2$, and when $t = 3$, $C(t) = 1$.

$$2 = C_0 e^{-k(0)}$$

$$C_0 = 2$$

$$= 2e^{-3k}$$

$$= e^{-3k}$$

$$-3k = \ln \frac{1}{2} = \ln 2^{-1} = -\ln 2$$

$$k = \frac{\ln 2}{3}$$

$$= -\frac{1}{k} \ln \frac{C_2}{C_1}$$

$$T = \frac{1}{\ln 2} \ln \frac{5C_1}{C_1} = 3 \ln 5 \approx 7.0$$

(b) $\ln \frac{V_2}{V_1} = \ln(1+r)^{t_2 - t_1}$

$$\ln V_2 - \ln V_1 = (t_2 - t_1)\ln(1+r)$$

$$\frac{\ln V_2 - \ln V_1}{t_2 - t_1} = \ln(1+r)$$

(c) Substitute $\ln(1+r) = \frac{\ln V_2 - \ln V_1}{t_2 - t_1}$ into $t = \frac{\ln 2}{\ln(1+r)}$, then simplify.

$$t = \frac{\ln 2}{\ln(1+r)} = \frac{\ln 2}{\frac{\ln V_2 - \ln V_1}{t_2 - t_1}} = \frac{(t_2 - t_1)\ln 2}{\ln V_2 - \ln V_1}$$

Use the formula derived in part (c) with $t_1 = 0$, $t_2 = 4.5$, and $V_2 = 1.55V_1$.

$$t = \frac{(4.5 - 0)\ln 2}{\ln(1.55V_1) - \ln V_1} = \frac{4.5 \ln 2}{\ln \frac{1.55V_1}{V_1}}$$

$$= \frac{4.5 \ln 2}{\ln 1.55} \approx 7.12$$

The doubling time for the tumor is about 7.12 years.

89. (a) $y(t) = y_0 e^{kt}$

$$\frac{y(t)}{y_0} = e^{kt}$$

$$\ln \frac{y(t)}{y_0} = kt$$

$$\frac{1}{t} \ln \frac{y(t)}{y_0} = k$$, which is a constant.

(b) The data in Section 2.1, exercise 40 is as follows:

Year	Demand for Physicians (in thousands)
	680.5
	758.6
	805.8
	859.3

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3 $C1$ ln 2

The drug should be given about every 7 hours.

(a) $V_1 = V_0(1+r)^{t_1}$

$V_2 = V_0(1+r)^{t_2}$

$$\frac{V_2}{V_1} = \frac{V_0(1+r)^{t_2}}{V_0(1+r)^{t_1}} = \frac{(1+r)^{t_2}}{(1+r)^{t_1}} = (\quad)^{t_2 - t_1}$$

2006

2015

2020

2025

Evaluate the expression for the years 2015 (t = 9), 2020 (t = 14), and 2025 t = 19)

(*continued on next page*)

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(*continued*)

2015: $\frac{1}{9}\ln\frac{758.6}{680.5} \approx 0.01207$

2020: $\frac{1}{14}\ln\frac{805.8}{680.5} \approx 0.01207$

2025: $\frac{1}{19}\ln\frac{859.3}{680.5} \approx 0.01228$

90. (a) From the given graph, when $x = 10$ g, $y \approx 1.3 \text{ cm}^3$/g/hr, and when $x = 1000$ g, $\approx 0.41 \text{ cm}^3$/g/hr.

If $y = ax^b$, then

$\ln y = \ln(ax^b) = \ln a + b \ln x.$

Thus, there is a linear relationship between $\ln y$ and $\ln x$.

$$1.3 = a(10)^b$$
$$0.41 = a(1000)^b$$
$$\frac{1.3}{0.41} = \frac{a(10)^b}{a(1000)^b} \Rightarrow \frac{1.3}{0.41} = \left(\frac{10}{1000}\right)^b$$
$$\ln\frac{1.3}{0.41} = \ln(0.01)^b$$
$$\ln\frac{1.3}{0.41} = b\ln(0.01)$$
$$b = \frac{\ln\left(\frac{1.3}{0.41}\right)}{\ln(0.01)} \approx -0.25$$

Substituting this value into $1.3 = a(10)^b$, $1.3 = a(10)^{-0.25}$

$$a = \frac{1.3}{(10)^{-0.25}} \approx 2.3.$$

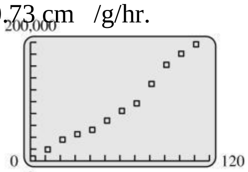
Therefore, $y = 2.3x^{-0.25}$.

(d) If $x = 100$,

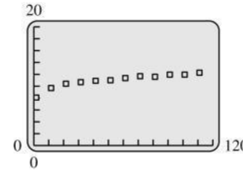
$y = 2.3(100)^{-0.25} \approx 0.73.$

We predict that the basal metabolism for a marsupial carnivore with a body mass of 100 g will be about 0.73 cm^3/g/hr.

91. (a)



(b)



Yes, the graph appears to be more linear, especially if the first point is eliminated.

Using a TI-84 Plus, the least squares line for the data in part (b) is $Y = 0.02940x + 9.348$.

Using a TI-84 Plus, the exponential regression function for the data in part (a) is $Y = 11,471(1.0298)^x$.

$$\ln Y = \ln\left(11,471(1.0298)^x\right)$$
$$\ln 11,471 + \ln\left(1.0298^x\right)$$
$$\ln 11,471 + x\ln 1.0298$$
$$9.348 + 0.2940x$$

For concert pitch A, $f = 440$.

$= 69 + 12\log_2(440/440) = 69 + 12\log_2(1) = 69+12\cdot 0 = 69$

For one octave higher than concert pitch A, $= 880$.

$P = 69 + 12\log_2(880/440) = 69 + 12\log_2(2) = 69+12\cdot 1 = 69+12 = 81$

$N(r) = -5000\ln r$

$N(0.9) = -5000\ln(0.9) \approx 530$

$N(0.5) = -5000\ln(0.5) \approx 3500$

$N(0.3) = -5000\ln(0.3) \approx 6000$

$N(0.7) = -5000\ln(0.7) \approx 1800$

About 1800 years have elapsed since the split if 70% of the words of the ancestral language are common to both languages today.

(e) $-5000\ln r = 1000$

$$\ln r = \frac{1000}{-5000}$$
$$\ln r = -\frac{1}{5}$$

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5

$r = e^{-1/5} \approx 0.8$

No, the data do not appear to lie along a straight line.

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Section 2.2 LOGARITHMIC FUNCTIONS

94. Decibel rating: $10\log\frac{I}{I_0}$

Intensity, $I = 115I_0$

$$10\log\frac{115I_0}{I_0} = 10\log 115 \approx 21$$

(b) $I = 9,500,000I_0$

$$10\log\frac{9.5\times10^6 I_0}{I_0} = 10\log 9.5\times10^6 \approx 70$$

$I = 1,200,000,000I_0$

$$10\log\frac{1.2\times10^9 I_0}{I_0} = 10\log 1.2\times10^9 \approx 91$$

$I = 895,000,000,000I_0$

$$10\log\frac{8.95\times10^{11} I_0}{I_0} = 10\log 8.95\times10^{11} \approx 120$$

$I = 109,000,000,000,000I_0$

$$10\log\frac{1.09\times10^{14} I_0}{I_0} = 10\log 1.09\times10^{14} \approx 140$$

$I_0 = 0.0002$ microbars

$1,200,000,000I_0 = 1,200,000,000(0.0002) = 240,000$ microbars

$895,000,000,000I_0 = 895,000,000,000(0.0002) = 179,000,000$ microbars

95. Let I_1 be the intensity of the sound whose decibel rating is 85.

$$10\log\frac{I_1}{I_0} = 85$$

$$\log\frac{I_1}{I_0} = 8.5$$

$$\log I_1 - \log I_0 = 8.5$$

$$10\log\frac{I_2}{I_0} = 75$$

$$\log\frac{I_2}{I_0} = 7.5$$

$$\log I_2 - \log I_0 = 7.5$$

$$\log I_0 = \log I_2 - 7.5$$

Substitute for I_0 in the equation for $\log I_1$.

$$\log I_1 = 8.5 + \log I_0 = 8.5 + \log I_2 - 7.5 = 1 + \log I_2$$

$$\log I_1 - \log I_2 = 1$$

$$\log\frac{I}{I_2} = 1$$

Then $\frac{I_1}{I_2} = 10$, so $I_2 = \frac{1}{10}I_1$.

intensity of the sound that had a rating of 75 decibels is $\frac{1}{10}$ as intense as the sound that had a rating of 85 decibels.

$$R(I) = \log\frac{I}{I_0}$$

$$R(1,000,000I_0) = \log\frac{1,000,000I_0}{I_0} = \log 1,000,000 = 6$$

$$R(1,000,000,000I_0) = \log\frac{1,000,000,000I_0}{I_0} = \log 100,000,000 = 8$$

$$R(I) = \log\frac{I}{I_0}$$

$$6.7 = \log\frac{I}{I_0}$$

$$10^{6.7} = \frac{I}{I_0} \Rightarrow I \approx 5,000,000I_0$$

(d) $R(I) = \log\frac{I}{I_0}$

$$8.1 = \log\frac{I}{I_0}$$

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$$\log I_1 = 8.5 + \log I_0$$

Let I_2 be the intensity of the sound whose decibel rating is 75.

$$10^{8.1} = \frac{I}{I_0} \Rightarrow I \approx 126,000,000 I_0$$

(e) $\frac{\text{1985 quake}}{\text{1999 quake}} = \frac{126,000,000 I_0}{5,000,000 I_0} \approx 25$

The 1985 earthquake had an amplitude more than 25 times that of the 1999 earthquake.

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(f) $R(E) = \frac{2}{3}\log\frac{E}{E_0}$

For the 1999 earthquake

$6.7 = \frac{2}{3}\log\frac{E}{E_0} \Rightarrow 10.05 = \log\frac{E}{E_0} \Rightarrow$

$\frac{E}{E_0} = 10^{10.05} \Rightarrow E = 10^{10.05} E_0$

For the 1985 earthquake,

$8.1 = \frac{2}{3}\log\frac{E}{E_0}$

$12.15 = \log\frac{E}{E_0}$

$\frac{E}{E_0} = 10^{12.15} \Rightarrow E = 10^{12.15} E_0$

The ratio of their energies is

$\frac{10^{12.15}E_0}{10^{10.05}E_0} = 10^{2.1} \approx 126$

The 1985 earthquake had an energy about 126 times that of the 1999 earthquake.

Find the energy of a magnitude 6.7 earthquake. From part f, we have

$= E_0 10^{10.05}$. For an earthquake that releases 15 times this much energy, $E = E_0(15)10^{10.05}$.

$R(E_0(15)10^{10.05}) = \frac{2}{3}\log\frac{E_0(15)10^{10.05}}{E_0}$

$= \frac{2}{3}\log(15\cdot 10^{10.05}) \approx 7.5$

So, it's true that a magnitude 7.5 earthquake releases 15 times more energy than one of magnitude 6.7.

97. $\text{pH} = -\log[H^+]$

For pure water:

$= -\log[H^+] \Rightarrow -7 = \log[H^+]$

For laundry solution:

$11 = -\log[H^+] \Rightarrow 10^{-11} = [H^+]$

For black coffee:

$5 = -\log[H^+]$

$10^{-5} = [H^+]$

$\frac{10^{-5}}{10^{-11}} = 10^6 = 1{,}000{,}000$

The coffee has a hydrogen ion concentration 1,000,000 times greater than the laundry mixture.

2.3 Applications: Growth and Decay

y_0 represents the initial quantity; k represents the rate of growth or decay.

k is positive in the exponential growth function.
k is negative in the exponential decay function.

The half-life of a quantity is the time period for the quantity to decay to one-half of the initial amount.

Assume that $y = y_0e^{kt}$ represents the amount remaining of a radioactive substance decaying with a half-life of T. Since $y = y_0$ is the amount of the substance at time $t = 0$, then

$y = \frac{y_0}{2}$ is the amount at time $t = T$.

Therefore, $\frac{y_0}{2} = y_0e^{kT}$.

Solving for k yields

$\frac{1}{2} = e^{kT}$

$\ln\frac{1}{2} = kT$

$\ln\left(\frac{1}{2}\right) = \ln(2^{-1}) = -\ln 2$.

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$\Rightarrow 10^{-7} = [H^+]$

For acid rain:

$$4 = -\log[H^+]$$
$$-4 = \log[H^+]$$
$$10^{-4} = [H^+]$$
$$\frac{10^{-4}}{10^{-7}} = 10^3 = 1000$$

The acid rain has a hydrogen ion concentration 1000 times greater than pure water.

Assume that $y = y_0 e^{kt}$ is the amount left of a radioactive substance decaying with a half-life of T. From Exercise 4, we know $k = -\dfrac{\ln 2}{T}$, so

$$= y_0 e^{(-\ln 2 \cdot t/T)}$$
$$= y_0 2^{-t/T} = y_0 \left(\frac{1}{2}\right)^{-1 \cdot -t/T} = y_0 \left(\frac{1}{2}\right)^{t/T}$$

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6. (a) $P = P_0 e^{kt}$

When $t = 1650$, $P = 500$.
When $t = 2010$, $P = 6756$.

$$= P_0 e^{1650k}$$
$$6756 = P_0 e^{2010k}$$
$$\frac{6756}{500} = \frac{P_0 e^{2010k}}{P_0 e^{1650k}}$$
$$\frac{6756}{500} = e^{360k}$$
$$360k = \ln\frac{6756}{500}$$
$$k = \frac{\ln\left(\frac{6756}{500}\right)}{360} \approx 0.007232$$

Substitute this value (include all decimal places) into $500 = P_0 e^{1650k}$ to find P_0.

$$= P_0 e^{1650(0.007232)}$$
$$P_0 = \frac{500}{e^{1650(0.007232)}} \approx 0.003285$$

Therefore, $P(t) = 0.003285e^{0.007232t}$.

$$P(1) = 0.003286e^{0.007232}$$
0.0033 million, or 3300.

The exponential equation gives a world population of only 3300 in the year 1.

No, the answer in part (b) is too small. Exponential growth does not accurately describe population growth for the world over a long period of time.

(a) $y = 2y_0$ after 12 hours.

$$y = y_0 e^{kt}$$
$$2y_0 = y_0 e^{12k}$$
$$2 = e^{12k}$$
$$\ln 2 = \ln e^{12k}$$
$$12k = \ln 2$$
$$k = \frac{\ln 2}{12} \approx 0.05776$$
$$= y_0 e^{0.05776t}$$
$$y = y_0 e^{(\ln 2/12)t}$$
$$y_0 e^{(\ln 2)(t/12)}$$
$$y_0[e^{\ln 2}]^{t/12}$$

For 15 days, $t = 15 \cdot 24$ or 360.

$$= (1)2^{360/12} = 2^{30} = 1{,}073{,}741{,}824$$

$$y = y_0 e^{kt}$$
$y = 40{,}000$, $y_0 = 25{,}000$, $t = 10$

$$40{,}000 = 25{,}000e^{k(10)}$$
$$1.6 = e^{10k}$$
$$\ln 1.6 = 10k$$

The equation is $y = 25{,}000e^{0.047t}$.

$$y = 25{,}000e^{0.047t}$$
$$25{,}000(e^{0.047})^t$$
$$25{,}000(1.048)^t$$
$$y = 60{,}000$$
$$60{,}000 = 25{,}000e^{0.047t}$$
$$2.4 = e^{0.047t}$$
$$\ln 2.4 = 0.047t$$
$$18.6 \approx t$$

There will be 60,000 bacteria in about 18.6 hours.

9. $y = y_0 e^{kt}$

$y = 20{,}000$, $y_0 = 50{,}000$, $t = 9$

$$20{,}000 = 50{,}000e^{9k}$$
$$0.40 = e^{9k}$$
$$\ln 0.4 = 9k$$
$$-0.102 \approx k$$

The equation is $y = 50{,}000e^{-0.102t}$.

$$\frac{1}{2}(50{,}000) = 25{,}000$$
$$25{,}000 = 50{,}000e^{-0.102t}$$
$$0.5 = e^{-0.102t}$$

$y_0 2^{t/12}$ since $e^{\ln 2} = 2$

For 10 days, $t = 10 \cdot 24$ or 240.

$$y = (1)2^{240/12} = 2^{20} = 1{,}048{,}576$$

$$\ln 0.5 = -0.102t$$
$$6.8 \approx t$$

Half the bacteria remain after about 6.8 hours.

10. $f(t) = 500e^{0.1t}$

$f(t) = 3000$

$3000 = 500e^{0.1t}$

$6 = e^{0.1t}$

$\ln 6 = 0.1t \Rightarrow t \approx 17.9$

It will take 17.9 days.

If $t = 0$ corresponds to January 1, the date January 17 should be placed on the product. January 18 would be more than 17.9 days.

Use $y = y_0 e^{-kt}$.

When $t = 5$, $y = 0.37\, y_0$.

$0.37\, y_0 = y_0 e^{-5k}$

$0.37 = e^{-5k}$

$-5k = \ln(0.37) \Rightarrow k = \frac{\ln(0.37)}{-5} \approx 0.1989$

(a) From the graph, the risks of chromosomal abnormality per 1000 at ages 20, 35, 42, and 49 are 2, 5, 24, and 125, respectively.

(Note: It is difficult to read the graph accurately. If you read different values from the graph, your answers to parts (b)−(e) may differ from those given here.)

$y = Ce^{kt}$

When $t = 20$, $y = 2$, and when $t = 35$, $y = 5$.

$= Ce^{20k}$

$= Ce^{35k}$

$\frac{5}{} = \frac{Ce^{35k}}{Ce^{20k}}$

$2.5 = e^{15k}$

$15k = \ln 2.5$

$k = \frac{\ln 2.5}{15} \Rightarrow k \approx 0.061$

(c) $y = Ce^{kt}$

When $t = 42$, $y = 29$, and when $t = 49$, $y = 125$.

$24 = Ce^{42k}$

$125 = Ce^{49k}$

$\frac{125}{24} = \frac{Ce^{49k}}{Ce^{42k}} = e^{7k}$

$7k = \ln \frac{125}{24} \Rightarrow k = \frac{\ln\left(\frac{125}{24}\right)}{7} \approx 0.24$

The results are summarized in the following table.

n	Value of k for [20, 35]	Value of k for [42, 49]
2	0.00093	0.0017
3	2.3×10^{-5}	2.5×10^{-5}
4	6.3×10^{-7}	4.1×10^{-7}

The value of n should be somewhere between 3 and 4.

13. $A(t) = A_0 \left(\frac{1}{2}\right)^{t/5600}$

$A(43{,}000) = A_0 \left(\frac{1}{2}\right)^{43{,}000/5600} \approx 0.005 A_0$

About 0.5% of the original carbon 14 was present.

$P(t) = 100e^{-0.1t}$

$P(4) = 100e^{-0.1(4)} \approx 67\%$

$P(10) = 100e^{-0.1(10)} \approx 37\%$

$10 = 100e^{-0.1t}$

$0.1 = e^{-0.1t}$

$\ln(0.1) = -0.1t$

$-\frac{\ln(0.1)}{0.1} = t \Rightarrow 23 \approx t$

It would take about 23 days.

$1 = 100e^{-0.1t}$

$0.01 = e^{-0.1t}$

$\ln(0.01) = -0.1t$

$-\frac{\ln(0.01)}{0.1} = t \Rightarrow 46 \approx t$

It would take about 46 days.

$A(t) = A_0 e^{kt}$

First, find k. Let $A(t) = \frac{1}{}A_0$ and $t = 1.25$.

Since the values of k are different, we cannot assume the graph is of the form $y = Ce^{kt}$.

$$\frac{1}{2}A_0 = A_0 e^{1.25k}$$

$$\frac{1}{2} = e^{1.25k}$$

$$\ln\frac{1}{2} = 1.25k \Rightarrow k = \frac{\ln\frac{1}{2}}{1.25}$$

Now, let $A_0 = 1$ and let $t = 0.25$. (Note that the half-life is given in billions of years, so 250 million years is 0.25 billion year.)

(*continued on next page*)

(*continued*)

$$A(0.25) = 1\cdot e^{0.25 \ln\frac{1}{2}/1.25} \approx 0.87$$

Therefore, about 87% of the potassium-40 remains from a creature that died 250 million years ago.

Solve for r. Let $t = 1997 - 1980 = 17$

$$\frac{1}{84} = \frac{1}{250}(1+r)^{17}$$

$$\frac{250}{84} = (1+r)^{17}$$

$$\left(\frac{250}{84}\right)^{1/17} = 1+r$$

$$r = \left(\frac{250}{84}\right)^{1/17} - 1 \approx 0.066 = 6.6\%$$

No, these numbers do not represent a 4% increase annually. They represent a 6.6% increase.

$$A(t) = A_0e^{kt}$$

$$0.60A_0 = A_0e^{(-\ln 2/5600)t}$$

$$0.60 = e^{(-\ln 2/5600)t}$$

$$\ln 0.60 = -\frac{\ln 2}{5600}t$$

$$-\frac{5600(\ln 0.60)}{\ln 2} = t$$

$$4127 \approx t$$

The sample was about 4100 years old.

18. $\frac{1}{2}A_0 = A_0e^{-0.053t}$

$$\frac{1}{2} = e^{-0.053t}$$

$$\ln\frac{1}{2} = -0.053t$$

$$-\ln 2 = -0.053t$$

$$t = \frac{\ln 2}{0.053} \approx 13.1$$

The half-life of plutonium 241 is about 13 years.

20. (a) $A(t) = A_0\left(\frac{1}{2}\right)^{t/13}$

$$A(100) = 4.0\left(\frac{1}{2}\right)^{100/13}$$

$$A(100) \approx 0.0193$$

After 100 years, about 0.0193 gram will remain.

(b) $0.1 = 4.0\left(\frac{1}{2}\right)^{t/13}$

$$\frac{0.1}{4.0} = \left(\frac{1}{2}\right)^{t/13}$$

$$\ln 0.025 = \frac{t}{13}\ln\frac{1}{2}$$

$$t = \frac{13\ln 0.025}{\ln\left(\frac{1}{2}\right)} \approx 69.19$$

It will take about 69 years.

21. (a) $A(t) = A_0\left(\frac{1}{2}\right)^{t/1620}$

$$A(100) = 4.0\left(\frac{1}{2}\right)^{100/1620} \approx 3.83$$

After 100 years, about 3.8 grams will remain.

(b) $0.1 = 4.0\left(\frac{1}{2}\right)^{t/1620}$

$$\frac{0.1}{4} = \left(\frac{1}{2}\right)^{t/1620}$$

$$\ln 0.025 = \frac{t}{1620}\ln\frac{1}{2}$$

$$\frac{1}{2}A_0 = A_0e^{-0.00043t}$$

$$\frac{1}{2} = e^{-0.00043t}$$

$$\ln\frac{1}{2} = -0.00043t$$

$$-\ln 2 = -0.00043t$$

$$t = \frac{\ln 2}{0.00043} \approx 1612$$

The half-life of radium 226 is about 1600 years.

1620 2

$$t = \frac{1600 \ln 0.025}{\ln\left(\frac{1}{2}\right)} \approx 8515$$

The half-life is about 8600 years.

22. (a) $y = y_0 e^{kt}$

When $t = 0$, $y = 500$, so $y_0 = 500$.

When $t = 3$, $y = 386$.

$$= 500e^{3k} \Rightarrow \frac{386}{500} = e^{3k} \Rightarrow$$

$$e^{3k} = 0.772 \Rightarrow 3k = \ln 0.772 \Rightarrow$$

$$k = \frac{\ln 0.772}{3} \approx -0.0863$$

$$y = 500e^{-0.0863t}$$

(b) From part (a), we have $k = \frac{\ln\left(\frac{386}{500}\right)}{3}$.

$y = 500e^{kt}$

$= 500e^{[\ln(386/500)/3]t}$

$= 500e^{\ln(386/500)\cdot(t/3)}$

$= 500\,[e^{\ln(386/500)}]^{t/3}$

$= 500\left(\frac{386}{500}\right)^{t/3} = 500(0.722)^{t/3}$

(c) $\frac{1}{2}y_0 = y_0e^{-0.0863t}$

$\ln\frac{1}{2} = -0.0863t$

$t = \frac{\ln\left(\frac{1}{2}\right)}{-0.0863} \approx 8.0$

The half-life is about 8.0 days.

23. (a) $y = y_0e^{kt}$

When $t = 0$, $y = 25.0$, so $y_0 = 25.0$

When $t = 50$, $y = 19.5$

$19.5 = 25.0e^{50k}$

$\frac{19.5}{25.0} = e^{50k}$

$50k = \ln\frac{19.5}{25.0}$

$k = \frac{\ln\left(\frac{19.5}{25.0}\right)}{50} \approx -0.00497$

$= 25.0e^{-0.00497t}$

(b) From part (a), we have $k = \frac{\ln\left(\frac{19.5}{25.0}\right)}{50}$.

$y = 25.0e^{kt}$

$= 25.0e^{[(\ln 19.5/25.0)/50]t}$

$= 25.0e^{\ln(19.5/25.0)\cdot(t/50)}$

$= 25.0\,[e^{\ln(19.5/25.0)}]^{t/50}$

$= 25.0(19.5/25.0)^{t/50}$

$= 25(0.78)^{t/50}$

(c) $\frac{1}{2}y_0 = y_0e^{-0.00497t}$

$\frac{1}{}$

$y = 40e^{-0.004t}$

$t = 180$

$= 40e^{-0.004(180)} = 40e^{-0.72}$

≈ 19.5 watts

$20 = 20e^{-0.0004t}$

$\frac{}{2} = e^{-0.004t}$

$\ln\frac{1}{2} = -0.004t$

$t = \frac{\ln 2}{0.004} \approx 173.29$

It will take about 173 days.

The power will never be completely gone. The power will approach 0 watts but will never be exactly 0.

(a) Let t = the number of degrees Celsius.

$y = y_0 \cdot e^{kt}$

$y_0 = 10$ when $t = 0°$.

To find k, let $y = 11$ when $t = 10°$.

$= 10e^{10k}$

$e^{10k} = \frac{11}{10}$

$10k = \ln 1.1$

$k = \frac{\ln 1.1}{10} \approx 0.0095$

The equation is $y = 10e^{0.0095t}$.

(b) Let $y = 1.5$; solve for t.

$15 = 10e^{0.0095t}$

$\ln 1.5 = 0.0095t$

$t = \frac{\ln 1.5}{0.0095} \approx 42.7$

15 grams will dissolve at 42.7°C.

$t = 9$, $T_0 = 18$, $C = 5$, $k = 0.6$

$f(t) = T_0 + Ce^{-kt}$

$(t) = 18 + 5e^{-0.6(9)} = 18 + 5e^{-5.4} \approx$

18.02 The temperature is about 18.02°.

$f(t) = T_0 + Ce^{-kt}$

$= 20 + 100e^{-0.1t}$

$$2 = e^{-0.00497t}$$

$$-0.00497t = \ln\left(\frac{1}{2}\right)$$

$$t = \frac{\ln\left(\frac{1}{2}\right)}{-0.00497} \approx 139.47$$

The half-life is about 139 days.

$$= 100e^{-0.1t}$$

$$e^{-0.1t} = 0.05$$

$$-0.1t = \ln 0.05$$

$$t = \frac{\ln 0.05}{-0.1} \approx 30$$

It will take about 30 min.

$C = -14.6$, $k = 0.6$, $T_0 = 18°$,
$f(t) = 10°$

$$f(t) = T_0 + Ce^{-kt}$$
$$10 = 18 + (-14.6)e^{-0.6t}$$
$$-8 = -14.6e^{-0.6t}$$
$$0.5479 = e^{-0.6t}$$
$$\ln 0.5479 = -0.6t$$
$$t = -\frac{\ln 0.5479}{0.6} \approx 1$$

It would take about 1 hour for the pizza to thaw.

2.4 Trigonometric Functions

1. $60° = 60\left(\frac{\pi}{180}\right) = \frac{\pi}{3}$

2. $90° = 90\left(\frac{\pi}{180}\right) = \frac{\pi}{2}$

3. $150° = 150\left(\frac{\pi}{180}\right) = \frac{5\pi}{6}$

4. $135° = 135\left(\frac{\pi}{180}\right) = \frac{3\pi}{4}$

5. $270° = 270\left(\frac{\pi}{180}\right) = \frac{3\pi}{2}$

6. $320° = 320\left(\frac{\pi}{180}\right) = \frac{16\pi}{9}$

7. $495° = 495\left(\frac{\pi}{180}\right) = \frac{11\pi}{4}$

8. $510° = 510\left(\frac{\pi}{180}\right) = \frac{17\pi}{6}$

9. $\frac{5\pi}{4} = \frac{5\pi}{4}\left(\frac{180°}{\pi}\right) = 225°$

14. $\frac{5\pi}{9} = \frac{5\pi}{9}\left(\frac{180°}{\pi}\right) = 100°$

15. $\frac{7\pi}{12} = \frac{7\pi}{12}\left(\frac{180°}{\pi}\right) = 105°$

16. $5\pi = 5\pi\left(\frac{180°}{\pi}\right) = 900°$

Let α = the angle with terminal side through (–3, 4). Then $x = -3$, $y = 4$, and

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (4)^2} = \sqrt{25} = 5.$

$\sin\alpha = \frac{y}{r} = \frac{4}{5}$ $\cot\alpha = \frac{x}{y} = -\frac{3}{4}$

$\cos\alpha = \frac{x}{r} = -\frac{3}{5}$ $\sec\alpha = \frac{r}{x} = -\frac{5}{3}$

$\tan\alpha = \frac{y}{x} = -\frac{4}{3}$ $\csc\alpha = \frac{r}{y} = \frac{5}{4}$

Let α = the angle with terminal side through (–12, –5) Then $x = -12$, $y = -5$, and

$r = \sqrt{x^2 + y^2} = \sqrt{144 + 25} = \sqrt{169} = 13.$

$\sin\alpha = \frac{y}{r} = -\frac{5}{13}$ $\cot\alpha = \frac{x}{y} = \frac{12}{5}$

$\cos\alpha = \frac{x}{r} = -\frac{12}{13}$ $\sec\alpha = \frac{r}{x} = -\frac{13}{12}$

$\tan\alpha = \frac{y}{x} = \frac{5}{12}$ $\csc\alpha = \frac{r}{y} = -\frac{13}{5}$

Let α = the angle with terminal side through (7, −24). Then $x = 7$, $y = -24$, and

$r = \sqrt{x^2 + y^2} = \sqrt{49 + 576} = \sqrt{625} = 25.$

$\sin\alpha = \frac{y}{r} = -\frac{24}{25}$ $\cot\alpha = \frac{x}{y} = -\frac{7}{24}$

$\cos\alpha = \frac{x}{r} = \frac{7}{25}$ $\sec\alpha = \frac{r}{x} = \frac{25}{7}$

$\tan\alpha = \frac{y}{x} = -\frac{24}{7}$ $\csc\alpha = \frac{r}{y} = -\frac{25}{24}$

10. $\frac{2\pi}{3} = \frac{2\pi}{3} \cdot \frac{180°}{\pi} = 120°$

11. $-\frac{13\pi}{6} = -\frac{13\pi}{6} \cdot \frac{180°}{\pi} = -390°$

12. $-\frac{\pi}{4} = -\frac{\pi}{4} \cdot \frac{180°}{\pi} = -45°$

13. $\frac{8\pi}{5} = \frac{8\pi}{5} \cdot \frac{180°}{\pi} = 288°$

Let α = the angle with terminal side through (20, 15). Then $x = 20$, $y = 15$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{400 + 225} = \sqrt{625} = 25.$$

$\sin\alpha = \frac{y}{r} = \frac{3}{5}$ $\qquad \cot\alpha = \frac{x}{y} = \frac{4}{3}$

$\cos\alpha = \frac{x}{r} = \frac{4}{5}$ $\qquad \sec\alpha = \frac{r}{x} = \frac{5}{4}$

$\tan\alpha = \frac{y}{x} = \frac{3}{4}$ $\qquad \csc\alpha = \frac{r}{y} = \frac{5}{3}$

In quadrant I, all six trigonometric functions are positive, so their sign is +.

Section 2.4 TRIGONOMETRIC FUNCTIONS 115

In quadrant II, $x < 0$ and $y > 0$. Furthermore, $r > 0$.

$\sin\theta = \frac{y}{r} > 0$, so the sign is $+$.

$\cos\theta = \frac{x}{r} < 0$, so the sign is $-$.

$\tan\theta = \frac{y}{x} < 0$, so the sign is $-$.

$\cot\theta = \frac{x}{y} < 0$, so the sign is $-$.

$\sec\theta = \frac{r}{x} < 0$, so the sign is $-$.

$\csc\theta = \frac{r}{y} > 0$, so the sign is $+$.

In quadrant III, $x < 0$ and $y < 0$. Furthermore, $r > 0$.

$\sin\theta = \frac{y}{r} < 0$, so the sign is $-$.

$\cos\theta = \frac{x}{r} < 0$, so the sign is $-$.

$\tan\theta = \frac{y}{x} > 0$, so the sign is $+$.

$\cot\theta = \frac{x}{y} > 0$, so the sign is $+$.

$\sec\theta = \frac{r}{x} < 0$, so the sign is $-$.

$\csc\theta = \frac{r}{y} < 0$, so the sign is $-$.

In quadrant IV, $x > 0$ and $y < 0$. Also, $r > 0$.

$\sin\theta = \frac{y}{r} < 0$, so the sign is $-$.

$\cos\theta = \frac{x}{r} > 0$, so the sign is $+$.

$\tan\theta = \frac{y}{x} < 0$, so the sign is $-$.

$\cot\theta = \frac{x}{y} < 0$, so the sign is $-$.

$\sec\theta = \frac{r}{x} > 0$, so the sign is $+$.

$\csc\theta = \frac{r}{y} < 0$, so the sign is $-$.

When an angle θ of 30° is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (\sqrt{3}, 1)$. Then

$$r = \sqrt{x^2 + y^2} = \sqrt{3 + 1} = 2.$$

$$\tan\theta = \frac{y}{x} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\cot\theta = \frac{x}{y} = \sqrt{3}$$

$$\csc\theta = \frac{r}{y} = 2$$

When an angle θ of 45° is drawn in standard position, $(x, y) = (1,1)$ is one point on its terminal side. Then

$$r = \sqrt{1 + 1} = \sqrt{2}.$$

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sec\theta = \frac{r}{x} = \sqrt{2}$$

$$\csc\theta = \frac{r}{y} = \sqrt{2}$$

When an angle θ of 60° is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (1, \sqrt{3})$. Then

$$r = \sqrt{x^2 + y^2} = \sqrt{1 + 3} = 2.$$

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{3}}{2}$$

$$\cot\theta = \frac{x}{y} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

When an angle θ of 120° is drawn in standard position, $(x, y) = (-1, \sqrt{3})$ is one point on its terminal side. Then

$$r = \sqrt{1 + 3} = 2.$$

$$\cos\theta = \frac{x}{r} = -\frac{1}{2}$$

$$\cot\theta = \frac{x}{y} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\sec\theta = \frac{r}{x} = -2$$

When an angle θ of 135° is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (-1,1)$. Then

$$r = \sqrt{x^2 + y^2} = \sqrt{1 + 1} = \sqrt{2}.$$

$$\tan\theta = \frac{y}{x} = -1$$

$$\cot\theta = \frac{x}{y} = -1$$

When an angle θ of 150° is drawn in standard position, $(x, y) = (-\sqrt{3}, 1)$ is one point on its terminal side. Then

$r = \sqrt{3+1} = 2.$

$\sin\theta = \frac{y}{r} = \frac{1}{2}$

$\cot\theta = \frac{x}{y} = -\sqrt{3}$

$\sec\theta = \frac{r}{x} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

When an angle θ of 210° is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (-\sqrt{3}, -1)$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{3+1} = 2.$

$\cos\theta = \frac{x}{r} = -\frac{\sqrt{3}}{2}$

$\sec\theta = \frac{r}{x} = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

When an angle θ of 240° is drawn in standard position, $(x, y) = (-1, -\sqrt{3})$ is one point on its terminal side.

$\tan\theta = \frac{y}{x} = \sqrt{3}$

$\cot\theta = \frac{x}{y} = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}$

When an angle of $\frac{\pi}{3}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (1, \sqrt{3})$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{1+3} = 2.$

$\sin\frac{\pi}{3} = \frac{y}{\ } = \frac{\sqrt{3}}{\ }$

When an angle of $\frac{\pi}{3}$ is drawn in standard position, $(x, y) = (1, \sqrt{3})$ is one point on its terminal side.

$\cot\frac{\pi}{3} = \frac{x}{y} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

When an angle of $\frac{\pi}{6}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (\sqrt{3}, 1)$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{3+1} = 2.$

$\csc\frac{\pi}{6} = \frac{r}{y} = \frac{2}{1} = 2$

When an angle of $\frac{3\pi}{2}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (0, -1)$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{0+1} = 1.$

$\sin\frac{3\pi}{2} = \frac{y}{r} = \frac{-1}{1} = -1$

When an angle of 3π is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (-1, 0)$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{1} = 1.$

$\cos 3\pi = \frac{x}{r} = -1$

When an angle of π is drawn in standard position, $(x, y) = (-1, 0)$ is one point on its terminal side. Then $r = \sqrt{1+0} = 1.$

$\sec\pi = \frac{r}{x} = -1$

7

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3 r 2

When an angle of $\frac{\pi}{6}$ is drawn in standard position, $(x, y) = (\sqrt{3}, 1)$ is one point on its terminal side. Then

$r = \sqrt{3+1} = 2.$

$\cos\frac{\pi}{6} = \frac{x}{r} = \frac{\sqrt{3}}{2}$

When an angle of $\frac{\pi}{4}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (1,1)$.

$\tan\frac{\pi}{4} = \frac{y}{x} = 1$

When an angle of $\frac{7\pi}{4}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (1, -1)$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{1+1} = \sqrt{2}.$

$\sin\frac{7\pi}{4} = \frac{y}{r} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

When an angle of $\frac{5\pi}{2}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (0,1)$. Then $\tan\frac{5\pi}{2} = \frac{y}{x} = \frac{1}{0}$ is undefined.

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When an angle of $\frac{5\pi}{4}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (-1, -1)$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{1 + 1} = \sqrt{2}.$

$\frac{\pi}{4}$ — $\frac{x}{}$ -1 — $-\sqrt{}$

When an angle of 5π is drawn in standard position, $(x, y) = (-1, 0)$ is one point on its terminal side. Then $r = \sqrt{1 + 0} = 1$. sec5=r22=

$\cos 5\pi = \frac{x}{r} = -1$

When an angle of $-\frac{3\pi}{4}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (-1, -1)$. Then

$\cot\left(-\frac{3\pi}{4}\right) = \frac{x}{y} = \frac{-1}{-1} = 1$

46. When an angle of $-\frac{5\pi}{6}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (-\sqrt{3}, -1)$. Then

$\tan\left(-\frac{5\pi}{6}\right) = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}$

When an angle of $-\frac{7\pi}{6}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (-\sqrt{3}, 1)$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{3 + 1} = 2.$

$\sin\left(-\frac{7\pi}{6}\right) = \frac{y}{r} = \frac{1}{2}$

48. When an angle of $-\frac{\pi}{6}$ is drawn in standard position, $(x, y) = (\sqrt{3}, -1)$ is one point on its terminal side. Then $r = \sqrt{3 + 1} = 2$.

$\cos\left(-\frac{\pi}{6}\right) = \frac{x}{r} = \frac{\sqrt{3}}{2}$

The sine function is negative in quadrants III and IV. We know that $\sin(\pi/6) = 1/2$. The solution in quadrant III is $\pi + (\pi/6) = 7\pi/6$. The solution in quadrant IV is $2\pi - (\pi/6)$ $11\pi/6$. The two solutions of $\sin x = -1/2$ between 0 and 2π are $7\pi/6$ and $11\pi/6$.

The tangent function is negative in quadrants II and IV. We know that $\tan(\pi/4) = 1$. The solution in quadrant II is $\pi - (\pi/4) = 3\pi/4$. The solution in quadrant IV is $2\pi - (\pi/4)$ $7\pi/4$. The two solutions of $\tan x = -1$ between 0 and 2π are $3\pi/4$ and $7\pi/4$.

The tangent function is positive in quadrants I and III. We know that $\tan(\pi/3) = \sqrt{3}$ so the solution in quadrant I is $\pi/3$. The solution in quadrant III is $\pi + (\pi/3) = 4\pi/3$. The two solutions of $\tan x = \sqrt{3}$ between 0 and 2π are $\pi/3$ and $4\pi/3$.

The secant function is negative in quadrants II and III. We know that $\sec(\pi/6) = 2/\sqrt{3}$, so the solution in quadrant II is $\pi - (\pi/6) = 5\pi/6$. The solution in quadrant III is

The cosine function is positive in quadrants I and IV. We know that $\cos(\pi/3) = 1/2$, so the solution in quadrant I is $\pi/3$. The solution in quadrant IV is $2\pi - (\pi/3) = 5\pi/3$. The two solutions of $\cos x = 1/2$ between 0 and 2π are $\pi/3$ and $5\pi/3$.

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+ (π/6) = 7π/6. The two solutions of sec x = −2/ 3 between 0 and 2π are 5π/6 and 7π/6.

The secant function is positive in quadrants I and IV. We know that sec (π/4) = 2, so the solution in quadrant I is π/4. The solution in quadrant IV is 2π − (π/4) = 7π/4. The two solutions of sec x = 2 between 0 and 2π are π/4 and 7π/4.

√ √

sin 39° ≈ 0.6293

cos 67° ≈ 0.3907

tan 123° ≈ −1.5399

tan 54° ≈ 1.3764

sin 0.3638 ≈ 0.3558

tan 1.0123 ≈ 1.6004

cos 1.2353 ≈ 0.3292

sin 1.5359 ≈ 0.9994

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$f(x) = \cos(3x)$ is of the form

$f(x) = a\cos(bx)$ where $a = 1$ and $b = 3$.

Thus, $a = 1$ and $T = \frac{2\pi}{b} = \frac{2\pi}{3}$.

$f(x) = -\frac{1}{2}\sin(4\pi x)$ is of the form

$f(x) = a\sin(bx)$ where $a = -\frac{1}{2}$ and $b = 4\pi$.

Thus, the amplitude is $\frac{1}{2}$ and

$T = \frac{2\pi}{b} = \frac{2\pi}{4\pi} = \frac{1}{2}$.

$g(t) = -2\sin\left(\frac{\pi}{4}t + 2\right)$ is of the form

$g(t) = a\sin(bt + c)$ where $a = -2$, $b = \frac{\pi}{4}$, and $c = 2$. Thus, $|a| = |-2| = 2$ and

$T = \frac{2\pi}{b} = \frac{2\pi}{\frac{\pi}{4}} = 8$.

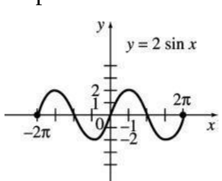
66. $s(x) = 3\sin(880\pi t - 7)$ is of the form

$s(x) = a\sin(bt + c)$ where $a = 3$, $b = 880\pi$,

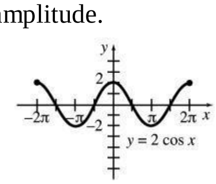
and $c = -7$. Thus, $a = 3$ and

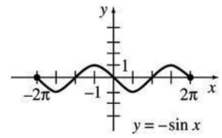
$T = \frac{2\pi}{b} = \frac{2\pi}{880\pi} = \frac{1}{440}$.

The graph of $y = 2\sin x$ is similar to the graph of $y = \sin x$ except that it has twice the amplitude.

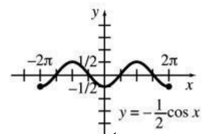


The graph of $y = 2\cos x$ is similar to the graph of $y = \cos x$ except that it has twice the amplitude.





The graph of $y = -\frac{1}{2}\cos x$ is similar to the graph of $y = \cos x$ except that it has half the amplitude and is reflected about the x-axis.



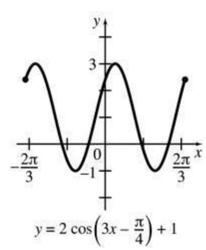
$y = 2\cos\left(3x - \frac{\pi}{4}\right) + 1$ has amplitude $a = 2$,

period $T = \frac{2\pi}{b} = \frac{2\pi}{3}$, phase shift

$\frac{c}{b} = \frac{-\pi/4}{3} = -\frac{\pi}{12}$, and vertical shift $d = 1$.

Thus, the graph of $y = 2\cos\left(3x - \frac{\pi}{4}\right) + 1$ is similar to the graph of $f(x) = \cos x$ except that it has 2 times the amplitude, a third of the period, and is shifted 1 unit vertically.

Also, $y = 2\cos\left(3x - \frac{\pi}{4}\right) + 1$ is shifted $\frac{\pi}{12}$ units to the right relative to the graph of graph of $g(x) = \cos 3x$.



$y = 4\sin\left(\frac{1}{2}x + \pi\right) + 2$ has amplitude $a = 4$,

period $T = \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$, phase shift

$\frac{c}{b} = \frac{\pi}{\frac{1}{2}} = 2\pi$, and vertical shift $d = 2$. Thus,

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The graph of $y = -\sin x$ is similar to the graph of $y = \sin x$ except that it is reflected about the x-axis.

the graph of $y = 4\sin\left(\frac{1}{2}x + \pi\right) + 2$ is similar to the graph of $f(x) = \sin x$ except that it has 4 times the amplitude, twice the period, and is shifted 2 units vertically. Also, $y = \sin\left(\frac{1}{2}x + \pi\right) + 2$ is shifted 2π units to the left relative to the graph of $g(x) = \sin\left(\frac{1}{2}x\right)$.

(*continued on next page*)

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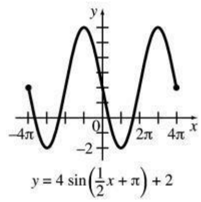
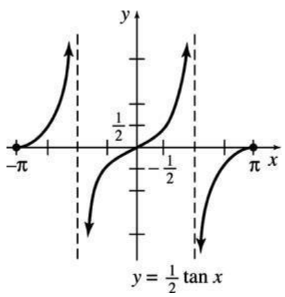
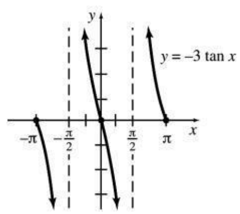
(*continued*)



The graph of $y = \frac{1}{2}\tan x$ is similar to the graph of $y = \tan x$ except that the y-values of points on the graph are one-half the y-values of points on the graph of $y = \tan x$.



The graph of $y = -3 \tan x$ is similar to the graph of $y = \tan x$ except that it is reflected about the x-axis and each ordinate value is three times larger in absolute value. Note that the points $\left(-\frac{\pi}{4}, 3\right)$ and $\left(\frac{\pi}{4}, -3\right)$ lie on the graph.



75. (a) Since the three angles θ are equal and their sum is 180°, each angle θ is 60°.

The base angle on the left is still 60°. The bisector is perpendicular to the base, so the other base angle is 90°. The angle formed by bisecting the original vertex angle θ is 30°.

Two sides of the triangle on the left are given in the diagram: the hypotenuse is 2, and the base is half of the original base of 2, or 1. The Pythagorean Theorem gives the length of the remaining side (the vertical bisector) as $\sqrt{2^2 - 1^2} = \sqrt{3}$.

(a) The Pythagorean Theorem gives the length of the hypotenuse as $\sqrt{1^2 + 1^2} = \sqrt{2}$.

The angle at the lower left is a right angle with measure 90°. Since the sum of all three angles is 180°, the measures of the remaining two angles sum to 90°. Since these two angles are the base angles of an isosceles triangle they are equal, and thus each has measure 45°.

(a) Since the amplitude is 2 and the period is 0.350, $a = 2$ and

$$0.350 = \frac{2\pi}{b} \Rightarrow \frac{1}{0.350} = \frac{b}{2\pi} \Rightarrow \frac{2\pi}{0.350} = b.$$

Therefore, the equation is $y = 2\sin(2\pi t/0.350)$, where t is the time in seconds.

(b) $2 = 2\sin\left(\frac{2\pi t}{0.350}\right)$

$1 = \sin\frac{2\pi t}{0.350}$

$$\frac{\pi}{2} = \frac{2\pi t}{0.350} \Rightarrow t = \frac{0.350\pi}{4\pi} = 0.0875$$

The image reaches its maximum amplitude after 0.0875 seconds.

(c) $t = 2$

$$y = 2\sin\left(\frac{2\pi(2)}{0.350}\right) \approx -1.95$$

The position of the object after 2 seconds is –1.95°.

78. (a) The period is $\frac{2\pi}{\frac{\pi}{14.77}} = 29.54$

There is a lunar cycle every 29.54 days.

(b) $y = 100 + 1.8\cos\left(\frac{(t-6)\pi}{\ldots}\right)$ reaches a

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14.77 maximum value when

$\cos\frac{(t-6)\pi}{14.77} = 1$ which occurs when

$t - 6 = 0 \Rightarrow t = 6$

Six days from January 16, 2014, is January 22, 2014.

$y = 100 + 1.8\cos\frac{(6-6)\pi}{14.77} = 101.8$

There is a percent increase of 1.8 percent.

On January 31, $t = 15$.

$y = 100 + 1.8\cos\frac{(15-6)\pi}{14.77} \approx 99.39$

The formula predicts that the number of consultations was 99.39% of the daily mean.

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$P(t) = 7(1 - \cos 2\pi t)(t + 10) + 100e^{0.2t}$

Since January 1 of the base year corresponds to $t = 0$, the pollution level

is $P(0) = 7(1 - \cos 0)(0 + 10) + 100e^{0}$
$7(0)(10) + 100 = 100.$

Since July 1 of the base year corresponds to $t = 0.5$, the pollution level is

$P(0.5) = 7(1 - \cos \pi)(0.5 + 10) + 100e^{0.1}$
$7(2)(10.5) + 100e^{0.1} \approx 258.$

Since January 1 of the following year corresponds to $t = 1$, the pollution level is

$P(1) = 7(1 - \cos 2\pi)(1 + 10) + 100e^{0.2}$

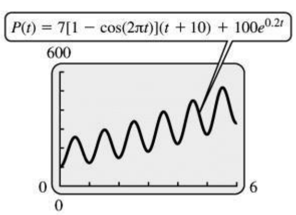
$7(0)(11) + 100e^{0.2} \approx 122$

Since July 1 of the following year corresponds to $t = 1.5$, the pollution level is

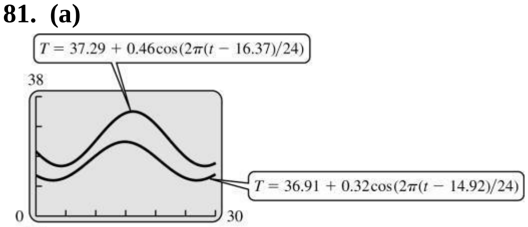
$P(1.5) = 7(1 - \cos 3\pi)(1.5 + 10) + 100e^{0.3}$

$= 7(2)(11.5) + 100e^{0.3} \approx 296.$

80.



81. (a)

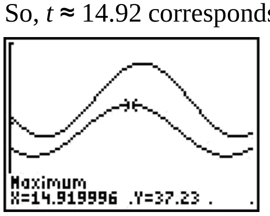


No, the functions never cross.

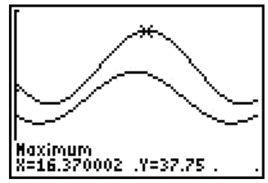
For a patient without Alzheimer's, the heights temperature occurs when $t \approx 14.92$. $14.92 - 12 = 2.92$ and

$0.92 \text{ hr} = 0.92 \cdot 60 \text{ min} \approx 55 \text{ min}.$

So, $t \approx 14.92$ corresponds to about 2:55 P.M.



For a patient without Alzheimer's, the heights temperature occurs when $t \approx 16.37$. $16.37 - 12 = 4.37$ and $0.37 \text{ hr} = 0.37 \cdot 60 \text{ min} \approx 22 \text{ min}.$ So, $t \approx 16.37$ corresponds to about 4:22 P.M.



(a) $\theta_2 = 23°$, $a = 10°$, $B_2 = 170°$

$\tan\theta = \tan 23° \left(\cos 10° - \cot 170° \sin 10°\right)$

$\theta = \tan^{-1}\left(\tan 23° \left(\cos 10° - \cot 170° \sin 10°\right)\right)$
$\approx 40°$

$\theta_2 = 20°$, $a = 10°$, $B_2 = 160°$

$\tan\theta = \tan 20° \left(\cos 10° - \cot 160° \sin 10°\right)$

$\theta = \tan^{-1}\left(\tan 20° \left(\cos 10° - \cot 160° \sin 10°\right)\right)$
$\approx 28°$

83. Solving $\dfrac{c_1}{c_2} = \dfrac{\sin\theta_1}{\sin\theta_2}$ for c_2 gives

$c_2 = \dfrac{c_1 \sin\theta_2}{\sin\theta_1}.$

When $\theta_1 = 39°$, $\theta_2 = 28°$, and $c_1 = 3 \times 10^8$,

$c_2 = \dfrac{(3 \times 10^8)(\sin 28°)}{\sin 39°} \approx 2.2 \times 10^8$ m/sec.

Solving $\dfrac{c_1}{c_2} = \dfrac{\sin\theta_1}{\sin\theta_2}$ for c_2 gives

$c_2 = \dfrac{c_1 \sin\theta_2}{\sin\theta_1}.$

When $c_1 = 3 \cdot 10^8$, $\theta_1 = 46°$, and $\theta_2 = 31°$,

$c_2 = \dfrac{3 \cdot 10^8 (\sin 31°)}{\sin 46°} = 214{,}796{,}150$

2.1×10^8 m/sec.

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Since the horizontal side of each square represents 30° and the sine wave repeats itself every 8 squares, the period of the sine wave is
$8 \cdot 30° = 240°$.

Since the horizontal side of each square represents 30° and the sine wave repeats itself every 4 squares, the period of the sine wave is
$4 \cdot 30° = 120°$.

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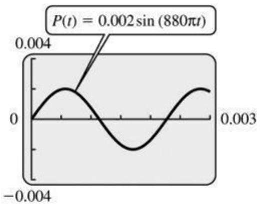
87. (a)



(b) Since the sine function is zero for multiples of π, we can determine the values (s) of t where $P = 0$ by setting $880\pi t = n\pi$, where n is an integer, and solving for t. After some algebraic manipulations, $t = \frac{n}{880}$ and $P = 0$ when $n = \ldots, -2, -1, 0, 1, 2, \ldots$. However, only values of $n = 0$, $n = 1$, or $n = 2$ produce values of t that lie in the interval $[0, 0.003]$. Thus, $P = 0$ when $t = 0$, $t = \frac{1}{880} \approx 0.0011$, and $t = \frac{1}{440} \approx 0.0023$. These values check with the graph in part (a).

(c) The period is $T = \frac{2\pi}{880\pi} = \frac{1}{440}$. Therefore, the frequency is 440 cycles per second.

88. $T(t) = 60 - 30\cos\frac{t}{2}$

$t = 1$ represents February, so the maximum afternoon temperature in February is $T(1) = 60 - 30\cos\frac{1}{2} \approx 34°\text{F}$.

$t = 3$ represents April, so the maximum afternoon temperature in April is $T(3) = 60 - 30\cos\frac{3}{2} \approx 58°\text{F}$.

$t = 8$ represents September, so the maximum afternoon temperature in September is $T(8) = 60 - 30\cos 4 \approx 80°\text{F}$.

$t = 6$ represents July, so the maximum afternoon temperature in July is $T(6) = 60 - 30\cos 3 \approx 90°\text{F}$.

$t = 11$ represents December, so the maximum afternoon temperature in December is

$T(11) = 60 - 30\cos\frac{11}{2}$

$T(x) = 37\sin\left[\frac{2\pi}{365}(t - 101)\right] + 25$

(a) $T(74) = 37\sin\left[\frac{2\pi}{365}(-27)\right] + 25 \approx 8°\text{F}$

(b) $T(121) = 37\sin\left[\frac{2\pi}{365}(20)\right] + 25 \approx 37°\text{F}$

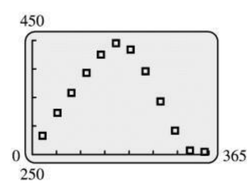
(c) $T(250) = 37\sin\left[\frac{2\pi}{365}(149)\right] + 25 \approx 45°\text{F}$

(d) $T(325) = 37\sin\left[\frac{2\pi}{365}(224)\right] + 25 \approx 1°\text{F}$

(e) The maximum and minimum values of the sine function are 1 and –1, respectively. Thus, the maximum value of T is $37(1) + 25 = 62°\text{F}$ and the minimum value of T is $37(-1) + 25 = -12°\text{F}$.

(f) The period is $\frac{2\pi}{\frac{2\pi}{365}} = 365$.

90. (a)



Yes; because of the cyclical nature of the days of the year, it is reasonable to assume that the times of the sunset are periodic.

The function $s(t)$, derived by a TI-84

Plus using the sine regression function under the STATCALC menu, is given by $s(t) = 94.0872\sin(0.0166t - 1.2213)$ 347.4158.

$s(60) = 94.0872\sin[0.0166(60) - 1.2213]$ 347.4158
326 minutes
5:26 P.M.

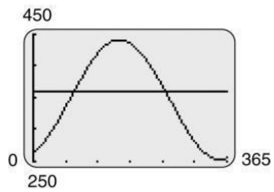
$s(120) = 94.0872\sin[0.0166(120) - 1.2213]$ 347.4158
413 minutes + 60 minutes (daylight savings)
7:53 P.M.

$s(240) = 94.0872\sin[0.0166(240) - 1.2213] + 347.4158$

$\approx 39°\text{F}$.

382 minutes + 60 minutes (daylight savings)	7:22 P.M.

(d) The following graph shows $s(t)$ and $y = 360$ (corresponding to a sunset at 6:00 P.M.). These graphs first intersect on day 82. However because of daylight savings time, to find the second value we find where the graphs of $s(t)$ and $y = 360 - 60 = 300$ intersect. These graphs intersect on day 295. Thus, the sun sets at approximately 6:00 P.M. on the 82nd and 295th days of the year.



91. Let h = the height of the building.

$\tan 42.8° = \frac{h}{65} \Rightarrow h = 65 \tan 42.8° \approx 60.2$

The height of the building is approximately 60.2 meters.

92. Let x be the distance to the opposite side of the canyon. Then

$\tan 27° = \frac{105}{x} \Rightarrow x = \frac{105}{\tan 27°} \approx 206$

The distance to the opposite side of the canyon is approximately 206 ft.

Chapter 2 Review Exercises

1. False; an exponential function has the form $f(x) = a^x$.

2. True

3. False; the logarithmic function $f(x) = \log_a x$ is not defined for $a = 1$.

4. False; $\ln(5 + 7) = \ln 12 \neq \ln 5 + \ln 7$

5. False; $(\ln 3)^4 \neq 4\ln 3$ since $(\ln 3)^4$ means $(\ln 3)(\ln 3)(\ln 3)(\ln 3)$.

6. False; $\log_{10} 0$ is undefined since $10^x = 0$ has no solution.

7. True

8. False; $\ln(-2)$ is undefined.

9. False; $\frac{\ln 4}{\ln 8} = 0.6667$ and

10. True

11. True

12. False; The period of cosine is 2π.

13. True

14. False. There's no reason to suppose that the Dow is periodic.

15. False; $\cos(a + b) = \cos a \cos b - \sin a \sin b$

16. True

17. A logarithm is the power to which a base must be raised in order to obtain a given number. It is the inverse of an exponential. We can write the definition mathematically as

$y = \log_a x \Leftrightarrow a^y = x$, for $a > 0$, $a \neq 1$, and $x > 0$.

18. Exponential growth functions grow without bound while limited growth functions reach a maximum size that is limited by some external constraint.

19. One degree is $\frac{1}{360}$ of a complete rotation, while one radian is the measure of the central angle in the unit circle that intercepts an arc with length 1. In a circle with radius r, an angle measuring 1 radian intercepts an arc with length r. To convert from degree measure to radian measure, multiply the number of degrees by $\frac{\pi}{180°}$. To convert from radian measure to degree measure, multiply the number of radians by $\frac{180°}{\pi}$.

20. There's a nice answer to the question of when to use radians vs degrees at http://mathwithbaddrawings.com/2013/05/02/degrees-vs-radians/

21. Let (x, y) is a point on the terminal side of an angle θ in standard position, and let r be the distance from the origin to (x, y). Then

$\sin\theta = \frac{y}{r}$ $\qquad \csc\theta = \frac{r}{y},\ y \neq 0$

$\cos\theta = \frac{x}{r}$ $\qquad \sec\theta = \frac{r}{x},\ y \neq 0$

$\tan\theta = \frac{y}{x},\ x \neq 0$ $\qquad \cot\theta = \frac{x}{y},\ y \neq 0$

22. The exact value for the trigonometric functions can be determined for any integer ln 4 − ln 8 = ln(1/2) ≈ −0.6931.

multiple of $\frac{\pi}{6}$ or $\frac{\pi}{4}$.

Chapter 2 REVIEW EXERCISES 123

$y = \ln\left(x^2 - 9\right)$

In order for the logarithm to be defined, $x^2 - 9 > 0$. So, $x^2 > 9 \Rightarrow x < -3$ or $x > 3$.

The domain is $(-\infty, -3) \cup (3, \infty)$.

24. $y = \dfrac{1}{e^x - 1}$

In order to the fraction to be defined, $e^x - 1 \neq 0$. So, $e^x - 1 \neq 0 \Rightarrow e^x \neq 1 \Rightarrow x \neq 0$.

The domain is $(-\infty, 0) \cup (0, \infty)$.

25. $y = \dfrac{1}{\sin x - 1}$

In order to the fraction to be defined, $\sin x - 1 \neq 0$. So, $\sin x - 1 \neq 0 \Rightarrow \sin x \neq 1$.

The domain is

$x \mid x \neq \frac{\pi}{2}, -\frac{3\pi}{2}, \frac{5\pi}{2}, -\frac{7\pi}{2}, \ldots$

$y = \tan x = \dfrac{\sin x}{\cos x}$

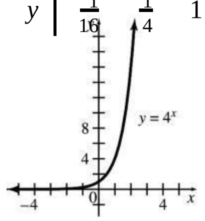
In order to the fraction to be defined, $\cos x \neq 0$.

The domain is

$x \mid x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

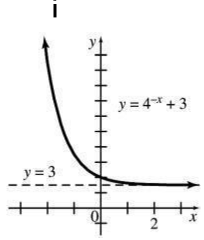
$y = 4^x$

x	-2	-1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16



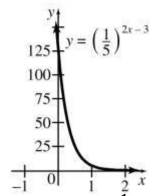
28. $y = 4^{-x} + 3$

x	-2	-1	0	1	2
				4	16



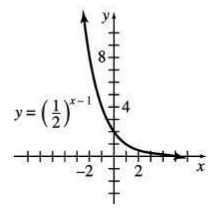
29. $y = \left(\frac{1}{5}\right)^{2x-3}$

x	0	1	2
y	125	5	$\frac{1}{5}$



30. $y = \left(\frac{1}{2}\right)^{x-1}$

x	-2	-1	0	1	2
y	8	4	2	1	$\frac{1}{2}$

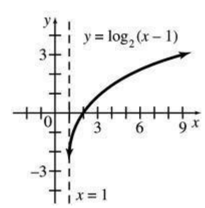


31. $y = \log_2 (x - 1)$

$2^y = x - 1$

$= 1 + 2^y$

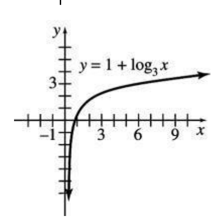
x	2	3	5	9
y	0	1	2	3



$y = 1 + \log_3 x$

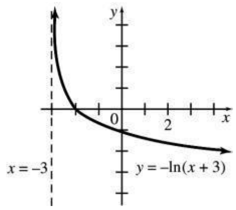
$1 = \log_3 xy$

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
y	-1	0	1	2	3



$y = -\ln(x + 3)$

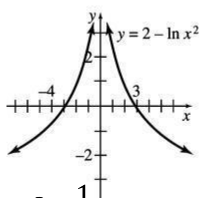
x	-2	-1	0	1	2
	0	-0.69	-1.1	-1.4	-1.6



$y = 2 - \ln x^2$

x	-4	-3	-2	-1
y	-0.8	-0.2	0.6	2

x	1	2	3	4
y	2	0.6	-0.2	-0.8



$2^{x+2} = \frac{1}{8}$

$2^{x+2} = \frac{1}{2^3}$

$2^{x+2} = 2^{-3}$

$x + 2 = -3$

$x = -5$

36. $\left(\frac{9}{16}\right)^x = \frac{3}{4}$

$\left(\frac{3}{4}\right)^{2x} = \left(\frac{3}{4}\right)^1$

$2x = 1$

$x = \frac{1}{2}$

$9^{2y+3} = 27^y$

$(3^2)^{2y+3} = (3^3)^y$

$3^{4y+6} = 3^{3y}$

$4y + 6 = 3y$

$y = -6$

38. $\frac{1}{2} = \left(\frac{b}{4}\right)^{1/4}$

$\left(\frac{1}{2}\right)^4 = \frac{b}{4}$

$4\cdot\left(\frac{1}{2}\right)^4 = b$

$4\cdot\frac{1}{16} = \frac{1}{4} = b$

$e^{0.8} = 2.22554$

The equation in logarithmic form is $\ln 2.22554 = 0.8$.

42. $10^{1.07918} = 12$

The equation in logarithmic form is $\log 12 = 1.07918$.

$\log_2 32 = 5$

The equation in exponential form is $2^5 = 32$.

$\log_9 3 = \frac{1}{2}$

The equation in exponential form is $9^{1/2} = 3$.

45. $\ln 82.9 = 4.41763$

The equation in exponential form is $e^{4.41763} = 82.9$.

$\log 3.21 = 0.50651$

The equation in exponential form is $10^{0.50651} = 3.21$.

Recall that $\log x$ means $\log_{10} x$.

47. $\log_3 81 = x$

$3^x = 81$

$3^x = 3^4$

$x = 4$

48. $\log_{32} 16 = x$

$32^x = 16$

$2^{5x} = 2^4$

$5x = 4$

$x = \frac{4}{5}$

49. $\log_4 8 = x$

$4^x = 8$

$(2^2)^x = 2^3$

$2x = 3$

$x = \frac{3}{2}$

50. $\log_{100} 1000 = x$

$100^x = 1000$

$(10^2)^x = 10^3$

$2x = 3$

$x = \frac{3}{2}$

51. $\log_5 3k + \log_5 7k^3 = \log_5 3k(7k^3)$

$= \log_5 (21k^4)$

52. $\log 2y^3 - \log 8y^2 = \log \frac{2y^3}{} = \log \frac{y}{}$

$3^5 = 243$

The equation in logarithmic form is $\log_3 243 = 5$.

40. $5^{1/2} = \sqrt{5}$

The equation in logarithmic form is $\log_5 \sqrt{5} = \frac{1}{2}$.

3 3 3 2 3

8y 4

53. $4\log_3 y - 2\log_3 x = \log_3 y^4 - \log_3 x^2$

$= \log_3 \frac{y^4}{x^2}$

54. $3\log_4 r^2 - 2\log_4 r = \log_4 (r^2)^3 - \log_4 r^2$

$= \log_4 \frac{r^6}{r^2} = \log_4(r^4)$

55. $6^p = 17$

$\ln 6^p = \ln 17$

$p \ln 6 = \ln 17$

$p = \frac{\ln 17}{\ln 6} \approx 1.581$

56. $3^{z-2} = 11$

$\ln 3^{z-2} = \ln 11$

$(z-2)\ln 3 = \ln 11$

$z - 2 = \frac{\ln 11}{\ln 3}$

$z = \frac{\ln 11}{\ln 3} + 2 \approx 4.183$

$2^{1-m} = 7$

$\ln 2^{1-m} = \ln 7$

$(1-m)\ln 2 = \ln 7$

$1 - m = \frac{\ln 7}{\ln 2}$

$-m = \frac{\ln 7}{\ln 2} - 1$

$m = 1 - \frac{\ln 7}{\ln 2} \approx -1.807$

$12^{-k} = 9$

$\ln 12^{-k} = \ln 9$

$-k \ln 12 = \ln 9$

$k = -\frac{\ln 9}{\ln 12} \approx -0.884$

$e^{-5-2x} = 5$

$\ln e^{-5-2x} = \ln 5$

$-5 - 2x = \ln 5$

$-2x = \ln 5 + 5$

$x = \frac{\ln 5 + 5}{-2} \approx -3.305$

$e^{3x-1} = 14$

$\ln (e^{3x-1}) = \ln 14$

$3x - 1 = \ln 14$

$x = \frac{1 + \ln 14}{3} \approx$

61. $\left(1 + \frac{m}{3}\right)^5 = 15$

$\left[\left(1 + \frac{m}{3}\right)^5\right]^{1/5} = 15^{1/5}$

$1 + \frac{m}{3} = 15^{1/5}$

$\frac{m}{3} = 15^{1/5} - 1$

$m = 3(15^{1/5} - 1) \approx 2.156$

62. $\left(1 + \frac{2p}{5}\right)^2 = 3$

$1 + \frac{2p}{5} = \pm\sqrt{3}$

$5 + 2p = \pm 5\sqrt{3}$

$2p = -5 \pm 5\sqrt{3}$

$p = \frac{-5 \pm 5\sqrt{3}}{2}$

$p = \frac{-5 + 5\sqrt{3}}{2} \approx 1.830$

or $p = \frac{-5 - 5\sqrt{3}}{2} \approx -6.830$

63. $\log_k 64 = 6$

$k^6 = 64$

$k^6 = 2^6$

$k = 2$

64. $\log_3 (2x + 5) = 5$

$3^5 = 2x + 5$

$243 = 2x + 5$

$= 2x$

$x = 119$

$\log (4p + 1) + \log p = \log 3$

$\log [p(4p + 1)] = \log 3$

$\log (4p^2 + p) = \log 3$

$4p^2 + p = 3$

$4p^2 + p - 3 = 0$

$(4p - 3)(p + 1) = 0$

$4p - 3 = 0$ or $p + 1 = 0$

3

Chapter 2 REVIEW EXERCISES 127

$p = 4$ $\quad p = -1$

1.213

p cannot be negative, so $p = 3$.

4

66. $\log_2 (5m - 2) - \log_2 (m + 3) = 2$

$$\log_2 \frac{5m-2}{m+3} = 2$$

$$\frac{5m-2}{m+3} = 2^2$$

$$5m - 2 = 4(m + 3)$$

$$5m - 2 = 4m + 12$$

$$m = 14$$

$f(x) = a^x$; $a > 0$, $a \neq 1$

The domain is $(-\infty, \infty)$.

The range is $(0, \infty)$.

The y-intercept is 1.

The x-axis, $y = 0$, is a horizontal asymptote.

The function is increasing if $a > 1$.

The function is decreasing if $0 < a < 1$.

68. $f(x) = \log_a x$; $a > 0$, $a \neq 1$

The domain is $(0, \infty)$.

The range is $(-\infty, \infty)$.

The x-intercept is 1.

The y-axis, $x = 0$, is a vertical asymptote.

f is increasing if $a > 1$.

f is decreasing if $0 < a < 1$.

The domain of $f(x) = a^x$ is the same as the range of $f(x) = \log_a x$ and the domain of $f(x) = \log_a x$ is the same as the range of $(x) = a^x$. Both functions are increasing if $a > 1$. Both functions are decreasing if $0 < a < 1$. The functions are asymptotic to different axes.

70. $90° = 90 \cdot \frac{\pi}{180} = \frac{90\pi}{180} = \frac{\pi}{2}$

$360° = 2\pi$

75. $405° = 405 \cdot \frac{\pi}{180} = \frac{9\pi}{4}$

76. $5\pi = 5\pi \cdot \frac{180°}{\pi} = 900°$

77. $\frac{3\pi}{4} = \frac{3\pi}{4} \cdot \frac{180°}{\pi} = 135°$

78. $\frac{9\pi}{20} = \frac{9\pi}{20} \cdot \frac{180°}{\pi} = 81°$

79. $\frac{3\pi}{10} = \frac{3\pi}{10} \cdot \frac{180°}{\pi} = 54°$

80. $\frac{13\pi}{20} = \frac{13\pi}{20} \cdot \frac{180°}{\pi} = 117°$

$\frac{13\pi}{15} = \frac{13\pi}{15} \cdot \frac{180°}{\pi} = 156°$

When an angle 60° is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (1, \sqrt{3})$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{1 + 3} = 2$, so

$\sin 60° = \frac{y}{r} = \frac{\sqrt{3}}{2}$.

When an angle of 120° is drawn in standard position, $(x, y) = (-1, \sqrt{3})$ is one point on its terminal side, so $\tan 120° = \frac{y}{x} = -\sqrt{3}$.

When an angle of $-45°$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (1, -1)$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{1 + 1} = \sqrt{2}$, so

$\cos(-45°) = \frac{x}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

When an angle of 150° is drawn in standard

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71. $160° = 160\dfrac{\pi}{180} = \dfrac{8\pi}{9}$

72. $225° = 225\dfrac{\pi}{180} = \dfrac{5\pi}{4}$

73. $270° = 270\dfrac{\pi}{180} = \dfrac{3\pi}{2}$

position, one choice of a point on its terminal side is $(x, y) = (-\sqrt{3}, 1)$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{3 + 1} = 2$, so

$\sec 150° = \dfrac{r}{x} = \dfrac{2}{-\sqrt{3}} = -\dfrac{2\sqrt{3}}{3}$.

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When an angle of 120° is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (-1, \sqrt{3})$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{1 + 3} = 2$, so

$\csc 120° = \frac{r}{y} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$.

When an angle of 300° is drawn in standard position, $(x, y) = (1, -\sqrt{3})$ is one point on its terminal side, so $\cot 300° = \frac{x}{y} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$.

When an angle of $\frac{\pi}{6}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (\sqrt{3}, 1)$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{3 + 1} = 2$, so $\sin \frac{\pi}{6} = \frac{y}{r} = \frac{1}{2}$.

When an angle of $\frac{7\pi}{3}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (1, \sqrt{3})$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{1 + 3} = 2$, so

$\cos \frac{7\pi}{3} = \frac{x}{r} = \frac{1}{2}$.

When an angle of $\frac{5\pi}{3}$ is drawn in standard position, one choice of a point on its terminal side is $(x, y) = (1, -\sqrt{3})$. Then

$r = \sqrt{x^2 + y^2} = \sqrt{1 + 3} = 2$, so

$\sec \frac{5\pi}{3} = \frac{r}{x} = \frac{2}{1} = 2$.

When an angle of $\frac{7\pi}{3}$ is drawn in standard position, $(x, y) = (1, \sqrt{3})$ is one point on its terminal side. Then $r = \sqrt{1 + 3} = 2$, so

$\csc \frac{7\pi}{3} = \frac{r}{y} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$.

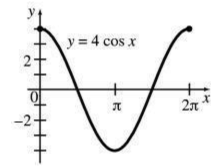
$\sin 47° \approx 0.7314$

$\cos 0.8215 \approx 0.6811$

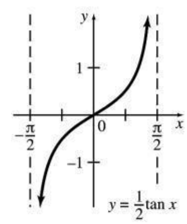
$\cos 0.5934 \approx 0.8290$

$\tan 1.2915 \approx 3.4868$

The graph of $y = \cos x$ appears in Figure 27 in Section 4 of this chapter. To get $y = 4 \cos x$, each value of y in $y = \cos x$ must be multiplied by 4. This gives a graph going through $(0, 4)$, $(\pi, -4)$ and $(2\pi, 4)$.



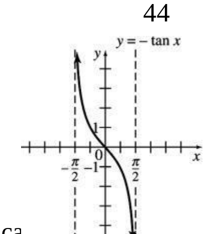
The graph of $y = \frac{1}{2} \tan x$ is similar to the graph of $y = \tan x$ except that each ordinate value is multiplied by a factor of $\frac{1}{2}$. Note that the points $\left(\frac{\pi}{4}, \frac{1}{2}\right)$, $(0, 0)$, and $\left(-\frac{\pi}{4}, -\frac{1}{2}\right)$ lie on the graph.



The graph of $y = \tan x$ appears in Figure 28 in Section 4 in this chapter. The difference between the graph of $y = \tan x$ and $y = -\tan x$ is that the y-values of points on the graph of $y = -\tan x$ are the opposites of the y-values of the corresponding points on the graph of $y = \tan x$.

A sample calculation:

When $x = \frac{\pi}{4}$, $y = -\tan \frac{\pi}{4} = -1$.



Chapter 2 REVIEW EXERCISES **129**

$\cos 72° \approx 0.3090$

$\tan 115° \approx -2.1445$

$\sin(-123°) \approx -0.8387$

$\sin 2.3581 \approx 0.7058$

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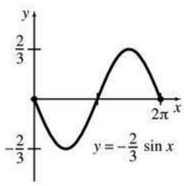
The graph of $y = -\frac{2}{3}\sin x$ is similar to the graph of $y = \sin x$ except that it has two-thirds the amplitude and is reflected about the x-axis.



$y = 17{,}000,\ y_0 = 15{,}000,\ t = 4$

$$y = y_0 e^{kt}$$
$$17{,}000 = 15{,}000e^{4k} \Rightarrow \frac{17}{15} = e^{4k} \Rightarrow$$
$$\ln\frac{17}{15} = 4k \Rightarrow k = \frac{\ln\frac{17}{15}}{4} \approx 0.313$$

So, $y = 15{,}000e^{0.0313t}$.

$$45{,}000 = 15{,}000e^{0.0313t}$$
$$3 = e^{0.0313t}$$
$$\ln 3 = 0.0313t$$
$$\frac{\ln 3}{0.0313} = k \approx 35.1$$

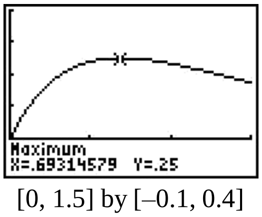
It would take about 35 years.

$I(x) \ge 1$

$$I(x) = 10e^{-0.3x}$$
$$10e^{-0.3x} \ge 1$$
$$e^{-0.3x} \ge 0.1$$
$$-0.3x \ge \ln 0.1$$
$$\frac{\ln 0.1}{-0.3} \approx 7.7$$

The greatest depth is about 7.7 m.

Graph $y = c(t) = e^{-t} - e^{-2t}$ on a graphing calculator and locate the maximum point. A calculator shows that the x-coordinate of the maximum point is about 0.69, and the y-coordinate is exactly 0.25. Thus, the maximum concentration of 0.25 occurs at about 0.69 minutes.



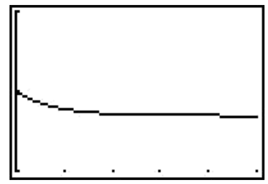
[0, 1.5] by [–0.1, 0.4]

107. $g(t) = \frac{c}{a} + \left(g_0 - \frac{c}{a}\right)e^{-at}$

If $g_0 = 0.08$, $c = 0.1$, and $a = 1.3$, the function becomes

$$g(t) = \frac{0.1}{1.3} + \left(0.08 - \frac{0.1}{1.3}\right)e^{-1.3t}.$$

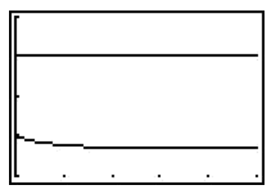
Graph this function on a graphing calculator.



[0, 5] by [0.07, 0.09]

From the graph, we see that the maximum value of g for $t \ge 0$ occurs at $t = 0$, the time when the drug is first injected. The maximum amount of glucose in the bloodstream, given by $G(0)$ is 0.08 gram.

From the graph, we see that the amount of glucose in the bloodstream decreases from the initial value of 0.08 gram, so it will never increase to 0.1 gram. We can also reach this conclusion by graphing $y_1 = G(t)$ and $y_2 = 0.1$ on the same screen with the window given in (a) and observing that the graphs of y_1 and y_2 do not intersect.



[0, 5] by [0.07, 0.11]

Using the TRACE function and the graph in part (a), we see that as t increases, the graph of $y_1 = G(t)$ becomes almost horizontal, and $G(t)$ approaches approximately 0.0769. Note that $\frac{c}{a} = \frac{0.1}{1.3} \approx 0.0769$. The amount of glucose in the bloodstream after a long time approaches 0.0769 grams.

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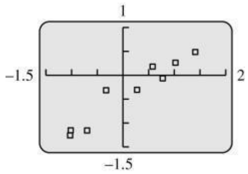
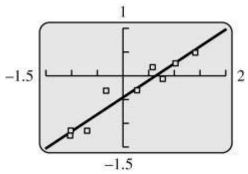
108. **(a)**



Yes, the data follows a linear trend.

(b)



The least squares line is $\log y = 0.7097 \log x - 0.4480.$

Solve the equation from part (b) for y.

$$\log y = 0.7097 \log x - 0.4480$$
$$\log y = \log x^{0.7097} - 0.4480$$
$$10^{\log y} = 10^{\log x^{0.7097} - 0.4480}$$
$$y = 10^{\log x^{0.7097}} \cdot 10^{-0.4480}$$
$$\approx 0.3565x^{0.7097}$$

Using a graphing calculator, the coefficient of correlation $r \approx 0.9625$.

(a) The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. The radius of a cancer cell is

$$\frac{2\left(10^{-5}\right)}{2} = 10^{-5} \text{ m. So,}$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(10^{-5}\right)^3$$

$$\approx 4.19 \times 10^{-15} \text{ cubic meters}$$

(b) The formula for the total volume of the cancer cells after t days is

$$V \approx \left(4.19 \times 10^{-15}\right) 2^t \text{ cubic meters.}$$

(c) The radius of a tumor with a diameter of 1 cm = 0.01 m is 0.005 m, so

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.005)^3.$$

Using the equation found in part (b), we have

$$\frac{4}{3}\pi(0.005)^3 = \left(4.19 \times 10^{-15}\right)2^t$$

$$\frac{\frac{4}{3}\pi(0.005)^3}{4.19 \times 10^{-15}} = 2^t$$

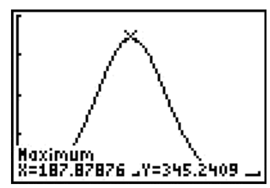
$$\ln\frac{\frac{4}{3}\pi(0.005)^3}{4.19 \times 10^{-15}} = t \ln 2$$

$$t = \frac{\ln\left(\frac{\frac{4}{3}\pi(0.005)^3}{4.19 \times 10^{-15}}\right)}{\ln 2}.$$

$$\approx 26.8969$$

It will take about 27 days for the cancer cell to grow to a tumor with a diameter of cm.

$$m(g) = e^{0.02 + 0.062g - 0.000165g^2}$$



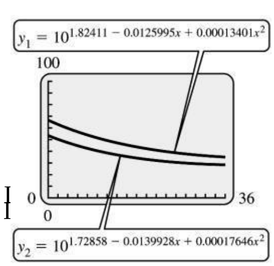
[0, 400] by [0, 400]

This function has a maximum value of $y \approx 345$ at $x \approx 187.9$. This is the largest value for which the formula gives a reasonable answer. The predicted mass of a polar bear with this girth is about 345 kg.

111. **(a)** The first three years of infancy corresponds to 0 months to 36 months, so the domain is [0, 36].

In both cases, the graph of the quadratic function in the exponent opens upward and the x coordinate of the vertex is greater than 36 ($x \approx 47$ for the awake infants and $x \approx 40$ for the sleeping infants). So the quadratic functions are both decreasing over this time. Therefore, both respiratory rates are decreasing.

(c)



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When $x = 12$, the waking respiratory rate is $y_1 \approx 49.23$ breaths per minute, and the sleeping respiratory rate is $y_2 \approx 38.55$.

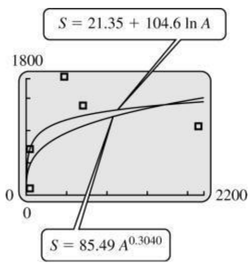
Therefore, for a 1-year-old infant in the 95th percentile, the waking respiratory rate is approximately 49.23 – 38.55

10.7 breaths per minute higher.

(a) $S = 21.35 + 104.6 \ln A$

$S = 85.49A^{0.3040}$

(c)



$S = 21.35 + 104.6 \ln(984.2) \approx 742.2$

$= 85.49(984.2)^{0.3040} \approx 694.7$

Neither number is close to the actual number of 421.

Answers will vary.

$P(t) = 90 + 15 \sin 144\pi t$

The maximum possible value of $\sin \alpha$ is 1, while the minimum possible value is –1. Replacing α with $144\pi t$ gives

$-1 \le \sin 144\pi t \le 1,$

$+ 15(-1) \le P(t) \le 90 +$

$15(1)\ 75 \le P(t) \le 105.$

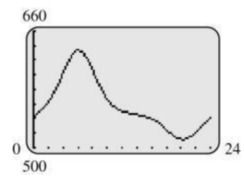
Therefore, the minimum value of $P(t)$ is 75 and the maximum value of $P(t)$ is 105.

Let $a = 546$, $C_0 = 511$, $C_1 = 634$,

$_0 = 20.27$, $t_1 = 6.05$, and $b = 24$. Then, the function is

$$C(t) = 546 + (634 - 546)e^{3\cos((2\pi/24)(t-6.05)) - 1} + (511 - 546)e^{3\cos((2\pi/24)(t-20.27)) - 1}$$

$$= 546 + 88e^{3\cos((\pi/12)(t-6.05)) - 1} - 35e^{3\cos((\pi/12)(t-20.27)) - 1}$$



(c)

$$C(20.27) = 546 + 88e^{3\cos((\pi/12)(20.27-6.05)) - 1} - 35e^{3\cos((\pi/12)(20.27-20.27)) - 1}$$

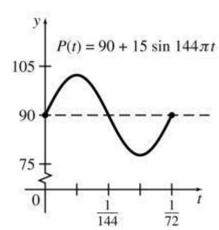
$\approx 511 \text{ ng/dl}$

This is approximately the value of C_0.

(d)

$$C(6.05) = 546 + 88e^{3\cos((\pi/12)(6.05-6.05)) - 1} - 35e^{3\cos((\pi/12)(6.05-20.27)) - 1}$$

634 ng/dl



2π __

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Chapter 2 REVIEW EXERCISES 133

This is approximately the value of C_1.

No, $C(t_0)$ is very close to C_0 because, when $t = t_0$, the last term in C_0 and the first term combine to yield C_0. The middle term in $C(t_0)$ is small but not 0. It has the value of about 0.3570.

Similarly, the first two terms in $C(t_1)$ combine to give C_1, while the last term is small, but not 0.

(a) The period is given by $\frac{2\pi}{b} = \text{period} = 24 \text{ hr}$, so $b = \frac{2\pi}{24}$

(a) The line passes through the points (60, 0.8) and (110, 2.2).

$$m = \frac{2.2 - 0.8}{110 - 60} = \frac{1.4}{50} = 0.028$$

Using the point (60, 0.8), we have $0.8 = 0.028(60) + b$

$0.8 = 1.68 + b$

$-0.88 = b$

Thus, the equation is $g(t) = 0.028t - 0.88$.

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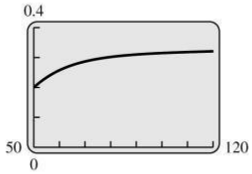
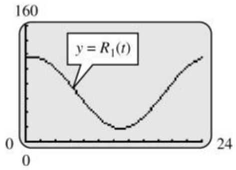
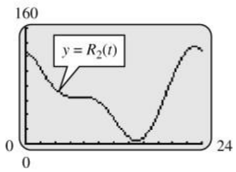
(b)

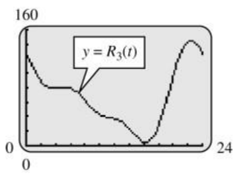


Per capita grain harvests have been increasing, but at a slower rate and are leveling off at about 0.32 ton per person.

116. (a)







As more terms are added, the function has greater complexity.

(b) $R_1(12) = 67.75 + 47.72\cos\frac{12\pi}{12} + 11.79\sin\frac{12\pi}{12}$

$67.75 + 47.72\cos\pi + 11.79\sin\pi$

$67.75 + 47.72(-1) + 11.79(0)$

20.0

$R_2(12) = R_1(12) + 14.29\cos\frac{12\pi}{6} - 21.09\sin\frac{12\pi}{6}$

$= 20.0 + 14.29\cos 2\pi - 21.09\sin 2\pi$

$= 20.0 + 14.29(1) - 21.09(0)$

$= 34.3$

$R_3(12) = R_2(12) - 2.86\cos\frac{12\pi}{4} - 14.31\sin\frac{12\pi}{}$

$p = \$6902$, $r = 6\%$, $t = 8$, $m = 2$

$A = P\left(1 + \frac{r}{m}\right)^{tm}$

$A = 6902\left(1 + \frac{0.06}{2}\right)^{8(2)}$

$6902(1.03)^{16}$

$\$11,075.68$

Interest $= A - P$

$= \$11,075.68 - \$6902 = \$4173.68$

118. $P = \$2781.36$, $r = 4.8\%$, $t = 6$, $m = 4$

$A = P\left(1 + \frac{r}{m}\right)^{tm}$

$A = 2781.36\left(1 + \frac{0.048}{4}\right)^{(6)(4)}$

$2781.36(1.012)^{24}$

$\$3703.31$

Interest $= \$3703.31 - \$2781.36 = \$921.95$

119. $P = \$12,104$, $r = 6.2\%$, $t = 2$

$A = Pe^{rt}$

$= 12,104e^{0.062(2)} = \$13,701.92$

120. $P = \$12,104$, $r = 6.2\%$, $t = 4$

$A = Pe^{rt}$

$A = 12,104e^{0.062(4)} = 12,104e^{0.248}$

$\approx \$15,510.79$

121. $A = \$1500$, $r = 0.06$, $t = 9$

$A = Pe^{rt}$

$= 1500e^{0.06(9)} = 1500e^{0.54} \approx$

$2574.01

122. $P = \$12,000$, $r = 0.05$, $t = 8$

$A = 12,000e^{0.05(8)} =$

$12,000e^{0.40} \approx \$17,901.90$

123. $1000 deposited at 6% compounded semiannually.

$A = P\left(1 + \frac{r}{m}\right)^{tm}$

To double:

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$$34.3 - 2.86\cos 3\pi - 14.31\sin\frac{3\pi}{4}$$
$$34.3 - 2.86(-1) - 14.31(0)$$
$$37.2$$

R_3 gives the most accurate value.

$$2(1000) = 1000\left(1 + \frac{0.06}{2}\right)^{t \cdot 2}$$
$$2 = 1.03^{2t}$$
$$\ln 2 = 2t \ln 1.03$$
$$t = \frac{\ln 2}{2 \ln 1.03} \approx 12 \text{ years}$$

(*continued on next page*)

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(*continued*)

To triple:

$3(1000) = 1000\left(1 + \frac{0.06}{2}\right)^{t \cdot 2}$

$3 = 1.03^{2t}$

$\ln 3 = 2t \ln 1.03$

$t = \frac{\ln 3}{2 \ln 1.03} \approx 19$ years

124. \$2100 deposited at 4% compounded quarterly.

$A = P\left(1 + \frac{r}{m}\right)^{tm}$

To double:

$2(2100) = 2100\left(1 + \frac{0.04}{4}\right)^{t \cdot 4}$

$2 = 1.01^{4t}$

$\ln 2 = 4t \ln 1.01$

$t = \frac{\ln 2}{4 \ln 1.01} \approx 17.4$

Because interest is compounded quarterly, round the result up to the nearest quarter, which is 17.5 years or 70 quarters. To triple:

$3(2100) = 2100\left(1 + \frac{0.04}{4}\right)^{t \cdot 4}$

$3 = 1.01^{4t}$

$\ln 3 = 4t \ln 1.01$

$t = \frac{\ln 3}{4 \ln 1.01}$

≈ 27.6

Because interest is compounded quarterly, round the result up to the nearest quarter, 27.75 years or 111 quarters.

$y = y_0 e^{-kt}$

$100{,}000 = 128{,}000e^{-k(5)}$

$128{,}000 = 100{,}000e^{5k}$

$\frac{128}{100} = e^{5k}$

$70{,}000 = 100{,}000e^{-0.05t}$

$\frac{7}{10} = e^{-0.05t}$

$\ln \frac{7}{10} = -0.05t$

$7.1 \approx t$

It will take about 7.1 years.

126. $t = (1.26 \times 10^9)\frac{\ln\left[1 + 8.33\left(\frac{A}{K}\right)\right]}{\ln 2}$

$A = 0, K > 0$

$= (1.26 \times 10^9)\frac{\ln[1 + 8.33(0)]}{\ln 2}$

$(1.26 \times 10^9)(0) = 0$ years

$t = (1.26 \times 10^9)\frac{\ln[1 + 8.33(0.212)]}{\ln 2}$

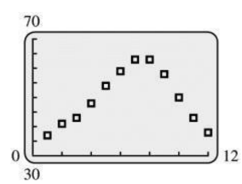
$(1.26 \times 10^9)\frac{\ln 2.76596}{\ln 2}$

$= 1{,}849{,}403{,}169$

or about 1.85×10^9 years

As r increases, t increases, but at a slower and slower rate. As r decreases, t decreases at a faster and faster rate

(a) Enter the data into a graphing calculator and plot.

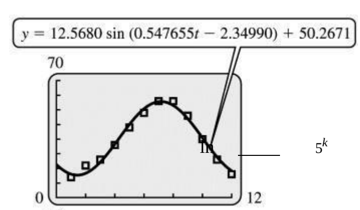


The sine regression is

$= 12.5680 \sin(0.54655t - 2.34990)$

50.2671

Graph the function with the data.



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100

$0.05 \approx k$

$= 100{,}000e^{-0.05t}$

$T = \frac{2\pi}{b} \approx 11.4729$

The period is about 11.5 months.

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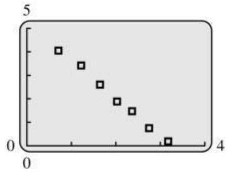
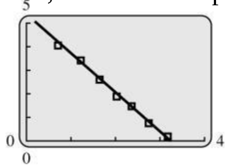
Extended Application: Power Functions

1. (a)



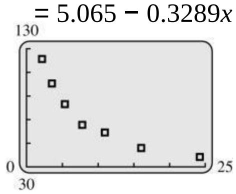
Yes, the relationship appears to be linear.

(b)



$= 5.065 - 0.3289x$

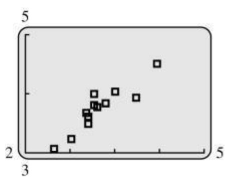
(c)



It is decreasing; the exponent is negative. As the price increases the demand decreases.

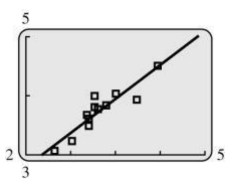
$Y = 158.4x^{-0.3289x}$

(a)



Yes, the relationship appears to be linear.

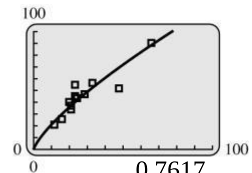
(b)



$= 0.7617x + 1.2874$

(c)

Species	Mass	BMR
P. boylii	23.20	54.29
P. californicus	47.55	51.97
P. crinitus	15.90	25.12
P. eremicus	21.50	33.11
P. gossypinus	21.50	36.98
P. leucopus	23.00	45.20
P. maniculatus	20.66	39.79
P. megalops	66.20	90.69
P. oreas	24.58	43.51
P. polionotus	12.00	21.48
P. sitkenisis	28.33	46.74
P. truei	33.25	56.70



$= 3.623x^{0.7617}$

An approximate value for the power is 0.76.

Chapter 2 Functions

a. Calculating Numerical Expressions

Spreadsheet can be used as a calculator once we use symbol "=". To evaluate numerical expression " 7.33^3 14

12

55.12

” we type “=7.33^3-14*(12+55.12)” in any cell. The copy of your expression will appear in formula bar, next to f_x . Enter. The result of calculation -545.847 will appear in the cell, as shown in ***Figure 22***.

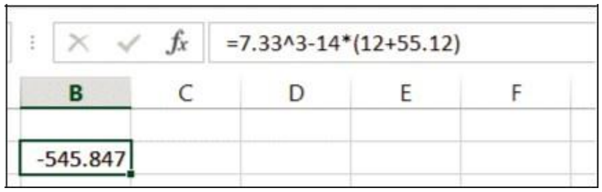


Figure 22

b. Using Function Notation

Excel allows us to create and use our own functions but it also has built-in functions that are performing various operations on data entered in cells.

To calculate value $f(0.9)$ for our own function $f(x)\ 1.8x^{5}$

$3x^3$ 7.3 we enter the value of x in B2 and we enter the function in cell C2 as “=-1.8*B2^5+3*B2^3-7.3”. ***Figure 23*** shows the final result f (0.9) 6.17588 . In case of long algebraic expressions it is easier to
type the function inside the formula bar. Notice that instead of x in the formula we are entering the value of cell B2. You can type “B2” or obtain the same by clicking on the cell B2.

Once your function is entered, you can evaluate it for a different value of x, say x 2.03 by changing cell B2 value to 2.03. That way y value will change automatically to -44.2553.

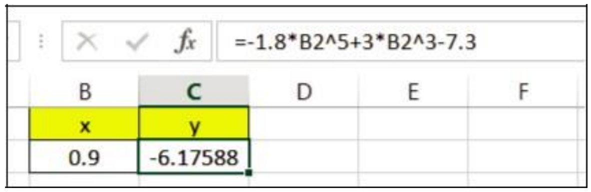


Figure 23

The list of built-in functions is found in the name box, to the left from the formula bar, after you type symbol “=” in any cell. Click on arrow next to the name box to obtain the list, like in ***Figure 24***. Click on “More Functions” to search over 100 built–in functions sorted in categories like “Statistics”, “Logical” etc., like in ***Figure 25***.

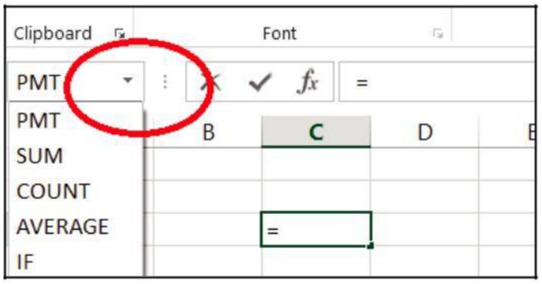


Figure 24

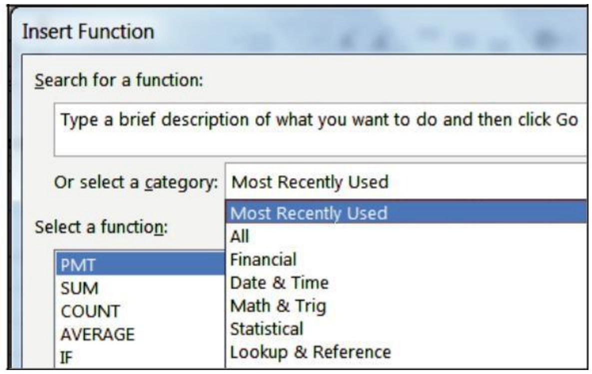


Figure 25

Another way to find and insert a build-in function is by clicking on the tab "Formulas" and selecting a function from the ribbon, like in ***Figure 26***. If you remember only the beginning of the name of the function, type it in a cell after "=". The spreadsheet will help you remember the rest of the name by offering possible choices.

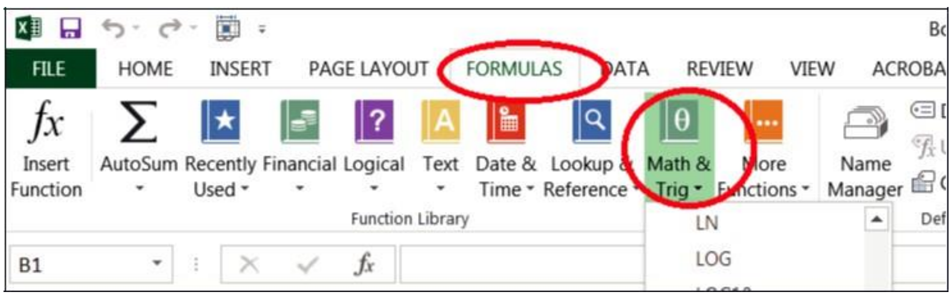


Figure 26

The built -in function "SUM" makes it easy to quickly sum columns, rows, or individual cells of data in a worksheet. To add numbers in cells A1:A4 we'll type "=" and select built-in function SUM as in ***Figure 27***. One easy way to tell Excel that you want the numbers in cells A1:A4 is by clicking on the small matrix to the right from the "Number1" box, and then selecting the array A1:A4.

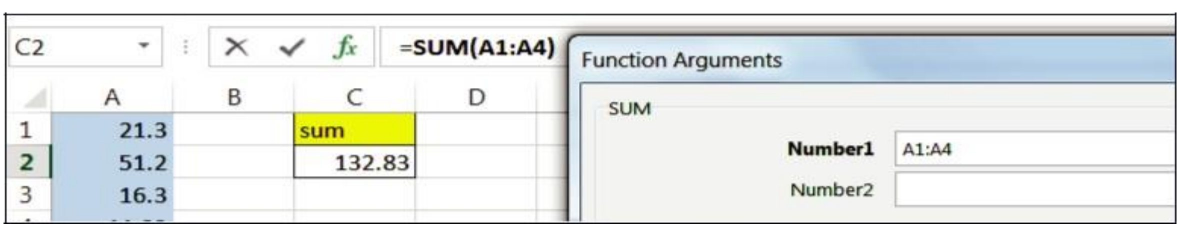


Figure 27

c. Creating Function Tables

Excel can graph the function y x^{3} $5x$

1 after the table of (x, y) values of interest is created. Say, we are interested in y values of the function for x 3.0, 2.5, 2.0,.....,2.5,3.0 .

Enter the first two values -3 and -2.5 in cells A2 and A3. One fast way to type the remaining *x* values in column A is to select the array A2:A3 and drag the handle from the lower right corner down, over column A, until cells A4:A14 are filled with the desired *x* values, as in ***Figure 28*** . Column B is reserved for the *y* values of our cubic function. A simple way to list the *y* values under observation is to click on B2 and enter the formula "=A2^3-5*A2+1". Drag the handle from the lower right corner over column B as in ***Figure 29*** to copy the formula down. Values - 11, -2.13, 3, etc. will appear in cells B2:B14.

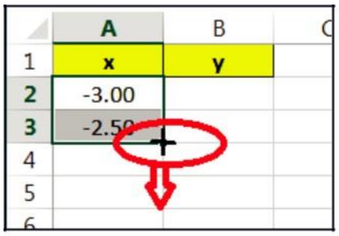


Figure 28

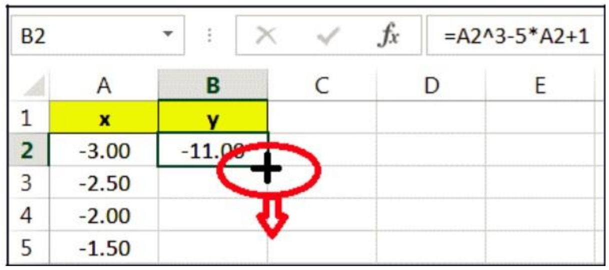


Figure 29

d. Graphing Function

Once we have table of (*x*, *y*) values listed in cells A2:B14 we can work on a graph. One of many ways to create a graph of our cubic function $y \quad x^{3} \quad 5x$

1 is to first select array A2:B14 to indicate *x* and *y* coordinates of points under observation. Select the “Insert” tab and then in the ribbon select the "Scatter" graph to the right. Select the chart sub-type “Scatter with smooth lines and markers” as shown in ***Figure 30***.

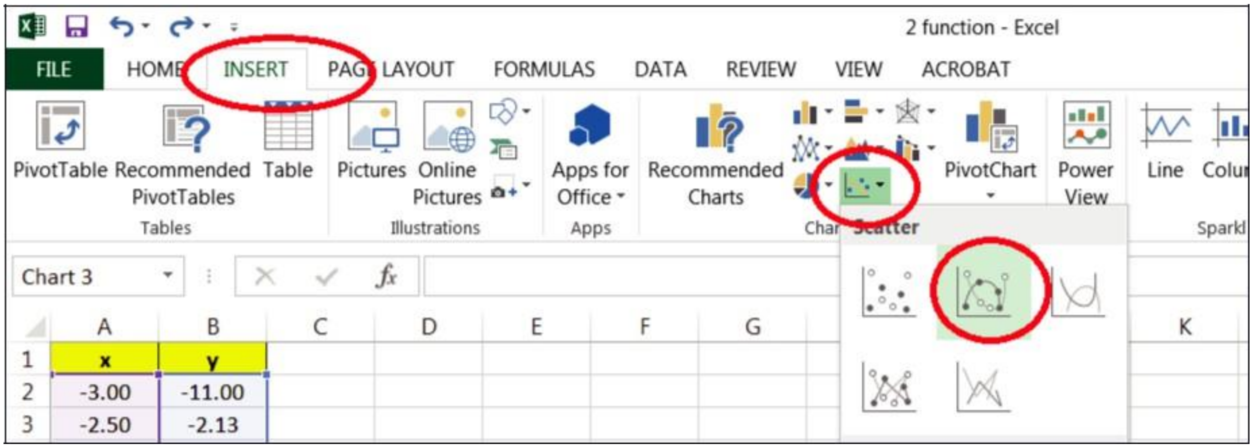


Figure 30

The final look of your graph is a matter of your personal taste. Click on your chart and the “Chart Tools” menu will appear as a new ribbon: use this to upgrade the style. Right-clicking on various sections of the final chart will let you select more options for gridlines, background color, text font, etc. Position your mouse exactly over one of the specially marked data points on your final graph, but do not click. After a short delay, the exact coordinates of that point will appear in a small yellow box next to the point, like in ***Figure 31***.

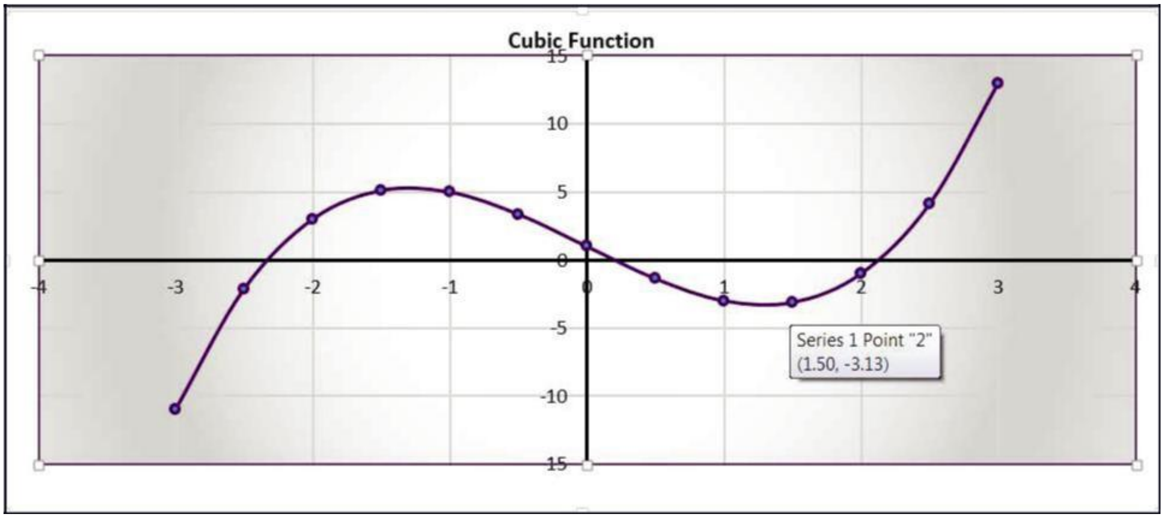


Figure 31

e. Piecewise Functions

To graph a piecewise defined function we'll create table of (*x, y*) values we are interested in, insert graph, and then define the domain of each. As an example we'll use the function:

$$f(x) = \begin{cases} -x^2 & \text{if } x < 2 \\ 6 & \text{if } x = 2 \\ 10 - x & \text{if } x > 2 \end{cases}$$

Figure 32 shows table of values of all three functions: $y = -x^2$, $y = 6$, and $y = 10 - x$ for $x = -4.0, -3.5, -3.0, \ldots$. To graph all three function together on $[-4, 6]$

we select A2:D22 and inserted scatter graph with smooth lines.

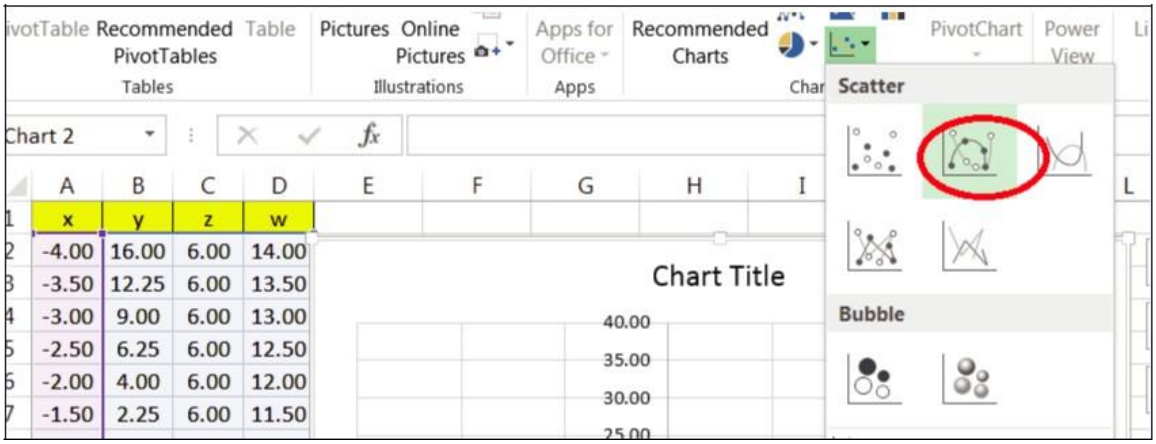


Figure 32

The chart we obtained shows all three functions, each on the entire domain (-4, 4), as shown in ***Figure 33***. We need to adjust the domain of each.

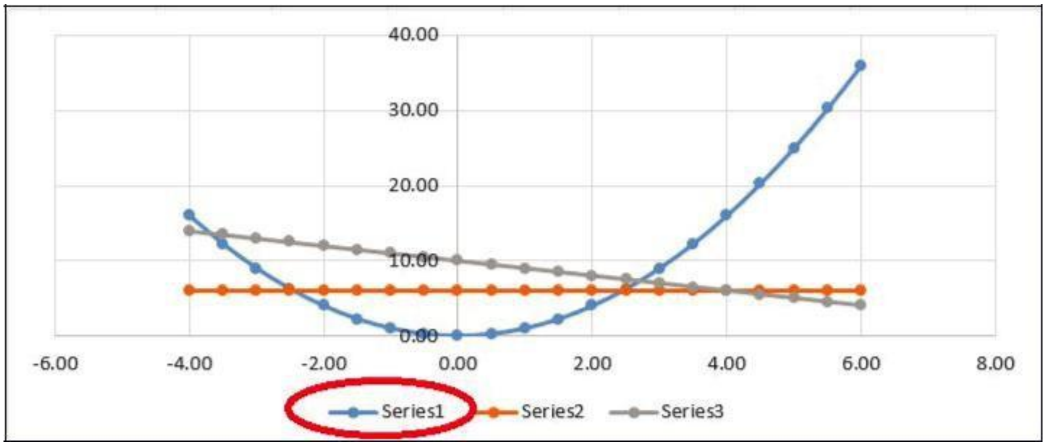


Figure 33

Right-click on “Series1” in legend under the graph and click on “Select Data”. The new pop-up screen will help change the visible domain of the first function $y\ x^2$ to

4, 2

. Select “Series1” and “Edit” as in ***Figure 34***.

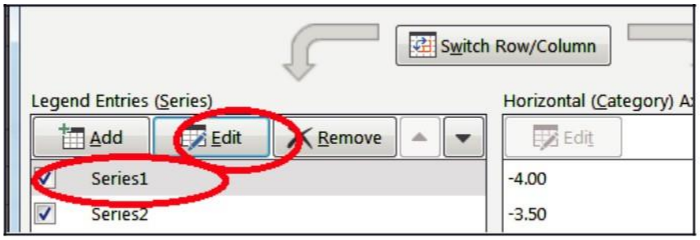


Figure 34

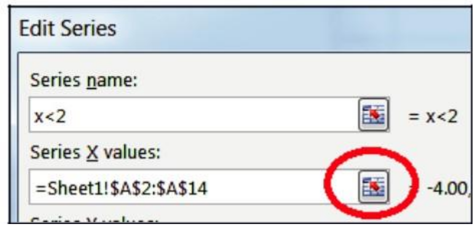


Figure 35

The new pop-up screen allowed us to change “Series1” name to “x<2” in the legend. Click to encircled box matrix in ***Figure 35.*** In the new pop-up screen select the array A2:A14 to indicate that the first function domain is

4, 2

. The final graph is shown in ***Figure 36***.

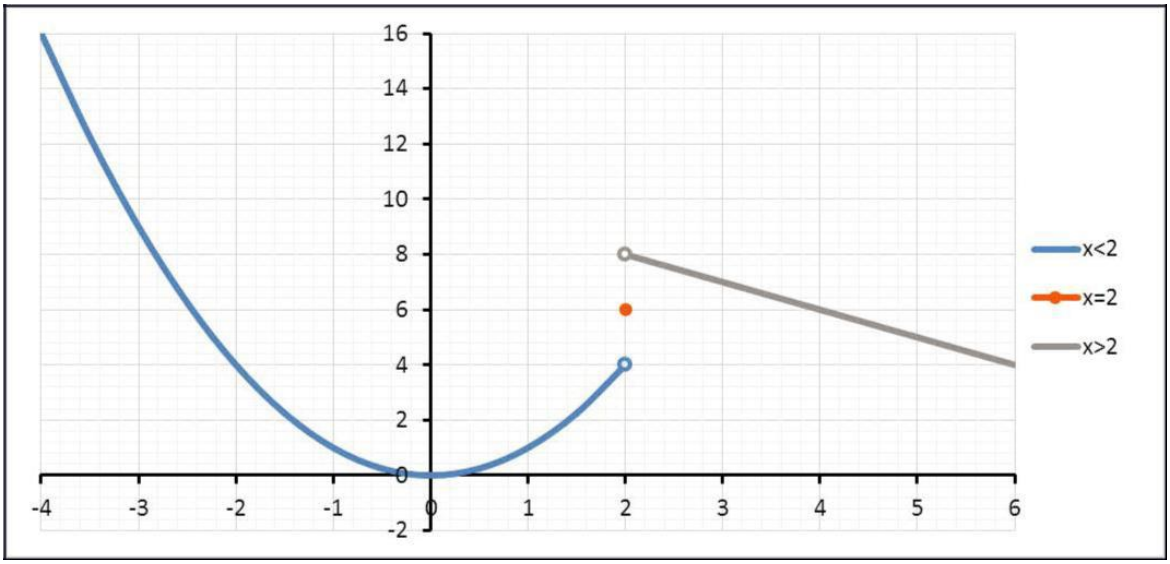


Figure 36

f. Finding Intersection Points

Suppose that the supply function is p 0.2 q

51 and the demand function is

3000 / (q

5) . We'd like to find the exact intersection point of these two functions (the equilibrium point).

One way to do it is to look at the table of values and examine *y* values for supply and demand functions. The first two values of *x* that are 0 and 5 are entered in cells A2 and A3, then the array A2:A3 is copied down to cells A4:A42 by dragging the fill handle from the lower right corner. The functions are entered in cells B2 and C2 as formulas "=0.2*A2+51" and "=3000/(A2+5)", and copied down by dragging the fill handle. ***Figure 37*** shows the equilibrium point in the table.

	A	B	C
1	x	supply	demand
8	30	57.00	85.71
9	35	58.00	75.00
10	40	59.00	66.67
11	45	60.00	60.00
12	50	61.00	54.55
13	55	62.00	50.00
14	60	63.00	46.15

Figure 37

It is even better if we graph the functions together, and find the intersection point on the graph. To obtain the graph select cells array A2:B42 and then "Scatter with Smooth Lines" from the "Insert" tab ribbon. Position the cursor exactly above the intersection point, but do not click. After a brief delay coordinates of the intersection point appear, like in ***Figure 38***.

.

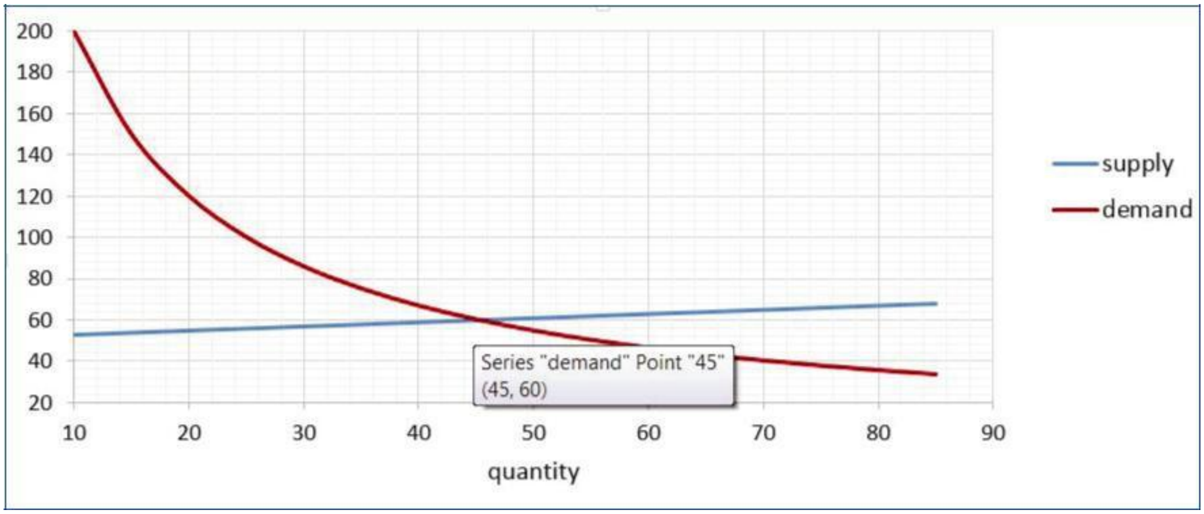


Figure 38

One more way to find the intersection point of functions p 0.2 q

51 and

p 3000 / (*q*

5) is to find zeros of the function y (0.2 *x*

51) 3000 / (*x*

5) . Type “0” in cell
H2 as shown in ***Figure 39*** , and type the formula “=(0.2*H2+51)-3000/(H2+5)” in cell I2. Select “What-If Analysis” in “Data” tab ribbon. Click on “Goal Seek” and fill the obtained pop-up screen as shown in ***Figure 40***. Cell H2 value will change to 45, that is the first coordinate of the intersection point.

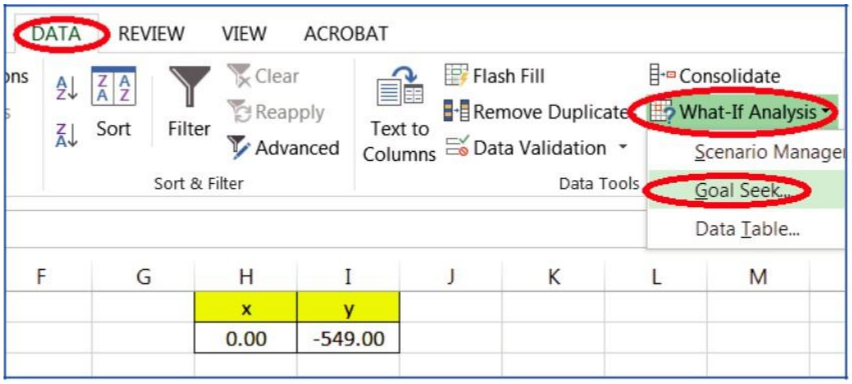


Figure 39

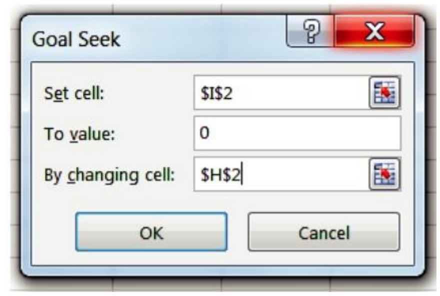


Figure 40

g. Finding Maximum and Minimum Points

We can estimate the maximum of the function
$0.0037\,x^{3}\;\;0.0591\,x^{2}$

2.534 x

38.21 by creating a spreadsheet table (see chapter 2 c) and examining *y* values. Once we have the table we might decide to graph the function (see chapter 2 d) and position the mouse over the point. Coordinates (11, 54.0082) will appear as in ***Figure 41***.

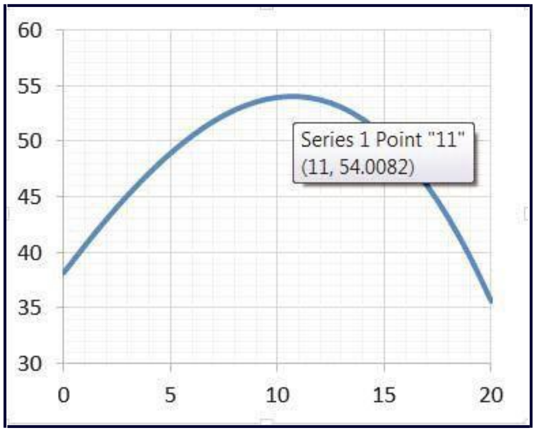


Figure 41

By looking at the *y* values of marked data points from the graph, we can estimate that the percentage will be maximized where the *x* -value is between 10 and 11. Assume we'd like to know the exact maximum of our function. Spreadsheet programs have a built-in Add-in called Solver. To use it in Excel you need to load it first. Click on “File” tab, select “Options” and then select ‘Add-Ins’ the Microsoft Office Button at the top left corner, and then click “Excel Options”. Select “Add-ins”. After you load “Solver Add-In” to your “Add-Ins”, the “Solver”, command is available in “Data” tab ribbon as one of “Analysis” tools.

Excel’s command “Solver” will calculate *y*’s for different *x* values in the formula $0.0037x^{3}\ \ 0.0591x^{2}$

$2.534x$

38.21 until the maximum or minimum value for *y* is found. Assign an initial *x*-value for the variable cell F2 as shown in ***Figure 42***. We selected “3”, but any value may be used here. Enter the quadratic function spreadsheet formula “=-0.0037*F2^3-0.0591*F2^2+2.534*F2+38.21” in the objective cell G2. Select “Solver” and fill the boxes. Click on “Solve” and the coordinates of the point where the function is maximized will be determined. In this example, the *x*-value 10.7 will appear in cell F2 and the *y*-value 54.02 will in cell C2 as the coordinates of the maximum.

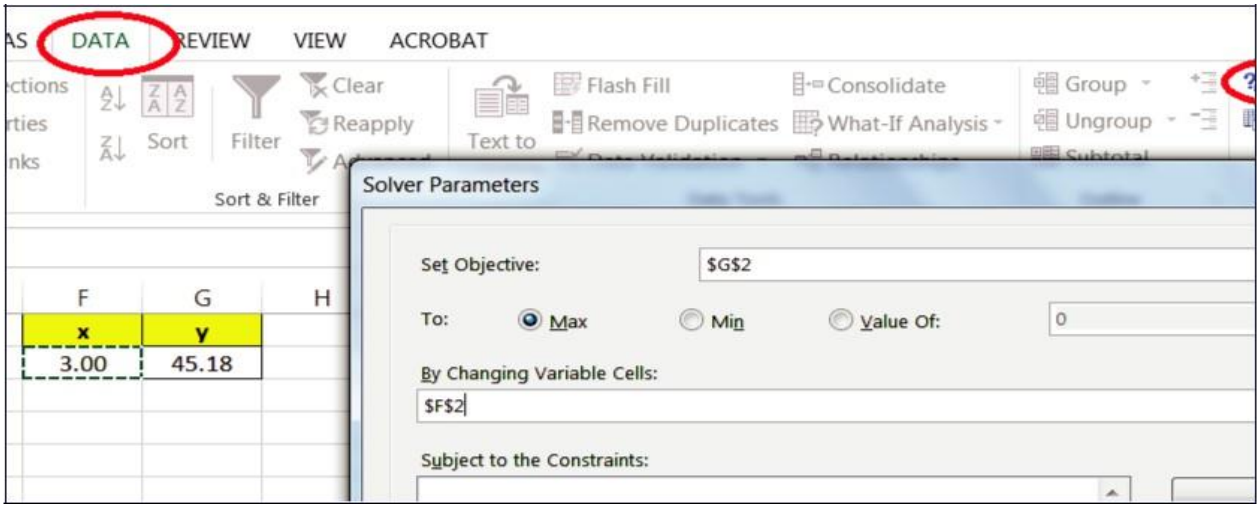


Figure 42