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Instructor's
Manual
to accompany
**Elements of Modern Algebra, Eighth
Edition**

Linda Gilbert and the late Jimmie Gilbert
University of South Carolina Upstate
Spartanburg, South Carolina

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Preface

This manual provides answers for the computational exercises and a few of the exercises requiring proofs in *Elements of Modern Algebra*, Eighth Edition, by Linda Gilbert and the late Jimmie Gilbert. These exercises are listed in the table of contents. In constructing proof of exercises, we have freely utilized prior results, including those results stated in preceding problems.

My sincere thanks go to Danielle Hallock and Lauren Crosby for their careful management of the production of this manual and to Eric Howe for his excellent work on the accuracy checking of all the answers.

Linda Gilbert

Answers to Selected Exercises

Section 1.1

1. True 2. True 3. False 4. True 5. True 6. False 7. True
 8. True 9. False 10. False

Exercises 1.1

1. a. $\square = \{\square \mid \square \text{ is a nonnegative even integer less than } 12\}$ b. $\square \mid \square^2 = 1$
 c. $\square = \{\square \mid \square \text{ is a negative integer}\}$ d. $\square \mid \square = \square^2 \text{ for } \square \in \mathbb{Z}^+$
2. a. False b. True c. False d. False e. False f. True
3. a. True b. True c. True d. True e. True f. False
 g. True h. True i. False j. False k. False l. True
4. a. False b. True c. True d. False e. True f. False
 g. False h. True i. False j. False k. False l. False
5. a. $\{0 \square 1 \square 2 \square 3 \square 4 \square 5 \square 6 \square 8 \square 10\}$ b. $\{2 \square 3 \square 5\}$ c. $\{0 \square 2 \square 4 \square 6 \square 7 \square 8 \square 9 \square 10\}$
 d. $\{2\}$
 e. \emptyset f. \square g. $\{0 \square 2 \square 3 \square 4 \square 5\}$ h. $\{6 \square 8 \square 10\}$ i. $\{1 \square 3 \square 5\}$
 j. $\{6 \square 8 \square 10\}$ k. $\{1 \square 2 \square 3 \square 5\}$ l. \square m. $\{3 \square 5\}$ n. $\{1\}$
6. a. \square b. \square c. \emptyset d. \square e. \square f. \emptyset g. \square h. \square i. \square
 j. \square k. \square l. \emptyset m. \square n. \emptyset
7. a. $\{\emptyset \square \square\}$ b. $\{\emptyset \square \{0\} \square \{1\} \square \square\}$
 c. $\{\emptyset \square \{\square\} \square \{\square\} \square \{\square\} \square \{\square \square\} \square \{\square \square\} \square \{\square \square \square\} \square \{\square \square \square\} \square \square\}$
 d. $\{\emptyset \square \{1\} \square \{2\} \square \{3\} \square \{4\} \square \{1 \square 2\} \square \{1 \square 3\} \square \{1 \square 4\} \square \{2 \square 3\} \square \{2 \square 4\} \square \{3 \square 4\} \square \{1 \square 2 \square 3\} \square \{1 \square 2 \square 4\} \square \{1 \square 3 \square 4\} \square \{2 \square 3 \square 4\} \square \square\}$
 e. $\{\emptyset \square \{1\} \square \{\{1\}\} \square \square\}$ f. $\{\emptyset \square \square\}$ g. $\{\emptyset \square \square\}$ h. $\{\emptyset \square \{\emptyset\} \square \{\{\emptyset\}\} \square \square\}$
8. a. Two possible partitions are:
 $\square_1 = \{\square \mid \square \text{ is a negative integer}\}$ and $\square_2 = \{\square \mid \square \text{ is a nonnegative integer}\}$ \square
 or
 $\square_1 = \{\square \mid \square \text{ is a negative integer}\}$ \square $\square_2 = \{\square \mid \square \text{ is a positive integer}\}$ \square $\square_3 = \{0\}$ \square

- b. One possible partition is $\mathcal{A}_1 = \{\{a, b, c\}\}$ and $\mathcal{A}_2 = \{\{a, b, c\}\}$. Another possible partition is $\mathcal{A}_1 = \{\{a\}\}$, $\mathcal{A}_2 = \{\{b, c\}\}$, $\mathcal{A}_3 = \{\{a, b, c\}\}$.
- c. One partition is $\mathcal{A}_1 = \{\{1, 5, 9\}\}$ and $\mathcal{A}_2 = \{\{1, 1, 15\}\}$. Another partition is $\mathcal{A}_1 = \{\{1, 15\}\}$, $\mathcal{A}_2 = \{\{1, 1\}\}$ and $\mathcal{A}_3 = \{\{5, 9\}\}$.
- d. One possible partition is $\mathcal{A}_1 = \{x \mid x = a + b, \text{ where } a \text{ is a positive real number, } b \text{ is a real number}\}$ and $\mathcal{A}_2 = \{x \mid x = a + b, \text{ where } a \text{ is a nonpositive real number, } b \text{ is a real number}\}$. Another possible partition is $\mathcal{A}_1 = \{x \mid x = ab, \text{ where } a \text{ is a real number}\}$, $\mathcal{A}_2 = \{x \mid x = ab, \text{ where } a \text{ is a nonzero real number}\}$ and $\mathcal{A}_3 = \{x \mid x = a + b, \text{ where } a \text{ and } b \text{ are both nonzero real numbers}\}$.

9. a. $\mathcal{A}_1 = \{1\}$, $\mathcal{A}_2 = \{2\}$, $\mathcal{A}_3 = \{3\}$

;

$\mathcal{A}_1 = \{1\}$, $\mathcal{A}_2 = \{2, 3\}$

;

$\mathcal{A}_1 = \{2\}$, $\mathcal{A}_2 = \{1, 3\}$

;

$\mathcal{A}_1 = \{3\}$, $\mathcal{A}_2 = \{1, 2\}$

b. $\mathcal{A}_1 = \{1\}$, $\mathcal{A}_2 = \{2\}$, $\mathcal{A}_3 = \{3\}$, $\mathcal{A}_4 = \{4\}$;

$\mathcal{A}_1 = \{1\}$, $\mathcal{A}_2 = \{2\}$, $\mathcal{A}_3 = \{3, 4\}$; $\mathcal{A}_1 = \{1\}$, $\mathcal{A}_2 = \{3\}$, $\mathcal{A}_3 = \{2, 4\}$;

$\mathcal{A}_1 = \{1\}$, $\mathcal{A}_2 = \{4\}$, $\mathcal{A}_3 = \{2, 3\}$; $\mathcal{A}_1 = \{2\}$, $\mathcal{A}_2 = \{3\}$, $\mathcal{A}_3 = \{1, 4\}$;

$\mathcal{A}_1 = \{2\}$, $\mathcal{A}_2 = \{4\}$, $\mathcal{A}_3 = \{1, 3\}$; $\mathcal{A}_1 = \{3\}$, $\mathcal{A}_2 = \{4\}$, $\mathcal{A}_3 = \{1, 2\}$;

$\mathcal{A}_1 = \{1, 2\}$, $\mathcal{A}_2 = \{3, 4\}$; $\mathcal{A}_1 = \{1, 3\}$, $\mathcal{A}_2 = \{2, 4\}$;

$\mathcal{A}_1 = \{1, 4\}$, $\mathcal{A}_2 = \{2, 3\}$; $\mathcal{A}_1 = \{1\}$, $\mathcal{A}_2 = \{2, 3, 4\}$;

$\mathcal{A}_1 = \{2\}$, $\mathcal{A}_2 = \{1, 3, 4\}$; $\mathcal{A}_1 = \{3\}$, $\mathcal{A}_2 = \{1, 2, 4\}$;

$\mathcal{A}_1 = \{4\}$, $\mathcal{A}_2 = \{1, 2, 3\}$

10. a. 2^n b. $\frac{n!}{n!(n-n)!}$

11. a. $A \subseteq B$ b. $A^0 \subseteq B$ or $A \cup B = B$ c. $A \subseteq B$

d. $A \cap B = \emptyset$ or $A \subseteq B^0$ e. $A = B = \emptyset$ f. $A^0 \subseteq B$ or $A \cup B = B$

g. $A = B$ h. $A = B$

36. Let $A = \{\{a, b, c\}\}$, $B = \{\{a\}\}$ and $C = \{\{a\}\}$. Then $A \cup B = A = A \cup C$ but $B = C$.

Answers to Selected Exercises

Answers to Selected Exercises

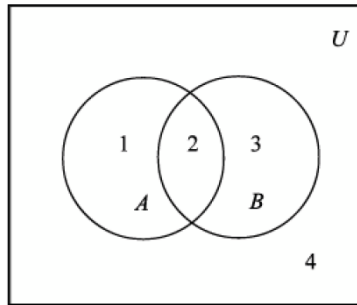
37. Let $A = \{a\}$, $B = \{a, b\}$ and $C = \{a, b, c\}$. Then $A \cap B = \{a\} = A \cap C$ but $A = B$.

38. Let $A = \{a, b, c\}$ and $B = \{a, b, c, d\}$. Then $A \cup B = \{a, b, c, d\}$ and $\{a, b, c, d\} \in P(A \cup B)$ but $\{a, b, c, d\} \notin P(A) \cup P(B)$.

40. Let $A = \{a, b, c\}$ and $B = \{a\}$. Then $A - B = \{b, c\}$ and $\emptyset \in P(A - B)$ but $\emptyset \notin P(A) - P(B)$.

41. $(A \cap B^0) \cup (A^0 \cap B) = (A \cup B) \cap (A^0 \cup B^0)$

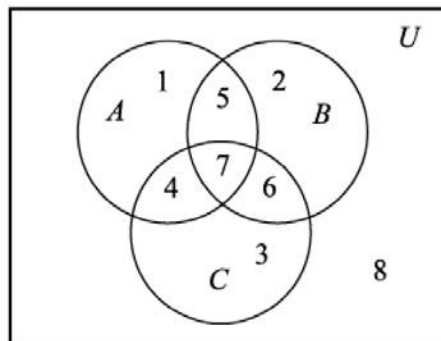
42. a.



- $A \cup B$: Regions 1,2,3
- $A \cap B$: Region 2
- $(A \cup B) - (A \cap B)$: Regions 1,3
- $A + B$: Regions 1,3
- $A - B$: Region 1
- $B - A$: Region 3
- $(A - B) \cup (B - A)$: Regions 1,3

Each of $A + B$ and $(A - B) \cup (B - A)$ consists of Regions 1,3.

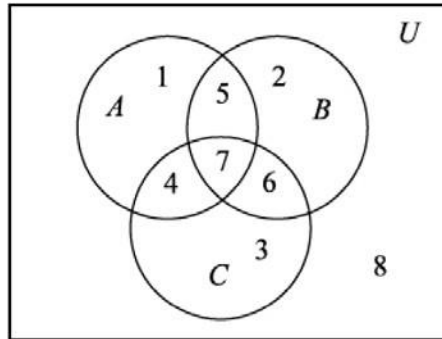
b.



- A : Regions 1,4,5,7
- $A + B$: Regions 1,2,4,6
- $A + C$: Regions 2,3,4,5
- $A \cup B \cup C$: Regions 3,4,6,7
- $A + (A + B)$: Regions 1,2,3,7
- $(A + B) + C$: Regions 1,2,3,7

Each of $A + (A + B)$ and $(A + B) + C$ consists of Regions 1,2,3,7.

c.



$A \cap C$: Regions 1,4,5,7

$A \cap B$: Regions 5,7

$A \cup B$: Regions 2,3,4,5

$A \cap C$: Regions 4,7

$A \cap (A \cup B)$: Regions 4,5

$(A \cap B) + (A \cap C)$: Regions 4,5

Each of $A \cap (A \cup B)$ and $(A \cap B) + (A \cap C)$ consists of Regions 4,5.

43. a. $A \cup B = (A \cup B) - (A \cap B) = A - B = A \cap B^c = \emptyset$

b. $A + \emptyset = (A \cup \emptyset) - (A \cap \emptyset) = A - \emptyset = A \cap \emptyset^c = A$

Section 1.2

1. False 2. False 3. False 4. False 5. False 6. True 7. True
 8. False 9. True

Exercises 1.2

1. a. $\{(0,0), (0,1), (1,0), (1,1)\}$ b. $\{(0,0), (0,1), (1,0), (1,1)\}$
 c. $\{(2,2), (4,2), (6,2), (8,2)\}$
 d. $\{(-1,1), (-1,5), (-1,9), (1,1), (1,5), (1,9)\}$
 e. $\{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$
2. a. Domain = E Codomain = Z Range = Z
 b. Domain = E Codomain = Z Range = E
 c. Domain = E Codomain = Z
 Range = $\{x \mid x \text{ is a nonnegative even integer}\} = (Z^+ \cap E) \cup \{0\}$
 d. Domain = E Codomain = Z Range = $Z - E$
3. a. $f(x) = \{1, 3, 5, \dots\} = Z^+ - E$ $f^{-1}(x) = \{-4, -3, -1, 1, 3, 4\}$
 b. $f(x) = \{1, 5, 9, \dots\}$ $f^{-1}(x) = Z$ c. $f(x) = \{0, 1, 4, \dots\}$ $f^{-1}(x) = \emptyset$

- d. $\varphi(\square) = \{0 \square 2 \square 14\}$ $\varphi^{-1}(\square) = \mathbb{Z}^+ \cup \{0 \square -1 \square -2\}$
4. a. The mapping φ is not onto, since there is no $\square \in \mathbb{Z}$ such that $\varphi(\square) = 1$. It is one-to-one.
- b. The mapping φ is not onto, since there is no $\square \in \mathbb{Z}$ such that $\varphi(\square) = 1$. It is one-to-one.
- c. The mapping φ is onto and one-to-one.
- d. The mapping φ is one-to-one. It is not onto, since there is no $\square \in \mathbb{Z}$ such that $\varphi(\square) = 2$.
- e. The mapping φ is not onto, since there is no $\square \in \mathbb{Z}$ such that $\varphi(\square) = -1$. It is not one-to-one, since $\varphi(1) = \varphi(-1)$ and $1 \neq -1$.
- f. We have $\varphi(3) = \varphi(2) = 0$ so φ is not one-to-one. Since $\varphi(\square)$ is always even, there is no $\square \in \mathbb{Z}$ such that $\varphi(\square) = 1$ and φ is not onto.
- g. The mapping φ is not onto, since there is no $\square \in \mathbb{Z}$ such that $\varphi(\square) = 3$. It is one-to-one.
- h. The mapping φ is not onto, since there is no $\square \in \mathbb{Z}$ such that $\varphi(\square) = 1$. Neither is φ one-to-one since $\varphi(0) = \varphi(1)$ and $0 \neq 1$.
- i. The mapping φ is onto. It is not one-to-one, since $\varphi(9) = \varphi(4)$ and $9 \neq 4$.
- j. The mapping φ is not onto, since there is no $\square \in \mathbb{Z}$ such that $\varphi(\square) = 4$. It is one-to-one.
5. a. The mapping is onto and one-to-one.
- b. The mapping is onto and one-to-one.
- c. The mapping is onto and one-to-one.
- d. The mapping is onto and one-to-one.
- e. The mapping is not onto, since there is no $\square \in \mathbb{R}$ such that $\varphi(\square) = -1$. It is not one-to-one, since $\varphi(1) = \varphi(-1)$ and $1 \neq -1$.
- f. The mapping is not onto, since there is no $\square \in \mathbb{R}$ such that $\varphi(\square) = 1$. It is not one-to-one, since $\varphi(0) = \varphi(1) = 0$ and $0 \neq 1$.
6. a. The mapping φ is onto and one-to-one.
- b. The mapping φ is one-to-one. Since there is no $\square \in \mathbb{E}$ such that $\varphi(\square) = 2$ the mapping is not onto.
7. a. The mapping φ is onto. The mapping ψ is not one-to-one, since $\psi(1) = \psi(-1)$ and $1 \neq -1$.
- b. The mapping φ is not onto, since there is no $\square \in \mathbb{Z}^+$ such that $\varphi(\square) = -1$. The mapping ψ is one-to-one.
- c. The mapping φ is onto and one-to-one.
- d. The mapping φ is onto. The mapping ψ is not one-to-one, since $\psi(1) = \psi(-1)$ and $1 \neq -1$.

8. a. The mapping f is not onto, since there is no $x \in \mathbb{Z}$ such that $|x + 4| = -1$.
The mapping f is not one-to-one, since $f(1) = f(-9) = 5$ but $1 \neq -9$.
- b. The mapping f is not onto, since there is no $x \in \mathbb{Z}^+$ such that $|x + 4| = 1$.
The mapping f is one-to-one.
9. a. The mapping f is not onto, since there is no $x \in \mathbb{Z}^+$ such that $2^x = 3$.
The mapping f is one-to-one.
- b. The mapping f is not onto, since there is no $x \in \mathbb{Z}^+ \cap \mathbb{E}$ such that $2^x = 6$.
The mapping f is one-to-one.
10. a. Let $f : \mathbb{E} \rightarrow \mathbb{E}$ where $f(x) = \begin{cases} x & \text{if } x \text{ is a multiple of 4} \\ 2x & \text{if } x \text{ is not a multiple of 4.} \end{cases}$
- b. Let $f : \mathbb{E} \rightarrow \mathbb{E}$ where $f(x) = \begin{cases} x & \text{if } x \text{ is a multiple of 4} \\ 2x & \text{if } x \text{ is not a multiple of 4.} \end{cases}$
- c. Let $f : \mathbb{E} \rightarrow \mathbb{E}$ where $f(x) = \begin{cases} x & \text{if } x \text{ is a multiple of 4} \\ 2x & \text{if } x \text{ is not a multiple of 4.} \end{cases}$
- d. Let $f : \mathbb{E} \rightarrow \mathbb{E}$ where $f(x) = x^2$.
11. a. For arbitrary $x \in \mathbb{Z}$, $2x$ is even and $f(2x) = \frac{2x}{2} = x$. Thus f is onto. But f is not one-to-one, since $f(1) = f(-1) = 0$.
- b. The mapping f is not onto, since there is no x in \mathbb{Z} such that $f(x) = 1$.
The mapping f is not one-to-one, since $f(0) = f(2) = 0$.
- c. For arbitrary x in \mathbb{Z} , $2x - 1$ is odd, and therefore

$$f(2x - 1) = \frac{(2x - 1) + 1}{2} = x$$

Thus f is onto. But f is not one-to-one, since $f(2) = 5$ and also $f(9) = 5$.

d. For arbitrary x in \mathbb{Z} , $2x$ is even and $f(2x) = \frac{2x}{2} = x$. Thus f is onto. But f is not one-to-one, since $f(4) = 2$ and $f(7) = 2$.

e. The mapping f is not onto, because there is no x in \mathbb{Z} such that $f(x) = 4$.

Since $f(2) = 6$ and $f(3) = 6$, then f is not one-to-one.

f. The mapping f is not onto, since there is no x in \mathbb{Z} such that $f(x) = 1$. Suppose that $f(x_1) = f(x_2)$. It can be seen from the definition of f that the image of an even integer is always an odd integer, and also that the image of an odd integer is always an even integer. Therefore, $f(x_1) = f(x_2)$ requires that either both x_1 and x_2 are even, or both x_1 and x_2 are odd. If both x_1 and x_2 are even,

$$f(x_1) = f(x_2) \Rightarrow 2x_1 - 1 = 2x_2 - 1 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$$

If both x_1 and x_2 are odd,

$$f(x_1) = f(x_2) \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$$

Hence, $f(x_1) = f(x_2)$ always implies $x_1 = x_2$ and f is one-to-one.

Answers to Selected Exercises

Answers to Selected Exercises

12. a. The mapping ϕ is not onto, because there is no $x \in \mathbb{R} - \{0\}$ such that $\phi(x) = 1$. If $x_1, x_2 \in \mathbb{R} - \{0\}$

$$\begin{aligned} \phi(x_1) &= \phi(x_2) & \frac{x_1 - 1}{x_1} &= \frac{x_2 - 1}{x_2} \\ \Rightarrow & & & \\ & \Rightarrow x_2(x_1 - 1) &= x_1(x_2 - 1) \\ & \Rightarrow x_2x_1 - x_2 &= x_1x_2 - x_1 \\ & \Rightarrow -x_2 &= -x_1 \\ & \Rightarrow x_2 &= x_1 \end{aligned}$$

Thus ϕ is one-to-one.

- b. The mapping ϕ is not onto, because there is no $x \in \mathbb{R} - \{0\}$ such that $\phi(x) = 2$. If $x_1, x_2 \in \mathbb{R} - \{0\}$

$$\begin{aligned} \phi(x_1) &= \phi(x_2) & \frac{2x_1 - 1}{x_1} &= \frac{2x_2 - 1}{x_2} \\ \Rightarrow & & & \\ & \Rightarrow 2 - \frac{1}{x_1} &= 2 - \frac{1}{x_2} \\ & \Rightarrow -\frac{1}{x_1} &= -\frac{1}{x_2} \\ & \Rightarrow x_1 &= x_2 \end{aligned}$$

Thus ϕ is one-to-one.

- c. The mapping ϕ is not onto, since there is no $x \in \mathbb{R} - \{0\}$ such that $\phi(x) = 0$. It is not one-to-one, since $\phi(2) = \frac{1}{5}$ and $\phi\left(\frac{1}{2}\right) = \frac{2}{5}$.
- d. The mapping ϕ is not onto, since there is no $x \in \mathbb{R} - \{0\}$ such that $\phi(x) = 1$. Since $\phi(1) = \phi(3) = \frac{1}{2}$ then ϕ is not one-to-one.

13. a. The mapping ϕ is onto, since for every $(z_1, z_2) \in \mathbb{Z} \times \mathbb{Z}$ there exists an $(x, y) \in \mathbb{Z} \times \mathbb{Z}$ such that $\phi(x, y) = (z_1, z_2)$. To show that ϕ is one-to-one, we assume $(x_1, y_1) \in \mathbb{Z} \times \mathbb{Z}$ and $(x_2, y_2) \in \mathbb{Z} \times \mathbb{Z}$ and

$$\phi(x_1, y_1) = \phi(x_2, y_2)$$

or

$$\begin{pmatrix} x_1 + y_1 \\ x_1 - y_1 \end{pmatrix} = \begin{pmatrix} x_2 + y_2 \\ x_2 - y_2 \end{pmatrix}$$

This means $x_1 + y_1 = x_2 + y_2$ and $x_1 - y_1 = x_2 - y_2$ and

$$\begin{pmatrix} x_1 + y_1 \\ x_1 - y_1 \end{pmatrix} = \begin{pmatrix} x_2 + y_2 \\ x_2 - y_2 \end{pmatrix}$$

b. For any $n \in \mathbb{Z}$ ($n \neq 0$) $\in \mathbb{Z}$ and $n(2n-3) = 2n^2 - 3n$. Thus f is onto. Since $f(2) = 3$
 $f(4) = 5$ f is not one-to-one.

Answers to Selected Exercises

Answers to Selected Exercises

- c. Since for every $x \in \mathbb{Z}$ there exists an $(a, b) \in \mathbb{Z} \times \mathbb{Z}$ such that $f(a, b) = x$ the mapping f is onto. However, f is not one-to-one, since $f(1, 0) = f(1, 1)$ and $f(0, 0) = f(1, 1)$.
- d. The mapping f is one-to-one since $f(a_1) = f(a_2) \Rightarrow (a_1, 1) = (a_2, 1) \Rightarrow a_1 = a_2$. Since there is no $(a, b) \in \mathbb{Z} \times \mathbb{Z}$ such that $f(a, b) = (0, 0)$ then f is not onto.
- e. The mapping f is not onto, since there is no $(a, b) \in \mathbb{Z} \times \mathbb{Z}$ such that $f(a, b) = 2$. The mapping f is not one-to-one, since $f(2, 0) = f(2, 1) = 4$ and $f(0, 0) = f(2, 1)$.
- f. The mapping f is not onto, since there is no $(a, b) \in \mathbb{Z} \times \mathbb{Z}$ such that $f(a, b) = 3$. The mapping is not one-to-one, since $f(1, 0) = f(-1, 0) = 1$ and $f(1, 0) = f(-1, 0)$.
- g. The mapping f is not onto, since there is no (a, b) in $\mathbb{Z}^+ \times \mathbb{Z}^+$ such that $f(a, b) = 0$. The mapping f is not one-to-one, since $f(2, 1) = f(4, 2) = 2$.
- h. The mapping f is not onto, since there is no (a, b) in $\mathbb{R} \times \mathbb{R}$ such that $f(a, b) = 2^{a+b} = 0$. The mapping f is not one-to-one, since $f(1, 0) = f(0, 1) = 2^1$.
14. a. The mapping f is obviously onto.
 b. The mapping f is not one-to-one, since $f(0) = f(2) = 1$.
 c. Let both a_1 and a_2 be even. Then $a_1 + a_2$ is even and $f(a_1 + a_2) = 1 = 1 \cdot 1 = f(a_1) f(a_2)$. Let both a_1 and a_2 be odd. Then $a_1 + a_2$ is even and $f(a_1 + a_2) = 1 = (-1)(-1) = f(a_1) f(a_2)$. Finally, if one of a_1 or a_2 is even and the other is odd, then $a_1 + a_2$ is odd and $f(a_1 + a_2) = -1 = (1)(-1) = f(a_1) f(a_2)$. Thus it is true that $f(a_1 + a_2) = f(a_1) f(a_2)$.
 d. Let both a_1 and a_2 be odd. Then $a_1 a_2$ is odd and $f(a_1 a_2) = -1 = (-1)(-1) = f(a_1) f(a_2)$.
15. a. The mapping f is not onto, since there is no $x \in \mathbb{Z}$ such that $f(x) = 9 \in \mathbb{Z}$. It is not one-to-one, since $f(-2) = f(2)$ and $-2 \neq 2$.
 b. $f^{-1}(\{1\}) = f^{-1}(\{1, 4\}) = \{-2, 1, 2\} = \mathbb{Z}$.
 c. With $A = \{4, 9\}$ $f^{-1}(A) = \{-2, 2\}$ and $f^{-1}(f^{-1}(A))^c = f^{-1}(\{-2, 2\}) = \{4\}$.
16. a. $f(A) = \{2, 4\}$ $f^{-1}(f(A)) = \{2, 3, 4, 7\}$
 b. $f^{-1}(A) = \{9, 6, 11\}$ $f^{-1}(f^{-1}(A))^c = \mathbb{Z}$
17. a. $f(A) = \{-1, 2, 3\}$ $f^{-1}(f(A)) = \mathbb{Z}$
 b. $f^{-1}(A) = \{0\}$ $f^{-1}(f^{-1}(A))^c = \{-1\}$
18. a. $(f \circ g)(x) = \begin{cases} 2x & \text{if } x \text{ is even} \\ 2(2x - 1) & \text{if } x \text{ is odd} \end{cases}$ b. $(f \circ g)(x) = 2x^3$

$$\begin{aligned}
 & \left\{ \begin{array}{l} \frac{\square + |\square|}{2} \text{ if } \square \text{ is even} \\ |\square| - \square \text{ if } \square \text{ is odd} \end{array} \right. \\
 \text{c. } (\square \circ \square)(\square) &= \frac{\square + |\square|}{2} & \text{d. } (\square \circ \square)(\square) &= \square \\
 & \left\{ \begin{array}{l} \frac{\square + |\square|}{2} \\ |\square| - \square \end{array} \right. \\
 \text{e. } (\square \circ \square)(\square) &= (\square - |\square|)^2 \\
 19. \text{ a. } (\square \circ \square)(\square) &= 2\square & \text{b. } (\square \circ \square)(\square) &= 8\square^3 & \text{c. } (\square \circ \square) &= \frac{\square + |\square|}{2} \\
 (\square) &= & & & & \\
 \text{d. } (\square \circ \square)(\square) &= \begin{cases} \frac{\square}{2} & \text{if } \square = 4k \text{ for } k \text{ an integer} \\ 1 & \text{otherwise} \end{cases} & \text{e. } (\square \circ \square)(\square) &= 0
 \end{aligned}$$

$$\begin{aligned}
 20. \square^\square & \quad 21. \square! & 22. \square(\square - 1)(\square - 2)\cdots(\square - \square + 1) &= \frac{\square!}{(\square - \square)!}
 \end{aligned}$$

28. Let $f : X \rightarrow Y$ where X and Y are nonempty.

Assume first that $f^{-1}(y) \neq \emptyset$ for every subset A of Y . For an arbitrary element x of X let $A = \{f(x)\}$. The equality $f^{-1}(A) \neq \emptyset$ implies that $f^{-1}(\{f(x)\})$ is not empty. For any $y \in f^{-1}(\{f(x)\})$ we have $f(y) = f(x)$. Thus f is onto. Assume now that f is onto. For an arbitrary $y \in f^{-1}(y) \neq \emptyset$ we have

$$\begin{aligned}
 y \in f^{-1}(y) \neq \emptyset & \Rightarrow y = f(x) \text{ for some } x \in X \\
 & \Rightarrow y = f(x) \text{ for some } x \in X \\
 & \Rightarrow y \in f(X)
 \end{aligned}$$

Thus $f^{-1}(y) \neq \emptyset$ for every $y \in f(X)$. For an arbitrary $x \in X$ there exists $y \in f(X)$ such that $f(x) = y$ since f is onto. Now

$$\begin{aligned}
 f(x) = y \in f(X) & \Rightarrow x \in f^{-1}(y) \\
 & \Rightarrow x \in f^{-1}(f(x)) \\
 & \Rightarrow x \in f^{-1}(f(x))
 \end{aligned}$$

$$\Rightarrow \emptyset \in \mathcal{P}(\mathcal{P}^{-1}(\emptyset))$$

Thus $\emptyset \subseteq \mathcal{P}^{-1}(\emptyset)$ and we have proved that $\mathcal{P}^{-1}(\emptyset) = \emptyset$ for an arbitrary subset \emptyset of \mathcal{P} . Answers to Selected Exercises

Section 1.3

1. False 2. True 3. False 4. False 5. False 6. False

Exercises 1.3

1. a. The mapping $\phi \circ \psi$ is not onto, since there is no $\phi \in Z$ such that $(\phi \circ \psi)(\psi) = 1$. It is not one-to-one, since $(\phi \circ \psi)(1) = (\phi \circ \psi)(-1)$ and $1 \neq -1$.

- b. The mapping $\square \circ \square$ is not onto, since there is no $\square \in \mathbb{Z}$ such that $(\square \circ \square)(\square) = 0$. The mapping $\square \circ \square$ is one-to-one.
 - c. The mapping $\square \circ \square$ is not onto, since there is no $\square \in \mathbb{Z}$ such that $(\square \circ \square)(\square) = 1$. The mapping $\square \circ \square$ is one-to-one.
 - d. The mapping $\square \circ \square$ is not onto, since there is no $\square \in \mathbb{Z}$ such that $(\square \circ \square)(\square) = 1$. The mapping $\square \circ \square$ is one-to-one.
 - e. The mapping $\square \circ \square$ is not onto, since there is no $\square \in \mathbb{Z}$ such that $(\square \circ \square)(\square) = 1$. It is not one-to-one, since $(\square \circ \square)(-2) = (\square \circ \square)(0) = 0$ and $-2 \neq 0$.
 - f. The mapping $\square \circ \square$ is both onto and one-to-one.
 - g. The mapping $\square \circ \square$ is not onto, since there is no $\square \in \mathbb{Z}$ such that $(\square \circ \square)(\square) = -1$. It is not one-to-one, since $(\square \circ \square)(1) = (\square \circ \square)(2) = 1$ and $1 \neq 2$.
- 2.
- a. The mapping $\square \circ \square$ is not onto, since there is no $\square \in \mathbb{Z}$ such that $(\square \circ \square)(\square) = -1$. It is not one-to-one since $(\square \circ \square)(0) = (\square \circ \square)(2) = 0$ and $0 \neq 2$.
 - b. The mapping $\square \circ \square$ is not onto, since there is no $\square \in \mathbb{Z}$ such that $(\square \circ \square)(\square) = 1$. The mapping $\square \circ \square$ is one-to-one.
 - c. The mapping $\square \circ \square$ is not onto, since there is no $\square \in \mathbb{Z}$ such that $(\square \circ \square)(\square) = 1$. The mapping $\square \circ \square$ is one-to-one.
 - d. The mapping $\square \circ \square$ is not onto, since there is no $\square \in \mathbb{Z}$ such that $(\square \circ \square)(\square) = 1$. The mapping $\square \circ \square$ is one-to-one.
 - e. The mapping $\square \circ \square$ is not onto, since there is no $\square \in \mathbb{Z}$ such that $(\square \circ \square)(\square) = -1$. It is not one-to-one, since $(\square \circ \square)(-1) = (\square \circ \square)(-2) = -1$ and $-1 \neq -2$.
 - f. The mapping $\square \circ \square$ is not onto, since there is no $\square \in \mathbb{Z}$ such that $(\square \circ \square)(\square) = 0$. The mapping $\square \circ \square$ is not one-to-one, since $(\square \circ \square)(1) = (\square \circ \square)(4) = 1$ and $1 \neq 4$.
 - g. The mapping $\square \circ \square$ is not onto, since there is no $\square \in \mathbb{Z}$ such that $(\square \circ \square)(\square) = 1$. It is not one-to-one, since $(\square \circ \square)(0) = (\square \circ \square)(1) = 0$ and $0 \neq 1$.
3. $\square(\square) = \square^2$, $\square(\square) = -\square$
4. Let $\square = \{0, 1\}$, $\square = \{-2, 1, 2\}$, $\square = \{1, 4\}$. Let $\square : \square \rightarrow \square$ be defined by $\square(\square) = \square + 1$ and $\square : \square \rightarrow \square$ be defined by $\square(\square) = \square^2$. Then \square is not onto, since $-2 \notin \square(\square)$. The mapping \square is onto. Also $\square \circ \square$ is onto, since $(\square \circ \square)(0) = \square(1) = 1$ and $(\square \circ \square)(1) = \square(2) = 4$.
5. Let \square and \square be defined as in Problem 1f. Then \square is not one-to-one, \square is one-to-one, and $\square \circ \square$ is one-to-one.
6. a. Let $\square : \mathbb{Z} \rightarrow \mathbb{Z}$ and $\square : \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by
- $$\square(\square) = \begin{cases} \frac{\square}{2} & \text{if } \square \text{ is even} \\ \square & \text{if } \square \text{ is odd.} \end{cases}$$

The mapping \square is one-to-one and the mapping \square is onto, but the composition $\square \circ \square = \square$ is not one-to-one, since $(\square \circ \square)(1) = (\square \circ \square)(2) = 1$ and $1 \neq 2$.

- b. Let $f : Z \rightarrow Z$ and $g : Z \rightarrow Z$ be defined by $f(x) = x^3$ and $g(x) = 2x$. The mapping f is one-to-one, the mapping g is onto, but the mapping $f \circ g$ given by $(f \circ g)(x) = 2x^3$ is not onto, since there is no $x \in Z$ such that $(f \circ g)(x) = 2$.

7. a. Let $f : Z \rightarrow Z$ and $g : Z \rightarrow Z$ be defined by

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ x & \text{if } x \text{ is odd} \end{cases} \quad g(x) = 2x$$

The mapping f is onto and the mapping g is one-to-one, but the composition $f \circ g = f$ is not one-to-one, since $(f \circ g)(1) = (f \circ g)(2) = 1$.

- b. Let $f : Z \rightarrow Z$ and $g : Z \rightarrow Z$ be defined by $f(x) = x$ and $g(x) = x^3$. The mapping f is onto, the mapping g is one-to-one, but the mapping $f \circ g$ given by $(f \circ g)(x) = x^3$ is not onto, since there is no $x \in Z$ such that $(f \circ g)(x) = 2$.

9. a. Let $f(x) = 2x$, $g(x) = x^2$ and $h(x) = |x|$ for all $x \in Z$.
 b. Let $f(x) = x^2$, $g(x) = x$ and $h(x) = -x$ for all $x \in Z$.

12. To prove that f is one-to-one, suppose $f(x_1) = f(x_2)$ for x_1 and x_2 in \mathbb{R} . Since $f \circ g$ is onto, there exist y_1 and y_2 in \mathbb{R} such that

$$x_1 = (f \circ g)(y_1) \quad \text{and} \quad x_2 = (f \circ g)(y_2)$$

Then $f((f \circ g)(y_1)) = f((f \circ g)(y_2))$ since $f(x_1) = f(x_2)$ or

$$(f \circ f)((f \circ g)(y_1)) = (f \circ f)((f \circ g)(y_2))$$

This implies that

$$f(y_1) = f(y_2)$$

since $f \circ f$ is one-to-one. Since f is a mapping, then

$$f(y_1) = f(y_2)$$

Thus

$$(f \circ g)(y_1) = (f \circ g)(y_2)$$

and

$$y_1 = y_2$$

Therefore f is one-to-one.

To show that f is onto, let $x \in \mathbb{R}$. Then $x \in \mathbb{R}$ and therefore $x = (f \circ g)(y)$ for some $y \in \mathbb{R}$ since $f \circ g$ is onto. It follows then that

$$\begin{aligned} (\square \circ \square)(\square) &= (\square \circ \square)(\square) \\ &(\square) \square \end{aligned}$$

Answers to Selected Exercises

Answers to Selected Exercises

Since $\square \circ \square$ is one-to-one, we have

$$\square = \square(\square)\square$$

and \square is onto.

Section 1.4

1. False 2. True 3. True 4. False 5. True 6. True 7. True
 8. True 9. True

Exercises 1.4

1.
 - a. The set \square is not closed, since $-1 \in \square$ and $-1 * -1 = 1 \notin \square$
 - b. The set \square is not closed, since $1 \in \square$ and $2 \in \square$ but $1 * 2 = 1 - 2 = -1 \notin \square$
 - c. The set \square is closed.
 - d. The set \square is closed.
 - e. The set \square is not closed, since $1 \in \square$ and $1 * 1 = 0 \notin \square$
 - f. The set \square is closed.
 - g. The set \square is closed.
 - h. The set \square is closed.
2.
 - a. Not commutative, Not associative, No identity element
 - b. Not commutative, Associative, No identity element
 - c. Not commutative, Not associative, No identity element
 - d. Commutative, Not associative, No identity element
 - e. Commutative, Associative, No identity element
 - f. Not commutative, Not associative, No identity element
 - g. Commutative, Associative, 0 is an identity element. 0 is the only invertible element and its inverse is 0
 - h. Commutative, Associative, -3 is an identity element. $-\square - 6$ is the inverse of \square
 - i. Not commutative, Not associative, No identity element
 - j. Commutative, Not associative, No identity element
 - k. Not commutative, Not associative, No identity element
 - l. Commutative, Not associative, No identity element
 - m. Not commutative, Not associative, No identity element
 - n. Commutative, Not associative, No identity element
3.
 - a. The binary operation $*$ is not commutative, since $\square * \square = \square * \square$

- b. There is no identity element.
4. a. The operation $*$ is commutative, since $a * b = b * a$ for all a, b in \mathbb{Q} .
 b. 1 is an identity element.
 c. The elements a and a^{-1} are inverses of each other and 1 is its own inverse.
5. a. The binary operation $*$ is not commutative, since $a * b = ab$ and $b * a = ba$.
 b. 1 is an identity element.
 c. The elements a and a^{-1} are inverses of each other and 1 is its own inverse.
6. a. The binary operation $*$ is commutative.
 b. 1 is an identity element.
 c. a is the only invertible element and its inverse is a^{-1} .
7. The set of nonzero integers is not closed with respect to division, since 1 and 2 are nonzero integers but $1 \div 2$ is not a nonzero integer.
8. The set of odd integers is not closed with respect to addition, since 1 is an odd integer but $1 + 1$ is not an odd integer.
10. a. The set of nonzero integers is not closed with respect to addition defined on \mathbb{Z} , since 1 and -1 are nonzero integers but $1 + (-1)$ is not a nonzero integer.
 b. The set of nonzero integers is closed with respect to multiplication defined on \mathbb{Z} .
11. a. The set \mathbb{Q} is not closed with respect to addition defined on \mathbb{Z} , since $1 \in \mathbb{Q}$ but $1 + 8 = 9 \notin \mathbb{Q}$.
 b. The set \mathbb{Q} is closed with respect to multiplication defined on \mathbb{Z} .
12. a. The set $\mathbb{Q} - \{0\}$ is closed with respect to multiplication defined on \mathbb{R} .
 b. The set $\mathbb{Q} - \{0\}$ is closed with respect to division defined on $\mathbb{R} - \{0\}$.

Section 1.5

1. True 2. False 3. False

Exercises 1.5

1. a. A right inverse does not exist, since f is not onto.
 b. A right inverse does not exist, since f is not onto.
 c. A right inverse $g : \mathbb{Z} \rightarrow \mathbb{Z}$ is defined by $g(n) = n - 2$.
 d. A right inverse $g : \mathbb{Z} \rightarrow \mathbb{Z}$ is defined by $g(n) = 1 - n$.
 e. A right inverse does not exist, since f is not onto.
 f. A right inverse does not exist, since f is not onto.

- g. A right inverse does not exist, since \square is not onto. h. A right inverse does not exist, since \square is not onto. i. A right inverse does not exist, since \square is not onto. j. A right inverse does not exist, since \square is not onto.

k. A right inverse $\square : Z \rightarrow Z$ is defined by $\square(\square) = \begin{cases} \square & \text{if } \square \text{ is even} \\ 2\square + 1 & \text{if } \square \text{ is odd.} \end{cases}$

- l. A right inverse does not exist, since \square is not onto.

m. A right inverse $\square : Z \rightarrow Z$ is defined by $\square(\square) = \begin{cases} 2\square & \text{if } \square \text{ is even} \\ \square - 2 & \text{if } \square \text{ is odd.} \\ 2\square - 1 & \text{if } \square \text{ is even} \end{cases}$

n. A right inverse $\square : Z \rightarrow Z$ is defined by $\square(\square) =$

2. a. A left inverse $\square : Z \rightarrow Z$ is defined by $\square(\square) = \begin{cases} \square - 1 & \text{if } \square \text{ is odd.} \\ \frac{\square}{2} & \text{if } \square \text{ is even} \\ 1 & \text{if } \square \text{ is odd.} \end{cases}$

b. A left inverse $\square : Z \rightarrow Z$ is defined by $\square(\square) = \begin{cases} \frac{\square}{3} & \text{if } \square \text{ is a multiple of 3} \\ 0 & \text{if } \square \text{ is not a multiple of 3.} \end{cases}$

c. A left inverse $\square : Z \rightarrow Z$ is defined by $\square(\square) = \square - 2\square$

d. A left inverse $\square : Z \rightarrow Z$ is defined by $\square(\square) = 1 - \square\square$

e. A left inverse $\square : Z \rightarrow Z$ is defined by $\square(\square) = \begin{cases} \square & \text{if } \square = \square^3 \text{ for some } \square \in Z \\ 0 & \text{if } \square = \square^3 \text{ for some } \square \in Z \end{cases}$

- f. A left inverse does not exist, since \square is not one-to-one.

g. A left inverse $\square : Z \rightarrow Z$ is defined by $\square(\square) = \begin{cases} \square & \text{if } \square \text{ is even} \\ \frac{\square + 1}{2} & \text{if } \square \text{ is odd.} \end{cases}$

- h. A left inverse does not exist, since \square is not one-to-one. i. A left inverse does not exist, since \square is not one-to-one. j. A left inverse does not exist, since \square is not one-to-one. k. A left inverse does not exist, since \square is not one-to-one.

l. A left inverse $\square : Z \rightarrow Z$ is defined by: $\square(\square) = \begin{cases} \square + 1 & \text{if } \square \text{ is odd} \\ \frac{\square}{2} & \text{if } \square \text{ is even.} \end{cases}$

- m. A left inverse does not exist, since \square is not one-to-one. n. A left inverse does not exist, since \square is not one-to-one.

3. $\square!$

4. Let $f : X \rightarrow Y$ where X is nonempty.

f has a left inverse $\Leftrightarrow f$ is one-to-one, by Lemma 1.24

$\Leftrightarrow f^{-1}(f(x)) = \{x\}$ for every subset A of X by Exercise 27 of Section 1.2.

5. Let $f : X \rightarrow Y$ where X is nonempty.

f has a right inverse $\Leftrightarrow f$ is onto, by Lemma 1.25

$\Leftrightarrow \bigcup_{x \in X} f^{-1}(y) \neq \emptyset$ for every subset A of Y by Exercise 28 of Section 1.2.

Section 1.6

1. True 2. False 3. False 4. False 5. False 6. False 7. True

8. False 9. False 10. False 11. True 12. True

Exercises 1.6

1. a. $f = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$ b. $f = \begin{bmatrix} -1 & -2 \\ 1 & 2 \end{bmatrix}$ c. $f = \begin{bmatrix} & & & \\ & & & \\ 1 & -1 & 1 & -1 \end{bmatrix}$

d. $f = \begin{bmatrix} & & & \\ 0 & 1 & 1 & 1 \\ & & & \\ & & & \end{bmatrix}$ e. $f = \begin{bmatrix} -1 & -2 \\ 1 & 2 \\ 2 & 0 & 0 \\ 3 & 4 & 0 \end{bmatrix}$ f. $f = \begin{bmatrix} & & & \\ & & & \\ -1 & 1 & -1 & 1 \\ & & & \\ & & & \\ & & & \\ 0 & 1 & 0 \\ & & & \\ & & & \end{bmatrix}$

$\begin{bmatrix} & & & \\ & & & \\ & & & \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 4 & 5 & 6 \\ 5 & 6 & 7 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} & & & \\ & & & \\ & & & \\ 3 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} & & & \\ & & & \\ & & & \\ 1 & 9 \end{bmatrix}$

2. a. $\begin{bmatrix} & - \\ 8 & -8 & 6 \end{bmatrix}$ b. $\begin{bmatrix} & \\ -3 & 2 \end{bmatrix}$ c. Not possible d. Not possible

Answers to Selected Exercises

3. a. $\begin{bmatrix} & \\ & \\ 8 & -1 \end{bmatrix}$ b. $\begin{bmatrix} & & \\ -10 & 2 & 1 \\ 14 & 6 & 21 \\ & - & - \\ 6 & -1 & -2 \end{bmatrix}$ c. Not possible d. $\begin{bmatrix} & \\ 7 & -11 \\ 12 & 6 \\ -2 & 20 \end{bmatrix}$

Answers to Selected Exercises

$$\begin{array}{l}
 \begin{bmatrix} 4 & 2 \\ 3 & 7 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 \\ -4 & 10 \end{bmatrix} \\
 \text{e. } \begin{bmatrix} 3 & 7 \\ 3 & 7 \end{bmatrix} \quad \text{f. } \begin{bmatrix} 1 & 3 \\ -4 & 10 \end{bmatrix} \quad \text{g. Not possible} \quad \text{h. Not possible} \\
 \text{i. } [4] \quad \text{j. } \begin{bmatrix} -12 & 8 & -4 \\ -15 & 10 & -5 \\ 18 & -12 & 6 \end{bmatrix}
 \end{array}$$

$$\begin{aligned}
 4. \sum_{x=1}^3 (x+1)(2x-x) \\
 &= (1+1)(2-1) + (2+1)(4-2) + (3+1)(6-3) \\
 &= 12 - 6 - 3 + 28
 \end{aligned}$$

$$6. \begin{bmatrix} 1 & 6 & -3 & 2 \\ 4 & -7 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

$$\begin{array}{l}
 7. \quad \text{a. } x \quad \text{b. } x(x-1) \quad \text{c. } 0 \\
 \text{d. } x^2 \text{ if } 1 \leq x \leq 1 \leq x \leq x; 0 \text{ if } x < 1 \text{ or } x > x
 \end{array}$$

8.

.	x	x	x	x
x	x	x	x	x
x	x	x	x	x
x	x	x	x	x
x	x	x	x	x

$$\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$9. \text{ (Answer not unique) } x = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} \quad y = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$$

10. A trivial example is with $x = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ and y an arbitrary 2×2 matrix. Another

example is provided by $x = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $y = \begin{bmatrix} 3 & 2 \\ 6 & -6 \end{bmatrix}$

$$11. \text{ (Answer not unique) } x = \begin{bmatrix} 1 & 2 \\ 6 & -6 \end{bmatrix}$$

$$\int_{-3}^3 x^2 dx = \int_{-3}^3 x^3 dx$$

Answers to Selected Exercises

Answers to Selected Exercises

$$12. \begin{pmatrix} 10 & 1 \\ 2 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 2 & 6 \\ -4 & 9 \end{pmatrix} \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} (\square - \square)(\square + \square) = \square^2 - \square^2$$

$$13. (\square + \square)^2 = \begin{pmatrix} 22 & 5 \\ 30 & 7 \end{pmatrix} \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} + 2 \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} + \square^2 = \begin{pmatrix} 30 & -1 \\ 36 & -1 \end{pmatrix} \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} (\square + \square) = \square^2 + 2 \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} + \square^2$$

$$14. \square = \square^{-1} \square$$

$$15. \square = \square^{-1} \square \square^{-1}$$

$$22. \text{ b. For each } \square \text{ in } \begin{pmatrix} \square & \square \\ 0 & 0 \end{pmatrix} \text{ of the form } \begin{pmatrix} \square & \square \\ 0 & 0 \end{pmatrix} \text{ then } \square = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \text{ For each } \square \text{ in } \begin{pmatrix} \square & \square \\ 0 & 0 \end{pmatrix}$$

$$\text{of the form } \begin{pmatrix} \square & \square \\ 0 & 0 \end{pmatrix} \text{ then } \square = \begin{pmatrix} \square & \square \\ 0 & 0 \end{pmatrix}$$

$$25. \text{ Let } \square = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \text{ and } \square = \begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 0 & 7 \end{pmatrix} \text{ Then the product } \square \square = \begin{pmatrix} 2 & 7 \\ 2 & 7 \end{pmatrix} \text{ is not}$$

diagonal even though \square is diagonal.

$$26. \text{ Let } \square = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \text{ and } \square = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \text{ Then the product } \square \square = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} \text{ is}$$

diagonal but neither \square nor \square is diagonal.

$$27. \text{ c. Let } \square = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \text{ and } \square = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \text{ Then the product } \square \square = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

is upper triangular but neither \square nor \square is upper triangular.

30. (Answer not unique) $\square = \lfloor \begin{matrix} \square \\ 0 \end{matrix} \rfloor$ $\square = \lfloor \begin{matrix} \square \\ 0 \end{matrix} \rfloor$ $\square = \lfloor \begin{matrix} \square \\ 4 \end{matrix} \rfloor$ $\square = \lfloor \begin{matrix} \square \\ 5 \end{matrix} \rfloor$ $\square = \lfloor \begin{matrix} \square \\ 6 \end{matrix} \rfloor$ $\square = \lfloor \begin{matrix} \square \\ 7 \end{matrix} \rfloor$

Section 1.7

Answers to Selected Exercises 4. False 5. True Answers to Selected Exercises 6. False

Exercises 1.7

1.
 - a. This is a mapping, since for every $\square \in \square$ there is a unique $\square \in \square$ such that $(\square \square)$ is an element of the relation.
 - b. This is a mapping, since for every $\square \in \square$ there is $1 \in \square$ such that $(\square 1)$ is an element of the relation.

- c. This is not a mapping, since the element 1 is related to three different values; $1 \square 1 \square 1 \square 3 \square$ and $1 \square 5 \square$
- d. This is a mapping, since for every $\square \in \square$ there is a unique $\square \in \square$ such that $(\square \square \square)$ is an element of the relation.
- e. This is a mapping, since for every $\square \in \square$ there is a unique $\square \in \square$ such that $(\square \square \square)$ is an element of the relation.
- f. This is not a mapping, since the element 5 is related to three different values: $5 \square 1 \square 5 \square 3 \square$ and $5 \square 5 \square$
2. a. The relation \square is not reflexive, since $2 \not\sqsubset 2 \square$ It is not symmetric, since $4R2$ but $2 \not\sqsubset 4 \square$ It is not transitive, since $4R2$ and $2R1$ but $4 \not\sqsubset 1 \square$
- b. The relation \square is not reflexive, since $2 \not\sqsubset 2 \square$ It is symmetric, since $\square = -\square \Rightarrow \square = -\square \square$ It is not transitive, since $2R(-2)$ and $(-2)R2$, but $2 \not\sqsubset 2 \square$
- c. The relation \square is reflexive and transitive, but not symmetric, since for arbitrary $\square \square \square \square$ and \square in Z we have:
- (1) $\square = \square \cdot 1$ with $1 \in Z$
 - (2) $6 = 3(2)$ with $2 \in Z$ but $3 = 6 \square$ where $\square \in Z$
 - (3) $\square = \square \square_1$ for some $\square_1 \in Z$ and $\square = \square \square_2$ for some $\square_2 \in Z$ imply $\square = \square \square_2 = \square (\square_1 \square_2)$ with $\square_1 \square_2 \in Z \square$
- d. The relation \square is not reflexive, since $1 \not\sqsubset 1 \square$ It is not symmetric, since $1R2 \square$ but $2 \not\sqsubset 1 \square$ It is transitive, since $\square \square \square$ and $\square \square \square \Rightarrow \square \square \square$ for all $\square \square \square \square$ and $\square \in Z \square$
- e. The relation \square is reflexive, since $\square \geq \square$ for all $\square \in Z \square$ It is not symmetric, since $5 \square 3$ but $3 \not\geq 5 \square$ It is transitive, since $\square \geq \square$ and $\square \geq \square$ imply $\square \geq \square$ for all $\square \square \square \square$ in $Z \square$
- f. The relation \square is not reflexive, since $(-1) \not\sqsubset (-1) \square$ It is not symmetric, since $1R(-1)$ but $(-1) \not\sqsubset 1 \square$ It is transitive, since $\square = |\square|$ and $\square = |\square|$ implies $\square = |\square| = ||\square|| = |\square|$ for all $\square \square \square \square$ and $\square \in Z \square$
- g. The relation \square is not reflexive, since $(-6) \not\sqsubset (-6) \square$ It is not symmetric, since $3R5$ but $5 \not\sqsubset 3 \square$ It is not transitive, since $4R3$ and $3R2$, but $4 \not\sqsubset 2 \square$
- h. The relation \square is reflexive, since $\square^2 \geq 0$ for all \square in $Z \square$ It is also symmetric, since $\square \square \geq 0$ implies that $\square \square \geq 0 \square$ It is not transitive, since $(-2) \square 0$ and $0 \square 4$ but $(-2) \not\sqsubset 4 \square$
- i. The relation \square is not reflexive, since $2 \not\sqsubset 2 \square$ It is symmetric, since $\square \square \leq 0$ implies $\square \square \leq 0$ for all $\square \square \square \square \in Z \square$ It is not transitive, since $-1 \square 2$ and $2 \square (-3)$ but $(-1) \not\sqsubset (-3) \square$
- j. The relation \square is not reflexive, since $|\square - \square| = 0 = 1 \square$ It is symmetric, since $|\square - \square| = 1 \Rightarrow |\square - \square| = 1 \square$ It is not transitive, since $|2 - 1| = 1$ and $|1 - 2| = 1$ but $|2 - 2| = 0 = 1 \square$

- k. The relation \sim is reflexive, symmetric and transitive, since for arbitrary a, b
and c in \mathbb{Z} we have:

- (1) $|x - y| = |0| \leq 1$
- (2) $|x - y| \leq 1 \Rightarrow |x - y| \leq 1$
- (3) $|x - y| \leq 1$ and $|x - y| \leq 1 \Rightarrow x = y$ and $x = y \Rightarrow |x - y| \leq 1$

- 3. a. $\{-3 \leq 3\}$ b. $\{-5 \leq -1 \leq 3 \leq 7 \leq 11\} \subseteq [3]$
- 4. b. $[0] = \{x \mid x \equiv -10 \pmod{5}\} = \{\dots, -10, -5, 0, 5, 10, \dots\}$ $[1] = \{x \mid x \equiv -9 \pmod{6}\} = \{\dots, -9, -4, 1, 6, 11, \dots\}$
 $[2] = \{x \mid x \equiv -8 \pmod{7}\} = \{\dots, -8, -3, 2, 7, 12, \dots\}$ $[8] = [3] = \{x \mid x \equiv -7 \pmod{3}\} = \{\dots, -7, -2, 3, 8, 13, \dots\}$
 $[-4] = [1] = \{x \mid x \equiv -9 \pmod{6}\} = \{\dots, -9, -4, 1, 6, 11, \dots\}$
- 5. b. $[0] = \{x \mid x \equiv -14 \pmod{7}\} = \{\dots, -14, -7, 0, 7, 14, \dots\}$ $[1] = \{x \mid x \equiv -13 \pmod{8}\} = \{\dots, -13, -6, 1, 8, 15, \dots\}$
 $[3] = \{x \mid x \equiv -11 \pmod{10}\} = \{\dots, -11, -4, 3, 10, 17, \dots\}$ $[9] = [2] = \{x \mid x \equiv -12 \pmod{5}\} = \{\dots, -12, -5, 2, 9, 16, \dots\}$
 $[-2] = [5] = \{x \mid x \equiv -9 \pmod{5}\} = \{\dots, -9, -2, 5, 12, 19, \dots\}$
- 6. $[0] = \{x \mid x \equiv -2 \pmod{4}\} = \{\dots, -2, 2, 6, 10, \dots\}$ $[1] = \{x \mid x \equiv -3 \pmod{3}\} = \{\dots, -3, -1, 1, 3, \dots\}$
- 7. $[0] = \{0 \leq 5 \leq 10 \leq \dots\}$ $\{\pm 1 \leq \pm 4 \leq \pm 6 \leq \pm 9\} \subseteq [1]$ $\{\pm 2 \leq \pm 3 \leq \pm 7 \leq \pm 8\} \subseteq [2]$
- 8. $[0] = \{x \mid x \equiv -4 \pmod{4}\} = \{\dots, -4, 0, 4, 8, \dots\}$ $[1] = \{x \mid x \equiv -7 \pmod{5}\} = \{\dots, -7, -3, 1, 5, \dots\}$
 $[2] = \{x \mid x \equiv -6 \pmod{6}\} = \{\dots, -6, -2, 2, 6, \dots\}$ $[3] = \{x \mid x \equiv -5 \pmod{3}\} = \{\dots, -5, -1, 3, 7, \dots\}$
- 9. $[0] = \{x \mid x \equiv -7 \pmod{7}\} = \{\dots, -7, 0, 7, 14, \dots\}$ $[1] = \{x \mid x \equiv -13 \pmod{8}\} = \{\dots, -13, -6, 1, 8, \dots\}$
 $[2] = \{x \mid x \equiv -12 \pmod{9}\} = \{\dots, -12, -5, 2, 9, \dots\}$ $[3] = \{x \mid x \equiv -11 \pmod{10}\} = \{\dots, -11, -4, 3, 10, \dots\}$ $[4] = \{x \mid x \equiv -10 \pmod{4}\} = \{\dots, -10, -3, 4, 11, \dots\}$
 $[5] = \{x \mid x \equiv -9 \pmod{5}\} = \{\dots, -9, -2, 5, 12, \dots\}$ $[6] = \{x \mid x \equiv -8 \pmod{6}\} = \{\dots, -8, -1, 6, 13, \dots\}$
- 10. $[-1] = \{x \mid x \equiv -3 \pmod{4}\} = \{\dots, -3, -1, 1, 3, \dots\}$ $[0] = \{x \mid x \equiv -2 \pmod{4}\} = \{\dots, -2, 2, 6, 10, \dots\}$

11. The relation \sim is symmetric but not reflexive or transitive, since for arbitrary integers a and b , we have the following:

- (1) $a + b = 2a$ is not odd;
- (2) $a + b$ is odd implies $a + a$ is odd;
- (3) $a + b$ is odd and $a + a$ is odd does not imply that $a + b$ is odd. For example, take $a = 1$, $b = 2$ and $a = 3$.

Thus \sim is not an equivalence relation on \mathbb{Z} .

12. a. The relation \sim is symmetric but not reflexive or transitive, since for arbitrary lines l_1 , l_2 and l_3 in a plane, we have the following:

- (1) l_1 is not parallel to l_1 since parallel lines have no points in common;
- (2) l_1 is parallel to l_2 implies that l_2 is parallel to l_1 ;

- (3) \square_1 is parallel to \square_2 and \square_2 is parallel to \square_3 does not imply that \square_1 is parallel to \square_3 . For example, take $\square_3 = \square_1$ with \square_1 parallel to \square_2 .

Thus \square is not an equivalence relation on \mathbb{Z} .

- b. The relation \perp is symmetric but not reflexive or transitive, since for arbitrary lines ℓ_1, ℓ_2 and ℓ_3 in a plane, we have the following:
- (1) ℓ_1 is not perpendicular to ℓ_1 ;
 - (2) ℓ_1 is perpendicular to ℓ_2 implies that ℓ_2 is perpendicular to ℓ_1 ;
 - (3) ℓ_1 is perpendicular to ℓ_2 and ℓ_2 is perpendicular to ℓ_3 does not imply that ℓ_1 is perpendicular to ℓ_3 .

Thus \perp is not an equivalence relation.

13. a. The relation \subseteq is reflexive and transitive but not symmetric, since for arbitrary nonempty subsets A, B, C and D of S we have:
- (1) A is a subset of A ;
 - (2) A is a subset of B does not imply that B is a subset of A ;
 - (3) A is a subset of B and B is a subset of C imply that A is a subset of C .
- b. The relation \subset is not reflexive and not symmetric, but it is transitive, since for arbitrary nonempty subsets A, B, C, D and E of S we have:
- (1) A is not a proper subset of A ;
 - (2) A is a proper subset of B implies that B is not a proper subset of A ;
 - (3) A is a proper subset of B and B is a proper subset of C imply that A is a proper subset of C .
- c. The relation \sim is reflexive, symmetric and transitive, since for arbitrary nonempty subsets A, B, C, D and E of S we have:
- (1) A and A have the same number of elements;
 - (2) If A and B have the same number of elements, then B and A have the same number of elements;
 - (3) If A and B have the same number of elements and B and C have the same number of elements, then A and C have the same number of elements.
14. a. The relation R is reflexive and symmetric but not transitive, since if x, y, z and w are human beings, we have:
- (1) x lives within 400 miles of x ;
 - (2) x lives within 400 miles of y implies that y lives within 400 miles of x ;
 - (3) x lives within 400 miles of y and y lives within 400 miles of z do not imply that x lives within 400 miles of z .
- b. The relation F is not reflexive, not symmetric, and not transitive, since if x, y, z, w and v are human beings we have:
- (1) x is not the father of x ;
 - (2) x is the father of y implies that y is not the father of x ;
 - (3) x is the father of y and y is the father of z imply that x is not the father of z .

- c. The relation is symmetric but not reflexive and not transitive. Let x, y, z and w be human beings, and we have:
- (1) x is a first cousin of y is not a true statement;
 - (2) x is a first cousin of y implies that y is a first cousin of x ;
 - (3) x is a first cousin of y and y is a first cousin of z do not imply that x is a first cousin of z .
- d. The relation R is reflexive, symmetric, and transitive, since if x, y, z and w are human beings we have:
- (1) x and x were born in the same year;
 - (2) if x and y were born in the same year, then y and x were born in the same year;
 - (3) if x and y were born in the same year and if y and z were born in the same year, then x and z were born in the same year.
- e. The relation R is reflexive, symmetric, and transitive, since if x, y, z and w are human beings, we have:
- (1) x and x have the same mother;
 - (2) x and y have the same mother implies y and x have the same mother;
 - (3) x and y have the same mother and y and z have the same mother imply that x and z have the same mother.
- f. The relation R is reflexive, symmetric and transitive, since if x, y, z and w are human beings we have:
- (1) x and x have the same hair color;
 - (2) x and y have the same hair color implies that y and x have the same hair color;
 - (3) x and y have the same hair color and y and z have the same hair color imply that x and z have the same hair color.
15. a. The relation R is an equivalence relation on $\mathbb{R} \times \mathbb{R}$. Let $(x, y), (u, v), (w, z)$ and (a, b) be arbitrary elements of $\mathbb{R} \times \mathbb{R}$.
- (1) $(x, y) R (x, y)$ since $x - x = 0$ and $y - y = 0$;
 - (2) $(x, y) R (u, v) \Rightarrow x - u = 0$ and $y - v = 0 \Rightarrow (u, v) R (x, y)$;
 - (3) $(x, y) R (u, v)$ and $(u, v) R (w, z) \Rightarrow x - u = 0$ and $y - v = 0$ and $u - w = 0$ and $v - z = 0$
 $\Rightarrow x - w = 0$ and $y - z = 0$
 $\Rightarrow (x, y) R (w, z)$.
- b. The relation R is an equivalence relation on $\mathbb{R} \times \mathbb{R}$. Let $(x, y), (u, v), (w, z)$ and (a, b) be arbitrary elements of $\mathbb{R} \times \mathbb{R}$.
- (1) $(x, y) R (x, y)$ since $x - x = 0$ and $y - y = 0$;

- (2) $(a \sim b) \wedge (b \sim c) \Rightarrow a = b \Rightarrow a = c \Rightarrow (a \sim c) \wedge (a \sim a) \wedge (a \sim a) \wedge (a \sim a)$
- (3) $(a \sim b) \wedge (b \sim c) \text{ and } (c \sim d) \wedge (d \sim e) \Rightarrow a = b \text{ and } c = d \Rightarrow a = c \Rightarrow a = d \Rightarrow a = e \Rightarrow (a \sim e) \wedge (a \sim a) \wedge (a \sim a) \wedge (a \sim a)$

c. The relation \sim is an equivalence relation on $\mathbb{R} \times \mathbb{R}$. Let $(a, b) \sim (c, d) \wedge (c, d) \sim (e, f)$ and (a, b) and (e, f) be arbitrary elements of $\mathbb{R} \times \mathbb{R}$.

- (1) $(a, b) \sim (c, d)$ since $a^2 + b^2 = c^2 + d^2$
- (2) $(a, b) \sim (c, d) \Rightarrow a^2 + b^2 = c^2 + d^2 \Rightarrow c^2 + d^2 = e^2 + f^2 \Rightarrow (c, d) \sim (e, f) \wedge (c, d) \sim (c, d)$
- (3) $(a, b) \sim (c, d)$ and $(c, d) \sim (e, f) \Rightarrow a^2 + b^2 = c^2 + d^2$ and $c^2 + d^2 = e^2 + f^2 \Rightarrow a^2 + b^2 = e^2 + f^2 \Rightarrow (a, b) \sim (e, f) \wedge (a, b) \sim (a, b)$

d. The relation \sim is an equivalence relation on $\mathbb{R} \times \mathbb{R}$. Let $(a, b) \sim (c, d) \wedge (c, d) \sim (e, f)$ and (a, b) and (e, f) be arbitrary elements of $\mathbb{R} \times \mathbb{R}$.

- (1) $(a, b) \sim (c, d)$ since $a - b = c - d$
- (2) $(a, b) \sim (c, d) \Rightarrow a - b = c - d \Rightarrow c - d = e - f \Rightarrow (c, d) \sim (e, f) \wedge (c, d) \sim (c, d)$
- (3) $(a, b) \sim (c, d)$ and $(c, d) \sim (e, f) \Rightarrow a - b = c - d$ and $c - d = e - f \Rightarrow a - b = e - f \Rightarrow (a, b) \sim (e, f) \wedge (a, b) \sim (a, b)$

16. The relation \sim is reflexive and symmetric but not transitive.
17. a. The relation is symmetric but not reflexive and not transitive. Let A, B, C and D be arbitrary elements of the power set $P(S)$ of the nonempty set S .
- (1) $A \cap B = \emptyset$ is not true if $A = S$
 - (2) $A \cap B = \emptyset$ implies that $B \cap A = \emptyset$
 - (3) $A \cap B = \emptyset$ and $B \cap C = \emptyset$ do not imply that $A \cap C = \emptyset$. For example, let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 3, 4\}$, $C = \{3, 4, 5\}$ and $D = \{1, 2, 3\}$. Then $A \cap B = \emptyset$, $B \cap C = \emptyset$ but $A \cap C = \{2, 3, 4\} \neq \emptyset$.
- b. The relation \sim is reflexive and transitive but not symmetric, since for arbitrary subsets A, B, C, D of S we have:
- (1) $A \subseteq B$;
 - (2) $\emptyset \subseteq A$ but $A \not\subseteq \emptyset$;
 - (3) $A \subseteq B$ and $B \subseteq C$ imply $A \subseteq C$.
18. The relation is reflexive, symmetric, and transitive. Let A, B, C, D and E be arbitrary elements of the power set $P(S)$ and F a fixed subset of S .
- (1) $A \cap B = A \cap B$ since $A \cap B = A \cap B$
 - (2) $A \cap B = A \cap B \Rightarrow A \cap B = C \cap D \Rightarrow C \cap D = A \cap B \Rightarrow C \cap D = C \cap D$