Solution Manual for College Algebra 10th Edition Larson ISBN 1337282294 9781337282291

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CHAPTER 2
Functions and Their Graphs

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Functions and Their Graphs

Section 2.1 Linear Equations in Two Variables

1. linear

2. slope

3. point-slope

4. parallel

5. perpendicular

6. rate or rate of change

7. linear extrapolation

8. general

9. (a) $m = \frac{2}{3}$. Because the slope is positive, the line rises. Matches L_2.

(b) m is undefined. The line is vertical. Matches L_3.

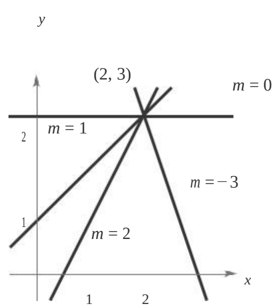
(c) $m = -2$. The line falls. Matches L_1.

10. (a) $m = 0$. The line is horizontal. Matches L_2.

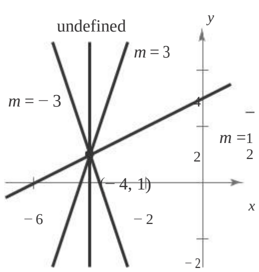
(b) $m = -\frac{3}{4}$. Because the slope is negative, the line falls. Matches L_1.

(c) $m = 1$. Because the slope is positive, the line rises. Matches L_3.

11.



12.



13. Two points on the line: $(0, 0)$ and $(4, 6)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6}{4} = \frac{3}{2}$$

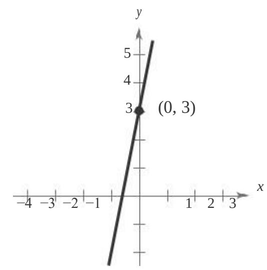
14. The line appears to go through $(0, 7)$ and $(7, 0)$.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 7}{7 - 0} = -1$$

15. $y = 5x + 3$

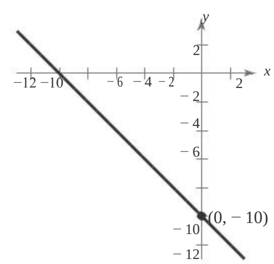
Slope: $m = 5$

y-intercept: $(0, 3)$



16. Slope: $m = -1$

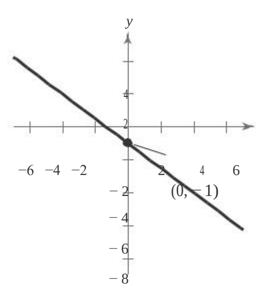
y-intercept: $(0, -10)$



17. $y = -\frac{3}{4}x - 1$

Slope: $m = -\frac{3}{4}$

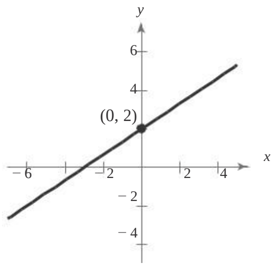
y-intercept: $(0, -1)$



18. $y = \frac{2}{3}x + 2$

Slope: $m = \frac{2}{3}$

y-intercept: $(0, 2)$

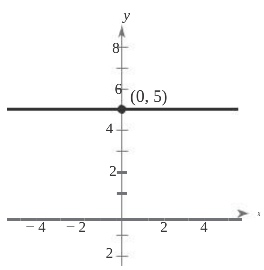


19. $y - 5 = 0$

$y = 5$

Slope: $m = 0$

y-intercept: $(0, 5)$

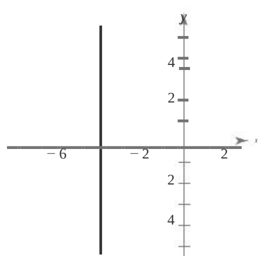


$+4=0$

$x = -4$

Slope: undefined (vertical line)

y-intercept: none

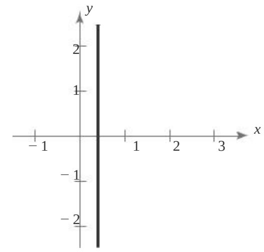


$5x - 2 = 0$

$= \frac{2}{5}$, vertical line

Slope: undefined

y-intercept: none



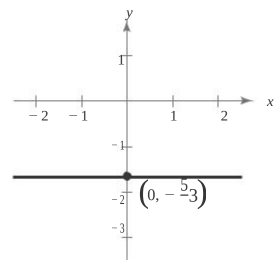
22. $3y + 5 = 0$

$3y = -5$

$= -\frac{5}{3}$

Slope: $m = 0$

y-intercept: $\left(0, -\frac{5}{3}\right)$



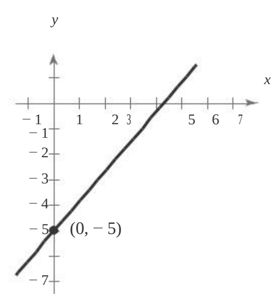
23. $7x - 6y = 30$

$-6y = -7x + 30$

$y = \frac{7}{6}x - 5$

$\frac{7}{}$

y-intercept: $(0, -5)$



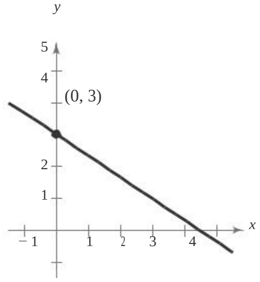
24. $2x + 3y = 9$

$3y = -2x + 9$

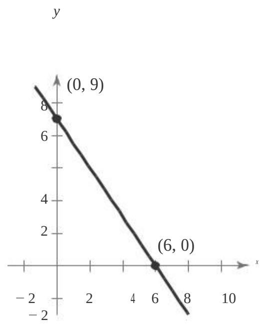
$y = -\frac{2}{3}x + 3$

Slope: $m = -\frac{2}{3}$

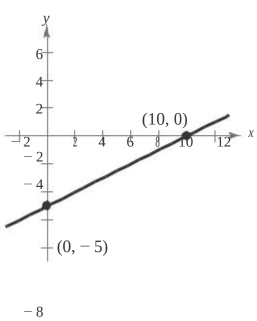
y-intercept: $(0, 3)$



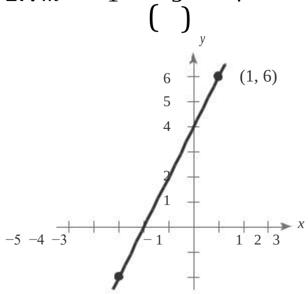
25. $m = \frac{0 - 9}{6 - 0} = \frac{-9}{6} = -\frac{3}{2}$



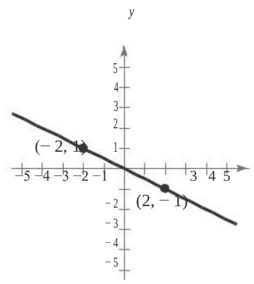
26. $m = \frac{-5-0}{0-10} = \frac{-5}{-10} = \frac{1}{2}$



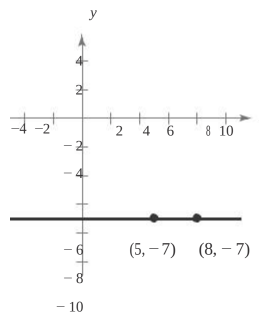
27. $m = \frac{6 - (-2)}{1 - (-3)} = \frac{8}{4} = 2$



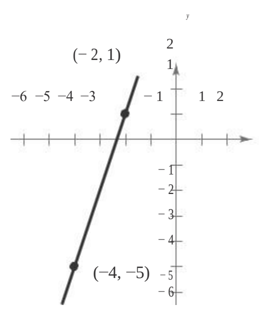
28. $m = \frac{1 - (-1)}{-2-2} = \frac{2}{-4} = -\frac{1}{2}$



29. $m = \frac{-7 - (-7)}{8 - 5} = \frac{0}{3} = 0$

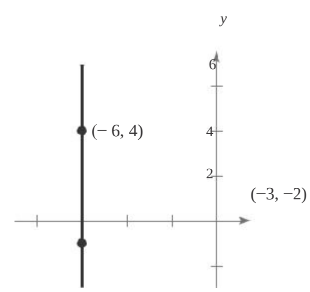


30. $m = \frac{-5-1}{-4 - (-2)} = 3$



31. $m = \frac{4 - (-1)}{-6 - (-6)} = \frac{5}{0}$

m is undefined.

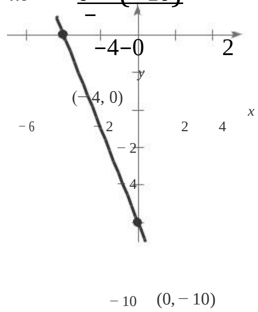


x

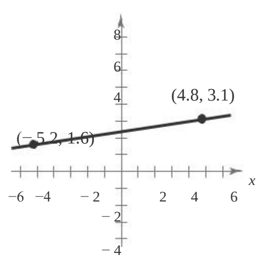
−8 (−6, −1) −2

−2

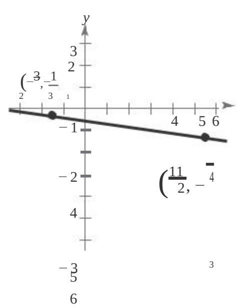
32. $m = \dfrac{0 - (-10)}{-4 - 0} = -\dfrac{5}{2}$



33. $m = \dfrac{1.6 - 3.1}{-5.2 - 4.8} = \dfrac{-1.5}{-10} = 0.15$



34. $m = \dfrac{-\frac{1}{3} - \left(-\frac{4}{3}\right)}{-\frac{3}{2} - \left(\frac{11}{2}\right)} = -\dfrac{1}{7}$



Point: $(5, 7)$, Slope: $m = 0$

Because $m = 0$, y does not change. Three other points are $(-1, 7)$, $(0, 7)$, and $(4, 7)$.

Point: $(3, -2)$, Slope: $m = 0$

Because $m = 0$, y does not change. Three other points are $(1, -2)$, $(10, -2)$, and $(-6, -2)$.

Point: $(-5, 4)$, Slope: $m = 2$

Because $m = 2 = \frac{2}{1}$, y increases by 2 for every one unit increase in x. Three additional points are $(-4, 6)$, $(-3, 8)$, and $(-2, 10)$.

Point: $(0, -9)$, Slope: $m = -2$

Because $m = -2$, y decreases by 2 for every one unit increase in x. Three other points are $(-2, -5)$, $(1, -11)$, and $(3, -15)$.

Point: $(4, 5)$, Slope: $m = -\frac{1}{3}$

Because $m = -\frac{1}{3}$, y decreases by 1 unit for every three unit increase in x. Three additional points are $(-2, 7)$, $\left(0, -\frac{19}{4}\right)$, and $(1, 6)$.

40. Point: $(3, -4)$, Slope: $m = \frac{1}{4}$

Because $m = \frac{1}{4}$, y increases by 1 unit for every four unit increase in x. Three additional points are $(-1, -5)$, $(1, -11)$, and $(3, -15)$.

Point: $(-4, 3)$, Slope is undefined.

Because m is undefined, x does not change. Three points are $(-4, 0)$, $(-4, 5)$, and $(-4, 2)$.

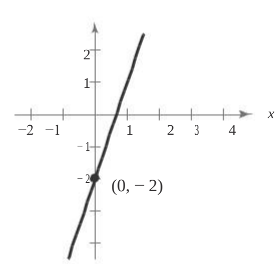
Point: $(2, 14)$, Slope is undefined.

Because m is undefined, x does not change. Three other points are $(2, -3)$, $(2, 0)$, and $(2, 4)$.

43. Point: $(0, -2)$; $m = 3$

$y + 2 = 3(x - 0)$

$y = 3x - 2$

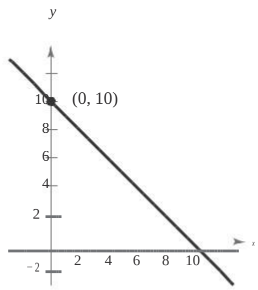


44. Point: $(0, 10)$; $m = -1$

$y - 10 = -1(x - 0)$

$y - 10 = -x$

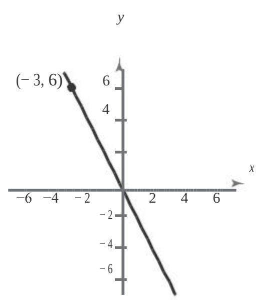
$y = -x + 10$



45. Point: $(-3, 6)$; $m = -2$

$y - 6 = -2(x + 3)$

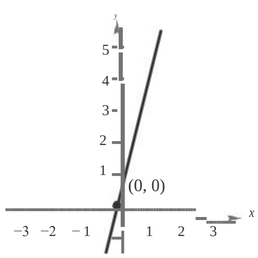
$y = -2x$



46. Point: $(0, 0)$; $m = 4$

$y - 0 = 4(x - 0)$

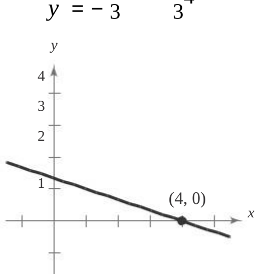
$y = 4x$



Point: $(4, 0)$; $m = -\frac{1}{3}$

$y - 0 = -\frac{1}{3}(x - 4)$

$y = -\frac{1}{3}x + \frac{4}{3}$

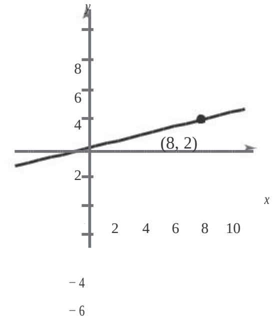


48. Point: $(8, 2)$; $m = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x - 8)$

$y - 2 = \frac{1}{4}x - 2$

$y = \frac{1}{4}x$

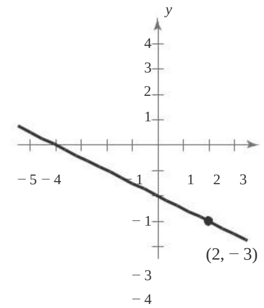


Point: $(2, -3)$; $m = -\frac{1}{2}$

$-(-3) = -\frac{1}{2}(x - 2)$

$+3 = -\frac{1}{2}x + 1$

$= -\frac{1}{2}x - 2$



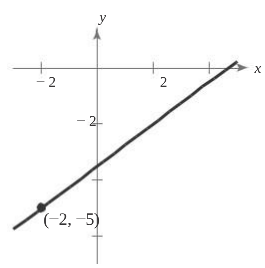
50. Point: $(-2, -5)$; $m = \frac{3}{4}$

$y + 5 = \frac{3}{4}(x + 2)$

$4y + 20 = 3x + 6$

$4y = 3x - 14$

$y = \frac{3}{4}x - \frac{7}{2}$

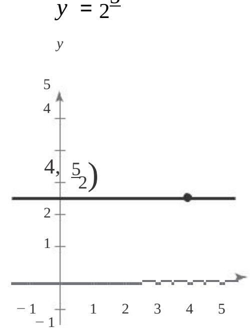


Point: $\left(4, \frac{5}{2}\right)$; $m = 0$

$-\frac{5}{2} = 0(x - 4)$

$y - \frac{5}{2} = 0$

$y = \frac{5}{2}$



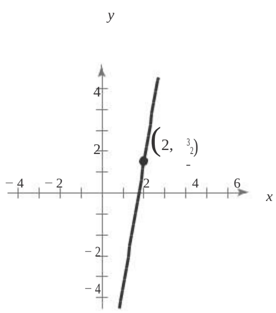
−1 1 2 3 4
−1
−2

52. Point: $\left(2, \frac{3}{2}\right)$; $m = 6$

$$y - \frac{3}{2} = 6(x - 2)$$

$$y - \frac{3}{2} = 6x - 12$$

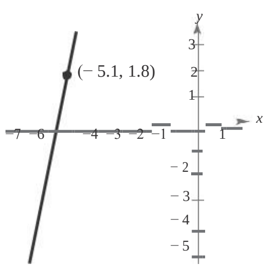
$$y = 6x - \frac{21}{2}$$



53. Point: $(-5.1, 1.8)$; $m = 5$

$$y - 1.8 = 5\left(x - (-5.1)\right)$$

$$y = 5x + 27.3$$

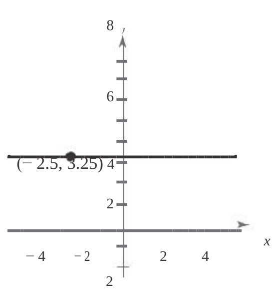


54. Point: $(-2.5, 3.25)$; $m = 0$

$$y - 3.25 = 0\left(x - (-2.5)\right)$$

$$y - 3.25 = 0$$

$$y = 3.25$$

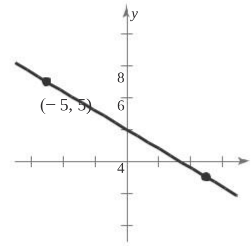


55. $(5, -1)$, $(-5, 5)$

$$y + 1 = \frac{5 + 1}{-5 - 5}(x - 5)$$

$$y = -\frac{3}{5}(x - 5) - 1$$

$$y = -\frac{3}{5}x + 2$$



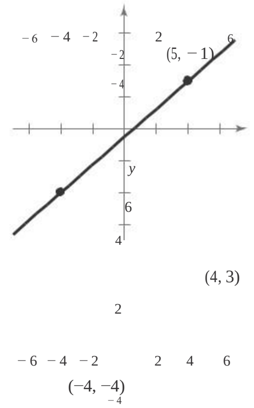
56. $(4, 3)$, $(-4, -4)$

$$y - 3 = \frac{-4 - 3}{-4 - 4}(x - 4)$$

$$y - 3 = \frac{7}{8}(x - 4)$$

$$y - 3 = \frac{7}{8}x - \frac{7}{2}$$

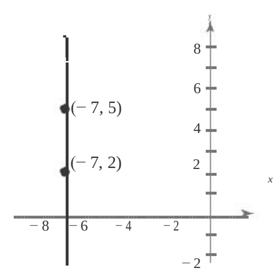
$$y = \frac{7}{8}x - \frac{1}{2}$$



57. $(-7, 2)$, $(-7, 5)$

$$m = \frac{5 - 2}{-7 - (-7)} = \frac{3}{0}$$

m is undefined.



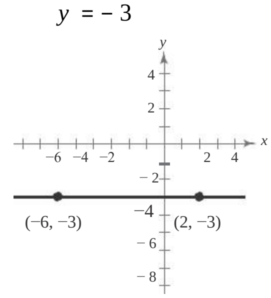
58. $(-6, -3)$, $(2, -3)$

$$y - 4 = \frac{-3 - (-3)}{2 - (-6)}(x + 6)$$

$$y + 3 = 0x + 6$$

$$y + 3 = 0$$

$$y = -3$$

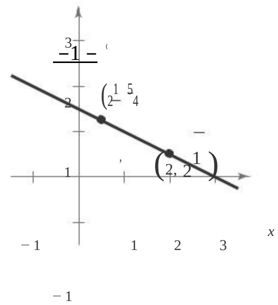


59. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$

$y - \frac{1}{2} = \dfrac{\frac{5}{4} - \frac{1}{2}}{\frac{1}{2} - 2}(x - 2)$

$y = -\frac{1}{2}(x - 2) + \frac{1}{2}$

$y = -\frac{1}{2}x + \frac{3}{2}$

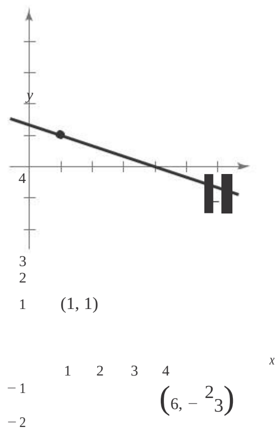


60. $(1, 1), \left(6, -\frac{2}{3}\right)$

$y - 1 = \dfrac{-\frac{2}{3} - 1}{6 - 1}(x - 1)$

$y - 1 = -\frac{1}{3}(x - 1)$

$y = -\frac{1}{3}x + \frac{4}{3}$

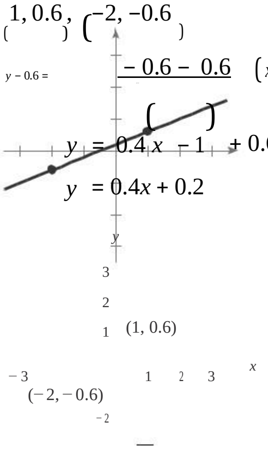


61. $(1, 0.6), (-2, -0.6)$

$y - 0.6 = \dfrac{-0.6 - 0.6}{-2 - 1}(x - 1)$

$y = 0.4(x - 1) + 0.6$

$y = 0.4x + 0.2$



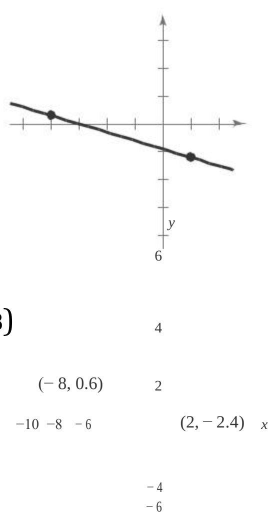
62. $(-8, 0.6), (2, -2.4)$

$y - 0.6 = \dfrac{-2.4 - 0.6}{2 - (-8)}(x + 8)$

$y - 0.6 = -\frac{3}{10}(x + 8)$

$10y - 6 = -3(x + 8)$

$10y - 6 = -3x - 24$



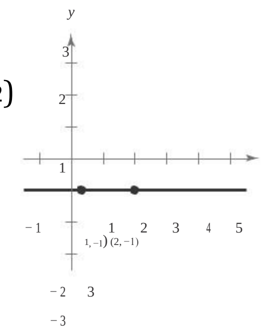
63. $(2, -1), \left(\frac{1}{3}, -1\right)$

$y + 1 = \dfrac{-1 - (-1)}{\frac{1}{3} - 2}(x - 2)$

$y + 1 = 0$

$y = -1$

The line is horizontal.

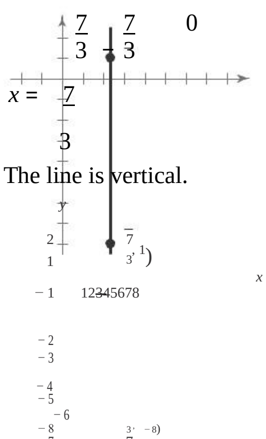


64. $\left(\frac{7}{3}, -8\right), \left(\frac{7}{3}, 1\right)$

$m = \dfrac{1 - (-8)}{\frac{7}{3} - \frac{7}{3}} = \dfrac{9}{0}$ and is undefined.

$x = \frac{7}{3}$

The line is vertical.



65. $L_1: y = -\frac{2}{3}x - 3$

$m_1 = -\frac{2}{3}$

$L_2: y = -\frac{2}{3}x - 1$

$m_2 = -\frac{2}{3}$

The slopes are equal, so the lines are parallel.

66. $L_1: y = \frac{1}{4}x - 1$

$m_1 = \frac{1}{4}$

$L_2: y = 4x + 7$

$m_2 = 4$

The lines are neither parallel nor perpendicular.

67. $L_1: y = \frac{1}{2}x - 3$

$m_1 = \frac{1}{2}$

$L_2: y = -\frac{1}{2} + 1$

$10y = -3x - 18$

$y = -\frac{3}{10}x - \frac{9}{5}$ or $y = -0.3x - 1.8$

$m_2 = -\frac{1}{2}$

The lines are neither parallel nor perpendicular.

$L_1: y = -\frac{4}{5}x - 5$

$m_1 = -\frac{4}{5}$

$L_2: y = \frac{5}{4}x + 1$

The slopes are negative reciprocals, so the lines are perpendicular.

$L_1: (0, -1), (5, 9)$

$m_1 = \frac{9 + 1}{5 - 0} = 2$

$L_2: (0, 3), (4, 1)$

$m_2 = \frac{1 - 3}{4 - 0} = -\frac{1}{2}$

The slopes are negative reciprocals, so the lines are perpendicular.

$L_1: (-2, -1), (1, 5)$

$m_1 = \frac{5 - (-1)}{1 - (-2)} = \frac{6}{3} = 2$

$L_2: (1, 3), (5, -5)$

$m_2 = \frac{-5 - 3}{5 - 1} = \frac{-8}{4} = -2$

The lines are neither parallel nor perpendicular.

Ā Ā

$L_1: (-6, -3), (2, -3)$

$m_1 = \frac{-3 - (-3)}{2 - (-6)} = \frac{0}{8} = 0$

$L_2: \left(3, -\frac{1}{2}\right), \left(6, -\frac{1}{2}\right)$

$m_2 = \frac{-\frac{1}{2} - \left(-\frac{1}{2}\right)}{6 - 3} = \frac{0}{3} = 0$

L_1 and L_2 are both horizontal lines, so they are parallel.

$L_1: (4, 8), (-4, 2)$

$m_1 = \frac{2 - 8}{-4 - 4} = \frac{-6}{-8} = \frac{3}{4}$

$L_2: (3, -5), \left(-1, \frac{1}{3}\right)$

1

73. $4x - 2y = 3$

$y = 2x - \frac{3}{2}$

Slope: $m = 2$

$(2, 1), m = 2$

$y - 1 = 2(x - 2)$

$y = 2x - 3$

$(2, 1), m = -\frac{1}{2}$

$y - 1 = -\frac{1}{2}(x - 2)$

$y = -\frac{1}{2}x + 2$

$x + y = 7$

$y = -x + 7$

Slope: $m = -1$

$m = -1, (-3, 2)$

$y - 2 = -1(x + 3)$

$y - 2 = -x - 3$

$y = -x - 1$

$m = 1, (-3, 2)$

$y - 2 = 1(x + 3)$

$y = x + 5$

Ā $3x + 4y = 7$ Ā Ā

$y = -\frac{3}{4}x + \frac{7}{4}$

Slope: $m = -\frac{3}{4}$

$\left(-\frac{2}{3}, \frac{7}{8}\right), m = -\frac{3}{4}$

$y - \frac{7}{8} = -\frac{3}{4}\left(x - \left(-\frac{2}{3}\right)\right)$

$y = -\frac{3}{4}x + \frac{3}{8}$

$\left(-\frac{2}{3}, \frac{7}{8}\right), m = \frac{4}{3}$

$y - \frac{7}{8} = \frac{4}{3}\left(x - \left(-\frac{2}{3}\right)\right)$

$y = \frac{4}{3}x + \frac{127}{72}$

$$m_2 = \frac{\frac{1}{3} - (-5)}{-1-3} = \frac{\frac{16}{3}}{-4} = -\frac{4}{3}$$

The slopes are negative reciprocals, so the lines are perpendicular.

$5x + 3y = 0$

$3y = -5x$

$= -\frac{5}{3}x$

Slope: $m = -\frac{5}{3}$

$m = -3 \quad -8, 4)$

$-\frac{3}{4} = -\frac{5}{3}\left(x - \frac{7}{8}\right)$

$24y - 18 = -40\left(x - \frac{7}{8}\right)$ $24y - 18 = -40x + 35$

$24y = -40x + 53$

$y = -\frac{5}{3}x + \frac{53}{24}$

$m = 5 \quad 8, \; 4)$ 7

$-\frac{3}{4} = \frac{3}{5}\left(x - 8\right)$

$40y - 30 = 24\left(x - \frac{7}{8}\right)$

$40y = 24x + 9$

$y = \frac{3}{5}x + \frac{9}{40}$

$y + 5 = 0$ y

$= -5$

Slope: $m = 0$

$(-2, 4)$, $m =$ 0 $y = 4$

$(-2, 4)$, m is undefined. $x = -2$

$x - 4 = 0$

$x = 4$

Slope: m is undefined.

$(3, -2)$, m is undefined.

$= 3$

$(3, -2)$, $m = 0$

$= -2$

$x - y = 4$

$y = x - 4$

Slope: $m = 1$

$(2.5, 6.8)$, $m = 1$

$y - 6.8 = 1(x - 2.5)$

$6x + 2y = 9$

$2y = -6x + 9$

$= -3x + \frac{9}{2}$

Slope: $m = -3$

$(-3.9, -1.4)$, $m = -3$

$y - (-1.4) = -3(x - (-3.9))$

$+ 1.4 = -3x - 11.7$

$= -3x - 13.1$

$(-3.9, -1.4)$, $m = \frac{1}{3}$

$-(-1.4) = \frac{1}{3}(x - (-3.9))$

$y + 1.4 = \frac{1}{3}x + 1.3$

$= \frac{1}{3}x - 0.1$

$\frac{x}{3} + \frac{y}{5} = 1$

$15\left(\frac{x}{3} + \frac{y}{5}\right) = 115$

$5x + 3y - 15 = 0$

$(-3, 0)$, $(0, 4)$

$\frac{x}{-3} + \frac{y}{4} = 1$

$(-12)\frac{x}{-3} + (-12)\frac{y}{4} = (-12) \cdot 1$

$4x - 3y + 12 = 0$

83. $\frac{x}{-1/6} + \frac{y}{-2/3} = 1$

$6x + \frac{3}{2}y = -1$

$12x + 3y + 2 = 0$

$\left(\frac{2}{3}, 0\right)$, $(0, -2)$

$\frac{x}{2/3} + \frac{y}{-2} = 1$

$\frac{3x}{2} - \frac{y}{2} = 1$

$3x - y - 2 = 0$

$y = x + 4.3$

$(2.5, 6.8)$, $m = -1$

$$y - 6.8 = (-1)(x - 2.5)$$
$$y = -x + 9.3$$

85. $\frac{x}{c} + \frac{y}{c} = 1, c \neq 0$

$$x + y = c$$
$$1 + 2 = c$$
$$3 = c$$
$$x + y = 3$$
$$x + y - 3 = 0$$

$(d, 0), (0, d), (-3, 4)$

$\frac{x}{d} + \frac{y}{d} = 1$

$x + y = d$

$-3 + 4 = d$

$1 = d$

$x + y - 1 = 0$

(a) $m = 135$. The sales are increasing 135 units per year.

$m = 0$. There is no change in sales during the year.

$m = -40$. The sales are decreasing 40 units per year.

(a) greatest increase = largest slope

$(14, 182.20), (15, 233.72)$

$m_1 = \frac{233.72 - 182.20}{15 - 14} = 51.52$

So, the sales increased the greatest between the years 2014 and 2015.

least increase = smallest slope

$(13, 170.91), (14, 182.20)$

$m_2 = \frac{182.20 - 170.91}{14 - 13} = 11.29$

So, the sales increased the least between the years 2013 and 2014.

(b) $(9, 42.91), (15, 233.72)$

$m = \frac{233.72 - 42.91}{15 - 9} = \frac{190.81}{6} \approx 31.80$

The slope of the line is about 31.8.

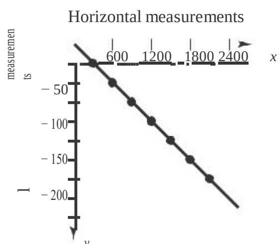
The sales increased an average of about $31.8 billion each year between the years 2009 and 2015.

$y = \frac{6}{100}x$

$= \frac{6}{100}(200) = 12$ feet

(a) and (b)

x	300	600	900	1200	1500	1800	2100
y	–25	–50	–75	–100	–125	–150	–175



$m = \frac{-50 - (-25)}{600 - 300} = \frac{-25}{300} = -\frac{1}{12}$

$y - (-50) = -\frac{1}{12}(x - 600)$

$y + 50 = -\frac{1}{12}x + 50$

$y = -\frac{1}{12}x$

Because $m = -\frac{1}{12}$, for every change in the horizontal measurement of 12 feet, the vertical measurement decreases by 1 foot.

$\frac{1}{12} \approx 0.083 = 8.3\%$ grade

$(16, 3000), m = -150$

$V - 3000 = -150(t - 16)$

$V - 3000 = -150t + 2400$

$V = -150t + 5400, 16 \le t \le 21$

$(16, 200), m = 6.50$

$V - 200 = 6.50(t - 16)$

$V - 200 = 6.50t + 104$

$V = 6.5t + 96, 16 \le t \le 21$

The C-intercept measures the fixed costs of manufacturing when zero bags are produced.

The slope measures the cost to produce one laptop bag.

Monthly wages = 7% of the Sales plus the Monthly Salary

$W = 0.07S + 5000$

Using the points $(0, 875)$ and $(5, 0)$, where the first coordinate represents the year t and the second coordinate represents the value V, you have

$= \frac{0 - 875}{} = -175$ $5 - 0$

$= -175t + 875,\ 0 \le t \le 5.$

Using the points $(0, 24{,}000)$ and $(10, 2000)$, where the first coordinate represents the year t and the second coordinate represents the value V, you have

$$m = \frac{2{,}000 - 24{,}000}{10 - 0} = \frac{-22{,}000}{10} = -2200.$$

Since the point $(0, 24{,}000)$ is the V-intercept, $b = 24{,}000$, the equation is

$= -2200t + 24{,}000,\ 0 \le t \le 10.$

Using the points $(0, 32)$ and $(100, 212)$, where the first coordinate represents a temperature in degrees Celsius and the second coordinate represents a temperature in degrees Fahrenheit, you have

$$m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}.$$

Since the point $(0, 32)$ is the F-intercept, $b = 32$, the equation is $F = 1.8C + 32$ or $C = \frac{5}{9}F - \frac{160}{9}$.

(a) Using the points $(1, 970)$ and $(3, 1270)$, you have

$$m = \frac{1270 - 970}{3 - 1} = \frac{300}{2} = 150.$$

Using the point-slope form with $m = 150$ and the point $(1, 970)$, you have

$- y_1 = m(t - t_1)$ y

$- 970 = 150(t - 1)$ $y -$

$970 = 150t - 150$

$y = 150t + 820.$

The slope is $m = 150$. The slope tells you the amount of increase in the weight of an average male child's brain each year.

Let $t = 2$:

$= 150(2) + 820$

$y = 300 + 820$

$y = 1120$

The average brain weight at age 2 is 1120 grams.

Answers will vary.

Answers will vary. *Sample answer:* No. The brain stops growing after reaching a certain age.

99. (a) Total Cost = cost for fuel and maintainance + cost for operator + cost purchase

$C = 9.5t + 11.5t + 42{,}000$

$C = 21t + 42{,}000$

(b) Revenue = Rate per hour · Hours

$R = 45t$

(c) $P = R - C$

$P = 45t - (21t + 42{,}000)$

$P = 24t - 42{,}000$

(d) Let $P = 0$, and solve for t.

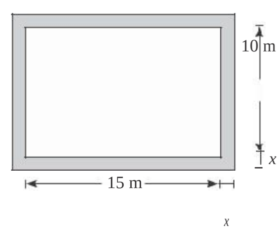
$0 = 24t - 42{,}000$

$42{,}000 = 24t$

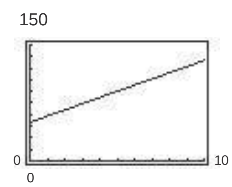
$1750 = t$

The equipment must be used 1750 hours to yield a profit of 0 dollars.

100. (a)



$y = 2(15 + 2x) + 2(10 + 2x) = 8x + 50$



Because $m = 8$, each 1-meter increase in x will increase y by 8 meters.

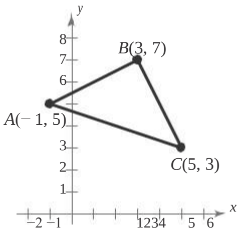
False. The slope with the greatest magnitude corresponds to the steepest line.

102. $(-8, 2)$ and $(-1, 4)$: $m_1 = \frac{4 - 2}{-1 - (-8)} = \frac{2}{7}$

$(0, -4)$ and $(-7, 7)$: $m_2 = \frac{7 - (-4)}{-7 - 0} = \frac{11}{-7}$

False. The lines are not parallel.

103. Find the slope of the line segments between the points A and B, and B and C.



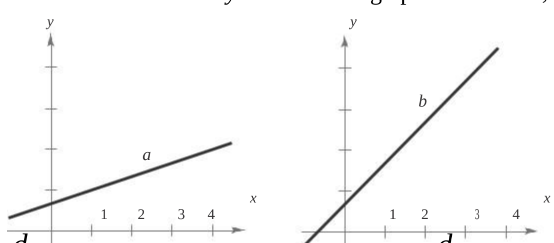
$$m_{AB} = \frac{7-5}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$$

$$m_{BC} = \frac{3-7}{5-3} = \frac{-4}{2} = -2$$

Since the slopes are negative reciprocals, the line segments are perpendicular and therefore intersect to form a right angle. So, the triangle is a right triangle.

104. On a vertical line, all the points have the same x-value, so when you evaluate $m = \frac{y_2 - y_1}{x_2 - x_1}$, you would have a zero in the denominator, and division by zero is undefined.

Since the scales for the y-axis on each graph is unknown, the slopes of the lines cannot be determined.



106. $d_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad$ $d_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1-0)^2 + (m_1 - 0)^2}$ $\qquad$ $= \sqrt{(1-0)^2 + (m_2 - 0)^2}$

$= \sqrt{1 + (m_1)^2}$ $\qquad$ $= \sqrt{1 + (m_2)^2}$

Using the Pythagorean Theorem:

$$(d_1)^2 + (d_2)^2 = \left(\text{distance between } (1, m_1), \text{ and } (1, m_2)\right)^2$$

$$\left(\sqrt{1 + (m_1)^2}\right)^2 + \left(\sqrt{1 + (m_2)^2}\right)^2 = \left(\sqrt{(1-1)^2 + (m_2 - m_1)^2}\right)^2$$

$$1 + (m_1)^2 + 1 + (m_2)^2 = (m_2 - m_1)^2$$

$$(m_1)^2 + (m_2)^2 + 2 = (m_2)^2 - 2m_1m_2 + (m_1)^2$$

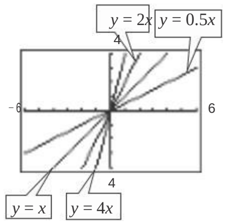
$$2 = -2m_1m_2$$

$$-\frac{1}{m_2} = m_1$$

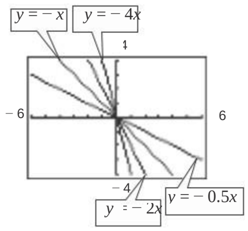
107. No, the slopes of two perpendicular lines have opposite signs. (Assume that neither line is vertical or horizontal.)

108. Because $|-4| > \left|\frac{5}{2}\right|$, the steeper line is the one with a slope of – 4. The slope with the greatest magnitude corresponds to the steepest line.

109. The line $y = 4x$ rises most quickly.



The line $y = -4x$ falls most quickly.



The greater the magnitude of the slope (the absolute value of the slope), the faster the line rises or falls.

(a) Matches graph (ii).

The slope is –20, which represents the decrease in the amount of the loan each week. The y-intercept is $(0, 200)$, which represents the original amount of the loan.

Matches graph (iii).

The slope is 2, which represents the increase in the hourly wage for each unit produced. The y-intercept is $(0, 12.5)$, which represents the hourly rate if the employee produces no units.

Matches graph (i).

The slope is 0.32, which represents the increase in travel cost for each mile driven. The y-intercept is $(0, 32)$, which represents the fixed cost of $30 per day for meals. This amount does not depend on the number of miles driven.

Matches graph (iv).

The slope is –100, which represents the amount by which the computer depreciates each year. The y-intercept is $(0, 750)$, which represents the original purchase price.

Set the distance between $(4, -1)$ and (x, y) equal to the distance between $(-2, 3)$ and (x, y).

$$\sqrt{(x-4)^2 + [y-(-1)]^2} = \sqrt{[x-(-2)]^2 + (y-3)^2}$$

$$(x-4)^2 + (y+1)^2 = (x+2)^2 + (y-3)^2$$

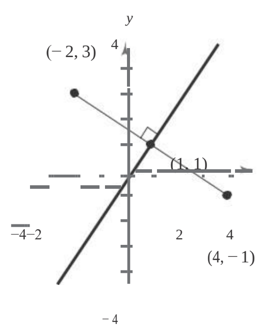
$$x^2 - 8x + 16 + y^2 + 2y + 1 = x^2 + 4x + 4 + y^2 - 6y + 9$$

$$-8x + 2y + 17 = 4x - 6y + 13$$

$$-12x + 8y + 4 = 0$$

$$-4(3x - 2y - 1) = 0$$

$$3x - 2y - 1 = 0$$



This line is the perpendicular bisector of the line segment connecting $(4, -1)$ and $(-2, 3)$.

Set the distance between $(6, 5)$ and (x, y) equal to the distance between $(1, -8)$ and (x, y).

$$\sqrt{(x-6)^2 + (y-5)^2} = \sqrt{(x-1)^2 + (y-(-8))^2}$$

$$(x-6)^2 + (y-5)^2 = (x-1)^2 + (y+8)^2$$

$$x^2 - 12x + 36 + y^2 - 10y + 25 = x^2 - 2x + 1 + y^2 + 16y + 64$$

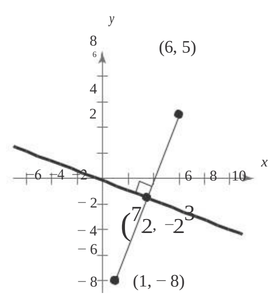
$$x^2 + y^2 - 12x - 10y + 61 = x^2 + y^2 - 2x + 16y + 65$$

$$-12x - 10y + 61 = -2x + 16y + 65$$

$$-10x - 26y - 4 = 0$$

$$-2(5x + 13y + 2) = 0$$

$$5x + 13y + 2 = 0$$



113. Set the distance between $\left(3, \frac{5}{2}\right)$ and (x, y) equal to the distance between $(-7, 1)$ and (x, y).

$$\sqrt{(x-3)^2 + \left(y-\frac{5}{2}\right)^2} = \sqrt{\left[x-(-7)\right]^2 + (y-1)^2}$$

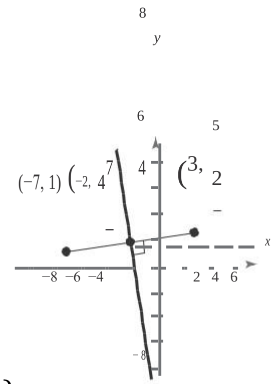
$$(x-3)^2 + \left(y-\frac{5}{2}\right)^2 = (x+7)^2 + (y-1)^2$$

$$x^2 - 6x + 9 + y^2 - 5y + \frac{25}{4} = x^2 + 14x + 49 + y^2 - 2y + 1$$

$$-6x - 5y + \frac{61}{4} = 14x - 2y + 50$$

$$-24x - 20y + 61 = 56x - 8y + 200$$

$$80x + 12y + 139 = 0$$



This line is the perpendicular bisector of the line segment connecting $\left(3, \frac{5}{2}\right)$ and $(-7, 1)$.

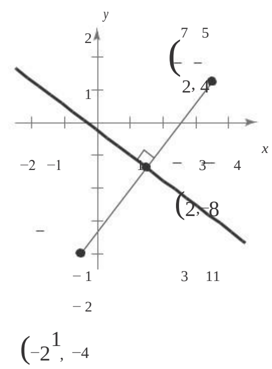
Set the distance between $\left(-\frac{1}{2}, -4\right)$ and (x, y) equal to the distance between $\left(\frac{7}{2}, \frac{5}{4}\right)$ and (x, y).

$$\sqrt{\left(x-\left(-\frac{1}{2}\right)\right)^2 + \left(y-(-4)\right)^2} = \sqrt{\left(x-\frac{7}{2}\right)^2 + \left(y-\frac{5}{4}\right)^2}$$

$$\left(x+\frac{1}{2}\right)^2 + (y+4)^2 = \left(x-\frac{7}{2}\right)^2 + \left(y-\frac{5}{4}\right)^2$$

$$x^2 + x + \frac{1}{4} + y^2 + 8y + 16 = x^2 - 7x + \frac{49}{4} + y^2 - \frac{5}{2}y + \frac{25}{16}$$

$$x^2 + y^2 + x + 8y + \frac{65}{4} = x^2 + y^2 - 7x - \frac{5}{2}y + \frac{221}{16}$$



$$x + 8y + \frac{65}{4} = -7x - \frac{5}{2}y + \frac{221}{16}$$

$$8x + \frac{21}{2}y + \frac{39}{16} = 0$$

$$128x + 168y + 39 = 0$$

Section 2.2 Functions

1. domain; range; function

2. independent; dependent

3. implied domain

4. difference quotient

5. Yes, the relationship is a function. Each domain value is matched with exactly one range value.

6. No, the relationship is not a function. The domain value of –1 is matched with two output values.

7. No, it does not represent a function. The input values of 10 and 7 are each matched with two output values.

8. Yes, the table does represent a function. Each input value is matched with exactly one output value.

Input, x	–2	0	2	4	6
Output, y	1	1	1	1	1

9. (a) Each element of A is matched with exactly one element of B, so it does represent a function.

(b) The element 1 in A is matched with two elements, –2 and 1 of B, so it does not represent a function.

Each element of A is matched with exactly one element of B, so it does represent a function.

The element 2 in A is not matched with an element of B, so the relation does not represent a function.

(a) The element c in A is matched with two elements, 2 and 3 of B, so it is not a function.

Each element of A is matched with exactly one element of B, so it does represent a function.

This is not a function from A to B (it represents a function from B to A instead).

Each element of A is matched with exactly one element of B, so it does represent a function.

$x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$

No, y *is not* a function of x.

$x^2 - y = 9 \Rightarrow y = x^2 - 9$

Yes, y *is* a function of x.

$y = \sqrt{16 - x^2}$

Yes, y *is* a function of x.

14. $y = \sqrt{x+5}$

Yes, *y is* a function of *x*.

$y = 4 - |x|$

Yes, *y is* a function of *x*.

16. $|y| = 4 - x \Rightarrow y = 4 - x$ or $y = -(4 - x)$

No, *y is not* a function of *x*.

17. $y = -75$ or $y = -75 + 0x$

Yes, *y is* a function of *x*.

18. $x - 1 = 0$

$x = 1$

No, this *is not* a function of *x*.

$f(x) = 3x - 5$

$f(1) = 3(1) - 5 = -2$

$f(-3) = 3(-3) - 5 = -14$

$f(x+2) = 3(x+2) - 5$

$3x + 6 - 5$

$3x + 1$

$V(r) = \frac{4}{3}\pi r^3$

$V(3) = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi$

$V\left(\frac{3}{2}\right) = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3\pi}\left(\frac{27}{8}\right) = \frac{9}{2\pi}$

$V(2r) = \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi(8r^3) = \frac{32}{3}\pi r^3$

$g(t) = 4t^2 - 3t + 5$

$g(2) = 4(2)^2 - 3(2) + 5$

15

$g(t-2) = 4(t-2)^2 - 3(t-2) + 5$

$4t^2 - 19t + 27$

$g(t) - g(2) = 4t^2 - 3t + 5 - 15$

$4t^2 - 3t - 10$

$h(t) = -t^2 + t + 1$

$h(2) = -(2)^2 + (2) + 1 = -4 + 2 + 1 = -1$

$h(-1) = -(-1)^2 + (-1) + 1 = -1 - 1 + 1 = -1$

$h(x+2) = -(x+1)^2 + (x+1) + 1$

23. $f(y) = 3 - \sqrt{y}$

(a) $f(4) = 3 - \sqrt{4} = 1$

(b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$

(c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

24. $f(x) = \sqrt{x+8} + 2$

$f(-8) = \sqrt{(-8) + 8} + 2 = 2$

$f(1) = \sqrt{(1) + 8} + 2 = 5$

(c) $f(x-8) = \sqrt{(x-8)+8} + 2 = \sqrt{x} + 2$

25. $q(x) = \frac{1}{x^2 - 9}$

(a) $q(0) = \frac{1}{0^2 - 9} = -\frac{1}{9}$

(b) $q(3) = \frac{1}{3^2 - 9}$ is undefined.

(c) $q(y+3) = \frac{1}{(y+3)^2 - 9} = \frac{1}{y^2 + 6y}$

26. $q(t) = \frac{2t^2 + 3}{t^2}$

(a) $q(2) = \frac{2(2)^2 + 3}{(2)^2} = \frac{8+3}{4} = \frac{11}{4}$

$q(0) = \frac{2(0)^2 + 3}{(0)^2}$

Division by zero is undefined.

(c) $q(-x) = \frac{2(-x)^2 + 3}{(-x)^2} = \frac{2x^2 + 3}{x^2}$

27. $f(x) = \frac{|x|}{x}$

(a) $f(2) = \frac{|2|}{2} = 1$

(b) $f(-2) = \frac{|-2|}{-2} = -1$

(c) $f(x-1) = \frac{|x-1|}{x-1} = \begin{cases} -1, & \text{if } x < 1 \end{cases}$

$\left(x^2 + 2x + 1\right)$

$+ x + 1 + 1$

$- x^2 - x + 1$

$x - 1$ $\quad 1, \quad \text{if } x > 1$

$f(x) = |x| + 4$

$f(2) = |2| + 4 = 6$

$f(-2) = |-2| + 4 = 6$

$f(x^2) = |x^2| + 4 = x^2 + 4$

29. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \ge 0 \end{cases}$

$f(-1) = 2(-1) + 1 = -1$

$f(0) = 2(0) + 2 = 2$

$f(2) = 2(2) + 2 = 6$

30. $f(x) = \begin{cases} -3x - 3, & x < -1 \\ x^2 + 2x - 1, & x \ge -1 \end{cases}$

$f(-2) = -3(-2) - 3 = 3$

$f(-1) = (-1)^2 + 2(-1) - 1 = -2$

(c) $f(1) = 1^2 + 2(1) - 1 = 2$

$f(x) = -x^2 + 5$

$f(-2) = -(-2)^2 + 5 = 1$

$f(-1) = -(-1)^2 + 5 = 4$

$f(0) = -(0)^2 + 5 = 5$

$f(1) = -(1)^2 + 5 = 4$

31. $f(2) = -(2)^2 + 5 = 1$

x	-2	-1	0	1	2
$f(x)$	1	4	5	-4	1

32. $h(t) = \frac{1}{2}\lvert t + 3 \rvert$

$h(-5) = \frac{1}{2}\lvert -5+3 \rvert = 1$

$h(-4) = \frac{1}{2}\lvert -4+3 \rvert = \frac{1}{2}$

$h(-3) = \frac{1}{2}\lvert -3+3 \rvert = 0$

$h(-2) = \frac{1}{2}\lvert -2+3 \rvert = \frac{1}{2}$

$h(-1) = \frac{1}{2}\lvert -1 + 3 \rvert = 1$

t	-5	-4	-3	-2	-1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

33. $f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \le 0 \\ (x - 2)^2, & x > 0 \end{cases}$

$f(-2) = -\frac{1}{2}(-2) + 4 = 5$

$f(-1) = -\frac{1}{2}(-1) + 4 = 4\frac{1}{2} = \frac{9}{2}$

$f(0) = -\frac{1}{2}(0) + 4 = 4$

$f(1) = (1 - 2)^2 = 1$

$f(2) = (2 - 2)^2 = 0$

x	-2	-1	0	1	2
$f(x)$	5	$\frac{9}{2}$	4	1	0

$f(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \ge 3 \end{cases}$

$f(1) = 9 - (1)^2 = 8$

$f(2) = 9 - (2)^2 = 5$

$f(3) = (3) - 3 = 0$

$f(4) = (4) - 3 = 1$

$f(5) = (5) - 3 = 2$

x	1	2	3	4	5
$f(x)$	8	5	0	1	2

$15 - 3x = 0$

$3x = 15$

$= 5$

$f(x) = 4x + 6$

$4x + 6 = 0$

$4x = -6$

$x = -\frac{3}{2}$

36.

$\frac{3x - 4}{5} = 0$

$3x - 4 = 0$

$x = \frac{4}{3}$

$f(x) = \frac{12 - x^2}{8}$

$\frac{12 - x^2}{8} = 0$

8

$x^2 = 12$

38. $x = \pm\sqrt{12} = \pm 2\sqrt{3}$

$f(x) = x^2 - 81$

$x^2 - 81 = 0$

$x^2 = 81$

$x = \pm 9$

$f(x) = x^2 - 6x - 16$

$x^2 - 6x - 16 = 0$

$(x - 8)(x + 2) = 0$

$-8 = 0 \Rightarrow x = 8$

$+2 = 0 \Rightarrow x = -2$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x + 1)(x - 1) = 0$

$x = 0$, $x = -1$, or $x = 1$

$f(x) = x^3 - x^2 - 3x + 3$

$x^3 - x^2 - 3x + 3 = 0$

$x^2(x - 1) - 3(x - 1) = 0$

42. $(x - 1)(x^2 - 3) = 0$

$x - 1 = 0 \Rightarrow x = 1$

$x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}$

$f(x) = g(x)$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x - 2 = 0$ $\quad x + 1 = 0$

$x = 2$ $\quad x = -1$

$f(x) = g(x)$

$x^2 + 2x + 1 = 5x + 19$

$x^2 - 3x - 18 = 0$

$(x - 6)(x + 3) = 0$

$x - 6 = 0$ $\quad x + 3 = 0$

$x = 6$ $\quad x = -3$

45. $f(x) = g(x)$

$x^4 - 2x^2 = 2x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x^2(x + 2)(x - 2) = 0$

$x^2 = 0 \Rightarrow x = 0$

$x + 2 = 0 \Rightarrow x = -2$

$x - 2 = 0 \Rightarrow x = 2$

46. $f(x) = g(x)$

$\sqrt{x} - 4 = 2 - x$

$x + \sqrt{x} - 6 = 0$

$(\sqrt{x} + 3)(\sqrt{x} - 2) = 0$

$\sqrt{x} + 3 = 0 \Rightarrow \sqrt{x} = -3$, which is a contradiction, since $\sqrt{\ }$ represents the principal square root.

$\sqrt{x} - 2 = 0 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$

$f(x) = 5x^2 + 2x - 1$

Because $f(x)$ is a polynomial, the domain is all real numbers x.

$f(x) = 1 - 2x^2$

Because $f(x)$ is a polynomial, the domain is all real numbers x.

49. $g(y) = \sqrt{y + 6}$

$y + 6 \geq 0$

$y \geq -6$

Domain:

The domain is all real numbers y such that $y \geq -6$.

$f(t) = \sqrt[3]{t + 4}$

Because $f(t)$ is a cube root, the domain is all real numbers t.

51. $g(x) = \dfrac{1}{x} - \dfrac{3}{x + 2}$

The domain is all real numbers x except $x = 0$, $x = -2$.

52. $h(x) = \dfrac{6}{x^2 - 4x}$

$x^2 - 4x \neq 0$

$x(x - 4) \neq 0$

$x \neq 0$

$x - 4 \neq 0 \Rightarrow x \neq 4$

The domain is all real numbers x except $x = 0, x = 4.$

53. $f(s) = \frac{\sqrt{s-1}}{s-4}$

Domain: $s - 1 \geq 0 \Rightarrow s \geq 1$ and $s \neq 4$

The domain consists of all real numbers s, such that $s \geq 1$ and $s \neq 4$.

54. $f(x) = \frac{\sqrt{x+6}}{6+x}$

Domain: $x + 6 \geq 0 \Rightarrow x \geq -6$ and $x \neq -6$

The domain is all real numbers x such that $x > -6$ or $(-6, \infty)$.

$f(x) = \frac{x-4}{\sqrt{x}}$

The domain is all real numbers x such that $x > 0$ or $(0, \infty)$.

56. $f(x) = \frac{x+2}{\sqrt{x-10}}$

$x - 10 > 0$

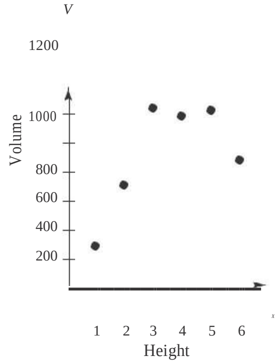
$x > 10$

The domain is all real numbers x such that $x > 10$.

57. (a)

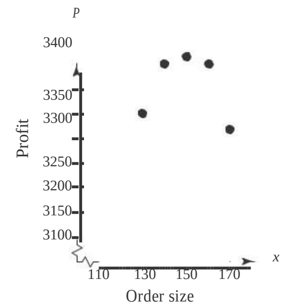
Height, x	Volume, V
1	484
2	800
3	972
4	1024
5	980
6	864

The volume is maximum when $x = 4$ and $V = 1024$ cubic centimeters.



(a) The maximum profit is $3375.

(b)



Yes, P is a function of x.

(c) Profit = Revenue − Cost

$= \begin{bmatrix} \begin{pmatrix} \text{price} \\ \text{per unit} \end{pmatrix} \begin{pmatrix} \text{number} \\ \text{of units} \end{pmatrix} \end{bmatrix} - (\text{cost}) \begin{pmatrix} \text{number} \\ \text{of units} \end{pmatrix}$

$= [90 - (x - 100)(0.15)]x - 60x, \; x > 100$

$= (90 - 0.15x + 15)x - 60x$

$= (105 - 0.15x)x - 60x$

$= 105x - 0.15x^2 - 60x$

$= 45x - 0.15x^2, \; x > 100$

59. $A = s^2$ and $P = 4s \Rightarrow \frac{P}{4} = s$

$A = \left(\frac{P}{4}\right)^2 = \frac{P^2}{16}$

$A = \pi r^2, \; C = 2\pi r$

$r = \frac{C}{2\pi}$

$A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$

$y = -\frac{1}{10}x^2 + 3x + 6$

$y(25) = -\frac{1}{10}(25)^2 + 3(25) + 6 = 18.5$ feet

If the child holds a glove at a height of 5 feet, then the ball *will* be over the child's head because it will be at a height of 18.5 feet.

62. (a) $V = l \cdot w \cdot h = x \cdot y \cdot x = x^2 y$ where $4x + y = 108$. So, $y = 108 - 4x$ and

$V = x^2(108 - 4x) = 108x^2 - 4x^3.$

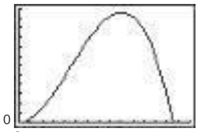
Domain: $0 < x < 27$

(b) 12,000

V is a function of *x*.

$V = x(24 - 2x)^2$

Domain: $0 < x < 12$



The dimensions that will maximize the volume of the package are 18 × 18 × 36. From the graph, the maximum volume occurs when $x = 18$. To find the dimension for y, use the equation $y = 108 - 4x$.

$$= 108 - 4x = 108 - 4(18) = 108 - 72 = 36$$

$$A = \frac{1}{2}bh = \frac{1}{2}xy$$

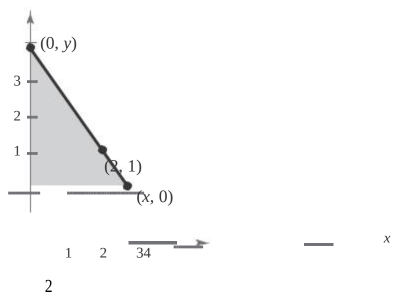
Because $(0, y)$, $(2, 1)$, and $(x, 0)$ all lie on the same line, the slopes between any pair are equal.

$$2 - 0 \qquad x - 2$$

$$2 \qquad x - 2$$

$$y = \frac{2}{x-2} + 1$$

$$y = \frac{x}{x-2}$$



So, $A = \frac{1}{2}x\left(\frac{x}{x-2}\right) = \frac{x^2}{2(x-2)}$.

The domain of A includes x-values such that $x^2/[2(x-2)] > 0$. By solving this inequality, the domain is $x > 2$.

$A = l \cdot w = (2x)y = 2xy$

But $y = \sqrt{36 - x^2}$, so $A = 2x\sqrt{36 - x^2}$. The domain is $0 < x < 6$.

For 2008 through 2011, use

$$p(t) = 2.77t + 45.2.$$

2008: $p(8) = 2.77(8) + 45.2 = 67.36\%$

2009: $p(9) = 2.77(9) + 45.2 = 70.13\%$

2010: $p(10) = 2.77(10) + 45.2 = 72.90\%$

For 2011 through 2014, use

$$p(t) = 1.95t + 55.9.$$

2012: $p(12) = 1.95(12) + 55.9 = 79.30\%$

2013: $p(13) = 1.95(13) + 55.9 = 81.25\%$

2014: $p(14) = 1.95(14) + 55.9 = 83.20\%$

66. For 2000 through 2006, use

$$p(t) = -0.757t^2 + 20.80t + 127.2.$$

2002: $p(2) = -0.757(2)^2 + 20.80(2) + 127.2 = \$165{,}722$

2003: $p(3) = -0.757(3)^2 + 20.80(3) + 127.2 = \$182{,}787$

2004: $p(4) = -0.757(4)^2 + 20.80(4) + 127.2 = \$198{,}288$

2005: $p(5) = -0.757(5)^2 + 20.80(5) + 127.2 = \$212{,}275$

2006: $p(6) = -0.757(6)^2 + 20.80(6) + 127.2 = \$224{,}748$

For 2007 through 2011, use

$$p(t) = 3.879t^2 - 82.50t + 605.8.$$

2007: $p(7) = 3.879(7)^2 - 82.50(7) + 605.8 = \$218{,}371$

2008: $p(8) = 3.879(8)^2 - 82.50(8) + 605.8 = \$194{,}056$

2009: $p(9) = 3.879(9)^2 - 82.50(9) + 605.8 = \$177{,}499$

2010: $p(10) = 3.879(10)^2 - 82.50(10) + 605.8 = \$168{,}700$

2011: $p(11) = 3.879(11)^2 - 82.50(11) + 605.8 = \$167{,}659$

For 2012 through 2014, use

$$p(t) = -4.171t^2 + 124.34t - 714.2$$

2012: $p(12) = -4.171(12)^2 + 124.34(12) - 714.2 = \$177{,}256$

2013: $p(13) = -4.171(13)^2 + 124.34(13) - 714.2 = \$197{,}321$

2014: $p(14) = -4.171(14)^2 + 124.34(14) - 714.2 = \$209{,}044$

(a) Cost = variable costs + fixed costs

$= 12.30x + 98{,}000$

Revenue = price per unit × number of units

$= 17.98x$

Profit = Revenue − Cost

$= 17.98x - (12.30x + 98{,}000)$

$P = 5.68x - 98{,}000$

(a) *Model:*

(Total cost) = (Fixed costs) + (Variable costs)

Labels: Total cost $= C$

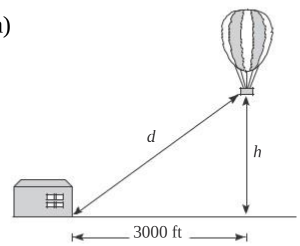
Fixed cost $= 6000$

Variable costs $= 0.95x$

Equation: $C = 6000 + 0.95x$

(b) $\overline{C} = \frac{C}{x} = \frac{6000 + 0.95x}{x} = \frac{6000}{x} + 0.95$

69. (a)



(b) $(3000)^2 + h^2 = d^2$

$h = \sqrt{d^2 - (3000)^2}$

Domain: $d \geq 3000$ (because both $d \geq 0$ and $d^2 - (3000)^2 \geq 0$)

70. $F(y) = 149.76\sqrt{10}\, y^{5/2}$

(a)

y	5	10	20	30	40
$F(y)$	26,474.08	149,760.00	847,170.49	2,334,527.36	4,792,320

The force, in tons, of the water against the dam increases with the depth of the water.

(b) It appears that approximately 21 feet of water would produce 1,000,000 tons of force.

(c) $1{,}000{,}000 = 149.76\sqrt{10}\, y^{5/2}$

$\frac{1{,}000{,}000}{149.76\sqrt{10}} = y^{5/2}$

$2111.56 \approx y^{5/2}$

$21.37 \text{ feet} \approx y$

71. (a) $R = n(\text{rate}) = n[8.00 - 0.05(n - 80)], \; n \geq 80$

$= 12.00n - 0.05n^2 = 12n - \frac{n^2}{20} = \frac{240n - n^2}{20}, \; n \geq 80$

(b)

n	90	100	110	120	130	140	150
$R(n)$	\$675	\$700	\$715	\$720	\$715	\$700	\$675

The revenue is maximum when 120 people take the trip.

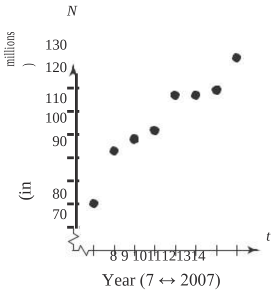
(a) $\dfrac{f(2014) - f(2007)}{2014 - 2007} = \dfrac{125.8 - 80.0}{7}$

$= \dfrac{45.8}{7}$

≈ 6.54

Approximately 6.54 million more tax returns were made through e-file each year from 2007 to 2014.

(b)



$N = 6.54t + 34.2$

(d)

t	7	8	9	10
N	80.0	86.5	93.1	99.6

t	11	12	13	14
N	106.1	112.7	119.2	125.8

The algebraic model is a good fit to the actual data.

$y = 6.05x + 40.0$; The models are similar.

73. $f(x) = x^2 - 2x + 4$

$f(2 + h) = (2 + h)^2 - 2(2 + h) + 4$

$4 + 4h + h^2 - 4 - 2h + 4$

$h^2 + 2h + 4$

$f(2) = (2)^2 - 2(2) + 4 = 4$

$f(2 + h) - f(2) = h^2 + 2h$

$\dfrac{f(2 + h) - f(2)}{h} = \dfrac{h^2 + 2h}{h} = h + 2, h \neq 0$

74. $f(x) = 5x - x^2$

$f(5 + h) = 5(5 + h) - (5 + h)^2$

$25 + 5h - (25 + 10h + h^2)$

$25 + 5h - 25 - 10h - h^2$

$-h^2 - 5h$

$f(5) = 5(5) - (5)^2$

$25 - 25 = 0$

$\dfrac{f(5 + h) - f(5)}{h} = \dfrac{-h^2 - 5h}{h}$

$\dfrac{-h(h + 5)}{h} = -(h + 5), h \neq 0$

75. $f(x) = x^3 + 3x$

$f(x + h) = (x + h)^3 + 3(x + h)$

$= x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h$

$\dfrac{f(x + h) - f(x)}{h} = \dfrac{(x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h) - (x^3 + 3x)}{h}$

$= \dfrac{h(3x^2 + 3xh + h^2 + 3)}{h}$

$= 3x^2 + 3xh + h^2 + 3, h \neq 0$

$f(x) = 4x^3 - 2x$

$f(x + h) = 4(x + h)^3 - 2(x + h)$

$= 4(x^3 + 3x^2h + 3xh^2 + h^3) - 2x - 2h$

$= 4x^3 + 12x^2h + 12xh^2 + 4h^3 - 2x - 2h$

$\dfrac{f(x + h) - f(x)}{h} = \dfrac{(4x^3 + 12x^2h + 12xh^2 + 4h^3 - 2x - 2h) - (4x^3 - 2x)}{h}$

$\dfrac{12x^2h + 12xh^2 + 4h^3 - 2h}{h}$

$\dfrac{h(12x^2 + 12xh + 4h^2 - 2)}{h}$

$= 12x^2 + 12xh + 4h^2 - 2, h \neq 0$

$g(x) = \frac{1}{x^2}$

$$\frac{g(x) - g(3)}{x - 3} = \frac{\frac{1}{x^2} - \frac{1}{9}}{x - 3}$$

$$= \frac{9 - x^2}{9x^2(x - 3)}$$

$$= \frac{-(x + 3)(x - 3)}{9x^2(x - 3)}$$

$$= -\frac{x + 3}{9x^2}, x \neq 3$$

$f(t) = \frac{1}{t - 2}$

$f(1) = \frac{1}{1 - 2} = -1$

$$\frac{f(t) - f(1)}{t - 1} = \frac{\frac{1}{t - 2} - (-1)}{t - 1}$$

$$= \frac{1 + (t - 2)}{(t - 2)(t - 1)}$$

$$= \frac{t - 1}{(t - 2)(t - 1)}$$

$$= \frac{1}{t - 2}, t \neq 1$$

79. $f(x) = \sqrt{5x}$

$$\frac{f(x) - f(5)}{x - 5} = \frac{\sqrt{5x} - 5}{x - 5}, x \neq 5$$

80. $f(x) = x^{2/3} + 1$

$f(8) = 8^{2/3} + 1 = 5$

$$\frac{f(x) - f(8)}{x - 8} = \frac{x^{2/3} + 1 - 5}{x - 8} = \frac{x^{2/3} - 4}{x - 8}, x \neq 8$$

By plotting the points, we have a parabola, so $g(x) = cx^2$. Because $(-4, -32)$ is on the graph, you have $-32 = c(-4)^2 \Rightarrow c = -2$. So, $g(x) = -2x^2$.

By plotting the data, you can see that they represent a line, or $f(x) = cx$. Because $(0, 0)$ and $\left(1, \frac{1}{4}\right)$ are on the line, the slope is $\frac{1}{4}$. So, $f(x) = \frac{1}{4}x$.

Because the function is undefined at 0, we have $r(x) = c/x$. Because $(-4, -8)$ is on the graph, you have $-8 = c/-4 \Rightarrow c = 32$. So, $r(x) = 32/x$.

By plotting the data, you can see that they represent $h(x) = c\sqrt{|x|}$. Because $\sqrt{|-4|} = 2$ and $\sqrt{|-1|} = 1$, and the corresponding y-values are 6 and 3, $c = 3$ and $h(x) = 3\sqrt{|x|}$.

85. False. The equation $y^2 = x^2 + 4$ is a relation between x and y. However, $y = \pm\sqrt{x^2 + 4}$ does not represent a function.

True. The set represents a function. Each x-value is mapped to exactly one y-value.

The domain of $f(x) = \sqrt{x - 1}$ includes $x = 1$, $x \geq 1$ and the domain of $g(x) = \frac{1}{\sqrt{x - 1}}$ does not include $x = 1$ because you cannot divide by 0. The domain of $g(x) = \frac{1}{\sqrt{x - 1}}$ is $x > 1$. So, the functions do not have the same domain.

90. Because $f(x)$ is a function of an even root, the radicand cannot be negative. $g(x)$ is an odd root, therefore the radicand can be any real number. So, the domain of g is all real numbers x and the domain of f is all real numbers x such that $x \geq 2$.

91. No; x is the independent variable, f is the name of the function.

True. A function is a relation by definition.

False. The range is $[-1, \infty)$.

92. (a) The height h is a function of t because for each value of t there is a corresponding value of h for $0 \leq t \leq 2.6$.

Using the graph when $t = 0.5$, $h \approx 20$ feet and when $t = 1.25$, $h \approx 28$ feet.

The domain of h is approximately $0 \leq t \leq$

2.6. No, the time t is not a function of the

height h
because some values of h correspond to more than one value of t.

(a) Yes. The amount that you pay in sales tax will increase as the price of the item purchased increases.

No. The length of time that you study the night before an exam does not necessarily determine your score on the exam.

(a) No. During the course of a year, for example, your salary may remain constant while your savings account balance may vary. That is, there may be two or more outputs (savings account balances) for one input (salary).

Yes. The greater the height from which the ball is dropped, the greater the speed with which the ball will strike the ground.

Section 2.3 Analyzing Graphs of Functions

Vertical Line Test

zeros

decreasing

maximum

average rate of change; secant

odd

Domain: $(-2, 2]$; Range: $[-1, 8]$

$f(-1) = -1$

$f(0) = 0$

$f(1) = -1$

$f(2) = 8$

Domain : $[-1, \infty)$; Range: $(-\infty, 7]$

$f(-1) = 4$

$f(0) = 3$

$f(1) = 6$

$f(3) = 0$

Domain: $(-\infty, \infty)$; Range: $(-2, \infty)$

$f(2) = 0$

$f(1) = 1$

$f(3) = 2$

$f(-1) = 3$

Domain: $(-\infty, \infty)$; Range: $(-\infty, 1]$

$f(-2) = -3$

$f(1) = 0$

$f(0) = 1$

$f(2) = -3$

A vertical line intersects the graph at most once, so *y is* a function of *x*.

y is not a function of *x*. Some vertical lines intersect the graph twice.

A vertical line intersects the graph more than once, so *y is not* a function of *x*.

A vertical line intersects the graph at most once, so *y is* a function of *x*.

$f(x) = 3x + 18$
$3x + 18 = 0$
$3x = -18$
$= -6$

$f(x) = 15 - 2x$
$15 - 2x = 0$
$2x = -15$
$= \frac{15}{2}$

$f(x) = 2x^2 - 7x - 30$
$2x^2 - 7x - 30 = 0$
$(2x + 5)(x - 6) = 0$
$2x + 5 = 0$ or $x - 6 = 0$
$x = -\frac{5}{2}$ $\quad x = 6$

$f(x) = 3x^2 + 22x - 16$
$3x^2 + 22x - 16 = 0$
$(3x - 2)(x + 8) = 0$
$3x - 2 = 0 \Rightarrow x = \frac{2}{3}$

19. $f(x) = \dfrac{x + 3}{2x^2 - 6}$

$\dfrac{x + 3}{2x^2 - 6} = 0$
$x + 3 = 0$
$x = -3$

20. $f(x) = \dfrac{x^2 - 9x + 14}{4x}$

$\dfrac{x^2 - 9x + 14}{4x} = 0$
$(x - 7)(x - 2) = 0$
$x - 7 = 0 \Rightarrow x = 7$
$x - 2 = 0 \Rightarrow x = 2$

21. $f(x) = \frac{1}{3}x^3 - 2x$

$\frac{1}{3}x^3 - 2x = 0$

$(3)\left(\frac{1}{3}x^3 - 2x\right) = 0(3)$

$x^3 - 6x = 0$

$x\left(x^2 - 6\right) = 0$

$x = 0$ or $x^2 - 6 = 0$

$x^2 = 6$

$x = \pm\sqrt{6}$

$f(x) = -25x^4 + 9x^2$

$x^2\left(25x^2 - 9\right) = 0$

$-x^2 = 0$ or $25x^2 - 9 = 0$

$x = 0$ $\quad 25x^2 = 9$

$x^2 = \frac{9}{25}$

$x = \pm\frac{3}{5}$

23. $f(x) = x^3 - 4x^2 - 9x + 36$

$x^3 - 4x^2 - 9x + 36 = 0$

$x^2(x - 4) - 9(x - 4) = 0$

$(x - 4)\left(x^2 - 9\right) = 0$

$x - 4 = 0 \Rightarrow x = 4$

$x^2 - 9 = 0 \Rightarrow x = \pm 3$

24. $f(x) = 4x^3 - 24x^2 - x + 6$

$4x^3 - 24x^2 - x + 6 = 0$

$4x^2(x - 6) - 1(x - 6) = 0$

$(x - 6)\left(4x^2 - 1\right) = 0$

$(x - 6)(2x + 1)(2x - 1) = 0$

$x - 6 = 0$ or $2x + 1 = 0$ or $2x - 1 = 0$

$x = 6$ $\quad x = -\frac{1}{2}$ $\quad x = \frac{1}{2}$

25. $f(x) = \sqrt{2x} - 1$

$\sqrt{2x} - 1 = 0$

$\sqrt{2x} = 1$

$2x = 1$

$x = \frac{1}{2}$

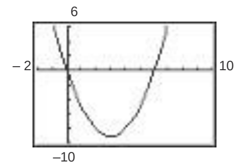
26. $f(x) = \sqrt{3x + 2}$

$\sqrt{3x + 2} = 0$

$3x + 2 = 0$

$-\frac{2}{3} = x$

27. (a)



Zeros: $x = 0, 6$

$f(x) = x^2 - 6x$

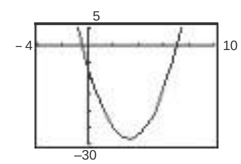
$x^2 - 6x = 0$

$x(x - 6) = 0$

$x = 0 \Rightarrow x = 0$

$x - 6 = 0 \Rightarrow x = 6$

28. (a)



Zeros: $x = -0.5, 7$

$f(x) = 2x^2 - 13x - 7$

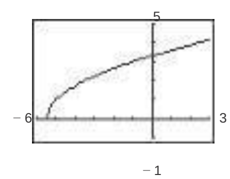
$2x^2 - 13x - 7 = 0$

$(2x + 1)(x - 7) = 0$

$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$

$x - 7 = 0 \Rightarrow x = 7$

29. (a)



Zero: $x = -5.5$

(b) $f(x) = \sqrt{2x + 11}$

$\sqrt{2x + 11} = 0$

$2x + 11 = 0$

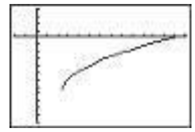
$x = -\frac{11}{2}$

30. (a)

4

−4 28

12



Zero: $x = 26$

$f(x) = \sqrt{3x - 14} - 8$

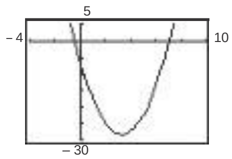
$\sqrt{3x - 14} - 8 = 0$

$\sqrt{3x - 14} = 8$

$3x - 14 = 64$

$x = 26$

31. (a)



Zero: $x = 0.3333$

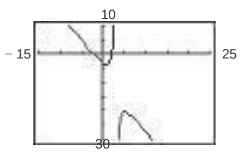
$f(x) = \frac{3x - 1}{6}$

$\frac{3x - 1}{x - 6} = 0$

$3x - 1 = 0$

$x = \frac{1}{3}$

32. (a)



Zeros: $x = \pm 2.1213$

$f(x) = \frac{2x^2 - 9}{3 - x}$

$\frac{2x^2 - 9}{3 - x} = 0$

$2x^2 - 9 = 0 \Rightarrow x = \pm\frac{3\sqrt{2}}{2} = \pm 2.1213$

$f(x) = -\frac{1}{2}x^3$

The function is decreasing on $(-\infty, \infty)$.

$f(x) = x^2 - 4x$

The function is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

35. $f(x) = \sqrt{x^2 - 1}$

The function is decreasing on $(-\infty, -1)$ and increasing on $(1, \infty)$.

$f(x) = x^3 - 3x^2 + 2$

The function is increasing on $(-\infty, 0)$ and $(2, \infty)$ and decreasing on $(0, 2)$.

$f(x) = |x + 1| + |x - 1|$

The function is increasing on $(1, \infty)$.

The function is constant on $(-1, 1)$.

The function is decreasing on $(-\infty, -1)$.

The function is decreasing on $(-2, -1)$ and $(-1, 0)$ and increasing on $(-\infty, -2)$ and $(0, \infty)$.

39. $f(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$

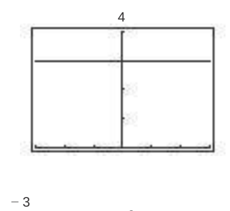
The function is decreasing on $(-1, 0)$ and increasing on $(-\infty, -1)$ and $(0, \infty)$.

40. $f(x) = \begin{cases} x + 3, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x + 1, & x > 2 \end{cases}$

The function is increasing on $(-\infty, 0)$ and $(2, \infty)$.

The function is constant on $(0, 2)$.

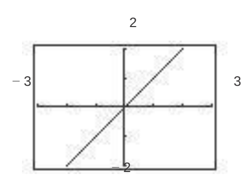
41. $f(x) = 3$



Constant on $(-\infty, \infty)$

x	–2	–1	0	1	2
$f(x)$	3	3	3	3	3

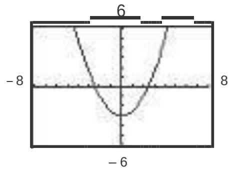
42. $g(x) = x$



Increasing on $(-\infty, \infty)$

x	–2	–1	0	1	2
$g(x)$	–2	–1	0	1	2

43. $g(x) = \frac{1}{2}x^2 - 3$

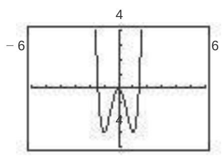


Decreasing on $(-\infty, 0)$.

Increasing on $(0, \infty)$.

x	–2	−1	0	1	2
$g(x)$	−1	$-\frac{5}{2}$	− 3	$-\frac{5}{2}$	−1

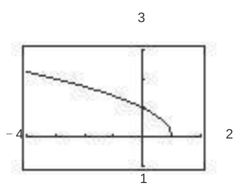
$f(x) = 3x^4 - 6x^2$



Increasing on $(-1, 0)$, $(1, \infty)$; Decreasing on $(-\infty, -1)$, $(0, 1)$

x	–2	–1	0	1	2
$f(x)$	24	–3	0	–3	24

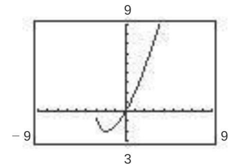
45. $f(x) = \sqrt{1 - x}$



Decreasing on $(-\infty, 1)$

x	–3	-2	–1	0	1
$f(x)$	2	$\sqrt{3}$	$\sqrt{2}$	1	0

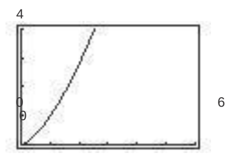
46. $f(x) = x\sqrt{x + 3}$



Increasing on $(-2, \infty)$; Decreasing on $(-3, -2)$

x	–3	–2	–1	0	1
$f(x)$	0	–2	–1.414	0	2

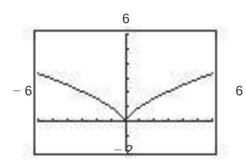
$f(x) = x^{3/2}$



Increasing on $(0, \infty)$

x	0	1	2	3	4
$f(x)$	0	1	2.8	5.2	8

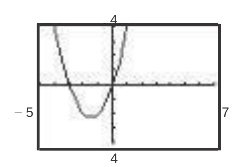
$f(x) = x^{2/3}$



Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

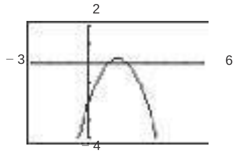
x	–2	–1	0	1	2
$f(x)$	1.59	1	0	1	1.59

$f(x) = x(x + 3)$



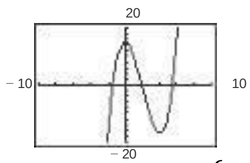
Relative minimum: $(-1.5, -2.25)$

50. $f(x) = -x^2 + 3x - 2$



Relative maximum: $(1.5, 0.25)$

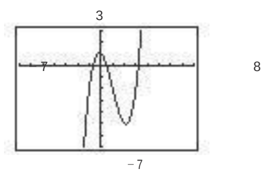
51. $h(x) = x^3 - 6x^2 + 15$



Relative minimum: $(4, -17)$

Relative maximum: $(0, 15)$

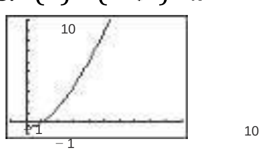
52. $f(x) = x^3 - 3x^2 - x + 1$



Relative maximum: $(-0.15, 1.08)$

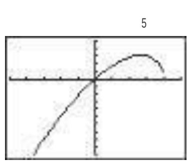
Relative minimum: $(2.15, -5.08)$

53. $h(x) = (x - 1)\sqrt{x}$



Relative minimum: $(0.33, -0.38)$

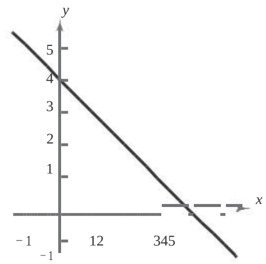
54. $g(x) = x\sqrt{4 - x}$



Relative maximum: $(2.67, 3.08)$

55. $f(x) = 4 - x$

$f(x) \ge 0$ on $(-\infty, 4]$



56. $f(x) = 4x + 2$

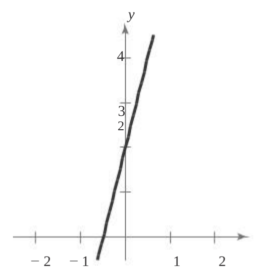
$f(x) \ge 0$ on $\left[-\frac{1}{2}, \infty\right)$

$4x + 2 \ge 0$

$4x \ge -2$

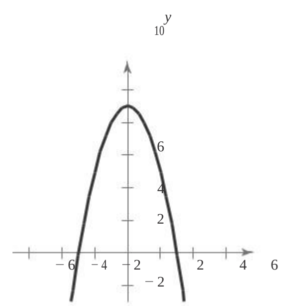
$x \ge -\frac{1}{2}$

$\left[-\frac{1}{2}, \infty\right)$



57. $f(x) = 9 - x^2$

$f(x) \ge 0$ on $[-3, 3]$



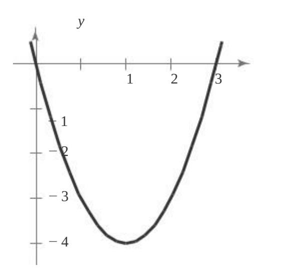
58. $f(x) = x^2 - 4x$

$f(x) \ge 0$ on $(-\infty, 0]$ and $[4, \infty)$

$x^2 - 4x \ge 0$

$x(x - 4) \ge 0$

$(-\infty, 0], [4, \infty)$



59. $f(x) = \sqrt{x - 1}$

$f(x) \geq 0$ on $[1, \infty)$

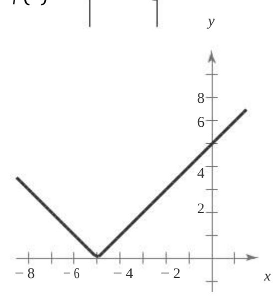
$\sqrt{x - 1} \geq 0$

$x - 1 \geq 0$

$x \geq 1$

$[1, \infty)$

60. $f(x) = |x + 5|$



$f(x)$ is always greater than or equal to 0. $f(x) \geq 0$ for all x.

$(-\infty, \infty)$

61. $f(x) = -2x + 15$

$$\frac{f(3) - f(0)}{3 - 0} = \frac{9 - 15}{3} = -2$$

The average rate of change from $x_1 = 0$ to $x_2 = 3$ is -2.

62. $f(x) = x^2 - 2x + 8$

$$\frac{f(5) - f(1)}{5 - 1} = \frac{23 - 7}{4} = \frac{16}{4} = 4$$

The average rate of change from $x_1 = 1$ to $x_2 = 5$ is 4.

63. $f(x) = x^3 - 3x^2 - x$

$$\frac{f(2) - f(-1)}{2 - (-1)} = \frac{-6 - (-3)}{3} = \frac{-3}{3} = -1$$

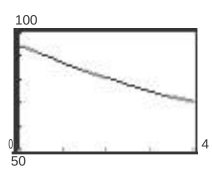
The average rate of change from $x_1 = -1$ to $x_2 = 2$ is -1.

64. $f(x) = -x^3 + 6x^2 + x$

$$\frac{f(6) - f(1)}{6 - 1} = \frac{6 - 6}{5} = 0$$

The average rate of change from $x_1 = 1$ to $x_2 = 6$ is 0.

65. (a)



To find the average rate of change of the amount the U.S. Department of Energy spent for research and development from 2010 to 2014, find the average rate of change from $(0, f(0))$ to $(4, f(4))$.

$\frac{f(4) - f(0)}{4 - 0}$ [redacted] $= \frac{\text{[redacted]}}{4} = \frac{\text{[redacted]}}{4} = -6.1364$

The amount the U.S. Department of Energy spent on research and development for defense decreased by about $6.14 billion each year from 2010 to 2014.

66. Average rate of change $= \frac{s(t_2) - s(t_1)}{t_2 - t_1}$

$$= \frac{s(9) - s(0)}{9 - 0}$$

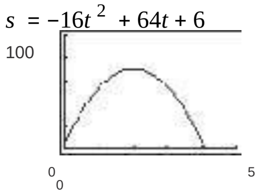
$$= \frac{540 - 0}{9 - 0}$$

$= 60$ feet per second.

As the time traveled increases, the distance increases rapidly, causing the average speed to increase with each time increment. From $t = 0$ to $t = 4$, the average speed is less than from $t = 4$ to $t = 9$. Therefore, the overall average from $t = 0$ to $t = 9$ falls below the average found in part (b).

$s_0 = 6, v_0 = 64$

$s = -16t^2 + 64t + 6$



$\frac{s(3) - s(0)}{3 - 0} = \frac{54 - 6}{3} = 16$

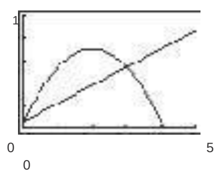
The slope of the secant line is positive.

$s(0) = 6, m = 16$

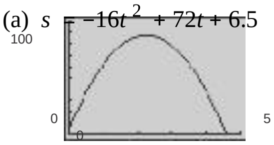
Secant line: $y - 6 = 16(t - 0)$

$y = 16t + 6$

(f)



(a) $s = -16t^2 + 72t + 6.5$



(c) The average rate of change from $t = 0$ to $t = 4$:

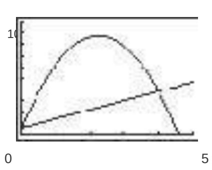
$\frac{s(4) - s(0)}{4 - 0} = \frac{38.5 - 6.5}{4} = \frac{32}{4} = 8$ feet per second

The slope of the secant line through $(0, s(0))$ and $(4, s(4))$ is positive.

The equation of the secant line:

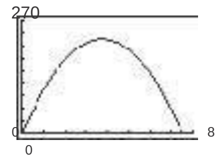
$m = 8, y = 8t + 6.5$

(f)



$v_0 = 120, s_0 = 0$

$s = -16t^2 + 120t$



(c) The average rate of change from $t = 3$ to $t = 5$:

$\frac{s(5) - s(3)}{5 - 3} = \frac{\ }{2} = -\frac{\ }{2} = -8$ feet per second

The slope of the secant line through $(3, s(3))$ and $(5, s(5))$ is negative.

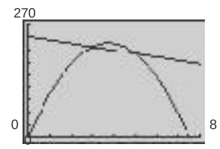
The equation of the secant line: $m = -8$

Using $(5, s(5)) = (5, 200)$ we have

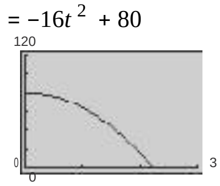
$y - 200 = -8(t - 5)$

$y = -8t + 240.$

(f)



(a) $s = -16t^2 + 80$



(c) The average rate of change from $t = 1$ to $t = 2$:

$\frac{s(2) - s(1)}{2 - 1} = \frac{16 - 64}{1} = -\frac{48}{1} = -48$ feet per second

The slope of the secant line through $(1, s(1))$ and $(2, s(2))$ is negative.

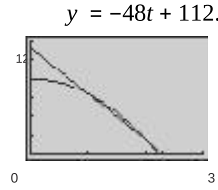
The equation of the secant line: $m = -48$

Using $(1, s(1)) = (1, 64)$ we have

$y - 64 = -48(t - 1)$

$y = -48t + 112.$

(f)



$f(x) = x^6 - 2x^2 + 3$

$f(-x) = (-x)^6 - 2(-x)^2 + 3$

$x^6 - 2x^2 + 3$

$f(x)$

The function is even. y-axis symmetry.

$g(x) = x^3 - 5x$

$g(-x) = (-x)^3 - 5(-x)$

$-x^3 + 5x$

$-g(x)$

The function is odd. Origin symmetry.

$h(x) = x\sqrt{x + 5}$

$h(-x) = (-x)\sqrt{-x + 5}$

$= -x\sqrt{5 - x}$

$\neq h(x)$

$\neq -h(x)$

The function is neither odd nor even. No symmetry.

74. $f(x) = x\sqrt{1 - x^2}$

$f(-x) = -x\sqrt{1 - (-x)^2}$

$= -x\sqrt{1 - x^2}$

$= -f(x)$

The function is odd. Origin symmetry.

75. $f(s) = 4s^{3/2}$

$= 4(-s)^{3/2}$

$\neq f(s)$

$\neq -f(s)$

The function is neither odd nor even. No symmetry.

76. $g(s) = 4s^{2/3}$

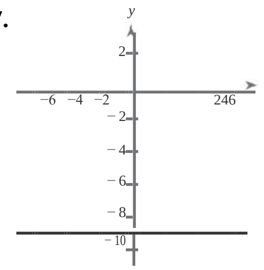
$g(-s) = 4(-s)^{2/3}$

$= 4s^{2/3}$

$= g(s)$

The function is even. y-axis symmetry.

77.



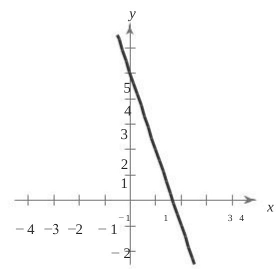
The graph of $f(x) = -9$ is symmetric to the y-axis, which implies $f(x)$ is even.

$f(-x) = -9$

$f(x)$

The function is even.

78. $f(x) = 5 - 3x$



The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$f(-x) = 5 - 3(-x)$

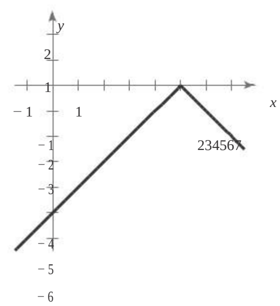
$= 5 + 3x$

$\neq f(x)$

$\neq -f(x)$

The function is neither even nor odd.

79. $f(x) = -|x - 5|$



The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$f(x) = -|(-x) - 5|$

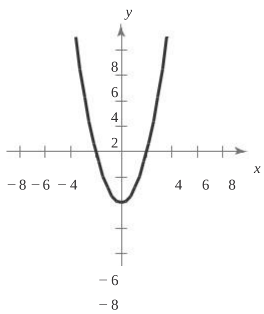
$= -|-x - 5|$

$\neq f(x)$

$\neq -f(x)$

The function is neither even nor odd.

$h(x) = x^2 - 4$

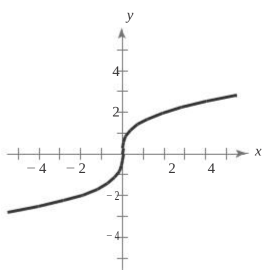


The graph displays y-axis symmetry, which implies $h(x)$ is even.

$h(-x) = (-x)^2 - 4 = x^2 - 4 = h(x)$

The function is even.

81. $f(x) = \sqrt[3]{4x}$



The graph displays origin symmetry, which implies $f(x)$ is odd.

$f(-x) = \sqrt[3]{4(-x)}$

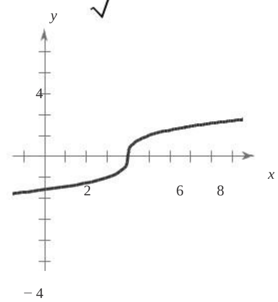
$\sqrt[3]{-4x}$

$-\sqrt[3]{4x}$

$-f(x)$

The function is odd.

82. $f(x) = \sqrt[3]{x - 4}$



The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$f(-x) = \sqrt[3]{(-x) - 4}$

$\sqrt[3]{-x-4}$

$\sqrt[3]{-(x+4)}$

$-\sqrt[3]{x+4}$

$f(x)$

$-f(x)$

The function is neither even nor odd.

$h = \text{top} - \text{bottom}$

$3 - (4x - x^2)$

$3 - 4x + x^2$

$h = \text{top} - \text{bottom}$

$(4x - x^2) - 2x$

$2x - x^2$

$L = \text{right} - \text{left}$

$2 - \sqrt[3]{2y}$

$L = \text{right} - \text{left}$

$\frac{2}{y} - 0$

$\frac{2}{y}$

The error is that $-2x^3 - 5 \neq -(2x^3 - 5)$. The correct process is as follows.

$f(x) = 2x^3 - 5$

$f(-x) = 2(-x)^3 - 5$

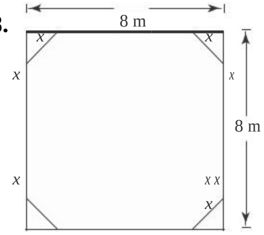
$-2x^3 - 5$

$-(2x^3 + 5)$

$f(-x) \neq -f(x)$ and $f(-x) \neq f(x)$, so the function

$f(x) = 2x^3 - 5$ is neither odd nor even.

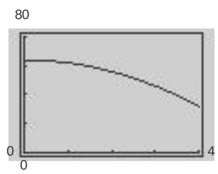
88.



$A = (8)(8) - 4\left(\frac{1}{2}\right)(x)(x) = 64 - 2x^2$

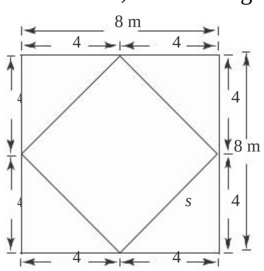
Domain: $0 \le x \le 4$

(b)



Range: $32 \le A \le 64$

(c) When $x = 4$, the resulting figure is a square.



By the Pythagorean Theorem,

$4^2 + 4^2 = s^2 \Rightarrow s = \sqrt{32} = 4\sqrt{2}$ meters.

(a) For the average salary of college professors, a scale of $10,000 would be appropriate.

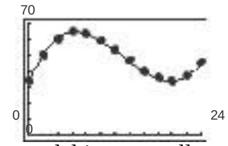
For the population of the United States, use a scale of 10,000,000.

For the percent of the civilian workforce that is unemployed, use a scale of 10%.

For the number of games a college football team wins in a single season, single digits would be appropriate.

For each of the graphs, using the suggested scale would show yearly changes in the data clearly.

90. (a)



The model is an excellent fit.

The temperature was increasing from 6 A.M. until noon $(x = 0$ to $x = 6)$. Then it decreases until 2 A.M. $(x = 6$ to $x = 20)$. Then the temperature increases until 6 A.M. $(x = 20$ to $x = 24)$.

The maximum temperature according to the model is about 63.93°F. According to the data, it is 64°F. The minimum temperature according to the model is about 33.98°F. According to the data, it is 34°F.

Answers may vary. Temperatures will depend upon the weather patterns, which usually change from day to day.

91. False. The function $f(x) = \sqrt{x^2 + 1}$ has a domain of all real numbers.

False. An odd function is symmetric with respect to the origin, so its domain must include negative values.

True. A graph that is symmetric with respect to the *y*-axis cannot be increasing on its entire domain.

94. (a) Domain: $[-4, 5)$; Range: $[0, 9]$

$(3, 0)$

Increasing: $(-4, 0) \cup (3, 5)$; Decreasing: $(0, 3)$

Relative minimum: $(3, 0)$

Relative maximum: $(0, 9)$

Neither

$\left(-\frac{5}{3}, -7\right)$

If f is even, another point is $\left(\frac{5}{3}, -7\right)$.

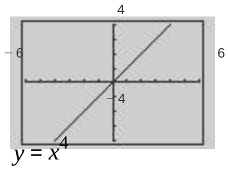
If f is odd, another point is $\left(\frac{5}{3}, 7\right)$.

$(2a, 2c)$

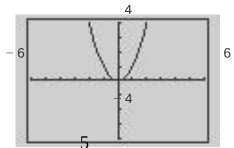
$(-2a, 2c)$

$(-2a, -2c)$

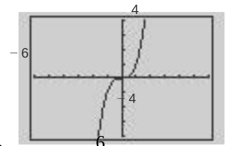
97. (a) $y = x$



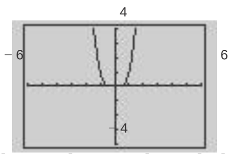
(b) $y = x^2$



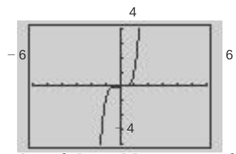
(c) $y = x^3$



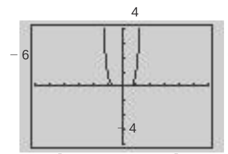
(d) $y = x^4$



(e) $y = x^5$

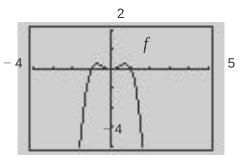


(f) $y = x^6$

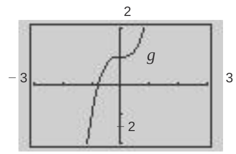


All the graphs pass through the origin. The graphs of the odd powers of x are symmetric with respect to the origin and the graphs of the even powers are symmetric with respect to the y-axis. As the powers increase, the graphs become flatter in the interval $-1 < x < 1$.

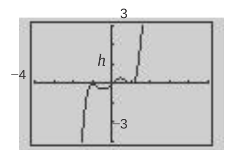
98.



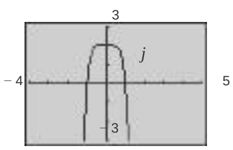
$f(x) = x^2 - x^4$ is even.



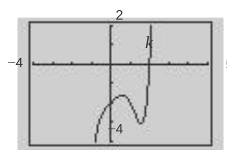
$g(x) = 2x^3 + 1$ is neither.



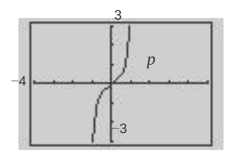
$h(x) = x^5 - 2x^3 + x$ is odd.



$j(x) = 2 - x^6 - x^8$ is even.



$k(x) = x^5 - 2x^4 + x - 2$ is neither.



$p(x) = x^9 + 3x^5 - x^3 + x$ is odd.

Equations of odd functions contain only odd powers of x. Equations of even functions contain only even powers of x. Odd functions have all variables raised to odd powers and even functions have all variables raised to even powers. A function that has variables raised to even and odd powers is neither odd nor even.

(a) Even. The graph is a reflection in the x-axis.

Even. The graph is a reflection in the y-axis.

Even. The graph is a vertical translation of f.

Neither. The graph is a horizontal translation of f.

Section 2.4 A Library of Parent Functions

1. Greatest integer function

2. Identity function

3. Reciprocal function

4. Squaring function

5. Square root function

6. Constant function

7. Absolute value function

8. Cubic function

9. Linear function

10. linear

(a) $f(1) = 4, f(0) = 6$

$(1, 4), (0, 6)$

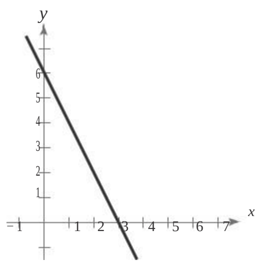
$m = \dfrac{6 - 4}{0 - 1} = -2$

$- 6 = -2(x - 0)$

$y = -2x + 6$

$f(x) = -2x + 6$

(b)



(a) $f(-3) = -8, f(1) = 2$

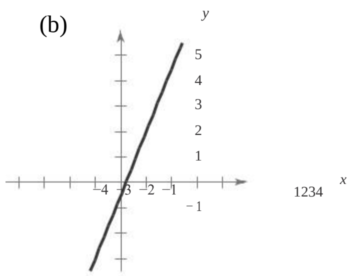
$(-3, -8), (1, 2)$

$m = \dfrac{2 - (-8)}{1 - (-3)} = \dfrac{10}{4} = \dfrac{5}{2}$

$f(x) - 2 = \dfrac{5}{2}(x - 1)$

$f(x) = \dfrac{5}{2}x - \dfrac{1}{2}$

(b)



13. (a) $f\left(\frac{1}{2}\right) = -\frac{5}{3}, f(6) = 2$

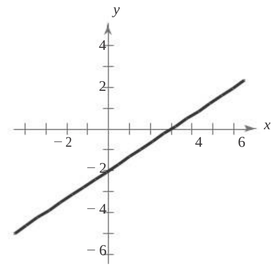
$\left(\frac{1}{2}, -\frac{5}{3}\right), (6, 2)$

$m = \dfrac{2 - \left(-\frac{5}{3}\right)}{6 - \left(\frac{1}{2}\right)} = \dfrac{\frac{11}{3}}{\frac{11}{2}} = \left(\frac{11}{3}\right)\left(\frac{2}{11}\right) = \frac{2}{3}$

$f(x) - 2 = \frac{2}{3}(x - 6)$

$f(x) - 2 = \frac{2}{3}x - 4$

(b)



(a) $f\left(\frac{3}{5}\right) = \frac{1}{2}, f(4) = 9$

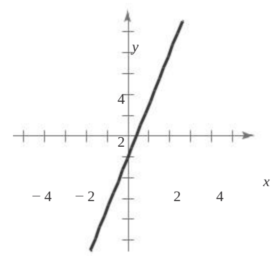
$\left(\frac{3}{5}, \frac{1}{2}\right), (4, 9)$

$m = \dfrac{9 - \left(\frac{1}{2}\right)}{4 - \left(\frac{3}{5}\right)} = \dfrac{\frac{17}{2}}{\frac{17}{5}} = \left(\frac{17}{2}\right)\left(\frac{5}{17}\right) = \frac{5}{2}$

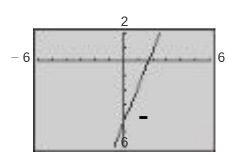
$f(x) - 9 = \frac{5}{2}(x - 4)$

$f(x) - 9 = \frac{5}{2}x - 10$

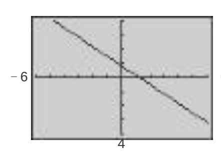
(b)



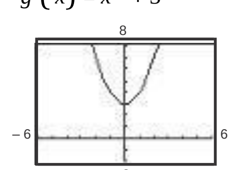
$f(x) = 2.5x - 4.25$



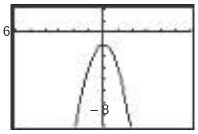
16. $f(x) = \frac{5}{6} - \frac{2}{3}x$



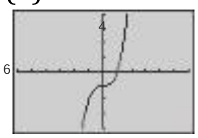
$g(x) = x^2 + 3$



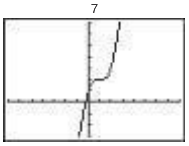
$g(x) = -2x^2 - 1$



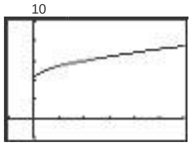
$f(x) = x^3 - 1$



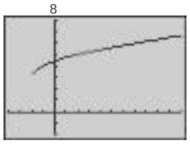
$f(x) = (x-1)^3 + 2$



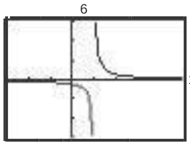
21. $f(x) = \sqrt{x} + 4$



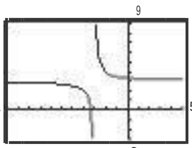
$h(x) = \sqrt{x+2} + 3$



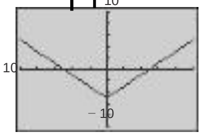
23. $f(x) = \dfrac{1}{x-2}$



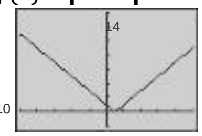
24. $k(x) = 3t + \dfrac{1}{x+3}$



25. $g(x) = |x| - 5$



26. $f(x) = |x - 1|$



$f(x) = [\![x]\!]$

$f(2.1) = 2$

$f(2.9) = 2$

$f(-3.1) = -4$

$\frac{\ }{2}\big) = 3$

$h(x) = [\![x + 3]\!]$

$h(-2) = [\![1]\!] = 1$

$h\left(\frac{1}{2}\right) = [\![3.5]\!] = 3$

$h(4.2) = [\![7.2]\!] = 7$

$h(-21.6) = [\![-18.6]\!] = -19$

$k(x) = [\![2x + 1]\!]$

(a) $k\left(\frac{1}{3}\right) = [\![2\left(\frac{1}{3}\right) + 1]\!] = [\![\frac{5}{3}]\!] = 1$

(b) $k(-2.1) = [\![2(-2.1) + 1]\!] = [\![-3.1]\!] = -4$

$k(1.1) = [\![2(1.1) + 1]\!] = [\![3.2]\!] = 3$

(d) $k\left(\frac{2}{3}\right) = [\![2\left(\frac{2}{3}\right) + 1]\!] = [\![\frac{7}{3}]\!] = 2$

$g(x) = -7[\![x + 4]\!] + 6$

$g\left(\frac{1}{8}\right) = -7[\![\frac{1}{8} + 4]\!] + 6$

$-7[\![4\frac{1}{8}]\!] + 6 = -7(4) + 6 = -22$

$g(9) = -7[\![9 + 4]\!] + 6$

$-7[\![13]\!] + 6 = -7(13) + 6 = -85$

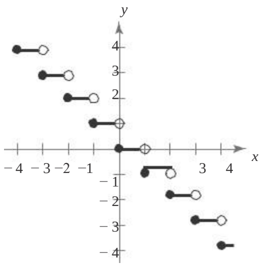
$g(-4) = -7[\![-4 + 4]\!] + 6$

$-7[\![0]\!] + 6 = -7(0) + 6 = 6$

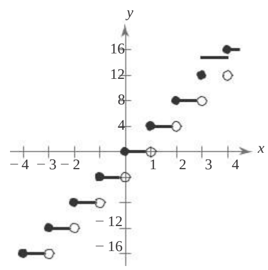
$g\left(\frac{3}{2}\right) = -7[\![\frac{3}{2} + 4]\!] + 6$

$-7[\![5\frac{1}{2}]\!] + 6 = -7(5) + 6 = -29$

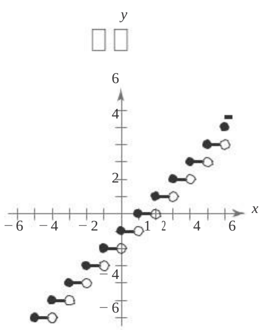
$g(x) = -[\![x]\!]$



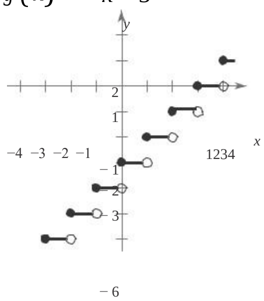
32. $g(x) = 4[\![x]\!]$



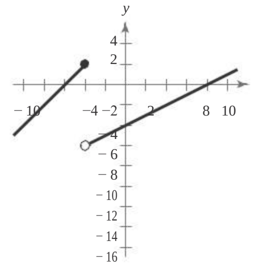
33. $g(x) = [\![x]\!] - 1$



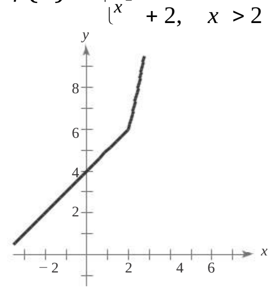
34. $g(x) = [\![x - 3]\!]$



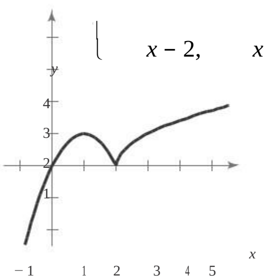
35. $g(x) = \begin{cases} x + 6, & x \le -4 \\ \frac{1}{2}x - 4, & x > -4 \end{cases}$



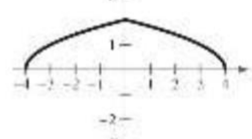
36. $f(x) = \begin{cases} 4 + x, & x \le 2 \\ x^2 + 2, & x > 2 \end{cases}$



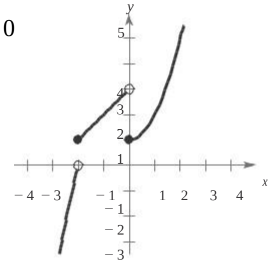
37. $f(x) = \begin{cases} 1 - (x - 1)^2, & x \le 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$



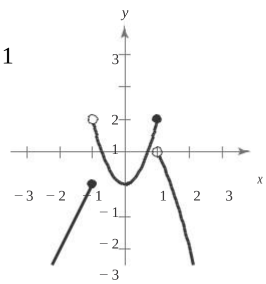
38. $f(x) =$ { [redacted]+[redacted], $x < 0$; $\sqrt{4 - x}$, $x \ge 0$ }



39. $h(x) = \begin{cases} 4 - x^2, & x < -2 \\ 3 + x, & -2 \le x < 0 \\ x^2 + 1, & x \ge 0 \end{cases}$

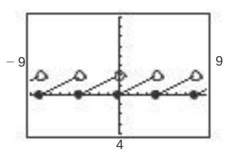


40. $k(x) = \begin{cases} 2x + 1, & x \le -1 \\ 2x^2 - 1, & -1 < x \le 1 \\ 1 - x^2, & x > 1 \end{cases}$



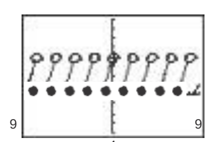
41. $s(x) = 2\left(\frac{1}{4}x - \left[\!\left[\frac{1}{4}x\right]\!\right]\right)$

(a)



Domain: $(-\infty, \infty)$; Range: $[0, 2)$

$k(x) = 4\left(\frac{1}{2}x - \left[\!\left[\frac{1}{2}x\right]\!\right]\right)^2$



Domain: $(-\infty, \infty)$; Range: $[0, 4)$

(a) $W(30) = 14(30) = 420$

$W(40) = 14(40) = 560$

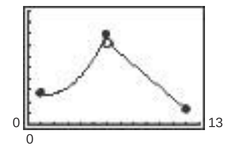
$W(45) = 21(45 - 40) + 560 = 665$

$W(50) = 21(50 - 40) + 560 = 770$

(b) $W(h) = \begin{cases} 14h, & 0 < h \le 36 \\ 21(h - 36) + 504, & h > 36 \end{cases}$

(c) $W(h) = \begin{cases} 16h, & 0 < h \le 40 \\ 24(h - 40) + 640, & h > 40 \end{cases}$

44. (a)



The domain of $f(x) = -1.97x + 26.3$ is $6 < x \le 12$. One way to see this is to notice that this is the equation of a line with negative slope, so the function values are decreasing as x increases, which matches the data for the corresponding part of the table. The domain of

$f(x) = 0.505x^2 - 1.47x + 6.3$ is then $1 \le x \le 6$.

$f(5) = 0.505(5)^2 - 1.47(5) + 6.3$

$= 0.505(25) - 7.35 + 6.3 = 11.575$

$f(11) = -1.97(11) + 26.3 = 4.63$

These values represent the revenue in thousands of dollars for the months of May and November, respectively.

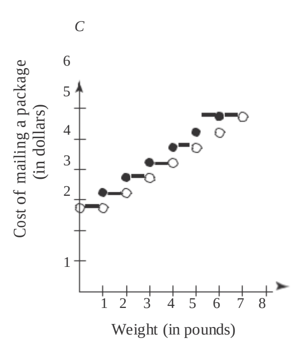
These values are quite close to the actual data values.

Answers will vary. *Sample answer:*

Interval	*Input Pipe*	*Drain Pipe 1*	*Drain Pipe 2*
[0,5]	Open	Closed	Closed
[5, 10]	Open	Open	Closed
[10,20]	Closed	Closed	Closed
[20,30]	Closed	Closed	Open
[30,40]	Open	Open	Open
[40,45]	Open	Closed	Open
[45,50]	Open	Open	Open
[50,60]	Open	Open	Closed

46. (a) $C = 0.5\left[\!\left[x\right]\!\right] + 2.72$

(b)



For the first two hours, the slope is 1. For the next six hours, the slope is 2. For the final hour, the slope is $\frac{1}{2}$.

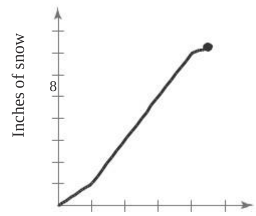
$$f(t) = \begin{cases} t, & 0 \le t \le 2 \\ 2t - 2, & 2 < t \le 8 \\ \frac{1}{2}t + 10, & 8 < t \le 9 \end{cases}$$

To find $f(t) = 2t - 2$, use $m = 2$ and $(2, 2)$.

$y - 2 = 2(t - 2) \Rightarrow y = 2t - 2$

To find $f(t) = \frac{1}{2}t + 10$, use $m = \frac{1}{2}$ and $(8, 14)$.

$y - 14 = \frac{1}{2}(t - 8) \Rightarrow y = \frac{1}{2}t + 10$



48. $f(x) = x^2$

(a) Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

(b) x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

(c) Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$

(d) Even; the graph has y-axis symmetry.

$f(x) = x^3$

(a) Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

(b) x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

(c) Increasing: $(-\infty, \infty)$

(d) Odd; the graph has origin symmetry.

49. False. A piecewise-defined function is a function that is defined by two or more equations over a specified domain. That domain may or may not include x- and y-intercepts.

50. False. The vertical line $x = 2$ has an x-intercept at the point $(2, 0)$ but does not have a y-intercept. The horizontal line $y = 3$ has a y-intercept at the point $(0, 3)$ but does not have an x-intercept.

Section 2.5 Transformations of Functions

1. rigid

$-f(x); f(-x)$

3. vertical stretch; vertical shrink

4. (a) iv

(b) ii

(c) iii

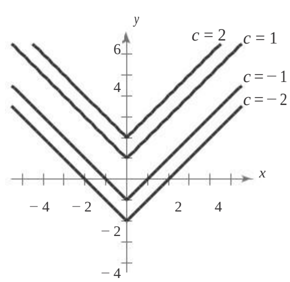
(d) i

5. (a) $f(x) = |x| + c$ Vertical shifts

$c = -2$: $f(x) = |x| - 2$ 2 units down

$c = -1$: $f(x) = |x| - 1$ 1 unit down

$c = 1$: $f(x) = |x| + 1$ 1 unit up



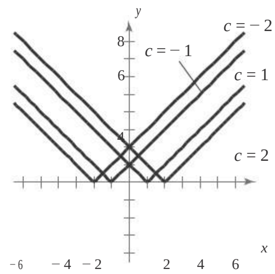
(b) $f(x) = |x - c|$ Horizontal shifts

$c = -2$: $f(x) = |x - (-2)| = |x + 2|$ 2 units left

$c = -1$: $f(x) = |x - (-1)| = |x + 1|$ 1 unit left

$c = 1$: $f(x) = |x - (1)| = |x - 1|$ 1 unit right

$c = 2$: $f(x) = |x - (2)| = |x - 2|$ 2 units right



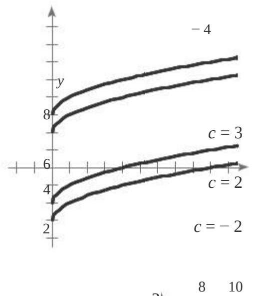
6. (a) $f(x) = \sqrt{x} + c$ Vertical shifts

$c = -3$: $f(x) = \sqrt{x} - 3$ 3 units down

$c = -2$: $f(x) = \sqrt{x} - 2$ 2 units down

$c = 2$: $f(x) = \sqrt{x} + 2$ 2 units up

$c = 3$: $f(x) = \sqrt{x} + 3$ 3 units up



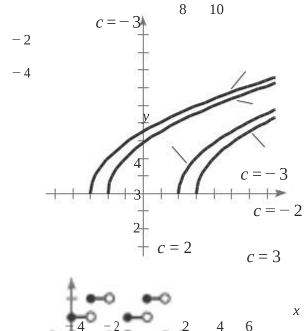
(b) $f(x) = \sqrt{x - c}$ Horizontal shifts

$c = -3$: $f(x) = \sqrt{x - (-3)} = \sqrt{x + 3}$ 3 units left

$c = -2$: $f(x) = \sqrt{x - (-2)} = \sqrt{x + 2}$ 2 units left

$c = 2$: $f(x) = \sqrt{x - 2}$ 2 units right

$c = 3$: $f(x) = \sqrt{x - 3}$ 3 units right



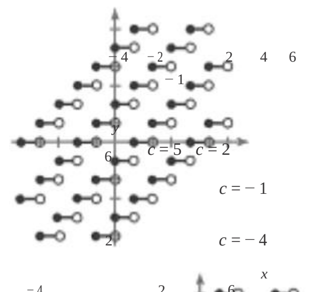
7. (a) $f(x) = [\![x]\!] + c$ Vertical shifts

$c = -4$: $f(x) = [\![x]\!] - 4$ 4 units down

$c = -1$: $f(x) = [\![x]\!] - 1$ 1 unit down

$c = 2$: $f(x) = [\![x]\!] + 2$ 2 units up

$c = 5$: $f(x) = [\![x]\!] + 5$ 5 units up



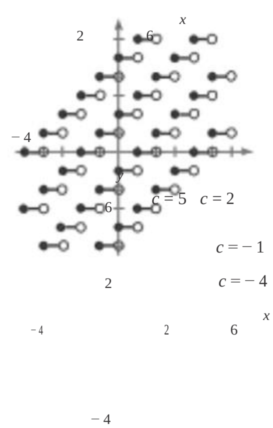
(b) $f(x) = [\![x + c]\!]$ Horizontal shifts

$c = -4$: $f(x) = [\![x - (-4)]\!] = [\![x + 4]\!]$ 4 units left

$c = -1$: $f(x) = [\![x - (-1)]\!] = [\![x + 1]\!]$ 1 unit left

$c = 2$: $f(x) = [\![x - (2)]\!] = [\![x - 2]\!]$ 2 units right

$c = 5$: $f(x) = [\![x - (5)]\!] = [\![x - 5]\!]$ 5 units right



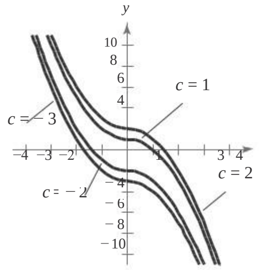
8. (a) $f(x) = \begin{cases} x^2 + c, & x < 0 \\ -x^2 + c, & x \ge 0 \end{cases}$ Vertical shifts

$c = -3$: $f(x) = \begin{cases} x^2 - 3, & x < 0 \\ -x^2 - 3, & x \ge 0 \end{cases}$ 3 units down

$c = -2$: $f(x) = \begin{cases} x^2 - 2, & x < 0 \\ -x^2 - 2, & x \ge 0 \end{cases}$ 2 units down

$c = 1$: $f(x) = \begin{cases} x^2 + 1, & x < 0 \\ -x^2 + 1, & x \ge 0 \end{cases}$ 1 unit up

$c = 2$: $f(x) = \begin{cases} x^2 + 2, & x < 0 \\ -x^2 + 2, & x \ge 0 \end{cases}$ 2 units up



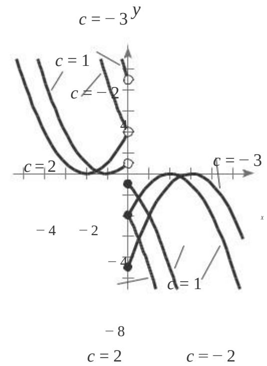
(b) $f(x) = \begin{cases} (x + c)^2, & x < 0 \\ -(x + c)^2, & x \ge 0 \end{cases}$ Horizontal shifts

$c = -3$: $f(x) = \begin{cases} (x - 3)^2, & x < 0 \\ -(x - 3)^2, & x \ge 0 \end{cases}$ 3 units right

$c = -2$: $f(x) = \begin{cases} (x - 2)^2, & x < 0 \\ -(x - 2)^2, & x \ge 0 \end{cases}$ 2 units right

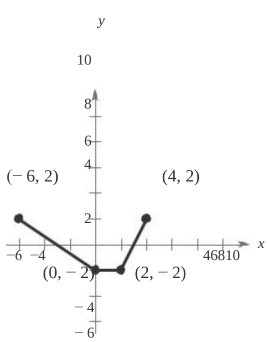
$c = 1$: $f(x) = \begin{cases} (x + 1)^2, & x < 0 \\ -(x + 1)^2, & x \ge 0 \end{cases}$ 1 unit left

$c = 2$: $f(x) = \begin{cases} (x + 2)^2, & x < 0 \\ -(x + 2)^2, & x \ge 0 \end{cases}$ 2 units left



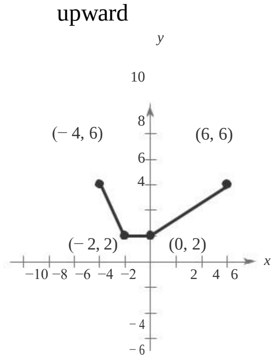
9. (a) $y = f(-x)$

Reflection in the y-axis



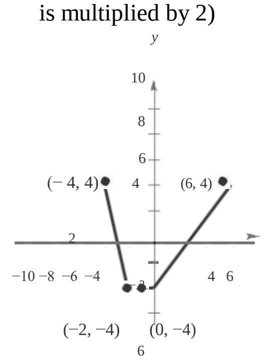
(b) $y = f(x) + 4$

Vertical shift 4 units upward



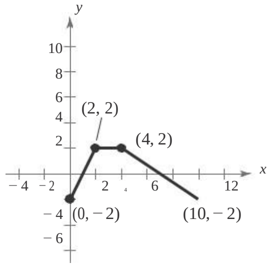
$y = 2f(x)$

Vertical stretch (each y-value is multiplied by 2)



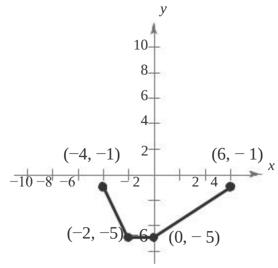
$y = -f(x-4)$

Reflection in the x-axis and a horizontal shift 4 units to the right



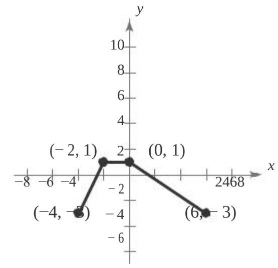
(e) $y = f(x) - 3$

Vertical shift 3 units downward



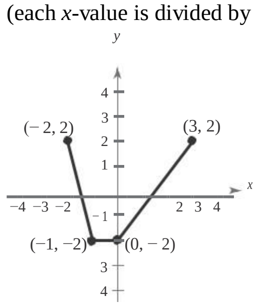
(f) $y = -f(x) - 1$

Reflection in the x-axis and a vertical shift 1 unit downward



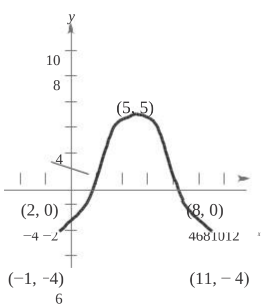
(g) $y = f(2x)$

Horizontal shrink
(each x-value is divided by 2)



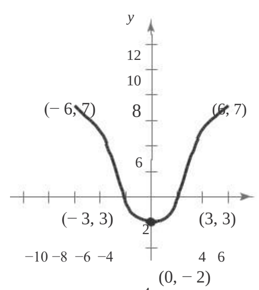
(a) $y = f(x-5)$

Horizontal shift 5 units to the right



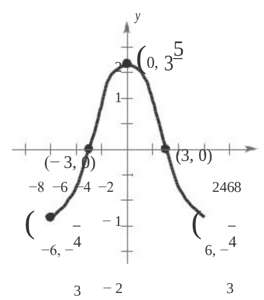
$y = -f(x) + 3$

Reflection in the x-axis and a vertical shift 3 units upward



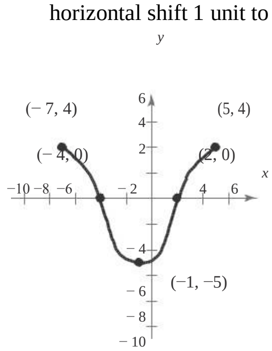
$y = \frac{1}{3} f(x)$

Vertical shrink
(each y-value is multiplied by $\frac{1}{3}$)



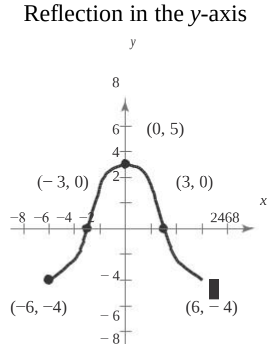
$y = -f(x+1)$

Reflection in the x-axis and a horizontal shift 1 unit to the left



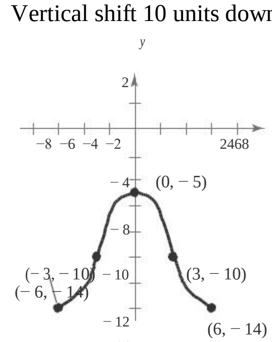
$y = f(-x)$

Reflection in the y-axis



(f) $y = f(x) - 10$

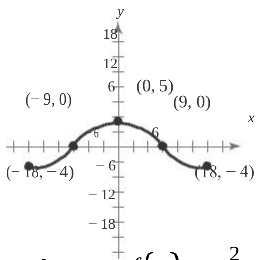
Vertical shift 10 units downward



(g) $y = f\left(\frac{1}{3}x\right)$

Horizontal stretch

(each x-value is multiplied by 3)



11. Parent function: $f(x) = x^2$

Vertical shift 1 unit downward $g(x) = x^2 - 1$

Reflection in the x-axis, horizontal shift 1 unit to the left, and a vertical shift 1 unit upward

$g(x) = -(x + 1)^2 + 1$

Parent function: $f(x) = x^3$

Shifted upward 1 unit

$g(x) = x^3 + 1$

Reflection in the x-axis, shifted to the left 3 units and down 1 unit

$g(x) = -(x + 3)^3 - 1$

Parent function: $f(x) = |x|$

Reflection in the x-axis and a horizontal shift 3 units to the left

$g(x) = -|x + 3|$

Horizontal shift 2 units to the right and a vertical shift 4 units downward

$g(x) = |x - 2| - 4$

14. Parent function: $f(x) = \sqrt{x}$

A vertical shift 7 units downward and a horizontal shift 1 unit to the left

$g(x) = \sqrt{x + 1} - 7$

Reflection in the x- and y-axis and shifted to the right 3 units and downward 4 units

$g(x) = -\sqrt{-x + 3} - 4$

Parent function: $f(x) = x^3$

Horizontal shift 2 units to the right

$y = (x - 2)^3$

Parent function: $y = x$

Vertical shrink (each y value is multiplied by $\frac{1}{2}$)

$= \frac{1}{2}x$

Parent function: $f(x) = x^2$

Reflection in the x-axis

$= -x^2$

18. Parent function: $y = [\![x]\!]$

Vertical shift 4 units upward

$y = [\![x]\!] + 4$

19. Parent function: $f(x) = \sqrt{x}$

Reflection in the x-axis and a vertical shift 1 unit upward

$y = -\sqrt{x} + 1$

20. Parent function: $y = |x|$

Horizontal shift 2 units to the left

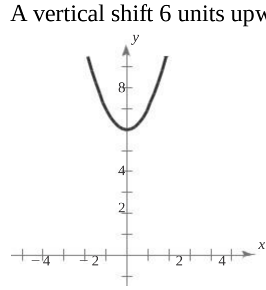
$y = |x + 2|$

21. $g(x) = x^2 + 6$

(a) Parent function: $f(x) = x^2$

(b) A vertical shift 6 units upward

(c)



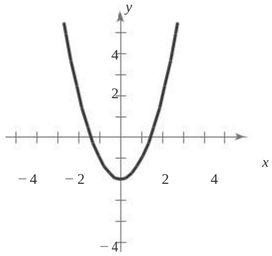
(d) $g(x) = f(x) + 6$

22. $g(x) = x^2 - 2$

(a) Parent function: $f(x) = x^2$

(b) A vertical shift 2 units downward

(c)



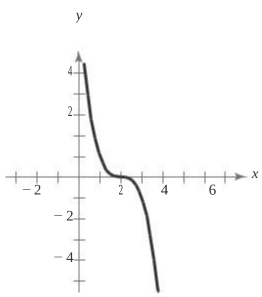
(d) $g(x) = f(x) - 2$

23. $g(x) = -(x - 2)^3$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift of 2 units to the right and a reflection in the *x*-axis

(c)



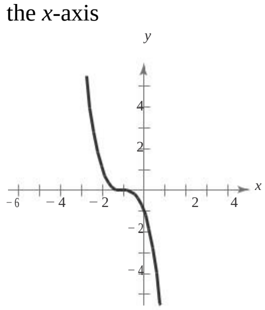
(d) $g(x) = -f(x - 2)$

24. $g(x) = -(x + 1)^3$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift 1 unit to the left and a reflection in the *x*-axis

(c)



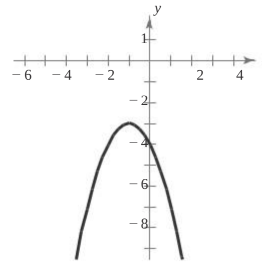
(d) $g(x) = -f(x + 1)$

$g(x) = -3 - (x + 1)^2$

Parent function: $f(x) = x^2$

Reflection in the *x*-axis, a vertical shift 3 units downward and a horizontal shift 1 unit left

(c)



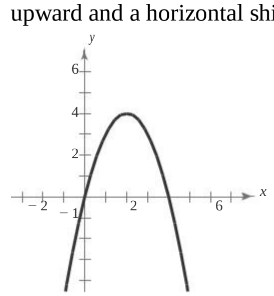
(d) $g(x) = -f(x + 1) - 3$

26. $g(x) = 4 - (x - 2)^2$

(a) Parent function: $f(x) = x^2$

(b) Reflection in the *x*-axis, a vertical shift 4 units upward and a horizontal shift 2 units right

(c)



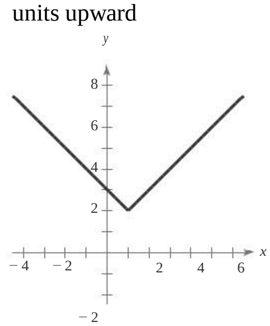
$g(x) = -f(x - 2) + 4$

$g(x) = |x - 1| + 2$

Parent function: $f(x) = |x|$

A horizontal shift 1 unit right and a vertical shift 2 units upward

(c)



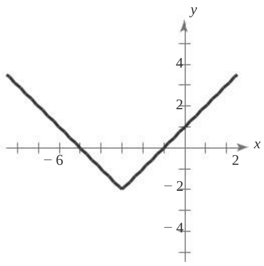
(d) $g(x) = f(x - 1) + 2$

$g(x) = |x + 3| - 2$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shift 3 units left and a vertical shift 2 units downward

(c)



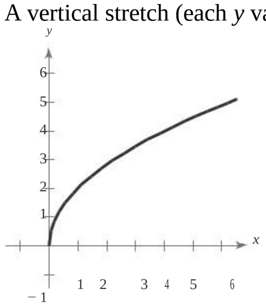
(d) $g(x) = f(x + 3) - 2$

29. $g(x) = 2\sqrt{x}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) A vertical stretch (each *y* value is multiplied by 2)

(c)



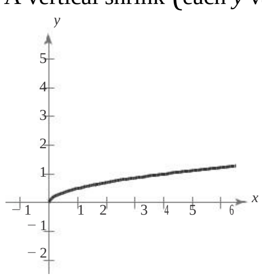
(d) $g(x) = 2f(x)$

30. $g(x) = \frac{1}{2}\sqrt{x}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) A vertical shrink (each *y* value is multiplied by $\frac{1}{2}$)

(c)



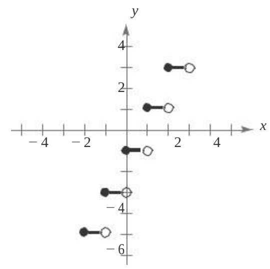
(d) $g(x) = \frac{1}{2}f(x)$

$g(x) = 2[\![x]\!] - 1$

(a) Parent function: $f(x) = [\![x]\!]$

(b) A vertical shift of 1 unit downward and a vertical stretch (each *y* value is multiplied by 2)

(c)



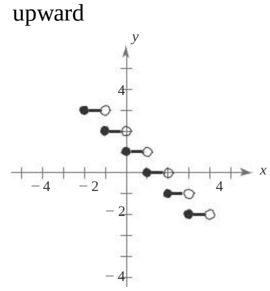
(d) $g(x) = 2f(x) - 1$

32. $g(x) = -[\![x]\!] + 1$

(a) Parent function: $f(x) = [\![x]\!]$

(b) Reflection in the *x*-axis and a vertical shift 1 unit upward

(c)



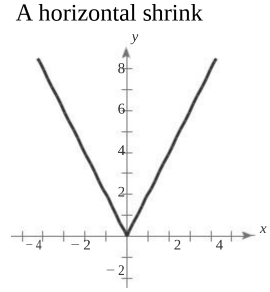
(d) $g(x) = -f(x) + 1$

33. $g(x) = |2x|$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shrink

(c)

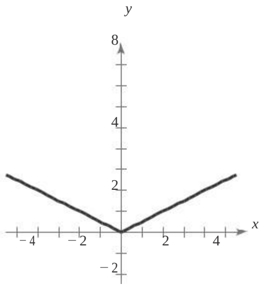


(d) $g(x) = f(2x)$

34. $g(x) = \left|\frac{1}{2}x\right|$

(a) Parent function: $f(x) = |x|$

(b) A horizontal stretch



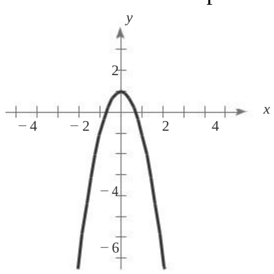
(d) $g(x) = f\left(\frac{1}{2}x\right)$

35. $g(x) = -2x^2 + 1$

(a) Parent function: $f(x) = x^2$

(b) A vertical stretch, reflection in the x-axis and a vertical shift 1 unit upward

(c)



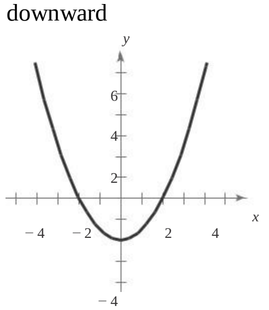
(d) $g(x) = -2f(x) + 1$

36. $g(x) = \frac{1}{2}x^2 - 2$

(a) Parent function: $f(x) = x^2$

(b) A vertical shrink and a vertical shift 2 units downward

(c)



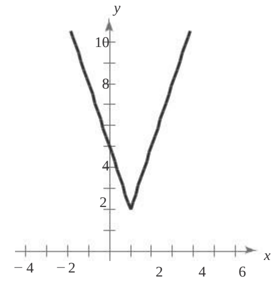
(d) $g(x) = \frac{1}{2}f(x) - 2$

37. $g(x) = |3x - 1| + 2$

(a) Parent function: $f(x) = |x|$

A horizontal shift of 1 unit to the right, a vertical stretch, and a vertical shift 2 units upward

(c)



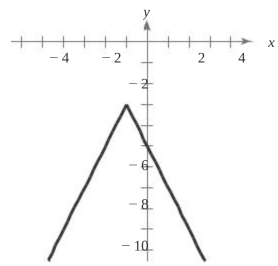
$g(x) = 3f(x - 1) + 2$

$g(x) = -2|x + 1| - 3$

Parent function: $f(x) = |x|$

A reflection in the x-axis, a vertical stretch, a horizontal shift 1 unit to the left, and a vertical shift 3 units downward

(c)



(d) $g(x) = -2f(x + 1) - 3$

39. $g(x) = (x - 3)^2 - 7$

40. $g(x) = -(x + 2)^2 + 9$

41. $f(x) = x^3$ moved 13 units to the right

$g(x) = (x - 13)^3$

42. $f(x) = x^3$ moved 6 units to the left, 6 units downward, and reflected in the y-axis (in that order)

$g(x) = (-x + 6)^3 - 6$

$g(x) = -|x| + 12$

$g(x) = |x + 4| - 8$

$f(x) = \sqrt{x}$ moved 6 units to the left and reflected in both the x- and y-axes

$g(x) = -\sqrt{-x + 6}$

$f(x) = \sqrt{x}$ moved 9 units downward and reflected in both the x-axis and the y-axis

$g(x) = -\left(\sqrt{-x} - 9\right)$

$-\sqrt{-x} + 9$

$f(x) = x^2$

Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 3)

$g(x) = -3x^2$

Vertical shift 3 units upward and a vertical stretch (each y-value is multiplied by 4)

$g(x) = 4x^2 + 3$

$f(x) = x^3$

Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$

$g(x) = \frac{1}{4}x^3$

Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 2)

$g(x) = -2x^3$

$f(x) = |x|$

Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = -\frac{1}{2}|x|$

Vertical stretch (each y-value is multiplied by 3) and a vertical shift 3 units downward

$g(x) = 3|x| - 3$

50. $f(x) = \sqrt{x}$

Vertical stretch (each y-value is multiplied by 8)

$g(x) = 8\sqrt{x}$

Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$

$g(x) = -\frac{1}{4}\sqrt{x}$

Parent function: $f(x) = x^3$

Vertical stretch (each y-value is multiplied by 2)

$g(x) = 2x^3$

Parent function: $f(x) = |x|$

Vertical stretch (each y-value is multiplied by 6)

$g(x) = 6|x|$

Parent function: $f(x) = x^2$

Reflection in the x-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = -\frac{1}{2}x^2$

54. Parent function: $y = [\![x]\!]$

Horizontal stretch (each x-value is multiplied by 2)

$g(x) = [\![\frac{1}{2}x]\!]$

55. Parent function: $\sqrt{x}$

Reflection in the y-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = \frac{1}{2}\sqrt{-x}$

Parent function: $f(x) = |x|$

Reflection in the x-axis, vertical shift of 2 units downward, vertical stretch (each y-value is multiplied by 2)

$g(x) = -2|x| - 2$

Parent function: $f(x) = x^3$

Reflection in the x-axis, horizontal shift 2 units to the right and a vertical shift 2 units upward

$g(x) = -(x - 2)^3 + 2$

Parent function: $f(x) = |x|$

Horizontal shift of 4 units to the left and a vertical shift of 2 units downward

$g(x) = |x + 4| - 2$

59. Parent function: $f(x) = \sqrt{x}$

Reflection in the x-axis and a vertical shift 3 units downward

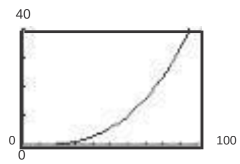
$g(x) = -\sqrt{x} - 3$

Parent function: $f(x) = x^2$

Horizontal shift of 2 units to the right and a vertical shift of 4 units upward

$g(x) = (x - 2)^2 + 4$

61. (a)



$H(x) = 0.00004636x^3$

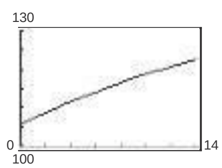
$H\left(\frac{x}{1.6}\right) = 0.00004636\left(\frac{x}{1.6}\right)^3$

$= 0.00004636\left(\frac{x^3}{4.096}\right)$

$= 0.0000113184x^3 = 0.00001132x^3$

The graph of $H\left(\frac{x}{1.6}\right)$ is a horizontal stretch of the graph of $H(x)$.

62. (a) The graph of $N(x) = -0.023(x - 33.12)^2 + 131$ is a reflection in the x-axis, a vertical shrink, a horizontal shift 33.12 units to the right and a vertical shift 131 units upward of the parent graph $f(x) = x^2$.



The average rate of change from $t = 0$ to $t = 14$ is given by the following.

$$\frac{N(14) - N(0)}{14 - 0} = \frac{122.591 - 105.770}{14}$$

$$= \frac{16.821}{14}$$

$$\approx 1.202$$

The number of households in the United States increased by an average of 1.202 million or 1,202,000 households each year from 2000 to 2014.

(c) Let $t = 22$:

$N(22) = -0.023(22 - 33.12)^2 + 131$

≈ 128.156

In 2022, the number of households in the United States will be about 128.2 million households. Answers will vary. *Sample answer:* Yes, because the number of households has been increasing on average.

63. False. $y = f(-x)$ is a reflection in the y-axis.

64. False. $y = -f(x)$ is a reflection in the x-axis.

65. True. Because $|x| = |-x|$, the graphs of $f(x) = |x| + 6$ and $f(-x) = |-x| + 6$ are identical.

66. False. The point $(-2, -61)$ lies on the transformation.

$y = f(x + 2) - 1$

Horizontal shift 2 units to the left and a vertical shift 1 unit downward

$(0,1) \to (0 - 2, 1 - 1) = (-2, 0)$

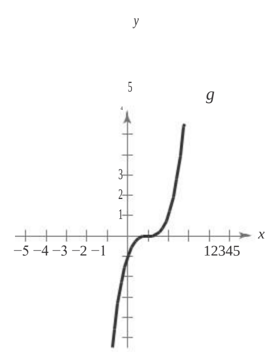
$(1, 2) \to (1 - 2, 2 - 1) = (-1, 1)$

$(2,3) \to (2 - 2, 3 - 1) = (0, 2)$

(a) Answers will vary. *Sample answer*: To graph $f(x) = 3x^2 - 4x + 1$ use the point-plotting method since it is not written in a form that is easily identified by a sequence of translations of the parent function $y = x^2$.

(b) Answers will vary. *Sample answer*: To graph $f(x) = 2(x - 1)^2 - 6$ use the method of translating the parent function $y = x^2$ since it is written in a form such that a sequence of translations is easily identified.

Since the graph of $g(x)$ is a horizontal shift one unit to the right of $f(x) = x^3$, the equation should be $g(x) = (x - 1)^3$ and not $g(x) = (x + 1)^3$.



70. (a) Increasing on the interval $(-2, 1)$ and decreasing on the intervals $(-\infty, -2)$ and $(1, \infty)$

Increasing on the interval $(-1, 2)$ and decreasing on the intervals $(-\infty, -1)$ and $(2, \infty)$

Increasing on the intervals $(-\infty, -1)$ and $(2, \infty)$ and decreasing on the interval $(-1, 2)$

Increasing on the interval $(0, 3)$ and decreasing on the intervals $(-\infty, 0)$ and $(3, \infty)$

Increasing on the intervals $(-\infty, 1)$ and $(4, \infty)$ and decreasing on the interval $(1, 4)$

71. (a) The profits were only $\frac{3}{4}$ as large as expected:

$g(t) = \frac{3}{4} f(t)$

The profits were \$10,000 greater than predicted:

$g(t) = f(t) + 10{,}000$

There was a two-year delay: $g(t) = f(t-2)$

72. No. $g(x) = -x^4 - 2$. Yes. $h(x) = -(x-3)^4$

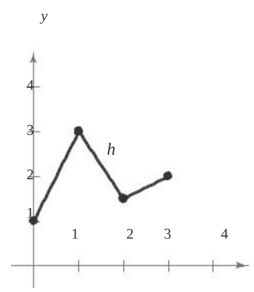
Section 2.6 Combinations of Functions: Composite Functions

addition; subtraction; multiplication; division

composition

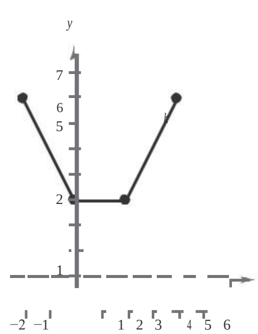
3.

x	0	1	2	3
f	2	3	1	2
g	–1	0	$\frac{1}{2}$	0
f + g	1	3	$\frac{3}{2}$	2



4.

x	–2	0	1	2	4
f	2				0124
g	4				2102
f + g	6				2226



5. $f(x) = x + 2,\ g(x) = x - 2$

(a) $(f+g)(x) = f(x) + g(x)$

$= (x+2) + (x-2)$
$= 2x$

$(f-g)(x) = f(x) - g(x)$

$(x+2) - (x-2)$

4

$f(x) \cdot g(x)\ (fg)(x) =$

$(x+2)(x-2)$

$x^2 - 4$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x+2}{x-2}$

Domain: all real numbers x except $x = 2$

6. $f(x) = 2x - 5,\ g(x) = 2 - x$

(a) $(f+g)(x) = 2x - 5 + 2 - x = x - 3$

(b) $(f-g)(x) = 2x - 5 - (2-x)$

$= 3x - 7$

(c) $(fg)(x) = (2x - 5)(2 - x)$

$= 4x - 2x^2 - 10 + 5x$

$= -2x^2 + 9x - 10$

(d) $\left(\frac{f}{g}\right)(x) = \frac{2x-5}{2-x}$

Domain: all real numbers x except $x = 2$

7. $f(x) = x^2,\ g(x) = 4x - 5$

(a) $(f+g)(x) = f(x) + g(x)$

$= x^2 + (4x - 5)$

$= x^2 + 4x - 5$

$= x^2 - (4x - 5)$

$= x^2 - 4x + 5$

(c) $(fg)(x) = f(x) \cdot g(x)$

$= x^2(4x - 5)$

$= 4x^3 - 5x^2$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{4x - 5}$

Domain: all real numbers x except $x = \frac{5}{4}$

8. $f(x) = 3x + 1,\ g(x) = x^2 - 16$

(a) $(f + g)(x) = f(x) + g(x)$

$= 3x + 1 + x^2 - 16$

$= x^2 + 3x - 15$

(b) $(f - g)(x) = f(x) - g(x)$

$= 3x + 1 - (x^2 - 16)$

$= -x^2 + 3x + 17$

$(fg)(x) = f(x) \cdot g(x)$

$(3x + 1)(x^2 - 16)$

$3x^3 + x^2 - 48x - 16$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x + 1}{x^2 - 16}$

Domain: all real numbers x, except $x \neq \pm 4$

9. $f(x) = x^2 + 6,\ g(x) = \sqrt{1 - x}$

(a) $(f + g)(x) = f(x) + g(x) = x^2 + 6 + \sqrt{1 - x}$

(b) $(f - g)(x) = f(x) - g(x) = x^2 + 6 - \sqrt{1 - x}$

$(fg)(x) = f(x) \cdot g(x) = (x^2 + 6)\sqrt{1 - x}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2 + 6}{\sqrt{1 - x}} = \frac{(x^2 + 6)\sqrt{1 - x}}{1 - x}$

Domain: $x < 1$

10. $f(x) = \sqrt{x^2 - 4},\ g(x) = \frac{x^2}{x^2 + 1}$

(a) $(f + g)(x) = \sqrt{x^2 - 4} + \frac{x^2}{x^2 + 1}$

(b) $(f - g)(x) = \sqrt{x^2 - 4} - \frac{x^2}{x^2 + 1}$

(c) $(fg)(x) = \sqrt{x^2 - 4}\left(\frac{x^2}{x^2 + 1}\right) = \frac{x^2\sqrt{x^2 - 4}}{x^2 + 1}$

(d) $\left(\frac{f}{g}\right)(x) = \sqrt{x^2 - 4} \div \frac{x^2}{x^2 + 1}$

$= \frac{(x^2 + 1)\sqrt{x^2 - 4}}{x^2}$

Domain: $x^2 - 4 \geq 0$

$x^2 \geq 4 \Rightarrow x \geq 2$ or $x \leq -2$

$|x| \geq 2$

11. $f(x) = \frac{x}{x + 1},\ g(x) = x^3$

(a) $(f + g)(x) = \frac{x}{x + 1} + x^3 = \frac{x^4 + x^3 + x}{x + 1}$

(b) $(f - g)(x) = \frac{x}{x + 1} - x^3 = \frac{x - x^4 - x^3}{x + 1}$

(c) $(fg)(x) = \frac{x}{x + 1} \cdot x^3 = \frac{x^4}{x + 1}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{x}{x + 1} \div x^3 = \frac{x}{x + 1} \cdot \frac{1}{x^3} = \frac{1}{x^2(x + 1)}$

Domain: all real numbers x except $x = 0$ and $x = -1$

12. $f(x) = \frac{2}{x},\ g(x) = \frac{1}{x^2 - 1}$

(a) $(f + g)(x) = \frac{2}{x} + \frac{1}{x^2 - 1} = \frac{2(x^2 - 1) + x}{x(x^2 - 1)} = \frac{2x^2 + x - 2}{\ldots}$

$f + g \quad x \qquad x \qquad x^2 - 1 \qquad x\left(x^2 - 1\right) \qquad x\left(x^2 - 1\right)$

(b) $(f - g)(x) = \frac{2}{x} - \frac{1}{x^2 - 1} = \frac{2\left(x^2 - 1\right) - x}{x\left(x^2 - 1\right)} = \frac{2x^2 - x - 2}{x\left(x^2 - 1\right)}$

(c) $(fg)(x) = \left(\frac{2}{x}\right)\left(\frac{1}{x^2 - 1}\right) = \frac{2}{x\left(x^2 - 1\right)}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{2}{x} \div \frac{1}{x^2 - 1}$

$= \frac{\frac{2}{x}}{\frac{1}{x^2-1}}$

$= \left(\frac{2}{x}\right)\left(\frac{x^2-1}{1}\right)$

$= \frac{2(x^2-1)}{x}$

Domain: all real numbers x, $x \neq 0, \pm 1$.

For Exercises 13–24, $f(x) = x + 3$ and $g(x) = x^2 - 2$.

$(f+g)(2) = f(2) + g(2)$

$(2+3)+(2^2-2)$

7

$(f+g)(-1) = f(-1) + g(-1)$

$(-1+3) + ((-1)^2 - 2)$

1

$(f-g)(0) = f(0) - g(0)$

$(0+3) - ((0)^2 - 2)$

5

$(f-g)(1) = f(1) - g(1)$

$(1+3) - ((1)^2 - 2)$

5

$(f-g)(3t) = f(3t) - g(3t)$

$((3t)+3) - ((3t)^2 - 2)$

$3t + 3 - (9t^2 - 2)$

$-9t^2 + 3t + 5$

$(f+g)(t-2) = f(t-2) + g(t-2)$

$((t-2)+3) + ((t-2)^2 - 2)$

$t + 1 + (t^2 - 4t + 4 - 2)$

$t^2 - 3t + 3$

$(fg)(6) = f(6)\,g(6)$

$((6)+3)((6)^2 - 2)$

$(9)(34)$

306

20. $(fg)(-6) = f(-6)\,g(-6)$

$= ((-6)+3)((-6)^2 - 2)$

$= (-3)(34)$

21. $(f/g)(5) = f(5)/g(5)$

$((5)+3)/((5)^2 - 2)$

$\frac{8}{23}$

22. $(f/g)(0) = f(0)/g(0)$

$((0)+3)/((0)^2 - 2)$

$-\frac{3}{2}$

23. $(f/g)(-1) - g(3) = f(-1)/g(-1) - g(3)$

$((-1)+3)/((-1)^2 - 2) - ((3)^2 - 2)$

$(2/-1) - 7$

$-2-7=-9$

$(fg)(5) + f(4) = f(5)g(5) + f(4)$

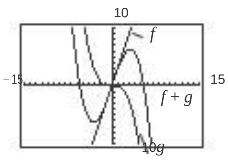
$((5) + 3)((5)^2 - 2) + ((4) + 3)$

$(8)(23) + 7$

$=184+7=191$

25. $f(x) = 3x,\ g(x) = -\frac{x^3}{10}$

$(f + g)(x) = 3x - \frac{x^3}{10}$

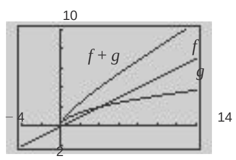


For $0 \le x \le 2$, $f(x)$ contributes most to the magnitude.

For $x > 6$, $g(x)$ contributes most to the magnitude.

26. $f(x) = \frac{x}{2},\ g(x) = \sqrt{x}$

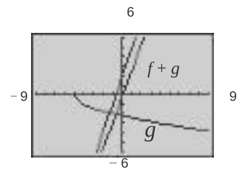
$(f + g)(x) = \frac{x}{2} + \sqrt{x}$



$g(x)$ contributes most to the magnitude of the sum for $\le x \le 2$. $f(x)$ contributes most to the magnitude of the sum for $x > 6$.

27. $f(x) = 3x + 2,\ g(x) = -\sqrt{x + 5}$

$(f + g)x = 3x - \sqrt{x + 5} + 2$

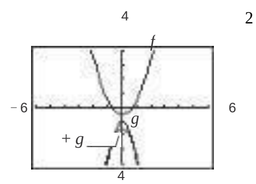


For $0 \le x \le 2$, $f(x)$ contributes most to the magnitude.

For $x > 6$, $f(x)$ contributes most to the magnitude.

28. $f(x) = x^2 - \frac{1}{2},\ g(x) = -3x^2 - 1$

$(f + g)(x) = -2x^2 - \frac{3}{2}$



For $0 \le x \le 2$, $g(x)$ contributes most to the magnitude.

For $x > 6$, $g(x)$ contributes most to the magnitude.

$f(x) = x + 8,\ g(x) = x - 3$

$(f \circ g)(x) = f(g(x)) = f(x - 3) = (x - 3) + 8 = x + 5$

(b) $(g \circ f)(x) = g(f(x)) = g(x + 8) = (x + 8) - 3 = x + 5$

$(g \circ g)(x) = g(g(x)) = g(x - 3) = (x - 3) - 3 = x - 6$

$f(x) = -4x,\ g(x) = x + 7$

(a) $(f \circ g)(x) = f(g(x)) = f(x + 7) = -4(x + 7) = -4x - 28$

(b) $(g \circ f)(x) = g(f(x)) = g(-4x) = (-4x) + 7 = -4x + 7$

(c) $(g \circ g)(x) = g(g(x)) = g(x + 7) = (x + 7) + 7 = x + 14$

$f(x) = x^2,\ g(x) = x - 1$

$(f \circ g)(x) = f(g(x)) = f(x-1) = (x-1)^2$

$(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 1$

$(g \circ g)(x) = g(g(x)) = g(x-1) = x - 2$

$f(x) = 3x,\ g(x) = x^4$

$(f \circ g)(x) = f(g(x)) = f(x^4) = 3(x^4) = 3x^4$

$(g \circ f)(x) = g(f(x)) = g(3x) = (3x)^4 = 81x^4$

$(g \circ g)(x) = g(g(x)) = g(x^4) = (x^4)^4 = x^{16}$

$f(x) = \sqrt[3]{x-1},\ g(x) = x^3 + 1$

$(f \circ g)(x) = f(g(x))$

$f(x^3 + 1)$

$\sqrt[3]{(x^3+1) - 1}$

$\sqrt[3]{x^3} = x$

$(g \circ f)(x) = g(f(x))$

$g(\sqrt[3]{x-1})$

$(\sqrt[3]{x-1})^3 + 1$

$(x-1) + 1 = x$

$(g \circ g)(x) = g(g(x))$

$g(x^3 + 1)$

$(x^3+1)^3 + 1$

$x^9 + 3x^6 + 3x^3 + 2$

$f(x) = x^3,\ g(x) = \dfrac{1}{x}$

(a) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right) = \left(\dfrac{1}{x}\right)^3 = \dfrac{1}{x^3}$

$(g \circ f)(x) = g(f(x)) = g(x^3) = \dfrac{1}{x^3}$

(c) $(g \circ g)(x) = g(g(x)) = g\left(\dfrac{1}{x}\right) = x$

35. $f(x) = \sqrt{x+4}$ Domain: $x \ge -4$

$g(x) = x^2$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x^2) =$

$(g \circ f)(x) = g(f(x))$

$g(\sqrt{x+4}) = (\sqrt{x+4})^2 = x + 4$

Domain: $x \ge -4$

$f(x) = \sqrt[3]{x-5}$ Domain: all real numbers x

$g(x) = x^3 + 1$ Domain: all real numbers x

$(f \circ g)(x) = f(g(x))$

$f(x^3 + 1)$

$\sqrt[3]{x^3 + 1 - 5}$

$\sqrt[3]{x^3 - 4}$

Domain: all real numbers x

$(g \circ f)(x) = g(f(x))$

$g(\sqrt[3]{x-5})$

$(\sqrt[3]{x-5})^3 + 1$

$x - 5 + 1 = x - 4$

Domain: all real numbers x

37. $f(x) = x^3$ Domain: all real numbers x

$g(x) = x^{2/3}$ Domain: all real numbers x

$(f \circ g)(x) = f(g(x)) = f(x^{2/3}) = (x^{2/3})^3 = x^2$

Domain: all real numbers x.

$(g \circ f)(x) = g(f(x)) = g(x^3) = (x^3)^{2/3} = x^2$

Domain: all real numbers x.

$f(x) = x^5$ Domain: all real numbers x

$g(x) = \sqrt[4]{x} = x^{1/4}$ Domain: all real numbers $x \ge 0$

$(f \circ g)(x) = f(g(x)) = f(x^{1/4}) = (x^{1/4})^5 = x^{5/4}$

Domain: all real numbers $x \ge 0$.

$(g \circ f)(x) = g(f(x)) = g(x^5) = (x^5)^{1/4} = x^{5/4}$

Domain: all real numbers $x \ge 0$.

$f(x) = |x|$ Domain: all real numbers x

$g(x) = x + 6$ Domain: all real numbers x

$(f \circ g)(x) = f(g(x)) = f(x+6) = |x+6|$

Domain: all real numbers x

$x^2 + 4$ $\sqrt{}$

Domain: all real numbers x

$(g \circ f)(x) = g(f(x)) = g(|x|) = |x| + 6$

Domain: all real numbers x

$f(x) = |x - 4|$ Domain: all real numbers x

$g(x) = 3 - x$ Domain: all real numbers x

$(f \circ g)(x) = f(g(x)) = f(3 - x) = |(3 - x) - 4| = |-x - 1| = |x + 1|$

Domain: all real numbers x

$(g \circ f)(x) = g(f(x)) = g(|x - 4|) = 3 - (|x - 4|) = 3 - |x - 4|$

Domain: all real numbers x

41. $f(x) = \frac{1}{x}$ Domain: all real numbers x except $x = 0$

$g(x) = x + 3$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 3) = \frac{1}{x + 3}$

Domain: all real numbers x except $x = -3$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{x} + 3$

Domain: all real numbers x except $x = 0$

42. $f(x) = \frac{3}{x^2 - 1}$ Domain: all real numbers x except $x = \pm 1$

$g(x) = x + 1$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 1) = \frac{3}{(x + 1)^2 - 1} = \frac{3}{x^2 + 2x + 1 - 1} = \frac{3}{x^2 + 2x}$

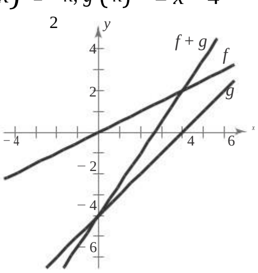
Domain: all real numbers x except $x = 0$ and $x = -2$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\frac{3}{x^2 - 1}\right) = \frac{3}{x^2 - 1} + 1 = \frac{3 + x^2 - 1}{x^2 - 1} = \frac{x^2 + 2}{x^2 - 1}$

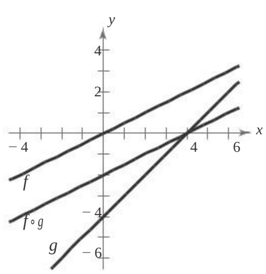
Domain: all real numbers x except $x = \pm 1$

43. $f(x) = \frac{1}{2}x,\ g(x) = x - 4$

(a)

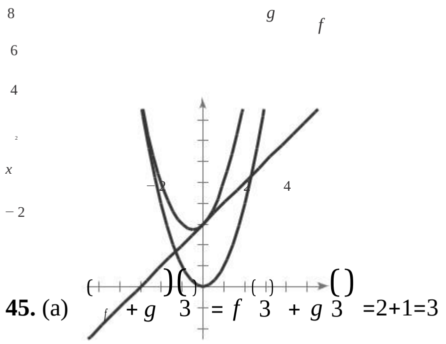


(b)

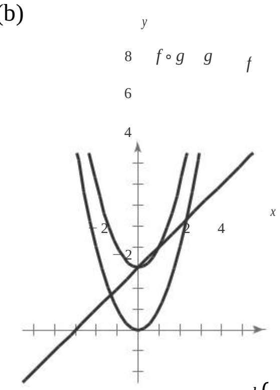


$f(x) = x + 3,\ g(x) = x^2$

(a)



(b)



45. (a) $(f + g)(3) = f(3) + g(3) = 2 + 1 = 3$

(b) $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{0}{2} = 0$

46. (a) $(f - g)(1) = f(1) - g(1) = 2 - 3 = -1$

(b) $(fg)(4) = f(4) \cdot g(4) = 4 \cdot 0 = 0$

47. (a) $(f \circ g)(2) = f(g(2)) = f(2) = 0$

(b) $(g \circ f)(2) = g(f(2)) = g(0) = 4$

48. (a) $(f \circ g)(1) = f(g(1)) = f(3) = 2$

(b) $(g \circ f)(3) = g(f(3)) = g(2) = 2$

49. $h(x) = (2x^2 + 1)^2$

One possibility: Let $f(x) = x^2$ and $g(x) = 2x + 1$, then $(f \circ g)(x) = h(x)$.

50. $h(x) = (1 - x)^3$

One possibility: Let $g(x) = 1 - x$ and $f(x) = x^3$, then $(f \circ g)(x) = h(x)$.

51. $h(x) = \sqrt[3]{x^2 - 4}$

One possibility: Let $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - 4$, then $(f \circ g)(x) = h(x)$.

54. $h(x) = \frac{4}{(5x + 2)^2}$

One possibility: Let $g(x) = 5x + 2$ and $f(x) = \frac{4}{x^2}$, then $(f \circ g)(x) = h(x)$.

55. $h(x) = \frac{-x^2 + 3}{4 - x^2}$

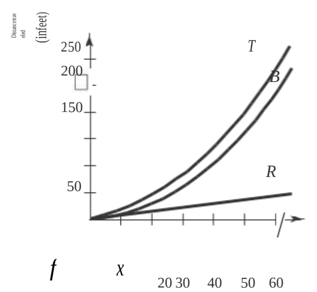
One possibility: Let $f(x) = \frac{x + 3}{4 + x}$ and $g(x) = -x^2$, then $(f \circ g)(x) = h(x)$.

56. $h(x) = \frac{27x^3 + 6x}{10 - 27x^3}$

One possibility: Let $g(x) = x^3$ and $f(x) = \frac{27x + 6\sqrt[3]{x}}{10 - 27x}$, then $(f \circ g)(x) = h(x)$.

57. (a) $T(x) = R(x) + B(x) = \frac{3}{4}x + \frac{1}{15}x^2$

(b)



52. $h(x) = \sqrt{9 - x}$

One possibility: Let $g(x) = 9 - x$ and $f(x) = \sqrt{x}$, then $(f \circ g)(x) = h(x)$.

then $(f \circ g)(x) = h(x)$.

53. $h(x) = \dfrac{1}{x+2}$.

One possibility: Let $f(x) = 1/x$ and $g(x) = x + 2$, Speed (in miles per hour)

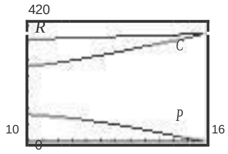
(c) $B(x)$; As x increases, $B(x)$ increases at a faster rate.

58. (a) $P = R - C$

$= (341 + 3.2t) - (254 - 9t + 1.1t^2)$

$= -1.1t^2 + 12.2t + 87$

(b)



59. (a) $c(t) = \dfrac{b(t) - d(t)}{p(t)} \times 100$

$c(16)$ represents the percent change in the population due to births and deaths in the year 2016.

(a) $p(t) = d(t) + c(t)$

$p(16)$ represents the number of dogs and cats in 2016.

(c) $h(t) = \dfrac{p(t)}{n(t)} = \dfrac{d(t) + c(t)}{n(t)}$

$h(t)$ represents the number of dogs and cats per capita.

61. (a) $r(x) = \dfrac{x}{2}$

(b) $A(r) = \pi r^2$

(c) $(A \circ r)(x) = A(r(x)) = A\left(\dfrac{x}{2}\right) = \pi\left(\dfrac{x}{2}\right)^2$

$(A \circ r)(x)$ represents the area of the circular base of the tank on the square foundation with side length x.

(a) $N(T(t)) = N(3t + 2)$

$10(3t + 2)^2 - 20(3t + 2) + 600$

$10(9t^2 + 12t + 4) - 60t - 40 + 600$

$90t^2 + 60t + 600$

$30(3t^2 + 2t + 20), \quad 0 \le t \le 6$

This represents the number of bacteria in the food as a function of time.

Use $t = 0.5$.

$N(T(0.5)) = 30\left(3(0.5)^2 + 2(0.5) + 20\right) = 652.5$

After half an hour, there will be about 653 bacteria.

(c) $30(3t^2 + 2t + 20) = 1500$

$3t^2 + 2t + 20 = 50$

$3t^2 + 2t - 30 = 0$

By the Quadratic Formula, $t \approx -3.513$ or 2.846.

Choosing the positive value for t, you have ≈ 2.846 hours.

(a) $f(g(x)) = f(0.03x) = 0.03x - 500{,}000$

$g(f(x)) = g(x - 500{,}000) = 0.03(x - 500{,}000)$

$(f(x))$ represents your bonus of 3% of an amount over $500,000.

factory rebate.

(b) $S(p) = 0.9p$ the cost of the car with the dealership discount.

(c) $(R \circ S)(p) = R(0.9p) = 0.9p - 2000$

$(S \circ R)(p) = S(p - 2000)$

$0.9(p - 2000) = 0.9p - 1800$

$(R \circ S)(p)$ represents the factory rebate *after* the dealership discount.

$(S \circ R)(p)$ represents the dealership discount after the factory rebate.

$(R \circ S)(p) = (R \circ S)(20{,}500)$

$0.9(20{,}500) - 2000 = \$16{,}450$

$(S \circ R)(p) = (S \circ R)(20{,}500)$

$0.9(20{,}500) - 1800 = \$16{,}650$

$(R \circ S)(20{,}500)$ yields the lower cost because 10% of the price of the car is more than $2000.

65. False. $(f \circ g)(x) = 6x + 1$ and $(g \circ f)(x) = 6x + 6$

True. $(f \circ g)(c)$ is defined only when $g(c)$ is in the domain of f.

67. Let O = oldest sibling, M = middle sibling, Y = youngest sibling.

Then the ages of each sibling can be found using the equations:

$O = 2M$

$M = \frac{1}{2}Y + 6$

(a) $O(M(Y)) = 2\left(\frac{1}{2}Y + 6\right) = 12 + Y$; Answers will vary.

Oldest sibling is 16: $O = 16$

Middle sibling: $O = 2M$

$16 = 2M$

$M = 8$ years old

Youngest sibling: $M = \frac{1}{2}Y + 6$

$8 = \frac{1}{2}Y + 6$

$2 = \frac{1}{2}Y$

$Y = 4$ years old

(a) $Y(M(O)) = 2\left(\frac{1}{2}O\right) - 12 = O - 12$; Answers will vary.

Youngest sibling is 2 → $Y = 2$

Middle sibling: $M = \frac{1}{2}Y + 6$

$= \frac{1}{2}(2) + 6$

$M = 7$ years old

Oldest sibling: $O = 2M$

$= 2(7)$

$= 14$ years old

Let $f(x)$ and $g(x)$ be two odd functions and define $h(x) = f(x)g(x)$. Then

$h(-x) = f(-x)g(-x)$

$= \left[-f(x)\right]\left[-g(x)\right]$ because f and g are odd

$f(x)g(x)$

$h(x)$.

So, $h(x)$ is even.

Let $f(x)$ and $g(x)$ be two even functions and define $h(x) = f(x)g(x)$. Then

$h(-x) = f(-x)g(-x)$

$f(x)g(x)$ because f and g are even

$h(x)$.

So, $h(x)$ is even.

Let $f(x)$ be an odd function, $g(x)$ be an even function, and define $h(x) = f(x)g(x)$. Then

$h(-x) = f(-x)g(-x)$

$= \left[-f(x)\right]g(x)$ because f is odd and g is even

$= -f(x)g(x)$

$= -h(x)$.

So, h is odd and the product of an odd function and an even function is odd.

(a) Answer not unique. *Sample answer*:

$f(x) = x + 3,\ g(x) = x + 2$

$(f \circ g)(x) = f(g(x)) = (x + 2) + 3 = x + 5$

$(g \circ f)(x) = g(f(x)) = (x + 3) + 2 = x + 5$

Answer not unique. *Sample answer*: $f(x) = x^2$, $g(x) = x^3$

$(f \circ g)(x) = f(g(x)) = (x^3)^2 = x^6$

$(g \circ f)(x) = g(f(x)) = (x^2)^3 = x^6$

(a) $f(p)$: matches L_2; For example, an original price of $p = \$15.00$ corresponds to a sale price of $S = \$7.50$.

(b) $g(p)$: matches L_1; For example an original price of $= \$20.00$ corresponds to a sale price of $S = \$15.00$.

$(g \circ f)(p)$: matches L_4; This function represents applying a 50% discount to the original price p, then subtracting a \$5 discount.

(d) $(f \circ g)(p)$ matches L_3; This function represents subtracting a \$5 discount from the original price p, then applying a 50% discount.

73. (a) $g(x) = \frac{1}{2}\left[f(x) + f(-x)\right]$

To determine if $g(x)$ is even, show $g(-x) = g(x)$. ✓

$g(-x) = \frac{1}{2}\left[f(-x) + f(-(-x))\right] = \frac{1}{2}\left[f(-x) + f(x)\right] = \frac{1}{2}\left[f(x) + f(-x)\right] = g(x)$

$h(x) = \frac{1}{2}\left[f(x) - f(-x)\right]$

To determine if $h(x)$ is odd show $h(-x) = -h(x)$.

$\frac{1}{2}\left[\ \ f(-(-x))\right] = \frac{1}{2}\ \ \frac{1}{2}\ \ f(x)\Big]\ \ h(x)$

(b) Let $f(x) = a$ function

$f(x)$ = even function + odd function.

Using the result from part (a) $g(x)$ is an even function and $h(x)$ is an odd function.

$$f(x) = g(x) + h(x) = \frac{1}{2}\left[f(x) + f(-x)\right] + \frac{1}{2}\left[f(x) - f(-x)\right] = \frac{1}{2}f(x) + \frac{1}{2}f(-x) + \frac{1}{2}f(x) - \frac{1}{2}f(-x) = f(x)$$

(c) $f(x) = x^2 - 2x + 1$

$f(x) = g(x) + h(x)$

$$g(x) = \frac{1}{2}\left[f(x) + f(-x)\right] = \frac{1}{2}\left[x^2 - 2x + 1 + (-x)^2 - 2(-x) + 1\right]$$

$$= \frac{1}{2}\left[x^2 - 2x + 1 + x^2 + 2x + 1\right] = \frac{1}{2}\left[2x^2 + 2\right] = x^2 + 1$$

$$h(x) = \frac{1}{2}\left[f(x) - f(-x)\right] = \frac{1}{2}\left[x^2 - 2x + 1 - \left((-x)^2 - 2(-x) + 1\right)\right]$$

$$= \frac{1}{2}\left[x^2 - 2x + 1 - x^2 - 2x - 1\right] = \frac{1}{2}\left[-4x\right] = -2x$$

$f(x) = (x^2 + 1) + (-2x)$

$k(x) = \dfrac{1}{x + 1}$

$k(x) = g(x) + h(x)$

$$g(x) = \frac{1}{2}\left[k(x) + k(-x)\right] = \frac{1}{2}\left[\frac{1}{x + 1} + \frac{1}{-x + 1}\right]$$

$$= \frac{1}{2}\left[\frac{1 - x + x + 1}{(x + 1)(1 - x)}\right] = \frac{1}{2}\left[\frac{2}{(x + 1)(1 - x)}\right]$$

$$= \frac{1}{(x + 1)(1 - x)} = \frac{-1}{(x + 1)(x - 1)}$$

$$h(x) = \frac{1}{2}\left[k(x) - k(-x)\right] = \frac{1}{2}\left[\frac{1}{x + 1} - \frac{1}{1 - x}\right]$$

$$= \frac{1}{2}\left[\frac{1 - x - (x + 1)}{(x + 1)(1 - x)}\right] = \frac{1}{2}\left[\frac{-2x}{(x + 1)(1 - x)}\right]$$

$$= \frac{-x}{(x + 1)(1 - x)} = \frac{x}{(x + 1)(x - 1)}$$

$$k(x) = \left(\frac{-1}{(x + 1)(x - 1)}\right) + \left(\frac{x}{(x + 1)(x - 1)}\right)$$

Section 2.7 Inverse Functions

1. inverse

4. $y = x$

2. f^{-1}

3. range; domain $\cdot$ **6**

5. one-to-one

Horizontal

$f(x) = 6x$

$f^{-1}(x) = \frac{x}{6} = \frac{1}{6}x$

$f(f^{-1}(x)) = f\left(\frac{x}{6}\right) = 6\left(\frac{x}{6}\right) = x$

$f^{-1}(f(x)) = f^{-1}(6x) = \frac{6x}{6} = x$

$f(x) = 3x + 1$

$f^{-1}(x) = \frac{x-1}{3}$

$f(f^{-1}(x)) = f\left(\frac{x-1}{3}\right) = 3\left(\frac{x-1}{3}\right) + 1 = x$

$f^{-1}(f(x)) = f^{-1}(3x+1) = \frac{(3x+1)-1}{3} = x$

8. $f(x) = \frac{1}{3}x$

$f^{-1}(x) = 3x$

$f(f^{-1}(x)) = f(3x) = \frac{1}{3}(3x) = x$

$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{3}x\right) = 3\left(\frac{1}{3}x\right) = x$

$f(x) = \frac{x-3}{2}$

$f^{-1}(x) = 2x + 3$

$f(f^{-1}(x)) = f(2x+3) = \frac{(2x+3)-3}{2} = \frac{2x}{2} = x$

$f^{-1}(f(x)) = f^{-1}\left(\frac{x-3}{2}\right) = 2\left(\frac{x-3}{2}\right) + 3 = x - 3 + 3 = x$

11. $f(x) = x^2 - 4,\ x \ge 0$

$f^{-1}(x) = \sqrt{x+4}$

$f(f^{-1}(x)) = f\left(\sqrt{x+4}\right) = \left(\sqrt{x+4}\right)^2 - 4 = (x+4) - 4 = x$

$f^{-1}(f(x)) = f^{-1}(x^2 - 4) = \sqrt{(x^2 - 4) + 4} = \sqrt{x^2} = x$

12. $f(x) = x^2 + 2,\ x \ge 0$

$f^{-1}(x) = \sqrt{x-2}$

$f(f^{-1}(x)) = f\left(\sqrt{x-2}\right) = \left(\sqrt{x-2}\right)^2 + 2 = (x-2) + 2 = x$

$f^{-1}(f(x)) = f^{-1}(x^2 + 2) = \sqrt{(x^2 + 2) - 2} = \sqrt{x^2} = x$

$f(x) = x^3 + 1$

$f^{-1}(x) = \sqrt[3]{x-1}$

$f(f^{-1}(x)) = f\left(\sqrt[3]{x-1}\right) = \left(\sqrt[3]{x-1}\right)^3 + 1 = (x-1) + 1 = x$

$f^{-1}(f(x)) = f^{-1}(x^3 + 1) = \sqrt[3]{(x^3 + 1) - 1} = \sqrt[3]{x^3} = x$

$f(x) = \frac{x^5}{4}$

$f^{-1}(x) = \sqrt[5]{4x}$

$f(f^{-1}(x)) = f\left(\sqrt[5]{4x}\right) = \frac{\left(\sqrt[5]{4x}\right)^5}{4} = \frac{4x}{4} = x$

$f^{-1}(f(x)) = f^{-1}\left(\frac{x^5}{4}\right) = \sqrt[5]{4\left(\frac{x^5}{4}\right)} = \sqrt[5]{x^5} = x$

15. $(f \circ g)(x) = f(g(x)) = f(4x + 9) = \dfrac{4x + 9 - 9}{4} = \dfrac{4x}{4} = x$

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{x - 9}{4}\right) = 4\left(\dfrac{x - 9}{4}\right) + 9 = x - 9 + 9 = x$

16. $f(g(x)) = f\left(-\dfrac{2x + 8}{3}\right) = -\dfrac{3}{2}\left(-\dfrac{2x + 8}{3}\right) - 4$

$= \dfrac{1}{2}\left(2(x + 4)\right) - 4$

$= x + 4 - 4 = x$

$g(f(x)) = g\left(-\dfrac{3}{2}x - 4\right) = -\dfrac{2\left(-\frac{3}{2}x - 4\right) + 8}{3}$

$= -\dfrac{-3x - 8 + 8}{3}$

$= -\dfrac{-3x}{3}$

$= x$

17. $f(g(x)) = f\left(\sqrt[3]{4x}\right) = \dfrac{\left(\sqrt[3]{4x}\right)^3}{4}$

$= \dfrac{4x}{4}$

$= x$

$g(f(x)) = g\left(\dfrac{x^3}{4}\right) = \sqrt[3]{4\left(\dfrac{x^3}{4}\right)}$

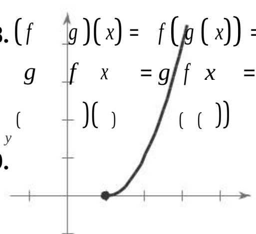
$= \sqrt[3]{x^3}$

$= x$

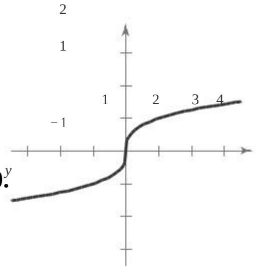
18. $(f \circ g)(x) = f(g(x)) = f\left(\sqrt[3]{x - 5}\right) = \left(\sqrt[3]{x - 5}\right)^3 + 5 = x - 5 + 5 = x$

$(g \circ f)(x) = g(f(x)) = g\left(x^3 + 5\right) = \sqrt[3]{x^3 + 5 - 5} = \sqrt[3]{x^3} = x$

19.



20.

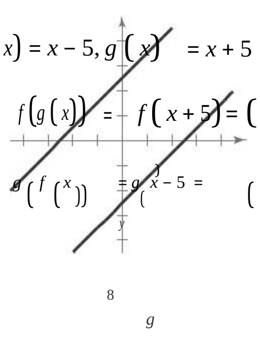


21. $f(x) = x - 5,\ g(x) = x + 5$

(a) $f(g(x)) = f(x + 5) = (x + 5) - 5 = x$

$g(f(x)) = g(x - 5) = (x - 5) + 5 = x$

(b)

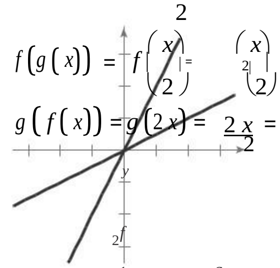


22. $f(x) = 2x,\ g(x) = \dfrac{x}{2}$

(a) $f(g(x)) = f\left(\dfrac{x}{2}\right) = 2\left(\dfrac{x}{2}\right) = x$

$g(f(x)) = g(2x) = \dfrac{2x}{2} = x$

(b)

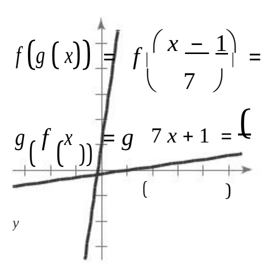


23. $f(x) = 7x + 1,\ g(x) = \dfrac{x - 1}{7}$

(a) $f(g(x)) = f\left(\dfrac{x - 1}{7}\right) = 7\left(\dfrac{x - 1}{7}\right) + 1 = x$

$g(f(x)) = g(7x + 1) = \dfrac{(7x + 1) - 1}{7} = x$

(b)



x

1 2 3 4 5

−4 f

−8 f

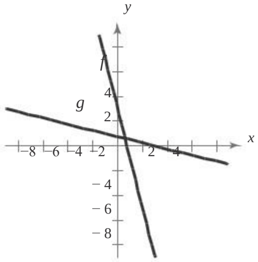
24. $f(x) = 3 - 4x,\ g(x) = \dfrac{3 - x}{4}$

(a) $f(g(x)) = f\left(\dfrac{3-x}{4}\right) = 3 - 4\left(\dfrac{3-x}{4}\right)$

$= 3 - (3 - x) = x$

$g(f(x)) = g(3 - 4x) = \dfrac{3 - (3 - 4x)}{4} = \dfrac{4x}{4} = x$

(b)

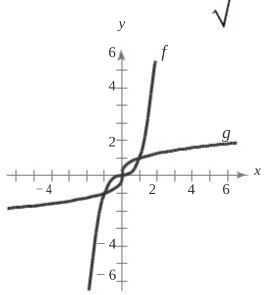


25. $f(x) = x^3,\ g(x) = \sqrt[3]{x}$

(a) $f(g(x)) = f(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$

$g(f(x)) = g(x^3) = \sqrt[3]{x^3} = x$

(b)

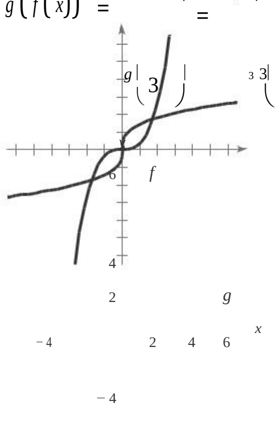


26. $f(x) = \dfrac{x^3}{3},\ g(x) = \sqrt[3]{3x}$

(a) $f(g(x)) = f(\sqrt[3]{3x}) = \dfrac{(\sqrt[3]{3x})^3}{3} = \dfrac{3x}{3} = x$

$g(f(x)) = g\left(\dfrac{x^3}{3}\right) = \sqrt[3]{3\left(\dfrac{x^3}{3}\right)} = \sqrt[3]{x^3} = x$

(b)



27. $f(x) = \sqrt{x + 5},\ g(x) = x^2 - 5,\ x \ge 0$

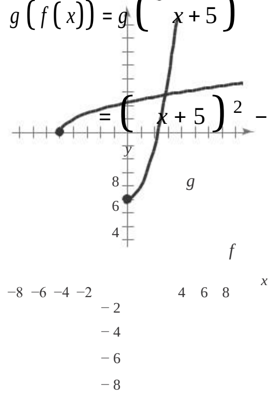
(a) $f(g(x)) = f(x^2 - 5),\ x \ge 0$

$= \sqrt{(x^2 - 5) + 5} = x$

$g(f(x)) = g(\sqrt{x + 5})$

$= (\sqrt{x + 5})^2 - 5 = x$

(b)



28. $f(x) = 1 - x^3,\ g(x) = \sqrt[3]{1 - x}$

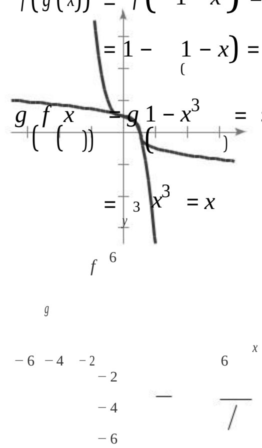
(a) $f(g(x)) = f(\sqrt[3]{1 - x}) = 1 - (\sqrt[3]{1 - x})^3$

$= 1 - (1 - x) = x$

$g(f(x)) = g(1 - x^3) = \sqrt[3]{1 - (1 - x^3)}$

$= \sqrt[3]{x^3} = x$

(b)

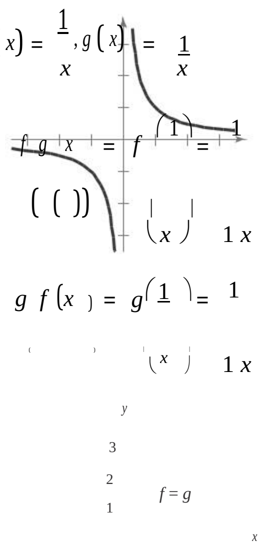


29. $f(x) = \dfrac{1}{x},\ g(x) = \dfrac{1}{x}$

(a) $f(g(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$

$g(f(x)) = g\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$

(b)

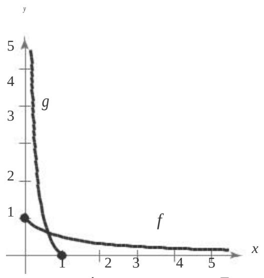


30. $f(x) = \frac{1}{1+x},\ x \ge 0;\ g(x) = \frac{1-x}{x},\ 0 < x \le 1$

(a) $f(g(x)) = f\left(\frac{1-x}{x}\right) = \frac{1}{1+\left(\frac{1-x}{x}\right)} = \frac{1}{\frac{x+1-x}{x}} = \frac{1}{\frac{1}{x}} = x$

$g(f(x)) = g\left(\frac{1}{1+x}\right) = \frac{1-\left(\frac{1}{1+x}\right)}{\left(\frac{1}{1+x}\right)} = \frac{\frac{1+x}{1+x}-\frac{1}{1+x}}{\frac{1}{1+x}} = \frac{\frac{x}{1+x}}{\frac{1}{1+x}} = \frac{x}{1+x}\cdot\frac{x+1}{1} = x$

(b)

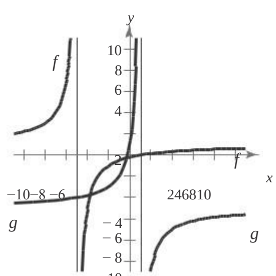


31. $f(x) = \frac{x-1}{x+5},\ g(x) = -\frac{5x+1}{x-1}$

(a) $f(g(x)) = f\left(-\frac{5x+1}{x-1}\right) = \frac{\left(-\frac{5x+1}{x-1}\right)-1}{\left(-\frac{5x+1}{x-1}\right)+5}\cdot\frac{x-1}{x-1} = \frac{-(5x+1)-(x-1)}{-(5x+1)+5(x-1)} = \frac{-6x}{-6} = x$

$g(f(x)) = g\left(\frac{x-1}{x+5}\right) = -\frac{\left[5\left(\frac{x-1}{x+5}\right)+1\right]}{\left[\frac{x-1}{x+5}-1\right]}\cdot\frac{x+5}{x+5} = -\frac{5(x-1)+(x+5)}{(x-1)-(x+5)} = -\frac{6x}{-6} = x$

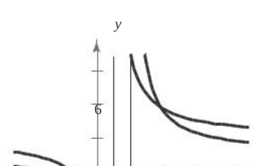
(b)



$f(x) = \frac{x+3}{x-2},\ g(x) = \frac{2x+3}{x-1}$

(a) $f(g(x)) = f\left(\frac{2x+3}{x-1}\right) = \frac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2} = \frac{\frac{2x+3+3x-3}{x-1}}{\frac{2x+3-2x+2}{x-1}} = \frac{5x}{5} = x$

(b)



$$g(f(x)) = g\left(\frac{x+3}{x-2}\right) = \frac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1} = \frac{\frac{2x+6+3x-6}{x-2}}{\frac{x+3-x+2}{x-2}} = \frac{5x}{5} = x$$

f g 4 2 −4 −2 4 g x

−4

−6

f

6 8

No, $\{(-2, -1), (1, 0), (2, 1), (1, 2), (-2, 3), (-6, 4)\}$ does not represent a function. −2 and 1 are paired with two different values.

Yes, $\{(10, -3), (6, -2), (4, -1), (1, 0), (-3, 2), (10, 2)\}$ does represent a function.

35.

x	3	5	7	9	11	13
$f^{-1}(x)$	–10123					4

36.

x	10	5	0	–5	–10	–15
$f^{-1}(x)$	–3–2–10				1	2

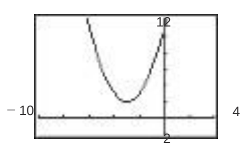
37. Yes, because no horizontal line crosses the graph of *f* at more than one point, *f has* an inverse.

38. No, because some horizontal lines intersect the graph of *f* twice, *f does not* have an inverse.

39. No, because some horizontal lines cross the graph of *f* twice, *f does not* have an inverse.

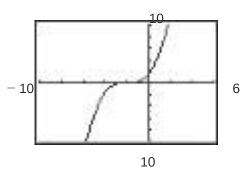
40. Yes, because no horizontal lines intersect the graph of *f* at more than one point, *f has* an inverse.

41. $g(x) = (x + 3)^2 + 2$



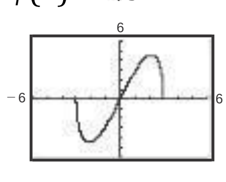
g does not pass the Horizontal Line Test, so *g does not* have an inverse.

42. $f(x) = \frac{1}{5}(x + 2)^3$



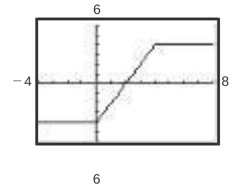
f does pass the Horizontal Line Test, so *f does* have an inverse.

43. $f(x) = x\sqrt{9 - x^2}$



f does not pass the Horizontal Line Test, so *f does not* have an inverse.

44. $h(x) = |x| - |x - 4|$



h does not pass the Horizontal Line Test, so *h does not* have an inverse.

45. (a) $f(x) = x^5 - 2$

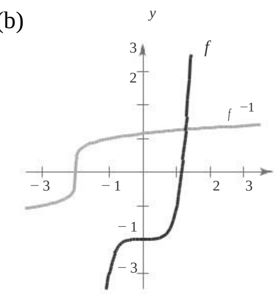
$y = x^5 - 2$

$x = y^5 - 2$

$y = \sqrt[5]{x + 2}$

$f^{-1}(x) = \sqrt[5]{x + 2}$

(b)



(c) The graph of f^{-1} is the reflection of the graph of *f* in the line $y = x$.

(d) The domains and ranges of *f* and f^{-1} are all real numbers.

46. (a) $f(x) = x^3 + 8$

$y = x^3 + 8$

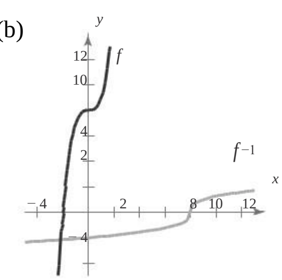
$x = y^3 + 8$

$x - 8 = y^3$

$\sqrt[3]{x - 8} = y$

$f^{-1}(x) = \sqrt[3]{x - 8}$

(b)



The graph of f^{-1} is the reflection of *f* in the line $y = x$.

The domains and ranges of *f* and f^{-1} are all real numbers.

47. (a) $f(x) = \sqrt{4 - x^2},\ 0 \le x \le 2$

$y = \sqrt{4 - x^2}$

$x = \sqrt{4 - y^2}$

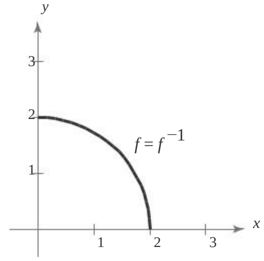
$x^2 = 4 - y^2$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

$f^{-1}(x) = \sqrt{4 - x^2},\ 0 \le x \le 2$

(b)



The graph of f^{-1} is the same as the graph of f.

The domains and ranges of f and f^{-1} are all real numbers x such that $0 \le x \le 2$.

48. (a) $f(x) = x^2 - 2,\ x \le 0$

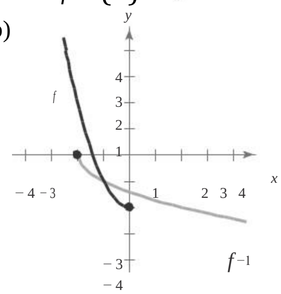
$y = x^2 - 2$

$x = y^2 - 2$

$\pm\sqrt{x + 2} = y$

$f^{-1}(x) = -\sqrt{x + 2}$

(b)



The graph of f^{-1} is the reflection of f in the line $y = x$.

(d) $[-2, \infty)$ is the range of f and domain of f^{-1}.

$(-\infty, 0]$ is the domain of f and the range of f^{-1}.

49. (a) $f(x) = \dfrac{4}{x}$

$y = \dfrac{4}{x}$

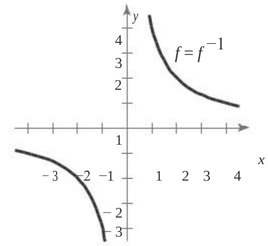
$x = \dfrac{4}{y}$

$xy = 4$

$y = \dfrac{4}{x}$

$f^{-1}(x) = \dfrac{4}{x}$

(b)



The graph of f^{-1} is the same as the graph of f.

The domains and ranges of f and f^{-1} are all real numbers except for 0.

50. (a) $f(x) = -\dfrac{2}{x}$

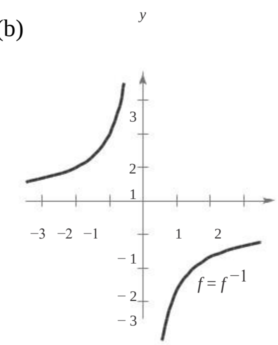
$y = -\dfrac{2}{x}$

$x = -\dfrac{2}{y}$

$y = -\dfrac{2}{x}$

$f^{-1}(x) = -\dfrac{2}{x}$

(b)



(c) The graphs are the same.

(d) The domains and ranges of f and f^{-1} are all real numbers except for 0.

51. (a) $f(x) = \dfrac{x + 1}{x - 2}$

$y = \dfrac{x + 1}{x - 2}$

$x = \dfrac{y + 1}{y - 2}$

$x(y - 2) = y + 1$

$xy - 2x = y + 1$

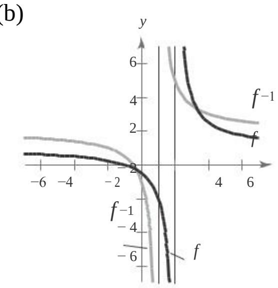
$xy - y = 2x + 1$

$y(x - 1) = 2x + 1$

$y = \dfrac{2x + 1}{x - 1}$

$f^{-1}(x) = \dfrac{2x + 1}{x - 1}$

(b)



(c) The graph of f^{-1} is the reflection of graph of f in the line $y = x$.

The domain of f and the range of f^{-1} is all real numbers except 2.

The range of f and the domain of f^{-1} is all real numbers except 1.

52. (a) $f(x) = \frac{x-2}{3x+5}$

$y = \frac{x-2}{3x+5}$

$x = \frac{y-2}{3y+5}$

$3xy + 5x - y + 2 = 0$

$3xy - y = -5x - 2$

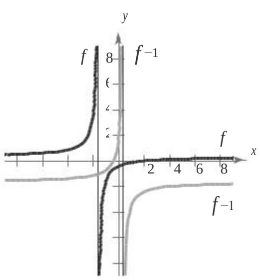
$y(3x-1) = -5x - 2$

$= \frac{-5x-2}{3x-1}$

$= -\frac{5x+2}{3x-1}$

$f^{-1}(x) = -\frac{5x+2}{3x-1}$

(b)



The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

The domain of f and the range of f^{-1} is all real numbers except $x = -\frac{5}{3}$.

The range of f and the domain of f^{-1} is all real numbers x except $x = \frac{1}{3}$.

53. (a) $f(x) = \sqrt[3]{x-1}$

$y = \sqrt[3]{x-1}$

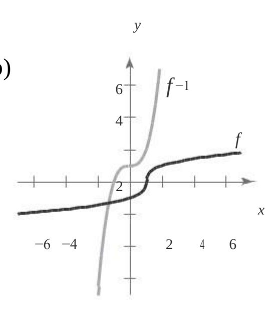
$x = \sqrt[3]{y-1}$

$x^3 = y - 1$

$y = x^3 + 1$

$f^{-1}(x) = x^3 + 1$

(b)



The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

The domains and ranges of f and f^{-1} are all real numbers.

54. (a) $f(x) = x^{3/5}$

$y = x^{3/5}$

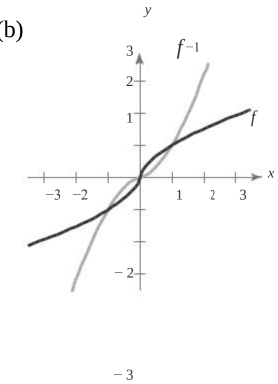
$x = y^{3/5}$

$x^{5/3} = \left(y^{3/5}\right)^{5/3}$

$x^{5/3} = y$

$f^{-1}(x) = x^{5/3}$

(b)



The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

The domains and ranges of f and f^{-1} are all real numbers.

$f(x) = x^4$

$= x^4$

$= y^4$

$= \pm\sqrt[4]{x}$

This does not represent y as a function of x. f does not have an inverse.

$f(x) = \frac{1}{x^2}$

$y = \frac{1}{x^2}$

$x = \frac{1}{y^2}$

$y^2 = \frac{1}{x}$

$y = \pm\sqrt{\frac{1}{x}}$

This does not represent y as a function of x. f does not have an inverse.

$g(x) = \frac{x+1}{6}$

$y = \frac{x+1}{6}$

$x = \frac{y+1}{6}$

$6x = y + 1$

$= 6x - 1$

This is a function of x, so g has an inverse.

$g^{-1}(x) = 6x - 1$

58. $f(x) = 3x + 5$

$y = 3x + 5$

$x = 3y + 5$

$x - 5 = 3y$

$\frac{x-5}{3} = y$

This is a function of x, so f has an inverse.

$f^{-1}(x) = \frac{x-5}{3}$

59. $p(x) = -4$

$y = -4$

Because $y = -4$ for all x, the graph is a horizontal line and fails the Horizontal Line Test. p does not have an inverse.

60. $f(x) = 0$

$y = 0$

Because $y = 0$ for all x, the graph is a horizontal line and fails the Horizontal Line Test. f does not have an inverse.

61. $f(x) = (x+3)^2,\ x \ge -3 \Rightarrow y \ge 0$

$y = (x+3)^2,\ x \ge -3,\ y \ge 0$

$= (y+3)^2,\ y \ge -3,\ x \ge 0$

$\sqrt{x} = y + 3,\ y \ge -3,\ x \ge 0$

$y = \sqrt{x} - 3,\ x \ge 0,\ y \ge -3$

This is a function of x, so f has an inverse.

$f^{-1}(x) = \sqrt{x} - 3,\ x \ge 0$

$q(x) = (x-5)^2$

$= (x-5)^2$

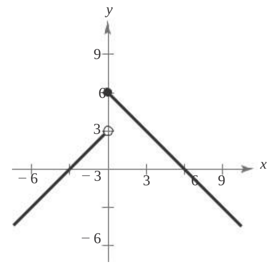
$= (y-5)^2$

$\sqrt{x} = y - 5$

$5 \pm \sqrt{x} = y$

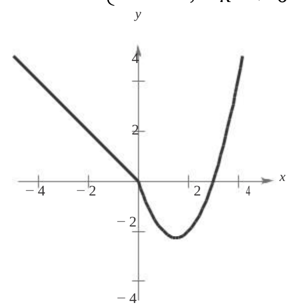
This does not represent y as a function of x, so q does not have an inverse.

63. $f(x) = \begin{cases} x+3, & x < 0 \\ 6-x, & x \ge 0 \end{cases}$



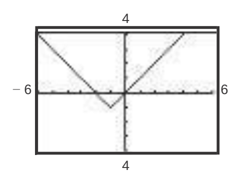
This graph fails the Horizontal Line Test, so f does not have an inverse.

64. $f(x) = \begin{cases} -x, & x \le 0 \\ x^2 - 3x, & x > 0 \end{cases}$



The graph fails the Horizontal Line Test, so f does not have an inverse.

65. $h(x) = |x+1| - 1$



The graph fails the Horizontal Line Test, so h does not have an inverse.

66. $f(x) = |x-2|,\ x \le 2 \Rightarrow y \ge 0$

$y = |x-2|,\ x \le 2,\ y \ge 0$

$x = |y-2|,\ y \le 2,\ x \ge 0$

$x = y - 2$ or $-x = y - 2$

$2 + x = y$ or $2 - x = y$

The portion that satisfies the conditions $y \le 2$ and ≥ 0 is $2 - x = y$. This is a function of x, so f has an inverse.

$^{-1}(x) = 2 - x,\ x \ge 0$

67. $f(x) = \sqrt{2x+3} \Rightarrow x \ge -\frac{3}{2},\ y \ge 0$

$y = \sqrt{2x+3},\ x \ge -\frac{3}{2},\ y \ge 0$

$x = \sqrt{2y+3},\ y \ge -\frac{3}{2},\ x \ge 0$

$x^2 = 2y + 3,\ x \ge 0,\ y \ge -\frac{3}{2}$

$y = \frac{x^2 - 3}{2},\ x \ge 0,\ y \ge -\frac{3}{2}$

This is a function of x, so f has an inverse.

$f^{-1}(x) = \frac{x^2 - 3}{2},\ x \ge 0$

$f(x) = \sqrt{x-2} \Rightarrow x \ge 2,\ y \ge 0$

$y = \sqrt{x-2},\ x \ge 2,\ y \ge 0$

$x = \sqrt{y-2},\ y \ge 2,\ x \ge 0$

$x^2 = y - 2,\ x \ge 0,\ y \ge 2$

$x^2 + 2 = y,\ x \ge 0,\ y \ge 2$

This is a function of x, so f has an inverse.

$f^{-1}(x) = x^2 + 2,\ x \ge 0$

$f(x) = \frac{6x+4}{4x+5}$

$y = \frac{6x+4}{4x+5}$

$x = \frac{6y+4}{4y+5}$

$x(4y+5) = 6y + 4$

$4xy + 5x = 6y + 4$

$4xy - 6y = -5x + 4$

$y(4x - 6) = -5x + 4$

$y = \frac{-5x+4}{4x-6}$

$= \frac{5x-4}{6-4x}$

This is a function of x, so f has an inverse.

$f^{-1}(x) = \frac{5x-4}{6-4x}$

The graph of f passes the Horizontal Line Test. So, you know f is one-to-one and has an inverse function.

$f(x) = \frac{5x-3}{2x+5}$

$y = \frac{5x-3}{2x+5}$

$x = \frac{5y-3}{2y+5}$

$x(2y+5) = 5y - 3$

$2xy + 5x = 5y - 3$

$2xy - 5y = -5x - 3$

$y(2x-5) = -(5x+3)$

$y = -\frac{5x+3}{2x-5}$

$f^{-1}(x) = -\frac{5x+3}{2x-5}$

$f(x) = |x+2|$

domain of $f: x \ge -2$, range of $f: y \ge 0$

$f(x) = |x+2|$

$y = |x+2|$

$x = y + 2$

$x - 2 = y$

So, $f^{-1}(x) = x - 2$.

domain of $f^{-1}: x \ge 0$, range of $f^{-1}: y \ge -2$

$f(x) = |x-5|$

domain of $f: x \ge 5$, range of $f: y \ge 0$

$f(x) = |x-5|$

$y = x - 5$

$x = y - 5$

$x + 5 = y$

So, $f^{-1}(x) = x + 5$.

domain $f^{-1}: x \ge 0$, range of $f^{-1}: y \ge 5$

$f(x) = (x+6)^2$

domain of $f: x \geq -6$, range of $f: y \geq 0$

$f(x) = (x+6)^2$

$= (x+6)^2$

$x = (y+6)^2$

$\sqrt{x} = y + 6$

$\sqrt{\ } - 6 = y$

So, $f^{-1}(x) = \sqrt{x} - 6$.

domain of $f^{-1}: x \geq 0$, range of $f^{-1}: y \geq -6$

$f(x) = (x-4)^2$

domain of $f: x \geq 4$, range of $f: y \geq 0$

$f(x) = (x-4)^2$

$= (x-4)^2$

$= (y-4)^2$

$\sqrt{\ } = y - 4$

$\sqrt{x} + 4 = y$

So, $f^{-1}(x) = \sqrt{x} + 4$.

domain of $f^{-1}: x \geq 0$, range of $f^{-1}: y \geq 4$

$f(x) = -2x^2 + 5$

domain of $f: x \geq 0$, range of $f: y \leq 5$

$f(x) = -2x^2 + 5$

$= -2x^2 + 5\ x$

$= -2y^2 + 5$

$x - 5 = -2y^2$

$5 - x = 2y^2$

$\sqrt{\frac{5-x}{2}} = y$

$\frac{\sqrt{5-x}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = y$

$\sqrt{\frac{[\ ]}{2}} = y$

So, $f^{-1}(x) = \frac{\sqrt{-2(x-5)}}{2}$.

domain of $f^{-1}(x): x \leq 5$, range of $f^{-1}(x): y \geq 0$

$f(x) = \frac{1}{2}x^2 - 1$

domain of $f: x \geq 0$, range of $f: y \geq -1$

$f(x) = \frac{1}{2}x^2 - 1$

$= \frac{1}{2}x^2 - 1$

$x = \frac{1}{2}y^2 - 1$

$x + 1 = \frac{1}{2}y^2$

$2x + 2 = y^2$

$\sqrt{2x+2} = y$

So, $f^{-1}(x) = \sqrt{2x+2}$.

domain of $f^{-1}: x \geq -1$, range of $f^{-1}: y \geq 0$

$f(x) = |x - 4| + 1$

domain of $f: x \geq 4$, range of $f: y \geq 1$

$f(x) = |x-4| + 1$

$y = x - 3$

$x = y - 3$

$x + 3 = y$

So, $f^{-1}(x) = x + 3$.

domain of $f^{-1}: x \geq 1$, range of $f^{-1}: y \geq 4$

$f(x) = -|x - 1| - 2$

domain of $f: x \geq 1$, range of $f: y \leq -2$

$f(x) = -|x-1| - 2$

$= -(x-1) - 2$

$x = -(y-1) - 2$

$x = -y - 1$

$x - 1 = y$

So, $f^{-1}(x) = -x - 1$.

domain of $f^{-1}: x \leq -2$, range of $f^{-1}: y \geq 1$

In Exercises 79–84, $f(x) = \frac{1}{8}x - 3$, $f^{-1}(x) = 8(x+3)$, $g(x) = x^3$, $g^{-1}(x) = \sqrt[3]{x}$.

79. $(f^{-1} \circ g^{-1})(1) = f^{-1}(g^{-1}(1))$

$= f^{-1}(\sqrt[3]{1})$

$= 8(\sqrt[3]{1} + 3) = 32$

80. $\left(g^{-1} \circ f^{-1}\right)(-3) = g^{-1}\left(f^{-1}(-3)\right)$

$g^{-1}\left(8(-3+3)\right)$

$g^{-1}(0) = \sqrt[3]{0} = 0$

81. $\left(f^{-1} \circ f^{-1}\right)(4) = f^{-1}\left(f^{-1}(4)\right)$

$f^{-1}\left(8[4+3]\right)$

$8\left[8(4+3)+3\right]$

$8\left[8(7)+3\right]$

$8(59) = 472$

$\left(g^{-1} \circ g^{-1}\right)(-1) = g^{-1}\left(g^{-1}(-1)\right)$

$g^{-1}\left(\sqrt[3]{-1}\right)$

$\sqrt[3]{\sqrt[3]{-1}}$

$\sqrt[3]{-1}$

-1

83. $(f \circ g)(x) = f(g(x)) = f(x^3) = \frac{1}{8}x^3 - 3$

$= \frac{1}{8}x^3 - 3$

$x = \frac{1}{8}y^3 - 3$

$x + 3 = \frac{1}{8}y^3$

$8(x+3) = y^3$

$\sqrt[3]{8(x+3)} = y$

$(f \circ g)^{-1}(x) = 2\sqrt[3]{x+3}$

84. $g^{-1} \circ f^{-1} = g^{-1}\left(f^{-1}(x)\right)$

$g^{-1}\left(8(x+3)\right)$

$\sqrt[3]{8(x+3)}$

$2\sqrt[3]{x+3}$

In Exercises 85–88, $f(x) = x + 4$, $f^{-1}(x) = x - 4$, $g(x) = 2x - 5$, $g^{-1}(x) = \frac{x+5}{2}$.

85. $\left(g^{-1} \circ f^{-1}\right)(x) = g^{-1}\left(f^{-1}(x)\right)$

$= g^{-1}(x-4)$

$= \frac{(x-4)+5}{2}$

$= \frac{x+1}{2}$

86. $\left(f^{-1} \circ g^{-1}\right)(x) = f^{-1}\left(g^{-1}(x)\right)$

$f^{-1}\left(\frac{x+5}{2}\right)$

$= \frac{x+5}{2} - 4$

$= \frac{x+5-8}{2}$

$= \frac{x-3}{2}$

$(f \circ g)(x) = f(g(x))$

$f(2x-5)$

$(2x-5)+4$

$2x-1$

$(f \circ g)^{-1}(x) = \frac{x+1}{2}$

Note: Comparing Exercises 85 and 87, $(f \circ g)^{-1}(x) = \left(g^{-1} \circ f^{-1}\right)(x)$.

$(g \circ f)(x) = g(f(x))$

$g(x+4)$

$2(x+4) - 5$

$2x + 8 - 5$

$2x + 3$

$= 2x + 3$

$x = 2y + 3$

$x - 3 = 2y$

$\frac{-3}{2} = y$

$(g \circ f)^{-1}(x) = \frac{x-3}{2}$

(a) $y = 10 + 0.75x$

$= 10 + 0.75y$

$-10 = 0.75y$

$\frac{x-10}{0.75} = y$

So, $f^{-1}(x) = \frac{x-10}{0.75}$.

x = hourly wage, y = number of units produced

(b) $y = \frac{24.25-10}{} = 19$

So, 19 units are produced.

90. (a) $y = 0.03x^2 + 245.50,\ 0 < x < 100$

$\Rightarrow 245.50 < y < 545.50$

$x = 0.03y^2 + 245.50$

$x - 245.50 = 0.03y^2$

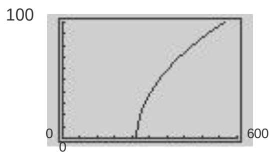
$\dfrac{x - 245.50}{0.03} = y^2$

$\sqrt{\dfrac{x - 245.50}{0.03}} = y,\ 245.50 < x < 545.50$

$f^{-1}(x) = \sqrt{\dfrac{x - 245.50}{0.03}}$

= temperature in degrees Fahrenheit

= percent load for a diesel engine



(c) $0.03x^2 + 245.50 \le 500$

$0.03x^2 \le 254.50$

$x^2 \le 8483.33$

≤ 92.10

Thus, $0 < x \le 92.10$.

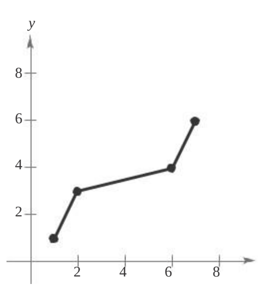
False. $f(x) = x^2$ is even and does not have an inverse.

True. If $f(x)$ has an inverse and it has a y-intercept at $(0, b)$, then the point $(b, 0)$, must be a point on the graph of $f^{-1}(x)$.

93.

x	1	3	4	6
f	1	2	6	7

x	1	2	6	7
$f^{-1}(x)$	1	3	4	6



94.

x	−4	−2	0	3
f	3	4	0	−1

The graph does not pass the Horizontal Line Test, so $f^{-1}(x)$ does not exist.

Let $(f \circ g)(x) = y$. Then $x = (f \circ g)^{-1}(y)$. Also,

$(f \circ g)(x) = y \Rightarrow f(g(x)) = y$

$g(x) = f^{-1}(y)$

$x = g^{-1}(f^{-1}(y))$

$x = (g^{-1} \circ f^{-1})(y)$.

Because f and g are both one-to-one functions, $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

Let $f(x)$ be a one-to-one odd function. Then $f^{-1}(x)$ exists and $f(-x) = -f(x)$. Letting (x, y) be any point on the graph of $f(x) \Rightarrow (-x, -y)$ is also on the graph of $f(x)$ and $f^{-1}(-y) = -x = -f^{-1}(y)$. So, $f^{-1}(x)$ is also an odd function.

97. If $f(x) = k(2 - x - x^3)$ has an inverse and $f^{-1}(3) = -2$, then $f(-2) = 3$. So,

$f(-2) = k(2 - (-2) - (-2)^3) = 3$

$k(2 + 2 + 8) = 3$

$12k = 3$

$k = \dfrac{3}{12} = \dfrac{1}{4}$.

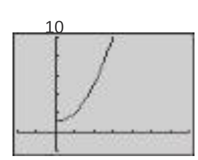
So, $k = \dfrac{1}{4}$.

98.

x	−10	0	7	45
$f(f^{-1}(x))$	−10	0	7	45
$f^{-1}(f(x))$	−10	0	7	45

$f(x)$ and $f^{-1}(x)$ are inverses of each other.

99.



the domain of f
is equal to the
range of f^{-1} and
the range of f is
equal to the
domain of f^{-1}.

(a) $C(x)$ is represented by graph m and $C^{-1}(x)$ is represented by graph n.

$C(x)$ represents the cost of making x units of personalized T-shirts. $C^{-1}(x)$ represents the number of personalized T-shirts that can be made for a given cost.

This situation could be represented by a one-to-one function if the runner does not stop to rest. The inverse function would represent the time in hours for a given number of miles completed.

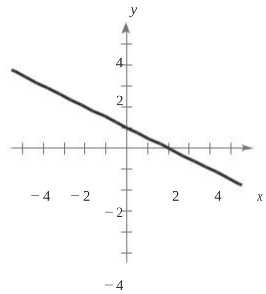
This situation cannot be represented by a one-to-one function because it oscillates.

Review Exercises for Chapter 2

1. $y = -\frac{1}{2}x + 1$

Slope: $m = -\frac{1}{2}$

y-intercept: $(0, 1)$



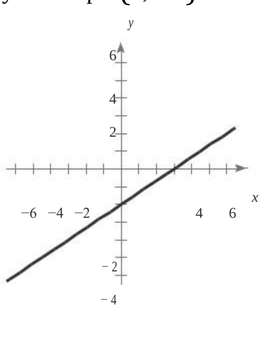
2. $2x - 3y = 6$

$-3y = -2x + 6$

$y = \frac{2}{3}x - 2$

Slope: $m = \frac{2}{3}$

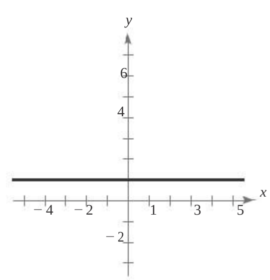
y-intercept: $(0, -2)$



3. $y = 1$

Slope: $m = 0$

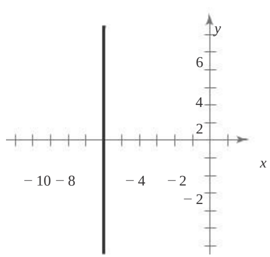
y-intercept: $(0, 1)$



4. $x = -6$

Slope: m is undefined.

y-intercept: none



$(5, -2), (-1, 4)$

$= \frac{4-(-2)}{-1-5} = \frac{6}{-6} = -1$

$(-1, 6), (3, -2)$

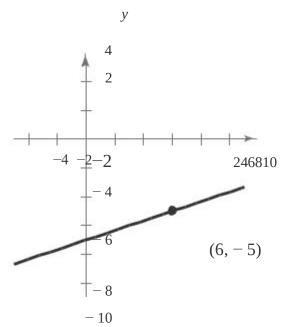
$m = \frac{-2-6}{3-(-1)} = \frac{-8}{4} = -2$

7. $(6, -5), m = \frac{1}{3}$

$y - (-5) = \frac{1}{3}(x - 6)$

$y + 5 = \frac{1}{3}x - 2$

$y = \frac{1}{3}x - 7$

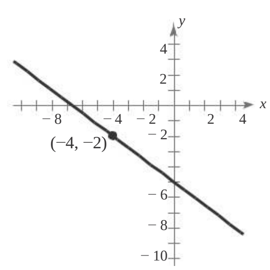


$(-4, -2), m = -\frac{3}{4}$

$-(-2) = -\frac{3}{4}(x - (-4))$

$y + 2 = -\frac{3}{4}(x + 4)$

$y + 2 = -\frac{3}{4}x - 3$



− 4

− 6

$(-6, 4), (4, 9)$

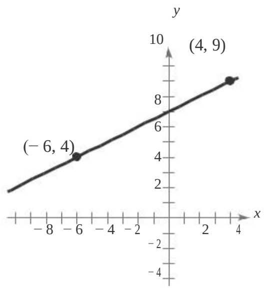
$$m = \frac{9 - 4}{4 - (-6)} = \frac{5}{10} = \frac{1}{2}$$

$$y - 4 = \frac{1}{2}\left(x - (-6)\right)$$

$$y - 4 = \frac{1}{2}(x + 6)$$

$$y - 4 = \frac{1}{2}x + 3$$

$$y = \frac{1}{2}x + 7$$



10. $(-9, -3), (-3, -5)$

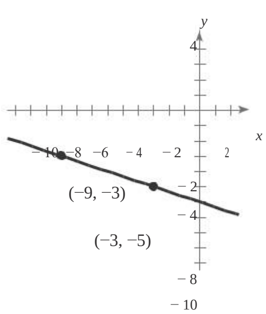
$$m = \frac{-5 - (-3)}{-3 - (-9)} = \frac{-2}{6} = -\frac{1}{3}$$

$$y - (-3) = -\frac{1}{3}\left(x - (-9)\right)$$

$$y + 3 = -\frac{1}{3}(x + 9)$$

$$y + 3 = -\frac{1}{3}x - 3$$

$$y = -\frac{1}{3}x - 6$$



Point: $(3, -2)$

$$5x - 4y = 8$$

$$y = \frac{5}{4}x - 2$$

(a) Parallel slope: $m = \frac{5}{4}$

$$y - (-2) = \frac{5}{4}(x - 3)$$

$$y + 2 = \frac{5}{4}x - \frac{15}{4}$$

$$y = \frac{5}{4}x - \frac{23}{4}$$

Perpendicular slope: $m = -\frac{4}{5}$

$$y - (-2) = -\frac{4}{5}(x - 3)$$

$$y + 2 = -\frac{4}{5}x + \frac{12}{5}$$

$$y = -\frac{4}{5}x + \frac{2}{5}$$

Point: $(-8, 3)$, $2x + 3y = 5$

$$3y = 5 - 2x$$

$$y = \frac{5}{3} - \frac{2}{3}x$$

(a) Parallel slope: $m = -\frac{2}{3}$

$$y - 3 = -\frac{2}{3}(x + 8)$$

$$3y - 9 = -2x - 16$$

$$3y = -2x - 7$$

$$y = -\frac{2}{3}x - \frac{7}{3}$$

(b) Perpendicular slope: $m = \frac{3}{2}$

$$y - 3 = \frac{3}{2}(x + 8)$$

$$2y - 6 = 3x + 24$$

$$2y = 3x + 30$$

$$y = \frac{3}{2}x + 15$$

13. *Verbal Model:* Sale price = (List price) − (Discount)

Labels: Sale price $= S$

List price $= L$

Discount = 20% of $L = 0.2L$

Equation: $S = L - 0.2L$

$S = 0.8L$

Verbal Model: Amount earned = (starting fee) + (per page rate) · (number of *pages*)

Labels: Hourly wage = A Starting fee = 50 Per page rate = 2.50 Number of pages = p

Equation: $A = 50 + 2.5p$

$16x - y^4 = 0$

$y^4 = 16x$

$= \pm 2\sqrt[4]{x}$

No, y is not a function of x. Some x-values correspond to two y-values.

$2x - y - 3 = 0$

$2x - 3 = y$

Yes, the equation represents y as a function of x.

17. $y = \sqrt{1 - x}$

Yes, the equation represents y as a function of x. Each x-value, $x \le 1$, corresponds to only one y-value.

18. $|y| = x + 2$ corresponds to $y = x + 2$ or $y = x + 2$.

No, y is not a function of x. Some x-values correspond to two y-values.

$g(x) = x^{4/3}$

$g(8) = 8^{4/3} = 2^4 = 16$

$g(t + 1) = (t + 1)^{4/3}$

(c) $(-27)^{4/3} = (-3)^4 = 81$

(d) $g(-x) = (-x)^{4/3} = x^{4/3}$

20. $h(x) = |x - 2|$

(a) $h(-4) = |-4 - 2| = |-6| = 6$

(b) $h(-2) = |-2 - 2| = |-4| = 4$

(c) $h(0) = |0 - 2| = |-2| = 2$

(d) $h(-x + 2) = |-x + 2 - 2| = |-x| = |x|$

21. $f(x) = \sqrt{25 - x^2}$

Domain: $25 - x^2 \ge 0$

$(5 + x)(5 - x) \ge 0$

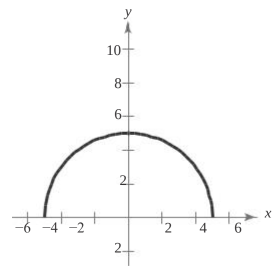
Critical numbers: $x = \pm 5$

Test intervals: $(-\infty, -5), (-5, 5), (5, \infty)$

Test: Is $25 - x^2 \ge 0$?

Solution set: $-5 \le x \le 5$

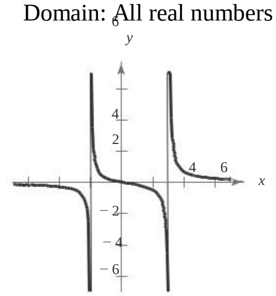
Domain: all real numbers x such that $-5 \le x \le 5$, or $[-5, 5]$



22. $h(x) = \dfrac{x}{x^2 - x - 6}$

$= \dfrac{x}{(x + 2)(x - 3)}$

Domain: All real numbers x except $x = -2, 3$



23. $v(t) = -32t + 48$

$v(1) = 16$ feet per second

24. $0 = -32t + 48$

$t = \frac{48}{32} = 1.5$ seconds

25. $f(x) = 2x^2 + 3x - 1$

$$\frac{f(x + h) - f(x)}{h} = \frac{\left[2(x + h)^2 + 3(x + h) - 1\right] - (2x^2 + 3x - 1)}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 + 3x + 3h - 1 - 2x^2 - 3x + 1}{h}$$

$$= \frac{h(4x + 2h + 3)}{h}$$

$$= 4x + 2h + 3, \quad h \ne 0$$

26. $f(x) = x^3 - 5x^2 + x$

$f(x+h) = (x+h)^3 - 5(x+h)^2 + (x+h)$

$= x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h$

$\dfrac{f(x+h) - f(x)}{h}$ [redacted] $= \dfrac{x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h - x^3 + 5x^2 - x}{h}$

$= \dfrac{3x^2h + 3xh^2 + h^3 - 10xh - 5h^2 + h}{h}$

$= \dfrac{h(3x^2 + 3xh + h^2 - 10x - 5h + 1)}{h}$

$= 3x^2 + 3xh + h^2 - 10x - 5h + 1,\ h \neq 0$

27. $y = (x - 3)^2$

A vertical line intersects the graph no more than once, so *y is* a function of *x*.

$x = -|4 - y|$

A vertical line intersects the graph more than once, so *y is not* a function of *x*.

$f(x) = 5x^2 + 4x - 1$

$5x^2 + 4x - 1 = 0$

$(5x - 1)(x + 1) = 0$

$5x - 1 = 0 \Rightarrow x = \frac{1}{5}$

$f(x) = \dfrac{8x + 3}{11 - x}$

$\dfrac{8x + 3}{11 - x} = 0$

$8x + 3 = 0$

$= -\frac{3}{8}$

31. $f(x) = \sqrt{2x + 1}$

$\sqrt{2x + 1} = 0$

$2x + 1 = 0$

$2x = -1$

$= -\frac{1}{2}$

$f(x) = x^3 - x^2$

$x^3 - x^2 = 0$

$x^2(x - 1) = 0$

$x^2 = 0$ or $x - 1 = 0$

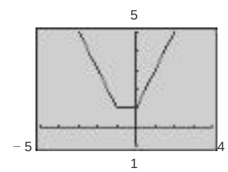
$x = 0$ $\quad x = 1$

33. $f(x) = |x| + |x + 1|$

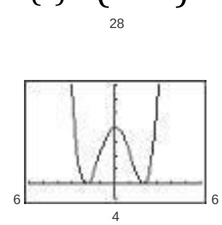
is increasing on $(0, \infty)$.

is decreasing on $(-\infty, -1)$.

is constant on $(-1, 0)$.



$f(x) = (x^2 - 4)^2$

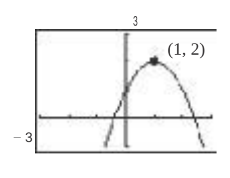


f is increasing on $(-2, 0)$ and $(2, \infty)$.

is decreasing on $(-\infty, -2)$ and $(0, 2)$.

35. $f(x) = -x^2 + 2x + 1$

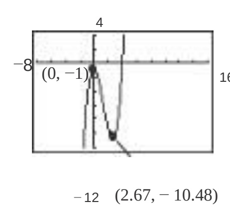
Relative maximum: $(1, 2)$



36. $f(x) = x^3 - 4x^2 - 1$

Relative minimum: $(2.67, -10.48)$

Relative maximum: $(0, -1)$



37. $f(x) = -x^2 + 8x - 4$

$\frac{f(4) - f(0)}{4 - 0} = \frac{12 - [redacted])}{4} = 4$

The average rate of change of f from $x_1 = 0$ to $x_2 = 4$ is 4.

38. $f(x) = x^3 + 2x + 1$

$\frac{f(3) - f(1)}{3 - 1} = \frac{34 - 4}{2} = \frac{30}{2} = 15$

The average rate of change of f from $x_1 = 1$ to $x_2 = 3$ is 15.

$f(x) = x^5 + 4x - 7$

$f(-x) = (-x)^5 + 4(-x) - 7$

$-x^5 - 4x - 7$

$f(x)$

$-f(x)$

The function is neither even nor odd, so the graph has no symmetry.

40. $f(x) = x^4 - 20x^2$

$f(-x) = (-x)^4 - 20(-x)^2 = x^4 - 20x^2 = f(x)$

The function is even, so the graph has y-axis symmetry.

41. $f(x) = 2x\sqrt{x^2 + 3}$

$f(-x) = 2(-x)\sqrt{(-x)^2 + 3}$

$= -2x\sqrt{x^2 + 3}$

$= -f(x)$

The function is odd, so the graph has origin symmetry.

42. $f(x) = \sqrt[5]{6x^2}$

$f(-x) = \sqrt[5]{6(-x)^2} = \sqrt[5]{6x^2} = f(x)$

The function is even, so the graph has y-axis symmetry.

43. (a) $f(2) = -6, f(-1) = 3$

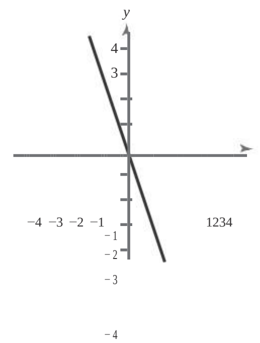
Points: $(2, -6), (-1, 3)$

$m = \frac{3 - (-6)}{-1 - 2} = \frac{9}{-3} = -3$

$y - (-6) = -3(x - 2)$

$y + 6 = -3x + 6$

(b)



(a) $f(0) = -5, f(4) = -8$

$(0, -5), (4, -8)$

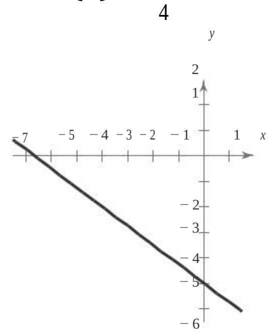
$m = \frac{-8 - (-5)}{4 - 0} = -\frac{3}{4}$

$y - (-5) = -\frac{3}{4}(x - 0)$

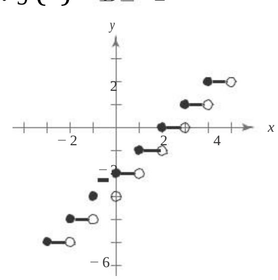
$y = -\frac{3}{4}x - 5$

$f(x) = -\frac{3}{4}x - 5$

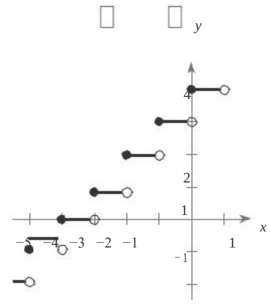
(b)



45. $g(x) = [\![x]\!] - 2$



46. $g(x) = [\![x + 4]\!]$

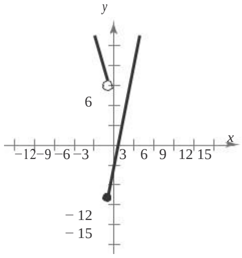


$$y = -3x$$

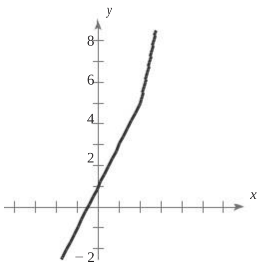
$$f(x) = -3x$$

-2

47. $f(x) = \begin{cases} 5x - 3, & x \geq -1 \\ -4x + 5, & x < -1 \end{cases}$



48. $f(x) = \begin{cases} 2x + 1, & x \leq 2 \\ x^2 + 1, & x > 2 \end{cases}$

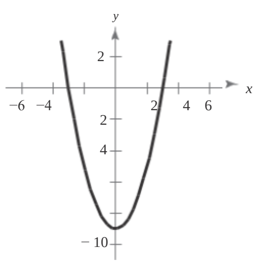


49. (a) $f(x) = x^2$

(b) $h(x) = x^2 - 9$

Vertical shift 9 units downward

(c)



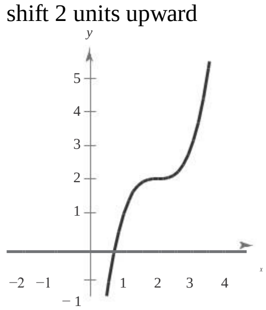
$h(x) = f(x) - 9$

(a) $f(x) = x^3$

$h(x) = (x - 2)^3 + 2$

Horizontal shift 2 units to the right; vertical shift 2 units upward

(c)



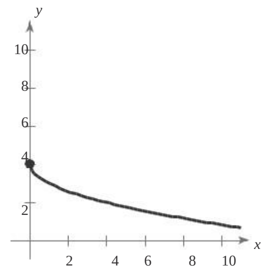
$h(x) = f(x - 2) + 2$

51. (a) $f(x) = \sqrt{x}$

$h(x) = -\sqrt{x} + 4$

Reflection in the x-axis and a vertical shift 4 units upward

(c)



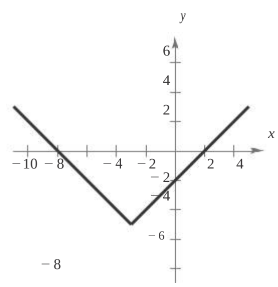
(d) $h(x) = -f(x) + 4$

(a) $f(x) = |x|$

$h(x) = |x + 3| - 5$

Horizontal shift 3 units to the left and a vertical

(c)



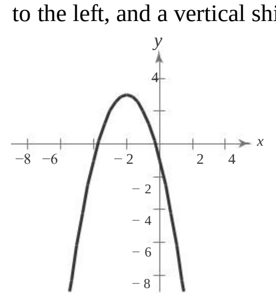
(d) $h(x) = f(x + 3) - 5$

(a) $f(x) = x^2$

$h(x) = -(x + 2)^2 + 3$

Reflection in the x-axis, a horizontal shift 2 units to the left, and a vertical shift 3 units upward

(c)



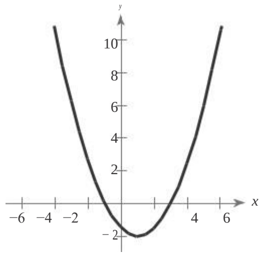
$h(x) = -f(x + 2) + 3$

54. (a) $f(x) = x^2$

(b) $h(x) = \frac{1}{2}(x-1)^2 - 2$

Horizontal shift one unit to the right, vertical shrink, and a vertical shift 2 units downward

(c)



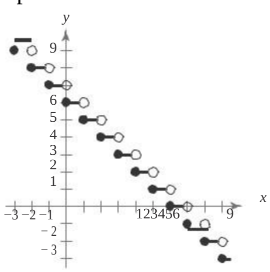
(d) $h(x) = \frac{1}{2}f(x-1) - 2$

55. (a) $f(x) = [\![x]\!]$

(b) $h(x) = -[\![x]\!] + 6$

Reflection in the x-axis and a vertical shift 6 units upward

(c)



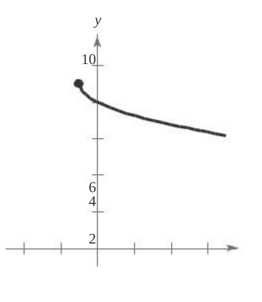
(d) $h(x) = -f(x) + 6$

56. (a) $f(x) = \sqrt{x}$

(b) $h(x) = -\sqrt{x+1} + 9$

Reflection in the x-axis, a horizontal shift 1 unit to the left, and a vertical shift 9 units upward

(c)



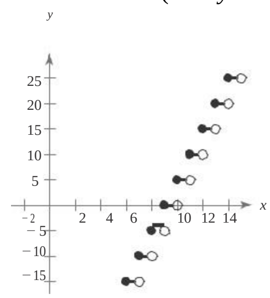
(d) $h(x) = -f(x+1) + 9$

57. (a) $f(x) = [\![x]\!]$

(b) $h(x) = 5[\![x-9]\!]$

Horizontal shift 9 units to the right and a vertical stretch (each y-value is multiplied by 5)

(c)



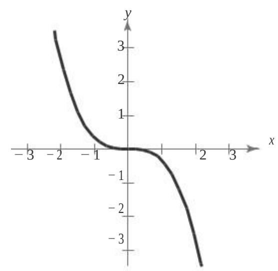
(d) $h(x) = 5f(x-9)$

58. (a) $f(x) = x^3$

(b) $h(x) = -\frac{1}{3}x^3$

Reflection in the x-axis and a vertical shrink (each y-value is multiplied by $\frac{1}{3}$)

(c)



(d) $h(x) = -\frac{1}{3}f(x)$

59. $f(x) = x^2 + 3,\ g(x) = 2x - 1$

(a) $(f+g)(x) = (x^2 + 3) + (2x - 1) = x^2 + 2x + 2$

(b) $(f-g)(x) = (x^2 + 3) - (2x - 1) = x^2 - 2x + 4$

(c) $(fg)(x) = (x^2 + 3)(2x - 1) = 2x^3 - x^2 + 6x - 3$

(d) $\left(\frac{f}{g}\right)(x) = \frac{x^2 + 3}{2x - 1}$, Domain: $x \neq \frac{1}{2}$

60. $f(x) = x^2 - 4,\ g(x) = \sqrt{3 - x}$

(a) $(f+g)(x) = f(x) + g(x) = x^2 - 4 + \sqrt{3 - x}$

(b) $(f-g)(x) = f(x) - g(x) = x^2 - 4 - \sqrt{3 - x}$

(c) $(fg)(x) = f(x)g(x) = (x^2 - 4)\sqrt{3 - x}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2 - 4}{[redacted]}$, Domain: $x < 3$

$(g)\; x \qquad g(x)\;\sqrt{3 - x^{2}}$

61. $f(x) = \frac{1}{3}x - 3,\ g(x) = 3x + 1$

The domains of f and g are all real numbers.

(a) $(f \circ g)(x) = f(g(x))$

$= f(3x + 1)$

$= \frac{1}{3}(3x + 1) - 3$

$= x + \frac{1}{3} - 3$

$= x - \frac{8}{3}$

Domain: all real numbers

(b) $(g \circ f)(x) = g(f(x))$

$= g\left(\frac{1}{3}x - 3\right)$

$= 3\left(\frac{1}{3}x - 3\right) + 1$

$= x - 9 + 1$

$x - 8$

Domain: all real numbers

62. $f(x) = \sqrt{x + 1},\ g(x) = x^2$

The domain of f is all real numbers x such that $x \geq -1$. The domain of g is all real numbers.

(a) $(f \circ g)(x) = f(g(x))$

$= f(x^2)$

$= \sqrt{x^2 + 1}$

Domain: all real numbers

(b) $(g \circ f)(x) = g(f(x))$

$= g\left(\sqrt{x + 1}\right)$

$= \left(\sqrt{x + 1}\right)^2$

$= x + 1$

Domain: all real numbers x such that $x \geq -1$

In Exercises 63-64 use the following functions. $f(x) = x - 100,\ g(x) = 0.95x$

63. $(f \circ g)(x) = f(0.95x) = 0.95x - 100$ represents the sale price if first the 5% discount is applied and then the $100 rebate.

$(g \circ f)(x) = g(x - 100) = 0.95(x - 100) =$ $0.95x - 95$ represents the sale price if first the $100 rebate is applied and then the 5% discount.

$f(x) = \frac{x - 4}{5}$

$y = \frac{x - 4}{5}$

$x = \frac{y - 4}{5}$

$5x = y - 4$

$= 5x + 4$

So, $f^{-1}(x) = 5x + 4.$

$f\left(f^{-1}(x)\right) = f(5x + 4) = \frac{5x + 4 - 4}{5} = \frac{5x}{5}$ x

$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{x - 4}{5}\right) = 5\left(\frac{x - 4}{5}\right) + 4 =$ $-4 + 4 = x$

$f(x) = x^3 - 1$

$y = x^3 - 1$ x

$= y^3 - 1$

$+ 1 = y^3$

$\sqrt[3]{x + 1} = y$

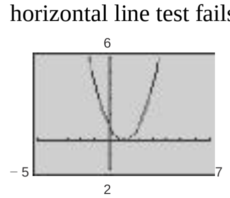
$f^{-1}(x) = \sqrt[3]{x + 1}$

$f\left(f^{-1}(x)\right) = \left(\sqrt[3]{x + 1}\right)^3 - 1 =$

x $f^{-1}\left(f(x)\right) = \sqrt[3]{(x^3 - 1) + 1} = x$

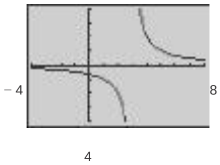
$f(x) = (x - 1)^2$

No, the function does not have an inverse because the horizontal line test fails.



68. $h(t) = \frac{2}{t-3}$

Yes, the function has an inverse because no horizontal lines intersect the graph at more than one point. The function has an inverse.



69. (a) $f(x) = \frac{1}{2}x - 3$ (b)

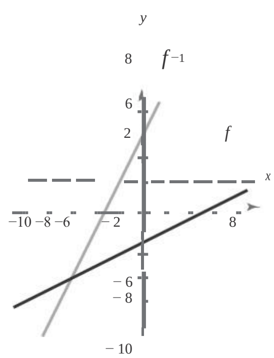
$y = \frac{1}{2}x - 3$

$x = \frac{1}{2}y - 3$

$x + 3 = \frac{1}{2}y$

$2(x + 3) = y$

$f^{-1}(x) = 2x + 6$



The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

The domains and ranges of f and f^{-1} are the set of all real numbers.

70. (a) $f(x) = \sqrt{x+1}$

$y = \sqrt{x+1}$

$x = \sqrt{y+1}$

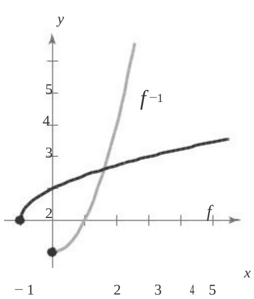
$x^2 = y + 1$

$x^2 - 1 = y$

$f^{-1}(x) = x^2 - 1, \ x \ge 0$

Note: The inverse must have a restricted domain.

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domain of f and the range of f^{-1} is $[-1, \infty)$.

The range of f and the domain of f^{-1} is $[0, \infty)$.

$f(x) = 2(x-4)^2$ is increasing on $(4, \infty)$.

Let $f(x) = 2(x-4)^2$, $x > 4$ and $y > 0$.

$= 2(x-4)^2$

$x = 2(y-4)^2$, $x > 0$, $y > 4$

$\frac{x}{2} = (y-4)^2$

$\sqrt{\frac{x}{2}} = y - 4$

$\sqrt{\frac{x}{2}} + 4 = y$

$f^{-1}(x) = \sqrt{\frac{x}{2}} + 4, x > 0$

| |

$f(x) = x - 2$ is increasing on $(2, \infty)$.

Let $f(x) = x - 2$, $x > 2$, $y > 0$.

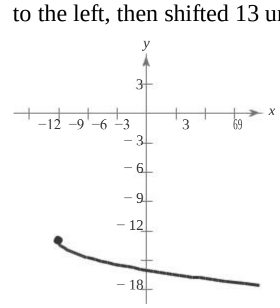
$y = x - 2$

$x = y - 2$, $x > 0$, $y > 2$

$x + 2 = y$, $x > 0$, $y > 2$

$f^{-1}(x) = x + 2, x > 0$

False. The graph is reflected in the x-axis, shifted 9 units to the left, then shifted 13 units downward.

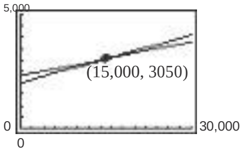


74. True. If $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$, then the domain of g is all real numbers, which is equal to the range of f and vice versa

Problem Solving for Chapter 2

(a) $W_1 = 0.07S + 2000$

$W_2 = 0.05S + 2300$



Point of intersection: $(15{,}000,\ 3050)$

Both jobs pay the same, $3050, if you sell $15,000 per month.

No. If you think you can sell $20,000 per month, keep your current job with the higher commission rate. For sales over $15,000 it pays more than the other job.

Mapping numbers onto letters is *not* a function. Each number between 2 and 9 is mapped to more than one letter.

$\{(2, A), (2, B), (2, C), (3, D), (3, E), (3, F), (4, G), (4, H), (4, I), (5, J), (5, K), (5, L),$
$(6, M), (6, N), (6, O), (7, P), (7, Q), (7, R), (7, S), (8, T), (8, U), (8, V), (9, W), (9, X), (9, Y), (9, Z)\}$

Mapping letters onto numbers *is* a function. Each letter is only mapped to one number.

$\{(A, 2), (B, 2), (C, 2), (D, 3), (E, 3), (F, 3), (G, 4), (H, 4), (I, 4), (J, 5), (K, 5), (L, 5),$
$(M, 6), (N, 6), (O, 6), (P, 7), (Q, 7), (R, 7), (S, 7), (T, 8), (U, 8), (V, 8), (W, 9), (X, 9), (Y, 9), (Z, 9)\}$

3. (a) Let $f(x)$ and $g(x)$ be two even functions.

Then define $h(x) = f(x) \pm g(x)$.

$h(-x) = f(-x) \pm g(-x)$
$= f(x) \pm g(x)$ because f and g are even
$= h(x)$

So, $h(x)$ is also even.

(b) Let $f(x)$ and $g(x)$ be two odd functions.

Then define $h(x) = f(x) \pm g(x)$.

$h(-x) = f(-x) \pm g(-x)$
$= -f(x) \pm g(x)$ because f and g are odd
$= -h(x)$

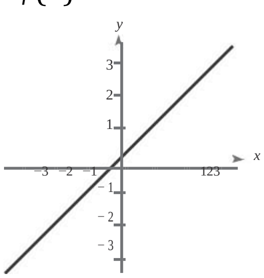
So, $h(x)$ is also odd. (If $f(x) \neq g(x)$)

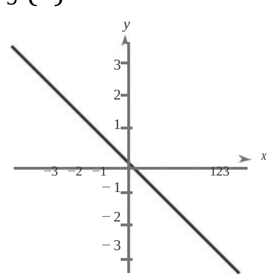
(c) Let $f(x)$ be odd and $g(x)$ be even. Then define $h(x) = f(x) \pm g(x)$.

$h(-x) = f(-x) \pm g(-x)$
$-f(x) \pm g(x)$ because f is odd and g is even
$\neq h(x)$
$\neq -h(x)$

So, $h(x)$ is neither odd nor even.

4. $f(x) = x$ $g(x) = -x$





$(f \circ f)(x) = x$ and $(g \circ g)(x) = x$

These are the only two linear functions that are their own inverse functions since m has to equal $1/m$ for this to be true.

General formula: $y = -x + c$

5. $f(x) = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$

$f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0 = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 = f(x)$

So, $f(x)$ is even.

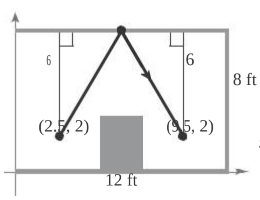
6. It appears, from the drawing, that the triangles are equal; thus $(x, y) = (6, 8)$.

The line between $(2.5, 2)$ and $(6, 8)$ is $y = \frac{12}{7}x - \frac{16}{7}$.

The line between $(9.5, 2)$ and $(6, 8)$ is $y = -\frac{12}{7}x + \frac{128}{7}$.

The path of the ball is:

$$f(x) = \begin{cases} \frac{12}{7}x - \frac{16}{7}, & 2.5 \le x \le 6 \\ -\frac{12}{7}x + \frac{128}{7}, & 6 < x \le 9.5 \end{cases}$$



7. (a) April 11: 10 hours

April 12: 24 hours

April 13: 24 hours

April 14: $23\frac{1}{2}$ hours

Total: $81\frac{1}{2}$ hours

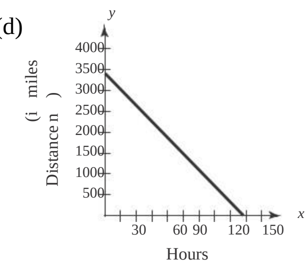
(b) Speed $= \frac{\text{distance}}{\text{time}} = \frac{2100}{81\frac{1}{2}} = \frac{180}{7} = 25\frac{5}{7}$ mph

(c) $D = -\frac{180}{7}t + 3400$

Domain: $0 \le t \le \frac{1190}{9}$

Range: $0 \le D \le 3400$

(d)



8. (a) $\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(2) - f(1)}{2 - 1}$ [redacted] $= \frac{\quad}{\quad} = 1$

(b) $\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1.5) - f(1)}{1.5 - 1} = \frac{\quad}{0.5}$

$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1.25) - f(1)}{1.25 - 1} = \frac{0.4375 - 0}{0.25} = 1.75$

$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1.125) - f(1)}{1.125 - 1} = \frac{0.234375 - 0}{0.125} = 1.875$

$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1.0625) - f(1)}{1.0625 - 1} = \frac{0.12109375 - 0}{0.0625} = 1.9375$

(f) Yes, the average rate of change appears to be approaching 2.

(g) a. $(1, 0)$, $(2, 1)$, $m = 1$, $y = x - 1$

b. $(1, 0)$, $(1.5, 0.75)$, $m = \frac{0.75}{0.5} = 1.5$, $y = 1.5x - 1.5$

c. $(1, 0)$, $(1.25, 0.4375)$, $m = \frac{0.4375}{0.25} = 1.75$, $y = 1.75x - 1.75$

d. $(1, 0)$, $(1.125, 0.234375)$, $m = \frac{0.234375}{0.125} = 1.875$, $y = 1.875x - 1.875$

e. $(1, 0)$, $(1.0625, 0.12109375)$, $m = \frac{0.12109375}{0.0625} = 1.9375$, $y = 1.9375x - 1.9375$

(h) $(1, f(1)) = (1, 0)$, $m \to 2$, $y = 2(x - 1)$, $y = 2x - 2$

9. (a)–(d) Use $f(x) = 4x$ and $g(x) = x + 6$.

(a) $(f \circ g)(x) = f(x + 6) = 4(x + 6) = 4x + 24$

(b) $(f \circ g)^{-1}(x) = \frac{x - 24}{4} = \frac{1}{4}x - 6$

(c) $f^{-1}(x) = \frac{1}{4}x$

$g^{-1}(x) = x - 6$

(d) $(g^{-1} \circ f^{-1})(x) = g^{-1}\left(\frac{1}{4}x\right) = \frac{1}{4}x - 6$

(e) $f(x) = x^3 + 1$ and $g(x) = 2x$

$(f \circ g)(x) = f(2x) = (2x)^3 + 1 = 8x^3 + 1$

$(f \circ g)^{-1}(x) = \sqrt[3]{\frac{x - 1}{8}} = \frac{1}{2}\sqrt[3]{x - 1}$

$f^{-1}(x) = \sqrt[3]{x - 1}$

$g^{-1}(x) = \frac{1}{2}x$

$(g^{-1} \circ f^{-1})(x) = g^{-1}\left(\sqrt[3]{x - 1}\right) = \frac{1}{2}\sqrt[3]{x - 1}$

(f) Answers will vary.

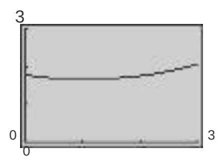
(g) Conjecture: $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$

10. (a) The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$.

The total time is

$$T(x) = \frac{\sqrt{4 + x^2}}{2} + \frac{\sqrt{1 + (3 - x)^2}}{4} = \frac{1}{2}\sqrt{4 + x^2} + \frac{1}{4}\sqrt{x^2 - 6x + 10}.$$

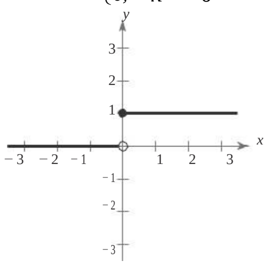
Domain of $T(x)$: $0 \le x \le 3$



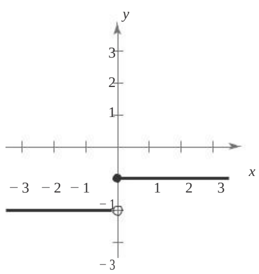
$T(x)$ is a minimum when $x = 1$.

Answers will vary. *Sample answer:* To reach point Q in the shortest amount of time, you should row to a point one mile down the coast, and then walk the rest of the way. The distance $x = 1$ yields a time of 1.68 hours.

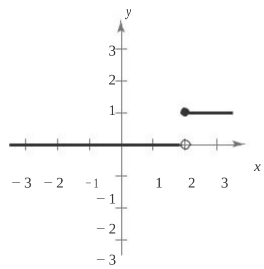
11. $H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$



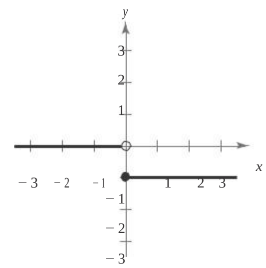
(a) $H(x) - 2$



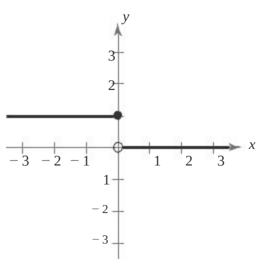
(b) $H(x-2)$



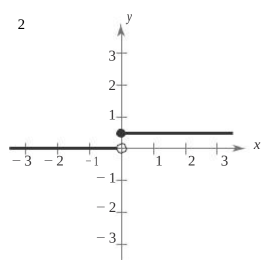
(c) $-H(x)$



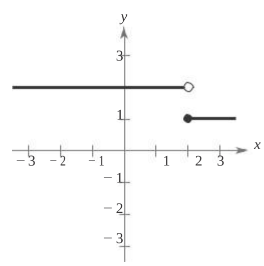
(d) $H(-x)$



(e) $\frac{1}{2}H(x)$



(f) $-H(x-2) + 2$



12. $f(x) = y = \dfrac{1}{1-x}$

(a) Domain: all real numbers x except $x = 1$

Range: all real numbers y except $y = 0$

(b) $f(f(x)) = f\left(\dfrac{1}{1-x}\right)$

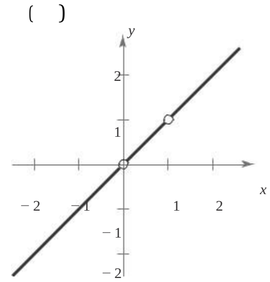
$= \dfrac{1}{1 - \left(\dfrac{1}{1-x}\right)} = \dfrac{1}{\dfrac{1-x-1}{1-x}}$

$= \dfrac{1-x}{-x} = \dfrac{x-1}{x}$

Domain: all real numbers x except $x = 0$ and $x = 1$

(c) $f(f(f(x))) = f\left(\dfrac{x-1}{x}\right) = \dfrac{1}{1 - \left(\dfrac{x-1}{x}\right)} = \dfrac{1}{\dfrac{1}{x}} = x$

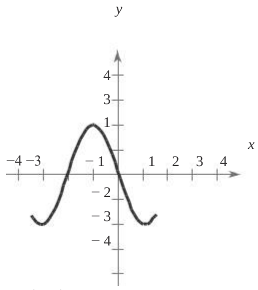
The graph is not a line. It has holes at $(0, 0)$ and $(1, 1)$.



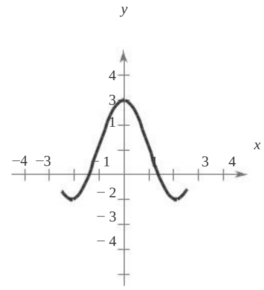
$(f\circ(g\circ h))(x) = f((g\circ h)(x)) = (g(h(x))) = (f\circ g\circ h)(x)$

$((f\circ g)\circ h)(x) = (f\circ g)(h(x)) = f(g(h(x))) = (f\circ g\circ h)(x)$

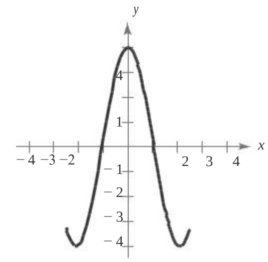
14. (a) $f(x+1)$



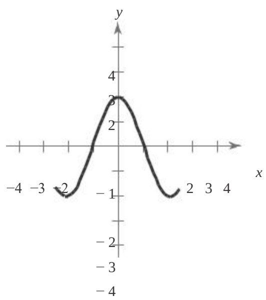
(b) $f(x)+1$



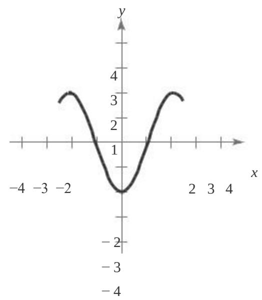
(c) $2f(x)$



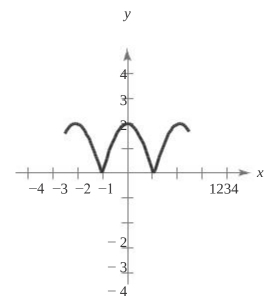
(d) $f(-x)$



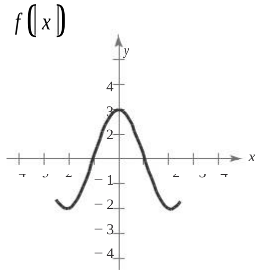
(e) $-f(x)$



(f) $|f(x)|$



(g) $f(|x|)$



15.

x	$f(x)$	$f^{-1}(x)$
-4	—	2
-3	4	1
-2	1	0
-1	0	—
0	-2	-1
1	-3	-2
2	-4	—
3	—	—
4	—	-3

(a)

x	$f(f^{-1}(x))$
-4	$f(f^{-1}(-4)) = f(2) = -4$
-2	$f(f^{-1}(-2)) = f(0) = -2$
0	$f(f^{-1}(0)) = f(-1) = 0$
4	$f(f^{-1}(4)) = f(-3) = 4$

(b)

x	$(f + f^{-1})(x)$
-3	$f(-3) + f^{-1}(-3) = 4 + 1 = 5$
-2	$f(-2) + f^{-1}(-2) = 1 + 0 = 1$
0	$f(0) + f^{-1}(0) = -2 + (-1) = -3$
1	$f(1) + f^{-1}(1) = -3 + (-2) = -5$

(c)

x	$(f\cdot f^{-1})(x)$
-3	$f(-3)f^{-1}(-3) = (4)(1) = 4$
-2	$f(-2)f^{-1}(-2) = (1)(0) = 0$
0	$f(0)f^{-1}(0) = (-2)(-1) = 2$
1	$f(1)f^{-1}(1) = (-3)(-2) = 6$

(d)

x	$\lvert f^{-1}(x)\rvert$
-4	$\lvert f^{-1}(-4)\rvert = \lvert 2\rvert = 2$
-3	$\lvert f^{-1}(-3)\rvert = \lvert 1\rvert = 1$
0	$\lvert f^{-1}(0)\rvert = \lvert -1\rvert = 1$
4	$\lvert f^{-1}(4)\rvert = \lvert -3\rvert = 3$

Practice Test for Chapter 2

Find the equation of the line through $(2, 4)$ and $(3, -1)$.

Find the equation of the line with slope $m = 4/3$ and y-intercept $b = -3$.

3. Find the equation of the line through $(4, 1)$ perpendicular to the line $2x + 3y = 0$.

If it costs a company $32 to produce 5 units of a product and $44 to produce 9 units, how much does it cost to produce 20 units? (Assume that the cost function is linear.)

Given $f(x) = x^2 - 2x + 1$, find $f(x - 3)$.

6. Given $f(x) = 4x - 11$, find $\dfrac{f(x) - f(3)}{x - 3}$.

7. Find the domain and range of $f(x) = \sqrt{36 - x^2}$.

Which equations determine y as a function of x?

$6x - 5y + 4 = 0$

$x^2 + y^2 = 9$

$y^3 = x^2 + 6$

Sketch the graph of $f(x) = x^2 - 5$.

Sketch the graph of $f(x) = |x + 3|$.

11. Sketch the graph of $f(x) = \begin{cases} 2x + 1, & \text{if } x \ge 0, \\ x^2 - x, & \text{if } x < 0. \end{cases}$

Use the graph of $f(x) = |x|$ to graph the following:

$f(x + 2)$

$-f(x) + 2$

Given $f(x) = 3x + 7$ and $g(x) = 2x^2 - 5$, find the following:

$(g - f)(x)$

$(fg)(x)$

Given $f(x) = x^2 - 2x + 16$ and $g(x) = 2x + 3$, find $f(g(x))$.

Given $f(x) = x^3 + 7$, find $f^{-1}(x)$.

Which of the following functions have inverses?

$f(x) = |x - 6|$

$f(x) = ax + b, a \ne 0$

$f(x) = x^3 - 19$

17. Given $f(x) = \sqrt{\frac{3 - x}{x}}$, $0 < x \leq 3$, find $f^{-1}(x)$.

Exercises 18–20, true or false?

18. $y = 3x + 7$ and $y = -\frac{1}{3}x - 4$ are perpendicular.

$$(f \circ g)^{-1} = g^{-1} \circ f^{-1}$$