

**Solution Manual for College Algebra 11th Edition by Gustafson and Hughes
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This manual contains detailed solutions to all of the exercises of the text *College Algebra*, eleventh edition, by R. David Gustafson and Jeff Hughes.

Many of the exercises in the text may be solved using more than one method, but it is not feasible to list all possible solutions in this manual. Also, some of the exercises may have been solved in this manual using a method that differs slightly from that presented in the text. There are a few exercises in the text whose solutions may vary from person to person. Some of these solutions may not have been included in this manual. For the solution to an exercise like this, the notation "answers may vary" has been included.

If you are a student using this manual, please remember that only reading a solution does not teach you how to solve a problem. To repeat a commonly used phrase, mathematics is not a spectator sport. You **MUST** make an honest attempt to solve each exercise in the text without using this manual first. This manual should be viewed more or less as a last resort. Above all, **DO NOT** simply copy the solution from this manual onto your own paper. Doing so will not help you learn how to do the exercise, nor will it help you to do better on quizzes or tests.

I would like to thank Paul McCombs from Rock Valley College and Cynthia Ashton of Brooks/Cole Publishing Company for their help and support. This solutions manual was prepared using EXP 5.1.

This book is dedicated to John, who helps me to realize that mathematics cannot describe everything in life.

May your study of this material be successful and rewarding.

Michael G. Welden

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SECTION 0.1

Exercises 0.1 (page 13)

- | | | | |
|---|--|-----------------------|-----------------|
| 1. set | 2. subset | 3. union | 4. intersection |
| 5. decimal | 6. variable | 7. # | 8. even |
| 9. composite | 10. rational | 11. decimals | 12. \square |
| 13. negative | 14. 0 | 15. $B \times \Delta$ | 16. XB |
| 17. $\&7 \ \& \ \square$
| 18. commutative,
multiplication | 19. interval | 20. no |
| 21. two | 22. half-open | 23. positive | 24. distance |
| 25. Every natural number is a whole number,
so $N \clubsuit W$. <input type="text" value="TRUE"/> | 26. Every rational number is a real number,
so $Q \clubsuit R$. <input type="text" value="TRUE"/> | | |
| 27. The rational number $\frac{-}{\sqrt{}}$ is not a natural
number, so $Q \S N$. <input type="text" value="FALSE"/> | 28. Every integer is a rational number,
so $Z \clubsuit Q$. <input type="text" value="TRUE"/> | | |
| 29. Every whole number is an integer,
so $W \clubsuit Z$. <input type="text" value="TRUE"/> | 30. The real number $\cup \#$ is not an integer,
so $R \clubsuit Z$. <input type="text" value="FALSE"/> | | |
| 31. $E \cup \Phi \square \varepsilon a, b, c, d, e, f, g\phi$ | 32. $E \cap \Phi \square \varepsilon d, e\phi$ <input type="text" value=""/> | | |
| 33. $E \cap \Gamma \square \varepsilon a, c, e\phi$ <input type="text" value=""/> | 34. $\Phi \cup \Gamma \square \varepsilon a, c, d, e, f, g\phi$ | | |
| 35. $\frac{*}{\square} \square ! \Rightarrow \& \# \& ;$
terminates
$\forall \exists$ | 36. $\exists \square ! \Rightarrow \exists (\& \# \text{terminates})$
$\exists \square ! \Rightarrow \# (\# (\Rightarrow \Rightarrow \Rightarrow ; \text{repeats})$ | | |
| 37. <input type="text" value=""/> | | | |

38. $\& \square \Rightarrow \% \forall \exists \exists \exists \Rightarrow \Rightarrow \Rightarrow;$



repeats
 $\forall \forall$

SECTION 0.1 $\forall \#$

39. natural: $\forall, \#, \exists, ($

40. whole: $!, \forall, \#, \exists, ($

41. integers: $\&, \%, !, \forall, \#, \exists, ($

42. rational: $\&, \%, \#, !, \forall, \#, \# \Rightarrow (\&, \exists, ($

43. irrational: $\cup \#$
_

44. prime: $\#, ($

45. composite: \exists \exists

46. even: $\%, !, \#, \exists$

47. odd: $\&, \forall, ($

48. negative: $\&, \%, \#$

49. $\circ \theta \rangle \theta \rangle \theta \rangle \theta \pi$

50. $\circ \rangle \theta \theta \rangle \theta \theta \rangle \theta \rangle \pi$

$\# \exists \%$
 $\%$

$\exists \quad) \quad *$

SECTION 0.1

51. $\circ \rangle \theta \rangle \theta \theta \rangle \theta \rangle \pi$

$\forall \forall$ \forall \forall^*
 $\forall \exists$

52. $\circ \rangle \theta \rangle \theta \rangle \theta \rangle \theta \rangle \theta \rangle \pi$

\forall ! \forall # \exists %

53. $\circ \rangle \theta \rangle \theta \rangle \theta \rangle \pi$

% \exists # \forall

54. $\circ \rangle \theta \rangle \theta \rangle \theta \rangle \pi$

) \exists % #

55. $\circ \rangle \theta \rangle \theta \rangle \theta \rangle \theta \rangle \pi$

& \exists \forall \forall \exists

56. $\circ \rangle \theta \theta \theta \rangle \theta \theta \theta \theta \rangle \theta \rangle \pi$

$! \Rightarrow$ ($\forall \Rightarrow$ (& \exists)

57. $B \# \pi \# \infty$

58. $B ! \% \pi \infty, \%$

59. $!!B! \& \pi !, \&$

$\circ \theta \theta \angle | | \cup$
#

$\{ | | \nabla \theta \theta \pi$
%

$\circ \theta \angle | | \nabla \theta \pi$
! &

60. $\# ! B ! \exists \pi \# \exists$

61. $B \% \pi \%, \infty$

62. $B ! \exists \pi \infty, \exists$

$\circ \theta \angle | | \nabla \theta \pi$
\exists

$\circ \theta \theta \angle | | \cup$
%

$\{ | | \nabla \theta \theta \pi$
 \exists

63. $\# \square B ! \# \pi \chi \#, \#$

64. $\% ! B \square \forall \pi \%, \forall \delta$

65. $B \square \& \pi \infty, \& \delta$

$\circ \theta \textcircled{R} | | \nabla \theta \pi$
#

$\circ \theta \angle | | \textcircled{C} \theta \pi$
% \forall

$\{ | | \textcircled{C} \theta \theta \pi$
&

66. $B \square \forall \pi \chi \forall, \infty$

67. $\& ! B \square ! \pi \&, ! \delta$

68. $\exists \square B ! \% \pi \chi \exists, \%$

$\circ \theta \theta \textcircled{R} | | \cup$
 \forall

$\circ \theta \angle | | \textcircled{C} \theta \pi$
& !
%

$\circ \theta \textcircled{R} | | \nabla \theta \pi$
 \exists

69. $\# \square B \square \exists \pi \chi \#, \exists \delta$

70. $\% \square B \square \% \pi \chi \%, \% \delta$

$\circ \theta \textcircled{R} | | \textcircled{C} \theta \pi$
\exists

$\circ \theta \textcircled{R} | | \textcircled{C} \theta \pi$
%

71. $\exists \square B \square \# \pi \# \square B \square \exists \pi \chi \#, \exists \delta$

72. $\exists \square B \square \# \pi \# \square B \square \exists \pi \chi \#, \exists \delta$

$\circ \theta \textcircled{R} | | \textcircled{C} \theta \pi$
\exists

$\circ \theta \textcircled{R} | | \textcircled{C} \theta \pi$
\exists

73. $B \& \text{ and } B ! \% \pi \&, \infty \cap \infty, \%$

&, ∞

oθ∠|||l

∞, %

{|||∇θπ SECTION 0.1

&, ∞ ∩ ∞, %

oθ∠||∇θπ
& %

SECTION 0.1

74. $B \square \exists$ and $B \square \exists \pi \chi \exists, \infty \cap \infty, \exists$

$$\frac{\begin{array}{l} \chi \exists, \infty \\ \infty, \exists \end{array} \quad \begin{array}{l} \circ\theta\textcircled{R}||| \llcorner \\ \{||| \nabla\theta\pi \\ \exists \quad \exists \end{array}}{\chi \exists, \infty \cap \infty, \exists \quad \begin{array}{l} \circ\theta\textcircled{R}||| \nabla\theta\pi \\ \exists \quad \exists \end{array}}$$

75. $B \square)$ and $B \square \exists \pi \chi), \infty \cap \infty, \exists \delta$

$$\frac{\begin{array}{l} \chi), \infty \\ \infty, \exists \delta \end{array} \quad \begin{array}{l} \circ\theta\textcircled{R}||| \llcorner \\ \{||| \textcircled{C}\theta\pi \\ \exists \quad \exists \end{array}}{\chi), \infty \cap \infty, \exists \delta \quad \begin{array}{l} \circ\theta\textcircled{R}||| \textcircled{C}\theta\pi \\) \quad \exists \end{array}}$$

76. $B \square \forall$ and $B \square (\pi \forall, \infty \cap \infty, (\delta$

$$\frac{\begin{array}{l} \forall, \infty \\ \infty, (\delta \end{array} \quad \begin{array}{l} \circ\theta\angle||| \llcorner \\ \{||| \textcircled{C}\theta\pi \\ \forall \quad (\end{array}}{\forall, \infty \cap \infty, (\delta \quad \begin{array}{l} \circ\theta\angle||| \textcircled{C}\theta\pi \\ \forall \quad (\end{array}}$$

77. $B \square \#$ or $B \square \# \pi \infty, \# \cup \#, \infty$

$$\{|\nabla\theta\theta\angle| \llcorner \\ \# \quad \#$$

79. $B \square \forall$ or $B \square \exists \pi \infty, \forall \delta \cup \chi \exists, \infty$

$$\{|\textcircled{C}\theta\theta\textcircled{R}| \llcorner \\ \forall \quad \exists$$

81. Since $\forall \exists \square !, \kappa \forall \exists \kappa \square \forall \exists$.

78. $B \square \&$ or $B \square ! \pi \infty, \& \delta \cup !, \infty$

$$\{|\textcircled{C}\theta\theta\angle| \llcorner \\ \& \quad !$$

80. $B \square \exists$ or $B \square \# \pi \infty, \exists \cup \chi \#, \infty$

$$\{|\nabla\theta\theta\textcircled{R}| \llcorner \\ \exists \quad \#$$

82. Since $\forall (! !, \kappa \forall (\kappa \square \forall (\square \forall ($.

83. Since $\exists x \neg (x \wedge \neg x)$.

84. Since $\exists x \neg (x \wedge \neg x)$,
 $\neg \forall x (x \wedge \neg x)$.

85. Since $\exists x (x \wedge \neg x)$,
 $\neg \forall x (x \wedge \neg x)$.

SECTION 0.1
86. Since $\exists x (x \wedge \neg x)$, $\neg \forall x (x \wedge \neg x)$.

SECTION 0.1

87. Since $\exists x \neg x$, $\neg \exists x x$.
88. Since $\exists x \neg x$, $\neg \exists x x$.
89. Since $1 \& 1$,
 $\neg(1 \& \neg 1) \rightarrow 1 \& 1$.
90. Since $\neg(1 \& \neg 1)$, $\neg(1 \& \neg 1)$.
91. $\neg(1 \& \neg 1) \rightarrow 1 \& 1$.
92. Since $\neg(1 \& \neg 1)$, $\neg(1 \& \neg 1)$.
93. If $B \rightarrow \neg B$, then $B \rightarrow \neg B$. Then
 $\neg B \rightarrow B$.
94. If $B \rightarrow \neg B$, then $B \rightarrow \neg B$. Then
 $\neg B \rightarrow B$.
95. If $B \rightarrow \neg B$, then $B \rightarrow \neg B$. Then
 $\neg B \rightarrow B$.
96. If $B \rightarrow \neg B$, then $B \rightarrow \neg B$. Then
 $\neg B \rightarrow B$.
97. distance $\rightarrow \neg$ $\exists x \neg x$ & \neg
98. distance $\rightarrow \neg$ $\exists x \neg x$ & \neg $\exists x \neg x$
99. distance $\rightarrow \neg$ $\exists x \neg x$ & \neg
100. distance $\rightarrow \neg$ $\exists x \neg x$ & \neg $\exists x \neg x$
101. Since population must be positive and never has a fractional part, the set of **natural numbers** should be used.
102. Since the subdivisions on a ruler are measured in fractions of an inch, the set of **rational numbers** should be used.
103. Since temperatures are usually reported without fractional parts and may be either positive or negative (or zero), the set of **integers** should be used.
104. Since the financial condition of a business is usually described in terms of dollars and cents (fractional parts of a dollar), the set of **rational numbers** should be used.
105. B will represent a positive number if B itself is negative. For instance, if $B \rightarrow \neg B$, then $B \rightarrow \neg B$, which is a positive number.
106. Every integer is a rational number because every integer is equal to itself over 1.
107. The statement is always true.
108. The statement is always true.
109. The statement is not always true.
- For example, let \neg & and \neg #.

110. The statement will be true if $\forall x \exists y$ and $\exists x \forall y$, or if $\forall x \forall y$ and $\exists x \exists y$.

111. The statement $\forall x \exists y, y - x$ could be interpreted to mean that $\forall x \exists y, y = -x$, when this is not necessarily true.

SECTION 0.1
 11.1. $\forall x \exists y, x + y = 0$, $\exists x \forall y, x + y = 0$, $\forall x \forall y, x + y = 0$, $\exists x \exists y, x + y = 0$

Exercises 0.2 (page 24)

- 1. factor
- 2. natural
- 3. $\exists, \#B$
- 4. exponential

SECTION 0.2

5. scientific, integer 6. **Answers may vary.** 7. $B^7 B^8 \square B^{78}$ 8. $B^{7^8} \square B^{78}$
9. $BX^8 \square B^8 X^8$ 10. $\frac{B^7}{B^8} \square B^{78}$ 11. $B^! \square \forall$ 12. $B^8 \square \frac{\forall}{B^8}$
13. $\forall \exists \# \square \forall \exists \square \forall \exists \square \forall \exists *$ 14. $\forall ! \exists \square \forall ! \square \forall ! \square \forall ! \square \forall , !!!$
15. $\& \# \square \forall \square \& \square \& \square \# \&$ 16. $\& \# \square \& \& \square \# \&$
17. $\% B^3 \square \% \square B \square B \square B$ 18. $\% B^3 \square \% B \% B \% B$
19. $\& B \% \square \& B \& B \& B \& B$ 20. $\exists B \# \square \exists \square B \square B$
21. $) B \% \square) \square B \square B \square B \square B$ 22. $) B \% \square) B) B) B) B$
23. $(B B B \square (B^3$ 24. $) X X X X \square) X \%$
25. $B \quad B \square \quad \forall \quad \forall B \# \square B \#$ 26. $\# + \quad \# + \quad \# + \square \# \square \# \square \# \square +^3 \square) +^3$
27. $\exists > \quad \exists > \quad \exists > \square \exists \exists \exists >^3 \square \# (>^3$ 28. $\#, \#, \#, \#, \square \forall \square \# \square \# \square \# \square \# \square , \% \square \forall \exists , \%$
29. $B B B X X \square B^3 X \#$ 30. $+++ , , , , \square +^3 , \%$ 31. $\# \Rightarrow \#^3 \square \forall ! \Rightarrow \exists \%)$
32. $(\Rightarrow \forall \% \square$ 33. $! \Rightarrow \& \% \square$ 34. $! \Rightarrow \# \% \square ! \Rightarrow !! \forall \exists$
35. $\# \& \% \forall \Rightarrow \forall \exists) \forall$ 36. $! \Rightarrow ! \exists \# \& X \exists X \% \square$ 37. $\Delta \#^3 \square \Delta \# \square \square \Delta^3$
- $B \# B^3 \square B \#^3 \square B \&$ $X \exists \% \square X ($
38. $>^3 (\square >^3 \square (\square > \% \#$ 39. $X \& X \#^3 \square X (\exists \square X \# \forall$ 40. $+^3 +^3 + \% \square + * + \% \square + \forall \exists$
41. $\Delta \#^3 \Delta \% \& \square \Delta^3 \Delta \# ! \square \Delta \#^3$ 42. $> \exists \quad \% \& \quad \# \square > \forall \# \square \forall ! \square > \# \#$
43. $+ \#^3 + \% \# \square + \exists +) \square + \forall \%$ 44. $+ \#^3 + \exists \exists \square +) + * \square + \forall ($
45. $\exists B^3 \square \exists^3 B^3 \square \# (B^3$ 46. $\# X \% \square \# \% X \% \square \forall \exists X \%$

$$47. \quad B^{\#}X^{\exists} \square B^{\#}X^{\exists} \square \\ B^{\exists}X^{\exists}$$

$$48. \quad B^{\exists}\Delta^{\% \exists} \square B^{\exists} \exists \square B^{\forall}\Delta^{\#\%} \\ \Delta^{\%}$$

SECTION 0.2

$$49. \quad \square \pm^{\# \exists} \square \square \square \frac{\pm^{\# \exists}}{\pm^{\exists}} \square \pm^{\exists} \\ , \quad , \exists \quad , \exists$$

$$50. \quad \square \underline{B}^{\%} \square \square \square \underline{B}^{\%} \square \underline{B}^{\%} \\ X^{\exists} \quad X^{\exists \%} \quad X^{\forall \#}$$

$$51. \quad B^{\exists} \square \forall$$

$$52. \quad \%B^{\exists} \square \% \square \forall \square \%$$

$$53. \quad \%B^{\exists} \square \forall$$

SECTION 0.2

54. $\#B' \square \# \square \forall \square \#$

55. $\Delta \% \square \frac{\forall}{\Delta \%}$

56. $\frac{\forall}{> \#} \square > \#$

57. $X \# X \exists \square X \& \square \frac{\forall}{X \&}$

58. $7 \# 7 \exists \square 7 \forall \square 7$

59. $B \exists B \% \square B \forall \square B \#$
 $\# \quad \#$

60. $X \# X \exists \square X \forall \square X \% \square \frac{\forall}{X \%}$
 $\% \quad \%$

61. $\frac{B}{B \exists} \square B (\exists \square B \%$

62. $\frac{\leq \&}{< \#} \square < \& \# \square < \exists$

63. $\frac{\pm \# \forall}{+ \forall} \square + \# \forall \forall (\square + \%$

64. $\frac{\geq \exists}{> \%} \square > \exists \% \square > *$

65. $\frac{B \#}{B \exists} \# \square \frac{B \%}{B \exists} \square B \% \exists \square B \forall \square B$

66. $\frac{\equiv \#}{\equiv \#} \exists \square \equiv \forall \# \square = \forall \# \% \square =$
 $\equiv \# \quad \equiv \%$

67. $\square \frac{7}{8 \#} \exists \exists \square \frac{7}{8 \#} \exists \exists \square \frac{7}{8} *$

68. $\square \frac{\geq}{> \exists} \square \square \square > \% \exists \square \square \square > \forall \square \square > \exists$

69. $\frac{+ \exists \#}{++ \#} \square \pm \exists \square \square + \exists \exists \square + * \square \frac{\forall}{< \# \exists}$

70. $\frac{* \leq \exists}{< \exists} \square \leq \exists \square \square < \exists \square \square < \forall \#$

71. $\square \frac{\pm \exists \%}{, \forall} \square \square \frac{+ \exists \%}{, \forall \%} \square \pm \forall \#$
 $, \forall \quad , \forall \% \quad , \%$

72. $\square \frac{\geq \% \#}{> \exists} \square \square \frac{> \% \#}{> \exists \#} \square \geq \square > \#$

73. $\square \frac{\% \exists}{< \exists < \exists} \square \square \square \frac{\# \#}{< !} \square \square \square < \# \square \square < \%$

74. $\frac{B B}{B} \square \frac{B}{B} \square \frac{B}{B} \square B \forall \forall \square \frac{\forall}{\#}$

$\forall \square \prec \%$

$\frac{B \# B}{\& \exists}$

$\frac{B \exists \exists}{B^*}$

$B^{\forall \forall}$

SECTION 0.2

$\frac{B \& X \#}{\%}$

$\frac{B \& B \exists}{\%}$

$B) \% B \exists \#$

$B (X \& \exists$

$X \& X \%$

$X^* \exists$

$X \#($

75. $\square \frac{B \exists X \#}{\square} \square \frac{X \# X \#}{\square} \square \frac{X \%}{\square} \square \frac{\overline{X \forall}}{\square}$

76. $\square \frac{\overline{B (X \% \square)}}{\square} \square \frac{B (B (\square)}{\square} \square \frac{\overline{B^{\forall \% \square}}}{\square} \square \frac{\overline{B^{\forall \% \#}}}{\square}$

$\& B \exists X \# \#$

$\exists B \# X \exists \#$

$\frac{\overline{\exists B \# B \exists X \#}}{\#}$

$\exists B \& * B^{\forall !}$

77. $\square \frac{\overline{\exists B \# X \exists}}{\square} \square \frac{\overline{\& B \exists X \#}}{\square} \square \frac{\overline{\& X}}{\square} \square \frac{\& X}{\square} \square \frac{\# \& X \#}{\square}$

$\exists B \# X \& \exists$

$\# B \# X \exists \exists$

$\# X \&$

$\# \exists)$

78. $\square \frac{\# B \# X \exists}{\square} \square \frac{\overline{\exists B \# X \&}}{\square} \square \frac{\exists B \# B \# X \exists}{\square} \square \frac{\exists B \% X}{\square} \square \frac{\# (B^{\forall \# X \exists}}{\square}$

$\exists B \& X \exists \#$

$\exists B \& X \exists \#$

$\# X \exists X \exists$

$\frac{\overline{\# X \exists \#}}{\#} \% X^{\forall \#}$

79. $\square \frac{\overline{\exists B \& X \exists}}{\square} \square \frac{\overline{\exists B \& X \exists}}{\square} \square \frac{\overline{\forall B \& B \&}}{\square} \square \frac{B^{\forall !}}{\square} \square \frac{B \# !}{\square}$

SECTION 0.2

$$\frac{\forall \# B \% X \exists \Delta \&}{\exists \exists X \exists X \exists} \quad \exists X \exists \quad \exists \quad \#(X \forall)$$

80. $\frac{\% B \% X \exists}{\Delta \&}$ $\frac{\forall B \% B \% \Delta \& \Delta \&}{B) \Delta \forall !}$ $\frac{\exists \exists \exists \exists \exists}{B \# \% \Delta \exists !}$

81. $\frac{) \Delta X \# \exists \forall}{\& X \# \Delta \# \exists \& X \Delta \# \forall}$ $\frac{\# \exists \exists \Delta X}{\& \forall X \forall \Delta \#}$ $\frac{\exists \forall}{\& \# X \& \Delta}$ $\frac{\exists \% \Delta \Delta \exists \%}{\& \# X \&}$ $\frac{\exists \% \Delta}{\# \& X \exists}$ (

82. $\frac{78: \# \exists \% \#}{78 \# : \exists \% 78 \# : \forall}$ $\frac{\# \exists \%}{\% (78:) \# 78:) \forall \#}$ $\frac{78: \forall \% \%}{7 \& 8 \exists : \forall \exists}$ $\frac{77: \& \% \forall \exists}{8 \exists 8 \forall \%}$ $\frac{7: \forall \exists}{8 \# !}$

83. $\frac{\& \gamma \exists \#}{\% \# \exists \#}$ $\frac{* \& \delta}{\% \forall \#}$ $\frac{\& \gamma \exists \exists \% \delta}{\% \forall}$ $\frac{\& \gamma \% ! \delta}{\% !}$

84. $\frac{\exists \exists \exists \% (\# \exists)}{\& \# \% \#}$ $\frac{\exists \exists \exists \exists \# \exists}{\& \# \forall \exists}$ $\frac{\exists \gamma \exists * \delta}{\& \forall \%}$ $\frac{\exists \gamma \exists \delta}{(!)}$ $\frac{\exists \exists}{(!)}$ $\frac{\forall}{\exists \&}$

85. $B \# \square \quad \# \# \square$
%

86. $B \# \square \quad \# \# \square \quad \forall \square \% \square \%$

87. $B \exists \square \quad \# \exists \square$
)

88. $B \exists \square \quad \# \exists \square \quad \forall \square \quad) \square)$

89. $B \Delta \exists \square \quad \chi \forall \square \quad \# \square \exists \delta \exists \square \quad \exists \exists \square$
\forall \exists

90. $B \Delta \exists \square \quad \forall \square \quad \# \square \exists \exists \square \quad \# \square \# (\square \& \% \%$

91. $\frac{B \Delta \# \exists}{\Delta \# X \#}$ $\frac{\square \# \square \exists \square \exists}{\exists \# ! \#}$ $\frac{\chi \% \square \# (\delta)}{* !}$ $\frac{\forall !}{*}$ $\forall \#$

92. $\frac{\Delta \# B \# X \#}{B \exists \Delta}$ $\frac{\exists \# \# \#}{\# \exists \exists}$ $\frac{\# \# * \% !}{\# \# * \% !}$ $\frac{\exists \exists}{\# \%}$ $\frac{\exists}{\# \%}$ #

93. $\& B \# \quad \exists X \exists \Delta \square \& \# \# \quad \exists ! \exists \exists \square \& \% \exists ! \quad \exists \square \# ! \quad ! \square \# !$

94. $\exists B \Delta^{\#} \# X \Delta^{\exists} \square \exists \# \exists^{\#} \# ! \exists^{\exists} \square \exists \& \# \exists^{\exists} \square \exists \# \& \# \# (\square (\& \& \% \square \# \forall$

95. $\frac{\exists B \Delta^{\exists} \#}{\exists B^{\#} \Delta^{\exists}} \square \frac{\forall \Delta^{\exists} \Delta}{\# \# B^{\exists} \Delta} \square \frac{\Delta}{\# B \# \# \& \# \exists \#} \square \frac{\exists}{\exists \% \exists \%} \square \frac{\exists}{\exists} \square \frac{\exists}{\exists} \square$

SECTION 0.2

96. $\frac{\& B \Delta^{\#} \exists^{\#}}{\& B \Delta^{\#}} \square \frac{\# \& B \Delta^{\%}}{\& B \Delta^{\#}} \square \frac{\& B^{\%}}{B \Delta^{\#} \Delta^{\exists}} \square \frac{\& B^{\exists}}{\Delta^{\%}} \square \frac{\& \#}{\Delta^{\%}} \square \frac{\&)}{\Delta^{\%}} \square \frac{\% !}{\Delta^{\%}} \square \frac{\% !}{\Delta^{\%}}$

97. $\exists (\#, !!! \square \exists \Rightarrow (\# \square \forall ! \&$

98. $) \#, \& ! ! \square) \Rightarrow * \& \square \forall ! \%$

99. $\forall ((, !!!, !!! \square \forall \Rightarrow ((\square \forall !$

100. $\# \exists, \% (!, !!!, !!! \square \# \Rightarrow \exists \% (\square \forall ! \forall !$

101. $! \Rightarrow !!! (\square (\square \forall ! \exists$

102. $! \Rightarrow !!! \& \# \square \& \Rightarrow \# \square \forall ! \%$

$B^{\%}$, the **136.**
base of
the
exponent
is B .

$\exists\#$
 \square
 $\forall\!^{\#}$
SECTION 0.2
not
in
scie
ntifi
c
nota
tion
bec
aus
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 $\exists\#$
is
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n 1
and
10.

$\exists \% \forall \in \#$
 \supseteq not a
 real
 number

22.

\forall
 $\#$
 $\&$
SECTION 0.2

\exists
 \square
 \square
 $\&$
 $\exists \square$
 $\forall \in$
 \exists
 \square
 $\&$

23. $\forall \in \# \quad \square \square \% + \square \square \quad \forall \in \# \quad \square \square \quad \% \kappa + \kappa$

24. $\# \& + \% \quad \square \square \& + \# \quad \square \square \& \kappa + \# \kappa \quad \square \square \& + \#$

25. $\forall \in \% \quad \square \square \# + \square \square \quad \forall \in \% \quad \square \square \quad \# \kappa + \kappa$
 $\forall \in \exists$
 $\forall \in$

26. $\exists \% + \exists \quad \square \square \% + \square \square \quad \% +$
 $\exists \forall \in \exists$

27. $\exists \# + \& \quad \square \square \# + \& \square \forall \in \& \quad \square \square \# +$

28. $\exists \% + \exists \quad \forall \in \square \square \# + \exists \square \forall \in \exists \quad \square \square \# \kappa + \kappa$

29. $\# \forall \exists, \exists \quad \forall \in \exists \quad \square \square \exists, \# \quad \forall \in \exists \quad \square \square \exists, \#$

30. $\# \& \exists > \quad \forall \in \% \quad \square \square \% > \# \quad \forall \in \% \quad \square \square \% \kappa > \# \kappa \quad \square \square \% > \#$
 \square

- 48
- SECTION 0.3
47. $\forall x \exists y (x \neq y) \wedge \exists x \forall y (x \neq y) \wedge \exists x \forall y (x \neq y) \wedge \exists x \forall y (x \neq y)$
49. $\forall x \exists y (x \neq y) \wedge \exists x \forall y (x \neq y) \wedge \exists x \forall y (x \neq y) \wedge \exists x \forall y (x \neq y)$
50. $\forall x \exists y (x \neq y) \wedge \exists x \forall y (x \neq y) \wedge \exists x \forall y (x \neq y) \wedge \exists x \forall y (x \neq y)$
51. $\forall x \exists y (x \neq y) \wedge \exists x \forall y (x \neq y) \wedge \exists x \forall y (x \neq y) \wedge \exists x \forall y (x \neq y)$
52. $\forall x \exists y (x \neq y) \wedge \exists x \forall y (x \neq y) \wedge \exists x \forall y (x \neq y) \wedge \exists x \forall y (x \neq y)$

SECTION 0.3

$$53. \exists X \forall \Delta \& \forall \epsilon \& \square \frac{\forall}{\exists X \forall \Delta \& \forall \epsilon \&}} \square \frac{\forall}{\exists \# \forall \epsilon \& X \forall \forall \epsilon \& \Delta \&}} \square \frac{\forall}{\# X \# \Delta}$$

$$54. \exists \# \& + \%) \forall \epsilon \% \square \frac{\forall}{\forall \epsilon \%}} \square \frac{\forall}{\forall \epsilon \%}} \square \frac{\forall}{\forall \epsilon \%}} \square \#$$

$$\exists \# \& + , \quad \exists \# \& + , \quad \& + ,$$

$$\%) \quad \forall \epsilon \% \quad \%)$$

$$55. B \forall X \& \square B \exists \epsilon \& X \forall \epsilon \& \square B \exists X \exists$$

$$56. \exists \% + \exists , \forall \# \& \epsilon \exists \square \exists \% \& \epsilon \exists + \exists ! \epsilon \exists \square \square \exists \% \forall \epsilon \exists \& + \& , \forall ! \square \# \& + \& , \forall ! \square \exists \# + \& , \forall !$$

$$57. \square \langle \rangle = \forall \exists \square \exists \epsilon \% \square \langle \# \% \epsilon \% = \%) \epsilon \% \square \langle \exists = \forall \# \square \frac{\forall}{\langle \exists = \forall \#}$$

$$58. \square \rangle B * X \forall \# \square \# \epsilon \exists \square \rangle \# \epsilon \exists B \forall \epsilon \exists X \# \% \epsilon \exists \square \frac{\forall}{\# \epsilon \exists} B \exists X \rangle \square \frac{\forall}{\# \# B \exists X \rangle} \square \frac{\forall}{\% B \exists X \rangle}$$

$$59. \square \rangle \pm \square \square \frac{\# \epsilon \exists}{\forall \# \& , *}} \square \frac{\forall \# \epsilon}{\forall \# \& \# \epsilon \exists , \forall)}} \square \frac{\# \% + \%}{\# + \% \pm}} \square \frac{\%}{\% \pm}}$$

$$\forall \exists B \% \square \exists \epsilon \% \frac{\forall \exists \epsilon \% B \forall \# \epsilon \%}{\# \exists B \exists} \rangle B \exists$$

$$60. \square \frac{\forall \exists \epsilon \% B \forall \# \epsilon \%}{\exists \# \& \exists \epsilon \% X \# \% \epsilon \%}} \square \square \frac{\# \exists B \exists}{\& \exists X \exists} \square \forall \# \& X \exists$$

$$\# \langle \exists \quad \# \epsilon \exists \quad \forall , ! ! ! = \forall \# \quad \# \epsilon \exists \quad \forall , ! ! ! \# \epsilon \exists = \% \& \epsilon \quad \forall ! \# = \rangle \quad \forall ! ! = \rangle$$

$$61. \square \quad \square \quad \square \quad \square$$

$$\frac{\forall , ! ! ! = \forall \#}{\# \langle \exists} \quad \square \quad \# \langle \exists \quad \square \quad \# \langle \# \epsilon \exists \langle \forall \# \epsilon \exists \quad \square \quad \exists \# \langle \% \quad \square \quad * \langle \%$$

$$62. \square \frac{\exists \# 7}{\# \% \exists 8 \forall \&}} \square \square \frac{\# \% \exists 8}{\exists \# 7 \forall}} \square \square \frac{\# \% \exists 8}{\exists \# \epsilon \& 7 \# ! \epsilon \&}} \square \frac{\# \exists 8}{\# 7 \%}} \square \frac{* 8}{\% 7 \%}}$$

63. $\frac{+\# \& \% \in \&}{+} \quad \frac{\exists \in \&}{\square} \quad \frac{\& \in \square}{+}$
 $\frac{+\forall \in \&}{-} \quad \frac{+\forall \in}{-\&}$

64. $\frac{B \ B \exists \in (\exists B (}{\square} \quad \frac{* \in (}{\square} \ B \# \in ($

SECTION 0.3 $\frac{\# \in (B \& \in}{(\quad} \quad \frac{B \in (}{-}$

65. $\cup \% * \square \cup (\# \overline{\square} ($

66. $\cup) \overline{\forall \square} \cup * \# \square *$

67. $\cup \overline{\forall \# \& \square} \cup \exists \& \exists \square \&$

68. $\cup \quad \exists \% \square \exists \quad \% \exists \square \%$

69. $\cup \overline{\forall \# \& \square} \exists \& \exists \square$
 $\&$

70. $\cup \quad \# \% \exists \square \exists \quad \exists \& \square \exists$

71. $\frac{\exists \# \quad \& \quad \# \quad \&}{\#} \quad \frac{\forall}{-}$
 $\frac{\&}{\exists} \quad \frac{\forall \{!, !!!\}}{\square} \quad \frac{\& \square}{\forall \{!, !!!\}} \quad \frac{\forall \{!, !!!\}}{\square} \quad \frac{\forall \{!, !!!\}}{\&}$
 $\frac{\& \# \&}{\exists \# \&}$

72. $\frac{\# \& \exists}{\square} \quad \frac{\% \quad \% \quad \%}{-} \quad \frac{\%}{-}$
 $\frac{\supset}{\square} \quad \frac{\square \& \square}{\square} \quad \frac{\square \& \square}{\square} \quad \&$

73. $\cup \exists B \# \square \supset \exists B \# \square \kappa \exists B \kappa \square \exists \kappa B \kappa$

74. $\cup \# \& X \# \square \supset \& X \# \square \kappa \& X \kappa \square \& \kappa X \kappa$

$\overline{U \&}$ $\overline{U \&}$ $U \&$ & UX UX UX X

107. **SECTION 108.3**

 $\forall!$ $\forall!$ $\overline{U \&}$ $\forall! U \&$ $\# U \&$ E E UX $\exists UX$

109.

 $\overline{U *}$ $\overline{U *}$ $\overline{U \#}$ $\overline{U \#}$ E \forall

E E $\overline{U \#}$ $\overline{U \#}$ $\overline{U \#}$ $\overline{U \#}$

110.

 $\overline{U \forall \exists, \#}$ $\overline{U \forall \exists, \#}$ $\overline{U \%,}$ $\overline{U \%,}$ $\%,$,

 $\forall \exists$ $\overline{U \%,}$ $\forall \exists \overline{U \%,}$ $\forall \exists \overline{U \%,}$ $\% \overline{U \%,}$

 $\exists \forall$

111.

 $\overline{U \forall \exists, \#}$ $\overline{U \forall \exists, \#}$ $\overline{U \#, \#}$ $\overline{U \#, \#}$ $\#,$,

 $\exists \% +$ $\overline{U \#, \#}$ $\exists \% + \overline{U \#, \#}$ $\exists \% + \overline{U \#, \#}$ $\exists \# + \overline{U \#, \#}$

 $\exists \% +$

$\exists \# \in \% \quad \exists \forall \in \# \quad \cup \exists$

118. $\cup \# (\overline{\cup \exists \forall \in \# \quad \exists \exists \forall \in \# \quad \exists \exists \in \# \quad \cup \exists})$ **SECTION 0.3**

119. $\cup \overline{\forall \exists B^3} \quad \forall \exists B^3 \quad \forall \in \forall ! \quad \# \% B^3 \quad \forall \in \forall ! \quad \# \% \in \forall ! \quad B^3 \in \forall ! \quad \# \# \in \& \quad B^3 \in \& \quad \# \# B^3 \quad \cup \% B^3$
 $\forall \in \&$

120. $\cup \# (\overline{B^*} \quad \# (B^* \quad \forall \in \exists \quad \exists \exists B^* \quad \forall \in \exists \quad \exists \exists \in \exists \quad B^* \in \exists \quad \exists \forall \in \# \quad B^3 \in \# \quad \cup \exists \overline{B^3} \quad B \cup \exists B$
 $\exists B^3)$

121. $\cup \# \overline{\cup \#} \quad \# \overline{\forall \in \#} \quad \# \overline{\forall \in \exists} \quad \# \overline{\exists \in \exists} \quad \# \# \overline{\in \exists} \quad \cup \# \overline{\exists \cup \#} \quad \cup \overline{\cup \%} \quad \cup \exists \#$

122. $\cup \overline{\exists \cup \&} \quad \overline{\exists \forall \in \# \&} \quad \forall \in \exists \quad \exists \exists \in \exists \quad \# \in \exists \quad \cup \overline{\exists \exists \cup \&} \quad \cup \# (\overline{\cup \# \&} \quad \cup \exists \&)$

SECTION 0.3

123. $\frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%}$
124. $\frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%}$
125. $\frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%}$
126. $\frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%} \quad \frac{\exists \forall \in \%}{\exists \forall \in \%}$

127. $\frac{B^{7 \in 8}}{X} \quad \frac{B^{7 \in 8}}{X^{7 \in 8}} \quad \frac{B^{7 \in 8}}{X^{7 \in 8}} \quad \frac{B^{7 \in 8} X^{7 \in 8}}{B^{7 \in 8} X^{7 \in 8}} \quad \frac{X^{7 \in 8}}{B^{7 \in 8}} \quad \frac{X^{7 \in 8}}{B^{7 \in 8}} \quad \frac{X^{7 \in 8}}{B^{7 \in 8}}$

128. Consider the case when 8 is even, 7 is odd and B is negative. Then $B^{7 \in 8} \square \square B^{\forall \in 8} \square \square \square B^{\forall \in 8}$. Thus, $\exists B$ must be a real number for the expression to be defined.

129. $\# ! B \square \& \supseteq \# , \& \delta$
130. If B %, then $\exists B ! !$. Then $\kappa \exists B \kappa \square \exists B \square \exists B \square B \square \exists$.
131. $B^{\#} X^{\#} \square \#^{\#} \exists^{\#} \square \% * \square \&$
132. $\frac{BX \% X}{B} \square \frac{\# \exists \% \exists}{\#} \square \frac{\exists \forall \#}{\#} \square \exists \square \exists$
133. $\exists \forall (, !!!, !!!) \square \exists \Rightarrow \forall (\square \forall !)$
134. $! \Rightarrow !! \# \exists \& \square \forall ! \% \square \# \exists \Rightarrow \&$

Exercises 0.4 (page 50)

1. monomial, 2. degree, 3. trinomial 4. binomial
- variables variables
5. one 6. zero 7. like 8. degree

9. coefficients, variables 10. $\exists \cup \bar{B} \#$ 11. yes, trinomial, 2nd degree
12. yes, binomial, 3rd degree 13. no **SECTION 0.3** 14. no
15. yes, binomial, 3rd degree 16. yes, monomial, 5th degree 17. yes, monomial, 0th degree
18. no 19. yes, monomial, no degree 20. yes, none, 3rd degree
21. $B^3 \quad \exists B^{\#} \ \& B^3 \quad)B \square B^3 \quad \exists B^{\#} \ \& B^3 \quad)B \square B^3 \quad \& B^3 \ \exists B^{\#} \quad)B \square \exists B^3 \quad \exists B^{\#}$
 $)B$

SECTION 0.4

32. $\exists 787 \#8 \exists 7 \exists 78 \forall \#8 \%78 \forall$
 $\square \exists 787 \exists 78 \#8 \exists 7 \exists 78 \exists 7 \forall \#8 \%78 \#8 \forall$
 $\square \exists 7\#8 \exists 78\# \forall)7\#8 \exists 7)78\# \#8$
 $\square \exists 7\#8 \forall)7\#8 \exists 78\#)78\# \exists 7 \#8 \square \forall \&7\#8 \#78\# \exists 7 \#8$
33. $\#B\#X\exists \%BX\% \square \# \% B\#BX\exists X\% \square$
 $)B\exists X($
34. $\forall \&+\exists, \square \#+\exists, \exists \square \square \forall \& \# +\exists+\exists, \exists$
 $\square \exists !+\&, \%$

35. $\exists 7\#8 \square \#78\# \square \square \frac{78}{\square} \square \square \exists \# \square \frac{\forall}{\square} \square 7\#7788\#8 \square \exists 7\%8\% \square \frac{7\%8\%}{\square}$
 $\forall \# \quad \forall \# \quad \forall \# \quad \#$

36. $\frac{\exists < \#}{\square} \exists \frac{\# < \#}{\square} \equiv \# \square \square \square \square \frac{\exists}{\square} \# \square < \# < \# < \# \equiv \# \square \exists < \& \equiv$
 $\square \frac{\forall \& < \#}{\square} \square \square$
 $\& \exists \# \quad \& \exists \#$

37. $\% < \# < \# \equiv \# \square \% < \# < \# \% < \# \equiv \# \square \% < \# \equiv \% < \# \equiv$

38. $\exists ? \# \equiv \# ? \# \equiv \# X \square \exists ? \# \equiv \# ? \# \equiv \# \exists ? \# \equiv X \square \forall \# ? \# \equiv \# \exists ? \# \equiv X$

39. $\exists +, \# - \square \# + - \exists, - \# \quad \% +, \# - \square \square \exists +, \# - \# + - \exists +, \# - \square \exists, - \# \quad \exists +, \# - \square \% +, \# - \square$
 $\square \forall \# +, \# - \# \quad \forall) +, \exists - \exists \quad \# \% +, \# - \#$

40. $\frac{78}{\#} \# \square \%78 \exists 7\#) \square \square \frac{78\#}{\#} \%78 \frac{78\#}{\#} \square \exists 7\# \square \frac{78}{\#} \#$
 $\square \#7\#8\exists \exists 7\#8\# \%78\#$

41. $+ \# + \# \square + \# \# + \# +$
 $\%$

42. $X \& X \& \square X\# \& X \& X$
 $\# \&$

43. $+ \exists \# \square + \exists + \exists$
 $\square + \# \quad \% + \quad \%$

44. $> * \# \square > * > *$
 $\square X\# \quad \forall ! X \# \&$

45. $\square + \# \quad \exists + \quad \exists + \quad \exists \exists$
 $) \forall \square + \# \quad \forall \# + \quad \exists \exists$
 $) \forall$

46. $> \# \quad * > \quad * >$
 $\square > \# \quad \forall) >$

45. $B \quad \% \quad B \quad \% \square B\# \quad \% B \quad \% B$
 $\forall \exists$

46. $\Delta \quad (\quad \Delta \quad (\square \Delta\# \quad (\Delta \quad (\Delta \quad \%*$

47. $B \quad \exists \quad B \quad \& \square B\# \quad \& B \quad \exists B$
 $\forall \&$
 $\square B\# \quad \# B \quad \forall \&$

48. $\Delta \quad \% \quad \Delta \quad \exists \square \Delta\# \quad \exists \Delta \quad \% \Delta \quad \# \%$
 $\square \Delta\# \quad \# \Delta \quad \# \%$

49. ? # ∃? # □ ∃?# #? ∃? %

50. %B ∨ #B ∃ □)B# ∨#B #B
∃

□ ∃?# %? %

SECTION 0.4

□)B# ∨!B ∃

51. &B ∨ #B ∃ □ ∨!B# ∨&B #B
∃

52. %B ∨ #B (□)B# #)B #B (

□ ∨!B# ∨∃B ∃

□)B# ∃!B (

SECTION 0.4

53. $\exists+ \#, \# \square \exists+ \#, \exists+ \#, \square *+^{\#} \ni+, \ni+, \%, \# \square *+^{\#} \forall \#+, \%, \#$
54. $\%+ \&, \%+ \&, \square \forall \ni+^{\#} \#!+, \#!+, \# \&, \# \square \forall \ni+^{\#} \# \&, \#$
55. $\exists 7 \ \%8 \ \exists 7 \ \%8 \square *7^{\#} \ \forall \#78 \ \forall \#78 \ \forall \ni 8^{\#} \square *7^{\#} \ \forall \ni 8^{\#}$
56. $\%< \ \exists =^{\#} \square \ \%< \ \exists = \ \%< \ \exists = \square \forall \ni <^{\#} \ \forall \#< = \ \forall \#< = \ * =^{\#} \square \forall \ni <^{\#} \ \# \%< = \ * =^{\#}$
57. $\#X \ \%B \ \exists X \ \#B \square \ni X^{\#} \ \%BX \ \forall \#BX \)B^{\#} \square \ni X^{\#} \ \forall \ni BX \)B^{\#}$
58. $\#B \ \exists X \ \exists B \ X \square \ \ni B^{\#} \ \#BX \ *BX \ \exists X^{\#} \square \ \ni B^{\#} \ (BX \ \exists X^{\#}$
59. $*B \ X \ B^{\#} \ \exists X \square *B^{\exists} \ \#(BX \ B^{\#}X \ \exists X^{\#} \square *B^{\exists} \ B^{\#}X \ \#(BX \ \exists X^{\#}$
60. $)+^{\#} \ , \ + \ \#, \square)+^{\exists} \ \forall \ni +^{\#}, \ +, \ \#, \#$
61. $\&\Delta \ \#> \ \Delta^{\#} \ > \square \&\Delta^{\exists} \ \&>\Delta \ \#>\Delta^{\#} \ \#>^{\#} \square \&\Delta^{\exists} \ \#>\Delta^{\#} \ \&>\Delta \ \#>^{\#}$
62. $X \ \#B^{\#} \ B^{\#} \ \exists X \square B^{\#}X \ \exists X^{\#} \ \#B^{\%} \ \ni B^{\#}X \square \ \#B^{\%} \ \&B^{\#}X \ \exists X^{\#}$
63. $\square \cup \bar{\&} \ \exists B \square \square \# \ \bar{\cup} \&B \square \square \# \bar{\cup} \& \ \&B \ \ni B \ \bar{\exists} \cup \&B^{\#} \square \ \bar{\exists} \cup \&B^{\#} \ B^{-} \ \# \cup \&$
64. $\square \cup \bar{\#} \ B \square \square \exists \ \bar{\cup} \#B \square \square \exists \cup \bar{\#} \ \#B \ \exists B \ \bar{\cup} \#B^{\#} \square \bar{\cup} \#B^{\#} \ \&B \ \exists \bar{\cup} \#$
65. $\exists B \ \forall^{\exists} \square \exists B \ \forall \exists B \ \forall \exists B \ \forall$
 $\square \square *B^{\#} \ \exists B \ \exists B \ \forall \square \exists B \ \forall$
 $\square \square *B^{\#} \ \ni B \ \forall \square \exists B \ \forall$
 $\square *B^{\#} \exists B \ *B^{\#} \ \forall \ \ni B \ \exists B \ \ni B \ \forall \ \forall \exists B \ \forall \ \forall$
 $\square \#(B^{\exists} \ *B^{\#} \ \forall)B^{\#} \ \ni B \ \exists B \ \forall \square \#(B^{\exists} \ \#(B^{\#} \ *B \ \forall$
66. $\#B \ \exists^{\exists} \square \#B \ \exists \#B \ \exists \#B \ \exists$
 $\square \square \%B^{\#} \ \ni B \ \ni B \ * \square \#B \ \exists$
 $\square \square \%B^{\#} \ \forall \#B \ * \square \#B \ \exists$
 $\square \%B^{\#} \#B \ \%B^{\#} \ \exists \ \forall \#B \#B \ \forall \#B \ \exists \ * \#B \ * \ \exists$
 $\square)B^{\exists} \ \forall \#B^{\#} \ \#\%B^{\#} \ \exists \ni B \ \forall)B \ \#(\square)B^{\exists} \ \exists \ni B^{\#} \ \&\%B \ \#($
67. $\exists B \ \forall \square \#B^{\#} \ \%B \ \exists \square \square \exists B \square \#B^{\#} \square \ \exists B \ \%B \ \exists B \ \exists \ \forall \square \#B^{\#} \square \ \forall \%B \ \forall \ \exists$
 $\square \ni B^{\exists} \ \forall \#B^{\#} \ *B \ \#B^{\#} \ \%B \ \exists \square \ni B^{\exists} \ \forall \%B^{\#}$
 $\&B \ \exists$
68. $\#B \ \& \square B^{\#} \ \exists B \ \# \square \square \#B \square B^{\#} \square \ \#B \ \exists B \ \#B \ \# \ \& \square B^{\#} \square \ \& \ \exists B \ \& \ \#$
 $\square \#B^{\exists} \ \ni B^{\#} \ \%B \ \&B^{\#} \ \forall \&B \ \forall ! \square \#B^{\exists} \ \forall \forall \forall B^{\#}$
 $\forall *B \ \forall !$

SECTION 0.4

69. $\exists B \#X \square \#B^{\#} \exists BX \%X^{\#} \square$
 $\square \exists B \square \#B^{\#} \square \exists B \exists BX \exists B \square \%X^{\#} \square \#X \square \#B^{\#} \square \#X \exists BX \#X \square \%X^{\#} \square$
 $\square \exists B^{\exists} *B^{\#}X \forall \#BX^{\#} \%B^{\#}X \exists BX^{\#})X^{\exists} \square \exists B^{\exists} \&B^{\#}X \exists BX^{\#}$
 $)X^{\exists}$

70. $\%< \exists = \square \#<^{\#} \%< = \# = \# \square$
 $\square \%< \square \#<^{\#} \square \%< \%< = \%< \square \# = \# \square \exists = \square \#<^{\#} \square \exists = \%< = \exists = \square \# = \# \square$
 $\square)<^{\exists} \forall \exists <^{\#} =)< = \# \exists <^{\#} = \forall \#< = \# \exists = \exists \square)<^{\exists} \forall !<^{\#} = \# !< = \#$
 $\exists = \exists$

71. $\#X^8 \exists X^8 X^8 \square \#X^8 \exists X^8 \#X^8 X^8 \square \exists X^8 \#X^8 \square \exists X^8 \#X^8 \square \exists X^8 \#X^8 \square \exists X^8 \#X^8 \square$

72. $\exists +^8 \square \# +^8 \exists +^8 \forall \square \exists +^8 \# +^8 \exists +^8 \exists +^8 \forall \square \exists +^8 \# +^8 * +^8 \forall \square \exists +^8 ! +^8 \forall$
 $\square \exists +^8 \# +^8$

73. $\&B^{\#8} X^8 \square \#B^{\#8} X^8 \exists B^{\#8} X^8 \square \&B^{\#8} X^8 \square \#B^{\#8} X^8 \square \&B^{\#8} X^8 \square \exists B^{\#8} X^8 \square$
 $\square \forall !B^{\#8} \#8 X^8 \#8 \forall \&B^{\#8} \#8 X^8 \#8$
 $\square \forall !B^{\#8} X^8 ! \forall \&B^{\#8} X^8 \square \forall !B^{\#8} \forall \&X^{\#8}$

74. $\# +^{\exists8}, \#8 \square \& +^{\exists8}, +, \#8 \square \square \# +^{\exists8}, \#8 \square \& +^{\exists8}, \square \# +^{\exists8}, \#8 \square +, \#8 \square$
 $\square \forall ! +^{\exists8} \exists8, \#8 \forall \# +^{\exists8} \forall, \#8 \#8$
 $\square \forall ! +^{\exists8} \forall, \#8 \forall \# +^{\exists8} \forall, ! \square \forall !, \#8 \forall \# +^{\exists8} \forall$

75. $B^8 \exists B^8 \% B^8 B^8 \% B^8 \exists B^8 \forall \# \square B^8 B^8 \forall \#$

76. $+^8 \& +^8 \exists \square +^8 +^8 \exists +^8 \& +^8 \forall \& \square +^8 +^8) +^8 \forall \&$

77. $\#<^8 (\exists <^8 \# \square \#<^8 \exists <^8 \#<^8 \# (\exists <^8 \forall \% \square \exists <^{\#8} \%<^8 \# \forall <^8 \forall \% \square \exists <^{\#8} \# \& <^8 \forall \%$

78. $\% \Delta^8 \exists \exists \Delta^8 \forall \square \% \Delta^8 \exists \Delta^8 \% \Delta^8 \forall \exists \exists \Delta^8 \exists \square \forall \# \Delta^{\#8} \% \Delta^8 * \Delta^8 \exists \square \forall \# \Delta^{\#8} \forall \exists \Delta^8 \exists$

79. $B^{\forall \in \#} \square B^{\forall \in \#} X BX^{\forall \in \#} \square \square B^{\forall \in \#} B^{\forall \in \#} X B^{\forall \in \#} BX^{\forall \in \#} \square B^{\in \#} X B^{\exists \in \#} X^{\forall \in \#} \square BX B^{\exists \in \#} X^{\forall \in \#}$

80. $+ , \forall \in \# \square + \forall \in \# , \forall \in \# , \forall \in \# \square \square + , \forall \in \# + \forall \in \# , \forall \in \# + , \forall \in \# , \forall \in \# \square + \exists \in \# , \# \in \# + , \# \in \# \square + \exists \in \# , + ,$

81. $\square + \forall \in \# , \forall \in \# \square \square + \forall \in \# , \forall \in \# \square \square + \forall \in \# + \forall \in \# + \forall \in \# , \forall \in \# + \forall \in \# , \forall \in \# , \forall \in \# \square + \# \in \# , \# \in \# \square +$

82. $\square B^{\exists \in \#} \# \square \square B^{\exists \in \#} X^{\forall \in \#} \square \square B^{\exists \in \#} X^{\forall \in \#} \square \square B^{\exists \in \#} B^{\exists \in \#} X^{\forall \in \#} B^{\exists \in \#} X^{\forall \in \#}$
 $X^{\forall \in \#} \square X^{\forall \in \#} X^{\forall \in \#}$

$\square B^{\exists \in \#} \# B^{\exists \in \#} X^{\forall \in \#} X^{\# \in \#}$

□

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SECTION 0.4

\exists 110.
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SECTION 0.4 $\&B$ $\exists\&$
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$\exists B^{\#} \quad \forall \forall B \quad \forall! \exists \exists B^{\%} \quad (\#B^{\exists} \quad \forall \# \forall B^{\#}$
 $\quad \quad \quad \forall \% \# B \quad \forall \#!$
 $\quad \quad \quad \underline{\exists \exists B^{\%} \quad \exists \exists B^{\exists} \quad \exists! B^{\#}}$
 $\quad \quad \quad \exists B^{\exists} \quad \exists \forall B^{\#}$
SECTION 0.4
 $\quad \quad \quad \forall \% \# B$
 $\quad \quad \quad \underline{\exists B^{\exists} \quad \forall \forall B^{\#}}$
 $\quad \quad \quad \forall! B$
 $\quad \quad \quad (\#B^{\#} \quad \forall \exists \# B \quad \forall \#!$
 $\quad \quad \quad \underline{(\#B^{\#} \quad \forall \exists \# B \quad \forall \#!$
 $\quad \quad \quad !$

SECTION 0.4

122.
$$\begin{array}{r} \exists B^\# \quad B \left[\begin{array}{l} \frac{\exists B^\# \quad \forall \forall B \quad \forall!}{\forall \# \exists B^\% \quad (\#B^\exists \quad \forall \# \forall B^\#} \\ \frac{\forall \% \# B \quad \forall \#!}{\exists B^\% \quad \exists B^\exists} \\ \frac{(\#B^\#}{\exists \exists B^\exists \quad \% * B^\# \quad \forall \% \# B} \\ \frac{\exists \exists B^\exists \quad \forall \forall B^\# \quad \forall \exists \# B}{\exists ! B^\# \quad \forall ! B \quad \forall \#!} \\ \frac{\exists ! B^\# \quad \forall ! B \quad \forall \#!}{\quad} \end{array} \right. \end{array}$$

123. Area \square length \square width \square B & B # ft# \square B# #B &B $\forall!$ ft# \square B# $\exists B$ $\forall!$ ft#

124. Area \square $\frac{\forall}{\#}$ \square base \square height

$$B^\# \quad \exists B \quad \% ! \square \frac{\forall}{\#} \quad B) \square$$

$$\# \square B^\# \quad \exists B \quad \% ! \square \square B) \square$$

$$\# B^\# \quad \exists B \quad) ! \square B) \square \text{ height}$$

$$\frac{\# B^\# \square \exists B !}{B)} \square \text{ height}$$

$$B \left[\begin{array}{l} \frac{\# B \quad \forall!}{\# B^\# \quad \exists B !} \\ \frac{\# B^\# \quad \forall \exists B}{\forall ! B \quad \forall ! B} \\ \frac{\quad}{\quad} \end{array} \right. \downarrow$$

The height is #B $\forall!$ ft.

125. Volume \square 6 \square A \square 2

$$\square \quad \forall \# \quad \# B \quad \forall \# \quad \# B B$$

$$\text{in.}^\exists \square \square \forall \% \% \quad \%) B \quad \% B^\# \square B$$

$$\text{in.}^\exists \square \square \forall \% \% B \quad \%) B^\#$$

$$\% B^\exists \square \text{in.}^\exists$$

126.
$$\begin{array}{r} > \square < \square \frac{\exists B^\# \square \forall * B \quad \# !}{\exists B \quad \%} \\ \exists B \quad \frac{B \quad \&}{\% \exists B^\# \quad \forall * B \quad \# !} \\ \frac{\exists B^\# \quad \% B}{\forall \& B \quad \# !} \\ \frac{\forall \& B \quad \# !}{\forall \& B \quad \# !} \\ \downarrow \\ > \square B \quad \& \end{array}$$

127. + , - \square + , - + , - \square ++ , - , + , - - + , - \square + $\#$

+ , +- + , # , - + , - -# \square + $\#$

, # -# #+, #, - #+-

128. + , - .# \square + , - . + , - .

\square ++ , - . , + , - . - + , - . . + , - .

\square + $\#$ + , +- +. + , # , - ,. + , - -# - . + . ,. - .

.#

\square + $\#$, # -# .# #+, #+- #+. #, - #, . #-.

129. Answers may vary.

130. Multiply the numerator and denominator by the conjugate of the numerator $\frac{a+b\sqrt{c}}{d+e\sqrt{c}}$.

131. Check the formula with $+$, \forall and $,$.

132. Check the formula with $+$, \exists and $,$.

133. $\forall x \in \mathbb{R} \exists y \in \mathbb{R} (x+y=1)$

SECTION 0.4

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134. $\forall x \exists y (x + y = 0) \wedge \exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0)$
135. $\exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0)$
136. $\exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0)$
137. $\exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0)$
138. $\exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0) \wedge \exists x \forall y (x + y = 0)$

Exercises 0.5 (page 61)

1. factor
2. integer, prime
3. $x + y = y + x, B \cap B = B, \dots$
4. $B \cap X = X \cap B, B \cap X = X \cap B$
5. $B \cap X = X \cap B, B \cap X = X \cap B, B \cap X = X \cap B$
6. $X \cap B = B \cap X, X \cap B = B \cap X, X \cap B = B \cap X$
7. $B \cap X = X \cap B, B \cap X = X \cap B, B \cap X = X \cap B$
- 8.
9. $\exists B \ni \exists B \#$
10. $\&X \forall \& \square \& X \exists$
11. $)B \# \%B \square \%B \# B$
12. $*X \ni \ni X \square \exists X \exists X \#$
13. $(B \# X \# \forall \%B \ni X \# \square (B \# X \# \forall \#B$
14. $\# \& X \# \Delta \forall \& X \Delta \# \square \& X \Delta \& X \exists \Delta$
15. $+ B X, B X \square B X +,$
16. $, B X + B X \square B X, +$
17. $\%+ , \forall \# + \# \exists +, \square \%+ , \exists + \%+ , \square \forall \%+ , \exists + \%+ , \square \%+ , \forall \exists +$
18. $B \# \%B BX \%X \square BB \% XB \% \square B \% B X$

19. $\forall x \exists y (x \neq y \wedge x \neq z) \rightarrow \exists z (x \neq z \wedge x \neq y)$ 20. $\exists x \exists y (x \neq y \wedge x \neq z) \rightarrow (\exists z (x \neq z \wedge x \neq y) \wedge \exists z (x \neq z \wedge x \neq y))$
21. $\forall x \exists y (x \neq y \wedge x \neq z) \rightarrow \exists z (x \neq z \wedge x \neq y)$ 22. $\forall x \exists y (x \neq y \wedge x \neq z) \rightarrow (\exists z (x \neq z \wedge x \neq y) \wedge \exists z (x \neq z \wedge x \neq y))$
- SECTION 0.4
23. $\forall x \exists y (x \neq y \wedge x \neq z) \rightarrow \exists z (x \neq z \wedge x \neq y)$ $\forall x \exists y (x \neq y \wedge x \neq z) \rightarrow \exists z (x \neq z \wedge x \neq y)$
24. $\forall x \exists y (x \neq y \wedge x \neq z) \rightarrow \exists z (x \neq z \wedge x \neq y)$ $\forall x \exists y (x \neq y \wedge x \neq z) \rightarrow \exists z (x \neq z \wedge x \neq y)$
25. $\forall x \exists y (x \neq y \wedge x \neq z) \rightarrow \exists z (x \neq z \wedge x \neq y)$ 26. $\forall x \exists y (x \neq y \wedge x \neq z) \rightarrow \exists z (x \neq z \wedge x \neq y)$

SECTION 0.5

27. $B^{\#})B \forall \exists \square B \% B \% \square B$
 $\% \#$
28. $+^{\#} \forall \# + \exists \exists \square + \exists + \exists \square +$
 $\exists \#$
29. $, \# \forall !, \# \& \square , \& , \& \square ,$
30. $X^{\#} \forall \% X \% * \square X (X (\square X$
31. $\& \#$
32. $(\#$
33. $7^{\#} \% 78 \% 8^{\#} \square 7 \# 8 7 \# 8$
 $\square 7 \# 8^{\#}$
34. $<^{\#}) < = \forall \exists = \# \square < \% = < \% =$
 $\square < \% = \#$
35. $\forall \# B^{\#} BX \exists X^{\#} \square \% B \exists X \exists B$
 $\# X$
36. $) B^{\#} \forall ! B X \exists X^{\#} \square \% B X \# B \exists X$
 $B^{\#} (B \forall ! : + \square \forall , , \square (, - \square \forall !$
 key number $\square + - \square \forall \forall ! \square \forall !$
 $B^{\#} (B \forall ! \square B^{\#} \& B \# B \forall !$
 $\square B B \& \# B \&$
37. $B^{\#} \forall ! B \# \forall \square B^{\#} (B \exists B \# \forall$
 $\square B B (\exists B ($
 $\square B (B \exists$
38. $\square B \& B \#$
39. $B^{\#} \% B \forall \# : + \square \forall , , \square \% , - \square \forall \#$
 key number $\square + - \square \forall \forall \# \square \forall \#$
 $B^{\#} \% B \forall \# \square B^{\#} \exists B \# B$
 $\forall \#$
40. $\square B B \exists \# B \exists$
 $\square B \exists B \#$
 $\exists : \# (: \exists + \square \exists , , \square (, - \square \exists$
 key number $\square + - \square \exists \exists \square \forall$
 $\exists : \# (: \exists \square \exists : \# * : \# : \exists$
 $\square \exists : \# : \exists \# : \exists$
 $\square \# : \exists \exists : \forall$
41. $>^{\exists} \exists \% \exists \square >^{\exists} (\square > (\chi >^{\#} > ((\# \delta \square > (>^{\#} (> \% *$
42. $<^{\exists}) = \exists \square <^{\exists} \# = \exists \square < \# = \square <^{\#} < \# = \# = \# \square \square < \# = <^{\#} \# < = \% = \#$
43. $) \Delta^{\exists} \# (\square \# \Delta^{\exists} \exists \exists \square \# \Delta \exists \square \# \Delta^{\#} \# \Delta \exists \exists \# \square \square \# \Delta \exists \% \Delta^{\#} \exists \Delta *$
44. $\forall \# \& +^{\exists} \exists \% \square \& +^{\exists} \% \exists \square \& + \% \square \& +^{\#} \& + \% \% \# \square \square \& + \% \# \& +^{\#} \# ! +$
 $\forall \exists$
45. $\exists +^{\#} , - \exists + , \# - * + , -^{\#} \square \exists + , - + \# , \exists -$
46. $\& B^{\exists} X^{\exists} \Delta^{\exists} \# \& B^{\#} X^{\#} \Delta^{\#} \forall \# \& B X \Delta \square \& B X \Delta B^{\#} X^{\#} \Delta^{\#} \& B X \Delta \# \&$
47. $\exists B^{\exists} \exists B^{\#} B \forall \square \exists B^{\#} B \forall \forall B \forall \square B \forall \exists B^{\#} \forall$

48. %B эBX *X э□#B# ЭX ЭЭX #□ ЭX # #B Э

49. #>BX #->B Э>X Э->□>#BXSECTION 05 Э-□>#BX - ЭX - δ□>X - #B
Э

SECTION 0.5

50. $\# + B \quad \% + X \quad , B \quad \# , X \square \# + B \quad \# X \quad , B \quad \# X \square \quad B \quad \# X \quad \# + \quad ,$
51. $+ B \quad , B \quad + X \quad , X \quad + \Delta \quad , \Delta \square B + \quad , X + \quad , \Delta + \quad , \square + \quad , B \quad X \quad \Delta$
52. $\exists B^{\#} X^{\exists} \quad \forall) B X \quad \exists B^{\#} X^{\#} \quad * B \square \exists B \square \# B X^{\exists} \quad \exists X \quad B X^{\#} \quad \exists \square \square \exists B \square \# X \square B X^{\#} \quad \exists \square$
 $\forall \square B X^{\#} \quad \exists \square \square$
53. $B^{\#} \quad X \quad \Delta^{\#} \square \chi B \quad X \quad \Delta \delta \chi B$
 $X \quad \Delta \delta$
 $\square \quad B \quad X \quad \Delta \quad B \quad X \quad \Delta$
54. $\Delta^{\#} \quad X \quad \exists^{\#} \square \chi \Delta \quad X \quad \exists \delta \chi \Delta$
 $X \quad \exists \delta$
 $\square \quad \Delta \quad X \quad \exists \quad \Delta \quad X \quad \exists$
55. $B \quad X^{\#} \quad B \quad X^{\#} \square \chi B \quad X \quad B \quad X \delta \chi B \quad X \quad B \quad X \delta \square B \quad X \quad B \quad X \quad B \quad X \quad B \quad X$
 $\square \quad \# B \quad \# X \square \quad \% B X$
56. $\# + \quad \exists^{\#} \quad \# + \quad \exists^{\#} \square \chi \# + \quad \exists \# + \quad \exists \delta \chi \# + \quad \exists \# + \quad \exists \delta$
 $\square \quad \# + \quad \exists \quad \# + \quad \exists \quad \# + \quad \exists \quad \# + \quad \exists \square \quad \% + \quad \exists \square \# \% +$
57. $B^{\%} \quad X^{\%} \square \quad B^{\#} \quad X^{\#} \square \quad B^{\#} \quad X^{\#} \quad B^{\#} \quad X^{\#} \square \quad B^{\#} \quad X^{\#} \quad B \quad X \quad B \quad X$
58. $\Delta^{\%} \quad) \forall \square \quad \Delta^{\#} \quad \#^{\#} \square \quad \Delta^{\#} \quad * \quad \Delta^{\#} \quad * \square \quad \Delta^{\#} \quad * \quad \Delta^{\#} \quad \exists^{\#} \square \quad \Delta^{\#} \quad * \quad \Delta \quad \exists \quad \Delta \quad \exists$
59. $\exists B^{\#} \quad \forall \# \square \exists B^{\#} \quad \% \square \exists B \quad \# \quad B \quad \#$
60. $\exists B^{\exists} X \quad \exists B X \square \exists B X \square B^{\#} \quad \forall \square$
 $\square \exists B X \quad B \quad \forall \quad B \quad \forall$
61. $\forall) B X^{\#} \quad) B \square \# B \square * X^{\#} \quad \% \square$
 $\square \# B \exists X \quad \# \quad \exists X \quad \#$
62. $\# (B^{\#} \quad \forall \# \square \exists \square * B^{\#} \quad \% \square$
 $\square \exists \exists B \quad \# \quad \exists B \quad \#$
63. $B^{\#} \quad \# B \quad \forall \& \supseteq$
 prime
64. $B^{\#} \quad B \quad \# \supseteq \text{prime}$
65. $\forall \& \quad \# + \quad \# \% +^{\#} \square \# \% +^{\#} \quad \# +$
 $\forall \&$
 $\square \quad \exists + \quad \& \quad \% +$
 \exists
66. $\exists \# \quad \exists) B \quad * B^{\#} \square * B^{\#} \quad \exists) B \quad \exists \#$
 $\square \quad * B \quad \% \quad B \quad)$
67. $\exists B^{\#} \quad \# * B X \quad \exists \& X^{\#} \square \exists B \quad (X \quad \# B$
 $\& X$
68. $\forall ! B^{\#} \quad \forall (B X \quad \exists X^{\#} \square \quad \& B \quad \exists X \quad \# B \quad X$
69. $\forall \# :^{\#} \quad \&) ; ; \quad (! ;^{\#} \square \# \exists :^{\#} \quad \# * : ; \quad \exists \& ;^{\#} \square \# \exists : \quad \exists \& ; \quad ; \quad ;$
70. $\exists B^{\#} \quad \exists B X \quad * X^{\#} \square \exists B^{\#} \quad \# B X \quad \exists X^{\#} \square \exists B \quad \exists X \quad B \quad X$

71. $\exists 7^{\#} \quad \% (78 \quad \exists \& 8^{\#} \square \quad \exists 7^{\#} \quad \% (78 \quad \exists \& 8^{\#} \square \quad \exists 7 \quad \& 8 \quad 7 \quad (8$

72. $\forall \% <^{\#} \quad \forall \forall < = \quad \forall \& =^{\#} \square \quad \forall \% <^{\#} \quad \forall \forall < = \quad \forall \& =^{\#} \square \quad (< \quad \& = \quad \# < \quad \exists =$

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73. $\exists B^{\exists} \quad \# \exists B^{\#} \quad \exists \& B \square \quad B \exists B^{\#} \quad \# \exists B \quad \exists \& \square \quad B \exists B \quad (B \quad \&$

74. $X^{\exists} \quad X^{\#} \quad *! X \square \quad X X^{\#} \quad X \quad *! \square \quad X X \quad \forall! \quad X \quad *$

SECTION 0.5

75. $\exists B^{\%} \quad \forall \forall B^{\exists} \quad \exists \& B^{\#} \square B^{\#} \exists B^{\#} \quad \forall \forall B \quad \exists \& \square B^{\#} \# B \quad (\exists B \&$
76. $\forall \# B \quad \forall (B^{\#} \quad (B^{\exists} \square \quad (B^{\exists} \quad \forall (B^{\#} \quad \forall \# B \square \quad B (B^{\#} \quad \forall (B \quad \forall \# \square \quad B B \quad \exists (B \quad \% \quad \# B^{\#} \quad \forall \& \square \quad B^{\#} \quad \& B^{\#} \quad 77. \quad B^{\%} \quad \# B^{\#} \quad \forall \& \square \quad B^{\#} \quad \& B^{\#} \quad 78. \quad B^{\%} \quad B^{\#} \quad \exists \square \quad B^{\#} \quad \exists \quad B^{\#} \quad \# \quad \exists$
79. $+^{\#8} \quad \# +^{\#8} \quad \exists \square \quad +^{\#8} \quad \exists \quad +^{\#8} \quad \forall$
80. $+^{\#8} \quad \exists +^{\#8} \quad) \square \quad +^{\#8} \quad \% \quad +^{\#8} \quad \#$
81. $\exists B^{\#8} \quad (B^{\#8} \quad \# \square \quad \exists B^{\#8} \quad \# \# B^{\#8} \quad \forall$
82. $*B^{\#8} \quad *B^{\#8} \quad \# \square \quad \exists B^{\#8} \quad \# \exists B^{\#8} \quad \forall$
83. $\% B^{\#8} \quad *X^{\#8} \square \quad \# B^{\#8} \quad \exists X^{\#8} \quad \# \quad \square \quad \# B^{\#8} \quad \exists X^{\#8} \quad \# B^{\#8}$
84. $)B^{\#8} \quad \# B^{\#8} \quad \exists \square \quad \% B^{\#8} \quad \exists \quad \# B^{\#8} \quad \forall$
85. $\exists X^{\#8}$
86. $\forall \exists X^{\%8} \quad \# \& X^{\#8} \square \quad X^{\#8} \square \forall \exists X^{\#8} \quad \# \& \square \quad \square \quad X^{\#8} \square \quad \% X^{\#8} \quad \# \& \square \quad \square \quad X^{\#8} \quad \% X^{\#8} \quad \& \quad \% X^{\#8} \quad \&$
87. $\# B^{\exists} \quad \#, !!! \square \# B^{\exists} \quad \forall, !!! \square \# B^{\exists} \quad \forall !^{\exists} \square \# B \quad \forall ! B^{\#} \quad \forall ! B \quad \forall !!$
88. $\exists X^{\exists} \quad \exists \% \square \exists X^{\exists} \quad \# \forall \exists \square \exists X^{\exists} \quad \exists^{\exists} \square \exists X \quad \exists \quad X^{\#} \quad \exists X \quad \exists \exists$
89. $B \quad X^{\exists} \quad \exists \% \square \quad B \quad X^{\exists} \quad \%^{\exists} \square \chi B \quad X \quad \% \delta \square B \quad X^{\#} \quad \% B \quad X \quad \% \# \square \quad \square \quad B \quad X \quad \% \square B^{\#} \quad \# B X \quad X^{\#} \quad \% B \quad \% X \quad \forall \exists \square$
90. $B \quad X^{\exists} \quad \# (\square \quad B \quad X^{\exists} \quad \exists^{\exists} \square \chi B \quad X \quad \exists \delta \square B \quad X^{\#} \quad \exists B \quad X \quad \exists \# \square \quad \square \quad B \quad X \quad \exists \square B^{\#} \quad \# B X \quad X^{\#} \quad \exists B \quad \exists X \quad * \square$
91. $\exists \% +^{\exists} \quad X^{\exists} \square \square +^{\exists} \square^{\#} \quad \square X^{\exists} \square^{\#} \square \square +^{\exists} \quad X^{\exists} \square \square +^{\exists} \quad X^{\exists} \square \quad \square \quad \# + \quad X \square \% +^{\#} \quad \# + X \quad X^{\#} \square \# + \quad X \square \% +^{\#} \quad \# + X \quad X^{\#} \square \quad \square \quad \# + \quad X \quad \# + \quad X \square \% +^{\#} \quad \# + X \quad X^{\#} \square \square \% +^{\#} \quad \# + X \quad X^{\#} \square$
92. $+^{\exists} \quad , \exists \square \quad +^{\#} \exists \quad , \# \exists \square \quad +^{\#} \quad , \# \square +^{\#} \# \quad +^{\#} \# \quad , \# \square = +^{\#} \quad , \# \quad +^{\#} \quad +^{\#} \# \quad , \%$
93. $+^{\exists} \quad , \exists \quad + \quad , \square \quad + \quad , \quad +^{\#} \quad + \quad , \# \quad + \quad , \forall \square \quad + \quad , \quad +^{\#} \quad + \quad , \# \quad \forall$
94. $+^{\#} \quad X^{\#} \quad \& + \quad X \square \quad + \quad X \quad + \quad X \quad \& + \quad X \square \quad + \quad X \quad + \quad X \quad \&$
95. $\exists \% B^{\exists} \quad X^{\exists} \square \square \% B^{\#} \square^{\exists} \quad \square X^{\#} \square^{\exists} \square \square \% B^{\#} \quad X^{\#} \square \square \% B^{\#} \square^{\#} \quad \% B^{\#} X^{\#} \quad \square X^{\#} \square^{\#} \square \quad \square \square \% B^{\#} \quad X^{\#} \square \square \forall \exists B^{\%} \quad \% B^{\#} X^{\#} \quad X^{\%} \square$
96. $\Delta^{\#} \quad \exists \Delta \quad * \quad \# \# \& X^{\#} \square \quad \Delta \quad \exists \quad \Delta \quad \exists \quad \# \# \& X^{\#} \square \quad \Delta \quad \exists \quad \# \quad \forall \& X^{\#} \quad \square \quad \Delta \quad \exists \quad \forall \& X \quad \Delta \quad \exists \quad \forall \& X$

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97. $B^{\#} \ni B * \forall \% \% X^{\#} \square B \exists B \exists \forall \% \% X^{\#} \square B \exists^{\#} \forall \# X^{\#}$
 $\square B \exists \forall \# X B \exists$
 $\forall \# X$

98. $B^{\#} \# B * X^{\#} \forall \square B^{\#} \# B \forall * X^{\#} \square B \forall B \forall * X^{\#}$
 $\square B \forall^{\#} \exists X^{\#} \square B \forall \exists X B \forall$
 $\exists X$

99. $+ ,^{\#} \exists + , \forall ! \square \chi + , \& \delta \chi + , \# \delta \square + , \& + , \#$

100. $\# + ,^{\#} \& + , \exists \square \chi \# + , \forall \delta \chi + , \exists \delta \square \# + \# , \forall + , \exists$

101. $B^{\exists} (B^{\exists}) \square B^{\exists}) B^{\exists} \forall \square B \# B^{\#} \# B \% B \forall B^{\#} B \forall$

102. $B^{\exists} \forall \exists B^{\%} \exists \ni B^{\#} \square B^{\#} B^{\%} \forall \exists B^{\#} \exists \ni \square B^{\#} B^{\#} * B^{\#} \% \square B^{\#} B \exists B \exists B \# B$
 $\#$

103. $B^{\%} B^{\#} \forall \square B^{\%} \# B^{\#} \forall B^{\#}$ 104. $B^{\%} \exists B^{\#} \% \square B^{\%} \% B^{\#} \% B^{\#}$
 $\square \square B^{\#} \forall \square \square B^{\#} \forall \square B^{\#}$ $\square \square B^{\#} \# \square \square B^{\#} \# \square B^{\#}$
 $\square \square B^{\#} \forall \square^{\#} B^{\#}$ $\square \square B^{\#} \# \square^{\#} B^{\#}$
 $\square \square B^{\#} \forall B \square \square B^{\#} \forall$ $\square \square B^{\#} \# B \square \square B^{\#} \# B \square$
 $B \square$

105. $B^{\%} (B^{\#} \forall \ni \square B^{\%}) B^{\#} \forall \ni B^{\#}$ 106. $X^{\%} \# X^{\#} * \square X^{\%} \ni X^{\#} * \% X^{\#}$
 $\square \square B^{\#} \% \square \square B^{\#} \% \square$ $\square \square X^{\#} \exists \square \square X^{\#} \exists \square \% X^{\#}$
 $B^{\#}$ $\square \square X^{\#} \exists \square^{\#} \# X^{\#}$
 $\square \square B^{\#} \% \square^{\#} B^{\#}$ $\square \square X^{\#} \exists \# X \square \square X^{\#} \exists$
 $\square \square B^{\#} \% B \square \square B^{\#} \%$ $\# X \square$
 $B \square$

107. $\% + \% \forall \exists + \# \square \% + \% \% + \# \forall + \# \square \square \# + \# \forall \square \square \# + \# \forall \square + \# \square \square \# + \# \forall \square^{\#}$
 $+ \#$
 $\square \square \# + \# \forall + \square \square \# + \# \forall + \square$

108. $B^{\%} \# \& \ni B^{\#} \square B^{\%} \forall ! B^{\#} \# \& \% B^{\#} \square \square B^{\#} \& \square \square B^{\#} \& \square \% B^{\#}$
 $\square \square B^{\#} \& \square^{\#} \# B^{\#} \square \square B^{\#} \& \# B \square \square B^{\#} \&$
 $\# B \square$

109. $Z \square \frac{\%}{1} < \exists 1 < \exists$
 $\exists \forall \exists \#$
 $\% \exists \#$
 $\square \frac{1}{1} \square < \exists \# < \exists \square$
 $\exists \exists \#$
 $\frac{\%}{\#}$

110. $0 \square \forall \% \% \forall \ni >^{\#}$
 $\square \forall \ni * >^{\#} \square$
 $\square \forall \ni \exists > \exists >$

□ 1 <v <# □ <v <v <# <# □

111-114. Answers may vary.

115. ∃B # □ # □ ∃B - # □ □ # □ ∃B
v □

#

117. B# #B % □ # #B % □
□ B

□ # □ v B# B # □
#

SECTION 0.5

116. &B ∃ □ □ & □ &B ∃ □ □ & □ B ∃ □

& & &

118. ∃B# #B & □ # #B - & □
∃ □ ∃B

∃ ∃ ∃
□ ∃ □ B# #B & □
∃ ∃

SECTION 0.5

119. $+ , \square + \square \pm - \cdot \square \square + \square \forall -$
 $, \square$

120. $+ , \square , \square^+ \cdot \square \square , \square^+ \forall \square$

121. $B \ B \ B^{\forall \in \#} \square B^{\forall \in \#} \square B^{\forall \in \#} \square B^{\forall \in \#} \square$
 $B^{\forall \in \#} \square B^{\forall \in \#} \square$
 $\square B^{\forall \in \#} \square B^{\forall \in \#} \square$
 $\forall \square$

122. $B^{\exists \in \#} \ B^{\forall \in \#} \square B^{\forall \in \#} \square B^{\exists \in \#} \forall \in \#$
 $B^{\forall \in \#} \square B^{\forall \in \#} \square$
 $\square B^{\forall \in \#} \ B \ \forall$

123. $\#B \ \cup \#X \square \cup \# \ \#B \ \cup \#X$
 $\square \cup \# \ \cup \# \square$
 $\square \cup \# \square \cup \#B \ X \square$

124. $\cup \exists + \ \exists , \square \cup \exists - \ \cup \exists \pm \ \exists -$
 $\square \cup \exists - \ - \cup \exists \square$
 $\square \cup \exists \square + \ \cup \exists , \square$

125. $+ , \exists \in \# \ + \exists \in \# , \square + , \square \ \frac{+ , \exists \in \#}{+} \ \frac{+ , \exists \in \#}{+} \square$
 $\square + , \square , \forall \in \# \ + \forall \in \# \square$

126. $+ , \# \ , \square , \forall \ \frac{\pm}{\square} \# \ \frac{\pm}{\square} \square$
 $\square , \forall \square + , \exists \ , \# \square$

127. $B^{\#} \ B \ \ni \ BX \ \#X \square \ B \ \exists \ B \ \# \ XB \ \# \square \ B \ \# \ B \ \exists \ X$

128. $\#B^{\#} \ \&B \ \# \ BX \ \#X \square \ \#B \ \forall \ B \ \# \ XB \ \# \square \ B \ \# \ \#B \ \forall \ X$

129. $+^{\%} \ \# + \exists \ \ +^{\#} \ + \ \forall \square +^{\#} \square +^{\#} \ \# + \ \forall \square \ + \ \forall \square +^{\#} \ + \ \forall \ + \ \forall \ \forall \ + \ \forall$
 $\square \ + \ \forall \square +^{\#} \ + \ \forall \ \forall \square$
 $\square \ + \ \forall \square +^{\exists} \ +^{\#}$
 $\forall \square$

130. $+^{\%} \ \ + \exists \ \ \# +^{\#} \ \ + \ \forall \square +^{\#} \square +^{\#} \ \ + \ \# \square \ \ + \ \forall \square +^{\#} \ \ \# \ + \ \forall \ \forall \ + \ \forall$
 $\square \ \ + \ \forall \square +^{\#} \ \ + \ \# \ \forall \square$
 $\square \ \ + \ \forall \square +^{\exists} \ \ \# +^{\#}$
 $\forall \square$

131. The number 1 is neither prime nor composite.

132. $\chi \# , \exists \supseteq \circ \theta \textcircled{R} \mid \mid \nabla \theta \pi$
 $\# \ \exists$

133. $B^{\exists} B^{\#} \ \square \ B \& \ \square \ B^{\#}!$
 $\% \ \%$

134. $\frac{\pm \exists \pm}{\pm} \ \# \ \% \ \frac{*}{\pm} \) \ \pm \ \forall (\ \square \ +^{\#}$

$+ \# + \exists \exists$

+& ∃

+∇&

135. $\exists B^{\exists} \exists !$
 $\exists B \# B \% \square \square \forall$

SECTION 6.5 $\overline{\# ! B \&} \square \cup \% \overline{B \%} \cup \overline{\& B} \square \# B^{\#} \cup \overline{\& B}$

137. $\cup \# ! B \quad \cup \overline{\forall \# \& B} \square \cup \% \cup \overline{\& B} \quad \cup \# \& \cup \overline{\& B} \square \# \cup \overline{\& B} \quad \& \cup \overline{\& B} \square \exists \cup \overline{\& B}$

138. $\frac{\exists \square \exists \square \cup *}{\exists \square} \quad \frac{\exists \cup *}{\exists \square} \quad \frac{\exists \cup *}{\exists \square} \quad \frac{\exists \cup *}{\exists \square} \quad -$

$\forall \& 7 \% > \#$

$\forall \& 7$
 $\% > \#$
 7%

SECTION 0.6

23. $\frac{\#B}{B\# \%} \frac{\%}{B} \frac{\#B \#}{B \# B \#} \frac{\#}{B \#}$

24. $\frac{B\# \forall \exists}{B\#)B \forall \exists} \frac{B \% B \%}{B \% B \%} \frac{B \%}{B \%}$

25. $\frac{\% B \#}{B\# \& B} \frac{\# B \# B}{B \exists B \#} \frac{B \#}{B \exists}$

26. $\frac{\# \& B \#}{B\# \forall ! B \# \&} \frac{\& B \& B}{B \& B \&}$
 $\frac{\& B}{B \&} \frac{B \&}{B \&}$

27. $\frac{\exists B \exists B \# \forall \# B}{\% B \exists \% B \#} \frac{B \exists B \# B \forall \#}{\exists B B \% B \#} \frac{B \# B \exists \exists B \%}{B \# B \exists \# B \forall} \frac{\exists B \%}{\# B \forall}$

SECTION 0.6

28. $\frac{\exists B^{\%} \& B \exists B}{\#B^{\exists} \quad (B^{\#} \quad \forall \& B \quad B \# B^{\#} \quad (B \quad B \# B \quad \exists B \quad \& \quad \# B \quad \exists B \quad \& \quad \forall \&)} \quad \# \quad \frac{B \# B \quad \# \& B \quad \exists}{B \# B \quad \exists B \quad \# \quad B \# B \quad \exists B \quad \#} \quad \frac{B \# B \quad \exists B \quad \#}{B \# B \quad \exists B \quad \#}$
29. $\frac{B \quad) \quad \exists}{B^{\#} \quad + B} \quad \frac{B \quad \# \quad \exists}{\# B \quad \# + \quad B B \quad + \quad \# B \quad +} \quad \frac{B \quad \# \quad B \quad \# B \quad \%}{B \quad + \quad B \quad \#} \quad \frac{B \# B \quad \%}{B \quad +}$
30. $\frac{B X \quad \# B \quad \exists X}{B^{\exists} \quad \#(\quad \exists^{\exists}} \quad \frac{B X \quad \# \quad \exists X \quad \#}{B^{\exists} \quad \exists B \quad \# \quad \exists B \quad *}$
31. $\frac{B^{\#} \quad \forall}{B \quad B} \quad \frac{B \quad \#}{B^{\#} \quad \# B} \quad \frac{B \quad \forall \quad B \quad \forall}{\forall} \quad \frac{B}{B} \quad \frac{B^{\#} B \quad \forall}{\forall B \quad \forall} \quad B \quad \forall$
32. $\frac{X^{\#} \quad \# X}{X \quad X} \quad \frac{\forall}{X^{\#} \quad X \quad \#} \quad \frac{X \quad \#}{X} \quad \frac{X \quad \forall \quad X \quad \forall}{X \quad \# \quad X \quad \forall} \quad \frac{X \quad \#}{X} \quad \frac{X \quad \forall}{X}$
33. $\frac{\exists B^{\#} \quad (B \quad \#}{B^{\#} \quad \# B} \quad \frac{B^{\#} B}{\exists B^{\#} \quad B} \quad \frac{\exists B \quad \forall \quad B \quad \#}{B B} \quad \frac{B B \quad \forall}{B \exists B \quad \forall} \quad \frac{B \quad \forall}{B}$
34. $\frac{B B \quad \#}{\# B^{\#} \quad \exists B} \quad \frac{\# B \quad B \quad \exists}{B^{\#} \quad \forall} \quad \frac{B B \quad \forall}{B \# B \quad \exists} \quad \frac{\# B \quad \exists \quad B \quad \forall}{B \quad \forall \quad B \quad \forall}$
35. $\frac{B^{\#} B \quad B \quad \#}{B \quad \forall \quad B \quad \#} \quad \frac{B B \quad \forall}{B \quad \forall} \quad \frac{B \quad \forall \quad B \quad \forall}{B \quad \#} \quad \frac{B B \quad \forall}{B \quad \#} \quad B \quad \#$
36. $\frac{B^{\#} \quad \& B \quad \exists}{B^{\#} \quad \exists B \quad * \quad B^{\#} \quad B \quad \exists \quad B \quad \exists \quad B \quad \# \quad B \quad \# \quad B \quad \exists \quad B \quad \#} \quad \frac{B \quad \#}{B \quad \#} \quad \frac{B \quad \# \quad B \quad \exists}{B \quad \#} \quad \frac{B \quad \#}{B \quad \#}$
37. $\frac{\# B^{\#} \quad \exists \#}{) \quad f} \quad \frac{B \quad \# \quad \forall \quad \exists}{\#} \quad \frac{\# B \quad \# \#}{) \quad \#} \quad \frac{\#}{B^{\#} \quad \forall \quad \exists} \quad \frac{\# B \quad \# \quad \exists}{) \quad B^{\#} \quad \forall \quad \exists} \quad \frac{\#}{B^{\#} \quad \forall \quad \exists} \quad \#$
38. $\frac{B^{\#} \quad B \quad \exists}{B^{\#} \quad \exists B \quad * \quad B^{\#} \quad * \quad B^{\#} \quad \exists B \quad * \quad B^{\#}} \quad \frac{B \quad \%}{B^{\#} \quad \%} \quad \frac{B^{\#} \quad B \quad \exists}{B^{\#} \quad * \quad B \quad \exists \quad B \quad \# \quad B \quad \exists \quad B \quad \exists} \quad \frac{B \quad \exists \quad B \quad \# \quad B \quad \exists \quad B \quad \exists}{B \quad \exists \quad B \quad \#}$

39. $\frac{\Delta^\# \Delta \#!}{f} \frac{\Delta \#&}{\Delta \Delta \#!} \square \frac{\Delta \&}{\Delta \# \Delta \#} \square \frac{\Delta \&}{\Delta \& \Delta \%} \square \frac{\Delta \&}{\Delta \&}$

$\frac{\Delta^\#}{\#&} \% \Delta \& \Delta^\#$ **SECTION 0.6** $\frac{\% \Delta^\# \Delta \# \Delta \# \Delta \# \Delta \& \Delta \&}{\Delta \%}$

$\square \frac{\Delta \# \Delta \#}{\Delta \# \Delta \#}$

SECTION 0.6

$$40. \frac{+B \ , B \ +}{+ \# \ #+ , \ #} \cdot f \frac{B \ \# \ \forall}{B \ \# \ \#B} \square \frac{+B \ , B \ +}{+ \# \ #+ , \ #} \cdot \frac{B \ \# \#B}{B \ \#} \frac{\forall}{\forall}$$

$$\square \frac{B \ + \ , \ \forall \ + \ ,}{+ \ , \ + \ ,} \square \frac{B \ \forall \ B \ \forall}{B \ \forall \ B \ \forall}$$

$$\square \frac{+ \ , \ B \ \forall}{+ \ , \ + \ ,} \square \frac{B \ \forall \ B \ \forall}{B \ \forall \ B \ \forall} \square \frac{B \ \forall}{+ \ ,}$$

$$41. \frac{\exists B \ \& \ B \ \#}{B \ \exists \ \#B \ \#} f \frac{\exists B \ \forall \ \exists \ B \ \&}{\#B \ \exists \ \& \ B \ \#} \square \frac{\exists B \ \& \ B \ \#}{B \ \exists \ \#B \ \#} \square \frac{\# \ \#B \ \& \ B}{\exists B \ \forall \ \exists B \ \&}$$

$$\square \frac{\exists B \ \forall \ B \ \#}{\forall B \ \# \ B \ \#} \frac{B \ \# \ \#B \ \&}{\exists B \ \forall \ \#B \ \&}$$

$$42. \frac{B \ \# \ \forall \ \exists B \ \forall \ \#}{)B \ \# \ \exists B \ \& \ (B \ \#} f \frac{\#B \ B \ \# \ \exists}{\forall \%B \ \& \)B \ \# \ \exists B \ \& \ \#B \ \# \ B \ \exists}$$

$$\square \frac{B \ \# \ \forall \ \exists B \ \forall \ \#}{\forall \%B \ \& \)B \ \# \ \exists B \ \& \ \#B \ \# \ B \ \exists} \square \frac{)B \ \# \ \forall \%B}{\#B \ \forall \ \# \ \#B}$$

$$\square \frac{\#B \ \# \ B \ \exists}{\#B \ \forall \ \# \ \#B}$$

$$43. \frac{B \ \# \ (B \ \forall \ \# \ B \ \# \ \exists B \ \forall \ ! \ B \ \exists \%B \ \# \ B \ B \ \exists)}{B \ \# \ \# \ ! \ \exists B \ \# \ \#B \ \exists \ B \ \# \ B} \square \frac{B \ \exists \ B \ \% \ B \ \& \ B \ \#}{\forall B \ B \ \exists \ B \ \# \ B \ \exists \ B \ \forall \ B \ \& \ B \ \%}$$

$$44. \frac{B \ B \ \# \ \exists}{B \ B \ (\ \exists B \ \forall} \square \frac{B \ B \ \forall \ \#}{B \ B \ (\ \exists B \ \forall} \square \frac{B \ \# \ B \ \#}{B \ \# \ (B \ \exists B \ \exists \ B \ \# \ (B \ \exists B \ \exists} \square \frac{B \ B \ \#}{B \ \# \ \#B \ \exists \ B \ \# \ B \ \#} \square \frac{B \ \# \ \%B}{B \ \# \ \%B \ \exists} \square \frac{B \ \exists \ B \ \forall}{B \ \# \ B \ \forall \ B \ \#} \square \frac{B \ \exists \ B \ \forall}{B \ \exists \ B \ \forall \ B \ \exists}$$

$$45. \frac{B \ \# \ B \ \exists}{\# \ \forall B \ \# \ \& \ !B \ \forall \ \exists \ B \ \exists} \square \frac{\exists B}{\exists B \ \&} \cdot f \frac{B \ \exists B \ \&}{(B \ \exists \ \exists B \ \forall \ ! \ \# \ \forall B \ \# \ \& \ !B \ \forall \ \exists \ B \ \exists \ B \ \#} \square \frac{B \ \# \#B \ \exists \ \exists B}{B \ \exists \ B \ \forall \ \exists B} \square \frac{B \ \# \ B \ \&}{(B \ \# \ B \ \&}$$

$$\square \frac{B \ \exists \ B \ \forall \ \exists B}{(B \ \# \ \exists B)} \square \frac{B \ \&}{B \ \&}$$

SECTION 0.6

+ % + % + % +
 %
 + %
 □ $\frac{\quad}{\quad}$ SECTION 0.6 $\frac{\quad}{\quad}$
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 □ $\frac{\# + \%}{\quad}$ □ $\frac{\# + \#}{\quad}$
 + % + % + % + %

□ >> & > &

SECTION 0.6

SECTION 0.6

61. $\frac{\#}{\forall X} \exists \forall$ $X \frac{\#}{X \forall X \forall} \exists \forall \square$ $\forall X \forall$

$\square \frac{\#}{X \forall X \forall} \exists X \forall X \forall$ $\frac{\forall X \forall}{X \forall X \forall}$

$\forall \square \frac{\#}{X \forall X} \forall \frac{\exists X \# \exists}{X \forall X} \exists$ $\frac{X}{\forall X \forall X \forall}$ \forall

$\square \frac{\exists X \# X \#}{X \forall X} \square \frac{\forall X \# X \forall}{X \forall X} \square$ $\frac{\exists X \#}{X \forall}$

62. $\# \frac{\%}{\forall} \forall$ $\# \frac{\%}{\forall} \square$

$\forall \# \% > \# > \# > \# > \# > \#$

$\square \frac{\# > \# > \#}{\forall > \# > \#} \frac{\%}{\# > \# > \#} \frac{\forall > \#}{> \# > \#}$

$\square \frac{\#}{> \# > \#} \frac{\#}{\# > \# > \#} \frac{\#}{> \# > \#}$

$\square \frac{\# > \# > \#}{> \# > \#} \frac{\# > \# > \#}{\# > \# > \#} \frac{\# > \# > \#}{\# > \# > \#}$

$\square > \# > \# > \# > \# > \# > \#$

63. $\forall \exists$ $\frac{\exists B \#}{\exists} \square$ $\frac{\#}{\forall} \frac{\exists B \#}{\exists}$

$B \# B \# B \# \% B \# B B \# B \#$ $\frac{\exists B \#}{\exists B \#}$ $\frac{\exists B \#}{\exists B \#}$

$\forall B \#$

$\square \frac{B \# B \#}{B \# B \#} \frac{\#}{B \# B \#} \frac{B \# B \#}{B \# B \#}$

$\square \frac{B \#}{B \# B \#} \frac{\exists B \exists}{B \# B \#} \frac{\exists B \#}{B \# B \#}$

$\forall \square \frac{B \#}{B \# B \#} \square \frac{\#}{B \#}$

64. $\underline{B} \ \&$ $\exists \exists B \forall \square \underline{B} \ \&$ $\frac{*B \exists}{\exists}$

$B \exists B \exists B \# * B \exists B B \exists B \exists$ $\frac{*B \exists}{\& B \exists}$ $\frac{*B \exists}{\& B \exists}$

$B B \exists$

$$\begin{array}{l}
\boxed{\frac{B \exists B \exists}{B^\# \exists B} \quad \frac{B \exists B \exists}{\&B} \quad \frac{B \exists B \exists}{*B \exists}} \\
\boxed{\frac{\text{SECTION 0.6}}{B \exists B \exists}} \\
\frac{B^\#(B \forall \#)}{B \exists B \exists} \quad \frac{B \exists B \%}{B \exists B \exists} \quad \frac{B \%}{B \exists B \exists} \\
\boxed{B \exists B \exists} \quad \boxed{B \exists B \exists} \quad \boxed{B \exists B \exists}
\end{array}$$

65. $\frac{\forall}{B \# B \exists} \quad \frac{\forall}{B \# B \exists} \quad \frac{B \exists}{B \# B \exists} \quad \frac{\forall B \exists}{B \# B \exists} \quad \frac{\forall B \#}{B \# B \exists} \quad \frac{B \exists}{B \# B \exists}$

$$\begin{array}{l}
\boxed{\frac{B \exists}{B \# B \exists} \quad \frac{B \#}{B \# B \exists} \quad \frac{B \exists}{\#B \&}} \\
\boxed{\frac{\&}{B \# B \exists} \quad \frac{\&}{\#B} \quad \frac{\&}{\#B B \#}}
\end{array}$$

$$\square \frac{B \# B \forall B \exists B \# B \forall B \exists \quad B \# B \forall B \exists}{B \exists \#B \% B \forall \quad \square \forall \quad ! \quad \square ! \exists} \frac{B \# B \forall B \exists \quad B \# B \forall B \exists}{B \# B \forall B \exists}$$

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70.
$$\frac{\# \quad \#}{B \quad X \quad B} \quad \frac{\# \Delta \quad \# X}{X \quad B \quad \Delta \quad B} \quad \frac{\#}{X \quad B \quad \Delta \quad B} \quad \frac{\# \Delta \quad \# X}{X \quad B \quad \Delta \quad B}$$

$$\frac{\# \Delta \quad \# X}{\# \Delta \quad B} \quad \frac{\# \Delta \quad \# X}{\# X \quad B} \quad \frac{\# \Delta \quad \# X}{\# \Delta \quad \# X}$$

$$\frac{X \quad B \quad \Delta \quad B}{\# \Delta \quad \# B} \quad \frac{X \quad B \quad \Delta \quad B}{\# X \quad \# B} \quad \frac{X \quad B \quad \Delta \quad B}{\# \Delta}$$

$$\frac{X \quad B \quad \Delta \quad B}{\# \Delta \quad \# B} \quad \frac{X \quad B \quad \Delta \quad B}{\# X \quad \# B} \quad \frac{X \quad B \quad \Delta \quad B}{\# \Delta}$$

$$\frac{\# \Delta \quad \# B \quad \# X \quad \# B \quad \# \Delta \quad \# X}{X \quad B \quad \Delta \quad B \quad X \quad B \quad \Delta \quad B} \quad \frac{\# \Delta \quad \# X}{\# \Delta \quad \# X} \quad ! \quad !$$

71.
$$\frac{\exists B \quad \#}{B \quad \#} \quad \frac{\%B \quad \#}{B \quad \#} \quad \frac{\exists B \quad \# \& B \#}{B \quad \# \& B \#}$$

$$\frac{\exists B \quad \#}{B \quad \& B \quad \%} \quad \frac{\%B \#}{B \quad \& B \quad \%} \quad \frac{\exists B \#}{B \quad \& B \quad \%}$$

$$\frac{\exists B \quad \# \quad B \quad \& B \quad \%}{B \quad \& B \quad \%} \quad \frac{\%B \# \# \quad B \quad \% \quad B \quad \%}{B \quad \& B \quad \%} \quad \frac{\exists B \# \& B \& B \&}{B \quad \& B \quad \%} \quad \frac{B \quad \% \quad B \quad \% \quad B \quad \& B \quad \&}{B \quad \& B \quad \%}$$

$$\frac{\exists B \exists \quad \& B \# \quad \&) B \quad \% !}{B \quad \& B \quad \%} \quad \frac{\% B \% \quad \exists \# B \#}{B \quad \& B \quad \%} \quad \frac{\exists \#}{B \quad \& B \quad \%}$$

$$\frac{\exists B \exists \quad \& B \# \quad \&) B \quad \% ! \quad \% B \% \quad \exists \# B \# \quad \exists \#}{B \quad \& B \quad \%} \quad \frac{\exists B \% \quad \forall ! B \# \quad \exists \# \&}{B \quad \& B \quad \%} \quad \frac{\exists \# \&}{B \quad \& B \quad \%}$$

$$\frac{\exists B \exists \quad \& B \# \quad \&) B \quad \% ! \quad \% B \% \quad \exists \# B \# \quad \exists \# \quad \exists B \% \quad \forall ! B \# \quad \exists \# \&}{B \quad \& B \quad \%} \quad \frac{\exists \# \&}{B \quad \& B \quad \%}$$

$$\frac{B \% \quad \exists B \exists \quad \% \exists B \# \quad \&) B \quad \exists * (}{B \quad \& B \quad \%} \quad \frac{\exists B \exists \quad \% \exists B \# \quad \&) B \quad \exists * (}{B \quad \& B \quad \%}$$

72.
$$\frac{\exists B \#}{B \#} \quad \frac{B \%}{\forall ! B} \quad \frac{\forall}{\exists \exists B \#} \quad \frac{\forall \forall \forall B}{\exists \exists B \#} \quad \frac{\exists \% B \quad \forall}{B \quad \%}$$

$$\frac{\exists B \quad \# \quad \exists B \quad \forall}{\exists B \quad \# \quad \exists B \quad \forall} \quad \frac{B \quad \% \quad \% B}{B \quad \% \quad \% B} \quad \frac{\forall \# B \quad \exists \exists B \quad \forall}{\forall \# B \quad \exists \exists B \quad \forall}$$

$$\frac{\% B \quad \forall \quad \# B \quad \exists \quad \exists B \quad \forall \quad \exists B \quad \forall \quad \# B \quad \exists \quad \% B \quad \forall \quad \% B \quad \forall \quad \# B \quad \exists \quad \exists B}{\% B \quad \forall \quad \# B \quad \exists \quad \exists B \quad \forall \quad \exists B \quad \forall \quad \# B \quad \exists \quad \% B \quad \forall \quad \% B \quad \forall \quad \# B \quad \exists \quad \exists B}$$

$$\frac{* B \# \quad \exists B \quad \#}{* B \# \quad \exists B \quad \#} \quad \frac{\% B \# \quad \forall (B \quad \% \quad \exists B \#}{\% B \# \quad \forall (B \quad \% \quad \exists B \#}$$

$$\frac{\% B \quad \forall \quad \# B \quad \exists \quad \exists B}{\% B \quad \forall \quad \# B \quad \exists \quad \exists B} \quad \frac{\forall \quad \% B \quad \forall \quad \# B \quad \exists \quad \exists B \quad \forall \quad \% B \quad \forall \quad \# B \quad \exists \quad \exists B}{\forall \quad \% B \quad \forall \quad \# B \quad \exists \quad \exists B \quad \forall \quad \% B \quad \forall \quad \# B \quad \exists \quad \exists B}$$

$$\frac{\exists x \forall y (P(x,y) \rightarrow Q(x,y))}{\forall x \exists y (P(x,y) \rightarrow Q(x,y))}$$

73. $\frac{\exists x \forall y (P(x,y) \rightarrow Q(x,y))}{\forall x \exists y (P(x,y) \rightarrow Q(x,y))}$

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74. $\frac{\exists x \forall y (P(x,y) \rightarrow Q(x,y))}{\forall x \exists y (P(x,y) \rightarrow Q(x,y))}$

75. $\frac{\exists x \forall y (P(x,y) \rightarrow Q(x,y))}{\forall x \exists y (P(x,y) \rightarrow Q(x,y))}$

76. $\frac{\exists x \forall y (P(x,y) \rightarrow Q(x,y))}{\forall x \exists y (P(x,y) \rightarrow Q(x,y))}$

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$$77. \frac{\frac{B X}{+X B} X}{+} \frac{B}{+} \frac{X}{+} \frac{X}{+} \frac{B}{+} \frac{B}{+} \square \vee$$

$$78. \frac{\frac{B^{\#} \& B \exists}{\# B^{\#} X} \square}{\square} \frac{B^{\#} \& B}{\exists} \frac{B^{\#}}{f} \frac{*}{\square} \frac{\# B \& B}{\exists} \frac{\# B X}{\square}$$

$$\frac{B^{\#} *}{\# B^{\#} X} \quad \# B^{\#} X \quad \# B^{\#} X \quad \# B^{\#} X \quad B^{\#} \quad *$$

$$\frac{B \exists B \#}{\# B^{\#} X} \square \frac{\# B^{\#} X}{B \exists B} \frac{B}{\exists} \frac{\#}{B \exists}$$

$$79. \frac{\frac{B X}{B X} \frac{X}{B X}}{\square} \frac{\frac{B X}{B X} \frac{X}{B X}}{\square} \frac{X B}{B^{\#} X^{\#}} \square$$

$$80. \frac{\frac{B X}{\forall \forall} \frac{B X B X}{\forall \forall}}{\square} \frac{B X}{\square} \frac{\# \#}{\square} \frac{B X}{\square} \frac{\# \#}{\square} \frac{B X^{\#} \#}{\forall \forall X \forall \forall B} \frac{\# \#}{\forall \forall X B}$$

$$81. \frac{\frac{B X}{B X} \frac{X}{B X}}{\square} \frac{\frac{B X}{B X} \frac{X}{B X}}{\square} \frac{X B}{X B} \square$$

$$82. \frac{\frac{B X}{B X} \frac{X}{B X}}{\square} \frac{\frac{B X}{B X} \frac{X}{B X}}{\square} \frac{X B}{X B} \square$$

$$83. \frac{\frac{\exists \pm \#}{-B} \square}{\square} \frac{+, B \square \exists \pm \quad \% \pm \square}{, \quad B} \frac{+, B \square \exists \pm \square \quad +, B \square \% \pm \# \square}{, \quad B} \frac{\# \quad \exists \quad \#}{\exists + B \quad \% \pm \quad , \quad + \exists B \quad \% \pm ,}$$

$$84. \frac{\frac{B}{X'} \vee}{\square} \frac{X^{\#} \square \vee}{\square} \frac{B \square}{\square} \frac{X^{\#} \vee}{\square} \frac{X^{\#} \square B \square}{\square} \frac{X^{\#}}{\square} \frac{B X}{\square} \frac{X X}{\square} \frac{B}{\square} \frac{X}{\square}$$

$$85. \frac{B \vee \exists}{B \square} \frac{B \square B \vee \exists \square}{B \square} \frac{B B \quad B \vee B \square \exists \square}{B \square} \frac{B^{\#} B \exists}{\square} \frac{B \exists B \#}{\square} \frac{B \#}{\square}$$

B & \exists B B & \exists B B B & B \exists B[#] & B B # B \exists B #

86. $\frac{\# \Delta}{\Delta} \square \frac{\Delta \# \Delta}{\Delta} \square \frac{\# \Delta}{\Delta} \#$ SECTION 0.6 BX BX \exists BX \exists BX # #

$\forall \frac{\exists}{\Delta} \Delta \square \forall \exists \square \Delta \forall \Delta \square \exists \square$ $\forall \frac{\exists}{\text{BX}} \text{BX} \square \forall \frac{\exists}{\text{BX}} \text{BX} \square \forall \text{BX} \square \forall \text{BX} \square$

$\square \frac{\# \Delta}{\Delta} \exists$ $\square \frac{\exists \text{BX}}{\text{BX}} \forall$

88. $\frac{B \exists \forall}{\square} \frac{B \square B \exists \forall}{\square} \frac{B^{\#} \exists B \forall}{\square} \frac{B^{\#} \exists B \forall}{\square}$

$\frac{B \exists \forall}{\square} \frac{B \exists \forall}{\square} \frac{B \exists \forall}{\square} \forall B^{\#} \exists B \forall B^{\#} \exists B \forall$

89. $\frac{\exists B}{\square} \frac{B \exists B}{\square} \frac{\exists B^{\#}}{\square}$

$B \forall \frac{\exists}{\square} B \square B \forall \square B^{\#} \forall$

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90.
$$\frac{\#B\#}{\# \%B} \% \square \frac{\&\#B}{\&\square\# \%B} \% \square \frac{\forall!B\#}{\forall! \%B} \# \square \frac{\# \&B\#}{\# \& \#B} \forall! \square \frac{\&B\forall\#}{\& \#B}$$

$$\frac{\&}{B \#} \frac{\&}{B \#} B \forall \square \# \square \frac{B}{B} \# B \forall \square \square B \# B \forall \square \# \square$$

91.
$$\frac{B\#}{\exists B} \frac{B\forall}{B} \square \frac{B\#}{B} \frac{B\forall}{B} \square \frac{B\#}{B} \frac{B\forall}{B} \square \frac{B\#}{B} \frac{B\forall}{B} \square \frac{B\#}{B} \frac{B\forall}{B} \square$$

$$B\# B\forall \quad B\# B\forall \quad \frac{B\#}{B \forall B} \frac{B\#}{B \# \#} B\forall \quad B\forall$$

$$\square \frac{B \forall \exists B \# B}{B\# B \#B \% B\# \exists B \%} \square \frac{\exists B \exists B\#}{\exists B \exists B\#} \square \frac{\%}{\#B} \frac{B\#}{B\#} \exists B \% \square$$

92.
$$\frac{\#B}{\exists B} \frac{\forall}{B} \square \frac{B}{\#B} \exists B \# \square \forall \square \frac{B}{B} \exists B \# \square \#B \square \frac{B}{B} \exists B \# \square \forall \square$$

92.
$$\frac{B\exists}{\exists B} \frac{B\#}{B} \square \frac{B\exists}{B} \frac{B\#}{B} \square \frac{B\exists}{B} \frac{B\#}{B} \square \frac{B\exists}{B} \frac{B\#}{B} \square \frac{B\exists}{B} \frac{B\#}{B} \square$$

$$B\exists B\# \quad B\exists B\# \quad \frac{B\exists}{B \# \#B} \frac{B\exists}{B \exists \forall} B\# \quad B\#$$

$$\square \frac{B \# \exists B \exists B}{\#B\# \%B B \exists} \square \frac{\exists B \exists B\#}{\exists B \exists B\#} \square \frac{\#B\#}{B\#} \frac{\exists B}{\exists B} \square$$

$$\square \frac{\#B\#}{B\#} \frac{\exists B}{\exists B} \square \frac{\#B\#}{B\#} \frac{\exists B}{\exists B} \square$$

93.
$$\frac{\forall}{B\forall} \frac{\forall \square}{\forall \#B} \square \frac{B\forall}{B\forall} \frac{B}{B} \square \frac{B}{B} \forall$$

94.
$$\frac{X}{B\forall X\forall} \frac{\forall}{B\forall X\forall} \square \frac{\forall}{B\forall X\forall} \frac{\forall}{B\forall X\forall} \square \frac{BX\forall \square}{B\forall X\forall} \frac{B}{B} \square$$

95.
$$\frac{\exists B \#}{B \# \forall} \frac{\forall \#B \forall}{\#B \#B\forall} \square \frac{\exists \#}{\#B \#B\forall} \square \frac{B \# B \forall \square \exists \square \#}{B \# B \forall \square \forall \square \#}$$

$$B \# \forall \quad \forall \#B \quad B \# B \forall \square \forall \square \square$$

$$\square \frac{B \# B \forall \square \exists \square}{B\#} \frac{B \# B \forall \square \# \square}{B\forall} \square$$

$$\square \frac{B \forall \exists B \# \#}{B \forall} \square \frac{B \forall}{\exists B \exists \#B \% \&B \forall} \square \frac{\&B \forall}{B \forall} \square$$

96.
$$\frac{\#B B \exists \forall \exists B \#}{B \exists \forall B \#} \square \frac{\#B}{B\exists B\#} \square \frac{B \exists B \# \square \#B \square \exists}{B\exists B\#} \square \frac{B \exists B \#}{B\exists B\#} \square$$

$$B \exists \forall B \# \quad \frac{\#B}{B\exists B\#} \quad \frac{B \exists B \#}{B\exists B\#} \quad \frac{B \exists B \#}{B\exists B\#}$$

B ∃ B # □
∇ □

□ B # #B B ∃ ∃
∇

□ #B# %B ∃B * □ #B# B *

SECTION 0.6

97. $\frac{\nabla}{\nabla} \frac{\square}{\nabla} \frac{5 \nabla 5 \# \nabla}{\nabla \# 5} \frac{\square}{\nabla} \frac{5 \nabla 5 \#}{\nabla \# 5} \frac{\square}{\nabla} \frac{5 \nabla 5 \#}{\nabla \# 5} \frac{5}{\#} \frac{5}{\nabla}$

112. $\forall x \in B, \exists y \in B, x \neq y$ () $\exists x \in B, \forall y \in B, x \neq y$ ()

Chapter 0 Review (page 75) SECTION 0.6

1. natural: $\exists, \exists,)$

2. whole: $!, \exists, \exists,)$

3. integers: $\exists, \exists, !, \exists, \exists,)$

4. rational: $\exists, \exists, !, \forall, \exists, \exists,)$

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- 5. irrational: $1, \cup, \bar{\&}$
- 6. real: $\exists, \exists, !, \forall, \exists_{\neq} 1, \cup \&, \exists, \bar{\&}$
- 7. prime: \exists
- 8. composite: $\exists,)$
- 9. even integers: $\exists, !, \exists,)$
- 10. odd integers: \exists, \exists
- 11. associative property of addition
- 12. commutative property of addition
- 13. associative property of multiplication
- 14. distributive property
- 15. commutative property of multiplication
- 16. commutative property of addition

17. double negative rule

18.
$$\begin{array}{c} \circ \rangle \theta \rangle \theta \rangle \theta \rangle \pi \\ \forall \forall \quad \forall (\quad \forall * \\ \forall \exists \end{array}$$

19.
$$\begin{array}{c} \circ \rangle \theta \rangle \theta \rangle \theta \rangle \pi \\ \exists \quad) \quad \forall ! \quad \forall \# \quad \forall \% \end{array}$$

20. $\exists ! B \square \&$

21. $B \square ! \text{ or } B ! \quad \forall$

$$\begin{array}{c} \circ \theta \angle | | \circ \theta \pi \\ \exists \quad \& \end{array}$$

$$\left\{ \begin{array}{c} | \nabla \theta \theta \textcircled{R} | | \\ \forall \quad ! \end{array} \right.$$

22. $\angle \#, \% \textcircled{\&}$

$$\begin{array}{c} \circ \theta \angle | | \circ \theta \pi \\ \# \quad \% \end{array}$$

23. $\infty, \# \cap \&, \infty$
 $\infty, \#$

$$\left\{ \begin{array}{c} | | | \nabla \theta \pi \\ \# \end{array} \right.$$

$$\begin{array}{c} \&, \infty \\ \hline \begin{array}{c} \circ \theta \angle | | \nabla \theta \pi \\ \& \quad \# \\ \circ \theta \angle | | \nabla \theta \pi \\ \& \quad \# \end{array} \end{array}$$

24. $\infty, \% \cup \textcircled{R} \exists, \infty \nabla$

$$\left\{ \begin{array}{c} | \nabla \theta \theta \textcircled{R} | | \\ \% \quad \exists \end{array} \right.$$

25. Since $\exists \square !, \kappa \& \square \exists$.

26. Since $\# \& ! !, \kappa \# \& \kappa \square \# \& \square \# \&$.

27. Since $\forall x \exists y (x \neq y)$,
 $\exists x \forall y (x \neq y) \rightarrow \exists x \forall y (x = y)$
 $\forall x \exists y (x \neq y) \rightarrow \exists x \forall y (x = y)$

28. Since $\exists x \forall y (x \neq y)$,
 $\exists x \forall y (x \neq y) \rightarrow \exists x \forall y (x = y)$
 $\exists x \forall y (x \neq y) \rightarrow \exists x \forall y (x = y)$

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29. distance $\square \kappa$ & $\kappa \square \kappa \forall \# \kappa \square \forall \#$

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30. $\&+^{\exists} \square \&++++$

31. $\&+^{\#} \square \&+ \&+$

32. $\exists >>> \square \exists >^{\exists}$

33. $\#, \exists, \square \quad \# \exists, \square \quad \exists, \#$

34. $8^{\#}8^{\%} \square 8^{\#} \% \square 8^{\exists}$

35. $:\exists \# \square :\exists \square \# \square :\exists$

36. $B^{\exists}X^{\#} \square B^{\exists} \square B^{\forall\#}X^{\exists}$
 $X^{\#}$
 $\% \quad \%$
 $\%$

37. $\square \begin{matrix} \pm \\ \square \end{matrix} \% \exists \quad \begin{matrix} \pm \\ \square \end{matrix} \% \exists \quad \pm^{\forall\#}$
 $, \# \quad , \# \exists \quad , \exists$

38. $\square 7^{\exists}8^{\exists} \square \square \square 7^{\exists} \square \forall \square \# \square 7^{\exists} \square \quad \forall$
 7^{\exists}

39. $\square \begin{matrix} \# \\ \square \end{matrix} \begin{matrix} \# \\ \square \end{matrix} \exists \quad \square \square \begin{matrix} \# \\ \square \end{matrix} \exists \quad \square \square \quad \text{---} \begin{matrix} \# \\ \square \end{matrix} \exists \quad \exists$
 $\# \quad \# \# \quad \# \# \exists \quad) :^{\exists}$

40. $\begin{matrix} \& \\ \pm \\ + \end{matrix} \square (+\&) \square +^{\exists} \square \quad \forall$
 $+^{\exists}$

41. $\begin{matrix} \# \\ \square \end{matrix} \begin{matrix} \# \\ \square \end{matrix} \exists \quad \begin{matrix} \# \\ \square \end{matrix} \begin{matrix} \# \\ \square \end{matrix} \exists$
 $\text{---} \begin{matrix} \pm \\ \square \end{matrix} \begin{matrix} \# \\ \square \end{matrix} \exists \quad \begin{matrix} \pm \\ \square \end{matrix} \begin{matrix} \# \\ \square \end{matrix} \exists \quad \begin{matrix} \pm \\ \square \end{matrix} \begin{matrix} \# \\ \square \end{matrix} \exists$

$\frac{\exists B^{\#}X^{\#}}{\#} \quad \# \quad \frac{B^{\#}X^{\#}}{\#} \quad B^{\#}X^{\#}X^{\#} \quad X^{\%} \# \quad \neg X^{\exists}$

42. $\square B^{\#}X^{\#} \square \square \square \exists B^{\#}X^{\#} \square \square \square \quad \exists B^{\#} \square \square \square \exists \square \square \square \quad \text{---}$
 $*$

$+^{\exists}, \# \# \quad +^{\exists}, \# \# \quad ++^{\exists} \# \quad +^{\%} \# \quad +^{\exists}$

43. $\square \frac{\square}{+^{\exists}} \square \quad \square \square \frac{\square}{+^{\exists}, \#} \square \quad \square \square, \# \exists \square \quad \square \square, \& \square \quad \square \square, \forall!$

$\frac{\exists B^{\exists}X^{\#}}{\#} \quad \frac{BX^{\exists}}{\#} \quad \text{---} X^{\#} \# \quad \text{---} X^{\%}$

44. $\square BX^{\exists} \square \square \square \exists B^{\exists}X^{\#} \square \square \square \frac{\square}{\exists B^{\#}} \square \quad \square *B^{\%} \quad \text{---}$

$\#7^{\#}8^{\exists} \exists \quad \%7^{\#}8^{\forall} \exists \quad \text{---} \#7^{\#}7^{\#} \exists \quad \text{---} \#7^{\%} \exists \quad)7^{\forall\#}$

45. $\square \frac{\square}{\%7^{\#}8^{\forall}} \square \quad \square \square \frac{\square}{\#7^{\#}8^{\exists}} \square \quad \square \square \quad 8^{\forall}8^{\exists} \square \square \square \quad 8 \square \square \quad 8^{\exists}$

46. $B^{\#} \quad BX^{\#} \quad \exists^{\#}$ $\exists \quad \exists^{\#} \quad * \quad \exists \quad * \quad * \quad \#(\quad * \quad \#(\quad \forall)$
 47. $\exists, (\&! \quad \exists \Rightarrow (\&! \quad \forall!^{\exists}$ 48. $! \Rightarrow !!! \# \exists \quad \# \Rightarrow \exists \quad \forall!^{\%}$

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49. $\% \Rightarrow) \quad \forall!^{\#} \quad \%$
 $\%)!$ 50. $! \Rightarrow \# \& \quad \forall!^{\exists} \quad \% \Rightarrow !!! \# \&$

51. $\frac{\% \& , !!! \quad \exists \& ! , !!!}{\% \Rightarrow \& \quad \forall!^{\%} \quad \exists \Rightarrow \& \quad \forall!^{\&}}$ $\frac{\% \Rightarrow \& \quad \exists \Rightarrow \& \quad \forall!^{\%} \quad \forall}{! \&}$ $\forall \& \Rightarrow (\& \quad \forall!^{\#}$
 $\forall \Rightarrow ! \& \quad \forall!^{\%}$ $\forall \Rightarrow ! \& \quad \forall!^{\%}$ $\forall \Rightarrow ! \& \quad \forall!^{\%}$
 $\forall \Rightarrow ! \& \quad \forall!^{\%}$ $\forall \Rightarrow ! \& \quad \forall!^{\%}$
 $\forall \Rightarrow ! \& \quad \forall!^{\%}$ $\forall \Rightarrow ! \& \quad \forall!^{\%}$

52. $\forall \# \forall \forall \in \# \quad \forall \forall \# \quad \forall \in \#$ 53. $\#(\quad \forall \in \exists \quad \exists \quad \exists \quad \forall \in \exists \quad \exists$
 $\forall \# \& \quad \forall \# \& \quad \forall \# \&$

54. $\exists \# B \& \quad \forall \in \& \quad \forall \in \& \quad \# B$
 $\exists \# \forall \in \& \quad B \&$ 55. $) \forall + \% \quad \forall \in \% \quad \forall \in \% \quad \exists \kappa + \kappa$
 $) \forall \forall \in \% \quad + \%$

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56. $\forall x \exists y (x \neq y) \wedge \forall x \exists y (x < y) \wedge \forall x \exists y (x > y)$
 $\forall x \exists y (x \neq y)$

57. $\exists x \forall y (x \neq y) \wedge \exists x \forall y (x < y) \wedge \exists x \forall y (x > y)$
 \supseteq not a real number

58. $\forall x \exists y (x \neq y) \wedge \forall x \exists y (x < y) \wedge \forall x \exists y (x > y)$
 $\forall x \exists y (x \neq y)$

59. $\exists x \forall y (x \neq y) \wedge \exists x \forall y (x < y) \wedge \exists x \forall y (x > y)$
 $\exists x \forall y (x \neq y)$

60. $\forall x \exists y (x \neq y) \wedge \forall x \exists y (x < y) \wedge \forall x \exists y (x > y)$
 $\forall x \exists y (x \neq y)$

61. $\forall x \exists y (x \neq y) \wedge \forall x \exists y (x < y) \wedge \forall x \exists y (x > y)$
 $\forall x \exists y (x \neq y)$

62. $\exists x \forall y (x \neq y) \wedge \exists x \forall y (x < y) \wedge \exists x \forall y (x > y)$
 $\forall x \exists y (x \neq y)$

63. $\exists x \forall y (x \neq y) \wedge \exists x \forall y (x < y) \wedge \exists x \forall y (x > y)$
 $\forall x \exists y (x \neq y)$

64. $\forall x \exists y (x \neq y) \wedge \forall x \exists y (x < y) \wedge \forall x \exists y (x > y)$
 $\forall x \exists y (x \neq y)$

65. $\exists x \forall y (x \neq y) \wedge \exists x \forall y (x < y) \wedge \exists x \forall y (x > y)$
 $\forall x \exists y (x \neq y)$

66. $\forall x \exists y (x \neq y) \wedge \forall x \exists y (x < y) \wedge \forall x \exists y (x > y)$
 $\forall x \exists y (x \neq y)$

67. $\exists x \forall y (x \neq y) \wedge \exists x \forall y (x < y) \wedge \exists x \forall y (x > y)$
 $\forall x \exists y (x \neq y)$

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$$103. \frac{\# \cup \bar{A}}{\cup \bar{A}} \frac{\# \cup \bar{A}}{\cup \bar{A}} \frac{\# \cup \bar{A}}{\cup \bar{A}} \frac{\# \cup \bar{A}}{\cup \bar{A}} \frac{\# \cup \bar{A}}{\cup \bar{A}} \frac{\# \cup \bar{A}}{\cup \bar{A}} \frac{\# \cup \bar{A}}{\cup \bar{A}} \frac{\# \cup \bar{A}}{\cup \bar{A}}$$

$$104. \frac{\infty \# \infty \infty}{\cup \bar{E} \cup \#} \frac{\infty \# \infty \infty}{\cup \bar{E} \cup \#} \frac{\infty \# \infty \infty}{\cup \bar{E} \cup \#} \frac{\infty \# \infty \infty}{\cup \bar{E} \cup \#} \frac{\infty \# \infty \infty}{\cup \bar{E} \cup \#} \frac{\infty \# \infty \infty}{\cup \bar{E} \cup \#} \frac{\infty \# \infty \infty}{\cup \bar{E} \cup \#} \frac{\infty \# \infty \infty}{\cup \bar{E} \cup \#}$$

$$105. \frac{\#B}{\cup B} \frac{\#B}{\cup B} \frac{\cup B}{\cup B} \frac{\#B \cup \bar{B}}{\cup B} \frac{\#B \cup \bar{B}}{\cup B} \frac{\#B \cup \bar{B}}{\cup B} \frac{\#B \cup \bar{B}}{\cup B} \frac{\#B \cup \bar{B}}{\cup B} \frac{\#B \cup \bar{B}}{\cup B}$$

$$106. \frac{\cup \bar{B} \cup X}{\cup \bar{B} \cup X} \frac{\cup \bar{B} \cup X}{\cup \bar{B} \cup X} \frac{\cup \bar{B} \cup X}{\cup \bar{B} \cup X} \frac{\cup \bar{B} \cup X}{\cup \bar{B} \cup X} \frac{\cup \bar{B} \cup X}{\cup \bar{B} \cup X} \frac{\cup \bar{B} \cup X}{\cup \bar{B} \cup X} \frac{\cup \bar{B} \cup X}{\cup \bar{B} \cup X} \frac{\cup \bar{B} \cup X}{\cup \bar{B} \cup X}$$

$$107. \frac{\cup \bar{B} \#}{\cup \bar{B} \#} \frac{\cup \bar{B} \#}{\cup \bar{B} \#} \frac{\cup \bar{B} \#}{\cup \bar{B} \#} \frac{\cup \bar{B} \#}{\cup \bar{B} \#} \frac{\cup \bar{B} \#}{\cup \bar{B} \#} \frac{\cup \bar{B} \#}{\cup \bar{B} \#} \frac{\cup \bar{B} \#}{\cup \bar{B} \#} \frac{\cup \bar{B} \#}{\cup \bar{B} \#}$$

$$108. \frac{\forall \cup +}{\forall \cup +} \frac{\forall \cup +}{\forall \cup +} \frac{\forall \cup +}{\forall \cup +} \frac{\forall \cup +}{\forall \cup +} \frac{\forall \cup +}{\forall \cup +} \frac{\forall \cup +}{\forall \cup +} \frac{\forall \cup +}{\forall \cup +} \frac{\forall \cup +}{\forall \cup +}$$

$$109. \frac{\exists B^* X}{\exists B^* X} \frac{X}{\exists B^* X}$$

$$110. \frac{\% \# \exists +}{\% \# \exists +} \frac{\% \# \exists +}{\% \# \exists +} \frac{\% \# \exists +}{\% \# \exists +} \frac{\% \# \exists +}{\% \# \exists +} \frac{\% \# \exists +}{\% \# \exists +} \frac{\% \# \exists +}{\% \# \exists +} \frac{\% \# \exists +}{\% \# \exists +} \frac{\% \# \exists +}{\% \# \exists +}$$

$$111. \frac{B^{\#} \#B}{\#B \exists \#B^{\exists} (B^{\#})B \exists} \frac{\#B^{\exists} \exists B^{\#}}{\%B^{\#}})B$$

$$112. \frac{B^{\exists} \#B \exists B^{\exists} B^{\exists} \forall}{\forall B^{\&} !B^{\%} B^{\exists} \exists B^{\#} \#B} \frac{B^{\&}}{\#B^{\exists}} B^{\exists} \exists B^{\#} \#B$$

$$\frac{\%B^{\#} \exists B}{\#B} \exists$$

$$\frac{\#B^{\exists} \#B}{\exists B^{\#}} \exists$$

$$\frac{\#B \exists}{\dagger}$$

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$$\frac{\exists B^{\#} \exists}{\exists}$$

113. $\exists >^{\exists} \quad \exists > \square \exists > >^{\#} \quad \forall \square \exists > > \quad \forall > \forall$

114. $\& <^{\exists} \quad \& \square \& <^{\exists} \quad \forall \square \& <^{\exists} \quad \forall^{\exists} \square \& < \forall <^{\#} < \forall$

115. $\exists B^{\#} (B \ \% \ \square \ \exists B) \ \#B \ \exists$

116. $\exists +^{\#} \quad +B \ \exists + \quad B \ \square + \exists + \quad B \ \forall \exists + \quad B \ \square \ \exists + \quad B + \ \forall$

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117. $\exists B \forall \# \& \square \# B \exists \& \exists \square \# B \& \square \# B \# B \& \& \# \square \square \# B \& \% B \# \forall ! B \# \&$

118. $\exists B \# ! B \forall \exists \square \# \exists B \# \forall ! B) \square \# \exists B \# B \%$

119. $B \# \exists B * > \# \square B \exists B \exists > \# \square B \exists \# > \# \square B \exists > B \exists >$

120. $\exists B \# \forall \& B \square \exists B \# \& B \forall \supseteq \text{prime}$

121. $\Delta \exists \exists \% \exists \square \# \Delta \exists (\exists \square \# \Delta (\square \# \Delta \# \Delta ((\# \square \square \# \Delta (\% \Delta \# \forall \% \Delta \% * \# \&$

122. $\forall \forall \%, \% *, \# \square \% *, \# \forall \%, \forall \square (, \forall (, \forall \square (, \forall \#$

123. $\forall \# \forall \Delta \# \% \% \Delta \square \forall \# \forall \Delta \# \% \Delta \% \square \forall \forall \Delta \# \forall \forall \Delta \# \square \forall \forall \Delta \# \#$

124. $\exists \% X \exists \forall , ! ! \square) X \exists \forall \# \& \square) \square \# X \exists \& \exists \square \square) \# X \& \% X \# \forall ! X \# \&$

125. $\# B X \% \Delta B A X \# \Delta A \square \# B X \# \Delta A X \# \Delta \square X \# \Delta \# B A$

126. $B \# B \% \forall \square B \# B \% \forall B \% \square \square B \% \forall \square \square B \% \forall \square B \%$

$\square \square B \% \forall \square \#$

$\square B \# \square \#$

$\square \square B \% \forall B \# \square \square B \% \forall B \# \square$

$\square \square B \% \# B \# \forall B \# \square \square B \% B \# \forall \square$

$\square \square \square B \# \forall \square \square B \# \forall \square B \# \square \square B \% B \# \forall \square$

$\square \square B \# \forall B \square \square B \# \forall B \square \square B \% B \# \forall \square$

127. $\frac{\# B}{B \%} \frac{B \#}{B \# B \#} \frac{\forall \square}{B \#}$ 128. $\frac{+ \#}{+ \# \exists + *} \frac{*}{+ \exists + \exists} \frac{+ \exists}{+ \exists}$

129. $\frac{B \# \% B}{B \#} \frac{\% \# B \& B}{B \#} \exists \square \frac{B \# B \#}{B \#} \frac{B \# B \exists}{B \#} \square B \# B \exists$

130. $\frac{\# X \# \forall \forall X \forall \&}{X \# \exists X)} \frac{\# X \# X}{X \# X}) \frac{\# X \& X \exists}{X \% X \#} \frac{X \% X \# \# X}{X \exists X} \frac{\# X}{X \#} \&$

131. $\frac{\# \# > \exists}{\exists \# (> \% \exists \#} f \frac{\forall ! > \forall \&}{> \% \exists \# (> \% \forall ! >} \square \frac{\# \# > \exists}{\exists \# (> \% \forall ! >} \square \frac{\exists \# > \%}{\# > \exists > \forall \square \exists > \%} > \frac{\# > \exists > \forall \square \exists > \%}{\forall \square} >$

∃ > % > ∇ & # > ∃ &

132.

$\frac{:\# (\:\forall\#}{:\exists \):\# \%:$
 f
 $\frac{\# \cdot *}{_ _}$
 \square
 $\frac{:\# (\:\forall\#}{:\exists \):\# \%:$
 $\frac{:\# \cdot \# \cdot \exists \cdot \%}{_ _}$
 $\frac{\#}{_ _}$

~~CHAPTER 0 REVIEW~~

$\frac{:\# (\:\forall\#}{:\exists \):\# \%:$
 $\frac{:\# \cdot \# \cdot \exists \cdot \%}{_ _}$
 $\frac{\#}{_ _}$

\square
 $\frac{:\# (\:\forall\#}{:\exists \):\# \%:$
 $\frac{:\# \cdot \# \cdot \exists \cdot \%}{_ _}$
 $\frac{\#}{_ _}$

B ∇ B # B # B ∇ B # B #
 □ $\frac{B}{B \nabla} \frac{B \varepsilon}{B \#}$
 □ $\frac{B \nabla B \# B \varepsilon B \nabla}{B \nabla B \# B \nabla}$
 □ $\frac{B \# \# B \varepsilon B \# (B \nabla)}{B \nabla B \# B \nabla}$ &B ε
 □ $\frac{B \nabla B \# B \nabla B \#}{B \nabla B \#}$ □ B ∇ B #

CHAPTER 0 REVIEW

139.
$$\frac{\exists B \vee \& B \exists \# B}{B \quad B^\# \quad B \vee \quad B^\# B \vee \quad B^\# B \vee \quad B^\# B \vee}$$

$$\frac{\exists B^\exists \quad \exists B^\# \quad \exists B \quad \& B^\exists \quad \& B^\# \quad \vee \& B \quad \vee \&}{B^\# B \vee \quad B^\# B \vee \quad B^\# B \vee} \quad B^\exists$$

$$\frac{B^\exists \quad B^\# \quad \vee \# B \quad \vee \&}{B^\# B \vee}$$

140.
$$\frac{\exists B \quad B^\# \% B \quad \exists \quad B^\# B \quad \exists \quad \exists B \quad B \exists \quad B \vee \quad B \exists \quad B \#}{B \vee \quad B^\# \quad \exists B \quad \# \quad B^\# \% \quad B \quad B \vee \quad B \quad \# \quad B \quad \# \quad B \quad \#}$$

$$\frac{\exists B \quad B \quad \exists \quad B \quad \exists}{B \vee \quad B \quad \# \quad B \quad \#}$$

141.
$$\frac{\& B \quad \& B \quad \exists B^\# \quad \& B \quad)}{\# \quad \# \quad) \quad \# \quad \exists B^\# \quad \exists B}$$

142.
$$\frac{\exists B \quad \exists B \quad \exists B \quad \exists B \quad \exists B \quad \exists B}{X \quad X \quad X \quad X \quad X \quad X}$$

143.
$$\frac{\vee \quad \vee \quad BX \vee \quad \vee \quad BX \vee \quad \vee \quad BX \vee \quad \vee \quad X \quad B}{B \quad X \quad BX \quad B \quad BX \quad B \quad X \quad BX \quad B \quad X}$$

144.
$$\frac{\vee \quad \vee \quad \vee \quad \vee \quad BX \vee \quad \vee \quad BX \vee \quad \vee \quad BX \vee \quad \vee \quad X \quad B}{X \vee \quad B \vee \quad \vee \quad \vee \quad BX \vee \quad \vee \quad BX \vee \quad \vee \quad BX \vee \quad \vee \quad B \quad X}$$

Chapter 0 Test (page 82)

1. odd integers: $(, \vee, \exists$
2. prime numbers: \exists
3. commutative property of addition
4. distributive property
5. $\% ! B \square \# \supseteq \circ \theta \angle | | \odot \theta \pi$
6. $\angle \infty, \exists \vee \cup \otimes \exists, \infty \vee \supseteq \{ | \nabla \theta \theta \otimes | | \}$

7. Since $\forall x (P(x) \rightarrow Q(x))$, $\forall x (Q(x) \rightarrow P(x))$

8. If $\forall x (P(x) \rightarrow Q(x))$, then $\exists x (P(x) \wedge \neg Q(x))$. Then $\neg \exists x (P(x) \wedge \neg Q(x))$.

CHAPTER 0 REVIEW

9. distance $\leq \sqrt{2} \cdot \text{width}$

10. distance $\leq \sqrt{2} \cdot \text{width}$

11. $B^{\forall} B^{\exists} B^{\forall} \leq B^{\exists} B^{\forall} B^{\exists}$

12. $\frac{\leq \exists}{< \exists} \leq \frac{\leq \exists}{< \exists} \leq \frac{\leq \exists}{< \exists}$

13. $\frac{\forall \# \#}{++} \leq \frac{\forall \# \#}{++} \leq \frac{\forall \# \#}{++}$

14. $\frac{\exists \# \#}{B} \leq \frac{\exists \# \#}{B} \leq \frac{\exists \# \#}{B}$

15. $\forall x (P(x) \rightarrow Q(x)) \rightarrow \exists x (P(x) \wedge Q(x))$

16. $\exists x (P(x) \wedge Q(x)) \rightarrow \forall x (P(x) \rightarrow Q(x))$

$$\begin{array}{l}
 B \quad \exists \overline{\exists B^{\#} \quad B} \quad \# \exists \\
 \quad \exists B^{\#} \quad \forall B \\
 \quad \quad \forall *B \quad \# \exists \\
 \quad \quad \forall *B \quad \& (\\
 \quad \quad \quad \exists \\
 \quad \quad \quad \%
 \end{array}$$

$$\begin{array}{l}
 \#B \quad \forall \overline{\#B^{\exists} \quad \exists B^{\#} \quad !B} \quad \forall \\
 \quad \#B^{\exists} \quad B^{\#} \\
 \quad \quad \%B^{\#} \quad !B \\
 \quad \quad \%B^{\#} \quad \#B \\
 \quad \quad \quad \#B \quad \forall \\
 \quad \quad \quad \#B \quad \forall \\
 \quad \quad \quad \quad \downarrow
 \end{array}$$

35. $\exists B \exists X \square \exists B \#X$

37. $\forall ! >^{\#} \quad \forall * > A \quad \exists A^{\#} \square \& > \#A \# >$
 $\exists A$

36. $B^{\#} \quad \forall ! ! \square B^{\#} \quad \forall !^{\#} \square \quad B \quad \forall ! \quad B \quad \forall !$

38. $\exists +^{\exists} \quad \exists \% \square \exists \square +^{\exists} \quad \# \forall \exists \square$
 $\square \exists + \quad \exists \square +^{\#} \quad \exists + \quad \exists \square$

$\frac{B \ \$ \ B^\# \ \bullet}{B \ \$ \ \infty \ \frac{\&B}{B \ \% \ B \ \%}}$	$B \ " \ B \ \% \ B \ \frac{B^\# \ \$B \ \% \ B}{\ " \ \infty \ - \ \#}$
<p>B \$ Á ! B % Á ! B % Á ! B Á \$ B Á % B Á % B Á \$, B Á %, B Á %</p>	<p>SECTION 1.1 B " Á ! B % Á ! B Á ! B Á " B Á % B Á ", B Á %, B Á !</p>

SECTION 1.1

17. $\#B \ \& \ \in \ "&$
 $\#B \ \& \ \& \ \in \ "& \ \&$
 $\#B \ \in \ "!$
 $\#B \ \in \ "!$
 $\#B \ \in \ \&\#$

conditional equation

18. $\$B \ \# \ \in \ B \)$
 $\$B \ B \ \# \ \in \ B \ B \)$
 $\#B \ \# \ \in \)$
 $\#B \ \# \ \# \ \in \) \ \#$
 $\#B \ \in \ \#$

$\frac{_}{\#} \ \in \ \frac{_}{\#}$
 $B \ \in \ \$$
conditional equation

19. $\#8 \ \# \ \& \ \in \ \#8$
 $\#8 \ \% \ \& \ \in \ \#8$
 $\#8 \ " \ \in \ \#8$
 $\#8 \ \#8 \ " \ \in \ \#8 \ \#8$
contradiction

20. $\$7 \ \# \ \in \ \#7 \ \$ \ 7$
 $\$7 \ \# \ \in \ \#7 \ \# \ 7$
 $\$7 \ \# \ \in \ \$7 \ \#$

identity

21. $\frac{B}{\#} \ \in \ ($
 $\frac{BA}{\#} \ \in \ ($
 $\# \ \# \ \in \ \# \ ($
 $B \ \in \ (\ \in \ "%$
 $B \ \in \ (\ \in \ "% \ ($
 $B \ \in \ ($

conditional equation

22. $\frac{B}{\#} \ \in \ (\ \in \ "%$
 $\frac{B}{\#} \ \in \ (\ \in \ "% \ ($
 $\frac{B}{\#} \ \in \ \#"$
 $\# \ \# \ \in \ \# \ \#"$
 $B \ \in \ \% \ \#$

conditional equation

23. $\# + \ " \ \in \ \$ + \ \# +$
 $\# + \ \# \ \in \ \$ + \ \# +$
 $\# + \ \# \ \in \ \# + \ \# +$
 $\# + \ \# + \ \# \ \in \ \# + \ \# + \ \# +$
 $\# \ \# \ \in \ \#$

contradiction

24. $B^\# \ \in \ B \ \% \ B \ \% \ " \ \#$
 $B^\# \ \in \ B^\# \ " \ \# \ " \ \#$
 $B^\# \ \in \ B^\#$

identity

25. $\$B \ \$ \ \in \ \frac{\#B \ ")}{\#}$
 $\$B \ * \ \in \ \frac{\#B \ ")}{\#}$
 $\frac{\#B \ ")}{\#}$

$\# \$B \ * \ \in \ \# \ \#$
 $\#B \ " \ \in \ \#B \ " \)$
identity

26. $B B \ \# \ \in \ B \ " \ \#$
 $B^\# \ \#B \ \in \ B \ " \ B \ "$
 $B^\# \ \#B \ \in \ B^\# \ \#B \ "$
 $B^\# \ B^\# \ \#B \ \in \ B^\# \ B^\# \ \#B \ "$

$\#B \ \in \ \#B \ "$
 $\#B \ \#B \ \in \ \#B \ \#B \ "$
 $! \ \# \ "$
contradiction

SECTION 1.1

27.
$$\frac{\$}{\$} \alpha "$$

$$, \$ \dagger \frac{\$}{\$} \alpha , \$ "$$

$$\$ \alpha , \$$$

$$\$ \$ \alpha , \$ \$$$

$$' \alpha ,$$
 conditional equation

28.
$$B^\#)B "& \alpha B \$ B \&$$

$$B^\#)B "& \alpha B^\# \#B "&$$

$$B^\# B^\#)B "& \alpha B^\# B^\# \#B "&$$

$$)B "& \alpha \#B "&$$

$$)B)B "& \alpha \#B)B "&$$

$$"& \alpha " !B "&$$

$$"& "& \alpha " !B "& "&$$

$$\$! \alpha " !B$$

$$\frac{\$!}{" !} \alpha \frac{" !B}{" !}$$

$$\$ \alpha B$$
 conditional equation

29.
$$\#B^\# \&B \$ \alpha \#B " B \$$$

$$\#B^\# \&B \$ \alpha \#B^\# \&B \$$$
 identity

30.
$$\#B^\# \&B \$ \alpha \#B \square B \frac{"*}{\#} \square$$

$$\#B^\# \&B \$ \alpha \#B^\# "*"B$$

$$\#B^\# \#B^\# \&B \$ \alpha \#B^\# \#B^\# "*"B$$

$$\&B \$ \alpha "*"B$$

$$\&B \&B \$ \alpha "*"B \&B$$

$$\$ \alpha "%B$$

$$\frac{\$}{"%} \alpha \frac{"%}{"%}$$

$$\frac{\$}{"%} \alpha B$$
 conditional equation

31.
$$\#B (\alpha " ! B$$

$$\$B (\alpha " !$$

$$\$B \alpha \$$$

$$B \alpha "$$

32.
$$*+ \$ \alpha "& \$+$$

$$' + \$ \alpha "&$$

$$' + \alpha "$$

$$+ \alpha \$$$

33.
$$\&B \# \alpha \#B \%$$

$$\&B " ! \alpha \#B)$$

$$\$B " ! \alpha)$$

$$\$B \alpha ")$$

$$B \alpha ' "$$

34.
$$\& < \% \alpha \& < \%$$

$$\& < \# ! \alpha \& < \# !$$

$$" ! < \# ! \alpha \# !$$

$$" ! < \alpha \% !$$

$$< \alpha \%$$

35.
$$(\#B \& 'B) \alpha ($$

$$"%B \$\& 'B \%) \alpha ($$

$$)B "$ \alpha ($$

$$)B \alpha \# !$$

$$B \alpha \# ! \alpha \frac{\&}{\#}$$

$$) \alpha \frac{\&}{\#}$$

36.
$$'B \& \% B \# \alpha "$$

$$'B \$! \%B) \alpha "$$

$$\#B \$) \alpha "$$

$$\#B \alpha \$ ($$

$$B \alpha \$ ($$

$$) \alpha \#$$

SECTION 1.1

37. $\frac{\&}{\$} D) \alpha ($
 $\$ \frac{\&}{\$} D \alpha " \&$
 $\$ \dagger \frac{\&}{\$} D \alpha \$ " \&$
 $\& D \alpha \% \&$
 $D \alpha *$

38. $\frac{\%}{\$} C "# \alpha \%$
 $\$ \frac{\%}{\$} C \alpha " \cdot$
 $\$ \dagger \frac{\%}{\$} C \alpha \$ " \cdot$
 $\% C \alpha \%)$
 $C \alpha "#$

39. $\frac{D}{\&} \# \alpha \%$
 $\& \frac{D}{\&} \alpha \#$
 $\& \dagger \frac{D}{\&} \alpha \& \#$
 $D \alpha " !$

40. $\frac{\$}{:} : \alpha \%$
 $(\square \frac{\$}{:} : \square \alpha (\%$
 $\$: (: \alpha \#)$
 $\% : \alpha \#)$
 $: \alpha ($

41. $\frac{\$B}{\&} \# \alpha \#B \<$
 $\$ \dagger \frac{\$B}{\&} \# \alpha \$ \square \#B \<$
 $\$B \# \alpha \cdot B ($
 $\$B \# \alpha ($
 $\$B \alpha *$
 $B \alpha \$$

42. $\<_B \& \alpha B \frac{"}{\&}$
 $\# \<_B \& \alpha \#_B \frac{"}{\&}$
 $\# \square \<_B \& \alpha \# \square B \frac{"}{\&}$
 $(B " ! \alpha \#B " \&$
 $\&B " ! \alpha " \&$
 $\&B \alpha \&$
 $B \alpha "$

43. $\frac{\$B}{\&} " \alpha "$
 $\# ! \dagger \frac{\$B}{\&} " \alpha \# ! \dagger \frac{"}{\#}$
 $\$B " \alpha " !$
 $\$B \alpha *$
 $B \alpha \$$

44. $\#B \< \frac{B}{\&} \alpha \%B \$$
 $\cdot \cdot \cdot$
 $\cdot \square \#B \< \frac{B}{\&} \alpha \cdot \dagger \frac{\%B}{\&} \$$
 $"\#B (B \alpha \%B \$$
 $"\$B (\alpha \%B \$$
 $*B (\alpha \$$
 $*B \alpha " !$
 $B \alpha \frac{"}{*}$

45. $\frac{\$ B}{\&} \frac{B}{\&} \< \alpha \%B "$
 $\cdot \square \frac{\$ B}{\&} \frac{B}{\&} \< \alpha \cdot \%B "$
 $\# \$ B \$ B (\alpha \# \%B \cdot$
 $\cdot \#B \$B \# " \alpha \# \%B \cdot$
 $\&B \# (\alpha \# \%B \cdot$
 $" *B \# (\alpha \cdot$
 $" *B \alpha \# "$
 $\frac{\#}{\&}$
 $B \alpha " *$

46. $\# \#B " \alpha \frac{\$B}{\&} \frac{\$ \% B}{\&}$
 $\# \square \# \#B " \frac{\$B}{\&} \bullet \alpha \# \dagger \frac{\$ \% B}{\&}$
 $\% \#B " \$B \alpha \$ \% B$
 $)B \% \$B \alpha "# \B
 $\&B \% \alpha "# \$B$
 $)B \% \alpha "#$
 $)B \alpha " \cdot$
 $B \alpha \#$

SECTION 1.1

47.
$$\frac{\$}{\#} \$B \# " !B \% \alpha !$$

$$\frac{\$}{\#} \$B \# " !B \% \bullet \alpha \# !$$

$$\$ \$B \# # !B) \alpha !$$

$$*B \cdot \# !B) \alpha !$$

$$""B \% \alpha !$$

$$""B \alpha ""\%$$

$$B \alpha \frac{""\%}{""}$$

48.
$$\frac{++ \$ \&}{7} \alpha \frac{+ " \#}{7}$$

$$\frac{++ \$ \&}{7} \bullet \alpha 7 \square \frac{+ " \#}{7} \square$$

$$++ \$ \& \alpha + " + "$$

$$+\# \$+ \& \alpha +\# \#+ "$$

$$\$+ \& \alpha \#+ "$$

$$\& \alpha + "$$

$$\% \alpha +$$

49.
$$\frac{C \# \#}{\$} \alpha C \# \frac{C\#}{\$}$$

$$\frac{C \# \#}{\$} \alpha \$ \square C \# \frac{C\#}{\$ \square}$$

$$C \# \# \alpha \$C \cdot C\#$$

$$C\# \%C \% \alpha C\# \$C \cdot$$

$$\%C \% \alpha \$C \cdot$$

$$C \% \alpha \cdot$$

$$C \alpha \#$$

50.
$$> " > " \alpha > \# > \$ \%$$

$$> \# " \alpha > \# > \cdot \%$$

$$" \alpha > \#$$

$$> " \alpha \#$$

$$> \alpha "$$

51.
$$BB \# \alpha B " \# "$$

$$B\# \#B \alpha B " B " "$$

$$B\# \#B \alpha B\# \#B " "$$

$$B\# \#B \alpha B\# \#B$$

$$! \alpha ! \hat{E} \text{ identity}$$

52.
$$B \# B \$ \alpha B \$ B \%$$

$$B\# \&B \cdot \alpha B\# (B " \#$$

$$\&B \cdot \alpha (B " \#$$

$$" \#B \cdot \alpha " \#$$

$$" \#B \alpha \cdot$$

$$B \alpha \frac{\cdot}{\#} \alpha "$$

$$\#$$

53.
$$\# = \# = \$ \alpha = = \& \# \frac{""}{\#} = \square$$

$$\# = \% = \# \cdot = * \alpha = \# \& = "(\# =$$

$$= \#) = "$ \alpha = \# (= "($$

$$) = "$ \alpha (= "($$

$$= \alpha \%$$

SECTION 1.1

54.
$$\begin{aligned} & \frac{\$}{\#} \frac{-}{\#} \frac{\%}{\#} \\ & \frac{B}{\#} \frac{\#}{\#} \frac{B}{\#} \\ & \frac{\$}{\#} \frac{-}{\#} \frac{\%}{\#} \\ & \#B \square \frac{-}{\#} \frac{\%}{\#} \#B \dagger \\ & \frac{B}{\#} \frac{\#}{\#} \frac{B}{\#} \\ & \cdot B \alpha) \\ & B \alpha \# \end{aligned}$$

55.
$$\begin{aligned} & \frac{\#}{\#} \frac{-}{\#} \frac{-}{\#} \\ & \frac{B}{\#} \frac{-}{\#} \frac{\$}{\#} \frac{B}{\#} \frac{-}{\#} \\ & \$ B \frac{-}{\#} \frac{\#}{\#} \frac{-}{\#} \frac{\alpha}{\#} \$ B \frac{-}{\#} \frac{\dagger}{\#} \frac{-}{\#} \\ & \frac{B}{\#} \frac{-}{\#} \frac{\$}{\#} \frac{B}{\#} \frac{-}{\#} \\ & \cdot \frac{-}{\#} \frac{B}{\#} \frac{-}{\#} \frac{\alpha}{\#} \frac{\$}{\#} \\ & \cdot \frac{B}{\#} \frac{-}{\#} \frac{\alpha}{\#} \frac{\$}{\#} \\ & \frac{B}{\#} \frac{-}{\#} \frac{(\alpha}{\#} \frac{\$}{\#} \\ & \frac{B}{\#} \frac{-}{\#} \frac{\alpha}{\#} \frac{\%}{\#} \end{aligned}$$

56.
$$\begin{aligned} & \frac{\$}{\#} \frac{-}{\#} \frac{\alpha}{\#} \frac{\$}{\#} \\ & \frac{B}{\#} \frac{\#}{\#} \frac{B}{\#} \frac{B}{\#} \frac{\#}{\#} \\ & \frac{\$}{\#} \frac{-}{\#} \frac{-}{\#} \frac{\alpha}{\#} \frac{B}{\#} \frac{B}{\#} \frac{\dagger}{\#} \\ & \frac{B}{\#} \frac{\#}{\#} \frac{B}{\#} \frac{B}{\#} \\ & \$B \frac{-}{\#} \frac{B}{\#} \frac{\#}{\#} \frac{\alpha}{\#} \$B \\ & \$B \frac{B}{\#} \frac{\#}{\#} \frac{\alpha}{\#} \$B \\ & \%B \frac{\#}{\#} \frac{\alpha}{\#} \$B \\ & B \alpha \# \end{aligned}$$

57.
$$\begin{aligned} & \frac{*}{\#} \frac{>}{\#} \frac{\cdot}{\#} \frac{\alpha}{\#} \frac{(\#}{\#} \\ & >> \$ > \$ \\ & >> \$ \frac{*}{\#} \frac{>}{\#} \frac{\cdot}{\#} \frac{\alpha}{\#} >> \$ \dagger \frac{(\#}{\#} \\ & >> \$ >> \$ \\ & * > \frac{\cdot}{\#} \frac{\alpha}{\#} \frac{(\#}{\#} \\ & \# > \frac{\cdot}{\#} \frac{\alpha}{\#} ! \\ & \# > \frac{\alpha}{\#} \frac{\cdot}{\#} \\ & > \frac{\alpha}{\#} \frac{\$}{\#} \end{aligned}$$

The answer does not check. \hat{E} no solution

The answer does not check. \hat{E} no solution

58.
$$\begin{aligned} & \frac{B}{\#} \frac{\#}{\#} \frac{\#B}{\#} \frac{-}{\#} \frac{\alpha}{\#} \frac{\$B\#}{\#} \\ & \frac{\$B}{\#} \frac{\&}{\#} \frac{\square B}{\#} \frac{\#}{\#} \frac{\#B}{\#} \frac{-}{\#} \frac{\alpha}{\#} \frac{\$B}{\#} \frac{\&}{\#} \frac{\dagger}{\#} \frac{\$B}{\#} \frac{\&}{\#} \\ & B \$B \frac{\&}{\#} \frac{\#}{\#} \frac{\#B}{\#} \frac{-}{\#} \frac{\alpha}{\#} \frac{\$B}{\#} \\ & \$B \frac{\#}{\#} \frac{\&}{\#} \frac{\%B}{\#} \frac{\#}{\#} \frac{\alpha}{\#} \frac{\$B}{\#} \\ & B \frac{\#}{\#} \frac{\alpha}{\#} ! \\ & B \alpha \# \end{aligned}$$

59.
$$\begin{aligned} & \frac{\#}{+} \frac{(\#}{+} \frac{\alpha}{\#} \frac{\%}{+} \frac{\$}{+} \frac{\#}{+} \\ & + (\# + \# + \$ \dagger \frac{\#}{+} \frac{(\#}{+} \frac{\#}{+} \frac{\alpha}{\#} \frac{(\#}{+} \frac{\#}{+} + \$ \dagger \frac{\%}{+} \frac{\$}{+} \frac{\#}{+} \\ & \# + \$ \alpha \% + (\# \\ & \# + \cdot \alpha \% + \#) \\ & \# + \alpha \$ \\ & + \alpha "(\end{aligned}$$

SECTION 1.1

60.

$$\frac{\frac{\#}{8} \frac{''}{\#}}{\frac{\#}{8} \frac{''}{\#}} \propto \frac{\frac{''}{8} \frac{''}{\#}}{\frac{''}{8} \frac{''}{\#}}$$

$$8 \# 8 '' \square \frac{\#}{8} \frac{''}{\#} \square \propto 8 \# 8 '' \dagger \frac{''}{8} \frac{''}{\#}$$

$$\begin{aligned} \# 8 '' 8 \# \propto '' \\ \# 8 \# 8 \# \propto '' \\ \$ 8 \propto '' \\ 8 \propto \frac{''}{\$} \end{aligned}$$

61.

$$\frac{\frac{\#B}{\&B} \frac{\$}{\cdot}}{\frac{\#B}{\&B} \frac{\$}{\cdot}} \frac{\frac{\$B}{\#} \frac{\#}{\cdot}}{\frac{\$B}{\#} \frac{\#}{\cdot}} \propto \frac{\frac{\&B}{\#} \frac{\#B}{\%}}{\frac{\&B}{\#} \frac{\#B}{\%}}$$

B # #B \$ B # \$B # \propto B \$ &B # \epsilon multiply by common denominator \phi

$$\begin{aligned} \#B^{\#} B \cdot \$B^{\#} \%B \% \propto \&B^{\#} " \$B \cdot \\ \&B^{\#} \$B " ! \propto \&B^{\#} " \$B \cdot \\ \$B " ! \propto " \$B \cdot \\ " ! B \propto \% \\ B \propto \frac{\%}{"!} \propto \frac{\#}{\&} \end{aligned}$$

62.

$$\frac{\frac{\$B}{B^{\#}} \frac{B}{\&B}}{\frac{\$B}{B} \frac{B}{\&B}} \frac{\frac{B}{\#} \frac{\#}{\cdot}}{\frac{B}{\#} \frac{\#}{\cdot}} \frac{\frac{B}{\&B}}{\frac{B}{\&B}}$$

$$\frac{\$}{B} \frac{\#}{\&} \propto \frac{B}{B} \frac{\#}{\&B}$$

\$ B & # B '' \propto B # \epsilon multiply by common denominator \phi

$$\begin{aligned} \$B "& \#B \# \propto B \# \\ B "$ \propto B \# \end{aligned}$$

"\$ \hat{A} \# \hat{E} no solution

SECTION 1.1

63.
$$\frac{\$B \&}{B^{\$})} \frac{\$}{B^{\#} \%} \frac{\# \$B \#}{B \# B^{\#} \#B \%}$$

$$\frac{\$B \&}{B \# B^{\#} \#B \%} \frac{\$}{B \# B \#} \frac{\# \$B \#}{B \# B^{\#} \#B \%}$$

B # \$B & □ B# #B % □ \$ α # B # \$B # ε multiply by common denominator φ

$$\frac{\$B^{\#} B \ " !}{\ "B^{\#} (B \ # \ \alpha \ "B^{\#})B \)} \frac{\ " \&B \ \alpha \ \ " !}{B \ \alpha \ \ \ " ! \ \ \alpha \ \ \$}$$

64.
$$\frac{\ "}{8 \)} \frac{\ \$8 \ \%}{\&8^{\#} \% \#8} \frac{\ "}{\ " \ \alpha \ \&8 \ \#}$$

$$\frac{\ "}{8 \)} \frac{\ \$8 \ \%}{\&8 \ \# \ 8 \)} \frac{\ "}{\ \alpha \ \&8 \ \#}$$

&8 # " \$8 % α 8) ε multiply by common denominator φ

$$\frac{\&8 \ \# \ \$8 \ \% \ \alpha \ 8 \)}{\&8 \ \# \ \$8 \ \% \ \alpha \ 8 \)}$$

$$\frac{\#8 \ \cdot \ \alpha \ 8 \)}{8 \ \alpha \ \#}$$

65.
$$\frac{\ "}{\ " \ \# \ 8} \frac{\ \# \ \$8 \ \ "}{(8^{\#} \ (\%8 \ \$\$ \ \alpha \ (8 \ \$)}$$

$$\frac{\ "}{8 \ \ " \ \#} \frac{\ \# \ \$8 \ \ "}{(8^{\#} \ (\%8 \ \$\$ \ \alpha \ (8 \ \$)}$$

$$\frac{\ "}{8 \ \ " \ \#} \frac{\ \# \ \$8 \ \ "}{(8 \ \$ \ 8 \ \ " \ \#} \frac{\ "}{\ \alpha \ (8 \ \$}$$

(8 \$ '8 # α 8 "" " ε multiply by common denominator φ

$$\frac{(8 \ \$ \ '8 \ \# \ \alpha \ 8 \ \ " \)}{(8 \ \$ \ '8 \ \# \ \alpha \ 8 \ \ " \)}$$

$$\frac{8 \ \& \ \alpha \ 8 \ \ " \)}{\#8 \ \alpha \ \cdot}$$

$$\frac{8 \ \alpha \ \$}{8 \ \alpha \ \$}$$

66.
$$\frac{\ \%}{\ + \ \# \ \ " \$ + \ \%)} \frac{\ \#}{\ + \ \# \ \ ") +} \frac{\ \ "}{\ \alpha \ \ + \ \# \ \ + \ \cdot}$$

$$\frac{\ \%}{\ + \ \ " \ \cdot \ +} \frac{\ \$}{\ + \ \ " \ \cdot \ + \ \#} \frac{\ \alpha \ \ \$ \ + \ \#}{\ + \ \# \ \# \ + \ \$ \ \alpha \ \ " \ + \ \ " \cdot}$$

ε multiply by common denominator φ

$$\frac{\ \% \ + \) \ \# \ + \ \cdot \ \alpha \ \ + \ \ " \cdot}{\# \ + \ \ " \% \ \alpha \ \ + \ \ " \cdot}$$

$$\frac{\ + \ \alpha \ \ \#}{\ + \ \alpha \ \ \#}$$

SECTION 1.1

$$67. \frac{\frac{\&}{C} \cdot \frac{\#}{C} \cdot \frac{\cdot}{\#} \cdot \frac{\cdot}{\#}}{\frac{\&}{C} \cdot \frac{\#}{C} \cdot \frac{\cdot}{\#} \cdot \frac{\cdot}{\#}} = \frac{\frac{\&}{C} \cdot \frac{\#}{C} \cdot \frac{\cdot}{\#} \cdot \frac{\cdot}{\#}}{\frac{\&}{C} \cdot \frac{\#}{C} \cdot \frac{\cdot}{\#} \cdot \frac{\cdot}{\#}}$$

& C # ⋅ % C % " εmultiply by common denominatorφ
 &C " ! ⋅ %C " ⋅ " "
 &C " ! ⋅ %C " & "
 C ⋅ &

$$68. \frac{\frac{\cdot}{\#} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$}}{\frac{\cdot}{\#} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$}} = \frac{\frac{\cdot}{\#} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$}}{\frac{\cdot}{\#} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$}}$$

\$ + " " + \$ ⋅ " " εmultiply by common denominatorφ
 \$+ \$ + \$ ⋅ "
 %+ ⋅ ⋅ "
 %+ ⋅ ("
 ("
 + ⋅ ⋅ "

$$69. \frac{\frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$}}{\frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$}} = \frac{\frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$}}{\frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$}}$$

C # C C # C ⋅) εmultiply by common denominatorφ
 #C C# #C C# ⋅)
 %C ⋅)

C ⋅ # Ê The solution does not check, so the equation has no solution.

$$70. \frac{\frac{\cdot}{\#} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$}}{\frac{\cdot}{\#} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$}} = \frac{\frac{\cdot}{\#} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$}}{\frac{\cdot}{\#} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$} \cdot \frac{\cdot}{\$}}$$

+ # #+ \$ + # \$+ # ⋅ + \$ &+ # εmultiply by common denominatorφ
 #+ # + ⋅ \$+ # %+ % ⋅ &+ # "\$+ ⋅
 &+ # \$+ " ! ⋅ &+ # "\$+ ⋅

$$\begin{aligned} & \text{"!} + \alpha \% \\ & + \alpha \frac{\%}{\text{"!}} \quad \alpha \frac{\#}{\&} \\ & \text{SECTION 1.1} \end{aligned}$$

SECTION 1.1

71.
$$\frac{\frac{+}{+ \#} \cdot \frac{+}{+ \#}}{\frac{+}{+ \#}} \cong \frac{\frac{\$+ \#}{+ \#} \cdot \frac{\%+ \%}{+ \#}}{\frac{\$+ \#}{+ \#} + \frac{\%+ \%}{+ \#}}$$

$$\frac{+ \# +}{\# \square} \cdot \frac{+}{+ \#} \cong \frac{+ \# + \# \dagger \square}{+ \# + \#} \cdot \frac{\$+ \#}{+ \# + \#}$$

$$\frac{++ \# + \# + \# \cong \$+ \#}{+ \# \# + \square + \# \%+ \% \square \cong \$+ \#}$$

$$\frac{+ \# \# + + \# \%+ \% \cong \$+ \#}{\# + \% \cong \$+ \#}$$

$$+ \cong \#$$

72.
$$\frac{\frac{B}{B} \cdot \frac{\$}{\$}}{\frac{B}{B} \cdot \frac{\$}{\$}} \cong \frac{\frac{\#B}{\$} \cdot \frac{B}{\$}}{\frac{\#B}{\$} \cdot \frac{B}{\$}}$$

B \$ B " B \$ B # B \$ #B " multiply by common denominator
 B# %B \$ B# B ' B# &B \$
 #B# \$B \$ B# &B \$
)B !
 B !)

73.
$$\frac{\frac{\$}{\#} \cdot \frac{\#}{\#}}{\frac{\$}{\#}} \cong \frac{\frac{\$}{\#} \cdot \frac{\#}{\#}}{\frac{\$}{\#}}$$

74.
$$+B \cdot \frac{\$}{\$} \cong \frac{\$}{\$}$$

75.
$$\frac{\frac{\$}{\#} \cdot \frac{\#}{\#}}{\frac{\$}{\#}} \cong \frac{\frac{\$}{\#} \cdot \frac{\#}{\#}}{\frac{\$}{\#}}$$

76.
$$\frac{\frac{\$}{\$} \cdot \frac{\#}{\#}}{\frac{\$}{\$}} \cong \frac{\frac{\$}{\$} \cdot \frac{\#}{\#}}{\frac{\$}{\$}}$$

$$\frac{\$Z}{1 \#} \cong \frac{\$}{1 \#}$$

$$\frac{\$Z}{1 \#} \cong \frac{\$}{1 \#}$$

$$\frac{\$Z}{1 \#} \cong \frac{\$}{1 \#}$$

$$\frac{\$Z}{1 \#} \cong \frac{\$}{1 \#}$$

77.
$$\frac{\frac{\$}{\$} \cdot \frac{\#}{\#}}{\frac{\$}{\$}} \cong \frac{\frac{\$}{\$} \cdot \frac{\#}{\#}}{\frac{\$}{\$}}$$

$$\frac{\$Z}{1 \#} \cong \frac{\$}{1 \#}$$

$$\frac{\$Z}{1 \#} \cong \frac{\$}{1 \#}$$

$$\frac{\$Z}{1 \#} \cong \frac{\$}{1 \#}$$

$$\frac{\$Z}{1 \#} \cong \frac{\$}{1 \#}$$

78.
$$\frac{\frac{\$}{\$} \cdot \frac{\#}{\#}}{\frac{\$}{\$}} \cong \frac{\frac{\$}{\$} \cdot \frac{\#}{\#}}{\frac{\$}{\$}}$$

$$D5 \cong \frac{B}{5} \dagger 5$$

$$D5 \cong B \cdot$$

$$D5 \cong B$$

$$\cdot \cong B \quad D5$$

SECTION 1.1

79.
$$\begin{aligned} T &\propto P^{\frac{3}{8}} \\ T_8 &P \propto \frac{0}{0} \\ 0T_8 &P \propto 0 \uparrow \frac{3}{0} \\ 0T_8 &P \propto =3 \\ \frac{0T_8 P}{0} &\propto \frac{3}{3} \\ \frac{0T_8 P}{3} &\propto = \end{aligned}$$

80.
$$\begin{aligned} T &\propto P^{\frac{3}{8}} \\ T_8 &P \propto \frac{0}{0} \\ 0T_8 &P \propto 0 \uparrow \frac{3}{0} \\ 0T_8 &P \propto =3 \\ \frac{0T_8 P}{0} &\propto \frac{=3}{=} \\ T_8 P &T_8 P \\ 0 &\propto \frac{=3}{T_8 P} \end{aligned}$$

81.
$$\begin{aligned} J &\propto \frac{7Q1}{\#} \\ J &< \propto \frac{7Q1}{\#} \uparrow < \\ J &< \propto \frac{7Q1}{\#} \\ J &\leq \propto \frac{7Q1}{Q1} \\ Q1 &\propto \frac{7Q1}{Q1} \\ J &\leq \propto 7 \\ Q1 & \end{aligned}$$

82.
$$\begin{aligned} 0 &: \uparrow \frac{0}{0} \propto 0 : \square \frac{0}{0} \\ &: \propto 0 ; 0 : \\ &: \propto 0 ; : \\ &: \propto \frac{0}{0} \\ &: \propto 0 \end{aligned}$$

83.
$$\begin{aligned} B &C \propto " \\ + &\frac{C}{\square} \propto " \frac{B}{\square} \\ , \uparrow &\frac{C}{\square} \propto , \square " \frac{B}{\square} \\ , &C \propto , \square " \frac{B}{\square} \\ &+ \end{aligned}$$

84.
$$\begin{aligned} B &C \propto " \\ +, \square &\frac{B}{\square} \frac{C}{\square} \propto +, \uparrow " \\ , B &+ C \propto +, \\ , B &\propto +, +C \\ , B &\propto +, C \\ \frac{B}{\square} &\propto \frac{+C}{\square} \\ , C &\propto , C \\ , \frac{B}{\square} &\propto + \end{aligned}$$

85.
$$\begin{aligned} &< <" <\# \\ <<" <\# \uparrow &\frac{0}{0} \propto <<" <\# \square \frac{0}{0} \\ &<\# \end{aligned}$$

86.
$$\begin{aligned} &< <" <\# \\ <<" <\# \uparrow &\frac{0}{0} \propto <<" <\# \square \frac{0}{0} \\ &<\# \end{aligned}$$

87.
$$6 \propto + 8 \text{ " .}$$

88.
$$6 \propto + 8 \text{ " .}$$

$$6x + 8 = 8x$$

$$\frac{6x + 8}{8} = \frac{8x}{8}$$

$$\frac{6x + 8}{8} = x$$

$$\frac{6x + 8}{8} = x$$

SECTION 1.3

$$\frac{6x + 8}{8} = x$$

SECTION 1.1

$$97. \frac{(\# \& B)^{\$} \cdot \# \hat{I} \$}{\square \)C' \square} \propto \square \ \# \& B^{\$} \square$$

$$\frac{\)A^{\hat{I} \$} \)A^{\hat{I} \$}}{\square} \propto \frac{\% C \%}{\square}$$

$$\# \& \# \hat{I} \$ B^{\$} \# \hat{I} \$ \quad \# \& B^{\#}$$

$$98. \frac{\# C C^{\$} \cdot \# \hat{I} \$}{\square \ \#, !!! B' \square} \propto \frac{\# C^{\hat{I} \$} \ \$ \ \# \hat{I} \$}{\square \ \#, !!! \hat{I} \$ B' \ \# \hat{I} \$}$$

$$\frac{\)A^{\hat{I} \$} \)A^{\hat{I} \$}}{\square} \propto \frac{\$ C}{\square} \propto \frac{\$ C}{\square}$$

$$\# \hat{I} B^{\#} \quad \# \hat{I} B^{\#}$$

$$99. \cup \# \& C^{\#} \propto \supset \& C^{\#} \propto \& \& C \& \kappa$$

$$100. \cup \# \& C^{\#} \propto \supset \& C^{\#} \propto \& C^{\#}$$

$$101. \supset \frac{\% \ +, \% \ \#}{\square} \propto \square \ \frac{\% \ +, \$ \ \%}{\square} \propto \frac{\% \ +, \$ \ \%}{\square}$$

$$D) \quad D^{\#} \quad D^{\#}$$

$$102. \supset \frac{B^{\#} C^{\&}}{\square} \propto \square \ \frac{B^{\#} C^{\&}}{\square} \propto \frac{B^{\#} C^{\&}}{\square}$$

$$D^{\#} \quad D^{\#} \quad D^{\#}$$

Exercises 1.2 (page "!"")

1. add
2. perimeter
3. amount
4. break point
5. rate, time
6. !\& \\$! \propto "\&
7. Let B \propto the score on the first exam. Then B \& \propto the score on the midterm, and B "\\$ \propto the score on the final.

$$\frac{\text{Sum of scores}}{\$} \propto *!$$

$$\frac{B \ B \ \& \ B \ \#\$}{\$} \propto *!$$

$$\frac{\$ B \ \#)}{\$} \propto *!$$

$$\$ B \ \#) \propto \#(!$$

$$\$ B \propto \# \& \#$$

$$B \propto)\%$$
 His score on the first exam was 84.
8. Let B \propto the score on the first exam. Then her scores on the following tests were B \\$, B ' and B *.

$$\frac{\text{Sum of scores}}{\%} \propto \# \cdot \# \&$$

$$\frac{B \ B \ \$ \ B \ ' \ B \ *}{\%} \propto \# \cdot \# \&$$

$$\frac{\% B \ \#)}{\%} \propto \# \cdot \# \&$$

$$\% B \ \#) \propto \#(!$$

$$\% B \propto \# \cdot !$$

$$B \propto \# \&$$
 Her score on the first exam was 65%.

$$9. \text{ Let } B \propto \text{ the program development score.}$$

$$\frac{\text{Sum of scores}}{\%} \propto) \cdot$$

$$\frac{) \# \ *! \ B \ ()}{\%} \propto) \cdot$$

$$\frac{B \ \# \& !}{\%}$$

$$\frac{\text{Sum of scores}}{\%} \propto \# \cdot \# \&$$

$$\frac{B \ B \ \$ \ B \ ' \ B \ *}{\%} \propto \# \cdot \# \&$$

$$\frac{\% B \ \#)}{\%} \propto \# \cdot \# \&$$

$$\% B \ \#) \propto \#(!$$

$$\% B \propto \# \cdot !$$

$$B \propto \# \&$$

The program development score was 94.

10.

Let B be the set of all possible scores in the final round. She needs to shoot 74 on the final round.

SECTION 1.1

score on the final round.

Sum

of

scores

Let B be the set of all possible scores in the final round.

;

!

B

Let B be the set of all possible scores in the final round.

Let B be the set of all possible scores in the final round.

SECTION 1.2

11. Let B be the number of locks replaced.

$$\% \# \uparrow \boxed{\text{Number of locks}} \in \# \$ \cdot$$

$$\begin{aligned} \% \# \#) B \in \# \$ \cdot \\ \#) B \in " * \cdot \\ B \in (\end{aligned}$$

7 locks can be changed for \$236.

12. Let B be the number of interviews.

$$\# \# \# \uparrow \boxed{\text{Number of interviews}} \in \& \cdot$$

$$\begin{aligned} \# \# \# \#) B \in \& \cdot \\ \# \#) B \in \$ \cdot \\ B \in \% \end{aligned}$$

He interviewed 48 people.

13. Let B be the width.

Then B # * be the height.

$$\frac{\text{Perimeter} \in * \#}{\# B \# B \# \cdot \in * \#}$$

$$\begin{aligned} \# B \# B \& \# \in * \# \\ \% B \in \% ! \\ B \in " ! \end{aligned}$$

The dimensions are 10 ft by 36 ft.

14. Let B be the width.

Then B "" & be the length.

$$\frac{\text{Perimeter} \in \& (!)}{\# B \# B "" \& \in \& (!)}$$

$$\begin{aligned} \# B \# B \# \$! \in \& (!) \\ \% B \in \$ \% ! \\ B \in) \& \end{aligned}$$

The dimensions are 85 ft by 200 ft.

15. $\boxed{\text{Perimeter}} \in " \%$

$$\begin{aligned} B B \# B B \# \in " \% \\ \% B \% \in " \% \\ \% B \in " ! \\ B \in \frac{\&}{\#} \in \# \frac{''}{\#} \hat{E} \text{ The width is } 2 \frac{''}{\#} \text{ feet.} \end{aligned}$$

16. $\boxed{\text{Total Fence Length}} \in \# \uparrow \boxed{\text{Square Fence Length}}$

$$\begin{aligned} B B \# \% B B \# \% \in \# \uparrow B B B B \\ \% B \% \in) B \\ \% \in \% B \\ B \in " \# \end{aligned}$$

The total fencing required is $\% B \% \in \% " \# \% \in * \cdot$ feet.

17. $\boxed{\text{Total Area}} \in \# \uparrow \boxed{\text{Triangular Area}}$

$$\begin{aligned} \# ! B \frac{''}{\#} \cdot \# ! \in \# \uparrow \frac{''}{\#} \cdot \# ! \\ \# ! B " \cdot ! \in \$ \# ! \\ \# ! B \in " \cdot ! \\ B \in) \end{aligned}$$

The dimensions are 8 feet by 20 feet.

18. $\boxed{\text{Sum of angles}} \in " !$

$$\begin{aligned} B B \$! B \$! \in " ! \\ \$ B \cdot ! \in " ! \\ \$ B \in " \# ! \\ B \in \% ! \end{aligned}$$

The angles measure $\% !^\circ$, $(!^\circ$ and $(!^\circ$.

SECTION 1.2

19. $\frac{\text{New Area}}{\text{Old Area}} \approx \frac{\text{Old Area}}{\text{New Area}}$

"# B " ! "#B \approx "# B " ! !b&! † "# B " !
 "#B "#! "#B \approx "#B "#! 'B '!
 #%B "#! \approx ")B ")!
 'B \approx '!

B \approx " ! Ê The length of the living room is B " ! \approx # ! feet.

20. $\frac{\text{Area}}{\text{Area}} \approx \&\%$

" #) \approx &%
 " ! . \approx &%
 . \approx &b% Ê The depth is 5.4 inches

21. Let B \approx the amount invested at 7%. Then ##!!! B \approx the amount invested at 6%.

$\frac{\text{Interest at 7\%}}{\text{Interest at 6\%}} \approx \text{Total interest}$

!p!(B !p!' ##!!! B \approx "%#!
 !p!(B "\$#! !p!'B \approx "%#!
 !p!"B \approx "!!
 B \approx "!!!!

\$10,000 was invested at 7% and \$12,000 was invested at 6%.

22. Let B \approx the amount invested at 7%.

$\frac{\text{Interest at 7\%}}{\text{Interest at 9\%}} \approx \text{Total interest}$

!p!(B !p!* #!!!! \approx &!!!
 !p!(B ")!! \approx &!!!
 !p!(B \approx \$#!!
 B \approx %&(" %b#*

She needs to invest \$45,714.29 at 7% to reach her goal.

23. Let B \approx the amount invested at each rate.

$\frac{\text{Interest at 6\%}}{\text{Interest at 7\%}} \approx \frac{\text{Interest at 8\%}}{\text{Total interest}}$

!p!'B !p!(B !p!)B \approx #!\$(
 !p#"B \approx #!\$(
 B \approx *(!!

\$9,700 was invested at each rate, for a total investment of \$#,100.

SECTION 1.2

24. Let B be the amount invested at 8%. Then C be the amount invested at $9\frac{1}{2}\%$.

$$\text{Interest at } 9\frac{1}{2}\% \text{ } \alpha \text{ } \text{Interest at } 8\% \text{ } \% \& \# \& !$$

$$\begin{aligned} & \text{Interest at } 9\frac{1}{2}\% \text{ } \alpha \text{ } \text{Interest at } 8\% \text{ } \% \& \# \& ! \\ & \$ \& \# \& ! \text{ } \alpha \text{ } \text{Interest at } 8\% \text{ } \% \& \# \& ! \\ & \$ \& \# \& ! \text{ } \alpha \text{ } \text{Interest at } 8\% \text{ } \% \& \# \& ! \\ & \text{Interest at } 8\% \text{ } \% \& \# \& ! \end{aligned}$$

\$17,500 is invested at 8% and \$19,500 is invested at $9\frac{1}{2}\%$.

25. Let B be the number of full-price tickets sold. Then C be the number of student tickets sold.

$$\begin{aligned} & \text{# of full-price tickets} \text{ } \alpha \text{ } \text{# of student tickets} \text{ } \alpha \text{ } \text{# of student tickets} \\ & \text{# of full-price tickets} \text{ } \alpha \text{ } \text{# of student tickets} \text{ } \alpha \text{ } \text{# of student tickets} \\ & \text{# of full-price tickets} \text{ } \alpha \text{ } \text{# of student tickets} \text{ } \alpha \text{ } \text{# of student tickets} \\ & \text{# of full-price tickets} \text{ } \alpha \text{ } \text{# of student tickets} \text{ } \alpha \text{ } \text{# of student tickets} \end{aligned}$$

B be C There were 327 student tickets sold.

26. Let B be the cost of a student ticket.

$$\begin{aligned} & \text{Cost of full-price} \text{ } \alpha \text{ } \text{# of full-price} \text{ } \alpha \text{ } \text{Cost of student} \text{ } \alpha \text{ } \text{# of student} \text{ } \alpha \text{ } \% \& \# \& ! \\ & \% \& \# \& ! \text{ } \alpha \text{ } \text{Cost of student} \text{ } \alpha \text{ } \text{# of student} \text{ } \alpha \text{ } \% \& \# \& ! \\ & \$ \& \# \& ! \text{ } \alpha \text{ } \text{Cost of student} \text{ } \alpha \text{ } \text{# of student} \text{ } \alpha \text{ } \% \& \# \& ! \\ & \$ \& \# \& ! \text{ } \alpha \text{ } \text{Cost of student} \text{ } \alpha \text{ } \text{# of student} \text{ } \alpha \text{ } \% \& \# \& ! \end{aligned}$$

B be C A student ticket cost \$5.

27. Let C be the original price.

$$\begin{aligned} & \text{Original price} \text{ } \alpha \text{ } \text{Discount} \text{ } \alpha \text{ } \text{New price} \\ & \text{Original price} \text{ } \alpha \text{ } \text{Discount} \text{ } \alpha \text{ } \text{New price} \\ & \text{Original price} \text{ } \alpha \text{ } \text{Discount} \text{ } \alpha \text{ } \text{New price} \\ & \text{Original price} \text{ } \alpha \text{ } \text{Discount} \text{ } \alpha \text{ } \text{New price} \end{aligned}$$

The original price was \$79.95.

28. Let A be the wholesale cost.

$$\begin{aligned} & \text{Wholesale cost} \text{ } \alpha \text{ } \text{Markup} \text{ } \alpha \text{ } \text{Selling price} \\ & \text{Wholesale cost} \text{ } \alpha \text{ } \text{Markup} \text{ } \alpha \text{ } \text{Selling price} \\ & \text{Wholesale cost} \text{ } \alpha \text{ } \text{Markup} \text{ } \alpha \text{ } \text{Selling price} \\ & \text{Wholesale cost} \text{ } \alpha \text{ } \text{Markup} \text{ } \alpha \text{ } \text{Selling price} \end{aligned}$$

The wholesale cost is \$453.

29. Let B be # of plates for equal costs.

$$\begin{aligned} & \text{Cost of 1st machine} \text{ } \alpha \text{ } \text{Cost of 2nd machine} \\ & \text{Cost of 1st machine} \text{ } \alpha \text{ } \text{Cost of 2nd machine} \\ & \text{Cost of 1st machine} \text{ } \alpha \text{ } \text{Cost of 2nd machine} \\ & \text{Cost of 1st machine} \text{ } \alpha \text{ } \text{Cost of 2nd machine} \end{aligned}$$

The break point is 200 plates.

30. Let B be # of fasteners for equal costs.

$$\begin{aligned} & \text{Cost of 1st machine} \text{ } \alpha \text{ } \text{Cost of 2nd machine} \\ & \text{Cost of 1st machine} \text{ } \alpha \text{ } \text{Cost of 2nd machine} \\ & \text{Cost of 1st machine} \text{ } \alpha \text{ } \text{Cost of 2nd machine} \\ & \text{Cost of 1st machine} \text{ } \alpha \text{ } \text{Cost of 2nd machine} \end{aligned}$$

The break point is about 85,714 fasteners.

SECTION 1.2

31. Let B = # of computers to break even.

Income	Expenses
$1000B$	$10000 + 200B$
$1000B$	$10000 + 200B$
B	B

21 computers need to be sold to break even.

32. Let B = # of meals to break even.

Income	Expenses
$1.50B$	$1000 + 0.50B$
$1.50B$	$1000 + 0.50B$
B	B

More than 110 meals need to be sold to make a profit.

33. Let B = days for both working together.

Man in	Robin in	Total in
$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{B}$
$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{B}$
$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{B}$

They can roof the house in $2\frac{6}{11}$ days.

35. Let B = hours for both working together.

Woman	Man in	Total in
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{B}$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{B}$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{B}$

They can mow the lawn in $1\frac{2}{5}$ hours.

34. Let B = hours for both working together.

1st hose	2nd hose	Total in
$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{B}$
$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{B}$
$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{B}$

They can seal the parking lot in $4\frac{2}{3}$ hours.

36. Let B = days for both hoses to fill the pool.

1st hose	2nd hose	Total in
$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{B}$
$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{B}$
$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{B}$

The pool can be filled in $1\frac{1}{3}$ days.