Test Bank for College Algebra 12th Edition by Lial Hornsby Schneider and Daniels ISBN 0134217454 9780134217451

Chapter 2 Graphs and Functions

2.1 Rectangular Coordinates and Graphs

■ Ordered Pairs ■ The Rectangular Coordinate System ■ The Distance Formula ■ The Midpoint Formula ■ Equations in Two Variables

Key Terms: ordered pair, origin, *x*-axis, *y*-axis, rectangular (Cartesian) coordinate system, coordinate plane (*xy*-plane), quadrants, coordinates, conditional statement, collinear, graph of an equation, *x*-intercept, *y*-intercept

<u>Ordered Pairs</u>

CLASSROOM EXAMPLE 1 Writing Ordered Pairs

Use the table to write ordered pairs to express the relationship between each category and the amount spent on it.

Category	Amount Spent
food	\$ 8506
housing	\$21,374
transportation	\$12,153
health care	\$ 4917
apparel and services	\$ 2076
entertainment	\$ 3240

Source: U.S. Bureau of Labor Statistics.

(a) transportation

(b) health care

The Rectangular Coordinate System

The *x*-axis and *y*-axis together make up a __________________________ system, or

______________ ______________ system. The plane into which the coordinate system is introduced is the __________________________, or __________________________. The *x*-axis and *y*-axis divide the plane into four regions, or ______________.

The Distance Formula

Distance Formula

Suppose that $P\left(x_1, y_1\right)$ and $R(x_2, y_2)$ are two points in a coordinate plane. The distance between P and R, written $d(P, R)$, is given by the following formula.

$$d\left(P, R\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

The distance between two points in a coordinate plane is the ____________ ____________ of the of the square of the difference between their x-coordinates and the square of the difference between their y-coordinates.

CLASSROOM EXAMPLE 2 Using the Distance Formula

Find the distance between $P\left(3, -5\right)$ and $Q(-2, 8)$.

If the sides a, b, and c of a triangle satisfy $a^2 + b^2 = c^2$ ***then the triangle is a ____________***

____________ with ____________having lengths a and b and having length c.

CLASSROOM EXAMPLE 3 Applying the Distance Formula

Determine whether the points $R(0, -2)$, $S(5, 1)$, and $T(-4, 3)$ are the vertices of a right triangle.

CLASSROOM EXAMPLE 4 Applying the Distance Formula

Determine whether the points $P(-2, 5)$, $Q(0, 3)$, and $R(8, -5)$ are collinear.

The Midpoint Formula

MIDPOINT FORMULA

The coordinates of the midpoint M of the line segment with endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$ are given by the following.

$$M = \left(\underline{\quad}, \quad \right).$$

That is, the x-coordinate of the midpoint of a line segment is the________________ of the x-coordinates of the segment's endpoints, and the y-coordinate is the________________ of the y-coordinates of the segment's endpoints.

CLASSROOM EXAMPLE 5 **Using the Midpoint Formula**

Use the midpoint formula to do each of the following.

(a) Find the coordinates of the midpoint *M* of the line segment with endpoints $(-7, -5)$ and $(-2, 13)$.

(b) Find the coordinates of the other endpoint *Q* of a line segment with one endpoint $P(8, -20)$ and midpoint $M(4, -4)$.

CLASSROOM EXAMPLE 6 **Applying the Midpoint Formula**

Total revenue of full-service restaurants in the United States increased from $124.0 billion in 2009 to $137.4 billion in 2013. Use the midpoint formula to estimate the total revenue for 2011, and compare this to the actual figure of $130.6 billion. (*Source*: National Restaurant Association.)

Equations in Two Variables

CLASSROOM EXAMPLE 7 Finding Ordered-Pair Solutions of Equations For each equation, find at least three ordered pairs that are solutions.

(a) $y = -2x + 5$ **(b)** $x = \sqrt[3]{y+1}$ **(c)** $y = -x^2 + 1$

Graphing an Equation by Point Plotting

Step 1 Find the intercepts.

Step 2 Find as many additional ordered pairs as needed.

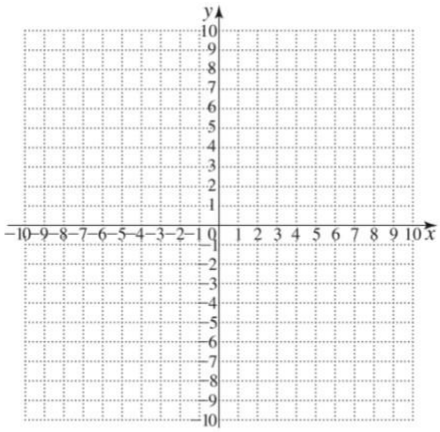
Step 3 Plot the ordered pairs from Steps 1 and 2.

Step 4 Join the points from Step 3 with a smooth line or curve.

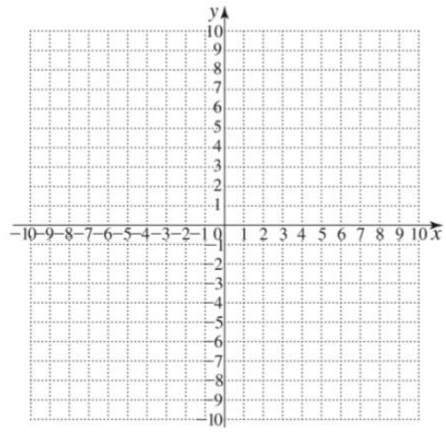
CLASSROOM EXAMPLE 8 GRAPHING EQUATIONS

Graph each of the equations here, from **Classroom Example 7**.

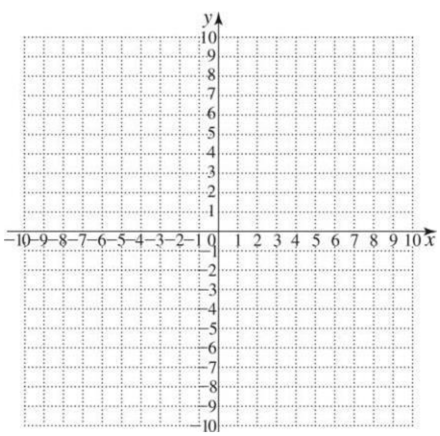
(a) $y = -2x + 5$



(b) $x = \sqrt[3]{y + 1}$



(c) $y = -x^2 + 1$



2.2 Circles

■ Center-Radius Form ■ General Form ■ An Application

Key Terms: circle, radius, center of a circle

Center-Radius Form

By definition, a **circle** is the set of all points in a plane that lie a given distance from a given point. The given distance is called the____________________and the given point is the ____________________.

Center-Radius Form of the Equation of a Circle

A circle with center (h, k) and radius r has the equation

$$(x-h)^2+(y-k)^2=r^2,$$

which is the **center-radius form** of the equation of a circle. A circle with center (0, 0) and radius r has the following equation.

$$x^2+y^2=r^2$$

CLASSROOM EXAMPLE 1 Finding the Center-Radius Form

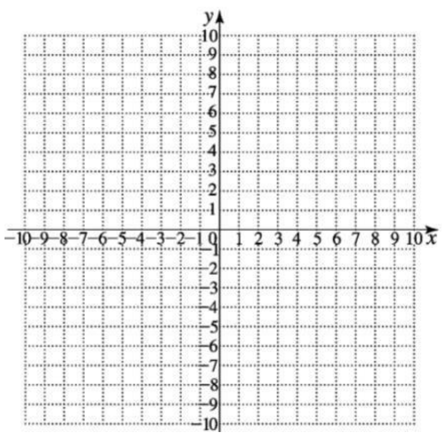
Find the center-radius form of the equation of each circle described.

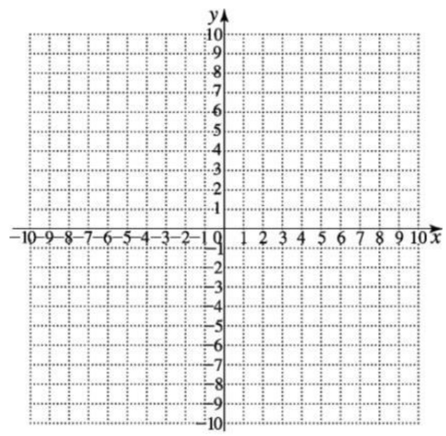
(a) center (1, −2), radius 3 **(b)** center (0, 0), radius 2

CLASSROOM EXAMPLE 2 **Graphing Circles**

Graph each circle discussed in **Classroom Example 1.**

(a) $(x-1)^2 + (y+2)^2 = 9$ **(b)** $x^2 + y^2 = 4$





<u>General Form</u>

General Form of the Equation of a Circle

For some real numbers D, E, and F, the equation

$$x^2 + y^2 + Dx + Ey + F = 0$$

can have a graph that is a circle or a point, or is nonexistent.

CLASSROOM EXAMPLE 3 **Finding the Center and Radius by Completing the Square**

Show that $x^2 + 4x + y^2 - 8y - 44 = 0$ has a circle as its graph. Find the center and radius.

CLASSROOM EXAMPLE 4 **Finding the Center and Radius by Completing the Square**

Show that $2x^2 + 2y^2 + 2x - 6y = 45$ has a circle as its graph. Find the center and radius.

CLASSROOM EXAMPLE 5 Determining Whether a Graph is a Point or Nonexistent

The graph of the equation $x^2 - 6x + y^2 + 2y + 12 = 0$ is either a point or is nonexistent. Which is it?

An Application

CLASSROOM EXAMPLE 6 Locating the Epicenter of an Earthquake

If three receiving stations at (1, 4), (−6, 0), and (5, −2) record distances to an earthquake epicenter of 4 units, 5 units, and 10 units, respectively, show algebraically that the epicenter lies at (−3, 4).

2.3 Functions

■ Relations and Functions ■ Domain and Range
■ Determining Whether Relations are Functions ■ Function Notation
■ Increasing, Decreasing, and Constant Functions

Key Terms: dependent variable, independent variable, relation, function, input, output, input-output (function machine), domain, range, increasing function, decreasing function, constant function

Relations and Functions

One quantity can sometimes be described in terms of another.

- The letter grade a student receives in a mathematics course depends on a ____________________.
- The amount paid for gas at a gas station depends on the number of___________________.

Relation and Function
A **relation** is a set of. A **function** is a relation in which, for each distinct value of the first component of the ordered pairs, there is ___________ value of the second component.

If the value of the second component *y* depends on the value of the first component *x*, then *y* is the_____________________________________and *x* is the _____________________ ________________.

CLASSROOM EXAMPLE 1 Deciding Whether Relations Define Functions

Decide whether each relation defines a function.

$$M = \{(-4, 0), (-3, 1), (3,1)\}$$
$$N = \{(2, 3), (3, 2), (4, 5), (5, 4)\}$$
$$P = \{(-4, 3), (0, 6), (2, 8), (-4, -3)\}$$

In a function, no two ordered pairs can have the same _________________ __________________ and different_____________________________________.

In a function, there is exactly one value of the ______. __________variable, the second component, for each value of the __________________ variable, the first component.

<u>Domain and Range</u>

Domain and Range
For every relation consisting of ordered pairs (x, y), there are two important sets of elements.

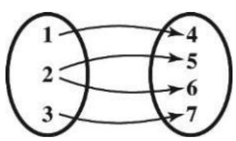
- The set of all values of the independent variable (x) is the ______________________.
- The set of all values of the dependent variable (y) is the ______________________.

CLASSROOM EXAMPLE 2 Finding Domains and Ranges of Relations

Give the domain and range of each relation. Tell whether the relation defines a function.

(a) $\{(-4, -2), (-1, 0), (1, 2), (3, 5)\}$

(b)



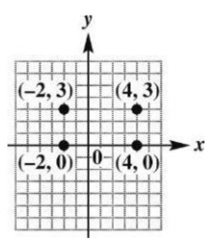
(c)

x	y
-3	5
0	5
3	5
5	5

CLASSROOM EXAMPLE 3 Finding Domains and Ranges from Graphs

Give the domain and range of each relation.

(a)



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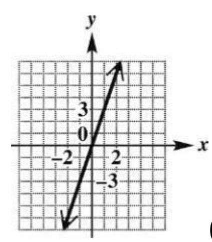

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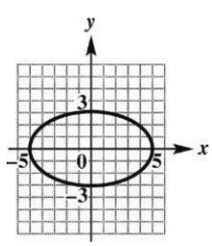

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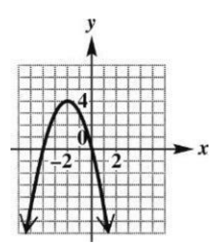


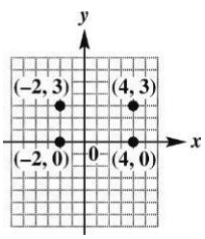
Agreement on Domain
Unless specified otherwise, the domain of a relation is assumed to be all real numbers that produce real numbers when substituted for the independent variable.

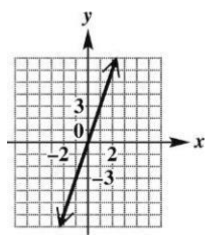
In general, the domain of a function defined by an algebraic expression is ______________________________, except those numbers that lead to ______________________________ or ______________________________.

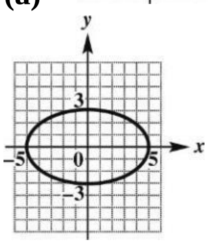
Determining Whether Relations Are Functions

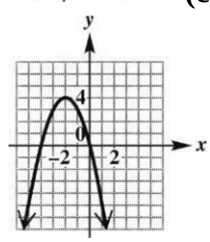
Vertical Line Test
If every vertical line intersects the graph of a relation in no more __________ __________than , then the relation is a function.

CLASSROOM EXAMPLE 4 Using the Vertical Line Test
Use the vertical line test to determine whether each relation graphed in **Classroom Example 3** is a function.

(a) 

(b) 

(c) 



(d)

CLASSROOM EXAMPLE 5 Identifying Functions, Domains, and Ranges
Decide whether each relation defines y as a function of x, and give the domain and range.

(a) $y = 2x - 5$ **(b)** $y = x^2 + 3$ ***(c)*** $x = |y|$

(d) $y \geq -x$ **(e)** $y = \frac{3}{x+2}$

Variations of the Definition of Function

1. A **function** is a relation in which, for each distinct value of the first component of the ordered pairs, there is exactly one value of the second component.
2. A **function** is a set of ordered pairs in which no first component is repeated.
3. A **function** is a rule or correspondence that assigns exactly one range value to each distinct domain value.

Function Notation

When a function *f* is defined with a rule or an equation using *x* and *y* for the independent and dependent variables, we say, "*y is a function of x*" to emphasize that *y depends on x*. We use the notation

$$y = f(x)$$

called **function notation,** to express this and read $f(x)$ as **"*f* of *x*,"** or **"*f* at *x*."**

Note that f (x) is just another name for the ______________________________ y.

The symbol $f(x)$ does not indicate "*f* times *x*," but represents the ______________

______________ associated with the indicated ______________________________.

CLASSROOM EXAMPLE 6 Using Function Notation

Let $f(x) = -x^2 - 6x + 4$ and $g(x) = 3x + 1$. Find each of the following.

(a) $f(-3)$ **(b)** $f(r)$ **(c)** $g(r+2)$

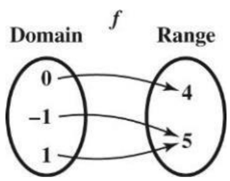
CLASSROOM EXAMPLE 7 Using Function Notation

For each function , find $f(-1)$.

(a) $f(x) = 2x^2 - 9$

(b) $f = \{(-4, 0), (-1, 6), (0, 8), (2, -2)\}$

(c)



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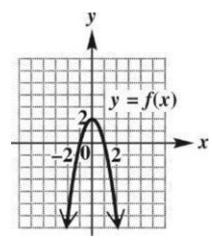


Finding an Expression for *f* (*x*)

Consider an equation involving *x* and *y*. Assume that *y* can be expressed as a function *f* of *x*. To find an expression for $f(x)$, use the following steps.

Step 1 Solve the equation for ____________ .

Step 2 Replace ____________ with ____________.

CLASSROOM EXAMPLE 8 Writing Equations Using Function Notation

Assume that *y* is a function *f* of *x*. Rewrite each equation using function notation. Then find $f(-5)$ and $f(t)$.

(a) $y = x^2 + 2x - 3$

(b) $2x - 3y = 6$

Increasing, Decreasing, and Constant Functions

Increasing, Decreasing, and Constant Functions

Suppose that a function *f* is defined over an *open* interval *I* and x_1 and x_2 are in *I*.

(a) *f* **increases** over *I* if, whenever $x_1 < x_2$, $f(x_1) < f(x_2)$.

(b) *f* **decreases** over *I* if, whenever $x_1 < x_2$, $f(x_1) > f(x_2)$.

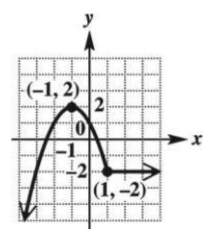
(c) *f* is **constant** over *I* if, for every x_1 and x_2, $f(x_1) = f(x_2)$.

To decide whether a function is increasing, decreasing, or constant over an interval, ask yourself,____

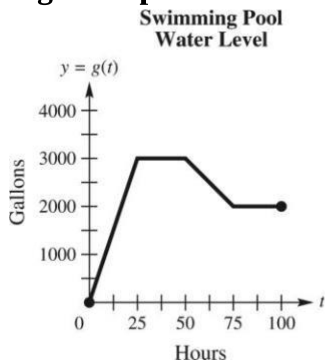
Our definition of *increasing*, *decreasing*, and *constant* function behavior applies to open intervals of the____________, not to________________________.

CLASSROOM EXAMPLE 9 Determining Intervals over Which a Function Is Increasing, Decreasing, or Constant

The figure shows the graph of a function. Determine the largest open intervals of the domain over which the function is increasing, decreasing, or constant.



CLASSROOM EXAMPLE 10 Interpreting a Graph



The figure shows the relationship between the number of gallons, $g(t)$, of water in a small swimming pool and time in hours, t. By looking at this graph of the function, we can answer questions about the water level in the pool at various times. For example, at time 0 the pool is empty. The water level then increases, stays constant for a while, decreases, then becomes constant again. Use the graph to respond to the following.

(a) Over what period of time is the water level changing most rapidly?

(b) After how many hours does the water level start to decrease?

(c) How many gallons of water are in the pool after 75 hr?

2.4 Linear Functions

■ Basic Concepts of Linear Functions ■ Standard Form *Ax* + *By* = *C* ■ Slope
■ Average Rate of Change ■ Linear Models

Key Terms: linear function, constant function, standard form, relatively prime, slope, average rate of change, mathematical modeling, linear cost function, cost, fixed cost, revenue function, profit function

Basic Concepts of Linear Functions

Linear Function
A function *f* is a **linear function** if, for real numbers *a* and *b*,

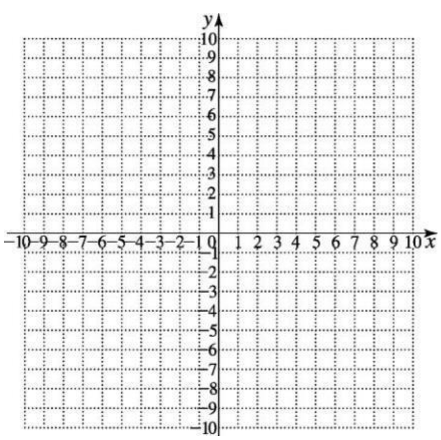
$$\boldsymbol{f(x) = ax + b.}$$

If $a \neq 0$, the domain and range of a linear function are both $(-\infty, \infty)$.

(d)

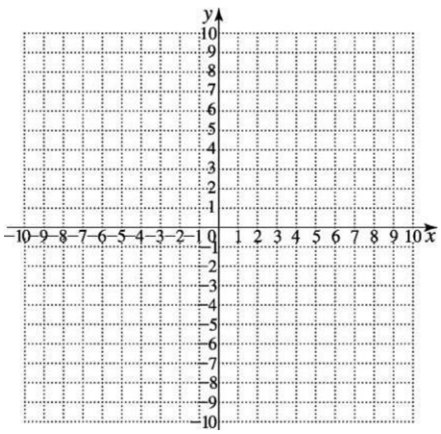
CLASSROOM EXAMPLE 1 (e) **Graphing a Linear Function Using**

Graph $f(x) = \bar{2}3\,x + 6$. Give the domain and range.



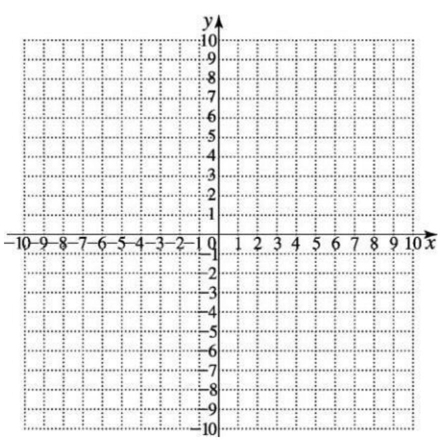
CLASSROOM EXAMPLE 2 Graphing a Horizontal Line

Graph $f(x) = 2$. Give the domain and range.



CLASSROOM EXAMPLE 3 Graphing a Vertical Line

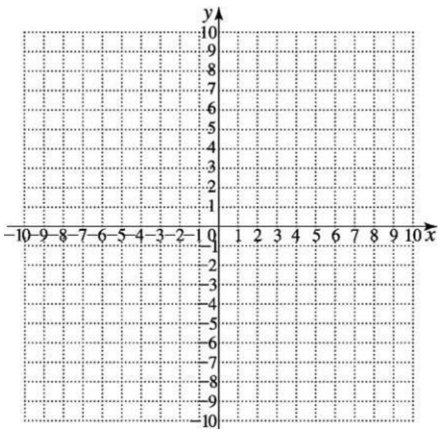
Graph $x = 5$. Give the domain and range of this relation.



Standard Form *Ax* + *By* = *C*

CLASSROOM EXAMPLE 4 Graphing *Ax* + *By* = *C* (*C* = 0)

Graph $3x + 4y = 0$. Give the domain and range.



Slope

Slope

The **slope** m of the line through the points (x_1, y_1) and (x_2, y_2) is given by the following.

$$m = \frac{\textbf{rise}}{\textbf{run}} = \frac{\otimes y}{\otimes x} = \frac{y_2 - y_1}{x_2 - x_1}, \qquad \text{where } \otimes x \neq 0$$

That is, the slope of a line is the change in __________ divided by the corresponding change in __________ , where the change in __________ is not 0.

When using the slope formula, it makes no difference which point is (x_1, y_1) or (x_2, y_2). However, be consistent. Be sure to write the difference of the __________ in the numerator and the difference of the __________ in the denominator.

Undefined Slope

The slope of a vertical line is ________________________.

CLASSROOM EXAMPLE 5 Finding Slopes With The Slope Formula

Find the slope of the line through the given points.

(a) $(-2, 4), (2, -6)$ **(b)** $(-3, 8), (5, 8)$ **(c)** $(-4, -10), (-4, 10)$

Slope Equal to Zero
The slope of a horizontal line is______ .

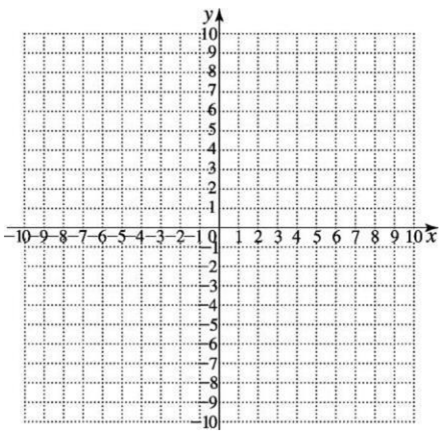
Slope is the same no matter which pair of distinct points on the line are used to find it.

CLASSROOM EXAMPLE 6 Finding Slope from an Equation

Find the slope of the line $2x - 5y = 10$.

CLASSROOM EXAMPLE 7 **Graphing a Line Using a Point and the Slope**

Graph the line passing through the point (–2, – 3) and having slope 43. –



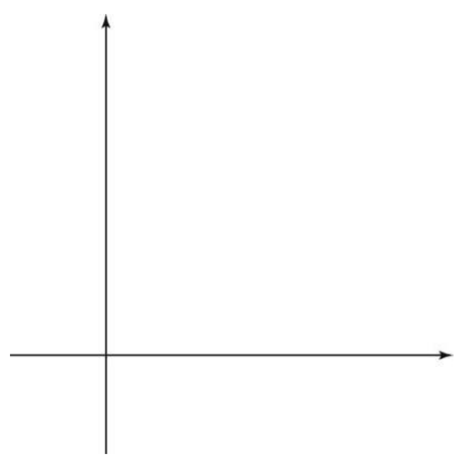
- *A line with a positive slope rises from* ______ *to* ______ .
- *A line with a negative slope falls from* ______ *to* ______ .
- *A line with* ______________________ *neither rises nor falls.*
- *The slope of a* ______________________ *is undefined.*

Slope gives the average rate of change in ______ ***per unit of change in*** ______***, where the value of y depends on the value of x.***

Average Rate of Change

CLASSROOM EXAMPLE 8 **Interpreting Slope as Average Rate of Change**

In 2010, the federal government spent $9547 million on research and development for general science. In 2013, $8658 million was spent. Assume a linear relationship, and find the average rate of change in the amount of money spent on R&D per year. Graph as a line segment, and interpret the result. (*Source:* National Science Foundation.)



Linear Models

A **linear cost function** has the form ____________________, where x represents the number of items produced, m represents the ____________________ per item and b represents the ____________________ .

The **revenue function** for selling a product depends on the price per item p and the number of items sold x. It is given by the function ____________________ .

The **profit function** is____________________.

CLASSROOM EXAMPLE 9 Writing Linear Cost, Revenue, and Profit Functions

Assume that the cost to produce an item is a linear function and all items produced are sold. The fixed cost is \$2400, the variable cost per item is \$120, and the item sells for \$150. Write linear functions to model each of the following.

(a) cost **(b)** revenue **(c)** profit

(d) How many items must be sold for the company to make a profit?

2.5 Equations of Lines and Linear Models

■ **Point-Slope Form** ■ **Slope-Intercept Form** ■ **Vertical and Horizontal Lines**
■ **Parallel and Perpendicular Lines** ■ **Modeling Data**
■ **Graphical Solution of Linear Equations in One Variable**

Key Terms: point-slope form, slope-intercept form, negative reciprocals, scatter diagram linear regression, zero (of a function)

Point-Slope Form

Point-Slope Form
The **point-slope form** of the equation of the line with slope *m* passing through the point (x_1, y_1) is given as follows.

$$\boldsymbol{y - y_1 = m(x - x_1)}$$

CLASSROOM EXAMPLE 1 Using the Point-Slope Form (Given a Point and the Slope)

Write an equation of the line through the point $(3, -5)$ having slope -2.

CLASSROOM EXAMPLE 2 Using the Point-Slope Form (Given Two Points)

Write an equation of the line through the points $(-4, 3)$ and $(5, -1)$. Write the result in standard form $Ax + By = C$.

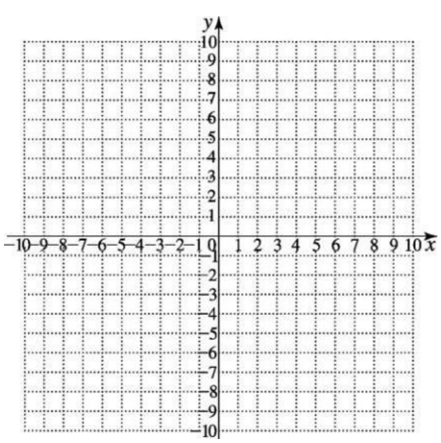
Slope-Intercept Form

Slope-Intercept Form
The **slope-intercept form** of the equation of the line with slope *m* and *y*-intercept (0, *b*) is given as follows.

$$\boldsymbol{y = mx + b}$$

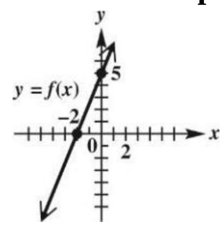
CLASSROOM EXAMPLE 3 Finding Slope and *y*-Intercept from an Equation of a Line
Find the slope and *y*-intercept of the line with equation $3x - 4y = 12$. .

CLASSROOM EXAMPLE 4 Using the Slope-Intercept Form (Given Two Points) Write an equation of the line through the points (−2, 4) and (2, 2). Then graph the line using the slope-intercept form.



CLASSROOM EXAMPLE 5 **Finding an Equation from a Graph**

Use the graph of the linear function *f* shown in the figure to complete the following.



(a) Identify the slope, *y*-intercept, and *x*-intercept.

(b) Write an equation that defines *f*.

Vertical and Horizontal Lines

Equations of Vertical and Horizontal Lines

An equation of the **vertical line** through the point (a,b) is ______________ .

An equation of the **horizontal line** through the point (a,b) is ______________.

Parallel and Perpendicular Lines

Parallel Lines

Two distinct nonvertical lines are parallel if and only if they have the same __________.

Perpendicular Lines

Two lines, neither of which is vertical, are perpendicular if and only if their slopes have a product of________. Thus, the slopes of perpendicular lines, neither of which is vertical, are ________________ ________________.

CLASSROOM EXAMPLE 6 **Finding Equations of Parallel and Perpendicular Lines**

Write an equation in both slope-intercept and standard form of the line that passes through the point (2, –4) and satisfies the given condition.

(a) parallel to the line $2x + 5y = 4$

(b) perpendicular to the line $2x + 5y = 4$

Summary of Forms of Linear Equations

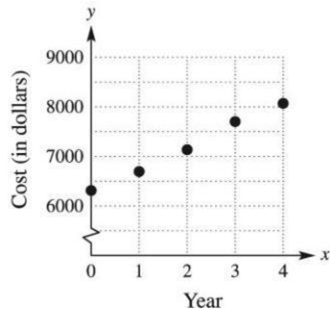
Equation	Description	When to Use
$\boldsymbol{y = mx + b}$	**Slope-Intercept Form** Slope is______. y-intercept is _____.	The slope and y-intercept can be easily identified and used to quickly graph the equation. This form can also be used to find the equation of a line given a point and the slope.
$\boldsymbol{y - y_1 = m(x - x_1)}$	**Point-Slope Form** Slope is______. Line passes through_______ .	This form is ideal for finding the equation of a line if the slope and a point on the line or two points on the line are known.
$\boldsymbol{Ax + By = C}$	**Standard Form** (If the coefficients and constant are rational, then A, B, and C are expressed as relatively prime integers, with $A \geq 0$) Slope is $(B \neq 0)$. x-intercept is _____ $(A \neq 0)$. y-intercept is _____ $(B \neq 0)$.	The x- and y-intercepts can be found quickly and used to graph the equation. The slope must be calculated.
$\boldsymbol{y = b}$	**Horizontal Line** Slope is . y-intercept is _______ .	If the graph intersects only the y-axis, then y is the only variable in the equation.
$\boldsymbol{x = a}$	**Vertical Line** Slope is _____________. x-intercept is _______ .	If the graph intersects only the x-axis, then x is the only variable in the equation.

Modeling Data

CLASSROOM EXAMPLE 7 Finding an Equation of a Line That Models Data
Average annual tuition and fees for in-state students at public four-year colleges are shown in the table for selected years and graphed as ordered pairs of points in the figure, where x = 0 represents 2009, x = 1 represents 2010, and so on, and y represents the cost in dollars. This graph of ordered pairs of data is called a **scatter diagram**.

Year	Cost (in dollars)
2009	6312
2010	6695
2011	7136
2012	7703
2013	8070

Source: National Center for Education Statistics.



(a) Find an equation that models the data using the data points (1, 6695) and (3, 7703).

(b) Use the equation from part (a) to predict the cost of tuition and fees at public four-year colleges in 2015.

Guidelines for Modeling

Step 1 Make a of the data.

Step 2 Find an that models the data. For a line, this involves selecting data points and finding the of the through them.

Graphical Solution of Linear Equations in One Variable

CLASSROOM EXAMPLE 8 Solving an Equation with a Graphing Calculator Use a graphing calculator to solve $-3x + 2(5 - x) = 2x + 38$.

2.6 Graphs of Basic Functions

■ Continuity ■ The Identity, Squaring, and Cubing Functions
■ The Square Root and Cube Root Functions ■ The Absolute Value Function
■ Piecewise-Defined Functions ■ The Relation $x = y^2$

Key Terms: continuous function, parabola, vertex, piecewise-defined function, step function

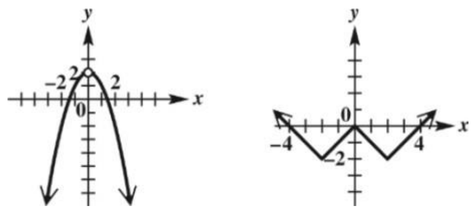
Continuity

Continuity (Informal Definition)
A function is **continuous** over an interval of its domain if its hand-drawn graph over that interval can be sketched without lifting the ____________ from the ____________.

If a function is not continuous at a ____________, then it has a ____________ there.

CLASSROOM EXAMPLE 1 Determining Intervals of Continuity

Describe the intervals of continuity for each function.

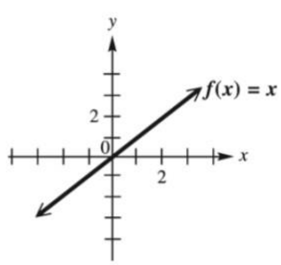


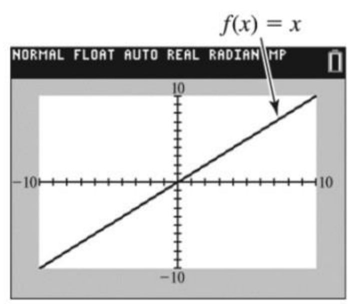
(a) (b)

The Identity, Squaring, and Cubing Functions

Identity Function $f(x) = x$

x	y
-2	-2
-1	-1
0	0
1	1
2	2





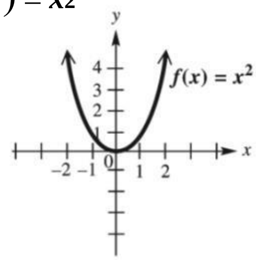
Domain: ____________

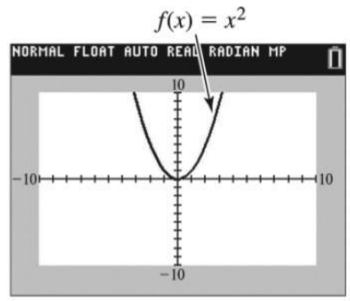
Range: ____________

- $f(x) = x$ is ____________ on its entire domain.
- It is ____________ on its entire domain.

Squaring Function $f(x) = x^2$

x	y
−2	4
−1	1
0	0
1	1
2	4





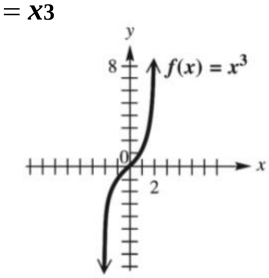
Domain:

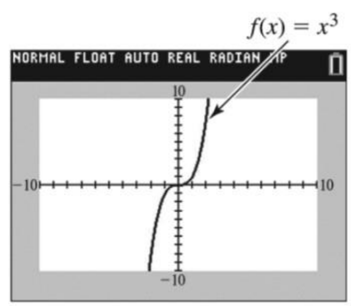
Range: __________

- $f(x) = x^2$ decreases on the open interval __________ and increases on the open interval ______.
- It is __________ on its entire domain.

Cubing Function $f(x) = x^3$

x	y
−2	−8
−1	−1
0	0
1	1
2	8





Domain:

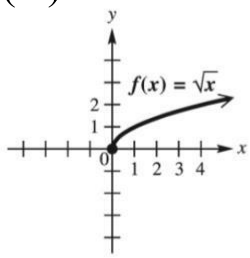
Range: __________

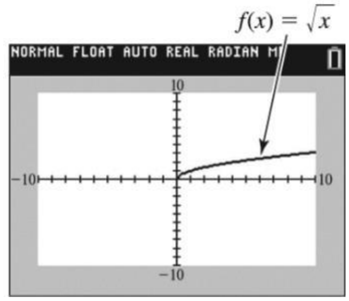
- $f(x) = x^3$ __________ on its entire domain.
- It is __________ on its entire domain.

The Square Root and Cube Root Functions

Square Root Function $f(x)=\sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4





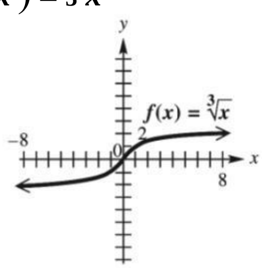
Domain:

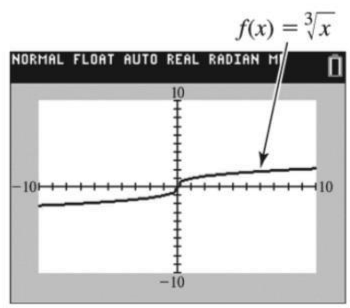
Range: ___________

- $f(x)=\sqrt{x}$ ____________ on the open interval ____________ .
- It is __________ on its entire domain.

Cube Root Function $f(x)=\sqrt[3]{x}$

x	y
−8	−2
−1	−1
0	0
1	1
8	2





Domain:

Range: ___________

- $f(x)=\sqrt[3]{x}$ ____________ on its entire domain.
- It is __________ on its entire domain ____________ .

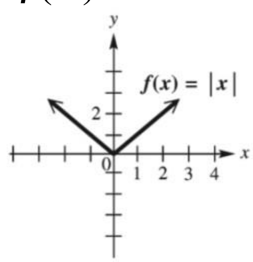
The Absolute Value Function

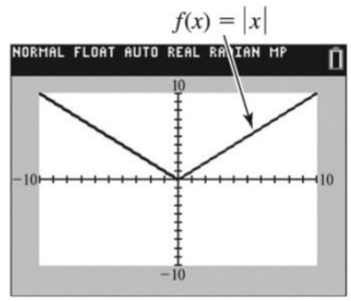
The **absolute value function** is defined as follows.

$$f(x) = |x| = \begin{cases} \_\_\_\_\_\_ & \textbf{if } x \ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \textbf{if } x \ \_\_\_\_\_\_ \end{cases}$$

Absolute Value Function $f(x) = |x|$

x	y
−2	2
−1	1
0	0
1	1
2	2





Domain: __________

Range: __________

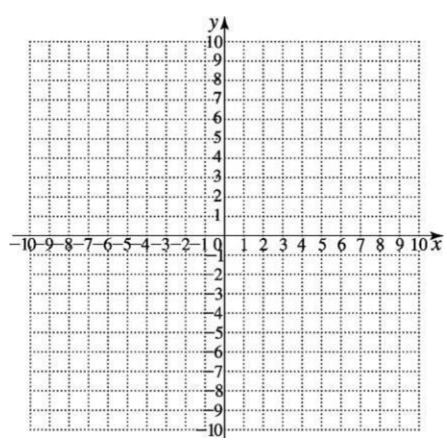
- $f(x) = |x|$ decreases on the open interval __________ and increases on the open interval __________ . __________
- It is __________ on its entire domain __________ .

Piecewise-Defined Functions

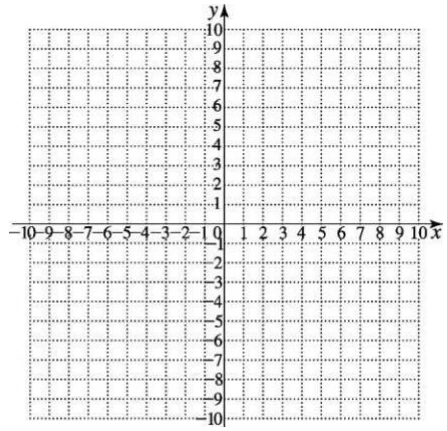
CLASSROOM EXAMPLE 2 Graphing Piecewise-Defined Functions

Graph each function.

(a) $f(x) = \begin{cases} 2x + 4 & \text{if } x < 1 \\ 4 - x & \text{if } x \geq 1 \end{cases}$



(b) $f(x)=\begin{cases} -x-2 & \text{if } x \le 0 \\ \;\;2 \\ \;\;- & >x \end{cases}$

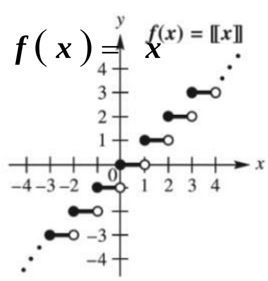


$f(x) = x$

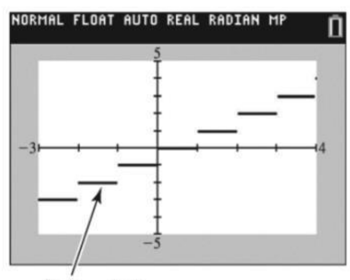
The **greatest integer function,** $f(x) = x$, pairs every real number x with the greatest integer or x .

Greatest Integer Function $f(x) = x$

x	y
−2	−2
−1.5	−2
−0.99	−1
0	0
0.001	0
3	3
3.99	3



The dots indicate that the graph continues indefinitely in the same pattern.



$f(x) = [\![x]\!]$

Domain:

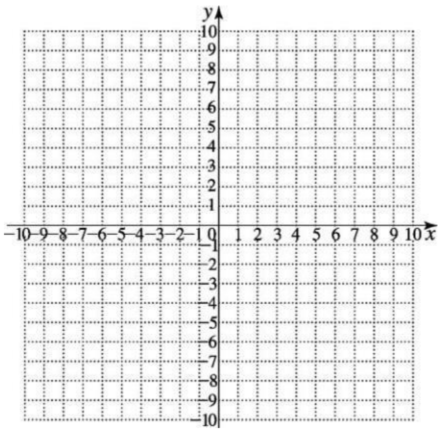
Range:

- $f(x) = x$ is constant on the open intervals …, , …
- It is at all integer values in its domain .

CLASSROOM EXAMPLE 3 Graphing a Greatest Integer Function

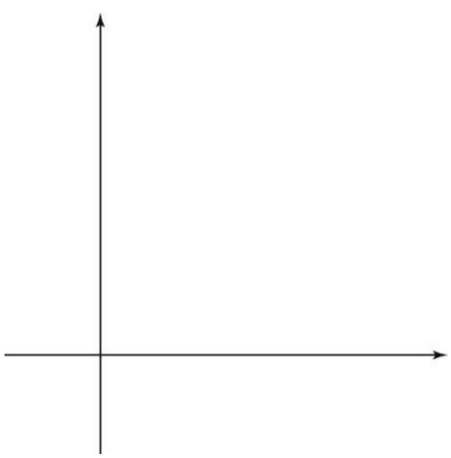
Graph $f(x = x^{1} - 2)$.

3



CLASSROOM EXAMPLE 4 Applying a Greatest Integer Function

An express mail company charges \$20 for a package weighing up to 2 lb. For each additional pound or fraction of a pound, there is an additional charge of \$2. Let $y = C(x)$ represent the cost to send a package weighing x pounds. Graph $y = C(x)$ for x in the interval $(0,6]$.



The Relation $x = y2$

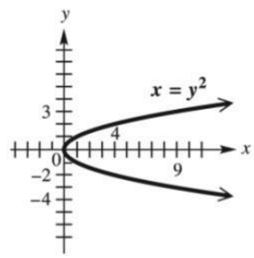
Recall that a function is a relation where every domain value is paired with ________ ________ ________ ________ range value.

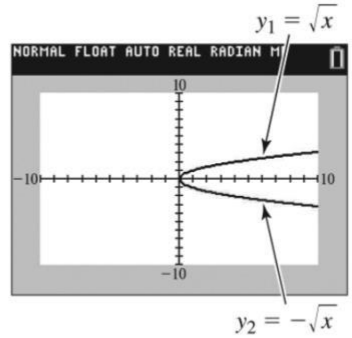
$x = y2$

Selected Ordered Pairs for $x = y^2$

x	y
0	0
1	± 1
4	± 2
9	± 3

There are two different y-values for the same x-value.





Domain: ____________

Range: ____________

2.7 Graphing Techniques

■ Stretching and Shrinking ■ Reflecting ■ Symmetry ■ Even and Odd Functions ■ Translations

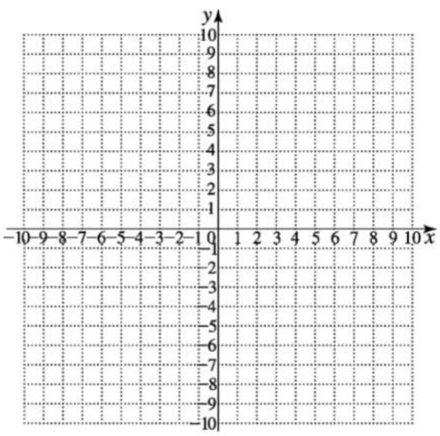
Key Terms: symmetry, even function, odd function, vertical translation, horizontal translation

Stretching and Shrinking

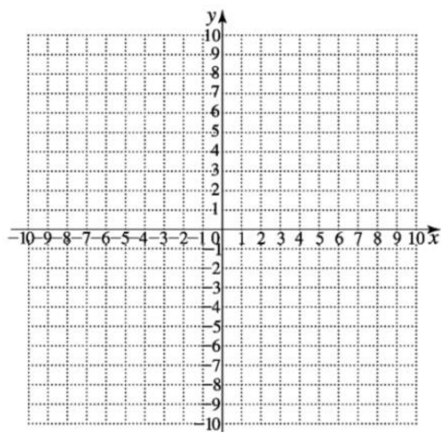
CLASSROOM EXAMPLE 1 Stretching or Shrinking Graphs

Graph each function.

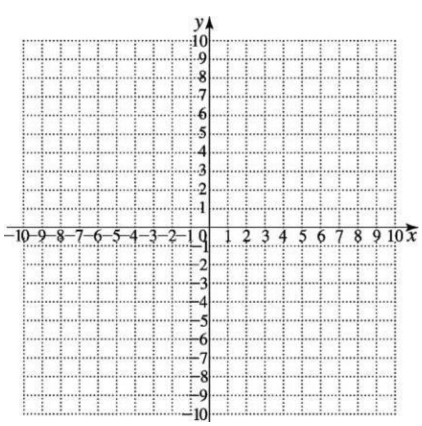
(a) $g(x) = 2x^2$



(b) $h(x) = \frac{1}{2}x^2$



(c) $k(x) = \left|\frac{1}{2}x\right|$



Vertical Stretching or Shrinking of the Graph of a Function

Suppose that $a > 0$. If a point (x, y) lies on the graph of $y = f(x)$, then the point (x, ay) lies on the graph of $\boldsymbol{y = af(x)}$.

(a) If $a > 1$, then the graph of $y = af(x)$ is a ______________________________ of the graph of $y = f(x)$.

(b) If $0 < a < 1$, then the graph of $y = af(x)$ is a ______________________________ of the graph of $y = f(x)$.

Horizontal Stretching or Shrinking of the Graph of a Function

Suppose that $a > 0$. If a point (x, y) lies on the graph of $y = f\ x$, then the point (, y lies on the graph of $\boldsymbol{y = f(ax)}$.

If $0 < a < 1$, then the graph of $y = f(ax)$ is a _ of the graph of $y = f(x)$.

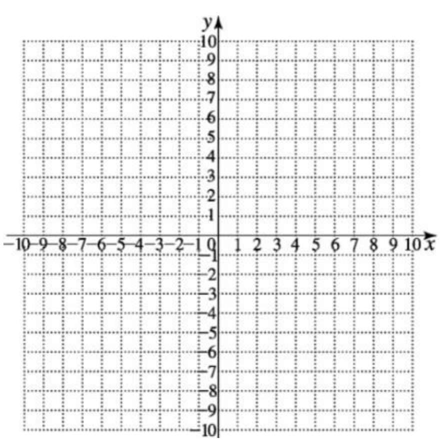
If $a > 1$, then the graph of $y = f(ax)$ is a of the graph of $y = f(x)$.

Reflecting

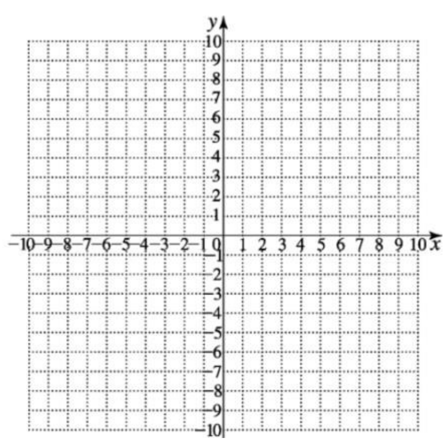
CLASSROOM EXAMPLE 2 Reflecting Graphs Across Axes

Graph each function.

(a) $g(x) = -|x|$



(b) $h(x) = |-x|$



Reflecting across an Axis

The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$ reflected across the ________. (If a point (x, y) lies on the graph of $y = f(x)$, then ________ lies on this reflection.)

The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$ reflected across the ________. (If a point (x, y) lies on the graph of $y = f(x)$, then ________ lies on this reflection.)

Symmetry

For a graph to be symmetric with respect to the y-axis, the point ______ ______ must be on the graph whenever the point (x, y) is on the graph.

For a graph to be symmetric with respect to the x-axis, the point ______ ______ must be on the graph whenever the point (x, y) is on the graph.

Symmetry with Respect to an Axis

The graph of an equation is **symmetric with respect to the** ______-______ if the replacement of x with $-x$ results in an equivalent equation.

The graph of an equation is **symmetric with respect to the** ______-______ if the replacement of y with $-y$ results in an equivalent equation.

CLASSROOM EXAMPLE 3 Testing for Symmetry with Respect to an Axis Test for symmetry with respect to the x-axis and the y-axis.

(a) $x = |y|$

(b) $y = |x| - 3$

(c) $2x - y = 6$

(d) $x^2 + y^2 = 25$

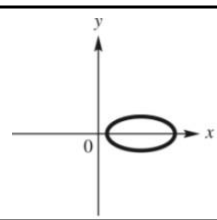
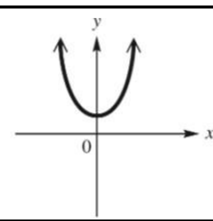
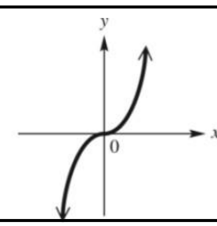
Symmetry with Respect to the Origin
The graph of an equation is **symmetric with respect to the origin** if the replacement of both *x* with –*x* and *y* with –*y* at the same time results in an____________________________.

CLASSROOM EXAMPLE 4 Testing for Symmetry with Respect to the Origin

Determine whether the graph of each equation is symmetric with respect to the origin?

(a) $y = -2x^3$ **(b)** $y = -2x^2$

Summary of Tests for Symmetry

	Symmetry with Respect to:		
	***x*-axis**	***y*-axis**	**Origin**
Equation is unchanged if:	_______ is replaced with _______	_______ is replaced with _______	____ ___ is replaced with _______and ____ ___ is replaced with _______
Example:			

A graph symmetric with respect to the___________-_____________does not represent a function.

Even and Odd Functions

Even and Odd Functions

A function f is called an **even function** if for all x in the domain of f. (Its graph is symmetric with respect to the .)

A function f is called an **odd function** if for all x in the domain of f. (Its graph is symmetric with respect to the .)

CLASSROOM EXAMPLE 5 Determining Whether Functions Are Even, Odd, or Neither

Determine whether each function defined is *even, odd*, or *neither*.

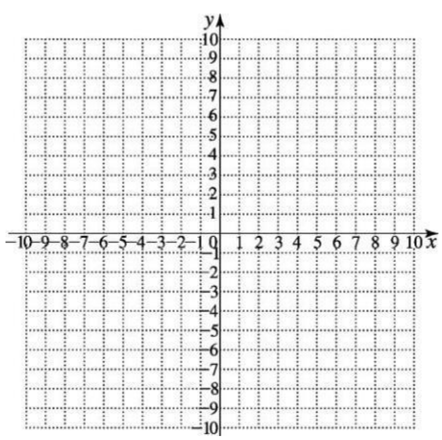
(a) $g(x) = x^5 + 2x^3 - 3x$ **(b)** $h(x) = 2x^2 - 3$ **(c)** $k(x) = x^2 + 6x + 9$

If a function defined by a polynomial in *x* has only *even* exponents on *x* (including the case of a constant where x^0 is understood to have the ____________________ exponent ____________________), it will *always* be an ____________________ function. Similarly, if only *odd* exponents appear on *x*, the function will be an ____________________ function.

Translations

CLASSROOM EXAMPLE 6 Translating a Graph

Vertically Graph $f(x) = x^2 + 2$.



Vertical Translations

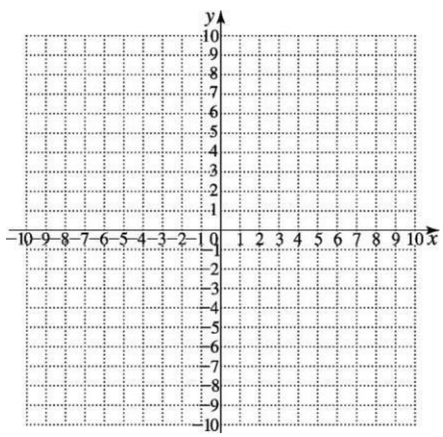
Given a function g defined by $\boldsymbol{g(x) = f(x) + c}$, where c is a real number:

- For every point (x, y) on the graph of f, there will be a corresponding ____________________ point on the graph of g.
- The graph of g will be the same as the graph of f, but translated c units ________________ if c is positive or $|c|$ units ________________ if c is negative.

The graph of g is a ____ of the graph of f.

CLASSROOM EXAMPLE 7 Translating a Graph

Horizontally Graph $f(x) = (x+4)^2$.



Horizontal Translations

Given a function g defined by $\boldsymbol{g(x) = f(x - c)}$, where c is a real number:

- For every point (x, y) on the graph of f, there will be a corresponding ____________________ point on the graph of g.
- The graph of g will be the same as the graph of f, but translated c units ________________ if c is positive or $|c|$ units ________________ if c is negative.

The graph of g is a ____ of the graph of f.

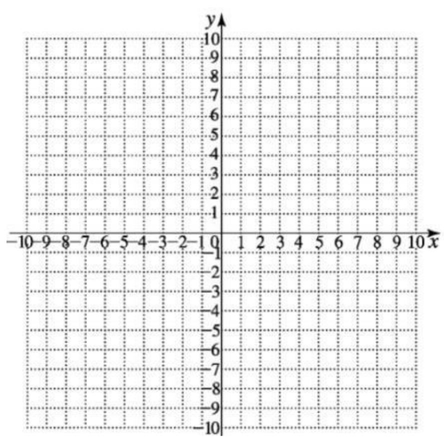
Summary of Translations

($c > 0$) **To Graph**:	**Shift the Graph of $y = f(x)$ by *c* Units:**
$y = f(x) + c$	
$y = f(x) - c$	
$y = f(x + c)$	
$y = f(x - c)$	

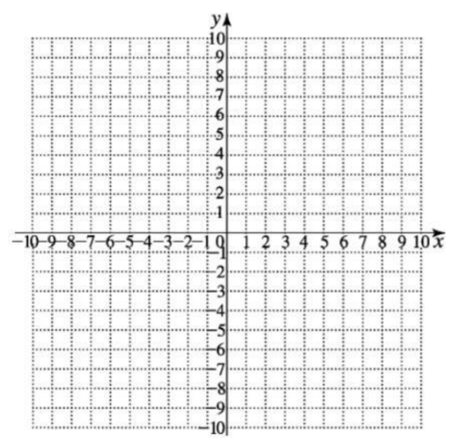
CLASSROOM EXAMPLE 8 Using More Than One Transformation

Graph each function.

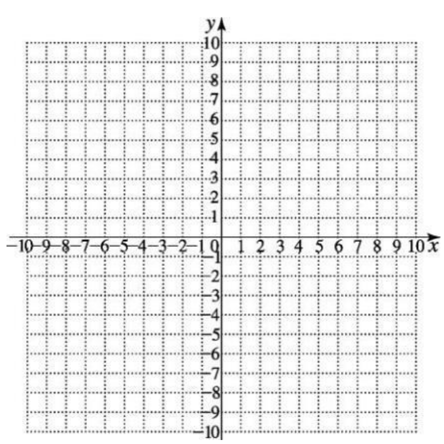
(a) $f(x) = -(x-1)^2 + 4$



(b) $g(x) = -|2x| + 3$

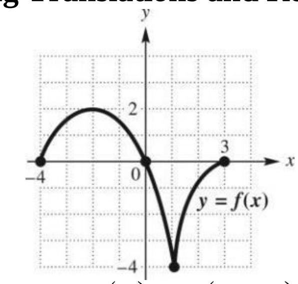


(c) $h(x) = \frac{1}{2}\sqrt{x+2} - 3$



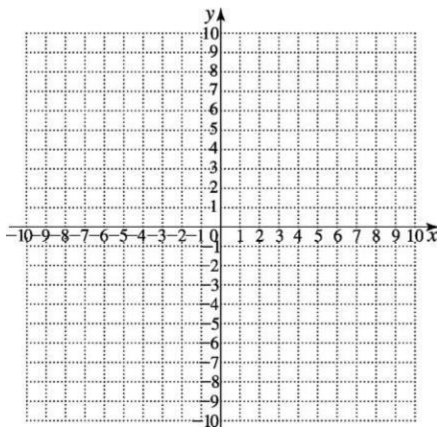
CLASSROOM EXAMPLE 9 Graphing Translations and Reflections of a Given Graph

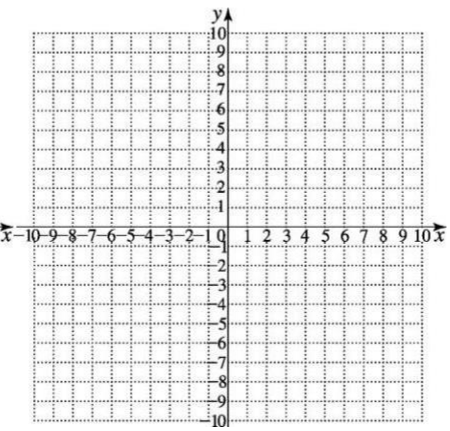
A graph of a function defined by $y = f(x)$ is shown in the figure. Use this graph to sketch each of the following graphs.



(a) $g(x) = f(x) - 2$

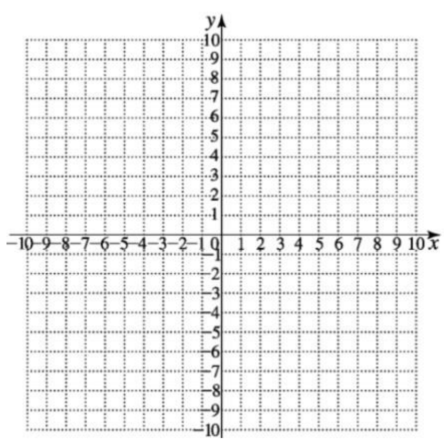
(b) $h(x) = f(x-2)$

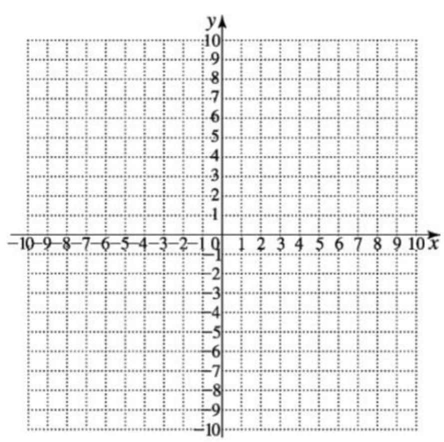




(c) $k(x) = f(x+1) + 2$

(d) $F(x) = f(-x)$





Summary of Graphing Techniques

In the descriptions that follow, assume that $a > 0$, $h > 0$, and $k > 0$. In comparison with the graph of $y = f(x)$:

1. The graph of $y = f(x) + k$ is translated ______ units ______.
2. The graph of $y = f(x) - k$ is translated ______ units ______.
3. The graph of $y = f(x + h)$ is translated ______ units ______.
4. The graph of $y = f(x - h)$ is translated ______ units ______.
5. The graph of $y = af(x)$ is a ______ of the graph of $y = f(x)$ if $a > 1$. It is a ______ if $0 < a < 1$.
6. The graph of $y = f(ax)$ is a ______ of the graph of $y = f(x)$ if $0 < a < 1$. It is a ______ if $a > 1$.
7. The graph of $y = -f(x)$ is ______ across the ______-axis.
8. The graph of $y = f(-x)$ is ______ across the ______-axis.

2.8 Function Operations and Composition

■ **Arithmetic Operations on Functions** ■ **The Difference Quotient**
■ **Composition of Functions and Domain**

Key Terms: difference quotient, secant line, composite function (composition)

Arithmetic Operations on Functions

Operations on Functions and Domains

Given two functions *f* and *g*, then for all values of *x* for which both $f(x)$ and $g(x)$ are defined, the functions $f+g$, $f-g$, fg, and $\frac{f}{g}$ are defined as follows.

$(f+g)(x) = f(x) + g(x)$	Sum function
$(f-g)(x) = f(x) - g(x)$	Difference function
$(fg)(x) = f(x) \cdot g(x)$	Product function
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0$	Quotient function

The **domains of $f+g$, $f-g$,** and ***fg*** include all real numbers in the intersection of the domains of *f* and *g*, while the **domain of** $\frac{f}{g}$ includes those real numbers in the intersection of the domains of *f* and *g* for which $g(x) \neq 0$.

CLASSROOM EXAMPLE 1 Using Operations on Functions

Let $f(x) = 3x - 4$ and $g(x) = 2x^2 - 1$. Find each of the following.

(a) $(f+g)(0)$

(b) $(f-g)(4)$

(c) $(fg)(-2)$

(d) $\left(\frac{f}{g}\right)(3)$

CLASSROOM EXAMPLE 2 Using Operations on Functions and Determining Domains

Let $f(x) = x^2 - 3x$ and $g(x) = 4x + 5$. Find each of the following.

(a) $(f+g)(x)$ ***(b)*** $(f-g)(x)$

(c) $(fg)(x)$ **(d)** $\left(\dfrac{f}{g}\right)(x)$

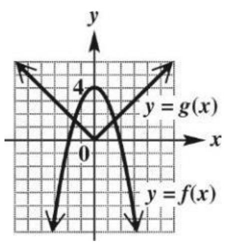
(e) Give the domains of the functions in parts (a)–(d).

CLASSROOM EXAMPLE 3 Evaluating Combinations of Functions

If possible, use the given representations of functions f and g to evaluate

$$(f+g)(1),\quad (f-g)(0),\quad (fg)(-1),\quad \text{and}\quad \left(\frac{f}{g}\right)(-2).$$

(a)



(b)

x	$f(x)$	$g(x)$
-2	-5	0
-1	-3	2
0	-1	4
1	1	6

(c) $f(x) = 3x + 4,\ g(x) = -|x|$

The Difference Quotient

CLASSROOM EXAMPLE 4 Finding The Difference Quotient

Let $f(x) = 3x^2 - 2x + 4$. Find and simplify the expression for the difference quotient.

Composition of Functions and Domain

Composition of Functions and Domain
If *f* and *g* are functions, then the **composite function,** or **composition,** of *f* and *g* is defined by

$$(f\ g)(x) = f(g(x)).$$

The **domain of** $(f g)$ is the set of all numbers *x* in the domain of *g* such that *g*(*x*) is in the domain of *f* .

CLASSROOM EXAMPLE 5 Evaluating Composite Functions

Functions Let $f(x) = \sqrt{x + 4}$ and $g(x) = \frac{2}{x}$.

(a) Find $(f\ g)(2)$ **(b)** Find $(g\ f)(5)$

CLASSROOM EXAMPLE 6 Determining Composite Functions and Their Domains

Given that $f(x) = \sqrt{x-1}$ and $g(x) = 2x+5$, find each of the following.

(a) $(f \circ g)(x)$ and its domain **(b)** $(g \circ f)(x)$ and its domain

CLASSROOM EXAMPLE 7 Determining Composite Functions and Their Domains

Given that $f(x) = \frac{x+4}{5}$ and $g(x) = -\frac{2}{x}$, find each of the following.

(a) $(f \circ g)(x)$ and its domain **(b)** $(g \circ f)(x)$ and its domain

CLASSROOM EXAMPLE 8 Showing That $(g \circ f)(x)$ Is Not Equivalent to $(f \circ g)(x)$

Let $f(x) = 2x - 5$ and $g(x) = 3x^2 + x$. Show that $(g \circ f)(x) \neq (f \circ g)(x)$.

CLASSROOM EXAMPLE 9 Finding Functions That Form a Given Composite Find functions f and g such that

$$(f \circ g)(x) = 4(3x + 2)^2 - 5(3x + 2) - 8.$$