Solution Manual for College Algebra 5th Edition by Beecher Penna and Bittinger ISBN 032196957X 9780321969576

Chapter 2

More on Functions

Exercise Set 2.1

1. a) For x-values from −5 to 1, the y-values increase from −3 to 3. Thus the function is increasing on the interval (−5, 1).

   b) For x-values from 3 to 5, the y-values decrease from 3 to 1. Thus the function is decreasing on the inter-val (3, 5).

   c) For x-values from 1 to 3, y is 3. Thus the function is constant on (1, 3).

2. a) For x-values from 1 to 3, the y-values increase from 1 to 2. Thus, the function is increasing on the interval (1, 3).

   b) For x-values from −5 to 1, the y-values decrease from 4 to 1. Thus the function is decreasing on the interval (−5, 1).

   c) For x-values from 3 to 5, y is 2. Thus the function is constant on (3, 5).

3. a) For x-values from −3 to −1, the y-values increase from −4 to 4. Also, for x-values from 3 to 5, the y-values increase from 2 to 6. Thus the function is increasing on (−3, −1) and on (3, 5).

   b) For x-values from 1 to 3, the y-values decrease from 3 to 2. Thus the function is decreasing on the inter-val (1, 3).

   c) For x-values from −5 to −3, y is 1. Thus the func-tion is constant on (−5, −3).

4. a) For x-values from 1 to 2, the y-values increase from 1 to 2. Thus the function is increasing on the interval (1, 2).

   c) For x-values from −5 to −2, the y-values decrease from 3 to 1. For x-values from −2 to 1, the y-values decrease from 3 to 1. And for x-values from 3 to 5, the y-values decrease from 2 to 1. Thus the function is decreasing on (−5, −2), on (−2, 1), and on (3, 5).

6. a) For x-values from 1 to 4, the y-values increase from 2 to 11. Thus the function is increasing on the interval (1, 4).

   b) For x-values from −1 to 1, the y-values decrease from 6 to 2. Also, for x-values from 4 to ∞, the y-values decrease from 11 to −∞. Thus the function is decreasing on (−1, 1) and on (4, ∞).

   c) For x-values from −∞ to −1, y is 3. Thus the func-tion is constant on (−∞, −1).

7. The x-values extend from −5 to 5, so the domain is [−5, 5]. The y-values extend from −3 to 3, so the range is [−3, 3].

8. Domain: [−5, 5]; range: [1, 4]

9. The x-values extend from −5 to −1 and from 1 to 5, so the domain is [−5, −1] ∪ [1, 5].

   The y-values extend from −4 to 6, so the range is [−4, 6].

10. Domain: [−5, 5]; range: [1, 3]

11. c) For x-values from 2 to 3, y is 2. Thus the function is The x-values extend from −∞ to ∞, so the domain is

    constant on (2, 3).

5. a) For x-values from −∞ to −8, the y-values increase from −∞ to 2. Also, for x-values from −3 to −2, the y-values increase from −2 to 3. Thus the function is increasing on (−∞, −8) and on (−3, −2).

   b) For x-values from −8 to −6, the y-values decrease from 2 to −2. Thus the function is decreasing on the interval (−8, −6).

   c) For x-values from −6 to −3, y is −2. Also, for x-values from −2 to ∞, y is 3. Thus the function is constant on (−6, −3) and on (−2, ∞).

$(-\infty,$
$\infty)$.

The y-values extend from $-\infty$ to 3, so the range is $(-\infty, 3]$.

12. Domain: $(-\infty, \infty)$; range: $(-\infty, 11]$

13. From the graph we see that a relative maximum value of the function is 3.25. It occurs at $x = 2.5$. There is no relative minimum value.

    The graph starts rising, or increasing, from the left and stops increasing at the relative maximum. From this point, the graph decreases. Thus the function is increasing on $(-\infty, 2.5)$ and is decreasing on $(2.5, \infty)$.

14. From the graph we see that a relative minimum value of 2 occurs at $x = 1$. There is no relative maximum value.

    The graph starts falling, or decreasing, from the left and stops decreasing at the relative minimum. From this point, the graph increases. Thus the function is increasing on $(1, \infty)$ and is decreasing on $(-\infty, 1)$.

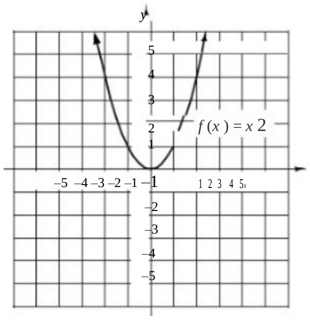
15. From the graph we see that a relative maximum value of the function is 2.370. It occurs at $x = -0.667$. We also see that a relative minimum value of 0 occurs at $x = 2$.

    The graph starts rising, or increasing, from the left and stops increasing at the relative maximum. From this point it decreases to the relative minimum and then increases again. Thus the function is increasing on $(-\infty, -0.667)$ and on $(2, \infty)$. It is decreasing on $(-0.667, 2)$.

16. From the graph we see that a relative maximum value of 2.921 occurs at $x = 3.601$. Arelative minimum value of 0.995 occurs at $x = 0.103$.

The graph starts decreasing from the left and stops de-creasing at the relative minimum. From this point it in-creases to the relative maximum and then decreases again. Thus the function is increasing on (0.103, 3.601) and is de-creasing on (−∞, 0.103) and on (3.601, ∞).

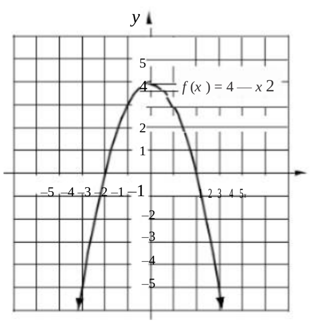
17.



The function is increasing on (0, ∞) and decreasing on (−∞, 0). We estimate that the minimum is 0 at $x = 0$. There are no maxima.

18.

3



Increasing: (−∞, 0)

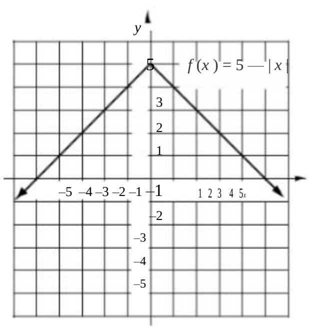
Decreasing: (0, ∞)

Maximum: 4 at $x = 0$

Minima: none

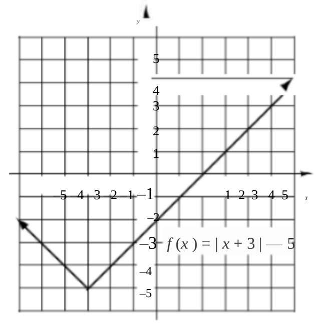
19.

4



The function is increasing on (−∞, 0) and decreasing on (0, ∞). We estimate that the maximum is 5 at $x = 0$. There are no minima.

20.



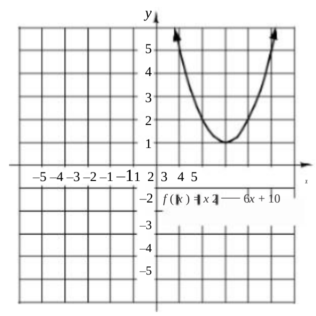
Increasing: (−3, ∞)

Decreasing: (−∞, −3)

Maxima: none

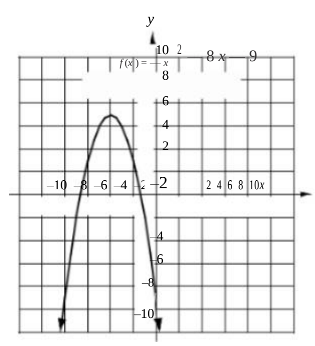
Minimum: −5 at $x = −3$

21.



The function is decreasing on (−∞, 3) and increasing on (3, ∞). We estimate that the minimum is 1 at $x = 3$. There are no maxima.

22.



Increasing: (−∞, −4)

Decreasing: (−4, ∞)

Maximum: 7 at $x = −4$

Minima: none

23. If x = the length of the rectangle, in meters, then the width is $\frac{480 - 2x}{2}$, or $240 - x$. We use the formula Area = length × width:

$$A(x) = x(240 - x), \text{ or}$$

$$A(x) = 240x - x^2$$

Copyright © 2016 Pearson Education, Inc.

24. Let h = the height of the scarf, in inches. Then the length of the base = $2h - 7$.

$A(h) = \frac{1}{2}(2h-7)(h)$

$A(h) = h^2 - \frac{7}{2}h$

25. We use the Pythagorean theorem.

$[h(d)]^2 + 3500^2 = d^2$

$[h(d)]^2 = d^2 - 3500^2$

$h(d) = \sqrt{d^2 - 3500^2}$

We considered only the positive square root since distance must be nonnegative.

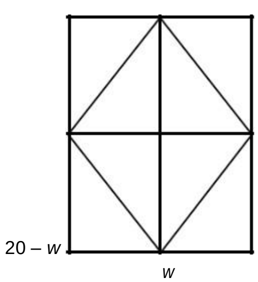
26. After t minutes, the balloon has risen $120t$ ft. We use the Pythagorean theorem.

$[d(t)]^2 = (120t)^2 + 400^2$

$d(t) = \sqrt{(120t)^2 + 400^2}$

We considered only the positive square root since distance must be nonnegative.

27. Let w = the width of the rectangle. Then the length = $\frac{40-2w}{2}$, or $20 - w$. Divide the rectangle into quadrants as shown below.



In each quadrant there are two congruent triangles. One triangle is part of the rhombus and both are part of the rectangle. Thus, in each quadrant the area of the rhombus is one-half the area of the rectangle. Then, in total, the area of the rhombus is one-half the area of the rectangle.

$A(w) = \frac{1}{2}(20-w)(w)$

$A(w) = 10w - \frac{w^2}{2}$

28. Let w = the width, in feet. Then the length = $\frac{46-2w}{2}$, or $23 - w$.

$A(w) = (23-w)w$

$A(w) = 23w - w^2$

29. We will use similar triangles, expressing all distances in feet. 6 in. = $\frac{1}{2}$ ft, s in. = $\frac{s}{12}$ ft, and d yd = $3d$ ft. We

$\frac{3d}{7} = \frac{\frac{1}{2}}{\frac{s}{12}}$

$\frac{s}{12} \cdot 3d = 7 \cdot \frac{1}{2}$

$\frac{sd}{4} = \frac{7}{2}$

$d = \frac{4}{s} \cdot \frac{7}{2}$, so

$d(s) = \frac{14}{s}$.

30. The volume of the tank is the sum of the volume of a sphere with radius r and a right circular cylinder with radius r and height 6 ft.

$V(r) = \frac{4}{3}\pi r^3 + 6\pi r^2$

31. a) After 4 pieces of float line, each of length x ft, are used for the sides perpendicular to the beach, there remains $(240-4x)$ ft of float line for the side parallel to the beach. Thus we have a rectangle with length $240 - 4x$ and width x. Then the total area of the three swimming areas is

$A(x) = (240-4x)x$, or $240x - 4x^2$.

b) The length of the sides labeled x must be positive and their total length must be less than 240 ft, so $4x < 240$, or $x < 60$. Thus the domain is $\{x | 0 < x < 60\}$, or $(0, 60)$.

c) We see from the graph that the maximum value of the area function on the interval $(0, 60)$ appears to be 3600 when $x = 30$. Thus the dimensions that yield the maximum area are 30 ft by $240 - 4 \cdot 30$, or $240 - 120$, or 120 ft.

32. a) If the length = x feet, then the width = $24 - x$ feet.

$A(x) = x(24-x)$

$A(x) = 24x - x^2$

b) The length of the rectangle must be positive and less than 24 ft, so the domain of the function is $\{x | 0 < x < 24\}$, or $(0, 24)$.

c) We see from the graph that the maximum value of the area function on the interval $(0, 24)$ appears to be 144 when $x = 12$. Then the dimensions that yield the maximum area are length = 12 ft and width = $24 - 12$, or 12 ft.

Copyright © 2016 Pearson Education, Inc.

have 2 12

33. a) When a square with sides of length x is cut from each corner, the length of each of the remaining sides of the piece of cardboard is $12 - 2x$. Then the dimensions of the box are x by $12 - 2x$ by $12 - 2x$. We use the formula Volume = length × width × height to find the volume of the box:

$$V(x) = (12 - 2x)(12 - 2x)(x)$$
$$V(x) = (144 - 48x + 4x^2)(x)$$
$$V(x) = 144x - 48x^2 + 4x^3$$

This can also be expressed as $V(x) = 4x(x - 6)^2$, or $V(x) = 4x(6 - x)^2$.

Copyright C 2016 Pearson Education, Inc.

b) The length of the sides of the square corners that are cut out must be positive and less than half the length of a side of the piece of cardboard. Thus, the domain of the function is $\{x \mid 0 < x < 6\}$, or (0, 6).

c) We see from the graph that the maximum value of the area function on the interval (0, 6) appears to be 128 when $x = 2$. When $x = 2$, then $12 - 2x = 12 - 2 \cdot 2 = 8$, so the dimensions that yield the maximum volume are 8 cm by 8 cm by 2 cm.

34. a) $V(x) = 8x(14 - 2x)$, or $112x - 16x^2$

b) The domain is $\left\{x \mid 0 < x < \frac{14}{2}\right\}$, or $\{x \mid 0 < x < 7\}$, or (0, 7).

c) The maximum occurs when $x = 3.5$, so the file should be 3.5 in. tall.

35. $g(x) = \begin{cases} x + 4, & \text{for } x \le 1, \\ 8 - x, & \text{for } x > 1 \end{cases}$

Since $-4 \le 1$, $g(-4) = -4 + 4 = 0$.

Since $3 > 1$, $g(3) = 8 - 3 = 5$.

3,

f

or $x \le -2$,

2 $x + 6$, for $x > -2$

$f(-5) = 3$

$f(-2) = 3$

$f(0) = \frac{1}{2} \cdot 0 + 6 = 6$

$f(2) = 2 \cdot 2 + 6 = 7$

37. $h(x) =$ 1, for $-5 \le x < 1$,

Since -5 is in the interval $[-5, 1)$, $h(-5) = 1$.

Since $1 \ge 1$, $h(1) = 1 + 2 = 3$.

Since $4 \ge 1$, $h(4) = 4 + 2 = 6$.

38. $f(x) = \begin{cases} -5x - 8, & \text{for } x < -2, \\ \frac{1}{2}x + 5, & \text{for } -2 \le x \le 4, \\ 10 - 2x, & \text{for } x > 4 \end{cases}$

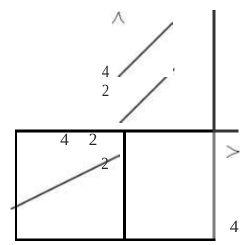
Since $-4 < -2$, $f(-4) = -5(-4) - 8 = 12$.

Since 4 is in the interval $[-2, 4]$, $f(4) = \frac{1}{2} \cdot 4 + 5 = 7$.

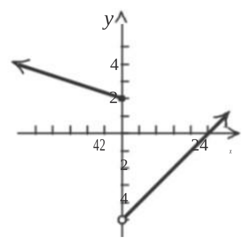
Since $6 > 4$, $f(6) = 10 - 2 \cdot 6 = -2$.

39. $f(x) = \begin{cases} \frac{1}{2}x, & \text{for } x < 0, \\ x + 3, & \text{for } x \ge 0 \end{cases}$

We create the graph in two parts. Graph $f(x) = \frac{1}{2}x$ for inputs x less than 0. Then graph $f(x) = x + 3$ for inputs x greater than or equal to 0.

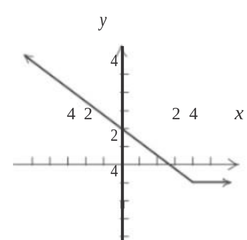


40. $f(x) = \begin{cases} -\frac{1}{3}x + 2, & \text{for } x \le 0, \\ x - 5, & \text{for } x > 0 \end{cases}$

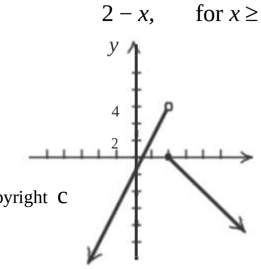


41. $f(x) = \begin{cases} -\frac{3}{4}x + 2, & \text{for } x < 4, \\ -1, & \text{for } x \ge 4 \end{cases}$

We create the graph in two parts. Graph $f(x) = -\frac{3}{4}x + 2$ for inputs x less than 4. Then graph $f(x) = -1$ for inputs x greater than or equal to 4.



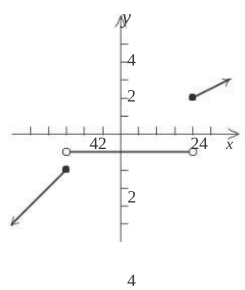
42. $h(x) = \begin{cases} 2x - 1, & \text{for } x < 2 \\ 2 - x, & \text{for } x \ge 2 \end{cases}$



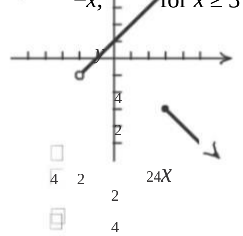
4 2 2 4x
2
4

43. $f(x) = \begin{cases} x + 1, & \text{for } x \le -3, \\ -1, & \text{for } -3 < x < 4 \\ \frac{1}{2}x, & \text{for } x \ge 4 \end{cases}$

We create the graph in three parts. Graph $f(x) = x + 1$ for inputs x less than or equal to -3. Graph $f(x) = -1$ for inputs greater than -3 and less than 4. Then graph $f(x) = \frac{1}{2}x$ for inputs greater than or equal to 4.

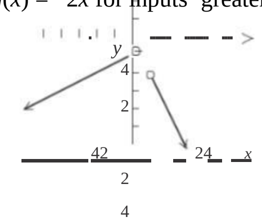


44. $f(x) = \begin{cases} 4, & \text{for } x \le -2, \\ x + 1, & \text{for } -2 < x < 3 \\ -x, & \text{for } x \ge 3 \end{cases}$

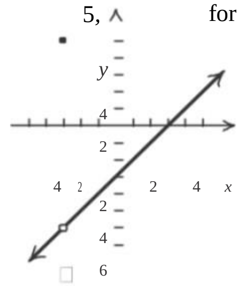


45. $g(x) = \begin{cases} \frac{1}{2}x - 1, & \text{for } x < 0, \\ 3, & \text{for } 0 \le x \le 1 \\ -2x, & \text{for } x > 1 \end{cases}$

We create the graph in three parts. Graph $g(x) = \frac{1}{2}x - 1$ for inputs less than 0. Graph $g(x) = 3$ for inputs greater than or equal to 0 and less than or equal to 1. Then graph $g(x) = -2x$ for inputs greater than 1.



46. $f(x) = \begin{cases} \dfrac{x^2 - 9}{x + 3}, & \text{for } x \ne -3, \\ 5, & \text{for } x = -3 \end{cases}$



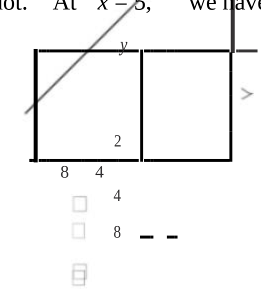
47. $f(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5}, & \text{for } x \ne 5, \\ 2, & \text{for } x = 5 \end{cases}$

When $x \ne 5$, the denominator of $(x^2 - 25)/(x - 5)$ is nonzero so we can simplify:

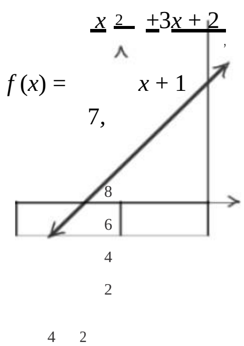
$$\frac{x^2 - 25}{x - 5} = \frac{(x + 5)(x - 5)}{x - 5} = x + 5.$$

Thus, $f(x) = x + 5$, for $x \ne 5$.

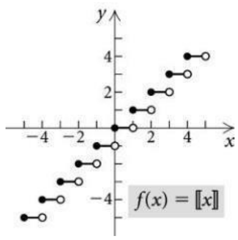
The graph of this part of the function consists of a line with a "hole" at the point (5, 10), indicated by an open dot. At $x = 5$, we have $f(5) = 2$, so the point (5, 2) is



48. $f(x) = \begin{cases} \dfrac{x^2 + 3x + 2}{x + 1}, & \text{for } x \ne -1, \\ 7, & \text{for } x = -1 \end{cases}$



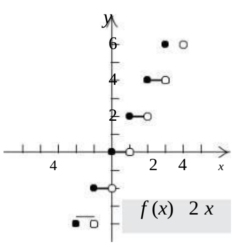
49. $f(x) = [[x]]$ See Example 9.



50. $f(x) = 2[[x]]$

This function can be defined by a piecewise function with an infinite number of statements:

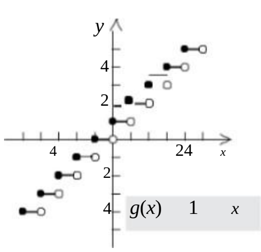
$$f(x) = \begin{cases} \vdots \\ -4, & \text{for } -2 \le x < -1, \\ -2, & \text{for } -1 \le x < 0, \\ 0, & \text{for } 0 \le x < 1, \\ 2, & \text{for } 1 \le x < 2, \\ \vdots \end{cases}$$



51. $f(x) = 1 + [[x]]$

This function can be defined by a piecewise function with an infinite number of statements:

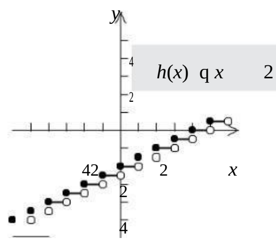
$$f(x) = \begin{cases} \vdots \\ -1, & \text{for } -2 \le x < -1, \\ 0, & \text{for } -1 \le x < 0, \\ 1, & \text{for } 0 \le x < 1, \\ 2, & \text{for } 1 \le x < 2, \\ \vdots \end{cases}$$



52. $f(x) = \frac{1}{2}[[x]] - 2$

This function can be defined by a piecewise function with an infinite number of statements:

$$f(x) = \begin{cases} \vdots \\ -2\frac{1}{2}, & \text{for } -1 \le x < 0, \\ -2, & \text{for } 0 \le x < 1, \\ -1\frac{1}{2}, & \text{for } 1 \le x < 2, \\ -1, & \text{for } 2 \le x < 3, \\ \vdots \end{cases}$$



53. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $(-\infty, 0) \cup [3, \infty)$.

54. Domain: $(-\infty, \infty)$; range: $(-5, \infty)$

55. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $[-1, \infty)$.

56. Domain: (∞, ∞); range: $(-\infty, 3)$

57. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $\{y \mid y \le -2 \text{ or } y = -1 \text{ or } y \ge 2\}$.

58. Domain: $(-\infty, \infty)$; range: $(-\infty, -3] \cup (-1, 4]$

59. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $\{-5, -2, 4\}$. An equation for the function is:

$$f(x) = \begin{cases} -2, & \text{for } x < 2, \\ -5, & \text{for } x = 2, \\ 4, & \text{for } x > 2 \end{cases}$$

60. Domain: $(-\infty, \infty)$; range: $\{y \mid y = -3 \text{ or } y \ge 0\}$

$$a(x) = \begin{cases} -3, & \text{for } x < 0, \\ x, & \text{for } x \ge 0 \end{cases}$$

61. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $(-\infty, -1] \cup [2, \infty)$. Finding the slope of each segment and using the slope-intercept or point-slope for-mula, we find that an equation for the function is:

$$g(x) = \begin{cases} x, & \text{for } x \le -1, \\ 2, & \text{for } -1 < x \le 2, \\ x, & \text{for } x > 2 \end{cases}$$

This can also be expressed as follows:

$$g(x) = \begin{cases} x, & \text{for } x \le -1, \\ 2, & \text{for } -1 < x < 2, \\ x, & \text{for } x \ge 2 \end{cases}$$

62. Domain: $(-\infty, \infty)$; range: $\{y \mid y = -2 \text{ or } y \geq 0\}$. An equation for the function is:

$$h(x) = \begin{cases} |x|, & \text{for } x < 3, \\ -2, & \text{for } x \geq 3 \end{cases}$$

This can also be expressed as follows:

$$h(x) = \begin{cases} -x, & \text{for } x \leq 0, \\ x, & \text{for } 0 < x < 3, \\ -2, & \text{for } x \geq 3 \end{cases}$$

It can also be expressed as follows:

$$h(x) = \begin{cases} -x, & \text{for } x < 0, \\ x, & \text{for } 0 \leq x < 3, \\ -2, & \text{for } x \geq 3 \end{cases}$$

63. From the graph we see that the domain is $[-5, 3]$ and the range is $(-3, 5)$. Finding the slope of each segment and using the slope-intercept or point-slope formula, we find that an equation for the function is:

$$h(x) = \begin{cases} x + 8, & \text{for } -5 \leq x < -3, \\ 3, & \text{for } -3 \leq x \leq 1, \\ 3x - 6, & \text{for } 1 < x \leq 3 \end{cases}$$

64. Domain: $[-4, \infty)$; range: $[-2, 4]$

$$f(x) = \begin{cases} -2x - 4, & \text{for } -4 \leq x \leq -1, \\ x - 1, & \text{for } -1 < x < 2, \\ 2, & \text{for } x \geq 2 \end{cases}$$

This can also be expressed as:

$$f(x) = \begin{cases} -2x - 4, & \text{for } -4 \leq x < -1, \\ x - 1, & \text{for } -1 \leq x < 2, \\ 2, & \text{for } x \geq 2 \end{cases}$$

65. $f(x) = 5x^2 - 7$

a) $f(-3) = 5(-3)^2 - 7 = 5 \cdot 9 - 7 = 45 - 7 = 38$

b) $f(3) = 5 \cdot 3^2 - 7 = 5 \cdot 9 - 7 = 45 - 7 = 38$

c) $f(a) = 5a^2 - 7$

d) $f(-a) = 5(-a)^2 - 7 = 5a^2 - 7$

66. $f(x) = 4x^3 - 5x$

a) $f(2) = 4 \cdot 2^3 - 5 \cdot 2 = 4 \cdot 8 - 5 \cdot 2 = 32 - 10 = 22$

b) $f(-2) = 4(-2)^3 - 5(-2) = 4(-8) - 5(-2) = -32 + 10 = -22$

c) $f(a) = 4a^3 - 5a$

d) $f(-a) = 4(-a)^3 - 5(-a) = 4(-a^3) - 5(-a) = -4a^3 + 5a$

67. First find the slope of the given line.

$8x - y = 10$

$8x = y + 10$

$8x - 10 = y$

The slope of the given line is 8. The slope of a line perpendicular to this line is the opposite of the reciprocal of 8, or $-\frac{1}{8}$.

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{1}{8}[x - (-1)]$

$y - 1 = -\frac{1}{8}(x + 1)$

$y - 1 = -\frac{1}{8}x - \frac{1}{8}$

$y = -\frac{1}{8}x + \frac{7}{8}$

68. $2x - 9y + 1 = 0$

$2x + 1 = 9y$

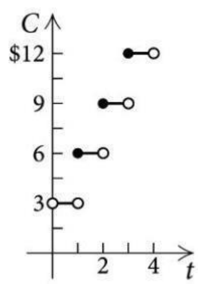
$\frac{2}{9}x + \frac{1}{9} = y$

Slope: $\frac{2}{9}$; y-intercept: $\left(0, \frac{1}{9}\right)$

69. a) The function $C(t)$ can be defined piecewise.

$$C(t) = \begin{cases} 3, & \text{for } 0 < t < 1, \\ 6, & \text{for } 1 \leq t < 2, \\ 9, & \text{for } 2 \leq t < 3, \\ \vdots & \end{cases}$$

We graph this function.



b) From the definition of the function in part (a), we see that it can be written as

$C(t) = 3[[t]] + 1,\ t > 0.$

70. If $[[x + 2]] = -3$, then $-3 \leq x + 2 < -2$, or $-5 \leq x < -4$. The possible inputs for x are $\{x \mid -5 \leq x < -4\}$.

71. If $[[x]]^2 = 25$, then $[[x]] = -5$ or $[[x]] = 5$. For $-5 \leq x < -4$, $[[x]] = -5$. For $5 \leq x < 6$, $[[x]] = 5$. Thus, the possible inputs for x are $\{x \mid -5 \leq x < -4 \text{ or } 5 \leq x < 6\}$.

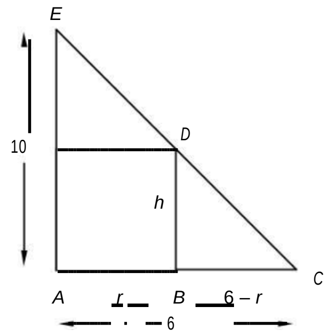
72. a) The distance from A to S is $4 - x$.

Using the Pythagorean theorem, we find that the distance from S to C is $\sqrt{1 + x^2}$.

Then $C(x) = 3000(4 - x) + 5000\sqrt{1 + x^2}$, or $12,000 - 3000x + 5000\sqrt{1 + x^2}$.

b) Use a graphing calculator to graph $y = 12,000 - 3000x + 5000\sqrt{1 + x^2}$ in a window such as $[0, 5, 10,000, 20,000]$, Xscl = 1, Yscl = 1000. Using the MINIMUM feature, we find that cost is minimized when $x = 0.75$, so the line should come to shore 0.75 mi from B.

73. a) We add labels to the drawing in the text.



We write a proportion involving the lengths of the sides of the similar triangles BCD and ACE. Then we solve it for h.

$$\frac{h}{6-r} = \frac{10}{6}$$

$$h = \frac{10}{6}(6-r) = \frac{5}{3}(6-r)$$

$$h = \frac{30-5r}{3}$$

Thus, $h(r) = \dfrac{30-5r}{3}$.

b) $V = \pi r^2 h$

$V(r) = \pi r^2 \left(\dfrac{30-5r}{3}\right)$ Substituting for h

c) We first express r in terms of h. $h = \dfrac{30-5r}{3}$

$3h = 30 - 5r$

$5r = 30 - 3h$

$r = \dfrac{30-3h}{5}$

$V = \pi r^2 h$

$V(h) = \pi \left(\dfrac{30-3h}{5}\right)^2 h$ Substituting for r

We can also write $V(h) = \pi h \left(\dfrac{30-3h}{5}\right)^2$.

Exercise Set 2.2

1. $(f+g)(5) = f(5) + g(5)$
$= (5^2 - 3) + (2\cdot 5 + 1)$
$= 25 - 3 + 10 + 1$
$= 33$

2. $(fg)(0) = f(0)\cdot g(0)$
$= (0^2 - 3)(2\cdot 0 + 1)$
$= -3(1) = -3$

3. $(f-g)(-1) = f(-1) - g(-1)$
$= ((-1)^2 - 3) - (2(-1) + 1)$
$= -2 - (-1) = -2 + 1$
$= -1$

4. $(fg)(2) = f(2)\cdot g(2)$
$= (2^2 - 3)(2\cdot 2 + 1)$
$= 1\cdot 5 = 5$

5. $(f/g)\left(-\frac{1}{2}\right) = \dfrac{f\left(-\frac{1}{2}\right)}{g\left(-\frac{1}{2}\right)}$
$= \dfrac{\left(-\frac{1}{2}\right)^2 - 3}{2\left(-\frac{1}{2}\right) + 1}$
$= \dfrac{\frac{1}{4} - 3}{-1 + 1}$
$= \dfrac{-\frac{11}{4}}{0}$

Since division by 0 is not defined, $(f/g)\left(-\frac{1}{2}\right)$ does not exist.

6. $(f-g)(0) = f(0) - g(0)$
$= (0^2 - 3) - (2\cdot 0 + 1)$
$= -3 - 1 = -4$

7. $(fg)\left(-\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)\cdot g\left(-\frac{1}{2}\right)$
$= \left[\left(-\frac{1}{2}\right)^2 - 3\right]\left[2\left(-\frac{1}{2}\right) + 1\right]$
$= -\frac{11}{4}\cdot 0 = 0$

8. $(f/g)(-\sqrt{3}) = \dfrac{f(-\sqrt{3})}{g(-\sqrt{3})}$
$= \dfrac{(-\sqrt{3})^2 - 3}{2(-\sqrt{3}) + 1}$
$= \dfrac{0}{-2\sqrt{3} + 1} = 0$

9. $(g-f)(-1) = g(-1) - f(-1)$
$= [2(-1) + 1] - [(-1)^2 - 3]$

$= (-2 + 1) - (1 - 3)$
$= -1 - (-2)$

$= -1 + 2$
$= 1$

10. $(g/f)\left(-\frac{1}{2}\right) = \frac{g\left(-\frac{1}{2}\right)}{f\left(-\frac{1}{2}\right)}$

$= \frac{-\frac{1}{2}+1}{2\left(-\frac{1}{2}\right)-3}$

$= \frac{\frac{1}{2}}{-4}$

$= 0$

11. $(h-g)(-4) = h(-4) - g(-4)$

$= (-4+4) - \sqrt{-4-1}$

$= 0 - \sqrt{-5}$

Since $\sqrt{-5}$ is not a real number, $(h-g)(-4)$ does not exist.

12. $(gh)(10) = g(10) \cdot h(10)$

$= \sqrt{10-1}(10+4)$

$= \sqrt{9}(14)$

$= 3 \cdot 14 = 42$

13. $(g/h)(1) = \frac{g(1)}{h(1)}$

$= \frac{\sqrt{1-1}}{1+4}$

$= \frac{\sqrt{0}}{5}$

$= \frac{0}{5} = 0$

14. $(h/g)(1) = \frac{h(1)}{g(1)}$

$= \frac{1+4}{\sqrt{1-1}}$

$= \frac{5}{0}$

Since division by 0 is not defined, $(h/g)(1)$ does not exist.

15. $(g+h)(1) = g(1) + h(1)$

$= \sqrt{1-1} + (1+4)$

$= \sqrt{0} + 5$

$= 0 + 5 = 5$

16. $(hg)(3) = h(3) \cdot g(3)$

$= (3+4)\sqrt{3-1}$

$= 7\sqrt{2}$

17. $f(x) = 2x+3$, $g(x) = 3-5x$

a) The domain of f and of g is the set of all real numbers, or $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, ff, and fg is also $(-\infty, \infty)$. For f/g we must exclude $\frac{3}{5}$ since $g\left(\frac{3}{5}\right) = 0$. Then the domain of f/g is $\left(-\infty, \frac{3}{5}\right) \cup \left(\frac{3}{5}, \infty\right)$. For g/f we must exclude $-\frac{3}{2}$ since $f\left(-\frac{3}{2}\right) = 0$. The domain of g/f is $\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, \infty\right)$.

b) $(f+g)(x) = f(x) + g(x) = (2x+3) + (3-5x) = -3x+6$

$(f-g)(x) = f(x) - g(x) = (2x+3) - (3-5x) = 2x+3-3+5x = 7x$

$(fg)(x) = f(x) \cdot g(x) = (2x+3)(3-5x) = 6x - 10x^2 + 9 - 15x = -10x^2 - 9x + 9$

$(ff)(x) = f(x) \cdot f(x) = (2x+3)(2x+3) = 4x^2 + 12x + 9$

$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{2x+3}{3-5x}$

$(g/f)(x) = \frac{g(x)}{f(x)} = \frac{3-5x}{2x+3}$

18. $f(x) = -x+1$, $g(x) = 4x-2$

a) The domain of f, g, $f+g$, $f-g$, fg, and ff is $(-\infty, \infty)$. Since $g\left(\frac{1}{2}\right) = 0$, the domain of f/g is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$. Since $f(1) = 0$, the domain of g/f is $(-\infty, 1) \cup (1, \infty)$.

b) $(f+g)(x) = (-x+1) + (4x-2) = 3x-1$

$(f-g)(x) = (-x+1) - (4x-2) = -x+1-4x+2 = -5x+3$

$(fg)(x) = (-x+1)(4x-2) = -4x^2 + 6x - 2$

$(ff)(x) = (-x+1)(-x+1) = x^2 - 2x + 1$

$(f/g)(x) = \frac{-x+1}{4x-2}$

$(g/f)(x) = \frac{4x-2}{-x+1}$

19. $f(x) = x - 3$, $g(x) = \sqrt{x+4}$

a) Any number can be an input in f, so the domain of f is the set of all real numbers, or $(-\infty, \infty)$.

The domain of g consists of all values of x for which $x+4$ is nonnegative, so we have $x+4 \geq 0$, or $x \geq -4$.

Thus, the domain of g is $[-4, \infty)$. The domain of $f+g$, $f-g$, and fg is the set of all numbers in the domains of both f and g. This is $[-4, \infty)$.

The domain of ff is the domain of f, or $(-\infty, \infty)$. The domain of f/g is the set of all numbers in the domains of f and g, excluding those for which $g(x) = 0$. Since $g(-4) = 0$, the domain of f/g is $(-4, \infty)$.

The domain of g/f is the set of all numbers in the domains of g and f, excluding those for which $f(x) = 0$. Since $f(3) = 0$, the domain of g/f is $[-4, 3) \cup (3, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = x - 3 + \sqrt{x+4}$
$(f-g)(x) = f(x) - g(x) = x - 3 - \sqrt{x+4}$
$(fg)(x) = f(x)\cdot g(x) = (x-3)\sqrt{x+4}$
$(ff)(x) = [f(x)]^2 = (x-3)^2 = x^2 - 6x + 9$
$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{x-3}{\sqrt{x+4}}$
$(g/f)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{x+4}}{x-3}$

20. $f(x) = x + 2$, $g(x) = \sqrt{x-1}$

a) The domain of f is $(-\infty, \infty)$. The domain of g consists of all the values of x for which $x - 1$ is nonnegative, or $[1, \infty)$. Then the domain of $f+g$, $f-g$, and fg is $[1, \infty)$. The domain of ff is $(-\infty, \infty)$. Since $g(1) = 0$, the domain of f/g is $(1, \infty)$. Since $f(-2) = 0$ and -2 is not in the domain of g, the domain of g/f is $[1, \infty)$.

b) $(f+g)(x) = x + 2 + \sqrt{x-1}$
$(f-g)(x) = x + 2 - \sqrt{x-1}$
$(fg)(x) = (x+2)\sqrt{x-1}$
$(ff)(x) = (x+2)(x+2) = x^2 + 4x + 4$
$(f/g)(x) = \frac{x+2}{\sqrt{x-1}}$
$(g/f)(x) = \frac{\sqrt{x-1}}{x+2}$

21. $f(x) = 2x - 1$, $g(x) = -2x^2$

a) The domain of f and of g is $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty, \infty)$. For f/g, we must exclude 0 since $g(0) = 0$. The domain of f/g is $(-\infty, 0) \cup (0, \infty)$. For g/f, we must exclude $\frac{1}{2}$ since $f\left(\frac{1}{2}\right) = 0$. The domain of g/f is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

b) $(f+g)(x) = f(x) + g(x) = (2x-1) + (-2x^2) = -2x^2 + 2x - 1$
$(f-g)(x) = f(x) - g(x) = (2x-1) - (-2x^2) = 2x^2 + 2x - 1$
$(fg)(x) = f(x)\cdot g(x) = (2x-1)(-2x^2) = -4x^3 + 2x^2$
$(ff)(x) = f(x)\cdot f(x) = (2x-1)(2x-1) = 4x^2 - 4x + 1$
$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{2x-1}{-2x^2}$
$(g/f)(x) = \frac{g(x)}{f(x)} = \frac{-2x^2}{2x-1}$

22. $f(x) = x^2 - 1$, $g(x) = 2x + 5$

a) The domain of f and of g is the set of all real numbers, or $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, 5

b) $(f+g)(x) = x^2 - 1 + 2x + 5 = x^2 + 2x + 4$
$(f-g)(x) = x^2 - 1 - (2x+5) = x^2 - 2x - 6$
$(fg)(x) = (x^2-1)(2x+5) = 2x^3 + 5x^2 - 2x - 5$
$(ff)(x) = (x^2-1)^2 = x^4 - 2x^2 + 1$
$(f/g)(x) = \frac{x^2-1}{2x+5}$
$(g/f)(x) = \frac{2x+5}{x^2-1}$

23. $f(x) = \sqrt{x-3}$, $g(x) = \sqrt{x+3}$

a) Since $f(x)$ is nonnegative for values of x in $[3, \infty)$, this is the domain of f. Since $g(x)$ is nonnegative for values of x in $[-3, \infty)$, this is the domain of g. The domain of $f+g$, $f-g$, and fg is the intersection of the domains of f and g, or $[3, \infty)$. The domain of ff is the same as the domain of f, or $[3, \infty)$. For f/g, we must exclude -3 since $g(-3) = 0$. This is not in $[3, \infty)$, so the domain of f/g is $[3, \infty)$. For g/f, we must exclude 3 since $f(3) = 0$. The domain of g/f is $(3, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = \sqrt{x-3} + \sqrt{x+3}$
$(f-g)(x) = f(x) - g(x) = \sqrt{x-3} - \sqrt{x+3}$
$(fg)(x) = f(x)\cdot g(x) = \sqrt{x-3}\cdot\sqrt{x+3} = \sqrt{x^2-9}$
$(ff)(x) = f(x)\cdot f(x) = \sqrt{x-3}\cdot\sqrt{x-3} = |x-3|$
$(f/g)(x) = \frac{\sqrt{x-3}}{\sqrt{x+3}}$
$(g/f)(x) = \frac{\sqrt{x+3}}{\sqrt{x-3}}$

24. $f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$

a) The domain of f is $[0, \infty)$. The domain of g is $(-\infty, 2]$. Then the domain of $f+g$, $f-g$, and fg is $[0, 2]$. The domain of ff is the same as the domain of f, $[0, \infty)$. Since $g(2) = 0$, the domain of f/g is $[0, 2)$. Since $f(0) = 0$, the domain of g/f is $(0, 2]$.

b) $(f+g)(x) = \sqrt{x} + \sqrt{2-x}$
$(f-g)(x) = \sqrt{x} - \sqrt{2-x}$
$(fg)(x) = \sqrt{x}\cdot\sqrt{2-x} = \sqrt{2x-x^2}$
$(ff)(x) = \sqrt{x}\cdot\sqrt{x} = \sqrt{x^2} = |x|$
$(f/g)(x) = \frac{\sqrt{x}}{\sqrt{2-x}}$
$(g/f)(x) = \frac{\sqrt{2-x}}{\sqrt{x}}$

25. $f(x) = x + 1$, $g(x) = |x|$

a) The domain of f and of g is $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty, \infty)$. For f/g, we must

fg and ff is $(-\infty, \infty)$. Since $g\left(-\frac{2}{5}\right) = 0$, the domain of f/g is $\left(-\infty, -\frac{2}{5}\right) \cup \left(-\frac{2}{5}, \infty\right)$. Since $f(1) = 0$ and $f(-1) = 0$, the domain of g/f is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

exclude 0 since $g(0) = 0$. The domain of f/g is $(-\infty, 0) \cup (0, \infty)$. For g/f, we must exclude -1 since $f(-1) = 0$. The domain of g/f is $(-\infty, -1) \cup (-1, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = x + 1 + |x|$ $(f-g)(x) = f(x) - g(x) = x + 1 - |x|$ $(fg)(x) = f(x) \cdot g(x) = (x+1)|x|$

$(ff)(x) = f(x) \cdot f(x) = (x+1)(x+1) = x^2 + 2x + 1$

$(f/g)(x) = \frac{x+1}{|x|}$

$(g/f)(x) = \frac{|x|}{x+1}$

26. $f(x) = 4|x|$, $g(x) = 1 - x$

a) The domain of f and of g is $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty, \infty)$. Since $g(1) = 0$, the domain of f/g is $(-\infty, 1) \cup (1, \infty)$. Since $f(0) = 0$, the domain of g/f is $(-\infty, 0) \cup (0, \infty)$.

b) $(f+g)(x) = 4|x| + 1 - x$

$(f-g)(x) = 4|x| - (1-x) = 4|x| - 1 + x$
$(fg)(x) = 4|x|(1-x) = 4|x| - 4x|x|$

$(ff)(x) = 4|x| \cdot 4|x| = 16x^2$

$(f/g)(x) = \frac{4|x|}{1-x}$

$(g/f)(x) = \frac{1-x}{4|x|}$

27. $f(x) = x^3$, $g(x) = 2x^2 + 5x - 3$

a) Since any number can be an input for either f or g, the domain of f, g, $f+g$, $f-g$, fg, and ff is the set of all real numbers, or $(-\infty, \infty)$.

Since $g(-3) = 0$ and $g\left(\frac{1}{2}\right) = 0$, the domain of f/g is $(-\infty, -3) \cup \left(-3, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

Since $f(0) = 0$, the domain of g/f is $(-\infty, 0) \cup (0, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = x^3 + 2x^2 + 5x - 3$ $(f-g)(x) = f(x) - g(x) = x^3 - (2x^2 + 5x - 3) =$

$x^3 - 2x^2 - 5x + 3$

$(fg)(x) = f(x) \cdot g(x) = x^3(2x^2 + 5x - 3) =$

$2x^5 + 5x^4 - 3x^3$

$(ff)(x) = f(x) \cdot f(x) = x^3 \cdot x^3 = x^6$

$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{x^3}{2x^2 + 5x - 3}$

$(g/f)(x) = \frac{g(x)}{f(x)} = \frac{2x^2 + 5x - 3}{x^3}$

28. $f(x) = x^2 - 4$, $g(x) = x^3$

a) The domain of f and of g is $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty, \infty)$. Since $g(0) = 0$, the domain of f/g is $(-\infty, 0) \cup (0, \infty)$. Since f

b) $(f+g)(x) = x^2 - 4 + x^3$, or $x^3 + x^2 - 4$

$(f-g)(x) = x^2 - 4 - x^3$, or $-x^3 + x^2 - 4$ $(fg)(x) = (x^2 - 4)(x^3) = x^5 - 4x^3$

$(ff)(x) = (x^2 - 4)(x^2 - 4) = x^4 - 8x^2 + 16$

$(f/g)(x) = \frac{x^2 - 4}{x^3}$

$(g/f)(x) = \frac{x^3}{x^2 - 4}$

29. $f(x) = \frac{4}{x+1}$, $g(x) = \frac{1}{6-x}$

a) Since $x + 1 = 0$ when $x = -1$, we must exclude -1 from the domain of f. It is $(-\infty, -1) \cup (-1, \infty)$. Since $6 - x = 0$ when $x = 6$, we must exclude 6 from the domain of g. It is $(-\infty, 6) \cup (6, \infty)$. The domain of $f+g$, $f-g$, and fg is the intersection of the domains of f and g, or $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$.
The domain of ff is the same as the domain of f, or $(-\infty, -1) \cup (-1, \infty)$. Since there are no values of x for which $g(x) = 0$ or $f(x) = 0$, the domain of f/g and g/f is $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = \frac{4}{x+1} + \frac{1}{6-x}$

$(f-g)(x) = f(x) - g(x) = \frac{4}{x+1} - \frac{1}{6-x}$

$(fg)(x) = f(x) \cdot g(x) = \frac{4}{x+1} \cdot \frac{1}{6-x} = \frac{4}{(x+1)(6-x)}$

$(ff)(x) = f(x) \cdot f(x) = \frac{4}{x+1} \cdot \frac{4}{x+1} = \frac{16}{(x+1)^2}$, or $\frac{16}{x^2 + 2x + 1}$

$\frac{4}{x+1} \qquad \frac{4}{6-x} \qquad 4(6-x)$

$(f/g)(x) = \frac{1}{x+1} = x+1$. 1 =

6 − x

$(g/f)(x) = \frac{\frac{1}{6-x}}{4} = \frac{1}{6-x} \cdot \frac{x+1}{4} = \frac{x+1}{4(6-x)}$

=

x + 1

(−2) = 0 and f (2) = 0, the domain of g/f is (−∞, −2) ∪ (−2, 2) ∪ (2, ∞).

30. $f(x) = 2x^2$, $g(x) = \frac{2}{x-5}$

a) The domain of f is (−∞, ∞). Since x − 5 = 0 when x = 5, the domain of g is (−∞, 5)∪(5, ∞). Then the domain of f + g, f − g , and f g is (−∞ , 5) ∪ (5, ∞). The domain of f f is (−∞, ∞). Since there are no values of x for which g(x) = 0, the domain of f/g is (−∞, 5) ∪ (5, ∞). Since f (0) = 0, the domain of g/f is (−∞, 0) ∪ (0, 5) ∪ (5, ∞).

b) $(f+g)(x) = 2x^2 + \frac{2}{x-5}$

$(f-g)(x) = 2x^2 - \frac{2}{x-5}$

$(fg)(x) = 2x^2 \cdot \frac{2}{x-5} = \frac{4x^2}{x-5}$

$(ff)(x) = 2x^2 \cdot 2x^2 = 4x^4$

$(f/g)(x) = \frac{2x^2}{\frac{x-5}{2}} = 2x^2 \cdot \frac{5}{2} = x^2(x-5) = x^3 - 5x^2$

$(g/f)(x) = \frac{x-5}{2x^2} = \frac{2}{x-5} \cdot \frac{1}{2x^2} = \frac{1}{x^2(x-5)} = \frac{1}{x^3-5x^2}$

31. $f(x) = \frac{1}{x}$, $g(x) = x - 3$

a) Since $f(0)$ is not defined, the domain of f is $(-\infty, 0) \cup (0, \infty)$. The domain of g is $(-\infty, \infty)$. Then the domain of $f + g$, $f - g$, fg, and ff is $(-\infty, 0) \cup (0, \infty)$. Since $g(3) = 0$, the domain of f/g is $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$. There are no values of x for which $f(x) = 0$, so the domain of g/f is $(-\infty, 0) \cup (0, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = \frac{1}{x} + x - 3$

$(f-g)(x) = f(x) - g(x) = \frac{1}{x} - (x-3) = \frac{1}{x} - x + 3$

$(fg)(x) = f(x) \cdot g(x) = \frac{1}{x} \cdot (x-3) = \frac{x-3}{x}$, or $1 - \frac{3}{x}$

$(ff)(x) = f(x) \cdot f(x) = \frac{1}{x} \cdot \frac{1}{x} = \frac{1}{x^2}$

$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{\frac{1}{x}}{x-3} = \frac{1}{x} \cdot \frac{1}{x-3} = \frac{1}{x(x-3)}$

$(g/f)(x) = \frac{g(x)}{f(x)} = \frac{x-3}{\frac{1}{x}} = (x-3) \cdot \frac{x}{1} = x(x-3)$, or $x^2 - 3x$

32. $f(x) = \sqrt{x+6}$, $g(x) = \frac{1}{x}$

a) The domain of $f(x)$ is $[-6, \infty)$. The domain of $g(x)$ is $(-\infty, 0) \cup (0, \infty)$. Then the domain of $f + g$, $f - g$, and fg is $[-6, 0) \cup (0, \infty)$. The domain of ff is $[-6, \infty)$. Since there are no values of x for which $g(x) = 0$, the domain of f/g is $[-6, 0) \cup (0, \infty)$. Since $f(-6) = 0$, the domain of g/f is $(-6, 0) \cup (0, \infty)$.

b) $(f+g)(x) = \sqrt{x+6} + \frac{1}{x}$

$(f-g)(x) = \sqrt{x+6} - \frac{1}{x}$

$(fg)(x) = \sqrt{x+6} \cdot \frac{1}{x} = \frac{\sqrt{x+6}}{x}$

33. $f(x) = \frac{3}{x-2}$, $g(x) = \sqrt{x-1}$

a) Since $f(2)$ is not defined, the domain of f is $(-\infty, 2) \cup (2, \infty)$. Since $g(x)$ is nonnegative for val-ues of x in $[1, \infty)$, this is the domain of g. The domain of $f + g$, $f - g$, and fg is the intersection of the domains of f and g, or $[1, 2) \cup (2, \infty)$. The domain of ff is the same as the domain of f, or $(-\infty, 2) \cup (2, \infty)$. For f/g, we must exclude 1 since $g(1) = 0$, so the domain of f/g is $(1, 2) \cup (2, \infty)$. There are no values of x for which $f(x) = 0$, so the domain of g/f is $[1, 2) \cup (2, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = \frac{3}{x-2} + \sqrt{x-1}$

$(f-g)(x) = f(x) - g(x) = \frac{3}{x-2} - \sqrt{x-1}$

$(fg)(x) = f(x) \cdot g(x) = \frac{3}{x-2}(\sqrt{x-1})$, or $\frac{3\sqrt{x-1}}{x-2}$

$(ff)(x) = f(x) \cdot f(x) = \frac{3}{x-2} \cdot \frac{3}{x-2} = \frac{9}{(x-2)^2}$

$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{\frac{3}{x-2}}{\sqrt{x-1}} = \frac{3}{(x-2)\sqrt{x-1}}$

$(g/f)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{x-1}}{\frac{3}{x-2}} = \frac{(x-2)\sqrt{x-1}}{3}$

34. $f(x) = \frac{2}{4-x}$, $g(x) = \frac{5}{x-1}$

a) The domain of f is $(-\infty, 4) \cup (4, \infty)$. The domain of g is $(-\infty, 1) \cup (1, \infty)$. The domain of $f + g$, $f - g$, and fg is $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$. The domain of ff is $(-\infty, 4) \cup (4, \infty)$. The domain of f/g and g/f is $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$.

b) $(f+g)(x) = \frac{2}{4-x} + \frac{5}{x-1}$

$(f-g)(x) = \frac{2}{4-x} - \frac{5}{x-1}$

$(fg)(x) = \frac{2}{4-x} \cdot \frac{5}{x-1} = \frac{10}{(4-x)(x-1)}$

$(ff)(x) = \frac{2}{4-x} \cdot \frac{2}{4-x} = \frac{4}{(4-x)^2}$

$$(ff)(x) = \sqrt{x+6}\cdot\sqrt{x+6} = |x+6|$$

$$(f/g)(x) = \frac{\sqrt{x+6}}{\frac{1}{x}} = \sqrt{x+6}\cdot\frac{x}{1} = x\sqrt{x+6}$$

$$(g/f)(x) = \frac{\frac{1}{x}}{\sqrt{x+6}} = \frac{1}{x}\cdot\frac{1}{\sqrt{x+6}} = \frac{1}{x\sqrt{x+6}}$$

$$(f/g)(x) = \frac{\frac{2}{4-x}}{\frac{5}{x-1}} = \frac{2(x-1)}{5(4-x)}$$

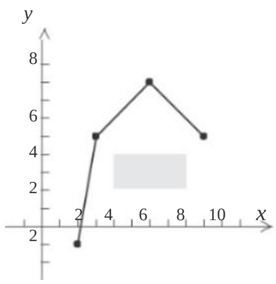
$$(g/f)(x) = \frac{\frac{5}{x-1}}{\frac{2}{4-x}} = \frac{5(4-x)}{2(x-1)}$$

35. From the graph we see that the domain of F is [2, 11] and the domain of G is [1, 9]. The domain of $F + G$ is the set of numbers in the domains of both F and G. This is [2, 9].

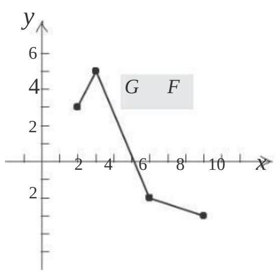
36. The domain of $F - G$ and FG is the set of numbers in the domains of both F and G. (See Exercise 35.) This is [2, 9].

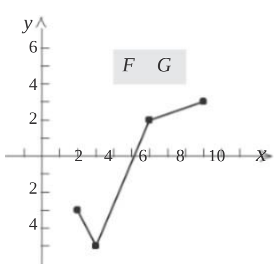
The domain of *F/G* is the set of numbers in the domains of both *F* and *G*, excluding those for which $G = 0$. Since $G > 0$ for all values of *x* in its domain, the domain of *F/G* is [2, 9].

37. The domain of *G/F* is the set of numbers in the domains of both *F* and *G* (See Exercise 35.), excluding those for which $F = 0$. Since $F(3) = 0$, the domain of *G/F* is $[2, 3) \cup (3, 9]$.



39.





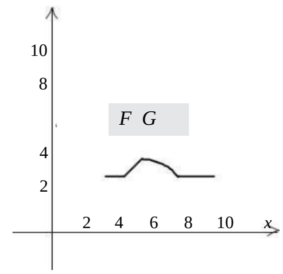
41. From the graph, we see that the domain of *F* is [0, 9] and the domain of *G* is [3, 10]. The domain of $F + G$ is the set of numbers in the domains of both *F* and *G*. This is [3, 9].

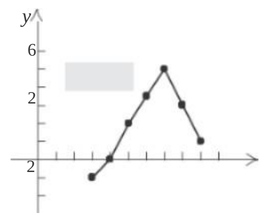
42. The domain of $F - G$ and $F\,G$ is the set of numbers in the domains of both *F* and *G*. (See Exercise 41.) This is [3, 9].

The domain of *F/G* is the set of numbers in the domains of both *F* and *G*, excluding those for which $G = 0$. Since $G > 0$ for all values of *x* in its domain, the domain of *F/G* is [3, 9].

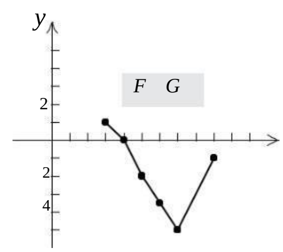
43. The domain of *G/F* is the set of numbers in the domains of both *F* and *G* (See Exercise 41.), excluding those for which $F = 0$. Since $F(6) = 0$ and $F(8) = 0$, the domain of *G/F* is $[3, 6) \cup (6, 8) \cup (8, 9]$.

44. $(F + G)(x) = F(x) + G(x)$





46.



47. a) $P(x) = R(x) - C(x) = 60x - 0.4x^2 - (3x + 13) =$

$6000 - 4000 = 2000$

$C(100) = 3 \cdot 100 + 13 = 300 + 13 = 313$

$P(100) = R(100) - C(100) = 2000 - 313 = 1687$

48. a) $P(x) = 200x - x^2 - (5000 + 8x) =$

$200x - x^2 - 5000 - 8x = -x^2 + 192x - 5000$

b) $R(175) = 200(175) - 175^2 = 4375$

$C(175) = 5000 + 8 \cdot 175 = 6400$

$P(175) = R(175) - C(175) = 4375 - 6400 = -2025$

(We could also use the function found in part (a) to find $P(175)$.)

49. $f(x) = 3x - 5$

$f(x + h) = 3(x + h) - 5 = 3x + 3h - 5$

$$\frac{f(x+h) - f(x)}{h} = \frac{3x + 3h - 5 - (3x - 5)}{h}$$

$$= \frac{3x + 3h - 5 - 3x + 5}{h}$$

$$= \frac{3h}{h} = 3$$

50. $f(x) = 4x - 1$

$$\frac{f(x+h)-f(x)}{h} = \frac{4(x+h)-1-(4x-1)}{h} = \frac{4x+4h-1-4x+1}{h} = \frac{4h}{h} = 4$$

51. $f(x) = 6x + 2$

$f(x+h) = 6(x+h) + 2 = 6x + 6h + 2$

$$\frac{f(x+h)-f(x)}{h} = \frac{6x+6h+2-(6x+2)}{h} = \frac{6x+6h+2-6x-2}{h} = \frac{6h}{h} = 6$$

52. $f(x) = 5x + 3$

$$\frac{f(x+h)-f(x)}{h} = \frac{5(x+h)+3-(5x+3)}{h} = \frac{5x+5h+3-5x-3}{h} = \frac{5h}{h} = 5$$

53. $f(x) = \frac{1}{3}x + 1$

$f(x+h) = \frac{1}{3}(x+h) + 1 = \frac{1}{3}x + \frac{1}{3}h + 1$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{3}x+\frac{1}{3}h+1-\left(\frac{1}{3}x+1\right)}{h} = \frac{\frac{1}{3}x+\frac{1}{3}h+1-\frac{1}{3}x-1}{h} = \frac{\frac{1}{3}h}{h} = \frac{1}{3}$$

54. $f(x) = -\frac{1}{2}x + 7$

$$\frac{f(x+h)-f(x)}{h} = \frac{-\frac{1}{2}(x+h)+7-\left(-\frac{1}{2}x+7\right)}{h} = \frac{-\frac{1}{2}x-\frac{1}{2}h+7+\frac{1}{2}x-7}{h} = \frac{-\frac{1}{2}h}{h} = -\frac{1}{2}$$

55. $f(x) = \frac{1}{3x}$

$f(x+h) = \frac{1}{3(x+h)}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{3(x+h)}-\frac{1}{3x}}{h} = \frac{\frac{1}{3(x+h)}\cdot\frac{x}{x}-\frac{1}{3x}\cdot\frac{x+h}{x+h}}{h} = \frac{\frac{x}{3x(x+h)}-\frac{x+h}{3x(x+h)}}{h}$$

$$= \frac{\frac{x-x-h}{3x(x+h)}}{h} = \frac{\frac{-h}{3x(x+h)}}{h} = \frac{-h}{3x(x+h)}\cdot\frac{1}{h} = \frac{-1\cdot h}{3x(x+h)\cdot h} = \frac{-1}{3x(x+h)}, \text{ or } -\frac{1}{3x(x+h)}$$

56. $f(x) = \frac{1}{2x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{2(x+h)}-\frac{1}{2x}}{h} = \frac{\frac{1}{2(x+h)}\cdot\frac{x}{x}-\frac{1}{2x}\cdot\frac{x+h}{x+h}}{h} = \frac{\frac{x-x-h}{2x(x+h)}}{h} = \frac{\frac{-h}{2x(x+h)}}{h}$$

$$= \frac{-h}{2x(x+h)}\cdot\frac{1}{h} = \frac{-1}{2x(x+h)}, \text{ or } -\frac{1}{2x(x+h)}$$

57. $f(x) = -\frac{1}{4x}$

$f(x+h) = -\frac{1}{4(x+h)}$

f (x + h) – f (x)

4(x + h) 4x h =

$$= \frac{\frac{1}{4(x+h)} \cdot \frac{x}{x} - \frac{1}{4x} \cdot \frac{x+h}{x+h}}{h}$$

$$= \frac{\frac{x}{4x(x+h)} + \frac{x+h}{4x(x+h)}}{h}$$

$$= \frac{-x + x + h}{4x(x+h)} = \frac{h}{4x(x+h)}$$

h

$$= \frac{h}{4x(x+h)} \cdot \frac{1}{h} = \frac{\not{h} \cdot 1}{4x(x+h)\,\not{h}} = \frac{1}{4x(x+h)}$$

58. $f(x) = -\frac{1}{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{-\frac{1}{x+h}-\left(-\frac{1}{x}\right)}{h} = \frac{-\frac{1}{x+h}+\frac{1}{x}}{h} = \frac{\frac{-x+x+h}{x(x+h)}}{h} = \frac{\frac{h}{x(x+h)}}{h} = \frac{h}{x(x+h)}\cdot\frac{1}{h} = \frac{1}{x(x+h)}$$

59. $f(x) = x^2 + 1$

$f(x+h) = (x+h)^2 + 1 = x^2 + 2xh + h^2 + 1$

$$\frac{f(x+h)-f(x)}{h} = \frac{x^2+2xh+h^2+1-(x^2+1)}{h} = \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h} = \frac{h}{h}\cdot\frac{2x+h}{1} = 2x+h$$

60. $f(x) = x^2 - 3$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-3-(x^2-3)}{h} = \frac{x^2+2xh+h^2-3-x^2+3}{h} = \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h} = 2x+h$$

61. $f(x) = 4 - x^2$

$f(x+h) = 4-(x+h)^2 = 4-(x^2+2xh+h^2) = 4-x^2-2xh-h^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{4-x^2-2xh-h^2-(4-x^2)}{h} = \frac{4-x^2-2xh-h^2-4+x^2}{h}$$

63. $f(x) = 3x^2 - 2x + 1$

$f(x+h) = 3(x+h)^2 - 2(x+h) + 1 = 3(x^2+2xh+h^2) - 2(x+h) + 1 = 3x^2+6xh+3h^2-2x-2h+1$

$$\frac{f(x+h)-f(x)}{h} = \frac{(3x^2+6xh+3h^2-2x-2h+1)-(3x^2-2x+1)}{h} = \frac{3x^2+6xh+3h^2-2x-2h+1-3x^2+2x-1}{h} = \frac{6xh+3h^2-2h}{h} = \frac{h(6x+3h-2)}{h\cdot 1} = \frac{h}{h}\cdot\frac{6x+3h-2}{1} = 6x+3h-2$$

64. $f(x) = 5x^2 + 4x$

$$\frac{f(x+h)-f(x)}{h} = \frac{(5x^2+10xh+5h^2+4x+4h)-(5x^2+4x)}{h} = \frac{10xh+5h^2+4h}{h} = 10x+5h+4$$

65. $f(x) = 4 + 5|x|$

$f(x+h) = 4 + 5|x+h|$

$$\frac{f(x+h)-f(x)}{h} = \frac{4+5|x+h|-(4+5|x|)}{h} = \frac{4+5|x+h|-4-5|x|}{h} = \frac{5|x+h|-5|x|}{h}$$

66. $f(x) = 2|x| + 3x$

$$\frac{f(x+h)-f(x)}{h} = \frac{(2|x+h|+3x+3h)-(2|x|+3x)}{h} = \frac{2|x+h|-2|x|+3h}{h}$$

67. $f(x) = x^3$

2 − 2xh − h 2 −

$f(x+h) = (x+h)^3 = x^3 + 3x^2 h + 3xh^2 + h^3$ $f(x)$

$= x^3$

$= \frac{-2xh - h^2}{h} = \frac{h(-2x - h)}{h}$

$= -2x - h$

62. $f(x) = 2 - x^2$

$\frac{f(x+h) - f(x)}{h} = \frac{2 - (x+h)^2 - (2 - x^2)}{h} = \frac{2 - x^2 - 2xh - h^2 - 2 + x^2}{h}$ ... $2xh - h$

$\frac{f(x+h) - f(x)}{h} = \frac{x^3 + 3x^2 h + 3xh^2 + h^3 - x^3}{h} =$

$\frac{3x^2 h + 3xh^2 + h^3}{h} = \frac{h(3x^2 + 3xh + h^2)}{h \cdot 1} =$

$\frac{h}{} \cdot 3x^2 + 3xh + h^2 = 3x^2 + 3xh + h^2$

2 + x 2 − 2

$h1$

68. $f(x) = x^3 - 2x$

$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - 2(x+h) - (x^3 - 2x)}{h}$

$= \quad h \quad =$

h

$\frac{(-2x - h)}{}$

$-2x - h$

$= h$

$=$

$h \qquad h$

$\frac{3 + 3x^2 h + 3xh^2 + h^3 - 2x - 2h - x^3}{}$

$\frac{+2x}{h} \quad =$

$\frac{3x^2 h + 3xh^2 + h^3 - 2h}{h} = \frac{(3x^2 + 3xh + h^2 - 2)}{h} =$

$3x^2 + 3xh + h^2 - 2$