Solution Manual for College Algebra 7th Edition Stewart Redlin Watson ISBN 1305115546 9781305115545

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PROLOGUE: Principles of Problem Solving

Let r be the rate of the descent. We use the formula time $\frac{\text{distance}}{\text{rate}}$; the ascent takes $\frac{1}{15}$ h, the descent takes $\frac{1}{r}$ h, and the total trip should take $\frac{2}{30}$ $\frac{1}{15}$ h. Thus we have $\frac{1}{15}$ $\frac{1}{r}$ $\frac{1}{15}$ $\frac{1}{r}$ 0, which is impossible. So the car cannot go fast enough to average 30 mi/h for the 2-mile trip.

Let us start with a given price P. After a discount of 40%, the price decreases to 0 6P. After a discount of 20%, the price decreases to 0 8P, and after another 20% discount, it becomes 0 8 0 8P 0 64P. Since 0 6P 0 64P, a 40% discount is better.

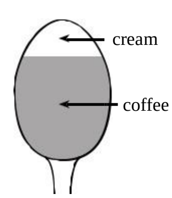
We continue the pattern. Three parallel cuts produce 10 pieces. Thus, each new cut produces an additional 3 pieces. Since the first cut produces 4 pieces, we get the formula f n 4 3 n 1 , n 1. Since f 142 4 3 141 427, we see that 142 parallel cuts produce 427 pieces.

By placing two amoebas into the vessel, we skip the first simple division which took 3 minutes. Thus when we place two amoebas into the vessel, it will take 60 3 57 minutes for the vessel to be full of amoebas.
The statement is false. Here is one particular counterexample:

	Player A	Player B
First half	1 hit in 99 at-bats: average $\frac{1}{99}$	0 hit in 1 at-bat: average $\frac{0}{1}$
Second half	1 hit in 1 at-bat: average $\frac{1}{1}$	98 hits in 99 at-bats: average $\frac{98}{99}$
Entire season	2 hits in 100 at-bats: average $\frac{2}{100}$	

Method 1: After the exchanges, the volume of liquid in the pitcher and in the cup is the same as it was to begin with. Thus, any coffee in the pitcher of cream must be replacing an equal amount of cream that has ended up in the coffee cup.

Method 2: Alternatively, look at the drawing of the spoonful of coffee and cream mixture being returned to the pitcher of cream. Suppose it is possible to separate the cream and the coffee, as shown. Then you can see that the coffee going into the cream occupies the same volume as the cream that was left in the coffee.



Method 3 (an algebraic approach): Suppose the cup of coffee has y spoonfuls of coffee. When one spoonful of cream is added to the coffee cup, the resulting mixture has the following ratios: $\frac{\text{cream}}{\text{mixture}}$ $\frac{1}{y\ 1}$ and $\frac{\text{coffee}}{\text{mixture}}$ $\frac{y}{y\ 1}$.

So, when we remove a spoonful of the mixture and put it into the pitcher of cream, we are really removing $\frac{1}{y+1}$ of a spoonful of cream and $\frac{y}{y+1}$ spoonful of coffee. Thus the amount of cream left in the mixture (cream in the coffee) is $\frac{y}{y+1}$ of a spoonful. This is the same as the amount of coffee we added to the cream.

Let r be the radius of the earth in feet. Then the circumference (length of the ribbon) is $2\pi r$. When we increase the radius by 1 foot, the new radius is $r+1$, so the new circumference is $2\pi(r+1)$. Thus you need $2\pi(r+1) - 2\pi r = 2\pi$ extra feet of ribbon.

The north pole is such a point. And there are others: Consider a point a_1 near the south pole such that the parallel passing through a_1 forms a circle C_1 with circumference exactly one mile. Any point P_1 exactly one mile north of the circle C_1 along a meridian is a point satisfying the conditions in the problem: starting at P_1 she walks one mile south to the point a_1 on the circle C_1, then one mile east along C_1 returning to the point a_1, then north for one mile to P_1. That's not all. If a point a_2 (or a_3, a_4, a_5,) is chosen near the south pole so that the parallel passing through it forms a circle C_2 (C_3, C_4, C_5,) with a circumference of exactly $\frac{1}{2}$ mile ($\frac{1}{3}$ mi, $\frac{1}{4}$ mi, $\frac{1}{5}$ mi,), then the point P_2 (P_3, P_4, P_5,) one mile north of a_2 (a_3, a_4, a_5,) along a meridian satisfies the conditions of the problem: she walks one mile south from P_2 (P_3, P_4, P_5,) arriving at a_2 (a_3, a_4, a_5,) along the circle C_2 (C_3, C_4, C_5,), walks east along the circle for one mile thus traversing the circle twice (three times, four times, five times,) returning to a_2 (a_3, a_4, a_5,), and then walks north one mile to P_2 (P_3, P_4, P_5,).

PREREQUISITES

P.1 MODELING THE REAL WORLD WITH ALGEBRA

Using this model, we find that if S 12, L 4S 4 12 48. Thus, 12 sheep have 48 legs.

If each gallon of gas costs \$3 50, then x gallons of gas costs \$3 5x. Thus, C 3 5x.

If x \$120 and T 0 06x, then T 0 06 120 7 2. The sales tax is \$7 20.

If x 62,000 and T 0 005x, then T 0 005 62,000 310. The wage tax is \$310.

If70, t 3 5, and dt, then d 70 3 5 245. The car has traveled 245 miles.

Vr^2h3^2 5 45141 4 in^3

7. (a) M $\frac{N}{G}$ $\frac{240}{8}$ 30 miles/gallon

(b) 25 $\frac{175}{G}$ G $\frac{175}{25}$ 7 gallons

8. (a) T 70 0 003h 70 0 003 1500 65 5 F

(b) 64 70 0 003h 0 003h 6 h 2000 ft

9. (a) V 9 5S 9 5 4 km^3 38 km^3

(b) 19 km^3 9 5S S 2 k m^3

10. (a) P 0 06s^3 0 06 12^{3}103 7 hp

(b) 7 5 0 06 s^3 s^3 125 so s 5 knots

11. (a)

Depth (ft)	Pressure (lb/in^2)
0	045 0 147 147
10	045 10 147 19 2
20	045 2 147 237
30	045 3 147 282
40	045 4 147 327
50	045 5 147 372
60	045 6 147 417

(b) We know that P 30 and we want to find d, so we solve the equation 30 14 7 0 45d 15 3 0 45d

d $\frac{15\ 3}{0\ 45}$ 34 0. Thus, if the pressure is 30 lb/in^2, the depth is 34 ft.

12. (a)

Population	Water use (gal)
0	0
1000	40 1000 40,000
2000	40 2000 80,000
3000	40 3000 120,000
4000	40 4000 160,000
5000	40 5000 200,000

We solve the equation 40x 120,000 x $\frac{120{,}000}{40}$ 3000. Thus, the population is about 3000.

The number N of cents in q quarters is N 25q.

The average A of two numbers, a and b, is A $\frac{a\ b}{2}$.

The cost C of purchasing x gallons of gas at \$3 50 a gallon is C 3 5x.

The amount T of a 15% tip on a restaurant bill of x dollars is T 0 15x.

The distance d in miles that a car travels in t hours at 60 mi/h is d 60t.

The speed r of a boat that travels d miles in 3 hours is $r = \frac{d}{3}$.

(a) $12 3 $1 $12 $3 $15

The cost C, in dollars, of a pizza with n toppings is C 12 n.

Using the model C 12 n with C 16, we get 16 12 n n 4. So the pizza has four toppings.

(a) 3 30 280 0 10 90 28 $118

(b) The cost is $\frac{\text{daily}}{\text{rental}}$ $\frac{\text{days}}{\text{rented}}$ $\frac{\text{cost}}{\text{per mile}}$ $\frac{\text{miles}}{\text{driven}}$, so C 30n 0 1m.

(c) We have C 140 and n 3. Substituting, we get 140 30 3 0 1m 140 90 0 1m 50 0 1m m 500. So the rental was driven 500 miles.

21. (a) **(i)** For an all-electric car, the energy cost of driving x miles is C_e 0 04x.

(ii) For an average gasoline powered car, the energy cost of driving x miles is C_g 0 12x.

(b) **(i)** The cost of driving 10,000 miles with an all-electric car is C_e 0 04 10,000 $400.

(ii) The cost of driving 10,000 miles with a gasoline powered car is C_g 0 12 10,000 $1200.

22. (a) If the width is 20, then the length is 40, so the volume is 20 20 40 16,000 in^3.

(b) In terms of width, V x x 2x 2x^3.

23. (a) The GPA is $\frac{4a\ \ 3b\ \ 2c\ \ 1d\ \ 0f}{a\ \ b\ \ c\ \ d\ \ f}$ $\frac{4a\ \ 3b\ \ 2c\ \ d}{a\ \ b\ \ c\ \ d\ \ f}$.

(b) Using a 2 3 6, b 4, c 3 3 9, and d f 0 in the formula from part (a), we find the GPA to be $\frac{46\ 34\ 29}{6\ \ 4\ \ 9}$ $\frac{54}{19}$ 2 84.

P.2 THE REAL NUMBERS

1. (a) The natural numbers are 1 2 3 .

(b) The numbers 3 2 1 0 are integers but not natural numbers.

(c) Any irreducible fraction $\frac{p}{q}$ with q 1 is rational but is not an integer. Examples: $\frac{3}{2}$, $\frac{5}{12}$, $\frac{1729}{23}$.

(d) Any number which cannot be expressed as a ratio $\frac{p}{q}$ of two integers is irrational. Examples are 2, 3, , and e.

2. (a) ab ba; Commutative Property of Multiplication

(b) a b c a b c; Associative Property of Addition

(c) a b c ab ac; Distributive Property

3. The set of numbers between but not including 2 and 7 can be written as **(a)** x 2 x 7 in interval notation, or **(b)** 2 7 in interval notation.

4. The symbol x stands for the *absolute value* of the number x. If x is not 0, then the sign of x is always *positive*.

5. The distance between a and b on the real line is d a b b a. So the distance between 5 and 2 is 2 5 7.

6. (a) Yes, the sum of two rational numbers is rational: $\frac{a}{b}$ $\frac{c}{d}$ $\frac{ad\ \ bc}{bd}$.

No, the sum of two irrational numbers can be irrational (2) or rational (0).

(a) No: a bb a b a in general.

No; by the Distributive Property, 2 a 52a2 52a 102a 10.

(a) Yes, absolute values (such as the distance between two different numbers) are always positive.

Yes, b aa b .

(a) Natural number: 100

Integers: 0, 100, 8

Rational numbers: 1 5, 0, $\frac{5}{2}$, 2 71, 3 14, 100, 8

Irrational numbers: 7,

ommutative Property of addition

Associative Property of addition

Distributive Property

Commutative Property of multiplication

x 3 3 x

4*AB* 4*A*4*B*

3 x y 3x 3y

4 2m4 2 m 8m

$\frac{5}{2}$ 2x 4y $\frac{5}{2}$ 2x $\frac{5}{2}$ 4y 5x 10y

(a) $\frac{3}{10}$ $\frac{4}{15}$ $\frac{9}{30}$ $\frac{8}{30}$ $\frac{17}{30}$

$\frac{1}{4}$ $\frac{1}{5}$ $\frac{5}{20}$ $\frac{4}{20}$ $\frac{9}{20}$

31. (a) $\frac{2}{3}$ 6 $\frac{3}{2}$ $\frac{2}{3}$ 6 $\frac{2}{3}$ $\frac{3}{2}$ 4 1 3

1 4 12 1 5 4 13 1 13

(b) 3 4 1 5 4 4 5 5 4 5 20

(a) 2 3 6 and 2 $\frac{7}{2}$ 7, so 3 $\frac{7}{2}$

67

35 $\frac{7}{2}$

(a) False

(b) True

37. (a) True **(b)** False

39. (a) x 0 **(b)** t 4

(c) a p 3 5 **(d)** 5 x $\frac{1}{3}$

(a) A B 1 2 3 4 5 6 7 8

(a) Natural number: 16 4

Integers: 500, 16, $\frac{20}{4}$

Rational numbers: 1 3, 1 3333, 5 34, 500, 1 $\frac{2}{3}$, 16, $\frac{246}{579}$, $\frac{20}{5}$

Irrational number: 5

Commutative Property of multiplication

Distributive Property

Distributive Property

Distributive Property

7 3x7 3 x

5x 5y 5 x y

24. a b 8 8a 8b

26. $\frac{4}{3}$ 6y $\frac{4}{3}$ 6 y 8y

3a b c 2d 3ab 3ac 6ad

a) $\frac{2}{3}$ $\frac{3}{5}$ $\frac{10}{15}$ $\frac{9}{15}$ $\frac{1}{15}$

1 $\frac{5}{8}$ 1 6 $\frac{24}{24}$ $\frac{15}{24}$ 24 4 $\frac{35}{24}$

32. (a) 2 2 2 3 3 2 2 3 2 3 3 1 3 9 3 1 3 8 3

(b) $\frac{2}{5}$ $\frac{1}{2}$ ... 10 15 10 5 10 5 $\frac{10}{10}$ 14 25 $\frac{9}{3}$ 3

(a) 3 $\frac{2}{3}$ 2 and 3 0 67 2 01, so $\frac{2}{3}$ 0 67

$\frac{2}{3}$ 067

AB 246

067067

(a) False: 3 1 73205 1 7325.

−

False

38.(a) True **(b)** True

40.(a) *y* 0 **(b)** *z* 1

(c) *b* 8 **(d)** 017

*y*2

(a) *B* *C*2 4 6 7 8 9 10

B *C* 8

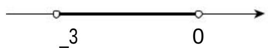
(a) A C1 2 3 4 5 6 7 8 9 10

A C 7

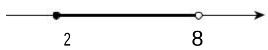
(a) B Cx x 5

B C x 1 x 4

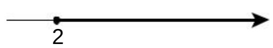
3 0x 3 x 0



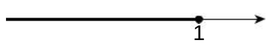
[2 8x 2 x 8



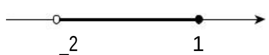
[2x x 2



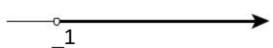
x 1 x1]



2 x 1 x2 1]



x1 x1



59. (a) [3 5] (b) 3 5]

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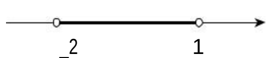
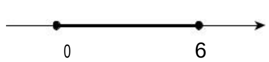


[46] [08 [06]



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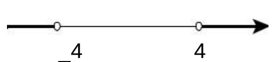


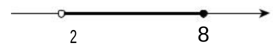
(a) A B C1 2 3 4 5 6 7 8 9 10

$ABC\varnothing$

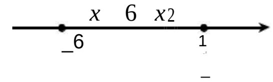
(a) A Cx 1 x 5

A B x 2 x 4

2 8] x 2 x 8

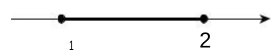


50. 6 $\frac{1}{2}$ x 6 x2 $\frac{1}{}$

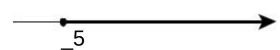


1x x 1

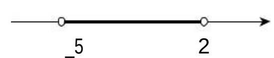
1 x 2 x [1 2]



x5 x [5



5 x 2 x5 2



60. (a) [0 2 (b) 2 0]

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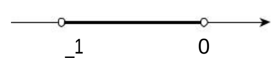
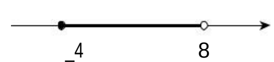
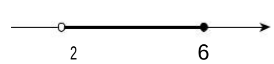


[46] [08 [48



66. 6] 210 26]



67. (a) 100 100

(b) 73 73

68. (a) 5 5 5 555, since 5 5.

(b) 10 , since 10.

69. (a) 646 42 2

(b) 1 1 11 1

70. (a) 2122 1210 10

(b) 1 111 1 11 01

(b) 1 15 5 5

71. (a) 2 6 3 12 12 **(b)**

72. (a) 7 12 246 12 7 5 1 4 5 1 4 1 1

73. 2355

74. 251544

75. (a) 17 2 15

(b) 21 3 21 3 24 24

(c) 3 11 12 55 67 67

76. (a) 7 15 1 21 49 105 5 105 54 105 18 35 18 35

(b) 38 38 57

(c) 2 61826 1808 08.

(a) Let x 0 777 . So 10x 7 7777x 0 77779x 7. Thus, x 7 9 .

Let x 0 2888 . So 100x 28 888810x 2 888890x 26. Thus, x 26 90 13 45 .

Let x 0 575757 . So 100x 57 5757x 0 575799x 57. Thus, x 57 99 19 33 .

(a) Let x 5 2323 . So 100x 523 23231x 5 232399x 518. Thus, x 518 99 .

Let x 1 3777 . So 100x 137 777710x 13 777790x 124. Thus, x 124 90 62 45 .

Let x 2 13535 . So 1000x 2135 353510x 21 3535990x 2114. Thus, x 2114 990 1057 495 .

79. 3, so33.

80. 2 1, so 1 2 2 1.

81. a b, so a ba b b a.

82. a b a b a b b a 2b

(a) a is negative because a is positive.

bc is positive because the product of two negative numbers is positive.

a bab is positive because it is the sum of two positive numbers.

ab ac is negative: each summand is the product of a positive number and a negative number, and the sum of two negative numbers is negative.

(a) b is positive because b is negative.

a bc is positive because it is the sum of two positive numbers.

c a ca is negative because c and a are both negative.

ab^2 is positive because both a and b^2 are positive.

Distributive Property

86.

Day	T_O	T_G	$T_O \quad T_G$	$T_O \quad T_G$
Sunday	68	77	9	9
Monday	72	75	3	3
Tuesday	74	74	0	0
Wednesday	80	75	5	5
Thursday	77	69	8	8
Friday	71	70	1	1
Saturday	70	71	1	1

$T_O \quad T_G$ gives more information because it tells us which city had the higher temperature.

(a) When L 60, x 8, and y 6, we have L 2 x y60 2 8 6 60 28 88. Because 88 108 the post office will accept this package.

When L 48, x 24, and y 24, we have L 2 x y 48 2 24 24 48 96 144, and since 144 108, the post office will *not* accept this package.

If x y 9, then L 2 9 9 108 L 36 108 L 72. So the length can be as long as 72 in. 6 ft.

88. Let x $\frac{m_1}{n_1}$ and y $\frac{m_2}{n_2}$ be rational numbers. Then x y $\frac{m_1}{n_1}$ $\frac{m_2}{n_2}$ $\frac{m_1n_2 \quad m_2n_1}{n_1n_2}$,

y $\frac{m_1}{n_1}$ $\frac{m_2}{n_2}$ $\frac{m_1n_2 \quad m_2n_1}{n_2n_1}$, and x y $\frac{m_1}{n_1}$ $\frac{m_2}{n_2}$ $\frac{m_1m_2}{n_1n_2}$. This shows that the sum, difference, and product of two rational numbers are again rational numbers. However the product of two irrational numbers is not necessarily irrational; for example, $\sqrt{2}$ $\sqrt{2}$ 2, which is rational. Also, the sum of two irrational numbers is not necessarily irrational; for example, $\sqrt{2}$ $\sqrt{2}$ 0 which is rational.

89. $\frac{1}{2}$ $\sqrt{2}$ is irrational. If it were rational, then by Exercise 6(a), the sum $\frac{1}{2}$ $\sqrt{2}$ $\frac{1}{2}$ $\sqrt{2}$ would be rational, but this is not the case.

Similarly, $\frac{1}{2}$ $\sqrt{2}$ is irrational.

Following the hint, suppose that r t q, a rational number. Then by Exercise 6(a), the sum of the two rational numbers r t and r is rational. But r t r t, which we know to be irrational. This is a contradiction, and hence our original premise—that r t is rational—was false.

r is rational, so r $\frac{a}{b}$ for some integers a and b. Let us assume that r t q, a rational number. Then by definition, q $\frac{c}{d}$ for some integers c and d. But then r t q $\frac{a}{b}$ t $\frac{c}{d}$, whence t $\frac{bc}{ad}$, implying that t is rational. Once again we have arrived at a contradiction, and we conclude that the product of a rational number and an irrational number is irrational.

90.

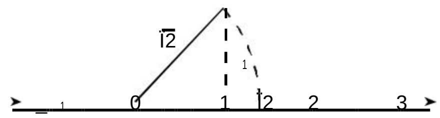
x	1	2	10	100	1000
$\frac{1}{x}$	1	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

As x gets large, the fraction 1 x gets small. Mathematically, we say that 1 x goes to zero.

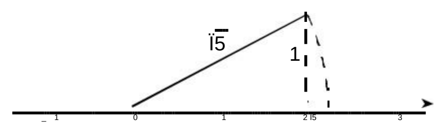
x	1	0 5	0 1	0 01	0 001
$\frac{1}{x}$	1	$\frac{1}{0\ 5}$ 2	$\frac{1}{0\ 1}$ 10	$\frac{1}{0\ 01}$ 100	$\frac{1}{0\ 001}$ 1000

As x gets small, the fraction 1 x gets large. Mathematically, we say that 1 x goes to infinity.

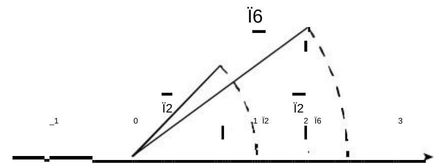
(a) Construct the number 2 on the number line by transferring the length of the hypotenuse of a right triangle with legs of length 1 and 1.



(b) Construct a right triangle with legs of length 1 and 2. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{1^2 \; 2^2}$ 5. Then transfer the length of the hypotenuse to the number line.



(c) Construct a right triangle with legs of length 2 and 2 [construct 2 as in part (a)]. By the Pythagorean Theorem, the length of the hypotenuse is 2 ^{2} 22 6. Then transfer the length of the hypotenuse to the number line.



(a) Subtraction is not commutative. For example, 5 1 1 5.

Division is not commutative. For example, 5 1 1 5.

Putting on your socks and putting on your shoes are not commutative. If you put on your socks first, then your shoes, the result is not the same as if you proceed the other way around.

Putting on your hat and putting on your coat are commutative. They can be done in either order, with the same result.

Washing laundry and drying it are not commutative.

Answers will vary.

Answers will vary.

Answers will vary.

(a) If *x* 2 and *y* 3, then *x* *y*2 35 5 and *xy*2 3 5.

If *x*2 and *y*3, then *x* *y*5 5 and *xy* 5.

If *x*2 and *y* 3, then *x* *y*2 3 1 and *xy* 5.

In each case, *x* *yxy* and the Triangle Inequality is satisfied.

(b) *Case 0:* If either *x* or *y* is 0, the result is equality, trivially.

Case 1: If *x* and *y* have the same sign, then *x* *y* *x* *y* if *x* and *y* are positive *x* *y* .
x *y* if *x* and *y* are negative

Case 2: If *x* and *y* have opposite signs, then suppose without loss of generality that *x* 0 and *y* 0. Then *x* *y* *x* *y* *x* *y* .

P.3 INTEGER EXPONENTS AND SCIENTIFIC NOTATION

1. Using exponential notation we can write the product 5 5 5 5 5 5 as 5^6.

2. Yes, there is a difference: 5^4 5 5 5 5 625, while 5^4 5 5 5 5 625.

3. In the expression 3^4, the number 3 is called the *base* and the number 4 is called the *exponent*.

4. When we multiply two powers with the same base, we *add* the exponents. So 3^4 3^5 3^9.

5. When we divide two powers with the same base, we *subtract* the exponents. So $\frac{3^5}{3^2}$ 3^3.

6. When we raise a power to a new power, we *multiply* the exponents. So $3^{4\;2}$ 3^8.

7. (a) 2 1 2 1 **(b)** 2 3 8 1 **(c)** 2 2 1 1 **(d)** 2 3 2^3 8 1

Scientists express very large or very small numbers using *scientific* notation. In scientific notation, 8,300,000 is 8 3 10^6 and 0 0000327 is 3 27 10 5.

9. (a) No, 2/3 2 3/2 2 9/4. **(b)** Yes, 5^4 625 and 5^4 5^4 625.

No, $2x^4$ 3 2^3 x^4 3 $8x^{12}$.

10. (a) No, x^2 3 $x^{2\,3}$ x^6.

2 3

1 2 3 1 3 27

6 6

11. (a) 2 64 **(b)** 2 64 **(c)** 5 3 5^2 25

2 2 2 2 2

12. (a) $5^3$125 **(b)** $5^3$125 8 **(c)** 5 2 5 5^2 4

5 0 1 2 3 1 1 1 2

13. (a) 3 2 1 2

3

(a) 2 3 2 0 1 3 1 28 – 3 0 3 2 3 3 27

(b) 2 2 28 **(c)** 3 2 3 8

2 0 2 6

(a) 5^3 5 5^4 625 **(b)** 3 3 3 9 **(c)** 2^2 3 2 64

2 8

(a) 3^8 3^5 3^{13} 1,594,323 **(b)** 6^0 6 6 **(c)** 5^4 5 390,625

10^7

(a) 5^4 5 2 5^2 25 **(b)** 10 10^3 1000 **(c)** 2 1 1 2 9

4 3 4 1 1

5^4 5

1 1 4

18. (a) 3 3 3 1 3 4 3 81 **(b)** 5 3 5 125 **(c)** 7 7^3 343

(a) $x^{2\,3}x$ 2 3 x 5 x **(b)** x^2 3 1 3 x 2 3 6 x **(c)** t x 3 5 t t x 3 5 t 2 x

(a) y^5 y^2 y^5 2 y^7 **(b)** $8x^2$ 8^2x^2 $64x^2$ **(c)** 4 x 3 4 3

(a) x^5 x^3 $x^{5\,3}$ x^2 1/x^2 1 **(b)** 2 4 52 4 51

$y^{10}y^0$ / y^7y 10 0 7 y^3

(c)

22. (a) y^2 y^5 $y^{2\,5}$ y^3 1/y^3 **(b)** z^5z 3z 4 z^5 3 4 z^2 z 1/2 **(c)** x^6/x^{10} $x^{6\,10}$ x^4 1/x^4

23. (a) $\frac{a^9 a^{-2}}{a} = a^{9-2-1} = a^6$

(b) $(a^2a^4)^3 = (a^{2+4})^3 = (a^6)^3 = a^{18}$

(c) $(2x)^2(5x^6) = 2^2x^2 \cdot 5x^6 = 20x^{2+6} = 20x^8$

24. (a) $\frac{z^2z^4}{z^3z^{-1}} = \frac{z^{2+4}}{z^{3-1}} = \frac{z^6}{z^2} = z^{6-2} = z^4$

(b) $(2a^3a^2)^4 = (2a^{3+2})^4 = (2a^5)^4 = 2^4a^{5\cdot 4} = 16a^{20}$

(c) $(3z^2)^3(2z^3) = 3^3z^{2\cdot 3}\,2z^3 = 54z^{6+3} = 54z^9$

25. (a) $(3x^2y)(2x^3) = 3\cdot 2x^{2+3}y = 6x^5y$

(b) $(2a^2b^{-1})(3a^{-2}b^2) = 2\cdot 3a^{2-2}b^{-1+2} = 6b$

(c) $4y^2 \; x^4 y^2 \quad 4y^2 x^{4\,2} y^2 \quad 4x^8 y^{2\,2} \quad 4x^8 y^4$

26. (a) $4x^3 y^2 \; 7y^5 4 \; 7x^3 y^{2\,5} \quad 28x^3 \; y^7$

(b) $9y^{\;2}z^2 \;\; 3y^3 z 9 \;\; 3y^{\;2\;3} z^{2\;1} \quad 27yz^3$

(c) $8x^7 y^2 \quad \frac{1}{2} x^3 y \quad 2 \quad \frac{8x^7 y^2}{\frac{1}{2} x^3 y\;2} \quad \frac{2^2\;8x^7 y^2}{x^{3\,2} y^2} \quad \frac{32x^7 y^2}{x^6 y^2} \quad 32x^{7\;6} y^{2\;2} \quad 32x$

(a) $2x^2 y^{3\;2} \;\; 3y \quad 2^2 x^{2\;2} y^{3\;2} \;\; 3y \quad 12 x^4 y^7$

$\frac{x^2 y}{y^5}^{1} \; x^{2\;5} \; y^{\;1} \quad x^7 \; y^{\;1} \quad \frac{x^7}{y}$

(c) $\frac{x}{2}^3 \quad \frac{x^{2\,3}}{3} \quad \frac{x^6 y^3}{27}$

28. (a) $5x^{\;4} y^3 \;\; 8x^{3\;2} \quad 5x^{\;4} y^3 \; 8^2 x^{3\;2} \quad 5 \; 8^2 x^{\;4\;6} y^3 \quad 320x^2 y^3$

(b) $\frac{y^{\;2} z^{\;3}}{y^{\;1}} \quad \frac{y}{y^2 z^3} \quad \frac{1}{yz^3}$

(c) $\frac{a^3 b}{b^2}^{\;2} \quad \frac{a^6 b^{\;4}}{} \quad \frac{a^6}{}$

(a) $x^3 y^{3\;\;1} \quad \frac{1}{x^3 y^3}$

(c) $a^2 b^{\;2\;3} \; a^{3\;2} \quad a^{2\;3} b^{\;2\;3} a^{3\;2} a^{\;6} b^6 a^{\;6} \quad \frac{b^6}{a^{12}}$

$\frac{x^2}{2}^{\;2} \quad \frac{2y}{3}^{\;3} \qquad \frac{\;\;8}{}$

30. (a) $x^{\;2} y^4 \;\; 3 \quad \frac{x^2}{y^4}\;3 \quad \frac{y^4}{x^2}\;3 \quad \frac{y^{12}}{x^6}$

(b) $y^2 \;\; \frac{1}{} \quad 2x^{\;3} y^4 \;\; \frac{3}{} \;\; y^{\;2} \quad 2^{\;3} x^{\;3\;3} y^{4\;3} \quad \frac{x^9}{8y^{14}}$

(c) $\frac{2a}{}^{1}\;3 \quad \frac{b}{1}\;2 \qquad \frac{1}{}$

$\frac{3x^{\;2} y^5}{2} \quad \frac{xy^3}{} \quad x$

31. (a) $9x^{\;3} y \quad 3^2 y^{\;3} \quad 2^2 x^{3\,2} \quad 4x^6$

(b) $\frac{3}{y}^{\;1}\;2 \quad \frac{3}{}\;2 \quad \frac{y^{\;3\,2}}{} \quad \frac{y^6}{}$

(c) $x \;\; 2 \quad y^2 \quad y^{\;1\;1} x^{2\;1} \; 3 \; 2x^{\;3\;2} y^{\;2\;2} \qquad 9$

$\frac{y}{1}\;1 \quad \frac{3x}{3}\;2 \qquad \frac{}{x^4 y^5}$

32. (a) $\frac{\frac{1}{2} a^{\;3} b^{\;4}}{5} \quad 1 \quad 2a \qquad \frac{1}{} \qquad \frac{a^2}{}$

1 2 2 1*a* 35 *b*
41
4 a^2b 3
3

b
4*b*

(b) $\left(\frac{x^2y}{5x^4}\right)^{-2} = \left(\frac{5x^4}{x^2y}\right)^2 = \left(\frac{5x^2}{y}\right)^2 = \frac{25x^4}{y^2}$

(c) $\left(\frac{2y^{-1}z}{z^2}\right)^{-1}\left(\frac{y}{3z^2}\right)^2 = \left(\frac{2}{yz}\right)^{-1}\frac{y^2}{9z^4} = \frac{y^3}{18z^3}$

33. (a) $\frac{3a}{b^3}^{-1} = 3^{-1}a^{-1}b^{3} = \frac{b^3}{3a}$

(b) $\left(\frac{q^{-1}r^{-1}s^{-2}}{r^{-5}sq^{-8}}\right)^{-1} = \frac{r^{-5}sq^{-8}}{q^{-1}r^{-1}s^{-2}} = q^{-8+1}r^{-5+1}s^{1+2} = \frac{s^3}{q^7r^4}$

34. (a)

$\frac{s^2t^{-4}}{}^{2}$ $\frac{25t^{10}}{}$

(b) $\left(\frac{x^2y^3z^{-4}}{xy^{-2}z^3}\right)^{-3} = x^{-3}y^{-2\cdot 3-3\cdot 3}z^{3\cdot 3+4\cdot 3} = \frac{z^3}{x^3y^{15}}$

35. (a) $69{,}300{,}000 = 6.93 \times 10^7$

(b) $7{,}200{,}000{,}000{,}000 = 7.2 \times 10^{12}$

(c) $0.000028536 = 2.8536 \times 10^{-5}$

(d) $0.0001213 = 1.213 \times 10^{-4}$

36. (a) $129{,}540{,}000 = 1.2954 \times 10^8$

(b) $7{,}259{,}000{,}000 = 7.259 \times 10^9$

(c) $0.0000000014 = 1.4 \times 10^{-9}$

(d) $0.0007029 = 7.029 \times 10^{-4}$

37. (a) $3.19 \times 10^5 = 319{,}000$

(b) $2.721 \times 10^8 = 272{,}100{,}000$

(c) $2.670 \times 10^{-8} = 0.00000002670$

(d) $9.999 \times 10^{-9} = 0.000000009999$

38. (a) $7.1 \times 10^{14} = 710{,}000{,}000{,}000{,}000$

(b) $6 \times 10^{12} = 6{,}000{,}000{,}000{,}000$

(c) $8.55 \times 10^{-3} = 0.00855$

(d) $6.257 \times 10^{-10} = 0.0000000006257$

(a) $5{,}900{,}000{,}000{,}000$ mi $= 5.9 \times 10^{12}$ mi

0.0000000000004 cm $= 4 \times 10^{-13}$ cm

33 billion billion molecules $= 33 \times 10^9 \times 10^9 = 3.3 \times 10^{19}$ molecules

(a) $93{,}000{,}000$ mi $= 9.3 \times 10^7$ mi

$0.0000000000000000000000053$ g $= 5.3 \times 10^{-23}$ g

$5{,}970{,}000{,}000{,}000{,}000{,}000{,}000{,}000$ kg $= 5.97 \times 10^{24}$ kg

41. $7.2 \times 10^{-9} \times 1.806 \times 10^{-12} = 7.2 \times 1.806 \times 10^{-9} \times 10^{-12} \approx 13.0 \times 10^{-21} \approx 1.3 \times 10^{-20}$

42. $1.062 \times 10^{24} \times 8.61 \times 10^{19} = 1.062 \times 8.61 \times 10^{24} \times 10^{19} \approx 9.14 \times 10^{43}$

43. $\frac{1.295643 \times 10^9}{3.610 \times 10^{-17} \times 2.511 \times 10^6} = \frac{1.295643}{3.610 \times 2.511} \times 10^{9+17-6} \approx 0.1429 \times 10^{19} \approx 1.429 \times 10^{19}$

44. $\frac{73.1 \times 1.6341 \times 10^{28}}{0.0000000019} = \frac{7.31 \times 10^1 \times 1.6341 \times 10^{28}}{1.9 \times 10^{-9}} = \frac{7.31 \times 1.6341}{1.9} \times 10^{1+28+9} \approx 6.3 \times 10^{38}$

45. $\frac{0.0000162 \times 0.01582}{594621000 \times 0.0058} = \frac{1.62 \times 10^{-5} \times 1.582 \times 10^{-2}}{5.94621 \times 10^8 \times 5.8 \times 10^{-3}} = \frac{1.62 \times 1.582}{5.94621 \times 5.8} \times 10^{-5-2-8+3} \approx 0.074 \times 10^{-12} = 7.4 \times 10^{-14}$

46. 3542 10 6 9 — 3542 9 10 54 / 05 4 12 505 12 48 10 - 87747 96 / 275103767 10 54 48 / 10 3 19 10 4 102 / 10 3 19 106 / 10

47. 550 / 10 10 / 10 10 50, whereas 10 101 10 100 10 100 / 101 910 100 10 50. So 10 10 is closer to 10 than 50

10^{100} is to 10^{101}.

48. (a) b^5 is negative since a negative number raised to an odd power is negative.

(b) b^{10} is positive since a negative number raised to an even power is positive.

(c) ab^2c^3 we have positive negative ^{2} negative ^{3} positive positive negative which is negative.

(d) Since b a is negative, b a ^{3} negative ^{3} which is negative.

(e) Since b a is negative, b a ^{4} negative ^{4} which is positive.

(f) $\frac{a^3c^3}{b^6c^6}$ $\frac{\text{positive}^3\ \text{negative}^3}{\text{negative}^6\ \text{negative}^6}$ $\frac{\text{positive negative}}{\text{positive positive}}$ $\frac{\text{negative}}{\text{positive}}$ which is negative.

49. Since one light year is 5 9 10^{12} miles, Centauri is about 4 3 5 9 10^{12} 2 54 10^{13} miles away or 25,400,000,000,000 miles away.

9 3 10^7 mi 186 000 $\frac{\text{mi}}{\text{s}}$ t s t $\frac{9\ 3\ 10^7}{186\ 000}$ s 500 s 8 $\frac{1}{3}$ min.

Volume average depth area 3 7 10^3 m 3 6 10^{14} m^2 $\frac{10^3 \text{ liters}}{m^3}$ 1 33 10^{21} liters

52. Each person's share is equal to $\frac{\text{national debt}}{\text{population}}$ $\frac{1\ 674\ 10^{13}}{3\ 164\ 10^8}$ \$52,900.

The number of molecules is equal to

volume $\frac{\text{liters}}{m^3}$ $\frac{\text{molecules}}{22\ 4 \text{ liters}}$ 5103 10 ^{3} $\frac{6\ 02\ 10^{23}}{22\ 4}$ 4 03 10 ^{27}

54. (a)

Person	Weight	Height		BMI 703 $\frac{W}{H^2}$	Result
Brian	295 lb	5 ft 10 in.	70 in.	42 32	obese
Linda	105 lb	5 ft 6 in.	66 in.	16 95	underweight
Larry	220 lb	6 ft 4 in.	76 in.	26 78	overweight
Helen	110 lb	5 ft 2 in.	62 in.	20 12	normal

Answers will vary.

55.

Year	Total interest
1	\$152 08
2	308 79
3	470 26
4	636 64
5	808 08

Since 10^6 10^3 10^3 it would take 1000 days 2 74 years to spend the million dollars.

Since 10^9 10^3 10^6 it would take 10^6 1,000,000 days 2739 72 years to spend the billion dollars.

57. (a) $\frac{18^5}{9^5} = \left(\frac{18}{9}\right)^5 = 2^5 = 32$

$20^6 \cdot (0.5)^6 = (20 \cdot 0.5)^6 = 10^6 = 1{,}000{,}000$

(a) We wish to prove that $\frac{a^m}{a^n} = a^{m-n}$ for positive integers $m \geq n$. By definition, $a^k = \underbrace{a \cdot a \cdots a}_{k \text{ factors}}$. Thus, $\frac{a^m}{a^n} = \frac{\overbrace{a \cdot a \cdots a}^{m \text{ factors}}}{\underbrace{a \cdot a \cdots a}_{n \text{ factors}}}$. Because $m \geq n$, $m - n \geq 0$, so we can write

$$\frac{a^m}{a^n} = \frac{\overbrace{a \cdot a \cdots a}^{n \text{ factors}}\ \overbrace{a \cdot a \cdots a}^{m-n \text{ factors}}}{\underbrace{a \cdot a \cdots a}_{n \text{ factors}}} = \frac{a \cdot a \cdots a}{1} = a^{m-n}.$$

(b) We wish to prove that $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ for positive integers m and n. By definition,

$$\left(\frac{a}{b}\right)^n = \underbrace{\frac{a}{b} \cdot \frac{a}{b} \cdots \frac{a}{b}}_{n \text{ factors}} = \frac{\overbrace{a \cdot a \cdots a}^{n}}{\underbrace{b \cdot b \cdots b}_{n}} = \frac{a^n}{b^n}.$$

59. (a) We wish to prove that $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$. By definition, and using the result from Exercise 58(b),

$$\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = \frac{1}{\frac{a^n}{b^n}} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n.$$

(b) We wish to prove that $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$. By definition, $\frac{a^{-n}}{b^{-m}} = \frac{\frac{1}{a^n}}{\frac{1}{b^m}} = \frac{1}{a^n} \cdot \frac{b^m}{1} = \frac{b^m}{a^n}$.

P.4 RATIONAL EXPONENTS AND RADICALS

Using exponential notation we can write $\sqrt[3]{5}$ as $5^{1/3}$.

Using radicals we can write $5^{1/2}$ as $\sqrt{5}$.

No. $\left(\sqrt{5}\right)^2 = \left(5^{1/2}\right)^2 = 5^{2 \cdot 1/2} = 5$ and $\left(\sqrt{5}\right)^2 = \left(5^{1/2}\right)^2 = 5^{1/2 \cdot 2} = 5$.

$\left(4^{1/2}\right)^3 = 2^3 = 8$; $\left(4^3\right)^{1/2} = 64^{1/2} = 8$

Because the denominator is of the form $\sqrt{a}$, we multiply numerator and denominator by $\sqrt{a}$: $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

$5^{1/3}\,5^{2/3} = 5^1 = 5$

No. If a is negative, then $\sqrt{4a^2} = -2a$.

No. For example, if $a = -2$, then $\sqrt{a^2 + 4} = \sqrt{8} = 2\sqrt{2}$, but $a + 2 = 0$.

9. $\frac{1}{\sqrt{3}} = 3^{-1/2}$

10. $\sqrt[3]{7^2} = 7^{2/3}$

11. $4^{2/3} = \sqrt[3]{4^2} = \sqrt[3]{16}$

12. $10^{-3/2} = \frac{1}{10^{3/2}} = \frac{1}{\sqrt{10^3}}$

13. $\sqrt[5]{5^3} = 5^{3/5}$

14. $2^{-1.5} = 2^{-3/2} = \frac{1}{\sqrt{2^3}} = \frac{1}{\sqrt{8}}$

a2 5 5 a2 —

17. 3 y4 y4 3

(a) 164 2 4 —

(b) 4 16 4 24 2

1 1 4 1

(c) 4 164 2 2

(a) 3 3 16 3 3 2 23 6 3 2

18 18 2 2

(b) 81 81 9 3

27 33 2 33

42 2 2

(a) 7 287 28 196 14

48 48

(b) 3 3 16 4

4 24 4 54 4 24 54 4 1296 6

216 216

25. (a) 6 6 36 6

3 2 3 32 3 64 4

1 1 1 1 1

(c) 4 44 64 4 256 4 2564

4 x4 x

5 32y6 5 25 y6 2 5 y6 2y 5 y

4 16x8 4 24 x8 2x2

3 x3 y x3 1 3 y1 3 x 3 y

36r2 t4 6r t2 2 6 r t2

3 64x6 8 x3 1 3 2 x

32 18 16 2 9 2 4 2 2 3 2 2 42 32 72

75 48 25 3 16 3 5 2 3 4 2 3 53 43 93

125 45 25 5 9 5 5 2 5 3 2 5 55 35 25

16. 1 1 x

x5 x5 2 5 2

18. y 5 3 1 1

(a) 64 8 2 y5 3 3 y 5

8

(b) 3 64 3 4 3 4

(c) 5 32 5 2 5 2

22. (a) 2 3 81 2 3 3 33 6 3 3

2

12 3 2 2 3

7

(b) 25 5 5

18 3 2 3 2

(c) 49 2 2 7

(a) 12 24 12 24 288 2 12 2 12 2

54 54

(b) 6 6 9 3

(c) 3 15 3 75 3 15 75 3 1125 3 125 9 5 3 9

1 1 1 1

26. (a) 5 8 5 4 5 32 2

1

(b) 6 2 6 128 6 64 2

3 4 4 1 1 1 3 3

3 108 108 27 3 27 3

5 x10 x10 1 5 x2

3 8a5 3 23 a3 a2 2a 3 a2

3 x3 y6 x3 y6 1 3 x y2

x4 y4 x4 y4 1 2 x2 y2

4 48a7 b4 4 4 4 3 3

4 2 a b 3a 2 ab 4 3a

3 54 3 16 3 2 33 3 23 2 3 3 2 2 3 2 3 2

9a3 a 3 2 a2 a a 3a a a 3a 1 a

16x x5 4 2 x x2 2 x 4 x x2 x x2

4 x

$\sqrt[3]{x^4}$ $\sqrt[3]{8x}$ $\sqrt[3]{x^3 x}$ $\sqrt[3]{2^3 x}$ $x \sqrt[3]{x}$ $2 \sqrt[3]{x}$ x $2 \sqrt[3]{x}$ $\sqrt[4]{x^4 y^2 z^2}$ $\sqrt[4]{x^4}$ $\sqrt[4]{y^2 z^2}$ x $\sqrt[4]{y^2 z^2}$

$\sqrt[3]{2y^4}$ $\sqrt[3]{2y^3 2y}$ $\sqrt[3]{y^3}$ $\sqrt[3]{2y}$ $\sqrt[3]{2y}$ $\sqrt[3]{y^3}$ $\sqrt[3]{2y}$ $y \sqrt[3]{2y}$

$81x^2$ 81 $81\ x^2$ 1 81 x^2 1 9 x^2 1

$36x^2$ $36y^2 36\ x^2$ y^2 36 x^2 y^2 6 x^2 y^2

49. (a) $16^{1\ 4}$ 2 **(b)** $125^{1\ 3}$ 5 **(c)** 9 $1\ 2$ $\frac{1}{9^{1\ 2}}$ $\frac{1}{3}$

50. (a) $27^{1\ 3}$ 3 2^2 4 **(b)** 8 $1\ 3$ 2 4 2 **(c)** 8 1 $1\ 3$ $\frac{1}{2}$ 27

51.

25 3 2 5 3 125 1

52. (a) $125^{2\ 3}$ 5^2 25 **(b)** 64 8 512 **(c)** 27 $4\ 3$ 3 4 $\frac{}{81}$

53. (a) $5^{2\ 3}$ $5^{1\ 3}$ $5^{2\ 3\ \ 1\ 3}$ 5^1 5 **(b)** $\frac{3^{3\ 5}}{3^{2\ 5}}$ $3^{35\ 25}$ $5\ 3$ **(c)** $^{3}4$ 3 $4^{1\ 3\ 3}$ 4

54. (a) $3^{2\ 7}$ $3^{12\ 7}$ $3^{2\ 7\ \ 12\ 7}$ 3^2 9 **(b)** $\frac{7^{2\ 3}}{7^{5\ 3}}$ $7^{23\ 53}$ $\frac{1}{7}$ **(c)** $^{5}6$ 10 $6^{15\ \ 10}$ $\frac{1}{36}$

When x 3, y 4, z 1 we have x^2 y^2 3^2 4^2 9 16 25 5.

3 3 4

When x 3, y 4, z 1 we have ^{4} x $14y$ $2z$ $^4 3$ $14\ 4$ 2 1 $^4 27$ 56 2 $^4 81$ $^4 3$ 3.

When x 3, y 4, z 1 we have

$9x^{2\ 3}2y^{2\ 3}$ $z^{2\ 3}$ 9 $3^{2\ 3}$ 2 $4^{2\ 3}$ $1^{2\ 3}$ 3^3 $^{2\ 3}$ 2^3 $^{2\ 3}$ $1^{1\ 3}$

$3^2 2^2 1$ 9 4 1 14.

When x 3, y 4, z 1 we have $x\ y$ ^{2z} 3 4^2 1 12^2 144^1 .

59. (a) $x^{3\ 4}x^{5\ 4}$ $x^{3\ 4\ \ 5\ 4}$ x^2 **(b)** $y^{2\ 3}y^{4\ 3}$ $y^{2\ 3\ \ 4\ 3}$ y^2

60. (a) $r^{1\ 6}r^{5\ 6}$ $r^{1\ 6\ \ 5\ 6}$ r **(b)** $a^{3\ 5}a^{3\ 10}$ $a^{3\ 5\ \ 3\ 10}$ $a^{9\ 10}$

61. (a) 5 3 4 3 2 3 **(b)** $\frac{a^{5\ 4}\ \ 2a^{3\ 4}}{a}$ 3 $8a^{13\ 4}$ y

62. (a) $\frac{x^{3\ 4}}{x^{7\ 4}}$ $x^{3\ 4\ \ 7\ 4}$ $5\ 4$ $x^{5\ 4}$ **(b)** $2y^{4\ 3}$ 2 $y^{2\ 3}$ $7\ 3$ $4y^{8\ 3\ \ 2\ 3\ \ 7\ 3}$ $4y^{1\ 3}$ $\frac{4}{3}$

63. (a) $8a^6b^{3\ 2}$ $^{2\ 3}$ $8^{2\ 3}a^{6\ 2\ 3}b^{3\ 2\ \ 2\ 3}4a^4b$ **(b)** $4a^6b^{8\ 3\ 2}$ $4^{3\ 2}a^{6\ 3\ 2}b^{8\ 3\ 2}8a^9b^{12}$

64. (a) $64a^6b^3$ $^{2\ 3}$ $64^{2\ 3}a^{6\ 2\ 3}b^{3\ 2\ 3}16a^4b^2$ **(b)** 16 $8z^{3\ 2}$ $^{3\ 4}$ $16^{3\ 4}$ $8^{3\ 4}$ $z^{3\ 2\ \ 3\ 4}8$ $6z^{9\ 8}$

65. (a) 2 3 1 1 3 1

66. (a) x $^5 y^{1\ 3}$ $3\ 5$ $1\ 2$ x $^{5\ 3\ 5}y^{1\ 3\ \ 3\ 5}$ $1\ 5$ $\frac{x^3}{y^{1\ 5}}$ 32 $^{1\ 5}t$ $^{5\ 4}$ $^{1\ 5}2r^4t$ $^{1\ 4}$ 2 ^{1} $t^{1\ 4}$ r^4t^0 r^4

67. (a) $y^{1\ 2}$ y 3 x $^{2\ 32}$ $^{1\ 6}y$ $^{1\ 23}$ $^{1\ 6}$ x

$x^{2\ 3}$ x^2 $1\ 6$ 1

(b) $x^2 y^{1} \; y \; 4x \; y \quad 2y$ $\quad x^{1\,2} \; 4y^{2\,4\,2} \; 1\,4\,y \; 1\,4 \; 4\,4\,1\,2\,x \; 2\,1\,2\,y \; 4\,1\,2\,y \; 2\,1\,2$

2 2 4 2 4 1 2

68. (a) $16y^{4\,3}$ 1 $\quad x^2 y^8 2 \; 4\, y \; 1 \quad 2x \; 1\, y \; 2\, y \; 1 \; 2 \; 4 \; 1x^2 \; 1\, y^8 \; 1 \; 2 \; 1 \; \frac{x\,y^4}{8}$

$x^8 y \; 4 \; 4$ $\quad 2y^{4\,3}$

(b) $y^3 z^6 \quad 8 \; 1\,3\, y^{3\,4} \; 1\,3 \; 3 \; 1\,3\, z \; 6 \; 1\,3 \quad 2$

$8y^{3\,4} \; 1\,3 \quad y^{3\,4} z^2$

69. $\sqrt[3]{x} \; x^{3\,2}$

70. $x^5 \; x^{5\,2}$

71. 9 5 5 9 x

72. 5 $\frac{1}{x^3} \quad \frac{1}{x^{3\,5}} \quad x \; 3\,5$

73. 6 $y^5 \quad 3 \; y^2 \quad y^{5\,6} \; y^{2\,3} \quad y^{5\,6\;2\,3} \quad y^{3\,2}$

74. $4 \; b^3 \quad b \; b^{3\,4\;1\,2} \quad b^{5\,4}$

75. $5\; 3\; x \quad 2\; 4\; x \quad 5 \; 2x^{1\,3\;1\,4} \quad 10x^{7\,12}$

76. $2 \; a \quad a^2 \quad 2a^{1\,2\;2\,3} \quad 2a^{7\,6}$

77. 4 $x^7 4$ 4 x^3

78. 3 $8x^2$ 2 3 1 2 1 6 6

79. 2 3 $16u \quad 16u \; 2 \quad 4u$

80. 3 x 3 $3xy \quad 54x \; 2\, y^4 \; 3 \; 27y$

81. 4 $16x\,y \quad 16 \; 1\,4\, x^{1\,2} \; 1\,4\, y^{1\,2} \; 1\,4 \quad x^{1\,4} y^{1\,4} \quad 2$

82. 4 3 $a \; b \quad a^{3\,2\;3\,4} b^{1\,2\;2\,4}$

$y^{1\,1\,2\;1\,3}$ $\quad s^{1\,3\,2\;1\,2}$

83. 3 $y \; y \quad y^{3\,2\;1\,3} y^{1\,2}$

84. $s \; s^3 \quad a^{3\,4}$

1 1

86. (a) 12 12 3 $s^{5\,4}$

85. (a) 6 6 6

3 3 12 3 4 2 3

3 2 6 1 2 12 5 60 15

(c) 9 9 $2^{3\,4}$ $9^{4}\,8$ $\quad$ **(c)** 8 8 $5^{1\,3}$ $8^{3}\,5$

87. 4 2 $2^{1\,4}$ $2^{3\,2}$ 2 $\quad$ **88.** $3 \; 5^2 \quad 5^{2\,3} \quad 5^{1\,3} \quad 5$

1 1 $5x$ $5x$ $\quad s \quad s \quad 3t \quad 3st$

(b) 5 5 5 5 $\quad$ **(b)** $6\; b^2 \quad b^{1\,3} \quad b^{2\,3} \quad b$

x 5 $5x$ $\quad a \quad a \quad b^{2\,3} \quad ab^{2\,3}$

(c) 5 $x^3 \; x \quad 3\,5 \; x \; 2 \; x$ $\quad$ **(c)** $c^{3\,5} \quad c^{3\,1\,5} \quad c^{2\,5} \quad c \; c$

1 1 x^{2} $x^{2\,5}$ $\quad$ 1 $c^{2\,5}$ 2 5

1 1 $3\; x$ $3 \; x^2$ $\quad$ 1 1 3 x x 3

90. (a) 3 3 3 $x^2 \quad x^2 \quad x \quad x$

89. (a) $3\,x \quad 3\,x \quad 3\,x^2 \quad x$

1 1 $6x$ $6x$

(b) $6x^5$ 6 x^5 6 x x

1 7 x^4 7 x^4

(c) 1

1 1 4 x 4 x

(b) $4x^3$ 4 x^3 4 x x

1 1 1 1 3 x^2

(c)

x

$3x^2$ $3\ x^2$

$x^3\ x3$ x^2

(a) Since $\frac{1}{2} \; \frac{1}{3}$, $2^{1\,2} \; 2^{1\,3}$.
(b) 2 2 1 2 and 2 2 1 3 . Since 2 3, we have 2 2 .
1 1 2 1 1 3 1 1 1 1 2 1 1 3

(a) We find a common root: $7^{1\,4}$ $7^{3\,12}$ 7^{3} $^{1\,12}$ $343^{1\,12}$; $4^{1\,3}$ $4^{4\,12}$ 4^{4} $^{1\,12}$ $256^{1\,12}$. So $7^{1\,4}$ $4^{1\,3}$.

We find a common root: $^{3}5$ $5^{1\,3}$ $5^{2\,6}$ 5^{2} $^{1\,6}$ $25^{1\,6}$; 3 $3^{1\,2}$ $3^{3\,6}$ 3^{3} $^{1\,6}$ $27^{1\,6}$. So $^{3}5$ 3.

First convert 1135 feet to miles. This gives 1135 ft 1135 $\frac{\text{mile}}{5280 \text{ feet}}$ 0 215 mi. Thus the distance you can see is given by D 2r h h^2 2 3960 0 215 0 215 ^{2} 1702 8 41 3 miles.

(a) Using f 0 4 and substituting d 65, we obtain s 30 f d 30 0 4 65 28 mi/h.

Using f 0 5 and substituting s 50, we find d. This gives s 30 f d 50 30 0 5 d 50 15d 2500 15d d $\frac{500}{3}$ 167 feet.

(a) Substituting, we get 0 30 60 0 38 3400 $^{1\,2}$ 3 650 $^{1\,3}$ 18 0 38 58 31 3 8 66 18 22 16 25 98 14 18. Since this value is less than 16, the sailboat qualifies for the race.

Solve for A when L 65and V 600. Substituting, we get 0 30 65 0 38$A^{1\,2}$ 3 600 $^{1\,3}$ 16 195 038$A^{1\,2}$ 2530 16 038$A^{1\,2}$ 580 16 038$A^{1\,2}$ 2180 A $^{1\,2}$ 5738 A 32920. Thus, the largest possible sail is 3292 ft^2.

96. (a) Substituting the given values we get V 1 486 $\frac{75 \quad 0\,050}{24\;12\;3 \quad 0\,040}$ $^{2\,3}$ $^{1\,2}$ 17 707 ft/s.

(b) Since the volume of the flow is V A, the canal discharge is 17 707 75 1328 0 ft^3 s.

97. (a)

n	1	2	5	10	100
$2^{1\,n}$	$2^{1\,1}$ 2	$2^{1\,2}$ 1414	$2^{1\,5}$ 1149	$2^{1\,10}$ 1 072	$2^{1\,100}$ 1 007

So when n gets large, $2^{1\,n}$ decreases toward 1.

(b)

n	1	2	5	10	100
1 1 n	1 1 1	1 1 2	1 1 5	1 1 10	1 1 100

So when n gets large, $\frac{1}{2}$ $^{1\,n}$ increases toward 1.

P.5 ALGEBRAIC EXPRESSIONS

(a) $2x^3$ $\frac{1}{2}$ x3 is a polynomial. (The constant term is not an integer, but all exponents are integers.)

x^2 $\frac{1}{2}$ 3 x x^2 $\frac{1}{2}$ 3$x^{1\,2}$ is not a polynomial because the exponent $\frac{1}{2}$ is not an integer.

$\frac{1}{x^2 \; 4x \; 7}$ is not a polynomial. (It is the reciprocal of the polynomial x^2 4x 7.)

x^5 7x^2 x 100 is a polynomial.

^{3} 8x^6 5x^3 7x 3 is not a polynomial. (It is the cube root of the polynomial 8x^6 5x^3 7x 3.)

3x^4 5x^2 15x is a polynomial. (Some coefficients are not integers, but all exponents are integers.)

2. To add polynomials we add *like* terms. So

$3x^2$ $2x$ $48x^2$ x 13 8 x^2 2 1 x 4 1 $11x^2$ x 5.

3. To subtract polynomials we subtract *like* terms. So

$2x^3$ $9x^2$ x $10x^3$ x^2 $6x$ 82 1 x^3 9 1 x^2 1 6 x 10 8 x^3 $8x^2$ $5x$ 2.

We use FOIL to multiply two polynomials: x 2 x 3 x x x 3 2 x 2 3 x^2 $5x$ 6.

5. The Special Product Formula for the "square of a sum" is A B ^{2} A^2 $2AB$ B^2. So $2x$ 3 ^{2} $2x$ ^{2} 2 $2x$ 3 3 ^{2}

$4x^2$ $12x$ 9.

The Special Product Formula for the "product of the sum and difference of terms" is A B A B A^2 B^2. So 5 x 5 x 5^2 x^2 25 x^2.

(a) No, x 5 ^{2} x^2 $10x$ 25 x^2 25.

Yes, if a 0, then x a ^{2} x^2 $2ax$ a^2.

(a) Yes, x 5 x 5 x^2 $5x$ $5x$ 25 x^2 25.

Yes, if a 0, then x a x a x^2 ax ax a^2 x^2 a^2.

9. Binomial, terms $5x^3$ and 6, degree 3

10. Trinomial, terms $2x^2$, $5x$, and 3, degree 2

11. Monomial, term 8, degree 0

12. Monomial, term $\frac{1}{2}x^7$, degree 7

13. Four terms, terms x, x^2, x^3, and x^4, degree 4

14. Binomial, terms 2x and 3, degree 1

$6x$ $33x$ $76x$ $3x3$ 7 $9x$ 4

3 $7x11$ $4x7x$ $4x3$ $1111x$ 8

17. $2x^2$ $5xx^2$ $8x$ $32x^2$ x^2[$5x8x$]3 x^2 $3x$ 3

18. $2x^2$ $3x$ $13x^2$ $5x$ $42x^2$ $3x^2$ $3x$ $5x1$ 4 x^2 $2x$ 3

3 x 1 4 x 2 $3x$ 3 $4x$ 8 $7x$ 5

8 $2x$ 5 7 x 9 $16x$ 40 $7x$ 63 $9x$ 103

21. $5x^3$ $4x^2$ $3x$ x^2 $7x$ 2 $5x^3$ $4x^2$ x^2 $3x$ $7x$ 2 $5x^3$ $3x^2$ $10x$ 2

22. 4 x^2 $3x$ 53 x^2 $2x$ $14x^2$ $12x$ 20 $3x^2$ $6x$ 3 x^2 $6x$ 17

23. $2x$ x 1 $2x^2$ $2x$

24. $3y$ $2y$ 5 $6y^2$ $15y$

25. x^2 x 3 x^3 $3x^2$

26. y y^2 $2y^3$ $2y$

27. 2 2 $5t$ t t 10 4 $10t$ t^2 $10t$ t^2 4

28. 5 $3t$ 4 $2t$ t $32t^2$ $21t$ 20

29. r r^2 9 $3r^2$ $2r^3$ 1 r^3 $9r$ $6r^2$ $3r^3$ $2r^2$

$7r^3$ $3r^2$ $9r$

30. x^3 92 x^2 22 x 5 4 x 3 2 t

31. x^2 $2x^2$ x $12x^4$ x^3 x^2

32. $3x^3$ x^4 $4x^2$ $53x^7$ $12x^5$ 15 x^3

33. x 3 x 5 x^2 $5x$ $3x$ 15 x^2 $2x$ 15

34. 4 x 2 x 8 $4x$ $2x$ x^2 x^2 $6x$ 8

35. $s \quad 6 \;\; 2s \quad 3 \quad 2s^2 \;\; 3s \quad 12s \quad 18 \;\; 2s^2 \quad 15s \quad 18$

$\qquad\qquad\qquad 2 \qquad\qquad\qquad\qquad 2$

36. $2t \quad 3 \;\; t \quad 1 \quad 2^2 \quad 2t \quad 3t \quad 3 \quad 2^2 \quad t \quad 3$

$\qquad\qquad\qquad 2$

37. $3t \quad 2 \;\; 7t \quad 4 \quad 21t \quad 12t \;\; 14t \;\; 8 \quad 21t \quad 26t \;\; 8$

38. $4s \quad 1 \;\; 2s \quad 5 \quad 8s \quad 18s \quad 5$

x

t

39. $3x \quad 5 \quad 2x \quad 1 \quad 6x^2 \quad 10x \quad 3x \quad 5 \quad 6x^2 \quad 7x \quad 5$

40. $7y \quad 3 \quad 2y \quad 1 \quad 14y^2 \quad 13y \quad 3$

41. $x \quad 3y \; 2x \quad y \quad 2x^2 \quad 5xy \quad 3^2$
$2 \quad y \quad 2$

42. $4x \quad 5y \; 3x \quad y \quad 12x^2 \quad 19xy \quad 5y^2$
$2 \quad 2$

43. $2r \quad 5s \; 3r \quad 2s \quad 6r \quad 19rs \quad 10s$

44. $6u \quad 5 \quad u \quad 26u \quad 7u \quad 10$

45. $5x \quad 1^2 \quad 25x^2 \quad 10x \quad 1$

46. $2 \quad 7y^2 \quad 49y^2 \quad 28y \quad 4$
$2 \quad 2 \quad 2 \quad 2 \quad 2$

47. $3y \quad 1^2 \quad 3y^2 \quad 2\,3y \quad 1 \quad 1^2 \quad 9y \quad 6y \quad 1$

48. $2y \quad 5 \quad 2y \quad 2\,2y \quad 5 \quad 5 \quad 4y \quad 20y \quad 25$
$2 \quad 2$

49. $2u^2 \quad 4u^2 \quad 4u \quad 2$

50. $x \quad 3y^2 \quad x \quad 6xy \quad 9y$ s

51. $2x \quad 3y^2 \quad 4x^2 \quad 12xy \quad 9y^2$

52. $r \quad 2s^2 \quad r^2 \quad 4rs \quad 4^2$

53. $x^2 \quad 1^2 \quad x^4 \quad 2x^2 \quad 1$

54. $2 \quad y^3{}^2 \quad y^6 \quad 4y^3 \quad 4$
$2 \quad 2$

55. $x \quad 6 \; x \quad 6 \quad x \quad 36$

56. $5 \quad y \; 5 \quad y \quad 25 \quad y$
y

57. $3x \quad 4 \; 3x \quad 4\,3x^2 \quad 4^2 \quad 9x^2 \quad 16$

58. $2y \quad 5 \; 2y \quad 5 \quad 4^2 \quad 25$

59. $x \quad 3y \; x \quad 3y \quad x^2 \quad 3y^2 \quad x^2 \quad 9y^2$

60. $2u\,2u\,4u^2 \quad 2$

61.

62.

63. $y \quad 2^3 \quad y^3 \quad 3y^2 \quad 2 \quad 3y \; 2^2 \quad 2^3 \quad y^3 \quad 6y^2 \quad 12y \quad 8$
$3 \quad 2 \quad 2 \quad 3 \quad 3 \quad 2$

64. $x \quad 3^3 \quad x \quad 3x \quad 3 \; 3x \quad 3 \quad 3 \quad x \quad 9x \quad 27x \quad 27$

$3 \quad 2 \quad 2 \quad 3 \quad 3 \quad 2$

65. $1 \quad 2r^3 \quad 1^3 \quad 3 \quad 1 \quad 2r \quad 3\,1 \quad 2r \quad 2r \quad 8r \quad 12r \quad 6r \quad 1$

$3 \quad 2 \quad 2 \quad 3 \quad 3 \quad 2$

66. $3 \quad 2y^3 \quad 3 \quad 3 \quad 3 \quad 2y \quad 3\,3 \quad 2y \quad 2y \quad 8y \quad 36y \quad 54y \quad 27$

$2 \quad 3 \quad 2 \quad 2 \quad 3 \quad 2$

67. $x \quad 2 \; x \quad 2x \quad 3x \quad 2x \quad 3x \quad 2x \quad 4x \quad 6 \quad x \quad 4x \quad 7x \quad 6$

$2 \quad 3 \quad 2 \quad 2 \quad 3 \quad 2$

68. $x \quad 1 \; 2x \quad x \quad 12x \quad x \quad x \quad 2x \quad x \quad 1 \quad 2x \quad x \quad 1$

$2 \quad 3 \quad 2 \quad 2 \quad 3 \quad 2$

69. $2x \quad 5 \; x \quad x \quad 12x \quad 2x \quad 2x \quad 5x \quad 5x \quad 5 \quad 2x \quad 7x \quad 7x \quad 5$

$2 \quad 2 \quad 3 \quad 2 \quad 3 \quad 2$

70. $1 \quad 2x \quad x \quad 3x \quad 1x \quad 3x \quad 1 \quad 2x \quad 6x \quad 2x \quad 2x \quad 5x \quad x \quad 1$

71. $x \quad x \quad x \quad x^2 \quad x \quad x\,x \quad y \quad y$

72. $x^{3\,2} \quad x \quad 1 \quad x \quad x^2 \quad x$

73. $y \quad y \quad y \quad y \quad y$ 2 3 13 53 2

74. $x \quad 2x \quad x \quad 2x$ 1 4 3 4 1 4

75. $x^2 \quad y^2{}^2 \quad x^2{}^2 \quad y^2{}^2 \quad 2x^2y^2 \quad x^4 \quad y^4 \quad 2x^2y^2$

76.

1 2 1

77. $x^2 \quad a^2 \; x^2 \quad a^2 x^4 \quad a^4$

78. $x^{1\,2} \quad y^{1\,2} \quad x^{1\,2} \quad y^{1\,2} x \quad y$

81. $1 \quad x^2 \, 3 \quad 1 \quad x^2 \, 3 \, 1 \quad x^4 \, 3$

82. $1 \quad b^2 \quad 1 \quad b^2 \quad b^4 \quad 2b^2 \quad 1$

83. $x \quad 1 x^2 \quad x \quad 1 \quad x^2 \quad x \quad 1 \, ^2 \quad x^2 \; ^2$

$x^2 \quad 2x \quad 1 \quad x^4 x^4 \quad x^2 \quad 2x \quad 1$

84. $x2 \quad x^2 \quad x \quad 2 \quad x^2 \quad x^4 \quad 3x^2 \quad 4$

85. $2x \quad y \quad 3 \quad 2x \quad y \quad 32x \quad y^2 \quad 3^2 \quad 4x^2 \quad 4xy \quad y^2 \quad 9$

86. $x \quad y \quad z \; x \quad y \quad z \quad x^2 \quad y^2 \quad z^2 \quad 2yz$

87. (a) RHS $\frac{1}{2}$ $a \quad b\ {}^2 a^2 \quad b^2$ $\frac{1}{2}$ $a^2 \quad b^2 \quad 2ab a^2 \quad b^2$ $\frac{1}{2}$ $2ab \quad ab$ LHS

RHS

(b) LHS $a^2 \; b^2$ $2\; a^2 \; b^2$ 2 a^2 2 b^2 $^2\; 2a_2b_2$ a^2 $2\; b^2$ $^2\; 2a_2b_2$ $4a^2b^2$

88. LHS $a^2 \quad b^2 \quad c^2 \quad d^2 \quad a^2c^2 \quad a^2d^2 \quad b^2c^2 \quad b^2d^2$

$a^2c^2 \quad b^2d^2 \quad 2abcd a^2d^2 \quad b^2c^2 \quad 2abcd ac \quad bd^2 \quad ad \quad bc^2$ RHS

(a) The height of the box is x, its width is $6 \quad 2x$, and its length is $10 \quad 2x$. Since Volume height width length, we have $V \quad x\,6 \quad 2x \quad 10 \quad 2x$.

(b) $V \quad x$ $60 \quad 32x \quad 4x^2$ $60x \quad 32 \; x^2 \quad 4 \; x^3$, degree 3.

(c) When $x \quad 1$, the volume is $V \quad 601 \quad 32 \quad 1^2 4 \quad 1^3 32$, and when $x \quad 2$, the volume is

$V \; 602 \; 32 \quad 2^2 4 \quad 2^3 \quad 24$.

(a) The width is the width of the lot minus the setbacks of 10 feet each. Thus width x 20 and length y 20. Since Area width length, we get $A\; x\; 20\; y\; 20$.

$A \quad x \quad 20 \; y \quad 20 \quad xy \quad 20x \quad 20y \quad 400$

For the 100 400 lot, the building envelope has A 100 20 400 20 80 380 30,400. For the 200 200, lot the building envelope has A 200 20 200 20 180 180 32,400. The 200 200 lot has a larger building envelope.

91. (a) $A \quad 2000\,1 \quad r^3 \quad 2000 \quad 1 \quad 3r \quad 3r^2 \quad r^3 2000 \quad 6000r \quad 6000r^2 \quad 2000r^3$, degree 3.

Remember that % means divide by 100, so 2% 0 02.

Interest rate r	2%	3%	4 5%	6%	10%
Amount A	\$2122 42	\$2185 45	\$2282 33	\$2382 03	\$2662 00

92. (a) $P \quad R \quad C 50$ $x \quad 0\;05x^2 50 \quad 30x \quad 0\;1x^2 50x \quad 0\;05x^2 \quad 50 \quad 30x \quad 0\;1x^2 \quad 0\;05x^2 \quad 20x \quad 50$.

(b) The profit on 10 calculators is $P \quad 0\;05 \quad 10^2 20\;10 \quad 50$ \$155. The profit on 20 calculators is

$P \quad 0\;05 \quad 20^2 \quad 20\;20 \quad 50$ \$370 .

(a) When $x \quad 1$, $x \quad 5^2 \quad 1 \quad 5^2 \quad 36$ and $x^2 \quad 25 \quad 1^2 \quad 25 \quad 26$.

2

$x \quad 5^2 \quad x \quad 10x \quad 25$

(a) The degree of the product is the sum of the degrees of the original polynomials.

The degree of the sum could be lower than either of the degrees of the original polynomials, but is at most the largest of the degrees of the original polynomials.

(c) Product: $2x \quad {}^3x \quad 32x \quad {}^3 \quad x \quad 74x\,{}^6 2x \quad 4 \quad 14x^3 \; 2 \quad x\,4 \quad x^2 \quad 7x \quad 6x^3 \quad 3x \quad 21$

$4x \quad 4x \quad 20x \quad x \quad 10x \quad 21$

Sum: $2x^3 \quad x \quad 3 \quad 2x^3 \quad x \quad 7 \quad 4$.

P.6 FACTORING

1. The polynomial $2x^5$ $6x^4$ $4x^3$ has three terms: $2x^5$, $6x^4$, and $4x^3$.

2. The factor $2x^3$ is common to each term, so $2x^5$ $6x^4$ $4x^3$ $2x^3$ x^2 $3x$ 2.

[In fact, the polynomial can be factored further as $2x^3$ x 2 x 1.]

To factor the trinomial x^2 $7x$ 10 we look for two integers whose product is 10 and whose sum is 7. These integers are 5 and 2, so the trinomial factors as x 5 x 2 .

4. The greatest common factor in the expression 4 x 1^2 x x 1^2 is x 1^2, and the expression factors as 4 x 1^2 x x 1^2 x 1^2 4 x.

5. The Special Factoring Formula for the "difference of squares" is A^2 B^2 A B A B. So $4x^2$ 25 $2x$ 5 $2x$ 5 .

. ^{2} $10x$ 25 x 5^2.

7. $5a$ 20 5 a 4

8. $3b$ 12 $3b$ 4 3 b 4

9. $2x^3$ x x $2x^2$ 1

10. $3x^4$ $6x^3$ x^2 x^2 $3x^2$ $6x$ 1

11. $2x^2y$ $6xy^2$ $3xy$ xy $2x$ $6y$ 3

12. $7x^4y^2$ $14xy^3$ $21xy^4$ $7xy^2$ x^3 $2y$ $3y^2$

y y 6 9 y $6y$ 6 y 9

z 2^2 5 z $2z$ 2 [z 2 5] z 2 z 3

15. x^2 $8x$ 7 x 7 x 1

16. x^2 $4x$ 5 x 5 x 1

17. x^2 $2x$ 15 x 5 x 3

18. $2x^2$ $5x$ 7 x 1 $2x$ 7

19. $3x^2$ $16x$ 5 $3x$ 1 x 5

20. 5^2 $7x$ 6 $5x$ 3 x 2

$3x$ 2^2 8 $3x$ 2 12 [$3x$ 2 2] [$3x$ 2 6] $3x$ 4 $3x$ 8

2 a b^2 5 a b 3 [a b 3] [2 a b 1] a b 3 $2a$ $2b$ 1 x

23. x^2 25 x 5 x 5

24. 9 y^2 3 y 3 y

25. 49 $4z^2$ 7 $2z$ 7 $2z$

26. $9a^2$ 16 $3a$ 4 $3a$ 4

27. $16y^2$ z^2 $4y$ z $4y$ z

28. a^2 $36b^2$ a $6b$ a $6b$

2

x 3^2 y x 3 y x 3 yx y 3 x y 3

x^2 y 5^2 x y 5 x y $5x$ y 5 x y 5

31. x^2 $10x$ 25 x 5^2

32. 9 $6y$ y^2 3 y^2

2 2

2 2

33. z $12z$ 36 z 6

34. 16648

2 2

35. $4t^2 \quad 20t \quad 25 \quad 2t \quad 5^2$

36. $16a^2 \quad 24a \quad 9 \quad 4a \quad 3^2$

37. $9u^2 \quad 6u^2 \; 3u^2$

38. $x^2 \quad 10xy \quad 25y \quad x \quad 5y$

39. $x^3 \; 27 \quad x \quad 3 \; x^2 \quad 3x \quad 9$

40. $y^3 \quad 64 \quad y \quad 4 \quad y^2 \quad 4y \quad 16$

41. $8a^3 \quad 1 \quad 2a \quad 1 \; 4a^2 \quad 2a \quad 1$

42. $827^3 \; 23469^2$

43. $27x^3 \quad y^3 \quad 3x \quad y \; 9x^2 \quad 3xy \quad y^2$

44. $1 \quad 1000y^3 \quad 1 \quad 10y \; 1 \quad 10y \quad 100y^2$

45. $u^{36} \; u^{32} \; ^3 \qquad u^2 \; u^2 \; u^{24}$

46. $8r^3 \; 64t^6 2r \; 4t^2 \quad 4r^2 \quad 8rt^2 \quad 16t^4$

47. $x^3 \quad 4x^2 \quad x \quad 4 \; x^2 \; x \quad 4 \quad 1 \; x \quad 4x \quad 4 \qquad x^2 \; 1$

48. $3x^3 \quad x^2 \quad 6x \quad 2 \quad x^2 \quad 3x \quad 1 \quad 2 \; 3x \quad 13x^2 \quad 1 \; x \quad 2$

49. $5x^3 \quad x^2 \quad 5x \quad 1 \quad x^2 \quad 5x \quad 15x \quad 1x^2 \quad 1 \; 5x \quad 1$

50. $18x^3 \quad 9x^2 \quad 2x \quad 1 \quad 9x^2 \quad 2x \quad 12x \quad 19x^2 \quad 1 \; 2x \quad 1$

51. $x^3 \quad x^2 \quad x \quad 1 \quad x^2 \quad x \quad 1 \quad 1 \; x \quad 1x \quad 1 \; x^2 \quad 1$

52. $x^5 \quad x^4 \quad x \quad 1 \quad x^4 \quad x \quad 1 \quad 1 \; x \quad 1x \quad 1 \; x^4 \quad 1$

53. $x^{5\,2} \; x^{1\,2} \; x^{1\,2} \; x^2 \; 1 \qquad x \quad x \quad 1 \; x \quad 1$

54.

1

55. Start by factoring out the power of x with the smallest exponent, that is, $x^{3\,2}$. So

$$x^{3\,2} \quad 2x^{1\,2} \quad x^{1\,2} \quad x^{3\,2} \quad 1 \quad 2x \quad x^2 \quad \frac{1 \quad x^2}{x^{3\,2}}.$$

56. $x \quad 1^{7\,2} \quad x \quad 1^{3\,2} x \quad 1^{3\,2} \quad x \quad 1^2 \quad 1x \quad 1^{3\,2} [\,x \quad 1 \quad 1]\,[\,x \quad 1 \quad 1]$

$$x \quad 1^{3\,2} \quad x \quad 2 \; x$$

57. Start by factoring out the power of $x^2 \quad 1$ with the smallest exponent, that is, $x^2 \; 1^{1\,2}$. So

$$x^2 \; 1^{1\,2} \; 2\,x^2 \; 1^{1\,2} x^2 \quad 1^{1\,2} \quad x^2 \qquad 1 \quad 2 \frac{x}{x^2 \; 1} \quad \frac{3}{2}.$$

58. $x^{1\,2} \quad x^{1\,2} \quad 1 \quad x^{1\,2} \quad x \quad 1^{1\,2} \quad x^{1\,2} \quad x \quad 1^{1\,2} \; [\,x \quad 1 \quad x\,] \quad \frac{2x \quad 1}{x \quad x \quad 1}$

$2x^{1\,3} \quad x \quad 2^{2\,3} \quad 5x^{4\,3} \quad x \quad 2^{1\,3} \quad x^{1\,3} \quad x \quad 2^{1\,3} [2\,x \quad 2 \quad 5x] \quad x^{1\,3} \quad x \quad 2^{1\,3} \; 2x \quad 4 \quad 5x$

$$x^{1\,3} \; x \quad 2^{1\,3} \quad 3x \quad 4 \quad \frac{3x \quad 4}{3\,x^2} \; \frac{3}{x}$$

60. $3x^{1\,2} \; x^2 \quad 1^{5\,4} \quad x^{3\,2} \; x^2 \quad 1^{1\,4} \quad x^{1\,2} \; x^2 \quad 1^{1\,4} \; 3 \quad x^2 \quad 1x^2 \quad 1 \qquad 3 \quad \frac{{}_4x^2 \; 1 \quad 2x^2 \quad 3}{x}$

61. $12x^3 \quad 18x \quad 6x \; 2x^2 \quad 3$

62. $30x^3 \quad 15x^4 \quad 15x^3 \; 2 \quad x$

63. $6y^4 \quad 15y^3 \quad 3y^3 \; 2y \quad 5$

64. $5ab \quad 8abc \quad ab \; 5 \quad 8c$

65. $x^2 \quad 2x \quad 8 \quad x \quad 4 \; x \quad 2$

66. $x^2 \quad 14x \quad 48 \quad x \quad 8 \; x \quad 6$

67. $y^2 \;\; 8y \;\; 15 \;\; y \;\; 3 \; y \;\; 5$

68. $z^2 \;\; 6z \;\; 16 \;\; z \;\; 2 \; z \;\; 8$

69. $2x^2 \;\; 5x \;\; 3 \;\; 2x \;\; 3 \; x \;\; 1$

70. $2x^2 \;\; 7x \;\; 4 \;\; 2x \;\; 1 \; x \;\; 4$

71. $9x^2 \;\; 36x \;\; 45 \;\; 9 \;\; x^2 \;\; 4x \;\; 5 \; 9 \; x \;\; 5 \; x \;\; 1$

72. $8x^2 \;\; 10x \;\; 3 \;\; 4x \;\; 3 \; 2x \;\; 1$

73. $6x^2 \;\; 5x \;\; 6 \;\; 3x \;\; 2 \; 2x \;\; 3$

74. $6 \;\; 5t \;\; 6t^2 \;\; 3 \;\; 2t \; 2 \;\; 3t$

75. $x^2 \;\; 36 \;\; x \;\; 6 \; x \;\; 6$

76. $4x^2 \;\; 25 \;\; 2x \;\; 5 \; 2x \;\; 5$

77. $49 \;\; 4y^2 \;\; 7 \;\; 2y \; 7 \;\; 2y$

78. $4t^2 \;\; 9s^2 \;\; 2t \;\; 3s \; 2t \;\; 3s$

79. $t^2 \;\; 6t \;\; 9 \;\; t \;\; 3^2$

80. $x^2 \;\; 10x \;\; 25 \;\; x \;\; 5^2$

81. $4x^2 \;\; 4xy \;\; y^2 \;\; 2x \;\; y^2$

82. $r^2 \;\; 6rs \;\; 9s^2 \;\; r \;\; 3s^2$

83. $t^3 \;\; 1 \;\; t \;\; 1 \; t^2 \;\; t \;\; 1$

84. $x^3 \;\; 27 \;\; x^3 \;\; 3^3 \;\; x \;\; 3 \; x^2 \;\; 3x \;\; 9$

85. $8x^3 \;\; 125 \;\; 2x^3 \;\; 5^3 \;\; 2x \;\; 5 \; 2x^2 \;\; 2x \; 5 \;\; 5^2 \;\; 2x \;\; 5 \; 4x^2 \;\; 10x \;\; 25$

86. $125 \;\; 27y^3 \;\; 5^3 \;\; 3y^3 \;\; 5 \;\; 3y \; 5^2 \;\; 5 \; 3y \; 3y^2 \; 3y \;\; 5 \;\; 9y^2 \;\; 15y \;\; 25$

87. $x^3 \;\; 2x^2 \;\; x \;\; x \; x^2 \;\; 2x \;\; 1 \; x \; x \;\; 1^2$

88. $3x^3 \;\; 27x \;\; 3x \; x^2 \;\; 9 \; 3x \; x \;\; 3 \; x \;\; 3$

89. $x^4 \;\; 2x^3 \;\; 3x^2 \;\; x^2 \; x^2 \;\; 2x \;\; 3x^2 \;\; x \;\; 1 \; x \;\; 3$

90. $3^5 5^4 2^3 3^3 2^2 5 \; 2 \;\; 3^3 3 \; 1 \;\; 2$

$x^4 y^3 \;\; x^2 y^5 \;\; x^2 y^3 \; x^2 \;\; y^2 \; x^2 y^3 \;\; x \;\; y \; x \;\; y$

$18y^3 x^2 \;\; 2xy^4 \;\; 2xy^3 \; 9x \;\; y$ $\quad 9$

94. $\dfrac{x^6}{27a^3} \;\; \dfrac{8y^3}{b^6} \;\; \dfrac{x^2}{3a}^3 \;\; \dfrac{2y}{b^2}^3 \;\; \dfrac{x^2}{3a} \;\; \dfrac{2y}{b^2} \;\; \dfrac{x^2}{3a}^2 \;\; \dfrac{x^2}{3a} \; \dfrac{2y}{b^2} \;\; \dfrac{2y}{b^2}^2 \;\; \dfrac{x^2}{3a} \;\; \dfrac{2y}{b^2} \;\; \dfrac{x^4}{a^2} \;\; \dfrac{2x^2 y}{3ab^2} \;\; \dfrac{4y^2}{b^4}$

95. $y^3 \;\; 3y^2 \;\; 4y \;\; 12 \;\; y^3 \;\; 3y^2 \; 4y \;\; 12 \;\; y^2 \; y \;\; 3 \; 4 \; y \;\; 3 \; y \;\; 3 \; y^2 \;\; 4$

$y \;\; 3 \; y \;\; 2 \; y \;\; 2$ (factor by grouping)

96. $y^3 \;\; y^2 \;\; y \;\; 1 \;\; y^2 \;\; y \;\; 1 \;\; 1 \; y \;\; 1 \; y^2 \;\; 1 \; y \;\; 1$

97. $3x^3 \quad x^2 \quad 12x \quad 4 \quad 3x^3 \quad 12x \quad x^2 \quad 4 \quad 3x \; x^2 \quad 4x^2 \quad 43x \quad 1 \; x^2 \quad 43x \quad 1 \; x \quad 2 \; x \quad 2$

(factor by grouping)

98. $9x^3 \quad 18x^2 \quad x \quad 2 \quad 9x^2 \quad x \quad 2x \quad 29x^2 \quad 1 \; x \quad 23x \quad 1 \; 3x \quad 1 \; x \quad 2$

99. $a \quad b^2 \quad a \quad b^2 \quad [a \quad ba \quad b][a \quad ba \quad b] \quad 2b \; 2a \quad 4ab$

100. 1 *x* 1 *x* 1 *x*

1 2 1 2 1

1

101. x^2 x^2 19 x^2 1x^2 1 x^2 9

102. a^2 1 b^2 4 a^2 1a^2 1 b^2 4

x 1 *x* 2 2 *x* 1 2 *x* 2*x* 1 *x* 2 [*x* 2*x* 1] 3 *x* 1 *x* 2

104. *x* 1 3 *x* 2 *x* 1 2 2 *x* *x* 3 *x* 1 *xx* 1 *x* 1 2 2 *x* 1 *x* 2 *xx* 1 [*x* 1 *x*] 2

x

xx 1 1 2 *xx* 1

y 4 *y* 2 3 *y* 5 *y* 2 4 *y* 4 *y* 2 3 1 *yy* 2*y* 4 *y* 2 3 *y* 2 2*y* 1*y* 4 *y* 2 3 *y* 1 2

n x *yn* 1 *y* *x* *n x* *yn* 1 *x* *yx* *y* [*n* *n* 1] *x* *y*

Start by factoring y^2 7*y* 10, and then substitute a^2 1 for *y*. This gives

2

a 2 1 7 *a* 2 1 10 *a* 2 1 2 *a* 2 1 5 *a* 2 1 *a* 2 4 *a* 1 *a* 1 *a* 2 *a* 2

2 **108.** *a* 2 2*a* 2 *a* 2 2*a* 3 *a* 2 2*a* 3 *a* 2 2*a* 1 *a* 2 2*a* 3 *a* 2 2*a* 1

a 1 *a* 3 *a* 1 2

3*x* 2 4*x* 12 2 *x* 3 2 4*x* 12 4 2 4*x* 12 [3 4*x* 12 *x* 2 4] 4 2 *x* 3 12*x* 36 8*x* 4 2 *x* 3 20*x* 36 16*x* 2 *x* 3 5*x* 9

x *x* *x*

110. 5x^2 4 4 2*x x* 2 4x^2 4 5 4 *x* 2 3 2 x^2 4 4 *x* 2 3 5 *x x* 2 x^2 4 2

2 x^2 4 4 *x* 2 3 5x^2 10*x* 2x^2 8 2 x^2 4 4 *x* 2 3 7x^2 10*x* 8

111. 3 2*x* 1 2 2 *x* 3 1 2 2*x* 1 3 2 1 *x* 3 1 2 2*x* 1 2 *x* 3 1 2 6 *x* 3 2*x* 1 2 1

2*x* 1 2 *x* 3 1 2 6*x* 18 *x* 1 2 2*x* 1 2 *x* 3 1 2 7*x* 35 2

1 3 *x* 6 2 3 2*x* 3 2 *x* 6 1 3 2 2*x* 3 2 1 3 *x* 6 2 3 2*x* 3 [2*x* 3 3 *x* 6 4]

1 3 *x* 6 2 3 2*x* 3 [2*x* 3 12*x* 72] 1 3 *x* 6 2 3 2*x* 3 14*x* 69

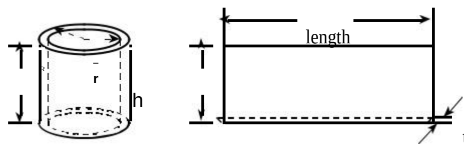
113. *x* 2 3 1 3 2 3*x* 2 *x* 2 3 4 3 *x* 2 3 4 3 *x* 2 3 2 3*x* 2 *x* 2 3 4 3 1 3*x* 2 3 1 3 *x* 2 x^2 3 3 4 3

1 2 *x* 1 2 3*x* 4 1 2 3 2 *x* 1 2 3*x* 4 1 2 1 2 *x* 1 2 3*x* 4 1 2 [3*x* 4 3*x*] 1 2 *x* 1 2 3*x* 4 1 2 6*x* 4

x 1 2 3*x* 4 1 2 3*x* 2

The volume of the shell is the difference between the volumes of the outside cylinder (with radius R) and the inside cylinder (with radius r). Thus V $R^2 h$ $r^2 h$ R^2 r^2 h R r R r h 2 $\frac{R\ r}{2}$ h R r. The average radius is $\frac{R\ r}{2}$ and 2 $\frac{R\ r}{2}$ is the average circumference (length of the rectangular box), h is the height, and R r is the thickness of the rectangular box. Thus V $R^2 h$ $r^2 h$ 2 $\frac{R\ r}{2}$ h R r 2 average radius height thickness

h



(a) Mowed portion field habitat

Using the difference of squares, we get b^2 b $2x^2$ $[b$ b $2x]$ $[b$ b $x]$ $2x$ $2b$ $2x$ $4x$ b x.

(a) 528^2 527^2 528 527 528 527 1 1055 1055

122^2 120^2 122 120 122 120 2 242 484

1020^2 1010^2 1020 1010 1020 1010 10 2030 20,300

(a) 501 499 500 1 500 1 500^2 1 250,000 1 249,999

79 61 70 9 70 9 70^2 9^2 4900 81 4819

2007 1993 2000 7 2000 7 2000^2 7^2 4,000,000 49 3,999,951

(a) A^4 B^4 A^2 B^2 A^2 B^2 A B A B A^2 B^2

A^6 B^6 A^3 B^3 A^3 B^3 (difference of squares)

A B A^2 AB B^2 A B A^2 AB B^2 (difference and sum of cubes)

(b) 12^4 7^4 20,736 2,401 18,335; 12^6 7^6 2,985,984 117,649 2,868,335

(c) 18,335 12^4 7^4 12 7 12 7 12^2 7^2 5 19 144 49 5 19 193

2,868,335 12^6 7^6 12 7 12 7 12^2 12 7 7^2 12^2 12 7 7^2 5 19 144 84 49 144 84 49 5

19 277 109

120. (a) A 1 A 1 A^2 A A 1 A^2 1

A 1 A^2 A 1 A^3 A^2 A A^2 A 1 A^3 1

A 1 A^3 A^2 A 1 A^4 A^3 A^2 A A^3 A^2 A 1

(b) We conjecture that A^5 1 A 1 A^4 A^3 A^2 A 1. Expanding the right-hand side, we have

A 1 A^4 A^3 A^2 A 1 A^5 A^4 A^3 A^2 A A^4 A^3 A^2 A

1 A^5 1, verifying our

conjecture. Generally, $A^{n} \quad 1 \qquad A \quad 1 \quad A^{n\ 1} \quad A^{n\ 2} \qquad\qquad A \quad 1$ for any positive integer *n*.

121. (a)

$A \quad 1$ / $A \quad 1$; $A \quad 1$; $A^2 \quad A$ / $A^2 \quad 1$

$A^2 A \quad 1$ / $A \quad 1$; $A^2 A \quad 1$; $A^3 \quad A^2 A$ / $A^3 \quad 1$

$A^3 A^2 A 1$ / $A \quad 1$; $A^3 A^2 A 1$; $A^4 A^3 A^2 A$ / $A^4 \quad 1$

(b) Based on the pattern in part (a), we suspect that $A^5 \quad 1 A \quad 1 \qquad A^4 \quad A^3 \quad A^2 \quad A \quad 1$. Check:

$A^4 A^3 \quad A^2 A 1$ / $A \quad 1$

$A^4 A^3 \quad A^2 A 1$

$A^5 \quad A^4 A^3 \quad A^2 \quad A$

$A^5 \qquad 1$

The general pattern is $A^n \quad 1 \quad A \quad 1 \quad A^{n\ 1} \quad A^{n\ 2} \qquad A^2 \quad A \quad 1$, where n is a positive integer.

P.7 RATIONAL EXPRESSIONS

1. (a) $3x$ / $x^2 \quad 1$ is a rational expression.

$x \quad 1$ / $2x \quad 3$ is not a rational expression. A rational expression must be a polynomial divided by a polynomial, and the numerator of the expression is $x \quad 1$, which is not a polynomial.

$x\, x^2 \quad 1$ / $x^3 \quad x$ is a rational expression.

$3x \quad 3$

To simplify a rational expression we cancel factors that are common to the *numerator* and *denominator*. So, the expression $\frac{x \ 1 \ x \ 2}{x \ 3 \ x \ 2}$ simplifies to $\frac{x \ 1}{x \ 3}$.

Ā Ā Ā Ā Ā Ā Ā

o multiply two rational expressions we multiply their *numerators* together and multiply their *denominators* together. So $\frac{2}{x \ 1} \quad \frac{x}{x \ 3}$ is the same as $\frac{2 \ x}{x \ 1 \ x \ 3} \quad \frac{2x}{x^2 \ 4x \ 3}$.

$1 \quad 2 \quad x$

4. (a) $x \quad x \ 1 \qquad x 1^2$ has three terms.

The least common denominator of all the terms is $x \ x \ 1^2$.

(c) $\frac{1}{x} \quad \frac{2}{x \ 1} \quad \frac{x}{x \ 1^2} \quad \frac{x \ 1^2}{x\, x \ 1^2} \quad \frac{2x\, x \ 1}{x \ 1} \quad \frac{x\, x}{x \ 1^2} \quad \frac{x \ 1^2 \ 2x\, x \ 1 \ x^2}{x\, x \ 1^2}$

$x^2 \ 2x \ 1 \ 2x^2 \ 2x \ x^2 \qquad \frac{2x^2 \ 1}{x\, x \ 1^2}$

$x\, x \ 1^2$

5. (a) Yes. Cancelling $x \quad 1$, we have $\frac{x\, x \ 1}{x \ 1^2} \quad \frac{x}{x \ 1}$.

(b) No; $x 5^2 \quad x^2 \qquad 10x \quad 25 x^2 \quad 25$, so $x \quad 5 \qquad x^2 \quad 10x \quad 25 \qquad x^2 \quad 25$.

6. (a) Yes, $3 \quad 3 \quad 3 \qquad 1 3$. x

No. We cannot “separate” the denominator in this way; only the numerator, as in part (a). (See also Exercise 101.)

7. The domain of $4x^2 \quad 10x \quad 3$ is all real numbers.

8. The domain of $x^4 \quad 3 \quad 9x$ is all real numbers.

9. Since x 3 0 we have x 3. Domain: x x 3

10. Since 3t 6 0 we have t2. Domain: t t2

11. Since x 3 0, x3. Domain; x x3

12. Since x 1 0, x 1. Domain; x x 1

13. x^2 x 2 x 1 x 2 0 x1 or 2, so the domain is x x1 2 .

14. 2x 0 and x 1 0 x 0 and x1, so the domain is x x 0 .

15. 5 x 3 2x 1 / 10 x 3   5 x 3 2x 1 / 5 x 3   2x 1 / 2 x 3

16. 4 x^2 1 / 12 x 2 x 1   4 x 1 x 1 / 12 x 2 x 1 x 1 / 3 x 2

17. x 2 / x^2 4 x 2 1 / x 2 x 2 x 2

18. x^2 x 2 / x^2 1 x 2 x 1 / x 1 x 1 x 2 / x 1

19. x^2 5x 6 / x^2 8x 15 x 2 x 3 / x 5 x 3 x 2 / x 5

20. x^2 x 12 / x^2 5x 6 x 4 x 3 / x 2 x 3 x 4 / x 2

21. y^2 y / y^2 1 y y 1 / y 1 y 1 y / y 1

22. y^2 3y 18 / 2y^2 7y 3 y 6 y 3 / 2y 1 y 3 y 6 / 2y 1

23. 2x3 x2 6x / 2 2x 7x 6 x 2x2 x 6 / 2x 3 x 2 x 2x 3 x 2 / 2x 3 x 2x 3 / 2x 3

24. 1 x^2 / x^3 1   1 x 1 x / x 1 x^2 x 1 x 1 1 x / x 1 x^2 x 1 x 1 / x^2 x 1

25. 4x / x^2 4 x 2 / 16x 4x / x 2 x 2 x 2 / 16x 1 / 4 x 2

26. x^2 25 x 4 / x^2 16 x 5 x 5 x 5 x 4 / x 4 x 4 x 5 x 5 / x 4

27. x^2 2x 15 x 5 / x^2 25 x 2 x 5 x 3 x 5 / x 5 x 5 x 2 x 3 / x 2 x 1   1 x

28. x^2 2x 3 3 x / x^2 2x 3 3 x x 3 x 1x 3 / x 3 x 1 x 3 x 1 / x 1 x 1   1 x

29. t 3 t 3 / t^2 9 t^2 9 t 3 t 3 / t^2 9 t 3 t 3   1 / t^2 9

30. x^2 x 6 / x^2 2x x^3 x^2 / x^2 2x 3 x 3 x 2 / x x 2 x^2 x 1 / x 3 x 1 x x 4

31. x^2 7x 12 x^2 5x 6 / x^2 3x 2 x^2 6x 9 x 3 x 4 x 2 x 3 / x 1 x 2 x 3 x 3 x 4 / x 1

32. x^2 2xy y^2 / x^2 y 2 x^2 xy y^2 / x xy 2y 2 2 x y x y x y 2x y / x y x y x 2y x y 2x y / x 2y

33. x 3 / 4x^2 9 x^2 7x^2 12 / 2x^2 7x 15 x 3 / 4x^2 9   2x^2 7x 15 / x^2 7x 12 x 3 / 2x 3 2x 3 x 5 2x 3 / x 3 x 4 x 5 / 2x 3 x 4

34. 2x 1 / 2x^2 x 15   6x x 2 / x 3   2x 1 / x 3 2x 5 x 3 / 2x 1 3x 2   1 / 2x 5 3x 2

3

35. x / x 1 x 1 x^3 x^2 2x 1   3 x 1 x 1 x^2 x 1

$x^2 \quad 2x \quad 1$ $\qquad x \qquad x \quad 1\, x$

$2 \qquad 2$

$2x \quad 3x \quad 2 \qquad 2x \quad 3x \quad 2 \qquad x^2 \quad x \quad 2 \qquad x \quad 2 \;\; 2x \quad 1 \qquad x \quad 1 \;\; x \quad 2 \qquad x \quad 2$

36. $\dfrac{x^2 \quad 1}{2x^2 \quad 5x \quad 2}$

2

$x \quad x \quad 2$

$x^2 \quad 1$ $\quad 2x^2 \quad 5x \quad 2x \quad 1 \quad x \quad 1x \quad 2 \quad 2x \quad 1x \quad 1$

x y x 1x
zy z yz
y x z xz

y z x z 1 yy
x 31x 4
1 x 3 x 3 x 3 x 3

3x 2 3x 2 2x 1 3x 2 2x 2 x 4
40. x 1 2 x 1 x 1 x 1 x 1

1 2 x 3 2x 5 x 3 2x 10 3x 7
41. x 5 x 3 x 5 x 3 x 5 x 3 x 5 x 3 x 5 x 3

1 1 x 1 x 1 x 1 x 1 2x
42. x 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1

3 1 3x 2 x 1 3x 6 x 1 2x 5
43. x 1 x 2 x 1 x 2 x 1 x 2 x 1 x 2 x 1 x 2

x 3 xx 6 3x 4 x^2 6x 3x 12 x^2 3x 12
44. x 4 x 6 x 4 x 6 x 4 x 6 x 4 x 6 x 4 x 6

5 3 5 2x 3 3 10x 15 3 10x 18 2 5x 9
45. 2x 3 $2x\ 3^2$ $2x\ 3^2$ $2x\ 3^2$ $2x\ 3^2$ $2x\ 3^2$ $2x\ 3^2$

x 2 x 2x 1 x 2x 2 3x 2
46. $x\ 1^2$ x 1 $x\ 1^2$ x 1 x 1 $x\ 1^2$ $x\ 1^2$

u 1 u u 1 u 1 u u^2 2u 1 u u^2 3u 1
47. u 1 u 1 u 1 u 1 u 1

2 3 4 $2b^2$ 3ab $4a^2$ $2b^2$ 3ab 4 a^2
48. a^2 ab b^2 a^2b^2 a^2b^2 a^2b^2 a^2b^2

1 1 1 1 x 1 x 2x 1
49. x^2 x^2 x x^2 xx 1 x^2 x 1 x^2 x 1 x^2 x 1
x

1 1 1 x^2 x 1 x^2 x 1
50. x x^2 x^3 x^3 x^3 x^3 x^3

2 1 2 1 2x 4 1
51. x 3 x^2 7x 12 x 3 x 3 x 4 x 3 x 4 x 3 x 4

2x 8 1 2x 7
x 3 x 4x 3 x 4

x 1 x 1 x x 2
52. x^2 4 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2
2x 2 2x 1
x 2 x 2x 2 x 2

1 1 1 1 x 3 1 x 2
53. x 3 x^2 9 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3

x 2 x 2
54. x^2 x 2 x^2 5x 4 x 1 x 2 x 1 x 4
xx 4 2x 2 x^2 4x 2x 4 x^2 6x 4
x 1 x 2 x 4x 1 x 2 x 4x 1 x 2 x 4x 1 x 2 x 4

2 3 4 2 3 4 2x 1 3x 4 2x 2 3x 4 5x 6
55. x x 1 x^2 x x x 1 xx 1 xx 1 xx 1 xx 1 xx 1 xx 1

x 1 2 x 1 2
56. x^2 x 6 x 2 x 3 x 3 x 2 x 2 x 3
x 1x 3 2x 2 x x 3 2x 4 2x 1

x 3 x 2 x 3 x 2 x 3 x 2 x 3 x 2 x 3 x 2

57. $\frac{1}{x^2 \; 3x \; 2} \quad \frac{1}{x^2 \; 2x \; 3} \quad \frac{1}{x \; 2 \; x \; 1} \quad \frac{1}{x \; 3 \; x \; 1}$

$\frac{x \; 3}{x \; 3 \; x \; 2 \; x \; 1} \quad \frac{x \; 2}{x \; 3 \; x \; 2 \; x \; 1} \quad \frac{x \; 3 \; x \; 2}{x \; 3 \; x \; 2 \; x \; 1} \quad \frac{5}{x \; 3 \; x \; 2 \; x \; 1}$

58. $\frac{1}{x \; 1} \quad \frac{2}{x \; 1^2} \quad \frac{3}{x^2 \; 1} \quad \frac{1}{x \; 1} \quad \frac{2}{x \; 1^2} \quad \frac{3}{x \; 1 \; x \; 1}$

$\frac{x \; 1 \; x \; 1}{x \; 1 \; x \; 1^2} \quad \frac{2 \; x \; 1}{x \; 1 \; x \; 1^2} \quad \frac{3 \; x \; 1}{x \; 1 \; x \; 1^2}$

$\frac{x^2 \; 1}{x \; 1 \; x \; 1^2} \quad \frac{2x \; 2}{x \; 1 \; x \; 1^2} \quad \frac{3x \; 3}{x \; 1 \; x \; 1^2} \quad \frac{x^2 \; 1 \; 2x \; 2 \; 3x \; 3}{x \; 1 \; x \; 1^2} \quad \frac{x^2 \; x \; 4}{x \; 1 \; x \; 1^2}$

59. $1_1 \, x$ $\quad x \; 1 \quad 1$

60. $x \; 2 \quad x \quad 2 \quad 1 \; 2x$

$13 \; y \quad 3 \quad y \quad y \; 2$

$y \; 1 \quad 2$

$y \; 1 \quad y \quad y \quad 1 \quad 3 \; y \; 1$

61. $1 \quad x \; 2 \; 1 \quad x \; 2 \quad x \; 2 \; 1 \quad x \; 2 \; 1 \quad x \; 1 \quad x \; 3$

$1 \quad 1$

1

62. $\frac{1}{c \; 1} \quad c \; 1 \; 1 \quad c$

$1 \quad 1 \quad c \; 1 \; 1 \; c \; 2$

63. $x \quad c \; 1 \quad 1 \; 3 \quad x \; 1 \; x \; 3 \; x \; 1 \; x \; 3 \quad x \; 3x \; 1 \quad 2 \; x \; 1$

$1 \quad 1$

$x \; 1 \quad x \; 1 \; x \; 3 \; x \; 1 \quad x \; 1 \; x \; 1 \; x \; 3 \quad x \; 1 \; x \; 1 \; x \; 3$

$2 \quad x \; 1 \; x \; 3$

64. $x \; 4 \; x \; 1 \quad x \; 3 \; x \; 1x \; 2 \; x \; 4 \quad x \; 2x \; 3 \; x \quad 2x \; 8 \quad 5$

$x \; 3 \quad x \; 2 \quad 2 \quad 2$

$x \; 3 \quad x \; 4 \; x \; 3 \; x \; 1 \quad x \; 4 \; x \; 3 \; x \; 1 \quad x \; 4 \; x \; 3 \; x \; 1$

65. $x \quad x$

66. $y \; x \quad xy \; y \; x \quad xy \; y \; 2 \quad y \; x \; 1$

$x \quad x \quad xy \; x \quad x \quad x^2 \; y \quad y^2 \quad y \; y \quad x^2$

$y \quad y \quad xy \; y \quad y \quad xy \quad x \quad x \; x \; y$

$y \quad x \; y^2 \quad 2 \; 2 \quad 2 \; 2$

$y \; x \quad xy \quad x \; y \quad x \; y \quad xy$

$1 \quad 1$

$2y^2 \quad y^2 \; x^2 \; 2 \; 2 \; y^2 \quad xy \quad y^2 \; x^2 \quad 1 \quad xy.$ An alternative method is to multiply the numerator and denominator by the common denominator of both the numerator and denominator, in this case x^2

$x \quad y \qquad x \quad y \qquad\qquad y^2:$

$1 \quad 1 \qquad 1 \quad 1 \qquad 2 \; 2 \qquad 2 \quad 2 \qquad 2 \quad 2 \qquad xy.$

$y \qquad y \qquad y$

$x^2 \quad y^2 \qquad x^2 \quad y^2 \, x \qquad x \qquad x$

68. $\dfrac{\frac{x}{y}-\frac{y}{x}}{\frac{1}{y^2}-\frac{1}{x^2}} = \dfrac{\frac{x^2-y^2}{xy}}{\frac{x^2-y^2}{x^2y^2}} = \dfrac{x^2-y^2}{xy}\cdot\dfrac{x^2y^2}{x^2-y^2} = xy$

69. $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} = \dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}+\frac{1}{y}} = \dfrac{\frac{y^2-x^2}{x^2y^2}}{\frac{y+x}{xy}} = \dfrac{(y-x)(y+x)}{x^2y^2}\cdot\dfrac{xy}{y+x} = \dfrac{y-x}{xy}$.

Alternatively, $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} = \dfrac{(x^{-1}-y^{-1})(x^{-1}+y^{-1})}{x^{-1}+y^{-1}} = \dfrac{1}{x}-\dfrac{1}{y} = \dfrac{y-x}{xy}$

70. $\dfrac{x^{-1}+y^{-1}}{(x+y)^{-1}} = \dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x+y}} = \dfrac{\frac{y+x}{xy}}{\frac{1}{x+y}} = \dfrac{(x+y)(x+y)}{xy} = \dfrac{xy+y^2+x^2+xy}{xy} = \dfrac{x^2+2xy+y^2}{xy} = \dfrac{(x+y)^2}{xy}$

71. $1-\dfrac{1}{1-\frac{1}{x}} = 1-\dfrac{x}{x-1} = \dfrac{x-1-x}{x-1} = \dfrac{1}{1-x}$

72. $1+\dfrac{1}{1+\frac{1}{1+x}} = 1+\dfrac{1+x}{1+x+1} = 1+\dfrac{x+1}{x+2} = \dfrac{x+2+x+1}{x+2} = \dfrac{2x+3}{x+2}$

73. $\dfrac{\frac{1}{1+x+h}-\frac{1}{1+x}}{h} = \dfrac{1+x-(1+x+h)}{h(1+x)(1+x+h)} = \dfrac{-1}{(1+x)(1+x+h)}$

In calculus it is necessary to eliminate the h in the denominator, and we do this by rationalizing the numerator:

$\dfrac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h} = \dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\dfrac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}} = \dfrac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)} = \dfrac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$

75. $\dfrac{(x+h)^{-2}-x^{-2}}{h} = \dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \dfrac{x^2-(x+h)^2}{hx^2(x+h)^2} = \dfrac{x^2-(x^2+2xh+h^2)}{hx^2(x+h)^2} = \dfrac{-2xh-h^2}{hx^2(x+h)^2} = \dfrac{-(2x+h)}{x^2(x+h)^2}$

76. $\dfrac{(x+h)^3-7(x+h)-(x^3-7x)}{h} = \dfrac{x^3+3x^2h+3xh^2+h^3-7x-7h-x^3+7x}{h} = \dfrac{3x^2h+3xh^2+h^3-7h}{h} = \dfrac{h(3x^2+3xh+h^2-7)}{h} = 3x^2+3xh+h^2-7$

77. $\sqrt{1+\left(\dfrac{x}{\sqrt{1-x^2}}\right)^2} = \sqrt{1+\dfrac{x^2}{1-x^2}} = \sqrt{\dfrac{1-x^2+x^2}{1-x^2}} = \sqrt{\dfrac{1}{1-x^2}} = \dfrac{1}{\sqrt{1-x^2}}$

16 6

78. 1 4x 3 1 x⁶

1 2

2x 3 16x⁶ 1

1 x₀ 16x⁶ 1

x⁶ 2 1 1

x3 4x

3 x 3 4x3

1 2 1

3 x 2 2 x 3 2 x 2 3 2 x 3 x 2 2 x 3 [3 x 3x 2 2]

79. x 3 4 x 3 4

2 2

x 2 3x 9 2x 4 x 2 x 13

x 3 3 x 3 3

80. 8 8 5 5 5

2x x 6 4 x 2 4 x 6 3 x 6 3 2x x 6 4x 2 2x 2 12x 4x 2 12x 2x 2 2x 6 x

x 6 x 6 x 6 x 6 x 6

2 1 x 1 2 x 1 x 1 2 1 x 1 2 [2 1 x x] x 2

81. 1 x 1 x 1 x 3 2

2 3 2

2 1 2 x 2 1 2 2 1 2 2 2 1

1 x 2 1 x 1 x 1 x x

2

82. 1 3 1 2 3 3 x 1 x 2

2

3 1 x x 1 x 1 x [3 1 x x] 2x 3

83. 1 x 2 3 1 x 2 3 1 x 4 3

84. 7 3x 2 x 7 2 2 3 2

3 3

1 2 3x 1 2 7 3x 1 2 7 3x x 7 3 x

7 3x 7 3x 7 3x

1 15 3 5 3 5 3

85. 5 3 5 3 5 3 25 3 22

2 3 6 3

86. 3 5 5 6 3 5

2 5 2 5 2 5 4 5

87. 2 2 2 7 2 2 7 2 2 7 2 7 2

2 7 2 2 7 2 7 5 5

1 1 x 1 x 1

88. x 1 x 1 x 1 x 1

y y 3 y 3 y y 3 y

89. y y

3 y 3 y 3 y 3 y 3 y

90. x x y y y xy 2 x 2 y

2 x y 2 x y x 2 x y x

1 5 1 5 1 5 1 5 4

91. 3 3 1 5 3 1 5 3 1 5

3 5 3 3 5 3 5 2 1

92. 2 2 3 5 2 3 5 2 3 5 3 5

r 2

5

93.

r 2 r 2 r 2

r 2 5 r 2

x x h x x h x x h x x h

94. h x x h h x x h h x x h

h2 x h x x h 2 h x x1

95. x 1 x

2

x 1 x x 1 x x 2 1 x x^2 1 x

x 1 x x 1 x 1

96. $\dfrac{x}{x+1} - \dfrac{x+1}{x} \Big/ \dfrac{1}{x+1} \; ... \; x \quad 1 \quad x$ (expression fragments: x 1 x ; 1 ; x 1 ; x 1 x ; x 1 ; x ; 1 ; x 1 ; x)

97. (a) $R = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} \cdot \dfrac{R_1 R_2}{R_1 R_2} = \dfrac{R_1 R_2}{R_2 + R_1}$

(b) Substituting $R_1 = 10$ ohms and $R_2 = 20$ ohms gives $R = \dfrac{10 \cdot 20}{20 + 10} = \dfrac{200}{30} \approx 6\ 7$ ohms.

98. (a) The average cost $A = \dfrac{\text{Cost}}{\text{number of shirts}} = \dfrac{500 + 6x + 0\ 01x^2}{x}$.

(b)

x	10	20	50	100	200	500	1000
Average cost	$56 10	$31 20	$16 50	$12 00	$10 50	$12 00	$16 50

99.

x	2 80	2 90	2 95	2 99	2 999	3	3 001	3 01	3 05	3 10	3 20
$\dfrac{x^2-9}{x-3}$	5 80	5 90	5 95	5 99	5 999	?	6 001	6 01	6 05	6 10	6 20

From the table, we see that the expression $\dfrac{x^2-9}{x-3}$ approaches 6 as x approaches 3. We simplify the expression:

$\dfrac{x^2-9}{x-3} = \dfrac{(x-3)(x+3)}{x-3} = x + 3$, $x \neq 3$. Clearly as x approaches 3, $x + 3$ approaches 6. This explains the result in the table.

100. No, squaring $\dfrac{2}{x}$ changes its value by a factor of $\dfrac{2}{x}$.

101. Answers will vary.

Algebraic Error	Counterexample
$\dfrac{1}{a} + \dfrac{1}{b} \neq \dfrac{1}{a+b}$	$\dfrac{1}{2} + \dfrac{1}{2} \neq \dfrac{1}{2+2}$
$(a+b)^2 \neq a^2 + b^2$	$(1+3)^2 \neq 1^2 + 3^2$
$\sqrt{a^2+b^2} \neq a + b$	$\sqrt{5^2+12^2} \neq 5 + 12$
$\dfrac{a+b}{a} \neq b$	$\dfrac{2+6}{2} \neq 6$
$\dfrac{a}{a+b} \neq \dfrac{1}{b}$	$\dfrac{1}{1+1} \neq 1$
$\dfrac{a^m}{a^n} \neq a^{m/n}$	$\dfrac{3^5}{3^2} \neq 3^{5/2}$

102. (a) $5\ a \quad 5 \quad a \quad a$

$\overline{5}$ $\overline{5}$ $\overline{5}\,1$ $\overline{5}$, so the statement is true.

(b) This statement is false. For example, take $x = 5$ and $y = 2$. Then LHS $= \dfrac{x+1}{y+1} = \dfrac{5+1}{2+1} = \dfrac{6}{3} = 2$, while RHS $= \dfrac{x}{y} = \dfrac{5}{2}$, and $2 \neq \dfrac{5}{2}$.

This statement is false. For example, take $x = 0$ and $y = 1$. Then LHS $= \dfrac{x}{x+y} = \dfrac{0}{0+1} = 0$, while RHS $= \dfrac{1}{1+y} = \dfrac{1}{1+1} = \dfrac{1}{2}$, and $0 \neq \dfrac{1}{2}$.

(d) This statement is false. For example, take x 1 and y 1. Then LHS 2 $\frac{a}{b}$ 2 $\frac{1}{1}$ 2, while

RHS $\frac{2a}{2b}$ $\frac{2}{2}$ 1, and 2 1.

This statement is true: $\frac{a}{b}$ $\frac{1}{b}$ $\frac{1}{b}$ $\frac{a}{b}$ $\frac{1}{b}$ $\frac{1}{b}$ $\frac{a}{b}$.

(f) This statement is false. For example, take x 2. Then LHS $\frac{2}{4\ x}$ $\frac{2}{4\ 2}$ $\frac{2}{6}$ $\frac{1}{3}$, while

RHS $\frac{1}{2}$ $\frac{2}{x}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$, and $\frac{1}{3}$ $\frac{3}{2}$.

103. (a)

x	1	3	$\frac{1}{2}$	$\frac{9}{10}$	$\frac{99}{100}$	$\frac{999}{1000}$	$\frac{9999}{10{,}000}$
$x\ \frac{1}{x}$	2	3 333	2 5	2 011	2 0001	2 000001	2 00000001

It appears that the smallest possible value of x $\frac{1}{x}$ is 2.

(b) Because x 0, we can multiply both sides by x and preserve the inequality: x $\frac{1}{x}$ 2 x x $\frac{1}{x}$ $2x$ x^2 1 $2x$ x^2 $2x$ 1 $0x$ 1^2 0. The last statement is true for all x 0, and because each step is reversible, we have shown that x $\frac{1}{x}$ 2 for all x 0.

P.8 SOLVING BASIC EQUATIONS

Substituting x 3 in the equation $4x$ 2 10 makes the equation true, so the number 3 is a *solution* of the equation.

Subtracting 4 from both sides of the given equation, $3x$ 4 10, we obtain $3x$ 4 4 10 4 $3x$ 6. Multiplying by $\frac{1}{3}$, we have $\frac{1}{3}$ $3x$ $\frac{1}{3}$ 6 x 2, so the solution is x 2.

3. (a) $\frac{x}{2}$ $2x$ 10 is equivalent to $\frac{5}{2}x$ 10 0, so it is a linear equation.

$\frac{2}{x}$ $2x$ 1 is not linear because it contains the term $\frac{2}{x}$, a multiple of the reciprocal of the variable.

x 7 5 $3x$ $4x$ 2 0, so it is linear.

(a) x x 1 6 x^2 x 6 is not linear because it contains the square of the variable.

x 2 $\sqrt{x}$ is not linear because it contains the square root of x 2.

$3x^2$ $2x$ 1 0 is not linear because it contains a multiple of the square of the variable.

(a) This is true: If a b, then a x b x.

This is false, because the number could be zero. However, it is true that multiplying each side of an equation by a *nonzero* number always gives an equivalent equation.

This is false. For example, 5 5 is false, but 5^2 5^2 is true.

To solve the equation x^3 125 we take the *cube* root of each side. So the solution is x $\sqrt[3]{125}$ 5.

(a) When x2, LHS 4 2 7 8 7 1 and RHS 9 2 3 18 3 21. Since LHS RHS, x 2 is not a solution.

When x 2, LHS 4 2 7 8 7 15 and RHS 9 2 3 18 3 15. Since LHS RHS, x 2 is a solution.

(a) When x1, LHS 2 5 1 2 5 7 and RHS 81 7. Since LHS RHS, x1 is a solution.

When x 1, LHS 2 5 1 2 53 and RHS 8 1 9. Since LHS RHS, x 1 is not a solution.

(a) When x 2, LHS 1 [2 3 2] 1 [2 1] 1 1 0 and RHS 4 26 2 8 8 0. Since LHS RHS, x 2 is a solution.

When x 4 LHS 1 [2 3 4] 1 [2 1] 1 3 2 and RHS 4 4 6 4 16 10 6. Since LHS RHS, x 4 is not a solution.

10. (a) When x 2, LHS $\frac{1}{2}$ $\frac{1}{2\ 4}$ $\frac{1}{2}$ $\frac{1}{2}$. $\frac{1}{2}$ $\frac{1}{2}$ 1 and RHS 1. Since LHS RHS, x 2 is a solution.

(b) When x 4 the expression $\frac{1}{4\ 4}$ is not defined, so x 4 is not a solution.

11. (a) When x 1, LHS 2 $1^{1\,3}$ 3 2 1 32 35. Since LHS 1, x1 is not a solution.

(b) When x 8 LHS 2 $8^{1\,3}$ 3 2 2 3 4 3 1 RHS. So x 8 is a solution.

12. (a) When x 4, LHS $\frac{4^{3\,2}}{4\ 6}$ $\frac{2^3}{2}$ $\frac{8}{2}$ 4 and RHS 4 84. Since LHS RHS, x 4 is a solution.

(b) When x 8, LHS $\frac{8^{3\,2}}{8\ 6}$ $\frac{2^{3\,3\,2}}{2}$ $\frac{2^{9\,2}}{2}$ $2^{7\,2}$ and RHS 8 8 0. Since LHS RHS, x 8 is not a solution.

13. (a) When x 0, LHS $\frac{0\ a}{0\ b}$ $\frac{a}{b}$ $\frac{a}{b}$ RHS. So x 0 is a solution.

When x b, LHS $\frac{b\ a\ b\ a}{b\ b}$ 0 is not defined, so x b is not a solution.

14. (a) When x $\frac{b}{2}$, LHS $\frac{b}{2}$ 2 b $\frac{b}{2}$ $\frac{1}{4}b^2$ $\frac{b^2}{4}$ $\frac{b^2}{2}$ $\frac{b^2}{2}$ 4 0 RHS. So x $\frac{b}{2}$ is a solution.

(b) When x $\frac{1}{b}$, LHS $\frac{1}{b}$ 2 b $\frac{1}{b}$ $\frac{1}{4}b^2$ $\frac{1}{b^2}$ 1 $\frac{b}{4}$, so x $\frac{1}{b}$ is not a solution.

15. $5x$ 6 14 $5x$ 20 x 4

16. $3x$ 4 7 $3x$ 3 x 1

17. 7 $2x$ 15 $2x$8 x4

18. $4x$ 95 1 $4x$ 96 x 24

19. $\frac{1}{2}$ x 7 3 $\frac{1}{2}x$4 x8

20. $\frac{1}{2}$ $\frac{1}{3}x$4 $\frac{1}{3}x$6 x18

21. $3x$ 3 $5x$ 3 0 $8x$ x 0

22. $2x$ 3 5 $2x$ $4x$ 2 x $\frac{1}{2}$

23. $7x$ 1 4 $2x$ $9x$ 3 x $\frac{1}{3}$

24. 1 x x 43 $2x$ x $\frac{3}{2}$

25. x 3 $4x$ 3 $5x$ x $\frac{3}{5}$

26. $2x$ 3 7 $3x$ $5x$ 4 x $\frac{4}{5}$

27. $\frac{x}{3}$ $1\frac{5}{3}$ x 7 x 3 $5x$ 21 $4x$24 x6

28. $\frac{2}{5}x$ 1 $\frac{3}{10}x$ 3 $4x$ 10 $3x$ 30 x 40

29. 2 1 x 3 1 $2x$ 5 2 $2x$ 3 $6x$ 5 2 $2x$ 8 $6x$6 $8x$ x $\frac{3}{4}$

30. 5 x 3 92 x 2 1 $5x$ 15 92x 4 1 $5x$ 242x 3 7x21 x3

31. 4 y $\frac{1}{2}$ y 6 5 y4y 2 y 30 $6y$ $3y$ 2 30 $6y$ $9y$ 32 y $\frac{32}{9}$

32. r 2 [1 3 2r 4] 61 r 2 1 6r 12 61 r 2 6r 11 61 r 12r 22 61 13r 39
r 3

33. x $\frac{1}{3}$ x $\frac{1}{2}x$ 5 0 $6x$ $2x$ $3x$ 30 0 (multiply both sides by 6) x 30

34. $\frac{2}{3}y$ $\frac{1}{2}$ y 3 $\frac{y\ 1}{4}$ 8y 6 y 3 3 y 1 8y 6y 18 3y 3 14y 18 3y 3 11y 21

y $\frac{21}{11}$

2*x* x x 1 6*x* 8*x* 2*x* *x* 1 24*x* 7*x* 1 24*x* 1 17*x* *x* 1

24 17

5*x* *x* 1 1

36. 3*x* 2 3 6 18*x* 15*x* 2 *x* 1 1 3*x* 2*x* 1 *x* 1

x

x 1 *x* 2*x* 2 *x* 3*x* 2 *x* 2 2 5*x* 6 *x* 25*x* 6 6*x* 8 *x* 4 3

2 2

x x 1*x* 3 2 *x* *x x* 6*x* 9 *x* 6*x* 95*x* 9 *x*

2 2 14

x 1 4*x* 52*x* 3 2 4*x* *x* 5 4*x* 12*x* 9 *x* 512*x* 9 13*x* 14 *x* 13

2

t 4 2 *t* 4 2 32 *t* 2 8*t* 16 *t* 8*t* 16 3216*t* 32 *t*2

1 4 1

41. *x* 3*x* 1 3 4 3*x* (multiply both sides by the LCD, 3*x*)1 3*x* *x* 3

2 5 6 4 2 5*x* 6 4*x*4 9*x* 4 *x*

9

xx

2*x* 1 4 13

43. *x* 2 5 5 2*x* 1 4 *x* 2 10*x* 5 4*x* 8 6*x* 13 *x* 6

2*x* 7 2 2*x* 7 3 2 2*x* 4 (cross multiply) 6*x* 21 4*x* 8 2*x* 29 *x* 29

2*x* 4 32

2 3

45. *t* 6 *t* 1 2 *t* 1 3 *t* 6 [multiply both sides by the LCD, *t* 1 *t* 6] 2*t* 2 3*t* 1820 *t*

5

x 3 *x* 4 6 *x* 4 5 *x* 36*x* 24 5*x* 15 *x*39

3 1 1

47. *x* 1 2 3*x* 3 3 63*x* 3 2 [multiply both sides by 6 *x* 1] 18 3*x* 3 23*x* 15 2

13

3*x*13 *x* 3

12*x* 5 5 2 2

48. 6*x* 3 2 *x* 12*x* 5 *x* 2*x* 6*x* 3 5 6*x* 3 12*x* 5*x* 12*x* 6*x* 30*x* 15

12*x* 2 5*x* 12*x* 2 24*x* 15 19*x*15 *x* 15/19

1 1 1 10

49. *z* 2*z* 5*z* *z* 1 10 *z* 1 5 *z* 1 2 *z* 110 10*z* [multiply both sides by 10*z z* 1]

3 *z* 1 100*z* 3*z* 3 100*z* 3 97*z* 3/97 *z*

1 4 15

50. 3 *t* 3 *t* 9 *t* 2 03 *t* 4 3 *t* 15 0 3 *t* 12 4*t* 15 03*t* 30 03*t*30

t 10

x 1

51. 2*x* 4 2*x* 2 *x* 2 2*x* 4 2 [multiply both sides by 2 *x* 2] *x* 4*x* 8 23*x*6 *x* 2.

But substituting *x* 2 into the original equation does not work, since we cannot divide by 0. Thus there is no solution.

1 5 2

52. *x* 3 *x* 2 9 *x* 3 *x* 3 5 2 *x* 3*x* 2 2*x* 6 *x*4

1 6*x* 12

x 4 *x x*2 4*x* 3 *x x* 4 6*x* 12 (multiply both sides by *x x* 4] 3*x* 7*x* 16 4*x* 16 *x* 4. But substituting *x* 4 into the original equation does not work, since we cannot divide by 0. Thus, there is no solution.

1 2 1 1

54. *x* 2*x* 1 2*x* 2 *x* 2*x* 1 2 *x* 1 1 1. This is an identity for *x* 0 and *x* 2, so the solutions are all real numbers except 0 and $\frac{1}{2}$.

x^2 25 x5

3x^2 48 x^2 16 x4

2 2 −

5x 15 x 3 x3

58. x^2 1000 x100010 10

59. 8x^2 64 0 x ^{2}8 0 x ^{2} 8 x82 2

60. 5x^2 125 0 5 x^2 250 x^2 25 x5
2 2

61. x 16 0 x 16 which has no real solution.

62. 6x^2 100 0 6$x^2$100 x^2 50 3, which has no real solution.
2

63. x 35 x 35 x 3 5
2 4 7

64. 3x 4 7 3x 47 3x 4 7 x 3

65. x^3 27 x 27$^{1\ 3}$ 3
5 5 1 5

66. x 32 0 x 32 x32 2

67. 0 x^4 16x^2 4 x^2 4 x^2 4 x 2 x 2 x^2 4 0 has no real solution. If x 2 0, then x 2.

If x 2 0, then x2. The solutions are 2.

68. 64x^6 27 x^6 27 / 64 x 27 1 6 / 64 27$^{1\ 6}$ / 64$^{1\ 6}$ 3 / 2
4 4

x 64 0 x 64 which has no real solution.

x 1^3 8 0x 1^{3}8 x 18$^{1\ 3}$2 x1.

71. x 2^4 81 0x 2 4 81x 2 4 $^{1\ 4}$ 81$^{1\ 4}$ x 23. So x 2 3, then x 1. If x 23, then x 5. The solutions are 5 and 1.

x 1^4 16 0x 1^{4}16, which has no real solution.

3 x 3^3 375x 3^3 125x 3 125$^{1\ 3}$ 5 x 3 5 8

4 x 2^5 1x 2^5 1 4 x 2 5 1 4 x2 5 1 4

3 x 5 x 5^3 125

$x^{4\ 3}$ 16 0$x^{4\ 3}$ 16 2^4 $x^{4\ 3}$ ^{3} 2^4 ^{3} 2^{12} x^4 2^{12} x 2^{12} $^{1\ 4}$ 2^3 8
5 3 5 3 5 3 3 5 5 1 5 3
2x 64 0 2x 64 x 32 x32 2 2 8

6$x^{2\ 3}$ 216 06$x^{2\ 3}$ 216 $x^{2\ 3}$ 36 6 2 $x^{2\ 3}$ $^{3\ 2}$ 6 2 $^{3\ 2}$ x 6 3 216

79. 3 02x 1 48 10 92 3 02x 9 44 x 9 44 / 3 02 3 13

80. 8 36 0 95x 9 970 95x 1 61 x 1 61 / 0 95 1 69

81. 2 15x 4 63 x 1 19 1 15x 5 82 x 5 82 / 1 19 5 06

82. 3 95 x 2 32x 2 00 1 95 3 32x x 1 95 / 3 32 0 59

83. 3 16 x 4 63 4 19 x 7 243 16x 14 63 4 19x 30 34 44 97 1 03x x 44 97 / 1 03 43 66
2 14 x 4 06 2 27 0 11x 2 14x 8 6684 2 27 0 11x 2 25x 10 9584 x 4 8704 4 87
0 26x 1 94

7 27

1 76 0 26x 1 94 1 76 3 03 2 44x 0 26x 1 94 5 33 4 29x 4 55x 7 27 3 03 2 44x

x 4 55 1 60

1 73x 3 20

86. 2 12 x 1 51 1 73x 1 51 2 12 x1 73x 3 20 1 51x 0 22x 3 20 x 0 22 14 55

87. r $\frac{12}{M}$ M $\frac{12}{r}$

88. d $r T H$ T $\frac{d}{rH}$ 2

89. $P V$ $n R T$ R $\frac{PV}{nT}$

90. F G $\frac{mM}{r^2}$ m $\frac{Fr}{GM}$

P 2l 22 P 2l P 2l 2

1 1 1

92. $\overline{R}$ $\overline{R1}$ $\overline{R2}$ $R1$ $R2$ R $R2$ R $R1$(multiply both sides by the LCD, R $R1$ $R2$). Thus $R1$ $R2$ R $R1$ R $R2$

$R1$ $R2$ $RRR2$ $R1$ $\frac{R R2}{R2 \ R}$.

93. V $\frac{1}{3} r^2 h$ r^2 $\frac{3V}{}$ $3V$ h r h

94. F G $\frac{mM}{r^2}$ r^2 G $\frac{mM}{F}$ mM r GF

95. V $\frac{4}{3}$ r 3 r 3 $\frac{3V}{4}$ 3 r $\frac{3V}{4}$

96. a^2 b^2 c^2 b^2 c^2 a^2 b $\overline{c^2 \ a^2}$

100 i 2 A_P i 100 2 i 100 A_P i 100 1 P A A

98. $a^2 x$ a 1a 1 x $a^2 x$ a 1 xa 1a^2 a 1 xa 1a^2 a 1 xa 1

x $\frac{a \ 1}{c^2 \ a \ 1}$

99. $\frac{ax \ b}{cx \ d}$ 2 ax b 2 cx d ax b 2cx 2d ax 2cx 2d ba 2 2c x 2d b x $\frac{2d \ b}{a \ 2c}$

100. $\frac{a \ 1}{b}$ $\frac{a \ 1}{b}$ $\frac{b \ 1}{a}$ a a 1 a a 1 b b 1 a^2 a a a^2 b b^2 2a b b^2

a $\frac{1}{2} b^2$ b

101. (a) The shrinkage factor when 250 is S $\frac{0032\ 250 \quad 25}{10{,}000}$ $\frac{8 \quad 2\ 5}{10{,}000}$ 0 00055. So the beam shrinks

0 00055 12 025 0 007 m, so when it dries it will be 12 025 0 007 12 018 m long.

(b) Substituting S 0 00050 we get 0 00050 $\frac{003225}{10{,}000}$ 5 0 032 2 5 7 5 0 032

$\frac{7\ 5}{0\ 032}$ 234 375. So the water content should be 234 375 kg/m^3.

3
1
5
0

102. Substituting C 3600 we get 3600 450 3 75x 3150 3 75x x $\overline{3\ 75}$ 840. So the toy manufacturer can manufacture 840 toy trucks.

103. (a) Solving for when P 10,000 we get 10,000 15 6 ^{33} 641 028 6 km/h.

(b) Solving for when P 50,000 we get 50,000 15 6 ^{33} 3205 1314 7 km/h.

104. Substituting F 300 we get 300 0 3$x^{3\ 4}$ 1000 10^3 $x^{3\ 4}$ $x^{1\ 4}$ 10 x 10^4 10,000 lb.

105. (a) 3 0 k 5 k 0 k 1 k 5k 1 2k 6 k 3

(b) $3\,1 \quad k \quad 5 \quad k\,1 \quad k \quad 1 \quad 3 \quad k \quad 5 \quad k \quad k \quad 1 \quad k \quad 2 \quad 1 \quad k \quad 3$

(c) $3\,2 \quad k \quad 5 \quad k\,2 \quad k \quad 1 \quad 6 \quad k \quad 5 \quad 2k \quad k \quad 1 \quad k \quad 1 \quad k \quad 1$. $x \quad 2$ is a solution for every value of k.

That is, $x \quad 2$ is a solution to every member of this family of equations.

106. When we multiplied by x, we introduced $x \quad 0$ as a solution. When we divided by $x \quad 1$, we are really dividing by 0, since

$x \quad 1 \quad x \quad 1 \quad 0.$

P.9 MODELING WITH EQUATIONS

An equation modeling a real-world situation can be used to help us understand a real-world problem using mathematical methods. We translate real-world ideas into the language of algebra to construct our model, and translate our mathematical results back into real-world ideas in order to interpret our findings.

In the formula $I \quad Pr\,t$ for simple interest, P stands for *principal*, r for *interest rate*, and t for *time (in years)*.

(a) A square of side x has area $A \quad x^2$.
A rectangle of length l and width has area $A \quad l$.
A circle of radius r has area Ar^2.

Balsamic vinegar contains 5% acetic acid, so a 32 ounce bottle of balsamic vinegar contains 32 5% 32 $\frac{5}{100}$ 1 6 ounces of acetic acid.

5. A painter paints a wall in x hours, so the fraction of the wall she paints in one hour is $\frac{1 \text{ wall}}{x \text{ hours}} \quad \frac{1}{x}$.

6. Solving $d \quad r\,t$ for r, we find $\frac{d}{t} \quad \frac{rt}{t}\, r \quad \frac{d}{t}$. Solving $d \quad r\,t$ for t, we find $\frac{d}{r} \quad \frac{rt}{r}\, t \quad \frac{d}{r}$.

7. If n is the first integer, then $n \quad 1$ is the middle integer, and $n \quad 2$ is the third integer. So the sum of the three consecutive integers is $n \quad n \quad 1n \quad 2 \quad 3n \quad 3$.

8. If n is the middle integer, then $n \quad 1$ is the first integer, and $n \quad 1$ is the third integer. So the sum of the three consecutive integers is $n \quad 1 \quad n \quad n \quad 1 \quad 3n$.

9. If n is the first even integer, then $n \quad 2$ is the second even integer and $n \quad 4$ is the third. So the sum of three consecutive even integers is $n \quad n \quad 2n \quad 4 \quad 3n \quad 6$.

10. If n is the first integer, then the next integer is $n \quad 1$. The sum of their squares is

$$n^2 \quad n \quad 1^2 \quad n^2 \quad n^2 \quad 2n \quad 1 \qquad 2n^2 \quad 2n \quad 1.$$

If s is the third test score, then since the other test scores are 78 and 82, the average of the three test scores is 78 $\frac{82\ s\ 160\ s}{33}$.

12. If q is the fourth quiz score, then since the other quiz scores are 8, 8, and 8, the average of the four quiz scores is $\frac{8 \quad 8 \quad 8 \quad q}{44} \cdot \frac{24 \quad q}{}$

If x dollars are invested at $2\frac{1}{2}$ % simple interest, then the first year you will receive 0 025x dollars in interest.

If n is the number of months the apartment is rented, and each month the rent is \$795, then the total rent paid is 795n.

Since is the width of the rectangle, the length is four times the width, or 4 . Then

$$\text{area} \quad \text{length} \quad \text{width} \quad 4 \qquad 4^{\;2}\ \text{ft}^2$$

Since is the width of the rectangle, the length is4. Then

$$\text{perimeter } 2 \quad \text{length} \quad 2 \quad \text{width} \quad 2 \quad 4 \quad 2 \quad 4 \quad 8 \text{ ft}$$

If d is the given distance, in miles, and distance rate time, we have time $\frac{\text{distance}}{\text{rate}} \frac{d}{55}$.

18. Since distance rate time we have distance s 45 min $\frac{1 \text{ h}}{60 \text{ min}}$ $\frac{3}{4}s$ mi.

If x is the quantity of pure water added, the mixture will contain 25 oz of salt and 3 x gallons of water. Thus the concentration is $\frac{25}{3 \quad x}$.

20. If p is the number of pennies in the purse, then the number of nickels is 2p, the number of dimes is 4 2 p, and the number of quarters is 2 p 4 2 p 4 p 4. Thus the value (in cents) of the change in the purse is 1 p 5 2 p 10 4 2 p 25 4 p 4 p 10 p 40 20 p 100 p 100 131 p 140.

If d is the number of days and m the number of miles, then the cost of a rental is C 65d 0 20m. In this case, d 3

80

and C 275, so we solve for m: 275 65 3 0 20m 275 195 0 2m 0 2m 80 m 0 2 400. Thus, Michael drove 400 miles.

22. If m is the number of messages, then a monthly cell phone bill (above $10) is B 10 0 10 m 1000 . In this case,

28 5

B 38 5 and we solve for m: 38 5 10 0 10 m 10000 10 m 1000 28 5m 1000 0 1 285

m 1285. Thus, Miriam sent 1285 text messages in June.

If x is Linh's score on her final exam, then because the final counts twice as much as each midterm, her average score

is 82 75 71 2x / 3 100 200 228 2x / 500 114 x / 250 . For her to average 80%, we must have 114 x / 250 80% 0 8

114 x 250 0 8 200 x 86. So Linh scored 86% on her final exam.

24. Six students scored 100 and three students scored 60. Let x be the average score of the remaining 25 6 3 16 students.

Because the overall average is 84% 0 84, we have 6 100 3 60 16x / 25 100 0 84 780 16x 0 84 2500 2100

16x 1320 x 1320 / 16 82 5. Thus, the remaining 16 students' average score was 82 5%.

Let m be the amount invested at 4 1/2 %. Then 12,000 m is the amount invested at 4%.

Since the total interest is equal to the interest earned at 4 1/2 % plus the interest earned at 4%, we have

45

525 0 045m 0 04 12,000 m 525 0 045m 480 0 04m 45 0 005m m 0 005 9000. Thus $9000 is invested at 4 1/2 %, and

$12,000 9000 $3000 is invested at 4%.

Let m be the amount invested at 5 1/2 %. Then 4000 m is the total amount invested. Thus

4 1/2 % of the total investment interest earned at 4% interest earned at 5 1/2 %

20

So 0 045 4000 m 0 04 4000 0 055m 180 0 045m 160 0 055m 20 0 01m m 0 01 2000.

Thus $2,000 needs to be invested at 5 1/2 %.

262 5

27. Using the formula I Pr t and solving for r, we get

28. If $1000 is invested at an interest rate a%, then 2000 is invested at

a ***a*** **1**

percentage, the total interest is I 1000 100 1 2000 100 2 1 10a 20a 10 30a 10. Since the total interest is $190, we have 190 30a 10 180 30a a 6. Thus, the $1000 is invested at 6% interest.

29. Let x be her monthly salary. Since her annual salary ***Christmas bonus we have 97,300 12x 8,500 88,800 12x x 7,400. Her monthly salary is $7,400.***

30. Let s be the husband's annual salary. Then her annual salary is 1 15s. Since husband's annual salary wife's annual salary total annual income, we have s 1 15s 69,875 2 15s 69,875 s 32,500. Thus the husband's annual salary is $32,500.

31. Let x be the overtime hours Helen works. ***Since gross pay regular salary overtime pay, we obtain the equation 90***

352 50 7 50 35 7 50 1 5 x 352 50 262 50 11 25x 90 11 25x x ***11 25*** *8. Thus Helen*

worked 8 hours of overtime.

32. Let x be the hours the assistant worked. Then 2x is the hours the plumber worked. Since the labor charge is equal to the plumber's labor plus the assistant's labor, we have 4025 45 2x 25x 4025 90x 25x 4025 115x x 4025 / 115 35. Thus the assistant works for 35 hours, and the plumber works for 2 35 70 hours.

262 50 3500 r 1 r 3500 0 075 or 7 5%.

$a \quad \frac{1}{2}$ %, so, remembering that a is expressed as a

All ages are in terms of the daughter's age 7 years ago. Let y be age of the daughter 7 years ago. Then 11y is the age of the movie star 7 years ago. Today, the daughter is y 7, and the movie star is 11y 7. But the movie star is also 4 times his daughter's age today. So 4 y 7 11y 7 4y 28 11y 7 21 7y y 3. Thus the movie star's age today is 11 3 7 40 years.

Let h be number of home runs Babe Ruth hit. Then h 41 is the number of home runs that Hank Aaron hit. So 1469 h h 41 1428 2h h 714. Thus Babe Ruth hit 714 home runs.

Let p be the number of pennies. Then p is the number of nickels and p is the number of dimes. So the value of the coins in the purse is the value of the pennies plus the value of the nickels plus the value of the dimes. Thus

1 44 0 01 p 0 05 p 0 10 p 1 44 0 16 p p $\frac{1\ 44}{0\ 16}$ ~~9. S~~o the purse contains 9 pennies, 9 nickels, and 9 dimes.

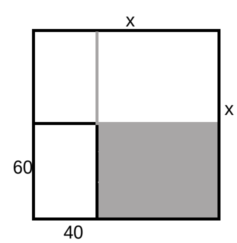
Let q be the number of quarters. Then 2q is the number of dimes, and 2q 5 is the number of nickels. Thus 3 00 value of the nickels value of the dimes value of the quarters. So

3 00 0 05 2q 5 0 10 2q 0 25q 3 00 0 10q 0 25 0 20q 0 25q 2 75 0 55q q $\frac{2\ 75}{0\ 55}$ 5. Thus Mary has 5 quarters, 2 5 ~~10~~ dimes, and 2 5 5 15 nickels.

Let l be the length of the garden. Since area width length, we obtain the equation 1125 25l l $\frac{1125}{25}$ 45 ft. So the garden is 45 feet long.

Let be the width of the pasture. Then the length of the pasture is 2 . Since area length width we have 115,200 2 2 2 2 57,600 240. Thus the width of the pasture is 240 feet.

39. Let x be the length of a side of the square plot. As shown in the figure, area of the plot area of the building area of the parking lot. Thus, x^2 60 40 12,000 2,400 12,000 14,400 x120. So the plot of land measures 120 feet by 120 feet.

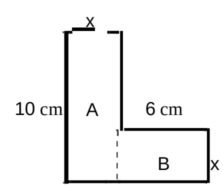


Let be the width of the building lot. Then the length of the building lot is 5 . Since a half-acre is $\frac{1}{2}$ 43,560 21,780 and area is length times width, we have 21,780 5 5 2 2 4,356 66. Thus the width of the building lot is 66 feet and the length of the building lot is 5 66 330 feet.

41. The figure is a trapezoid, so its area is $\frac{\text{base1} \quad \text{base2}}{2}$ height . Putting in the known quantities, we have

120 $\frac{y \quad 2y}{2}$ y $\frac{3}{2}y^2$ y^2 80 y 80 4 5. Since length is positive, y 4 5 8 94 inches.

42. First we write a formula for the area of the figure in terms of x. Region A has dimensions 10 cm and x cm and region B has dimensions 6 cm and x cm. So the shaded region has area 10 x6 x 16x cm^2. We are given that this is equal to 144 cm^2, so 144 16x x $\frac{144}{16}$ 9 cm.



Let x be the width of the strip. Then the length of the mat is 20 2x, and the width of the mat is 15 2x. Now the perimeter is twice the length plus twice the width, so 102 2 20 2x 2 15 2x 102 40 4x 30 4x 102 70 8x 32 8x x 4. Thus the strip of mat is 4 inches wide.

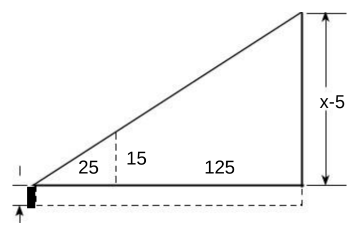
Let x be the width of the strip. Then the width of the poster is 100 2x and its length is 140 2x. The perimeter of the printed area is 2 100 2 140 480, and the perimeter of the poster is 2 100 2x 2 140 2x . Now we use the fact that the perimeter of the poster is 1 $\frac{1}{2}$ times the perimeter of the printed area: 2 100 2x 2 140 2x $\frac{3}{2}$ 480 480 8x 720 8x 240 x 30. The blank strip is thus 30 cm wide.

Let x be the length of the man's shadow, in meters. Using similar triangles, $\frac{10 \quad x \quad x}{62}$ _ 20 2x 6x 4x 20

5. Thus the man's shadow is 5 meters long.

46. Let x be the height of the tall tree. Here we use the property that corresponding sides in similar triangles are proportional. The base of the similar triangles starts at eye level of the woodcutter, 5 feet. Thus we obtain the proportion $\frac{x\ 5\ 150}{15} \quad \overline{25}$

25 x 5 15 150 25x 125 2250 25x 2375 x 95. Thus the tree is 95 feet tall.



Let x be the amount (in mL) of 60% acid solution to be used. Then 300 x mL of 30% solution would have to be used to yield a total of 300 mL of solution.

	60% acid	30% acid	Mixture
mL	x	300 x	300
Rate (% acid)	0 60	0 30	0 50
Value	0 60x	0 30 300 x	0 50 300

Thus the total amount of pure acid used is 0 60x 0 30 300 x0 50 3000 3x 90 150 x $\frac{60}{0\ 3}$ 200.

So 200 mL of 60% acid solution must be mixed with 100 mL of 30% solution to get 300 mL of 50% acid solution.

48. The amount of pure acid in the original solution is 300 50% 150. Let x be the number of mL of pure acid added. Then x. Because its concentration is to be 60%, we must have $\frac{150\ x}{}$ 60% 0 6 300 x

30

150 x 0 6 300 x 150 x 180 0 6x 0 4x 30 x 0 4 75. Thus, 75 mL of pure acid must be added.

Let x be the number of grams of silver added. The weight of the rings is 5 18 g 90 g.

	5 rings	Pure silver	Mixture
Grams	90	x	90 x
Rate (% gold)	0 90	0	0 75
Value	090 90	0x	0 75 90 x

So 0 90 90 0x 0 75 90 x 81 67 5 0 75x 0 75x 13 5 x $\frac{13\ \ 5}{075}$ 18. Thus 18 grams of silver must be added to get the required mixture.

Let x be the number of liters of water to be boiled off. The result will contain 6 x liters.

	Original	Water	Final
Liters	6	x	6 x
Concentration	120	0	200
Amount	120 6	0	200 6 x

So 120 6 0 200 6 x 720 1200 200x 200x 480 x 2 4. Thus 2 4 liters need to be boiled off.

Let x be the number of liters of coolant removed and replaced by water.

	60% antifreeze	60% antifreeze (removed)	Water	Mixture
Liters	3 6	x	x	3 6
Rate (% antifreeze)	0 60	0 60	0	0 50
Value	060 36	0 60x	0x	050 36

So 0 60 3 6 0 60x 0x 0 50 3 6 2 160 6x 1 8 0 6x 0 36 x 0 36 0 6 0 6. Thus 0 6 liters must be removed and replaced by water.

Let x be the number of gallons of 2% bleach removed from the tank. This is also the number of gallons of pure bleach added to make the 5% mixture.

	Original 2%	Pure bleach	5% mixture
Gallons	100 x	x	100
Concentration	0 02	1	0 05
Bleach	0 02 100 x	1x	0 05 100

So 0 02 100 x x 0 05 100 2 0 02x x 5 0 98x 3 x 3 06. Thus 3 06 gallons need to removed and replaced with pure bleach.

Let c be the concentration of fruit juice in the cheaper brand. The new mixture that Jill makes will consist of 650 mL of the original fruit punch and 100 mL of the cheaper fruit punch.

	Original Fruit Punch	Cheaper Fruit Punch	Mixture
mL	650	100	750
Concentration	0 50	c	0 48
Juice	0 50 650	100c	0 48 750

So 0 50 650 100c 0 48 750 325 100c 360 100c 35 c 0 35. Thus the cheaper brand is only 35% fruit juice.

Let x be the number of ounces of $3 00 oz tea Then 80 x is the number of ounces of $2 75 oz tea.

	$3 00 tea	$2 75 tea	Mixture
Pounds	x	80 x	80
Rate (cost per ounce)	3 00	2 75	2 90
Value	3 00x	2 75 80 x	290 80

So 3 00x 2 75 80 x 2 90 80 3 00x 220 2 75x 232 0 25x 12 x 48. The mixture uses 48 ounces of $3 00 oz tea and 80 48 32 ounces of $2 75 oz tea.

Let t be the time in minutes it would take Candy and Tim if they work together. Candy delivers the papers at a rate of $\frac{1}{70}$ of the job per minute, while Tim delivers the paper at a rate of $\frac{1}{80}$ of the job per minute. The sum of the fractions of the job that each can do individually in one minute equals the fraction of the job they can do working together. So we have $\frac{1}{t}$ $\frac{1}{70}$ $\frac{1}{80}$ 560 8t 7t 560 15t t 37 $\frac{1}{3}$ minutes. Since $\frac{1}{3}$ of a minute is 20 seconds, it would take them 37 minutes 20 seconds if they worked together.

Let t be the time, in minutes, it takes Hilda to mow the lawn. Since Hilda is twice as fast as Stan, it takes Stan 2t minutes to mow the lawn by himself. Thus 40 $\frac{1}{t}$ 40 $\frac{1}{2t}$ 1 40 20 t t 60. So it would take Stan 2 60 120 minutes to mow the lawn.

Let t be the time, in hours, it takes Karen to paint a house alone. Then working together, Karen and Betty can paint a house in $\frac{2}{3}t$ hours. The sum of their individual rates equals their rate working together, so $\frac{1}{t} + \frac{1}{6} = \frac{1}{\frac{2}{3}t} \Leftrightarrow \frac{1}{t} + \frac{1}{6} = \frac{3}{2t} \Leftrightarrow$ $6 + t = 9 \Leftrightarrow t = 3$. Thus it would take Karen 3 hours to paint a house alone.

Let h be the time, in hours, to fill the swimming pool using Jim's hose alone. Since Bob's hose takes 20% less time, it uses only 80% of the time, or $0.8h$. Thus $18 \cdot \frac{1}{h} + 18 \cdot \frac{1}{0.8h} = 1 \Leftrightarrow 18(0.8) + 18 = 0.8h \Leftrightarrow 14.4 + 18 = 0.8h \Leftrightarrow 32.4 = 0.8h \Leftrightarrow$ $h = 40.5$. Jim's hose takes 40.5 hours, and Bob's hose takes 32.4 hours to fill the pool alone.

59. Let t be the time in hours that Wendy spent on the train. Then $\frac{11}{2} - t$ is the time in hours that Wendy spent on the bus. We construct a table:

	Rate	Time	Distance
By train	40	t	$40t$
By bus	60	$\frac{11}{2} - t$	$60\left(\frac{11}{2} - t\right)$

The total distance traveled is the sum of the distances traveled by bus and by train, so $300 = 40t + 60\left(\frac{11}{2} - t\right) \Leftrightarrow$ $300 = 40t + 330 - 60t \Leftrightarrow -30 = -20t \Leftrightarrow t = \frac{-30}{-20} = 1.5$ hours. So the time spent on the train is $5.5 - 1.5 = 4$ hours.

Let r be the speed of the slower cyclist, in mi/h. Then the speed of the faster cyclist is $2r$.

	Rate	Time	Distance
Slower cyclist	r	2	$2r$
Faster cyclist	$2r$	2	$4r$

When they meet, they will have traveled a total of 90 miles, so $2r + 4r = 90 \Leftrightarrow 6r = 90 \Leftrightarrow r = 15$. The speed of the slower cyclist is 15 mi/h, while the speed of the faster cyclist is $2(15) = 30$ mi/h.

Let r be the speed of the plane from Montreal to Los Angeles. Then $r + 0.20r = 1.20r$ is the speed of the plane from Los Angeles to Montreal.

	Rate	Time	Distance
Montreal to L.A.	r	$\frac{2500}{r}$	2500
L.A. to Montreal	$1.2r$	$\frac{2500}{1.2r}$	2500

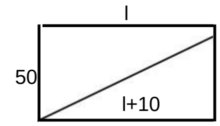
The total time is the sum of the times each way, so $9\frac{1}{6} = \frac{2500}{r} + \frac{2500}{1.2r} \Leftrightarrow \frac{55}{6} = \frac{2500}{r} + \frac{2500}{1.2r} \Leftrightarrow$ $55 \cdot 1.2r = 2500 \cdot 6 \cdot 1.2 + 2500 \cdot 6 \Leftrightarrow 66r = 18{,}000 + 15{,}000 \Leftrightarrow 66r = 33{,}000 \Leftrightarrow r = \frac{33{,}000}{66} = 500$. Thus the plane flew at a speed of 500 mi/h on the trip from Montreal to Los Angeles.

Let x be the speed of the car in mi/h. Since a mile contains 5280 ft and an hour contains 3600 s, 1 mi/h $= \frac{5280 \text{ ft}}{3600 \text{ s}} = \frac{22}{15}$ ft/s. The truck is traveling at $50 \cdot \frac{22}{15} = \frac{220}{3}$ ft/s. So in 6 seconds, the truck travels $6 \cdot \frac{220}{3} = 440$ feet. Thus the back end of the car must travel the length of the car, the length of the truck, and the 440 feet in 6 seconds, so its speed must be $\frac{14 + 30 + 440}{6} = \frac{242}{3}$ ft/s. Converting to mi/h, we have that the speed of the car is $\frac{242}{3} \cdot \frac{15}{22} = 55$ mi/h.

Let x be the distance from the fulcrum to where the mother sits. Then substituting the known values into the formula given, we have $100(8) = 125x \Leftrightarrow 800 = 125x \Leftrightarrow x = 6.4$. So the mother should sit 6.4 feet from the fulcrum.

Let be the largest weight that can be hung. In this exercise, the edge of the building acts as the fulcrum, so the 240 lb man is sitting 25 feet from the fulcrum. Then substituting the known values into the formula given in Exercise 43, we have $240(25) = 5 \Leftrightarrow 6000 = 5 \Leftrightarrow = 1200$. Therefore, 1200 pounds is the largest weight that can be hung.

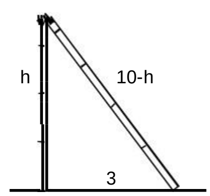
65. Let l be the length of the lot in feet. Then the length of the diagonal is l 10. We apply the Pythagorean Theorem with the hypotenuse as the diagonal. So l^2 50^2 l 10^2 l^2 2500 l^2 20l 100 20l 2400 l 120.



Let r be the radius of the running track. The running track consists of two semicircles and two straight sections 110 yards long, so we get the equation 2 r 220 440 2 r 220 r $\frac{110}{}$ 35 03. Thus the radius of the semicircle is about 35 yards.

Let h be the height in feet of the structure. The structure is composed of a right cylinder with radius 10 and height $\frac{2}{3}h$ and a cone with base radius 10 and height $\frac{1}{3}h$. Using the formulas for the volume of a cylinder and that of a cone, we obtain the equation 140010 ^{2} $\frac{2}{3}h$ $\frac{1}{3}10^2$ $\frac{1}{3}h$1400 $\frac{200}{3}h$ $\frac{100}{9}h$ 126 6h h (multiply both sides by $\frac{9}{100}$) 126 7h h 18. Thus the height of the structure is 18 feet.

68. Let h be the height of the break, in feet. Then the portion of the bamboo above the break is 10 h. Applying the Pythagorean Theorem, we obtain h^2 3^2 10 h^2 h^2 9 100 20h h^2 91 20h h $\frac{91}{20}$ 4 55. Thus the break is 4 55 ft above the ground.



Pythagoras was born about 569 BC in Samos, Ionia and died about 475 BC. Euclid was born about 325 BC and died about 265 BC in Alexandria, Egypt. Archimedes was born in 287 BC in Syracuse, Sicily and died in 212 BC in Syracuse.

Answers will vary.

CHAPTER P REVIEW

(a) Since there are initially 250 tablets and she takes 2 tablets per day, the number of tablets T that are left in the bottle after she has been taking the tablets for x days is T 250 2x.

After 30 days, there are 250 2 30 190 tablets left.

We set T 0 and solve: T 250 2x 0 250 2x x 125. She will run out after 125 days.

(a) The total cost is \$2 per calzone plus the \$3 delivery charge, so C 2x 3.

4 calzones would be 2 4 3 \$11.

We solve C 2x 3 15 2x 12 x 6. You can order six calzones.

(a) 16 is rational. It is an integer, and more precisely, a natural number.

16 is rational. It is an integer, but because it is negative, it is not a natural number.

16 4 is rational. It is an integer, and more precisely, a natural number.

2 is irrational.

$\frac{8}{3}$ is rational, but is neither a natural number nor an integer.

$\frac{8}{24}$ is rational. It is an integer, but because it is negative, it is not a natural number.

(a) 5 is rational. It is an integer, but not a natural number.

$\frac{25}{6}$ is rational, but is neither an integer nor a natural number.

25 5 is rational, a natural number, and an integer.

3 is irrational.

24 16 3 2 is rational, but is neither a natural number nor an integer.

20

10 is rational, a natural number, and an integer.

5. Commutative Property of addition.

6. Commutative Property of multiplication.

7. Distributive Property.

8. Distributive Property.

9. (a) 5 6 2 3 5 6 4 6 9 6 3 2
5 2 5 4 1

(b) 6 3 6 6 6
15 12 15 12 3 3 9

10. (a) 7 10 11 15 21 30 22 30 1 30
7 11 21 22 43

(b) 10 15 30 30 30
30 12 30 35 5 5 25

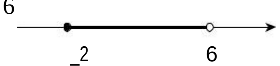
11. (a) 8 5 8 5 2 1 2

(b) 15 8 12 5 15 5 8 12 5 5 8 4 25 32

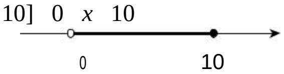
12. (a) 7 35 7 12 12 2

(b) 30 7 12 35 30 12 7 35 6 12 7 7 72 49

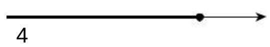
13. *x* [2 62 *x* 6



14. *x* 0 10] 0 *x* 10

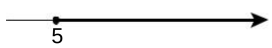


15. *x* 4] *x* 4



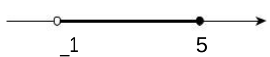
16. *x* [2 2 *x*

x 5 *x* [5

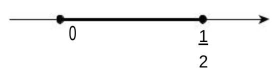


18. *x* 3 *x* 3

1 *x* 5 *x*1 5]



20. 0 *x* 1 2 *x* 0 1 2



(a) *C* *D*1 2]

(b) *C* *D* 0 1]

(a) *A* *B*1 0 1 2 1
2 3 4
A *B* 1

(a) *A* *D* 0 1
BC 1 2 1

393966

23. (a) *A* *C* 1 2

(b) *B* *D* 1 2

2 3 3 2 1 8 1 9 8 1
9 72 72 72

1

71033

64 23 4 3 23 4 2 16

2 12 8 12 2 816

2 50 100 10

4

(a) 4 04 4

29. 216 13 1 13 3 216 1 216 1 6

(b) 4 48 8

242 242

31.

2
2 121 11
(a) 5
32
2

(b)
5
38
8

35. (a) 3 7 7 1 3

5 4 5

4 4

37. (a) 6 x^5 $x^{5\,6}$ 9 38. (a) y^3 $y^{3\;1\,2}$ $y^{3\,2}$

2

(b) x 9 $x^{1\,2}$ $x^{9\,2}$ (b) 8 y 2 $y^{1\,8}$ $y^{1\,4}$

3 2 $3x$ 1 2 $4x^6 y^2$ $3x$ 1 y^2 4 $3x^6$ 1 y^2 2 2 3 a^3 2 b^3 4 a 6 $a^6 b^2$ b^{12}

40.

39. $2x$ y y 5 4 a b 6 6 2 12 14

41. x^3 x 3 $12x$ y 42. r r 1 3 s $r^2 s^6$ a b b

x^4 $3x^2$ x^4 $9x^2$ 2 $s^{4\,3}$ 6 $r^{12} s^8$

43. 3 $x^3 y^2 y^4$ 3 $x^6 y^4 y^2$ 3 $x^6 y^6$ $x^2 y^2$ 44. $x^2 y^4$ x^2 y^2 2 $x y^2$

1

2 3 5 2

45. 8r s 2r s $4r^{1\,2}$ $2s^{3\,4}$ $4r^{5\,2} s^7$ 4r s

$ab^2 c^3$ 2 $a^2 b^4 c^6$ $22a$ 2 6 b 4 8c6 $4a^4 b$ $12c^6$ $4a^4 c^6$

$2a^3 b^4 2$ $2a$ $6b^8 b^{12}$

47. 78,250,000,000 7 825 10^{10}

48. 2 08 10^8 0 0000000208

49. $\frac{ab}{c}$ 0 00000293 1 582 10^{14} 12 293 10^6 1 582 10^{14} 12 2 93 1 582 2 8064 6 14 12 10

10

165 10^{32}

80 times minute 60 minutes hour 24 hours day 365 days year 90 years 3 8 10^9 times

51. $2x^2 y$ $6x y^2$ $2x y$ x $3y$

52. $12x^2 y^4$ $3x y^5$ $9x^3 y^2$ $3x y^2$ $4x y^2$ y^3 $3x^2$
2

53. x 5x 14 x 7 x 2

54. x^4 x^2 $2x^2$ 2 x^2 $2x^2$ 2 x^2 1 x^2 2 x 1 x 1

$3x^2$ $2x$ 1 $3x$ 1 x 1

$6x^2$ x 12 $3x$ 4 $2x$ 3

$4t^2$ $13t$ 12 $4t$ 3 t 4

x^4 $2x^2$ $1x^2$ 1^2 [x 1 x 1 $]^2$ x 1^2 x 1^2

59. 16 $4t^2$ 4 4 t^2 4 t 2 t 2

60. $2y^6$ $32y^2$ $2y^2 y^4$ $16 2y^2$ y^2 4 y^2 4 $2y^2$ y^2 4 y 2 y 2

61. $x^6\ 1x^3\ 1\ x^3\ 1\ x\ 1\ x^2\ x\ 1\ x\ 1\ x^2\ x\ 1$

62. $a^4b^2\ ab^5\ ab^2 a^3\ b^3 ab^2\ a\ b\ a^2\ ab\ b^2$

3 2

63. $x\ 27\ x\ 3\ x\ 3x\ 9$

64. $3y^3$ $81x^3$ $3y^3$ $27x^3$ 3 y $3x$ y^2 $3xy$ $9x^2$

65. $4x^3 \quad 8x^2 \quad 3x \quad 6 \quad 4x^2 \quad x \quad 2 \quad 3 \; x \quad 24x^2 \quad 3 \; x \quad 2$

3 2 2 2

66. $3x \quad 2x \quad 18x \quad 12 \quad x \quad 3x \quad 2 \quad 6 \; 3x \quad 23x \quad 2 \; x \quad 6$

2 **67.** x y 2 7x y 6x y 6 x y 1x y 6 x y 1

68. $a \quad b3 \; a \quad b \quad 10 \quad a \quad b \quad 5 \; a \quad b \quad 2$

69. $2y \quad 7 \; 2y \quad 7 \quad 4y^2 \quad 49$

70. $1 \quad x \; 2 \quad x3 \quad x \; 3 \quad x \quad 2 \quad x \quad x^2 9 \quad x^2 2 \quad x \quad x^2 \quad 9 \quad x^2 7 \quad x$ x

71. $x^2 \quad x \quad 2 \quad x \, x \quad 2^2 \quad x^3 \quad 2x^2 \quad x \; x^2 \quad 4x \quad 4x^3 \quad 2x^2 \quad x^3 \quad 4x^2 \quad 4x \quad 2^3 \quad 6^2 \quad 4x$

3 2 x

73. $x \quad x \quad 1 \quad 2 \; x \quad 1 \quad x \quad x \quad 2 \quad x \quad 1 \quad 2x \quad x \quad x \quad 2x \quad x \quad 2x \quad x \, x \quad x$

74. $2x \quad 1 \quad 2x \quad 3 \; 2x \quad 1 \quad 3 \; 2x \; 1 \quad 1 \quad 8x \quad 12x \quad 6x \quad 1$

75. $\dfrac{x^2 \quad 2x \quad 3}{2x^2 \quad 5x \quad 3} \quad \dfrac{x \quad 3 \; x \quad 1}{2x \quad 3 \; x \quad 1} \quad \dfrac{x \quad 3}{2x \quad 3}$

76. $t^2 \quad 1 \quad t \quad 1 \; t \; 2 \; t \quad 1 \quad t \quad t \quad 1$

$\dfrac{3}{t \quad 1} \quad t \quad 1 \; t \quad 1 \quad \dfrac{2}{t \quad 1}$

77. $\dfrac{x^2 \quad 2x \quad 3 \; 3x \quad 12}{x^2 \quad 8x \quad 16} \quad x \quad 1 \quad \dfrac{x \quad 3 \; x \quad 1 \; 3 \; x \quad 4}{x \quad 4 \; x \quad 4} \quad x \quad 1 \quad \dfrac{3 \; x \quad 3}{x \quad 4}$

78. $\dfrac{x^2 \quad 2x \quad 15}{x^2 \quad 6x \quad 5} \quad \dfrac{x^2 \quad x \quad 12}{x^2 \quad 1} \quad \dfrac{x \quad 5 \; x \quad 3 \quad x \quad 1 \; x \quad 1}{x \quad 5 \; x \quad 1} \quad \dfrac{x \quad 1}{x \quad 4 \; x \quad 3} \quad x \quad 4$

79. $x \quad \dfrac{1}{x \quad 1} \quad \dfrac{x \, x \quad 1}{x \quad 1} \quad \dfrac{1}{x \quad 1} \quad \dfrac{x^2 \quad x \quad 1}{x \quad 1}$

$1 \quad x \quad x^2 \quad 1 \quad x \, x \quad 1 \quad 2 \quad 2 \quad x \quad 1 \quad x \quad x \quad x \quad 1$

80. $x \quad 1 \quad x^2 \quad 1 \quad x \quad 1 \; x^2 \quad 1 \; 2 \quad x \quad 1 \quad x^2 \quad 1 \quad x \quad 1 \quad x^2 \quad 1 \quad x \quad 1 \; x^2 \quad 1$

81. $\dfrac{2}{x} \quad \dfrac{1}{x \quad 2} \quad \dfrac{3}{x \; x \quad 2^2} \quad \dfrac{2x \quad 2}{x \, x \quad 2^2} \quad \dfrac{x \, x \quad 2}{x \; x \quad 2^2} \quad \dfrac{3x}{x \, x \quad 2^2}$

2

$2 \; x^2 \quad 4x \quad 4 \quad x \quad 2 \quad 2x \quad 3x \quad 2x^2 \quad 8x \quad 8 \quad x^2 \quad 2x \quad 3x \quad 3x^2 \quad 7x \quad 8$

1 $x \, x \quad 2$ 1 $x \, x \quad 2$ $x \, x \quad 2$

82. $\dfrac{1}{x \quad 2} \quad \dfrac{}{x^2 \quad 4} \quad \dfrac{2}{x^2 \quad x \quad 2} \quad \dfrac{1}{x \quad 2} \quad \dfrac{}{x \quad 2 \; x \quad 2} \quad \dfrac{2}{x \quad 2 \; x \quad 1}$

$\dfrac{x \quad 2 \; x \quad 1}{x \quad 2 \; x \quad 1 \; x \quad 2} \quad \dfrac{x \quad 1}{x \quad 2 \; x \quad 1 \; x \quad 2} \quad \dfrac{2 \; x \quad 2}{x \quad 2 \; x \quad 1 \; x \quad 2}$

$x^2 \quad x \quad 2 \quad x \quad 1 \quad 2x \quad 4 \quad \dfrac{}{x \quad 2 \; x \quad 1 \; x \quad 2} \quad \dfrac{x^2 \quad 2x \quad 5}{x \quad 2 \; x \quad 1 \; x \quad 2}$

83. $\dfrac{1}{x} \quad \dfrac{1}{2} \quad \dfrac{2}{2x} \quad \dfrac{x}{2x} \quad 2 \quad x \quad 1 \quad 1 \; x \quad 2 \quad 1 \quad 1$

$x \quad 2 \quad x \quad 2 \quad 2x \quad x \quad 2 \quad 2x \quad x \quad 2 \quad 2x$

1 1 1 1

84. $\dfrac{x \quad x+1}{\frac{1}{x} \quad \frac{1}{x+1}} \quad \dfrac{x \quad x+1}{\frac{1}{x} \quad \frac{1}{x+1}} \quad \dfrac{x\,x+1}{} \quad \dfrac{x+1 \quad x}{} \quad \dfrac{1}{}$

$x\,x \quad 1x \quad 1 \quad x \quad 2x \quad 1$

85. $3x h^2 \; 5x \; h \; 3x^2 \; 5x$ over h $\quad 3x^2 \; 6xh \; 3h^2 \; 5x \; 5h \; 3x^2 \; 5x$ over h $\quad 6xh \; 3h^2 \; 5h$ over h

$h \; 6x \; 3h \; 5$ over h $\quad 6x \; 3h \; 5$

86. $x \; h \quad x \quad x \; h \quad x \quad x \; h \quad x \quad x \; h \; x \quad h \quad 1$

$1 \quad h \; 1 \quad 11 \quad 11 \quad x \; h \quad x \quad h \quad x \; h \quad x \quad h \quad x \; h \quad x \quad x \; h \quad x$

87. 11 11 11 11

36366

66662

10 10 2 1 10 102

89. 21212 1 2 1 10 10 2

14 14 3 2 42 14 2 42 14 2

90. 2 62 2

91. 3 2 3 2 3 2 322 x 7 4

$x \quad x \quad 2 \quad x \quad 2x \quad x \; x$ 4 $x \; x$ 2

$x \; 2 \quad x \; 2 \quad x \; 2 \quad x \; 4 \; x$

92. $x \; 2x \; 2x \; 2$ $\quad x \; 4$

$\frac{x\;5}{10}$ is defined whenever $x \; 10 \; 0 \; x10$, so its domain is $x \; x10$.

94. $\frac{2x}{x^2 \; 9}$ is defined whenever $x^2 \; 9 \; 0 \; x^2 \; 9 \; x3$, so its domain is $x \; x3$ and $x \; 3$.

x

2

95. $x^2 \; 3x \; 4$ is defined whenever $x \; 0$ (so that x is defined) and x 2 $3x \; 4 \; x \; 1 \; x \; 4 \; 0 \; x1$ and

$x \; 4$. Thus, its domain is $x \; x \; 0$ and $x \; 4$.

2

96. $x^2 \; 4x \; 4$ is defined whenever $x \; 3 \; 0 \; x \; 3$ and $x \; 4x \; 4 \; x \; 2 \; 0 \; x2$. Thus, its domain is $x \; x \; 3$.

97. This statement is false. For example, take $x \; 1$ and $y \; 1$. Then LHS $x \; y^3 \; 1 \; 1^3 \; 2^3 \; 8$, while

RHS $x^3 \; y^3 \; 1^3 \; 1^3 \; 1 \; 1 \; 2$, and 8 2.

98. This statement is true for $a \; 1$: $\frac{1 \; a}{1 \; a} \; \frac{1 \; a}{1 \; a} \; 1 \; a \; \frac{1 \; a}{1 \; a \; 1 \; a} \; \frac{1}{1 \; a}$.

This statement is true: $\frac{12 \; y}{yy} \; \frac{12 \; y}{yy} \; 12 \; 1$.

This statement is false. For example, take a 1 and b 1. Then LHS $^3 a \; b \; ^3 1 \; 1 \; ^3 2$, while RHS $^3 a \; ^3 b \; ^3 1 \; ^3 1 \; 1 \; 1 \; 2$, and $^3 2$ 2.

101. This statement is false. For example, take a 1. Then LHS $a2 \; \frac{2}{1} \; 1 \; 1$, which does not equal 1. The true statement is $a^2 \; a$.

102. This statement is false. For example, take x 1. Then LHS $\frac{1}{x \; 4} \; \frac{1}{1 \; 4} \; \frac{1}{5}$, while RHS $\frac{1}{x} \; \frac{1}{4} \; \frac{1}{1} \; \frac{1}{4} \; \frac{5}{4}$, $\frac{1}{5} \; \frac{5}{4}$

and 5 4 .

103. $3x \; 12 \; 24 \; 3x \; 12 \; x \; 4$

104. $5x \; 7 \; 42 \; 5x \; 49 \; x \; \frac{49}{5}$

105. 7*x* 6 4*x* 9 3*x* 15 *x* 5

106. 8 2*x* 14 *x*3*x* 6 *x*2

107. 1/3 *x* 1/2 2 2*x* 3 12 2*x* 15 *x* 15/2

108. 2/3 *x* 3/5 1/5 2*x* 10*x* 9 3 30*x* 40*x*6 *x* 6/40 3/20

109. 2 *x* 3 4 *x* 5 8 5*x* 2*x* 6 4*x* 20 8 5*x*2*x* 26 8 5*x* 3*x*18 *x*6

110. (*x* 5)/2 (2*x* 5)/3 5/6 3 *x* 5 2 2*x* 5 5 3*x* 15 4*x* 10 5*x* 30 *x*30

111. (*x* 1)/(*x* 1) (2*x* 1)/(2*x* 1) *x* 1 2*x* 1 2*x* 1 *x* 1 2*x* ^{2} 3*x* 1 2*x* ^{2} 3*x* 1 6*x* 0 *x* 0

112. *x*/(*x* 2) 3 1/(*x* 2) *x* 3 *x* 2 1 *x* 3*x* 6 12*x* 7 *x* 7/2

113. (*x* 1)/(*x* 1). 3*x*/(3*x* 6) 3*x*/(3 *x* 2) *x*/(*x* 2) *x* 1 *x* 2 *x x* 1*x*^{2} *x* 2 *x*^{2} *x*2 0. Since this

last equation is never true, there is no real solution to the original equation.

114. *x* 2^2 *x* 4^2 *x* 2^2 *x* 4^2 0[*x* 2*x* 4] [*x* 2*x* 4]0

[*x* 2 *x* 4] [*x* 2 *x* 4] 6 2*x* 2 0 2*x* 2 0 *x* 1.

x^2 144 *x*12

4*x*^{2} 49 *x*^{2} 49/4 *x* 7/2

3 3

x 27 0 *x* 27 *x* 3.

6*x*^{4} 15 0 6*x*^{4} 15 *x*^{4} 5/2. Since *x*^{4} must be nonnegative, there is no real solution.

x 1 ^{3}64 *x* 14 *x*1 45.

x 2^2 2 0*x* 2^2 2 *x* 22 *x*22.

^{3} *x*3 *x*3 ^{3}27.

x$^{2\ 3}$ 4 0*x*$^{1\ 3\ 2}$ 4 *x*$^{1\ 3}$2 *x*8.

4*x*$^{3\ 4}$ 500 0 4*x*$^{3\ 4}$ 500 *x*$^{3\ 4}$ 125 *x* 125$^{4\ 3}$ 5^4 625.
5

x 2$^{1\ 5}$ 2 *x* 2 2 32 *x* 2 32 34.

125. *A* (*x* *y*)/2 2*A* *x* *y* *x* 2*A* *y*.

126. *V* *x y* *yz* *x z* *V* *y x* *z* *x z* *V* *x z* *y x* *zy* (*V* *x z*)/(*x* *z*).

127. Multiply through by *t*: *J* 1/*t* 1/2*t* 1/3*t* *t J* 1 1/2 1/3 11/6 *t* 11/6*J* , *J* 0.

128. *F* *k* *q*1*q*2/*r*^{2} *r*^{2} *k* *q*1*q*2/*F* *rk* *q*1*q*2/*F*. (In real-world applications, *r* represents distance, so we would take the positive root.)

Let *x* be the number of pounds of raisins. Then the number of pounds of nuts is 50 *x*.

	Raisins	Nuts	Mixture
Pounds	*x*	50 *x*	50
Rate (cost per pound)	3 20	2 40	2 72

So 3 20x 2 40 50 x 2 72 50 3 20x 120 2 40x 136 0 8x 16 x 20. Thus the mixture uses 20 pounds of raisins and 50 20 30 pounds of nuts.

Let t be the number of hours that Anthony drives. Then Helen drives for t $\frac{1}{4}$ hours.

	Rate	Time	Distance
Anthony	45	t	$45t$
Helen	40	t $\frac{1}{4}$	40 t $\frac{1}{4}$

When they pass each other, they will have traveled a total of 160 miles. So $45t$ 40 t $\frac{1}{4}$ 160 $45t$ $40t$ 10 160

$85t$ 170 t 2. Since Anthony leaves at 2:00 P.M. and travels for 2 hours, they pass each other at 4:00 P.M.

Let x be the amount invested in the account earning 1 5% interest. Then the amount invested in the account earning 2 5% is 7000 x.

	1 5% Account	2 5% Account	Total
Amount invested	x	7000 x	7000
Interest earned	0 015x	0 025 7000 x	120 25

From the table, we see that 0 015x 0 025 7000 x 120 25 0 015x 175 0 025x 120 25 54 75 0 01x x 5475. Thus, Luc invested $5475 in the account earning 1 5% interest and $1525 in the account earning 2 5% interest.

The amount of interest Shania is currently earning is 6000 0 03 $180. If she wishes to earn a total of $300, she must earn another $120 in interest at a rate of 1 25% per year. If the additional amount invested is x, we have the equation 0 0125x 120 x 9600. Thus, Shania must invest an additional $9600 at 1 25% simple interest to earn a total of $300 interest per year.

Let t be the time it would take Abbie to paint a living room if she works alone. It would take Beth $2t$ hours to paint the living room alone, and it would take $3t$ hours for Cathie to paint the living room. Thus Abbie does $\frac{1}{t}$ of the job per hour, Beth does $\frac{1}{2t}$ of the job per hour, and Cathie does $\frac{1}{3t}$ of the job per hour. So $\frac{1}{t}$ $\frac{1}{2t}$ $\frac{1}{3t}$ 1 6 3 2 $6t$ $6t$ 11– t $\frac{11}{6}$. So it would Abbie 1 hour 50 minutes to paint the living room alone.

Let be width of the pool. Then the length of the pool is 2 , and its volume is 828464 16 ^{2} 8464 ^{2} 529 23. Since 0, we reject the negative value. The pool is 23 feet wide, 2 23 46 feet long, and 8 feet deep.

CHAPTER P TEST

(a) The cost is C 9 1 5x.

There are four extra toppings, so x 4 and C 9 1 5 4 $15.

(a) 5 is rational. It is an integer, and more precisely, a natural number.
5 is irrational.

$\frac{9}{3}$3 is rational, and it is an integer.
1,000,000 is rational, and it is an integer.

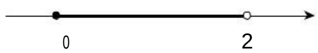
(a) A B 0 1 5

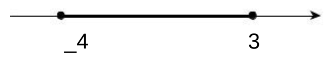
A B20$\frac{1}{2}$1357

(a)

_4 2 0 3
[42 [03]

(b)





[42 [03] [02 [
42 [03] [
43]

4266

5. **(a)** 2^6 64 **(b)** 2^6 64 **(c)** 2 6 1 1 2 2664 **(d)** 7^{10} 7 2 1 7^{12} 49

3 2 2 2 4 5 32 2 1 4 3^8 2 9 3 4 1

3

(e) 2 3 9 **(f)** 4 2 **(g)** 16 4 16 **(h)** 81 3 4 3^4 3 3 27

6. **(a)** 186,000,000,000 1 86 10^{11} **(b)** 0 0000003965 3 965 10 7

7. **(a)** $\frac{a^3b^2}{ab^3}$ $\frac{a^2}{b}$

$2x^3y$ 2 2 $4^y x^4$ 6

2

(c) $2x^{1\,2}y^2$ $3x^{1\,4}$ y 1 2 3^2x 1 2 2 1 4 y^2 2 1 18x

2012545255 25 5535

$18x^3y^{49}$ 2 x^2 x y^2 2 $3xy^2$ $2x$

(f) x 3 y 1 2 2 2x2 23 2y 2 1 2 2 4x10y

$2x^2y$ 2 1

8. **(a)** 3x 6 4 2x 5 3x 18 8x 20 11x 2

(b) x 3 4x 5 4x^2 5x 12x 15 4 2 7x 15

(c) a b a b a 2 b 2a b

2 2 x 2 2

(d) 2x 3 2x2 2x 33 4x 12x 9

(e) x 2^3 x^3 3x^2 2 3x 2^2 2^3 x^3 6x^2 12x 8

(f) x^2 x 3 x 3 x^2 x^2 9x^4 9x^2

2

(a) 4x 25 2x 5 2x 5

2

2x 5x 12 2x 3 x 4

(c) x^3 3x^2 4x 12 x^2 x 3 4 x 3x 3 2 4x 3 x 2 x 2

4 3 2 x

(d) x 27x x x 27x x 3 x 3x 9

(e) 2x y^2 10 2x y 25 2x y^2 2 5 2x y 5^2 2x y 5 2

(f) x^3y 4xy xy x^2 4xy x 2 x 2

10. **(a)** $\frac{x^2 \; 3x \; 2}{x^2 \; x \; 2}$ $\frac{x \; 1 \; x \; 2}{x \; 1 \; x \; 2}$ $\frac{x \; 2}{x \; 2}$

2

(b) (c)

$$\frac{2x}{x\ \ 3} \quad x\ \ 1\ \ \frac{3}{x^2\ \ 9} \quad \frac{x\ \ 1}{2x\ \ 1} \quad \frac{}{x\ \ 3\ \ x\ \ 3} \quad \frac{}{2x\ \ 1} \quad \frac{}{x\ \ 3}$$

$$\frac{2x\ \ 1\ \ x}{1\ \ \ \ x} \quad \frac{x^2}{x^2\ \ 4} \quad \frac{x\ \ 1}{x\ \ 2} \quad \frac{x^2}{x\ \ 2\ \ x\ \ 2} \quad \frac{x\ \ 1}{x\ \ 2} \quad \frac{x^2}{x\ \ 2\ \ x\ \ 2} \quad \frac{x\ \ 1\ \ x\ \ 2}{x\ \ 2\ \ x\ \ 2}$$

$$x \quad \frac{x^2\ \ \ x^2\ \ x\ \ 2}{2\ \ x\ \ \ 2} \quad \frac{x\ \ 2}{x\ \ \ 2\ \ x\ \ \ 2} \quad \frac{1}{x\ \ \ 2}$$

y x y x

x y x y xy y^2 x^2 y x y x x y y x

(d) 1 1 1 1 xy x y x y x y y x

y x y x

6 6 6 3 2 6 3 2 3

11. (a) 343223223 2 2 3 2

(b) 10 525252 10 5 2 10 5 2 / 5 4 50 2 10 5 2 2 10

1 11x 1x

(c) 1x 1x 1x 1 x

(a) 4x 3 2x 7 4x 2x 732x 10 x 5.

$8x^3$ 125 3 $8x^3$ 3 125 2x5 x 5/2 .

$x^{2\,3}$ 64 0 $x^{2\,3}$ 64 $x^{2\,3}$ 3 2 $64^{3\,2}$ x 8^3 512.

x x 3

(d) 2x 5 2x 1 x 2x 1 x 3 2x 5 2x x^2 2x $5x^2$ 6x 15x x 15 2x 15

x 15/2 .

3 x 1^2 18 0 3 x 1^2 18x 1^2 6 x 16 x6 1

E mc^2 E c^2 c E/m. (We take the positive root because c represents the speed of light, which is positive.)

14. Let *d* be the distance in km, between Bedingfield and Portsmouth.

Direction	Distance	Rate	Time
Bedingfield Portsmouth	*d*	100	*d* / 100
Portsmouth Bedingfield	*d*	75	*d* / 75

We have used time = distance / rate to fill in the time column of the table. We are given that the sum of the times is 3 5 hours.

Thus we get the equation 100 d 75 d 35 300 100 d 75 d 300 3 5 3d 4d 1050 d 1050 / 7 150 km.

FOCUS ON MODELING Making the Best Decisions

1. (a) The total cost is cost of copier maintenance cost number of months copy cost number of months. Each month the copy cost is 8000 0 03 240. Thus we get C_1 5800 25n 240n 5800 265n.

(b) In this case the cost is rental cost number of months copy cost number of months. Each month the copy cost is 8000 0 06 480. Thus we get C_2 95n 480n 575n.

(c)

Years	n	Purchase	Rental
1	12	8,980	6,900
2	24	12,160	13,800
3	36	15,340	20,700
4	48	18,520	27,600
5	60	21,700	34,500
6	72	24,880	41,400

(d) The cost is the same when C_1 C_2 are equal. So 5800 265n 575n 5800 310n n 18 71 months.

(a) The cost of Plan 1 is 3 daily cost cost per mile number of miles 3 65 0 15x 195 0 15x.

3 90 270.

The cost of Plan 2 is 3 daily cost

When x 400, Plan 1 costs 195 0 15 400 \$255 and Plan 2 costs \$270, so Plan 1 is cheaper. When x 800, Plan 1 costs 195 0 15 800 \$315 and Plan 2 costs \$270, so Plan 2 is cheaper.

The cost is the same when 195 0 15x 270 0 15 75x x 500. So both plans cost \$270 when the businessman drives 500 miles.

(a) The total cost is setup cost cost per tire number of tires. So C 8000 22x.

The revenue is price per tire number of tires. So R 49x.

Profit Revenue Cost. So P R C 49x 8000 22x 27x 8000.

Break even is when profit is zero. Thus 27x 8000 0 27x 8000 x 296 3. So they need to sell at least 297 tires to break even.

(a) *Option 1:* In this option the width is constant at 100. Let x be the increase in length. Then the additional area is

width increase in length 100x. The cost is the sum of the costs of moving the old fence, and of installing the

new one. The cost of moving is \$6 100 \$600 and the cost of installation is 2 10x 20x, so the total cost is

C 20x 600. Solving for x, we get C 20x 600 20x C 600 x $\frac{C\ 600}{20}$. Substituting in the area

we have A_1 100 $\frac{C\ 600}{20}$ 5C 600 5C 3,000.

Option 2: In this option the length is constant at 180. Let y be the increase in the width. Then the additional area is

length increase in width 180y. The cost of moving the old fence is 6 180 \$1080 and the cost of installing the new

one is 2 10 y 20x, so the total cost is C 20y 1080. Solving for y, we get C 20y 1080 20y C 1080

y $\frac{C\ 1080}{20}$. Substituting in the area we have A_2 180 $\frac{C\ 1080}{20}$ 9 C 1080 9C 9,720.

(b)

Cost, C	Area gain A_1 from Option 1	Area gain A_2 from Option 2
\$1100	2,500 ft^2	180 ft^2
\$1200	3,000 ft^2	1,080 ft^2
\$1500	4,500 ft^2	3,780 ft^2
\$2000	7,000 ft^2	8,280 ft^2
\$2500	9,500 ft^2	12,780 ft^2
\$3000	12,000 ft^2	17,280 ft^2

If the farmer has only \$1200, Option 1 gives him the greatest gain. If the farmer has only \$2000, Option 2 gives him the greatest gain.

(a) Design 1 is a square and the perimeter of a square is four times the length of a side. 24 4x, so each side is x 6 feet long. Thus the area is 6^2 36 ft^2.

Design 2 is a circle with perimeter 2 r and area r^2. Thus we must solve 2 r 24 r 12. Thus, the area is 12 2 144

45 8 ft^2. Design 2 gives the largest area.

(b) In Design 1, the cost is \$3 times the perimeter p, so 120 3 p and the perimeter is 40 feet. By part (a), each side is then $\frac{40}{4}$ 10 feet long. So the area is 10^2 100 ft^2.

In Design 2, the cost is \$4 times the perimeter p. Because the perimeter is 2 r, we get 120 4 2 r so

$\frac{120}{8}$ 15. The area is r^2 15 2 225 71 6 ft^2. Design 1 gives the largest area.

(a) Plan 1: Tomatoes every year. Profit acres Revenue cost 100 1600 300 130,000. Then for n years the profit is P_1 130,000n.

Plan 2: Soybeans followed by tomatoes. The profit for two years is Profit acres

soybean revenue tomato revenue 100 1200 1600 280,000. Remember that no fertilizer is

needed in this plan. Then for 2k years, the profit is P_2 280,000k.

When n 10, P_1 130,000 10 1,300,000. Since 2k 10 when k 5, P_2 280,000 5 1,400,000. So Plan B is more profitable.

7. (a)

Data (GB)	Plan A	Plan B	Plan C
1	$25	$40	$60
1 5	25 5 200 $35	40 5 150 $4750	60 5 100 $65
2	25 10 2 00 $45	40 10 1 50 $55	60 10 1 00 $70
2 5	25 15 2 00 $55	40 15 1 50 $62 50	60 15 1 00 $75
3	25 20 2 00 $65	40 20 1 50 $70	60 20 1 00 $80
3 5	25 25 2 00 $75	40 25 1 50 $77 50	60 25 1 00 $85
4	25 30 2 00 $85	40 30 1 50 $85	60 30 1 00 $90

For Plan A: C_A 25 2 10x 1020x 5. For Plan B: C_B 40 1 5 10x 1015x 25.

For Plan C: C_C 60 1 10x 10 10x 50. Note that these equations are valid only for x 1.

If Gwendolyn uses 2 2 GB, Plan A costs 25 12 2 $49, Plan B costs 40 12 1 5 $58, and Plan C costs 60 12 1 $72.

If she uses 3 7 GB, Plan A costs 25 27 2 $79, Plan B costs 40 27 1 5 $80 50, and Plan C costs 60 27 1 $87.

If she uses 4 9 GB, Plan A costs 25 39 2 $103, Plan B costs 40 39 1 5 $98 50, and Plan C costs 60 39 1 $99.

(i) We set C_A C_B 20x 5 15x 25 5x 20 x 4. Plans A and B cost the same when 4 GB are used.

We set C_A C_C 20x 5 10x 50 10x 45 x 4 5. Plans A and C cost the same when 4 5 GB are used.

We set C_B C_C 15x 25 10x 50 5x 25 x 5. Plans B and C cost the same when 5 GB are used.

(a) In this plan, Company A gets $3 2 million and Company B gets $3 2 million. Company A's investment is $1 4 million, so they make a profit of 3 2 1 4 $1 8 million. Company B's investment is $2 6 million, so they make a profit of 3 2 2 6 $0 6 million. So Company A makes three times the profit that Company B does, which is not fair.

The original investment is 1 4 2 6 $4 million. So after giving the original investment back, they then share the profit of $2 4 million. So each gets an additional $1 2 million. So Company A gets a total of 1 4 1 2 $2 6 million and Company B gets 2 6 1 2 $3 8 million. So even though Company B invests more, they make the same profit as Company A, which is not fair.

The original investment is $4 million, so Company A gets $\frac{1\ 4}{4}$ 6 4 $2 24 million and Company B gets $\frac{2\ 6}{4}$ 6 4 $4 16 million. This seems the fairest.

EQUATIONS AND GRAPHS

1.1 THE COORDINATE PLANE

The point that is 2 units to the left of the *y*-axis and 4 units above the *x*-axis has coordinates 2 4 .

If *x* is positive and *y* is negative, then the point *x y* is in Quadrant IV.

The distance between the points *a b* and *c d* is $c\ a^2\ d\ b^2$. So the distance between 1 2 and 7 10 is $7\ 1^2\ 10\ 2^2\ 6^2\ 8^2$ 36

64 100 10.

4. The point midway between *a b* and *c d* is $\frac{a\ c}{2}\ \frac{b\ d}{2}$. So the point midway between 1 2 and 7 10 is

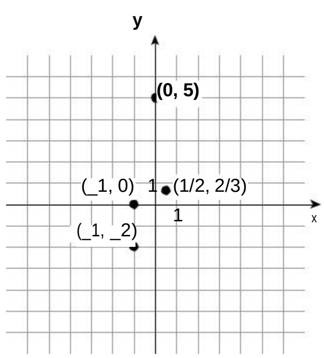
4 6 .

17 2 10 8 12

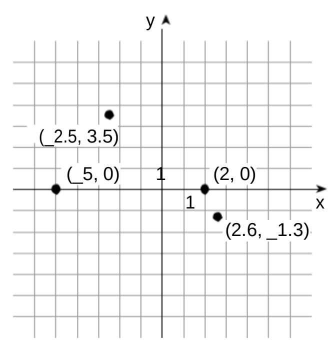
*A*51,*B*12,*C* 26,*D* 62,*E* 4 1,*F* 20,*G* 1 3,*H*2 2

Points *A* and *B* lie in Quadrant 1 and points *E* and *G* lie in Quadrant 3.

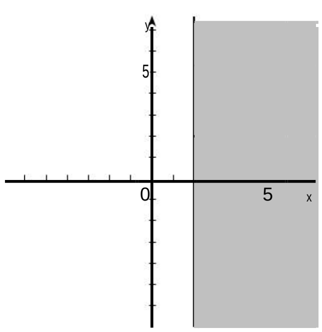
7. 0 5 , 1 0 , 1 2 , and $\frac{1}{2}\ \frac{2}{3}$



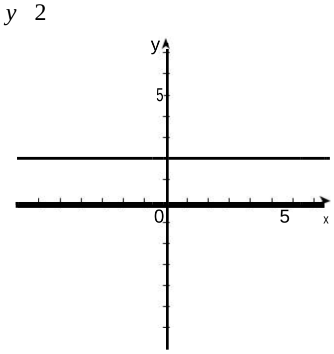
8. 5 0 , 2 0 , 2 6 1 3 , and 2 5



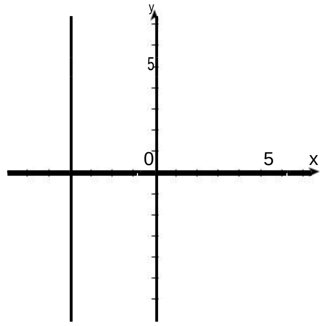
9. *x y x* 2



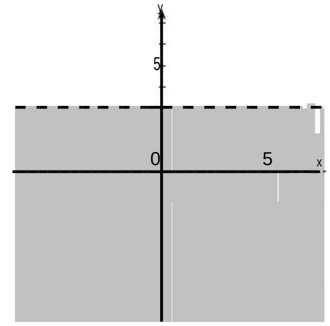
10. *x y y* 2



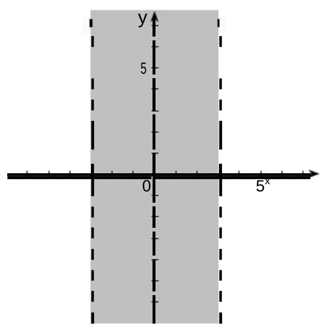
11. *x y x*4



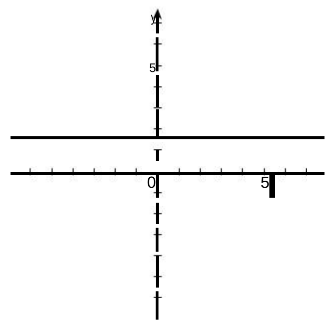
12. *x y y* 3



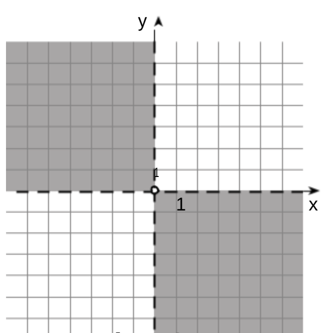
13. *x y*3 *x* 3



14. *x y* 0 *y* 2

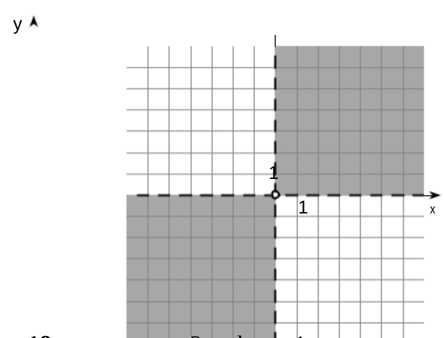


15. *x y* *x y* 0
x y

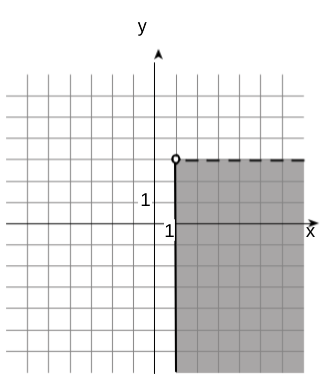


16. *x y* *x y* 0

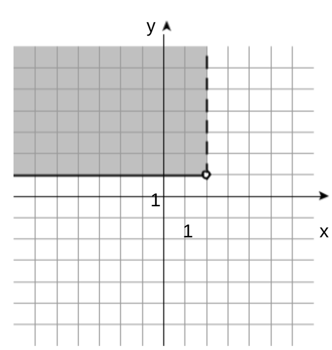
x 0 and *y* 0 or *x*
0 and *y* 0 *x y*
x 0 and *y* 0 or *x*
0 and *y* 0



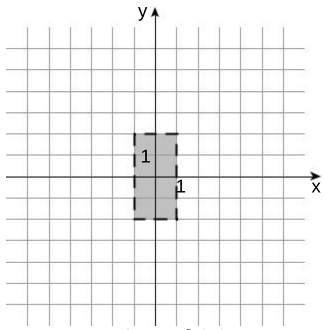
17. *x y* *x* 1 and *y* 3



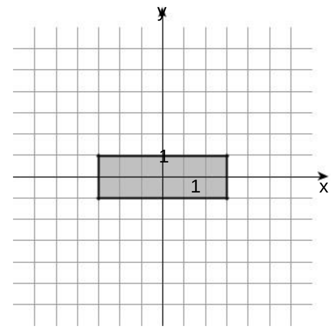
18. *x y* *x* 2 and *y* 1



19. $x\ y1\ \ x\ \ 1$ and $2\ \ y\ \ 2$



20. $x\ y3\ \ x\ \ 3$ and $1\ \ y\ \ 1$



21. The two points are 0 2 and 3 0 .

(a) $d3\ \ 0^2\ \ 0\ \ 2^2\ \ 3^2\ \ 2^2\ \ 9413$

(b) midpoint: $\frac{3\ \ 0}{2}\ \ \frac{0\ 2}{2}\ \ \frac{3}{2}\ \ 1$

22. The two points are 2 1 and 2 2 .

(a) $d2\ \ 2^2\ 1\ \ 2^2\ \ 4^2\ 3^2\ 169255$

(b) midpoint: $\frac{2\ \ 2}{2}\ \ \frac{1\ \ 2}{2}\ \ 0\ 2^1$

23. The two points are 3 3 and 5 3 .

(a) $d3\ \ 5^2\ \ 33^2\ \ 8^2\ 6^2 64\ 36100\ 10$

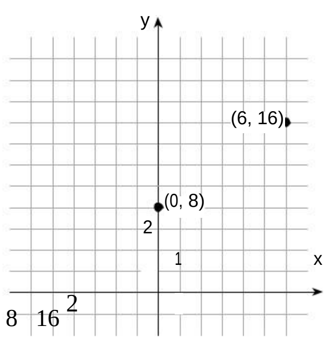
(b) midpoint: $\frac{3}{2}\ \ 5\ 3\ \ 2\ 3\ \ 1\ 0$

24. The two points are 2 3 and 4 1 .

(a) $d2\ \ 4^2 31^2 6^2 2^2 36\ \ 450\ \ 2\ 10$

(b) midpoint: $\frac{2}{2}\ \ 4\ \ 3\ \ \frac{1}{2}\ \ 1\ \ 2$

25. (a)

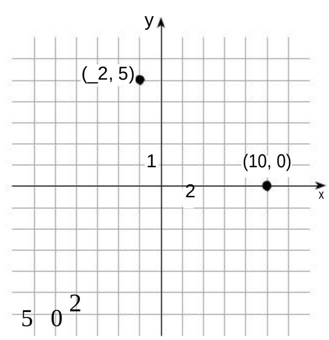


(b) $d0\ \ 6^2\ \ 8\ \ 16^2$

$6^2 8\ \ 100\ \ 10$

(c) Midpoint: $\frac{0\ \ 6}{2}\ \ \frac{8\ \ 16}{2}\ \ 3\ 12$

26. (a)

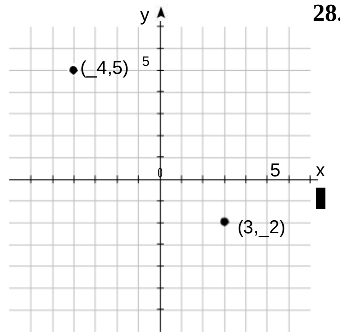


(b) $d2\ \ 10^2\ \ 5\ \ 0^2$

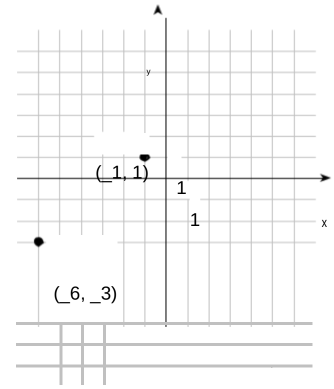
$12^2\ \ 5^2\ 169\ \ 13$

(c) Midpoint: $\frac{2\ \ 10}{2}\ \ \frac{5\ \ 0}{2}\ \ 4\ \frac{5}{2}$

27. (a)



28. (a)



4

(b) *d* 3 2 25 2

7 2 2 7

7 49 49 98 2

4 3 5 2 1 3

Midpoint:

(c) Midpoint: 2 2 2 2

(b) *d* 16 2 1 3 2

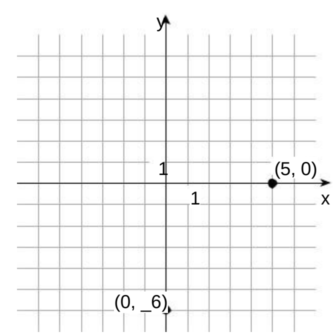
2 2

5 4 41

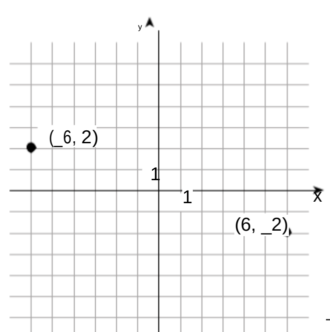
(c) 6 1 3 1 7 1

2 2 2

30. (a)



29. (a)



(b) *d* 6 6 2 2 2 2 12 2 4 2

144 16160 4 10

6 6 2 2

(c) Midpoint: 2 2 0 0

*d*0 5 2 6 0 2

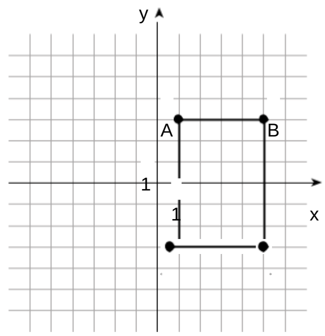
5 2 6 2 25 36 61

0 5 6 0 5

(c) Midpoint: 2 2 2 3

31. $d\ A\ B$ $\quad 1 \quad 5^2 \quad 3 \quad 3^2 \quad 4^2 \quad 4.$

$d\ A\ C\ 1\ 1^2\ 3\ 3^2\ 6^2$ 6. So the area is 4 6 24.



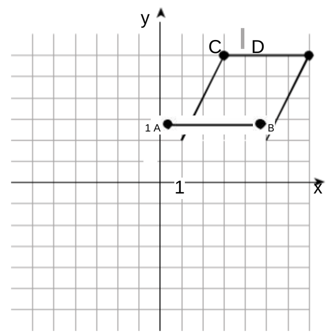
The area of a parallelogram is its base times its height. Since two sides are parallel to the *x*-axis, we use the length of one of these as the base. Thus, the base is

$d\ A\ B$ $\quad 15^2 \quad 2 \quad 4^2$ 4. The

22

height is the

is 6 2 4. So the area of the parallelogram is base height 4 4 16.



33. From the graph, the quadrilateral *ABC D* has a pair of parallel sides, so *ABC D* is a trapezoid. The area is

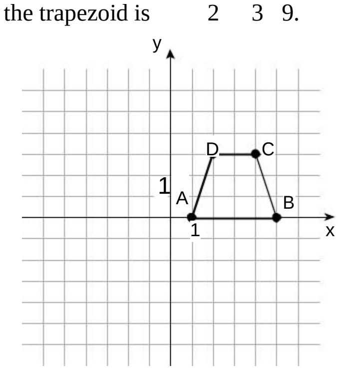
$\frac{b1 \quad b2}{2}$ *h*. From the graph we see that

$b1 \quad d\ A\ B1 \quad 5^2 \quad 0 \quad 0^2\ 4^2 \quad 4;$

$b2 \quad d\ C\ D4 \quad 2^2 \quad 3 \quad 3^2\ 2^2 \quad 2;$ and

h is the difference in *y*-coordinates is 3 0 3. Thus

the area of the trapezoid is $\frac{4 \quad 2}{2}$ 3 9.



The point *S* must be located at 0 4 . To find the area, we find the length of one side and square it. This gives

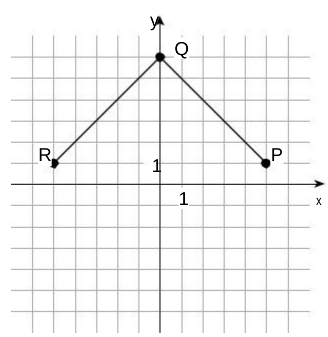
$d\ Q\ R$ $\quad 5 \quad 0^2 \quad 1 \quad 6^2$

$5^2 5^2$

25 2550

2

So the area is $\overline{50}$ 50.



$d\ 0\ A6 \quad 0^2 \quad 7 \quad 0^2\ 6^2 \quad 7^2\ 36 \quad 4985.$

$d\ 0\ B$ $\quad 5 \quad 0^2 \quad 8 \quad 0^2 \quad 5^2 \quad 8^2 \quad \overline{25 \quad 64} \quad \overline{89}.$

Thus point *A* 6 7 is closer to the origin.

36. $d\,E\,C$ $62^2 31^2$ $4^2\ 2$ 16 4 20.

2

$d\,E\,D$ $32^2\ 2$ 2 2 25 1 26.

01 5 1

Thus point C is closer to point E.

37. $d\,P\,R$ $13^2 11^2$ $4^2 2^2$ 16 4 20 2 5.

$d\,Q\,R$ $11^2\ 2$ 2 16 4. Thus point Q 1 3 is closer to point R.

13 04

(a) The distance from 7 3 to the origin is 7 0^2 3 0^2 7^2 3^2 49 958. The distance from

3 7 to the origin is $30^2 70^2$ $3^2\ 7^2$ 9 49 58. So the points are the same distance from

the origin.

(b) The distance from $a\ b$ to the origin is a 0^2 b 0^2 a^2 b^2. The distance from $b\ a$ to the origin is

b 0^2 a 0^2 b^2 a^2 a^2 b^2. So the points are the same distance from the origin.

Since we do not know which pair are isosceles, we find the length of all three sides.

$d\,A\,B$ 3 0^2 1 2^2 3^2 3^2 9 9 18 3 2.

$d\,C\,B$ 3 4^2 1 3^2 1^2 4^2 1 16 17.

$d\,A\,C$ 0 4^2 2 3^2 4^2 1^2 16 1 17. So sides AC and $C\,B$ have the same length.

40. Since the side AB is parallel to the x-axis, we use this as the base in the formula area $\frac{1}{2}$ base height . The height is the

change in the y-coordinates. Thus, the base is 2 4 6 and the height is 4 1 3. So the area is $\frac{1}{2}$ 6 3 9.

41. (a) Here we have A 2 2 , B 3 1 , and C3 3 . So

12 2 1 9 10;

$d\,C\,B$3 3 ^{2}13 ^{2}6^2 2^2 36 4 40 2 10;

$d\,A\,C$ 3 2^2 3 2^2 5^2 5^2 25 25 50 5 2.

The area of the triangle is $\frac{1}{2}$ $d\,C\,B$ $d\,A\,B$ $\frac{1}{2}$ 10 2 10 10.

42. $d\,A\,B$ 116^{2}37^2 2 4^2 25 16 41;

2 2 5 2 2

$d\,A\,C$ 26 27 4 5 16 25 41;

2 2 2

$d\,B\,C$ 211^{2}23^2 9^2 1^2 81 1 82.

Since $[d\ A\,B][d\ A\,C]$ $[d\ B\,C]$, we conclude that the triangle is a right triangle. The area is

2 41 41 2.

41

43. We show that all sides are the same length (its a rhombus) and then show that the diagonals are equal. Here we have $A(-2,9)$, $B(4,6)$, $C(1,0)$, and $D(-5,3)$. So

$d(A,B) = \sqrt{(4-(-2))^2 + (6-9)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45}$;

$d(B,C) = \sqrt{(1-4)^2 + (0-6)^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{9+36} = \sqrt{45}$;

$d(C,D) = \sqrt{(-5-1)^2 + (3-0)^2} = \sqrt{(-6)^2 + 3^2} = \sqrt{36+9} = \sqrt{45}$;

$d(D,A) = \sqrt{(-2-(-5))^2 + (9-3)^2} = \sqrt{3^2 + 6^2} = \sqrt{9+36} = \sqrt{45}$. So the points form a rhombus. Also $d(A,C) = \sqrt{(1-(-2))^2 + (0-9)^2} = \sqrt{3^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$,

and $d(B,D) = \sqrt{(-5-4)^2 + (3-6)^2} = \sqrt{(-9)^2 + (-3)^2} = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$. Since the diagonals are equal, the rhombus is a square.

44. $d(A,B) = \sqrt{(3-(-1))^2 + (11-3)^2} = \sqrt{4^2 + 8^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$.

$d(B,C) = \sqrt{(5-3)^2 + (15-11)^2} = \sqrt{2^2 + 4^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$.

$d(A,C) = \sqrt{(5-(-1))^2 + (15-3)^2} = \sqrt{6^2 + 12^2} = \sqrt{36+144} = \sqrt{180} = 6\sqrt{5}$. So $d(A,B) + d(B,C) = d(A,C)$, and the points are collinear.

45. Let $P(0,y)$ be such a point. Setting the distances equal we get

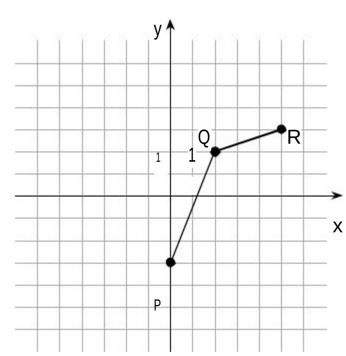
$\sqrt{(0-5)^2 + (y-(-5))^2} = \sqrt{(0-1)^2 + (y-1)^2} \Leftrightarrow 25 + y^2 + 10y + 25 = 1 + y^2 - 2y + 1 \Rightarrow 10y + 50 = -2y + 2 \Leftrightarrow 12y = -48 \Leftrightarrow y = -4$. Thus, the point is $P(0,-4)$. Check:

$\sqrt{(0-5)^2 + (-4-(-5))^2} = \sqrt{(-5)^2 + 1^2} = \sqrt{25+1} = \sqrt{26}$;

$\sqrt{(0-1)^2 + (-4-1)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}$.

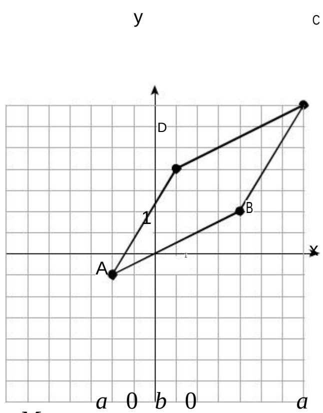
46. The midpoint of AB is $C' = \left(\frac{1+3}{2}, \frac{0+6}{2}\right) = (2,3)$. So the length of the median CC' is $d(C,C') = \sqrt{(2-8)^2 + (3-2)^2} = \sqrt{36+1} = \sqrt{37}$. The midpoint of AC is $B' = \left(\frac{1+8}{2}, \frac{0+2}{2}\right) = \left(\frac{9}{2}, 1\right)$. So the length of the median BB' is $d(B,B') = \sqrt{\left(\frac{9}{2}-3\right)^2 + (1-6)^2} = \sqrt{\frac{109}{4}} = \frac{\sqrt{109}}{2}$. The midpoint of BC is $A' = \left(\frac{3+8}{2}, \frac{6+2}{2}\right) = \left(\frac{11}{2}, 4\right)$. So the length of the median AA' is $d(A,A') = \sqrt{\left(\frac{11}{2}-1\right)^2 + (4-0)^2} = \frac{\sqrt{145}}{2}$.

47. As indicated by Example 3, we must find a point $S(x_1, y_1)$ such that the midpoints of PR and of QS are the same. Thus $\left(\frac{-1+4}{2}, \frac{-4+2}{2}\right) = \left(\frac{x_1+1}{2}, \frac{y_1+1}{2}\right)$. Setting the x-coordinates equal, we get $\frac{-1+4}{2} = \frac{x_1+1}{2} \Leftrightarrow -1+4 = x_1+1 \Leftrightarrow x_1 = 2$. Setting the y-coordinates equal, we get $\frac{-4+2}{2} = \frac{y_1+1}{2} \Leftrightarrow -4+2 = y_1+1 \Leftrightarrow y_1 = -3$. Thus $S(2,-3)$.



48. We solve the equation 6 $\frac{2 \quad x}{2}$ to find the x coordinate of B. This gives 6 $\frac{2 \quad x}{2}$ 12 2 x x 10. Likewise, 8 $\frac{3 \quad y}{2}$ 16 3 y y 13. Thus, B 1013.

49. (a)



(b) The midpoint of AC is $\frac{2 \quad 7}{} \frac{1 \quad 7}{}$ $\frac{5}{2}$ 3 , the midpoint of B D is $\frac{4 \quad 1}{} \frac{2 \quad 4}{}$ $\frac{5}{}$ 3 .

(c) Since the they have the same midpoint, we conclude that the diagonals bisect each other.

50. We have M $\frac{a \quad 0}{2} \frac{b \quad 0}{2}$ $\frac{a}{2} \frac{b}{2}$. Thus,

d C M $\frac{a}{2}$ 0 2 $\frac{b}{2}$ 0 2 $\frac{a}{}^2 \frac{b}{}^2$ $\frac{a^2 \; b^2}{2}$;

d A M $\frac{a}{2}$ a 2 $\frac{b}{2}$ 0 2 $\frac{a}{2}$ 2 $\frac{b}{2}$ 2 $\frac{a}{}^2$ 2 $\sqrt{a^2 \quad b^2}$

d B M $\frac{a}{2}$ 0 2 $\frac{b}{2}$ b 2 $\frac{a}{2}$ 2 $\frac{b}{2}$ 2 $\frac{a}{}^2 \frac{b}{}^2$ $\frac{a^2 \quad b^2}{}$

51. (a) The point 5 3 is shifted to 5 3 3 28 5 .

(b) The point a b is shifted to a 3 b 2 .

(c) Let x y be the point that is shifted to 3 4 . Then x 3 y 23 4 . Setting the x-coordinates equal, we get x 3 3 x 0. Setting the y-coordinates equal, we get y 2 4 y 2 So the point is 0 2 .

(d) A5 1 , so A 5 3 1 22 1 ; B3 2 , so B 3 3 2 20 4 ; and C 2 1 , so C2 3 1 25 3 .

52. (a) The point 3 7 is reflected to the point 3 7 .

(b) The point a b is reflected to the point a b .

(c) Since the point a b is the reflection of a b , the point 4 1 is the reflection of 4 1 .

(d) A 3 3 , so A3 3 ; B 6 1 , so B 6 1 ; and C 1 4 , so C1 4 .

53. (a) d A B $\sqrt{3^2 \quad 4^2}$ $\sqrt{25}$ 5. 11 26 . The walking distance is

We want the

4 11 2 26 7 24 31 blocks. Straight-line distance is

$\sqrt{4 \quad 11^2 \quad 2 \quad 26^2}$ $\sqrt{7^2 \quad 24^2}$ $\sqrt{625}$ 25 blocks.

(c) The two points are on the same avenue or the same street.

54. (a) The midpoint is at $\frac{3 \quad 27}{2}$ $\frac{7 \quad 17}{2}$ 15 12 , which is at the intersection of 15th Street and 12th Avenue.

(b) They each must walk 15 312 7 12 5 17 blocks.

55. The midpoint of the line segment is 66 45 . The pressure experienced by an ocean diver at a depth of 66 feet is 45 lb/in^2.

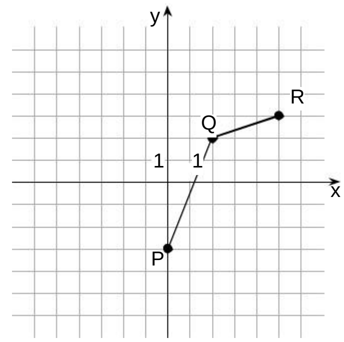
We solve the equation $6 \quad \frac{2 \quad x}{2}$ to find the x coordinate of B: $6 \quad \frac{2 \quad x}{2} \quad 12 \quad 2 \quad x \quad x \quad 10$. Likewise, for the y coordinate of B, we have $8 \quad \frac{3 \quad y}{2} \quad 16 \quad 3 \quad y \quad y \quad 13$. Thus B $\quad 10 \; 13$.

57. We need to find a point $S \; x_1 \; y_1$ such that $P \; Q \; RS$ is a parallelogram. As indicated by Example 3, ~~this w~~ill be the case if the diagonals $P \; R$ and QS bisect each other. So the midpoints of $P \; R$ and QS are the same. Thus

$\frac{0 \quad 5}{2} \; \frac{3 \quad 3}{2} \qquad \frac{x_1 \quad 2}{2} \; \frac{y_1 \quad 2}{2}$. Setting the x-coordinates equal, we get $\frac{0 \quad 5}{2} \quad \frac{x_1 \quad 2}{2} \quad 0 \quad 5 \quad x_1 \quad 2 \quad x_1 \quad 3$.

Setting the y-coordinates equal, we get $\frac{3 \quad 3}{2} \quad \frac{y_1 \quad 2}{2} \quad 3 \quad 3 \quad y_1 \quad 2$

$y_1 2$. Thus $S \quad 3 \quad 2$.



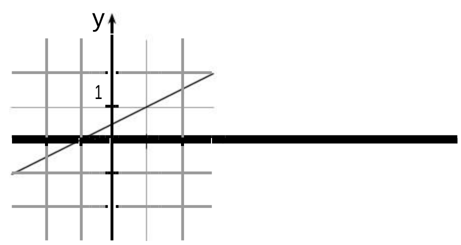
1.2 GRAPHS OF EQUATIONS IN TWO VARIABLES: CIRCLES

If the point 2 3 is on the graph of an equation in x and y, then the equation is satisfied when we replace x by 2 and y by 3.

We check whether $2 \; 3216 \overset{?}{} \quad \overset{?}{} \; 3$. This is false, so the point 2 3 is not on the graph of the equation $2y \quad x \quad 1$.

To complete the table, we express y in terms of x: $2y \quad x \quad 1 \quad y \quad \frac{1}{2} x \quad 1 \quad \frac{1}{2} x \quad \frac{1}{2}$.

x	y	$x \; y$
2	$\frac{1}{2}$	2 $\frac{1}{2}$
1	0	1 0
	1	1
1 $_2$	1 $_2$ $\frac{}{3}$	1 1 $_{2\,2}$ $\frac{}{3}$



2. To find the x-intercept(s) of the graph of an equation we set y equal to 0 in the equation and solve for x: $2 \; 0 \quad x \quad 1$ 1, so the x-intercept of $2y \quad x \quad 1$ is 1.

To find the y-intercept(s) of the graph of an equation we set x equal to 0 in the equation and solve for y: $2y \quad 0 \quad 1$ $\frac{1}{2}$, so the y-intercept of $2y \quad x \quad 1$ is $\frac{1}{2}$.

The graph of the equation $x \quad 1^2 \quad y \quad 2^2 \quad 9$ is a circle with center 1 2 and radius $\sqrt{9} \quad 3$.

(a) If a graph is symmetric with respect to the x-axis and $a \; b$ is on the graph, then $a \quad b$ is also on the graph.

(b) If a graph is symmetric with respect to the y-axis and $a \; b$ is on the graph, then $a \; b$ is also on the graph.

(c) If a graph is symmetric about the origin and $a \; b$ is on the graph, then $a \quad b$ is also on the graph.

(a) The x-intercepts are 3 and 3 and the y-intercepts are 1 and 2.

(b) The graph is symmetric about the y-axis.

Yes, this is true. If for every point $x \; y$ on the graph, $x \; y$ and $x \; y$ are also on the graph, then $x \; y$ must be on the graph as well, and so it is symmetric about the origin.

No, this is not necessarily the case. For example, the graph of $y \quad x$ is symmetric about the origin, but not about either axis.

9. y 3 4x. For the point 0 3 : 3 ? 3 4 03 3. Yes. For 4 0 : 0 ? 3440 ? 13. No. For 1 1 : 1 ? 3 4 111. Yes. ?

So the points 0 3 and 1 1 are on the graph of this equation.

10. y 1 x. For the point 2 1 : 1 ? 1 2 ? 1 1. No. For 3 2 : 2 ? 132 ? 4. Yes. For 0 1 : 1 ? 1 0. Yes.

So the points 3 2 and 0 1 are on the graph of this equation.

11. x 2y 1 0. For the point 0 0 : 0 2 0 ? 1 0 ? 1 0. No. For 1 0:1 20 ? 1 01 ? 1 0. Yes. For 1 1 : 1 2 1 1 01 2 1 ? ? 0. Yes.

So the points 1 0 and 1 1 are on the graph of this equation.

12. y x^2 11. For the point 1 1 : 1 1 ^{2} 1 1 1 2 1. No. For 1 2^1 : 21121 1212 1.

Yes. For1 21 : 21121 1212 2 1. Yes.

So the points 1 $\frac{1}{2}$ and 1 $\frac{1}{2}$ are on the graph of this equation.

13. x^2 2x y y^2 1. For the point 0 1 : 0^2 2 0 1 1^2 ? 1 1 ? 1. Yes. For 2 1 : 2^2 2 2 11^2 ? 1 4 4 1 1 1. Yes. For 2 3 : 2 2 2233 $_2$? 1 4 12 9 1 1 1. Yes.

So the points 0 1 , 2 1 , and 2 3 are on the graph of this equation.

14. 01: 0^2 1^2 ? 1 0 0 ? 1 1 0. Yes.

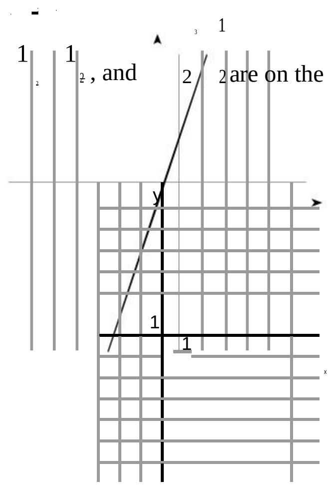
2 2 : 2 2 2 ? 1 0 4 4 ? 1 0. Yes.

3 1 3 2 1 2 ? 3 1 ?

So the points 0 1 , 1 1 , and 2 2 are on the graph of this equation.

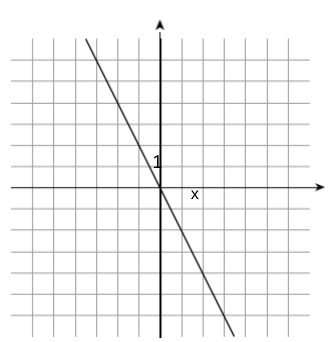
15. y 3x

x	y
3	9
2	6
1	3
0	0
1	3
2	6
3	9



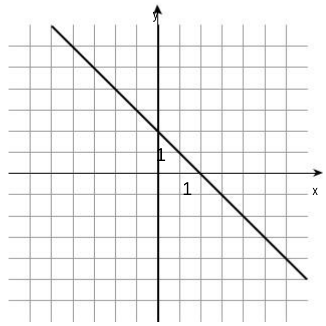
16. y 2x

x	y
3	6
2	4
1	2
0	0
1	2
2	4
3	6



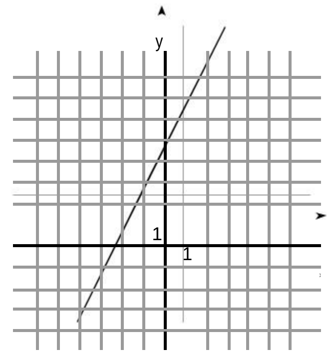
y 2 *x*

	y
46	
2	4
0	2
2	0
4	2



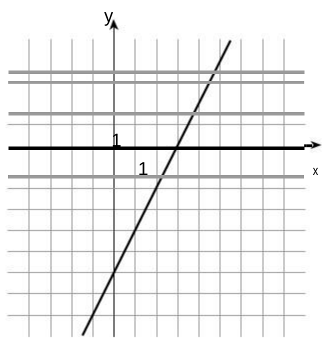
18. *y* 2*x* 3

x	*y*
4	5
2	1
0	3
2	7
4	11



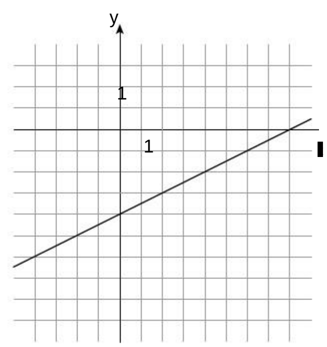
Solve for *y*: 2*x* *y* 6 *y* 2*x* 6.

x	*y*
2	10
0	6
2	2
	2
	6



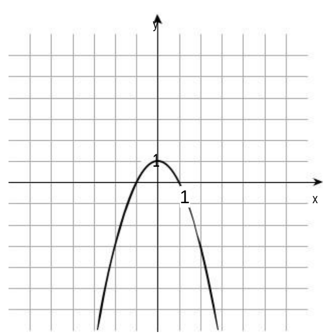
20. Solve for *x*: *x* 4*y* 8 *x* 4*y* 8.

x	*y*
4	3
2	$\frac{5}{2}$
0	2
2	$\frac{3}{2}$
4	1
6	$\frac{1}{2}$
8	0
10	$\frac{1}{2}$



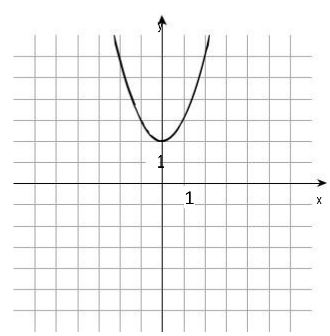
21. *y* 1 x^2

x	*y*
3	8
2	3
1	0
0	1
1	0
2	3
3	8



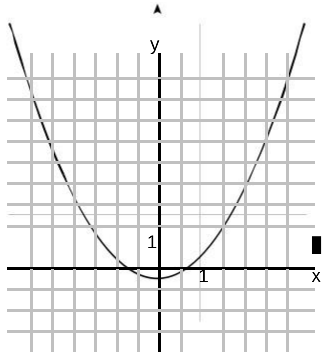
y x^2 2

	y
3	11 2
6	
1	3
0	2
1	3
2	6
3	11



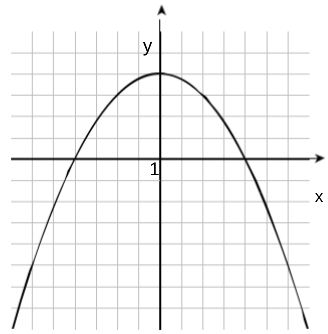
23. y x^2 2

x	y
3	7
2	2
1	1
0	2
1	1
2	2
3	7



24. yx^2 4

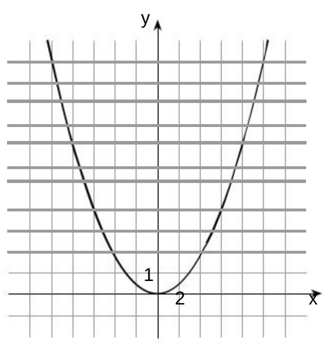
x	y
3	5
2	0
1	3
0	4
1	3
2	0
3	5



9y x^2. To make a table, we rewrite the equation as y

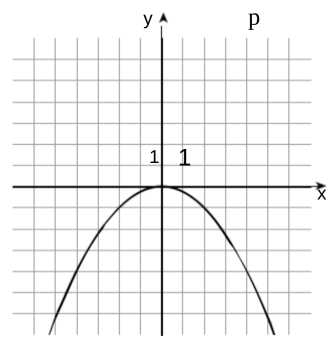
$\frac{1}{9}x^2$.

x	y
9	9
3	1
0	0
3	1
9	9



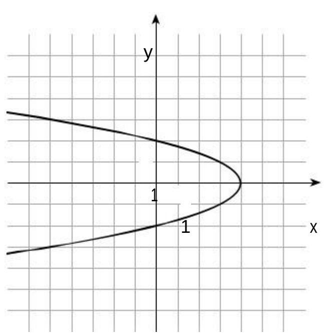
4yx^2.

	y
4 4	2 1
0	0
2	1
4	4



x y^2 4.

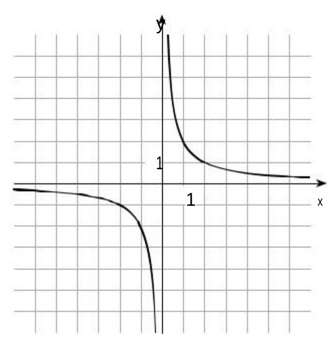
x	y
12	4
5	3
0	2
3	1
4	0
3	1
0	2
5	3
12	4



28. $x\,y$ 2 y x^2.

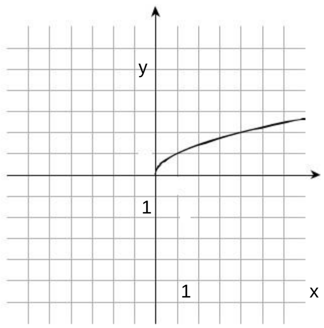
–

x	y
4	$\frac{1}{2}$
2	1
1	2
$\frac{1}{2}$	4
$\frac{1}{4}$	8
$\frac{1}{4}$	8
$\frac{1}{2}$	4
1	2
2	1
4	$\frac{1}{2}$



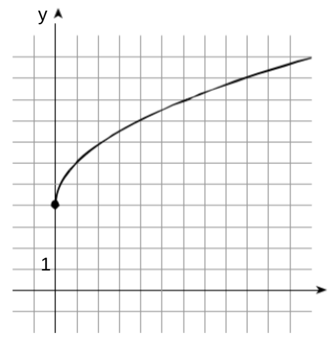
yx.

x	*y*
0	0
$\frac{1}{4}$	$\frac{1}{2}$
1	1
2	2
4	2
9	3
16	4



30. y 2 x .

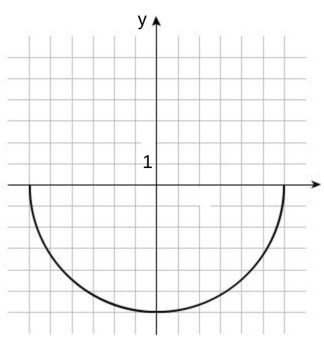
x	*y*
0	2
1	3
2	2 2
4	4
9	5



31. y 9 x^2. Since the radicand (the expression inside the square root) cannot be negative, we must have

9 x^2 0 x^2 9 x 3.

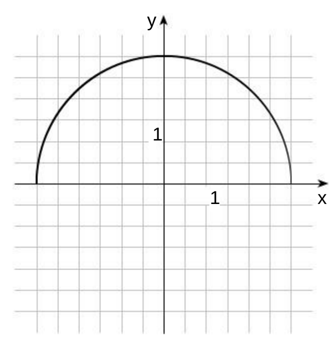
x	*y*
3	0
2	5
1	2 2
1	2 2
2	5
3	0



*y*9 x^2.

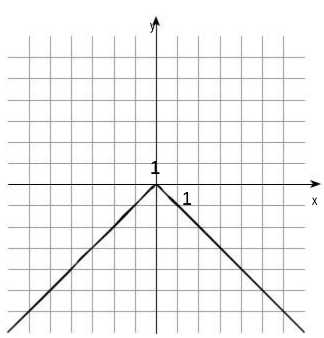
Since the radicand (the expression inside the square root) cannot be negative, we must have 9 x^2 0 x^2 9 x 3.

x	*y*
3	0
2	5
1	2 2
0	3
1	2 2
2	5
3	0



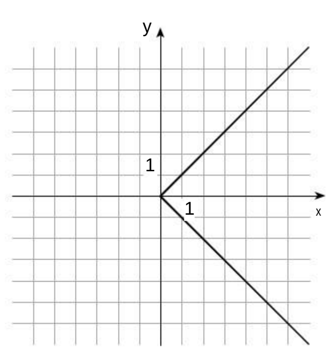
33. y x .

x	*y*
6	6
4	4
2	2
0	0
2	2
4	4
6	6



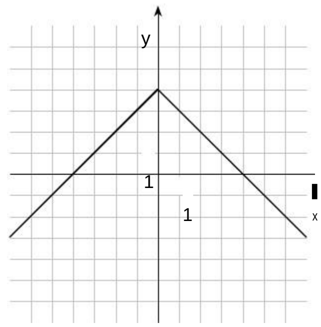
x y . In the table below, we insert values of *y* and find the corresponding value of *x*.

x	*y*
3	3
2	2
1	1
0	0
1	1
2	2
3	3



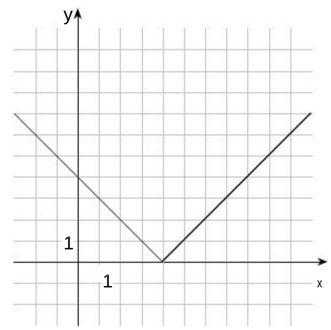
35. $y \quad 4 \quad x$.

x	y
6	2
4	0
2	2
0	4
2	2
4	0
6	2



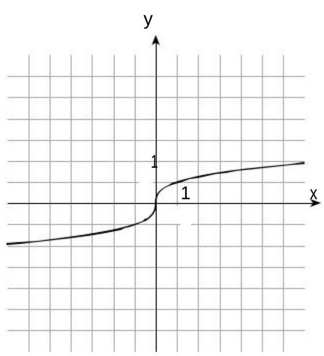
$y \ 4 \ x$.

	y
6	10 4
8	
	2 6
0	4
2	2
4	0
6	2
8	4
10	6



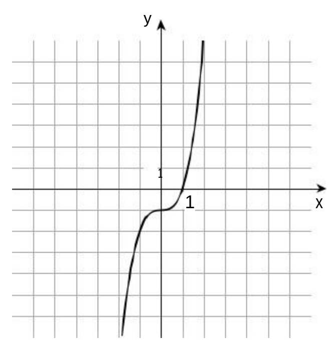
$x \, y^3$. Since $x \, y^3$ is solved for x in terms of y, we insert values for y and find the corresponding values of x in the table below.

x	y
27	3
8	2
1	1
0	0
1	1
8	2
27	3



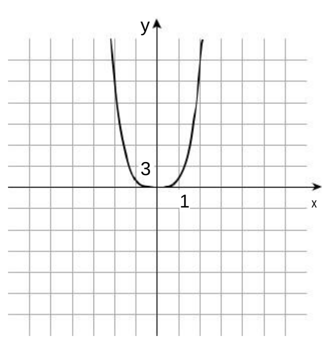
38. $y \quad x^3 \quad 1$.

x	y
3	28
2	9
1	2
0	1
1	1
2	7
3	26



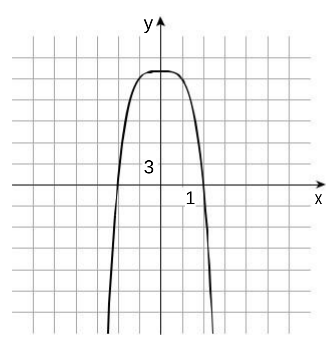
39. $y \quad x^4$.

x	y
3	81
2	16
1	1
0	0
1	1
2	16
3	81

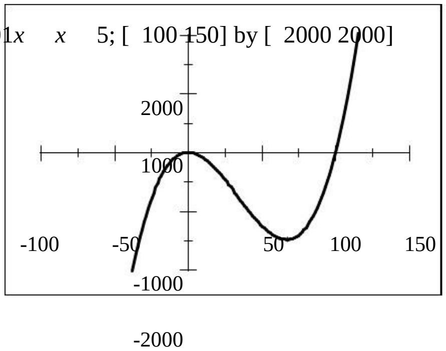


40. y

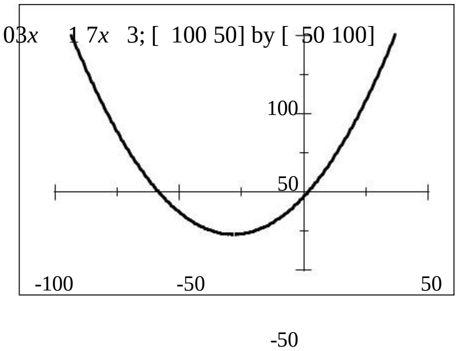
16 x	x^4. y
3	65
2	0
1	15
0	16
1	15
2	0
3	65



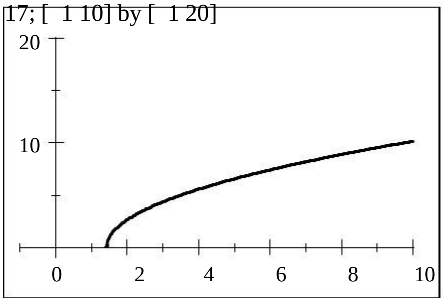
41. y 0 01x^3 x^2 5; [100 150] by [2000 2000]



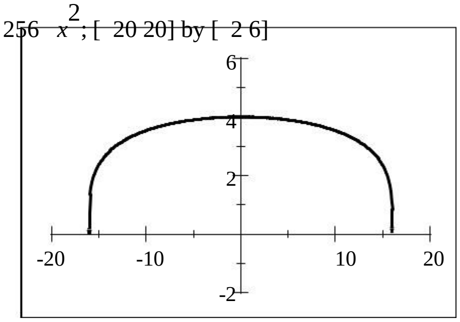
42. y 0 03x^2 1 7x 3; [100 50] by [50 100]



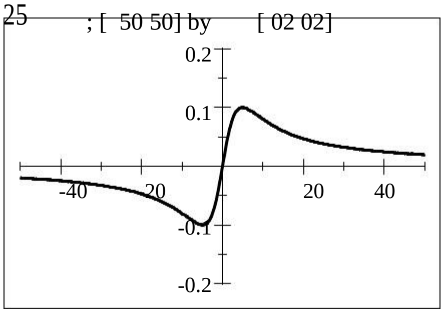
43. y 12x 17; [1 10] by [1 20]



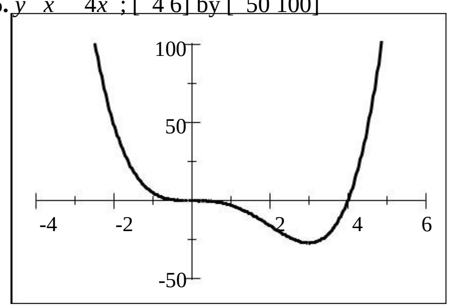
44. y ^{4} 256 x^2; [20 20] by [2 6]



45. y $\frac{x}{x^2 \; 25}$; [50 50] by [02 02]



46. y x^4 4x^3; [4 6] by [50 100]



y x 6. To find x-intercepts, set y 0. This gives 0 x 6 x 6, so the x-intercept is 6. To find y-intercepts, set x 0. This gives y 0 6 y 6, so the y-intercept is 6.

2x 5y 40. To find x-intercepts, set y 0. This gives 2x 5 0 40 2x 40 x 20, and the x-intercept is 20. To find y-intercepts, set x 0. This gives 2 0 5y 40 y 8, so the y-intercept is 8.

y x^2 5. To find x-intercepts, set y 0. This gives 0 x^2 5 x^2 5 x 5, so the x-intercepts are 5. To find y-intercepts, set x 0. This gives y 0^2 5 5, so the y-intercept is 5.

y^2 9 x^2. To find x-intercepts, set y 0. This gives 0^2 9 x^2 x^2 9 x 3, so the x-intercepts are 3. To find y-intercepts, set x 0. This gives y^2 9 0^2 9 y 3, so the y-intercepts are 3.

y 2x y 2x 1. To find x-intercepts, set y 0. This gives 0 2x 0 2x 1 2x 1 x $\frac{1}{2}$, so the x-intercept is $\frac{1}{2}$.

To find y-intercepts, set x 0. This gives y 2 0 y 2 0 1 y 1, so the y-intercept is 1.

3 2 2

x^2 x y y 1. To find *x*-intercepts, set *y* 0. This gives x^2 x 0 0 1 x^2 1 *x* 1, so the *x*-intercepts are 1 and 1.

To find *y*-intercepts, set *x* 0. This gives *y* 0^2 0 *y* *y* 1 *y* 1, so the *y*-intercept is 1.

y x 1. ~~To find~~ *x*-intercepts, set *y* 0. This gives 0 *x* 1 0 *x* 1 *x* 1, so ~~the x-~~intercept is 1. To find *y*-intercepts, set *x* 0. This gives *y* 0 1 *y* 1, so the *y*-intercept is 1.

x y 5. To find *x*-intercepts, set *y* 0. This gives *x* 0 5 0 5, which is impossible, so there is no *x*-intercept. To find *y*-intercepts, set *x* 0. This gives 0 *y* 5 0 5, which is again impossible, so there is no *y*-intercept.

55. 4x^2 25y^2 100. To find *x*-intercepts, set *y* 0. This gives 4x^2 25 0^2 100 x^2 25 *x* 5, so the *x*-intercepts are 5 and 5.

To find *y*-intercepts, set *x* 0. This gives 4 0^2 25y^2 100 y^2 4 *y* 2, so the *y*-intercepts are 2 and 2.

25x^2 y^2 100. To find *x*-intercepts, set *y* 0. This gives 25x^2 0^2 100 x^2 4 *x* 2, so the *x*-intercepts are 2 and 2.

To find *y*-intercepts, set *x* 0. This gives 25 0^2 y^2 100 y^2 100, which has no solution, so there is no *y*-intercept.

57. *y* 4*x* x^2. To find *x*-intercepts, set *y* 0. This gives 0 4*x* x^2 0 *x* 4 *x* 0 *x* or *x* 4, so the *x*-intercepts are 0 and 4.

To find *y*-intercepts, set *x* 0. This gives *y* 4 0 0^2 *y* 0, so the *y*-intercept is 0.

58. $\frac{x^2}{9}$ $\frac{y^2}{4}$ 1. To find *x*-intercepts, set *y* 0. This gives $\frac{x^2}{9}$ $\frac{0^2}{4}$ 1 $\frac{x^2}{9}$ 1 x^2 9 *x* 3, so the *x*-intercepts are 3 and 3.

To find *y*-intercepts, set *x* 0. This gives $\frac{0^2}{9}$ $\frac{y^2}{4}$ 1 $\frac{y^2}{4}$ 1 y^2 4 *x* 2, so the *y*-intercepts are 2 and 2.

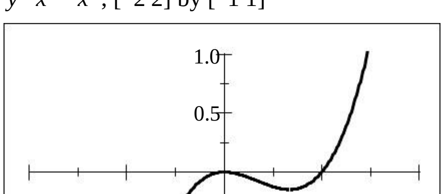
x^4 y^2 x y 16. To find *x*-intercepts, set *y* 0. This gives x^4 0^2 x 0 16 x^4 16 *x* 2. So the *x*-intercepts are 2 and 2.

To find *y*-intercepts, set *x* 0. This gives 0^4 y^2 0 *y* 16 y^2 16 *y* 4. So the *y*-intercepts are 4 and 4.

x^2 y^3 x^2 y^2 64. To find *x*-intercepts, set *y* 0. This gives x^2 0^3 x^2 0^2 64 x^2 64 *x* 8. So the *x*-intercepts are 8 and 8.

To find *y*-intercepts, set *x* 0. This gives 0^2 y^3 0^2 y^2 64 y^3 64 *y* 4. So the *y*-intercept is 4.

61. (a) *y* x^3 x^2; [2 2] by [1 1]



(b) From the graph, it appears that the *x*-intercepts are 0 and 1 and the *y*-intercept is 0.

(c) To find *x*-intercepts, set *y* 0. This gives 0 x^3 x^2 x^2 *x* 1 0 *x* 0 or 1. So

3 2

-2 -1 1 2

-0.5

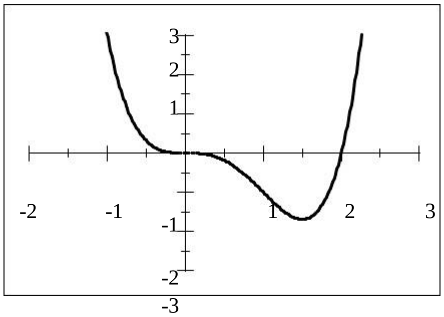
-1.0

2

the x-intercepts are 0 and 1.

To find y-intercepts, set x 0. This gives y 0^3 0^2 0. So the y-intercept is 0.

62. (a) $y \;\; x^4 \;\; 2x^3$; [2 3] by [3 3]

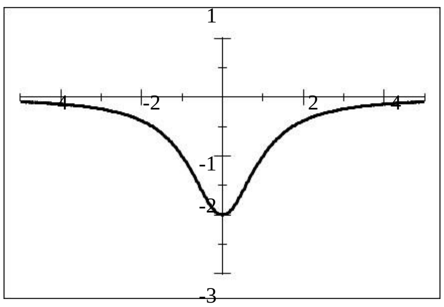


(b) From the graph, it appears that the x-intercepts are 0 and 2 and the y-intercept is 0.

(c) To find x-intercepts, set y 0. This gives $0 \;\; x^4 \;\; 2x^3 \;\; x^3 \;\; x \;\; 2 \;\; 0 \;\; x$ 0 or 2. So the x-intercepts are 0 and 2.

To find y-intercepts, set x 0. This gives $y \;\; 0^4 \;\; 2\,0^3$ 0. So the y-intercept is 0.

63. (a) $y \;\; \frac{2}{x^2 \;\; 1}$; [5 5] by [3 1]

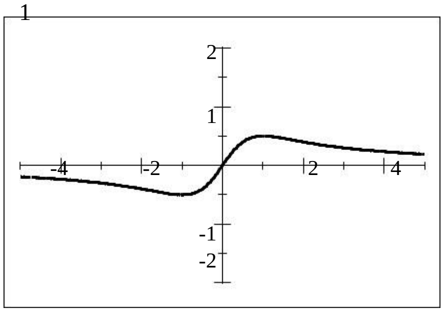


(b) From the graph, it appears that there is no x-intercept and the y-intercept is 2.

(c) To find x-intercepts, set y 0. This gives $0 \;\; \frac{2}{x^2 \;\; 1}$, which has no solution. So there is no x-intercept.

To find y-intercepts, set x 0. This gives $y \;\; \frac{2}{0^2 \;\; 1}$ 2. So the y-intercept is 2.

64. (a) $y \;\; \frac{x}{x^2 \;\; 1}$; [5 5] by [2 2]

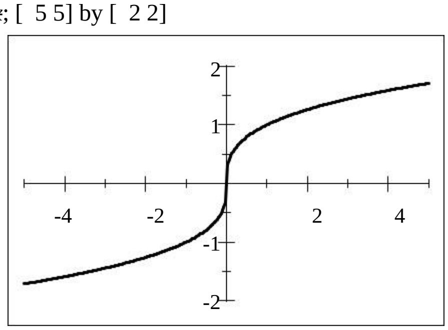


(b) From the graph, it appears that the x- and y-intercepts are 0.

(c) To find x-intercepts, set y 0. This gives $0 \;\; \frac{x}{x^2 \;\; 1} \;\; x \;\; 0$. So the x-intercept is 0.

To find y-intercepts, set x 0. This gives $y \;\; \frac{0}{0^2 \;\; 1}$ 0. So the y-intercept is 0.

65. (a) $y \;\; \sqrt[3]{x}$; [5 5] by [2 2]

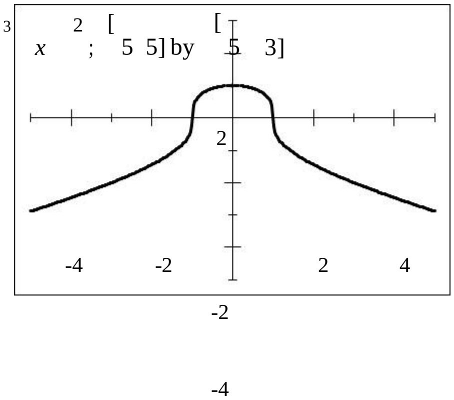


(b) From the graph, it appears that and the x- and y-intercepts are 0.

(c) To find x-intercepts, set y 0. This gives $0 \;\; \sqrt[3]{x} \;\; x$ 0. So the x-intercept is 0.
To find y-intercepts, set x 0. This gives $y \;\; \sqrt[3]{0}$ 0. So the y-intercept is 0.

4 3

66. (a) $y_1 = \sqrt[3]{1 - x^2}$; $[-5, 5]$ by $[-5, 3]$

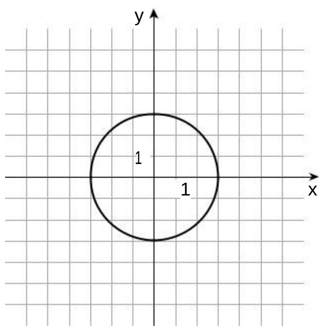


(b) From the graph, it appears that the x-intercepts are -1 and 1 and the y-intercept is 1.

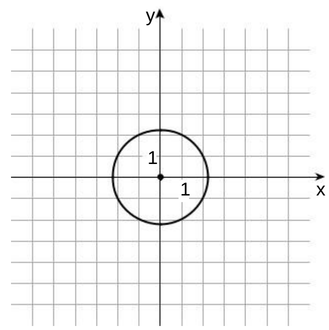
(c) To find x-intercepts, set $y = 0$. This gives $0 = \sqrt[3]{1 - x^2} \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$. So the x-intercepts are -1 and 1.

To find y-intercepts, set $x = 0$. This gives $y = \sqrt[3]{1 - 0^2} = 1$. So the y-intercept is 1.

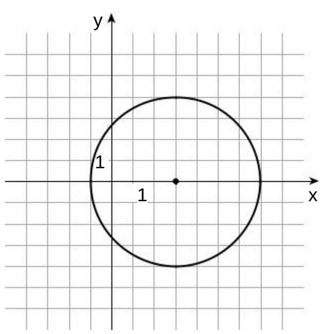
67. $x^2 + y^2 = 9$ has center $(0, 0)$ and radius 3.



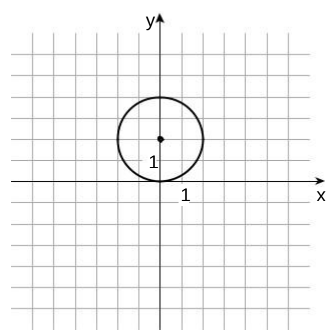
68. $x^2 + y^2 = 5$ has center $(0, 0)$ and radius $\sqrt{5}$.



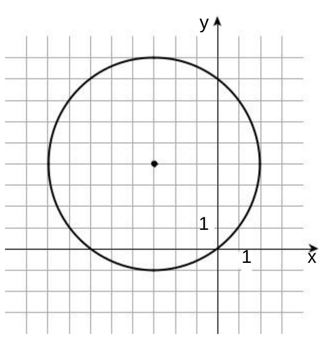
69. $(x - 3)^2 + y^2 = 16$ has center $(3, 0)$ and radius 4.



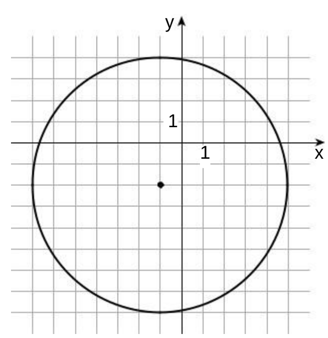
70. $x^2 + (y - 2)^2 = 4$ has center $(0, 2)$ and radius 2.



71. $(x + 3)^2 + (y - 4)^2 = 25$ has center $(-3, 4)$ and radius 5.



72. $(x + 1)^2 + (y + 2)^2 = 36$ has center $(-1, -2)$ and radius 6.



73. Using h 3, k 2, and r 5, we get x 3 ^{2} y 2 ^{2} 5^2 x 3 ^{2} y 2 ^{2} 25.

74. Using h 1, k 3, and r 3, we get x 1 ^{2} y 3 ^{2} 3^2 x 1 ^{2} y 3 ^{2} 9.

75. The equation of a circle centered at the origin is x^2 y^2 r^2. Using the point 4 7 we solve for r^2. This gives

4^2 7^2 r^2 16 49 65 r^2. Thus, the equation of the circle is x^2 y^2 65.

76. Using h 1 and k 5, we get x 1 ^{2} y 5 ^{2} r^2 x 1 ^{2} y 5 ^{2} r^2. Next, using the point

4 6 , we solve for r^2. This gives 4 1 ^{2} 6 5 ^{2} r^2 130 r^2. Thus, an equation of the circle is x 1 ^{2} y 5 ^{2} 130.

77. The center is at the midpoint of the line segment, which is $\frac{1\ 5}{2}$ $\frac{1\ 9}{2}$ 2 5 . The radius is one half the diameter, so r $\frac{1}{2}$ 1 5 ^{2} 1 9 ^{2} $\frac{1}{2}$ 36 64 $\frac{1}{2}$ 100 5. Thus, an equation of the circle is x 2 ^{2} y 5 ^{2} 5^2 x 2 y 5 25.

78. The center is at the midpoint of the line segment, which is $\frac{1\ 7}{2}$ $\frac{3\ 5}{2}$ 3 1 . The radius is one half the diameter, so r $\frac{1}{2}$ 17 ^{2} ^{2} 35 42. Thus, an equation of the circle is x 3 ^{2} y 1 ^{2} 32.

79. Since the circle is tangent to the x-axis, it must contain the point 7 0 , so the radius is the change in the y-coordinates. That is, r 3 0 3. So the equation of the circle is x 7 ^{2} y 3 ^{2} 3^2, which is x 7 ^{2} y 3 ^{2} 9.

80. Since the circle with r 5 lies in the first quadrant and is tangent to both the x-axis and the y-axis, the center of the circle is at 5 5 . Therefore, the equation of the circle is x 5 ^{2} y 5 ^{2} 25.

81. From the figure, the center of the circle is at 2 2 . The radius is the change in the y-coordinates, so r 2 0 2. Thus the equation of the circle is x 2 ^{2} y 2 ^{2} 2^2, which is x 2 ^{2} y 2 ^{2} 4.

82. From the figure, the center of the circle is at 1 1 . The radius is the distance from the center to the point 2 0 . Thus r 1 2 ^{2} ^{2} 10 9 1 10, and the equation of the circle is x 1 ^{2} y 1 ^{2} 10.

x^2 2x 1 y^2 4y 4 1 1 4 x 1 ^{2} y 2 ^{2} 4. Thus, the center is 1 2 , and the radius is 2.

84. Completing the square gives x^2 y^2 2x 2y 2 x^2 2x $\frac{2}{2}^2$ y^2 2y $\frac{2}{2}^2$ 2 $\frac{2}{2}^2$ $\frac{2}{2}^2$ x^2 2x 1 y^2 2y 1 2 1 1 x 1 ^{2} y 1 ^{2} 4. Thus, the center is 1 1 , and the radius is 2.

85. Completing the square gives x^2 y^2 4x 10y 13 0 x^2 4x $\frac{4}{2}^2$ y^2 10y $\frac{10}{2}^2$ 13 $\frac{4}{2}^2$ $\frac{10}{2}^2$

x^2 4x 4 y^2 10y 25 13 4 25 x 2 ^{2} y 5 ^{2} 16.

Thus, the center is 2 5 , and the radius is 4.

Completing the square gives x^2 y^2 6y 2 0 x^2 y^2 6y $\frac{6}{2}^2$ 2 $\frac{6}{2}^2$ x^2 y^2 6y 9 2 9 x^2 y 3 ^{2} 7. Thus, the circle has center 0 3 and radius 7.

2

87. Completing the square gives $x^2 \quad y^2 \quad x \quad 0 \quad x^2 \quad x \quad \frac{1}{2}\ 2 \quad y^2 \quad \frac{1}{2}\ 2 \quad x^2 \quad x \quad \frac{1}{4} \quad y^2 \quad \frac{1}{4}$

$x \quad \frac{1}{2}\ ^2 \quad y^2 \quad \frac{1}{4}$. Thus, the circle has center $\frac{1}{2}\ \ 0$ and radius $\frac{1}{2}$.

Completing the square gives $x^2 \quad y^2 \quad 2x \quad y \quad 1 \quad 0 \quad x^2 \quad 2x \quad \frac{2}{2}\ 2 \quad y \quad y \quad \frac{1}{2}\ 2 \quad \frac{1}{2}\ 1 \quad 1 \quad \frac{1}{2}\ 2$

$2 \quad 2 \quad \frac{1}{} \quad \frac{1}{} \quad 2 \quad \frac{1}{} \quad 2 \quad \frac{1}{} \quad \frac{1}{}$

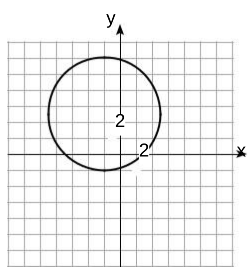
$4\frac{1}{}$. Thus, the circle has center

89. Completing the square gives $x^2 + y^2 - \frac{1}{2}x + \frac{1}{2}y - \frac{1}{8} = 0 \Rightarrow x^2 - \frac{1}{2}x + \frac{1}{16} + y^2 + \frac{1}{2}y + \frac{1}{16} = \frac{1}{8} + \frac{1}{16} + \frac{1}{16} \Rightarrow \left(x - \frac{1}{4}\right)^2 + \left(y + \frac{1}{4}\right)^2 = \frac{1}{4}$. Thus, the circle has center $\left(\frac{1}{4}, -\frac{1}{4}\right)$ and radius $\frac{1}{2}$.

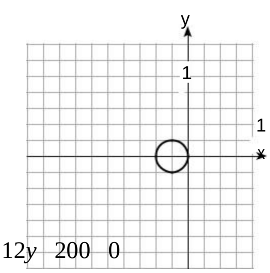
90. Completing the square gives $x^2 + y^2 + \frac{1}{2}x + 2y + \frac{1}{16} = 0 \Rightarrow x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 + y^2 + 2y + 1 = -\frac{1}{16} + \frac{1}{16} + 1 \Rightarrow \left(x + \frac{1}{4}\right)^2 + (y + 1)^2 = 1$. Thus, the circle has center $\left(-\frac{1}{4}, -1\right)$ and radius 1.

91. Completing the square gives $x^2 + y^2 + 4x - 10y = 21 \Rightarrow x^2 + 4x + \left(\frac{4}{2}\right)^2 + y^2 - 10y + \left(\frac{10}{2}\right)^2 = 21 + \left(\frac{4}{2}\right)^2 + \left(\frac{10}{2}\right)^2 \Rightarrow (x + 2)^2 + (y - 5)^2 = 21 + 4 + 25 = 50.$

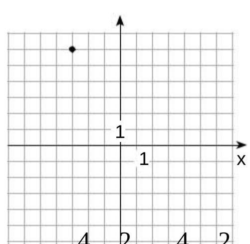
Thus, the circle has center $(-2, 5)$ and radius $\sqrt{50} = 5\sqrt{2}$.



92. First divide by 4, then complete the square. This gives $4x^2 + 4y^2 + 2x = 0 \Rightarrow x^2 + y^2 + \frac{1}{2}x = 0 \Rightarrow x^2 + \frac{1}{2}x + y^2 = 0 \Rightarrow x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 + y^2 = \left(\frac{1}{4}\right)^2 \Rightarrow \left(x + \frac{1}{4}\right)^2 + y^2 = \frac{1}{16}$. Thus, the circle has center $\left(-\frac{1}{4}, 0\right)$ and radius $\frac{1}{4}$.



93. Completing the square gives $x^2 + y^2 + 6x - 12y + 45 = 0 \Rightarrow (x + 3)^2 + (y - 6)^2 = -45 + 9 + 36 = 0$. Thus, the center is $(-3, 6)$, and the radius is 0. This is a degenerate circle whose graph consists only of the point $(-3, 6)$.



94. $x^2 + y^2 + 16x + 12y + 200 = 0 \Rightarrow x^2 + 16x + \left(\frac{16}{2}\right)^2 + y^2 + 12y + \left(\frac{12}{2}\right)^2 = -200 + \left(\frac{16}{2}\right)^2 + \left(\frac{12}{2}\right)^2 \Rightarrow (x + 8)^2 + (y + 6)^2 = -200 + 64 + 36 = -100$. Since completing the square gives $r^2 = -100$, this is not the equation of a circle. There is no graph.

95. x-axis symmetry: $(-y) = x^4 - x^2 \Rightarrow y = -x^4 + x^2$, which is not the same as $y = x^4 - x^2$, so the graph is not symmetric with respect to the x-axis.

y-axis symmetry: $y = (-x)^4 - (-x)^2 = x^4 - x^2$, so the graph is symmetric with respect to the y-axis.

Origin symmetry: $(-y) = (-x)^4 - (-x)^2 \Rightarrow -y = x^4 - x^2$, which is not the same as $y = x^4 - x^2$, so the graph is not symmetric with respect to the origin.

96. x-axis symmetry: $x = (-y)^4 - (-y)^2 = y^4 - y^2$, so the graph is symmetric with respect to the x-axis.

y-axis symmetry: $(-x) = y^4 - y^2$, which is not the same as $x = y^4 - y^2$ ... y-axis.

2 y , so the graph is not symmetric with respect to the

Origin symmetry: $xy^4 y^2 x y^4 y^2$, which is not the same as $x y^4 y^2$, so the graph is not symmetric with respect to the origin.

x-axis symmetry: $y\ x^3\ 10x\ y\ x^3\ 10x$, which is not the same as $y\ x^3\ 10x$, so the graph is not symmetric with respect to the *x*-axis.

3

y-axis symmetry: $y\ x^3\ 10\ x\ y\ x^3\ 10x$, which is not the same as $y\ x\ \ 10x$, so the graph is not symmetric with respect to the *y*-axis.

Origin symmetry: $y\ x^3\ 10\ x\ y\ x^3\ 10x\ y\ x^3\ 10x$, so the graph is symmetric with respect to the origin.

x-axis symmetry: $y\ x^2\ x\ y\ x^2\ x$, which is not the same as $y\ x^2\ x$, so the graph is not symmetric with respect to the *x*-axis.

y-axis symmetry: $yx^2xy\ \ x^2\ \ x$, so the graph is symmetric with respect to the *y*-axis. Note that xx .

Origin symmetry: $y\ x^2\ x\ y\ x^2\ x\ y\ x^2\ x$, which is not the same as $y\ x^2\ x$, so the graph is not symmetric with respect to the origin.
1

x-axis symmetry: $x^4\ \ y^4\ \ x^2\ \ y^2\ \ 1\ \ x^4y^4\ \ x^2y^2\ \ 1$, so the graph is symmetric with respect to the *x*-axis.

y-axis symmetry: $x^4y^4x^2y^2\ \ \ x^4y^4\ \ x^2y^2\ \ 1$, so the graph is symmetric with respect to the *y*-axis.

4 4 2 2

Origin symmetry: $x^4y^4x^2y^2\ 1\ x\ \ y\ \ x\ \ y\ \ 1$, so the graph is symmetric with respect to the origin.

x

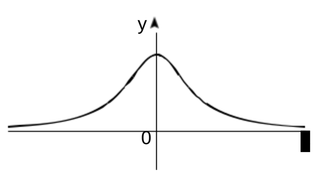
x-axis symmetry: $x^2y^2\ x\ y\ 1\ x^2y^2\ x\ y\ 1$, which is not the same as $^2y^2\ x\ y\ 1$, so the graph is not symmetric with respect to the *x*-axis.

2 2 2 2 2

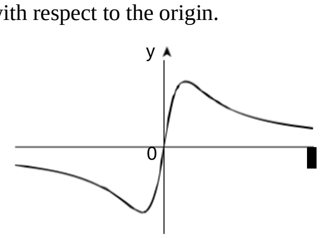
y-axis symmetry: $x^2y\ \ x\ y\ 1\ x\ \ y\ \ x\ y\ 1$, which is not the same as $x\ \ y\ \ x\ y\ 1$, so the graph is not symmetric with respect to the *y*-axis.

Origin symmetry: $x^2\ \ y^2xy\ \ 1\ \ x^2y^2\ \ x\ y\ \ 1$, so the graph is symmetric with respect to the origin.

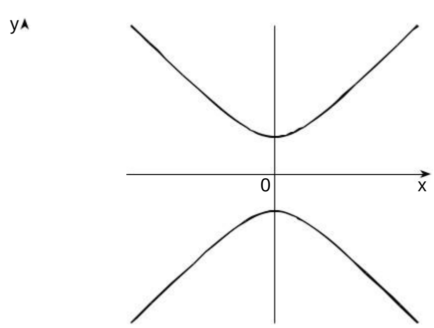
101. Symmetric with respect to the *y*-axis.



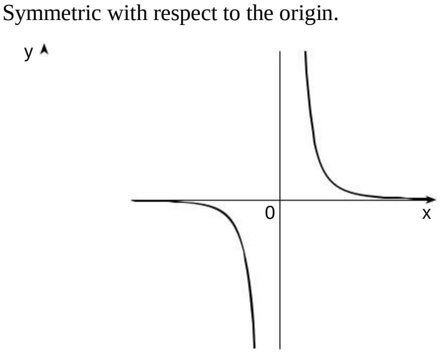
Symmetric with respect to the origin.



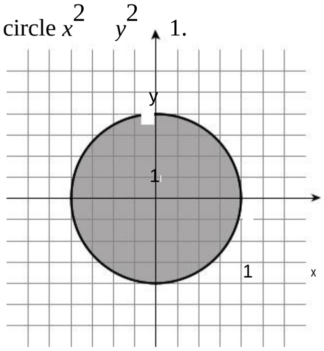
102. Symmetric with respect to the *x*-axis.



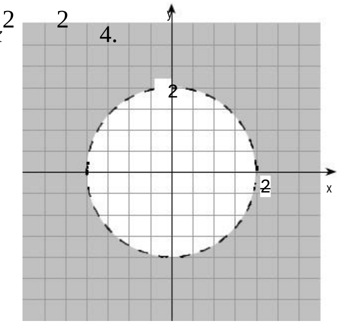
Symmetric with respect to the origin.



$x\ y\ \ x^2\ \ y^2\ \ 1$. This is the set of points inside (and on) the circle $x^2\ \ y^2\ \ 1$.



$x\ y\ \ x^2\ \ y^2\ \ 4$. This is the set of points outside the circle $x^2\ \ y^2\ \ 4$.

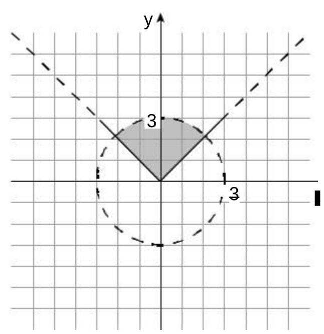


107. Completing the square gives $x^2\ \ y^2\ \ 4y\ \ 12\ \ 0$

$$x^2\quad y^2\quad 4y\quad \frac{4}{2}^2\quad 12\quad \frac{4}{2}^2$$

$x^2\ y\ 2^2$ 16. Thus, the center is 0 2 , and the radius is 4. So the circle $x^2\ y^2$ 4, with center 0 0 and radius 2 sits completely inside the larger circle. Thus, the area is $4^2\ 2^2$ 16 4 12 .

108. This is the top quarter of the circle of radius 3. Thus, the area is $4^1\ 9\frac{9}{4}$.



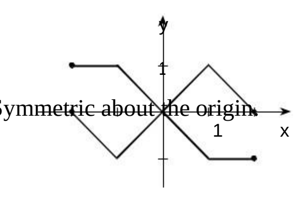
(a) The point 5 3 is shifted to 5 3 3 28 5 .

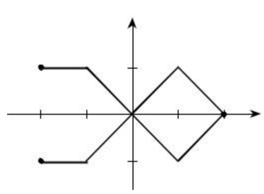
The point $a\ b$ is shifted to $a\ \ 3\ b\ \ 2$.

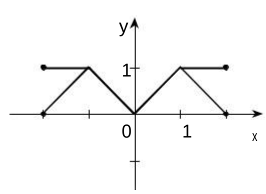
Let $x\ y$ be the point that is shifted to 3 4 . Then x 3 y 23 4 . Setting the x-coordinates equal, we get x 3 3 x 0. Setting the y-coordinates equal, we get y 2 4 y 2 So the point is 0 2 .

A 5 1 , so A 5 3 1 2 2 1 ; B 3 2 , so B 3 3 2 2 0 4 ; and C 2 1 , so C 2 3 1 2 5 3 .

110. (a) Symmetric about the x-axis. **(b)** Symmetric about the y-axis. Symmetric about the origin.







(a) In 1980 inflation was 14%; in 1990, it was 6%; in 1999, it was 2%.

Inflation exceeded 6% from 1975 to 1976 and from 1978 to 1982.

Between 1980 and 1985 the inflation rate generally decreased. Between 1987 and 1992 the inflation rate generally increased.

The highest rate was about 14% in 1980. The lowest was about 1% in 2002.

(a) Closest: 2 Mm. Farthest: 8 Mm.

(b) When y 2 we have $\frac{x \; 3^2}{25}$ $\frac{2^2}{16}$ 1 $\frac{x \; 3^2}{25}$ $\frac{1}{4}$ 1 $\frac{x \; 3^2}{25}$ $\frac{3}{4}$ x 3 ^{2} $\frac{75}{4}$. Taking the square root of both sides we get x 3 $\frac{75}{4}$ $\frac{5}{2}$ $\sqrt{3}$ x 3 $\frac{5}{2}\sqrt{3}$. So x 3 $\frac{5}{2}\sqrt{3}$ 1 33 or x 3 $\frac{5}{2}\sqrt{3}$ 7 33. The distance from 1 33 2 to the center 0 0 is d $\sqrt{1\,33 \quad 0^2 \quad 2 \quad 0^2}$ $\sqrt{5\,7689}$ 2 40. The distance from 7 33 2 to the center 0 0 is d $\sqrt{7\,33 \quad 0^2 \quad 2 \quad 0^2}$ $\sqrt{57\,7307}$ 7 60.

Completing the square gives x^2 y^2 ax by c 0 x^2 ax $\frac{a}{2}^2$ y^2 by $\frac{b}{2}^2$ c $\frac{a}{2}^2$ $\frac{b}{2}^2$ x $\frac{a}{2}^2$ y $\frac{b}{2}^2$ c $\frac{a^2 \; b^2}{4}$. This equation represents a circle only when c $\frac{a^2 \; b^2}{4}$ 0. This equation represents a point when c $\frac{a^2 \; b^2}{4}$ 0, and this equation represents the empty set when c $\frac{a^2 \; b^2}{4}$ 0.

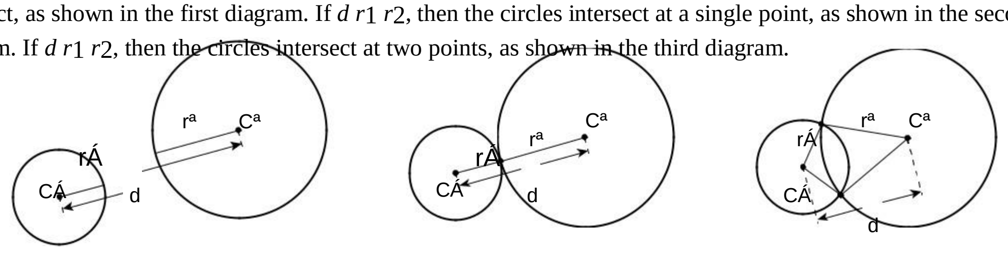
When the equation represents a circle, the center is $\frac{a}{2}$ $\frac{b}{2}$, and the radius is $\sqrt{c \quad \frac{a^2 \; b^2}{4}}$ $\frac{1}{2}\sqrt{a^2 \; b^2 \quad 4ac}$.

114. (a) (i) x 2 ^{2} y 1 ^{2} 9, the center is at 2 1 , and the radius is 3. x 6 ^{2} y 4 ^{2} 16, the center is at 6 4 , and the radius is 4. The distance between centers is $\sqrt{2 \; 6^2 \quad 1 \; 4^2}$ $\sqrt{4^2 \; 3^2}$ $\sqrt{16 \quad 9}$ $\sqrt{25}$ 5. Since 5 3 4, these circles intersect.

(ii) x^2 y 2 ^{2} 4, the center is at 0 2 , and the radius is 2. x 5 ^{2} y 14 ^{2} 9, the center is at 5 14 , and the radius is 3. The distance between centers is $\sqrt{0 \; 5^2 \; 2 \; 14^2}$ $\sqrt{5^2 \; 12^2}$ $\sqrt{25 \quad 144}$ $\sqrt{169}$ 13. Since 13 2 3, these circles do not intersect.

(iii) x 3 ^{2} y 1 ^{2} 1, the center is at 3 1 , and the radius is 1. x 2 ^{2} y 2 ^{2} 25, the center is at 2 2 , and the radius is 5. The distance between centers is $\sqrt{3 \; 2^2 \quad 1 \; 2^2}$ $\sqrt{1^2 \; 3^2}$ $\sqrt{1 \quad 9}$ $\sqrt{10}$. Since $\sqrt{10}$ 1 5, these circles intersect.

If the distance d between the centers of the circles is greater than the sum r_1 r_2 of their radii, then the circles do not intersect, as shown in the first diagram. If d r_1 r_2, then the circles intersect at a single point, as shown in the second diagram. If d r_1 r_2, then the circles intersect at two points, as shown in the third diagram.



Case 1 d r_1 r_2 **Case 2** d r_1 r_2 **Case 3** d r_1 r_2

1.3 LINES

1. We find the "steepness" or slope of a line passing through two points by dividing the difference in the y-coordinates of these points by the difference in the x-coordinates. So the line passing through the points 0 1 and 2 5 has slope

$$\frac{5 \quad 1}{2 \quad 0} \quad 2.$$

(a) The line with equation $y \quad 3x \quad 2$ has slope 3.

Any line parallel to this line has slope 3.

Any line perpendicular to this line has slope $\frac{1}{3}$.

The point-slope form of the equation of the line with slope 3 passing through the point 1 2 is y 2 3 x 1 .

For the linear equation 2x 3y 12 0, the x-intercept is 6 and the y-intercept is 4. The equation in slope-intercept form is y $\frac{2}{3}$ x 4. The slope of the graph of this equation is $\frac{2}{3}$.

The slope of a horizontal line is 0. The equation of the horizontal line passing through 2 3 is y 3.

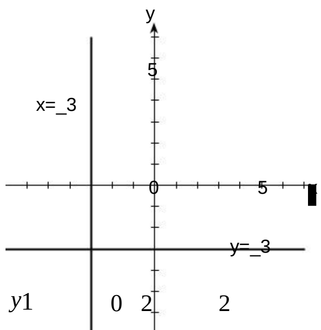
The slope of a vertical line is undefined. The equation of the vertical line passing through 2 3 is x 2.

(a) Yes, the graph of y3 is a horizontal line 3 units below the x-axis.

Yes, the graph of x3 is a vertical line 3 units to the left of the y-axis.

No, a line perpendicular to a horizontal line is vertical and has undefined slope.

Yes, a line perpendicular to a vertical line is horizontal and has slope 0.

8. 

Yes, the graphs of y3 and x3 are perpendicular lines.

9. $m \frac{y_2 \; y_1}{x_2 \; x_1}$ 0 2 / 0 1 1 $\frac{2}{2}$ 2

10. $m \frac{y_2 \; y_1}{x_2 \; x_1}$ $\frac{1 \; 0}{3 \; 0}$ 3 $\frac{1}{}$ $\frac{\_1}{3}$

11. $m \frac{y_2 \; y_1}{x_2 \; x_1}$ $\frac{12}{7 \; 2}$ $\frac{1}{5}$

12. $m \frac{y_2 \; y_1}{x_2 \; x_1}$ $\frac{2 \; 1}{3 \; 5 \; 3}$ $\frac{3}{}$ $\frac{\_3}{8}$

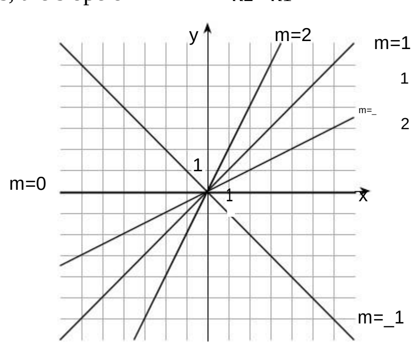
13. $m \frac{y_2 \; y_1}{x_2 \; x_1}$ $\frac{4 \; 4}{0 \; 5}$ 0

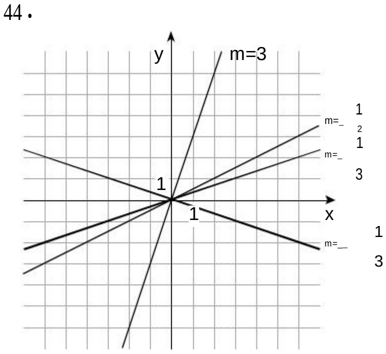
14. $m \frac{y_2 \; y_1}{x_2 \; x_1}$ $\frac{1 \; 3}{1 \; 4}$ $\frac{4}{3}$ $\frac{4}{3}$

15. $m \frac{y_2 \; y_1}{x_2 \; x_1}$ $\frac{52}{6 \; 10}$ $\frac{3}{4}$ $\frac{3}{4}$

16. $m \frac{y_2 \; y_1}{x_2 \; x_1}$ $\frac{22}{6 \; 3}$ 0

17. For 1, we find two points, 1 2 and 0 0 that lie on the line. Thus the slope of 1 is $m \frac{y_2 \; y_1}{x_2 \; x_1}$ $\frac{2 \; 0}{1 \; 0}$ 2. For 2, we find two points 0 2 and 2 3 . Thus, the slope of 2 is $m \frac{y_2 \; y_1}{x_2 \; x_1}$ $\frac{3 \; 2}{2 \; 0}$ $\frac{1}{2}$. For 3 we find the points 2 2 and 3 1 . Thus, the slope of 3 is $m \frac{y_2 \; y_1}{x_2 \; x_1}$ $\frac{}{3 \; 2}$ 3. For 4, we find the points 2 1 and 2 2 . Thus, the slope of 4 is $m \frac{y_2 \; y_1}{x_2 \; x_1}$ $\frac{21}{22}$ $\frac{1}{4}$ $\frac{\_1}{4}$.

18. **(a)** 

(b) 

19. First we find two points 0 4 and 4 0 that lie on the line. So the slope is m $\frac{0\ 4}{4\ 0}$ 1. Since the y-intercept is 4, the equation of the line is y mx b1x 4. So $y$$x$ 4, or x y 4 0.

20. We find two points on the graph, 0 4 and 2 0 . So the slope is m $\frac{0\ 4}{2\ 0}$ 2. Since the y-intercept is 4, the equation of the line is y mx b 2x 4, so y 2x 4 2x y 4 0.

21. We choose the two intercepts as points, 0 3 and 20 . So the slope is m $\frac{03}{2\ 0}$ $\frac{3}{2}$. Since the y-intercept is 3, the equation of the line is y mx b $\frac{3}{2}$ x 3, or 3x 2y 6 0.

We choose the two intercepts, 0 4 and 3 0 . So the slope is m $\frac{04}{3\ 0}$ $\frac{4}{3}$. Since the y-intercept is 4, the equation of the line is y mx b $\frac{4}{3}$ x 4 4x 3y 12 0.

Using y mx b, we have y 3x2 or 3x y 2 0.

Using y mx b, we have y $\frac{2}{5}$ x 4 2x 5y 20 0.

Using the equation y y_1 m x x_1 , we get y 3 5 x 25x y7 5x y 7 0.

Using the equation y y_1 m x x_1 , we get y 41 x2y 4x 2 x y 2 0.

Using the equation y y_1 m x x_1 , we get y 7 $\frac{2}{3}$ x 1 3y 21 2x 2 2x 3y 19 2x 3y 19 0.

Using the equation y y_1 m x x_1 , we get y5 $\frac{7}{2}$ x 3 2y 10 7x 21 7x 2y 31 0.

29. First we find the slope, which is m $\frac{y_2\ y_1}{x_2\ x_1}$ $\frac{6\ 1}{1\ 2}$ $\frac{5}{1}$ 5. Substituting into y y_1 m x x_1 , we get y 6 5 x 1 y 6 5x 5 5x y 11 0.

First we find the slope, which is m $\frac{y_2\ y_1}{x_2\ x_1}$ $\frac{3\ 2}{4\ 1}$ $\frac{5}{5}$ 1. Substituting into y y_1 m x x_1 , we get y 3 1 x 4y 3 x 4 x y 1 0.

31. We are given two points, 2 5 and 1 3 . Thus, the slope is m $\frac{y_2\ y_1}{x_2\ x_1}$ $\frac{3\ 5}{12}$ $\frac{8}{1}$ 8. Substituting into y y_1 m x x_1 , we get y 58 [x2] y8x 11 or 8x y 11 0.

32. We are given two points, 1 7 and 4 7 . Thus, the slope is m $\frac{y_2\ y_1}{x_2\ x_1}$ $\frac{7\ 7}{4\ 1}$ 0. Substituting into y y_1 m x x_1 , we get y 7 0 x 1 y 7 or y 7 0.

We are given two points, 1 0 and 0 3 . Thus, the slope is m $\frac{y_2\ y_1}{x_2\ x_1}$ $\frac{3\ 0}{10}$ $\frac{3}{11}$ 3. Using the y-intercept, we have y 3x3 or y 3x 3 or 3x y 3 0.

34. We are given two points, 8 0 and 0 6 . Thus, the slope is m $\frac{y_2\ y_1}{x_2\ x_1}$ $\frac{6\ 0}{08}$ $\frac{6}{8}$ $\frac{3}{4}$. Using the y-intercept we have y $\frac{3}{4}$ x 6 3x 4y 24 0.

Since the equation of a line with slope 0 passing through a b is y b, the equation of this line is y 3.

Since the equation of a line with undefined slope passing through a b is x a, the equation of this line is x1.

Since the equation of a line with undefined slope passing through a b is x a, the equation of this line is x 2.

Since the equation of a line with slope 0 passing through a b is y b, the equation of this line is y 1.

Any line parallel to y 3x 5 has slope 3. The desired line passes through 1 2 , so substituting into y y_1 m x x_1 , we get y 2 3 x 1 y 3x 1 or 3x y 1 0.

40. Any line perpendicular to y $\frac{1}{2}$ x 7 has slope $\frac{1}{1\ 2}$ 2. The desired line passes through 3 2 , so substituting into $y \quad y_1 \quad m\,x \quad x_1$, we get $y \quad 2 \quad 2\,[x \quad 3\,] \quad y \quad 2x \quad 8$ or $2x \quad y \quad 8 \quad 0$.

Since the equation of a horizontal line passing through a b is y b, the equation of the horizontal line passing through 4 5 is y 5.

Any line parallel to the y-axis has undefined slope and an equation of the form x a. Since the graph of the line passes through the point 4 5 , the equation of the line is x 4.

43. Since x 2y 6 2yx 6 y $\frac{1}{2}x$ 3, the slope of this line is $\frac{1}{2}$. Thus, the line we seek is given by y6 $\frac{1}{2}x$ 12y 12x 1 x 2y 11 0.

44. Since 2x 3y 4 0 3y2x 4 y $\frac{2}{3}x$ $\frac{4}{3}$, the slope of this line is m $\frac{2}{3}$. Substituting m $\frac{2}{3}$ and b 6 into the slope-intercept formula, the line we seek is given by y $\frac{2}{3}x$ 6 2x 3y 18 0.

Any line parallel to x 5 has undefined slope and an equation of the form x a. Thus, an equation of the line is x1.

Any line perpendicular to y 1 has undefined slope and an equation of the form x a. Since the graph of the line passes through the point 2 6 , an equation of the line is x 2.

First find the slope of 2x 5y 8 0. This gives 2x 5y 8 0 5y2x 8 y $\frac{2}{5}x$ $\frac{8}{5}$. So the slope of the line that is perpendicular to 2x 5y 8 0 is m $\frac{1}{2\,5}$ $\frac{5}{2}$. The equation of the line we seek is y 2 $\frac{5}{2}x$ 1 2y 4 5x 5 5x 2y 1 0.

First find the slope of the line 4x 8y 1. This gives 4x 8y 18y4x 1 y $\frac{1}{2}x$ $\frac{1}{8}$. So the slope of the

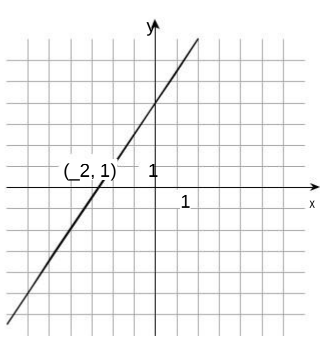
$\frac{2}{3}$2 x $\frac{1}{2}$1 linethatisperpendicularto4x8y1is$m$122.Theequationofthelineweseekis$y$

y $\frac{2}{3}$ 2x 1 6x 3y 1 0.

49. First find the slope of the line passing through 2 5 and 2 1 . This gives m $\frac{1\;5}{2\;2}$ $\frac{4}{4}$ 1, and so the equation of the line we seek is y 7 1 x 1x y 6 0.

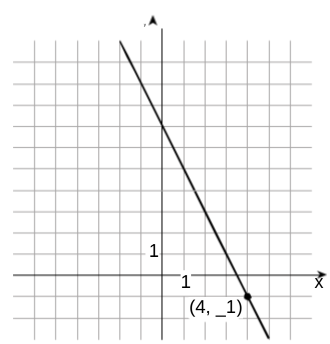
50. First find the slope of the line passing through 1 1 and 5 1 . This gives m $\frac{1\;1}{5\;1}$ $\frac{2}{4}$ $\frac{1}{2}$, and so the slope of the line that is perpendicular is m $\frac{1}{1\;2}$ 2. Thus the equation of the line we seek is y 11 2 x 2 2x y 7 0.

51. (a)



y 1 $\frac{3}{2}$ x22y 2 3 x 2

52. (a)

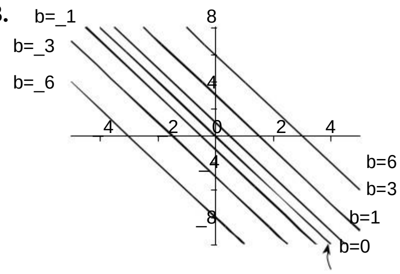


(b) y 1 2 x 4 y 1 2x 8

2*y* 2 3*x* 6 3*x* 2*y* 8 0.

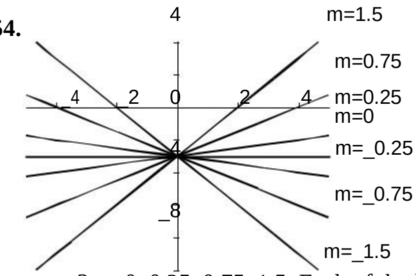
2*x* *y* 7 0.

53.



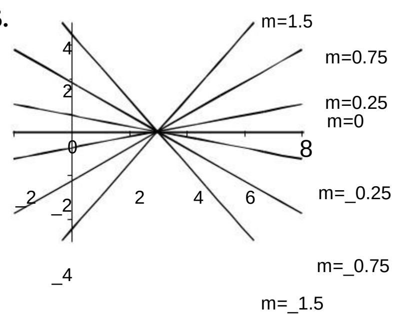
*y*2*x* *b*, *b* 0, 1, 3, 6. They have the same slope, so they are parallel.

54.



y mx 3, *m* 0, 0 25, 0 75, 1 5. Each of the lines contains the point 0 3 because the point 0 3 satisfies each equation *y mx* 3. Since 0 3 is on the *y*-axis, they all have the same *y*-intercept.

55.

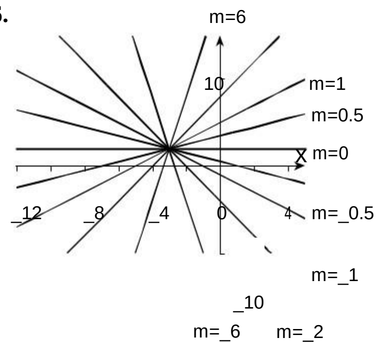


m x 3 , *m* 0, 0 25, 0 75, 1 5. Each of the lines contains

the point 3 0 because the point 3 0 satisfies each equation *y m x* 3 . Since 3 0 is on the *x*-axis, we could also say that they all have the same

-intercept.

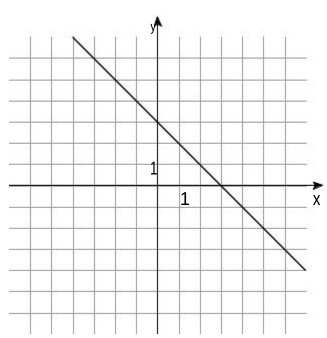
56.



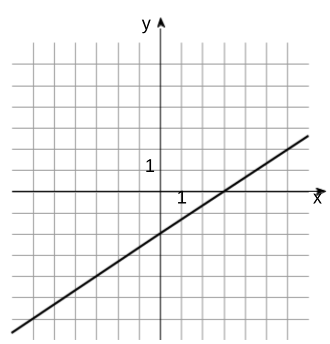
2 *m x* 3 , *m* 0, 0 5, 1, 2, 6. Each of the lines contains the point 3 2 because the point 3 2 satisfies each equation *y* 2 *m x* 3 .

y 3 *x x* 3. So the slope is 1 and the *y*-intercept is

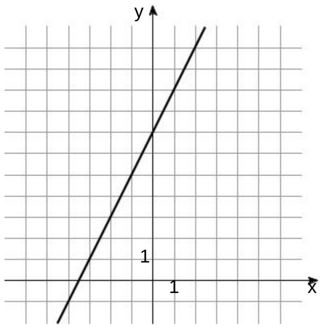
3.



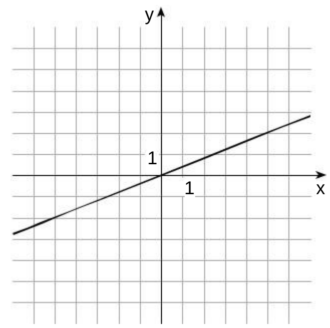
58. *y* $\frac{2}{3}$ *x* 2. So the slope is $\frac{2}{3}$ and the *y*-intercept is 2.



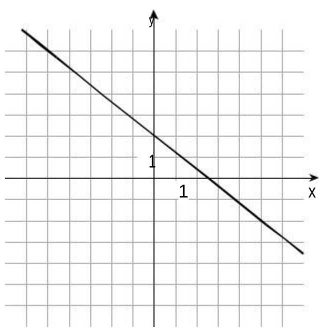
59. $2x \quad y \quad 7 \quad y \quad 2x \quad 7$. So the slope is 2 and the *y*-intercept is 7.



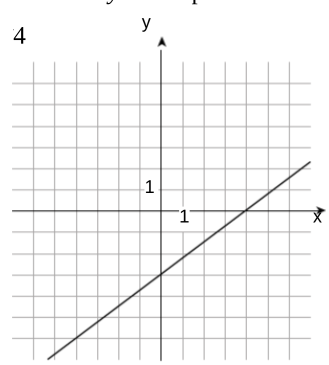
60. $2x \quad 5y \quad 0 5y 2x \quad y \quad \frac{2}{5} x$. So the slope is $\frac{2}{5}$ and the *y*-intercept is 0.



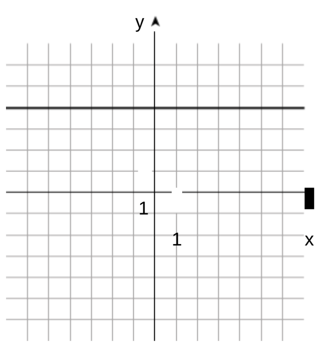
$4x$ $5y$ 10 $5y$ $4x$ 10 y $\frac{4}{5}x$ 2. So the slope is $\frac{4}{5}$ and the *y*-intercept is 2.



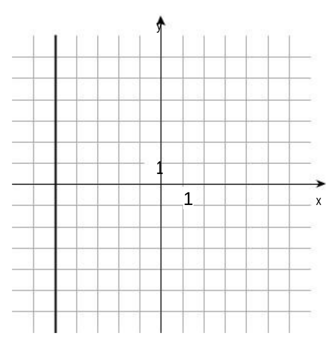
62. $3x \quad 4y \quad 12 \quad 4y \quad 3x \quad 12 \quad y \quad \frac{3}{4}x$ 3. So the slope is $\frac{3}{4}$ and the *y*-intercept is 3.



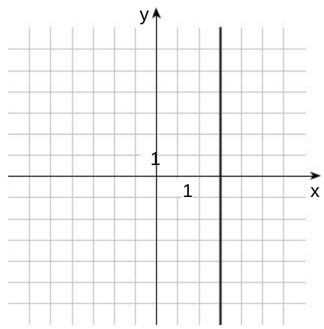
y 4 can also be expressed as y $0x$ 4. So the slope is 0 and the *y*-intercept is 4.



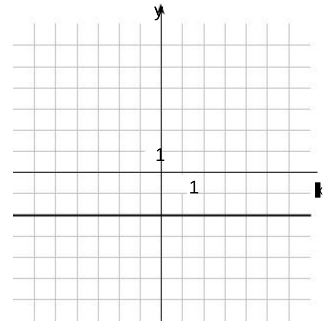
x 5 cannot be expressed in the form y mx b. So the slope is undefined, and there is no *y*-intercept. This is a vertical line.



x 3 cannot be expressed in the form *y* *mx* *b*. So the slope is undefined, and there is no *y*-intercept. This is a vertical line.



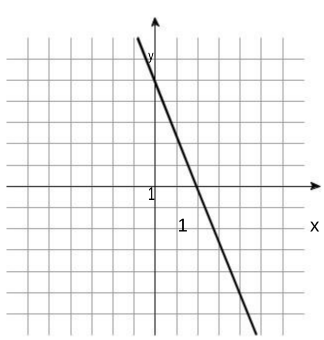
66. *y*2 can also be expressed as *y* 0*x* 2. So the slope is 0 and the *y*-intercept is 2.



5*x* 2*y* 10 0. To find *x*-intercepts, we set *y* 0 and solve for *x*: 5*x* 2 0 10 0 5*x* 10 *x* 2, so the *x*-intercept is 2.

To find *y*-intercepts, we set *x* 0 and solve for *y*:

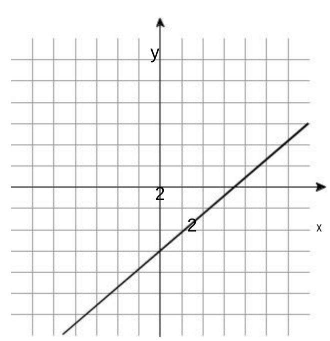
5 0 2*y* 10 0 2*y* 10 *y* 5, so the *y*-intercept is 5.



68. 6*x* 7*y* 42 0. To find *x*-intercepts, we set *y* 0 and solve for *x*: 6*x* 7 0 42 0 6*x* 42 *x* 7, so the *x*-intercept is 7.

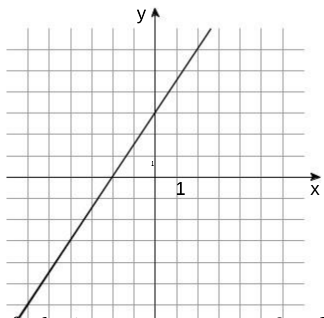
To find *y*-intercepts, we set *x* 0 and solve for *y*:

6 0 7*y* 42 0 7*y* 42 *y* 6, so the *y*-intercept is 6.



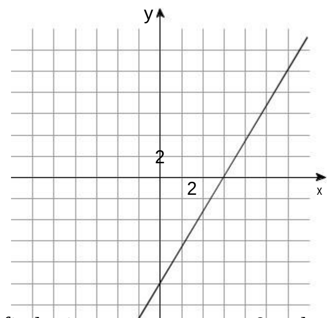
$\frac{1}{2}x \; \frac{1}{3}y \; 1 \; 0$. To find x-intercepts, we set $y \; 0$ and solve for x: $\frac{1}{2}x \; \frac{1}{3}0 \; 1 \; 0 \; \frac{1}{2}x 1 \; x 2$, so the x-intercept is 2.

To find y-intercepts, we set $x \; 0$ and solve for y: $\frac{1}{2}0 \; \frac{1}{3}y \; 1 \; 0 \; \frac{1}{3} \; y \; 1 \; y \; 3$, so the y-intercept is 3.



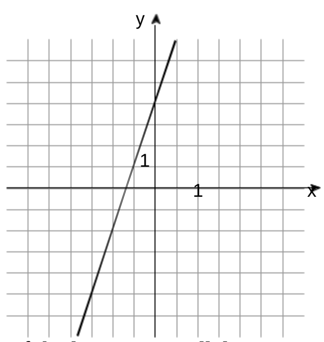
70. $\frac{1}{3}x \; \frac{1}{5}y \; 2 \; 0$. To find x-intercepts, we set $y \; 0$ and solve for x: $\frac{1}{3}x \; \frac{1}{5}0 \; 2 \; 0 \; \frac{1}{3}x \; 2 \; x \; 6$, so the x-intercept is 6.

To find y-intercepts, we set $x \; 0$ and solve for y: $\frac{1}{3}0 \; \frac{1}{5}y \; 2 \; 0 \; \frac{1}{5}y 2 \; y 10$, so the y-intercept is 10.



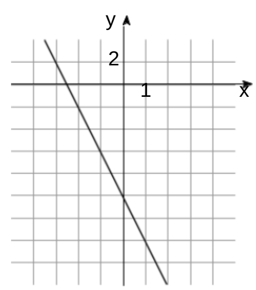
$y \; 6x \; 4$. To find x-intercepts, we set $y \; 0$ and solve for x: $0 \; 6x \; 4 \; 6x \; 4 \; x \; \frac{2}{3}$, so the x-intercept is $\frac{2}{3}$.

To find y-intercepts, we set $x \; 0$ and solve for y: $6 \; 0 \; 4 \; 4$, so the y-intercept is 4.



$y \; 4x \; 10$. To find x-intercepts, we set $y \; 0$ and solve for x: $0 \; 4x \; 10 \; 4x \; 10 \; x \; \frac{5}{2}$, so the x-intercept is $\frac{5}{2}$.

To find y-intercepts, we set $x \; 0$ and solve for y: $4 \; 0 \; 10 10$, so the y-intercept is 10.



To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $y \; 2x \; 3$ has slope 2. The line with equation $2y \; 4x \; 5 \; 0 \; 2y \; 4x \; 5 \; y \; 2x \; \frac{5}{2}$ also has slope 2, and so the lines are parallel.

To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $y \; \frac{1}{2}x \; 4$ has slope $\frac{1}{2}$. The line with equation $2x \; 4y \; 1 \; 4y \; 2x \; 1 \; y \; \frac{1}{2}x \; \frac{1}{4}$ has slope $\frac{1}{2} \; 1 \; \frac{1}{2}$, and so the lines are neither parallel nor perpendicular.

To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $3x \; 4y \; 4$ $4y \; 3x \; 4 \; y \; \frac{3}{4}x \; 1$ has slope $\frac{3}{4}$. The line with equation $4x \; 3y \; 5 \; 3y \; 4x \; 5 \; y \; \frac{4}{3}x \; \frac{5}{3}$ has slope $\frac{4}{3} \; 3 \; \frac{1}{4}$, and so the lines are perpendicular.

To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $2x \; 3y \; 10$ $3y \; 2x \; 10 \; y \; \frac{2}{3}x \; \frac{10}{3}$ has slope $\frac{2}{3}$. The line with equation $3y \; 2x \; 7 \; 0 \; 3y \; 2x \; 7 \; y \; \frac{2}{3}x \; \frac{7}{3}$ also has slope $\frac{2}{3}$, and so the lines are parallel.

To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $7x$ $3y$ 2

– – -3 . The line with equation $9y$ $21x$ 1 $9y$ $21x$ 1 y – -9 has slope -3 $\frac{1}{3}$, and so the lines are

neither parallel nor perpendicular.

To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $6y$ $2x$ 5

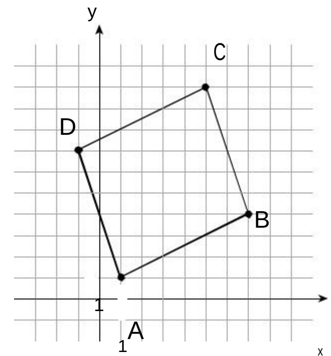
$6y$ $2x$ 5 y $\frac{1}{3}x$ $\frac{5}{6}$ has slope $\frac{1}{3}$. The line with equation $2y$ $6x$ 1 $2y6x$ 1 $y3x$ $\frac{1}{2}$ has

slope 3 $1\frac{1}{3}$, and so the lines are perpendicular.

79. We first plot the points to find the pairs of points that determine each side. Next we find the slopes of opposite sides. The slope of AB is 4 1 3 1 , and the 7162

slope of DC is 10 7 3 1 516 2 Since these slope are equal, these two sides are parallel. The slope of AD is 7 1 6 3, and the slope of BC is 1 1 2

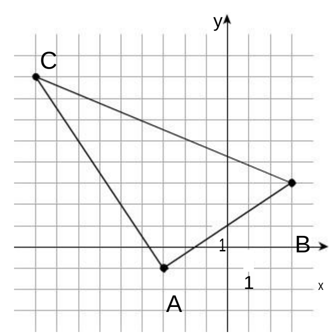
10 4 6 5 7 23. Since these slope are equal, these two sides are parallel. Hence ABC D is a parallelogram.



80. We first plot the points to determine the perpendicular sides. Next find the slopes of the sides. The slope of AB is 31 4 2 , and the slope of AC is

81 9362 . Since 9 3 3363

slope of ABslope of AC $3\frac{2}{2}\frac{3}{}$ 1 the sides are perpendicular, and ABC is a right triangle.



81. We first plot the points to find the pairs of points that determine each side. Next we find the slopes of opposite sides. The slope of AB is 3 1 111105 2 1 and the

slope of DC is 6 8 0 10 2 10 1 5 . Since these slope are equal, these two sides

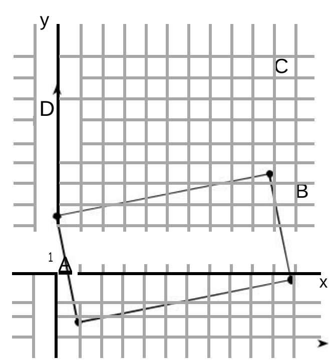
are parallel. Slope of AD is 6 1 5 5, and the slope of BC is

3 8 5 0 1 1

11 10 1 5. Since these slope are equal, these two sides are parallel.

Since slope of ABslope of AD $\frac{1}{5}$ 51, the first two sides are

each perpendicular to the second two sides. So the sides form a rectangle.



9 1 8

82. (a) The slope of the line passing through 1 1 and 3 9 is 3 1 2 4. The slope of the line passing through 1 1

2
1

1 20

and 6 21 is 6 1 5 4. Since the slopes are equal, the points are collinear.

(b) The slope of the line passing through 1 3 and 1 7 is $\frac{7 \quad 3}{1 \quad 1}$ $\frac{4}{2}$ 2. The slope of the line passing through 1 3 and 4 15 is $\frac{15 \quad 3}{4 \quad 1}$ $\frac{12}{5}$. Since the slopes are not equal, the points are not collinear.

83. We need the slope and the midpoint of the line *AB*. The midpoint of *AB* is $\frac{1 \quad 7}{2}$ $\frac{4 \quad 2}{2}$ 4 1 , and the slope of *AB* is m $\frac{2\,4}{7 \quad 1}$ $\frac{6}{6}$ 1. The slope of the perpendicular bisector will have slope $\frac{1}{m}$ $\frac{1}{1}$ 1. Using the point-slope form, the equation of the perpendicular bisector is y 1 1 x 4 or x y 3 0.

84. We find the intercepts (the length of the sides). When x 0, we have 2y 3 0 6 0 2y 6 y 3, and when 0, we have 2 0 3x 6 0 3x 6 x 2. Thus, the area of the triangle is $\frac{1}{2}$ 3 2 3.

85. (a) We start with the two points a 0 and 0 b . The slope of the line that contains them is $\frac{b \quad 0}{0 \quad a}$ $\frac{b}{a}$. So the equation of the line containing them is y $\frac{b}{a}x$ b (using the slope-intercept form). Dividing by b (since b 0) gives $\frac{y}{b}$ $\frac{x}{a}$ 1 $\frac{x}{a}$ $\frac{y}{b}$ 1.

Setting a 6 and b 8, we get $\frac{x}{6}$ $\frac{y}{8}$ 1 4x 3y 24 4x 3y 24 0.

(a) The line tangent at 3 4 will be perpendicular to the line passing through the points 0 0 and 3 4 . The slope of this line is $\frac{4 \quad 0}{3 \quad 0}$ $\frac{4}{3}$. Thus, the slope of the tangent line will be $\frac{1}{4\,3}$ $\frac{3}{4}$. Then the equation of the tangent line is y 4 $\frac{3}{4}x$ 3 4y 4 3x 3 3x 4y 25 0.

Since diametrically opposite points on the circle have parallel tangent lines, the other point is 3 4 .

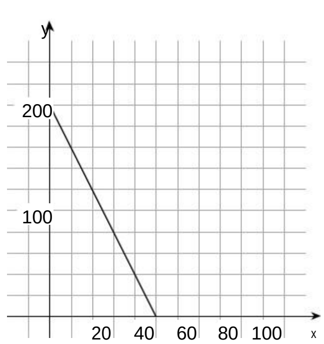
(a) The slope represents an increase of 0 02 C every year. The T -intercept is the average surface temperature in 1950, or 15 C.

In 2050, t 2050 1950 100, so T 0 02 100 15 17 degrees Celsius.

(a) The slope is 0 0417D 0 0417 200 8 34. It represents the increase in dosage for each one-year increase in the child's age.

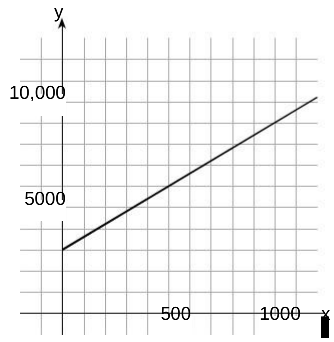
When a 0, c 8 34 0 1 8 34 mg.

89. (a)



The slope, 4, represents the decline in number of spaces sold for each \$1 increase in rent. The y-intercept is the number of spaces at the flea market, 200, and the x-intercept is the cost per space when the manager rents no spaces, \$50.

90. (a)



The slope is the cost per toaster oven, \$6. The *y*-intercept, \$3000, is the monthly fixed cost—the cost that is incurred no matter how many toaster ovens are produced.

91. (a)

C	30	20	10	0	10	20	30
F	22	4	14	32	50	68	86

(b) Substituting *a* for both F and C, we have $a \; \frac{9}{5} a \; 32 \; \frac{4}{5} a \; 32$

$a 40$. Thus both scales agree at 40 .

92. (a) Using *n* in place of *x* and *t* in place of *y*, we find that the slope is $\frac{t_2 \; t_1}{n_2 \; n_1} \; \frac{80 \; 70}{168 \; 120} \; \frac{10}{48} \; \frac{5}{24}$. So the linear equation is $t \; 80 \; \frac{5}{24} n \; 168 \; t \; 80 \; \frac{5}{24} n \; 35 \; t \; \frac{5}{24} n \; 45$.

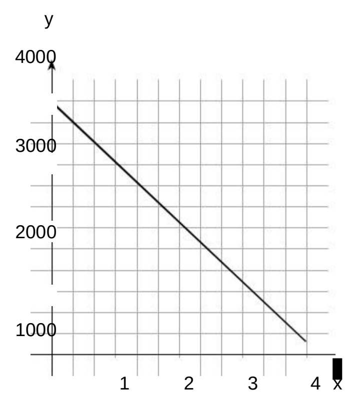
(b) When $n \; 150$, the temperature is approximately given by $t \; \frac{5}{24} \; 150 \; 45 \; 76\;25$ F 76 F.

93. (a) Using *t* in place of *x* and *V* in place of *y*, we find the slope of the line using the points 0 4000 and 4 200 . Thus, the slope is

$$m \; \frac{200 \; 4000}{4 \; 0} \; \frac{3800}{4} \; 950.$$

Using the *V* -intercept, the linear equation is $V 950 t \; 4000$.

(b)

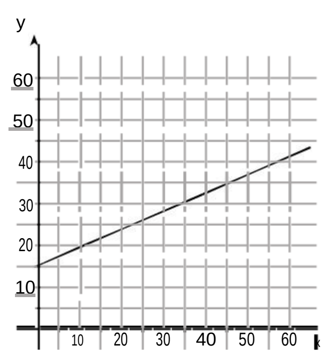


(c) The slope represents a decrease of \$950 each year in the value of the computer. The *V* -intercept represents the cost of the computer.

(d) When $t \; 3$, the value of the computer is given by $V 950 \; 3 \; 4000 \; 1150$.

94. (a) We are given $\frac{\text{change in pressure}}{10 \text{ feet change in depth}} \; \frac{4\;34}{10} \; 0\;434$. Using *P* for pressure and *d* for depth, and using the point $P \; 15$ when $d \; 0$, we have $P \; 15 \; 0\;434 \; d \; 0 P \; 0\;434 d \; 15$.

(b)



(c) The slope represents the increase in pressure per foot of descent. The *y*-intercept represents the pressure at the surface.

(d) When $P \; 100$, then $100 \; 0\;434 d \; 15 \; 0\;434 d \; 85$ $d \; 195\;9$ ft. Thus the pressure is 100 lb/in^3 at a depth of approximately 196 ft.

The temperature is increasing at a constant rate when the slope is positive, decreasing at a constant rate when the slope is negative, and constant when the slope is 0.

We label the three points *A*, *B*, and *C*. If the slope of the line segment $\overline{AB}$ is equal to the slope of the line segment $\overline{BC}$, then the points *A*, *B*, and *C* are collinear. Using the distance formula, we find the distance between *A* and *B*, between *B* and *C*, and between *A* and *C*. If the sum of the two smaller distances equals the largest distance, the points *A*, *B*, and *C* are collinear.

Another method: Find an equation for the line through *A* and *B*. Then check if *C* satisfies the equation. If so, the points are collinear.

1.4 SOLVING QUADRATIC EQUATIONS

1. (a) The Quadratic Formula states that $x \quad \frac{b \quad b^2 \quad 4ac}{2a}$.

(b) In the equation $\frac{1}{2}x^2 \quad x \quad 4 \quad 0$, $a \quad \frac{1}{2}$, $b 1$, and $c 4$. So, the solution of the equation is

$$x \quad \frac{1 \quad 1^2 \quad 4 \frac{1}{2} 4}{2 \frac{1}{2}} \quad \frac{1 \quad 3}{1} \quad 2 \text{ or } 4.$$

(a) To solve the equation $x^2 \; 4x \; 5 \; 0$ by factoring, we write $x^2 \; 4x \; 5 x \; 5 \; x \; 1 \; 0$ and use the Zero-Product Property to get $x \; 5$ or $x \; 1$.

(b) To solve by completing the square, we add 5 to both sides to get $x^2 \quad 4x \quad 5$, and then add $\frac{4}{2}^2$ to both sides to get $x^2 \quad 4x \quad 4 \quad 5 \quad 4 x \quad 2^2 \quad 9 \quad x \quad 2 3 \quad x \quad 5$ or $x 1$.

To solve using the Quadratic Formula, we substitute $a \quad 1$, $b 4$, and $c 5$, obtaining

$$x \quad \frac{4 \quad 4^2 \; 4 1 5}{2 1} \quad \frac{4 \quad 36}{2} \quad 2 \quad 3 \quad x \quad 5 \text{ or } x 1.$$

For the quadratic equation $ax^2 \quad bx \; c \; 0$ the discriminant is $D \; b^2 \quad 4ac$. If $D \; 0$, the equation has two real solutions; if $D \; 0$, the equation has one real solution; and if $D \; 0$, the equation has no real solution.

There are many possibilities. For example, $x^2 \; 1$ has two solutions, $x^2 \; 0$ has one solution, and $x^2 \; 1$ has no solution.

$x^2 \quad 8x \quad 15 \quad 0 x \quad 3 \; x \quad 5 \quad 0 \quad x \quad 3 \quad 0$ or $x \quad 5 \quad 0$. Thus, $x \quad 3$ or $x \quad 5$.

$x^2 \quad 5x \quad 6 \quad 0 x \quad 3 \; x \quad 2 \quad 0 \quad x \quad 3 \quad 0$ or $x \quad 2 \quad 0$. Thus, $x 3$ or $x 2$.

$x^2 \quad x \quad 6 \quad x^2 \quad x \quad 6 \quad 0 x \quad 2 \; x \quad 3 \quad 0 \quad x \quad 2 \quad 0$ or $x \quad 3 \quad 0$. Thus, $x 2$ or $x \quad 3$.

$x^2 \quad 4x \quad 21 \quad x^2 \quad 4x \quad 21 \quad 0 x \quad 3 \; x \quad 7 \quad 0 \quad x \quad 3 \quad 0$ or $x \quad 7 \quad 0$. Thus, $x 3$ or $x \quad 7$.

$5x^2 \quad 9x \quad 2 \quad 0 5x \quad 1 \; x \quad 2 \quad 0 \quad 5x \quad 1 \quad 0$ or $x \quad 2 \quad 0$. Thus, $x \frac{1}{5}$ or $x \quad 2$.

$6x^2 \quad x \quad 12 \quad 0 3x \quad 4 \; 2x \quad 3 \quad 0 \quad 3x \quad 4 \quad 0$ or $2x \quad 3 \quad 0$. Thus, $x \frac{4}{3}$ or $x \quad \frac{3}{2}$.

$2s^2 \quad 5s \quad 3 \quad 2s^2 \quad 5s \quad 3 \quad 0 2s \quad 1 \; s \quad 3 \quad 0 \quad 2s \quad 1 \quad 0$ or $s \quad 3 \quad 0$. Thus, $s \frac{1}{2}$ or $s \quad 3$.

$4y^2 \quad 9y \quad 28 \quad 4y^2 \quad 9y \quad 28 \quad 0 4y \quad 7 \; y \quad 4 \quad 0 \quad 4y \quad 7 \quad 0$ or $y \quad 4 \quad 0$. Thus, $y \frac{7}{4}$ or $y \quad 4$.

$12z^2 \quad 44z \quad 45 \quad 12z^2 \quad 44z \quad 45 \quad 0 6z \quad 5 \; 2z \quad 9 \quad 0 \quad 6z \quad 5 \quad 0$ or $2z \quad 9 \quad 0$. Thus, $z \frac{5}{6}$ or $z \quad \frac{9}{2}$.

$4^2 \; 4 3 \; 4^2 \; 4 \; 3 \; 0 \; 2 \; 1 \; 2 \; 3 \; 0 \; 2 \; 1 \; 0$ or $2 \; 3 \; 0$. If $2 \; 1 \; 0$, then $\frac{1}{2}$; if $2 \; 3 \; 0$, then $\frac{3}{2}$.

$x^2 \; 5 \; x \; 100 \; x^2 \; 5x \; 500 \; x^2 \; 5x \; 500 \; 0 \; x \; 25 \; x \; 20 \; 0 \; x \; 25 \; 0$ or $x \; 20 \; 0$. Thus, $x \; 25$ or $x \; 20$.

6x x 1 21 x 6x^2 6x 21 x 6x^2 5x 21 0 2x 3 3x 7 0 2x 3 0 or 3x 7 0. If 2x 3 0, then x $\frac{3}{2}$; if 3x 7 0, then x $\frac{7}{3}$.

x^2 8x 1 0 x^2 8x1 x^2 8x 161 16x 4^2 15 x 415 x 415.

2 2 2 2

x 6x 2 0 x 6x 2 x 6x 9 2 9x 3 11 x 311 x311.

2 2 2 2

x 6x 11 0 x 6x 11 x 6x 9 11 9x 3 20 x 32 5 x 3 2 5.

2 7 2 7 2 9 7 9 3 2 16 3 3

20. x 3x 4 0 x 3x 4 x 3x 4 4 4 x 2 4 4 x 2 2 x 2 2

1 2 or x 2 2.

21. x^2 x $\frac{3}{4}$ 0 x^2 x 3 x^2 x $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{4}$ x $\frac{1}{2}$ 1 x $\frac{1}{2}$ 1 x $\frac{1}{2}$ 1. So

22. x $\frac{}{2}$ 1 $\frac{}{2}$ or x $\frac{}{2}$ 1 $\frac{}{2}$. 25 4x 2 5 21 4 x 2 521 4 2 21

2

521.

22

2 2 2 2 2 2

23. x 22x 21 0 x 22x 21 x 22x 11 21 11 21 121 x 11 100 x 11 10 x 11 10. Thus, x 1 or x 21.

2 2 2 2 2

x 18x 19 x 18x 9 19 9 19 81 x 9 100 x 9 10 x 9 10, so x 1 or x 19.

x

25. 5x^2 10x 7 0 2 2x $\frac{7}{5}$ 0 x^2 2x $\frac{7}{5}$ x^2 2x 1 $\frac{7}{5}$ 1 x 1^2 $\frac{12}{5}$ x 1 $\frac{12}{5}$

1 215

5

26. 2x^2 16x 5 0 x^2 8x $\frac{5}{2}$ 0 x^2 8x $\frac{5}{2}$ x^2 8x 16 $\frac{5}{2}$ 16x 4^2 $\frac{27}{2}$

27. 2x^2 7x 4 0 x^2 $\frac{7}{2}$$x$ 2 0 x^2 $\frac{7}{2}$ x2 x^2 $\frac{7}{2}$ x $\frac{49}{16}$ 2 $\frac{49}{16}$ x $\frac{7}{4}$ $\frac{17}{16}$

x $\frac{7}{4}$ $\frac{\sqrt{17}}{4}$ x $\frac{7}{4}$ $\frac{\sqrt{17}}{4}$

28. 4x^2 5x 8 0 x^2 $\frac{5}{4}$ x 2 0 x^2 $\frac{5}{4}$ x 2 x^2 $\frac{5}{4}$ x $\frac{25}{64}$ 2 $\frac{25}{64}$ x $\frac{5}{8}$^{2} $\frac{153}{64}$ x $\frac{5}{8}$ $\frac{153}{64}$

x $\frac{5}{8}$ $\frac{3\sqrt{17}}{8}$.

29. x^2 8x 12 0 x 2 x 6 0 x 2 or x 6.

30. x^2 3x 18 0 x 3 x 6 0 x3 or x 6.

31. x^2 8x 20 0 x 10 x 2 0 x10 or x 2.

32. 10x^2 9x 7 0 5x 7 2x 1 0 x $\frac{7}{5}$ or x $\frac{1}{2}$.

33. 2x^2 x 3 0 x 1 2x 3 0 x 1 0 or 2x 3 0. If x 1 0, then x 1; if 2x 3 0, then x $\frac{3}{2}$.

34. 3x^2 7x 4 0 3x 4 x 1 0 3x 4 0 or x 1 0. Thus, x $\frac{4}{3}$ or x1.

35. 3x^2 6x 5 0 x^2 2x $\frac{5}{3}$ 0 x^2 2x $\frac{5}{3}$ x 2 2x 1 $\frac{5}{3}$ 1 x 1^2 $\frac{8}{3}$ x 1 $\frac{8}{3}$

x1 $\frac{2\sqrt{6}}{3}$.

36. $x^2 - 6x + 1 = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \dfrac{6 \pm \sqrt{36 - 4}}{2} = \dfrac{6 \pm \sqrt{32}}{2} = \dfrac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}.$

37. $x^2 - \frac{4}{3}x + \frac{4}{9} = 0 \Rightarrow 9x^2 - 12x + 4 = 0 \Rightarrow (3x - 2)^2 = 0 \Rightarrow x = \frac{2}{3}.$

38. $2x^2 - 3x + \frac{1}{2} = 0 \Leftrightarrow 4x^2 - 6x + 1 = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{6^2 - 4(4)(1)}}{2(4)} = \frac{6 \pm \sqrt{52}}{8} = \frac{3 \pm \sqrt{13}}{4}$.

$4x^2 + 16x - 9 = 0 \Leftrightarrow (2x - 1)(2x + 9) = 0 \Leftrightarrow 2x - 1 = 0$ or $2x + 9 = 0$. If $2x - 1 = 0$, then $x = \frac{1}{2}$; if $2x + 9 = 0$, then $x = -\frac{9}{2}$.

40. $0 = x^2 - 4x + 1 \Leftrightarrow x = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$.

41. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4(1)(3)}}{2(1)} = \frac{-3 \pm \sqrt{-3}}{2}$. Since the discriminant is less than 0, the equation has no real solution.

$3 + 5z + z^2 = 0 \Leftrightarrow z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)} = \frac{-5 \pm \sqrt{25 - 12}}{2} = \frac{-5 \pm \sqrt{13}}{2}$.

$10y^2 - 16y + 5 = 0 \Leftrightarrow$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{16 \pm \sqrt{(-16)^2 - 4(10)(5)}}{2(10)} = \frac{16 \pm \sqrt{256 - 200}}{20} = \frac{16 \pm \sqrt{56}}{20} = \frac{8 \pm \sqrt{14}}{10}$.

$25x^2 + 70x + 49 = 0 \Leftrightarrow (5x + 7)^2 = 0 \Leftrightarrow 5x + 7 = 0 \Leftrightarrow 5x = -7 \Leftrightarrow x = -\frac{7}{5}$.

$3x^2 + 2x + 2 = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(3)(2)}}{2(3)} = \frac{-2 \pm \sqrt{-20}}{6}$. Since the discriminant is less than 0, the equation has no real solution.

$5x^2 - 7x + 5 = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{(-7)^2 - 4(5)(5)}}{2(5)} = \frac{7 \pm \sqrt{49 - 100}}{10} = \frac{7 \pm \sqrt{-51}}{10}$. Since the discriminant is less than 0, the equation has no real solution.

$x^2 - 0.011x - 0.064 = 0 \Rightarrow$

$x = \frac{-(-0.011) \pm \sqrt{(-0.011)^2 - 4(1)(-0.064)}}{2(1)} = \frac{0.011 \pm \sqrt{0.000121 + 0.256}}{2} = \frac{0.011 \pm 0.506}{2}$.

Thus, $x = \frac{0.011 + 0.506}{2} \approx 0.259$ or $x = \frac{0.011 - 0.506}{2} \approx -0.248$.

$x^2 - 2.450x + 1.500 = 0 \Rightarrow$

$x = \frac{-(-2.450) \pm \sqrt{(-2.450)^2 - 4(1)(1.500)}}{2(1)} = \frac{2.450 \pm \sqrt{6.0025 - 6}}{2} = \frac{2.450 \pm \sqrt{0.0025}}{2} = \frac{2.450 \pm 0.050}{2}$. Thus, $x = \frac{2.450 + 0.050}{2} = 1.250$ or $x = \frac{2.450 - 0.050}{2} = 1.200$.

$x^2 - 2.450x + 1.501 = 0 \Rightarrow$

$x = \frac{-(-2.450) \pm \sqrt{(-2.450)^2 - 4(1)(1.501)}}{2(1)} = \frac{2.450 \pm \sqrt{6.0025 - 6.004}}{2} = \frac{2.450 \pm \sqrt{-0.0015}}{2}$.

$x^2 - 1.800x + 0.810 = 0 \Rightarrow$

$x = \frac{-(-1.800) \pm \sqrt{(-1.800)^2 - 4(1)(0.810)}}{2(1)} = \frac{1.800 \pm \sqrt{0}}{2} = \frac{1.800}{2} = 0.900$. Thus the only solution is $x = 0.900$.

51. h $\frac{1}{2}gt^2$ $0t$ $\frac{1}{2}gt^2$ $0t$ h 0. Using the Quadratic Formula,

t 0 2 1 00^2 $2gh$.

2 $\frac{1}{2}g$ g

52. S $\frac{n\,n\,1}{2}$ $2S$ n ^{2} n n ^{2} n $2S0$. Using the Quadratic Formula,

n 1 1^2 41 $2S$ 11 $8S$.

2 1 2

A $2x^2$ $4xh$ $2x^2$ $4xh$ A 0. Using the Quadratic Formula,

x $4h$ $4h^2$ 4 2 A $4h$ $16h^2$ $8A$ $4h$ 4 $4h^2$ $2A$ $4h$ 2 $4h^2$ $2A$

2 2 4 4 4

2 $2h$ $4h^2$ $2A$ $2h$ $4h^2$ $2A$

4 2

2 2

54. A 2 r 2 $r\,h$ 2 r 2 $r\,hA$ 0. Using the Quadratic Formula,

r 2 h 2 h^2 4 $2A$ 2 h 4 $^2h^2$ 8 A h $^2h^2$ 2 A.

2 2 4 2

55. $\frac{1}{s\ a}$ $\frac{1}{s\ b}$ $\frac{1}{c}$ $c\,s$ b $c\,s$ as a s bcs bc cs ac s^2 as bs ab

2

s a b $2c\,s$ ab ac bc 0. Using the Quadratic Formula,

s a b $2c$ a b $2c^2$ 4 1 ab ac bc

2 1

2 2 2

a b $2ca$ b $4c$ ~~$2ab$ $4ac$ $4bc$ $4ab$ $4ac$ $4bc$~~

2

a b $2ca^2$ b^2 $4c^2$ ~~$2ab$~~

2

56. r 1 r r 2 r^2 1 r r 1 r r^2 1 r r^2 r 1 r $2r^2$ 4 1 rr r^2 $2r^2$ 4 $4r$

1 2 4 1 2 4

r^2 $5r$ 4 0. Using the Quadratic Formula, r $5\,5^2\,414$ / 2 1 5 25 16 / 2 $5\,41$ / 2 .

D b^2 $4ac6^2$ 4 1 1 32. Since D is positive, this equation has two real solutions.

x^2 $6x$ 9 x^2 $6x$ 9, so D b^2 $4ac$ 6 ^{2} 4 1 9 36 36 0. Since D 0, this equation has one real solution.

D b^2 $4ac$ 2 20 ^{2} 4 1 1 21 4 84 4 84 0. Since D 0, this equation has one real solution.

D b^2 $4ac$ 2 21 ^{2} 4 1 1 21 4 8841 4 84 0 0441. Since D 0, this equation has two real solutions.

D b^2 $4ac^2$ 4 4 $\frac{13}{8}$ 25 26 1. Since D is negative, this equation has no real solution.5

D b^2 $4ac$ r^2 4 1 s r^2 $4s$. Since D is positive, this equation has two real solutions.

a^2x^2 $2ax$ 1 $0ax$ 1^2 0 ax 1 0. So ax 1 0 then $ax1$ x_a^1.

2 a 1

a

ax 2a 1 x a 1 0 [ax a 1] x 1 0 ax a 1 0 or x 1 0. If ax a 1 0, then x ; if x 1 0, then x 1.

We want to find the values of k that make the discriminant 0. Thus k^2 4 4 25 0 k^2 400 k20 2 2

We want to find the values of k that make the discriminant 0. Thus D 36^2 4 k k 0 4k 36 2k 36 k 18.

67. Let n be one number. Then the other number must be 55 n since n55 n 55. Because the product is 684, we have n 55 n68455n n^2 684 n^2 55n 6840

n 55 55^2 41 684 / 2 1 55 3025 2736 / 2 55 289 / 2 55 17 / 2. So n 55 17 / 2 72 / 2 36 or

n 55 17 / 2 38 / 2 19. In either case, the two numbers are 19 and 36.

68. Let n be one even number. Then the next even number is n 2. Thus we get the equation n^2 n 2^2 1252

n^2 n^2 4n 4 1252 0 2n^2 4n 1248 2 n^2 2n 624 2 n 24 n 26 . So n 24 or n26.

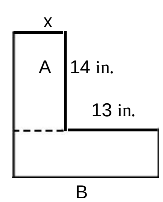
Let be the width of the garden in feet. Then the length is 10. Thus 875 10 ^{2} 10 875 0 35 25 0. So 35 0 in which case 35 which is not possible, or 25 0 and so 25. Thus the width is 25 feet and the length is 35 feet.

Let be the width of the bedroom. Then its length is7. Since area is length times width, we have

2287 ^{2} 7 ^{2} 7 228 0 19 12 0 19 0 or 12 0. Thus 19 or 12. Since the width must be positive, the width is 12 feet.

Let be the width of the garden in feet. We use the perimeter to express the length l of the garden in terms of width. Since the perimeter is twice the width plus twice the length, we have 200 2 2l 2l 200 2 l 100 . Using the formula for area, we have 2400 100 100 ^{2} ^{2} 100 2400 0 40 60 0. So 40 0 40, or 60 0 60. If 40, then l 100 40 60. And if 60, then 100 60 40. So the length is 60 feet and the width is 40 feet.

72. First we write a formula for the area of the figure in terms of x. Region A has dimensions 14 in. and x in. and region B has dimensions 13 x in. and x in. So the area of the figure is 14 x [13 x x] 14x 13x x^2 x^2 27x. We are given that this is equal to 160 in^2, so 160 x^2 27x x^2 27x 160 0



x 32 x 5x32 or x 5. x must be positive, so x 5 in.

The shaded area is the sum of the area of a rectangle and the area of a triangle. So A y 1 $\frac{1}{2}$ y y $\frac{1}{2}$ y^2 y. We are given that the area is 1200 cm^2, so 1200 $\frac{1}{2}$ y^2 y y^2 2y 2400 0 y 50 y 48 0. y is positive, so y 48 cm.

74. Setting P 1250 and solving for x, we have 1250 $\frac{1}{10}$$x$ 300 x30x $\frac{1}{10}x^2$ $\frac{1}{10}x^2$ 30x 1250 0.

Using the Quadratic Formula, x 30 30^2 4 $\frac{1}{10}$ 1250 / 2 $\frac{1}{10}$ 30 900 500 / 0 2 30 20 / 0 2. Thus

x 30 20 / 0 2 50 or x 30 20 / 0 2 250. Since he must have 0 x 200, he should make 50 ovens per week.

Let x be the length of one side of the cardboard, so we start with a piece of cardboard x by x. When 4 inches are

, ^{2} 16x 64 25 x^2

16x 39 0 x 3 x 13 0. So x 3 or x 13. But x 3 is not possible, since then the length of the base would be 3 8 5 and all lengths must be positive. Thus x 13, and the piece of cardboard is 13 inches by 13 inches.

Let r be the radius of the can. Now using the formula $V\ r^2h$ with V 40 cm^3 and h 10, we solve for r. Thus 40 r^2 10 4 r^2 r

2. Since r represents radius, r 0. Thus r 2, and the diameter is 4 cm.

Let be the width of the lot in feet. Then the length is6. Using the Pythagorean Theorem, we have

2 6^2 174^2 2 2 12 36 30,276 2 ^{2} 12 30240 0 ^{2} 6 15120 0 126 120 0. So either 126 0 in which case 126 which is not

possible, or 120 0 in which case 120. Thus the width is 120 feet and the length is 126 feet.

Let h be the height of the flagpole, in feet. Then the length of each guy wire is h 5. Since the distance between the points where the wires are fixed to the ground is equal to one guy wire, the triangle is equilateral, and the flagpole is the perpendicular bisector of the base. Thus from the Pythagorean Theorem, we get

$\frac{1}{2}h$ 5 ^{2} h^2 h 5 ^{2} h^2 10h 25 4h^2 4h^2 40h 100 h^2 30h 75 0

h 30 30 41 75 30 900 300 30 1200 30 20 3 2 1 2 2 2. Since h 30 20 3 2 0, we reject it. Thus 30 20 3

the height is h 2 15 10 3 32 32 ft 32 ft 4 in.

Let x be the rate, in mi/h, at which the salesman drove between Ajax and Barrington.

Direction	Distance	Rate	Time
Ajax Barrington	120	x	$\frac{120}{x}$
Barrington Collins	150	x 10	$\frac{150}{x\ 10}$

We have used the equation time $\frac{\text{distance}}{\text{rate}}$ to fill in the "Time" column of the table. Since the second part of the trip took 6 minutes (or $\frac{1}{10}$ hour) more than the first, we can use the time column to get the equation $\frac{120}{x}$ $\frac{1}{10}$ $\frac{150}{x\ 10}$

120 10 x 10 x x 10 150 10x 1200x 12,000 x^2 10x 1500x ^{2} 290x 12,000 0

x 290 290 ^{2} 4 1 12,000 2 290 100 48 000 84, 2 , 290 36 100 2 ' 290 2 190 145 95. Hence, the salesman drove either 50 mi/h or 240 mi/h between Ajax and Barrington. (The first choice seems more likely!)

Let x be the rate, in mi/h, at which Kiran drove from Tortula to Cactus.

Direction	Distance	Rate	Time
Tortula Cactus	250	x	$\frac{250}{x}$
Cactus Dry Junction	360	x 10	$\frac{360}{x\ 10}$

We have used time $\frac{\text{distance}}{\text{rate}}$ to fill in the time column of the table. We are given that the sum of the times is 11 hours. Thus we get the equation $\frac{250}{x}$ $\frac{360}{x\ 10}$ 11250 x 10360x

11x x 10250x 2500 360x11x^2 110x11x^2 500x 2500 0

$x = \frac{500 \pm \sqrt{500^2 - 4(11)(-250)}}{2(11)} = \frac{500 \pm \sqrt{250{,}000 + 110{,}000}}{22} = \frac{500 \pm \sqrt{360{,}000}}{22} = \frac{500 \pm 600}{22}$. Hence, Kiran drove either -4.54 mi/h (impossible) or 50 mi/h between Tortula and Cactus.

Let r be the rowing rate in km/h of the crew in still water. Then their rate upstream was r 3 km/h, and their rate downstream was r 3 km/h.

Direction	Distance	Rate	Time
Upstream	6	r 3	$\frac{6}{r\ 3}$
Downstream	6	r 3	$\frac{6}{r\ 3}$

Since the time to row upstream plus the time to row downstream was 2 hours 40 minutes $\frac{8}{3}$ hour, we get the equation
$\frac{6}{r\ 3}$ $\frac{6}{r\ 3}$ $\frac{8}{3}$ 6 3 r 3 6 3 r 38 r 3 r 318r 54 18r 54 8r^2 72

0 8r^2 36r 72 4 2r^2 9r 184 2r 3 r 62r 3 0 or r 6 0. If 2r 3 0, then r $\frac{3}{2}$,

which is impossible because the rowing rate is positive. If r 6 0, then r 6. So the rate of the rowing crew in still water is 6 km/h.

Let r be the speed of the southbound boat. Then r 3 is the speed of the eastbound boat. In two hours the southbound boat has traveled 2r miles and the eastbound boat has traveled 2 r 3 2r 6 miles. Since they are traveling is directions with are 90 apart, we can use the Pythagorean Theorem to get 2r^2 2r 6^2 30^2 4r^2 4r^2 24r 36 900

8r^2 24r 864 0 8 r^2 3r 108 0 8 r 12 r 9 0. So r 12 or r 9. Since speed is positive, the speed of the southbound boat is 9 mi/h.

Using h_0 288, we solve 0 16t^2 288, for t 0. So 0 16t^2 288 16t^2 288 t^2 18

t 183 2. Thus it takes 3 2 4 24 seconds for the ball the hit the ground.

84. (a) Using h_0 96, half the distance is 48, so we solve the equation 48 16t^2 96 48 16t^2 3 t^2
t $\sqrt{3}$. Since t 0, it takes $\sqrt{3}$ 1 732 s.

The ball hits the ground when h 0, so we solve the equation 0 16t^2 96 16t^2 96 t^2 6 t 6. Since t 0, it takes 6 2 449 s.

85. We are given $_o$ 40 ft/s. 16t^2 40t 16t^2 40t 24 0 8 2t^2 5t 3 0 8 2t 3 t 1 0

(a) Setting h 24, we
t 1 or t 1 $\frac{1}{2}$. Therefore, the ball reaches 24 feet in 1 second (ascending) and again after 1 $\frac{1}{2}$ seconds (descending).

(b) Setting h 48, we have 48 16t^2 40t 16t^2 40t 48 0 2t^2 5t 6 0

t $\frac{5 \quad \sqrt{25\ 48}}{4}$ $\frac{5 \quad 23}{4}$. However, since the discriminant D 0, there is no real solution, and hence the ball never reaches a height of 48 feet.

(c) The greatest height h is reached only once. So h 16t^2 40t 16t^2 40t h 0 has only one solution. Thus D 40^2 4 16 h 0

1600 64h 0 h 25. So the greatest height reached by the ball is 25 feet.

(d) Setting h 25, we have 25 16t^2 40t 16t^2 40t 25 0 4t 5^2 0 t 1 $\frac{1}{4}$. Thus the ball reaches the highest point of its path after 1 $\frac{1}{4}$ seconds.

0 (ground level), we have 0 16t^2 40t 2t^2 5t 0 t 2t 5 0 t 0 (start) or t 2 $\frac{1}{2}$.

86. If the maximum height is 100 feet, then the discriminant of the equation, $16t^2$ ot 100 0, must equal zero. So 0 b^2 $4ac$ o^2 4 16 100 o^2 6400 o 80. Since o 80 does not make sense, we must have o 80 ft/s.

87. (a) The fish population on January 1, 2002 corresponds to t 0, so F 1000 30 17 00 ^{2} 30 000. To find when the population will again reach this value, we set F 30 000, giving

30000 1000 30 17t t^2 30000 17000t 1000t^2 0 17000t 1000t^2 1000t 17 tt 0 or t 17. Thus the fish population will again be the same 17 years later, that is, on January 1, 2019.

(b) Setting F 0, we have 01000 30 17t t^2 t^2 17t 30 0

t 17 289 120 / 2 7 409 / 2 17 20 ?2 / 2. Thus t1 612 or t 18 612. Since t 0 is inadmissible, it follows that the fish in the lake will have died out 18 612 years after January 1, 2002, that is on August 12, 2020.

88. Let y be the circumference of the circle, so 360 y is the perimeter of the square. Use the circumference to find the radius, r, in terms of y: y 2 r r y 2. Thus the area of the circle is y 2 2 y^2 4. Now if the perimeter of the square is 360 y, the length of each side is 1/4 2360 y and the area of the square is 1/4 360 y 2.

Setting these areas equal, we obtain y 4 1/4 360 y y 2 1/4 360 y2y 360 y 2 y 360. Therefore, y 360 2 169 1. Thus one wire is 169 1 in. long and the other is 1909 in. long.

89. Let be the uniform width of the lawn. With cut off each end, the area of the factory is 240 2 180 2. Since the lawn and the factory are equal in size this area, is 1/2 240 180. So 21,600 43,200 4803604 ^{2} 0 4 ^{2} 84021,600 4 2 2105400 43018030 or180. Since 180 ft is too wide, the width of the lawn is 30 ft, and the factory is 120 ft by 180 ft.

90. Let h be the height the ladder reaches (in feet). Using the Pythagorean Theorem we have 7 1/2 2 h 2 19 1/2 2 15/2 h^2 39/4 2 h^2 39/4 2 15/2 2 1521/4 225/4 1296/4 324. So h324 18 feet.

Let t be the time, in hours it takes Irene to wash all the windows. Then it takes Henry t 3/2 hours to wash all the windows, and the sum of the fraction of the job per hour they can do individually equals the fraction of the job they can do together. Since 1 hour 48 minutes 1 48/60 1 4/5 9/5, we have 1/t 1/(t 3/2) 1/(9/5)

1/t 2/(2t 3) 5/9 9 2t 3 2 9t 5t 2t 3 18t 27 18t 10t^2 15t 10t 21t 27 0

t 21 21 4 10 27 / 2 10 21 441 1080 / 20 21 39 / 20. So t 21 39 / 20 9/10 or t 21 39 / 20 3. Since t 0 is impossible, all the windows are washed by Irene alone in 3 hours and by Henry alone in 3 3/2 4 1/2 hours.

Let t be the time, in hours, it takes Kay to deliver all the flyers alone. Then it takes Lynn t 1 hours to deliver all the flyers alone, and it takes the group 0 4t hours to do it together. Thus 1 1/t 1/(t 1) 1/(0 4t) 1 0 4t 1/t 0 4t 1/(t 1) 0 4t 1

t 4 4t 10 tt 1 4t 1 4t 10t 1 t^2 t 4t 4 4t 10t 10 t^2 t 6 0 *t* 1 *t* 3 *t* 2 0. So *t* 3 or *t* 2. Since *t* 2 is impossible, it takes Kay 3 hours to deliver all the flyers alone.

Let x be the distance from the center of the earth to the dead spot (in thousands of miles). Now setting

F 0, we have 0 $\frac{K}{x^2}$ $\frac{0\,012K}{239\ x^2}$ $\frac{K}{x^2}$ $\frac{0\,012K}{239\ x^2}$ K 239 x^2 0 012K x^2

57121 478x x^2 0 012x^2 0 988x^2 478x 57121 0. Using the Quadratic Formula, we obtain

x $\frac{478}{}$ $\frac{478^2\ 4\,0\,988\ 57121}{20988}$ $\frac{478\quad 228484\ 225742\,192}{1\,976}$ $\frac{478\quad 2741\,808}{1\,976}$ $\frac{478\ 52\,362}{1\,976}$ 241 903 26 499.

So either x 241 903 26 499 268 or x 241 903 26 499 215. Since 268 is greater than the distance from the earth to the moon, we reject it; thus x 215,000 miles.

If we have x^2 9x 20 x 4 x 5 0, then x 4 or x 5, so the roots are 4 and 5. The product is 4 5 20, and the sum is 4 5 9. If we have x^2 2x 8 x 4 x 2 0, then x 4 or x 2, so the roots are 4 and 2. The product is 4 2 8, and the sum is 4 2 2. Lastly, if we have x^2 4x 2 0, then using the Quadratic Formula, we have x $\frac{4}{2\ 1}$ $\frac{4}{2}$ $\frac{4\ 2}{2}$ 2 2. The roots are 2 2 and 2 2. The product is 2 2 2 2 4 2 2, and the sum is 2 2 2 2 4. In general, if x r_1 and x r_2 are roots, then x bx c x r_1 x r_2 x r_1x r_2x r_1r_2 x r_1 r_2 x r_1r_2. Equating the coefficients, we get c r_1r_2 and b r_1 r_2 .

Let x equal the original length of the reed in cubits. Then x 1 is the piece that fits 60 times along the length of the field, that is, the length is 60 x 1 . The width is 30x. Then converting cubits to ninda, we have 375 60 x 1 30x $\frac{1}{12^2}$ $\frac{25}{2}$ x x 1 30 x^2 x x^2 x 30 0 x 6 x 5 0. So x 6 or 5. Since x must be positive, the original length of the reed is 6 cubits.

1.5 COMPLEX NUMBERS

The imaginary number i has the property that i^2 1.

For the complex number 3 4i the real part is 3 and the imaginary part is 4.

(a) The complex conjugate of 3 4i is $\overline{3\ 4i}$ 3 4i.

3 4i 3 4i 3^2 4^2 25

If 3 4i is a solution of a quadratic equation with real coefficients, then $\overline{3\ 4i}$ 3 4i is also a solution of the equation.

Yes, every real number a is a complex number of the form a 0i.

Yes. For any complex number z, z $\bar{z}$ a bi a bi a bi a bi 2a, which is a real number.

7. 5 7i: real part 5, imaginary part 7.

8. 6 4i: real part 6, imaginary part 4.

9. $\frac{2\ 5i}{3}$ $\frac{2}{3}$ $\frac{5}{3}$ i: real part $\frac{2}{3}$, imaginary part $\frac{5}{3}$.

10. $\frac{4\ 7i}{2}$ 2 $\frac{7}{2}$ i: real part 2, imaginary part $\frac{7}{2}$.

11. 3: real part 3, imaginary part 0.

12. $\frac{1}{2}$: real part $\frac{1}{2}$, imaginary part 0.

13. $\frac{2}{3}$ i: real part 0, imaginary part $\frac{2}{3}$.

14. i 3: real part 0, imaginary part 3.

15. 3 4 3 2i: real part 3, imaginary part 2.

16. 2 5 2 i 5: real part 2, imaginary part 5.

17. 3 2i 5i 3 2 5 i 3 7i

18. 3i 2 3i2 [33] i 2 6i

5 3*i*4 7*i*5 43 7 *i* 1 10*i* **20.** 3 4*i* 2 5*i*3 2 [45] *i*5 9*i*

21. 6 6*i*9 *i*6 9 6 1 *i* 3 5*i* **22.** 3 2*i*5 $\frac{1}{3}$*i*3 52 $\frac{1}{3}$ *i*2 $\frac{7}{3}$*i*

23. 7 $\frac{1}{2}$*i*5 $\frac{3}{2}$*i*7 5 $\frac{1}{2}$ $\frac{3}{2}$*i* 2 2*i*

4 i2 5i4 i 2 5i4 21 5 i6 6i

12 8i7 4i12 8i 7 4i12 78 4 i19 4i

6i 4 i 6i 4 i46 1 i4 7i

27. 4 1 2i 4 8i

7 i 4 2i 28 14i 4i 2i2 28 214 4 i 30 10i

5 3i 1 i 5 5i 3i 3i2 5 35 3 i 8 2i

6 5i 2 3i 12 18i 10i 15i2 12 1518 10 i 27 8i

2 i 3 7i6 14i 3i 7i2 6 714 3 i 1 17i

2 5i 2 5i 22 5i 2 4 25 1 29

3 7i 3 7i 32 7i 2 58

2 5i 2 22 5i 2 2 2 5i 4 25 20i21 20i

3 7i 2 32 7i 2 2 3 7i40 42i

28. 2 3 4i 6 8i

1 1 i i i

37. i i i i2 1 i

1 1 1 i 1 i1 i 1 i 1 1

38. 1 i 1 i 1 i 1 i2 1 1 2 2 2i

2 3i 2 3i 1 2i 2 4i 3i 6i2 2 64 3 i 8 i 8 1

1

39. 1 2i 1 2i 1 2i 1 4i2 4 5 or 5 5 i

5 i 5 i 3 4i 15 20i 3i 4i2 15 420 3 i 11 23i 11 23

9

1 2

40. 3 4i 3 4i 3 4i 9 16i2 6 5 25 25 i

10i 10i 1 2i 10i 20i2 20 10i 5 4 2i

1

41. 1 2i 1 2i 1 2i 1 4i2 4 5 4 2i

1 1 1 2 3i 2 3i 2 3i 2 3i 2 3

42. 2 3i 2 3i 2 3i 2 3i 4 9i 2 4 9 13 13 13 i

4 6i 4 6i 3i 12i 18i2 18 12i 18 12 4

2

43. 3i 3i 3i 9i 9 9 9 i 2 3 i

3 5i 3 5i 15i 45i 75i2 75 45i 75 45 1 1

2

2

44. 15i 15i 15i 225i2 5 225 225 i 3 5 i

1 1 1 1 i 1 1 i 1 i 1 i 1 i 1 i

45. 1 i 1 i 1 i 1 i 1 i 1 i 1 i2 1 i2 i2 2 i

1 2i 3 i 3 i 6i 2i2 5 5i 2 i 10 5i 10i 5 2 10 55 10 i

46. 2 i 2 i 2 i 2 i 4 i2 5

15 5i 15 5

5 5 5 i 3 i

47. i3 i2 ii

48. i10 i2 5 1 5 1

49. $3i^5 \quad 3^5\, i^2\, {}^2\, i \quad 243 \quad 1^2\, i \quad 243i$

50. $2i^4 \quad 2^4 i^4 \quad 16\; 1 \quad 16$

51. $i^{1000} i^4\; {}^{250} \quad 1^{250} \quad 1$

52. $i^{1002} i^4\; {}^{250}\, i^2 \quad 1 i^2 \quad 1$

53. 49 49 1 7i

54. 16 81 9i 4

55. 3 12 i 3 2i 3 6i 2 6

56. $\frac{1}{3}$ 27 $\frac{1}{3}$ 3i 3 3i

57. 35 1 1 3 i 5 1 i 3 3i i 5 i2 53535 i

58. 2i 2i 2i 2i 4i2

32 4 2 2 6 2 6 i 2 4i 6

59. 2 8 2 2i 2 2 1 i 2 2

1 2 1 i 2 1 i 2

3

6 6i 2 i 2 2i 2 i 2

60. 2 9 i 2 3i i 2 i 2 2i2 1 i 2

2 2

61. x 49 0 x 49 x7i

62. 3x2 1 0 3x2 1 x2 x 3 3i

63. x2 x 2 0 x 2 2 i

b b2 4ac 1 1 2 4 1 2 1 7 1 7

2a 2 1 2

64. x 2 2x 2 0 2 2 2 4 1 2 2 4 8 2 4 2 2i 1 i

65. x 2 1 2 2 2 i 2

2 3 32 4 1 7 3 19 3 19

66. x 2 6x 10 0 x 6 62 4 1 10 6 36 40 6 4 6 2i 3 i

2 1 2 2 2

[redacted] i

1 1 2 4 1 1 1 1 4 1 3 1 3

2 1 2 2 2 1 3

68. x2 3x 3 0 2 2 i

3 3 2 4 1 3 3 9 12 3 3 3 i 3 3 3

2

69. 2x2 2x 1 0 x 1 2 2 2 2 2 i

2 2 2 4 2 1 2 4 8 2 4 2 2i 1 1

2

2 4 4 4

3 3 3 2 1 3

4

71. 6x2 12x 7 0 2i

12 12 2 4 6 7 12 144 168 12 24 12 2i 6 12 6 6

x 2 6 12 12 12 12 12 1 6 i

2 1

72. x x 1 0

1 1 2 4 1 1 1 1 4 1 15

2 2 4 2 4

x

2 1 2 2 2

z 3 4i 5 2i 3 4i 5 2i 8 2i

z 3 4i 5 2i 8 2i 8 2i

z z 3 4i 3 4i 3^2 4^2 25

z 3 4i 5 2i 15 6i 20i $8i^2$ 23 14i

LHS z a bi c di a bi c di a c b d i a c b d i. RHS z a bi c di a c b d i a c b d i.

Since LHS RHS, this proves the statement.

LHS $\neq a\ bi\ c\ di\ ac\ adi\ bci\ bdi^2\ ac\ bd\ ad\ bc\ i\ ac\ bd\ ad\ bc\ i$. RHS $\neq a\ bi\ c\ di\ a\ bi\ c\ di\ ac\ adi\ bci\ bdi^2\ ac\ bd\ ad\ bc\ i$.

Since LHS RHS, this proves the statement.

79. LHS $\bar{z}^2$ $\overline{a\ bi}^2$ $a\ bi^2\ a^2\ 2abi\ b^2 i^2 a^2\ b^2\ 2abi$.

RHS $\overline{z^2}$ $\overline{a\ bi^2}$ $\overline{a^2\ 2abi\ b^2 i^2}$ $\overline{a^2\ b^2\ 2abi}$ $a^2\ b^2\ 2abi$.

Since LHS RHS, this proves the statement.

80. $\bar{\bar{z}}$ $\overline{\overline{a\ bi}}$ $\overline{a\ bi}$ $a\ bi\ z$.

81. $z\ \bar{z}\ a\ bi$ $\overline{a\ bi}$ $a\ bi\ a\ bi\ 2a$, which is a real number.

82. $z\ \bar{z}\ a\ bi$ $\overline{a\ bi}$ $a\ bi\ a\ bi\ a\ bi\ a\ bi\ 2bi$, which is a pure imaginary number.

83. $z\ \bar{z}\ a\ bi$ $\overline{a\ bi}$ $a\ bi\ a\ bi\ a^2\ b^2 i^2\ a^2\ b^2$, which is a real number.

84. Suppose $z\ \bar{z}$. Then we have $a\ bi$ $\overline{a\ bi}$ $a\ bi\ a\ bi\ 0\ 2bi\ b\ 0$, so z is real. Now if z is real, then $z\ a\ 0i$ (where a is real). Since $\bar{z}\ a\ 0i$, we have $z\ \bar{z}$.

85. Using the Quadratic Formula, the solutions to the equation are x $\frac{b\ \sqrt{b^2\ 4ac}}{2a}$. Since both solutions are imaginary, we have $b^2\ 4ac\ 0\ 4ac\ b^2\ 0$, so the solutions are x $\frac{b}{2a}$ $\frac{\sqrt{4ac\ b^2}}{2a}\ i$, where $\sqrt{4ac\ b^2}$ is a real number. Thus the solutions are complex conjugates of each other.

86. $i^5\ i^4 i,\ i^9\ i^8\ i\ i,\ i\ i$; $i^{26}\ i^2\ 1,\ i^{2\ 4}\ i^4\ 1$, $i^{10\ 8}\ i^2$ $i\ i\ i\ 1$;

$i^7\ i^{4\ 3}\ i^{11\ 8\ 3}$ $i^3\ i,\ i\ i\ i\ i,\ i\ i\ i\ i$; $i^8\ i^{4\ 4}\ i^{12\ 8\ 4}$ $i^4\ 1,\ i\ i\ i\ 1,\ i\ i\ i\ 1$.

Because $i^4\ 1$, we have $i^n\ i^r$, where r is the remainder when n is divided by 4, that is, $n\ 4\ k\ r$, where k is an integer and $0\ r\ 4$. Since $4446\ 4\ 1111\ 2$, we must have $i^{4446}\ i^2\ 1$.

1.6 SOLVING OTHER TYPES OF EQUATIONS

Note: In cases where both sides of an equation are squared, the implication symbol is sometimes used loosely. For example, $\sqrt{x}\ x\ 1$ "" $x^2\ x\ 1^2$ is valid only for positive x. In these cases, inadmissible solutions are identified later in the solution.

(a) To solve the equation $x^3\ 4x^2\ 0$ we *factor* the left-hand side: $x^2\ x\ 4\ 0$, as above.

The solutions of the equation $x^2\ x\ 4\ 0$ are $x\ 0$ and $x\ 4$.

(a) Isolating the radical in $\sqrt{2x}\ x\ 0$, we obtain $\sqrt{2x}\ x$.

Now square both sides: $\sqrt{2x}^2\ x^2$ $2x\ x^2$.

Solving the resulting quadratic equation, we find $2x\ x^2$ $x^2\ 2x\ x\ x\ 2\ 0$, so the solutions are $x\ 0$ and $x\ 2$.

We substitute these possible solutions into the original equation: $\sqrt{2\ 0}\ 0\ 0$, so $x\ 0$ is a solution, but $\sqrt{2\ 2}\ 2\ 4\ 0$, so $x\ 2$ is not a solution. The only real solution is $x\ 0$.

The equation $x\ 1^2\ 5\ x\ 1\ 6\ 0$ is of *quadratic* type. To solve the equation we set $W\ x\ 1$. The resulting quadratic equation is $W^2\ 5W\ 6\ 0$ $W\ 3\ W\ 2\ 0$ $W\ 2$ or $W\ 3$ $x\ 1\ 2$ or $x\ 1\ 3$ $x\ 1$ or $x\ 2$. You can verify that these are both solutions to the original equation.

The equation x^6 $7x^3$ 8 0 is of *quadratic* type. To solve the equation we set W 3. The resulting quadratic equation is W^2 $7W$ 8 0.

2

x x 0 x x 1 0 x 0 or x 1 0. Thus, the two real solutions are 0 and 1.

$3x^3$ $6x^2$ 0 $3x^2$ x 2 0 x 0 or x 2 0. Thus, the two real solutions are 0 and 2.

3 32

x 25x x 25 0 x x 5 x 5 0 x 0 or x 5 0 or x 5 0. The three25x0xx

real solutions are 5, 0, and 5.

x^5 $5x^3$ x^5 $5x^3$ 0 x^3 x^2 50 x 0 or x^2 5 0. The solutions are 0 and 5.

x^5 $3x^2$ 0 x^2 x^3 30 x 0 or x^3 3 0. The solutions are 0 and ^{3}3.

10. $6x^5$ $24x$ 0 $6x$ x^4 4 0 $6x$ x^2 2 x^2 20. Thus, x 0, or x^2 2 0 (which has no solution), or

x 2 0. The solutions are 0 and 2.

11. 0 $4z^5$ $10z^2$ $2z^2$ $2z^3$ 5. If $2z^2$ 0, then z 0. If $2z^3$ 5 0, then $2z^3$ 5 z ^{3}5/2. The solutions are 0

0 $125t^{10}$ $2t^7$ t^7 $125t^3$ 2. If t^7 0, then t 0. If $125t^3$ 2 0, then t^3 125 2 ^{3}5 2. The solutions are 0 and ^{3}5

13. and 2/5 x^3 $8x^2$ x 2 x^2 $2x$ 4 x^2 0, x 2 0, or x^2 $2x$ 4 0. If x^2 0, then

x 0; if x 2 0, then x 2, and x^2 $2x$ 4 0 has no real solution. Thus the solutions are x 0 and x 2.

14. 0 x^4 $64x$ x x^3 $64x$ 0 or x^3 64 0. If x^3 64 0, then x^3 64 x 4. The solutions are 0 and 4.

15. 0 x^3 $5x^2$ $6x$ x x^2 $5x$ $6x$ x 2 x $3x$ 0, x 2 0, or x 3 0. Thus x 0, or x 2, or

x 3. The solutions are x 0, x 2, and x 3.

16. 0 x^4 x^3 $6x^2$ x^2 x^2 x 6 x^2 x 3 x 2. Thus either x^2 0, so x 0, or x 3, or x 2. The

solutions are 0, 3, and 2.

17. 0 x^4 $4x^3$ $2x^2$ x^2 x^2 $4x$ 2. So either x^2 0 x 0, or using the Quadratic Formula on x^2 $4x$ 2 0,

we have x 4 4^2 4 1 2 / 2 1 4 16 8 / 2 4 8 / 2 4 2 2 / 2 2 2. The solutions are 0, 2 2, and 2 2.

18. 0 y^5 $8y^4$ $4y^3$ y^3 y^2 $8y$ 4. If y^3 0, then y 0. If y^2 $8y$ 4 0, then using the Quadratic Formula, we

have y 8 8^2 4 1 4 / 2 1 8 48 / 2 4 2 3. Thus, the three solutions are 0, 4 2 3, and 4 2 3.

19. $3x$ 5 ^{4} $3x$ 5 ^{3} 0. Let y $3x$ 5. The equation becomes y^4 y^3 0 y^3 y 1 0. If y 0, then $3x$ 5 0

x 5/3. If y 1 0, then $3x$ 5 ^{2} $3x$ 5 1 0 $9x^2$ $33x$ 31 0. The discriminant is

b^2 $4ac$ 33 ^{2} 4 9 31 27 0, so this case gives no real solution. The solutions are x 5/3 and x 4/3.

20. x 5 ^{4} 16 x 5 ^{2} 0. Let y x 5. The equation becomes y^4 $16y^2$ y^2 y 4 y 4 0. If y^2 0, then

x 5 0 and x 5. If y 4 0, then x 5 4 0 and x 1. If y 4 0, then x 5 4 0 and x 9. Thus, the solutions are 9, 5, and 1.

21. 0 x^3 $5x^2$ $2x$ 10 x^2 x 5 2 x 5 x 5 x^2 2. If x 5 0, then x 5. If x^2 2 0, then

$x\ \ 2\ \ x$ 2. The solutions are 5 and 2.

22. $0\ \ 2x^3\ \ x^2\ \ 18x\ \ 9\ \ x^2\ \ 2x\ \ 1\ \ 9\ 2x\ \ 1\ \ 2x\ \ 1\ \ x^2\ \ 9\ \ 2x\ \ 1\ \ x\ \ 3\ \ x\ \ 3$. The solutions are $\frac{1}{2}$, 3, and 3.

23. $x^3\ \ x^2\ \ x\ \ 1\ \ x^2\ \ 1\ \ 0\ \ x^3\ \ 2x^2\ \ x\ \ 2\ \ x^2\ \ x\ \ 2x\ \ 2x\ \ 2\ \ x^2\ \ 1$. Since $x^2\ \ 1\ \ 0$ has no real solution, the only solution comes from $x\ \ 2\ \ 0\ \ x\ \ 2$.

24. $7x^3$ x 1 x^3 $3x^2$ x 0 $6x^3$ $3x^2$ $2x$ 1 $3x^2$ $2x$ 1 $2x$ 1 $3x^2$ 1 $2x$ 1 0

or $3x^2$ 1 0. If $2x$ 1 0, then x $\frac{1}{2}$. If $3x^2$ 1 0, then $3x^2$ 1 x^2 $\frac{1}{3}$ x $\frac{1}{\sqrt{3}}$. The solutions are $\frac{1}{2}$ and $\frac{1}{\sqrt{3}}$.

25. z 1 $3z^2$ z 4 $3z$ 3 z^2 $2z$ 1 0 z 1^2 0. The solution is z 1. We must check the original equation to make sure this value of z does not result in a zero denominator.

26. $\frac{10}{m}$ 5 15 $3m$ m 5 $\frac{10}{m}$ 5 $15m$ 5 $3m$ 10 $15m$ 75 $3m^2$ $15m$ $3m^2$ 85 0

m $\sqrt{\frac{85}{3}}$. Verifying that neither of these values of m results in a zero denominator in the original equation, we see that the solutions are $\sqrt{\frac{85}{3}}$ and $\sqrt{\frac{85}{3}}$.

27. $\frac{1}{x\ 1}$ $\frac{1}{x\ 2}$ $\frac{5}{4}$ 4 x 1 x 2 4 x 2 4 x 1 5 x 1 x 2 $4x$ 8 $4x$ 4 $5x^2$ $5x$ 10 $5x^2$ $3x$ 14 0

28. $5x$ 7 x 2 0. If $5x$ 7 0, then x $\frac{7}{5}$; if x 2 0, then x 2. The solutions are $\frac{7}{5}$ and 2.

$\frac{10}{x}$ $\frac{12}{x\ 3}$ 4 0 10 x 3 $12x$ $4x$ x 3 0 $10x$ 30 $12x$ $4x^2$ $12x$ 0 $4x^2$ $14x$ 30. Using the Quadratic Formula, we have

x $\frac{14}{}$ 14^2 4 4 30 $\frac{}{2\ 4}$ $\frac{14\ \sqrt{196\ 480}}{8}$ $\frac{14\ \sqrt{676}}{8}$ $\frac{14\ 26}{8}$. So the solutions are 5 and $\frac{3}{2}$.

29. x $\frac{50}{x}$ 100 $\frac{x^2}{2}$ ... 50 x 100 $50x$ 5000 x^2 $50x$ 5000 0 x 100 x 50 0 x 100 0

or x 50 0. Thus x 100 or x 50. The solutions are 100 and 50.

30. $\frac{2x}{x^2\ 1}$ $\frac{x}{}$ 1 1 $2x$ x^2 $2x$ 1 x 1^2 0, so x 1. This is indeed a solution to the original equation.

31. 1 $\frac{1}{x\ 1\ \ x\ 2}$ $\frac{2}{x\ 1}$ $\frac{x}{x\ 2}$ 1 x 1 x 2 1 2 x 2 x 1 x $3x$ 2 1 $2x$ 4 x 1

x^2 2 0 x $\sqrt{2}$. We verify that these are both solutions to the original equation.

32. $\frac{x}{x\ 3}$ $\frac{2}{x\ 3}$ $\frac{1}{x^2\ 9}$ x x 3 2 x 3 1 x^2 $3x$ $2x$ 6 1 x^2 $5x$ 5 0. Using the Quadratic Formula, x $\frac{5\ \sqrt{5^2\ 4\ 1\ 5}}{2}$ $\frac{5\ 3\sqrt{5}}{2}$. We verify that both are solutions to the original equation.

33. $\frac{x}{2x\ 7}$ $\frac{x\ 1}{x\ 3}$ 1 x x 3 x 1 $2x$ 7 $2x$ 7 x 3 x^2 $3x$ $2x^2$ $9x$ 7 $2x^2$ $13x$ 21

$3x^2$ $19x$ 28 0 $3x$ 7 x 4 0. Thus either $3x$ 7 0, so x $\frac{7}{3}$, or x 4. The solutions are $\frac{7}{3}$ and 4.

34. $\frac{1}{x-1} - \frac{2}{x^2} = 0 \Rightarrow x^2 - 2(x-1) = 0 \Rightarrow x^2 - 2x + 2 = 0$

$x = \frac{2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2}$. Since the radicand is negative, there is no real solution.

35. 3 4^x 5x 3 4^x x 3x 4 5x x^2 2 5x 3x 4x^2 2 15x^2 20x 0 14x^2 20x 2

x 20 20^2 414 2 20 400 112 20 512 20 16 2 5 4 2 . The

2 14 2 8 2 28 8 7

solutions are $\frac{5\ 4\ 2}{7}$.

36. 2 4x x x 2 4x x 2 x^4 x 3x 1 2x^2 4x 2x^2 7x 1 0. Using the Quadratic

3 1 3 1

Formula, we find x $\frac{7\ 7^2\ 4 2 1}{2 2}$ $\frac{7\ 57}{4}$. Both are admissible, so the solutions are $\frac{7\ 57}{4}$.

37. 5 4x 3 5^2 4x 3 2 25 4x 3 4x 28 x 7 is a potential solution. Substituting into the

original equation, we get 5 4 7 3 5 which is true, so the solution is x 7.

38. $\frac{5}{8x\ 1}$ 3 8x 1 2 3^2 8x 1 9 x $\frac{5}{4}$. Substituting into the original equation, we get

8 4 1 3 9 3, which is true, so the solution is x $\frac{5}{4}$.

39. 2x 1 3x 5 2x 1 2 3x 5 2 2x 1 3x 5 x 4. Substituting into the original equation,

which is true, so the solution is x 4.

40.

x 1 or x 2. Substituting into the original equation, we get 3 1 1 ^{2} 1 2 2, which is true, and 3 2 2^2 1, which is also true. So the solutions are x 1 and x 2.

x 2 xx 2^2 x^2 x 2 x^2 x^2 x 2 x 1 x 2 0 x1 or x 2. Substituting

into the original equation, we get 1 21 1 1, which is false, and 2 2 2 4 2, which is true. So x 2 is the only real solution.

42. 4 6x 2x 4 6x 2^2 $2x^2$ 4 6x 4x^2 x^2 3x 2x 2 2x 10 x2

or x $\frac{1}{2}$. Substituting into the original equation, we get 4 6 2 2 2 16 4, which is false, and

4 6 $\frac{1}{2}$ 2 $\frac{1}{2}$ 1 1, which is true. So x $\frac{1}{2}$ is the only real solution.

43. 2x 1 1 x 2x 1 x 1 2x 1 x 1^2 2x 1 x^2 2x 1 0 x^2 4x x x 4 .

Potential solutions are x 0 and x 4 x 4. These are only potential solutions since squaring is not a reversible operation. We must check each potential solution in the original equation.

Checking x 0: 2 0 1 1 0 1 1 0 is false.

Checking x 4: 2 4 1 1 4 9 1 4 3 1 4 is true. The only solution is x 4.

44. x 9 3x 0 x 9 3x x 9 3x 0 x 3x 9. Using the Quadratic Formula to find the potential

3 $3^2$419 3 45 3 3 5

solutions, we have x 2 1 2 2 . Substituting each of these solutions into the original equation, we see that x 3 3 5 2 is a solution, but x 3 3 5 2 is not. Thus x 3 3 5 2 is the only solution.

2 2

xx 1 3 x 3x 1x 3 2 x 1 2 x 6x 9 x 1 x 7x 10 0

x 2 x 5 0. Potential solutions are x 2 and x 5. We must check each potential solution in the original equation. Checking x 2: 2 2 1 3, which is false, so x 2 is not a solution. Checking x 5: 5 5 1 3 5 2 3, which is true, so x 5 is the only solution.

3 x 2 1 x3 x1 x3 x^2 1 x^2 3 x x^2 2x 1 x^2 3x 2 0. Using

3 $3^2 4 1 2$ 3 17

the Quadratic Formula to find the potential solutions, we have x 2 1 2 . Substituting each of these solutions into the original equation, we see that x $\frac{3\ 17}{2}$ is a solution, but x $\frac{3\ 17}{2}$ is not. Thus x $\frac{3\ 17}{2}$ is the only solution.

3x 1 2 x 1 3x 1 ^{2} 2 x 1 ^{2} 3x 1 4 4 x 1 x 1 2x 4 4 x 1 x 2 2 x 1 x 2 ^{2} 2 x 1 ^{2} x^2 4x 4 4 x 1 x^2 8x 0 x x 8 0 x 0 or x 8.

Substituting each of these solutions into the original equation, we see that x 0 is not a solution but x 8 is a solution. Thus, x 8 is the only solution.

48. 1 x 1 x 4 1x 1x 2 1 x 1 x 1x 1x 2 2^2 1 x1 x 2 1x 1x 4

2 2 1 1 x 1 x1 1 x^2 1 x^2 0, so x 0. We verify that this is a solution to the original equation.

x^4 4x^2 3 0. Let y x^2. Then the equation becomes y^2 4y 3 0y 1 y 3 0, so y 1 or y 3. If y 1, then x^2 1 x 1, and if y 3, then x^2 3 x 3.

x^4 5x^2 6 0. Let y x^2. Then the equation becomes y^2 5y 6 0y 2 y 3 0, so y 2 or y 3. If y 2, then x^2 2 x 2, and if y 3, then x^2 3 x 3.

2x^4 4x^2 1 0. The LHS is the sum of two nonnegative numbers and a positive number, so 2x^4 4x^2 1 1 0. This equation has no real solution.

0 x^6 2x^3 3x^3 3 x^3 1 . If x^3 3 0, then x 3 3 x 3 3, or if x^3 1 x1. Thus x 3 3 or x1. The solutions are 3 3 and 1.

53. 0 x^6 26x^3 27x^3 27 x^3 1 . If x^3 27 0 x^3 27, so x 3. If x^3 1 0 x^3 1, so x1.

The solutions are 3 and 1.

54. x^8 15x^4 16 4 0 x^8 15x^4 4 16x^4 16 x^4 1 . If x^4 16 0, then x^4 16 which is impossible (for real numbers). If x 1 0 x 1, so x1. The solutions are 1 and 1.

0 x 5 ^{2} 3 x 5 10 [x 5 5] [x 5 2] x x 7x 0 or x 7. The solutions are 0 and 7.

Let x 1 . Then 0 xxx x 1 2 4 x 1 3 becomes 0 2 4 313 . Now if 1 0,

then x 1 x 1 0 x 1 x 1 x 1 x x 1 2, and if 3 0, then x 1 x 3 0 x 1 x 3 x 13x x 1 4. The solutions are 1 2 and 1 4 .

57. Let 1 x 1 . Then x 1 1 2 2 x 1 1 8 0 becomes 2 28 04 2 0. So 4 0 1 3

4, and 2 02. When4, we have x 1 4 1 4x 43 4x x 4. When

2, we have x 1 1 2 12x 2 32x x x x 2 4x $\frac{3}{2}$. Solutions are $\frac{3}{4}$ and $\frac{3}{2}$.

58. Let $\overline{x\ 2}$. Then $\overline{x\ 2}$ $\overline{x\ 2}$ x 4 becomes 2 2 2 44 0 4 42 . Now if

2 0, then x 2 $\overline{2\ 0}$ x 2 2 x $\overline{2x\ 4}$ x4. The solution is 4.

2 3 4 3 2 3 2 2 3 2 3 3 2

Let u x . Then 0 x 5x 6 becomes u 5u 6 0 u 3 u 2 0 u 3 0 or u 2 0. If u 3 0, then x 3 0 x 3 x 3 3 3. If u 2

0, then $x^{2\,3}$ 2 0 $x^{2\,3}$ 2 x $2^{3\,2}$ 2 2. The solutions are 3 3 and 2 2.

60. Let u 4 x; then0 x 3 4 x 4 u^2 3u 4 u 4 u 1 . So u 4 4 x 4 0 4 x 4

4^4 256, or u 1 4 x 1 0 4 x 1. However, 4 x is the positive fourth root, so this cannot equal 1. The only solution is 256.

61. 4 x $1^{1\,2}$ 5 x $1^{3\,2}$ x $1^{5\,2}$ 0 x 1 4 5 x 1 x 1^2 0

2 2

x 1 4 5x 5 x 2x 1 0 x 1 x 3x 0 x 1 x x 3 0 x 1 or x 0 or

3. The solutions are 1, 0, and 3.

62. Let u x 4; then 0 2 x $4^{7\,3}$ x $4^{4\,3}$ x $4^{1\,3}$ 2$u^{7\,3}$ $u^{4\,3}$ $u^{1\,3}$ $u^{1\,3}$ 2u 1 u 1 . So

x 4 0 x 4, or 2u 1 2 x 4 1 2x 7 0 2x 7 x $\frac{7}{2}$, or u 1 x 4 1 x 5 0 x 5. The solutions are 4, $\frac{7}{2}$,and 5.

63. $x^{3\,2}$ 10$x^{1\,2}$ 25$x^{1\,2}$ 0 $x^{1\,2}$ x^2 10x 250 $x^{1\,2}$ x 5^2 0. Now $x^{1\,2}$ 0, so the only

solution is x 5.

64. $x^{1\,2}$ $x^{1\,2}$ 6$x^{3\,2}$ 0 $x^{3\,2}$ x^2 x 60 $x^{3\,2}$ x 2 x 3 0. Now $x^{1\,2}$ 0, and furthermore

the original equation cannot have a negative solution. Thus, the only solution is x 3.

1 3 1 6

65. Let u $x^{1\,6}$. (We choose the exponent $\frac{1}{6}$ because the LCD of 2, 3, and 6 is 6.) Then $x^{1\,2}$ 3x 3x 9

2 1 6 1 6 6 2

u

$x^{3\,6}$ 3$x^{2\,6}$ 3 $x^{1\,6}$ 9 u^3 3u^2 3u 9 0 u^3 u^2 3u 9 2 u 3 3 u 3 u 3 u^2 3 .

So u 3 0 or u 3 0. If u 3 0, then x 3 0 x 3 x 3 729. If u 3 0, then

$x^{1\,3}$ 3 0 $x^{1\,3}$ 3 x 3^3 27. The solutions are 729 and 27.

Let u ~~x. Then 0 x 5 x 6 becomes u^2 5u 6 u 3 u 2 0. If u 3 0, then x 3 0 x 3 x 9. If u 2 0, then x 2 0 x 2 x 4. The solutions are 9 and 4.~~

1 4 4

67. x^3 x^2 x 0 1 4x 4x^2 01 2x^2 0 1 2x 0 2x1 x $\frac{1}{2}$. The solution is $\frac{1}{2}$.

x^4

4 2 2

68. 0 4x^4 16x^2 4. Multiplying by $\frac{}{4}$ we get 0 1 4x 2 x . Substituting u x , we get 0 1 4u u , and

4 4 4 16 4 4 12 4 2 3

using the Quadratic Formula, we get u 2 411 2 1 2 2 2 23. Substituting

back, we have x 2 3, and since 2 3 and 2 3 are both positive we have x 2 3 or x 2 3.

Thus the solutions are2 3, 2 3,2 3, and 2 3.

x 5 x 5. Squaring both sides, we get x 5 x 25x 5 25 x. Squaring both sides again, we

get x 5 25 x^2 x 5 625 50x x^2 0 x^2 51x 620 x 20 x 31 . Potential solutions are x 20 and x 31. We must check each potential solution in the original equation.

Checking x 20: ~~20 5 20~~ 5 25 20 5 5 20 5, which is true, and hence x 20 is a solution.

Checking x 31: ~~31 5 31~~ 5 36 31 5 37 5, which is false, and hence x 31 is not a solution. The only real solution is x 20.

70. $\sqrt[3]{4x^2 - 4x} = x \Leftrightarrow 4x^2 - 4x = x^3 \Leftrightarrow 0 = x^3 - 4x^2 + 4x = x\left(x^2 - 4x + 4\right) = x\left(x - 2\right)^2$. So $x = 0$ or $x = 2$. The solutions are 0 and 2.

x^2 x $3x$ 3 3 2 0 x^2 x 3 x 3 3 2 0 x 3 x^2 x 3 0 x 3 x^2 x 3 .

If x 3 1 2 0, then x 3 0 x 3. If x^2 x 3 0, then using the Quadratic Formula x $\frac{1}{2}$ 13. The solutions are 3 and $\frac{1}{2}$ 13.

72. Let u 11 x^2. By definition of u we require it to be nonnegative. Now 11 x^2 11 2 x^2 1 2 1. Multiplying both sides by u we obtain u^2 2 u 0 u^2 u 2 u 2 u 1 . So u 2 or u 1. But since u must be nonnegative, we only have u 2 11 x^2 2 11 x^2 4 x^2 7 x 7. The solutions are 7.

73. x x 2 2 2. Squaring both sides, we get x x 2 4 x 2 4 x. Squaring both sides again, we get x 2 2 16 8x x^2 0 x^2 9x 14 0 x 7 x 2 . If x 7 0, then x 7. If x 2 0, then x 2. So x 2 is a solution but x 7 is not, since it does not satisfy the original equation.

74. 1 x 2x 1 5 x . We square both sides to get 1 x 2x 1 5 x 2x 1 x 2 16 8 x 2x 1 16 8 x. Again, squaring both sides, we obtain 2x 1 16 8 x 2 256 256 x 64x 62x 255 256 x. We could continue squaring both sides until we found possible solutions; however, consider the last equation. Since we are working with real numbers, for x to be defined, we must have x 0. Then 62x 255 0 while 256 x 0, so there is no solution.

75. 0 x^4 5ax^2 4a^2 a x^2 4a x^2 . Since a is positive, a x^2 0 x^2 a x a. Again, since a is positive, 4a x^2 0 x^2 4a x 2 a. Thus the four solutions are a and 2 a.

76. x ab ab^2 4 a^2 b^2 ab 3a^2b^2 2a^2 b, but this gives no real solution. Thus, the solution is x b.

x ax a 2 x 6. Squaring both sides, we have x a 2 x ax a x a 2 x 6 2x 2 x ax a 2x 12 2 x ax a 12 x ax a 6. Squaring both sides again we have x a x a 36 x^2 a^2 36 x^2 a^2 36 x 2 36. Checking these answers, we see that x a^2 36 is not a solution (for example, try substituting a 8), but x a 36 is a solution.

16 2 12 3

78. Let x . Then x and x , and so b a 3x b x 3 3 2 2 2 3 6 a . So 6 x a 0 a x x a is one solution. Setting the first factor equal to zero, we have x b 0 x b xb . However, the original equation includes the term b 6 x, and we cannot take the sixth root of a negative number, so this is not a solution. The only solution is x a^6.

79. Let x be the number of people originally intended to take the trip. Then originally, the cost of the trip is $\frac{900}{x}$. After 5 people cancel, there are now x 5 people, each paying $\frac{900}{x}$ 2. Thus 900 x 5 $\frac{900}{x}$ 2900 900 2x $\frac{4500}{x}$ 10 0 2x 10 $\frac{4500}{x}$ 0 2x^2 10x 4500 2x 100 x 45 . Thus either 2x 100 0, so x 50, or

x 45 0, x 45. Since the number of people on the trip must be positive, originally 50 people intended to take the trip.

80. Let n be the number of people in the group, so each person now pays $\frac{120,000}{n}$. If one person joins the group, then there would

be n 1 members in the group, and each person would pay $\frac{120,000}{n}$ 6000. So n 1 $\frac{120,000}{n}$ 6000120,000

6000 n 6000 n 1 6000 120,00020 n n 1 $20nn^2$ $19n$ 20 $20n$

n 120,000 n

0 n^2 n 20 n 4 n 5 . Thus n 4 or n 5. Since n must be positive, there are now 4 friends in the group.

We want to solve for t when P 500. Letting ut and substituting, we have 500 $3t$ 10 t 140

500 $3u^2$ $10u$ 140 0 $3u^2$ $10u$ 360 u 5 1105 3 . Since u t, we must have u 0. So

t u 5 1105 9 414 t 88 62. So it will take 89 days for the fish population to reach 500. 3

Let d be the distance from the lens to the object. Then the distance from the lens to the image is d 4. So substituting

F 4 8, x d, and y d 4, and then solving for x, we have $\frac{1}{4\,8}$ $\frac{1}{d}$ $\frac{1}{d\ 4}$. Now we multiply by the

LCD, 4 8d d 4 , to get d d 4 4 8 d 4 4 8d d^2 $4d$ 9 6d 19 2 0 d^2 13 6d 19 2

13 6 10 4 . So d 1 6 or d 12. Since d 4 must also be positive, the object is 12 cm from the lens. 2

Let x be the height of the pile in feet. Then the diameter is $3x$ and the radius is $\frac{3}{2}x$ feet. Since the volume of the cone is 1000 ft^3, we have $3x$ 2 x 1000 $3x^3$ 1000 x^3 4000 x^3 4000 7 52 feet.

32433

84. Let r be the radius of the tank, in feet. The volume of the spherical tank is $\frac{4}{3}r^3$ and is also 750 0 1337 100 275. So

$\frac{4}{3}r^3$ 100 275 r^3 23 938 r 2 88 feet.

85. Let r be the radius of the larger sphere, in mm. Equating the volumes, we have $\frac{4}{3}r^3$ $\frac{4}{3}2^3$ 3^3 4^3

r^3 2^3 3^3 4^4 r^3 99 r 99 4 63. Therefore, the radius of the larger sphere is about 4 63 mm.

86. We have that the volume is 180 ft^3, so x x 4 x 9 180 x^3 $5x^2$ $36x$ 180 x^3 $5x^2$ $36x$ 180 0

x^2 x 5 36 x 5 0x 5 x^2 $360x$ 5 x 6 x 6 0 x 6 is the only positive

solution. So the box is 2 feet by 6 feet by 15 feet.

Let x be the length, in miles, of the abandoned road to be used. Then the length of the abandoned road not used

is 40 x, and the length of the new road is 10^2 40 x^2 miles, by the Pythagorean Theorem. Since the the road is cost per mile number of miles, we have 100,000x 200,000 6,800,000

cost of

2 80x 1700 68 x. Squaring both sides, we get $4x^2$ 320x 6800 4624 136x x^2

2

6 6 3 3

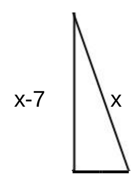
road, 16 miles of the abandoned road should be used. A completely new road would have length 10^2 40^2 (let x 0) and would cost 1700 200,000 8 3 million dollars. So no, it would not be cheaper.

88. Let x be the distance, in feet, that he goes on the boardwalk before veering off onto the sand. The distance along the boardwalk from where he started to the point on the boardwalk closest to the umbrella is $\sqrt{750^2 - 210^2} = 720$ ft. Thus the distance that he walks on the sand is $\sqrt{(720 - x)^2 + 210^2} = \sqrt{518{,}400 - 1440x + x^2 + 44{,}100} = \sqrt{x^2 - 1440x + 562{,}500}$.

	Distance	Rate	Time
Along boardwalk	x	4	$\frac{x}{4}$
Across sand	$\sqrt{x^2 - 1440x + 562{,}500}$	2	$\frac{\sqrt{x^2 - 1440x + 562{,}500}}{2}$

Since 4 minutes 45 seconds = 285 seconds, we equate the time it takes to walk along the boardwalk and across the sand to the total time to get $285 = \frac{x}{4} + \frac{\sqrt{x^2 - 1440x + 562{,}500}}{2} \Leftrightarrow 1140 - x = 2\sqrt{x^2 - 1440x + 562{,}500}$. Squaring both sides, we get $(1140 - x)^2 = 4(x^2 - 1440x + 562{,}500) \Leftrightarrow 1{,}299{,}600 - 2280x + x^2 = 4x^2 - 5760x + 2{,}250{,}000$ $\Leftrightarrow 0 = 3x^2 - 3480x + 950{,}400 = 3(x^2 - 1160x + 316{,}800) = 3(x - 720)(x - 440)$. So $x - 720 = 0$ $\Leftrightarrow x = 720$, and $x - 440 = 0 \Leftrightarrow x = 440$. Checking $x = 720$, the distance across the sand is 210 feet. So $\frac{720}{4} + \frac{210}{2} = 180 + 105 = 285$ seconds. Checking $x = 440$, the distance across the sand is $\sqrt{(720 - 440)^2 + 210^2} = 350$ feet. So $\frac{440}{4} + \frac{350}{2} = 110 + 175 = 285$ seconds. Since both solutions are less than or equal to 720 feet, we have two solutions: he walks 440 feet down the boardwalk and then heads towards his umbrella, or he walks 720 feet down the boardwalk and then heads toward his umbrella.

Let x be the length of the hypotenuse of the triangle, in feet. Then one of the other sides has length $x - 7$ feet, and since the perimeter is 392 feet, the remaining side must have length $392 - x - (x - 7) = 399 - 2x$. From the Pythagorean Theorem, we get $(x - 7)^2 + (399 - 2x)^2 = x^2 \Leftrightarrow 4x^2 - 1610x + 159250 = 0$. Using the Quadratic Formula, we get



$x = \frac{1610 \pm \sqrt{1610^2 - 4(4)(159250)}}{2(4)} = \frac{1610 \pm \sqrt{44100}}{8} = \frac{1610 \pm 210}{8}$, and so $x = 227.5$ or $x = 175$. But if $x = 227.5$, then the side of length $x - 7$ combined with the hypotenuse already exceeds the perimeter of 392 feet, and so we must have $x = 175$. Thus the other sides have length $175 - 7 = 168$ and $399 - 2(175) = 49$. The lot has sides of length 49 feet, 168 feet, and feet.

Let h be the height of the screens in inches. The width of the smaller screen is $h + 7$ inches, and the width of the bigger screen is $1.8h$ inches. The diagonal measure of the smaller screen is $\sqrt{h^2 + (h + 7)^2}$, and the diagonal measure of the larger screen is $\sqrt{h^2 + (1.8h)^2} = \sqrt{4.24h^2} \approx 2.06h$. Thus $\sqrt{h^2 + (h + 7)^2} + 3 = 2.06h \Leftrightarrow \sqrt{h^2 + (h + 7)^2} = 2.06h - 3$. Squaring both sides gives $h^2 + h^2 + 14h + 49 = 4.24h^2 - 12.36h + 9 \Leftrightarrow 0 = 2.24h^2 - 26.36h - 40$. Applying the Quadratic Formula, we obtain $h = \frac{26.36 \pm \sqrt{26.36^2 - 4(2.24)(-40)}}{2(2.24)} = \frac{26.36 \pm \sqrt{1053.2496}}{4.48} \approx \frac{26.36 \pm 32.45}{4.48}$. So $h \approx \frac{26.36 + 32.45}{4.48} \approx 13.13$. Thus, the screens are approximately 13.1 inches high.

d d 1 1 2 1 2 1

91. Since the total time is 3 s, we have 3 4 1090 . Letting d, we have 3 4 1090 1090 4 3 0

2 5456540 0 545 591 054 . Since 0, we have d 11 51, so d 132 56. The well 4

is 132 6 ft deep.

(a) *Method 1:* Let ~~$u x$, so u^2 x. Thus $x\ x$ 2 0 becomes u^2 u 2 0 u 2 u 1 0. So u 2 or u 1. If u 2, then x 2 x 4. If u 1, then x 1 x 1. So the possible solutions are 4 and 1. Checking x 4 we have 4 4 2 4 2 2 0. Checking x 1 we have 1 1 2 1 1 2 0. The only solution is 4.~~

Method 2: $x\ x$ 2 0 x 2 $x\ x^2$ 4x 4 $x\ x^2$ 5x 4 0 x 4 x 1 0. So the possible solutions are 4 and 1. Checking will result in the same solution.

(b) *Method 1:* Let u 1, so u^2 1. Thus 12 2 10 1 0 becomes 12u^2 10u 1 0. Using

x 3 x 3 x 3 x 3

the Quadratic Formula, we have u 10 10 412 1 / 2 12 10 52 / 24 10213 / 24 5 13 / 12. If u 5 13 / 12,

then 1 / x 3 5 13 / 12 x 3 12 / 5 13 12 5 13 / 12 5 13. So x 2 13.

If u 5 13 / 12, then 1 / x 3 5 13 / 12 x 3 12 / 5 13 12 5 13 / 12 5 13. So 2 13.

The solutions are 2 13.

Method 2: Multiplying by the LCD, x 3^2, we get x 3^2 12 / x 3^2 10 / x 3 10 x 3^2

12 10 x 3x 3^2 0 12 10x 30 x^2 6x 9 0 x^2 4x 9 0. Using the Quadratic

Formula, we have u 4 4^2 41 9 / 2 4 52 / 2 4 2 13 / 22 2 13. The solutions are 2 13.

1.7 SOLVING INEQUALITIES

1. (a) If x 5, then x 3 5 3 x 3 2.

(b) If x 5, then 3 x 3 5 3x 15.

(c) If x 2, then 3 x 3 2 3x 6.

(d) If x 2, then x 2.

2. To solve the nonlinear inequality $\frac{x\ 1}{x\ 2}$ 0 we first observe that the numbers 1 and 2 are zeros of the numerator and denominator. These numbers divide the real line into the three intervals 1, 1 2, and 2.

Interval	1	1 2	2
Sign of x 1 **Sign of** x 2			
Sign of x 1 x 2			

The endpoint 1 satisfies the inequality, because $\frac{1\ 1}{1\ 2}$ 0 0, but 2 fails to satisfy the inequality because $\frac{2\ 1}{2\ 2}$ is not defined.

Thus, referring to the table, we see that the solution of the inequality is [1 2 .

(a) No. For example, if x2, then $x\ x$ 12 1 2 0.

No. For example, if x 2, then $x\ x$ 1 2 3 6.

(a) To solve 3x 7, start by dividing both sides of the inequality by 3.

To solve 5x 2 1, start by adding 2 to both sides of the inequality.

5.

x	$2 \quad 3x \quad \frac{1}{3}$
5	17 $\frac{1}{3}$; no
1	5 $\frac{1}{3}$; no
0	2 0; no
$\frac{2}{3}$	0 $\frac{1}{3}$; no
$\frac{5}{6}$	$\frac{1}{2}$ $\frac{1}{3}$; yes
1	1 $\frac{1}{3}$; yes
$\sqrt{5}$	4 7 $\frac{1}{3}$; yes
3	7 $\frac{1}{3}$; yes
5	13 $\frac{1}{3}$; yes

6.

	$1 \quad 2x \quad 5x$
5	11 25; yes
1	3 5; yes
0	1 0; yes
$\frac{2}{3}$	$\frac{1}{3}$ $\frac{10}{3}$; no
$\frac{5}{6}$	$\frac{2}{3}$ $\frac{25}{6}$; no
1	1 5; no
$\sqrt{5}$	3 47 11 18; no
3	5 15; no
5	9 25; no

7.

x	$1 \quad 2x \quad 4 \quad 7$
5	11 4 7; no
1	1 6 7; no
0	1 4 7; no
$\frac{2}{3}$	1 $\frac{8}{3}$ 7; no
$\frac{5}{6}$	1 $\frac{7}{3}$ 7; no
1	1 2 7; no
$\sqrt{5}$	1 0 47 7; no
3	1 2 7; yes
5	1 6 7; yes

The elements 3 and 5 satisfy the inequality.

8.

	$2 \quad 3 \quad x \quad 2$
5	2 8 2; no
1	2 4 2; no
0	2 3 2; no
$\frac{2}{3}$	2 $\frac{7}{3}$ 2; no
$\frac{5}{6}$	2 $\frac{13}{6}$ 2; no
1	2 2 2; no
$\sqrt{5}$	2 0 76 2; yes
3	2 0 2; yes
5	2 2 2; yes

The elements $\sqrt{5}$, 3, and 5 satisfy the inequality.

x	$\frac{1}{x} \quad \frac{1}{2}$
5	$\frac{1}{5}$ $\frac{1}{2}$; yes
1	1 $\frac{1}{2}$; yes
0	$\frac{1}{0}$ is undefined; no
$\frac{2}{3}$	$\frac{3}{2}$ $\frac{1}{2}$; no
$\frac{5}{6}$	$\frac{6}{5}$ $\frac{1}{2}$; no
1	1 $\frac{1}{2}$; no
$\sqrt{5}$	0 45 $\frac{1}{2}$; yes
3	$\frac{1}{3}$ $\frac{1}{2}$; yes
5	$\frac{1}{5}$ $\frac{1}{2}$; yes

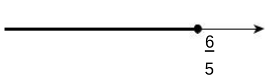
The elements 5, 1, $\sqrt{5}$, 3, and 5 satisfy the inequality.

x	$x^2 \quad 2 \quad 4$
5	27 4; no
1	3 4; yes
0	2 4; yes
$\frac{2}{3}$	$\frac{22}{9}$ 4; yes
$\frac{5}{6}$	$\frac{97}{36}$ 4; yes
1	3 4; yes
$\sqrt{5}$	7 4; no
3	11 4; no
5	27 4; no

The elements 1, 0, $\frac{2}{3}$, $\frac{5}{6}$, and 1 satisfy the inequality.

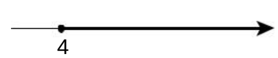
11. $5x \quad 6 \quad x \quad \frac{6}{5}$. Interval: $\frac{6}{5}$

Graph:



12. $2x \quad 8 \quad x \quad 4$. Interval: [4

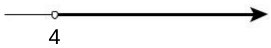
Graph:



13. 2*x* 5 3 2*x* 8 *x* 4

Interval: 4

Graph:

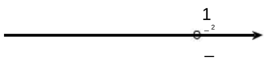


2 3*x* 8 3*x* 2 8 *x* 2 Interval: 2

Graph:

2*x* 1 0 2*x*1 *x*

Graph:



19. 1 4*x* 5 2*x* 6*x* 4 *x* $\frac{2}{3}$

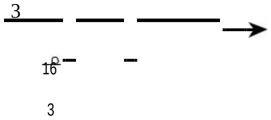
Interval: $\frac{2}{3}$

Graph:

21. $\frac{1}{2}x$ $\frac{2}{3}$ 2 $\frac{1}{2}x$ $\frac{8}{3}$ *x* $\frac{16}{3}$

Interval: $\frac{16}{3}$

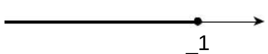
Graph:



4 3*x*1 8*x*4 3*x*1 8*x* 5*x*5

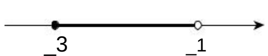
x 1 Interval: 1]

Graph:



2 *x* 5 4 3 *x* 1 Interval: [3 1

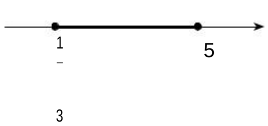
Graph:



27. 6 3*x* 7 8 1 3*x* 15 $\frac{1}{3}$ *x* 5

Interval: $\frac{1}{3}$ 5

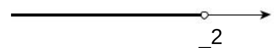
Graph:



14. 3*x* 11 5 3*x* 6 *x* 2

Interval: 2

Graph:



1 5 2*x* 2*x* 5 1 *x* 2 Interval: 2

Graph:

18. 0 5 2*x* 2*x* 5 *x* $\frac{5}{2}$

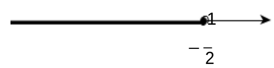
Interval: $\frac{5}{2}$

Graph:

5 3*x* 2 9*x* 6*x*3 *x* $\frac{1}{2}$

Interval: $\frac{1}{2}$

Graph:

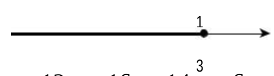


$\frac{2}{3}$ $\frac{1}{2}x$ $\frac{1}{6}$ *x* (multiply both sides by 6)

4 3*x* 1 6*x* 3 9*x* $\frac{1}{3}$ *x*

Interval: $\frac{1}{3}$

Graph:

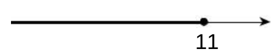


24. 2 7*x* 3 12*x* 16 14*x* 6 12*x* 16

2*x* 22 *x* 11

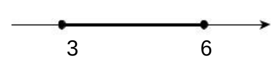
Interval: 11]

Graph:



5 3*x* 4 14 9 3*x* 18 3 *x* 6 Interval: [3 6]

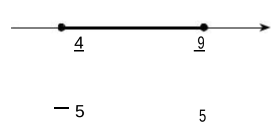
Graph:



28. 8 5*x* 4 5 4 5*x* 9 $\frac{4}{5}$ *x* $\frac{9}{5}$

Interval: $\frac{4}{5}$ $\frac{9}{5}$

Graph:

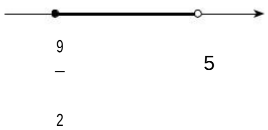


29. 2 8 2x1102x9 5 x $\frac{9}{2}$

$\frac{9}{2}$ x 5

Interval: $\frac{9}{2}$ 5

Graph:

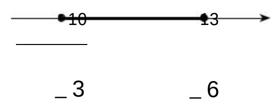


30. 3 3x 7 $\frac{1}{2}$ 10 3x $\frac{13}{2}$

$\frac{10}{3}$ x $\frac{13}{6}$

Interval: $\frac{10}{3}$ $\frac{13}{6}$

Graph:



31. $\frac{2}{3}$ $\frac{2x\ \ 3}{12}$ $\frac{1}{6}$ 8 2x 3 2 (multiply each expression by 12) 11 2x 5 $\frac{11}{2}$ x $\frac{5}{2}$

$\frac{5}{13}$ $\frac{11}{}$

Interval: 2 2

Graph:

32. $\frac{1}{2}$ $\frac{4\ \ 3x}{5}$ $\frac{1}{4}$ (multiply each expression by 20)

10 4 4 3x 510 16 12x 5

2612x11 6 $\frac{13}{}$ x 12 $\frac{11}{12}$ $\frac{11}{}$ x 6

Interval: $\frac{11}{12}$ $\frac{13}{6}$

Graph:

x 2 x 3 0. The expression on the left of the inequality changes sign where x 2 and where x 3. Thus we must check the intervals in the following table.

Interval	2	2 3	3
Sign of x 2 Sign of x 3			
Sign of x 2 x 3			

From the table, the solution set is x 2 x 3 . Interval: 2 3 .

Graph:

x 5 x 4 0. The expression on the left of the inequality changes sign when x 5 and x 4. Thus we must check the intervals in the following table.

Interval	4	4 5	5
Sign of x 5 Sign of x 4			
Sign of x 5 x 4			

From the table, the solution set is x x 4 or 5 x . Interval: 4] [5 .

Graph:

x 2x 7 0. The expression on the left of the inequality changes sign where x 0 and where x $\frac{7}{2}$. Thus we must check the intervals in the following table.

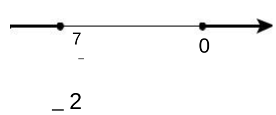
Interval	$\frac{7}{2}$	$\frac{7}{2}$ 0	0
Sign of x Sign of 2x 7			
Sign of x 2x 7			

From the table, the solution set is

$\frac{7}{2}$ or 0 x .xx

Interval: $\frac{7}{2}$ [0 .

Graph:

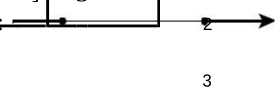


x 2 3x 0. The expression on the left of the inequality changes sign when x 0 and x $\frac{2}{3}$. Thus we must check the intervals in the following table.

Interval	0	0 $\frac{2}{3}$	$\frac{2}{3}$
Sign of x Sign of 2 3x			
Sign of x 2 3x			

From the table, the solution set is x 0 or $\frac{2}{3}$ x . Interval:0] $\frac{2}{3}$.

Graph:

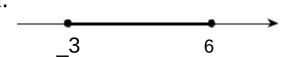


x^2 3x 18 0 x 3 x 6 0. The expression on the left of the inequality changes sign where x 6 and where x 3. Thus we must check the intervals in the following table.

Interval	3	3 6	6
Sign of x 3 Sign of x 6			
Sign of x 3 x 6			

From the table, the solution set is x 3 x 6 . Interval: [3 6].

Graph:

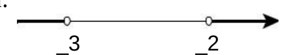


38. x^2 5x 6 0 x 3 x 2 0. The expression on the left of the inequality changes sign when x 3 and 2. Thus we must check the intervals in the following table.

Interval	3	3 2	2
Sign of x 3 Sign of x 2			
Sign of x 3 x 2			

From the table, the solution set is x x 3 or 2 x . Interval: 3 2 .

Graph:



2x^2 x 1 2x^2 x 1 0x 1 2x 1 0. The expression on the left of the inequality changes sign where x 1 and where x $\frac{1}{2}$. Thus we must check the intervals in the following table.

Interval	1	1 $\frac{1}{2}$	$\frac{1}{2}$
Sign of x 1 Sign of 2x 1			
Sign of x 1 2x 1			

From the table, the solution set is x x1 or $\frac{1}{2}$ x .

Interval:1] $\frac{1}{2}$.

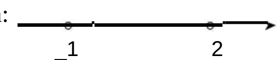
Graph:

40. x^2 x 2 x^2 x 2 0 x 1 x 2 0. The expression on the left of the inequality changes sign when 1 and x 2. Thus we must check the intervals in the following table.

Interval	1	1 2	2
Sign of x 1 Sign of x 2			
Sign of x 1 x 2			

From the table, the solution set is x 1 x 2 . Interval: 1 2 .

Graph:

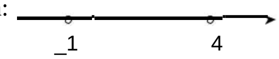


$3x^2$ $3x$ $2x^2$ 4 x^2 $3x$ 4 0 x 1 x 4 0. The expression on the left of the inequality changes sign where x 1 and where x 4. Thus we must check the intervals in the following table.

Interval	1	1 4	4
Sign of x 1 Sign of x 4			
Sign of x 1 x 4			

From the table, the solution set is x 1 x 4 . Interval: 1 4 .

Graph:



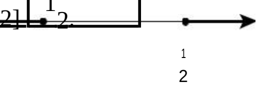
$5x^2$ $3x$ $3x^2$ 2 $2x^2$ $3x$ 2 0 $2x$ 1 x 2 0. The expression on the left of the inequality changes sign when x $\frac{1}{2}$ and x 2. Thus we must check the intervals in the following table.

Interval	2	2 $\frac{1}{2}$	$\frac{1}{2}$
Sign of $2x$ 1 Sign of x 2			
Sign of $2x$ 1 x 2			

From the table, the solution set is x2 or $-\frac{1}{2}$ x .

Interval:2] $\frac{1}{2}$.

Graph:



x^2 3 x 6 x^2 $3x$ 18 0 x 3 x 6 0. The expression on the left of the inequality changes sign where x 6 and where x 3. Thus we must check the intervals in the following table.

Interval	3	3 6	6
Sign of x 3 Sign of x 6			
Sign of x 3 x 6			

From the table, the solution set is x x 3 or 6 x .

Interval: 3 6 .

Graph:

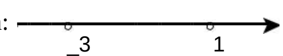
44. x^2 $2x$ 3 x^2 $2x$ 3 0 x 3 x 1 0. The expression on the left of the inequality changes sign when x 3 and x 1. Thus we must check the intervals in the following table.

Interval	3	3 1	1
Sign of x 3 Sign of x 1			
Sign of x 3 x 1			

From the table, the solution set is x x 3 or 1 x .

Interval: 3 1 .

Graph:

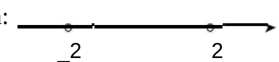


x^2 4 x^2 4 0 x 2 x 2 0. The expression on the left of the inequality changes sign where x 2 and where x 2. Thus we must check the intervals in the following table.

Interval	2	2 2	2
Sign of x 2 Sign of x 2			
Sign of x 2 x 2			

From the table, the solution set is x 2 x 2 . Interval: 2 2 .

Graph:



x^2 9 x^2 9 0 *x* 3 *x* 3 0. The expression on the left of the inequality changes sign when *x* 3 and *x* 3. Thus we must check the intervals in the following table.

Interval	3	3 3	3
Sign of *x* 3 Sign of *x* 3			
Sign of *x* 3 *x* 3			

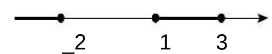
From the table, the solution set is *x* *x* 3 or 3 *x* . Interval: 3] [3 .

Graph:

x 2 *x* 1 *x* 3 0. The expression on the left of the inequality changes sign when *x* 2, *x* 1, and *x* 3. Thus we must check the intervals in the following table.

Interval	2	2 1	1 3	3
Sign of *x* 2 Sign of *x* 1 Sign of *x* 3				
Sign of *x* 2 *x* 1 *x* 3				

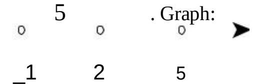
From the table, the solution set is *x* *x*2 or 1 *x* 3 . Interval:2] [1 3]. Graph:

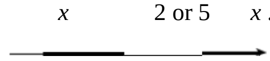


x 5 *x* 2 *x* 1 0. The expression on the left of the inequality changes sign when *x* 5, *x* 2, and *x* 1. Thus we must check the intervals in the following table.

Interval	1	1 2	2 5	5
Sign of *x* 5 Sign of *x* 2 Sign of *x* 1				
Sign of *x* 5 *x* 2 *x* 1				

From the table, the solution set is *x* 1 *x* 2 or 5 *x* . Interval: 1 2 5 . Graph:





x 4 *x* 2 ^{2} 0. Note that *x* 2 ^{2} 0 for all *x* 2, so the expression on the left of the original inequality changes sign only when *x* 4. We check the intervals in the following table.

Interval	2	2 4	4
Sign of *x* 4 Sign of *x* 2 ^{2}			
Sign of *x* 4 *x* 2 ^{2}			

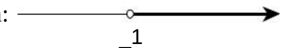
From the table, the solution set is *x* *x* 2 and *x* 4 . We exclude the endpoint 2 since the original expression cannot be 0. Interval: 2 2 4 .

Graph:

$x\ 3^2\ x\ 1\ 0$. Note that $x\ 3^2\ 0$ for all $x\ 3$, so the expression on the left of the original inequality changes sign only when $x\ 1$. We check the intervals in the following table.

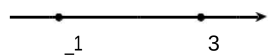
Interval	3	3 1	1
Sign of $x\ \ 3^2$ Sign of $x\ \ 1$			
Sign of $x\ \ 3^2\ \ x\ \ 1$			

From the table, the solution set is $x\ x\ 1$. (The endpoint 3 is already excluded.) Interval: 1 .

Graph: 

$x\ 2^2\ x\ 3\ x\ 1\ 0$. Note that $x\ 2^2\ 0$ for all x, so the expression on the left of the original inequality changes sign only when $x\ 1$ and $x\ 3$. We check the intervals in the following table.

Interval	1	1 2	2 3	3
Sign of $x\ \ 2^2$ Sign of $x\ \ 3$ Sign of $x\ \ 1$				
Sign of $x\ \ 2^2\ \ x\ \ 3\ \ x\ \ 1$				

From the table, the solution set is $x\ \ 1\ \ x\ \ 3$. Interval: [1 3]. Graph: 

52. $x^2\ \ x^2\ \ 1\ \ 0\ \ x^2\ x\ \ 1\ \ x\ \ 1\ \ 0$. The expression on the left of the inequality changes sign when x 1 and

0. Thus we must check the intervals in the following table.

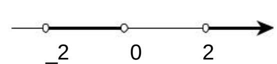
Interval	1	1 0	0 1	1
Sign of x^2 Sign of $x\ \ 1$ Sign of $x\ \ 1$				
Sign of $x^2\ \ x^2\ 1$				

From the table, the solution set is $x\ x\ 1$, $x\ 0$, or $1\ x$. (The endpoint 0 is included since the original expression is allowed to be 0.) Interval: 1] 0 [1 . Graph: 

53. $x^3\ \ 4x\ \ 0\ \ x\ x^2\ \ 4\ \ 0\ \ x\ x\ \ 2\ \ x\ \ 2\ \ 0$. The expression on the left of the inequality changes sign where

0, $x2$ and where x 4. Thus we must check the intervals in the following table.

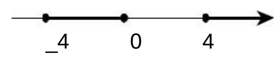
Interval	2	2 0	0 2	2
Sign of x Sign of $x\ \ 2$ Sign of $x\ \ 2$				
Sign of $x\ x\ \ 2\ \ x\ \ 2$				

From the table, the solution set is $x\ \ 2\ \ x\ \ 0$ or $x\ \ 2$. Interval: 2 0 2. Graph: 

54. $16x\ x^3\ 0\ x^3\ 16x\ x\ x^2\ 16\ x\ x\ 4\ x\ 4$. The expression on the left of the inequality changes sign when x 4, x 0, and x 4. Thus we must check the intervals in the following table.

Interval	4	4 0	0 4	4
Sign of x 4 Sign of x Sign of x 4				
Sign of $x\ x$ 4 x 4				

From the table, the solution set is x 4 x 0 or 4 x . Interval: [4 0] [4. Graph:



55. $\frac{x\ 3}{2x\ 1}$ 0. The expression on the left of the inequality changes sign where x3 and where x $\frac{1}{2}$. Thus we must check the intervals in the following table.

Interval	3	3 $\frac{1}{2}$	$\frac{1}{2}$
Sign of x 3			
Sign of 2x 1			
Sign of $\frac{x\ 3}{2x\ 1}$			

From the table, the solution set is

x3 or x $\frac{1}{2}$. Since the denominator cannot equal 0, x $\frac{1}{2}$.

Interval:3] $\frac{1}{2}$.

Graph:

$\frac{4\ x}{}$ 0. The expression on the left of the inequality changes sign when x4 and x 4. Thus we must check the x 4 intervals in the following table.

Interval	4	4 4	4
Sign of 4 x Sign of x 4			
$\frac{4\ x}{}$ Sign of x 4			

From the table, the solution set is x x 4 or x 4 .

Interval: 4 4 .

Graph:

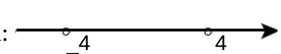
$\frac{4\ x}{}$ 0. The expression on the left of the inequality changes sign where x4. Thus we must check the intervals in x 4 the following table.

Interval	4	4 4	4
Sign of 4 x Sign of x 4			
Sign of $\frac{4\ x}{x\ 4}$			

From the table, the solution set is x x 4 or x 4 .

Interval: 4 4 .

Graph:



58. $2 \quad \frac{x \quad 1}{x \quad 3} \quad 0 \qquad \frac{x \quad 1}{x \quad 3} \quad 2 \quad 0 \qquad \frac{x \quad 1}{x \quad 3} \qquad \frac{2\,x \quad 3}{x \quad 3} \quad 0 \qquad \frac{3x \quad 5}{x \quad 3}$ The expression on the left of the inequality changes sign when $x \quad \frac{5}{3}$ and $x \quad 3$. Thus we must check the intervals in the following table.

Interval	$\frac{5}{3}$	$\frac{5}{3}\ 3$	3
Sign of $3x \quad 5$ Sign of $x \quad 3$			
Sign of $\frac{3x \quad 5}{x \quad 3}$			

From the table, the solution set is

$x \qquad x$

or 3 $\qquad x$

3

Interval: $\quad 5 \quad 3 \quad .$

3

Graph: 

59. $\frac{2x \quad 1}{x \quad 5} \quad 3 \qquad \frac{2x \quad 1}{x \quad 5} \quad 3 \quad 0 \qquad \frac{2x \quad 1 \quad 3\,x \quad 5}{x \quad 5} \qquad \frac{}{x \quad 5} \quad 0 \qquad \frac{x \quad 16}{x \quad 5}$ 0. The expression on the left of the inequality changes sign where $x \quad 16$ and where x 5. Thus we must check the intervals in the following table.

Interval	5	5 16	16
Sign of $x \quad 16$ Sign of $x \quad 5$			
Sign of $\frac{x \quad 16}{x \quad 5}$			

From the table, the solution set is $x\ x$ 5 or x 16 . Since the denominator cannot equal 0, we must have x 5. Interval: 5 [16 .

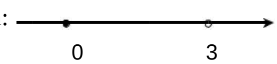
Graph: 

60. $\frac{3 \quad x}{3 \quad x} \quad 1 \qquad \frac{3 \quad x}{3 \quad x} \quad 1 \quad 0 \qquad \frac{3 \quad x}{3 \quad x} \qquad \frac{3 \quad x}{3 \quad x} \quad 0 \qquad \frac{2x}{3 \quad x}$ 0. The expression on the left of the inequality changes sign when $x \quad 0$ and $x \quad 3$. Thus we must check the intervals in the following table.

Interval	0	0 3	3
Sign of $3 \quad x$ Sign of $2x$			
Sign of $\frac{2x}{3 \quad x}$			

Since the denominator cannot equal 0, we must have x 3. The solution set is $x \quad 0 \quad x \quad 3$.

Interval: [0 3 .

Graph: 

61. $x \quad \frac{4}{x} \qquad x \quad \frac{4}{x} \quad 0 \qquad \frac{4 \quad x\,x}{x} \quad 0 \qquad \frac{4 \quad x^2}{x} \quad 0 \qquad \frac{2 \quad x\ 2 \quad x}{x}$ 0. The expression on the left of the inequality changes sign where x 0, where x 2, and where x 2. Thus we must check the intervals in the following table.

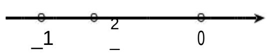
Interval	2	2 0	0 2	2
Sign of $2 \quad x$ Sign of x Sign of $2 \quad x$				
Sign of $\frac{2 \quad x\ 2 \quad x}{x}$				

From the table, the solution set is $x \quad 2 \quad x \quad 0$ or $2 \quad x$. Interval: 2 0 2. Graph: _2 0 2

62. $\frac{3}{x-1} - \frac{4}{x} \ge 1 \Leftrightarrow \frac{3}{x-1} - \frac{4}{x} - 1 \ge 0 \Leftrightarrow \frac{3x}{x(x-1)} - \frac{4(x-1)}{x(x-1)} - \frac{x(x-1)}{x(x-1)} \ge 0 \Leftrightarrow \frac{3x - 4x + 4 - x^2 + x}{x(x-1)} \ge 0 \Leftrightarrow \frac{2x - 3x^2}{x(x-1)} \ge 0 \Leftrightarrow \frac{x(2 - 3x)}{x(x-1)} \ge 0$. The expression on the left of the inequality changes sign when $x = 0$, $x = \frac{2}{3}$, and $x = 1$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, -\frac{2}{3})$	$(-\frac{2}{3}, 0)$	$(0, \infty)$
Sign of x				
Sign of $2 - 3x$				
Sign of $x - 1$				
Sign of $\frac{(2 - x)(2 + x)}{x}$				

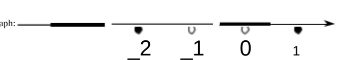
From the table, the solution set is $\{x \mid x - 1$ or $\frac{2}{3}$ x $0\}$. Interval: 1 $\frac{2}{3}$ 0.

Graph: 

63. $\frac{2}{x+1} \le \frac{2}{x} \le \frac{2}{x - 1} \le \frac{2}{x} \Leftrightarrow \frac{x(x-1)}{x(x-1)} \ldots \frac{2x}{\ldots} \ldots \frac{2(x+1)}{\ldots} \ldots \frac{x^2 - x - 2x - 2x - 2}{x(x+1)} \ldots 0$

$\frac{x^2 - x - 2}{x(x+1)} \ldots 0 \Leftrightarrow \frac{(x-2)(x+1)}{x(x+1)} \ldots 0$. The expression on the left of the inequality changes sign where $x = 2$, where -1, where $x = 0$, and where $x = 1$. Thus we must check the intervals in the following table.

Interval	2	$2\ \ 1$	$1\ 0$	$0\ 1$	1
Sign of $x - 2$ Sign of $x - 1$ Sign of x Sign of $x - 1$					
Sign of $\frac{(x-2)(x-1)}{x(x-1)}$					

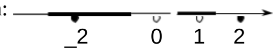
Since x 1 and x 0 yield undefined expressions, we cannot include them in the solution. From the table, the solution set is x 2 x 1 or 0 x 1. Interval: $[2\ \ 1$ $0\ 1]$. Graph: 

64. $\frac{3}{x \; 1} \quad \frac{4}{x} \quad 1 \quad \frac{3}{x \; 1} \quad \frac{4}{x} \quad 1 \; 0 \quad \frac{3x}{x x \; 1} \quad \frac{4 \, x \; 1}{x \, x \; 1} \quad \frac{x x \; 1}{x x \; 1} \; 0 \quad \frac{3x \; 4x \; 4 \; x^2 \; x}{x \, x \; 1} \quad 0$

$\frac{4 \; x^2}{x x \; 1} \quad 0 \quad \frac{2 \; x \; 2 \; x}{x \; x \; 1} \quad 0$ The expression on the left of the inequality changes sign when x 2, x2, 0, and x 1. Thus we must check the intervals in the following table.

Interval	2	2 0	0 1	1 2	2
Sign of 2 x					
Sign of 2 x					
Sign of x					
Sign of x 1					
Sign of $\frac{2 \; x \; 2 \; x}{x \; x \; 1}$					

Since x 0 and x 1 give undefined expressions, we cannot include them in the solution. From the table, the solution set is x 2 x 0 or 1 x 2 . Interval: [2 0 1 2]. Graph:

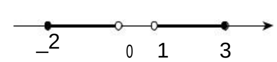


$\frac{6666x6 \; x \; 1x \; x \; 1}{x \; 1 \; x} \; 1 \quad \frac{}{x \; 1 \; x^2} \; 1 \; 0 \quad \frac{}{x \; x \; 1x \; x \; 1x \; x \; 1^2} \; 0$

$\frac{6x \; 6x \; 6 \; x \; x}{x \; x \; 1} \quad 0 \quad \frac{x \; x \; 6}{x \; x \; 1} \quad 0 \quad \frac{x \; 3 \; x \; 2}{x x \; 1} \quad 0.$ The expression on the left of the inequality changes sign where x 3, where x 2, where x 0, and where x 1. Thus we must check the intervals in the following table.

Interval	2	2 0	0 1	1 3	3
Sign of x 3					
Sign of x 2					
Sign of x					
Sign of x 1					
Sign of $\frac{x \; 3 \; x \; 2}{x \; x \, 1}$					

From the table, the solution set is x 2 x 0 or 1 x 3 . The points x 0 and x 1 are excluded from the solution set because they make the denominator zero. Interval: [2 0 1 3]. Graph:



3 4 3 4 3*x* 4 *x* 1*x x* 1 3*x* 4*x* 4 x^2 *x*

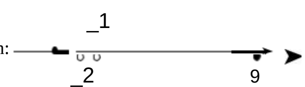
x 5 *x* 5 *x x* 1 2 5 4 2 *x* 1 x^2 *x* 10 8*x* 8

66. 2 *x* 1 4 2 *x* 1 4 0 2 *x* 1 2 *x* 1 2 *x* 1 0 2 *x* 1 0
2

x 7*x* 18 0 *x* 9 *x* 2 0. The expression on the left of the inequality changes sign when *x* 9, *x*2,
2 *x* 1 2 *x* 1

and *x*1. Thus we must check the intervals in the following table.

Interval	2	2 1	1 9	9
Sign of *x* 9 Sign of *x* 2 Sign of *x* 1				
Sign of *x* 9 / 2 *x* 2				

From the table, the solution set is *x* 2 *x* 1 or 9 *x* . The point *x* 1 is excluded from the solution set because it makes the expression undefined. Interval: [2 1 [9 . Graph: 

x 2 *x* 1 *x* 2 *x* 1 *x* 2 *x* 2 *x* 1 *x* 3

67. *x* 3 *x* 2 *x* 3 *x* 2 0 *x* 3 *x* 2*x* 2 *x* 3 0

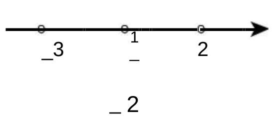
x^2 4 x^2 2*x* 3 2*x* 1

x 3 *x* 2 0 *x* 3 *x* 2 0. The expression on the left of the inequality

changes sign where *x* $\frac{1}{2}$, where *x* 3, and where *x* 2. Thus we must check the intervals in the following table.

Interval	3	3 $\frac{1}{2}$	$\frac{1}{2}$ 2	2
Sign of 2*x* 1 Sign of *x* 3 Sign of *x* 2				
Sign of 2*x* 1 / *x* 3 *x* 2				

From the table, the solution set is *x* 3 *x* $\frac{1}{2}$ or 2 *x* . Interval: 3 $\frac{1}{2}$ 2.

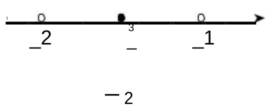
Graph: 

68. $\frac{1}{x\ 1}\quad \frac{1}{x\ 2}\quad 0\quad \frac{x\ 2}{x\ 1\ x\ 2}\quad \frac{x\ 1}{x\ 1\ x\ 2}\quad 0\quad \frac{x\ 2\ x\ 1}{x\ 1\ x\ 2}\ 0\quad \frac{2x\ 3}{x\ 1\ x\ 2}\quad 0$. The expression on the left of the inequality changes sign when $x\ \frac{3}{2}$, $x1$, and $x2$. Thus we must check the intervals in the following table.

Interval	2	2 $\frac{3}{2}$	$\frac{3}{2}$ 1	1
Sign of $2x\ 3$				
Sign of $x\ 1$				
Sign of $x\ 2$				
Sign of $\frac{2x\ 3}{x\ 1\ x\ 2}$				

From the table, the solution set is $x\ x2$ or $\frac{3}{2}$ $x1$. The points $x\ 2$ and $x1$ are excluded from the solution because the expression is undefined at those values. Interval:2 $\frac{3}{2}$ 1 .

Graph:

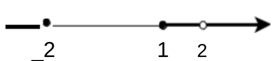


$\frac{x\ 1\ x\ 2}{x\ 2^2}$ 0. Note that $x\ 2^2$ 0 for all x. The expression on the left of the original inequality changes sign when x 2 and x 1. We check the intervals in the following table.

Interval	2	2 1	1 2	2
Sign of $x\ 1$				
Sign of $x\ 2$				
Sign of $x\ 2^2$				
Sign of $\frac{x\ 1\ x\ 2}{x\ 2^2}$				

From the table, and recalling that the point x 2 is excluded from the solution because the expression is undefined at those values, the solution set is $x\ x$ 2 or x 1 and x 2 . Interval: 2] [1 2 2 .

Graph:

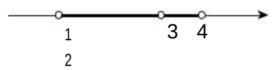


70. $\frac{2x\ \ 1\ x\ \ 3^2}{x\ \ 4}$ 0. Note that x 3^2 0 for all x 3. The expression on the left of the inequality changes sign

when x $\frac{1}{2}$ and x 4. We check the intervals in the following table.

Interval	$\frac{1}{2}$	$\frac{1}{2}$ 3	3 4	4
Sign of 2x 1 Sign of x 3^2 Sign of x 4				
Sign of $\frac{2x\ \ 1\ x\ \ 3^2}{x\ \ 4}$				

From the table, the solution set is x x 3 and $\frac{1}{2}$ x 4 . We exclude the endpoint 3 because the original expression cannot be 0. Interval: $\frac{1}{2}$ 3 3 4 . Graph:



71. x^4 x^2 x^4 x^2 0 x^2 x^2 10 x^2 x 1 x 10. The expression on the left of the inequality

changes sign where x 0, where x 1, and where x1. Thus we must check the intervals in the following table.

Interval	1	1 0	0 1	1
Sign of x^2 Sign of x 1 Sign of x 1				
Sign of x^2 x 1 x 1				

From the table, the solution set is x x1 or 1 x . Interval:11. Graph:

72. x^5 x^2 x^5 x^2 0 x^2 x^3 10 x^2 x 1 x^2 x 10. The expression on the left of the inequality

changes sign when x 0 and x 1. But the solution of x^2 x 1 0 are x $\frac{1\ \ \sqrt{1^2\ 411}}{21}$ $\frac{1\ \ 3}{2}$.

Since these are not real solutions. The expression x^2 x 1 does not change signs, so we must check the intervals in the following table.

Interval	0	0 1	1
Sign of x^2 Sign of x 1 Sign of x^2 x 1			
Sign of x^2 x 1 x^2 x 1			

From the table, the solution set is x 1 x . Interval: 1 . Graph:

For $16 \quad 9x^2$ to be defined as a real number we must have $16 \;\; 9x^2 \;\; 0 4 \;\; 3x \;\; 4 \;\; 3x \;\; 0$. The expression in the inequality changes sign at $x \;\; \frac{4}{3}$ and $x \;\; \frac{4}{3}$.

Interval	$\frac{4}{3}$	$\frac{4}{3} \; \frac{4}{3}$	$\frac{4}{3}$
Sign of $4 \;\; 3x$ Sign of $4 \;\; 3x$			
Sign of $4 \;\; 3x \;\; 4 \;\; 3x$			

Thus $\frac{4}{3} \;\; x \;\; \frac{4}{3}$.

For $3x^2 \;\; 5x \; 2$ to be defined as a real number we must have $3x^2 \;\; 5x \; 2 \; 0 \; 3x \; 2 \; x \; 1 \; 0$. The expression on the left of the inequality changes sign when $x \; \frac{2}{3}$ and x 1. Thus we must check the intervals in the following table.

Interval	$\frac{2}{3}$	$\frac{2}{3}$ 1	1
Sign of $3x$ Sign of $x \;\; 1$			
Sign of $3x \;\; 2 \;\; x \;\; 1$			

Thus $x \;\; \frac{2}{3}$ or $1 \;\; x$.

75. For $\dfrac{1}{x^2 \;\; 5x \;\; 14}$ 1 2 to be defined as a real number we must have $x^2 \;\; 5x \;\; 14 \;\; 0x \;\; 7 \;\; x \;\; 2 0$. The expression in the inequality changes sign at $x \;\; 7$ and $x 2$.

Interval	2	2 7	7
Sign of $x \;\; 7$ Sign of $x \;\; 2$			
Sign of $x \;\; 7 \;\; x \;\; 2$			

Thus $x 2$ or $7 \;\; x$, and the solution set is 27.

76. For 4 $\dfrac{1 \;\; x}{2 \;\; x}$ to be defined as a real number we must have $\dfrac{1 \;\; x}{2 \;\; x}$ 0. The expression on the left of the inequality changes sign when $x \;\; 1$ and $x 2$. Thus we must check the intervals in the following table.

Interval	2	2 1	1
Sign of $1 \;\; x$ Sign of $2 \;\; x$			
$\dfrac{1 \;\; x}{\text{Sign of } 2 \;\; x}$			

Thus $2 \; x \; 1$. Note that $x \; 2$ has been excluded from the solution set because the expression is undefined at that value.

77. $a \; bx \;\; c \;\; bc$ (where $a, b, c \;\; 0$) $\quad bx \;\; c \quad \frac{bc}{a} \; bx \quad \frac{bc}{a} \; c \; x \quad \frac{1}{b} \; \frac{bc}{a} \; c \quad \frac{c}{a} \; \frac{c}{b} \; x \quad \frac{c}{a} \; \frac{c}{b}$.

We have $a \;\; bx \;\; c \;\; 2a$, where $a, b, c \;\; 0 \;\; a \;\; c \;\; bx \;\; 2a \;\; c \;\; \frac{a \;\; c}{b} \; x \; \frac{2a \;\; c}{b}$.

Inserting the relationship C $\frac{5}{9}$ F 32 , we have 20 C 30 20 $\frac{5}{9}$ F 32 30 36 F 32 54 F 86.

Inserting the relationship F $\frac{9}{5}$ C 32, we have 50 F 95 50 $\frac{9}{5}$ C 32 95 18 $\frac{9}{5}$ C 63 10 C 35.

Let x be the average number of miles driven per day. Each day the cost of Plan A is 30 0 10x, and the cost of Plan B is Plan B saves money when 50 30 0 10x 20 0 1x 200 x. So Plan B saves money when you average more than 200 miles a day.

Let m be the number of minutes of long-distance calls placed per month. Then under Plan A, the cost will be 0 05m, and under Plan B, the cost will be 5 0 12m. To determine when Plan B is advantageous, we must solve 0 05m 5 0 12m 20 0 07m 285 7 m. So Plan B is advantageous if a person places fewer than minutes of long-distance calls during a month.

We need to solve 6400 0 35m 2200 7100 for m. So 6400 0 35m 2200 7100 4200 0 35m 4900 12,000 m 14,000. She plans on driving between 12,000 and 14,000 miles.

(a) T 20 $\frac{h}{100}$, where T is the temperature in C, and h is the height in meters.

Solving the expression in part (a) for h, we get h 100 20 T . So 0 h 5000 0 100 20 T 5000 0 20 T 50 20 T 30 20 T 30. Thus the range of temperature is from 20 C down to 30 C.

(a) Let x be the number of \$3 increases. Then the number of seats sold is 120 x. So P 200 3x 3x P 200 x $\frac{1}{3}$ P 200 . Substituting for x we have that the number of seats sold is 120 x 120 $\frac{1}{3}$ P 200 $\frac{1}{3}$ P $\frac{560}{3}$.

90 $\frac{1}{3}$ P $\frac{560}{3}$ 115 270 360 P 200 345 270 P 560 345 290 P 215 290 P 215. Putting this into standard order, we have 215 P 290. So the ticket prices are between \$215 and \$290.

If the customer buys x pounds of coffee at \$6 50 per pound, then his cost c will be 6 50x. Thus x $\frac{c}{6\ 5}$. Since the scale's accuracy is 0 03 lb, and the scale shows 3 lb, we have 3 0 03 x 3 0 03 2 97 $\frac{c}{6\ 5}$ 3 03 6 50 2 97 c 6 50 3 03 19 305 c 19 695. Since the customer paid \$19 50, he could have been over- or undercharged by as much as 19 5 cents.

87. 0 0004 $\frac{4{,}000{,}000}{d^2}$ 0 01. Since d^2 0 and d0, we can multiply each expression by d^2 to obtain 0 0004d^2 4,000,000 0 01d^2. Solving each pair, we have 0 0004d^2 4,000,000 d^2 10,000,000,000 d 100,000 (recall that d represents distance, so it is always nonnegative). Solving 4,000,000 0 01d^2 400,000,000 d^2 20,000 d. Putting these together, we have 20,000 d 100,000.

88. $\frac{600{,}000}{x^2 \quad 300}$ 500 600,000 500 x^2 300 (Note that x^2 300 300 0, so we can multiply both sides by the denominator and not worry that we might be multiplying both sides by a negative number or by zero.) 1200 x^2 300

0 x^2 900 0 x 30 x 30 . The expression in the inequality changes sign at x 30 and x30. However, since x represents distance, we must have x 0.

Interval	0 30	30
Sign of x 30 Sign of x 30		
Sign of x 30 x 30		

So x 30 and you must stand at least 30 meters from the center of the fire.

89. 128 16t 16t^2 3216t^2 16t 96 016 t^2 t 6016 t 3 t 2 0. The expression on the left of the inequality changes sign at x2, at t 3, and at t2. However, t 0, so the only endpoint is t 3.

Interval	0 3	3
Sign of 16 Sign of t 3 Sign of t 2		
Sign of 16 t 3 t 2		

So 0 t 3.

Solve 30 10 0 9 0 01 ^{2} for 1075. We have 30 10 0 9 0 01 ^{2} 0 01 ^{2} 0 9 20 0
0 1 4 0 1 5 0. The possible endpoints are 0 1 4 0 0 14 40 and 0 1 5 0 0 1 5 50.

Interval	10 40	40 50	50 75
Sign of 0 1 4 Sign of 0 1 5			
Sign of 0 1 4 0 1 5			

Thus he must drive between 40 and 50 mi/h.

91. 240 $\frac{2}{20}$ $\frac{}{20}$ 2 2400 $\frac{1}{20}$ 3800. The expression in the inequality changes sign at 60 and80. However, since represents the speed, we must have0.

Interval	0 60	60
Sign of $\frac{1}{20}$3 Sign of80		
Sign of $\frac{1}{20}$380		

So Kerry must drive between 0 and 60 mi/h.

92. Solve 2400 20x 2000 8x 0 0025x^2 2400 20x 2000 8x 0 0025x^2 0 0025x^2 12x 4400 0

0 0025x 1 x 4400 0. The expression on the left of the inequality changes sign when$x$ 400 and x 4400. Since the manufacturer can only sell positive units, we check the intervals in the following table.

Interval	0 400	400 4400	4400
Sign of 0 0025x 1 Sign of x 4400			
Sign of 0 0025x 1 x 4400			

So the manufacturer must sell between 400 and 4400 units to enjoy a profit of at least $2400.

Let x be the length of the garden and its width. Using the fact that the perimeter is 120 ft, we must have 2x 2120

60 x. Now since the area must be at least 800 ft^2, we have 800 x 60 x 800 60x x^2

x^2 60x 800 0 x 20 x 40 0. The expression in the inequality changes sign at x 20 and x 40. However, since x represents length, we must have x 0.

Interval	0 20	20 40	40
Sign of x 20 Sign of x 40			
Sign of x 20 x 40			

The length of the garden should be between 20 and 40 feet.

Case 1: a b 0 We have a a a b, since a 0, and b a b b, since b 0. So a^2 a b b^2, that is a b 0 a^2 b^2. Continuing, we have a

a^2 a b^2, since a 0 and b^2 a b^2 b, since b^2 0. So a^3 ab^2 b^3. Thus a b 0 a^3 b^3. So a b 0 a^n b^n , if n is even, and a^n b, if n is odd.

Case 2: 0 a b We have a a a b, since a 0, and b a b b, since b 0. So a^2 a b b^2. Thus 0 a b a^2 b^2. Likewise, a^2 a a^2 b and b

a^2 b b^2, thus a^3 b^3. So 0 a b a^n b^n , for all positive integers n.

Case 3: a 0 b If n is odd, then a^n b^n , because a^n is negative and b^n is positive. If n is even, then we could have either a^n

b^n or a^n b^n . For example, 1 2 and 1 ^{2} 2^2, but 3 2 and 3 ^{2} 2^2.

The rule we want to apply here is "a b ac bc if c 0 and a b ac bc if c 0 ". Thus we cannot simply multiply by x, since we don't yet know if x is positive or negative, so in solving 1 $\frac{3}{x}$, we must consider two cases. *Case 1:* x 0 Multiplying both

sides by x, we have x 3. Together with our initial condition, we have 0 x 3. *Case 2:* x 0 Multiplying both sides by x, we have x 3. But x 0 and x 3 have no elements in common, so this gives no additional solution.

Hence, the only solutions are 0 x 3.

a b, so by Rule 1, a c b c. Using Rule 1 again, b c b d, and so by transitivity, a c b d.

$\frac{a}{b}$ $\frac{c}{d}$, so by Rule 3, d $\frac{a}{b}$ d $\frac{c}{d}$ $\frac{ad}{b}$ c. Adding a to both sides, we have $\frac{ad}{b}$ a c a. Rewriting the left-hand

side as $\frac{ad}{b}$ $\frac{ab}{b}$ $\frac{a\ b\ d}{b}$ and dividing both sides by b d gives $\frac{a}{b}$ $\frac{a\ c}{b\ d}$.

Similarly, a c $\frac{cb}{c}$ $\frac{c\ b\ d}{}$, so $\frac{a\ c}{}$ $\frac{c}{}$.

d *d* *b* *d* *d*

1.8 SOLVING ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

The equation x 3 has the two solutions 3 and 3.

(a) The solution of the inequality x 3 is the interval [3 3].

The solution of the inequality x 3 is a union of two intervals3] [3.

(a) The set of all points on the real line whose distance from zero is less than 3 can be described by the absolute value inequality x 3.

The set of all points on the real line whose distance from zero is greater than 3 can be described by the absolute value inequality x 3.

(a) $2x$ 1 5 is equivalent to the two equations $2x$ 1 5 and $2x$ 15.

$3x$ 2 8 is equivalent to 8 $3x$ 2 8.

$5x$ 20 $5x$20 x4.

$3x$ 103x10 $x\frac{10}{3}$.

5 x 3 28 5 x 25x 5 x5.

$\frac{1}{2}$ x 7 2 $\frac{1}{2}$ x 9x 18 x18.

x 3 2 is equivalent to x 32 x 3 2 x 1 or x 5.

$2x$ 3 7 is equivalent to either $2x$ 3 7 $2x$ 10 x 5; or $2x$ 3 7 $2x$ 4 x 2. The two solutions are x 5 and x 2.

x 4 0 5 is equivalent to x 40 5 x4 0 5 x4 5 or x3 5.

x 43. Since the absolute value is always nonnegative, there is no solution.

$2x$ 3 11 is equivalent to either $2x$ 3 11 $2x$ 14 x 7; or $2x$ 3 11 $2x$ 8 x 4. The two solutions are x 7 and x 4.

2 x 11 is equivalent to either 2 x 11 x 9; or 2 x 11 x 13. The two solutions are x 9 and x 13.

4 $3x$ 6 1 $3x$ 6 3 $3x$ 6 3, which is equivalent to either $3x$ 6 3 $3x$ 3 x 1; or $3x$ 6 3 $3x$ 9 x 3. The two solutions are x 1 and x 3.

5 $2x$ 6 145 $2x$ 8 which is equivalent to either 5 $2x$ 8 $2x$ 3 x $\frac{3}{2}$; or 5 $2x$ 8 $2x$ 13 x $\frac{13}{2}$. The two solutions are x $\frac{3}{2}$ and x $\frac{13}{2}$.

3 x 5 6 15 3 x 5 9x 5 3, which is equivalent to either x 5 3 x 2; or x 5 3 x 8. The two solutions are x 2 and x 8.

20 $2x$ 4 152x 45. Since the absolute value is always nonnegative, there is no solution.

19. 8 5 $\frac{1}{2}x$ $\frac{5}{6}$ 3 33 5 $\frac{1}{3}x$ $\frac{5}{6}$25 $\frac{1}{2}$ x $\frac{5}{2}$5, which is equivalent to either $\frac{1}{2}x$ $\frac{5}{2}$ 5 $\frac{1}{2}x$ $\frac{35}{2}$ x 35; or $\frac{1}{2}x$ $\frac{5}{2}$ 5 $\frac{1}{2}x$ $\frac{25}{2}$ x 25. The two solutions are x 25 and x 35.

20. $\frac{3}{5}$ x 2 $\frac{1}{2}$ 4 $\frac{3}{5}$ x 2 $\frac{9}{2}$ which is equivalent to either $\frac{3}{5}$ x 2$\frac{9}{2}$ $\frac{3}{5}$ x $\frac{5}{2}$ x $\frac{25}{6}$; or $\frac{3}{5}$ x 2 $\frac{9}{2}$ $\frac{3}{5}$ x $\frac{13}{2}$ x $\frac{65}{6}$. The two solutions are x $\frac{25}{6}$ and x $\frac{65}{6}$.

21. x 1$3x$ 2 , which is equivalent to either x 1 $3x$ 2$2x$ 3 x $\frac{3}{2}$; or x 1$3x$ 2 $4x$1 $x\frac{1}{4}$. The two solutions are x $\frac{3}{2}$ and x $\frac{1}{4}$.

x 3$2x$ 1 is equivalent to either x 3 $2x$ 1 x 2 x 2; or x 3 $2x$ 1 x 3 $2x$ 1 $3x$ 4 x $\frac{4}{3}$. The two solutions are x 2 and x $\frac{4}{3}$.

x 55 x 5. Interval: [5 5].

24. 2*x* 2020 2*x* 2010 *x* 10. Interval: [10 10].

25. 2*x* 7 is equivalent to 2*x* 7 *x* 7 2; or 2*x*7 *x* 7 2. Interval: 7 2 7 2.

1 2 *x* 1*x* 2 is equivalent to *x* 2 or *x*2. Interval:2] [2.

x 4 10 is equivalent to 10 *x* 4 106 *x* 14. Interval: [6 14].

x 3 9 is equivalent to *x* 39 *x*6; or *x* 3 9 *x* 12. Interval:612.

x 1 1 is equivalent to *x* 1 1 *x* 0; or *x* 11 *x*2. Interval:2] [0.

x 4 0 is equivalent to *x* 4 0 *x* 4 0 *x*4. The only solution is *x*4.

2*x* 1 3 is equivalent to 2*x* 13 2*x*4 *x*2; or 2*x* 1 3 2*x* 2 *x* 1. Interval: 2][1.

32. 3*x* 2 7 is equivalent to 3*x* 2 7 3*x* 5 *x* 5 3; or 3*x* 2 7 3*x* 9 *x* 3. Interval:

5 3 3 .

2*x* 3 0 40 4 2*x* 3 0 4 2 6 2*x* 3 4 1 3 *x* 1 7. Interval: [1 3 1 7].

4 5*x* 8 4 5 *x* 8 5. Interval: 4 5 8 5. **34.** 5*x*2665*x*26

35. *x* 2 3 22 *x* 2 3 26 *x* 2 64 *x* 8. Interval: 4 8 .

1 8 *x* 9. Interval: 9] [7 .

x 6 0 0010 001 *x* 6 0 0016 001 *x*5 999. Interval: 6 001 5 999 .

x *a* *dd* *x* *a* *d* *a* *d* *x* *a* *d*. Interval: *a* *d* *a* *d* .

4 *x* 2 3 13 4 *x* 2 16*x* 2 44 *x* 2 46 *x* 2. Interval: 6 2 .

3 2*x* 4 1 2*x* 4 2 2*x* 4 2 which is equivalent to either 2*x* 4 2 2*x* 2 *x* 1; or 2*x* 4 2 2*x* 6 *x* 3. Interval: 3] [1 .

41. 8 2*x* 1 62*x* 122*x* 1 22 2*x* 1 21 2*x* 3

Interval: 1 2 3 2 .

7 *x* 2 5 4 7 *x* 21*x* 2 1 7. Since the absolute value is always nonnegative, the inequality is true for all real numbers. In interval notation, we have .

43. 4*x*2 *x* 1 . Interval: 1 1 .

44. 2 1 *x* 3 3 51 2 1 *x* 3 48 1 *x* 3 2424 1 *x* 3 2427 1 *x* 2154 *x* 42.

Interval: [54 42].

45. 1 *x* 4. If *x* 0, then this is equivalent to 1 *x* 4. If *x* 0, then this is equivalent to 1*x* 41 *x*4 4 *x*1. Interval: [4 1] [1 4].

46. 0 *x* 5 1. For *x* 5, this is equivalent to 1 *x* 5 1 9 *x* 11. Since *x* 5 is excluded, the solution is 9 2 55 11 2 .

47. *x* 7 1 2 1 2 *x* 7 (*x*7)*x* 7 1 2 1 2 *x* 7 1 2 15 2 *x* 13 2 and *x*7.

Interval: 15 2 7 7 13 2 .

1 1 3 3

48. $2x$ 3 5 5 $2x$ 3 , since $2x$ 30, provided $2x$ 3 0 x 2 . Now for x 2 , we have

1 $2x$ 3 is equivalent to either 1 $2x$ 3 16 $2x$ 8 x; or $2x$ 3 1 $2x$ 14 x 7 .

Interval: $\frac{7}{5}$ $\frac{8}{5}$.

49. x 3 **50.** x 2 **51.** x 7 5 **52.** x 2 4

53. x 2 **54.** x 1 **55.** x 3 **56.** x 4

(a) Let x be the thickness of the laminate. Then x 0 020 0 003.

x 0 020 0 0030 003 x 0 020 0 003 0 017 x 0 023.

h 68 2 22 h 68 2 25 8 h 68 2 5 8 62 4 h 74 0. Thus 95% of the adult males are 2929

between 62 4 in and 74 0 in.

x 1 is the distance between x and 1; x 3 is the distance between x and 3. So x 1 x 3 represents those points closer to 1 than to 3, and the solution is x 2, since 2 is the point halfway between 1 and 3. If a b, then the solution to x a x b is x $\frac{a\ b}{2}$.

1.9 SOLVING EQUATIONS AND INEQUALITIES GRAPHICALLY

The solutions of the equation x^2 $2x$ 3 0 are the x-intercepts of the graph of y x^2 $2x$ 3.

The solutions of the inequality x^2 $2x$ 3 0 are the x-coordinates of the points on the graph of y x^2 $2x$ 3 that lie *above* the x-axis.

(a) From the graph, it appears that the graph of y x^4 $3x^3$ x^2 $3x$ has x-intercepts 1, 0, 1, and 3, so the solutions to the equation x^4 $3x^3$ x^2 $3x$ 0 are x 1, x 0, x 1, and x 3.

From the graph, we see that where 1 x 0 or 1 x 3, the graph lies below the x-axis. Thus, the inequality x^4 $3x^3$ x^2 $3x$ 0 is satisfied for x 1 x 0 or 1 x 3 [1 0] [1 3].

(a) The graphs of y $5x$ x^2 and y 4 intersect at x 1 and at x 4, so the equation $5x$ x^2 4 has solutions x 1 and x 4.

The graph of y $5x$ x^2 lies strictly above the graph of y 4 when 1 x 4, so the inequality $5x$ x^2 4 is satisfied for those values of x, that is, for x 1 x 4 1 4 .

5. Algebraically: x 4 $5x$ 1216 $4x$ x4.

Graphically: We graph the two equations y_1 x 4 and y_2 $5x$ 12 in the viewing rectangle [6 4] by [10 2]. Zooming in, we see that the solution is x4.

6. Algebraically: $\frac{1}{2}x$ 3 6 $2x$9 $\frac{3}{2}$ x x6.

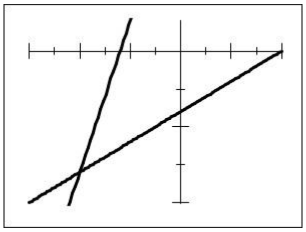
Graphically: We graph the two equations y_1 $\frac{1}{2}x$ 3 and y_2 6 $2x$ in the viewing rectangle [10 5] by [10 5]. Zooming in, we see that the solution is x 6.

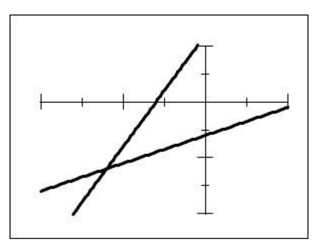
5

-10 -5 5

-5

-10





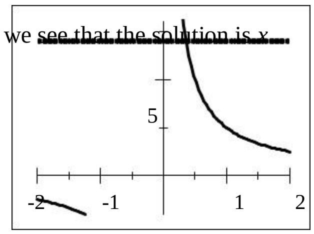
7. Algebraically: $x \; 2x \; 7 \; 2x$ $\frac{2}{\,} \; \frac{1}{\,}$ $x \; 2x$ $2x \; 7$

5 .

4 1 $14x \; x \; 14$ 2 1

Graphically: We graph the two equations $y_1 \; x \; 2x$ and $y_2 \; 7$ in the viewing rectangle [2 2] by [2 8].

Zooming in, we see that the solution is x 0 36.

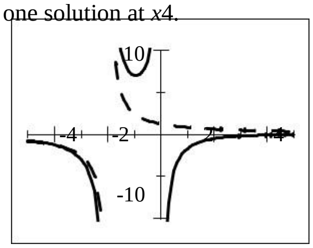


8. Algebraically: $\frac{4}{x \; 2} \; \frac{6}{2x} \; \frac{5}{2x \; 4}$

$\frac{4}{2x \; x \; 2} \; \frac{6}{x \; 2 \; 2x}$ $\frac{5}{2x \; x \; 2 \; 2x \; 4}$

$2x \; 4x \; 2 \; 6 \; x \; 58x \; 6x \; 12 \; 5x$
$12 \; 3x4 \; x$.

Graphically: We graph the two equations $y_1 \; \frac{4}{x \; 2} \; \frac{6}{2x}$ and $y_2 \; \frac{5}{2x \; 4}$ in the viewing rectangle [5 5] by [10 10]. Zooming in, we see that there is only one solution at $x4$.

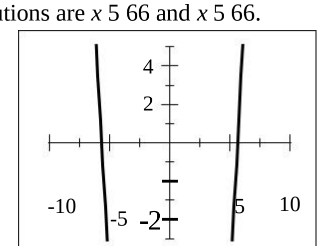


9. Algebraically: $x^2 \; 32 \; 0 \; x^2 \; 32$

$x324$ 2.

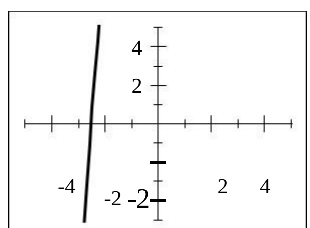
2

Graphically: We graph the equation $y_1 \; x \; 32$ and determine where this curve intersects the x-axis. We use the viewing rectangle [10 10] by [5 5]. Zooming in, we see that solutions are x 5 66 and x 5 66.



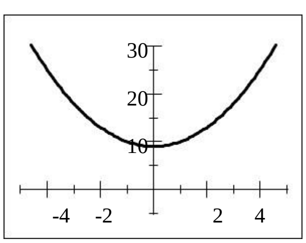
10. Algebraically: $x^3 \; 16 \; 0 \; x^3 16 \; x2^3$ 2.

Graphically: We graph the equation $y \; x^3 \; 16$ and determine where this curve intersects the x-axis. We use the viewing rectangle [5 5] by [5 5]. Zooming in, we see that the solution is x 2 52.



Algebraically: x^2 9 0 x^2 9, which has no real solution.

Graphically: We graph the equation $y \; x^2$ 9 and see that this curve does not intersect the x-axis. We use the viewing rectangle [5 5] by [5 30].

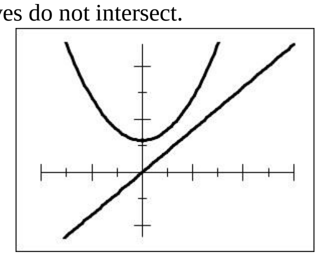


Algebraically: $x^2 \; 3 \; 2x \; x^2 \; 2x \; 3 \; 0$

$x \; \frac{2 \quad 22 \; 413}{2 \; 1} \; \frac{2 \quad 8}{2 \; 1}$.

Because the discriminant is negative, there is no real solution.

Graphically: We graph the two equations $y_1 \; x^2 \; 3$ and $y_2 \; 2x$ in the viewing rectangle [4 6] by [6 12], and see that the two curves do not intersect.

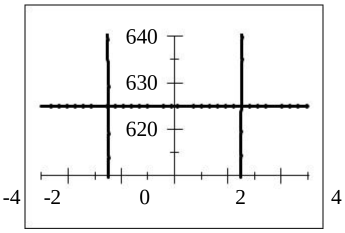


Algebraically: $16x^4$ 625 x^4 $\frac{625}{16}$

5

225.

Graphically: We graph the two equations y_1 $16x^4$ and y_2 625 in the viewing rectangle [5 5] by [610 640]. Zooming in, we see that solutions are x 2 5.

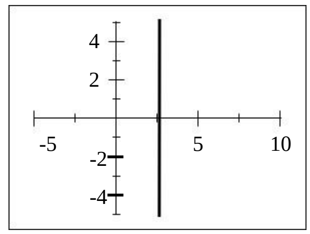


14. Algebraically: $2x^5$ 243 0 $2x^5$ 243 x^5 $\frac{243}{2}$

x 5 $\frac{243}{2}$ $\frac{3}{2}$ 5 16.

Graphically: We graph the equation y $2x^5$ 243 and determine where this curve intersects the x-axis. We use the viewing rectangle [5 10] by [5 5].

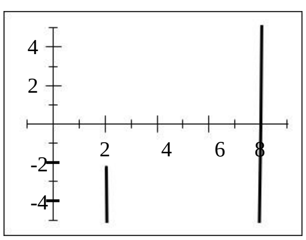
Zooming in, we see that the solution is x 2 61.



Algebraically: x 5^4 80 0x 5^4 80

5^4 80 2^4 5 x 5 2^4 5. Graphically: We graph the equation y_1 x 5^4 80

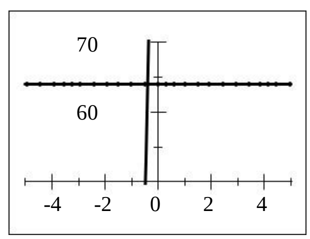
and determine where this curve intersects the x-axis. We use the viewing rectangle [1 9] by [5 5]. Zooming in, we see that solutions are x 2 01 and x 7 99.



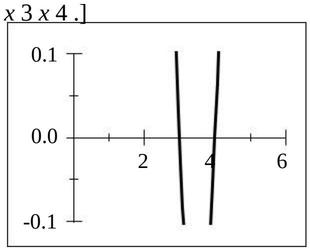
Algebraically: 6 x 2^5 64x 2^5 $\frac{64}{6}$ $\frac{32}{3}$

$\frac{32}{3}$ $\frac{2}{3}$ 5 81 x2 $\frac{2}{3}$ 5 81.x2 5

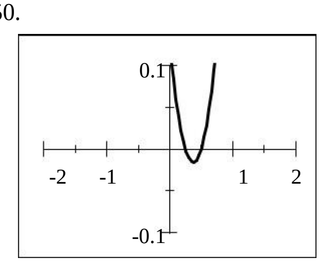
Graphically: We graph the two equations y_1 6 x 2^5 and y_2 64 in the viewing rectangle [5 5] by [50 70]. Zooming in, we see that the solution is x 0 39



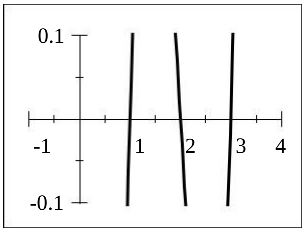
We graph y x^2 7x 12 in the viewing rectangle [0 6] by [0 1 0 1]. The solutions appear to be exactly x 3 and x 4. [In fact x^2 7x 12 x 3 x 4 .]



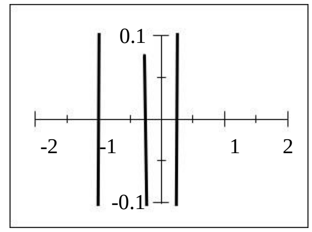
We graph y x^2 0 75x 0 125 in the viewing rectangle [2 2] by [0 1 0 1]. The solutions are x 0 25 and x 0 50.



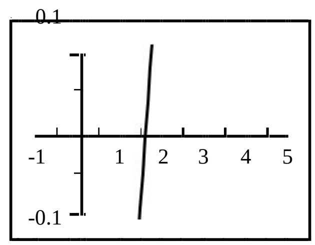
We graph $y\ x^3\ 6x^2\ 11x\ 6$ in the viewing rectangle [1 4] by [0 1 0 1]. The solutions are x 1 00, x 2 00,and x 3 00.



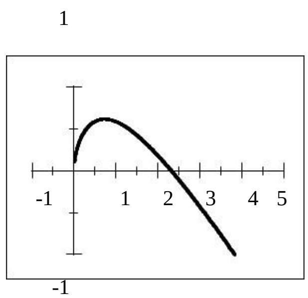
Since $16x^3\ 16x^2\ x\ 1\ 16\ ^3\ 16\ ^2\ x\ 1\ 0$, we graph $y\ 16\ ^3$ 2 x x x $16x\ \ x\ 1$ in the viewing rectangle [2 2] by [0 1 0 1]. The solutions are:
x1 00, x0 25, and x 0 25.

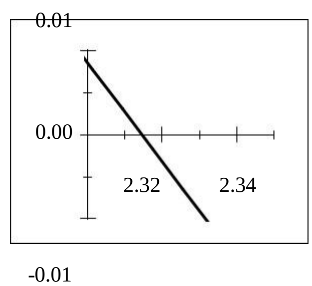


21. We first graph $y\ \ x\ \ \ \underline{x\ \ 1}$ in the viewing rectangle [1 5] by [0 1 0 1] and find that the solution is near 1 6. Zooming in, we see that solutions is x 1 62.

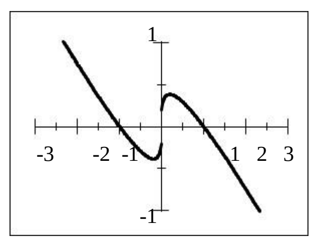


22. 1x1 x2 2 1x 1 x 0 Since x is only defined for x 0, [1 5] by [1 1]. In this rectangle, there appears to be an exact solution at x 0 and another solution between x 2 and x 2 5. We then use the viewing rectangle [2 3 2 35] by [0 01 0 01], and isolate the second solution as 2 314. Thus the solutions are x 0 and 2 31.

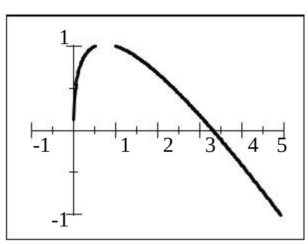


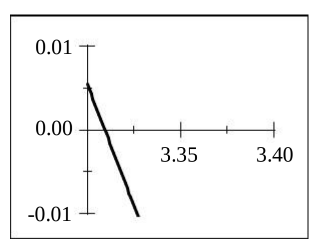


23. We graph $y\ \ x^{1\ 3}\ \ x$ in the viewing rectangle [3 3] by [1 1]. The solutions are x1, x 0, and x 1, as can be verified by substitution.

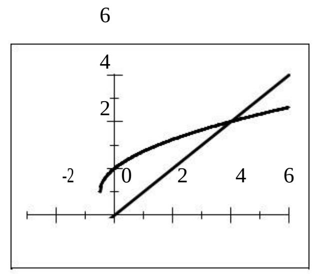


24. Since $x^{1\ 2}$ is defined only for x 0, we start by graphing $y\ \ x^{1\ 2}\ \ x^{1\ 3}\ \ x$ in the viewing rectangle [1 5] by [1 1] We see a solution at x 0 and another one between x 3 and x 3 5. We then use the viewing rectangle [3 3 3 4] by [0 01 0 01], and isolate the second solution as x 3 31. Thus, the solutions are x 0 and x 3 31.

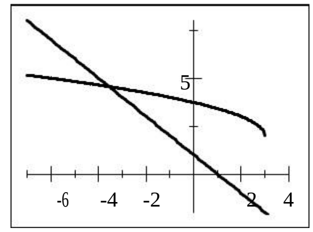




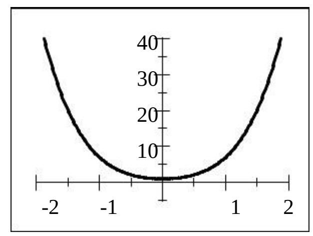
We graph *y* 2*x* 1 1 and *y* *x* in the viewing rectangle [3 6] by [0 6] and see that the only solution to the equation 2*x* 1 1 *x* is *x* 4, which can be verified by substitution.



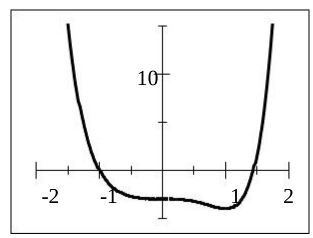
We graph *y* 3 *x* 2 and *y* 1 *x* in the viewing rectangle [7 4] by [2 8] and see that the only solution to the equation 3 *x* 2 1 *x* is *x* 3 56, which can be verified by substitution.



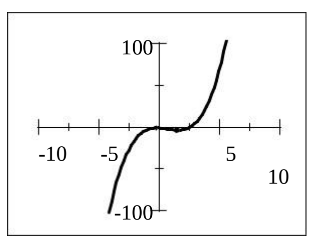
We graph $y\ 2x^4\ 4x^2$ 1 in the viewing rectangle [2 2] by [5 40] and see that the equation $2x^4\ \ 4x^2$ 1 0 has no solution.

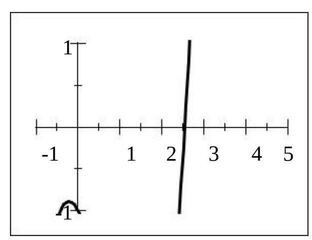


We graph $y\ x^6\ 2x^3$ 3 in the viewing rectangle [2 2] by [5 15] and see that the equation $x^6\ \ 2x^3$ 3 0 has solutions *x* 1 and *x* 1 44, which can be verified by substitution.

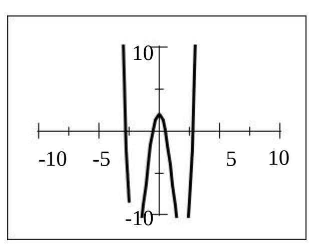


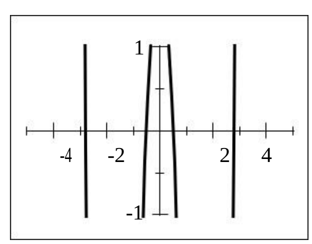
$x^3\ 2x^2$ *x* 1 0, so we start by graphing the function $y\ x^3\ 2x^2$ *x* 1 in the viewing rectangle [10 10] by [100 100]. There appear to be two solutions, one near *x* 0 and another one between *x* 2 and *x* 3. We then use the viewing rectangle [1 5] by [1 1] and zoom in on the only solution, *x* 2 55.





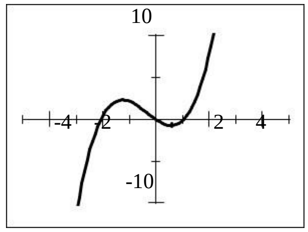
$x^4\ 8x^2$ 2 0. We start by graphing the function $y\ x^4\ 8x^2$ 2 in the viewing rectangle [10 10] by [10 10]. There appear to be four solutions between *x* 3 and *x* 3. We then use the viewing rectangle [5 5] by [1 1], and zoom to find the four solutions *x*2 78, *x*0 51, *x* 0 51, and *x* 2 78.

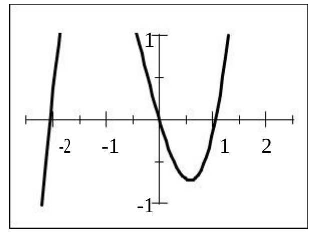




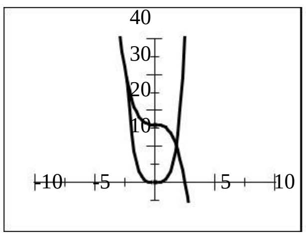
$x\,x\ \ 1\ \ x\ \ 2\frac{1}{6}x$

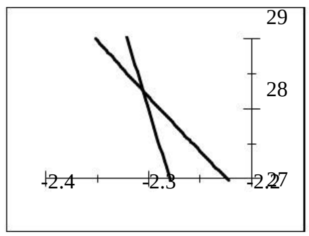
$x\,x\ \ 1\ \ x\ \ 2\ \ \frac{1}{6}x\ \ 0$. We start by graphing the function $y\ x\ x\ 1\ x\ 2\ \frac{1}{6}\ x$ in the viewing rectangle [5 5] by [10 10]. There appear to be three solutions. We then use the viewing rectangle [2 5 2 5] by [1 1] and zoom into the solutions at x 2 05, x 0 00, and x 1 05.

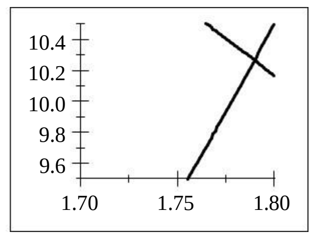




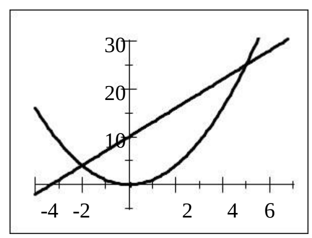
x^4 16 x^3. We start by graphing the functions $y_1\ x^4$ and y_2 16 x^3 in the viewing rectangle [10 10] by [5 40]. There appears to be two solutions, one near x 2 and another one near x 2. We then use the viewing rectangle [2 4 2 2] by [27 29], and zoom in to find the solution at x 2 31. We then use the viewing rectangle [1 7 1 8] by [9 5 10 5], and zoom in to find the solution at x 1 79.



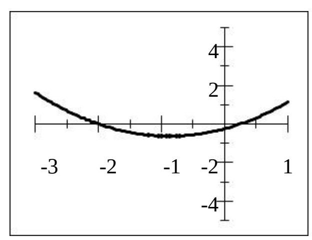




We graph $y\ x^2$ and y 3x 10 in the viewing rectangle [4 7] by [5 30]. The solution to the inequality is [2 5].

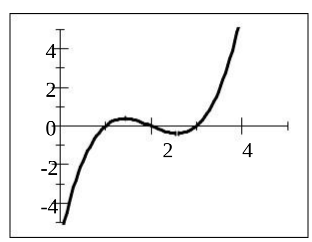


Since 0 5x^2 0 875x 0 25 0 5x^2 0 875x 0 25 0, we graph
2
y 0 5x 0 875x 0 25 in the viewing rectangle [3 1] by [5 5]. Thus the solution to the inequality is [2 0 25].



Since x^3 11x 6x^2 6 x^3 6x^2 11x 6 0, we graph

$y\ x^3$ 6x^2 11x 6 in the viewing rectangle [0 5] by [5 5]. The solution set is 1 0] [2 0 3 0].



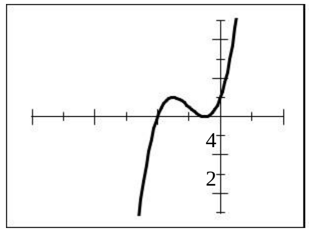
36. Since $16x^3$ 24 ^{2} x 1

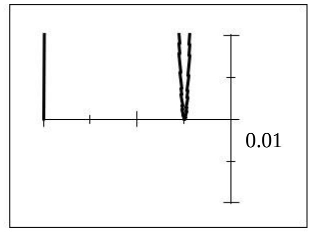
x 9

$16x^3$ $24x^2$ $9x$ 1 0, we graph

3 2

y $16x$ $24x$ $9x$ 1 in the viewing





0.01

0.00

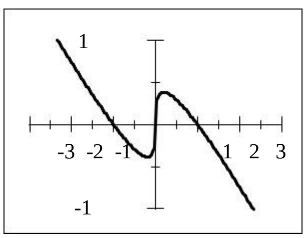
-3 -2 -1 -2 1 -1.0 -0.5

-4

-0.01

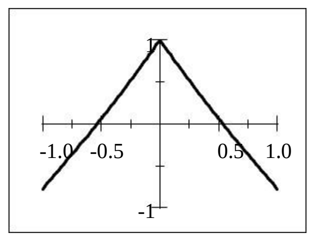
rectangle [3 1] by [5 5]. From this rectangle, we see that x1 is an x-intercept, but it is unclear what is occurring between x0 5 and x 0. We then use the viewing rectangle [1 0] by [0 01 0 01]. It shows y 0 at x0 25. Thus in interval notation, the solution is 1 0 250 25.

Since $x^{1\ 3}$ x $x^{1\ 3}$ x 0, we graph y $x^{1\ 3}$ x in the viewing rectangle [3 3] by [1 1]. From this, we find that the solution set is 1 0 1 .

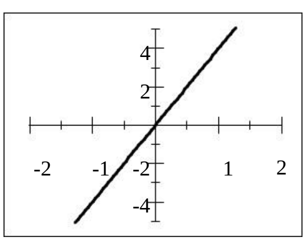


Since 0 5 x^2 1 2 x 0 5x^2 1 2 x 0, we graph y 0 5 x^2 1 2 x in the viewing rectangle [1 1] by [1 1]. We locate the x-intercepts at

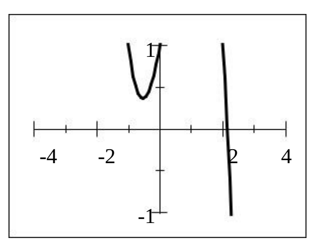
x 0 535. Thus in interval notation, the solution is approximately 0 535] [0 535 .



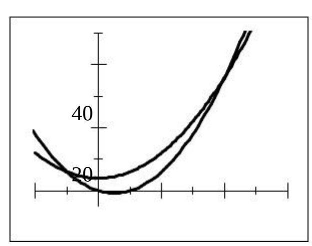
Since x 1 ^{2} x 1 ^{2} x 1 ^{2} x 1 ^{2} 0, we graph y x 1 ^{2} x 1 ^{2} in the viewing rectangle [2 2] by [5 5]. The solution set is 0 .



Since x 1 ^{2} x^3 x 1 ^{2} x^3 0, we graph y x 1 ^{2} x^3 in the viewing rectangle [4 4] by [1 1]. The x-intercept is close to x 2. Using a trace function, we obtain x 2 148. Thus the solution is [2 148 .



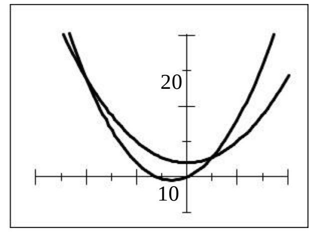
We graph the equations y $3x^2$ $3x$ and y $2x^2$ 4 in the viewing rectangle [2 6] by [5 50]. We see that the two curves intersect at x 1 and at x 4, and that the first curve is lower than the second for 1 x 4. Thus, we see that the inequality $3x^2$ $3x$ $2x^2$ 4 has the solution set 1 4 .



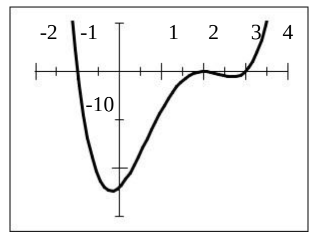
-2 2 4 6

42. We graph the equations y $5x^2$ $3x$ and y $3x^2$ 2 in the viewing rectangle [3 2] by [5 20]. We see that the two curves intersect at x2 and at x $2^{\frac{1}{}}$, which can be verified by substitution. The first curve is larger than the second for x2 and for x 2^1, so the solution set of the inequality $5x^2$ $3x$ $3x^2$ 2 is

2]$2^{\frac{1}{}}$.

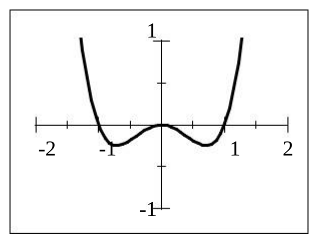


We graph the equation y x 2^2 x 3 x 1 in the viewing rectangle [2 4] by [15 5] and see that the inequality x 2^2 x 3 x 1 0 has the solution set [1 3].



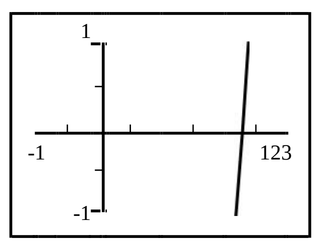
44. We graph the equation y x^2 x^2 1 in the viewing rectangle [2 2] by [1 1] and see that the inequality x^2 x^2 10 has the solution set

1]0[1.



45. To solve 5 3x 8x 20 by drawing the graph of a single equation, we isolate all terms on the left-hand side: 5 3x 8x 20
5 3x 8x 20 8x 20 8x 2011x 25 0 or 11x 25 0.

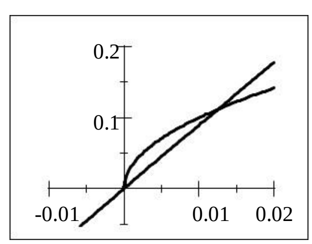
We graph y 11x 25, and see that the solution is x 2 27, as in Example 2.



Graphing y x^3 $6x^2$ 9x by [0 and y ~~x in the viewing rectangle [0 01 0 02] 0 and x 0 01 are solutions of the equation~~

05 0 2], we see that ~~x~~ x^3 $6x^2$

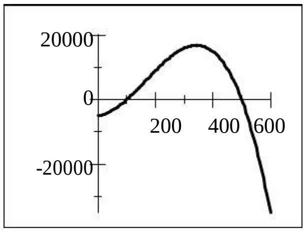
9x ~~x~~.



(a) We graph the equation

$10x$ 0 $5x^2$ 0 $001x^3$ 5000 in the viewing

rectangle [0 600] by [30000 20000].



From the graph it appears that

0 $10x$ 0 $05x^2$ 0 $001x^3$ 5000 for

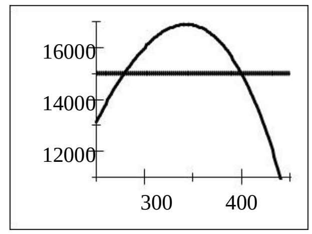
100 x 500, and so 101 cooktops must be produced to *begin* to make a profit.

We graph the equations y 15,000 and

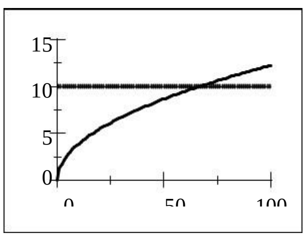
y $10x$ 0 $5x^2$ 0 $001x^3$ 5000 in the viewing rectangle

[250 450] by [11000 17000]. We use a zoom or trace

company's profits are greater than $15,000 for 279 x 400.



48. (a)



Using a zoom or trace function, we find that y 10 for x 66 7. We could estimate this since if x 100, then $\frac{x}{5280}$ ^{2} 0 00036. So for x 100 we have 1 $5x$ 5280 1 $5x$. Solving 1 $5x$ 10 we get 1 5 100 or x 1 5 66 7 mi.

Answers will vary.

Calculators perform operations in the following order: exponents are applied before division and division is applied before addition. Therefore, `Y_1=x^1/3` is interpreted as y x^1 x, which is the equation of a line. Likewise, `Y_2=x/x+4` is 3 3 interpreted as y $\frac{x}{x}$ 4 1 4 5. Instead, enter the following: `Y_1=x^(1/3)`, `Y_2=x/(x+4)`.

1.10 MODELING VARIATION

If the quantities x and y are related by the equation y $3x$ then we say that y is *directly proportional* to x, and the constant of *proportionality* is 3.

If the quantities x and y are related by the equation y $\frac{3}{x}$ then we say that y is *inversely proportional* to x, and the constant of *proportionality* is 3.

If the quantities x, y, and z are related by the equation z $3\frac{x}{y}$ then we say that z is *directly proportional* to x and *inversely proportional* to y.

4. Because z is jointly proportional to x and y, we must have z kxy. Substituting the given values, we get 10 k 4 5 $20k$ k $\frac{1}{2}$. Thus, x, y, and z are related by the equation z $\frac{1}{2}xy$.

(a) In the equation y $3x$, y is directly proportional to x.

In the equation y $3x$ 1, y is not proportional to x.

6. (a) In the equation y $\frac{3}{x\ 1}$, y is not proportional to x.

3

(b) In the equation *y* *x* , *y* is inversely proportional to *x*.

$T\ \ kx$, where k is constant.

$P\ \ k$, where k is constant.

$\frac{k}{z}$, where k is constant.

kmn, where k is constant.

$y\ \ \frac{ks}{t}$, where k is constant.

$P\ \ \frac{k}{T}$, where k is constant.

$z\ \ k\ y$, where k is constant.

14. $A\ \ \frac{kx^2}{t^3}$, where k is constant.

15. $V\ \ kl\ \ h$, where k is constant.

16. $S\ \ kr^2\ \ ^2$, where k is constant.

17. $R\ \ \frac{kP^2t^2}{b^3}$, where k is constant.

18. $A\ \ k\ \ xy$, where k is constant.

19. Since y is directly proportional to x, $y\ \ kx$. Since $y\ \ 42$ when $x\ \ 6$, we have $42\ \ k\,6\ \ k\ \ 7$. So $y\ \ 7x$.

20. is inversely proportional to t, so $\frac{k}{t}$. Since 3 when $t\ \ 8$, we have $3\ \ \frac{k}{8}\ \ k\ \ 24$, so $\frac{24}{t}$.

A varies inversely as r, so $A\ \ \frac{k}{r}$. Since $A\ \ 7$ when $r\ \ 3$, we have $7\ \ \frac{k}{3}\ \ k\ \ 21$. So $A\ \ \frac{21}{r}$.

P is directly proportional to T, so $P\ \ kT$. Since $P\ \ 20$ when $T\ \ 300$, we have $20\ \ k\,300\ \ k\ \ \frac{1}{15}$. So $P\ \ \frac{1}{15}T$.

Since A is directly proportional to x and inversely proportional to t, $A\ \ \frac{kx}{t}$. Since $A\ \ 42$ when $x\ \ 7$ and $t\ \ 3$, we have $42\ \ \frac{k\,7}{3}\ \ k\ \ 18$. Therefore, $A\ \ \frac{18x}{t}$.

$S\ \ kpq$. Since $S\ \ 180$ when $p\ \ 4$ and $q\ \ 5$, we have $180\ \ k\,4\ \ 5\,180\ \ 20k\ \ k\ \ 9$. So $S\ \ 9pq$.

25. Since W is inversely proportional to the square of r, $W\ \ \frac{k}{r^2}$. Since $W\ \ 10$ when $r\ \ 6$, we have $10\ \ \frac{k}{6^2}\ \ k\ \ 360$. So $W\ \ \frac{360}{r^2}$.

26. $t\ \ k\ \ \frac{xy}{r}$. Since $t\ \ 25$ when $x\ \ 2$, $y\ \ 3$, and $r\ \ 12$, we have $25\ \ k\ \ \frac{2\ \ 3}{12}\ \ k\ \ 50$. So $t\ \ 50\ \frac{xy}{r}$.

Since C is jointly proportional to l, , and h, we have $C\ kl\ h$. Since C 128 when $l\ h$ 2, we have 128 k 2 2 2 128 $8k$ k 16. Therefore, C 16$l\ h$.

$H\ \ kl^2\ \ ^2$. Since $H\ \ 36$ when $l\ \ 2$ and $\frac{1}{3}$, we have $36\ \ k\,2^2\ \frac{1}{3}^2\ \ 36\ \frac{4}{9}k\ \ k\ \ 81$. So $H\ \ 81l^2\ \ ^2$.

29. $R\ \ \frac{k}{x}$. Since $R\ \ 2\ 5$ when $x\ \ 121$, $2\ 5\ \ \frac{k}{121}\ \ \frac{k}{11}\ \ k\ \ 27\ 5$. Thus, $R\ \ \frac{27\ 5}{x}$.

30. $M\ \ k\ \frac{abc}{d}$. Since $M\ \ 128$ when $a\ \ d$ and $b\ \ c\ \ 2$, we have $128\ \ k\ \frac{a\ 2\ 2}{a}\ \ 4k\ \ k\ \ 32$. So $M\ \ 32\ \frac{abc}{d}$.

31. (a) $z\ \ k\ \frac{x^3}{y^2}$

(b) If we replace x with $3x$ and y with $2y$, then $z\ \ k\ \frac{3x^3}{2y^2}\ \ \frac{27}{4}k\ \frac{x^3}{y^2}$, so z changes by a factor of $\frac{27}{4}$.

32. (a) $z\ \ k\ \frac{x^2}{y^4}$

(b) If we replace x with $3x$ and y with $2y$, then $z\ \ k\ \frac{3x^2}{2y^4}\ \ \frac{9}{16}k\ \frac{x^2}{y^4}$, so z changes by a factor of $\frac{9}{16}$.

(a) $z = kx^3 y^5$

If we replace x with $3x$ and y with $2y$, then $z = k(3x)^3(2y)^5 = 864kx^3y^5$, so z changes by a factor of 864.

34. (a) $z = \dfrac{k}{x^2 y^3}$

(b) If we replace x with $3x$ and y with $2y$, then $z = \dfrac{k}{(3x)^2 (2y)^3} = \dfrac{1}{72} \dfrac{k}{x^2 y^3}$, so z changes by a factor of $\frac{1}{72}$.

35. (a) The force F needed is $F = kx$.

(b) Since $F = 30$ N when $x = 9$ cm and the spring's natural length is 5 cm, we have $30 = k(9 - 5) \Leftrightarrow k = 7.5$.

(c) From part (b), we have $F = 7.5x$. Substituting $x = 11 - 5 = 6$ into $F = 7.5x$ gives $F = 7.5(6) = 45$ N.

36. (a) $C = kpm$

(b) Since $C = 60{,}000$ when $p = 120$ and $m = 4000$, we get $60{,}000 = k(120)(4000) \Leftrightarrow k = \frac{1}{8}$. So $C = \frac{1}{8}pm$.

(c) Substituting $p = 92$ and $m = 5000$, we get $C = \frac{1}{8}(92)(5000) = \$57{,}500$.

37. (a) $P = ks^3$.

(b) Since $P = 96$ when $s = 20$, we get $96 = k \cdot 20^3 \Leftrightarrow k = 0.012$. So $P = 0.012s^3$.

(c) Substituting $x = 30$, we get $P = 0.012(30^3) = 324$ watts.

38. (a) The power P is directly proportional to the cube of the speed s, so $P = ks^3$.

(b) Because $P = 80$ when $s = 10$, we have $80 = k \cdot 10^3 \Leftrightarrow k = \frac{80}{1000} = \frac{2}{25} = 0.08$.

(c) Substituting $k = \frac{2}{25}$ and $s = 15$, we have $P = \frac{2}{25}(15^3) = 270$ hp.

39. $D = ks^2$. Since $D = 150$ when $s = 40$, we have $150 = k(40)^2$, so $k = 0.09375$. Thus, $D = 0.09375s^2$. If $D = 200$, then $200 = 0.09375s^2 \Leftrightarrow s^2 = 2133.3$, so $s \approx 46$ mi/h (for safety reasons we round down).

40. $L = ks^2A$. Since $L = 1700$ when $s = 50$ and $A = 500$, we have $1700 = k(50^2)(500) \Leftrightarrow k = 0.00136$. Thus $L = 0.00136s^2A$. When $A = 600$ and $s = 40$ we get the lift is $L = 0.00136(40^2)(600) = 1305.6$ lb.

41. $F = kAs^2$. Since $F = 220$ when $A = 40$ and $s = 5$. Solving for k we have $220 = k(40)(5)^2 \Leftrightarrow 220 = 1000k \Leftrightarrow k = 0.22$. Now when $A = 28$ and $F = 175$ we get $175 = 0.220(28)s^2 \Leftrightarrow 28.4090 = s^2$ so $s = \sqrt{28.4090} = 5.33$ mi/h.

(a) $T^2 = kd^3$

(b) Substituting $T = 365$ and $d = 93 \times 10^6$, we get $365^2 = k(93 \times 10^6)^3 \Leftrightarrow k = 1.66 \times 10^{-19}$.
$T^2 = 1.66 \times 10^{-19}(2.79 \times 10^9)^3 = 3.60 \times 10^9 \Rightarrow T = 6.00 \times 10^4$. Hence the period of Neptune is 6.00×10^4 days ≈ 164 years.

(a) $P = \dfrac{kT}{V}$.

(b) Substituting $P = 33.2$, $T = 400$, and $V = 100$, we get $33.2 = \dfrac{k(400)}{100} \Leftrightarrow k = 8.3$. Thus $k = 8.3$ and the equation is $P = \dfrac{8.3T}{V}$.

(c) Substituting $T = 500$ and $V = 80$, we have $P = \dfrac{8.3(500)}{80} = 51.875$ kPa. Hence the pressure of the sample of gas is about 51.9 kPa.

(a) $F = k\dfrac{s^2}{r}$

1600 60^2 2500 s2^2 16 60^2 2

we have k r k r 25 s2, so s2 48 mi/h.

(a) The loudness L is inversely proportional to the square of the distance d, so $L = \frac{k}{d^2}$.

(b) Substituting $d = 10$ and $L = 70$, we have $70 = \frac{k}{10^2} \Leftrightarrow k = 7000$.

(c) Substituting $2d$ for d, we have $L = \frac{k}{(2d)^2} = \frac{1}{4}\frac{k}{d^2}$, so the loudness is changed by a factor of $\frac{1}{4}$.

(d) Substituting $\frac{1}{2}d$ for d, we have $L = \frac{k}{\left(\frac{1}{2}d\right)^2} = 4\frac{k}{d^2}$, so the loudness is changed by a factor of 4.

46. (a) The power P is jointly proportional to the area A and the cube of the velocity v, so $P = kAv^3$.

(b) Substituting $2v$ for v and $\frac{1}{2}A$ for A, we have $P = k\left(\frac{1}{2}A\right)(2v)^3 = 4kAv^3$, so the power is changed by a factor of 4.

(c) Substituting $\frac{1}{2}v$ for v and $3A$ for A, we have $P = k(3A)\left(\frac{1}{2}v\right)^3 = \frac{3}{8}kAv^3$, so the power is changed by a factor of $\frac{3}{8}$.

47. (a) $R = \frac{kL}{d^2}$

(b) Since $R = 140$ when $L = 1.2$ and $d = 0.005$, we get $140 = \frac{k(1.2)}{(0.005)^2} \Leftrightarrow k = \frac{7}{2400} = 0.002916$.

(c) Substituting $L = 3$ and $d = 0.008$, we have $R = \frac{7}{2400} \cdot \frac{3}{(0.008)^2} = \frac{4375}{32} \approx 137\ \Omega$.

If we substitute $2d$ for d and $3L$ for L, then $R = \frac{k(3L)}{(2d)^2} = \frac{3}{4}\frac{kL}{d^2}$, so the resistance is changed by a factor of $\frac{3}{4}$.

48. Let S be the final size of the cabbage, in pounds, let N be the amount of nutrients it receives, in ounces, and let c be the number of other cabbages around it. Then $S = k\frac{N}{c}$. When $N = 20$ and $c = 12$, we have $S = 30$, so substituting, we have $30 = k\frac{20}{12} \Leftrightarrow k = 18$. Thus $S = 18\frac{N}{c}$. When $N = 10$ and $c = 5$, the final size is $S = 18\left(\frac{10}{5}\right) = 36$ lb.

49. (a) For the sun, $E_S = k6000^4$ and for earth $E_E = k300^4$. Thus $\frac{E_S}{E_E} = \frac{k6000^4}{k300^4} = \left(\frac{6000}{300}\right)^4 = 20^4 = 160{,}000$. So the sun produces 160,000 times the radiation energy per unit area than the Earth.

$\frac{4\pi\,435{,}000^2}{4\pi\,3{,}960^2} = \left(\frac{435{,}000}{3{,}960}\right)^2$ times the surface area of the Earth. Thus the total radiation emitted by the sun is $160{,}000 \times \left(\frac{435{,}000}{3{,}960}\right)^2 \approx 1{,}930{,}670{,}340$ times the total radiation emitted by the Earth.

Let V be the value of a building lot on Galiano Island, A the area of the lot, and q the quantity of the water produced. Since V is jointly proportional to the area and water quantity, we have $V = kAq$. When $A = 200 \cdot 300 = 60{,}000$ and $q = 10$, we have $V = \$48{,}000$, so $48{,}000 = k(60{,}000)(10) \Leftrightarrow k = 0.08$. Thus $V = 0.08Aq$. Now when $A = 400 \cdot 400 = 160{,}000$ and $q = 4$, the value is $V = 0.08(160{,}000)(4) = \$51{,}200$.

(a) Let T and l be the period and the length of the pendulum, respectively. Then $T = k\sqrt{l}$.

(b) $T = k\sqrt{l} \Rightarrow T^2 = k^2 l \Leftrightarrow l = \frac{T^2}{k^2}$. If the period is doubled, the new length is $\frac{(2T)^2}{k^2} = 4\frac{T^2}{k^2} = 4l$. So we would quadruple the length l to double the period T.

Let H be the heat experienced by a hiker at a campfire, let A be the amount of wood, and let d be the distance from campfire. So $H = k\frac{A}{d^3}$. When the hiker is 20 feet from the fire, the heat experienced is $H = k\frac{A}{20^3}$, and when the amount of wood is doubled, the heat experienced is $H = k\dfrac{2A}{d^3}$. So $k\dfrac{A}{8\,000} = k\dfrac{2A}{d^3} \Leftrightarrow d^3 = 16\,000 \Leftrightarrow d = 20\sqrt[3]{2} \approx 25.2$ feet.

(a) Since f is inversely proportional to L, we have $f \quad \frac{k}{L}$, where k is a positive constant.

(b) If we replace L by $2L$ we have $\frac{k}{2L} \quad \frac{1}{2} \frac{k}{L} \quad \frac{1}{2} f$. So the frequency of the vibration is cut in half.

(a) Since r is jointly proportional to x and $P \quad x$, we have $r \quad kx \; P \quad x$, where k is a positive constant.

(b) When 10 people are infected the rate is $r \quad k10\; 5000 \quad 10 \quad 49{,}900k$. When 1000 people are infected the rate is $k\; 1000\; 5000 \quad 1000 \quad 4{,}000{,}000k$. So the rate is much higher when 1000 people are infected. Comparing these rates, we find that $\frac{\text{1000 people infected}}{\text{10 people infected}} \quad \frac{4{,}000{,}000k}{49{,}900k} \quad 80$. So the infection rate when 1000 people are infected is about 80 times as large as when 10 people are infected.

When the entire population is infected the rate is $r\; k\; 5000\; 5000\; 5000\; 0$. This makes sense since there are no more people who can be infected.

55. Using $B \quad k \frac{L}{d^2}$ with $k \quad 0\,080$, $L \quad 25\; 10^{26}$, and $d \quad 2\,4 \quad 10^{19}$, we have $B \quad 0\,080 \frac{2\,5 \quad 10^{26}}{24 \quad 10^{19\,2}} \quad 347 \quad 10^{14}$.

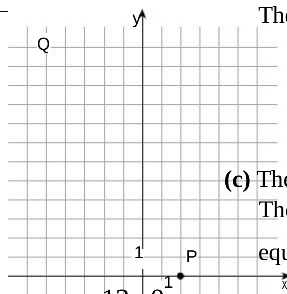
The star's apparent brightness is about $3\,47 \quad 10^{14}$ W m^2.

56. First, we solve $B \quad k \frac{L}{d^2}$ for d: $d^2 \quad k \frac{L}{B} \quad d \quad \sqrt{k \frac{L}{B}}$ because d is positive. Substituting $k \quad 0\,080$, $L \quad 5\,8 \quad 10^{30}$, and $B \quad 8\,2 \quad 10^{16}$, we find $d \quad \sqrt{0\,080 \frac{5\,8 \quad 10^{30}}{8\,2 \quad 10^{16}}} \quad 2\,38 \quad 10^{22}$, so the star is approximately $2\,38 \quad 10^{22}$ m from earth.

Examples include radioactive decay and exponential growth in biology.

CHAPTER 1 REVIEW

1. (a)



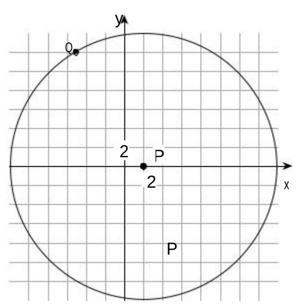
The distance from P to Q is

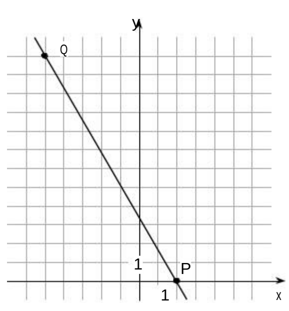
$$d\; P\,Q \quad \sqrt{5 \quad 2^2 \quad 12 \quad 0^2} \quad \sqrt{49 \quad 144} \quad \sqrt{193}$$

(c) The midpoint is $\frac{5 \quad 2}{2}, \frac{12 \quad 0}{2} \quad \frac{3}{2}, 6$.

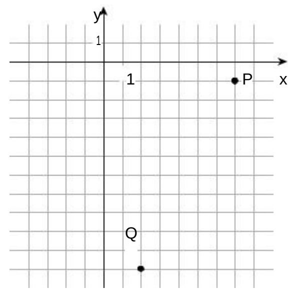
The line has slope $m \quad \frac{12 \quad 0}{5 \quad 2} \quad \frac{12}{7}$, and has equation $y\; 0 \quad \frac{12}{7}\, x \quad 2 \quad y \quad \frac{12}{7} x \quad \frac{24}{7} \quad 12x \quad 7y \quad 24 \quad 0$.

The radius of this circle was found in part (b). It is $r \quad d\; P\,Q \quad \sqrt{193}$. So an equation is $x \quad 2^2 \quad y \quad 0^2 \quad \sqrt{193}^2 \quad x \quad 2^2 \quad y^2 \quad 193$.





2. (a)

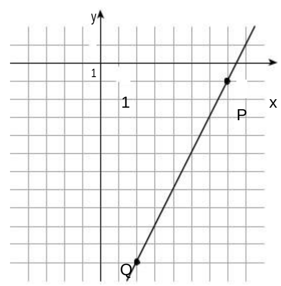


(b) The distance from P to Q is

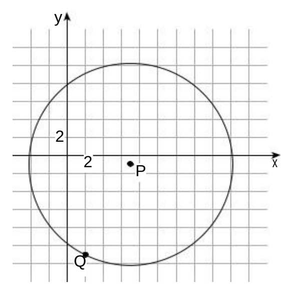
$$d(P,Q) = \sqrt{(2-7)^2 + (-11+1)^2} = \sqrt{25+100} = \sqrt{125} = 5\sqrt{5}$$

(c) The midpoint is $\left(\frac{2+7}{2}, \frac{-11-1}{2}\right) = \left(\frac{9}{2}, -6\right)$.

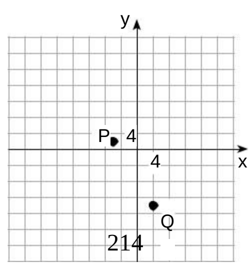
(d) The line has slope $m = \frac{-11-(-1)}{2-7} = \frac{-10}{-5} = 2$, and its equation is $y-(-11) = 2(x-2) \Leftrightarrow y+11 = 2x-4 \Leftrightarrow y = 2x-15$.



(e) The radius of this circle was found in part (b). It is $r = d(P,Q) = 5\sqrt{5}$. So an equation is $(x-7)^2 + (y+1)^2 = (5\sqrt{5})^2 \Leftrightarrow (x-7)^2 + (y+1)^2 = 125$.



3. (a)

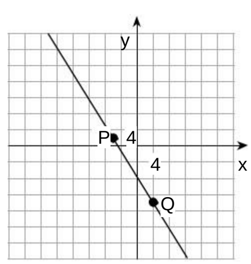


(b) The distance from P to Q is

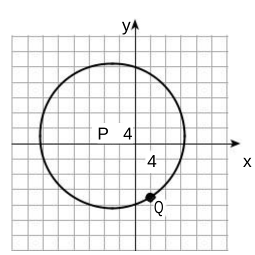
$$d(P,Q) = \sqrt{(-6-4)^2 + [2-(-14)]^2} = \sqrt{100+256} = \sqrt{356} = 2\sqrt{89}$$

(c) The midpoint is $\left(\frac{-6+4}{2}, \frac{2-14}{2}\right) = (-1, -6)$.

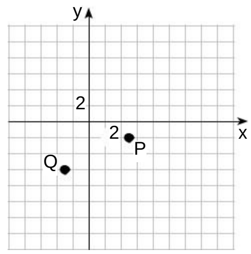
(d) The line has slope $m = \frac{2-(-14)}{-6-4} = \frac{16}{-10} = -\frac{8}{5}$ and equation $y-2 = -\frac{8}{5}(x+6) \Leftrightarrow y-2 = -\frac{8}{5}x - \frac{48}{5} \Leftrightarrow y = -\frac{8}{5}x - \frac{38}{5}$.



(e) The radius of this circle was found in part (b). It is $r = d(P,Q) = 2\sqrt{89}$. So an equation is $[x-(-6)]^2 + (y-2)^2 = (2\sqrt{89})^2 \Leftrightarrow (x+6)^2 + (y-2)^2 = 356$.



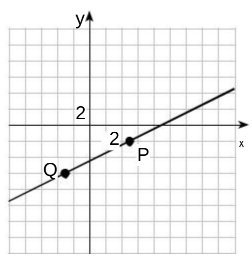
4. **(a)**



The line has slope m 2 6 4 1 , and 53 82

has equation y 2 1 2 x 5 y 2 1 2 x 5 2 y

1 2 x 9 2 .



(b) The distance from P to Q is

d P Q[53]^2 [26]^2

64 1680 45.

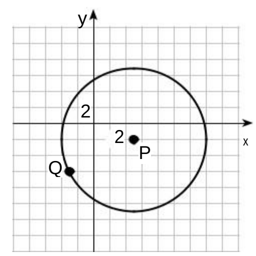
(c) The midpoint is 5 2 3 2 2 6 1 4 .

The radius of this circle was found in part (b). It is

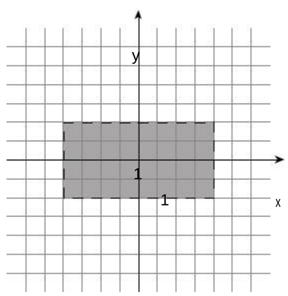
r d P Q 4 5. So an equation is

x 5 2y2 2 4 5 ^{2}

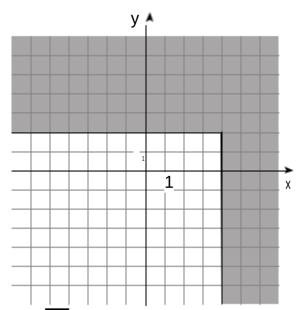
x 5 ^{2} y 2 ^{2} 80.



5.



6. x y x 4 or y 2



7. d A C 41^{2}43^2 41^{2}43^2 74 and

d B C 51^{2}33^2 51^{2}33^2 72. Therefore, B is closer to C.

The circle with center at 2 5 and radius 2 has equation x 2 ^{2} y 5 ^{2} 2 ^{2} x 2 ^{2} y 5 ^{2} 2.

9. The center is C5 1 , and the point P0 0 is on the circle. The radius of the circle is

r d 2P C2 0 5 ^{2} 0 1 ^{2} =0 5^{2}01^2 26. Thus, the equation of the circle is

x 5 y 1 26.

10. The midpoint of segment P Q is 2 1 2 3 8 2 1 2 11 2 , and the radius is 1 2 of the distance from P to Q, or

r^2 1 2 d P, Q ^{2} 1 2 2 2 1 2 3 8 ^{2} 1 2 2 3 8 ^{2} r 1 34. Thus the equation is

1 11 17

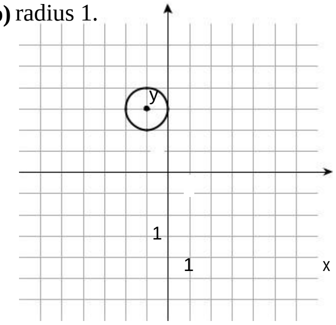
x 2 y 2 2 .

11. (a) $x^2 + y^2 - 2x + 6y + 9 = 0 \Leftrightarrow x^2 - 2x + y^2 + 6y = -9$

$x^2 - 2x + 1 + y^2 + 6y + 9 = -9 + 1 + 9$

$(x-1)^2 + (y+3)^2 = 1$, an equation of a circle.

(b) The circle has center $(1, -3)$ and radius 1.

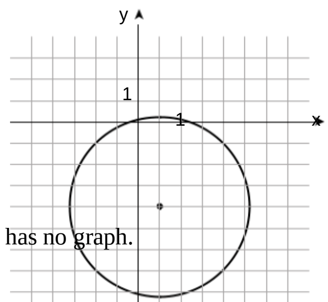


(a) $2x^2 + 2y^2 - 2x + 8y = \frac{1}{2} \Leftrightarrow x^2 - x + y^2 + 4y = \frac{1}{4}$

$x^2 - x + \frac{1}{4} + y^2 + 4y + 4 = \frac{1}{4} + \frac{1}{4} + 4$

$\left(x - \frac{1}{2}\right)^2 + (y+2)^2 = \frac{9}{2}$, an equation of a circle.

(b) The circle has center $\left(\frac{1}{2}, -2\right)$ and radius $\frac{3\sqrt{2}}{2}$.



13. (a) $x^2 + y^2 + 72 = 12x \Leftrightarrow x^2 - 12x + y^2 = -72 \Leftrightarrow x^2 - 12x + 36 + y^2 = -72 + 36 \Leftrightarrow (x-6)^2 + y^2 = -36.$

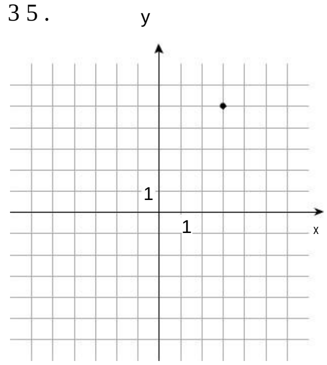
Since the left side of this equation must be greater than or equal to zero, this equation has no graph.

14. (a) $x^2 + y^2 + 6x - 10y + 34 = 0 \Leftrightarrow x^2 + 6x + y^2 - 10y = -34$

$x^2 + 6x + 9 + y^2 - 10y + 25 = -34 + 9 + 25$

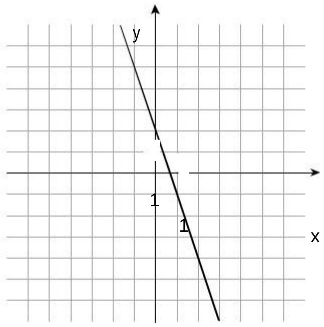
$(x+3)^2 + (y-5)^2 = 0$, an equation of a point.

(b) This is the equation of the point $(-3, 5)$.



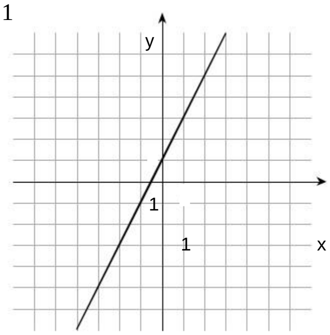
15. $y \quad 2 \quad 3x$

x	y
2	8
0	2
$\frac{2}{3}$	0



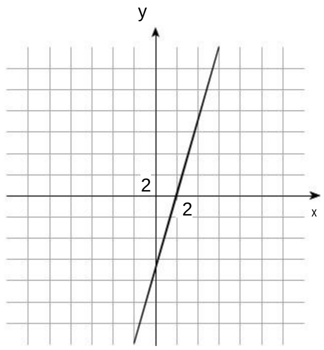
16. $2x \quad y \quad 1 \quad 0 \quad y \quad 2x \quad 1$

x	y
2	3
0	1
$\frac{1}{2}$	0



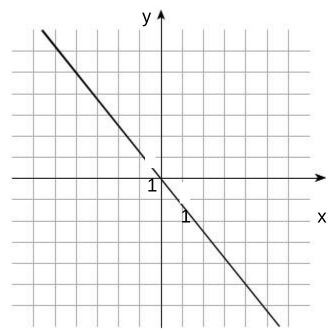
17. $\frac{x}{2} \quad \frac{y}{7} \quad 1 \quad y \quad \frac{7}{2}x \quad 7$

x	y
2	14
0	7
2	0



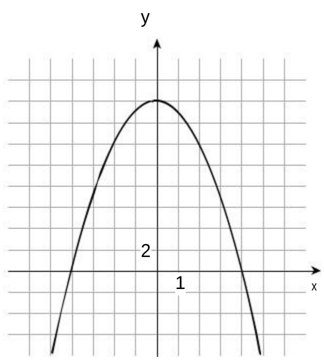
18. $\frac{x}{4} \quad \frac{y}{5} \quad 0 \quad 5x \quad 4y \quad 0$

x	y
4	5
0	0
4	5



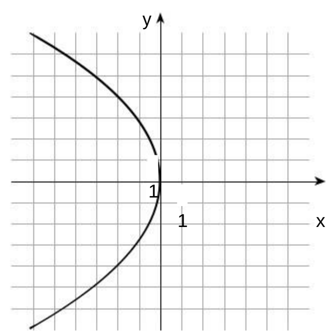
19. $y \quad 16 \quad x^2$

x	y
3	7
1	15
0	16
1	15
3	7



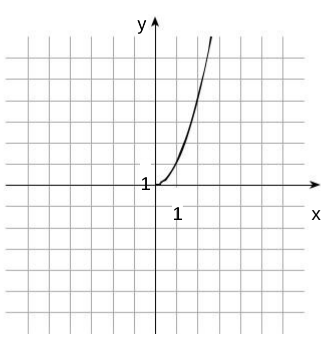
20. $8x \quad y^2 \quad 0 \quad y^2 \quad 8x$

x	y
8	8
2	4
0	0



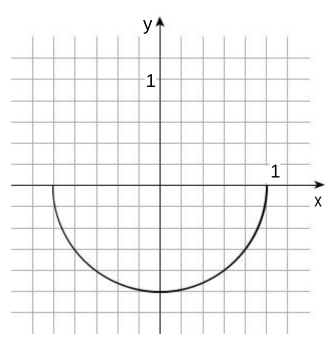
21. $x \quad \sqrt{y}$

x	y
0	0
1	1
2	4
3	9



22. $y \quad \sqrt{1 \quad x^2}$

x	y
1	0
$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
0	1
1	0



y 9 x^2

x-axis symmetry: replacing y by y gives y 9 x^2, which is not the same as the original equation, so the graph is not symmetric about the x-axis.

y-axis symmetry: replacing x by x gives y 9 x^2 9 x^2, which is the same as the original equation, so the graph is symmetric about the y-axis.

Origin symmetry: replacing x by x and y by y gives y 9 x^2 y 9 x^2, which is not the same as the original equation, so the graph is not symmetric about the origin.

To find x-intercepts, we set y 0 and solve for x: 0 9 x^2 x^2 9 x3, so the x-intercepts are 3 and 3.

To find y-intercepts, we set x 0 and solve for y: y 9 0^2 9, so the y-intercept is 9.

$6x$ y^2 36

x-axis symmetry: replacing y by y gives $6x$ y^2 36 $6x$ y^2 36, which is the same as the original equation, so the graph is symmetric about the x-axis.

y-axis symmetry: replacing x by x gives 6 x y^2 36 $6x$ y^2 36, which is not the same as the original equation, so the graph is not symmetric about the y-axis.

Origin symmetry: replacing x by x and y by y gives 6 x y^2 36 $6x$ y^2 36, which is not the same as the original equation, so the graph is not symmetric about the origin.

To find x-intercepts, we set y 0 and solve for x: $6x$ 0^2 36 x 6, so the x-intercept is 6.

To find y-intercepts, we set x 0 and solve for y: 6 0 y^2 36 y6, so the y-intercepts are 6 and 6.

x^2 y 1^2 1

x-axis symmetry: replacing y by y gives x^2 y 1^2 1 x^2 y 1^2 1, so the graph is not symmetric about the x-axis.

y-axis symmetry: replacing x by x gives x^2 y 1^2 1 x^2 y 1^2 1, so the graph is symmetric about the y-axis.

Origin symmetry: replacing x by x and y by y gives x^2 y 1^2 1 x^2 y 1^2 1, so the graph is not symmetric about the origin.

To find x-intercepts, we set y 0 and solve for x: x^2 0 1^2 1 x^2 0, so the x-intercept is 0.

To find y-intercepts, we set x 0 and solve for y: 0^2 y 1^2 1 y 1 1 y 0 or 2, so the y-intercepts are 0 and 2.

x^4 16 y

x-axis symmetry: replacing y by y gives x^4 16 y x^4 16 y, so the graph is not symmetric about the x-axis.

y-axis symmetry: replacing x by x gives x^4 16 y x^4 16 y, so the graph is symmetric about the y-axis.

Origin symmetry: replacing x by x and y by y gives x^4 16 y x^4 16 y, so the graph is not symmetric about the origin.

To find x-intercepts, we set y 0 and solve for x: x^4 16 0 x^4 16 x 2, so the x-intercepts are 2 and 2.

To find y-intercepts, we set x 0 and solve for y: 0^4 16 y y16, so the y-intercept is 16.

$9x^2$ $16y^2$ 144

x-axis symmetry: replacing y by y gives $9x^2$ 16 y^2 144 $9x^2$ $16y^2$ 144, so the graph is symmetric about the x-axis.

y-axis symmetry: replacing x by x gives $9\,x^2\ 16y^2\ 144\ 9x^2\ 16y^2\ 144$, so the graph is symmetric about the *y*-axis.

Origin symmetry: replacing x by x and y by y gives $9\,x^2\ 16\,y^2\ 144\ 9x^2\ 16y^2\ 144$, so the graph is symmetric about the origin.

(b) To find x-intercepts, we set y 0 and solve for x: $9x^2$ 16 0 ^{2} 144 $9x^2$ 144 x 4, so the x-intercepts are 4 and 4.

To find y-intercepts, we set x 0 and solve for y: 9 0 ^{2} 16y^2 144 16 y^2 144, so there is no y-intercept.

28. y $\frac{4}{}$

x-axis symmetry: replacing y by y gives y $\frac{4}{x}$, which is different from the original equation, so the graph is not symmetric about the x-axis.

y-axis symmetry: replacing x by x gives y $\frac{4}{x}$, which is different from the original equation, so the graph is not symmetric about the y-axis.

Origin symmetry: replacing x by x and y by y gives y $\frac{4}{x}$ y $\frac{4}{x}$, so the graph is symmetric about the origin.

To find x-intercepts, we set y 0 and solve for x: 0 $\frac{4}{x}$ has no solution, so there is no x-intercept.

To find y-intercepts, we set x 0 and solve for y. But we cannot substitute x 0, so there is no y-intercept.

x^2 4xy y^2 1

x-axis symmetry: replacing y by y gives x^2 4x y y^2 1, which is different from the original equation, so the graph is not symmetric about the x-axis.

y-axis symmetry: replacing x by x gives x^2 4 x y y^2 1, which is different from the original equation, so the graph is not symmetric about the y-axis.

Origin symmetry: replacing x by x and y by y gives x^2 4 x y y^2 1 x^2 4x y y^2 1, so the graph is symmetric about the origin.

To find x-intercepts, we set y 0 and solve for x: x^2 4x 0 0^2 1 x^2 1 x1, so the x-intercepts are 1 and 1.

To find y-intercepts, we set x 0 and solve for y: 0^2 4 0 y y^2 1 y^2 1 y1, so the y-intercepts are 1 and 1.

x^3 xy^2 5

x-axis symmetry: replacing y by y gives x^3 x y^2 5 x^3 xy^2 5, so the graph is symmetric about the x-axis.

y-axis symmetry: replacing x by x gives x^3 x y^2 5, which is different from the original equation, so the graph is not symmetric about the y-axis.

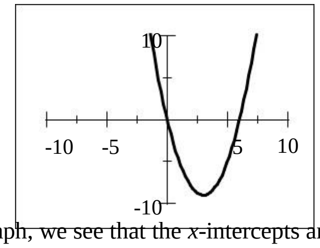
Origin symmetry: replacing x by x and y by y gives x^3 $x\,y^2$ 5, which is different from the original equation, so the graph is not symmetric about the origin.

To find x-intercepts, we set y 0 and solve for x: x^3 $x\,0^2$ x^3 5 x $\sqrt[3]{5}$, so the x-intercept is $\sqrt[3]{5}$.

2

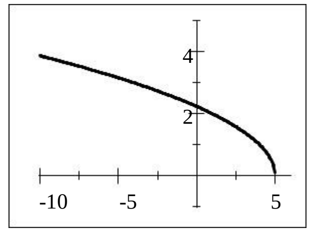
To find y-intercepts, we set x 0 and solve for y: 0^3 $0y$ 5 has no solution, so there is no y-intercept.

(a) We graph $y\ x^2\ 6x$ in the viewing rectangle [10 10] by [10 10].



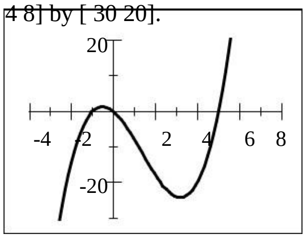
From the graph, we see that the x-intercepts are 0 and 6 and the y-intercept is 0.

(a) We graph $y 5\ x$ in the viewing rectangle [10 6] by [1 5].



From the graph, we see that the x-intercept is 5 and the y-intercept is approximately 2 24.

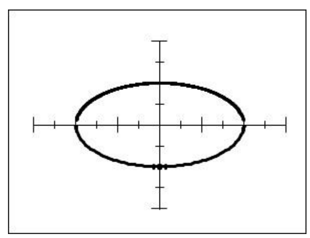
(a) We graph $y\ x^3\ 4x^2\ 5x$ in the viewing rectangle [4 8] by [30 20].



From the graph, we see that the x-intercepts are 1, 0, and 5 and the y-intercept is 0.

(a) We graph $\frac{x^2}{4}\ y^2\ 1\ y^2\ 1\ \frac{x^2}{4}$

$y\ \ 1\ \frac{x^2}{4}$ in the viewing rectangle [3 3] by [2 2].

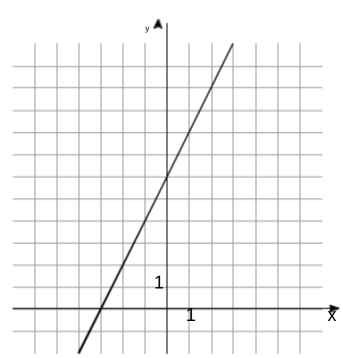


From the graph, we see that the x-intercepts are 2 and 2 and the y-intercepts are 1 and 1.

35. (a) The line that has slope 2 and y-intercept 6 has the slope-intercept equation $2x\ \ 6$.

An equation of the line in general form is $2x\ \ y\ \ 6\ \ 0$.

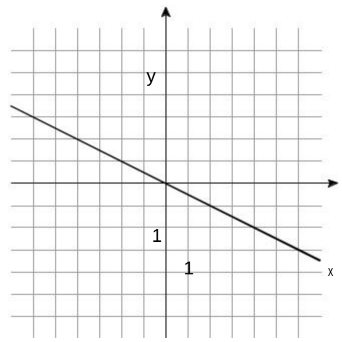
(c)



36. (a) The line that has slope $2\frac{1}{}$ and passes through the point 6 3 has equation $y 3 2\frac{1}{} x \ 6 \ y \ 3 \ 2\frac{1}{} x \ 6 \ y \ 2\frac{1}{} x$.

(b) $2\frac{1}{} x \ 3 \ y \ 3 \ x \ 6 2y \ 6 \ x \ 2y \ 0$.

(c)

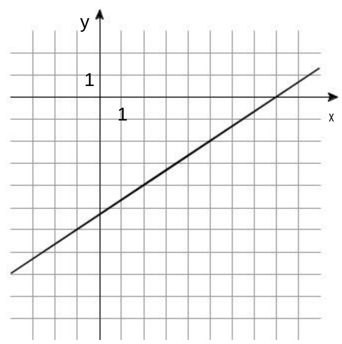


37. (a) The line that passes through the points 1 6 and 2 4 has slope $m \frac{46}{213}$, so y 2 6 $\frac{2}{3}[x 1$] y 6 $\frac{2}{3} x \frac{2}{3}$

$y \ 3\frac{2}{} x \ \frac{16}{3}$.

(b) $y \ 3\frac{2}{} x \ \frac{16}{3} \ 3y \ 2x \ 16 \ 2x \ 3y \ 16 \ 0$.

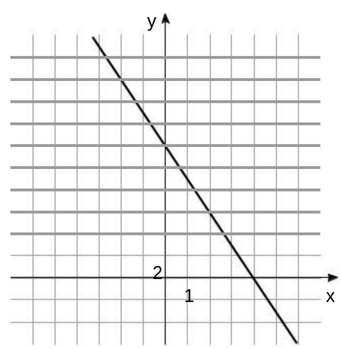
(c)



38. (a) The line that has x-intercept 4 and y-intercept 12 passes through the points 4 0 and 0 12 , so $m \frac{12 \ 0}{0 \ 4}$ 3 and the equation is $y \ 0 3 x \ 4 y 3x \ 12$.

(b) $y 3x \ 12 \ 3x \ y \ 12 \ 0$.

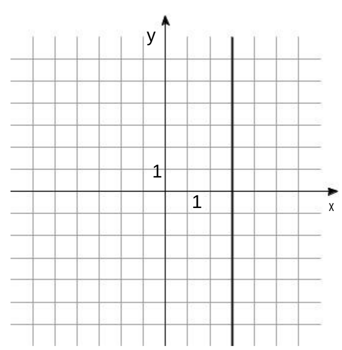
(c)



(a) The vertical line that passes through the point 3 2 has equation x 3.

$x \ 3 \ x \ 3 \ 0$.

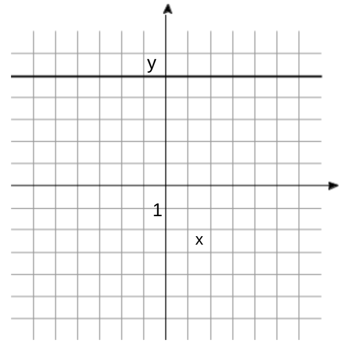
(c)



40. (a) The horizontal line with y-intercept 5 has equation y 5.

y 5 y 5 0.

(c)

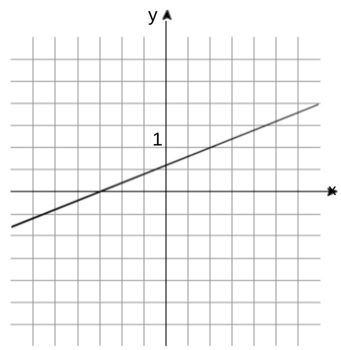


41. (a) $2x$ $5y$ 10 $5y$ $2x$ 10 y $\frac{2}{5}x$ 2, so the given line has slope

$\frac{2}{5}$. Thus, an equation of the line passing through 1 1 parallel to this line is y 1 $\frac{2}{5}x$ 1 y $\frac{2}{5}x$ $\frac{3}{5}$

y $\frac{2}{5}x$ $\frac{3}{5}$ $5y$ $2x$ 3 $2x$ $5y$ 3 0.

(c)

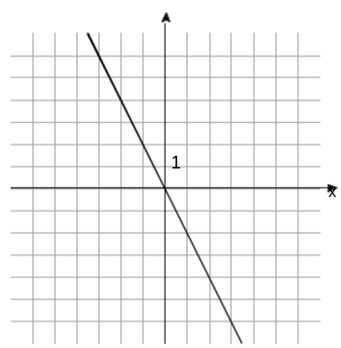


42. (a) The line containing 2 4 and 4 4 has slope

m $\frac{4 \quad 4}{4 \quad 2}$ $\frac{8}{2}$ 4, and the line passing through the origin with this slope has equation y $4x$.

$y4x$ $4x$ y 0.

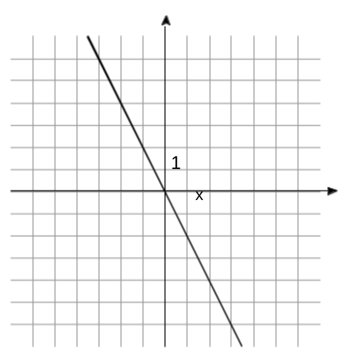
(c)



43. (a) The line y $\frac{1}{2}x$ 10 has slope $\frac{1}{2}$, so a line perpendicular to this one has slope $\frac{1}{1\ 2}$ 2. In particular, the line passing through the origin perpendicular to the given line has equation $y2x$.

(b) $y2x$ $2x$ y 0.

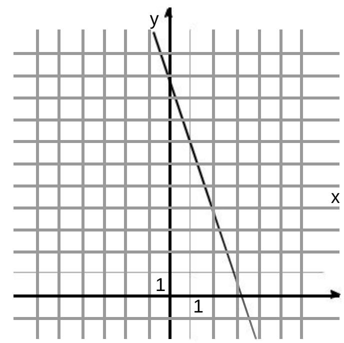
(c)



44. (a) x 3y 16 0 3y x 16 y $\frac{1}{3}x$ $\frac{16}{3}$, so the given line has slope $\frac{1}{3}$. The line passing through 1 7 perpendicular to the given line has equation y 7 $\frac{1}{1\ 3}$ x 1y 73 x 1y3x 10.

(b) y3x 10 3x y 10 0.

(c)



45. The line with equation y $\frac{1}{3}x$ 1 has slope $\frac{1}{3}$. The line with equation 9y 3x 3 0 9y 3x 3 $\frac{1}{3}x$ $\frac{1}{3}$ also has slope $\frac{1}{3}$, so the lines are parallel.

The line with equation 5x 8y 3 8y 5x 3 y $\frac{5}{8}x$ $\frac{3}{8}$ has slope $\frac{5}{8}$. The line with equation 10y 16x 1 10y 16x 1 y $\frac{8}{5}x$ $\frac{1}{10}$ has slope $\frac{8}{5}$ $\frac{1}{5\ 8}$, so the lines are perpendicular.

(a) The slope represents a stretch of 0 3 inches for each one-pound increase in weight. The s-intercept represents the length of the unstretched spring.

(b) When5, s 0 3 5 2 5 1 5 2 5 4 0 inches.

(a) We use the information to find two points, 0 60000 and 3 70500 . Then the slope is

m $\frac{70{,}500\ 60{,}000}{3\ 0}$ $\frac{10{,}500}{3}$ 3,500. So S 3,500t 60,000.

The slope represents an annual salary increase of \$3500, and the S-intercept represents her initial salary.

When t 12, her salary will be S 3500 12 60,000 42,000 60,000 \$102,000.

x^2 9x 14 0x 7 x 2 0 x 7 or x 2.

x^2 24x 144 0x 12^2 0 x 12 0 x12.

51. 2x^2 x 1 2x^2 x 1 0 2x 1 x 1 0. So either 2x 1 0 2x 1 x $\frac{1}{2}$; or x 1 0 x1.

52. 3x^2 5x 2 03x 1 x 2 0 x $\frac{1}{3}$ or x2.

53. 0 4x^3 25x x 4x^2 25 5 x 2x 5 2x 5 0. So either x 0 or 2x 5 0 2x 5 x $\frac{5}{2}$; or

2x 5 0 2x5 x $\frac{5}{2}$.

54. x^3 2 2x^2 5x 10 0 x^2 x 2 5 x 2 0x 2 x2 50 x 2 or x 5.

55. 3x 4x 1 0 $\frac{b \quad b^2 \quad 4ac}{2}$ $\frac{4 \quad 4^2 \quad 4\ 3\ 1}{2\ 3}$ $\frac{4}{6}$ $\frac{6}{}$ $\frac{4 \quad 2}{6}$ $\frac{6}{}$ $\frac{1}{6}$ $\frac{4\ 2\ 7}{2\ 2}$ $\frac{6}{}$ $\frac{2}{}$ $\frac{3}{}$ $\frac{7}{}$.

56. x^2 3x 9 0 x $\frac{b \quad b^2 \quad 4ac}{2a}$ $\frac{3 \quad 3^2 \quad 4\ 1\ 9}{2\ 1}$ $\frac{3 \quad 9\ 36}{2}$ $\frac{3 \quad 27}{2}$, which are not real numbers. There is no real solution.

57. x $\frac{1}{}$ $\frac{2}{}$ x 1 3 x 1 2 x3 x x 1x 1 2x $\frac{2}{}$ $\frac{2}{}$ 3x 3x0 3x 6x 1 $\frac{x}{2a}$ $\frac{6 \quad 6^2 \quad 4\ 3\ 1}{2\ 3}$ $\frac{6}{6}$ $\frac{3}{6}$ $\frac{6}{6}$ $\frac{6}{2\ 3}$ $\frac{3}{6}$

b b 4ac 6 1 2 . 6 2 4 . 6 2 6 . 3 6

x 1 8

2 2

58. x 2 x 2 x^2 4 x x 2x 2 8 x 2x x 2 8 x 3x 10 0x 2 x 5 0

x 2 or x 5. However, since x 2 makes the expression undefined, we reject this solution. Hence the only solution is x

5.

 x 2 or x 1. However, since x 2 makes the expression undefined, we reject this solution. Hence the only solution is x

59. x^4 8x^2 9 0x^2 9 x^2 1 0 x 3 x 3 x^2 1 0 x 3 0 x 3, or x 3 0

2

x3, however x 1 0 has no real solution. The solutions are x3.

x 4 x 32. Let u x. Then u^2 4u u 4 0. If u 8 0, 32 u^2 4u 32 0 u 8 u 4 0 So either u 8 0 or 8 x 64. If u 4 0, then u 4 x 4, then u 8 x solution. So the only solution is x which has no real 64.

61. x 1 2 2x1 2 x3 2 0 x 1 2 1 2x x^2 0 x 1 2 1 x^2 0. Since x 1 2 1 x is never 0, the

2

only solution comes from 1 x 0 1 x 0 x 1.

62. 1 x^2 2 1 x 15 0. Let u 1 x, then the equation becomes u^2 2u 15 0 u 5 u 3 0 u 5 0 or u 3 0. If u 5 0, then u 5 1 x 5 x 4 x 16. If u 3 0, then u 3 1 x 3 x 4, which has no real solution. So the only solution is x 16.

63. x 7 4 x 74 x 7 4, so x 11 or x 3.

64. 2x 5 9 is equivalent to 2x 59 2x 5 9 x $\frac{5\ 9}{2}$. So x2 or x 7.

65. (a) 2 3i 1 4i 2 13 4 i 3 i

(b) 2 i 3 2i 6 4i 3i 2i^2 6 i 2 8 i

66. (a) 3 6i 6 4i 3 6i 6 4i 3 66 4 i 3 2i

(b) 4i 2 $\frac{1}{2}$ i8i 2i^2 8i 2 2 8i 2

67. (a) $\frac{4\ 2i}{2i}$ $\frac{4\ 2i}{2\ i}$ $\frac{2\ i}{2\ i}$ $\frac{8\ 8i\ 2i}{4\ i^2}$ $\frac{8\ 8i\ 2}{4\ 1}$ $\frac{6\ 8i}{5}$ $\frac{6}{5}$ $\frac{8}{5}i$

(b) 1 1 1 1 1 i 1 i 1 i 2 i i^2 1 1 2

68. (a) $\frac{8\ 3i}{4\ 3i}$ $\frac{8\ 3i}{4\ 3i}$ $\frac{4\ 3i}{4\ 3i}$ $\frac{32\ 12i\ 9i}{16\ 9i^2}$ $\frac{32\ 12i\ 9}{16\ 9}$ $\frac{41\ 12i}{25}$ $\frac{41}{25}$ $\frac{12}{25}i$

2

(b) 10 40 i 10 2i 10 20i 20

x^2 16 0 x^2 16 x4i

2

x 12 x122 3i

x^2 6x 10 0x $\frac{bb2\ 4ac}{2a}$ $\frac{6\ 6^2\ 4\ 1\ 10}{2}$ $\frac{6\ 36\ 40}{12}$ 3 i

2x^2 3x 2 0x $\frac{3\ 3^2\ 4\ 2\ 2}{2\ 2}$ $\frac{3\ 7}{4}$ i 44

73. x^4 256 0x^2 16 x^2 16 0 x4 or x4i

74. x^3 2x^2 4x 8 0x 2 x^2 4x 2 or x2i

Let r be the rate the woman runs in mi/h. Then she cycles at r 8 mi/h.

	Rate	Time	Distance
Cycle	r 8	$\frac{4}{r\ 8}$	4
Run	r	$\frac{2\ 5}{r}$	2 5

Since the total time of the workout is 1 hour, we have $\frac{4}{r\ 8}$ $\frac{2\ 5}{r}$ 1. Multiplying by $2r$ r 8 , we get 4 $2r$2 5 2 r 8 2 $2r$ r 8 2 8r 5r 402r 16r 02r 3r 40

r 3 3 2 42 40 / 2 2 3 9 320 / 4 3 329 / 4 . Since r 0, we reject the negative value. She runs at

r 3 4 329 3 78 mi/h.

76. Substituting 75 for d, we have 75 x $\frac{x^2}{20}$ 1500 20x x^2 x^2 20x 1500 0x 30 x 50 0. So x 30 or x 50. The speed of the car was 30 mi/h.

77. Let x be the length of one side in cm. Then 28 x is the length of the other side. Using the Pythagorean Theorem, we have x^2 28 x^2 20^2 x^2 784 56x x^2 400 $2x^2$ 56x 384 0 2 x^2 28x 1920

2x 12 x 16 0. So x 12 or x 16. If x 12, then the other side is 28 12 16. Similarly, if x 16, then the other side is 12. The sides are 12 cm and 16 cm.

78. Let l be length of each garden plot. The width of each plot is then $\frac{80}{l}$ and the total amount of fencing material is 4l 6 $\frac{80}{l}$ 88. Thus 4l $\frac{480}{l}$ 88 $4l^2$ 480 88l $4l^2$ 88l 480 0 4 l^2 22l 120 0

4l 10 l 12 0. So l 10 or l 12. If l 10 ft, then the width of each plot is $\frac{80}{10}$ 8 ft. If l 12 ft, then the width of each plot is $\frac{80}{12}$ 6 67 ft. Both solutions are possible.

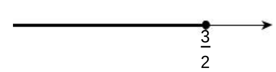
79. 3x 211 3x9 x3.
Interval: 3 .

Graph:

80. 12 x 7x 12 8x $\frac{3}{2}$ x.

Interval: $\frac{3}{2}$

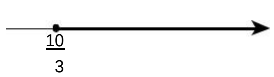
Graph:



3 x 2x 7 10 3x $\frac{10}{3}$ x

Interval: $\frac{10}{3}$

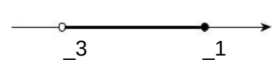
Graph:



82. 1 2x 5 36 2x23 x1

Interval: 3 1].

Graph:



x^2 4x 12 0 x 2 x 6 0. The expression on the left of the inequality changes sign where x 2 and where x 6. Thus we must check the intervals in the following table.

Interval	6	6 2	2
Sign of x 2			
Sign of x 6			
Sign of x 2 x 6			

Interval:62

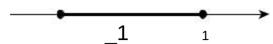
Graph:

$\frac{6}{2}$

x^2 1 x^2 1 0 x 1 x 1 0. The expression on the left of the inequality changes sign when x 1 and x 1. Thus we must check the intervals in the following table.

Interval	1	1 1	1
Sign of x 1 Sign of x 1			
Sign of x 1 x 1			

Interval: [1 1]

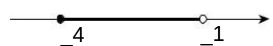
Graph: 

85. $\frac{2x\ 5}{x\ 1}$ 1 $\frac{2x\ 5}{x\ 1}$ 1 0 $\frac{2x\ 5}{x\ 1}$ $\frac{x\ 1}{x\ 1}$ 0 $\frac{x\ 4}{x\ 1}$ 0. The expression on the left of the inequality

changes sign where x 1 and where x 4. Thus we must check the intervals in the following table.

Interval	4	4 1	1
Sign of x 4 Sign of x 1			
Sign of $\frac{x\ 4}{x\ 1}$			

We exclude x 1, since the expression is not defined at this value. Thus the solution is [4 1.

Graph: 

$2x^2$ x 3 $2x^2$ x 3 0$2x$ 3 x 1 0. The expression on the left of the inequality changes sign when 1 and $\frac{3}{2}$. Thus we must check the intervals in the following table.

Interval	1	1 $\frac{3}{2}$	$\frac{3}{2}$
Sign of $2x$ 3 Sign of x 1			
Sign of $2x$ 3 x 1			

Interval:1] $\frac{3}{2}$

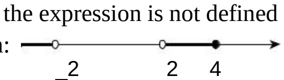
Graph: 

87. $\frac{x\ 4}{x^2\ 4}$ 0 $\frac{x\ 4}{x\ 2\ x\ 2}$ 0. The expression on the left of the inequality changes sign where x2, where x 2,

and where x 4. Thus we must check the intervals in the following table.

Interval	2	2 2	2 4	4
Sign of x 4 Sign of x 2 Sign of x 2				
Sign of $\frac{x\ 4}{x\ 2\ x\ 2}$				

Since the expression is not defined when x 2 we exclude these values and the solution is 2 2 4].

Graph: 

88. $\frac{5}{x^3 \quad x^2 \quad 4x \quad 4} \quad 0 \qquad \frac{5}{x^2 \quad x \quad 1 \quad 4x \quad 1} \quad 0 \qquad \frac{5}{x \quad 1 x^2 \quad 4} \quad 0 \qquad \frac{5}{x \quad 1 \quad x \quad 2 \quad x \quad 2} \quad 0$. The

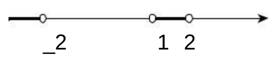
expression on the left of the inequality changes sign when 2 1 and 2.

table.

Interval	2	2 1	1 2	2
Sign of x 1 Sign of x 2 Sign of x 2				
5 Sign of x 1 x 2 x 2				

Interval: 2 1 2

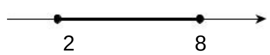
Graph:



89. x 5 3 3 x 5 3 2 x 8.

Interval: [2 8]

Graph:

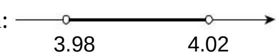


90. x 4 0 02 0 02 x 4 0 02

3 98 x 4 02

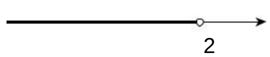
Interval: 3 98 4 02

Graph:



91. 2x 1 1 is equivalent to 2x 1 1 or 2x 1 1. *Case 1:* 2x 1 1 2x 0 x 0. *Case 2:* 2x 1 1 2x 2 x 1. Interval: 1] [0 . Graph:

92. x 1 is the distance between x and 1 on the number line, and x 3 is the distance between x and 3. We want those points that are closer to 1 than to 3. Since 2 is midway between 1 and 3, we get x2 as the solution. Graph:

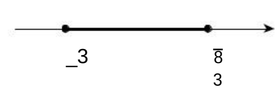


93. (a) For $\sqrt{24 \quad x \quad 3x^2}$ to define a real number, we must have 24 x 3 x^2 08 3x 3 x 0. The expression on the left of the inequality changes sign where 8 3x 03x8 x $3\frac{8}{}$; or where x3. Thus we must check the intervals in the following table.

Interval	3	3$\frac{8}{3}$	$\frac{8}{3}$
Sign of 8 3x			
Sign of 3 x			
Sign of 8 3x 3 x			

Interval: $3\frac{8}{3}$.

Graph:



(b) For $\frac{1}{4\ x\ x^4}$ to define a real number we must have $x\ x^4\ \ 0\ \ x\ 1\ \ x^3 0\ \ x\ 1\ \ x\ 1\ \ x\ \ x^2 0$.

expression on the left of the inequality changes sign where x 0; or where x 1; or where 1 x 2 0

The $x\ \ 1\ \ \frac{1\ 4\ 1\ 1}{2}\ \ \frac{1\ 2\ 1\ 4}{}$ which is imaginary. We check the intervals in the following table.

Interval	0	0 1	1
Sign of x Sign of 1 x Sign of 1 $x\ x^2$			
Sign of x 1 x 1 x x^2			

Interval: 0 1 .

Graph:

0 1

94. We have $8\ \ 3^4\ r^3\ \ 12r^3$ $\frac{6}{}$ $\frac{9\ \ _3 6\ \ _3 9}{}$ r . Thus r $_3 6\ \ \overline{_3 9}$.

From the graph, we see that the graphs of $y\ x^2\ 4x$ and $y\ x\ 6$ intersect at x 1 and x 6, so these are the solutions of the equation $x^2\ \ 4x\ x\ 6$.

From the graph, we see that the graph of $y\ x^2\ 4x$ crosses the x-axis at x 0 and x 4, so these are the solutions of the equation $x^2\ \ 4x\ 0$.

From the graph, we see that the graph of $y\ \ x^2\ \ 4x$ lies below the graph of $y\ \ x\ \ 6$ for $1\ \ x\ \ 6$, so the inequality $^2\ \ 4x\ \ x\ \ 6$ is satisfied on the interval [1 6].

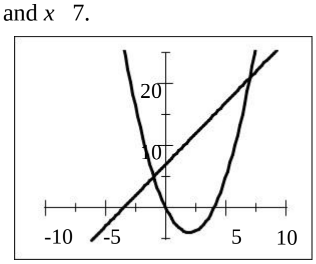
From the graph, we see that the graph of $y\ x^2\ 4x$ lies above the graph of $y\ x\ 6$ for x 1 and 6 x , so the inequality $x^2\ 4x\ x\ 6$ is

satisfied on the intervals 1] and [6 .

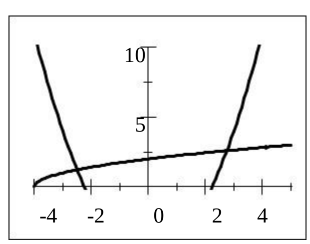
From the graph, we see that the graph of $y\ \ x^2\ \ 4x$ lies above the x-axis for $x\ \ 0$ and for $x\ \ 4$, so the inequality $^2\ \ 4x\ \ 0$ is satisfied on the intervals0] and [4.

From the graph, we see that the graph of $y\ x^2\ 4x$ lies below the x-axis for 0 x 4, so the inequality $x^2\ 4x\ 0$ is satisfied on the interval [0 4].

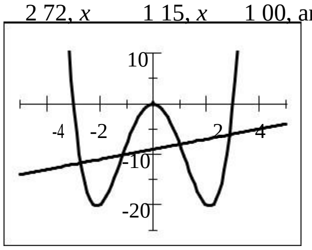
$x^2\ \ 4x\ \ 2x\ \ 7$. We graph the equations $y_1\ \ x^2\ \ 4x$ and $y_2\ \ 2x\ \ 7$ in the viewing rectangle [10 10] by [5 25]. Using a zoom or trace function, we get the solutions x1 and $x\ \ 7$.



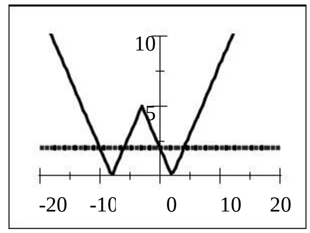
102. $x\ \ 4\ \ x^2\ \ 5$. We graph the equations $y_1 x\ \ 4$ and $y_2\ \ \ x^2\ \ 5$ in the viewing rectangle [4 5] by [0 10]. Using a zoom or trace function, we get the solutions $x\ \ \ 2\ 50$ and $x\ \ \ 2\ 76$.



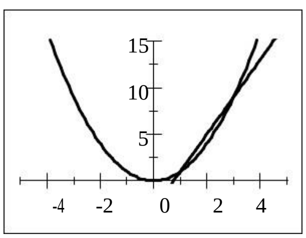
x^4 $9x^2$ x 9. We graph the equations y_1 x^4 $9x^2$ and y_2 x 9 in the viewing rectangle [5 5] by [25 10]. Using a zoom or trace function, we get the solutions x 2 72, x 1 15, x 1 00, and x 2 87.



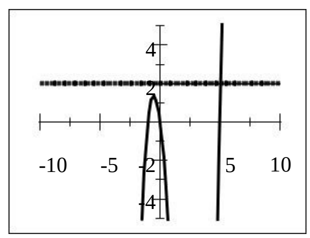
104. x 3 5 2. We graph the equations y_1 x 3 5 and y_2 2 in the viewing rectangle [20 20] by [0 10]. Using Zoom and/or Trace, we get the solutions x 10, x 6, x 0, and x 4.



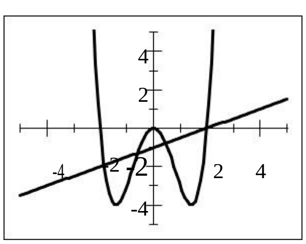
105. $4x$ 3 x^2. We graph the equations y_1 $4x$ 3 and y_2 x^2 in the viewing rectangle [5 5] by [0 15]. Using a zoom or trace function, we find the points of intersection are at x 1 and x 3. Since we want $4x$ 3 x^2, the solution is the interval [1 3].



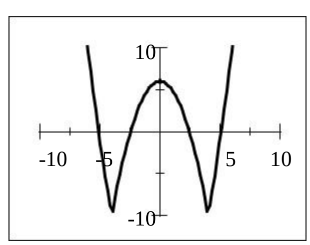
106. x^3 $4x^2$ $5x$ 2. We graph the equations y_1 x^3 $4x^2$ $5x$ and y_2 2 in the viewing rectangle [10 10] by [5 5]. We find that the point of intersection is at x 5 07. Since we want x^3 $4x^2$ $5x$ 2, the solution is the interval 5 07 .



x^4 $4x^2$ $\frac{1}{2}x$ 1. We graph the equations y_1 x^4 $4x^2$ and y_2 $\frac{1}{2}x$ 1 in the viewing rectangle [5 5] by [5 5]. We find the points of intersection are at x 1 85, x 0 60, x 0 45, and x 2 00. Since we want x^4 $4x^2$ $\frac{1}{2}x$ 1, the solution is 185 060045 200.



x^2 16 10 0. We graph the equation x^2 16 10 in the viewing rectangle [10 10] by [10 10]. Using a zoom or trace function, we find that the x-intercepts are x 5 10 and x 2 45. Since we want x^2 16 10 0, the solution is approximately 510] [245 245] [510 .



109. Here the center is at 0 0 , and the circle passes through the point 5 12 , so the radius is

r 22 5 0 120 25 144 169 13. The equation of the circle is x^2 y^2 13^2

x y 169. The line shown is the tangent that passes through the point 5 12 , so it is perpendicular to the line through the points 0 0 and 5 12 . This line has slope m_1 $\frac{12 \quad 0}{5 \quad 0}$ $\frac{12}{5}$. The slope of the line we seek is

m_2 $\frac{1}{}$ $\frac{1}{}$ $\frac{5}{}$. Thus, an equation of the tangent line is y 12 $\frac{5}{12}$ x 5y 12 $\frac{5}{12}$ $\frac{25}{12}x$

y $\frac{5}{12}$ x $\frac{169}{12}$ 5x 12y 169 0.

Because the circle is tangent to the x-axis at the point 5 0 and tangent to the y-axis at the point 0 5 , the center is at 5 5 and the radius is 5. Thus an equation is x 5 ^{2} y 5 ^{2} 5^2 x 5 ^{2} y 5 ^{2} 25. The slope of the line passing through the points 8 1 and 5 5 is m $\frac{5 \quad 1}{5 \quad 8}$ $\frac{4}{3}$ $\frac{4}{3}$, so an equation of the line we seek is 1 $\frac{4}{3}$ x 8 4x 3y 35 0.

Since M varies directly as z we have M kz. Substituting M 120 when z 15, we find 120 k 15k 8.

Therefore, M 8z.

112. Since z is inversely proportional to y, we have z $\frac{k}{y}$. Substituting z 12 when y 16, we find 12 $\frac{k}{16}$ k 192.

Therefore z $\frac{192}{y}$.

113. (a) The intensity I varies inversely as the square of the distance d, so I $\frac{k}{d^2}$.

(b) Substituting I 1000 when d 8, we get 1000 $\frac{k}{8^2}$ k 64,000.

(c) From parts (a) and (b), we have I $\frac{64{,}000}{d^2}$. Substituting d 20, we get I $\frac{64{,}000}{20^2}$ 160 candles.

Let f be the frequency of the string and l be the length of the string. Since the frequency is inversely proportional to the length, we have f $\frac{k}{l}$. Substituting l 12 when k 440, we find 440 $\frac{k}{12}$ k 5280. Therefore f $\frac{5280}{l}$. For

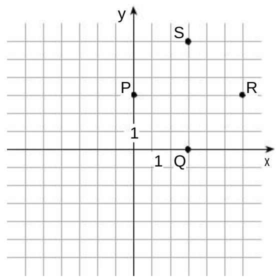
660, we must have 660 $\frac{5280}{l}$ l $\frac{5280}{660}$ 8. So the string needs to be shortened to 8 inches.

Let be the terminal velocity of the parachutist in mi/h and be his weight in pounds. Since the terminal velocity is directly proportional to the square root of the weight, we have k . Substituting 9 when 160, we solve for k. This gives 9 k 160 k $\frac{9}{160}$ 0 712. Thus0 712 . When240, the terminal velocity is 0 712 240 11 mi/h.

Let r be the maximum range of the baseball and be the velocity of the baseball. Since the maximum range is directly proportional to the square of the velocity, we have r l^2. Substituting 60 and r 242, we find 242 k 60^2 k 0 0672. If 70, then we have a maximum range of r 0 0672 70^2 329 4 feet.

CHAPTER 1 TEST

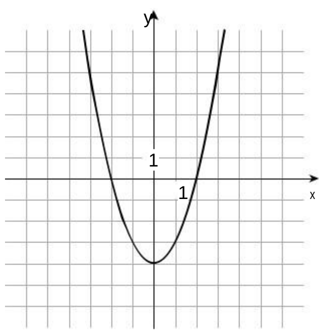
1. (a)



There are several ways to determine the coordinates of *S*. The diagonals of a square have equal length and are perpendicular. The diagonal *P R* is horizontal and has length is 6 units, so the diagonal *Q S* is vertical and also has length 6. Thus, the coordinates of *S* are 3 6 .

(b) The length of *P Q* is $0 \quad 3^2 \quad 3 \quad 0^2$ $\overline{18}$ 3 $\overline{2}$. So the area of *P Q R S* is $3 \quad \overline{2}^{\,2}$ 18.

2. (a)

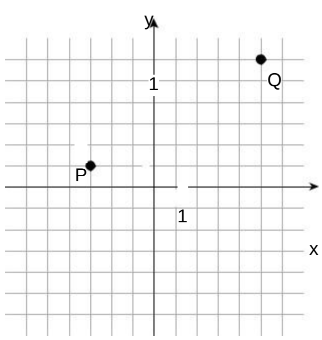


The *x*-intercept occurs when *y* 0, so 0 x^2 4 x^2 4 *x*2. The *y*-intercept occurs when *x* 0, so *y* 4.

x-axis symmetry: *y* x^2 4 *y* x^2 4, which is not the same as the original equation, so the graph is not symmetric with respect to the *x*-axis.

y-axis symmetry: *y* x^2 4 *y* x^2 4, which is the same as the original equation, so the graph is symmetric with respect to the *y*-axis. Origin symmetry: *y* x^2 4 *y* x^2 4, which is not the same as the original equation, so the graph is not symmetric with respect to the origin.

3. (a)



(b) The distance between *P* and *Q* is

*d P Q*3 5^2 $\overline{1 \quad 6^2}$ 64 25 $\overline{89}$.

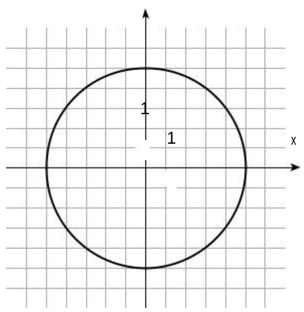
(c) The midpoint is $\dfrac{3 \quad 5}{2}$ $\dfrac{1 \quad 6}{2}$ 1 $\dfrac{7}{2}$.

(d) The slope of the line is $\dfrac{1 \quad 6}{3 \quad 5}$ $\dfrac{5}{8}$ $\dfrac{5}{8}$.

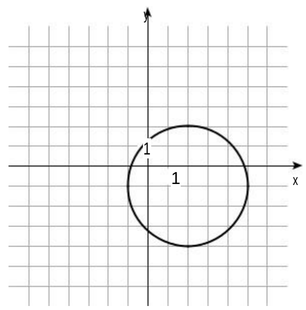
(e) The perpendicular bisector of *P Q* contains the midpoint, 1 $\frac{7}{2}$, and it slope is the negative reciprocal of $\frac{5}{8}$. Thus the slope is $\frac{1}{5 \quad 8}$ $\frac{8}{5}$. Hence the equation is *y* $\frac{7}{2}$ $\frac{8}{5}$ *x* 1 *y* $\frac{8}{5}$ *x* $\frac{8}{5}$ $\frac{7}{2}$ $\frac{8}{5}$ *x* $\frac{51}{10}$. That is, $\frac{8}{5}$ *x* $\frac{51}{10}$.

The center of the circle is the midpoint, 1 $\frac{7}{2}$, and the length of the radius is $\frac{1}{2}$ $\overline{89}$. Thus the equation of the circle whose diameter is *P Q* is *x* 1^2 *y* $\frac{7}{2}^2$ $\frac{1}{2}$ $\overline{89}^{\,2}$ *x* 1^2 *y* $\frac{7}{2}^2$ $\frac{89}{4}$.

(a) $x^2 \; y^2 \; 25 \; 5^2$ has center 0 0 and radius 5.



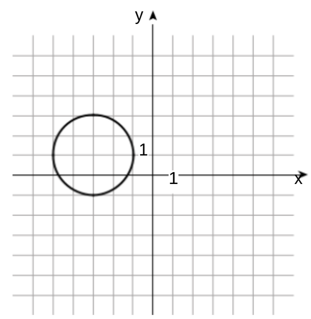
(b) $x \; 2^2 \; y \; 1^2 \; 9 \; 3^2$ has center 2 1 and radius 3.



(c) $x^2 \; 6x \; y^2 \; 2y \; 6 \; 0$

$x^2 \; 6x \; 9 \; y^2 \; 2y \; 1 \; 4$

$x \; 3^2 \; y \; 1^2 \; 4 \; 2^2$ has center 3 1 and radius 2.

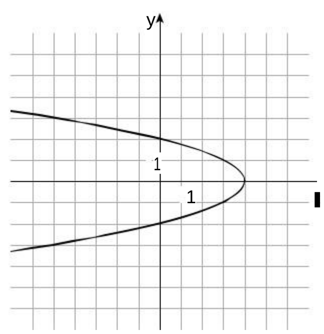


(a) $x \; 4 \; y^2$. To test for symmetry about the *x*-axis, we replace *y* with *y*:

$4 \; y^2 \; x \; 4 \; y^2$, so the graph is symmetric about the *x*-axis. To test for symmetry about the *y*-axis, we replace *x* with *x*:

$x \; 4 \; y^2$ is different from the original equation, so the graph is not symmetric about the *y*-axis.

For symmetry about the origin, we replace *x* with *x* and *y* with *y*: $x \; 4 \; y^2 \; x \; 4 \; y^2$, which is different from the original equation, so the graph is not symmetric about the origin.



To find *x*-intercepts, we set *y* 0 and solve for *x*: $x \; 4 \; 0^2 \; 4$, so the *x*-intercept is 4.

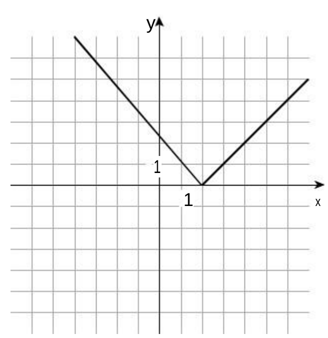
To find *y*-intercepts, we set *x* 0 and solve for *y*:: $0 \; 4 \; y^2 \; y^2 \; 4 \; y2$, so the *y*-intercepts are 2 and 2.

y x 2 . To test for symmetry about the *x*-axis, we replace *y* with *y*: *y x* 2 is different from the original equation, so the graph is not symmetric about the *x*-axis.

To test for symmetry about the *y*-axis, we replace *x* with *x*:

y x 2 *x* 2 is different from the original equation, so the graph is not symmetric about the *y*-axis.

To test for symmetry about the origin, we replace *x* with *x* and *y* with *y*: *y x* 2 *y x* 2 , which is different from the original equation, so the graph is not symmetric about the origin.



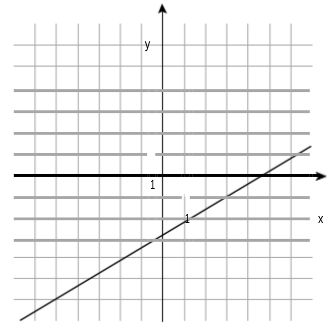
To find *x*-intercepts, we set *y* 0 and solve for *x*: 0 *x* 2 *x* 2 0 *x* 2, so the *x*-intercept is 2.

To find *y*-intercepts, we set *x* 0 and solve for *y*: *y* 0 2 2 2, so the *y*-intercept is 2.

6. (a) To find the x-intercept, we set y 0 and solve for x: $3x$ 5 0 15 3x 15 x 5, so the x-intercept is 5.

To find the y-intercept, we set x 0 and solve for y: 3 0 5y 15 5y 15 y3, so the y-intercept is 3.

(b)



(c) 3x 5y 15 5y 3x 15 y $\frac{3}{5}x$ 3.

(d) From part (c), the slope is $\frac{3}{5}$.

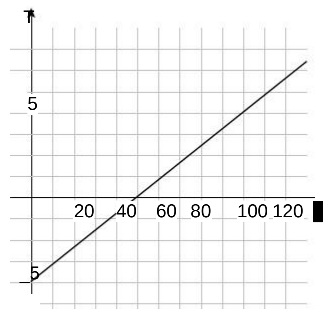
(e) The slope of any line perpendicular to the given line is the negative reciprocal of its slope, ~~that~~ is, 3 1 5 5 3 .

7. **(a)** 3x y 10 0 y3x 10, so the slope of the line we seek is 3. Using the point-slope, y63 x 3 y 63x 9 3x y 3 0.

(b) Using the intercept form we get $\frac{x}{6}$ $\frac{y}{4}$ 1 2x 3y 12 2x 3y 12 0.

8. **(a)** When x 100 we have T 0 08 100 4 8 4 4, so the temperature at one meter is 4 C.

(b)



(c) The slope represents an increase of 0 08 C for each one-centimeter increase in depth, the x-intercept is the depth at which the temperature is 0 C, and the T -intercept is the temperature at ground level.

9. **(a)** x^2 x 12 0x 4 x 3 0. So x 4 or x3.

(b) 2x 4x 1 0 x 2 4 4^2 4 2 1 4 16 8 48 4 2 2 22 4 4 2 2 2

(c) 3 x 3 x3 xx 33 x^2 3 x^2 x^2 6x 9 3 x 2

x 5x 6 x 2 x 3 0. Thus, x 2 and x 3 are potential solutions. Checking in the original equation, we see that only x 3 is valid.

$x^{1\ 2}$ 3$x^{1\ 4}$ 2 0. Let u $x^{1\ 4}$, then we have u^2 3u 2 0 u 2 u 1 0. So either u 2 0 or u 1 0. If u 2 0, then u 2 x 1 4 2 x 2^4 16. If u 1 0, then u 1 x 1 4 1 x 1. So x 1 or x 16.

x^4 3x^2 2 0x^2 1 x^2 20. So x^2 1 0 x1 or x^2 2 0 x2. Thus the solutions are x1, x 1, x2, and x 2.

(f) 3 x 4 10 0 3x 4 10x 4 $\frac{10}{3}$ x 4 3 x 4 $\frac{10}{3}$. So x 4 $\frac{10}{3}$ $\frac{2}{3}$ or x 4 $\frac{10}{3}$ $\frac{22}{3}$. Thus the solutions are x $\frac{2}{3}$ and x $\frac{22}{3}$.

(a) 3 2i4 3i 3 42i 3i 7 i

3 2i4 3i3 42i 3i1 5i

3 2i 4 3i 3 4 3 3i 2i 4 2i 3i 12 9i 8i 6i^2 12 i 6 1 18 i

(d) $\frac{3\ 2i}{4\ 3i}$ $\frac{3\ 2i}{4\ 3i}$ $\frac{4\ 3i}{4\ 3i}$ $\frac{12\ 17i\ 6i^2}{16\ 9i^2}$ $\frac{12\ 17i\ 6}{16\ 9}$ $\frac{6}{25}$ $\frac{17}{25}i$

(e) $i^{48}i^{2}$ ^{24} 1^{24} 1

(f) 2282 2 8 2 2 2 8 2 24 2i 4i2 6 2i

Using the Quadratic Formula, 2x2 4x 3 0x 442 4 2 348 12 i.

2242

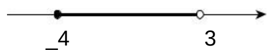
12. Let be the width of the parcel of land. Then70 is the length of the parcel of land. Then 270 2 1302

22 140 4900 16,900 2 2 140 12,000 0 2 70 6000 050120 0.

So50 or120. Since0, the width is50 ft and the length is70 120 ft.

13. (a) 4 5 3x 1793x 12 3 x4. Expressing in standard form we have: 4 x 3.

Interval: [4 3 . Graph:

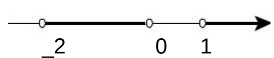


(b) x x 1 x 2 0. The expression on the left of the inequality changes sign when x 0, x 1, and x2. Thus we must check the intervals in the following table.

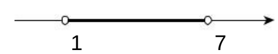
Interval	2	2 0	0 1	1
Sign of x Sign of x 1 Sign of x 2				
Sign of x x 1 x 2				

From the table, the solution set is x 2 x 0 or 1 x . Interval: 2 01.

Graph:



(c) x 4 3 is equivalent to 3 x 4 3 1 x 7. Interval: 1 7 . Graph:



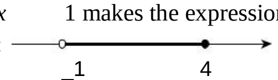
(d) $\frac{2x\ 3}{x\ 1}$ 1 $\frac{2x\ 3}{x\ 1}$ 1 0 $\frac{2x\ 3}{x\ 1}$ $\frac{x\ 1}{x\ 1}$ 0 $\frac{x\ 4}{x\ 1}$ 0. The expression on the left of the inequality

changes sign where x4 and where x1. Thus we must check the intervals in the following table.

Interval	1	1 4	4
Sign of x 4 Sign of x 1			
Sign of $\frac{x\ 4}{x\ 1}$			

Since x 1 makes the expression in the inequality undefined, we exclude this value. Interval: 1 4].

Graph:



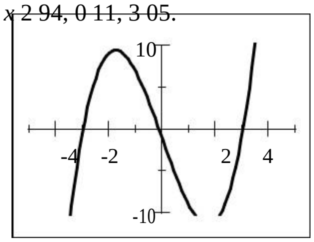
5 $\frac{5}{9}$ F 32 10 9 F 32 18 41 F 50. Thus the medicine is to be stored at a temperature between 41 F and 50 F.

For $6x\ x^2$ to be defined as a real number $6x\ x^2$ 0 x 6 x 0. The expression on the left of the inequality changes sign when x 0 and x 6. Thus we must check the intervals in the following table.

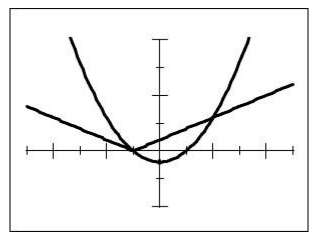
Interval	0	0 6	6
Sign of x Sign of 6 x			
Sign of x 6 x			

From the table, we see that $6x\ \ x^2$ is defined when 0 x 6.

16. (a) x^3 $9x$ 1 0. We graph the equation x^3 $9x$ 1 in the viewing rectangle [5 5] by [10 10]. We find that the points of intersection occur at x 2 94, 0 11, 3 05.



(b) x 1 x^2 1 . We graph the equations y_1 x 1 and y_2 x^2 1 in the viewing rectangle [5 5] by [5 10]. We find that the points of intersection occur at x 1 and x 2. Since we want x 1 x^2 1 , the solution is the interval [1 2].



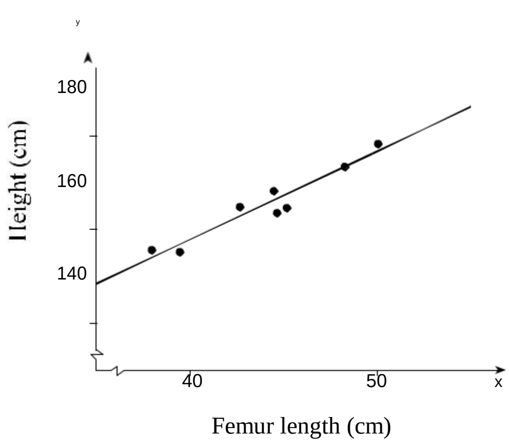
17. (a) M k $\dfrac{h^2}{L}$

(b) Substituting 4, h 6, L 12, and M 4800, we have 4800 $k\ \dfrac{4\ 6^2}{12}$ k 400. Thus M 400 $\dfrac{h^2}{L}$.

(c) Now if L 10, 3, and h 10, then M 400 $\dfrac{3\ 10^2}{10}$ 12,000. So the beam can support 12,000 pounds.

FOCUS ON MODELING Fitting Lines to Data

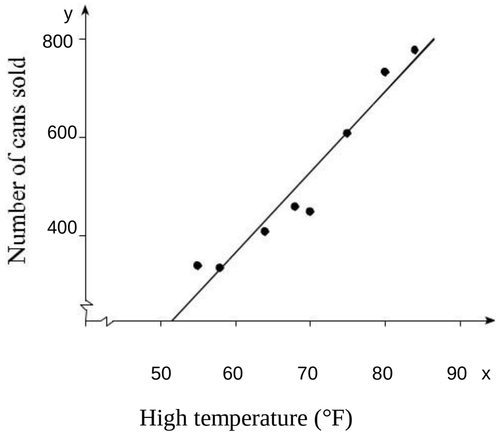
1. (a)



Using a graphing calculator, we obtain the regression line y 1 8807x 82 65.

Using x 58 in the equation y 1 8807x 82 65, we get y 1 8807 58 82 65 191 7 cm.

2. (a)



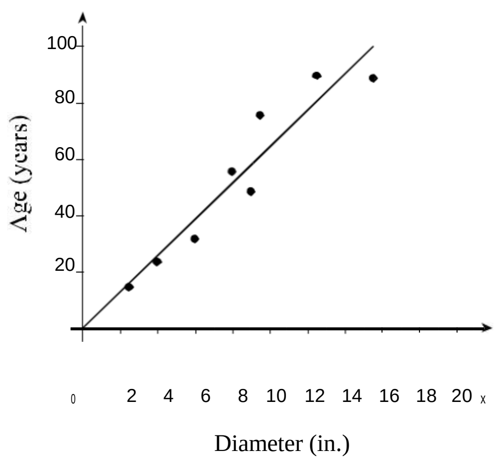
Using a graphing calculator, we obtain the regression line *y* 16 4163*x* 621 83.

Using *x* 95 in the equation

y 16 4163*x* 621 83, we get

y 16 4163 95 621 83 938 cans.

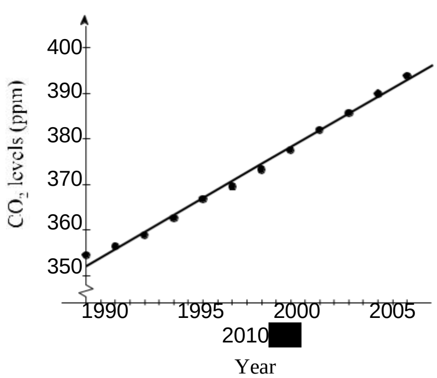
3. (a)



Using a graphing calculator, we obtain the regression line *y* 6 451*x* 0 1523.

Using *x* 18 in the equation *y* 6 451*x* 0 1523, we get *y* 6 451 18 0 1523 116 years.

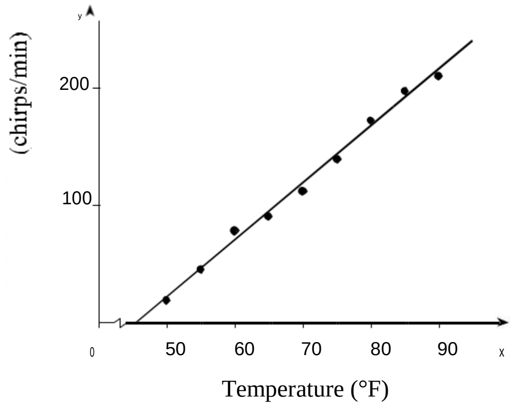
4. (a)



Letting *x* 0 correspond to 1990, we obtain the regression line *y* 1 8446*x* 352 2.

Using *x* 21 in the equation *y* 1 8446*x* 352 2, we get *y* 1 8446 21 352 2 390 9 ppm CO2, slightly lower than the measured value.

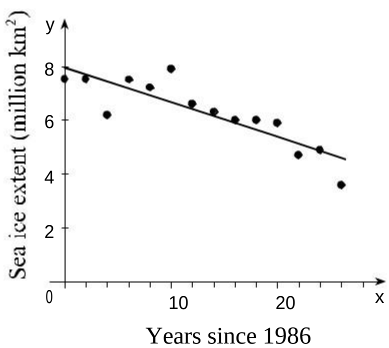
5. (a)



Using a graphing calculator, we obtain the regression line *y* 4 857*x* 220 97.

Using *x* 100 F in the equation *y* 4 857*x* 220 97, we get *y* 265 chirps per minute.

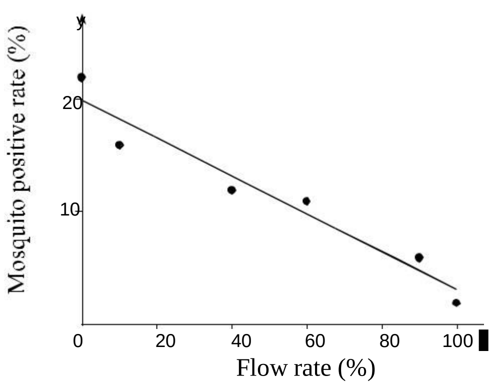
6. (a)



Using a graphing calculator, we obtain the regression line *y* 0 1275*x* 7 929.

Using *x* 30 in the regression line equation, we get *y* 0 1275 30 7 929 4 10 million km^2.

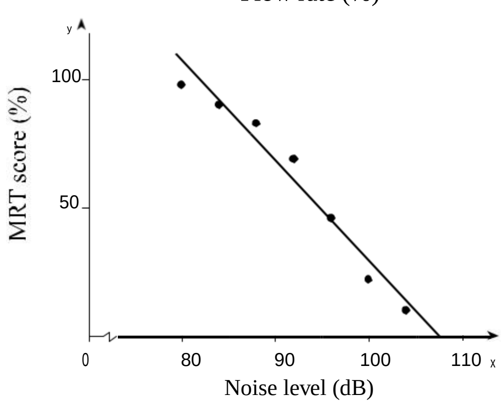
7. (a)



(b) Using a graphing calculator, we obtain the regression line *y*0 168*x* 19 89.

(c) Using the regression line equation *y*0 168*x* 19 89, we get *y* 8 13% when *x* 70%.

8. (a)

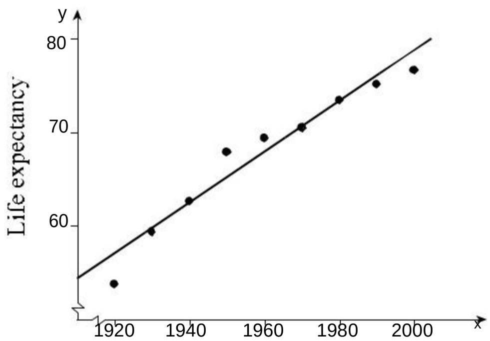


Using a graphing calculator, we obtain *y* 3 9018*x* 419 7.

The correlation coefficient is *r* 0 98, so linear model is appropriate for *x* between 80 dB and 104 dB.

Substituting *x* 94 into the regression equation, we get *y* 3 9018 94 419 7 53. So the intelligibility is about 53%.

9. (a)



Using a graphing calculator, we obtain *y* 0 27083*x* 462 9.

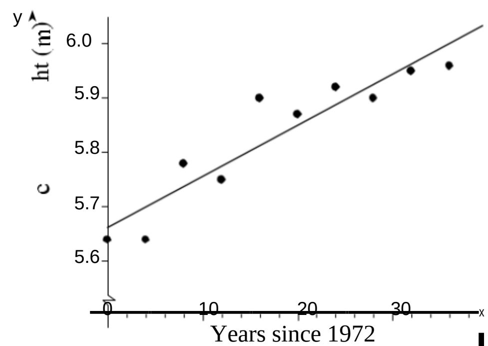
We substitute *x* 2006 in the model *y* 0 27083*x* 462 9 to get *y* 80 4, that is, a life expectancy of 80 4 years.

The life expectancy of a child born in the US in 2006 was 77 7 years, considerably less than our estimate in part (b).

10. (a)

Year	*x*	Height (m)
1972	0	5 64
1976	4	5 64
1980	8	5 78
1984	12	5 75
1988	16	5 90
1992	20	5 87
1996	24	5 92
2000	28	5 90
2004	32	5 95
2008	36	5 96

Using a graphing calculator, we obtain the regression line *y* 5 664 0 00929*x*.



The regression line provides a good model.

The regression line predicts the winning pole vault height in 2012 to be *y* 0 00929 2012 1972 5 664 6 04 meters.

Students should find a fairly strong correlation between shoe size and height.

Results will depend on student surveys in each class.