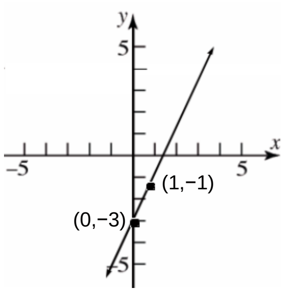
Solution Manual for College Algebra Concepts Through Functions 3rd Edition by Sullivan ISBN 0321925742 9780321925749

Chapter 2
Linear and Quadratic Functions

Section 2.1

From the equation $y = 2x - 3$, we see that the y-intercept is -3. Thus, the point $(0, -3)$ is on the graph. We can obtain a second point by choosing a value for x and finding the corresponding value for y. Let $x = 1$, then $y = 2(1) - 3 = -1$. Thus, the point $(1, -1)$ is also on the graph. Plotting the two points and connecting with a line yields the graph below.



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{-1 - 2} = \frac{-2}{-3} = \frac{2}{3}$$

$f(2) = 3(2)^2 - 2 = 10$
$f(4) = 3(4)^2 - 2 = 46$

$$\frac{\Delta y}{\Delta x} = \frac{f(4) - f(2)}{4 - 2} = \frac{46 - 10}{4 - 2} = \frac{36}{2} = 18$$

$60x - 900 = -15x + 2850$
$75x - 900 = 2850$
$75x = 3750$
$x = 50$

The solution set is $\{50\}$.

slope; y-intercept

-4; 3

$$f(-2) = (-2)^2 - 4 = 4 - 4 = 0$$

True

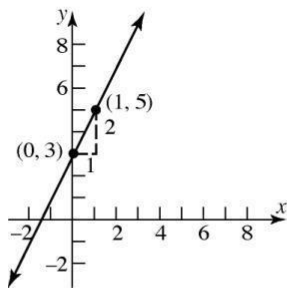
False. If x increases by 3, then y increases by 2.

False. The y-intercept is 8. The average rate of change is 2 (the slope).

$f(x) = 2x + 3$

Slope = 2; y-intercept = 3

Plot the point (0, 3). Use the slope to find an additional point by moving 1 unit to the right and 2 units up.



average rate of change = 2

positive

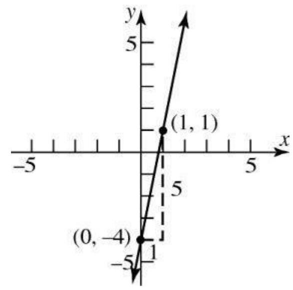
True

increasing

$g(x) = 5x - 4$

Slope = 5; y-intercept = −4

Plot the point (0, −4) . Use the slope to find an additional point by moving 1 unit to the right and 5 units up.



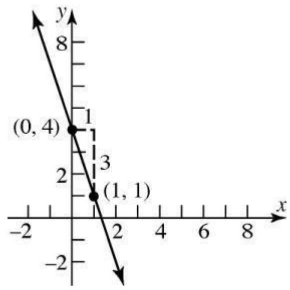
average rate of change = 5

increasing

15. $h(x) = -3x + 4$

a. Slope = -3 ; y-intercept = 4

b. Plot the point (0, 4). Use the slope to find an additional point by moving 1 unit to the right and 3 units down.



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verage rate of change = -3

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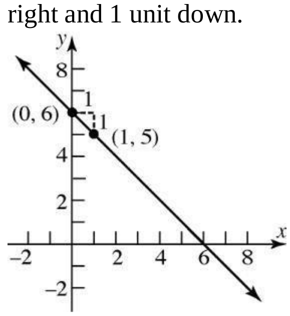
ecreasing

$p(x) = -x + 6$

Slope = -1 ; y-intercept = 6

Plot the point (0, 6). Use the slope to find an additional point by moving 1 unit to the right and 1 unit down.



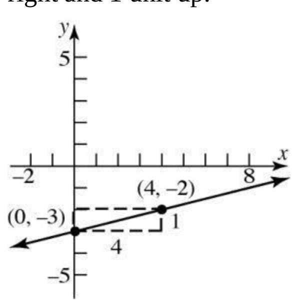
c. average rate of change = -1

d. decreasing

17. $f(x) = \frac{1}{4}x - 3$

a. Slope = $\frac{1}{4}$; y-intercept = -3

b. Plot the point $(0, -3)$. Use the slope to find an additional point by moving 4 units to the right and 1 unit up.



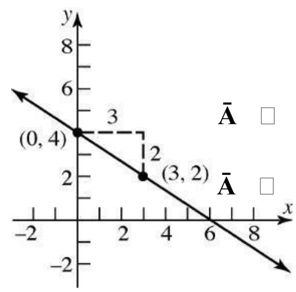
c. average rate of change = $\frac{1}{4}$

d. increasing

18. $h(x) = -\frac{2}{3}x + 4$

a. Slope = $-\frac{2}{3}$; y-intercept = 4

b. Plot the point (0, 4). Use the slope to find an additional point by moving 3 units to the right and 2 units down.



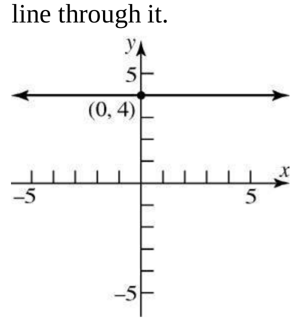
c. average rate of change = $-\frac{2}{3}$

d. decreasing

19. $F(x) = 4$

a. Slope = 0; y-intercept = 4

b. Plot the point (0, 4) and draw a horizontal line through it.



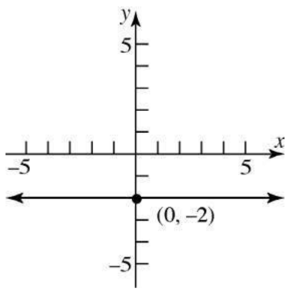
c. average rate of change = 0

d. constant

$G(x) = -2$

Slope = 0; y-intercept = -2

Plot the point $(0, -2)$ and draw a horizontal line through it.



average rate of change = 0

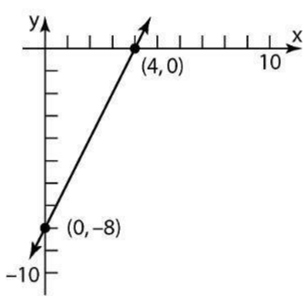
constant

$g(x) = 2x - 8$

zero: $0 = 2x - 8$: y-intercept = -8

$x = 4$

Plot the points $(4,0),(0,-8)$.

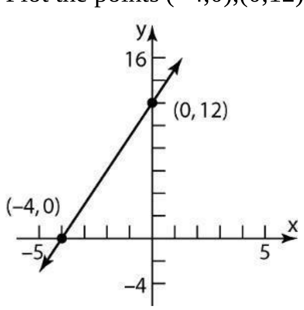


$g(x) = 3x + 12$

a. zero: $0 = 3x + 12$: y-intercept = 12

$x = -4$

b. Plot the points $(-4,0),(0,12)$.

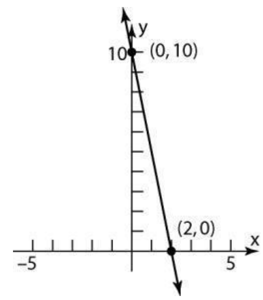


$f(x) = -5x + 10$

a. zero: $0 = -5x + 10$: y-intercept = 10

$x = 2$

Plot the points 1 unit to the right and 5 units down.



$f(x) = -6x + 12$

a. zero: $0 = -6x + 12$: y-intercept = 12

$x = 2$

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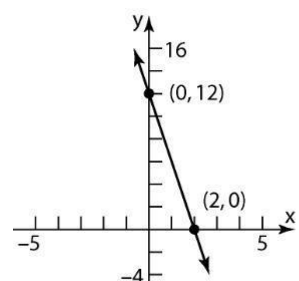
lot the points $(2,0),(0,12)$.

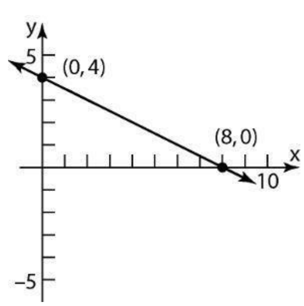


$H(x) = = \frac{1}{2}x + 4$

a. zero: $0 = -\frac{1}{2}x + 4$: y-intercept = 4

$x = 8$

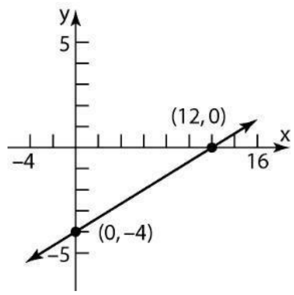
b. Plot the points $(8,0),(0,4)$.



$G(x) = \frac{1}{3}x - 4$

a. zero: $0 = \frac{1}{3}x - 4$; y-intercept = −4
$= 12$

b. Plot the points (12,0),(0, −4) .



27.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
−2	4	
−1	1	$\frac{1-4}{-1-(-2)} = \frac{-3}{1} = -3$
0	−2	$\frac{-2-1}{0-(-1)} = \frac{-3}{1} = -3$
1	−5	$\frac{-5-(-2)}{1-0} = \frac{-3}{1} = -3$
2	−8	$\frac{-8-(-5)}{2-1} = \frac{-3}{1} = -3$

Since the average rate of change is constant at 3 , this is a linear function with slope = –3. The y-intercept is (0, −2) , so the equation of the line is $y = -3x - 2$.

28.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
−2	$\frac{1}{4}$	
−1	$\frac{1}{2}$	$\frac{\frac{1}{2}-\frac{1}{4}}{-1-(-2)} = \frac{\frac{1}{4}}{1} = \frac{1}{4}$

29.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
−2	−8	
−1	−3	$\frac{-3-(-8)}{-1-(-2)} = \frac{5}{1} = 5$
0	0	$\frac{0-(-3)}{0-(-1)} = \frac{3}{1} = 3$
1	1	
2	0	

Since the average rate of change is not constant, this is not a linear function.

30.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
−2	−4	
−1	0	$\frac{0-(-4)}{-1-(-2)} = \frac{4}{1} = 4$
0	4	$\frac{4-0}{0-(-1)} = \frac{4}{1} = 4$
1	8	$\frac{8-4}{1-0} = \frac{4}{1} = 4$
2	12	$\frac{12-8}{2-1} = \frac{4}{1} = 4$

this is a linear function with slope = 4. The y-intercept is (0, 4) , so the equation of the line is $= 4x + 4$.

31.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
−2	−26	
−1	−4	$\frac{-4-(-26)}{-1-(-2)} = \frac{22}{1} = 22$
0	2	$\frac{2-(-4)}{0-(-1)} = \frac{6}{1} = 6$
1	–2	
2	–10	

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0	1
1	2
2	4

0 − (−1) = 1 = 2

constant, this is not a linear function.

Since the average rate of change is not constant, this is not a linear function.

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32.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
−2	−4	
−1	−3.5	$\frac{-3.5-(-4)}{-1-(-2)} = \frac{0.5}{1} = 0.5$
0	−3	$\frac{-3-(-3.5)}{0-(-1)} = \frac{0.5}{1} = 0.5$
1	−2.5	$\frac{-2.5-(-3)}{1-0} = \frac{0.5}{1} = 0.5$
2	−2	$\frac{-2-(-2.5)}{2-1} = \frac{0.5}{1} = 0.5$

Since the average rate of change is constant at 0.5, this is a linear function with slope = 0.5. The y-intercept is $(0, -3)$, so the equation of the line is $y = 0.5x - 3$.

33.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
−2	8	
−1	8	$\frac{8-8}{-1-(-2)} = \frac{0}{1} = 0$
0	8	$\frac{8-8}{0-(-1)} = \frac{0}{1} = 0$
1	8	$\frac{8-8}{1-0} = \frac{0}{1} = 0$
2	8	$\frac{8-8}{2-1} = \frac{0}{1} = 0$

Since the average rate of change is constant at 0, this is a linear function with slope = 0. The y-intercept is $(0, 8)$, so the equation of the line is $y = 0x + 8$ or $y = 8$.

34.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
−2	0	
−1	1	$\frac{1-0}{-1-(-2)} = \frac{1}{1} = 1$
0	4	$\frac{4-1}{0-(-1)} = \frac{3}{1}$
1	9	
2	16	

35. $f(x) = 4x - 1$; $g(x) = -2x + 5$

$f(x) = 0$

$4x - 1 = 0$

$= \frac{1}{4}$

$f(x) > 0$

$4x - 1 > 0$

$> \frac{1}{4}$

The solution set is $\left\{x \mid x > \frac{\ }{4}\right\}$ or $\left(\frac{\ }{4}, \infty\right)$

c. $f(x) = g(x)$

$4x - 1 = -2x + 5$

$6x = 6$

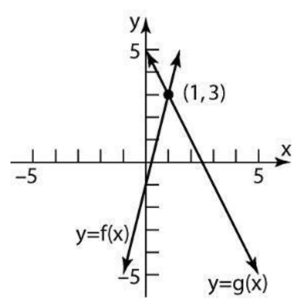
$x = 1$

d. $f(x) \le g(x)$

$4x - 1 \le -2x + 5$

$x \le 1$ $\{ \quad | \quad \}$ $(\quad]$

e.



36. $f(x) = 3x + 5$; $g(x) = -2x + 15$

a. $f(x) = 0$

$3x + 5 = 0$

$x = -\frac{5}{3}$

b. $f(x) < 0$

$3x + 5 < 0$

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$x < -\frac{5}{3}$

Since the average rate of change is not constant, this is not a linear function.

The solution set is $\left\{ x \,\middle|\, x < -\frac{5}{3} \right\}$ or $\left(-\infty, -\frac{5}{3} \right)$.

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c. $f(x) = g(x)$

$3x + 5 = -2x + 15$

$5x = 10$

$x = 2$

d. $f(x) \geq g(x)$

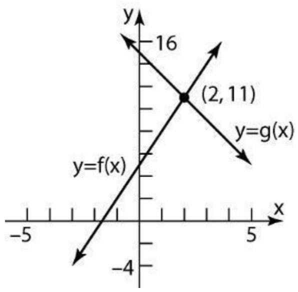
$3x + 5 \geq -2x + 15$

$5x \geq 10$

$x \geq 2$

The solution set is $\{x \mid x \geq 2\}$ or $[2, \infty)$.

e.



a. The point (40, 50) is on the graph of $y = f(x)$, so the solution to $f(x) = 50$ is $= 40$.

The point (88, 80) is on the graph of $= f(x)$, so the solution to $f(x) = 80$ is $=88$.

The point (−40, 0) is on the graph of $= f(x)$, so the solution to $f(x) = 0$ is $=-40$.

The *y*-coordinates of the graph of $y = f(x)$ are above 50 when the *x*-coordinates are larger than 40. Thus, the solution to $f(x) > 50$ is $\{x \mid x > 40\}$ or $(40, \infty)$.

The *y*-coordinates of the graph of $y = f(x)$ are below 80 when the *x*-coordinates are smaller than 88. Thus, the solution to $(x) \leq 80$ is $\{x \mid x \leq 88\}$ or $(-\infty, 88]$.

The *y*-coordinates of the graph of $y = f(x)$ are between 0 and 80 when the *x*-coordinates are between −40 and 88. Thus, the solution to $0 < f(x) < 80$ is $\{x \mid -40 < x < 88\}$ or $(-40, 88)$.

a. The point (5, 20) is on the graph of $y = g(x)$, so the solution to $g(x) = 20$ is $x = 5$.

The point (−15, 60) is on the graph of $= g(x)$, so the solution to $g(x) = 60$ is $x = -15$.

The point (15, 0) is on the graph of $y = g(x)$, so the solution to $g(x) = 0$ is $x = 15$.

The *y*-coordinates of the graph of $y = g(x)$ are above 20 when the *x*-coordinates are smaller than 5. Thus, the solution to $g(x) > 20$ is $\{x \mid x < 5\}$ or $(-\infty, 5)$.

The *y*-coordinates of the graph of $y = f(x)$ are below 60 when the *x*-coordinates are larger than −15. Thus, the solution to $g(x) \leq 60$ is $\{x \mid x \geq -15\}$ or $[-15, \infty)$.

The *y*-coordinates of the graph of $y = f(x)$ are between 0 and 60 when the *x*-coordinates are between −15 and 15. Thus, the solution to $0 < f(x) < 60$ is $\{x \mid -15 < x < 15\}$ or $(-15, 15)$.

a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = -4$.

$f(x) \leq g(x)$ when the graph of *f* is above the graph of *g*. Thus, the solution is $\{x \mid x < -4\}$ or $(-\infty, -4)$.

a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = 2$.

$f(x) \leq g(x)$ when the graph of *f* is below or intersects the graph of *g*. Thus, the solution is $\{x \mid x \leq 2\}$ or $(-\infty, 2]$.

a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = -6$.

$g(x) \leq f(x) < h(x)$ when the graph of *f* is above or intersects the graph of *g* and below the graph of *h*. Thus, the solution is $\{x \mid -6 \leq x < 5\}$ or $[-6, 5)$.

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a. $f(x) = g(x)$ when their graphs intersect.
Thus, $x = 7$.
$g(x) \le f(x) < h(x)$ when the graph of f is above or intersects the graph of g and below the graph of h. Thus, the solution is $\{x \mid -4 \le x < 7\}$ or $[-4, 7)$.

$C(x) = 0.35x + 45$
$C(40) = 0.35(40) + 45 \approx \$59$.

Solve $C(x) = 0.25x + 35 =$
80 $0.35x + 45 = 108$
$0.35x = 63$
$= \frac{63}{0.35} = 180$ miles

Solve $C(x) = 0.35x + 45 \le$
150 $0.35x + 45 \le 150$
$0.35x \le 105$
$\le \frac{105}{0.35} = 300$ miles

The number of mile driven cannot be negative, so the implied domain for C is $\{x \mid x \ge 0\}$ or $[0, \infty)$.
The cost of renting the moving truck for a day increases \$0.35 for each mile driven, or there is a charge of \$0.35 per mile to rent the truck in addition to a fixed charge of \$45.
It costs \$45 to rent the moving truck if 0 miles are driven, or there is a fixed charge of \$45 to rent the truck in addition to a charge that depends on mileage.

$C(x) = 2.06x + 1.39$
$C(50) = 2.06(50) + 1.39 = \$104.39$.

Solve $C(x) = 2.06x + 1.39 = 133.23$
$2.06x + 1.39 = 133.23$
$2.06x = 131.84$
$x = \frac{131.84}{2.06} = 64$ minutes

The number of minutes cannot be negative, so $x \ge 0$. If there are 30 days in the month, then the number of minutes can be at most $30 \cdot 24 \cdot 60 = 43,200$. Thus, the implied domain for C is $\{x \mid 0 \le x \le 43,200\}$ or $[0, 43200]$.
The monthly cost of the plan increases \$2.06 for each minute used, or there is a charge of \$2.06 per minute to use the phone in addition to a fixed charge of \$1.39.
It costs \$1.39 per month for the plan if 0 minutes are used, or there is a fixed charge of \$1.39 per month for the plan in addition to a charge that depends on the number of minutes used.

45. $S(p) = -600 + 50p$; $D(p) = 1200 - 25p$

Solve $S(p) = D(p)$.
$600 + 50p = 1200 - 25p$
$75p = 1800$
$p = \frac{1800}{75} = 24$

c. Solve $C(x) = 2.06x + 1.39 \le 100$
$2.06x + 1.39 \le 100$
$2.06x \le 98.61$
$\frac{98.61}{2.06} \approx 47$ minutes

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$S(24) = -600 + 50(24) = 600$

Thus, the equilibrium price is \$24, and the equilibrium quantity is 600 T-shirts.

Solve $D(p) > S(p)$. $1200 - 25p > -600 + 50p$

$$1800 > 75p$$

$$\frac{1800}{75} > p$$

$$24 > p$$

The demand will exceed supply when the price is less than \$24 (but still greater than \$0). That is, $\$0 < p < \$24$.

The price will eventually be increased.

46. $S(p) = -2000 + 3000p$; $D(p) = 10000 - 1000p$ **a.** Solve $S(p) = D(p)$.

$$-2000 + 3000p = 10000 - 1000p$$

$$4000p = 12000$$

$$p = \frac{12000}{4000} = 3$$

$(3) = -2000 + 3000(3) = 7000$

Thus, the equilibrium price is \$3, and the equilibrium quantity is 7000 hot dogs.

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b. Solve $D(p) < S(p)$.

$$10000 - 1000p < -2000 + 3000p$$
$$12000 < 4000p$$
$$\frac{12000}{4000} < p$$
$$3 < p$$

The demand will be less than the supply when the price is greater than $3.

c. The price will eventually be decreased.

47. a. We are told that the tax function T is for adjusted gross incomes x between $8,925 and $36,250, inclusive. Thus, the domain is $\{x \mid 8{,}925 \le x \le 36{,}250\}$ or $[8925, 36250]$.

b. $T(20{,}000) = 0.15(20{,}000 - 8925) + 892.5 = 2553.75$

If a single filer's adjusted gross income is $20,000, then his or her tax bill will be $2553.75.

c. The independent variable is adjusted gross income, x. The dependent variable is the tax bill, T.

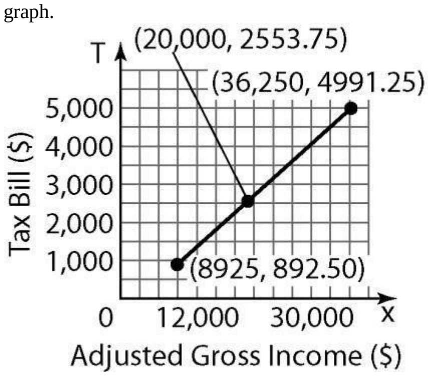
d. Evaluate T at $x = 8925$, 20000, and 36250.

$$T(8925) = 0.15(8925 - 8925) + 892.5 = 892.5$$

$$T(20{,}000) = 0.15(20{,}000 - 8925) + 892.5 = 2553.75$$

$$T(36{,}250) = 0.15(36250 - 8925) + 892.5 = 4991.25$$

Thus, the points $(8925, 892.5)$, $(20000, 2553.75)$, and $(36250, 4991.25)$ are on the graph.



e. We must solve $T(x) = 3693.75$.

$$0.15(x - 8925) + 892.5 = 3693.75$$
$$0.15x - 1338.75 + 892.5 = 3693.75$$
$$0.15x - 446.25 = 3693.75$$
$$0.15x = 4140$$
$$x = 27600$$

A single filer with an adjusted gross income of $27,600 will have a tax bill of $3693.75.

f. For each additional dollar of taxable income between $8925 and $36,250, the tax bill of a single person in 2013 increased by $0.15.

48. a. The independent variable is payroll, p. The payroll tax only applies if the payroll exceeds $178 million. Thus, the domain of T is $\{p \mid p > 178\}$ or $(178, \infty)$.

b. $T(222.5) = 0.425(222.5 - 178) = 18.9125$

The luxury tax for the New York Yankees was $18.9125 million.

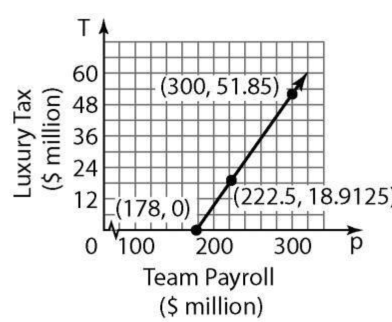
c. Evaluate T at $p = 178$, 222.5, and 300 million.

$$T(178) = 0.425(178 - 178) = 0 \text{ million}$$

$$T(222.5) = 0.425(222.5 - 178) = 18.9125 \text{ million}$$

$$T(300) = 0.425(300 - 178) = 51.85 \text{ million}$$

Thus, the points $(178 \text{ million}, 0 \text{ million})$, $(222.5 \text{ million}, 18.9125 \text{ million})$, and $(300 \text{ million}, 51.85 \text{ million})$ are on the graph.



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d. We must solve $T(p) = 27.2$.

$$\begin{aligned} 0.425(p - 178) &= 27.2 \\ 0.425p - 75.65 &= 27.2 \\ 0.425p &= 102.85 \\ p &= 242 \end{aligned}$$

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If the luxury tax is \$27.2 million, then the payroll of the team is \$242 million. For each additional million dollars of payroll in excess of \$178 million in 2011, the luxury tax of a team increased by \$0.425 million.

$R(x) = 8x$; $C(x) = 4.5x + 17{,}500$

Solve $R(x) = C(x)$.

$8x = 4.5x + 17{,}500$

$3.5x = 17{,}500$

$x = 5000$

The break-even point occurs when the company sells 5000 units.

Solve $R(x) > C(x)$

$8x > 4.5x + 17{,}500$

$3.5x > 17{,}500$

> 5000

The company makes a profit if it sells more than 5000 units.

$R(x) = 12x$; $C(x) = 10x +$ 15,000 **a.** Solve $R(x) = C(x)$

$12x = 10x + 15{,}000$

$2x = 15{,}000$

$x = 7500$

The break-even point occurs when the

company sells 7500 units.

Solve $R(x) > C(x)$

$12x > 10x + 15{,}000$

$2x > 15{,}000$

$x > 7500$

The company makes a profit if it sells more than 7500 units.

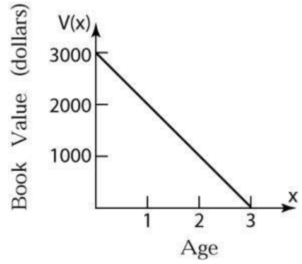
a. Consider the data points (x, y), where $x =$ the age in years of the computer and $y =$ the value in dollars of the computer. So we have the points $(0,3000)$ and $(3,0)$. The slope formula yields:

$$m = \frac{y}{x} = \frac{0-3000}{3-0} = \frac{-3000}{3} = -1000$$

The y-intercept is $(0,3000)$, so $b = 3000$. Therefore, the linear function is

\$0 after 3 years. Thus, the implied domain for V is $\{x \mid 0 \le x \le 3\}$ or $[0, 3]$.

The graph of $V(x) = -1000x + 3000$



$V(2) = -1000(2) + 3000 = 1000$

The computer's book value after 2 years will be \$1000.

Solve $V(x) = 2000$

$-1000x + 3000 = 2000$

$-1000x = -1000$

$= 1$

The computer will have a book value of \$2000 after 1 year.

a. Consider the data points (x, y), where $x =$ the age in years of the machine and $y =$ the value in dollars of the machine. So we have the points $(0,120000)$ and $(10,0)$. The slope formula yields:

$$m = -y = \frac{0-120000}{} = \frac{-120000}{} = -12000$$

$V(x) = mx + b = -1000x + 3000$

.

The age of the computer cannot be negative, and the book value of the computer will be

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x 10−0 10

The y-intercept is $(0,120000)$, so

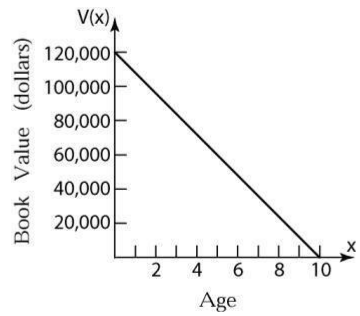
$= 120,000$.

Therefore, the linear function is

$(x) = mx + b = -12,000x + 120,000$.

The age of the machine cannot be negative, and the book value of the machine will be
$0 after 10 years. Thus, the implied domain for V is $\{x \mid 0 \le x \le 10\}$ or [0, 10].

The graph of $V(x) = -12,000x + 120,000$



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d. $V(4) = -12000(4) + 120000 = 72000$
The machine's value after 4 years is given by \$72,000.

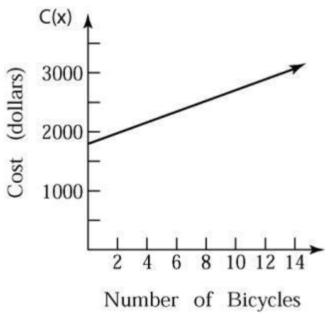
Solve $V(x) = 72000$.

$$-12000x + 120000 = 72000$$
$$-12000x = -48000$$
$$x = 4$$

The machine will be worth \$72,000 after 4 years.

a. Let x = the number of bicycles manufactured.
We can use the cost function $C(x) = mx + b$, with $m = 90$ and $b = 1800$. Therefore
$C(x) = 90x + 1800$

The graph of $C(x) = 90x + 1800$



The cost of manufacturing 14 bicycles is given by $C(14) = 90(14) + 1800 = \$3060$.

Solve $C(x) = 90x + 1800 = 3780$

+ 1800 =

3780 90 x =

1980

$x = 22$

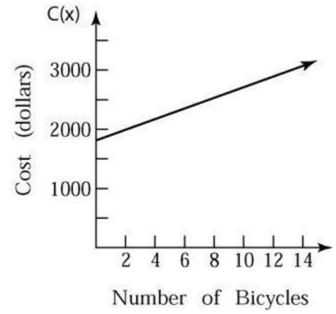
So 22 bicycles could be manufactured for \$3780.

a. The new daily fixed cost is

$$1800 + \frac{100}{20} = \$1805$$

Let x = the number of bicycles manufactured. We can use the cost function $C(x) = mx + b$, with $m = 90$ and b = 1805. Therefore $C(x) = 90x + 1805$

c. The graph of $C(x) = 90x + 1805$



The cost of manufacturing 14 bicycles is given by $C(14) = 90(14) + 1805 = \$3065$.

Solve $C(x) = 90x + 1805 = 3780$

$$x + 1805 = 3780$$
$$90x = 1975$$
$$x \approx 21.94$$

So approximately 21 bicycles could be manufactured for \$3780.

a. Let x = number of miles driven, and let C = cost in dollars. Total cost = (cost per mile)(number of miles) + fixed cost
$C(x) = 0.89x + 31.95$

$C(110) = (0.89)(110) + 31.95 = \$129.85$

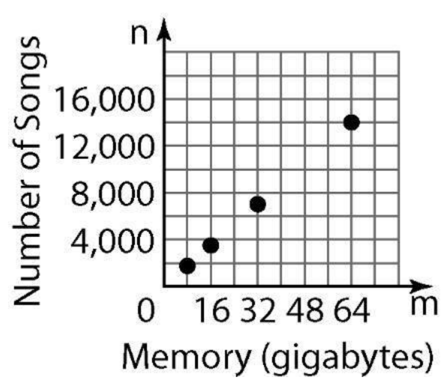
$C(230) = (0.89)(230) + 31.95 = \$236.65$

a. Let x = number of minutes used, and let C = cost in dollars. Total cost = (cost per minute)(number of minutes) + fixed cost x
$C(x) = 0.50x - 10$

$C(105) = (0.50)(105) - 10 = \$42.50$

$C(180) = (0.50)(120) - 10 = \$50$

a.



b.

m	n	Avg. rate of change = $\frac{\Delta n}{\Delta m}$
8	1750	
16	3500	$\frac{3500-1750}{16-8}=\frac{1750}{8}=\frac{875}{4}$
32	7000	$\frac{7000-3500}{32-16}=\frac{3500}{16}=\frac{875}{4}$
64	14000	$\frac{14000-7000}{64-32}=\frac{7000}{32}=\frac{875}{4}$

Since each input (memory) corresponds to a

single output (number or songe), we know that the number of songs is a function of

change is constant at 218.75 per gigabyte, the function is linear.

From part (b), we know *slope* = 218.75 . Using $(m_1, n_1) = (8, 1750)$, we get the equation:

$$n - n_1 = s(m - m_1)$$
$$n - 1750 = 218.75(m - 8)$$
$$n - 1750 = 218.75m - 1750$$
$$n = 218.75m$$

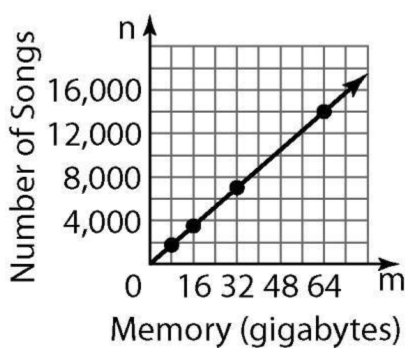
Using function notation, we have $n(m) = 218.75m$.

ᴵĀ □ Ā □

he price cannot be negative, so $m \ge 0$. Likewise, the quantity cannot be negative, so, $n(m) \ge 0$.

$$218.75m \ge 0$$
$$m \ge 0$$

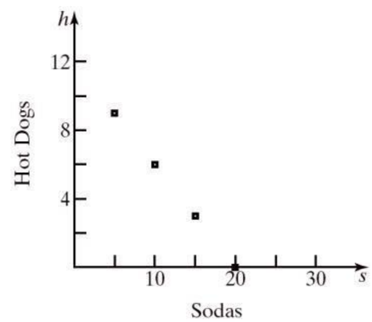
Thus, the implied domain for n(m) is $\{m \mid m \ge 0\}$ or $[0, \infty)$.



e.

If memory increases by 1 GB, then the number of songs increases by 218.75.

58. a.



b.

s	h	Avg. rate of change = $\frac{\Delta h}{\Delta s}$
20	0	
15	3	$\frac{3-0}{15-20}=\frac{3}{-5}=-0.6$
10	6	$\frac{6-3}{10-15}=\frac{3}{-5}=-0.6$
5	9	$\frac{9-6}{5-10}=\frac{3}{-5}=-0.6$

Since each input (soda) corresponds to a single output (hot dogs), we know that number of hot dogs purchased is a function of number of sodas purchased. Also, because the average rate of change is constant at −0.6 hot dogs per soda, the

Ā □ function is linear. Ā □ Ā □

From part (b), we know $m = -0.6$. Using $(s_1, h_1) = (20, 0)$, we get the equation:

$$h - h_1 = m(s - s_1)$$
$$h - 0 = -0.6(s - 20)$$
$$= -0.6s + 12$$

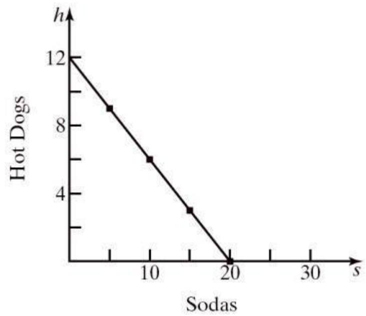
Using function notation, we have $h(s) = -0.6s + 12$.

The number of sodas cannot be negative, so $s \ge 0$. Likewise, the number of hot dogs cannot be negative, so, $h(s) \ge 0$.

$$-0.6s + 12 \ge 0$$
$$-0.6s \ge -12$$
$$s \le 20$$

Thus, the implied domain for h(s) is $\{s \mid 0 \le s \le 20\}$ or $[0, 20]$.

e.



If the number of hot dogs purchased increases by \$1, then the number of sodas purchased decreases by 0.6.

s-intercept: If 0 hot dogs are purchased, then 20 sodas can be purchased.

h-intercept: If 0 sodas are purchased, then 12 hot dogs may be purchased.

The graph shown has a positive slope and a positive *y*-intercept. Therefore, the function from (d) and (e) might have the graph shown.

The graph shown has a negative slope and a positive *y*-intercept. Therefore, the function from (b) and (e) might have the graph shown.

61. A linear function $f(x) = mx + b$ will be odd provided $f(-x) = -f(x)$.

That is, provided $m(-x) + b = -(mx + b)$.

$$-mx + b = -mx - b$$
$$b = -b$$
$$2b = 0$$
$$b = 0$$

So a linear function $f(x) = mx + b$ will be odd provided $b = 0$.

A linear function $f(x) = mx + b$ will be even provided $f(-x) = f(x)$.

That is, provided $m(-x) + b = mx + b$.

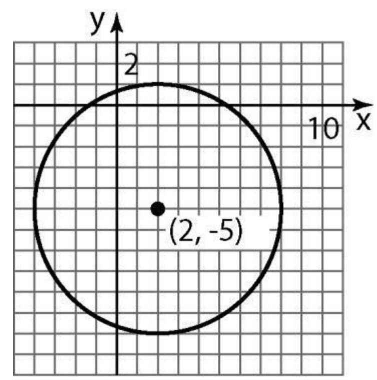
$$-mx + b = mx + b$$
$$-mxb = mx$$
$$0 = 2mx$$
$$m = 0$$

So, yes, a linear function $f(x) = mx + b$ cab be even provided $m = 0$.

62. If you solve the linear function $f(x) = mx + b$ for 0 you are actually finding the x-intercept. Therefore using x-intercept of the graph of $f(x) = mx + b$ would be same x-value as solving $mx + b > 0$ for x. Then the appropriate interval could be determined

$$x^2 - 4x + y^2 + 10y - 7 = 0$$
$$x^2 - 4x + 4) + (y^2 + 10y + 25) = 7 + 4$$
$$+ 25 \; (x-2)^2 + (y+5)^2 = 6^2$$

Center: (2, -5); Radius = 6



$$f(x) = \frac{2x + B}{-3}$$

$$f(5) = 8 = \frac{2(5) + B}{5-3}$$

$$8 = \frac{10 + B}{2}$$

$$16 = 10 + B$$

$$B = 6$$

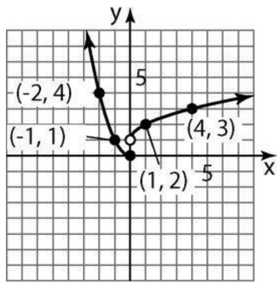
65. $\frac{f(3) - f(1)}{3-1}$

$$= \frac{12 - (-2)}{2}$$

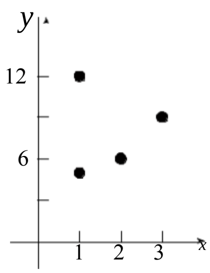
$$= \frac{14}{2}$$

$$= 7$$

66.



Section 2.2



No, the relation is not a function because an input, 1, corresponds to two different outputs, 5 and 12.

Let $(x_1, y_1) = (1, 4)$ and $(x_2, y_2) = (3, 8)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{3-1} = \frac{4}{2} = 2$$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 2(x - 1)$$
$$y - 4 = 2x - 2$$
$$= 2x + 2$$

scatter diagram

decrease; 0.008

Linear relation, $m > 0$

Nonlinear relation

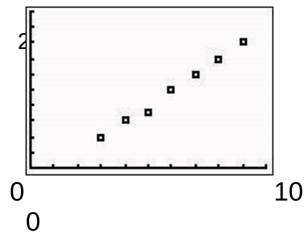
Linear relation, $m < 0$

Linear relation, $m > 0$

Nonlinear relation

Nonlinear relation

11. a.

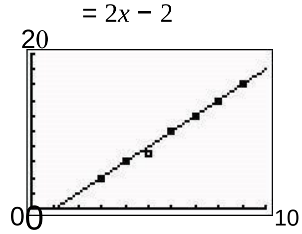


Answers will vary. We select (4, 6) and (8, 14). The slope of the line containing these points is:

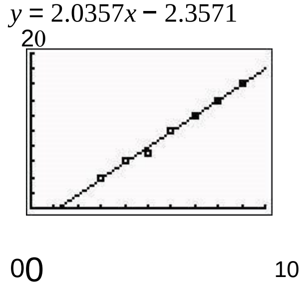
$$m = \frac{14-6}{8-4} = \frac{8}{4} = 2$$

The equation of the line is:

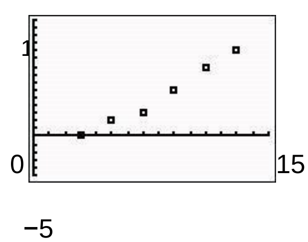
$$y - y_1 = m(x - x_1)$$
$$y - 6 = 2(x - 4)$$
$$y - 6 = 2x - 8$$
$$= 2x - 2$$



Using the LINear REGression program, the line of best fit is:
$y = 2.0357x - 2.3571$



12. a.



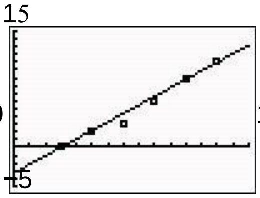
Answers will vary. We select (5, 2) and (11, 9). The slope of the line containing these points is: $m = \frac{9-2}{11-5} = \frac{7}{6}$

The equation of the line is:

$y - y_1 = m(x - x_1)$

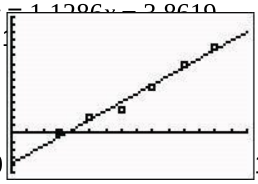
$y - 2 = \frac{7}{6}(x - 5)$

$y - 2 = \frac{7}{6}x - \frac{35}{6}$

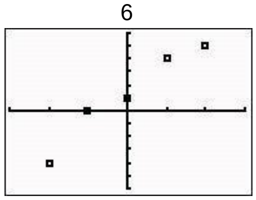


Using the LINear REGression program, the line of best fit is:

$y = 1.1286x - 3.8619$



13. a.



Answers will vary. We select (–2,–4) and (2, 5). The slope of the line containing these points is: $m = \frac{5-(-4)}{2-(-2)} = \frac{9}{4}$.

The equation of the line is:

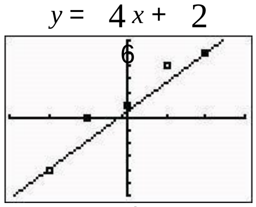
$y - y_1 = m(x - x_1)$

$y - (-4) = \frac{9}{4}(x - (-2))$

$y + 4 = \frac{9}{4}x + \frac{9}{2}$

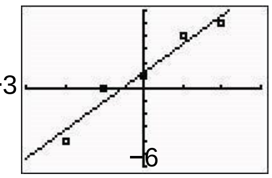
$y = \frac{9}{4}x + \frac{1}{2}$

c.

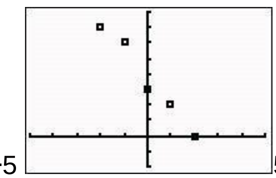


Using the LINear REGression program, the line of best fit is:

$y = 2.2x + 1.2$



14. a.



Answers will vary. We select (–2, 7) and (2, 0). The slope of the line containing these points is: $m = \frac{0-7}{2-(-2)} = \frac{-7}{4} = -\frac{7}{4}$.

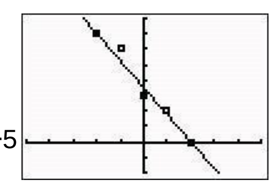
The equation of the line is:

$y - y_1 = m(x - x_1)$

$y - 7 = -\frac{7}{4}(x - (-2))$

$y = -\frac{7}{4}x + \frac{7}{2}$

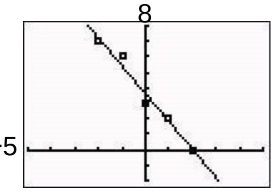
c.



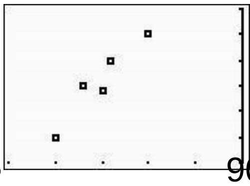
Using the LINear REGression program, the line of best fit is:

$y = -1.8x + 3.6$

e.



15. a.



Answers will vary. We select (–20,100) and (–10,140). The slope of the line containing these points is:

$$m = \frac{140 - 100}{-10 - (-20)} = \frac{40}{10} = 4$$

The equation of the line is:

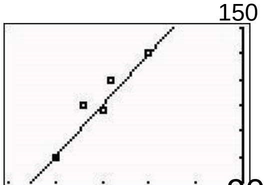
$$y - y_1 = m(x - x_1)$$

$$y - 100 = 4(x - (-20))$$

$$y - 100 = 4x + 80$$

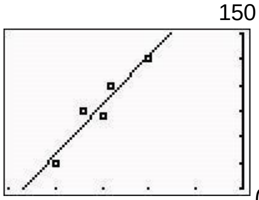
$$y = 4x + 180$$

c.

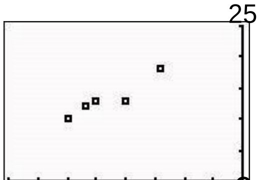


Using the LINear REGression program, the line of best fit is: $y = 3.8613x + 180.2920$

e.



16. a.



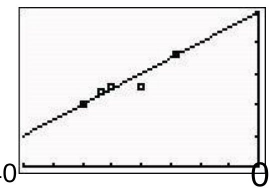
$$y - y_1 = m(x - x_1)$$

$$y - 10 = \frac{1}{2}(x - (-30))$$

$$y - 10 = \frac{1}{2}x + 15$$

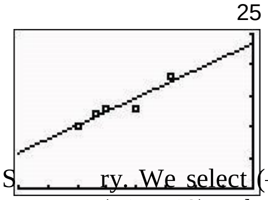
$$y = \frac{1}{2}x + 25$$

c.



Using the LINear REGression program, the line of best fit is: $y = 0.4421x + 23.4559$

e.



Selection of points will vary. We select (–30, 10) and (–14, 18). The slope of the line containing these points is:

18 − 10   8   1

160

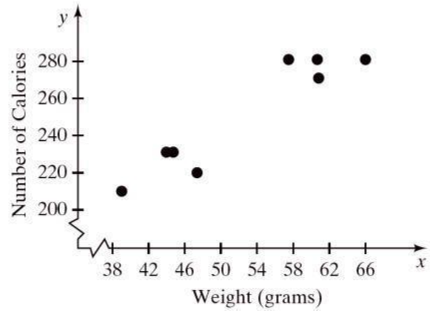
−40 6.93

0 0

− 210 = 2.5993316*x* − 102.7255848

y = 2.599*x* + 107.274

17. a.



Linear.

Answers will vary. We will use the points (39.52, 210) and (66.45, 280) .

$m = \frac{280 - 210}{66.45 - 39.52} = \frac{70}{2} \approx 2.5993316$

6.93

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$m = \frac{}{-14-(-30)} = \frac{}{16} = \frac{}{2}$

The equation of the line is:

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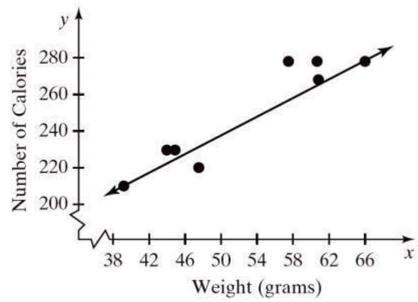
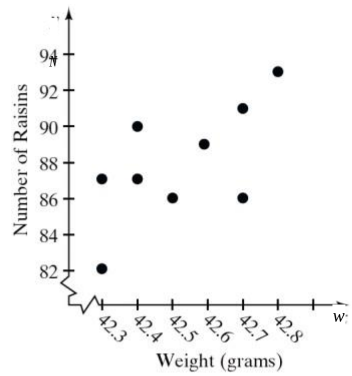


d. $x = 62.3 : y = 2.599(62.3) + 107.274 \approx 269$

We predict that a candy bar weighing 62.3 grams will contain 269 calories.

If the weight of a candy bar is increased by one gram, then the number of calories will increase by 2.599.

18. a.



Linear with positive slope.

Answers will vary. We will use the points (42.3, 82) and (42.8, 93).

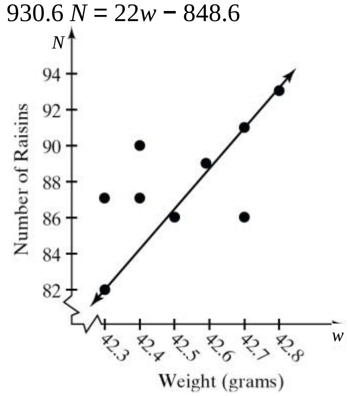
$$m = \frac{93-82}{42.8-42.3} = \frac{11}{0.5} = 22$$

$$N - N_1 = m(w - w_1)$$

$$N - 82 = 22(w - 42.3)$$

$$N - 82 = 22w - 930.6$$

$$N = 22w - 848.6$$



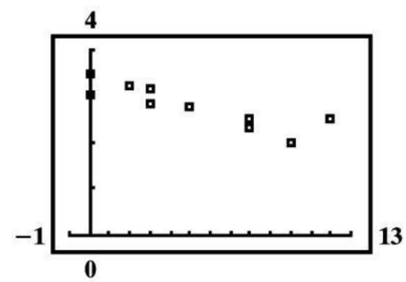
$N(42.5) = 22(42.5) - 848.6 = 86.4$

We predict that approximately 86 raisins will be in a box weighing 42.5 grams.

If the weight is increased by one gram, then the number of raisins will increase by 22.

a. The independent variable is the number of hours spent playing video games and cumulative grade-point average is the dependent variable because we are using number of hours playing video games to predict (or explain) cumulative grade-point average.

b.



Using the LINear REGression program, the line of best fit is: $G(h) = -0.0942h + 3.2763$

If the number of hours playing video games in a week increases by 1 hour, the cumulative grade-point average decreases 0.09, on average.

$G(8) = -0.0942(8) + 3.2763 = 2.52$

We predict a grade- point average of approximately 2.52 for a student who plays 8 hours of video games each week.

$$2.40 = -0.0942(h) + 3.2763$$

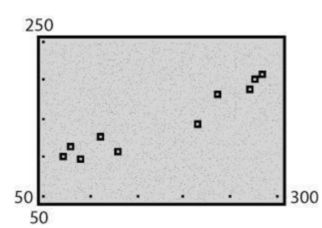
$$2.40 - 3.2763 = -0.0942h$$

$$-0.8763 = -0.0942h$$

$$9.3 = h$$

A student who has a grade-point average of 2.40 will have played approximately 9.3 hours of video games.

a.



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Using the LINear REGression program, the line of best fit is:
$P(t) = 0.4755t + 64.0143$

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If the flight time increases by 1 minute, the ticket price increases by about \$0.4755, on average.

$P(90) = 0.4755(90) + 64.0143 = \$107$

To find the time, we solve the following equation:

$180 = 0.4755t + 64.0143$

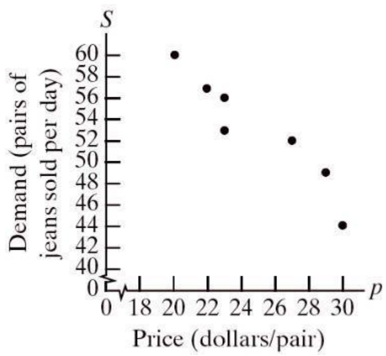
$115.9857 = 0.4755t$

$244 \approx t$

An airfare of \$180 would be for a flight time of about 244 minutes.

a. The relation is not a function because 23 is paired with both 56 and 53.

b.



Using the LINear REGression program, the line of best fit is: $D = -1.3355\,p + 86.1974$.

The correlation coefficient is: $r \approx -0.9491$.

If the price of the jeans increases by \$1, the demand for the jeans decreases by about 1.34 pairs per day.

$D(p) = -1.3355\,p + 86.1974$

Domain: $\{\,p \mid 0 < p \le 64\,\}$

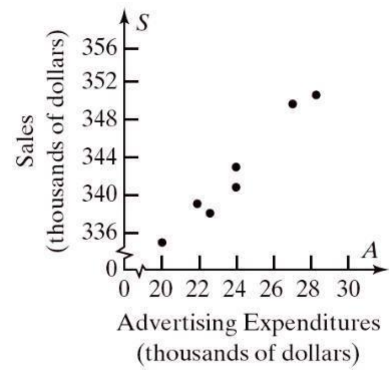
Note that the p-intercept is roughly 64.54 and that the number of pairs of jeans in demand cannot be negative.

$D(28) = -1.3355(28) + 86.1974 \approx$

48.8034 Demand is about 49 pairs.

a. The relation is not a function because 24 is paired with both 343 and 341.

b.



Using the LINear REGression program, the line of best fit is: $S = 2.0667\,A + 292.8869$.

The correlation coefficient is: $r \approx 0.9833$.

As the advertising expenditure increases by \$1000, the sales increase by about \$2067.

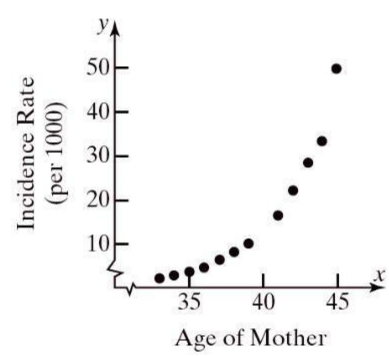
$S(A) = 2.0667A + 292.8869$

Domain: $\{\,A \mid A \ge 0\,\}$

$S(25) = 2.0667(25) + 292.8869 \approx$

345 Sales are about \$345 thousand.

23.



The data do not follow a linear pattern so it would not make sense to find the line of best fit.

Using the LINear REGression program, the line of best fit is: $y = 1.5\,x + 3.5$ and the correlation coefficient is: $r = 1$. The linear relation between two points is perfect.

If the correlation coefficient is 0 then there is no linear relation.

The y-intercept would be the calories of a candy bar with weight 0 which would not be meaningful in this problem.

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$G(0) = -0.0942(0) + 3.2763 = 3.2763$. The approximate grade-point average of a student

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who plays 0 hours of video games per week would be 3.28.

28. $m = \frac{-3-5}{3-(-1)} = \frac{-8}{4} = -2$

$y - y_1 = m(x - x_1)$

$-5 = -2(x + 1)$

$y - 5 = -2x - 2$

$y = -2x + 3$ or

$x + y = 3$

The domain would be all real numbers except those that make the denominator zero.

$^2 - 25 = 0$

$x^2 = 25 \rightarrow x = \pm 5$

So the domain is: $\{x \mid x \neq 5, -5\}$

30. $f(x) = 5x - 8$ and $g(x) = x^2 - 3x + 4$

$g - f)(x) = (x^2 - 3x + 4) - (5x - 8)$

$x^2 - 3x + 4 - 5x + 8$

$x^2 - 8x + 12$

31. Since y is shifted to the left 3 units we would use

$= (x + 3)^2$. Since y is also shifted down

4 units,we would use $y = (x + 3)^2 - 4$.

Section 2.3

a. $x^2 - 5x - 6 = (x - 6)(x + 1)$ **b.**

$2x^2 - x - 3 = (2x - 3)(x + 1)$

2. $\sqrt{8^2 - 4 \cdot 2 \cdot 3} = \sqrt{64 - 24}$

$\sqrt{40} = \sqrt{4 \cdot 10} = 2\sqrt{10}$

$(x - 3)(3x + 5) = 0$

$x - 3 = 0$ or $3x + 5 = 0$

$x = 3$ $\quad$ $3x = -5$

$x = -\frac{5}{3}$

The solution set is $\left\{-\frac{5}{3}, 3\right\}$.

If $f(4) = 10$, then the point (4, 10) is on the graph of *f*.

$f(-3) = (-3)^2 + 4(-3) + 3$

$9 - 12 + 3 = 0$

-3 is a zero of $f(x)$.

repeated; multiplicity 2

discriminant; negative

A quadratic functions can have either 0, 1 or 2 real zeros.

1. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$f(x) = 0$

$x^2 - 9x = 0$

$x(x - 9) = 0$

$x = 0$ or $x - 9 = 0$

$x = 9$

The zeros of $f(x) = x^2 - 9x$ are 0 and 9. The *x*-intercepts of the graph of *f* are 0 and 9.

$f(x) = 0$

$x^2 + 4x = 0$

$x(x + 4) = 0$

$x = 0$ or $x + 4 = 0$

$x = -4$

The zeros of $f(x) = x^2 + 4x$ are -4 and 0. The *x*-intercepts of the graph of *f* are -4 and 0.

$g(x) = 0$

$^2 - 25 = 0$

$(x + 5)(x - 5) = 0$

$x + 5 = 0$ or $x - 5 = 0$

$x = -5$ $\quad$ $x = 5$

The zeros of $g(x) = x^2 - 25$ are -5 and 5. The *x*-intercepts of the graph of *g* are -5 and 5.

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4. add; $\left(\frac{1}{2}\cdot 6\right)^2 = 9$

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$G(x) = 0$

$^2 - 9 = 0$

$(x + 3)(x - 3) = 0$

$x + 3 = 0$ or $x - 3 = 0$

$x = -3$ $\quad x = 3$

The zeros of $G(x) = x^2 - 9$ are −3 and 3. The x-intercepts of the graph of G are −3 and 3.

$F(x) = 0$

$^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x + 3 = 0$ or $x - 2 = 0$

$x = -3$ $\quad x = 2$

The zeros of $F(x) = x^2 + x - 6$ are −3 and 2. The x-intercepts of the graph of F are −3 and 2.

$H(x) = 0$

$^2 + 7x + 6 = 0$

$(x + 6)(x + 1) = 0$

$x + 6 = 0$ or $x + 1 = 0$

$x = -6$ $\quad x = -1$

The zeros of $H(x) = x^2 + 7x + 6$ are −6 and −1. The x-intercepts of the graph of H are −6 and −1.

$g(x) = 0$

$x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

$2x + 1 = 0$ or $x - 3 = 0$

$x = -\frac{1}{2}$ $\quad x = 3$

The zeros of $g(x) = 2x^2 - 5x - 3$ are $-\frac{1}{2}$ and 3.

The x-intercepts of the graph of g are $-\frac{1}{2}$ and 3.

$f(x) = 0$

$\frac{2}{3}$. The x-intercepts of the graph of f are −1 and $-\frac{2}{3}$.

$P(x) = 0$

$x^2 - 48 = 0$

$3(x^2 - 16) = 0$

$3(x + 4)(x - 4) = 0$

$t + 4 = 0$ or $t - 4 = 0$

$t = -4$ $\quad t = 4$

The zeros of $P(x) = 3x^2 - 48$ are −4 and 4.

The x-intercepts of the graph of P are −4 and 4.

$H(x) = 0$

$2x^2 - 50 = 0$

$2(x^2 - 25) = 0$

$2(x + 5)(x - 5) = 0$

$y + 5 = 0$ or $y - 5 = 0$

$y = -5$ $\quad y = 5$

The zeros of $H(x) = 2x^2 - 50$ are −5 and 5.

The x-intercepts of the graph of H are −5 and 5.

$g(x) = 0$

$(x + 8) + 12 = 0$

$^2 + 8x + 12 = 0$

$(x + 6)(x + 2) = 0$

$= -6$ or $x = -2$

The zeros of $g(x) = x(x + 8) + 12$ are −6 and −2. The x-intercepts of the graph of g are −6 and −2.

$f(x) = 0$

$3x^2 + 5x + 2$
$= 0$ $(3x + 2)(x$
$+ 1) = 0$

$3x + 2 = 0$ or $x + 1 = 0$

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$x = -\frac{2}{3}$ $\quad x = -1$

The zeros of $f(x) = 3x^2 + 5x + 2$ are -1 and

$(x - 4) - 12 = 0$

${}^2 - 4x - 12 = 0$

$(x - 6)(x + 2) = 0$

$= -2$ or $x = 6$

The zeros of $f(x) = x(x - 4) - 12$ are -2 and 6.
The x-intercepts of the graph of f are -2 and 6.

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$$G(x) = 0$$
$$4x^2 + 9 - 12x = 0$$
$$4x^2 - 12x + 9 = 0$$
$$(2x - 3)(2x - 3) = 0$$
$$2x - 3 = 0 \quad \text{or} \quad 2x - 3 = 0$$
$$x = \frac{3}{2} \qquad x = \frac{3}{2}$$

The only zero of $G(x) = 4x^2 + 9 - 12x$ is $\frac{3}{2}$.

The only x-intercept of the graph of G is $\frac{3}{2}$.

$$F(x) = 0$$
$$25x^2 + 16 - 40x = 0$$
$$25x^2 - 40x + 16 = 0$$
$$(5x - 4)(5x - 4) = 0$$
$$5x - 4 = 0 \quad \text{or} \quad 5x - 4 = 0$$
$$x = \frac{4}{5} \qquad x = \frac{4}{5}$$

The only zero of $F(x) = 25x^2 + 16 - 40x$ is $\frac{4}{5}$.

The only x-intercept of the graph of F is $\frac{4}{5}$.

$$f(x) = 0$$
$$x^2 - 8 = 0$$
$$x^2 = 8$$
$$x = \pm\sqrt{8} = \pm 2\sqrt{2}$$

The zeros of $f(x) = x^2 - 8$ are $-2\sqrt{2}$ and $2\sqrt{2}$. The x-intercepts of the graph of f are $-2\sqrt{2}$ and $2\sqrt{2}$.

$$g(x) = 0$$
$$x^2 - 18 = 0$$
$$x^2 = 18$$
$$x = \pm\sqrt{18} = \pm 3\sqrt{3}$$

The zeros of $g(x) = x^2 - 18$ are $-3\sqrt{3}$ and $3\sqrt{3}$. The x-intercepts of the graph of g are $-3\sqrt{3}$ and $3\sqrt{3}$.

$$g(x) = 0$$
$$(x - 1)^2 - 4 = 0$$
$$(x - 1)^2 = 4$$
$$x - 1 = \pm\sqrt{4}$$
$$x - 1 = \pm 2$$
$$x - 1 = 2 \text{ or } x - 1 = -2$$
$$x = 3 \qquad x = -1$$

The zeros of $g(x) = (x - 1)^2 - 4$ are -1 and 3. The x-intercepts of the graph of g are -1 and 3.

$$G(x) = 0$$
$$(x + 2)^2 - 1 = 0$$
$$(x + 2)^2 = 1$$
$$x + 2 = \pm\sqrt{1}$$
$$x + 2 = \pm 1$$
$$x + 2 = 1 \quad \text{or} \quad x + 2 = -1$$
$$x = -1 \qquad x = -3$$

The zeros of $G(x) = (x + 2)^2 - 1$ are -3 and -1. The x-intercepts of the graph of G are -3 and -1.

$$F(x) = 0$$
$$(2x + 3)^2 - 32 = 0$$
$$(2x + 3)^2 = 32$$
$$2x + 3 = \pm\sqrt{32}$$
$$2x + 3 = \pm 4\sqrt{2}$$
$$2x = -3 \pm 4\sqrt{2}$$
$$x = \frac{-3 \pm 4\sqrt{2}}{2}$$

The zeros of $F(x) = (2x + 3)^2 - 32$ are $\frac{-3+4\sqrt{2}}{2}$ and $\frac{-3-4\sqrt{2}}{2}$. The x-intercepts of the graph of F are $\frac{-3+4\sqrt{2}}{2}$ and $\frac{-3-4\sqrt{2}}{2}$.

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$F(x) = 0$

$(3x-2)^2 - 75 = 0$

$(3x-2)^2 = 75$

$3x - 2 = \pm\sqrt{75}$

$3x - 2 = \pm 5\sqrt{3}$

$3x = 2 \pm 5\sqrt{3}$

$x = \frac{2 \pm 5\sqrt{3}}{3}$

The zeros of $G(x) = (3x-2)^2 - 75$ are $\frac{2+5\sqrt{3}}{3}$ and $\frac{2-5\sqrt{3}}{3}$. The x-intercepts of the graph of G are $\frac{2-5\sqrt{3}}{3}$ and $\frac{2+5\sqrt{3}}{3}$.

$f(x) = 0$

$^2 + 4x - 8 = 0$

$^2 + 4x = 8$

$^2 + 4x + 4 = 8 + 4$

$(x+2)^2 = 12$

$+2 = \pm\sqrt{12}$

$+2 = \pm 2\sqrt{3}$

$x = -2 \pm 2\sqrt{3}$

$x = -2 + 2\sqrt{3}$ or $x = -2 - 2\sqrt{3}$

The zeros of $f(x) = x^2 + 4x - 8$ are $-2 + 2\sqrt{3}$ and $-2 - 2\sqrt{3}$. The x-intercepts of the graph of f are $-2 + 2\sqrt{3}$ and $-2 - 2\sqrt{3}$

$f(x) = 0$

$^2 - 6x - 9 = 0$

$^2 - 6x + 9 = 9 + 9$

$g(x) = 0$

$^2 - \frac{1}{2}x - \frac{3}{16} = 0$

$^2 - \frac{1}{2}x = \frac{3}{16}$

$^2 - \frac{1}{2}x + \frac{1}{16} = \frac{3}{16} + \frac{1}{16}$

$\left(x - \frac{1}{4}\right)^2 = \frac{1}{4}$

$x - \frac{1}{4} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$x = \frac{1}{4} \pm \frac{1}{2}$

$x = \frac{3}{4}$ or $x = -\frac{1}{4}$

The zeros of $g(x) = x^2 - \frac{1}{2}x - \frac{3}{16}$ are $-\frac{1}{4}$ and $\frac{3}{4}$.

The x-intercepts of the graph of g are $-\frac{1}{4}$ and $\frac{3}{4}$.

$g(x) = 0$

$x^2 + \frac{2}{3}x - \frac{1}{3} = 0$

$x^2 + \frac{2}{3}x = \frac{1}{3}$

$x^2 + \frac{2}{3}x + \frac{1}{9} = \frac{1}{3} + \frac{1}{9}$

$\left(x + \frac{1}{3}\right)^2 = \frac{4}{9}$

$x + \frac{1}{3} = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

$x = -\frac{1}{3} \pm \frac{2}{3}$

$x = \frac{1}{3}$ or $x = -1$

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$$(x-3)^2 = 18$$
$$-3 = \pm\sqrt{18}$$
$$x = 3 \pm 3\sqrt{2}$$

The zeros of $f(x) = x^2 - 6x - 9$ are $3 - 3\sqrt{2}$ and $3 + 3\sqrt{2}$. The x-intercepts of the graph of f are $3 - 3\sqrt{2}$ and $3 + 3\sqrt{2}$.

The zeros of $g(x) = x^2 + \frac{2}{3}x - \frac{1}{3}$ are -1 and $\frac{1}{3}$.

The x-intercepts of the graph of g are -1 and $\frac{1}{3}$.

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$$F(x) = 0$$
$$3x^2 + x - \frac{1}{2} = 0$$
$$x^2 + \frac{1}{3}x - \frac{1}{6} = 0$$
$$x^2 + \frac{1}{3}x = \frac{1}{6}$$
$$x^2 + \frac{1}{3}x + \frac{1}{36} = \frac{1}{6} + \frac{1}{36}$$
$$\left(x + \frac{1}{6}\right)^2 = \frac{7}{36}$$
$$x + \frac{1}{6} = \pm\sqrt{\frac{7}{36}} = \pm\frac{\sqrt{7}}{6}$$
$$x = \frac{-1 \pm \sqrt{7}}{6}$$

The zeros of $F(x) = 3x^2 + x - \frac{1}{2}$ are $\frac{-1-\sqrt{7}}{6}$ and $\frac{-1+\sqrt{7}}{6}$. The x-intercepts of the graph of F are $\frac{-1-\sqrt{7}}{6}$ and $\frac{-1+\sqrt{7}}{6}$.

$$G(x) = 0$$
$$2x^2 - 3x - 1 = 0$$
$$x^2 - \frac{3}{2}x - \frac{1}{2} = 0$$
$$x^2 - \frac{3}{2}x = \frac{1}{2}$$
$$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{1}{2} + \frac{9}{16}$$
$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$
$$x - \frac{3}{4} = \pm\sqrt{\frac{17}{16}} = \pm\frac{\sqrt{17}}{4}$$
$$x = \frac{3 \pm \sqrt{17}}{4}$$

The zeros of $G(x) = 2x^2 - 3x - 1$ are $\frac{3-\sqrt{17}}{4}$ and $\frac{3+\sqrt{17}}{4}$. The x-intercepts of the graph of G are $\frac{3-\sqrt{17}}{4}$ and $\frac{3+\sqrt{17}}{4}$.

$$f(x) = 0$$
$$x^2 - 4x + 2 = 0$$
$$a = 1,\ b = -4,\quad c = 2$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} = \frac{4 \pm \sqrt{16-8}}{2}$$
$$= \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

The zeros of $f(x) = x^2 - 4x + 2$ are $2 - \sqrt{2}$ and $2 + \sqrt{2}$. The x-intercepts of the graph of f are $2 - \sqrt{2}$ and $2 + \sqrt{2}$.

$$f(x) = 0$$
$$x^2 + 4x + 2 = 0$$
$$a = 1,\ b = 4,\quad c = 2$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)} = \frac{-4 \pm \sqrt{16-8}}{2}$$
$$= \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

The zeros of $f(x) = x^2 + 4x + 2$ are $-2 - \sqrt{2}$ and $-2 + \sqrt{2}$. The x-intercepts of the graph of f are $-2 - \sqrt{2}$ and $-2 + \sqrt{2}$.

$$g(x) = 0$$
$$x^2 - 4x - 1 = 0$$
$$a = 1,\ b = -4,\ c = -1$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)} = \frac{4 \pm \sqrt{16+4}}{2}$$
$$= \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

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2 2

The zeros of $g(x) = x^2 - 4x - 1$ are $2 - \sqrt{5}$ and $2 + \sqrt{5}$. The x-intercepts of the graph of g are $2 - \sqrt{5}$ and $2 + \sqrt{5}$.

$$g(x) = 0$$

$$x^2 + 6x + 1 = 0$$

$$a = 1, \quad b = 6, \quad c = 1$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(1)}}{2(1)} = \frac{-6 \pm \sqrt{36-4}}{2}$$

$$= \frac{-6 \pm \sqrt{32}}{2} = \frac{-6 \pm 4\sqrt{2}}{2} = -3 \pm 2\sqrt{2}$$

The zeros of $g(x) = x^2 + 6x + 1$ are $-3 - 2\sqrt{2}$

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and $-3+2\sqrt{2}$. The x-intercepts of the graph of g are $-3-2\sqrt{2}$ and $-3+2\sqrt{2}$.

$$F(x)=0$$
$$x^2-5x+3=0$$
$$a=2,\ b=-5,\ c=3$$
$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(3)}}{2(2)}=\frac{5\pm\sqrt{25-24}}{4}$$
$$\frac{5\pm 1}{4}=\frac{3}{2}\text{ or }1$$

The zeros of $F(x)=2x^2-5x+3$ are 1 and $\frac{3}{2}$.

$$g(x)=0$$
$$x^2+5x+3=0$$
$$a=2,\ b=5,\ c=3$$
$$x=\frac{-5\pm\sqrt{5^2-4(2)(3)}}{2(2)}=\frac{-5\pm\sqrt{25-24}}{4}$$
$$\frac{-5\pm 1}{4}=-1\text{ or }-\frac{3}{2}$$

The zeros of $g(x)=2x^2+5x+3$ are $-\frac{3}{2}$ and -1.

$$P(x)=0$$
$$4x^2-x+2=0$$
$$a=4,\ b=-1,\ c=2$$
$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(4)(2)}}{2(4)}=\frac{1\pm\sqrt{-32}}{8}$$
$$\frac{1\pm\sqrt{-31}}{8}=\text{not real}$$

The function $P(x)=4x^2-x+2$ has no real zeros, and the graph of P has no x-intercepts.

$$H(x)=0$$
$$4x^2+x+1=0$$
$$a=4,\ b=1,\ c=1$$

The function $H(x)=4x^2+x+1$ has no real zeros, and the graph of H has no x-intercepts.

$$f(x)=0$$
$$x^2-1+2x=0$$
$$x^2+2x-1=0$$
$$a=4,\ b=2,\ c=-1$$
$$x=\frac{-2\pm\sqrt{2^2-4(4)(-1)}}{2(4)}=\frac{-2\pm\sqrt{4+16}}{8}$$
$$=\frac{-2\pm\sqrt{20}}{8}=\frac{-2\pm 2\sqrt{5}}{8}=\frac{-1\pm\sqrt{5}}{4}$$

The zeros of $f(x)=4x^2-1+2x$ are $\frac{-1-\sqrt{5}}{4}$ and $\frac{-1+\sqrt{5}}{4}$. The x-intercepts of the graph of f are $\frac{-1-\sqrt{5}}{4}$ and $\frac{-1+\sqrt{5}}{4}$.

$$f(x)=0$$
$$x^2-1+2x=0$$
$$x^2+2x-1=0$$
$$a=2,\ b=2,\ c=-1$$
$$x=\frac{-2\pm\sqrt{2^2-4(2)(-1)}}{2(2)}=\frac{-2\pm\sqrt{4+8}}{4}$$
$$=\frac{-2\pm\sqrt{12}}{4}=\frac{-2\pm 2\sqrt{3}}{4}=\frac{-1\pm\sqrt{3}}{2}$$

The zeros of $f(x)=2x^2-1+2x$ are $\frac{-1-\sqrt{3}}{2}$ and $\frac{-1+\sqrt{3}}{2}$. The x-intercepts of the graph of f are $\frac{-1-\sqrt{3}}{2}$ and $\frac{-1+\sqrt{3}}{2}$.

$$G(x)=0$$
$$2x(x+2)-3=0$$
$$x^2+4x-3=0$$
$$a=2,\ b=4,\ c=-3$$
$$x=\frac{-(4)\pm\sqrt{(4)^2-4(2)(-3)}}{2(2)}=\frac{-4\pm\sqrt{16+24}}{4}$$

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$$t = \frac{-1 \pm \sqrt{1^2 - 4(4)(1)}}{2(4)} = \frac{-1 \pm \sqrt{1-16}}{8}$$

$$\frac{-1 \pm \sqrt{-15}}{8} = \text{not real}$$

$$= \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}$$

The zeros of $G(x) = 2x(x+2) - 3$ are $\frac{-2 + \sqrt{10}}{2}$

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and $\frac{-2-\sqrt{10}}{2}$. The x-intercepts of the graph of G are $\frac{-2+\sqrt{10}}{2}$ and $\frac{-2-\sqrt{10}}{2}$.

$$F(x) = 0$$
$$3x(x+2) - 1 = 0 \Rightarrow 3x^2 + 6x - 1 = 0$$
$$a = 3,\ b = 6,\ c = -1$$
$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(3)(-1)}}{2(3)} = \frac{-6 \pm \sqrt{36+12}}{6}$$
$$= \frac{-6 \pm \sqrt{48}}{6} = \frac{-6 \pm 4\sqrt{3}}{6} = \frac{-3 \pm 2\sqrt{3}}{3}$$

The zeros of $F(x) = 3x(x+2) - 2$ are $\frac{-3+2\sqrt{3}}{3}$ and $\frac{-3-2\sqrt{3}}{3}$. The x-intercepts of the graph of G are $\frac{-3+2\sqrt{3}}{3}$ and $\frac{-3-2\sqrt{3}}{3}$.

$$p(x) = 0$$
$$9x^2 - 6x + 1 = 0$$
$$a = 9,\ b = -6,\ c = 1$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)} = \frac{6 \pm \sqrt{36-36}}{18}$$
$$\frac{6 \pm 0}{18} = \frac{1}{3}$$

The only real zero of $p(x) = 9x^2 - 6x + 1$ is $\frac{1}{3}$.

The only x-intercept of the graph of g is $\frac{1}{3}$.

$$q(x) = 0$$
$$4x^2 + 20x + 25 = 0$$
$$a = 4,\ b = 20,\ c = 25$$
$$x = \frac{-20 \pm \sqrt{(20)^2 - 4(4)(25)}}{2(4)} = \frac{-20 \pm \sqrt{400-400}}{8}$$
$$= \frac{-20 \pm 0}{8} = -\frac{20}{8} = -\frac{5}{2}$$

The only real zero of $q(x) = 4x^2 + 20x + 25$ is $-\frac{5}{2}$. The only x-intercept of the graph of F is $-\frac{5}{2}$.

$$f(x) = g(x)$$
$$^2 + 6x + 3 = 3$$
$$x^2 + 6x = 0 \Rightarrow x(x+6) = 0$$
$$x = 0 \text{ or } x + 6 = 0$$
$$= -6$$

The x-coordinates of the points of intersection are -6 and 0. The y-coordinates are $g(-6) = 3$ and $g(0) = 3$. The graphs of the f and g intersect at the points $(-6,3)$ and $(0,3)$.

$$f(x) = g(x)$$
$$^2 - 4x + 3 = 3$$
$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$x = 0 \text{ or } x - 4 = 0$$
$$x = 4$$

The x-coordinates of the points of intersection are 0 and 4. The y-coordinates are $g(0) = 3$ and $g(4) = 3$. The graphs of the f and g intersect at the points $(0,3)$ and $(4,3)$.

$$f(x) = g(x)$$
$$-2x^2 + 1 = 3x + 2$$
$$0 = 2x^2 + 3x + 1$$
$$0 = (2x+1)(x+1)$$
$$2x + 1 = 0 \text{ or } x + 1 = 0$$
$$x = -\frac{1}{2} \qquad x = -1$$

The x-coordinates of the points of intersection are -1 and $-\frac{1}{2}$. The y-coordinates are $g(-1) = 3(-1) + 2 = -3 + 2 = -1$ and

$$g\left(-\frac{1}{2}\right) = 3\left(-\frac{1}{2}\right) + 2 = -\frac{3}{2} + 2 = \frac{1}{2}.$$

$(-1,-1)$ and $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

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$f(x) = g(x)$
$3x^2 - 7 = 10x + 1$
$3x^2 - 10x - 8 = 0$
$(3x + 2)(x - 4) = 0$
$3x + 2 = 0$ or $x - 4 = 0$
$x = -\frac{2}{3}$ $\quad x = 4$

The x-coordinates of the points of intersection are $-\frac{2}{3}$ and 4. The y-coordinates are

$g\left(-\frac{2}{3}\right) = 10\left(-\frac{2}{3}\right) + 1 = -\frac{20}{3} + 1 = -\frac{17}{3}$ and

$(4) = 10(4) + 1 = 40 + 1 = 41.$

The graphs of the f and g intersect at the points $\left(-\frac{2}{3}, -\frac{17}{3}\right)$ and $(4, 41)$.

$f(x) = g(x)$
$^2 - x + 1 = 2x^2 - 3x - 14$
$= x^2 - 2x - 15$
$= (x + 3)(x - 5)$
$x + 3 = 0$ or $x - 5 = 0$
$x = -3$ $\quad x = 5$

The x-coordinates of the points of intersection are -3 and 5. The y-coordinates are

$(-3) = (-3)^2 - (-3) + 1 = 9 + 3 + 1 =$

13 and $f(5) = 5^2 - 5 + 1 = 25 - 5 + 1 = 21$.

The graphs of the f and g intersect at the points $(-3, 13)$ and $(5, 21)$.

$f(x) = g(x)$
$^2 + 5x - 3 = 2x^2 + 7x - 27$
$= x^2 + 2x - 24$
$= (x + 6)(x - 4)$
$x + 6 = 0$ or $x - 4 = 0$
$x = -6$ $\quad x = 4$

$P(x) = 0$
$x^4 - 6x^2 - 16 = 0$
$x^2 + 2)(x^2 - 8) = 0$
$x^2 + 2 = 0$ or $x^2 - 8 = 0$
$x^2 = -2$ $\quad x^2 = 8$
$x = \pm\sqrt{-2}$ $\quad x = \pm\sqrt{8}$
$=$ not real $\quad = \pm 2\sqrt{2}$

The zeros of $P(x) = x^4 - 6x^2 - 16$ are $-2\sqrt{2}$ and $2\sqrt{2}$. The x-intercepts of the graph of P are

$-2\sqrt{2}$ and $2\sqrt{2}$.

$H(x) = 0$
$x^4 - 3x^2 - 4 = 0$
$x^2 + 1)(x^2 - 4) = 0$
$x^2 + 1 = 0$ or $x^2 - 4 = 0$
$x^2 = -1$ $\quad x^2 = 4$
$x = \pm\sqrt{-1}$ $\quad x = \pm\sqrt{4}$
$=$ not real $\quad = \pm 2$

The zeros of $H(x) = x^4 - 3x^2 - 4$ are -2 and 2. The x-intercepts of the graph of H are -2 and 2.

$f(x) = 0$
$x^4 - 5x^2 + 4 = 0$
$(x^2 - 4)(x^2 - 1) = 0$

The x-coordinates of the points of intersection are -6 and 4. The y-coordinates are

$(-6) = (-6)^2 + 5(-6) - 3 = 36 - 30 - 3 =$

3 and $f(4) = 4^2 + 5(4) - 3 = 16 + 20 - 3 = 33$.

The graphs of the f and g intersect at the points $(-6, 3)$ and $(4, 33)$.

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$x^2 - 4 = 0$ or $x^2 - 1 = 0$

$x = \pm 2$ or $x = \pm 1$

The zeros of $f(x) = x^4 - 5x^2 + 4$ are -2, -1, 1, and 2. The x-intercepts of the graph of f are -2, -1, 1, and 2.

$$f(x) = 0$$
$$x^4 - 10x^2 + 24 = 0$$
$$(x^2 - 4)(x^2 - 6) = 0$$

$x^2 - 4 = 0$ or $x^2 - 6 = 0$

$x^2 = 4$ $\quad$ $x^2 = 6$

$x = \pm 2$ $\quad$ $x = \pm\sqrt{6}$

The zeros of $f(x) = x^4 - 10x^2 + 24$ are $-\sqrt{6}$, $\sqrt{6}$, 2 and -2. The x-intercepts of the graph of f are $-\sqrt{6}$, $\sqrt{6}$, 2 and -2.

$G(x) = 0$

$3x^4 - 2x^2 - 1 = 0$

$(3x^2 + 1)(x^2 - 1) = 0$

$3x^2 + 1 = 0$ or $x^2 - 1 = 0$

$x^2 = -\frac{1}{3}$ $\quad x^2 = 1$

$x = \pm\sqrt{-\frac{1}{3}}$ $\quad x = \pm\sqrt{1} = \pm 1$

$x =$ not real

The zeros of $G(x) = 3x^4 - 2x^2 - 1$ are -1 and 1. The x-intercepts of the graph of G are -1 and 1.

$F(x) = 0$

$2x^4 - 5x^2 - 12 = 0$

$(2x^2 + 3)(x^2 - 4) = 0$

$x^2 + 3 = 0$ or $x^2 - 4 = 0$

$2 = -\frac{3}{2}$ $\quad x^2 = 4$

$x = \pm\sqrt{-\frac{3}{2}}$ $\quad x = \pm\sqrt{4} = \pm 2$

$=$ not real

The zeros of $F(x) = 2x^4 - 5x^2 - 12$ are -2 and 2. The x-intercepts of the graph of F are -2 and 2.

$g(x) = 0$

$x^6 + 7x^3 - 8 = 0$

$x^3 + 8)(x^3 - 1) = 0$

$x^3 + 8 = 0$ or $x^3 - 1 = 0$

$x^3 = -8$ $\quad x^3 = 1$

$x = -2$ $\quad x = 1$

The zeros of $g(x) = x^6 + 7x^3 - 8$ are -2 and 1. The x-intercepts of the graph of g are -2 and 1.

$g(x) = 0$

$x^6 - 7x^3 - 8 = 0$

$x^3 - 8)(x^3 + 1) = 0$

$x^3 - 8 = 0$ or $x^3 + 1 = 0$

$x^3 = 8$ $\quad x^3 = -1$

$x = 2$ $\quad x = -1$

65. $G(x) = 0$

$x + 2)^2 + 7(x + 2) + 12 = 0$

Let $u = x + 2 \rightarrow u^2 = (x + 2)^2$

$u^2 + 7u + 12 = 0$

$u + 3)(u + 4) = 0$

$u + 3 = 0$ or $u + 4 = 0$

$u = -3$ $\quad u = -4$

$x + 2 = -3$ $\quad x + 2 = -4$

$x = -5$ $\quad x = -6$

The zeros of $G(x) = (x + 2)^2 + 7(x + 2) + 12$ are -6 and -5 . The x-intercepts of the graph of G are -6 and -5 .

66. $f(x) = 0$

$2x + 5)^2 - (2x + 5) - 6 = 0$

Let $u = 2x + 5 \rightarrow u^2 = (2x + 5)^2$

$u^2 - u - 6 = 0$

$u - 3)(u + 2) = 0$

$u - 3 = 0$ or $u + 2 = 0$

$u = 3$ $\quad u = -2$

$2x + 5 = 3$ $\quad 2x + 5 = -2$

$x = -1$ $\quad x = -\frac{7}{2}$

The zeros of $f(x) = (2x + 5)^2 - (2x + 5) - 6$ are $\frac{7}{2}$ and -1 . The x-intercepts of the graph of f are $-\frac{7}{2}$ and -1 .

67. $f(x) = 0$

$3x + 4)^2 - 6(3x + 4) + 9 = 0$ Let u $= 3x + 4 \rightarrow u^2 = (3x + 4)^2$

$u^2 - 6u + 9 = 0$

$u - 3)^2 = 0$

$u - 3 = 0$

$= 3$

$3x + 4 = 3$

$= -\frac{1}{3}$

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The zeros of $g(x) = x^6 - 7x^3 - 8$ are -1 and 2. The x-intercepts of the graph of g are -1 and 2.

The only zero of $f(x) = (3x+4)^2 - 6(3x+4) + 9$ is $-\frac{1}{3}$. The x-intercept of the graph of f is $-\frac{1}{3}$.

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68. $H(x) = 0$

$(2 - x)^2 + (2 - x) - 20 = 0$

Let $u = 2 - x \rightarrow u^2 = (2 - x)^2$

$u^2 + u - 20 = 0$

$(u + 5)(u - 4) = 0$

$u + 5 = 0$ or $u - 4 = 0$

$u = -5$ $\quad u = 4$

$2 - x = -5$ $\quad 2 - x = 4$

$x = 7$ $\quad x = -2$

The zeros of $H(x) = (2 - x)^2 + (2 - x) - 20$ are -2 and 7. The x-intercepts of the graph of H are -2 and 7.

69. $P(x) = 0$

$2(x + 1)^2 - 5(x + 1) - 3 = 0$

Let $u = x + 1 \rightarrow u^2 = (x + 1)^2$

$2u^2 - 5u - 3 = 0$

$(2u + 1)(u - 3) = 0$

$2u + 1 = 0$ or $u - 3 = 0$

$u = -\frac{1}{2}$ $\quad u = 3$

$x + 1 = -\frac{1}{2}$ $\quad x + 1 = 3$

$x = -\frac{3}{2}$ $\quad x = 2$

The zeros of $P(x) = 2(x + 1)^2 - 5(x + 1) - 3$ are $-\frac{3}{2}$ and 2. The x-intercepts of the graph of P are $-\frac{3}{2}$ and 2.

70. $H(x) = 0$

$3(1 - x)^2 + 5(1 - x) + 2 = 0$

Let $u = 1 - x \rightarrow u^2 = (1 - x)^2$

$3u^2 + 5u + 2 = 0$

$(3u + 2)(u + 1) = 0$

$3u + 2 = 0$ or $u + 1 = 0$

$u = -\frac{2}{3}$ $\quad u = -1$

$1 - x = -\frac{2}{3}$ $\quad 1 - x = -1$

$x = \frac{5}{3}$ $\quad x = 2$

The zeros of $H(x) = 3(1 - x)^2 + 5(1 - x) + 2$ are $\frac{5}{3}$ and 2. The x-intercepts of the graph of H are $\frac{5}{3}$ and 2.

$G(x) = 0$

$x - 4\sqrt{x} = 0$

Let $u = \sqrt{x} \rightarrow u^2 = x$

$u^2 - 4u = 0$

$u(u - 4) = 0$

$u = 0$ or $u - 4 = 0$

$u = 4$

$\sqrt{x} = 0$ $\quad \sqrt{x} = 4$

$x = 0^2 = 0$ $\quad x = 4^2 = 16$

Check:

$G(0) = 0 - 4\sqrt{0} = 0$

$G(16) = 16 - 4\sqrt{16} = 16 - 16 = 0$

The zeros of $G(x) = x - 4\sqrt{x}$ are 0 and 16. The x-intercepts of the graph of G are 0 and 16.

$f(x) = 0$

$x + 8\sqrt{x} = 0$

Let $u = \sqrt{x} \rightarrow u^2 = x$

$u^2 + 8u = 0$

$u(u + 8) = 0$

$u = 0$ or $u + 8 = 0$

$u = -8$

$\sqrt{x} = 0$ $\quad \sqrt{x} = -8$

$x = 0^2 = 0$ $\quad x =$ not real

Check: $f(0) = 0 + 8\sqrt{0} = 0$

The only zero of $f(x) = x + 8\sqrt{x}$ is 0. The only x-intercept of the graph of f is 0.

73. $g(x) = 0$

$x + \sqrt{x} - 20 = 0$

Let $u = \sqrt{x} \rightarrow u^2 = x$

2

3

$u + u - 20 = 0$

$(u + 5)(u - 4) = 0$

$u+5=0$ or $u-4=0$

$u=-5$ $u=4$

$\sqrt{x}=-5$ $\sqrt{x}=4$

$x=$ not real $x=4^2=16$

Check: $g(16)=16+\sqrt{16}-20=16+4-20=0$

The only zero of $g(x)=x+\sqrt{x}-20$ is 16. The only x-intercept of the graph of g is 16.

$f(x)=0$

$x+\sqrt{x}-2=0$

Let $u=\sqrt{x}\rightarrow u^2=x$

$u^2+u-2=0$

$(u-1)(u+2)=0$

$u-1=0$ or $u+2=0$

$u=1$ $u=-2$

$\sqrt{x}=1$ $\sqrt{x}=-2$

$x=1^2=1$ $x=$ not real

Check: $f(1)=1+\sqrt{1}-2=1+1-2=0$

The only zero of $f(x)=x+\sqrt{x}-2$ is 1. The only x-intercept of the graph of f is 1.

$f(x)=0$

$^2-50=0$

$x^2=50\Rightarrow x=\pm\sqrt{50}=\pm5\sqrt{2}$

The zeros of $f(x)=x^2-50$ are $-5\sqrt{2}$ and $\sqrt{2}$. The x-intercepts of the graph of f are $-5\sqrt{2}$ and $5\sqrt{2}$

$f(x)=0$

$^2-20=0$

$x^2=20\Rightarrow x=\pm\sqrt{20}=\pm2\sqrt{5}$

The zeros of $f(x)=x^2-6$ are $-2\sqrt{5}$ and $\sqrt{5}$. The x-intercepts of the graph of f are

The only real zero of $g(x)=16x^2-8x+1$ is $\frac{1}{4}$.

The only x-intercept of the graph of g is $\frac{1}{4}$.

$F(x)=0$ $4x^2-$

$12x+9=0$ $(2x-$

$3)^2=0$

$2x-3=0\Rightarrow x=\frac{3}{2}$

The only real zero of $F(x)=4x^2-12x+9$ is $\frac{3}{2}$.

The only x-intercept of the graph of F is $\frac{3}{2}$.

$G(x)=0$

$10x^2-19x-15=0$

$(5x+3)(2x-5)=0$

$5x+3=0$ or $2x-5=0$

$x=-\frac{3}{5}$ $x=\frac{5}{2}$

The zeros of $G(x)=10x^2-19x-15$ are $-\frac{3}{5}$ and $\frac{5}{2}$. The x-intercepts of the graph of G are $-\frac{3}{5}$ and $\frac{5}{2}$.

$f(x)=0$

$x^2+7x-20=0$

$(3x-4)(2x+5)=0$

$3x-4=0$ or $2x+5=0$

$x=\frac{4}{3}$ $x=-\frac{5}{2}$

The zeros of $f(x)=6x^2+7x-20$ are $-\frac{5}{2}$ and $\frac{4}{3}$.

$-2\sqrt{5}$ and $2\sqrt{5}$.

$g(x)=0$

$x^2-8x+1=0$

$(4x-1)^2=0$

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$4x - 1 = 0 \Rightarrow x = \frac{1}{4}$

The x-intercepts of the graph of f are $-\frac{5}{2}$ and $\frac{4}{3}$.

$$P(x) = 0$$
$$x^2 - x - 2 = 0$$
$$(3x - 2)(2x + 1) = 0$$
$$3x - 2 = 0 \text{ or } 2x + 1 = 0$$
$$x = \frac{2}{3} \qquad x = -\frac{1}{2}$$

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The zeros of $P(x) = 6x^2 - x - 2$ are $-\frac{1}{2}$ and $\frac{2}{3}$.

The x-intercepts of the graph of P are $-\frac{1}{2}$ and $\frac{2}{3}$.

$H(x) = 0$

$x^2 + x - 2 = 0$

$(3x+2)(2x-1) = 0$

$3x + 2 = 0$ or $2x - 1 = 0$

$x = -\frac{2}{3}$ $\quad x = \frac{1}{2}$

The zeros of $H(x) = 6x^2 + x - 2$ are $-\frac{2}{3}$ and $\frac{1}{2}$.

The x-intercepts of the graph of H are $-\frac{2}{3}$ and $\frac{1}{2}$.

$G(x) = 0$

$x^2 + \sqrt{2}x - \frac{1}{2} = 0$

$2\left(x^2 + \sqrt{2}x - \frac{1}{2}\right) = (0)(2)$

$2x^2 + 2\sqrt{2}x - 1 = 0$

$a = 2,\ b = 2\sqrt{2},\ c = -1$

$x = \dfrac{-(2\sqrt{2}) \pm \sqrt{(2\sqrt{2})^2 - 4(2)(-1)}}{2(2)}$

$= \dfrac{-2\sqrt{2} \pm \sqrt{8+8}}{4} = \dfrac{-2\sqrt{2} \pm \sqrt{16}}{4}$

$= \dfrac{-2\sqrt{2} \pm 4}{4} = \dfrac{-\sqrt{2} \pm 2}{2}$

The zeros of $G(x) = x^2 + \sqrt{2}x - \frac{1}{2}$ are $\frac{-\sqrt{2}-2}{2}$ and $\frac{-\sqrt{2}+2}{2}$. The x-intercepts of the graph of G are $\frac{-\sqrt{2}-2}{2}$ and $\frac{-\sqrt{2}+2}{2}$.

$F(x) = 0$

$\frac{1}{2}x^2 - \sqrt{2}x - 1 = 0$

$x = \dfrac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$

$= \dfrac{2\sqrt{2} \pm \sqrt{16}}{2} = \dfrac{2\sqrt{2} \pm 4}{2} = \dfrac{\sqrt{2} \pm 2}{1}$

The zeros of $F(x) = \frac{1}{2}x^2 - \sqrt{2}x - 1$ are $\sqrt{2} - 2$ and $\sqrt{2} + 2$. The x-intercepts of the graph of F are $\sqrt{2} - 2$ and $\sqrt{2} + 2$.

$f(x) = 0$

$x^2 + x - 4 = 0$

$a = 1,\ b = 1,\ c = -4$

$= \dfrac{-(1) \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)}$

$\dfrac{-1 \pm \sqrt{1+16}}{2} = \dfrac{-1 \pm \sqrt{17}}{2}$

The zeros of $f(x) = x^2 + x - 4$ are $\frac{-1-\sqrt{17}}{2}$ and $\frac{-1+\sqrt{17}}{2}$. The x-intercepts of the graph of f are $\frac{-1-\sqrt{17}}{2}$ and $\frac{-1+\sqrt{17}}{2}$.

$g(x) = 0$

$x^2 + x - 1 = 0$

$a = 1,\ b = 1,\ c = -1$

$x = \dfrac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)} = \dfrac{-1 \pm \sqrt{5}}{2}$

The zeros of $g(x) = x^2 + x - 1$ are $\frac{-1-\sqrt{5}}{2}$ and $\frac{-1+\sqrt{5}}{2}$. The x-intercepts of the graph of g are $\frac{-1-\sqrt{5}}{2}$ and $\frac{-1+\sqrt{5}}{2}$.

a. $g(x) = (x-1)^2 - 4$

Using the graph of $y = x^2$, horizontally shift to

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$$2\left(\frac{1}{2}x^2-\sqrt{2}x-1\right)=(0)(2)$$

$$x^2-2\sqrt{2}x-2=0$$

$a=1,\ b=-2\sqrt{2},\quad c=-2$

the right 1 unit, and then vertically shift

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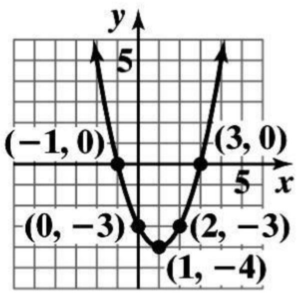
downward 4 units.



$g(x) = 0$

$(x - 1)^2 - 4 = 0$

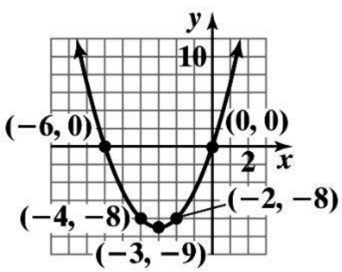
$^2 - 2x + 1 - 4 = 0$

$x^2 - 2x - 3 = 0$

$x + 1)(x - 3) = 0 \Rightarrow x = -1 \text{ or } x = 3$

a. $F(x) = (x + 3)^2 - 9$

Using the graph of $y = x^2$, horizontally shift to the left 3 units, and then vertically shift downward 9 units.



$F(x) = 0$

$(x + 3)^2 - 9 = 0$

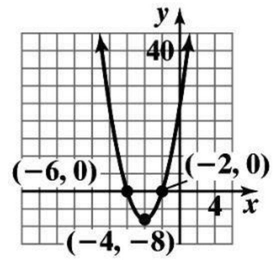
$^2 + 6x + 9 - 9 = 0$

$x^2 + 6x = 0$

$x(x + 6) = 0 \Rightarrow x = 0 \text{ or } x = -6$

a. $f(x) = 2(x + 4)^2 - 8$

Using the graph of $y = x^2$, horizontally shift to the left 4 units, vertically stretch by a factor of 2, and then vertically shift downward 8 units.



b. $f(x) = 0$

$2(x + 4)^2 - 8 = 0$

$2(x^2 + 8x + 16) - 8 = 0$

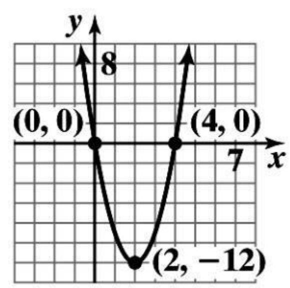
$x^2 + 16x + 32 - 8 = 0$

$2x^2 + 16x + 24 = 0$

$2(x + 2)(x + 6) = 0 \Rightarrow x = -2 \text{ or } x = -6$

a. $h(x) = 3(x - 2)^2 - 12$

Using the graph of $y = x^2$, horizontally shift to the right 2 units, vertically stretch by a factor of 3, and then vertically shift downward 12 units.



b. $h(x) = 0$

$3(x - 2)^2 - 12 = 0$

$3(x^2 - 4x + 4) - 12 = 0$

$3x^2 - 12x + 12 - 12 = 0$

$3x^2 - 12x = 0$

$3x(x - 4) = 0 \Rightarrow x = 0 \text{ or } x = 4$

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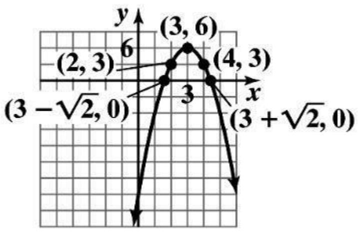
a. $H(x) = -3(x-3)^2 + 6$

Using the graph of $y = x^2$, horizontally shift to the right 3 units, vertically stretch by a factor of 3, reflect about the x-axis, and then vertically shift upward 6 units.



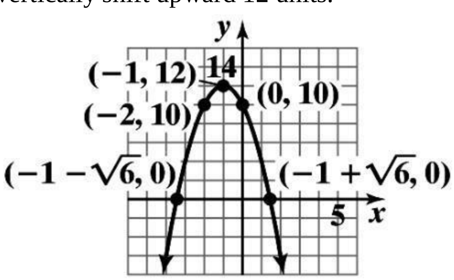
b.

$$H(x) = 0$$
$$-3(x-3)^2 + 6 = 0$$
$$-3(x^2 - 6x + 9) + 6 = 0$$
$$-3x^2 + 18x - 27 + 6 = 0$$
$$-3x^2 + 18x - 21 = 0$$
$$-3(x^2 - 6x + 7) = 0$$
$$a = 1,\ b = -6,\ c = 7$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)} = \frac{6 \pm \sqrt{36-28}}{2}$$
$$= \frac{6 \pm \sqrt{8}}{2} = \frac{6 \pm 2\sqrt{2}}{2} = 3 \pm \sqrt{2}$$

a. $f(x) = -2(x+1)^2 + 12$

Using the graph of $y = x^2$, horizontally shift to the left 1 unit, vertically stretch by a factor of 2, reflect about the x-axis, and then vertically shift upward 12 units.



b.

$$f(x) = 0$$
$$-2(x+1)^2 + 12 = 0$$
$$-2(x^2 + 2x + 1) + 12 = 0$$
$$-2x^2 - 4x - 2 + 12 = 0$$
$$-2x^2 - 4x + 10 = 0$$
$$-2(x^2 + 2x - 5) = 0$$
$$a = 1,\ b = 2,\ c = -5$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-5)}}{2(1)} = \frac{-2 \pm \sqrt{4+20}}{2}$$
$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 \pm 2\sqrt{6}}{2} = -1 \pm \sqrt{6}$$

$f(x) = g(x)$

$$5x(x-1) = -7x^2 + 2$$
$$5x^2 - 5x = -7x^2 + 2$$
$$x^2 - 5x - 2 = 0$$

$$(3x-2)(4x+1) = 0 \Rightarrow x = \frac{2}{3} \text{ or } x = -\frac{1}{4}$$

$$f\left(\frac{2}{3}\right) = 5\left(\frac{2}{3}\right)\left[\left(\frac{2}{3}\right) - 1\right] = \left(\frac{10}{3}\right)\left(-\frac{1}{3}\right) = -\frac{10}{9}$$

$$f\left(-\frac{1}{4}\right) = 5\left(-\frac{1}{4}\right)\left[\left(-\frac{1}{4}\right) - 1\right] = \left(-\frac{5}{4}\right)\left(-\frac{5}{4}\right) = \frac{25}{16}$$

The points of intersection are: $\left(\frac{2}{3}, -\frac{10}{9}\right)$ and $\left(-\frac{1}{4}, \frac{25}{16}\right)$

$f(x) = g(x)$

$$10x(x+2) = -3x + 5$$
$$x^2 + 20x = -3x +$$

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$5\ 10x^2 + 23x - 5 = 0$

$(2x + 5)(5x - 1) = 0 \Rightarrow x = -\frac{5}{2}$ or $x = \frac{1}{5}$

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$$f\left(-\frac{5}{2}\right)=10\left(-\frac{5}{2}\right)\left[\left(-\frac{5}{2}\right)+2\right]$$

$$(-25)\left(-\frac{1}{2}\right)=\frac{25}{2}$$

$$f\left(\frac{1}{5}\right)=10\left(\frac{1}{5}\right)\left[\left(\frac{1}{5}\right)+2\right]$$

$$(2)\left(\frac{11}{5}\right)=\frac{22}{5}$$

The points of intersection are:

$$\left(-\frac{5}{2},\frac{25}{2}\right) \text{ and } \left(\frac{1}{5},\frac{22}{5}\right)$$

95. $f(x)=g(x)$

$$3(x^2-4)=3x^2+2x+4$$
$$3x^2-12=3x^2+2x+4$$
$$-12=2x+4$$
$$-16=2x \Rightarrow x=-8$$
$$f(-8)=3\left[(-8)^2-4\right]$$
$$=3[64-4]=180$$

The point of intersection is: $(-8,180)$

96. $f(x)=g(x)$

$$4(x^2+1)=4x^2-3x-8$$
$$4x^2+4=4x^2-3x-8$$
$$4=-3x-8$$
$$12=-3x \Rightarrow x=-4$$
$$f(-4)=4\left[(-4)^2+1\right]$$
$$=4[16+1]=68$$

The point of intersection is: $(-4, 68)$

$$x=\frac{5}{3} \text{ or } \cancel{x=-1}$$

$$f\left(\frac{5}{3}\right)=\frac{3\left(\frac{5}{3}\right)}{\left(\frac{5}{3}\right)+2}-\frac{5}{\left(\frac{5}{3}\right)+1}$$

$$=\frac{5}{\left(\frac{11}{3}\right)}-\frac{5}{\left(\frac{8}{3}\right)}$$

$$=\frac{15}{11}-\frac{15}{8}$$

$$=-\frac{45}{88}$$

The point of intersection is: $\left(\frac{5}{3},-\frac{45}{88}\right)$

98. $f(x)=g(x)$

$$\frac{2x}{x-3}-\frac{3}{x+1}=\frac{2x+18}{x^2-2x-3}$$

$$\frac{2x}{x-3}-\frac{3}{x+1}=\frac{2x+18}{(x-3)(x+1)}$$

$$2x(x+1)-3(x-3)=2x+18$$
$$2x^2+2x-3x+9=2x+18$$
$$2x^2-3x-9=0$$
$$(2x+3)(x-3)=0$$

$$x=-\frac{3}{2} \text{ or } \cancel{x=3}$$

$$f\left(-\frac{3}{2}\right)=\frac{2\left(-\frac{3}{2}\right)}{\left(-\frac{3}{2}\right)-3}-\frac{3}{\left(-\frac{3}{2}\right)+1}$$

$$=\frac{-3}{\left(-\frac{9}{2}\right)}-\frac{[illegible]}{\left(-\frac{1}{2}\right)}$$

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6 2 20

97. $f(x) = g(x)$

$$\frac{3x}{x+2} - \frac{5}{x+1} = \frac{-5}{x^2+3x+2}$$

$$\frac{3x}{x+2} - \frac{5}{x+1} = \frac{-5}{(x+2)(x+1)}$$

$$3x(x+1) - 5(x+2) = -5$$

$$3x^2 + 3x - 5x - 10 = -5$$

$$3x^2 - 2x - 5 = 0$$

$$(3x-5)(x+1) = 0$$

$= 9 + 6 = 3 + 6 = 3$

The point of intersection is: $\left(-\frac{3}{2}, \frac{20}{3}\right)$

99. a. $(f+g)(x) =$

$$= x^2 + 5x - 14 + x^2 + 3x - 4$$

$$= 2x^2 + 8x - 18$$

$$2x^2 + 8x - 18 = 0$$

$$x^2 + 4x - 9 = 0$$

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$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-9)}}{2(1)} = \frac{-4 \pm \sqrt{16+36}}{2}$

$= \frac{-4 \pm \sqrt{52}}{2} = \frac{-4 \pm 2\sqrt{13}}{2} = -2 \pm \sqrt{13}$

$(f - g)(x) =$

$(x^2 + 5x - 14) - (x^2 + 3x - 4)$

$x^2 + 5x - 14 - x^2 - 3x + 4$

$2x - 10$

$x - 10 = 0 \Rightarrow x =$

5 **c.** $(f \cdot g)(x) =$

$(x^2 + 5x - 14)(x^2 + 3x - 4)$

$(x + 7)(x - 2)(x + 4)(x - 1)$

$f \cdot g)(x) = 0$

$= (x + 7)(x - 2)(x + 4)(x - 1)$

$x = -7$ or $x = 2$ or $x = -4$ or $x = 1$

a. $(f + g)(x) =$

$x^2 - 3x - 18 + x^2 + 2x - 3$

$2x^2 - x - 21$

$x^2 - x - 21 = 0$

$2x - 7)(x + 3) = 0 \Rightarrow x = \frac{7}{2}$ or $x = -3$

b. $(f - g)(x) =$

$(x^2 - 3x - 18) - (x^2 + 2x - 3)$

$x^2 - 3x - 18 - x^2 - 2x + 3$

$-5x - 15$

$-5x - 15 = 0 \Rightarrow x = -3$

$(f \cdot g)(x) =$

$(x^2 - 3x - 18)(x^2 + 2x - 3)$

$(x + 3)(x - 6)(x + 3)(x - 1)$

$f \cdot g)(x) = 0$

$= (x + 3)(x - 6)(x + 3)(x - 1)$

$x = -3$ or $x = 6$ or $x = 1$

$A(x) = 143 \; x(x + 2) =$

143

2

$x \quad + 2x - 143 = 0$

$x + 13)(x - 11) = 0$

~~= −13~~ or $x = 11$

Discard the negative solution since width cannot be negative. The width of the rectangular window is 11 feet and the length is 13 feet.

$A(x) = 306 \; x(x + 1) =$

306

$x^2 + x - 306 = 0$

$x + 18)(x - 17) = 0$

~~= −18~~ or $x = 17$

Discard the negative solution since width cannot be negative. The width of the rectangular window is 17 cm and the length is 18 cm.

$V(x) = 4$

$x - 2)^2 = 4$

$x - 2 = \pm\sqrt{4}$

$x - 2 = \pm 2$

$= 2 \pm 2$

$= 4$ or $x = 0$

Discard $x = 0$ since that is not a feasible length

for the original sheet. Therefore, the original sheet should measure 4 feet on each side.

$V(x) = 4$

$(x - 2)^2 = 16$

$-2 = \pm\sqrt{16}$

$-2 = \pm 4$

$= 2 \pm 4$

$x = 6$ or ~~$x = -2$~~

Discard $x = -2$ since width cannot be negative. Therefore, the original sheet should measure 6 feet on each side.

a. When the ball strikes the ground, the distance from the ground will be 0. Therefore, we solve

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$$s = 0$$
$$96 + 80t - 16t^2 = 0$$
$$-16t^2 + 80t + 96 = 0$$
$$t^2 - 5t - 6 = 0$$
$$(t - 6)(t + 1) = 0$$
$t = 6$ or ~~$t = -1$~~

Discard the negative solution since the time of flight must be positive. The ball will strike the ground after 6 seconds.

When the ball passes the top of the building, it will be 96 feet from the ground. Therefore, we solve

$$s = 96$$
$$96 + 80t - 16t^2 = 96$$
$$-16t^2 + 80t = 0$$
$$t^2 - 5t = 0$$
$$t(t - 5) = 0$$
$t = 0$ or $t = 5$

The ball is at the top of the building at time $t = 0$ seconds when it is thrown. It will pass the top of the building on the way down after 5 seconds.

a. To find when the object will be 15 meters above the ground, we solve

$$s = 15$$
$$-4.9t^2 + 20t = 15$$
$$-4.9t^2 + 20t - 15 = 0$$
$a = -4.9$, $b = 20$, $c = -15$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-15)}}{2(-4.9)}$$
$$= \frac{-20 \pm \sqrt{106}}{-9.8}$$
$$= \frac{20 \pm \sqrt{106}}{9.8}$$
$t \approx 0.99$ or $t \approx 3.09$

The object will be 15 meters above the ground after about 0.99 seconds (on the way up) and about 3.09 seconds (on the way down).

The object will strike the ground when the distance from the ground is 0. Thus, we solve

$$s = 0$$
$$-4.9t^2 + 20t = 0$$
$$t(-4.9t + 20) = 0$$
$t = 0$ or $-4.9t + 20 = 0$
$$-4.9t = -20$$
$$t \approx 4.08$$

The object will strike the ground after about 4.08 seconds.

c.
$$s = 100$$
$$-4.9t^2 + 20t = 100$$
$$-4.9t^2 + 20t - 100 = 0$$
$a = -4.9$, $b = 20$, $c = -100$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-100)}}{2(-4.9)}$$
$$= \frac{-20 \pm \sqrt{-1560}}{-9.8}$$

There is no real solution. The object never reaches a height of 100 meters.

For the sum to be 210, we solve

$$S(n) = 210$$
$$\frac{1}{2} n(n + 1) = 210$$
$$n(n + 1) = 420$$
$$n^2 + n - 420 = 0$$
$$(n - 20)(n + 21) = 0$$
$n - 20 = 0$ or $n + 21 = 0$

$n = 20$ ~~$n = -21$~~

Discard the negative solution since the number of consecutive integers must be positive. For a sum of 210, we must add the 20 consecutive integers, starting at 1.

To determine the number of sides when a polygon has 65 diagonals, we solve

$$D(n) = 65$$
$$\frac{1}{2} n(n - 3) = 65$$
$$n(n - 3) = 130$$
$$n^2 - 3n - 130 = 0$$
$$(n + 10)(n - 13) = 0$$
$n + 10 = 0$ or $n - 13 = 0$

~~$n = -10$~~ $n = 13$

Discard the negative solution since the number of sides must be positive. A polygon with 65 diagonals will have 13 sides.

To determine the number of sides if a polygon has 80 diagonals, we solve

$D(n) = 80$

$\underline{1}$

$n(n-3) = 160$

$n^2 - 3n - 160 = 0$

$a = 1, b = -3, c = -160$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-160)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{649}}{2}$$

Since the solutions are not integers, a polygon with 80 diagonals is not possible.

The roots of a quadratic equation are

$$x_1 = \frac{-b - \sqrt{}}{2a} \quad \text{and} \quad x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a},$$

so the sum of the roots is

$$x_1 + x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} + \frac{-b + \sqrt{b^2 - 4ac}}{2a}.$$

$$= \frac{-b - \sqrt{b^2 - 4ac} - b + \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2b}{2a} = -\frac{b}{a}$$

The roots of a quadratic equation are

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a},$$

so the product of the roots is

$$x_1 \cdot x_2 = \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{(-b)^2 - \left(\sqrt{b^2 - 4ac}\right)^2}{(2a)^2} = \frac{b^2 - (b^2 - 4ac)}{4a^2}$$

$$\frac{b^2 - b^2 + 4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

In order to have one repeated real zero, we need the discriminant to be 0.

$b^2 - 4ac = 0$

$1^2 - 4(k)(k) = 0$

$1 - 4k^2 = 0$

$4k^2 = 1$

$k^2 = \frac{1}{4}$

$k = \pm\sqrt{\frac{1}{4}}$

$k = \frac{1}{2}$ or $k = -\frac{1}{2}$

In order to have one repeated real zero, we need the discriminant to be 0.

$b^2 - 4ac = 0$

$(-k)^2 - 4(1)(4) = 0$

$k^2 - 16 = 0$

$(k - 4)(k + 4) = 0$

$k = 4$ or $k = -4$

For $f(x) = ax^2 + bx + c = 0$:

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

For $f(x) = ax^2 - bx + c = 0$:

$$x_1^* = \frac{-(-b) - \sqrt{(-b)^2 - 4ac}}{2a}$$

$$= \frac{b - \sqrt{b^2 - 4ac}}{2a} = -\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right) = -x_2$$

and

$$x_2^* = \frac{-(-b) + \sqrt{(-b)^2 - 4ac}}{2a}$$

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$$= \frac{b + \sqrt{b^2 - 4ac}}{2a} = -\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} \right) = -x_1$$

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For $f(x) = ax^2 + bx + c = 0$:

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

For $f(x) = cx^2 + bx + a = 0$:

$$x_1^* = \frac{-b - \sqrt{b^2 - 4(c)(a)}}{2c} = \frac{-b - \sqrt{b^2 - 4ac}}{2c}$$

$$= \frac{-b - \sqrt{b^2 - 4ac}}{2c} \cdot \frac{-b + \sqrt{b^2 - 4ac}}{-b + \sqrt{b^2 - 4ac}}$$

$$= \frac{b^2 - \left(b^2 - 4ac\right)}{2c\left(-b + \sqrt{b^2 - 4ac}\right)} = \frac{4ac}{2c\left(-b + \sqrt{b^2 - 4ac}\right)}$$

$$= \frac{2a}{-b + \sqrt{b^2 - 4ac}} = \frac{1}{x_2}$$

and

$$x_2^* = \frac{-b + \sqrt{b^2 - 4(c)(a)}}{2c} = \frac{-b + \sqrt{b^2 - 4ac}}{2c}$$

$$= \frac{-b + \sqrt{b^2 - 4ac}}{2c} \cdot \frac{-b - \sqrt{b^2 - 4ac}}{-b - \sqrt{b^2 - 4ac}}$$

$$= \frac{b^2 - \left(b^2 - 4ac\right)}{2c\left(-b - \sqrt{b^2 - 4ac}\right)} = \frac{4ac}{2c\left(-b - \sqrt{b^2 - 4ac}\right)}$$

$$= \frac{2a}{-b - \sqrt{b^2 - 4ac}} = \frac{1}{x_1}$$

a. $x^2 = 9$ and $x = 3$ are not equivalent because they do not have the same solution set. In the first equation we can also have $x = -3$.

$x = \sqrt{9}$ and $x = 3$ are equivalent because $\sqrt{9} = 3$.

$(x-1)(x-2) = (x-1)^2$ and $x - 2 = x - 1$ are not equivalent because they do not have the same solution set. The first equation has the solution set $\{1\}$ while the second equation has no solutions.

Answers may vary. Methods discussed in this section include factoring, the square root method, completing the square, and the quadratic formula.

Answers will vary. Knowing the discriminant allows us to know how many real solutions the equation will have.

Answers will vary. One possibility:

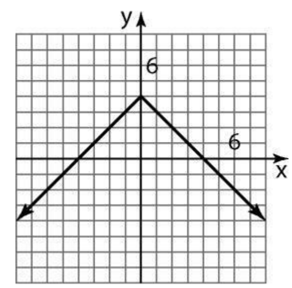
Two distinct: $f(x) = x^2 - 3x - 18$

One repeated: $f(x) = x^2 - 14x + 49$ No real: $f(x) = x^2 + x + 4$

Answers will vary.

Two quadratic functions can intersect 0, 1, or 2 times.

The graph is shifted vertically by 4 units and is reflected about the x-axis.



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122. Domain: $\{-3, -1, 1, 3\}$ Range: $\{2, 4\}$

123. $x = \frac{-10 + 2}{2} = \frac{-8}{2} = -4$

$y = \frac{4 + (-1)}{2} = \frac{3}{2}$

So the midpoint is: $\left(-4, \frac{3}{2}\right)$.

If the graph is symmetric with respect to the y-axis then x and –x are on the graph. Thus if $(-1,4)$ is on the graph, then so is $(1, 4)$.

Section 2.4

$y = x^2 - 9$

To find the y-intercept, let $x = 0$:

$y = 0^2 - 9 = -9$.

To find the x-intercept(s), let $y = 0$:

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm\sqrt{9}$

$= \pm 3$

The intercepts are $(0, -9)$, $(-3, 0)$, and $(3, 0)$.

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$2x^2 + 7x - 4 = 0$

$(2x - 1)(x + 4) = 0$

$2x - 1 = 0$ or $x + 4 = 0$

$2x = 1$ or $x = -4$

$x = \frac{1}{2}$ or $x = -4$

The solution set is $\left\{-4, \frac{1}{2}\right\}$.

3. $\left(\frac{1}{2}\cdot(-5)\right)^2 = \frac{25}{4}$

right; 4

parabola

axis (or axis of symmetry)

$-\frac{b}{2a}$

True; $a = 2 > 0$.

True; $-\frac{b}{2a} = -\frac{4}{2(-1)} = 2$

True

C

E

F

A

G

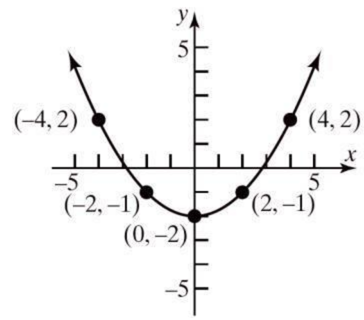
B

H

D

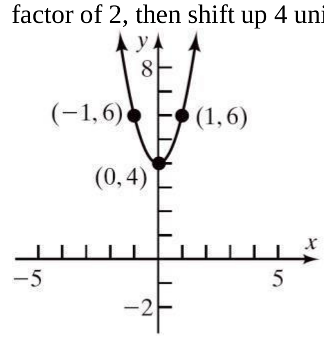
$f(x) = \frac{1}{4}x^2 - 2$

by a factor of $\frac{1}{4}$, then shift down 2 units.



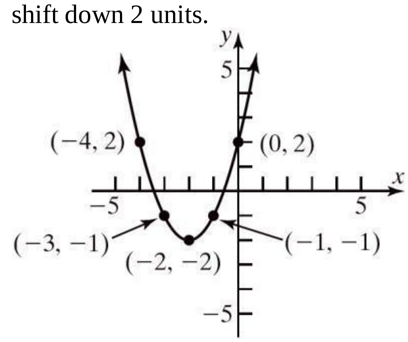
$f(x) = 2x^2 + 4$

Using the graph of $y = x^2$, stretch vertically by a factor of 2, then shift up 4 units.



$f(x) = (x + 2)^2 - 2$

Using the graph of $y = x^2$, shift left 2 units, then shift down 2 units.



$f(x) = (x - 3)^2 - 10$

Using the graph of $y = x^2$, shift right 3 units,

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Using the graph of $y = x^2$, compress vertically

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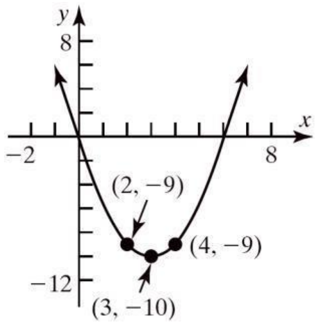
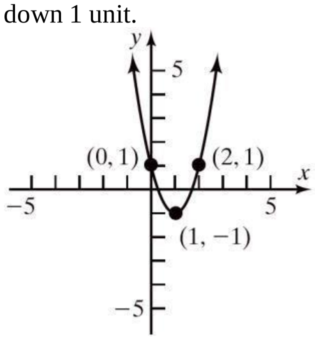
then shift down 10 units.



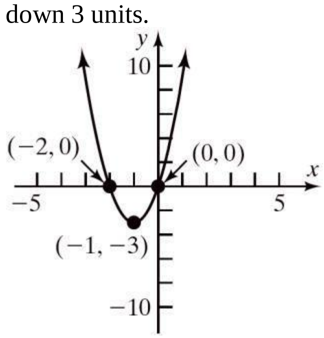
$f(x) = 2(x - 1)^2 - 1$

Using the graph of $y = x^2$, shift right 1 unit, stretch vertically by a factor of 2, then shift down 1 unit.



$f(x) = 3(x + 1)^2 - 3$

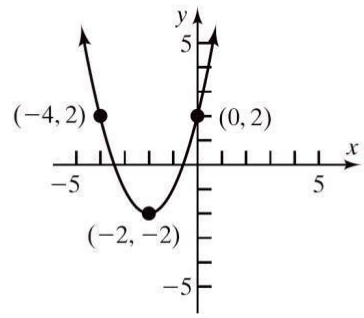
Using the graph of $y = x^2$, shift left 1 unit, stretch vertically by a factor of 3, then shift down 3 units.



$f(x) = x^2 + 4x + 2$

$(x^2 + 4x + 4) + 2 - 4$

$(x + 2)^2 - 2$

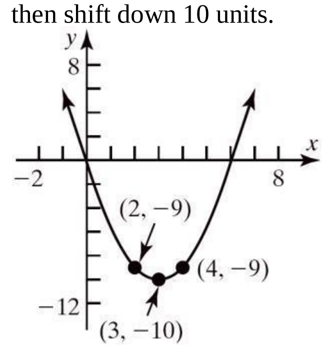


$f(x) = x^2 - 6x - 1$

$(x^2 - 6x + 9) - 1 - 9$

$(x - 3)^2 - 10$

Using the graph of $y = x^2$, shift right 3 units, then shift down 10 units.



$f(x) = -x^2 - 2x$

Using the graph of $y = x^2$, shift left 2 units, then shift down 2 units.

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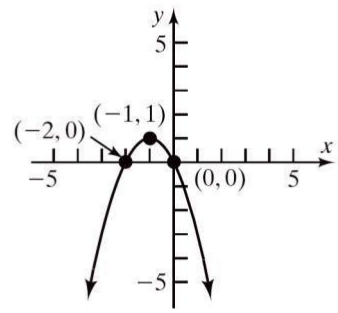
$-\left(x^2 + 2x\right)$

$-(x^2 + 2x + 1) + 1$

$-(x + 1)^2 + 1$

Using the graph of $y = x^2$, shift left 1 unit, reflect across the x-axis, then shift up 1 unit.



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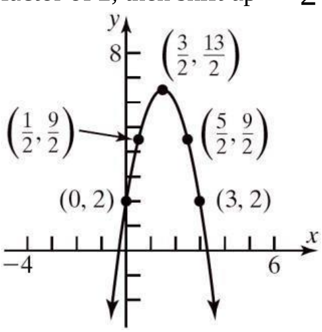
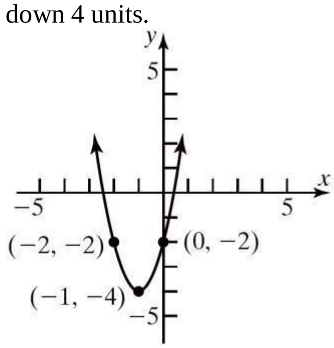
$$f(x) = -2x^2 + 6x + 2$$
$$-2\left(x^2 - 3x\right) + 2$$
$$-2\left(x^2 - 3x + \frac{9}{4}\right) + 2 + \frac{9}{2}$$
$$= -2\left(x - \frac{3}{2}\right)^2 + \frac{13}{2}$$

Using the graph of $y = x^2$, shift right $\frac{3}{2}$ units, reflect about the x-axis, stretch vertically by a factor of 2, then shift up $\frac{13}{2}$ units.



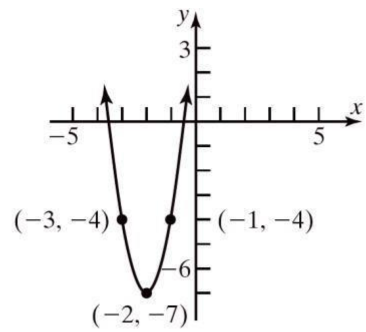
$$f(x) = 2x^2 + 4x - 2$$
$$2\left(x^2 + 2x\right) - 2$$
$$2\left(x^2 + 2x + 1\right) - 2 - 2$$
$$2(x + 1)^2 - 4$$

Using the graph of $y = x^2$, shift left 1 unit, stretch vertically by a factor of 2, then shift down 4 units.



$$f(x) = 3x^2 + 12x + 5$$

stretch vertically by a factor of 3, then shift down 7 units.



31. a. For $f(x) = x^2 + 2x$, $a = 1$, $b = 2$, $c = 0$. Since $a = 1 > 0$, the graph opens up. The x-coordinate of the vertex is $x = \frac{-b}{2a} = \frac{-(2)}{2(1)} = \frac{-2}{2} = -1$.

The y-coordinate of the vertex is $f\left(\frac{-b}{2a}\right) = f(-1) = (-1)^2 + 2(-1) = 1 - 2 = -1.$

$$3\left(x^2 + 4x\right) + 5$$
$$3\left(x^2 + 4x + 4\right) + 5 - 12$$
$$3(x + 2)^2 - 7$$

Using the graph of $y = x^2$, shift left 2 units,

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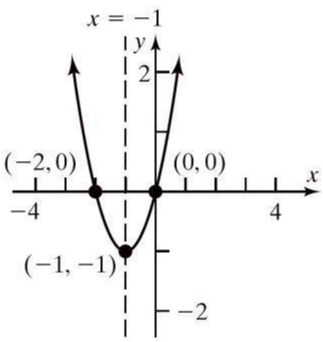
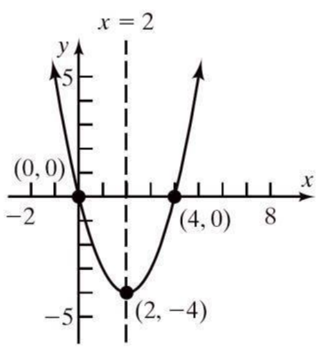
Thus, the vertex is $(-1, -1)$.
The axis of symmetry is the line $x = -1$. The discriminant is
$b^2 - 4ac = (2)^2 - 4(1)(0) = 4 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:
$x^2 + 2x = 0$
$x(x + 2) = 0$
$x = 0$ or $x = -2$
The x-intercepts are –2 and 0. The y-intercept is $f(0) = 0$.



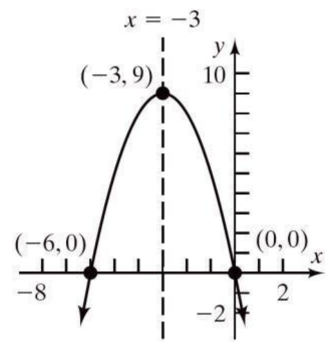
The domain is $(-\infty, \infty)$. The range is $[-1, \infty)$. Decreasing on $(-\infty, -1)$.
Increasing on $(-1, \infty)$.

32. a. For $f(x) = x^2 - 4x$, $a = 1$, $b = -4$, $c = 0$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f(2) = (2)^2 - 4(2) = 4 - 8 = -4.$$
Thus, the vertex is $(2, -4)$.
The axis of symmetry is the line $x = 2$.
The discriminant is:
$b^2 - 4ac = (-4)^2 - 4(1)(0) = 16 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:
$$x^2 - 4x = 0$$
$$x(x - 4) = 0$$
$$x = 0 \text{ or } x = 4.$$
The x-intercepts are 0 and 4.
The y-intercept is $f(0) = 0$.



The domain is $(-\infty, \infty)$.
The range is $[-4, \infty)$.
Decreasing on $(-\infty, 2)$.
Increasing on $(2, \infty)$.

33. a. For $f(x) = -x^2 - 6x$, $a = -1$, $b = -6$, $c = 0$. Since $a = -1 < 0$, the graph opens down. The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-(-6)}{2(-1)} = \frac{6}{-2} = -3.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f(-3) = -(-3)^2 - 6(-3)$$
$$= -9 + 18 = 9.$$
Thus, the vertex is $(-3, 9)$.
The discriminant is:
$b^2 - 4ac = (-6)^2 - 4(-1)(0) = 36 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:
$$x^2 - 6x = 0$$
$$x(x + 6) = 0$$
$$x = 0 \text{ or } x = -6.$$
The x-intercepts are -6 and 0.
The y-intercepts are $f(0) = 0$.



The domain is $(-\infty, \infty)$.
The range is $(-\infty, 9]$.
Increasing on $(-\infty, -3)$.
Decreasing on $(-3, \infty)$.

a. For $f(x) = -x^2 + 4x$, $a = -1$, $b = 4$, $c = 0$.
Since $a = -1 < 0$, the graph opens down.
The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f(2)$$
$$= -(2)^2 + 4(2)$$
$$= 4.$$
Thus, the vertex is $(2, 4)$.
The axis of symmetry is the line $x = 2$.
The discriminant is:
$b^2 - 4ac = 4^2 - 4(-1)(0) = 16 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:
$$x^2 + 4x = 0$$
$$x(x - 4) = 0$$
$$x = 0 \text{ or } x = 4.$$

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The axis of symmetry is the line $x = -3$.

The x-intercepts are 0 and 4.
The y-intercept is $f(0) = 0$.

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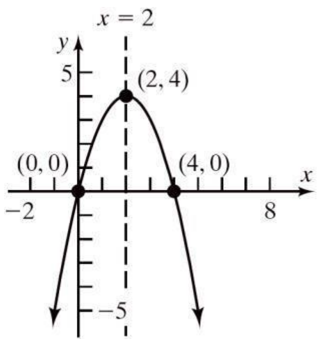


The domain is $(-\infty, \infty)$.

The range is $(-\infty, 4]$.

Increasing on $(-\infty, 2)$.

Decreasing on $(2, \infty)$.

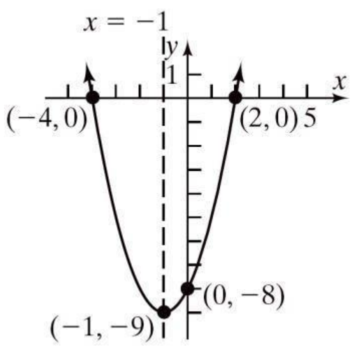
35. a. For $f(x) = x^2 + 2x - 8$, $a = 1$, $b = 2$, $c = -8$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f(-1) = (-1)^2 + 2(-1) - 8$$
$$= 1 - 2 - 8 = -9.$$

Thus, the vertex is $(-1, -9)$.
The axis of symmetry is the line $x = -1$.
The discriminant is:
$b^2 - 4ac = 2^2 - 4(1)(-8) = 4 + 32 = 36 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving: $x^2 + 2x - 8 = 0$
$(x + 4)(x - 2) = 0$
$x = -4$ or $x = 2$.
The x-intercepts are -4 and 2.
The y-intercept is $f(0) = -8$.



The domain is $(-\infty, \infty)$.

The range is $[-9, \infty)$.

Decreasing on $(-\infty, -1)$.

Increasing on $(-1, \infty)$.

36. a. For $f(x) = x^2 - 2x - 3$, $a = 1$, $b = -2$, $c = -3$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f(1) = 1^2 - 2(1) - 3 = -4.$$

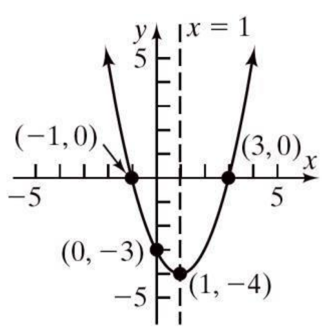
Thus, the vertex is $(1, -4)$.
The axis of symmetry is the line $x = 1$.
The discriminant is:

$b^2 - 4ac = (-2)^2 - 4(1)(-3) = 4 + 12 = 16 > 0$, so the graph has two x-intercepts.

The x-intercepts are found by solving:
$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$
$x = -1$ or $x = 3$.

The x-intercepts are -1 and 3.
The y-intercept is $f(0) = -3$.



The domain is $(-\infty, \infty)$.

The range is $[-4, \infty)$.

Decreasing on $(-\infty, 1)$.

Increasing on $(1, \infty)$.

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37. a. For $f(x) = x^2 + 2x + 1$, $a = 1$, $b = 2$, $c = 1$.

Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1.$$

The y-coordinate of the vertex is

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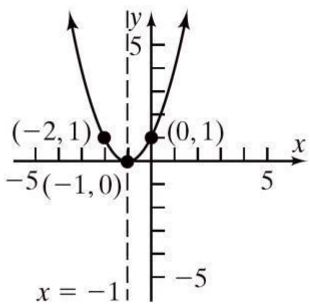
$$f\left(\frac{-b}{2a}\right) = f(-1) = (-1)^2 + 2(-1) + 1 = 1 - 2 + 1 = 0.$$

Thus, the vertex is $(-1, 0)$.
The axis of symmetry is the line $x = -1$.

The discriminant is:
$b^2 - 4ac = 2^2 - 4(1)(1) = 4 - 4 = 0$,
so the graph has one x-intercept.
The x-intercept is found by solving:

$$x^2 + 2x + 1 = 0$$
$$(x+1)^2 = 0$$
$$x = -1.$$

The x-intercept is -1.
The y-intercept is $f(0) = 1$.



The domain is $(-\infty, \infty)$. The range is $[0, \infty)$.

Decreasing on $(-\infty, -1)$.

Increasing on $(-1, \infty)$.

38. a. For $f(x) = x^2 + 6x + 9$, $a = 1$, $b = 6$, $c = 9$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is

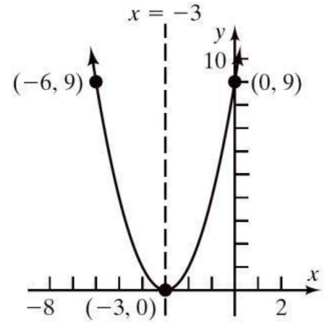
$$x = \frac{-b}{2a} = \frac{-6}{2(1)} = \frac{-6}{2} = -3.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f(-3) = (-3)^2 + 6(-3) + 9 = 9 - 18 + 9 = 0.$$

Thus, the vertex is $(-3, 0)$.

The x-intercept is -3.
The y-intercept is $f(0) = 9$.



The domain is $(-\infty, \infty)$. The range is $[0, \infty)$.

Decreasing on $(-\infty, -3)$.

Increasing on $(-3, \infty)$.

39. a. For $f(x) = 2x^2 - x + 2$, $a = 2$, $b = -1$, $c = 2$.
Since $a = 2 > 0$, the graph opens up.
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-(-1)}{2(2)} = \frac{1}{4}.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{4}\right) = 2\left(\frac{1}{4}\right)^2 - \frac{1}{4} + 2 = \frac{1}{8} - \frac{1}{4} + 2 = \frac{15}{8}.$$

Thus, the vertex is $\left(\frac{1}{4}, \frac{15}{8}\right)$.
The axis of symmetry is the line $x = \frac{1}{4}$.

$b^2 - 4ac = (-1)^2 - 4(2)(2) = 1 - 16 = -15$,

so the graph has no x-intercepts.

The y-intercept is $f(0) = 2$.

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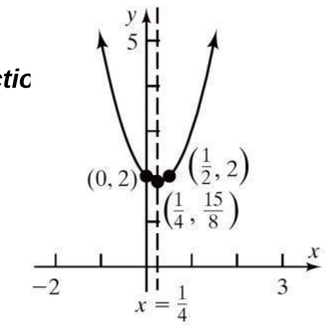
The axis of symmetry is the line $x = -3$.
The discriminant is:
$b^2 - 4ac = 6^2 - 4(1)(9) = 36 - 36 = 0$,
so the graph has one x-intercept.
The x-intercept is found by solving:
$x^2 + 6x + 9 = 0$
$x + 3)^2 = 0$
$= -3.$



The domain is $(-\infty, \infty)$.
The range is $\left[\frac{15}{8}, \right)$.

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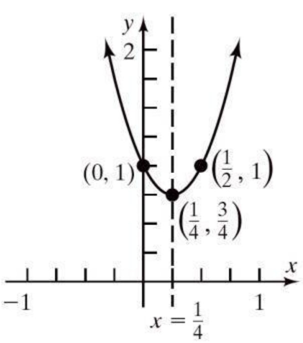
Decreasing on $\left(-\infty, \frac{1}{4}\right)$.

Increasing on $\left(\frac{1}{4}, \infty\right)$.

40. a. For $f(x) = 4x^2 - 2x + 1$, $a = 4$, $b = -2$, $c = 1$.
Since $a = 4 > 0$, the graph opens up.
The x-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-(-2)}{2(4)} = \frac{2}{8} = \frac{1}{4}$.
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right) + 1 = \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}.$$
Thus, the vertex is $\left(\frac{1}{4}, \frac{3}{4}\right)$.
The axis of symmetry is the line $x = \frac{1}{4}$.

$b^2 - 4ac = (-2)^2 - 4(4)(1) = 4 - 16 = -12$,
so the graph has no x-intercepts.
The y-intercept is $f(0) = 1$.



The domain is $(-\infty, \infty)$.

The range is $\left[\frac{3}{4}, \infty\right)$.

Decreasing on $\left(-\infty, \frac{1}{4}\right)$.

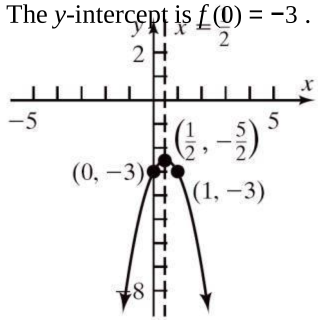
$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{2}\right) = -2\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 3$$

1 5

Thus, the vertex is $\left(\frac{1}{2}, -\frac{5}{2}\right)$.
The axis of symmetry is the line $x = \frac{1}{2}$.

$b^2 - 4ac = 2^2 - 4(-2)(-3) = 4 - 24 = -20$,
so the graph has no x-intercepts.

The y-intercept is $f(0) = -3$.



The domain is $(-\infty, \infty)$. The range is $\left(-\infty, -\frac{5}{2}\right]$.

Increasing on $\left(-\infty, \frac{1}{2}\right)$.

Decreasing on $\left(\frac{1}{2}, \infty\right)$.

42. a. For $f(x) = -3x^2 + 3x - 2$, $a = -3$, $b = 3$, $c = -2$. Since $a = -3 < 0$, the graph opens down.
The x-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-3}{2(-3)} = \frac{-3}{-6} = \frac{1}{2}$.

The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{2}\right) = -3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 2$$

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Increasing on $\left(\frac{1}{4}, \infty\right)$.

41. a. For $f(x) = -2x^2 + 2x - 3$, $a = -2$, $b = 2$, $c = -3$. Since $a = -2 < 0$, the graph opens down.

The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-(2)}{2(-2)} = \frac{-2}{-4} = \frac{1}{2}.$$

The y-coordinate of the vertex is

$$\left(2a\right)\left(2\right)\quad\left(2\right)\quad\left(2\right)$$

$$= -\frac{3}{4} + \frac{3}{2} - 2 = -\frac{5}{4}.$$

Thus, the vertex is $\left(\frac{1}{2}, -\frac{5}{4}\right)$.

The axis of symmetry is the line $x = \frac{1}{2}$.

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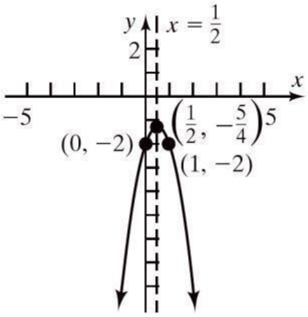
$b^2 - 4ac = 3^2 - 4(-3)(-2) = 9 - 24 = -15$, so the graph has no x-intercepts.

The y-intercept is $f(0) = -2$.



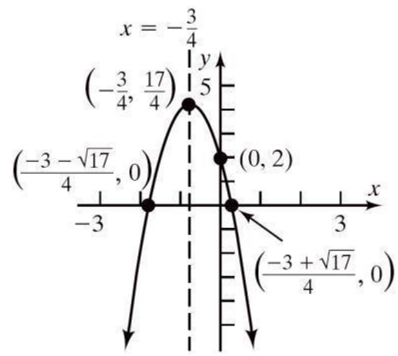
The domain is $(-\infty, \infty)$. The range is $\left(-\infty, -\frac{5}{4}\right]$.

Increasing on $\left(-\infty, \frac{1}{2}\right)$.

Decreasing on $\left(\frac{1}{2}, \infty\right)$.

The x-intercepts are $\frac{-3+\sqrt{17}}{4}$ and $\frac{-3-\sqrt{17}}{4}$.

The y-intercept is $f(0) = 2$.



The domain is $(-\infty, \infty)$. The range is $\left(-\infty, \frac{17}{4}\right]$.

Decreasing on $\left(-\frac{3}{4}, \infty\right)$.

Increasing on $\left(-\infty, -\frac{3}{4}\right)$.

43. a. For $f(x) = -4x^2 - 6x + 2$, $a = -4$, $b = -6$, $c = 2$. Since $a = -4 < 0$, the graph opens down.

The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(-4)} = \frac{6}{-8} = -\frac{3}{4}.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f\left(-\frac{3}{4}\right) = -4\left(-\frac{3}{4}\right)^2 - 6\left(-\frac{3}{4}\right) + 2$$

$$= -\frac{9}{4} + \frac{9}{2} + 2 = \frac{17}{4}.$$

Thus, the vertex is $\left(-\frac{3}{4}, \frac{17}{4}\right)$.

The axis of symmetry is the line $x = -\frac{3}{4}$.

$b^2 - 4ac = (-6)^2 - 4(-4)(2) = 36 + 32 = 68$, so the graph has two x-intercepts.

The x-intercepts are found by solving:

$-4x^2 - 6x + 2 = 0$

44. a. For $f(x) = 3x^2 - 8x + 2$, $a = 3$, $b = -8$, $c = 2$. Since $a = 3 > 0$, the graph opens up.

The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-(-8)}{2(3)} = \frac{8}{6} = \frac{4}{3}.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f\left(\frac{4}{3}\right) = 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 2$$

$$\frac{16}{3} - \frac{32}{3} + 2 = -\frac{10}{3}.$$

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{68}}{2(-4)}$$

$$= \frac{6 \pm \sqrt{68}}{-8} = \frac{6 \pm 2\sqrt{17}}{-8} = \frac{3 \pm \sqrt{17}}{-4}$$

Thus, the vertex is $\left(\frac{4}{3}, -\frac{10}{3} \right)$.

The axis of symmetry is the line $x = \frac{4}{3}$.

The discriminant is:
$b^2 - 4ac = (-8)^2 - 4(3)(2) = 64 - 24 = 40$, so the graph has two x-intercepts. The x-intercepts are found by solving: $3x^2 - 8x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{40}}{2(3)}$$

$$= \frac{8 \pm \sqrt{40}}{6} = \frac{8 \pm 2\sqrt{10}}{6} = \frac{4 \pm \sqrt{10}}{3}$$

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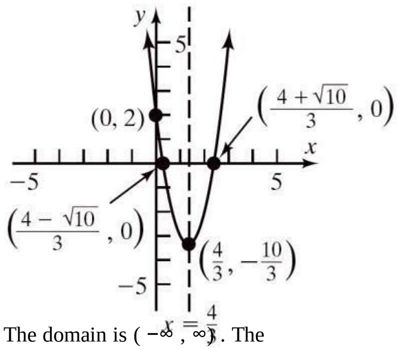
The x-intercepts are $\frac{4+\sqrt{10}}{3}$ and $\frac{4-\sqrt{10}}{3}$.

The y-intercept is $f(0) = 2$.



The domain is $(-\infty, \infty)$. The range is $\left[-\frac{10}{3}, \infty\right)$.

Decreasing on $\left(-\infty, \frac{4}{3}\right)$.

Increasing on $\left(\frac{4}{3}, \infty\right)$.

45. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(-1, -2)$ so we have $h = -1$ and $k = -2$. The graph also passes through the point $(x, y) = (0, -1)$. Substituting these values for x, y, h, and k, we can solve for a:

$$-1 = a(0-(-1))^2 + (-2)$$
$$-1 = a(1)^2 - 2$$
$$-1 = a - 2$$
$$1 = a$$

The quadratic function is $f(x) = (x+1)^2 - 2 = x^2 + 2x - 1$.

Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(2,1)$ so we have $h = 2$ and $k = 1$. The graph also passes through the point $(x, y) = (0, 5)$. Substituting

$$= a(0-2)^2 + 1$$
$$= a(-2)^2 + 1$$
$$= 4a + 1$$
$$= 4a$$
$$1 = a$$

The quadratic function is $f(x) = (x-2)^2 + 1 = x^2 - 4x + 5$.

Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(-3, 5)$ so we have $h = -3$ and $k = 5$. The graph also passes through the point $(x, y) = (0, -4)$. Substituting these values for x, y, h, and k, we can solve for a:

$$-4 = a(0-(-3))^2 + 5$$
$$-4 = a(3)^2 + 5$$
$$-4 = 9a + 5$$
$$-9 = 9a$$
$$-1 = a$$

The quadratic function is $f(x) = -(x+3)^2 + 5 = -x^2 - 6x - 4$.

Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(2, 3)$ so we have $h = 2$ and $k = 3$. The graph also passes through the point $(x, y) = (0, -1)$. Substituting these values for x, y, h, and k, we can solve for a:

$$-1 = a(0-2)^2 + 3$$
$$-1 = a(-2)^2 + 3$$
$$-1 = 4a + 3$$
$$-4 = 4a$$
$$-1 = a$$

The quadratic function is $f(x) = -(x-2)^2 + 3 = -x^2 + 4x - 1$.

Consider the form $y = a(x-h)^2 + k$. From the

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these values for *x*, *y*, *h*, and *k*, we can solve for *a*:

graph we know that the vertex is $(1, -3)$ so we have $h = 1$ and $k = -3$. The graph also passes through the point $(x, y) = (3, 5)$. Substituting these values for *x*, *y*, *h*, and *k*, we can solve for *a*:

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$= a(3-1)^2 + (-3)$
$= a(2)^2 - 3$
$= 4a - 3$
$= 4a$

$2 = a$
The quadratic function is
$(x) = 2(x-1)^2 - 3 = 2x^2 - 4x - 1.$

Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(-2, 6)$ so we have $h = -2$ and $k = 6$. The graph also passes through the point $(x, y) = (-4, -2)$. Substituting these values for $x, y, h,$ and k, we can solve for a:

$-2 = a(-4-(-2))^2 + 6$
$-2 = a(-2)^2 + 6$
$-2 = 4a + 6$
$-8 = 4a$

$-2 = a$
The quadratic function is

$(x) = -2(x+2)^2 + 6 = -2x^2 - 8x - 2$.

For $f(x) = 2x^2 + 12x$, $a = 2$, $b = 12$, $c = 0$. Since $a = 2 > 0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at $x = \frac{-b}{2a} = \frac{-12}{2(2)} = \frac{-12}{4} = -3.$

The minimum value is
$(-3) = 2(-3)^2 + 12(-3) = 18 - 36 = -18$.

52. For $f(x) = -2x^2 + 12x$, $a = -2$, $b = 12$, $c = 0$, . Since $a = -2 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \frac{-b}{2a} = \frac{-12}{2(-2)} = \frac{-12}{-4} = 3.$

54. For $f(x) = 4x^2 - 8x + 3$, $a = 4$, $b = -8$, $c = 3$. Since $a = 4 > 0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at $x = \frac{-b}{2a} = \frac{-(-8)}{2(4)} = \frac{8}{8} = 1$. The minimum value is
$(1) = 4(1)^2 - 8(1) + 3 = 4 - 8 + 3 = -1.$

For $f(x) = -x^2 + 10x - 4$, $a = -1$, $b = 10$, $c = -4$. Since $a = -1 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \frac{-b}{2a} = \frac{-10}{2(-1)} = \frac{-10}{-2} = 5$. The maximum value is
$(5) = -(5)^2 + 10(5) - 4 = -25 + 50 - 4 = 21.$

56. For $f(x) = -2x^2 + 8x + 3$, $a = -2$, $b = 8$, $c = 3$. Since $a = -2 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \frac{-b}{2a} = \frac{-8}{2(-2)} = \frac{-8}{-4} = 2$. The maximum value is
$(2) = -2(2)^2 + 8(2) + 3 = -8 + 16 + 3 = 11.$

57. For $f(x) = -3x^2 + 12x + 1$, $a = -3$, $b = 12$, $c = 1$. Since $a = -3 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \frac{-b}{2a} = \frac{-12}{2(-3)} = \frac{-12}{-6} = 2$. The maximum value is

$(2) = -3(2)^2 + 12(2) + 1 = -12 + 24 + 1 = 13.$

For $f(x) = 4x^2 - 4x$, $a = 4$, $b = -4$, $c = 0$. Since $a = 4 > 0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at $= \frac{-b}{2a} = \frac{-(-4)}{2(4)} = \frac{4}{8} = \frac{1}{2}$. The minimum value is

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$2a \quad 2(-2) \quad -4$

The maximum value is

$(3) = -2(3)^2 + 12(3) = -18 + 36 = 18$.

53. For $f(x) = 2x^2 + 12x - 3$, $a = 2$, $b = 12$, $c = -3$. Since $a = 2 > 0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at

$x = \frac{-b}{2a} = \frac{-12}{2(2)} = \frac{-12}{4} = -3$. The minimum value is

$f(-3) = 2(-3)^2 + 12(-3) - 3 = 18 - 36 - 3 = -21$.

$\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) = 1 - 2 = -1$.

59. a. For $f(x) = x^2 - 2x - 15$, $a = 1$, $b = -2$, $c = -15$. Since $a = 1 > 0$, the graph opens up. The x-coordinate of the vertex is $x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$.

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The y-coordinate of the vertex is

$f\left(\frac{-b}{2a}\right) = f(1) = (1)^2 - 2(1) - 15$

$1 - 2 - 15 = -16.$

Thus, the vertex is $(1, -16)$

. The discriminant is:

$^2 - 4ac = (-2)^2 - 4(1)(-15) = 4 + 60 = 64$

> 0 , so the graph has two x-intercepts.

The x-intercepts are found by solving:

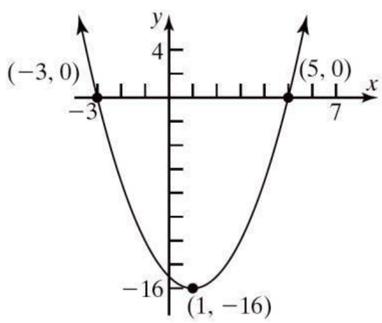
$x^2 - 2x - 15 = 0$

$x + 3)(x - 5) = 0$

$= -3$ or $x = 5$

The x-intercepts are -3 and 5 .

The y-intercept is $f(0) = -15$.



The domain is $(-\infty, \infty)$.

The range is $[-16, \infty)$.

Decreasing on $(-\infty, 1)$.

Increasing on $(1, \infty)$.

60. a. For $f(x) = x^2 - 2x - 8$, $a = 1$, $b = -2$, $c = -8$. Since $a = 1 > 0$, the graph opens up.

The x-coordinate of the vertex is

$x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$.

The y-coordinate of the vertex is

$f\left(\frac{-b}{2a}\right) = f(1) = (1)^2 - 2(1) - 8 = 1 - 2 - 8 = -9.$

Thus, the vertex is $(1, -9)$.

The discriminant is:

$b^2 - 4ac = (-2)^2 - 4(1)(-8) = 4 + 32 = 36 >$

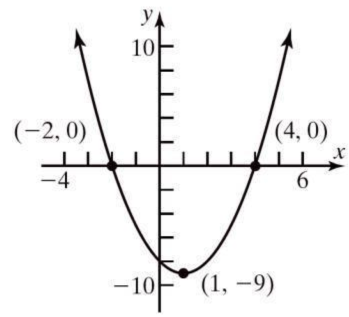
0 , so the graph has two x-intercepts.

The x-intercepts are found by solving:

$x^2 - 2x - 8 = 0$

The x-intercepts are -2 and 4 .

The y-intercept is $f(0) = -8$.



The domain is $(-\infty, \infty)$. The range is $[-9, \infty)$.

Decreasing on $(-\infty, 1)$. Increasing on $(1, \infty)$.

61. a. $F(x) = 2x - 5$ is a linear function.

The x-intercept is found by solving:

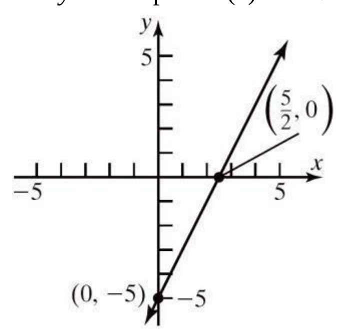
$x - 5 = 0$

$2x = 5$

$x = \frac{5}{2}$

The x-intercept is $\frac{5}{2}$.

The y-intercept is $F(0) = -5$.



The domain is $(-\infty, \infty)$.

The range is $(-\infty, \infty)$.

Increasing on $(-\infty, \infty)$.

$x + 2)(x - 4) = 0$

$= -2$ or $x = 4$

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a. $f(x) = \frac{3}{2}x - 2$ is a linear function.

The x-intercept is found by solving:

$$\frac{3}{2}x - 2 = 0$$

$$\frac{3}{2}x = 2$$

$$x = 2 \cdot \frac{2}{3} = \frac{4}{3}$$

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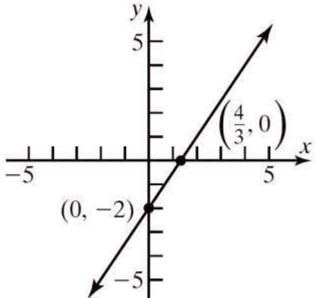
The x-intercept is $\frac{4}{3}$.

The y-intercept is $f(0) = -2$.



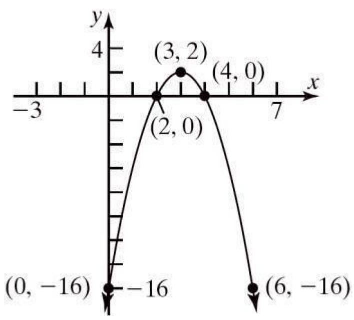
The domain is $(-\infty, \infty)$.

The range is $(-\infty, \infty)$.

Increasing on $(-\infty, \infty)$.

a. $g(x) = -2(x-3)^2 + 2$

Using the graph of $y = x^2$, shift right 3 units, reflect about the x-axis, stretch vertically by a factor of 2, then shift up 2 units.



The domain is $(-\infty, \infty)$.

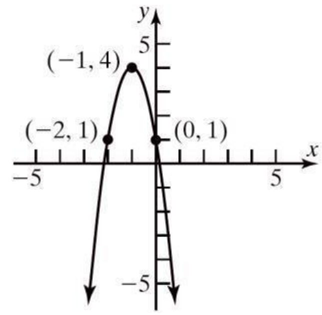
The range is $(-\infty, 2]$.

Increasing on $(-\infty, 3)$.

Decreasing on $(3, \infty)$.

a. $h(x) = -3(x+1)^2 + 4$

Using the graph of $y = x^2$, shift left 1 unit, reflect about the x-axis, stretch vertically by a factor of 3, then shift up 4 units.



The domain is $(-\infty, \infty)$.

The range is $(-\infty, 4]$.

Increasing on $(-\infty, -1)$.

Decreasing on $(-1, \infty)$.

65. a. For $f(x) = 2x^2 + x + 1$, $a = 2$, $b = 1$, $c = 1$.
Since $a = 2 > 0$, the graph opens up.
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-1}{2(2)} = \frac{-1}{4} = -\frac{1}{4}.$$

The y-coordinate of the vertex is

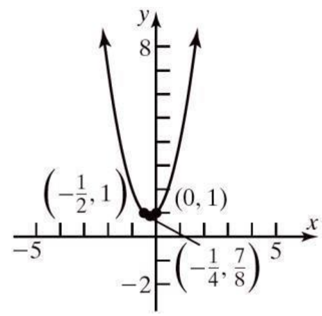
$$f\left(\frac{-b}{2a}\right) = f\left(-\frac{1}{4}\right) = 2\left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right) + 1$$

$$= \frac{1}{8} - \frac{1}{4} + 1 = \frac{7}{8}.$$

Thus, the vertex is $\left(-\frac{1}{4}, \frac{7}{8}\right)$.

The discriminant is:

$b^2 - 4ac = 1^2 - 4(2)(1) = 1 - 8 = -7$,
so the graph has no x-intercepts.
The y-intercept is $f(0) = 1$.



The domain is $(-\infty, \infty)$.

The range is $\left[\frac{7}{8}, \infty\right)$.

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Decreasing on $\left(-\infty, -\frac{1}{4}\right)$.

Increasing on $\left(-\frac{1}{4}, \infty\right)$.

66. a. For $G(x) = 3x^2 + 2x + 5$, $a = 3$, $b = 2$, $c = 5$.

Since $a = 3 > 0$, the graph opens up.

The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-2}{2(3)} = \frac{-2}{6} = -\frac{1}{3}.$$

The y-coordinate of the vertex is

$$G\left(\frac{-b}{2a}\right) = G\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right)^2 + 2\left(-\frac{1}{3}\right) + 5$$

$$\frac{1}{3} - \frac{2}{3} + 5 = \frac{14}{3}.$$

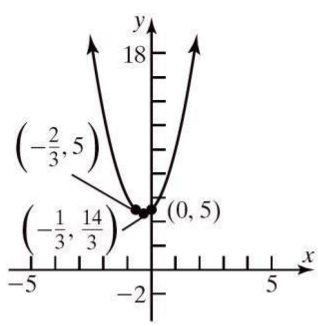
Thus, the vertex is $\left(-\frac{1}{3}, \frac{14}{3}\right)$.

The discriminant is:

$b^2 - 4ac = 2^2 - 4(3)(5) = 4 - 60 = -56$,

so the graph has no x-intercepts.

The y-intercept is $G(0) = 5$.



The domain is $(-\infty, \infty)$.

The range is $\left[\frac{14}{3}, \infty\right)$.

Decreasing on $\left(-\infty, -\frac{1}{3}\right)$.

Increasing on $\left(-\frac{1}{3}, \infty\right)$.

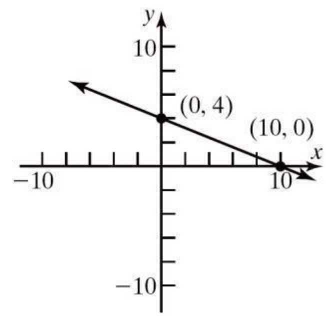
$$-\frac{2}{5}x + 4 = 0$$

$$-\frac{2}{5}x = -4$$

$$x = -4\left(-\frac{5}{2}\right) = 10$$

The x-intercept is 10.

The y-intercept is $h(0) = 4$.



The domain is $(-\infty, \infty)$.

The range is $(-\infty, \infty)$.

Decreasing on $(-\infty, \infty)$.

a. $f(x) = -3x + 2$ is a linear function.

The x-intercept is found by solving:

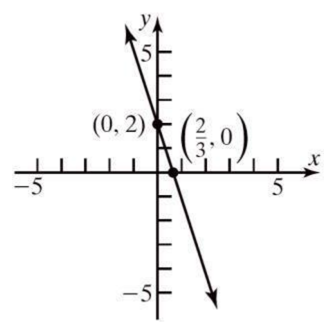
$$-3x + 2 = 0$$

$$-3x = -2$$

$$x = \frac{-2}{-3} = \frac{2}{3}$$

The x-intercept is $\frac{2}{3}$.

The y-intercept is $f(0) = 2$.



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67. a. $h(x) = -\frac{2}{5}x + 4$ is a linear function.

The x-intercept is found by solving:

The domain is $(-\infty, \infty)$.

The range is $(-\infty, \infty)$.

Decreasing on $(-\infty, \infty)$.

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a. For $H(x) = -4x^2 - 4x - 1$, $a = -4$, $b = -4$, $c = -1$. Since $a = -4 < 0$, the graph opens down. The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-(-4)}{2(-4)} = \frac{4}{-8} = -\frac{1}{2}.$$

The y-coordinate of the vertex is

$$H\left(\frac{-b}{2a}\right) = H\left(-\frac{1}{2}\right) = -4\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) - 1$$

$$= -1 + 2 - 1 = 0$$

Thus, the vertex is $\left(-\frac{1}{2}, 0\right)$.

The discriminant is:

$b^2 - 4ac = (-4)^2 - 4(-4)(-1) = 16 - 16 = 0$,

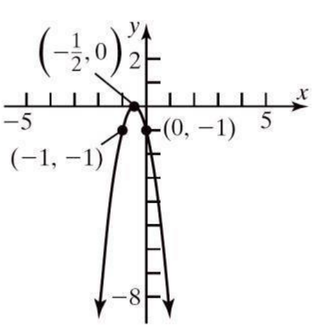
so the graph has one x-intercept.

The x-intercept is found by solving:

$$-4x^2 - 4x - 1 = 0$$
$$4x^2 + 4x + 1 = 0$$
$$(2x + 1)^2 = 0$$
$$x + 1 = 0$$
$$x = -\frac{1}{2}$$

The x-intercept is $-\frac{1}{2}$.

The y-intercept is $H(0) = -1$.



The domain is $(-\infty, \infty)$.

The range is $(-\infty, 0]$.

The y-coordinate of the vertex is

$$F\left(\frac{-b}{2a}\right) = F\left(\frac{5}{2}\right) = -4\left(\frac{5}{2}\right)^2 + 20\left(\frac{5}{2}\right) - 25$$

$$-25 + 50 - 25 = 0$$

Thus, the vertex is $\left(\frac{5}{2}, 0\right)$.

The discriminant is:

$$b^2 - 4ac = (20)^2 - 4(-4)(-25)$$

$$400 - 400 = 0,$$

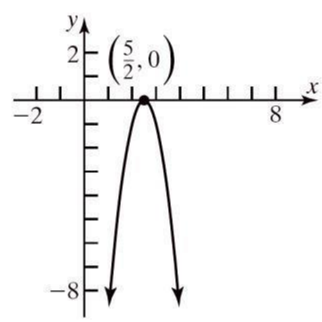
so the graph has one x-intercept.

The x-intercept is found by solving: $-4x^2 + 20x - 25 = 0$

$$4x^2 - 20x + 25 = 0$$
$$(2x - 5)^2 = 0$$
$$x - 5 = 0$$
$$x = \frac{5}{2}$$

The x-intercept is $\frac{5}{2}$.

The y-intercept is $F(0) = -25$.



The domain is $(-\infty, \infty)$.

The range is $(-\infty, 0]$.

Increasing on $\left(-\infty, \frac{5}{2}\right)$.

Decreasing on $\left(\frac{5}{2}, \infty\right)$.

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Increasing on $\left(-\infty, -\frac{1}{2}\right)$.

Decreasing on $\left(-\frac{1}{2}, \infty\right)$.

70. a. For $F(x) = -4x^2 + 20x - 25$, $a = -4$, $b = 20$, $c = -25$. Since $a = -4 < 0$, the graph opens down. The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-20}{2(-4)} = \frac{-20}{-8} = \frac{5}{2}.$$

71. Use the form $f(x) = a(x-h)^2 + k$.

The vertex is $(0, 2)$, so $h = 0$ and $k = 2$.

$$f(x) = a(x-0)^2 + 2 = ax^2 + 2.$$

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Since the graph passes through (1, 8) , $f(1) = 8$.

$(x) = ax^2 + 2$

$= a(1)^2 + 2$

$= a + 2$

$6 = a$

$(x) = 6x^2 + 2$.

$a = 6$, $b = 0$, $c = 2$

Use the form $f(x) = a(x - h)^2 + k$.

The vertex is (1, 4) , so $h = 1$ and $k = 4$.

$(x) = a(x - 1)^2 + 4$.

Since the graph passes through (−1, − 8) ,

$(-1) = -8$.

$-8 = a(-1 - 1)^2 + 4$

$-8 = a(-2)^2 + 4$

$-8 = 4a + 4$

$-12 = 4a$

$-3 = a$

$(x) = -3(x - 1)^2 + 4$

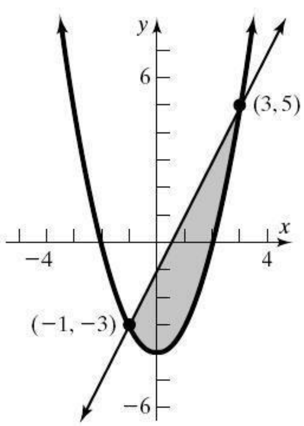
$-3(x^2 - 2x + 1) + 4$

$-3x^2 + 6x - 3 + 4$

$-3x^2 + 6x + 1$

$a = -3$, $b = 6$, $c = 1$

73. a and **d.**



$f(x) = g(x)$

$2x - 1 = x^2 - 4$

$0 = x^2 - 2x - 3$

$0 = (x + 1)(x - 3)$

$x + 1 = 0$ or $x - 3 = 0$

$x = -1$ $\quad$ $x = 3$

The solution set is {−1, 3}.

$f(-1) = 2(-1) - 1 = -2 - 1 =$

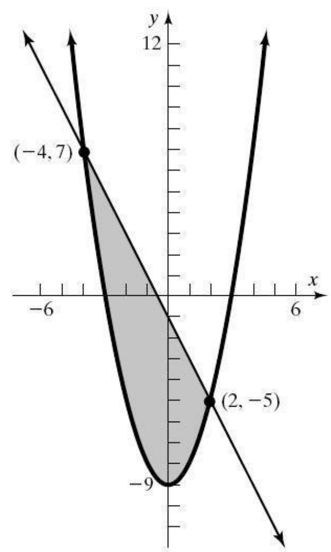
-3 $g(-1) = (-1)^2 - 4 = 1 - 4 =$

-3 $f(3) = 2(3) - 1 = 6 - 1 = 5$

$g(3) = (3)^2 - 4 = 9 - 4 = 5$

Thus, the graphs of f and g intersect at the points (−1, −3) and (3, 5) .

74. a and **d.**



$f(x) = g(x)$

$-2x - 1 = x^2 - 9$

$0 = x^2 + 2x - 8$

$0 = (x + 4)(x - 2)$

$x + 4 = 0$ or $x - 2 = 0$

$x = -4$ $\quad$ $x = 2$

The solution set is {−4, 2}.

$f(-4) = -2(-4) - 1 = 8 - 1 = 7$

$g(-4) = (-4)^2 - 9 = 16 - 9 = 7$ f

$(2) = -2(2) - 1 = -4 - 1 = -5$

$g(2) = (2)^2 - 9 = 4 - 9 = -5$

Thus, the graphs of f and g intersect at the points $(-4, 7)$ and $(2, -5)$.

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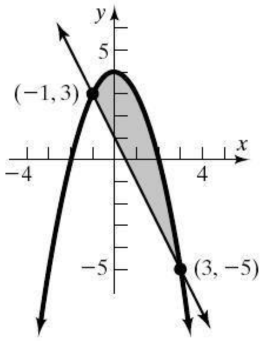
75. a and **d.**



$f(x) = g(x)$

$x^2 + 4 = -2x + 1$

$= x^2 - 2x - 3$

$= (x + 1)(x - 3)$

$x + 1 = 0$ or $x - 3 = 0$

$x = -1$ $\quad$ $x = 3$

The solution set is $\{-1, 3\}$.

$f(1) = -(-1)^2 + 4 = -1 + 4 = 3$

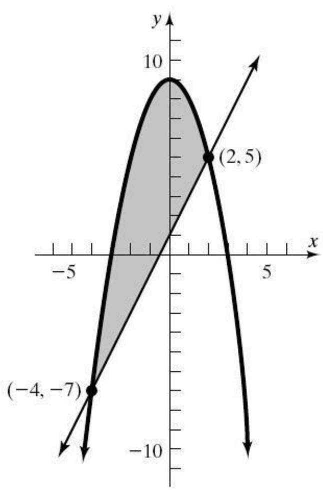
$g(1) = -2(-1) + 1 = 2 + 1 = 3$

$f(3) = -(3)^2 + 4 = -9 + 4 = -5$

$(3) = -2(3) + 1 = -6 + 1 = -5$

Thus, the graphs of f and g intersect at the points $(-1, 3)$ and $(3, -5)$.

76. a and **d.**



$f(x) = g(x)$

$x^2 + 9 = 2x + 1$

$= x^2 + 2x - 8$

$= (x + 4)(x - 2)$

$x + 4 = 0$ or $x - 2 = 0$

$x = -4$ $\quad$ $x = 2$

The solution set is $\{-4, 2\}$.

$f(-4) = -(-4)^2 + 9 = -16 + 9 = -7$

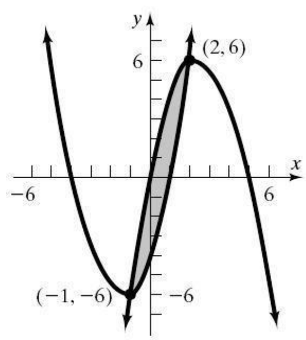
$g(-4) = 2(-4) + 1 = -8 + 1 = -7$

$f(2) = -(2)^2 + 9 = -4 + 9 = 5$

$g(2) = 2(2) + 1 = 4 + 1 = 5$

Thus, the graphs of f and g intersect at the points $(-4, -7)$ and $(2, 5)$.

77. a and **d.**



$f(x) = g(x)$

$x^2 + 5x = x^2 + 3x - 4\,0$

$= 2x^2 - 2x - 4$

$0 = x^2 - x - 2$

$= (x + 1)(x - 2)$

$x + 1 = 0$ or $x - 2 = 0$

$x = -1$ $\quad$ $x = 2$

The solution set is $\{-1, 2\}$.

$f(-1) = -(-1)^2 + 5(-1) = -1 - 5 = -6$

$g(-1) = (-1)^2 + 3(-1) - 4 = 1 - 3 - 4 =$

$-6\ f(2) = -(2)^2 + 5(2) = -4 + 10 = 6$

$g(2) = 2_2 + 3(2) - 4 = 4 + 6 - 4 = 6$

Thus, the graphs of f and g intersect at the points $(-1, -6)$ and $(2, 6)$.

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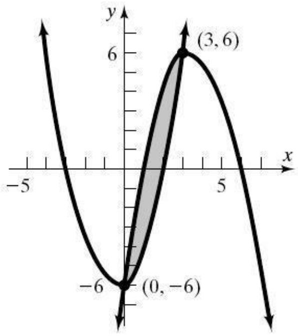
78. **a** and **d.**



$$f(x) = g(x)$$

$$x^2 + 7x - 6 = x^2 + x - 6$$
$$0 = 2x^2 - 6x$$
$$= 2x(x - 3)$$
$$2x = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = 0 \qquad x = 3$$

The solution set is {0, 3}.

$$f(0) = -(0)^2 + 7(0) - 6 = -6$$
$$g(0) = 0^2 + 0 - 6 = -6$$
$$(3) = -(3)^2 + 7(3) - 6 = -9 + 21 - 6 = 6$$
$$g(3) = 3^2 + 3 - 6 = 9 + 3 - 6 = 6$$

Thus, the graphs of f and g intersect at the points $(0, -6)$ and $(3, 6)$.

79. **a.** For $a = 1$:

$(x) = a(x - r_1)(x - r_2)$
$1(x - (-3))(x - 1)$
$(x + 3)(x - 1) = x^2 + 2x$
$- 3$ For $a = 2$:
$f(x) = 2(x - (-3))(x - 1)$
$2(x + 3)(x - 1)$
$2(x^2 + 2x - 3) = 2x^2 + 4x$
$- 6$ For $a = -2$:
$f(x) = -2(x - (-3))(x - 1)$
$-2(x + 3)(x - 1)$
$-2(x^2 + 2x - 3) = -2x^2 - 4x$
$+ 6$ For $a = 5$:
$f(x) = 5(x - (-3))(x - 1)$

The x-intercepts are not affected by the value of a. The y-intercept is multiplied by the value of a.

The axis of symmetry is unaffected by the value of a. For this problem, the axis of symmetry is $x = -1$ for all values of a.

The x-coordinate of the vertex is not affected by the value of a. The y-coordinate of the vertex is multiplied by the value of a.

The x-coordinate of the vertex is the mean of the x-intercepts.

a. For $a = 1$:

$(x) = 1(x - (-5))(x - 3)$
$(x + 5)(x - 3) = x^2 + 2x$
-15 For $a = 2$:
$f(x) = 2(x - (-5))(x - 3)$
$2(x + 5)(x - 3)$
$2(x^2 + 2x - 15) = 2x^2 + 4x -$
30 For $a = -2$:
$f(x) = -2(x - (-5))(x - 3)$
$-2(x + 5)(x - 3)$
$-2(x^2 + 2x - 15) = -2x^2 - 4x$
$+ 30$ For $a = 5$:
$f(x) = 5(x - (-5))(x - 3)$
$5(x + 5)(x - 3)$
$5(x^2 + 2x - 15) = 5x^2 + 10x - 75$

The x-intercepts are not affected by the value of a. The y-intercept is multiplied by the value of a.
The axis of symmetry is unaffected by the value of a. For this problem, the axis of symmetry is $x = -1$ for all values of a.
The x-coordinate of the vertex is not affected by the value of a. The y-coordinate of the vertex is multiplied by the value of a. The x-coordinate of the vertex is the mean of the x-intercepts.

81. **a.** $x = -\dfrac{b}{2a} = -\dfrac{4}{2(1)} = -2$

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$5(x+3)(x-1)$

$5(x^2+2x-3) = 5x^2+10x-15$

$= f(-2) = (-2)^2 + 4(-2) - 21$

$= -25$ The vertex is $(-2, -25)$.

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b. $(x) = 0$

$^2 + 4x - 21 = 0$

$x + 7)(x - 3) = 0$

$x + 7 = 0 \quad \text{or} \quad x - 3 = 0$

$x = -7 \qquad x = 3$

The x-intercepts of f are −7 and 3.

c. $(x) = -21$

$^2 + 4x - 21 =$

$-21\ x^2 + 4x =$

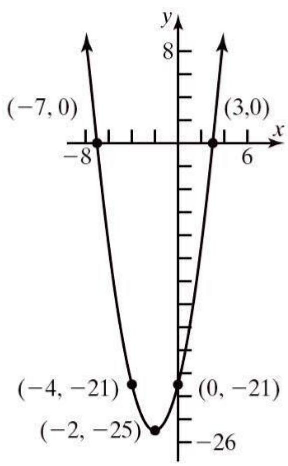
$0\ x(x + 4) = 0$

$x = 0 \quad \text{or} \quad x = -4$

The solutions $f(x) = -21$ are −4 and 0.

Thus, the points $(-4, -21)$ and $(0, -21)$ are on the graph of *f*.

d.



a. $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$

$8 = -9$ The vertex is $(-1, -9)$.

b. $(x) = 0$

$^2 + 2x - 8 = 0$

$x + 4 = 0 \quad \text{or} \quad x - 2 = 0$

$x = -4 \qquad x = 2$

The x-intercepts of f are −4 and 2.

$f(x) = -8$

$^2 + 2x - 8 = -8$

$x^2 + 2x = 0$

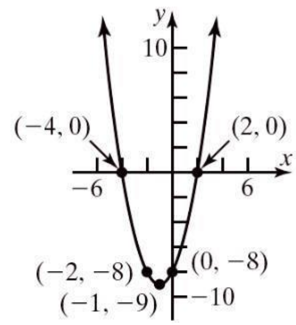
$x(x + 2) = 0$

$x = 0 \quad \text{or} \quad x + 2 = 0$

$x = -2$

The solutions $f(x) = -8$ are −2 and 0. Thus, the points $(-2, -8)$ and $(0, -8)$ are on the graph of *f*.

d.



$R(p) = -4p^2 + 4000p$, $a = -4$, $b = 4000$, $c = 0$. Since $a = -4 < 0$ the graph is a parabola that opens down, so the vertex is a maximum point. The maximum occurs at $p = \frac{-b}{2a} = \frac{-4000}{2(-4)} = 500$. Thus, the unit price should be \$500 for maximum revenue. The maximum revenue is

$R(500) = -4(500)^2 + 4000(500)$

$-1000000 + 2000000$

\$1,000,000

$R(p) = -\frac{1}{2}p^2 + 1900p$, $a = -\frac{1}{2}$, $b = 1900$, $c = 0$.

Since $a = -\frac{1}{2} < 0$, the graph is a parabola that opens down, so the vertex is a maximum point.

$p = \frac{-b}{2a} = \frac{-1900}{2(-1/2)} = \frac{-1900}{-1} = 1900$.

Thus, the unit price should be \$1900 for maximum revenue. The maximum revenue is

$\frac{1}{\ }$

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$$R(1900) = -\ 2\,(1900)^2 + 1900(1900)$$
$$-1805000 + 3610000$$
$$\$1,805,000$$

a. $C(x) = x^2 - 140x + 7400$,
$a = 1, b = -140, c = 7400$. Since $a = 1 > 0$,

the graph opens up, so the vertex is a

minimum point. The minimum marginal cost

occurs at $x = \frac{-b}{2a} = \frac{-(-140)}{2(1)} = \frac{140}{2} = 70$

thousand mp3 players produced.
The minimum marginal cost is

$f\left(\frac{-b}{2a}\right) = f(70) = (70)^2 - 140(70) + 7400$

$4900 - 9800 + 7400$
$\$2500$

$a = 5, b = -200, c = 4000$. Since $a = 5 > 0$,

minimum point. The minimum marginal

cost occurs at
$x = \frac{-b}{2a} = \frac{-(-200)}{2(5)} = \frac{200}{10} = 20$ thousand
cell phones manufactured.
The minimum marginal cost is

$f\left(\frac{-b}{2a}\right) = f(20) = 5(20)^2 - 200(20) + 4000$

$2000 - 4000 + 4000$
$\$2000$

a. $d(v) = 1.1v + 0.06v^2$

$(45) = 1.1(45) + 0.06(45)^2$
$49.5 + 121.5 = 171$ ft.

b. $200 = 1.1v + 0.06v^2$
$= -200 + 1.1v + 0.06v^2$

$= \frac{-(1.1) \pm \sqrt{(1.1)^2 - 4(0.06)(-200)}}{2(0.06)}$

$= \frac{-1.1 \pm \sqrt{49.21}}{0.12}$

$\frac{-1.1 \pm 7.015}{}$

The 1.1v term might represent the reaction time.

88. a. $a = \frac{-b}{2a} = \frac{-16.54}{2(-0.31)} = \frac{-16.54}{-0.62} \approx 26.7$ years old

$B(26.7) = -0.31(26.7)^2 + 16.54(26.7) - 151.04$
69.6 births per 1000 unmarried women

$B(40) = -0.31(40)^2 + 16.54(40) - 151.04$

14.6 births per 1000 unmarried women

a. $R(x) = 75x - 0.2x^2$
$= -0.2, b = 75, c = 0$

$x = \frac{-b}{2a} = \frac{-75}{2(-0.2)} = \frac{-75}{-0.4} = 187.5$

The maximum revenue occurs when
$x = 187$ or $x = 188$ watches.
The maximum revenue is:
$R(187) = 75(187) - 0.2(187)^2 = \$7031.20$
$R(188) = 75(188) - 0.2(188)^2 = \$7031.20$

$P(x) = R(x) - C(x)$

$75x - 0.2x^2 - (32x + 1750)$
$-0.2x^2 + 43x - 1750$

$P(x) = -0.2x^2 + 43x - 1750$
$a = -0.2, b = 43, c = -1750$

$x = \frac{-b}{2a} = \frac{-43}{2(-0.2)} = \frac{-43}{-0.4} = 107.5$

0
.
1
2

$v \approx 49$ or $v \approx -68$
Disregard the negative value since we are talking about speed. So the maximum speed you can be traveling would be approximately

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49 mph.

The maximum profit occurs when $x = 107$ or $x = 108$ watches.
The maximum profit is:

$$P(107) = -0.2(107)^2 + 43(107) - 1750$$
$$\$561.20$$

$$P(108) = -0.2(108)^2 + 43(108) - 1750$$
$$\$561.20$$

d. Answers will vary.

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a. $R(x) = 9.5x - 0.04x^2$

$= -0.04,\ b = 9.5,\ c = 0$

The maximum revenue occurs when

$x = \frac{-b}{2a} = \frac{-9.5}{2(-0.04)} = \frac{-9.5}{-0.08}$

118.75 ≈ 119 boxes of candy

The maximum revenue is:

$R(119) = 9.5(119) - 0.04(119)^2 = \$564.06$

$P(x) = R(x) - C(x)$

$9.5x - 0.04x^2 - (1.25x + 250)$

$-0.04x^2 + 8.25x - 250$

$P(x) = -0.04x^2 + 8.25x - 250$

$a = -0.04,\ b = 8.25,\ c = -250$

The maximum profit occurs when

$x = \frac{-b}{2a} = \frac{-8.25}{2(-0.04)} = \frac{-8.25}{-0.08}$

103.125 ≈ 103 boxes of

candy The maximum profit is:

$P(103) = -0.04(103)^2 + 8.25(103) - 250$

\$175.39

d. Answers will vary.

$f(x) = a(x - r_1)(x - r_2)$

$a(x + 4)(x - 2)$

$ax^2 + 2ax - 8a$

The x value of the vertex is $x = \frac{-b}{} = \frac{-2a}{} = -1$.

The y value of the vertex is 18.

$-18 = a(-1)^2 + 2a(-1) - 8a$

$-18 = -9a$

$a = 2$

So the function is $f(x) = 2(x +$

$f(x) = a(x - r_1)(x - r_2)$

$a(x + 1)(x - 5)$

$ax^2 - 4ax - 5a$

The x value of the vertex is

$= - \frac{b}{} = - \frac{(-4a)}{} =$

$= a(2)^2 - 4a(2) - 5a$

$= -9a$

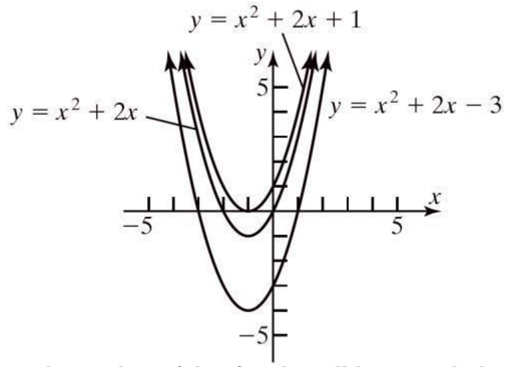
$a = -1$

So the function is $f(x) = -(x + 1)(x - 5)$

If x is even, then ax^2 and bx are even. When two even numbers are added to an odd number the result is odd. Thus, $f(x)$ is odd. If x is odd, then ax^2 and bx are odd. The sum of three odd numbers is an odd number. Thus, $f(x)$ is odd.

Answers will vary.

$y = x^2 + 2x - 3$; $y = x^2 + 2x + 1$; $y = x^2 + 2x$



Each member of this family will be a parabola with the following characteristics:

opens upwards since a > 0;

vertex occurs at $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$;

2 .

2a2a

The y value of the vertex is 9.

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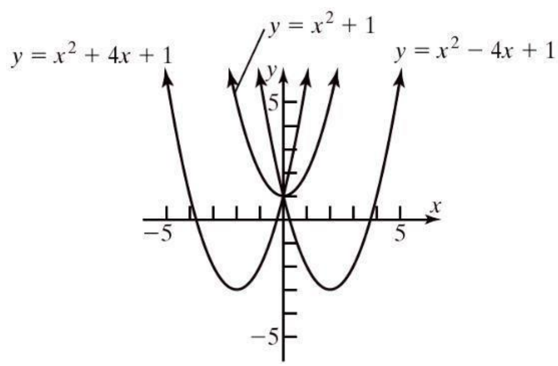
$2a$ $2a$

$4)(x - 2)$

There is at least one x-intercept since $b^2 - 4ac \geq 0$.

$y = x^2 - 4x + 1$; $y = x^2 + 1$; $y = x^2 + 4x + 1$



Each member of this family will be a parabola with the following characteristics:

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opens upwards since $a > 0$
y-intercept occurs at (0, 1).

The graph of the quadratic function $f(x) = ax^2 + bx + c$ will not have any x-intercepts whenever $b^2 - 4ac < 0$.

By completing the square on the quadratic function $f(x) = ax^2 + bx + c$ we obtain the equation $y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$. We can then draw the graph by applying transformations to the graph of the basic parabola $y = x^2$, which opens up. When $a > 0$, the basic parabola will either be stretched or compressed vertically. When $a < 0$, the basic parabola will either be stretched or compressed vertically as well as reflected across the x-axis. Therefore, when $a > 0$, the graph of $f(x) = ax^2 + bx + c$ will open up, and when $a < 0$, the graph of $f(x) = ax^2 + bx + c$ will open down.

No. We know that the graph of a quadratic function $f(x) = ax^2 + bx + c$ is a parabola with vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. If $a > 0$, then the vertex is a minimum point, so the range is $\left[f\left(-\frac{b}{2a}\right), \infty\right)$. If $a < 0$, then the vertex is a maximum point, so the range is $\left(-\infty, f\left(-\frac{b}{2a}\right)\right]$. Therefore, it is impossible for the range to be $(-\infty, \infty)$.

Two quadratic functions can intersect 0, 1, or 2 times.

$x^2 + 4y^2 = 16$

To check for symmetry with respect to the x-axis, replace y with –y and see if the equations are equivalent.

$x^2 + 4(-y)^2 = 16$

$x^2 + 4y^2 = 16$

So the graph is symmetric with respect to the x-axis.

axis, replace x with –x and see if the equations

$(-x)^2 + 4y^2 = 16$

$x^2 + 4y^2 = 16$

So the graph is symmetric with respect to the y-axis.

To check for symmetry with respect to the origin, replace x with –x and y with –y and see if the equations are equivalent.

$(-x)^2 + 4(-y)^2 = 16$

$x^2 + 4y^2 = 16$

So the graph is symmetric with respect to the origin.

$27 - x \ge 5x + 3$

$-6x \ge -24$

$x \le 4$

So the solution set is: $(-\infty, 4]$ or $\{x \mid x \le 4\}$.

$x^2 + y^2 - 10x + 4y + 20 = 0$

$x^2 - 10x + y^2 + 4y = -20$

$(x^2 - 10x + 25) + (y^2 + 4y + 4) = -20 + 25 + 4$

$(x - 5)^2 + (y + 2)^2 = 3^2$

Center: (5, -2); Radius = 3

104. $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$

$= \frac{6 \pm \sqrt{36 - 16}}{2}$

$= \frac{6 \pm \sqrt{20}}{2}$

$= \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$

So the zeros of the function are: $3 + \ 5, 3 - \ 5$

. The x-intercepts are: $3 + \ 5, 3 - \ 5$

Section 2.5

1. $-3x - 2 < 7$

$-3x < 9$

$x > -3$

The solution set is $\{x \mid x > -3\}$ or $(-3, \infty)$.

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$(-2, 7]$ represents the numbers between -2 and 7, including 7 but not including -2. Using inequality notation, this is written as $-2 < x \le 7$.

a. $f(x) > 0$ when the graph of f is above the x-axis.

Thus, $\{x \mid x < -2 \text{ or } x > 2\}$ or, using interval notation, $(-\infty, -2) \cup (2, \infty)$.

$f(x) \le 0$ when the graph of f is below or intersects the x-axis. Thus, $\{x \mid -2 \le x \le 2\}$ or, using interval notation, $[-2, 2]$.

a. $g(x) < 0$ when the graph of g is below the x-axis. Thus, $\{x \mid x < -1 \text{ or } x > 4\}$ or, using interval notation, $(-\infty, -1) \cup (4, \infty)$.

$g(x) \ge 0$ when the graph of f is above or intersects the x-axis. Thus, $\{x \mid -1 \le x \le 4\}$ or, using interval notation, $[-1, 4]$.

a. $g(x) \ge f(x)$ when the graph of g is above or intersects the graph of f. Thus $\{x \mid -2 \le x \le 1\}$ or, using interval notation, $[-2, 1]$.

$f(x) > g(x)$ when the graph of f is above the graph of g. Thus, $\{x \mid x < -2 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, -2) \cup (1, \infty)$

.

a. $f(x) < g(x)$ when the graph of f is below the graph of g. Thus, $\{x \mid x < -3 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, -3) \cup (1, \infty)$.

$f(x) \ge g(x)$ when the graph of f is above or intersects the graph of g. Thus, $\{x \mid -3 \le x \le 1\}$ or, using interval notation, $[-3, 1]$.

$x^2 - 3x - 10 \le 0$

We graph the function $f(x) = x^2 - 3x - 10$. The intercepts are

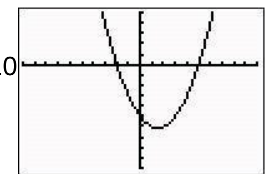
x-intercepts: $x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

$x = 5, x = -2$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-3)}{2(1)} = \frac{3}{2}$.

Since $f\left(\frac{3}{2}\right) = -\frac{49}{4}$, the vertex is $\left(\frac{3}{2}, -\frac{49}{4}\right)$.



The graph is below the x-axis for $-2 < x < 5$. Since the inequality is not strict, the solution set is $\{x \mid -2 \le x \le 5\}$ or, using interval notation, $[-2, 5]$.

$x^2 + 3x - 10 > 0$

We graph the function $f(x) = x^2 + 3x - 10$. The intercepts are

y-intercept: $f(0) = -10$

x-intercepts: $x^2 + 3x - 10 = 0$

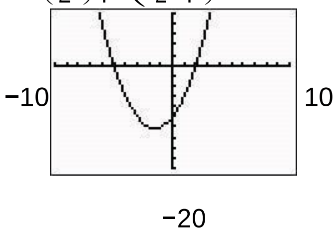
$(x + 5)(x - 2) = 0$

$x = -5, x = 2$

The vertex is at $x = \frac{-b}{2a} = \frac{-(3)}{2(1)} = -\frac{3}{2}$. Since $f\left(-\frac{3}{2}\right) = -\frac{49}{4}$, the vertex is $\left(-\frac{3}{2}, -\frac{49}{4}\right)$.

y-intercept: $f(0) = -10$

(2)4 (2 4)



The graph is above the x-axis when $x < -5$ or $x > 2$. Since the inequality is strict, the solution set is $\{ x | x < -5 \text{ or } x > 2 \}$ or, using interval notation, $(-\infty, -5) \cup (2, \infty)$.

9. $x^2 - 4x > 0$

We graph the function $f(x) = x^2 - 4x$. The intercepts are

y-intercept: $f(0) = 0$

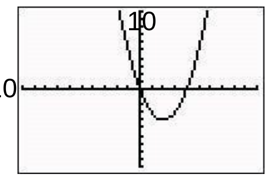
x-intercepts: $x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0, x = 4$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$. Since

$(2) = -4$, the vertex is $(2, -4)$.



The graph is above the x-axis when $x < 0$ or $x > 4$. Since the inequality is strict, the solution set is $\{ x | x < 0 \text{ or } x > 4 \}$ or, using interval notation, $(-\infty, 0) \cup (4, \infty)$.

10. $x^2 + 8x \le 0$

We graph the function $f(x) = x^2 + 8x$. The intercepts are

y-intercept: $f(0) = 0$

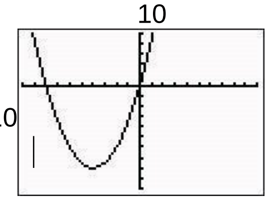
x-intercepts: $x^2 + 8x = 0$

$x(x+8) = 0$

$x = 0, x = -8$

The vertex is at $x = \frac{-b}{2a} = \frac{-(8)}{2(1)} = \frac{-8}{2} = -4$.

Since $f(-4) = -16$, the vertex is $(-4, -16)$.



$x^2 + x > 12$

$x^2 + x - 12 > 0$

We graph the function $f(x) = x^2 + x - 12$.

y-intercept: $f(0) = -12$

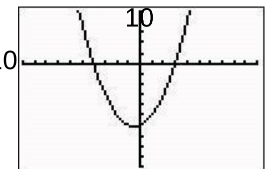
x-intercepts: $x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4, x = 3$

The vertex is at $x = \frac{-b}{2a} = \frac{-(1)}{2(1)} = -\frac{1}{2}$. Since

$f\left(-\frac{1}{2}\right) = -\frac{49}{4}$, the vertex is $\left(-\frac{1}{2}, -\frac{49}{4}\right)$.



The graph is above the x-axis when $x < -4$ or $x > 3$. Since the inequality is strict, the solution set is $\{ x | x < -4 \text{ or } x > 3 \}$ or, using interval notation, $(-\infty, -4) \cup (3, \infty)$.

$x^2 + 7x < -12$

$x^2 + 7x + 12 < 0$

We graph the function $f(x) = x^2 + 7x + 12$.

y-intercept: $f(0) = 12$

x-intercepts: $x^2 + 7x + 12 = 0$

$x + 4)(x + 3) = 0$

$x = -4, x = -3$

The vertex is at $x = \frac{-b}{2a} = \frac{-(7)}{2(1)} = -\frac{7}{2}$. Since

$f\left(-\frac{7}{2}\right) = -\frac{1}{4}$, the vertex is $\left(-\frac{7}{2}, -\frac{1}{4}\right)$.

The graph is below the x-axis when $-8 < x < 0$. Since the inequality is not strict, the solution set is $\{ x \; -8 \le x \le 0 \}$ or, using interval notation,

$-8, 0]$.

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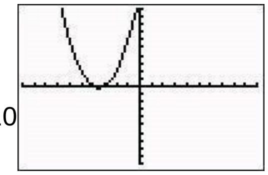
(2)4(2 4)



The graph is below the *x*-axis when $-4 < x < -3$. Since the inequality is strict, the solution set is $\{ x \mid -4 < x < -3 \}$ or, using interval notation, $(-4, -3)$.

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$x^2+6x+9\le 0$

We graph the function $f(x)=x^2+6x+9$.

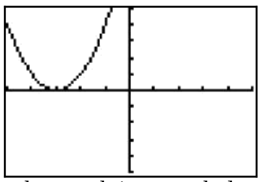
y-intercept: $f(0)=9$

x-intercepts: $x^2+6x+9=0$

$(x+3)(x+3)=0$

$x=-3$

The vertex is at $x=\frac{-b}{2a}=\frac{-(6)}{2(1)}=-3$. Since $f(-3)=0$, the vertex is $(-3,0)$.



Since the graph is never below the x-axis and only touches at $x=-3$ then the solution is $\{-3\}$.

$x^2-4x+4\le 0$

We graph the function $f(x)=x^2-4x+4$.

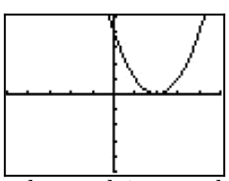
y-intercept: $f(0)=4$

x-intercepts: $x^2-4x+4=0$

$(x-2)(x-2)=0$

$x=2$

The vertex is at $x=\frac{-b}{2a}=\frac{-(-4)}{2(1)}=2$. Since $f(2)=0$, the vertex is $(2,0)$.

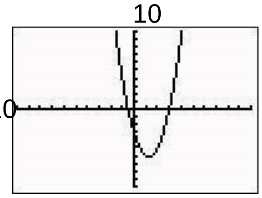


Since the graph is never below the x-axis and only touches at $x=2$ then the solution is $\{2\}$.

$2x^2\le 5x+3$

$2x^2-5x-3\le 0$

The vertex is at $x=\frac{-b}{2a}=\frac{-(-5)}{2(2)}=\frac{5}{4}$. Since $f\left(\frac{5}{4}\right)=-\frac{49}{8}$, the vertex is $\left(\frac{5}{4},-\frac{49}{8}\right)$.



The graph is below the x-axis when $-\frac{1}{2}<x<3$. Since the inequality is not strict, the solution set is $\left\{x \mid -\frac{1}{2}\le x\le 3\right\}$ or, using interval notation, $\left[-\frac{1}{2},3\right]$.

16. $6x^2\le 6+5x$

$6x^2-5x-6\le 0$

We graph the function $f(x)=6x^2-5x-6$. The intercepts are

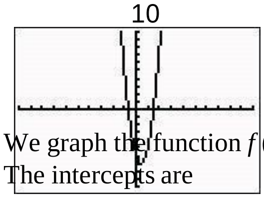
y-intercept: $f(0)=-6$

x-intercepts: $6x^2-5x-6=0$

$(3x+2)(2x-3)=0$

$x=-\frac{2}{3},\ x=\frac{3}{2}$

The vertex is at $x=\frac{-b}{2a}=\frac{-(-5)}{2(6)}=\frac{5}{12}$. Since $f\left(\frac{5}{12}\right)=-\frac{169}{24}$, the vertex is $\left(\frac{5}{12},-\frac{169}{24}\right)$.



We graph the function $f(x)=2x^2-5x-3$. The intercepts are

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−10 10

y-intercept: $f(0) = -3$

x-intercepts: $2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

$x = -\frac{1}{2}, x = 3$

−10

The graph is below the x-axis when $-\frac{2}{3} < x < \frac{3}{2}$.

Since the inequality is not strict, the solution set

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is $\left\{ x \middle| -\frac{2}{3} \le x \le \frac{3}{2} \right\}$ or, using interval notation, $\left[-\frac{2}{3}, \frac{3}{2} \right]$.

17. $x(x-7) > 8$

$x^2 - 7x > 8$

$x^2 - 7x - 8 > 0$

We graph the function $f(x) = x^2 - 7x - 8$. The intercepts are

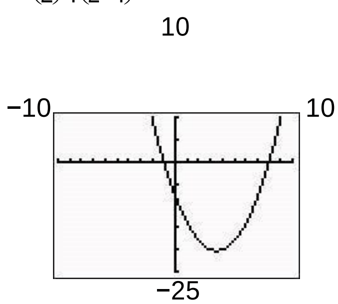
y-intercept: $f(0) = -8$

x-intercepts: $x^2 - 7x - 8 = 0$

$(x+1)(x-8) = 0$

$x = -1, x = 8$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-7)}{2(1)} = \frac{7}{2}$. Since $f\left(\frac{7}{2}\right) = -\frac{81}{4}$, the vertex is $\left(\frac{7}{2}, -\frac{81}{4}\right)$.



The graph is above the x-axis when $x < -1$ or $x > 8$. Since the inequality is strict, the solution set is $\{ x | x < -1 \text{ or } x > 8 \}$ or, using interval notation, $(-\infty, -1) \cup (8, \infty)$.

$x(x+1) > 20$

$^2 + x > 20$

$x^2 + x - 20 > 0$

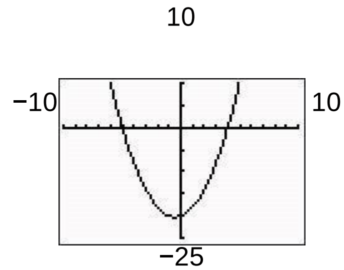
We graph the function $f(x) = x^2 + x - 20$.

y-intercept: $f(0) = -20$

x-intercepts: $x^2 + x - 20 = 0$

$(x+5)(x-4) = 0$

$x = -5, x = 4$



The graph is above the x-axis when $x < -5$ or $x > 4$. Since the inequality is strict, the solution set is $\{ x | x < -5 \text{ or } x > 4 \}$ or, using interval notation, $(-\infty, -5) \cup (4, \infty)$.

$4x^2 + 9 < 6x$

$x^2 - 6x + 9 < 0$

We graph the function $f(x) = 4x^2 - 6x + 9$.

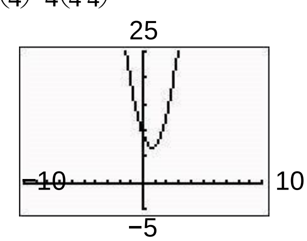
y-intercept: $f(0) = 9$

x-intercepts: $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$

$= \frac{6 \pm \sqrt{-108}}{8}$ (not real)

Therefore, f has no x-intercepts.

The vertex is at $x = \frac{-b}{2a} = \frac{-(-6)}{2(4)} = \frac{6}{8} = \frac{3}{4}$. Since $f\left(\frac{3}{4}\right) = \frac{27}{4}$, the vertex is $\left(\frac{3}{4}, \frac{27}{4}\right)$.



The graph is never below the x-axis. Thus, there is no real solution.

$25x^2 + 16 < 40x$

$25x^2 - 40x + 16 < 0$

We graph the function $f(x) = 25x^2 - 40x + 16$.

y-intercept: $f(0) = 16$

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x-intercepts: $25x^2 - 40x + 16 = 0$

$$(5x-4)^2 = 0$$
$$5x - 4 = 0$$
$$x = \frac{4}{5}$$

The vertex is at $x = -\frac{b}{2a} = -\frac{(1)}{2(1)} = -\frac{1}{2}$. Since $f\left(-\frac{1}{2}\right) = -\frac{81}{4}$, the vertex is $\left(-\frac{1}{2}, -\frac{81}{4}\right)$.

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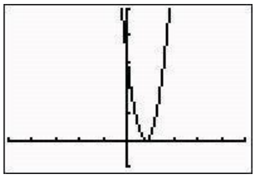
The vertex is at $x=\frac{-b}{2a}=\frac{-(-40)}{2(25)}=\frac{40}{50}=\frac{4}{5}$.

Since $f\left(\frac{4}{5}\right)=0$, the vertex is $\left(\frac{4}{5},0\right)$.



The graph is never below the x-axis. Thus, there is no real solution.

$6(x^2+1)>5x$

$x^2+6>5x$

$6x^2-5x+6>0$

We graph the function $f(x)=6x^2-5x+6$.

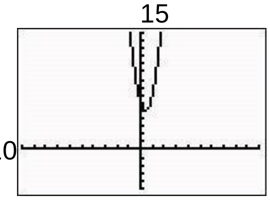
y-intercept: $f(0)=6$

x-intercepts: $x=\frac{-(-5)\pm\sqrt{(-5)^2-4(6)(6)}}{2(6)}$

$=\frac{6\pm\sqrt{-119}}{12}$ (not real)

Therefore, f has no x-intercepts.

The vertex is at $x=\frac{-b}{2a}=\frac{-(-5)}{2(6)}=\frac{5}{12}$. Since $f\left(\frac{5}{12}\right)=\frac{119}{24}$, the vertex is $\left(\frac{5}{12},\frac{119}{24}\right)$.

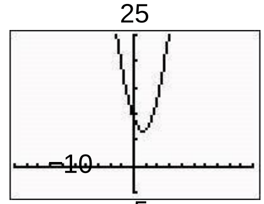


x-intercepts: $x=\frac{-(-6)\pm\sqrt{(-6)^2-4(4)(9)}}{2(4)}$

$=\frac{6\pm\sqrt{-108}}{8}$ (not real)

Therefore, f has no x-intercepts.

The vertex is at $x=\frac{-b}{2a}=\frac{-(-6)}{2(4)}=\frac{6}{8}=\frac{3}{4}$. Since $f\left(\frac{3}{4}\right)=\frac{27}{4}$, the vertex is $\left(\frac{3}{4},\frac{27}{4}\right)$.



The graph is always above the x-axis. Thus, the solution set is all real numbers or, using interval notation, $(-\infty,\infty)$.

23. The domain of the expression $f(x)=\sqrt{x^2-16}$ includes all values for which $x^2-16\ge 0$.

We graph the function $p(x)=x^2-16$. The intercepts of p are

y-intercept: $p(0)=-6$

x-intercepts: $x^2-16=0$

$(x+4)(x-4)=0$

$x=-4,\ x=4$

The vertex of p is at $x=\frac{-b}{2a}=\frac{-(0)}{2(1)}=0$. Since $p(0)=-16$, the vertex is $(0,-16)$.

The graph is always above the x-axis. Thus, the solution set is all real numbers or, using interval notation, $(-\infty,\infty)$.

$2(2x^2-3x)>-9\,4x$

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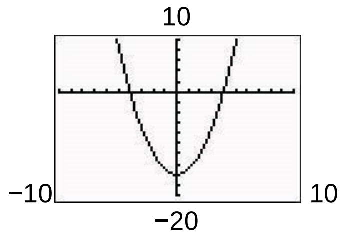
$^2 - 6x > -9$

$x^2 - 6x + 9 > 0$

We graph the function $f(x) = 4x^2 - 6x + 9$.

y-intercept: $f(0) = 9$



The graph of p is above the x-axis when $x < -4$ or $x > 4$. Since the inequality is not strict, the solution set of $x^2 - 16 \geq 0$ is $\{x \mid x \leq -4 \text{ or } x \geq 4\}$.

Thus, the domain of f is also $\{x \mid x \leq -4 \text{ or } x \geq 4\}$ or, using interval notation, $(-\infty, -4] \cup [4, \infty)$.

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24. The domain of the expression $f(x) = \sqrt{x - 3x^2}$ includes all values for which $x - 3x^2 \geq 0$.

We graph the function $p(x) = x - 3x^2$. The intercepts of p are

y-intercept: $p(0) = -6$

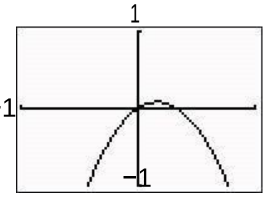
x-intercepts: $x - 3x^2 = 0$

$x(1 - 3x) = 0$

$x = 0, x = \frac{1}{3}$.

The vertex of p is at $x = \frac{-b}{2a} = \frac{-(1)}{2(-3)} = \frac{-1}{-6} = \frac{1}{6}$.

Since $p\left(\frac{1}{6}\right) = \frac{1}{12}$, the vertex is $\left(\frac{1}{6}, \frac{1}{12}\right)$.



The graph of p is above the x-axis when $0 < x < \frac{1}{3}$. Since the inequality is not strict, the solution set of $x - 3x^2 \geq 0$ is $\left\{ x \,\middle|\, 0 \leq x \leq \frac{1}{3} \right\}$.

Thus, the domain of f is also $\left\{ x \,\middle|\, 0 \leq x \leq \frac{1}{3} \right\}$ or, using interval notation, $\left[0, \frac{1}{3}\right]$.

25. $f(x) = x^2 - 1$; $g(x) = 3x + 3$

$f(x) = 0$

$x^2 - 1 = 0$

$(x - 1)(x + 1) = 0$

$x = 1; x = -1$

Solution set: $\{-1, 1\}$.

$g(x) = 0$

$3x + 3 = 0$

$3x = -3$

$x = -1$

Solution set: $\{-1\}$.

$f(x) = g(x)$

$x^2 - 1 = 3x + 3$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

$x = 4; x = -1$

Solution set: $\{-1, 4\}$.

$f(x) > 0$

We graph the function $f(x) = x^2 - 1$.

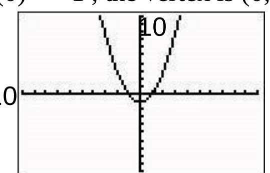
y-intercept: $f(0) = -1$

x-intercepts: $x^2 - 1 = 0$

$(x + 1)(x - 1) = 0$

$x = -1, x = 1$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(1)} = 0$. Since $f(0) = -1$, the vertex is $(0, -1)$.



The graph is above the x-axis when $x < -1$ or $x > 1$. Since the inequality is strict, the solution set is $\{ x \mid x < -1 \text{ or } x > 1 \}$ or, using interval notation, $(-\infty, -1) \cup (1, \infty)$.

$g(x) \leq 0$

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$3x + 3 \le 0$

$3x \le$
-3

$x \le$
-1

The solution set is $\{x \mid x \le -1\}$ or,

using interval notation, $(-\infty, -1]$.

$f(x) > g(x)$ $x^2 - 1 >$

$3x$
$+ 3$

$x^2 - 3x - 4 > 0$

We graph the function $p(x) = x^2 - 3x -$
4. The intercepts of p are

y-intercept: $p(0) = -4$

x-intercepts: $x^2 - 3x - 4 = 0$

$x - 4)(x + 1) = 0$

$= 4, x =$
-1

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The vertex is at $x = \frac{-b}{2a} = \frac{-(-3)}{2(1)} = \frac{3}{2}$. Since $p\left(\frac{3}{2}\right) = -\frac{25}{4}$, the vertex is $\left(\frac{3}{2}, -\frac{25}{4}\right)$.

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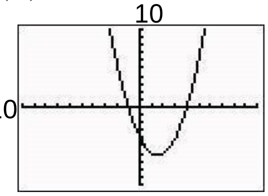
−10  10

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The graph of p is above the x-axis when $x < -1$ or $x > 4$. Since the inequality is strict, the solution set is $\{x \mid x < -1 \text{ or } x > 4\}$ or, using interval notation, $(-\infty, -1) \cup (4, \infty)$.

$f(x) \ge 1$

$x^2 - 1 \ge 1$

$^2 - 2 \ge 0$

We graph the function $p(x) = x^2 - 2$. The intercepts of p are

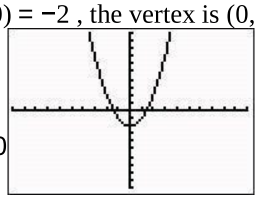
y-intercept: $p(0) = -2$

x-intercepts: $x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(1)} = 0$. Since $p(0) = -2$, the vertex is (0, −2).

−10  10

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The graph of p is above the x-axis when $x < -\sqrt{2}$ or $x > \sqrt{2}$. Since the inequality is not strict, the solution set is $\{x \mid x \le -\sqrt{2} \text{ or } x \ge \sqrt{2}\}$ or, using interval

26. $f(x) = -x^2 + 3$; $g(x) = -3x + 3$

$f(x) = 0$

$x^2 + 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

Solution set: $\{-\sqrt{3}, \sqrt{3}\}$.

$g(x) = 0$

$-3x + 3 = 0$

$-3x = -3$

$x = 1$

Solution set: {1}.

$f(x) = g(x)$

$x^2 + 3 = -3x + 3$

$0 = x^2 - 3x$

$0 = x(x - 3)$

$= 0; x = 3$

Solution set: $\{0, 3\}$.

d. $f(x) > 0$

We graph the function $f(x) = -x^2 + 3$.

y-intercept: $f(0) = 3$

x-intercepts: $-x^2 + 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since

notation, $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$.

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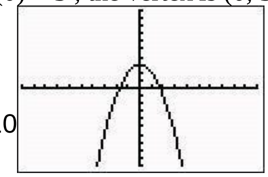
$f(0) = 3$, the vertex is (0, 3).



The graph is above the x-axis when

$3 < x < \; 3$. Since the inequality is strict, the solution set is $\{ x \; -3 < x < 3 \}$ or, using interval notation, $(-3, 3)$.

$$g(x) \le 0$$
$$-3x + 3 \le 0$$
$$-3x \le -3$$
$$x \ge 1$$

The solution set is $\{ x \; x \ge 1 \}$ or, using interval notation, $[1, \infty)$.

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$f(x) > g(x)$
$x^2 + 3 > -3x + 3$
$x^2 + 3x > 0$

We graph the function $p(x) = -x^2 + 3x$. The intercepts of p are

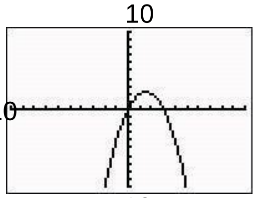
y-intercept: $p(0) = 0$

x-intercepts: $-x^2 + 3x = 0$
$x(x - 3) = 0$
$= 0; x = 3$

The vertex is at $x = \frac{-b}{2a} = \frac{-(3)}{2(-1)} = \frac{-3}{-2} = \frac{3}{2}$.

Since $p\left(\frac{3}{2}\right) = \frac{9}{4}$, the vertex is $\left(\frac{3}{2}, \frac{9}{4}\right)$.

10

−10  10

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The graph of p is above the x-axis when $0 < x < 3$. Since the inequality is strict, the solution set is $\{x \mid 0 < x < 3\}$ or, using interval notation, $(0, 3)$.

$f(x) \geq 1$
$x^2 + 3 \geq 1$
$-x^2 + 2 \geq 0$

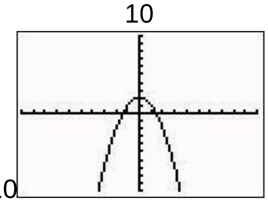
We graph the function $p(x) = -x^2 + 2$. The intercepts of p are

y-intercept: $p(0) = 2$

x-intercepts: $-x^2 + 2 = 0$
$x^2 = 2$
$x = \pm\sqrt{2}$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $p(0) = 2$, the vertex is $(0, 2)$.

10

−10  10

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The graph of p is above the x-axis when

27. $f(x) = -x^2 + 1$; $g(x) = 4x + 1$

$f(x) = 0$
$x^2 + 1 = 0$
$1 - x^2 = 0$
$(1 - x)(1 + x) = 0$
$x = 1; x = -1$

Solution set: $\{-1, 1\}$.

$g(x) = 0$
$4x + 1 = 0$
$4x = -1$

$x = -\frac{1}{4}$

Solution set: $\left\{-\frac{1}{4}\right\}$.

$f(x) = g(x)$
$x^2 + 1 = 4x + 1$ 0
$= x^2 + 4x$ 0 $=$
$x(x + 4)$ x
$= 0; x - 4$

Solution set: $\{-4, 0\}$.

$f(x) > 0$

We graph the function $f(x) = -x^2 + 1$.

y-intercept: $f(0) = 1$

x-intercepts: $-x^2 + 1 = 0$
$x^2 - 1 = 0$
$(x + 1)(x - 1) = 0$
$x = -1; x = 1$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since

$2 < x < 2$. Since the inequality is not strict, the solution set is $\{x$
$- 2 \leq x \leq 2\}$

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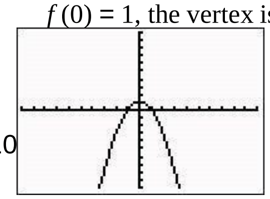
$f(0) = 1$, the vertex is (0, 1).



or, using interval notation, $\left[-\sqrt{2}, \sqrt{2}\right]$.

−10

The graph is above the x-axis when $-1 < x < 1$. Since the inequality is strict, the solution set is $\{x \mid -1 < x < 1\}$ or, using interval notation, $(-1, 1)$.

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e. $g(x) \le 0$

$4x + 1 \le 0$

$4x \le -1$

$x \le -\frac{1}{4}$

The solution set is $\left\{ x \,\middle|\, x \le -\frac{1}{4} \right\}$ or, using interval notation, $\left(-\infty, -\frac{1}{4} \right]$.

$f(x) > g(x)$

$x^2 + 1 > 4x + 1$

$x^2 - 4x > 0$

We graph the function $p(x) = -x^2 - 4x$.

The intercepts of p are

y-intercept: $p(0) = 0$

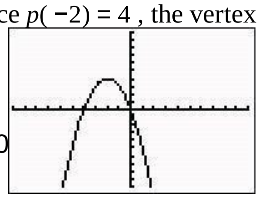
x-intercepts: $-x^2 - 4x = 0$

$x(x + 4) =$

$0\ x = 0; x = -4$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-4)}{2(-1)} = \frac{4}{-2} = -2$.

Since $p(-2) = 4$, the vertex is $(-2, 4)$.



The graph of p is above the x-axis when $-4 < x < 0$. Since the inequality is strict, the solution set is $\{ x | -4 < x < 0 \}$ or, using interval notation, $(-4, 0)$.

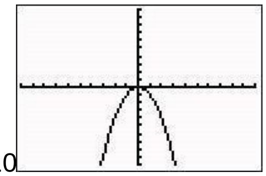
$f(x) \ge 1$

$x^2 + 1 \ge 1$

$x^2 \ge 0$

We graph the function $p(x) = -x^2$. The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since

$p(0) = 0$, the vertex is $(0, 0)$. Since $a = -1 < 0$, the parabola opens downward.



The graph of p is never above the x-axis, but it does touch the x-axis at $x = 0$. Since the inequality is not strict, the solution set is $\{0\}$.

28. $f(x) = -x^2 + 4$; $g(x) = -x - 2$

$f(x) = 0$

$x^2 + 4 =$

$0\ x^2 - 4 = 0\ (x +$

$2)(x - 2) = 0$

$= -2; x = 2$

Solution set: $\{-2, 2\}$.

$g(x) = 0$

$x - 2 = 0$

$-2 = x$

Solution set: $\{-2\}$.

$f(x) = g(x)$

$x^2 + 4 = -x - 2$

$0 = x^2 - x - 6$

$= (x - 3)(x + 2)$

$x = 3; x = -2$

Solution set: $\{-2, 3\}$.

d. $f(x) > 0$

$-x^2 + 4 > 0$

We graph the function $f(x) = -x^2 + 4$.

y-intercept: $f(0) = 4$

x-intercepts: $-x^2 + 4 = 0$

$x^2 - 4 = 0$

$x + 2)(x - 2) =$

$0\ x = -2; x = 2$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since

$f(0) = 4$, the vertex is $(0, 4)$.

10

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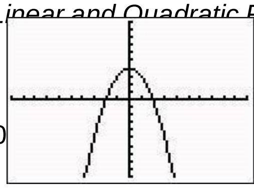

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The graph is above the x-axis when $-2 < x < 2$. Since the inequality is strict, the solution set is $\{x \mid -2 < x < 2\}$ or, using interval notation, $(-2, 2)$.

$$g(x) \le 0$$
$$-x - 2 \le 0$$
$$-x \le 2$$
$$x \ge -2$$

The solution set is $\{x \mid x \ge -2\}$ or, using interval notation, $[-2, \infty)$.

f.
$$f(x) > g(x)$$
$$-x^2 + 4 > -x - 2$$
$$-x^2 + x + 6 > 0$$

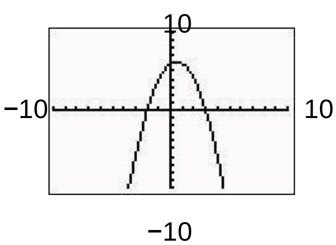
We graph the function $p(x) = -x^2 + x + 6$. The intercepts of p are

y-intercept: $p(0) = 6$

x-intercepts:
$$-x^2 + x + 6 = 0$$
$$x^2 - x - 6 = 0$$
$$(x+2)(x-3) = 0$$
$$x = -2;\ x = 3$$

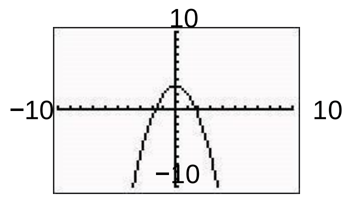
The vertex is at $x = \frac{-b}{2a} = \frac{-(1)}{2(-1)} = \frac{-1}{-2} = \frac{1}{2}$.

Since $p\left(\frac{1}{2}\right) = \frac{25}{4}$, the vertex is $\left(\frac{1}{2}, \frac{25}{4}\right)$.



The graph of p is above the x-axis when $-2 < x < 3$. Since the inequality is strict, the solution set is $\{x \mid -2 < x < 3\}$ or, using interval notation, $(-2, 3)$.

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $p(0) = 3$, the vertex is (0, 3).



The graph of p is above the x-axis when $-\sqrt{3} < x < \sqrt{3}$. Since the inequality is not strict, the solution set is $\{x \mid -\sqrt{3} \le x \le \sqrt{3}\}$ or, using interval notation, $[-\sqrt{3}, \sqrt{3}]$.

29. $f(x) = x^2 - 4$; $g(x) = -x^2 + 4$

a.
$$f(x) = 0$$
$$x^2 - 4 = 0$$
$$(x-2)(x+2) = 0$$
$$x = 2;\ x = -2$$

Solution set: $\{-2, 2\}$.

b.
$$g(x) = 0$$
$$-x^2 + 4 = 0$$
$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x = -2;\ x = 2$$

Solution set: $\{-2, 2\}$.

c.
$$f(x) = g(x)$$
$$x^2 - 4 = -x^2 + 4$$
$$2x^2 - 8 = 0$$
$$2(x-2)(x+2) = 0$$
$$x = 2;\ x = -2$$

Solution set: $\{-2, 2\}$.

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g. $f(x) \geq 1$

$-x^2 + 4 > 1$

$-x^2 + 3 > 0$

We graph the function $p(x) = -x^2 + 3$. The intercepts of p are

y-intercept: $p(0) = 3$

x-intercepts: $-x^2 + 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

d. $f(x) > 0$

$x^2 - 4 > 0$

We graph the function $f(x) = x^2 - 4$.

y-intercept: $f(0) = -4$

x-intercepts: $x^2 - 4 = 0$

$(x + 2)(x - 2) = 0$

$x = -2;\ x = 2$

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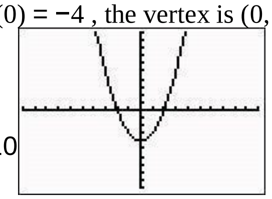
$\overline{2a}$ $\overline{2(-1)}$

$f(0) = -4$, the vertex is $(0, -4)$.



−10 10

−10

The graph is above the x-axis when $x < -2$ or $x > 2$. Since the inequality is strict, the solution set is $\{x \mid x < -2 \text{ or } x > 2\}$ or, using interval notation, $(-\infty, -2) \cup (2, \infty)$.

$g(x) \le 0$
$-x^2 + 4 \le 0$

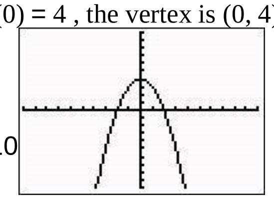
We graph the function $g(x) = -x^2 + 4$.
y-intercept: $g(0) = 4$

x-intercepts: $-x^2 + 4 = 0$
$x^2 - 4 = 0$

$x + 2)(x - 2) =$
0 $x = -2; x = 2$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since

$g(0) = 4$, the vertex is $(0, 4)$.



−10 10

−10

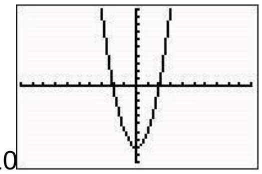
The graph is below the x-axis when $x < -2$ or $x > 2$. Since the inequality is not strict, the solution set is $\{x \mid x \le -2 \text{ or } x \ge 2\}$ or, using interval notation, $(-\infty, -2] \cup [2, \infty)$.

$f(x) > g(x)$ x
$^2 - 4 > -x^2 + 4$

-2 − − $\underline{(0)}$ =

0. Since $p(0) = -8$, the vertex is $(0, -8)$.

10



−10 10

−10

The graph is above the x-axis when $x < -2$ or $x > 2$. Since the inequality is strict, the solution set is $\{x \mid x < -2 \text{ or } x > 2\}$ or, using interval notation, $(-\infty, -2) \cup (2, \infty)$.

$f(x) \ge 1$
$x^2 - 4 \ge 1$
$x^2 - 5 \ge 0$

We graph the function $p(x) = x^2 - 5$

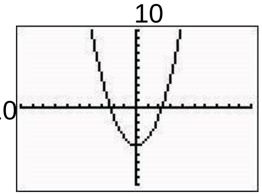
. y-intercept: $p(0) = -5$ x-intercepts:
$x^2 - 5 = 0$
$x^2 = 5$
$x = \pm\sqrt{5}$

$\underline{(0)}$

The vertex is at $x = \frac{-b}{2a} = \frac{-}{2(1)} =$

0. Since $p(0) = -5$, the vertex is $(0, -5)$.

10



−10 10

−10

The graph of p is above the x-axis when $x < -\sqrt{5}$ or $x > \sqrt{5}$. Since the inequality is not strict, the solution set is $\{x \mid x \le -\sqrt{5} \text{ or } x \ge \sqrt{5}\}$ or, using interval notation, $(-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty)$.

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$x^2 - 8 > 0$

We graph the function $p(x) = 2x^2 - 8$.

y-intercept: $p(0) = -8$

x-intercepts: $2x^2 - 8 = 0$

$2(x+2)(x-2) = 0$

$x = -2;\ x = 2$

30. $f(x) = x^2 - 2x + 1;\ g(x) = -x^2 + 1$

$f(x) = 0$

$x^2 - 2x + 1 = 0$

$x - 1)^2 = 0$

$-1 = 0$

$= 1$

Solution set: $\{1\}$.

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$g(x) = 0$

$x^2 + 1 = 0$

$x^2 - 1 = 0$

$x + 1)(x - 1) = 0$

$= -1; x = 1$ Solution

set: $\{-1, 1\}$.

$f(x) = g(x)$

$^2 - 2x + 1 = -x^2$

$+ 1\ 2x^2 - 2x = 0$

$2x(x - 1) = 0$

$= 0, x = 1$ Solution

set: $\{0, 1\}$.

d. $f(x) > 0\ x^2 - 2$

$x + 1 > 0$

We graph the function $f(x) = x^2 - 2x + 1$.

y-intercept: $f(0) = 1$

x-intercepts: $x^2 - 2x + 1 = 0$

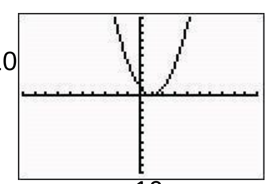
$x - 1)^2 = 0$

$-1 = 0$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$.

Since $f(1) = 0$, the vertex is (1, 0).

10

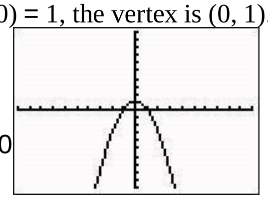
−10  10

−10

The graph is above the x-axis when $x < 1$ or $x > 1$. Since the inequality is strict, the solution set is $\{x \mid x < 1 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, 1) \cup (1, \infty)$.

$g(x) \le 0$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $g(0) = 1$, the vertex is (0, 1).

−10  10

−10

The graph is below the x-axis when $x < -1$ or $x > 1$. Since the inequality is not strict, the solution set is $\{x \mid x \le -1 \text{ or } x \ge 1\}$ or, using interval notation, $(-\infty, -1] \cup [1, \infty)$.

$f(x) > g(x)\ x^2 - 2x +$

$1 > -x^2 + 1$

$x^2 - 2x > 0$

We graph the function $p(x) = 2x^2 - 2x$.

y-intercept: $p(0) = 0$

x-intercepts: $2x^2 - 2x = 0$

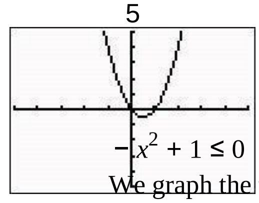
$x(x - 1) = 0$

$= 0; x = 1$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-2)}{2(2)} = \frac{2}{4} = \frac{1}{2}$.

Since $p\left(\frac{1}{2}\right) = -\frac{1}{2}$, the vertex is $\left(\frac{1}{2}, -\frac{1}{2}\right)$.

5



$-x^2 + 1 \le 0$

We graph the function $g(x) = -x^2 + 1$.

y-intercept: $g(0) = 1$

x-intercepts: $-x^2 + 1 = 0$

$x^2 - 1 = 0$

$x + 1)(x - 1) =$

$0\ x = -1; x = 1$

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−5 5

−5

The graph is above the x-axis when $x < 0$ or $x > 1$.

Since the inequality is strict, the solution set is $\{x \mid x < 0 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, 0) \cup (1, \infty)$.

$f(x) \geq 1$ $x^2 - 2x + 1 \geq 1$

$x^2 - 2x \geq 0$

We graph the function $p(x) = x^2 - 2x$.

y-intercept: $p(0) = 0$

x-intercepts: $x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0; x = 2$

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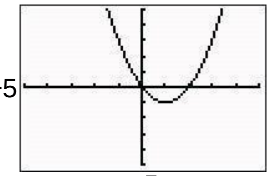
The vertex is at $x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$.

Since $p(1) = -1$, the vertex is $(1, -1)$.



The graph of p is above the x-axis when $x < 0$ or $x > 2$. Since the inequality is not strict, the solution set is $\{x \mid x \le 0 \text{ or } x \ge 2\}$ or, using interval notation, $(-\infty, 0] \cup [2, \infty)$.

31. $f(x) = x^2 - x - 2$; $g(x) = x^2 + x - 2$

$f(x) = 0$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2, x = -1$ Solution set: $\{-1, 2\}$.

b. $g(x) = 0$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2; x = 1$ Solution set: $\{-2, 1\}$.

$f(x) = g(x)$

$x^2 - x - 2 = x^2 + x - 2$

$-2x = 0$

$x = 0$

Solution set: $\{0\}$.

$f(x) > 0$

$x^2 - x - 2 > 0$

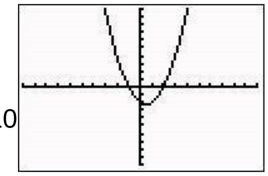
We graph the function $f(x) = x^2 - x - 2$.

y-intercept: $f(0) = -2$

x-intercepts: $x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2; x = -1$



The graph is above the x-axis when $x < -1$ or $x > 2$. Since the inequality is strict, the solution set is $\{x \mid x < -1 \text{ or } x > 2\}$ or, using interval notation, $(-\infty, -1) \cup (2, \infty)$.

$g(x) \le 0$

$x^2 + x - 2 \le 0$

We graph the function $g(x) = x^2 + x - 2$.

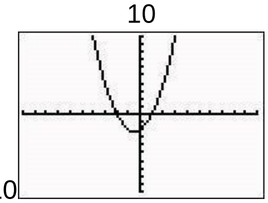
y-intercept: $g(0) = -2$

x-intercepts: $x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2; x = 1$

The vertex is at $x = \frac{-b}{2a} = \frac{-(1)}{2(1)} = -\frac{1}{2}$. Since $g\left(-\frac{1}{2}\right) = -\frac{7}{4}$, the vertex is $\left(-\frac{1}{2}, -\frac{7}{4}\right)$.



The graph is below the x-axis when $-2 < x < 1$. Since the inequality is not strict, the solution set is $\{x \mid -2 \le x \le 1\}$ or, using interval notation, $[-2, 1]$.

$f(x) > g(x)$

$x^2 - x - 2 > x^2 + x - 2$

$-2x > 0$

$x < 0$

The solution set is $\{x \mid x < 0\}$ or, using interval notation, $(-\infty, 0)$.

$f(x) \ge 1$

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The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$.

Since $p\left(\frac{1}{2}\right) = -\frac{9}{4}$, the vertex is $\left(\frac{1}{2}, -\frac{9}{4}\right)$.

$x^2 - x - 2 \geq 1$

$x^2 - x - 3 \geq 0$

We graph the function $p(x) = x^2 - x - 3$.

y-intercept: $p(0) = -3$

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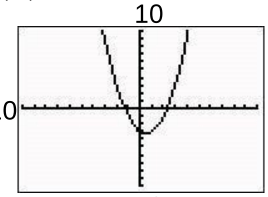
x-intercepts: $x^2 - x - 3 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1+12}}{2} = \frac{1 \pm \sqrt{13}}{2}$$

$x \approx -1.30$ or $x \approx 2.30$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$. Since $p\left(\frac{1}{2}\right) = -\frac{13}{4}$, the vertex is $\left(\frac{1}{2}, -\frac{13}{4}\right)$.



The graph of p is above the x-axis when $x < \frac{1-\sqrt{13}}{2}$ or $x > \frac{1+\sqrt{13}}{2}$. Since the inequality is not strict, the solution set is $\left\{ x \,\middle|\, x \le \frac{1-\sqrt{13}}{2} \text{ or } x \ge \frac{1+\sqrt{13}}{2} \right\}$ or, using interval notation, $\left(-\infty, \frac{1-\sqrt{13}}{2}\right] \cup \left[\frac{1+\sqrt{13}}{2}, \infty\right)$.

32. $f(x) = -x^2 - x + 1; \quad g(x) = -x^2 + x + 6$

a. $f(x) = 0$

$-x^2 - x + 1 = 0$

$x^2 + x - 1 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$f(x) = g(x)$

$-x^2 - x + 1 = -x^2 + x + 6$

$-2x - 5 = 0$

$-2x = 5$

$x = -\frac{5}{2}$

Solution set: $\left\{-\frac{5}{2}\right\}$.

d. $f(x) > 0$

$-x^2 - x + 1 > 0$

We graph the function $f(x) = -x^2 - x + 1$.

y-intercept: $f(0) = -1$

x-intercepts: $-x^2 - x + 2 = 0$

$x^2 + x - 2 = 0$

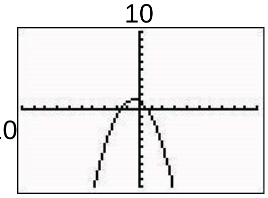
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$x \approx -1.62$ or $x \approx 0.62$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(-1)} = \frac{1}{-2} = -\frac{1}{2}$.

Since $f\left(-\frac{1}{2}\right) = \frac{5}{4}$, the vertex is $\left(-\frac{1}{2}, \frac{5}{4}\right)$.



The graph is above the x-axis when $\frac{-1-\sqrt{5}}{2} < x < \frac{-1+\sqrt{5}}{2}$. Since the inequality is strict, the solution set is $\left\{ x \,\middle|\, \frac{-1-\sqrt{5}}{2} < x < \frac{-1+\sqrt{5}}{2} \right\}$ or, using interval

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Solution set: $\left\{\dfrac{-1-\sqrt{5}}{2}, \dfrac{-1+\sqrt{5}}{2}\right\}$.

b. $g(x) = 0$

$-x^2 + x + 6 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3;\ x = -2$

Solution set: $\{-2, 3\}$.

notation, $\left(\dfrac{-1-\sqrt{5}}{2}, \dfrac{-1+\sqrt{5}}{2}\right)$.

e. $g(x) \le 0$

$-x^2 + x + 6 \le 0$

We graph the function $g(x) = -x^2 + x + 6$.

y-intercept: $g(0) = 6$

x-intercepts: $-x^2 + x + 6 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3;\ x = -2$

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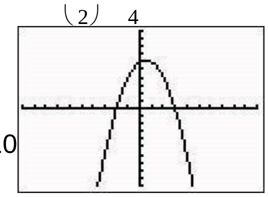
The vertex is at $x = \frac{-b}{2a} = \frac{-(1)}{2(-1)} = \frac{-1}{-2} = \frac{1}{2}$.

Since $f\left(\frac{1}{2}\right) = \frac{25}{4}$, the vertex is $\left(\frac{1}{2}, \frac{25}{4}\right)$.



−10 10

−10

The graph is below the x-axis when $x < -2$ or $x > 3$. Since the inequality is not strict, the solution set is $\{x \mid x \le -2 \text{ or } x \ge 3\}$ or, using interval notation, $(-\infty, 2] \cup [3, \infty)$.

$$f(x) > g(x)$$
$$x^2 - x + 1 > -x^2 + x + 6 - 2x > 5$$
$$x < -\frac{5}{2}$$

The solution set is $\left\{x \mid x < -\frac{5}{2}\right\}$ or, using interval notation, $\left(-\infty, -\frac{5}{2}\right)$.

g.
$$f(x) \ge 1$$
$$-x^2 - x + 1 \ge 1$$
$$-x^2 - x \ge 0$$

We graph the function $p(x) = -x^2 - x$.

y-intercept: $p(0) = 0$

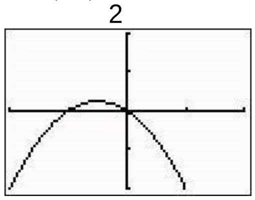
x-intercepts: $-x^2 - x = 0$

$$-x(x + 1) = 0$$
$$x = 0; x = -1$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(-1)} = \frac{1}{-2} = -\frac{1}{2}$.

Since $p\left(-\frac{1}{2}\right) = \frac{1}{4}$, the vertex is $\left(-\frac{1}{2}, \frac{1}{4}\right)$.

2



−2 2

−2

The graph of p is above the x-axis when $-1 < x < 0$. Since the inequality is not strict, the solution set is $\{x \mid -1 \le x \le 0\}$ or, using interval notation, $[-1, 0]$.

a. The ball strikes the ground when $s(t) = 80t - 16t^2 = 0$.

$$80t - 16t^2 = 0$$
$$16t(5 - t) = 0$$
$$t = 0, t = 5$$

The ball strikes the ground after 5 seconds.

Find the values of t for which $80t - 16t^2 > 96$

$$-16t^2 + 80t - 96 > 0$$

We graph the function $f(t) = -16t^2 + 80t - 96$. The intercepts are y-intercept: $f(0) = -96$

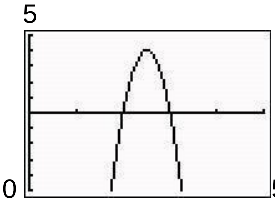
t-intercepts: $-16t^2 + 80t - 96 = 0$

$$-16(t^2 - 5t + 6) = 0$$
$$16(t - 2)(t - 3) = 0$$
$$t = 2, t = 3$$

The vertex is at $t = \frac{-b}{2a} = \frac{-(80)}{2(-16)} = 2.5$.

Since $f(2.5) = 4$, the vertex is $(2.5, 4)$.

5



0 5

−5

The graph of f is above the t-axis when $2 < t < 3$. Since the inequality is strict, the solution set is $\{t \mid 2 < t < 3\}$ or, using interval notation, $(2, 3)$. The ball is more than 96 feet above the ground for times between 2 and 3 seconds.

a. The ball strikes the ground when $s(t) = 96t - 16t^2 = 0$.

$$96t - 16t^2 = 0$$
$$16t(6 - t) = 0$$
$$t = 0, t = 6$$

The ball strikes the ground after 6 seconds.

Find the values of t for which

$$96t - 16t^2 > 128$$

$$-16t^2 + 96t - 128 > 0$$

We graph $f(t) = -16t^2 + 96t - 128$. The intercepts are

y-intercept: $f(0) = -128$

t-intercepts: $-16t^2 + 96t - 128 = 0$

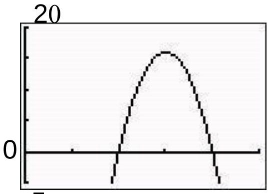
$16(t^2 - 6t + 8) = 0$

$-16(t-4)(t-2) = 0$

$t = 4, t = 2$

The vertex is at $t = \frac{-b}{2a} = \frac{-(96)}{2(-16)} = 3$. Since $f(3) = 16$, the vertex is $(3, 16)$.

20



0 5

−5

The graph of f is above the t-axis when $2 < t < 4$. Since the inequality is strict, the solution set is $\{t \mid 2 < t < 4\}$ or, using interval notation, $(2, 4)$. The ball is more than 128 feet above the ground for times between 2 and 4 seconds.

a. $R(p) = -4p^2 + 4000p = 0$

$4p(p - 1000) =$

$0\ p = 0, p = 1000$

Thus, the revenue equals zero when the price is \$0 or \$1000.

Find the values of p for which

$4p^2 + 4000p > 800{,}000$

$4p^2 + 4000p - 800{,}000 > 0$

We graph $f(p) = -4p^2 + 4000p - 800{,}000$.

The intercepts are

y-intercept: $f(0) = -800{,}000$

p-intercepts:

$-4p^2 + 4000p - 800000 = 0$

$p^2 - 1000p + 200000 = 0$

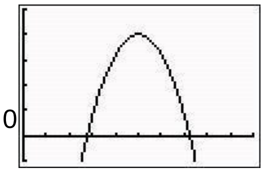
$= \frac{-(-1000) \pm \sqrt{(-1000)^2 - 4(1)(200000)}}{2(1)}$

$\frac{1000 \pm \sqrt{200000}}{2}$

$\frac{1000 \pm 200\sqrt{5}}{2}$

Since $f(500) = 200{,}000$, the vertex is $(500, 200000)$.

250,000



0 1000

−50,000

The graph of f is above the p-axis when $276.39 < p < 723.61$. Since the inequality is strict, the solution set is $\{p \mid 276.39 < p < 723.61\}$ or, using interval notation, $(276.39, 723.61)$. The revenue is more than \$800,000 for prices between \$276.39 and \$723.61.

a. $R(p) = -\frac{1}{2}p^2 + 1900p = 0$

$\frac{1}{2}p(p - 3800) = 0$

$p = 0, p = 3800$

Thus, the revenue equals zero when the price is \$0 or \$3800.

Find the values of p for which

$\frac{1}{2}p^2 + 1900p > 1200000$

$\frac{1}{2}p^2 + 1900p - 1200000 > 0$

We graph $f(p) = -\frac{1}{2}p^2 + 1900p - 1200000$.

y-intercept: $f(0) = -1{,}200{,}000$

p-intercepts: $-\frac{1}{2}p^2 + 1900p - 1200000$

$= 0\ p \quad - 3800p + 2400000 = 0$

$p - 800)(p - 3000)$

$= 0\ p = 800;\ p = 3000$

$500 \pm 100\ 5$

$p \approx 276.39;\ p \approx 723.61$.

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$$= \frac{}{2(1/2)} = 1900.$$

The vertex is at $p = \frac{-b}{2a} = \frac{-(4000)}{2(-4)} = 500.$

Since $f(1900) = 605{,}000$, the vertex is $(1900, 605000)$.

The vertex is at $p = \frac{-b}{2a} = \frac{-(-1900)}{}$

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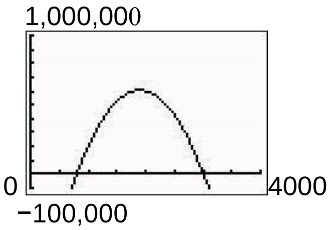


The graph of f is above the p-axis when $800 < p < 3000$. Since the inequality is strict, the solution set is $\{ p | 800 < p < 3000 \}$ or, using interval notation, $(800, 3000)$. The revenue is more than $1,200,000 for prices between $800 and $3000.

$(x-4)^2 \le 0$

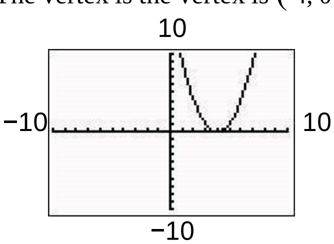
We graph the function $f(x) = (x-4)^2$.
y-intercept: $f(0) = 16$

x-intercepts: $(x-4)^2 = 0$

$x - 4 = 0$

$x = 4$

The vertex is the vertex is $(4, 0)$.

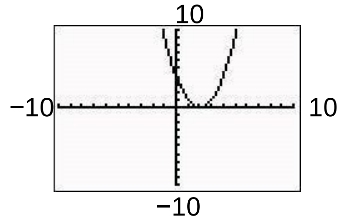


The graph is never below the x-axis. Since the inequality is not strict, the only solution comes from the x-intercept. Therefore, the given inequality has exactly one real solution, namely $x = 4$.

$(x-2)^2 > 0$

We graph the function $f(x) = (x-2)^2$.
y-intercept: $f(0) = 4$
x-intercepts: $(x-2)^2 = 0$
$x - 2 = 0$
$x = 2$

The vertex is the vertex is $(2, 0)$.



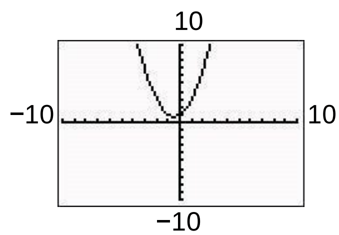
The graph is above the x-axis when $x < 2$ or $x > 2$. Since the inequality is strict, the solution set is $\{ x | x < 2 \text{ or } x > 2 \}$. Therefore, the given inequality has exactly one real number that is not a solution, namely $x \ne 2$.

Solving $x^2 + x + 1 > 0$
We graph the function $f(x) = x^2 + x + 1$.
y-intercept: $f(0) = 1$

x-intercepts: $b^2 - 4ac = 1^2 - 4(1)(1) = -3$,

so f has no x-intercepts.

The vertex is at $x = \frac{-b}{2a} = \frac{-(1)}{2(1)} = -\frac{1}{2}$. Since $f\left(-\frac{1}{2}\right) = \frac{3}{4}$, the vertex is $\left(-\frac{1}{2}, \frac{3}{4}\right)$.



The graph is always above the x-axis. Thus, the solution is the set of all real numbers or, using interval notation, $(-\infty, \infty)$.

Solving $x^2 - x + 1 < 0$
We graph the function $f(x) = x^2 - x + 1$.
y-intercept: $f(0) = 1$
x-intercepts: $b^2 - 4ac = (-1)^2 - 4(1)(1) = -3$, so f has no x-intercepts.
The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$. Since $f\left(\frac{1}{2}\right) = \frac{3}{4}$

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, the vertex is

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| - ,
|.

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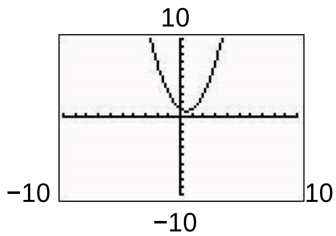


The graph is never below the x-axis. Thus, the inequality has no solution. That is, the solution set is { } or ∅ .

If the inequality is not strict, we include the x-intercepts in the solution.

Since the radical cannot be negative we determine what makes the radicand a nonnegative number.

$$10 - 2x \geq 0$$
$$-2x \geq -10$$
$$x \leq 5$$

So the domain is: $\{ x \mid x \leq 5 \}$.

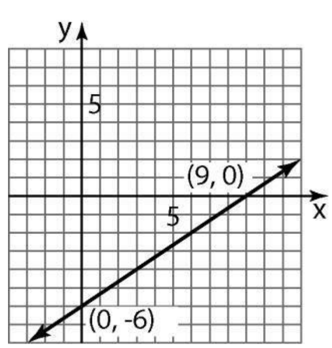
$$f(-x) = \frac{-(-x)}{(-x)^2 + 9}$$
$$= -\frac{-x}{x^2 + 9} = -f(x)$$

Since $f(-x) = -f(x)$ then the function is odd.

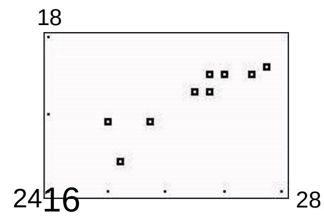
44. a. $0 = \frac{2}{3}x - 6$

$x = 9$

b.



45. a.



Using the LINear REGression program, the line of best fit is: $C(H) = 0.3734H + 7.3268$

If height increases by one inch, the head circumference increases by 0.3734 inch.

$C(26) = 0.3734(26) + 7.3268 \approx 17.0$ inches

To find the height, we solve the following equation:

$$17.4 = 0.3734H + 7.3268$$
$$10.0732 = 0.3734H$$
$$26.98 \approx H$$

A child with a head circumference of 17.4 inches would have a height of about 26.98 inches.

Section 2.6

$R = 3x$

Use LIN REGression to get

$y = 1.7826x + 4.0652$

a. $R(x) = x\left(-\frac{1}{6}x + 100\right) = -\frac{1}{6}x^2 + 100x$

The quantity sold price cannot be negative, so $x \geq 0$. Similarly, the price should be positive, so $p > 0$.

$$\frac{1}{6}x + 100 > 0$$
$$\frac{1}{6}x > -100$$
$$x < 600$$

Thus, the implied domain for R is $\{ x \mid 0 \leq x < 600\}$ or $[0, 600)$.

c. $R(200) = -\frac{1}{6}(200)^2 + 100(200)$

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$$= -\frac{20000}{3} + 20000$$

$$\frac{40000}{3} \approx \$13,333.33$$

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$x = \frac{-b}{2a} = \frac{-100}{2\left(-\frac{1}{6}\right)} = \frac{-100}{\left(-\frac{1}{3}\right)} = \frac{300}{1} = 300$

The maximum revenue is

$R(300) = -\frac{1}{6}(300)^2 + 100(300)$

$-15000 + 30000$

$15,000

$p = -\frac{1}{6}(300) + 100 = -50 + 100 = \$50$

a. $R(x) = x\left(-\frac{1}{3}x + 100\right) = -\frac{1}{3}x^2 + 100x$

The quantity sold price cannot be negative, so $x \geq 0$. Similarly, the price should be positive, so $p > 0$.

$-\frac{1}{3}x + 100 > 0$

$-\frac{1}{3}x > -100$

$x < 300$

Thus, the implied domain for R is $\{x \mid 0 \leq x < 300\}$ or $[0, 300)$.

c. $R(100) = -\frac{1}{3}(100)^2 + 100(100)$

$= \frac{-10000}{3} + 10000$

$\frac{20000}{3} \approx \$6,666.67$

$x = \frac{-b}{2a} = \frac{-100}{2\left(-\frac{1}{3}\right)} = \frac{-100}{\left(-\frac{2}{3}\right)} = \frac{300}{2} = 150$

The maximum revenue is

$R(150) = -\frac{1}{3}(150)^2 + 100(150)$

$-7500 + 15000 = \$7,500$ **e.**

c. $x = \frac{-b}{2a} = \frac{-20}{2\left(-\frac{1}{5}\right)} = \frac{-20}{\left(-\frac{2}{5}\right)} = \frac{100}{2} = 50$

The maximum revenue is

$R(50) = -\frac{1}{5}(50)^2 + 20(50)$

$= -500 + 1000 = \$500$

d. $p = \frac{100 - 50}{5} = \frac{50}{5} = \$10$

Graph $R = -\frac{1}{5}x^2 + 20x$ and $R = 480$. Find where the graphs intersect by solving

$480 = -\frac{1}{5}x^2 + 20x$.

$\frac{1}{5}x^2 - 20x + 480 = 0$

$x - 40)(x - 60) = 0$

$= 40, x = 60$

Solve for price.

$= -5p + 100$

$40 = -5p + 100 \Rightarrow p = \$12$

$60 = -5p + 100 \Rightarrow p = \$8$

The company should charge between \$8 and \$12.

a. If $x = -20p + 500$, then $p = \frac{500 - x}{20}$.

$\left(500 - x\right)$ 1 2

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$p = -\frac{1}{3}(150) + 100 = -50 + 100 = \$50$

5. a. If $x = -5p + 100$, then $p = \frac{100 - x}{5}$.

$R(x) = x\left(\frac{100 - x}{5}\right) = -\frac{1}{5}x^2 + 20x$

b. $R(15) = -\frac{1}{5}(15)^2 + 20(15)$

$= -45 + 300 = \$255$

$R(x) = x\left(\frac{\quad}{20}\right) = -\frac{\quad}{20}x + 25x$

b. $R(20) = -\frac{1}{20}(20)^2 + 25(20)$

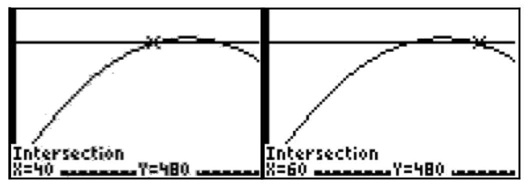
$= -20 + 500 = \$480$

c. $x = \frac{-b}{2a} = \frac{-25}{2\left(-\frac{1}{20}\right)} = \frac{-25}{\left(-\frac{1}{10}\right)} = \frac{250}{1} = 250.$

The maximum revenue is

$R(250) = -\frac{1}{20}(250)^2 + 25(250)$

$= -3125 + 6250 = \$3125$



225
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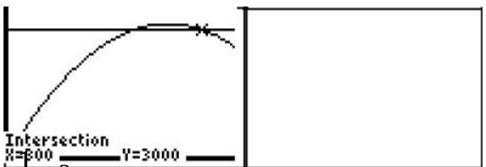
$p\$12.50 \frac{500 - 250}{20} = \frac{250}{20} =$

Graph $R = -\frac{1}{20}x^2 + 25x$ and $R = 3000$.

Find where the graphs intersect by solving

$3000 = -\frac{1}{20}x^2 + 25x$.



$\frac{1}{20}x^2 - 25x + 3000 = 0$

$x - 200)(x - 300) = 0$

$= 200, x = 300$

Solve for price.

$= -20p + 500$

$300 = -20p + 500 \Rightarrow p = \$10$

$200 = -20p + 500 \Rightarrow p = \$15$

The company should charge between \$10 and \$15.

a. Let w = width and l = length of the rectangular area.
Solving $P = 2w + 2l = 400$ for l:

$l = \frac{400 - 2w}{2} = 200 - w$.

Then $A(w) = (200 - w)w = 200w - w^2$

$-w^2 + 200w$

b. $w = \frac{-b}{2a} = \frac{-200}{2(-1)} = \frac{-200}{-2} = 100$ yards

$A(100) = -100^2 + 200(100)$

$-10000 + 20000$

$10,000 \text{ yd}^2$

a. Let x = width and y = width of the rectangle.
Solving $P = 2x + 2y = 3000$ for y:

$y = \frac{3000 - 2x}{2} = 1500 - x$.

Then $A(x) = (1500 - x)x$

$= 1500x - x^2$

$= -x^2 + 1500x$.

b. $x = \frac{-b}{2a} = \frac{-1500}{2(-1)} = \frac{-1500}{-2} = 750$ feet

c. $A(750) = -750^2 + 1500(750)$

$= -562500 + 1125000$

$= 562,500 \text{ ft}^2$

9. Let x = width and y = length of the rectangle.
Solving $P = 2x + y = 4000$ for y:

$y = 4000 - 2x$.

Then $A(x) = (4000 - 2x)x$

$= 4000x - 2x^2$

$= -2x^2 + 4000x$

$x = \frac{-b}{2a} = \frac{-4000}{2(-2)} = \frac{-4000}{-4} = 1000$ meters maximizes area.

$A(1000) = -2(1000)^2 + 4000(1000)$.

$-2000000 + 4000000$

$2,000,000$

The largest area that can be enclosed is 2,000,000 square meters.

Let x = width and y = length of the rectangle.

$2x + y = 2000$

$= 2000 - 2x$

Then $A(x) = (2000 - 2x)x$

$= 2000x - 2x^2$

$= -2x^2 + 2000x$

$x = \frac{-b}{2a} = \frac{-2000}{2(-2)} = \frac{-2000}{-4} = 500$ meters maximizes area.

$A(500) = -2(500)^2 + 2000(500)$

$-500,000 + 1,000,000$

$500,000$

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The largest area that can be enclosed is 500,000 square meters.

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11. $h(x) = \frac{-32x^2}{(50)^2} + x + 200 = -\frac{8}{625}x^2 + x + 200$

$a = -\frac{8}{625}$, $b = 1$, $c = 200$.

The maximum height occurs when

$x = \frac{-b}{2a} = \frac{-1}{2(-8/625)} = \frac{625}{16} \approx 39$ feet from base of the cliff.

The maximum height is

$h\left(\frac{625}{16}\right) = \frac{-8}{625}\left(\frac{625}{16}\right)^2 + \frac{625}{16} + 200$

$= \frac{7025}{32} \approx 219.5$ feet.

c. Solving when $h(x) = 0$:

$-\frac{8}{625}x^2 + x + 200 = 0$

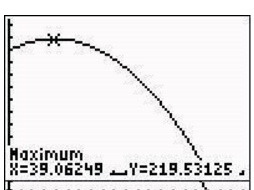
$x = \frac{-1 \pm \sqrt{1^2 - 4(-8/625)(200)}}{2(-8/625)}$

$x \approx \frac{-1 \pm \sqrt{11.24}}{-0.0256}$

$x \approx -91.90$ or $x \approx 170$

Since the distance cannot be negative, the projectile strikes the water approximately 170 feet from the base of the cliff.

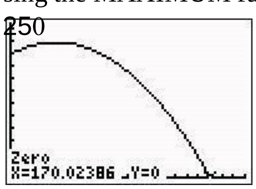
250



0 0 200

Using the MAXIMUM function

250



$-\frac{8}{625}x^2 + x + 200 = 100$

$-\frac{8}{625}x^2 + x + 100 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(-8/625)(100)}}{2(-8/625)} = \frac{-1 \pm \sqrt{6.12}}{-0.0256}$

$x \approx -57.57$ or $x \approx 135.70$

Since the distance cannot be negative, the projectile is 100 feet above the water when it is approximately 135.7 feet from the base of the cliff.

a. $h(x) = \frac{-32x^2}{(100)^2} + x = -\frac{2}{625}x^2 + x$

$a = -\frac{2}{625}$, $b = 1$, $c = 0$.

The maximum height occurs when

$x = \frac{-b}{2a} = \frac{-1}{2(-2/625)} = \frac{625}{4} = 156.25$ feet

The maximum height is

$h\left(\frac{625}{4}\right) = \frac{-2}{625}\left(\frac{625}{4}\right)^2 + \frac{625}{4}$

$= \frac{625}{8} = 78.125$ feet

c. Solving when $h(x) = 0$:

$-\frac{2}{625}x^2 + x = 0$

$x\left(-\frac{2}{625}x + 1\right) = 0$

0 0 200

Using the ZERO function

250

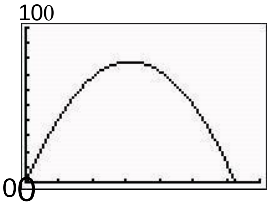
00 200

$x = 0$ or $-\frac{2}{625}x + 1 = 0$

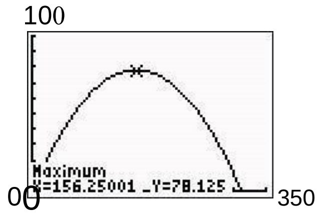
$x = 0$ or $1 = \frac{2}{625}x$

$x = 0$ or $x = \frac{625}{2} = 312.5$

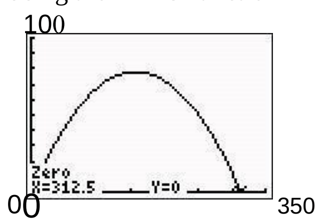
Since the distance cannot be zero, the projectile lands 312.5 feet from where it was fired.



Using the MAXIMUM function



Using the ZERO function



Solving when $h(x) = 50$:

$$-\frac{2}{625}x^2 + x = 50$$

$$-\frac{2}{625}x^2 + x - 50 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(-2/625)(-50)}}{2(-2/625)}$$

$$= \frac{-1 \pm \sqrt{0.36}}{-0.0064} \approx \frac{-1 \pm 0.6}{-0.0064}$$

$x = 62.5$ or $x = 250$

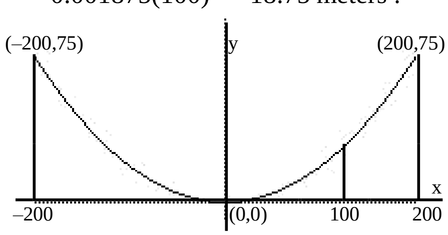
The projectile is 50 feet above the ground 62.5 feet and 250 feet from where it was fired.

Locate the origin at the point where the cable touches the road. Then the equation of the parabola is of the form: $y = ax^2$, where $a > 0$. Since the point (200, 75) is on the parabola, we can find the constant a:

Since $75 = a(200)^2$, then $a = \frac{75}{200^2} = 0.001875$.

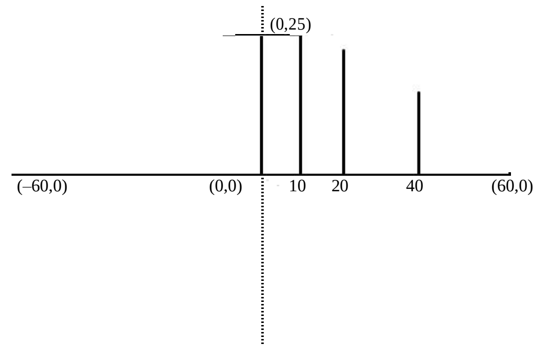
When $x = 100$, we have:

$= 0.001875(100)^2 = 18.75$ meters.



Locate the origin at the point directly under the highest point of the arch. Then the equation of the parabola is of the form: $y = -ax^2 + k$, where $a > 0$. Since the maximum height is 25

feet, when $x = 0$, $y = k = 25$. Since the point (60, 0) is on the parabola, we can find the constant a: Since $0 = -a(60)^2 + 25$ then $a = \frac{25}{60^2}$. The equation of the parabola is:



At $x = 10$:

$$h(10) = -\frac{25}{60^2}(10)^2 + 25 = -\frac{25}{36} + 25 \approx 24.3 \text{ ft.}$$

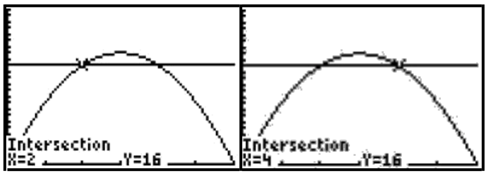
$$h(20) = -\frac{25}{60^2}(20)^2 + 25 = -\frac{25}{9} + 25 \approx 22.2 \text{ ft.}$$

At $x = 40$:

$$h(40) = -\frac{25}{60^2}(40)^2 + 25 = -\frac{100}{9} + 25 \approx 13.9 \text{ ft.}$$

a. Let x = the depth of the gutter and y the width of the gutter. Then $A = xy$ is the cross- sectional area of the gutter. Since the aluminum sheets for the gutter are 12 inches wide, we have $2x + y = 12$. Solving for y :
$y = 12 - 2x$. The area is to be maximized, so:
$A = xy = x(12 - 2x) = -2x^2 + 12x$. This equation is a parabola opening down; thus, it has a maximum when $x = \frac{-b}{2a} = \frac{-12}{2(-2)} = \frac{-12}{-4} = 3$.
Thus, a depth of 3 inches produces a maximum cross-sectional area.

Graph $A = -2x^2 + 12x$ and $A = 16$. Find where the graphs intersect by solving $16 = -2x^2 + 12x$.



$$2x^2 - 12x + 16 = 0$$
$$x^2 - 6x + 8 = 0$$
$$(x - 4)(x - 2) = 0$$
$$x = 4,\ x = 2$$

The graph of $A = -2x^2 + 12x$ is above the graph of $A = 16$ where the depth is between 2 and 4 inches.

16. Let x = width of the window and y = height of the rectangular part of the window. The perimeter of the window is: $x + 2y + \frac{\pi x}{2} = 20$.

Solving for y: $y = \frac{40 - 2x - \pi x}{4}$.

The area of the window is:

$$A(x) = x\left(\frac{40 - 2x - \pi x}{4}\right) + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$
$$= 10x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8}$$
$$= \left(-\frac{1}{2} - \frac{\pi}{8}\right)x^2 + 10x.$$

This equation is a parabola opening down; thus, it has a maximum when

$$x = \frac{-b}{2a} = \frac{-10}{2\left(-\frac{1}{2} - \frac{\pi}{8}\right)} = \frac{10}{\left(1 + \frac{\pi}{4}\right)} \approx 5.6 \text{ feet}$$

$$y = \frac{40 - 2(5.60) - \pi(5.60)}{4} \approx 2.8 \text{ feet}$$

Solving for x:

$$\pi x + 2y = 1500$$
$$\pi x = 1500 - 2y$$
$$x = \frac{1500 - 2y}{\pi}$$

The area of the rectangle is:

$$A = xy = \left(\frac{1500 - 2y}{\pi}\right)y = \frac{-2}{\pi}y^2 + \frac{1500}{\pi}y.$$

This equation is a parabola opening down; thus, it has a maximum when

$$y = \frac{-b}{2a} = \frac{-\frac{1500}{\pi}}{2\left(-\frac{2}{\pi}\right)} = \frac{-\frac{1500}{\pi}}{-\frac{4}{\pi}} = 375.$$

Thus, $x = \frac{1500 - 2(375)}{\pi} = \frac{750}{\pi} \approx 238.73$

The dimensions for the rectangle with maximum area are $\frac{750}{\pi} \approx 238.73$ meters by 375 meters.

Let x = width of the window and y = height of the rectangular part of the window. The perimeter of the window is:

$$3x + 2y = 16$$
$$y = \frac{16 - 3x}{2}$$

The area of the window is

$$A(x) = x\left(\frac{16 - 3x}{2}\right) + \frac{\sqrt{3}}{4}x^2$$
$$= 8x - \frac{3}{2}x^2 + \frac{\sqrt{3}}{4}x^2$$
$$= \left(-\frac{3}{2} + \frac{\sqrt{3}}{4}\right)x^2 + 8x$$

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The width of the window is about 5.6 feet and the height of the rectangular part is approximately 2.8 feet. The radius of the semicircle is roughly 2.8 feet, so the total height is about 5.6 feet.

Let x = the width of the rectangle or the diameter of the semicircle and let y = the length of the rectangle. The perimeter of each semicircle is $\frac{\pi x}{2}$. The perimeter of the track is given by: $\frac{\pi x}{2} + \frac{\pi x}{2} + y + y = 1500$.

This equation is a parabola opening down; thus, it has a maximum when

$$x = \frac{-b}{2a} = \frac{-8}{2\left(-\frac{3}{2} + \frac{\sqrt{3}}{4}\right)} = \frac{-8}{-3 + \frac{\sqrt{3}}{2}} = \frac{-16}{-6 + \sqrt{3}} \approx 3.75 \text{ ft.}$$

The window is approximately 3.75 feet wide.

$$y = \frac{16 - 3\left(\frac{-16}{-6+\sqrt{3}}\right)}{2} = \frac{16 + \frac{48}{-6+\sqrt{3}}}{2} = 8 + \frac{24}{-6+\sqrt{3}}$$

The height of the equilateral triangle is

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$\frac{\sqrt{3}}{2}\left(\frac{-16}{-6+\sqrt{3}}\right)=\frac{-8\sqrt{3}}{-6+\sqrt{3}}$ feet, so the total height is

$8+\frac{24}{-6+\sqrt{3}}+\frac{-8\sqrt{3}}{-6+\sqrt{3}}\approx 5.62$ feet.

We are given: $V(x)=kx(a-x)=-kx^2+akx$. The reaction rate is a maximum when:

$x=\frac{-b}{2a}=\frac{-ak}{2(-k)}=\frac{ak}{2k}=\frac{a}{2}$.

We have:

$a(-h)^2+b(-h)+c=ah^2-bh+c=y_0$

$a(0)^2+b(0)+c=c=y_1$

$a(h)^2+b(h)+c=ah^2+bh+c=y_2$

Equating the two equations for the area, we have:

$y_0+4y_1+y_2=ah^2-bh+c+4c+ah^2+bh+c$

$=2ah^2+6c.$

Therefore,

Area $=\frac{h}{3}\left(2ah^2+6c\right)=\frac{h}{3}\left(y_0+4y_1+y_2\right)$ sq. units.

$f(x)=-5x^2+8,\ h=1$

Area $=\frac{h}{3}\left(2ah^2+6c\right)=\frac{1}{3}\left(2(-5)(1)^2+6(8)\right)$

$\frac{1}{3}(-10+48)=\frac{38}{3}$ sq. units

22. $f(x)=2x^2+8,\quad h=2$

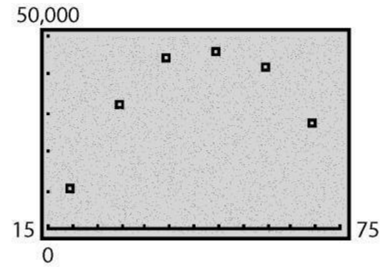
Area $=\frac{h}{3}(2ah^2+6c)=\frac{2}{3}\left(2(2)(2)^2+6(8)\right)$

$\frac{2}{3}(16+48)=\frac{2}{3}(64)=\frac{128}{3}$ sq. units

$f(x)=x^2+3x+5,\ h=4$

Area $=\frac{h}{3}\left(2ah^2+6c\right)=\frac{4}{3}\left(2(1)(4)^2+6(5)\right)$

25. a.



From the graph, the data appear to follow a quadratic relation with $a<0$.

Using the QUADratic REGression program

QuadReg
y=ax²+bx+c
a=-43.33464286
b=4184.883214
c=-54062.43902

$I(x)=-43.335x^2+4184.883x-54{,}062.439$

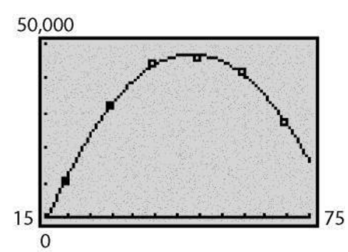
c. $x=\frac{-b}{2a}=\frac{-4184.883}{2(-43.335)}\approx 48.3$

An individual will earn the most income at about 48.3 years of age.

The maximum income will be: $I(48.3)=$ $-43.335(48.3)^2+4184.883(48.3)-54{,}062.439$

\$46,972

e.



26. a. 80

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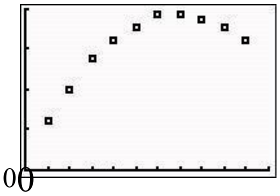
$$\frac{4}{3}(32+30)=\frac{248}{3} \text{ sq. units}$$

24. $f(x)=-x^2+x+4,\ h=1$

$$\text{Area}=\frac{h}{3}\left(2ah^2+6c\right)=\frac{1}{3}\left(2(-1)(1)^2+6(4)\right)$$

$$=\frac{1}{3}\left(-2+24\right)=\frac{1}{3}(22)=\frac{22}{3} \text{ sq. units}$$


0 0 220

From the graph, the data appear to follow a quadratic relation with $a<0$.

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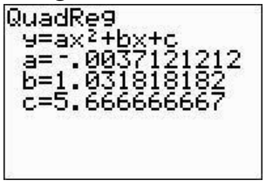
Using the QUADratic REGression program



$h(x) = -0.0037x^2 + 1.0318x + 5.6667$

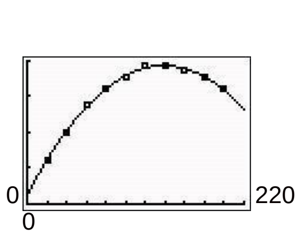
c. $x = \dfrac{-b}{2a} = \dfrac{-1.0318}{2(-0.0037)} \approx 139.4$

The ball will travel about 139.4 feet before it reaches its maximum height.

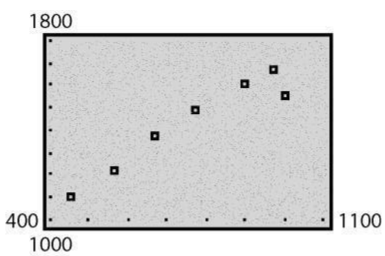
The maximum height will be: $h(139.4) =$

$-0.0037(139.4)^2 + 1.0318(139.4) + 5.6667$
77.6 feet

e.

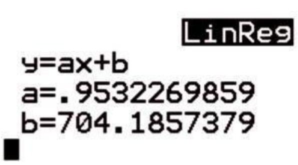


27. a.



From the graph, the data appear to be linearly related with $m > 0$.

Using the LINear REGression program

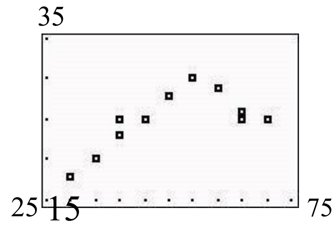


$R(x) = 0.953x + 704.186$

$R(850) = 0.953(850) + 704.186 \approx 1514$

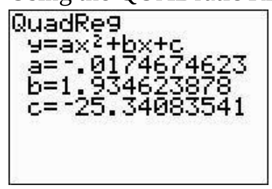
The rent for an 850 square-foot apartment in San Diego will be about $1514 per month.

28. a.



From the graph, the data appear to follow a quadratic relation with $a < 0$.

Using the QUADratic REGression program

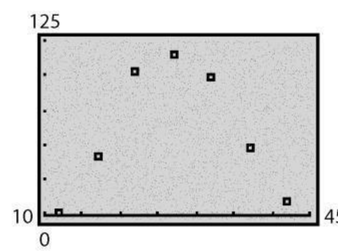


$M(s) = -0.017s^2 + 1.935s - 25.341$

$M(63) = -0.017(63)^2 + 1.935(63) - 25.341$
29.1

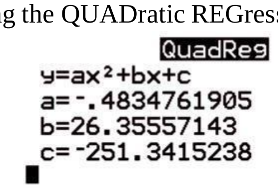
A Camry traveling 63 miles per hour will get about 29.1 miles per gallon.

29. a.



From the graph, the data appear to follow a quadratic relation with $a < 0$.

Using the QUADratic REGression program



$B(a) = -0.483a^2 + 26.356a - 251.342$

c.

$B(35) = -0.483(35)^2 + 26.356(35) - 251.342$
79.4

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The birthrate of 35-year-old women is about 79.4 per 1000.

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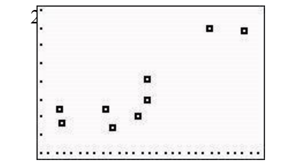
30. a.



From the graph, the data appear to be linearly related with $m > 0$.

Using the LINear REGression program

```
LinReg
 y=ax+b
 a=.2330507161
 b=-2.037230647
 r²=.7610474345
 r=.8723803267
```

$C(x) = 0.233x - 2.037$

$C(80) = 0.233(80) - 2.037 \approx 16.6$

When the temperature is 80°F , there will be about 16.6 chirps per second.

Answers will vary. One possibility follows: If the price is \$140, no one will buy the calculators, thus making the revenue \$0.

$\sqrt{-225} = \sqrt{(-1)(225)} = 15i$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{((-1) - 4)^2 + (5 - (-7))^2}$

$\sqrt{(-5)^2 + (12)^2}$

$= \sqrt{25 + 144} = \sqrt{169} = 13$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - (-6))^2 + (y - 0)^2 = (\sqrt{7})^2$

$(x + 6)^2 + y^2 = 7$

35. $x = \dfrac{-(8) \pm \sqrt{8^2 - 4(5)(-3)}}{2(5)} = \dfrac{-8 \pm \sqrt{64+60}}{10}$

$= \dfrac{-8 \pm \sqrt{124}}{10} = \dfrac{-8 \pm 2\sqrt{31}}{10} = \dfrac{-4 \pm \sqrt{31}}{5}$

So the zeros are: $\dfrac{-4 + \sqrt{31}}{5}, \dfrac{-4 - \sqrt{31}}{5}$

Section 2.7

True; the set of real numbers consists of all rational and irrational numbers.

$10 - 5i$

$2 - 5i$

True

$9i$

$2 + 3i$

True

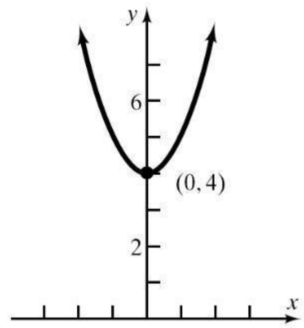
$f(x) = 0$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4} = \pm 2i$

The zero are $-2i$ and $2i$.



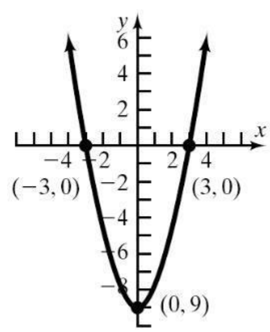
$f(x) = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm\sqrt{9} = \pm 3$

The zeros are -3 and 3.



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Integers: $\{-3, 0\}$

Rationals: $\left\{-3, 0, \frac{6}{5}\right\}$

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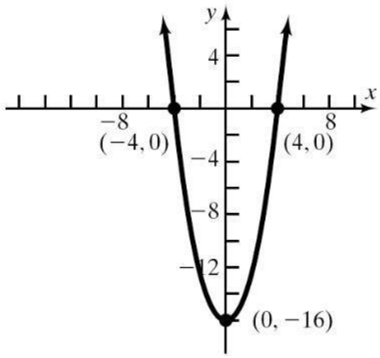
$f(x) = 0$

$^2 - 16 = 0$

$^2 = 16$

$x = \pm\sqrt{16} = \pm 4$

The zeros are -4 and 4.



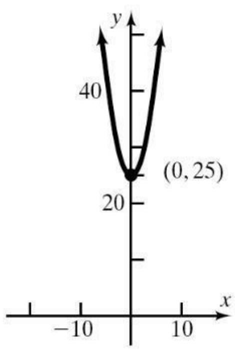
$f(x) = 0$

$^2 + 25 = 0$

$^2 = -25$

$= \pm\sqrt{-25} = \pm 5i$

The zeros are $-5i$ and $5i$.



$f(x) = 0$

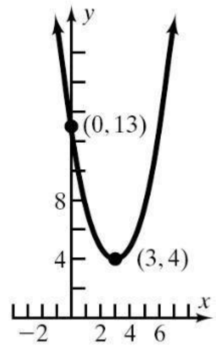
$x^2 - 6x + 13 = 0$

$= 1, b = -6, c = 13,$

$^2 - 4ac = (-6)^2 - 4(1)(13) = 36 - 52 = -16$

$x = \frac{-(-6) \pm \sqrt{-16}}{2(1)} = \frac{6 \pm 4i}{2} = 3 \pm 2i$

The zeros are $3 - 2i$ and $3 + 2i$.



$f(x) = 0$

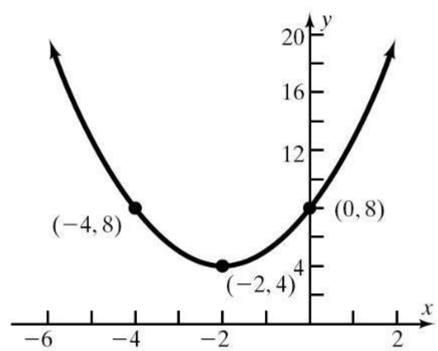
$x^2 + 4x + 8 = 0$

$a = 1, b = 4, c = 8$

$b^2 - 4ac = 4^2 - 4(1)(8) = 16 - 32 = -16$

$x = \frac{-4 \pm \sqrt{-16}}{2(1)} = \frac{-4 \pm 4i}{2} = -2 \pm 2i$

The zeros are $-2 - 2i$ and $-2 + 2i$.



$f(x) = 0$

$x^2 - 6x + 10 = 0$

$= 1, b = -6, c = 10$

$b^2 - 4ac = (-6)^2 - 4(1)(10) = 36 - 40 = -4$

$x = \frac{-(-6) \pm \sqrt{-4}}{2(1)} = \frac{6 \pm 2i}{2} = 3 \pm i$

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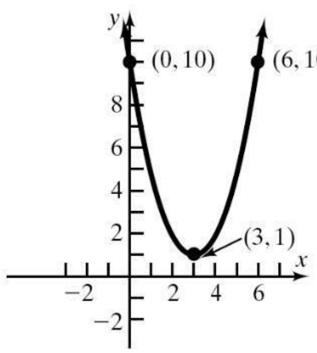
The zeros are $3 - i$ and $3 + i$.



$f(x) = 0$

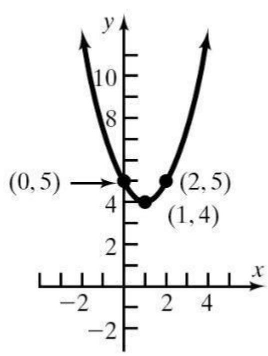
$x^2 - 2x + 5 = 0$

$= 1, b = -2, c = 5$

$b^2 - 4ac = (-2)^2 - 4(1)(5) = 4 - 20 = -16$

$x = \frac{-(-2) \pm \sqrt{-16}}{2(1)} = \frac{2 \pm 4i}{2} = 1 \pm 2i$

The zeros are $1 - 2i$ and $1 + 2i$.



$f(x) = 0$

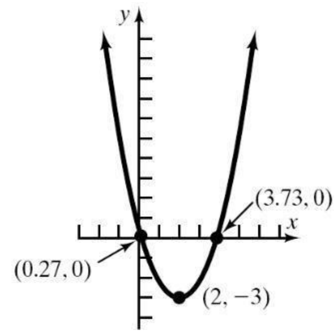
$x^2 - 4x + 1 = 0$

$= 1, b = -4, c = 1$

$b^2 - 4ac = (-4)^2 - 4(1)(1) = 16 - 4 = 12$

$x = \frac{-(-4) \pm \sqrt{12}}{2(1)} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

The zeros are $2 - \sqrt{3}$ and $2 + \sqrt{3}$, or approximately 0.27 and 3.73.



$f(x) = 0$

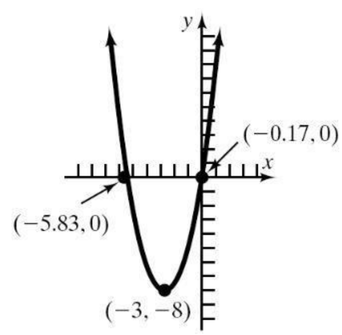
$x^2 + 6x + 1 = 0$

$= 1, b = 6, c = 1$

$b^2 - 4ac = 6^2 - 4(1)(1) = 36 - 4 = 32$

$x = \frac{-6 \pm \sqrt{32}}{2(1)} = \frac{-6 \pm 4\sqrt{2}}{2} = -3 \pm 2\sqrt{2}$

The zeros are $-3 - 2\sqrt{2}$ and $-3 + 2\sqrt{2}$, or approximately -5.83 and -0.17 .



$f(x) = 0$

$x^2 + 2x + 1 = 0$

$a = 2, b = 2, c = 1$

$b^2 - 4ac = (2)^2 - 4(2)(1) = 4 - 8 = -4$

$x = \frac{-2 \pm \sqrt{-4}}{2(2)} = \frac{-2 \pm 2i}{4} = -\frac{1}{2} \pm \frac{1}{2}i$

The zeros are $-\frac{1}{2} - \frac{1}{2}i$ and $-\frac{1}{2} + \frac{1}{2}i$.

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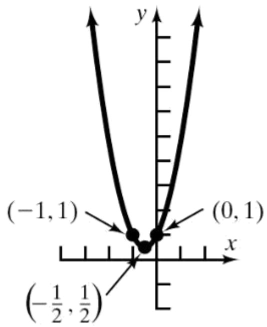


$f(x) = 0$

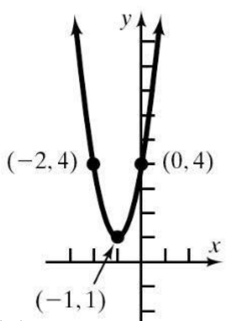
$3x^2 + 6x + 4 = 0$

$a = 3, b = 6, c = 4$

$b^2 - 4ac = (6)^2 - 4(3)(4) = 36 - 48 = -12$

$x = \frac{-6 \pm \sqrt{-12}}{2(3)} = \frac{-6 \pm 2\sqrt{3}i}{6} = -1 \pm \frac{\sqrt{3}}{3}i$

The zeros are $-1 - \frac{\sqrt{3}}{3}i$ and $-1 + \frac{\sqrt{3}}{3}i$.



$f(x) = 0$

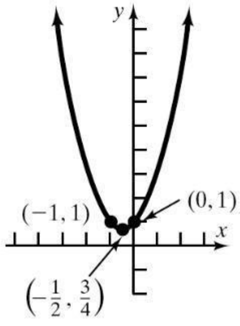
$x^2 + x + 1 = 0$

$= 1, b = 1, c = 1,$

$^2 - 4ac = 1^2 - 4(1)(1) = 1 - 4 = -3$

$x = \frac{-1 \pm \sqrt{-3}}{2(1)} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

The zeros are $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$ and $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$.



$f(x) = 0$

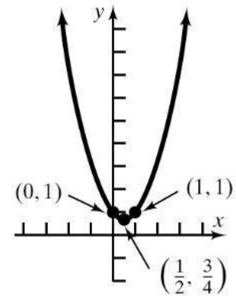
$x^2 - x + 1 = 0$

$= 1, b = -1, c = 1$

$^2 - 4ac = (-1)^2 - 4(1)(1) = 1 - 4 = -3$

$x = \frac{-(-1) \pm \sqrt{-3}}{2(1)} = \frac{1 \pm \sqrt{3}i}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

The zeros are $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ and $\frac{1}{2} + \frac{\sqrt{3}}{2}i$.



$f(x) = 0$

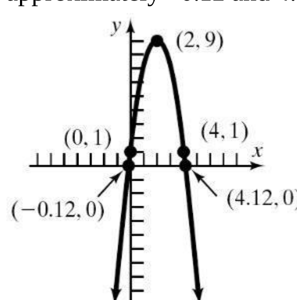
$-2x^2 + 8x + 1 = 0$

$= -2, b = 8, c = 1$

$^2 - 4ac = 8^2 - 4(-2)(1) = 64 + 8 = 72$

$x = \frac{-8 \pm \sqrt{72}}{2(-2)} = \frac{-8 \pm 6\sqrt{2}}{-4} = \frac{4 \pm 3\sqrt{2}}{2} = 2 \pm \frac{3\sqrt{2}}{2}$

The zeros are $\frac{4 - 3\sqrt{2}}{2}$ and $\frac{4 + 3\sqrt{2}}{2}$, or approximately -0.12 and 4.12.



$f(x) = 0$

$-3x^2 + 6x + 1 = 0$

$= -3, b = 6, c = 1$

$^2 - 4ac = 6^2 - 4(-3)(1) = 36 + 12 = 48$

$x = \frac{-6 \pm \sqrt{48}}{} = \frac{-6 \pm 4\sqrt{3}}{} = \frac{3 \pm 2\sqrt{3}}{} = 1 \pm \frac{2\sqrt{3}}{}$

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$2(-3)$ -6 3 3

The zeros are $\frac{3-2\sqrt{3}}{3}$ and $\frac{3+2\sqrt{3}}{3}$, or approximately -0.15 and 2.15.

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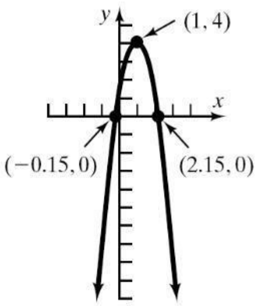

$3x^2 - 3x + 4 = 0$
$a = 3, b = -3, c = 4$

$b^2 - 4ac = (-3)^2 - 4(3)(4) = 9 - 48 = -39$

The equation has two complex solutions that are conjugates of each other.

$2x^2 - 4x + 1 = 0$
$a = 2, b = -4, c = 1$
$b^2 - 4ac = (-4)^2 - 4(2)(1) = 16 - 8 = 8$
The equation has two unequal real number solutions.

$2x^2 + 3x - 4 = 0$
$a = 2, b = 3, c = -4$

$b^2 - 4ac = 3^2 - 4(2)(-4) = 9 + 32 = 41$

The equation has two unequal real solutions.

$x^2 + 2x + 6 = 0$ $a =$

1, $b = 2, c = 6$

$b^2 - 4ac = (2)^2 - 4(1)(6) = 4 - 24 = -20$
The equation has two complex solutions that are conjugates of each other.

$9x^2 - 12x + 4 = 0$

$a = 9, b = -12, c = 4$
$b^2 - 4ac = (-12)^2 - 4(9)(4) = 144 - 144 =$
0 The equation has a repeated real solution.

$4x^2 + 12x + 9 = 0$ $a =$

$t^4 - 16 = 0$
$(t^2 - 4)(t^2 + 4) = 0$
$t^2 = 4 \quad t^2 = -4$
$t = \pm 2 \quad t = \pm 2i$

$y^4 - 81 = 0$
$(y^2 - 9)(y^2 + 9) = 0$
$y^2 = 9 \quad y^2 = -9$
$y = \pm 3 \quad y = \pm 3i$

$F(x) = x^6 - 9x^3 + 8 =$

$0\ (x^3 - 8)(x^3 - 1) = 0$
$(x - 2)(x^2 + 2x + 4)(x - 1)(x^2 + x + 1) = 0$

$^2 + 2x + 4 = 0 \rightarrow a = 1, b = 2, c = 4$
$x = \frac{-2 \pm \sqrt{2^2 - 4(4)}}{2(1)} = \frac{-2 \pm \sqrt{12}}{2} = \frac{-2 \pm 2i\sqrt{3}}{2}$
$= -1 \pm \sqrt{3}i$

$x^2 + x + 1 = 0 \rightarrow a = 1, b = 1, c = 1$
$x = \frac{-1 \pm \sqrt{1^2 - 4\,1}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$

$- \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

The solution set is $\left\{-1 \pm i\sqrt{3}, -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}, 2, 1\right\}$

$P(z) = z^6 + 28z^3 + 27 =$

$0\ (z^3 + 27)(z^3 + 1) = 0$

4, $b = 12, c = 9$
$b^2 - 4ac = 12^2 - 4(4)(9) = 144 - 144 =$
0 The equation has a repeated real solution.

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$(z+3)(z^2-3z+9)(z+1)(z^2-z+1)=0$

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$z^2 - 3z + 9 = 0$

$= 1,\ b = -3,\ c = 9$

$z = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(9)}}{2(1)} = \dfrac{3 \pm \sqrt{-27}}{2}$

$= \dfrac{3 \pm 3\sqrt{3}i}{2} = \dfrac{3}{2} \pm \dfrac{3\sqrt{3}}{2} i$

$z^2 - z + 1 = 0 \rightarrow a = 1, b = -1, c = 1 = \dfrac{-1 \pm \sqrt{-1^2 - 4\,1}}{2(1)} = \dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2} i$

$\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2} i$

The solution set is $\left\{ \dfrac{3}{2} \pm \dfrac{3\sqrt{3}}{2} i, \dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2} i, -3, -1 \right\}$

35. $f(x) = \dfrac{x}{x+1} \quad g(x) = \dfrac{x+2}{x}$

$(g - f)(x) = \dfrac{x+2}{x} - \dfrac{x}{x+1}$

$= \dfrac{(x+2)(x+1)}{x(x+1)} - \dfrac{x(x)}{x(x+1)}$

$= \dfrac{x^2 + 3x + 2}{x(x+1)} - \dfrac{x^2}{x(x+1)}$

$= \dfrac{x^2 + 3x + 2 - x^2}{x(x+1)}$

$= \dfrac{3x + 2}{x(x+1)}$

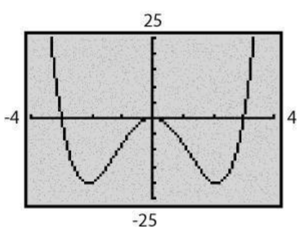
36. a. Domain: $[-3, 3]$ Range: $[-2, 2]$

Intercepts: $(-3,0), (0,0), (3,0)$

Symmetric with respect to the orgin.

The relation is a function. It passes the vertical line test.

37.



Local maximum: (0,0)

Local Minima: (-2.12,-20.25), (2.12,-20.25)

Increasing: (-2.12,0), (2.12,4)

Decreasing: (-4, -2.12), (0,2.12)

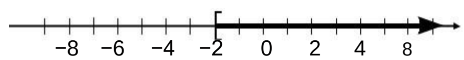
$y = \dfrac{k}{x^2}$

$24 = \dfrac{k}{5^2} = \dfrac{k}{25}$

$y = \dfrac{600}{x^2}$

Section 2.8

$x \geq -2$



The distance on a number line from the origin to a is $|a|$ for any real number a.

$4x - 3 = 9$

$4x = 12$

$x = 3$

The solution set is {3}.

$3x - 2 > 7$

$3x > 9$

$x > 3$

$-1 < 2x + 5 < 13$

$-6 < 2x < 8$

$-3 < x < 4$

The solution set is $\{x \mid -3 < x < 4\}$ or, using interval notation, $(-3, 4)$.

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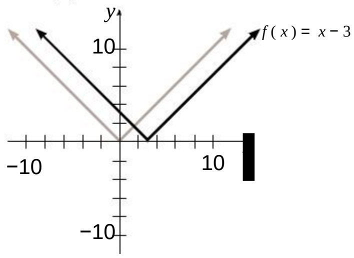
6. To graph $f(x) = |x - 3|$, shift the graph of $y = |x|$ to the right 3 units.



$-a$; a

$-a < u < a$

≤

True

False. Any real number will be a solution of $|\ | > -2$ since the absolute value of any real number is positive.

False. $|u| > a$ is equivalent to $u < -a$ or $u > a$.

a. Since the graphs of f and g intersect at the points $(-9, 6)$ and $(3, 6)$, the solution set of $(x) = g(x)$ is $\{-9, 3\}$.

Since the graph of f is below the graph of g when x is between -9 and 3 , the solution set of $f(x) \le g(x)$ is $\{x \mid -9 \le x \le 3\}$ or , using interval notation, $[-9, 3]$.

Since the graph of f is above the graph of g to the left of $x = -9$ and to the right of $= 3$, the solution set of $f(x) > g(x)$ is $\{x \mid x < -9 \text{ or } x > 3\}$ or , using interval notation, $(-\infty, -9) \cup (3, \infty)$.

a. Since the graphs of f and g intersect at the points $(0, 2)$ and $(4,2)$, the solution set of $(x) = g(x)$ is $\{0, 4\}$.

Since the graph of f is below the graph of g when x is between 0 and 4, the solution set of $f(x) \le g(x)$ is $\{x \mid 0 \le x \le 4\}$ or , using interval notation, $[0, 4]$.

$\{x \mid x < 0 \text{ or } x > 4\}$ or , using interval notation, $(-\infty, 0) \cup (4, \infty)$.

15. a. Since the graphs of f and g intersect at the points $(-2,5)$ and $(3,5)$, the solution set of $f(x) = g(x)$ is $\{-2, 3\}$.

b. Since the graph of f is above the graph of g to the left of $x = -2$ and to the right of $x = 3$, the solution set of $f(x) \ge g(x)$ is $\{x \mid x \le -2 \text{ or } x \ge 3\}$ or , using interval notation, $(-\infty, -2] \cup [3, \infty)$.

Since the graph of f is below the graph of g when x is between -2 and 3, the solution set of $f(x) < g(x)$ is $\{x \mid -2 < x < 3\}$ or , using interval notation, $(-2, 3)$.

a. Since the graphs of f and g intersect at the

Since the graph of f is above the graph of g to the left of $x = 0$ and to the right of $x = 4$, the solution set of $f(x) > g(x)$ is

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points $(-4,7)$ and $(3,7)$, the solution set of $f(x) = g(x)$ is $\{-4, 3\}$.
Since the graph of f is above the graph of g to the left of $x = -4$ and to the right of $x = 3$, the solution set of $f(x) \geq g(x)$ is $\{x \mid x \leq -4 \text{ or } x \geq 3\}$ or, using interval notation, $(-\infty, -4] \cup [3, \infty)$.
Since the graph of f is below the graph of g when x is between -4 and 3, the solution set of $f(x) < g(x)$ is $\{x \mid -4 < x < 3\}$ or, using interval notation, $(-4, 3)$.

$|x| = 6$
$x = 6$ or $x = -6$
The solution set is $\{-6, 6\}$.

$|x| = 12$
$x = 12$ or $x = -12$
The solution set is $\{-12, 12\}$.

$|2x + 3| = 5$
$2x + 3 = 5$ or $2x + 3 = -5$
$2x = 2$ or $2x = -8$
$x = 1$ or $x = -4$
The solution set is $\{-4, 1\}$.

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20. $|3x-1|=2$

$3x-1=2$ or $3x-1=-2$

$3x=3$ or $3x=-1$

$x=1$ or $x=-\frac{1}{3}$

The solution set is $\left\{-\frac{1}{3}, 1\right\}$.

21. $|1-4t|+8=13$

$|1-4t|=5$

$1-4t=5$ or $1-4t=-5$

$-4t=4$ or $-4t=-6$

$t=-1$ or $t=\frac{3}{2}$

The solution set is $\left\{-1, \frac{3}{2}\right\}$.

22. $|1-2z|+6=9$

$|1-2z|=3$

$1-2z=3$ or $1-2z=-3$

$-2z=2$ or $-2z=-4$

$z=-1$ or $z=2$

The solution set is $\{-1, 2\}$.

23. $|-2x|=8$

$-2x=8$ or $-2x=-8$

$x=-4$ or $x=4$

The solution set is {–4, 4}.

24. $|-x|=1$

$-x=1$ or $-x=-1$

The solution set is {–1, 1}.

26. $5-\left|\frac{1}{2}x\right|=3$

$-\left|\frac{1}{2}x\right|=-2$

$\left|\frac{1}{2}x\right|=2$

$\frac{1}{2}x=2$ or $\frac{1}{2}x=-2$

$x=4$ or $x=-4$

The solution set is $\{-4, 4\}$.

27. $\frac{2}{3}|x|=9$

$|x|=\frac{27}{2}$

$x=\frac{27}{2}$ or $x=-\frac{27}{2}$

The solution set is $\left\{-\frac{27}{2}, \frac{27}{2}\right\}$.

28. $\frac{3}{4}|x|=9$

$|x|=12$

$x=12$ or $x=-12$

The solution set is {–12, 12}.

29. $\left|\frac{x}{3}+\frac{2}{5}\right|=2$

$\frac{x}{3}+\frac{2}{5}=2$ or $\frac{x}{3}+\frac{2}{5}=-2$

$5x+6=30$ or $5x+6=-30$

$5x=24$ or $5x=-36$

$x=\frac{24}{5}$ or $x=-\frac{36}{5}$

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The solution set is $\left\{-\frac{36}{5}, \frac{24}{5}\right\}$.

25. $4-|2x|=3$

$|2x|=-1$

$|2x|=1$

$2x=1$ or $2x=-1$ $x=\frac{1}{2}$ or $x=-\frac{1}{2}$

The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$.

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30. $\left|\frac{x}{2} - \frac{1}{3}\right| = 1$

$\frac{x}{2} - \frac{1}{3} = 1$ or $\frac{x}{2} - \frac{1}{3} = -1$

$3x - 2 = 6$ or $3x - 2 = -6$

$3x = 8$ or $3x = -4$

$x = \frac{8}{3}$ or $x = -\frac{4}{3}$

The solution set is $\left\{-\frac{4}{3}, \frac{8}{3}\right\}$.

$|u + 2| = -\frac{1}{2}$

No solution, since absolute value always yields a non-negative number.

$|2 - v| = -1$

No solution, since absolute value always yields a non-negative number.

$|x^2 - 9| = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

The solution set is $\{-3, 3\}$.

$|x^2 - 16| = 0$

$x^2 - 16 = 0$

$x^2 = 16$

$x = \pm 4$

The solution set is $\{-4, 4\}$.

$|x^2 - 2x| = 3$

$x^2 - 2x = 3$ or $x^2 - 2x = -3$

$x^2 - 2x - 3 = 0$ or $x^2 - 2x + 3 = 0$

$(x - 3)(x + 1) = 0$ or $x = \frac{2 \pm \sqrt{4 - 12}}{2}$

$|x^2 + x| = 12$

$x^2 + x = 12$ or $x^2 + x = -12$

$x^2 + x - 12 = 0$ or $x^2 + x + 12 = 0$

$(x - 3)(x + 4) = 0$ or $x = \frac{-1 \pm \sqrt{1 - 48}}{2}$

$= \frac{-1 \pm \sqrt{-47}}{2}$ no real sol.

$x = 3$ or $x = -4$

The solution set is $\{-4, 3\}$.

$|x^2 + x - 1| = 1$

$x^2 + x - 1 = 1$ or $x^2 + x - 1 = -1$

$x^2 + x - 2 = 0$ or $x^2 + x = 0$

$(x - 1)(x + 2) = 0$ or $x(x + 1) = 0$

$x = 1, x = -2$ or $x = 0, x = -1$

The solution set is $\{-2, -1, 0, 1\}$.

$|x^2 + 3x - 2| = 2$

$x^2 + 3x - 2 = 2$ or $x^2 + 3x - 2 = -2$

$x^2 + 3x = 4$ or $x^2 + 3x = 0$

$x^2 + 3x - 4 = 0$ or $x(x + 3) = 0$

$(x + 4)(x - 1) = 0$ or $x = 0, x = -3$

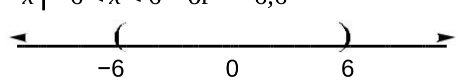
$x = -4, x = 1$

The solution set is $\{-4, -3, 0, 1\}$.

$|x| < 6$

$-6 < x < 6$

$\{x \mid -6 < x < 6\}$ or $(-6, 6)$



$|x| < 9$

$-9 < x < 9$

$\{x \mid -9 < x < 9\}$ or $(-9, 9)$

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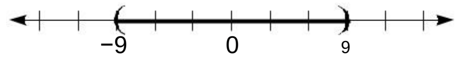
$= \frac{2 \pm \sqrt{-8}}{2}$ no real sol.

$x = 3$ or $x = -1$

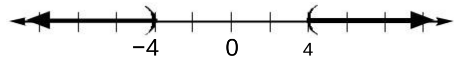
The solution set is $\{-1, 3\}$.



$|x| > 4$

$x < -4$ or $x > 4$

$\{x \mid x < -4 \text{ or } x > 4\}$ or $(-\infty, -4) \cup (4, \infty)$



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42. $|x| > 1$

$x < -1$ or $x > 1$

$\{x \mid x < -1 \text{ or } x > 1\}$ or $(-\infty, -1) \cup (1, \infty)$

−1 0 1

43. $|2x| < 8$

$-8 < 2x < 8$

$-4 < x < 4$

$\{x \mid -4 < x < 4\}$ or $(-4, 4)$

−4 0 4

44. $|3x| < 15$

$-15 < 3x < 15$

$-5 < x < 5$

$\{x \mid -5 < x < 5\}$ or $(-5, 5)$

−5 0 5

45. $|3x| > 12$

$3x < -12$ or $3x > 12$

$x < -4$ or $x > 4$

$\{x \mid x < -4 \text{ or } x > 4\}$ or $(-\infty, -4) \cup (4, \infty)$

−4 0 4

$|2x| > 6$

$2x < -6$ or $2x > 6$

$x < -3$ or $x > 3$

$\{x \mid x < -3 \text{ or } x > 3\}$ or $(-\infty, -3) \cup (3, \infty)$

−3 0 3

47. $|x-2| + 2 < 3$

$|x-2| < 1$

$-1 < x - 2 < 1$

$1 < x < 3$

$\{x \mid 1 < x < 3\}$ or $(1, 3)$

48. $|x+4| + 3 < 5$

$|x+4| < 2$

$-2 < x + 4 < 2$

$-6 < x < -2$

$\{x \mid -6 < x < -2\}$ or $(-6, -2)$

−6 −2 0

49. $|3t-2| \le 4$

$-4 \le 3t - 2 \le 4$

$-2 \le 3t \le 6$

$-\frac{2}{3} \le t \le 2$

$\left\{t \,\middle|\, -\frac{2}{3} \le t \le 2\right\}$ or $\left[-\frac{2}{3}, 2\right]$

$-\frac{2}{3}$ 0 2

50. $|2u+5| \le 7$

$-7 \le 2u + 5 \le 7$

$-12 \le 2u \le 2$

$-6 \le u \le 1$

$\{u \mid -6 \le u \le 1\}$ or $[-6, 1]$

−6 0 1

$|x-3| \ge 2$

$x - 3 \le -2$ or $x - 3 \ge 2$

$x \le 1$ or $x \ge 5$

$\{x \mid x \le 1 \text{ or } x \ge 5\}$ or $(-\infty, 1] \cup [5, \infty)$

0 1 5

52. $|x+4| \ge 2$

$x + 4 \le -2$ or $x + 4 \ge 2$

$x \le -6$ or $x \ge -2$

$\{x \mid x \le -6 \text{ or } x \ge -2\}$ or $(-\infty, -6] \cup [-2, \infty)$

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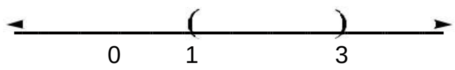

−6 −2 0

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53. $1-|4x-7|<-2$

$|4x-7|<5$

$-5<4x-7<5$

$-6<-4x<4$

$\frac{-6}{-4}>x>\frac{4}{-4}$

$\frac{3}{2}>x>-1$ or $-1<x<\frac{3}{2}$

$\left\{x \mid -1<x<\frac{3}{2}\right\}$ or $\left(-1,\frac{3}{2}\right)$

−1 0 $\frac{3}{2}$

54. $1-|2x-4|<-1$

$|2x-4|>2$ ... $1-2x<3$

$-3<1-2x<3$

$-4<-2x<2$

$\frac{-4}{-2}>x>\frac{2}{-2}$

$2>x>-1$ or $-1<x<2$

$\{x \mid -1<x<2\}$ or $(-1,2)$

−1 0 2

55. $|1-2x|>-3$

$|1-2x|>3$

$1-2x<-3$ or $1-2x>3$

$-2x<-4$ or $-2x>2$

$x>2$ or $x<-1$

$\{x \mid x<-1 \text{ or } x>2\}$ or $(-\infty,-1)\cup(2,\infty)$

−1 0 2 4

56. $|2-3x|>-1$

$|2-3x|>1$

$2-3x<-1$ or $2-3x>1$

$-3x<-3$ or $-3x>-1$

$x>1$ or $x<\frac{1}{3}$

$\left\{x \,\middle|\, x<\frac{1}{3} \text{ or } x>1\right\}$ or $\left(-\infty,\frac{1}{3}\right)\cup(1,\infty)$

57. $|2x+1|<-1$

No solution since absolute value is always non-negative.

0

58. $|3x-4|\geq 0$

All real numbers since absolute value is always non-negative.

$\{x \mid x \text{ is any real number}\}$ or $(-\infty,\infty)$

0

59. $|(3x-2)-7|<\frac{1}{2}$

$|3x-9|<\frac{1}{2}$

$-\frac{1}{2}<3x-9<\frac{1}{2}$

$\frac{17}{2}<3x<\frac{19}{2}$

$\frac{17}{6}<x<\frac{19}{6}$

$\left\{x \,\middle|\, \frac{17}{6}<x<\frac{19}{6}\right\}$ or $\left(\frac{17}{6},\frac{19}{6}\right)$

$\frac{17}{6}$ $\frac{19}{6}$

60. $|(4x-1)-11|<\frac{1}{4}$

$|4x-12|<\frac{1}{4}$

$-\frac{1}{4}<4x-12<\frac{1}{4}$

$\frac{47}{4}<4x<\frac{49}{4}$

$\frac{47}{16}<x<\frac{49}{16}$

$\left\{x \,\middle|\, \frac{47}{16}<x<\frac{49}{16}\right\}$ or $\left(\frac{47}{16},\frac{49}{16}\right)$

$\frac{47}{16}$ $\frac{49}{16}$

Section 2.8: *Equations and Inequalities Involving the Absolute Value Function*

0 $\frac{1}{3}$ 1

$5-|x-1|>2$

$|x-1|>-3$

$|x-1|<3$

$-3<x-1<3$

$-2<x<4$

$\{x \mid -2<x<4\}$ or $(-2,4)$

$6-|x+3|\ge 2$

$|x+3|\ge -4$

$|x+3|\le 4$

$-4\le x+3\le 4$

$-7\le x\le 1$

$\{x \mid -7\le x\le 1\}$ or $[-7,1]$

a. $f(x)=g(x)$

$-3|5x-2|=-9$

$|5x-2|=3$

$5x-2=3$ or $5x-2=-3$

$5x=5$ or $5x=-1$

$x=1$ or $x=-\frac{1}{5}$

$f(x)>g(x)$

$-3|5x-2|>-9$

$|5x-2|<3$

$-3<5x-2<3$

$-1<5x<5$

$-\frac{1}{5}<x<1$

$\left\{x \mid -\frac{1}{5}<x<1\right\}$ or $\left(-\frac{1}{5},1\right)$

$\left\{x \mid x\le -\frac{1}{5} \text{ or } x\ge 1\right\}$ or $\left(-\infty,-\frac{1}{5}\right]\cup[1,\infty)$

a. $f(x)=g(x)$

$-2|2x-3|=-12$

$|2x-3|=6$

$2x-3=6$ or $2x-3=-6$

$2x=9$ or $2x=-3$

$x=\frac{9}{2}$ or $x=-\frac{3}{2}$

$f(x)=g(x)$

$-2|2x-3|\ge -12$

$|2x-3|\le 6$

$-6\le 2x-3\le 6$

$-3\le 2x\le 9$

$-\frac{3}{2}\le x\le \frac{9}{2}$

$\left\{x \mid -\frac{3}{2}\le x\le \frac{9}{2}\right\}$ or $\left[-\frac{3}{2},\frac{9}{2}\right]$

c. $f(x)=g(x)$

$-2|2x-3|<-12$

$|2x-3|>6$

$2x-3>6$ or $2x-3<-6$

$2x>9$ or $2x<-3$

$x>\frac{9}{2}$ or $x<-\frac{3}{2}$

$\left\{x \mid x<-\frac{3}{2} \text{ or } x>\frac{9}{2}\right\}$ or $\left(-\infty,-\frac{3}{2}\right)\cup\left(\frac{9}{2},\infty\right)$

65. a. $f(x)=g(x)$

$|-3x+2|=x+10$

$-3x+2=x+10$ or $-3x+2=-(x+10)$

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$x = -2$ or $-2x = -12$

$x = 6$

$\left(-\frac{1}{5}, 1 \right)$

c. $f(x) \le g(x)$

$-3|5x - 2| \le -9$

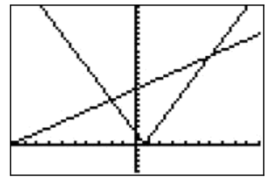
$|5x - 2| \ge 3$

$5x - 2 \ge 3$ or $5x - 2 \le -3$

$5x \ge 5$ or $5x \le -1$

$x \ge 1$ or $x \le -\frac{1}{5}$

b.



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Look at the graph of $f(x)$ and $g(x)$ and see where the graph of $f(x) \ge g(x)$. We see that this occurs where $x \le -2$ or $x \ge 6$. So the solution set is: $\{x \mid x \le -2 \text{ or } x \ge 6\}$ or $(-\infty, -2] \cup [6, \infty)$.

c. Look at the graph of $f(x)$ and $g(x)$ and see where the graph of $f(x) < g(x)$. We see that this occurs where x is between -2 and 6. So the solution set is:

$\{x \mid -2 < x < 6\}$ or $(-2, 6)$.

a. $f(x) = g(x)$

$|4x - 3| = x + 2$

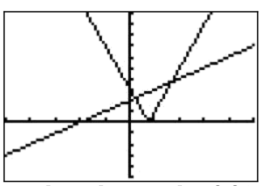
$4x - 3 = x + 2$ or $4x - 3 = -(x + 2)$

$3x = 5$ or $4x - 3 = -x - 2$

$x = \frac{5}{3}$ or $5x = 1$

or $x = \frac{1}{5}$

b.



Look at the graph of $f(x)$ and $g(x)$ and see where the graph of $f(x) > g(x)$. We see that this occurs where $x < \frac{1}{5}$ or $x > \frac{5}{3}$.

So the solution set is: $\left\{x \mid x < \frac{1}{5} \text{ or } x > \frac{5}{3}\right\}$ or $\left(-\infty, \frac{1}{5}\right) \cup \left(\frac{5}{3}, \infty\right)$

Look at the graph of $f(x)$ and $g(x)$ and see where the graph of $f(x) \le g(x)$. We see that this occurs where x is between $\frac{1}{5}$ and $\frac{5}{3}$. So the solution set is:

$|x - 10| < 2$

$-2 < x - 10 < 2$

$8 < x < 12$

Solution set: $\{x \mid 8 < x < 12\}$ or $(8,12)$

68. $|x - (-6)| < 3$

$|x + 6| < 3$

$-3 < x + 6 < 3$

$-9 < x < -3$

Solution set: $\{x \mid -9 < x < -3\}$ or $(-9, -3)$

69. $|2x - (-1)| > 5$

$|2x + 1| > 5$

$2x + 1 < -5$ or $2x + 1 > 5$

$2x < -6$ or $2x > 4$

$x < -3$ or $x > 2$

Solution set: $\{x \mid x < -3 \text{ or } x > 2\}$ or $(-\infty, -3) \cup (2, \infty)$

$|2x - 3| > 1$

$2x - 3 < -1$ or $2x - 3 > 1$

$2x < 2$ or $2x > 4$

$x < 1$ or $x > 2$

Solution set: $\{x \mid x < 1 \text{ or } x > 2\}$ or $(-\infty, 1) \cup (2, \infty)$

$|x - 5.7| \le 0.0005$

$-0.0005 < x - 5.7 < 0.0005$

$5.6995 < x < 5.7005$

The acceptable lengths of the rod is from 5.6995 inches to 5.7005 inches.

$|x - 6.125| \le 0.0005$

$-0.0005 < x - 6.125 < 0.0005$

$6.1245 < x < 6.1255$

The acceptable lengths of the rod is from

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$\left\{x \mid \frac{1}{5} \le x \le \frac{5}{3}\right\}$ or $\left[\frac{1}{5}, \frac{5}{3}\right]$.

6.1245 inches to 6.1255 inches.

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73. $\left|\frac{x-100}{15}\right| > 1.96$

$\frac{x-100}{15} < -1.96$ or $\frac{x-100}{15} > 1.96$

$x-100 < -29.4$ or $x-100 > 29.4$

$x < 70.6$ or $x > 129.4$

Since IQ scores are whole numbers, any IQ less than 71 or greater than 129 would be considered unusual.

74. $\left|\frac{x-266}{16}\right| > 1.96$

$\frac{x-266}{16} < -1.96$ or $\frac{x-266}{16} > 1.96$

$x-266 < -31.36$ or $x-266 > 31.36$

$x < 234.64$ or $x > 297.36$

Pregnancies less than 235 days long or greater than 297 days long would be considered unusual.

$5x|+1+|7 = 5$

$|5x+1| = -2$

No matter what real number is substituted for x, the absolute value expression on the left side of the equation must always be zero or larger. Thus, it can never equal -2 .

$2x|+5+|3 > 1 \Rightarrow 2x| + 5 > |-2$

No matter what real number is substituted for x, the absolute value expression on the left side of the equation must always be zero or larger. Thus, it will always be larger than -2 . Thus, the solution is the set of all real numbers.

$2\,x|-1 \le|0$

No matter what real number is substituted for x, the absolute value expression on the left side of the equation must always be zero or larger. Thus, the only solution to the inequality above will be when the absolute value expression equals 0:

$|\;x-1| = 0$

$2x-1 = 0$

$2x = 1$

$x = 2$

$f(x) = 2|x-7|$

$(-4) = |2(-4)-7|$

$|-8-7| = |-15| = 15$

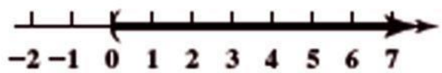
$2(x+4) + x < 4(x+2)$

$x+8+x < 4x+8$

$3x+8 < 4x+8$

$x < 0$

$x > 0$



$(5-i)(3+2i) = 15 +$

$10i - 3i - 2i^2 =$

$15 + 7i + 2 = 17 + 7i$

a. Intercepts: (0,0), (4,0)

Domain: $[-2,5]$, Range: $[-2,4]$

Increasing: $(3,5)$:Decreasing: $(-2,1)$

Constant: $(1,3)$

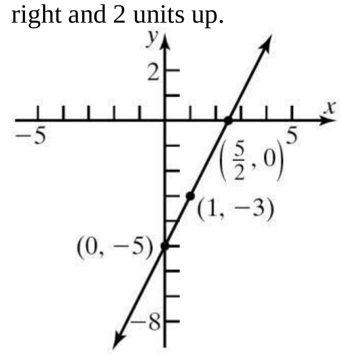
Neither

Chapter 2 Review Exercises

$f(x) = 2x - 5$

Slope = 2; y-intercept = -5

Plot the point $(0, -5)$. Use the slope to find an additional point by moving 1 unit to the right and 2 units up.



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c. Domain and Range: $(-\infty, \infty)$

1

Thus, the solution set is $\left\{ \right\}$.

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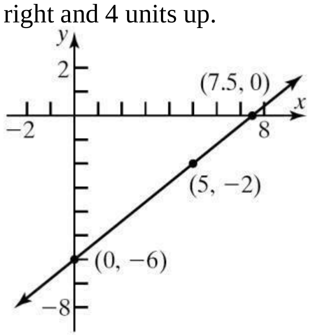
d. Average rate of change = slope = 2

e. Increasing

$h(x) = \frac{4}{5}x - 6$

Slope = $\frac{4}{5}$; y-intercept = -6

Plot the point $(0, -6)$. Use the slope to find an additional point by moving 5 units to the right and 4 units up.



c. Domain and Range: $(-\infty, \infty)$

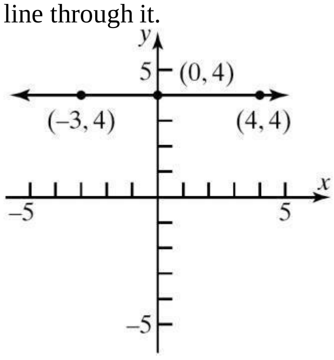
d. Average rate of change = slope = $\frac{4}{5}$

e. Increasing

3. $G(x) = 4$

a. Slope = 0; y-intercept = 4

b. Plot the point (0, 4) and draw a horizontal line through it.



c. Domain: $(-\infty, \infty)$

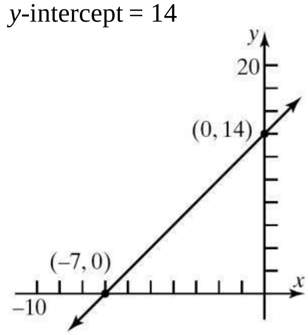
Range: $\{y \mid y = 4\}$

d. Average rate of change = slope = 0

$2x = -14$

$x = -7$

y-intercept = 14



5.

x	$y = f(x)$	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
−2	−7	
0	3	$\frac{3-(-7)}{0-(-2)} = \frac{10}{2} = 5$
1	8	$\frac{8-3}{1-0} = \frac{5}{1} = 5$
3	18	$\frac{18-8}{3-1} = \frac{10}{2} = 5$
6	33	$\frac{33-18}{6-3} = \frac{15}{3} = 5$

This is a linear function with slope = 5, since the average rate of change is constant at 5. To find the equation of the line, we use the point-slope

e. Constant

-

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formula and one of the points.

$y - y_1 = m(x - x_1)$

$y - 3 = 5(x - 0)$

$y = 5x + 3$

6.

x	$y = f(x)$	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
−1	−3	
0	4	$\frac{4-(-3)}{0-(-1)} = \frac{7}{1} = 7$
1	7	$\frac{7-4}{1-0} = \frac{3}{1} = 3$
2	6	
3	1	

This is not a linear function, since the average rate of change is not constant.

4. $f(x) = 2x + 14$

zero: $f(x) = 2x + 14 = 0$

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$f(x)=0$

$x^2+x-72=0$

$(x+9)(x-8)=0$

$x+9=0$ or $x-8=0$

$x=-9$ $\quad x=8$

The zeros of $f(x)=x^2+x-72$ are -9 and 8.

The x-intercepts of the graph of f are -9 and 8.

$P(t)=0$

$6t^2-13t-5=0$

$(3t+1)(2t-5)=0$

$3t+1=0$ or $2t-5=0$

$t=-\frac{1}{3}$ $\quad t=\frac{5}{2}$

The zeros of $P(t)=6t^2-13t-5$ are $-\frac{1}{3}$ and $\frac{5}{2}$.

The t-intercepts of the graph of P are $-\frac{1}{3}$ and $\frac{5}{2}$.

$g(x)=0$

$(x-3)^2-4=0$

$(x-3)^2=4$

$x-3=\pm\sqrt{4}$

$x-3=\pm 2$

$=3\pm 2$

$x=3-2=1$ or $x=3+2=5$

The zeros of $g(x)=(x-3)^2-4$ are 1 and 5.

The x-intercepts of the graph of g are 1 and 5.

$h(x)=0$

$x^2+6x+1=0$

$(3x+1)(3x+1)=0$

$3x+1=0$ or $3x+1=0$

$x=-\frac{1}{3}$ $\quad x=-\frac{1}{3}$

$\underline{1}$

$G(x)=0$

$2x^2-4x-1=0$

$^2-2x-\frac{1}{2}=0$

$^2-2x=\frac{1}{2}$

$\underline{1}$

$x^2-2x+1=\ 2+1$

$x-1)^2=\frac{3}{2}$

$x-1=\pm\sqrt{\frac{3}{2}}=\pm\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\pm\frac{\sqrt{6}}{2}$

$x=1\pm\frac{\sqrt{6}}{2}=\frac{2\pm\sqrt{6}}{2}$

The zeros of $G(x)=2x^2-4x-1$ are $\frac{2-\sqrt{6}}{2}$ and $\frac{2+\sqrt{6}}{2}$. The x-intercepts of the graph of G are $\frac{2-\sqrt{6}}{2}$ and $\frac{2+\sqrt{6}}{2}$.

$f(x)=0$

2

$-2x\quad +x+1=0$

$x^2-x-1=0$

$(2x+1)(x-1)=0$

$2x+1=0$ or $x-1=0$

$x=-\frac{1}{2}$ $\quad x=1$

The zeros of $f(x)=-2x^2+x+1$ are $-\frac{1}{2}$ and 1.

The x-intercepts of the graph of f are $-\frac{1}{2}$ and 1.

$f(x)=g(x)\,(x$

$-3)^2=16$

$-3=\pm\sqrt{16}=\pm 4$

$=3\pm 4$

$x=3-4=-1$ or $x=3+4=7$

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The only zero of $h(x) = 9x^2 + 6x + 1$ is $-$ 3 .

The only x-intercept of the graph of h is $-\frac{1}{3}$.

The solution set is $\{-1, 7\}$.

The x-coordinates of the points of intersection are -1 and 7. The y-coordinates are $g(-1) = 16$ and $g(7) = 16$. The graphs of the f and g intersect at

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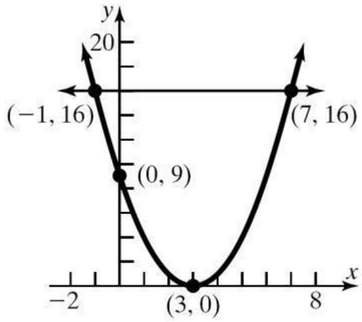
the points $(-1, 16)$ and $(7, 16)$.



$f(x) = g(x)$

$^2 + 4x - 5 = 4x - 1$

$x^2 - 4 = 0$

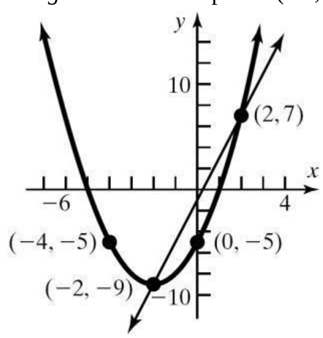
$x + 2)(x - 2) = 0$

$x + 2 = 0$ or $x - 2 = 0$

$x = -2$ $\quad x = 2$

The solution set is $\{-2, 2\}$.

The x-coordinates of the points of intersection are -2 and 2. The y-coordinates are $g(-2) = 4(-2) - 1 = -8 - 1 = -9$ and $g(2) = 4(2) - 1 = 8 - 1 = 7$. The graphs of the f and g intersect at the points $(-2, -9)$ and $(2, 7)$.



$f(x) = 0$

$x^4 - 5x^2 + 4 = 0$

$x^2 - 4)(x^2 - 1) = 0$

$^2 - 4 = 0$ or $x^2 - 1 =$

0 $x = \pm 2$ or $x = \pm 1$

The zeros of $f(x) = x^4 - 5x^2 + 4$ are -2, -1, 1, and 2. The x-intercepts of the graph of f are -2, -1, 1, and 2.

16. $F(x) = 0$

$x - 3)^2 - 2(x - 3) - 48 = 0$

Let $u = x - 3 \rightarrow u^2 = (x - 3)^2$

$u^2 - 2u - 48 = 0$

$u + 6)(u - 8) = 0$

$u + 6 = 0$ or $u - 8 = 0$

$u = -6$ $\quad u = 8$

$x - 3 = -6$ $\quad x - 3 = 8$

$x = -3$ $\quad x = 11$

The zeros of $F(x) = (x - 3)^2 - 2(x - 3) - 48$ are -3 and 11. The x-intercepts of the graph of F are -3 and 11.

$h(x) = 0$

$3x - 13\sqrt{x} - 10 = 0$

Let $u = \sqrt{x} \rightarrow u^2 = x$

$3u^2 - 13u - 10 = 0$

$3u + 2)(u - 5) = 0$

$3u + 2 = 0$ or $u - 5 = 0$

$u = -\frac{2}{3}$ $\quad u = 5$

$\sqrt{x} = -\frac{2}{3}$ $\quad \sqrt{x} = 5$

$x =$ not real $\quad x = 5^2 = 25$

Check: $h(25) = 3(25) - 13\sqrt{25} - 10$

$3(25) - 13(5) - 10$

$75 - 65 - 10 = 0$

The only zero of $h(x) = 3x - 13\sqrt{x} - 10$ is 25. The only x-intercept of the graph of h is 25.

18. $f(x) = 0$

$\left(\frac{1}{x}\right)^2 - 4\left(\frac{1}{x}\right) - 12 = 0$

Let $u = \frac{1}{x} \rightarrow u^2 = \left(\frac{1}{x}\right)^2$

$u^2 - 4u - 12 = 0$

$(u + 2)(u - 6) = 0$

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$u+2=0$ or $u-6=0$

$u=-2$ $\quad$ $u=6$

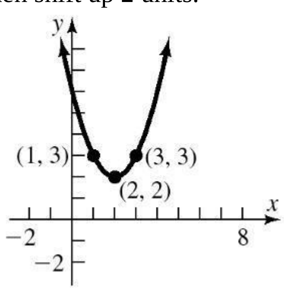
$\frac{1}{x}=-2$ $\quad$ $\frac{1}{x}=6$

$x=-\frac{1}{2}$ $\quad$ $x=\frac{1}{6}$

The zeros of $f(x)=\left(\frac{1}{x}\right)^2-4\left(\frac{1}{x}\right)-12$ are $-\frac{1}{2}$ and $\frac{1}{6}$. The x-intercepts of the graph of f are $-\frac{1}{2}$ and $\frac{1}{6}$.

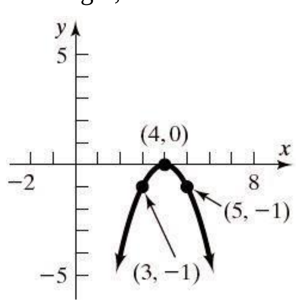
$f(x)=(x-2)^2+2$

Using the graph of $y=x^2$, shift right 2 units, then shift up 2 units.



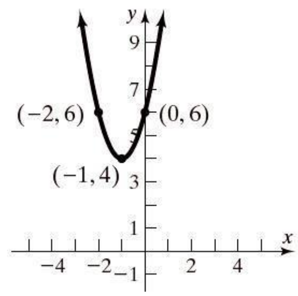
$f(x)=-(x-4)^2$

Using the graph of $y=x^2$, shift the graph 4 units right, then reflect about the x-axis.



$f(x)=2(x+1)^2+4$

Using the graph of $y=x^2$, stretch vertically by a factor of 2, then shift 1 unit left, then shift 4 units up.



a. $f(x)=(x-2)^2+2$

$x^2-4x+4+2$

x^2-4x+6

$a=1$, $b=-4$, $c=6$. Since $a=1>0$, the graph opens up. The x-coordinate of the vertex is $x=-\frac{b}{2a}=-\frac{-4}{2(1)}=\frac{4}{2}=2$.

$f\left(-\frac{b}{2a}\right)=f(2)=(2)^2-4(2)+6=2$.

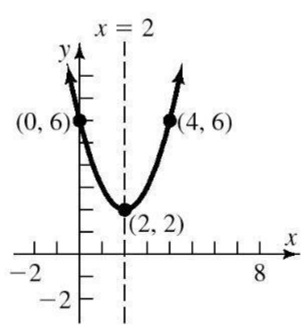
Thus, the vertex is (2, 2).

The axis of symmetry is the line $x=2$.

The discriminant is:

$b^2-4ac=(-4)^2-4(1)(6)=-8<0$, so the graph has no x-intercepts.

The y-intercept is $f(0)=6$.



Domain: $(-\infty, \infty)$. Range: $[2, \infty)$.

Decreasing on $(-\infty, 2)$; increasing on $2,\infty)$.

1

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a. $f(x) = \frac{1}{4}x^2 - 16$

$a = \frac{1}{4}$, $b = 0$, $c = -16$. Since $a = \frac{1}{4} > 0$,

the graph opens up. The *x*-coordinate of the

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vertex is $x = -\frac{b}{2a} = -\frac{0}{2\left(\frac{1}{4}\right)} = -\frac{0}{\frac{1}{2}} = 0$.

The y-coordinate of the vertex is

$f\left(-\frac{b}{2a}\right) = f(0) = \frac{1}{4}(0)^2 - 16 = -16$.

Thus, the vertex is (0, –16).

The axis of symmetry is the line $x = 0$.

The discriminant is:

$b^2 - 4ac = (0)^2 - 4\left(\frac{1}{4}\right)(-16) = 16 > 0$, so the graph has two x-intercepts.

The x-intercepts are found by solving:

$\frac{1}{4}x^2 - 16 = 0$

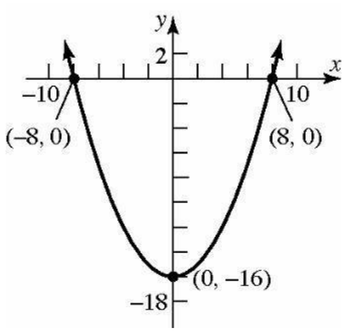
$x^2 - 64 = 0$

$x^2 = 64$

$x = 8$ or $x = -8$

The x-intercepts are –8 and 8.

The y-intercept is $f(0) = -16$.



Domain: $(-\infty, \infty)$. Range: $[-16, \infty)$.

Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.

a. $f(x) = -4x^2 + 4x$

$a = -4$, $b = 4$, $c = 0$. Since $a = -4 < 0$, the graph opens down. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{4}{2(-4)} = -\frac{4}{-8} = \frac{1}{2}$.

The axis of symmetry is the line $x = \frac{1}{2}$.

$b^2 - 4ac = 4^2 - 4(-4)(0) = 16 > 0$, so the graph has two x-intercepts.

The x-intercepts are found by solving:

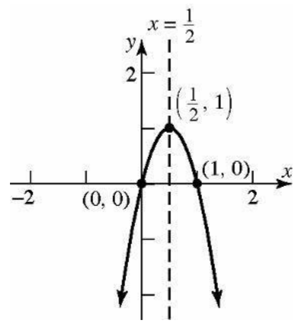
$4x^2 + 4x = 0$

$4x(x - 1) = 0$

$= 0$ or $x = 1$

The x-intercepts are 0 and 1.

The y-intercept is $f(0) = -4(0)^2 + 4(0) = 0$.



Domain: $(-\infty, \infty)$. Range: $(-\infty, 1]$.

Increasing on $\left(-\infty, \frac{1}{2}\right)$; decreasing on $\left(\frac{1}{2}, \infty\right)$.

a. $f(x) = \frac{9}{2}x^2 + 3x + 1$

$a = \frac{9}{2}$, $b = 3$, $c = 1$. Since $a = \frac{9}{2} > 0$, the graph opens up. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{3}{2\left(\frac{9}{2}\right)} = -\frac{3}{9} = -\frac{1}{3}$.

The y-coordinate of the vertex is

$f\left(-\frac{b}{2a}\right) = f\left(-\frac{1}{3}\right) = \frac{9}{2}\left(-\frac{1}{3}\right)^2 + 3\left(-\frac{1}{3}\right) + 1$

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The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{2}\right) = -4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)$$

$$= -1 + 2 = 1$$

Thus, the vertex is $\left(\frac{1}{2}, 1\right)$.

$= 2 \quad -1+1 = \quad 2$

Thus, the vertex is $\left(-\frac{1}{3}, \frac{1}{2}\right)$.

The axis of symmetry is the line $x = -\frac{1}{3}$.

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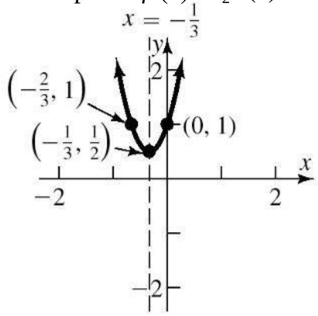
$b^2 - 4ac = 3^2 - 4\left(\frac{9}{2}\right)(1) = 9 - 18 = -9 < 0$,

so the graph has no x-intercepts. The y-intercept is $f(0) = \frac{9}{2}(0)^2 + 3(0) + 1 = 1$.



b. Domain: $(-\infty, \infty)$. Range: $\left[\frac{1}{2}, \infty\right)$.

c. Decreasing on $\left(-\infty, -\frac{1}{3}\right)$; increasing on $\left(-\frac{1}{3}, \infty\right)$.

26. a. $f(x) = 3x^2 + 4x - 1$

$a = 3$, $b = 4$, $c = -1$. Since $a = 3 > 0$, the graph opens up. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{4}{2(3)} = -\frac{4}{6} = -\frac{2}{3}$.

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right)^2 + 4\left(-\frac{2}{3}\right) - 1 = \frac{4}{3} - \frac{8}{3} - 1 = -\frac{7}{3}$$

Thus, the vertex is $\left(-\frac{2}{3}, -\frac{7}{3}\right)$.

The axis of symmetry is the line $x = -\frac{2}{3}$.

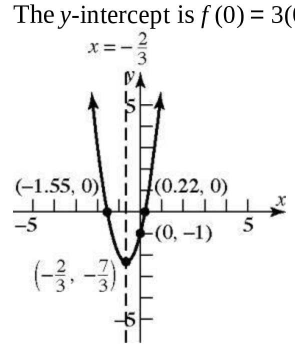
The discriminant is:

$b^2 - 4ac = (4)^2 - 4(3)(-1) = 28 > 0$, so the graph has two x-intercepts.

The x-intercepts are found by solving: $3x^2 + 4x - 1 = 0$.

The x-intercepts are $\frac{-2-\sqrt{7}}{3} \approx -1.55$ and $\frac{-2+\sqrt{7}}{3} \approx 0.22$.

The y-intercept is $f(0) = 3(0)^2 + 4(0) - 1 = -1$.



b. $\left[\ \frac{7}{3}\ \right)$

c. Decreasing on $\left(-\infty, -\frac{2}{3}\right)$; increasing on $\left(-\frac{2}{3}, \infty\right)$.

27. $f(x) = 3x^2 - 6x + 4$

$a = 3$, $b = -6$, $c = 4$. Since $a = 3 > 0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at

$x = -\frac{b}{2a} = -\frac{-6}{2(3)} = \frac{6}{6} = 1$.

The minimum value is

$f\left(-\frac{b}{2a}\right) = f(1) = 3(1)^2 - 6(1) + 4 = 3 - 6 + 4 = 1$

28. $f(x) = -x^2 + 8x - 4$

$a = -1$, $b = 8$, $c = -4$. Since $a = -1 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at

$x = -\frac{b}{2a} = -\frac{8}{2(-1)} = -\frac{8}{-2} = 4$.

The maximum value is

$f\left(-\frac{b}{2a}\right) = f(4) = -(4)^2 + 8(4) - 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{28}}{2(3)}$$

$$= \frac{-4 \pm 2\sqrt{7}}{6} = \frac{-2 \pm \sqrt{7}}{3}$$

6
2
4
7

$\left(2a \right)$

$= -16 + 32 - 4 = 12$

29. $f(x) = -3x^2 + 12x + 4$

$a = -3$, $b = 12$, $c = 4$. Since $a = -3 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at

$x = -\frac{b}{2a} = -\frac{12}{2(-3)} = -\frac{12}{-6} = 2$.

The maximum value is

$f\left(-\frac{b}{2a}\right) = f(2) = -3(2)^2 + 12(2) + 4$

$= -12+24+4 = 16$

30. Consider the form $y = a(x-h)^2 + k$. The vertex is $(2, -4)$ so we have $h = 2$ and $k = -4$. The function also contains the point $(0, -16)$. Substituting these values for *x*, *y*, *h*, and *k*, we can solve for *a*:

$-16 = a(0-(2))^2 + (-4)$

$-16 = a(-2)^2 - 4$

$-16 = 4a - 4$

$-12 = 4a$

The quadratic function is

$f(x) = -3(x-2)^2 - 4 = -3x^2 + 12x - 16$.

31. Use the form $f(x) = a(x-h)^2 + k$.

The vertex is $(-1, 2)$, so $h = -1$ and $k = 2$.

$f(x) = a(x+1)^2 + 2$.

Since the graph passes through $(1, 6)$, $f(1) = 6$.

$6 = a(1+1)^2 + 2$

$6 = a(2)^2 + 2$

$= 4a + 2$

$4 = 4a$

$1 = a$

$(x) = (x+1)^2 + 2$

$(x^2 + 2x + 1) + 2$

Interval	$(-\infty, -8)$	$(-8, 2)$	$(2, \infty)$
Test Number	−9	0	3
Value of *f*	11	−16	11
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid -8 < x < 2\}$ or, using interval notation, $(-8, 2)$.

33. $3x^2 \geq 14x + 5$

$3x^2 - 14x - 5 \geq 0$

$f(x) = 3x^2 - 14x - 5$

$3x^2 - 14x - 5 = 0$

$(3x+1)(x-5) = 0$

$x = -\frac{1}{3}$, $x = 5$ are the zeros of f.

Interval	$\left(-\infty, -\frac{1}{3}\right)$	$\left(-\frac{1}{3}, 5\right)$	$(5, \infty)$
Test Number	−1	0	2
Value of *f*	12	−5	19
Conclusion	Positive	Negative	Positive

using interval notation, $\left(-\infty, -\frac{1}{3}\right] \cup [5, \infty)$.

34. $f(x) = 0$

$x^2 + 8 = 0$

$x^2 + 2x + 3$

$x^2 + 6x - 16 < 0$

$(x) = x^2 + 6x - 16$

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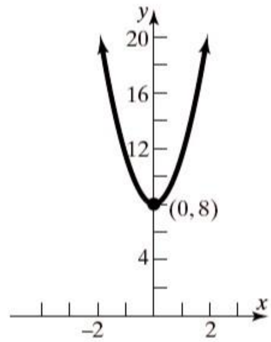
$x^2 + 6x - 16 = 0$
$x + 8)(x - 2) = 0$
$= -8$, $x = 2$ are the zeros of f.

$x^2 = -8$
$x = \pm\sqrt{-8} = \pm 2\sqrt{2}\, i$
The zero are $-2\sqrt{2}\, i$ and $2\sqrt{2}\, i$.



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$g(x) = 0$

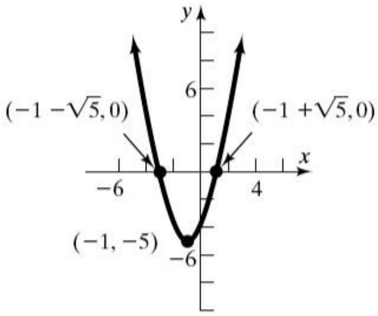
$x^2 + 2x - 4 = 0$

$a = 1,\ b = 2,\ c = -4$

$b^2 - 4ac = 2^2 - 4(1)(-4) = 4 + 16 = 20$

$x = \dfrac{-2 \pm \sqrt{20}}{2(1)} = \dfrac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$

The zeros are $-1 - \sqrt{5}$ and $-1 + \sqrt{5}$.



$p(x) = 0$

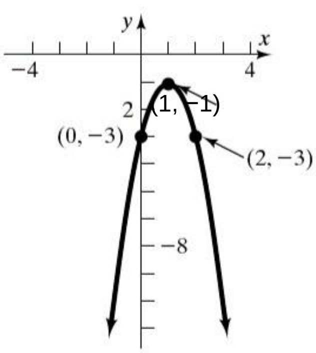
$-2x^2 + 4x - 3 = 0$

$= -2,\ b = 4,\ c = -3$

${}^2 - 4ac = 4^2 - 4(-2)(-3) = 16 - 24 = -8$

$x = \dfrac{-4 \pm \sqrt{-8}}{2(-2)} = \dfrac{-4 \pm 2\sqrt{2}\, i}{-4} = 1 \pm \dfrac{\sqrt{2}}{2} i$

The zeros are $1 - \dfrac{\sqrt{2}}{2} i$ and $1 + \dfrac{\sqrt{2}}{2} i$.



$f(x) = 0$

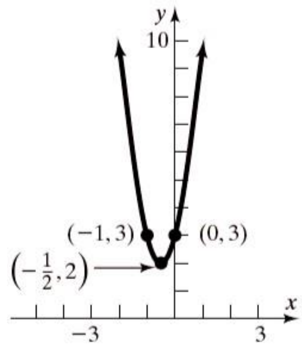
$x^2 + 4x + 3 = 0$

$a = 4,\ b = 4,\ c = 3$

$b^2 - 4ac = 4^2 - 4(4)(3) = 16 - 48 = -32$

$x = \dfrac{-4 \pm \sqrt{-32}}{2(4)} = \dfrac{-4 \pm 4\sqrt{2}\, i}{8} = -\dfrac{1}{2} \pm \dfrac{\sqrt{2}}{2} i$

The zeros are $-\dfrac{1}{2} - \dfrac{\sqrt{2}}{2} i$ and $-\dfrac{1}{2} + \dfrac{\sqrt{2}}{2} i$.



$|2x + 3| = 7$

$2x + 3 = 7$ or $2x + 3 = -7$

$2x = 4$ or $2x = -10$

$x = 2$ or $x = -5$

The solution set is $\{-5, 2\}$.

$|2 - 3x| + 2 = 9$

$|2 - 3x| = 7$

$2 - 3x = 7$ or $2 - 3x = -7$

$-3x = 5$ or $-3x = -9$

$x = -\dfrac{5}{3}$ or $x = 3$

The solution set is $\left\{-\dfrac{5}{3}, 3\right\}$.

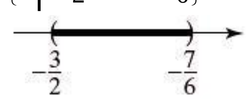
$|3x + 4| < \dfrac{1}{2}$

$-\dfrac{1}{2} < 3x + 4 < \dfrac{1}{2}$

$-\dfrac{9}{2} < 3x < -\dfrac{7}{2}$

$-\dfrac{3}{2} < x < -\dfrac{7}{6}$

$\left\{x \,\middle|\, -\dfrac{3}{2} < x < -\dfrac{7}{6}\right\}$ or $\left(-\dfrac{3}{2}, -\dfrac{7}{6}\right)$



$|2x - 5| \geq 9$

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$$2x - 5 \le -9 \text{ or } 2x - 5 \ge 9$$
$$2x \le -4 \text{ or } \quad 2x \ge 14$$
$$x \le -2 \text{ or } \quad x \ge 7$$

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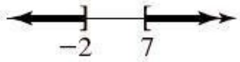
$\{x \mid x \le -2 \text{ or } x \ge 7\}$ or $(-\infty, -2] \cup [7, \infty)$



$2 + |2 - 3x| \le 4$

$|2 - 3x| \le 2$

$-2 \le 2 - 3x \le$

$2 - 4 \le -3x \le 0$

$\frac{4}{3} \ge x \ge 0$

$\left\{x \,\middle|\, 0 \le x \le \frac{4}{3}\right\}$ or $\left[0, \frac{4}{3}\right]$

$1 - |2 - 3x| < -4$

$|2 - 3x| < -5$

$|2 - 3x| > 5$

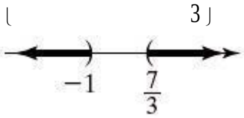
$2 - 3x < -5$ or $2 - 3x > 5$

$7 < 3x$ or $-3 > 3x$

$\frac{7}{3} < x$ or $-1 > x$

$x < -1$ or $x > \frac{7}{3}$

$\left\{x \,\middle|\, x < -1 \text{ or } x > \frac{7}{3}\right\}$ or $(-\infty, -1) \cup \left(\frac{7}{3}, \infty\right)$



a. Company A: $C(x) = 0.06x + 7.00$

Company B: $C(x) = 0.08x$

$0.06x + 7.00 = 0.08x$

$7.00 = 0.02x$

$350 = x$

The bill from Company A will equal the bill from Company B if 350 minutes are used.

$0.08x < 0.06x + 7.00$

$0.02x < 7.00$

$x < 350$

The bill from Company B will be less than the bill from Company A if fewer than 350 minutes are used. That is, $0 \le x < 350$.

a. If $x = 1500 - 10p$, then $p = \frac{1500 - x}{10}$.

$R(p) = px = p(1500 - 10p) = -10p^2 + 1500p$

Domain: $\{p \mid 0 < p \le 150\}$

c. $p = \frac{-b}{2a} = \frac{-1500}{2(-10)} = \frac{-1500}{-20} = \$75$

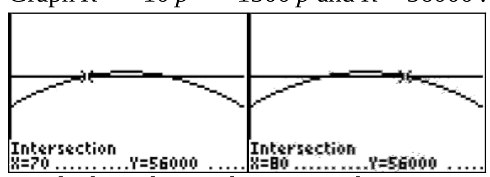
The maximum revenue is

$R(75) = -10(75)^2 + 1500(75)$

$-56250 + 112500 = \$56,250$

$x = 1500 - 10(75) = 1500 - 750 = 750$

Graph $R = -10p^2 + 1500p$ and $R = 56000$.



Find where the graphs intersect by solving $56000 = -10p^2 + 1500p$.

$p^2 - 1500p + 56000 =$

$0\ p^2 - 150p + 5600 = 0$

$(p - 70)(p - 80) = 0$

$p = 70, p = 80$

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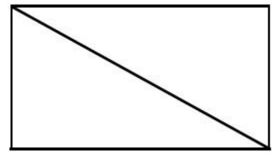
The company should charge between \$70 and \$80.

Let w = the width. Then $w + 2$ = the length.



in.

w

$w + 2$

By the Pythagorean Theorem we have:

$$w^2 + (w+2)^2 = (10)^2$$

$$w^2 + w^2 + 4w + 4 = 100$$

$$2w^2 + 4w - 96 = 0$$

$$w^2 + 2w - 48 = 0$$

$$w + 8)(w - 6) = 0$$

$$w = -8 \quad \text{or} \quad w = 6$$

Disregard the negative answer because the width

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of a rectangle must be positive. Thus, the width is 6 inches, and the length is 8 inches

$C(x) = 4.9x^2 - 617.4x + 19{,}600$;
$a = 4.9$, $b = -617.4$, $c = 19{,}600$. Since $a = 4.9 > 0$, the graph opens up, so the vertex is a minimum point.

The minimum marginal cost occurs at

$$x = -\frac{b}{2a} = -\frac{-617.40}{2(4.9)} = \frac{617.40}{9.8} = 63.$$

Thus, 63 golf clubs should be manufactured in order to minimize the marginal cost.
The minimum marginal cost is

$$C\left(-\frac{b}{2a}\right) = C(63)$$

$$4.9(63)^2 - (617.40)(63) + 19600$$
$151.90

Since there are 200 feet of border, we know that $2x + 2y = 200$. The area is to be maximized, so $A = x \cdot y$. Solving the perimeter formula for y: $2x + 2y = 200$

$$2y = 200 - 2x$$
$$= 100 - x$$

The area function is:

$$A(x) = x(100 - x) = -x^2 + 100x$$

The maximum value occurs at the vertex:

$$x = -\frac{b}{2a} = -\frac{100}{2(-1)} = -\frac{100}{-2} = 50$$

The pond should be 50 feet by 50 feet for maximum area.

The area function is:

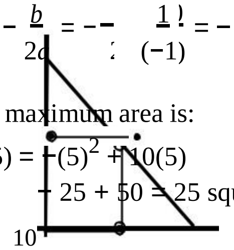
$$A(x) = x(10 - x) = -x^2 + 10x$$

The maximum value occurs at the vertex:

$$x = -\frac{b}{2a} = -\frac{10}{2(-1)} = -\frac{10}{-2} = 5$$

The maximum area is:

$$A(5) = -(5)^2 + 10(5)$$
$$= -25 + 50 = 25 \text{ square units}$$



Locate the origin at the point directly under the highest point of the arch. Then the equation is in the form: $y = -ax^2 + k$, where $a > 0$. Since the maximum height is 10 feet, when $x = 0$, $y = k = 10$. Since the point (10, 0) is on the parabola, we can find the constant:

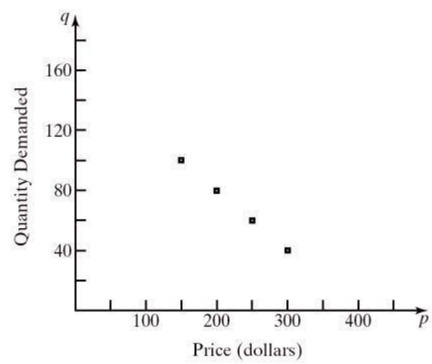
$$0 = -a(10)^2 + 10$$
$$a = \frac{10}{10^2} = \frac{1}{10} = 0.10$$

The equation of the parabola is:

$$y = -\frac{1}{10}x^2 + 10$$

At $x = 8$:

$$y = -\frac{1}{10}(8)^2 + 10 = -6.4 + 10 = 3.6 \text{ feet}$$



a.

b.

p	q	Avg. rate of change $= \frac{\Delta q}{\Delta p}$
150	100	
200	80	$\frac{80-100}{200-150} = \frac{-20}{50} = -0.4$
250	60	$\frac{60-80}{250-200} = \frac{-20}{50} = -0.4$
300	40	$\frac{40-60}{300-250} = \frac{-20}{50} = -0.4$

(0,10-x) (x,10-x) (x,0)

10

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Since each input (price) corresponds to a single output (quantity demanded), we know that the quantity demanded is a function of price. Also, because the average rate of change is constant at −$0.4 per LCD monitor, the function is linear.

From part (b), we know $m = -0.4$. Using (p_1, q_1) = (150,100), we get the equation:

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$q - q_1 = m(p - p_1)$
$q - 100 = -0.4(p - 150)$
$q - 100 = -0.4p + 60$
$= -0.4p + 160$

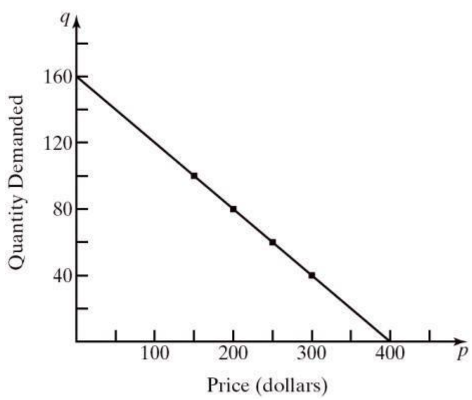
Using function notation, we have
$q(p) = -0.4p + 160$.

The price cannot be negative, so $p \ge 0$. Likewise, the quantity cannot be negative, so, $q(p) \ge 0$.

$-0.4p + 160 \ge 0$
$-0.4p \ge -160$
$p \le 400$

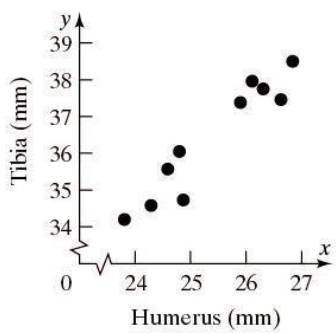
Thus, the implied domain for $q(p)$ is $\{p \mid 0 \le p \le 400\}$ or $[0, 400]$.

e.

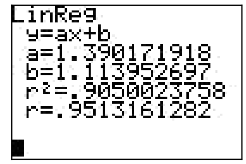


If the price increases by \$1, then the quantity demanded of LCD monitors decreases by 0.4 monitor.
p-intercept: If the price is \$0, then 160 LCD monitors will be demanded.
q-intercept: There will be 0 LCD monitors demanded when the price is \$400.

a.

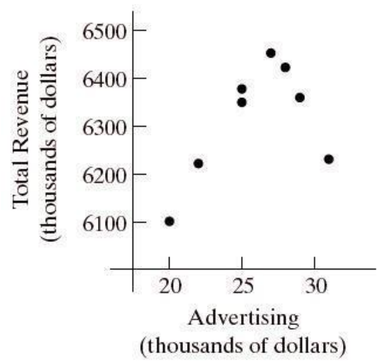


Yes, the two variables appear to have a linear relationship.
Using the LINear REGression program, the line of best fit is:
$y = 1.390171918x + 1.113952697$



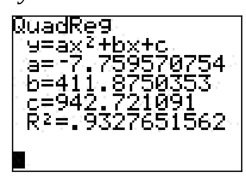
$y = 1.390171918(26.5) + 1.113952697$
38.0 mm

a.



The data appear to be quadratic with $a < 0$.

Using the QUADratic REGression program, the quadratic function of best fit is:
$y = -7.76x^2 + 411.88x + 942.72$.



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The maximum revenue occurs at

$$A = \frac{-b}{2a} = \frac{-(411.88)}{2(-7.76)} = \frac{-411.88}{-15.52} \approx \$26.5$$

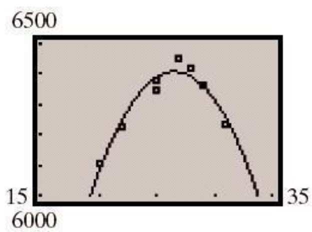
thousand **c.** The maximum revenue is

$$R\left(\frac{-b}{2a}\right) = R(26.53866)$$

$$= -7.76(26.5)^2 + (411.88)(26.5) + 942.72$$

$$\approx \$6408 \text{ thousand}$$

d.



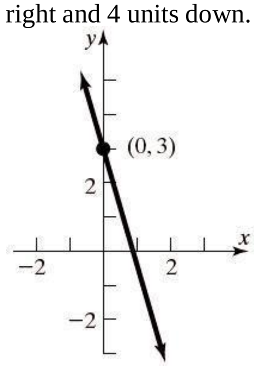
Chapter 2 Test

$f(x) = -4x + 3$

The slope f is -4.

The slope is negative, so the graph is decreasing.

Plot the point (0, 3) . Use the slope to find an additional point by moving 1 unit to the right and 4 units down.



2.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
−2	12	
−1	7	$\frac{7-12}{-1-(-2)} = \frac{-5}{1} = -5$
0	2	$\frac{2-7}{0-(-1)} = \frac{-5}{1} = -5$
1	−3	$\frac{-3-2}{1-0} = \frac{-5}{1} = -5$
2	−8	$\frac{-8-(-3)}{2-1} = \frac{-5}{1} = -5$

Since the average rate of change is constant at −5 , this is a linear function with slope = −5 . To find the equation of the line, we use the point-slope formula and one of the points.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -5(x - 0)$$

$$y = -5x + 2$$

$f(x) = 0$

$3x^2 - 2x - 8 = 0$

$(3x + 4)(x - 2) = 0$

$3x + 4 = 0$ or $x - 2 = 0$

$x = -\frac{4}{3}$ $\quad x = 2$

The zeros of f are $-\frac{4}{3}$ and 2.

$G(x) = 0$

$-2x^2 + 4x + 1 = 0$

$a = -2, b = 4, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(-2)(1)}}{2(-2)}$$

$$= \frac{-4 \pm \sqrt{24}}{-4} = \frac{-4 \pm 2\sqrt{6}}{-4} = \frac{2 \pm \sqrt{6}}{2}$$

The zeros of G are $\frac{2-\sqrt{6}}{2}$ and $\frac{2+\sqrt{6}}{2}$.

$f(x) = g(x)$

$x^2 + 3x = 5x + 3$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x + 1 = 0$ or $x - 3 = 0$

$x = -1$ $\quad x = 3$

The solution set is $\{-1, 3\}$.

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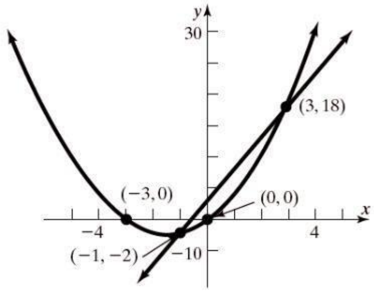


6. $f(x) = 0$

$(x-1)^2 + 5(x-1) + 4 = 0$

Let $u = x - 1 \rightarrow u^2 = (x-1)^2$

$u^2 + 5u + 4 = 0$

$(u+4)(u+1) = 0$

$u + 4 = 0$ or $u + 1 = 0$

$u = -4$ $\quad u = -1$

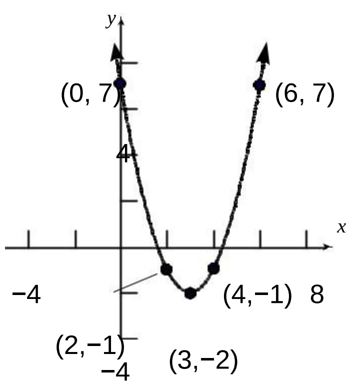
$x - 1 = -4$ $\quad x - 1 = -1$

$x = -3$ $\quad x = 0$

The zeros of G are −3 and 0.

$f(x) = (x-3)^2 - 2$

Using the graph of $y = x^2$, shift right 3 units, then shift down 2 units.



8. a. $f(x) = 3x^2 - 12x + 4$

$a = 3, b = -12, c = 4.$ Since $a = 3 > 0$, the graph opens up.

b. The x-coordinate of the vertex is

$b \quad -12 \quad -12$

The axis of symmetry is the line $x = 2$.

The discriminant is:

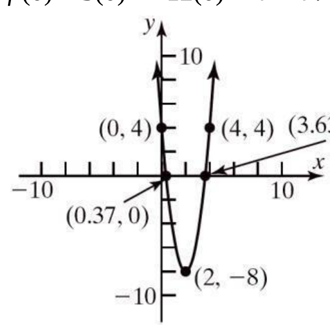
$b^2 - 4ac = (-12)^2 - 4(3)(4) = 96 > 0$, so the graph has two x-intercepts. The x-intercepts are found by solving: $3x^2 - 12x + 4 = 0$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-12) \pm \sqrt{96}}{2(3)}$

$= \frac{12 \pm 4\sqrt{6}}{6} = \frac{6 \pm 2\sqrt{6}}{3}$

The x-intercepts are $\frac{6-2\sqrt{6}}{3} \approx 0.37$ and $\frac{6+2\sqrt{6}}{3} \approx 3.63$. The y-intercept is $f(0) = 3(0)^2 - 12(0) + 4 = 4$.

e.



f. The domain is $(-\infty, \infty)$.

The range is $[-8, \infty)$.

g. Decreasing on $(-\infty, 2)$.

Increasing on $(2, \infty)$.

9. a. $g(x) = -2x^2 + 4x - 5$

$a = -2, b = 4, c = -5.$ Since $a = -2 < 0$, the graph opens down.

b. The x-coordinate of the vertex is

$x = -\frac{b}{2a} = -\frac{4}{2(-2)} = -\frac{4}{-4} = 1.$

The y-coordinate of the vertex is

$g\left(-\frac{b}{2a}\right) = g(1) = -2(1)^2 + 4(1) - 5$

$= -2 + 4 - 5 = -3$

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$x = -\frac{}{2a} = -\frac{}{2(3)} = -\frac{}{6} = 2$.

The *y*-coordinate of the vertex is

$f\left(-\frac{b}{2a}\right) = f(2) = 3(2)^2 - 12(2) + 4$

$= 12 - 24 + 4 = -8$

Thus, the vertex is $(2, -8)$.

Thus, the vertex is $(1, -3)$.

c. The axis of symmetry is the line $x = 1$.

d. The discriminant is:

$b^2 - 4ac = (4)^2 - 4(-2)(-5) = -24 < 0$, so the

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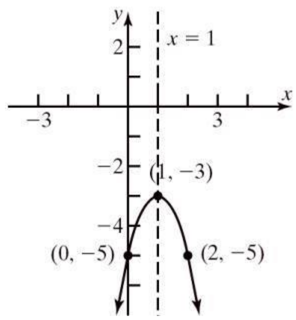
graph has no x-intercepts. The y-intercept is $g(0) = -2(0)^2 + 4(0) - 5 = -5$.

e.



The domain is $(-\infty, \infty)$.

The range is $(-\infty, -3]$.

Increasing on $(-\infty, 1)$.

Decreasing on $(1, \infty)$.

Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(1, -32)$ so we have $h = 1$ and $k = -32$. The graph also passes through the point $(x, y) = (0, -30)$. Substituting these values for x, y, h, and k, we can solve for a: $-30 = a(0-1)^2 + (-32)$ The quadratic function is $-30 = a(-1)^2 - 32$

$-30 = a -$

$32\ 2 = a$

$f(x) = 2(x-1)^2 - 32 = 2x^2 - 4x - 30$.

$f(x) = -2x^2 + 12x + 3$

$a = -2$, $b = 12$, $c = 3$. Since $a = -2 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at

$$x = -\frac{b}{2a} = -\frac{12}{2(-2)} = -\frac{12}{-4} = 3.$$

The maximum value is

$$f(3) = -2(3)^2 + 12(3) + 3 = -18 + 36 + 3 = 21.$$

$x^2 - 10x + 24 \ge 0$

$f(x) = x^2 - 10x + 24$

$x^2 - 10x + 24 = 0$

$(x-4)(x-6) = 0$

$x = 4$, $x = 6$ are the zeros of f.

Interval	$(-\infty, 4)$	$(4, 6)$	$(6, \infty)$
Test Number	0	5	7
Value of f	24	−1	3
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x \le 4 \text{ or } x \ge 6\}$ or, using interval notation, $(-\infty, 4] \cup [6, \infty)$.

$f(x) = 0$

$x^2 + 4x + 5 = 0$

$a = 2$, $b = 4$, $c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{-4 \pm \sqrt{-24}}{4} = \frac{-4 \pm 2\sqrt{6}\,i}{4} = -1 \pm \frac{\sqrt{6}}{2}i$$

The complex zeros of f are $-1 - \frac{\sqrt{6}}{2}i$ and $-1 + \frac{\sqrt{6}}{2}i$.

$|3x + 1| = 8$

$3x + 1 = 8$ or $3x + 1 = -8$

$3x = 7$ or $3x = -9$

$x = \frac{7}{3}$ or $x = -3$

The solution set is $\left\{-3, \frac{7}{3}\right\}$.

15. $\left|\frac{x+3}{4}\right| < 2$

$$-2 < \frac{x+3}{4} < 2$$

$-8 < x + 3 < 8$

$-11 < x < 5$

$\{x \mid -11 < x < 5\}$ or $(-11, 5)$

−11 0 5

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$2|x+3|-4 \ge 3$

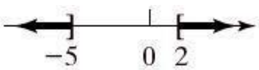
$|x+3| \ge 7$

$2x+3 \le -7$ or $2x+3 \ge 7$

$2x \le -10$ or $2x \ge 4$

$x \le -5$ or $x \ge 2$

$\{x \mid x \le -5 \text{ or } x \ge 2\}$ or $(-\infty, -5] \cup [2, \infty)$



a. $C(m) = 0.15m + 129.50$

b. $C(860) = 0.15(860) + 129.50$
$= 129 + 129.50 = 258.50$

If 860 miles are driven, the rental cost is $258.50.

$C(m) = 213.80$
$0.15m + 129.50 = 213.80$
$0.15m = 84.30$
$m = 562$

The rental cost is $213.80 if 562 miles were driven.

a. $R(x) = x\left(-\frac{1}{10}x + 1000\right) = -\frac{1}{10}x^2 + 1000x$

b. $R(400) = -\frac{1}{10}(400)^2 + 1000(400)$
$= -16,000 + 400,000$
$= \$384,000$

c. $x = \frac{-b}{2a} = \frac{-1000}{2\left(-\frac{1}{10}\right)} = \frac{-1000}{\left(-\frac{1}{5}\right)} = 5000$

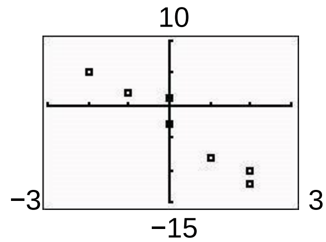
The maximum revenue is

$R(5000) = -\frac{1}{10}(5000)^2 + 1000(5000)$
$= -250,000 + 5,000,000$
$= \$2,500,000$

Thus, 5000 units maximizes revenue at $2,500,000.

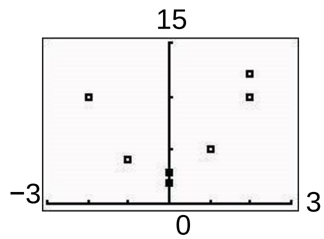
$p = -\frac{1}{10}(5000) + 1000$

19. a. Set A:



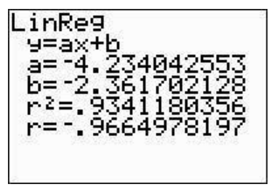
The data appear to be linear with a negative slope.

Set B:

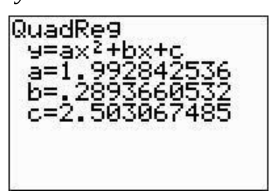


The data appear to be quadratic and opens up.

Using the LINear REGression program, the linear function of best fit is: $y = -4.234x - 2.362$.



Using the QUADratic REGression program, the quadratic function of best fit is: $y = 1.993x^2 + 0.289x + 2.503$.



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−500 + 1000
$500

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Chapter 2 Cumulative Review

$P = (-1,3); Q = (4,-2)$

Distance between P and Q:

$d(P,Q) = \sqrt{(4-(-1))^2 + (-2-3)^2}$

$= \sqrt{(5)^2 + (5)^2}$

$= \sqrt{25+25}$

$= \sqrt{50} = 5\sqrt{2}$

Midpoint between P and Q:

$\left(\frac{-1+4}{2}, \frac{3-2}{2}\right) = \left(\frac{3}{2}, \frac{1}{2}\right) = (1.5, 0.5)$

$y = x^3 - 3x + 1$

a. $(-2,-1)$: $-1 = (-2)^3 - 3(-2) + 1$

$-1 = -8+6+1$

$-1 = -1$

Yes, $(-2,-1)$ is on the graph.

$(2,3)$: $3 = (2)^3 - 3(2) + 1$

$3 = 8-6+1$

$3 = 3$

Yes, $(2,3)$ is on the graph.

$(3,1)$: $1 = (3)^3 - 3(3) + 1$

$1 = -27-9+1$

$1 \neq -35$

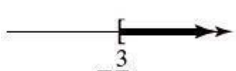
No, $(3,1)$ is not on the graph.

$5x + 3 \geq 0$

$5x \geq -3$

$x \geq -\frac{3}{5}$

The solution set is $\left\{x \,\middle|\, x \geq -\frac{3}{5}\right\}$ or $\left[-\frac{3}{5}, +\infty\right)$.



$(-1,4)$ and $(2,-2)$ are points on the line.

Slope $= \frac{-2-4}{2-(-1)} = \frac{-6}{3} = -2$

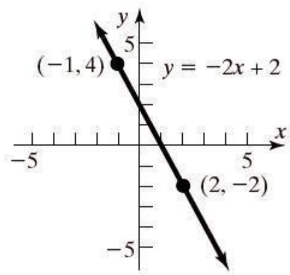
$y - y_1 = m(x - x_1)$

$y - 4 = -2(x - (-1))$

$y - 4 = -2(x + 1)$

$y - 4 = -2x - 2$

$y = -2x + 2$



Perpendicular to $y = 2x + 1$;
Containing $(3,5)$

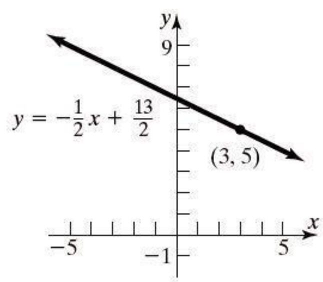
Slope of perpendicular $= -\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{2}(x - 3)$

$y - 5 = -\frac{1}{2}x + \frac{3}{2}$

$y = -\frac{1}{2}x + \frac{13}{2}$



$x^2 + y^2 - 4x + 8y - 5 = 0$

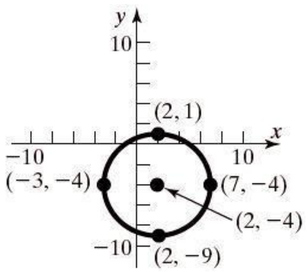
$x^2 - 4x + y^2 + 8y = 5$

$x^2 - 4x + 4) + (y^2 + 8y + 16) = 5 + 4$

$+ 16 (x - 2)^2 + (y + 4)^2 = 25$

$x - 2)^2 + (y + 4)^2 = 5^2$

Center: (2,–4) Radius = 5



Yes, this is a function since each x-value is paired with exactly one y-value.

$f(x) = x^2 - 4x + 1$

$f(2) = 2^2 - 4(2) + 1 = 4 - 8 + 1 = -3$

$f(x) + f(2) = x^2 - 4x + 1 + (-3)$

$x^2 - 4x - 2$

$f(-x) = (-x)^2 - 4(-x) + 1 = x^2 + 4x + 1$

$-f(x) = -(x^2 - 4x + 1) = -x^2 + 4x - 1$

$f(x+2) = (x+2)^2 - 4(x+2) + 1$

$x^2 + 4x + 4 - 4x - 8 + 1$

$x^2 - 3$

f. $\dfrac{f(x+h) - f(x)}{h}$

$\dfrac{(x+h)^2 - 4(x+h) + 1 - (x^2 - 4x + 1)}{h}$

$\dfrac{x^2 + 2xh + h^2 - 4x - 4h + 1 - x^2 + 4x - 1}{h}$

$\dfrac{2xh + h^2 - 4h}{h}$

$= \dfrac{h(2x + h - 4)}{h} = 2x + h - 4$

9. $h(z) = \dfrac{3z - 1}{6z - 7}$

The denominator cannot be zero:

$z - 7 \neq 0$

$6z \neq 7$

$z \neq \dfrac{7}{6}$

Domain: $\left\{ z \,\middle|\, z \neq \dfrac{7}{6} \right\}$

Yes, the graph represents a function since it passes the Vertical Line Test.

$f(x) = \dfrac{x}{x + 4}$

a. $f(1) = \dfrac{1}{1+4} = \dfrac{1}{5} \neq \dfrac{1}{4}$, so $\left(1, \dfrac{1}{4}\right)$ is not on the graph of f.

b. $f(-2) = \dfrac{-2}{-2+4} = \dfrac{-2}{2} = -1$, so $(-2, -1)$ is a point on the graph of f.

Solve for x:

$2 = \dfrac{x}{x+4}$

$2x + 8 = x$

$x = -8$

So, $(-8, 2)$ is a point on the graph of f.

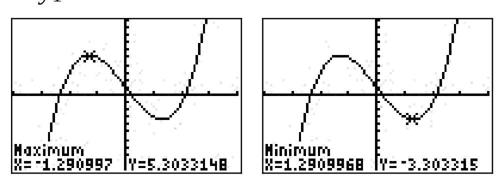
$f(x) = \dfrac{x^2}{2x + 1}$

$f(-x) = \dfrac{(-x)^2}{2(-x) + 1} = \dfrac{x^2}{-2x + 1} \neq f(x)$ or $-f(x)$

Therefore, f is neither even nor odd.

$f(x) = x^3 - 5x + 4$ on the interval $(-4, 4)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = x^3 - 5x + 4$.



Local maximum is 5.30 and occurs at $x \approx -1.29$;
Local minimum is –3.30 and occurs at $x \approx 1.29$;
f is increasing on $(-4, -1.29)$ or $(1.29, 4)$;
f is decreasing on $(-1.29, 1.29)$.

14. $f(x) = 3x + 5$; $g(x) = 2x + 1$

$f(x) = g(x)$

$3x + 5 = 2x + 1$

$3x + 5 = 2x + 1$

$x = -4$

b. $f(x) > g(x)$

$3x + 5 > 2x + 1$
$3x + 5 > 2x + 1$
$x > -4$

The solution set is $\{x \mid x > -4\}$ or $(-4, \infty)$.

a. Domain: $\{x \mid -4 \le x \le 4\}$ or $[-4, 4]$

Range: $\{y \mid -1 \le y \le 3\}$ or $[-1, 3]$

b. Intercepts: $(-1, 0)$, $(0, -1)$, $(1, 0)$
x-intercepts: -1, 1
y-intercept: -1

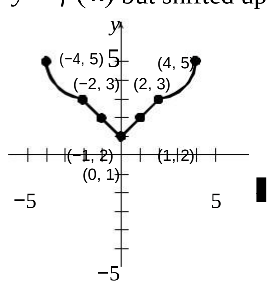
c. The graph is symmetric with respect to the y-axis.

d. When $x = 2$, the function takes on a value of 1. Therefore, $f(2) = 1$.

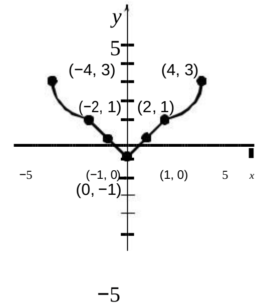
e. The function takes on the value 3 at $x = -4$ and $x = 4$.

f. $f(x) < 0$ means that the graph lies below the x-axis. This happens for x values between -1 and 1. Thus, the solution set is $\{x \mid -1 < x < 1\}$ or $(-1, 1)$.

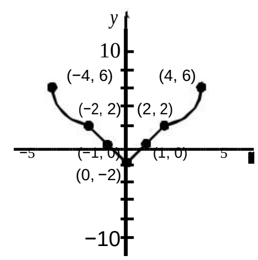
g. The graph of $y = f(x) + 2$ is the graph of $y = f(x)$ but shifted up 2 units.



h. The graph of $y = f(-x)$ is the graph of $y = f(x)$ but reflected about the y-axis.



i. The graph of $y = 2f(x)$ is the graph of $y = f(x)$ but stretched vertically by a factor of 2. That is, the coordinate of each point is multiplied by 2.



j. Since the graph is symmetric about the y-axis, the function is even.

k. The function is increasing on the open interval $(0,4)$.

Chapter 2 Projects

Project I – Internet-based Project

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Answers will vary.

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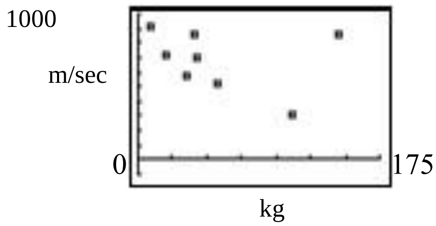
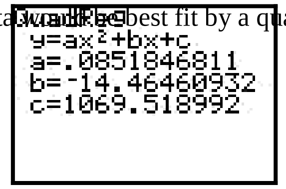
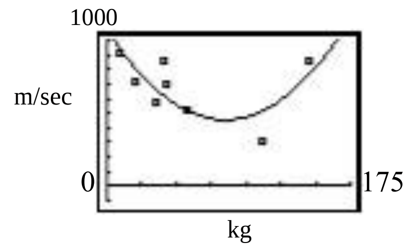
Project II



b. The data would be best fit by a quadratic function.



$y = 0.085x^2 - 14.46x + 1069.52$



These results seem reasonable since the function fits the data well.

c. $s_0 = 0$m

Type	Weight kg	Velocity m/sec	Equation in the form: $s(t) = -4.9t^2 + \frac{\sqrt{2}}{2} v_0 t + s_0$
MG 17	10.2	905	$s(t) = -4.9t^2 + 639.93t$ Best. (It goes the highest)
MG 131	19.7	710	$s(t) = -4.9t^2 + 502.05t$
MG 151	41.5	850	$s(t) = -4.9t^2 + 601.04t$
MG 151/20	42.3	695	$s(t) = -4.9t^2 + 491.44t$
MG/FF	35.7	575	$s(t) = -4.9t^2 + 406.59t$
MK 103	145	860	$s(t) = -4.9t^2 + 608.11t$
MK 108	58	520	$s(t) = -4.9t^2 + 367.70t$
WGr 21	111	315	$s(t) = -4.9t^2 + 222.74t$

$s_0 = 200$m

Type	Weight kg	Velocity m/sec	Equation in the form: $s(t) = -4.9t^2 + \frac{\sqrt{2}}{2} v_0 t + s_0$
MG 17	10.2	905	$s(t) = -4.9t^2 + 639.93t + 200$ Best. (It goes the
MG 131	19.7	710	highest) $s(t) = -4.9t^2 + 502.05t + 200$
MG 151	41.5	850	$s(t) = -4.9t^2 + 601.04t + 200$
MG 151/20	42.3	695	$s(t) = -4.9t^2 + 491.44t + 200$ s
MG/FF	35.7	575	$(t) = -4.9t^2 + 406.59t + 200$ $s(t$
MK 103	145	860	$) = -4.9t^2 + 608.11t + 200$ $s(t)$
MK 108	58	520	$= -4.9t^2 + 367.70t + 200$ $s(t) =$

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WGr 21	111	315	$-4.9t^2 + 222.74t + 200$

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$s_0 = 30$m Type	Weight kg	Velocity m/sec	Equation in the form: $s(t) = -4.9t^2 + v_0 t + s_0$
MG 17	10.2	905	$s(t) = -4.9t^2 + 639.93t + 30$ Best. (It goes the highest)
MG 131	19.7	710	$s(t) = -4.9t^2 + 502.05t + 30$
MG 151	41.5	850	$s(t) = -4.9t^2 + 601.04t + 30$
MG 151/20	42.3	695	$s(t) = -4.9t^2 + 491.44t + 30$
MG/FF	35.7	575	$s(t) = -4.9t^2 + 406.59t + 30$
MK 103	145	860	$s(t) = -4.9t^2 + 608.11t + 30$
MK 108	58	520	$s(t) = -4.9t^2 + 367.70t + 30$
WGr 21	111	315	$s(t) = -4.9t^2 + 222.74t + 30$

Notice that the gun is what makes the difference, not how high it is mounted necessarily. The only way to change the true maximum height that the projectile can go is to change the angle at which it fires.

Project III

a.

x	1	2	3	4	5
$y = -2x + 5$	3	1	−1	−3	−5

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{1} = -2$$

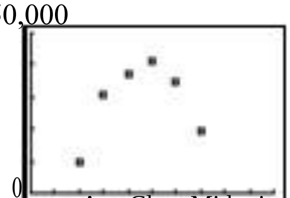
$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-1}{1} = -2$$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-1)}{1} = -2$$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-(-3)}{1} = -2$$

All of the values of $\frac{\Delta y}{\Delta x}$ are the same.

c.



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d. $\frac{I}{x} = \frac{30633 - 9548}{10} = 2108.50$

$= \frac{37088 - 30633}{x10} = 645.50$

$= \frac{41072 - 37088}{x10} = 398.40$

$= \frac{34414 - 41072}{x10} = -665.80$

$\frac{I}{x} = \frac{19167 - 34414}{10} = -1524.70$

These $\frac{I}{x}$ values are not all equal. The data are not linearly related.

e.

x	-2	-1	0	1	2	3	4
y	23	9	3	5	15	33	59
$\frac{y}{x}$		-14	-6	2	10	18	26

As x increases, $\frac{y}{x}$ increases. This makes sense because the parabola is increasing (going up) steeply as x increases.

f.

x	-2	-1	0	1	2	3	4
y	23	9	3	5	15	33	59
$\frac{^2y}{x^2}$			8	8	8	8	8

The second differences are all the same.

The paragraph should mention at least two observations:

The first differences for a linear function are all the same.

The second differences for a quadratic function are the same.

Project IV

a. – i. Answers will vary , depending on where the CBL is located above the bouncing ball.

The ratio of the heights between bounces will be the same.

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Mini-Lecture 2.8
Equations and Inequalities Involving the Absolute Value Function

Learning Objectives:

Solve Absolute Value Equations (p. 179)
Solve Absolute Value Inequalities (p. 179)

Examples:

1. Solve each equation.

   $(a)\ |5x-10| = 15$ $(b)\ \left|\frac{2}{3}x+6\right| = 12$ $(c)\ |4-3x|-4=1$ $(d)\ |3-x| = -7$

2. Solve each absolute value inequality.

   $(a)\ |3x| \le 21$ $(b)\ |4x-3| \ge 9$ $(c)\ |2-6x|-5 < 1$

Teaching Notes:

When solving absolute value equations, students will sometimes forget that there are two solutions.
Students will often not isolate the absolute value expression before trying to solve, such as examples 1c and 2c above.
Some students try to combine two intervals that cannot be combined, such as $-3 < x > 2$.

Answers:

1. $(a)\ x = 5, x = -1$ $(b)\ x = 9, x = -27$ $(c)\ x = -\frac{1}{3}, x = 3$ *(d) No solution*

2. $(a)\ -7 \le x \le 7$ $(b)\ x \le -\frac{3}{2}$ *or* $x \ge 3$ $(c)\ -\frac{2}{3} < x < \frac{4}{3}$

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Mini-Lecture 2.1

Properties of Linear Functions and Linear Models

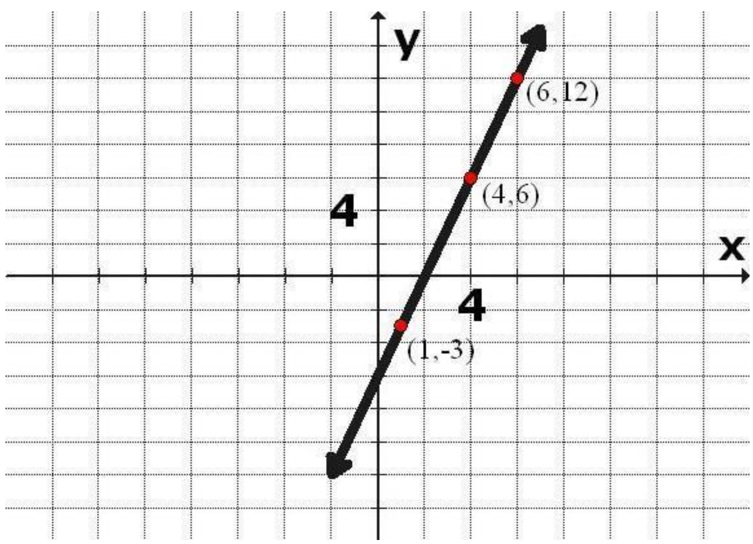
Learning Objectives:

Graph Linear Functions (p. 119)
Use Average Rate of Change to Identify Linear Functions (p. 119)
Determine Whether a Linear function Is Increasing, Decreasing, or Constant (p. 122)
Find the Zero of a Linear Function (p. 123)
4. Build Linear Models from Verbal Descriptions (p. 124)

Examples:

1. Suppose that $f(x) = 5x - 9$ and $g(x) = -3x + 7$. Solve $f(x) = g(x)$. Then graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution to the equation $(x) = g(x)$.

In parts (a) and (b) using the following figure,



(a) Solve $f(x) = 12$.   (b) Solve $0 < f(x) < 12$.

The monthly cost C, in dollars, for renting a full-size car for a day from a particular agency is modeled by the function $C(x) = 0.12x + 40$, where x is the number of miles driven. Suppose that your budget for renting a car is $100. What is the maximum number of miles that you can drive in one day?

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Find a firm's break-even point if $R(x) = 10x$ and $C(x) = 7x + 6000$. (Before working this problem, go over the explanation above Problems 49 and 50 on page 128.)

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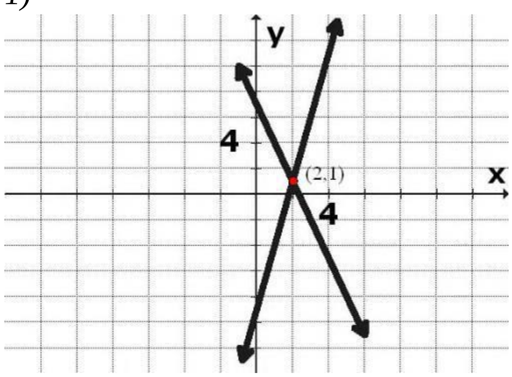
Teaching Notes:

Review the slope-intercept form of the equation of a line.
Emphasize the theorem for Average Rate of Change of a Linear Function in the book.
Emphasize the need to express the answer to a verbal problem in terms of the units given in the problem.
Discuss depreciation, supply and demand, and break-even analysis in more depth before working either the examples in the book or the examples above.

***Answers*:** *1) x=2; 2) a) x=6, b) 2 < x < 6; 3) 500 miles; 4) 2000 units*

1)



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Mini-Lecture 2.2
Building Linear Models from Data

Learning Objectives:

Draw and Interpret Scatter Diagrams (p. 130)
Distinguish between Linear and Nonlinear Relations (p. 131)
Use a Graphing Utility to Find the Line of Best Fit (p. 132)

Examples:

x	3	7	8	9	11	15
y	2	4	7	8	6	10

Draw a scatter diagram. Select two points from the scatter diagram and find the equation of the line containing the two points.

Graph the line on the scatter diagram.

The marketing manager for a toy company wishes to find a function that relates the demand *D* for a doll and *p* the price of the doll. The following data were obtained based on a price history of the doll. The Demand is given in thousands of dolls sold per day.

Price	9.00	10.50	11.00	12.00	12.50	13
Demand	12	11	9	10	9.5	8

Use a graphing utility to draw a scatter diagram. Then, find and draw the line of best fit.

How many dolls will be demanded if the price is $11.50?

Teaching Notes:

Many students have trouble deciding what scale to use on the *x*- and *y*-axes of their scatter plots. Remind them that the scale does not have to be the same on both axes and that the axes may show a break between zero and the first labeled tick mark.
Use a set of data and ask different students to pick two points and find the equation of the line through the points. Use the graphing utility to draw a scatter diagram and plot each student's line on the scatter diagram. Then find the line of best fit using the graphing utility and graph it on the scatter diagram. This exercise will give students a better understanding of the line of best fit.
To help students understand of the correlation coefficient show scatter diagrams for different data sets. Show the students data sets with correlations coefficients close to 1, -1, and 0.

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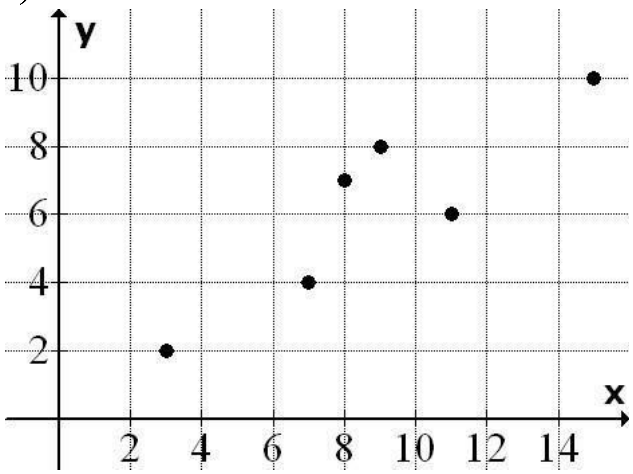
Answers: *1)* $y = \frac{1}{2}x + \frac{1}{2}$ *for points (3,2) and (7,4); 4) D=9.77 thousand dolls*

1)



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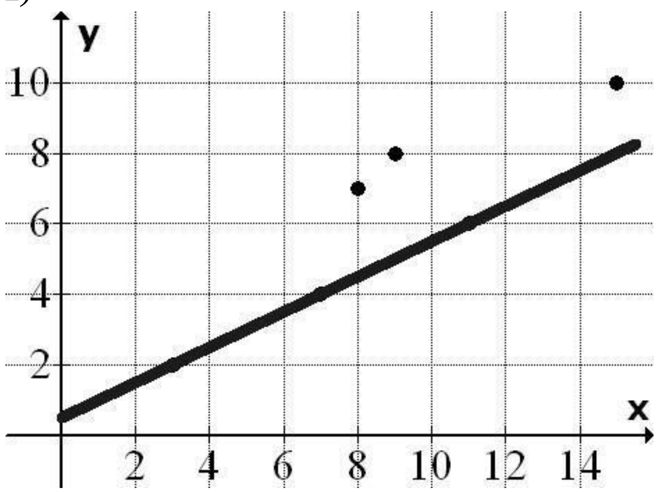

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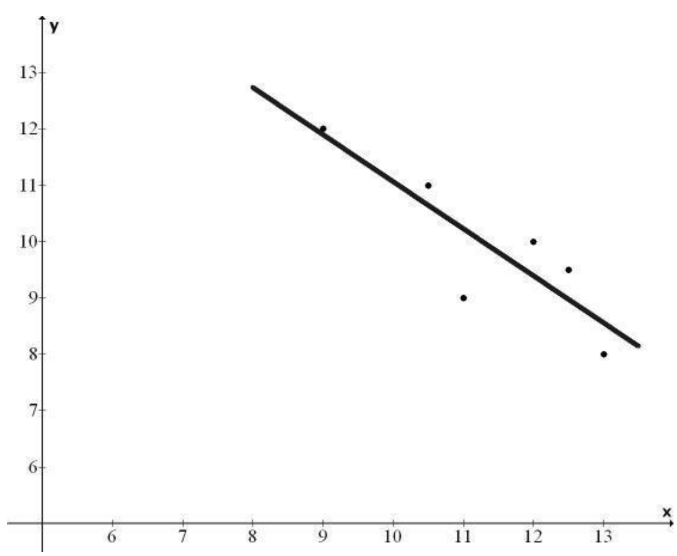


Mini-Lecture 2.3
Quadratic Functions and Their Zeros

Learning Objectives:

Find the Zeros of a Quadratic Function by Factoring (p. 138)
Find the Zeros of a Quadratic Function Using the Square Root Method (p. 138)
Find the Zeros of a Quadratic Function by Completing the Square (p. 140)
Find the Zeros of a Quadratic Function Using the Quadratic Formula (p. 141)
Find the Point of Intersection of Two Functions (p. 143)
Solve Equations That are Quadratic in Form (p. 144)

Examples:

Find the zeros by factoring: $f(x) = 3x^2 + 4x - 4$

Find the zeros by using the square root method: $f(x) = (4x - 1)^2 - 16$

Find the zeros by completing the square: $f(x) = x^2 + 4x - 10$

Find the zeros by using the quadratic formula: $f(x) = 3x^2 - 5x - 7$

Find the real zeros of: $f(x) = x^4 - 11x^2 + 18$

Find the points of intersection: $f(x) = x^2 - 4$ & $g(x) = 3 - x^2$

Teaching Notes:

Students that do not have good skills will struggle with this section. Most students can factor pretty well, but they will commit many types of algebraic mistakes when using the other methods.

When students use the quadratic formula, they will have trouble simplifying the rational expression. For example, reducing like this $\frac{10 \pm 5\sqrt{10}}{10} = 1 \pm 5\sqrt{10}$ is a common error.

Completing the square will shine a light on the difficulties that students have with fractions.

***Answers*:**

1) $\frac{2}{3}; -2$ *2)* $\frac{5}{4}; -\frac{3}{4}$ *3)* $-2 \pm \sqrt{14}$ *4)* $\frac{5 \pm \sqrt{109}}{6}$

5) $x = \pm\sqrt{2}; \pm 3$ *6)* $\pm\frac{\sqrt{14}}{2}, -\frac{1}{2}$

Mini-Lecture 2.4
Properties of Quadratic Functions

Learning Objectives:

Graph a Quadratic Function Using Transformations (p. 149)
Identify the Vertex and Axis of Symmetry of a Quadratic Function (p. 151)
Graph a Quadratic Function Using Its Vertex, Axis, and Intercepts (p. 152)
Find a Quadratic Function Given Its Vertex and One Other Point (p. 155)
Find the Maximum and Minimum Value of a Quadratic Function (p. 156)

Examples:

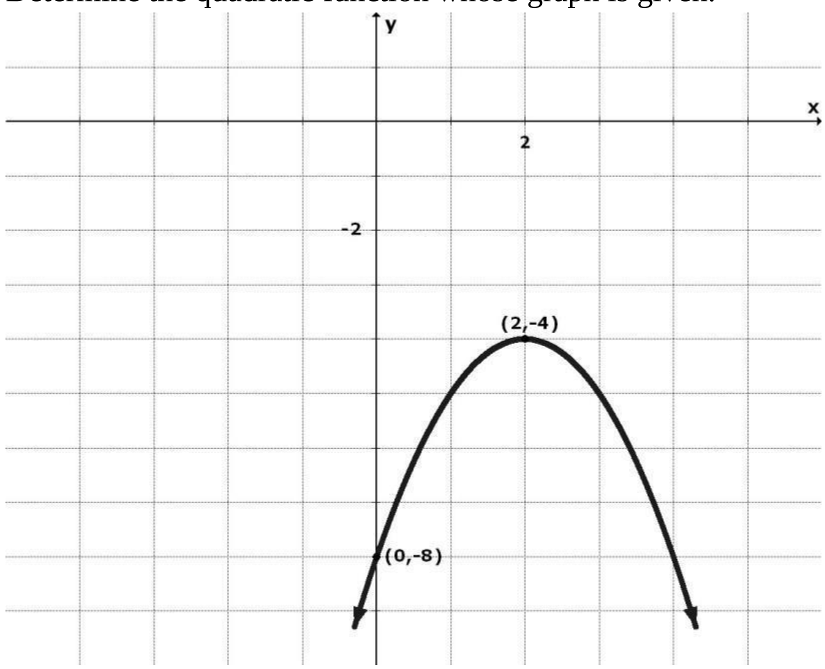
Find the coordinates of the vertex for the parabola defined by the given quadratic function. $f(x) = -3x^2 + 5x - 4$

Sketch the graph of the quadratic function by determining whether it opens up or down and by finding its vertex, axis of symmetry, y-intercepts, and x-intercepts, if any. $f(x) = 6 - 5x + x^2$

For the quadratic function, $f(x) = 4x^2 - 8x$,

determine, without graphing, whether the function has a minimum value or a maximum value,
find the minimum or maximum value.

Determine the quadratic function whose graph is given.



Teaching Notes:

Remind students to review transformations of graphs before beginning to graph quadratic functions.

Emphasize from the book “Steps for Graphing a Quadratic Function $f(x) = ax^2 + bx + c$, $a \neq 0$, by Hand.”

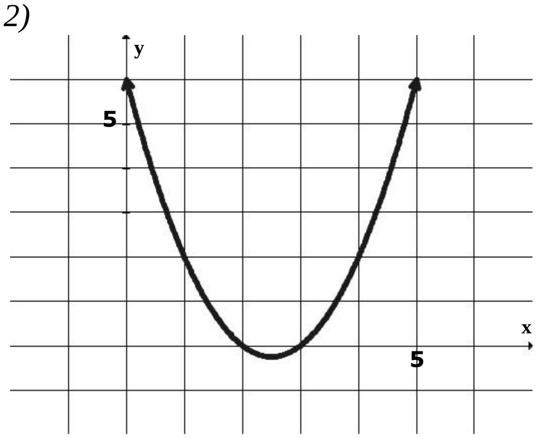
Stress the use of *a* from the standard form to determine the direction the parabola is opening before beginning to graph it. Students need to recognize early on the benefits of knowing as much about a graph as possible before beginning to draw it.

In addition to the intercepts, encourage students to use symmetry to find additional points on the graph of a quadratic function.

Many students will give the *x*-value found with $x = -\frac{b}{2a}$ as the maximum or minimum value of the quadratic function. Emphasize that finding the maximum or minimum is a two-step process. First, find <u>where</u> it occurs (*x*), then find <u>what</u> it is (*y*).

***Answers*:** *1)* $\left(\frac{5}{6}, -\frac{23}{12}\right)$ *; 2) opens up, vertex* $\left(\frac{5}{2}, -\frac{1}{4}\right)$ *, axis* $x = \frac{5}{2}$*, x-intercepts 2 and 3, y-intercept 6*

2)



3) a. minimum, b. minimum of -4; 4) $f(x) = -x^2 + 4x - 8$

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Mini-Lecture 2.5

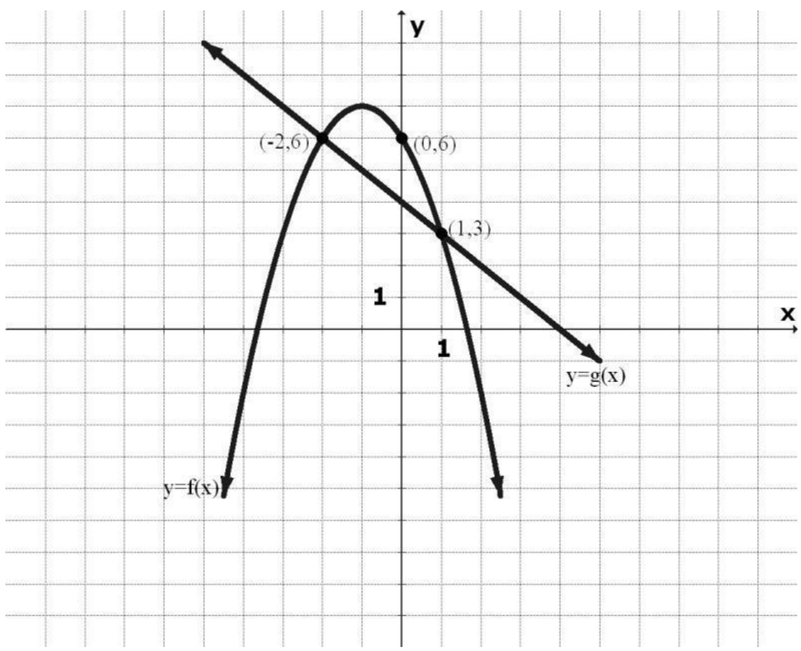
Inequalities Involving Quadratic Functions

Learning Objectives:

1. Solve Inequalities Involving a Quadratic Function (p. 160)

Examples:

1. Use the figure to solve the inequality $f(x) \geq g(x)$.



Solve and express the solution in interval notation. $9x^2 - 6x + 1 < 0$

Solve the inequality. $x(x+2) > 15$

4. Solve $f(x) > g(x)$. $f(x) = -x^2 + 2x + 3$; $g(x) = -x^2 - 2x + 8$

Teaching Notes:

Suggest that students review interval notation. Caution them to check for the correct use of brackets or parentheses in solutions written in interval notation.
Suggest that students review factoring a trinomial.
Be sure to show Method I and Method II in Example 2 in the book.

Answers: *1)* $[-2,1]$ *; 2)* $\varnothing$ *; 3)* $(-\infty, -5) \cup (3, \infty)$ *; 4)* $\frac{5}{4}, \infty$

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Mini-Lecture 2.6

Building Quadratic Models from Verbal Descriptions and from Data

Learning Objectives:

Build Quadratic Models from Verbal Descriptions (p. 165)
Build Quadratic Models from Data (p. 169)

Examples:

Among all pairs of numbers whose sum is 50, find a pair whose product is as large as possible. What is the maximum product?

A person standing close to the edge of the top of a 180-foot tower throws a ball vertically upward. The quadratic function $s(t) = -16t^2 + 64t + 180$ models the ball's height above ground, $s(t)$, in feet, t seconds after it was thrown. After how many seconds does the ball reach its maximum height? What is the maximum height?

The price p (in dollars) and the quantity x sold of a certain product obey the demand equation $p = -\frac{1}{4}x + 120$. Find the model that expresses the revenue R as a function of x. What quantity x maximizes revenue? What is the maximum revenue?

The following data represent the percentage of the population in a certain country aged 40 or older whose age is x who do not have a college degree of some type.

Age, x	40	45	50	55	60	65
No college	25.4	23.2	21.8	24.5	26.1	29.8

Find a quadratic model that describes the relationship between age and percentage of the population that do not have a college degree. Use the model to predict the percentage of 53-year-olds that do not have a college degree.

Teaching Notes:

Show students how to use MAXIMUM and MINIMUM on the graphing utility.
Show students how to use the QUADratic REGression program on the graphing utility.
Encourage students to review the discriminant.

Answers: *1) (25, 25), 625; 2) 2 sec., 244 ft; 3)* $R(x) = -\frac{1}{4}x^2 + 120x$,

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x=240, R(240)=$14,400; 4) $P(x) = .0296x - 2.9216x + 94.6550$, *23.0%*

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Mini-Lecture 2.7

Complex Zeros of a Quadratic Function

Learning Objectives:

1. Find the Complex Zeros of a Quadratic Function (p. 175)

Examples:

1. Find the complex zeros; graph the function; label the intercepts.

   $(a)\ f(x) = x^2 + 5$ $(b)\ f(x) = x^2 + 2x + 7$

   $(c)\ f(x) = 2x^2 - 4x - 5$ $(d)\ f(x) = x^2 - 2x + 5$

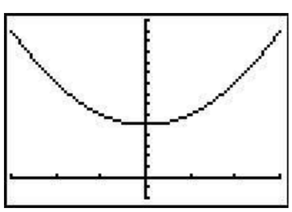
Teaching Notes:

Have students review the quadratic formula.
Students will need to know how to reduce radical and rational expressions.
Remind the students that *i* is outside the radical when simplifying a radical expression.

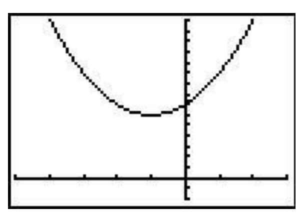
Answers:

1. (a) Complex zeros $x = \pm i\sqrt{5}$ *; y-intercept = 5; no x-intercepts.*
(b) Complex zeros $x = -1 \pm i\sqrt{6}$ *; y-intercept = 7; no x-intercepts*
(c) Complex zeros $x = \dfrac{2 \pm \sqrt{14}}{2}$ *; y-intercept = -5; x-intercepts* $x = \dfrac{2 \pm \sqrt{14}}{2}$
(d) Complex zeros $x = 1 \pm 2i$ *; y-intercept = 5; no x-intercepts*

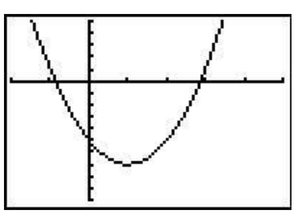
(a)



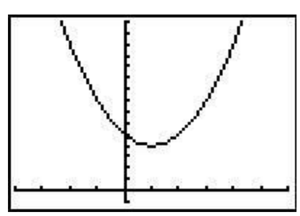
(b)



(c)



(d)



Cannons The velocity of a projectile depends upon many factors, in particular, the weight of the ammunition. Plot a scatter diagram of the data in the table below. Let x be the weight in kilograms and let y be the velocity in meters per second.

Type	Weight (kg)	Initial Velocity (m/ sec)
MG 17	10.2	905
MG 131	19.7	710
MG 151	41.5	850
MG 151>20	42.3	695
MG> FF	35.7	575
MK 103	145	860
MK 108	58	520
WGr 21	111	315

(Data and information taken from "Flugzeug-Handbuch, Ausgabe Dezember 1996: Guns and Cannons of the Jagdwaffe" at www.xs4all.nl/~rhorta/jgguns.htm*)*

Determine which type of function would fit this data the best: linear or quadratic. Use a graphing utility to find the function of best fit. Are the results reasonable?

Based on velocity, we can determine how high a projectile will travel before it begins**1** to come back down. If a cannon is fired at an angle of 45° to the horizon-tal, then the function for the height of the projectile is given

by s1t2 = $-16t^2 + \frac{2}{2} v_0t + s_0$, where v_0 is the

velocity at which the shell leaves the cannon (initial velocity), and s_0 is the initial height of the nose of the cannon (because cannons are not very long, we may assume that the nose and the firing pin at the back are at the same height for simplicity). Graph the func-tion s = s1t2 for each of the guns described in the table. Which gun would be the best for anti-aircraft if the gun were sitting on the ground? Which would be the best to have mounted on a hilltop or on the top of a tall building? If the guns were on the turret of a ship, which would be the most effective?

3. Suppose $f(x) = \sin x$.

(a) Build a table of values for $f(x)$ where $x = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}, 2\pi$. Use exact values.

(b) Find the **first differences** for each consecutive pair of values in part (a). That is, evaluate $g(x_i) = \frac{\Delta f(x_i)}{\Delta x_i} = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$, where $x_1 = 0$, $x_2 = \frac{\pi}{6}, \ldots,$ $x_{17} = 2\pi$. Use your calculator to approximate each value rounded to three decimal places.

(c) Plot the points $(x_i, g(x_i))$ for $i = 1, \ldots, 16$ on a scatter diagram. What shape does the set of points give? What function does this resemble? Fit a sine curve of best fit to the points. How does that relate to your guess?

(d) Find the first differences for each consecutive pair of values in part (b). That is, evaluate $h(x_i) = \frac{\Delta g(x_i)}{\Delta x_i} = \frac{g(x_{i+1}) - g(x_i)}{x_{i+1} - x_i}$ where $x_1 = 0$, $x_2 = \frac{\pi}{6}, \ldots, x_{16} = \frac{11\pi}{6}$. This is the set of **second differences** of $f(x)$. Use your calculator to approximate each value rounded to three decimal places. Plot the points $(x_i, h(x_i))$ for $i = 1, \ldots, 15$ on a scatter diagram. What shape does the set of points give? What func-tion does this resemble? Fit a sine curve of best fit to the points. How does that relate to your guess?

(e) Find the first differences for each consecutive pair of values in part (d). That is, evaluate $k(x_i) = \frac{\Delta h(x_i)}{\Delta x_i} = \frac{h(x_{i+1}) - h(x_i)}{x_{i+1} - x_i}$, where $x_1 = 0, x_2 = \frac{\pi}{6}, \ldots, x_{15} = \frac{7\pi}{4}$. This is the set of **third differences** of $f(x)$. Use your calculator to approximate each value rounded to three decimal places. Plot the points $(x_i, k(x_i))$ for $i = 1, \ldots, 14$ on a scatter diagram. What shape does the set of points give? What func-tion does this resemble? Fit a sine curve of best fit to the points. How does that relate to your guess?

(f) Find the first differences for each consecutive pair of values in part (e). That is, evaluate $m(x_i) = \frac{\Delta k(x_i)}{\Delta x_i} = \frac{k(x_{i+1}) - k(x_i)}{x_{i+1} - x_i}$, where $x_1 = 0$, $x_2 = \frac{\pi}{6}, \ldots, x_{14} = \frac{5\pi}{3}$. This is the set of **fourth differences** of $f(x)$.

Use your calculator to approximate each value rounded to three decimal places. Plot the points $(x_i, m(x_i))$ for $i = 1, \ldots, 13$ on a scatter diagram. What shape does the set of points give? What func-tion does this resemble? Fit a sine curve of best fit to the points. How does that relate to your guess?

What pattern do you notice about the curves that you found? What happened in part (f)? Can you make a generalization about what happened as you com-puted the differences? Explain your answers.

CBL Experiment Locate the motion detector on a Calculator Based Laboratory (CBL) or a Calculator Based Ranger (CBR) above a bouncing ball.

Plot the data collected in a scatter diagram with time as the independent variable.

Find the quadratic function of best fit for the sec-ond bounce.

Find the quadratic function of best fit for the third bounce.

Find the quadratic function of best fit for the fourth bounce.

Compute the maximum height for the second bounce.

Compute the maximum height for the third bounce.

Compute the maximum height for the fourth bounce.

Compute the ratio of the maximum height of the third bounce to the maximum height of the second bounce.

Compute the ratio of the maximum height of the fourth bounce to the maximum height of the third bounce.

(j) Compare the results from parts (h) and (i). What do you conclude?