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Chapter 2

More on Functions

Exercise Set 2.1

1. a) For x-values from -5 to 1, the y-values increase from -3 to 3. Thus the function is increasing on the interval $(-5, 1)$.

b) For x-values from 3 to 5, the y-values decrease from 3 to 1. Thus the function is decreasing on the interval $(3,5)$.

c) For x-values from 1 to 3, y is 3. Thus the function is constant on $(1, 3)$.

2. a) For x-values from 1 to 3, the y-values increase from 1 to 2. Thus, the function is increasing on the interval $(1, 3)$.

b) For x-values from -5 to 1, the y-values decrease from 4 to 1. Thus the function is decreasing on the interval $(-5, 1)$.

c) For x-values from 3 to 5, y is 2. Thus the function is constant on $(3, 5)$.

3. a) For x-values from -3 to -1, the y-values increase from -4 to 4. Also, for x-values from 3 to 5, the y-values increase from 2 to 6. Thus the function is increasing on $(-3, -1)$ and on $(3, 5)$.

b) For x-values from 1 to 3, the y-values decrease from 3 to 2. Thus the function is decreasing on the interval $(1,3)$.

c) For x-values from -5 to -3, y is 1. Thus the function is constant on $(-5, -3)$.

4. a) For x-values from 1 to 2, the y-values increase from 1 to 2. Thus the function is increasing on the interval $(1, 2)$.

b) For x-values from -5 to -2, the y-values decrease from 3 to 1. For x-values from -2 to 1, the y-values decrease from 3 to 1. And for x-values from 3 to 5, the y-values decrease from 2 to 1. Thus the function is decreasing on $(-5, -2)$, on $(-2, 1)$, and on $(3, 5)$.

c) For x-values from 2 to 3, y is 2. Thus the function isconstant on $(2, 3)$.

5. a) For x-values from $-\infty$ to -8, the y-values increase from
$-\infty$ to 2. Also, for x-values from -3 to -2, the y-values increase from -2 to 3. Thus the function is increasing on $(-\infty, -8)$ and on $(-3, -2)$.

b) For x-values from -8 to -6, the y-values decrease from 2to -2. Thus the function is decreasing on the interval$(-8, -6)$.

c) For x-values from -6 to -3, y is -2. Also, for x- values from -2 to ∞, y is 3. Thus the function is constant on $(-6, -3)$ and on $(-2, \infty)$.

6. a) For x-values from 1 to 4, the y-values increase from 2 to 11. Thus the function is increasing on the interval $(1, 4)$.

   b) For x-values from -1 to 1, the y-values decrease from 6 to 2. Also, for x-values from 4 to ∞, the y- values decrease from 11 to $-\infty$. Thus the function is decreasing on $(-1, 1)$ and on $(4, \infty)$.

   c) For x-values from $-\infty$ to -1, y is 3. Thus the func- tion is constant on $(-\infty, -1)$.

7. The x-values extend from -5 to 5, so the domain is $[-5, 5]$. The y-values extend from -3 to 3, so the range is $[-3, 3]$.

8. Domain: $[-5, 5]$; range: $[1, 4]$

9. The x-values extend from -5 to -1 and from 1 to 5, so the domain is $[-5, -1] \cup [1, 5]$.

   The y-values extend from -4 to 6, so the range is $[-4, 6]$.

10. Domain: $[-5, 5]$; range: $[1, 3]$

11. The x-values extend from $-\infty$ to ∞, so the domain is $(-\infty, \infty)$.

    The y-values extend from $-\infty$ to 3, so the range is $(-\infty, 3]$.

12. Domain: $(-\infty, \infty)$; range: $(-\infty, 11]$

13. From the graph we see that a relative maximum value of the function is 3.25. It occurs at $x = 2.5$. There is no relative minimum value.

    The graph starts rising, or increasing, from the left and stops increasing at the relative maximum. From this point, the graph decreases. Thus the function is increasing on $(-\infty, 2.5)$ and is decreasing on $(2.5, \infty)$.

14. From the graph we see that a relative minimum value of 2 occurs at $x = 1$. There is no relative maximum value.

    The graph starts falling, or decreasing, from the left and stops decreasing at the relative minimum. From this point, the graph increases. Thus the function is increasing on $(1, \infty)$ and is decreasing on $(-\infty, 1)$.

15. From the graph we see that a relative maximum value of the function is 2.370. It occurs at $x = 0.667$. We also see that a relative minimum value of 0 occurs at $x = 2$.

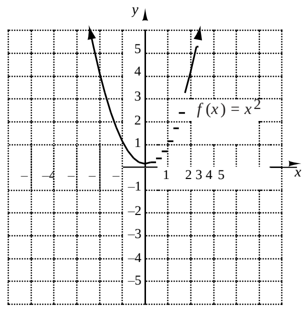
    The graph starts rising, or increasing, from the left and stops increasing at the relative maximum. From this point it decreases to the relative minimum and then increases again. Thus the function is increasing on $(-\infty, -0.667)$ and on $(2, \infty)$. It is decreasing on $(-0.667, 2)$.

    –

16. From the graph we see that a relative maximum value of 2.921 occurs at $x = 3.601$. A relative minimum value of 0.995 occurs at $x = 0.103$.

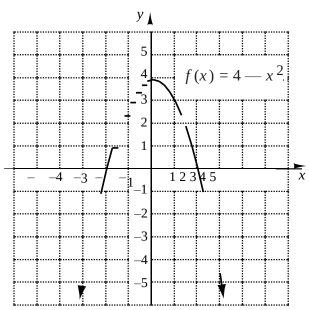
The graph starts decreasing from the left and stops decreasing at the relative minimum. From this point it increases to the relative maximum and then decreases again. Thus the function is increasing on (0.103,3.601) and is decreasing on $(-\infty, 0.103)$ and on $(3.601, \infty)$.

17.



The function is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$. We estimate that the minimum is 0 at $x = 0$. There are no maxima.

18.



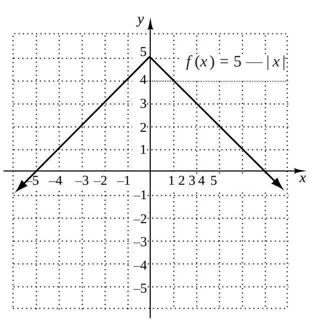
Increasing: $(-\infty, 0)$

Decreasing: $(0, \infty)$

Maximum: 4 at $x = 0$

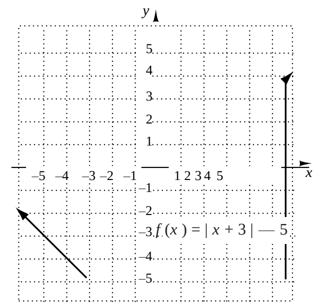
Minima: none

19.



The function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. We estimate that the maximum is 5 at $x = 0$. There are no minima.

20.



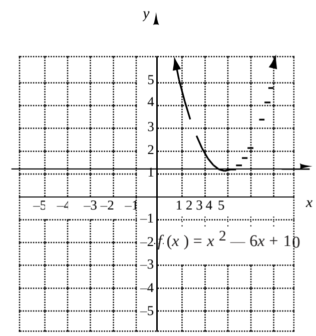
Increasing: $(-3, \infty)$

Decreasing: $(-\infty, -3)$

Maxima: none

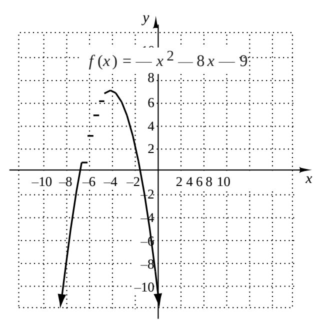
Minimum: -5 at $x = -3$

21.



The function is decreasing on $(-\infty, 3)$ and increasing on $(3, \infty)$. We estimate that the minimum is 1 at $x = 3$. There are no maxima.

22.



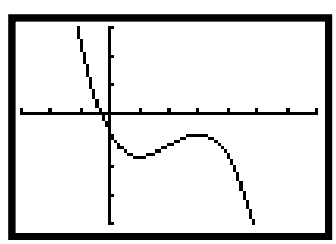
Increasing: $(-\infty, -4)$

Decreasing: $(-4, \infty)$

Maximum: 7 at $x = -4$

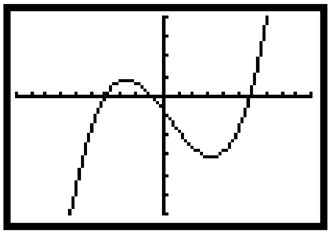
Minima: none

23.



Beginning at the left side of the window, the graph first drops as we move to the right. We see that the function is decreasing on $(-\infty, 1)$. We then find that the function is increasing on $(1, 3)$ and decreasing again on $(3, \infty)$. The MAXIMUM and MINIMUM features also show that the relative maximum is -4 at $x = 3$ and the relative minimum is -8 at $x = 1$.

24.



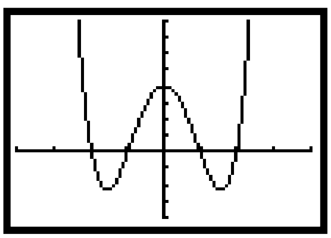
Increasing: $(-\infty, -2.573)$, $(3.239, \infty)$

Decreasing: $(-2.573, 3.239)$

Relative maximum: 4.134 at $x = -2.573$

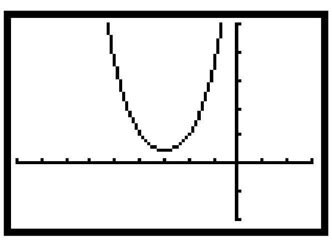
Relative minimum: -15.497 at $x = 3.239$

25.



We find that the function is increasing on $(-1.552, 0)$ and on $(1.552, \infty)$ and decreasing on $(-\infty, -1.552)$ and on $(0, 1.552)$. The relative maximum is 4.07 at $x = 0$ and the relative minima are -2.314 at $x = -1.552$ and -2.314 at $x = 1.552$.

26.



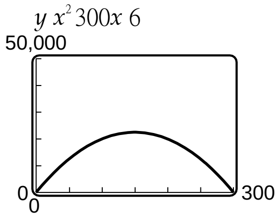
Increasing: $(-3, \infty)$

Decreasing: $(-\infty, -3)$

Relative maxima: none

Relative minimum: 9.78 at $x = -3$

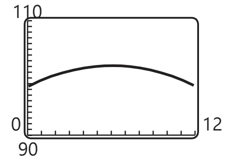
27. a) $y\ x^2\ 300x\ 6$



b) $22,506$ at $a = 150$

c) The greatest number of fruit trees will be sold when $150 thousand is spent on advertising. For that

28. a) $y\ 0.1x^2\ 1.2x\ 98.6$



b) Using the MAXIMUM feature we find that the relative maximum is 102.2 at $t = 6$. Thus, we know that the patient's temperature was the highest at $t = 6$, or 6 days after the onset of the illness and that the highest temperature was 102.2°F.

29. Graph $y = \dfrac{8x}{x^2+1}$.

Increasing: $(-1, 1)$

Decreasing: $(-\infty, -1)$, $(1, \infty)$

30. Graph $y = \dfrac{-4}{x^2+1}$.

Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$

31. Graph $y = x\sqrt{4-x^2}$, for $-2 \le x \le 2$.

Increasing: $(-1.414, 1.414)$

Decreasing: $(-2, -1.414)$, $(1.414, 2)$

32. Graph $y = -0.8x\sqrt{9-x^2}$, for $-3 \le x \le 3$.

Increasing: $(-3, -2.121)$, $(2.121, 3)$

Decreasing: $(-2.121, 2.121)$

33. If x = the length of the rectangle, in meters, then the width is $\dfrac{480-2x}{2}$, or $240 - x$. We use the formula Area = length × width:

$$A(x) = x(240 - x), \text{ or}$$
$$A(x) = 240x - x^2$$

34. Let h = the height of the scarf, in inches. Then the length of the base $= 2h - 7$.

$$A(h) = \frac{1}{2}(2h - 7)(h)$$
$$A(h) = h^2 - \frac{7}{2}h$$

35. We use the Pythagorean theorem.

$$[h(d)]^2 + 3500^2 = d^2$$
$$[h(d)]^2 = d^2 - 3500^2$$
$$h(d) = \sqrt{d^2 - 3500^2}$$

We considered only the positive square root since distance must be nonnegative.

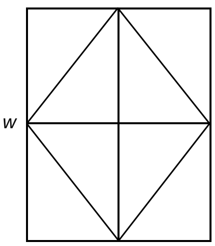
36. After t minutes, the balloon has risen $120t$ ft. We use the Pythagorean theorem.

$$[d(t)]^2 = (120t)^2 + 400^2$$
$$d(t) = \sqrt{(120t)^2 + 400^2}$$

amount, 22,506 fruit trees will be sold.

We considered only the positive square root since distance must be nonnegative.

37. Let w = the width of the rectangle. Then the length $= \frac{40 - 2w}{2}$, or $20 - w$. Divide the rectangle into quadrants as shown below.



In each quadrant there are two congruent triangles. One triangle is part of the rhombus and both are part of the rectangle. Thus, in each quadrant the area of the rhombus is one-half the area of the rectangle. Then, in total, the area of the rhombus is one-half the area of the rectangle.

$$A(w) = \frac{1}{2}(20 - w)(w)$$
$$A(w) = 10w - \frac{w^2}{2}$$

38. Let w = the width, in feet. Then the length $= \frac{46 - 2w}{2}$, or $23 - w$.

$$A(w) = (23 - w)w$$
$$A(w) = 23w - w^2$$

39. We will use similar triangles, expressing all distances in feet. $6 \text{ in.} = \frac{1}{2}$ ft, s in. $= \frac{s}{12}$ ft, and d yd $= 3d$ ft We have

$$\frac{3d}{7} = \frac{\frac{1}{2}}{\frac{s}{12}}$$
$$\frac{s}{12} \cdot 3d = 7 \cdot \frac{1}{2}$$
$$\frac{sd}{4} = \frac{7}{2}$$
$$d = \frac{4}{s} \cdot \frac{7}{2}, \text{ so}$$
$$d(s) = \frac{14}{s}.$$

40. The volume of the tank is the sum of the volume of a sphere with radius r and a right circular cylinder with radius r and height 6 ft.

$$V(r) = \frac{4}{3}\pi r^3 + 6\pi r^2$$

41. a) After 4 pieces of float line, each of length x ft, are used for the sides perpendicular to the beach, there remains $(240-4x)$ ft of float line for the side parallel to the beach. Thus we have a rectangle with length $240-4x$ and width x. Then the total area of the three swimming areas is

$$A(x) = (240 - 4x)x, \text{ or } 240x - 4x^2.$$

b) The length of the sides labeled x must be positive and their total length must be less than 240 ft, so $4x < 240$, or $x < 60$. Thus the domain is $\{x|0 < x < 60\}$, or $(0, 60)$.

c) We see from the graph that the maximum value of the area function on the interval $(0, 60)$ appears to be 3600 when $x = 30$. Thus the dimensions that yield the maximum area are 30 ft by $240 - 4 \cdot 30$, or $240 - 120$, or 120 ft.

42. a) If the length = x feet, then the width = $24 - x$ feet.

$$A(x) = x(24 - x)$$
$$A(x) = 24x - x^2$$

b) The length of the rectangle must be positive and less than 24 ft, so the domain of the function is $\{x|0 < x < 24\}$, or $(0, 24)$.

c) We see from the graph that the maximum value of the area function on the interval $(0, 24)$ appears to be 144 when $x = 12$. Then the dimensions that yield the maximum area are length = 12 ft and width = $24 - 12$, or 12 ft.

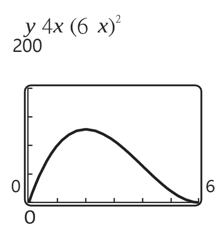
43. a) When a square with sides of length x is cut from each corner, the length of each of the remaining sides of the piece of cardboard is $12 - 2x$. Then the dimensions of the box are x by $12 - 2x$ by $12 - 2x$. We use the formula Volume = length × width × height to find the volume of the box:

$$V(x) = (12 - 2x)(12 - 2x)(x)$$
$$V(x) = (144 - 48x + 4x^2)(x)$$
$$V(x) = 144x - 48x^2 + 4x^3$$

This can also be expressed as $V(x) = 4x(x - 6)^2$, or $V(x) = 4x(6 - x)^2$.

b) The length of the sides of the square corners that are cut out must be positive and less than half the length of a side of the piece of cardboard. Thus, the domain of the function is $\{x|0 < x < 6\}$, or $(0, 6)$.

c)



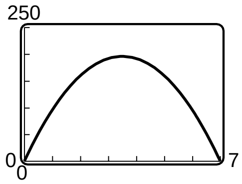
d) Using the MAXIMUM feature, we find that the maximum value of the volume occurs when $x = 2$. When $x = 2$, $12 - 2x = 12 - 2 \cdot 2 = 8$, so the dimensions that yield the maximum volume are 8 cm by 8 cm by 2 cm.

44. a) If the height of the file is x inches, then the width is $14 - 2x$ inches and the length is 8 in. We use the formula Volume = length $\times$ width $\times$ height to find the volume of the file.

$$V(x) = 8(14 - 2x)x, \text{ or}$$
$$V(x) = 112x - 16x^2$$

b) The height of the file must be positive and less than half of the measure of the long side of the piece of plastic. Thus, the domain is $\left\{x \middle| 0 < x < \frac{14}{2}\right\}$, or $\{x|0 < x < 7\}$.

c) $y = 112x - 16x^2$



d) Using the MAXIMUM feature, we find that the maximum value of the volume function occurs when $x = 3.5$, so the file should be 3.5 in. tall.

45. a) The length of a diameter of the circle (and a diagonal of the rectangle) is $2 \cdot 8$, or 16 ft. Let l = the length of the rectangle. Use the Pythagorean theorem to write l as a function of x.

$$x^2 + l^2 = 16^2$$
$$x^2 + l^2 = 256$$
$$l^2 = 256 - x^2$$
$$l = \sqrt{256 - x^2}$$

Since the length must be positive, we considered only the positive square root.

Use the formula Area = length × width to find the area of the rectangle:

$$A(x) = x\sqrt{256 - x^2}$$

b) The width of the rectangle must be positive and less than the diameter of the circle. Thus, the domain of the function is $\{x|0 < x < 16\}$, or $(0, 16)$.

46. a) Let $h(x)$ = the height of the box.

$$320 = x \cdot x \cdot h(x)$$
$$\frac{320}{x^2} = h(x)$$

Area of the bottom: x^2

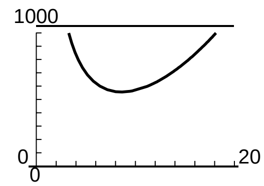
Area of each side: $x\frac{320}{x^2}$, or $\frac{320}{x}$

Area of the top: x^2

$$C(x) = 1.5x^2 + 4(2.5)\frac{320}{x} + 1 \cdot x^2$$
$$C(x) = 2.5x^2 + \frac{3200}{x}$$

b) The length of the base must be positive, so the domain of the function is $\{x|x > 0\}$, or $(0, \infty)$.

c) $y = 2.5x^2 + \frac{3200}{x}$

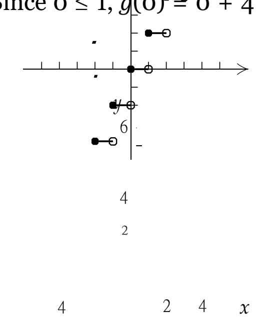


d) Using the MIMIMUM feature, we find that the minimum cost occurs when $x \approx 8.618$. Thus, the dimensions that minimize the cost are about 8.618 ft by 8.618 ft by $\frac{320}{(8.618)^2}$, or about 4.309 ft.

47. $g(x) = \begin{cases} x + 4, & \text{for } x \le 1, \\ 8 - x, & \text{for } x > 1 \end{cases}$

Since $-4 \le 1$, $g(-4) = -4 + 4 = 0$.

Since $0 \le 1$, $g(0) = 0 + 4 = 4$.



$f(x)\ 2\ x$

Exercise Set 2.1

When $x \neq 5$, the denominator of $(x^2 - 25)/(x - 5)$ is nonzero so we can simplify:

$$\frac{x^2 - 25}{x - 5} = \frac{(x + 5)(x - 5)}{x - 5} = x + 5.$$

Thus, $f(x) = x + 5$, for $x = 5$.

can also be expressed as:

$$f(x) = \begin{cases} -2x-4, & \text{for } -4 \le x < -1, \\ x-1, & \text{for } -1 \le x < 2, \\ 2, & \text{for } x \ge 2 \end{cases}$$

67. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $[-1, \infty)$.

68. Domain: (∞, ∞); range: $(-\infty, 3)$

69. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $\{y|y \le -2 \text{ or } y = -1 \text{ or } y \ge 2\}$.

70. Domain: $(-\infty, \infty)$; range: $(-\infty, -3] \cup (-1, 4]$

71. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $\{-5, -2, 4\}$. An equation for the function is:

$$f(x) = \begin{cases} -2, & \text{for } x < 2, \\ -5, & \text{for } x = 2, \\ 4, & \text{for } x > 2 \end{cases}$$

72. Domain: $(-\infty, \infty)$; range: $\{y|y = -3 \text{ or } y \ge 0\}$

$$g(x) = \begin{cases} -3, & \text{for } x < 0, \\ x, & \text{for } x \ge 0 \end{cases}$$

73. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $(-\infty, -1] \cup [2, \infty)$. Finding the slope of each segment and using the slope-intercept or point-slope formula, we find that an equation for the function is:

$$g(x) = \begin{cases} x, & \text{for } x \le -1, \\ 2, & \text{for } -1 < x \le 2, \\ x, & \text{for } x > 2 \end{cases}$$

This can also be expressed as follows:

$$g(x) = \begin{cases} x, & \text{for } x \le -1, \\ 2, & \text{for } -1 < x < 2, \\ x, & \text{for } x \ge 2 \end{cases}$$

77. $f(x) = 5x^2 - 7$

a) $f(-3) = 5(-3)^2 - 7 = 5 \cdot 9 - 7 = 45 - 7 = 38$

b) $f(3) = 5 \cdot 3^2 - 7 = 5 \cdot 9 - 7 = 45 - 7 = 38$

c) $f(a) = 5a^2 - 7$

d) $f(-a) = 5(-a)^2 - 7 = 5a^2 - 7$

78. $f(x) = 4x^3 - 5x$

a) $f(2) = 4 \cdot 2^3 - 5 \cdot 2 = 4 \cdot 8 - 5 \cdot 2 = 32 - 10 = 22$

b) $f(-2) = 4(-2)^3 - 5(-2) = 4(-8) - 5(-2) = -32 + 10 = -22$

c) $f(a) = 4a^3 - 5a$

d) $f(-a) = 4(-a)^3 - 5(-a) = 4(-a^3) - 5(-a) = -4a^3 + 5a$

79. First find the slope of the given line.

$$8x - y = 10$$
$$8x = y + 10$$
$$8x - 10 = y$$

The slope of the given line is 8. The slope of a line perpendicular to this line is the opposite of the reciprocal of 8, or $-\frac{1}{8}$.

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{1}{8}[x - (-1)]$

$y - 1 = -\frac{1}{8}(x + 1)$

$y - 1 = -\frac{1}{8}x - \frac{1}{8}$

$y = -\frac{1}{8}x + \frac{7}{8}$

80. $2x - 9y + 1 = 0$

$2x + 1 = 9y$

$\frac{2}{9}x + \frac{1}{9} = y$

Slope: $\frac{2}{9}$; y-intercept: $0, \frac{1}{9}$

81. Graph $y = x^4 + 4x^3 - 36x^2 - 160x + 400$

Increasing: $(-5, -2)$, $(4, \infty)$

Decreasing: $(-\infty, -5)$, $(-2, 4)$

Relative maximum: 560 at $x = -2$

Relative minima: 425 at $x = -5$, -304 at $x = 4$

82. Graph $y = 3.22x^5 - 5.208x^3 - 11$

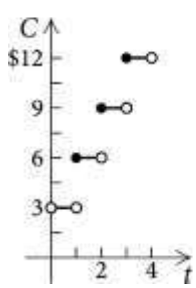
Increasing: $(-\infty, -0.985)$, $(0.985, \infty)$

Decreasing: $(-0.985, 0.985)$

Relative maximum: -9.008 at $x = -0.985$

Relative minimum: -12.992 at $x = 0.985$

83. a) The function $C(t)$ can be defined piecewise.



b) From the definition of the function in part (a), we see that it can be written as

$C(t) = 3[[t]] + 1,\ t > 0.$

84. If $[[x + 2]] = -3$, then $-3 \le x + 2 < -2$, or $-5 \le x < -4$. The possible inputs for x are $\{x \mid -5 \le x < -4\}$.

85. If $[[x]]^2 = 25$, then $[[x]] = -5$ or $[[x]] = 5$. For

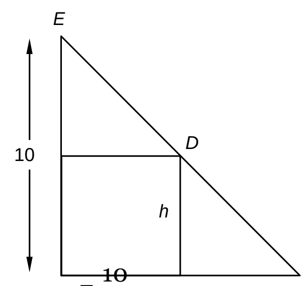
$-5 \le x < -4$, $[[x]] = -5$. For $5 \le x < 6$, $[[x]] = 5$. Thus, the possible inputs for x are $\{x \mid -5 \le x < -4 \text{ or } 5 \le x < 6\}$.

86. a) The distance from A to S is $4 - x$. Using the Pythagorean theorem, we find that the distance from S to C is $\sqrt{1 + x^2}$.

Then $C(x) = 3000(4 - x) + 5000\sqrt{1 + x^2}$, or $12{,}000 - 3000x + 5000\sqrt{1 + x^2}$.

b) Use a graphing calculator to graph $y = 12{,}000 - 3000x + 5000\sqrt{1 + x^2}$ in a window such as $[0, 5, 10{,}000, 20{,}000]$, Xscl = 1, Yscl = 1000. Using the MINIMUM feature, we find that cost is minimized when $x = 0.75$, so the line should come to shore 0.75 mi from B.

87. a) We add labels to the drawing in the text.



$\frac{h}{6 - r} = \frac{10}{6}$ 10 5

$h = \frac{10}{6}(6 - r) = \frac{5}{3}(6 - r)$

$h = \frac{30 - 5r}{3}$

Thus, $h(r) = \frac{30 - 5r}{3}$.

b) $V = \pi r^2 h$

$V(r) = \pi r^2 \left(\frac{30 - 5r}{3}\right)$ Substituting for h

c) We first express r in terms of h.

$$h = \frac{30-5r}{3}$$
$$3h = 30-5r$$
$$5r = 30-3h$$
$$r = \frac{30-3h}{5}$$
$$V = \pi r^2 h$$
$$V(h) = \pi\left(\frac{30-3h}{5}\right)^2 h$$

Substituting for r

We can also write $V(h) = \pi h\left(\frac{30-3h}{5}\right)^2$.

Exercise Set 2.2

1. $(f+g)(5) = f(5)+g(5)$
$= (5^2-3)+(2\cdot 5+1)$
$= 25-3+10+1$
$= 33$

2. $(fg)(0) = f(0)\cdot g(0)$
$= (0^2-3)(2\cdot 0+1)$
$= -3(1) = -3$

3. $(f-g)(-1) = f(-1)-g(-1)$
$= ((-1)^2-3)-(2(-1)+1)$
$= -2-(-1) = -2+1$
$= -1$

4. $(fg)(2) = f(2)\cdot g(2)$
$= (2^2-3)(2\cdot 2+1)$
$= 1\cdot 5 = 5$

5. $(f/g)\left(-\frac{1}{2}\right) = \frac{f\left(-\frac{1}{2}\right)}{g\left(-\frac{1}{2}\right)}$
$= \frac{\left(-\frac{1}{2}\right)^2-3}{2\left(-\frac{1}{2}\right)+1}$
$= \frac{\frac{1}{4}-3}{-1+1}$
$-\frac{11}{4}$

7. $(fg)\left(-\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)\cdot g\left(-\frac{1}{2}\right)$
$= \left[\left(-\frac{1}{2}\right)^2-3\right]\left[2\left(-\frac{1}{2}\right)+1\right]$
$= -\frac{11}{4}\cdot 0 = 0$

8. $(f/g)(-\sqrt{3}) = \frac{f(-\sqrt{3})}{g(-\sqrt{3})}$
$= \frac{(-\sqrt{3})^2-3}{2(-\sqrt{3})+1}$
$= \frac{0}{-2\sqrt{3}+1} = 0$

9. $(g-f)(-1) = g(-1)-f(-1)$
$= [2(-1)+1]-[(-1)^2-3]$
$= (-2+1)-(1-3)$
$= -1-(-2)$
$= -1+2$
$= 1$

10. $(g/f)\left(-\frac{1}{2}\right) = \frac{g\left(-\frac{1}{2}\right)}{f\left(-\frac{1}{2}\right)}$
$= \frac{2\left(-\frac{1}{2}\right)+1}{\left(-\frac{1}{2}\right)^2-3}$
$= \frac{0}{-\frac{11}{4}}$
$= 0$

11. $(h-g)(-4) = h(-4)-g(-4)$
$= (-4+4)-\sqrt{-4-1}$
$= 0-\sqrt{-5}$

Since $\sqrt{-5}$ is not a real number, $(h-g)(-4)$ does not exist.

12. $(gh)(10) = g(10)\cdot h(10)$
$= \sqrt{10-1}(10+4)$
$= \sqrt{9}(14)$
$= 3\cdot 14 = 42$

13. $(g/h)(1) = \frac{g(1)}{h(1)}$
$= \frac{\sqrt{1-1}}{1+4}$
$\sqrt{0}$

$= \frac{-4}{0}$

Since division by 0 is not defined, $(f/g)\left(-\frac{1}{2}\right)$ does not exist.

6. $(f-g)(0) = f(0) - g(0)$

$= (0^2 - 3) - (2 \cdot 0 + 1)$

$= -3 - 1 = -4$

$= \frac{0}{5}$

$= \frac{5}{5} = 0$

14. $(h/g)(1) = \frac{h(1)}{g(1)}$

$= \frac{\sqrt{1+4}}{1-1}$

$= \frac{5}{0}$

Since division by 0 is not defined, $(h/g)(1)$ does not exist.

15. $(g+h)(1) = g(1) + h(1)$
$= \sqrt{1-1} + (1+4)$
$= \sqrt{0} + 5$
$= 0 + 5 = 5$

16. $(hg)(3) = h(3) \cdot g(3)$
$= (3+4)\sqrt{3-1}$
$= 7\sqrt{2}$

17. $f(x) = 2x + 3$, $g(x) = 3 - 5x$

a) The domain of f and of g is the set of all real numbers, or $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, ff, and fg is also $(-\infty, \infty)$. For f/g we must exclude $\frac{3}{5}$ since $g\left(\frac{3}{5}\right) = 0$. Then the domain of f/g is $\left(-\infty, \frac{3}{5}\right) \cup \left(\frac{3}{5}, \infty\right)$. For g/f we must exclude $-\frac{3}{2}$ since $f\left(-\frac{3}{2}\right) = 0$. The domain of g/f is $\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, \infty\right)$.

b) $(f+g)(x) = f(x) + g(x) = (2x+3) + (3-5x) = -3x + 6$

$(f-g)(x) = f(x) - g(x) = (2x+3) - (3-5x) = 2x + 3 - 3 + 5x = 7x$

$(fg)(x) = f(x) \cdot g(x) = (2x+3)(3-5x) = 6x - 10x^2 + 9 - 15x = -10x^2 - 9x + 9$

$(ff)(x) = f(x) \cdot f(x) = (2x+3)(2x+3) = 4x^2 + 12x + 9$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x+3}{3-5x}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{3-5x}{2x+3}$

18. $f(x) = -x + 1$, $g(x) = 4x - 2$

a) The domain of f, g, $f+g$, $f-g$, fg, and ff is $(-\infty, \infty)$. Since $g\left(\frac{1}{2}\right) = 0$, the domain of f/g is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$. Since $f(1) = 0$, the domain of g/f is $(-\infty, 1) \cup (1, \infty)$.

b) $(f+g)(x) = (-x+1) + (4x-2) = 3x - 1$

$(f-g)(x) = (-x+1) - (4x-2) = -x + 1 - 4x + 2 = -5x + 3$

19. $f(x) = x - 3$, $g(x) = \sqrt{x+4}$

a) Any number can be an input in f, so the domain of f is the set of all real numbers, or $(-\infty, \infty)$.

The domain of g consists of all values of x for which $x+4$ is nonnegative, so we have $x+4 \geq 0$, or $x \geq -4$. Thus, the domain of g is $[-4, \infty)$.

The domain of $f+g$, $f-g$, and fg is the set of all numbers in the domains of both f and g. This is $[-4, \infty)$.

The domain of ff is the domain of f, or $(-\infty, \infty)$.

The domain of f/g is the set of all numbers in the domains of f and g, excluding those for which $g(x) = 0$. Since $g(-4) = 0$, the domain of f/g is $(-4, \infty)$.

The domain of g/f is the set of all numbers in the domains of g and f, excluding those for which $f(x) = 0$. Since $f(3) = 0$, the domain of g/f is $[-4, 3) \cup (3, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = x - 3 + \sqrt{x+4}$

$(f-g)(x) = f(x) - g(x) = x - 3 - \sqrt{x+4}$

$(fg)(x) = f(x) \cdot g(x) = (x-3)\sqrt{x+4}$

$(ff)(x) = [f(x)]^2 = (x-3)^2 = x^2 - 6x + 9$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x-3}{\sqrt{x+4}}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{\sqrt{x+4}}{x-3}$

20. $f(x) = x + 2$, $g(x) = \sqrt{x-1}$

a) The domain of f is $(-\infty, \infty)$. The domain of g consists of all the values of x for which $x-1$ is nonnegative, or $[1, \infty)$. Then the domain of $f+g$, $f-g$, and fg is $[1, \infty)$. The domain of ff is $(-\infty, \infty)$. Since $g(1) = 0$, the domain of f/g is $(1, \infty)$. Since $f(-2) = 0$ and -2 is not in the domain of g, the domain of g/f is $[1, \infty)$.

b) $(f+g)(x) = x + 2 + \sqrt{x-1}$

$(f-g)(x) = x + 2 - \sqrt{x-1}$

$(fg)(x) = (x+2)\sqrt{x-1}$

$(ff)(x) = (x+2)(x+2) = x^2 + 4x + 4$

$(f/g)(x) = \dfrac{x+2}{\sqrt{x-1}}$

$(g/f)(x) = \dfrac{\sqrt{x-1}}{x+2}$

$(fg)(x) = (-x+1)(4x-2) = -4x^2 + 6x - 2$

$(ff)(x) = (-x+1)(-x+1) = x^2 - 2x + 1$

$(f/g)(x) = \dfrac{-x+1}{4x-2}$

$(g/f)(x) = \dfrac{4x-2}{-x+1}$

21. $f(x) = 2x - 1$, $g(x) = -2x^2$

a) The domain of f and of g is $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty, \infty)$. For f/g, we must exclude 0 since $g(0) = 0$. The domain of f/g is $(-\infty, 0) \cup (0, \infty)$. For g/f, we must exclude $\frac{1}{2}$ since $f\left(\frac{1}{2}\right) = 0$. The domain of g/f is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

b) $(f+g)(x) = f(x)+g(x) = (2x-1)+(-2x^2) = -2x^2+2x-1$

$(f-g)(x) = f(x)-g(x) = (2x-1)-(-2x^2) = 2x^2+2x-1$

$(fg)(x) = f(x)\cdot g(x) = (2x-1)(-2x^2) = -4x^3+2x^2$

$(ff)(x) = f(x)\cdot f(x) = (2x-1)(2x-1) = 4x^2-4x+1$

$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{2x-1}{-2x^2}$

$(g/f)(x) = \frac{g(x)}{f(x)} = \frac{-2x}{2x-1}$

22. $f(x) = x^2-1$, $g(x) = 2x+5$

a) The domain of f and of g is the set of all real numbers, or $(-\infty,\infty)$. Then the domain of $f+g$, $f-g$, fg and ff is $(-\infty,\infty)$. Since $g\left(-\frac{5}{2}\right)=0$, the domain of f/g is $\left(-\infty,-\frac{5}{2}\right)\cup\left(-\frac{5}{2},\infty\right)$. Since $f(1)=0$ and $f(-1)=0$, the domain of g/f is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

b) $(f+g)(x) = x^2-1+2x+5 = x^2+2x+4$

$(f-g)(x) = x^2-1-(2x+5) = x^2-2x-6$

$(fg)(x) = (x^2-1)(2x+5) = 2x^3+5x^2-2x-5$

$(ff)(x) = (x^2-1)^2 = x^4-2x^2+1$

$(f/g)(x) = \frac{x^2-1}{2x+5}$

$(g/f)(x) = \frac{2x+5}{x^2-1}$

23. $f(x) = \sqrt{x-3}$, $g(x) = \sqrt{x+3}$

a) Since $f(x)$ is nonnegative for values of x in $[3,\infty)$, this is the domain of f. Since $g(x)$ is nonnegative for values of x in $[-3,\infty)$, this is the domain of g. The domain of $f+g$, $f-g$, and fg is the intersection of the domains of f and g, or $[3,\infty)$. The domain of ff is the same as the domain of f, or $[3,\infty)$. For f/g, we must exclude -3 since $g(-3)=0$. This is not in $[3,\infty)$, so the domain of f/g is $[3,\infty)$. For g/f, we must exclude 3 since $f(3)=0$. The domain of g/f is $(3,\infty)$.

b) $(f+g)(x) = f(x)+g(x) = \sqrt{x-3}+\sqrt{x+3}$

$(f-g)(x) = f(x)-g(x) = \sqrt{x-3}-\sqrt{x+3}$

$(fg)(x) = f(x)\cdot g(x) = \sqrt{x-3}\cdot\sqrt{x+3} = \sqrt{x^2-9}$

24. $f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$

a) The domain of f is $[0,\infty)$. The domain of g is $(-\infty,2]$. Then the domain of $f+g$, $f-g$, and fg is $[0,2]$. The domain of ff is the same as the domain of f, $[0,\infty)$. Since $g(2)=0$, the domain of f/g is $[0,2)$. Since $f(0)=0$, the domain of g/f is $(0,2]$.

b) $(f+g)(x) = \sqrt{x}+\sqrt{2-x}$

$(f-g)(x) = \sqrt{x}-\sqrt{2-x}$

$(fg)(x) = \sqrt{x}\cdot\sqrt{2-x} = \sqrt{2x-x^2}$

$(ff)(x) = \sqrt{x}\cdot\sqrt{x} = \sqrt{x^2} = |x|$

$(f/g)(x) = \frac{\sqrt{x}}{\sqrt{2-x}}$

$(g/f)(x) = \frac{\sqrt{2-x}}{\sqrt{x}}$

25. $f(x) = x+1$, $g(x) = |x|$

a) The domain of f and of g is $(-\infty,\infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty,\infty)$. For f/g, we must exclude 0 since $g(0)=0$. The domain of f/g is $(-\infty,0)\cup(0,\infty)$. For g/f, we must exclude -1 since $f(-1)=0$. The domain of g/f is $(-\infty,-1)\cup(-1,\infty)$.

b) $(f+g)(x) = f(x)+g(x) = x+1+|x|$

$(f-g)(x) = f(x)-g(x) = x+1-|x|$

$(fg)(x) = f(x)\cdot g(x) = (x+1)|x|$

$(ff)(x) = f(x)\cdot f(x) = (x+1)(x+1) = x^2+2x+1$

$(f/g)(x) = \frac{x+1}{|x|}$

$(g/f)(x) = \frac{|x|}{x+1}$

26. $f(x) = 4|x|$, $g(x) = 1-x$

a) The domain of f and of g is $(-\infty,\infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty,\infty)$. Since $g(1)=0$, the domain of f/g is $(-\infty,1)\cup(1,\infty)$. Since $f(0)=0$, the domain of g/f is $(-\infty,0)\cup(0,\infty)$.

b) $(f+g)(x) = 4|x|+1-x$

$(f-g)(x) = 4|x|-(1-x) = 4|x|-1+x$

$(fg)(x) = 4|x|(1-x) = 4|x|-4x|x|$

$(ff)(x) = 4|x|\cdot 4|x| = 16x^2$

$(f/g)(x) = \frac{4|x|}{1-x}$

$(ff)(x) = f(x) \cdot f(x) = \sqrt{x-3} \cdot \sqrt{x-3} = |x-3|$

$(f/g)(x) = \dfrac{\sqrt{x-3}}{\sqrt{x+3}}$

$(g/f)(x) = \dfrac{\sqrt{x+3}}{\sqrt{x-3}}$

$(g/f)(x) = \dfrac{1-x}{4|x|}$

27. $f(x) = x^3$, $g(x) = 2x^2 + 5x - 3$

a) Since any number can be an input for either f or g, the domain of f, g, $f + g$, $f - g$, fg, and ff is the set of all real numbers, or $(-\infty, \infty)$.

Since $g(-3) = 0$ and $g\left(\frac{1}{2}\right) = 0$, the domain of f/g

Since $f(0) = 0$, the domain of g/f is $(-\infty, 0) \cup (0, \infty)$.

b) $(f+g)(x) = f(x)+g(x) = x^3 + 2x^2 + 5x - 3$

$(f-g)(x) = f(x)-g(x) = x^3-(2x^2+5x-3) = x^3 - 2x^2 - 5x + 3$

$(fg)(x) = f(x)\cdot g(x) = x^3(2x^2+5x-3) = 2x^5 + 5x^4 - 3x^3$

$(ff)(x) = f(x)\cdot f(x) = x^3 \cdot x^3 = x^6$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^3}{2x^2+5x-3}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{2x^2+5x-3}{x^3}$

28. $f(x) = x^2 - 4$, $g(x) = x^3$

a) The domain of f and of g is $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty, \infty)$. Since $g(0) = 0$, the domain of f/g is $(-\infty, 0) \cup (0, \infty)$.

Since $f(-2) = 0$ and $f(2) = 0$, the domain of g/f is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

b) $(f+g)(x) = x^2 - 4 + x^3$, or $x^3 + x^2 - 4$

$(f-g)(x) = x^2 - 4 - x^3$, or $-x^3 + x^2 - 4$

$(fg)(x) = (x^2-4)(x^3) = x^5 - 4x^3$

$(ff)(x) = (x^2-4)(x^2-4) = x^4 - 8x^2 + 16$

$(f/g)(x) = \dfrac{x^2-4}{x^3}$

$(g/f)(x) = \dfrac{\quad}{x^2-4}$

29. $f(x) = \dfrac{4}{x+1}$, $g(x) = \dfrac{1}{6-x}$

a) Since $x+1 = 0$ when $x = -1$, we must exclude -1 from the domain of f. It is $(-\infty, -1) \cup (-1, \infty)$. Since $6-x = 0$ when $x = 6$, we must exclude 6 from the domain of g. It is $(-\infty, 6) \cup (6, \infty)$. The domain of $f+g$, $f-g$, and fg is the intersection of the domains of f and g, or $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$. The domain of ff is the same as the domain of f, or $(-\infty, -1) \cup (-1, \infty)$. Since there are no values of x for which $g(x) = 0$ or $f(x) = 0$, the domain of f/g and g/f is $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = \dfrac{4}{x+1} + \dfrac{1}{6-x}$

$(f-g)(x) = f(x) - g(x) = \dfrac{4}{x+1} - \dfrac{1}{6-x}$

$(f/g)(x) = \dfrac{\frac{4}{x+1}}{\frac{1}{6-x}} = \dfrac{4}{x+1}\cdot\dfrac{6-x}{1} = \dfrac{4(6-x)}{x+1}$

$(g/f)(x) = \dfrac{\frac{1}{6-x}}{\frac{4}{x+1}} = \dfrac{1}{6-x}\cdot\dfrac{x+1}{4} = \dfrac{x+1}{4(6-x)}$

30. $f(x) = 2x^2$, $g(x) = \dfrac{2}{x-5}$

a) The domain of f is $(-\infty, \infty)$. Since $x-5 = 0$ when $x = 5$, the domain of g is $(-\infty, 5) \cup (5, \infty)$. Then the domain of $f+g$, $f-g$, and fg is $(-\infty, 5) \cup (5, \infty)$. The domain of ff is $(-\infty, \infty)$. Since there are no values of x for which $g(x) = 0$, the domain of f/g is $(-\infty, 5) \cup (5, \infty)$. Since $f(0) = 0$, the domain of g/f is $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$.

b) $(f+g)(x) = 2x^2 + \dfrac{2}{x-5}$

$(f-g)(x) = 2x^2 - \dfrac{2}{x-5}$

$(fg)(x) = 2x^2 \cdot \dfrac{2}{x-5} = \dfrac{4x^2}{x-5}$

$(ff)(x) = 2x^2 \cdot 2x^2 = 4x^4$

$(f/g)(x) = \dfrac{2x^2}{\frac{2}{x-5}} = 2x^2 \cdot \dfrac{x-5}{2} = x^2(x-5)$, or $x^3 - 5x^2$

$(g/f)(x) = \dfrac{\frac{2}{x-5}}{2x^2} = \dfrac{2}{x-5}\cdot\dfrac{1}{2x^2} = \dfrac{1}{x^2(x-5)} = \dfrac{1}{x^3-5x^2}$

31. $f(x) = \dfrac{1}{x}$, $g(x) = x - 3$

a) Since $f(0)$ is not defined, the domain of f is $(-\infty, 0) \cup (0, \infty)$. The domain of g is $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty, 0) \cup (0, \infty)$. Since $g(3) = 0$, the domain of f/g is $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$. There are no values of x for which $f(x) = 0$, so the domain of g/f is $(-\infty, 0) \cup (0, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = \dfrac{1}{x} + x - 3$

$(f-g)(x) = f(x) - g(x) = \dfrac{1}{x} - (x-3) = \dfrac{1}{x} - x + 3$

$(fg)(x) = f(x)\cdot g(x) = \dfrac{1}{x}\cdot(x-3) = \dfrac{x-3}{x}$, or $1 - \dfrac{3}{x}$

$$(fg)(x) = f(x) \cdot g(x) = \frac{4}{x+1} \cdot \frac{1}{6-x} = \frac{4}{(x+1)(6-x)}$$

$$(ff)(x) = f(x) \cdot f(x) = \frac{4}{x+1} \cdot \frac{4}{x+1} = \frac{16}{(x+1)^2}, \text{ or } \frac{16}{x^2+2x+1}$$

$$(ff)(x) = f(x) \cdot f(x) = \frac{1}{x} \cdot \frac{1}{x} = \frac{1}{x^2}$$

$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{\frac{1}{x}}{x-3} = \frac{1}{x} \cdot \frac{1}{x-3} = \frac{1}{x(x-3)}$$

$$(g/f)(x) = \frac{g(x)}{f(x)} = \frac{x-3}{\frac{1}{x}} = (x-3) \cdot \frac{x}{1} = x(x-3), \text{ or } x^2 - 3x$$

32. $f(x) = \sqrt{x+6}$, $g(x) = \dfrac{1}{x}$

a) The domain of $f(x)$ is $[-6, \infty)$. The domain of $g(x)$ is $(-\infty, 0) \cup (0, \infty)$. Then the domain of $f+g$, $f-g$, and fg is $[-6, 0) \cup (0, \infty)$. The domain of ff is $[-6, \infty)$. Since there are no values of x for which $g(x) = 0$, the domain of f/g is $[-6, 0) \cup (0, \infty)$. Since $f(-6) = 0$, the domain of g/f is $(-6, 0) \cup (0, \infty)$.

b) $(f+g)(x) = \sqrt{x+6} + \dfrac{1}{x}$

$(f-g)(x) = \sqrt{x+6} - \dfrac{1}{x}$

$(fg)(x) = \sqrt{x+6} \cdot \dfrac{1}{x} = \dfrac{\sqrt{x+6}}{x}$

$(ff)(x) = \sqrt{x+6} \cdot \sqrt{x+6} = |x+6|$

$(f/g)(x) = \dfrac{\sqrt{x+6}}{\frac{1}{x}} = \sqrt{x+6} \cdot \dfrac{x}{1} = x\sqrt{x+6}$

$(g/f)(x) = \dfrac{\frac{1}{x}}{\sqrt{x+6}} = \dfrac{1}{x} \cdot \dfrac{1}{\sqrt{x+6}} = \dfrac{1}{x\sqrt{x+6}}$

33. $f(x) = \dfrac{3}{x-2}$, $g(x) = \sqrt{x-1}$

a) Since $f(2)$ is not defined, the domain of f is $(-\infty, 2) \cup (2, \infty)$. Since $g(x)$ is nonnegative for values of x in $[1, \infty)$, this is the domain of g. The domain of $f+g$, $f-g$, and fg is the intersection of the domains of f and g, or $[1, 2) \cup (2, \infty)$. The domain of ff is the same as the domain of f, or $(-\infty, 2) \cup (2, \infty)$. For f/g, we must exclude 1 since $g(1) = 0$, so the domain of f/g is $(1, 2) \cup (2, \infty)$. There are no values of x for which $f(x) = 0$, so the domain of g/f is $[1, 2) \cup (2, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = \dfrac{3}{x-2} + \sqrt{x-1}$

$(f-g)(x) = f(x) - g(x) = \dfrac{3}{x-2} - \sqrt{x-1}$

$(fg)(x) = f(x) \cdot g(x) = \dfrac{3}{x-2}\left(\sqrt{x-1}\right)$, or $\dfrac{3\sqrt{x-1}}{x-2}$

$(ff)(x) = f(x) \cdot f(x) = \dfrac{3}{x-2} \cdot \dfrac{3}{x-2} = \dfrac{9}{(x-2)^2}$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{3}{x-2}}{\sqrt{x-1}} = \dfrac{3}{(x-2)\sqrt{x-1}}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{\sqrt{x-1}}{\frac{3}{x-2}} = \dfrac{(x-2)\sqrt{x-1}}{3}$

34. $f(x) = \dfrac{2}{4-x}$, $g(x) = \dfrac{5}{x-1}$

a) The domain of f is $(-\infty, 4) \cup (4, \infty)$. The domain of g is $(-\infty, 1) \cup (1, \infty)$. The domain of $f+g$, $f-g$, and fg is $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$. The domain of ff is $(-\infty, 4) \cup (4, \infty)$. The domain of f/g and of g/f is $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$.

b) $(f \pm g)(x) = \dfrac{2}{4-x} \pm \dfrac{5}{x-1}$

$(fg)(x) = \dfrac{2}{4-x} \cdot \dfrac{5}{x-1} = \dfrac{10}{(4-x)(x-1)}$

$(ff)(x) = \dfrac{2}{4-x} \cdot \dfrac{2}{4-x} = \dfrac{4}{(4-x)^2}$

$(f/g)(x) = \dfrac{\frac{2}{4-x}}{\frac{5}{x-1}} = \dfrac{2(x-1)}{5(4-x)}$

$(g/f)(x) = \dfrac{\frac{5}{x-1}}{\frac{2}{4-x}} = \dfrac{5(4-x)}{2(x-1)}$

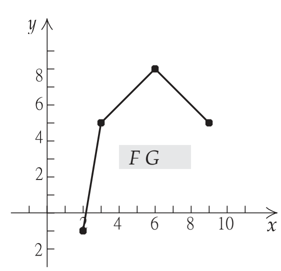
35. From the graph we see that the domain of F is $[2, 11]$ and the domain of G is $[1, 9]$. The domain of $F + G$ is the set of numbers in the domains of both F and G. This is $[2, 9]$.

36. The domain of $F - G$ and FG is the set of numbers in the domains of both F and G. (See Exercise 33.) This is $[2, 9]$.

The domain of F/G is the set of numbers in the domains of both F and G, excluding those for which $G = 0$. Since $G > 0$ for all values of x in its domain, the domain of F/G is $[2, 9]$.

37. The domain of G/F is the set of numbers in the domains of both F and G (See Exercise 33.), excluding those for which $F = 0$. Since $F(3) = 0$, the domain of G/F is $[2, 3) \cup (3, 9]$.

38.



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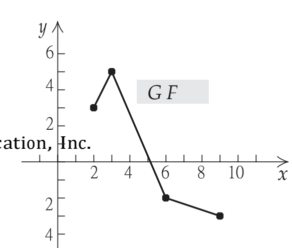

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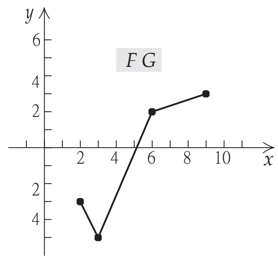


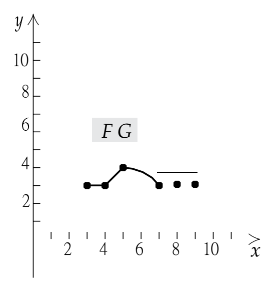
41. From the graph, we see that the domain of F is $[0, 9]$ and the domain of G is $[3, 10]$. The domain of $F + G$ is the set of numbers in the domains of both F and G. This is $[3,9]$.

42. The domain of $F - G$ and FG is the set of numbers in the domains of both F and G. (See Exercise 39.) This is $[3,9]$.

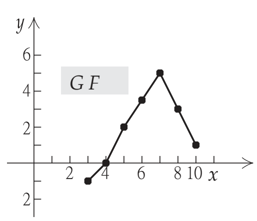
The domain of F/G is the set of numbers in the domains of both F and G, excluding those for which $G = 0$. Since $G > 0$ for all values of x in its domain, the domain of F/G is $[3, 9]$.

43. The domain of G/F is the set of numbers in the domains of both F and G (See Exercise 39.), excluding those for which $F = 0$. Since $F(6) = 0$ and $F(8) = 0$, the domain of G/F is $[3,6)\cup(6,8)\cup(8,9]$.

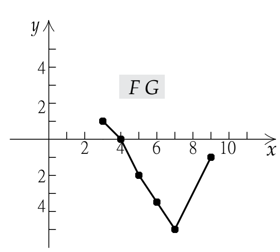
44. $(F + G)(x) = F(x) + G(x)$



45.



46.



47. a) $P(x) = R(x) - C(x) = 60x - 0.4x^2 - (3x + 13) =$

$60x - 0.4x^2 - 3x - 13 = -0.4x^2 + 57x - 13$

b) $R(100) = 60\cdot 100 - 0.4(100)^2 = 6000 - 0.4(10{,}000) =$

$6000 - 4000 = 2000$

$C(100) = 3\cdot 100 + 13 = 300 + 13 = 313$

$P(100) = R(100) - C(100) = 2000 - 313 = 1687$

48. a) $P(x) = 200x - x^2 - (5000 + 8x) =$

$200x - x^2 - 5000 - 8x = -x^2 + 192x - 5000$

b) $R(175) = 200(175) - 175^2 = 4375$

$C(175) = 5000 + 8\cdot 175 = 6400$

$P(175) = R(175) - C(175) = 4375 - 6400 = -2025$

(We could also use the function found in part (a) to find $P(175)$.)

49. $f(x) = 3x - 5$

$f(x+h) = 3(x+h) - 5 = 3x + 3h - 5$

$$\frac{f(x+h)-f(x)}{h} = \frac{3x+3h-5-(3x-5)}{h}$$

$$= \frac{3x+3h-5-3x+5}{h}$$

$$= \frac{3h}{h} = 3$$

50. $f(x) = 4x - 1$

$$\frac{f(x+h)-f(x)}{h} = \frac{4(x+h)-1-(4x-1)}{h} =$$

$$\frac{4x+4h-1-4x+1}{h} = \frac{4h}{h} = 4$$

51. $f(x) = 6x + 2$

$f(x+h) = 6(x+h) + 2 = 6x + 6h + 2$

$$\frac{f(x+h)-f(x)}{h} = \frac{6x+6h+2-(6x+2)}{h}$$

$$= \frac{6x+6h+2-6x-2}{h}$$

$$= \frac{6h}{h} = 6$$

52. $f(x) = 5x + 3$

$$\frac{f(x+h)-f(x)}{h} = \frac{5(x+h)+3-(5x+3)}{h} =$$

$$\frac{5x+5h+3-5x-3}{h} = \frac{5h}{h} = 5$$

53. $f(x) = \frac{1}{3}x + 1$

$f(x+h) = \frac{1}{3}(x+h) + 1 = \frac{1}{3}x + \frac{1}{3}h + 1$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{3}x + \frac{1}{3}h + 1 - \left(\frac{1}{3}x + 1\right)}{h}$$

$$= \frac{\frac{1}{3}x + \frac{1}{3}h + 1 - \frac{1}{3}x - 1}{h}$$

$$= \frac{\frac{1}{3}h}{h} = \frac{1}{3}$$

54. $f(x) = -\frac{1}{2}x + 7$

$$\frac{f(x+h)-f(x)}{h} = \frac{-\frac{1}{2}(x+h)+7-\left(-\frac{1}{2}x+7\right)}{h} = \frac{-\frac{1}{2}x-\frac{1}{2}h+7+\frac{1}{2}x-7}{h} = \frac{-\frac{1}{2}h}{h} = -\frac{1}{2}$$

55. $f(x) = \frac{1}{3x}$

$$f(x+h) = \frac{1}{3(x+h)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{3(x+h)} - \frac{1}{3x}}{h}$$

$$= \frac{\frac{1}{3(x+h)}\cdot\frac{x}{x} - \frac{1}{3x}\cdot\frac{x+h}{x+h}}{h}$$

$$= \frac{\frac{x}{3x(x+h)} - \frac{x+h}{3x(x+h)}}{h}$$

$$= \frac{\frac{x-(x+h)}{3x(x+h)}}{h} = \frac{\frac{x-x-h}{3x(x+h)}}{h}$$

$$= \frac{\frac{-h}{3x(x+h)}}{h} = \frac{-h}{3x(x+h)}\cdot\frac{1}{h}$$

$$= \frac{-h}{3x(x+h)\cdot h} = \frac{-1\cdot\cancel{h}}{3x(x+h)\cdot\cancel{h}}$$

$$= \frac{-1}{3x(x+h)}, \text{ or } -\frac{1}{3x(x+h)}$$

56. $f(x) = \frac{1}{2x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{2(x+h)}-\frac{1}{2x}}{h} = \frac{\frac{1}{2(x+h)}\cdot\frac{x}{x}-\frac{1}{2x}\cdot\frac{x+h}{x+h}}{h} =$$

$$\frac{\frac{x}{2x(x+h)}-\frac{x+h}{2x(x+h)}}{h} = \frac{\frac{x-x-h}{2x(x+h)}}{h} = \frac{\frac{-h}{2x(x+h)}}{h} =$$

$$\frac{-h}{2x(x+h)}\cdot\frac{1}{h} = \frac{-1}{2x(x+h)}, \text{ or } -\frac{1}{2x(x+h)}$$

57. $f(x) = -\frac{1}{4x}$

$$f(x+h) = -\frac{1}{4(x+h)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-\frac{1}{4(x+h)} - \left(-\frac{1}{4x}\right)}{h}$$

$$= \frac{-\frac{1}{4(x+h)}\cdot\frac{x}{x} - \left(-\frac{1}{4x}\right)\cdot\frac{x+h}{x+h}}{h}$$

$$= \frac{-\frac{x}{4x(x+h)}+\frac{x+h}{4x(x+h)}}{h}$$

$$= \frac{\frac{-x+x+h}{4x(x+h)}}{h} = \frac{\frac{h}{4x(x+h)}}{h}$$

$$= \frac{h}{4x(x+h)}\cdot\frac{1}{h} = \frac{\cancel{h}\cdot 1}{4x(x+h)\cdot\cancel{h}} = \frac{1}{4x(x+h)}$$

58. $f(x) = -\frac{1}{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{-\frac{1}{x+h}-\left(-\frac{1}{x}\right)}{h} =$$

$$\frac{-\frac{1}{x+h}\cdot\frac{x}{x}-\left(-\frac{1}{x}\right)\cdot\frac{x+h}{x+h}}{h} = \frac{-\frac{x}{x(x+h)}+\frac{x+h}{x(x+h)}}{h} =$$

$$\frac{\frac{-x+x+h}{x(x+h)}}{h} = \frac{\frac{h}{x(x+h)}}{h} = \frac{h}{x(x+h)}\cdot\frac{1}{h} = \frac{1}{x(x+h)}$$

59. $f(x) = x^2 + 1$

$$f(x+h) = (x+h)^2+1 = x^2+2xh+h^2+1$$

$$\frac{f(x+h)-f(x)}{h} = \frac{x^2+2xh+h^2+1-(x^2+1)}{h}$$

$$= \frac{x^2+2xh+h^2+1-x^2-1}{h}$$

$$= \frac{2xh+h^2}{h}$$

$$= \frac{h(2x+h)}{h}$$

$$= \frac{h}{h}\cdot\frac{2x+h}{1}$$

$$= 2x+h$$

60. $f(x) = x^2 - 3$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-3-(x^2-3)}{h} =$$

$$\frac{x^2 + 2xh + h^2 - 3 - x^2 + 3}{h} = \frac{2xh + h^2}{h} = \frac{h(2x + h)}{h} = 2x + h$$

61. $f(x) = 4 - x^2$

$f(x+h) = 4 - (x+h)^2 = 4 - (x^2 + 2xh + h^2) = 4 - x^2 - 2xh - h^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{4 - x^2 - 2xh - h^2 - (4 - x^2)}{h}$$

$$= \frac{4 - x^2 - 2xh - h^2 - 4 + x^2}{h}$$

$$= \frac{-2xh - h^2}{h} = \frac{\cancel{h}(-2x - h)}{\cancel{h}}$$

$$= -2x - h$$

62. $f(x) = 2 - x^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{2 - (x+h)^2 - (2 - x^2)}{h} =$$

$$\frac{2 - x^2 - 2xh - h^2 - 2 + x^2}{h} = \frac{-2xh - h^2}{h} =$$

$$\frac{h(-2x - h)}{h} = -2x - h$$

63. $f(x) = 3x^2 - 2x + 1$

$f(x+h) = 3(x+h)^2 - 2(x+h) + 1 =$

$3(x^2 + 2xh + h^2) - 2(x+h) + 1 =$

$3x^2 + 6xh + 3h^2 - 2x - 2h + 1$

$f(x) = 3x^2 - 2x + 1$

$$\frac{f(x+h) - f(x)}{h} =$$

$$\frac{(3x^2 + 6xh + 3h^2 - 2x - 2h + 1) - (3x^2 - 2x + 1)}{h} =$$

$$\frac{3x^2 + 6xh + 3h^2 - 2x - 2h + 1 - 3x^2 + 2x - 1}{h} =$$

$$\frac{6xh + 3h^2 - 2h}{h} = \frac{h(6x + 3h - 2)}{h \cdot 1} =$$

$$\frac{h}{h} \cdot \frac{6x + 3h - 2}{1} = 6x + 3h - 2$$

64. $f(x) = 5x^2 + 4x$

$$\frac{f(x+h) - f(x)}{h} = \frac{(5x^2 + 10xh + 5h^2 + 4x + 4h) - (5x^2 + 4x)}{h} =$$

$$\frac{10xh + 5h^2 + 4h}{h} = 10x + 5h + 4$$

65. $f(x) = 4 + 5|x|$

$f(x+h) = 4 + 5|x+h|$

$$\frac{f(x+h) - f(x)}{h} = \frac{4 + 5|x+h| - (4 + 5|x|)}{h}$$

66. $f(x) = 2|x| + 3x$

67. $f(x) = x^3$

$f(x+h) = (x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$f(x) = x^3$

$$\frac{f(x+h) - f(x)}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} =$$

$$\frac{3x^2h + 3xh^2 + h^3}{h} = \frac{h(3x^2 + 3xh + h^2)}{h \cdot 1} =$$

$$\frac{h}{h} \cdot \frac{3x^2 + 3xh + h^2}{1} = 3x^2 + 3xh + h^2$$

68. $f(x) = x^3 - 2x$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - 2(x+h) - (x^3 - 2x)}{h} =$$

$$\frac{x^3 + 3x^2h + 3xh^2 + h^3 - 2x - 2h - x^3 + 2x}{h} =$$

$$\frac{3x^2h + 3xh^2 + h^3 - 2h}{h} = \frac{h(3x^2 + 3xh + h^2 - 2)}{h} =$$

$$3x^2 + 3xh + h^2 - 2$$

69. $f(x) = \dfrac{x - 4}{x + 3}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\dfrac{x+h-4}{x+h+3} - \dfrac{x-4}{x+3}}{h} =$$

$$\frac{\dfrac{x+h-4}{x+h+3} - \dfrac{x-4}{x+3}}{h} \cdot \frac{(x+h+3)(x+3)}{(x+h+3)(x+3)} =$$

$$\frac{(x+h-4)(x+3) - (x-4)(x+h+3)}{h(x+h+3)(x+3)} =$$

$$\frac{x^2 + hx - 4x + 3x + 3h - 12 - (x^2 + hx + 3x - 4x - 4h - 12)}{h(x+h+3)(x+3)} =$$

$$\frac{x^2 + hx - x + 3h - 12 - x^2 - hx + x + 4h + 12}{h(x+h+3)(x+3)} =$$

$$\frac{7h}{h(x+h+3)(x+3)} = \frac{h}{h} \cdot \frac{7}{(x+h+3)(x+3)} =$$

$$\frac{7}{(x+h+3)(x+3)}$$

70. $f(x) = \dfrac{x}{2 - x}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\dfrac{x+h}{2-(x+h)} - \dfrac{x}{2-x}}{h} =$$

$$\frac{(x+h)(2-x) - x(2-x-h)}{h}$$

$$= \frac{4 + 5|x+h| - 4 - 5|x|}{h}$$

$$= \frac{5|x+h| - 5|x|}{h}$$

$$\frac{\frac{x}{2-x-h} - \frac{x}{2-x}}{h} \cdot \frac{(2-x-h)(2-x)}{(2-x-h)(2-x)} =$$

$$\frac{\frac{2x - x^2 + 2h - hx - 2x + x^2 + hx}{(2-x-h)(2-x)}}{h} =$$

$$\frac{\frac{2h}{(2-x-h)(2-x)}}{h} =$$

$$\frac{2h}{(2-x-h)(2-x)} \cdot \frac{1}{h} = \frac{2}{(2-x-h)(2-x)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(2|x+h|+3x+3h)-(2|x|+3x)}{h} =$$

$$\frac{2|x+h|-2|x|+3h}{h}$$

71. Graph $y = 3x - 1$.

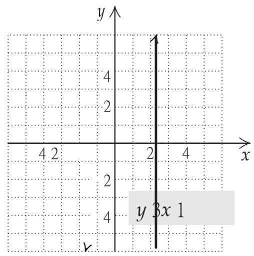
We find some ordered pairs that are solutions of the equation, plot these points, and draw the graph.

When $x = -1$, $y = 3(-1) - 1 = -3 - 1 = -4$.

When $x = 0$, $y = 3 \cdot 0 - 1 = 0 - 1 = -1$.

When $x = 2$, $y = 3 \cdot 2 - 1 = 6 - 1 = 5$.

x	y
-1	-4
0	-1
2	5



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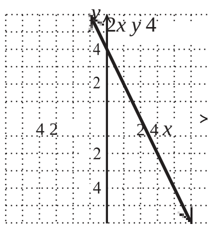


73. Graph $x - 3y = 3$.

First we find the x- and y-intercepts.

$$x - 3 \cdot 0 = 3$$
$$x = 3$$

The x-intercept is $(3, 0)$.

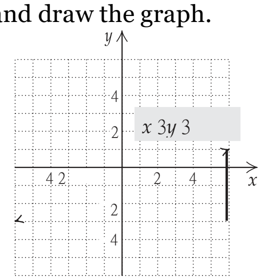
$$0 - 3y = 3$$
$$-3y = 3$$
$$y = -1$$

The y-intercept is $(0, -1)$.

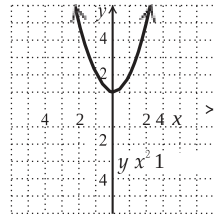
We find a third point as a check. We let $x = -3$ and solve for y.

$$-3 - 3y = 3$$
$$-3y = 6$$
$$y = -2$$

Another point on the graph is $(-3, -2)$. We plot the points and draw the graph.



74.



75. Answers may vary; $f(x) = \dfrac{1}{x+7}$, $g(x) = \dfrac{1}{x-3}$

76. The domain of $h+f$, $h-f$, and hf consists of all numbers that are in the domain of both h and f, or $\{-4, 0, 3\}$.

The domain of h/f consists of all numbers that are in the domain of both h and f, excluding any for which the value of f is 0, or $\{-4, 0\}$.

77. The domain of $h(x)$ is $\{x \mid x \neq -\frac{7}{3}\}$, and the domain of $g(x)$ is $\{x \mid x \neq 3\}$, so $-\frac{7}{3}$ and 3 are not in the domain of $(h/g)(x)$.

We must also exclude the value of x for which $g(x) = 0$.

$$\frac{x^4 - 1}{5x - 15} = 0$$
$$x^4 - 1 = 0 \quad \text{Multiplying by } 5x - 15$$
$$x^4 = 1$$
$$x = \pm 1$$

Then the domain of $(h/g)(x)$ is

$\{x \mid x \neq -\frac{7}{3} \text{ and } x \neq 3 \text{ and } x \neq -1 \text{ and } x \neq 1\}$, or

$(-\infty, -\frac{7}{3}) \cup (-\frac{7}{3}, -1) \cup (-1, 1) \cup (1, 3) \cup (3, \infty)$.

Exercise Set 2.3

1. $(f \circ g)(-1) = f(g(-1)) = f((-1)^2 - 2(-1) - 6) = f(1 + 2 - 6) = f(-3) = 3(-3) + 1 = -9 + 1 = -8$

2. $(g \circ f)(-2) = g(f(-2)) = g(3(-2) + 1) = g(-5) = (-5)^2 - 2(-5) - 6 = 25 + 10 - 6 = 29$

3. $(h \circ f)(1) = h(f(1)) = h(3 \cdot 1 + 1) = h(3 + 1) = h(4) = 4^3 = 64$

4. $(g \circ h)\left(\frac{1}{2}\right) = g\left(h\left(\frac{1}{2}\right)\right) = g\left(\left(\frac{1}{2}\right)^3\right) = g\left(\frac{1}{8}\right) = \left(\frac{1}{8}\right)^2 - 2 \cdot \frac{1}{8} - 6 = \frac{1}{64} - \frac{1}{4} - 6 = -\frac{399}{64}$

5. $(g \circ f)(5) = g(f(5)) = g(3 \cdot 5 + 1) = g(15 + 1) = g(16) = 16^2 - 2 \cdot 16 - 6 = 218$

6. $(f \circ g)\left(\frac{1}{3}\right) = f\left(g\left(\frac{1}{3}\right)\right) = f\left(\left(\frac{1}{3}\right)^2 - 2 \cdot \frac{1}{3} - 6\right) = f\left(\frac{1}{9} - \frac{2}{3} - 6\right) = f\left(-\frac{59}{9}\right) = 3\left(-\frac{59}{9}\right) + 1 = -\frac{56}{3}$

7. $(f \circ h)(-3) = f(h(-3)) = f((-3)^3) = f(-27) =$

$$3(-27)+1=-81+1=-80$$

8. $(h \circ g)(3) = h(g(3)) = h(3^2 - 2 \cdot 3 - 6) = h(9 - 6 - 6) = h(-3) = (-3)^3 = -27$

9. $(g \circ g)(-2) = g(g(-2)) = g((-2)^2 - 2(-2) - 6) = g(4 + 4 - 6) = g(2) = 2^2 - 2 \cdot 2 - 6 = 4 - 4 - 6 = -6$

10. $(g \circ g)(3) = g(g(3)) = g(3^2 - 2 \cdot 3 - 6) = g(9 - 6 - 6) = g(-3) = (-3)^2 - 2(-3) - 6 = 9 + 6 - 6 = 9$

11. $(h \circ h)(2) = h(h(2)) = h(2^3) = h(8) = 8^3 = 512$

12. $(h \circ h)(-1) = h(h(-1)) = h((-1)^3) = h(-1) = (-1)^3 = -1$

13. $(f \circ f)(-4) = f(f(-4)) = f(3(-4) + 1) = f(-12 + 1) = f(-11) = 3(-11) + 1 = -33 + 1 = -32$

14. $(f \circ f)(1) = f(f(1)) = f(3 \cdot 1 + 1) = f(3 + 1) = f(4) = 3 \cdot 4 + 1 = 12 + 1 = 13$

15. $(h \circ h)(x) = h(h(x)) = h(x^3) = (x^3)^3 = x^9$

16. $(f \circ f)(x) = f(f(x)) = f(3x + 1) = 3(3x + 1) + 1 = 9x + 3 + 1 = 9x + 4$

17. $(f \circ g)(x) = f(g(x)) = f(x - 3) = x - 3 + 3 = x$

$(g \circ f)(x) = g(f(x)) = g(x + 3) = x + 3 - 3 = x$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

18. $(f \circ g)(x) = f\left(\frac{4}{5}x\right) = \frac{5}{4} \cdot \frac{4}{5}x = x$

$(g \circ f)(x) = g\left(\frac{5}{4}x\right) = \frac{4}{5} \cdot \frac{5}{4}x = x$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

19. $(f \circ g)(x) = f(g(x)) = f(3x^2 - 2x - 1) = 3x^2 - 2x - 1 + 1 = 3x^2 - 2x$

$(g \circ f)(x) = g(f(x)) = g(x + 1) = 3(x + 1)^2 - 2(x + 1) - 1 = 3(x^2 + 2x + 1) - 2(x + 1) - 1 = 3x^2 + 6x + 3 - 2x - 2 - 1 = 3x^2 + 4x$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

20. $(f \circ g)(x) = f(x^2 + 5) = 3(x^2 + 5) - 2 = 3x^2 + 15 - 2 = 3x^2 + 13$

22. $(f \circ g)(x) = f(2x - 7) = 4(2x - 7)^2 - (2x - 7) + 10 = 4(4x^2 - 28x + 49) - (2x - 7) + 10 = 16x^2 - 112x + 196 - 2x + 7 + 10 = 16x^2 - 114x + 213$

$(g \circ f)(x) = g(4x^2 - x + 10) = 2(4x^2 - x + 10) - 7 = 8x^2 - 2x + 20 - 7 = 8x^2 - 2x + 13$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

23. $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{4}{1 - 5 \cdot \frac{1}{x}} = \frac{4}{1 - \frac{5}{x}} = \frac{4}{\frac{x - 5}{x}} = 4 \cdot \frac{x}{x - 5} = \frac{4x}{x - 5}$

$(g \circ f)(x) = g(f(x)) = g\left(\frac{4}{1 - 5x}\right) = \frac{1}{\frac{4}{1 - 5x}} = \frac{1 - 5x}{4}$

The domain of f is $\left\{x \mid x \neq \frac{1}{5}\right\}$ and the domain of g is $\{x \mid x \neq 0\}$. Consider the domain of $f \circ g$. Since 0 is not in the domain of g, 0 is not in the domain of $f \circ g$. Since $\frac{1}{5}$ is not in the domain of f, we know that $g(x)$ cannot be $\frac{1}{5}$. We find the value(s) of x for which $g(x) = \frac{1}{5}$.

$\frac{1}{x} = \frac{1}{5}$

$5 = x$ Multiplying by $5x$

Thus 5 is also not in the domain of $f \circ g$. Then the domain of $f \circ g$ is $\{x \mid x \neq 0 \text{ and } x \neq 5\}$, or $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$.

Now consider the domain of $g \circ f$. Recall that $\frac{1}{5}$ is not in the domain of f, so it is not in the domain of $g \circ f$. Now 0 is not in the domain of g but $f(x)$ is never 0, so the domain of $g \circ f$ is $\left\{x \mid x \neq \frac{1}{5}\right\}$, or $\left(-\infty, \frac{1}{5}\right) \cup \left(\frac{1}{5}, \infty\right)$.

24. $(f \circ g)(x) = f\left(\frac{1}{2x + 1}\right) = \frac{6}{\frac{1}{2x + 1}} = 6 \cdot \frac{2x + 1}{1} = 6(2x + 1)$, or $12x + 6$

$(g \circ f)(x) = g\left(\frac{6}{x}\right) = \frac{1}{2 \cdot \frac{6}{x} + 1} = \frac{1}{\frac{12}{x} + 1} = \frac{1}{\frac{12 + x}{x}} =$

$(g\circ f)(x)= g(3x-2)=(3x-2)^2 +5=9x^2 -12x+4+5=$

$9x^2 - 12x + 9$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

21. $(f\circ g)(x)=f(g(x))=f(4x-3)=(4x-3)^2-3=$
$16x^2 - 24x + 9 - 3 = 16x^2 - 24x + 6$

$(g\circ f)(x)=g(f(x)) =g(x^2-3)=4(x^2-3)-3=$

$4x^2 - 12 - 3 = 4x^2 - 15$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

$1 \cdot \frac{x}{12+x} = \frac{2x + 1}{12+x} \quad \frac{x+1}{x}$

The domain of f is $\{x | x \neq 0\}$ and the domain of g is $\{x | x \neq -\frac{1}{2}\}$. Since $-\frac{1}{2}$ is not in the domain of g, $-\frac{1}{2}$ is not in the domain of $f \circ g$. Consider the domain of $f \circ g$. Now 0 is not in the domain of f but $g(x)$ is never 0, so the domain of $f \circ g$ is $\{x | x \neq -\frac{1}{2}\}$, or $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

Now consider the domain of $g \circ f$. Since 0 is not in the domain of f, then 0 is not in the domain of $g \circ f$. Also, since $-\frac{1}{2}$ is not in the domain of g, we find the value(s) of x for which $f(x) = -\frac{1}{2}$.

$$\frac{6}{x} = -\frac{1}{2}$$
$$-12 = x$$

Then the domain of $g \circ f$ is $\{x \mid x \neq -12 \text{ and } x \neq 0\}$, or $(-\infty, -12) \cup (-12, 0) \cup (0, \infty)$.

25. $(f \circ g)(x) = f(g(x)) = f\left(\frac{x+7}{3}\right) = 3\left(\frac{x+7}{3}\right) - 7 = x + 7 - 7 = x$

$(g \circ f)(x) = g(f(x)) = g(3x - 7) = \frac{(3x-7)+7}{3} = \frac{3x}{3} = x$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

26. $(f \circ g)(x) = f(1.5x + 1.2) = \frac{2}{3}(1.5x + 1.2) - \frac{4}{5} = x + 0.8 - \frac{4}{5} = x$

$(g \circ f)(x) = g\left(\frac{2}{3}x - \frac{4}{5}\right) = 1.5\left(\frac{2}{3}x - \frac{4}{5}\right) + 1.2 = x - 1.2 + 1.2 = x$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

27. $(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = 2\sqrt{x} + 1$

$(g \circ f)(x) = g(f(x)) = g(2x + 1) = \sqrt{2x + 1}$

The domain of f is $(-\infty, \infty)$ and the domain of g is $\{x \mid x \geq 0\}$. Thus the domain of $f \circ g$ is $\{x \mid x \geq 0\}$, or $[0, \infty)$.

Now consider the domain of $g \circ f$. There are no restrictions on the domain of f, but the domain of g is $\{x \mid x \geq 0\}$. Since $f(x) \geq 0$ for $x \geq -\frac{1}{2}$, the domain of $g \circ f$ is $\left\{x \mid x \geq -\frac{1}{2}\right\}$, or $\left[-\frac{1}{2}, \infty\right)$.

28. $(f \circ g)(x) = f(2 - 3x) = \sqrt{2 - 3x}$

$(g \circ f)(x) = g(\sqrt{x}) = 2 - 3\sqrt{x}$

30. $(f \circ g)(x) = (\sqrt[4]{x})^4 = x$

$(g \circ f)(x) = \sqrt[4]{x^4} = |x|$

The domain of f is $(-\infty, \infty)$ and the domain of g is $\{x \mid x \geq 0\}$, so the domain of $f \circ g$ is $\{x \mid x \geq 0\}$, or $[0, \infty)$.

Now consider the domain of $g \circ f$. There are no restrictions on the domain of f and $f(x) \geq 0$ for all values of x, so the domain is $(-\infty, \infty)$.

31. $(f \circ g)(x) = f(g(x)) = f(x^2 - 5) = \sqrt{x^2 - 5 + 5} = \sqrt{x^2} = |x|$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x + 5}) = (\sqrt{x + 5})^2 - 5 = x + 5 - 5 = x$

The domain of f is $\{x \mid x \geq -5\}$ and the domain of g is $(-\infty, \infty)$. Since $x^2 \geq 0$ for all values of x, then $x^2 - 5 \geq -5$ for all values of x and the domain of $g \circ f$ is $(-\infty, \infty)$.

Now consider the domain of $f \circ g$. There are no restrictions on the domain of g, so the domain of $f \circ g$ is the same as the domain of f, $\{x \mid x \geq -5\}$, or $[-5, \infty)$.

32. $(f \circ g)(x) = (\sqrt[5]{x + 2})^5 - 2 = x + 2 - 2 = x$

$(g \circ f)(x) = \sqrt[5]{x^5 - 2 + 2} = \sqrt[5]{x^5} = x$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

33. $(f \circ g)(x) = f(g(x)) = f(\sqrt{3 - x}) = (\sqrt{3 - x})^2 + 2 = 3 - x + 2 = 5 - x$

$(g \circ f)(x) = g(f(x)) = g(x^2 + 2) = \sqrt{3 - (x^2 + 2)} = \sqrt{3 - x^2 - 2} = \sqrt{1 - x^2}$

The domain of f is $(-\infty, \infty)$ and the domain of g is $\{x \mid x \leq 3\}$, so the domain of $f \circ g$ is $\{x \mid x \leq 3\}$, or $(-\infty, 3]$.

Now consider the domain of $g \circ f$. There are no restrictions on the domain of f and the domain of g is $\{x \mid x \leq 3\}$, so we find the values of x for which $f(x) \leq 3$. We see that $x^2 + 2 \leq 3$ for $-1 \leq x \leq 1$, so the domain of $g \circ f$ is $\{x \mid -1 \leq x \leq 1\}$, or $[-1, 1]$.

34. $(f \circ g)(x) = f(\sqrt{x^2 - 25}) = 1 - (\sqrt{x^2 - 25})^2 = 1 - (x^2 - 25) = 1 - x^2 + 25 = 26 - x^2$

$(g \circ f)(x) = g(1 - x^2) = \sqrt{(1 - x^2)^2 - 25} = \sqrt{1 - 2x^2 + x^4 - 25} = \sqrt{x^4 - 2x^2 - 24}$

The domain of f is $\{x \mid x \geq 0\}$ and the domain of g is

The domain of f is $(-\infty, \infty)$ and the domain of g is $(-\infty, \infty)$. Since $g(x) \geq 0$ when $x \leq \frac{2}{3}$, the domain of $f \circ g$ is $\left(-\infty, \frac{2}{3}\right]$.

Now consider the domain of $g \circ f$. Since the domain of f is $\{x | x \geq 0\}$ and the domain of g is $(-\infty, \infty)$, the domain of $g \circ f$ is $\{x | x \geq 0\}$, or $[0, \infty)$.

29. $(f \circ g)(x) = f(g(x)) = f(0.05) = 20$

$(g \circ f)(x) = g(f(x)) = g(20) = 0.05$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

$\{x | x \leq -5 \quad or \quad x \geq 5\}$, so the domain of $f \circ g$ is $\{x | x \leq -5 \quad or \quad x \geq 5\}$, or $(-\infty, -5] \cup [5, \infty)$.

Now consider the domain of $g \circ f$. There are no restrictions on the domain of f and the domain of g is $\{x | x \leq -5 \quad or \quad x \geq 5\}$, so we find the values of x for which $f(x) \leq -5$ or $f(x) \geq 5$. We see that $1 - x^2 \leq -5$ when $x \leq -\sqrt{6}$ or $x \geq \sqrt{6}$ and $1 - x^2 \geq 5$ has no solution, so the domain of $g \circ f$ is $\{x | x \leq -\sqrt{6} \quad or \quad x \geq \sqrt{6}\}$, or $(-\infty, -\sqrt{6}] \cup [\sqrt{6}, \infty)$.

35. $(f\circ g)(x) = f(g(x)) = f\left(\frac{1}{1+x}\right) =$

$$\frac{1-\frac{1}{1+x}}{\frac{1}{1+x}} = \frac{\frac{1+x-1}{1+x}}{\frac{1}{1+x}} =$$

$$\frac{x}{1+x}\cdot\frac{1+x}{1} = x$$

$(g\circ f)(x) = g(f(x)) = g\left(\frac{1-x}{x}\right) =$

$$\frac{1}{1+\frac{1-x}{x}} = \frac{1}{\frac{x+1-x}{x}} = \frac{1}{\frac{1}{x}} = 1\cdot\frac{x}{1} = x$$

The domain of f is $\{x|x \neq 0\}$ and the domain of g is $\{x|x \neq -1\}$, so we know that -1 is not in the domain of $f\circ g$. Since 0 is not in the domain of f, values of x for which $g(x) = 0$ are not in the domain of $f\circ g$. But $g(x)$ is never 0, so the domain of $f\circ g$ is $\{x|x \neq -1\}$, or $(-\infty,-1)\cup(-1,\infty)$.

Now consider the domain of $g\circ f$. Recall that 0 is not in the domain of f. Since -1 is not in the domain of g, we know that $g(x)$ cannot be -1. We find the value(s) of x for which $f(x) = -1$.

$$\frac{1-x}{x} = -1$$

$1-x = -x$ Multiplying by x

$1 = 0$ False equation

We see that there are no values of x for which $f(x) = -1$, so the domain of $g\circ f$ is $\{x|x \neq 0\}$, or $(-\infty,0)\cup(0,\infty)$.

36. $(f\circ g)(x) = f\left(\frac{x+2}{x}\right) = \frac{1}{\frac{x+2}{x}-2}$

$$= \frac{1}{\frac{x+2-2x}{x}} = \frac{1}{\frac{-x+2}{x}}$$

$$= 1\cdot\frac{x}{-x+2} = \frac{x}{-x+2}, \text{ or } \frac{x}{2-x}$$

$(g\circ f)(x) = g\left(\frac{1}{x-2}\right) = \frac{\frac{1}{x-2}+2}{\frac{1}{x-2}} = \frac{\frac{1+2x-4}{x-2}}{\frac{1}{x-2}} =$

$$\frac{2x-3}{x-2}\cdot\frac{x-2}{1} = 2x-3$$

$$\frac{x+2}{x} = 2$$

$x+2 = 2x$

$2 = x$

Then the domain of $f\circ g$ is $(-\infty,0)\cup(0,2)\cup(2,\infty)$.

Now consider the domain of $g\circ f$. Since the domain of f is $\{x|x \neq 2\}$, we know that 2 is not in the domain of $g\circ f$. Since the domain of g is $\{x|x \neq 0\}$, we find the value of x for which $f(x) = 0$.

$$\frac{1}{x-2} = 0$$

$1 = 0$

We get a false equation, so there are no such values. Then the domain of $g\circ f$ is $(-\infty,2)\cup(2,\infty)$.

37. $(f\circ g)(x) = f(g(x)) = f(x+1) =$

$(x+1)^3 - 5(x+1)^2 + 3(x+1) + 7 =$

$x^3 + 3x^2 + 3x + 1 - 5x^2 - 10x - 5 + 3x + 3 + 7 =$

$x^3 - 2x^2 - 4x + 6$

$(g\circ f)(x) = g(f(x)) = g(x^3 - 5x^2 + 3x + 7) =$

$x^3 - 5x^2 + 3x + 7 + 1 = x^3 - 5x^2 + 3x + 8$

The domain of f and of g is $(-\infty,\infty)$, so the domain of $f\circ g$ and of $g\circ f$ is $(-\infty,\infty)$.

38. $(f\circ g)(x) = f(x+1) = (x+1)^3 + 2(x+1)^2 - 3(x+1) - 9 =$ ... $= x^3 + 2x^2 - 3x - 10$ ...

$(g\circ f)(x) = x^3 + 2x^2 - 3x - 9 - 1 =$

$x^3 + 2x^2 - 3x - 10$

$(g\circ f)(x) = (x-1)^3 + 2(x-1)^2 - 3(x-1) - 9 =$

$x^3 - 3x^2 + 3x - 1 + 2x^2 - 4x + 2 - 3x + 3 - 9 =$

$x^3 - x^2 - 4x - 5$

The domain of f and of g is $(-\infty,\infty)$, so the domain of $f\circ g$ and of $g\circ f$ is $(-\infty,\infty)$.

39. $h(x) = (4+3x)^5$

This is $4+3x$ to the 5th power. The most obvious answer is $f(x) = x^5$ and $g(x) = 4+3x$.

40. $f(x) = \sqrt[3]{x}$, $g(x) = x^2 - 8$

41. $h(x) = \frac{1}{(x-2)^4}$

This is 1 divided by $(x-2)$ to the 4th power. One obvious answer is $f(x) = \frac{1}{x^4}$ and $g(x) = x-2$. Another possibility

$\frac{x-2}{1}$ $=$

$x-2$

$\underline{2x-3}$

is $f(x) = 1$ and $g(x) = (x-2)^4$.

$\frac{x}{1}$

42. $f(x) = \sqrt{x}$, $g(x) = 3x + 7$

$$= \frac{x-2}{\frac{1}{x-2}} = \frac{x-2}{\frac{1}{x-2}}$$

$$= \frac{2x-3}{x-2} \cdot \frac{x-2}{1} = 2x - 3$$

The domain of f is $\{x|x = 2\}$ and the domain of g is $\{x|x = 0\}$, so 0 is not in the domain of $f \circ g$. We find the value of x for which $g(x) = 2$.

43. $f(x) = \frac{x-1}{x+1}$, $g(x) = x^3$

44. $f(x) = |x|$, $g(x) = 9x^2 - 4$

45. $f(x) = x^6$, $g(x) = \frac{2 + x^3}{2 - x^3}$

46. $f(x) = x^4$, $g(x) = \sqrt{x - 3}$

47. $f(x) = \sqrt{x}$, $g(x) = \dfrac{x-5}{x+2}$

48. $f(x) = \sqrt{1+x}$, $g(x) = \sqrt{1+x}$

49. $f(x) = x^3 - 5x^2 + 3x - 1$, $g(x) = x + 2$

50. $f(x) = 2x^{5/3} + 5x^{2/3}$, $g(x) = x - 1$, or $f(x) = 2x^5 + 5x^2$, $g(x) = (x-1)^{1/3}$

51. a) Use the distance formula, distance = rate × time. Substitute 3 for the rate and t for time.
$r(t) = 3t$

b) Use the formula for the area of a circle.
$A(r) = \pi r^2$

c) $(A \circ r)(t) = A(r(t)) = A(3t) = \pi(3t)^2 = 9\pi t^2$
This function gives the area of the ripple in terms of time t.

52. a) $h = 2r$
$S(r) = 2\pi r(2r) + 2\pi r^2$
$S(r) = 4\pi r^2 + 2\pi r^2$
$S(r) = 6\pi r^2$

b) $r = \dfrac{h}{2}$

$S(h) = 2\pi \left(\dfrac{h}{2}\right) h + 2\pi \left(\dfrac{h}{2}\right)^2$

$S(h) = \pi h^2 + \dfrac{\pi h^2}{2}$

$S(h) = \dfrac{3}{2}\pi h^2$

53. $f(x) = (t \circ s)(x) = t(s(x)) = t(x-3) = x - 3 + 4 = x + 1$
We have $f(x) = x + 1$.

54. The manufacturer charges $m+6$ per drill. The chain store sells each drill for $150\%(m+6)$, or $1.5(m+6)$, or $1.5m+9$. Thus, we have $P(m) = 1.5m + 9$.

55. Equations (a)-(f) are in the form $y = mx + b$, so we can read the y-intercepts directly from the equations. Equations (g) and (h) can be written in this form as $y = \frac{2}{3}x - 2$ and $y = -2x + 3$, respectively. We see that only equation (c) has y-intercept (0, 1).

56. None (See Exercise 55.)

57. If a line slopes down from left to right, its slope is negative. The equations $y = mx + b$ for which m is negative are (b), (d), (f), and (h). (See Exercise 55.)

58. The equation for which $|m|$ is greatest is the equation with the steepest slant. This is equation (b). (See Exercise 55.)

59. The only equation that has (0, 0) as a solution is (a).

60. Equations (c) and (g) have the same slope. (See Exer-cise 55.)

62. The only equations for which the product of the slopes is -1 are (a) and (f).

63. Only the composition $(c \circ p)(a)$ makes sense. It represents the cost of the grass seed required to seed a lawn with area a.

64. Answers may vary. One example is $f(x) = 2x + 5$ and $g(x) = \dfrac{x-5}{2}$. Other examples are found in Exercises 17, 18, 25, 26, 32 and 35.

Chapter 2 Mid-Chapter Mixed Review

1. The statement is true. See page 96 in the text.

2. The statement is false. See page 110 in the text.

3. The statement is true. See Examples 1 and 2 in Section 2.3.

4. a) For x-values from 2 to 4, the y-values increase from 2 to 4. Thus the function is increasing on the interval (2, 4).

   b) For x-values from -5 to -3, the y-values decrease from 5 to 1. Also, for x-values from 4 to 5, the y-values decrease from 4 to -3. Thus the function is decreasing on $(-5, -3)$ and on $(4, 5)$.

   c) For x-values from -3 to -1, y is 3. Thus the function is constant on $(-3, -1)$.

5. From the graph we see that a relative maximum value of 6.30 occurs at $x = -1.29$. We also see that a relative minimum value of -2.30 occurs at $x = 1.29$.
The graph starts rising, or increasing, from the left and stops increasing at the relative maximum. From this point it decreases to the relative minimum and then increases again. Thus the function is increasing on $(-\infty, -1.29)$ and on $(1.29, \infty)$. It is decreasing on $(-1.29, 1.29)$.

6. The x-values extend from -5 to -1 and from 2 to 5, so the domain is $[-5, -1] \cup [2, 5]$. The y-values extend from -3 to 5, so the range is $[-3, 5]$.

7. $A(h) = \dfrac{1}{2}(h+4)h$

   $A(h) = \dfrac{h^2}{2} + 2h$

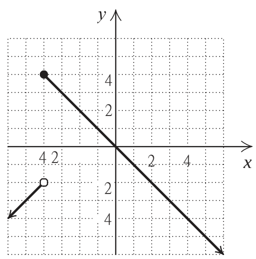
8. $$f(x) = \begin{cases} x - 5, & \text{for } x \le -3, \\ 2x + 3, & \text{for } -3 < x \le 0, \\ \frac{1}{2}x, & \text{for } x > 0, \end{cases}$$

61. Only equations (c) and (g) have the same slope and different y-intercepts. They represent parallel lines.

Since $-5 \le -3$, $f(-5) = -5 - 5 = -10$.
Since $-3 \le -3$, $f(-3) = -3 - 5 = -8$.
Since $-3 < -1 \le 0$, $f(-1) = 2(-1) + 3 = -2 + 3 = 1$.
Since $6 > 0$, $f(6) = \frac{1}{2} \cdot 6 = 3$.

9. $g(x) = \begin{cases} x+2, & \text{for } x < -4, \\ -x, & \text{for } x \geq -4 \end{cases}$

We create the graph in two parts. Graph $g(x) = x + 2$ for inputs less than -4. Then graph $g(x) = -x$ for inputs greater than or equal to -4.



10. $(f + g)(-1) = f(-1) + g(-1)$

$= [3(-1) - 1] + [(-1)^2 + 4]$

$= -3 - 1 + 1 + 4$

$= 1$

11. $(fg)(0) = f(0) \cdot g(0)$

$= (3 \cdot 0 - 1) \cdot (0^2 + 4)$

$= -1 \cdot 4$

$= -4$

12. $(g - f)(3) = g(3) - f(3)$

$= (3^2 + 4) - (3 \cdot 3 - 1)$

$= 9 + 4 - (9 - 1)$

$= 9 + 4 - 9 + 1$

$= 5$

13. $(g/f)\left(\frac{1}{3}\right) = \dfrac{g\left(\frac{1}{3}\right)}{f\left(\frac{1}{3}\right)}$

$= \dfrac{\left(\frac{1}{3}\right)^2 + 4}{3 \cdot \frac{1}{3} - 1}$

$= \dfrac{\frac{1}{9} + 4}{1 - 1}$

$= \dfrac{\frac{37}{9}}{0}$

Since division by 0 is not defined, $(g/f)\left(\frac{1}{3}\right)$ does not exist.

14. $f(x) = 2x + 5$, $g(x) = -x - 4$

a) The domain of f and of g is the set of all real numbers, or $(-\infty, \infty)$. Then the domain of $f + g$, $f - g$, fg, and ff is also $(-\infty, \infty)$.

For f/g we must exclude -4 since $g(-4) = 0$. Then the domain of f/g is $(-\infty, -4) \cup (-4, \infty)$.

For g/f we must exclude $-\frac{5}{2}$ since $f\left(-\frac{5}{2}\right) = 0$. Then the domain of g/f is

$\left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, \infty\right)$.

b) $(f + g)(x) = f(x) + g(x) = (2x + 5) + (-x - 4) = x + 1$

$(f - g)(x) = f(x) - g(x) = (2x + 5) - (-x - 4) = 2x + 5 + x + 4 = 3x + 9$

$(fg)(x) = f(x) \cdot g(x) = (2x + 5)(-x - 4) = -2x^2 - 8x - 5x - 20 = -2x^2 - 13x - 20$

$(ff)(x) = f(x) \cdot f(x) = (2x + 5) \cdot (2x + 5) = 4x^2 + 10x + 10x + 25 = 4x^2 + 20x + 25$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x + 5}{-x - 4}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{-x - 4}{2x + 5}$

15. $f(x) = x - 1$, $g(x) = \sqrt{x + 2}$

a) Any number can be an input for f, so the domain of f is the set of all real numbers, or $(-\infty, \infty)$.

The domain of g consists of all values for which $x + 2$ is nonnegative, so we have $x + 2 \geq 0$, or $x \geq -2$, or $[-2, \infty)$. Then the domain of $f + g$, $f - g$, and fg is $[-2, \infty)$.

The domain of ff is $(-\infty, \infty)$.

Since $g(-2) = 0$, the domain of f/g is $(-2, \infty)$.

Since $f(1) = 0$, the domain of g/f is $[-2, 1) \cup (1, \infty)$.

b) $(f + g)(x) = f(x) + g(x) = x - 1 + \sqrt{x + 2}$

$(f - g)(x) = f(x) - g(x) = x - 1 - \sqrt{x + 2}$

$(fg)(x) = f(x) \cdot g(x) = (x - 1)\sqrt{x + 2}$

$(ff)(x) = f(x) \cdot f(x) = (x - 1)(x - 1) = x^2 - x - x + 1 = x^2 - 2x + 1$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x - 1}{\sqrt{x + 2}}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{\sqrt{x + 2}}{x - 1}$

16. $f(x) = 4x - 3$

$\dfrac{f(x + h) - f(x)}{h} = \dfrac{4(x + h) - 3 - (4x - 3)}{h} =$

$$\frac{4x + 4h - 3 - 4x + 3}{h} = \frac{4h}{h} = 4$$

17. $f(x) = 6 - x^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{6-(x+h)^2-(6-x^2)}{h} =$$

$$\frac{6-(x^2+2xh+h^2)-6+x^2}{h} = \frac{6-x^2-2xh-h^2-6+x^2}{h} =$$

$$\frac{-2xh-h^2}{h} = \frac{\cancel{h}(-2x-h)}{\cancel{h}\cdot 1} = -2x-h$$

18. $(f\circ g)(1) = f(g(1)) = f(1^3+1) = f(1+1) = f(2) = 5\cdot 2 - 4 = 10 - 4 = 6$

19. $(g\circ h)(2) = g(h(2)) = g(2^2 - 2\cdot 2 + 3) = g(4 - 4 + 3) = g(3) = 3^3 + 1 = 27 + 1 = 28$

20. $(f\circ f)(0) = f(f(0)) = f(5\cdot 0 - 4) = f(-4) = 5(-4) - 4 = -20 - 4 = -24$

21. $(h\circ f)(-1) = h(f(-1)) = h(5(-1) - 4) = h(-5-4) = h(-9) = (-9)^2 - 2(-9) + 3 = 81 + 18 + 3 = 102$

22. $(f\circ g)(x) = f(g(x)) = f(6x+4) = \frac{1}{2}(6x+4) = 3x+2$

$(g\circ f)(x) = g(f(x)) = g\left(\frac{1}{2}x\right) = 6\cdot\frac{1}{2}x + 4 = 3x + 4$

The domain of f and g is $(-\infty, \infty)$, so the domain of $f\circ g$ and $g\circ f$ is $(-\infty, \infty)$.

23. $(f\circ g)(x) = f(g(x)) = f(\sqrt{x}) = 3\sqrt{x} + 2$

$(g\circ f)(x) = g(f(x)) = g(3x+2) = \sqrt{3x+2}$

The domain of f is $(-\infty, \infty)$ and the domain of g is $[0, \infty)$.

Consider the domain of $f\circ g$. Since any number can be an input for f, the domain of $f\circ g$ is the same as the domain of g, $[0, \infty)$.

Now consider the domain of $g\circ f$. Since the inputs of g must be nonnegative, we must have $3x+2 \ge 0$, or $x \ge -\frac{2}{3}$.

Thus the domain of $g\circ f$ is $\left[-\frac{2}{3}, \infty\right)$.

24. The graph of $y = (h-g)(x)$ will be the same as the graph of $y = h(x)$ moved down b units.

25. Under the given conditions, $(f+g)(x)$ and $(f/g)(x)$ have different domains if $g(x) = 0$ for one or more real numbers x.

26. If f and g are linear functions, then any real number can be an input for each function. Thus, the domain of $f\circ g$ = the domain of $g\circ f = (-\infty, \infty)$.

27. This approach is not valid. Consider Exercise 23 on page 120 in the text, for example. Since $(f\circ g)(x) = \frac{4x}{x-5}$, an examination of only this composed function would lead to the incorrect conclusion that the domain of $f\circ g$ is $(-\infty, 5)\cup(5, \infty)$. However, we must also exclude from the domain of $f\circ g$ those values of x that are not in the domain of g. Thus, the domain of $f\circ g$ is $(-\infty, 0)\cup(0, 5)\cup(5, \infty)$.

Exercise Set 2.4

1. If the graph were folded on the x-axis, the parts above and below the x-axis would not coincide, so the graph is not symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would coincide, so the graph is symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would not coincide with the original graph, so it is not symmetric with respect to the origin.

2. If the graph were folded on the x-axis, the parts above and below the x-axis would not coincide, so the graph is not symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would coincide, so the graph is symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would not coincide with the original graph, so it is not symmetric with respect to the origin.

3. If the graph were folded on the x-axis, the parts above and below the x-axis would coincide, so the graph is symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would not coincide, so the graph is not symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would not coincide with the original graph, so it is not symmetric with respect to the origin.

4. If the graph were folded on the x-axis, the parts above and below the x-axis would not coincide, so the graph is not symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would not coincide, so the graph is not symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would coincide with the original graph, so it is symmetric with respect to the origin.

5. If the graph were folded on the x-axis, the parts above and below the x-axis would not coincide, so the graph is not symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would not coincide, so the graph is not symmetric with respect to the y-axis.

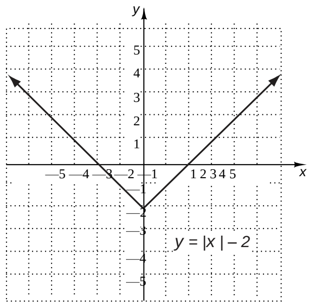
If the graph were rotated 180°, the resulting graph would coincide with the original graph, so it is symmetric with respect to the origin.

6. If the graph were folded on the x-axis, the parts above and below the x-axis would coincide, so the graph is symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would coincide, so the graph is symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would coincide with the original graph, so it is symmetric with respect to the origin.

7.



The graph is symmetric with respect to the y-axis. It is not symmetric with respect to the x-axis or the origin.

Test algebraically for symmetry with respect to the x-axis:

$y = |x| - 2$ Original equation

$-y = |x| - 2$ Replacing y by $-y$

$y = -|x| + 2$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$y = |x| - 2$ Original equation

$y = |-x| - 2$ Replacing x by $-x$

$y = |x| - 2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

$y = |x| - 2$ Original equation

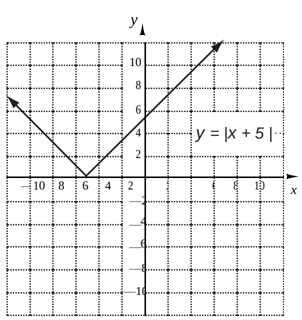
$-y = |-x| - 2$ Replacing x by $-x$ and y by $-y$

$-y = |x| - 2$ Simplifying

$y = -|x| + 2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

8.



The graph is not symmetric with respect to the x-axis, the y-axis, or the origin.

Test algebraically for symmetry with respect to the x-axis:

$y = |x + 5|$ Original equation

$-y = |x + 5|$ Replacing y by $-y$

$y = -|x + 5|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$y = |x + 5|$ Original equation

$y = |-x + 5|$ Replacing x by $-x$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

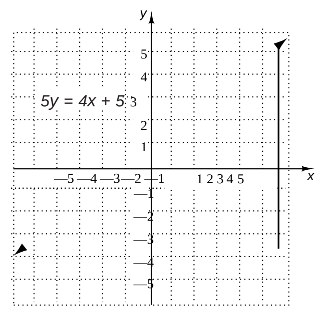
$y = |x + 5|$ Original equation

$-y = |-x + 5|$ Replacing x by $-x$ and y by $-y$

$y = -|-x + 5|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

9.



The graph is not symmetric with respect to the x-axis, the y-axis, or the origin.

Testalgebraically for symmetry with respect to the x-axis:

$5y = 4x + 5$ Original equation

$5(-y) = 4x + 5$ Replacing y by $-y$

$-5y = 4x + 5$ Simplifying

$5y = -4x - 5$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Testalgebraically for symmetry with respect to the y-axis:

$5y = 4x + 5$ Original equation

$5y = 4(-x) + 5$ Replacing x by $-x$

$5y = -4x + 5$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Testalgebraically for symmetry with respect to theorigin:

$5y = 4x + 5$ Original equation

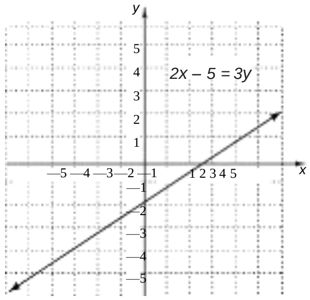
$5(-y) = 4(-x) + 5$ Replacing x by $-x$ and y by $-y$

$-5y = -4x + 5$ Simplifying

$5y = 4x - 5$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

10.



The graph is not symmetric with respect to the x-axis, the y-axis, or the origin.

Test algebraically for symmetry with respect to the x-axis:

$2x-5=3y$ Original equation

$2x-5=3(-y)$ Replacing y by $-y$

$-2x+5=3y$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$2x-5=3y$ Original equation

$2(-x)-5=3y$ Replacing x by $-x$

$-2x-5=3y$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

$2x-5=3y$ Original equation

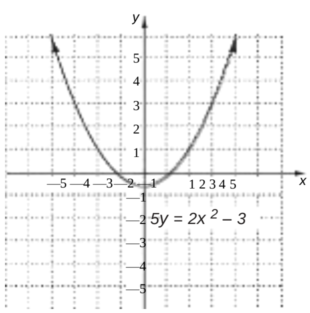
$2(-x)-5=3(-y)$ Replacing x by $-x$ and y by $-y$

$-2x-5=-3y$ Simplifying

$2x+5=3y$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

11.



The graph is symmetric with respect to the y-axis. It is not symmetric with respect to the x-axis or the origin.

Test algebraically for symmetry with respect to the x-axis:

$5y=2x^2-3$ Original equation

$5(-y)=2x^2-3$ Replacing y by $-y$

$-5y=2x^2-3$ Simplifying

$5y=-2x^2+3$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$5y=2x^2-3$ Original equation

$5y=2(-x)^2-3$ Replacing x by $-x$

$5y=2x^2-3$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

$5y=2x^2-3$ Original equation

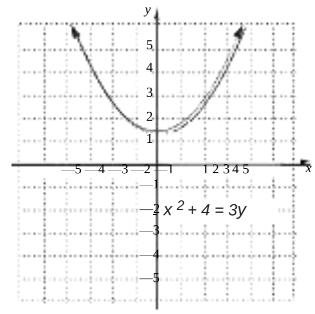
$5(-y)=2(-x)^2-3$ Replacing x by $-x$ and y by $-y$

$-5y=2x^2-3$ Simplifying

$5y=-2x^2+3$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

12.



The graph is symmetric with respect to the y-axis. It is not symmetric with respect to the x-axis or the origin.

Test algebraically for symmetry with respect to the x-axis:

$x^2+4=3y$ Original equation

$x^2+4=3(-y)$ Replacing y by $-y$

$-x^2-4=3y$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$x^2+4=3y$ Original equation

$(-x)^2+4=3y$ Replacing x by $-x$

$x^2+4=3y$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

$x^2+4=3y$ Original equation

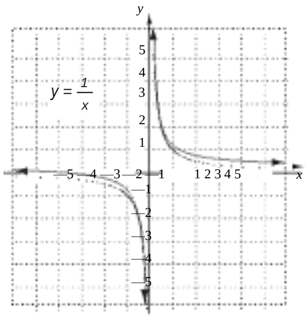
$(-x)^2+4=3(-y)$ Replacing x by $-x$ and y by $-y$

$x^2+4=-3y$ Simplifying

$-x^2-4=3y$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

13.



The graph is not symmetric with respect to the x-axis or the y-axis. It is symmetric with respect to the origin.
Test algebraically for symmetry with respect to the x-axis:

$y = \frac{1}{x}$ Original equation

$-y = \frac{1}{x}$ Replacing y by $-y$

$y = -\frac{1}{x}$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$y = \frac{1}{x}$ Original equation

$y = \frac{1}{-x}$ Replacing x by $-x$

$y = -\frac{1}{x}$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

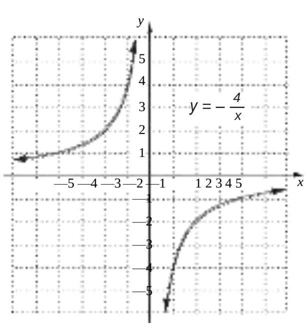
$y = \frac{1}{x}$ Original equation

$-y = \frac{1}{-x}$ Replacing x by $-x$ and y by $-y$

$y = \frac{1}{x}$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

14.



Test algebraically for symmetry with respect to the x-axis:

$y = -\frac{4}{x}$ Original equation

$-y = -\frac{4}{x}$ Replacing y by $-y$

$y = \frac{4}{x}$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$y = -\frac{4}{x}$ Original equation

$y = -\frac{4}{-x}$ Replacing x by $-x$

$y = \frac{4}{x}$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

$y = -\frac{4}{x}$ Original equation

$-y = -\frac{4}{-x}$ Replacing x by $-x$ and y by $-y$

$y = -\frac{4}{x}$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

15. Test for symmetry with respect to the x-axis:

$5x - 5y = 0$ Original equation

$5x - 5(-y) = 0$ Replacing y by $-y$

$5x + 5y = 0$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

The graph is not symmetric with respect to the x-axis or the y-axis. It is symmetric with respect to the origin.

$5 \quad 0$ Simplifying

$x \quad 5x + 5y = 0$

$-5y = 0$ Original equation

The last equation is not equivalent to the original equation,so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin: $5x - 5y = 0$ Original equation

$5(-x) - 5(-y) = 0$ Replacing x by $-x$ and y by $-y$

$-5x + 5y = 0$ Simplifying

$5x - 5y = 0$

The last equation is equivalent to the original equation, sothe graph is symmetric with respect to the origin.

16. Test for symmetry with respect to the x-axis: $6x + 7y = 0$ Original equation

$6x + 7(-y) = 0$ Replacing y by $-y$

$6x - 7y = 0$ Simplifying

$5(-x) - 5y = 0$ Replacing x by $-x$

$-5x - 5y =$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$6x + 7y = 0$ Original equation

$6(-x) + 7y = 0$ Replacing x by $-x$

$6x - 7y = 0$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$6x + 7y = 0$ Original equation

$6(-x) + 7(-y) = 0$ Replacing x by $-x$ and y by $-y$

$6x + 7y = 0$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

17. Test for symmetry with respect to the x-axis:

$3x^2 - 2y^2 = 3$ Original equation

$3x^2 - 2(-y)^2 = 3$ Replacing y by $-y$

$3x^2 - 2y^2 = 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$3x^2 - 2y^2 = 3$ Original equation

$3(-x)^2 - 2y^2 = 3$ Replacing x by $-x$

$3x^2 - 2y^2 = 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$3x^2 - 2y^2 = 3$ Original equation

$3(-x)^2 - 2(-y)^2 = 3$ Replacing x by $-x$ and y by $-y$

$3x^2 - 2y^2 = 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

18. Test for symmetry with respect to the x-axis:

$5y = 7x^2 - 2x$ Original equation

$5(-y) = 7x^2 - 2x$ Replacing y by $-y$

$5y = -7x^2 + 2x$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$5y = 7x^2 - 2x$ Original equation

$5y = 7(-x)^2 - 2(-x)$ Replacing x by $-x$

$5y = 7x^2 + 2x$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$5y = 7x^2 - 2x$ Original equation

$5(-y) = 7(-x)^2 - 2(-x)$ Replacing x by $-x$ and y by $-y$

$-5y = 7x^2 + 2x$ Simplifying

$5y = -7x^2 - 2x$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

19. Test for symmetry with respect to the x-axis:

$y = |2x|$ Original equation

$-y = |2x|$ Replacing y by $-y$

$y = -|2x|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$y = |2x|$ Original equation

$y = |2(-x)|$ Replacing x by $-x$

$y = |-2x|$ Simplifying

$y = |2x|$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$y = |2x|$ Original equation

$-y = |2(-x)|$ Replacing x by $-x$ and y by $-y$

$-y = |-2x|$ Simplifying

$-y = |2x|$

$y = -|2x|$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

20. Test for symmetry with respect to the x-axis:

$y^3 = 2x^2$ Original equation

$(-y)^3 = 2x^2$ Replacing y by $-y$

$-y^3 = 2x^2$ Simplifying

$y^3 = -2x^2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$y^3 = 2x^2$ Original equation

$y^3 = 2(-x)^2$ Replacing x by $-x$

$y^3 = 2x^2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$y^3 = 2x^2$ Original equation

$(-y)^3 = 2(-x)^2$ Replacing x by $-x$ and y by $-y$

$-y^3 = 2x^2$ Simplifying

$y^3 = -2x^2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

21. Test for symmetry with respect to the x-axis:

$2x^4 + 3 = y^2$ Original equation

$2x^4 + 3 = (-y)^2$ Replacing y by $-y$

$2x^4 + 3 = y^2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$2x^4 + 3 = y^2$ Original equation

$2(-x)^4 + 3 = y^2$ Replacing x by $-x$

$2x^4 + 3 = y^2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$2x^4 + 3 = y^2$ Original equation

$2(-x)^4 + 3 = (-y)^2$ Replacing x by $-x$ and y by $-y$

$2x^4 + 3 = y^2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

22. Test for symmetry with respect to the x-axis:

$2y^2 = 5x^2 + 12$ Original equation

$2(-y)^2 = 5x^2 + 12$ Replacing y by $-y$

$2y^2 = 5x^2 + 12$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$2y^2 = 5x^2 + 12$ Original equation

$2y^2 = 5(-x)^2 + 12$ Replacing x by $-x$

$2y^2 = 5x^2 + 12$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$2y^2 = 5x^2 + 12$ Original equation

$2(-y)^2 = 5(-x)^2 + 12$ Replacing x by $-x$ and y by $-y$

$2y^2 = 5x^2 + 12$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

23. Test for symmetry with respect to the x-axis:

$3y^3 = 4x^3 + 2$ Original equation

$3(-y)^3 = 4x^3 + 2$ Replacing y by $-y$

$-3y^3 = 4x^3 + 2$ Simplifying

$3y^3 = -4x^3 - 2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$3y^3 = 4x^3 + 2$ Original equation

$3y^3 = 4(-x)^3 + 2$ Replacing x by $-x$

$3y^3 = -4x^3 + 2$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$3y^3 = 4x^3 + 2$ Original equation

$3(-y)^3 = 4(-x)^3 + 2$ Replacing x by $-x$ and y by $-y$

$-3y^3 = -4x^3 + 2$ Simplifying

$3y^3 = 4x^3 - 2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

24. Test for symmetry with respect to the x-axis:

$3x = |y|$ Original equation

$3x = |-y|$ Replacing y by $-y$

$3x = |y|$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$3x = |y|$ Original equation

$3(-x) = |y|$ Replacing x by $-x$

$-3x = |y|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$3x = |y|$ Original equation

$3(-x) = |-y|$ Replacing x by $-x$ and y by $-y$

$-3x = |y|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

25. Test for symmetry with respect to the x-axis:

$xy = 12$ Original equation

$x(-y) = 12$ Replacing y by $-y$

$-xy = 12$ Simplifying

$xy = -12$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$xy = 12$ Original equation

$-xy = 12$ Replacing x by $-x$

$xy = -12$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$xy = 12$ Original equation

$-x(-y) = 12$ Replacing x by $-x$ and y by $-y$

$xy = 12$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

26. Test for symmetry with respect to the x-axis:

$xy - x^2 = 3$ Original equation

$x(-y) - x^2 = 3$ Replacing y by $-y$

$xy + x^2 = -3$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$xy - x^2 = 3$ Original equation

$-xy - (-x)^2 = 3$ Replacing x by $-x$

$xy + x^2 = -3$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$xy - x^2 = 3$ Original equation

$-x(-y) - (-x)^2 = 3$ Replacing x by $-x$ and y by $-y$

$xy - x^2 = 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

27. x-axis: Replace y with $-y$; $(-5, -6)$

y-axis: Replace x with $-x$; $(5, 6)$

Origin: Replace x with $-x$ and y with $-y$; $(5, -6)$

28. x-axis: Replace y with $-y$; $\left(\frac{7}{2}, 0\right)$

y-axis: Replace x with $-x$; $\left(-\frac{7}{2}, 0\right)$

Origin: Replace x with $-x$ and y with $-y$; $\left(-\frac{7}{2}, 0\right)$

29. x-axis: Replace y with $-y$; $(-10, 7)$

y-axis: Replace x with $-x$; $(10, -7)$

Origin: Replace x with $-x$ and y with $-y$; $(10, 7)$

30. x-axis: Replace y with $-y$; $\left(1, -\frac{3}{8}\right)$

y-axis: Replace x with $-x$; $\left(-1, \frac{3}{8}\right)$

Origin: Replace x with $-x$ and y with $-y$; $\left(-1, -\frac{3}{8}\right)$

31. x-axis: Replace y with $-y$; $(0, 4)$

y-axis: Replace x with $-x$; $(0, -4)$

Origin: Replace x with $-x$ and y with $-y$; $(0, 4)$

34. The graph is symmetric with respect to the y-axis, so the function is even.

35. The graph is symmetric with respect to the origin, so the function is odd.

36. The graph is not symmetric with respect to either the y-axis or the origin, so the function is neither even nor odd.

37. The graph is not symmetric with respect to either the y-axis or the origin, so the function is neither even nor odd.

38. The graph is not symmetric with respect to either the y-axis or the origin, so the function is neither even nor odd.

39. $f(x) = -3x^3 + 2x$

$f(-x) = -3(-x)^3 + 2(-x) = 3x^3 - 2x$

$-f(x) = -(-3x^3 + 2x) = 3x^3 - 2x$

$f(-x) = -f(x)$, so f is odd.

40. $f(x) = 7x^3 + 4x - 2$

$f(-x) = 7(-x)^3 + 4(-x) - 2 = -7x^3 - 4x - 2$

$-f(x) = -(7x^3 + 4x - 2) = -7x^3 - 4x + 2$

$f(x) \neq f(-x)$, so f is not even.

$f(-x) \neq -f(x)$, so f is not odd.

Thus, $f(x) = 7x^3 + 4x - 2$ is neither even nor odd.

41. $f(x) = 5x^2 + 2x^4 - 1$

$f(-x) = 5(-x)^2 + 2(-x)^4 - 1 = 5x^2 + 2x^4 - 1$

$f(x) = f(-x)$, so f is even.

42. $f(x) = x + \frac{1}{x}$

$f(-x) = -x + \frac{1}{-x} = -x - \frac{1}{x}$

$-f(x) = -\left(x + \frac{1}{x}\right) = -x - \frac{1}{x}$

$f(-x) = -f(x)$, so f is odd.

43. $f(x) = x^{17}$

$f(-x) = (-x)^{17} = -x^{17}$

$-f(x) = -x^{17}$

$f(-x) = -f(x)$, so f is odd.

44. $f(x) = \sqrt[3]{x}$

$f(-x) = \sqrt[3]{-x} = -\sqrt[3]{x}$

$-f(x) = -\sqrt[3]{x}$

$f(-x) = -f(x)$, so f is odd.

45. $f(x) = x - |x|$

32. x-axis: Replace y with $-y$; $(8, 3)$

y-axis: Replace x with $-x$; $(-8, -3)$

Origin: Replace x with $-x$ and y with $-y$; $(-8, 3)$

33. The graph is symmetric with respect to the y-axis, so the function is even.

$f(-x) = (-x) - |(-x)| = -x - |x|$

$-f(x) = -(x - |x|) = -x + |x|$ f

$(x) \div= f(-x)$, so f is not even.

$f(-x)$ $-f(x)$, so f is not odd.

Thus, $f(x) = x - |x|$ is neither even nor odd.

46. $f(x) = \frac{1}{x^2}$

$f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2}$

$f(x) = f(-x)$, so f is even.

47. $f(x) = 8$

$f(-x) = 8$

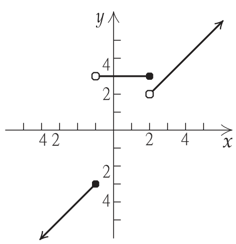
$f(x) = f(-x)$, so f is even.

48. $f(x) = \sqrt{x^2 + 1}$

$f(-x) = \sqrt{(-x)^2 + 1} = \sqrt{x^2 + 1}$

$f(x) = f(-x)$, so f is even.

49.



50. Let v = the number of volunteers from the University of Wisconsin - Madison. Then $v + 464$ = the number of volunteers from the University of California - Berkeley.

Solve: $v + (v + 464) = 6688$

$v = 3112$, so there were 3112 volunteers from the University of Wisconsin - Madison and 3112 + 464, or 3576 volunteers from the University of California - Berkeley.

51. $f(x) = x\sqrt{10 - x^2}$

$f(-x) = -x\sqrt{10 - (-x)^2} = -x\sqrt{10 - x^2}$

$-f(x) = -x\sqrt{10 - x^2}$

Since $f(-x) = -f(x)$, f is odd.

52. $f(x) = \frac{x^2 + 1}{x^3 + 1}$

$f(-x) = \frac{(-x)^2 + 1}{(-x)^3 + 1} = \frac{x^2 + 1}{-x^3 + 1}$

$-f(x) = -\frac{x^2 + 1}{x^3 + 1}$

Since $f(x) \neq f(-x)$, f is not even.

Since $f(-x) \neq -f(x)$, f is not odd.

Thus, $f(x) = \frac{x^2 + 1}{x^3 + 1}$ is neither even nor odd.

53. If the graph were folded on the x-axis, the parts above and below the x-axis would coincide, so the graph is symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left

54. If the graph were folded on the x-axis, the parts above and below the x-axis would not coincide, so the graph is not symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left

and right of the y-axis would not coincide, so the graph is not symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would coincide with the original graph, so it is symmetric with respect to the origin.

55. See the answer section in the text.

56. $O(-x) = \frac{f(-x) - f(-(-x))}{2} = \frac{f(-x) - f(x)}{2}$,

$-O(x) = -\frac{f(x) - f(-x)}{2} = \frac{f(-x) - f(x)}{2}$. Thus,

$O(-x) = -O(x)$ and O is odd.

57. a), b) See the answer section in the text.

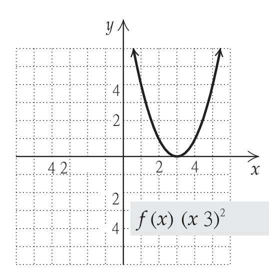
58. Let $f(x) = g(x) = x$. Now f and g are odd functions, but $(fg)(x) = x^2 = (fg)(-x)$. Thus, the product is even, so the statement is false.

59. Let $f(x)$ and $g(x)$ be even functions. Then by definition, $f(x) = f(-x)$ and $g(x) = g(-x)$. Thus, $(f+g)(x) = f(x) + g(x) = f(-x) + g(-x) = (f+g)(-x)$ and $f + g$ is even. The statement is true.

60. Let $f(x)$ be an even function, and let $g(x)$ be an odd function. By definition $f(x) = f(-x)$ and $g(-x) = -g(x)$, or $g(x) = -g(-x)$. Then $fg(x) = f(x) \cdot g(x) = f(-x) \cdot [-g(-x)] = -f(-x) \cdot g(-x) = -fg(-x)$, and fg is odd. The statement is true.

Exercise Set 2.5

1. Shift the graph of $f(x) = x^2$ right 3 units.



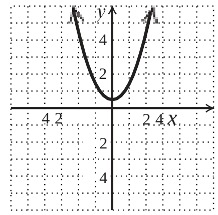
2. Shift the graph of $g(x) = x$ up $\frac{1}{2}$ unit.

and right of the y-axis would not coincide, so the graph is not symmetric with respect to the y-axis.

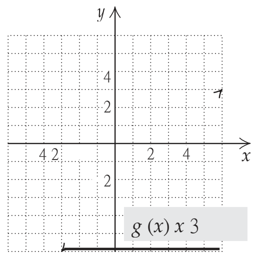
If the graph were rotated 180°, the resulting graph would not coincide with the original graph, so it is not symmetric with respect to the origin.

2 2

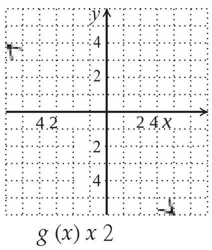
$g(x)\ x^{2}\ 1$



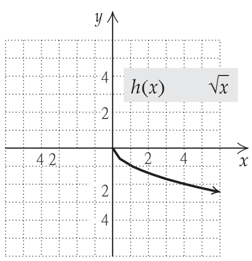
3. Shift the graph of $g(x) = x$ down 3 units.



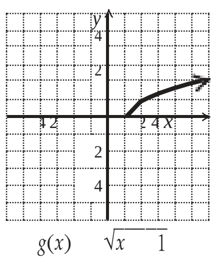
4. Reflect the graph of $g(x) = x$ across the x-axis and then shift it down 2 units.



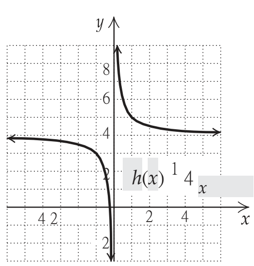
5. Reflect the graph of $h(x) = \sqrt{x}$ across the x-axis.



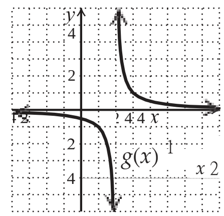
6. Shift the graph of $g(x) = \sqrt{x}$ right 1 unit.



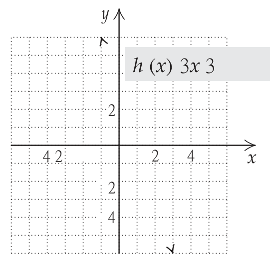
7. Shift the graph of $h(x) = \frac{1}{x}$ up 4 units.



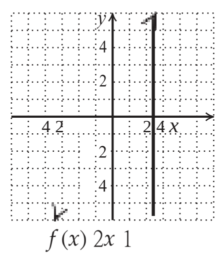
8. Shift the graph of $g(x) = \frac{1}{x}$ right 2 units.



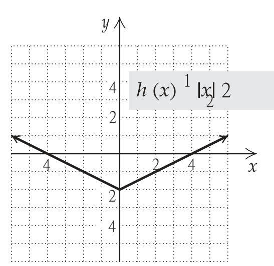
9. First stretch the graph of $h(x) = x$ vertically by multiplying each y-coordinate by 3. Then reflect it across the x-axis and shift it up 3 units.



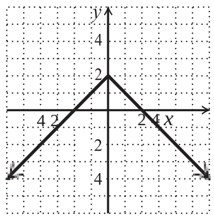
10. First stretch the graph of $f(x) = x$ vertically by multiplying each y-coordinate by 2. Then shift it up 1 unit.



11. First shrink the graph of $h(x) = |x|$ vertically by multiplying each y-coordinate by $\frac{1}{2}$. Then shift it down 2 units.

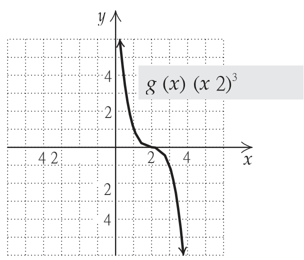


12. Reflect the graph of $g(x) = |x|$ across the x-axis and shift it up 2 units.

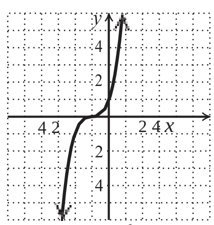


$g(x)\ x\ 2$

13. Shift the graph of $g(x) = x^3$ right 2 units and reflect it across the x-axis.

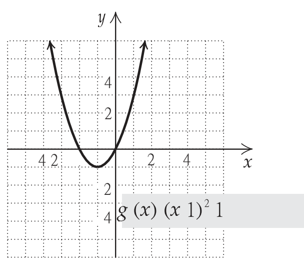


14. Shift the graph of $f(x) = x^3$ left 1 unit.

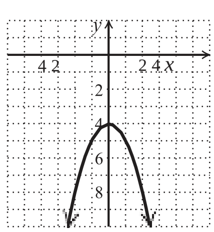


$f(x)\ (x\ 1)^3$

15. Shift the graph of $g(x) = x^2$ left 1 unit and down 1 unit.

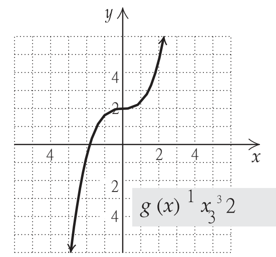


16. Reflect the graph of $h(x) = x^2$ across the x-axis and down 4 units.

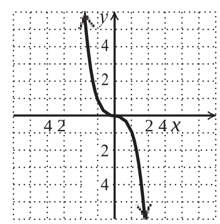


$h(x)\ x^2\ 4$

17. First shrink the graph of $g(x) = x^3$ vertically by multiplying each y-coordinate by $\frac{1}{3}$. Then shift it up 2 units.

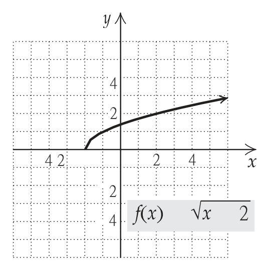


18. Reflect the graph of $h(x) = x^3$ across the y-axis.

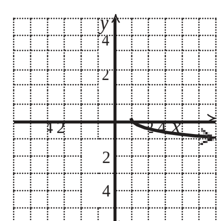


$h(x)\ (\ x\ )^3$

19. Shift the graph of $f(x) = \sqrt{x}$ left 2 units.

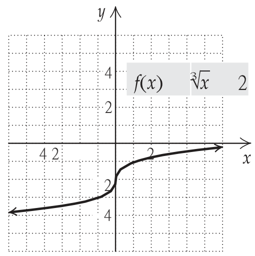


20. First shift the graph of $f(x) = \sqrt{x}$ right 1 unit. Shrink it vertically by multiplying each y-coordinate by $\frac{1}{2}$ and then reflect it across the x-axis.

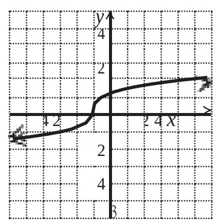


$f(x)\ \ \frac{1}{2}\sqrt{x\ \ 1}$

21. Shift the graph of $f(x) = \sqrt[3]{x}$ down 2 units.



22. Shift the graph of $h(x) = \sqrt[3]{x}$ left 1 unit.



$h(x) \quad \sqrt{x \quad 1}$

23. Think of the graph of $f(x) = |x|$. Since $g(x) = f(3x)$, the graph of $g(x) = |3x|$ is the graph of $f(x) = |x|$ shrunk horizontally by dividing each x-coordinate by 3 or multiplying each x-coordinate by $\frac{1}{3}$.

24. Think of the graph of $g(x) = \sqrt[3]{x}$. Since $f(x) = \frac{1}{2}g(x)$, the graph of $f(x) = \frac{1}{2}\sqrt[3]{x}$ is the graph of $g(x) = \sqrt[3]{x}$ shrunk vertically by multiplying each y-coordinate by $\frac{1}{2}$.

25. Think of the graph of $f(x) = \frac{1}{x}$. Since $h(x) = 2f(x)$, the graph of $h(x) = \frac{2}{x}$ is the graph of $f(x) = \frac{1}{x}$ stretched vertically by multiplying each y-coordinate by 2.

26. Think of the graph of $g(x) = |x|$. Since $f(x) = g(x-3) - 4$, the graph of $f(x) = |x-3| - 4$ is the graph of $g(x) = |x|$ shifted right 3 units and down 4 units.

27. Think of the graph of $g(x) = \sqrt{x}$. Since $f(x) = 3g(x) - 5$, the graph of $f(x) = 3\sqrt{x} - 5$ is the graph of $g(x) = \sqrt{x}$ stretched vertically by multiplying each y-coordinate by 3 and then shifted down 5 units.

28. Think of the graph of $g(x) = \frac{1}{x}$. Since $f(x) = 5 - \frac{1}{x}$, or $f(x) = -g(x) + 5$, the graph of $f(x) = 5 - \frac{1}{x}$ is the graph of $g(x) = \frac{1}{x}$ reflected across the x-axis and then shifted up 5 units.

30. Think of the graph of $g(x) = x^3$. Since $f(x) = \frac{2}{3}g(x) - 4$, the graph of $f(x) = \frac{2}{3}x^3 - 4$ is the graph of $g(x) = x^3$ shrunk vertically by multiplying each y-coordinate by $\frac{2}{3}$ and then shifted down 4 units.

31. Think of the graph of $g(x) = x^2$. Since $f(x) = -\frac{1}{4}g(x-5)$, the graph of $f(x) = -\frac{1}{4}(x-5)^2$ is the graph of $g(x) = x^2$ shifted right 5 units, shrunk vertically by multiplying each y-coordinate by $\frac{1}{4}$, and reflected across the x-axis.

32. Think of the graph of $g(x) = x^3$. Since $f(x) = g(-x) - 5$, the graph of $f(x) = (-x)^3 - 5$ is the graph of $g(x) = x^3$ reflected across the y-axis and shifted down 5 units.

33. Think of the graph of $g(x) = \frac{1}{x}$. Since $f(x) = g(x+3) + 2$, the graph of $f(x) = \frac{1}{x+3} + 2$ is the graph of $g(x) = \frac{1}{x}$ shifted left 3 units and up 2 units.

34. Think of the graph of $f(x) = \sqrt{x}$. Since $g(x) = f(-x) + 5$, the graph of $g(x) = \sqrt{-x} + 5$ is the graph of $f(x) = \sqrt{x}$ reflected across the y-axis and shifted up 5 units.

35. Think of the graph of $f(x) = x^2$. Since $h(x) = -f(x-3) + 5$, the graph of $h(x) = -(x-3)^2 + 5$ is the graph of $f(x) = x^2$ shifted right 3 units, reflected across the x-axis, and shifted up 5 units.

36. Think of the graph of $g(x) = x^2$. Since $f(x) = 3g(x+4) - 3$, the graph of $f(x) = 3(x+4)^2 - 3$ is the graph of $g(x) = x^2$ shifted left 4 units, stretched vertically by multiplying each y-coordinate by 3, and then shifted down 3 units.

37. The graph of $y = g(x)$ is the graph of $y = f(x)$ shrunk vertically by a factor of $\frac{1}{2}$. Multiply the y-coordinate by $\frac{1}{2}$: $(-12, 2)$.

38. The graph of $y = g(x)$ is the graph of $y = f(x)$ shifted right 2 units. Add 2 to the x-coordinate: $(-10, 4)$.

39. The graph of $y = g(x)$ is the graph of $y = f(x)$ reflected across the y-axis, so we reflect the point across the y-axis: $(12, 4)$.

40. The graph of $y = g(x)$ is the graph of $y = f(x)$ shrunk horizontally. The x-coordinates of $y = g(x)$ are $\frac{1}{4}$ the corresponding x-coordinates of $y = f(x)$, so we divide the x-coordinate by 4 or multiply it by $\frac{1}{4}$: $(-3, 4)$.

41. The graph of $y = g(x)$ is the graph of $y = f(x)$ shifted dow n 2 units. Subtra ct 2 from the y-coordinate: $(-12,2)$.

29. Think of the graph of $f(x) = |x|$. Since $g(x) = \frac{1}{3}x - 4 = f\left(\frac{1}{3}x\right) - 4$, the graph of $g(x) = \frac{1}{3}x - 4$ is the graph of $f(x) = |x|$ stretched horizontally by multiplying each x-coordinate by 3 and then shifted down 4 units.

42. The graph of $y = g(x)$ is the graph of $y = f(x)$ stretched horizontally. The x-coordinates of $y = g(x)$ are twice the corresponding x-coordinates of $y = f(x)$, so we multiply the x-coordinate by 2 or divide it by $\frac{1}{2}$: $(-24, 4)$.

43. The graph of $y = g(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 4. Multiply the y-coordinate by 4: $(-12, 16)$.

44. The graph of $y = g(x)$ is the graph $y = f(x)$ reflected across the x-axis. Reflect the point across the x-axis: $(-12, -4)$.

45. $g(x) = x^2 + 4$ is the function $f(x) = x^2 + 3$ shifted up 1 unit, so $g(x) = f(x) + 1$. Answer B is correct.

46. If we substitute $3x$ for x in f, we get $9x^2 + 3$, so $g(x) = f(3x)$. Answer D is correct.

47. If we substitute $x - 2$ for x in f, we get $(x-2)^3 + 3$, so $g(x) = f(x-2)$. Answer A is correct.

48. If we multiply $x^2 + 3$ by 2, we get $2x^2 + 6$, so $g(x) = 2f(x)$. Answer C is correct.

49. Shape: $h(x) = x^2$

Turn $h(x)$ upside-down (that is, reflect it across the x-axis): $g(x) = -h(x) = -x^2$

Shift $g(x)$ right 8 units: $f(x) = g(x-8) = -(x-8)^2$

50. Shape: $h(x) = \sqrt{x}$

Shift $h(x)$ left 6 units: $g(x) = h(x+6) = \sqrt{x+6}$

Shift $g(x)$ down 5 units: $f(x) = g(x) - 5 = \sqrt{x+6} - 5$

51. Shape: $h(x) = |x|$

Shift $h(x)$ left 7 units: $g(x) = h(x+7) = |x+7|$

Shift $g(x)$ up 2 units: $f(x) = g(x) + 2 = |x+7| + 2$

52. Shape: $h(x) = x^3$

Turn $h(x)$ upside-down (that is, reflect it across the x-axis): $g(x) = -h(x) = -x^3$

Shift $g(x)$ right 5 units: $f(x) = g(x-5) = -(x-5)^3$

53. Shape: $h(x) = \frac{1}{x}$

Shrink $h(x)$ vertically by a factor of $\frac{1}{2}$ (that is, multiply each function value by $\frac{1}{2}$):

$g(x) = \frac{1}{2}h(x) = \frac{1}{2}\cdot\frac{1}{x}$, or $\frac{1}{2x}$

Shift $g(x)$ down 3 units: $f(x) = g(x) - 3 = \frac{1}{2x} - 3$

54. Shape: $h(x) = x^2$

Shift $h(x)$ right 6 units: $g(x) = h(x-6) = (x-6)^2$

Shift $g(x)$ up 2 units: $f(x) = g(x) + 2 = (x-6)^2 + 2$

55. Shape: $m(x) = x^2$

Turn $m(x)$ upside-down (that is, reflect it across the x-axis): $h(x) = -m(x) = -x^2$

Shift $h(x)$ right 3 units: $g(x) = h(x-3) = -(x-3)^2$

Shift $g(x)$ up 4 units: $f(x) = g(x) + 4 = -(x-3)^2 + 4$

56. Shape: $h(x) = |x|$

Stretch $h(x)$ horizontally by a factor of 2 (that is, multiply each x-value by $\frac{1}{2}$): $g(x) = h\left(\frac{1}{2}x\right) = \left|\frac{1}{2}x\right|$

Shift $g(x)$ down 5 units: $f(x) = g(x) - 5 = \left|\frac{1}{2}x\right| - 5$

57. Shape: $m(x) = \sqrt{x}$

Reflect $m(x)$ across the y-axis: $h(x) = m(-x) = \sqrt{-x}$

Shift $h(x)$ left 2 units: $g(x) = h(x+2) = \sqrt{-(x+2)}$

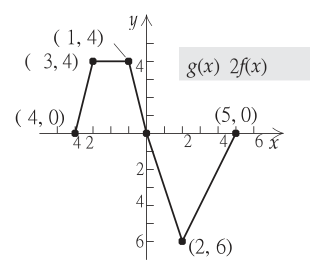
Shift $g(x)$ down 1 unit: $f(x) = g(x) - 1 = \sqrt{-(x+2)} - 1$

58. Shape: $h(x) = \frac{1}{x}$

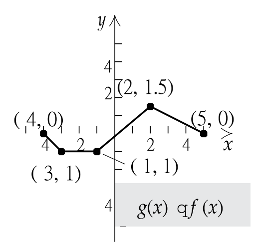
Reflect $h(x)$ across the x-axis: $g(x) = -h(x) = -\frac{1}{x}$

Shift $g(x)$ up 1 unit: $f(x) = g(x) + 1 = -\frac{1}{x} + 1$

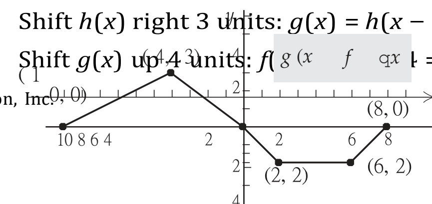
59. Each y-coordinate is multiplied by -2. We plot and connect $(-4, 0)$, $(-3, 4)$, $(-1, 4)$, $(2, -6)$, and $(5, 0)$.



60. Each y-coordinate is multiplied by $\frac{1}{2}$. We plot and connect $(-4, 0)$, $(-3, -1)$, $(-1, -1)$, $(2, 1.5)$, and $(5, 0)$.

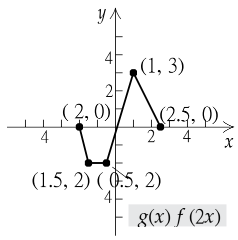


61. The graph is reflected across the y-axis and stretched horizontally by a factor of 2. That is, each x-coordinate is multiplied by -2 or divided by $-\frac{1}{2}$. We plot and con-

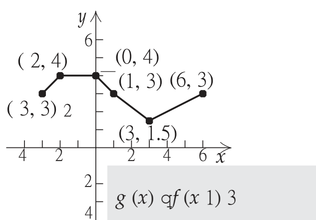


nect $(8, 0)$, $(6, -2)$, $(2, -2)$, $(-4, 3)$, and $(-10, 0)$.

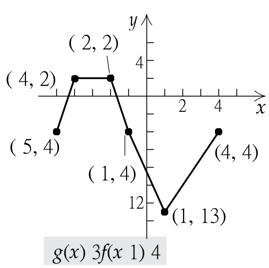
62. The graph is shrunk horizontally by a factor of 2. That is, each x-coordinate is divided by 2 or multiplied by $\frac{1}{2}$. We plot and connect $(-2, 0)$, $(-1.5, -2)$, $(-0.5, -2)$, $(1, 3)$, and $(2.5, 0)$.



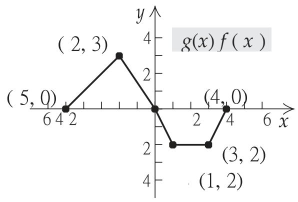
63. The graph is shifted right 1 unit so each x-coordinate is increased by 1. The graph is also reflected across the x-axis, shrunk vertically by a factor of 2, and shifted up 3 units. Thus, each y-coordinate is multiplied by $-\frac{1}{2}$ and then increased by 3. We plot and connect $(-3, 3)$, $(-2, 4)$, $(0, 4)$, $(3, 1.5)$, and $(6, 3)$.



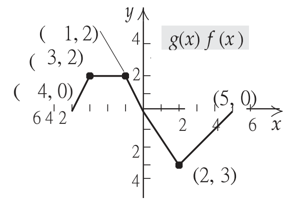
64. The graph is shifted left 1 unit so each x-coordinate is decreased by 1. The graph is also reflected across the x-axis, stretched vertically by a factor of 3, and shifted down 4 units. Thus, each y-coordinate is multiplied by -3 and then decreased by 4. We plot and connect $(-5, -4)$, $(-4, 2)$, $(-2, 2)$, $(1, -13)$, and $(4, -4)$.



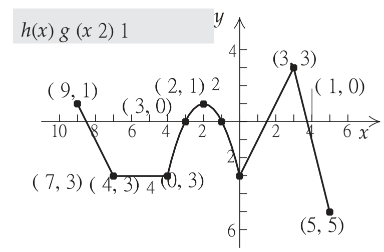
65. The graph is reflected across the y-axis so each x-coordinate is replaced by its opposite.



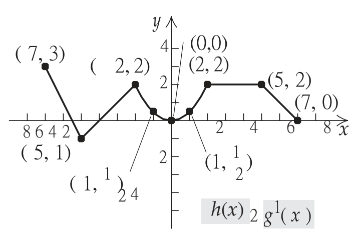
66. The graph is reflected across the x-axis so each y-coordinate is replaced by its opposite.



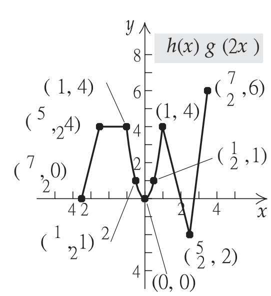
67. The graph is shifted left 2 units so each x-coordinate is decreased by 2. It is also reflected across the x-axis so each y-coordinate is replaced with its opposite. In addition, the graph is shifted up 1 unit, so each y-coordinate is then increased by 1.



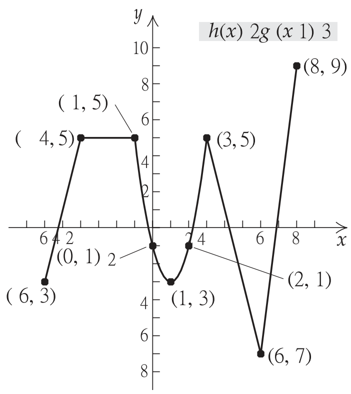
68. The graph is reflected across the y-axis so each x-coordinate is replaced with its opposite. It is also shrunk vertically by a factor of $\frac{1}{2}$, so each y-coordinate is multiplied by $\frac{1}{2}$ (or divided by 2).



69. The graph is shrunk horizontally. The x-coordinates of $y = h(x)$ are one-half the corresponding x-coordinates of $y = g(x)$.



70. The graph is shifted right 1 unit, so each x-coordinate is increased by 1. It is also stretched vertically by a factor of 2, so each y-coordinate is multiplied by 2 or divided by $\frac{1}{2}$. In addition, the graph is shifted down 3 units, so each y-coordinate is decreased by 3.



71. $g(x) = f(-x)+3$

The graph of $g(x)$ is the graph of $f(x)$ reflected across the y-axis and shifted up 3 units. This is graph (f).

72. $g(x) = f(x)+3$

The graph of $g(x)$ is the graph of $f(x)$ shifted up 3 units. This is graph (h).

73. $g(x) = -f(x)+3$

The graph of $g(x)$ is the graph of $f(x)$ reflected across the x-axis and shifted up 3 units. This is graph (f).

74. $g(x) = -f(-x)$

The graph of $g(x)$ is the graph of $f(x)$ reflected across the x-axis and the y-axis. This is graph (a).

75. $g(x) = \frac{1}{3}f(x-2)$

The graph of $g(x)$ is the graph of $f(x)$ shrunk vertically by a factor of 3 that is, each y-coordinate is multiplied by $\frac{1}{3}$ and then shifted right 2 units. This is graph (d).

76. $g(x) = \frac{1}{3}f(x) - 3$

The graph of $g(x)$ is the graph of $f(x)$ shrunk vertically by a factor of 3 that is, each y-coordinate is multiplied by $\frac{1}{3}$ and then shifted down 3 units. This is graph (e).

77. $g(x) = \frac{1}{3}f(x+2)$

The graph of $g(x)$ is the graph of $f(x)$ shrunk vertically by a factor of 3 that is, each y-coordinate is multiplied by $\frac{1}{3}$ and then shifted left 2 units. This is graph (c).

78. $g(x) = -f(x+2)$

The graph of $g(x)$ is the graph $f(x)$ reflected across the x-axis and shifted left 2 units. This is graph (b).

80. $f(-x) = \frac{1}{4}(-x)^4 + \frac{1}{5}(-x)^3 - 81(-x)^2 - 17 = \frac{1}{4}x^4 - \frac{1}{5}x^3 - 81x^2 - 17 = g(x)$

81. The graph of $f(x) = x^3 - 3x^2$ is shifted up 2 units. A formula for the transformed function is $g(x) = f(x) + 2$, or $g(x) = x^3 - 3x^2 + 2$.

82. Each y-coordinate of the graph of $f(x) = x^3 - 3x^2$ is multiplied by $\frac{1}{2}$. A formula for the transformed function is $h(x) = \frac{1}{2}f(x)$, or $h(x) = \frac{1}{2}(x^3 - 3x^2)$.

83. The graph of $f(x) = x^3 - 3x^2$ is shifted left 1 unit. A formula for the transformed function is $k(x) = f(x+1)$, or $k(x) = (x+1)^3 - 3(x+1)^2$.

84. The graph of $f(x) = x^3 - 3x^2$ is shifted right 2 units and up 1 unit. A formula for the transformed function is $t(x) = f(x-2) + 1$, or $t(x) = (x-2)^3 - 3(x-2)^2 + 1$.

85. Test for symmetry with respect to the x-axis.

$y = 3x^4 - 3$ Original equation

$-y = 3x^4 - 3$ Replacing y by $-y$

$y = -3x^4 + 3$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis.

$y = 3x^4 - 3$ Original equation

$y = 3(-x)^4 - 3$ Replacing x by $-x$

$y = 3x^4 - 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$y = 3x^4 - 3$

$-y = 3(-x)^4 - 3$ Replacing x by $-x$ and y by $-y$

$-y = 3x^4 - 3$

$y = -3x^4 + 3$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

86. Test for symmetry with respect to the x-axis.

$y^2 = x$ Original equation

$(-y)^2 = x$ Replacing y by $-y$

$y^2 = x$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

79. $f(-x) = 2(-x)^4 - 35(-x)^3 + 3(-x) - 5 = 2x^4 + 35x^3 - 3x - 5 = g(x)$

Test for symmetry with respect to the y-axis: $y^2 = x$ Original equation

$y^2 = -x$ Replacing x by $-x$

Thelast equation is not equivalent to the original equation,so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$y^2 = x$ Original equation
$(-y)^2 = -x$ Replacing x by $-x$ and y by $-y$
$y^2 = -x$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

87. Test for symmetry with respect to the x-axis:

$2x - 5y = 0$ Original equation
$2x - 5(-y) = 0$ Replacing y by $-y$
$2x + 5y = 0$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$2x - 5y = 0$ Original equation
$2(-x) - 5y = 0$ Replacing x by $-x$
$-2x - 5y = 0$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$2x - 5y = 0$ Original equation
$2(-x) - 5(-y) = 0$ Replacing x by $-x$ and y by $-y$

$-2x + 5y = 0$
$2x - 5y = 0$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

88. Let w = the average annual wages of a 64-year-old person with only a high school diploma.

Solve: $w + 0.537w = 67,735$

$w \approx \$44,070$

89. ***Familiarize***. Let c = the cost of knee replacement surgery in India in 2014.

Translate.

Cost in U.S.	was	$4000	more than	four times	cost in India
34,000	=	4000	+	4	· c

Carry out. We solve the equation.

$34,000 = 4000 + 4 \cdot c$
$30,000 = 4c$
$7500 = c$

Check. $4000 more than 4 times $7500 is $4000+4·$7500 = $4000+$30,000 = $34,000. The answer checks.

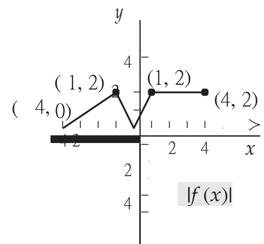
State. In 2014, the cost of knee replacement surgery in India was $7500.

90. Let c = the number of students Canada sent to the U.S. to study in universities in 2013-2014. Then $c + 25,615$ = the number of students Saudi Arabia sent.

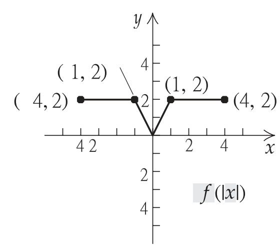
Solve: $c + (c + 25,615) = 82,223$

$c = 28,304$ students, and $c + 25,615 = 53,919$ students.

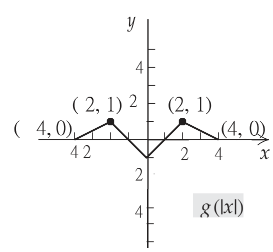
91. Each point for which $f(x) < 0$ is reflected across the x-axis.



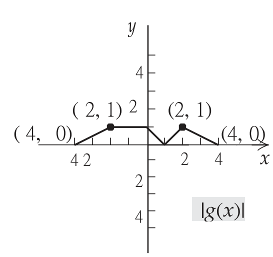
92. The graph of $y = f(|x|)$ consists of the points of $y = f(x)$ for which $x \geq 0$ along with their reflections across the y-axis.



93. The graph of $y = g(|x|)$ consists of the points of $y = g(x)$ for which $x \geq 0$ along with their reflections across the y-axis.



94. Each point for which $g(x) < 0$ is reflected across the x-axis.



95. Think of the graph of $g(x) = \text{int}(x)$. Since

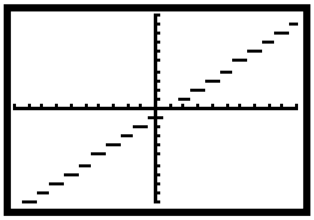
$f(x) = g\left(x - 1\right)$, the graph of $f(x) = \text{int}\left(x - 1\right)$ is the

2

graph of $g(x)$ = int(x) shifted right $\frac{1}{2}$ unit. The domain

2

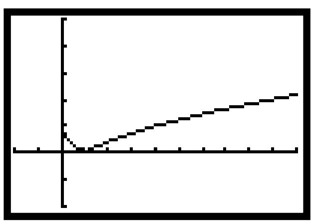
is the set of all real numbers; the range is the set of all integers.



96. This function can be defined piecewise as follows:

$$f(x) = \begin{cases} -(\sqrt{x} - 1), & \text{for } 0 \le x < 1, \\ \sqrt{x} - 1, & \text{for } x \ge 1. \end{cases}$$

Think of the graph of $g(x) = \sqrt{x}$. First shift it down 1 unit. Then reflect across the x-axis the portion of the graph for which $0 < x < 1$. The domain and range are both the set of nonnegative real numbers, or $[0, \infty)$.



97. On the graph of $y = 2f(x)$ each y-coordinate of $y = f(x)$ is multiplied by 2, so $(3, 4 \cdot 2)$, or $(3, 8)$ is on the transformed graph.

On the graph of $y = 2 + f(x)$, each y-coordinate of $y = f(x)$ is increased by 2 (shifted up 2 units), so $(3, 4 + 2)$, or $(3, 6)$ is on the transformed graph.

On the graph of $y = f(2x)$, each x-coordinate of $y = f(x)$ is multiplied by $\frac{1}{2}$ (or divided by 2), so $\left(\frac{1}{2} \cdot 3, 4\right)$, or $\left(\frac{3}{2}, 4\right)$ is on the transformed graph.

98. Using a graphing calculator we find that the zeros are -2.582, 0, and 2.582.

The graph of $y = f(x-3)$ is the graph of $y = f(x)$ shifted right 3 units. Thus we shift each of the zeros of $f(x)$ 3 units right to find the zeros of $f(x-3)$. They are $-2.582 + 3$, or 0.418; $0 + 3$, or 3; and $2.582 + 3$, or 5.582.

The graph of $y = f(x+8)$ is the graph of $y = f(x)$ shifted 8 units left. Thus we shift each of the zeros of $f(x)$ 8 units left to find the zeros of $f(x+8)$. They are $-2.582 - 8$, or -10.582; $0 - 8$, or -8; and $2.582 - 8$, or -5.418.

Exercise Set 2.6

1. $y = kx$

$54 = k \cdot 12$

$\frac{54}{12} = k$

$\frac{9}{2} = k$

The variation constant is $\frac{9}{2}$, or 4.5. The equation of variation is $y = \frac{9}{2}x$, or $y = 4.5x$.

2. $y = kx$

$0.1 = k(0.2)$

$\frac{1}{2} = k$ Variation constant

Equation of variation: $y = \frac{1}{2}x$, or $y = 0.5x$.

3. $y = \frac{k}{x}$

$3 = \frac{k}{12}$

$36 = k$

The variation constant is 36. The equation of variation is $y = \frac{36}{x}$.

4. $y = \frac{k}{x}$

$12 = \frac{k}{5}$

$60 = k$ Variation constant

Equation of variation: $y = \frac{60}{x}$

5. $y = kx$

$1 = k \cdot \frac{1}{4}$

$4 = k$

The variation constant is 4. The equation of variation is $y = 4x$.

6. $y = \frac{k}{x}$

$0.1 = \frac{k}{0.5}$

$0.05 = k$ Variation constant

Equation of variation: $y = \frac{0.05}{x}$

7. $y = \frac{k}{x}$

$32 = \frac{k}{\frac{1}{8}}$

$\frac{1}{8} \cdot 32 = k$

$4 = k$

The variation constant is 4. The equation of variation is $y = \frac{4}{x}$.

$y = \frac{.}{x}$

8. $y = kx$

$_{12} = k$, or $k = {}_{2}$

$3 = k \cdot 33$

$\frac{1}{11} = k$ Variation constant

Equation of variation: $y = \frac{1}{11}x$

9.
$y = kx$

$\frac{3}{4} = k \cdot 2$

$\frac{1}{2} \cdot \frac{3}{4} = k$

$\frac{3}{8} = k$

The variation constant is $\frac{3}{8}$. The equation of variation is $y = \frac{3}{8}x$.

10. $y = \frac{k}{x}$

$\frac{1}{5} = \frac{k}{35}$

$7 = k$ Variation constant

Equation of variation: $y = \frac{7}{x}$

11. $y = \frac{k}{x}$

$1.8 = \frac{k}{0.3}$

$0.54 = k$

The variation constant is 0.54. The equation of variation is $y = \frac{0.54}{x}$.

12. $y = kx$

$0.9 = k(0.4)$

$\frac{9}{4} = k$ Variation constant

Equation of variation: $y = \frac{9}{4}x$, or $y = 2.25x$

13. Let W = the weekly allowance and a = the child's age.

$W = ka$

$5.50 = k \cdot 6$

$\frac{11}{12} = k$

$W = \frac{11}{12}x$

$W = \frac{11}{12} \cdot 9$

$W = \$8.25$

14. Let S = the sales tax and p = the purchase price.

$S = kp$ S varies directly as p.

$7.14 = k \cdot 119$ Substituting

15. $t = \frac{k}{r}$

$5 = \frac{k}{80}$

$400 = r$

$t = \frac{400}{r}$

$t = \frac{400}{70}$

$t = \frac{40}{7}$, or $5\frac{5}{7}$ hr

16. $W = \frac{k}{L}$ W varies inversely as L.

$1200 = \frac{k}{8}$ Substituting

$9600 = k$ Variation constant

$W = \frac{9600}{L}$ Equation of variation

$W = \frac{9600}{14}$ Substituting

$W \approx 686$

A 14-m beam can support about 686 kg.

17. Let F = the number of grams of fat and w = the weight.

$F = kw$ F varies directly as w.

$60 = k \cdot 120$ Substituting

$\frac{60}{120} = k$, or Solving for k

$\frac{1}{2} = k$ Variation constant

$F = \frac{1}{2}w$ Equation of variation

$F = \frac{1}{2} \cdot 180$ Substituting

$F = 90$

The maximum daily fat intake for a person weighing 180 lb is 90 g.

18. $N = kP$

$53 = k \cdot 38{,}333{,}000$ Substituting

$\frac{53}{38{,}333{,}000} = k$ Variation constant

$0.06 = k$ Variation constant

$N = \frac{53}{38,333,000}P$

$N = \frac{53}{38,333,000} \cdot 26,448,000$ Substituting

$S = 0.06p$ Equation of variation

$S = 0.06(21)$ Substituting

$S \approx 1.26$

The sales tax is $1.26.

$N \approx 37$

Texas has 37 representatives.

19. $T = \frac{k}{P}$ T varies inversely as P.

$5 = \frac{k}{7}$ Substituting

$35 = k$ Variation constant

$T = \frac{35}{P}$ Equation of variation

$T = \frac{35}{10}$ Substituting

$T = 3.5$

It will take 10 bricklayers 3.5 hr to complete the job.

20. $t = \frac{k}{r}$

$45 = \frac{k}{600}$

$27,000 = k$

$t = \frac{27,000}{r}$

$t = \frac{27,000}{1000}$

$t = 27$ min

21. $d = km$ d varies directly as m.

$40 = k \cdot 3$ Substituting

$\frac{40}{3} = k$ Variation constant

$d = \frac{40}{3}m$ Equation of variation

$d = \frac{40}{3} \cdot 5 = \frac{200}{3}$ Substituting

$d = 66\frac{2}{3}$

A 5-kg mass will stretch the spring $66\frac{2}{3}$ cm.

22. $f = kF$

$6.3 = k \cdot 150$

$0.042 = k$

$f = 0.042F$

$f = 0.042(80)$

$f = 3.36$

23. $P = \frac{k}{W}$ P varies inversely as W.

$330 = \frac{k}{3.2}$ Substituting

$1056 = k$ Variation constant

$P = \frac{1056}{W}$ Equation of variation

$550 = \frac{1056}{W}$ Substituting

24. $M = kE$ M varies directly as E.

$35.9 = k \cdot 95$ Substituting

$0.378 \approx k$ Variation constant

$M = 0.378E$ Equation of variation

$M = 0.378 \cdot 100$ Substituting

$M = 37.8$

A 100-lb person would weigh about 37.8 lb on Mars.

25. $y = \frac{k}{x^2}$

$0.15 = \frac{k}{(0.1)^2}$ Substituting

$0.15 = \frac{k}{0.01}$

$0.15(0.01) = k$

$0.0015 = k$

The equation of variation is $y = \frac{0.0015}{x^2}$.

26. $y = \frac{k}{x^2}$

$6 = \frac{k}{3^2}$

$54 = k$

$y = \frac{54}{x^2}$

27. $y = kx^2$

$0.15 = k(0.1)^2$ Substituting

$0.15 = 0.01k$

$\frac{0.15}{0.01} = k$

$15 = k$

The equation of variation is $y = 15x^2$.

28. $y = kx^2$

$6 = k \cdot 3^2$

$\frac{2}{3} = k$

$y = \frac{2}{3}x^2$

29. $y = kxz$

$56 = k \cdot 7 \cdot 8$ Substituting

$56 = 56k$

$1 = k$

The equation of variation is $y = xz$.

$$550W = 1056 \quad \text{Multiplying by } W$$

$$W = \frac{1056}{550} \quad \text{Dividing by 550}$$

$$W = 1.92 \quad \text{Simplifying}$$

A tone with a pitch of 550 vibrations per second has a wavelength of 1.92 ft.

30. $y = \dfrac{kx}{z}$

$$4 = \frac{k \cdot 12}{15}$$

$$5 = k$$

$$y = \frac{5x}{z}$$

31. $y = kxz^2$

$105 = k \cdot 14 \cdot 5^2$ Substituting

$105 = 350k$

$\frac{105}{350} = k$

$\frac{3}{10} = k$

The equation of variation is $y = \frac{3}{10}xz^2$.

32. $y = k \cdot \frac{xz}{w}$

$\frac{3}{2} = k \cdot \frac{2 \cdot 3}{4}$

$1 = k$

$y = \frac{xz}{w}$

33. $y = k\frac{xz}{wp}$

$\frac{3}{28} = k\frac{3 \cdot 10}{7 \cdot 8}$ Substituting

$\frac{3}{28} = k \cdot \frac{30}{56}$

$\frac{3}{28} \cdot \frac{56}{30} = k$

$\frac{1}{5} = k$

The equation of variation is $y = \frac{1}{5}\frac{xz}{wp}$, or $\frac{xz}{5wp}$.

34. $y = k \cdot \frac{xz}{w^2}$

$\frac{12}{5} = k \cdot \frac{16 \cdot 3}{5^2}$

$\frac{5}{4} = k$

$y = \frac{5}{4}\frac{xz}{w^2}$, or $\frac{5xz}{4w^2}$

35. $I = \frac{k}{d^2}$

$90 = \frac{k}{5^2}$ Substituting

$90 = \frac{k}{25}$

$2250 = k$

The equation of variation is $I = \frac{2250}{d^2}$.

36. $D = kAv$

$222 = k \cdot 37.8 \cdot 40$

$\frac{37}{252} = k$

$D = \frac{37}{252}Av$

$430 = \frac{37}{252} \cdot 51v$

$v \approx 57.4$ mph

37. $d = kr^2$

$200 = k \cdot 60^2$ Substituting

$200 = 3600k$

$\frac{200}{3600} = k$

$\frac{1}{18} = k$

The equation of variation is $d = \frac{1}{18}r^2$.

Substitute 72 for d and find r.

$72 = \frac{1}{18}r^2$

$1296 = r^2$

$36 = r$

A car can travel 36 mph and still stop in 72 ft.

38. $W = \frac{k}{d^2}$

$220 = \frac{k}{(3978)^2}$

$3,481,386,480 = k$

$W = \frac{3,481,386,480}{d^2}$

$W = \frac{3,481,386,480}{(3978 + 200)^2}$

$W \approx 199$ lb

39. $E = \frac{kR}{I}$

We first find k.

$1.77 = \frac{k \cdot 39}{198.1}$ Substituting

$1.77\left(\frac{198.1}{39}\right) = k$ Multiplying by $\frac{198.1}{39}$

Substitute 40 for I and find d.

$40 = 2250$

$$d^2$$
$$40d^2 = 2250$$
$$d^2 = 56.25$$
$$d = 7.5$$

The distance from 5 m to 7.5 m is 7.5-5, or 2.5 m, so it is 2.5 m further to a point where the intensity is 40 W/m^2.

$$9 \approx k$$

The equation of variation is $E = \frac{9R}{I}$.

Substitute 1.77 for E and 220 for I and solve for R.

$$1.77 = \frac{9R}{220}$$

$$\frac{1.77(220)}{9} = R \qquad \text{Multiplying by } \frac{220}{9}$$

$$43 \approx R$$

Clayton Kershaw would have given up about 43 earned runs if he had pitched 220 innings.

40. $V = \frac{kT}{P}$

$231 = \frac{k \cdot 42}{20}$

$110 = k$

$V = \frac{110T}{P}$

$V = \frac{110 \cdot 30}{15}$

$V = 220 \text{ cm}^3$

41. parallel

42. zero

43. relative minimum

44. odd function

45. inverse variation

46. a) $7xy = 14$

$y = \frac{2}{x}$

Inversely

b) $x - 2y = 12$

$y = \frac{x}{2} - 6$

Neither

c) $-2x + y = 0$

$y = 2x$

Directly

d) $x = \frac{3}{4}y$

$y = \frac{4}{3}x$

Directly

e) $\frac{x}{y} = 2$

$y = \frac{1}{2}x$

Directly

47. Let V represent the volume and p represent the price of a jar of peanut butter.

$V = kp$ — V varies directly as p.

$\pi\left(\frac{3}{2}\right)^2(5) = k(2.89)$ Substituting

$3.89\pi = k$ Variation constant

$V = 3.89\pi p$ Equation of variation

$\pi(1.625)^2(5.5) = 3.89\pi p$ Substituting

$3.73 \approx p$

If cost is directly proportional to volume, the larger jar should cost \$3.73.

Now let W represent the weight and p represent the price of a jar of peanut butter.

$W = kp$

$18 = k(2.89)$ Substituting

$6.23 \approx k$ Variation constant

$W = 6.23p$ Equation of variation

$28 = 6.23p$ Substituting

$4.49 \approx p$

If cost is directly proportional to weight, the larger jar should cost \$4.49. (Answers may vary slightly due to rounding differences.)

48. $Q = \frac{kp^2}{q^3}$

Q varies directly as the square of p and inversely as the cube of q.

49. We are told $A = kd^2$, and we know $A = \pi r^2$ so we have:

$kd^2 = \pi r^2$

$kd^2 = \pi\left(\frac{d}{2}\right)^2$ $\quad r = \frac{d}{2}$

$kd^2 = \frac{\pi d^2}{4}$

$k = \frac{\pi}{4}$ Variation constant

Chapter 2 Review Exercises

1. This statement is true by the definition of the greatest integer function.
2. Thes statement is false. See Example 2(b) in Section 2.3 in the text.
3. The graph of $y = f(x - d)$ is the graph of $y = f(x)$ shifted right d units, so the statement is true.
4. The graph of $y = -f(x)$ is the reflection of the graph of $y = f(x)$ across the x-axis, so the statement is true.
5. a) For x-values from -4 to -2, the y-values increase from 1 to 4. Thus the function is increasing on the

interval $(-4, -2)$.

b) For x-values from 2 to 5, the y-values decrease from4 to 3. Thus the function is decreasing on the inter-val $(2,5)$.

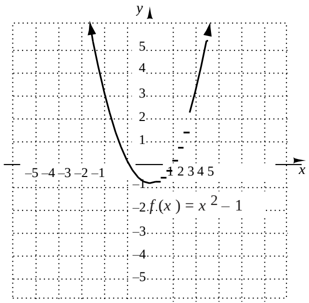
c) For x-values from -2 to 2, y is 4. Thus the function is constant on the interval $(-2, 2)$.

6. a) For x-values from -1 to 0, the y-values increase from3 to 4. Also, for x-values from 2 to ∞, the y-values increase from 0 to ∞. Thus the function is increas- ing on the intervals $(-1, 0)$, and $(2, \infty)$.

b) For x-values from 0 to 2, the y-values decrease from4 to 0. Thus, the function is decreasing on the in- terval $(0, 2)$.

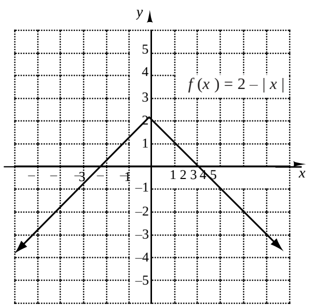
c) For x-values from $-\infty$ to -1, y is 3. Thus the function is constant on the interval $(-\infty, -1)$.

7.



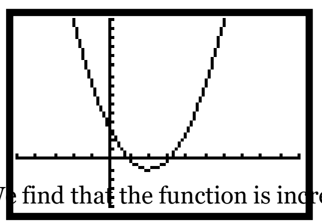
The function is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$. We estimate that the minimum value is -1 at $x = 0$. There are no maxima.

8.



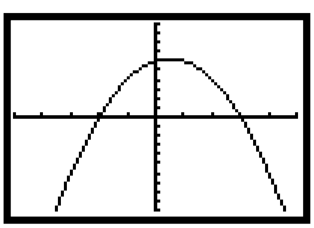
The function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. We estimate that the maximum value is 2 at $x = 0$. There are no minima.

9.



We find that the function is increasing on $(2, \infty)$ and decreasing on $(-\infty, 2)$. The relative minimum is -1 at $x = 2$. There are no maxima.

10.



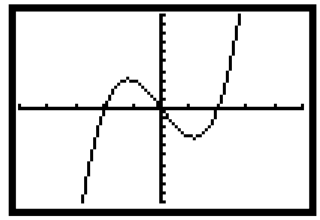
Increasing: $(-\infty, 0.5)$

Decreasing: $(0.5, \infty)$

Relative maximum: 6.25 at $x = 0.5$

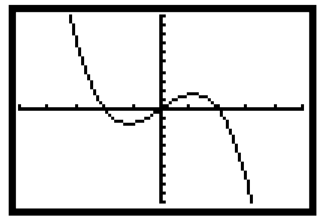
Relative minima: none

11.



We find that the function is increasing on $(-\infty, -1.155)$ and on $(1.155, \infty)$ and decreasing on $(-1.155, 1.155)$. The relative maximum is 3.079 at $x = -1.155$ and the relative minimum is -3.079 at $x = 1.155$.

12.



We find that the function is increasing on $(-1.155, 1.155)$ and decreasing on $(-\infty, -1.155)$ and on $(1.155, \infty)$. The relative maximum is 1.540 at $x = 1.155$ and the relative minimum is -1.540 at $x = -1.155$.

13. If two sides of the patio are each x feet, then the remaining side will be $(48 - 2x)$ ft. We use the formula Area = length × width.

$$A(x) = x(48 - 2x), \text{ or } 48x - 2x^2$$

14. The length of the rectangle is $2x$. The width is the second coordinate of the point (x, y) on the circle. The circle has center $(0, 0)$ and radius 2, so its equation is $x^2 + y^2 = 4$ and $y = \sqrt{4 - x^2}$. Thus the area of the rectangle is given by $A(x) = 2x\sqrt{4 - x^2}$.

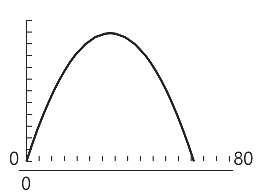
15. a) If the length of the side parallel to the garage is x feet long, then the length of each of the other two sides is $\frac{66 - x}{2}$, or $33 - \frac{x}{2}$. We use the formula Area = length × width.

$$A(x) = x\left(33 - \frac{x}{2}\right), \text{ or}$$

$$A(x) = 33x - \frac{x^2}{2}$$

b) The length of the side parallel to the garage must be positive and less than 66 ft, so the domain of the function is $\{x \mid 0 < x < 66\}$, or $(0, 66)$.

c) $y_1 = x\left(33 - \frac{x}{2}\right)$



d) By observing the graph or using the MAXIMUM feature, we see that the maximum value of the function occurs when $x = 33$. When $x = 33$, then $33 - \frac{x}{2} = 33 - \frac{33}{2} = 33 - 16.5 = 16.5$. Thus the dimensions that yield the maximum area are 33 ft by 16.5 ft.

16. a) Let h = the height of the box. Since the volume is 108 in^3, we have:

$$108 = x \cdot x \cdot h$$
$$108 = x^2 h$$
$$\frac{108}{x^2} = h$$

Now find the surface area.

$$S = x^2 + 4 \cdot x \cdot h$$
$$S(x) = x^2 + 4 \cdot x \cdot \frac{108}{x^2}$$
$$S(x) = x^2 + \frac{432}{x}$$

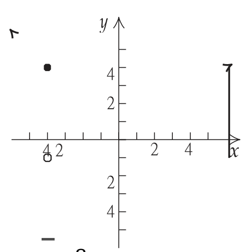
b) x must be positive, so the domain is $(0, \infty)$.

c) From the graph, we see that the minimum value of the function occurs when $x = 6$ in. For this value of x,

$$h = \frac{108}{x^2} = \frac{108}{6^2} = \frac{108}{36} = 3 \text{ in.}$$

17. $f(x) = \begin{cases} -x, & \text{for } x \le -4, \\ \frac{1}{2}x + 1, & \text{for } x > -4 \end{cases}$

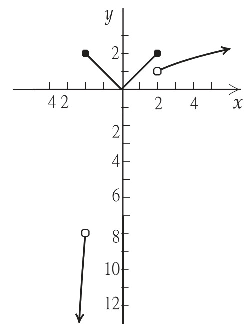
We create the graph in two parts. Graph $f(x) = -x$ for inputs less than or equal to -4. Then graph $f(x) = \frac{1}{2}x + 1$ for inputs greater than -4.



18. $f(x) = \begin{cases} x^3, & \text{for } x < -2, \\ |x|, & \text{for } -2 \le x \le 2, \\ \sqrt{x-1}, & \text{for } x > 2 \end{cases}$

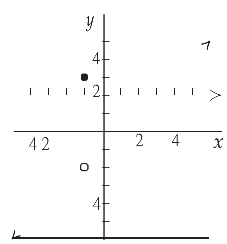
We create the graph in three parts. Graph $f(x) = x^3$ for inputs less than -2. Then graph $f(x) = |x|$ for inputs greater than or equal to -2 and less than or equal to 2.

Finally graph $f(x) = \sqrt{x-1}$ for inputs greater than 2.

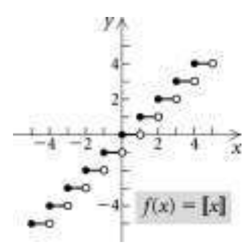


19. $f(x) = \begin{cases} \dfrac{x^2-1}{x+1}, & \text{for } x \ne -1, \\ 3, & \text{for } x = -1 \end{cases}$

We create the graph in two parts. Graph $f(x) = \frac{x^2-1}{x+1}$ for all inputs except -1. Then graph $f(x) = 3$ for $x = -1$.



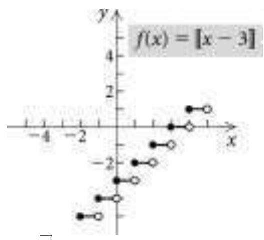
20. $f(x) = [[x]]$. See Example 9 on page 166 of the text.



21. $f(x) = [[x - 3]]$

This function could be defined by a piecewise function with an infinite number of statements.

$$f(x) = \begin{cases} \vdots \\ -4, & \text{for } -1 \le x < 0, \\ -3, & \text{for } 0 \le x < 1, \\ -2, & \text{for } 1 \le x < 2, \\ -1, & \text{for } 2 \le x < 3, \\ \vdots \end{cases}$$



22. $f(x) = \begin{cases} x^3, & \text{for } x < -2, \\ |x|, & \text{for } -2 \le x \le 2, \\ \sqrt{x-1}, & \text{for } x > 2 \end{cases}$

Since -1 is in the interval $[-2, 2]$, $f(-1) = |-1| = 1$.

Since $5 > 2$, $f(5) = \sqrt{5-1} = \sqrt{4} = 2$.

Since -2 is in the interval $[-2, 2]$, $f(-2) = |-2| = 2$.

Since $-3 < -2$, $f(-3) = (-3)^3 = -27$.

23. $f(x) = \begin{cases} \dfrac{x^2-1}{x+1}, & \text{for } x \neq -1, \\ 3, & \text{for } x = -1 \end{cases}$

Since $-2 \neq -1$, $f(-2) = \dfrac{(-2)^2-1}{-2+1} = \dfrac{4-1}{-1} = \dfrac{3}{-1} = -3$.

Since $x = -1$, we have $f(-1) = 3$.

Since $0 \neq -1$, $f(0) = \dfrac{0^2-1}{0+1} = \dfrac{-1}{1} = -1$.

Since $4 \neq -1$, $f(4) = \dfrac{4^2-1}{4+1} = \dfrac{16-1}{5} = \dfrac{15}{5} = 3$.

24. $(f-g)(6) = f(6) - g(6)$
$= \sqrt{6-2} - (6^2 - 1)$
$= \sqrt{4} - (36-1)$
$= 2 - 35$
$= -33$

25. $(fg)(2) = f(2) \cdot g(2)$
$= \sqrt{2-2} \cdot (2^2 - 1)$
$= 0 \cdot (4-1)$
$= 0$

26. $(f+g)(-1) = f(-1) + g(-1)$
$= \sqrt{-1-2} + ((-1)^2 - 1)$
$= \sqrt{-3} + (1-1)$

Since $\sqrt{-3}$ is not a real number, $(f+g)(-1)$ does not exist.

27. $f(x) = \dfrac{4}{x^2}$, $g(x) = 3 - 2x$

b) $(f+g)(x) = \dfrac{4}{x^2} + (3 - 2x) = \dfrac{4}{x^2} + 3 - 2x$

$(f-g)(x) = \dfrac{4}{x^2} - (3 - 2x) = \dfrac{4}{x^2} - 3 + 2x$

$(fg)(x) = \dfrac{4}{x^2}(3 - 2x) = \dfrac{12}{x^2} - \dfrac{8}{x}$

$(f/g)(x) = \dfrac{\frac{4}{x^2}}{(3-2x)} = \dfrac{4}{x^2(3-2x)}$

28. a) The domain of f, g, $f+g$, $f-g$, and fg is all real numbers, or $(-\infty, \infty)$. Since $g\left(\frac{1}{2}\right) = 0$, the domain of f/g is $\left\{x \mid x \neq \frac{1}{2}\right\}$, or $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

b) $(f+g)(x) = (3x^2 + 4x) + (2x - 1) = 3x^2 + 6x - 1$

$(f-g)(x) = (3x^2 + 4x) - (2x - 1) = 3x^2 + 2x + 1$

$(fg)(x) = (3x^2 + 4x)(2x - 1) = 6x^3 + 5x^2 - 4x$

$(f/g)(x) = \dfrac{3x^2 + 4x}{2x - 1}$

29. $P(x) = R(x) - C(x)$
$= (120x - 0.5x^2) - (15x + 6)$
$= 120x - 0.5x^2 - 15x - 6$
$= -0.5x^2 + 105x - 6$

30. $f(x) = 2x + 7$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{2(x+h) + 7 - (2x + 7)}{h} = \dfrac{2x + 2h + 7 - 2x - 7}{h} = \dfrac{2h}{h} = 2$

31. $f(x) = 3 - x^2$

$f(x+h) = 3 - (x+h)^2 = 3 - (x^2 + 2xh + h^2) = 3 - x^2 - 2xh - h^2$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{3 - x^2 - 2xh - h^2 - (3 - x^2)}{h}$

$= \dfrac{3 - x^2 - 2xh - h^2 - 3 + x^2}{h}$

$= \dfrac{-2xh - h^2}{h} = \dfrac{h(-2x - h)}{h}$

$= \dfrac{h}{h} \cdot \dfrac{-2x - h}{1} = -2x - h$

32. $f(x) = \dfrac{4}{x}$

4 4 4 4 +

x^2

a) Division by zero is undefined, so the domain of f is $\{x|x = 0\}$, or $(-\infty, 0) \cup (0, \infty)$. The domain of g is the set of all real numbers, or $(-\infty, \infty)$.

The domain of $f + g$, $f - g$ and fg is $\{x|x = 0\}$, or $(-\infty, 0) \cup (0, \infty)$. Since g $\frac{3}{2}$ $= 0$, the domain of f/g is $x\ x = 0\ and\ x \times \frac{3}{2}$ $= \frac{}{2}$, or

$(-\infty, 0) \cup$ $0, \frac{-}{2} \cup \frac{3}{2}, \infty$ $\frac{3}{}$.

$\frac{f(x+h)-f(x)}{h} = x + h \quad x = \frac{x + h \quad x \quad x \quad x + h}{h} =$

$\frac{4x - 4(x+h)}{x(x+h)} \over h$ $= \frac{\frac{4x - 4x - 4h}{x(x+h)}}{h} = \frac{\frac{-4h}{x(x+h)}}{h} =$

$\frac{-4h}{x(x+h)} \cdot \frac{1}{h} = \frac{-4 \cdot \cancel{h}}{x(x+h) \cdot \cancel{h}} = \frac{-4}{x(x+h)}$, or $-\frac{4}{x(x+h)}$

33. $(f \circ g)(1) = f(g(1)) = f(1^2 + 4) = f(1 + 4) = f(5) =$

$2 \cdot 5 - 1 = 10 - 1 = 9$

34. $(g \circ f)(1) = g(f(1)) = g(2 \cdot 1 - 1) = g(2 - 1) = g(1) = 1^2 + 4 = 1 + 4 = 5$

35. $(h \circ f)(-2) = h(f(-2)) = h(2(-2) - 1) = h(-4 - 1) = h(-5) = 3 - (-5)^3 = 3 - (-125) = 3 + 125 = 128$

36. $(g \circ h)(3) = g(h(3)) = g(3 - 3^3) = g(3 - 27) = g(-24) = (-24)^2 + 4 = 576 + 4 = 580$

37. $(f \circ h)(-1) = f(h(-1)) = f(3 - (-1)^3) = f(3 - (-1)) = f(3 + 1) = f(4) = 2 \cdot 4 - 1 = 8 - 1 = 7$

38. $(h \circ g)(2) = h(g(2)) = h(2^2 + 4) = h(4 + 4)$
$h(8) = 3 - 8^3 = 3 - 512 = -509$

39. $(f \circ f)(x) = f(f(x)) = f(2x - 1) = 2(2x - 1) - 1 = 4x - 2 - 1 = 4x - 3$

40. $(h \circ h)(x) = h(h(x)) = h(3 - x^3) = 3 - (3 - x^3)^3 = 3 - (27 - 27x^3 + 9x^6 - x^9) = 3 - 27 + 27x^3 - 9x^6 + x^9 = -24 + 27x^3 - 9x^6 + x^9$

41. a) $f \circ g(x) = f(3 - 2x) = \dfrac{4}{(3 - 2x)^2}$

$g \circ f(x) = g\left(\dfrac{4}{x^2}\right) = 3 - 2\left(\dfrac{4}{x^2}\right) = 3 - \dfrac{8}{x^2}$

b) The domain of f is $\{x \mid x \neq 0\}$ and the domain of g is the set of all real numbers. To find the domain of $f \circ g$, we find the values of x for which $g(x) = 0$.

Since $3 - 2x = 0$ when $x = \dfrac{3}{2}$, the domain of $f \circ g$ is $\left\{x \,\middle|\, x \neq \dfrac{3}{2}\right\}$, or $\left(-\infty, \dfrac{3}{2}\right) \cup \left(\dfrac{3}{2}, \infty\right)$. Since any real number can be an input for g, the domain of $g \circ f$ is the same as the domain of f, $\{x \mid x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$.

42. a) $f \circ g(x) = f(2x - 1)$
$= 3(2x - 1)^2 + 4(2x - 1)$
$= 3(4x^2 - 4x + 1) + 4(2x - 1)$
$= 12x^2 - 12x + 3 + 8x - 4$
$= 12x^2 - 4x - 1$

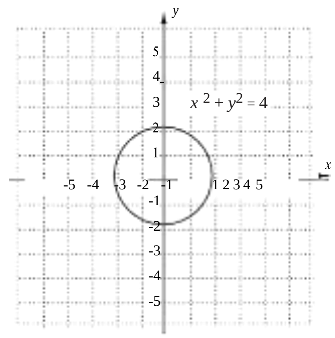
$(g \circ f)(x) = g(3x^2 + 4x)$
$= 2(3x^2 + 4x) - 1$
$= 6x^2 + 8x - 1$

b) Domain of f = domain of g = all real numbers, so domain of $f \circ g$ = domain of $g \circ f$ = all real numbers, or $(-\infty, \infty)$.

43. $f(x) = \sqrt{x}$, $g(x) = 5x + 2$. Answers may vary.

44. $f(x) = 4x^2 + 9$, $g(x) = 5x - 1$. Answers may vary.

45. $x^2 + y^2 = 4$



The graph is symmetric with respect to the x-axis, the y-axis, and the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the x-axis.

$x^2 + (-y)^2 = 4$
$x^2 + y^2 = 4$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Replace x with $-x$ to test algebraically for symmetry with respect to the y-axis.

$(-x)^2 + y^2 = 4$
$x^2 + y^2 = 4$

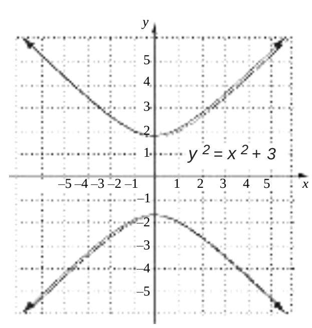
The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace x and $-x$ and y with $-y$ to test for symmetry with respect to the origin.

$(-x)^2 + (-y)^2 = 4$
$x^2 + y^2 = 4$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

46. $y^2 = x^2 + 3$



The graph is symmetric with respect to the x-axis, the y-axis, and the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the x-axis.

$$(-y)^2 = x^2 + 3$$
$$y^2 = x^2 + 3$$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Replace x with $-x$ to test algebraically for symmetry with respect to the y-axis.

$$y^2 = (-x)^2 + 3$$
$$y^2 = x^2 + 3$$

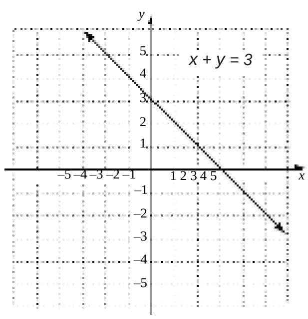
The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace x and $-x$ and y with $-y$ to test for symmetry with respect to the origin.

$$(-y)^2 = (-x)^2 + 3$$
$$y^2 = x^2 + 3$$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

47. $x + y = 3$



The graph is not symmetric with respect to the x-axis, the y-axis, or the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the x-axis.

$$x - y = 3$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ to test algebraically for symmetry with respect to the y-axis.

$$-x + y = 3$$

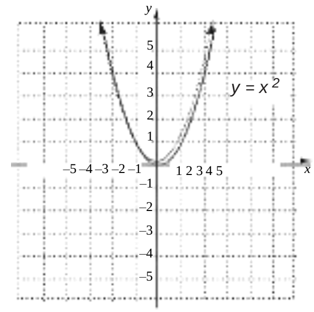
The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Replace x and $-x$ and y with $-y$ to test for symmetry with respect to the origin.

$$-x - y = 3$$
$$x + y = -3$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

48. $y = x^2$



The graph is symmetric with respect to the y-axis. It is not symmetric with respect to the x-axis or the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the x-axis.

$$-y = x^2$$
$$y = -x^2$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ to test algebraically for symmetry with respect to the y-axis.

$$y = (-x)^2$$
$$y = x^2$$

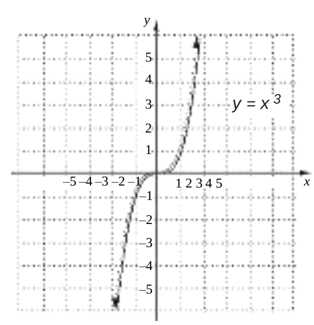
The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace x and $-x$ and y with $-y$ to test for symmetry with respect to the origin.

$$-y = (-x)^2$$
$$-y = x^2$$
$$y = -x^2$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

49. $y = x^3$



The graph is symmetric with respect to the origin. It is not symmetric with respect to the x-axis or the y-axis.

Replace y with $-y$ to test algebraically for symmetry with respect to the x-axis.

$$-y = x^3$$
$$y = -x^3$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ to test algebraically for symmetry with respect to the y-axis.

$y = (-x)^3$

$y = -x^3$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Replace x and $-x$ and y with $-y$ to test for symmetry with respect to the origin.

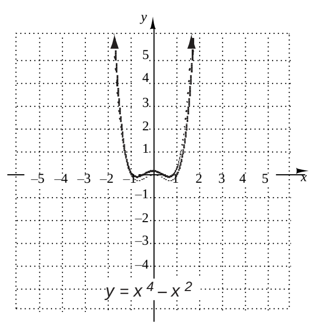
$-y = (-x)^3$

$-y = -x^3$

$y = x^3$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

50. $y = x^4 - x^2$



The graph is symmetric with respect to the y-axis. It is not symmetric with respect to the x-axis or the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the x-axis.

$-y = x^4 - x^2$

$y = -x^4 + x^2$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ to test algebraically for symmetry with respect to the y-axis.

$y = (-x)^4 - (-x)^2$

$y = x^4 - x^2$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace x and $-x$ and y with $-y$ to test for symmetry with respect to the origin.

$-y = (-x)^4 - (-x)^2$

$-y = x^4 - x^2$

$y = -x^4 + x^2$

51. The graph is symmetric with respect to the y-axis, so the function is even.

52. The graph is symmetric with respect to the y-axis, so the function is even.

53. The graph is symmetric with respect to the origin, so the function is odd.

54. The graph is symmetric with respect to the y-axis, so the function is even.

55. $f(x) = 9 - x^2$

$f(-x) = 9 - (-x^2) = 9 - x^2$

$f(x) = f(-x)$, so f is even.

56. $f(x) = x^3 - 2x + 4$

$f(-x) = (-x)^3 - 2(-x) + 4 = -x^3 + 2x + 4$

$f(x) \neq f(-x)$, so f is not even.

$-f(x) = -(x^3 - 2x + 4) = -x^3 + 2x - 4$

$f(-x) \neq -f(x)$, so f is not odd.

Thus, $f(x) = x^3 - 2x + 4$ is neither even or odd.

57. $f(x) = x^7 - x^5$

$f(-x) = (-x)^7 - (-x)^5 = -x^7 + x^5$

$f(x) \neq f(-x)$, so f is not even.

$-f(x) = -(x^7 - x^5) = -x^7 + x^5$

$f(-x) = -f(x)$, so f is odd.

58. $f(x) = |x|$

$f(-x) = |-x| = |x|$

$f(x) = f(-x)$, so f is even.

59. $f(x) = \sqrt{16 - x^2}$

$f(-x) = \sqrt{16 - (-x^2)} = \sqrt{16 - x^2}$

$f(x) = f(-x)$, so f is even.

60. $f(x) = \dfrac{10x}{x^2+1}$

$f(-x) = \dfrac{10(-x)}{(-x)^2+1} = -\dfrac{10x}{x^2+1}$

$f(x) \neq f(-x)$, so $f(x)$ is not even.

$-f(x) = -\dfrac{10x}{x^2+1}$

$f(-x) = -f(x)$, so f is odd.

61. Shape: $g(x) = x^2$

Shift $g(x)$ left 3 units: $f(x) = g(x+3) = (x+3)^2$

62. Shape: $t(x) = \sqrt{x}$

Turn $t(x)$ upside down (that is, reflect it across the x-axis):

$h(x) = -t(x) = -\sqrt{x}$.

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

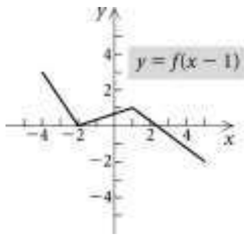
Shift $h(x)$ right 3 units: $g(x) = h(x-3) = -\sqrt{x-3}$. Shift $g(x)$ up 4 units: $f(x) = g(x) + 4 = -\sqrt{x-3} + 4$.

63. Shape: $h(x) = |x|$

Stretch $h(x)$ vertically by a factor of 2 (that is, multiplyeach function value by 2): $g(x) = 2h(x) = 2|x|$.

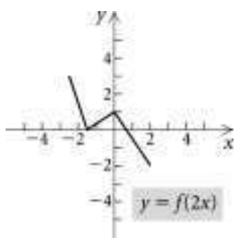
Shift $g(x)$ right 3 units: $f(x) = g(x-3) = 2|x-3|$.

64. The graph is shifted right 1 unit so each x-coordinate is increased by 1. We plot and connect $(-4, 3)$, $(-2, 0)$, $(1, 1)$ and $(5, -2)$.

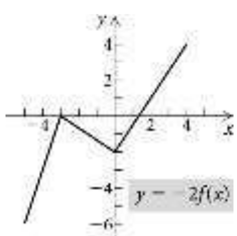


65. The graph is shrunk horizontally by a factor of 2. That is, each x-coordinate is divided by 2. We plot and connect

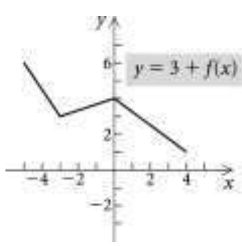
$\left(-\frac{5}{2}, 3\right)$, $\left(-\frac{3}{2}, 0\right)$, $(0, 1)$ and $(2, -2)$.



66. Each y-coordinate is multiplied by -2. We plot and connect $(-5, -6)$, $(-3, 0)$, $(0, -2)$ and $(4, 4)$.



67. Each y-coordinate is increased by 3. We plot and connect $(-5, 6)$, $(-3, 3)$, $(0, 4)$ and $(4, 1)$.



68.
$$y = kx$$
$$100 = 25x$$
$$4 = x$$
Equation of variation: $y = 4x$

69.
$$y = kx$$
$$6 = 9x$$
$$\frac{2}{3} = x \quad \text{Variation constant}$$
Equation of variation: $y = \frac{2}{3}x$

70.
$$y = \frac{k}{x}$$
$$100 = \frac{k}{25}$$
$$2500 = k$$
Equation of variation: $y = \frac{2500}{x}$

71.
$$y = \frac{k}{x}$$
$$6 = \frac{k}{9}$$
$$54 = k \quad \text{Variation constant}$$
Equation of variation: $y = \frac{54}{x}$

72.
$$y = \frac{k}{x^2}$$
$$12 = \frac{k}{2^2}$$
$$48 = k$$
$$y = \frac{48}{x^2}$$

73.
$$y = \frac{kxz^2}{w}$$
$$2 = \frac{k(16)\left(\frac{1}{2}\right)^2}{0.2}$$
$$2 = \frac{k(16)\frac{1}{4}}{0.2}$$
$$2 = \frac{4k}{0.2}$$
$$2 = 20k$$
$$\frac{1}{10} = k$$
$$y = \frac{1}{10}\frac{xz^2}{w}$$

74.
$$t = \frac{k}{r}$$
$$35 = \frac{k}{800}$$
$$28,000 = k$$
$$t = \frac{28,000}{r}$$
$$t = \frac{28,000}{1400}$$
$$t = 20 \text{ min}$$

75. $N = ka$

$87 = k \cdot 29$

$3 = k$

$N = 3a$

$N = 3 \cdot 25$

$N = 75$

Sam's score would have been 75 if he had answered 25 questions correctly.

76. $P = kC^2$

$180 = k \cdot 6^2$

$5 = k$ Variation constant

$P = 5C^2$ Variation equation

$P = 5 \cdot 10^2$

$P = 500$ watts

77. $f(x) = x + 1$, $g(x) = \sqrt{x}$

The domain of f is $(-\infty, \infty)$, and the domain of g is $[0, \infty)$. To find the domain of $(g \circ f)(x)$, we find the values of x for which $f(x) \geq 0$.

$x + 1 \geq 0$

$x \geq -1$

Thus the domain of $(g \circ f)(x)$ is $[-1, \infty)$. Answer A is correct.

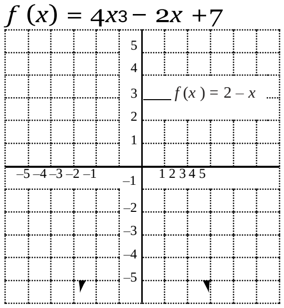
78. For $b > 0$, the graph of $y = f(x) + b$ is the graph of $y = f(x)$ shifted up b units. Answer C is correct.

79. The graph of $g(x) = -\frac{1}{2}f(x) + 1$ is the graph of $y = f(x)$ shrunk vertically by a factor of $\frac{1}{2}$, then reflected across the x-axis, and shifted up 1 unit. The correct graph is B.

80. Let $f(x)$ and $g(x)$ be odd functions. Then by definition, $f(-x) = -f(x)$, or $f(x) = -f(-x)$, and $g(-x) = -g(x)$, or $g(x) = -g(-x)$. Thus $(f + g)(x) = f(x) + g(x) = -f(-x) + [-g(-x)] = -[f(-x) + g(-x)] = -(f + g)(-x)$ and $f + g$ is odd.

81. Reflect the graph of $y = f(x)$ across the x-axis and then across the y-axis.

82. $f(x) = 4x^3 - 2x + 7$



a) $f(x) + 2 = 4x^3 - 2x + 7 + 2 = 4x^3 - 2x + 9$

b) $f(x + 2) = 4(x + 2)^3 - 2(x + 2) + 7$

$= 4(x^3 + 6x^2 + 12x + 8) - 2(x + 2) + 7$

$= 4x^3 + 24x^2 + 48x + 32 - 2x - 4 + 7$

83. In the graph of $y = f(cx)$, the constant c stretches or shrinks the graph of $y = f(x)$ horizontally. The constant c in $y = cf(x)$ stretches or shrinks the graph of $y = f(x)$ vertically. For $y = f(cx)$, the x-coordinates of $y = f(x)$ are divided by c; for $y = cf(x)$, the y-coordinates of $y = f(x)$ are multiplied by c.

84. The graph of $f(x) = 0$ is symmetric with respect to the x-axis, the y-axis, and the origin. This function is both even and odd.

85. If all of the exponents are even numbers, then $f(x)$ is an even function. If $a_0 = 0$ and all of the exponents are odd numbers, then $f(x)$ is an odd function.

86. Let $y(x) = kx^2$. Then $y(2x) = k(2x)^2 = k \cdot 4x^2 = 4 \cdot kx^2 = 4 \cdot y(x)$. Thus, doubling x causes y to be quadrupled.

87. Let $y = k_1x$ and $x = \frac{k_2}{z}$. Then $y = k_1 \cdot \frac{k_2}{z}$, or $y = \frac{k_1k_2}{z}$, so y varies inversely as z.

Chapter 2 Test

$= 4x^3 + 24x^2 + 46x + 35$

1. a) For x-values from -5 to -2, the y-values increase from -4 to 3. Thus the function is increasing on the interval $(-5, -2)$.

b) For x-values from 2 to 5, the y-values decrease from 2 to -1. Thus the function is decreasing on the interval $(2, 5)$.

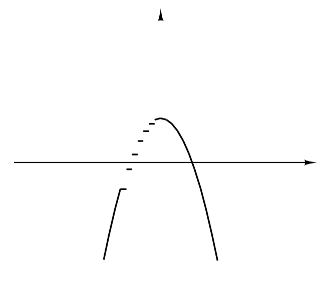
c) For x-values from -2 to 2, y is 2. Thus the function is constant on the interval $(-2, 2)$.

2.

y

The function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. The relative maximum is 2 at $x = 0$. There are no

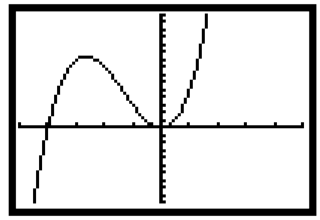
x



c) $$f(x) + f(2) = 4x^3 - 2x + 7 + 4 \cdot 2^3 - 2 \cdot 2 + 7$$
$$= 4x^3 - 2x + 7 + 32 - 4 + 7$$
$$= 4x^3 - 2x + 42$$

$f(x) + 2$ adds 2 to each function value; $f(x + 2)$ adds 2 to each input before the function value is found; $f(x) + f(2)$ adds the output for 2 to the output for x.

minima.

3. 

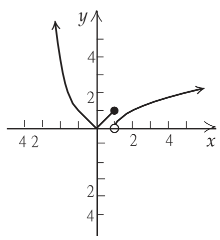
We find that the function is increasing on $(-\infty, -2.667)$ and on $(0, \infty)$ and decreasing on $(-2.667, 0)$. The relative maximum is 9.481 at -2.667 and the relative minimum is 0 at $x = 0$.

4. If b = the length of the base, in inches, then the height = $4b - 6$. We use the formula for the area of a triangle, $A = \frac{1}{2}bh$.

$A(b) = \frac{1}{2}b(4b - 6)$, or

$A(b) = 2b^2 - 3b$

5. $f(x) = \begin{cases} x^2, & \text{for } x < -1, \\ |x|, & \text{for } -1 \le x \le 1, \\ \sqrt{x-1}, & \text{for } x > 1 \end{cases}$



6. Since $-1 \le -\frac{1}{8} \le 1$, $f\left(-\frac{1}{8}\right) = \left|-\frac{1}{8}\right| = \frac{1}{8}$.

Since $5 > 1$, $f(5) = \sqrt{5-1} = \sqrt{4} = 2$.

Since $-4 < -1$, $f(-4) = (-4)^2 = 16$.

7. $(f+g)(-6) = f(-6) + g(-6) =$

$(-6)^2 - 4(-6) + 3 + \sqrt{3 - (-6)} =$

$36 + 24 + 3 + \sqrt{3 + 6} = 63 + \sqrt{9} = 63 + 3 = 66$

8. $(f-g)(-1) = f(-1) - g(-1) =$

$(-1)^2 - 4(-1) + 3 - \sqrt{3 - (-1)} =$

$1 + 4 + 3 - \sqrt{3+1} = 8 - \sqrt{4} = 8 - 2 = 6$

9. $(fg)(2) = f(2) \cdot g(2) = (2^2 - 4 \cdot 2 + 3)(\sqrt{3-2}) =$

$(4 - 8 + 3)(\sqrt{1}) = -1 \cdot 1 = -1$

10. $(f/g)(1) = \frac{f(1)}{g(1)} = \frac{1 - 4 \cdot 1 + 3}{\sqrt{3-1}} = \frac{1 - 4 + 3}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0$

11. Any real number can be an input for $f(x) = x^2$, so the domain is the set of real numbers, or $(-\infty, \infty)$.

12. The domain of $g(x) = \sqrt{x-3}$ is the set of real numbers for which $x - 3 \ge 0$, or $x \ge 3$. Thus the domain is $\{x | x \ge 3\}$, or $[3, \infty)$.

18. $(f-g)(x) = f(x) - g(x) = x^2 - \sqrt{x-3}$

19. $(fg)(x) = f(x) \cdot g(x) = x^2\sqrt{x-3}$

20. $(f/g)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{\sqrt{x-3}}$

21. $f(x) = \frac{1}{2}x + 4$

$f(x+h) = \frac{1}{2}(x+h) + 4 = \frac{1}{2}x + \frac{1}{2}h + 4$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{2}x + \frac{1}{2}h + 4 - \left(\frac{1}{2}x + 4\right)}{h}$$

$$= \frac{\frac{1}{2}x + \frac{1}{2}h + 4 - \frac{1}{2}x - 4}{h}$$

$$= \frac{\frac{1}{2}h}{h} = \frac{1}{2}h \cdot \frac{1}{h} = \frac{1}{2} \cdot \frac{h}{h} = \frac{1}{2}$$

22. $f(x) = 2x^2 - x + 3$

$f(x+h) = 2(x+h)^2 - (x+h) + 3 = 2(x^2 + 2xh + h^2) - x - h + 3 =$

$2x^2 + 4xh + 2h^2 - x - h + 3$

$$\frac{f(x+h) - f(x)}{h} = \frac{2x^2 + 4xh + 2h^2 - x - h + 3 - (2x^2 - x + 3)}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - x - h + 3 - 2x^2 + x - 3}{h}$$

$$= \frac{4xh + 2h^2 - h}{h}$$

$$= \frac{\cancel{h}(4x + 2h - 1)}{\cancel{h}}$$

$$= 4x + 2h - 1$$

23. $(g \circ h)(2) = g(h(2)) = g(3 \cdot 2^2 + 2 \cdot 2 + 4) =$

$g(3 \cdot 4 + 4 + 4) = g(12 + 4 + 4) = g(20) = 4 \cdot 20 + 3 =$

$80 + 3 = 83$

24. $(f \circ g)(-1) = f(g(-1)) = f(4(-1) + 3) = f(-4 + 3) =$

$f(-1) = (-1)^2 - 1 = 1 - 1 = 0$

25. $(h \circ f)(1) = h(f(1)) = h(1^2 - 1) = h(1 - 1) = h(0) =$

$3 \cdot 0^2 + 2 \cdot 0 + 4 = 0 + 0 + 4 = 4$

26. $(g \circ g)(x) = g(g(x)) = g(4x + 3) = 4(4x + 3) + 3 =$

13. The domain of f

$f + g$ is the intersection of the domains of f and g. This is $\{x|x \geq 3\}$, or $[3, \infty)$.

14. The domain of $f-g$ is the intersection of the domains of f and g. This is $\{x|x \geq 3\}$, or $[3, \infty)$.

15. The domain of fg is the intersection of the domains of f and g. This is $\{x|x \geq 3\}$, or $[3, \infty)$.

16. The domain of f/g is the intersection of the domains of f and g, excluding those x-values for which $g(x) = 0$. Since $x-3 = 0$ when $x = 3$, the domain is $(3, \infty)$.

17. $(f+g)(x) = f(x) + g(x) = x^2 + \sqrt{x-3}$

$16x + 12 + 3 = 16x + 15$

27. $(f \circ g)(x) = f(g(x)) = f(x^2 + 1) = \sqrt{x^2+1-5} = \sqrt{x^2-4}$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x-5}) = (\sqrt{x-5})^2 + 1 = x - 5 + 1 = x - 4$

28. The inputs for $f(x)$ must be such that $x-5 \geq 0$, or $x \geq 5$. Then for $(f \circ g)(x)$ we must have $g(x) \geq 5$, or $x^2+1 \geq 5$, or $x^2 \geq 4$. Then the domain of $(f \circ g)(x)$ is $(-\infty, -2] \cup [2, \infty)$. Since we can substitute any real number for x in g, the domain of $(g \circ f)(x)$ is the same as the domain of $f(x)$, $[5, \infty)$.

29. Answers may vary. $f(x) = x^4$, $g(x) = 2x - 7$

30. $y = x^4 - 2x^2$

Replace y with $-y$ to test for symmetry with respect to the x-axis.

$$-y = x^4 - 2x^2$$
$$y = -x^4 + 2x^2$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ to test for symmetry with respect to the y-axis.

$$y = (-x)^4 - 2(-x)^2$$
$$y = x^4 - 2x^2$$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace x with $-x$ and y with $-y$ to test for symmetry with respect to the origin.

$$-y = (-x)^4 - 2(-x)^2$$
$$-y = x^4 - 2x^2$$
$$y = -x^4 + 2x^2$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

31. $f(x) = \dfrac{2x}{x^2 + 1}$

$$f(-x) = \frac{2(-x)}{(-x)^2 + 1} = -\frac{2x}{x^2 + 1}$$

$f(x)$ $f(-x)$, so f is not even.

$$-f(x) = -\frac{2x}{x^2 + 1}$$

$f(-x) = -f(x)$, so f is odd.

32. Shape: $h(x) = x^2$

Shift $h(x)$ right 2 units: $g(x) = h(x - 2) = (x - 2)^2$

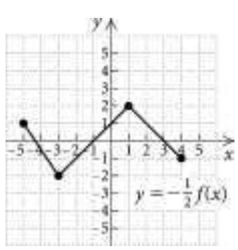
Shift $g(x)$ down 1 unit: $f(x) = (x - 2)^2 - 1$

33. Shape: $h(x) = x^2$

Shift $h(x)$ left 2 units: $g(x) = h(x + 2) = (x + 2)^2$

Shift $g(x)$ down 3 units: $f(x) = (x + 2)^2 - 3$

34. Each y-coordinate is multiplied by $-\frac{1}{2}$. We plot and connect $(-5, 1)$, $(-3, -2)$, $(1, 2)$ and $(4, -1)$.



35. $y = \dfrac{k}{x}$

$$5 = \frac{k}{6}$$

$30 = k$ Variation constant

Equation of variation: $y = \dfrac{30}{x}$

36. $y = kx$

$$60 = k \cdot 12$$

$5 = k$ Variation constant

Equation of variation: $y = 5x$

37. $y = \dfrac{kxz^2}{w}$

$$100 = \frac{k(0.1)(10)^2}{5}$$
$$100 = 2k$$

$50 = k$ Variation constant

$y = \dfrac{50xz^2}{w}$ Equation of variation

38. $d = kr^2$

$$200 = k \cdot 60^2$$

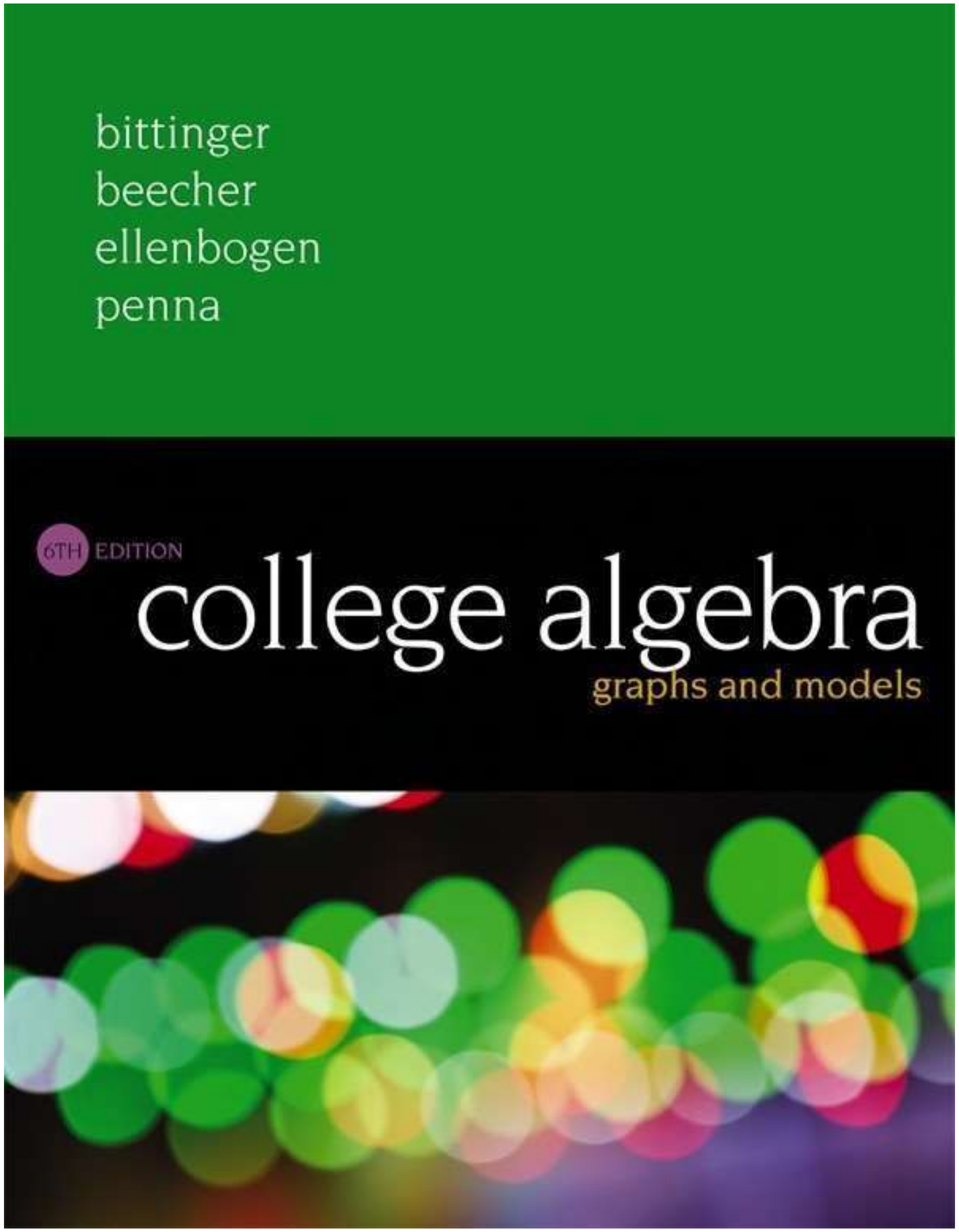
$\dfrac{1}{18} = k$ Variation constant

$d = \dfrac{1}{18}r^2$ Equation of variation

$$d = \frac{1}{18} \cdot 30^2$$
$$d = 50 \text{ ft}$$

39. The graph of $g(x) = 2f(x) - 1$ is the graph of $y = f(x)$ stretched vertically by a factor of 2 and shifted down 1 unit. The correct graph is C.

40. Each x-coordinate on the graph of $y = f(x)$ is divided by 3 on the graph of $y = f(3x)$. Thus the point $\left(\frac{-3}{3}, 1\right)$, or $(-1, 1)$ is on the graph of $f(3x)$.



Section R.2

Integer Exponents, Scientific Notation, and Order of Operations

R.2
Integer Exponents, Scientific Notation, and Order of Operations

- Simplify expressions with integer exponents.
- Solve problems using scientific notation.
- Use the rules for order of operations.

Integers as Exponents

When a positive integer is used as an *exponent*, it indicates the number of times a factor appears in a product. For example, 7^3 means $7 \cdot 7 \cdot 7$ and 5^1 means 5.

For any positive integer *n*,

$$\underbrace{a}_{base}{}^{n} = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}},$$

where *a* is the **base** and *n* is the **exponent.**

For any nonzero real number *a* and any integer *m*, $a^0 = 1$ and $a^{-m} = \dfrac{1}{a^m}$.

Properties of Exponents

Product rule

$$a^m \cdot a^n = a^{m+n}$$

Raising a product to a power

$$(ab)^m = a^m b^m$$

Quotient rule

$$\frac{a^m}{a^n} = a^{m-n} \qquad (a \neq 0)$$

Raising a quotient to a power

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \qquad (b \neq 0)$$

Power rule

$$(a^m)^n = a^{mn}$$

Examples

Simply each of the following.

a) $y^{-5} \bullet y^{3}$

$$y^{(-5+3)} = y^{-2}$$

b) $\dfrac{48x^{12}}{16x^{4}} = \dfrac{48}{\cancel{16}} x^{12-4} = 3x^{8}$

c) $(t^{-3})^{5} = t^{-3\cdot 5} = t^{-15}$, or $\dfrac{1}{t^{15}}$

d) $(2s^{-2})^{5} = 2^{5}(s^{-2})^{5}$

$$= 32s^{-10} \text{ or } \frac{32}{s^{10}}$$

e) $\left(\dfrac{45x^{-4}y^{2}}{\cancel{9}z^{-8}}\right)^{-3} = \left(\dfrac{5x^{-4}y^{2}}{z^{-8}}\right)^{-3}$

$$= \frac{5^{-3}x^{12}y^{-6}}{z^{24}}$$

$$= \frac{x^{12}}{5^{3}y^{6}z^{24}}, \text{ or } \frac{x^{12}}{125y^{6}z^{24}}$$

Scientific Notation

Use scientific notation to name very large and very small positive numbers and to perform computations.

Scientific notation for a number is an expression of the type

$$N \times 10^m,$$

where $1 \leq N < 10$, N is in decimal notation, and m is an integer.

Example

Views of a Video. In the first two months after having been uploaded to a popular video-sharing Web site, a video showing a baby laughing hysterically at ripping paper received approximately 12,630,000 views. Convert the number 12,630,000 to scientific notation.

Solution We want the decimal point to be positioned between the 1 and the 2, so we move it 7 places to the left. Since the number to be converted is greater than 10, the exponent must be positive. Thus we have

$$12{,}630{,}000 = 1.263 \times 10^7$$

Rules for Order of Operations

1. Do all calculations within grouping symbols before operations outside. When nested grouping symbols are present, work from the inside out.
2. Evaluate all exponential expressions.
3. Do all multiplications and divisions in order from left to right.
4. Do all additions and subtractions in order from left to right.

Examples

a) $8(5-3)^3 - 20 = 8(2)^3 - 20$

$= 8(8) - 20$

$= 64 - 20$

$= 44$

b) $\dfrac{10 \div (8-6) + 9 \cdot 4}{2^5 + 3^2} = \dfrac{10 \div 2 + 9 \cdot 4}{32 + 8}$

$= \dfrac{5 + 36}{41} = \dfrac{41}{41} = 1$