CHAPTER 2
Solving Equations and Inequalities

Section 2.1 Linear Equations and Problem Solving

1. equation

2. $ax + b = 0$

3. extraneous

4. formulas

5. The equation $x + 1 = 3$ is a conditional equation.

6. To clear the equation $\frac{x}{2} + 1 = \frac{1}{4}$ of fractions, multiply both sides of the equation by the least common denominator of all the fractions, which is 4.

7. $\frac{5}{2x} - \frac{4}{x} = 3$

(a)

(b)

8. $\frac{x}{2} + \frac{6x}{7} = \frac{19}{14}$

(a) $x = -2$

$\frac{-2}{2} + \frac{6(-2)}{7} \stackrel{?}{=} \frac{19}{14}$

$\frac{-14 - 24}{14} \stackrel{?}{=} \frac{19}{14}$

$\frac{-38}{14} \stackrel{?}{=} \frac{19}{14}$

$x = -2$ is not a solution.

(b) $x = 1$

$\frac{1}{2} + \frac{6(1)}{7} \stackrel{?}{=} \frac{19}{14}$

$2(-1\ 2) \quad (-1\ 2)$

$3 = 3$

$x = -\frac{1}{2}$ is a solution.

$\frac{5}{4}\ ?\ 3$

$\frac{5}{} - \frac{4}{}\ ?\ 3$

$\frac{1}{9}$

(c) $\frac{1}{9}$ $=$ — —

$\frac{1}{4}$

$\frac{1}{4}$

$x = 1$ is a solution.

$x = \frac{1}{2}$ –

$\underline{6}$ ($\underline{1}$ $\underline{2}$) /

$+$ —

$=$

$=$ $\frac{1/2}{2}$ $\overset{?}{}$ $\frac{19}{14}$

$2(4)$ 4 7

$-\frac{3}{8} \neq 3$ $\frac{7/2+6}{14} \overset{?}{=} \frac{19}{14}$

$x = 4$ is not a solution. $\frac{19}{} \neq \frac{19}{}$

(c) $\dfrac{5}{2(0)} - \dfrac{4}{0}$ is undefined.

$x = 0$ is not a solution.

(d) $\dfrac{5}{2(1/4)} - \dfrac{4}{1/4} \stackrel{?}{=} 3$

$-6 \neq 3$

$x = \frac{1}{4}$ is not a solution.

28 14

$x = \frac{1}{2}$ is not a solution.

(d) $x = 7$

$$\frac{7}{2} + \frac{6(7)}{7} = \frac{19}{14}$$

$$\frac{7}{2} + 6 = \frac{19}{14}$$

$$\frac{19}{2} \stackrel{?}{=} \frac{19}{14}$$

$x = 7$ is not a solution.

9. $\dfrac{\sqrt{x+4}}{6} + 3 = 4$

(a) $\dfrac{\sqrt{-3+4}}{6} + 3 \stackrel{?}{=} 4$

$\dfrac{19}{6} \neq 4$

$x = -3$ is not a solution.

(b) $\dfrac{\sqrt{0+4}}{6} + 3 \stackrel{?}{=} 4$

$\dfrac{10}{3} \neq 4$

$x = 0$ is not a solution.

(c) $\dfrac{\sqrt{21+4}}{6} + 3 \stackrel{?}{=} 4$

$\dfrac{23}{6} \neq 4$

$x = 21$ is not a solution.

(d) $\dfrac{\sqrt{32+4}}{6} + 3 \stackrel{?}{=} 4$

$4 = 4$

$x = 32$ is a solution.

10. $\dfrac{\sqrt[3]{x-8}}{3} = -1$

(a) $x = 4$

$\dfrac{\sqrt[3]{4-8}}{3} \stackrel{?}{=} -1$

$\dfrac{\sqrt[3]{-4}}{3} \neq -1$

$x = 4$ is not a solution.

(b) $x = 0$

$\dfrac{\sqrt[3]{0-8}}{3} \stackrel{?}{=} -1$

$\dfrac{\sqrt[3]{-8}}{3} \stackrel{?}{=} -1$

$\dfrac{-2}{3} \neq -1$

$x = 0$ is not a solution.

(c) $x = -19$

(d) $x = 16$

$\dfrac{\sqrt[3]{16-8}}{3} \stackrel{?}{=} -1$

$\dfrac{\sqrt[3]{8}}{3} \stackrel{?}{=} -1$

$\dfrac{2}{3} \neq -1$

$x = 16$ is not a solution.

11. $2(x-1) = 2x - 2$ is an *identity* by the Distributive Property. It is true for all real values of x.

12. $-5(x-1) = -5(x+1)$ is a *contradiction*. There are no real values of x for which it is true.

$-5x + 5 = -5x - 5$

$5 \neq -5$

13. $(x+3)(x-5) = x^2 - 2(x+7)$ is a *contradiction*. There are no real values of x for which it is true.

$x^2 - 2x - 15 = x^2 - 2x - 14$

$-15 \neq -14$

14. $x^2 - 8x + 5 = (x-4)^2 - 11$ is an *identity* since $(x-4)^2 - 11 = x^2 - 8x + 16 - 11 = x^2 - 8x + 5$. It is true for all real values of x.

15. $(x+6)^2 = (x+8)(x+2)$ is a *conditional*. There are real values of x for which the equation is not true (for example, $x = 0$).

16. $(x+1)(x-5) = (x+3)(x-1)$ is a *conditional*. There are real values of x for which the equation is not true (for example, $x = 0$).

17. $3 + \dfrac{1}{x+1} = \dfrac{4x}{x+1}$ is *conditional*. There are real values of x for which the equation is not true (for example, $x = 0$).

18. $\dfrac{5}{x} + \dfrac{3}{x} = 24$ is *conditional*. There are real values of x for which the equation is not true (for example, $x = 1$).

$$\sqrt[3]{\frac{-19-8}{3}} \stackrel{?}{=} -1$$

$$\frac{\sqrt[3]{-27}}{3} \stackrel{?}{=} -1$$

$$\frac{-3}{3} = -1$$

$x = -19$ is a solution.

19. Method 1:

$$\frac{3x}{8}-\frac{4x}{3}=4$$
$$\frac{9x-32x}{24}=4$$
$$-23x=96$$
$$x=-\frac{96}{23}$$

Method 2: Graph $y_1=\frac{3x}{8}-\frac{4x}{3}$ and $y_2=4$ in the same viewing window. These lines intersect at $x\approx-4.1739\approx-\frac{96}{23}$.

20. Method 1:

$$\frac{3z}{8}-\frac{z}{10}=6$$
$$z\left(\frac{3}{8}-\frac{1}{10}\right)=6$$
$$z\left(\frac{22}{80}\right)=6$$
$$z=\frac{6(80)}{22}=\frac{240}{11}\approx21.8182$$

Method 2: Graph $y_1=\frac{3x}{8}-\frac{x}{10}$ and $y_2=6$ in the same viewing window. The lines intersect at $x\approx21.8182\approx\frac{240}{11}$.

21. Method 1:

$$\frac{2x}{5}+5x=\frac{4}{3}$$
$$\frac{2x+25x}{5}=\frac{4}{3}$$
$$27x=\frac{20}{3}$$
$$x=\frac{20}{3(27)}=\frac{20}{81}$$

Method 2: Graph $y_1=\frac{2x}{5}+5x$ and $y_2=\frac{4}{3}$ in the same viewing window. These lines intersect at $x\approx0.2469\approx\frac{20}{81}$.

22. Method 1:

$$\frac{4y}{3}-2y=\frac{16}{5}$$
$$\frac{4y-6y}{3}=\frac{16}{5}$$

48

23.

$$3x-5=2x+7$$
$$3x-2x=7+5$$
$$x=12$$

24.

$$5x+3=6-2x$$
$$5x+2x=6-3$$
$$7x=3$$
$$x=\frac{3}{7}$$

25.

$$3(y-5)=3+5y$$
$$3y-15=3+5y$$
$$-18=2y$$
$$y=-9$$

26.

$$4(z-3)+3z=1+8z$$
$$4z-12+3z=1+8z$$
$$7z-12=1+8z$$
$$-z=13$$
$$z=-13$$

27.

$$\frac{x}{5}-\frac{x}{2}=3$$
$$\frac{2x-5x}{10}=3$$
$$-3x=30$$
$$x=-10$$

28.

$$\frac{3x}{4}+\frac{x}{2}=-5$$
$$4\left(\frac{3x}{4}+\frac{x}{2}\right)=4(-5)$$
$$3x+2x=-20$$
$$5x=-20$$
$$x=-4$$

29.

$$\frac{5x-4}{5x+4}=\frac{2}{3}$$
$$3(5x-4)=2(5x+4)$$
$$15x-12=10x+8$$
$$5x=20$$
$$x=4$$

30.

$$\frac{10x+3}{5x+6}=\frac{1}{\ }$$

5

$$y = -\frac{24}{5}$$

Method 2: Graph $y_1 = \frac{4x}{3} - 2x$ and $y_2 = \frac{16}{5}$ in the same viewing window. These lines intersect at $x = -4.8 = -\frac{24}{5}$.

$$20x + 6 = 5x + 6$$
$$15x = 0$$
$$x = 0$$

31.
$$\begin{aligned}
\frac{2}{5}(z-4)+\frac{3z}{10}&=4z\\
\frac{2}{5}z-\frac{8}{5}+\frac{3z}{10}&=4z\\
\frac{7z}{10}-\frac{8}{5}&=4z\\
-\frac{8}{5}&=\frac{33}{10}z\\
-\frac{16}{33}&=z
\end{aligned}$$

32.
$$\begin{aligned}
\frac{3x}{2}+\frac{1}{4}(x-2)&=10\\
(4)\left(\frac{3x}{2}\right)+(4)\frac{1}{4}(x-2)&=(4)10\\
6x+(x-2)&=40\\
7x-2&=40\\
7x&=42\\
x&=6
\end{aligned}$$

33.
$$\begin{aligned}
\frac{17+y}{y}+\frac{32+y}{y}&=100\\
(y)\frac{17+y}{y}+(y)\frac{32+y}{y}&=100(y)\\
17+y+32+y&=100y\\
49+2y&=100y\\
49&=98y\\
\frac{1}{2}&=y
\end{aligned}$$

34.
$$\begin{aligned}
\frac{x-11}{x}&=\frac{x-9}{x}+2\\
\frac{x-11}{x}&=\frac{x-9+2x}{x}\\
\frac{x-11}{x}&=\frac{3x-9}{x}\\
x-11&=3x-9\\
-2&=2x\\
-1&=x
\end{aligned}$$

36.
$$\begin{aligned}
\frac{1}{x-2}+\frac{3}{x+3}&=\frac{4}{x^2+x-6}\\
\left(x^2+x-6\right)\frac{1}{x-2}+\left(x^2+x-6\right)\frac{3}{x+3}&=\left(x^2+x-6\right)\frac{4}{x^2+x-6}\\
(x+3)+3(x-2)&=4\\
x+3+3x-6&=4\\
4x-3&=4\\
4x&=7\\
x&=\frac{7}{4}
\end{aligned}$$

37.
$$\begin{aligned}
\frac{1}{x}+\frac{2}{x-5}&=0\\
1(x-5)+2x&=0\\
3x-5&=0\\
3x&=5\\
x&=\frac{5}{3}
\end{aligned}$$

38.
$$\begin{aligned}
3&=2+\frac{2}{z+2}\\
1&=\frac{2}{z+2}\\
z+2&=2\\
z&=0
\end{aligned}$$

39.
$$\begin{aligned}
\frac{2}{(x-4)(x-2)}&=\frac{1}{x-4}+\frac{2}{x-2}\\
(x-4)(x-2)\left[\frac{2}{(x-4)(x-2)}\right]&=\left[\frac{1}{x-4}+\frac{2}{x-2}\right](x-4)(x-2)\\
2&=1(x-2)+2(x-4)\\
2&=x-2+2x-8\\
12&=3x\\
4&=x
\end{aligned}$$

In the original equation, $x=4$ yields a denominator of 0. So, $x=4$ is an extraneous solution, and the original equation has no solution.

40.
$$\frac{2}{x(x-2)}+\frac{5}{x}=\frac{1}{x-2}$$

35. $\dfrac{1}{x-3} + \dfrac{1}{x+3} = \dfrac{10}{x^2-9}$

$$\frac{(x+3)+(x-3)}{x^2-9} = \frac{10}{x^2-9}$$

$$2x = 10$$
$$x = 5$$

$$x(x-2)\left[\frac{2}{x(x-2)} + \frac{5}{x}\right] = \left[\frac{1}{x-2}\right]x(x-2)$$

$$2 + 5(x-2) = x$$
$$2 + 5x - 10 = x$$

$$4x = 8$$
$$x = 2$$

In the original equation, $x = 2$ yields a denominator of 0. So, $x = 2$ is an extraneous solution, and the original equation has no solution.

41. $\dfrac{3}{x(x-3)} + \dfrac{4}{x} = \dfrac{1}{x-3}$

$3 + 4(x-3) = x$

$3 + 4x - 12 = x$

$3x = 9$

$x = 3$

In the original equation, $x = 3$ yields a denominator of 0. So, $x = 3$ is an extraneous solution, and the original equation has no solution.

42. $\dfrac{6}{x} - \dfrac{2}{x+3} = \dfrac{3(x+5)}{x(x+3)}$

$x(x+3)\dfrac{6}{x} - x(x+3)\dfrac{2}{x+3} = x(x+3)\dfrac{3(x+5)}{x(x+3)}$

$6(x+3) - 2x = 3(x+5)$

$6x + 18 - 2x = 3x + 15$

$4x + 18 = 3x + 15$

$x = -3$

In the original equation, $x = -3$ yields a denominator of 0. Thus, $x = -3$ is an extraneous solution, and the original equation has no solution.

43. $A = \dfrac{1}{2}bh$

$2A = bh$

$\dfrac{2A}{b} = h$

44. $A = \dfrac{1}{2}(a+b)h$

$2A = ah + bh$

$2A - ah = bh$

$\dfrac{2A - ah}{h} = b$

45. $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

$P = \dfrac{A}{\left(1 + \dfrac{r}{n}\right)^{nt}}$

47. $V = \pi r^2 h$

$h = \dfrac{V}{\pi r^2}$

48. $V = \dfrac{1}{3}\pi r^2 h$

$3V = \pi r^2 h$

$h = \dfrac{3V}{\pi r^2}$

49. Female: $y = 0.386x - 19.20$

For $y = 43$,

$43 = 0.386x - 19.20$

$62.2 = 0.386x$

$161.14 \approx x.$

The height of the female is about 161.14 centimeters.

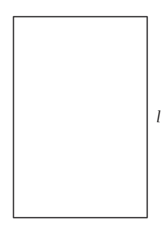
50. Male: $y = 0.442x - 29.37$

$48 \stackrel{?}{=} 0.442(175) - 29.37$

$48 \approx 47.98$

Yes, the estimated height of a male with a 48-centimeter thigh bone is about 175 centimeters.

51. (a)



(b) $l = 1.5w$

$P = 2l + 2w = 2(1.5w) + 2w = 5w$

(c) $25 = 5w \Rightarrow w = 5$ m and $l = (1.5)(5) = 7.5$ m

Dimensions: $7.5 \text{ m} \times 5 \text{ m}$

52. (a)

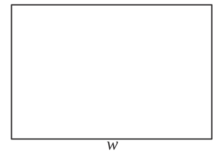
h

$$P = A\left(1 + \frac{r}{n}\right)^{-nt}$$

46. $A = P + Prt$

$$A - P = Prt$$

$$r = \frac{A - P}{Pt}$$



(b) $h = \frac{2}{3}w$

$$P = 2h + 2w = 2\left(\frac{2}{3}w\right) + 2w = \frac{4}{3}w + 2w = \frac{10}{3}w$$

(c) $P = 3 = \frac{10}{3}w \Rightarrow w = \frac{9}{10} = 0.9 \text{ m}$ and

$h = \frac{2}{3}(0.9) = 0.6 \text{ m}$

Dimensions: $0.6 \text{ m} \times 0.9 \text{ m}$

53. (a) $\text{Test Average} = \dfrac{\text{test 1} + \text{test 2} + \text{test 3} + \text{test 4}}{4}$

(b) $\text{Test Average} = \dfrac{\text{test 1} + \text{test 2} + \text{test 3} + \text{test 4}}{4}$

$$90 = \frac{93 + 91 + 84 + x}{4}$$

$$90 = \frac{268 + x}{4}$$

$$360 = 268 + x$$

$$92 = x$$

You must earn at least 92 points on the fourth test to earn an A in the course.

54. Sales: Monday, \$150

Tuesday, \$125

Wednesday, \$75

Thursday, \$180

Friday, \$$x$

Average: $\dfrac{150 + 125 + 75 + 180 + x}{5} = 150$

$$\frac{530 + x}{5} = 150$$

$$530 + x = 750$$

$$x = \$220$$

55. *Model:* $\text{distance} = (\text{rate})(\text{time})$

The salesperson drove 50 km in a half hour, therefore the rate is $\dfrac{50\text{ km}}{1/2\text{ hr}} = 100\dfrac{\text{km}}{\text{hr}}$.

Since the salesperson continues at the same rate to travel a total distance of 250 km, the time required is $\dfrac{\text{distance}}{\text{rate}} = \dfrac{250\text{ km}}{100\text{ km/hr}} = 2.5$ hours.

56. *Model:* $\text{distance} = (\text{rate})(\text{time})$

$\text{Total distance} = (\text{rate \#1})(\text{time \#1}) + (\text{rate \#2})(\text{time \#2})$

$$336 = \left(58\frac{\text{mi}}{\text{hr}}\right)t + \left(52\frac{\text{mi}}{\text{hr}}\right)(6 - t)$$

$$336 = 58t + 312 - 52t$$

$$24 = 6t$$

$$4 = t$$

The salesperson traveled for 4 hours at 58 mph and then 2 hours at 52 mph.

57. *Model:* $(\text{Distance}) = (\text{rate})(\text{time}_1 + \text{time}_2)$

Labels: Distance $= 2 \cdot 200 = 400$ miles, rate $= r$,

$\text{time}_1 = \dfrac{\text{distance}}{\text{rate}_1} = \dfrac{200}{55}$ hours,

$\text{time}_2 = \dfrac{\text{distance}}{\text{rate}_2} = \dfrac{200}{40}$ hours

Equation: $400 = r\left(\dfrac{200}{55} + \dfrac{200}{40}\right)$

$$400 = r\left(\frac{1600}{440} + \frac{2200}{440}\right) = \frac{3800}{440}r$$

$$43.6 \approx r$$

The average speed for the round trip was approximately 46.3 miles per hour.

58. $\text{Rate} = \dfrac{\text{Distance}}{\text{Time}} = \dfrac{50\text{ kilometers}}{\frac{1}{2}\text{ hours}} = 100\text{ kilometers/hour}$

$\text{Total time} = \dfrac{\text{Total distance}}{\text{Rate}} = \dfrac{300\text{ kilometers}}{100\text{ kilometers/hour}}$

$= 3$ hours

59. Let x = height of the pine tree.

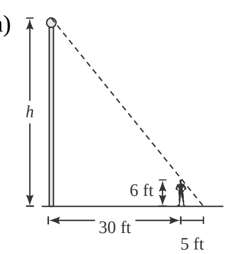
$$\frac{x}{20} = \frac{36}{24}$$

$$24x = 720$$

$$x = 30\text{ feet}$$

The pine tree is approximately 30 feet tall.

60. (a)



(b) *Model:* $\dfrac{(\text{height of pole})}{(\text{height of pole's shadow})} = \dfrac{(\text{height of person})}{\text{height of person's shadow}}$

Labels: height of pole $= h$,

height of pole's shadow $= 30 + 5 = 35$ feet,

height of person $= 6$ feet,

height of person's shadow $= 5$ feet

Equation: $\dfrac{h}{35} = \dfrac{6}{5}$

$h = \dfrac{6}{5} \cdot 35 = 42$

The pole is 42 feet tall.

61. $I = Prt$

$$200 = 8000r(2)$$
$$200 = 16{,}000r$$
$$r = \frac{200}{16{,}000}$$
$$r = 0.0125, \text{ or } 1.25\%$$

62. Let $x =$ amount in $4\frac{1}{2}\%$ fund.

Then $12{,}000 - x =$ amount in 5% fund.

$$560 = 0.045x + 0.05(12{,}000 - x)$$
$$560 = 0.045 + 600 - 0.05x$$
$$0.005x = 40$$
$$x = 8000$$

You must invest \$8000 in the $4\frac{1}{2}\%$ fund and

$12{,}000 - 8000 = \$4000$ in the 5% fund.

63. *Model*: Total pounds at \$5.25 = pounds at \$2.50 + pounds at \$8.00

Labels: $x =$ pounds of peanuts at \$2.50

$100 - x =$ pounds of walnuts at \$8.00

Equation: $100(5.25) = x(2.50) + (100 - x)(8.00)$

$$525 = 2.5x + 800 - 8x$$
$$275 = 5.5x$$
$$50 = x$$

The mixture contains 50 pounds at \$2.50 and $100 - 50 = 50$ pounds at \$8.00.

64. Initially, the forester has $\frac{64}{33}$ gallons of gas and $\frac{2}{33}$ gallons of oil.

$$\frac{64}{33} + \frac{2}{33} = 2$$
$$\frac{64/33}{2/33} = 32$$

Suppose she adds x gallons of gas.

$$\frac{64/33 + x}{2/33} = \frac{40}{1}$$

which gives $x = \frac{16}{33}$ gallon.

66. Let $x =$ amount invested in 8×10 frames, and

$y =$ amount invested in 5×7 frames.

$$x + y = 4500 \Rightarrow y = 4500 - x$$
$$0.25x + 0.22y = 0.24(4500) = 1080$$
$$0.25x + 0.22(4500 - x) = 1080$$
$$0.03x = 90$$
$$x = 3000$$

So, \$3000 is invested in 8×10 frames and $\$4500 - \$3000 = \$1500$ is invested in 5×7 frames.

67. $A = \frac{1}{2}bh$

$$h = \frac{2A}{b} = \frac{2(182.25)}{13.5} = 27 \text{ ft}$$

68. Let $x =$ length of side of square I, and

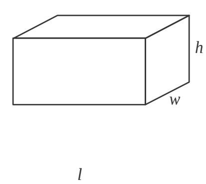
$y =$ length of side of square II.

$$4x = 20 \Rightarrow x = 5$$
$$4y = 32 \Rightarrow y = 8$$

Hence, square III has side of length $5 + 8 = 13$.

Area $= 13^2 = 169$ square inches

69. (a)



(b) $l = 3w,\ h = \left(1\frac{1}{2}\right)w$

$$V = lwh = (3w)(w)\left(\tfrac{3}{2}w\right) = 2304$$
$$\tfrac{9}{2}w^3 = 2304$$
$$w^3 = 512$$
$$w = 8 \text{ inches}$$

Dimensions: $24 \times 8 \times 12$ inches

70. $V = \frac{4}{3}\pi r^3$

$$\frac{4}{3}\pi r^3 = 6255$$
$$r^3 = \frac{18{,}765}{4\pi}$$

65.

M

odel: Total profit = profit on notebooks + profit on tablet

Labels:

x = amount invested in notebook computers

$40{,}000 - x$ = amount invested in tablet computers

Equation: $(0.24)(40{,}000) = 0.20x + 0.25(40{,}000 - x)$

$$\begin{aligned} 9600 &= 0.2x + 10{,}000 - 0.25x \\ -400 &= -0.05x \\ 8000 &= x \end{aligned}$$

So, \$8000 is invested in notebook computers and $\$40{,}000 - \$8000 = \$32{,}000,$ invested in tablet computers.

$$r = \sqrt[3]{\frac{18{,}765}{4\pi}} \approx 11.43 \text{ cm}$$

71. Solve the temperature for C.

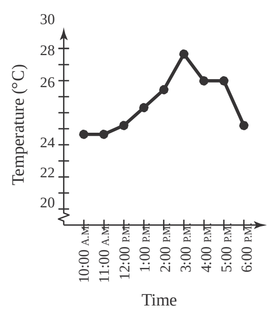
$$F = \frac{9}{5}C + 32$$
$$F - 32 = \frac{9}{5}C$$
$$\frac{5}{9}(F - 32) = C$$

$$F = 73°: C = \frac{5}{9}(73 - 32) \Rightarrow C \approx 22.8°$$

$$F = 74°: C = \frac{5}{9}(74 - 32) \Rightarrow C \approx 23.3°$$

9

$$F = 76°: C = \frac{5}{9}(76 - 32) \Rightarrow C \approx 24.4°$$
$$F = 78°: C = \frac{5}{9}(78 - 32) \Rightarrow C \approx 25.6°$$
$$F = 82°: C = \frac{5}{9}(82 - 32) \Rightarrow C \approx 27.8°$$
$$F = 79°: C = \frac{5}{9}(79 - 32) \Rightarrow C \approx 26.1°$$
$$F = 74°: C = \frac{5}{9}(74 - 32) \Rightarrow C \approx 23.3°$$



72. Solve the temperature for C.

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C$$

$$\frac{5}{9}(F - 32) = C$$

$$C = \frac{5}{9}(74.3 - 32)$$

$$C = \frac{5}{9}(42.3)$$

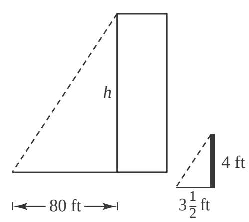
$$C = 23.5°$$

73. Let $x =$ the wind speed, then the rate to the city $= 600 + x$, the rate from the city $= 600 - x$, the distance to the city $= 1500$ kilometers, the distance traveled so far in the return trip $= 1500 - 300 = 1200$ kilometers.

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$
$$\frac{1500}{600 + x} = \frac{1200}{600 - x}$$
$$1500(600 - x) = 1200(600 + x)$$
$$900{,}000 - 1500x = 720{,}000 + 1200x$$
$$180{,}000 = 2700x$$
$$\frac{200}{3} = x$$

Wind speed: $\frac{200}{3}$ km/h

74. Let $h =$ height of the building in feet.



Not drawn to scale

$$\frac{h \text{ feet}}{80 \text{ feet}} = \frac{4 \text{ feet}}{3.5 \text{ feet}}$$
$$\frac{h}{80} = \frac{4}{3.5}$$
$$3.5h = 320$$
$$h \approx 91.4 \text{ feet}$$

75.
$$W_1 x = W_2(L - x)$$
$$50x = 75(10 - x)$$
$$50x = 750 - 75x$$
$$125x = 750$$
$x = 6$ feet from the 50-pound child

76.
$$W_1 x = W_2(L - x)$$
$W_1 = 200$ pounds

$W_2 = 550$ pounds

$L = 5$ feet
$$200x = 550(5 - x)$$
$$200x = 2750 - 550x$$

$750x = 2750$

x

3

=

1

1

f

e

e

t

77. False. $x(3-x)=10$ is a quadratic equation.
$-x^2+3x=10$ or $x^2-3x+10=0$

78. False.
Volume of cube $=(9.5)^3=857.375$ cubic inches
Volume of sphere $=\frac{4}{3}\pi(5.9)^3\approx 860.29$ cubic inches

79. You need $a(-3)+b=c(-3)$ or $b=(c-a)(-3)=3(a-c)$. One answer is $a=2$, $c=1$ and $b=3$ $(2x+3=x)$ and another is $a=6, c=1,$ and $b=15$ $(6x+15=x)$.

80. You need $a(0)+b=c$ or $b=c$. So, a can be any real number except 0. One answer is $a=1$ and $b=c=4: x+4=4$.

81. You need $a\left(\frac{1}{4}\right)+b=c$ or $a+4b=4c$. One answer is $a=2$ and $b=\frac{1}{2}$. So, $2+4\left(\frac{1}{2}\right)=4=4c\Rightarrow c=1$.

The equation is $2x+\frac{1}{2}=1$.

82. You need $a(-2.5)+b=c$. One answer is $a=-1$ and $b=2$. So, $-1(-2.5)+2=4.5=c$.

The equation is $-x+2=4.5$.

83. In the original equation, $x=1$ yields a denominator of zero. So, $x=1$ is an extraneous solution and therefore cannot be a solution to the equation.

84. (a)

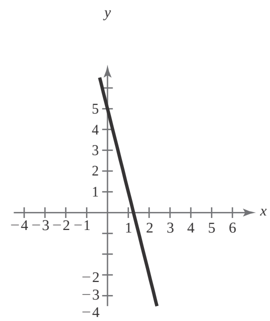
$$\frac{\text{Height of building}}{\text{Length of building's shadow}}=\frac{\text{Height of pole}}{\text{Length of pole's shadow}}$$

(b) $\frac{x}{30}=\frac{4}{3}$

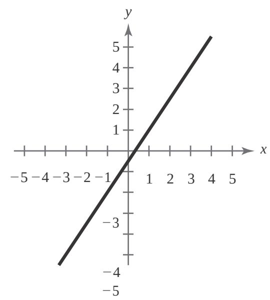
85. $\frac{6}{(x-3)(x-1)}=\frac{3}{x-3}+\frac{4}{x-1}$

To clear this equation of fractions, find the least common denominator (LCD) of all terms in the equation and multiply every term by this LCD. It is possible to introduce an extraneous solution because you are multiplying by a

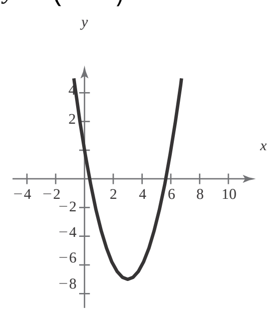
87. $y=5-4x$



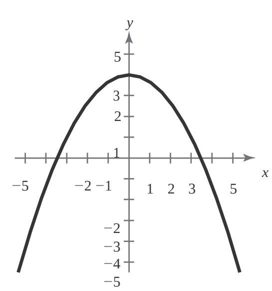
88. $y=\frac{3x-5}{2}+2=\frac{3}{2}x-\frac{5}{2}+2=\frac{3}{2}x-\frac{1}{2}$



89. $y=(x-3)^2-7$



90. $y=4-\frac{1}{3}x^2$



91.

86.

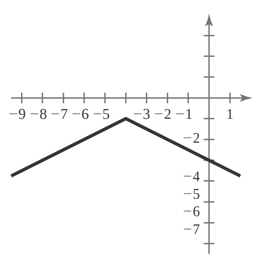
variable. $y = -\,_2\, x + 4 - 1$

y
3
2
1
x

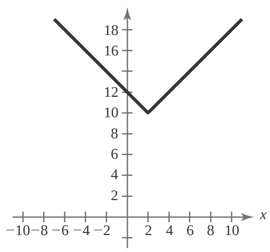
To determine whether a solution is extraneous, substitute the answer for the variable in the original equation or graph the original equation.

$5x + 2c = 12 + 4x - 2c,\ x = 2$

$$\begin{aligned} 5(2) + 2c &= 12 + 4(2) - 2c \\ 10 + 2c &= 20 - 2c \\ 4c &= 10 \\ c &= \tfrac{5}{2} \end{aligned}$$



92. $y = |x-2| + 10$



Section 2.2 Solving Equations Graphically

1. x-intercept, y-intercept

2. zero

3. The x-intercepts of $y = f(x)$ are $(-1, 0)$ and $(1, 0)$.

4. The y-intercept of $y = g(x)$ is $(0, -1)$.

5. The zeros of the function f are $x = -1$ and $x = 1$.

6. The solutions of the equation $f(x) = g(x)$ are

$x = -\frac{1}{2}$ and $x = 1$.

7. $y = x - 5$

Let $y = 0$: $0 = x - 5 \Rightarrow (5, 0)$ x-intercept

Let $x = 0$: $y = 0 - 5 \Rightarrow y = -5 \Rightarrow (0, -5)$ y-intercept

8. $y = -\frac{3}{4}x - 3$

Let $y = 0$:

$0 = -\frac{3}{4}x - 3 \Rightarrow \frac{3}{4}x = -3 \Rightarrow x = -4 \Rightarrow (-4, 0)$ x-intercept

Let $x = 0$:

$y = -\frac{3}{4}(0) - 3 = -3 \Rightarrow (0, -3)$ y-intercept

9. $y = x^2 + 2x + 2$

Let $y = 0$: $x^2 + 2x + 2 = 0 \Rightarrow$ no x-intercepts

Let $x = 0$:

$y = (0)^2 + 2(0) + 2 = 2 \Rightarrow (0, 2)$ y-intercept

11. $y = x\sqrt{x+2}$

Let $y = 0$:

$0 = x\sqrt{x+2} \Rightarrow x = 0, -2 \Rightarrow (0, 0), (-2, 0)$ x-intercepts

Let $x = 0$:

$y = 0\sqrt{0+2} = 0 \Rightarrow (0, 0)$ y-intercept

12. $y = -\frac{1}{2}x\sqrt{x+3} + 1$

Let $y = 0$:

$0 = -\frac{1}{2}x\sqrt{x+3} + 1 \Rightarrow \frac{1}{2}x\sqrt{x+3} = 1 \Rightarrow x\sqrt{x+3} = 2$

$\Rightarrow x^2(x+3) = 4 \Rightarrow x^3 + 3x^2 - 4 = 0$

$\Rightarrow (x-1)(x^2 + 4x + 4) = 0 \Rightarrow (x-1)(x+2)^2 = 0$

$\Rightarrow x = 1 \Rightarrow (1, 0)$ ($x = -2$ is impossible)

Let $x = 0 \Rightarrow y = 1 \Rightarrow (0, 1)$ y-intercept

13. $y = \dfrac{4x - 8}{x}$

Let $y = 0$:

$0 = \dfrac{4x - 8}{x}$

$0 = 4x - 8$

$8 = 4x$

$2 = x \Rightarrow (2, 0)$ x-intercept

Let $x = 0$:

$y = \dfrac{4(0) - 8}{0}$ is impossible. No y-intercepts

10. $y = 4 - x^2$

Let $y = 0$:

$0 = 4 - x^2 \Rightarrow x = 2, \ -2 \Rightarrow (2,\ 0),\ (-2,\ 0)$ x-intercepts

Let $x = 0$:

$y = 4 - 0^2 = 4 \Rightarrow (0,\ 4)$ y-intercept

14. $y = \frac{3x - 1}{4x}$

Let $y = 0$:

$0 = 3x - 1 \Rightarrow x = \frac{1}{3} \Rightarrow \left(\frac{1}{3},\ 0\right)$ x-intercept

Let $x = 0$: $0 = -1$ is impossible. No y-intercepts

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15. $xy - 2y - x + 1 = 0$

Let $y = 0$:

$0 = -x + 1 = 0 \Rightarrow x = 1 \Rightarrow (1,\ 0)$ x-intercept

Let $x = 0$:

$-2y + 1 = 0 \Rightarrow y = \frac{1}{2} \Rightarrow \left(0,\ \frac{1}{2}\right)$ y-intercept

16. $xy - x + 4y = 3$

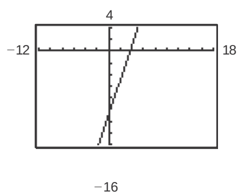
Let $y = 0$:

$$x(0) - x + 4(0) = 3$$
$$-x = 3$$
$$x = -3 \Rightarrow (-3, 0)\ x\text{-intercept}$$

Let $x = 0$:

$$(0)y - (0) + 4y = 3$$
$$4y = 3$$
$$y = \frac{3}{4} \Rightarrow \left(0, \frac{3}{4}\right)\ y\text{-intercept}$$

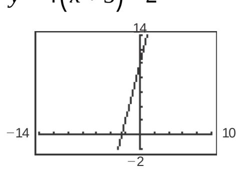
17. $y = 3(x - 2) - 5$



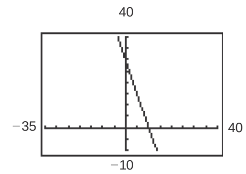
x-intercept: $3(x - 2) - 5 = 0$
$$3x - 6 - 5 = 0$$
$$3x = 11$$
$$x = \frac{11}{3} \Rightarrow \left(\frac{11}{3}, 0\right)$$

y-intercept: $y = 3(0 - 2) - 5$
$$y = -6 - 5$$
$$y = -11 \Rightarrow (0, -11)$$

18. $y = 4(x + 3) - 2$



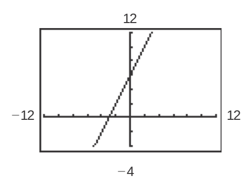
19. $y = 20 - (3x - 10)$



x-intercept: $0 = 20 - (3x - 10)$
$$0 = 20 - 3x + 10$$
$$0 = 30 - 3x$$
$$3x = 30$$
$$x = 10 \Rightarrow (10,\ 0)$$

y-intercept: $y = 20 - [3(0) - 10]$
$$y = 30 \Rightarrow (0,\ 30)$$

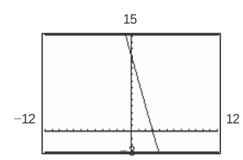
20. $y = 10 + 2(x - 2)$



x-intercept: $0 = 10 + 2(x - 2)$
$$0 = 10 + 2x - 4$$
$$0 = 6 + 2x$$
$$-2x = 6$$
$$x = -3 \Rightarrow (-3, 0)$$

y-intercept: $y = 10 + 2(0 - 2)$
$$y = 6 \Rightarrow (0,\ 6)$$

21. $f(x) = 4(3 - x)$
$$4(3 - x) = 0$$
$$3 - x = 0$$
$$x = 3$$



x-intercept: $0 = 4(x+3) - 2$

$0 = 4x + 10$

$4x = -10$

$x = -\frac{5}{2} \Rightarrow \left(-\frac{5}{2},\ 0\right)$

y-intercept: $y = 4(0+3) - 2$

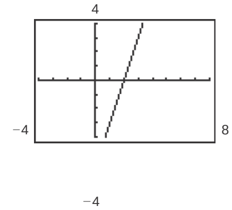
$y = 10 \Rightarrow (0,\ 10)$

22. $f(x) = 3(x-5) + 9$

$3x - 15 + 9 = 0$

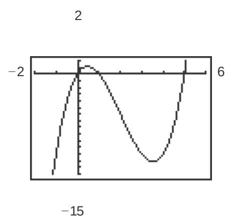
$3x = 6$

$x = 2$



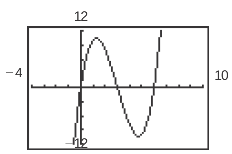
23. $f(x) = x^3 - 6x^2 + 5x$

$$x^3 - 6x^2 + 5x = 0$$
$$x(x^2 - 6x + 5) = 0$$
$$x(x-5)(x-1) = 0$$
$$x = 0,\ 5,\ 1$$



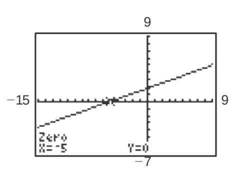
24. $f(x) = x^3 - 9x^2 + 18x$

$$x^3 - 9x^2 + 18x = 0$$
$$x(x-3)(x-6) = 0$$
$$x = 0,\ 3,\ 6$$



25. $f(x) = \dfrac{x+1}{2} - \dfrac{x-2}{7} + 1$

$$\frac{x+1}{2} - \frac{x-2}{7} + 1 = 0$$
$$7(x+1) - 2(x-2) + 14 = 0$$
$$7x + 7 - 2x + 4 + 14 = 0$$
$$5x + 25 = 0$$
$$5x = -25$$
$$x = -5$$



26. $f(x) = x - 3 - \dfrac{10}{x}$

$$x - 3 - \frac{10}{x} = 0$$
$$x^2 - 3x - 10 = 0$$
$$(x-5)(x+2) = 0$$
$$x = 5,\ -2$$

4

−6 6

27. $2.7x - 0.4x = 1.2$

$$2.3x = 1.2$$
$$x = \frac{1.2}{2.3} = \frac{12}{23} \approx 0.522$$
$$f(x) = 2.7x - 0.4x - 1.2 = 0$$
$$2.3x - 1.2 = 0$$
$$x \approx 0.522$$

28. $3.6x - 8.2 = 0.5x$

$$3.1x = 8.2$$
$$x = \frac{8.2}{3.1} = \frac{82}{31}$$
$$f(x) = 3.6x - 8.2 - 0.5x = 0$$
$$3.1x - 8.2 = 0$$
$$x = \frac{82}{31}$$

29. $12(x+2) = 15(x-4) - 3$

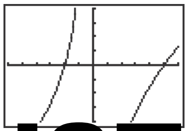
$$12x + 24 = 15x - 60 - 3$$
$$-3x = -87$$
$$x = 29$$
$$f(x) = 12(x+2) - 15(x-4) + 3 = 0$$
$$-3x + 87 = 0$$
$$x = 29$$

30. $1200 = 300 + 2(x - 500)$

$$900 = 2x - 1000$$
$$1900 = 2x$$
$$x = 950$$
$$f(x) = 300 + 2(x-500) - 1200 = 0$$
$$300 + 2x - 1000 - 1200 = 0$$
$$2x - 1900 = 0$$
$$x = 950$$

31. $\dfrac{3x}{2} + \dfrac{1}{4}(x+2) = 10$

$$\frac{6x}{4} + \frac{x}{4} = 10 - \frac{1}{2}$$
$$\frac{7x}{4} = \frac{19}{2}$$
$$x = \frac{38}{7}$$
$$f(x) = \frac{3x}{2} + \frac{1}{4}(x+2) - 10 = 0$$
$$\frac{3}{2}x + \frac{1}{4}x + \frac{1}{2} - 10 = 0$$
$$\frac{7}{4}x - \frac{19}{2} = 0$$
$$7x - 38 = 0$$
$$x = \frac{38}{7}$$



32. $\frac{2x}{3} + \frac{1}{2}(x-5) = 6$

$$\left(\frac{2}{3} + \frac{1}{2}\right)x = \frac{5}{2} + 6$$
$$\frac{7}{6}x = \frac{17}{2}$$
$$x = \frac{51}{7} \approx 7.286$$

$$f(x) = \frac{2}{3}x + \frac{1}{2}(x-5) - 6 = 0$$
$$4x + 3(x-5) - 36 = 0$$
$$4x + 3x - 15 - 36 = 0$$
$$7x - 51 = 0$$
$$x \approx 7.286$$

33. $0.60x + 0.40(100 - x) = 1.2$

$$0.60x + 40 - 0.40x = 1.2$$
$$0.20x = -38.8$$
$$x = -194$$

$$f(x) = 0.60x + 0.40(100 - x) - 1.2 = 0$$
$$0.60x + 40 - 0.40x - 1.2 = 0$$
$$0.20x + 38.8 = 0$$
$$x = -194$$

34. $0.75x + 0.2(80 - x) = 3.9$

$$0.75x + 16 - 0.2x - 3.9 = 0$$
$$0.55x + 12.1 = 0$$
$$55x = -1210$$
$$x = -22$$

$$f(x) = 0.75x + 0.2(80 - x) - 3.9 = 0$$
$$0.55x + 12.1 = 0$$
$$55x + 1210 = 0$$
$$x = -22$$

35. $\frac{x-3}{3} = \frac{3x-5}{2}$

$$2(x-3) = 3(3x-5)$$
$$2x - 6 = 9x - 15$$
$$9 = 7x$$
$$x = \frac{9}{7} \approx 1.286$$

$$f(x) = \frac{x-3}{3} - \frac{3x-5}{2} = 0$$

36. $\frac{x-3}{25} = \frac{x-5}{12}$

$$12(x-3) = 25(x-5)$$
$$12x - 36 = 25x - 125$$
$$13x = 89$$
$$x = \frac{89}{13} \approx 6.846$$

$$f(x) = \frac{x-3}{25} - \frac{x-5}{12} = 0$$
$$12(x-3) - 25(x-5) = 0$$
$$12x - 36 - 25x + 125 = 0$$
$$-13x + 89 = 0$$
$$13x - 89 = 0$$
$$x \approx 6.846$$

37. $\frac{x-5}{4} + \frac{x}{2} = 10$

$$(x-5) + 2x = 40$$
$$3x = 45$$
$$x = 15$$

$$f(x) = \frac{x-5}{4} + \frac{x}{2} - 10 = 0$$
$$x - 5 + 2x - 40 = 0$$
$$3x - 45 = 0$$
$$x = 15$$

38. $\frac{x-5}{10} - \frac{x-3}{5} = 1$

$$(x-5) - 2(x-3) = 10$$
$$x - 5 - 2x + 6 = 10$$
$$-x = 9$$
$$x = -9$$

$$f(x) = \frac{x-5}{10} - \frac{x-3}{5} - 1 = 0$$
$$x - 5 - 2(x-3) - 10 = 0$$
$$x - 5 - 2x + 6 - 10 = 0$$
$$-x - 9 = 0$$
$$x + 9 = 0$$
$$x = -9$$

39. $(x+2)^2 = x^2 - 6x + 1$

$$x^2 + 4x + 4 = x^2 - 6x + 1$$
$$10x = -3$$
$$x = -\frac{3}{10}$$

$$x = -\tfrac{3}{} = -0.3$$

$$f(x) = (x+2)^2 - x^2 + 6x - 1 = 0$$

$$2(x-3) - 3(3x-5) = 0$$

$$\begin{aligned} 2x - 6 - 9x + 15 &= 0 \\ -7x + 9 &= 0 \\ 7x - 9 &= 0 \\ x &\approx 1.286 \end{aligned}$$

$$\begin{aligned} x^2 + 4x + 4 - x^2 + 6x - 1 &= 0 \\ 10x + 3 &= 0 \\ x &= -0.3 \end{aligned}$$

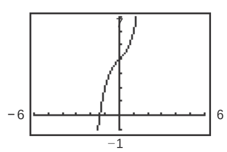
40. $(x+1)^2 + 2(x-2) = (x+1)(x-2)$

$$\begin{aligned} x^2 + 2x + 1 + 2x - 4 &= x^2 - x - 2 \\ 5x &= 1 \\ x &= \tfrac{1}{5} = 0.2 \end{aligned}$$

$$\begin{aligned} f(x) = (x+1)^2 + 2(x-2) - (x+1)(x-2) &= 0 \\ x^2 + 2x + 1 + 2x - 4 - (x^2 - x - 2) &= 0 \\ x^2 + 4x - 3 - x^2 + x + 2 &= 0 \\ 5x - 1 &= 0 \\ x &= 0.2 \end{aligned}$$

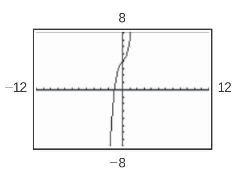
41. $x^3 + x + 4 = 0$

$x \approx -1.379$



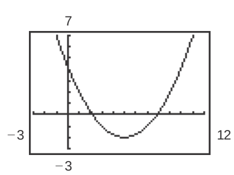
42. $2x^3 + x + 4 = 0$

$x \approx -1.128$



43. $\frac{1}{4}(x^2 - 10x + 17) = 0$

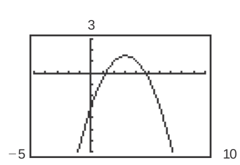
$x \approx 2.172,\ 7.828$



44. $-\frac{1}{2}(x^2 - 6x + 6) =$

0

$x \approx 1.268,\ 4.732$

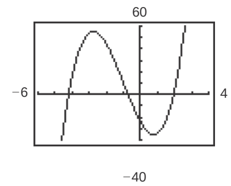


45. $2x^3 - x^2 - 18x + 9 = 0$

$x = -3.0,\ 0.5,\ 3.0$

46. $4x^3 + 12x^2 - 26x - 24 = 0$

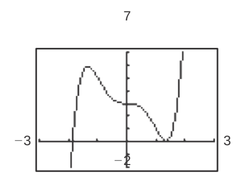
$x \approx -4.206,\ -0.735,\ 1.941$



47. $x^5 = 3x^3 - 3$

$x^5 - 3x^3 + 3 = 0$

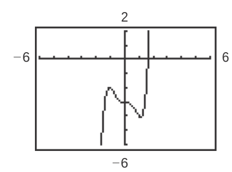
$x \approx -1.861$



48. $x^5 = 3 + 2x^3$

$x^5 - 3 - 2x^3 = 0$

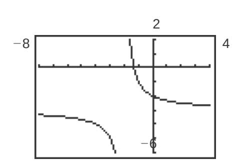
$x \approx 1.638$



49. $\dfrac{2}{x+2} = 3$

$\dfrac{2}{x+2} - 3 = 0$

$x \approx -1.333$

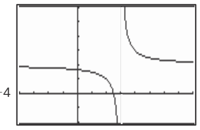


50. $\dfrac{1}{x-3} = -2$

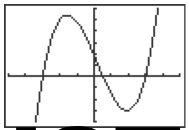
$\dfrac{1}{x-3} + 2 = 0$

$x = 2.5$

30



−4
8
−2

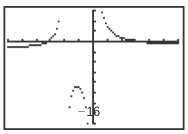


6
−5
5

51. $\frac{5}{x} = 1 + \frac{3}{x+2}$

$\frac{5}{x} - 1 - 3\frac{3}{x+2} = 0$

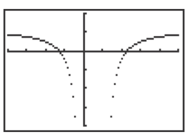
$x \approx -3.162,\ 3.162$



52. $\frac{3}{x} + 1 = \frac{3}{x+2}$

$\frac{3}{x} + 1 - \frac{3}{x-1} = 0$

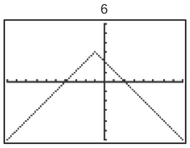
$x \approx -1.303,\ 2.303$



53. $-|x + 1| = -3$

$-|x + 1| + 3 = 0$

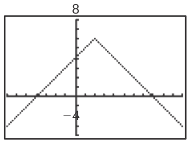
$x = -4, 2$



54. $-|x - 2| = -6$

$-|x - 2| + 6 = 0$

$x = -4, 8$



55. $|3x - 2| - 1 = 4$

$|3x - 2| - 5 = 0$

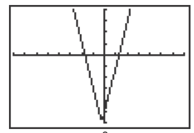
$x \approx -1.0,\ 2.333$

2

56. $|4x + 1| + 2 = 8$

$|4x + 1| - 6 = 0$

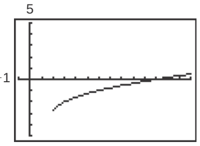
$x = -1.75,\ 1.25$



57. $\sqrt{x - 2} = 3$

$\sqrt{x - 2} - 3 = 0$

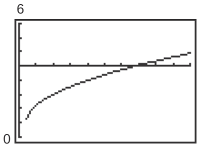
$x = 11$



58. $\sqrt{x - 4} = 8$

$\sqrt{x - 4} - 8 = 0$

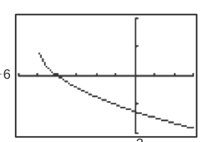
$x = 68$



59. $2 - \sqrt{x + 5} = 1$

$1 - \sqrt{x + 5} = 0$

$x = -4$

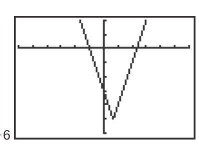


60. $8 - \sqrt{x + 9} = 6$

$2 - \sqrt{x + 9} = 0$

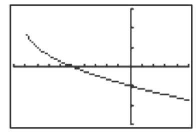
$x = -5$

3



−10 5

−3



61. (a)

x	-1	0	1	2	3	4
$3.2x - 5.8$	-9	-5.8	-2.6	0.6	3.8	7.0

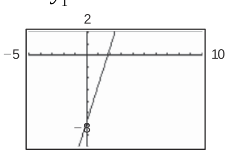
Because of the sign change, $1 < x < 2$.

(b)

x	1.5	1.6	1.7
$3.2x - 5.8$	-1	-0.68	-0.36

x	1.8	1.9	2.0
$3.2x - 5.8$	-0.04	0.28	0.6

Because of the sign change, $1.8 < x < 1.9$. To improve accuracy, evaluate the expression for values in this interval and determine where the sign changes.
Let $y_1 = 3.2x - 5.8$. The graph of y_1 crosses the x-axis at $x = 1.8125$.



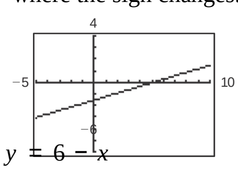
62. $0.3(x - 1.8) - 1 = 0$

x	2	3	4	5	6	7	8
$0.3(x - 1.8) - 1$	-0.94	-0.64	-0.34	-0.04	0.26	0.56	0.86

Because of the sign change, $5 < x < 6$.

x	5.0	5.1	5.2	5.3	5.4
$0.3(x - 1.8) - 1$	-0.04	-0.01	0.02	0.05	0.08

Because of the sign change, $5.1 < x < 5.2$. To improve accuracy, evaluate the expression in this interval, and determine where the sign changes. Let $y_1 = 0.3(x - 1.8) - 1$. The graph of y_1 crosses the x-axis at $x \approx 5.13$.



63. $y = 6 - x$

$y = 3x - 2$

$6 - x = 3x - 2$

$8 = 4x$

$x = 2 \Rightarrow y = 6 - 2 = 4$

$(x, y) = (2, 4)$

64. $y = 2x - 3$

$y = 9 - x$

$2x - 3 = 9 - x$

$3x = 12$

$x = 4 \Rightarrow y = 9 - 4 = 5$

$(x, y) = (4, 5)$

65. $2x + y = 6 \Rightarrow y = 6 - 2x$

$-x + y = 0 \Rightarrow y = x$

$6 - 2x = x$

$6 = 3x$

$x = 2 \Rightarrow y = x = 2$

$(x, y) = (2,\ 2)$

66. $x - y = -4 \Rightarrow x = y - 4$

$x + 2y = 5 \Rightarrow x = -2y + 5$

$y - 4 = -2y + 5$

$3y = 9$

$y = 3 \Rightarrow x = 3 - 4 = -1$

$(x, y) = (-1, 3)$

67. $x - y = 10 \Rightarrow y = x - 10$
$x + 2y = 4 \Rightarrow y = -\frac{1}{2}x + 2$

$x - 10 = -\frac{1}{2}x + 2$
$2x - 20 = -x + 4$
$3x = 24$
$x = 8 \Rightarrow y = 8 - 10 = -2$

$(x, y) = (8, -2)$

68. $4x - y = 4 \Rightarrow y = 4x - 4$
$x - 4y = 1 \Rightarrow y = \frac{1}{4}x - \frac{1}{4}$

$4x - 4 = \frac{1}{4}x - \frac{1}{4}$
$16x - 16 = x - 1$
$15x = 15$
$x = 1 \Rightarrow y = 4x - 4 = 0$
$(x, y) = (1, 0)$

69. $y = x^2 - x + 1$
$y = x^2 + 2x + 4$

$x^2 - x + 1 = x^2 + 2x + 4$
$-3 = 3x$
$x = -1$

$y = (-1)^2 - (-1) + 1 = 3$
$(x, y) = (-1, 3)$

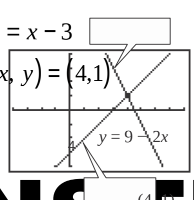
70. $y = -x^2 + 3x + 1$
$y = -x^2 - 2x - 4$
$-x^2 + 3x + 1 = -x^2 - 2x - 4$
$3x + 1 = -2x - 4$

$5x = -5$
$x = -1$
$y = -(-1)^2 + 3(-1) + 1 = -3$

$(x, y) = (-1, -3)$

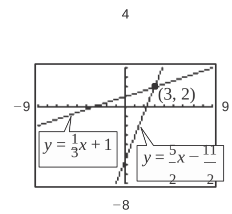
71. $y = 9 - 2x$

$y = x - 3$
$(x, y) = (4,1)$



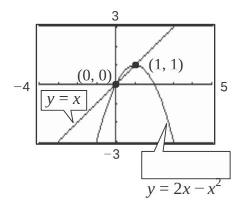
72. $x - 3y = -3 \Rightarrow y = \frac{1}{3}x + 1$

$5x - 2y = 11 \Rightarrow y = \frac{5}{2}x - \frac{11}{2}$
$(x, y) = (3, 2)$



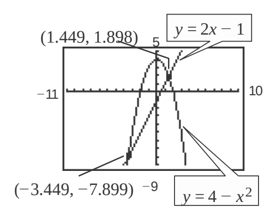
73. $y = x$

$y = 2x - x^2$
$(x, y) = (0, 0), (1, 1)$

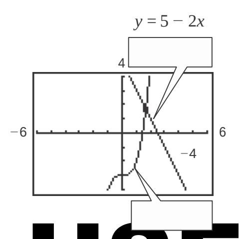


74. $y = 4 - x^2$
$y = 2x - 1$

$(x,y) = (1.449, 1.898),$
$(-3.449, -7.899)$



75. $x^3 - y = 3 \Rightarrow y = x^3 - 3$
$2x + y = 5 \Rightarrow y = 5 - 2x$
$(x,y) = (1.670, 1.660)$



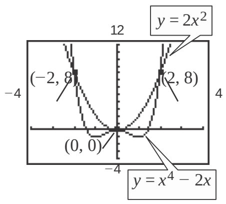
$-4 \quad y = x^3 - 3$

$-4 \quad y = x - 3$

76. $y = 2x^2$

$y = x^4 - 2x^2$

$(x, y) = (0, 0), (2, 8), (-2, 8)$



77. (a) $\dfrac{1 + 0.73205}{1 - 0.73205} = \dfrac{1.73205}{0.26795} \approx 6.464079 \approx 6.46$

(b) $\dfrac{1 + 0.73205}{1 - 0.73205} = \dfrac{1.73205}{0.26795} \approx \dfrac{1.73}{0.27} \approx 6.407407 \approx 6.41$

Yes, the more rounding performed, the less accurate the result.

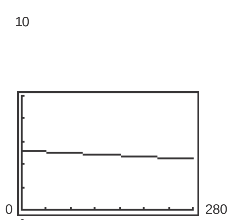
78. (a) $\dfrac{1 + 0.86603}{1 - 0.86603} = \dfrac{1.86603}{0.13397} \approx 13.92871538 \approx 13.93$

(b) $\dfrac{1 + 0.86603}{1 - 0.86603} = \dfrac{1.86603}{0.13397} \approx \dfrac{1.87}{0.13} \approx 14.38461538 \approx 14.38$

Yes, the more rounding performed, the less accurate the result.

79. (a) $t = \dfrac{x}{63} + \dfrac{(280 - x)}{54}$

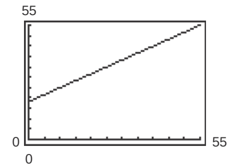
(b) Domain: $0 \le x \le 280$



(c) If the time was 4 hours and 45 minutes, then $t = 4\frac{3}{4}$ and $x = 164.5$ miles.

80. (a) $A = x + 0.33(55 - x)$

(b) Domain: $0 \le x \le 55$

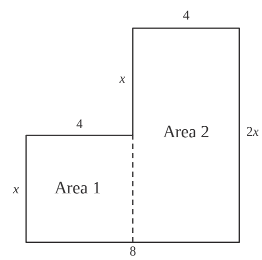


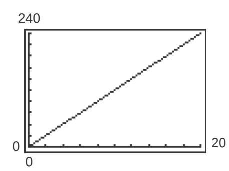
(c) If the final mixture is 60% concentrate, then $A = 0.6(55) = 33$ and $x = 22.2$ gallons.

81. (a) Divide into two regions. Then find the area of each region and add.

$$\text{Total area} = \text{Area } 1 + \text{Area } 2$$
$$A(x) = 4 \cdot x + 4 \cdot 2x$$
$$= 4x + 8x$$
$$= 12x$$

(b)



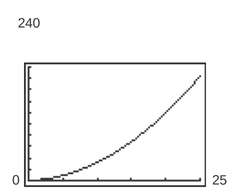


(c) If the area is 180 square units, then $x = 15$ units.

82. (a) $A = \frac{1}{2}bh$

$A(x) = \frac{1}{2}(x)\left(\frac{2}{3}x + 1\right) = \frac{1}{3}x^2 + \frac{1}{2}x$

(b)



(c) If the area is 180 square units, then $x = 22.5$ units.

83. (a) $T = I + S = x + 10{,}000 - \frac{1}{2}x = 10{,}000 + \frac{1}{2}x$

(b) If $S = 6600 = 10{,}000 - \frac{1}{2}x \Rightarrow \frac{1}{2}x = 3400$

$\Rightarrow x = \$6800$

(c) If $T = 13{,}800 = 10{,}000 + \frac{1}{2}x \Rightarrow 3800 = \frac{1}{2}x$

$\Rightarrow x = \$7600$

(d) If $T = 12{,}500 = 10{,}000 + \frac{1}{2}x$ then

$x = 5000$. Thus, $S = 10{,}000 - \frac{1}{\,}x = \$7500$.

INSTRUCTOR USE ONLY

84. $y = 16.9t + 574, \ 0 \le t \le 12$

(a) Let $t = 0$ and find y.

$y = 16.9(0) + 574 = 574$

The median weekly earnings of full-time workers was $574 in 2000.

(b) The slope m is 16.9. The median weekly earnings of full-time workers increases by $16.90 every year.

(c) Answers will vary.

(d) Answers will vary. Sample answer:

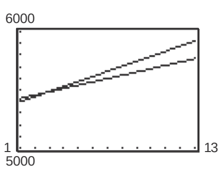
Algebraically: Let $y = 800$ and solve for t.

Graphically: Using the *zoom* and *trace* features, find t when $y = 800$.

85. $M = 43.4t + 5355, \ 1 \le t \le 13$

$W = 28.4t + 5398, \ 1 \le t \le 13$

(a)



The point of intersection is approximately $(2.9, 5479.4)$. So, in 2002, both states had the same population.

(b) $43.4t + 5355 = 28.4t + 5398$

$15t = 43$

$t \approx 2.9$

The point of intersection is approximately $(2.9, 5479.4)$. So, in 2002, both states had the same population.

(c) The slopes of the linear models represent the change in population per year. Since the slope of the model for Maryland is greater than that of Wisconsin, the population of Maryland is growing faster.

(d) Find $t = 16$.

$M = 43.4(16) + 5355 = 6049.4$

$W = 28.4(16) + 5398 = 5852.4$

86. (a) $V_{\text{TOTAL}} = V_{\text{RECTANGULAR SIDEWALL}} + V_{\text{TRIANGULAR SIDEWALL}}$

$= l \cdot w \cdot (\text{pool width}) + \frac{1}{2} \cdot b \cdot h \cdot (\text{pool width})$

$= (4)(40)(20) + \left(\frac{1}{2}\right)(40)(5)(20)$

$= 5200$ cubic feet

(b) Number of gallons $= \left(5200 \text{ ft}^3\right)\left(7.48 \dfrac{\text{gallons}}{\text{ft}^3}\right)$

$= 38{,}896$ gallons

(c) The base of the pool passes through the points $(0, 0)$ and $(40, 5)$.

$m = \dfrac{5 - 0}{40 - 0} = \dfrac{1}{8}$

$y - 0 = \dfrac{1}{8}(x - 0)$

$y = \dfrac{1}{8}x$

(d) For $0 \le d \le 5$, by similar triangle, $\dfrac{d}{5} = \dfrac{b}{40} \Rightarrow b = 8d$.

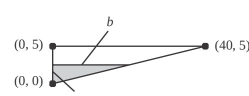
So, $V = \dfrac{1}{2}bd(20) = \dfrac{1}{2}(8d)d(20) = 80d^2, \ 0 \le d \le 5$.

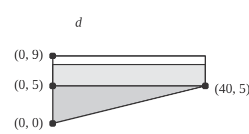
For $5 < d \le 9$,

$V = \dfrac{1}{2}(5)(40)(20) + (d - 5)(40)(20)$

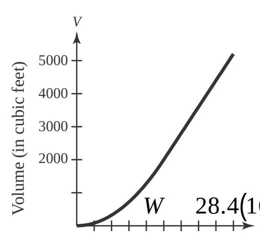
$= 2000 + 800d - 4000$

$= 800d - 2000, \ 5 < d \le 9$.





(e)



1000

d

1 2
3 4
5 6
7 8
9

Depth (in feet)

The population of Maryland will be 6,049,400 and the population of Wisconsin will be 5,852,400.

Answers will vary.

(f)

d	3	5	7	9
V	720	2000	3600	5200

(g) $V = 4800: \quad 800d - 2000 = 4800$

$$800d = 6800$$

$$d = 8.5 \text{ feet}$$

87. True.

88. True. A line must intersect at least one axis.

89. $\dfrac{x}{x-1} = \dfrac{99}{100}$

$100x = 99x - 99$

$x = -99$

The approximate answer −99.1 is not a good answer, even though the substitution yields a small error.

90. (a) From the table, $f(x) = 0$ for $x = 3$.

(b) From the table, $g(x) = 0$ for $x = -2$.

(c) From the table, $g(x) = -f(x)$ for $x = 1$. In this case, $f(x) = -6$ and $g(x) = 6$.

(d) From the table, $f(x) = -6g(x)$. In this case,

$f(x) = -12$ and $g(x) = 2$, for $x = -1$.

91. (a) $y = 2x + 2$

Algebraically: Let $y = 0$: $2x + 2 = 0$

$2x = -1$

$x = -1$

$(-1, 0)$ x-intercept

Let $x = 0$: $y = 2(0) + 2$

$y = 2$

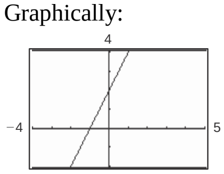
$(0, 2)$ y-intercept

Numerically:

x	−2	−1	0	1	2
$y = 2x + 2$	−2	0	2	4	6

The x-intercept is $(-1, 0)$, The y-intercept is $(0, 2)$.

Graphically:



(b) $f(x) = x^2 - 1$

Algebraically: $x = -1$ and $x = 1$ are zeros.

$f(-1) = (-1)^2 - 1 = 0$

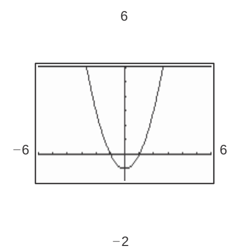
$f(1) = (1)^2 - 1 = 0$

Numerically:

x	−2	−1	0	1	2
$y = x^2 - 1$	3	0	−1	0	3

So, the zeros are −1 and 1.

Graphically:



(c) $y = 2x + 2,\ y = x^2 - 1$

Algebraically: $2x + 2 = x^2 - 1$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x - 3 = 0$ $x + 1 = 0$

$x = 3$ $x = -1$

$x = 3 \Rightarrow y = 2(3) + 2 = 8$

$(3, 8)$

$x = -1 \Rightarrow y = 2(-1) + 2 = 0$

$(-1, 0)$

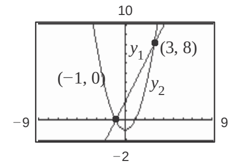
Numerically:

x	−1	0	1	2	3
$y = 2x + 2$	0	2	4	6	8
$y = x^2 - 1$	0	−1	0	3	8

The points of intersection are $(-1, 0)$ and $(3, 8)$.

Graphically:

−2



10
y_1
(3, 8)
(−1, 0)
y_2
−9
9
−2

92. $\dfrac{12}{5\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{12\sqrt{3}}{5(3)} = \dfrac{4\sqrt{3}}{5}$

93. $\dfrac{10}{\sqrt{14} - 2} = \dfrac{10}{\sqrt{14} - 2} \cdot \dfrac{\sqrt{14} + 2}{\sqrt{14} + 2}$

$= \dfrac{10\left(\sqrt{14} + 2\right)}{\left(\sqrt{14}\right)^2 - (2)^2}$

$= \dfrac{10\left(\sqrt{14} + 2\right)}{14 - 4}$

$= \dfrac{10\left(\sqrt{14} + 2\right)}{10}$

$= 2 + \sqrt{14}$

94. $\dfrac{3}{8 + \sqrt{11}} \cdot \dfrac{8 - \sqrt{11}}{8 - \sqrt{11}} = \dfrac{3\left(8 - \sqrt{11}\right)}{64 - 11} = \dfrac{3\left(8 - \sqrt{11}\right)}{53}$

95. $\dfrac{14}{3\sqrt{10} - 1} \cdot \dfrac{3\sqrt{10} + 1}{3\sqrt{10} + 1} = \dfrac{14\left(3\sqrt{10} + 1\right)}{90 - 1} = \dfrac{14}{89}\left(3\sqrt{10} + 1\right)$

96. $(x - 6)(3x - 5) = 3x^2 - 5x - 18x + 30$

$= 3x^2 - 23x + 30$

97. $(3x + 13)(4x - 7) = 12x^2 + 31x - 91$

98. $(2x - 9)(2x + 9) = 4x^2 - 81$

99. $(4x + 1)^2 = (4x + 1)(4x + 1) = 16x^2 + 8x + 1$

Section 2.3 Complex Numbers

1. (a) ii (b) iii (c) i

2. $\sqrt{-1}$, -1

3. complex, $a + bi$

4. To multiply two complex numbers, $(a + bi)(c + di)$, the FOIL Method can be used;

$(a + bi)(c + di) = ac + adi + bci + bdi^2$

$= (ac - bd) + (ad + bc)i.$

5. The additive inverse of $2 - 4i$ is $-2 + 4i$.

6. The complex conjugate of $2 - 4i$ is $2 + 4i$.

7. $a + bi = -9 + 4i$

$a = -9$

$b = 4$

8. $a + bi = 12 + 5i$

$a = 12$

$b = 5$

11. $4 + \sqrt{-9} = 4 + \sqrt{9}i$

$= 4 + 3i$

12. $7 - \sqrt{-25} = 7 - \sqrt{25}i$

$= 7 - 5i$

13. $12 = 12 + 0i$

$= 12$

14. $-3 = -3 + 0i$

$= -3$

15. $-8i - i^2 = -8i - (-1)$

$= 1 - 8i$

16. $2i^2 - 6i = 2(-1) - 6i$

$= -2 - 6i$

17. $\left(\sqrt{-16}\right)^2 + 5 = \left(\sqrt{16}i\right)^2 + 5$

9. $(a-1)+(b+3)i=5+8i$

$a-1=5 \Rightarrow a=6$

$b+3=8 \Rightarrow b=5$

10. $(a+6)+2bi=6-5i$

$2b=-5$

$b=-\frac{5}{2}$

$a+6=6$

$a=0$

$= (4i)^2 + 5$

$= 16i^2 + 5$

$= 16(-1) + 5$

$= -11$

18. $-i-\left(\sqrt{-23}\right)^2 = -i-\left(\sqrt{23}\,i\right)^2$

$= -i - 23i^2$

$= -i - 23(-1)$

$= 23 - i$

19. $\sqrt{-0.09} = \sqrt{0.09}i = 0.3i$

20. $\sqrt{-0.0004} = 0.02i$

21. $(4+i)-(7-2i) = (4-7)+(1+2)i$
$= -3+3i$

22. $(11-2i)-(-3+6i) = (11+3)+(-2-6)i$
$= 14-8i$

23. $13i-(14-7i) = 13i-14+7i = -14+20i$

24. $22+(-5+8i)-9i = (22-5)+(8-9)i$
$= 17-i$

25. $\left(\frac{3}{2}+\frac{5}{2}i\right)+\left(\frac{5}{3}+\frac{11}{3}i\right) = \left(\frac{3}{2}+\frac{5}{3}\right)+\left(\frac{5}{2}+\frac{11}{3}\right)i$
$= \frac{9+10}{6}+\frac{15+22}{6}i$
$= \frac{19}{6}+\frac{37}{6}i$

26. $\left(\frac{3}{4}+\frac{7}{5}i\right)-\left(\frac{5}{6}-\frac{1}{6}i\right) = \left(\frac{3}{4}-\frac{5}{6}\right)+\left(\frac{7}{5}+\frac{1}{6}\right)i$
$= -\frac{1}{12}+\frac{47}{30}i$

27. $(1.6+3.2i)+(-5.8+4.3i) = -4.2+7.5i$

28. $-(-3.7-12.8i)-(6.1-16.3i) = 3.7+12.8i-6.1+16.3i$
$= (3.7-6.1)+(12.8+16.3)i$
$= -2.4+29.1i$

29. $\left(5+\sqrt{-27}\right)-\left(-12+\sqrt{-48}\right) = \left(5+3\sqrt{3}i\right)-\left(-12+4\sqrt{3}i\right)$
$= 5+3\sqrt{3}i+12-4\sqrt{3}i$
$= 17+\left(3\sqrt{3}-4\sqrt{3}\right)i$
$= 17-\sqrt{3}i$

30. $\left(7+\sqrt{-18}\right)+\left(3+\sqrt{-32}\right) = \left(7+3\sqrt{2}i\right)+\left(3+4\sqrt{2}i\right)$
$= (7+3)+\left(3\sqrt{2}+4\sqrt{2}\right)i$
$= 10+7\sqrt{2}i$

31. $\sqrt{-6}\cdot\sqrt{-2} = \left(\sqrt{6}i\right)\left(\sqrt{2}i\right) = \sqrt{12}i^2$
$= \left(2\sqrt{3}\right)(-1) = -2\sqrt{3}$

32. $\sqrt{-5}\cdot\sqrt{-10} = \left(\sqrt{5}i\right)\left(\sqrt{10}i\right)$

38. $(6-2i)(2-3i) = 12-18i-4i+6i^2$
$= 12-22i-6$
$= 6-22i$

39. $4i(8+5i) = 32i+20i^2$
$= 32i+20(-1)$
$= -20+32i$

40. $-3i(6-i) = -18i-3 = -3-18i$

$= \sqrt{50}i^2 = 5\sqrt{2}(-1) = -5\sqrt{2}$

33. $\left(\sqrt{-10}\right)^2 = \left(\sqrt{10}i\right)^2 = 10i^2 = -10$

34. $\left(\sqrt{-75}\right)^2 = \left(\sqrt{75}i\right)^2 = 75i^2 = -75$

35. $4(3+5i) = 4(3) + 4(5)i$

$= 12 + 20i$

36. $-6(5-3i) = (-6)(5) + (-6)(-3i)$

$= -30 + 18i$

37. $(1+i)(3-2i) = 3 - 2i + 3i - 2i^2$

$= 3 + i + 2$

$= 5 + i$

41. $\left(\sqrt{14} + \sqrt{10}i\right)\left(\sqrt{14} - \sqrt{10}i\right) = 14 - 10i^2$

$= 14 + 10 = 24$

42. $\left(\sqrt{3} + \sqrt{15}i\right)\left(\sqrt{3} - \sqrt{15}i\right)$

$= \left(\sqrt{3}\right)\left(\sqrt{3}\right) - \sqrt{3}\sqrt{15}i + \sqrt{3}\sqrt{15}i - \left(\sqrt{15}i\right)\left(\sqrt{15}i\right)$

$= 3 - 15i^2 = 3 + 15 = 18$

43. $(6+7i)^2 = (6)^2 + 2(6)(7i) + (7i)^2$

$= 36 + 84i + 49i^2$

$= (36 - 49) + 84i$

$= -13 + 84i$

44. $(5-4i)^2 = (5)^2 - 2(5)(4i) + (4i)^2$

$= 25 - 40i + 16i^2$

$= (25 - 16) - 40i$

$= 9 - 40i$

45. $(4+5i)^2 - (4-5i)^2$

$= [(4+5i)+(4-5i)][(4+5i)-(4-5i)]$

$= 8(10i) = 80i$

46. $(1-2i)^2 - (1+2i)^2 = 1 - 4i + 4i^2 - (1 + 4i + 4i^2)$

$= 1 - 4i + 4i^2 - 1 - 4i - 4i^2$

$= -8i$

47. $6 + 2i$ is the complex conjugate of $6 - 2i$.

$(6-2i)(6+2i) = 36 - 4i^2$

$= 36 + 4 = 40$

48. $3 - 5i$ is the complex conjugate of $3 + 5i$.

$(3+5i)(3-5i) = 9 - 25i^2$

$= 9 + 25 = 34$

49. $-1 - \sqrt{7}\,i$ is the complex conjugate of $-1 + \sqrt{7}\,i$.

$(-1+\sqrt{7}\,i)(-1-\sqrt{7}\,i) = 1 - 7i^2$

$= 1 + 7 = 8$

50. $-4 + \sqrt{3}\,i$ is the complex conjugate of $-4 - \sqrt{3}\,i$.

$(-4-\sqrt{3}\,i)(-4+\sqrt{3}\,i) = 16 - 3i^2$

$= 16 + 3 = 19$

51. $\sqrt{-29} = \sqrt{29}\,i$

$-\sqrt{29}\,i$ is the complex conjugate of $\sqrt{29}\,i$.

$(-\sqrt{29}\,i)(\sqrt{29}\,i) = -29i^2$

$= 29$

52. $\sqrt{-10} = \sqrt{10}\,i$

55. $\frac{6}{i} = \frac{6}{i} \cdot \frac{-i}{-i} = \frac{-6i}{-i^2} = \frac{-6i}{1} = -6i$

56. $\frac{-5}{2i} \cdot \frac{i}{i} = \frac{-5i}{-2} = \frac{5}{2}i$

57. $\frac{2}{4-5i} = \frac{2}{4-5i} \cdot \frac{4+5i}{4+5i} = \frac{8+10i}{16+25} = \frac{8}{41} + \frac{10}{41}i$

58. $\frac{3}{1-i} \cdot \frac{1+i}{1+i} = \frac{3+3i}{1-i^2} = \frac{3+3i}{2} = \frac{3}{2} + \frac{3}{2}i$

59. $\frac{3-i}{3+i} = \frac{3-i}{3+i} \cdot \frac{3-i}{3-i}$

$= \frac{9 - 6i + i^2}{9 - i^2}$

$= \frac{9 - 6i - 1}{9 + 1}$

$= \frac{8 - 6i}{10}$

$= \frac{4}{5} - \frac{3}{5}i$

60. $\frac{8-7i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{8 + 16i - 7i - 14i^2}{1 - 4i^2}$

$= \frac{22+9i}{5} = \frac{22}{5} + \frac{9}{5}i$

61. $\frac{i}{(4-5i)^2} = \frac{i}{16 - 25 - 40i}$

$= \frac{i}{-9-40i} \cdot \frac{-9+40i}{-9+40i}$

$= \frac{-40-9i}{81+40^2}$

$= -\frac{40}{1681} - \frac{9}{1681}i$

$-\sqrt{10}\,i$ is the complex conjugate of $\sqrt{10}\,i$.

$$\left(-\sqrt{10}\,i\right)\left(\sqrt{10}\,i\right) = -10i^2$$
$$= 10$$

53. $9 + \sqrt{6}\,i$ is the complex conjugate of $9 - \sqrt{6}\,i$.

$$\left(9 + \sqrt{6}\,i\right)\left(9 - \sqrt{6}\,i\right) = 81 - 6i^2$$
$$= 81 + 6 = 87$$

54. $-8 - \sqrt{15}\,i$ is the complex conjugate of $-8 + \sqrt{15}\,i$.

$$\left(-8 + \sqrt{15}\,i\right)\left(-8 - \sqrt{15}\,i\right) = 64 - 15i^2$$
$$= 64 + 15 = 79$$

1681 1681

62. $$\frac{5i}{(2+3i)^2} = \frac{5i}{-5+12i}\cdot\frac{-5-12i}{-5-12i}$$
$$= \frac{-25i+60}{25+144}$$
$$= \frac{60}{169} - \frac{25}{169}i$$

63. $$\frac{2}{1+i} - \frac{3}{1-i} = \frac{2(1-i) - 3(1+i)}{(1+i)(1-i)}$$
$$= \frac{2-2i-3-3i}{1+1}$$
$$= \frac{-1-5i}{2} = -\frac{1}{2} - \frac{5}{2}i$$

64. $\dfrac{2i}{2+i}+\dfrac{5}{2-i}=\dfrac{2i(2-i)}{(2+i)(2-i)}+\dfrac{5(2+i)}{(2+i)(2-i)}$

$=\dfrac{4i-2i^2+10+5i}{4-i^2}$

$=\dfrac{12+9i}{5}=\dfrac{12}{5}+\dfrac{9}{5}i$

65. $\dfrac{i}{3-2i}+\dfrac{2i}{3+8i}=\dfrac{3i+8i^2+6i-4i^2}{(3-2i)(3+8i)}$

$=\dfrac{-4+9i}{9+18i+16}$

$=\dfrac{-4+9i}{25+18i}\cdot\dfrac{25-18i}{25-18i}$

$=\dfrac{-100+72i+225i+162}{25^2+18^2}$

$=\dfrac{62+297i}{949}$

$=\dfrac{62}{949}+\dfrac{297}{949}i$

66. $\dfrac{1+i}{i}-\dfrac{3}{4-i}=\dfrac{1+i}{i}\cdot\dfrac{-i}{-i}-\dfrac{3}{4-i}\cdot\dfrac{4+i}{4+i}$

$=\dfrac{-i+1}{1}-\dfrac{12+3i}{16+1}$

$=\dfrac{5}{17}-\dfrac{20}{17}i$

67. $-6i^3+i^2=-6i^2i+i^2$

$=-6(-1)i+(-1)$

$=6i-1$

$=-1+6i$

68. $4i^2-2i^3=-4+2i$

73. (a) $i^{20}=\left(i^4\right)^5=(1)^5=1$

(b) $i^{45}=\left(i_4\right)^{11}i=(1)^{11}i=i$

(c) $i^{67}=\left(i^4\right)^{16}i^3=(1)^{16}i^3=i^3=-i$

(d) $i^{114}=\left(i^4\right)^{28}i^2=(1)^{28}i^2=i^2=-1$

74. (a) $z_1=5+2i$

$z_2=3-4i$

$\dfrac{1}{z}=\dfrac{1}{z_1}+\dfrac{1}{z_2}=\dfrac{1}{5+2i}+\dfrac{1}{3-4i}$

$=\dfrac{(3-4i)+(5+2i)}{(5+2i)(3-4i)}$

$=\dfrac{8-2i}{23-14i}$

$z=\dfrac{23-14i}{8-2i}\left(\dfrac{8+2i}{8+2i}\right)$

$=\dfrac{212-66i}{68}\approx 3.118-0.971i$

(b) $z_1=16i+9$

$z_2=20-10i$

$\dfrac{1}{z}=\dfrac{1}{z_1}+\dfrac{1}{z_2}=\dfrac{1}{9+16i}+\dfrac{1}{20-10i}$

$=\dfrac{(20-10i)+(9+16i)}{(9+16i)(20-10i)}$

$=\dfrac{29+6i}{340+230i}$

$z=\dfrac{340+230i}{29+6i}\left(\dfrac{29-6i}{29-6i}\right)=\dfrac{11{,}240+4630i}{877}$

69. $\left(\sqrt{-75}\right)^3 = \left(5\ \ 3i\right)^3 = 5^3\left(\sqrt{3}\right)^3 i^3$

$= 125\left(3\sqrt{3}\right)(-i)$
$= -375\sqrt{3}i$

70. $\left(\sqrt{-2}\right)_6 = \left(\sqrt{2}i\right)_6 = 8i^6 = 8i^4 i^2 = -8$

71. $\frac{1}{i^3} = \frac{1}{i^3}\cdot\frac{i}{i} = \frac{i}{i^4} = \frac{i}{1} =$

72. $\frac{1}{(2i)^3} = \frac{1}{8i^3} = \frac{1}{-8i}\cdot\frac{\overline{8i}}{8i} = \frac{8i}{-64i^2} = \frac{1}{8}$

$\approx 12.816 + 5.279i$

75. False. A real number $a + 0i = a$ is equal to its conjugate.

76. False.
$i^{44} + i^{150} - i^{74} - i^{109} + i^{61} = 1 - 1 - i + i = 1$

77. False. For example, $(1+2i)+(1-2i)=2,$ which is not an imaginary number.

78. True. Let $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$. Then

$z_1 z_2 = (a_1 + b_1 i)(a_2 + b_2 i)$
$= (a_1a_2 - b_1b_2) + (a_1b_2 + b_1a_2)i$

$= (a_1a_2 - b_1b_2) - (a_1b_2 + b_1a_2)i$
$= (a_1 - b_1 i)(a_2 - b_2 i)$
$= a_1 + b_1 i\ \ a_2 + b_2 i$
$= z_1\ z_2.$

79. True. Let $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$. Then

$$\begin{aligned} z_1 + z_2 &= (a_1 + b_1 i) + (a_2 + b_2 i) \\ &= (a_1 + a_2) + (b_1 + b_2)i \\ &= (a_1 + a_2) - (b_1 + b_2)i \\ &= (a_1 - b_1 i) + (a_2 - b_2 i) \\ &= a_1 + b_1 i + a_2 + b_2 i \\ &= z_1 + z_2. \end{aligned}$$

80. (i) $2 = 2 + 0i$ is the point $(2, 0)$ that matches D.

(ii) $2i = 0 + 2i$ is the point $(0, 2)$ that matches C.

(iii) $-2 + i$ is the point $(-2, 1)$ that matches A.

(iv) $1 - 2i$ is the point $(1, -2)$ that matches B.

81. The error is not simplifying before multiplying.

The correct method is $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = 6i^2 = -6$.

82. Given the binomials: $x + 5$ and $2x - 1$ and the complex numbers: $1 + 5i$ and $2 - i$

(a) Sum of the binomials:
$(x + 5) + (2x - 1) = 3x + 4$
Sum of the complex numbers:
$(1 + 5i) + (2 - i) = 3 + 4i$
Answers will vary.
Sample answer:
The coefficient of the x-terms of the binomials are the same as the real part of the complex numbers. The constant terms of the binomials are the same as the coefficients of the imaginary part of the complex numbers.

(b) Product of the binomials:

$$\begin{aligned} (x + 5)(2x - 1) &= 2x^2 - x + 10x - 5 \\ &= 2x^2 + 9x - 5 \end{aligned}$$

Product of the complex numbers:

$$\begin{aligned} (1 + 5i)(2 - i) &= 2 - i + 10i - 5i^2 \\ &= 7 + 9i \end{aligned}$$

The product of the binomials results in a second-degree trinomial. The product of the complex numbers is a complex number because of the property $i^2 = 1$.

(c) Answers will vary.

83. $(4x - 5)(4x + 5) = 16x^2 - 20x + 20x - 25$
$= 16x^2 - 25$

84. $(x + 2)^3 = x^3 + 3x^2(2) + 3x(2)^2 + 2^3$
$= x^3 + 6x^2 + 12x + 8$

85. $(3x - \frac{1}{2})(x + 4) = 3x^2 - \frac{1}{2}x + 12x - 2$
$= 3x^2 + \frac{23}{2}x - 2$

86. $(2x - 5)^2 = 4x^2 - 20x + 25$

Section 2.4 Solving Quadratic Equations Algebraically

1. quadratic equation

2. discriminant

3. Four methods to solve a quadratic equation are: factoring, extracting square roots, completing the square, and using the Qualitative Formula.

4. The height of an object that is falling is given by the equation $s = -16t^2 + v_0 t + s_0$, where s is the height, v_0 is

7. $\frac{1}{5}(3x^2 - 10) = 12x$
$3x^2 - 10 = 60x$
Standard form: $3x^2 - 60x - 10 = 0$

8.
$$\begin{aligned} x(x + 2) &= 3x^2 + 1 \\ x^2 + 2x &= 3x^2 + 1 \\ -2x^2 + 2x - 1 &= 0 \end{aligned}$$

0 0 o

the initial velocity, and s_o is the initial height.

5. $2x^2 = 3 - 5x$

Standard form; $2x^2 + 5x - 3 = 0$

6. $x^2 = 25x + 26$

Standard form**:** $x^2 - 25x - 26 = 0$

$(-1)(-2x^2 + 2x - 1) = -1(0)$

Standard form: $2x^2 - 2x + 1 = 0$

9. $15x^2 + 5x = 0$

$5x(3x + 1) = 0$

$5x = 0 \Rightarrow x = 0$

$3x + 1 = 0 \Rightarrow x = -\frac{1}{3}$

10. $9x^2 - 21x = 0$
$3x(3x - 7) = 0$
$3x = 0 \Rightarrow x = 0$
$3x - 7 = 0 \Rightarrow x = \frac{7}{3}$

11. $x^2 - 10x + 21 = 0$
$(x - 7)(x - 3) = 0$
$x - 7 = 0 \Rightarrow x = 7$
$x - 3 = 0 \Rightarrow x = 3$

12. $x^2 - 10x + 9 = 0$
$(x - 9)(x - 1) = 0$
$x - 9 = 0 \Rightarrow x = 9$
$x - 1 = 0 \Rightarrow x = 1$

13. $x^2 - 8x + 16 = 0$
$(x - 4)^2 = 0$
$x - 4 = 0$
$x = 4$

14. $4x^2 + 12x + 9 = 0$
$(2x + 3)(2x + 3) = 0$
$2x + 3 = 0$
$2x = -3$
$x = -\frac{3}{2}$

15. $3x^2 = 8 - 2x$
$3x^2 + 2x - 8 = 0$
$(3x - 4)(x + 2) = 0$
$3x - 4 = 0 \Rightarrow x = \frac{4}{3}$
$x + 2 = 0 \Rightarrow x = -2$

16. $2x^2 = 19x + 33$
$2x^2 - 19x - 33 = 0$
$(2x + 3)(x - 11) = 0$
$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$
$x - 11 = 0 \Rightarrow x = 11$

18. $-x^2 - 11x = 30$
$-x^2 - 11x - 30 = 0$
$x^2 + 11x + 30 = 0$
$(x + 5)(x + 6) = 0$
$x + 5 = 0 \Rightarrow x = -5$
$x + 6 = 0 \Rightarrow x = -6$

19. $(x + a)^2 - b^2 = 0$
$[(x + a) + b][(x + a) - b] = 0$
$x + a + b = 0 \Rightarrow x = -a - b$
$x + a - b = 0 \Rightarrow x = -a + b$

20. $x^2 + 2ax + a^2 = 0$
$(x + a)^2 = 0$
$x + a = 0$
$x = -a$

21. $x^2 = 49$
$x = \pm\sqrt{49} = \pm 7$

22. $x^2 = 144$
$x = \pm\sqrt{144} = \pm 12$

23. $3x^2 = 81$
$x^2 = 27$
$x = \pm\sqrt{27}$
$x = \pm 3\sqrt{3} \approx \pm 5.20$

24. $9x^2 = 36$
$x^2 = 4$
$x = \pm\sqrt{4}$
$x = \pm 2$

25. $x^2 + 12 = 112$
$x^2 = 100$
$x = \pm\sqrt{100}$
$x = \pm 10$

17. $-x^2 - 11x = 28$

$$-x^2 - 11x - 28 = 0$$
$$x^2 + 11x + 28 = 0$$
$$(x + 4)(x + 7) = 0$$
$$x + 4 = 0 \Rightarrow x = -4$$
$$x + 7 = 0 \Rightarrow x = -7$$

26. $x^2 - 3 = 78$

$$x^2 = 81$$
$$x = \pm\ \ 81$$
$$x = \pm 9$$

27. $(x-12)^2 = 16$

$x - 12 = \pm\sqrt{16} = \pm 4$

$x = 12 \pm 4$

$x = 8,\ 16$

28. $(x-5)^2 = 25$

$x - 5 = \pm 5$

$x = 5 \pm 5$

$x = 0,\ 10$

29. $(3x-1)^2 + 6 = 0$

$(3x-1)^2 = -6$

$3x - 1 = \pm\sqrt{-6} = \pm\sqrt{6}i$

$x = \dfrac{1}{3} \pm \dfrac{\sqrt{6}}{3} i \approx 0.33 \pm 0.82i$

30. $(2x+3)^2 + 25 = 0$

$(2x+3)^2 = -25$

$2x + 3 = \pm\sqrt{-25} = \pm 5i$

$x = -\frac{3}{2} \pm \frac{5}{2} i = -1.50 \pm 2.50i$

31. $(x-7)^2 = (x+3)^2$

$x - 7 = \pm(x+3)$

$x - 7 = +(x+3) \Rightarrow$ impossible

$x - 7 = -(x+3) \Rightarrow 2x = 4$

$x = 2$

32. $(x+5)^2 = (x+4)^2$

$(x+5) = \pm(x+4)$

33.

35. $x^2 - 6x + 2 = 0$

$x^2 - 6x = -2$

$x^2 - 6x + 3^2 = -2 + 3^2$

$(x-3)^2 = 7$

$x - 3 = \pm\sqrt{7}$

$x = 3 \pm \sqrt{7}$

36. $x^2 + 8x + 14 = 0$

$x^2 + 8x = -14$

$x^2 + 8x + 4^2 = -14 + 16$

$(x+4)^2 = 2$

$x + 4 = \pm\sqrt{2}$

$x = -4 \pm \sqrt{2}$

37. $x^2 - 4x + 13 = 0$

$x^2 - 4x + 4 = -13 + 4$

$(x-2)^2 = -9$

$x - 2 = \pm 3i$

$x = 2 \pm 3i$

38. $x^2 - 6x + 34 = 0$

$x^2 - 6x + 9 = -34 + 9$

$(x-3)^2 = -25$

$x - 3 = \pm 5i$

$x = 3 \pm 5i$

39. $x^2 + 8x + 32 = 0$

$x^2 + 8x + (4)^2 = -32 + 16$

$(x+4)^2 = -16$

$x + 5 = x + 4$, not possible

$x + 5 = -(x+4)$

$2x = -9$

$x = -\frac{9}{2} = -4.50$

$x^2 + 4x = 32$

$x^2 + 4x + 4 = 32 + 4$

$(x+2)^2 = 36$

$x + 2 = \pm 6$

$x = -2 \pm 6$

$x = -8,\ 4$

34. $x^2 - 2x - 3 = 0$

$x^2 - 2x + 1 = 3 + 1$

$(x-1)^2 = 4$

$x - 1 = \pm 2$

$x = 1 \pm 2$

$x = -1,\ 3$

40.

$x + 4 = \pm\sqrt{-16}$

$x + 4 = \pm 4i$

$x = -4 \pm 4i$

$x^2 + 18x + 117 = 0$

$x^2 + 18x + 81 = -117 + 81$

$(x+9)^2 = -36$

$x + 9 = \pm 6i$

$x = -9 \pm 6i$

41. $-6 + 2x - x^2 = 0$

$(x^2 - 2x + 1) = -6 + 1$

$(x-1)^2 = -5$

$x - 1 = \pm\sqrt{-5}$

$= \pm\sqrt{5}i$

$x = 1 \pm \sqrt{5}i$

42.
$$-x^2 + 6x - 16 = 0$$
$$x^2 - 6x + 16 = 0$$
$$x^2 - 6x + (3)^2 = -16 + 9$$
$$(x-3)^2 = -7$$
$$x - 3 = \pm\sqrt{-7}$$
$$x - 3 = \pm\sqrt{7}i$$
$$x = 3 \pm \sqrt{7}i$$

43.
$$9x^2 - 18x + 3 = 0$$
$$x^2 - 2x + \frac{1}{3} = 0$$
$$x^2 - 2x = -\frac{1}{3}$$
$$x^2 - 2x + 1^2 = -\frac{1}{3} + 1^2$$
$$(x-1)^2 = \frac{2}{3}$$
$$x - 1 = \pm\sqrt{\frac{2}{3}}$$
$$x = 1 \pm \sqrt{\frac{2}{3}}$$
$$x = 1 \pm \frac{\sqrt{6}}{3}$$

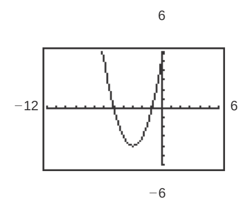
44.
$$4x^2 - 16x - 5 = 0$$
$$x^2 - 4x - \frac{5}{4} = 0$$
$$x^2 - 4x + (2)^2 = \frac{5}{4} + 4$$
$$(x-2)^2 = \frac{21}{4}$$
$$x - 2 = \pm\sqrt{\frac{21}{4}}$$
$$x - 2 = \pm\frac{\sqrt{21}}{2}$$
$$x = 2 \pm \frac{\sqrt{21}}{2}$$

45.
$$2x^2 + 5x - 8 = 0$$
$$x^2 + \frac{5}{2}x = 4$$

46.
$$9x^2 - 12x = 14$$
$$x^2 - \frac{4}{3}x = \frac{14}{9}$$
$$x^2 - \frac{4}{3}x + \left(\frac{2}{3}\right)^2 = \frac{14}{9} + \frac{4}{9}$$
$$\left(x - \frac{2}{3}\right)^2 = \frac{18}{9}$$
$$\left(x - \frac{2}{3}\right)^2 = 2$$
$$x - \frac{2}{3} = \pm\sqrt{2}$$
$$x = \frac{2}{3} \pm \sqrt{2}$$

47. $y = (x+3)^2 - 4$

(a)



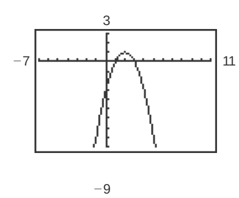
(b) The x-intercepts are $(-1, 0)$ and $(-5, 0)$.

(c)
$$0 = (x+3)^2 - 4$$
$$4 = (x+3)^2$$
$$\pm\sqrt{4} = x + 3$$
$$-3 \pm 2 = x$$
$$x = -1 \text{ or } x = -5$$

The x-intercepts are $(-1, 0)$ and $(-5, 0)$.

48. $y = 1 - (x-2)^2$

(a)



(b) The x-intercepts are $(1, 0)$ and $(3, 0)$.

(c)
$$0 = 1 - (x-2)^2$$
$$(x-2)^2 = 1$$

$$x^2 + \frac{5}{2}x + \frac{25}{16} = 4 + \frac{25}{16}$$

$$\left(x + \frac{5}{4}\right)^2 = \frac{89}{16}$$

$$x + \frac{5}{4} = \pm\frac{\sqrt{89}}{4}$$

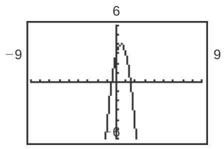
$$x = -\frac{5}{4} \pm \frac{\sqrt{89}}{4}$$

$$x - 2 = \pm\sqrt{1}$$

$$x = 2 \pm 1 = 3,\ 1$$

The x-intercepts are $(3,\ 0)$ and $(1,\ 0)$.

49. $y = -4x^2 + 4x + 3$

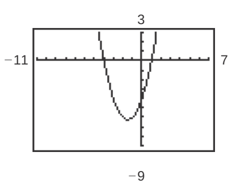
(a) 

(b) The *x*-intercepts are $(-0.5,\ 0)$ and $(1.5,\ 0)$.

(c)
$$\begin{aligned} 0 &= -4x^2 + 4x + 3 \\ 4x^2 - 4x &= 3 \\ 4(x^2 - x) &= 3 \\ x^2 - x &= \tfrac{3}{4} \\ x^2 - x + \left(\tfrac{1}{2}\right)^2 &= \tfrac{3}{4} + \left(\tfrac{1}{2}\right)^2 \\ \left(x - \tfrac{1}{2}\right)^2 &= 1 \\ x - \tfrac{1}{2} &= \pm\sqrt{1} \\ x &= \tfrac{1}{2} \pm 1 \\ x &= \tfrac{3}{2} \text{ or } x = -\tfrac{1}{2} \end{aligned}$$

The *x*-intercepts are $\left(\tfrac{3}{2},\ 0\right)$ and $\left(-\tfrac{1}{2},\ 0\right)$.

50. $y = x^2 + 3x - 4$

(a) 

(b) The *x*-intercepts are $(1,\ 0)$ and $(-1,\ 0)$.

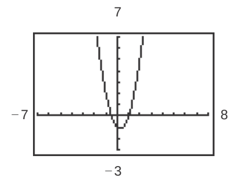
(c)
$$\begin{aligned} 0 &= x^2 + 3x - 4 \\ 0 &= (x + 4)(x - 1) \\ 0 &= x + 4 \Rightarrow x = -4 \\ 0 &= x - 1 \Rightarrow x = 1 \end{aligned}$$

The *x*-intercepts are $(-4,\ 0)$ and $(1,\ 0)$.

51. $y = x^2 - 4x + 4 = 0$

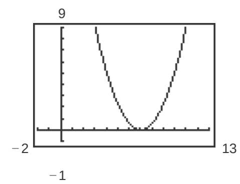
8

52. $y = 2x^2 - x - 1 = 0$



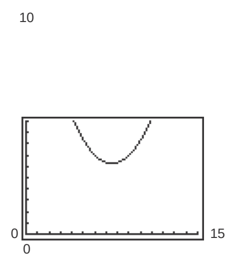
Two real solutions

53. $y = \frac{4}{7}x^2 - 8x + 28 = 0$



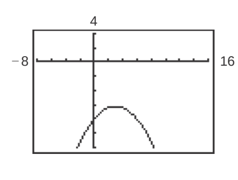
One real solution

54. $y = \frac{1}{3}x^2 - 5x + 25 = 0$



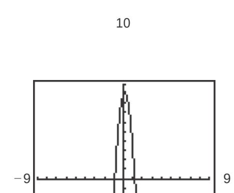
No real solutions

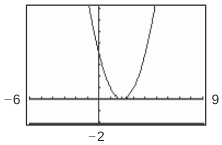
55. $y = -0.2x^2 + 1.2x - 8 = 0$



No real solution

56. $y = 9 + 2.4x - 8.3x^2 = 0$





One real solution

Two real solution

57. $x^2 - 9x + 19 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(19)}}{2(1)}$$

$$= \frac{9 \pm \sqrt{81 - 76}}{2}$$

$$= \frac{9 \pm \sqrt{5}}{2}$$

INSTRUCTOR USE ONLY

58. $x^2 - 10x + 22 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(22)}}{2(1)}$$
$$= \frac{10 \pm \sqrt{100 - 88}}{2}$$
$$= \frac{10 \pm 2\sqrt{3}}{2} = 5 \pm \sqrt{3}$$

59. $x^2 + 3x + 8 = 0$

$$x = \frac{-3 \pm \sqrt{9 - 4(8)}}{2}$$
$$= \frac{-3 \pm \sqrt{-23}}{2}$$
$$= -\frac{3}{2} \pm \frac{\sqrt{23}i}{2}$$

60. $x^2 + 5x + 16 = 0$

$$x = \frac{-5 \pm \sqrt{25 - 4(16)}}{2}$$
$$= \frac{-5 \pm \sqrt{-39}}{2}$$
$$= -\frac{5}{2} \pm \frac{\sqrt{39}}{2}i$$

61. $4x = 8 - x^2$

$x^2 + 4x - 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-8)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{16 + 32}}{2}$$
$$= \frac{-4 \pm \sqrt{48}}{2}$$
$$= \frac{-4 \pm 4\sqrt{3}}{2}$$
$$= -2 \pm 2\sqrt{3}$$

62. $8x = 4 - x^2$

$x^2 + 8x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(-4)}}{2(1)}$$
$$= \frac{-8 \pm \sqrt{64 + 16}}{2}$$
$$= \frac{-8 \pm \sqrt{80}}{2}$$
$$= \frac{-8 \pm 4\sqrt{5}}{2}$$
$$= -4 \pm 2\sqrt{5}$$

63. $20x^2 - 20x + 5 = 0$

$4x^2 - 4x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2(4)}$$
$$= \frac{4 \pm \sqrt{16 - 16}}{8}$$
$$= \frac{4 \pm 0}{8}$$
$$= \frac{1}{2}$$

64. $9x^2 - 18x + 9 = 0$

$x^2 - 2x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{0}}{2}$$
$$= 1$$

65. $16x^2 + 24x + 9 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(24) \pm \sqrt{(24)^2 - 4(16)(9)}}{2(16)}$$
$$= \frac{-24 \pm \sqrt{576 - 576}}{32}$$
$$= \frac{-24 \pm 0}{32}$$
$$= -\frac{3}{4}$$

66. $9x^2 + 30x + 25 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(30) \pm \sqrt{(30)^2 - 4(9)(25)}}{2(9)}$$
$$= \frac{-30 \pm \sqrt{900 - 900}}{18}$$
$$= \frac{-30 \pm 0}{18}$$
$$= \frac{5}{3}$$

67. $4x^2 + 16x + 17 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-16 \pm \sqrt{16^2 - 4(4)(17)}}{2(4)}$$
$$= \frac{-16 \pm \sqrt{-16}}{8}$$
$$= \frac{-16 \pm 4i}{8}$$
$$= -2 \pm \frac{1}{2}i$$

68. $9x^2 - 6x + 37 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{}$$

70. $11x^2 + 33x = 0$

$$11(x^2 + 3x) = 0$$
$$x(x + 3) = 0$$
$$x = 0$$
$$x + 3 = 0 \Rightarrow x = -3$$

71. $(x + 3)^2 = 81$

$$x + 3 = \pm 9$$
$$x + 3 = 9 \Rightarrow x = 6$$
$$x + 3 = -9 \Rightarrow x = -12$$

72. $(x - 1)^2 = -1$

$$x - 1 = \pm\sqrt{-1} = \pm i$$
$$x = 1 \pm i$$

73. $x^2 - 2x = -\frac{13}{4}$

$$x^2 - 2x + \frac{13}{4} = 0$$
$$x = \frac{-b \pm \sqrt{b^3 - 4ac}}{2a}$$
$$= \frac{2 \pm \sqrt{4 - 4(13/4)}}{2}$$
$$= \frac{2 \pm \sqrt{-9}}{2}$$
$$= 1 \pm \frac{3}{2}i$$

74. $x^2 + 4x = -\frac{19}{4}$

$$x^2 + 4x + 4 = -\frac{19}{4} + 4$$
$$(x + 2)^2 = -\frac{3}{4}$$
$$x + 2 = \pm\sqrt{-\frac{3}{4}}$$
$$x + 2 = \pm\frac{\sqrt{3}}{2}i$$
$$x = -2 \pm \frac{\sqrt{3}}{2}i$$

$$2a$$

$$= \frac{6 \pm \sqrt{36 - 1332}}{18}$$

$$= \frac{1}{3} \pm 2i$$

69.

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x - 4 = 0 \Rightarrow x = 4$$

$$x + 1 = 0 \Rightarrow x = -1$$

75.

$$5x^2 = 3x + 1$$

$$5x^2 - 3x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-1)}}{2(5)}$$

$$= \frac{3 \pm \sqrt{9 + 20}}{10}$$

$$= \frac{3 \pm \sqrt{29}}{10}$$

76. $4x^2 = 7x + 3$

$4x^2 - 7x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(-3)}}{2(4)}$$
$$= \frac{7 \pm \sqrt{49 + 48}}{8}$$
$$= \frac{7 \pm \sqrt{97}}{8}$$

77. $2x^2 + 7x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-7 \pm \sqrt{7^2 - 4(2)(-3)}}{2(2)}$$
$$= \frac{-7 \pm \sqrt{73}}{4}$$
$$= -\frac{7}{4} \pm \frac{\sqrt{73}}{4}$$

78. $-10x^2 + 11x - 3 = 0$

$10x^2 - 11x + 3 = 0$

$(5x - 3)(2x - 1) = 0$

$5x - 3 = 0$ or $2x - 1 = 0$

$5x = 3$ $\quad$ $2x = 1$

$x = \frac{3}{5}$ $\quad$ $x = \frac{1}{2}$

79. $-4x^2 + 12x - 9 = 0$

$4x^2 - 12x + 9 = 0$

$(2x - 3)^2 = 0$

$2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

80. $16x^2 - 24x + 9 = 0$

81. $(x + 6)(x - 5) = 0$

$x^2 + x - 30 = 0$

Multiply by 2 to find a second equation.

$2(x^2 + x - 30) = 2x^2 + 2x - 60 = 0$

(other answers possible)

82. $(x + 2)(x - 1) = 0$

$x^2 + x - 2 = 0$

Multiply by 2 to find a second equation.

$2(x^2 + x - 2) = 2x^2 + 2x - 4 = 0$

(other answers possible)

83. $\left(x + \frac{7}{3}\right)\left(x - \frac{6}{7}\right) = 0$

$\frac{(3x + 7)(7x - 6)}{21} = 0$

$21x^2 + 31x - 42 = 0$

Multiply by $\frac{1}{7}$ to find a second equation.

$\frac{1}{7}(21x^2 + 31x - 42) = 3x^2 + \frac{31}{7}x - 6 = 0$

(other answers possible)

84. $\left(x + \frac{2}{3}\right)\left(x - \frac{4}{3}\right) = 0$

$x^2 - \frac{2}{3}x - \frac{8}{9} = 0$

Multiply by 9 to find a second equation.

$9\left(x^2 - \frac{2}{3}x - \frac{8}{9}\right) = 9x^2 - 6x - 8 = 0$

(other answers possible)

85. $(x - 5\sqrt{3})(x + 5\sqrt{3}) = 0$

$x^2 - (5\sqrt{3})^2 = 0$

$x^2 - 75 = 0$

Multiply by $\frac{1}{5}$ to find a second equation.

$\frac{1}{5}(x^2 - 75) = \frac{1}{5}x^2 - 15 = 0$

$$(4x - 3) = 0$$
$$4x - 3 = 0$$
$$4x = 3$$
$$x = \frac{3}{4}$$

(other answers possible)

86. $\left(x - 2\sqrt{5}\right)\left(x + 2\sqrt{5}\right) = 0$

$$x^2 - \left(2\sqrt{5}\right)^2 = 0$$
$$x^2 - 20 = 0$$

Multiply by $\frac{1}{2}$ to find a second equation.

$$\frac{1}{2}\left(x^2 - 20\right) = \frac{1}{2}x^2 - 10 = 0$$

(other answers possible)

87. $\left(x - 1 - 2\sqrt{3}\right)\left(x - 1 + 2\sqrt{3}\right) = 0$

$\left(\left(x - 1\right) - 2\sqrt{3}\right)\left(\left(x - 1\right) + 2\sqrt{3}\right) = 0$

$\left(x - 1\right)^2 - \left(2\sqrt{3}\right)^2 = 0$

$x^2 - 2x + 1 - 12 = 0$

$x^2 - 2x - 11 = 0$

Multiply by 5 to find a second equation.

$5\left(x^2 - 2x - 11\right) = 5x^2 - 10x - 55 = 0$

(other answers possible)

88. $\left(x - 2 - 3\sqrt{5}\right)\left(x - 2 + 3\sqrt{5}\right) = 0$

$\left(x - 2\right)^2 - \left(3\sqrt{5}\right)^2 = 0$

$x^2 - 4x + 4 - 45 = 0$

$x^2 - 4x - 41 = 0$

Multiply by 2 to find a second equation.

$2\left(x^2 - 4x - 41\right) = 2x^2 - 8x - 82 = 0$

(other answers possible)

89. $\left[x - \left(2 + i\right)\right]\left[x - \left(2 - i\right)\right] = 0$

$\left[\left(x - 2\right) - i\right]\left[\left(x - 2\right) + i\right] = 0$

$\left(x - 2\right)^2 + 1 = 0$

$x^2 - 4x + 5 = 0$

Multiply by -1 to find a second equation.

$-\left(x^2 - 4x + 5\right) = -x^2 + 4x - 5 = 0$

(other answers possible)

90. $\left[x - \left(3 + 4i\right)\right]\left[x - \left(3 - 4i\right)\right] = 0$

$\left[\left(x - 3\right) - 4i\right]\left[\left(x - 3\right) + 4i\right] = 0$

$\left(x - 3\right)^2 + 16 = 0$

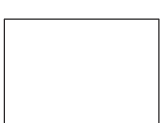
$x^2 - 6x + 25 = 0$

Multiply by -1 to find a second equation.

$-\left(x^2 - 6x + 25\right) = -x^2 + 6x - 25 = 0$

(other answers possible)

91. (a)



92. $S = x^2 + 4xh$

$561 = x^2 + 4x(4)$

$561 = x^2 + 16x$

$561 + 64 = x^2 + 16x + 64$

$625 = (x + 8)^2$

$\pm\sqrt{625} = x + 8$

$-8 \pm 25 = x$

Because $x > 0$, $x = 17$.

The dimensions are 17 feet by 17 feet by 4 feet.

93. (a) $4x + 3y = 100$ (amount of fence)

$y = \dfrac{1}{3}\left(100 - 4x\right)$

Domain: $0 < x < 25$

Area $= A\left(x\right) = \left(2x\right)y = 2x\dfrac{1}{3}\left(100 - 4x\right)$

$= \dfrac{8}{3}x\left(25 - x\right)$

$= -\dfrac{8}{3}x^2 + \dfrac{200}{3}x$

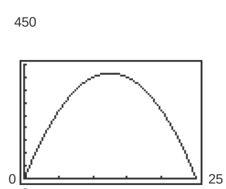
(b)

x	y	Area
2	$\frac{92}{3}$	$\frac{368}{3} \approx 123$
4	28	224
6	$\frac{76}{3}$	304
8	$\frac{68}{3}$	$\frac{1088}{3} \approx 363$
10	20	400
12	$\frac{52}{3}$	416
14	$\frac{44}{3}$	$\frac{1232}{3} \approx 411$

Approximate dimensions for maximum area:

$x = 12$, $y = \frac{52}{3}$, or $24 \text{ m} \times \frac{52}{3}$ m

(c)



w

Approximate dimensions for maximum area:

$w + 14$

(b) $w(w+14) = 1632$

$w^2 + 14w - 1632 = 0$

(c) $(w+48)(w-34) = 0$

$w = 34,$ length $= w + 14 = 48$

Width: 34 feet, length: 48 feet

$x = 12.5,\ y = 16.67,$ or $25 \text{ m} \times \frac{50}{3}$ m

(d) The graphs of $y_1 = \frac{8}{3}x(25-x)$ and $y_2 = 350$ intersect at $x = 7.5$ and $x = 17.5$. The dimensions are therefore $15 \text{ m} \times 23\frac{1}{3}$ m and $35 \text{ m} \times 10$ m.

(e)
$$\frac{8}{3}x(25-x) = 350$$
$$-8x^2 + 200x = 1050$$
$$4x^2 - 100x + 525 = 0$$
$$(2x-35)(2x-15) = 0$$
$$x = \frac{35}{2} \text{ or } x = \frac{15}{2}$$

Dimensions: $35 \text{ m} \times 10$ m and $15 \text{ m} \times 23\frac{1}{3}$ m

94. $V = \text{Length} \cdot \text{Width} \cdot \text{Height} = 576 \text{ cm}^3$

$$\begin{aligned}(x)(x)(4) &= 576\\ 4x^2 &= 576\\ x^2 &= 144\\ x &= 12 \text{ cm}\end{aligned}$$

Because $x = 12$ cm, the original piece of material is $12 + 4 + 4 = 20$ cm by 20 cm.

95. (a) $$\begin{aligned}s &= -16t^2 + v_0 t + s_0\\ &= -16t^2 + (0)t + 2080\\ &= -16t^2 + 2080\end{aligned}$$

(b)

t	0	2	4	6	8	10	12
s	2080	2016	1824	1504	1056	480	−224

(c) From the table, the object reaches the ground between 10 and 12 seconds, $(10, 12)$.

$$\begin{aligned}0 &= -16t^2 + 2080\\ 16t^2 &= 2080\\ t^2 &= 130\\ t &= \pm\sqrt{130}\\ t &= \pm 11.40\end{aligned}$$

The object reaches the ground after 11.40 seconds.

96. (a) $s = -16t^2 + v_0 t + s_0$

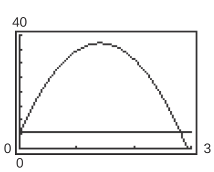
$v_0 = 45,\ s_0 = 5.5$

$s = -16t^2 + 45t + 5.5$

(b) $s\left(\frac{1}{2}\right) = -16\left(\frac{1}{2}\right)^2 + 45\left(\frac{1}{2}\right) + 5.5 = 24$ feet

(c) $$\begin{aligned}-16t^2 + 45t + 5.5 &= 6\\ 16t^2 - 45t + 0.5 &= 0\end{aligned}$$

Using the Quadratic Formula, $t \approx 2.801$ seconds.



The curves $y = -16t^2 + 45t + 5.5$ and $y = 6$ intersect at $t \approx 2.801$.

97. (a) $$\begin{aligned}s &= -16t^2 + v_0 t + s_0\\ 0 &= -16t^2 + 8000\\ 16t^2 &= 8000\\ t^2 &= 500\\ t &= 10\sqrt{5} \approx 22.36 \text{ seconds.}\end{aligned}$$

(b) $$\begin{aligned}\text{Distance} &= \frac{(600 \text{ miles/hour})(10\sqrt{5} \text{ seconds})}{(3600 \text{ seconds/hour})}\\ &= \frac{1}{6}\left(10\sqrt{5}\right) \text{ miles}\\ &\approx 3.73 \text{ miles} \approx 19{,}677.4 \text{ feet}\end{aligned}$$

98. (a) $s(t) = -16t^2 + 1100$

(b) The pellets hit the ground when $16t^2 = 1100 \Rightarrow t \approx 8.29$ seconds.

95 miles per hour $= \dfrac{95}{3600}$ miles per second.

The plane travels

$\dfrac{95}{3600} \cdot 8.29 \approx 0.22$ mile, or 1162 feet.

99. (a) $S = -0.143t^2 + 3.73t + 32.5, \quad 5 \le t \le 13$

Find t when $S = 50$.

$$50 = -0.143t^2 + 3.73t + 32.5$$
$$0 = -0.143t^2 + 3.73t - 17.5$$
$$t = \frac{-(3.73) \pm \sqrt{(3.73)^2 - 4(-0.143)(-17.5)}}{2(-0.143)}$$
$$t = \frac{-3.73 \pm \sqrt{13.9129 - 10.01}}{-0.286}$$
$$t = \frac{-3.73 \pm \sqrt{3.9029}}{-0.286}$$

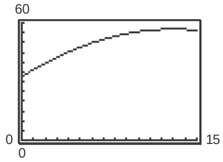
$t \approx 6.13$ and $t \approx 19.95$ (not in the domain of the model)

In 2006, the average salary was $50,000.

(b)

t	S
5	47.46
6	49.73
7	51.60
8	53.19
9	54.49
10	55.50
11	56.23
12	56.67
13	56.82

(c)



(d)
$$55.5 = -0.143t^2 + 3.73t + 32.5$$
$$0 = -0.143t^2 + 3.73t - 23$$
$$t = \frac{-(3.73) \pm \sqrt{(3.73)^2 - 4(-0.143)(-23)}}{2(-0.143)}$$
$$t = \frac{-3.73 \pm \sqrt{13.9129 - 13.156}}{-0.286}$$
$$t = \frac{-3.73 \pm \sqrt{0.7569}}{-0.286}$$

$t = 10$ and $t \approx 16.1$ (not in the domain of the model)

In 2010, the average salary was $55,500.

(e) Answers will vary. Sample answer: For some years, the model may be used to predict the average salaries for years beyond 2013. However, the model eventually would yield values that begin to decrease and eventually become negative.

100. (a) $D = 0.051t^2 + 0.20t + 5.0, \ 5 \le t \le 14$

Find t when $D = 10$.

$10 = 0.051t^2 + 0.20t + 5.0$

$0 = 0.051t^2 + 0.20t - 5.0$

$$t = \frac{-(0.20) \pm \sqrt{(0.20)^2 - 4(0.051)(-5.0)}}{2(0.051)}$$

$$t = \frac{-0.20 \pm \sqrt{0.04 + 1.02}}{0.102}$$

$$t = \frac{-0.20 \pm \sqrt{1.06}}{0.102}$$

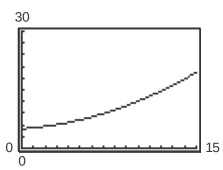
$t \approx 8.13$, and $t \approx -12.05$ (not in the domain of the model)

In 2008, total public debt reached \$10 trillion.

(b)

t	D
5	7.28
6	8.04
7	8.90
8	9.86
9	10.93
10	12.10
11	13.37
12	14.74
13	16.22
14	17.80

(c)



(d) $20 = 0.051t^2 + 0.20t + 5.0$

$0 = 0.051t^2 + 0.20t - 15.0$

$$t = \frac{-(0.20) \pm \sqrt{(0.20)^2 - 4(0.051)(-15.0)}}{2(0.051)}$$

$$t = \frac{-0.20 \pm \sqrt{0.04 + 3.06}}{0.102}$$

$$t = \frac{-0.20 \pm \sqrt{3.10}}{0.102}$$

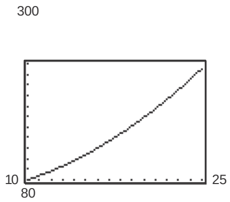
$t \approx 15.3$, and $t \approx -19.2$ (not in the domain of the model)

In 2015, the total public debt will reach \$20 trillion.

(e) Answers will vary. Sample answer: For some years, the model may be used to predict the total public debt for years beyond 2014. However, the model eventually would yield values that are very high.

101. $C = 0.45x^2 - 1.73x + 52.65,\ 10 \le x \le 25$

(a)

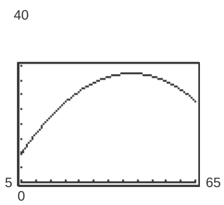


(b) When $C = 150$, $x = 16.797°$ C.

(c) If the temperature is increased from $10°$ C to $20°$ C, the oxygen consumption increases by a factor of approximately 2.5, from 80.35 to 198.05.

102. $F = -0.091s^2 + 1.639s + 2.20,\ 5 \le s \le 65$

(a)



(b) Use the *maximum* feature of a graphing utility to find the greatest fuel efficiency. So, the car should travel at a speed of 42.91 miles per hour for the greatest fuel efficiency of 37.36 miles per gallon.

(c) When the average speed of the car is increased from 20 miles per hour to 30 miles per hour, the fuel efficiency will increase from 27.34 miles per gallon to 34.18 miles per gallon, which is a factor of approximately 1.25.

103. (a) $x^2 + 15^2 = l^2$

$x^2 + 225 = l^2$

(b)
$$
\begin{aligned}
x^2 + 225 &= (75)^2\\
x^2 + 225 &= 5625\\
x^2 &= 5400\\
x &= \pm\sqrt{5400}\\
x &= \pm 30\sqrt{6}\\
x &\approx \pm 73.5
\end{aligned}
$$

Because $x > 0$, the distance is about 73.5 feet.

104. Let u be the speed of the eastbound plane. Then $u + 50 =$ speed of northbound plane.

$$
\begin{aligned}
\left[u(3)\right]^2 + \left[(u+50)3\right]^2 &= 2440^2\\
9u^2 + 9(u+50)^2 &= 2440^2\\
18u^2 + 900u + 22{,}500 &= 2440^2\\
18u^2 + 900u - 5{,}931{,}100 &= 0
\end{aligned}
$$

$$u = \frac{-900 \pm \sqrt{(900)^2 - 4(18)(-5{,}931{,}100)}}{2(18)}$$

105. False. The solutions are complex numbers.

106. False. You can only draw a conclusion about the factors if their product is 0, not 8.

107. False. The solutions are either both imaginary or both real.

108. (a)
$$
\begin{aligned}
ax^2 + bx &= 0\\
x(ax + b) &= 0\\
x &= 0\\
ax + b = 0 \Rightarrow ax &= -b\\
x &= -\frac{b}{a}
\end{aligned}
$$

(b)
$$
\begin{aligned}
ax^2 - ax &= 0\\
ax(x - 1) &= 0\\
ax &= 0\\
x &= 0\\
x - 1 = 0 \Rightarrow x &= 1
\end{aligned}
$$

109. Add the two solutions and the radicals cancel.

$$S = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \ b^2 - 4ac}{2a}$$

$= -$

$$= \frac{-900 \pm \sqrt{427{,}849{,}200}}{36}$$

$$\approx 549.57$$

$$u + 50 = 599.57$$

$\overline{a}$

So, the eastband plane is traveling at about 550 mph and the northband plane is traveling at about 600 mph.

110. Multiply the two solutions and the radicals disappear.

$$P = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$
$$= \frac{(-b)^2 - (b^2 - 4ac)}{4a^2}$$
$$= \frac{4ac}{4a^2} = \frac{c}{a}$$

111. (a) $3x^2 + 5x - 11 = 0$: Quadratic Formula; The equation does not factor and it is not easily solved using completing the square, nor is it the type to extract square roots.

(b) $x^2 + 10x = 3$: Complete the square; The equation is easily solved by completing the square since the leading coefficient is 1, and the linear term is even.

(c) $x^2 - 16 + 64 = 0$: Factoring; The equation is easily factored, since it is a perfect square trinomial.

(d) $x^2 - 15 = 0$: Extracting square roots; The equation is of the type $x^2 - d = 0$, and is easily solved by extracting square roots.

112. (a) Because the equation $0 = (x - 1)^2 + 2$ has no real solutions, its graph has no x-intercepts. So, the equation matches graph (ii).

(b) Because the equation $0 = (x + 1)^2 - 2$ has two real solutions, its graph has two x-intercepts. So, the equation matches graph (i).

113. $x^5 - 27x^2 = x^2(x^3 - 27) = x^2(x - 3)(x^2 + 3x + 9)$

114.
$$x^3 - 5x^2 - 14x = x(x^2 - 5x - 14)$$
$$= x(x - 7)(x + 2)$$

115.
$$x^3 + 5x^2 - 2x - 10 = x^2(x + 5) - 2(x + 5)$$
$$= (x^2 - 2)(x + 5)$$
$$= (x + \sqrt{2})(x - \sqrt{2})(x + 5)$$

116.
$$5(x + 5)x^{1/3} + 4x^{4/3} = x^{1/3}\left[5(x + 5) + 4x\right]$$
$$= x^{1/3}(9x + 25)$$

117. Answers will vary. (Make a Decision)

Section 2.5 Solving Other Types of Equations Algebraically

1. polynomial

2. $x(x - 3)$

3. To eliminate or remove the radical from the equation $\sqrt{x + 2} = x$, square each side of the equation to produce the equation $x + 2 = x^2$.

4. The equation $x^4 - 2x + 4 = 0$ is *not* of quadratic type.

5.
$$4x^4 - 16x^2 = 0$$
$$4x^2(x^2 - 4) = 0$$
$$4x^2(x - 2)(x + 2) = 0$$
$$x = 0, \ \pm 2$$

6.
$$8x^4 - 18x^2 = 0$$
$$2x^2(4x^2 - 9) = 0$$
$$2x^2(2x + 3)(2x - 3) = 0$$
$$x = 0,$$

7.
$$7x^3 + 63x = 0$$
$$7x(x^2 + 9) = 0$$
$$7x = 0 \Rightarrow x = 0$$
$$x^2 + 9 = 0 \Rightarrow x^2 = -9$$
$$x = \pm\sqrt{-9}$$
$$x = \pm 3i$$

8.
$$x^3 + 512 = 0$$
$$(x + 8)(x^2 - 8x + 64) = 0$$
$$x + 8 = 0 \Rightarrow x = -8$$
$$x^2 - 8x + 64 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-8) \pm}{}$$

± 3

$(-8)^2$

$2(1)$

2

$\sqrt{-4(1)(64)}$

$$= \frac{8 \pm \sqrt{64 - 256}}{2}$$

$$= \frac{8 \pm \sqrt{-192}}{2}$$

$$= \frac{8 \pm 8\sqrt{3}i}{2}$$

$$= 4 \pm 4\sqrt{3}i$$

9.
$$5x^3 + 30x^2 + 45x = 0$$
$$5x\left(x^2 + 6x + 9\right) = 0$$
$$5x\left(x + 3\right)^2 = 0$$
$$5x = 0 \Rightarrow x = 0$$
$$x + 3 = 0 \Rightarrow x = -3$$

10.
$$9x^4 - 24x^3 + 16x^2 = 0$$
$$x^2\left(9x^2 - 24x + 16\right) = 0$$
$$x^2\left(3x - 4\right)^2 = 0$$
$$x^2 = 0 \Rightarrow x = 0$$
$$3x - 4 = 0 \Rightarrow x = \tfrac{4}{3}$$

11.
$$x^3 + 5 = 5x^2 + x$$
$$x^3 - 5x^2 - x + 5 = 0$$
$$x^2\left(x - 5\right) - \left(x - 5\right) = 0$$
$$\left(x - 5\right)\left(x^2 - 1\right) = 0$$
$$\left(x - 5\right)\left(x + 1\right)\left(x - 1\right) = 0$$
$$x - 5 = 0 \Rightarrow x = 5$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$x - 1 = 0 \Rightarrow x = 1$$

12.
$$x^4 + 2x^3 - 8x - 16 = 0$$
$$x^3\left(x + 2\right) - 8\left(x + 2\right) = 0$$
$$\left(x^3 - 8\right)\left(x + 2\right) = 0$$
$$\left(x - 2\right)\left(x^2 + 2x + 4\right)\left(x + 2\right) = 0$$
$$x^2 + 2x + 4 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4 - 16}}{2} = -1 \pm \sqrt{3}i$$
$$x - 2 = 0 \Rightarrow x = 2$$
$$x + 2 = 0 \Rightarrow x = -2$$

13.
$$x^4 - 4x^2 + 3 = 0$$
$$\left(x^2 - 3\right)\left(x^2 - 1\right) = 0$$
$$\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)\left(x + 1\right)\left(x - 1\right) = 0$$
$$x + \sqrt{3} = 0 \Rightarrow x = -\sqrt{3}$$

15.
$$36t^4 + 29t^2 - 7 = 0$$
$$\left(36t^2 - 7\right)\left(t^2 + 1\right) = 0$$
$$\left(6t + \sqrt{7}\right)\left(6t - \sqrt{7}\right)\left(t^2 + 1\right) = 0$$
$$6t + \sqrt{7} = 0 \Rightarrow t = -\frac{\sqrt{7}}{6}$$
$$6t - \sqrt{7} = 0 \Rightarrow t = \frac{\sqrt{7}}{6}$$
$$t^2 + 1 = 0 \Rightarrow t = \pm i$$

16.
$$4x^4 - 65x^2 + 16 = 0$$
$$\left(4x^2 - 1\right)\left(x^2 - 16\right) = 0$$
$$\left(2x + 1\right)\left(2x - 1\right)\left(x + 4\right)\left(x - 4\right) = 0$$
$$2x + 1 = 0 \Rightarrow x = -\tfrac{1}{2}$$
$$2x - 1 = 0 \Rightarrow x = \tfrac{1}{2}$$
$$x + 4 = 0 \Rightarrow x = -4$$
$$x - 4 = 0 \Rightarrow x = 4$$

17.
$$3\sqrt{x} - 10 = 0$$
$$3\sqrt{x} = 10$$
$$9x = 100$$
$$x = \tfrac{100}{9}$$

18.
$$3\sqrt{x} - 6 = 0$$
$$3\sqrt{x} = 6$$
$$\sqrt{x} = 2$$
$$\left(\sqrt{x}\right)^2 = \left(2\right)^2$$
$$x = 4$$

19.
$$\sqrt{x - 10} - 4 = 0$$
$$\sqrt{x - 10} = 4$$
$$x - 10 = 16$$
$$x = 26$$

20.
$$\sqrt{2x + 5} + 3 = 0$$
$$\sqrt{2x + 5} = -3$$

No solution

$$x-\sqrt{3}=0 \Rightarrow x=\sqrt{3}$$
$$x+1=0 \Rightarrow x=-1$$
$$x-1=0 \Rightarrow x=1$$

14. $$x^4-5x^2-36=0$$

$$\left(x^2-9\right)\left(x^2+4\right)=0$$
$$\left(x+3\right)\left(x-3\right)\left(x^2+4\right)=0$$

$$x+3=0 \Rightarrow \quad x=-3$$
$$x-3=0 \Rightarrow \quad x=3$$
$$x^2+4=0 \Rightarrow \quad x^2=-4=\pm 2i$$

21. $$\sqrt[3]{6x}+9=0$$
$$\sqrt[3]{6x}=-9$$
$$\left(\sqrt[3]{6x}\right)^3=\left(-9\right)^3$$

$$6x=-729$$
$$x=-\frac{243}{2}$$

22. $2\sqrt[3]{x} - 10 = 0$

$2\sqrt[3]{x} = 10$

$\sqrt[3]{x} = 5$

$\left(\sqrt[3]{x}\right)^3 = (5)^3$

$x = 125$

23. $\sqrt[3]{2x+1} + 8 = 0$

$\sqrt[3]{2x+1} = -8$

$2x + 1 = -512$

$2x = -513$

$x = -\frac{513}{2}$

$= -256.5$

24. $\sqrt[3]{4x-3} + 2 = 0$

$(4x-3)^{1/3} = -2$

$4x - 3 = -8$

$4x = -5$

$x = -\frac{5}{4}$

25. $\sqrt{5x-26} + 4 = x$

$\sqrt{5x-26} = x - 4$

$5x - 26 = x^2 - 8x + 16$

$x^2 - 13x + 42 = 0$

$(x-6)(x-7) = 0$

$x - 6 = 0 \Rightarrow x = 6$

$x - 7 = 0 \Rightarrow x = 7$

26. $x - \sqrt{8x-31} = 5$

$x - 5 = \sqrt{8x-31}$

$x^2 - 10x + 25 = 8x - 31$

$x^2 - 18x + 56 = 0$

$(x-4)(x-14) = 0$

$x - 4 = 0 \Rightarrow x = 4,$ extraneous

28. $\sqrt{x+5} - 2x = 3$

$\sqrt{x+5} = 2x + 3$

$x + 5 = 4x^2 + 12x + 9$

$4x^2 + 11x + 4 = 0$

$x = \frac{-11 \pm \sqrt{121-64}}{8} = \frac{-11 \pm \sqrt{57}}{8}$

$= \frac{-11 - \sqrt{57}}{8}$ (extraneous) and

$\frac{-11 \pm \sqrt{57}}{8}$

29. $\sqrt{x+1} = \sqrt{3x+1}$

$x + 1 = 3x + 1$

$0 = 2x$

$x = 0$

30. $\sqrt{x+5} = \sqrt{2x-5}$

$x + 5 = 2x - 5$

$10 = x$

$x = 10$

31. $2x + 9\sqrt{x} - 5 = 0$

$\left(2\sqrt{x} - 1\right)\left(\sqrt{x} + 5\right) = 0$

$\sqrt{x} = \frac{1}{2} \Rightarrow x = \frac{1}{4}$

($\sqrt{x} = -5$ is not possible.**)**

Note: You can see graphically that there is only one solution.

32. $6x - 7\sqrt{x} - 3 = 0$

$6x - 3 = 7\sqrt{x}$

$(6x-3)^2 = \left(7\sqrt{x}\right)^2$

$36x^2 - 36x + 9 = 49x$

$36x^2 - 85x + 9 = 0$

27.

x
-1
$4 =$
0
$\Rightarrow$
x
$= 1$
4

$(9x-1)(4x-9) = 0$

$9x - 1 = 0$
$\Rightarrow x = \frac{1}{9}$, extraneous

$4x - 9 = 0$
$\Rightarrow x = \frac{9}{4}$

$\sqrt{}\quad\sqrt{}$

$x + 1$
$- 3x$
$= 1$
$\sqrt{}$

x
$+$
1
$=$
3
x
$+$
1
$\sqrt{}$

$$x + 1 = 9x^2 + 6x + 1$$
$$0 = 9x^2 + 5x$$
$$0 = x(9x + 5)$$
$$x = 0$$
$$9x + 5 = 0 \Rightarrow x = -\tfrac{5}{9}, \text{ extraneous}$$

33. $x - \quad x - 5 = 1$

$$\sqrt{x} = 1 + \sqrt{x-5}$$
$$\left(\sqrt{x}\right)^2 = \left(1 + \sqrt{x-5}\right)^2$$
$$x = 1 + 2\sqrt{x-5} + x - 5$$
$$4 = 2\sqrt{x-5}$$
$$2 = \sqrt{x-5}$$
$$4 = x - 5$$
$$9 = x$$

34. $\sqrt{x} + \sqrt{x-20} = 10$

$$\sqrt{x} = 10 - \sqrt{x-20}$$

$$\left(\sqrt{x}\right)^2 = \left(10 - \sqrt{x-20}\right)^2$$

$$x = 100 - 20\sqrt{x-20} + x - 20$$

$$-80 = -20\sqrt{x-20}$$

$$4 = \sqrt{x-20}$$

$$16 = x - 20$$

$$36 = x$$

35. $3\sqrt{x-5} + \sqrt{x-1} = 0$

$$3\sqrt{x-5} = -\sqrt{x-1}$$

$$\left(3\sqrt{x-5}\right)^2 = \left(-\sqrt{x-1}\right)^2$$

$$9(x-5) = x-1$$

$$9x - 45 = x - 1$$

$$8x = 44$$

$$x = \frac{11}{2}, \text{ extraneous}$$

No solution

36. $4\sqrt{x-3} - \sqrt{6x-17} = 13$

$$4\sqrt{x-3} = \sqrt{6x-17} + 3$$

$$16(x-3) = (6x-17) + 9 + 6\sqrt{6x-17}$$

$$10x - 40 = 6\sqrt{6x-17}$$

$$5x - 20 = 3\sqrt{6x-17}$$

$$25x^2 - 200x + 400 = 9(6x-17)$$

$$25x^2 - 254x + 553 = 0$$

$$(x-7)(25x-79) = 0$$

$$x = 7$$

($x = \frac{79}{25}$ is extraneous.)

37. $3x^{1/3} + 2x^{2/3} = 5$

39. $(x+6)^{3/2} = 1$

$$\left[(x+6)^{3/2}\right]^{2/3} = (1)^{2/3}$$

$$x + 6 = 1$$

$$x = -5$$

40. $(x-1)^{3/2} = 8$

$$x - 1 = 8^{2/3}$$

$$x - 1 = 4$$

$$x = 5$$

41. $(x-9)^{2/3} = 25$

$$\left[(x-9)^{2/3}\right]^{3/2} = (25)^{3/2}$$

$x - 9 = (5)^3$ $\quad$ $x - 9 = (-5)^3$

$x - 9 = 125$ $\quad$ $x - 9 = -125$

$x = 134$ $\quad$ $x = -116$

42. $(x-7)^{2/3} = 9$

$$\left[(x-7)^{2/3}\right]^{3/2} = (9)^{3/2}$$

$x - 7 = (3)^3$ $\quad$ $x - 7 = (-3)^3$

$x - 7 = 27$ $\quad$ $x - 7 = -27$

$x = 34$ $\quad$ $x = -20$

43. $(x^2 - 5x - 2)^{1/3} = -2$

$$x^2 - 5x - 2 = (-2)^3 = -8$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x - 3 = 0 \Rightarrow x = 3$$

$$x - 2 = 0 \Rightarrow x = 2$$

44. $(x^2 - x - 22)^{4/3} = 16$

$$2x^{2/3} + 3x^{1/3} - 5 = 0$$
$$\left(2x^{1/3} + 5\right)\left(x^{1/3} - 1\right) = 0$$
$$x^{1/3} = -\tfrac{5}{2} \Rightarrow x = -\tfrac{125}{8}$$
$$x^{1/3} = 1 \Rightarrow x = 1$$

38. $9t^{2/3} + 24t^{1/3} + 16 = 0$

$$\left(3t^{1/3} + 4\right)\left(3t^{1/3} + 4\right) = 0$$
$$3t^{1/3} + 4 = 0$$
$$3t^{1/3} = -4$$
$$t^{1/3} = -\frac{4}{3} \Rightarrow t = -\frac{64}{27}$$

$$x^2 - x - 22 = \pm 16^{3/4}$$
$$x^2 - x - 22 = \pm 8$$
$$x^2 - x - 30 = 0 \Rightarrow x = -5,\ 6$$
$$x^2 - x - 14 = 0 \Rightarrow x = \frac{1 \pm \sqrt{57}}{2}$$

45. $3x\left(x-1\right)^{1/2} + 2\left(x-1\right)^{3/2} = 0$

$$\left(x-1\right)^{1/2}\left[3x + 2\left(x-1\right)\right] = 0$$
$$\left(x-1\right)^{1/2}\left(5x-2\right) = 0$$
$$\left(x-1\right)^{1/2} = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1$$
$$5x - 2 = 0 \Rightarrow x = \tfrac{2}{5}\text{, extraneous}$$

46. $4x^2(x-1)^{1/3} + 6x(x-1)^{4/3} = 0$

$2x\left[2x(x-1)^{1/3} + 3(x-1)^{4/3}\right] = 0$

$2x(x-1)^{1/3}\left[2x + 3(x-1)\right] = 0$

$2x(x-1)^{1/3}(5x-3) = 0$

$2x = 0 \Rightarrow x = 0$

$x - 1 = 0 \Rightarrow x = 1$

$5x - 3 = 0 \Rightarrow x = \frac{3}{5}$

47. $x = \frac{3}{x} + \frac{1}{2}$

$(2x)(x) = (2x)\left(\frac{3}{x}\right) + (2x)\left(\frac{1}{2}\right)$

$2x^2 = 6 + x$

$2x^2 - x - 6 = 0$

$(2x+3)(x-2) = 0$

$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

$x - 2 = 0 \Rightarrow x = 2$

48. $\frac{4}{x} - \frac{5}{3} = \frac{x}{6}$

$(6x)\frac{4}{x} - (6x)\frac{5}{3} = (6x)\frac{x}{6}$

$24 - 10x = x^2$

$x^2 + 10x - 24 = 0$

$(x+12)(x-2) = 0$

$x + 12 = 0 \Rightarrow x = -12$

$x - 2 = 0 \Rightarrow x = 2$

49. $\frac{20-x}{x} = x$

$20 - x = x^2$

$0 = x^2 + x - 20$

$0 = (x+5)(x-4)$

51. $\frac{1}{x} - \frac{1}{x+1} = 3$

$x(x+1)\frac{1}{x} - x(x+1)\frac{1}{x+1} = x(x+1)(3)$

$x + 1 - x = 3x(x+1)$

$1 = 3x^2 + 3x$

$0 = 3x^2 + 3x - 1$

$a = 3,\ b = 3,\ c = -1$

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(3)(-1)}}{2(3)} = \frac{-3 \pm \sqrt{21}}{6}$

52. $\frac{4}{x+1} - \frac{3}{x+2} = 1$

$4(x+2) - 3(x+1) = (x+1)(x+2)$

$4x + 8 - 3x - 3 = x^2 + 3x + 2$

$x^2 + 2x - 3 = 0$

$(x-1)(x+3) = 0$

$x = 1,\ -3$

53. $\frac{1}{t^2} + \frac{8}{t} + 15 = 0$

$1 + 8t + 15t^2 = 0$

$(1+3t)(1+5t) = 0$

$1 + 3t = 0 \Rightarrow t = -\frac{1}{3}$

$1 + 5t = 0 \Rightarrow t = -\frac{1}{5}$

54. $6 - \frac{1}{x} - \frac{1}{x^2} = 0$

$6x^2 - x - 1 = 0$

$(3x+1)(2x-1) = 0$

$x = -\frac{1}{3},\ \frac{1}{2}$

$x + 5 = 0 \Rightarrow x = -5$

55.

3
2

x

−

2

−

1

=

0 x

x

+

2

$x - 4 = 0 \Rightarrow x = 4$

50. $4x + 1 = \frac{3}{x}$

$(x)4x + (x)1 = (x)\frac{3}{x}$

$4x^2 + x = 3$

$4x^2 + x - 3 = 0$

$(4x - 3)(x + 1) = 0$

$4x - 3 = 0 \quad \Rightarrow \quad x = \frac{3}{4}$

$x + 1 = 0 \quad \Rightarrow \quad x = -1$

$\frac{x}{2}$
1

$x = x + 2$

$(x + 2)(x - 2) = x$

$x^2 - 4 = x$

$x^2 - x - 4 = 0$

$x = \frac{1 \pm \sqrt{1 - 4(-4)}}{2}$

$x = \frac{1}{2} \pm \frac{\sqrt{17}}{2}$

56. $\dfrac{x}{x^2-4}+\dfrac{1}{x+2}=3$

$(x+2)(x-2)\dfrac{x}{x^2-4}+(x+2)(x-2)\dfrac{1}{x+2}=3(x+2)(x-2)$

$x+x-2=3x^2-12$

$3x^2-2x-10=0$

$a=3,\ b=-2,\ c=-10$

$x=\dfrac{-(-2)\pm\sqrt{(-2)^2-4(3)(-10)}}{2(3)}$

$=\dfrac{2\pm\sqrt{124}}{6}=\dfrac{2\pm2\sqrt{31}}{6}=\dfrac{1\pm\sqrt{31}}{3}$

57. $6\left(\dfrac{s}{s+1}\right)^2+5\left(\dfrac{s}{s+1}\right)-6=0$

Let $u=s/(s+1)$.

$6u^2+5u-6=0$

$(3u-2)(2u+3)=0$

$3u-2=0 \Rightarrow u=\dfrac{2}{3}$

$2u+3=0 \Rightarrow u=-\dfrac{3}{2}$

$\dfrac{s}{s+1}=\dfrac{2}{3} \Rightarrow s=2$

$\dfrac{s}{s+1}=-\dfrac{3}{2} \Rightarrow s=-\dfrac{3}{5}$

58. $8\left(\dfrac{t}{t-1}\right)^2-2\left(\dfrac{t}{t-1}\right)-3=0$

Let $\dfrac{t}{t-1}=x$. Then:

$8x^2-2x-3=0$

$(2x+1)(4x-3)=0$

$x=-\dfrac{1}{2},\ x=\dfrac{3}{4}$

$\dfrac{t}{t-1}=-\dfrac{1}{2} \Rightarrow 2t=-t+1 \Rightarrow t=\dfrac{1}{3}$

$\dfrac{t}{t-1}=\dfrac{3}{4} \Rightarrow 4t=3t-3 \Rightarrow t=-3$

59. $|2x-5|=11$

$2x-5=11$ or $2x-5=-11$

$2x=16$ $\quad 2x=-6$

$x=8$ $\quad x=-3$

60. $|3x+2|=7$

$3x+2=7$ or $3x+2=-7$

$3x=5$ $\quad 3x=-9$

$x=\dfrac{5}{3}$ $\quad x=-3$

61. $|x|=x^2+x-24$

$x=x^2+x-24$ or $-x=x^2+x-24$

$0=x^2-24$ $\quad 0=x^2+2x-24$

$x^2=24$ $\quad 0=(x+6)(x-4)$

$x=\pm\sqrt{24}$ $\quad x+6=0 \Rightarrow x=-6$

$x=\pm2\sqrt{6}$ $\quad x=-2\sqrt{6}$, extraneous

$x = 2\sqrt{6}$ $\quad\quad\quad$ $x - 4 = 0 \Rightarrow x = 4$, extraneous

62. $\left|x^2 + 6x\right| = 3x + 18$

$$\begin{aligned} x^2 + 6x &= 3x + 18 \\ x^2 + 3x - 18 &= 0 \\ (x + 6)(x - 3) &= 0 \\ x + 6 = 0 &\Rightarrow x = -6 \\ x - 3 = 0 &\Rightarrow x = 3 \end{aligned}$$

or

$$\begin{aligned} x^2 + 6x &= -(3x + 18) \\ x^2 + 6x &= -3x - 18 \\ x^2 + 9x + 18 &= 0 \\ (x + 3)(x + 6) &= 0 \\ x + 3 = 0 &\Rightarrow x = -3 \\ x + 6 = 0 &\Rightarrow x = -6 \end{aligned}$$

63. $|x+1| = x^2 - 5$

$$x+1 = x^2 - 5 \quad \text{or} \quad -(x+1) = x^2 - 5$$

$$\begin{aligned} x^2 - x - 6 &= 0 \\ (x-3)(x+2) &= 0 \\ x &= 3 \\ x &= -2, \text{ is extraneous} \end{aligned}$$

$$\begin{aligned} x^2 + x - 4 &= 0 \\ x &= \frac{-1 \pm \sqrt{1 - 4(-4)}}{2} \\ x &= -\frac{1}{2} - \frac{\sqrt{17}}{2} \\ x &= -\frac{1}{2} + \frac{\sqrt{17}}{2}, \text{ extraneous} \end{aligned}$$

64. $|x - 15 = x^2 - 15x$

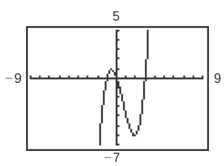
$$x - 15| = x^2 - 15x \quad \text{or} \quad x - 5 = -x^2 + 15x$$

$$\begin{aligned} 0 &= x^2 - 16x + 15 \\ 0 &= (x - 15)(x + 1) \\ x - 15 = 0 &\Rightarrow x = 15 \\ x + 1 = 0 &\Rightarrow x = -1 \end{aligned}$$

$$\begin{aligned} 0 &= x^2 - 14x - 5 \\ x &= \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(-5)}}{2} \\ x &= \frac{14 \pm \sqrt{216}}{2} \\ x &= 7 \pm 3\sqrt{6}, \text{ extraneous} \end{aligned}$$

65. $y = x^3 - 2x^2 - 3x$

(a)



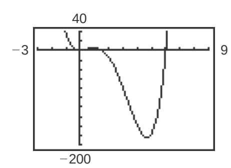
(b) x-intercepts: $(-1, 0)$, $(0, 0)$, $(3, 0)$

(c)
$$\begin{aligned} 0 &= x^3 - 2x^2 - 3x \\ 0 &= x(x+1)(x-3) \\ x &= 0 \\ x + 1 = 0 &\Rightarrow x = -1 \\ x - 3 = 0 &\Rightarrow x = 3 \end{aligned}$$

(d) The x-intercepts are the same as the solutions.

66. $y = 2x^4 - 15x^3 + 18x^2$

(a)



(b) x-intercepts: $(0, 0)$, $\left(\frac{3}{2}, 0\right)$, $(6, 0)$

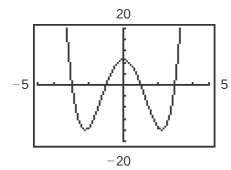
(c)
$$\begin{aligned} 0 &= 2x^4 - 15x^3 + 18x^2 \\ &= x^2\left(2x^2 - 15x + 18\right) \\ &= x^2(2x - 3)(x - 6) \\ 0 = x^2 &\Rightarrow x = 0 \\ 0 = 2x - 3 &\Rightarrow x = \tfrac{3}{2} \\ 0 = x - 6 &\Rightarrow x = 6 \end{aligned}$$

x-intercepts: $(0, 0)$, $\left(\frac{3}{2}, 0\right)$, $(6, 0)$

(d) The x-intercepts are the same as the solutions.

67. $y = x^4 - 10x^2 + 9$

(a)



(b) x-intercepts: $(\pm 1,\ 0),\ (\pm 3,\ 0)$

(c) $0 = x^4 - 10x^2 + 9$

$0 = (x^2 - 1)(x^2 - 9)$

$0 = (x + 1)(x - 1)(x + 3)(x - 3)$

$x + 1 = 0 \Rightarrow x = -1$

$x - 1 = 0 \Rightarrow x = 1$

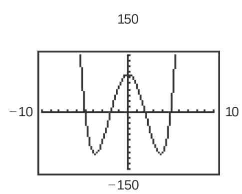
$x + 3 = 0 \Rightarrow x = -3$

$x - 3 = 0 \Rightarrow x = 3$

(d) The x-intercepts are the same as the solutions.

68. $y = x^4 - 29x^2 + 100$

(a)



(b) x-intercepts: $(\pm 2,\ 0),\ (\pm 5,\ 0)$

(c) $0 = x^4 - 29x^2 + 100$

$= (x^2 - 4)(x^2 - 25)$

$= (x + 2)(x - 2)(x + 5)(x - 5)$

$0 = x + 2 \Rightarrow x = -2$

$0 = x - 2 \Rightarrow x = 2$

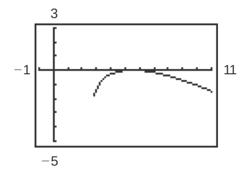
$0 = x + 5 \Rightarrow x = -5$

$0 = x - 5 \Rightarrow x = 5$

x-intercepts: $(-2,\ 0),\ (2,\ 0),\ (-5,\ 0),\ (5,\ 0)$

69. $y = \sqrt{11x - 30} - x$

(a)



(b) x-intercepts: $(5,\ 0),\ (6,\ 0)$

(c) $0 = \sqrt{11x - 30} - x$

$x = \sqrt{11x - 30}$

$x^2 = 11x - 30$

$x^2 - 11x + 30 = 0$

$(x - 5)(x - 6) = 0$

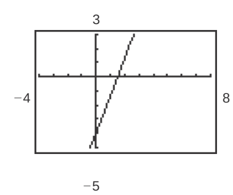
$x - 5 = 0 \Rightarrow x = 5$

$x - 6 = 0 \Rightarrow x = 6$

(d) The x-intercepts and the solutions are the same.

70. $y = 2x - \sqrt{15 - 4x}$

(a)



(b) x-intercept: $\left(\frac{3}{2},\ 0\right)$

(c) $0 = 2x - \sqrt{15 - 4x}$

$\sqrt{15 - 4x} = 2x$

$15 - 4x = 4x^2$

$0 = 4x^2 + 4x - 15$

$0 = (2x + 5)(2x - 3)$

$0 = 2x + 5 \Rightarrow x = -\frac{5}{2}$

$0 = 2x - 3 \Rightarrow x = \frac{3}{2}$

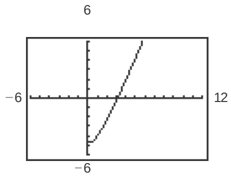
$x = -\frac{5}{2}$ is extraneous. The x-intercept is $\left(\frac{3}{2},\ 0\right)$.

(d) The x-intercepts are the same as the solutions.

(d) The x-intercept and the solution are the same.

71. $y = 3x - 3\sqrt{x} - 4$

(a)



(b) x-intercept: $(3.09164, 0)$

(c) $3x - 3\sqrt{x} - 4 = 0$. Let $y = \sqrt{x}$.

$3y^2 - 3y - 4 = 0$

$$y = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-4)}}{2(3)}$$

$$y = \frac{3 \pm \sqrt{57}}{6}$$

Because $y = \sqrt{x} = \frac{3 \pm \sqrt{57}}{6}$, then

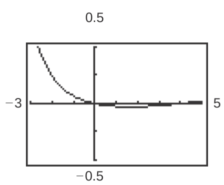
$$x = \frac{\left(3 + \sqrt{57}\right)^2}{36} \approx 3.09164$$

$$x = \frac{\left(3 - \sqrt{57}\right)^2}{6} \approx 0.57503 \text{ extraneous}$$

(d) The x-intercept and the solution are the same.

72. $y = \sqrt{7x + 36} - \sqrt{5x + 16} - 2$

(a)

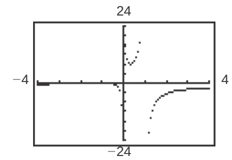


(b) x-intercepts: $(0, 0)$, $(4, 0)$

(c)
$$0 = \sqrt{7x + 36} - \sqrt{5x + 16} - 2$$
$$\sqrt{7x + 36} = 2 + \sqrt{5x + 16}$$
$$\left(\sqrt{7x + 36}\right)^2 = \left(2 + \sqrt{5x + 16}\right)^2$$
$$7x + 36 = 4 + 4\sqrt{5x + 16} + 5x + 16$$
$$7x + 36 = 5x + 20 + 4\sqrt{5x + 16}$$
$$2x + 16 = 4\sqrt{5x + 16}$$

73. $y = \frac{1}{x} - \frac{4}{x - 1} - 1$

(a)



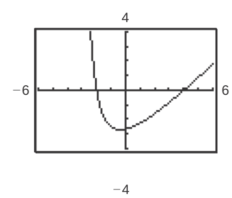
(b) x-intercept: $(-1, 0)$

(c)
$$0 = \frac{1}{x} - \frac{4}{x - 1} - 1$$
$$0 = (x - 1) - 4x - x(x - 1)$$
$$0 = x - 1 - 4x - x^2 + x$$
$$0 = -x^2 - 2x + 1$$
$$0 = x^2 + 2x + 1$$
$$x + 1 = 0 \Rightarrow x = -1$$

(d) The x-intercept and the solution are the same.

74. $y = x - 5 + \frac{7}{x + 3}$

(a)



(b) x-intercepts: $(-2, 0)$, $(4, 0)$

(c)
$$x - 5 + \frac{7}{x + 3} = 0$$
$$(x - 5)(x + 3) + 7 = 0$$
$$x^2 - 2x - 15 + 7 = 0$$
$$x^2 - 2x - 8 = 0$$
$$(x - 4)(x + 2) = 0$$
$$x - 4 = 0 \Rightarrow x = 4$$
$$x + 2 = 0 \Rightarrow x = -2$$

x-intercepts: $(-2, 0)$, $(4, 0)$

$x + 8 = 2 \quad 5x + 16 \; x^2$

$+ 16x + 64 = 4(5x + 16) \; x^2 +$

$16x + 64 = 20x + 64$

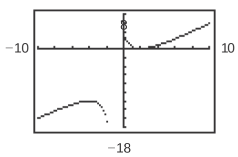
$$
\begin{aligned}
x^2 - 4x &= 0 \\
x(x-4) &= 0 \\
x &= 0 \\
x - 4 = 0 \Rightarrow x &= 4
\end{aligned}
$$

(d) The x-intercepts and the solutions are the same.

(d) The x-intercepts and the solutions are the same.

75. $y = x + \dfrac{9}{x+1} - 5$

(a)



(b) x-intercept: $(2, 0)$

(c) $0 = x + \dfrac{9}{x+1} - 5$

$0 = x(x+1) + (x+1)\dfrac{9}{x+1} - 5(x+1)$

$0 = x^2 + x + 9 - 5x - 5$

$0 = x^2 - 4x + 4$

$0 = (x-2)(x-2)$

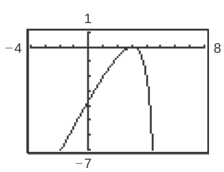
$0 = x - 2 \Rightarrow x = 2$

x-intercept: $(2, 0)$

(d) The x-intercept and the solution are the same.

76. $y = 2x + \dfrac{8}{x-5} - 2$

(a)



(b) x-intercept: $(3, 0)$

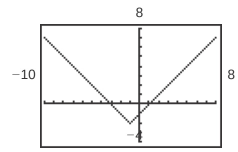
$$\begin{aligned} 2x + \frac{8}{x-5} - 2 &= 0 \\ 2x(x-5) + 8 - 2(x-5) &= 0 \\ 2x^2 - 10x + 8 - 2x + 10 &= 0 \\ 2x^2 - 12x + 18 &= 0 \\ x^2 - 6x + 9 &= 0 \\ (x-3)^2 &= 0 \\ x &= 3 \end{aligned}$$

x-intercept: $(3, 0)$

(c) The x-intercept and the solution are the same.

77. $y = |x+1| - 2$

(a)



(b) x-intercepts: $(1, 0)$, $(-3, 0)$

(c) $0 = |x+1| - 2$

$2 = |x+1| - 2$

$x + 1 = 2$ or $-(x+1) = 2$

$x = 1$ $\qquad -x - 1 = 2$

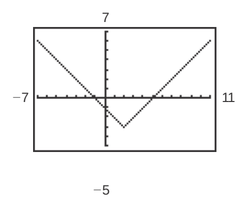
$-x = 3$

$x = -3$

(d) The x-intercepts and the solutions are the same.

78. $y = |x-2| - 3$

(a)



(b) x-intercepts: $(5, 0)$, $(-1, 0)$

(c) $0 = |x-2| - 3$

$3 = |x-2|$

First equation: $x - 2 = 3 \Rightarrow x = 5$

Second equation: $-(x-2) = 3$

$-(x-2) = 3$

$-x + 2 = 3 \Rightarrow x = -1$

(d) The x-intercepts and the solutions are the same.

79. Let x = the number of students. Then $\frac{1700}{x}$ is the original cost per student. So, $\frac{1700}{x} - 7.50$ is the reduced cost per student after 6 more students join the trip.

Verbal model: Cost per student × Number of students = Total cost

$$\left(\frac{1700}{x} - 7.50\right)(x + 6) = 1700$$
$$1700 + \frac{10{,}200}{x} - 7.5x - 45 = 1700$$
$$-7.5x - 45 + \frac{10{,}200}{x} = 0$$
$$7.5x^2 + 45x - 10{,}200 = 0$$
$$x^2 + 6x - 1360 = 0$$
$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-1360)}}{2(1)}$$
$$x = \frac{-6 \pm \sqrt{36 + 5440}}{2}$$
$$x = \frac{-6 \pm \sqrt{5476}}{2}$$
$$x = \frac{-6 - 74}{2}, \text{ extraneous}$$
$$x = \frac{-6 + 74}{2} = 34 \text{ students in the original group}$$

80. Let x = monthly rent for the apartment. The original rent for each student is $x/3$. By adding a fourth student, the rent is $x/4$. So,

$$\frac{x}{3} = \frac{x}{4} + 75$$
$$4x = 3x + 900$$
$$x = 900.$$

The monthly rent is \$900.

81. $A = P\left(1 + \frac{r}{n}\right)^{nt}, n = 12, t = 10$

$$11{,}752.45 = 7500\left(1 + \frac{r}{12}\right)^{(12)(10)}$$
$$\left(1 + \frac{r}{12}\right)^{120} = 1.566993$$
$$1 + \frac{r}{12} = 1.00375$$
$$\frac{r}{12} = 0.00375$$
$$r \approx 0.045, \text{ or } 4.5\%$$

82. $A = P\left(1 + \frac{r}{n}\right)^{nt}, n = 2, t = 20$

$$8055.19 = 3000\left(1 + \frac{r}{2}\right)^{(2)(20)}$$
$$\left(1 + \frac{r}{2}\right)^{40} = 2.685063$$
$$1 + \frac{r}{2} = 1.025000$$
$$\frac{r}{2} = 0.025000$$
$$r \approx 0.050, \text{ or } 5.0\%$$

83. $T = 75.82 - 2.11x + 43.51\sqrt{x},\ 5 \le x \le 40$

Find x when $T = 212°F$.

$$212 = 75.82 - 2.11x + 43.51\sqrt{x}$$

$$2.11x - 43.51\sqrt{x} + 136.18 = 0$$

Let $y = \sqrt{x}$.

$$2.11y^2 - 43.51y + 136.18 = 0$$

$$y = \frac{-(-43.51) \pm \sqrt{(-43.51)^2 - 4(2.11)(136.18)}}{2(2.11)}$$

$$= \frac{43.51 \pm \sqrt{743.7609}}{4.22}$$

$$y = \sqrt{x} = \frac{43.51 \pm \sqrt{743.7609}}{4.22}$$

$$x = \left(\frac{43.51 \pm \sqrt{743.7609}}{4.22}\right)^2$$

$x \approx 14.806$ pounds per square inch

$(x \approx 281.332$ is not in the given domain, $5 \le x \le 40.)$

84. $\sqrt{0.2x + 1} = C = 2.5$

$$0.2x + 1 = 6.25$$

$$0.2x = 5.25$$

$$x = 26.25, \text{ or } 26{,}250 \text{ passengers}$$

85. The hypotenuse of the right triangle is

$$\sqrt{x^2 + \left(\tfrac{3}{4}\right)^2} = \sqrt{x^2 + \frac{9}{16}} = \sqrt{\frac{16x^2 + 9}{4}}.$$

(a) Total cost = Cost of powerline over land + Cost of powerline under water

$$C = \frac{24 \text{ dollars}}{\text{foot}} \cdot 5280\frac{\text{feet}}{\text{mile}} \cdot (8 - x)\text{ miles} + \frac{30 \text{ dollars}}{\text{foot}} \cdot 5280\frac{\text{feet}}{\text{mile}} \cdot \frac{\sqrt{16x^2 + 9}}{4}\text{ miles}$$

$$C = 1{,}013{,}760 - 126{,}720x + 39{,}600\sqrt{16x^2 + 9}$$

(b) Find C when $x = 3$ (miles).

$$C = 1{,}013{,}760 - 126{,}720(3) + 39{,}600\sqrt{16(3)^2 + 9}$$

$$= \$1{,}123{,}424.95$$

(c) Find x, when $C = 1{,}098{,}662.40$.

$$1{,}013{,}760 - 126{,}720x + 39{,}600\sqrt{16x^2 + 9} = 1{,}098{,}662.40$$

$$39{,}600\sqrt{16x^2 + 9} - 126{,}720x - 84{,}902.4 = 0$$

$$39{,}600\sqrt{16x^2 + 9} = 126{,}720 + 84{,}902.4$$

$$\sqrt{16x^2 + 9} = 3.2x + 2.144$$

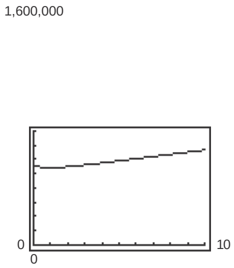
$$16x^2 + 9 = 10.24x^2 + 13.7216x + 4.596736$$

$$5.76x^2 - 13.7216x + 4.403264 = 0$$

Use the Quadratic Formula.

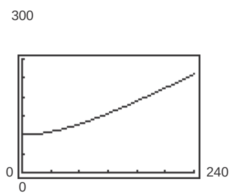
$x \approx 1.997 \Rightarrow 2$ miles

$x \approx 0.385 \Rightarrow 0.382$ mile

(d) 

(e) Using the graph from part (d), if $x \approx 1$ mile, the cost is minimized, $C \approx \$1{,}085{,}340$.

86. $d = \sqrt{100^2 + h^2}$

(a) 

If $d = 200$, $h \approx 173$.

(b)

h	160	165	170	175	180	185
d	188.68	192.94	197.23	201.56	205.91	210.3

When $d = 200$, $170 < h < 175$.

(c)

$$\sqrt{100^2 + h^2} = 200$$
$$100^2 + h^2 = 200^2$$
$$h^2 = 200^2 - 100^2$$
$$h \approx 173.205$$

(d) Solving graphically or numerically yields an approximate solution. An exact solution is obtained algebraically.

87. (a)

Year	2008	2009	2010	2011	2012
Crimes (in millions)	11.17	10.73	10.41	10.23	10.20

(b) According to the table, in 2008, the number of crimes committed fell below 11 million.

(c) $C = \sqrt{1.49145t^2 - 35.034t + 309.6}$, $8 \le t \le 12$

Find t when $C = 11$.

$$11 = \sqrt{1.49145t^2 - 35.034t + 309.6}$$

$$(11)^2 = \left(\sqrt{1.49145t^2 - 35.034t + 309.6}\right)^2$$

$$121 = 1.49145t^2 - 35.034t + 309.6$$
$$0 = 1.49145t^2 - 35.034t + 188.6$$
$$t = \frac{-(-35.034) \pm \sqrt{(-35.034)^2 - 4(1.49145)(188.6)}}{2(1.49145)}$$

$$t = \frac{35.034 \pm \sqrt{1227.381156 - 1125.14988}}{}$$

829

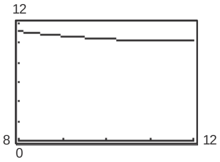
$$t = \frac{35.034 \pm \sqrt{102.231276}}{2.9829}$$

$$t \approx 8.36, 15.13$$

Because $t \approx 15.13$ is not in the domain of the model, $t \approx 8.36$.

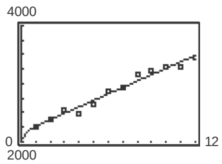
So, in 2008, the number of crimes committed fell below 1 million.

(d)



In 2008, the number of crimes fell below 11 million.

88. (a) and (b)



The model is a good fit for the data.

(c) Find t when $S = 4$.

$$S = 110.386t - \frac{16.4988}{t} + 2167.7, \quad 1 \le t \le 12$$

$$4000 = 110.386t - \frac{16.4988}{t} + 2167.7$$

$$0 = 110.386t - \frac{16.4988}{t} - 1832.3$$

$$0 = 110.386t^2 - 1832.3t - 16.4988$$

$$t = \frac{-(-1832.3) \pm \sqrt{(-1832.3)^2 - 4(110.386)(-16.4988)}}{2(110.386)}$$

$$t = \frac{1832.3 \pm \sqrt{3{,}357{,}323.29 + 7284.9461472}}{220.772}$$

$$t = \frac{1832.3 \pm \sqrt{3{,}364{,}608.23615}}{220.772}$$

$$t \approx 16.61, -0.01$$

Since $t \approx -0.01$ is not in the domain of the model, $t \approx 16.61$.

In 2016, the average MLB player salary will exceed $4 million.

89. False. An equation can have any number of extraneous solutions. For example, see Example 7.

90. False. Consider $|x| = 0 \Rightarrow x = 0$.

91. (a) The distance between $(1, 2)$ and $(x, -10)$ is 13.

$$\sqrt{(x-1)^2 + (-10-2)^2} = 13$$

$$(x-1)^2 + (-12)^2 = 13^2$$

$$x^2 - 2x + 1 + 144 = 169$$

$$x^2 - 2x - 24 = 0$$

$$(x+4)(x-6) = 0$$

(b) The distance between $(-8, 0)$ and $(x, 5)$ is 13.

$$\sqrt{(x+8)^2 + (5-0)^2} = 13$$

$$(x+8)^2 + 5^2 = 13^2$$

$$x^2 + 16x + 64 + 25 = 169$$

$$x^2 + 16x - 80 = 0$$

$$(x+20)(x-4) = 0$$

$x + 4 = 0 \Rightarrow x = -4$
$x - 6 = 0 \Rightarrow x = 6$

$x + 20 = 0 \quad \Rightarrow \quad x = -20$
$x - 4 = 0 \quad \Rightarrow \quad x = 4$

92. $x + \sqrt{x - a} = b,\ x = 20$

$20 + \sqrt{20 - a} = b$

One solution is $a = 19$ and $b = 21$. Another solution is $a = b = 20$.

93. Dividing each side of the equation by x loses the solution $x = 0$. The correct method is to first factor:

$$x^3 - 25x = 0$$
$$x(x^2 - 25) = 0$$
$$x = 0$$
$$x^2 = 25$$
$$x = \pm 5$$

94. (a) The expression represents the volume of water in the tank.

(b) Solve the equation for x and substitute that value into the expression x^3 to find the volume of the cube.

95. $\dfrac{8}{3x} + \dfrac{3}{2x} = \dfrac{16}{6x} + \dfrac{9}{6x} = \dfrac{25}{6x}$

96.
$$\frac{2}{x^2 - 4} - \frac{1}{x^2 - 3x + 2} = \frac{2}{(x-2)(x+2)} - \frac{1}{(x-2)(x-1)}$$
$$= \frac{2(x-1)}{(x-2)(x+2)(x-1)} - \frac{(x+2)}{(x-2)(x-1)(x+2)}$$
$$= \frac{2(x-1) - (x+2)}{(x-2)(x+2)(x-1)}$$
$$= \frac{x-4}{(x-2)(x+2)(x-1)}$$

97.
$$\frac{2}{z+2} - \left(3 - \frac{2}{z}\right) = \frac{2}{z+2} - 3 + \frac{2}{z}$$
$$= \frac{2z - 3(z+2)z + 2(z+2)}{z(z+2)}$$
$$= \frac{2z - 3z^2 - 6z + 2z + 4}{z(z+2)}$$
$$= \frac{-3z^2 - 2z + 4}{z(z+2)}$$

98.
$$25y^2 \div \frac{xy}{5} = 25y^2\left(\frac{5}{xy}\right)$$
$$= 125\left(\frac{y}{x}\right),\ y \neq 0$$

99.
$$x^2 - 22x + 121 = 0$$
$$(x - 11)^2 = 0$$
$$x = 11$$

100.
$$x(x - 20) + 3(x - 20) = 0$$
$$(x + 3)(x - 20) = 0$$
$$x = -3,\ 20$$

Section 2.6 Solving Inequalities Algebraically and Graphically

1. double

2. $-a \le x \le a$

3. $x \le -a,\ x \ge a$

4. ..., undefined valu...

8. $x \le 2$

Matches (a).

9. $-2 < x < 2$

5. The inequalities $x - 4 < 5$ and $x > 9$ are not equivalent. The first simplifies to $x - 4 < 5 \Rightarrow x < 9$, which is not equivalent to the second, $x > 9$.

6. The Transitive Property of Inequalities is as follows: $a < b$ and $b < c \Rightarrow a < c$.

7. $x < 2$
Matches (d).

Matches (f).

10. $-2 < x \leq 2$
Matches (b).

11. $-2 \leq x < 2$
Matches (e).

12. $-2 \leq x \leq 2$
Matches (c).

13. $5x - 12 > 0$

(a) $x = 3$

$5(3) - 12 \stackrel{?}{>} 0$

$3 > 0$

Yes, $x = 3$ is a solution.

(b) $x = -3$

$5(-3) - 12 \stackrel{?}{>} 0$

$-27 \not> 0$

No, $x = -3$ is not a solution.

(c) $x = \frac{5}{2}$

$5\left(\frac{5}{2}\right) - 12 \stackrel{?}{>} 0$

$\frac{1}{2} > 0$

Yes, $x = \frac{5}{2}$ is a solution.

(d) $x = \frac{3}{2}$

$5\left(\frac{3}{2}\right) - 12 \stackrel{?}{>} 0$

$-\frac{9}{2} \not> 0$

No, $x = \frac{3}{2}$ is not a solution.

14. $-5 < 2x - 1 \le 1$

(a) $x = 2$

$-5 \stackrel{?}{<} 2(2) - 1 \stackrel{?}{\le} 1$

$-5 < 3 \not\le 1$

No, $x = 2$ is not a solution.

(b) $x = -2$

$-5 \stackrel{?}{<} 2(-2) - 1 \stackrel{?}{\le} 1$

$-5 \not< -5 \le 1$

No, $x = -2$ is not a solution.

(c) $x = 0$

15. $-1 < \dfrac{3 - x}{2} \le 1$

(a) $x = -1$

$-1 \stackrel{?}{<} \dfrac{3 - (-1)}{2} \stackrel{?}{\le} 1$

$-1 < 2 \le 1$

No, $x = -1$ is not a solution.

(b) $x = \sqrt{5}$

$-1 \stackrel{?}{<} \dfrac{3 - \sqrt{5}}{2} \stackrel{?}{\le} 1$

$-1 < \dfrac{3 - \sqrt{5}}{2} \le 1$

Note: $\dfrac{3 - \sqrt{5}}{2} \approx 0.382$

Yes, $x = \sqrt{5}$ is a solution.

(c) $x = 1$

$-1 \stackrel{?}{<} \dfrac{3 - 1}{2} \stackrel{?}{\le} 1$

$-1 < 1 \le 1$

Yes, $x = 1$ is a solution.

(d) $x = 5$

$-1 \stackrel{?}{<} \dfrac{3 - 5}{2} \stackrel{?}{\le} 1$

$-1 < -1 \le 1$

No, $x = 5$ is not a solution.

16. $|x - 10| \ge 3$

(a) $x = 13$

$|13 - 10| \stackrel{?}{\ge} 3$

$|3| \ge 3$

Yes, $x = 13$ is a solution.

(b) $x = -1$

$-5 \stackrel{?}{<} 2(0) - 1 \le 1$

$-5 < -1 \le$

Yes, $x = 0$ is a solution.

(c)

(d) $x = -\frac{1}{2}$

$$-5 \overset{?}{<} 2\left(-\frac{1}{2}\right) - 1 \overset{?}{\leq} 1$$

$$-5 < -2 \leq 1$$

Yes, $x = -\frac{1}{2}$ is a solution.

$$|-1 - 10| \geq 3$$

$$|-11| \geq 3$$

Yes, $x = -1$ is a solution.

$x = 14$

$$|14 - 10| \overset{?}{\geq} 3$$

$$|4| \geq 3$$

Yes, $x = 14$ is a solution.

(d) $x = 8$

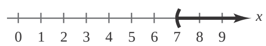
$$|8 - 10| \overset{?}{\geq} 3$$

$$|-2| \geq 3$$

No, $x = 8$ is not a solution.

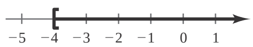
17. $6x > 42$

$x > 7$



18. $-10x \le 40$

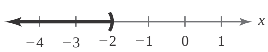
$x \ge -4$



19. $4x + 7 < 3 + 2x$

$2x < -4$

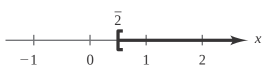
$x < -2$



20. $3x + 1 \ge 2 + x$

$2x \ge 1$

$x \ge \frac{1}{2}$

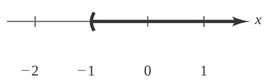


21. $2(1 - x) < 3x + 7$

$2 - 2x < 3x + 7$

$-5x < 5$

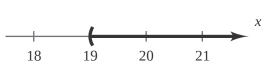
$x > -1$



22. $2x + 7 < 3(x - 4)$

$2x + 7 < 3x - 12$

$19 < x \Rightarrow x > 19$

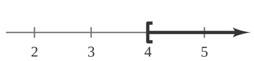


23. $\frac{3}{4}x - 6 \le x - 7$

$1 \le \frac{1}{4}x$

$4 \le x$

$x \ge 4$

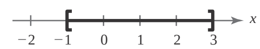


24. $3 + \frac{2}{7}x > x - 2$

25. $1 \le 2x + 3 \le 9$

$-2 \le 2x \le 6$

$-1 \le x \le 3$



26. $-8 \le -3x + 5 < 13$

$-13 \le -3x < 8$

$-\frac{8}{3} < x \le \frac{13}{3}$

27. $-8 \le 1 - 3(x - 2) < 13$

$-8 \le 1 - 3x + 6 < 13$

$-8 \le -3x + 7 < 13$

$-15 \le -3x < 6$

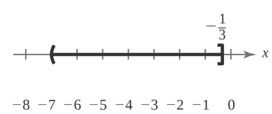
$5 \ge x > -2 \Rightarrow -2 < x \le 5$

28. $0 \le 2 - 3(x + 1) < 20$

$0 \le -3x - 1 < 20$

$1 \le -3x < 21$

$-\frac{1}{3} \ge x > -7 \Rightarrow -7 < x \le -\frac{1}{3}$

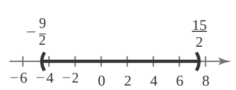


29. $-4 < \frac{2x - 3}{3} < 4$

$-12 < 2x - 3 < 12$

$-9 < 2x < 15$

$-\frac{9}{2} < x < \frac{15}{2}$



$21 + 2x > 7x - 14$

$35 > 5x$

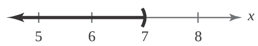
$7 > x\ x < 7$

$x \quad 0$

$+ \quad \leq$

3

2^{x}



$+ 3 < 10$

$-3 \leq x < 7$

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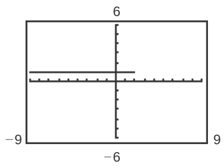
31. $5 - 2x \geq 1$

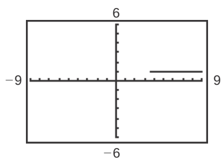
$-2x \geq -4$

$x \leq 2$



32. $20 < 6x - 1$

$x > \frac{7}{2}$

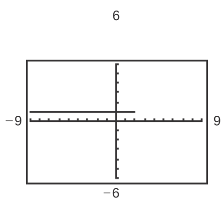


33. $3(x+1) < x + 7$

$3x + 3 < x + 7$

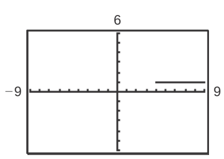
$2x < 4$

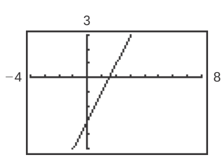
$x < 2$



34. $4(x-3) > 8 - x$

$x > 4$



35. 

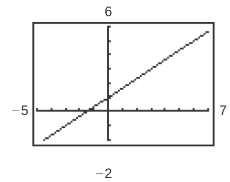
Using the graph, (a) $y \geq 1$ for $x \geq 2$ and (b) $y \leq 0$ for

(b) $y \leq 0$

$2x - 3 \leq 0$

$2x \leq 3$

$x \leq \frac{3}{2}$

36. 

Using the graph, (a) $y \leq 5$ for $x \leq 6$, and (b) $y \geq 0$ for $x \geq -\frac{3}{2}$.

Algebraically:

(a) $y \leq 5$

$\frac{2}{3}x + 1 \leq 5$

$\frac{2}{3}x \leq 4$

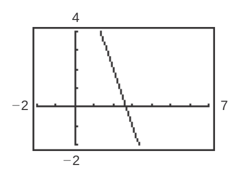
$x \leq 6$

(b) $y \geq 0$

$\frac{2}{3}x + 1 \geq 0$

$\frac{2}{3}x \geq -1$

$x \geq -\frac{3}{2}$

37. 

Using the graph, (a) $-1 \leq y \leq 3$ for $\frac{5}{3} \leq x \leq 3$, and (b) $y \leq 0$ for $x \geq \frac{8}{3}$.

Algebraically:

(a) $-1 \leq y \leq 3$

$-1 \leq -3x + 8 \leq 3$

$-9 \leq -3x \leq -5$

$3 \geq x \geq \frac{5}{3} \Rightarrow \frac{5}{3} \leq x \leq 3$

(b) $y \leq 0$

$-3x + 8 \leq 0$

$8 \leq 3x$

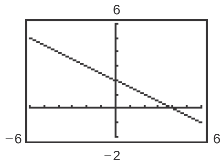
$\frac{8}{3} \leq x \Rightarrow x \geq \frac{8}{3}$

$x \le \frac{3}{2}$.

Algebraically:

(a)
$$\begin{aligned} y &\ge 1 \\ 2x - 3 &\ge 1 \\ 2x &\ge 4 \\ x &\ge 2 \end{aligned}$$

38.



Using the graph, (a) $0 \le y \le 3$ for $-2 \le x \le 4$ and

(b) $y \ge 0$ for $x \le 4$.

Algebraically:

(a) $0 \le y \le 3$

$0 \le -\frac{1}{2}x + 2 \le 3$

$-2 \le -\frac{1}{2}x \le 1$

$4 \ge x \ge -2 \Rightarrow -2 \le x \le 4$

(b) $y \ge 0$

$-\frac{1}{2}x + 2 \ge 0$

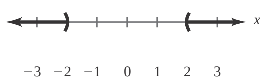
$2 \ge \frac{1}{2}x$

$4 \ge x \Rightarrow x \le 4$

39. $|5x| > 10$

$5x < -10$ or $5x > 10$

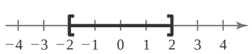
$x < -2$ or $x > 2$



40. $\left|\frac{x}{2}\right| \le 1$

$|x| \le 2$

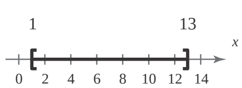
$-2 \le x \le 2$



41. $|x - 7| \le 6$

$-6 \le x - 7 \le 6$

$1 \le x \le 13$



42. $|x - 20| > 4$

$x - 20 > 4$ or $x - 20 < -4$

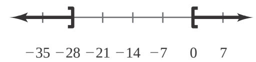
$x > 24$ or $x < 16$

44. $|x + 14| + 3 \ge 17$

$|x + 14| \ge 14$

$x + 14 \le -14$ or $x + 14 \ge 14$

$x \le -28$ or $x \ge 0$



45. $10|1 - x| < 5$

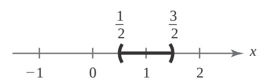
$|1 - x| < \frac{1}{2}$

$-\frac{1}{2} < 1 - x < \frac{1}{2}$

$-\frac{3}{2} < -x < -\frac{1}{2}$

$\frac{3}{2} > x > \frac{1}{2}$

$\frac{1}{2} < x < \frac{3}{2}$



46. $3|4 - 5x| < 9$

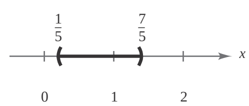
$|4 - 5x| < 3$

$-3 < 4 - 5x < 3$

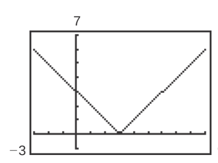
$-7 < -5x < -1$

$\frac{7}{5} > x > \frac{1}{5}$

$\frac{1}{5} < x < \frac{7}{5}$



47. $y = |x - 3|$



Graphically, (a) $y \le 2$ for $1 \le x \le 5$ and (b) $y \ge 4$ for $x \le -1$ or $x \ge 7$.

Algebraically:

(a) $y \le 2$

$|x - 3| \le 2$

$-2 \le x - 3 \le 2$

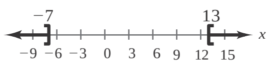
14 16 18 20 22 24 26

43. $\left|\dfrac{x-3}{2}\right| \geq 5$

$|x-3| \geq 10$

$x-3 \geq 10$ or $x-3 \leq -10$

$x \geq 13$ or $x \leq -7$



$1 \leq x \leq 5$

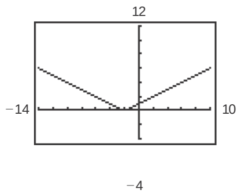
(b) $y \geq 4$

$|x-3| \geq 4$

$x-3 \leq -4$ or $x-3 \geq 4$

$x \leq -1$ or $x \geq 7$

48. $y = \left|\frac{1}{2}x + 1\right|$



Graphically, (a) $y \le 4$ for $-10 \le x \le 6$ and (b) $y \ge 1$ for $x \le -4$ or $x \ge 0$.

Algebraically:

(a)
$$y \le 4$$
$$\left|\tfrac{1}{2}x + 1\right| \le 4$$
$$-4 \le \tfrac{1}{2}x \le 4$$
$$-5 \le \tfrac{1}{2}x \le 3$$
$$-10 \le x \le 6$$

(b)
$$y \ge 1$$
$$\left|\tfrac{1}{2}x + 1\right| \ge 1$$
$$\tfrac{1}{2}x + 1 \le -1 \quad \text{or} \quad \tfrac{1}{2}x + 1 \ge 1$$
$$\tfrac{1}{2}x \le -2 \quad \text{or} \quad \tfrac{1}{2}x \ge 0$$
$$x \le -4 \quad \text{or} \quad x \ge 0$$

49. The midpoint of the interval $[-3, 3]$ is 0. The interval represents all real numbers x no more than three units from 0.
$$|x - 0| \le 3$$
$$|x| \le 3$$

50. The midpoint of the interval $(-1, 1)$ is 0. The two intervals represent all real numbers x more than one unit from 0.
$$|x - 0| > 1$$
$$|x| > 1$$

51. The midpoint of the interval $(-5, 3)$ is -1. The two intervals represent all real numbers x at least four units from -1.

55. All real numbers more than 3 units from -1

$|x + 1| > 3$

56. All real numbers at least 5 units from 3

$|x - 3| \ge 5$

57.
$$x^2 - 4x - 5 > 0$$
$$(x - 5)(x + 1) > 0$$

Key numbers: $-1,\ 5$

Testing the intervals $(-\infty, -1)$, $(-1, 5)$ and $(5, \infty)$, we have $x^2 - 4x - 5 > 0$ on $(-\infty, -1)$ and $(5, \infty)$.

Similarly, $x^2 - 4x - 5 < 0$ on $(-1, 5)$.

58.
$$x^2 - 3x - 4 > 0$$
$$(x - 4)(x + 1) > 0$$

Key numbers: $-1,\ 4$

Testing the intervals $(-\infty, -1)$, $(-1, 4)$, and $(4, \infty)$, we have $x^2 - 3x - 4 > 0$ on $(-\infty, -1)$ and $(4, \infty)$.

Similarly, $x^2 - 3x - 4 < 0$ on $(-1, 4)$.

59. $2x^2 - 4x - 3 > 0$

Key numbers: $x = \dfrac{4 \pm \sqrt{16 + 24}}{4} = 1 \pm \dfrac{\sqrt{10}}{2}$

Testing the intervals $\left(-\infty, 1 - \dfrac{\sqrt{10}}{2}\right)$, $\left(1 - \dfrac{\sqrt{10}}{2}, 1 + \dfrac{\sqrt{10}}{2}\right)$, and $\left(1 + \dfrac{\sqrt{10}}{2}, \infty\right)$, you have $2x^2 - 4x - 3 > 0$ on $\left(-\infty, 1 - \dfrac{\sqrt{10}}{2}\right) \cup \left(1 + \dfrac{\sqrt{10}}{2}, \infty\right)$. Similarly, $2x^2 - 4x - 3 < 0$ on $\left(1 - \dfrac{\sqrt{10}}{2}, 1 + \dfrac{\sqrt{10}}{2}\right)$.

$(\quad 2 \;) \; (\quad 2 \quad)$

$|x - (-1)| \geq 4$

$|x + 1| \geq 4$

52. The midpoint of the interval $(4, 10)$ is 7. The interval represents all real numbers x less than three units from 7.

$|x - 7| < 3$

53. All real numbers less than 10 units from 6

$|x - 6| < 10$

54. All real numbers no more than 8 units from -5

$|x + 5| \leq 8$

60. $-2x^2 + x + 5 = 0$

$2x^2 - x - 5 = 0$

Key numbers: $x = \dfrac{1 \pm \sqrt{1 - 4(2)(-5)}}{2(2)} = \dfrac{1 \pm \sqrt{41}}{4}$

Test intervals:

$\left(-\infty, \dfrac{1}{4} - \dfrac{\sqrt{41}}{4}\right)$, $\left(\dfrac{1}{4} - \dfrac{\sqrt{41}}{4}, \dfrac{1}{4} + \dfrac{\sqrt{41}}{4}\right)$, and

$\left(\dfrac{1}{4} + \dfrac{\sqrt{41}}{4}, \infty\right)$

$2x^2 - x - 5 > 0$ on $\left(-\infty, \dfrac{1}{4} - \dfrac{\sqrt{41}}{4}\right)$ and $\left(\dfrac{1}{4} + \dfrac{\sqrt{41}}{4}, \infty\right)$, and $2x^2 - x - 5 < 0$ on $\left(\dfrac{1}{4} - \dfrac{\sqrt{41}}{4}, \dfrac{1}{4} + \dfrac{\sqrt{41}}{4}\right)$.

61. There are no key numbers because $-x^2 + 6x - 10 \neq 0$. The only test interval is $(-\infty, \infty)$. $-x^2 + 6x - 10 < 0$ for all x.

62. There are no key numbers because $3x^2 + 8x + 6 \neq 0$. The only test interval is $(-\infty, \infty)$. $3x^2 + 8x + 6 > 0$ for all real numbers x.

63. $x^2 + 4x + 4 \geq 9$

$x^2 + 4x - 5 \geq 0$

$(x + 5)(x - 1) \geq 0$

Key numbers: $x = -5$, $x = 1$

Test intervals: $(-\infty, -5)$, $(-5, 1)$, $(1, \infty)$

Test: Is $(x + 5)(x - 1) \geq 0$?

Solution set: $(-\infty, -5 \cup [1, \infty)$

64. $x^2 - 6x + 9 < 16$

$x^2 - 6x - 7 < 0$

$(x + 1)(x - 7) < 0$

Key numbers: $x = -1$, $x = 7$

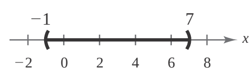
Test intervals: $(-\infty, -1) \Rightarrow (x + 1)(x - 7) > 0$

$(-1, 7) \Rightarrow (x + 1)(x - 7) < 0$

$(7, \infty) \Rightarrow (x + 1)(x - 7) > 0$

Test: Is $(x + 1)(x - 7) < 0$?

Solution set: $(-1, 7)$



65. $(x + 2)^2 < 25$

$x^2 + 4x + 4 < 25$

$x^2 + 4x - 21 < 0$

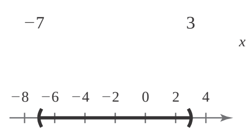
$(x + 7)(x - 3) < 0$

Key numbers: $x = -7$, $x = 3$

Test intervals: $(-\infty, -7)$, $(-7, 3)$, $(3, \infty)$

Test: Is $(x + 7)(x - 3) < 0$?

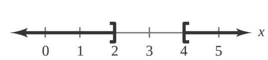
Solution set: $(-7, 3)$



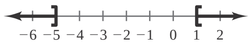
66. $(x - 3)^2 \geq 1$

$x - 3 \geq 1$ or $x - 3 \leq -1$

$x \geq 4$ $\qquad$ $x \leq 2$



67. $x^3 - 4x^2 \geq 0$



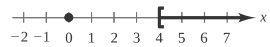
$x^2(x - 4) \geq 0$

Key numbers: $x = 0, x = 4$

Test intervals: $(-\infty, 0), (0, 4), (4, \infty)$

Test: Is $x^2(x - 4) \geq 0$?

Solution set: $x = 0, x = 4$ and $(4, \infty) \Rightarrow x = 0 \cup [4, \infty)$



68. $x^5 - 3x^4 \le 0$

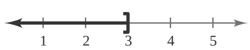
$x^4(x - 3) \le 0$

Key numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0), (0, 3), (3, \infty)$

Test: Is $x^4(x - 3) \le 0$?

Solution set: $(-\infty, 0)$, $x = 0$, $(0, 3)$, and $x = 3 \Rightarrow (-\infty, 3]$



69. $2x^3 + 5x^2 - 6x - 9 > 0$

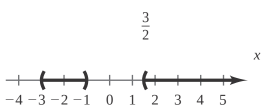
$(x + 1)(x + 3)(2x - 3) > 0$

Key numbers: $x = -3, x = -1, x = \frac{3}{2}$

Test intervals: $(-\infty, 3), (-3, -1), \left(-1, \frac{3}{2}\right), \left(\frac{3}{2}, \infty\right)$

Test: Is $(x + 1)(x + 3)(2x - 3) > 0$?

Solution set: $(-3, -1), \left(\frac{3}{2}, \infty\right)$



70. $2x^3 + 3x^2 - 11x - 6 < 0$

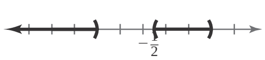
$(x - 2)(x + 3)(2x + 1) < 0$

Key numbers: $x = -3, x = -\frac{1}{2}, x = 2$

Test intervals: $(-\infty, -3), \left(-3, -\frac{1}{2}\right), \left(-\frac{1}{2}, 2\right), (2, \infty)$

Test: Is $(x - 2)(x + 3)(2x + 1) < 0$?

Solution set: $(-\infty, -3), \left(-\frac{1}{2}, 2\right)$



72. $2x^3 + 13x^2 - 8x - 46 \ge 6$

$2x^3 + 13x^2 - 8x - 52 \ge 0$

$x^2(2x + 13) - 4(2x + 13) \ge 0$

$(2x + 13)(x^2 - 4) \ge 0$

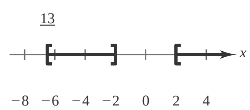
$(2x + 13)(x + 2)(x - 2) \ge 0$

Key numbers: $x = -\frac{13}{2}, x = -2, x = 2$

Test intervals: $\left(-\infty, -\frac{13}{2}\right), \left(-\frac{13}{2}, -2\right), (-2, 2), (2, \infty)$

Test: Is $(2x + 13)(x + 2)(x - 2) \ge 0$?

Solution set: $\left[-\frac{13}{2}, -2\right], [2, \infty)$



73. $3x^2 - 11x + 16 \le 0$

Since $b^2 - 4ac = -71 < 0$, there are no real solutions of $3x^2 - 11x + 16 = 0$. In fact, $3x^2 - 11x + 16 > 0$ for all x.

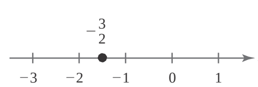
No solution

74. $4x^2 + 12x + 9 \le 0$

$(2x + 3)^2 \le 0$

$2x + 3 = 0$

$x = -\frac{3}{2}$



75. $4x^2 - 4x + 1 > 0$

$(2x - 1)^2 > 0$

Key number: $x = \frac{1}{2}$

71. $x^3 - 3x^2 - x > -3$

$x^3 - 3x^2 - x + 3 > 0$

$x^2(x-3) - (x-3) > 0$

$(x-3)(x^2-1) > 0$

$(x-3)(x+1)(x-1) > 0$

Key numbers: $x = 3,\ x = -1,\ x = 1$

Test intervals: $(-\infty,\ -1),\ (-1,\ 1),\ (1,\ 3),\ (3,\ \infty)$

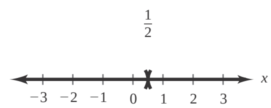
Test: Is $(x-3)(x+1)(x-1) > 0$?

Solution set: $(-1,\ 1),\ (3,\ \infty)$

Test intervals: $\left(-\infty, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

Test: Is $(2x - 1)^2 > 0$?

Solution set: $\left(-\infty, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

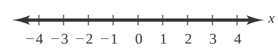


76. $x^2 + 3x + 8 > 0$

Because $x^2 + 3x + 8 \neq 0$, there are no real key numbers.

$x^2 + 3x + 8$ is always positive.

Solution set: all real numbers x



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77. (a) $f(x) = g(x)$ when $x = 1$.

(b) $f(x) \geq g(x)$ when $x \geq 1$.

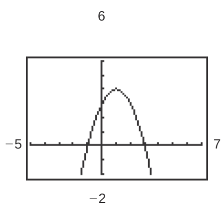
(c) $f(x) > g(x)$ when $x > 1$.

78. (a) $f(x) = g(x)$ when $x = -1$ or $x = 3$.

(b) $f(x) \geq g(x)$ when $x \leq -1$ or $x \geq 3$.

(c) $f(x) > g(x)$ when $x < -1$ or $x > 3$.

79. $y = -x^2 + 2x + 3$



Using the graph, (a) when $x \leq -1$ or $x \geq 3$ and

(b) $y \geq 3$ when $0 \leq x \leq 2$.

Algebraically:

(a) $-x^2 + 2x + 3 \leq 0$

$x^2 - 2x - 3 \geq 0$

$(x-3)(x+1) \geq 0$

Key numbers: $x = -1,\ x = 3$

Testing the intervals $(-\infty, -1)$, $(-1, 3)$, and $(3, \infty)$, you obtain $x \leq -1$ or $x \geq 3$.

(b) $-x^2 + 2x + 3 \geq 3$

$-x^2 + 2x \geq 0$

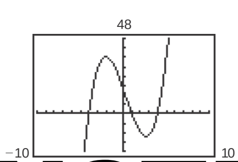
$x^2 - 2x \leq 0$

$x(x-2) \leq 0$

Key numbers: $x = 0,\ x = 2$

Testing the intervals $(-\infty, 0)$, $(0, 2)$ and $(2, \infty)$, you obtain $0 \leq x \leq 2$.

80. $y = x^3 - x^2 - 16x + 16$



(b) $y \geq 36$

$x^3 - x^2 - 16x + 16 \geq 36$

$x^3 - x^2 - 16x - 20 \geq 0$

$(x+2)(x-5)(x+2) \geq 0$

$y \geq 36$ when $x = -2,\ 5 \leq x < \infty$.

81. $\dfrac{1}{x} - x > 0$

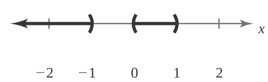
$\dfrac{1 - x^2}{x} > 0$

Key numbers: $x = 0,\ x = \pm 1$

Test intervals: $(-\infty, -1)$, $(-1, 0)$, $(0, 1)$, $(1, \infty)$

Test: Is $\dfrac{1 - x^2}{x} \geq 0$?

Solution set: $(-\infty, -1) \cup (0, 1)$



82. $\dfrac{1}{x} - 4 < 0$

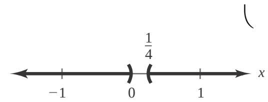
$\dfrac{1 - 4x}{x} < 0$

Key numbers: $x = 0,\ x = \dfrac{1}{4}$

Test intervals: $(-\infty, 0)$, $\left(0, \dfrac{1}{4}\right)$, $\left(\dfrac{1}{4}, \infty\right)$

Test: Is $\dfrac{1 - 4x}{x} < 0$?

Solution set: $(-\infty, 0) \cup \left(\dfrac{1}{4}, \infty\right)$



83. $\dfrac{x+6}{x+1} - 2 \leq 0$

$\dfrac{x + 6 - 2(x+1)}{x+1} \leq 0$

$\dfrac{4 - x}{x+1} \leq 0$

Using the graph, (a) $y \le 0$ when $-\infty < x \le -4$ and $1 \le x \le 4$, and (b) $y \ge 36$ when $x = -2$ and $5 \le x < \infty$.

Algebraically:

(a) $y \le 0$

$$x^3 - x^2 - 16x + 16 \le 0$$

$$x^2(x-1) - 16(x-1) \le 0$$

$$(x-1)(x^2-16) \le 0$$

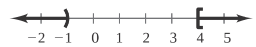
$y \le 0$ when $-\infty < x \le -4,\ 1 \le x \le 4$.

Key numbers: $x = -1,\ x = 4$

Test intervals: $(-\infty, -1),\ (-1, 4),\ (4, \infty)$

Test: Is $\dfrac{4-x}{x+1} \le 0$?

Solution set: $(-\infty, -1) \cup [4, \infty)$



84. $\dfrac{x+12}{x+2} - 3 \ge 0$

$\dfrac{x+12-3(x+2)}{x+2} \ge 0$

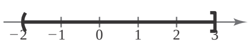
$\dfrac{6-2x}{x+2} \ge 0$

Key numbers: $x = -2,\ x = 3$

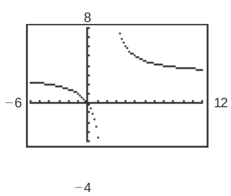
Test intervals: $(-\infty,\ -2),\ (-2,\ 3),\ (3,\ \infty)$

Test: Is $\dfrac{6-2x}{x+2} \ge 0$?

Solution set: $(-2,\ 3$



85. $y = \dfrac{3x}{x-2}$



Using the graph, (a) $y \le 0$ when $0 \le x < 2$ and (b) $y \ge 6$ when $2 < x \le 4$.

Algebraically:

(a) $y \le 0$

$\dfrac{3x}{x-2} \le 0$

Key numbers: $x = 0,\ x = 2$

Test intervals: $(-\infty,\ 0),\ (0,\ 2),\ (2,\ \infty)$

Test: Is $\dfrac{3x}{x-2} \le 0$?

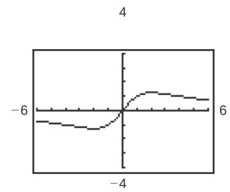
Solution set: $[0,\ 2)$

(b) $y \ge 6$

$\dfrac{3x}{x-2} \ge 6$

$\dfrac{3x-6(x-2)}{x-2} \ge 0$

86. $y = \dfrac{5x}{x^2+4}$



Using the graph, $y \ge 1$ when $1 \le x \le 4$ and $y \le 0$ when $-\infty < x \le 0$.

Algebraically:

(a) $y \ge 1$

$\dfrac{5x}{x^2+4} \ge 1$

$\dfrac{5x-(x^2+4)}{x^2+4} \ge 0$

$\dfrac{(x-4)(x-1)}{x^2+4} \le 0$

Key numbers: $x = 1,\ x = 4$

Test intervals: $(-\infty,\ 1),\ (1,\ 4),\ (4,\ \infty)$

Test: Is $\dfrac{(x-4)(x-1)}{x^2+4} \le 0$?

Solution set: $[1,\ 4$

(b) $y \le 0$

$\dfrac{5x}{x^2+4} \le 0$

Key number: $x = 0$

Test intervals: $(-\infty,\ 0),\ (0,\ \infty)$

Test: Is $\dfrac{5x}{x^2+4} \le 0$?

Solution set: $(-\infty,\ 0$

87. $\sqrt{x-5}$

$x - 5 \ge 0$

$x \ge 5$

Domain: $[5,\ \infty)$

88. $\sqrt{6x + 15}$

$$\frac{-3x+12}{x-2} \geq 0$$

Key numbers: $x = 2,\ x = 4$

Test intervals: $(-\infty,\ 2),\ (2,\ 4),\ (4,\ \infty)$

Test: Is $\dfrac{-3x+12}{x-2} \geq 0$?

Solution set: $(2,\ 4$

$$6x + 15 \geq 0$$
$$6x \geq -15$$
$$x \geq -\frac{5}{2}$$

Domain: $\left[-\frac{5}{2}, \infty\right)$

89. $\sqrt{-x^2 + x + 12}$

$$-x^2 + x + 12 \ge 0$$
$$x^2 - x - 12 \le 0$$
$$(x - 4)(x + 3) \le 0$$

Key numbers: $x = -3, x = 4$

Test intervals: $(-\infty, -3) > 0$

$(-3, 4) < 0$

$(4, \infty) > 0$

Domain: $[-3, 4]$

90. $\sqrt{2x^2 - 8}$

$$2x^2 - 8 \ge 0$$
$$2(x - 2)(x + 2) \ge 0$$

Key numbers: $x = -2, x = 2$

Test intervals: $(-\infty, -2) > 0$

$(-2, 2) < 0$

$(2, \infty) > 0$

Domain: $(-\infty, -2] \cup [2, \infty)$

91. $\sqrt[4]{3x^2 - 20x - 7}$

$$3x^2 - 20x - 7 \ge 0$$
$$(3x + 1)(x - 7) \ge 0$$

Key numbers: $x = -\frac{1}{3}, x = 7$

Test intervals: $\left(-\infty, -\frac{1}{3}\right) > 0$

$\left(-\frac{1}{3}, 7\right) < 0$

$(7, \infty) > 0$

Domain: $\left(-\infty, -\frac{1}{3}\right] \cup [7, \infty)$

92. $\sqrt[4]{2x^2 + 4x + 3}$

$$2x^2 + 4x + 3 \ge 0$$

93. (a) Find t when $P(t) = 450$. Using the graph,

$P(t) = 450$ at about $t = 5$. So, in 2005, the population was 450,000.

(b) Find the interval of t when $P(t) < 450$. Using the graph, $P(t) < 450$ for $0 < t < 5$. So, from 2003 to 2005, the population was less than 450,000.

Find the interval of t when $P(t) > 450$. Using the graph, $P(t) > 450$ for $5 < t < 12$. So, from 2005 to 2012, the population was greater than 450,000.

94. (a) Find t when $P(t) = 400$. Using the graph, $P(t) = 400$ at about $t = 9$. So, in 2009, the population was 400,000.

(b) Find the interval of t when $P(t) < 400$. Using the graph, $P(t) < 400$ for $9 < t < 12$. So, from 2009 to 2012, the population was less than 400,000.

Find the interval of t when $P(t) > 400$. Using the graph, $P(t) > 400$ for $3 < t < 9$. So, from 2003 to 2009, the population was greater than 400,000.

95. (a)
$$s = -16t^2 + v_0t + s_0$$
$$s = -16t^2 + 160t$$
$$s = 16t(10 - t)$$

$s = 0$ when $t = 10$ seconds.

(b)
$$s = -16t^2 + 160t > 384$$
$$16t^2 - 160t + 384 < 0$$
$$16(t - 6)(t - 4) < 0$$

Key numbers: $t = 6,\ 4$

$s > 384$ when $4 < t < 6$.

96. (a)
$$s = -16t^2 + v_0t + s_0$$
$$= -16t^2 + 128t$$
$$= 16t(8 - t)$$

$t = 8$ seconds

There are no key numbers. Test interval: $(-\infty, \infty) > 0$

Domain: $(-\infty, \infty)$

(b) $-16t^2 + 128t < 128$

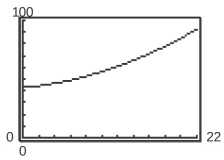
$16t^2 - 128t + 128 > 0$

Key numbers: $t \approx 1.17, 6.83$

$(0, 1.17), (6.83, 8)$

97. $D = 0.0743t^2 + 0.628t + 42.61, 0 \le t \le 22$

(a)



(b) The number of doctorate degrees was between 50 and 60 thousand for about $6 \le t \le 11$, or $(1996, 2001)$.

(c) $50 \le 0.0743t^2 + 0.628t + 42.61 \le 60$

Find the key numbers.

$$50 = 0.0743t^2 + 0.628t + 42.61$$
$$0 = 0.0743t^2 + 0.628t - 7.39$$

and

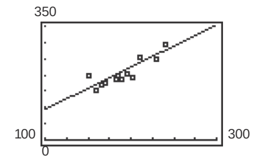
$$0.0743t^2 + 0.628t + 42.61 = 60$$
$$0.0743t^2 + 0.628t - 17.39 = 0$$

Key numbers: $t \approx 6.6, t \approx -15.1, t \approx -20.1$ and $t \approx 11.6$

Note: $t \approx -15.1$ and $t = -20.1$ are not in the domain.

Solution set: $6.6 < t < 11.6$, which corresponds to $(1996, 2001)$

98. (a) and (b)



(c) For $y \ge 200$, $x \ge 181.5$ pounds

(d) The model is not accurate. The data are not linear. Other factors include muscle strength, height, physical condition, etc.

In Exercises 99–102, use the following equations.

Bed Bath & Beyond: $B = 86.5t + 342, 0 \le t \le 13$

Williams-Sonoma:

$$W = -2.92t^2 + 52.0t + 381, 0 \le t \le 13$$

99.
$$B(t) \ge 900$$
$$86.5t + 342 \ge 900$$
$$86.5t \ge 558$$
$$t \ge 6.5$$

During the year 2006, there were at least 900 Bed Bath & Beyond stores.

100.
$$W(t) \le 600$$
$$-2.92t^2 + 52.0t + 381 \le 600$$
$$-2.92t^2 + 52.0t - 219 \le 0$$

Key numbers: $t \approx 6.83, \quad t \approx 10.97$

After testing the intervals, $t \le 6.83$ and $t \ge 10.97$.

From 2000 to 2006 and 2011 to 2013, there were at most 600 Williams-Sonoma stores.

101.
$$B(t) = W(t)$$
$$86.5t + 342 = -2.92t^2 + 52.0t + 381$$
$$2.92t^2 + 34.5t - 39 = 0$$
$$t \approx 1.04, t \approx -12.85 \text{ (not in the domain of the models)}$$

In 2001, there were about the same number of Bed Bath & Beyond stores as Williams-Sonoma stores.

102.
$$B(t) \ge W(t)$$
$$86.5t + 342 \ge -2.92t^2 + 52.0t + 381$$
$$2.92t^2 + 34.5t - 39 \ge 0$$
$$t \ge 1.04$$

Note: $t \approx -12.85$ is not in the domain.

During the year 2001, the number of Bed Bath & Beyond stores exceeded the number of Williams- Sonoma stores.

105. When $200 \le v \le 400$, $1.2 < t < 2.4$.

106. For $t < 3$, $0 < v < 500$.

107. False. If $-10 \le x \le 8$, then $10 \ge -x$ and $-x \le -8$.

103. When $t = 2$: $v \approx 333$ vibrations per second.

[illegible]4. [illegible]or [illegible] = [illegible] t [illegible]m[illegible].

108. True. $\frac{3}{2}x^2 + 3x + 6 \ge 0$ for all x.

109. False. Cube roots have no restrictions on the domain.

110. (iv) $a < b$
(ii) $2a < 2b$
(iii) $2a < a + b < 2b$
(i) $a < \dfrac{a+b}{2} < b$

111. (a) When solving this inequality, you do not need to reverse the inequality symbol. So, in the solution, the symbol will be $\le$. Matches (iv).
(b) When solving this inequality, you need to reverse the inequality symbol. So, in the solution, the symbol will be $\ge$. Matches (ii).
(c) When solving this inequality, you will use a double inequality that results in a bounded interval. Matches (iii).
(d) When solving this inequality, you will use two separate inequalities. The solution will contain two unbounded intervals. Matches (i).

112. (a) The polynomial equals zero at $x = a$ and at $x = b$.
(b) $f(x) > 0$ on the intervals $(-\infty, a)$ and (b, ∞), and $f(x) < 0$ on the interval (a, b).

When $x < a$ or $x > b$, the factors have the same sign so the product is positive. When $a < x < b$, the factors have opposite signs so the product is negative.
(c) The polynomial changes signs at the critical numbers $x = a$ and $x = b$.

113.
$$y = 12x$$
$$x = 12y$$
$$\frac{x}{12} = y$$
$$f^{-1}(x) = \frac{x}{12}$$

114.
$$y = 5x + 8$$
$$x = 5y + 8$$
$$x - 8 = 5y$$
$$\frac{1}{5}(x - 8) = y$$
$$f^{-1}(x) = \frac{1}{5}(x - 8)$$

115.
$$y = x^3 + 7$$
$$x = y^3 + 7$$
$$x - 7 = y^3$$
$$\sqrt[3]{x - 7} = y$$
$$f^{-1}(x) = \sqrt[3]{x - 7}$$

116.
$$y = \sqrt[3]{y - 7}$$
$$x = \sqrt[3]{y - 7}$$
$$x^3 = y - 7$$
$$x^3 + 7 = y$$
$$f^{-1}(x) = x^3 + 7$$

117. Answers will vary. (Make a Decision)

Section 2.7 Linear Models and Scatter Plots

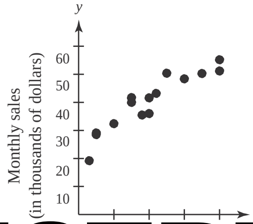
1. positive

2. regression or linear regression

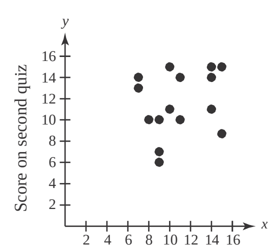
3. negative

4. No. The closer the correlation coefficient $|r|$ is to 1, the better the fit.

5. (a)



6. (a)



x

1 2 3 4
Years of experience

Score on first quiz

(b) No. Quiz scores are dependent on several variables, such as study time, class attendance, etc.

(b) Yes, the data appears somewhat linear. The more experience x corresponds to higher sales y.

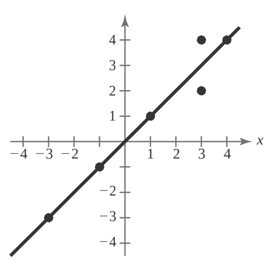
7. Negative correlation

8. No correlation

9. No correlation

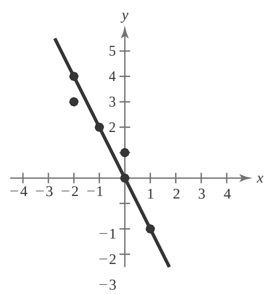
10. Positive correlation

11. (a) and (b)



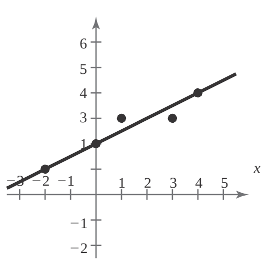
(c) Using the points $(1, 1)$ and $(4, 4)$, an equation of the line is $y = x$.

12. (a) and (b)



(c) Using the points $(0, 0)$ and $(1, -2)$, an equation of the line is $y = -2x$.

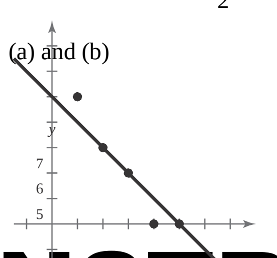
13. (a) and (b)



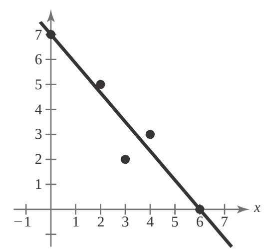
(c) Using the points $(0, 2)$ and $(-2, 1)$, an equation of

the line is $y = \frac{1}{2}x + 2$.

14. (a) and (b)

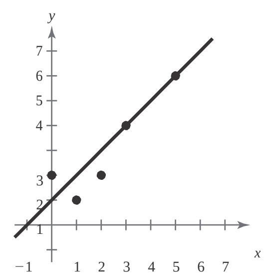


15. (a) and (b)



(c) Using the points (0, 7) and (6, 0), an equation of the line is $y = -\frac{7}{6}x + 7$.

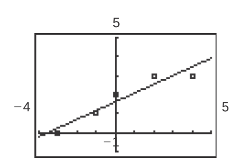
16. (a) and (b)



(c) Using the points (3, 4) and (5, 6), an equation of the line is $y = x + 1$.

17. $y = 0.46x + 1.6$

(a)



(b)

x	–3	–1	0	2	4
Linear equation	0.22	1.14	1.6	2.52	3.44
Given data	0	1	2	3	3

The model fits the data well.

18. $y = -1.3x + 2.8$

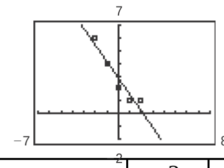
(a)

(b)

x

-1 -1 1 2 3 4 5 6 7

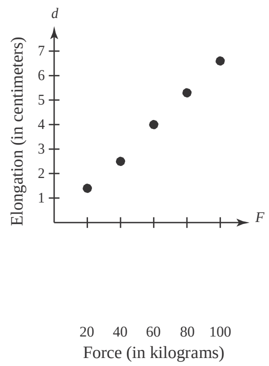
(c) Using the points $(2, 3)$ and $(3, 2)$, an equation of the line is $y = -x + 5$.



x	–2	–1	0	1	2
Linear equation	5.4	4.1	2.8	1.5	0.2
Given data	6	4	2	1	1

The model fits the data fairly well.

19. (a)

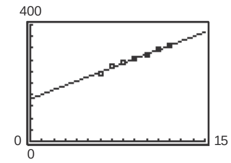


(b) $d = 0.07F - 0.3$

(c) $d = 0.066F$ or $F = 15.13d + 0.096$

(d) If $F = 55$, $d = 0.066(55) \approx 3.63$ centimeters.

20. (a) and (c)



The model fits the data well.

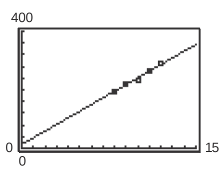
(b) $S = 15.28t + 145.8$

(d) For $2020, t = 20$ and

$S = 15.28(20) + 145.8$
$= 451.4$ million subscribers.

Answers will vary.

21. (a) and (c)



The model fits the data well.

(b) $V = 22.8t + 12.8$

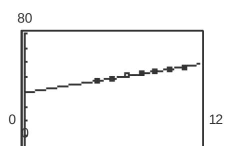
(d) For 2015, $t = 15$ and $V = 22.8(15) + 12.8 = \$354.8$ million.

For 2020, $t = 20$ and $V = 22.8(20) + 12.8 = \$468.8$ million.

Answers will vary.

(e) 22.8; The slope represents the average annual increase in salaries (in millions of dollars).

22. (a) and (c)



The model fits the data well.

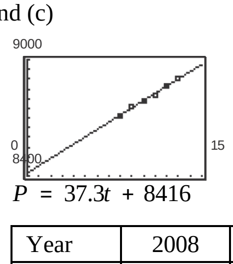
(b) $S = 1.61t + 39.6$

(d) For 2020, $t = 20$ and $S = 1.61(20) + 39.6 = \$71.8$ thousand.

For 2022, $t = 22$ and $S = 16.1(22) + 39.6 = \$75.82$ thousand.

Answers will vary.

23. (a) and (c)



(b) $P = 37.3t + 8416$

(d)

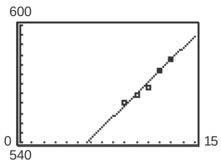
Year	2008	2009	2010	2011	2012	2013
Actual	8711	8756	8792	8821	8868	8899
Model	8714.4	8751.7	8789	8826.3	8863.6	8900.9

Th[illegible]der[illegible]s th[illegible]ua[illegible] w[illegible]ll.

(e) For 2050, $t = 50$ and $P = 37.3(50) + 8416 = 10{,}280.8$ hundred people or 10,280,800 people.

Answers will vary.

24. (a) and (c)



(b) $P = 5.9t + 506$

(d)

Year	2009	2010	2011	2012	2013
Actual	560	564	568	577	583
Model	559.1	565	570.9	576.8	582.7

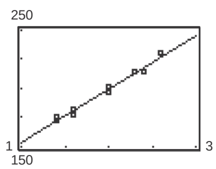
The model fits the data well.

(e) For 2050, $t = 50$ and $P = 5.9(50) + 506 = 800.5$ thousand people or 800,500 people.

Answers will vary.

25. (a) $y = 45.70x + 108.0$

(b)

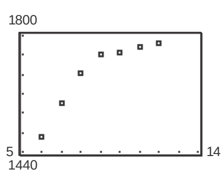


(c) The slope represents the increase in sales due to increased advertising.

(d) For \$1500, $x = 1.5$, and

$y = 45.70(1.5) + 108.0 = 176.55$ or \$176,550.

26. (a)



The first three points and the last four points are approximately linear.

(b) $T_1 = 97t + 908$

$T_2 = 12.7t + 1624$

(c) $T = \begin{cases} 97t + 908, & 6 \le t \le 8 \\ 12.7t + 1624, & 8 < t \le 12 \end{cases}$

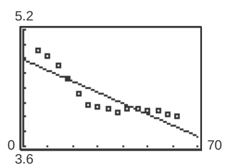
1800

27. (a) $T = -0.015t + 4.79$

$r \approx -0.866$

(b) The negative slope means that the winning times are generally decreasing over time.

(c)



(d)

Year	1956	1960	1964	1968	1972
Actual	4.91	4.84	4.72	4.53	4.32
Model	4.70	4.64	4.58	4.52	4.46

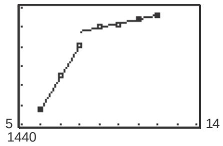
Year	1976	1980	1984	1988	1992
Actual	4.16	4.15	4.12	4.06	4.12
Model	4.40	4.34	4.28	4.22	4.16

Year	1996	2000	2004	2008	2012
Actual	4.12	4.10	4.09	4.05	4.02
Model	4.10	4.04	3.98	3.92	3.86

The model does not fit the data well.

(e) The closer $|r|$ is to 1, the better the model fits the data.

(f) No; The winning times have leveled off in recent years, but the model values continue to decrease to unrealistic times.



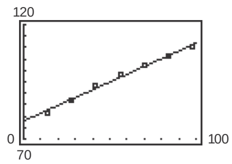
(d) Answers will vary.

28. (a) $l = 0.34d + 77.9$

$r \approx 0.993$

(b) Yes, $|r|$ is close to 1.

(c)



The data fits the model well.

(d) For $d = 112$, $l = 0.34(112) + 77.9 = 115.98$ or about 116 centimeters.

29. True. To have positive correlation, the y-values tend to increase as x increases.

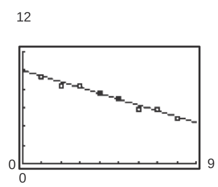
30. False. The closer the correlation coefficient is to –1 or 1, the better a line fits the data.

31. Answers will vary.

32. (a) (i) $y = -0.62x + 10.0$

$r = -0.986$

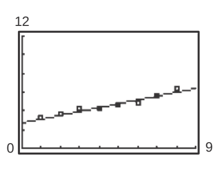
The data are decreasing, so the slope and correlation coefficient are negative.



(ii) $y = 0.41x + 2.7$

$r = 0.973$

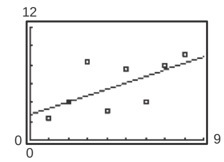
The slope is less steep than the slope of the line in (iii). The line is a better fit for the data than the line in (iii), so the correlation coefficient will be greater.



(iii) $y = 0.68x + 2.7$

$r = 0.62$

The slope is steeper than the slope of the line in (ii). The line is not a good fit for the data, so the correlation coefficient will not be close to 1.



(b) Model **(i)** is the best fit for its data because its r-value is -0.986, and therefore $|r|$ is closest to 1.

33. $f(x) = 2x^2 - 3x + 5$

(a) $f(-1) = 2 + 3 + 5 = 10$

(b) $f(w+2) = 2(w+2)^2 - 3(w+2) + 5$

$= 2w^2 + 5w + 7$

34. $g(x) = 5x^2 - 6x + 1$

(a) $g(-2) = 5(4) - 6(-2) + 1 = 33$

(b) $g(z-2) = 5(z-2)^2 - 6(z-2) + 1$

$= 5z^2 - 26z + 33$

35. $6x + 1 = -9x - 8$

$15x = -9$

$x = -\dfrac{9}{15} = -\dfrac{3}{5}$

36. $3(x-3) = 7x + 2$

$-11 = 4x$

$x = -\dfrac{11}{4}$

37. $8x^2 - 10x - 3 = 0$

$(4x+1)(2x-3) = 0$

$x = -\dfrac{1}{4}, \dfrac{3}{2}$

38. $10x^2 - 23x - 5 = 0$

$(2x-5)(5x+1) = 0$

$x = \dfrac{5}{2}, -\dfrac{1}{5}$

Chapter 2 Review

1. $6+\dfrac{3}{x-4}=5$

(a) $x=\dfrac{11}{2}$

$6+\dfrac{3}{\frac{11}{2}-4}\stackrel{?}{=}5$

$6+\dfrac{3}{\frac{3}{2}}\stackrel{?}{=}5$

$6+2\neq5$

No, $x=\frac{11}{2}$ is not a solution.

(b) $x=0$

$6+\dfrac{3}{0-4}\stackrel{?}{=}5$

$6-\dfrac{3}{4}\stackrel{?}{=}5$

$5.25\neq5$

No, $x=0$ is not a solution.

(c) $x=-2$

$6+\dfrac{3}{-2-4}\stackrel{?}{=}5$

$6-\dfrac{1}{2}\stackrel{?}{=}5$

$5.5\neq5$

No, $x=-2$ is not a solution.

(d) $x=1$

$6+\dfrac{3}{1-4}\stackrel{?}{=}5$

$6-1\stackrel{?}{=}5$

$5=5$

Yes, $x=1$ is a solution.

2. $6+\dfrac{2}{x+3}=\dfrac{6x+1}{3}$

(a) $x=-3$

$6+\dfrac{2}{-3+3}$ is undefined.

No, $x=-3$ is not a solution.

(b) $x=3$

$6+\dfrac{2}{3+3}\stackrel{?}{=}\dfrac{6(3)+1}{3}$

$\dfrac{38}{6}\stackrel{?}{=}\dfrac{19}{3}$

Yes, $x=3$ is a solution.

(c) $x=0$

$6+\dfrac{2}{0+3}\stackrel{?}{=}\dfrac{6(0)+1}{3}$

$\dfrac{20}{3}\stackrel{?}{=}\dfrac{1}{3}$

No, $x=0$ is not a solution.

(d) $x=-\dfrac{2}{3}$

$6+\dfrac{2}{(-2/3)+3}\stackrel{?}{=}\dfrac{6(-2/3)+1}{3}$

$6+\dfrac{6}{7}\stackrel{?}{=}-1$

No, $x=-\frac{2}{3}$ is not a solution.

3. $\dfrac{x}{18}+\dfrac{x}{10}=7$

$5x+9x=630$

$14x=630$

$x=45$

4. $\dfrac{x}{2}+\dfrac{x}{7}=9$

$7x+2x=126$

$9x=126$

$x=14$

5. $\dfrac{5}{x-2}=\dfrac{13}{2x-3}$

$$\begin{aligned} 10x - 15 &= 13x - 26 \\ 11 &= 3x \\ x &= \frac{11}{3} \end{aligned}$$

6. $\dfrac{10}{x+1} = \dfrac{12}{3x-2}$

$$10(3x-2) = 12(x+1)$$
$$30x - 20 = 12x + 12$$
$$18x = 32$$
$$x = \frac{32}{18} = \frac{16}{9}$$

7. $14 + \dfrac{2}{x-1} = 10$

$$\frac{2}{x-1} = -4$$
$$2 = -4(x-1)$$
$$2 = -4x + 4$$
$$4x = 2$$
$$x = \frac{1}{2}$$

8. $10 + \dfrac{2}{x-1} = 4$

$$10(x-1) + 2 = 4(x-1)$$
$$10x - 10 + 2 = 4x - 4$$
$$6x = 4$$
$$x = \frac{4}{6} = \frac{2}{3}$$

9. $6 - \dfrac{4}{x} = 6 + \dfrac{5}{x}$

$$6x - 4 = 6x + 5$$
$$-4 \neq 5$$

No solutions

10. $2 - \dfrac{1}{x^2} = 2 + \dfrac{4}{x^2}$

$$2x^2 - 1 = 2x^2 + 4$$
$$-1 \neq 4$$

No solutions

11. $\dfrac{9x}{3x-1} - \dfrac{4}{3x+1} = 3$

$$9x(3x+1) - 4(3x-1) = 3(3x-1)(3x+1)$$
$$27x^2 + 9x - 12x + 4 = 3(9x^2 - 1)$$
$$27x^2 - 3x + 4 = 27x^2 - 3$$
$$-3x = -7$$

12. $\dfrac{5}{x-5} + \dfrac{1}{x+5} = \dfrac{2}{x^2-25}$

$$\frac{5(x+5) + (x-5)}{x^2-25} = \frac{2}{x^2-25}$$
$$6x + 20 = 2$$
$$6x = -18$$
$$x = -3$$

13. September's profit + October's profit = 689,000

Let x = September's profit. Then

$x + 0.12x$ = October's profit.

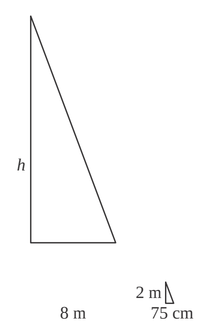
$$x + (x + 0.12x) = 689{,}000$$
$$2.12x = 689{,}000$$
$$x = 325{,}000$$
$$x + 0.12x = 364{,}000$$

September: $325,000; October: $364,000

14. Let x = the number of quarts of pure antifreeze.

$$30\% \text{ of } (10 - x) + 100\% \text{ of } x = 50\% \text{ of } 10$$
$$0.30(10 - x) + 1.00x = 0.50(10)$$
$$3 - 0.30x + 1.00x = 5$$
$$0.70x = 2$$
$$x = \frac{2}{0.70}$$
$$= \frac{20}{7} = 2\frac{6}{7} \text{ liters}$$

15. (a)



(b) $\dfrac{h}{\ } = \dfrac{2}{/} = \dfrac{8}{\ }$

x

$=$

7

16. 3

8
3
4
3

$h = \frac{6}{4} = 2\frac{1}{1}$ meters

3

3

$12,000(0.025) + 10,000x = 870$

$10,000x = 570$

$x = 0.057$, or 5.7%

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17. $$F = \frac{9}{5}C + 32$$
$$F - 32 = \frac{9}{5}C$$
$$\frac{5}{9}(F - 32) = C$$
Let $F = 28.3$.
$$\frac{5}{9}(28.3 - 32) = C$$
$$\frac{5}{9}(-3.7) = C$$
$$-2.06 \approx C$$
The average temperature is about $-2.06°$ C.

18. Basketball: $2\pi r = 30 \Rightarrow r = \dfrac{15}{\pi}$
$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{15}{\pi}\right)^3 \approx 455.95 \text{ in.}^3$$
Baseball: $2\pi r = 9.25 \Rightarrow r = \dfrac{9.25}{2\pi}$
$$V = \frac{4}{3}\pi\left(\frac{9.25}{2\pi}\right)^3 \approx 13.4 \text{ in.}^3$$

19. $-x + y = 3$

Let $x = 0$: $y = 3$, y-intercept: $(0, 3)$

Let $y = 0$: $x = -3$, x-intercept: $(-3, 0)$

20. $x - 5y = 20$

Let $x = 0$: $-5y = 20 \Rightarrow y = -4$, y-intercept: $(0, -4)$

Let $y = 0$: $x - 5(0) = 20 \Rightarrow x = 20$, x-intercept: $(20, 0)$

21. $y = x^2 - 9x + 8 = (x - 8)(x - 1)$

Let $x = 0$: $y = 0^2 - 9(0) + 8 \Rightarrow y = 8$,

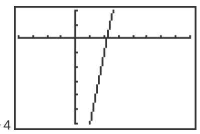
y-intercept: $(0, 8)$

Let $y = 0$: $0 = x^2 - 9x + 8 \Rightarrow 0 = (x - 8)(x - 1)$

$\Rightarrow x = 1, 8$, x-intercepts: $(1, 0)$, $(8, 0)$

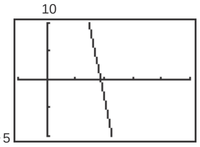
22. $y = 25 - x^2$

23. $5(x - 2) - 1 = 0$



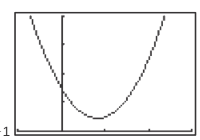
Solution: $x = 2.2$

24. $12 - 5(x - 7) = 0$



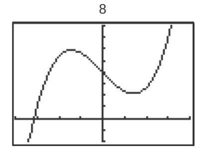
Solution: $x = 9.4$

25. $$3x^2 + 3 = 5x$$
$$3x^2 - 5x + 3 = 0$$



No real solution

26. $$\frac{1}{3}x^3 + 4 = 2x$$
$$\frac{1}{3}x^3 - 2x + 4 = 0$$



Solution: $x \approx -3.135$

27. $$0.3x^4 = 5x + 2$$

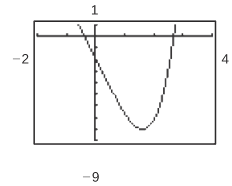
Let $x = 0$: $y = 25 - (0)^2 \Rightarrow y = 25$,

y-intercept: $(0, 25)$

Let $y = 0$: $0 = 25 - x^2 = (5 - x)(5 + x)$

$\Rightarrow x = \pm 5$, x-intercepts: $(5, 0)$, $(-5, 0)$

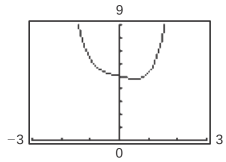
$0.3x^4 - 5x - 2 = 0$



Solutions: $x \approx -0.402, 2.676$

28. $0.8x^4 = 0.5x - 5$

$0.8x^4 - 0.5x + 5 = 0$



No real solution

29. $3x + 5y = -7$

$-x - 2y = 3 \Rightarrow x = -2y - 3$

$3(-2y - 3) + 5y = 7$
$-y - 9 = -7$

$y = -2$
$x = -2(-2) - 3 = 1$

$(x, y) = (1, -2)$

30. $x - y = 3$

$2x + y = 12 \Rightarrow y = -2x + 12$
$x - (-2x + 12) = 3$
$x + 2x - 12 = 3$

$3x - 12 = 3$

$3x = 15$
$x = 5$
$5 - y = 3 \Rightarrow y = 2$

$(x, y) = (5, 2)$

31. $4x^2 + 2y = 14$

$2x + y = 3 \Rightarrow y = -2x + 3$

$4x^2 + 2(-2x + 3) = 14$
$4x^2 - 4x + 6 = 14$

$4x^2 - 4x - 8 = 0$
$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$

32. $y = -x + 7$
$y = 2x^3 - x + 9$

$2x^3 - x + 9 = -x + 7$
$2x^3 + 2 = 0$
$x^3 + 1 = 0$
$(x + 1)(x^2 - x + 1) = 0 \Rightarrow x = -1$
$\Rightarrow y = -(-1) + 7 = 8$
$(x, y) = (-1, 8)$

33. $3 - \sqrt{-16} = 3 - 4i$

34. $\sqrt{-50} + 8 = 8 + 5\sqrt{2}i$

35. $-i + 4i^2 = -i + 4(-1)$
$= -4 - i$

36. $7i - 9i^2 = 7i - 9(-1)$
$= 9 + 7i$

37. $(2 + 13i) + (6 - 5i) = (2 + 6) + (13 - 5)i$
$= 8 + 8i$

38. $\left(\frac{1}{2} + \frac{\sqrt{3}}{4}i\right) - \left(\frac{1}{2} - \frac{\sqrt{3}}{4}i\right) = \left(\frac{1}{2} - \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}\right)i$
$= 0 + \frac{2\sqrt{3}}{4}i$

$= \frac{\sqrt{3}}{2}i$

39. $5i(13 - 8i) = 65i - 40i^2 = 40 + 65i$

40. $(1 + 6i)(5 - 2i) = 5 - 2i + 30i + 12 = 17 + 28i$

41. $\left(\sqrt{-16} + 3\right)\left(\sqrt{-25} - 2\right) = (4i + 3)(5i - 2)$
$= -20 - 8i + 15i - 6$
$= -26 + 7i$

$$x - 2 = 0 \Rightarrow x = 2$$
$$y = -2(2) + 3 = -1$$
$$(x, y) = (2, -1)$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$y = -2(-1) + 3 = 5$$
$$(x, y) = (-1, 5)$$

42. $\left(5 - \quad -4\right)\left(5 + \quad -4\right) = (5 - 2i)(5 + 2i)$
$= 25 + 4$
$= 29$

43. $\sqrt{}$
$-9 + 3 + \quad -36 = 3i + 3 + 6i$
$= 3 + 9i$

44. $7 - \sqrt{-81} + \sqrt{-49} = 7 - 9i + 7i$
$= 7 - 2i$

45. $(10 - 8i)(2 - 3i) = 20 - 30i - 16i + 24i^2$
$= -4 - 46i$

46. $i(6+i)(3-2i) = i(18+3i-12i+2)$
$= i(20-9i) = 9+20i$

47. $(3+7i)^2 + (3-7i)^2 = (9+42i-49)+(9-42i-49)$
$= -80$

48. $(4-i)^2 - (4+i)^2 = (16-8i-1)-(16+8i-1)$
$= -16i$

49. $\frac{6+i}{i} = \frac{6+i}{i}\cdot\frac{-i}{-i} = \frac{-6i-i^2}{-i^2}$

$= \frac{-6i+1}{1} = 1-6i$

50. $\frac{4}{-3i} = \frac{-4}{3i}\cdot\frac{-i}{-i} = \frac{4i}{3} = \frac{4}{3}i$

51. $\frac{3+2i}{5+i}\cdot\frac{5-i}{5-i} = \frac{15+10i-3i+2}{25+1}$
$= \frac{17}{26}+\frac{7}{26}i$

52. $\frac{1-7i}{2+3i}\cdot\frac{2-3i}{2-3i} = \frac{2-21-17i}{4+9}$
$= -\frac{19}{13}-\frac{17}{13}i$

53. $9x^2 = 49$

$x^2 = \frac{49}{9}$

$x = \pm\sqrt{\frac{49}{9}}$

$x = \pm\frac{7}{3}$

54. $8x = 2x^2$
$0 = 2x^2 - 8x$

$0 = 2x(x-4)$
$2x = 0 \Rightarrow x = 0$

57. $x^2 - 7x - 8 = 0$
$(x-8)(x+1) = 0$
$x - 8 = 0 \Rightarrow x = 8$

$x + 1 = 0 \Rightarrow x = -1$

58. $x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$
$x + 6 = 0 \Rightarrow x = -6$
$x - 3 = 0 \Rightarrow x = 3$

59. $(x+4)^2 = 18$

$x+4 = \pm\sqrt{18}$
$x = -4 \pm 3\sqrt{2}$

60. $(x+1)^2 = 24$
$x+1 = \pm\sqrt{24} = \pm 2\sqrt{6}$

$x = -1 \pm 2\sqrt{6}$

61. $(3x-1)^2 + 4 = 0$
$(3x-1)^2 = -4$

$3x - 1 = \pm\sqrt{-4}$
$3x - 1 = \pm 2i$
$3x = 1 \pm 2i$
$x = \frac{1}{3} \pm \frac{2}{3}i$

62. $(5x-3)^2 + 16 = 0$

$(5x-3)^2 = -16$

$5x - 3 = \pm\sqrt{-16}$
$5x - 3 = \pm 4i$
$5x = 3 \pm 4i$
$x = \frac{3}{5} \pm \frac{4}{5}i$

$x - 4 = 0 \Rightarrow x = 4$

$x = \pm i$ 5 5

63.

$x^2 + 4x - 9 = 0$

$x^2 + 4x + 4 = 9 + 4$

$(x + 2)^2 = 13$

$x + 2 = \pm\sqrt{13}$

$x = -2 \pm \sqrt{13}$

55. $6x = 3x^2$

$0 = 3x^2 - 6x$

$0 = 3x(x - 2)$

$3x = 0 \Rightarrow x = 0$

$x - 2 = 0 \Rightarrow x = 2$

56. $16x^2 = 25$

$x^2 = \frac{25}{16}$

$x = \pm\sqrt{\frac{25}{16}} = \pm\frac{5}{4}$

64. $x^2 - 6x - 5 = 0$
$x^2 - 6x = 5$
$x^2 - 6x + 9 = 5 + 9$
$(x - 3)^2 = 14$
$x - 3 = \pm\sqrt{14}$
$x = 3 \pm \sqrt{14}$

65. $x^2 - 12x + 30 = 0$
$x^2 - 12x = -30$
$x^2 - 12x + 36 = -30 + 36$
$(x - 6)^2 = 6$
$x - 6 = \pm\sqrt{6}$
$x = 6 \pm \sqrt{6}$

66. $x^2 + 6x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(1)(-3)}}{2(1)} = \frac{-6 \pm \sqrt{48}}{2} = -3 \pm 2\sqrt{3}$$

67. $x^2 - 3x = 28$
$x^2 - 3x - 28 = 0$
$(x - 7)(x + 4) = 0$
$x - 7 = 0 \Rightarrow x = 7$
$x + 4 = 0 \Rightarrow x = -4$

68. $x^2 + 3x = 40$
$x^2 + 3x - 40 = 0$
$(x - 5)(x + 8) = 0$
$x - 5 = 0 \Rightarrow x = 5$
$x + 8 = 0 \Rightarrow x = -8$

69. $x^2 - 10x = 9$
$x^2 - 10x + 25 = 9 + 25$
$(x - 5)^2 = 34$
$x - 5 = \pm\sqrt{34}$

71. $2x^2 + 9x - 5 = 0$
$(2x - 1)(x + 5) = 0$
$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$
$x + 5 = 0 \Rightarrow x = -5$

72. $4x^2 + x - 5 = 0$
$(4x - 5)(x - 1) = 0$
$4x + 5 = 0 \Rightarrow x = -\frac{5}{4}$
$x - 1 = 0 \Rightarrow x = 1$

73. $-x^2 - x + 15 = 0$
$x^2 + x - 15 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 4(-15)}}{2} = \frac{-1 \pm \sqrt{61}}{2}$$

74. $-x^2 - 3x + 2 = 0$
$x^2 + 3x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-2)}}{2(1)} = \frac{-3 \pm \sqrt{9 + 8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$$

75. $x^2 + 4x + 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 40}}{2} = \frac{-4 \pm \sqrt{-24}}{2} = -2 \pm \sqrt{6}i$$

76. $x^2 + 6x - 1 = 0$

$$x = 5 \pm \sqrt{34}$$

70.

$$x^2 + 8x = 7$$

$$x^2 + 8x + 16 = 7 + 16$$

$$(x + 4)^2 = 23$$

$$x + 4 = \pm\sqrt{23}$$

$$x = -4 \pm \sqrt{23}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{(6)^2 - 4(-1)}}{2}$$

$$= \frac{-6 \pm \sqrt{40}}{2}$$

$$= -3 \pm \sqrt{10}$$

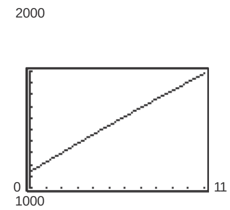
77. $2x^2 - 6x + 21 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{6 \pm \sqrt{36 - 168}}{4}$$
$$= \frac{6 \pm \sqrt{-132}}{4}$$
$$= \frac{3}{2} \pm \frac{\sqrt{33}}{2} i$$

78. $2x^2 - 8x + 11 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{8 \pm \sqrt{(-8)^2 - 4(2)(11)}}{2(2)}$$
$$= \frac{8 \pm \sqrt{-24}}{4}$$
$$= \frac{8 \pm 2\sqrt{6}i}{4}$$
$$= 2 \pm \frac{\sqrt{6}}{2} i$$

79. (a)



(b) In 2008, the average cost per day reached $1800.

(c) $1800 = -0.54t^2 + 82.6t + 1136$

$0 = -0.54t^2 + 82.6t - 664$

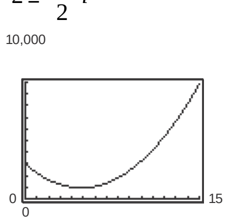
$$t = \frac{-(82.6) \pm \sqrt{(82.6)^2 - 4(-0.54)(-664)}}{2(-0.54)}$$

$t \approx 8.51, 144.45$ (not in the domain of the model)

In 2008, the average cost per day reached $1800.

(d) Answers will vary.

80. (a)



(b) In 2010, the revenues reached $4 billion dollars.

(c) $4000 = 86.727t^2 - 839.83t + 2967.9$

$0 = 86.727t^2 - 839.83t - 1032.1$

$$t = \frac{-(-839.83) \pm \sqrt{(-839.83)^2 - 4(86.727)(-1032.1)}}{2(86.727)}$$

$t \approx -1.10$ (not in the domain of the model), 10.79

In 2010, the revenue reached $4 billion.

(d) $10{,}000 = 86.727t^2 - 839.83t + 2967.9$

$0 = 86.727t^2 - 839.83t - 7032.1$

$$t = \frac{-(-839.83) \pm \sqrt{(-839.83)^2 - 4(86.727)(-7032.1)}}{2(86.727)}$$

$t \approx -5.38$ (not in the domain of the model), 15.07

In 2015, the revenue will reach \$10 billion.

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81. $3x^3 - 26x^2 + 16x = 0$

$x(3x^2 - 26x + 16) = 0$

$x(3x - 2)(x - 8) = 0$

$x = 0$

$3x - 2 = 0 \Rightarrow x = \frac{2}{3}$

$x - 8 = 0 \Rightarrow x = 8$

82. $36x^3 - x = 0$

$x(36x^2 - 1) = 0$

$x(6x + 1)(6x - 1) = 0$

$x = 0$

$6x + 1 = 0 \Rightarrow x = -\frac{1}{6}$

$6x - 1 = 0 \Rightarrow x = \frac{1}{6}$

83. $5x^4 - 12x^3 = 0$

$x^3(5x - 12) = 0$

$x^3 = 0$ or $5x - 12 = 0$

$x = 0$ $\quad x = \frac{5}{12}$

84. $4x^3 - 6x^2 = 0$

$x^2(4x - 6) = 0$

$x^2 = 0 \Rightarrow x = 0$

$4x - 6 = 0 \Rightarrow x = \frac{3}{2}$

85. $x^4 - x^2 - 12 = 0$

$(x^2 - 4)(x^2 + 3) = 0$

$x^2 - 4 = 0$ or $x^2 + 3 = 0$

$x^2 = 4$ $\quad x^2 = -3$

$x = \pm\sqrt{4} = \pm 2$ $\quad x = \pm\sqrt{3}i$

86. $x^4 - 4x^2 - 5 = 0$

$(x^2 - 5)(x^2 + 1) = 0$

$x^2 - 5 = 0$ or

$x^2 = 5$

$x = \pm\sqrt{5}$

88. $3x^4 + 18x^2 + 24 = 0$

$x^4 + 6x^2 + 8 = 0$

$(x^2 + 4)(x^2 + 2) = 0$

$x^2 + 4 = 0$ or $x^2 + 2 = 0$

$x^2 = -4$ $\quad x^2 = -2$

$x = \pm\sqrt{-4} = \pm 2i$ $\quad x = \pm\sqrt{-2} = \pm\sqrt{2}i$

89. $\sqrt{x + 4} = 3$

$(\sqrt{x + 4})^2 = (3)^2$

$x + 4 = 9$

$x = 5$

90. $\sqrt{x - 2} - 8 = 0$

$\sqrt{x - 2} = 8$

$x - 2 = 64$

$x = 66$

91. $2x - 5\sqrt{x} + 3 = 0$

$2x + 3 = 5\sqrt{x}$

$(2x + 3)^2 = (5\sqrt{x})^2$

$4x^2 + 12x + 9 = 25x$

$4x^2 - 13x + 9 = 0$

$(4x - 9)(x - 1) = 0$

$4x - 9 = 0 \Rightarrow x = \frac{9}{4}$

$x - 1 = 0 \Rightarrow x = 1$

92. $\sqrt{3x - 2} = 4 - x$

$3x - 2 = (4 - x)^2$

$3x - 2 = 16 - 8x + x^2$

$0 = 18 - 11x + x^2$

$$x^2 + 1 = 0$$
$$x^2 = -1$$
$$x = \pm\sqrt{-1} = \pm i$$

$$0 = (x - 9)(x - 2)$$

$$0 = x - 9 \Rightarrow x = 9, \text{ extraneous}$$
$$0 = x - 2 \Rightarrow x = 2$$

87. $2x^4 - 22x^2 + 56 = 0$

$$\left(x^2 - 4\right)\left(2x^2 - 14\right) = 0$$

$x^2 - 4 = 0$ or $2x^2 - 14 = 0$

$x^2 = 4$ $2x^2 = 14$

$x = \pm\sqrt{4} = \pm 2$ $x^2 = 7$

$x = \pm\sqrt{7}$

93. $\sqrt{2x+3} + \sqrt{x-2} = 2$

$$\left(\sqrt{2x+3}\right)^2 = \left(2 - \sqrt{x-2}\right)^2$$

$$2x + 3 = 4 - 4\sqrt{x-2} + x - 2$$

$$x + 1 = -4\sqrt{x-2}$$

$$(x+1)^2 = \left(-4\sqrt{x-2}\right)^2$$

$$x^2 + 2x + 1 = 16(x-2)$$

$$x^2 - 14x + 33 = 0$$

$$(x-3)(x-11) = 0$$

$x = 3$, extraneous or $x = 11$, extraneous
No solution (You can verify that the graph of $y = \sqrt{2x+3} + \sqrt{x-2} - 2$ lies above the x-axis.)

94. $5\sqrt{x} - \sqrt{x-1} = 6$

$$5\sqrt{x} = 6 + \sqrt{x-1}$$

$$25x = 36 + 12\sqrt{x-1} + x - 1$$

$$24x - 35 = 12\sqrt{x-1}$$

$$576x^2 - 1680x + 1225 = 144(x - 1)$$

$$576x^2 - 1824x + 1369 = 0$$

$$x = \frac{-(-1824) \pm \sqrt{(-1824)^2 - 4(576)(1369)}}{2(576)}$$

$$= \frac{1824 \pm \sqrt{172{,}800}}{1152} = \frac{1824 \pm 240\sqrt{3}}{1152}$$

$$x = \frac{38 + 5\sqrt{3}}{24}$$

$x = \dfrac{38 - 5\sqrt{3}}{24}$, extraneous

95. $(x-1)^{2/3} - 25 = 0$

97. $(x+4)^{1/2} + 5x(x+4)^{3/2} = 0$

$$(x+4)^{1/2}\left[1 + 5x(x+4)\right] = 0$$

$$(x+4)^{1/2}\left(5x^2 + 20x + 1\right) = 0$$

$(x+4)^{1/2} = 0$ or $5x^2 + 20x + 1 = 0$

$x = -4$

$$x = \frac{-20 \pm \sqrt{400 - 20}}{10}$$

$$x = \frac{-20 \pm 2\sqrt{95}}{10}$$

$$x = -2 \pm \frac{\sqrt{95}}{5}$$

98. $8x^2\left(x^2 - 4\right)^{1/3} + \left(x^2 - 4\right)^{4/3} = 0$

$$\left(x^2 - 4\right)^{1/3}\left[8x^2 + x^2 - 4\right] = 0$$

$$\left(x^2 - 4\right)^{1/3}\left(9x^2 - 4\right) = 0$$

$$(x-2)^{1/3}(x+2)^{1/3}(3x-2)(3x+2) = 0$$

$$x - 2 = 0 \Rightarrow x = 2$$

$$x + 2 = 0 \Rightarrow x = -2$$

$$3x - 2 = 0 \Rightarrow x = \tfrac{2}{3}$$

$$3x + 2 = 0 \Rightarrow x = -\tfrac{2}{3}$$

99. $\dfrac{x}{8} + \dfrac{3}{8} = \dfrac{1}{2x}$

$$x + 3 = \frac{4}{x}$$

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$x + 4 = 0 \Rightarrow x = -4$$

$$x - 1 = 0 \Rightarrow x = 1$$

100. $\dfrac{3x}{\ } = \dfrac{1}{\ } - \dfrac{5}{\ }$

$(x-1) \quad = 25$

$^{2} \quad ^{3}$

$(x-1) \quad = 25$

$x - 1 = \pm\sqrt{25^3}$

$x = 1 \pm 125$

$x = 126 \text{ or } x = -124$

96. $(x+2)^{3/4} = 27$

$x + 2 = 27^{4/3}$

$x + 2 = 81$

$x = 79$

2 $\quad x \quad$ 2

$$3x = \frac{2}{x} - 5$$

$3x^2 + 5x - 2 = 0$

$(x+2)(3x-1) = 0$

$x + 2 = 0 \Rightarrow x = -2$

$3x - 1 = 0 \Rightarrow x = \frac{1}{3}$

101. $\frac{5}{x} = 1 + \frac{3}{x+2}$

$$\begin{aligned}
5(x + 2) &= x(x + 2) + 3(x) \\
5x + 10 &= x^2 + 2x + 3x \\
5x + 10 &= x^2 + 5x + 5 \\
10 &= x^2 \\
\pm\sqrt{10} &= x
\end{aligned}$$

102. $\dfrac{6}{x} + \dfrac{8}{x+5} = 3$

$$6(x+5) + 8(x) = 3(x)(x+5)$$
$$6x + 30 + 8x = 3x^2 + 15x$$
$$0 = 3x^2 + x - 30$$
$$0 = (3x+10)(x-3)$$
$$3x + 10 = 0 \Rightarrow x = -\frac{10}{3}$$
$$x - 3 = 0 \Rightarrow x = 3$$

103. $3 + \dfrac{2}{x} = \dfrac{16}{x^2}$

$$3x^2 + 2x = 16$$
$$3x^2 + 2x - 16 = 0$$
$$(3x+8)(x-2) = 0$$
$$3x + 8 = 0 \Rightarrow x = -\frac{8}{3}$$
$$x - 2 = 0 \Rightarrow x = 2$$

104. $\dfrac{2x}{x^2-1} - \dfrac{3}{x+1} = 1$

$$\frac{2x}{(x-1)(x+1)} - \frac{3}{x+1} = 1$$
$$2x - 3(x-1) = (x-1)(x+1)$$
$$2x - 3x + 3 = x^2 - 1$$
$$x^2 + x - 4 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{17}}{2}$$

105. $|x-5| = 10$

$x - 5 = -10$ or $x - 5 = 10$

$x = -5$ $\qquad$ $x = 15$

106. $|2x+3| = 7$

$2x + 3 = 7$ or $2x + 3 = -7$

$2x = 4$ $\qquad$ $2x = -10$

$x = 2$ $\qquad$ $x = -5$

107. $|x^2 - 3| = 2x$

$x^2 - 3 = 2x$ or $x^2 - 3 = -2x$

$x^2 - 2x - 3 = 0$ $\qquad$ $x^2 + 2x - 3 = 0$

$(x-3)(x+1) = 0$ $\qquad$ $(x+3)(x+1) = 0$

$x = 3$ or $x = -1$ $\qquad$ $x = -3$ or $x = 1$

The only solutions to the original equation are $x = 3$ or $x = 1$. $(x = -3$ and $x = -1$ are extraneous.)

108. $|x^2 - 6| = x$

$x^2 - 6 = x$ or $x^2 - 6 = -x$

$x^2 - x - 6 = 0$ $\qquad$ $x^2 + x - 6 = 0$

$(x-3)(x+2) = 0$ $\qquad$ $(x+3)(x-2) = 0$

$x - 3 = 0 \Rightarrow x = 3$ $\qquad$ $x - 2 = 0 \Rightarrow x = 2$

$x + 2 = 0 \Rightarrow x = -2$, extraneous $\qquad$ $x + 3 = 0 \Rightarrow x = -3$, extraneous

109. Let x = number of investors.

$$\frac{240{,}000}{x} = \frac{240{,}000}{x+2} + 20{,}000$$

$$\left(x(x+2)\right)\left(\frac{240{,}000}{x}\right) = \left(\frac{240{,}000}{x+2} + 20{,}000\right)\left(x(x+2)\right)$$

$$240{,}000(x+2) = 240{,}000x + 20{,}000x(x+2)$$

$$240{,}000x + 480{,}000 = 240{,}000x + 20{,}000x^2 + 40{,}000x$$

$$20{,}000x^2 + 40{,}000x - 480{,}000 = 0$$

$$x^2 + 2x - 24 = 0$$

$$(x+6)(x-4) = 0$$

$$x + 6 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = -6 \qquad x = 4$$

($x = -6$ is not in the original domain.)

There are 4 investors currently in the group.

110. Let x = number of students.

$$\frac{1700}{x} = \frac{1700}{x+6} + 7.50$$

$$\left(x(x+6)\right)\left(\frac{1700}{x}\right) = \left(\frac{1700}{x+6} + 7.50\right)\left(x(x+6)\right)$$

$$1700(x+6) = 1700x + 7.50x(x+6)$$

$$1700x + 10{,}200 = 1700x + 7.5x^2 + 45x$$

$$7.5x^2 + 45x - 10{,}200 = 0$$

$$75x^2 + 450x - 102{,}000 = 0$$

$$3x^2 + 18x - 4080 = 0$$

$$x^2 + 6x - 1360 = 0$$

$$(x+40)(x-34) = 0$$

$$x + 40 = 0 \quad \text{or} \quad x - 34 = 0$$

$$x = -40 \qquad x = 34$$

($x = -40$ is not in the original domain.)

There are 34 students currently in the group.

111. Let x = average speed originally from Portland to Seattle.

$$\frac{145}{x} = \frac{145}{x+40} + \frac{12}{60}$$

$$5x(x+40)\left(\frac{145}{x}\right) = \left(\frac{145}{x+40} + \frac{1}{5}\right)\left(5x(x+40)\right)$$

$$725x + 29000 = 725x + x^2 + 40x$$

$$x^2 + 40x - 29{,}000 = 0$$

Using the Quadratic Formula:

$$x = \frac{-40 \pm \sqrt{(40)^2 - 4(1)(-29{,}000)}}{2(1)}$$

$= \dfrac{-40 \pm \sqrt{117{,}600}}{2} \approx 151.5 \text{ mi/h}$

So, on the return trip from Seattle to Portland, the average speed is $x + 40 = 191.5$ mph.

112. Let x = average speed on the first trip.

$$\frac{56}{x} = \frac{56}{x+8} + \frac{1}{6}$$

$$56(x+8) = 56x + \frac{1}{6}x(x+8)$$

$$448 = \frac{1}{6}x(x+8)$$

$$x^2 + 8x - 2688 = 0$$

$$(x-48)(x+56) = 0$$

$$x = 48$$

So, the average speed on the return trip is

$48 + 8 = 56$ mph.

113.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$1196.95 = 1000\left(1+\frac{r}{12}\right)^{12(6)}$$

$$1.19695 = \left(1+\frac{r}{12}\right)^{72}$$

$$1+\frac{r}{12} = (1.19695)^{1/72}$$

$$r = 0.03,\ 3\%$$

114.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$2465.43 = 1500\left(1+\frac{r}{4}\right)^{4(10)}$$

$$1.64362 = \left(1+\frac{r}{4}\right)^{40}$$

$$1+\frac{r}{4} = 1.64362^{1/40}$$

$$r = 0.05,\ 5\%$$

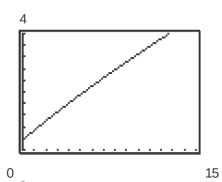
115. (a)

Year	2000	2001	2002	2003	2004
Enrollment (in millions)	3.09	3.17	3.25	3.33	3.40

Year	2005	2006	2007	2008	2009
Enrollment (in millions)	3.48	3.55	3.62	3.69	3.76

Year	2010	2011	2012
Enrollment (in millions)	3.83	3.89	3.96

(b)



(c) In 2005, the number of students reached 3.5 million.

(d) Find t when $S = 3.5$.

$$\sqrt{0.51049t + 9.5287} = 3.5$$

$$\left(\sqrt{0.51049t + 9.5287}\right)^2 = (3.5)^2$$

$$0.51049t + 9.5287 = 12.25$$

$$0.51049t = 2.7213$$

$$t \approx 5.33$$

In 2005, enrollment reached 3.5 million.

(e) Find t when $S = 6$.

$$\sqrt{0.51049t + 9.5287} = 6$$
$$\left(\sqrt{0.51049t + 9.5287}\right)^2 = (6)^2$$
$$0.51049t + 9.5287 = 36$$
$$0.51049t = 26.4713$$
$$t \approx 51.85$$

In 2051, enrollment will reach 6 million.
Answers will vary.

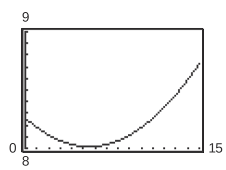
(f) Answers will vary.

116. (a)

Year	2003	2004	2005	2006	2007
Population (in millions)	8.07	8.03	8.02	8.01	8.03

Year	2008	2009	2010	2011	2012
Population (in millions)	8.06	8.11	8.18	8.26	8.36

(b)



(c) In 2010, the population of New York reached 8.2 million.

(d) Find t when $P = 8.2$.

$$\sqrt{0.13296t^2 - 1.4650t + 68.243} = 8.2$$
$$\left(\sqrt{0.13296t^2 - 1.4650t + 68.243}\right)^2 = (8.2)^2$$
$$0.13296t^2 - 1.4650t + 68.243 = 67.24$$
$$0.13296t^2 - 1.4650t + 1.003 = 0$$
$$t = \frac{-(-1.4650) \pm \sqrt{(-1.4650)^2 - 4(0.13296)(1.003)}}{2(0.13296)}$$
$t \approx 0.73$ (not in the domain model), 10.28

In 2010, the population of New York reached 8.2 million.

(e) Find t when $P = 8.5$.

$$\sqrt{0.13296t^2 - 1.4650t + 68.243} = 8.5$$
$$\left(\sqrt{0.13296t^2 - 1.4650t + 68.243}\right)^2 = (8.5)^2$$
$$0.13296t^2 - 1.4650t + 68.243 = 72.25$$
$$0.13296t^2 - 1.4650t - 4.007 = 0$$

$-(-1.4650) \pm$

$$t = \frac{\sqrt{(-1.4650)^2 - 4(0.13296)(-4.007)}}{2(0.13296)}$$

$$t \approx -2.27 \text{ (not in the domain model)}, 13.29$$

In 2013, the population will reach 8.5 million.

Answers will vary.

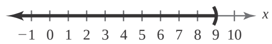
(f) Answers will vary.

117. $8x - 3 < 6x + 15$

$2x < 18$

$x < 9$

$(-\infty, 9)$

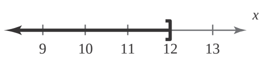


118. $9x - 8 \le 7x + 16$

$2x \le 24$

$x \le 12$

$(-\infty, 12]$



119. $\frac{1}{2}(3 - x) > \frac{1}{3}(2 - 3x)$

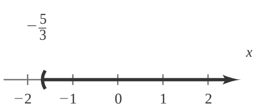
$3(3 - x) > 2(2 - 3x)$

$9 - 3x > 4 - 6x$

$3x > -5$

$x > -\frac{5}{3}$

$\left(-\frac{5}{3}, \infty\right)$



120. $4(5 - 2x) \ge \frac{1}{2}(8 - x)$

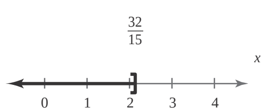
$8(5 - 2x) \ge 8 - x$

$40 - 16x \ge 8 - x$

$32 \ge 15x$

$x \le \frac{32}{15}$

$\left(-\infty, \frac{32}{15}\right]$



121. $-2 < -x + 7 \le 10$

$-9 < -x \le 3$

$9 > x \ge -3$

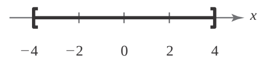
$-3 \le x < 9$

$[-3, 9)$

123. $|x| \le 4$

$-4 \le x \le 4$

$[-4, 4]$



124. $|x - 2| < 1$

$-1 < x - 2 < 1$

$1 < x < 3$

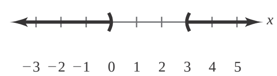
$(1, 3)$

125. $\left|x - \frac{3}{2}\right| > \frac{3}{2}$

$x - \frac{3}{2} < -\frac{3}{2}$ or $x - \frac{3}{2} > \frac{3}{2}$

$x < 0$ or $x > 3$

$(-\infty, 0) \cup (3, \infty)$

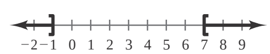


126. $|x - 3| \ge 4$

$x - 3 \ge 4$ or $x - 3 \le 4$

$x \ge 7$ $\quad$ $x \le -1$

$(-\infty, -1] \cup [7, \infty)$



127. $4|3 - 2x| \le 16$

$|3 - 2x| \le 4$

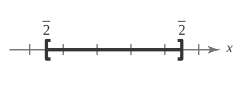
$-4 \le 3 - 2x \le 4$

$-7 \le -2x \le 1$

$\frac{7}{2} \ge x \ge -\frac{1}{2}$

$-\frac{1}{2} \le x \le \frac{7}{2}$

$\left[-\frac{1}{2}, \frac{7}{2}\right]$



128. $-6 < 13 - 2x < 14$

122. $-6 \le 3 - 2(x \ldots$

$x + 9$
$+ 7$
> 19

$x + 9 < -12$

$x < -21$

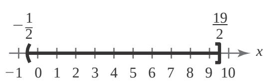
$x + 9 > 12$

$x + 9 > 12$ or

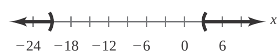
$x > 3$

$-\frac{1}{2} < x \le \frac{19}{2}$

$\left(-\frac{1}{2}, \frac{19}{2}\right]$



$\left(-\infty, -21\right) \cup \left(3, \infty\right)$



INSTRUCTOR USE ONLY

129. $x^2 - 2x > 3$

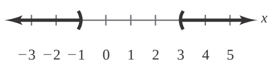
$x^2 - 2x - 3 > 0$

$(x-3)(x+1) > 0$

Key numbers: $x = -1,\ x = 3$

Test intervals: $(-\infty, -1),\ (-1, 3),\ (3, \infty)$

Solution set: $(-\infty, -1) \cup (3, \infty)$



130. $x^2 - 6x - 27 < 0$

$(x-9)(x+3) < 0$

Key numbers: $x = -3,\ x = 9$

Test intervals: $(-\infty, -3),\ (-3, 9),\ (9, \infty)$

Test: Is $(x-9)(x+3) < 0$?

Solution set: $(-3, 9)$

131. $4x^2 - 23x \le 6$

$4x^2 - 23x - 6 \le 0$

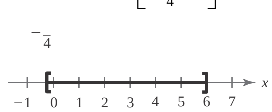
$(x-6)(4x+1) \le 0$

Key numbers: $x = 6,\ x = -\frac{1}{4}$

Test intervals: $(-\infty, -\frac{1}{4}),\ (-\frac{1}{4}, 6),\ (6, \infty)$

Test: Is $(x-6)(4x+1) \le 0$?

Solution set: $\left[-\frac{1}{4}, 6\right]$



132. $6x^2 + 5x \ge 4$

$x^2 \quad 5x - \quad \ge$

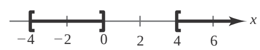
133. $x^3 - 16x \ge 0$

$x(x-4)(x+4) \ge 0$

Key numbers: $x = 0, x = 4, x = -4$

Test intervals: $(-\infty, -4),\ (-4, 0),\ (0, 4),\ (4, \infty)$

Test: Is $x(x-4)(x+4) \ge 0$?



134. $12x^3 - 20x^2 < 0$

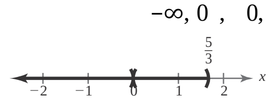
$4x^2(3x-5) < 0$

Key numbers: $x = 0,\ x = \frac{5}{3}$

Test intervals: $(-\infty, 0),\ (0, \frac{5}{3}),\ (\frac{5}{3}, \infty)$

Test: Is $4x^2(3x-5) < 0$?

Solution set: $(-\infty, 0),\ (0, \frac{5}{3})$



135. $\frac{1}{x} + 3 > 0$

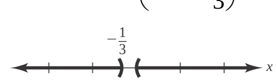
$\frac{1 + 3x}{x} > 0$

Key numbers: $x = -\frac{1}{3}, x = 0$

Intervals: $\left(-\infty, -\frac{1}{3}\right), \left(-\frac{1}{3}, 0\right), (0, \infty)$

Test: Is $\frac{1+3x}{x} > 0$?

Solution set: $\left(-\infty, -\frac{1}{3}\right) \cup (0, \infty)$



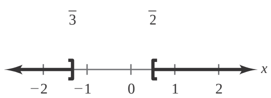
$(3x + 4)(2x - 1) \geq 0$

Key numbers: $x = -\frac{4}{3}, x = \frac{1}{2}$

Test intervals: $\left(-\infty, -\frac{4}{3}\right), \left(-\frac{4}{3}, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

Test: Is $(3x+4)(2x-1) \geq 0$?

Solution set: $\left(-\infty, -\frac{4}{3}\right] \cup \left[\frac{1}{2}, \infty\right)$



136. $\frac{}{x} \geq x$

$\frac{9}{x} - x \geq 0$

$\frac{9 - x^2}{x} \geq 0$

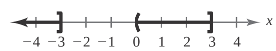
$\frac{(3 - x)(3 + x)}{x} \geq 0$

Key numbers: $x = -3, x = 0, x = 3$

Test intervals: $(-\infty, -3), (-3, 0), (0, 3), (3, \infty)$

Test: Is $4x \frac{(3-x)(3+x)}{x} \geq 0$?

Solution set: $(-\infty, -3] \cup (0, 3]$



137. $\dfrac{3x+8}{x-3} - 4 \le 0$

$\dfrac{3x+8-4(x-3)}{(x-3)} \le 0$

$\dfrac{20-x}{x-3} \le 0$

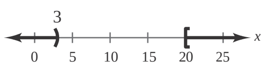
$\dfrac{x-20}{x-3} \ge 0$

Key numbers: $x = 3,\ x = 20$

Test intervals: $(-\infty, 3),\ (3, 20),\ (20, \infty)$

Test: Is $\dfrac{x-20}{x-3} \ge 0$?

Solution set: $(-\infty,\ 3) \cup [20,\ \infty)$



138. $\dfrac{x+8}{x+5} - 2 < 0$

$\dfrac{x+8-2(x+5)}{x+5} < 0$

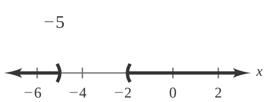
$\dfrac{-x-2}{x+5} < 0$

Key numbers: $x = -5,\ x = -2$

Test intervals: $(-\infty,\ -5),\ (-5,\ -2),\ (-2, \infty)$

Test: Is $\dfrac{-x-2}{x+5} < 0$?

Solution set: $(-\infty,\ -5) \cup (-2, \infty)$



139. $\sqrt{16 - x^2}$

$16 - x^2 \ge 0$

$(4 - x)(4 + x) \ge 0$

Key numbers: $x = -4, 4$

Test intervals: $(-\infty, -4), (-4, 4), (4, \infty)$

$(4 - x)(4 + x) \ge 0$ on $[-4, 4]$

Domain: $[-4,$

141. $(0.1)(3.69) = 0.369$

$\approx \$0.37$ per gallon

You may be overcharged $\$0.37 \times 15$ gallons $\approx \$5.55$.

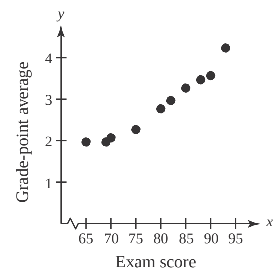
142. $|h - 50| \le 30$

$-30 \le h - 50 \le 30$

$20 \le h \le 80$

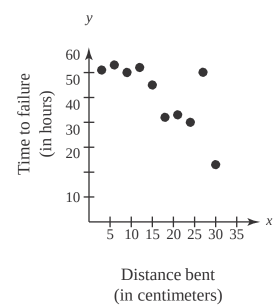
Minimum 20, maximum 80

143. (a)



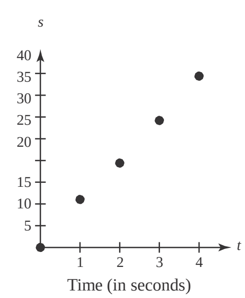
(b) Yes, the relationship is approximately linear. Higher entrance exam scores, x, are associated with higher grade-point averages, y.

144. (a)



(b) Yes. Answers will vary.

145. (a)



(b) $s \approx 10t - 0.4$

Approximations will vary.

Speed (ir

(c) $s = 9.7t + 0.4;\ r \approx 0.999$

(d) $s(2.5) = 9.7(2.5) + 0.4 \approx 24.7$ m/sec

140. $\sqrt[4]{x^2 - 5x - 14}$

$x^2 - 5x - 14 \geq 0$

$(x + 2)(x - 7) \geq 0$

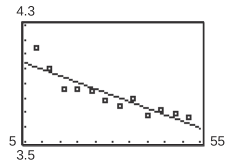
Key numbers: $x = -2, 7$

Test intervals: $(-\infty, -2), (-2, 7), (7, \infty)$

$(x + 2)(x - 7) \geq 0$ on $(-\infty, -2] \cup [7, \infty)$

Domain: $(-\infty, -2] \cup [7, \infty)$

146. (a) $y = -0.0089x + 4.086$

(b)


(c) Answers will vary. Sample answer: Yes, the model fits the data well.

(d) Answers will vary. Sample answer: No, eventually the model will yield results including negative times that would be physically impossible.

147. False. A function can have only one y-intercept. (Vertical Line Test)

148. False. $(1+2i)+(1-2i)=2$, a real number.

149. False. The slope can be positive, negative, or 0.

150. An identity is an equation that is true for every real number in the domain of the variable. A conditional equation is true for just some (or even none) of the real numbers in the domain.

151. They are the same. A point $(a,\ 0)$ is an x-intercept if it is a solution point of the equation. In other words, a is a zero of the function.

152. $ax + b = 0$. $x = -\frac{b}{a}$. Then:

(a) If $ab > 0$, then $x < 0$.

(b) If $ab < 0$, then $x > 0$.

153. The error is $\sqrt{-8}\sqrt{-8} \neq \sqrt{(-8)(-8)}$.

In fact, $\sqrt{-8}\sqrt{-8} = \sqrt{8}i\sqrt{8}i = -8$.

154. The error is $\sqrt{-4} \neq 4i$. In fact,

$-i\left(\sqrt{-4}-1\right) = -i(2i-1) = 2+i.$

155. (a) $i^{40} = \left(i^4\right)^{10} = 1^{10} = 1$

(b) $i^{25} = i\left(i^{24}\right) = i(1) = i$

(c) $i^{50} = i^2\left(i^{48}\right) = (-1)(1) = -1$

(d) $i^{67} = i^3\left(i^{64}\right) = -i(1) = -i$

Chapter 2 Test

1. $\frac{12}{x} - 7 = -\frac{27}{x} + 6$

$\frac{39}{x} = 13$

$39 = 13x$

$3 = x \Rightarrow x = 3$

2. $\frac{4}{3x-2} - \frac{9x}{3x+2} = -3$

$4(3x+2) - 9x(3x-2) = -3(3x-2)(3x+2)$

$12x + 8 - 27x^2 + 18x = -3\left(9x^2 - 4\right)$

$-27x^2 + 30x + 8 = -27x^2 + 12$

$30x = 4$

$x = \frac{2}{}$

7. $\frac{8+5i}{i} = \frac{8+5i}{i} \cdot \frac{-i}{-i}$

$= \frac{-8i - 5i^2}{-i^2}$

$= \frac{-8i - 5(-1)}{-(-1)}$

$= 5 - 8i$

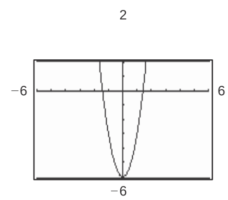
8. $\frac{(2i-1)}{(3i+2)} \cdot \frac{2-3i}{2-3i} = \frac{6-2+4i+3i}{4+9}$

$= \frac{4}{13} + \frac{7}{13}i$

9. $f(x) = 3x^2 - 6 = 0$

15

$x \approx \pm 1.414$



3. $(-8-3i)+(-1-15i) = -9-18i$

4. $\left(10+\sqrt{-20}\right)-\left(4-\sqrt{-14}\right) = 6+2\sqrt{5}i+\sqrt{14}i$

$= 6+\left(2\sqrt{5}+\sqrt{14}\right)i$

5. $(2+i)(6-i) = 12+6i-2i+1 = 13+4i$

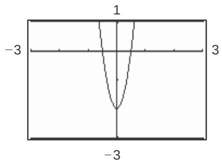
6. $(4+3i)^2-(5+i)^2 = (16+24i-9)-(25+10i-1)$

$= -17+14i$

INSTRUCTOR USE ONLY

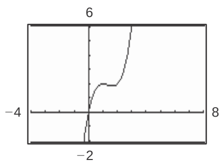
10. $f(x) = 8x^2 - 2 = 0$

$x = \pm 0.5$



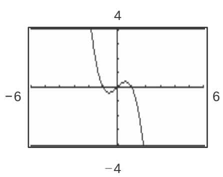
11. $f(x) = x^3 - 4x^2 + 5x = 0$

$x = 0$



12. $f(x) = x - x^3 = 0$

$x = 0, \ \pm 1$



13. $x^2 - 15x + 56 = 0$

$(x - 7)(x - 8) = 0$

$x - 7 = 0 \Rightarrow x = 7$

$x - 8 = 0 \Rightarrow x = 8$

14. $x^2 + 12x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-12 \pm \sqrt{12^2 - 4(-2)}}{2}$$

$$= \frac{-12 + \sqrt{152}}{2}$$

$$= -6 \pm \sqrt{38}$$

15. $4x^2 - 81 = 0$

$4x^2 = 81$

$x^2 = \frac{81}{4}$

$x = \pm \frac{9}{2}$

16. $5x^2 + 7x + 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(7) \pm \sqrt{(7)^2 - 4(5)(6)}}{2(5)}$$

$$= \frac{-7 \pm \sqrt{49 - 120}}{10}$$

$$= \frac{-7 \pm \sqrt{-71}}{10}$$

$$= \frac{-7 \pm \sqrt{71}i}{10}$$

17. $3x^3 - 4x^2 - 12x + 16 = 0$

$x^2(3x - 4) - 4(3x - 4) = 0$

$(x^2 - 4)(3x - 4) = 0$

$x^2 - 4 = 0 \Rightarrow x^2 = 4 = \pm 2$

$3x - 4 = 0 \Rightarrow x = \frac{4}{3}$

18. $x + \sqrt{22 - 3x} = 6$

$\sqrt{22 - 3x} = 6 - x$

$22 - 3x = (6 - x)^2$

$22 - 3x = 36 - 12x + x^2$

$x^2 - 9x + 14 = 0$

$(x - 2)(x - 7) = 0$

$x - 2 = 0 \Rightarrow x = 2$

$x - 7 = 0 \Rightarrow x = 7$, extraneous

19. $(x^2 + 6)^{2/3} = 16$

$x^2 + 6 = 16^{3/2} = 64$

$x^2 = 58$

$x = \pm\sqrt{58} \approx \pm 7.616$

20. $|8x - 1| = 21$

$8x - 1 = 21$ or $-(8x - 1) = 21$

$8x = 22$ $\quad$ $-8x = 20$

$x = \frac{11}{4}$ $\quad$ $x = -\frac{5}{2}$

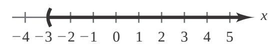
2 4 2

21. $6x - 1 > 3x - 10$

$$3x > -9$$

$$x > -3$$

$(-3, \infty)$



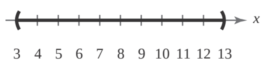
22. $2|x-8|<10$

$|x-8|<5$

$-5<x-8<5$

$3<x<13$

Solution set: $(3, 13)$



23. $6x^2+5x+1\ge 0$

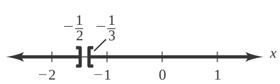
$(3x+1)(2x+1)\ge 0$

Key numbers: $x=-\frac{1}{3}, x=-\frac{1}{2}$

Test intervals: $\left(-\infty, -\frac{1}{2}\right), \left(-\frac{1}{2}, -\frac{1}{3}\right), \left(-\frac{1}{3}, \infty\right)$

Test: Is $(3x+1)(2x+1)\ge 0$?

Solution set: $\left(-\infty, -\frac{1}{2}\right], \left[-\frac{1}{3}, \infty\right)$



24. $\dfrac{8-5x}{2+3x} \le -2$

$\dfrac{8-5x}{2+3x} + 2 \le 0$

$\dfrac{8-5x+2(2+3x)}{2+3x} \le 0$

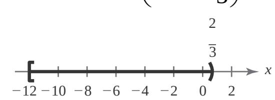
$\dfrac{x+12}{2+3x} \le 0$

Key numbers: $x=-12, x=-\frac{2}{3}$

Test intervals: $(-\infty, -12), \left(-12, -\frac{2}{3}\right), \left(-\frac{2}{3}, \infty\right)$

Test: Is $\dfrac{x+12}{2+3x} \le 0$?

Solution set: $\left(-\infty, -\frac{2}{3}\right)$



25. $-16t^2+224t>350$

$-16t^2+224t-350>0$

Key numbers: $t=\dfrac{-(224)\pm\sqrt{(224)^2-4(-16)(-350)}}{2(-16)}$

$t\approx 1.8, t\approx 12.2$

Test intervals: $(-\infty, 1.8), (1.8, 12.2), (12.2, \infty)$

Test: Is $-16t^2+224t-350>0$?

$-16t^2+224t-350>0$ on $(1.8, 12.2)$

The projectile exceeds 350 feet over the interval (1.8 seconds, 12.2 seconds).

Chapters P–2 Cumulative Test

26.

$$C = 2.485t + 30.36$$

$$85 = 2.485t + 30.36$$

$$54.64 = 2.485t$$

$$t \approx 21.99$$

In 2021, the average monthly cost of table will be $85.

14x y 1. 7x

$, x \neq 0$

5. $(x$
$-$
$2)(x^2$
$+ x -$
$3) = x^3$
$+ x^2 -$
$3x -$
$2x^2 -$
$2x + 6$

$32x^{-1}y^2 \quad 16y^5$

$= x^3 - x^2 - 5x + 6$

2. $8\sqrt{60} - 2\sqrt{135} - \sqrt{15} = 16\sqrt{15} - 6\sqrt{15} - \sqrt{\quad}$
15
$= 9\sqrt{15}$

3. $\sqrt{28x^4y^3} = 2x^2y\sqrt{7y}$

4. $4x - [2x + 5(2 - x)] = 4x - [-3x + 10$
$= -10 + 7x = 7x - 10$

6. $\dfrac{2}{x+3} - \dfrac{1}{x+1} = \dfrac{2(x+1) - (x+3)}{(x+3)(x+1)}$

$= \dfrac{x-1}{(x+3)(x+1)}$

INSTRUCTOR USE ONLY

7. $$\begin{aligned}36 - (x-4)^2 &= [6 - (x-4)][6 + (x-4)] \\ &= [6 - x + 4][6 + x - 4] \\ &= (-x + 10)(x + 2) \\ &= -(x - 10)(x + 2)\end{aligned}$$

8. $$\begin{aligned}x - 5x^2 - 6x^3 &= -x(6x^2 + 5x - 1) \\ &= -x(6x - 1)(x + 1) \\ &= x(x + 1)(1 - 6x)\end{aligned}$$

9. $$\begin{aligned}54 - 16x^3 &= 2(27 - 8x^3) \\ &= 2(3 - 2x)(9 + 6x + 4x^2)\end{aligned}$$

10. $$\text{Midpoint} = \left(\frac{(-7/2) + (5/2)}{2}, \frac{4 + (-8)}{2}\right) = \left(-\frac{1}{2}, -2\right)$$

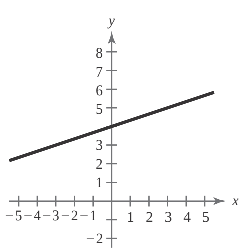
$$\begin{aligned}\text{Distance} &= \sqrt{\left(\frac{5}{2} - \left(-\frac{7}{2}\right)\right)^2 + (-8 - 4)^2} \\ &= \sqrt{36 + 144} \\ &= \sqrt{180} \\ &= 6\sqrt{5} \approx 13.42\end{aligned}$$

11. Center: $\left(-\frac{1}{2}, 8\right) = (h, k)$

Radius: $r = 4$

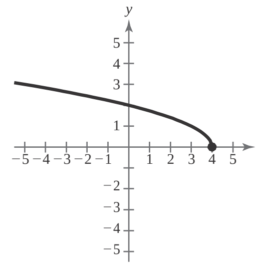
$$\left(x + \frac{1}{2}\right)^2 + (y + 8)^2 = 16$$

12. $$\begin{aligned}x - 3y + 12 &= 0 \\ -3y &= -x - 12 \\ y &= \frac{x}{3} + 4\end{aligned}$$



13. $y = x^2 - 9$

14. $y = \sqrt{4 - x}$



15. (a) $\text{Slope} = \dfrac{8 - 4}{-5 - (-1)} = -1$

$$\begin{aligned}y - 8 &= -1(x + 5) \\ y &= -x + 3 \\ x + y &= 3\end{aligned}$$

(b) Three additional points: $(-1, 4)$, $(0, 3)$, $(1, 2)$

(Answers not unique.)

16. (a) $$\begin{aligned}y - 1 &= -2\left(x + \frac{1}{2}\right) \\ y - 1 &= -2x - 1 \\ y &= -2x \\ 2x + y &= 0\end{aligned}$$

(b) Three additional points: $(0, 0)$, $(1, -2)$, $(2, -4)$

(Answers not unique.)

17. (a) Vertical line: $x = -\frac{3}{7}$ or $x + \frac{3}{7} = 0$

(b) Three additional points:

$\left(-\frac{3}{7}, 0\right)$, $\left(-\frac{3}{7}, 1\right)$, $\left(-\frac{3}{7}, 2\right)$

(Answers not unique.)

18. $6x - y = 4$ has slope $m = 6$.

(a) Parallel line has slope $m = 6$.

$$\begin{aligned}y - 3 &= 6(x - 2) \\ y &= 6x - 9\end{aligned}$$

(b) Perpendicular line has slope $m = -\dfrac{1}{6}$.

$$\begin{aligned}y - 3 &= -\tfrac{1}{6}(x - 2) \\ y &= -\tfrac{1}{6}x + \tfrac{10}{3}\end{aligned}$$

19. $f(x) = \dfrac{x}{x - 2}$

(a) (b) $f(5) = \dfrac{5}{5 - 2} = \dfrac{5}{3}$

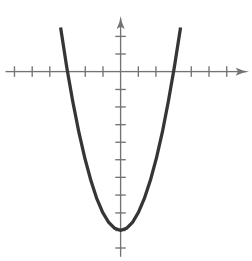
$f(2)$ is undefined. $5 + 4s$

$5 + 4s$

$-6\ -5\ -4\quad -1\quad 1\ 2\quad 4\ 5\ 6\quad x$

-2
-3
-4
-5
-6

(c) $f(5 + 4s) = \dfrac{}{(5 + 4s) - 2} = \dfrac{}{3 + 4s}$



20. $f(x) = \begin{cases} 3x - 8, & x < 0 \\ x^2 + 4, & x \ge 0 \end{cases}$

(a) $f(-8) = 3(-8) - 8 = -32$

(b) $f(0) = 0^2 + 4 = 4$

(c) $f(4) = 4^2 + 4 = 20$

21. $(-\infty, \infty)$

22. $5 + 7t \ge 0$

$7t \ge -5$

$t \ge -\frac{5}{7}$

$\left[-\frac{5}{7}, \infty\right)$

23. $9 - s^2 \ge 0$

$9 \ge s^2$

$[-3, 3]$

24. $\left(-\infty, -\frac{3}{5}\right) \cup \left(-\frac{2}{5}, \infty\right)$

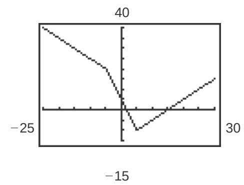
25. $g(-x) = 3(-x) - (-x)^3$

$= -3x + x^3 = -g(x)$

Odd function.

26. No, for some x-values, correspond two values of y.

27.



Decreasing on $(-\infty, 5)$, increasing on $(5, \infty)$

28. (a) $r(x) = \frac{1}{2}\sqrt{x} = \frac{1}{2}f(x)$ is a vertical shrink of f.

(b) $h(x) = \sqrt{x} + 2 = f(x) + 2$ a vertical shift two units upward of f.

(c) $g(x) = -\sqrt{x + 2} = -f(x + 2)$ is a horizontal shift two units to the left, followed by a reflection in the x-axis of f.

29. $(f + g)(x) = (x^2 + x) + (3x - 2) = x^2 + 4x - 2$

30. $(g - f)(x) = (x^2 + x)(3x - 2) - (x^2 + x) = -x^2 + 2x - 2$

31. $(g \circ f)(x) = g(f(x)) = [3(x^2 + x) - 2] = 3x^2 + 3x - 2$

32. $(fg)(x) = (x^2 + x)(3x - 2) = 3x^3 - 2x^2 + 3x^2 - 2x = 3x^3 + x^2 - 2x$

33. $f(x) = -5x + 4$ has an inverse function.

$y = -5x + 4$

$x = -5y + 4$

$x - 4 = -5y$

$y = \frac{x - 4}{-5}$

$f^{-1}(x) \quad \frac{1}{5}x + \frac{4}{5}$

36. $4x^3 - 12x^2 + 8x = 0$

$4x(x^2 - 3x + 2) = 0$

$4x(x - 2)(x - 1) = 0$

$4x = 0 \Rightarrow x = 0$

$x - 2 = 0 \Rightarrow x = 2$

$x - 1 = 0 \Rightarrow x = 1$

37.

$$\frac{x}{5} = \frac{x-3}{10}$$

$$5(x-3) = 10x$$

$$\begin{aligned} 5x - 15 &= 10x \\ -5x &= 15 \\ x &= -3 \end{aligned}$$

34. $f(x) = (x-1)^2$ is not one-to-one, so $f^{-1}(x)$ does

not exist.

35. $f(x) = \sqrt[3]{x} + 2$ has an inverse function.

$$\begin{aligned} y &= \sqrt[3]{x} + 2 \\ x &= \sqrt[3]{y} + 2 \end{aligned}$$

$$\begin{aligned} x - 2 &= \sqrt[3]{y} \\ y &= (x-2)^3 \\ f^{-1}(x) &= (x-2)^3 \end{aligned}$$

38. $|3x + 4| - 2 = 0$

$|3x + 4| = 2$

$3x + 4 = -2$ or $3x + 4 = 2$

$3x = -6$ $\quad$ $3x = -2$

$x = -2$ $\quad$ $x = -\frac{2}{3}$

39. $\sqrt{x^2 + 1} + x - 9 = 0$

$\sqrt{x^2 + 1} = -x + 9$

$\left(\sqrt{x^2 + 1}\right)^2 = (-x + 9)^2$

$x^2 + 1 = x^2 - 18x + 81$

$18x = 80$

$x = \frac{40}{9}$

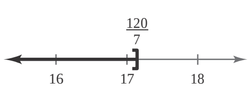
40. $\frac{x}{5} - 6 \le \frac{-x}{2} + 6$

$\frac{x}{2} + \frac{x}{5} \le 12$

$\frac{7x}{10} \le 12$

$x \le \frac{120}{7}$

$\left(-\infty, \frac{120}{7}\right]$



41. $2x^2 + x \ge 15$

$2x^2 + x - 15 \ge 0$

$(2x - 5)(x + 3) \ge 0$

Key numbers: $x = -3, \frac{5}{2}$

Test intervals: $(-\infty, -3), \left(-3, \frac{5}{2}\right), \left(\frac{5}{2}, \infty\right)$

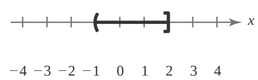
42. $|7 + 8x| > 5$

$7 + 8x > 5$ or $7 + 8x < -5$

$8x > -2$ $\quad$ $8x < -12$

$x > -\frac{1}{4}$ $\quad$ $x < -\frac{3}{2}$

$\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{1}{4}, \infty\right)$



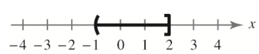
43. $\frac{2(x - 2)}{x + 1} \le 0$

Key numbers: $x = -1, 2$

Test intervals: $(-\infty, -1), (-1, 2), (2, \infty)$

Test: Is $\frac{2(x - 2)}{x + 1} \le 0$?

Solution set: $(-1, 2]$



44. $V = \frac{4}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{3}{4\pi} V}$

$= \sqrt[3]{\frac{3}{4\pi}(370.7)}$

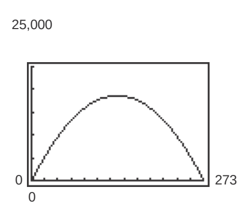
≈ 4.456 inches

45. (a) Let x and y be the lengths of the sides.

$2x + 2y = 546 \Rightarrow y = 273 - x$

$A = xy = x(273 - x)$

(b)



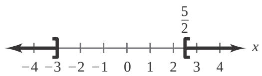
Domain: $0 < x < 273$

(c) If $A = 15{,}000$, then $x = 76.23$ or 196.77.

Dimensions in feet:

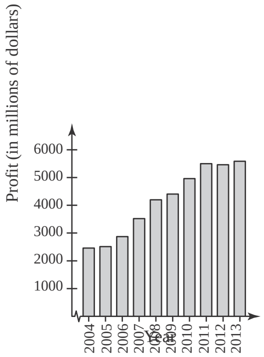
Test: Is $(2x-5)(x+3) \ge 0$?

Solution set: $(-\infty, \ -3] \cup \left[\frac{5}{2}, \infty\right)$



76.23×196.77 or 196.77×76.23

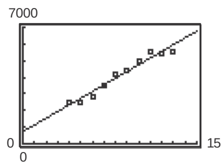
46. (a)



McDonald's profits appear to be increasing at a fairly constant rate.

(b) $P = 403.05t + 722.69$; $r \approx 0.9811$

(c)



(d) 2015: $P(15) = 403.05(15) + 722.69 = \$6768.44$ million

2018: $P(18) = 403.05(18) + 722.69 = \$7977.59$ million

(e) Answers will vary. Sample answer: Yes, if the net profits continue to follow the model, it can be used to predict future years.