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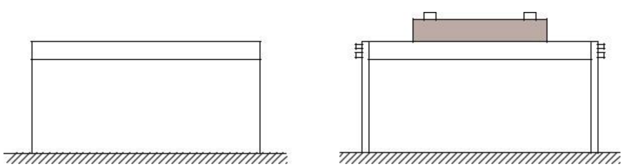
CHAPTER 2

Problem 2.1

A heavy table is supported by flat steel legs (Fig. P2.1). Its natural period in lateral vibration is 0.5 sec. When a 50-lb plate is clamped to its surface, the natural period in lateral vibration is lengthened to 0.75 sec. What are the weight and the lateral stiffness of the table?



$T_n = 0.5$ sec $T_n = 0.75$ sec **Figure P2.1**

Solution:

Given:

$$T_n = 2\pi\sqrt{\frac{m}{k}} = 0.5 \text{ sec} \qquad \text{(a)}$$

$$T_n' = 2\pi\sqrt{\frac{m + 50/g}{k}} = 0.75 \text{ sec} \qquad \text{(b)}$$

1. *Determine the weight of the table.*

Taking the ratio of Eq. (b) to Eq. (a) and squaring the result gives

$$\left(\frac{T_n'}{T_n}\right)^2 = \frac{m + 50/g}{m} \Rightarrow 1 + \frac{50}{mg} = \left(\frac{0.75}{0.5}\right)^2 = 2.25$$

or

$$mg = \frac{50}{1.25} = 40 \text{ lbs}$$

2. *Determine the lateral stiffness of the table.*

Substitute for m in Eq. (a) and solve for *k*:

$$k = 16\pi^2 m = 16\pi^2 \left(\frac{40}{386} \right) = 16.4 \text{ lbs/in.}$$

Problem 2.2

An electromagnet weighing 400 lb and suspended by a spring having a stiffness of 100 lb/in. (Fig. P2.2a) lifts 200 lb of iron scrap (Fig. P2.2b). Determine the equation describing the motion when the electric current is turned off and the scrap is dropped (Fig. P2.2c).

[redacted]

Figure P2.2

Solution:

1. *Determine the natural frequency.*

$$k = 100 \text{ lb/in.} \qquad m = \frac{400}{386} \text{ lb} - \text{sec}^2/\text{in.}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{400/386}} = 9.82 \text{ rads/sec}$$

2. *Determine initial deflection.*

Static deflection due to weight of the iron scrap

$$u(0) = \frac{200}{100} = 2 \text{ in.}$$

3. *Determine free vibration.*

$$u(t) = u(0) \cos \omega_n t = 2 \cos (9.82t)$$

Problem 2.3

A mass m is at rest, partially supported by a spring and partially by stops (Fig. P2.3). In the posi-tion shown, the spring force is $mg/2$. At time $t = 0$ the stops are rotated, suddenly releasing the mass. Determine the motion of the mass.

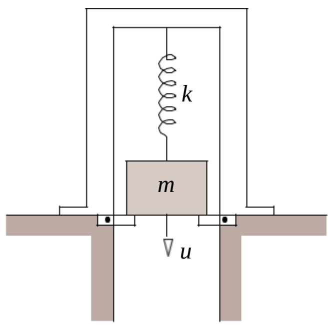
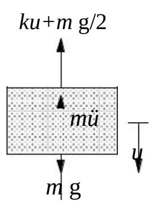


Figure P2.3

Solution:

1. *Set up equation of motion.*



$$m\ddot{u} + ku = \frac{mg}{2}$$

2. *Solve equation of motion.*

$$u(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{mg}{2k}$$

At $t = 0$, $u(0) = 0$ and $\dot{u}(0) = 0$

$$\therefore A = -\frac{mg}{2k}, \quad B = 0$$

$$u(t) = \frac{mg}{2k}(1 - \cos\omega_n t)$$

Problem 2.4

The weight of the wooden block shown in Fig. P2.4 is 10 lb and the spring stiffness is 100 lb/in. A bullet weighing 0.5 lb is fired at a speed of 60 ft/sec into the block and becomes embedded in the block. Determine the resulting motion $u(t)$ of the block.

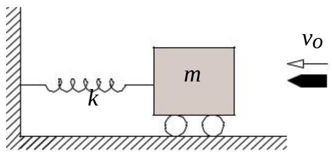
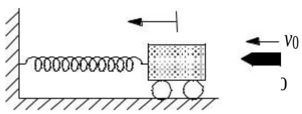


Figure P2.4

Solution:



$$m = \frac{10}{386} = 0.0259 \text{ lb} - \text{sec}^2/\text{in.}$$

$$m_0 = \frac{0.5}{386} = 1.3 \times 10^{-3} \text{ lb} - \text{sec}^2/\text{in.}$$

$$k = 100 \text{ lb/in.}$$

Conservation of momentum implies

$$m_0 v_0 = (m + m_0)\,\dot{u}(0)$$

$$\dot{u}(0) = \frac{m_0 v_0}{m + m_0} = 2.857 \text{ ft/sec} = 34.29 \text{ in./sec}$$

After the impact the system properties and initial conditions are

$$\text{Mass} = m + m_0 = 0.0272 \text{ lb} - \text{sec}^2/\text{in.}$$

$$\text{Stiffness} = k = 100 \text{ lb/in.}$$

Natural frequency:

$$\omega_n = \sqrt{\frac{k}{m + m_0}} = 60.63 \text{ rads/sec}$$

Initial conditions: $u(0) = 0$, $\dot{u}(0) = 34.29$ in./sec

The resulting motion is

$$u(t) = \frac{\dot{u}(0)}{\omega_n} \sin \omega_n t = 0.565 \sin(60.63t) \text{ in.}$$

Problem 2.5

A mass m_1 hangs from a spring k and is in static equilibrium. A second mass m_2 drops through a height h and sticks to m_1 without rebound (Fig. P2.5). Determine the subsequent motion $u(t)$ measured from the static equilibrium position of m_1 and k.

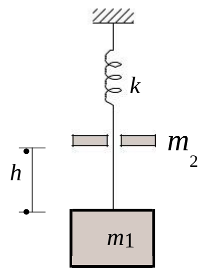
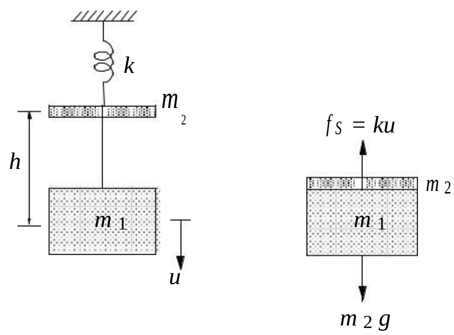


Figure P2.5

Solution:



With u measured from the static equilibrium position of m_1 and k, the equation of motion after impact is

$$(m_1 + m_2)\ddot{u} + ku = m_2 g \qquad \text{(a)}$$

The general solution is

$$u(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{m_2 g}{k} \qquad \text{(b)}$$

$$\omega_n = \sqrt{\frac{k}{m_1 + m_2}} \qquad \text{(c)}$$

The initial conditions are

$$u(0) = 0 \qquad \dot{u}(0) = \frac{m_2}{m_1 + m_2}\sqrt{2gh} \qquad \text{(d)}$$

The initial velocity in Eq. (d) was determined by conservation of momentum during impact:

$$m_2\dot{u}_2 = (m_1 + m_2)\dot{u}(0)$$

where

$$\dot{u}_2 = \sqrt{2gh}$$

Impose initial conditions to determine A and B:

$$u(0) = 0 \quad \Rightarrow A = -\frac{m_2 g}{k} \qquad \text{(e)}$$

$$\dot{u}(0) = \omega_n B \Rightarrow B = \frac{m_2}{m_1 + m_2}\frac{\sqrt{2gh}}{\omega_n} \qquad \text{(f)}$$

Substituting Eqs. (e) and (f) in Eq. (b) gives

$$u(t) = \frac{m_2 g}{k}(1 - \cos\omega_n t) + \frac{\sqrt{2gh}}{\omega_n}\frac{m_2}{m_1 + m_2}\sin\omega_n t$$

Problem 2.6

Problem 2.7

The packaging for an instrument can be modeled as shown in Fig. P2.6, in which the instrument of mass m is restrained by springs of total stiffness k inside a container; m = 10 lb/g and k = 50 lb/in. The container is accidentally dropped from a height of 3 ft above the ground. Assuming that it does not bounce on contact, determine the maximum deformation of the packaging within the box and the maximum acceleration of the instrument.

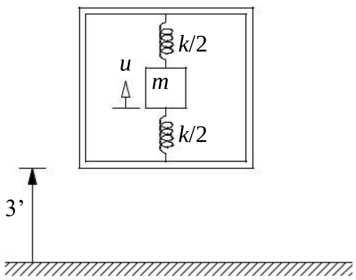


Figure P2.6

Solution:

1. *Determine deformation and velocity at impact.*

$$u(0) = \frac{mg}{k} = \frac{10}{50} = 0.2 \text{ in.}$$

$$\dot{u}(0) = -\sqrt{2gh} = -\sqrt{2(386)(36)} = -166.7 \text{ in./sec}$$

2. *Determine the natural frequency.*

$$\omega_n = \sqrt{\frac{kg}{w}} = \sqrt{\quad}$$

3. *Compute the maximum deformation.*

$$u(t) = u(0)\cos\omega_n t + \frac{\dot{u}(0)}{\omega_n}\sin\omega_n t$$

$$= (0.2)\cos 316.8t - \left(\frac{166.7}{43.93}\right)\sin 316.8t$$

$$u_o = \sqrt{[u(0)]^2 + \left[\frac{\dot{u}(0)}{\omega_n}\right]^2}$$

$$= \sqrt{0.2^2 + (-3.795)^2} = 3.8 \text{ in.}$$

4. *Compute the maximum acceleration.*

$$\ddot{u}_o = \omega_n^2 u_o = (43.93)^2\,(3.8)$$

$$= 7334 \text{ in./sec}^2 = 18.98g$$

Problem 2.8

Consider a diver weighing 200 lbs at the end of a diving board that cantilevers out 3 ft. The diver oscillates at a frequency of 2 Hz. What is the flexural rigidity *EI* of the diving board?

Solution:

Given:

$$m = \frac{200}{32.2} = 6.211 \text{ lb} - \text{sec}^2/\text{ft} \quad f_n = 2 \text{ Hz}$$

Determine *EI*:

$$k = \frac{3EI}{L^3} = \frac{3EI}{3^3} = \frac{EI}{9} \text{ lb/ft}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow 2 = \frac{1}{2\pi}\sqrt{\frac{EI}{55.90}} \Rightarrow$$

$$EI = (4\pi)^2\, 55.90 = 8827 \text{ lb} - \text{ft}^2$$

Problem 2.9

Show that the motion of a critically damped system due to initial displacement $u(0)$ and initial velocity $\dot{u}(0)$ is

$$u(t) = \{u(0) + [\dot{u}(0) + \omega_n u(0)]\, t\}\, e^{-\omega_n t}$$

Solution:

Equation of motion:

$$m\ddot{u} + c\dot{u} + ku = 0 \qquad \text{(a)}$$

Dividing Eq. (a) through by m gives

$$\ddot{u} + 2\zeta\omega_n \dot{u} + \omega_n^2 u = 0 \qquad \text{(b)}$$

where $\zeta = 1$.

Equation (b) thus reads

$$\ddot{u} + 2\omega_n \dot{u} + \omega_n^2 u = 0 \qquad \text{(c)}$$

Assume a solution of the form $u(t) = e^{st}$.
Substituting this solution into Eq. (c) yields

$$(s^2 + 2\omega_n s + \omega_n^2)\, e^{st} = 0$$

Because e^{st} is never zero, the quantity within parentheses must be zero:

$$s^2 + 2\omega_n s + \omega_n^2 = 0$$

or

$$s = \frac{-2\omega_n \pm \sqrt{(2\omega_n)^2 - 4\omega_n^2}}{2} = -\omega_n$$

(double root)

The general solution has the following form:

$$u(t) = A_1 e^{-\omega_n t} + A_2 t e^{-\omega_n t} \qquad \text{(d)}$$

where the constants A_1 and A_2 are to be determined from the initial conditions: $u(0)$ and $\dot{u}(0)$.

Evaluate Eq. (d) at $t = 0$:

$$u(0) = A_1 \Rightarrow A_1 = u(0) \qquad \text{(e)}$$

Differentiating Eq. (d) with respect to t gives

$$\dot{u}(t) = -\omega_n A_1 e^{-\omega_n t} + A_2(1 - \omega_n t)e^{-\omega_n t} \qquad \text{(f)}$$

Evaluate Eq. (f) at $t = 0$:

$$\dot{u}(0) = -\omega_n A_1 + A_2(1 - 0)$$

$$\therefore A_2 = \dot{u}(0) + \omega_n A_1 = \dot{u}(0) + \omega_n u(0) \qquad \text{(g)}$$

Substituting Eqs. (e) and (g) for A_1 and A_2 in Eq. (d) gives

$$u(t) = \{u(0) + [\dot{u}(0) + \omega_n u(0)]\, t\} e^{-\omega_n t} \qquad \text{(h)}$$

Problem 2.

Show that the motion of an overcritically damped system due to initial displacement $u(0)$ and initial velocity $\dot{u}(0)$ is

$$u(t) = e^{-\zeta\omega_n t}\left(A_1 e^{-\omega_D t} + A_2 e^{\omega_D t}\right)$$

where $\omega_D = \omega_n\sqrt{\zeta^2 - 1}$ and

$$A_1 = \frac{-\dot{u}(0) + \left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)}{2\omega_D}$$

$$A_2 = \frac{\dot{u}(0) + \left(\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)}{2\omega_D}$$

Solution:

Equation of motion:

$$m\ddot{u} + c\dot{u} + ku = 0 \qquad \text{(a)}$$

Dividing Eq. (a) through by m gives

$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = 0 \qquad \text{(b)}$$

where $\zeta > 1$.

Assume a solution of the form $u(t) = e^{st}$. Substituting this solution into Eq. (b) yields

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)e^{st} = 0$$

Because e^{st} is never zero, the quantity within parentheses must be zero:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

or

$$s = \frac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2} = \left(-\zeta \pm \sqrt{\zeta^2-1}\right)\omega_n$$

The general solution has the following form:

Differentiating Eq. (c) with respect to t gives

$$\dot{u}(t) = A_1\left(-\zeta - \sqrt{\zeta^2-1}\right)\omega_n \exp\left[\left(-\zeta-\sqrt{\zeta^2-1}\right)\omega_n t\right] + A_2\left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n \exp\left[\left(-\zeta+\sqrt{\zeta^2-1}\right)\omega_n t\right] \qquad \text{(e)}$$

Evaluate Eq. (e) at $t = 0$:

$$\dot{u}(0) = A_1\left(-\zeta - \sqrt{\zeta^2-1}\right)\omega_n + A_2\left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n$$
$$= [u(0) - A_2]\left(-\zeta - \sqrt{\zeta^2-1}\right)\omega_n + A_2\left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n$$

or

$$A_2\omega_n\left[-\zeta + \sqrt{\zeta^2-1} + \zeta + \sqrt{\zeta^2-1}\right] = \dot{u}(0) + \left(\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)$$

or

$$A_2 = \frac{\dot{u}(0) + \left(\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)}{2\sqrt{\zeta^2-1}\,\omega_n} \qquad \text{(f)}$$

Substituting Eq. (f) in Eq. (d) gives

Problem 2.

$$u(t) = A_1 \exp\left[\left(-\zeta - \sqrt{\zeta^2-1}\right)\omega_n t\right] + A_2 \exp\left[\left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n t\right] \tag{c}$$

where the constants A_1 and A_2 are to be determined from the initial conditions: $u(0)$ and $\dot{u}(0)$.

Evaluate Eq. (c) at $t = 0$:

$$u(0) = A_1 + A_2 \Rightarrow A_1 + A_2 = u(0) \tag{d}$$

$$A_1 = u(0) - \frac{\dot{u}(0) + \left(\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)}{2\sqrt{\zeta^2-1}\,\omega_n}$$

$$= \frac{2\sqrt{\zeta^2-1}\,\omega_n u(0) - \dot{u}(0) - \left(\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)}{2\sqrt{\zeta^2-1}\,\omega_n}$$

$$= \frac{-\dot{u}(0) + \left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)}{2\sqrt{\zeta^2-1}\,\omega_n} \tag{g}$$

The solution, Eq. (c), now reads:

$$u(t) = e^{-\zeta\omega_n t}\left(A_1 e^{-\omega'_D t} + A_2 e^{\omega'_D t}\right)$$

where

$$\omega'_D = \sqrt{\zeta^2 - 1}\,\omega_n$$

$$A_1 = \frac{-\dot{u}(0) + \left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n u(0)}{2\omega'_D}$$

$$A_2 = \frac{\dot{u}(0) + \left(\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n u(0)}{2\omega'_D}$$

Problem 2.10

Derive the equation for the displacement response of a viscously damped SDF system due to initial velocity $\dot{u}(0)$ for three cases: **(a)** underdamped systems; **(b)** critically damped systems; and **(c)** overdamped systems. Plot $u(t) \div \dot{u}(0)/\omega_n$ against t/T_n for $\zeta = 0.1$, 1, and 2.

Solution:

Equation of motion:

$$\ddot{u} + 2\zeta\omega_n \dot{u} + \omega_n^2 u = 0 \qquad \text{(a)}$$

Assume a solution of the form

$$u(t) = e^{st}$$

Substituting this solution into Eq. (a) yields:

$$\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right) e^{st} = 0$$

Because e^{st} is never zero

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \qquad \text{(b)}$$

The roots of this characteristic equation depend on ζ.

(a) Underdamped Systems, $\zeta<1$

The two roots of Eq. (b) are

$$s_{1,2} = \omega_n\left(-\zeta \pm i\sqrt{1-\zeta^2}\right) \qquad \text{(c)}$$

Hence the general solution is

$$u(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

which after substituting in Eq. (c) becomes

$$u(t) = e^{-\zeta\omega_n t}\left(A_1 e^{i\omega_D t} + A_2 e^{-i\omega_D t}\right) \qquad \text{(d)}$$

where

$$\omega_D = \omega_n\sqrt{1-\zeta^2} \qquad \text{(e)}$$

Rewrite Eq. (d) in terms of trigonometric functions:

$$u(t) = e^{-\zeta\omega_n t}\left(A\cos\omega_D t + B\sin\omega_D t\right) \qquad \text{(f)}$$

Determine A and B from initial conditions $u(0) = 0$ and $\dot{u}(0)$:

Substituting A and B into Eq. (f) gives

$$u(t) = \frac{\dot{u}(0)}{\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\, t\right) \qquad \text{(g)}$$

(b) Critically Damped Systems, $\zeta = 1$

The roots of the characteristic equation [Eq. (b)] are:

$$s_1 = -\omega_n \qquad s_2 = -\omega_n \qquad \text{(h)}$$

The general solution is

$$u(t) = A_1 e^{-\omega_n t} + A_2 t e^{-\omega_n t} \qquad \text{(i)}$$

Determined from the initial conditions $u(0) = 0$ and $\dot{u}(0)$:

$$A_1 = 0 \qquad A_2 = \dot{u}(0) \qquad \text{(j)}$$

Substituting in Eq. (i) gives

$$u(t) = \dot{u}(0)\, t e^{-\omega_n t} \qquad \text{(k)}$$

(c) Overdamped Systems, $\zeta>1$

The roots of the characteristic equation [Eq. (b)] are:

$$s_{1,2} = \omega_n\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right) \qquad \text{(l)}$$

The general solution is:

$$u(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \text{(m)}$$

which after substituting Eq. (l) becomes

$$u(t) = A_1 e^{\left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n t} + A_2 e^{\left(-\zeta - \sqrt{\zeta^2-1}\right)\omega_n t} \qquad \text{(n)}$$

Determined from the initial conditions $u(0) = 0$ and $\dot{u}(0)$:

$$-A_1 = A_2 = \frac{\dot{u}(0)}{2\omega_n\sqrt{\zeta^2 - 1}} \qquad \text{(o)}$$

$$A = 0 \qquad B = \dot{u}(0)$$

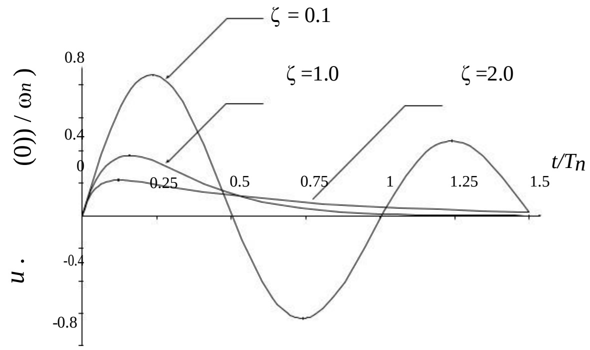
Substituting in Eq. (n) gives

$$u(t) = \frac{\dot{u}(0)\, e^{-\zeta\omega_n t}}{2\omega_n\sqrt{\zeta^2 - 1}}\left(e^{\omega_n t\sqrt{\zeta^2-1}} - e^{-\omega_n t\sqrt{\zeta^2-1}}\right) \qquad \text{(p)}$$

ω_D

(d) Response Plots

Plot Eq. (g) with $\zeta = 0.1$; Eq. (k), which is for $\zeta = 1$; and Eq. (p) with $\zeta = 2$.



Problem 2.11

For a system with damping ratio ζ, determine the number of free vibration cycles required to reduce the displacement amplitude to 10% of the initial amplitude; the initial velocity is zero.

Solution:

$$\frac{1}{j}\ln\left(\frac{u_1}{u_{j+1}}\right) \approx 2\pi\zeta \Rightarrow \frac{1}{j_{10\%}}\ln\left(\frac{1}{0.1}\right) \approx 2\pi\zeta$$

$$\therefore j_{10\%} \approx \ln(10)/2\pi\zeta \approx 0.366/\zeta$$

Problem 2.12

What is the ratio of successive amplitudes of vibration if the viscous damping ratio is known to be **(a)** $\zeta = 0.01$, **(b)** $\zeta = 0.05$, or **(c)** $\zeta = 0.25$?

Solution:

$$\frac{u_i}{u_{i+1}} = \exp\left(\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

(a) $\zeta = 0.01$: $\quad \dfrac{u_i}{u_{i+1}} = 1.065$

(b) $\zeta = 0.05$: $\quad \dfrac{u_i}{u_{i+1}} = 1.37$

(c) $\zeta = 0.25$: $\quad \dfrac{u_i}{u_{i+1}} = 5.06$

Problem 2.13

The supporting system of the tank of Example 2.6 is enlarged with the objective of increasing its seismic resistance. The lateral stiffness of the modified system is double that of the original system. If the damp-ing coefficient is unaffected (this may not be a realistic assumption), for the modified tank determine
(a) the natural period of vibration T_n, and **(b)** the damping ratio ζ.

Solution:

Given:

w = 20.03 kips (empty); m = 0.0519 kip-sec^2/in.

k = 2 (8.2) = 16.4 kips/in.

c = 0.0359 kip-sec/in.

$$\text{(a) } T_n = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.0519}{16.4}} = 0.353 \text{ sec}$$

$$\text{(b) } \zeta = \frac{c}{2\sqrt{k\,m}} = \frac{0.0359}{2\sqrt{(16.4)(0.0519)}} = 0.0194$$

$$= 1.94\%$$

Problem 2.14

The vertical suspension system of an automobile is idealized as a viscously damped SDF system. Under the 3000-lb weight of the car, the suspension system deflects 2 in. The suspension is designed to be critically damped.

(a) Calculate the damping and stiffness coefficients of the suspension.
(b) With four 160-lb passengers in the car, what is the effective damping ratio?
(c) Calculate the natural frequency of damped vibration for case (b).

Solution:

(a) The stiffness coefficient is $k = \frac{3000}{2} = 1500$ lb/in.

The damping coefficient is

$$c = c_{cr} = 2\sqrt{km}$$

$$c = 2\sqrt{1500\,\frac{3000}{386}} = 215.9 \text{ lb - sec / in.}$$

(b) With passengers the weight is w = 3640 lb. The damping ratio is

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{215.9}{2\sqrt{1500\,\frac{3640}{386}}} = 0.908$$

(c) The natural vibration frequency for case (b) is

$$\omega_D = \omega_n\sqrt{1-\zeta^2}$$

$$= \sqrt{\frac{1500}{3640/386}}\sqrt{1-(0.908)^2}$$

$$= 12.61 \times 0.419$$

$$= 5.28 \text{ rads / sec}$$

Problem 2.15

The stiffness and damping properties of a mass–spring– damper system are to be determined by a free vibration test; the mass is given as m = 0.1 lb-sec^2/in. In this test, the mass is displaced 1 in. by a hydraulic jack and then suddenly released. At the end of 20 complete cycles, the time is 3 sec and the amplitude is 0.2 in. Determine the stiffness and damping coefficients.

Solution:

1. *Determine ζ and ω_n.*

$$\zeta \approx \frac{1}{2\pi j}\ln\left(\frac{u_1}{u_{j+1}}\right) = \frac{1}{2\pi(20)}\ln\left(\frac{1}{0.2}\right) = 0.0128 = 1.28\%$$

Therefore the assumption of small damping implicit in the above equation is valid.

$$T_D = \frac{3}{20} = 0.15 \text{ sec} ; \quad T_n \approx T_D = 0.15 \text{ sec} ;$$

$$\omega_n = \frac{2\pi}{0.15} = 41.89 \text{ rads/sec}$$

2. *Determine stiffness coefficient.*

$$k = \omega_n^2 m = (41.89)^2\, 0.1 = 175.5 \text{ lbs/in.}$$

3. *Determine damping coefficient.*

$$c_{cr} = 2m\omega_n = 2\,(0.1)\,(41.89) = 8.377 \text{ lb} - \text{sec/in.}$$

$$c = \zeta c_{cr} = 0.0128\ (8.377) = 0.107 \text{ lb} - \text{sec/in.}$$

Problem 2.16

A machine weighing 250 lbs is mounted on a supporting system consisting of four springs and four dampers. The vertical deflection of the supporting system under the weight of the machine is measured as 0.8 in. The dampers are designed to reduce the amplitude of vertical vibration to one-eighth of the initial amplitude after two complete cycles of free vibration. Find the following properties of the system:
(a) undamped natural frequency, **(b)** damping ratio, and **(c)** damped natural frequency. Comment on the effect of damping on the natural frequency.

Solution:

(a) $k = \dfrac{250}{0.8} = 312.5 \text{ lbs/in.}$

$$m = \frac{w}{g} = \frac{250}{386} = 0.647 \text{ lb} - \text{sec}^2/\text{in.}$$

$$\omega_n = \sqrt{\frac{k}{m}} = 21.98 \text{ rads/sec}$$

(b) Assuming small damping,

$$\ln\left(\frac{u_1}{u_{j+1}}\right) \approx 2j\pi\zeta \Rightarrow$$

$$\ln\left(\frac{u_0}{u_0/8}\right) = \ln(8) \approx 2(2)\pi\zeta \Rightarrow \zeta = 0.165$$

This value of ζ may be too large for small damping assumption; therefore, we use the exact equation:

$$\ln\left(\frac{u_1}{u_{j+1}}\right) = \frac{2j\pi\zeta}{\sqrt{1-\zeta^2}}$$

or,

$$\ln(8) = \frac{2(2)\pi\zeta}{\sqrt{1-\zeta^2}} \Rightarrow \frac{\zeta}{\sqrt{1-\zeta^2}} = 0.165 \Rightarrow$$

$$\zeta^2 = 0.027(1-\zeta^2) \Rightarrow$$

$$\zeta = \sqrt{0.0267} = 0.163$$

(c) $\omega_D = \omega_n\sqrt{1-\zeta^2} = 21.69 \text{ rads/sec}$

Damping decreases the natural frequency.

Problem 2.17

Determine the natural vibration period and damping ratio of the aluminum frame model (Fig. 1.1.4a) from the acceleration record of its free vibration shown in Fig. 1.1.4b.

Solution:

Reading values directly from Fig. 1.1.4b:

Peak	Time, t_i (sec)	Peak, $\ddot{u}_i$ (g)
1	0.80	0.78
31	7.84	0.50

$$T_D = \frac{7.84 - 0.80}{30} = 0.235 \text{ sec}$$

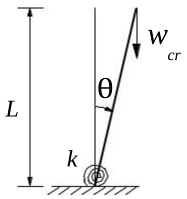
$$\zeta = \frac{1}{2\pi(30)} \ln\left(\frac{0.78g}{0.50g}\right) = 0.00236 = 0.236\%$$

Problem 2.18

Show that the natural vibration frequency of the system in Fig. E1.6a is $\omega_n = \omega_n (1 - w/w_{cr})^{1/2}$, where ω_n is the natural vibration frequency computed neglecting the action of gravity, and w_{cr} is the buckling weight.

Solution:

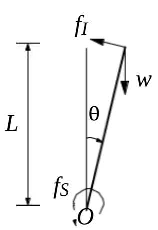
1. *Determine buckling load.*



$$w_{cr} (L\,\theta) = k\,\theta$$

$$w_{cr} = \frac{k}{L}$$

2. *Draw free-body diagram and set up equilibrium equation.*



$$\sum M_O = 0 \Rightarrow \quad f_I L + f_S = w L\theta \qquad \text{(a)}$$

where

$$f_I = \frac{w}{g} L^2 \ddot{\theta} \qquad f_S = k\,\theta \qquad \text{(b)}$$

Substituting Eq. (b) in Eq. (a) gives

$$\frac{w}{g} L^2 \ddot{\theta} + (k - wL)\,\theta = 0 \qquad \text{(c)}$$

3. *Compute natural frequency.*

$$\omega_n' = \sqrt{\frac{k - wL}{(w/g)\,L^2}} = \sqrt{\frac{k}{(w/g)\,L^2}\left(1 - \frac{wL}{k}\right)}$$

or

$$\omega'_n = \omega_n \sqrt{1 - \frac{w}{w_{cr}}} \qquad \text{(d)}$$

Problem 2.19

An impulsive force applied to the roof slab of the building of Example 2.8 gives it an initial velocity of 20 in./sec to the right. How far to the right will the slab move? What is the maximum displacement of the slab on its return swing to the left?

Solution:

For motion of the building from left to right, the governing equation is

$$m\ddot{u} + ku = -F \tag{a}$$

for which the solution is

$$u(t) = A_2 \cos \omega_n t + B_2 \sin \omega_n t - u_F \tag{b}$$

With initial velocity of $\dot{u}(0)$ and initial displacement $u(0) = 0$, the solution of Eq. (b) is

$$u(t) = \frac{\dot{u}(0)}{\omega_n} \sin \omega_n t + u_F (\cos \omega_n t - 1) \tag{c}$$

$$\dot{u}(t) = \dot{u}(0) \cos \omega_n t - u_F \omega_n \sin \omega_n t \tag{d}$$

At the extreme right, $\dot{u}(t) = 0$; hence from Eq. (d)

$$\tan \omega_n t = \frac{\dot{u}(0)}{\omega_n} \frac{1}{u_F}. \tag{e}$$

Substituting $\omega_n = 4\pi$, $u_F = 0.15$ in. and $\dot{u}(0) =$ 20 in./sec in Eq. (e) gives

$$\tan \omega_n t = \frac{20}{4\pi} \frac{1}{0.15} = 10.61$$

or

$$\sin \omega_n t = 0.9956; \quad \cos \omega_n t = 0.0938$$

Substituting in Eq. (c) gives the displacement to the right:

$$u = \frac{20}{4\pi}(0.9956) + 0.15(0.0938 - 1) = 1.449 \text{ in.}$$

After half a cycle of motion the amplitude decreases by

$$2u_F = 2 \times 0.15 = 0.3 \text{ in.}$$

Maximum displacement on the return swing is

$$u = 1.449 - 0.3 = 1.149 \text{ in.}$$

Problem 2.20

An SDF system consisting of a weight, spring, and friction device is shown in Fig. P2.20. This device slips at a force equal to 10% of the weight, and the natural vibration period of the system is 0.25 sec. If this system is given an initial displacement of 2 in. and released, what will be the displacement amplitude after six cycles? In how many cycles will the system come to rest?

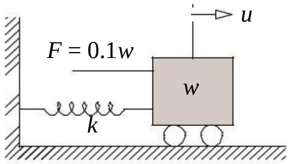


Figure P2.20

Solution:

Given:

$F = 0.1w$, $T_n = 0.25$ sec

$$u_F = \frac{F}{k} = \frac{0.1w}{k} = \frac{0.1mg}{k} = \frac{0.1g}{\omega_n^2} = \frac{0.1g}{(2\pi/T_n)^2}$$

$$= \frac{0.1g}{(8\pi)^2} = 0.061 \text{ in.}$$

The reduction in displacement amplitude per cycle is

$4u_F = 0.244$ in.

The displacement amplitude after 6 cycles is

$2.0 - 6\,(0.244) = 2.0 - 1.464 = 0.536$ in.

Motion stops at the end of the half cycle for which the displacement amplitude is less than u_F. Displacement amplitude at the end of the 7th cycle is 0.536 – 0.244 = 0.292 in.; at the end of the 8th cycle it is 0.292 – 0.244 = 0.048 in.; which is less than u_F. Therefore, the motion stops after 8 cycles.