Chapter 2 Solutions

Section 2.1 Introduction

2.1 Current source

2.2 Voltage source

2.3 Resistor

2.4 Capacitor

2.5 Inductor

Section 2.2 Charge and Current

2.6 b)

The current direction is designated as the direction of the movement of positive charges.

2.7 The relationship of charge and current is

$$q(t) = \int_{t_0}^{t} i(t)dt + q(t_0)$$

so

$$q(t) = \int_{t_0}^{t} 2\sin(10\pi t)dt + q(t_0)$$

$$q(t) = \left[-\frac{2}{10\pi}\cos(10\pi t)\right]_{t_0}^{t} + q(t_0)$$

2.8 The coulomb of one electron is denoted by e and

$$q(t) = \int_{t_0}^{t} i(t)dt + q(t_0)$$

So

$$n(t) = q(t)/e = \frac{1}{e}\int_{0}^{t} 12t\,dt + q(t_0)$$

If $t_0 = 0$ and $q(t_0) = 0$,

$$n(t)=\frac{6}{e}t^{2}$$

2.9

$$q(t)=\int_{t} i\,dt$$

$$q(t)=\int_{0} 5dt$$

$$q(t)=5t$$

2.10

$$q(t)=_{0}^{5}\left[5t\right]=5(5)-5(0)=25 \text{ Coulombs}$$

2.11 Using the definition of current-charge relationship, the equation can be rewritten as:

$$i=\frac{dq}{\ }=\frac{-n}{\ }e$$

Thus, the current flow within t_1 and t_2 time interval is,

$$i=\frac{(5.75-2)\times 10^{19}(-1.6\times 10^{-19})}{2}=-3A$$

The negative sign shows the current flow in the opposite direction with respect to the electric charge.

2.12 Assuming the area of the metal surface is S, The mass of the nickel with depth d = 0.15mm is

$$m = \rho \times d \times S$$

Meanwhile, using the electro-chemical equivalent, the mass of the nickel can be expressed as

$$m = k \times I \times t$$

where $I = \sigma \times S$.

Equating the two expressions of the mass, the coating time is found:

$$t = \rho \times d / \sigma = 1.24 \times 10^{5} \text{ s} \approx 34.4 \text{ hour}$$

Section 2.3 Voltage

2.13 By the definition of voltage, when a positive charge moves from high voltage to low voltage, its potential energy decreases.
So a is "+", b is "-". In other words, u_{ab}=1V.

2.14 The current $i(t)$ is defined as:

$$i(t)=\begin{cases} 3 & 0 < t \leq 1 \\ 0 & elsewhere \end{cases}$$

Therefore, the charge is

$$g = \int_0^1 3dt = 3 \text{ C}$$

The energy in Joules is given by:

$$J = V \times C = 5 \times 3 = 15J$$

2.15

$$1 \text{ electron} = -1.6 \times 10^{-19} \text{ Coulombs.}$$

Therefore, there are 6.25×10^{18} electrons in a coulomb.

$$\text{Coulombs of } 5 \times 10^{16} \text{ electrons} = \frac{5 \times 10^{16}}{6.25 \times 10^{18}} = 8 \times 10^{-3}$$

Therefore, the voltage is

$$V = \frac{J}{C} = \frac{15}{8 \times 10^{-3}} = 1875 \text{ V}$$

2.16

$$q(t) = \int_0^1 2\sin\left(\frac{3}{2}\pi t\right)dt$$

$$= \left[-\frac{4}{3\pi}\cos\left(\frac{3\pi}{2}t\right)\right]_0^1 = 0.4244C$$

$$J = V \times C = 5 \times 0.4244 = 2.122\text{J}$$

2.17

$$q = \frac{20J}{2V} = 10C$$

$$i = \frac{dq}{dt} = \frac{10C}{4s} = 2.5A$$

Section 2.4 Respective Direction of Voltage and Current 2.18 True

2.19 False

2.20 True

Section 2.5 Kirchoff's Current Law

2.21 According to KCL

$$0 = 7 ! i_2 ! 3$$

Therefore,

$$i_2 = 4A$$

2.22 Notice that

$$0 = 3 ! 2 +$$

Therefore,

$$i_4 \; i_4 = !1A$$

It can be seen that

$$i_5 = !2A$$

i_6 can now be found.

$$0 = 5 + 2 + 1!$$

$$i_6 \; i_6 = 8A$$

2.23

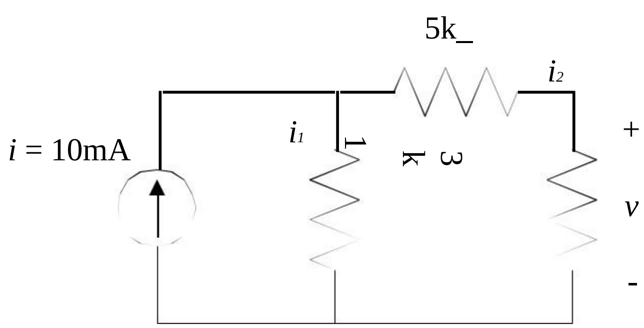

Figure S2.23: Circuit for Problem 2.23.

According to KCL,

$$10mA = i_1 + i_2$$

and

$$2ki_1 = 18ki_2$$

Therefore,

$$i_1 = 9mA \,, \; i_2 \; = 1mA$$
$$V = I ! R = 13k !1mA = 13V$$

2.24 Using the KCL method, we have,

$$i_1 = i_2 + i_3$$

Also, at the other end of the node, we have,

$$i_4 = i_2 + i_3$$

Then, combine these two equations we obtain,

$$i_1 = i_4$$

2.25 The algebraic sum of currents flowing into the node in Figure 2.11 is

$$i_1 + i_2 - i_3 - i_4$$

where i_1 and i_2 flow in, and i_3 and i_4 flow out

As current is defined as the rate of variation of the charge,

$$i_1 + i_2 - i_3 - i_4 = dq/dt$$

By law of the conservation of charge, charges cannot be stored in the node and charges can neither be destroyed nor created. Therefore the variation rate of the charge is zero and further the KCL law holds,

$$i_1 + i_2 - i_3 - i_4 = 0$$

2.26 The resistance across the voltage source is

$$R = 20 + 40\,//120 = 20 + \frac{40 \times 120}{40 + 120} = 50\Omega$$

So the current flowing through the voltage source is

$$I = \frac{V}{R} = \frac{5V}{50\Omega} = 0.1A$$

Therefore,

$$i = \frac{40}{160} \times 0.1 = 25mA$$

2.27 By KCL,

$$0 = \frac{V - 3}{5} + \frac{V}{3} - 2I_x$$

Note that,

$$I_x = \frac{V}{3} \Rightarrow V = 3I_x$$

Therefore,

$$0 = \frac{3I_x - 3}{5} + I_x - 2I_x$$

$$0 = 3I_x - 3 - 5I_x$$

$$3 = !2I_x \ \forall\ I_x = !1.5\text{A}$$

2.28 The voltage across node C and D (node C is positive) is $U_2 - U_1$, the voltage across node C and B is $U_{AB} - U_1$, and the voltage across node D and B is $U_{AB} - U_2$ The KCL equation for node C is

$$U_1 /R = (U_2 - U_1) /R + (U_{AB} - U_1) /2R$$

which is

$$5U_1 - 2U_2 = U_{AB}$$

The KCL equation for node D is

$$U_2 /2R + (U_2 - U_1) /R = (U_{AB} - U_2) /R$$

which is

$$5U_2 - 2U_1 = 2U_{AB}$$

Solving U_2 and U_1 , $U_1 = 9U_{AB} /21$, $U_2 = 12U_{AB} /21$

Thus, by the KCL equation at node A

$$I_{AB} = U_1 /R + U_2 /2R$$

$$R_{AB} = U_{AB}/I_{AB} = 7R /5$$

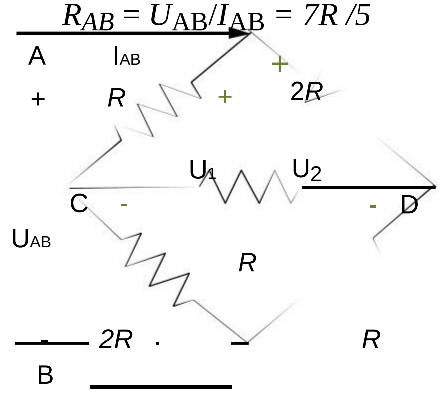

Figure S2.28: Circuit for Problem 2.28.

Section 2.6 Kirchoff's Voltage Law

2.29 By KVL,

$$0 = !v_A \ ! \ v_B \ ! \ v_C + v_D$$

$$0 = !v_A \ ! \ 2 + 6 + 6$$

Then

$$v_A = 10V$$

2.30 Using KVL,

$$0 = !12 + v_R \ ! \ 6 + 14$$

Then

$$v_R = 4 \text{ V}$$

2.31 Using KVL,

$$0 = !v_D + 3 + 5$$

Then

$$v_D = 8 \text{ V}$$

Using KVL,

$$0 = !8 ! 2 + v_C + 8$$

Then

$$v_C = 2 \text{ V}$$

2.32 Using KVL,

$$0 = !9 + 4 ! v_C$$

Then

$$v_C = !5 \text{ V}$$

Using KVL,

$$0 = !5 + 3 ! v_E ! 5 ! 6$$

Then

$$v_E = !13\text{V}$$

2.33 Using KVL,

$$0 = !v_D + 4 ! 3$$

Then

$$v_D = 1\text{V}$$

Using KVL,

$$0 = 1 + 3 + 1 + v_C$$

Then

$$v_C = !5 \text{ V}$$

Using KVL,

$$5 + 3 ! v_E = 0$$

Then

$$v_E = 8 \text{ V}$$

2.34 Using KVL,

$$0 = 7 ! v_E + 1$$

Then

$$v_E = 8 \text{ V}$$

Using KVL,

$$0 = !v_B + 5 + 8$$

Then

$$v_B = 13 \text{ V}$$

Using KVL,

$$0 = 5 ! v_D ! 7$$

Then

$$v_D = !2 \text{ V}$$

Using KVL

$$0 = v_A + 13 \; ! \; 2$$

Then

$$v_A = !11 \text{ V}$$

2.35 Using KVL,

$$10 = V_H + 2 ! V_H = 8V$$
$$2 + V_H = 5 + V_I \; ! V_I = 5V$$
$$V_C + 4 = 6 ! V_C = 2V$$
$$5 + V_D = 4 + V_C \; ! V_D = 1V$$

2.36

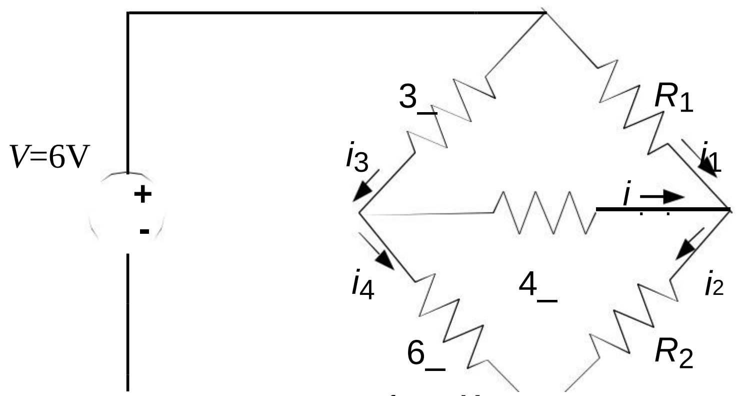

Figure S2.36: Circuit for Problem 2.36.

a) $R_1 = 6\Omega$, $R_2 = 3\Omega$
By KCL,

$$i_1 + i = i_2$$
$$i_4 + i = i_3$$

Applying KVL on the loop made by 6V source, R_1 and R_2, $6i_1 + 3i_2 = 6$

Applying KVL on the loop made by 6V source, 3Ω resistor and 6Ω resistor,

$$3i_3 + 6i_4 = 6$$

Applying KVL on the loop made by 3Ω resistor, 4Ω resistor and R_1, $3i_3 + 4i = 6i_1$

Now we have 5 equations for 5 variables and i can be solved using Cramer's rule or variable elimination. Then we get

$$i = 1/4\text{A}, \; i_1 = 7/12\text{A}, \; i_2 = 5/6\text{A}, \; i_3 = 5/6\text{A}, \; i_4 = 7/12\text{A}$$

b) $i = 0 \Rightarrow i_1 = i_2, \; i_3 = i_4.$

Moreover, the voltage across the 4Ω resistor is zero. This also leads to the voltage across R_1 equals to the voltage across the 3Ω resistor and the voltage across R_2 equals to the voltage across the 6Ω resistor,

$$R_1 i_1 = 3 i_3$$
$$R_2 i_2 = 6 i_4$$

Therefore,

$$R_1/R_2 = 3/6 = 1/2$$

2.37 The current from node A to node C is I_{AB} - I_1, the current from node D to node B is I_1 - I_2, and the current from node C to node B is I_{AB} - I_1 + I_2.
Then the KVL equation for loop ADC is

$$I_1 2R + I_2 R = (I_{AB} - I_1)R$$

which is

$$3I_1 + I_2 = I_{AB}$$

The KVL equation for loop DBC is

$$(I_1 - I_2)R = I_2 R + (I_{AB} - I_1 + I_2)2R$$

which is

$$3I_1 - 4I_2 = 2I_{AB}$$

Solving I_1 and I_2, $I_1 = 2I_{AB}/5$, $I_2 = -I_{AB}/5$
Thus,

$$U_{AB} = I_1 2R + (I_1 - I_2)R = 7I_{AB}R/5$$
$$R_{AB} = U_{AB}/I_{AB} = 7R/5$$

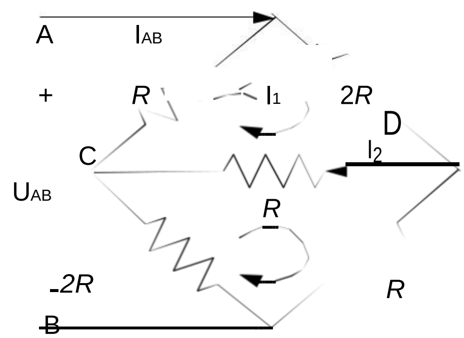

Figure S2.37: Circuit for Problem 2.37.

Section 2.7 Ohm's Law

2.38 By KCL, the current through the 4Ω resistor is 3A. Using Ohm's law,

$$V = I!R = 4!3 = 12V$$

2.39 By KVL , the voltage across the resistor is 5V.

Using Ohm's law:

$$R=\frac{V}{I}=\frac{5}{2}=2.5!$$

2.40 Using Ohm's law:

$$I=\frac{V}{R}=\frac{15}{5}=3A$$

2.41 The resistivity of copper is,

$$\exists = 1.72\ \#10^{!8}\ \forall\ !\ m$$

The resistance of the copper wire is,

$$R=\frac{\exists L}{A}=\frac{1.72\ \forall 10^{\#8}\ \forall\ 0.5}{1.2286\ \forall 10^{\#10}}=70!$$

Using the definition of Ohm's Law, we can obtain the maximum allowable voltage, which is:

$$V_{max}=I_{max}\,R=1!\ 70=70V$$

2.42 Using Ohm's law, the resistance of the tissue is

$$R=\frac{10}{12.43\forall 10^{\#6}}=804.5k!$$

Because,

$$R=\frac{\tilde{n}L}{A}$$

Therefore,

$$804.5k\grave{U}=\frac{175\grave{U}\ \#m\ \forall L}{3.468\ \forall 10^{!3}\ m^{2}}$$

$$L=15.94\text{ m}$$

2.43 The resistance across the voltage source is

$$R=R_1+R_2//R_3=R_1+\frac{R_2!R_3}{R_2+R_3}$$

Using Ohm's law $I=\frac{V}{R}$, and

$$i=\frac{R_2}{R_2+R_3}!I=\frac{R_2}{R_2+R_3}!\frac{V_S!(R_2+R_3)}{R_1(R_2+R_3)+R_2R_3}$$

$$=\frac{V_S!R_2}{R_1(R_2+R_3)+R_2R_3}$$

2.44 By KCL:

$$I_{20} = 5\text{A}$$

Therefore, by Ohm's Law:

$$V_A = 20!5 = 100\text{V}$$
$$V_B = 3!10 = 30\text{V}$$
$$V_C = 2!15 = 30\text{V}$$

By KVL:

$$V_S = 130\text{V}$$

2.45 Using Ohm's law:

$$I = \frac{240}{200!10^3} = 1.2 \text{ mA}$$

2.46 For short circuit,

$$v(t) = 0$$

Using ohm's law

$$\text{R}_{short} = v(t)/i(t) = 0$$

For open circuit

$$i(t) = 0$$

Using ohm's law

$$\text{R}_{open} = v(t)/i(t) = \text{infinity}$$

2.47

$$V_{out} = Vs(Rs/(Rs + Rc)) - Vs(Rb/(Ra+Rb))$$

By product rule of derivation

$$\text{d}V_{out}/\text{d}Rs = VsRc/(Rs+Rc)^2$$

When $Rs = 0$, the above expression is minimum. However, you cannot control over sensor resistance, but you can choose Rc to achieve the maximum slope.

We differentiate $\text{d}V_{out}/\text{d}Rs$ with respect to Rc,

$$\text{d}(\text{d}V_{out}/\text{d}Rs)/\text{d}Rc = Vs(Rs-Rc)/(Rs+Rc)^3$$

The maximum slope is achieved when $Rc = Rs$.

Rc can be chosen to have the same value as the nominal resistance of the sensor.

2.48 Using data pairs in the table, we get three equations in A, B, C

$$(1/273) = A + B\ln(16330) + C(\ln(16330))^3$$

$$(1/298) = A + B\ln(5000) + C(\ln(5000))^3$$

$$(1/323) = A + B\ln(1801) + C(\ln(1801))^3$$

Solving those equations, we get

$$A = 0.001284$$
$$B = 2.364 \times 10^{-4}$$
$$C = 9.304 \times 10^{-8}$$

Section 2.8 Power and Energy

2.49

a) The voltage and the current are in associated direction,
$P = u \times i = -3 \times 1 = -3W$ => active element

b) The voltage and the current are not in associated direction,
$P = -u \times i = 2 \times 2 = 4W$ => passive element

c) The voltage and the current are not in associated direction,
$P = -u \times i = -2 \times 3 = -6W$ => active element

2.50

a) The voltage and the current are in associated direction,
$P = u \times i = 20 \times 2.5e^{-2t} = 50e^{-2t}$ W => active element

b) The voltage and the current are not in associated direction,
$P = -u \times i = -20 \times 2\sin t = -40\sin t$ W => $t > 0$, active element; $t < 0$, passive element

2.51 The current can be found using the power formula:

$$I = \frac{P}{V} = \frac{5}{10} = 0.5\text{A}$$

And the resistance can then be found using Ohm's law:

$$R = \frac{V}{I} = \frac{10}{0.5} = 20\Omega$$

2.52

$$P = V \times I = R \times I^2$$

$$= 20 \times 5^2 = 500 \text{ W}$$

2.53 Applying KCL, the current through each resistor is 4mA.
Given $P = R \times I^2$,

$$P_1 = 5 \times 10^3 \times \left(4 \times 10^{-3}\right)^2 = 80mW$$

$$P_2 = 14 \times 10^3 \times \left(4 \times 10^{-3}\right)^2 = 224mW$$

$$P_3 = 32 \times 10^3 \times \left(4 \times 10^{-3}\right)^2 = 512mW$$

2.54 According to KVL:

$$1.5ki_1 = 3ki_2$$

By KCL:

$$3mA = i_1 + i_2$$

$$i_2 = 1mA$$

$$V = 1mA\forall 1k! = 1V$$

2.55 From the problem 2.54, we have $i_1 = 2\text{mA}$ and $i_2 = 1\text{mA}$.
Recall $P = I^2 R$. Therefore,

$$P_{1.5k} = \left(2!10^{\#3}\right)^2 \forall 1.5!10^3 = 6\text{mW}$$
$$P_{2k} = \left(1!10^{\#3}\right)^2 \forall 2!10^3 = 2\text{mW}$$
$$P_{1k} = \left(1!10^{\#3}\right)^2 \forall 1!10^3 = 1\text{mW}$$

2.56

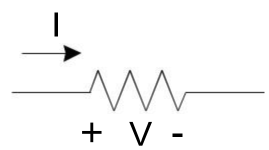

Figure S2.56: Resistor with current direction and voltage polarity.

Consider Figure S2.56, the power absorbed by the resistor is

$$P=VI$$

By Ohm's law,

$$V=RI$$

Thus

$$P=I^2 R=V^2/R$$

Therefore P is always a nonnegative number when R is positive.

The voltage and the current are in associated direction, so
$P = u\times i$
and the waveform of the power is shown below

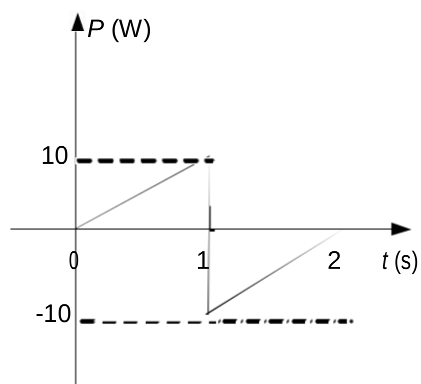

Figure S2.57: Power curve.

The consumed energy is

$$W=\int_0^2 P=0$$

which is resulted from the negative symmetry of P around $t = 1$.

2.58

a) Let the current through the -5V voltage source be in associated direction with the voltage, which is from the positive side to the negative side.
By KCL at the node below the -5V voltage source, the current through it equals

$$2A+4A=6A.$$

Therefore the power of the -5V voltage source is

$$P = -5V \times 6A = 30W$$

b) To zero the power above, the current through the -5V voltage has to be zero. So the 4A current source needs to be changed to -2A by KCL.

Section 2.9 Independent and Dependent Sources

2.59 In Figure S2.59, i axis is the current through the source and u axis is the voltage across the source. u_S denotes the voltage provided by the ideal voltage source and i_S denotes the current provided by the ideal current source. It is shown that for ideal voltage source the voltage does not change with the current through it; for ideal current source the current does not change with the voltage.

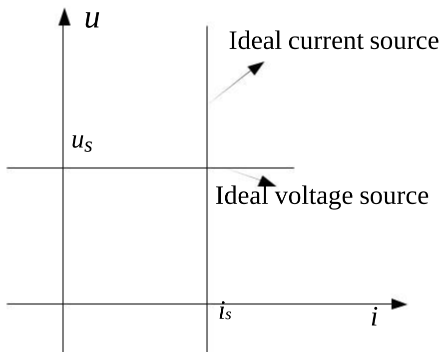

Figure S2.59: Voltage VS current curves.

2.60 Because the two current sources are in series, the currents through them are the same,

$$i = 1A$$

2.61 Because the 3Ω resistor is in series with the current source, the current through the resistor is 1A and, by Ohm's law, its voltage is

$$u = 3\Omega \times 1A = 3V$$

By KVL, the voltage across the current source is

$$u + 2V = 5V$$

The power supplied by the current source is (the current and the voltage are not in associated direction)

$$P = -5V \times 1A = -5W$$

2.62 Using KVL:

$$0 = !10 + V_x + 3V_x$$

Therefore,

$$V_x = 2.5$$

2.63 Using KVL and Ohm's law,

$$0 = \forall 6 \forall \left(I_x !3.5!10^3\right) +$$

$$20 I_x 6 = \forall 3.48!10^3 I_x$$

$$I_x = !1.724 mA$$

2.64 The voltage across the 2Ω resistor is -1V, where the voltage polarity is associated with the current direction. So by Ohm's law

$$I = -1V/2\Omega = -0.5A$$

Therefore, the current of the dependent source is

$$2I = -1A$$

2.65 According to KCL, the current through R3 is $! i_x$

According to Ohm's law,

$$v_0 = ! i_x R_3$$

i_x is given by:

$$i_x = \frac{v_s}{R_1 + R_2}$$

Therefore,

$$v_0 = ! R_3 \frac{v_s}{R_1 + R_2}$$

2.66 According to Ohm's law, the current flow (in associated direction with the voltage) through the 5Ω resistor is

$$i_1 = 4.9V/5\Omega = 0.98A.$$

Because the CCCS is in series with the 5Ω resistor,

$$i_1 = 0.98i \Rightarrow i = 1A$$

which is the current through the 6Ω.

By KCL, the current through the 0.1Ω resistor is i-

$$0.98i = 0.02A.$$

So the voltage across the 0.1Ω resistor is

$$u_1 = 0.1\Omega \times 0.02\text{A} = 0.002\text{V}.$$

Applying KVL in the loop made by u_s , 0.1Ω and 6Ω

$$u_s = u_1 + 6i = 0.002 + 6 = 6.002\text{V}$$

The power supplied by the dependent source is

$$\begin{aligned} P &= (0.98i) \times u = 0.98 \times (0.002-4.9) \\ &= 0.98 \times (-4.898) \\ &= -4.8\text{W} \end{aligned}$$

Section 2.10 Analysis of Circuits using PSpice

2.67 A PSpice circuit is shown in Figure S2.67. Choose the Bias Point for the simulation analysis.

$$R_{TOTAL} = 20 + \left(\frac{1}{40} + \frac{1}{120}\right)^{-1} = 50\Omega$$

$$I_{TOTAL} = \frac{V_{TOTAL}}{R_{TOTAL}} = 0.1A$$

Using Ohm's law, we obtain:

$$R_1 i_1 = R_2 i_2$$

$$i_1 = \frac{R_2}{R_1} i_2$$

Apply the KCL:

$$I = i_1 + i_2$$

The i can be calculated as:

$$0.1 = i + \frac{120}{40} i$$

$$i = 0.025A$$

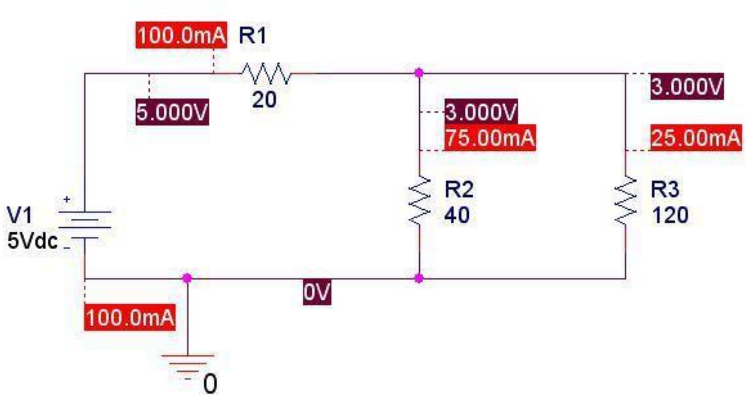

Figure S2.67: PSpice Circuit for Problem 2.67.

2.68 Applying KCL:

$$i_1 = i_2 + i_3 = 10A$$

For parallel voltage, the current relationship is given by:

$$i_2 = \frac{R_3}{R_2} i_3 = \frac{2}{3} i_3$$

Therefore

$$i_2 = 4A$$
$$i_3 = 6A$$

The total voltage across the circuit is:

$$V_{TOTAL} = i_i R_i + i_2 R_2 = 52V$$

The PSpice setup with current source is shown in Figure S2.68 a. Figure S2.68 b shows the current source is replaced by voltage source that produces the same current and voltage across each resistance as those in Figure S2.68 a.

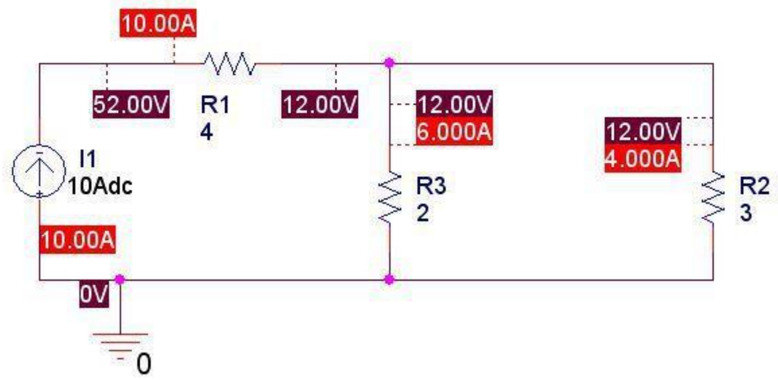

Figure S2.68 a: PSpice Circuit with Current source for Problem 2.68.

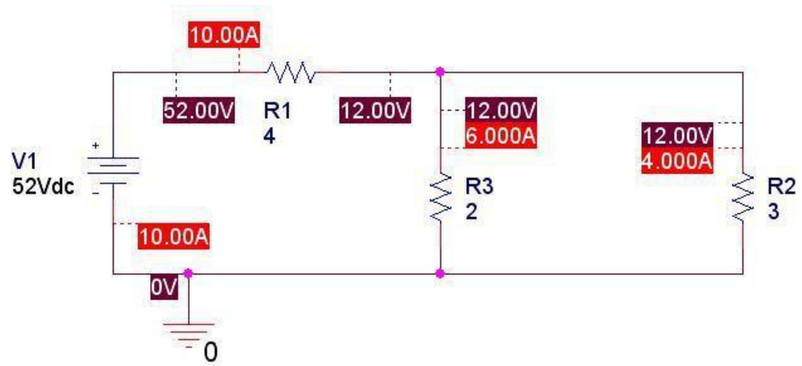

Figure S2.68 b: PSpice Circuit with Current source for Problem 2.68.

2.69 Applying KVL equation, we have:

(2.1)
(2.2)
(2.3)

Applying KCL equation, we have:

(2.4)
(2.5)
(2.6)

From equation (2.3), the current relationship can be written as:

Using the relationship obtained above and substitute into equation (2.2), we obtain:

(2.7)

Expressing the equation (2.1) for i_1 in term of i_3:

(2.8)

Substitute equation (2.8) into equation (2.4), i_3 is calculated:

(2.9)

Substitute equation (2.9) into equation (2.7), R_6 and i_5 can be determined:

(2.10)
(2.11)

A PSpice schematic is shown in Figure S2.69 to verify the answers obtained:

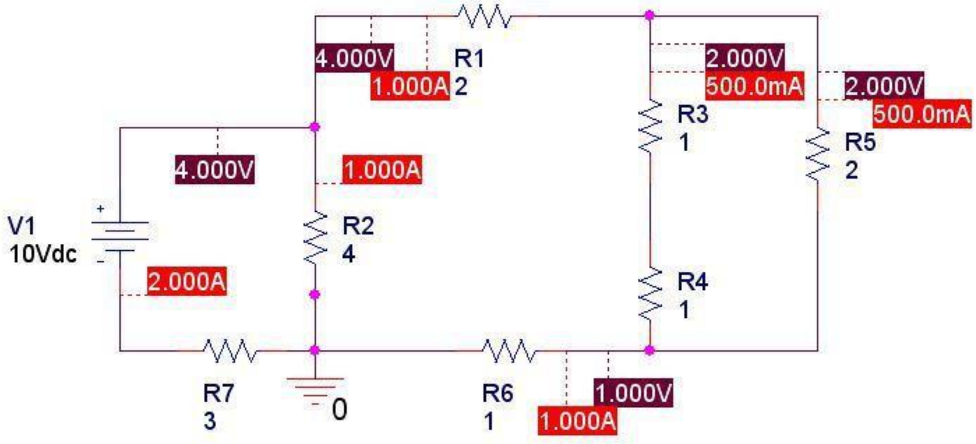

Figure S2.69: PSpice Circuit with Current source for Problem 2.69.

2.70 Applying the KCL euqation:

(2.12)
(2.13)
(2.14)
(2.15)

Applying the KVL equation:

(2.16)
(2.17)
(2.18)
(2.19)

Substituting equation (2.13) into equation (2.12), we obtain:

(2.20)

Substituting equation (2.20) into equation (2.16), we obtain:

(2.21)

Using the current relationship in equation (2.13) and substituting it into equation (2.17), we get:

(2.22)

Substituting equation (2.14) and equation (2.18), we obtain:

(2.23)

Applying the current relationship in equation (2.14) and (2.15) into equation (2.19), and it can be rewritten as:

(2.24)

We can express the equations (2.21) to (2.24) by using a matrix equation ,
of which is given below:

Matrix X can obtained,

A PSpice schematic is shown in Figure S2.70:

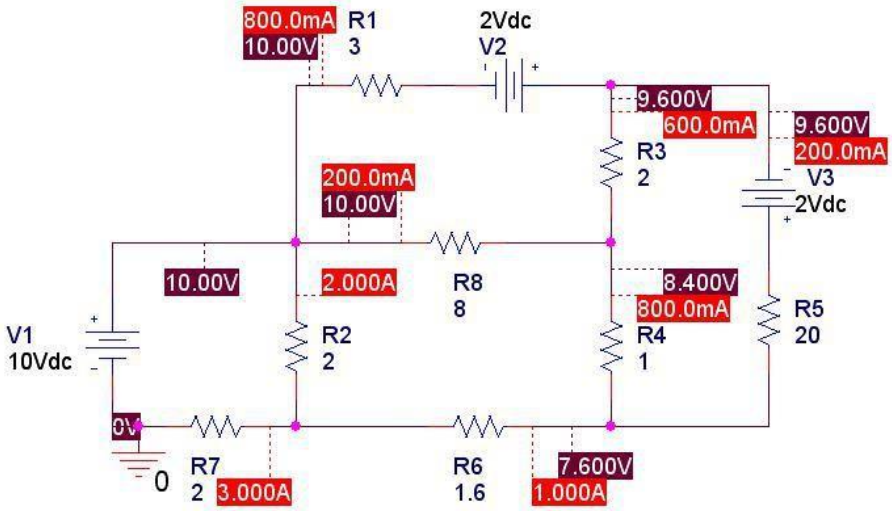

Figure S2.70: PSpice Circuit with Current source for Problem 2.70.