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# INSTRUCTOR'S SOLUTIONS MANUAL

TONI GARCIA

Arizona State University

# ELEMENTARY STATISTICS

NINTH EDITION

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### **PEARSON**

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#### CHAPTER 1 SOLUTIONS

#### Exercises 1.1

- (a) The *population* is the collection of all individuals or items under consideration in a statistical study.
- (b) A sample is that part of the population from which information is obtained.

The two major types of statistics are descriptive and inferential statistics. Descriptive statistics consists of methods for organizing and summarizing information. Inferential statistics consists of methods for drawing and measuring the reliability of conclusions about a population based on information obtained from a sample of the population.

Descriptive methods are used for organizing and summarizing information and include graphs, charts, tables, averages, measures of variation, and percentiles.

Descriptive statistics are used to organize and summarize information from a sample before conducting an inferential analysis. Preliminary descriptive analysis of a sample may reveal features of the data that lead to the appropriate inferential method.

- (a) An observational study is a study in which researchers simply observe characteristics and take measurements.
- (b) A designed experiment is a study in which researchers impose treatments and controls and then observe characteristics and take measurements.

Observational studies can reveal only association, whereas designed experiments can help establish causation.

This study is inferential. Data from a sample of Americans are used to make an estimate of (or an inference about) average TV viewing time for all Americans.

This study is descriptive. It is a summary of the average salaries in professional baseball, basketball, and football for 2005 and 2011.

This study is descriptive. It is a summary of information on all homes sold in different cities for the month of September 2012.

This study is inferential. National samples are used to make estimates of (or inferences about) drug use throughout the entire nation.

This study is descriptive. It is a summary of the annual final closing values of the Dow Jones Industrial Average at the end of December for the years 2004-2013.

This study is inferential. Survey results were used to make percentage estimates on which college majors were in demand among U.S firms for all graduating college students.

- (a) This study is inferential. It would have been impossible to survey all U.S. adults about their opinions on Darwinism. Therefore, the data must have come from a sample. Then inferences were made about the opinions of all U.S. adults.
- (b) The population consists of all U.S. adults. The sample consists only of those U.S. adults who took part in the survey.
- (a) The population consists of all U.S. adults. The sample consists of the 1000 U.S. adults who were surveyed.
- (b) The percentage of 50% is a descriptive statistic since it describes the opinion of the U.S. adults who were surveyed.
- (a) The statement is descriptive since it only tells what was said by the respondents of the survey.

- (b) Then the statement would be inferential since the data has been used to provide an estimate of what all Americans believe.
- (a) To change the study to a designed experiment, one would start with a randomly chosen group of men, then randomly divide them into two groups, an experimental group in which all of the men would have vasectomies and a control group in which the men would not have them. This would enable the researcher to make inferences about vasectomies being a cause of prostate cancer.
  - (b) This experiment is not feasible, since, in the vasectomy group there would be men who did not want one, and in the control group there would be men who did want one. Since no one can be forced to participate in the study, the study could not be done as planned.
- Designed experiment. The researchers did not simply observe the two groups of children, but instead randomly assigned one group to receive the Salk vaccine and the other to get a placebo.
- Observational study. The researchers at Harvard University and the National Institute of Aging simply observed the two groups.
- Observational study. The researchers simply collected data from the men and women in the study with a questionnaire.
- Designed experiment. The researchers did not simply observe the two groups of women, but instead randomly assigned one group to receive aspirin and the other to get a placebo.
- Designed experiment. The researchers did not simply observe the three groups of patients, but instead randomly assigned some patients to receive optimal pharmacologic therapy, some to receive optimal pharmacologic therapy and a pacemaker, and some to receive optimal pharmacologic therapy and a pacemaker-defibrillator combination.
- Observational studies. The researchers simply collected available information about the starting salaries of new college graduates.
- (a) This statement is inferential since it is a statement about all Americans based on a poll. We can be reasonably sure that this is the case since the time and cost of questioning every single American on this issue would be prohibitive. Furthermore, by the time everyone could be questioned, many would have changed their minds.
  - (b) To make it clear that this is a descriptive statement, the new statement could be, "Of 1032 American adults surveyed, 73% favored a law that would require every gun sold in the United States to be test-fired first, so law enforcement would have its fingerprint in case it were ever used in a crime." To rephrase it as an inferential statement, use "Based on a sample of 1032 American adults, it is estimated that 73% of American adults favor a law that would require every gun sold in the United States to be test-fired first, so law enforcement would have its fingerprint in case it were ever used in a crime."
- Descriptive statistics. The U.S. National Center for Health Statistics collects death certificate information from each state, so the rates shown reflect the causes of all deaths reported on death certificates, not just a sample.
- (a) The population consists of all Americans between the ages of 18 and 29.
  - (b) The sample consists only of those Americans who took part in the survey.
  - (c) The statement in quotes is inferential since it is a statement about all Americans based on a survey.
  - (d) "Based on a sample of Americans between the ages of 18 and 29, it is estimated that 59% of Americans oppose medical testing on animals."

- (a) The \$5.36 billion lobbying expenditure figure would be a descriptive figure if it was based on the results of all lobbying expenditures during the period from 1998 through 2012.
- (b) The \$5.36 billion lobbying expenditure figure would be an inferential figure if it was an estimate based on the results of a sample of lobbying expenditures during the period from 1998 through 2012.

#### Exercises 1.2

A census is generally time consuming, costly, frequently impractical, and sometimes impossible.

Sampling and experimentation are two alternative ways to obtain information without conducting a complete census.

The sample should be representative so that it reflects as closely as possible the relevant characteristics of the population under consideration.

There are many possible answers. Surveying people regarding political candidates as they enter or leave an upscale business location, surveying the readers of a particular publication to get information about the population in general, polling college students who live in dormitories to obtain information of interest to all students are all likely to produce samples unrepresentative of the population under consideration.

- (a) Probability sampling consists of using a randomizing device such as tossing a coin or consulting a random number table to decide which members of the population will constitute the sample.
- (b) No. It is possible for the randomizing device to randomly produce a sample that is not representative.
- (c) Probability sampling eliminates unintentional selection bias, permits the researcher to control the chance of obtaining a non-representative sample, and guarantees that the techniques of inferential statistics can be applied.
- (a) Simple random sampling is a procedure for which each possible sample of a given size is equally likely to be the one obtained.
- (b) A simple random sample is one that was obtained by simple random sampling.
- (c) Random sampling may be done with or without replacement. In sampling with replacement, it is possible for a member of the population to be chosen more than once, i.e., members are eligible for re-selection after they have been chosen once. In sampling without replacement, population members can be selected at most once.

Simple random sampling.

One method would be to place the names of all members of the population under consideration on individual slips of paper, place the slips in a container large enough to allow them to be thoroughly shuffled by shaking or spinning, and then draw out the desired number of slips for the sample while blindfolded. A second method, which is much more practical when the population size is large, is to assign a number to each member of the population, and then use a random number table, random number generating device, or computer program to determine the numbers of those members of the population who are chosen.

The acronym used for simple random sampling without replacement is SRS.

- **1.36** (a) 123, 124, 125, 134, 135, 145, 234, 235, 245, 345
  - (b) There are 10 samples, each of size three. Each sample has a one in 10 chance of being selected. Thus, the probability that a sample of three is 1, 3, and 5 is 1/10.

- (c) Starting in Line 05 and column 20, reading single digit numbers down the column and then up the next column, the first digit that is a one through five is a 5. Ignoring duplicates and skipping digits 6 and above and also skipping zero, the second digit found that is a one through five is a 4. Continuing down column 20 and then up column 21, the third digit found that is a one through five is a 1. Thus the SRS of 1,4, and 5 is obtained.
- 1.37 (a) 12, 13, 14, 23, 24, 34
  - (b) There are 6 samples, each of size two. Each sample has a one in six chance of being selected. Thus, the probability that a sample of two is 2 and 3 is 1/6.
  - (c) Starting in Line 17 and column 07 (notice there is a column 00), reading single digit numbers down the column and then up the next column, the first digit that is a one through four is a 1. Continue down column 07 and then up column 08. Ignoring duplicates and skipping digits 5 and above and also skipping zero, the second digit found that is a one through four is a 4. Thus the SRS of 1 and 4 is obtained.
  - (a) Starting in Line 15 and reading two digits numbers in columns 25 and 26 going down the table, the first two digit number between 01 and 90 is 06. Continuing down the columns and ignoring duplicates and numbers 91-99, the next two numbers are 33 and 61. Then, continuing up columns 27 and 28, the last two numbers selected are 56 and 20. Therefore the SRS of size five consists of observations 06, 33, 61, 56, and 20.
    - (b) There are many possible answers.
  - (a) Starting in Line 10 and reading two digits numbers in columns 10 and 11 going down the table, the first two digit number between 01 and 50 is 43. Continuing down the columns and ignoring duplicates and numbers 51-99, the next two numbers are 45 and 01. Then, continuing up columns 12 and 13, the last three numbers selected are 42, 37, and 47. Therefore the SRS of size six consists of observations 43, 45, 01, 42, 37, and 47.
    - (b) There are many possible answers.
  - The online poll clearly has a built-in non-response bias. Since it was taken over the Memorial Day weekend, most of those who responded were people who stayed at home and had access to their computers. Most people vacationing outdoors over the weekend would not have carried their computers with them and would not have been able to respond.
  - Dentists form a high-income group whose incomes are not representative of the incomes of Seattle residents in general.
  - (a) The five possible samples of size one are G, L, S, A, and T.
    - (b) There is no difference between obtaining a SRS of size 1 and selecting one official at random.
    - (c) The one possible sample of size five is GLSAT.
    - (d) There is no difference between obtaining a SRS of size 5 and taking a census of the five officials.
  - (a) GLS, GLA, GLT, GSA, GST, GAT, LSA, LST, LAT, SAT.
    - (b) There are 10 samples, each of size three. Each sample has a one in 10 chance of being selected. Thus, the probability that a sample of three officials is the first sample on the list presented in part (a) is 1/10. The same is true for the second sample and for the tenth sample.
  - (a) E,M E,A M,L P,L L,A E,P
    E,B M,A P,A L,B E,L M,PM,B
    P,B A,B

- (b) One procedure for taking a random sample of two representatives from the six is to write the initials of the representatives on six separate pieces of paper, place the six slips of paper into a box, and then, while blindfolded, pick two of the slips of paper. Or, number the representatives 1-6, and use a table of random numbers or a randomnumber generator to select two different numbers between 1 and 6.
- (c) 1/15; 1/15
- (a) E,M,P,L E,M,L,B E,P,A,B M,P,A,B E,M,P,A E,M,A,B E,L,A,B M,L,A,BE,M,P,B E,P,L,A M,P,L,A P,L,A,BE,M,L,A E,P,L,B M,P,L,B
- (b) One procedure for taking a random sample of four representatives from the six is to write the initials of the representatives on six separate pieces of paper, place the six slips of paper into a box, and then, while blindfolded, pick four of the slips of paper. Or, number the representatives 1-6, and use a table of random numbers or a randomnumber generator to select four different numbers between 1 and 6.
- (c) 1/15; 1/15
- (a) E,M,P E,P,A M,P,L M,A,B
  E,M,L E,P,B M,P,A P,L,A
  E,M,A E,L,A M,P,B P,L,B
  E,M,B E,L,B M,L,A P,A,B
  E,P,L E,A,B M,L,B L,A,B
- (b) One procedure for taking a random sample of three representatives from the six is to write the initials of the representatives on six separate pieces of paper, place the six slips of paper into a box, and then, while blindfolded, pick three of the slips of paper. Or, number the representatives 1-6, and use a table of random numbers or a randomnumber generator to select three different numbers between 1 and 6.
- (c) 1/20; 1/20
- (a) F,T F,G F,H F,L F,B F,A T,B T,G T, HT,L T,A G,H G,L G,B G,A H,L H,B H,A L,B L,A B,A
- (b) 1/21; 1/21
- (a) I am using Table I to obtain a list of 20 different random numbers between 1 and 80 as follows.

I start at the two digit number in line number 5 and column numbers 31-32, which is the number 86. Since I want numbers between 1 and 80 only, I throw out numbers between 81 and 99, inclusive. I also discard the number 00.

I now go down the table and record the two-digit numbers appearing directly beneath 86.

After skipping 86, I record 39, 03, skip 97, record 28, 58, 59, skip 81, record 09, 36, skip 81, record 52, skip 94, record 24 and 78.

Now that I've reached the bottom of the table, I move directly rightward to the adjacent column of two-digit numbers and go up.

I skip 84, record 57, 40, skip 89, record 69, 25, skip 95, record 51, 20, 42, 77, skip 89, skip 40(duplicate), record 14, and 34. I've finished recording the 20 random numbers. In summary, these are

39	03	28	58	59
09	36	52	24	78
57	40	69	25	51
20	42	77	14	34

- (b) We can use Minitab to generate random numbers. Following the instructions in The Technology Center, our results are 55, 47, 66, 2, 72, 56, 10, 31, 5, 19, 39, 57, 44, 60, 23, 34, 43, 9, 49, and 62. Your result may be different from ours.
- (a) I am using Table I to obtain a list of 10 random numbers between 1 and 500 as follows.

I start at the three digit number in line number 14 and column numbers 10-12, which is the number 452.

I now go down the table and record the three-digit numbers appearing directly beneath 452. Since I want numbers between 1 and 500 only, I throw out numbers between 501 and 999, inclusive. I also discard the number 000.

After 452, I skip 667, 964, 593, 534, and record 016.

Now that I've reached the bottom of the table, I move directly rightward to the adjacent column of three-digit numbers and go up.

I record 343, 242, skip 748, 755, record 428, skip 852, 794, 596, record 378, skip 890, record 163, skip 892, 847, 815, 729, 911, 745, record 182, 293, and 422.

I've finished recording the 10 random numbers. In summary, these are:

452 016 343 242 428 378 163 182 293 422

- (b) We can use Minitab to generate random numbers. Following the instructions in The Technology Center, our results are 489, 451, 61, 114, 389, 381, 364, 166, 221, and 437. Your result may be different from ours.
- (a) First assign the digits 0 though 9 to the ten cities as listed in the exercise. Select a random starting point in Table I of Appendix A and read in a pre-selected direction until you have encountered 5 different digits. For example, if we start at the top of the fifth column of digits and read down, we encounter the digits 4,1,5,2,5,6. We ignore the second '5'. Thus our sample of five cities consists of Osaka, Tokyo, Miami, San Francisco, and New York. Your answer may be different from this one.
  - (b) We can use Minitab to generate random numbers. Following the instructions in The Technology Center, our results are 3, 8, 6, 5, 9. Thus our sample of 5 cities is Los Angeles, Manila, New York, Miami, and London. Your result may be different from ours.
- (a) First re-assign the elements 93 though 118 as elements 01 to 26.

Select a random starting point in Table I of Appendix A and read in a pre-selected direction until you have encountered 8 different elements. For example, if we start at the top of the column 10 and read two digit numbers down and then up in the following columns, we encounter the elements 04, 01, 03, 08, 11, 18, 22, and 15. This corresponds to a sample of the elements Cm, Np, Am, Fm, Lr, Ds, Fl, and Bh. Your answer may be different from this one.

- (b) We can use Minitab to generate random numbers. Following the instructions in The Technology Center, our results are 8, 2, 9, 20, 24, 19, 21, and 13. Thus our sample of 8 elements is Fm, Pu, Md, Cn, Lv, Rg, Uut, and Db. Your result may be different from ours.
- (a) One of the biggest reasons for undercoverage in household surveys is that respondents do not correctly indicate all who are living in a household maybe due to deliberate concealment or irregular household structure or living arrangements. The household residents are only partially listed.
- (b) A telephone survey of Americans from a phone book will likely have bias due to undercoverage because many people have unlisted phone numbers and also it is becoming more popular that many people do not even have home phones. This would cause the phone book to be an incomplete list of the population.
- (a) One of the dangers of nonresponse is that the individuals who do not respond may have a different observed value than the individuals that do respond causing a nonresponse bias in the estimate. Nonresponse bias may make the measured value too small or too large.
- (b) The lower the response rate, the more likely there is a nonresponse bias in the estimate. Therefore the estimate will either under or over estimate the generalized results to the entire population.
- (a) The respondent may wish to please the questioner by answering what is morally or legally right. The respondent might not be willing to admit to the questioner that they smoke marijuana and the measured value of the percentage of people that smoke marijuana would then be underestimated due to response bias.
- (b) Another situation that might be conducive to response bias is perhaps a woman questioning men on their opinion of domestic violence, or an environmentalist questioning people on their recycling habits.
- (c) The wording of a question could lead to response bias. Whether the survey is anonymous or not could lead to response bias. The characteristics of the questioner could lead to response bias. It could also happen if the questioner obviously favors and is pushing for one particular answer.

#### Exercises 1.3

Systematic random sampling is easier to execute than simple random sampling and usually provides comparable results. The exception is the presence of some kind of <a href="cyclical pattern">cyclical pattern</a> in the listing of the members of the population.

Ideally, in cluster sampling, each cluster should  $\underline{pattern}$  the entire population.

Ideally, in stratified sampling, the members of each stratum should be homogeneous relative to the characteristic under consideration.

Surveys that combine one or more of simple random sampling, systematic random sampling, cluster sampling, and stratified sampling employ what is called multistage sampling.

(a) Answers will vary, but here is the procedure: (1) Divide the population size, 372, by the sample size, 5, and round down to the nearest whole number if necessary; this gives 74. Use a table of random numbers (or a similar device) to select a number between 1 and 74, call it k. (3) List every  $74^{\text{th}}$  number, starting with k, until 5 numbers are obtained; thus, the first number of the required list of 5 numbers is k, the second is k + 74, the third is k + 148, and so forth.

- (b) Following part (a) with k=10, the first number of the sample is 10, the second is 10+74=84. The remaining three numbers in the sample would be 158, 232, and 306. Thus, the sample of 5 would be 10, 84, 158, 232, and 306.
- (a) Answers will vary, but here is the procedure: (1) Divide the population size, 500, by the sample size, 9, and round down to the nearest whole number if necessary; this gives 55. Use a table of random numbers (or a similar device) to select a number between 1 and 55, call it k. (3) List every  $55^{th}$  number, starting with k, until 9 numbers are obtained; thus, the first number of the required list of 9 numbers is k, the second is k + 55, the third is k + 110, and so forth.
  - (b) Following part (a) with k=48, the first number of the sample is 48, the second is 48+55=103. The remaining seven numbers in the sample would be 158, 213, 268, 323, 378, 433, and 488. Thus, the sample of 9 would be 48, 103, 158, 213, 268, 323, 378, 433, and 488.
- (a) Answers will vary, but here is the procedure: (1) The population of size 50 is already divided into five clusters of size 10. (2) Since the required sample size is 20, we will need to take a SRS of 2 clusters. Use a table of random numbers (or a similar device) to select two numbers between 1 and 5. These are the two clusters that are selected. (3) Use all the members of each cluster selected in part (2) as the sample.
  - (b) Following part (a) with clusters #1 and #3 selected, we would select all the members in cluster 1, which are 1-10, and all the members in cluster 3, which are 21-30.
- (a) Answers will vary, but here is the procedure: (1) The population of size 100 is already divided into ten clusters of size 10. (2) Since the required sample size is 30, we will need to take a SRS of 3 clusters. Use a table of random numbers (or a similar device) to select three numbers between 1 and 10. These are the three clusters that are selected. (3) Use all the members of each cluster selected in part (2) as the sample.
  - (b) Following part (a) with clusters #2, #6, and #9 selected, we would select all the members in cluster 2 (11-20), all the members in cluster 6 (51-60), and all the members in cluster 9 (81-90). Therefore, our sample would consist of 11-20, 51-60, and 81-90.
- (a) From each strata, we need to obtain a SRS of a size proportional to the size of the stratum. Therefore, since strata #1 is 30% of the population, a SRS equal to 30% of 20, or 6, should be sampled from strata #1. Since strata #2 is 20% of the population, a SRS equal to 20% of 20, or 4, should be sampled from strata #2. Similarly, a SRS of size 8 should be sampled from strata #3 and a SRS of size 2 should be sampled from strata #4. The sample sizes from stratum #1 through #4 are 6, 4, 8, and 2 respectively.
  - (b) Answers will vary following the procedure in part (a).
- (a) From each strata, we need to obtain a SRS of a size proportional to the size of the stratum. Therefore, since strata #1 is 40% of the population, a SRS equal to 40% of 10, or 4, should be sampled from strata #1. Since strata #2 is 30% of the population, a SRS equal to 30% of 10, or 3, should be sampled from strata #2. Similarly, a SRS of size 3 should be sampled from strata #3. The sample sizes from stratum #1 through #3 are 4, 3, and 3 respectively.
  - (b) Answers will vary following the procedure in part (a).

Stratified Sampling. The entire population is naturally divided into subpopulations, one from each lake, and random sampling is done from each lake. The stratified sampling is not with proportional allocation since that would require knowing how many fish were in each lake.

Stratified Sampling. The entire population is naturally divided into four subpopulations, and random sampling is done from each and then combined into a single sample.

Systematic Random Sampling. Kennedy selected his sample using the fixed periodic interval of every  $50^{\text{th}}$  letter, which is the similar to the method presented in procedure 1.1.

Cluster Sampling. The clusters of this sampling design are the 1285 journals. A random sample of 26 clusters was selected and then all articles from the selected journals for a particular year were examined.

Cluster Sampling. The clusters of this sampling design are the 46 schools. A random sample of 10 clusters was selected and then all of the parents of the nonimmunized children at the 10 selected schools were sent a questionnaire.

Systematic Random Sampling. This sampling design follows procedure 1.1. First, dividing the population size of 8493 by 30, they arrived at k=283. Then, the randomly selected starting point was m=10. Then, the sampled stickers were m=10, m+k=293, m+2k=576, etc.

- (a) Answers will vary, but here is the procedure: (1) Divide the population size, 500, by the sample size, 10, and round down to the nearest whole number if necessary; this gives 50. (2) Use a table of random numbers (or a similar device) to select a number between 1 and 50, call it k. (3) List every 50th, starting with k, until 10 numbers are obtained; thus, the first number on the required list of 10 numbers is k, the second is k+50, the third is k+100, and so forth (e.g., if k=6, then the numbers on the list are 6, 56, 106, ...).
- (b) Systematic random sampling is easier.
- (c) The answer depends on the purpose of the sampling. If the purpose of sampling is not related to the size of the sales outside the U.S., systematic sampling will work. However, since the listing is a ranking by amount of sales, if k is low (say 2), then the sample will contain firms that, on the average, have higher sales outside the U.S. than the population as a whole. If the k is high, (say 49) then the sample will contain firms that, on the average, have lower sales than the population as a whole. In either of those cases, the sample would not be representative of the population in regard to the amount of sales outside the U.S.
- (a) Answers will vary, but here is the procedure: (1) Divide the population size, 80, by the sample size, 20, and round down to the nearest whole number if necessary; this gives 4. (2) Use a table of random numbers (or a similar device) to select a number between 1 and 4, call it k. (3) List every 4th number, starting with k, until 20 numbers are obtained; thus the first number on the required list of 20 numbers is k, the second is k+4, the third is k+8, and so forth (e.g., if k=3, then the numbers on the list are 3, 7, 11, 15, ...).
- (b) Systematic random sampling is easier.
- (c) No. In Keno, you want every set of 20 balls to have the same chance of being chosen. Systematic sampling would give each of 4 sets of balls [(1, 5, 9,...,77), (2, 6, 10,...,78), (3, 7, 11,...,79) and (4, 8, 12,...,80)], a 1/4 chance of occurring, while all of the other possible sets of balls would have no chance of occurring.

- (a) Number the suites from 1 to 48, use a table of random numbers to randomly select three of the 48 suites, and take as the sample the 24 dormitory residents living in the three suites obtained.
  - (b) Probably not, since friends are more likely to have similar opinions than are strangers.
  - (c) There are 384 students in total. Freshmen make up 1/3 of them. Sophomores make up 7/24 of them, Juniors 1/4, and Seniors 1/8. Multiplying each of these fractions by 24 yields the proportional allocation, which dictates that the number of freshmen, sophomores, juniors, and seniors selected should be, respectively, 8, 7, 6, and 3. Thus a stratified sample of 24 dormitory residents can be obtained as follows: Number the freshmen dormitory residents from 1 to 128 and use a table of random numbers to randomly select 8 of the 128 freshman dormitory residents; number the sophomore dormitory residents from 1 to 112 and use a table of random numbers to randomly select 7 of the 112 sophomore dormitory residents; and so forth.
- (a) Each category of "Percent free lunch" should be represented in the sample in the same proportion that it is present in the population of top 100 ranked high schools. Thus 50/100 of the sample of 25 schools should be from the 0 to under 10% free lunch category, 18/100 from the second category, 11/100 from the third, 8/100 from the fourth, and 13/100 from Multiplying each of these fractions by 25 gives us the sample sizes from each category. These sample sizes will not necessarily be integers, so we will need to make some minor adjustments of the results. The first category should have (50/100)(25) = 12.5. The second should have (18/100)(25) = 4.5. Similarly, the third, fourth, and fifth categories should have 2.75, 2, and 3.25 for their sample sizes. We round the third and fifth sample sizes each to 3. After flipping a coin, we round the first two categories to 12 and 5. Thus the sample sizes for the five Percent free lunch categories should be 12, 5, 3, 2, and 3 respectively. We would now use a random number generator to select 12 out of the 50 in the first category, 5 out of the 18 in the second, 3 out of the 11 in the third, 2 of the 8 in the fourth, and 3 of the 13 in the last category.
  - (b) From part (a), two schools would be selected from the strata with a percent free lunch value of 30-under 40.
- (a) Answers will vary, but here is the procedure: (1) Divide the population size, 435, by the sample size, 15, and round down to the nearest whole number if necessary; this gives 29. Use a table of random numbers (or a similar device) to select a number between 1 and 29, call it k. (3) List every  $29^{\text{th}}$  number, starting with k, until 15 numbers are obtained; thus, the first number of the required list of 15 numbers is k, the second is k + 29, the third is k + 58, and so forth.
  - (b) Following part (a) with k=12, the first number of the sample is 12, the second is 12+29=41. The third number selected is 12+58=70. The remaining twelve numbers are similarly selected. Thus, the sample of 15 would be 12, 41, 70, 99, 128, 157, 186, 215, 244, 273, 302, 331, 360, 389, and 418.
- (a) Each category of "Region" should be represented in the sample in the same proportion that it is present in the population. Thus 43% of the sample of 50 should be volunteers serving in Africa, 21% from Latin America, 15% from Eastern Europe/Central Asia, 10% from Asia, 4% from the Caribbean, 4% from North Africa/Middle East, and 3% from the Pacific Island. Finding each of these proportions of 50 gives us the sample sizes from each category. These sample sizes will not necessarily be integers, so we will need to make some minor adjustments of the results. Volunteers from Africa should have (0.43)(50) = 21.5. Volunteers from Latin America should have (0.21)(50) = 10.5.

- Similarly, the remaining categories should have 7.5, 5, 2, 2, and 1.5 for their sample sizes. After flipping a coin, we round the first two categories either up or down. Thus the sample sizes for the categories should be 21, 11, 7, 5, 2, 2, and 2 respectively. We would now use a random number generator to select the volunteers from each category.
- (b) From part (a), two volunteers would be selected from the strata with volunteers serving in the Caribbean.
- (a) This is a poll taken by calling randomly selected U.S. adults. Thus, the sampling design appears to be simple random sampling, although it is possible that a more complex design was used to ensure that various political, religious, educational, or other types of groups were proportionately represented in the sample.
- (b) The sample size for the second question was 78% of 1010 or 788.
- (c) The sample size for the third question was 28% of 788 or 221.

No. In your text, Example 1.10, only 48 different samples are possible. A sample containing students 5,6, and 7 is not possible at all. While the 48 possible samples are equally likely, there are other samples that could be obtained through simple random sampling that are not possible at all in systematic sampling. Thus not  $\underline{\text{all}}$  possible samples are equally likely. Nevertheless, if there is no pattern or cycle to the data, this method will tend to give about the same results as simple random sampling.

- (a) It is also true for systematic random sampling if the population size divided by the sample size results in an integer for m. The chance for each member to be selected is then still equal to the sample size divided by the population size. For example, suppose the population size is N=10 and the sample size is n=2. The chance that each member in simple random sampling to be selected is 2/10 = 1/5. In systematic random sampling for the same example, m=5. The possible samples of size two are 1 and 6, 2 and 7, 3 and 8, 4 and 9, and 5 and 10. Therefore, the chance that a member is selected is equal to the chance of one of those five samples being selected, which is the same as simple random sampling of 1/5.
- (b) It is not true for systematic random sampling if the population size divided by the sample size does not result in an integer for m. For example, suppose the population size is N=15 and the sample size is n=2. After dividing the population size by the sample size and rounding down to the nearest whole number, we get m=7. You would select every  $7^{\text{th}}$  member after a random starting place k, between 1 and 7, is determined. If k=1, you would select the first and eighth member. If k=7, you would select the seventh and fourteenth member. In this situation, the last member (fifteenth) can never be selected. Therefore, the last member of the sample does not have the same chance of being selected as any other member in the population.

Refer to example 1.14. If we approached this problem as a simple random sample each member would have a chance of being selected equal to the sample size divided by the population size: 20/250, or 2/25.

If we approached this same example as a stratified sample with proportional allocation, we would select 2 out of 25 households in the upper income group, 14 out of the 175 households in the middle income group, and 4 out of 50 households in the lower income group. Thus the chance that an upper income household is selected is 2/25. The chance that a middle income household is selected is 14/175 = 2/25. Finally, the chance that a lower income household is selected is 4/50 = 2/25. Thus, the chance that each member is selected is the same as a simple random sample.

#### Exercises 1.4

- (a) Experimental units are the individuals or items on which the experiment is performed.
  - (b) When the experimental units are humans, we call them subjects.
- The three basic principles of experimental design are control, randomization, and replication.

Control: Two or more treatments should be compared.

Randomization: The experimental units should be randomly divided into groups to avoid unintentional selection bias in constituting the groups.

Replication: A sufficient number of experimental units should be used to ensure that randomization creates groups that resemble each other closely and to increase the chances of detecting differences among the treatments.

- (a) The response variable is the characteristic of the experimental outcome that is to be measured or observed.
  - (b) A factor is a variable whose effect on the response variable is of interest in the experiment.
  - (c) The levels are the possible values of the factor.
  - (d) For a one-factor experiment, the treatments are the levels of the factor. For multifactor experiments, the treatments are the combinations of levels of the factors.
- One type of statistical design is a completely randomized design. In a completely randomized design, all the experimental units are assigned randomly among all the treatments. The second type of statistical design is a randomized block design. In a randomized block design, the experimental units are assigned randomly among all the treatments separately within each block.
- In a one-factor experiment, the number of treatments is equal to the number of levels of the factor. Therefore, there are four treatments.
- In a one-factor experiment, the number of treatments is equal to the number of levels of the factor. Therefore, there are five treatments.

#### **1.87** (a)

	В			
	$b_1$	$b_2$	$b_3$	$b_4$
$a_1$	$a_1b_1$	$a_1b_2$	$a_1b_3$	$a_1b_4$
$a_2$	$a_2b_1$	$a_2b_2$	$a_2b_3$	$a_2b_4$
$a_3$	$a_3b_1$	$a_3b_2$	$a_3b_3$	$a_3b_4$

 $\mathbf{A}$ 

- (b) There are twelve combinations of the levels of the factors. Therefore, there are twelve treatments.
- (c) Yes, you could have multiplied the number of levels in each factor. There are three levels of factor A and four levels of factor B. Therefore, there are (3)(4) = 12 treatments.

#### **1.88** (a)

		$\boldsymbol{\mathit{B}}$		
		$b_1$	$b_2$	
	$a_1$	$a_1b_1$	$a_1b_2$	
_	$a_2$	$a_2b_1$	$a_2b_2$	
¥	$a_3$	$a_3b_1$	$a_3b_2$	
	$a_4$	$a_4b_1$	$a_4b_2$	

- (b) There are eight combinations of the levels of the factors. Therefore, there are eight treatments.
- (c) Yes, you could have multiplied the number of levels in each factor. There are four levels of factor A and two levels of factor B. Therefore, there are (4)(2) = 8 treatments.

You can multiply the number of levels in each factor. There are m levels in the first factor and n levels in the second factor. Therefore, there are

- (m) (n) =  $m \times n$  treatments.
- (a) The treatment group consisted of the 2444 patients who took Prozac.
- (b) The control group consisted of the 1331 patients who received a placebo.
- (c) The treatments were administering Prozac and administering the placebo.
- (a) There were three treatments.
- (b) The first group, the one receiving only the pharmacologic therapy, would be considered the control group.
- (c) There were three treatment groups. The first received only pharmacologic therapy, the second received pharmacologic therapy plus a pacemaker, and the third received pharmacologic therapy plus a pacemaker-defibrillator combination.
- (d) The first group (control) contained 1/5 of the 1520 patients or 304. The other two groups each contained 2/5 of the 1520 patients or 608.
- (e) Each patient could be randomly assigned a number from 1 to 1520. Any patient assigned a number between 1 and 304 would be assigned to the control group; any patient assigned to the next 608 numbers (305 to 912) would be assigned to receive the pharmacologic therapy plus a pacemaker; and any patient assigned a number between 913 and 1520 would receive pharmacologic therapy plus a pacemaker-defibrillator combination. Each random number would be used only once to ensure that the resulting treatment groups were of the intended sizes.
- (a) Experimental units: batches of the product being sold
- (b) Response variable: the number of units of the product sold
- (c) Factors: two factors display type and pricing scheme
- (d) Levels of each factor: three types of display of the product and three pricing schemes
- (e) Treatments: the nine different combinations of display type and price resulting from testing each of the three pricing schemes with each of the three display types
- (a) Experimental units: the drivers
- (b) Response variable: the detection distance, in feet
- (c) Factors: two factors sign size and sign material

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- (d) Levels of each factor: three levels of sign size (small, medium, and large) and three levels of sign material (1, 2, and 3)
- (e) Treatments: the nine different combinations of sign size and sign material resulting from testing each of the three sign sizes with each of the three sign materials
- (a) Experimental units: fields of oats
  - (b) Response variable: crop yield of the oats per acre
  - (c) Factors: variety of oats and concentration of manure on the fields
  - (d) Levels of each factor: three varieties of oats and four concentrations of manure
  - (e) Treatments: the twelve combinations of oat variety and manure concentration resulting from testing each of the three oat varieties with each of the four concentration levels of the manure
- (a) Experimental units: female lions
  - (b) Response variable: whether or not the female lion approached the male lion dummy
  - (c) Factors: length and color of the mane on the male lion dummy
  - (d) Levels of each factor: two different mane lengths and two different mane colors
  - (e) Treatments: the four combinations of mane length and color
- (a) Experimental units: the women in the study
  - (b) Response variable: the color of the shirt chosen
  - (c) Factors: gender and attractiveness of the new acquaintance
  - (d) Levels of each factor: two different genders (male, female) and two different levels of attractiveness (attractive, unattractive)
  - (e) Treatments: the four combinations of gender and attractiveness
- (a) Experimental units: the children
  - (b) Response variable: IQ score
  - (c) Factor: Whether they were given dexamethasone (control or dexamethasone group)
  - (d) Levels of each factor: two levels of the single factor (control or dexamethasone group)
  - (e) Treatments: the two levels of the single factor
- (a) This is a completely randomized design since the flashlights were randomly assigned to the different battery brands.
  - (b) This is a randomized block design since the four different battery brands would be randomly assigned within each set of four flashlights from each of the five flashlight brands.
- (a) This is a randomized block design. The experiment first blocked by gender. All the experimental units are not randomly assigned among all the treatments.
  - (b) The blocks are the two genders (male and female).
- Double-blinding guards against bias, both in the evaluations and in the responses. In the Salk vaccine experiment, double-blinding prevented a doctor's evaluation from being influenced by knowing which treatment (vaccine or placebo) a patient received; it also prevented a patient's response to the treatment from being influenced by knowing which treatment he or she received.

- (a) Simple random sampling corresponds to completely randomized designs since selection is randomly made from the entire population.
- (b) Stratified sampling corresponds to randomized block designs since selection is randomly made from within each strata.

#### Review Problems for Chapter 1

- 1. Student exercise.
- 2. Descriptive statistics are used to display and summarize the data to be used in an inferential study. Preliminary descriptive analysis of a sample often reveals features of the data that lead to the choice or reconsideration of the choice of the appropriate inferential analysis procedure.
- 3. (a) An observational study is a study in which researchers simply observe characteristics and take measurements.
  - (b) A designed experiment is a study in which researchers impose treatments and controls and then observe characteristics and take measurements.
- 4. A literature search should be made before planning and conducting a study.
- 5. (a) A representative sample is one that reflects as closely as possible the relevant characteristics of the population under consideration.
  - (b) Probability sampling involves the use of a randomizing device such as tossing a coin or die, using a random number table, or using computer software that generates random numbers to determine which members of the population will make up the sample.
  - (c) A sample is a simple random sample if all possible samples of a given size are equally likely to be the actual sample selected.
- 6. (a) This method does not involve probability sampling. No randomizing device is being used and people who do not visit the campus cafeteria have no chance of being included in the sample.
  - (b) The dart throwing is a randomizing device that makes all samples of size 20 equally likely. This is probability sampling.
- 7. (a) Systematic random sampling is done by first dividing the population size by the sample size and rounding the result down to the next integer, say m. Then we select one random number, say k, between 1 and m inclusive. That number will be the first member of the sample. The remaining members of sample will be those numbered k+m, k+2m, k+3m,... until a sample of size n has been chosen. Systematic sampling will yield results similar to simple random sampling as long as there is nothing systematic about the way the members of the population were assigned their numbers.
  - (b) In cluster sampling, clusters of the population (such as blocks, precincts, wards, etc.) are chosen at random from all such possible clusters. Then every member of the population lying within the chosen clusters is sampled. This method of sampling is particularly convenient when members of the population are widely scattered and is most appropriate when the members of each cluster are representative of the entire population. Cluster sampling can save both time and expense in doing the survey, but can yield misleading results if individual clusters are made up of subjects with very similar views on the topic being surveyed.
  - (c) In stratified random sampling with proportional allocation, the population is first divided into subpopulations, called strata, and simple random sampling is done within each stratum. Proportional allocation means that the size of the sample from each stratum is proportional to the size of the population in that stratum. This type

of sampling may improve the accuracy of the survey by ensuring that those in each stratum are more proportionately represented than would be the case with cluster sampling or even simple random sampling. Ideally, the members of each stratum should be homogeneous relative to the characteristic under consideration. If they are not homogeneous within each stratum, simple random sampling would work just as well.

- 8. The three basic principles of experimental design are control, randomization, and replication. Control refers to methods for controlling factors other than those of primary interest. Randomization means randomly dividing the subjects into groups in order to avoid unintentional selection bias in constituting the groups. Replication means using enough experimental units or subjects so that groups resemble each other closely and so that there is a good chance of detecting differences among the treatments when such differences actually exist.
- 9. Descriptive study. It is a summary of the scores of major league baseball games on August 14, 2013.
- 10. (a) Descriptive study. It is a summary of the responses from those that participated in the poll.
  - (b) Inferential statement. It is an implied estimate of the responses of all adults in the U.S.
- 11. Inferential study. The results of a sample are used to make inferences about the age distribution of all British backpackers in South Africa.
- 12. (a) Descriptive study. It is a summary of the percentages of Jewish children sampled in Israel and Britain that have peanut allergies.
  - (b) Observational study. The researchers simply observed the two groups.
- 13. This is an observational study. To be a designed experiment, the researchers would have to have the ability to assign some children at random to live in persistent poverty during the first 5 years of life or to not suffer any poverty during that period. Clearly that is not possible.
- 14. This is a designed experiment since the researcher is imposing a treatment and then observing the results.
- 15. Because Yale is a very expensive school, incomes of parents of Yale students will not be representative of the incomes of all college students' parents.
- **16.** (a) H, Z, C H, Z, A H, Z, J H, C, A H, C, J H, A, J Z, C, A Z, C, J Z, A, J C, A, J
  - (b) Since each of the 10 samples of size three is equally likely, there is a 1/10 chance that the sample chosen is the first sample in the list, 1/10 chance that it is the second sample in the list, and 1/10 chance that it is the tenth sample in the list.
  - (c) (i) Make five slips of paper with each airline on one slip. Draw three slips at random. (ii) Make 10 slips of paper, each having one of the combinations in part (a). Draw one slip at random. (iii) Number the five airlines from 1 to 5. Use a random number table or random number generator to obtain three distinct random numbers between 1 and 5, inclusive.
  - (d) Your method and result may differ from ours. We rolled a die (ignoring 6's and duplicates) and got 2, 5, 2, 6, 4. Ignoring duplicates and numbers greater than five, our sample consists of Horizon, Jazz, and Alaska Airlines.
- 17. (a) Table I can be employed to obtain a sample of 15 random numbers between 1 and 100 as follows. First, I pick a random starting point by closing my eyes and putting my finger down on the table.

My finger falls on three digits located at the intersection of a line with three columns. (Notice that the first column of digits is labeled "00" rather than "01".) This is my starting point.

I now go down the table and record all three-digit numbers appearing directly beneath the first three-digit number that are between 001 and 100 inclusive. I throw out numbers between 101 and 999, inclusive. I also discard the number 0000. When the bottom of the column is reached, I move over to the next sequence of three digits and work my way back up the table. Continue in this manner. When 10 distinct three-digit numbers have been recorded, the sample is complete.

- (b) Starting in row 10, columns 7-9, we skip 484, 797, record 082, skip 586, 653, 452, 552, 155, record 008, skip 765, move to the right and record 016, skip 534, 593, 964, 667, 452, 432, 594, 950, 670, record 001, skip 581, 577, 408, 948, 807, 862, 407, record 047, skip 977, move to the right, skip 422 and all of the rest of the numbers in that column, move to the right, skip 732, 192, record 094, skip 615 and all of the rest of the numbers in that column, move to the right, record 097, skip 673, record 074, skip 469, 822, record 052, skip 397, 468, 741, 566, 470, record 076, 098, skip 883, 378, 154, 102, record 003, skip 802, 841, move to the right, skip 243, 198, 411, record 089, skip 701, 305, 638, 654, record 041, skip 753, 790, record 063.
  - The final list of numbers is 82, 8, 16, 1, 47, 94, 97, 74, 52, 76, 98, 3, 89, 41, 63.
- (c) Using Minitab, our results were the numbers 46, 99, 90, 31, 75, 98, 79, 14, 44, 13, 66, 49, 37, 87, 73, 26, 61, 71, 72, 2. Thus our sample consists of the first 15 numbers 46, 99, 90, 31, 75, 98, 79, 14, 44, 13, 66, 49, 37, 87, 73. Your sample may be different.
- 18. The statement under the vote is a disclaimer as to the validity of the survey. Since the vote reflects only the responses of volunteers who chose to vote, it cannot be regarded as representative of the public in general, some of which do not use the Internet, nor as representative of Internet users since the sample was not chosen at random from either group.
- 19. The data in this study were clearly not collected via a controlled experiment in which some participants were forced to do crossword puzzles, practice musical instruments, play board games, or read while others were not allowed to do any of those activities. Therefore, any data relative to these activities and dementia arose as a result of observing whether or not the subjects in the study carried out any of those activities and whether or no they had some form of dementia. Since this would be an observational study, no statement of cause and effect can rightfully be made.
- 20. The researchers did not impose or manipulate any of the conditions of this study. They didn't decide who had cancer, who didn't have cancer, who had hepatitis B, or who had hepatitis C. This study was an observational study and not a controlled experiment. Observational studies can only reveal an association, not causation. Therefore, the statement in quotes is valid. If the researchers wanted to establish causation, they would need a designed experiment.
- 21. (a) Answers will vary, but here is the procedure: (1) Divide the population size, 100, by the sample size 15, and round down to the nearest whole number; this gives 6. (2) Use a table of random numbers (or a similar device) to select a number between 1 and 6, call it k. (3) List every 6th number, starting with k, until 15 numbers are obtained; thus the first number on the required list of 15 numbers is k, the second is k+6, the third is k+12, and so forth (e.g., if k=4, then the numbers on the list are 4, 10, 16, ...).
  - (b) Yes, unless for some reason there is some kind of trend or a cyclical pattern in the listing of the athletes.

- 22. (a) Each category of "Distance from Plant" should be represented in the sample in the same proportion that it is present in the population of City of Durham's water distribution system. 1310/11707 = 0.112. Thus, 11.2% of the sample of 80 water samples should be from "Less than 1.5 miles", 27.0% from "1.5 - less than 3.0 miles", 24.1% from "3.0 - less than 4.5 miles", 13.6% from "4.5 - less than 6.0 miles", 11.5% from "6.0 - less than 7.5 miles", and 12.5% from "7.5 miles or greater". Multiplying each of these fractions by 80 gives us the sample sizes from each category. These sample sizes will not necessarily be integers, so we will need to make some minor adjustments of the results. The first category should have (11.2/100)(80) = 8.96. The second should have (27/100)(80) = 21.6. Similarly, the third, fourth, fifth, and sixth categories should have 19.28, 10.88, 9.2, and 10 for their sample sizes. We round the six sample sizes from the categories to 9, 22, 19, 11, 9, and 10 respectively. We would now randomly select water samples from each region.
- 23. (a) This is a designed experiment.
  - (b) The treatment group consists of the 158 patients who took AVONEX. The control group consists of the 143 patients who were given a placebo. The treatments were the AVONEX and the placebo.
- 24. (a) Experimental units: tomato plants
  - (b) Response variable: yield of tomatoes
  - (c) Factor(s): tomato variety and density of plants
  - (d) Levels of each factor: The four tomato varieties (Harvester, Pusa Early Dwarf, Ife No. 1, and Ibadan Local) would be the levels of variety. The four densities (10,000, 20,000, 30,000, and 40,000 plants/ha) would be the levels of the density.
  - (e) Treatments: Each treatment would be one of the combinations of a variety planted at a given plant density.
- 25. (a) Experimental Units: The children
  - (b) Response variable: Whether or not the child was able to open the bottle
  - (c) Factors: The container designs
  - (d) Levels of each factor: Three (types of containers)
  - (e) Treatments: The container designs
- 26. This is a completely randomized design. All of the experimental units (batches of doughnuts) were assigned at random to the four treatments (four different fats).
- 27. (a) This is a completely randomized design since the 24 cars were randomly assigned to the 4 brands of gasoline.
  - (b) This is a randomized block design. The four different gasoline brands are randomly assigned to the four cars in each of the six car model groups. The blocks are the six groups of four identical cars each.
  - (c) If the purpose is to learn about the mileage rating of one particular car model with each of the four gasoline brands, then the completely randomized design is appropriate. But if the purpose is to learn about the performance of the gasoline across a variety of cars (and this seems more reasonable), then the randomized block design is more appropriate and will allow the researcher to determine the effect of car model as well as of gasoline type on the mileage obtained.

#### Case Study: Top Films of All Time

- (a) The population of interest in the AFI survey is the population of film artists, critics, and historians.
- (b) The sample is the 1500 film artists, critics, and historians polled.
- (c) No. The population of all American moviegoers includes many people who are not film artists, critics, nor historians. Furthermore, these members of the film community have very specialized interests and possibly different viewpoints as to what constitutes a great actor or actress than many others in the American movie-going population.
- (d) Descriptive. It merely describes the opinion of those in the sample without trying to draw an inference about the opinions of all moviegoers.
- (e) Inferential. This statement would be an attempt to draw an inference about the opinion of all artists, historians, and critics based on the opinions of those 1500 people who were interviewed.

#### CHAPTER 2 SOLUTIONS

#### Exercises 2.1

- (a) Hair color, model of car, and brand of popcorn are qualitative variables.
- (b) Number of eggs in a nest, number of cases of flu, and number of employees are discrete, quantitative variables.
- (c) Temperature, weight, and time are quantitative continuous variables.
- (a) A qualitative variable is a nonnumerically valued variable. Its possible "values" are descriptive (e.g., color, name, gender).
- (b) A discrete, quantitative variable is one whose possible values can be listed. It is usually obtained by counting rather than by measuring.
- (c) A continuous, quantitative variable is one whose possible values form some interval of numbers. It usually results from measuring.
- (a) Qualitative data result from observing and recording values of a qualitative variable, such as, color or shape.
- (b) Discrete, quantitative data are values of a discrete quantitative variable. Values usually result from counting something.
- (c) Continuous, quantitative data are values of a continuous variable. Values are usually the result of measuring something such as temperature that can take on any value in a given interval.

The classification of data is important because it will help you choose the correct statistical method for analyzing the data.

Of qualitative and quantitative (discrete and continuous) types of data, only qualitative yields nonnumerical data.

- (a) The first column consists of *quantitative*, *discrete* data. This column provides ranks of the highest recorded temperature for each continent.
  - (b) The second column consists of *qualitative* data since continent names are nonnumerical.
  - (c) The fourth column consists of *quantitative*, *continuous* data. This column provides the highest recorded temperatures for the continents in degrees Farenheit.
  - (d) The information that Death Valley is in the United States is qualitative data since country in which a place is located is nonnumerical.
- (a) The first column consists of *quantitative*, *continuous* data. This column provides the time that the earthquake occurred.
  - (b) The second column consists of *quantitative*, *continuous* data. This column provides the magnitude of each earthquake.
  - (c) The third column consists of *quantitative*, *continuous* data. This column provides the depth of each earthquake in kilometers.
  - (d) The fourth column consists of *quantitative*, *discrete* data. This column provides the number of stations that reported activity on the earthquake.
  - (e) The fifth column consists of *qualitative* data since the region of the location of each earthquake is nonnumerical.
- (a) The first column consists of *quantitative*, *discrete* data. This column provides ranks of the top ten IPOs in the United States.

- (b) The second column consists of *qualitative* data since company names are nonnumerical.
- (c) The third column consists of quantitative, discrete data. Since money involves discrete units, such as dollars and cents, the data is discrete, although, for all practical purposes, this data might be considered quantitative continuous data.
- (d) The information that Facebook is a social networking business is qualitative data since type of business is nonnumerical.
- (a) The first column consists of quantitative, discrete data. This column provides the ranks of the deceased celebrities with the top 10 earnings.
  - (b) The second column consists of qualitative data since names are nonnumerical.
  - (c) The third column consists of *quantitative*, *discrete* data, the earnings of the celebrities. Since money involves discrete units, such as dollars and cents, the data is discrete, although, for all practical purposes, this data might be considered quantitative continuous data
- (a) The first column consists of *quantitative*, *discrete* data. This column provides the ranks of the top 10 universities for 2012-2013.
  - (b) The second column consists of *qualitative* data since names of the institutions are nonnumerical.
  - (c) The third column consists of quantitative, continuous data. This column provides the overall score of the top 10 universities for 2012-2013.
- (a) The first column contains types of products. They are *qualitative* data since they are nonnumerical.
  - (b) The second column contains number of units shipped in the millions. These are whole numbers and are quantitative, discrete.
  - (c) The third column contains money values. Technically, these are quantitative, discrete data since there are gaps between possible values at the cent level. For all practical purposes, however, these are quantitative, continuous data.
- Player name, team, and position are nonnumerical and are therefore qualitative data. The number of runs batted in, or RBI, are whole numbers and are therefore quantitative, discrete. Weight is quantitative, continuous.
- The first column contains quantitative, discrete data in the form of ranks. These are whole numbers. The second and third columns contain qualitative data in the form of names. The last column contains the rating of the program which is quantitative, continuous.
- The first column is *qualitative* since it is nonnumerical. The second and third columns are *quantitative*, *discrete* since they report the number of grants and applications received. The last column is *quantitative*, continuous since it reports the success rate of the grants.
- The first column is *quantitative*, *discrete* since it is reporting a rank. The second and third columns are *qualitative* since make/model and type are nonnumerical. The last column is *quantitative*, *continuous* since it is reporting mileage.
- Of the eight items presented, only high school class rank involves ordinal data. The rank is ordinal data.

#### Exercises 2.2

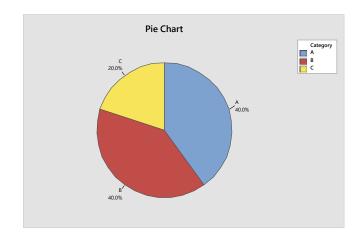
A frequency distribution of qualitative data is a table that lists the distinct values of data and their frequencies. It is useful to organize the data and make it easier to understand.

- (a) The frequency of a class is the number of observations in the class, whereas, the relative frequency of a class is the ratio of the class frequency to the total number of observations.
- (b) The percentage of a class is 100 times the relative frequency of the class. Equivalently, the relative frequency of a class is the percentage of the class expressed as a decimal.
- (a) True. Having identical frequency distributions implies that the total number of observations and the numbers of observations in each class are identical. Thus, the relative frequencies will also be identical.
- (b) False. Having identical relative frequency distributions means that the ratio of the count in each class to the total is the same for both frequency distributions. However, one distribution may have twice (or some other multiple) the total number of observations as the other. For example, two distributions with counts of 5, 4, 1 and 10, 8, 2 would be different, but would have the same relative frequency distribution.
- (c) If the two data sets have the same number of observations, either a frequency distribution or a relative-frequency distribution is suitable. If, however, the two data sets have different numbers of observations, using relative-frequency distributions is more appropriate because the total of each set of relative frequencies is 1, putting both distributions on the same basis for comparison.

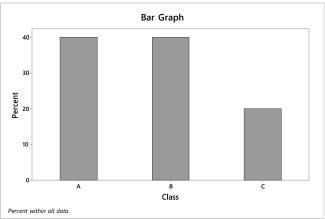
#### **2.20** (a) - (b)

The classes are presented in column 1. The frequency distribution of the classes is presented in column 2. Dividing each frequency by the total number of observations, which is 5, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

Class	Frequency	Relative Frequency
А	2	0.40
В	2	0.40
С	1	0.20
	5	1.00



(d) We use the bar chart to show the relative frequency with which each class occurs. The result is

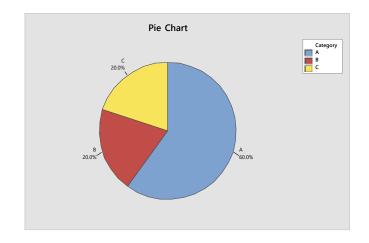


#### **2.21** (a) - (b)

The classes are presented in column 1. The frequency distribution of the classes is presented in column 2. Dividing each frequency by the total number of observations, which is 5, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

Class	Frequency	Relative Frequency
A	3	0.60
В	1	0.20
С	1	0.20
	5	1.00

25



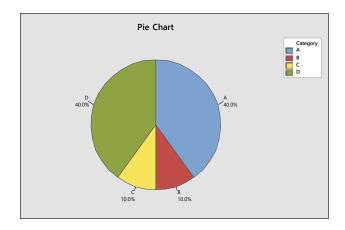
(d) We use the bar chart to show the relative frequency with which each class occurs. The result is  $\frac{1}{2}$ 



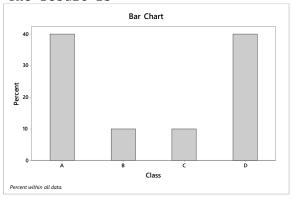
#### **2.22** (a) - (b)

The classes are presented in column 1. The frequency distribution of the classes is presented in column 2. Dividing each frequency by the total number of observations, which is 10, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

Class	Frequency	Relative Frequency
А	4	0.40
В	1	0.10
С	1	0.10
D	4	0.40
	10	1.00



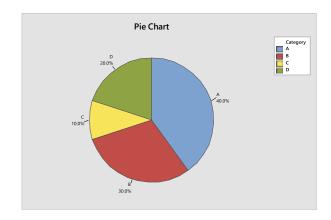
(d) We use the bar chart to show the relative frequency with which each class occurs. The result is  $\frac{1}{2}$ 



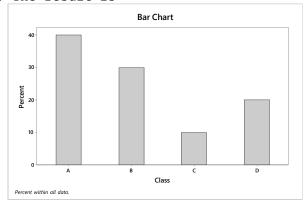
#### 2.23 (a) - (b)

The classes are presented in column 1. The frequency distribution of the classes is presented in column 2. Dividing each frequency by the total number of observations, which is 10, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

Class	Frequency	Relative Frequency
А	4	0.40
В	3	0.30
С	1	0.10
D	2	0.20
	10	1.00



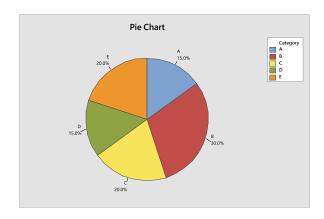
(d) We use the bar chart to show the relative frequency with which each class occurs. The result is  $\frac{1}{2}$ 



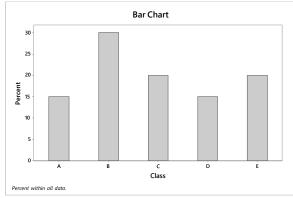
#### **2.24** (a) - (b)

The classes are presented in column 1. The frequency distribution of the classes is presented in column 2. Dividing each frequency by the total number of observations, which is 20, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

Class	Frequency	Relative Frequency
А	3	0.15
В	6	0.30
С	4	0.20
D	3	0.15
E	4	0.20
	20	1.00



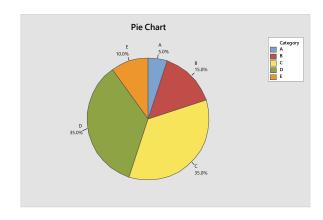
(d) We use the bar chart to show the relative frequency with which each class occurs. The result is



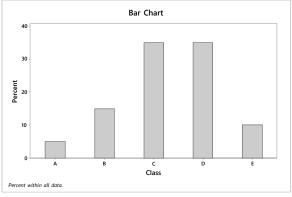
#### **2.25** (a) - (b)

The classes are presented in column 1. The frequency distribution of the classes is presented in column 2. Dividing each frequency by the total number of observations, which is 20, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

Class	Frequency	Relative Frequency
А	1	0.05
В	3	0.15
С	7	0.35
D	7	0.35
E	2	0.10
	20	1.00



(d) We use the bar chart to show the relative frequency with which each class occurs. The result is  $\frac{1}{2}$ 



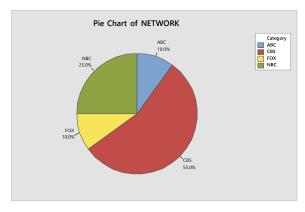
# **2.26** (a) - (b)

The classes are the networks and are presented in column 1. The frequency distribution of the networks is presented in column 2. Dividing each frequency by the total number of shows, which is 20, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

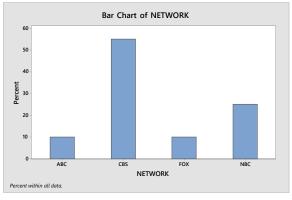
Network	Frequency	Relative Frequency
ABC	2	0.10
CBS	11	0.55
Fox	2	0.10
NBC	5	0.25
	20	1.00

(c) We multiply each of the relative frequencies by 360 degrees to obtain the portion of the pie represented by each network. The result is

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(d) We use the bar chart to show the relative frequency with which each network occurs. The result is



## **2.27** (a) - (b)

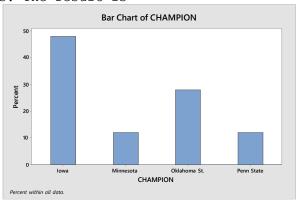
The classes are the NCAA wrestling champions and are presented in column 1. The frequency distribution of the champions is presented in column 2. Dividing each frequency by the total number of champions, which is 25, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

Champion	Frequency	Relative Frequency
Iowa	12	0.48
Penn State	3	0.12
Minnesota	3	0.12
Oklahoma St.	7	0.28
	25	1.00

(b) We multiply each of the relative frequencies by 360 degrees to obtain the portion of the pie represented by each team. The result is

31

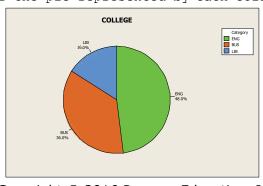
(c) We use the bar chart to show the relative frequency with which each TEAM occurs. The result is



2.28 (a)-(b) The classes are the colleges and are presented in column 1. The frequency distribution of the colleges is presented in column 2. Dividing each frequency by the total number of students in the section of Introduction to Computer Science, which is 25, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

College	Frequency	Relative Frequency
BUS	9	0.36
ENG	12	0.48
LIB	4	0.16
	25	1.00

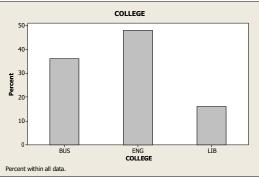
(c) We multiply each of the relative frequencies by 360 degrees to obtain the portion of the pie represented by each college. The result is



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(b) We use the bar chart to show the relative frequency with which each COLLEGE occurs. The result is

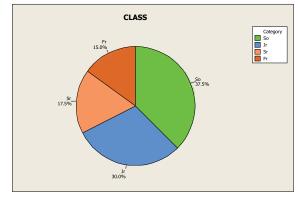


### **2.29** (a) - (b)

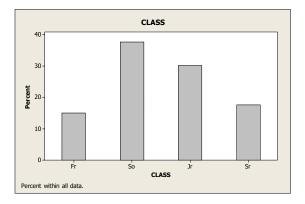
The classes are the class levels and are presented in column 1. The frequency distribution of the class levels is presented in column 2. Dividing each frequency by the total number of students in the introductory statistics class, which is 40, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

Class Level	Frequency	Relative Frequency
Fr	6	0.150
So	15	0.375
Jr	12	0.300
Sr	7	0.175
	40	1.000

(c) We multiply each of the relative frequencies by 360 degrees to obtain the portion of the pie represented by each class level. The result is



(d) We use the bar chart to show the relative frequency with which each CLASS level occurs. The result is

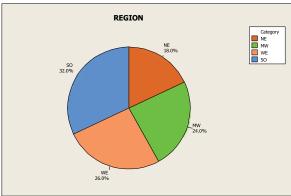


## **2.30** (a) - (b)

The classes are the regions and are presented in column 1. The frequency distribution of the regions is presented in column 2. Dividing each frequency by the total number of states, which is 50, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

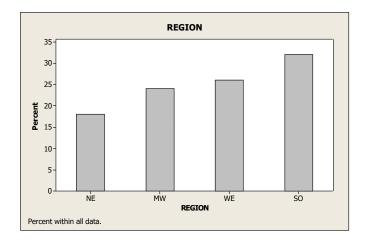
Class Level	Frequency	Relative Frequency
NE	9	0.18
MW	12	0.24
SO	16	0.32
WE	13	0.26
	50	1.00

(c) We multiply each of the relative frequencies by 360 degrees to obtain the portion of the pie represented by each region. The result is



(d) We use the bar chart to show the relative frequency with which each REGION occurs. The result is

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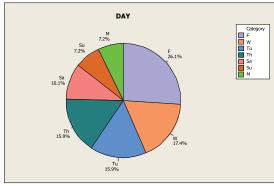


#### **2.31** (a) - (b)

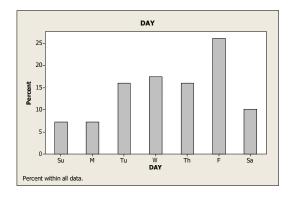
The classes are the days and are presented in column 1. The frequency distribution of the days is presented in column 2. Dividing each frequency by the total number road rage incidents, which is 69, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

Class Level	Frequency	Relative Frequency
Su	5	0.0725
M	5	0.0725
Tu	11	0.1594
W	12	0.1739
Th	11	0.1594
F	18	0.2609
Sa	7	0.1014
	69	1.0000

(c) We multiply each of the relative frequencies by 360 degrees to obtain the portion of the pie represented by each day. The result is



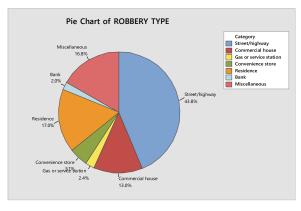
(d) We use the bar chart to show the relative frequency with which each DAY occurs. The result is



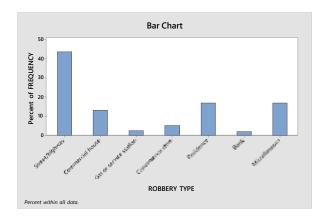
(a) We find each of the relative frequencies by dividing each of the frequencies by the total frequency of 291,176. Due to rounding, the sum of the relative frequency column is 0.9999.

Robbery Type	Frequency	Relative Frequency
Street/highway	127,403	0.4375
Commercial house	37 <b>,</b> 885	0.1301
Gas or service station	7,009	0.0241
Convenience store	14,863	0.0510
Residence	49,361	0.1695
Bank	5 <b>,</b> 777	0.0198
Miscellaneous	48,878	0.1679
	291,176	0.9999

(b) We multiply each of the relative frequencies by 360 degrees to obtain the portion of the pie represented by each robbery type. The result is



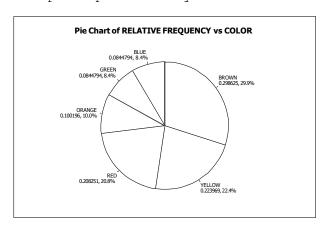
(c) We use the bar chart to show the relative frequency with which each robbery type occurs. The result is  $\frac{1}{2}$ 



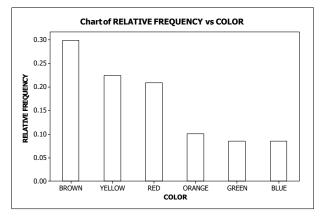
(a) We find the relative frequencies by dividing each of the frequencies by the total sample size of 509.

Color	Frequency	Relative Frequency
Brown	152	0.2986
Yellow	114	0.2240
Red	106	0.2083
Orange	51	0.1002
Green	43	0.0845
Blue	43	0.0845
	509	1.0000

(b) We multiply each of the relative frequencies by 360 degrees to obtain the portion of the pie represented by each color of M&M. The result is



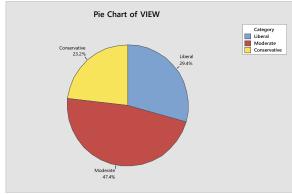
(c) We use the bar chart to show the relative frequency with which each color occurs. The result is  $\frac{1}{2}$ 



(a) We find the relative frequencies by dividing each of the frequencies by the total sample size of 500.

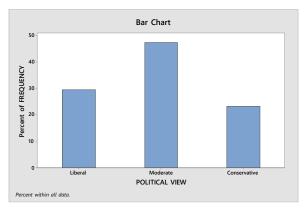
Political View	Frequency	Relative
		Frequency
Liberal	147	0.294
Moderate	237	0.474
Conservative	116	0.232
	500	1.000

(b) We multiply each of the relative frequencies by 360 degrees to obtain the portion of the pie represented by each political view. The result is



(c) We use the bar chart to show the relative frequency with which each political view occurs. The result is

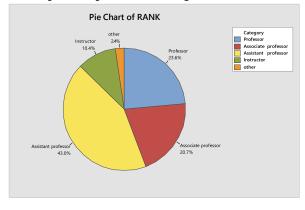
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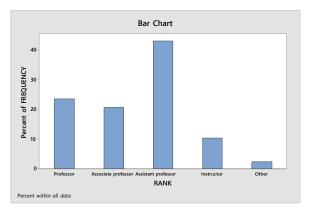
(a) We find the relative frequencies by dividing each of the frequencies by the total sample size of 137,925.

Rank	Frequency	Relative
		Frequency
Professor	32,511	0.2357
Associate professor	28 <b>,</b> 572	0.2072
Assistant professor	59 <b>,</b> 277	0.4298
Instructor	14,289	0.1036
Other	3 <b>,</b> 276	0.0238
	137,925	1.0000

(b) We multiply each of the relative frequencies by 360 degrees to obtain the portion of the pie represented by each rank. The result is



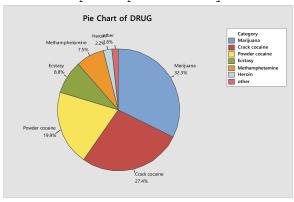
(c) We use the bar chart to show the relative frequency with which each rank occurs. The result is  $% \left( 1\right) =\left( 1\right) +\left( 1\right)$ 



(a) We find the relative frequencies by dividing each of the frequencies by the total sample size of 226. The sum of the relative frequency columns is 0.9999 due to rounding.

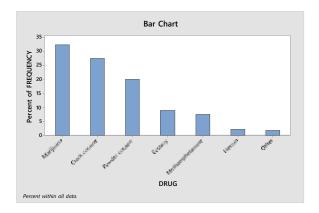
Drug Sold	Frequency	Relative Frequency
Marijuana	73	0.3230
Crack cocaine	62	0.2743
Powder cocaine	45	0.1991
Ecstasy	20	0.0885
Methamphetamine	17	0.0752
Heroin	5	0.0221
Other	4	0.0177
	226	0.9999

(b) We multiply each of the relative frequencies by 360 degrees to obtain the portion of the pie represented by each drug type. The result is



(c) We use the bar chart to show the relative frequency with which each rank occurs. The result is

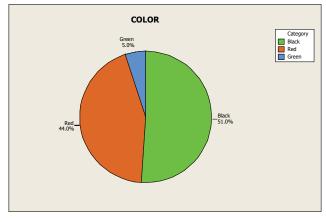
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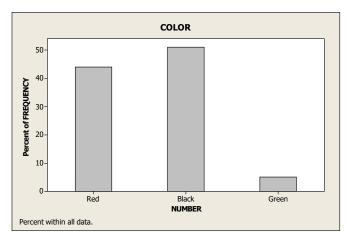
(a) We first find the relative frequencies by dividing each of the frequencies by the total sample size of 200.

Color	Frequency	Relative Frequency
Red	88	0.44
Black	102	0.51
Green	10	0.05
	200	1.00

(b) We multiply each of the relative frequencies by 360 degrees to obtain the portion of the pie represented by each color. The result is



(c) We use the bar chart to show the relative frequency with which each color occurs. The result is  $\frac{1}{2}$ 



(a) The classes are the different levels of health status and are presented in column 1. The frequency distribution of health status is presented in column 2. Dividing each frequency by the total number of people, which is 188.3 million for persons aged 19-64 and 41.5 million for persons aged 65 and over, results in each class's relative frequency. The relative frequency distribution is presented in column 3.

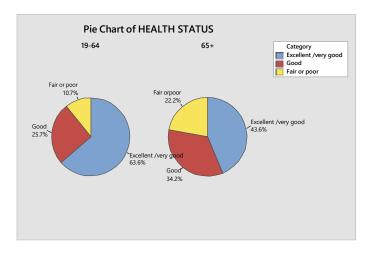
For People aged 19-64

Health Status	Frequency	Relative
Three librations are seen as a	110 7	Frequency
Excellent or very good	119.7	0.636
Good	48.4	0.257
Fair or poor	20.2	0.107
	188.3	1.000

For People aged 65 and over

	1	1
Health Status	Frequency	Relative
		Frequency
Excellent or very good	18.1	0.436
Good	14.2	0.342
Fair or poor	9.2	0.222
	41.5	1.000

(b) We multiply each of the relative frequencies by 360 degrees to obtain the portion of each pie represented by each health status. The results are



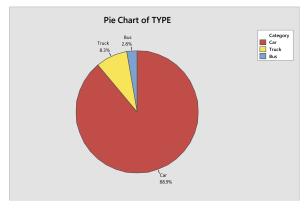
- (c) A bigger proportion of persons aged 19-64 have a health status of excellent or very good than persons aged 65 and over. A smaller proportion of persons aged 19-64 have a health status of fair or poor than persons aged 65 and over.
- (a) Using Minitab, retrieve the data from the WeissStats Resource Site.

  Column 1 contains the type of the vehicle. From the tool bar, select

Stat < Tables < Tally Individual Variables, double-click on TYPE in the first box so that TYPE appears in the Variables box, put a check mark next to Counts and Percents under Display, and click OK. The result is

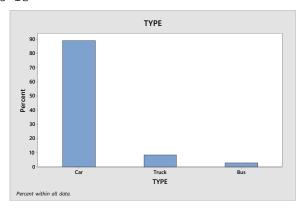
TYPE	Count	Percent
Bus	21	2.80
Car	667	88.93
Truck	62	8.27
N=	750	

- (b) The relative frequencies were calculated in part(a) by putting a check mark next to Percents. 2.8% of the vehicles were busses, 88.93% of the vehicles were cars, and 8.27% of the vehicles were trucks.
- (c) Using Minitab, select **Graph < Pie Chart**, check Chart counts of unique values, double-click on TYPE in the first box so that TYPE appears in the Categorical Variables box. Click Pie Options, check decreasing volume, click OK. Click Labels, enter TYPE in for the title, click Slice Labels, check Category Name, Percent, and Draw a line from label to slice, Click OK twice. The result is



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(d) Using Minitab, select Graph Sar Chart, select Counts of unique values, select Simple option, click OK. Double-click on TYPE in the first box so that TYPE appears in the Categorical Variables box. Select Chart Options, check decreasing Y, check show Y as a percent, click OK. Select Labels, enter in TYPE as the title. Click OK twice. The result is



(a) Using Minitab, retrieve the data from the WeissStats Resource Site. Column 2 contains the marital status and column 3 contains the number of drinks per

month

From the tool bar, select Stat < Tables < Tally

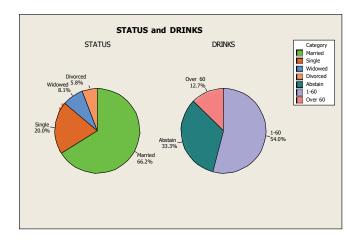
Individual Variables, double-click on STATUS and DRINKS in the first box so that both STATUS and DRINKS appear in the Variables box, put a check mark next to Counts and Percents under Display, and click OK.

The results are

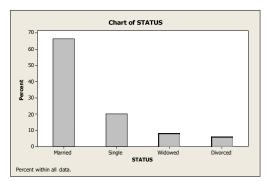
STATUS	Count	Percent	DRINKS	Count	Percent
Single	354	19.98	Abstain	590	33.30
Married	1173	66.20	1-60	957	54.01
Widowed	143	8.07	Over 60	225	12.70
Divorce	d 102	5.76	N=	1772	
N=	1772				

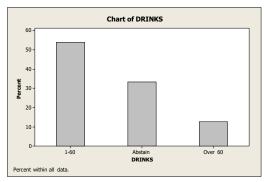
- (b) The relative frequencies were calculated in part(a) by putting a check mark next to Percents. For the STATUS variable; 19.98% of the US Adults are single, 66.20% are married, 8.07% are widowed, and 5.76% are divorced. For the DRINKS variable, 33.30% of US Adults abstain from drinking, 54.01% have 1-60 drinks per month, and 12.70% have over 60 drinks per month.
- (c) Using Minitab, select **Graph < Pie Chart**, check Chart counts of unique values, double-click on STATUS and DRINKS in the first box so that STATUS and DRINKS appear in the Categorical Variables box. Click Pie Options, check decreasing volume, click OK. Click Multiple Graphs, check On the Same Graphs, Click OK. Click Labels, click Slice Labels, check Category Name, Percent, and Draw a line from label to slice, Click OK twice. The results are

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(d) Using Minitab, select **Graph ≺ Bar Chart**, select Counts of unique values, select Simple option, click OK. Double-click on STATUS and DRINKS in the first box so that STATUS and DRINKS appear in the Categorical Variables box. Select Chart Options, check decreasing Y, check show Y as a percent, click OK. Click OK twice. The results are





(a) Using Minitab, retrieve the data from the WeissStats Resource Site.

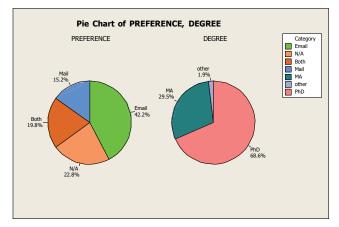
Column 2 contains the preference for how the members want to receive the ballots and column 3 contains the highest degree obtained by the

members. From the tool bar, select  $\mathsf{Stat} < \mathsf{Tables} < \mathsf{Tally}$  Individual Variables, double-click on PREFERENCE and DEGREE in the first box so that both PREFERENCE and DEGREE appear in the Variables box, put a check mark next to Counts and Percents under Display, and click  $\mathsf{OK}$ . The results are

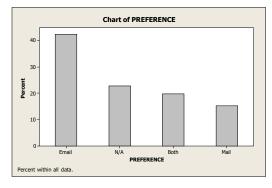
PREFERENCE	Count	Percent	DEGREE	Count	Percent
Both	112	19.79	MA	167	29.51
Email	239	42.23	Other	11	1.94
Mail	86	15.19	PhD	388	68.55
N/A	129	22.79	N=	566	
N=	566				

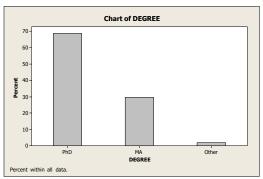
- (b) The relative frequencies were calculated in part(a) by putting a check mark next to Percents. For the PREFERENCE variable; 19.79% of the members prefer to receive the ballot by both e-mail and mail, 42.23% prefer e-mail, 15.19% prefer mail, and 22.79% didn't list a preference. For the Degree variable; 29.51% obtained a Master's degree, 68.55% obtained a PhD, and 1.94% received a different degree.
- (c) Using Minitab, select **Graph < Pie Chart**, check Chart counts of unique values, double-click on PREFERENCE and DEGREE in the first box so that PREFERENCE and DEGREE appear in the Categorical Variables box. Click

Pie Options, check decreasing volume, click OK. Click Multiple Graphs, check On the Same Graphs, Click OK. Click Labels, click Slice Labels, check Category Name, Percent, and Draw a line from label to slice, Click OK twice. The results are



(d) Using Minitab, select Graph < Bar Chart, select Counts of unique values, select Simple option, click OK. Double-click on PREFERENCE and DEGREE in the first box so that PREFERENCE and DEGREE appear in the Categorical Variables box. Select Chart Options, check decreasing Y, check show Y as a percent, click OK. Click OK twice. The results are





### Exercises 2.3

One important reason for grouping data is that grouping often makes a large and complicated set of data more compact and easier to understand.

For class limits, marks, cutpoints and midpoints to make sense, data must be numerical. They do not make sense for qualitative data classes because such data are nonnumerical.

The most important guidelines in choosing the classes for grouping a data set are: (1) the number of classes should be small enough to provide an effective summary, but large enough to display the relevant characteristics of the data; (2) each observation must belong to one, and only one, class; and (3) whenever feasible, all classes should have the same width.

In the first method for depicting classes called cutpoint grouping, we used the notation  $\bf a$  - under  $\bf b$  to mean values that are greater than or equal to  $\bf a$  and up to, but not including  $\bf b$ , such as 30 - under 40 to mean a range of values greater than or equal to 30, but strictly less than 40. In the alternate method called limit grouping, we used the notation  $\bf a$ - $\bf b$  to indicate a class that extends from  $\bf a$  to  $\bf b$ , including both. For example, 30-39 is a class that includes both 30 and 39. The alternate method is

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- especially appropriate when all of the data values are integers. If the data include values like 39.7 or 39.93, the first method is more advantageous since the cutpoints remain integers; whereas, in the alternate method, the upper limits for each class would have to be expressed in decimal form such as 39.9 or 39.99.
- (a) For continuous data displayed to one or more decimal places, using the cutpoint grouping is best since the description of the classes is simpler, regardless of the number of decimal places displayed.
  - (b) For discrete data with relatively few distinct observations, the single value grouping is best since either of the other two methods would result in combining some of those distinct values into single classes, resulting in too few classes, possibly less than 5.
- For limit grouping, we find the class mark, which is the average of the lower and upper class limit. For cutpoint grouping, we find the class midpoint, which is the average of the two cutpoints.
- A frequency histogram shows the actual frequencies on the vertical axis; whereas, the relative frequency histogram always shows proportions (between 0 and 1) or percentages (between 0 and 100) on the vertical axis.
- An advantage of the frequency histogram over a frequency distribution is that it is possible to get an overall view of the data more easily. A disadvantage of the frequency histogram is that it may not be possible to determine exact frequencies for the classes when the number of observations is large.
- By showing the lower class limits (or cutpoints) on the horizontal axis, the range of possible data values in each class is immediately known and the class mark (or midpoint) can be quickly determined. This is particularly helpful if it is not convenient to make all classes the same width. The use of the class mark (or midpoint) is appropriate when each class consists of a single value (which is, of course, also the midpoint). Use of the class marks (or midpoints) is not appropriate in other situations since it may be difficult to determine the location of the class limits (or cutpoints) from the values of the class marks (or midpoints), particularly if the class marks (or midpoints) are not evenly spaced. Class Marks (or midpoints) cannot be used if there is an open class.
- If the classes consist of single values, stem-and-leaf diagrams and frequency histograms are equally useful. If only one diagram is needed and the classes consist of more than one value, the stem-and-leaf diagram allows one to retrieve all of the original data values whereas the frequency histogram does not. If two or more sets of data of different sizes are to be compared, the relative frequency histogram is advantageous because all of the diagrams to be compared will have the same total relative frequency of 1.00. Finally, stem-and-leaf diagrams are not very useful with very large data sets and may present problems with data having many digits in each number.
- The histogram (especially one using relative frequencies) is generally preferable. Data sets with a large number of observations may result in a stem of the stem-and-leaf diagram having more leaves than will fit on the line. In that case, the histogram would be preferable.
- You can reconstruct the stem-and-leaf diagram using two lines per stem. For example, instead of listing all of the values from 10 to 19 on a '1' stem, you can make two '1' stems. On the first, you record the values from 10 to 14 and on the second, the values from 15 to 19. If there are still two few stems, you can reconstruct the diagram using five lines per stem, recording 10 and 11 on the first line, 12 and 13 on the second, and so on.

For the number of bedrooms per single-family dwelling, single-value grouping is probably the best because the data is discrete with relatively few distinct observations.

For the ages of householders, given as a whole number, limit grouping is probably the best because the data are given as whole numbers and there are probably too many distinct observations to list them as single-value grouping.

For additional sleep obtained by a sample of 100 patients by using a particular brand of sleeping pill, cutpoint grouping is probably the best because the data is continuous and the data was recorded to the nearest tenth of an hour.

For the number of automobiles per family, single-value grouping is probably the best because the data is discrete with relatively few distinct observations.

For gas mileages, rounded to the nearest number of miles per gallon, limit grouping is probably the best because the data are given as whole numbers and there are probably too many distinct observations to list them as single-value grouping.

For carapace length for a sample of giant tarantulas, cutpoint grouping is probably the best because the data is continuous and the data was recorded to the nearest hundredth of a millimeter.

(a) Since the data values range from 1 to 4, we construct a table with classes based on a single value. The resulting table follows.

Single Value Class	Frequency
1	2
2	0
3	5
4	3
	10

(b) To get the relative frequencies, divide each frequency by the sample size of 10.

Single Value Class	Relative Frequency
1	0.20
2	0.00
3	0.50
4	0.30
	1.00

Figure (a)

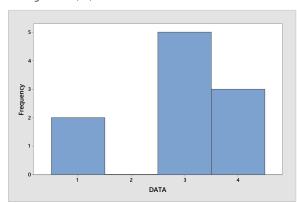
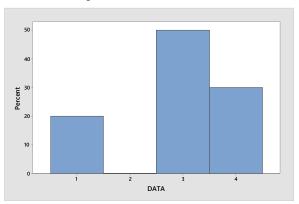


Figure (b)



- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The middle of each histogram bar is placed directly over the single numerical value represented by the class. Also, the height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.
- (a) Since the data values range from 1 to 4, we construct a table with classes based on a single value. The resulting table follows.

Single Value Class	Frequency
1	2
2	2
3	5
4	1
	10

(b) To get the relative frequencies, divide each frequency by the sample size of  $10\,.$ 

Single Value Class	Relative Frequency
1	0.20
2	0.20
3	0.50
4	0.10
	1.00

Figure (a)

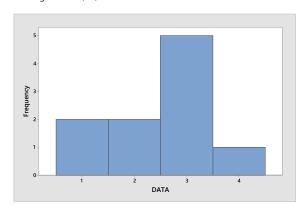
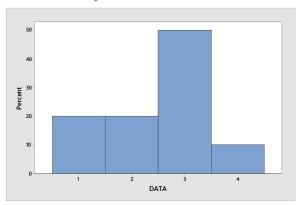


Figure (b)



- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The middle of each histogram bar is placed directly over the single numerical value represented by the class. Also, the height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.
- (a) Since the data values range from 0 to 4, we construct a table with classes based on a single value. The resulting table follows.

Single Value Class	Frequency
0	2
1	8
2	7
3	2
4	1
	20

(b) To get the relative frequencies, divide each frequency by the sample size of 20.

Single Value Class	Relative Frequency
0	0.10
1	0.40
2	0.35
3	0.10
4	0.05
	1.00

Figure (a)

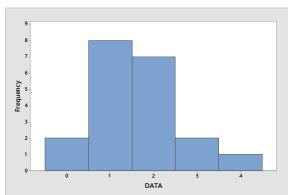
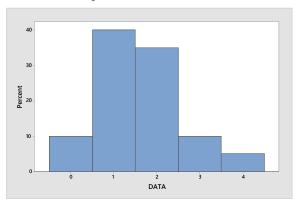


Figure (b)



- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The middle of each histogram bar is placed directly over the single numerical value represented by the class. Also, the height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.
- (a) Since the data values range from 0 to 4, we construct a table with classes based on a single value. The resulting table follows.

Single Value Class	Frequency
0	1
1	1
2	4
3	8
4	6
	20

(b) To get the relative frequencies, divide each frequency by the sample size of 20.

Single Value Class	Relative Frequency
0	0.05
1	0.05
2	0.20
3	0.40
4	0.30
	1.00

Figure (a)

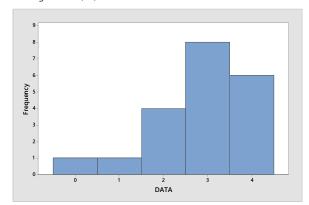
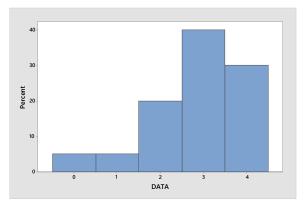


Figure (b)



- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The middle of each histogram bar is placed directly over the single numerical value represented by the class. Also, the height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.
- (a) The first class to construct is 0-9. The width of all the classes is 10, so the next class would be 10-19. The classes are presented in column 1. The last class to construct is 40-49, since the largest single data value is 41. The tallied results are presented in column 2, which lists the frequencies.

Limit Grouping Classes	Frequency
0-9	3
10-19	3
20-29	5
30-39	8
40-49	1
	20

(b) Dividing each frequency by the total number of observations, which is 20, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2. The resulting table follows.

Limit Grouping Classes	Relative Frequency
0-9	0.15
10-19	0.15
20-29	0.25
30-39	0.40
40-49	0.05
	1.00

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution presented in part (a) of this exercise. The lower class limits of column 1 are used to label the horizontal axis of the frequency histogram. The height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram.

The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

Figure (a)

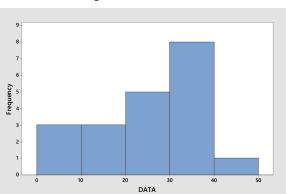
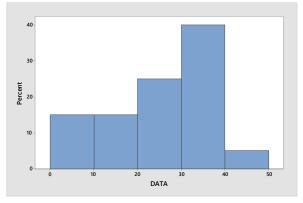


Figure (b)



(a) The first class to construct is 0-4. The width of all the classes is 5, so the next class would be 5-9. The classes are presented in column 1. The last class to construct is 25-29, since the largest single data value is 26. The tallied results are presented in column 2, which lists the frequencies.

Limit Grouping Classes	Frequency
0-4	5
5-9	5
10-14	1
15-19	4
20-24	2
25-29	3
	20

(b) Dividing each frequency by the total number of observations, which is 20, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2. The resulting table follows.

Limit Grouping Classes	Relative Frequency
0-4	0.25
5-9	0.25
10-14	0.05
15-19	0.20
20-24	0.10
25-29	0.15
	1.00

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution presented in part (a) of this exercise. The lower class limits of column 1 are used to label the horizontal axis of the frequency histogram. The height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

Figure (a)

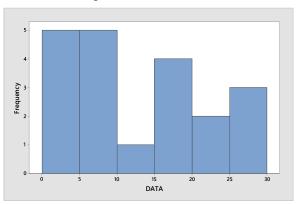
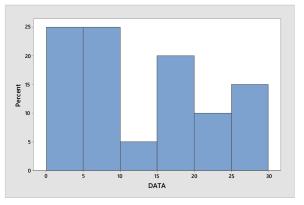


Figure (b)



(a) The first class to construct is 30-36. The width of all the classes is 7, so the next class would be 37-43. The classes are presented in column 1. The last class to construct is 72-78, since the largest single data value is 78. The tallied results are presented in column 2, which lists the frequencies.

Limit Grouping Classes	Frequency
30-36	7
37-43	2
44-50	5
51-57	4
58-64	3
65-71	2
72-78	2
	25

(b) Dividing each frequency by the total number of observations, which is 25, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2. The resulting table follows.

Limit Grouping Classes	Relative Frequency
30-36	0.28
37-43	0.08
44-50	0.20
51-57	0.16
58-64	0.12
65-71	0.08
72-78	0.08
	1.00

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution presented in part (a) of this exercise. The lower class limits of column 1 are used to label the horizontal axis of the frequency histogram. The height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

Figure (a)

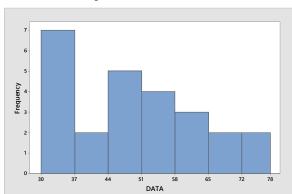
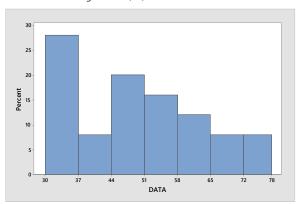


Figure (b)



(a) The first class to construct is 50-59. The width of all the classes is 10, so the next class would be 60-69. The classes are presented in column 1. The last class to construct is 90-99, since the largest single data value is 98. The tallied results are presented in column 2, which lists the frequencies.

Limit Grouping Classes	Frequency
50-59	6
60-69	3
70-79	6
80-89	7
90-99	3
	25

(b) Dividing each frequency by the total number of observations, which is 25, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2. The resulting table follows.

Limit Grouping Classes	Relative Frequency
50-59	0.24
60-69	0.12
70-79	0.24
80-89	0.28
90-99	0.12
	1.00

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution presented in part (a) of this exercise. The lower class limits of column 1 are used to label the horizontal axis of the frequency histogram. The height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

Figure (a)

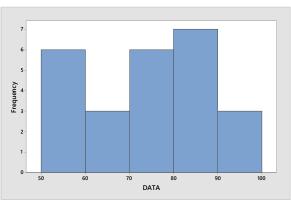
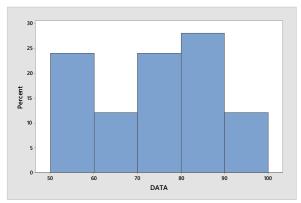


Figure (b)



(a) The first class to construct is 10 - under 15. Since all classes are to be of equal width 5, the second class is 15 - under 20. All of the classes are presented in column 1. The last class to construct is 35 - under 40, since the largest single data value is 38.8. The results of the tallying are presented in column 2, which lists the frequencies.

Cutpoint Grouping Classes	Frequency
10 - under 15	4
15 - under 20	5
20 - under 25	7
25 - under 30	3
30 - under 35	0
35 - under 40	1
	20

(b) Dividing each frequency by the total number of observations, which is 20, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2.

Cutpoint Grouping Classes	Relative Frequency
10 - under 15	0.20
15 - under 20	0.25
20 - under 25	0.35
25 - under 30	0.15
30 - under 35	0.00
35 - under 40	0.05
	1.00

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution obtained in part (a) of this exercise. The cutpoints are used to label the horizontal axis. Also, the height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution obtained in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

Figure (a)

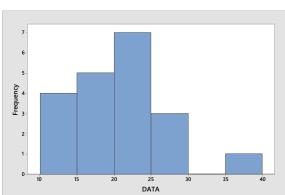
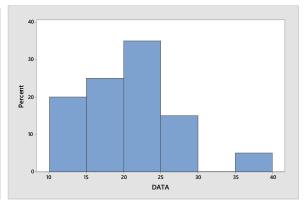


Figure (b)



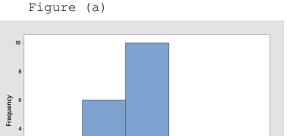
(a) The first class to construct is 40 - under 46. Since all classes are to be of equal width 6, the second class is 46 - under 52. All of the classes are presented in column 1. The last class to construct is 64 - under 70, since the largest single data value is 65.4. The results of the tallying are presented in column 2, which lists the frequencies.

Cutpoint Grouping Classes	Frequency
40 - under 46	3
46 - under 52	6
52 - under 58	10
58 - under 64	0
64 - under 70	1
	20

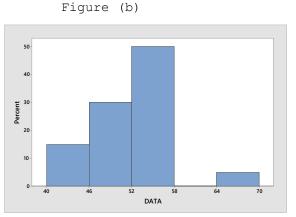
(b) Dividing each frequency by the total number of observations, which is 20, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2.

Cutpoint Grouping	Relative Frequency
40 - under 46	0.15
46 - under 52	0.30
52 - under 58	0.50
58 - under 64	0.00
64 - under 70	0.05
	1.00

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution obtained in part (a) of this exercise. The cutpoints are used to label the horizontal axis. Also, the height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution obtained in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.



DATA



(a) The first class' midpoint is 0.5 with a width of 1. Therefore, the first class to construct is 0 - under 1. Since all classes are to be of equal width 1, the second class is 1 - under 2. All of the classes are presented in column 1. The last class to construct is 7 - under 8, since the largest single data value is 7.69. The results of the tallying are presented in column 2, which lists the frequencies.

Cutpoint Grouping Classes	Frequency
0 - under 1	3
1 - under 2	11
2 - under 3	4
3 - under 4	1
4 - under 5	2
5 - under 6	1
6 - under 7	2
7 - under 8	1
	25

(b) Dividing each frequency by the total number of observations, which is 25, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2.

Cutpoint Grouping Classes	Relative Frequency
0 - under 1	0.12
1 - under 2	0.44
2 - under 3	0.16
3 - under 4	0.04
4 - under 5	0.08
5 - under 6	0.04
6 - under 7	0.08
7 - under 8	0.04
	1.00

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution obtained in part (a) of this exercise. The cutpoints are used to label the horizontal axis. Also, the height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution obtained in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.



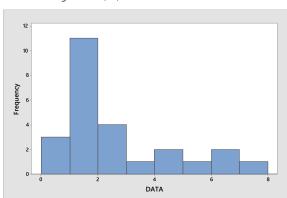
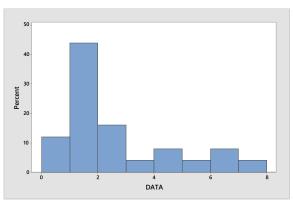


Figure (b)



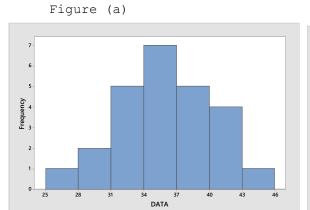
(a) The first class' cutpoint is 25 with a width of 3. Therefore, the first class to construct is 25 - under 28. Since all classes are to be of equal width 3, the second class is 28 - under 31. All of the classes are presented in column 1. The last class to construct is 43 - under 46, since the largest single data value is 43.01. The results of the tallying are presented in column 2, which lists the frequencies.

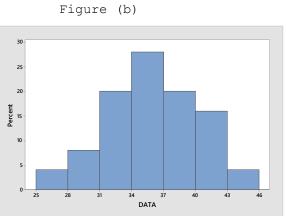
Cutpoint Grouping C	lasses Frequency
25 - under 28	1
28 - under 31	2
31 - under 34	5
34 - under 37	7
37 - under 40	5
40 - under 43	4
43 - under 46	1
	25

(b) Dividing each frequency by the total number of observations, which is 25, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2.

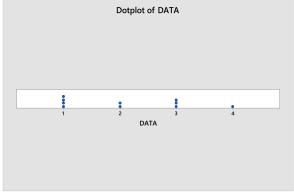
Cutpoint Grouping Classes	Relative Frequency
25 - under 28	0.04
28 - under 31	0.08
31 - under 34	0.20
34 - under 37	0.28
37 - under 40	0.20
40 - under 43	0.16
43 - under 46	0.04
	1.00

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution obtained in part (a) of this exercise. The cutpoints are used to label the horizontal axis. Also, the height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution obtained in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

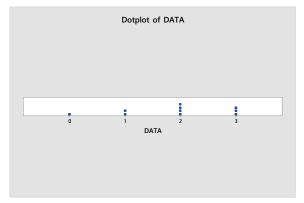




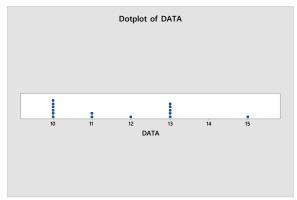
The horizontal axis of this dotplot displays a range of possible values. To complete the dotplot, we go through the data set and record data value by placing a dot over the appropriate value on the horizontal axis.



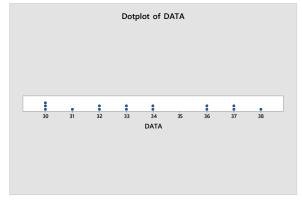
The horizontal axis of this dotplot displays a range of possible values. To complete the dotplot, we go through the data set and record data value by placing a dot over the appropriate value on the horizontal axis.



The horizontal axis of this dotplot displays a range of possible values. To complete the dotplot, we go through the data set and record data value by placing a dot over the appropriate value on the horizontal axis.



The horizontal axis of this dotplot displays a range of possible values. To complete the dotplot, we go through the data set and record data value by placing a dot over the appropriate value on the horizontal axis.



Each data value consists of 2 or 3 digit numbers ranging from 60 to 125. The last digit becomes the leaf and the remaining digits are the stems, so we have stems of 6 to 12. The resulting stem-and-leaf diagram is

6| 0

7 |

8 |

9| 79

10 | 068 11 | 278

12 | 5

Each data value consists of 2 digit numbers ranging from 20 to 62. The last digit becomes the leaf and the remaining digits are the stems, so we have stems of 2 to 6. The resulting stem-and-leaf diagram is

2 | 01

3| 2278

4 | 13

5 | 5

6 | 2

Each data value consists of 1 or 2 digit numbers ranging from 5 to 23. The last digit becomes the leaf and the remaining digits are the stems, so we have stems of 0 to 2. Splitting the stems into five lines per stem, the resulting stem-and-leaf diagram is

Each data value consists of 2 digit numbers ranging from 22 to 46. The last digit becomes the leaf and the remaining digits are the stems, so we have stems of 2 to 4. Splitting the stems into two lines per stem, the resulting stem-and-leaf diagram is

(a) Since the data values range from 0 to 4, we construct a table with classes based on a single value. The resulting table follows.

Number of Siblings	Frequency
0	8
1	17
2	11
3	3
4	1
	40

(b) To get the relative frequencies, divide each frequency by the sample size of 40.

Number of Siblings	Relative Frequency
0	0.200
1	0.425
2	0.275
3	0.075
4	0.025
	1.000

(c) The frequency histogram in Figure (a) is constructed using the frequency distribution presented in part (a) of this exercise. Column 1 demonstrates that the data are grouped using classes based on a single value. These single values in column 1 are used to label the horizontal axis of the frequency histogram. Suitable candidates for vertical axis units in the frequency histogram are the integers within the range 0 through 17, since these are representative of the magnitude and spread of the frequencies presented in column 2. When classes are based on a single value, the middle of each histogram bar is placed directly over the single numerical value represented by the class. Also, the height of each bar in the frequency histogram matches the respective frequency in column 2.

Figure (a)

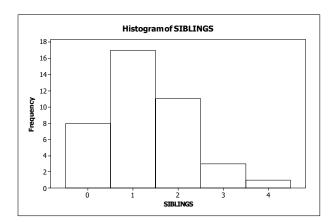
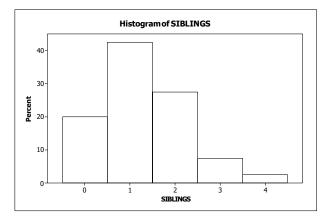


Figure (b)



- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. We notice that the relative frequencies presented in column 2 range in size from 0.025 to 0.425. Thus, suitable candidates for vertical-axis units in the relative-frequency histogram are increments of 0.05 (or 5%), from zero to 0.45 (or 45%). The middle of each histogram bar is placed directly over the single numerical value represented by the class. Also, the height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.
- (a) Since the data values range from 1 to 7, we construct a table with classes based on a single value. The resulting table follows.

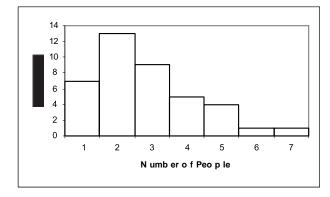
Number of Persons	Frequency
1	7
2	13
3	9
4	5
5	4
6	1
7	1
	40

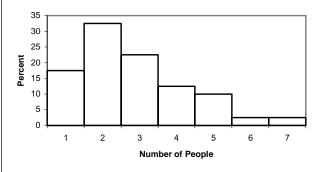
(b) To get the relative frequencies, divide each frequency by the sample size of 40.

Number of Persons	Relative Frequency
1	0.175
2	0.325
3	0.225
4	0.125
5	0.100
6	0.025
7	0.025
	1.000

(c) The frequency histogram in Figure (a) is constructed using the frequency distribution presented in part (a) of this exercise. Column 1 demonstrates that the data are grouped using classes based on a single value. These single values in column 1 are used to label the horizontal axis of the frequency histogram. Suitable candidates for vertical axis units in the frequency histogram are the integers within the range 0 through 13, since these are representative of the magnitude and spread of the frequencies presented in column 2. When classes are based on a single value, the middle of each histogram bar is placed directly over the single numerical value represented by the class. Also, the height of each bar in the frequency histogram matches the respective frequency in column 2.

Figure (a) Figure (b)





- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. We notice that the relative frequencies presented in column 2 range in size from 0.025 to 0.325. Thus, suitable candidates for vertical-axis units in the relative-frequency histogram are increments of 0.05 (or 5%), from zero to 0.35 (or 35%). The middle of each histogram bar is placed directly over the single numerical value represented by the class. Also, the height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.
- (a) Since the data values range from 1 to 8, we construct a table with classes based on a single value. The resulting table follows.

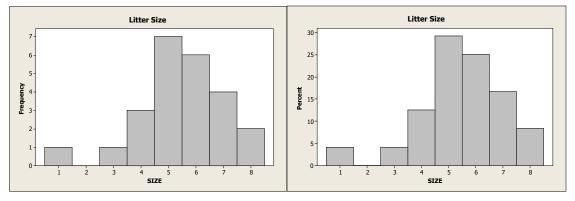
Litter Size	Frequency
1	1
2	0
3	1
4	3
5	7
6	6
7	4
8	2
	24

(b) To get the relative frequencies, divide each frequency by the sample size of 24.

Litter Size	Relative Frequency
1	0.042
2	0.000
3	0.042
4	0.125
5	0.292
6	0.250
7	0.167
8	0.083
	1.000

(c) The frequency histogram in Figure (a) is constructed using the frequency distribution presented in part (a) of this exercise. Column 1 demonstrates that the data are grouped using classes based on a single value. These single values in column 1 are used to label the horizontal axis of the frequency histogram. When classes are based on a single value, the middle of each histogram bar is placed directly over the single numerical value represented by the class. Also, the height of each bar in the frequency histogram matches the respective frequency in column 2.

Figure (a) Figure (b)



(d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this

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exercise. It has the same horizontal axis as the frequency histogram. We notice that the relative frequencies presented in column 2 range in size from 0.000 to 0.292. Thus, suitable candidates for vertical-axis units in the relative-frequency histogram are increments of 0.05 (or 5%), from zero to 0.30 (or 30%). The middle of each histogram bar is placed directly over the single numerical value represented by the class. Also, the height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

(a) Since the data values range from 0 to 5, we construct a table with classes based on a single value. The resulting table follows.

Number of Computers	Frequency
0	5
1	22
2	11
3	2
4	4
5	1
	45

(b) To get the relative frequencies, divide each frequency by the sample size of 45. The sum of the relative frequency column is 0.999 due to rounding

Number of Computers	Relative Frequency
0	0.111
1	0.489
2	0.244
3	0.044
4	0.089
5	0.022
	0.999

(c) The frequency histogram in Figure (a) is constructed using the frequency distribution presented in part (a) of this exercise. Column 1 demonstrates that the data are grouped using classes based on a single value. These single values in column 1 are used to label the horizontal axis of the frequency histogram. When classes are based on a single value, the middle of each histogram bar is placed directly over the single numerical value represented by the class. Also, the height of each bar in the frequency histogram matches the respective frequency in column 2.

Figure (a)

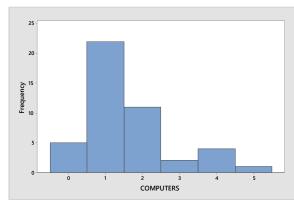
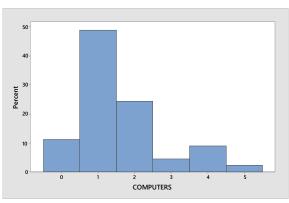


Figure (b)



(d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The middle

of each histogram bar is placed directly over the single numerical value represented by the class. Also, the height of each bar in the relative-frequency histogram matches the respective relative frequencies in column 2.

(a) The first class to construct is 40-49. Since all classes are to be of equal width, and the second class begins with 50, we know that the width of all classes is 50-40=10. All of the classes are presented in column 1. The last class to construct is 150-159, since the largest single data value is 155. Having established the classes, we tally the energy consumption figures into their respective classes. These results are presented in column 2, which lists the frequencies.

Consumption (mil. BTU)	Frequency
40-49	1
50-59	7
60-69	7
70-79	3
80-89	6
90-99	10
100-109	5
110-119	4
120-129	2
130-139	3
140-149	0
150-159	2
	50

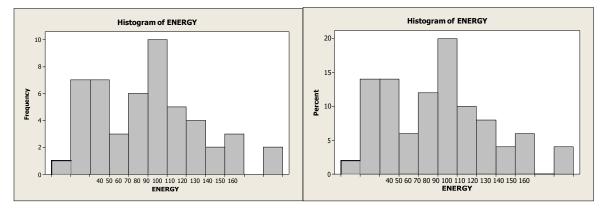
(b) Dividing each frequency by the total number of observations, which is 50, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2. The resulting table follows.

Consumption	(mil. BTU)	Relative Frequency
40-49		0.02
50-59		0.14
60-69		0.14
70-79		0.06
80-89		0.12
90-99		0.20
100-109		0.10
110-119		0.08
120-129		0.04
130-139		0.06
140-149		0.00
150-159		0.04
		1.00

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution presented in part (a) of this exercise. The lower class limits of column 1 are used to label the horizontal axis of the frequency histogram. Suitable candidates for vertical-axis units in the frequency histogram are the even integers 0 through 10, since these are representative of the magnitude and spread of the frequency presented in column 2. The height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this

exercise. It has the same horizontal axis as the frequency histogram. We notice that the relative frequencies presented in column 2 vary in size from 0.00 to 0.20. Thus, suitable candidates for vertical axis units in the relative-frequency histogram are increments of 0.05 (or 5%), from zero to 0.20 (or 20%). The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

Figure (a) Figure (b)



(a) The first class to construct is 40-44. Since all classes are to be of equal width, and the second class begins with 45, we know that the width of all classes is 45-40=5. All of the classes are presented in column 1. The last class to construct is 60-64, since the largest single data value is 61. Having established the classes, we tally the age figures into their respective classes. These results are presented in column 2, which lists the frequencies.

Age	Frequency
40-44	4
45-49	3
50-54	4
55-59	8
60-64	2
	21

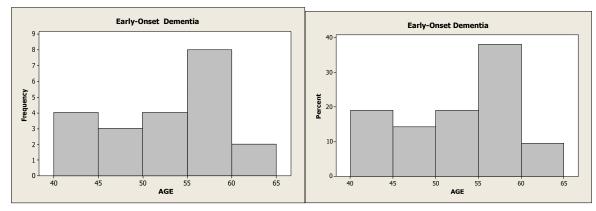
(b) Dividing each frequency by the total number of observations, which is 21, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2. The resulting table follows.

Age	Relative Frequency
40-44	0.190
45-49	0.143
50-54	0.190
55-59	0.381
60-64	0.095
	1.000

(c) The frequency histogram in Figure (a) is constructed using the frequency distribution presented in part (a) of this exercise. The lower class limits of column 1 are used to label the horizontal axis of the frequency histogram. Suitable candidates for vertical-axis units in the frequency histogram are the even integers 2 through 8,

- since these are representative of the magnitude and spread of the frequency presented in column 2. The height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. We notice that the relative frequencies presented in column 2 vary in size from 0.095 to 0.381. Thus, suitable candidates for vertical axis units in the relative-frequency histogram are increments of 0.10 (or 10%), from zero to 0.40 (or 40%). The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

Figure (a) Figure (b)



(a) The first class to construct is 20-22. Since all classes are to be of equal width, and the second class begins with 23, we know that the width of all classes is 23 - 20 = 3. All of the classes are presented in column 1. The last class to construct is 44-46, since the largest single data value is 46. Having established the classes, we tally the cheese consumption figures into their respective classes. These results are presented in column 2, which lists the frequencies.

Cheese Consumption	Frequency
20-22	1
23-25	3
26-28	5
29-31	3
32-34	5
35-37	9
38-40	2
41-43	4
44-46	3
	35

(b) Dividing each frequency by the total number of observations, which is 35, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2. The resulting table follows. The sum of the relative frequency column is 1.001 due to rounding.

Cheese Consumption	Relative Frequency
20-22	0.029
23-25	0.086
26-28	0.143
29-31	0.086
32-34	0.143
35-37	0.257
38-40	0.057
41-43	0.114
44-46	0.086
	1.001

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution presented in part (a) of this exercise. The lower class limits of column 1 are used to label the horizontal axis of the frequency histogram. Suitable candidates for vertical-axis units in the frequency histogram are the even integers 2 through 7, since these are representative of the magnitude and spread of the frequency presented in column 2. The height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. We notice that the relative frequencies presented in column 2 vary in size from 0.057 to 0.200. Thus, suitable candidates for vertical axis units in the relative-frequency histogram are increments of 0.05 (or 5%), from zero to 0.20 (or 20%). The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

Figure (a)

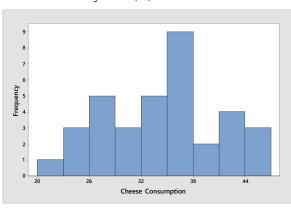
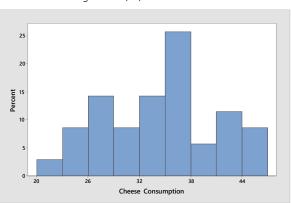


Figure (b)



(a) The first class to construct is 12-17. Since all classes are to be of equal width, and the second class begins with 18, we know that the width of all classes is 18 - 12 = 6. All of the classes are presented in column 1. The last class to construct is 60-65, since the largest single data value is 61. Having established the classes, we tally the anxiety questionnaire score figures into their respective classes. These results are presented in column 2, which lists the frequencies.

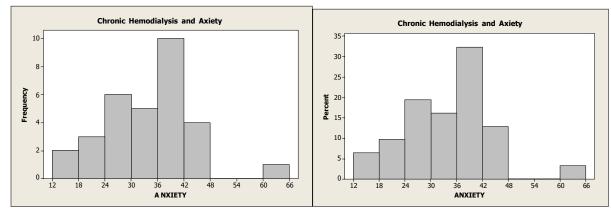
Anxiety	Frequency
12-17	2
18-23	3
24-29	6
30-35	5
36-41	10
42-47	4
48-53	0
54-59	0
60-65	1
	31

(b) Dividing each frequency by the total number of observations, which is 31, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2. The resulting table follows.

Anxiety	Relative Frequency
12-17	0.065
18-23	0.097
24-29	0.194
30-35	0.161
36-41	0.323
42-47	0.129
48-53	0.000
54-59	0.000
60-65	0.032
	1.000

- The frequency histogram in Figure (a) is constructed using the (C) frequency distribution presented in part (a) of this exercise. The lower class limits of column 1 are used to label the horizontal axis of the frequency histogram. Suitable candidates for vertical-axis units in the frequency histogram are the even integers 0 through 10, since these are representative of the magnitude and spread of the frequency presented in column 2. The height of each bar in the frequency histogram matches the respective frequency in column 2.
- The relative-frequency histogram in Figure (b) is constructed using (d) the relative-frequency distribution presented in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. We notice that the relative frequencies presented in column 2 vary in size from 0.000 to 0.323. Thus, suitable candidates for vertical axis units in the relative-frequency histogram are increments of 0.05 (or 5%), from zero to 0.35 (or 35%). The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

Figure (a) Figure (b)



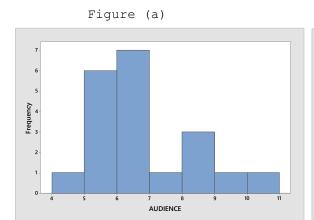
(a) The first class to construct is 4 -under 5. Since all classes are to be of equal width 1, the second class is 5 - under 6. All of the classes are presented in column 1. The last class to construct is 10 - under 11, since the largest single data value is 10.360. Having established the classes, we tally the audience sizes into their respective classes. These results are presented in column 2, which lists the frequencies.

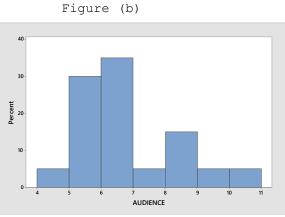
Audience (Millions)	Frequency
4 - under 5	1
5 - under 6	6
6 - under 7	7
7 - under 8	1
8 - under 9	3
9 - under 10	1
10 - under 11	1
	20

(b) Dividing each frequency by the total number of observations, which is 20, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2.

Audience (Millions)	Relative
4 - under 5	0.05
5 - under 6	0.30
6 - under 7	0.35
7 - under 8	0.05
8 - under 9	0.15
9 - under 10	0.05
10 - under 11	0.05
	1.00

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution obtained in part (a) of this exercise Column 1 demonstrates that the data are grouped using classes with class widths of 1. Also, the height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution obtained in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.





(a) The first class to construct is 52 - under 54. Since all classes are to be of equal width 2, the second class is 54 - under 56. All of the classes are presented in column 1. The last class to construct is 74 - under 76, since the largest single data value is 75.3. Having established the classes, we tally the cheetah speeds into their respective classes. These results are presented in column 2, which lists the frequencies.

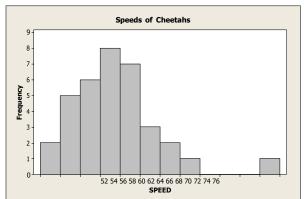
Speed	Frequency
52 - under 54	2
54 - under 56	5
56 - under 58	6
58 - under 60	8
60 - under 62	7
62 - under 64	3
64 - under 66	2
66 - under 68	1
68 - under 70	0
70 - under 72	0
72 - under 74	0
74 - under 76	1
	35

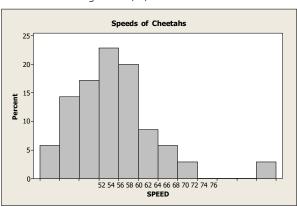
(b) Dividing each frequency by the total number of observations, which is 35, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2

Speed	Relative Frequency
52 - under 54	0.057
54 - under 56	0.143
56 - under 58	0.171
58 - under 60	0.229
60 - under 62	0.200
62 - under 64	0.086
64 - under 66	0.057
66 - under 68	0.029
68 - under 70	0.000
70 - under 72	0.000
72 - under 74	0.000
74 - under 76	0.029
	1.000

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution obtained in part (a) of this exercise Column 1 demonstrates that the data are grouped using classes with class widths of 2. Suitable candidates for vertical axis units in the frequency histogram are the integers within the range 0 through 8, since these are representative of the magnitude and spread of the frequencies presented in column 2. Also, the height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution obtained in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. We notice that the relative frequencies presented in column 3 range in size from 0.000 to 0.229. Thus, suitable candidates for vertical axis units in the relative-frequency histogram are increments of 0.05 (5%), from zero to 0.25 (25%). The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

Figure (a) Figure (b)





(a) The first class to construct is 12 - under 14. Since all classes are to be of equal width 2, the second class is 14 - under 66. All of the classes are presented in column 1. The last class to construct is 26 - under 28, since the largest single data value is 26.4. Having established the classes, we tally the fuel tank capacities into their respective classes. These results are presented in column 2, which lists the frequencies.

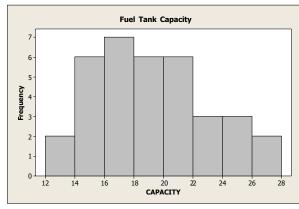
Fuel Tank Capacity	Frequency
12 - under 14	2
14 - under 16	6
16 - under 18	7
18 - under 20	6
20 - under 22	6
22 - under 24	3
24 - under 26	3
26 - under 28	2
	35

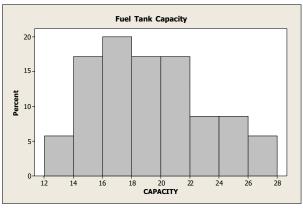
(b) Dividing each frequency by the total number of observations, which is 35, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2

Fuel Tank Capacity	Relative Frequency
12 - under 14	0.057
14 - under 16	0.171
16 - under 18	0.200
18 - under 20	0.171
20 - under 22	0.171
22 - under 24	0.086
24 - under 26	0.086
26 - under 28	0.057
	1.000

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution obtained in part (a) of this exercise Column 1 demonstrates that the data are grouped using classes with class widths of 2. Suitable candidates for vertical axis units in the frequency histogram are the integers within the range 2 through 7, since these are representative of the magnitude and spread of the frequencies presented in column 2. Also, the height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution obtained in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. We notice that the relative frequencies presented in column 3 range in size from 0.057 to 0.200. Thus, suitable candidates for vertical axis units in the relative-frequency histogram are increments of 0.05 (5%), from zero to 0.20 (20%). The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

Figure (a) Figure (b)





(a) The first class to construct is 0 -under 1. Since all classes are to be of equal width 1, the second class is 1 - under 2. All of the classes are presented in column 1. The last class to construct is 7 - under 8, since the largest single data value is 7.6. Having established the classes, we tally the fuel tank capacities into their respective classes. These results are presented in column 2, which lists the frequencies.

Oxygen Distribution	Frequency
0 - under 1	1
1 - under 2	10
2 - under 3	5
3 - under 4	4
4 - under 5	0
5 - under 6	0
6 - under 7	1
7 - under 8	1
	22

(b) Dividing each frequency by the total number of observations, which is 22, results in each class's relative frequency. The relative frequencies for all classes are presented in column 2

Oxygen Distribution	Relative Frequency
0 - under 1	0.045
1 - under 2	0.455
2 - under 3	0.227
3 - under 4	0.182
4 - under 5	0.000
5 - under 6	0.000
6 - under 7	0.045
7 - under 8	0.045
	1.000

- (c) The frequency histogram in Figure (a) is constructed using the frequency distribution obtained in part (a) of this exercise Column 1 demonstrates that the data are grouped using classes with class widths of 2. Suitable candidates for vertical axis units in the frequency histogram are the integers within the range 0 through 10, since these are representative of the magnitude and spread of the frequencies presented in column 2. Also, the height of each bar in the frequency histogram matches the respective frequency in column 2.
- (d) The relative-frequency histogram in Figure (b) is constructed using the relative-frequency distribution obtained in part (b) of this exercise. It has the same horizontal axis as the frequency histogram. We notice that the relative frequencies presented in column 3 range in size from 0.000 to 0.455. Thus, suitable candidates for vertical axis units in the relative-frequency histogram are increments of 0.05 (5%), from zero to 0.50 (50%). The height of each bar in the relative-frequency histogram matches the respective relative frequency in column 2.

Figure (a)

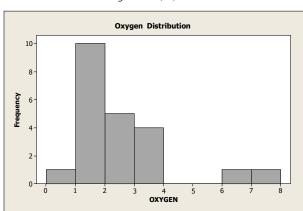
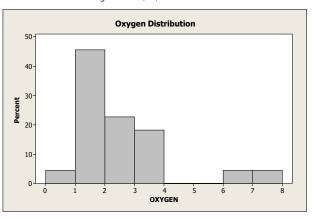
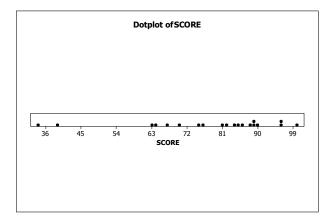


Figure (b)

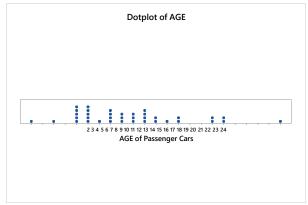


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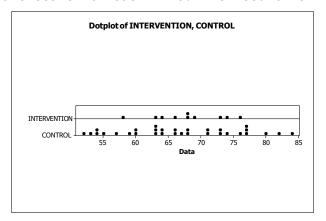
The horizontal axis of this dotplot displays a range of possible exam scores. To complete the dotplot, we go through the data set and record each exam score by placing a dot over the appropriate value on the horizontal axis.



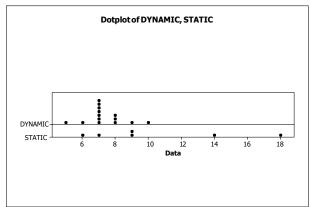
The horizontal axis of this dotplot displays of range of possible ages of the passenger cars. To complete the dotplot, we go through the data set and record each age by placing a dot over the appropriate value on the horizontal axis.



(a) The data values range from 52 to 84, so the scale must accommodate those values. We stack dots above each value on two different lines using the same scale for each line. The result is



- (b) The two sets of pulse rates are both centered near 68, but the Intervention data are more concentrated around the center than are the Control data.
- (a) The data values range from 7 to 18, so the scale must accommodate those values. We stack dots above each value on two different lines using the same scale for each line. The result is



(b) The Dynamic system does seem to reduce acute postoperative days in the hospital on the average. The Dynamic data are centered at about 7 days, whereas the Static data are centered at about 11 days and are much more spread out than the Dynamic data.

The data values consist of 2 digit numbers followed by a one digit decimal place ranging from 45.5 to 57.0. The last digit following the decimal will be the leaves and the stems will be the remaining digits. Therefore, the stem-and-leaf diagram will have stems of 45 through 57.

- 45| !
- 46| 4
- 47 | 2
- 48 | 0
- 491 1
- 50| 3578
- 51| 35589
- 521 011
- 53| 355689
- 54| 13
- 55| 033
- 56| 0
- 57| 0

Since each data value consists of 2 digits, each beginning with 1, 2, 3, or 4, we will construct the stem-and-leaf diagram with these four values as the stems. The result is

- 1 | 238
- 2| 1678899
- 31 34459
- 4 | 04
- (a) Since each data value consists of a 2 digit number with a one digit decimal, we will make the leaf the decimal digit and the stems the remaining two digit numbers of 28, 29, 30, and 31. The result is

```
28 | 8
29 | 368
30 | 1247
31 | 02
```

- (b) Splitting into two lines per stem, leafs of 0-4 belong in the first stem and leafs of 5-9 belong in the second stem. The result is
  - 28 | 8 29 | 3 29 | 68 30 | 124 30 | 7 31 | 02
- (c) The stem-and-leaf diagram in part (b) is more useful because by splitting the stems into two lines per stem, you have created more lines. Part (a) had too few lines.
- (a) Since each data value lies between 2 and 93, we will construct the stem-and-leaf diagram with one line per stem. The result is

```
0| 2234799

1| 11145566689

2| 023479

3| 004555

4| 19

5| 5

6| 9

7| 9

8|

9| 3
```

(b) Using two lines per stem, the same data result in the following diagram:

```
01 2234
0| 799
1| 1114
1| 5566689
2 | 0234
2| 79
31 004
31 555
4 | 1
4 | 9
5 I
5 | 5
6|
6| 9
7 |
7 |
    9
8 |
8 |
```

91 3

(c) The stem with one line per stem is more useful. One gets the same impression regarding the shape of the distribution, but the two lines per stem version has numerous lines with no data, making it take up

more space than necessary to interpret the data and giving it too many lines.

- (a) Since we have two digit numbers, the last digit becomes the leaf and the first digit becomes the stem. For this data, we have a stem of 7. Splitting the data into five lines per stem, we put the leaves 0-1 in the first stem, 2-3 in the second stem, 4-5 in the third stem, 6-7 in the fourth stem, and 8-9 in the fifth stem. The result is
  - 7| 001111111
  - 71 2222223333333
  - 7 | 444444445555
  - 71 66666666677
  - (b) Using one or two lines per stem would have given us too few lines.
- (a) Since we have two digit numbers, the last digit becomes the leaf and the first digit becomes the stem.

  For this data, we have stems of 6, 7, and 8.

  Splitting the data into five lines per stem, we put the leaves 0-1 in the first stem, 2-3 in the second stem, 4-5 in the third stem, 6-7 in the fourth stem, and 8-9 in the fifth stem.

  The result is
  - 61 8
  - 7 | 11111111
  - 7| 2222222233333
  - 7 | 4444555555
  - 7| 66666777777
  - 71 8
  - 8 | 0
  - (b) Using one or two lines per stem would have given us too few lines.

The heights of the bars of the relative-frequency histogram indicate that:

- (a) About 27.5% of the returns had an adjusted gross income between \$10,000 and \$19,999, inclusive.
- (b) About 37.5% were between \$0 and \$9,999; 27.5% were between \$10,000 and \$19,999; and 19% were between \$20,000 and \$29,999. Thus, about 84% (i.e., 37.5% + 27.5% + 19%) of the returns had an adjusted gross income less than \$30,000.
- (c) About 11% were between \$30,000 and \$39,999; and 5% were between \$40,000 and \$49,999. Thus, about 16% (i.e., 11% + 5%) of the returns had an adjusted gross income between \$30,000 and \$49,999. With 89,928,000 returns having an adjusted gross income less than \$50,000, the number of returns having an adjusted gross income between \$30,000 and \$49,999 was 14,388,480 (i.e., 0.16 x 89,928,000).

The graph indicates that:

- (a) 20% of the patients have cholesterol levels between 205 and 209, inclusive.
- (b) 20% are between 215 and 219; and 5% are between 220 and 224. Thus, 25% (i.e., 20% + 5%) have cholesterol levels of 215 or higher.
- (c) 35% of the patients have cholesterol levels between 210 and 214, inclusive. With 20 patients in total, the number having cholesterol levels between 210 and 214 is 7 (i.e.,  $0.35 \times 20$ ).

The graph indicates that:

- (a) 15 states had at least three but less than four hospital beds per 1000 people available.
- (b) 3 states had at least four and a half hospital beds per 1000 people available.

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The graph indicates that:

- (a) 1 of these 14 patients was under 45 years old at the onset of symptoms.
- (b) 3 of these 14 patients were at least 65 years old at the onset of symptoms.
- (c) 8 of these 14 patients were between 50 and 64 years old, inclusive, at the onset of symptoms.
- (a) Using Minitab, retrieve the data from the WeissStats Resource Site.

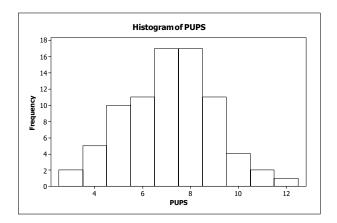
  Column 1 contains the numbers of pups borne in a lifetime for each of
  - 80 female Great White Sharks. From the tool bar, select  $\mathsf{Stat} < \mathsf{Tables}$

≺ Tally Individual Variables, double-click on PUPS in the first box so
that PUPS appears in the Variables box, put a check mark next to Counts
and Percents under Display, and click OK. The result is

PUPS	Count	Percent
3	2	2.50
4	5	6.25
5	10	12.50
6	11	13.75
7	17	21.25
8	17	21.25
9	11	13.75
10	4	5.00
11	2	2.50
12	1	1.25

(b) After retrieving the data from the WeissStats Resource Site, select

 ${\tt Graph} \mathrel{$<$} {\tt Histogram}, \; {\tt choose} \; {\tt Simple} \; {\tt and} \; {\tt click} \; {\tt OK}. \; {\tt Double} \; {\tt click} \; {\tt on} \; {\tt PUPS} \; {\tt in} \; {\tt the} \; {\tt first} \; {\tt box} \; {\tt to} \; {\tt enter} \; {\tt PUPS} \; {\tt in} \; {\tt the} \; {\tt Graphs} \; {\tt variables} \; {\tt box}, \; {\tt and} \; {\tt click} \; {\tt OK}. \; {\tt The} \; {\tt frequency} \; {\tt histogram} \; {\tt is} \; {\tt on} \;$ 

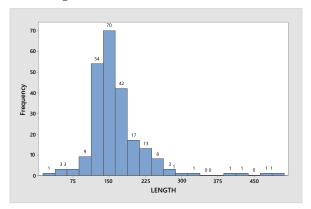


To change to a relative-frequency histogram, before clicking OK the second time, click on the **Scale** button and the **Y-Scale type** tab, and choose **Percent** and click **OK**. The graph will look like the frequency histogram, but will have relative frequencies on the vertical scale instead of counts.

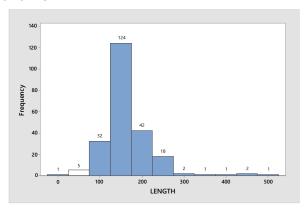
The numbers of pups range from 1 to 12 per female with 7 and 8 pups occurring more frequently than any other values.

(a) Using Minitab, there is not a direct way to get a grouped frequency distribution. However, you can use an option in creating your histogram that will report the frequencies in each of the classes, essentially creating a grouped frequency distribution. Retrieve the data from the WeissStats Resource Site. Column 2 contains the length

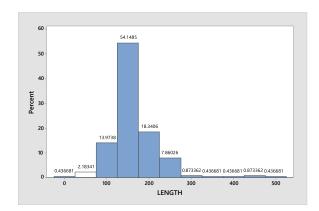
of the Beatles songs, in seconds. From the tool bar, select **Graph < Histogram**, choose **Simple** and click **OK**. Double click on LENGTH to enter LENGTH in the **Graph variables** box. Click **Labels**, click the **Data Labels** tab, then check **Use y-value labels**. Click **OK** twice. The result is



Above each bar is a label for each of the frequencies. Also, the labeling on the horizontal axis is the midpoint for class, where each class has a width of 25. To change the width of the bars, right Click on the bars and choose **Edit Bars**, click **Binning**, click **Midpoint/Cutpoint positions:**, type 0 50 100 150 200 250 300 350 400 450 500. The result is

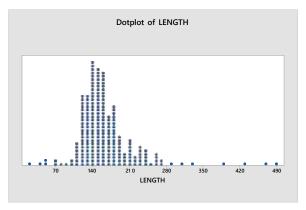


To get the relative-frequency distribution, follow the same steps as above, but also Click the **Scale** button, click the **Y-scale Type**, check **Percent**, then click **OK** twice. The result is



Above each bar is the percentage for that class, essentially creating a relative-frequency distribution. You could also transfer these results into a table.

- (b) A frequency histogram and a relative frequency distribution were created in part (a).
- (c) To obtain the dotplot, select Graph ✓ Dotplot, select Simple in the One Y row, and click OK. Double click on LENGTH to enter LEGNTH in the Graph variables box and click OK. The result is



- (d) The graphs are similar, but not identical. This is because the dotplot preserves the raw data by plotting individual dots and the histogram looses the raw data because it groups observations into grouped classes. The overall impression, however, remains the same. They both are generally the same shape with outliers to the right.
- (a) After entering the data from the WeissStats Resource Site, in Minitab,

select Graph < Stem-and-Leaf, double click on PERCENT to enter PERCENT in the  $Graph \ variables$  box and enter a  $\underline{10}$  in the Increment box, and click OK. The result is

Stem-and-leaf of PERCENT N = 51Leaf Unit = 1.0

- (35) 8 00122223334444455566777777888889999 16 9 000000001111122
- (b) Repeat part (a), but this time enter a  $\underline{5}$  in the **Increment** box. The result is

```
Stem-and-leaf of PERCENT N = 51
Leaf Unit = 1.0
     8 001222233344444
(20) 8 55566777777888889999
16 9 000000001111122
```

(c) Repeat part (a) again, but this time enter a 2 in the Increment box. The result is

```
Leaf Unit = 1.0
3 8 001
10 8 2222333
18 8 44444555
(8) 8 6677777
25 8 888889999
16 9 00000000011111
   9 22
```

Stem-and-leaf of PERCENT N = 51

- (d) The last graph is the most useful since it gives a better idea of the shape of the distribution. Typically, we like to have five to fifteen classes and this is the only one of the three graphs that satisfies that condition.
- (a) After entering the data from the WeissStats Resource Site, in Minitab, select

Graph < Stem-and-Leaf, double click on PERCENT to enter PERCE T in the Graph variables box and enter a 10 in the Increment box, and click OK. The result is

```
Stem-and-leaf of PERCENT N = 51
Leaf Unit = 1.0
   1 79
(32) 2 01122333444455555566666777778999
17 3 0001112223456668
```

(b) Repeat part (a), but this time enter a 5 in the Increment box. The result is

```
Leaf Unit = 1.0
2
    1 79
14 2 011223334444
(20) 2 55555566666777778999
17 3 00011122234
6 3 56668
1
    4
    4 9
```

Stem-and-leaf of PERCENT N = 51

(c) Repeat part (a) again, but this time enter a 2 in the Increment box. The result is

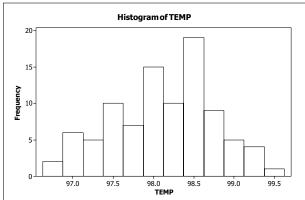
```
Stem-and-leaf of PERCENT N = 51
Leaf Unit = 1.0
```

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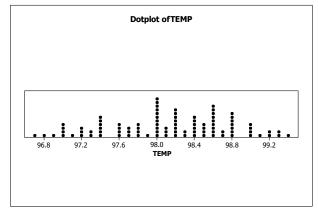
```
1 7
1 9
5
      2
        011
10
      2
         22333
         4444555555
20
      2
(10)
     2
        6666677777
21
         8999
17
         000111
      3
11
         2223
7
      3 45
5
      3
         666
2
      3
1
      4
1
      4
      4
      4
```

- (d) The second graph is the most useful. The third one has more classes than necessary to comprehend the shape of the distribution and has a number of empty stems. Typically, we like to have five to fifteen classes and the first and second diagrams satisfy that condition, but the second one provides a better idea of the shape of the distribution.
- (a) After entering the data from the WeissStats Resource Site, in Minitab,

select **Graph < Histogram**, select **Simple** and click **OK**. double click on TEMP to enter TEMP in the **Graph variables** box and click **OK**. The result is



(b) Now select Graph ✓ Dotplot, select Simple in the One Y row, and click OK. Double click on TEMP to enter TEMP in the Graph variables box and click OK. The result is



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(c) Now select **Graph < Stem-and-Leaf**, double click on TEMP to enter TEMP in the **Graph variables** box and click **OK**. Leave the **Increment** box blank to allow Minitab to choose the number of lines per stem. The result is

- (d) The dotplot shows all of the individual values. The stem-and-leaf diagram used five lines per stem and therefore each line contains leaves with possibly two values. The histogram chose classes of width 0.25. This resulted in, for example, the class with midpoint 97.0 including all of the values 96.9, 97.0, and 97.1, while the class with midpoint 97.25 includes only the two values 97.2 and 97.3. Thus the 'smoothing' effect is not as good in the histogram as it is in the stem-and-leaf diagram. Overall, the dotplot gives the truest picture of the data and allows recovery of all of the data values.
- (a) The classes are presented in column 1. With the classes established, we then tally the exam scores into their respective classes. These results are presented in column 2, which lists the frequencies. Dividing each frequency by the total number of exam scores, which is 20, results in each class's relative frequency. The relative frequencies for all classes are presented in column 3. The class mark of each class is the average of the lower and upper limits. The class marks for all classes are presented in column 4.

Score	Frequency	Relative Frequency	Class Mark
30-39	2	0.10	34.5
40-49	0	0.00	44.5
50-59	0	0.00	54.5
60-69	3	0.15	64.5
70-79	3	0.15	74.5
80-89	8	0.40	84.5
90-100	4	0.20	95.0
	20	1.00	_

- (b) The first six classes have width 10; the seventh class had width 11.
- (c) Answers will vary, but one choice is to keep the first six classes the same and make the next two classes 90-99 and 100-109. Another possibility is 31-40, 41-50, ..., 91-100.

Answers will vary, but by following the steps we first decide on the approximate number of classes. Since there are 40 observations, we should

have 7-14 classes. This exercise states we should have approximately seven classes. Step 2 says that we calculate an approximate class width as (99-36)/7=9. A convenient class width close to 9 would be a class width of 10. Step 3 says that we choose a number for the lower class limit which is less than or equal to our minimum observation of 36. Let's choose 35. Beginning with a lower class limit of 35 and width of 10, we have a first class of 35-44, a second class of 45-54, a third class of 55-64, a fourth class of 65-74, a fifth class of 75-84, a sixth class of 85-94, and a seventh class of 95-104. This would be our last class since the largest observation is 99.

Answers will vary, but by following the steps we first decide on the approximate number of classes. Since there are 37 observations, we should have 7-14 classes. This exercise states we should have approximately eight classes. Step 2 says that we calculate an approximate class width as (278.8 - 129.2)/8 = 18.7. A convenient class width close to 18.7 would be a class width of 20. Step 3 says that we choose a number for the lower cutpoint which is less than or equal to our minimum observation of 129.2. Let's choose 120. Beginning with a lower cutpoint of 120 and width of 20, we have a first class of 120 - under 140, a second class of 140 - under 160, a third class of 160 - under 180, a fourth class of 180 - under 200, a fifth class of 200 - under 220, a sixth class of 220 - under 240, a seventh class of 240 - under 260, and an eighth class of 260 - under 280. This would be our last class since the largest observation is 278.8.

- (a) Tally marks for all 50 students, where each student is categorized by age and gender, are presented in the contingency table given in part (b).
  - (b) Tally marks in each box appearing in the following chart are counted. These counts, or frequencies, replace the tally marks in the contingency table. For each row and each column, the frequencies are added, and their sums are recorded in the proper "Total" box.

Age (yrs)

Gender	Under 21	21 - 25	Over 25	Total
Male	11111 111	11111 11111 11	II	
Female	11111 11111 11	11111 11111 111	III	
Total				

Age (yrs)

Gender	Under 21	21-25	Over 25	Total
Male	8	12	2	22
Female	12	13	3	28
Total	20	25	5	50

(c) The row and column totals represent the total number of students in each of the corresponding categories. For example, the row total of 22 indicates that 22 of the students in the class are male.

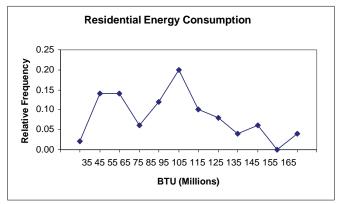
- (d) The sum of the row totals is 50, and the sum of the column totals is 50. The sums are equal because they both represent the total number of students in the class.
- (e) Dividing each frequency reported in part (b) by the grand total of 50 students results in a contingency table that gives relative frequencies.

Age (yrs)

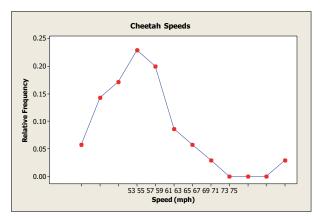
	Under 21	21-25	Over 25	Total
Gender				
Male	0.16	0.24	0.04	0.44
Female	0.24	0.26	0.06	0.56
Total	0.40	0.50	0.10	1.00

(f) The 0.16 in the upper left-hand cell indicates that 16% of the students in the class are males *and* under 21. The 0.40 in the lower left-hand cell indicates that 40% of the students in the class are under age 21. A similar interpretation holds for the remaining entries.

Consider columns 1 and 2 of the energy-consumption data given in Exercise 2.56 part (b). Compute the class mark for each class presented in column 1. Pair each class mark with its corresponding relative frequency found in column 2. Construct a horizontal axis, where the units are in terms of class marks and a vertical axis where the units are in terms of relative frequencies. For each class mark on the horizontal axis, plot a point whose height is equal to the relative frequency of the class. Then join the points with connecting lines. The result is a relative-frequency polygon.



Consider columns 1 and 2 of the Cheetah speed data given in Exercise 2.61 part (b). Compute the midpoint for each class presented in column 1. Pair each midpoint with its corresponding relative frequency found in column 2. Construct a horizontal axis, where the units are in terms of midpoints and a vertical axis where the units are in terms of relative frequencies. For each midpoint on the horizontal axis, plot a point whose height is equal to the relative frequency of the class. Then join the points with connecting lines. The result is a relative-frequency polygon.

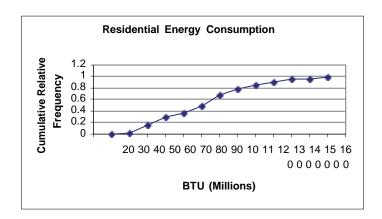


In single value grouping the horizontal axis would be labeled with the value of each class.

(a) Consider parts (a) and (b) of the energy-consumption data given in Exercise 2.56. The classes are now reworked to present just the lower class limit of each class. The frequencies are reworked to sum the frequencies of all classes representing values less than the specified lower class limit. These successive sums are the cumulative frequencies. The relative frequencies are reworked to sum the relative frequencies of all classes representing values less than the specified class limits. These successive sums are the cumulative relative frequencies. (Note: The cumulative relative frequencies can also be found by dividing the each cumulative frequency by the total number of data values.)

Less than	Cumulative Frequency	Cumulative Relative Frequency
	1 1	1 2
40	0	0.00
50	1	0.02
60	8	0.16
70	15	0.30
80	18	0.36
90	24	0.48
100	34	0.68
110	39	0.78
120	43	0.86
130	45	0.90
140	48	0.96
150	48	0.96
160	50	1.00

(b) Pair each class limit with its corresponding cumulative relative frequency found in column 3. Construct a horizontal axis, where the units are in terms of the class limits and a vertical axis where the units are in terms of cumulative relative frequencies. For each class limit on the horizontal axis, plot a point whose height is equal to the cumulative relative frequency. Then join the points with connecting lines. The result, presented in the following figure, is an ogive using cumulative relative frequencies. (Note: A similar procedure could be followed using cumulative frequencies.)

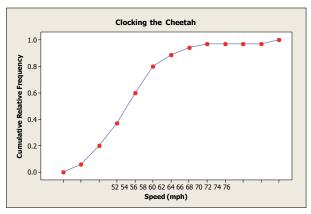


(a) Consider parts (a) and (b) of the Cheetah speed data given in Exercise 2.61. The classes are now reworked to present just the lower cutpoint of each class. The frequencies are reworked to sum the frequencies of all classes representing values less than the specified lower cutpoint. These successive sums are the cumulative frequencies. The relative frequencies are reworked to sum the relative frequencies of all classes representing values less than the specified cutpoints. These successive sums are the cumulative relative frequencies. (Note: The cumulative relative frequencies can also be found by dividing the each cumulative frequency by the total number of data values.)

Less than	Cumulative Frequency	Cumulative Relative Frequency
52	0	0.000
54	2	0.057
56	7	0.200
58	13	0.371
60	21	0.600
62	28	0.800
64	31	0.886
66	33	0.943
68	34	0.971
70	34	0.971
72	34	0.971
74	34	0.971
76	35	1.000

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Pair each cutpoint with its corresponding cumulative relative frequency found in column 3. Construct a horizontal axis, where the units are in terms of the cutpoints and a vertical axis where the units are in terms of cumulative relative frequencies. For each cutpoint on the horizontal axis, plot a point whose height is equal to the cumulative relative frequency. Then join the points with connecting lines. The result, presented in the following figure, is an ogive using cumulative relative frequencies. (Note: A similar procedure could be followed using cumulative frequencies.)



- (a) After rounding each observation to the nearest year, the stem-and-leaf diagram for the rounded ages is
  - 51 334469
  - 6|
  - 7 | 256689
  - 234678 8 |
  - 9|
  - (b) After truncating each weight by dropping the decimal part, the stemand-leaf diagram for the rounded weights is
    - 5| 223468
    - 6| 5
    - 7| 245688 8| 123678
  - (c) Although there are minor differences between the two diagrams, the overall impression of the distribution of weights is the same for both diagrams.

Minitab used truncation. Note that there was a data point of 5.8 in the sample. It would have been plotted with a stem of 0 and a leaf of 6 if it had been rounded. Instead Minitab plotted the observation with a stem of 0and a leaf of 5.

### Section 2.4

The distribution of a data set is a table, graph, or formula that provides the values of the observations and how often they occur.

Sample data are the values of a variable for a sample of the population.

Population data are the values of a variable for the entire population.

A sample distribution is the distribution of sample data.

A population distribution is the distribution of population data.

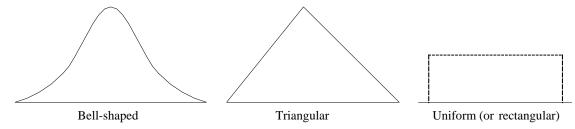
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A distribution of a variable is the same as a population distribution. A smooth curve makes it a little easier to see the shape of a distribution and to concentrate on the overall pattern without being distracted by minor differences in shape.

A large simple random sample from a bell-shaped distribution would be expected to have roughly a bell-shaped distribution since more sample values should be obtained, on average, from the middle of the distribution.

- (a) Yes. We would expect both simple random samples to have roughly a reverse J-shaped distribution.
- (b) Yes. We would expect some variation in shape between the two sample distributions since it is unlikely that the two samples would produce exactly the same frequency table. It should be noted, however, that as the sample size is increased, the difference in shape for the two samples should become less noticeable.

Three distribution shapes that are symmetric are bell-shaped, triangular, and Uniform (or rectangular), shown in that order below. It should be noted that there are others as well.



- (a) The shape of the distribution is unimodal.
- (b) The shape of the distribution is symmetric.
- (a) The shape of the distribution is unimodal.
- (b) The shape of the distribution is symmetric.
- (a) The shape of the distribution is unimodal.
- (b) The shape of the distribution is not symmetric.
- (c) The shape of the distribution is right-skewed.
- (a) The shape of the distribution is unimodal.
- (b) The shape of the distribution is not symmetric.
- (c) The shape of the distribution is left-skewed.
- (a) The shape of the distribution is unimodal.
- (b) The shape of the distribution is not symmetric.
- (c) The shape of the distribution is right-skewed.
- (a) The shape of the distribution is unimodal.
- (b) The shape of the distribution is not symmetric.
- (c) The shape of the distribution is left-skewed.
- (a) The shape of the distribution is bimodal.
- (b) The shape of the distribution is symmetric.
- (a) The shape of the distribution is multimodal.
- (b) The shape of the distribution is not symmetric.

# 92 Chapter 2

The overall shape of the distribution of the number of children of U.S. presidents is right skewed.

Except for the one data value between 74 and 76, this distribution is close to bell-shaped. That one value makes the distribution slightly right skewed.

The distribution of weights of the male Ethiopian born school children is roughly symmetric.

The distribution of depths of the burrows is left skewed.

The distribution of heights of the Baltimore Ravens is roughly symmetric.

The distribution of PCB concentration is symmetric.

The distribution of adjusted gross incomes is right skewed.

The distribution of cholesterol levels appears to be slightly left skewed.

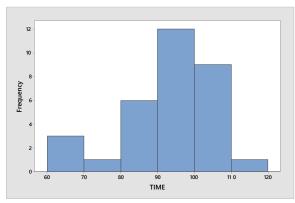
The distribution of hemoglobin levels for patients with sickle cell disease is roughly symmetric.

The distribution of length of stay is right skewed.

(a) The frequency distribution for this data is shown in the following table.

Time Between Eruptions (minutes)	Frequency
60 - under 70	3
70 - under 80	1
80 - under 90	6
90 - under 100	12
100 - under 110	9
110 - under 120	1

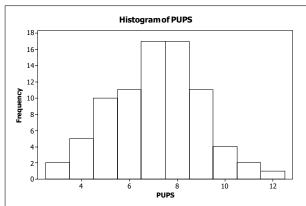
The histogram for the distribution is shown below.



- (b) This distribution is unimodal.
- (c) This distribution is not symmetric.
- (d) This distribution is left skewed.
- (a) The distributions for Year 1 and Year 2 are both unimodal.
  - (b) Both distributions are not symmetric.
  - (c) Both distributions are right skewed.
  - (d) The distribution for Year 1 has a longer right tail indicating more variation than the distribution for Year 2. They are also not centered in the same place.

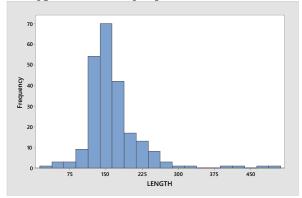
(a) After entering the data from the WeissStats Resource Site, in Minitab, select **Graph <Histogram**, select **Simple** and click **OK**. Double click on PUPS to enter PUPS in the **Graph variables** box and click **OK**. Our result is as follows. Results may vary depending on the type of technology used and graph obtained.

The overall shape of the distribution is unimodal and symmetric.



(a) After entering the data from the WeissStats Resource Site, in

Minitab, select **Graph < Histogram**, select **Simple** and click **OK**. Double click on LENGTH to enter LENGTH in the **Graph variables** box and click **OK**. Our result is as follows. Results may vary depending on the type of technology used and graph obtained.



The distribution of LENGTH is unimodal and not symmetric.

- (b) The distribution is right skewed.
- (a) In Exercise 2.108, we used Minitab to obtain a stem-and-leaf diagram using 5 lines per stem. That diagram is shown below  $\frac{1}{2}$

Stem-and-leaf of PERCENT N = 51Leaf Unit = 1.03 8 001 10 8 2222333 44444555 18 8 66777777 (8) 8 25 8 888889999 16 9 0000000011111 22

The overall shape of this distribution is unimodal and not symmetric. (b) The distribution is left skewed.

## **94** Chapter 2

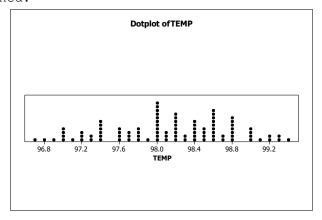
(a) In Exercise 2.109, we used Minitab to obtain a stem-and-leaf diagram using 2 lines per stem. That diagram is shown below.

Stem-and-leaf of PERCENT N = 51Leaf Unit = 1.0

- 2 1 79
- 14 2 011223334444
- (20) 2 55555566666777778999
- 17 3 00011122234
- 6 3 56668
- 1 4
- 1 4 9

The overall shape of this distribution is unimodal and roughly symmetric.

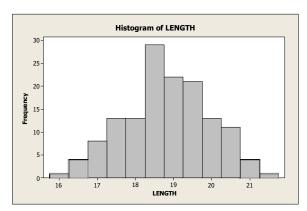
(a) After entering the data in Minitab, select **Graph < Dotplot**, select **Simple** in the **One Y** row, and click **OK**. Double click on TEMP to enter TEMP in the **Graph variables** box and click **OK**. Our result is as follows. Results may vary depending on the type of technology used and graph obtained.



The overall distribution of temperatures is roughly unimodal and roughly symmetric.

(a) After entering the data from the WeissStats Resource Site, in Minitab,

select **Graph < Histogram**, select **Simple** and click **OK**. Double click on LENGTH to enter LENGTH in the **Graph variables** box and click **OK**. Our result is as follows. Results may vary depending on the type of technology used and graph obtained.



The distribution of LENGTH is approximately unimodal and symmetric.

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Class Project. The precise answers to this exercise will vary from class to class.

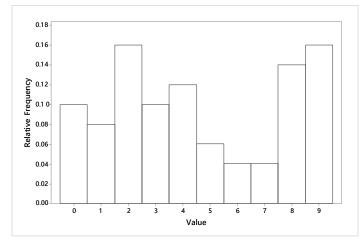
The precise answers to this exercise will vary from class to class or individual to individual. Thus your results will likely differ from our results shown below.

(a) We obtained 50 random digits from a table of random numbers. The digits were

4 5 4 6 8 9 9 7 7 2 2 2 9 3 0 3 4 0 0 8 8 4 4 5 3 9 2 4 8 9 6 3 0 1 1 0 9 2 8 1 3 9 2 5 8 1 8 9 2 2

- (b) Since each digit is equally likely in the random number table, we expect that the distribution would look roughly uniform.
- (c) Using single value classes, the frequency distribution is given by the following table. The relative frequency histogram is shown below.

Value	Frequency	Relative-Frequency
0	5	.10
1	4	.08
2	8	.16
3	5	.10
4	6	.12
5	3	.06
6	2	.04
7	2	.04
8	7	.14
9	8	.16



We did not expect to see this much variation.

- (d) We would have expected a histogram that was a little more 'even', more like a uniform distribution, but the relatively small sample size can result in considerable variation from what is expected.
- (e) We should be able to get a more uniformly distributed set of data if we choose a larger set of data.
- (f) Class project.
- (a-c) Your results will differ from the ones below which were obtained using Excel. Enter a name for the data in cell A1, say RANDNO. Click on

cell A2 and enter =RANDBETWEEN(0,9). Then copy this cell into cells A3 to A51. There are two ways to produce a histogram of the resulting data in Excel. The easier way is to highlight A1-A51 with the mouse, click on the toolbar, select Graphs and Plots, then choose Histogram in the Function type box. Now click on RANDNO in the Names and Columns box and drag the name into the Quantitative Variables box. Then click OK. A graph and a summary table will be produced. To get five more samples, simply go back to the spreadsheet and press the F9 key. This will generate an entire new sample in Column A and you can repeat the procedure. The only disadvantage of this method is that the graphs produced use white lines on a black background.

The second method is a bit more cumbersome and does not provide a summary chart, but yields graphs that are better for reproduction and that can be edited. Generate the data in the same way as was done above. In cells B1 to B10 enter the integers 0 to 9. These cells are called the BIN. Now click on Tools, Data Analysis, Histogram. Data Analysis is not in the Tools menu, you will have to add it from the original CD.) Click on the Input box and highlight cells A2-A51 with the mouse, then click on the Bin box and highlight cells B1-B10. Finally click on the Output box and enter C1. This will give you a frequency table in columns C and D. Now enter the integers 0 to 9 as text in cells E2 to E11 by entering each digit preceded by a single quote mark, i.e., '0, '1, etc. In cell F2, enter =D2, and copy this cell into F3 through F11. Now highlight the data in columns E and F with the mouse and click the chart icon, pick the Column graph type, pick the first sub-type, click on the Next button twice, enter any titles desired, remove the legend, and then click on the Next button and then the Finish button. The graph will appear on the spreadsheet as a bar chart with spaces between the bars. Use the mouse to point to any one of the bars and click with the right mouse button. Choose Format Data Series. Click on the Options tab and change the Gap Width to zero, and click OK. Repeat this sequence to produce additional histograms, but use different cells.

[If you would like to avoid repeating most of the above steps, click near the border of the graph and copy the graph to the Clipboard, then go to Microsoft Word or other word processor, and click on Edit on the Toolbar and Paste Special. Highlight Microsoft Excel Chart Object, and click OK. The graphs can be resized in the word processor if necessary. Now go back to Excel and hit the F9 key. This will produce a completely new set of random numbers. Click on Tools, Data Analysis, Histogram, leave all the boxes as they are and click OK. Then click OK to overwrite existing data. A new table will be created and the existing histogram will be updated automatically. We used this process for the following histograms.]



