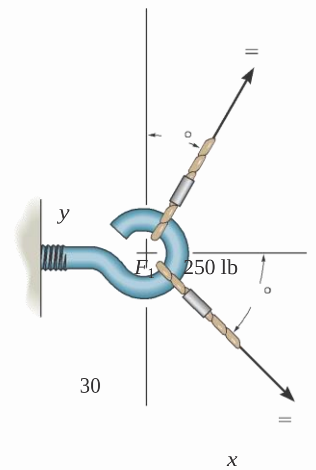
Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive $x$ axis.



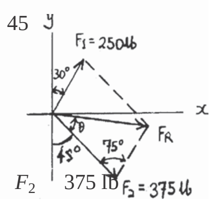

## SOLUTION

$$F_R = \sqrt{(250)^2 + (375)^2 - 2(250)(375)\cos 75^\circ} = 393.2 = 393 \text{ lb}$$ **Ans.**

$$\frac{393.2}{\sin 75^\circ} = \frac{250}{\sin \theta}$$

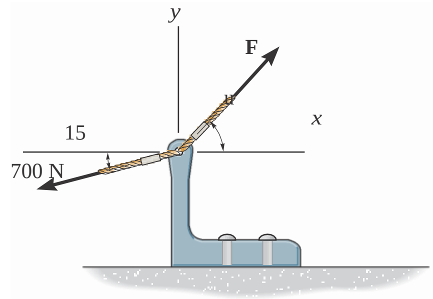
$$\theta = 37.89^\circ$$

$$\phi = 360^\circ - 45^\circ + 37.89^\circ = 353^\circ$$ **Ans.**



**2–2.**

resultant force and its direction, measured counterclockwise from the positive *x* axis.




## SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of consines to Fig. *b*,

$$F_R = \sqrt{700^2 + 450^2 - 2(700)(450)\cos 45^\circ}$$

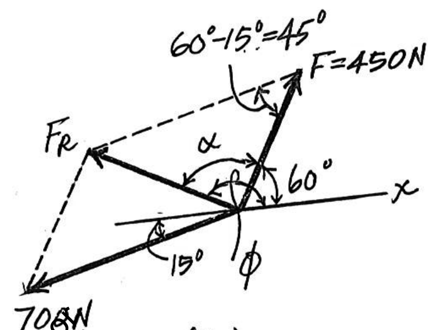
$$= 497.01 \text{ N} = 497 \text{ N} \qquad \textbf{Ans.}$$

This yields

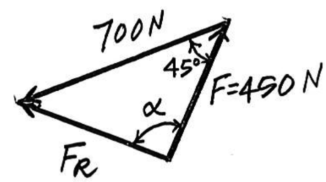
$$\frac{\sin \alpha}{700} = \frac{\sin 45^\circ}{497.01} \qquad \alpha = 95.19^\circ$$

Thus, the direction of angle $\phi$ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis, is

$$\phi = \alpha + 60^\circ = 95.19^\circ + 60^\circ = 155^\circ \qquad \textbf{Ans.}$$



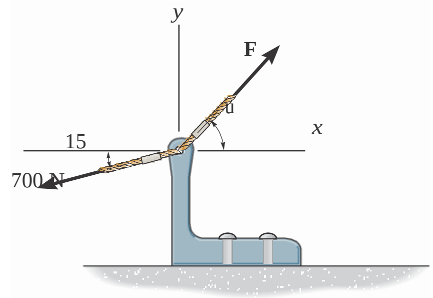

(a)




(b)

**2–3.**

If the magnitude of the resultant force is to be 500 N, directed along the positive *y* axis, determine the magnitude of force **F** and its direction u.




## SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

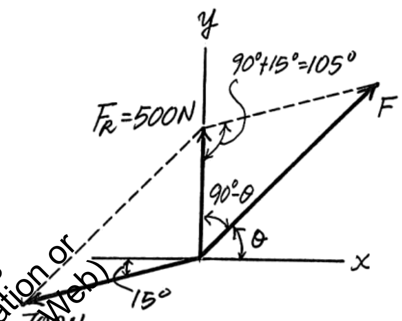
$$F = \sqrt{500^2 + 700^2 - 2(500)(700)\cos 105^\circ}$$

$$= 959.78 \text{ N} = 960 \text{ N}$$ **Ans.**

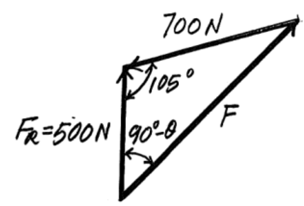
Applying the law of sines to Fig. *b*, and using this result, yields

$$\frac{\sin(90^\circ + u)}{700} = \frac{\sin 105^\circ}{959.78}$$

$$u = 45.2^\circ$$ **Ans.**



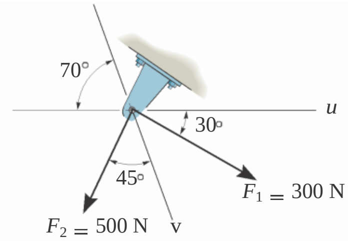

(a)




(b)

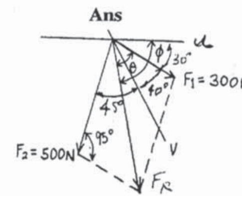
***2–4.**

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive $u$ axis.




SOLUTION

$F_R = \sqrt{(300)^2 + (500)^2 - 2(300)(500)\cos 95°} = 605.1 = 605\text{ N}$ **Ans.**



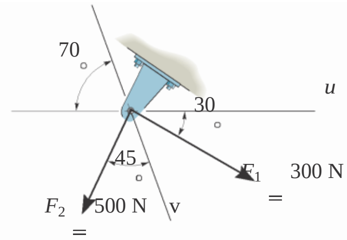

$$\frac{605.1}{\sin 95°} = \frac{500}{\sin u}$$

$u = 55.40°$

$\phi = 55.40° + 30° = 85.4°$ **Ans.**

**2–5.**

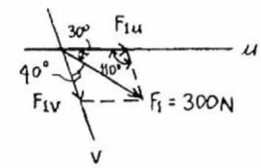
Resolve the force $\mathbf{F}_1$ into components acting along the *u* and v axes and determine the magnitudes of the components.




## SOLUTION

$$\frac{F_{1u}}{\sin 40^\circ} = \frac{300}{\sin 110^\circ}$$

$$F_{1u} = 205 \text{ N}$$ **Ans.**

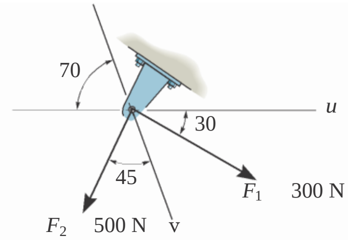



$$\frac{F_{1v}}{\sin 30^\circ} = \frac{300}{\sin 110^\circ}$$

$$F_{1v} = 160 \text{ N}$$ **Ans.**

**2–6.**

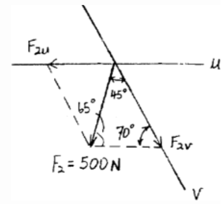
Resolve the force $\mathbf{F}_2$ into components acting along the $u$ and v axes and determine the magnitudes of the components.




## SOLUTION

$$\frac{F_{2u}}{\sin 45^\circ} = \frac{500}{\sin 70^\circ}$$

$$F_{2u} = 376 \text{ N}$$ **Ans.**

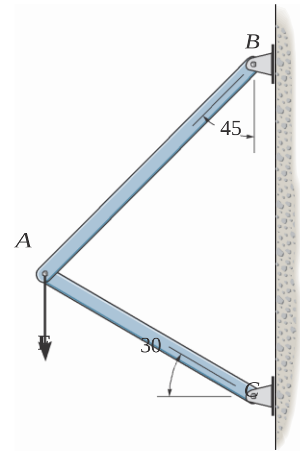



$$\frac{F_{2v}}{\sin 65^\circ} = \frac{500}{\sin 70^\circ}$$

$$F_{2v} = 482 \text{ N}$$ **Ans.**

**2–7.**

The vertical force **F** acts downward at A on the two-membered frame. Determine the magnitudes of the two components of **F** directed along the axes of AB and AC. Set F = 500 N.




## SOLUTION

***Parallelogram Law:*** The parallelogram law of addition is shown in Fig. *a*.

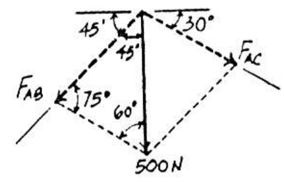
***Trigonometry:*** Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60°} = \frac{500}{\sin 75°}$$

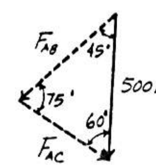
$$F_{AB} = 448 \text{ N} \qquad \textbf{Ans.}$$

$$\frac{F_{AC}}{\sin 45°} = \frac{500}{\sin 75°}$$

$$F_{AC} = 366 \text{ N} \qquad \textbf{Ans.}$$



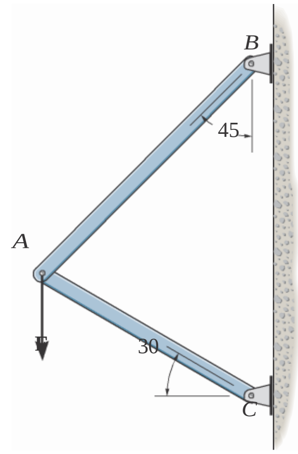
(a)



(b)

***2–8.**

Solve Prob. 2-7 with $F = 350$ lb.




## SOLUTION

***Parallelogram Law:*** The parallelogram law of addition is shown in Fig. *a*.

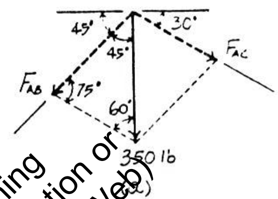
***Trigonometry:*** Using the law of sines (Fig. *b*), we have

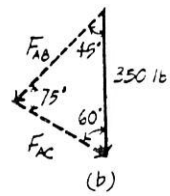
$$\frac{F_{AB}}{\sin 60^\circ} = \frac{350}{\sin 75^\circ}$$

$$F_{AB} = 314 \text{ lb}$$ **Ans.**

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{350}{\sin 75^\circ}$$

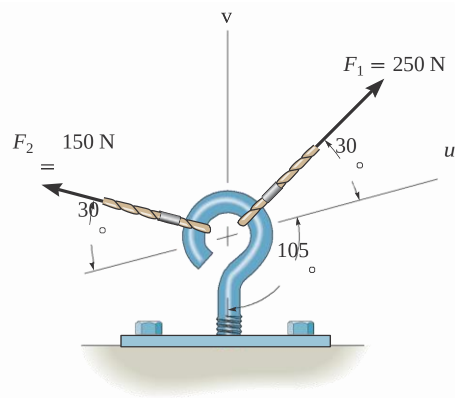
$$F_{AC} = 256 \text{ lb}$$ **Ans.**



**2–9.**

Resolve $\mathbf{F}_1$ into components along the $u$ and v axes and determine the magnitudes of these components.

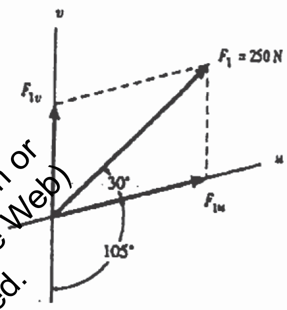


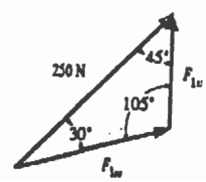

## SOLUTION

Sine law:

$$\frac{F_{1v}}{\sin 30^\circ} = \frac{250}{\sin 105^\circ} \qquad F_{1v} = 129 \text{ N} \qquad \textbf{Ans.}$$

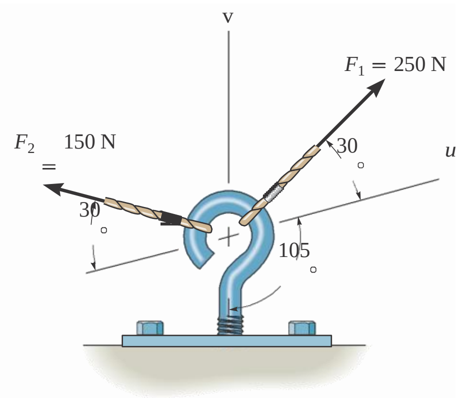
$$\frac{F_{1u}}{\sin 45^\circ} = \frac{250}{\sin 105^\circ} \qquad F_{1u} = 183 \text{ N} \qquad \textbf{Ans.}$$



**2–10.**

Resolve $\mathbf{F}_2$ into components along the $u$ and v axes and determine the magnitudes of these components.

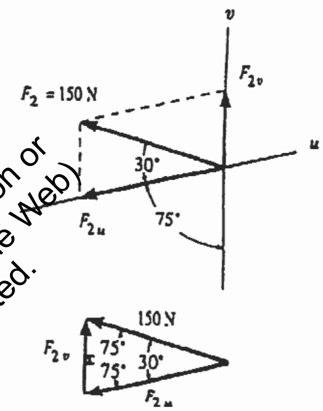



## SOLUTION

Sine law:

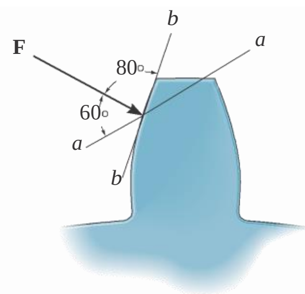
$$\frac{F_{2v}}{\sin 30^\circ} = \frac{150}{\sin 75^\circ} \qquad F_{2v} = 77.6 \text{ N} \qquad \textbf{Ans.}$$

$$\frac{F_{2u}}{\sin 75^\circ} = \frac{150}{\sin 75^\circ} \qquad F_{2u} = 150 \text{ N} \qquad \textbf{Ans.}$$



**2–11.**

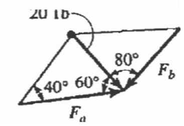
The force acting on the gear tooth is **F** = 20 lb. Resolve this force into two components acting along the lines *aa* and *bb*.




SOLUTION

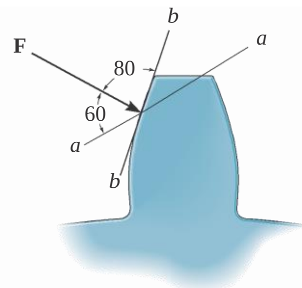
$$\frac{20}{\sin 40°} = \frac{F_a}{\sin 80°}; \qquad F_a = 30.6 \text{ lb}$$ **Ans.**

$$\frac{20}{\sin 40°} = \frac{F_b}{\sin 60°}; \qquad F_b = 26.9 \text{ lb}$$ **Ans.**



***2–12.**

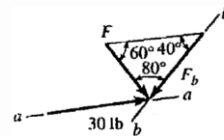
The component of force **F** acting along line *aa* is required to be 30 lb. Determine the magnitude of **F** and its component along line *bb*.




SOLUTION

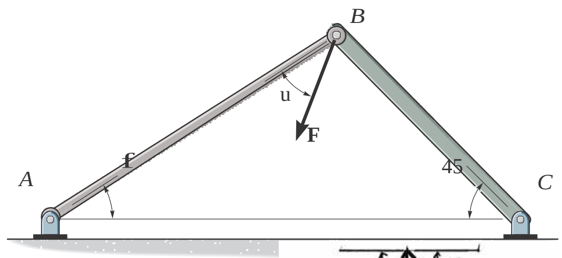
$$\frac{30}{\sin 80^\circ} = \frac{F}{\sin 40^\circ}; \qquad F = 19.6 \text{ lb}$$ **Ans.**

$$\frac{30}{\sin 80^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.4 \text{ lb}$$ **Ans.**



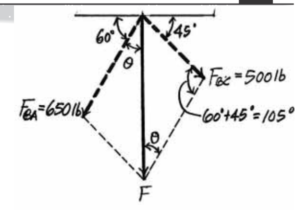
**2–13.**

Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A, and the component acting along member BC is 500 lb, directed from B towards C. Determine the magnitude of **F** and its direction u. Set f = 60°.




## SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.




(a)

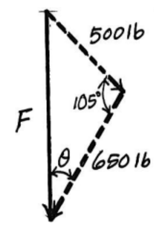
Applying the law of cosines to Fig. *b*,

$$F = \sqrt{500^2 + 650^2 - 2(500)(650)\cos 105°}$$

$$= 916.91 \text{ lb} = 917 \text{ lb}$$ **Ans.**

Using this result and applying the law of sines to Fig. *b*, yields

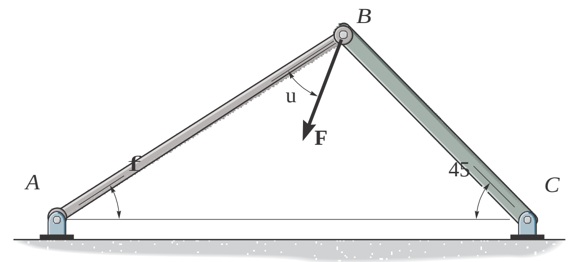
$$\frac{\sin u}{500} = \frac{\sin 105°}{916.91} \qquad u = 31.8°$$ **Ans.**




(b)

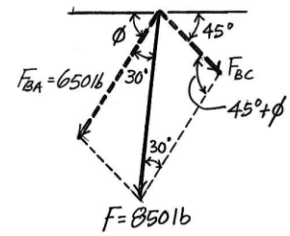
**2–14.**

Force **F** acts on the frame such that its component acting along member *AB* is 650 lb, directed from *B* towards *A*. Determine the required angle $\phi$ ($0° \leq \phi \leq 90°$) and the component acting along member BC. Set $F = 850$ lb and $\theta = 30°$.

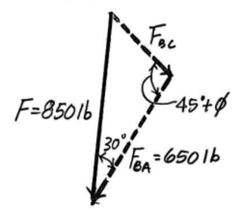



## SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



(a)



(b)

Applying the law of cosines to Fig. *b*,

$$F_{BC} = \sqrt{850^2 + 650^2 - 2(850)(650)\cos 30°}$$

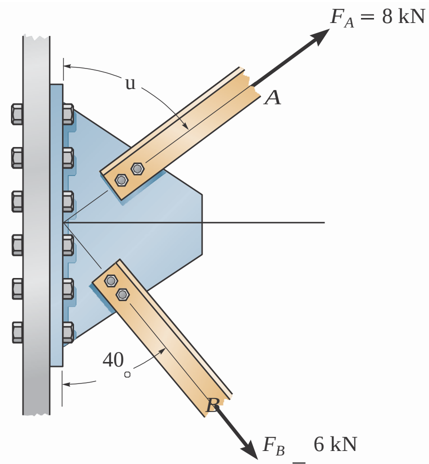
$$= 433.64 \text{ lb} = 434 \text{ lb}$$ **Ans.**

Using this result and applying the sine law to Fig. *b*, yields

$$\frac{\sin(45° + \phi)}{850} = \frac{\sin 30°}{433.64} \qquad \phi = 56.5°$$ **Ans.**

**2–15.**

The plate is subjected to the two forces at $A$ and $B$ as shown. If u = 60°, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.




## SOLUTION

***Parallelogram Law:*** The parallelogram law of addition is shown in Fig. *a*.

***Trigonometry:*** Using law of cosines (Fig. *b*), we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100°}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN} \qquad \textbf{Ans.}$$

The angle u can be determined using law of sines (Fig. *b*).

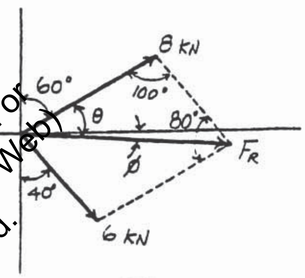
$$\frac{\sin u}{6} = \frac{\sin 100°}{10.80}$$

$$\sin u = 0.5470$$

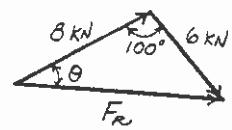
$$u = 33.16°$$

Thus, the direction f of $\mathbf{F}_R$ measured from the $x$ axis is

$$f = 33.16° - 30° = 3.16° \qquad \textbf{Ans.}$$



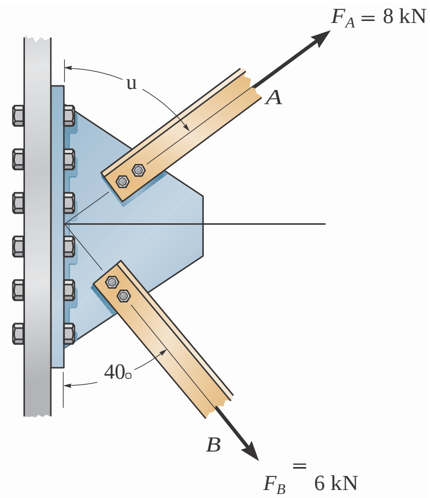

(a)




(b)

***2–16.**

Determine the angle of $\theta$ for connecting member $A$ to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?




## SOLUTION

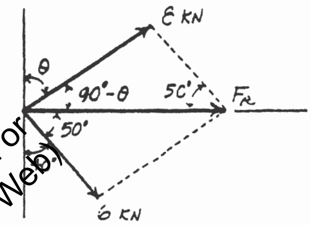
***Parallelogram Law:*** The parallelogram law of addition is shown in Fig. *a*.

***Trigonometry:*** Using law of sines (Fig .*b*), we have

$$\frac{\sin(90° - \theta)}{6} = \frac{\sin 50°}{8}$$

$$\sin(90° - \theta) = 0.5745$$

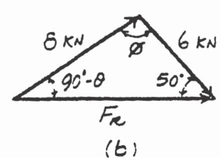
$$\theta = 54.93° = 54.9° \qquad \textbf{Ans.}$$




From the triangle, $\phi = 180° - (90° - 54.93°) - 50° = 94.93°$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

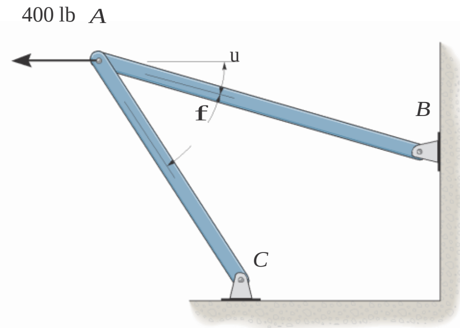
$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93°}$$

$$= 10.4 \text{ kN} \qquad \textbf{Ans.}$$



**2–17.**

Determine the design angle u (0° … u … 90°) for strut *AB* so that the 400-lb horizontal force has a component of 500 lb directed from *A* towards *C*. What is the component of force acting along member *AB*? Take **f** = 40°.




## SOLUTION

***Parallelogram Law:*** The parallelogram law of addition is shown in Fig. *a*.

***Trigonometry:*** Using law of sines (Fig. *b*), we have

$$\frac{\sin u}{500} = \frac{\sin 40°}{400}$$

$$\sin u = 0.8035$$

$$u = 53.46° = 53.5° \qquad \textbf{Ans.}$$

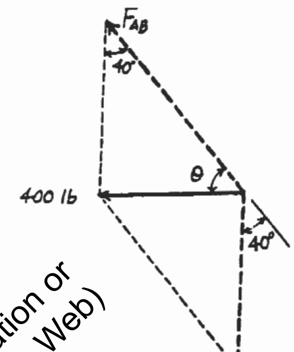
Thus,

$$c = 180° - 40° - 53.46° = 86.54°$$

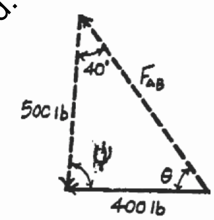
Using law of sines (Fig. *b*)

$$\frac{F_{AB}}{\sin 86.54°} = \frac{400}{\sin 40°}$$

$$F_{AB} = 621 \text{ lb} \qquad \textbf{Ans.}$$



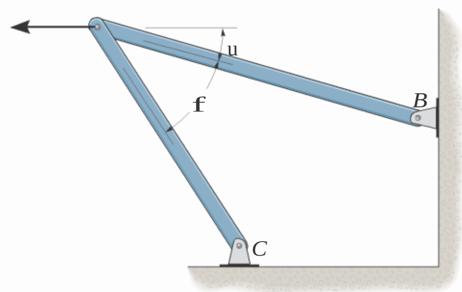

(a)




(b)

**2–18.**

Determine the design angle $\phi$ ($0° \le \phi \le 90°$) between struts *AB* and *AC* so that the 400-lb horizontal force has a component of 600 lb which acts up to the left, in the same direction as from *B* towards *A*. Take $\theta = 30°$.




## SOLUTION

***Parallelogram Law:*** The parallelogram law of addition is shown in Fig. *a*.

***Trigonometry:*** Using law of cosines (Fig. *b*), we have

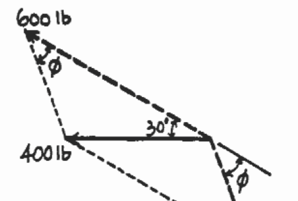
$$F_{AC} = \sqrt{400^2 + 600^2 - 2(400)(600)\cos 30°} = 322.97 \text{ lb}$$

The angle $\phi$ can be determined using law of sines (Fig. *b*).

$$\frac{\sin \phi}{400} = \frac{\sin 30°}{322.97}$$

$$\sin \phi = 0.6193$$

$$\phi = 38.3° \qquad \textbf{Ans.}$$

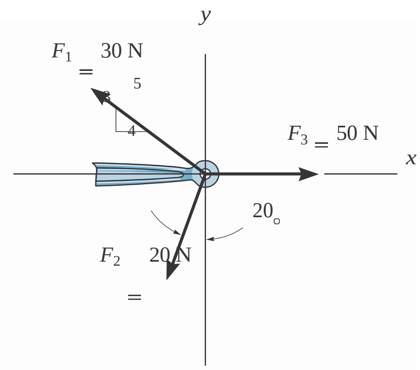


(a)

(b)

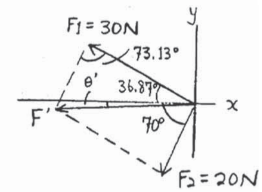
**2–19.**

Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.



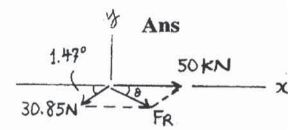

$$F' = \sqrt{(20)^2 + (30)^2 - 2(20)(30)\cos 73.13^\circ} = 30.85 \text{ N}$$

$$\frac{30.85}{\sin 73.13^\circ} = \frac{30}{\sin(70^\circ - \theta')}; \qquad \theta' = 1.47^\circ$$



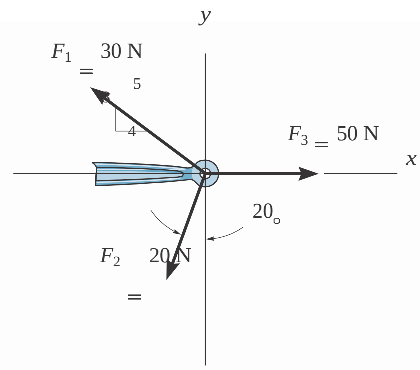

$$F_R = \sqrt{(30.85)^2 + (50)^2 - 2(30.85)(50)\cos 1.47^\circ} = 19.18 = 19.2 \text{ N}$$ **Ans.**

$$\frac{19.18}{\sin 1.47^\circ} = \frac{30.85}{\sin \theta}; \qquad \theta = 2.37^\circ$$ **Ans.**

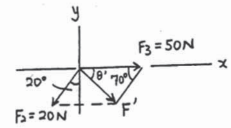


***2–20.**

Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.

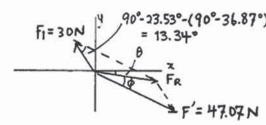



$$F' = \sqrt{(20)^2 + (50)^2 - 2(20)(50)\cos 70°} = 47.07 \text{ N}$$




$$\frac{20}{\sin \theta'} = \frac{47.07}{\sin 70°}; \qquad \theta' = 23.53°$$

$$F_R = \sqrt{(47.07)^2 + (30)^2 - 2(47.07)(30)\cos 13.34°} = 19.18 = 19.2 \text{ N} \qquad \textbf{Ans.}$$

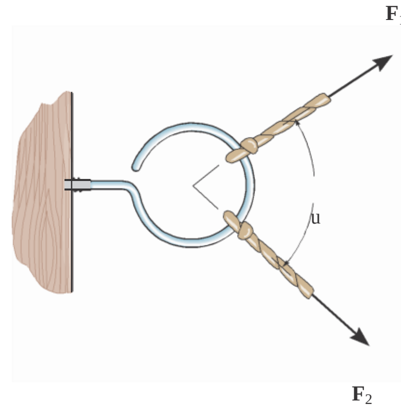



$$\frac{19.18}{\sin 13.34°} = \frac{30}{\sin \phi}; \qquad \phi = 21.15°$$

$$\theta = 23.53° - 21.15° = 2.37° \ ⦪ \qquad \textbf{Ans.}$$

**2–21.**

Two forces act on the screw eye. If $F_1 = 400$ N and $F_2 = 600$ N, determine the angle $u(0° \ldots u \ldots 180°)$ between them, so that the resultant force has a magnitude of $F_R = 800$ N.




## SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively. Applying law of cosines to Fig. *b*,

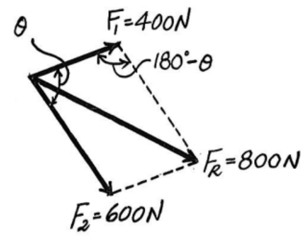
$$800 = \sqrt{400^2 + 600^2 - 2(400)(600)\cos(180° - u°)}$$

$$800^2 = 400^2 + 600^2 - 480000\cos(180° - u)$$

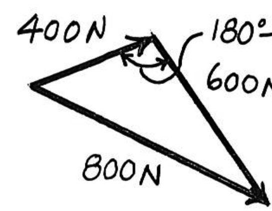
$$\cos(180° - u) = -0.25$$

$$180° - u = 104.48$$

$$u = 75.52° = 75.5°$$ **Ans.**



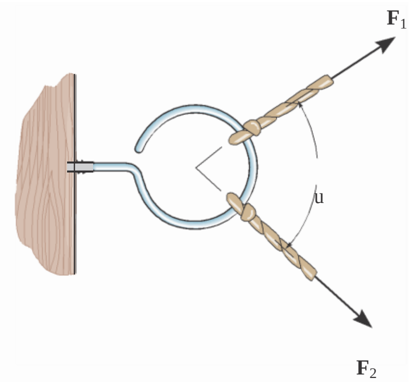
(a)



(b)

**2–22.**

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If their lines of action are at an angle u apart and the magnitude of each force is $F_1 = F_2 = F$, determine the magnitude of the resultant force $\mathbf{F}_R$ and the angle between $\mathbf{F}_R$ and $\mathbf{F}_1$.




## SOLUTION

$$\frac{F}{\sin f} = \frac{F}{\sin (u - f)}$$

$$\sin (u - f) = \sin f$$

$$u - f = f$$

$$f = \frac{u}{2}$$ **Ans.**

$$F_R = \sqrt{(F)^2 + (F)^2 - 2(F)(F)\cos (180° - u)}$$

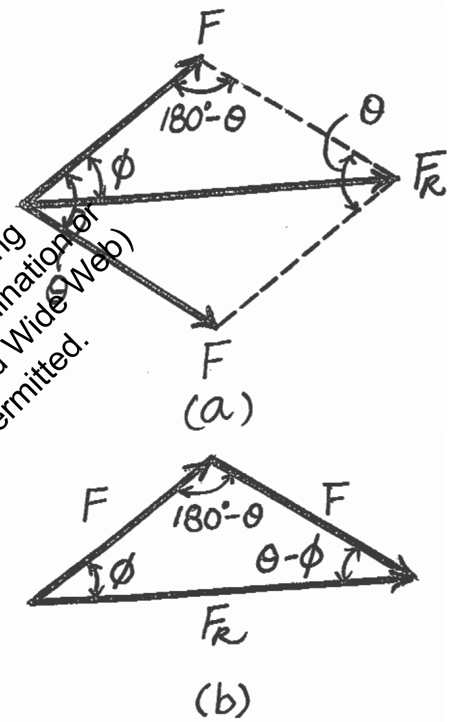
Since $\cos (180° - u) = -\cos u$

$$F_R = F\left(\sqrt{2}\right)\sqrt{1 + \cos u}$$

Since $\cos \left(\frac{u}{2}\right) = \sqrt{\frac{1 + \cos u}{2}}$

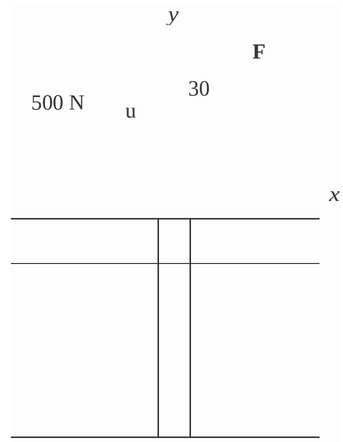
Then

$$F_R = 2F \cos \left(\frac{u}{2}\right)$$ **Ans.**



**2–23.**

Two forces act on the screw eye. If $F = 600$ N, determine the magnitude of the resultant force and the angle u if the resultant force is directed vertically upward.




## SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b* respectively. Applying law of sines to Fig. *b*,

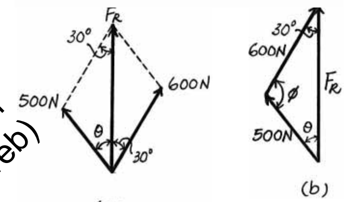
$$\frac{\sin u}{600} = \frac{\sin 30°}{500}; \quad \sin u = 0.6 \quad u = 36.87° = 36.9°$$ **Ans.**

Using the result of u,

$$f = 180° - 30° - 36.87° = 113.13°$$

Again, applying law of sines using the result of f,

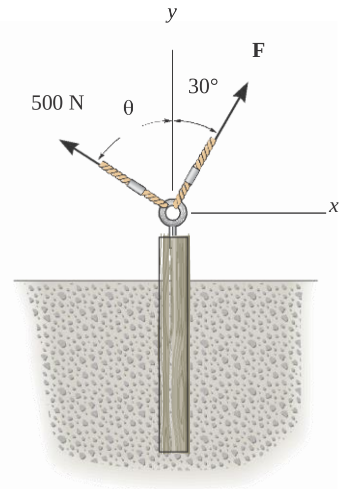
$$\frac{F_R}{\sin 113.13°} = \frac{500}{\sin 30°}; \quad F_R = 919.61 \text{ N} = 920 \text{ N}$$ **Ans.**




(a) (b)

***2–24.**

Two forces are applied at the end of a screw eye in order to remove the post. Determine the angle $\theta$ $(0° \le \theta \le 90°)$ and the magnitude of force **F** so that the resultant force acting on the post is directed vertically upward and has a magnitude of 750 N.




## SOLUTION

***Parallelogram Law:*** The parallelogram law of addition is shown in Fig. *a*.

***Trigonometry:*** Using law of sines (Fig. *b*), we have

$$\frac{\sin \phi}{750} = \frac{\sin 30°}{500}$$

$$\sin \phi = 0.750$$

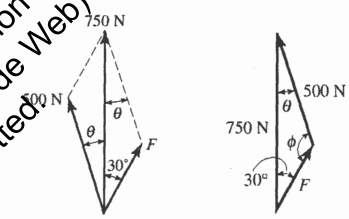
$$\phi = 131.41° \quad (\text{By observation, } \phi > 90°)$$

Thus,

$$\theta = 180° - 30° - 131.41° = 18.59° = 18.6°$$ **Ans.**

$$\frac{F}{\sin 18.59°} = \frac{500}{\sin 30°}$$

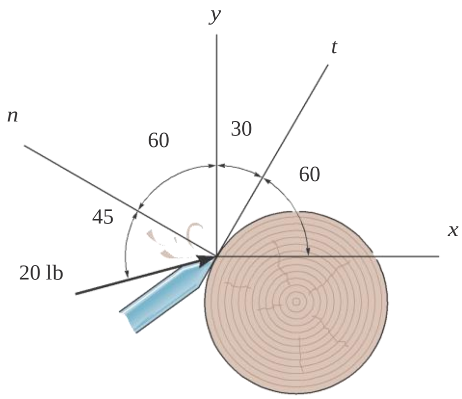
$$F = 319 \text{ N}$$ **Ans.**




(a) (b)

**2–25.**

The chisel exerts a force of 20 lb on the wood dowel rod which is turning in a lathe. Resolve this force into components acting (a) along the n and t axes and (b) along the x and y axes.



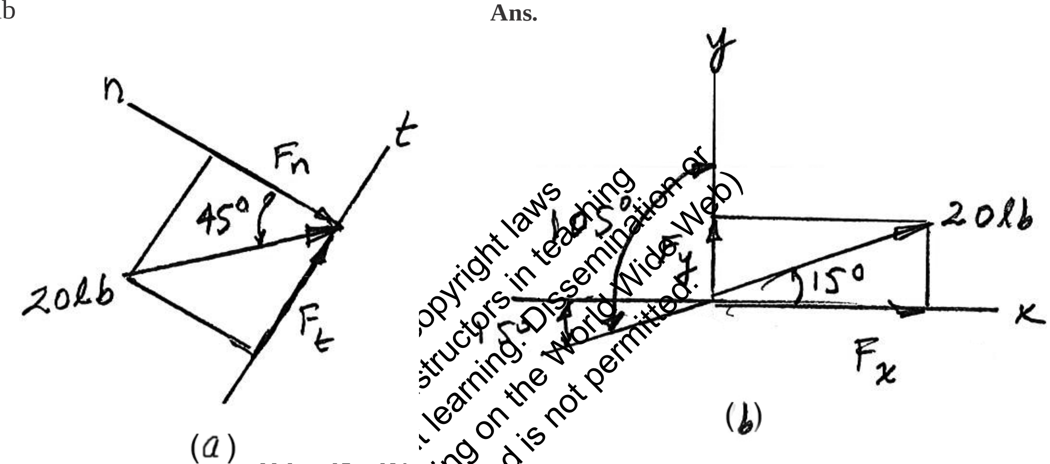

## SOLUTION

a) $F_n = -20 \cos 45° = -14.1$ lb **Ans.**

$F_t = 20 \sin 45° = 14.1$ lb **Ans.**

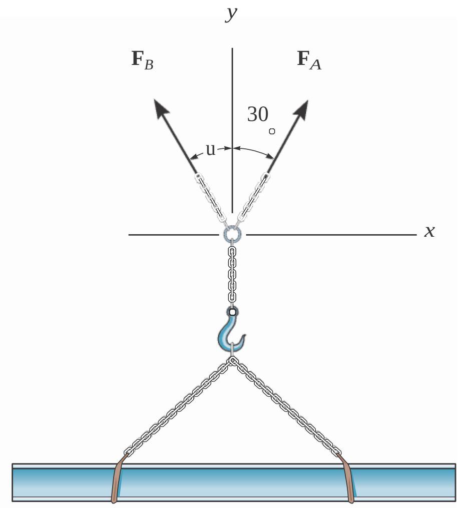
b) $F_x = 20 \cos 15° = 19.3$ lb **Ans.**

$F_y = 20 \sin 15° = 5.18$ lb **Ans.**



**2–26.**

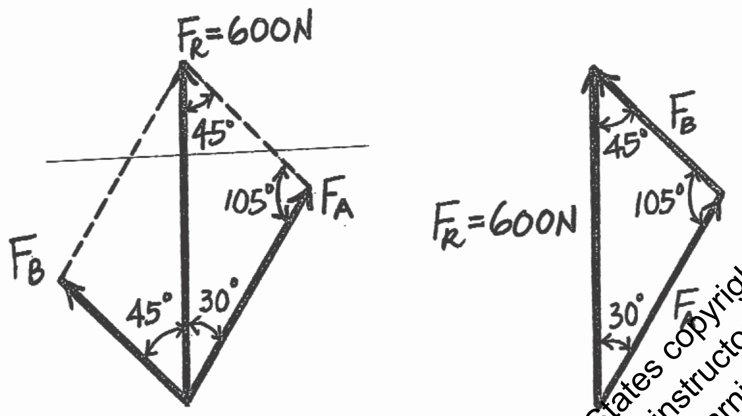
The beam is to be hoisted using two chains. Determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain in order to develop a resultant force of 600 N directed along the positive $y$ axis. Set $u = 45°$.




## SOLUTION

$$\frac{F_A}{\sin 45°} = \frac{600}{\sin 105°}; \qquad F_A = 439 \text{ N} \qquad \textbf{Ans.}$$

$$\frac{F_B}{\sin 30°} = \frac{600}{\sin 105°}; \qquad F_B = 311 \text{ N} \qquad \textbf{Ans.}$$




(a) (b)

**2–27.**

The beam is to be hoisted using two chains. If the resultant force is to be 600 N directed along the positive $y$ axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain and the angle u of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a *minimum*. $\mathbf{F}_A$ acts at 30° from the $y$ axis, as shown.

$y$

$\mathbf{F}_B$ $\mathbf{F}_A$

u 30

$x$

## SOLUTION

For minimum $F_B$, require

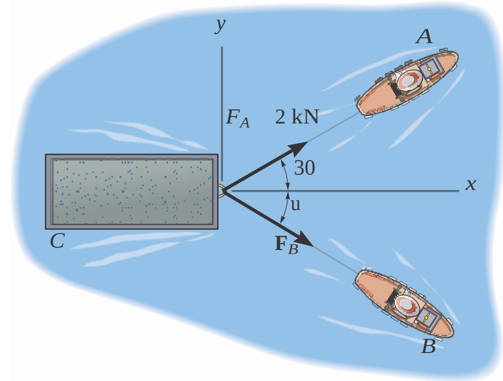
$$u = 60°$$ **Ans.**

$$F_A = 600 \cos 30° = 520 \text{ N}$$ **Ans.**

$$F_B = 600 \sin 30° = 300 \text{ N}$$ **Ans.**

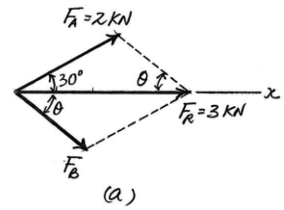
***2–28.**

If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force $\mathbf{F}_B$ and its direction u.




## SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



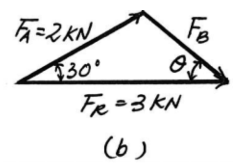

Applying the law of cosines to Fig. *b*,

$$F_B = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 30°}$$

$$= 1.615\text{kN} = 1.61 \text{ kN}$$ **Ans.**

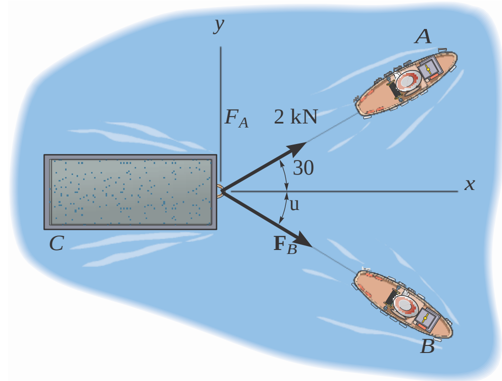
Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin u}{2} = \frac{\sin 30°}{1.615} \qquad u = 38.3°$$ **Ans.**



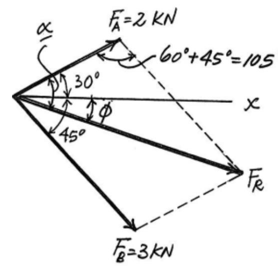
**2–29.**

If $F_B = 3$ kN and $\theta = 45°$, determine the magnitude of the resultant force of the two tugboats and its direction measured clockwise from the positive $x$ axis.

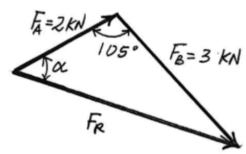



## SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.




(a)




(b)

Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 105°}$$

$$= 4.013 \text{ kN} = 4.01 \text{ kN}$$ **Ans.**

Using this result and applying the law of sines to Fig. *b*, yields

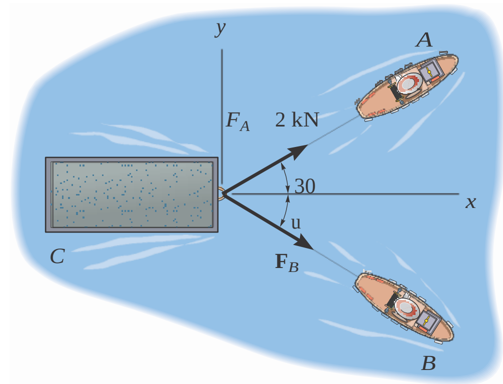
$$\frac{\sin \alpha}{3} = \frac{\sin 105°}{4.013} \qquad \alpha = 46.22°$$

Thus, the direction angle $\phi$ of $\mathbf{F}_R$, measured clockwise from the positive $x$ axis, is

$$\phi = \alpha - 30° = 46.22° - 30° = 16.2°$$ **Ans.**

**2–30.**

If the resultant force of the two tugboats is required to be directed towards the positive x axis, and $F_B$ is to be a minimum, determine the magnitude of $\mathbf{F}_R$ and $\mathbf{F}_B$ and the angle u.




## SOLUTION

For $\mathbf{F}_B$ to be minimum, it has to be directed perpendicular to $\mathbf{F}_R$. Thus,

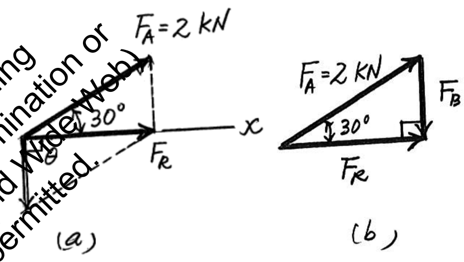
$$u = 90° \qquad \textbf{Ans.}$$

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

By applying simple trigonometry to Fig. *b*,

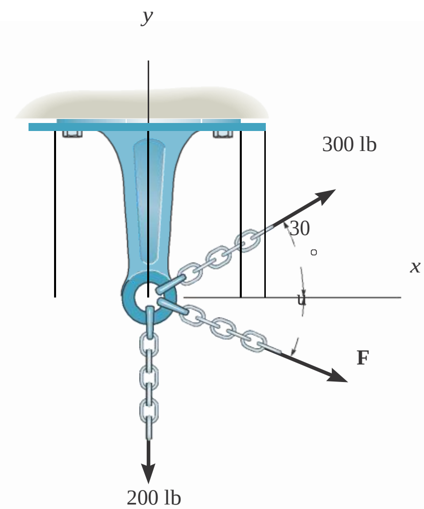
$$F_B = 2 \sin 30° = 1 \text{ kN} \qquad \textbf{Ans.}$$

$$F_R = 2 \cos 30° = 1.73 \text{ kN} \qquad \textbf{Ans.}$$



**2–31.**

Three chains act on the bracket such that they create a resultant force having a magnitude of 500 lb. If two of the chains are subjected to known forces, as shown, determine the angle u of the third chain measured clockwise from the positive *x* axis, so that the magnitude of force **F** in this chain is a *minimum*. All forces lie in the *x*–*y* plane. What is the magnitude of **F**? *Hint*: First find the resultant of the two known forces. Force **F** acts in this direction.




## SOLUTION

Cosine law:

$$F_{R1} = \sqrt{300^2 + 200^2 - 2(300)(200)\cos 60^\circ} = 264.6 \text{ lb}$$

Sine law:

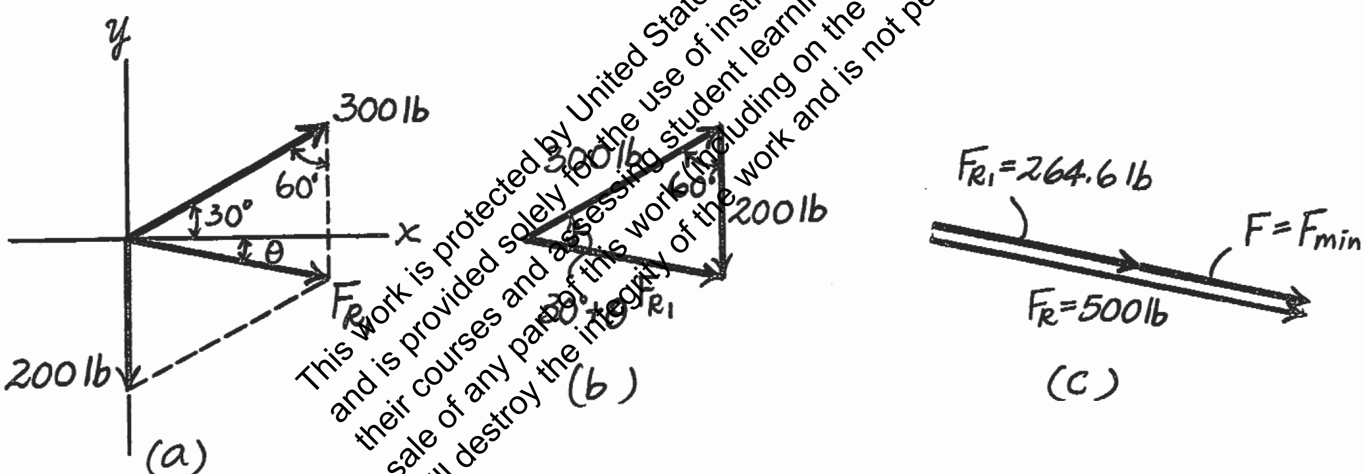
$$\frac{\sin(30^\circ + u)}{200} = \frac{\sin 60^\circ}{264.6} \qquad u = 10.9^\circ$$ **Ans.**

When **F** is directed along $\mathbf{F}_{R1}$, $F$ will be minimum to create the resultant force.

$$F_R = F_{R1} + F$$

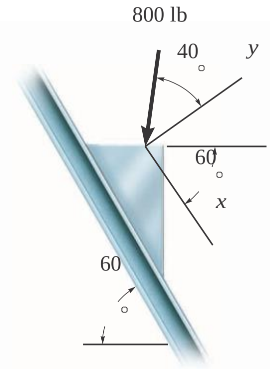
$$500 = 264.6 + F_{min}$$

$$F_{min} = 235 \text{ lb}$$ **Ans.**



***2–32.**

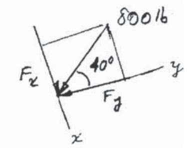
Determine the $x$ and $y$ components of the 800-lb force.




## SOLUTION

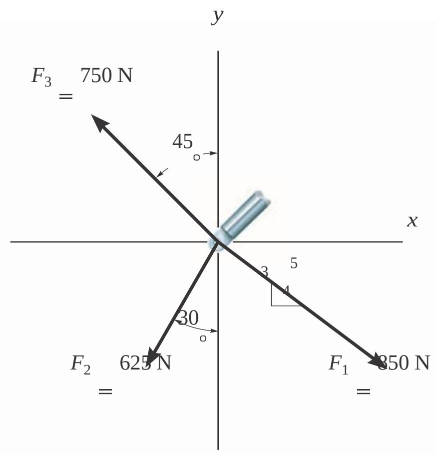
$F_x = 800 \sin 40° = 514 \text{ lb}$ **Ans.**

$F_y = -800 \cos 40° = -613 \text{ lb}$ **Ans.**



**2–33.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive $x$ axis.




## SOLUTION

$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = \frac{4}{5}(850) - 625 \sin 30° - 750 \sin 45° = -162.8 \text{ N}$

$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{Ry} = -\frac{3}{5}(850) - 625 \cos 30° + 750 \cos 45° = -520.9 \text{ N}$

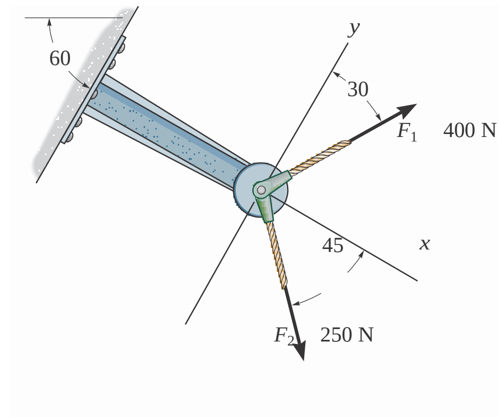
$F_R = \sqrt{(-162.8)^2 + (-520.9)^2} = 546 \text{ N}$ **Ans.**

$\phi = \tan^{-1}\left[\frac{-520.9}{-162.8}\right] = 72.64°$

$\theta = 180° + 72.64° = 253°$ **Ans.**

**2–34.**

Resolve $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x and y components.



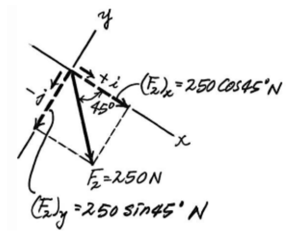

## SOLUTION

$$\mathbf{F}_1 = \{400 \sin 30°(+\mathbf{i}) + 400 \cos 30°(+\mathbf{j})\} \text{ N}$$

$$= \{200\mathbf{i} + 346\mathbf{j}\} \text{ N}$$ **Ans.**

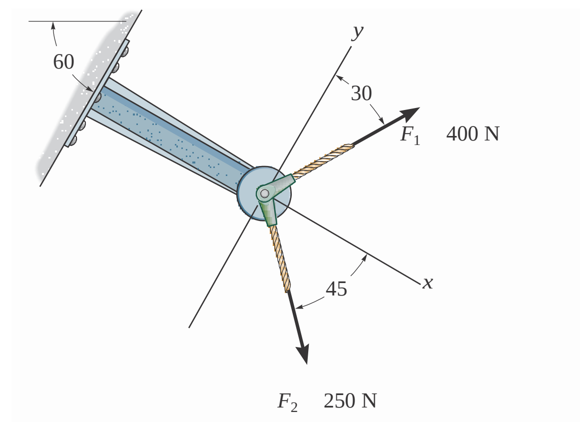
$$\mathbf{F}_2 = \{250 \cos 45°(+\mathbf{i}) + 250 \sin 45°(-\mathbf{j})\} \text{ N}$$

$$= \{177\mathbf{i} + 177\mathbf{j}\} \text{ N}$$ **Ans.**

**2–35.**

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive *x* axis.




## SOLUTION

***Rectangular Components:*** By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written as

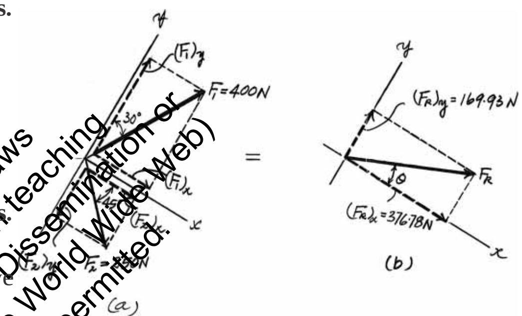
$$(F_1)_x = 400 \sin 30° = 200 \text{ N} \qquad (F_1)_y = 400 \cos 30° = 346.41 \text{ N}$$

$$(F_2)_x = 250 \cos 45° = 176.78 \text{ N} \qquad (F_2)_y = 250 \sin 45° = 176.78 \text{ N}$$

***Resultant Force:*** Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200 + 176.78 = 376.78 \text{ N} \rightarrow$$ **Ans.**

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 346.41 - 176.78 = 169.63 \text{ N} \uparrow$$




The magnitude of the resultant force $\mathbf{F}_R$ is

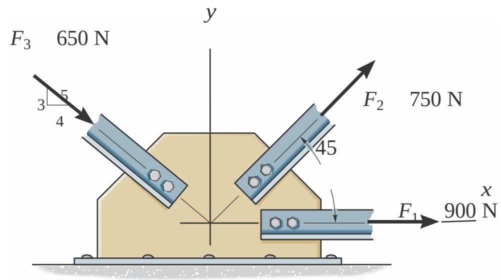
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{376.78^2 + 169.63^2} = 413 \text{ N}$$ **Ans.**

The direction angle $\theta$ of $\mathbf{F}_R$, Fig. *b*, measured counterclockwise from the positive axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{169.63}{376.78}\right) = 24.2°$$ **Ans.**

***2–36.**

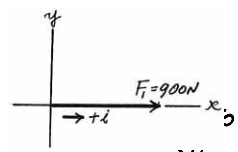
Resolve each force acting on the gusset plate into its x and y components, and express each force as a Cartesian vector.

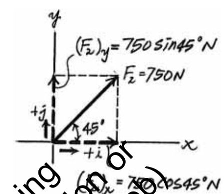


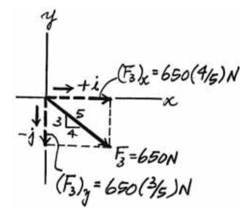

$$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_2 = \{750\cos 45°(+\mathbf{i}) + 750\sin 45°(+\mathbf{j})\}\text{ N}$$
$$= \{530\mathbf{i} + 530\mathbf{j}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_3 = \left\{650\left(\frac{4}{5}\right)(+\mathbf{i}) + 650\left(\frac{3}{5}\right)(-\mathbf{j})\right\}\text{ N}$$
$$= \{520\,\mathbf{i} - 390\mathbf{j})\}\text{ N}$$ **Ans.**

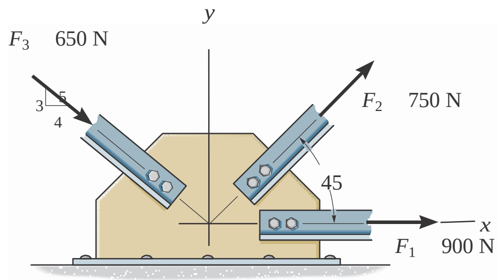


**2–37.**

Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive $x$ axis.




## SOLUTION

and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 900\text{ N} \qquad (F_1)_y = 0$$

$$(F_2)_x = 750\cos 45° = 530.33\text{ N} \qquad (F_2)_y = 750\sin 45° = 530.33\text{ N}$$

$$(F_3)_x = 650\left(\frac{4}{5}\right) = 520\text{ N} \qquad (F_3)_y = 650\left(\frac{3}{5}\right) = 390\text{ N}$$

***Resultant Force:*** Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900 + 530.33 + 520 = 1950.33\text{ N} \rightarrow$$

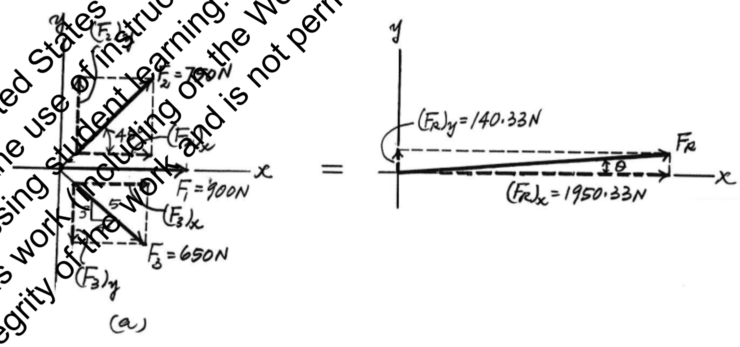
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 530.33 - 390 = 140.33\text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{1950.33^2 + 140.33^2} = 1955\text{ N} = 1.96\text{ kN} \quad \textbf{Ans.}$$

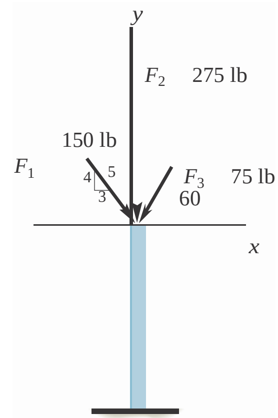
The direction angle $\theta$ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{140.33}{1950.33}\right) = 4.12° \quad \textbf{Ans.}$$



**2–38.**

Express each of the three forces acting on the column in Cartesian vector form and compute the magnitude of the resultant force.




## SOLUTION

$$\mathbf{F}_1 = 150\left(\frac{3}{5}\right)\mathbf{i} - 150\left(\frac{4}{5}\right)\mathbf{j}$$

$$\mathbf{F}_1 = \{90\mathbf{i} - 120\mathbf{j}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_2 = \{-275\mathbf{j}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_3 = -75\cos 60^\circ\mathbf{i} - 75\sin 60^\circ\mathbf{j}$$

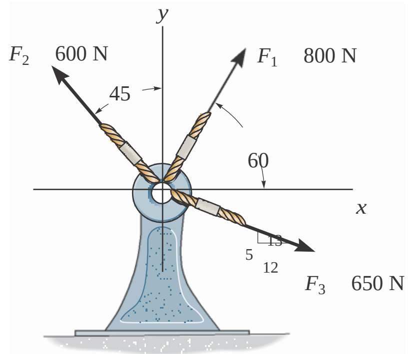
$$\mathbf{F}_3 = \{-37.5\mathbf{i} - 65.0\mathbf{j}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{52.5\mathbf{i} - 460\mathbf{j}\}\text{ lb}$$

$$F_R = \sqrt{(52.5)^2 + (-460)^2} = 463\text{ lb}$$ **Ans.**

**2–39.**

Resolve each force acting on the support into its $x$ and $y$ components, and express each force as a Cartesian vector.




## SOLUTION

$$\mathbf{F}_1 = \{800 \cos 60°(+\mathbf{i}) + 800 \sin 60°(+\mathbf{j})\} \text{ N}$$

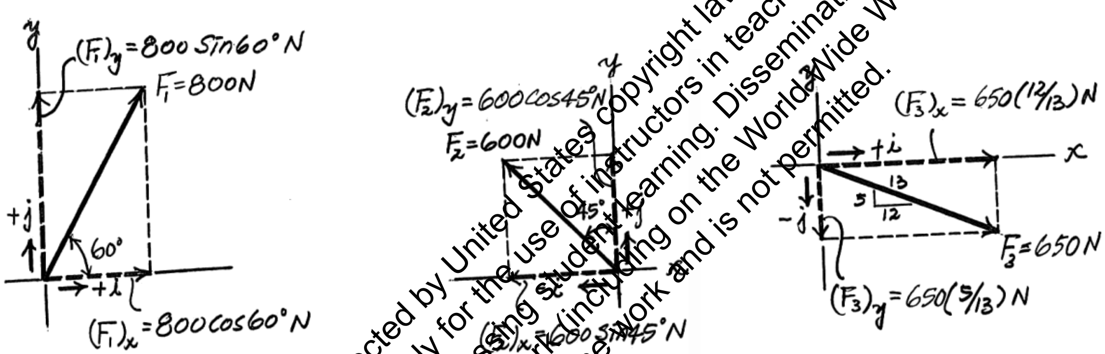
$$= \{400\mathbf{i} + 693\mathbf{j}\} \text{ N}$$ **Ans.**

$$\mathbf{F}_2 = \{600 \sin 45°(-\mathbf{i}) + 600 \cos 45°(+\mathbf{j})\} \text{ N}$$ **Ans.**

$$= \{-424\mathbf{i} + 424\mathbf{j}\} \text{ N}$$

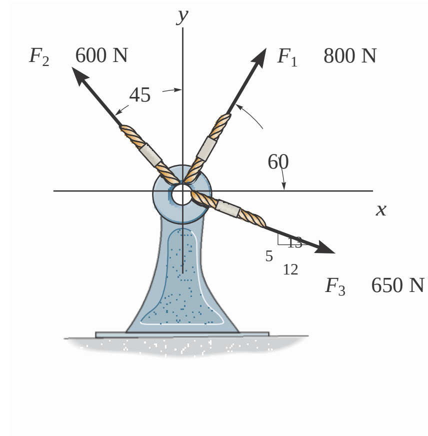
$$\mathbf{F}_3 = \left\{650\left(\frac{12}{13}\right)(+\mathbf{i}) + 650\left(\frac{5}{13}\right)(-\mathbf{j})\right\} \text{ N}$$ **Ans.**

$$= \{600\mathbf{i} - 250\mathbf{j}\} \text{ N}$$



***2–40.**

Determine the magnitude of the resultant force and its direction $\theta$, measured counterclockwise from the positive




## SOLUTION

***Rectangular Components:*** By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 800\cos 60^\circ = 400\text{ N} \qquad (F_1)_y = 800\sin 60^\circ = 692.82\text{ N}$$

$$(F_2)_x = 600\sin 45^\circ = 424.26\text{ N} \qquad (F_2)_y = 600\cos 45^\circ = 424.26\text{ N}$$

$$(F_3)_x = 650\left(\frac{12}{13}\right) = 600\text{ N} \qquad (F_3)_y = 650\left(\frac{5}{13}\right) = 250\text{ N}$$

***Resultant Force:*** Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 400 - 424.26 + 600 = 575.74\text{ N} \rightarrow$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 692.82 + 424.26 - 250 = 867.08\text{ N} \uparrow$$

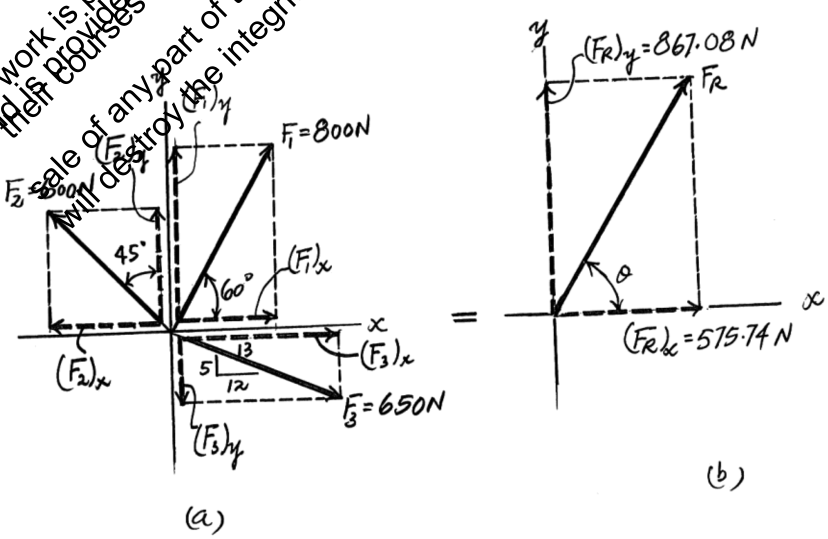
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{575.74^2 + 867.08^2} = 1041\text{ N} = 1.04\text{ kN} \qquad \textbf{Ans.}$$

The direction angle $\theta$ of $\mathbf{F}_R$, Fig. *b*, measured counterclockwise from the positive x axis, is

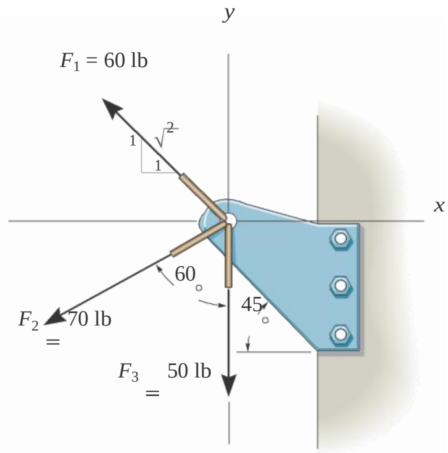
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{867.08}{575.74}\right) = 56.4^\circ \qquad \textbf{Ans.}$$





**2–41.**

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive $x$ axis.




## SOLUTION

$$\mathbf{F}_1 = -60\left(\frac{1}{\sqrt{2}}\right)\mathbf{i} + 60\left(\frac{1}{\sqrt{2}}\right)\mathbf{j} = \{-42.43\mathbf{i} + 42.43\mathbf{j}\}\ \text{lb}$$

$$\mathbf{F}_2 = -70\sin 60°\,\mathbf{i} - 70\cos 60°\,\mathbf{j} = \{-60.62\mathbf{i} - 35\mathbf{j}\}\ \text{lb}$$

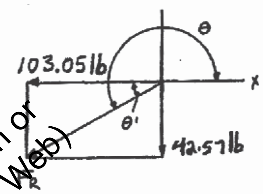
$$\mathbf{F}_3 = \{-50\mathbf{j}\}\ \text{lb}$$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{-103.05\mathbf{i} - 42.57\mathbf{j}\}\ \text{lb}$$

$$F_R = \sqrt{(-103.05)^2 + (-42.57)^2} = 111\ \text{lb} \qquad \textbf{Ans.}$$

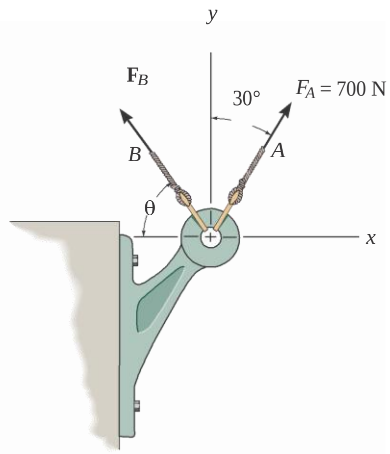
$$\theta' = \tan^{-1}\left(\frac{42.57}{103.05}\right) = 22.4°$$

$$\theta = 180° + 22.4° = 202° \qquad \textbf{Ans.}$$



**2–42.**

Determine the magnitude and orientation $\theta$ of $\mathbf{F}_B$ so that the resultant force is directed along the positive $y$ axis and has a magnitude of 1500 N.




## SOLUTION

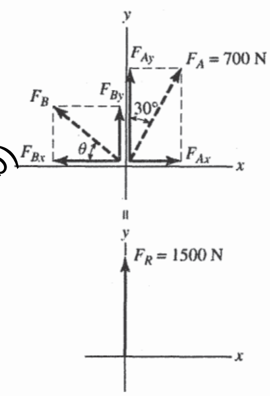
***Scalar Notation:*** Summing the force components algebraically, we have

$\xrightarrow{+} F_{R_x} = \Sigma F_x;\qquad 0 = 700 \sin 30° - F_B \cos\theta$

$$F_B \cos\theta = 350 \qquad \textbf{(1)}$$

$+\uparrow F_{R_y} = \Sigma F_y;\qquad 1500 = 700 \cos 30° + F_B \sin\theta$

$$F_B \sin\theta = 893.8 \qquad \textbf{(2)}$$

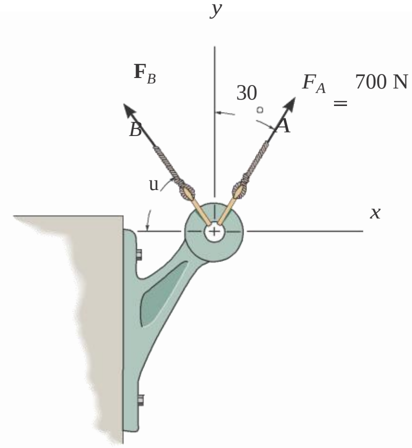



Solving Eq. (1) and (2) yields

$$\theta = 68.6° \quad F_B = 960 \text{ N} \qquad \textbf{Ans.}$$

**2–43.**

Determine the magnitude and orientation, measured counterclockwise from the positive *y* axis, of the resultant force acting on the bracket, if $F_B = 600$ N and $\theta = 20°$.




## SOLUTION

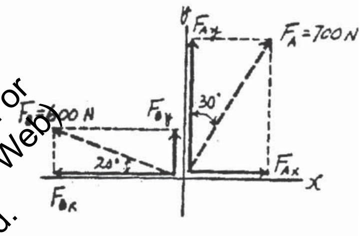
***Scalar Notation:*** Summing the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 700 \sin 30° - 600 \cos 20°$$

$$= -213.8 \text{ N} = 213.8 \text{ N} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 700 \cos 30° + 600 \sin 20°$$

$$= 811.4 \text{ N} \uparrow$$

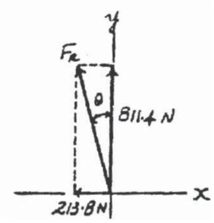



The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{213.8^2 + 811.4^2} = 839 \text{ N} \qquad \textbf{Ans.}$$

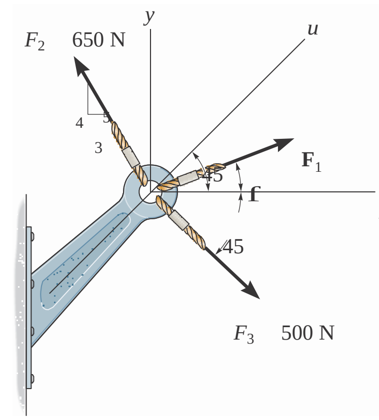
The direction angle θ measured counterclockwise from the positive *y* axis is

$$\theta = \tan^{-1}\frac{F_{R_x}}{F_{R_y}} = \tan^{-1}\left(\frac{213.8}{811.4}\right) = 14.8° \qquad \textbf{Ans.}$$



***2–44.**

The magnitude of the resultant force acting on the bracket is to be 400 N. Determine the magnitude of $\mathbf{F}_1$ if $\phi = 30°$.




## SOLUTION

***Rectangular Components:*** By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = F_1 \cos 30° = 0.8660F_1 \qquad (F_1)_y = F_1 \sin 30° = 0.5F_1$$

$$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N} \qquad (F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$$

$$(F_3)_x = 500 \cos 45° = 353.55 \text{ N} \qquad (F_3)_y = 500 \sin 45° = 353.55 \text{ N}$$

***Resultant Force:*** Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 0.8660F_1 - 390 + 353.55 = 0.8660F_1 - 36.45$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 0.5F_1 + 520 - 353.55 = 0.5F_1 + 166.45$$

Since the magnitude of the resultant force is $F_R = 400$ N, we can write

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

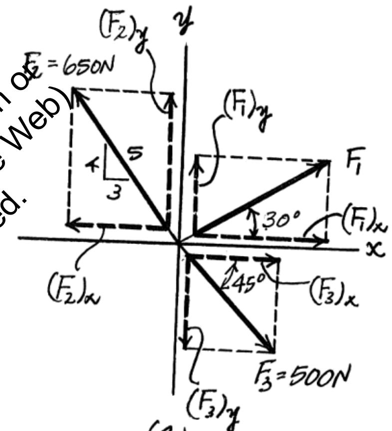
$$400 = \sqrt{(0.8660F_1 - 36.45)^2 + (0.5F_1 + 166.45)^2}$$

$$F_1^2 + 103.32F_1 - 130967.17 = 0 \qquad \textbf{Ans.}$$

Solving,

$$F_1 = 314 \text{ N} \qquad \text{or} \qquad F_1 = -417 \text{ N} \qquad \textbf{Ans.}$$

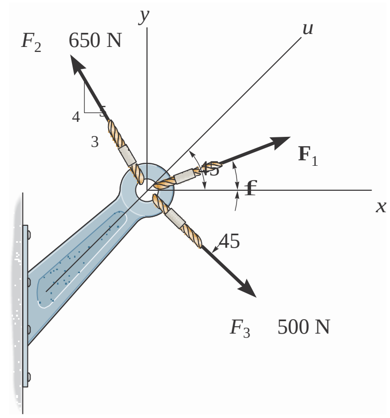
The negative sign indicates that $\mathbf{F}_1 = 417$ N must act in the opposite sense to that shown in the figure.



(a)

**2–45.**

If the resultant force acting on the bracket is to be directed along the positive $u$ axis, and the magnitude of $\mathbf{F}_1$ is required to be *minimum*, determine the magnitudes of the resultant force and $\mathbf{F}_1$.




## SOLUTION

***Rectangular Components:*** By referring to Figs. *a* and *b*, the *x* and *y* components of $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$$

$$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N} \qquad (F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$$

$$(F_3)_x = 500 \cos 45° = 353.55 \text{ N} \qquad (F_3)_y = 500 \sin 45° = 353.55 \text{ N}$$

$$(F_R)_x = F_R \cos 45° = 0.7071F_R \qquad (F_R)_y = F_R \sin 45° = 0.7071F_R$$

***Resultant Force:*** Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad 0.7071F_R = F_1 \cos\phi - 390 + 353.55 \qquad (1)$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad 0.7071F_R = F_1 \sin\phi + 520 - 353.55 \qquad (2)$$

Eliminating $F_R$ from Eqs. (1) and (2), yields

$$F_1 = \frac{202.89}{\cos\phi - \sin\phi} \qquad (3)$$

The first derivative of Eq. (3) is

$$\frac{dF_1}{d\phi} = \frac{\sin\phi + \cos\phi}{(\cos\phi - \sin\phi)^2} \qquad \mathbf{(4)}$$

The second derivative of Eq. (3) is

$$\frac{d^2F_1}{d\phi^2} = \frac{2(\sin\phi + \cos\phi)^2}{(\cos\phi - \sin\phi)^3} + \frac{1}{\cos\phi - \sin\phi} \qquad \mathbf{(5)}$$

For $\mathbf{F}_1$ to be minimum, $\frac{dF_1}{d\phi} = 0$. Thus, from Eq. (4)

$$\sin\phi + \cos\phi = 0$$

$$\tan\phi = -1$$

$$\phi = -45°$$

Substituting $\phi = -45°$ into Eq. (5), yields

$$\frac{d^2F_1}{d\phi^2} = 0.7071 > 0$$

(a)

This shows that $\phi = -45°$ indeed produces minimum $F_1$. Thus, from Eq. (3)

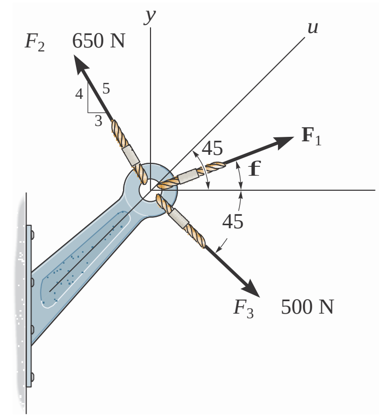
$$F_1 = \frac{202.89}{\cos(-45°) - \sin(-45°)} = 143.47 \text{ N} = 143 \text{ N} \qquad \textbf{Ans.}$$

Substituting $\phi = -45°$ and $F_1 = 143.47$ N into either Eq. (1) or Eq. (2), yields

$$F_R = 919 \text{ N} \qquad \textbf{Ans.}$$

**2–46.**

If the magnitude of the resultant force acting on the bracket is 600 N, directed along the positive u axis, determine the magnitude of **F** and its direction $\phi$.




## SOLUTION

***Rectangular Components:*** By referring to Figs. *a* and *b*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = F_1 \cos \phi \qquad (F_1)_y = F_1 \sin \phi$

$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N} \qquad (F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$

$(F_3)_x = 500 \cos 45° = 353.55 \text{ N} \qquad (F_3)_y = 500 \cos 45° = 353.55 \text{ N}$

$(F_R)_x = 600 \cos 45° = 424.26 \text{ N} \qquad (F_R)_y = 600 \sin 45° = 424.26 \text{ N}$

***Resultant Force:*** Summing the force components algebraically along the x and y axes, we have

$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad 424.26 = F_1 \cos \phi - 390 + 353.55$

$F_1 \cos \phi = 460.71 \qquad (1)$

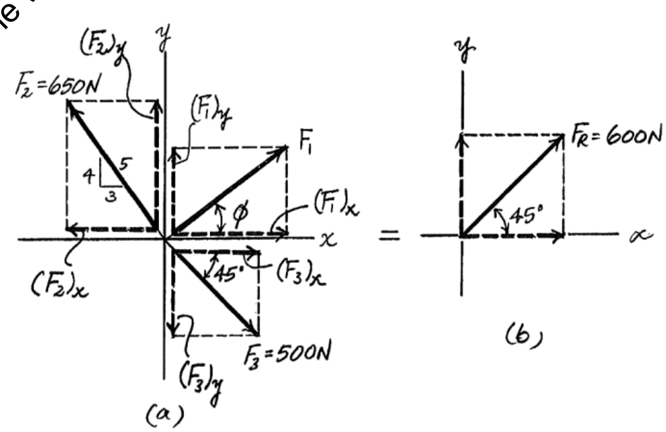
$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad 424.26 = F_1 \sin \phi + 520 - 353.55$

$F_1 \sin \phi = 257.82 \qquad (2)$

Solving Eqs. (1) and (2), yields

$\phi = 29.2° \qquad F_1 = 528 \text{ N}$ **Ans.**

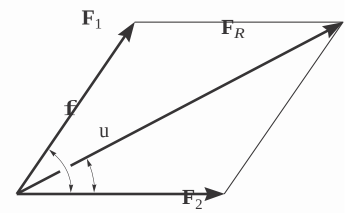




**2–47.**

Determine the magnitude and direction $\theta$ of the resultant force $\mathbf{F}_R$. Express the result in terms of the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ and the angle $\phi$.

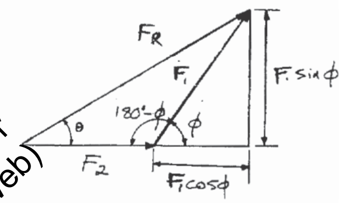
## SOLUTION




$$F_R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos(180^\circ - \phi)$$

Since $\cos(180^\circ - \phi) = -\cos\phi$,

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\phi}$$ **Ans.**



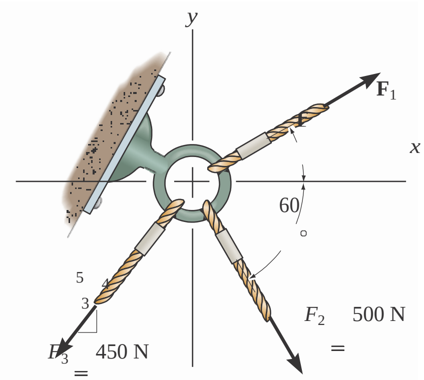

From the figure,

$$\tan\theta = \frac{F_1\sin\phi}{F_2 + F_1\cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{F_1\sin\phi}{F_2 + F_1\cos\phi}\right)$$ **Ans.**

***2–48.**

If $F_1 = 600$ N and $\phi = 30°$, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive $x$ axis.




## SOLUTION

***Rectangular Components:*** By referring to Fig. *a*, the $x$ and $y$ components of each force can be written as

$$(F_1)_x = 600\cos 30° = 519.62 \text{ N} \qquad (F_1)_y = 600\sin 30° = 300 \text{ N}$$

$$(F_2)_x = 500\cos 60° = 250 \text{ N} \qquad (F_2)_y = 500\sin 60° = 433.01 \text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270 \text{ N} \qquad (F_3)_y = 450\left(\frac{4}{5}\right) = 360 \text{ N}$$

***Resultant Force:*** Summing the force components algebraically along the $x$ and $y$ axes,

$$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 519.62 + 250 - 270 = 499.62 \text{ N} \rightarrow$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 300 - 433.01 - 360 = -493.01 \text{ N} = 493.01 \text{ N} \downarrow$$

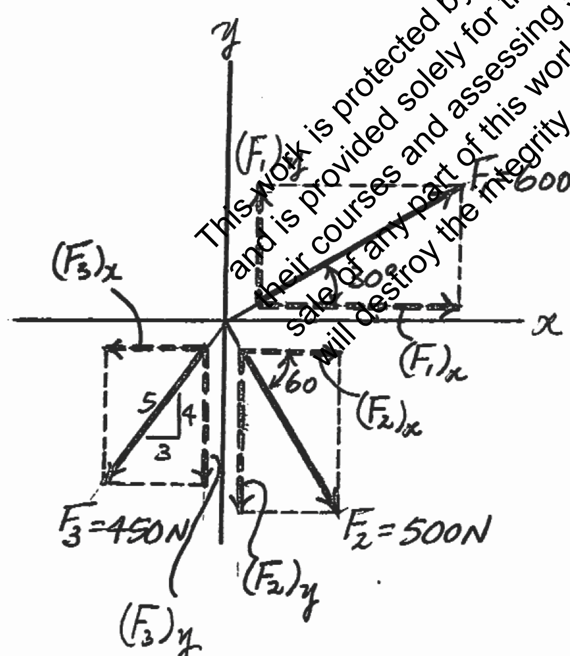
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{499.62^2 + 493.01^2} = 701.91 \text{ N} = 702 \text{ N} \qquad \textbf{Ans.}$$

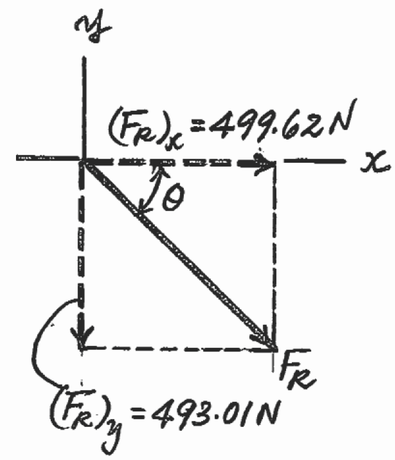
The direction angle $\theta$ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the $x$ axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{493.01}{499.62}\right) = 44.6° \qquad \textbf{Ans.}$$





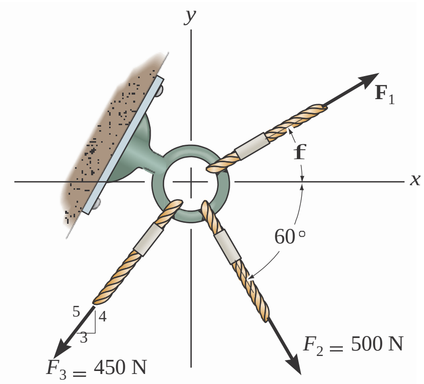

(a)




(b)

**2–49.**

If the magnitude of the resultant force acting on the eyebolt is 600 N and its direction measured clockwise from the positive $x$ axis is $\theta = 30°$, determine the magnitude of $\mathbf{F}_1$ and the angle $\phi$.




## SOLUTION

***Rectangular Components:*** By referring to Figs. *a* and *b*, the $x$ and $y$ components of $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$$

$$(F_2)_x = 500 \cos 60° = 250 \text{ N} \qquad (F_2)_y = 500 \sin 60° = 433.01 \text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270 \text{ N} \qquad (F_3)_y = 450\left(\frac{4}{5}\right) = 360 \text{ N}$$

$$(F_R)_x = 600 \cos 30° = 519.62 \text{ N} \qquad (F_R)_y = 600 \sin 30° = 300 \text{ N}$$

***Resultant Force:*** Summing the force components algebraically along the $x$ and $y$ axes,

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad 519.62 = F_1 \cos\phi + 250 - 270$$

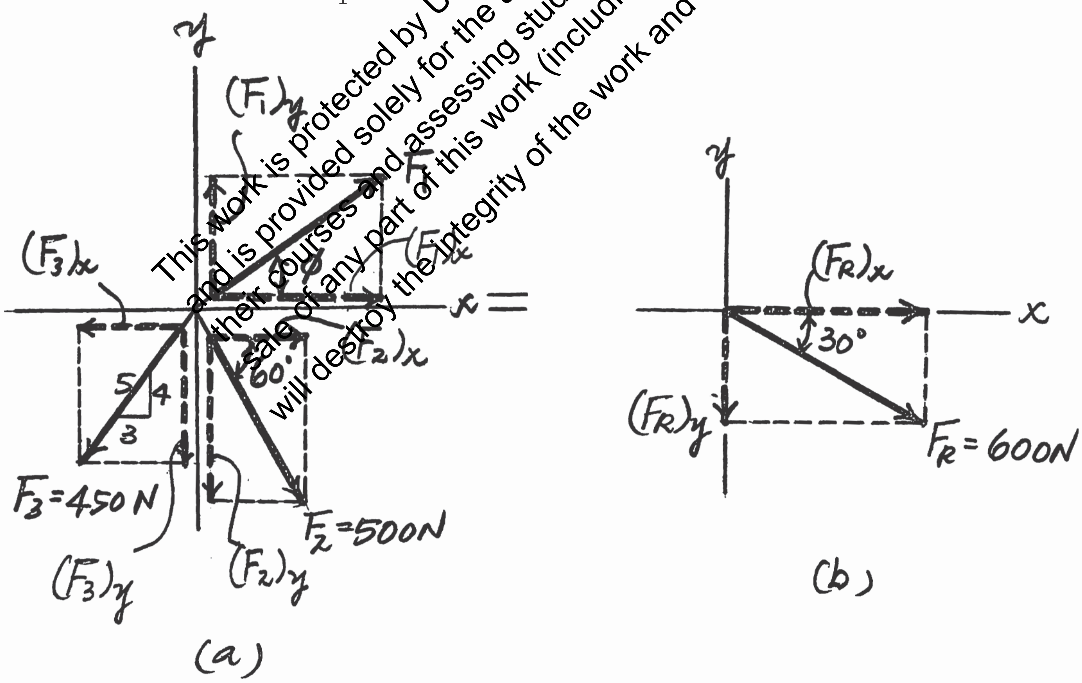
$$F_1 \cos\phi = 539.62 \qquad (1)$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad -300 = F_1 \sin\phi - 433.01 - 360$$

$$F_1 \sin\phi = 493.01 \qquad (2)$$

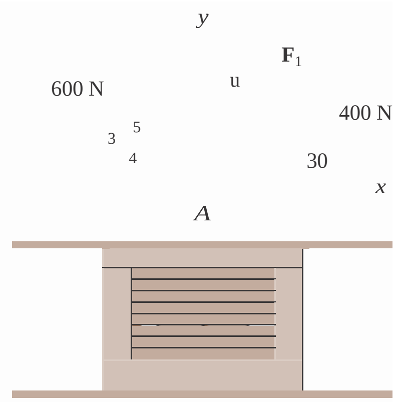
Solving Eqs. (1) and (2), yields

$$\phi = 42.4° \qquad F_1 = 731 \text{ N} \qquad \textbf{Ans.}$$



**2–50.**

Determine the magnitude of $\mathbf{F}_1$ and its direction u so that the resultant force is directed vertically upward and has a magnitude of 800 N.




## SOLUTION

***Scalar Notation:*** Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = ©F_x; \qquad F_{R_x} = 0 = F_1 \sin u + 400 \cos 30° - 600\left(\frac{4}{5}\right)$$

$$F_1 \sin u = 133.6 \qquad \textbf{(1)}$$

$$+c\ F_{R_y} = ©F_y; \qquad F_{R_y} = 800 = F_1 \cos u + 400 \sin 30° + 600\left(\frac{3}{5}\right)$$

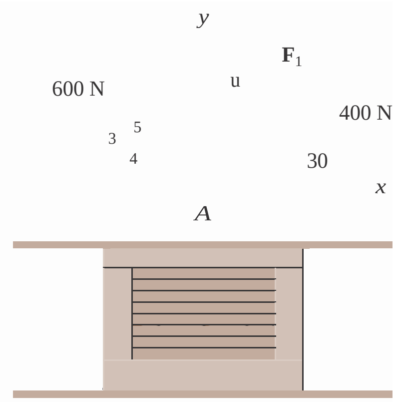
$$F_1 \cos u = 240 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$u = 29.1° \qquad F_1 = 275 \text{ N} \qquad \textbf{Ans.}$$

**2–51.**

Determine the magnitude and direction measured counterclockwise from the positive $x$ axis of the resultant force of the three forces acting on the ring $A$. Take $F_1 = 500$ N and $\theta = 20°$.




## SOLUTION

***Scalar Notation:*** Summing the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 500 \sin 20° + 400 \cos 30° - 600\left(\frac{4}{5}\right)$$

$$= 37.42 \text{ N} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 500 \cos 20° + 400 \sin 30° + 600\left(\frac{3}{5}\right)$$

$$= 1029.8 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

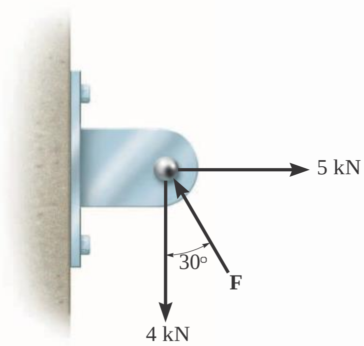
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{37.42^2 + 1029.8^2} = 1030.5 \text{ N} = 1.03 \text{ kN} \qquad \textbf{Ans.}$$

The direction angle $\theta$ measured counterclockwise from positive $x$ axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{1029.8}{37.42}\right) = 87.9° \qquad \textbf{Ans.}$$

***2–52.**

Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible. What is the minimum magnitude of $\mathbf{F}_R$?

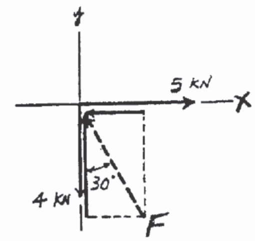



## SOLUTION

***Scalar Notation:*** Summing the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 5 - F\sin 30° = 5 - 0.50F \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = F\cos 30° - 4 = 0.8660F - 4 \uparrow$$




The magnitude of the resultant force $\mathbf{F}_R$ is

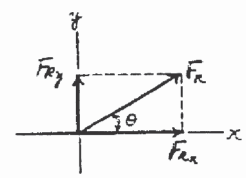
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2}$$

$$= \sqrt{(5 - 0.50F)^2 + (0.8660F - 4)^2}$$

$$= \sqrt{F^2 - 11.93F + 41} \qquad (1)$$

$$F_R^2 = F^2 - 11.93F + 41$$

$$2F_R \frac{dF_R}{dF} = 2F - 11.93 \qquad (2)$$




$$\left(F_R \frac{d^2F_R}{dF^2} + \frac{dF_R}{dF} \times \frac{dF_R}{dF}\right) = 1 \qquad (3)$$

In order to obtain the *minimum* resultant force $\mathbf{F}_R$, $\frac{dF_R}{dF} = 0$. From Eq. (2)

$$2F_R \frac{dF_R}{dF} = 2F - 11.93 = 0$$

$$F = 5.964 \text{ kN} = 5.96 \text{ kN} \qquad \textbf{Ans.}$$

Substituting $F = 5.964$ kN into Eq. (1), we have

$$F_R = \sqrt{5.964^2 - 11.93(5.964) + 41}$$

$$= 2.330 \text{ kN} = 2.33 \text{ kN} \qquad \textbf{Ans.}$$

Substituting $F_R = 2.330$ kN with $\frac{dF_R}{dF} = 0$ into Eq. (3), we have

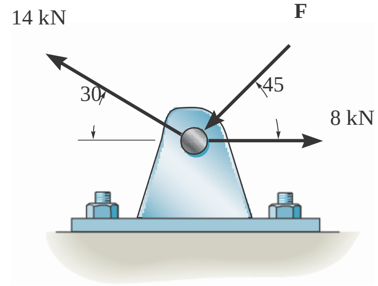
$$\left[(2.330)\frac{d^2F_R}{dF^2} + 0\right] = 1$$

$$\frac{d^2F_R}{dF^2} = 0.429 > 0$$

Hence, $\mathbf{F} = 5.96$ kN is indeed producing a minimum resultant force.

**2–53.**

Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?




## SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;$ $\quad F_{Rz} = 8 - F\cos 45° - 14\cos 30°$

$= -4.1244 - F\cos 45°$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = -F\sin 45° + 14\sin 30°$

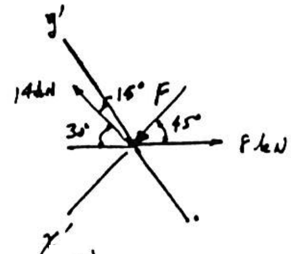
$= 7 - F\sin 45°$

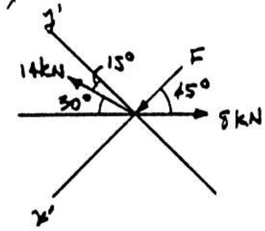
$$F_R^2 = (-4.1244 - F\cos 45°)^2 + (7 - F\sin 45°)^2 \qquad \textbf{(1)}$$

$$2F_R\frac{dF_R}{dF} = 2(-4.1244 - F\cos 45°)(-\cos 45°) + 2(7 - F\sin 45°)(-\sin 45°) = 0$$

$F = 2.03\text{ kN}$ **Ans.**

From Eq. (1); $\quad F_R = 7.87\text{ kN}$ **Ans.**




Also, from the figure require

$(F_R)_{x'} = 0 = \Sigma F_{x'};$ $\quad F + 14\sin 15° - 8\cos 45° = 0$

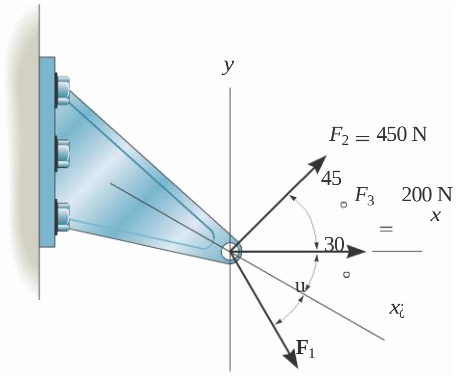
$F = 2.03\text{ kN}$ **Ans.**

$(F_R)_{y'} = \Sigma F_{y'};$ $\quad F_R = 14\cos 15° - 8\sin 45°$

$F_R = 7.87\text{ kN}$ **Ans.**

**2–54.**

Three forces act on the bracket. Determine the magnitude and direction u of $\mathbf{F}_1$ so that the resultant force is directed along the positive x¿ axis and has a magnitude of 1 kN.




## SOLUTION

± $F_{Rx} = ©F_x$;  $1000 \cos 30° = 200 + 450 \cos 45° + F_1 \cos(u + 30°)$

+ c $F_{Ry} = ©F_y$;  $-1000 \sin 30° = 450 \sin 45° - F_1 \sin(u + 30°)$

$F_1 \sin(u + 30°) = 818.198$

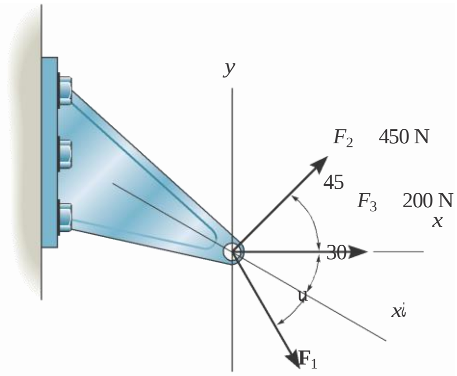
$F_1 \cos(u + 30°) = 347.827$

$u + 30° = 66.97°$,  $u = 37.0°$ **Ans.**

$F_1 = 889$ N **Ans.**

**2–55.**

If $F_1 = 300$ N and $\theta = 20°$, determine the magnitude and direction, measured counterclockwise from the $x'$ axis, of the resultant force of the three forces acting on the bracket.




## SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = 300\cos 50° + 200 + 450\cos 45° = 711.03\text{ N}$

$+\uparrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = -300\sin 50° + 450\sin 45° = 88.38\text{ N}$

$F_R = \sqrt{(711.03)^2 + (88.38)^2} = 717\text{ N}$ **Ans.**

$\phi'(\text{angle from x axis}) = \tan^{-1}\left[\frac{88.38}{711.03}\right]$

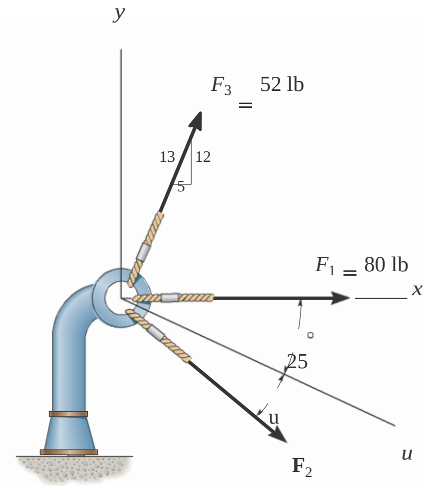
$\phi' = 7.10°$

$\phi\,(\text{angle from } x' \text{ axis}) = 30° + 7.10°$

$\phi = 37.1°$ **Ans.**

***2–56.**

Three forces act on the bracket. Determine the magnitude and direction $\theta$ of $\mathbf{F}_2$ so that the resultant force is directed along the positive $u$ axis and has a magnitude of 50 lb.




## SOLUTION

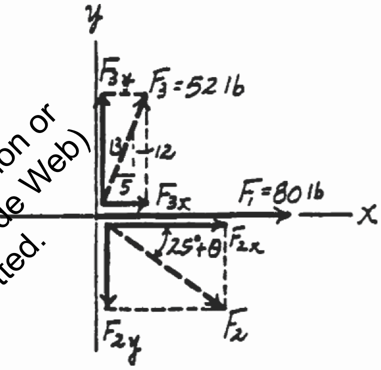
***Scalar Notation:*** Summing the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad 50\cos 25° = 80 + 52\left(\frac{5}{13}\right) + F_2\cos(25° + \theta)$$

$$F_2\cos(25° + \theta) = -54.684 \qquad \textbf{(1)}$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad -50\sin 25° = 52\left(\frac{12}{13}\right) - F_2\sin(25° + \theta)$$

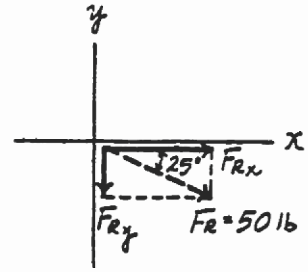
$$F_2\sin(25° + \theta) = 69.131 \qquad \textbf{(2)}$$




Solving Eqs. (1) and (2) yields

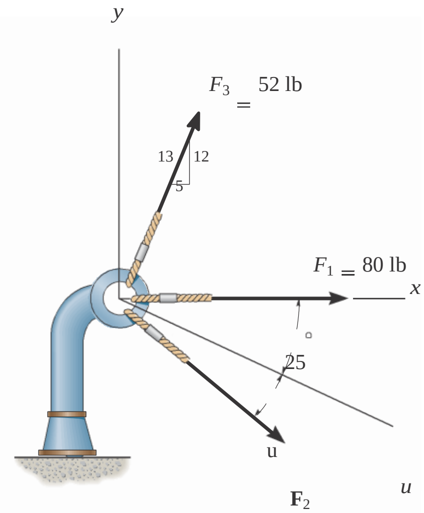
$$25° + \theta = 128.35° \qquad \theta = 103° \qquad \textbf{Ans.}$$

$$F_2 = 88.1 \text{ lb} \qquad \textbf{Ans.}$$



**2–57.**

If $F_2 = 150$ lb and $\theta = 55°$, determine the magnitude and direction, measured clockwise from the positive $x$ axis, of the resultant force of the three forces acting on the bracket.




## SOLUTION

***Scalar Notation:*** Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 80 + 52\left(\frac{5}{13}\right) + 150 \cos 80°$$

$$= 126.05 \text{ lb} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 52\left(\frac{12}{13}\right) - 150 \sin 80°$$

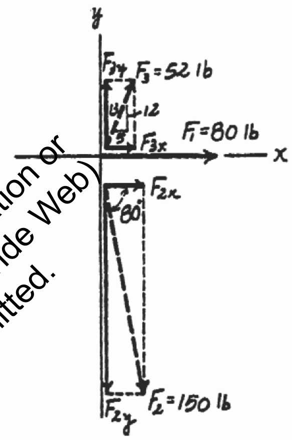
$$= -99.72 \text{ lb} = 99.72 \text{ lb} \downarrow$$

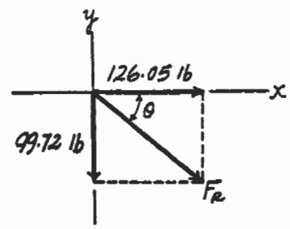
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{126.05^2 + 99.72^2} = 161 \text{ lb}$$ **Ans.**

The direction angle $\theta$ measured clockwise from positive $x$ axis is

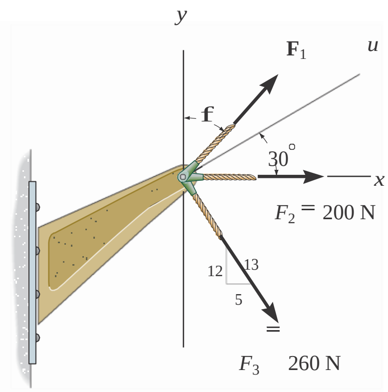
$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{99.72}{126.05}\right) = 38.3°$$ **Ans.**



**2–58.**

If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive $u$ axis, determine the magnitude of $\mathbf{F}_1$ and its direction $\phi$.




## SOLUTION

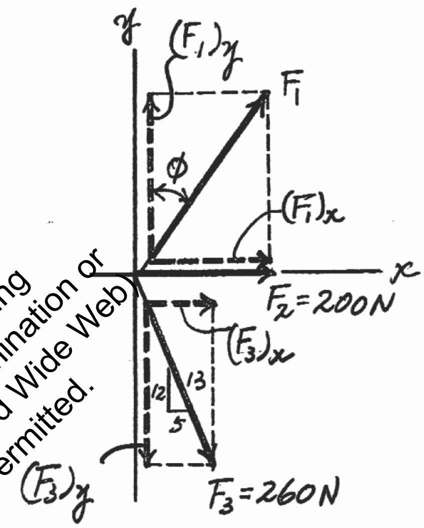
***Rectangular Components:*** By referring to Fig. *a*, the $x$ and $y$ components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = F_1 \sin \phi$ $\qquad$ $(F_1)_y = F_1 \cos \phi$

$(F_2)_x = 200 \text{ N}$ $\qquad$ $(F_2)_y = 0$

$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ N}$ $\qquad$ $(F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ N}$

$(F_R)_x = 450 \cos 30° = 389.71 \text{ N}$ $\qquad$ $(F_R)_y = 450 \sin 30° = 225 \text{ N}$




(a)

***Resultant Force:*** Summing the force components algebraically along the $x$ and $y$ axes,

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \quad 389.71 = F_1 \sin \phi + 200 + 100$$

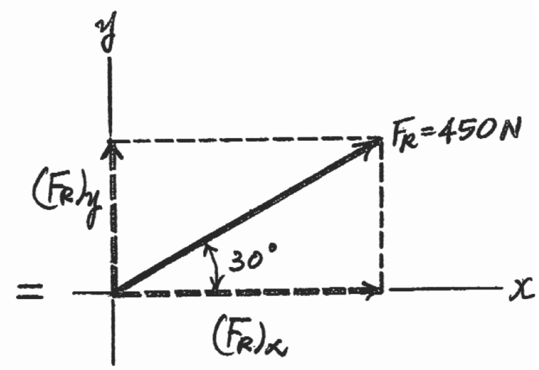
$$F_1 \sin \phi = 89.71 \qquad (1)$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad 225 = F_1 \cos \phi - 240$$

$$F_1 \cos \phi = 465 \qquad (2)$$

Solving Eqs. (1) and (2), yields

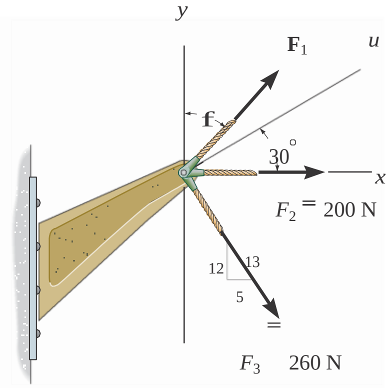
$\phi = 10.9°$ $\qquad$ $F_1 = 474 \text{ N}$ $\qquad$ **Ans.**




(b)

**2–59.**

If the resultant force acting on the bracket is required to be a minimum, determine the magnitudes of $\mathbf{F}_1$ and the resultant force. Set $\phi = 30°$.




## SOLUTION

***Rectangular Components:*** By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$(F_1)_x = F_1 \sin 30° = 0.5F_1$ $\qquad$ $(F_1)_y = F_1 \cos 30° = 0.8660F_1$

$(F_2)_x = 200 \text{ N}$ $\qquad$ $(F_2)_y = 0$

$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ N}$ $\qquad$ $(F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ N}$

***Resultant Force:*** Summing the force components algebraically along the *x* and *y* axes,

$$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 0.5F_1 + 200 + 100 = 0.5F_1 + 300$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 0.8660F_1 - 240$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(0.5F_1 + 300)^2 + (0.8660F_1 - 240)^2}$$

$$= \sqrt{F_1^2 - 115.69F_1 + 147\,600} \qquad (1)$$

Thus,

$$F_R^2 = F_1^2 - 115.69F_1 + 147\,600 \qquad \textbf{(2)}$$

The first derivative of Eq. (2) is

$$2F_R\frac{dF_R}{dF_1} = 2F_1 - 115.69 \qquad \textbf{(3)}$$

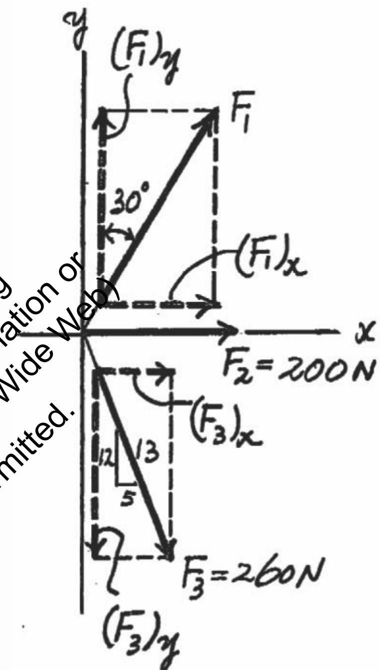
For $\mathbf{F}_R$ to be minimum, $\frac{dF_R}{dF_1} = 0$. Thus, from Eq. (3)

$$2F_R\frac{dF_R}{dF_1} = 2F_1 - 115.69 = 0$$

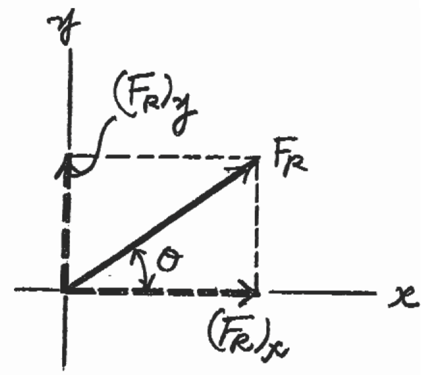
$$F_1 = 57.846 \text{ N} = 57.8 \text{ N} \qquad \textbf{Ans.}$$

from Eq. (1),

$$F_R = \sqrt{(57.846)^2 - 115.69(57.846) + 147\,600} = 380 \text{ N} \qquad \textbf{Ans.}$$



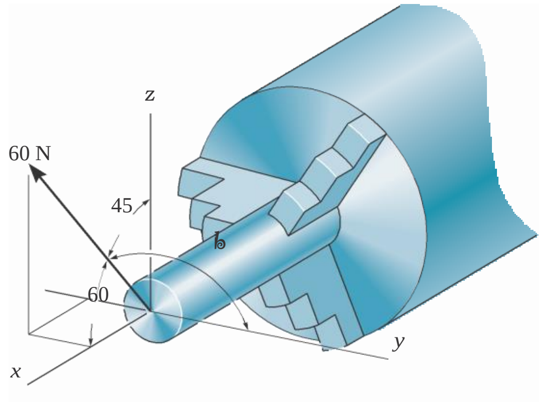
(a)



(b)

***2–60.**

The stock mounted on the lathe is subjected to a force of 60 N. Determine the coordinate direction angle b and express the force as a Cartesian vector.




## SOLUTION

$$1 = \sqrt{\cos^2 a + \cos^2 b + \cos^2 g}$$

$$1 = \cos^2 60° + \cos^2 b + \cos^2 45°$$

$$\cos b = \pm 0.5$$

$$b = 60°, 120°$$

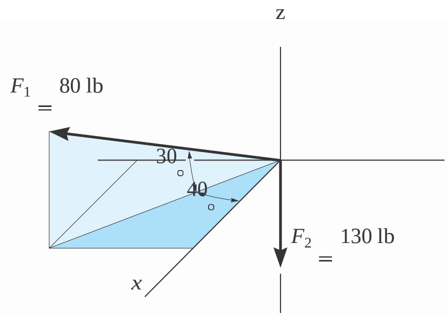
Use

$$b = 120° \qquad \textbf{Ans.}$$

$$\mathbf{F} = 60\ \text{N}(\cos 60°\mathbf{i} + \cos 120°\mathbf{j} + \cos 45°\mathbf{k})$$
$$= \{30\mathbf{i} - 30\mathbf{j} + 42.4\mathbf{k}\}\ \text{N} \qquad \textbf{Ans.}$$

**2–61.**

Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the




## SOLUTION

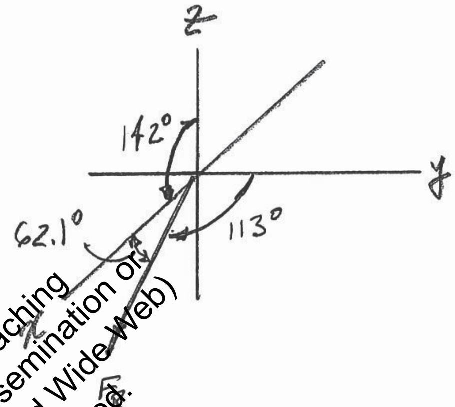
$\mathbf{F}_1 = \{80 \cos 30° \cos 40° \mathbf{i} - 80 \cos 30° \sin 40° \mathbf{j} + 80 \sin 30° \mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

$\mathbf{F}_R = \{53.1\mathbf{i} - 44.5\mathbf{j} - 90.0\mathbf{k}\}$ lb




$F_R = \sqrt{(53.1)^2 + (-44.5)^2 + (-90.0)^2} = 114$ lb **Ans.**

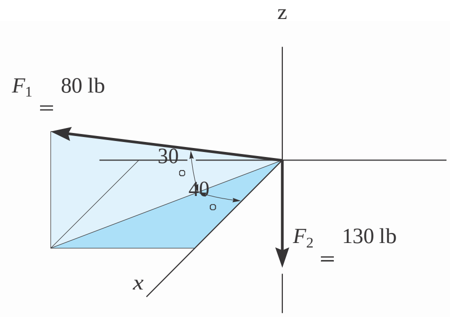
$\alpha = \cos^{-1}\left(\frac{53.1}{113.6}\right) = 62.1°$ **Ans.**

$\beta = \cos^{-1}\left(\frac{-44.5}{113.6}\right) = 113°$ **Ans.**

$\gamma = \cos^{-1}\left(\frac{-90.0}{113.6}\right) = 142°$ **Ans.**

**2–62.**

Specify the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_2$ and express each force as a Cartesian vector.




## SOLUTION

$\mathbf{F}_1 = \{80 \cos 30° \cos 40° \mathbf{i} - 80 \cos 30° \sin 40° \mathbf{j} + 80 \sin 30° \mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb **Ans.**

$\alpha_1 = \cos^{-1}\left(\frac{53.1}{80}\right) = 48.4°$ **Ans.**

$\beta_1 = \cos^{-1}\left(\frac{-44.5}{80}\right) = 124°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\frac{40}{80}\right) = 60°$ **Ans.**

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb **Ans.**

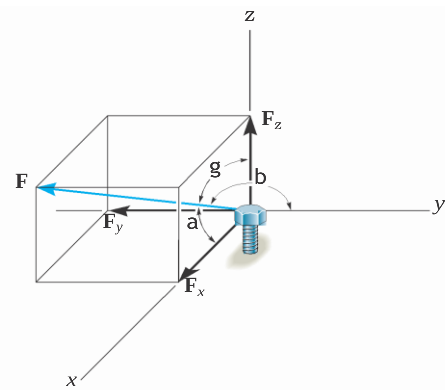
$\alpha_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$ **Ans.**

$\beta_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$ **Ans.**

$\gamma_2 = \cos^{-1}\left(\frac{-130}{130}\right) = 180°$ **Ans.**

**2–63.**

The bolt is subjected to the force **F**, which has components acting along the *x, y, z* axes as shown. If the magnitude of **F** is 80 N, and $a = 60°$ and $g = 45°$, determine the magnitudes of its components.




## SOLUTION

$$\cos b = \sqrt{1 - \cos^2 a - \cos^2 g}$$

$$= \sqrt{1 - \cos^2 60° - \cos^2 45°}$$

$$b = 120°$$

$$F_x = |80 \cos 60°| = 40 \text{ N}$$ **Ans.**

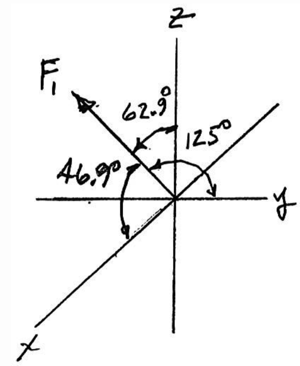
$$F_y = |80 \cos 120°| = 40 \text{ N}$$ **Ans.**

$$F_z = |80 \cos 45°| = 56.6 \text{ N}$$ **Ans.**

***2–64.**

Determine the magnitude and coordinate direction angles of $\mathbf{F}_1 = \{60\mathbf{i} - 50\mathbf{j} + 40\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-40\mathbf{i} - 85\mathbf{j} + 30\mathbf{k}\}$ N. Sketch each force on an $x, y, z$ reference frame.

## SOLUTION



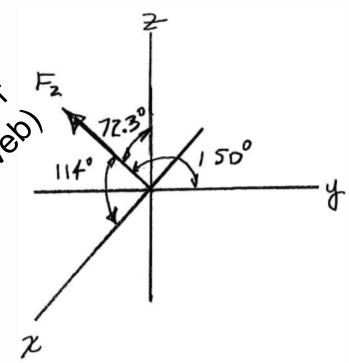

$\mathbf{F}_1 = 60\,\mathbf{i} - 50\,\mathbf{j} + 40\,\mathbf{k}$

$F_1 = \sqrt{(60)^2 + (-50)^2 + (40)^2} = 87.7496 = 87.7\text{ N}$ **Ans.**

$\alpha_1 = \cos^{-1}\left(\frac{60}{87.7496}\right) = 46.9°$ **Ans.**

$\beta_1 = \cos^{-1}\left(\frac{-50}{87.7496}\right) = 125°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\frac{40}{87.7496}\right) = 62.9°$ **Ans.**




$\mathbf{F}_2 = -40\,\mathbf{i} - 85\,\mathbf{j} + 30\,\mathbf{k}$

$F_2 = \sqrt{(-40)^2 + (-85)^2 + (30)^2} = 98.615 = 98.6\text{ N}$ **Ans.**

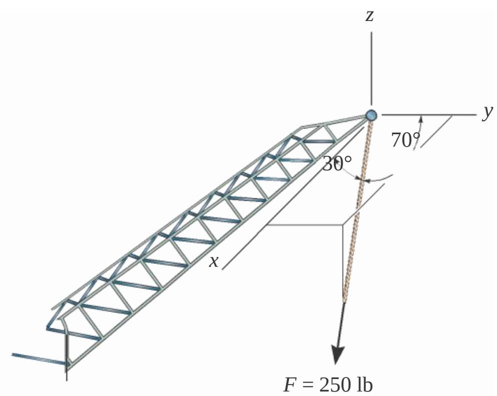
$\alpha_2 = \cos^{-1}\left(\frac{-40}{98.615}\right) = 114°$ **Ans.**

$\beta_2 = \cos^{-1}\left(\frac{-85}{98.615}\right) = 150°$ **Ans.**

$\gamma_2 = \cos^{-1}\left(\frac{30}{98.615}\right) = 72.3°$ **Ans.**

**2–65.**

The cable at the end of the crane boom exerts a force of 250 lb on the boom as shown. Express **F** as a Cartesian vector.




## SOLUTION

***Cartesian Vector Notation:*** With a = 30° and b = 70°, the third coordinate direction angle g can be determined using Eq. 2–8.

$$\cos^2 a + \cos^2 b + \cos^2 g = 1$$

$$\cos^2 30° + \cos^2 70° + \cos^2 g = 1$$

$$\cos g = ; 0.3647$$

$$g = 68.61° \text{ or } 111.39°$$

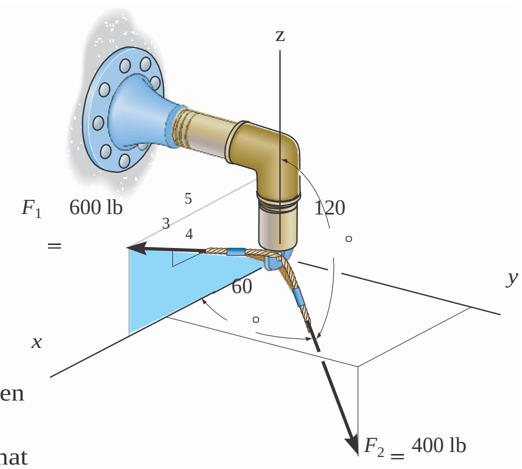
By inspection, g = 111.39° since the force **F** is directed in negative octant.

$$\mathbf{F} = 2505\cos 30°\mathbf{i} + \cos 70°\mathbf{j} + \cos 111.39°6 \text{ lb}$$

$$= \{217\mathbf{i} + 85.5\mathbf{j} - 91.2\mathbf{k}\} \text{ lb}$$ **Ans.**

**2–66.**

Express each force acting on the pipe assembly in Cartesian vector form.




## SOLUTION

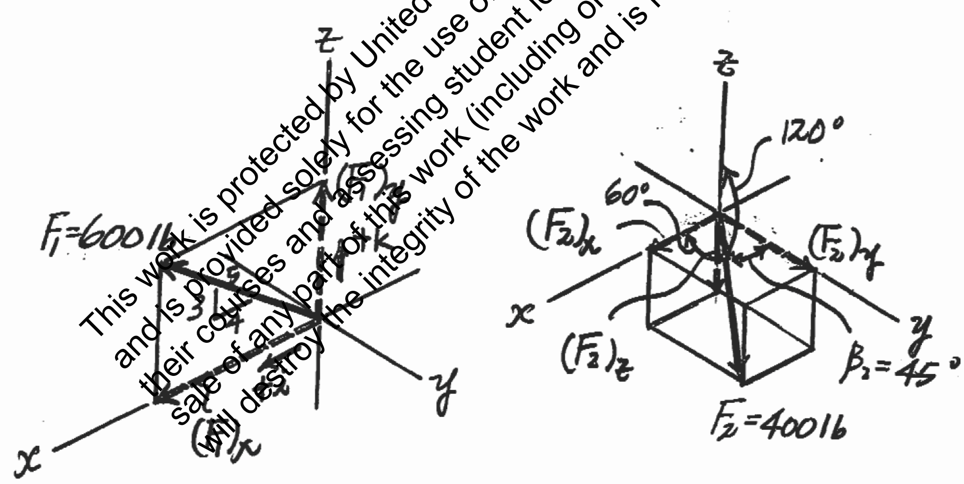
***Rectangular Components:*** Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\beta_2 = \pm\sqrt{1 - \cos^2 60° - \cos^2 120°} = \pm 0.7071$. However, it is required that $\beta_2 < 90°$, thus, $\beta_2 = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their $x$, $y$, and $z$ components, as shown in Figs. $a$ and $b$, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)(+\mathbf{i}) + 0\mathbf{j} + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

$$= [480\mathbf{i} + 360\mathbf{k}]\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 400\cos 60°\mathbf{i} + 400\cos 45°\mathbf{j} + 400\cos 120°\mathbf{k}$$

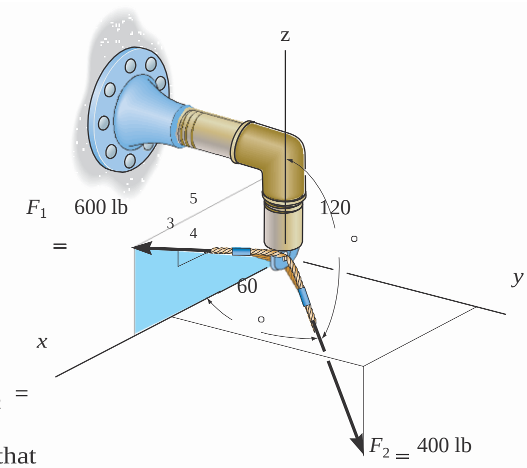
$$= [200\mathbf{i} + 283\mathbf{j} - 200\mathbf{k}]\text{ lb} \qquad \textbf{Ans.}$$




(a) (b)

**2–67.**

Determine the magnitude and direction of the resultant force acting on the pipe assembly.




## SOLUTION

***Force Vectors:*** Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_2 = \pm\sqrt{1 - \cos^2 60° - \cos^2 120°} = \pm 0.7071$. However, it is required that $\beta_2 < 90°$, thus, $\beta_2 = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their *x*, *y*, and *z* components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)(+\mathbf{i}) + 0\mathbf{j} + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

$$= \{480\mathbf{i} + 360\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = 400\cos 60°\mathbf{i} + 400\cos 45°\mathbf{j} + 400\cos 120°\mathbf{k}$$

$$= \{200\mathbf{i} + 282.84\mathbf{j} - 200\mathbf{k}\}\text{ lb}$$

***Resultant Force:*** By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (480\mathbf{i} + 360\mathbf{k}) + (200\mathbf{i} + 282.84\mathbf{j} - 200\mathbf{k})$$

$$= \{680\mathbf{i} + 282.84\mathbf{j} + 160\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$

$$= \sqrt{680^2 + 282.84^2 + 160^2} = 753.66\text{ lb} = 754\text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are:

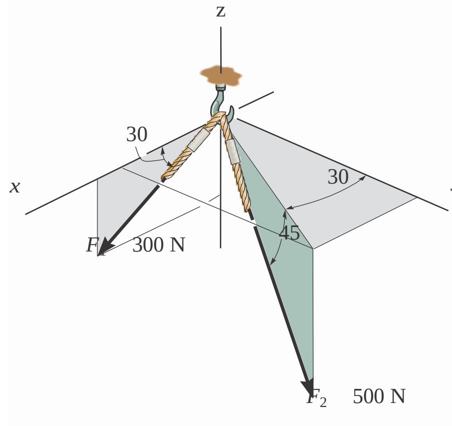
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{680}{753.66}\right) = 25.5° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{282.84}{753.66}\right) = 68.0° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{160}{753.66}\right) = 77.7° \qquad \textbf{Ans.}$$

***2–68.**

Express each force as a Cartesian vector.



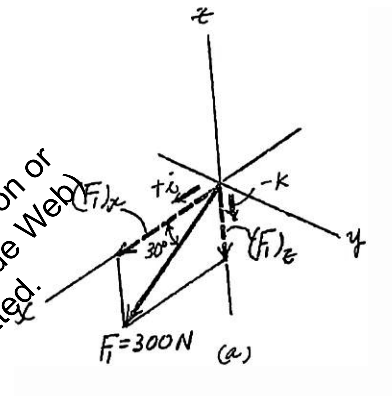

## SOLUTION

***Rectangular Components:*** By referring to Figs. *a* and *b*, the *x*, *y*, and *z* components of $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written as

$(F_1)_x = 300 \cos 30° = 259.8 \text{ N}$ $\quad$ $(F_2)_x = 500 \cos 45° \sin 30° = 176.78 \text{ N}$

$(F_1)_y = 0$ $\quad$ $(F_2)_y = 500 \cos 45° \cos 30° = 306.19 \text{ N}$

$(F_1)_t = 300 \sin 30° = 150 \text{ N}$ $\quad$ $(F_2)_z = 500 \sin 45° = 353.55 \text{ N}$




Thus, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written in Cartesian vector form as

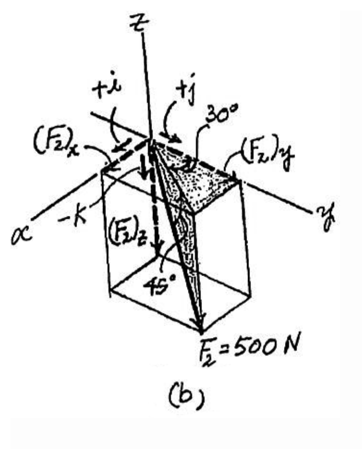
$$\mathbf{F}_1 = 259.81(+\mathbf{i}) + 0\mathbf{j} + 150(-\mathbf{k})$$

$$= \{260\mathbf{i} - 150\mathbf{k}\} \text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 176.78(+\mathbf{i}) + 306.19(+\mathbf{j}) + 353.55(-\mathbf{k})$$

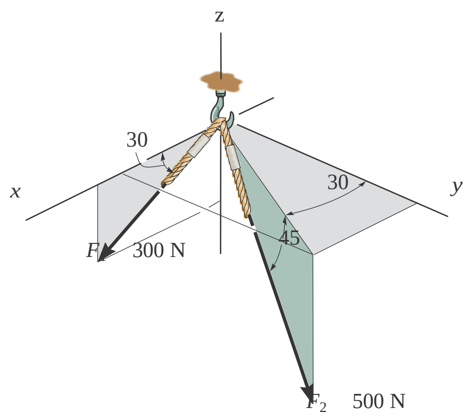
$$= 2\{177\mathbf{i} + 306\mathbf{j} - 354\mathbf{k}\} \text{ N} \qquad \textbf{Ans.}$$





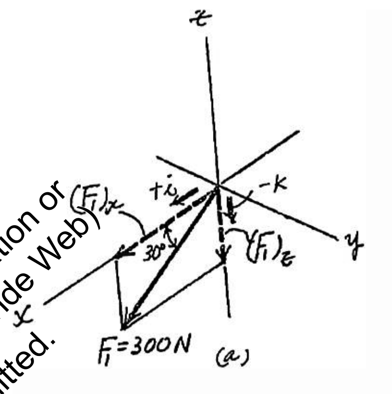
**2–69.**

Determine the magnitude and coordinate direction angles of the resultant force acting on the hook.




## SOLUTION

***Force Vectors:*** By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their $x$, $y$, and $z$ components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expessed in Cartesian vector form as




$$\mathbf{F}_1 = 300 \cos 30°(+\mathbf{i}) + 0\mathbf{j} + 300 \sin 30°(-\mathbf{k})$$

$$= \{259.81\mathbf{i} - 150\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 500 \cos 45° \sin 30°(+\mathbf{i}) + 500 \cos 45° \cos 30°(+\mathbf{j}) + 500 \sin 45°(-\mathbf{k})$$

$$= \{176.78\mathbf{i} - 306.19\mathbf{j} - 353.55\mathbf{k}\}\text{ N}$$

***Resultant Force:*** The resultant force acting on the hook can be obtained by vectorally adding $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (259.81\mathbf{i} - 150\mathbf{k}) + (176.78\mathbf{i} + 306.19\mathbf{j} - 353.55\mathbf{k})$$

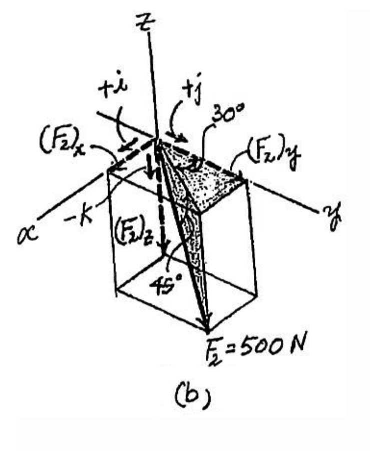
$$= \{436.58\mathbf{i}) + 306.19\mathbf{j} - 503.55\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$

$$= \sqrt{(436.58)^2 + (306.19)^2 + (-503.55)^2} = 733.43\text{ N} = 733\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are



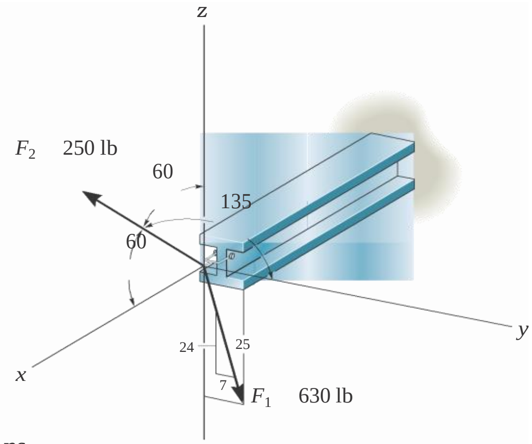

$$u_x = \cos^{-1}\left(\frac{(F_R)_x}{F_R}\right) = \cos^{-1}\left(\frac{436.58}{733.43}\right) = 53.5° \qquad \textbf{Ans.}$$

$$u_y = \cos^{-1}\left(\frac{(F_R)_y}{F_R}\right) = \cos^{-1}\left(\frac{306.19}{733.43}\right) = 65.3° \qquad \textbf{Ans.}$$

$$u_z = \cos^{-1}\left(\frac{(F_R)_z}{F_R}\right) = \cos^{-1}\left(\frac{-503.55}{733.43}\right) = 133° \qquad \textbf{Ans.}$$

**2–70.**

The beam is subjected to the two forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.




## SOLUTION

$$\mathbf{F}_1 = 630\left(\frac{7}{25}\right)\mathbf{j} - 630\left(\frac{24}{25}\right)\mathbf{k}$$

$$\mathbf{F}_1 = (176.4\,\mathbf{j} - 604.8\mathbf{k})$$

$$\mathbf{F}_1 = \{176\mathbf{j} - 605\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_2 = 250\cos 60°\mathbf{i} + 250\cos 135°\mathbf{j} + 250\cos 60°\mathbf{k}$$

$$\mathbf{F}_2 = (125\mathbf{i} - 176.777\mathbf{j} + 125\mathbf{k})$$

$$\mathbf{F}_2 = \{125\mathbf{i} - 177\mathbf{j} + 125\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\mathbf{F}_R = 125\mathbf{i} - 0.3767\mathbf{j} - 479.8\mathbf{k}$$

$$\mathbf{F}_R = \{125\mathbf{i} - 0.377\mathbf{j} - 480\mathbf{k}\}\text{ lb}$$

$$F_R = \sqrt{(125)^2 + (-0.3767)^2 + (-479.8)^2} = 495.82$$

$$= 496\text{ lb}$$ **Ans.**

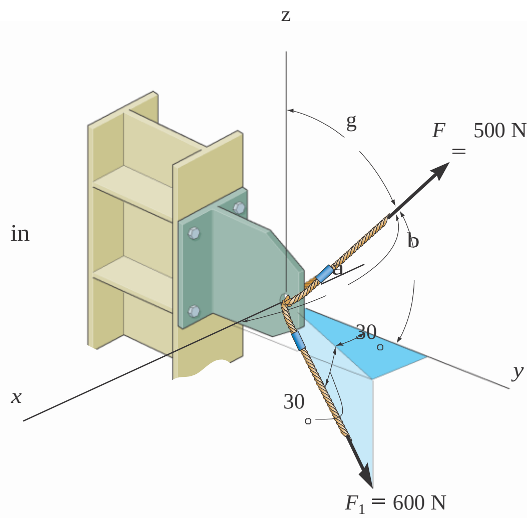
$$\alpha = \cos^{-1}\left(\frac{125}{495.82}\right) = 75.4°$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{-0.3767}{495.82}\right) = 90.0°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-479.8}{495.82}\right) = 165°$$ **Ans.**

**2–71.**

If the resultant force acting on the bracket is directed along the positive $y$ axis, determine the magnitude of the resultant force and the coordinate direction angles of **F** so that $\beta \leq 90°$.




## SOLUTION

***Force Vectors:*** By resolving $\mathbf{F}_1$ and **F** into their $x$, $y$, and $z$ components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and **F** can be expressed in Cartesian vector form as

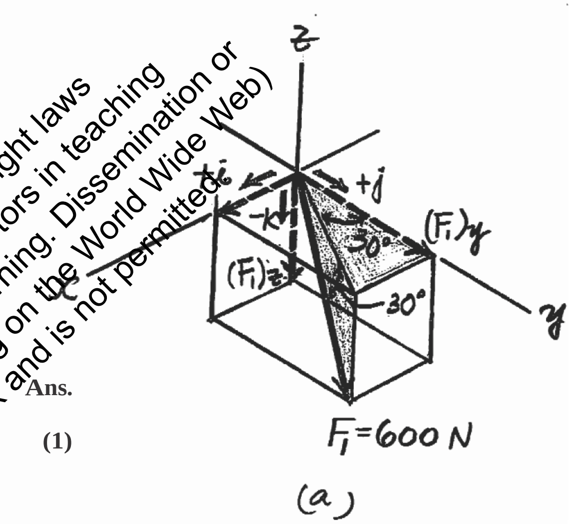
$$\mathbf{F}_1 = 600 \cos 30° \sin 30°(+\mathbf{i}) + 600 \cos 30° \cos 30°(+\mathbf{j}) + 600 \sin 30°(-\mathbf{k})$$

$$= \{259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}\} \text{ N}$$

$$\mathbf{F} = 500 \cos \alpha\mathbf{i} + 500 \cos \beta\mathbf{j} + 500 \cos \gamma\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed towards the positive $y$ axis, then

$$\mathbf{F}_R = F_R\,\mathbf{j}$$




***Resultant Force:***

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$

$$F_R\,\mathbf{j} = (259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}) + (500 \cos \alpha\mathbf{i} + 500 \cos \beta\mathbf{j} + 500 \cos \gamma\mathbf{k})$$

$$F_R\,\mathbf{j} = (259.81 + 500 \cos \alpha)\mathbf{i} + (450 + 500 \cos \beta)\mathbf{j} + (500 \cos \gamma - 300)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = 259.81 + 500 \cos \alpha$$

$$\alpha = 121.31° = 121° \qquad \textbf{Ans.}$$

$$F_R = 450 + 500 \cos \beta \qquad \textbf{(1)}$$

$$0 = 500 \cos \gamma - 300$$

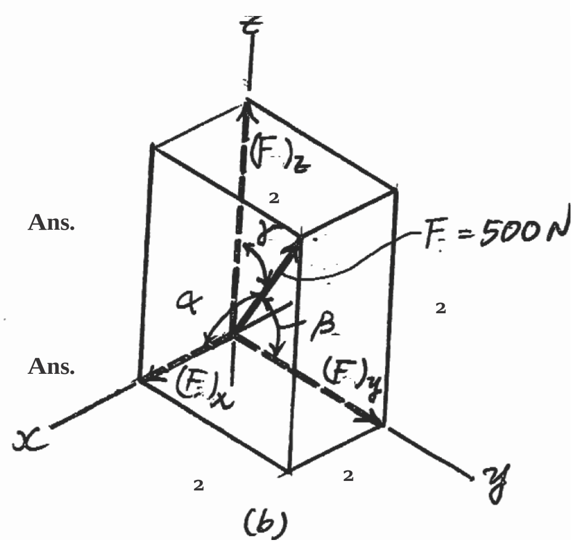
$$\gamma = 53.13° = 53.1° \qquad \textbf{Ans.}$$

However, since $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$, $\alpha = 121.31°$, and $\gamma = 53.13°$,

$$\cos \beta = \pm\sqrt{1 - \cos^2 121.31° - \cos^2 53.13°} = \pm 0.6083$$

If we substitute $\cos \beta = 0.6083$ into Eq. (1),

$$F_R = 450 + 500(0.6083) = 754 \text{ N} \qquad \textbf{Ans.}$$

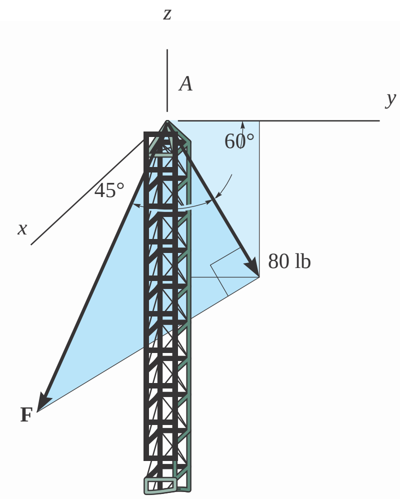



and

$$\beta = \cos^{-1}(0.6083) = 52.5° \qquad \textbf{Ans.}$$

***2–72.**

A force **F** is applied at the top of the tower at *A*. If it acts in the direction shown such that one of its components lying in the shaded *y-z* plane has a magnitude of 80 lb, determine its magnitude *F* and coordinate direction angles a, b, g.




## SOLUTION

***Cartesian Vector Notation:*** The magnitude of force **F** is

$$F \cos 45° = 80 \qquad F = 113.14 \text{ lb} = 113 \text{ lb}$$ **Ans.**

Thus,

$$\mathbf{F} = 5113.14 \sin 45°\mathbf{i} + 80 \cos 60°\mathbf{j} - 80 \sin 60°\mathbf{k}6 \text{ lb}$$

$$= 580.0\mathbf{i} + 40.0\mathbf{j} - 69.28\mathbf{k}6 \text{ lb}$$

The coordinate direction angles are

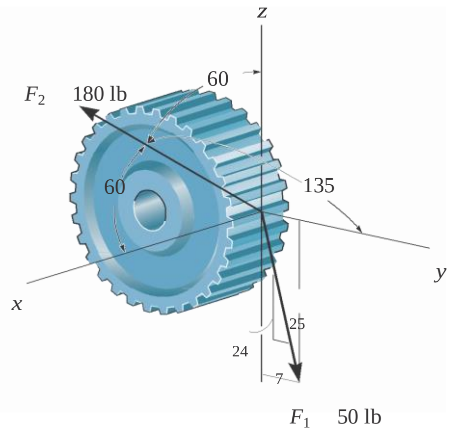
$$\cos a = \frac{F_x}{F} = \frac{80.0}{113.14} \qquad a = 45.0°$$ **Ans.**

$$\cos b = \frac{F_y}{F} = \frac{40.0}{113.14} \qquad b = 69.3°$$ **Ans.**

$$\cos g = \frac{F_z}{F} = \frac{-69.28}{113.14} \qquad g = 128°$$ **Ans.**

**2–73.**

The spur gear is subjected to the two forces caused by contact with other gears. Express each force as a Cartesian vector.




## SOLUTION

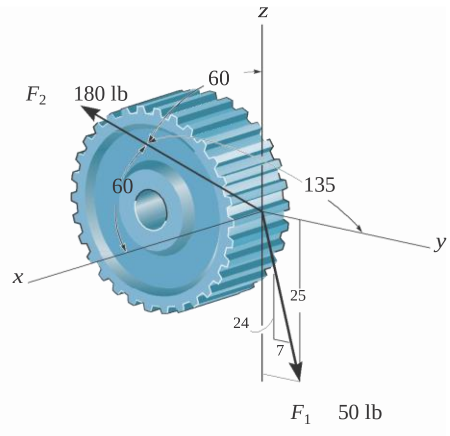
$$\mathbf{F}_1 = \frac{7}{25}(50)\mathbf{j} - \frac{24}{25}(50)\mathbf{k} = \{14.0\mathbf{j} - 48.0\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_2 = 180\cos 60°\mathbf{i} + 180\cos 135°\mathbf{j} + 180\cos 60°\mathbf{k}$$

$$= \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}\text{ lb}$$ **Ans.**

**2–74.**

The spur gear is subjected to the two forces caused by contact with other gears. Determine the resultant of the two forces and express the result as a Cartesian vector.




## SOLUTION

$$F_{Rx} = 180 \cos 60° = 90$$

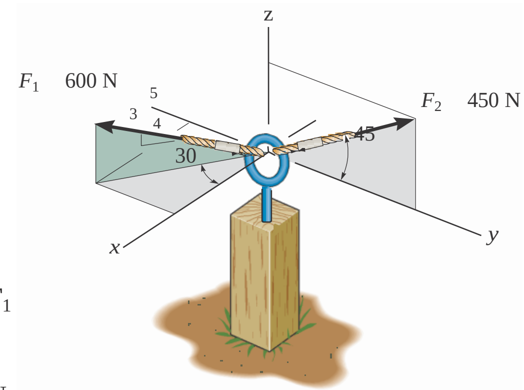
$$F_{Ry} = \frac{7}{25}(50) + 180 \cos 135° = -113$$

$$F_{Rz} = -\frac{24}{25}(50) + 180 \cos 60° = 42$$

$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\} \text{ lb}$$ **Ans.**

**2–75.**

Determine the coordinate direction angles of force $\mathbf{F}_1$.

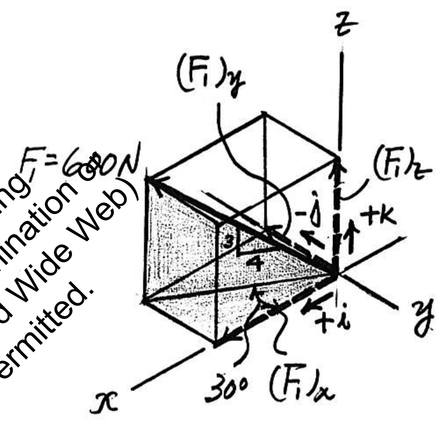



## SOLUTION

***Rectangular Components:*** By referring to Figs. *a*, the *x*, *y*, and *z* components of $\mathbf{F}_1$ can be written as

$$(F_1)_x = 600\left(\frac{4}{5}\right)\cos 30°\ \text{N} \qquad (F_1)_y = 600\left(\frac{4}{5}\right)\sin 30°\ \text{N} \qquad (F_1)_z = 600\left(\frac{3}{5}\right)\ \text{N}$$

Thus, $\mathbf{F}_1$ expressed in Cartesian vector form can be written as



(a)

$$\mathbf{F}_1 = 600\left\{\frac{4}{5}\cos 30°(+\mathbf{i}) + \frac{4}{5}\sin 30°(-\mathbf{j}) + \frac{3}{5}(+\mathbf{k})\right\}\ \text{N}$$

$$= 600[0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}]\ \text{N}$$

Therefore, the unit vector for $\mathbf{F}_1$ is given by

$$\mathbf{u}_{F_1} = \frac{\mathbf{F}_1}{F_1} = \frac{600(0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k})}{600} = 0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}$$

The coordinate direction angles of $\mathbf{F}_1$ are

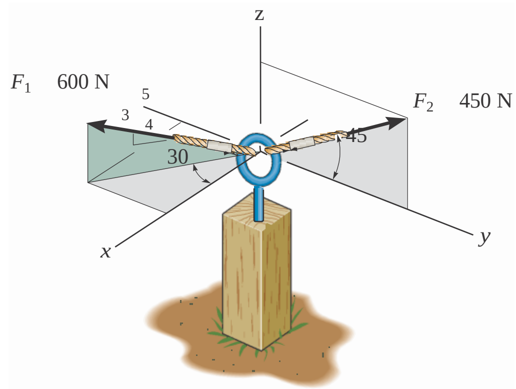
$$\alpha = \cos^{-1}(u_{F_1})_x = \cos^{-1}(0.6928) = 46.1° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}(u_{F_1})_y = \cos^{-1}(-0.4) = 114° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}(u_{F_1})_z = \cos^{-1}(0.6) = 53.1° \qquad \textbf{Ans.}$$

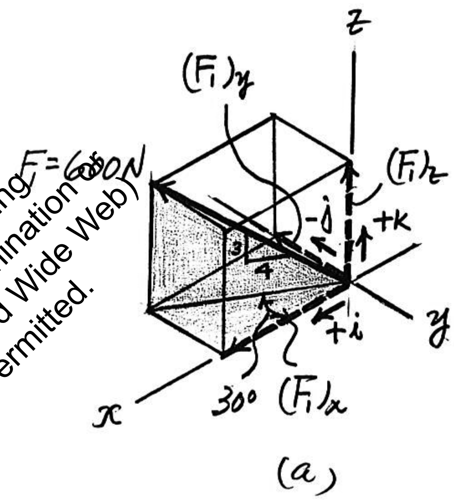
***2–76.**

Determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.




## SOLUTION

***Force Vectors:*** By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their $x$, $y$, and $z$ components, as shown in Figs. *a* and *b*, respectively, they are expressed in Cartesian vector form as




$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\sin 30°(-\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

$$= \{415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 450\cos 45°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$

$$= \{318.20\mathbf{j} + 318.20\mathbf{k}\}\text{ N}$$

***Resultant Force:*** The resultant force acting on the eyebolt can be obtained by vectorally adding $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}) + (318.20\mathbf{j} + 318.20\mathbf{k})$$

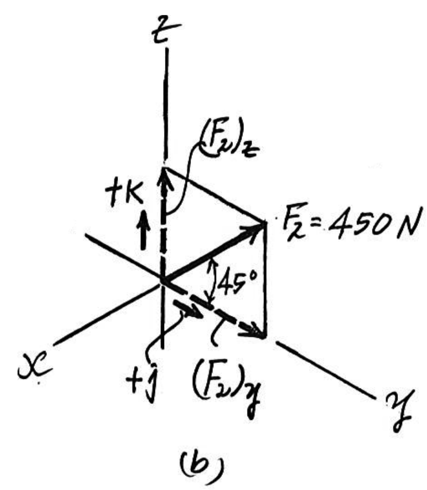
$$= \{415.69\mathbf{i} + 78.20\mathbf{j} + 678.20\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(415.69)^2 + (78.20)^2 + (678.20)^2} = 799.29\text{ N} = 799\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are



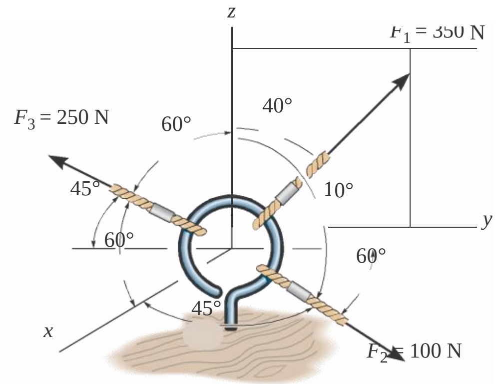

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{415.69}{799.29}\right) = 58.7° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{78.20}{799.29}\right) = 84.4° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{678.20}{799.29}\right) = 32.0° \qquad \textbf{Ans.}$$

**2–77.**

The cables attached to the screw eye are subjected to the three forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.




## SOLUTION

***Cartesian Vector Notation:***

$\mathbf{F}_1 = 3505\sin 40°\mathbf{j} + \cos 40°\mathbf{k}6$ N

$= 5224.98\mathbf{j} + 268.12\mathbf{k}6$ N

$= 5225\mathbf{j} + 268\mathbf{k}6$ N **Ans.**

$\mathbf{F}_2 = 1005\cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}6$ N

$= 570.71\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}6$ N

$= 570.7\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}6$ N **Ans.**

$\mathbf{F}_3 = 2505\cos 60°\mathbf{i} + \cos 135°\mathbf{j} + \cos 60°\mathbf{k}6$ N

$= 5125.0\mathbf{i} - 176.78\mathbf{j} + 125.0\mathbf{k}6$ N

$= 5125\mathbf{i} - 177\mathbf{j} + 125\mathbf{k}6$ N **Ans.**

***Resultant Force:***

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$= 5170.71 + 125.02\mathbf{i} + 1224.98 + 50.0 - 176.782\mathbf{j} + 1268.12 - 50.0 + 125.02\mathbf{k}6$ N

$= 5195.71\mathbf{i} + 98.20\mathbf{j} + 343.12\mathbf{k}6$ N

The magnitude of the resultant force is

$$F_R = 2F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2$$

$$= 2195.71^2 + 98.20^2 + 343.12^2$$

$$= 407.03 \text{ N} = 407 \text{ N}$$ **Ans.**

The coordinate direction angles are

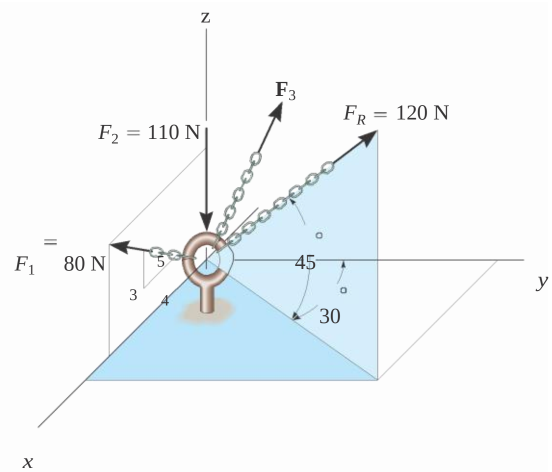
$\cos a = \dfrac{F_{R_x}}{F_R} = \dfrac{195.71}{407.03}$ $a = 61.3°$ **Ans.**

$\cos b = \dfrac{F_{R_y}}{F_R} = \dfrac{98.20}{407.03}$ $b = 76.0°$ **Ans.**

$\cos g = \dfrac{F_{R_z}}{F_R} = \dfrac{343.12}{407.03}$ $g = 32.5°$ **Ans.**

**2–78.**

Three forces act on the ring. If the resultant force $\mathbf{F}_R$ has a magnitude and direction as shown, determine the magnitude and the coordinate direction angles of force $\mathbf{F}_3$.




## SOLUTION

***Cartesian Vector Notation:***

$$\mathbf{F}_R = 120\{\cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k}\}\ \text{N}$$

$$= \{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_1 = 80\left\{\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right\}\ \text{N} = \{64.0\mathbf{i} + 48.0\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_2 = \{-110\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_3 = \{F_{3_x}\mathbf{i} + F_{3_y}\mathbf{j} + F_{3_z}\mathbf{k}\}\ \text{N}$$

***Resultant Force:***

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$\{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\} = \left\{\left(64.0 + F_{3_x}\right)\mathbf{i} + F_{3_y}\mathbf{j} + \left(48.0 - 110 + F_{3_z}\right)\mathbf{k}\right\}$$

Equating **i**, **j** and **k** components, we have

$$64.0 + F_{3_x} = 42.43 \qquad F_{3_x} = -21.57\ \text{N}$$

$$F_{3_y} = 73.48\ \text{N}$$

$$48.0 - 110 + F_{3_z} = 84.85 \qquad F_{3_z} = 146.85\ \text{N}$$

The magnitude of force $\mathbf{F}_3$ is

$$F_3 = \sqrt{F_{3_x}^2 + F_{3_y}^2 + F_{3_z}^2}$$

$$= \sqrt{(-21.57)^2 + 73.48^2 + 146.85^2}$$

$$= 165.62\ \text{N} = 166\ \text{N}$$ **Ans.**

The coordinate direction angles for $\mathbf{F}_3$ are

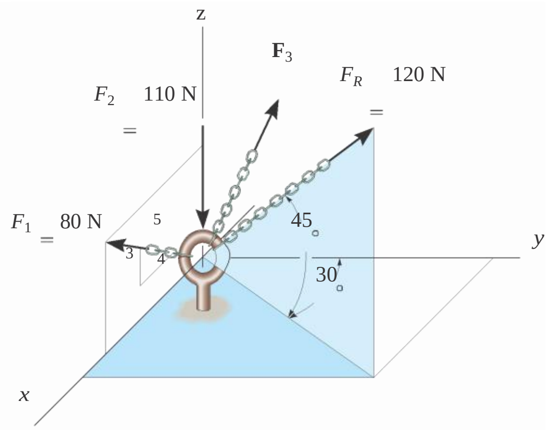
$$\cos\alpha = \frac{F_{3_x}}{F_3} = \frac{-21.57}{165.62} \qquad \alpha = 97.5°$$ **Ans.**

$$\cos\beta = \frac{F_{3_y}}{F_3} = \frac{73.48}{165.62} \qquad \beta = 63.7°$$ **Ans.**

$$\cos\gamma = \frac{F_{3_z}}{F_3} = \frac{146.85}{165.62} \qquad \gamma = 27.5°$$ **Ans.**

**2–79.**

Determine the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_R$.




# SOLUTION

***Unit Vector of* $\mathbf{F}_1$ *and* $\mathbf{F}_R$:**

$$\mathbf{u}_{F_1} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k} = 0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\mathbf{u}_R = \cos 45^\circ \sin 30^\circ \mathbf{i} + \cos 45^\circ \cos 30^\circ \mathbf{j} + \sin 45^\circ \mathbf{k}$$

$$= 0.3536\mathbf{i} + 0.6124\mathbf{j} + 0.7071\mathbf{k}$$

Thus, the coordinate direction angles $\mathbf{F}_1$ and $\mathbf{F}_R$ are

$\cos a_{F_1} = 0.8 \qquad a_{F_1} = 36.9^\circ$ **Ans.**

$\cos b_{F_1} = 0 \qquad b_{F_1} = 90.0^\circ$ **Ans.**

$\cos g_{F_1} = 0.6 \qquad g_{F_1} = 53.1^\circ$ **Ans.**

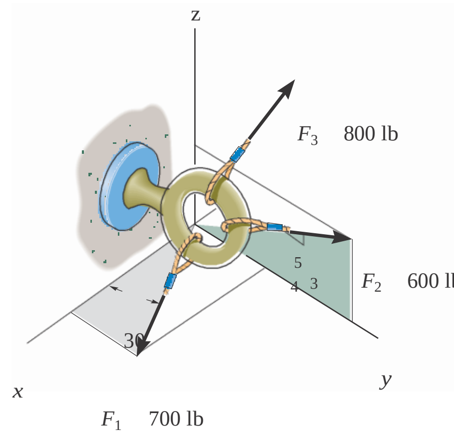
$\cos a_R = 0.3536 \qquad a_R = 69.3^\circ$ **Ans.**

$\cos b_R = 0.6124 \qquad b_R = 52.2^\circ$ **Ans.**

$\cos g_R = 0.7071 \qquad g_R = 45.0^\circ$ **Ans.**

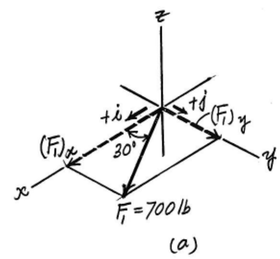
***2–80.**

If the coordinate direction angles for $\mathbf{F}_3$ are $\alpha_3 = 120°$, $\beta_3 = 45°$ and $\gamma_3 = 60°$, determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.




## SOLUTION

***Force Vectors:*** By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their *x*, *y*, and *z* components, as shown in Figs. *a*, *b*, and *c*, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ can be expressed in Cartesian vector form as




$$\mathbf{F}_1 = 700\cos 30°(+\mathbf{i}) + 700\sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\}\text{ lb}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\}\text{ lb}$$

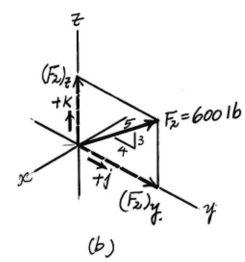
$$\mathbf{F}_3 = 800\cos 120°\mathbf{i} + 800\cos 45°\mathbf{j} + 800\cos 60°\mathbf{k} = \{-400\mathbf{i} + 565.69\mathbf{j} + 400\mathbf{k}\}\text{ lb}$$

***Resultant Force:*** By adding $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (606.22\mathbf{i} + 350\mathbf{j}) + (480\mathbf{j} + 360\mathbf{k}) + (-400\mathbf{i} + 565.69\mathbf{j} + 400\mathbf{k})$$

$$= \{206.22\mathbf{i} + 1395.69\mathbf{j} + 760\mathbf{k}\}\text{ lb}$$

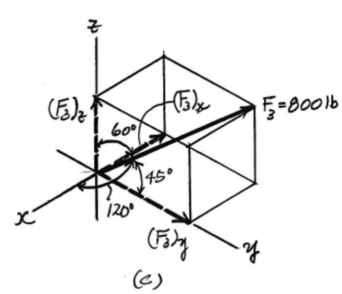



The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2} + (F_R)_z^{\,2}}$$

$$= \sqrt{(206.22)^2 + (1395.69)^2 + (760)^2} = 1602.52\text{ lb} = 1.60\text{ kip} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are



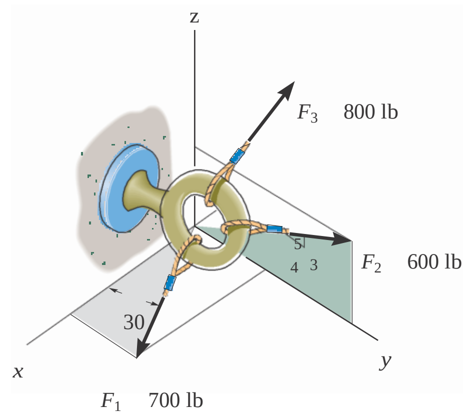

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{206.22}{1602.52}\right) = 82.6° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{1395.69}{1602.52}\right) = 29.4° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{760}{1602.52}\right) = 61.7° \qquad \textbf{Ans.}$$

**2–81.**

If the coordinate direction angles for $\mathbf{F}_3$ are $\alpha_3 = 120°$, $\beta_3 = 45°$ and $\gamma_3 = 60°$, determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.




## SOLUTION

***Force Vectors:*** By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their $x$, $y$, and $z$ components, as shown in Figs. $a$, $b$, and $c$, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 700 \cos 30°(+\mathbf{i}) + 700 \sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\} \text{ lb}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_3 = 800 \cos 120°\mathbf{i} + 800 \cos 45°\mathbf{j} + 800 \cos 60°\mathbf{k} = \{-400\mathbf{i} + 565.69\mathbf{j} + 400\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= 606.22\mathbf{i} + 350\mathbf{j} + 480\mathbf{j} + 360\mathbf{k} - 400\mathbf{i} + 565.69\mathbf{j} + 400\mathbf{k}$$

$$= \{206.22\mathbf{i} + 1395.69\mathbf{j} + 760\mathbf{k}\} \text{ lb}$$

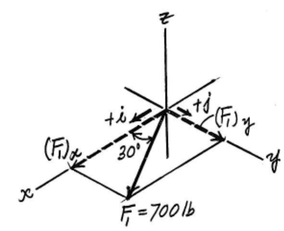
$$F_R = \sqrt{(206.22)^2 + (1395.69)^2 + (760)^2}$$

$$= 1602.52 \text{ lb} = 1.60 \text{ kip}$$ **Ans.**

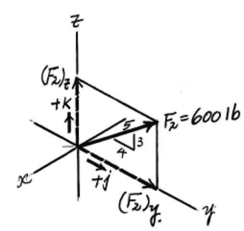
$$\alpha = \cos^{-1}\left(\frac{206.22}{1602.52}\right) = 82.6°$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{1395.69}{1602.52}\right) = 29.4°$$ **Ans.**

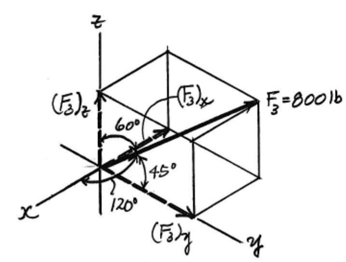
$$\gamma = \cos^{-1}\left(\frac{760}{1602.52}\right) = 61.7°$$ **Ans.**



(a)



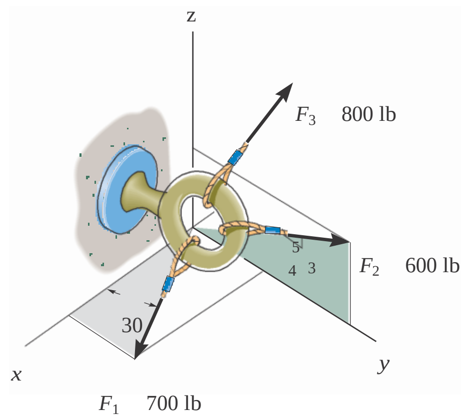
(b)



(c)

**2–82.**

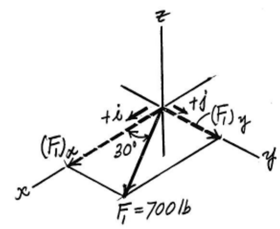
If the direction of the resultant force acting on the eyebolt is defined by the unit vector $\mathbf{u}_{F_R} = \cos 30°\mathbf{j} + \sin 30°\mathbf{k}$, determine the coordinate direction angles of $\mathbf{F}_3$ and the magnitude of $\mathbf{F}_R$.




## SOLUTION

***Force Vectors:*** By resolving $\mathbf{F}_1, \mathbf{F}_2$ and $\mathbf{F}_3$ into their *x*, *y*, and *z* components, as shown in Figs. *a*, *b*, and *c*, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 700 \cos 30°(+\mathbf{i}) + 700 \sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\} \text{ lb}$$



(a)

$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_3 = 800 \cos \alpha_3\mathbf{i} + 800 \cos \beta_3\,\mathbf{j} + 800 \cos \gamma_3\mathbf{k}$$

Since the direction of $\mathbf{F}_R$ is defined by $\mathbf{u}_{F_R} = \cos 30°\mathbf{j} + \sin 30°\mathbf{k}$, it can be written in Cartesian vector form as

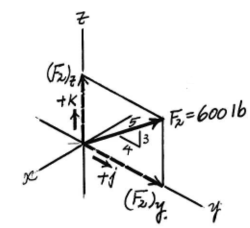
$$\mathbf{F}_R = F_R\mathbf{u}_{F_R} = F_R(\cos 30°\mathbf{j} + \sin 30°\mathbf{k}) = 0.8660F_R\,\mathbf{j} + 0.5F_R\,\mathbf{k}$$

***Resultant Force:*** By adding $\mathbf{F}_1, \mathbf{F}_2$, and $\mathbf{F}_3$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$0.8660F_R\,\mathbf{j} + 0.5F_R\,\mathbf{k} = (606.22\mathbf{i} + 350\mathbf{j}) + (480\mathbf{j} + 360\mathbf{k}) + (800 \cos \alpha_3\mathbf{i} + 800 \cos \beta_3\,\mathbf{j} + 800 \cos \gamma_3\mathbf{k})$$

$$0.8660F_R\,\mathbf{j} + 0.5F_R\,\mathbf{k} = (606.22 + 800 \cos \alpha_3)\mathbf{i} + (350 + 480 + 800 \cos \beta_3)\mathbf{j} + (360 + 800 \cos \gamma_3)\mathbf{k}$$



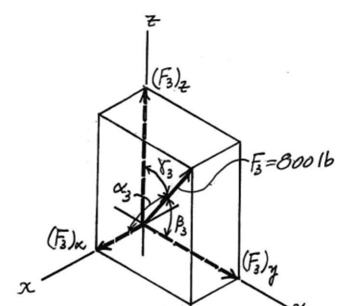
(b)

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components, we have

$$0 = 606.22 + 800 \cos \alpha_3$$
$$800 \cos \alpha_3 = -606.22 \qquad \textbf{(1)}$$

$$0.8660F_R = 350 + 480 + 800 \cos \beta_3$$
$$800 \cos \beta_3 = 0.8660F_R - 830 \qquad \textbf{(2)}$$

$$0.5F_R = 360 + 800 \cos \gamma_3$$
$$800 \cos \gamma_3 = 0.5F_R - 360 \qquad \textbf{(3)}$$



(c)

Squaring and then adding Eqs. (1), (2), and (3), yields

$$800^2\,[\cos^2 \alpha_3 + \cos^2 \beta_3 + \cos^2 \gamma_3] = F_R^{\,2} - 1797.60F_R + 1{,}186{,}000 \qquad \textbf{(4)}$$

However, $\cos^2 \alpha_3 + \cos^2 \beta_3 + \cos^2 \gamma_3 = 1$. Thus, from Eq. (4)

$$F_R^{\,2} - 1797.60F_R + 546{,}000 = 0$$

Solving the above quadratic equation, we have two positive roots

$$F_R = 387.09 \text{ N} = 387 \text{ N} \qquad \textbf{Ans.}$$

$$F_R = 1410.51 \text{ N} = 1.41 \text{ kN} \qquad \textbf{Ans.}$$

From Eq. (1),

$$\alpha_3 = 139° \qquad \textbf{Ans.}$$

Substituting $F_R = 387.09$ N into Eqs. (2), and (3), yields

$$\beta_3 = 128°$$

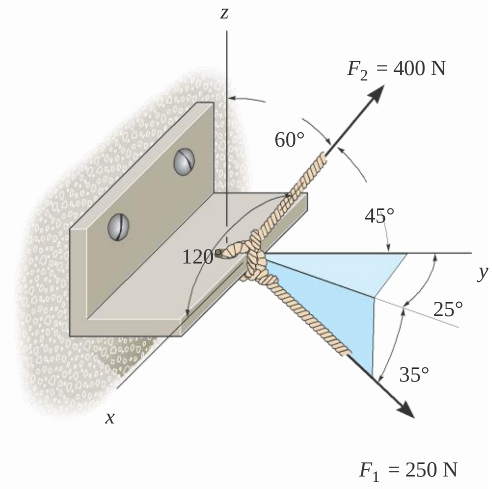
$g_3 = 102°$ **Ans.**

Substituting $F_R = 1410.51$ N into Eqs. (2), and (3), yields

$b_3 = 60.7°$ $g_3 = 64.4°$ **Ans.**

**2–83.**

The bracket is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force $\mathbf{F}_R$. Find the magnitude and coordinate direction angles of the resultant force.




## SOLUTION

***Cartesian Vector Notation:***

$\mathbf{F}_1 = 2505\cos 35° \sin 25°\mathbf{i} + \cos 35° \cos 25°\mathbf{j} - \sin 35°\mathbf{k}6$ N

$= 586.55\mathbf{i} + 185.60\mathbf{j} - 143.39\mathbf{k}6$ N

$= 586.5\mathbf{i} + 186\mathbf{j} - 143\mathbf{k}6$ N **Ans.**

$\mathbf{F}_2 = 4005\cos 120°\mathbf{i} + \cos 45°\mathbf{j} + \cos 60°\mathbf{k}6$ N

$= 5{-}200.0\mathbf{i} + 282.84\mathbf{j} + 200.0\mathbf{k}6$ N

$= 5{-}200\mathbf{i} + 283\mathbf{j} + 200\mathbf{k}6$ N **Ans.**

***Resultant Force:***

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

$= 5186.55 - 200.02\mathbf{i} + 1185.60 + 282.842\mathbf{j} + 1{-}143.39 + 200.02\mathbf{k}6$

$= 5{-}113.45\mathbf{i} + 468.44\mathbf{j} + 56.61\mathbf{k}6$ N

$= 5{-}113\mathbf{i} + 468\mathbf{j} + 56.6\mathbf{k}6$ N **Ans.**

The magnitude of the resultant force is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$$

$$= \sqrt{1{-}113.452^2 + 468.44^2 + 56.61^2}$$

$= 485.30$ N $= 485$ N **Ans.**

The coordinate direction angles are

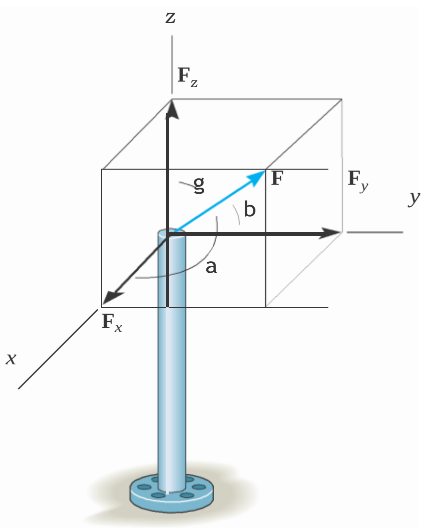
$\cos a = \frac{F_{R_x}}{F_R} = \frac{-113.45}{485.30}$ $a = 104°$ **Ans.**

$\cos b = \frac{F_{R_y}}{F_R} = \frac{468.44}{485.30}$ $b = 15.1°$ **Ans.**

$\cos g = \frac{F_{R_z}}{F_R} = \frac{56.61}{485.30}$ $g = 83.3°$ **Ans.**

***2–84.**

The pole is subjected to the force **F**, which has components acting along the *x*, *y*, *z* axes as shown. If the magnitude of **F** is 3 kN, $\beta = 30°$, and $\gamma = 75°$, determine the magnitudes of its three components.




## SOLUTION

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\cos^2 \alpha + \cos^2 30° + \cos^2 75° = 1$$

$$\alpha = 64.67°$$

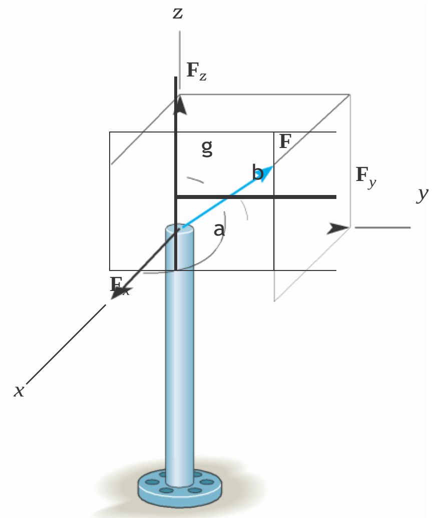
$$F_x = 3 \cos 64.67° = 1.28 \text{ kN}$$ **Ans.**

$$F_y = 3 \cos 30° = 2.60 \text{ kN}$$ **Ans.**

$$F_z = 3 \cos 75° = 0.776 \text{ kN}$$ **Ans.**

**2–85.**

The pole is subjected to the force **F** which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75°$, determine the magnitudes of **F** and $\mathbf{F}_y$.




## SOLUTION

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

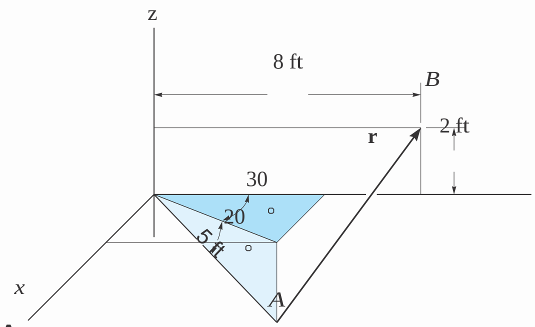
$$\left(\frac{1.5}{F}\right)^2 + \cos^2 75° + \left(\frac{1.25}{F}\right)^2 = 1$$

$$F = 2.02 \text{ kN}$$ **Ans.**

$$F_y = 2.02 \cos 75° = 0.523 \text{ kN}$$ **Ans.**

**2–86.**

Express the position vector **r** in Cartesian vector form; then determine its magnitude and coordinate direction angles.




## SOLUTION

$\mathbf{r} = (-5\cos 20^\circ \sin 30^\circ)\mathbf{i} + (8 - 5\cos 20^\circ \cos 30^\circ)\mathbf{j} + (2 + 5\sin 20^\circ)\mathbf{k}$

$\mathbf{r} = \{-2.35\mathbf{i} + 3.93\mathbf{j} + 3.71\mathbf{k}\}$ ft **Ans.**

$r = \sqrt{(-2.35)^2 + (3.93)^2 + (3.71)^2} = 5.89$ ft **Ans.**

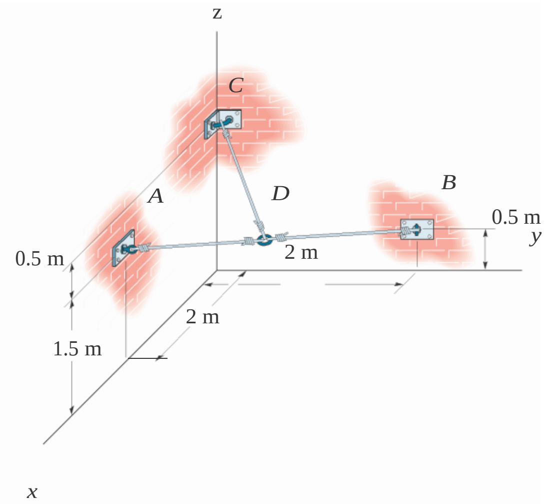
$\alpha = \cos^{-1}\left(\frac{-2.35}{5.89}\right) = 113^\circ$ **Ans.**

$\beta = \cos^{-1}\left(\frac{3.93}{5.89}\right) = 48.2^\circ$ **Ans.**

$\gamma = \cos^{-1}\left(\frac{3.71}{5.89}\right) = 51.0^\circ$ **Ans.**

**2–87.**

Determine the lengths of wires *AD*, *BD*, and *CD*. The ring at *D* is midway between *A* and *B*.




## SOLUTION

$$D\left(\frac{2+0}{2}, \frac{0+2}{2}, \frac{1.5+0.5}{2}\right) \text{m} = D(1,\ 1,\ 1)\ \text{m}$$

$$\mathbf{r}_{AD} = (1-2)\mathbf{i} + (1-0)\mathbf{j} + (1-1.5)\mathbf{k}$$

$$= -1\mathbf{i} + 1\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{r}_{BD} = (1-0)\mathbf{i} + (1-2)\mathbf{j} + (1-0.5)\mathbf{k}$$

$$= 1\mathbf{i} - 1\mathbf{j} + 0.5\mathbf{k}$$

$$\mathbf{r}_{CD} = (1-0)\mathbf{i} + (1-0)\mathbf{j} + (1-2)\mathbf{k}$$

$$= 1\mathbf{i} + 1\mathbf{j} - 1\mathbf{k}$$

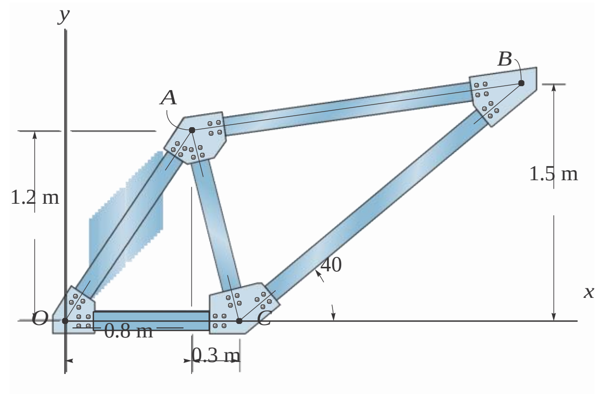
$$r_{AD} = \sqrt{(-1)^2 + 1^2 + (-0.5)^2} = 1.50\ \text{m}$$ **Ans.**

$$r_{BD} = \sqrt{1^2 + (-1)^2 + 0.5^2} = 1.50\ \text{m}$$ **Ans.**

$$r_{CD} = \sqrt{1^2 + 1^2 + (-1)^2} = 1.73\ \text{m}$$ **Ans.**

***2–88.**

Determine the length of member AB of the truss by first establishing a Cartesian position vector from A to B and




## SOLUTION

$$\mathbf{r}_{AB} = (1.1) = \frac{1.5}{\tan 40°} - 0.80)\mathbf{i} + (1.5 - 1.2)\mathbf{j}$$

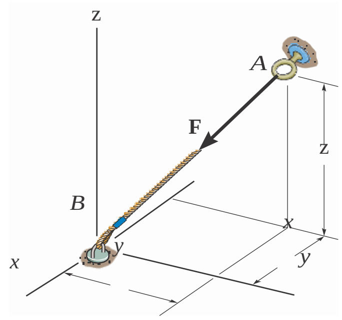
$$\mathbf{r}_{AB} = \{2.09\mathbf{i} + 0.3\mathbf{j}\}\ \text{m}$$

$$\mathbf{r}_{AB} = \sqrt{(2.09)^2 + (0.3)^2} = 2.11\ \text{m}$$

**Ans.**

**2–89.**

If $\mathbf{F} = 5350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}6$ N and cable $AB$ is 9 m long, determine the $x$, $y$, $z$ coordinates of point $A$.




## SOLUTION

***Position Vector:*** The position vector $\mathbf{r}_{AB}$, directed from point $A$ to point $B$, is given by

$$\mathbf{r}_{AB} = [0 - (-x)]\mathbf{i} + (0 - y)\mathbf{j} + (0 - z)\mathbf{k}$$

$$= x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$$

***Unit Vector:*** Knowing the magnitude of $\mathbf{r}_{AB}$ is 9 m, the unit vector for $\mathbf{r}_{AB}$ is given by

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9}$$

The unit vector for force $\mathbf{F}$ is

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \frac{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}}{\sqrt{350^2 + (-250)^2 + (-450)^2}} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Since force $\mathbf{F}$ is also directed from point $A$ to point $B$, then

$$\mathbf{u}_{AB} = \mathbf{u}_F$$

$$\frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

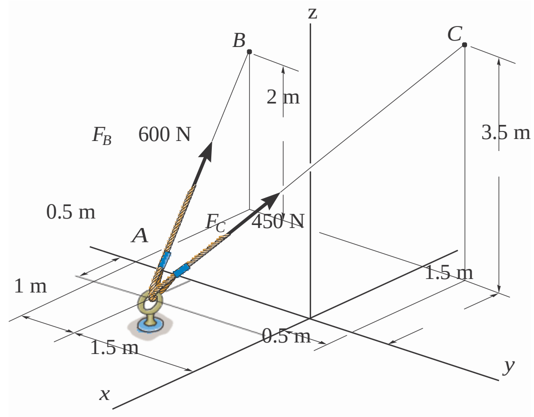
$$\frac{x}{9} = 0.5623 \qquad x = 5.06 \text{ m}$$ **Ans.**

$$\frac{-y}{9} = -0.4016 \qquad y = 3.61 \text{ m}$$ **Ans.**

$$\frac{-z}{9} = 0.7229 \qquad z = 6.51 \text{ m}$$ **Ans.**

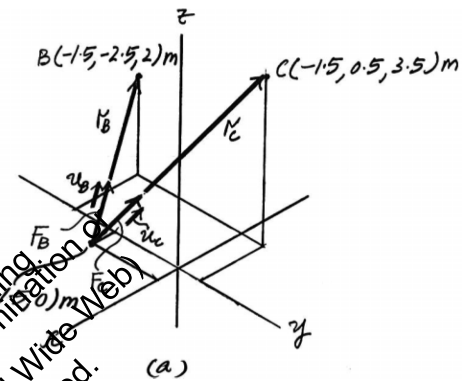
**2–90.**

Express $\mathbf{F}_B$ and $\mathbf{F}_C$ in Cartesian vector form.




## SOLUTION

***Force Vectors:*** The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*




$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5-0.5)\mathbf{i} + [-2.5-(-1.5)]\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(-1.5-0.5)^2 + [-2.5-(-1.5)]^2 + (2-0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5-0.5)\mathbf{i} + [0.5-(-1.5)]\mathbf{j} + (3.5-0)\mathbf{k}}{\sqrt{(-1.5-0.5)^2 + [0.5-(-1.5)]^2 + (3.5-0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$

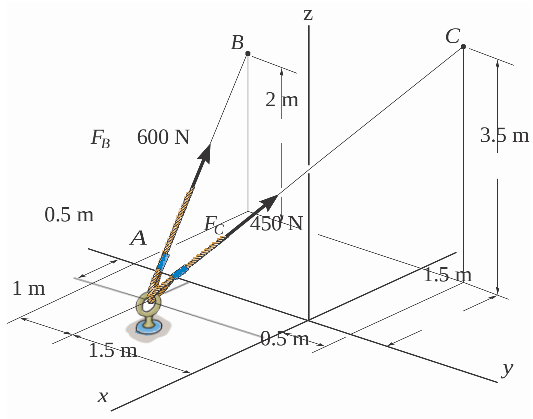
Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\,\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\ \text{N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_C = F_C\,\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\ \text{N} \qquad \textbf{Ans.}$$

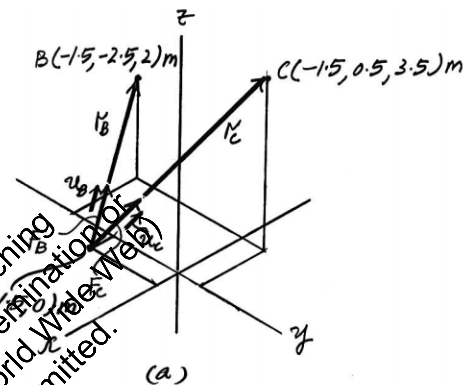
**2–91.**

Determine the magnitude and coordinate direction angles of the resultant force acting at A.




## SOLUTION

***Force Vectors:*** The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*




$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5 - 0.5)\mathbf{i} + [-2.5 - (-1.5)]\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [-2.5 - (-1.5)]^2 + (2 - 0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5 - 0.5)\mathbf{i} + [0.5 - (-1.5)]\mathbf{j} + (3.5 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [0.5 - (-1.5)]^2 + (3.5 - 0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\text{ N}$$

***Resultant Force:***

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}) + (-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k})$$

$$= \{-600\mathbf{i} + 750\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(-600)^2 + 0^2 + 750^2} = 960.47\text{ N} = 960\text{ N}$$

The coordinate direction angles of $\mathbf{F}_R$ are

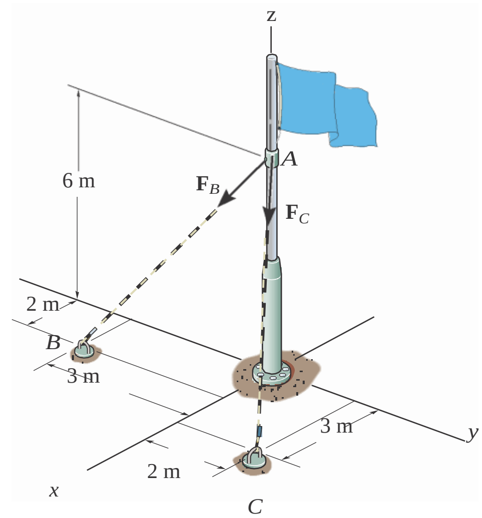
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{-600}{960.47}\right) = 129°$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{960.47}\right) = 90°$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{760}{960.47}\right) = 38.7°$$ **Ans.**

***2–92.**

If $F_B = 560$ N and $F_C = 700$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.




## SOLUTION

***Force Vectors:*** The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 560\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 700\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

***Resultant Force:***

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}) + (300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k})$$

$$= \{460\mathbf{i} - 40\mathbf{j} + 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2} + (F_R)_z^{\,2}}$$

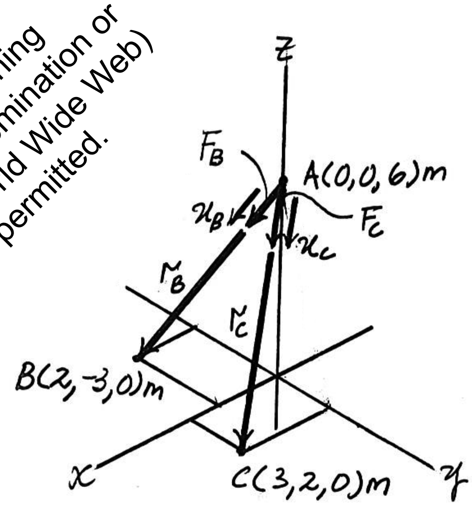
$$= \sqrt{(460)^2 + (-40)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{460}{1174.56}\right) = 66.9° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-40}{1174.56}\right) = 92.0° \qquad \textbf{Ans.}$$

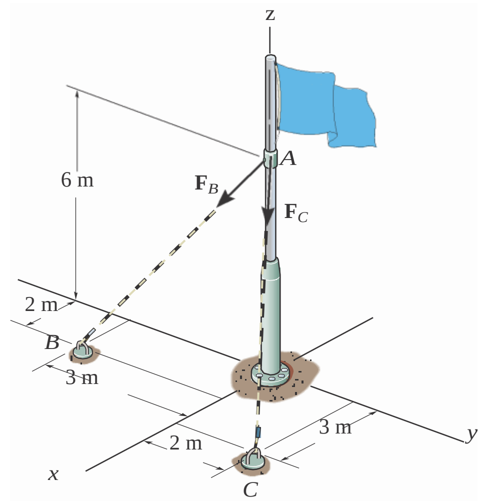
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157° \qquad \textbf{Ans.}$$




(a)

**2–93.**

If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.




## SOLUTION

***Force Vectors:*** The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 700\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 560\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

***Resultant Force:***

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}) + (240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k})$$

$$= \{440\mathbf{i} - 140\mathbf{j} - 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2} + (F_R)_z^{\,2}}$$

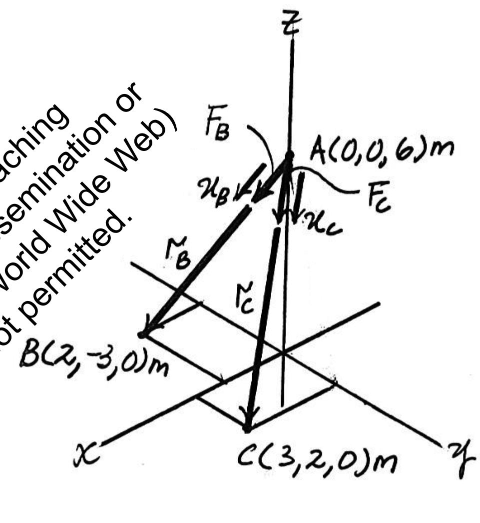
$$= \sqrt{(440)^2 + (-140)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{440}{1174.56}\right) = 68.0^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-140}{1174.56}\right) = 96.8^\circ$$ **Ans.**

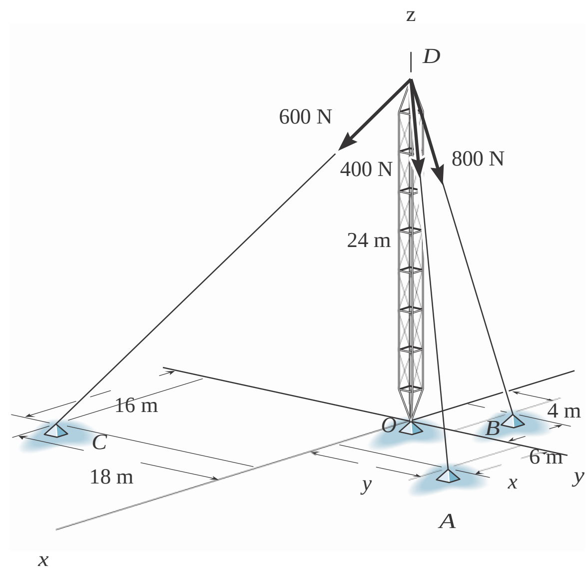
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157^\circ$$ **Ans.**



(a)

**2–94.**

The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles $\alpha, \beta, \gamma$ of the resultant force. Take $x = 20$ m, $y = 15$ m.




## SOLUTION

$$\mathbf{F}_{DA} = 400\left(\frac{20}{34.66}\,\mathbf{i} + \frac{15}{34.66}\,\mathbf{j} - \frac{24}{34.66}\,\mathbf{k}\right)\text{ N}$$

$$\mathbf{F}_{DB} = 800\left(\frac{-6}{25.06}\,\mathbf{i} + \frac{4}{25.06}\,\mathbf{j} - \frac{24}{25.06}\,\mathbf{k}\right)\text{ N}$$

$$\mathbf{F}_{DC} = 600\left(\frac{16}{34}\,\mathbf{i} - \frac{18}{34}\mathbf{j} - \frac{24}{34}\,\mathbf{k}\right)\text{ N}$$

$$\mathbf{F}_R = \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC}$$

$$= \{321.66\mathbf{i} - 16.82\mathbf{j} - 1466.71\mathbf{k}\}\text{ N}$$

$$F_R = \sqrt{(321.66)^2 + (-16.82)^2 + (-1466.71)^2}$$

$$= 1501.66\text{ N} = 1.50\text{ kN}$$ **Ans.**

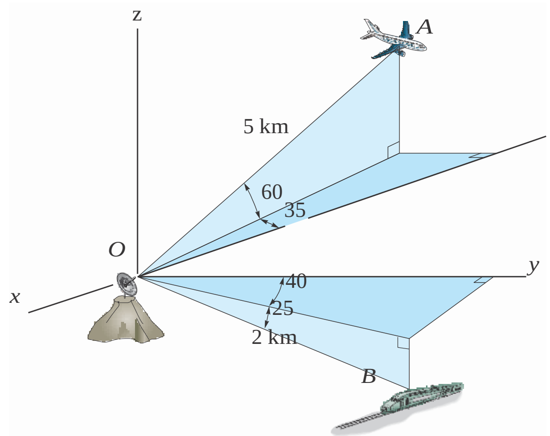
$$\alpha = \cos^{-1}\left(\frac{321.66}{1501.66}\right) = 77.6^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{-16.82}{1501.66}\right) = 90.6^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-1466.71}{1501.66}\right) = 168^\circ$$ **Ans.**

2–95.

At a given instant, the position of a plane at A and a train at B are measured relative to a radar antenna at O. Determine the distance d between A and B at this instant. To solve the problem, formulate a position vector, directed from A to B, and then determine its magnitude.




## SOLUTION

***Position Vector:*** The coordinates of points $A$ and $B$ are

$$A(-5\cos 60°\cos 35°, -5\cos 60°\sin 35°, 5\sin 60°)\text{ km}$$

$$= A(-2.048, -1.434, 4.330)\text{ km}$$

$$B(2\cos 25°\sin 40°, 2\cos 25°\cos 40°, -2\sin 25°)\text{ km}$$

$$= B(1.165, 1.389, -0.845)\text{ km}$$

The position vector $\mathbf{r}_{AB}$ can be established from the coordinates of points $A$ and $B$.

$$\mathbf{r}_{AB} = \{[1.165 - (-2.048)]\mathbf{i} + [1.389 - (-1.434)]\mathbf{j} + (-0.845 - 4.330)\mathbf{k}\}\text{ km}$$

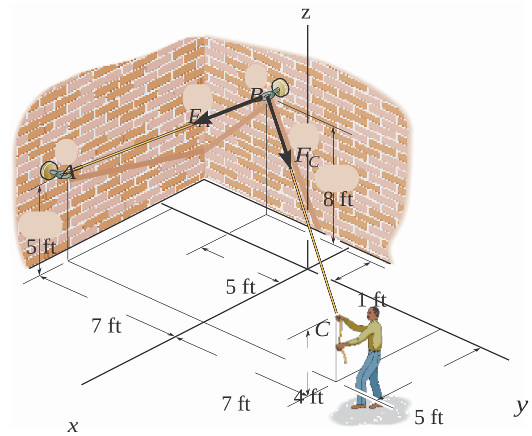
$$= \{3.213\mathbf{i} + 2.822\mathbf{j} - 5.175)\mathbf{k}\}\text{ km}$$

The distance between points $A$ and $B$ is

$$d = r_{AB} = \sqrt{3.213^2 + 2.822^2 + (-5.175)^2} = 6.71\text{ km} \qquad \textbf{Ans.}$$

***2–96.**

The man pulls on the rope at C with a force of 70 lb which causes the forces $\mathbf{F}_A$ and $\mathbf{F}_C$ at B to have this same magnitude. Express each of these two forces as Cartesian vectors.




## SOLUTION

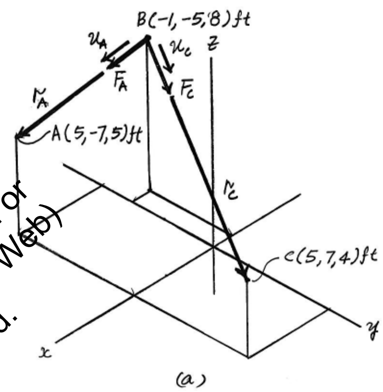
***Unit Vectors:*** The coordinate points *A*, *B*, and *C* are shown in Fig. *a*. Thus,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{[5-(-1)]\mathbf{i} + [-7-(-5)]\mathbf{j} + (5-8)\mathbf{k}}{3[5-(-1)]^2 + [-7-(-5)]^2 + (5-8)^2}$$

$$= \frac{6}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{[5-(-1)]\mathbf{i} + [-7(-5)]\mathbf{j} + (4-8)\mathbf{k}}{3[5-(-1)]^2 + [-7(-5)]^2 + (4-8)^2}$$

$$= \frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$




***Force Vectors:*** Multiplying the magnitude of the force with its unit vector,

$$\mathbf{F}_A = F_A\,\mathbf{u}_A = 70\text{a}\frac{6}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}\text{b}$$

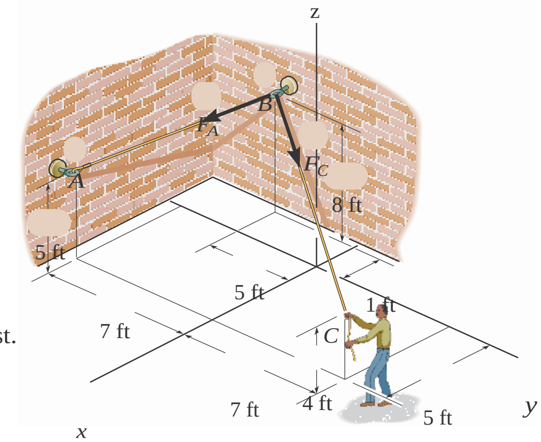
$$= 560\mathbf{i} - 20\mathbf{j} + 30\mathbf{k}6 \text{ lb}$$ **Ans.**

$$\mathbf{F}_C = F_C\,\mathbf{u}_C = 70\text{a}\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\text{b}$$

$$= 530\mathbf{i} + 60\mathbf{j} + 20\mathbf{k}6 \text{ lb}$$ **Ans.**

**2–97.**

The man pulls on the rope at C with a force of 70 lb which causes the forces $\mathbf{F}_A$ and $\mathbf{F}_C$ at B to have this same magnitude. Determine the magnitude and coordinate direction angles of the resultant force acting at B.




## SOLUTION

***Force Vectors:*** The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{[5-(-1)]\mathbf{i} + [-7(-5)]\mathbf{j} + (5-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7(-5)]^2 + (5-8)^2}}$$

$$= \frac{6}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{[5-(-1)]\mathbf{i} + [-7(-5)]\mathbf{j} + (4-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7(-5)]^2 + (4-8)^2}}$$

$$= \frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_A = F_A\,\mathbf{u}_A = 70\left(\frac{6}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}\right) = \{60\mathbf{i} - 20\mathbf{j} + 30\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_C = F_C\,\mathbf{u}_C = 70\left(\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) = \{30\mathbf{i} + 60\mathbf{j} + 20\mathbf{k}\}\text{ lb}$$

***Resultant Force:***

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_C = (60\mathbf{i} - 20\mathbf{j} - 30\mathbf{k}) + (30\mathbf{i} + 60\mathbf{j} + 20\mathbf{k})$$

$$= \{90\mathbf{i} + 40\mathbf{j} - 50\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2} + (F_R)_z^{\,2}}$$

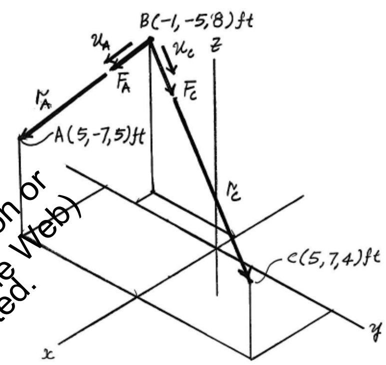
$$= \sqrt{(90)^2 + (40)^2 + (-50)^2} = 110.45 \text{ lb} = 110 \text{ lb}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{90}{110.45}\right) = 35.4^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{40}{110.45}\right) = 68.8^\circ$$ **Ans.**

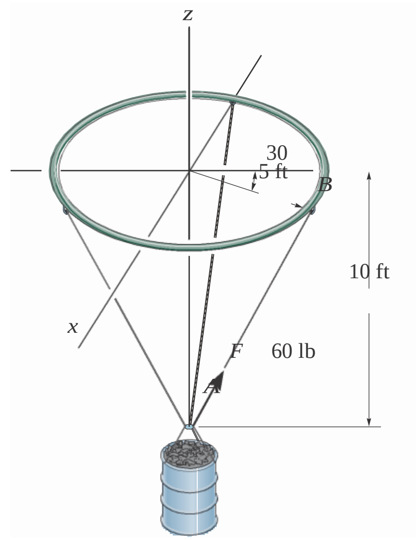
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-50}{110.45}\right) = 117^\circ$$ **Ans.**




(a)

**2–98.**

The load at A creates a force of 60 lb in wire AB. Express this force as a Cartesian vector acting on A and directed toward B as shown.




## SOLUTION

***Unit Vector:*** First determine the position vector $\mathbf{r}_{AB}$. The coordinates of point B are

B (5 sin 30°, 5 cos 30°, 0) ft = B (2.50, 4.330, 0) ft

Then

$$\mathbf{r}_{AB} = \{(2.50 - 0)\mathbf{i} + (4.330 - 0)\mathbf{j} + [0 - (-10)]\mathbf{k}\} \text{ ft}$$

$$= \{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}\} \text{ ft}$$

$$r_{AB} = \sqrt{2.50^2 + 4.330^2 + 10.0^2} = 11.180 \text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}}{11.180}$$

$$= 0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}$$

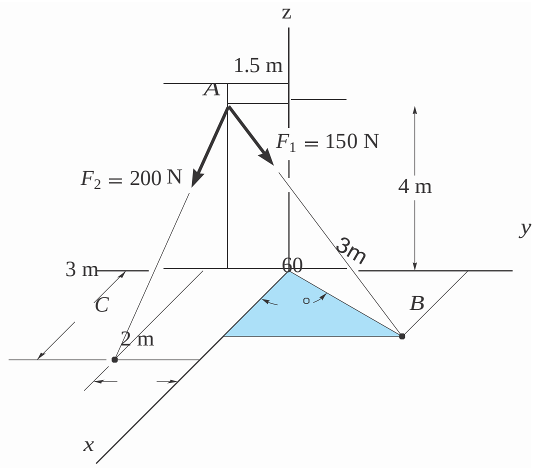
***Force Vector:***

$$\mathbf{F} = F\mathbf{u}_{AB} = 60\{0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}\} \text{ lb}$$

$$= \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\} \text{ lb}$$ **Ans.**

**2–99.**

Determine the magnitude and coordinate direction angles of the resultant force acting at point $A$.




## SOLUTION

$$\mathbf{r}_{AC} = \{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}\}\text{ m}$$

$$|\mathbf{r}_{AC}| = \sqrt{3^2 + (-0.5)^2 + (-4)^2} = \sqrt{25.25} = 5.02494$$

$$\mathbf{F}_2 = 200\left(\frac{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}}{5.02494}\right) = (119.4044\mathbf{i} - 19.9007\mathbf{j} - 159.2059\mathbf{k})$$

$$\mathbf{r}_{AB} = (3\cos 60°\mathbf{i} + (1.5 + 3\sin 60°)\mathbf{j} - 4\mathbf{k})$$

$$\mathbf{r}_{AB} = (1.5\mathbf{i} + 4.0981\mathbf{j} + 4\mathbf{k})$$

$$|\mathbf{r}_{AB}| = \sqrt{(1.5)^2 + (4.0981)^2 + (-4)^2} = 5.9198$$

$$\mathbf{F}_1 = 150\left(\frac{1.5\mathbf{i} + 4.0981\mathbf{j} - 4\mathbf{k}}{5.9198}\right) = (38.0079\mathbf{i} + 103.8396\mathbf{j} - 101.3545\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (157.4124\mathbf{i} + 83.9389\mathbf{j} - 260.5607\mathbf{k})$$

$$F_R = \sqrt{(157.4124)^2 + (83.9389)^2 + (-260.5604)^2} = 315.7786 = 316\text{ N} \qquad \textbf{Ans.}$$

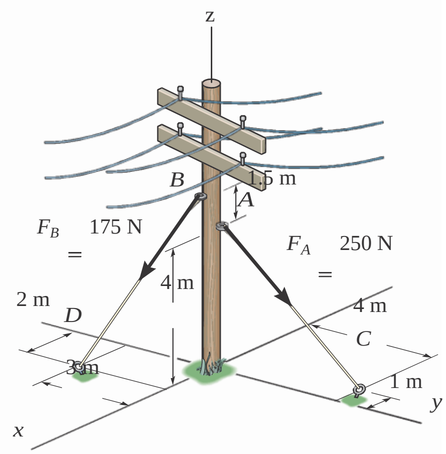
$$\alpha = \cos^{-1}\left(\frac{157.4124}{315.7786}\right) = 60.100° = 60.1° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{83.9389}{315.7786}\right) = 74.585° = 74.6° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{-260.5607}{315.7786}\right) = 145.60° = 146° \qquad \textbf{Ans.}$$

***2–100.**

The guy wires are used to support the telephone pole. Represent the force in each wire in Cartesian vector form. Neglect the diameter of the pole.




## SOLUTION

***Unit Vector:***

$$\mathbf{r}_{AC} = \{(-1 - 0)\mathbf{i} + (4 - 0)\mathbf{j} + (0 - 4)\mathbf{k}\}\text{ m} = \{-1\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}\}\text{ m}$$

$$r_{AC} = \sqrt{(-1)^2 + 4^2 + (-4)^2} = 5.745\text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-1\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}}{5.745} = -0.1741\mathbf{i} + 0.6963\mathbf{j} - 0.6963\mathbf{k}$$

$$\mathbf{r}_{BD} = \{(2 - 0)\,\mathbf{i} + (-3 - 0)\mathbf{j} + (0 - 5.5)\mathbf{k}\}\text{ m} = \{2\mathbf{i} - 3\mathbf{j} - 5.5\mathbf{k}\}\text{ m}$$

$$r_{BD} = \sqrt{2^2 + (-3)^2 + (-5.5)^2} = 6.576\text{ m}$$

$$\mathbf{u}_{BD} = \frac{\mathbf{r}_{BD}}{r_{BD}} = \frac{2\mathbf{i} - 3\mathbf{j} - 5.5\mathbf{k}}{6.576} = 0.3041\mathbf{i} - 0.4562\mathbf{j} - 0.8363\mathbf{k}$$

***Force Vector:***

$$\mathbf{F}_A = F_A\,\mathbf{u}_{AC} = 250\{-0.1741\mathbf{i} + 0.6963\mathbf{j} - 0.6963\mathbf{k}\}\text{ N}$$

$$= \{-43.52\mathbf{i} + 174.08\mathbf{j} - 174.08\mathbf{k}\}\text{ N}$$

$$= \{-43.5\mathbf{i} + 174\mathbf{j} - 174\mathbf{k}\}\text{ N}$$ **Ans.**

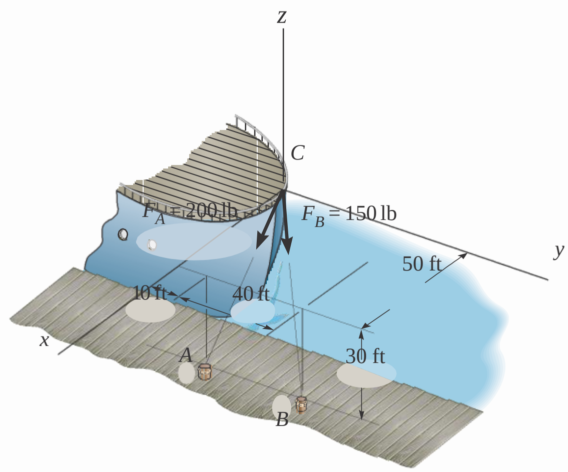
$$\mathbf{F}_B = F_B\mathbf{u}_{BD} = 175\{0.3041\mathbf{i} + 0.4562\mathbf{j} - 0.8363\mathbf{k}\}\text{ N}$$

$$= \{53.22\mathbf{i} - 79.83\mathbf{j} - 146.36\mathbf{k}\}\text{ N}$$

$$= \{53.2\mathbf{i} - 79.8\mathbf{j} - 146\mathbf{k}\}\text{ N}$$ **Ans.**

**2–101.**

The two mooring cables exert forces on the stern of a ship as shown. Represent each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant.




## SOLUTION

***Unit Vector:***

$\mathbf{r}_{CA}$ = 5150 – 02**i** + 110 – 02**j** + 1–30 – 02**k**6 ft = 550**i** + 10**j** – 30**k**6 ft

$r_{CA}$ = 250$^2$ + 10$^2$ + 1–302$^2$ = 59.16 ft

$\mathbf{u}_{CA} = \dfrac{\mathbf{r}_{CA}}{r_{CA}} = \dfrac{50\mathbf{i} + 10\mathbf{j} - 30\mathbf{k}}{59.16} = 0.8452\mathbf{i} + 0.1690\mathbf{j} - 0.5071\mathbf{k}$

$\mathbf{r}_{CB}$ = 5150 – 02**i** + 150 – 02**j** + 1–30 – 02**k**6 ft = 550**i** + 50**j** – 30**k**6 ft

$r_{CB}$ = 250$^2$ + 50$^2$ + 1–302$^2$ = 76.81 ft

$\mathbf{u}_{CB} = \dfrac{\mathbf{r}_{CA}}{r_{CA}} = \dfrac{50\mathbf{i} + 50\mathbf{j} - 30\mathbf{k}}{76.81} = 0.6509\mathbf{i} + 0.6509\mathbf{j} - 0.3906\mathbf{k}$

***Force Vector:***

$\mathbf{F}_A = F_A\,\mathbf{u}_{CA}$ = 20050.8452**i** + 0.1690**j** – 0.5071**k**6 lb

= 5169.03**i** + 33.81**j** – 101.42**k**6 lb

= 5169**i** + 33.8**j** – 101**k**6 lb **Ans.**

$\mathbf{F}_B = F_B\,\mathbf{u}_{CB}$ = 15050.6509**i** + 0.6509**j** – 0.3906**k**6 lb

= 597.64**i** + 97.64**j** – 58.59**k**6 lb

= 597.6**i** + 97.6**j** – 58.6**k**6 lb **Ans.**

***Resultant Force:***

$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B$

= 51169.03 + 97.642**i** + 133.81 + 97.642**j** + 1–101.42 – 58.592**k**6 lb

= 5266.67**i** + 131.45**j** – 160.00**k**6 lb

The magnitude of $\mathbf{F}_R$ is

$F_R$ = 2266.67$^2$ + 131.45$^2$ + 1–160.002$^2$

= 337.63 lb = 338 lb **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

cos a = $\dfrac{266.67}{337.63}$ a = 37.8°

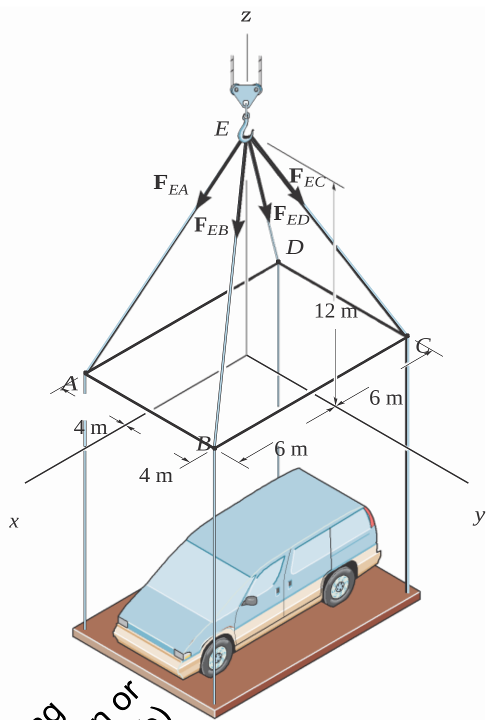
cos b = $\dfrac{131.45}{337.63}$ b = 67.1°

**Ans.** **Ans.**

$$\cos g = -\frac{160.00}{337.63} \qquad g = 118°$$ **Ans.**

**2–102.**

Each of the four forces acting at $E$ has a magnitude of 28 kN. Express each force as a Cartesian vector and determine the resultant force.




## SOLUTION

$$\mathbf{F}_{EA} = 28\left(\frac{6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{EA} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\text{ kN}$$ **Ans.**

$$\mathbf{F}_{EB} = 28\left(\frac{6}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{EB} = \{12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\}\text{ kN}$$ **Ans.**

$$\mathbf{F}_{EC} = 28\left(\frac{-6}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{EC} = \{-12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\}\text{ kN}$$ **Ans.**

$$\mathbf{F}_{ED} = 28\left(\frac{-6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

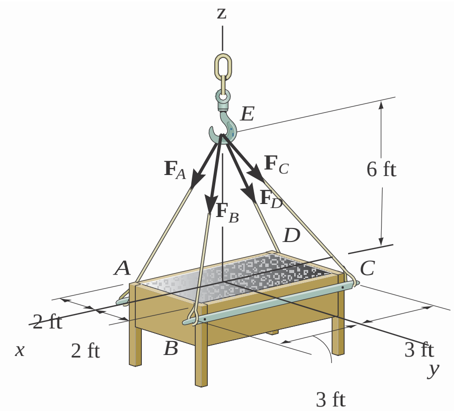
$$\mathbf{F}_{ED} = \{-12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\text{ kN}$$ **Ans.**

$$\mathbf{F}_R = \mathbf{F}_{EA} + \mathbf{F}_{EB} + \mathbf{F}_{EC} + \mathbf{F}_{ED}$$

$$= \{-96\mathbf{k}\}\text{ kN}$$ **Ans.**

**2–103.**

If the force in each cable tied to the bin is 70 lb, determine the magnitude and coordinate direction angles of the resultant force.




## SOLUTION

***Force Vectors:*** The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, $\mathbf{u}_C$, and $\mathbf{u}_D$ of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = 70\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}]\ \text{lb}$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = 70\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}]\ \text{lb}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 70\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [-30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}]\ \text{lb}$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = 70\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [-30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}]\ \text{lb}$$

***Resultant Force:***

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = (30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}) + (30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}) + (-30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}) + (-30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k})$$
$$= \{-240\mathbf{k}\}\ \text{N}$$

The magnitude of $\mathbf{F}_R$ is

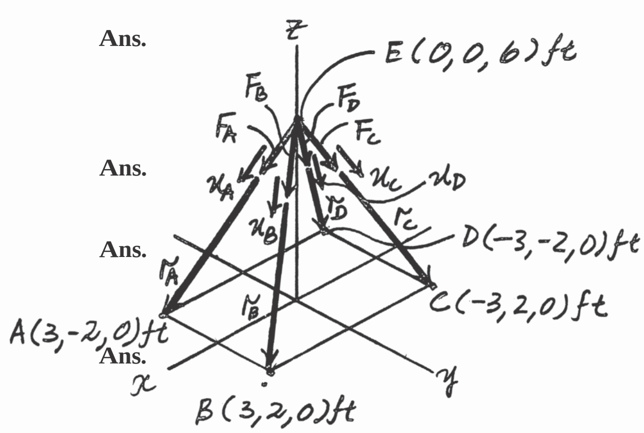
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{0 + 0 + (-240)^2} = 240\ \text{lb}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{0}{240}\right) = 90^\circ$$ **Ans.**

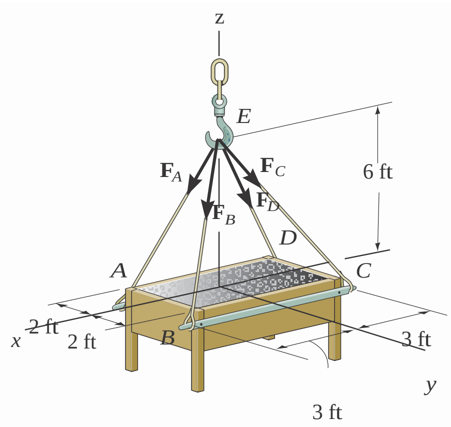
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{240}\right) = 90^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-240}{240}\right) = 180^\circ$$ **Ans.**



***2–104.**

If the resultant of the four forces is $\mathbf{F}_R = \{-360\mathbf{k}\}$ lb, determine the tension developed in each cable. Due to symmetry, the tension in the four cables is the same.




## SOLUTION

***Force Vectors:*** The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, $\mathbf{u}_C$, and $\mathbf{u}_D$ of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Since the magnitudes of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ are the same and denoted as $F$, the four vectors or forces can be written as

$$\mathbf{F}_A = F_A\mathbf{u}_A = F\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = F\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = F\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = F\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

***Resultant Force:*** The vector addition of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ is equal to $\mathbf{F}_R$. Thus,

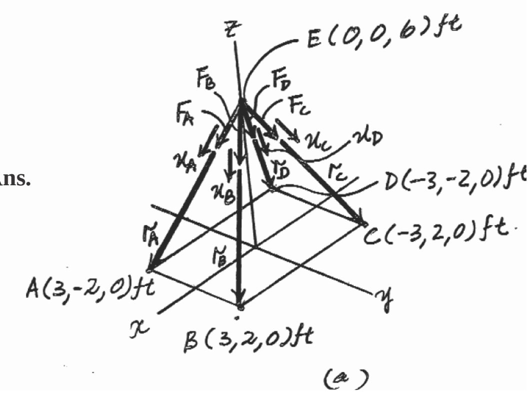
$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D$$

$$\{-360\mathbf{k}\} = \left[F\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right]$$

$$-360\mathbf{k} = -\frac{24}{7}F\mathbf{k}$$

Thus,

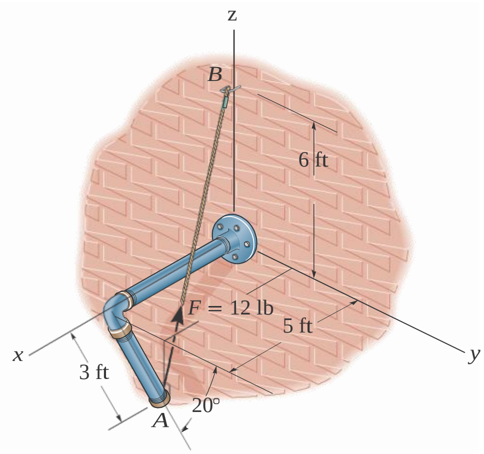
$$360 = \frac{24}{7}F \qquad F = 105\text{ lb} \qquad \textbf{Ans.}$$



(a)

**2–105.**

The pipe is supported at its end by a cord $AB$. If the cord exerts a force of $\mathbf{F} = 12$ lb on the pipe at $A$, express this force as a Cartesian vector.




## SOLUTION

***Unit Vector:*** The coordinates of point $A$ are

$$A(5,\ 3\cos 20°,\ -3\sin 20°)\text{ ft} = A(5.00,\ 2.819,\ -1.206)\text{ ft}$$

Then

$$\mathbf{r}_{AB} = \{(0 - 5.00)\mathbf{i} + (0 - 2.819)\mathbf{j} + [6-(-1.206)]\mathbf{k}\}\text{ ft}$$

$$= \{-5.00\mathbf{i} - 2.819\mathbf{j} + 7.026\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{(-5.00)^2 + (-2.819)^2 + 7.026^2} = 9.073\text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-5.00\mathbf{i} - 2.819\mathbf{j} + 7.026\mathbf{k}}{9.073}$$

$$= -0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}$$

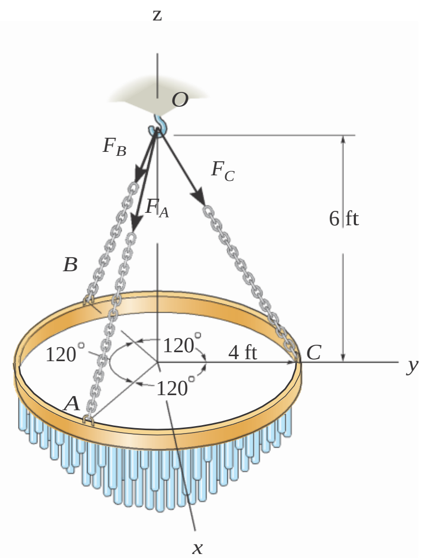
***Force Vector:***

$$\mathbf{F} = F\mathbf{u}_{AB} = 12\{-0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}\}\text{ lb}$$

$$= \{-6.61\mathbf{i} - 3.73\mathbf{j} + 9.29\mathbf{k}\}\text{ lb}$$ **Ans.**

**2–106.**

The chandelier is supported by three chains which are concurrent at point $O$. If the force in each chain has a magnitude of 60 lb, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.




## SOLUTION

$$\mathbf{F}_A = 60\,\frac{(4\cos 30^\circ\,\mathbf{i} - 4\sin 30^\circ\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(4\cos 30^\circ)^2 + (-4\sin 30^\circ)^2 + (-6)^2}}$$

$$= \{28.8\,\mathbf{i} - 16.6\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_B = 60\,\frac{(-4\cos 30^\circ\,\mathbf{i} - 4\sin 30^\circ\,\mathbf{j} - 6\mathbf{k})}{\sqrt{(-4\cos 30^\circ)^2 + (-4\sin 30^\circ)^2 + (-6)^2}}$$

$$= \{-28.8\,\mathbf{i} - 16.6\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_C = 60\,\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(4)^2 + (-6)^2}}$$

$$= \{33.3\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \{-149.8\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$F_R = 150\text{ lb}$$ **Ans.**

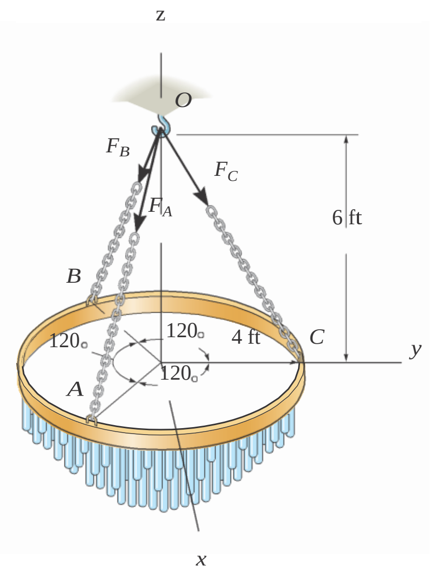
$$\alpha = 90^\circ$$ **Ans.**

$$\beta = 90^\circ$$ **Ans.**

$$\gamma = 180^\circ$$ **Ans.**

**2–107.**

The chandelier is supported by three chains which are concurrent at point *O*. If the resultant force at *O* has a magnitude of 130 lb and is directed along the negative *z* axis, determine the force in each chain.




## SOLUTION

$$\mathbf{F}_C = F\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{4^2 + (-6)^2}} = 0.5547\,F\mathbf{j} - 0.8321\,F\mathbf{k}$$

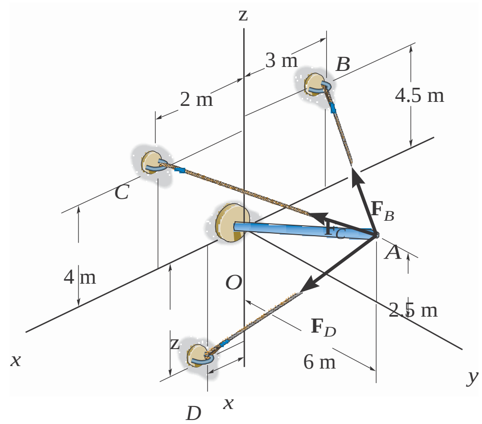
$$\mathbf{F}_A = \mathbf{F}_B = \mathbf{F}_C$$

$$F_{Rz} = \Sigma F_z; \qquad 130 = 3(0.8321F)$$

$$F = 52.1\text{ lb}$$ **Ans.**

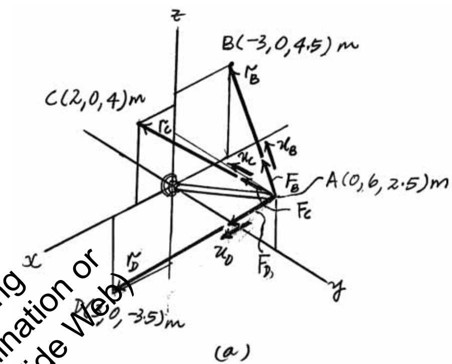
***2–108.**

Determine the magnitude and coordinate direction angles of the resultant force. Set $F_B = 630$ N, $F_C = 520$ N and $F_D = 750$ N, and $x = 3$ m and $z = 3.5$ m.




## SOLUTION

***Force Vectors:*** The unit vectors $\mathbf{u}_B$, $\mathbf{u}_C$, and $\mathbf{u}_D$ of $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ must be determined first. From Fig. *a*,




$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-3-0)\mathbf{i} + (0-6)\mathbf{j} + (4.5-2.5)\mathbf{k}}{\sqrt{(-3-0)^2 + (0-6)^2 + (4.5-2.5)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(2-0)\mathbf{i} + (0-6)\mathbf{j} + (4-2.5)\mathbf{k}}{\sqrt{(2-0)^2 + (0-6)^2 + (4-2.5)^2}} = \frac{4}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(3-0)\mathbf{i} + (0-6)\mathbf{j} + (-3.5-2.5)\mathbf{k}}{\sqrt{(0-3)^2 + (0-6)^2 + (-3.5-2.5)^2}} = \frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 630\left(-\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) = \{-270\mathbf{i} - 540\mathbf{j} + 180\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 520\left(\frac{4}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}\right) = \{160\mathbf{i} - 480\mathbf{j} + 120\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = 750\left(\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) = \{250\mathbf{i} - 500\mathbf{j} - 500\mathbf{k}\}\text{ N}$$

***Resultant Force:***

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = (-270\mathbf{i} - 540\mathbf{j} + 180\mathbf{k}) + (160\mathbf{i} - 480\mathbf{j} + 120\mathbf{k}) + (250\mathbf{i} - 500\mathbf{j} - 500\mathbf{k})$$
$$= [140\mathbf{i} - 1520\mathbf{j} - 200\mathbf{k}]\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{140^2 + (-1520)^2 + (-200)^2} = 1539.48\text{ N} = 1.54\text{ kN} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

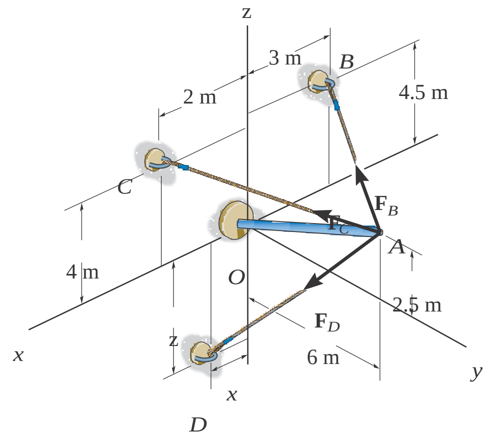
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{140}{1539.48}\right) = 84.8° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-1520}{1539.48}\right) = 171° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-200}{1539.48}\right) = 97.5° \qquad \textbf{Ans.}$$

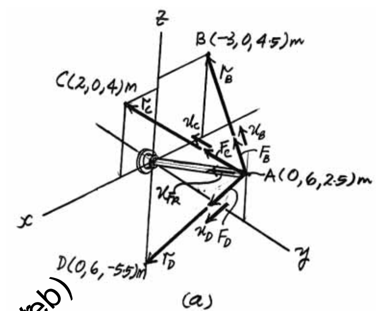
**2–109.**

If the magnitude of the resultant force is 1300 N and acts along the axis of the strut, directed from point A towards O, determine the magnitudes of the three forces acting on the strut. Set $x = 0$ and $z = 5.5$ m.




## SOLUTION

***Force Vectors:*** The unit vectors $\mathbf{u}_B$, $\mathbf{u}_C$, $\mathbf{u}_D$, and $\mathbf{u}_{F_R}$ of $\mathbf{F}_B$, $\mathbf{F}_C$, $\mathbf{F}_D$, and $\mathbf{F}_R$ must be determined first. From Fig. *a*,




$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-3-0)\mathbf{i} + (0-6)\mathbf{j} + (4.5-2.5)\mathbf{k}}{\sqrt{(-3-0)^2 + (0-6)^2 + (4.5-2.5)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(2-0)\mathbf{i} + (0-6)\mathbf{j} + (4-2.5)\mathbf{k}}{\sqrt{(2-0)^2 + (0-6)^2 + (4-2.5)^2}} = \frac{4}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(0-0)\mathbf{i} + (0-6)\mathbf{j} + (-5.5-2.5)\mathbf{k}}{\sqrt{(0-0)^2 + (0-6)^2 + (-5.5-2.5)^2}} = -\frac{3}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}$$

$$\mathbf{u}_{F_R} = \frac{\mathbf{r}_{AO}}{r_{AO}} = \frac{(0-0)\mathbf{i} + (0-6)\mathbf{j} + (0-2.5)\mathbf{k}}{\sqrt{(0-0)^2 + (0-6)^2 + (0-2.5)^2}} = -\frac{12}{13}\mathbf{j} + \frac{5}{13}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$, $\mathbf{F}_C$, $\mathbf{F}_D$, and $\mathbf{F}_R$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = -\frac{3}{7}F_B\mathbf{i} - \frac{6}{7}F_B\mathbf{j} + \frac{2}{7}F_B\mathbf{k}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = \frac{4}{13}F_C\mathbf{i} - \frac{12}{13}F_C\mathbf{j} + \frac{3}{13}F_C\mathbf{k}$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = -\frac{3}{5}F_D\mathbf{j} - \frac{4}{5}F_D\mathbf{k}$$

$$\mathbf{F}_R = F_R\mathbf{u}_R = 1300\left(-\frac{12}{13}\mathbf{j} - \frac{5}{13}\mathbf{k}\right) = [-1200\mathbf{j} - 500\mathbf{k}]\text{ N}$$



***Resultant Force:***

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D$$

$$-1200\mathbf{j} - 500\mathbf{k} = \left(-\frac{3}{7}F_B\mathbf{i} - \frac{6}{7}F_B\mathbf{j} + \frac{2}{7}F_B\mathbf{k}\right) + \left(\frac{4}{13}F_C\mathbf{i} - \frac{12}{13}F_C\mathbf{j} + \frac{3}{13}F_C\mathbf{k}\right) + \left(-\frac{3}{5}F_D\mathbf{j} - \frac{4}{5}F_D\mathbf{k}\right)$$

$$-1200\mathbf{j} - 500\mathbf{k} = \left(-\frac{3}{7}F_B + \frac{4}{13}F_C\right)\mathbf{i} + \left(-\frac{6}{7}F_B - \frac{12}{13}F_C - \frac{3}{5}F_D\right)\mathbf{j} + \left(\frac{2}{7}F_B + \frac{3}{13}F_C - \frac{4}{5}F_D\right)\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

$$0 = -\frac{3}{7}F_B + \frac{4}{13}F_C \qquad (1)$$

6 12 3

$$-1200 = -\frac{}{7}F_B - \frac{}{13}F_C - \frac{}{5}F_D\ \mathbf{j} \qquad (2)$$

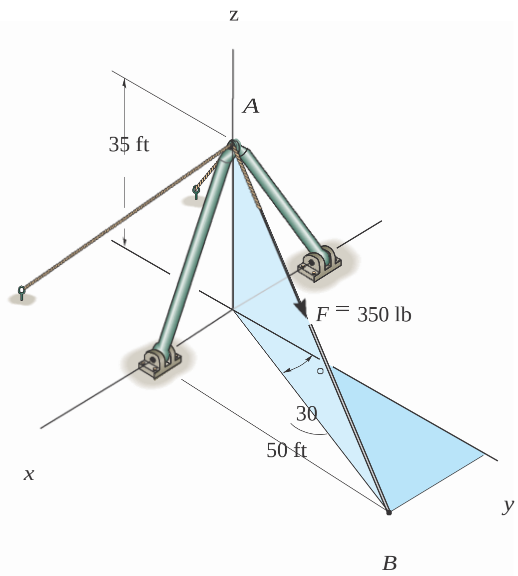
$$-500 = \frac{2}{7}F_B + \frac{3}{13}F_C - \frac{4}{5}F_D \qquad (3)$$

Solving Eqs. (1), (2), and (3), yields

$F_C = 442$ N $\qquad F_B = 318$ N $\qquad F_D = 866$ N **Ans.**

**2–110.**

The cable attached to the shear-leg derrick exerts a force on the derrick of F = 350 lb. Express this force as a Cartesian vector.




## SOLUTION

***Unit Vector:*** The coordinates of point $B$ are

$$B(50\sin 30°, 50\cos 30°, 0)\text{ ft} = B(25.0, 43.301, 0)\text{ ft}$$

Then

$$\mathbf{r}_{AB} = \{(25.0 - 0)\mathbf{i} + (43.301 - 0)\mathbf{j} + (0 - 35)\mathbf{k}\}\text{ ft}$$

$$= \{25.0\mathbf{i} + 43.301\mathbf{j} - 35.0\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{25.0^2 + 43.301^2 + (-35.0)^2} = 61.033\text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{25.0\mathbf{i} + 43.301\mathbf{j} - 35.0\mathbf{k}}{61.033}$$

$$= 0.4096\mathbf{i} + 0.7094\mathbf{j} - 0.5735\mathbf{k}$$

***Force Vector:***

$$\mathbf{F} = F\mathbf{u}_{AB} = 350\{0.4096\mathbf{i} + 0.7094\mathbf{j} - 0.5735\mathbf{k}\}\text{ lb}$$

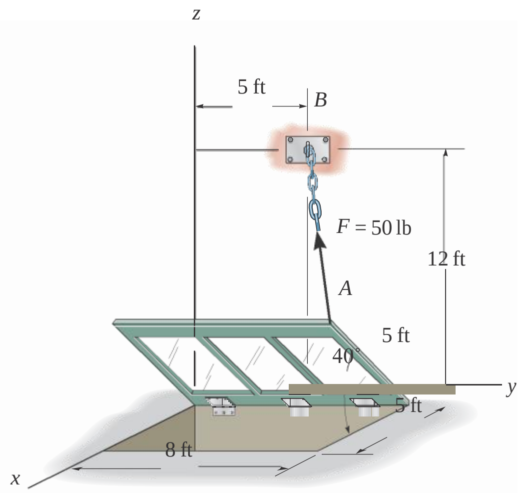
$$= \{143\mathbf{i} + 248\mathbf{j} - 201\mathbf{k}\}\text{ lb}$$

**Ans.**

**2–111.**

The window is held open by chain *AB*. Determine the length of the chain, and express the 50-lb force acting at *A*

coordinate direction angles.




## SOLUTION

***Unit Vector:*** The coordinates of point $A$ are

$$A15 \cos 40°, 8, 5 \sin 40°2 \text{ ft} = A13.830, 8.00, 3.2142 \text{ ft}$$

Then

$$\mathbf{r}_{AB} = 510 - 3.8302\mathbf{i} + 15 - 8.002\mathbf{j} + 112 - 3.2142\mathbf{k}6 \text{ ft}$$

$$= 5-3.830\mathbf{i} - 3.00\mathbf{j} + 8.786\mathbf{k}6 \text{ ft}$$

$$r_{AB} = 21-3.8302^2 + 1-3.002^2 + 8.786^2 = 10.043 \text{ ft} = 10.0 \text{ ft}$$ **Ans.**

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-3.830\mathbf{i} - 3.00\mathbf{j} + 8.786\mathbf{k}}{10.043}$$

$$= -0.3814\mathbf{i} - 0.2987\mathbf{j} + 0.8748\mathbf{k}$$

***Force Vector:***

$$\mathbf{F} = F\mathbf{u}_{AB} = 505-0.3814\mathbf{i} - 0.2987\mathbf{j} + 0.8748\mathbf{k}6 \text{ lb}$$

$$= 5-19.1\mathbf{i} - 14.9\mathbf{j} + 43.7\mathbf{k}6 \text{ lb}$$ **Ans.**

***Coordinate Direction Angles:*** From the unit vector $\mathbf{u}_{AB}$ obtained above, we have

$\cos a = -0.3814$ $\quad a = 112°$ **Ans.**

$\cos b = -0.2987$ $\quad b = 107°$ **Ans.**

$\cos g = 0.8748$ $\quad g = 29.0°$ **Ans.**

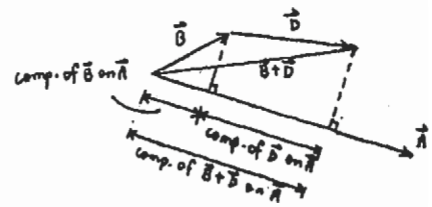
***2–112.**

Given the three vectors **A**, **B**, and **D**, show that $\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$.

# SOLUTION

Since the component of $(\mathbf{B} + \mathbf{D})$ is equal to the sum of the components of **B** and **D**, then

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{D} \qquad \textbf{(QED)}$$




Also,

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\mathbf{k}) \cdot [(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}]$$

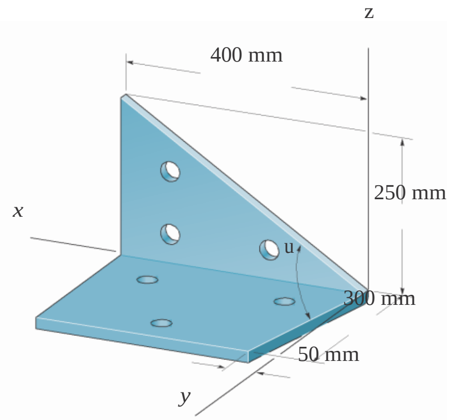
$$= A_x (B_x + D_x) + A_y (B_y + D_y) + A_z (B_z + D_z)$$

$$= (A_xB_x + A_yB_y + A_zB_z) + (A_xD_x + A_yD_y + A_zD_z)$$

$$= (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D}) \qquad \textbf{(QED)}$$

**2–113.**

Determine the angle $\theta$ between the edges of the sheet-metal bracket.




## SOLUTION

$\mathbf{r}_1 = \{400\mathbf{i} + 250\mathbf{k}\}$ mm ; $\qquad r_1 = 471.70$ mm

$\mathbf{r}_2 = \{50\mathbf{i} + 300\mathbf{j}\}$ mm ; $\qquad r_2 = 304.14$ mm

$\mathbf{r}_1 \cdot \mathbf{r}_2 = (400)(50) + 0(300) + 250(0) = 20\,000$

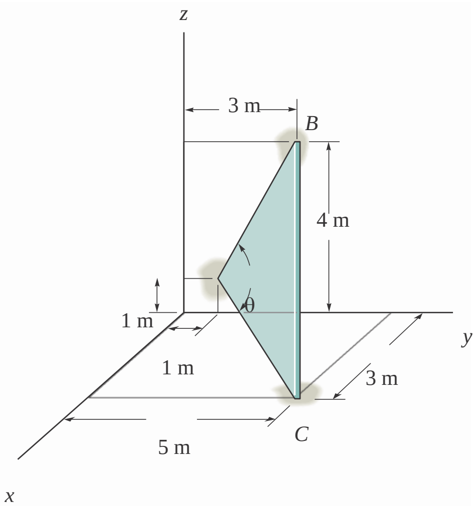
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{r_1 r_2}\right)$$

$$= \cos^{-1}\left(\frac{20\,000}{(471.70)(304.14)}\right) = 82.0^\circ$$

**Ans.**

**2–114.**

Determine the angle u between the sides of the triangular plate.




## SOLUTION

*A*

$\mathbf{r}_{AC} = 53\,\mathbf{i} + 4\,\mathbf{j} - 1\,\mathbf{k}6 \text{ m}$

$r_{AC} = 2 13 2^2 + 14 2^2 + 1-12^2 = 5.0990 \text{ m}$

$\mathbf{r}_{AB} = 52\,\mathbf{j} + 3\,\mathbf{k}6 \text{ m}$

$r_{AB} = 2 12 2^2 + 13 2^2 = 3.6056 \text{ m}$

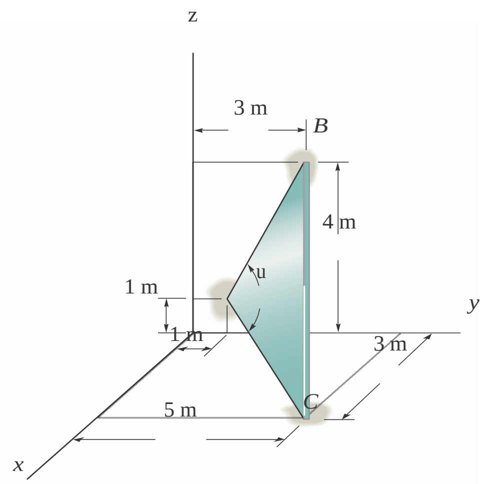
$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 0 + 4122 + 1-12132 = 5$

$$u = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC} r_{AB}}\right) = \cos^{-1}\frac{5}{15.0990213.60562}$$

$u = 74.219° = 74.2°$ **Ans.**

**2–115.**

Determine the length of side $BC$ of the triangular plate. Solve the problem by finding the magnitude of $\mathbf{r}_{BC}$; then check the result by first finding $\theta$, $r_{AB}$, and $r_{AC}$ and then using the cosine law.




## SOLUTION

$$\mathbf{r}_{BC} = \{3\,\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k}\}\text{ m}$$

$$r_{BC} = \sqrt{(3)^2 + (2)^2 + (-4)^2} = 5.39\text{ m}$$ **Ans.**

Also,

$$\mathbf{r}_{AC} = \{3\,\mathbf{i} + 4\,\mathbf{j} - 1\,\mathbf{k}\}\text{ m}$$

$$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990\text{ m}$$

$$\mathbf{r}_{AB} = \{2\,\mathbf{j} + 3\,\mathbf{k}\}\text{ m}$$

$$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056\text{ m}$$

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC}\, r_{AB}}\right) = \cos^{-1}\frac{5}{(5.0990)(3.6056)}$$

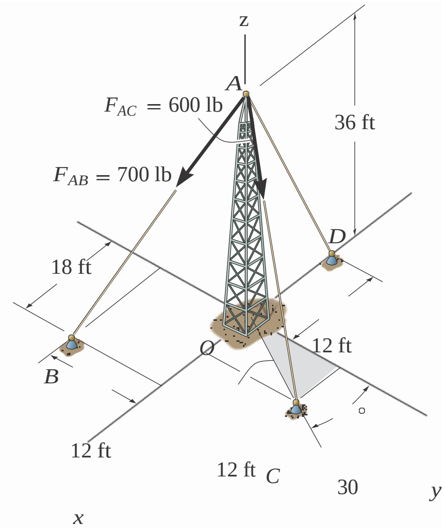
$$\theta = 74.219°$$

$$r_{BC} = \sqrt{(5.0990)^2 + (3.6056)^2 - 2(5.0990)(3.6056)\cos 74.219°}$$

$$r_{BC} = 5.39\text{ m}$$ **Ans.**

***2–116.**

Determine the magnitude of the projected component of force $\mathbf{F}_{AB}$ acting along the $z$ axis.




# SOLUTION

***Unit Vector:*** The unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(18-0)\mathbf{i} + (-12-0)\mathbf{j} + (0-36)\mathbf{k}}{\sqrt{(18-0)^2 + (-12-0)^2 + (0-36)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AB}$ is given by

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 700\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{300\mathbf{i} - 200\mathbf{j} - 600\mathbf{k}\}\text{ lb}$$

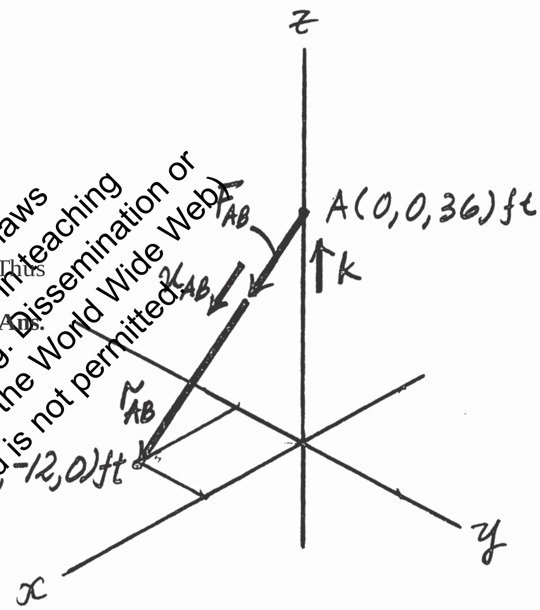
***Vector Dot Product:*** The projected component of $\mathbf{F}_{AB}$ along the $z$ axis is

$$(F_{AB})_z = \mathbf{F}_{AB}\cdot\mathbf{k} = (300\mathbf{i} - 200\mathbf{j} - 600\mathbf{k})\cdot\mathbf{k}$$

$$= -600\text{ lb}$$

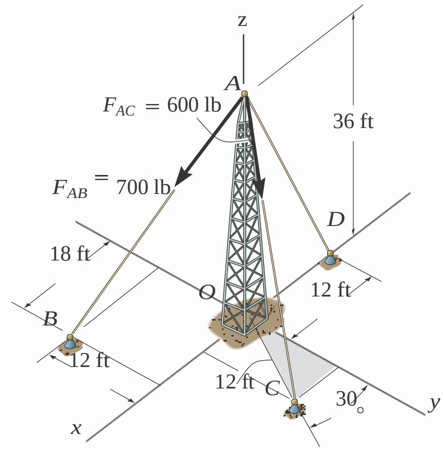
The negative sign indicates that $(\mathbf{F}_{AB})_z$ is directed towards the negative $z$ axis. Thus

$$(F_{AB})_z = 600\text{ lb} \qquad \textbf{Ans.}$$



***2–117.**

Determine the magnitude of the projected component of force $\mathbf{F}_{AC}$ acting along the $z$ axis.




## SOLUTION

***Unit Vector:*** The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(12\sin 30^\circ - 0)\mathbf{i} + (12\cos 30^\circ - 0)\mathbf{j} + (0 - 36)\mathbf{k}}{\sqrt{(12\sin 30^\circ - 0)^2 + (12\cos 30^\circ - 0)^2 + (0 - 36)^2}} = 0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AC}$ is given by

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 600(0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}) = \{94.87\mathbf{i} + 164.32\mathbf{j} - 569.21\mathbf{k}\}\text{ N}$$

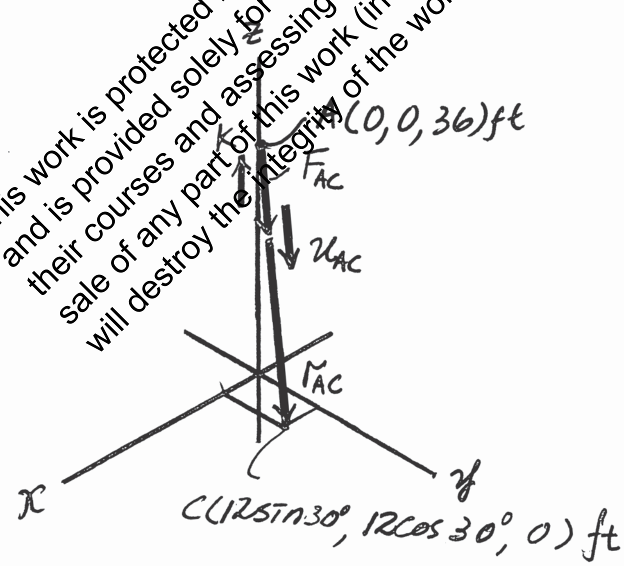
***Vector Dot Product:*** The projected component of $\mathbf{F}_{AC}$ along the $z$ axis is

$$(F_{AC})_z = \mathbf{F}_{AC} \cdot \mathbf{k} = (94.87\mathbf{i} + 164.32\mathbf{j} - 569.21\mathbf{k}) \cdot \mathbf{k}$$

$$= -569 \text{ lb}$$

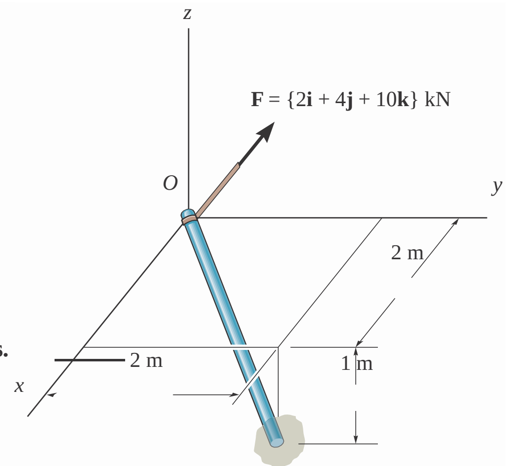
The negative sign indicates that $(\mathbf{F}_{AC})_z$ is directed towards the negative $z$ axis. Thus

$$(F_{AC})_z = 569 \text{ lb} \qquad \textbf{Ans.}$$



**2–118.**

Determine the projection of the force **F** along the pole.



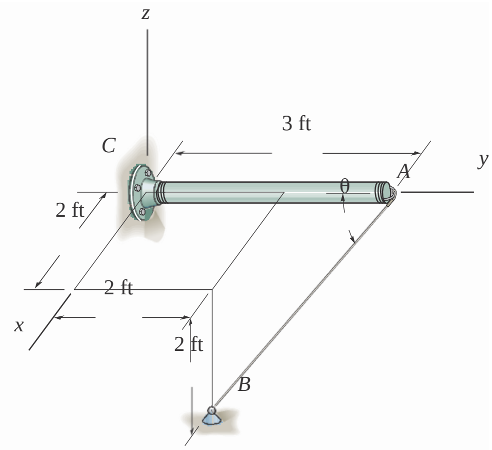

## SOLUTION

$$\text{Proj } F = \mathbf{F} \cdot \mathbf{u}_a = (2\,\mathbf{i} + 4\,\mathbf{j} + 10\,\mathbf{k}) \cdot \left(\frac{2}{3}\,\mathbf{i} + \frac{2}{3}\,\mathbf{j} - \frac{1}{3}\,\mathbf{k}\right)$$

$$\text{Proj } F = 0.667 \text{ kN}$$ **Ans.**

**2–119.**

Determine the angle u between the $y$ axis of the pole and the wire $AB$.




## SOLUTION

***Position Vector:***

$$\mathbf{r}_{AC} = 5-3\mathbf{j}6 \text{ ft}$$

$$\mathbf{r}_{AB} = 512 - 02\mathbf{i} + 12 - 32\mathbf{j} + 1-2 - 02\mathbf{k}6 \text{ ft}$$

$$= 52\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}6 \text{ ft}$$

The magnitudes of the position vectors are

$$r_{AC} = 3.00 \text{ ft} \qquad r_{AB} = 2\overline{2^2 + 1-12^2 + 1-22^2} = 3.00 \text{ ft}$$

***The Angles Between Two Vectors*** U***:*** The dot product of two vectors must be determined first.

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 1-3\mathbf{j}2 \cdot 12\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}2$$

$$= 0122 + 1-321-12 + 01-22$$

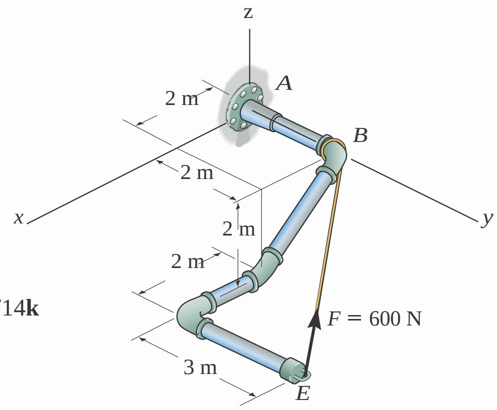
$$= 3$$

Then,

$$u = \cos^{-1}\left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AB}}{r_{AO} r_{AB}}\right) = \cos^{-1}\left[\frac{3}{3.00(3.00)}\right] = 70.5°$$ **Ans.**

***2–120.**

Determine the magnitudes of the components of $F = 600\text{ N}$ acting along and perpendicular to segment $DE$ of the pipe assembly.




## SOLUTION

***Unit Vectors:*** The unit vectors $\mathbf{u}_{EB}$ and $\mathbf{u}_{ED}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{EB} = \frac{\mathbf{r}_{EB}}{r_{EB}} = \frac{(0-4)\mathbf{i} + (2-5)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-4)^2 + (2-5)^2 + [0-(-2)]^2}} = -0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}$$

$$\mathbf{u}_{ED} = -\mathbf{j}$$

Thus, the force vector $\mathbf{F}$ is given by

$$\mathbf{F} = F\mathbf{u}_{EB} = 600(-0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}) = [-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}]\text{ N}$$

***Vector Dot Product:*** The magnitude of the component of $\mathbf{F}$ parallel to segment $DE$ of the pipe assembly is

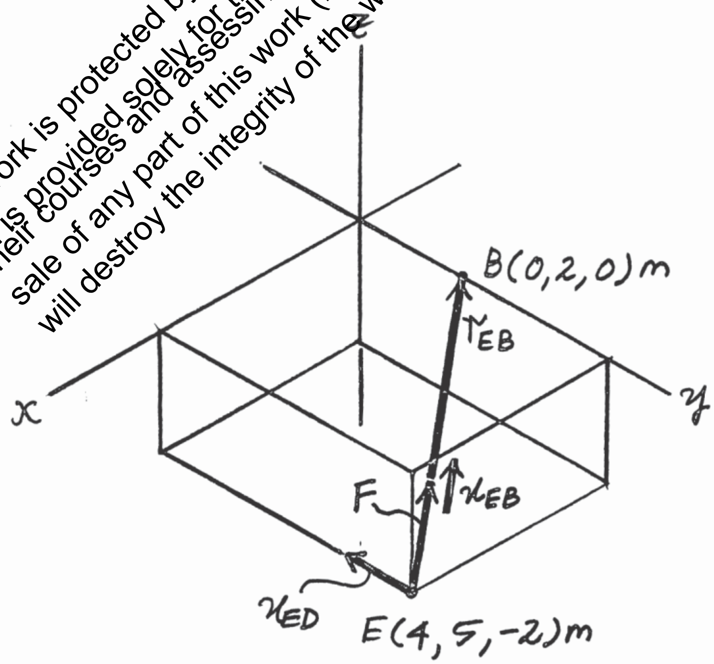
$$(F_{ED})_{paral} = \mathbf{F} \cdot \mathbf{u}_{ED} = (-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}) \cdot (-\mathbf{j})$$

$$= (-445.66)(0) + (-334.25)(-1) + (222.83)(0)$$

$$= 334.25 = 334\text{ N} \quad \textbf{Ans.}$$

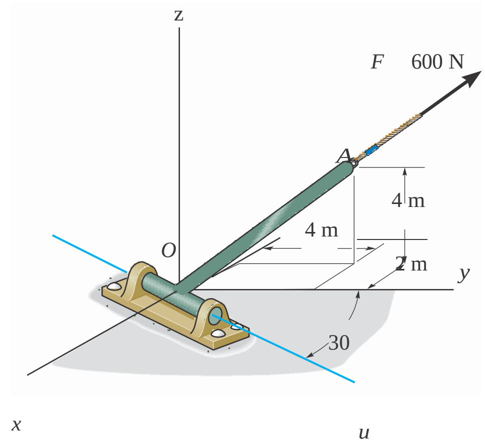
The component of $\mathbf{F}$ perpendicular to segment $DE$ of the pipe assembly is

$$(F_{ED})_{per} = \sqrt{F^2 - (F_{ED})_{paral}^2} = \sqrt{600^2 - 334.25^2} = 498\text{ N} \quad \textbf{Ans.}$$



**2–121.**

Determine the magnitude of the projection of force $F = 600$ N along the u axis.




## SOLUTION

***Unit Vectors:*** The unit vectors $\mathbf{u}_{OA}$ and $\mathbf{u}_u$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-2-0)\mathbf{i} + (4-0)\mathbf{j} + (4-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (4-0)^2 + (4-0)^2}} = -\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_u = \sin 30°\mathbf{i} + \cos 30°\mathbf{j}$$

Thus, the force vectors $\mathbf{F}$ is given by

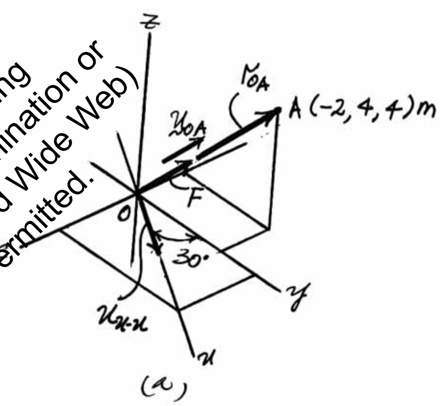
$$\mathbf{F} = F\mathbf{u}_{OA} = 600\left(-\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}\}\text{ N}$$

***Vector Dot Product:*** The magnitude of the projected component of $\mathbf{F}$ along the u axis is

$$F_u = \mathbf{F} \cdot \mathbf{u}_u = (-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}) \cdot (\sin 30°\mathbf{i} + \cos 30°\mathbf{j})$$

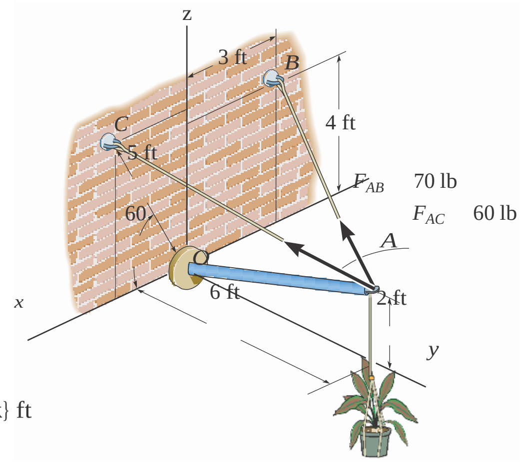
$$= (-200)(\sin 30°) + 400(\cos 30°) + 400(0)$$

$$= 246\text{ N}$$ **Ans.**



**2–122.**

Determine the angle $\theta$ between cables AB and AC.




## SOLUTION

***Position Vectors:*** The position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$ must be determined first. From Fig. *a*,

$$\mathbf{r}_{AB} = (-3 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (4 - 2)\mathbf{k} = \{-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AC} = (5\cos 60° - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (5\sin 60° - 2)\mathbf{k} = \{2.5\mathbf{i} - 6\mathbf{j} + 2.330\mathbf{k}\}\text{ ft}$$

The magnitudes of $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$ are

$$r_{AB} = \sqrt{(-3)^2 + (-6)^2} = 7\text{ ft}$$

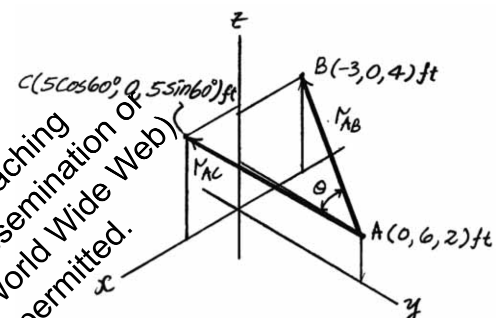
$$r_{AC} = \sqrt{2.5^2 + (-6)^2 + 2.330^2} = 6.905\text{ ft}$$

***Vector Dot Product:***

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AC} = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}) \cdot (2.5\mathbf{i} - 6\mathbf{j} + 2.330\mathbf{k})$$

$$= (-3)(2.5) + (-6)(-6) + (2)(2.330)$$

$$= 33.160\text{ ft}^2$$



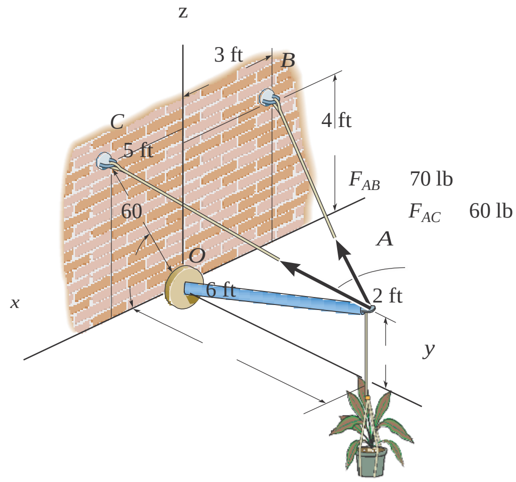
(a)

Thus,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AC}}{r_{AB}\, r_{AC}}\right) = \cos^{-1}\left[\frac{33.160}{7(6.905)}\right] = 46.7° \qquad \textbf{Ans.}$$

**2–123.**

Determine the angle $\phi$ between cable $AC$ and strut $AO$.

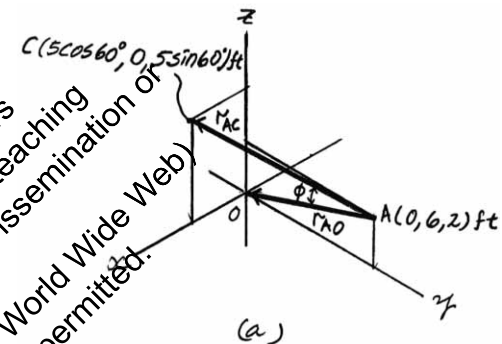



## SOLUTION

***Position Vectors:*** The position vectors $\mathbf{r}_{AC}$ and $\mathbf{r}_{AO}$ must be determined first. From Fig. *a*,

$$\mathbf{r}_{AC} = (5\cos 60^\circ - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (5\sin 60^\circ - 2)\mathbf{k} = \{2.5\mathbf{i} - 6\mathbf{j} + 2.330\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AO} = (0 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (0 - 2)\mathbf{k} = \{-6\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$




The magnitudes of $\mathbf{r}_{AC}$ and $\mathbf{r}_{AO}$ are

$$r_{AC} = \sqrt{2.5^2 + (-6)^2 + 2.330^2} = 6.905\text{ ft}$$

$$r_{AO} = \sqrt{(-6)^2 + (-2)^2} = \sqrt{40}\text{ ft}$$

***Vector Dot Product:***

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (2.5\mathbf{i} - 6\mathbf{j} + 2.330\mathbf{k}) \cdot (-6\mathbf{j} - 2\mathbf{k})$$

$$= (2.5)(0) + (-6)(-6) + (2)(2.330)(-2)$$

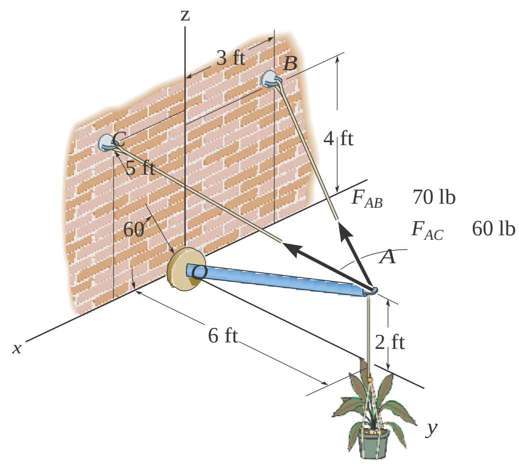
$$= 31.34\text{ ft}^2$$

Thus,

$$\phi = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC}\, r_{AO}}\right) = \cos^{-1}\left(\frac{31.34}{6.905\sqrt{40}}\right) = 44.1^\circ \quad \textbf{Ans.}$$

***2–124.**

Determine the projected component of force $\mathbf{F}_{AB}$ along the axis of strut AO. Express the result as a Cartesian vector.




## SOLUTION

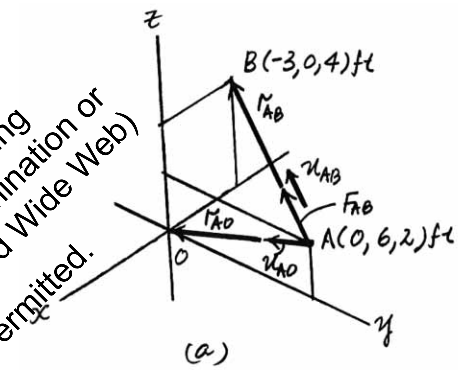
***Unit Vectors:*** The unit vectors $\mathbf{u}_{AB}$ and $\mathbf{u}_{AO}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{(-3-0)\mathbf{i} + (0-6)\mathbf{j} + (4-2)\mathbf{k}}{\sqrt{(-3-0)^2 + (0-6)^2 + (4-2)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_{AO} = \frac{(0-0)\mathbf{i} + (0-6)\mathbf{j} + (0-2)\mathbf{k}}{\sqrt{(0-0)^2 + (0-6)^2 + (0-2)^2}} = -0.9487\mathbf{j} - 0.3162\,\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_{AB}$ is

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 70\left(-\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) = \{-30\mathbf{i} - 60\mathbf{j} + 20\mathbf{k}\}\text{ lb}$$




***Vector Dot Product:*** The magnitude of the projected component of $\mathbf{F}_{AB}$ along strut AO is

$$(F_{AB})_{AO} = \mathbf{F}_{AB} \cdot \mathbf{u}_{AO} = (-30\mathbf{i} - 60\mathbf{j} + 20\mathbf{k}) \cdot (-0.9487\mathbf{j} - 0.3162\mathbf{k})$$

$$= (-30)(0) + (-60)(-0.9487) + (20)(-0.3162)$$

$$= 50.596\text{ lb}$$

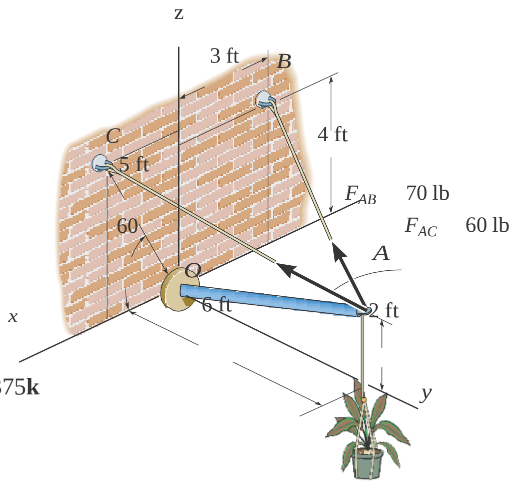
Thus, $(\mathbf{F}_{AB})_{AO}$ expressed in Cartesian vector form can be written as

$$(\mathbf{F}_{AB})_{AO} = (F_{AB})_{AO}\mathbf{u}_{AO} = 50.596(-0.9487\mathbf{j} - 0.3162\mathbf{k})$$

$$= \{-48\mathbf{j} - 16\mathbf{k}\}\text{ lb}$$ **Ans.**

**2–125.**

Determine the projected component of force $\mathbf{F}_{AC}$ along the axis of strut AO. Express the result as a Cartesian vector.




## SOLUTION

***Unit Vectors:*** The unit vectors $\mathbf{u}_{AC}$ and $\mathbf{u}_{AO}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AC} = \frac{(5\cos 60^\circ - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (5\sin 60^\circ - 2)\mathbf{k}}{\sqrt{(5\cos 60^\circ - 0)^2 + (0 - 6)^2 + (0 - 2)^2}} = 0.3621\mathbf{i} - 0.8689\mathbf{j} + 0.3375\mathbf{k}$$

$$\mathbf{u}_{AO} = \frac{(0 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (0 - 2)\mathbf{k}}{\sqrt{(0 - 0)^2 + (0 - 6)^2 + (0 - 2)^2}} = -0.9487\mathbf{j} - 0.3162\,\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_{AC}$ is given by

$$\mathbf{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = 60(0.3621\mathbf{i} - 0.8689\mathbf{j} + 0.3375\mathbf{k}) = \{21.72\mathbf{i} - 52.14\mathbf{j} + 20.25\mathbf{k}\}\text{ lb}$$

***Vector Dot Product:*** The magnitude of the projected component of $\mathbf{F}_{AC}$ along strut AO is

$$(F_{AC})_{AO} = \mathbf{F}_{AC}\cdot\mathbf{u}_{AO} = (21.72\mathbf{i} - 52.14\mathbf{j} + 20.25\mathbf{k})\cdot(-0.9487\mathbf{j} - 0.3162\mathbf{k})$$

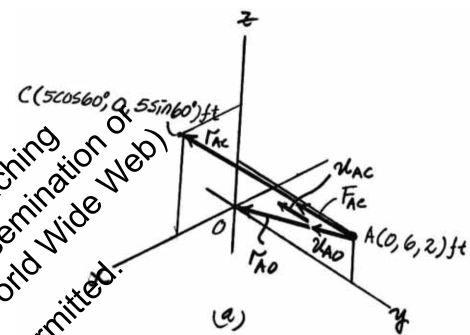
$$= (21.72)(0) + (-52.14)(-0.9487) + (20.25)(-0.3162)$$

$$= 43.057\text{ lb}$$

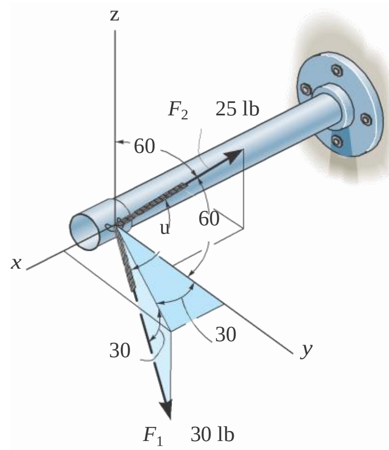
Thus, $(\mathbf{F}_{AC})_{AO}$ expressed in Cartesian vector form can be written as

$$(\mathbf{F}_{AC})_{AO} = (F_{AC})_{AO}\mathbf{u}_{AO} = 43.057(-0.9487\mathbf{j} - 0.3162\mathbf{k})$$

$$= \{-40.8\mathbf{j} - 13.6\mathbf{k}\}\text{ lb}$$ **Ans.**



**2–126.**




Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.

## SOLUTION

***Force Vector:***

$$\mathbf{u}_{F_1} = \cos 30° \sin 30°\mathbf{i} + \cos 30° \cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{F}_1 = F_R\mathbf{u}_{F_1} = 30(0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k})\text{ lb}$$

$$= \{12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}\}\text{ lb}$$

***Unit Vector:*** One can obtain the angle a = 135° for $\mathbf{F}_2$ using Eq. 2–8.

$\cos^2 a + \cos^2 b + \cos^2 g = 1$, with b = 60° and g = 60°. The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k} = -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

***Projected Component of $\mathbf{F}_1$ Along the Line of Action of $\mathbf{F}_2$:***

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

$$= (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5)$$

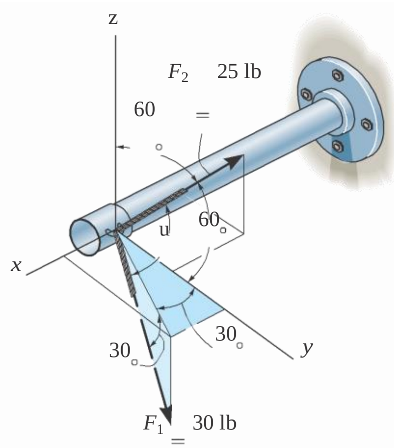
$$= -5.44\text{ lb}$$

Negative sign indicates that the projected component of $(F_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

The magnitude is $(F_1)_{F_2} = 5.44$ lb **Ans.**

**2–127.**

Determine the angle $\theta$ between the two cables attached to the pipe.




## SOLUTION

***Unit Vectors:***

$$\mathbf{u}_{F_1} = \cos 30° \sin 30° \mathbf{i} + \cos 30° \cos 30° \mathbf{j} - \sin 30° \mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 135° \mathbf{i} + \cos 60° \mathbf{j} + \cos 60° \mathbf{k}$$

$$= -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

***The Angles Between Two Vectors*** $\theta$:

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

$$= 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5)$$

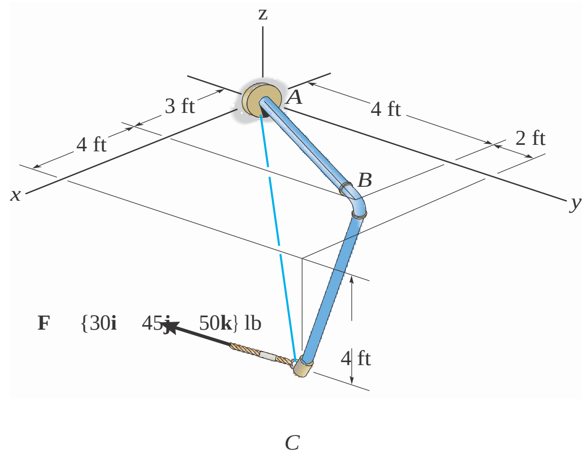
$$= -0.1812$$

Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1812) = 100° \qquad \textbf{Ans.}$$

***2–128.**

Determine the magnitudes of the components of **F** acting along and perpendicular to segment *BC* of the pipe assembly.




## SOLUTION

***Unit Vector:*** The unit vector $\mathbf{u}_{CB}$ must be determined first. From Fig. *a*

$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CB}}{r_{CB}} = \frac{(3-7)\mathbf{i} + (4-6)\mathbf{j} + [0-(-4)]\mathbf{k}}{\sqrt{(3-7)^2 + (4-6)^2 + [0-(-4)]^2}} = -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

***Vector Dot Product:*** The magnitude of the projected component of **F** parallel to segment *BC* of the pipe assembly is

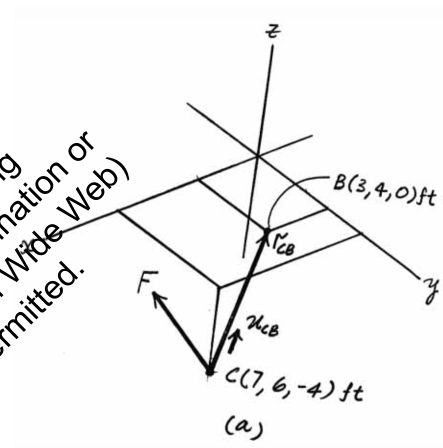
$$(F_{BC})_{pa} = \mathbf{F}\cdot\mathbf{u}_{CB} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k})\cdot\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$$= (30)\left(-\frac{2}{3}\right) + (-45)\left(-\frac{1}{3}\right) + 50\left(\frac{2}{3}\right)$$

$$= 28.33 \text{ lb} = 28.3 \text{ lb} \qquad \textbf{Ans.}$$

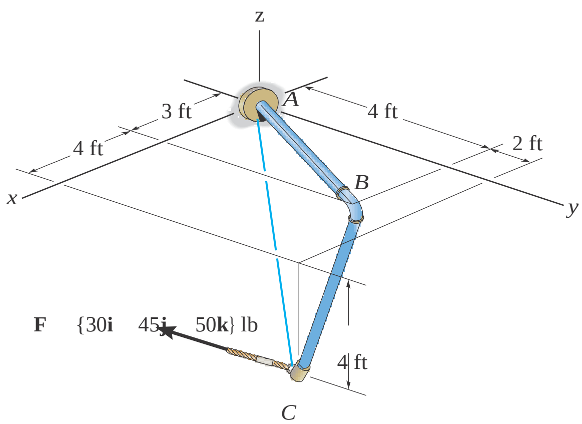
The magnitude of **F** is $F = \sqrt{30^2 + (-45)^2 + 50^2} = \sqrt{5425}$ lb. Thus, the magnitude of the component of **F** perpendicular to segment *BC* of the pipe assembly can be determined from

$$(F_{BC})_{pr} = \sqrt{F^2 - (F_{BC})_{pa}^{\,2}} = \sqrt{5425 - 28.33^2} = 68.0 \text{ lb} \qquad \textbf{Ans.}$$



**2–129.**

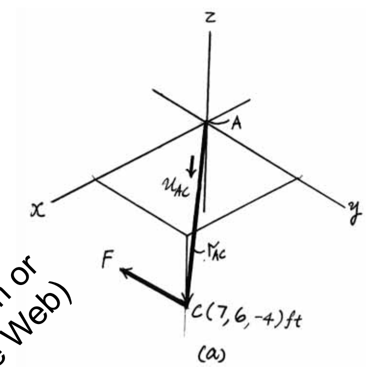
Determine the magnitude of the projected component of **F** along *AC*. Express this component as a Cartesian vector.




## SOLUTION

***Unit Vector:*** The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*

$$\mathbf{u}_{AC} = \frac{(7-0)\mathbf{i} + (6-0)\mathbf{j} + (-4-0)\mathbf{k}}{\sqrt{(7-0)^2 + (6-0)^2 + (-4-0)^2}} = 0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k}$$




***Vector Dot Product:*** The magnitude of the projected component of **F** along line *AC* is

$$F_{AC} = \mathbf{F} \cdot \mathbf{u}_{AC} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k}) \cdot (0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

$$= (30)(0.6965) + (-45)(0.5970) + 50(-0.3980)$$
$$= 25.87 \text{ lb}$$ **Ans.**

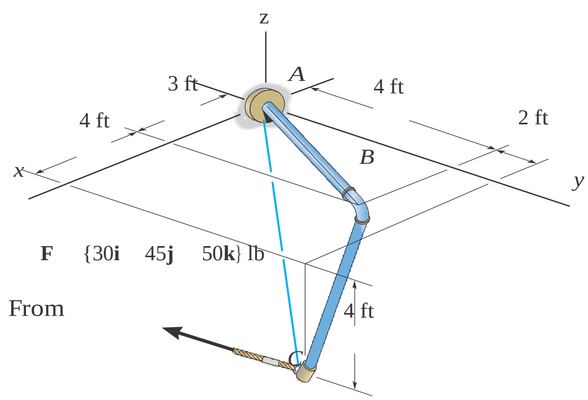
Thus, $\mathbf{F}_{AC}$ expressed in Cartesian vector form is

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = -25.87(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

$$= \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\} \text{ lb}$$ **Ans.**

**2–130.**

Determine the angle $\theta$ between the pipe segments *BA* and *BC*.

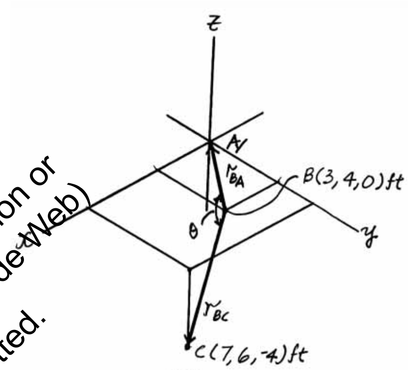



## SOLUTION

***Position Vectors:*** The position vectors $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ must be determined first. From Fig. *a*,

$$\mathbf{r}_{BA} = (0-3)\mathbf{i} + (0-4)\mathbf{j} + (0-0)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_{BC} = (7-3)\mathbf{i} + (6-4)\mathbf{j} + (-4-0)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}\text{ ft}$$




(a)

The magnitude of $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ are

$$r_{BA} = \sqrt{(-3)^2 + (-4)^2} = 5\text{ ft}$$

$$r_{BC} = \sqrt{4^2 + 2^2 + (-4)^2} = 6\text{ ft}$$

***Vector Dot Product:***

$$\mathbf{r}_{BA} \cdot \mathbf{r}_{BC} = (-3\mathbf{i} - 4\mathbf{j}) \cdot (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

$$= (-3)(4) + (-4)(2) + 0(-4)$$

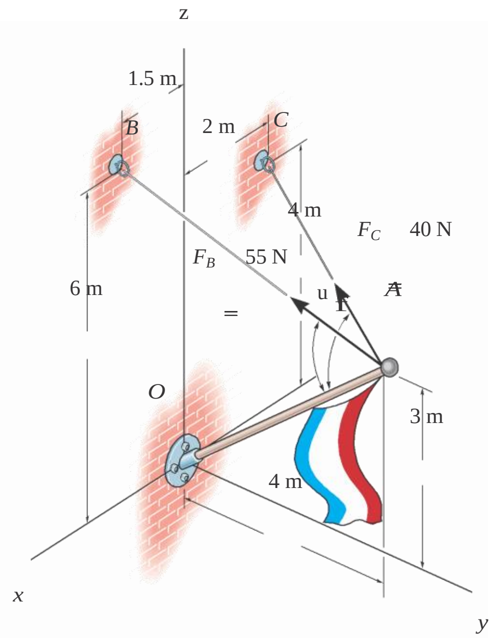
$$= -20\text{ ft}^2$$

Thus,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{r_{BA}\, r_{BC}}\right) = \cos^{-1}\left[\frac{-20}{5(6)}\right] = 132^\circ$$ **Ans.**

**2–131.**

Determine the angles u and **f** made between the axes *OA* of the flag pole and *AB* and *AC*, respectively, of each cable.




## SOLUTION

$\mathbf{r}_{AC} = \{-2\mathbf{i} - 4\mathbf{j} + 1\mathbf{k}\}$ m; $\quad r_{AC} = 4.58$ m

$\mathbf{r}_{AB} = \{1.5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\}$ m; $\quad r_{AB} = 5.22$ m

$\mathbf{r}_{AO} = \{-4\mathbf{j} - 3\mathbf{k}\}$ m; $\quad r_{AO} = 5.00$

$\mathbf{r}_{AB} \cdot \mathbf{r}_{AO} = (1.5)(0) + (-4)(-4) + (3)(-3) = 7$

$$u = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AO}}{r_{AB}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{7}{5.22(5.00)}\right) = 74.4°$$

**Ans.**

$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (-2)(0) + (-4)(-4) + (1)(-3) = 13$

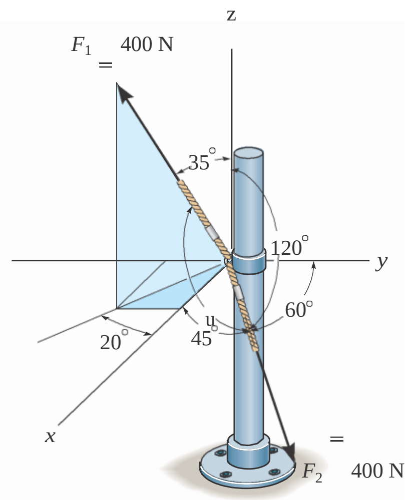
$$f = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{13}{4.58(5.00)}\right) = 55.4°$$

**Ans.**

***2–132.**

The cables each exert a force of 400 N on the post. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.




## SOLUTION

***Force Vector:***

$$\mathbf{u}_{F_1} = \sin 35° \cos 20° \mathbf{i} - \sin 35° \sin 20° \mathbf{j} + \cos 35° \mathbf{k}$$

$$= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{F}_1 = F_1 \mathbf{u}_{F_1} = 400(0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \text{ N}$$

$$= \{215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}\} \text{ N}$$

***Unit Vector:*** The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 45° \mathbf{i} + \cos 60° \mathbf{j} + \cos 120° \mathbf{k}$$

$$= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

***Projected Component of* $\mathbf{F}_1$ *Along Line of Action of* $\mathbf{F}_2$:**

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k})$$

$$= (215.59)(0.7071) + (-78.47)(0.5) + (327.66)(-0.5)$$

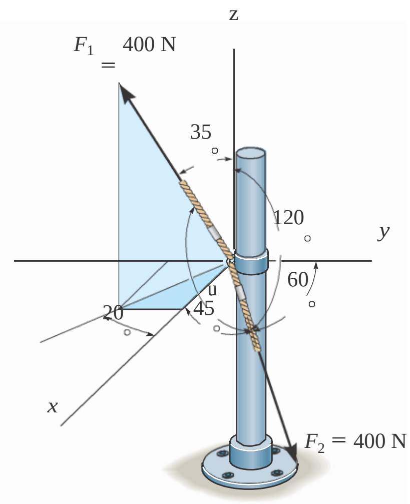
$$= -50.6 \text{ N}$$

Negative sign indicates that the force component $(\mathbf{F}_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F2}$.

thus the magnitude is $(F_1)_{F_2} = 50.6$ N **Ans.**

**2–133.**

Determine the angle θ between the two cables attached to the post.




## SOLUTION

***Unit Vector:***

$$\mathbf{u}_{F_1} = \sin 35° \cos 20°\mathbf{i} - \sin 35° \sin 20°\mathbf{j} + \cos 35°\mathbf{k}$$

$$= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}$$

$$= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

***The Angle Between Two Vectors*** θ: The dot product of two unit vectors must be determined first.

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k})$$

$$= 0.5390(0.7071) + (-0.1962)(0.5) + 0.8192(-0.5)$$

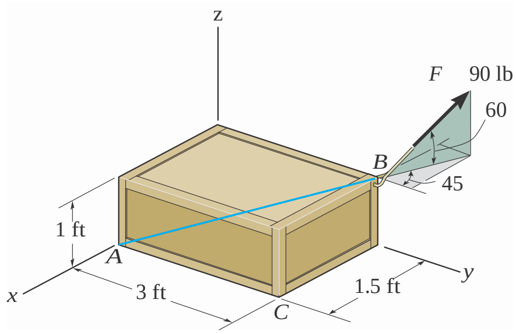
$$= -0.1265$$

Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1265) = 97.3° \qquad \text{Ans.}$$

**2–134.**

Determine the magnitudes of the components of force $F = 90$ lb acting parallel and perpendicular to diagonal $AB$ of the crate.



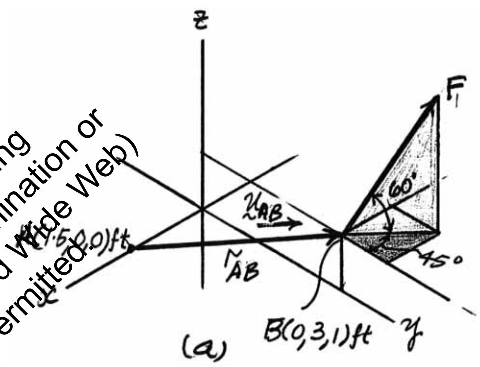

## SOLUTION

***Force and Unit Vector:*** The force vector **F** and unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*

$$\mathbf{F} = 90(-\cos 60° \sin 45°\mathbf{i} + \cos 60° \cos 45°\mathbf{j} + \sin 60°\mathbf{k})$$

$$= \{-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}\} \text{ lb}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(0-1.5)\mathbf{i} + (3-0)\mathbf{j} + (1-0)\mathbf{k}}{\sqrt{(0-1.5)^2 + (3-0)^2 + (1-0)^2}} = -\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$




***Vector Dot Product:*** The magnitude of the projected component of **F** parallel to the diagonal $AB$ is

$$[(F)_{AB}]_{pa} = \mathbf{F} \cdot \mathbf{u}_{AB} = (-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}) \cdot \left(-\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

$$= (-31.82)\left(-\frac{3}{7}\right) + 31.82\left(\frac{6}{7}\right) + 77.94\left(\frac{2}{7}\right)$$

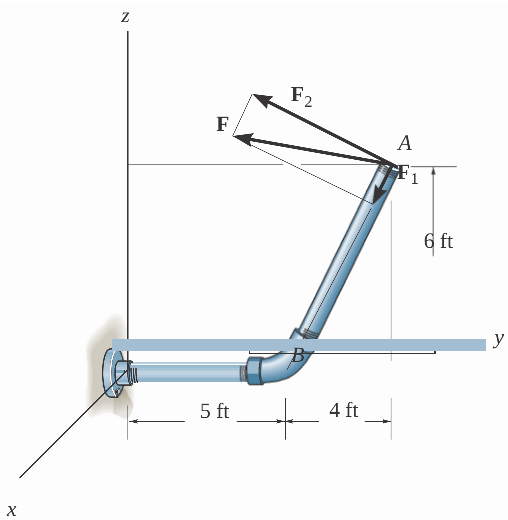
$$= 63.18 \text{ lb} = 63.2 \text{ lb}$$ **Ans.**

The magnitude of the component **F** perpendicular to the diagonal $AB$ is

$$[(F)_{AB}]_{pr} = \sqrt{F^2 - [(F)_{AB}]_{pa}^2} = \sqrt{90^2 - 63.18^2} = 64.1 \text{ lb}$$ **Ans.**

**2–135.**

The force $\mathbf{F} = 525\mathbf{i} - 50\mathbf{j} + 10\mathbf{k}6$ lb acts at the end $A$ of the pipe assembly. Determine the magnitude of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which act along the axis of $AB$ and perpendicular to it.




## SOLUTION

***Unit Vector:*** The unit vector along $AB$ axis is

$$\frac{10 - 02\mathbf{i} + 15 - 92\mathbf{j} + 10 - 62\mathbf{k}}{2 10 - 02^2 + 15 - 92^2 + 10 - 62^2}$$

***Projected Component of* F *Along AB Axis:***

$$F_1 = \mathbf{F} \cdot \mathbf{u}_{AB} = 125\mathbf{i} - 50\mathbf{j} + 10\mathbf{k}2 \cdot 1 - 0.5547\mathbf{j} - 0.8321\mathbf{k}2$$

$$= 1252102 + 1-5021-0.55472 + 11021-0.83212$$

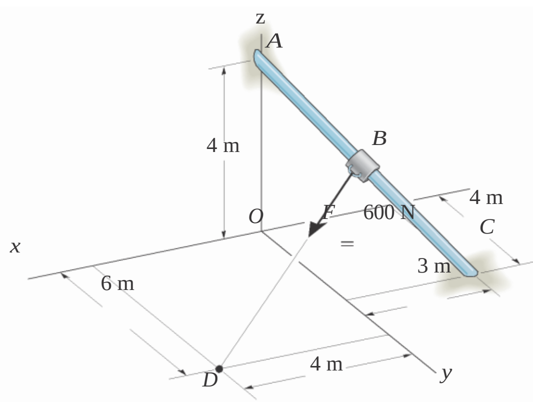
$$= 19.415 \text{ lb} = 19.4 \text{ lb}$$ **Ans.**

***Component of* F *Perpendicular to AB Axis:*** The magnitude of force F is $F = 2 25^2 + 1-502^2 + 10^2 = 56.789$ lb.

$$F_2 = \sqrt{F^2 - F_1^2} = \sqrt{56.789^2 - 19.414^2} = 53.4 \text{ lb}$$ **Ans.**

***2–136.**

Determine the components of **F** that act along rod *AC* and perpendicular to it. Point *B* is located at the midpoint of the rod.




## SOLUTION

$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \qquad r_{AC} = \sqrt{(-3)^2 + 4^2 + (-4)^2} = \sqrt{41}\text{ m}$

$$\mathbf{r}_{AB} = \frac{\mathbf{r}_{AC}}{2} = \frac{-3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}}{2} = -1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$

$\mathbf{r}_{AD} = \mathbf{r}_{AB} + \mathbf{r}_{BD}$

$\mathbf{r}_{BD} = \mathbf{r}_{AD} - \mathbf{r}_{AB}$

$= (4\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}) - (-1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$

$= \{5.5\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\}\text{ m}$

$r_{BD} = \sqrt{(5.5)^2 + (4)^2 + (-2)^2} = 7.0887\text{ m}$

$$\mathbf{F} = 600\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = 465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k}$$

Component of **F** along $\mathbf{r}_{AC}$ is $\mathbf{F}_{||}$

$$F_{||} = \frac{\mathbf{F} \cdot \mathbf{r}_{AC}}{r_{AC}} = \frac{(465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k}) \cdot (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$$

$F_{||} = 99.1408 = 99.1\text{ N}$ **Ans.**

Component of *F* perpendicular to $\mathbf{r}_{AC}$ is $F_{\perp}$

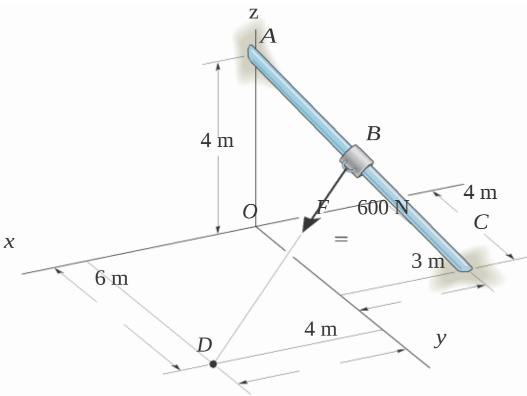
$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$

$F_{\perp}^2 = 600^2 - 99.1408^2$

$F_{\perp} = 591.75 = 592\text{ N}$ **Ans.**

**2–137.**

Determine the components of **F** that act along rod *AC* and perpendicular to it. Point *B* is located 3 m along the rod from end *C*.




## SOLUTION

$\mathbf{r}_{CA} = 3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$

$r_{CA} = 6.403124$

$\mathbf{r}_{CB} = \dfrac{3}{6.403124}(\mathbf{r}_{CA}) = 1.40556\mathbf{i} - 1.874085\mathbf{j} + 1.874085\mathbf{k}$

$\mathbf{r}_{OB} = \mathbf{r}_{OC} + \mathbf{r}_{CB}$

$= -3\mathbf{i} + 4\mathbf{j} + \mathbf{r}_{CB}$

$= -1.59444\mathbf{i} + 2.1259\mathbf{j} + 1.874085\mathbf{k}$

$\mathbf{r}_{OD} = \mathbf{r}_{OB} + \mathbf{r}_{BD}$

$\mathbf{r}_{BD} = \mathbf{r}_{OD} - \mathbf{r}_{OB} = (4\mathbf{i} + 6\mathbf{j}) - \mathbf{r}_{OB}$

$= 5.5944\mathbf{i} + 3.8741\mathbf{j} - 1.874085\mathbf{k}$

$r_{BD} = \sqrt{(5.5944)^2 + (3.8741)^2 + (-1.874085)^2} = 7.0582$

$\mathbf{F} = 600\left(\dfrac{\mathbf{r}_{BD}}{r_{BD}}\right) = 475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k}$

$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \quad r_{AC} = \sqrt{41}$

Component of **F** along $\mathbf{r}_{AC}$ is $\mathbf{F}_{||}$

$F_{||} = \dfrac{\mathbf{F} \cdot \mathbf{r}_{AC}}{r_{AC}} = \dfrac{(475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k}) \cdot (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$

$F_{||} = 82.4351 = 82.4 \text{ N}$ **Ans.**

Component of **F** perpendicular to $\mathbf{r}_{AC}$ is $\mathbf{F}_{\perp}$

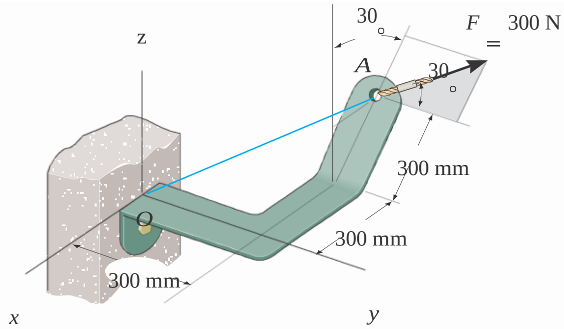
$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$

$F_{\perp}^2 = 600^2 - 82.4351^2$

$F = 594 \text{ N}$ **Ans.**

**2–138.**

Determine the magnitudes of the projected components of the force $\mathbf{F} = 300$ N acting along the $x$ and $y$ axes.




## SOLUTION

***Force Vector:*** The force vector **F** must be determined first. From Fig. *a*,

$$\mathbf{F} = -300 \sin 30° \sin 30° \mathbf{i} + 300 \cos 30° \mathbf{j} + 300 \sin 30° \cos 30° \mathbf{k}$$

$$= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}] \text{ N}$$

***Vector Dot Product:*** The magnitudes of the projected component of **F** along the $x$ and $y$ axes are

$$F_x = \mathbf{F} \cdot \mathbf{i} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{i}$$

$$= -75(1) + 259.81(0) + 129.90(0)$$

$$= -75 \text{ N}$$

$$F_y = \mathbf{F} \cdot \mathbf{j} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{j}$$

$$= -75(0) + 259.81(1) + 129.90(0)$$

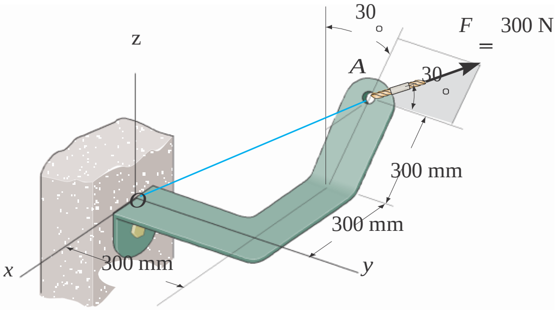
$$= 260 \text{ N}$$

The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative $x$ axis. Thus,

$$F_x = 75 \text{ N}, \qquad F_y = 260 \text{ N} \qquad \textbf{Ans.}$$

**2–139.**

Determine the magnitude of the projected component of the force $\mathbf{F} = 300$ N acting along line $OA$.




## SOLUTION

***Force and Unit Vector:*** The force vector **F** and unit vector $\mathbf{u}_{OA}$ must be determined first. From Fig. a

$$\mathbf{F} = (-300 \sin 30° \sin 30°\mathbf{i} + 300 \cos 30°\mathbf{j} + 300 \sin 30° \cos 30°\mathbf{k})$$

$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\} \text{ N}$$

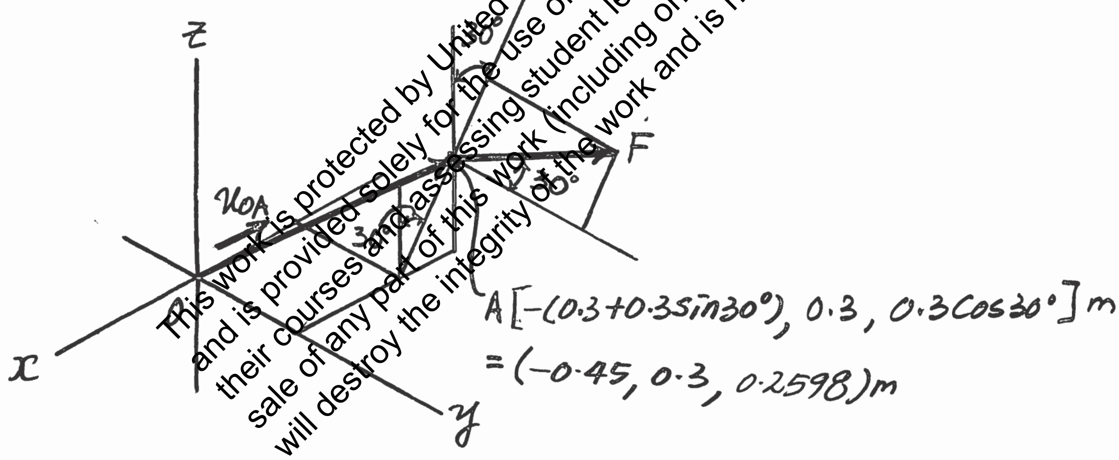
$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.2598 - 0)\mathbf{k}}{\sqrt{(-0.45 - 0)^2 + (0.3 - 0)^2 + (0.2598 - 0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

***Vector Dot Product:*** The magnitude of the projected component of **F** along line $OA$ is

$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot (-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k})$$

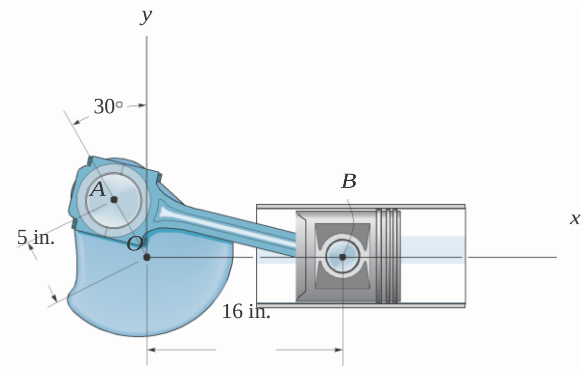
$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$

$$= 242 \text{ N}$$ **Ans.**



***2–140.**

Determine the length of the connecting rod *AB* by first formulating a Cartesian position vector from *A* to *B* and then determining its magnitude.




## SOLUTION

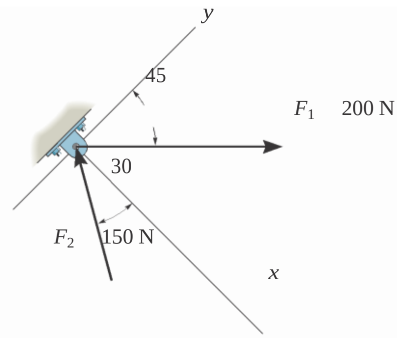
$$\mathbf{r}_{AB} = [16 - (-5\sin 30°)]\mathbf{i} + (0 - 5\cos 30°)\,\mathbf{j}$$

$$= \{18.5\,\mathbf{i} - 4.330\,\mathbf{j}\}\text{ in.}$$

$$r_{AB} = \sqrt{(18.5)^2 + (4.330)^2} = 19.0\text{ in.}$$ **Ans.**

**2–141.**

Determine the $x$ and $y$ components of $\mathbf{F}_1$ and $\mathbf{F}_2$.




## SOLUTION

$F_{1x} = 200 \sin 45° = 141$ N **Ans.**

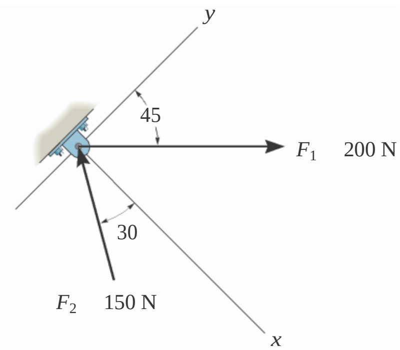
$F_{1y} = 200 \cos 45° = 141$ N **Ans.**

$F_{2x} = -150 \cos 30° = -130$ N **Ans.**

$F_{2y} = 150 \sin 30° = 75$ N **Ans.**

**2–142.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive *x* axis.




## SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = -150 \cos 30° + 200 \sin 45° = 11.518 \text{ N}$

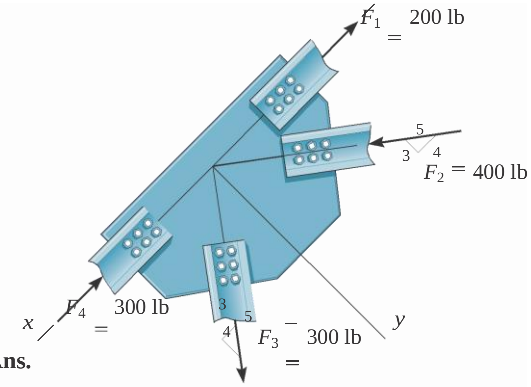
$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 150 \sin 30° + 200 \cos 45° = 216.421 \text{ N}$

$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217 \text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{216.421}{11.518}\right) = 87.0°$ **Ans.**

**2–143.**

Determine the $x$ and $y$ components of each force acting on the *gusset plate* of the bridge truss. Show that the resultant force is zero.




## SOLUTION

$F_{1x} = -200$ lb **Ans.**

$F_{1y} = 0$ **Ans.**

$F_{2x} = 400\left(\frac{4}{5}\right) = 320$ lb **Ans.**

$F_{2y} = -400\left(\frac{3}{5}\right) = -240$ lb **Ans.**

$F_{3x} = 300\left(\frac{3}{5}\right) = 180$ lb **Ans.**

$F_{3y} = 300\left(\frac{4}{5}\right) = 240$ lb **Ans.**

$F_{4x} = -300$ lb **Ans.**

$F_{4y} = 0$ **Ans.**

$F_{Rx} = F_{1x} + F_{2x} + F_{3x} + F_{4x}$

$F_{Rx} = -200 + 320 + 180 - 300 = 0$

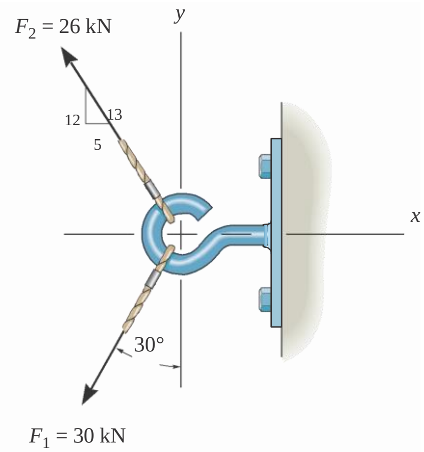
$F_{Ry} = F_{1y} + F_{2y} + F_{3y} + F_{4y}$

$F_{Ry} = 0 - 240 + 240 + 0 = 0$

Thus, $F_R = 0$

***2–144.**

Express $\mathbf{F}_1$ and $\mathbf{F}_2$ as Cartesian vectors.




## SOLUTION

$$\mathbf{F}_1 = -30 \sin 30° \,\mathbf{i} - 30 \cos 30° \,\mathbf{j}$$

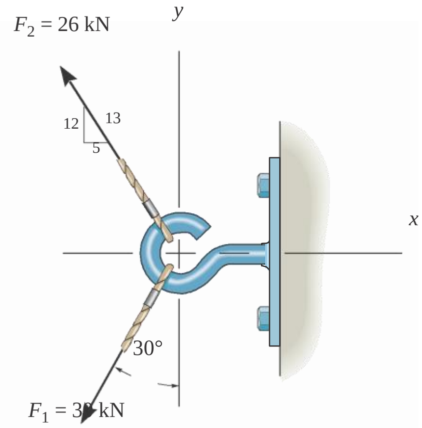
$$= \{-15.0\,\mathbf{i} - 26.0\,\mathbf{j}\} \text{ kN}$$ **Ans.**

$$\mathbf{F}_2 = -\frac{5}{13}(26)\,\mathbf{i} + \frac{12}{13}(26)\,\mathbf{j}$$

$$= \{-10.0\,\mathbf{i} + 24.0\,\mathbf{j}\} \text{ kN}$$ **Ans.**

**2–145.**

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive $x$ axis.

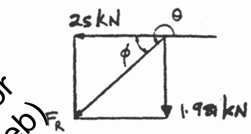



## SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = -30 \sin 30^\circ - \frac{5}{13}(26) = -25 \text{ kN}$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = -30 \cos 30^\circ + \frac{12}{13}(26) = -1.981 \text{ kN}$

$F_R = \sqrt{(-25)^2 + (-1.981)^2} = 25.1 \text{ kN}$ **Ans.**

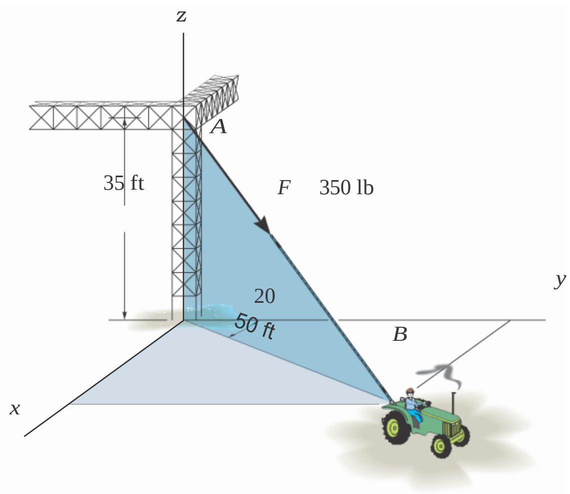



$\phi = \tan^{-1}\left(\frac{1.981}{25}\right) = 4.53^\circ$

$\theta = 180^\circ + 4.53^\circ = 185^\circ$ **Ans.**

**2–146.**

The cable attached to the tractor at *B* exerts a force of 350 lb on the framework. Express this force as a Cartesian vector.




## SOLUTION

$$\mathbf{r} = 50 \sin 20°\mathbf{i} + 50 \cos 20°\mathbf{j} - 35\mathbf{k}$$

$$\mathbf{r} = \{17.10\mathbf{i} + 46.98\mathbf{j} - 35\mathbf{k}\} \text{ ft}$$

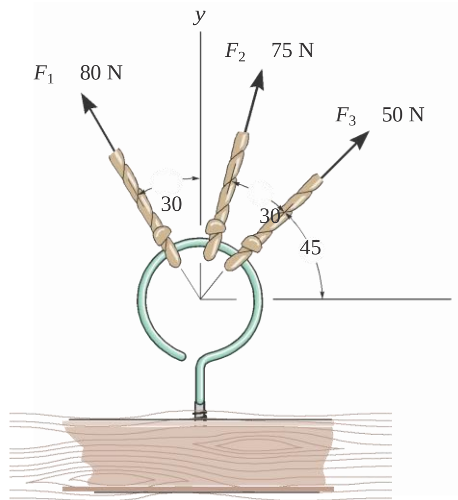
$$r = \sqrt{(17.10)^2 + (46.98)^2 + (-35)^2} = 61.03 \text{ ft}$$

$$\mathbf{u} = \frac{\mathbf{r}}{r} = (0.280\mathbf{i} + 0.770\mathbf{j} - 0.573\mathbf{k})$$

$$\mathbf{F} = F\mathbf{u} = \{98.1\mathbf{i} + 269\mathbf{j} - 201\mathbf{k}\} \text{ lb}$$ **Ans.**

**2–147.**

Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_2$. Specify its direction measured counterclockwise from the positive x axis.




## SOLUTION

$$F' = \sqrt{(80)^2 + (50)^2 - 2(80)(50)\cos 105°} = 104.7 \text{ N}$$

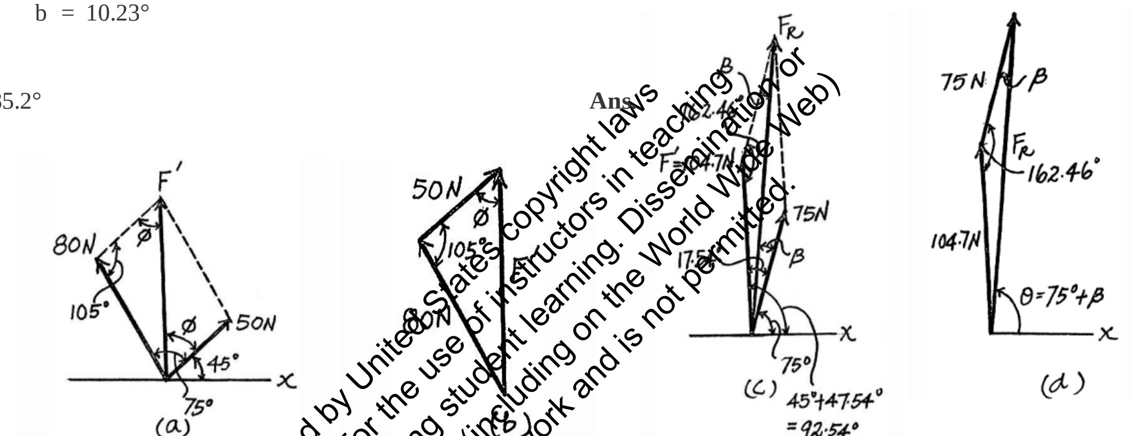
$$\frac{\sin \phi}{80} = \frac{\sin 105°}{104.7}; \qquad \phi = 47.54°$$

$$F_R = \sqrt{(104.7)^2 + (75)^2 - 2(104.7)(75)\cos 162.46°}$$

$$F_R = 177.7 = 178 \text{ N}$$ **Ans.**

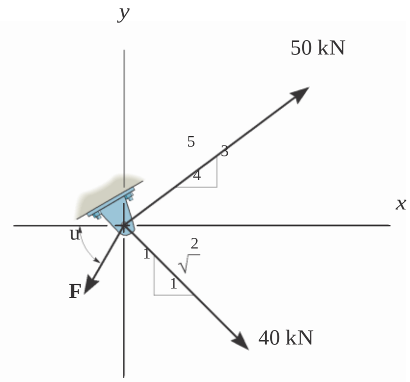
$$\frac{\sin \beta}{104.7} = \frac{\sin 162.46°}{177.7}; \qquad \beta = 10.23°$$

$$\theta = 75° + 10.23° = 85.2°$$ **Ans.**



***2–148.**

If $\theta = 60°$ and $F = 20$ kN, determine the magnitude of the resultant force and its direction measured clockwise from the positive $x$ axis.




## SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = 50\left(\frac{4}{5}\right) + \frac{1}{\sqrt{2}}(40) - 20\cos 60° = 58.28 \text{ kN}$

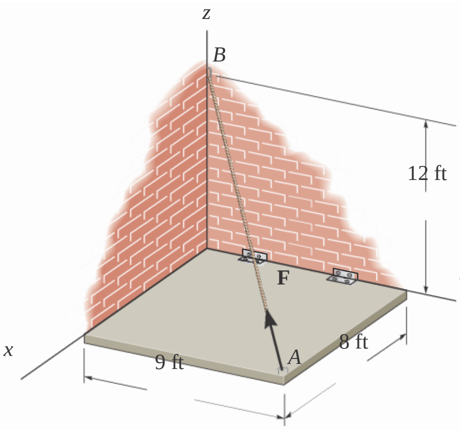
$+\uparrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 50\left(\frac{3}{5}\right) - \frac{1}{\sqrt{2}}(40) - 20\sin 60° = -15.60 \text{ kN}$

$F_R = \sqrt{(58.28)^2 + (-15.60)^2} = 60.3 \text{ kN}$ **Ans.**

$\phi = \tan^{-1}\left[\frac{15.60}{58.28}\right] = 15.0°$ **Ans.**

**2–149.**

The hinged plate is supported by the cord $AB$. If the force in the cord is $\mathbf{F} = 340$ lb, express this force, directed from $A$ toward $B$, as a Cartesian vector. What is the length of the cord?




## SOLUTION

***Unit Vector:***

$$\mathbf{r}_{AB} = \{(0 - 8)\mathbf{i} + (0 - 9)\mathbf{j} + (12 - 0)\mathbf{k}\}\text{ ft}$$

$$= \{-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{(-8)^2 + (-9)^2 + 12^2} = 17.0\text{ ft} \qquad \textbf{Ans.}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}}{17} = -\frac{8}{17}\mathbf{i} - \frac{9}{17}\mathbf{j} + \frac{12}{17}\mathbf{k}$$

***Force Vector:***

$$\mathbf{F} = F\mathbf{u}_{AB} = 340\left\{-\frac{8}{17}\mathbf{i} - \frac{9}{17}\mathbf{j} + \frac{12}{17}\mathbf{k}\right\}\text{ lb}$$

$$= \{-160\mathbf{i} - 180\mathbf{j} + 240\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$