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Solutions Manual

Engineering Mechanics: Statics

2nd Edition

Michael E. Plesha
University of Wisconsin–Madison

Gary L. Gray
The Pennsylvania State University

Francesco Costanzo
The Pennsylvania State University

With the assistance of:
Chris Punshon
Andrew J. Miller
Justin High
Chris O'Brien
Chandan Kumar
Joseph Wyne
Jonathan Fleischmann

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Contact the Authors

If you find any errors and/or have questions concerning a solution, please do not hesitate to contact the authors and editors via email at:

plesha@engr.wisc.edu, and
stat_solns@email.esm.psu.edu

We welcome your input.

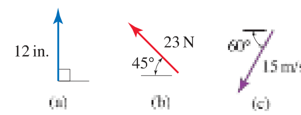
Accuracy of Numbers in Calculations

Throughout this solutions manual, we will generally assume that the data given for problems is accurate to *3* significant digits. When calculations are performed, all intermediate numerical results are reported to *4* significant digits. Final answers are usually reported with *3* or *4* significant digits. If you verify the calculations in this solutions manual using the rounded intermediate numerical results that are reported, you should obtain the final answers that are reported to *3* significant digits.

Chapter 2 Solutions

Problem 2.1

For each vector, write two expressions using polar vector representations, one using a positive value of & and the other a negative value, where & is measured counterclockwise from the right-hand horizontal direction.



Solution

Part (a)

$$\vec{r} = 12\,\text{in.} @ 90^\circ \measuredangle \quad \text{or} \quad \vec{r} = 12\,\text{in.} @ -270^\circ \measuredangle. \tag{1}$$

Part (b)

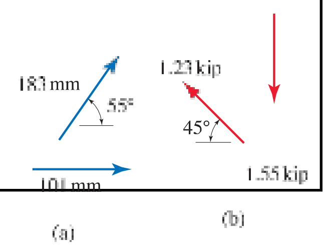
$$\vec{F} = 23\,\text{N} @ 135^\circ \measuredangle \quad \text{or} \quad \vec{F} = 23\,\text{N} @ -225^\circ \measuredangle. \tag{2}$$

Part (c)

$$\vec{v} = 15\,\text{m/s} @ 240^\circ \measuredangle \quad \text{or} \quad \vec{v} = 15\,\text{m/s} @ -120^\circ \measuredangle. \tag{3}$$

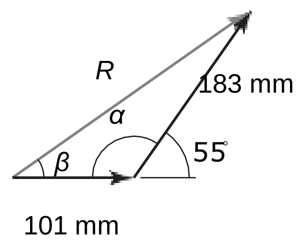
Problem 2.2

Add the two vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution

Part (a) The vector polygon shown at the right corresponds to the addition of the two position vectors to obtain a resultant position vector $\vec{R}$. Note that γ is given by $\gamma = 180° - 55° = 125°$. Knowing this angle, the law of cosines may be used to determine R



$$R = \sqrt{(101\ \text{mm})^2 + (183\ \text{mm})^2 - 2(101\ \text{mm})(183\ \text{mm})\cos 125°} = 254.7\ \text{mm}. \quad (1)$$

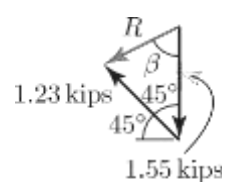
Next, the law of sines may be used to determine the angle β:

$$\frac{R}{\sin\gamma} = \frac{183\ \text{mm}}{\sin\beta} \quad \Rightarrow \quad \beta = \sin^{-1}\left(\frac{183\ \text{mm}}{254.7\ \text{mm}}\sin 125°\right) = 36.05°. \quad (2)$$

Using these results, we may report the vector $\vec{R}$ using polar vector representation as

$$\boxed{\vec{R} = 255\ \text{mm} \ @ \ 36.0° \ \measuredangle.} \quad (3)$$

Part (b) The vector polygon shown at the right corresponds to the addition of the two force vectors to obtain a resultant force vector $\vec{R}$. The law of cosines may be used to determine R



$$R = \sqrt{(1.23\ \text{kip})^2 + (1.55\ \text{kip})^2 - 2(1.23\ \text{kip})(1.55\ \text{kip})\cos 45°} = 1.104\ \text{kip}. \quad (4)$$

Using the law of sines, we find that

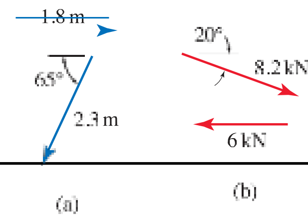
$$\frac{R}{\sin 45°} = \frac{1.23\ \text{kip}}{\sin\beta} \quad \Rightarrow \quad \beta = \sin^{-1}\left(\frac{1.23\ \text{kip}}{1.104\ \text{kip}}\sin 45°\right) = 51.97°. \quad (5)$$

The direction of $\vec{R}$ measured from the right-hand horizontal direction is $-90° - 51.97° = -142°$. Using these results, we may report $\vec{R}$ using polar vector representation as

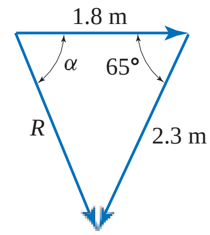
$$\boxed{\vec{R} = 1.10\ \text{kip} \ @ \ -142° \ \measuredangle.} \quad (6)$$

Problem 2.3

Add the two vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution



Part (a) The vector polygon shown at the right corresponds to the addition of the two position vectors to obtain a resultant position vector $\vec{R}$. The law of cosines may be used to determine R as

$$R = \sqrt{(1.8\text{ m})^2 + (2.3\text{ m})^2 - 2(1.8\text{ m})(2.3\text{ m})\cos 65^\circ}$$
$$= 2.243\text{ m}. \tag{1}$$

The law of sines may be used to determine the angle α as

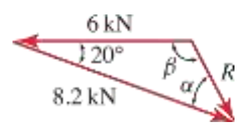
$$\frac{R}{\sin 65^\circ} = \frac{2.3\text{ m}}{\sin\alpha} \quad\Rightarrow\quad \alpha = \sin^{-1}\frac{2.3\text{ m}}{2.243\text{ m}}\sin 65^\circ = 68.34^\circ. \tag{2}$$

Using polar vector representation, the resultant is

$$\boxed{\vec{R} = 2.243\text{ m} @ -68.34^\circ \measuredangle.} \tag{3}$$

If desired, this resultant may be stated using a positive angle, where $360^\circ - 68.34^\circ = 291.7^\circ$, as

$$\boxed{\vec{R} = 2.243\text{ m} @ 291.7^\circ \measuredangle.} \tag{4}$$



Part (b) The vector polygon shown at the right corresponds to the addition of the two force vectors to obtain a resultant force vector $\vec{R}$. The law of cosines may be used to determine R as

$$R = \sqrt{(6\text{ kN})^2 + (8.2\text{ kN})^2 - 2(6\text{ kN})(8.2\text{ kN})\cos 20^\circ}$$
$$= 3.282\text{ kN}. \tag{5}$$

Noting that β appears to be an obtuse angle (see the Common Pitfall margin note in the text), we will use the law of sines to determine α as

$$\frac{6\text{ kN}}{\sin\alpha} = \frac{R}{\sin 20^\circ} \quad\Rightarrow\quad \alpha = \sin^{-1}\left[\frac{6\text{ kN}}{3.282\text{ kN}}\sin 20^\circ\right] = 38.70^\circ. \tag{6}$$

Angle β is obtained using

$$20^\circ + \alpha + \beta = 180^\circ; \tag{7}$$

$$\beta = 180^\circ - 20^\circ - 38.70^\circ = 121.3^\circ. \tag{8}$$

Using polar vector representation, the resultant is

$$\vec{R} = 3.282\,\mathrm{kN} \;@\; -(180^\circ - 121.3^\circ)\ \measuredangle \tag{9}$$

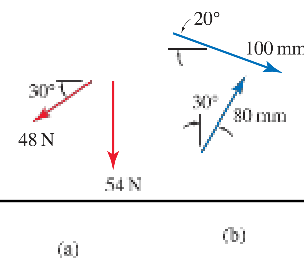
$$\boxed{\vec{R} = 3.282\,\mathrm{kN} \;@\; -58.70^\circ\ \measuredangle.} \tag{10}$$

If desired, the resultant may be stated using a positive angle, where $360^\circ - 58.70^\circ = 301.3^\circ$, as

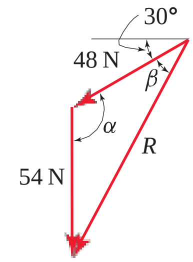
$$\boxed{\vec{R} = 3.282\,\mathrm{kN} \;@\; 301.3^\circ\ \measuredangle.} \tag{11}$$

Problem 2.4

Add the two vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution



Part (a) The vector polygon shown at the right corresponds to the addition of the two force vectors to obtain a resultant force vector $\vec{R}$. Since the 54 N force is vertical, the angle α may be obtained by inspection as $\alpha = 90^\circ + 30^\circ = 120^\circ$. The law of cosines may be used to determine R as

$$R = \sqrt{(48\text{ N})^2 + (54\text{ N})^2 - 2(48\text{ N})(54\text{ N})\cos 120^\circ} = 88.39\text{ N}. \tag{1}$$

The law of sines may be used to determine the angle β as

$$\frac{54\text{ N}}{\sin\beta} = \frac{R}{\sin\alpha} \quad \Rightarrow \quad \beta = \sin^{-1}\frac{54\text{ N}}{88.39\text{ N}}\sin 120^\circ = 31.95^\circ. \tag{2}$$

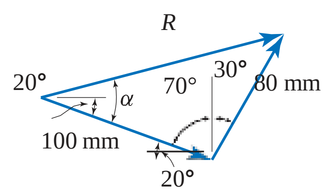
Using polar vector representation, the resultant is

$$\vec{R} = 88.39\text{ N} @ -(180^\circ - 30^\circ - \beta) \measuredangle \tag{3}$$

$$\boxed{\vec{R} = 88.39\text{ N} @ -118.1^\circ \measuredangle.} \tag{4}$$

If desired, this resultant may be stated using a positive angle, where $360^\circ - 118.1^\circ = 241.9^\circ$, as

$$\boxed{\vec{R} = 88.39\text{ N} @ 241.9^\circ \measuredangle.} \tag{5}$$



Part (b) The vector polygon shown at the right corresponds to the addition of the two position vectors to obtain a resultant position vector $\vec{R}$. Given the 20° and 30° angles provided in the problem statement, we determine the angle opposite R to be $70^\circ + 30^\circ = 100^\circ$. The law of cosines may be used to determine R as

$$R = \sqrt{(100\text{ mm})^2 + (80\text{ mm})^2 - 2(100\text{ mm})(80\text{ mm})\cos 100^\circ} = 138.5\text{ mm}. \tag{6}$$

The law of sines may be used to determine the angle α as

$$\frac{80\,\text{mm}}{\sin\alpha} = \frac{R}{\sin 100^\circ} \quad \Rightarrow \quad \alpha = \sin^{-1}\frac{80\,\text{mm}}{138.5\,\text{mm}}\sin 100^\circ = 34.67^\circ. \tag{7}$$

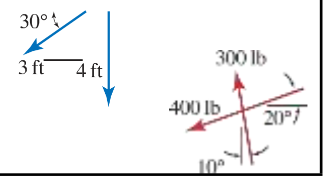
Using polar vector representation, the resultant is

$$\vec{R} = 138.5\,\text{mm} \;@\; 34.67^\circ - 20^\circ \measuredangle \tag{8}$$

$$\boxed{\vec{R} = 138.5\,\text{mm} \;@\; 14.67^\circ \measuredangle.} \tag{9}$$

Problem 2.5

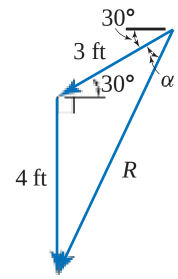
Add the two vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



(a) (b)

Solution

Part (a) The vector polygon shown at the right corresponds to the addition of the two position vectors to obtain the resultant position vector $\vec{R}$. The law of cosines may be used to determine R as



$$R = \sqrt{(3\text{ ft})^2 + (4\text{ ft})^2 - 2(3\text{ ft})(4\text{ ft})\cos 120^\circ} = 6.083\text{ ft}. \quad (1)$$

The law of sines may be used to determine the angle α as

$$\frac{4\text{ ft}}{\sin\alpha} = \frac{R}{\sin 120^\circ} \Rightarrow \alpha = \sin^{-1}\frac{4\text{ ft}}{6.083\text{ ft}}\sin 120^\circ = 34.72^\circ. \quad (2)$$

Using polar vector representation, the resultant is

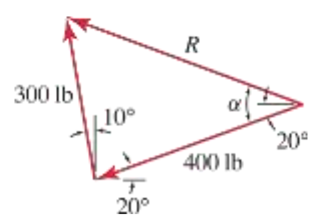
$$\vec{R} = 6.083\text{ ft} @ -(180^\circ - 30^\circ - \alpha) \measuredangle \quad (3)$$

$$\boxed{\vec{R} = 6.083\text{ ft} @ -115.3^\circ \measuredangle.} \quad (4)$$

If desired, this resultant may be stated using a positive angle, where $360^\circ - 115.3^\circ = 244.7^\circ$, as

$$\boxed{\vec{R} = 6.083\text{ ft} @ 244.7^\circ \measuredangle.} \quad (5)$$

Part (b) The vector polygon shown at the right corresponds to the addition of the two force vectors to obtain the resultant force vector $\vec{R}$. Given the 10° and 20° angles provided in the problem statement, we determine the angle opposite R to be $10^\circ + 90^\circ - 20^\circ = 80^\circ$. The law of cosines may be used to determine R as



$$R = \sqrt{(300\text{ lb})^2 + (400\text{ lb})^2 - 2(300\text{ lb})(400\text{ lb})\cos 80^\circ} = 456.4\text{ lb}. \quad (6)$$

The law of sines may be used to determine the angle α as

$$\frac{300\text{ lb}}{\sin\alpha} = \frac{R}{\sin 80^\circ} \Rightarrow \alpha = \sin^{-1}\frac{300\text{ lb}}{456.4\text{ lb}}\sin 80^\circ = 40.34^\circ. \quad (7)$$

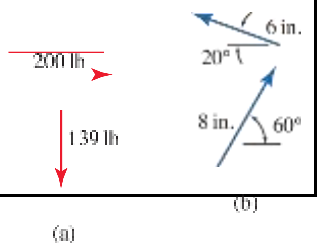
Using polar vector representation, the resultant is

$$\vec{R} = 456.4\,\text{lb} \;@\; 180^\circ + 20^\circ - \text{[illegible]} \;\measuredangle \qquad (8)$$

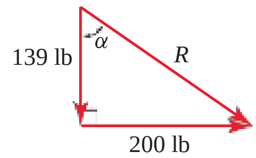
$$\boxed{\vec{R} = 456.4\,\text{lb} \;@\; 159.7^\circ \;\measuredangle.} \qquad (9)$$

Problem 2.6

Add the two vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution



Part (a) The vector polygon shown at the right corresponds to the addition of the two force vectors to obtain a resultant force vector $\vec{R}$. Since the two forces being added are perpendicular, basic trigonometry may be used to obtain R and α as

$$R = \sqrt{(139\text{ lb})^2 + (200\text{ lb})^2} = 243.6\text{ lb}, \tag{1}$$

$$\alpha = \tan^{-1}\frac{200\text{ lb}}{139\text{ lb}} = 55.20°. \tag{2}$$

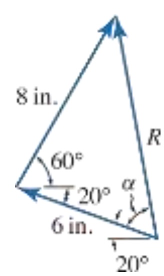
Using polar vector representation, the resultant is

$$\vec{R} = 243.6\text{ lb @ } -(90° - 55.20°) \measuredangle \tag{3}$$

$$\boxed{\vec{R} = 243.6\text{ lb @ } -34.80° \measuredangle.} \tag{4}$$

If desired, this resultant may be stated using a positive angle, where $360° - 34.80° = 325.2°$, as

$$\boxed{\vec{R} = 243.6\text{ lb @ } 325.2° \measuredangle.} \tag{5}$$



Part (b) The vector polygon shown at the right corresponds to the addition of the two position vectors to obtain a resultant position vector $\vec{R}$. The law of cosines may be used to determine R as

$$R = \sqrt{(6\text{ in.})^2 + (8\text{ in.})^2 - 2(6\text{ in.})(8\text{ in.})\cos 80°}$$
$$= 9.129\text{ in.} \tag{6}$$

The law of sines may be used to determine the angle α as

$$\frac{8\text{ in.}}{\sin\alpha} = \frac{R}{\sin 80°} \quad\Rightarrow\quad \alpha = \sin^{-1}\frac{8\text{ in.}}{9.129\text{ in.}}\sin 80° = 59.66°. \tag{7}$$

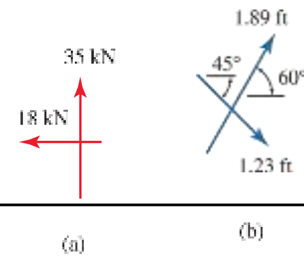
Using polar vector representation, the resultant is

$$\vec{R} = 9.129\text{ in. @ } 180° - 20° - \alpha \measuredangle \tag{8}$$

$$\vec{R} = 9.129 \text{ in. } @ \; 100.3^\circ \measuredangle. \tag{9}$$

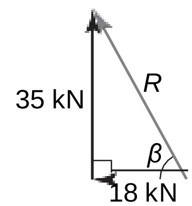
Problem 2.7

Add the two vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution

Part (a) The vector polygon shown at the right corresponds to the addition of the two force vectors to obtain a resultant force vector $\vec{R}$. Using this vector polygon, we determine R and β as

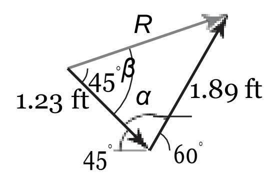


$$R = \sqrt{(35\text{ kN})^2 + (18\text{ kN})^2} = 39.36\text{ kN}; \quad \beta = \tan^{-1}\left(\frac{35\text{ kN}}{18\text{ kN}}\right) = 62.78^\circ. \quad (1)$$

The direction for $\vec{R}$ measured from the right-hand horizontal direction is $180^\circ - 62.78^\circ = 117.2^\circ$. Therefore, the polar vector representation for $\vec{R}$ is

$$\vec{R} = 39.4\text{ kN} @ 117^\circ \measuredangle. \quad (2)$$

Part (b) The vector polygon shown at the right corresponds to the addition of the two position vectors to obtain a resultant position vector $\vec{R}$. We observe from this vector polygon that $\gamma = 180^\circ - 60^\circ - 45^\circ = 75^\circ$. Using the law of cosines



$$R = \sqrt{(1.23\text{ ft})^2 + (1.89\text{ ft})^2 - 2(1.23\text{ ft})(1.89\text{ ft})\cos 75^\circ} = 1.970\text{ ft}. \quad (3)$$

Next, use the law of sines to find β, such that

$$\frac{1.89\text{ ft}}{\sin\beta} = \frac{R}{\sin\gamma} \quad \Rightarrow \quad \beta = \sin^{-1}\left[\frac{1.89\text{ ft}}{1.970\text{ ft}}\sin 75^\circ\right] = 67.91^\circ. \quad (4)$$

The direction of $\vec{R}$ measured from the right-hand horizontal direction is $67.91^\circ - 45^\circ = 22.91^\circ$. Therefore, the polar vector representation of $\vec{R}$ is

$$\vec{R} = 1.97\text{ ft} @ 22.9^\circ \measuredangle. \quad (5)$$

Problem 2.8

Let $\vec{A} = 2\,\mathrm{m} \angle 0^\circ$ and $\vec{B} = 6\,\mathrm{m} \angle 90^\circ$. Sketch the vector polygons and evaluate $\vec{R}$ for the following, reporting your answer using polar vector representation.

(a) $\vec{R} = \vec{A} + \vec{B}$,

(b) $\vec{R} = 2\vec{A} - \vec{B}$,

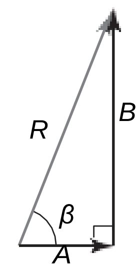
(c) $\vec{R} = |\vec{A}|\vec{B} + |\vec{B}|\vec{A}$,

(d) $\vec{R} = \dfrac{\vec{A}}{|\vec{A}|} + \dfrac{\vec{B}}{|\vec{B}|}$.

Solution

Part (a) The vector polygon is shown to the right. The magnitude R of vector $\vec{R}$ is given by

$$R = \sqrt{A^2 + B^2} = \sqrt{(2\,\mathrm{m})^2 + (6\,\mathrm{m})^2} = 6.325\,\mathrm{m}. \qquad (1)$$



Referring to the figure again, we find β in the following manner:

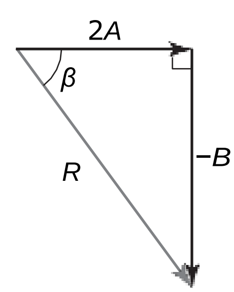
$$R\cos\beta = A \quad \Rightarrow \quad \beta = \cos^{-1}\left(\frac{2\,\mathrm{m}}{6.325\,\mathrm{m}}\right) = 71.57^\circ. \qquad (2)$$

The polar vector representation of $\vec{R}$ is

$$\boxed{\vec{R} = 6.32\,\mathrm{m} \angle 71.6^\circ.} \qquad (3)$$

Part (b) Referring to the vector polygon shown at the right, we determine the values for R and β as

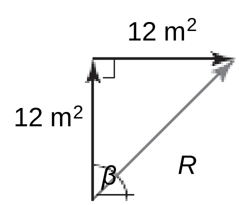
$$R = \sqrt{(2\cdot 2\,\mathrm{m})^2 + (6\,\mathrm{m})^2} = 7.211\,\mathrm{m}; \qquad \beta = \sin^{-1}\left(\frac{6\,\mathrm{m}}{7.211\,\mathrm{m}}\right) = 56.31^\circ. \qquad (4)$$



The polar vector representation of $\vec{R}$ is

$$\boxed{\vec{R} = 7.21\,\mathrm{m} \angle -56.3^\circ.} \qquad (5)$$

Part (c) Each vector $|\vec{A}|\vec{B}$ and $|\vec{B}|\vec{A}$ has a magnitude of $12\,\mathrm{m}^2$; since they are perpendicular to one another, it follows that $\beta = 45^\circ$ and $\gamma = 45^\circ$. The magnitude of R is given by

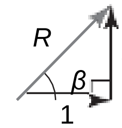


$$R = \sqrt{(12\ \mathrm{m}^2)^2 + (12\ \mathrm{m}^2)^2} = 16.97\ \mathrm{m}^2. \qquad (6)$$

θ

The polar vector representation of $\vec{R}$ is

$$\boxed{\vec{R} = 17.0\ \mathrm{m}^2\ @\ 45.0^\circ \measuredangle.} \qquad (7)$$



Part (d) Each vector $\vec{A}/|\vec{A}|$ and $\vec{B}/|\vec{B}|$ has a magnitude of one; since they are perpendicular to one another, it follows that $\beta = 45°$. The magnitude and polar vector representation of R are

$$R = \sqrt{(1)^2 + (1)^2} = 1.414. \tag{8}$$

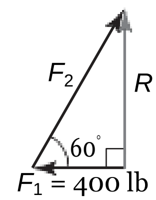
$$\boxed{\vec{R} = 1.41 \ @ \ 45.0° \ \measuredangle.} \tag{9}$$

Problem 2.9

A tow truck applies forces $\vec{F}_1$ and $\vec{F}_2$ to the bumper of an automobile where $\vec{F}_1$ is horizontal. Determine the magnitude of $\vec{F}_2$ that will provide a vertical resultant force, and determine the magnitude of this resultant.

Solution

The resultant force is defined as $\vec{R} = \vec{F}_1 + \vec{F}_2$, and this resultant is to be vertical. The force polygon is shown at the right. Since F_1 is given,



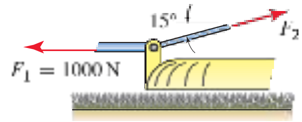
$$F_2 \cos 60^\circ = 400\,\text{lb} \quad \Rightarrow \quad F_2 = \frac{400\,\text{lb}}{\cos 60^\circ} = 800\,\text{lb}. \tag{1}$$

It then follows that R is given by

$$R = F_2 \sin 60^\circ = (800\ \text{lb}) \sin 60^\circ = 693\ \text{lb}. \tag{2}$$

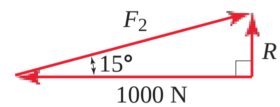
Problem 2.10

One of the support brackets for the lawn mowing deck of a garden tractor is shown where $\vec{F}_1$ is horizontal. Determine the magnitude of $\vec{F}_2$ so that the resultant of these two forces is vertical, and determine the magnitude of this resultant.



Solution

The vector polygon corresponding to the addition of $\vec{F}_1$ and $\vec{F}_2$ is shown at the right, where, as given in the problem statement, $\vec{R}$ is vertical. Thus,

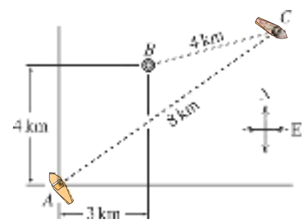


$$F_2 \cos 15^\circ = 1000\,\text{N} \quad \Rightarrow \quad F_2 = 1035\,\text{N} \tag{1}$$

$$R = F_2 \sin 15^\circ = (1035\,\text{N}) \sin 15^\circ = 267.9\,\text{N} \tag{2}$$

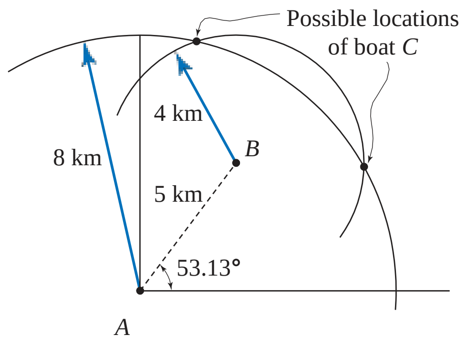
Problem 2.11

A buoy at point B is located 3 km east and 4 km north of boat A. Boat C is located 4 km from the buoy and 8 km from boat A. Determine the possible position vectors that give the position from boat A to boat C, $\vec{r}_{AC}$. State your answers using polar vector representation.



Solution

The locations of boat A and buoy B are shown. To determine the possible locations of boat C, we draw a circle with 8 km radius with center at A, and we draw a circle with 6 km radius with center at B; the intersections of these two circles are possible locations of boat C.



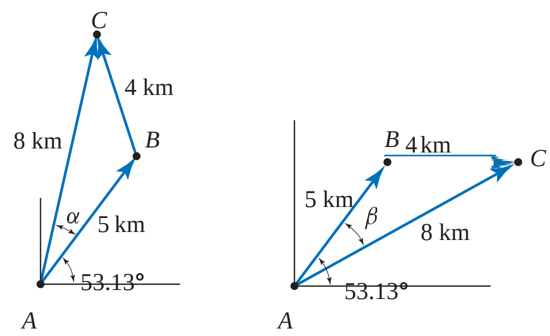
$$\tan^{-1}\frac{4\text{ km}}{3\text{ km}} = 53.13°;$$

$$r_{AB} = \sqrt{(3\text{ km})^2 + (4\text{ km})^2} = 5\text{ km}.$$

The two vector polygons corresponding to

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} \tag{1}$$

are shown below



For the vector polygon shown at the left, the law of cosines provides

$$(4\text{ km})^2 = (5\text{ km})^2 + (8\text{ km})^2 - 2(5\text{ km})(8\text{ km})\cos\alpha; \tag{2}$$

$$\alpha = \cos^{-1}\left[\frac{(4\text{ km})^2 - (5\text{ km})^2 - (8\text{ km})^2}{-2(5\text{ km})(8\text{ km})}\right] = 24.15°. \tag{3}$$

Hence, one of the possible position vectors from boat A to boat C is

$$r_{AC} = 8 \text{ km} @ \angle + 53.13^\circ \tag{4}$$

$$= 8 \text{ km} @ 77.28^\circ \tag{5}$$

For the vector polygon shown at the right, the law of cosines provides

$$(4\text{ km})^2 = (5\text{ km})^2 + (8\text{ km})^2 - 2(5\text{ km})(8\text{ km})\cos\theta; \tag{6}$$

$$\theta = \cos^{-1}\left[\frac{(4\text{ km})^2 - (5\text{ km})^2 - (8\text{ km})^2}{-2(5\text{ km})(8\text{ km})}\right] = 24.15^\circ. \tag{7}$$

Hence, the other possible position vector from boat A to boat C is

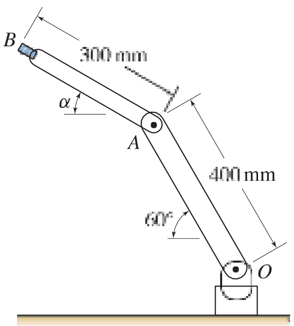
$$\vec{r}_{AC} = 8\text{ km} @ 53.13^\circ - \theta\,\measuredangle \tag{8}$$

$$= 8\text{ km} @ 28.98^\circ\,\measuredangle. \tag{9}$$

Remark: Equations (2) and (6) are identical, and hence $\alpha = \theta = 24.15^\circ$. In fact, Eq. (2) has multiple solutions, two of which are $\alpha = \pm 24.15^\circ$. Using this result, we could have arrived with both answers to this problem, namely Eqs. (5) and (9).

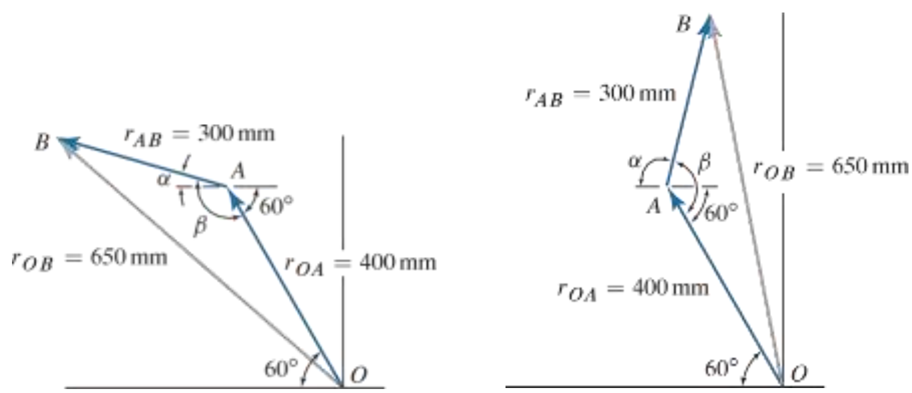
Problem 2.12

Arm OA of a robot is positioned as shown. Determine the value for angle α of arm AB so that the distance from point O to the actuator at B is 650 mm.



Solution

The two vector polygons shown below illustrate the addition $\vec{r}_{OB} = \vec{r}_{OA} + \vec{r}_{AB}$. These vector polygons show the two possible positions of arm AB such that the distance between points O and B is 650 mm.



First vector polygon: Applying the law of cosines, we obtain

$$650\text{ mm} = \sqrt{(300\text{ mm})^2 + (400\text{ mm})^2 - 2(300\text{ mm})(400\text{ mm})\cos\beta}. \tag{1}$$

By squaring both sides and solving for β, we find that

$$\beta = \cos^{-1}\left[\frac{(650\text{ mm})^2 - (300\text{ mm})^2 - (400\text{ mm})^2}{-2(300\text{ mm})(400\text{ mm})}\right] \tag{2}$$

$$= \cos^{-1}(-23/32) \tag{3}$$

$$= 136.0^\circ \tag{4}$$

To determine α, observe that

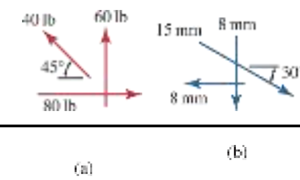
$$180^\circ = \beta - \alpha + 60^\circ \quad \Rightarrow \quad \alpha = \beta + 60^\circ - 180^\circ = 16.0^\circ. \tag{5}$$

Second vector polygon: Using the second vector polygon, Eq. (1) is still valid, which again provides $\beta = 136.0^\circ$. Thus,

$$180^\circ = \alpha + \beta - 60^\circ \quad \Rightarrow \quad \alpha = 180^\circ - \beta + 60^\circ = 104^\circ. \tag{6}$$

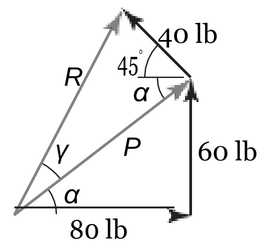
Problem 2.13

Add the three vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution

Part (a) The vector polygon shown at the right corresponds to the addition of the three force vectors to obtain a resultant force vector $\vec{R}$. Although our goal is to determine $\vec{R}$, we will begin by determining $\vec{P}$. The magnitude of $\vec{P}$ is given by



$$P = \sqrt{(60\ \text{lb})^2 + (80\ \text{lb})^2} = 100\ \text{lb}. \tag{1}$$

The angle α is found by

$$\tan\alpha = \frac{60\,\text{lb}}{80\,\text{lb}} \Rightarrow \alpha = \tan^{-1}\left(\frac{60\,\text{lb}}{80\,\text{lb}}\right) = 36.87^\circ. \tag{2}$$

Next, use the law of cosines to find R

$$R = \sqrt{P^2 + (40\ \text{lb})^2 - 2P(40\ \text{lb})\cos(45^\circ + \alpha)} = 102.3\ \text{lb}. \tag{3}$$

Use the law of sines to find γ

$$\frac{R}{\sin(45^\circ + \alpha)} = \frac{40\,\text{lb}}{\sin\gamma} \Rightarrow \gamma = \sin^{-1}\left(\frac{40\,\text{lb}\sin\beta}{R}\right) = 22.77^\circ. \tag{4}$$

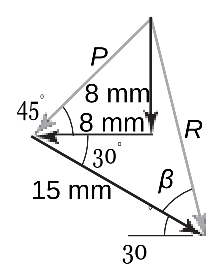
In polar vector representation, the direction of $\vec{R}$ measured from the right-hand horizontal direction is given by the sum of α and γ, such that

$$\boxed{\vec{R} = 102\,\text{lb} \ @\ 59.6^\circ \measuredangle.} \tag{5}$$

Part (b) The vector polygon shown at the right corresponds to the addition of the three position vectors to obtain a resultant position vector $\vec{R}$. By inspection, the angle of $\vec{P}$ is 45°, while P is given given by

$$P = \sqrt{(8\ \text{mm})^2 + (8\ \text{mm})^2} = 11.31\ \text{mm}. \tag{6}$$

The law of cosines is used to find R



$$R = \sqrt{P^2 + (15\ \text{mm})^2 - 2P(15\ \text{mm})\cos(45^\circ + 30^\circ)} = 16.28\ \text{mm}. \tag{7}$$

The law of sines is used to determine the angle β as

$$\frac{P}{\sin\beta} = \frac{R}{\quad} \qquad \sin^{-1}\left(\frac{11.31\,\text{mm}\sin(75^\circ)}{\quad}\right)$$

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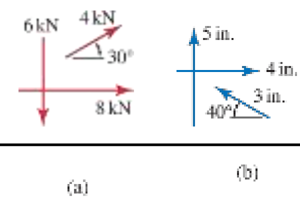
16:28 mm

The direction of $\vec{R}$ measured from the right-hand horizontal direction is given by $-\theta - 30^\circ = -72.15^\circ$, and the polar vector representation of $\vec{R}$ is

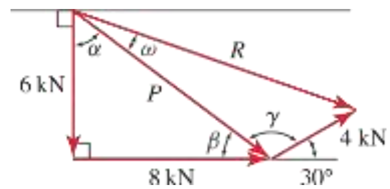
$$\vec{R} = 16.3 \text{ mm} @ -72.2^\circ \quad . \tag{9}$$

Problem 2.14

Add the three vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution



Part (a) The vector polygon shown at the right corresponds to the addition of the three force vectors to obtain a resultant force vector $\vec{R}$. Although our goal is to determine $\vec{R}$, we will begin by determining $\vec{P}$. The magnitude of $\vec{P}$ is

$$P = \sqrt{(6\text{ kN})^2 + (8\text{ kN})^2} = 10\text{ kN}. \qquad (1)$$

The angle α is found by

$$\tan\alpha = \frac{8\text{kN}}{6\text{kN}} \quad\Rightarrow\quad \alpha = \tan^{-1}\frac{8\text{ kN}}{6\text{ kN}} = 53.13^\circ; \qquad (2)$$

and then, noting that $\alpha + \beta + 90^\circ = 180^\circ$,

$$\beta = 180^\circ - \alpha - 90^\circ = 36.87^\circ. \qquad (3)$$

Considering the triangle formed by the 4 kN force, P, and R, the angle θ is obtained from $\beta + \theta + 30^\circ = 180^\circ$ as

$$\theta = 180^\circ - \beta - 30^\circ = 113.1^\circ. \qquad (4)$$

Using the law of cosines

$$R = \sqrt{P^2 + (4\text{ kN})^2 - 2P(4\text{ kN})\cos\theta} = 12.14\text{ kN}. \qquad (5)$$

Using the law of sines, the angle ω is obtained as follows

$$\frac{4\text{ kN}}{\sin\omega} = \frac{R}{\sin\theta}, \qquad (6)$$

$$\omega = \sin^{-1}\left(\frac{4\text{ kN}}{12.14\text{ kN}}\sin 113.1^\circ\right) = 17.64^\circ. \qquad (7)$$

Using polar vector representation, the resultant force vector is

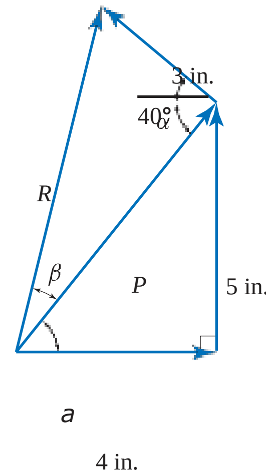
$$\vec{R} = 12.14\text{ kN} \,@\, -(90^\circ - \alpha - \omega)\,\measuredangle$$
$$= 12.14\text{ kN} \,@\, -19.23^\circ\,\measuredangle.$$

If desired, this resultant may be stated using a positive angle, where $360^\circ - 19.23^\circ = 340.8^\circ$, as

$$14\ \text{kN} \ @\ 340.8^\circ \quad . \tag{8}$$

(9)

(10)



Part (b) The vector polygon shown at the right corresponds to the addition of the three position vectors to obtain a resultant position vector $\vec{R}$. Although our goal is to determine $\vec{R}$, we will begin by determining $\vec{P}$. The magnitude of $\vec{P}$ is

$$P = \sqrt{(4\text{ in.})^2 + (5\text{ in.})^2} = 6.403\text{ in.} \tag{11}$$

The angle α is found by

$$\tan\alpha = \frac{5\text{ in.}}{4\text{ in.}} \quad\Rightarrow\quad \alpha = \tan^{-1}\frac{5\text{ in.}}{4\text{ in.}} = 51.34°. \tag{12}$$

Considering the triangle formed by the 3 in. position vector, P, and R, the law of cosines may be used to obtain

$$R = \sqrt{(3\text{ in.})^2 + P^2 - 2(3\text{ in.})P\cos(40° + \alpha)} = 7.134\text{ in.}, \tag{13}$$

and the law of sines may be used to determine the angle β as

$$\frac{3\text{ in.}}{\sin\beta} = \frac{R}{\sin(40° + \alpha)} \quad\Rightarrow\quad \beta = \sin^{-1}\left[\frac{3\text{ in.}}{7.134\text{ in.}}\sin(40° + 51.34°)\right] = 24.86°. \tag{14}$$

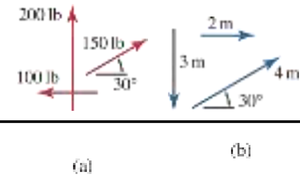
Using polar vector representation, the resultant position vector is

$$\vec{R} = 7.134\text{ in. @ } \alpha + \beta\ \measuredangle \tag{15}$$

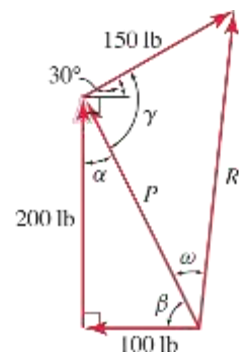
$$= 7.134\text{ in. @ } 76.20°\ \measuredangle. \tag{16}$$

Problem 2.15

Add the three vectors shown to form a resultant vector $\vec{R}$, and report your result using polar vector representation.



Solution



Part (a) The vector polygon shown at the right corresponds to the addition of the three force vectors to obtain a resultant force vector $\vec{R}$. Although our goal is to determine $\vec{R}$, we will begin by determining $\vec{P}$. The magnitude of $\vec{P}$ is

$$P = \sqrt{(100\ \text{lb})^2 + (200\ \text{lb})^2} = 223.6\ \text{lb}. \tag{1}$$

The angle α is found by

$$\tan\alpha = \frac{100\ \text{lb}}{200\ \text{lb}} \quad\Rightarrow\quad \alpha = \tan^{-1}\frac{100\ \text{lb}}{200\ \text{lb}} = 26.57°, \tag{2}$$

and then, noting that $\alpha + \gamma + 90° = 180°$,

$$\gamma = 180° - \alpha - 90° = 63.43°. \tag{3}$$

Considering the triangle formed by the 150 lb force, P, and R, the angle θ is obtained from $\theta + \alpha = 90° + 30°$ as

$$\theta = 90° + 30° - \alpha = 93.43°. \tag{4}$$

Using the law of cosines

$$R = \sqrt{(150\ \text{lb})^2 + P^2 - 2(150\ \text{lb})P\cos\theta} = 276.6\ \text{lb}. \tag{5}$$

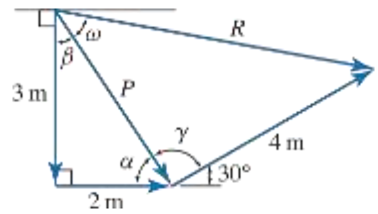
Using the law of sines, the angle ω is obtained from

$$\frac{150\ \text{lb}}{\sin\omega} = \frac{R}{\sin\theta} \quad\Rightarrow\quad \omega = \sin^{-1}\frac{150\ \text{lb}}{276.6\ \text{lb}}\sin 93.43° = 32.77°. \tag{6}$$

Using polar vector representation, the resultant force is

$$\vec{R} = 276.6\ \text{lb} \ @\ 180° - \gamma - \omega \ \angle \tag{7}$$

$$= 276.6\ \text{lb} \ @\ 83.79° \ \angle. \tag{8}$$



Part (b) The vector polygon shown at the right corresponds to the addition of the three position vectors to obtain a resultant position vector $\vec{R}$. Although our goal is to determine $\vec{R}$, we will begin by determining $\vec{P}$. The magnitude of $\vec{P}$ is

$$P = \sqrt{(2\text{ m})^2 + (3\text{ m})^2} = 3.606\text{ m}. \tag{9}$$

The angle α is found by

$$\tan\alpha = \frac{3\text{ m}}{2\text{ m}} \quad\Rightarrow\quad \alpha = \tan^{-1}\frac{3\text{ m}}{2\text{ m}} = 56.31°; \tag{10}$$

and then, noting that $\alpha + \beta + 90° = 180°$,

$$\beta = 180° - \alpha - 90° = 33.69°. \tag{11}$$

Considering the triangle formed by the 4 m position vector, P, and R, the angle γ is obtained from $\alpha + \gamma + 30° = 180°$ as

$$\gamma = 180° - \alpha - 30° = 93.69°. \tag{12}$$

Using the law of cosines

$$R = \sqrt{(4\text{ m})^2 + P^2 - 2(4\text{ m})P\cos\gamma} = 5.555\text{ m}. \tag{13}$$

Using the law of sines,

$$\frac{4\text{ m}}{\sin\omega} = \frac{R}{\sin\gamma} \quad\Rightarrow\quad \omega = \sin^{-1}\frac{4\text{ m}}{5.555\text{ m}}\sin 93.69° = 45.94°. \tag{14}$$

Using polar vector representation, the resultant position vector is

$$\vec{R} = 5.555\text{ m} @ -(90° - \beta - \omega)\ \angle \tag{15}$$

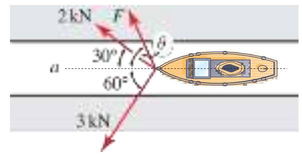
$$= 5.555\text{ m} @ -10.37°\ \angle. \tag{16}$$

If desired, the resultant may be stated using a positive angle, where $360° - 10.37° = 349.6°$ as

$$\vec{R} = 5.555\text{ m} @ 349.6°\ \angle. \tag{17}$$

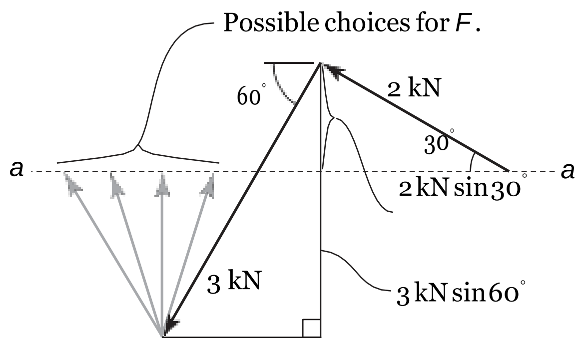
Problem 2.16

A ship is towed through a narrow channel by applying forces to three ropes attached to its bow. Determine the magnitude and orientation θ of the force $\vec{F}$ so that the resultant force is in the direction of line *a* and the magnitude of $\vec{F}$ is as small as possible.



Solution

The force polygon shown at the right corresponds to the addition of the forces applied by the three ropes to the ship. In sketching the force polygon, the known force vectors are sketched first (i.e., the 2 kN and 3 kN forces). There are many possible choices of $\vec{F}$ such that the resultant force will be parallel to line *a*. The smallest value of F occurs when $\vec{F}$ is perpendicular to line *a*; i.e., when



$$\theta = 90^\circ. \tag{1}$$

The magnitude of $\vec{F}$ is then found by using the force polygon to write

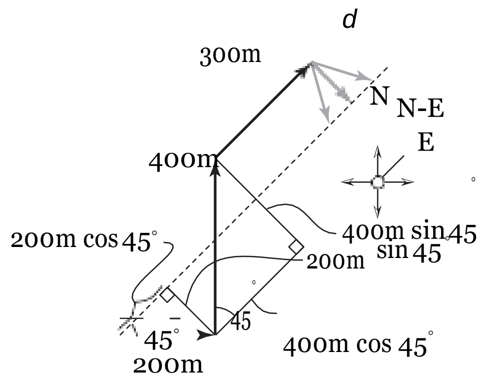
$$F = (3\text{ kN})\sin 60^\circ - (2\text{ kN})\sin 30^\circ = 1.60\text{ kN}. \tag{2}$$

Problem 2.17

A surveyor needs to plant a marker directly northeast from where she is standing. Because of obstacles, she walks a route in the horizontal plane consisting of *200* m east, followed by *400* m north, followed by *300* m northeast. From this position, she would like to take the shortest-distance route back to the line that is directly northeast of her starting position. What direction should she travel and how far, and what will be her final distance from her starting point?

Solution

The vector polygon shown at the right corresponds to the addition of the four position vectors corresponding to the path walked by the surveyor. The first three position vectors take the surveyor to the point at which she begins to travel back to the line that is directly north-east of her starting position (this direction is shown as a dashed line in the vector polygon). The path she takes to reach this line has distance d, and several possibilities are shown. By examining the vector polygon, the smallest value of d results when she travels directly south-east, in which case d is given by



$$d = (400\text{ m})\sin 45^\circ - (200\text{ m})\sin 45^\circ = 141\text{ m}. \tag{1}$$

To summarize,

$$\boxed{\text{The surveyor should walk } 141 \text{ m in the S-E direction.}} \tag{2}$$

The distance R from her starting point to her final position is given by

$$\boxed{R = (200\text{ m})\cos 45^\circ + (400\text{ m})\cos 45^\circ + 300\text{ m} = 724\text{ m}.} \tag{3}$$

Problem 2.18

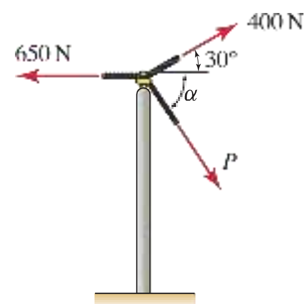
A utility pole supports a bundle of wires that apply the 400 and 650 N forces shown, and a guy wire applies the force $\vec{P}$.

(a) If $P = 0$, determine the resultant force applied by the wires to the pole and report your result using polar vector representation.

(b) Repeat Part (a) if $P = 500$ N and $\alpha = 60°$.

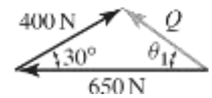
(c) With $\alpha = 60°$, what value of P will produce a resultant force that is vertical?

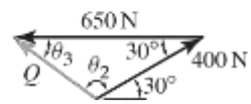
(d) If the resultant force is to be vertical and P is to be as small as possible, determine the value α should have and the corresponding value of P.



Solution

Part (a) Either of the force polygons shown at the right may be used to determine the resultant force Q. Regardless of which force polygon is used, the law of cosines provides





$$Q = \sqrt{(400\,\text{N})^2 + (650\,\text{N})^2 - 2(400\,\text{N})(650\,\text{N})\cos 30°} = 363.5\,\text{N} \quad (1)$$

Using the *first* force polygon shown, the law of sines is used to determine the angle θ_1 as

$$\frac{400\,\text{N}}{\sin\theta_1} = \frac{Q}{\sin 30°} \Rightarrow \theta_1 = \sin^{-1}\left(\frac{(400\,\text{N})\sin 30°}{363.5\,\text{N}}\right) = 33.38°. \quad (2)$$

The orientation of $\vec{Q}$ to be used for its polar vector representation is $180° - \theta_1 = 180° - 33.38° = 146.6°$, and hence the vector representation of $\vec{Q}$ is

$$\vec{Q} = 364\,\text{N} \;@\; 147° \measuredangle. \quad (3)$$

Alternatively, the *second* force polygon could be used. As discussed above, Eq. (1) still applies, and $Q = 363.3$ N. Because angle θ_2 appears to be obtuse, we will avoid using the law of sines to determine its value (see the discussion in the text regarding the pitfall when using the law of sines to determine an obtuse angle). Using the law of sines to determine angle θ_3 provides

$$\frac{400\,\text{N}}{\sin\theta_3} = \frac{Q}{\sin 30°} \Rightarrow \theta_3 = \sin^{-1}\left(\frac{(400\,\text{N})\sin 30°}{363.5\,\text{N}}\right) = 33.38°. \quad (4)$$

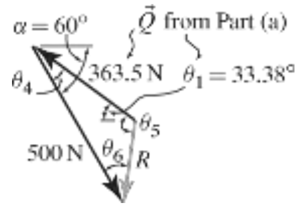
Once θ_3 is known, angle θ_2 is easily found as

$$\theta_2 = 180^\circ - \theta_3 - 30^\circ = 180^\circ - 33.38^\circ - 30^\circ = 116.6^\circ. \tag{5}$$

The orientation of $\vec{Q}$ to be used for its polar vector representation is $30^\circ + \theta_2 = 30^\circ + 116.6^\circ = 146.6^\circ$, and hence the vector representation of $\vec{Q}$ is

$$\boxed{\vec{Q} = 364\,\text{N} \,@\, 147^\circ \measuredangle.} \tag{6}$$

As expected, the same result for $\vec{Q}$ is obtained regardless of which force polygon was used.



Part (b) Our strategy will be to add the force vector $\vec{F}$ to the result for $\vec{Q}$ obtained in Part (a). Thus, the force polygon is shown at the right, where Q from Eq. (1) and θ_1 from Eq. (2) are used, such that

$$\theta_4 = 60^\circ - \theta_1 = 60^\circ - 33.38^\circ = 26.62^\circ. \tag{7}$$

The law of cosines may be used to find R:

$$R = \sqrt{(500\ \text{N})^2 + (363.5\ \text{N})^2 - 2(500\ \text{N})(363.3\ \text{N})\cos\theta_4} = 239.1\ \text{N}. \tag{8}$$

Since θ_5 is obtuse, we will avoid using the law of sines to determine it, and instead will use the law of sines to determine θ_6, as follows

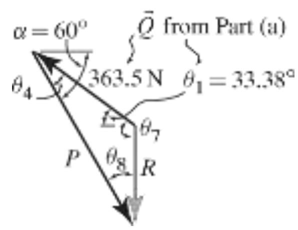
$$\frac{R}{\sin\theta_4} = \frac{363.5\,\text{N}}{\sin\theta_6} \quad\Rightarrow\quad \theta_6 = \sin^{-1}\left(\frac{(363.5\ \text{N})\sin\theta_4}{R}\right) = 42.95^\circ. \tag{9}$$

The angle θ_5 is given by

$$\theta_5 = 180^\circ - \theta_4 - \theta_6 = 110.4^\circ. \tag{10}$$

The orientation of $\vec{R}$ relative to the right-hand horizontal direction is the sum of the orientation of $\vec{Q}$ obtained in Part (a), namely 146.6°, plus θ_5. Thus

$$\boxed{\vec{R} = 239\,\text{N} \ @\ 257^\circ \measuredangle.} \tag{11}$$



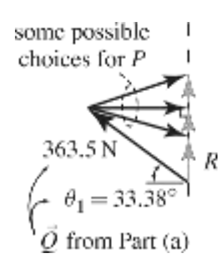
Part (c) The force polygon is shown at the right, where angle $\theta_4 = 26.62^\circ$ was determined in Eq. (7). For the resultant force $\vec{R}$ to be vertical, $\theta_7 = 90^\circ + \theta_1 = 90^\circ + 33.38^\circ = 123.4^\circ$ Thus

$$\theta_8 = 180^\circ - \theta_4 - \theta_7 = 30^\circ. \tag{12}$$

The law of sines is used to determine P as

$$\frac{363.5\,\text{N}}{\sin\theta_8} = \frac{P}{\sin\theta_7} \tag{13}$$

$$\boxed{\Rightarrow\quad P = (363.5\,\text{N})\frac{\sin 123.4^\circ}{\sin 30^\circ} = 607\,\text{N}.} \tag{14}$$



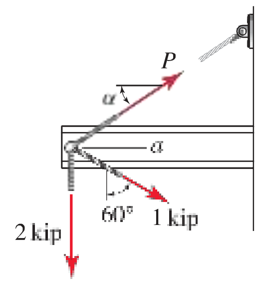
Part (d) Using the results for $\vec{Q}$ from Part (a), and if the resultant force is to be vertical, then the force polygon is as shown at the right; three possible choices (among many possibilities) for P along with the corresponding resultant force are shown. The smallest value of P occurs when $\vec{F}$ is perpendicular to $\vec{R}$, hence

$$\boxed{\alpha = 0^\circ.} \tag{15}$$

For this value of θ,

$$P = (363.5\ \text{N})\cos 33.38^\circ = 304\ \text{N}. \tag{16}$$

Problem 2.19



The end of a cantilever I beam supports forces from three cables.

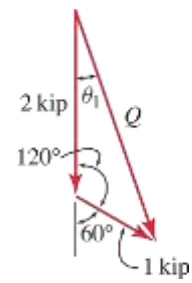
(a) If $P = 0$, determine the resultant force applied by the two cables to the I beam and report your result using polar vector representation.

(b) Repeat Part (a) if $P = 1.5$ kip and $\alpha = 30^\circ$.

(c) With $\alpha = 30^\circ$, what value of P will produce a resultant force that is horizontal?

(d) If the resultant force is to be horizontal and P is to be as small as possible, determine the value α should have and the corresponding value of P.

Solution



Part (a) The force polygon shown at the right may be used to determine the resultant force Q, Noting that the angle opposite Q is $180^\circ - 60^\circ = 120^\circ$; the law of cosines may be used to obtain

$$Q = \sqrt{(1\,\text{kip})^2 + (2\,\text{kip})^2 - 2(1\,\text{kip})(2\,\text{kip})\cos 120^\circ} = 2.646\,\text{kip}. \tag{1}$$

Using the law of sines, the angle θ_1, is obtained as follows

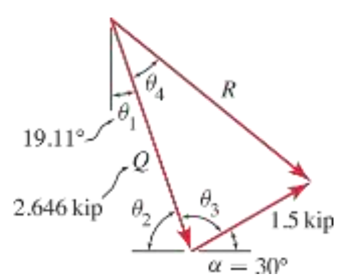
$$\frac{1\,\text{kip}}{\sin\theta_1} = \frac{Q}{\sin 120^\circ} \Rightarrow \theta_1 = \sin^{-1}\left[\frac{1\,\text{kip}}{2.646\,\text{kip}}\sin 120^\circ\right] = 19.11^\circ. \tag{2}$$

Using polar vector representation, the resultant force is

$$\vec{Q} = 2.646\,\text{kip} \ @ \ -(90^\circ - \theta_1) \measuredangle \tag{3}$$
$$= 2.646\,\text{kip} \ @ \ -70.89^\circ \measuredangle. \tag{4}$$

If desired, the resultant force may be stated using a positive angle, where $360^\circ - 70.89^\circ = 289.1^\circ$; as

$$\vec{Q} = 2.646\,\text{kip} \ @ \ 289.1^\circ \measuredangle. \tag{5}$$



Part (b) Our strategy will be to add the force vector $\vec{P}$ to the result for $\vec{Q}$ obtained in Part (a). Thus, the force polygon is shown at the right where Q from Eq. (1) and θ_1 from Eq. (2) are used, and θ_2 is obtained from $\theta_1 + \theta_2 + 90^\circ = 180^\circ$ which provides

$$\theta_2 = 180^\circ - \theta_1 - 90^\circ = 70.89^\circ. \tag{6}$$

The angle opposite force R is obtained by using $\theta_2 + \theta_3 + 30^\circ = 180^\circ$; which provides

$$\theta_3 = 180^\circ - \theta_2 - 30^\circ = 79.11^\circ. \tag{7}$$

Using the law of cosines, the resultant force is

$$R = \sqrt{(1.5\ \text{kip})^2 + (2.646\ \text{kip})^2 - 2(1.5\ \text{kip})(2.646\ \text{kip})\cos\theta_3} = 2.784\ \text{kip}. \tag{8}$$

Using the law of sines, angle θ_4 may be determined

$$\frac{1.5\,\text{kip}}{\sin\theta_4} = \frac{R}{\sin\theta_3} \;\Rightarrow\; \theta_4 = \sin^{-1}\left[\frac{1.5\,\text{kip}}{2.784\,\text{kip}}\sin 79.11^\circ\right] = 31.95^\circ. \tag{9}$$

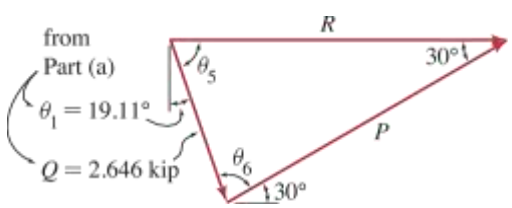
Using polar vector representation, the resultant force is

$$\vec{R} = 2.784\,\text{kip} @ -(90^\circ - \theta_1 - \theta_4) \circlearrowleft \tag{10}$$
$$= 2.784\,\text{kip} @ -38.95^\circ \circlearrowleft. \tag{11}$$

If desired, the resultant may be stated using a positive angle, where $360^\circ - 38.95^\circ = 321.1^\circ$, as

$$\vec{R} = 2.784\,\text{kip} @ 321.1^\circ \circlearrowleft. \tag{12}$$

Part (c) The force polygon is shown below



For the resultant force R to be horizontal, using $\theta_1 + \theta_5 = 90^\circ$, we obtain

$$\theta_5 = 90^\circ - \theta_1 = 70.89^\circ, \tag{13}$$

and noting that $\theta_5 + \theta_6 + 30^\circ = 180^\circ$, we obtain

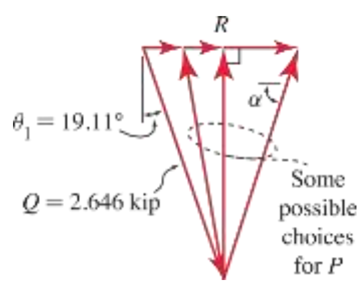
$$\theta_6 = 180^\circ - \theta_5 - 30^\circ = 79.11^\circ. \tag{14}$$

Using the law of sines, with $Q = 2.646$ kip from Part (a),

$$\frac{R}{\sin\theta_6} = \frac{Q}{\sin 30^\circ}, \tag{15}$$

$$R = 2.646\ \text{kip}\,\frac{\sin 79.11^\circ}{\sin 30^\circ} = 5.196\,\text{kip}. \tag{16}$$

Part (d) Using the results for Q from Part (a), and if the resultant force is to be horizontal, then the force polygon is shown at the right; three possible choices (among many possibilities) for P along with the corresponding resultant force R are shown. The smallest value of P occurs when $\vec{P}$ is perpendicular to $\vec{R}$, hence



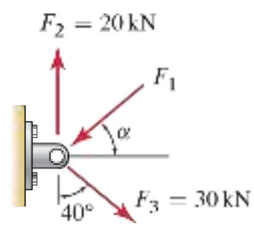
$$\phi = 90^\circ; \tag{17}$$

and

$$\boxed{P = (2.646\ \text{kip})\cos 19.11^\circ = 2.500\ \text{kip}.} \tag{18}$$

Problem 2.20

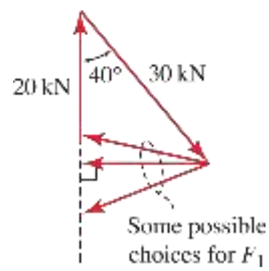
Determine the smallest force F_1 such that the resultant of the three forces F_1, F_2, and F_3 is vertical, and the angle α at which F_1 should be applied.



Solution

The force polygon, including various choices for $\vec{F}_1$, is shown at the right. The smallest value of F_1 occurs when the vector $\vec{F}_1$ is horizontal, hence

$$\alpha = 0^\circ; \tag{1}$$

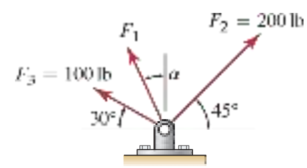


and the force is

$$F_1 = (30 \text{ kN}) \sin 40^\circ = 19.28 \text{ kN}. \tag{2}$$

Problem 2.21

Determine the smallest force F_1 such that the resultant of the three forces F_1, F_2, and F_3 is vertical, and the angle α at which F_1 should be applied.

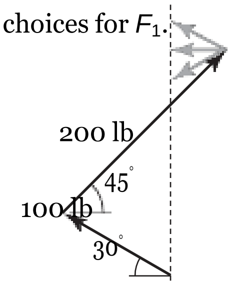


Solution

The force polygon, including various choices for $\vec{F}_1$, is shown to the right. The smallest value of F_1 occurs when the vector $\vec{F}_1$ is horizontal, i.e., when

$$\alpha = 90^\circ. \qquad (1)$$

Possible choices for F_1.

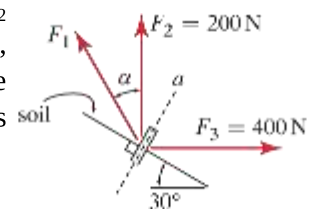


The value of F_1 is given by

$$F_1 = (200 \text{ lb}) \cos 45^\circ - (100 \text{ lb}) \cos 30^\circ = 54.8 \text{ lb}. \qquad (2)$$

Problem 2.22

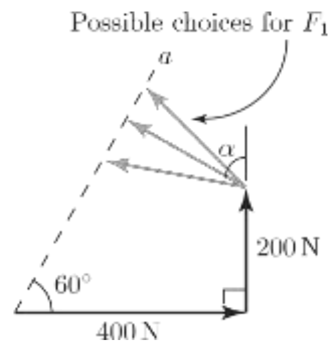
Forces F_1, F_2, and F_3 are applied to a soil nail to pull it out of a slope. If F_2 and F_3 are vertical and horizontal, respectively, with the magnitudes shown, determine the magnitude of the smallest force F_1 that can be applied and the angle α so that the resultant force applied to the nail is directed along the axis of the nail (direction a).



Solution

We begin by adding the two known forces, $\vec{F}_2$ and $\vec{F}_3$, as shown in the force polygon to the right. There are an infinite number of choices for $\vec{F}_1$, but we desire the one with the smallest magnitude. By examining the force polygon, F_1 is smallest when its direction is perpendicular to line a, i.e., when

$$\alpha = 60^\circ. \tag{1}$$

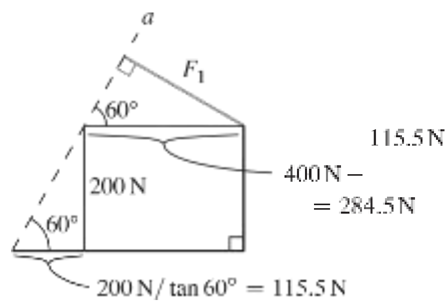


To determine the value of F_1, consider the sketch shown at the right. Noting that the hypotenuse of the upper triangle is given by

$$400\,\text{N} - \frac{200\,\text{N}}{\tan 60^\circ} = 400\,\text{N} - 115.5\,\text{N} = 284.5\,\text{N}; \tag{2}$$

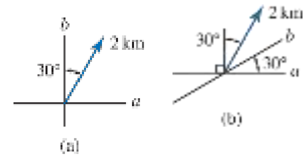
it follows that

$$F_1 = (284.5\,\text{N})\sin 60^\circ = 246\,\text{N}. \tag{3}$$



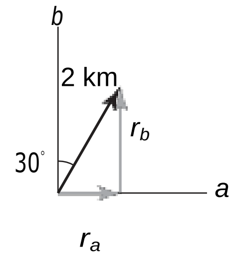
Problem 2.23

Determine the magnitudes of vectors $\vec{r}_a$ and $\vec{r}_b$ in the a and b directions, respectively, such that their sum is the 2 km position vector shown.



Solution

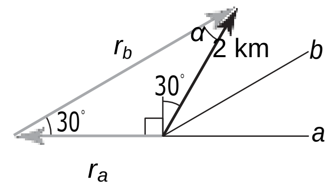
Part (a) Because the directions a and b of the two component vectors to be determined are orthogonal, determination of the magnitudes of the component vectors will be straightforward. The magnitudes r_a and r_b of vectors $\vec{r}_a$ and $\vec{r}_b$ are determined using



$$r_a = (2\,\text{km})\sin 30^\circ = 1.00\,\text{km}, \tag{1}$$

$$r_b = (2\,\text{km})\cos 30^\circ = 1.73\,\text{km}. \tag{2}$$

Part (b) Because the directions a and b of the two component vectors to be determined are not orthogonal, determination of the magnitudes of the component vectors will be slightly more work than for Part (a). Observe that



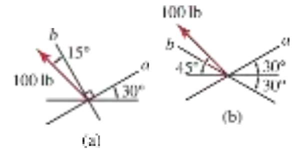
the angle $\alpha = 180^\circ - 30^\circ - 120^\circ = 30^\circ$. The magnitudes r_a and r_b of vectors $\vec{r}_a$ and $\vec{r}_b$ are determined using the law of sines to obtain

$$\frac{2\,\text{km}}{\sin 30^\circ} = \frac{r_b}{\sin 120^\circ} = \frac{r_a}{\sin 30^\circ} \quad \Rightarrow \quad r_a = -2.00\,\text{km}, \quad \text{and} \quad r_b = 3.46\,\text{km}, \tag{3}$$

where the negative sign is inserted for r_a since it acts in the negative a direction.

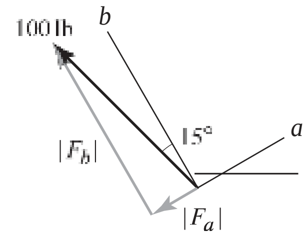
Problem 2.24

Determine the magnitudes of vectors $\vec{F}_a$ and $\vec{F}_b$ in the a and b directions, respectively, such that their sum is the 100 lb force vector shown.



Solution

Part (a) Let F_a and F_b be the components (scalars) of force vectors $\vec{F}_a$ and $\vec{F}_b$, respectively. These components are determined using



$$F_a = (-100\ \text{lb})\sin 15^\circ = -25.9\ \text{lb}; \tag{1}$$

$$F_b = (100\ \text{lb})\cos 15^\circ = 96.6\ \text{lb}; \tag{2}$$

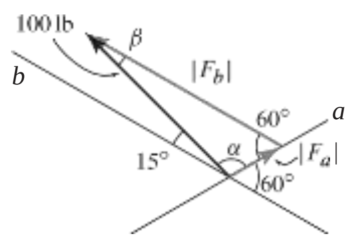
where F_a is negative since it acts in the negative a direction. Hence, the magnitudes of vectors $\vec{F}_a$ and $\vec{F}_b$ are

$$|\vec{F}_a| = 25.9\ \text{lb}; \tag{3}$$

$$|\vec{F}_b| = 96.6\ \text{lb}. \tag{4}$$

Part (b) It is necessary to determine α and β, by noting that

$$\alpha = 180^\circ - 15^\circ - 60^\circ = 105^\circ; \qquad \beta = 180^\circ - \alpha - 60^\circ = 15^\circ. \tag{5}$$



The law of sines may then be used to find the components F_a and F_b (scalars) of vectors $\vec{F}_a$ and $\vec{F}_b$ as

$$\frac{100\ \text{lb}}{\sin 60^\circ} = \frac{F_b}{\sin\alpha} = \frac{F_a}{\sin\beta} \quad \Rightarrow \quad F_a = 29.9\ \text{lb}; \quad F_b = 112\ \text{lb}. \tag{6}$$

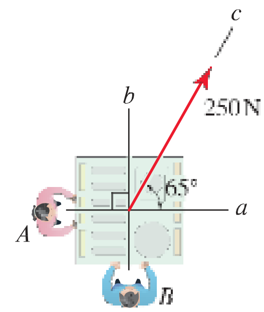
Hence, the magnitudes of vectors $\vec{F}_a$ and $\vec{F}_b$ are

$$|\vec{F}_a| = 29.9\ \text{lb}; \tag{7}$$

$$|\vec{F}_b| = 112\ \text{lb}. \tag{8}$$

Problem 2.25

The child's play structure from Examples 2.2 and 2.3 on pp. 38 and 39 is shown again. The woman at A applies a force in the a direction and the man at B applies a force in the b direction, with the goal of producing a resultant force of 250 N in the c direction. Determine the forces the two people must apply, expressing the results as vectors.

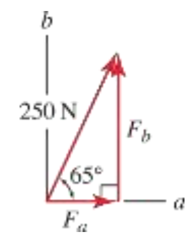


Solution

Let $\vec{F}$ denote the 250 N force vector acting in the c direction. Our objective is to determine the force vectors $\vec{F}_a$ acting in the a direction and $\vec{F}_b$ acting in the b direction such that

$$\vec{F} = \vec{F}_a + \vec{F}_b. \tag{1}$$

The force polygon corresponding to this addition is shown at the right. Since $\vec{F}_a$ and $\vec{F}_b$ are perpendicular, basic trigonometry provides



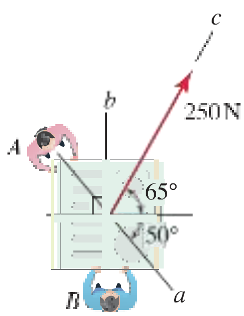
$$|F_a| = (250\text{ N})\cos 65^\circ = 105.7\text{ N} \tag{2}$$
$$|F_b| = (250\text{ N})\sin 65^\circ = 226.6\text{ N} \tag{3}$$

Using polar vector representation, the forces are

$$\vec{F}_a = 105.7\text{ N @ }0^\circ \measuredangle; \text{ and} \tag{4}$$
$$\vec{F}_b = 226.6\text{ N @ }90^\circ \measuredangle. \tag{5}$$

Problem 2.26

The child's play structure from Examples 2.2 and 2.3 on pp. 38 and 39 is shown again. The woman at *A* applies a force in the *a* direction and the man at *B* applies a force in the *b* direction, with the goal of producing a resultant force of 250 N in the *c* direction. Determine the forces the two people must apply, expressing the results as vectors.

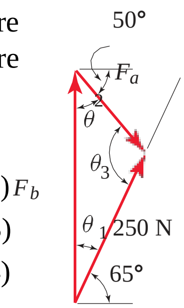


Solution

Let $\vec{F}$ denote the 250 N force vector acting in the *c* direction. Our objective is to determine the force vectors $\vec{F}_a$ acting in the *a* direction and $\vec{F}_b$ acting in the *b* direction such that

$$\vec{F} = \vec{F}_a + \vec{F}_b. \tag{1}$$

The force polygon corresponding to this addition is shown at the right. Since $\vec{F}_a$ and $\vec{F}_b$ are not perpendicular, the laws of sines and cosines must be used. The angles $\theta_1, \theta_2,$ and θ_3 are easily determined as



$$\theta_1 = 90^\circ - 65^\circ = 25^\circ, \tag{2}$$
$$\theta_2 = 90^\circ - 50^\circ = 40^\circ, \tag{3}$$
$$\theta_3 = 180^\circ - \theta_1 - \theta_2 = 115^\circ. \tag{4}$$

The law of sines provides

$$\frac{250\text{ N}}{\sin\theta_2} = \frac{|\vec{F}_a|}{\sin\theta_1} = \frac{|\vec{F}_b|}{\sin\theta_3}, \tag{5}$$

which yields

$$|\vec{F}_a| = 250\text{ N}\,\frac{\sin 25^\circ}{\sin 40^\circ} = 164.4\text{ N}, \tag{6}$$

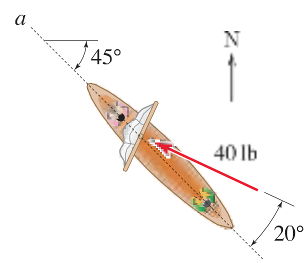
$$|\vec{F}_b| = 250\text{ N}\,\frac{\sin 115^\circ}{\sin 40^\circ} = 352.5\text{ N}. \tag{7}$$

Using polar vector representation, the forces are

$$\vec{F}_a = 164.4\text{ N} @ -50^\circ \measuredangle; \text{ and} \tag{8}$$
$$\vec{F}_b = 352.5\text{ N} @ 90^\circ \measuredangle. \tag{9}$$

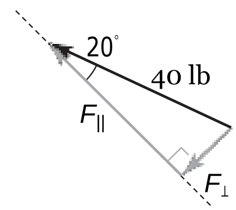
Problem 2.27

While canoes are normally propelled by paddle, if there is a favorable wind from the stern, adventurous users will sometimes employ a small sail. If a canoe is sailing north-west and the wind applies a 40 lb force perpendicular to the sail in the direction shown, determine the components of the wind force parallel and perpendicular to the keel of the canoe (direction *a*).



Solution

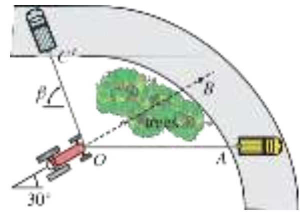
Let the force perpendicular to the keel be denoted by $F_\perp$ and the force parallel to the keel be denoted by $F_\parallel$. The sketch shown at the right illustrates the addition of these two forces to yield the 40 lb force applied to the sail. Thus,



$$F_\perp = (40\text{ lb})\sin 20^\circ = 13.7\text{ lb}, \quad (1)$$
$$F_\parallel = (40\text{ lb})\cos 20^\circ = 37.6\text{ lb}. \quad (2)$$

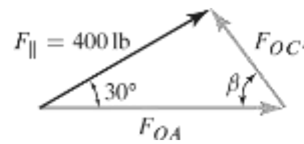
Problem 2.28

Repeat Part (b) of Example 2.5, using the optimization methods of calculus. *Hint:* Redraw the force polygon of Fig. 3 and rewrite Eq. (1) on p. 41 with the 45° angle shown there replaced by β, where β is defined in Fig. P2.28. Rearrange this equation to obtain an expression for $F_{OC'}$ as a function of β, and then determine the value of β that makes $dF_{OC'}/d\beta = 0$. While the approach described here is straightforward to carry out "by hand," you might also consider using symbolic algebra software such as Mathematica or Maple.



Solution

A relationship for $F_{OC'}$ in terms of $F_{\|}$ and β is needed, and this may be obtained using the force polygon shown at the right with the law of sines



$$\frac{400\text{ lb}}{\sin\beta} = \frac{F_{OC'}}{\sin 30^\circ} \quad\Rightarrow\quad F_{OC'} = (400\text{ lb})\frac{\sin 30^\circ}{\sin\beta}. \tag{1}$$

To determine the minimum value of $F_{OC'}$ as a function of β, we make $F_{OC'}$ stationary by setting its derivative with respect to β equal to zero; i.e.,

$$\frac{dF_{OC'}}{d\beta} = 0 = (400\text{ lb})(\sin 30^\circ)\frac{d}{d\beta}\left(\frac{1}{\sin\beta}\right) = (400\text{ lb})(\sin 30^\circ)\frac{\cos\beta}{\sin\beta\sin\beta}. \tag{2}$$

Satisfaction of Eq. (2) requires $\cos\beta = 0$, which gives $\beta = 90^\circ$. From Eq. (1) we obtain

$$\boxed{\beta = 90^\circ \quad\Rightarrow\quad F_{OC'} = (400\text{ lb})\frac{\sin 30^\circ}{\sin 90^\circ} = 200\text{ lb}.} \tag{3}$$

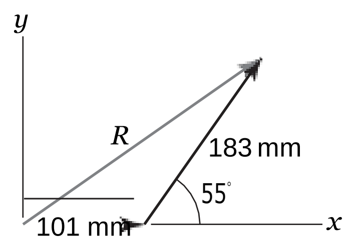
Problem 2.29

For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.2 on p. 43.

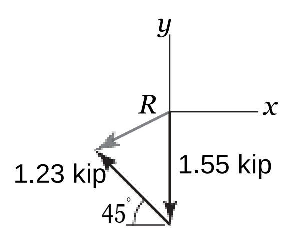
Solution

Part (a) The 101 mm and 183 mm position vectors are shown to the right with an xy Cartesian coordinate system, and the sum of these vectors is given by



$$\vec{R} = \left[101\,\hat{\imath} + 183(\cos 55^\circ\,\hat{\imath} + \sin 55^\circ\,\hat{\jmath})\right] \text{mm} = (206\,\hat{\imath} + 150\,\hat{\jmath})\ \text{mm}. \tag{1}$$

Part (b) The 1.23 kip and 1.55 kip force vectors are shown to the right with an xy Cartesian coordinate system, and the sum of these vectors is given by



$$\vec{R} = \left[-1.55\,\hat{\jmath} + 1.23(-\cos 45^\circ\,\hat{\imath} + \sin 45^\circ\,\hat{\jmath})\right] \text{kip}, = (-0.870\,\hat{\imath} - 0.680\,\hat{\jmath})\ \text{kip}. \tag{2}$$

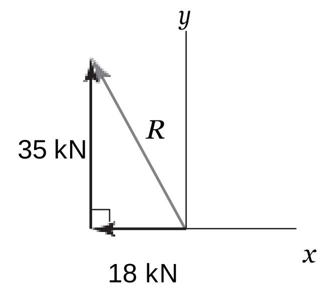
Problem 2.30

For the following problems, use an *xy* Cartesian coordinate system where *x* is horizontal, positive to the right, and *y* is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.3 on p. 43.

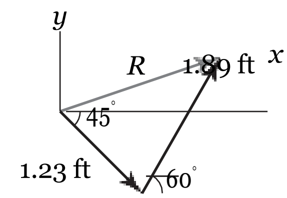
Solution

Part (a) The 18 kN and 35 kN force vectors are shown to the right with an *xy* Cartesian coordinate system, and the sum of these vectors is given by



$$\vec{R} = (-18\,\hat{\imath} + 35\,\hat{\jmath})\ \text{kN}. \quad (1)$$

The 1.23 ft and 1.89 ft position vectors are shown at the right with an *xy* Cartesian coordinate system, and the sum of these vectors is given by



$$\vec{R} = 1.23(\cos 45^\circ\,\hat{\imath} - \sin 45^\circ\,\hat{\jmath})\ \text{ft} + 1.89(\cos 60^\circ\,\hat{\imath} + \sin 60^\circ\,\hat{\jmath})\ \text{ft} \quad (2)$$

$$= (1.81\,\hat{\imath} + 0.767\,\hat{\jmath})\ \text{ft}. \quad (3)$$

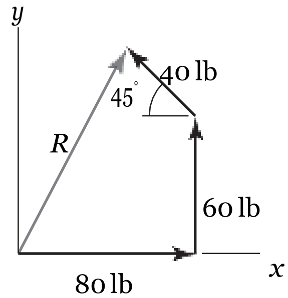
Problem 2.31

For the following problems, use an *xy* Cartesian coordinate system where *x* is horizontal, positive to the right, and *y* is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.13 on p. 45.

Solution

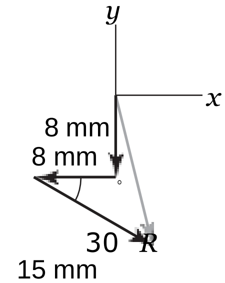
Part (a) The 40 lb, 60 lb, and 80 lb force vectors are shown at the right with an *xy* Cartesian coordinate system, and the sum of these vectors is given by



$$\vec{R} = 80\,\hat{\imath} + 60\,\hat{\jmath} + 40\left(-\cos 45^\circ\,\hat{\imath} + \sin 45^\circ\,\hat{\jmath}\right)\ \text{lb} \tag{1}$$

$$= (51.7\,\hat{\imath} + 88.3\,\hat{\jmath})\ \text{lb}. \tag{2}$$

Part (b) The 8 mm, 8 mm, and 15 mm position vectors are shown to the right with an *xy* Cartesian coordinate system, and the sum of these vectors is given by



$$\vec{R} = -8\,\hat{\jmath} - 8\,\hat{\imath} + 15\left(\cos 30^\circ\,\hat{\imath} - \sin 30^\circ\,\hat{\jmath}\right)\ \text{mm} \tag{3}$$

$$= (4.99\,\hat{\imath} - 15.5\,\hat{\jmath})\ \text{mm}. \tag{4}$$

Problem 2.32

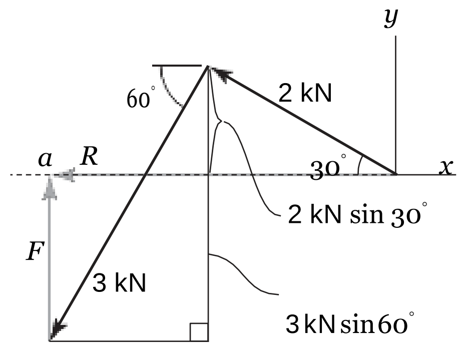
For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.16 on p. 45.

Solution

A force polygon shown at the right is constructed by selecting an xy Cartesian coordinate system, and then sketching the known force vectors (the 2 kN and 3 kN forces), followed by sketching the unknown force such that the resultant lies in the negative x direction (the a line in the problem statement). Based on this force polygon, $\vec{F}$ must act in the y direction, and its magnitude is given by

$$F = (3\,\text{kN})\sin 60^\circ - (2\,\text{kN})\sin 30^\circ = 1.598\,\text{kN}. \tag{1}$$



Therefore, using Cartesian representation, $\vec{F}$ may be written as

$$\boxed{\vec{F} = 1.60\,\hat{\jmath}\,\text{kN}.} \tag{2}$$

Problem 2.33

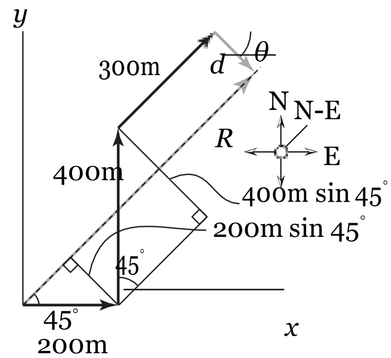
For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.17 on p. 45.

Solution

We sketch a vector polygon along with an xy Cartesian coordinate system. The vector $\vec{d}$ should be perpendicular to the dashed line shown. As such, $\theta = 45^\circ$ and the magnitude d of vector $\vec{d}$ is given by

$$d = (400\text{ m})\sin 45^\circ - (200\text{ m})\sin 45^\circ = 141.4\text{ m}; \tag{1}$$



hence,

$$\vec{d} = (141.4\text{ m})(\cos 45^\circ\,\hat{\imath} - \sin 45^\circ\,\hat{\jmath}) = (100\,\hat{\imath} - 100\,\hat{\jmath})\text{ m}, \tag{2}$$

where $\hat{\imath}$ and $\hat{\jmath}$ correspond to east and north, respectively.

The resultant vector $\vec{R}$ is given by the sum of the four vectors

$$\vec{R} = \left[200\,\hat{\imath} + 400\,\hat{\jmath} + 300(\cos 45^\circ\,\hat{\imath} + \sin 45^\circ\,\hat{\jmath}) + 100\,\hat{\imath} - 100\,\hat{\jmath}\right]\text{m} = \left[512.1(\hat{\imath} + \hat{\jmath})\right]\text{m}. \tag{3}$$

The magnitude of the above expression is the final distance from her starting point to ending point, thus

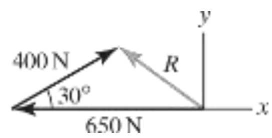
$$R = \sqrt{(512.1\text{ m})^2 + (512.1\text{ m})^2} = 724\text{ m}. \tag{4}$$

Problem 2.34

For the following problems, use an *xy* Cartesian coordinate system where *x* is horizontal, positive to the right, and *y* is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.18 on p. 45.

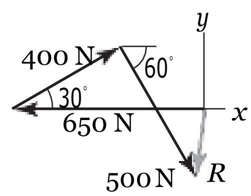
Solution



Part (a) Using the force polygon to the right, the resultant force $\vec{R}$ may be written as

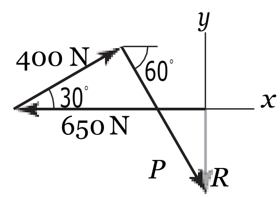
$$\vec{R} = \left[-650\,\hat{\imath} + 400(\cos 30^\circ\,\hat{\imath} + \sin 30^\circ\,\hat{\jmath})\right]\text{N} = (-304\,\hat{\imath} + 200\,\hat{\jmath})\,\text{N}. \tag{1}$$

Remark: In the solution to Prob. 2.18, the resultant force for Part (a) was called $\vec{Q}$.



Part (b) Using the force polygon to the right, the resultant force $\vec{R}$ may be written as

$$\vec{R} = \left[-650\,\hat{\imath} + 400(\cos 30^\circ\,\hat{\imath} + \sin 30^\circ\,\hat{\jmath}) + 500(\cos 60^\circ\,\hat{\imath} - \sin 60^\circ\,\hat{\jmath})\right]\text{N}$$
$$= (-53.6\,\hat{\imath} - 233\,\hat{\jmath})\,\text{N}. \tag{2}$$



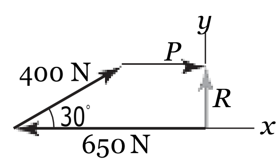
Part (c) The resultant force vector $\vec{R}$ is required to be vertical. Thus, we sketch the force polygon shown at the right. Using this force polygon, the *x* component of $\vec{P}$ is

and the *y* component is found by

$$\tan 60^\circ = |P_y/P_x| \quad \Rightarrow \quad P_y = (-303.6\,\text{N})\tan 60^\circ = -525.8\,\text{N}. \tag{4}$$

where the negative sign is inserted because the vertical component of $\vec{P}$ acts in the negative *y* direction. Thus it follows that

$$P = \sqrt{P_x^2 + P_y^2} = 607\,\text{N}. \tag{5}$$



Part (d) The 400 N and 650 N force vectors are shown in the force polygon at the right along with a vertical resultant force *R*. The smallest value of *P* occurs when this vector's direction is perpendicular to the resultant. Thus, it follows that

$$P = 650\,\text{N} - (400\,\text{N})\cos 30^\circ = 304\,\text{N} \quad \text{and} \quad \alpha = 0^\circ. \tag{6}$$

Problem 2.35

For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

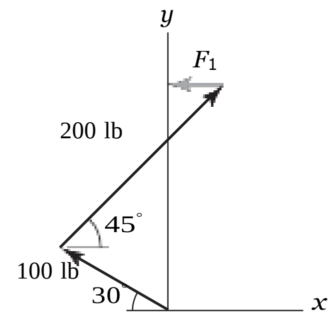
Repeat Prob. 2.21 on p. 46.

Solution

The force polygon shown at the right includes the 100 lb and 200 lb force vectors, along with the smallest possible force F_1 such that the resultant of the three vectors is vertical. Using this force polygon

$$F_1 = (200\ \text{lb})\cos 45^\circ - (100\ \text{lb})\cos 30^\circ = 54.8\ \text{lb} \quad \text{and} \quad \theta = 90^\circ.$$

(1)



Problem 2.36

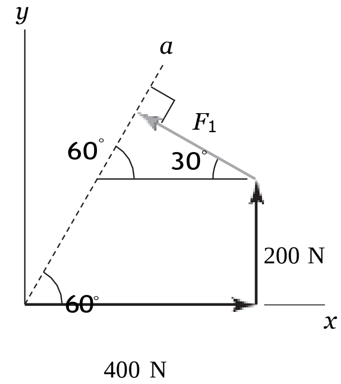
For the following problems, use an xy Cartesian coordinate system where x is horizontal, positive to the right, and y is vertical, positive upward. For problems where the answers require vector expressions, report the vectors using Cartesian representations.

Repeat Prob. 2.22 on p. 46.

Solution

The dashed line in the figure to the right represents the direction along which the resultant force vector $\vec{R}$ is required to act. The horizontal and vertical components of $\vec{R}$ are given by

$$R_x = 400\,\text{N} - F_1\cos 30^\circ; \quad \text{and} \quad R_y = 200\,\text{N} + F_1\sin 30^\circ. \tag{1}$$



It follows that

$$\tan 60^\circ = \frac{R_y}{R_x} = \frac{200\,\text{N} + F_1\sin 30^\circ}{400\,\text{N} - F_1\cos 30^\circ} \quad \Rightarrow \quad F_1 = 246\,\text{N}. \tag{2}$$

Note that θ equals 60° since F_1 is perpendicular to line a. Using Cartesian representation, $\vec{F}_1$ is given by

$$\vec{F}_1 = (246\,\text{N})(-\cos 30^\circ\,\hat{\imath} + \sin 30^\circ\,\hat{\jmath}) = (-213\,\hat{\imath} + 123\,\hat{\jmath})\,\text{N}. \tag{3}$$

Problem 2.37

Let $\vec{A} = (150\,\hat{\imath} - 200\,\hat{\jmath})$ lb and $\vec{B} = (200\,\hat{\imath} + 480\,\hat{\jmath})$ lb. Evaluate the following, and for Parts (a) and (b) state the magnitude of $\vec{R}$.

(a) $\vec{R} = \vec{A} + \vec{B}$.

(b) $\vec{R} = 2\vec{A} - (1/2)\vec{B}$.

(c) Find a scalar s such that $\vec{R} = s\vec{A} + \vec{B}$ has an x component only.

(d) Determine a dimensionless unit vector in the direction $\vec{B} - \vec{A}$.

Solution

Part (a)

$$\vec{R} = (150\,\hat{\imath} - 200\,\hat{\jmath})\,\text{lb} + (200\,\hat{\imath} + 480\,\hat{\jmath})\,\text{lb} = (150 + 200)\,\hat{\imath}\,\text{lb} + (-200 + 480)\,\hat{\jmath}\,\text{lb}; \qquad (1)$$

which simplifies to

$$\boxed{\vec{R} = (350\,\hat{\imath} + 280\,\hat{\jmath})\,\text{lb}.} \qquad (2)$$

The magnitude R is given by

$$\boxed{R = \sqrt{(350)^2 + (280)^2}\,\text{lb} = 448\,\text{lb}.} \qquad (3)$$

Part (b)

$$\vec{R} = (300\,\hat{\imath} - 400\,\hat{\jmath})\,\text{lb} - (100\,\hat{\imath} + 240\,\hat{\jmath})\,\text{lb} = (300 - 100)\,\hat{\imath}\,\text{lb} + (-400 - 240)\,\hat{\jmath}\,\text{lb}; \qquad (4)$$

which simplifies to

$$\boxed{\vec{R} = (200\,\hat{\imath} - 640\,\hat{\jmath})\,\text{lb}.} \qquad (5)$$

The magnitude R is given by

$$\boxed{R = \sqrt{(200)^2 + (-640)^2}\,\text{lb} = 671\,\text{lb}.} \qquad (6)$$

Part (c)

$$\vec{R} = s(150\,\hat{\imath} - 200\,\hat{\jmath})\,\text{lb} + (200\,\hat{\imath} + 480\,\hat{\jmath})\,\text{lb} = (150s + 200)\,\hat{\imath}\,\text{lb} + (-200s + 480)\,\hat{\jmath}\,\text{lb}; \qquad (7)$$

where, according to the problem statement,

$$R_y = 0 \quad\Rightarrow\quad -200s + 480 = 0 \quad\Rightarrow\quad \boxed{s = 480/200 = 2.40.} \qquad (8)$$

Applying this value of s to Eq. (8) yields

$$\boxed{\vec{R} = [150(2.40) + 200]\,\hat{\imath}\,\text{lb} = 560\,\hat{\imath}\,\text{lb}; \quad R = \sqrt{(560)^2 + 0^2}\,\text{lb} = 560\,\text{lb}.} \qquad (9)$$

Part (d)

$$\vec{R} = \left[(200\,\hat{\imath} + 480\,\hat{\jmath}) - (150\,\hat{\imath} - 200\,\hat{\jmath}) \right] \text{lb} = (50\,\hat{\imath} + 680\,\hat{\jmath})\ \text{lb} \tag{10}$$

$$R = \sqrt{(50)^2 + (680)^2}\ \text{lb} = 681.8\ \text{lb}. \tag{11}$$

The unit vector in the direction of $\vec{R}$ is $\hat{R}$, and it is given by

$$\boxed{\hat{R} = \frac{\vec{R}}{R} = \frac{(50\,\hat{\imath} + 680\,\hat{\jmath})\ \text{lb}}{681.8\ \text{lb}} = 0.0733\,\hat{\imath} + 0.997\,\hat{\jmath}.} \tag{12}$$