MOTION IN A STRAIGHT LINE **2**

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EXERCISES

Section 2.1 Average Motion

12. **INTERPRET** We need to find average speed, given distance and time.

DEVELOP From Equation 2.1, the average speed (velocity) is $\bar{v} = \otimes x/\otimes t$, where $\otimes x$ is the distance of the race, and $\otimes t$ is the time it took Ursain Bolt to finish.

EVALUATE Plugging in the values,

$$\bar{v} = (100\text{ m})/(9.58\text{ s}) = 10.4\text{ m/s}.$$

ASSESS This is equivalent to 23 mi/h.

13. **INTERPRET** We need to find the average runner speed, and use that to find how long it takes them to run the additional distance.

DEVELOP The average speed is $\bar{v} = \otimes x/\otimes t$ (Equation 2.1). Looking ahead to part **(b)**, we will express this answer in terms of yards per minute. That means converting miles to yards and hours to minutes. A mile is 1760 yards (see Appendix C). Once we know the average speed, we will use it to determine how long ($\otimes t = \otimes x/\bar{v}$) it would take a top runner to go the extra mile and 385 yards that was added to the marathon in 1908.

EVALUATE **(a)** First converting the marathon distance to yards and time to seconds

$$\otimes x = 26\text{ mi}\left(\frac{1760\text{ yd}}{1\text{ mi}}\right) + 385\text{ yd} = 46{,}145\text{ yd}$$

$$\otimes t = 2\text{ h}\left(\frac{60\text{ min}}{1\text{ h}}\right) + 3\text{ min} = 123\text{ min}$$

Dividing these quantities, the average velocity is $\bar{v} = 375$ yd/min.

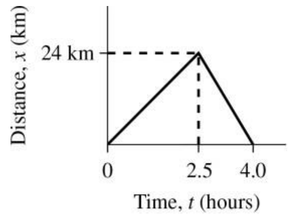
(b) The extra mile and 385 yards is equal to 2145 yd. The time to run this is

$$\otimes t = \frac{\otimes x}{v} = \frac{2145 \text{ yd}}{375 \text{ yd/ min}} = 5.72 \text{ min}$$

ASSESS The average speed that we calculated is equivalent to about 13 mi/h, which means top runners can run 26 mi marathons in roughly 2 hours. The extra distance is about 5% of the total distance, and correspondingly the extra time is about 5% of the total time, as it should be.

14. **INTERPRET** This is a one-dimensional kinematics problem that involves calculating your displacement and average velocity as a function of time. There are two different parts to the problem: in the first part we travel north and in the second part where we travel south.

DEVELOP It will help to plot our displacement as a function of time (see figure below). We are given three points: the point where we start $(t, y) = (0\text{ h}, 0\text{ km})$, the point where we stop after traveling north at $(t, y) = (2.5\text{ h}, 24\text{ km})$, and the point where we return home at $(t, y) = (4\text{ h}, 0\text{ km})$. We can use Equation 2.1, $\bar{v} = \otimes x/\otimes t$, to calculate the average velocity. To calculate the displacement we will subtract the initial position from the final position.



EVALUATE (a) After the first 2.5 hours, you have traveled north 24 km, so your change in position (i.e., your displacement) is $\Delta x = x - x_0 = 24\text{ km} - 0\text{ km} = 24\text{ km}$, where the x_0 is the initial position and x is the final position.

(b) The time it took for this segment of the trip is $\Delta t = t - t_0 = 2.5\text{ h} - 0\text{ h} = 2.5\text{ h}$. Inserting these quantities into Equation 2.1, we find the average velocity for this segment of the trip is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{24\text{ km}}{2.5\text{ h}} = 9.6\text{ km/h}$$

(c) For the homeward leg of the trip, $\Delta x = x - x_0 = 0\text{ km} - 24\text{ km} = -24\text{ km}$, and $\Delta t = t - t_0 = 4.0\text{ h} - 2.5\text{ h} = 1.5\text{ h}$, so your average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-24\text{ km}}{1.5\text{ h}} = -16\text{ km/h}.$$

(d) The displacement for the entire trip is $\Delta x = x - x_0 = 0\text{ km} - 0\text{ km} = 0\text{ km}$, because you finished at the same position as you started.

(e) For the entire trip, the displacement is 0 km, and the time is 4.0 h, so the average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0\text{ km}}{1.5\text{ h}} = 0\text{ km/h}$$

ASSESS We see that the average velocity for parts **(b)** and **(c)** differ in sign, which is because we are traveling in the opposite direction during these segments of the trip. Also, because we return to our starting point, the average velocity for the entire trip is zero—we would have finished at the same position had we not moved at all!

15. **INTERPRET** This problem asks for the time it will take a light signal to reach us from the edge of our solar system.

DEVELOP The time is just the distance divided by the speed: $\Delta t = \Delta x / v$. The speed of light is 3.00×10^8 m/s (recall Section 1.2).

EVALUATE Using the above equation

$$\Delta t = \frac{\Delta x}{v} = \frac{(14 \times 10^9\text{ mi})}{(3.00 \times 10^8\text{ m/s})}\left(\frac{1609\text{ m}}{1\text{ mi}}\right) = 7.5 \times 10^4\text{ s} = 21\text{ h}$$

ASSESS It takes light from the Sun 8.3 minutes to reach Earth. This means that the Voyager spacecraft will be 150 times further from us than the Sun.

16. **INTERPRET** We interpret this as a task of summing the distances for the various legs of the race and then dividing by the time to get the average speed.

DEVELOP The average speed is $\bar{v} = \Delta x / \Delta t$ (Equation 2.1). After summing the distances of the different legs, we will want to convert the time to units of seconds.

EVALUATE The three legs have a combined distance of $\Delta x = (1.5 + 40 + 10)\text{km} = 51.5\text{ km}$. The elapsed time is

$$\Delta t = 1\text{ h}\left(\frac{3600\text{ s}}{1\text{ h}}\right) + 58\text{ min}\left(\frac{60\text{ s}}{1\text{ min}}\right) + 27.66\text{ s} = 7107.66\text{ s}$$

Dividing these quantities, the average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{51500\text{ m}}{7107.66\text{ s}} = 7.25\text{ m/s}$$

ASSESS In common units, the triathlete's average speed is 16 mi/h. This is faster than the marathoner's pace in Problem 2.13, which might seem surprising, but we have to remember that part of the race is on a bike.

17. **INTERPRET** The problem asks for the Earth's speed around the Sun. We'll use the fact that the Earth completes a full revolution in a year.

DEVELOP The distance the Earth travels is approximately equal to the circumference ($2\pi r$) of a circle with radius equal to 1.5×10^8 km. It takes a year, or roughly $\pi\times10^7$ s, to complete this orbit.

EVALUATE (a) The average velocity in m/s is

$$\bar{v} = \frac{2\pi r}{\Delta t} = \frac{2\pi(1.5\times10^{11}\text{ m})}{\pi\times10^7\text{ s}} = 3.0\times10^4\text{ m/s}$$

(b) Using 1609 m = 1 mi gives $\bar{v} = 19$ mi/s.

ASSESS It's interesting that the Earth's orbital speed is $1/10^4$ of the speed of light.

18. **INTERPRET** This problem involves converting units from m/s to mi/h.

DEVELOP Using the data from Appendix C, we find that 1 mi = 1.609 km or 1 mi = 1609 m. We also know that there are 60 minutes in an hour and 60 seconds in a minute, so 1 h = (60 s/min)(60 min) = 3600 s, or 1 = 3600 s/h. We can use these formulas to convert an arbitrary speed in m/s to the equivalent speed in mi/h.

EVALUATE Using the conversion factors from above, we convert x from m/s to mi/h:

$$x\text{ m/s} = \left(x\,\frac{\text{m}}{\text{s}}\right)\left(\frac{1\text{ mi}}{1609\text{ m}}\right)\underbrace{\left(\frac{3600\text{ s}}{\text{h}}\right)}_{\text{conversion factor}} = x\text{ mi/h}$$

From this formula, we see that the conversion factor is (3600 mi · s)/(1609 km· h) = 2.237 mi · s · km^{-1}· h^{-1}.

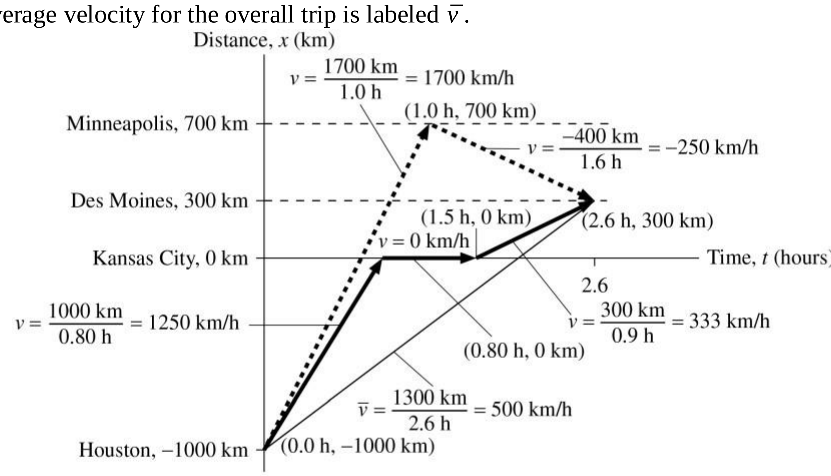
ASSESS Notice that we have retained 4 significant figures in the answer because the conversion factor from s to h is a definition, so it has infinite significant figures. Thus, the number of significant figures is determined by the number 1.609, which has 4 significant figures. Also notice that the conversion factor has the proper units so that the final result is in mi/h.

Section 2.2 Instantaneous Velocity

19. **INTERPRET** This problem asks us to plot the average and instantaneous velocities from the information in the text regarding the trip from Houston to Des Moines. The problem statement does not give us the times for the intermediate flights, nor the length of the layover in Kansas City, so we will have to assign these values ourselves.

DEVELOP We can use Equation 2.1, $\bar{v} = \Delta x/\Delta t$, to calculate the average velocities. Furthermore, because each segment of the trip involves a constant velocity, the instantaneous velocity is equivalent to the average velocity, so we can apply Equation 2.1 to these segments also. To calculate the Δ-values, we subtract the initial value from the final value (e.g., for the first segment from Houston to Minneapolis, $\Delta x = x - x_0 = 700\text{ km} - (-1000\text{ km}) = 1700\text{ km}$.

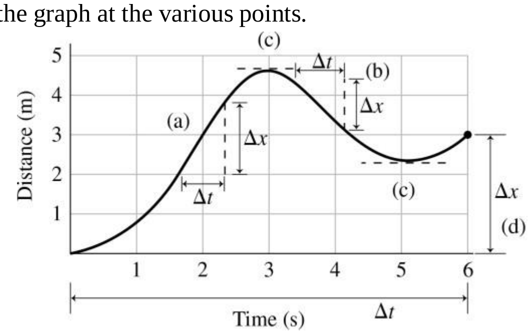
EVALUATE See the figure below, on which is labeled the coordinates for each point and the velocities for each segment. The average velocity for the overall trip is labeled $\bar{v}$.



ASSESS Although none of instantaneous velocities are equivalent to the average velocity, they arrive at the same point as if you traveled at the average velocity for the entire length of the trip.

20. **INTERPRET** This problem involves interpreting a graph of position vs. time to determine several key values. Recall that instantaneous velocity is the tangent to the graph at any point, and that the average velocity is simply the total distance divided by the total time.

DEVELOP We know that the largest instantaneous velocity corresponds to the steepest section of the graph because this is where the largest displacement in the least amount of time occurs [see region **(a)** of figure below]. For the instantaneous velocity to be negative, the slope of the tangent to a point on the graph must descend in going from left to right, so that the final position will be less than the initial position [see region **(b)** of figure below] A region of zero instantaneous velocity is where the tangent to the graph is horizontal, indicating that there is no displacement in time [see regions **(c)** of figure below]. Finally, we can apply Equation 2.1 to find the average velocity over the entire period [see **(d)** in figure below]. To estimate the instantaneous velocities, we need to estimate the slope dx/dt of the graph at the various points.



EVALUATE (a) The largest instantaneous velocity in the positive-x direction occurs at approximately t = 2 s and is approximately $v = dx/dt \approx \otimes x/\otimes t$ = (1.8 m)/(0.6 s) = 3 m/s.
(b) The largest negative velocity occurs at approximately t = 4 s and is approximately $v = dx/dt \approx \otimes x/\otimes t$ = −(1 m)/(0.7 s) = −1.4 m/s.
(c) The instantaneous velocity goes to zero at t = 3 s and t = 5 s, because the graph has extremums (i.e., maxima or minima) at these points, so the slope is horizontal.
(d) Applying Equation 2.1 to the total displacement, we find the average velocity is

$$\bar{v} = \frac{\otimes x}{\otimes t} = \frac{x - x_0}{t - t_0} = \frac{3\text{ m} - 0\text{ m}}{6\text{ s} - 0\text{ s}} = 0.5\text{ m/s}.$$

ASSESS The average velocity is positive, as expected, because the final position is greater than the initial position.

21. **INTERPRET** This problem involves using calculus to express velocity given position as a function of time. We must also understand that zero velocity occurs where the slope (i.e., the derivative) of the plot is zero.

DEVELOP The instantaneous velocity $v(t)$ can be obtained by taking the derivative of $y(t)$. The derivative of a function of the form bt^n can be obtained by using Equation 2.3.

EVALUATE (a) The instantaneous velocity as a function of time is

$$v = \frac{dy}{dt} = b - 2ct$$

(b) By using the general expression for velocity, we find that it goes to zero at

$$v = 0 = b - 2ct$$
$$t = \frac{b}{2c} = \frac{82\text{ m/s}}{4.9\text{ m/s}^2} = 8.4\text{ s}$$

ASSESS From part **(a)**, we see that at t = 0, the velocity is 82 m/s. This velocity decreases as time progresses due to the term −2ct, until the velocity reverses and the rocket falls back to Earth. Note also that the units for part **(b)** come out to be s, as expected for a time.

Section 2.3 Acceleration

22. **INTERPRET** Solar material is accelerated from rest (v = 0) to a high speed. We are asked to find the average acceleration.

DEVELOP Equation 2.4 gives the average acceleration $\bar{a} = \otimes v/\otimes t$.

EVALUATE Over 1 hour, the average acceleration is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{(450\text{ km/s}) - (0)}{1\text{ h}} = 125\text{ m/s}^2$$

ASSESS This is 13 times the gravitational acceleration on Earth.

23. **INTERPRET** The object of interest is the subway train that undergoes acceleration from rest, followed by deceleration through braking. The kinematics are one-dimensional, and we are asked to find the average acceleration over the braking period.

DEVELOP The average acceleration over a time interval Δt is given by Equation 2.4: $\bar{a} = \Delta v/\Delta t$.

EVALUATE Over a time interval $\Delta t = t_2 - t_1 = 48$ s , the velocity of the train (along a straight track) changes from $v_1 = 0$ (starting at rest) to $v_2 = 17$ m/s. The change in velocity is thus $\Delta v = v_2 - v_1 = 17\text{ m/s} - 0.0\text{ m/s} = 17$ m/s. Thus, the average acceleration is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{17\text{ m/s}}{48\text{ s}} = 0.35\text{ m/s}^2$$

ASSESS We find that the average acceleration only depends on the change of velocity between the starting point and the end point; the intermediate velocity is irrelevant.

24. **INTERPRET** This problem involves calculating an average acceleration given the initial and final times and velocities. We will also need to convert units from min to s (to express the quantities in consistent units) and from km to m (to express the answer in convenient units).

DEVELOP The average acceleration over a time interval Δt is given by Equation 2.4: $\bar{a} = \Delta v/\Delta t$. Because the space shuttle starts at rests, $v_1 = 0$, so $\Delta v = v_2 - v_1 = 7.6\text{ km/s} - 0.0\text{ km/s} = 7.6\text{ km/s} = 7600$ m/s. The time interval $\Delta t = (8.5\text{ min})(60\text{ s/min}) = 510$ s.

EVALUATE The average acceleration of the space shuttle during the given period is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{7600\text{ m/s}}{510\text{ s}} = 15\text{ m/s}^2$$

ASSESS The result is in m/s^2, as expected for an acceleration. The acceleration is positive, which means the velocity of the space shuttle increased during this period. Note that the magnitude of this acceleration is greater than that due to gravity, which is -9.8 m/s^2 (i.e., directed toward the Earth).

25. **INTERPRET** For this problem, the motion can be divided into two stages: (i) free fall, and (ii) stopping after striking the ground. We need to find the average acceleration for both stages.

DEVELOP We chose a coordinate system in which the positive direction is that of the egg's velocity. For stage (i), the initial velocity is $v_1^{(i)} = 0.0$ m/s, and the final velocity is $v_2^{(i)} = 11.0$ m/s, so the change in velocity is $\Delta v^{(i)} = v_2^{(i)} - v_1^{(i)} = 11.0\text{ m/s} - 0.0\text{ m/s} = 11.0$ m/s. The time interval for this stage is $\Delta t^{(i)} = 1.12$ s. For the second stage, the initial velocity is $v_1^{(ii)} = 11.0$ m/s, the final velocity is $v_2^{(ii)} = 0.0$ m/s, so the change in velocity is $\Delta v^{(ii)} = v_2^{(ii)} - v_1^{(ii)} = 0\text{ m/s} - 11\text{ m/s} = -11.0$ m/s. The time interval for the second stage is $\Delta t^{(ii)} = 0.131$ s. Insert these values into Equation 2.4, $\bar{a} = \Delta v/\Delta t$, to find the average acceleration for each stage.

EVALUATE (a) While undergoing free fall - stage (i), the average acceleration is

$$\bar{a}^{(i)} = \frac{\Delta v^{(i)}}{\Delta t^{(i)}} = \frac{11.0\text{ m/s}}{1.12\text{ s}} = 9.82\text{ m/s}^2$$

(b) For the stage (ii), where the egg breaks on the ground, the average acceleration is

$$\bar{a}^{(ii)} = \frac{\Delta v^{(ii)}}{\Delta t^{(ii)}} = \frac{-11.0\text{ m/s}}{0.131\text{ s}} = -84.0\text{ m/s}^2$$

ASSESS For stage (i), the acceleration is that due to gravity, and is directed downward toward the Earth. It is in the same direction as the velocity so the velocity increases during this stage. For stage (ii), the acceleration is in the opposite direction (i.e., upward away from the Earth) so the velocity decreases during this stage.

26. **INTERPRET** For this problem, we need to calculate the time it takes for the airplane to reach its take off speed given its acceleration. Notice that this is similar to the previous problems, except that we are given the velocity and acceleration and are solving for the time, whereas before we were given the velocity and time and solved for acceleration.

DEVELOP We can use Equation 2.4, $\bar{a} = \otimes v/\otimes t$, to solve this problem. We can assume the airplane's initial velocity is $v_1 = 0$ km/h, and we are given the final velocity ($v_2 = 320$ km/h), so the change in the airplane's velocity is $\otimes v = v_2 - v_1 = 320$ km/h. The average acceleration is given as $\bar{a} = 2.9\ \text{m/s}^2$. Notice that the velocity and the acceleration are given in different units, so we will convert km/h to m/s for the calculation.

EVALUATE Insert the known quantities into Equation 2.4 and solve for the time interval, $\otimes t$. This gives

$$\bar{a} = \frac{\otimes v}{\otimes t}$$

$$\otimes t = \frac{\otimes v}{\bar{a}} = \left(\frac{320\ \text{km/h}}{2.9\ \text{m/s}^2}\right)\left(\frac{10^3\ \text{m}}{\text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 31\ \text{s}$$

ASSESS With an average acceleration of $2.9\ \text{m/s}^2$, the airplane's velocity increases by just under 3 m/s each second. Given that 320 km/h is just under 90 m/s, the answer seems reasonable because if you increment the velocity by 3 m/s 30 times, it will attain 90 m/s.

27. **INTERPRET** The object of interest is the car, which we assume undergoes constant acceleration. The kinematics are one-dimensional.

DEVELOP We first convert the units km/h to m/s, using the conversion factor

$$1\ \text{km/h} = \left(1\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 0.278\ \text{m/s}$$

and then use Equation 2.4, $\bar{a} = \otimes v/\otimes t$, to find the average acceleration.

EVALUATE The speed of the car at 16 s is 1000 km/h, or 278 m/s. Therefore, the average acceleration is

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{(278\ \text{m/s}) - (0)}{16\ \text{s} - 0\ \text{s}} = 17\ \text{m/s}^2$$

ASSESS The magnitude of the average acceleration is about $1.8g$, where $g = 9.8\ \text{m/s}^2$ is the gravitational acceleration. An object undergoing free fall attains only a speed of 157 m/s after 16.0 s, compared to 278 m/s for the supersonic car. Given the supersonic nature of the vehicle, the value of a is completely reasonable.

Section 2.4 Constant Acceleration

28. **INTERPRET** The problem states that the acceleration of the car is *constant*, so we can use the constant-acceleration equations and techniques developed in this chapter. We're given initial and final speeds, and the time, and we're asked to find the distance.

DEVELOP Equation 2.9 relates distance to initial speed, final speed, and to time—that's just what we need. The distance traveled during the given time is the difference between x and x_0. We also need to be careful with our units because the problem gives us speeds in km/h and time in seconds, so we will convert everything to meters and seconds so that everything has consistent and convenient units.

EVALUATE First, convert the speeds to units of m/s. This gives

$$70\ \text{km/h} = \left(70\ \frac{\text{km}}{\text{h}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 19.4\ \text{m/s}$$

$$80\ \text{km/h} = \left(80\ \frac{\text{km}}{\text{h}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 22.2\ \text{m/s}$$

where we have retained more significant figures than warranted because this is an intermediate result. Insert these quantities into Equation 2.9 and solve for the distance, $x - x_0$. This gives

$$(x - x_0) = \frac{1}{2}(v - v_0)t = \frac{1}{2}(19.4\ \text{m/s} + 22.2\ \text{m/s})(6\ \text{s}) = 125\ \text{m}$$

Because we know the time to only a single significant figure (6 s), we should report our answer to a single significant, which is 100 m.

ASSESS This distance for passing seems reasonable. Note that the answer actually implies that the passing distance is 100 ± 50 m.

29. **INTERPRET** The problem is designed to establish a connection between the equation for displacement and the equation for velocity in one-dimensional kinematics.

DEVELOP Recall that the derivative of position with respect to time dx/dt is the instantaneous velocity (see Equation 2.2b, $dx/dt = v$). Thus, by differentiating the displacement $x(t)$ given in Equation 2.10 with respect to t, we obtain the corresponding velocity $v(t)$. We can use Equation 2.3 for evaluating the derivatives.

EVALUATE Differentiating Equation 2.10, we obtain

$$\frac{dx}{dt} = \frac{d}{dt}\left(x_0 + v_0 t + \frac{1}{2}at^2\right) = 0 + v_0 + \frac{1}{2}a\cdot(2t)$$

$$v = v_0 + at$$

which is Equation 2.7. Notice that we have used Equation 2.2b and that we have used the fact that the derivative (i.e., the change in) the initial position x_0 with respect to time is zero, or $dx_0/dt = 0$.

ASSESS Both Equations 2.7 and 2.10 describe one-dimensional kinematics with constant acceleration a, but whereas Equation 2.10 gives the displacement, Equation 2.7 gives the final velocity.

30. **INTERPRET** The acceleration is constant, so we can use equations from Table 2.1.

DEVELOP We're given the distance and the final velocity but no time, so Equation 2.11 seems appropriate for finding the acceleration

$$a = \frac{v^2 - v_0^2}{2(x - x_0)}$$

Once we have a, we can use Equation 2.7, 2.9 or 2.10 to find the time. Equation 2.7 would seem to be the simplest.

EVALUATE (a) We assume the electrons start at the origin ($x = 0$) and at rest ($v_0 = 0$).

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(1.2\times 10^7\ \text{m/s})^2 - (0)^2}{2(0.15\ \text{m} - 0)} \quad 4.8\ 10\ \text{m/s}$$

(b) Using this acceleration in Equation 2.7 allows us to solve for the time

$$t = \frac{v - v_0}{a} = \frac{1.2\times 10^7\ \text{m/s}}{4.8\times 10^{14}\ \text{m/s}^2} = 2.5\times 10^{-8}\ \text{s} = 25\ \text{ns}$$

ASSESS The electron has such a small mass that it can be accelerated rather easily. Here, it is accelerated to 4% of the speed of light in a few nanoseconds.

31. **INTERPRET** This is a one-dimensional kinematics problem with constant acceleration. We are asked to find the acceleration and the assent time for a rocket given its speed and the distance it travels.

DEVELOP The three quantities of interest; displacement, velocity, and acceleration, are related by Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$. Solve this equation for acceleration for part **(a)**. Once the acceleration is known, the time elapsed for the ascent can be calculated by using Equation 2.7, $v = v_0 + at$.

EVALUATE (a) Taking x to indicate the upward direction, we know that $x - x_0 = 85$ km = 85,000 m, $v_0 = 0$ (the rocket starts from rest), and $v = 2.8$ km/s = 2800 m/s. Therefore, from Equation 2.11, the acceleration is

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(2800\ \text{m/s})^2 - (0\ \text{m/s})^2}{2(85{,}000\ \text{m})} \quad 46\ \text{m/s}$$

(b) From Equation 2.7, the time of flight is

$$t = \frac{v - v_0}{a} = \frac{2800\ \text{m/s} - (0\ \text{m/s})}{46\ \text{m/s}^2} = 61\ \text{s}$$

ASSESS An acceleration of 46 m/s^2 or approximately 5g (g = 9.8 m/s^2), is typical for rockets during liftoff. This enables the rocket to reach a speed of 2.8 km/s in just about one minute.

32. **INTERPRET** This problem asks us to find the acceleration given the initial and final velocities and the time interval.

DEVELOP (a) From Table 2.1, we find Equation 2.7 $v = v_0 + at$ contains the acceleration, velocity (initial and final), and time. Thus, given the initial and final velocity and the time interval, we can solve for acceleration. The initial velocity $v_0 = 0$ because the car starts from rest, the final velocity v = 88 km/h, and the time interval is t = 12 s. We chose to convert the velocity to m/s, because these will be more convenient units for the calculation. By using the data in Appendix C, we find the final velocity is v = (88 km/h)(1000 m/1 km)(1 h/3600 s) = 24.4 m/s (where we keep more significant figures than warranted because this is an intermediate result). **(b)** To find the distance travled during the accleration period, use Equation 2.10, which relates distance to velocity (initial and final), acceleration, and time.

EVALUATE (a) Inserting the given quantities in Equation 2.7 gives

$$v = v_0 + at$$
$$a = \frac{v - v_0}{t} = \frac{24.4 \text{ m/s} - 0.0 \text{ m/s}}{12 \text{ s}} = 2.0 \text{ m/s}^2$$

where we have retained two significant figures in the answer, as warranted by the data.

(b) Inserting the acceleration just calculated into Equation 2.10, we find

$$x - x_0 = v_0 t + \frac{1}{2}at^2 = (0 \text{ m/s})(12 \text{ s}) + \frac{1}{2}(2.04 \text{ m/s}^2)(12 \text{ s})^2 = 150 \text{ m}$$

where we have retained 3 significant figures in the acceleration because it's now an intermediate result, but have retained only 2 significant figures in the final result because the data is given to only 2 significant figures.

ASSESS Is this answer reasonable? If we increase our velocity by 2 m/s every second, in 12 seconds we can expect to be moving at 12 × 2 m/s = 24 m/s, which agrees with the data. To see if 150 m is a reasonable distance, imagine traveling at the average velocity of about 12 m/s (how do we know it's 12 m/s?) for 12 s. In this case we would travel 12 s × 12 m/s = 144 m, which is close to our result.

33. **INTERPRET** The object of interest is the car that undergoes constant deceleration (via braking) and comes to a complete stop after traveling a certain distance.

DEVELOP The three quantities, displacement, velocity, and deceleration (negative acceleration), are related by Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$. This is the equation we shall use to solve for a. Since the distance to the light is in feet, we can convert the initial speed

$$v_0 = 50 \text{ mi/h}\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 73.3 \text{ ft/s}$$

EVALUATE Since the car stops ($v = 0$) after traveling $x - x_0$ = 100 ft from an initial speed of v_0 = 73.3 ft/s, Equation 2.11 gives

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (73.3 \text{ ft/s})^2}{2(100 \text{ ft})} \quad 27 \text{ ft/s}$$

The magnitude of the deceleration is the absolute value of a: $|a|$ = 27 ft/s^2.

ASSESS With this deceleration, it would take about $t = v_0/|a|$ = (73 ft/s)(27 ft/s^2) = 2.7 s for the car to come to a complete stop. The value is in accordance with our driving experience.

34. **INTERPRET** The electrons are accelerated to high-speed beforehand. We are only asked to consider the rapid deceleration that occurs when they slam into the tungsten target.

DEVELOP We are given the initial and final velocities, as well as the time duration of the deceleration. We are not asked what the deceleration is, but merely what distance the electrons penetrate the tungsten before stopping. Equation 2.9 is therefore what we will use.

EVALUATE Plugging in the given values we find the stopping distance is

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t = \tfrac{1}{2}(10^8\ \text{m/s} + 0)(10^{-9}\text{s}) = 0.05\ \text{m}$$

ASSESS The electrons are initially travelling close to the speed of light, but only a thin sheet of tungsten is needed to stop them. The X rays that are produced in this way are called *bremsstrahlung,* which means "braking radiation."

35. **INTERPRET** This question asks us to calculate the advance warning needed for the BART train to brake and come to a safe speed when the earthquake strikes.

DEVELOP The initial speed of the train is $v_0 = 112$ km/h = 31.1 m/s. The acceleration that brings the train to a complete stop in 24 s is $a = (0 - 31.1\ \text{m/s})/24\ \text{s} = -1.30\ \text{m/s}^2$. We want to apply this acceleration to reduce the train speed to $v = 42$ km/h = 11.7 m/s.

EVALUATE Using Eq. 2.11: $v = v_0 + at$, we find the time needed to be

$$t = \frac{v - v_0}{a} = \frac{11.7\ \text{m/s} - 31.1\ \text{m/s}}{-1.30\ \text{m/s}^2} = 15\ \text{s}$$

ASSESS The 15 s advance warning may not seem long, but it allows the train operator to slow down and take appropriate steps to ensure the safety of the passengers.

36. **INTERPRET** This question asks us to derive an expression for the acceleration needed to stop before hitting a moose with your car.

DEVELOP We are given the distance, d, and the initial velocity, v_0. Since we don't know the time, the equation to use is 2.11: $v^2 = v_0^2 + 2a(x - x_0)$, where $d = x - x_0$.

EVALUATE Since the goal is to stop before the moose, the final velocity is zero. Solving for a gives

$$a = \frac{-v_0^2}{2d}$$

ASSESS The acceleration is negative, reflecting the fact that the car is dropping in speed as it stops.

Section 2.5 The Acceleration of Gravity

37. **INTERPRET** This problem involves constant acceleration due to gravity. We are asked to calculate the distance traveled by the rock before it hit the water.

DEVELOP We chose a coordinate system where the positive-x axis is downward. We are given the rock's constant acceleration (gravity, $g = 9.8\ \text{m/s}^2$), its initial velocity $v_0 = 0.0$ m/s, and its travel time $t = 4.4$ s. Insert this data into Equation 2.10 and solve for the displacement $x - x_0$.

EVALUATE From Equation 2.10, we find

$$\begin{aligned} x - x_0 &= v_0 t + \frac{1}{2}at^2 = v_0 t + \frac{1}{2}gt^2 \\ &= (0.0\ \text{m/s})(4.4\ \text{s}) + \frac{1}{2}(9.8\ \text{m/s}^2)(4.4\ \text{s})^2 = 95\ \text{m} \end{aligned}$$

ASSESS When the travel time of the sound is ignored, the depth of the well is quadratic in t. The depth of the well is about the length of an American football field. If we use the speed of sound $s = 340$ m/s, how will that change our answer?

38. **INTERPRET** This problem involves the constant acceleration due to gravity. We are asked to calculate the initial velocity required for an object to travel a given distance under the influence of constant acceleration (directed opposite to the initial velocity).

DEVELOP We chose a coordinate system where the positive-x axis points upward. We are given the apple's constant acceleration (gravity, $g = -9.8\ \text{m/s}^2$), its final velocity v = 0.0 m/s, and the distance traveled $x - x_0 = 6.5$ m. These quantities are related to the initial velocity v_0 by Equation 2.11.

EVALUATE Insert this data into Equation 2.11 and solve for the initial velocity v_0. This gives

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$v_0 = \pm\sqrt{v^2 - 2a(x - x_0)} = \pm\sqrt{(0.0\text{ m/s})^2 - 2(-9.8\text{ m/s}^2)(6.5\text{ m})}$$
$$= 11\text{ m/s}$$

where we choose the positive square root because we throw the apple upwards, which is the positve-x direction in our chosen coordinate system.

ASSESS Is this a hard throw to make? Compare this velocity to an MLB pitcher's fastball, which is routinely clocked at 90 mi/h = (90 mi/h)(1609 m/mi)(1 h/3600 s) = 40 m/s. So, you only have to generate about 25% of the velocity of a major-league pitcher.

39. **INTERPRET** The problem involves constant acceleration due to gravity. We are asked to find the maximum altitude reached by a model rocket that is launched upward with the given velocity. In addition, we need to find the speed and altitude at three different times, counting from the launch time.

DEVELOP We choose a coordinate system in which the upward direction corresponds to the positive-x direction. We are given the initial velocity, $v_0 = 49$ m/s, and we know that the velocity at the peak of the rocket's flight is $v = 0$ m/s, the rocket's acceleration is $a = g = -9.8\text{ m/s}^2$ (i.e., it accelerates downward toward the Earth), and its initial position is $x_0 = 0$ m. Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$, relates these quantities to the rocket's displacement x. For parts **(b)**, **(c)**, and **(d)**, use Equation 2.7, $v = v_0 + at$, to find the rocket's speed at the different times, and then Equation 2.9, $x - x_0 = (v_0 + v)t/2$, to find its displacement (i.e., altitude).

EVALUATE (a) At the peak of the rocket's flight, Equation 2.11 gives

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$x = \frac{v^2 - v_0^2}{2a} + x_0 = \frac{(0.0\text{ m/s})^2 - (49\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} + 0.0\text{ m} = 123\text{ m}$$

(b) At $t = 1$ s, the speed and the altitude are

$$v = v_0 - gt = 49\text{ m/s} - (9.8\text{ m/s}^2)(1\text{ s}) = 39\text{ m/s}$$
$$x = x_0 + v_0 t - \frac{1}{2}gt^2 = 0.0\text{ m/s} + (49\text{ m/s})(1\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(1\text{ s})^2 = 44\text{ m}$$

The first quantity (39 m/s) is known to two significant figures because we know the intial velocity to this precision, so subtacting a less-precise quantity from it does not change its precision. The second quantity should be rounded to 40 m because both non-zero terms in Equation 2.9 are known to a single significant figure.

(c) At $t = 1$ s, the speed and the altitude are

$$v = v_0 - gt = 49\text{ m/s} - (9.8\text{ m/s}^2)(4\text{ s}) = 9.8\text{ m/s}$$
$$x = x_0 + v_0 t - \frac{1}{2}gt^2 = 0.0\text{ m/s} + (49\text{ m/s})(4\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(4\text{ s})^2 = 118\text{ m}$$

Again, we need to round the second result to a single significant figure, which gives 100 m as the final answer.

(d) At $t = 7$ s, the speed and the altitude are

$$v = v_0 - gt = 49\text{ m/s} - (9.8\text{ m/s}^2)(7\text{ s}) = -20\text{ m/s}$$
$$x = x_0 + v_0 t - \frac{1}{2}gt^2 = 0.0\text{ m/s} + (49\text{ m/s})(7\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(7\text{ s})^2 = 103\text{ m}$$

Again, we need to round the second result to a single significant figure, which gives 100 m as the final answer.

ASSESS As the rocket moves vertically upward, its velocity decreases due to gravitational acceleration, which is oriented downward. Upon reaching its maximum height, the velocity reduces to zero. It then falls back to Earth with a negative velocity. From **(c)** and **(d)**, we see that the velocities have different signs at $t = 4$ s and $t = 7$ s, so we conclude that the rocket reaches its maximum height between 4 and 7 s. Calculating the time it takes to reach its maximum height using Equation 2.7 gives $t = (v - v_0)/a = (0.0\text{ m/s} - 49\text{ m/s})/(-9.8\text{ m/s}^2) = 5.0\text{ s}$, in agreement with our expectation.

40. **INTERPRET** This problem involves one-dimensional motion under the influence of gravity. We are asked to calculate how high a ball will rise and how long it remains airborne given its initial velocity.

DEVELOP Choose a coordinate system in which the positive-x direction is upward. From the problem statement, we know that the ball's initial velocity is $v_0 = 23$ m/s. From physics, we know that the velocity of the ball at the summit of its flight is $v = 0$ m/s, and that during its flight it is accelerated by gravity at $a = g = -9.8$ m/s². To find how high the ball rises, use Equation 211, $v^2 = v_0^2 + 2a(x - x_0)$, and to find the total time the ball is airborne, use Equation 2.10, $x = x_0 + v_0 t + at^2/2$.

EVALUATE (a) Inserting the known quantities into Equation 2.11 gives

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$x = \frac{v^2 - v_0^2}{2a} + x_0 = \frac{(0.0\text{ m/s})^2 - (23\text{m/s})^2}{2(-9.8\text{ m/s}^2)} + 0.0\text{m} = 27\text{ m}$$

(b) Inserting the known quantities into Equation 2.10 gives

$$x - x_0 = 0 = v_0 t + \frac{1}{2}at^2$$
$$t = \frac{-2v_0}{a} = \frac{-2(23\text{ m/s})}{-9.8\text{ m/s}^2} = 4.7\text{ s}$$

where we have used the fact that $x = x_0$ because the ball returns to the level at which it left the bat.

ASSESS If the ball goes straight up as it leaves the bat and stays airborne for almost 5 s, what are the chances the catcher will catch the ball?

41. **INTERPRET** This problem involves one-dimensional motion under the influence of gravity. We are asked to calculate what initial velocity of the rock is needed so that it is traveling at 3 m/s when it reaches the Frisbee.

DEVELOP Choose a coordinate system in which the positive-x direction is upward. When the rock hits the Frisbee, its velocity and height are $v = 3$ m/s and $x = 6.5$ m, and the rocks initial position is $x_0 = 1.3$ m. These quantities are related by Equation 2.11:

$$v^2 = v_0^2 + 2a(x - x_0)$$

EVALUATE Solving this equation for the initial velocity, we obtain

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$v_0 = \pm\sqrt{v^2 - 2a(x - x_0)} = \pm\sqrt{(3\text{ m/s}) - 2(-9.8\text{ m/s}^2)(6.5\text{ m} - 1.3\text{ m})} = 11\text{ m/s}$$

where we have chosen the positive square root because the rock must be travelling upward.

ASSESS The initial velocity v_0 must be positive since the rock is thrown upward. In addition, v_0 must be greater than the final velocity 3 m/s. These conditions are met by our result.

42. **INTERPRET** This problem involves one-dimensional motion under the influence of gravity. We need to find the acceleration due to gravity on an unknown planet, and to identify the planet by comparing our result with the data in Appendix E.

DEVELOP Choose a coordinate system in which the positive-x direction is upward. We know the initial position of the watch is $x_0 = 1.70$ m, the final position is $x = 0$ m, and the time it takes to fall is 0.95 s. Furthermore, we know that the initial velocity of the watch is $v_0 = 0.0$ m/s, so we can use Equation 2.10, $x = x_0 + v_0 t + at^2/2$, to find the acceleration of the watch.

EVALUATE Solving this equation for the acceleration, we obtain

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$a = \frac{2(x - x_0 - v_0 t)}{t^2} = \frac{2[1.70\text{ m} - 0.00\text{ m} - (0.00\text{ m/s})(0.95\text{ m/s})]}{(0.95\text{ s})^2} = 3.8\text{ m/s}^2$$

This acceleration is closest to the gravity listed for Mars in Appendix E, so our earthling must be on Mars.

ASSESS This value for the acceleration due to gravity is approximately one-third of the gravitational acceleration on the surface of the Earth.

PROBLEMS

43. **INTERPRET** This is a one-dimensional problem involving two travel segments. We are asked to calculate the average velocity for the second segment of the trip.

DEVELOP The trip can be divided into two time intervals, t_1 and t_2 with $t = t_1 + t_2 = 40$ min = 2/3 h. The total distance traveled is $x = x_1 + x_2 = 25$ mi, where x_1 and x_2 are the distances covered in each time interval.

EVALUATE During the first time interval, $t_1 = 15$ min (or 0.25 h), and with an average speed of $v_1 \equiv 20$ mi/h, the distance traveled is

$$x_1 = \bar{v}_1 t_1 = (20\text{ mi/h})(0.25\text{ h}) = 5\text{ mi}$$

Therefore, the remaining distance $x_2 = x - x_1 = 25$ mi − 5mi = 20 mi must be covered in

$$t_2 = t - t_1 = 40\text{ min} - 15\text{ min} = 25\text{ min} = \frac{5}{12}\text{ h}$$

This implies an average speed of

$$\bar{v}_2 = \frac{x_2}{t_2} = \frac{20\text{ mi}}{5\text{ h}/12} = 48\text{ mi/h}$$

ASSESS The overall average speed was pre-determined to be

$$\bar{v} = \frac{x}{t} = \frac{25\text{ mi}}{2\text{ h}/3} = 37.5\text{ mi/h}$$

When you drive slower during the first segment, you make it up by driving faster during the second. In fact, the overall average speed equals the time-weighted average of the average speeds for the two parts of the trip:

$$\bar{v} = \frac{x}{t} = \frac{x_1 + x_2}{t} = \frac{\bar{v}_1 t_1 + \bar{v}_2 t_2}{t} = \left(\frac{t_1}{t}\right)\bar{v}_1 + \left(\frac{t_2}{t}\right)\bar{v}_2 = \left(\frac{15\text{ min}}{40\text{ min}}\right)(20\text{ mi/h}) + \left(\frac{25\text{ min}}{40\text{ min}}\right)(48\text{ mi/h}) = 37.5\text{ mi/h}$$

44. **INTERPRET** This problem involves calculating the time it takes the ball to travel from the pitcher to the catcher, then calculating how fast the catcher must throw the ball to get it to second base before the base runner.

DEVELOP We can break this problem into two segments: the time it takes the ball to travel from pitcher to catcher, and the time it takes the catcher to get the ball to second base. For the first segment, convert mi/h to ft/s to have consistent units. The conversion is (90 mi/h)(5280 ft/mi)(1 h/3600 s) = 132 ft/s. Therefore the time it takes the ball to reach the catcher is

$$t_1 = \frac{d}{v} = \frac{61\text{ ft}}{132\text{ ft/s}} = 0.462\text{ s}$$

(Note that were retaining more significant figures than warranted for the intermediate calculations.) Taking into account the time it takes the catcher to release the ball towards second plate, the ball must travel to second base in a time t_2 give by

$$t_2 = 3.4\text{ s} - t_1 - 0.45\text{ s} = 2.95 - 0.462 = 2.49\text{ s}$$

Now calculate the distance to second base and divide by the time t_2 to find the necessary speed with which the catcher must throw the ball.

EVALUATE The diagonal of a square 90 ft on a side is 90 $\sqrt{2}$ ft = 127.3 ft, so the catcher must throw the ball with a speed

$$\bar{v} = \frac{d}{t_2} = \frac{90\sqrt{2}\text{ ft}}{2.49\text{ s}} \equiv 51\text{ ft/s}$$

Because we know the size of the baseball diamond (90 ft) to a single significant figure, we must round our answer to a single significant figure, which give 50 ft/s as the average velocity for the catcher’s throw.

ASSESS This speed is about one-third the speed of the pitcher’s fast ball.

45. **INTERPRET** This is a one-dimensional kinematics problem that involves calculating the average velocity of two brothers. In particular, we are asked to calculate much sooner the slower brother must start to arrive at the finish line at the same time as the faster brother.

DEVELOP Because the brothers desire to have a tie race over 100 meters, they must both cover that distance. Thus, the head start must be in time, not distance. The average velocity of the fast brother is 20% greater than that of the slow brother, so

$$\bar{v}_{slow}(1.00 + 0.20) = \bar{v}_{fast}$$

$$\bar{v}_{slow} = \frac{\bar{v}_{fast}}{1.20} = \frac{9.0\ \text{m/s}}{1.20} = 7.5\ \text{m/s}$$

Knowing the speed of both brothers, calculate the difference in this time for them to cover 100 m. This time is the head start needed by the slower brother.

EVALUATE The time it takes for each brother to cover $\Delta x = 100$ m is

$$t_{fast} = \frac{\Delta x}{\bar{v}_{fast}} = \frac{100\ \text{m}}{9.0\ \text{m/s}} = 11.1\ \text{s}$$

$$t_{slow} = \frac{\Delta x}{\bar{v}_{slow}} = \frac{100\ \text{m}}{7.5\ \text{m/s}} = 13.3\ \text{s}$$

The difference between these times is the head start needed by the slower brother. This is $\Delta t = t_{slow} - t_{fast} = 13.3\ \text{s} - 11.1\ \text{s} = 2.2\ \text{s}$.

ASSESS What if both brothers started at the same time, but the slower one was given a head start in distance—what distance would be needed? The distance needed is simply the distance the slower brother covers in his 2.2-s head start, or $\Delta x = \bar{v}_{slow}\Delta t = (7.5\ \text{m/s})(2.222\ \text{s}) = 16.7\ \text{m} \approx 17\ \text{m}$ to two significant figures.

46. **INTERPRET** This is a one-dimensional kinematics problem that asks us to calculate the point at which two jetliners will meet given their starting points and average velocities.

DEVELOP Given the average speed, the distance traveled during a time interval can be calculated using Equation 2.1, $\Delta x = \bar{v}\Delta t$. An important point here is to recognize that at the instant the airplanes pass each other, the sum of the total distance traveled by both airplanes is $\Delta x = 4600$ km.

EVALUATE Suppose that the two planes pass each other after a time Δt from take-off. We then have

$$\Delta x = \Delta x_1 + \Delta x_2 = \bar{v}_1\Delta t + \bar{v}_2\Delta t = (\bar{v}_1 + \bar{v}_2)\Delta t$$

which yields

$$\Delta t = \frac{\Delta x}{\bar{v}_1 + \bar{v}_2} = \frac{4600\ \text{km}}{1100\ \text{km/h} + 700\ \text{km/h}} = 2.56\ \text{h} \approx 2.6\ \text{h}$$

Thus, the encounter occurs at a point about $\Delta x_1 = \bar{v}_1\Delta t = (1100\ \text{km/h})(2.56\ \text{h}) = 2811\ \text{km} \approx 2800\ \text{km}$ from San Francisco, or $\Delta x_2 = \bar{v}_2\Delta t = (700\ \text{km/h})(2.56\ \text{h}) = 1789\ \text{km} \approx 2000\ \text{km}$ from New York. The approximate results are those with the correct number of significant figures.

ASSESS The point of encounter is closer to New York than San Francisco. This makes sense because the plane that leaves from New York travels at a lower speed. If we sum the distances covered by the two airplanes when they encounter, we find $\Delta x = 2811\ \text{km} + 1789\ \text{km} = 4600\ \text{km}$, which is the distance from San Francisco to New York, as expected.

47. **INTERPRET** The goal of this problem is to gain an understanding of the limiting procedure at the root of calculus. We are to estimate an object's instantaneous velocity to ever-increasing precision without using calculus, then compare the results with the result obtained with calculus.

DEVELOP Use Equation 2.1, $\bar{v} = \Delta x/\Delta t$, to calculate the average speed for each time interval. To do this, we need to know the displacements, which we can calculate using the given formula for position as a function of time. This gives

$$\textbf{(a)}\ x_{1a} = (1.50\ \text{m/s})(1.00\ \text{s}) + (0.640\ \text{m/s}^3)(1.00\ \text{s})^3 = 2.140\ \text{m}$$
$$x_{2a} = (1.50\ \text{m/s})(3.00\ \text{s}) + (0.640\ \text{m/s}^3)(3.00\ \text{s})^3 = 21.78\ \text{m}$$
$$\textbf{(b)}\ x_{1b} = (1.50\ \text{m/s})(1.50\ \text{s}) + (0.640\ \text{m/s}^3)(1.50\ \text{s})^3 = 4.410\ \text{m}$$
$$x_{2b} = (1.50\ \text{m/s})(2.50\ \text{s}) + (0.640\ \text{m/s}^3)(2.50\ \text{s})^3 = 13.75\ \text{m}$$
$$\textbf{(c)}\ x_{1c} = (1.50\ \text{m/s})(1.95\ \text{s}) + (0.640\ \text{m/s}^3)(1.95\ \text{s})^3 = 7.671\ \text{m}$$
$$x_{2c} = (1.50\ \text{m/s})(2.05\ \text{s}) + (0.640\ \text{m/s}^3)(2.05\ \text{s})^3 = 8.589\ \text{m}$$

The instantaneous velocity may be found by differentiating the given formula for position (see Equation 2.3).

EVALUATE From Equation 2.1, we find the following average velocities:

$$\textbf{(a)}\ \bar{v}_a = \frac{\Delta x_a}{\Delta t_a} = \frac{x_{2a} - x_{1a}}{t_{2a} - t_{1a}} = \frac{21.78\ \text{m} - 2.140\ \text{m}}{3.00\ \text{s} - 1.00\ \text{s}} = 9.82\ \text{m/s}$$

$$\textbf{(b)}\ \bar{v}_b = \frac{\Delta x_b}{\Delta t_b} = \frac{x_{2b} - x_{1b}}{t_{2b} - t_{1b}} = \frac{13.75\ \text{m} - 4.410\ \text{m}}{2.50\ \text{s} - 1.50\ \text{s}} = 9.34\ \text{m/s}$$

$$\textbf{(c)}\ \bar{v}_c = \frac{\Delta x_c}{\Delta t_c} = \frac{x_{2c} - x_{1c}}{t_{2c} - t_{1c}} = \frac{8.589\ \text{m} - 7.671\ \text{m}}{2.05\ \text{s} - 1.95\ \text{s}} = 9.18\ \text{m/s}$$

(d) Differentiating the given formula for position, and evaluating it at $t = 2$ s give

$$v(t) = dx/dt = b + 3ct^2$$
$$v(2\ \text{s}) = 1.50\ \text{m/s} + 3(0.640\ \text{m/s}^3)(2\ \text{s})^2 = 9.18\ \text{m/s}$$

We find that the average velocity provides an ever-increasing precise estimation of the instantaneous velocity as the time interval over which the average velocity is calculated shrinks.

ASSESS As the interval surrounding 2 s gets smaller, the average and instantaneous velocities approach each other; the values in parts **(c)** and **(d)** differ by less than 0.02% (if you retain more significant figures).

48. **INTERPRET** This is a one-dimensional kinematics problem involving finding the instantaneous velocity as a function of time, given the position as a function of time. We must also show that the average velocity from $t = t_1 = 0$ to any arbitrary time $t = t_2$ is one-fourth of the instantaneous velocity at t_2.

DEVELOP The instantaneous velocity $v(t)$ can be obtained by taking the derivative of $x(t)$. The derivative of a function of the form bt^n can be obtained by using Equation 2.3. The average velocity for any arbitrary time interval $\Delta t = t_2 - t_1$ may be calculated by using Equation 2.1, $\bar{v} = \Delta x/\Delta t$, where Δx is determined by evaluating $x = bt^4$ for the two times t_1 and t_2.

EVALUATE The instantaneous velocity is $v(t) = dx/dt = d/dt\,(bt^4) = 4bt^3$. The average velocity over the time interval from $t = 0$ to any time $t > 0$ is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(t) - x(0)}{t - 0} = \frac{bt^4}{t} = bt^3$$

which is just ¼ of $v(t)$ from above.

ASSESS Note that $\bar{v}$ is not equal to the average of $v(0)$ and $v(t)$, as stated in Equation 2.8. That is applicable only when acceleration is constant, which is clearly not the case here.

49. **INTERPRET** This is a one-dimensional kinematics problem in which we need to use calculus to calculate the velocity and acceleration given the expression for position as a function of time. We must find the time at which the car passes two points, and calculate the average velocity for the car between these points from these measurements. Finally, we need to calculate the difference between this average velocity and the instantaneous velocity midway between the two points.

DEVELOP The instantaneous velocity $v(t)$ can be obtained by taking the derivative of $x(t) = bt^2$ (see Equation 2.2b). Thus we have

$$x(t) = bt^2$$

$$v(t) = \frac{dx}{dt} = 2bt = \pm 2\sqrt{xb}$$

Where we have used the $x(t)$ to eliminate t in the expression $v(t)$. The first equation will tell us the times at which the car passes the two observers, and we can use Equation 2.1 $\bar{v} = \Delta x / \Delta t$ to find the average velocity calculated by each observer. The instantaneous velocity at 200 m is given by the second equation.

EVALUATE (a) Using the expression $x(t)$, we find the time at which the car passes the two observers is

$$t = \pm\sqrt{\frac{x}{b}} = \begin{cases} \pm\sqrt{\dfrac{180\text{ m}}{2.000\text{ m/s}^2}} = 9.4868\text{ s (first observer)} \\ \pm\sqrt{\dfrac{220\text{ m}}{2.000\text{ m/s}^2}} = 10.488\text{ s (second observer)} \end{cases}$$

Using Equation 2.1, the observers find an average velocity of

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{220\text{ m} - 180\text{ m}}{10.488\text{ s} - 9.4868\text{ s}} = 39.95\text{ m/s}$$

(b) Using the expression $v(t)$ for the instantaneous velocity at x = 200 m is

$$v = \pm 2\sqrt{xb} = \pm 2\sqrt{(200\text{ m})(2.000\text{ m/s}^2)} = 40.00\text{ m/s}$$

which differs from the average velocity by (100%)(39.95 m/s − 40.00 m/s) / (40.00 m/s) = −0.13%.

ASSESS What would happen if the observers were not symmetrically positioned about the 200-m mark? How would that affect the result? At 220 m, we see that the instantaneous velocity is $v = 2\sqrt{(220\text{ m/s})(2\text{ m/s}^2)} = 41.95$ m/s, which is a 4.8% difference with respect to the average velocity.

50. **INTERPRET** This problem is a mathematical exercise desinged to familiarize us with the kinetic equations for one-dimensional motion with constant acceleration.

DEVELOP Equation 2.7 is $v = v_0 + at$ and Equation 2.11 is $v^2 = v_0^2 + 2a(x - x_0)$.

EVALUATE Squaring Equation 2.7 gives $v^2 = (v_0 + at)^2 = v_0^2 + 2v_0 at + a^2t^2$. Equating the result to Equation 2.11 gives $2v_0 at + a^2t^2 = 2a(x - x_0)$, or $x - x_0 = v_0 t + \frac{1}{2}at^2$ which is Equation 2.10.

ASSESS Can you derive other relationships between the equations of motion?

51. **INTERPRET** This problem deals with the landing of spacecraft *Curiosity* on Mars. We apply a simple one-dimensional kinematics with constant deceleration.

DEVELOP The initial speed of the *Curiosity* is v_0 = 32.0 m/s. Its speed then decreases steadily to v = 0.75 m/s as its altitude is dropped from 142 m to 23 m. We use Equation 2.11: $v^2 = v_0^2 + 2a(x - x_0)$ to solve for the acceleartion a.

EVALUATE Using Equation 2.11, we find the acceleration to be

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(0.75\text{ m/s})^2 - (32.0\text{ m/s})^2}{2(142\text{ m} - 23\text{ m})} \quad 4.3\text{ m/s}$$

Thus, the magnitude of the spacecraft's acceleration is 4.3 m/s^2.

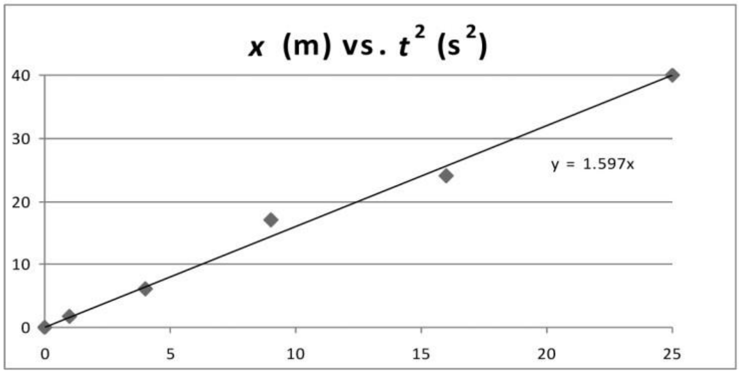
ASSESS This is about 1.16 times the surface gravity of Mars: $g_{\text{Mars}} = 3.71\text{ m/s}^2$. The duration of this CD phase can be calculated using Equation 2.7:

$$t = \frac{v - v_0}{a} = \frac{0.75\text{ m/s} - 32\text{ m/s}}{-4.3\text{ m/s}^2} = 7.3\text{ s.}$$

52. **INTERPRET** This is a data-analysis problem, where the position of a car in a drag race is given at various times. We analyze the data and look for a quantity, which when position is plotted against it, gives a straight line.

DEVELOP The car starts from rest (x_0 = 0, v_0 = 0) and undergoes constant acceleration. From one-dimensional kinematics, the position of the car as a function of time can be written as $x = at^2/2$, where a is the acceleration. Thus, plotting x against t^2 should give a straight line with slope $a/2$.

EVALUATE A plot of position versus t^2 is given below.



The plot yields a best-fit line with slope $a/2 = 1.6\text{ m/s}^2$. Thus, the acceleration of the car is approximately 3.2 m/s^2.

ASSESS This is about $0.3g$. For Formula One race, the initial acceleration is typically around $1.5g$.

53. **INTERPRET** The problem involves constant acceleration due to gravity. We have a fireworks rocket that explodes at a given height, with some fragments traveling upward and some downward. We want to know the time interval of the fragments hitting the ground.

DEVELOP The fragment that travels vertically downward will hi the ground first, while the one that move vertically upward will come down last. We choose a coordinate system in which the upward direction corresponds to the positive-y direction. For the first (downward) fragment, the initial height is $y_0 = 82$ m, and $v_{10} = -7.68$ m/s (the negative sign indicates that the fragment moves downward), Equation 2.10 gives

$$y_1 = y_0 + v_{10}t - \frac{1}{2}gt^2 = 82.0\text{ m} + (-7.68\text{ m/s})t - \frac{1}{2}(9.80\text{ m/s}^2)t^2$$

Setting $y_1 = 0$, and solving the quadratic equation, the time for the fragment to reach the ground is $t_1 = 3.382$ s. Similarly, for the upward traveling fragment, we have

$$y_2 = y_0 + v_{20}t - \frac{1}{2}gt^2 = 82.0\text{ m} + (16.7\text{ m/s})t - \frac{1}{2}(9.80\text{ m/s}^2)t^2$$

Setting $y_2 = 0$, and solving the quadratic equation, the time for the fragment to reach the ground is $t_2 = 6.136$ s.

EVALUATE The time interval over which the fragments hit the ground is $\otimes t = t_2 - t_1 \approx 2.75$ s, to three significant figures.

ASSESS A fragment that undergoes free fall would have reached the ground in $\sqrt{2y_0/g} = 5.79$ s. Travel time is longer for fragments having an upward velocity, but shorter for fragments with a downward velocity.

54. **INTERPRET** In this problem, we want to know how high a grasshopper can jump with a given initial velocity.

DEVELOP We choose a coordinate system in which the upward direction corresponds to the positive-y direction. We note that the grasshopper is momentarily at rest when it reaches the maximum height. We use Equation 2.11: $v^2 = v_0^2 + 2a(x - x_0)$ to solve for the maximum height.

EVALUATE Rewriting the equation as $v^2 = v_0^2 - 2gy_{max}$, where $v = 0$, we find the maximum height to be

$$y_{max} = \frac{v_0^2}{2g} = \frac{(3.0\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 0.46\text{ m}$$

ASSESS The body length of a grasshopper is between 1 and 7 cm, depending on the species. The maximum height calculated here means that grasshoppers can make jumps that are many times the length of their bodies, a task not possible for humans.

55. **INTERPRET** This as a one-dimensional problem involving a car subjected to constant deceleration. We need to relate the car's stopping distance to its stopping time.

DEVELOP For motion with constant acceleration, the stopping distance and the stopping time are related by Equation 2.9, $x - x_0 = (v_0 + v)t/2$.

EVALUATE Let v_0 be the initial velocity and $v = 0$ be the final velocity. Equation 2.9 can then be rewritten as

$$x - x_0 = \frac{1}{2}(v_0 + v)t = \frac{1}{2}v_0 t$$

Thus, we see that the stopping distance, $x - x_0$, is proportional to the stopping time, t, so both are reduced by the same amount (55%).

ASSESS Anti-lock brakes optimize the deceleration by controlling the wheels so that they roll just at the point of skidding.

56. **INTERPRET** This as a one-dimensional problem involving a car subjected to constant deceleration. We need to relate the car's stopping distance to its stopping time.

DEVELOP In this problem we must use $-\alpha$ for the acceleration in Table 2.1. Because we are given the acceleration, the displacement ($x - x_0 = 0$), and the initial velocity, we can use Equation 2.10 to find the time.

EVALUATE **(a)** A return to the initial position means that $x(t) = x_0$ for $t > 0$. From Equation 2.10, $x = x_0 + v_0 t + \frac{1}{2}(-\alpha)t^2$, or $2v_0 t = \alpha t^2$. Because $t \neq 0$, we can divide to get $t = 2v_0/\alpha$, which is the time at which the particle returns to the starting point.

(b) The speed, or magnitude of the velocity, can be found from Equation 2.7, $v = v_0 + at$. Taking the magnitude of the velocity gives $|v| = |v_0 + (-\alpha)t| = |v_0 - \alpha(2v_0/\alpha)| = |-v_0| = v_0$. The speed is the same, but the direction of motion is reversed.

ASSESS This means that if you throw a ball straight up in the air, it will return to the ground at the same speed at which it departed (ignoring air resistance).

57. **INTERPRET** We interpret this as a one-dimensional kinematics problem with the hockey puck being the object of interest.

DEVELOP We are told that the hockey puck undergoes constant deceleration while moving through the snow. Equation 2.9, $x = x_0 + \frac{1}{2}(v_0 + v)t$, provides the connection between the initial velocity $v_0 = 32$ m/s, the final velocity $v = 18$ m/s, the travel time t, and the distance traveled $x = 0.35$ m. For part **(b)**, we use Equation 11, $v^2 = v_0^2 + 2a(x - x_0)$ to find the acceleration, and then use the same equation again to find the minimum thickness of the snow, $x_{\min}$, needed to stop the hockey puck entirely ($v = 0$).

EVALUATE **(a)** Solving for the time

$$t = \frac{(x - x_0)}{\frac{1}{2}(v_0 + v)} = \frac{(0.35\text{ m} - 0)}{\frac{1}{2}(32\text{ m/s} + 18\text{ m/s})} = 0.014\text{ s}$$

(b) First we solve for the acceleration

$$a = \frac{(v^2 - v_0^2)}{2(x - x_0)} = \frac{\left((18\text{ m/s})^2 - (32\text{ m/s})^2\right)}{2(0.35\text{ m} - 0)} = -1000\text{ m/s}^2$$

Then we plug this back in to the same equation to find the minimum snow thickness for stopping the puck

$$x_{\min} = \frac{(v^2 - v_0^2)}{2a} = \frac{\left(0 - (32\text{ m/s})^2\right)}{2(-1000\text{ m/s}^2)} = 0.51\text{ m} = 51\text{ cm}$$

ASSESS We find the minimum thickness to be proportional to v_0^2 and inversely proportional to the deceleration $-a$. This agrees with our intuition: The greater the speed of the puck, the thicker the snow needed to bring it to a stop; similarly, less snow would be needed with increasing deceleration.

58. **INTERPRET** This is a one-dimensional kinematics problem in which we are asked to find the average acceleration of the train (magnitude and direction) and the distance required for it to stop.

DEVELOP We choose a coordinate system in which the positive-x axis indicates the direction in which the train is traveling. Because we are given the initial velocity ($v_0 = 110$ km/h), the final velocity ($v = 0$), and the time interval ($t = 1.2$ min $= 0.020$ h), we can use Equation 2.7, $v = v_0 + at$, to find the acceleration. Once we find the acceleration we can use Equation 2.9, $x - x_0 = (v_0 + v)t\,|2$, to find the stopping distance $x - x_0$.

EVALUATE **(a)** Inserting the given quantities into Equation 2.7 gives the acceleration as

$$v = v_0 + at$$
$$a = \frac{v - v_0}{t} = \frac{0.0\ \text{m/s} - 110\ \text{km/h}}{0.020\ \text{h}} = \left(-5500\ \text{km/h}^2\right)\left(\frac{10^3\ \text{m}}{\text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)^2 = -0.42\ \text{m/s}^2$$

(b) Because $a < 0$, the acceleration must be directed opposite to the train's motion. In other words, it's a deceleration.

(c) Using Equation 2.9, we find a stopping distance of

$$x - x_0 = \frac{1}{2}\left(v + v_0\right)t = \frac{\left(110\ \text{km/h} + 0.0\ \text{km/h}\right)}{2}\left(0.020\ \text{h}\right) = 1.1\ \text{km}$$

ASSESS Notice that we had to be careful to keep proper track of the initial and final speed to get the correct direction of acceleration. Had we inverted the two, we would have found an acceleration in the same direction as the train's motion, which would have meant that the train accelerated to hit the cow!

59. **INTERPRET** This is a one-dimensional kinematics problem. We assume the jetliner slows down on the runway with constant deceleration.

DEVELOP Equation 2.9, $x = x_0 + \frac{1}{2}(v_0 + v)t$, relates distance, initial velocity, and final velocity. The equation can be used to solve for the shortest runway.

EVALUATE With t = 29 s = (29/3600) h, and the final velocity v set to zero, Equation 2.9 gives

$$x - x_0 = \tfrac{1}{2}\left(v_0 + v\right)t = \tfrac{1}{2}\left(220\ \text{km/h}\right)\left(29 / 3600\ \text{h}\right) = 0.89\ \text{km}$$

ASSESS The length is a bit short compared to the typical minimum landing runway length of about 1.5 km for full-size jetliners.

60. **INTERPRET** This is a one-dimensional kinematics problem with constant deceleration. We are given the final velocity, the acceleration distance, and the acceleration distance, and we are asked to find the initial velocity and the acceleration time.

DEVELOP We choose a coordinate system in which the positive-x direction is in the direction of the car's initial velocity. Using the known quantities (v = 18 kh/h, a = $-6.3\ \text{m/s}^2$, $x - x_0$ = 34 m), solve Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$, for the initial velocity v_0. Then use the result for v_0 in Equation 2.7, $v = v_0 + at$, to find the acceleration time t. Converting the final velocity to m/s for the calculation, we have v = (18 km/h)(1 h/3600 s)(10^3 m/km) = 5.0 m/s.

EVALUATE **(a)** Inserting the known quantities into Equation 2.11 gives

$$v_0 = \sqrt{v^2 - 2a\left(x - x_0\right)} = \sqrt{\left(5.0\ \text{m/s}\right)^2 - 2\left(-6.3\ \text{m/s}^2\right)\left(34\ \text{m}\right)} = 21\ \text{m/s}$$

(**b**) Inserting this result for v_0 into Equation 2.7 gives

$$t = \frac{v - v_0}{a} = \frac{5.0\ \text{m/s} - 21.3\ \text{m/s}}{-6.3\ \text{m/s}^2} = 2.6\ \text{s}$$

where we have retained more significant figures for v_0 because it serves as an intermediate result for this part.

ASSESS In km/h, the initial velocity is v_0 = (21.3 m/s)(10^{-3} km/m)(3600 s/h) = 77 km/h.

61. **INTERPRET** This is a one-dimensional kinematics problem in which we are asked to find the initial velocity of a racing car given its initial velocity, it acceleration, the distance covered, and the time interval.

DEVELOP We chose a coordinate system in which the positive-x direction is in the direction of the car's velocity. We are told that the car undergoes constant acceleration, so we can use the equations from Table 2.1. For part **(a)**, we are given the distance, time, and final velocity, so we can use Equation 2.9, $x - x_0 = (v_0 + v)t\,|2$, to find the initial velocity. For part **(b)**, find the acceleration of the car and use the result in Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$, to solve for the distance travelled.

EVALUATE (a) The distance covered $x - x_0 = 140$ m, the time interval is $t = 3.6$ s, and the final velocity is $v = 53$ m/s. Inserting these quantities into Equation 2.9 and solving for the intial velociy v_0 gives

$$x - x_0 = \frac{1}{2}(v_0 + v)t$$

$$v_0 = \frac{2(x - x_0)}{t} - v = \frac{2(140\text{ m})}{3.6\text{ s}} - 53\text{ m/s} = 24.8\text{ m/s} = 25\text{ m/s}$$

(b) From Equation 2.7, we find the acceleration to be

$$a = \frac{v - v_0}{t} = \frac{53\text{ m/s} - 24.8\text{ m/s}}{3.6\text{ s}} = 7.84\text{ m/s}^2$$

Upon substituting the result into Equation 2.11, the distance traveled starting from rest ($v_0 = 0$) to a velocity $v = 53$ m/s is

$$x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{(53\text{ m/s})^2 - 0}{2(7.84\text{ m/s}^2)} = 180\text{ m}$$

to two significant figures.

ASSESS Comparing parts **(a)** and **(b)**, the car travels a distance of 179 m *from rest* to the end of the 140-m distance. Using Equation 2.11, we can show that the additional 39 m (=179 m − 140 m) is the distance traveled to bring the car from rest to an initial speed of $v_0 = 24.8$ m/s:

$$x - x_0 = \frac{v_0^2}{2a} = \frac{(24.8\text{ m/s})^2}{2(7.84\text{ m/s}^2)} = 39\text{ m}$$

62. INTERPRET This problem asks us to calculate the stopping distance for two cars given their acceleration and initial velocity, and to compare this distance with their initial separation to see if the cars will collide and, if so, at what speed. We are also asked to plot the cars' displacement as a function of time.

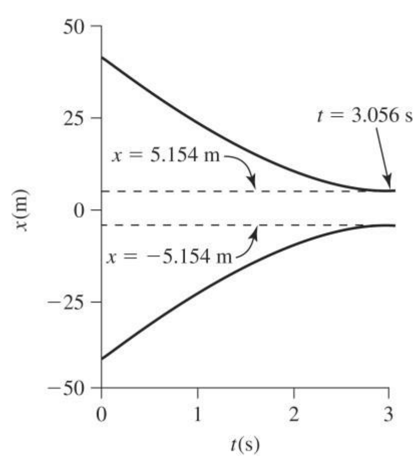
DEVELOP To find the stopping distance, use Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$ with $v_0 = (88\text{ km/h})(10^3\text{ m/km})(1\text{ h/3600 s}) = 24.4$ m/s, $v = 0.0$ m/s, and $a = -8\text{ m/s}^2$. If the result is less than 85/2 m = 42.5 m, the cars will not collide.

EVALUATE Inserting the given quantities into Equation 2.11 gives a stopping distance of

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{(0.0\text{ m/s})^2 - (24.4\text{ m/s})^2}{2(-8\text{ m/s}^2)} = 37.3\text{ m} < 42.5\text{ m}$$

so the cars will not collide. When they stop, they will be separated by 85 − 2(37.3 m) = 10.3 m. To plot x versus t, use Equation 2.10 for each car and choose the origin at the midpoint of the separation between the cars, with positive x in the direction of the initial velocity of the first car, and $t = 0$ when the brakes are applied. The graph of $x_1(t)$ and $x_2(t)$ is shown below



ASSESS Note that the accelration is negative for each car because each car is decelerating.

63. INTERPRET We interpret this as *two* problems involving one-dimensional kinematics with constant acceleration.

We are asked to find the acceleration needed so that the two runners arrive at the finish line simultaneously.

DEVELOP Calculate the speed of the runner B (the leader) from the distance she's already covered. This gives

$$v_B = \frac{\Delta x}{\Delta t} = \frac{(9\text{ km} + 0.1\text{ km})(10^3\text{ m/km})}{(35\text{ min})(60\text{ s/min})} = 4.33\text{ m/s}$$

The remaining 900 m will take her $t = (900\text{ m})/(4.33\text{ m/s}) = 207.7$ s to cover. The initial speed of the trailing runner A is

$$v_{0A} = \frac{\Delta x}{\Delta t} = \frac{9000\text{ m}}{(35\text{ min})(60\text{ s/min})} = 4.29\text{ m/s}$$

Use these results in Equation 2.10 to find the acceleration needed so that both runners finish at the same time.

EVALUATE The acceleration needed so that both runners finish simultaneously can be found by inserting the time into Equation 2.10, and solving for the acceleration, which gives

$$x_A = v_{0A}t + \frac{1}{2}at^2$$

$$a = \frac{2(x_A - v_{0A}t)}{t^2} = \frac{2[1000\text{ m} - (4.29\text{ m/s})(207.7\text{ s})]}{(207.7\text{ s})^2} = 0.0051\text{ m/s}^2$$

ASSESS For runner A to catch up to runner B, he must run faster than the speed at which he was initially running, so his acceleration is positive. When runner A crosses the finish line, his speed is $v_A = v_{0A} + at = 4.29\text{ m/s} + (0.0051\text{ m/s}^2)(207.7\text{ s}) = 5.34$ m/s, or an increase of about 25% with respect to his initial speed.

64. **INTERPRET** We are asked to calculate the minimum separation between two cars, one which moves at constant speed and the other which moves at constant acceleration. This change in this separation as a function of time (i.e., their relative velocity) is the time derivative of the difference Δx in the cars' positions, and this quantity will be zero when the cars are at their minimum separation.

DEVELOP The car in front has constant speed $v_{2,0}$ = (60 km/h)(1000 h/km)(1 h/3600 s) = 23.6 m/s, so its equation for position is $v_2 = \Delta x/\Delta t$, or $x_2 = x_{2,0} + v_{2,0}t$ where $x_{2,0} = 10$ m is the distance between the two cars at $t = 0$. At $t = 0$, the car coming from behind has initial position $x_{1,0} = 0$, initial velocity

$$v_{1,0} = (85\text{ km/h})(1000\text{ m/s})(1\text{ h/3600 s}) = 16.7\text{ m/s},$$

and acceleration $a_1 = -4.2\text{ m/s}^2$ and its equation of motion is $x_1 = x_{1,0} + v_{1,0}t + \frac{1}{2}a_1t^2 = v_{1,0}t + \frac{1}{2}a_1t^2$ ($x_{1,0} = 0$). The distance between the two cars is $\Delta x = x_2 - x_1 = x_{2,0} + v_{2,0}t - (v_{1,0}t + a_1t^2/2)$. The minimum separation between the cars occurs when their relative speed is zero, or $d\Delta x/dt = 0$. If this position is zero or less, the cars collide, if not, we can evaluate the separation Δx at the minimum-separation time to find how close the cars approach.

EVALUATE Evaluating the time derivative $d\Delta x/dt$ gives

$$\frac{d\Delta x}{dt} = \frac{dx_{2,0}}{dt} + \frac{d}{dt}(v_{2,0}t) - \frac{d}{dt}\left(v_{1,0}t + \frac{1}{2}a_1t^2\right)$$

$$= 0 + v_{2,0} - v_{1,0} - a_1t$$

$$t = \frac{v_{1,0} - v_{2,0}}{-a_1} = \frac{3.06\times10^8\text{ m/s} - 2.16\times10^8\text{ m/s}}{-(-4.2\text{ m/s}^2)} = 1.65\text{ s}$$

Insert this time into the equation for Δx to obtain their minimum separation $\Delta x_{min} = x_{2,0} + (v_{1,0} - v_{2,0})t - \frac{1}{2}a_1t^2 = 4.33\text{ m} \approx 4$ m, where we retain no figures to the right of the decimal point because $x_{2,0}$ has no figures to the right of the decimal point. Because the result is positive, the cars do not collide.

ASSESS The cars do not collide, and the minimum distance between them is 4.33 m, which occurs 1.65 s after the driver of the trailing car applies the brakes.

65. **INTERPRET** This as a one-dimensional kinematics problem in which we are asked to find the initial velocity of an object given its acceleration due to gravity (on Mars) and its maximum height.

DEVELOP Choose a coordinate system in which x indicates the upward direction from the surface of Mars, with the origin at the surface (i.e., $x_0 = 0$). Use Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$, to describe the vertical motion of the

Mars rover Spirit. Because the impact speed is the same as the rebound speed, both are given by v_0 (note that the impact velocity is opposite in sign to the rebound velocity). The spacecraft attains a maximum height of $x = 15$ m when $v = 0$. Note that the gravitational acceleration of Mars is $g_{\text{Mars}} = 3.71\ \text{m/s}^2$ (Appendix E).

EVALUATE Solving Equation 2.11 with $a = -g_{\text{Mars}} = -3.71\ \text{m/s}^2$, the impact speed is

$$v_0 = \sqrt{v^2 - 2a(x - x_0)} = \sqrt{2g_{\text{Mars}}(x - x_0)} = \sqrt{2(3.71\ \text{m/s}^2)(15\ \text{m})} = 10.55\ \text{m/s} = 11\ \text{m/s}$$

where we have retained two significant figures in the answer.

ASSESS We find the impact speed to be proportional to $\sqrt{x - x_0}$, which is the square root of the rebound height. This agrees with our expectation that the greater the impact speed, the higher the rover will rebound.

66. **INTERPRET** We are asked to find the speed at which an object should be tossed upward so that the entire up-and-down trajectory takes 1 second. This problem involves constant acceleration because the acceleration of the object is due to gravity at the surface of the Earth.

DEVELOP Choose a coordinate system in which the positive-x direction indicates the distance above the surface of the Earth. Define the initial and final positions of the atom cluster as $x_0 = x = 0$. The acceleration of the cluster is $a = g = -9.82$ m/s, and the time interval $t = 1.0$ s. Solve Equation 2.10, $x = x_0 + v_0 t + \frac{1}{2}at^2$, for the initial speed v_0.

EVALUATE Solving Equation 2.10 for v_0 gives

$$\overbrace{x}^{=0} = \overbrace{x_0}^{=0} + v_0 t + \frac{1}{2}at^2$$

$$v_0 = \frac{-gt}{2} = \frac{-(-9.82\ \text{m/s}^2)(1.0\ \text{s})}{2} = 4.9\ \text{m/s}$$

ASSESS Note that the answer is independent of what is thrown. Whether we throw a ball, or "throw" a cluster of atoms, the acceleration due to gravity is the same and they have the same behavior (ignoring air resistance and what-not).

67. **INTERPRET** This is a one-dimensional kinematics problem that involves finding the vertical distance of an object as a function of time.

DEVELOP Choose a coordinate system in which the positive-x direction is upward. Equation 2.10, $x(t) = x_0 + v_0 t + at^2/2$, describes the vertical position $x(t)$ of an object falling from x_0 as a function of time. Because the object was dropped from a stationary position, $v_0 = 0$ so $x(t) = x_0 + at^2/2$. Furthermore, we are free to choose the origin of the x axis where we like, so we let $x_0 = 0$, which gives $x(t) = at^2/2$. Finally, the acceleration is $a = -g = -9.8\ \text{m/s}^2$, which points downward, so our Equation 2.10 takes the form $x(t) = -gt^2/2$. The problem states that $x(t) - x(t-1) = x(t)/4$, from which we can solve for t, which we can insert into $x(t)$ to find x (i.e., the height from which it was dropped). Notice that x will be negative because the object's final position is below its initial position.

EVALUATE

$$x(t) - x(t-1) = \frac{x(t)}{4}$$

$$-\frac{1}{2}gt^2 - \left[-\frac{1}{2}g(t-1)^2\right] = -\frac{1}{8}gt^2$$

$$\frac{1}{2}g(1-2t) = -\frac{1}{8}gt^2$$

$$t^2 - 8t + 4 = 0$$

$$t = 4 \pm 2\sqrt{3}\ \text{m/s}$$

(We discarded the negative square root because $t > 1$ s.) Inserting this result into $x(t)$ gives

$$x(t) = -\frac{1}{2}gt^2 = -\frac{1}{2}(9.8\ \text{m/s}^2)\left[\left(4 + 2\sqrt{3}\right)\text{s}\right]^2 = -270\ \text{m}$$

to two significant figures. Thus, the object must be dropped from a height of 273 m.

ASSESS During a free fall, the vertical distance traveled is proportional to t^2. Therefore, we expect the object to travel a greater distance during the latter time interval. In general, we must also take into consideration air resistance.

68. **INTERPRET** We have to calculate the final velocity of an object falling from the given height above the surface of Io.
DEVELOP From Appendix E, the surface gravity of Io is $g = 1.8\ \text{m/s}^2$. We know the height $(y_0 = 100\ \text{m})$ at which the probe is at rest ($v_0 = 0$), so Equation 2.11 can tell us the final velocity when the probe hits the ground $(y = 0)$:

$$v = \sqrt{v_0^2 - 2g(y - y_0)} = \sqrt{2gy_0}$$

EVALUATE Plugging in the values

$$v = \sqrt{2(1.8\ \text{m/s}^2)(100\ \text{m})} = 19\ \text{m/s}$$

ASSESS This is approximately 43 mi/h. With special shock absorbers, it's reasonable to assume the probe could withstand a crash landing at this speed.

69. **INTERPRET** This is a gravitational acceleration problem where two balls are dropped at the same time, but they have different initial positions and velocities.
DEVELOP The first ball starts at a height of $y_{10} = h/2$ and velocity of $v_{10} = 0$. The second ball starts at a height of $y_{20} = h$, but we are asked to find its initial velocity. The goal is to have them hit the ground ($y_1 = y_2 = 0$) at the same time. We'll use Equation 2.10, $y = y_0 + v_0 t - \frac{1}{2} g t^2$, for each ball.
EVALUATE The time it takes the first ball to reach the ground is

$$t = \sqrt{\frac{-y_{10}}{-\frac{1}{2} g}} = \sqrt{\frac{2y_{10}}{g}} = \sqrt{\frac{h}{g}}$$

This is the same time for the second ball, so we can use this to find its initial velocity:

$$v_{20} = \tfrac{1}{2} gt - y_{20}/t = \tfrac{1}{2}\sqrt{hg} - \sqrt{hg} = -\tfrac{1}{2}\sqrt{hg}$$

The corresponding initial speed is $\frac{1}{2}\sqrt{hg}$.

ASSESS The velocity is negative since the second ball has to be thrown downwards to catch up with the first ball.

70. **INTERPRET** This is a one-dimensional, constant acceleration kinematics problem that asks us to calculate an object's final speed given its initial speed and acceleration.
DEVELOP Choose a coordinate system where the positive-x direction is upward, so $a = g = -9.8\ \text{m/s}^2$, and $x - x_0 = -15$ m, because the rock's final position is below its initial position. Use Equation 2.11 in the form of $v_T^2 = v_{0,T}^2 + 2a(x - x_0)$ and $v_D^2 = v_{0,D}^2 + 2a(x - x_0)$, with $v_{0,T} = -10$ m/s (for the rock thrown downward) and $v_{0,D} = 0.0$ m/s (for a rock that is dropped). Solve each equation for the final velocity and take the difference to find how much faster the thrown rock is moving when it reaches the ground.
EVALUATE For the thrown rock, we find

$$v_T^2 = v_{0,T}^2 + 2a(x - x_0)$$
$$v_T = \pm\sqrt{(-10\ \text{m/s})^2 - 2(9.8\ \text{m/s}^2)(-15\ \text{m})} = -19.85\ \text{m/s}$$

where we retain the negative solution because the rock is moving downward (negative-x direction). Repeating the calculation for the rock that is dropped gives

$$v_T^2 = v_{0,D}^2 + 2a(x - x_0)$$
$$v_D = \pm\sqrt{(0.0\ \text{m/s})^2 - 2(9.8\ \text{m/s}^2)(-15\ \text{m})} = -17.15\ \text{m/s}$$

The difference in speed is −19.85 m/s − (−17.15 m/s) = 2.7 m/s, where we retain two significant figures in our answer.

ASSESS The result would be the same if the rock is thrown upward with $v_0 = 10$ m/s, but then the attackers would have more time to get out of the way.

71. **INTERPRET** We interpret this as two problems involving one-dimensional kinematics with constant acceleration due to gravity. We are asked to find the final velocity of two divers given their initial speed, and to find which diver hits the water first and by how much time.

DEVELOP We choose a coordinate system in which the positive-x direction is upward. Let A be the diver who jumps upward at 1.80 m/s, and B be the one who steps off the platform. The velocity of diver A as he passes B on his way down is $v = -1.80$ m/s, which can be found by inserting $x = x_0$ in Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$ with $v_0 = 1.80$ m/s. Thus, the initial velocity of diver A for the remainder of his trajectory is $v_{0,\mathrm{A}} = -1.80$ m/s. The initial velocity of diver B is $v_{0,\mathrm{B}} = 0.00$ m/s. Applying Equation 2.11 to both divers gives

$$v_\mathrm{A}^2 = v_{0,\mathrm{A}}^2 - 2g(x - x_0)$$
$$v_\mathrm{B}^2 = v_{0,\mathrm{B}}^2 - 2g(x - x_0) = -2g(x - x_0)$$

which we can solve to find the speeds at the water. Note that the acceleration is $a = -g$, which points downward. For part **(b)**, use Equation 2.10, $x = x_0 + v_0 t + at^2/2$, to express the vertical position of the divers as a function of time.

EVALUATE (a) At the water's surface, $x = 0$, and the speeds of the divers are

$$v_\mathrm{A} = \sqrt{v_0^2 - 2g(x - x_0)} = \sqrt{(-1.80\ \mathrm{m/s})^2 - 2(9.82\ \mathrm{m/s^2})(0.00\ \mathrm{m} - 3.00\ \mathrm{m})} = 7.88\ \mathrm{m/s}$$

$$v_\mathrm{B} = \sqrt{-2g(x - x_0)} = \sqrt{-2(9.82\ \mathrm{m/s^2})(0.00\ \mathrm{m} - 3.00\ \mathrm{m})} = 7.67\ \mathrm{m/s}$$

(b) From Equation 2.10, the vertical position of the divers as a function of time is

$$x_\mathrm{A}(t) = x_0 + v_0 t + \frac{1}{2}at^2 = (3.00\ \mathrm{m}) + (-1.80\ \mathrm{m/s})t - \frac{1}{2}(9.82\ \mathrm{m/s^2})t^2$$
$$x_\mathrm{B}(t) = x_0 + \frac{1}{2}at^2 = (3.00\ \mathrm{m}) - \frac{1}{2}(9.82\ \mathrm{m/s^2})t^2$$

The divers hit the water when $x(t) = 0$. Solving the equations above, we find $t_\mathrm{A} = 1.61$ s and $t_\mathrm{B} = 0.782$ s. Therefore, diver A hits about $\otimes t = t_\mathrm{B} - t_\mathrm{A} = 0.782\ \mathrm{s} - 0.620\ \mathrm{s} = 0.162$ s before diver B.

ASSESS We expect diver A to hit the water first because he has a non-zero initial velocity for the trajectory from the platform to the water.

72. **INTERPRET** This is a one-dimensional, constant-acceleration problem. A ball is thrown upward by a person who is rising at 10 m/s. We must calculate how long the ball is in the air before the person catches it.

DEVELOP We choose a coordinate system in which the positive-x direction is upward. The initial velocity of the ball is 12 m/s relative to the passenger who throws it. Because the passenger is moving upward with a constant velocity of 10 m/s, the initial velocity of the ball relative to the ground is $v_{0,\mathrm{B}} = 22$ m/s. The acceleration of the ball is $a = -g = -9.82\ \mathrm{m/s^2}$. From Equation 2.10, the position of the ball is $x_\mathrm{B}(t) = x_{0,\mathrm{B}} + v_{0,\mathrm{B}}t + at^2/2 = v_{0,\mathrm{B}}t - gt^2/2$ because its initial position is $x_{0,\mathrm{B}} = 0$ m. The position of the passenger $x_\mathrm{P}(t)$ can be expressed using Equation 2.9, with $v_{0,\mathrm{P}} = v_\mathrm{P} = 10$ m/s because the balloon rises without acceleration. This gives $x_\mathrm{P}(t) = x_{0,\mathrm{P}} + (v_{0.\mathrm{P}} + v_\mathrm{P})t/2 = v_\mathrm{P}t$. When the passenger catches the ball, $x_B(t) = x_P(t)$, from which we can solve for the time t that the ball is in the air.

EVALUATE Inserting the given values gives

$$x_B(t) = x_P(t)$$
$$v_{0,B}t - gt^2/2 = v_P t$$
$$t = \frac{2(v_{0,B} - v_P)}{g} = \frac{2(22\ \mathrm{m/s} - 10\ \mathrm{m/s})}{9.8\ \mathrm{m/s^2}} = 2.4\ \mathrm{s}$$

ASSESS If the balloon were stuck to the ground, $v_{0,\mathrm{B}} = 12$ m/s and $v_\mathrm{P} = 0$, and the result would be identical. This is because when the balloon moves with constant velocity it still constitutes an inertial reference frame (i.e., a reference frame that does not accelerate). Consider tossing a ball up in the air in a car moving at constant speed down the highway—there is no difference between this and executing the same task while standing on the ground.

73. INTERPRET This is a one-dimensional kinematics problem involving a spacecraft that undergoes free fall under the influence of the gravitational acceleration of the Moon. We are asked to find the spacecraft's impact speed and the time of its fall given the height from which it falls.

DEVELOP We choose a coordinate system in which the positive-*x* direction is downward. Using Equation 2.10, $x = x_0 + v_0 t + at^2/2$, the vertical position of the spacecraft falling from x_0 as a function of time is

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2 = x_0 + \frac{1}{2}g_{\text{Moon}}t^2$$

because $v_0 = 0$ (the spacecraft falls from a stationary position), and the gravitational acceleration of the Moon is g_{Moon} is downward. Note that when the spacecraft impacts the Moon, it will have fallen $x - x_0 = 12$ m. From Appendix E, we find that $g_{\text{Moon}} = 1.62\ \text{m/s}^2$.

EVALUATE Solving this equation for the time *t*, we find that the amount of time it takes the spacecraft to drop 12 m from rest is

$$t = \sqrt{\frac{2(x - x_0)}{g_{\text{Moon}}}} = \sqrt{\frac{2(12\ \text{m})}{1.62\ \text{m/s}^2}} = 3.849\ \text{s} \approx 3.8\ \text{m/s}$$

to two significant figures. From Equation 2.7, the velocity at impact is

$$v = v_0 + g_{\text{Moon}}t = 0.00\ \text{m/s} + (1.62\ \text{m/s}^2)(3.85\ \text{s}) = 6.2\ \text{m/s}.$$

ASSESS Our result indicates that *t* is proportional to $g^{-1/2}$. Therefore, the greater the gravitational acceleration, the less time it takes for the free fall and the higher the velocity at impact. The same fall on the Earth would result in a velocity at impact of $v = (9.8\ \text{m/s}^2)[2(12\ \text{m})/(9.8\ \text{m/s}^2)]^{1/2} = 15\ \text{m/s}$.

74. INTERPRET The question is asking you how long the rocket would be inside the clouds, and thus out of sight.

DEVELOP The band of clouds extend between the altitudes of $y_B = 1.9$ km and $y_T = (1.9 + 5.3)\text{km} = 7.2$ km. The rocket's altitude is given by Equation 2.10: $y = \frac{1}{2}at^2$, where we assume $y_0 = v_0 = 0$. From this, the time can be solved for as a function of altitude

$$t(y) = \sqrt{\frac{2y}{a}}$$

EVALUATE The time spent in the clouds is then

$$t(y_T) - t(y_B) = \sqrt{\frac{2y_T}{a}} - \sqrt{\frac{2y_B}{a}} = \frac{\sqrt{2(7200\text{m})} - \sqrt{2(1900\text{m})}}{\sqrt{4.6\ \text{m/s}^2}} = 27\ \text{s}$$

This is less than 30 s, so yes, the rocket can launch.

ASSESS The rocket is accelerating against Earth's gravity. If it had the same thrust in outer space, it would accelerate at $a = (4.6 + 9.8)\text{m/s}^2 = 14.4\ \text{m/s}^2$.

75. INTERPRET We're asked to find the relative speed between the two subway trains when they collide. We can interpret this as two problems involving one-dimensional kinematics with constant acceleration.The two objects of interest are the two trains.

DEVELOP Let the fast train be A and the slow train be B. While B maintains a constant speed, A tries to slow down to avoid collision with a constant deceleration. We take the origin $x = 0$ and $t = 0$ at the point where A begins decelerating, with positive x in the direction of motion. Position as a function of time is given by Equation 2.10, $x = x_0 + v_0 t + \frac{1}{2}at^2$. We write two versions of this equation, one for x_A and one for x_B. The condition that both trains collide may be expressed as $x_A = x_B$.

EVALUATE We first rewrite the initial speeds of the trains as

$$v_{0A} = 80\ \text{km/h} = \left(80\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 22.22\ \text{m/s}$$

$$v_{0B} = 25\ \text{km/h} = \left(25\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 6.94\ \text{m/s}$$

0B h 1 km 3600 s

We can express the positions of trains A and B as

$$x_A = v_{0A}t + \tfrac{1}{2}a_A t^2 = (22.22 \text{ m/s})t + \tfrac{1}{2}(-2.1 \text{ m/s}^2)t^2$$

$$x_B = x_{0B} + v_{0B}t = 50 \text{ m} + (6.94 \text{ m/s})t$$

When the trains collide, $x_A = x_B$. The above equations then give

$$\tfrac{1}{2}a_A t^2 + (v_{0A} - v_{0B})t - x_{0B} = 0 \Rightarrow \quad (-1.05 \text{ m/s}^2)t^2 + (15.28 \text{ m/s})t - (50 \text{ m}) = 0$$

Using the quadratic formula to solve for the smaller root, we find $t = 4.97$ s. The velocity of train A at the time of the collision is

$$v_A = v_{0A} + a_A t = (22.22 \text{ m/s}) + (-2.1 \text{ m/s}^2)(4.97 \text{ s}) = 11.78 \text{ m/s}$$

Therefore, their relative speed at the collision is

$$v_{\text{rel}} = v_A - v_{0B} = 11.78 \text{ m/s} - 6.94 \text{ m/s} = 4.84 \text{ m/s}$$

or 17.4 km/h.

ASSESS The initial relative speed is $v_{\text{rel},0} = v_{A0} - v_{0B} = 22.22$ m/s − 6.94 m/s = 15.28 m/s. Braking reduces the speed of train A, and the relative speed between A and B, but the deceleration a = • 2.1 m/s^2 is not enough to prevent collision.

76. **INTERPRET** Although the book must have a horizontal component of velocity, this will remain constant, so we can consider this as a one-dimensional kinematics problem involving an object undergoing constant acceleration due to gravity. We need to find the (vertical) velocity of the book at a given height given its starting position, its acceleration, and the maximum height it attains.

DEVELOP We choose a coordinate system in which the positive-x direction is upward. Use Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$, to find the velocity v_0 with which the book leaves your hand. For part **(a)**, the final velocity is $v = 0$, because the book is at the top of its trajectory. The acceleration is $a = -g = -9.8$ m/s^2, and the displacement $x - x_0$ = 4.2 m – 1.5 m = 2.7 m. Insert the result for the initial velocity into Equation 2.10, $x = x_0 + v_0 t + at^2/2$, to find the time at which it hits the floor (x = 4.2 m – 0.87 m = 3.33 m).

EVALUATE (a) Solving Equation 2.11 for the initial velocity, we find

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$v_0 = \pm\sqrt{v^2 - 2a(x - x_0)} = \pm\sqrt{(0.0 \text{ m/s})^2 - 2(-9.8 \text{ m/s}^2)(2.7 \text{ m})} = 7.27 \text{ m/s} = 7.3 \text{ m/s}$$

where we have retained two significant figures.

(b) Inserting this result into Equation 2.10 and solving for the time t gives

$$x = x_0 + v_0 t + at^2/2$$
$$0 = \left(\frac{a}{2}\right)t^2 + (v_0)t + (x_0 - x)$$
$$t = \frac{-v_0 \pm \sqrt{v_0^2 + 2a(x - x_0)}}{a} = \frac{-7.27 \text{ m/s} - \sqrt{(7.27 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(3.33 \text{ m} - 1.50 \text{ m})}}{-9.8 \text{ m/s}^2}$$
$$= 1.2 \text{ s}$$

to two significant figures. We have taken the negative sign of the square root because we are looking for the longer of the two times at which the book passes the 3.33-m level (it passes once on its way up and once on its way down).

ASSESS We neglect air resistance and the size of the book in this problem. If we use the positive sign for the square root in part **(b)**, we find that the book passes the 3.33-m level at t = 0.32 s.

77. **INTERPRET** This is a one-dimensional kinematics problem involving two travel segments. The key concept here is the average speed.

DEVELOP The average speed is the total distance divided by the total time, or $\bar{v} = ⊗x/⊗t$. For both cases, we shall find the total distance traveled and the time taken.

EVALUATE (a) Let the distances traveled during the two time intervals be L_1 and L_2. The total distance is the sum of the distances covered at each speed:

$$L = L_1 + L_2 = v_1\left(\frac{t}{2}\right) + v_2\left(\frac{t}{2}\right) = \frac{1}{2}(v_1 + v_2)t$$

so

$$\bar{v} = \frac{L}{t} = \frac{1}{2}(v_1 + v_2)$$

(b) In this case, let t_1 and t_2 be the two time intervals. The total time is the sum of the times traveled at each speed:

$$t = t_1 + t_2 = \frac{L/2}{v_1} + \frac{L/2}{v_1} = \frac{L}{2}\left(\frac{v_1 + v_2}{v_1 v_2}\right)$$

Therefore, the average speed is

$$\bar{v}' = \frac{L}{t} = \frac{2v_1v_2}{v_1 + v_2}$$

(c) The difference between the two cases is

$$\bar{v} - \bar{v}' = \frac{1}{2}(v_1 + v_2) - \frac{2v_1v_2}{v_1 + v_2} = \frac{2v_1v_2}{2(v_1 + v_2)}\left[(v_1 + v_2)^2 - 2v_1v_2\right] = \frac{2v_1v_2}{2(v_1 + v_2)}\left(v_1^2 + v_2^2\right) > 0$$

So the first case gives a greater average speed.

ASSESS The average speed $\bar{v}$ is the time-weighted average of the separate speeds: $\bar{v} = (t_1/t)v_1 + (t_2/t)v_2$. With this in mind, the result in part **(a)** may be rewritten as

$$\bar{v} = (1/2)v_1 + (1/2)v_2$$

and for part **(b)**,

$$\bar{v} = \left(\frac{t_1}{t}\right)v_1 + \left(\frac{t_2}{t}\right)v_2 = \left(\frac{v_2}{v_1 + v_2}\right)v_1 + \left(\frac{v_1}{v_1 + v_2}\right)v_2 = \frac{2v_1v_2}{v_1 + v_2}$$

78. INTERPRET This problem involves calculating the instantaneous velocity and acceleration given the position as a function of time.

DEVELOP Using the formulas in Appendix A, we can differentiate the given formula with respect to time to get the instantaneous velocity. We then differentiate the resulting expression for velocity to find the instantaneous acceleration.

EVALUATE **(a)** For $x(t) = x_0\sin(\omega t)$, $dx/dt = v(t) = \omega x_0\cos(\omega t)$ and $dv/dt = d^2x/dt^2 = a(t) = -\omega^2 x_0\sin(\omega t) = -\omega^2 x(t)$.

(b) Because the maximum value of the sine or cosine functions is 1, $v_{max} = \omega x_0$ and $a_{max} = \omega^2 x_0$.

ASSESS The motion described by $x(t)$ is called simple harmonic motion; see Chapter 13.

79. INTERPRET This as a one-dimensional kinematics problem that involves finding the vertical position of a leaping person as a function of time.

DEVELOP We choose a coordinate system in which the positive-x direction is upward. Using Equation 2.10, the vertical position of a person as a function of time may be written as (setting $x_0 = 0$)

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

$$\frac{1}{2}gt^2 - v_0 t + x = 0$$

Note that the acceleration is $a = -g$, which points downward. The quadratic formula gives two times when the leaper passes a particular height:

$$v_0 \pm \sqrt{v_0^2 - 2gx}$$

$$t_{\pm} = \frac{0 \quad \sqrt{\quad 0 \qquad}}{g}$$

The smaller value, t_-, corresponds to the time for going up and the larger value, t_+, corresponds to the time for coming down. Therefore, the time spent above that height is

$$\Delta t(x) = t_+ - t_- = \frac{v_0 + \sqrt{v_0^2 - 2gx}}{g} - \frac{v_0 - \sqrt{v_0^2 - 2gx}}{g} = \frac{2\sqrt{v_0^2 - 2gx}}{g}$$

Using Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$, we find that in order to reach a maximum height h, the initial velocity must be $v_0 = \sqrt{2gh}$. Therefore, the above expression for $\Delta t(x)$ may be simplified as

$$\Delta t(x) = \frac{2\sqrt{2g(h-x)}}{g}$$

EVALUATE The total time spent in the air is the time spent above the ground. Setting $x = 0$, we have

$$\Delta t(0) = \frac{2\sqrt{2gh}}{g} = 2\sqrt{\frac{2h}{g}}$$

Similarly, the time spent in the upper half, above $x = h/2$, is

$$\Delta t(h/2) = \frac{2\sqrt{2g(h/2)}}{g} = 2\sqrt{\frac{h}{g}}$$

Therefore,

$$\frac{\Delta t(h/2)}{\Delta t(0)} = \frac{2\sqrt{h/g}}{2\sqrt{2h/g}} = \frac{1}{\sqrt{2}} = 0.707$$

or 70.7%.

ASSESS Our result indicates that while in the air, a person spends 70.7% of the time on the upper half of the height. Such a large fraction of time is what gives the illusion of "hanging" almost motionless near the top of the leap.

80. **INTERPRET** This problem considers a balloon falling under the influence of gravity.

DEVELOP If the balloon was dropped from height y_0 at time $t = 0$, then its height at any later time is $y = y_0 - \frac{1}{2}gt^2$. When it passes the top of the window, $y_1 = y_0 - \frac{1}{2}gt_1^2$, and when passing the bottom, $y_2 = y_0 - \frac{1}{2}gt_2^2$. We will use the length of the window ($y_1 - y_2 = 1.3$ m) and the time the balloon is in front of the window ($t_2 - t_1 = 0.22$ s) to solve for the initial height ($y_0 - y_1$).

EVALUATE Subtracting the equations for the window height gives

$$y_1 - y_2 = \frac{1}{2}g\left(t_2^2 - t_1^2\right) = \frac{1}{2}g\left(t_2 - t_1\right)\left(t_2 + t_1\right)$$

$$\Rightarrow t_2 + t_1 = \frac{2\left(y_1 - y_2\right)}{g\left(t_2 - t_1\right)} = \frac{2(1.3\text{ m})}{\left(9.8\text{ m/s}^2\right)\left(0.22\text{ s}\right)} = 1.21\text{ s}$$

Combining this result for $t_2 + t_1$ with $t_2 - t_1$ gives

$$t_1 = \left(1.21\text{s} - 0.22\text{s}\right)/2 = 0.495\text{ s}$$

Plugging this into the equation for y_1, we finally have the drop height

$$y_0 - y_1 = \frac{1}{2}gt_1^2 = \frac{1}{2}\left(9.8\text{ m/s}^2\right)\left(0.495\text{ s}\right)^2 = 1.2\text{ m}$$

ASSESS We had to assume the balloon was dropped from rest. The balloon could have been given an initial velocity by the thrower, and this would invalidate our result for the initial height.

81. **INTERPRET** This is a one-dimensional kinematics problem involving constant deceleration. We are asked to calculate an acceleration given the distance and the initial and final velocities.

DEVELOP Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$ relates the distance traveled to the initial speed, the final speed, and the acceleration. We shall use this equation to solve for the acceleration.

EVALUATE The motorist has to reduce his speed within $x - x_0 = 0.9$ km from $v_0 = 110$ km/h to $v = 70$ km/h. This requires a constant acceleration of

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(70\ \text{km/h})^2 - (110\ \text{km/h})^2}{2(0.9\ \text{km})}$$
$$= -4000\ \text{km/h}^2 = -1.11\ \text{km}\cdot\text{h}^{-1}\cdot\text{s}^{-1} = -0.31\ \text{m/s}^2$$

ASSESS The result means that the speed must be decreased by 1.11 km/h in each second. So, in 36 seconds, the speed is decreased from 110 km/h − $(1.11\ \text{km}\cdot\text{h}^{-1}\cdot\text{s}^{-1})(36\ \text{s})$ = 70 km/h.

82. **INTERPRET** Given an equation for a non-constant acceleration, we are asked to find equations for the instantaneous velocity and the position as a function of time.

DEVELOP We cannot use the constant-acceleration equations (2.7, 2.9–2.11), but we can use the definitions of instantaneous velocity ($v \equiv \frac{dx}{dt}$) and acceleration ($a \equiv \frac{dv}{dt}$) and work backward (i.e., integrate) to get the equations we need. For example:

$$a(t) = \frac{dv}{dt} \Rightarrow a(t)dt = dv \Rightarrow \int dv = \int a(t)dt \Rightarrow v(t) = \int a(t)dt$$

The initial position (at $t = 0$) is x_0 and the initial velocity is v_0.

EVALUATE (a) Integrating the given equation for acceleration gives us the velocity

$$v(t) = \int a\,dt = \int (a_0 + bt)dt = a_0 t + \tfrac{1}{2}bt^2 + C_1$$

To find the constant C_1, note that $v(t = 0) = C_1 = v_0$, so $v(t) = v_0 + a_0 t + \frac{1}{2}bt^2$.

(b) Use the same procedure to find an expression for position:

$$x(t) = \int v(t)dt = \int \left(v_0 + a_0 t + \tfrac{1}{2}bt^2\right)dt = v_0 t + \tfrac{1}{2}a_0 t^2 + \tfrac{1}{6}bt^3 + C_2$$

The position at $t = 0$ is x_0, so $C_2 = x_0$ and $x(t) = x_0 + v_0 t + \frac{1}{2}a_0 t^2 + \frac{1}{6}bt^3$.

ASSESS Note that the derivative of $a(t)$ for this problem is a constant. The derivative of acceleration is called *jerk*, so we have just derived the equations for constant-jerk motion.

83. **INTERPRET** This problem considers a car falling through a camera's field of view in a given time duration.

DEVELOP Let's assume the car starts at rest at the position y_0. Let's also define the top of the field of view as y_1 and the bottom as y_2. As the car falls, it reaches y_1 at time t_1 with velocity v_1, and similarly for y_2. By definition, $y_1 - y_2 = h$ and $t_2 - t_1 = \otimes t$. We are looking for the height the car is released above the top of the field of view, $y_0 - y_1 = H$. We can solve for H using the equations in Table 2.1.

EVALUATE From Equation 2.11, we have

$$v_1^2 = -2g(y_1 - y_0) \rightarrow H = \frac{v_1^2}{2g}$$

We need to relate v_1 to the variables we were given: h and $\otimes t$. We can do that with Equation 2.10:

$$y_2 = y_1 + v_1(t_2 - t_1) - \tfrac{1}{2}g(t_2 - t_1)^2$$
$$\Rightarrow v_1 = \frac{h}{\otimes t}\left(\frac{g\otimes t^2}{2h} - 1\right)$$

Plugging this into the above equation for H gives us

$$H = \frac{h^2}{2g\otimes t^2}\left(\frac{g\otimes t^2}{2h} - 1\right)^2$$

ASSESS This problem is actually the same as Problem 2.80, with the car and camera view replacing the balloon and window view. If you substitute the values from that problem ($h = 1.3$ m and $\otimes t = 0.22$ s) into the expression for H, you find the answer comes out right ($H = 1.2$ m).

84. **INTERPRET** This problem, like Example 2.6, involves constant acceleration of a ball due to gravity. We are asked to find the speed with which the ball hits the floor and the time that it hits the floor given several initial conditions.

DEVELOP The ball in Example 2.6 starts at a height of 1.5 meters (x_0 = 1.5 m), with an initial upward speed of v_0 = 7.3 m/s. The second ball starts at the same height with the same speed, but downward ($v_0' = -7.3$ m/s). We're asked to find the speed of both balls just before they hit the floor (x = 0.0 m) and the time at which the second ball hits the floor. We can use the constant-acceleration equations, because the only acceleration is due to gravity

($a = -g = -9.8\ \text{m/s}^2$). Start with Equation 2. 2.11, $v^2 = v_0^2 + 2a(x - x_0)$, to find the final velocities, then use Equation 2.10, $x = x_0 + v_0 t + \frac{1}{2}at^2$, to find the time.

EVALUATE (a) Inserting the given quantities into Equation 2.11 and solving for the initial final velocity gives

$$v^2 = v_0^2 + 2a\left(x - x_0\right)$$
$$v = \pm\sqrt{\left(7.3\ \text{m/s}\right)^2 + 2\left(-9.8\ \text{m/s}^2\right)\left(0.0\ \text{m} - 1.5\ \text{m}\right)} = \pm 9.1\ \text{m/s}$$

which is the speed with which the first ball hits the floor. The positive answer corresponds to the ball "hitting" the floor on the way up (i.e., at the point where the ball's trajectory crosses the point $x = 0$ for $t < 0$ in Figure 2.13). This answer is non-physical because the ball was not thrown up from the floor with a velocity of + 9.1 m/s, but was thrown upward from a height of 1.5 m. The negative answer corresponds to the ball hitting the floor on the way down, after it has executed the trajectory shown in Figure 2.13. This answer is physical and corresponds to the real velocity of the ball when it hits the floor.

(b) Repeating the calculation for the second ball gives

$$v^2 = v_0^2 + 2a\left(x - x_0\right)$$

$$v = \pm\sqrt{\left(-7.3\ \text{m/s}\right)^2 + 2\left(-9.8\ \text{m/s}^2\right)\left(0.0\ \text{m} - 1.5\ \text{m}\right)} = \pm 9.1\ \text{m/s}$$

where the positive sign corresponds to the ball passing through the floor on the way up, as if it had been thrown up from the floor with a velocity of +9.1 m/s. This answer is non-physical because the ball was thrown downward from a height of 1.5 m, not upward from the floor. The negative sign corresponds to the ball passing the floor on the way down, after being thrown downward from a height of 1.5 m. This answer is physical and corresponds to the real velocity of the ball when it hits the floor.

(c) To find when the ball hits the floor, insert the known quantities into Equation 2.10. This gives

$$x = x_0 + v_0' t + \frac{1}{2}at^2$$
$$0 = x_0 + v_0' t - \frac{1}{2}gt^2$$
$$t = \frac{-v_0' \pm \sqrt{v_0'^2 + 2x_0 g}}{g} = \frac{7.3\ \text{m/s} \pm \sqrt{\left(-7.3\ \text{m/s}\right)^2 + 2\left(-9.8\ \text{m/s}^2\right)\left(1.5\ \text{m}\right)}}{-9.8\ \text{m/s}^2} = -1.7\ \text{s},\ 0.18\ \text{s}$$

The negative solution corresponds to the positive solution of part **(b)**. In other words, it corresponds to the ball passing through the floor on the way up, as if it were thrown upward from the floor at a speed of 9.1 m/s. This result is non-physical because the ball was not thrown upward from the floor, but down from a height of 1.5 m. The positive solution corresponds to the ball hitting the floor on the way down after being thrown down from the height of 1.5 m. This result is physical and corresponds to the real time at which the ball hits the floor.

ASSESS Note that the answers to parts **(a)** and **(b)** are the same. This makes sense, because the speed of the ball when it comes back down to the 1.5-m level in part **(a)** is the same as the initial speed of the ball in part **(b)**.

85. **INTERPRET** This problem involves one-dimensional kinematics under constant acceleration. We are asked to find the frequency with which drops of water hit the sink given the initial conditions.

DEVELOP There are exactly three drops falling at any time: two partway down and one either hitting the sink or just leaving the faucet. Find the time it takes one drop to fall and divide that by three to get the time between drops. Use Equation 2.10, $x = x_0 + v_0 t + \frac{1}{2}at^2$ with $x = 0$, $x_0 = 19.6$ cm = 0.196 m, $v_0 = 0$, and $a = -g = -9.8\ \text{m/s}^2$. The question asks for drops per second, so convert seconds per drop to drops per second for the final answer.

EVALUATE From Equation 2.10, the time it takes one drop to fall is

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$0 = x_0 - \frac{1}{2}gt^2$$

$$\frac{2x_0}{} \quad \frac{2\left(0.196\ \text{m}\right)}{}$$

$$t = \sqrt{\frac{\quad}{g}} = \sqrt{\frac{\quad}{9.8\ \text{m/s}^2}} = 0.20\ \text{s}$$

There are three drops that hit the sink in this time interval, so the time between drops is (0.20 s)/(3 drops) = 0.067 s/drop. Thus, the frequency with which the drops hit the sink is 1/(0.067 s/drop) = 15 drops/s.

ASSESS This is pretty fast for a leaky faucet, but the time looks about right for the distance involved.

86. **INTERPRET** This problem involves calculating the time it takes a water balloon to reach the ground.

DEVELOP We know from previous problems that the balloon will reach the ground in a time of $t = \sqrt{2h/g}$, where h is the height from which it is released. In order for the balloon to hit its target, the distance, d, between the X and the impact point must be vt, where v here is the typical velocity of students entering the building.

EVALUATE Putting together the information above

$$d = vt = v\sqrt{\frac{2h}{g}} = (2\,\text{m/s})\sqrt{\frac{2(20\text{ m})}{(9.8\text{ m/s}^2)}} = 4\text{ m}$$

ASSESS Since not all the students will be walking at the average 2 m/s, a more effective strategy would be to use 2 X's on the ground farther out from the building. By measuring the time it takes a given student to walk between the X's, you can measure his/her speed. From that, you can more accurately predict when they will be underneath your window, and you will therefore know for sure when to release your balloon.

87. **INTERPRET** You are asked to integrate Equation 2.7 in order to derive Equation 2.10.

DEVELOP Recall the general formula for integrating a polynomial

$$\int t^n dt = \frac{1}{n+1}t^{n+1} + \text{constant}$$

EVALUATE Let's integrate Equation 2.7 over the time variable, t', from $t' = 0$ to $t' = t$

$$\int_0^t v\,dt = \int_0^t (v_0 + at')dt'$$

By definition, the time integral of $v(t)$ is $x(t)$, so the equation transforms to

$$x(t) - x(0) = \left(v_0 t' + \frac{1}{2}at'^2\right)\Big|_0^t = v_0 t + \frac{1}{2}at^2$$

Since $x(0) = x_0$ by definition, this is Equation 2.10.

ASSESS For those who want a challenge, it's also possible to derive Equation 2.11 by integrating $v = dx/dt$ over velocity.

88. **INTERPRET** This is a one-dimensional kinematics problem in which we need to use calculus to calculate the velocity and then position given the expression for acceleration as a function of time.

DEVELOP The instantaneous velocity $v(t)$ can be obtained by integrating over $a(t) = bt^2$. Thus we have

$$v(t) = \int a(t)dt = \int bt^2 dt = \frac{b}{3}t^3$$

where $v_0 = 0$ since we are told that the object starts from rest. Integrating over t one more time then gives $x(t)$.

EVALUATE Using the expression for $v(t)$, we integrate and obtain

$$x(t) = \int v(t)dt = \frac{b}{3}\int t^3 dt = \frac{b}{12}t^4$$

With $b = 0.041\text{ m/s}^4$, the distance traveled by the object in 6.3 s is

$$x(t = 6.3\text{ s}) = \int v(t)dt = \frac{b}{3}\int t^3 dt = \frac{(0.041\text{ m/s}^4)}{12}(6.3\text{ s})^4 = 5.4\text{ m}$$

ASSESS This problem involves non-constant acceleration. In physics, the rate of change of acceleration is called jerk, and the rate of change of jerk is called jounce or snap. So jerk is the third derivative, and jounce is the fourth derivative of the position vector with respect to time.

89. **INTERPRET** This is a one-dimensional kinematics problem in which acceleration is given as a function of time. We need to use calculus to calculate the velocity and then position.

DEVELOP The instantaneous velocity $v(t)$ can be obtained by integrating over $a(t) = -a_0 \cos\omega t$. Integrating $v(t)$ over t then gives $x(t)$.

EVALUATE (a) Integrating $a(t)$ over t leads to

$$v(t) = \int a(t)dt = -a_0\int \cos\omega t dt = -\frac{a_0}{\omega}\sin\omega t$$

where $v_0 = 0$ since we are told that the object starts from rest.

(b) Integrating $v(t)$ over t, we obtain position as a function of time:

$$x(t) = \int v(t)dt = -\frac{a_0}{\omega}\int \sin\omega t dt = \frac{a_0}{\omega^2}\cos\omega t$$

(c) From the expression for $v(t)$, we see that the magnitude is at a maximum when $|\sin\omega t| = 1$. Thus, $v_{max} = a_0/\omega$. Similarly, for $x(t)$, its magnitude is at a maximum when $|\cos\omega t| = 1$, leading to $x_{max} = a_0/\omega^2$.

ASSESS The type of motion described here is called simple harmonic motion. In this type of motion, the acceleration is proportional and in the opposite direction of the displacement: $a(t) = -\omega^2 x(t)$.

90. **INTERPRET** This is a one-dimensional kinematics problem in which acceleration decreases exponentially with time. We need to use calculus to calculate the velocity and then position.

DEVELOP The instantaneous velocity $v(t)$ can be obtained by integrating over $a(t) = a_0 e^{-bt}$. Integrating $v(t)$ over t then gives $x(t)$.

EVALUATE (a) Integrating $a(t)$ over t leads to

$$v(t) = \int a(t)dt = a_0\int e^{-bt}dt = -\frac{a_0}{b}e^{-bt} + v_0$$

The condition that $v(0) = 0$ implies $v_0 = a_0/b$. Therefore, $v(t) = \frac{a_0}{b}(1 - e^{-bt})$.

(b) No, the speed does not increase indefinitely. As $t \to \infty$, $v(t) \to a_0/b$.

(c) Integrating $v(t)$ over t tobtain position as a function of time, we have

$$x(t) = \int v(t)dt = \frac{a_0}{b}\int_0^\infty (1 - e^{-bt})dt \to \infty$$

Clearly, the object will continue to move indefinitely, and travel infinitely far from the origin.

ASSESS Since the acceleration $a(t)$ decreases exponentially, at large t, $a(t) \to 0$, and the object essentially moves with a constant speed $v_0 = a_0/b$.

91. **INTERPRET** This problem involves one-dimensional kinematics under constant acceleration. We have two balls, one dropped from height h_0, and the other launched upward simultaneously from the ground with speed v_0. We are interested in finding the condition on v_0 such that the two balls collide in mid-air.

DEVELOP We first consider just the ball that's dropped from rest at height h_0. Since $h_0 = gt_0^2/2$, the time for it to reach the ground is $t_0 = \sqrt{2h_0/g}$. Now, with the two balls described in the problem, suppose they collide in mid-air after t seconds, then the distances traveled are $h_1 = \frac{1}{2}gt^2$ and $h_2 = v_0t - \frac{1}{2}gt^2$ such that

$$h_0 = h_1 + h_2 = \frac{1}{2}gt^2 + \left(v_0t - \frac{1}{2}gt^2\right) = v_0t$$

The balls collide in mid-air if $t < t_0$.

EVALUATE (a) The condition $t < t_0$ implies $\frac{h_0}{v_0} < \sqrt{\frac{2h_0}{g}}$, or $v_0 > \sqrt{\frac{gh_0}{2}}$.

(b) Substituting $t = h_0/v_0$ into the expression for h_2, we find the height at which the balls collide to be

$$h_2 = v_0t - \frac{1}{2}gt^2 = v_0\left(\frac{h_0}{v_0}\right) - \frac{1}{2}g\left(\frac{h_0}{v_0}\right)^2 = h_0 - \frac{gh_0^2}{2v_0^2}$$

ASSESS The greater the speed v_0, the greater the height at which the two balls collide. In the limit where the launch speed is much greater than $\sqrt{gh_0/2}$, the height where they meet would be very close to h_0.

92. **INTERPRET** We're asked to interpret the graph of a tiger's velocity.

DEVELOP The tiger is at rest when the velocity is zero.

EVALUATE The velocity is zero at points A, E and H.

The answer is **(b)**.

ASSESS The tiger starts at rest, moves to the right (positive direction), stops (at point E), then turns and moves to the left (negative direction) before stopping.

93. **INTERPRET** We're asked to interpret the graph of a tiger's velocity.

DEVELOP The tiger has zero acceleration when the velocity is not changing, i.e., when the curve is flat.

EVALUATE The acceleration is zero at points C and F.

The answer is **(c)**.

ASSESS The tiger first accelerates to the right, but then at point C it starts to slow down and comes to a stop at point E. She then immediately begins to accelerate to the left, but then at point F it starts to slow down and comes to a stop at point H.

94. **INTERPRET** We're asked to interpret the graph of a tiger's velocity.

DEVELOP The tiger has greatest speed at the point in the graph farthest from zero.

EVALUATE The two points C and F are extreme points, but it appears that C is larger than F.

The answer is **(b)**.

ASSESS The point C is where the tiger is going the fastest to the right, whereas the point F is where the tiger is going the fastest to the left.

95. **INTERPRET** We're asked to interpret the graph of a tiger's velocity.

DEVELOP The tiger has greatest acceleration at the point in the graph where the velocity is changing the fastest, i.e., where the slope is greatest.

EVALUATE The slope appears to be the greatest at point D.

The answer is **(c)**.

ASSESS At point C, the tiger is moving quickly to the right, but it suddenly slows down at point D and comes to a stop at point E.

96. **INTERPRET** We're asked to interpret the graph of a tiger's velocity.

DEVELOP The tiger begins moving to the right, but it stops and comes back towards the left. Therefore the farthest it reaches away from its starting point must be the point where it stops.

EVALUATE The farthest point is E.

The answer is **(b)**.

ASSESS The distance traveled is the integral of velocity with respect to time: $x = \int v dt$. From A to E, this integral is positive, but after point E, it becomes negative, characterizing the fact that the tiger has turned around and is retracing its steps.