Section 1-1 Linear Equations and Inequalities

Goal: To solve linear equation and linear inequalities

Equality Properties:	1. If $x = y$ and a is any real number, then $x \pm a = y \pm a$.
	2. If $x = y$ and a is any nonzero real number, then $ax = ay$ and $\dfrac{x}{a} = \dfrac{y}{a}$.
Inequality Properties:	1. If $x > y$ and a is any real number, then $x \pm a > y \pm a$.
	2. If $x > y$ and a is any positive real number, then $ax > ay$ and $\dfrac{x}{a} > \dfrac{y}{a}$.
	3. If $x > y$ and a is any negative real number, then $ax < ay$ and $\dfrac{x}{a} < \dfrac{y}{a}$.
Interval Notation:	A bracket,] or [, is used if the endpoint is included. A parentheses,) or (, is used if the endpoint is not included. Infinity, either positive or negative, always uses a parentheses.

In problems 1–3, solve for the variable:

1.
$$5x + 7 = 9x - 13$$
$$5x + 7 - 5x = 9x - 13 - 5x$$
$$7 = 4x - 13$$
$$7 + 13 = 4x - 13 + 13$$
$$20 = 4x$$
$$\frac{20}{4} = \frac{4x}{4}$$
$$5 = x$$

2.
$$7y + 3(6y - 11) = 167$$
$$7y + 18y - 33 = 167$$
$$25y - 33 = 167$$
$$25y - 33 + 33 = 167 + 33$$
$$25y = 200$$
$$\frac{25y}{25} = \frac{200}{25}$$
$$y = 8$$

3.
$$\frac{m}{5} + 4 = \frac{m}{2} + 7$$
$$\frac{10m}{5} + 4(10) = \frac{10m}{2} + 7(10)$$
$$2m + 40 = 5m + 70$$
$$40 = 3m + 70$$
$$-30 = 3m$$
$$-10 = m$$

In problems 4–6, solve for the variable and place the final answer in interval notation.

4.
$$7x + 5 > 19$$
$$7x > 14$$
$$x > 2$$
$$(2, ¥)$$

5. $-7 < -3x - 1 \le 17$

$-6 < -3x \le 18$

$2 > x \ge -6$

$-6 \le x < 2$

$[-6, 2)$

6. $\dfrac{u}{3} + \dfrac{3}{4} < \dfrac{u}{2} - \dfrac{5}{4}$

$\dfrac{12u}{3} + \dfrac{3(12)}{4} < \dfrac{12u}{2} - \dfrac{5(12)}{4}$

$4u + 9 < 6u - 15$

$24 < 2u$

$12 < u$

$(12, \infty)$

7.   *Break-even Analysis*. A publisher for a promising new novel figures fixed costs (overhead, advances, promotion, copy, editing, typesetting, and so on) at \$67,000 and variable costs (printing, paper, binding, shipping) at \$3.50 for each book produced. If the book is sold to distributors for \$21 each, how many must be produced and sold for the publisher to break even?

Let $x =$ the number of books produced. Since the break even point is the point when cost is the same as the revenue:

$21x = 67,000 + 3.50x$

$17.50x = 67,000$

$x = 3828.571429$

Therefore, the publisher must produce 3829 books to break even.

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Goal: To find the equations of lines, x–intercepts, and y–intercepts

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Slope of a Line: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ where $P_1 : (x_1, y_1)$ and $P_2 : (x_2, y_2)$

Slope-Intercept Form of a Line: $y = mx + b,$ where m is the slope and $(0, b)$ is the y–intercept.

Equation of a line in standard form: $Ax + By = C$, where A and B are not both zero.

Horizontal Line: $y = b$, slope is zero.   Vertical Line: $x = a$, slope is undefined.

~~*y*–intercept: (0, *b*)~~   ~~*x*–intercept: (*a*, 0)~~

In problems 1–12, write the equation of the line in slope-intercept form with the given characteristics:

1. Slope is 6 and y-intercept is (0, 5).
       Since the slope and y-intercept are given, $y = 6x + 5$.

2. Slope is –5 and y-intercept is $(0, - 6)$.
       Since the slope and y-intercept are given, $y = - 5x - 6$.

3. Slope is $\dfrac{2}{5}$ and passes through the point (–10, 6).

$$y - y_1 = m(x - x_1)$$

$$y - 6 = \dfrac{2}{5}(x - (-10))$$

$$y - 6 = \dfrac{2}{5}x + 4$$

$$y = \dfrac{2}{5}x + 10$$

4. Slope is $\dfrac{-4}{5}$ and passes through the point (2, –3).

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = \dfrac{-4}{5}(x - 2)$$

$$y + 3 = \dfrac{-4}{5}x + \dfrac{8}{5}$$

$$y = \dfrac{-4}{5}x - \dfrac{7}{5}$$

5. Passes through the points (3, 6) and (6, 3).

$$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3 - 6}{6 - 3} = \dfrac{-3}{3} = -1$$

$$y - y_1 = m(x - x_1)$$

$$y - 6 = -1(x - 3)$$
$$y - 6 = -x + 3$$
$$y = -x + 9$$

6. Passes through the points (–1, 4) and (2, –2).

$$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2 - 4}{2 - (-1)} = \dfrac{-6}{3} = -2$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -2(x - (-1))$$
$$y - 4 = -2x - 2$$
$$y = -2x + 2$$

7. Passes through the points (0, 6) and (5, 0).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 6}{5 - 0} = \frac{-6}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-6}{5}(x - 5)$$

$$y = \frac{-6}{5}x + 6$$

8. Passes through the points (0, 4) and (–1, 0).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{-1 - 0} = \frac{-4}{-1} = 4$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 4(x - (-1))$$

$$y = 4x + 4$$

9. A horizontal line that passes through the point (–1, 7).
A horizontal line is parallel to the x-axis and in the form $y = b$, therefore the equation is $y = 7$.

10. A horizontal line that passes through the point (2, –5).
A horizontal line is parallel to the x-axis and in the form $y = b$, therefore the equation is $y = -5$.

11. A vertical line that passes through the point (–4, 5).
A vertical line is parallel to the y-axis and in the form $x = a$, therefore the equation is $x = -4$.

12. A vertical line that passes through the point (2, –8).

A vertical line is parallel to the y-axis and in the form $x = a$, therefore the equation is $x = 2$.

In problems 13–17, find the *x*-intercept and the *y*-intercept.

Solutions for the *x*-intercept will be found by setting $y = 0$, and the *y*-intercepts will be found by setting $x = 0$.

13. $y = -3x + 3$

$y = -3(0) + 3$

$y = 3$

(0, 3) *y*-intercept

$0 = -3x + 3$

$3x = 3$

$x = 1$

(1, 0) *x*-intercept

14. $y = -2x - 2$

$y = -2(0) - 2$

$y = -2$

(0, –2) *y*-intercept

$0 = -2x - 2$

$2x = -2$

$x = -1$

(–1, 0) *x*-intercept

15. $y = \dfrac{1}{2}x - 1$

$y = \dfrac{1}{2}(0) - 1$

$y = -1$

(0, –1) *y*-intercept

$0 = \dfrac{1}{2}x - 1$

$\dfrac{1}{\,} $

$-\dfrac{}{2}x = -1$

$x = 2$

(2, 0) *x*-intercept

16. $y = \dfrac{4}{3}x - 4$

$y = \dfrac{4}{3}(0) - 4$

$y = -4$

(0, –4) *y*-intercept

$0 = \dfrac{4}{3}x - 4$

$-\dfrac{4}{3}x = -4$

$x = 3$

(3, 0) *x*-intercept

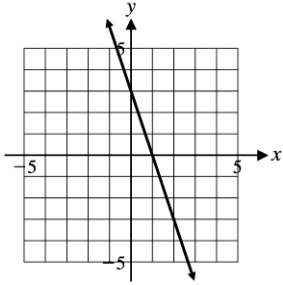17. $x - 3y = -3$

$0 - 3y = -3$

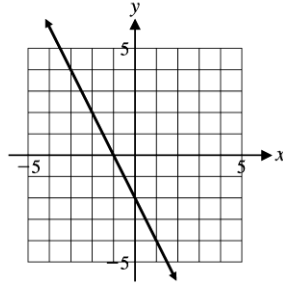$y = 1$

(0, 1) *y*-intercept

$x - 3(0) = -3$

$x = -3$

(–3, 0) *x*-intercept

18. Graph each line in problems 13–17.

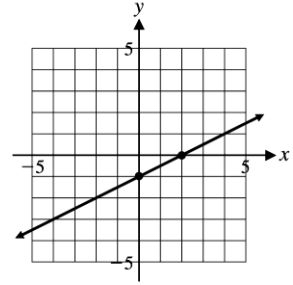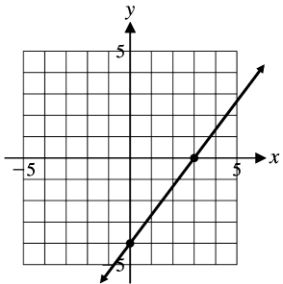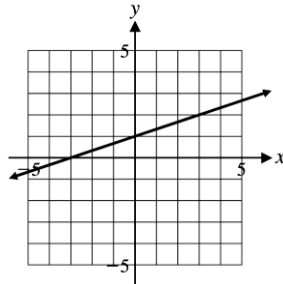Graph for 13

Graph for 14

Graph for 15

Graph for 16

Graph for 17

19. A piece of equipment used in a landfill has an original value of $100,000. After two
    years of use, the piece of equipment is valued at $75,000.

a) If the depreciation of the equipment is assumed to be linear, find an equation to relate the value (V) of the equipment over time (t).

b) What would the value of the piece of equipment be after 5 years?

c) In how many years would the value of the piece of equipment be $0?

Solution:

a)   Since the value started at $100,000 and after two (2) years it was worth $75,000, the equipment depreciated as follows:

$$m = \frac{75,000 - 100,000}{2} = \frac{-25,000}{2} = -12,500$$

Since the slope is –12,500 and the equipment had a starting value of $100,000, the equation is:

$$V = -12,500t + 100,000$$

b)   Substitute 5 in for t:

$$V = -12,500t + 100,000$$
$$V = -12,500(5) + 100,000$$
$$V = -62,500 + 100,000$$
$$V = 37,500$$

Therefore, the equipment will be worth $37,500 after 5 years.

c)   Find the value of t when V=0.

$$V = -12,500t + 100,000$$
$$0 = -12,500t + 100,000$$
$$12,500t = 100,000$$
$$t = 8$$

Therefore, the value will be $0 after 8 years.

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Section 1-3 Linear Regression

Goal: To interpret slopes and find linear regression equations

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Solving Real-World Problems

1. Construct a mathematical model.

2. Solve the mathematical model.

3. Interpret the solution.

Linear Regression on a Graphing Calculator

1. Enter the data in columns L_1 and L_2.

2. In the "STAT" mode, find the "LinReg" function.

3. Read the display to find the values of the slope and the y-intercept.

In problems 1–3, use the given information to answer the questions.

1. *Depreciation*. A new car worth $36,000 is depreciating in value by $4000 per year.

    a) Find the linear model for the current value of the car, v, and the number of years, y, after it was purchased.
    b) Interpret the slope of the model.
    c) If the car is 5 years old, what does the model predict for it's value?
    d) After how many years will the car be worth nothing?

    SOLUTION:

    a) $v = -4000y + 36,000$

    b) The value of the car decreases $4000 for every year after it was purchased.
    c) $v = -4000y + 36,000$

    $v = -4000(5) + 36,000$

    $v = -20,000 + 36,000$

    $v = 16,000$

    The car is worth $16,000 after it is 5 years old.

d) $$v = -4000y + 36,000$$
$$0 = -4000y + 36,000$$
$$4000y = 36,000$$
$$y = 9$$

The car will be worth nothing after 9 years.

2. *Health Club Membership*. A health club offers membership for a fee of $99 plus a monthly fee of $10 per month.

    a) Find the linear model for the membership fee, f, and the number of months, m, since you have been a member.
    b) Interpret the slope of the model.
    c) If you have been a member for 36 months, what does the model predict for the fee you have paid so far?
    d) After how many months will you have paid the health club $279?

SOLUTION:

a) $f = 10m + 99$

b) The fee will increase by $10 for every additional month of membership.

c) $f = 10m + 99$
$$f = 10(36) + 99$$
$$f = 360 + 99$$
$$f = 459$$

The fee after 36 months will be $459.

d) $f = 10m + 99$

$$279 = 10m + 99$$
$$180 = 10m$$
$$18 = m$$

After 18 months, you will have paid $279 for your membership.

3.   *Stress.* The table below shows the relationship between a stress test score and the diastolic blood pressure for 8 patients. A linear regression model for this data is:

$$y = 0.56x + 41.71$$

where x represents the stress test score and y represents the blood pressure.

Stress Test Score, x	55	62	58	78	92	88	75	80
Blood Pressure, y	70	85	72	85	96	90	82	85

a)   Interpret the slope of the model.
b)   Use the model to predict the blood pressure for a person with a stress test score of 75
c)   Use the model to estimate the stress test score for if the diastolic blood pressure was 90.

SOLUTION:

a)   For every 1 point increase in the stress test score, the diastolic blood pressure will increase by 0.56 points.

b) $y = 0.56x + 41.71$

$y = 0.56(75) + 41.71$

$y = 42 + 41.71$

$y = 83.71$

A person with a stress test score of 75 will have an approximate diastolic blood pressure of 84.

c) $y = 0.56x + 41.71$

$90 = 0.56x + 41.71$

$48.29 = 0.56x$

$86.23 = x$

A person with a diastolic blood pressure of 90 will have a stress test score of approximately 86.