Finite Mathematics for the Managerial Life and Social Sciences 11th Edition

Solution Manual for Finite Mathematics for the Managerial Life and Social Sciences 11th Edition by Tan ISBN 1285464656 9781285464657

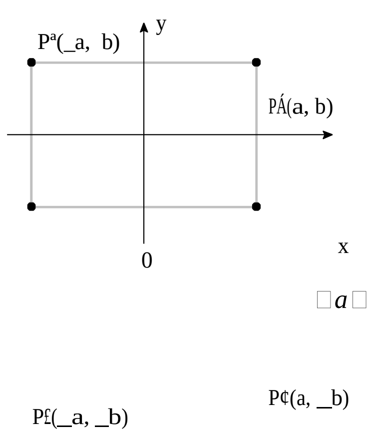
1 STRAIGHT LINES AND LINEAR FUNCTIONS

1.1 The Cartesian Coordinate System

Concept Questions page 6

1. a. a □ 0 and b □ 0 **b.** a □ 0 and b □ 0 **c.** a □ 0 and b □ 0

2. a.



b. d □P_1 □a□ b□ □ □0□ 0□□ □ □0 □ a□^{2} □ □0 □ b□^{2} □ a^2 □ b^2,

d □P_2 □□a□ b□ □ □0□ 0□□ □ [0 □ □□a□]^2 □ □0 □ b□^{2} □ a^2 □ b^2,

□a□ and d □P_4 b□ □ □0□ 0□□ □ 0 □ a□ [0 □ b□] □ a^2 b^2,

so the points P_1 □a□ b□, P_2 □□a□ b□, P_3 □□a□ □b□, and P_4 □a□ □b□ are all

Exercises page 7

1. The coordinates of *A* are □3□ 3□ and it is located in Quadrant I.

2. The coordinates of *B* are □□5□ 2□ and it is located in Quadrant II.

3. The coordinates of *C* are □2□ □2□ and it is located in Quadrant IV.

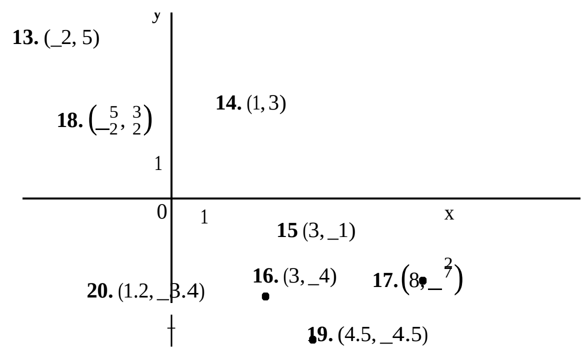
4. The coordinates of *D* are □□2□ 5□ and it is located in Quadrant II.

5. The coordinates of *E* are □□4□ □6□ and it is located in Quadrant III.

6. The coordinates of *F* are □8□ □2□ and it is located in Quadrant IV.

7. *A** **8.** □□5□ 4□ **9.** *E*, *F*, and *G* **10.** *E* **11.** *F* **12.** *D*

For Exercises 13–20, refer to the following figure.



21. Using the distance formula, we find that $\sqrt{(4-1)^2+(7-3)^2}=\sqrt{3^2+4^2}=\sqrt{25}=5$.

22. Using the distance formula, we find that $\sqrt{(4-1)^2+(4-0)^2}=\sqrt{3^2+4^2}=\sqrt{25}=5$.

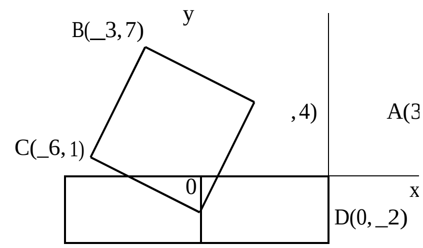
23. Using the distance formula, we find that $\sqrt{[4-(-1)]^2+(9-3)^2}=\sqrt{5^2+6^2}=\sqrt{25+36}=\sqrt{61}$.

24. Using the distance formula, we find that $\sqrt{[10-(-2)]^2+(6-1)^2}=\sqrt{12^2+5^2}=\sqrt{144+25}=\sqrt{169}=13$.

25. The coordinates of the points have the form $(x,-6)$. Because the points are 10 units away from the origin, we have $(x-0)^2+(-6-0)^2=10^2$, $x^2=64$, or $x=\pm 8$. Therefore, the required points are $(-8,-6)$ and $(8,-6)$.

26. The coordinates of the points have the form $(3,y)$. Because the points are 5 units away from the origin, we have $(3-0)^2+(y-0)^2=5^2$, $y^2=16$, or $y=\pm 4$. Therefore, the required points are $(3,4)$ and $(3,-4)$.

27. The points are shown in the diagram. To show that the four sides are equal, we compute



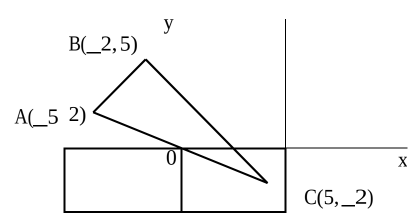
$d(A,B)=\sqrt{(-3-3)^2+(7-4)^2}=\sqrt{(-6)^2+3^2}=\sqrt{45}$,

$d(B,C)=\sqrt{[-6-(-3)]^2+(1-7)^2}=\sqrt{(-3)^2+(-6)^2}=\sqrt{45}$,

$d(C,D)=\sqrt{[0-(-6)]^2+[(-2)-1]^2}=\sqrt{6^2+(-3)^2}=\sqrt{45}$, and $d(A,D)=\sqrt{(0-3)^2+(-2-4)^2}=\sqrt{(-3)^2+(-6)^2}=\sqrt{45}$.

Next, to show that $\triangle ABC$ is a right triangle, we show that it satisfies the Pythagorean Theorem. Thus, $d(A,C)=\sqrt{(-6-3)^2+(1-4)^2}=\sqrt{(-9)^2+(-3)^2}=\sqrt{90}=3\sqrt{10}$ and $[d(A,B)]^2+[d(B,C)]^2=90=[d(A,C)]^2$. Similarly, $d(B,D)=\sqrt{90}=3\sqrt{10}$, so $\triangle BAD$ is a right triangle as well. It follows that $\angle B$ and $\angle D$ are right angles, and we conclude that $ADCB$ is a square.

28. The triangle is shown in the figure. To prove that $\triangle ABC$ is a right triangle, we show that $[d(A,C)]^2=[d(A,B)]^2+[d(B,C)]^2$ and the result will then follow from the Pythagorean Theorem. Now



$[d(A,C)]^2=(-5-5)^2+[2-(-2)]^2=100+16=116$.

Next, we find

$[d(A,B)]^2+[d(B,C)]^2=[-2-(-5)]^2+(5-2)^2+[5-(-2)]^2+(-2-5)^2=9+9+49+49=116$, and the result follows.

29. The equation of the circle with radius 5 and center $(2,-3)$ is given by $(x-2)^2+[y-(-3)]^2=5^2$, or

$(x-2)^2+(y+3)^2=25$.

30. The equation of the circle with radius 3 and center $(-2, -4)$ is given by $[x - (-2)]^2 + [y - (-4)]^2 = 9$, or $(x + 2)^2 + (y + 4)^2 = 9$.

31. The equation of the circle with radius 5 and center $(0, 0)$ is given by $(x - 0)^2 + (y - 0)^2 = 5^2$, or $x^2 + y^2 = 25$.

32. The distance between the center of the circle and the point $(2, 3)$ on the circumference of the circle is given by $d = \sqrt{(3 - 0)^2 + (2 - 0)^2} = \sqrt{13}$. Therefore $r = \sqrt{13}$ and the equation of the circle centered at the origin that passes through $(2, 3)$ is $x^2 + y^2 = 13$.

33. The distance between the points $(5, 2)$ and $(2, -3)$ is given by $d = \sqrt{(5-2)^2 + [2-(-3)]^2} = \sqrt{3^2 + 5^2} = \sqrt{34}$.

Therefore $r = \sqrt{34}$ and the equation of the circle passing through $(5, 2)$ and $(2, -3)$ is $(x-2)^2 + [y-(-3)]^2 = 34$, or $(x-2)^2 + (y+3)^2 = 34$.

34. The equation of the circle with center $(-a, a)$ and radius $2a$ is given by $[x-(-a)]^2 + (y-a)^2 = (2a)^2$, or

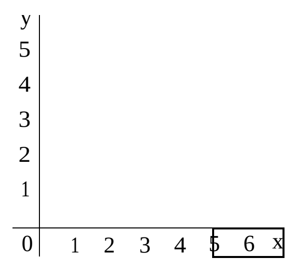
$(x+a)^2 + (y-a)^2 = 4a^2$.

35. **a.** The coordinates of the suspect's car at its final destination are $x = 4$ and $y = 4$.

b. The distance traveled by the suspect was $5 + 4 + 1$, or 10 miles.

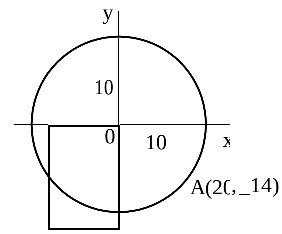
c. The distance between the original and final positions of the suspect's car was $d = \sqrt{(4-0)^2 + (4-0)^2} = \sqrt{32} = 4\sqrt{2}$,

or approximately 5.66 miles.

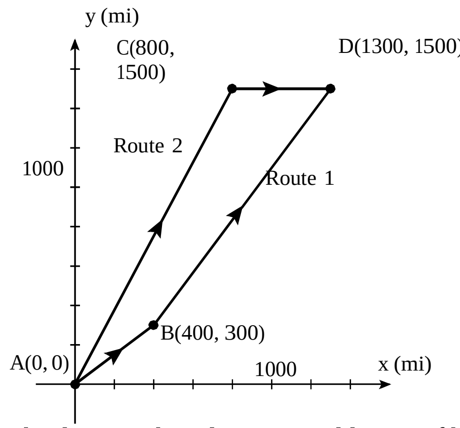


36. Referring to the diagram on page 8 of the text, we see that the distance from A to B is given by $d(A, B) = \sqrt{400^2 + 300^2} = \sqrt{250,000} = 500$. The distance from B to C is given by $d(B, C) = \sqrt{(-800-400)^2 + (800-300)^2} = \sqrt{(-1200)^2 + (500)^2} = \sqrt{1,690,000} = 1300$. The distance from C to D is given by $d(C, D) = \sqrt{[-800-(-800)]^2 + (800-0)^2} = \sqrt{0 + 800^2} = 800$. The distance from D to A is given by $d(D, A) = \sqrt{[(-800)-0]^2 + (0-0)^2} = \sqrt{640,000} = 800$. Therefore, the total distance covered on the tour is $d(A, B) + d(B, C) + d(C, D) + d(D, A) = 500 + 1300 + 800 + 800 = 3400$, or 3400 miles.

37. Suppose that the furniture store is located at the origin O so that your house is located at $A(20, -14)$. Because $d(O, A) = \sqrt{20^2 + (-14)^2} = \sqrt{596} \approx 24.4$, your house is
located within a 25-mile radius of the store and you will not incur a delivery charge.



38.



Referring to the diagram, we see that the distance the salesman would cover if he took Route 1 is given by

$$d(A, B) + d(B, D) = \sqrt{400^2 + 300^2} + \sqrt{(1300 - 400)^2 + (1500 - 300)^2}$$
$$= \sqrt{250{,}000} + \sqrt{2{,}250{,}000} = 500 + 1500 = 2000$$

or 2000 miles. On the other hand, the distance he would cover if he took Route 2 is given by

$$d(A, C) + d(C, D) = \sqrt{800^2 + 1500^2} + \sqrt{(1300 - 800)^2} = \sqrt{2{,}890{,}000} + \sqrt{250{,}000}$$
$$= 1700 + 500 = 2200$$

or 2200 miles. Comparing these results, we see that he should take Route 1.

39. The cost of shipping by freight train is $(0.66)(2000)(100) = 132{,}000$, or \$132,000.
The cost of shipping by truck is $(0.62)(2200)(100) = 136{,}400$, or \$136,400.
Comparing these results, we see that the automobiles should be shipped by freight train. The net savings are $136{,}400 - 132{,}000 = 4400$, or \$4400.

40. The length of cable required on land is $d(S, Q) = 10{,}000 - x$ and the length of cable required under water is $d(Q, M) = \sqrt{(x^2 - 0) + (0 - 3000)^2} = \sqrt{x^2 + 3000^2}$. The cost of laying cable is thus

$$3(10{,}000 - x) + 5\sqrt{x^2 + 3000^2}.$$

If $x = 2500$, then the total cost is given by $3(10{,}000 - 2500) + 5\sqrt{2500^2 + 3000^2} \approx 42{,}025.62$, or \$42,025.62.
If $x = 3000$, then the total cost is given by $3(10{,}000 - 3000) + 5\sqrt{3000^2 + 3000^2} \approx 42{,}213.20$, or \$42,213.20.

41. To determine the VHF requirements, we calculate $d = \sqrt{25^2 + 35^2} = \sqrt{625 + 1225} = \sqrt{1850} \approx 43.01$.
Models B, C, and D satisfy this requirement.
To determine the UHF requirements, we calculate $d = \sqrt{20^2 + 32^2} = \sqrt{400 + 1024} = \sqrt{1424} \approx 37.74$. Models C and D satisfy this requirement.
Therefore, Model C allows him to receive both channels at the least cost.

42. a. Let the positions of ships A and B after t hours be $A(0, y)$ and $B(x, 0)$, respectively. Then $x = 30t$ and $y = 20t$.

Therefore, the distance in miles between the two ships is $D = \sqrt{(30t)^2 + (20t)^2} = \sqrt{900t^2 + 400t^2} = 10\sqrt{13}t$.

b. The required distance is obtained by letting $t = 2$, giving $D = 10\sqrt{13}\,(2)$, or approximately 72.11 miles.

43. a. Let the positions of ships A and B be $(0, y)$ and $(x, 0)$, respectively. Then $y = 25\left(t + \frac{1}{2}\right)$ and $x = 20t$. The distance D in miles between the two ships is

$$D = \sqrt{(x-0)^2 + (0-y)^2} = \sqrt{x^2 + y^2} = \sqrt{400t^2 + 625\left(t+\frac{1}{2}\right)^2} \quad (1).$$

b. The distance between the ships 2 hours after ship A has left port is obtained by letting $t = \frac{3}{2}$ in Equation (1), yielding $D = \sqrt{400\left(\frac{3}{2}\right)^2 + 625\left(\frac{3}{2}+\frac{1}{2}\right)^2} = \sqrt{3400}$, or approximately 58.31 miles.

44. a. The distance in feet is given by $\sqrt{(4000)^2 + x^2} = \sqrt{16{,}000{,}000 + x^2}$.

b. Substituting the value $x = 20{,}000$ into the above expression gives $\sqrt{16{,}000{,}000 + (20{,}000)^2} \approx 20{,}396$, or 20,396 ft.

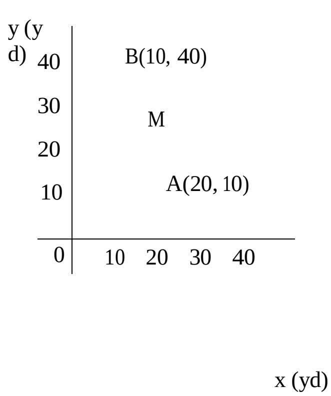
45. a. Suppose that $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are endpoints of the line segment and that the point $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ is the midpoint of the line segment PQ. The distance between P and Q is $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$. The distance between P and M is

$$\sqrt{\left(\frac{x_1+x_2}{2} - x_1\right)^2 + \left(\frac{y_1+y_2}{2} - y_1\right)^2} = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2} = \frac{1}{2}\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2},$$

which is one-half the distance from P to Q. Similarly, we obtain the same expression for the distance from M to P.

b. The midpoint is given by $\left(\frac{4-3}{2}, \frac{-5+2}{2}\right)$, or $\left(\frac{1}{2}, -\frac{3}{2}\right)$.

46. a.



b. The coordinates of the position of the prize are $x = \frac{20+10}{2}$ and $y = \frac{10+40}{2}$, or $x = 15$ yards and $y = 25$ yards.

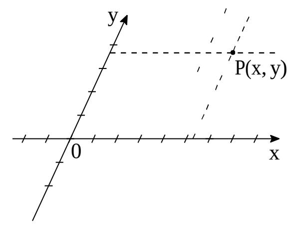
c. The distance from the prize to the house is $d(M, (0, 0)) = \sqrt{(15-0)^2 + (25-0)^2} = \sqrt{850} \approx 29.15$ (yards).

47. False. The distance between $P_1(a, b)$ and $P_3(kc, kd)$ is
$d = \sqrt{(kc - a)^2 + (kd - b)^2}$
$\neq |k| D = |k| \sqrt{(c - a)^2 + (d - b)^2} = \sqrt{k^2 (c - a)^2 + k^2 (d - b)^2} = \sqrt{[k(c - a)]^2 + [k(d - b)]^2}$.

48. True. $kx^2 + ky^2 = a^2$ gives $x^2 + y^2 = \frac{a^2}{k} < a^2$ if $k > 1$. So the radius of the circle with equation $kx^2 + ky^2 = a^2$ is a circle of radius smaller than a centered at the origin if $k > 1$. Therefore, it lies inside the circle of radius a with equation $x^2 + y^2 = a^2$.

49. Referring to the figure in the text, we see that the distance between the two points is given by the length of the hypotenuse of the right triangle. That is, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

50. a. Let $P(x, y)$ be any point in the plane. Draw a line through P parallel to the y-axis and a line through P parallel to the x-axis (see the figure). The x-coordinate of P is the number corresponding to the point on the x-axis at which the line through P crosses the x-axis. Similarly, y is the number that corresponds to the point on the y-axis at which the line parallel to the x-axis crosses the y-axis. To show the converse, reverse the process.



b. You can use the Pythagorean Theorem in the Cartesian coordinate system. This greatly simplifies the computations.

1.2 Straight Lines

Concept Questions page 19

1. The slope is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $P(x_1, y_1)$ and $P(x_2, y_2)$ are any two distinct points on the nonvertical line.

The slope of a vertical line is undefined.

2. a. $y - y_1 = m(x - x_1)$ **b.** $y = mx + b$ **c.** $ax + by + c = 0$, where a and b are not both zero.

3. a. $m_1 = m_2$ **b.** $m_2 = -\dfrac{1}{m_1}$

4. a. Solving the equation for y gives $By = -Ax - C$, so $y = -\dfrac{A}{B}x - \dfrac{C}{B}$. The slope of L is the coefficient of x, $-\dfrac{A}{B}$.

b. If $B = 0$, then the equation reduces to $Ax + C = 0$. Solving this equation for x, we obtain $x = -\dfrac{C}{A}$. This is an equation of a vertical line, and we conclude that the slope of L is undefined.

Exercises page 19

1. Referring to the figure shown in the text, we see that $m = \dfrac{2 - 0}{0 - (-4)} = \dfrac{1}{2}$.

2. Referring to the figure shown in the text, we see that $m = \dfrac{4 - 0}{0 - 2} = -2$.

3. This is a vertical line, and hence its slope is undefined.

4. This is a horizontal line, and hence its slope is 0.

5. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 3}{5 - 4} = 5$.

6. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 5}{3 - 4} = \dfrac{3}{-1} = -3$.

7. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 3}{4 - (-2)} = \dfrac{5}{6}$.

8. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4 - (-2)}{4 - (-2)} = \dfrac{-2}{6} = -\dfrac{1}{3}$.

9. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{d - b}{c - a}$, provided $a \neq c$.

10. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-b - (b + 1)}{-(a - 1) - (a + 1)} = \dfrac{-b - b - 1}{-a + 1 - a - 1} = \dfrac{1 + 2b}{2a}$.

11. Because the equation is already in slope-intercept form, we read off the slope $m = 4$.

a. If x increases by 1 unit, then y increases by 4 units.

b. If x decreases by 2 units, then y decreases by $4(-2) = -8$ units.

12. Rewrite the given equation in slope-intercept form: $2x + 3y = 4$, $3y = 4 - 2x$, and so $y = \frac{4}{3} - \frac{2}{3}x$.

a. Because $m = -\frac{2}{3}$, we conclude that the slope is negative.

b. Because the slope is negative, y decreases as x increases.

c. If x decreases by 2 units, then y increases by $\left(-\frac{2}{3}\right)(-2) = \frac{4}{3}$ units.

13. (e) **14.** (c) **15.** (a) **16.** (d) **17.** (f) **18.** (b)

19. The slope of the line through A and B is $\dfrac{-10 - (-2)}{-3 - 1} = \dfrac{-8}{-4} = 2$. The slope of the line through C and D is $\dfrac{1 - 5}{-1 - 1} = \dfrac{-4}{-2} = 2$. Because the slopes of these two lines are equal, the lines are parallel.

20. The slope of the line through A and B is $\dfrac{-2 - 3}{2 - 2}$. Because this slope is undefined, we see that the line is vertical. The slope of the line through C and D is $\dfrac{5 - 4}{-2 - (-2)}$. Because this slope is undefined, we see that this line is also vertical. Therefore, the lines are parallel.

21. The slope of the line through the point $(1, a)$ and $(4, -2)$ is $m_1 = \dfrac{-2 - a}{4 - 1}$ and the slope of the line through $(2, 8)$ and $(-7, a + 4)$ is $m_2 = \dfrac{a + 4 - 8}{-7 - 2}$. Because these two lines are parallel, m_1 is equal to m_2. Therefore, $\dfrac{-2 - a}{3} = \dfrac{a - 4}{-9}$, $-9(-2 - a) = 3(a - 4)$, $18 + 9a = 3a - 12$, and $6a = -30$, so $a = -5$.

22. The slope of the line through the point $(a, 1)$ and $(5, 8)$ is $m_1 = \dfrac{8 - 1}{5 - a}$ and the slope of the line through $(4, 9)$ and $(a + 2, 1)$ is $m_2 = \dfrac{1 - 9}{a + 2 - 4}$. Because these two lines are parallel, m_1 is equal to m_2. Therefore, $\dfrac{7}{5 - a} = \dfrac{-8}{a - 2}$, $7(a - 2) = -8(5 - a)$, $7a - 14 = -40 + 8a$, and $a = 26$.

23. We use the point-slope form of an equation of a line with the point $(3, -4)$ and slope $m = 2$. Thus

$y - y_1 = m(x - x_1)$ becomes $y - (-4) = 2(x - 3)$. Simplifying, we have $y + 4 = 2x - 6$, or $y = 2x - 10$.

24. We use the point-slope form of an equation of a line with the point $(2, 4)$ and slope $m = -1$. Thus $y - y_1 = m(x - x_1)$, giving $y - 4 = -1(x - 2)$, $y - 4 = -x + 2$, and finally $y = -x + 6$.

25. Because the slope $m = 0$, we know that the line is a horizontal line of the form $y = b$. Because the line passes through $(-3, 2)$, we see that $b = 2$, and an equation of the line is $y = 2$.

26. We use the point-slope form of an equation of a line with the point $(1, 2)$ and slope $m = -\frac{1}{2}$. Thus $y - y_1 = m(x - x_1)$ gives $y - 2 = -\frac{1}{2}(x - 1)$, $2y - 4 = -x + 1$, $2y = -x + 5$, and $y = -\frac{1}{2}x + \frac{5}{2}$.

27. We first compute the slope of the line joining the points $(2, 4)$ and $(3, 7)$ to be $m = \frac{7-4}{3-2} = 3$. Using the point-slope form of an equation of a line with the point $(2, 4)$ and slope $m = 3$, we find $y - 4 = 3(x - 2)$, or $y = 3x - 2$.

28. We first compute the slope of the line joining the points $(2, 1)$ and $(2, 5)$ to be $m = \frac{5-1}{2-2}$. Because this slope is undefined, we see that the line must be a vertical line of the form $x = a$. Because it passes through $(2, 5)$, we see that $x = 2$ is the equation of the line.

29. We first compute the slope of the line joining the points $(1, 2)$ and $(-3, -2)$ to be $m = \frac{-2-2}{-3-1} = \frac{-4}{-4} = 1$. Using the point-slope form of an equation of a line with the point $(1, 2)$ and slope $m = 1$, we find $y - 2 = x - 1$, or $y = x + 1$.

30. We first compute the slope of the line joining the points $(-1, -2)$ and $(3, -4)$ to be $m = \frac{-4-(-2)}{3-(-1)} = \frac{-2}{4} = -\frac{1}{2}$. Using the point-slope form of an equation of a line with the point $(-1, -2)$ and slope $m = -\frac{1}{2}$, we find $y - (-2) = -\frac{1}{2}[x - (-1)]$, $y + 2 = -\frac{1}{2}(x + 1)$, and finally $y = -\frac{1}{2}x - \frac{5}{2}$.

31. The slope of the line through A and B is $\frac{2-5}{4-(-2)} = -\frac{3}{6} = -\frac{1}{2}$. The slope of the line through C and D is $\frac{6-(-2)}{3-(-1)} = \frac{8}{4} = 2$. Because the slopes of these two lines are the negative reciprocals of each other, the lines are perpendicular.

32. The slope of the line through A and B is $\frac{-2-0}{1-2} = \frac{-2}{-1} = 2$. The slope of the line through C and D is $\frac{4-2}{-8-4} = \frac{2}{-12} = -\frac{1}{6}$. Because the slopes of these two lines are not the negative reciprocals of each other, the lines are not perpendicular.

33. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = 3$ and $b = 4$, the equation is $y = 3x + 4$.

34. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = -2$ and $b = -1$, the equation is $y = -2x - 1$.

35. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = 0$ and $b = 5$, the equation is $y = 5$.

36. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = -\frac{1}{2}$, and $b = \frac{3}{4}$, the equation is

$y \square \square_{\overline{2}} x \square \overline{4}$.

37. We first write the given equation in the slope-intercept form: $x \square 2y \square 0$, so $\square 2y \square \square x$, or $y \square \frac{1}{2}x$. From this equation, we see that $m \square \frac{1}{2}$ and $b \square 0$.

38. We write the equation in slope-intercept form: $y \square 2 \square 0$, so $y \square 2$. From this equation, we see that $m \square 0$ and $b \square 2$.

39. We write the equation in slope-intercept form: $2x - 3y - 9 = 0$, $-3y = -2x + 9$, and $y = \frac{2}{3}x - 3$. From this equation, we see that $m = \frac{2}{3}$ and $b = -3$.

40. We write the equation in slope-intercept form: $3x - 4y + 8 = 0$, $-4y = -3x - 8$, and $y = \frac{3}{4}x + 2$. From this equation, we see that $m = \frac{3}{4}$ and $b = 2$.

41. We write the equation in slope-intercept form: $2x + 4y = 14$, $4y = -2x + 14$, and $y = -\frac{2}{4}x + \frac{14}{4} = -\frac{1}{2}x + \frac{7}{2}$.

From this equation, we see that $m = -\frac{1}{2}$ and $b = \frac{7}{2}$.

42. We write the equation in the slope-intercept form: $5x + 8y - 24 = 0$, $8y = -5x + 24$, and $y = -\frac{5}{8}x + 3$. From this equation, we conclude that $m = -\frac{5}{8}$ and $b = 3$.

43. An equation of a horizontal line is of the form $y = b$. In this case $b = -3$, so $y = -3$ is an equation of the line.

44. An equation of a vertical line is of the form $x = a$. In this case $a = 0$, so $x = 0$ is an equation of the line.

45. We first write the equation $2x - 4y - 8 = 0$ in slope-intercept form: $2x - 4y - 8 = 0$, $4y = 2x - 8$, $y = \frac{1}{2}x - 2$. Now the required line is parallel to this line, and hence has the same slope. Using the point-slope form of an equation of a line with $m = \frac{1}{2}$ and the point $(-2, 2)$, we have $y - 2 = \frac{1}{2}[x - (-2)]$ or $y = \frac{1}{2}x + 3$.

46. The slope of the line passing through $(-2, -3)$ and $(2, 5)$ is $m = \frac{5 - (-3)}{2 - (-2)} = \frac{8}{4} = 2$. Thus, the required equation is $y - 3 = 2[x - (-1)]$, $y = 2x + 2 + 3$, or $y = 2x + 5$.

47. We first write the equation $3x + 4y - 22 = 0$ in slope-intercept form: $3x + 4y - 22 = 0$, so $4y = -3x + 22$ and $y = -\frac{3}{4}x + \frac{11}{2}$. Now the required line is perpendicular to this line, and hence has slope $\frac{4}{3}$ (the negative reciprocal of $-\frac{3}{4}$). Using the point-slope form of an equation of a line with $m = \frac{4}{3}$ and the point $(2, 4)$, we have $y - 4 = \frac{4}{3}(x - 2)$, or $y = \frac{4}{3}x + \frac{4}{3}$.

48. The slope of the line passing through $(-2, -1)$ and $(4, 3)$ is given by $m = \frac{3 - (-1)}{4 - (-2)} = \frac{3 + 1}{4 + 2} = \frac{4}{6} = \frac{2}{3}$, so the slope of the required line is $m = -\frac{3}{2}$ and its equation is $y - (-2) = -\frac{3}{2}(x - 1)$, $y = -\frac{3}{2}x + \frac{3}{2} - 2$, or $y = -\frac{3}{2}x - \frac{1}{2}$.

49. The midpoint of the line segment joining $P_1(-2, -4)$ and $P_2(3, 6)$ is $M\left(\frac{-2 + 3}{2}, \frac{-4 + 6}{2}\right)$ or $M\left(\frac{1}{2}, 1\right)$.

Using the point-slope form of the equation of a line with $m = -2$, we have $y - 1 = -2\left(x - \frac{1}{2}\right)$ or $y = -2x + 2$.

50. The midpoint of the line segment joining $P_1(-1, -3)$ and $P_2(3, 3)$ is $M_1\left(\frac{-1 + 3}{2}, \frac{-3 + 3}{2}\right)$ or $M_1(1, 0)$.

The midpoint of the line segment joining $P_3(-2, 3)$ and $P_4(2, -3)$ is $M_2\left(\frac{\;}{2}, \frac{\;}{2}\right)$ or $M_2(0, 0)$.

The slope of the required line is $m = \frac{0 - 0}{1 - 0} = 0$, so an equation of the line is $y - 0 = 0(x - 0)$ or $y = 0$.

51. A line parallel to the x-axis has slope 0 and is of the form $y = b$. Because the line is 6 units below the axis, it passes through $(0, -6)$ and its equation is $y = -6$.

52. Because the required line is parallel to the line joining $(2, 4)$ and $(4, 7)$, it has slope $m = \frac{7-4}{4-2} = \frac{3}{2}$. We also know that the required line passes through the origin $(0, 0)$. Using the point-slope form of an equation of a line, we find $y - 0 = \frac{3}{2}(x - 0)$, or $y = \frac{3}{2}x$.

53. We use the point-slope form of an equation of a line to obtain $y - b = 0(x - a)$, or $y = b$.

54. Because the line is parallel to the x-axis, its slope is 0 and its equation has the form $y = b$. We know that the line passes through $(-3, 4)$, so the required equation is $y = 4$.

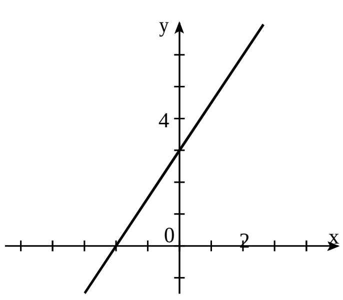
55. Because the required line is parallel to the line joining $(-3, 2)$ and $(6, 8)$, it has slope $m = \frac{8-2}{6-(-3)} = \frac{6}{9} = \frac{2}{3}$. We also know that the required line passes through $(-5, -4)$. Using the point-slope form of an equation of a line, we find $y - (-4) = \frac{2}{3}[x - (-5)]$, $y = \frac{2}{3}x + \frac{10}{3} - 4$, and finally $y = \frac{2}{3}x - \frac{2}{3}$.

56. Because the slope of the line is undefined, it has the form $x = a$. Furthermore, since the line passes through (a, b), the required equation is $x = a$.

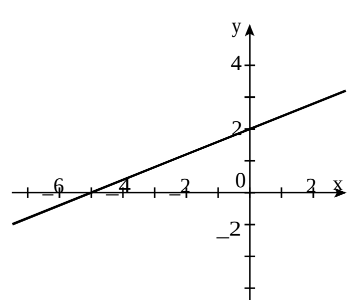
57. Because the point $(-3, 5)$ lies on the line $kx + 3y + 9 = 0$, it satisfies the equation. Substituting $x = -3$ and $y = 5$ into the equation gives $-3k + 15 + 9 = 0$, or $k = 8$.

58. Because the point $(2, -3)$ lies on the line $-2x + ky + 10 = 0$, it satisfies the equation. Substituting $x = 2$ and $y = -3$ into the equation gives $-2(2) + (-3)k + 10 = 0$, $-4 - 3k + 10 = 0$, $-3k = -6$, and finally $k = 2$.

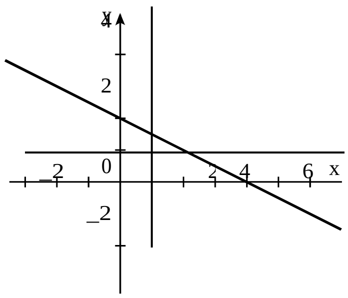
59. $3x - 2y + 6 = 0$. Setting $y = 0$, we have $3x + 6 = 0$ or $x = -2$, so the x-intercept is -2. Setting $x = 0$, we have $-2y + 6 = 0$ or $y = 3$, so the y-intercept is 3.



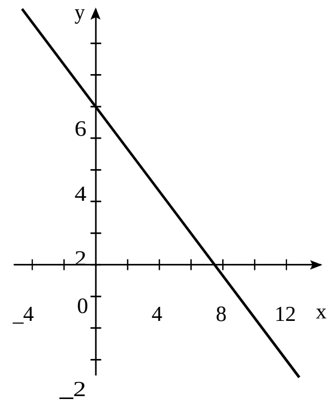
60. $2x - 5y + 10 = 0$. Setting $y = 0$, we have $2x + 10 = 0$ or $x = -5$, so the x-intercept is -5. Setting $x = 0$, we have $-5y + 10 = 0$ or $y = 2$, so the y-intercept is 2.



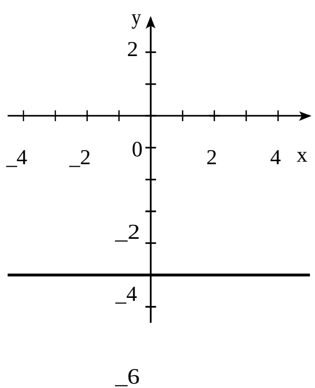
61. $x + 2y - 4 = 0$. Setting $y = 0$, we have $x - 4 = 0$ or $x = 4$, so the x-intercept is 4. Setting $x = 0$, we have $2y - 4 = 0$ or $y = 2$, so the y-intercept is 2.



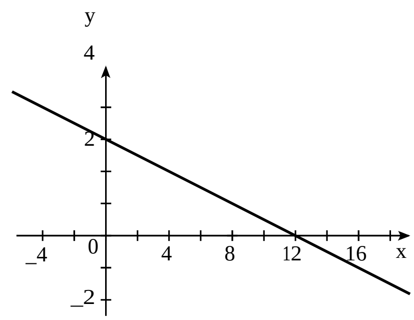
62. $2x + 3y - 15 = 0$. Setting $y = 0$, we have $2x - 15 = 0$, so the x-intercept is $\frac{15}{2}$. Setting $x = 0$, we have $3y - 15 = 0$, so the y-intercept is 5.



63. $y + 5 = 0$. Setting $y = 0$, we have $0 + 5 = 0$, which has no solution, so there is no x-intercept. Setting $x = 0$, we have $y + 5 = 0$ or $y = -5$, so the y-intercept is -5.



64. $-2x - 8y + 24 = 0$. Setting $y = 0$, we have $-2x + 24 = 0$ or $x = 12$, so the x-intercept is 12. Setting $x = 0$, we have $-8y + 24 = 0$ or $y = 3$, so the y-intercept is 3.



65. Because the line passes through the points $(a, 0)$ and $(0, b)$, its slope is $m = \frac{b - 0}{0 - a} = -\frac{b}{a}$. Then, using the point-slope form of an equation of a line with the point $(a, 0)$, we have $y - 0 = -\frac{b}{a}(x - a)$ or $y = -\frac{b}{a}x + b$, which may be written in the form $\frac{b}{a}x + y = b$. Multiplying this last equation by $\frac{1}{b}$, we have $\frac{x}{a} + \frac{y}{b} = 1$.

66. Using the equation $\frac{x}{a} + \frac{y}{b} = 1$ with $a = 3$ and $b = 4$, we have $\frac{x}{3} + \frac{y}{4} = 1$. Then $4x + 3y = 12$, so $3y = 12 - 4x$ and thus $y = -\frac{4}{3}x + 4$.

67. Using the equation $\frac{x}{a} + \frac{y}{b} = 1$ with $a = -2$ and $b = -4$, we have $-\frac{x}{2} - \frac{y}{4} = 1$. Then $-4x - 2y = 8$,

$2y = 8 - 4x$, and finally $y = -2x + 4$.

68. Using the equation $\frac{x}{a} + \frac{y}{b} = 1$ with $a = -\frac{1}{2}$ and $b = \frac{3}{4}$, we have $\frac{x}{-1/2} + \frac{y}{3/4} = 1$, $\frac{3}{4}x - \frac{1}{2}y = -\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)$, $-\frac{1}{2}y = -\frac{3}{4}x - \frac{3}{8}$, and finally $y = 2\left(\frac{3}{4}x + \frac{3}{8}\right) = \frac{3}{2}x + \frac{3}{4}$.

69. Using the equation $\frac{x}{a} + \frac{y}{b} = 1$ with $a = 4$ and $b = -\frac{1}{2}$, we have $\frac{x}{4} + \frac{y}{-1/2} = 1$, $-\frac{1}{4}x + 2y = -1$, $2y = \frac{1}{4}x - 1$, and so $y = \frac{1}{8}x - \frac{1}{2}$.

70. The slope of the line passing through A and B is $m = \frac{-2-7}{2-(-1)} = -\frac{9}{3} = -3$, and the slope of the line passing through B and C is $m = \frac{-9-(-2)}{5-2} = -\frac{7}{3}$. Because the slopes are not equal, the points do not lie on the same line.

71. The slope of the line passing through A and B is $m = \frac{7-1}{1-(-2)} = \frac{6}{3} = 2$, and the slope of the line passing through B and C is $m = \frac{13-7}{4-1} = \frac{6}{3} = 2$. Because the slopes are equal, the points lie on the same line.

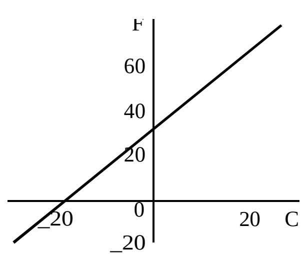
72. The slope of the line L passing through $P_1(1.2, -9.04)$ and $P_2(2.3, -5.96)$ is $m = \frac{-5.96-(-9.04)}{2.3-1.2} = 2.8$, so an equation of L is $y - (-9.04) = 2.8(x - 1.2)$ or $y = 2.8x - 12.4$.

Substituting $x = 4.8$ into this equation gives $y = 2.8(4.8) - 12.4 = 1.04$. This shows that the point $P_3(4.8, 1.04)$ lies on L. Next, substituting $x = 7.2$ into the equation gives $y = 2.8(7.2) - 12.4 = 7.76$, which shows that the point $P_4(7.2, 7.76)$ also lies on L. We conclude that John's claim is valid.

73. The slope of the line L passing through $P_1(1.8, -6.44)$ and $P_2(2.4, -5.72)$ is $m = \frac{-5.72-(-6.44)}{2.4-1.8} = 1.2$, so an equation of L is $y - (-6.44) = 1.2(x - 1.8)$ or $y = 1.2x - 8.6$.

Substituting $x = 5.0$ into this equation gives $y = 1.2(5) - 8.6 = -2.6$. This shows that the point $P_3(5.0, -2.72)$ does not lie on L, and we conclude that Alison's claim is not valid.

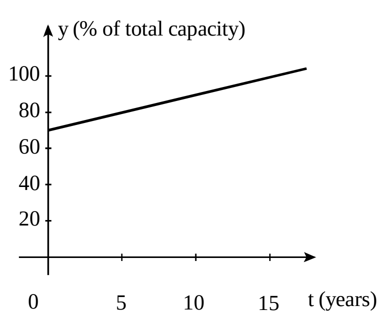
74. a.



b. The slope is $\frac{9}{5}$. It represents the change in °F per unit change in °C.

c. The F-intercept of the line is 32. It corresponds to 0°, so it is the freezing point in °F.

75. a.



b. The slope is 1.9467 and the y-intercept is 70.082.

c. The output is increasing at the rate of 1.9467% per year. The output at the beginning of 1990 was 70.082%.

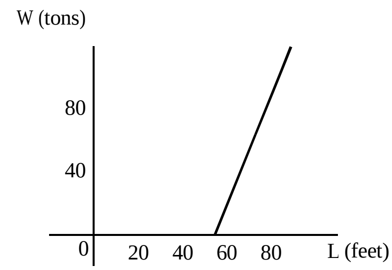
d. We solve the equation $1.9467t + 70.082 = 100$, obtaining $t = 15.37$. We conclude that the plants were generating at

76. **a.** $y = 0.0765x$ **b.** $0.0765 **c.** $0.0765(65{,}000) = 4972.50$, or $4972.50.

77. **a.** $y = 0.55x$ **b.** Solving the equation $1100 = 0.55x$ for x, we have $x = \frac{1100}{0.55} = 2000$.

78. a. Substituting L □ 80 into the given equation, we have
W □ 3□51 □80□ □ 192 □ 280□8 □ 192 □ 88□8, or 88□8 British tons.

b.



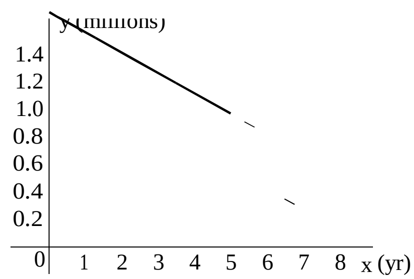
79. Using the points □0□ 0□68□ and □10□ 0□80□, we see that the slope of the required line is

m □ $\dfrac{0□80 □ 0□68}{10 □ 0}$ □ $\dfrac{0□12}{10}$ □ 0□012. Next, using the point-slope form of the equation of a line, we have

y □ 0□68 □ 0□012 □t □ 0□ or y □ 0□012t □ 0□68. Therefore, when t □ 14, we have y □ 0□012 □14□ □ 0□68 □ 0□848,

or 84□8%. That is, in 2004 women's wages were 84□8% of men's wages.

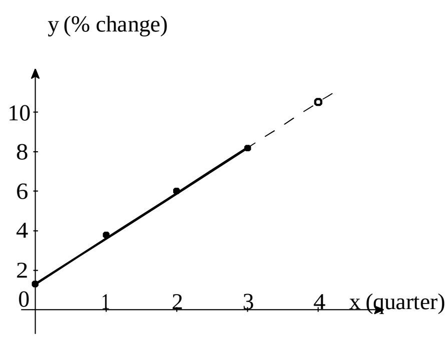
80. a, b.



c. The slope of L is m □ $\dfrac{0□56 □ 1□30}{5 \quad 0}$ □ □0□148, so an equation of L is y □ 1□3 □ □0□148 □x □ 0□ or y □ □0□148x □ 1□3.

d. The number of pay phones in 2012 is estimated to be □0□148 □8□ □ 1□3, or approximately 116,000.

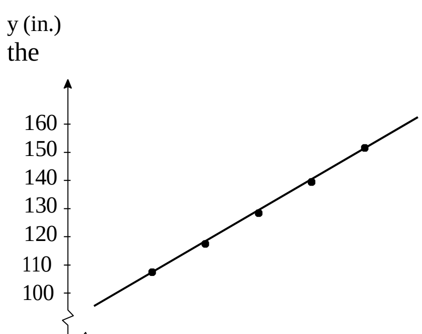
81. a, b.



c. The slope of L is m □ $\dfrac{}{3 \quad 0}$ □ 2□3, so an equation of L is

y □ 1□3 □ 2□3 □x □ 0□ or y □ 2□3x □ 1□3.

82. a, b.



c. Using the points □60□ 108□ and □72□ 152□, we see that the slope of the required line is m □ $\dfrac{152 □ 108}{72 □ 60}$ □ $\dfrac{44}{12}$ □ $\dfrac{11}{3}$.

Therefore, an equation is y □ 108 □ $\frac{11}{3}$ □x □ 60□, y □ $\frac{11}{3}x$ □ $\frac{11}{3}$ □60□ □ 108 □

y □ $\frac{11}{}$

$\frac{11}{3}x$ □ 220 □ 108, or

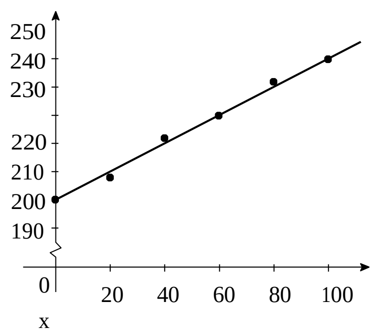
$_3$ x □ 112.

0 58 60 62 64 66 68 70 72 x (lb)
74

d. Using the equation from part c, we find

y □ $\frac{11}{3}$ □65□ □ 112 □ $126\frac{1}{3}$, or $126\frac{1}{3}$ pounds.

83. a, b.



c. Using the points □0□ 200□ and □100□ 250□, we see that the slope of the required line is m □ $\frac{250 □ 200}{100}$ □ $\frac{1}{2}$.

Therefore, an equation is y □ 200 □ $\frac{1}{2}x$ or y □ $\frac{1}{2}x$ □ 200.

d. The approximate cost for producing 54 units of the commodity is $\frac{1}{2}$ □54□ □ 200, or $227.

84. a. The slope of the line L passing through A □0□ 545□ and B □3□ 726□

726 □ 545 181

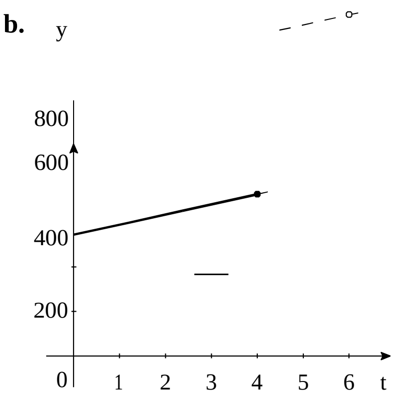
□ 4

y □ 545 □ $\frac{181}{4}$ □x □ 0□ or y □ $\frac{181}{4}$ x □ 545.

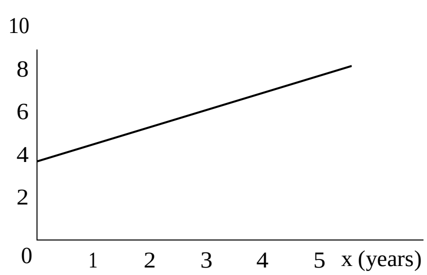
c. The number of corporate fraud cases pending at the beginning of

4

b.



85. a, b.



c. The slope of L is m □ $\frac{}{5 \quad 1}$ □ 4 □ 0□8. Using the

□

point-slope form of an equation of a line, we have

y □ 5□8 □ 0□8 □x □ 1□ □ 0□8x □ 0□8, or y □ 0□8x □ 5.

86. a. The slope of the line passing through P_1 □0□ 27□ and P_2 □1□ 29□ is $\frac{29 □ 27}{1 □ 0}$ □ 2, which is equal to the slope m_1 □

31 □ 29

of the line through P_2 □1□ 29□ and P_3 □2□ 31□, which is 1 0 □ 2. Thus, the three points lie on the line L. m_2 □

□

b. The percentage is of moviegoers who use social media to chat about movies in 2014 is estimated to be 31 □ 2 □2□, or 35%.

c. y □ 27 □ 2 □x □ 0□, so y □ 2x □ 27. The estimate for 2014 (t □ 4) is 2 □4□ □ 27 □ 35, as found in part (b).

87. Yes. A straight line with slope zero (m □ 0) is a horizontal line, whereas a straight line whose slope does not exist is a vertical line (m cannot be computed).

88. **a.** We obtain a family of parallel lines with slope m.

b. We obtain a family of straight lines passing through the point $(0, b)$.

89. True. The slope of the line is given by $-\frac{2}{4} = -\frac{1}{2}$.

90. True. If $(1, k)$ lies on the line, then $x = 1$, $y = k$ must satisfy the equation. Thus $3 - 4k = 12$, or $k = -\frac{9}{4}$. Conversely, if $k = -\frac{9}{4}$, then the point $(1, k) = \left(1, -\frac{9}{4}\right)$ satisfies the equation. Thus, $3(1) - 4\left(-\frac{9}{4}\right) = 12$, and so the point lies on the line.

91. True. The slope of the line $Ax \square By \square C \square 0$ is $\square\frac{A}{B}$. (Write it in slope-intercept form.) Similarly, the slope of the line $ax \square by \square c \square 0$ is $\square\frac{a}{b}$. They are parallel if and only if $\square\frac{A}{B} \square \square\frac{a}{b}$, that is, if $Ab \square aB$, or $Ab \square aB \square 0$.

92. False. Let the slope of L_1 be $m_1 \square 0$. Then the slope of L_2 is $m_2 \square \square\frac{1}{m_1} \square 0$.

93. True. The slope of the line $ax \square by \square c_1 \square 0$ is $m_1 \square \square\frac{a}{b}$. The slope of the line $bx \square ay \square c_2 \square 0$ is $m_2 \square \frac{b}{a}$. Because $m_1 m_2 \square \square 1$, the straight lines are indeed perpendicular.

94. True. Set $y \square 0$ and we have $Ax \square C \square 0$ or $x \square \square C \square A$, and this is where the line intersects the x-axis.

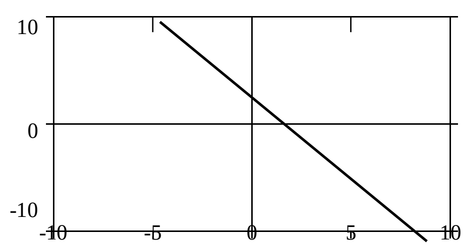
95. Writing each equation in the slope-intercept form, we have $y \square \square\frac{a_1}{b_1}x \square \frac{c_1}{b_1}$ $(b_1 \square 0)$ and $y \square \square\frac{a_2}{b_2}x \square \frac{c_2}{b_2}$ $(b_2 \square 0)$. Because two lines are parallel if and only if their slopes are equal, we see that the lines are parallel if and only if $\square\frac{a_1}{b_1} \square \square\frac{a_2}{b_2}$, or $a_1 b_2 \square b_1 a_2 \square 0$.

96. The slope of L_1 is $m_1 \square \frac{b \square 0}{1 \square 0} \square b$. The slope of L_2 is $m_2 \square \frac{c \square 0}{1 \square 0} \square c$. Applying the Pythagorean theorem to $\square OAC$ and $\square OCB$ gives $\square OA\square^2 \square 1^2 \square b^2$ and $\square OB\square^2 \square 1^2 \square c^2$. Adding these equations and applying the Pythagorean theorem to $\square OBA$ gives $\square AB\square^2 \square \square OA\square^2 \square \square OB\square^2 \square 1^2 \square b^2 \square 1^2 \square c^2 \square 2 \square b^2 \square c^2$. Also, $\square AB\square^2 \square \square b \square c\square^2$, so $\square b \square c\square^2 \square 2 \square b^2 \square c^2$, $b^2 \square 2bc \square c^2 \square 2 \square b^2 \square c^2$, and $\square 2bc \square 2$, $1 \square \square bc$. Finally, $m_1 m_2 \square b \square c \square bc \square \square 1$, as was to be shown.

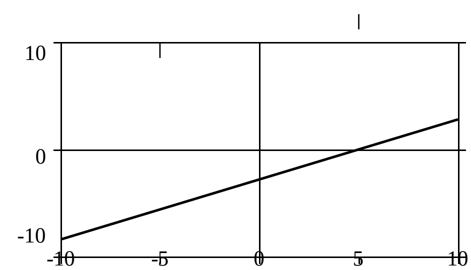
Technology Exercises page 28

Graphing Utility

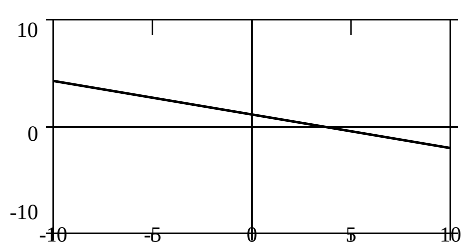
1.



2.



3.



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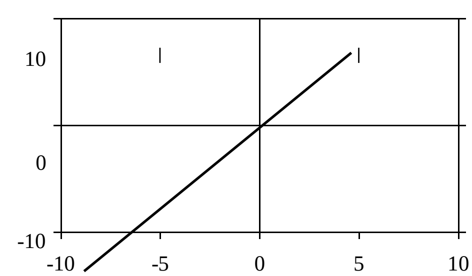

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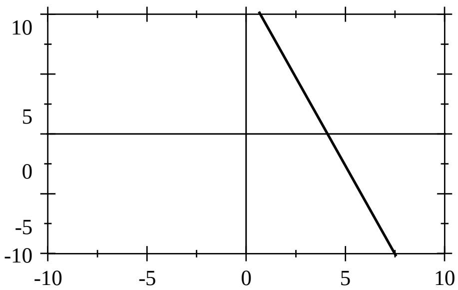

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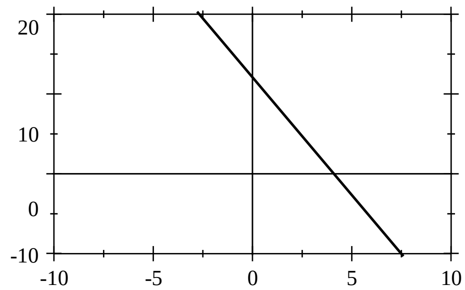

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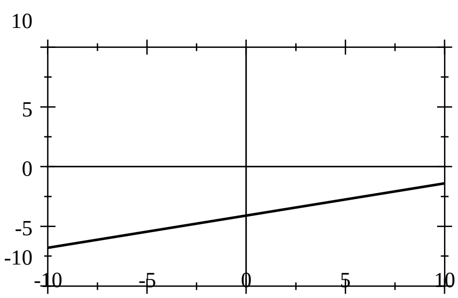

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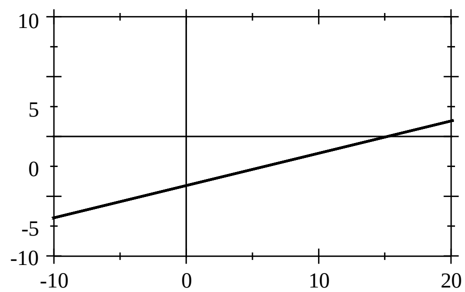

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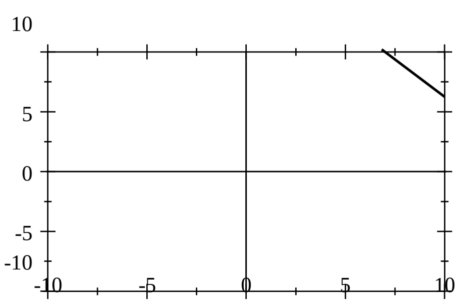

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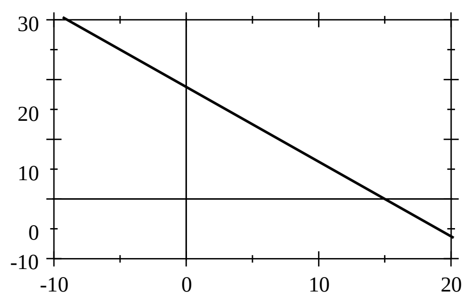

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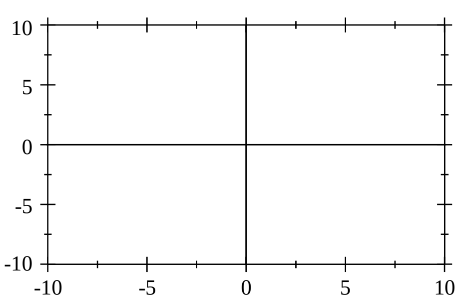

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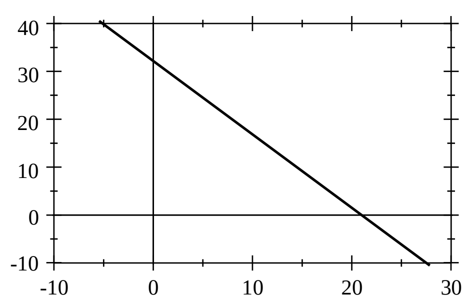

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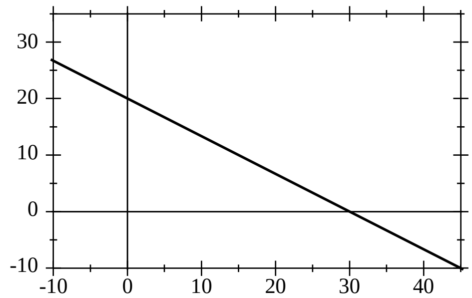

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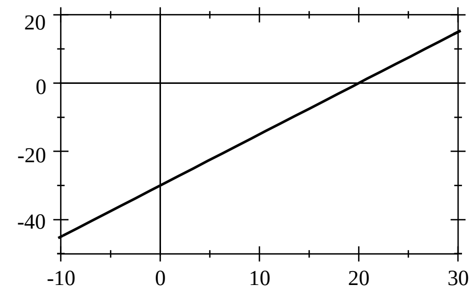

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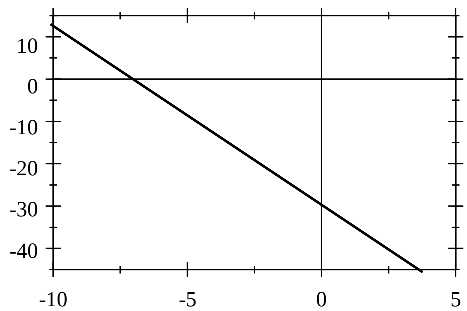

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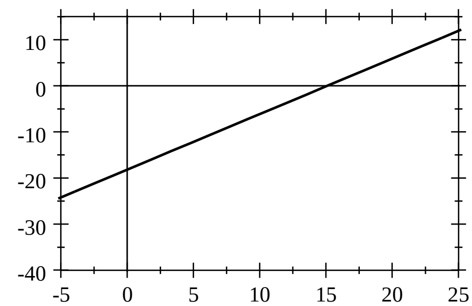

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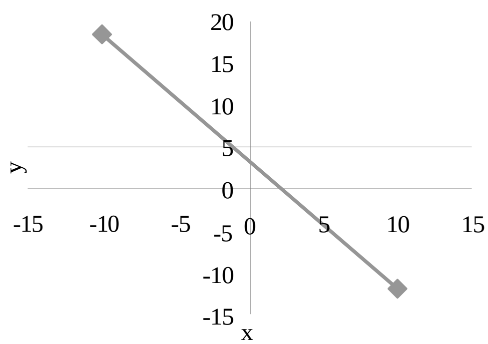

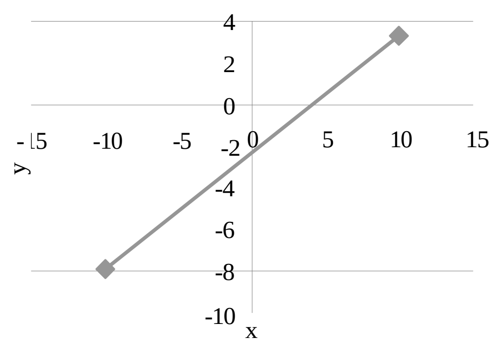

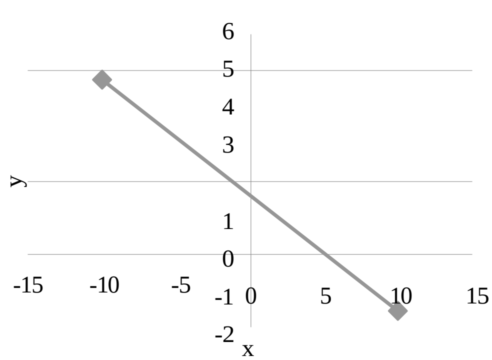

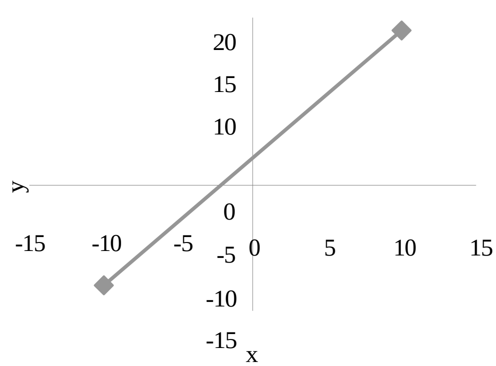

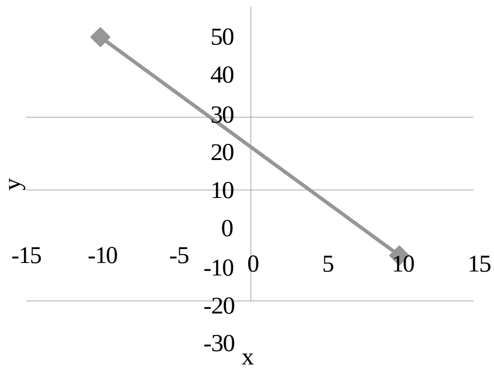
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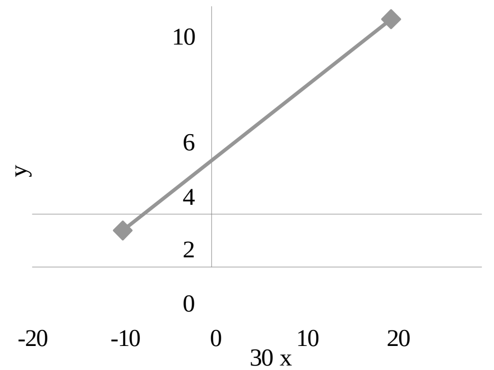
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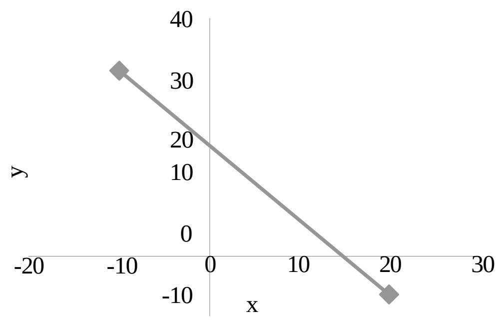
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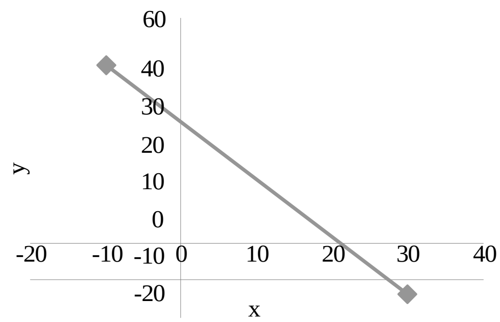
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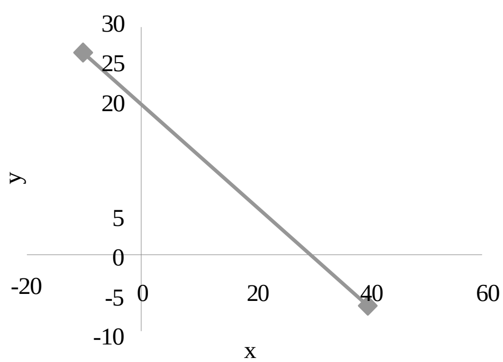
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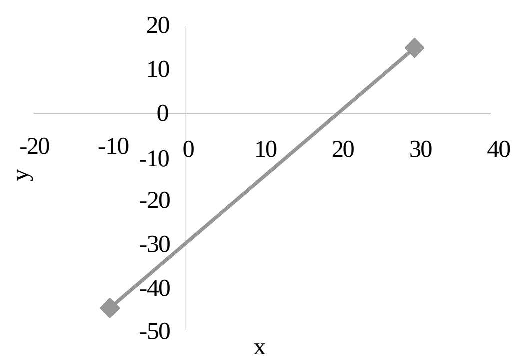
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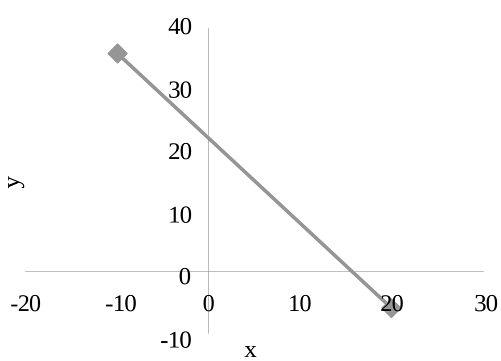
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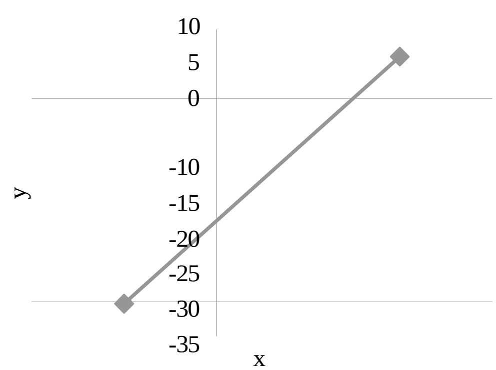
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1.3 Linear Functions and Mathematical Models

Concept Questions page 36

1. a. A function is a rule that associates with each element in a set A exactly one element in a set B.

b. A linear function is a function of the form $f(x) = mx + b$, where m and b are constants. For example, $f(x) = 2x + 3$ is a linear function.

c. The domain and range of a linear function are both $(-\infty, \infty)$.

d. The graph of a linear function is a straight line.

2. $C(x) = cx + F$, $R(x) = sx$, $P(x) = (s - c)x - F$

3. Negative, positive

4. a. The initial investment was $V(0) = 50{,}000 + 4000(0) = 50{,}000$, or \$50,000.

b. The rate of growth is the slope of the line with the given equation, that is, \$4000 per year.

Exercises page 36

1. Yes. Solving for y in terms of x, we find $3y = -2x + 6$, or $y = -\frac{2}{3}x + 2$.

2. Yes. Solving for y in terms of x, we find $4y = 2x + 7$, or $y = \frac{1}{2}x + \frac{7}{4}$.

3. Yes. Solving for y in terms of x, we find $2y = x + 4$, or $y = \frac{1}{2}x + 2$.

4. Yes. Solving for y in terms of x, we have $3y = 2x - 8$, or $y = \frac{2}{3}x - \frac{8}{3}$.

5. Yes. Solving for y in terms of x, we have $4y = 2x + 9$, or $y = \frac{1}{2}x + \frac{9}{4}$.

6. Yes. Solving for y in terms of x, we find $6y = 3x + 7$, or $y = \frac{1}{2}x + \frac{7}{6}$.

7. y is not a linear function of x because of the quadratic term $2x^2$.

8. y is not a linear function of x because of the nonlinear term $3\sqrt{x}$.

9. y is not a linear function of x because of the nonlinear term $-3y^2$.

10. y is not a linear function of x because of the nonlinear term $\sqrt{y}$.

11. a. $C(x) = 8x + 40{,}000$, where x is the number of units produced.

b. $R(x) = 12x$, where x is the number of units sold.

c. $P(x) = R(x) - C(x) = 12x - (8x + 40{,}000) = 4x - 40{,}000$.

d. $P(8000) = 4(8000) - 40{,}000 = -8000$, or a loss of \$8,000. $P(12{,}000) = 4(12{,}000) - 40{,}000 = 8000$, or a profit of \$8000.

12. a. $C(x) = 14x + 100{,}000$.

b. $R(x) = 20x$.

c. $P(x) = R(x) - C(x) = 20x - (14x + 100{,}000) = 6x - 100{,}000$.

d. $P(12{,}000) = 6(12{,}000) - 100{,}000 = -28{,}000$, or a loss of \$28,000.
$P(20{,}000) = 6(20{,}000) - 100{,}000 = 20{,}000$, or a profit of \$20,000.

13. $f(0) = 2$ gives $m(0) + b = 2$, or $b = 2$. Thus, $f(x) = mx + 2$. Next, $f(3) = -1$ gives $m(3) + 2 = -1$, or $m = -1$.

14. The fact that the straight line represented by $f(x) = mx + b$ has slope -1 tells us that $m = -1$ and so $f(x) = -x + b$. Next, the condition $f(2) = 4$ gives $f(2) = -1(2) + b = 4$, or $b = 6$.

15. Let V be the book value of the office building after 2008. Since $V = 1{,}000{,}000$ when $t = 0$, the line passes through $(0, 1000000)$. Similarly, when $t = 50$, $V = 0$, so the line passes through $(50, 0)$. Then the slope of the line is given by $m = \dfrac{0 - 1{,}000{,}000}{50 - 0} = -20{,}000$. Using the point-slope form of the equation of a line with the point $(0, 1000000)$, we have $V - 1{,}000{,}000 = -20{,}000(t - 0)$, or $V = -20{,}000t + 1{,}000{,}000$. In 2013, $t = 5$ and $V = -20{,}000(5) + 1{,}000{,}000 = 900{,}000$, or \$900,000.
In 2018, $t = 10$ and $V = -20{,}000(10) + 1{,}000{,}000 = 800{,}000$, or \$800,000.

16. Let V be the book value of the automobile after 5 years. Since $V = 24{,}000$ when $t = 0$, and $V = 0$ when $t = 5$, the slope of the line L is $m = \dfrac{0 - 24{,}000}{5 - 0} = -4800$. Using the point-slope form of an equation of a line with the point $(0, 5)$, we have $V - 0 = -4800(t - 5)$, or $V = -4800t + 24{,}000$. If $t = 3$, $V = -4800(3) + 24{,}000 = 9600$. Therefore, the book value of the automobile at the end of three years will be \$9600.

17. The consumption function is given by $C(x) = 0.75x + 6$. Thus, $C(0) = 6$, or 6 billion dollars; $C(50) = 0.75(50) + 6 = 43.5$, or 43.5 billion dollars; and $C(100) = 0.75(100) + 6 = 81$, or 81 billion dollars.

18. a. $T(x) = 0.06x$.

b. $T(200) = 0.06(200) = 12$, or \$12, and $T(5.60) = 0.06(5.60) = 0.336$, or approximately \$0.34.

19. a. $y = I(x) = 1.033x$, where x is the monthly benefit before adjustment and y is the adjusted monthly benefit.

b. His adjusted monthly benefit is $I(1220) = 1.033(1220) = 1260.26$, or \$1260.26.

20. $C(x) = 8x + 48{,}000$.

b. $R(x) = 14x$.

c. $P(x) = R(x) - C(x) = 14x - (8x + 48{,}000) = 6x - 48{,}000$.

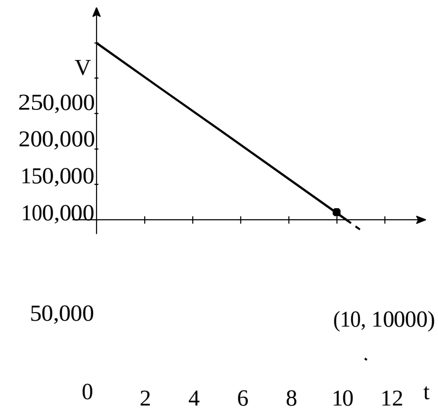
d. $P(4000) = 6(4000) - 48{,}000 = -24{,}000$, a loss of \$24,000.
$P(6000) = 6(6000) - 48{,}000 = -12{,}000$, a loss of \$12,000.
$P(10{,}000) = 6(10{,}000) - 48{,}000 = 12{,}000$, a profit of \$12,000.

21. Let the number of tapes produced and sold be x. Then $C(x) = 12{,}100 + 0.60x$, $R(x) = 1.15x$, and $P(x) = R(x) - C(x) = 1.15x - (12{,}100 + 0.60x) = 0.55x - 12{,}100$.

22. a. Let V denote the book value of the machine after t years. Since $V = 250{,}000$ when $t = 0$ and $V = 10{,}000$ when $t = 10$, the line passes through the points $(0, 250000)$ and $(10, 10000)$. The slope of the line through these points is given by $m = \frac{}{10 - 0} = -\frac{}{} = -24{,}000$.

Using the point-slope form of an equation of a line with the point $(10, 10000)$, we have $V - 10{,}000 = -24{,}000(t - 10)$, or $V = -24{,}000t + 250{,}000$.

b.



c. In 2014, $t = 4$ and $V = -24{,}000(4) + 250{,}000 = 154{,}000$, or \$154,000.

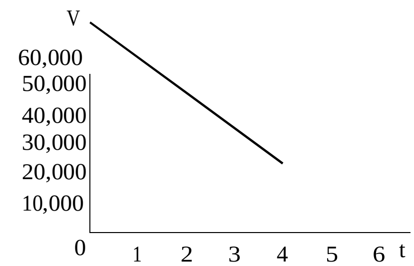
d. The rate of depreciation is given by $-m$, or \$24,000/yr.

23. Let the value of the workcenter system after t years be V. When $t = 0$, $V = 60{,}000$ and when $t = 4$, $V = 12{,}000$.

a. Since $m = \frac{}{4} = -\frac{}{4} = -12{,}000$, the rate of depreciation $(-m)$ is \$12,000/yr.

b. Using the point-slope form of the equation of a line with the point $(4, 12000)$, we have $V - 12{,}000 = -12{,}000(t - 4)$, or $V = -12{,}000t + 60{,}000$.

c.



d. When $t = 3$, $V = -12{,}000(3) + 60{,}000 = 24{,}000$, or \$24,000.

24. The slope of the line passing through the points $(0, C)$ and (N, S) is $m = \frac{S - C}{N - 0} = \frac{S - C}{N} = -\frac{C - S}{N}$. Using the point-slope form of an equation of a line with the point $(0, C)$, we have $V - C = -\frac{C - S}{N}t$, or $V = C - \frac{C - S}{N}t$.

25. The formula given in Exercise 24 is $V = C - \frac{C - S}{N}t$. When $C = 1{,}000{,}000$, $N = 50$, and $S = 0$, we have $V = 1{,}000{,}000 - \frac{1{,}000{,}000 - 0}{50}t$, or $V = 1{,}000{,}000 - 20{,}000t$. In 2013, $t = 5$ and $V = 1{,}000{,}000 - 20{,}000(5) = 900{,}000$, or \$900,000. In 2018, $t = 10$ and $V = 1{,}000{,}000 - 20{,}000(10) = 800{,}000$, or \$800,000.

26. The formula given in Exercise 24 is $V = C - \frac{C - S}{N}t$. When $C = 24{,}000$, $N = 5$, and $S = 0$, we have $V = 24{,}000 - \frac{24{,}000 - 0}{5}t = 24{,}000 - 4800t$. When $t = 3$, $V = 24{,}000 - 4800(3) = 9600$, or \$9600.

27. a. $D(S) = \frac{Sa}{1.7}$. If we think of D as having the form $D(S) = mS + b$, then $m = \frac{a}{1.7}$, $b = 0$, and D is a linear function of S.

b. $D(0.4) = \frac{500(0.4)}{1.7} = 117.647$, or approximately 117.65 mg.

28. a. $D(t) = \frac{(t+1)}{24}a = \frac{a}{24}t + \frac{a}{24}$. If we think of D as having the form $D(t) = mt + b$, then $m = \frac{a}{24}$, $b = \frac{a}{24}$, and D is a linear function of t.

b. If $a = 500$ and $t = 4$, $D(4) = \frac{4+1}{24}(500) = 104.167$, or approximately 104.2 mg.

29. a. The graph of f passes through the points $P_1(0, 17.5)$ and $P_2(10, 10.3)$. Its slope is $\frac{10.3 - 17.5}{10 - 0} = -0.72$.

An equation of the line is $y - 17.5 = -0.72(t - 0)$ or $y = -0.72t + 17.5$, so the linear function is $f(t) = -0.72t + 17.5$.

b. The percentage of high school students who drink and drive at the beginning of 2014 is projected to be $f(13) = -0.72(13) + 17.5 = 8.14$, or 8.14%.

30. a. The slope of the graph of f is a line with slope -13.2 passing through the point $(0, 400)$, so an equation of the line is $y - 400 = -13.2(t - 0)$ or $y = -13.2t + 400$, and the required function is $f(t) = -13.2t + 400$.

b. The emissions cap is projected to be $f(2) = -13.2(2) + 400 = 373.6$, or 373.6 million metric tons of carbon dioxide equivalent.

31. a. The line passing through $P_1(0, 61)$ and $P_2(4, 51)$ has slope $m = \frac{61 - 51}{0 - 4} = -2.5$, so its equation is

$y - 61 = -2.5(t - 0)$ or $y = -2.5t + 61$. Thus, $f(t) = -2.5t + 61$.

b. The percentage of middle-income adults in 2021 is projected to be $f(t) = -2.5(5) + 61$, or 48.5%.

32. a. The graph of f is a line through the points $P_1(0, 0.7)$ and $P_2(20, 1.2)$, so it has slope $\frac{1.2 - 0.7}{20 - 0} = 0.025$. Its equation is $y - 0.7 = 0.025(t - 0)$ or $y = 0.025t + 0.7$. The required function is thus $f(t) = 0.025t + 0.7$.

b. The projected annual rate of growth is the slope of the graph of f, that is, 0.025 billion per year, or 25 million per year.

c. The projected number of boardings per year in 2022 is $f(10) = 0.025(10) + 0.7 = 0.95$, or 950 million boardings per year.

33. a. Since the relationship is linear, we can write $F = mC + b$, where m and b are constants. Using the condition $C = 0$ when $F = 32$, we have $32 = b$, and so $F = mC + 32$. Next, using the condition $C = 100$ when $F = 212$, we have $212 = 100m + 32$, or $m = \frac{9}{5}$. Therefore, $F = \frac{9}{5}C + 32$.

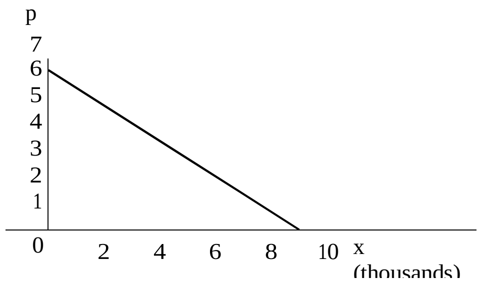
b. From part a, we have $F = \frac{9}{5}C + 32$. When $C = 20$, $F = \frac{9}{5}(20) + 32 = 68$, and so the temperature equivalent to 20° C is 68° F.

c. Solving for C in terms of F, we find $\frac{9}{5}C = F - 32$, or $C = \frac{5}{9}F - \frac{160}{9}$. When $F = 70$, $C = \frac{5}{9}(70) - \frac{160}{9} = \frac{190}{9}$, or approximately 21.1° C.

34. a. Since the relationship between T and N is linear, we can write $N = mT + b$, where m and b are constants. Using the points $(70, 120)$ and $(80, 160)$, we find that the slope of the line joining these points is $\frac{160-120}{80-70} = \frac{40}{10} = 4$.

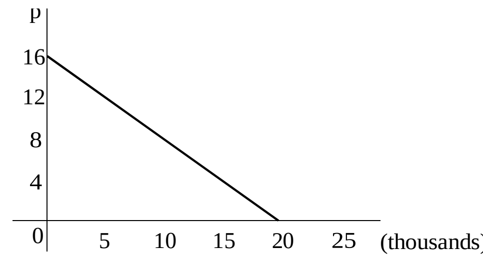
b. If $T = 70$, then $N = 120$, and this gives $120 = 70(4) + b$, or $b = -160$. Therefore, $N = 4T - 160$. If $T = 102$, we find $N = 4(102) - 160 = 248$, or 248 chirps per minute.

35. a. $2x$ □ $3p$ □ 18 □ 0, so setting x □ 0 gives $3p$ □ 18, or p □ 6. Next, setting p □ 0 gives $2x$ □ 18, or x □ 9.



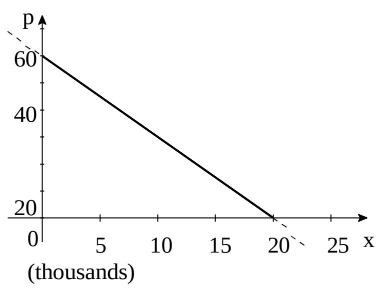
b. If p □ 4, then $2x$ □ 3 □4□ □ 18 □ 0, $2x$ □ 18 □ 12 □ 6, and x □ 3. Therefore, the quantity demanded when p □ 4 is 3000. (Remember that x is measured in units of a 1000.)

36. a. $5p$ □ $4x$ □ 80 □ 0, so setting x □ 0 gives p □ 16. Next, setting p □ 0 gives x □ 20.



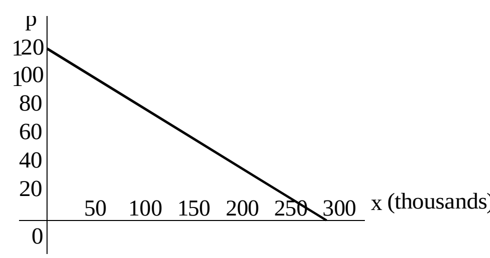
b. 5 □10□ □ $4x$ □ 80 □ 0, so $4x$ □ 80 □ 50 and x □ 7□5, or 7500 units (Remember that x represents the quantity demanded in units of 1000.)

37. a. p □ □$3x$ □ 60, so when x □ 0, p □ 60 and when p □ 0, □$3x$ □ □60, or x □ 20.



b. When p □ 30, 30 □ □$3x$ □ 60, $3x$ □ 30, and x □ 10. Therefore, the quantity demanded when p □ 30 is 10,000 units.

38. a. p □ □0□$4x$ □ 120, so when x □ 0, p □ 120, and when p □ 0, □0□$4x$ □ □120, or x □ 300.



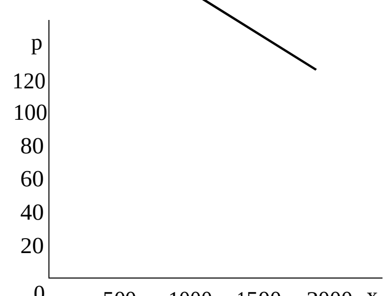
b. When p □ 80, 80 □ □0□$4x$ □ 120, 0□$4x$ □ 40, or x □ 100. Therefore, the quantity demanded when p □ 80 is 100,000.

39. When x □ 1000, p □ 55, and when x □ 600, p □ 85. Therefore, the graph of the linear demand equation is the straight line passing through the points □1000□ 55□ and □600□ 85□. The slope of the line is $\dfrac{85 □ 55}{600 □ 1000}$ □ □$\dfrac{3}{40}$.

Using this slope and the point □1000□ 55□, we find that the required equation is p □ 55 □ □$\frac{3}{40}$ □x □ 1000□, or p □ □$\frac{3}{40}x$ □ 130. When x □ 0, p □ 130, and this means that there will be no demand above $130. When p □ 0, x □ 1733□33, and this means that 1733 units is the maximum quantity demanded.

40. When x □ 200, p □ 90, and when x □ 1200, p □ 40. Therefore, the graph of the linear demand equation is the straight line passing through the points □200□ 90□ and □1200□ 40□. The slope is

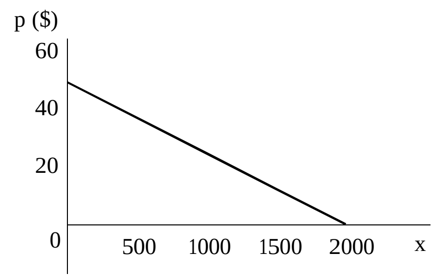
m □ $\dfrac{40 □ 90}{1200 ␣ 200}$ □ □$\dfrac{50}{1000}$ □ □0□05. Using the point-slope form of



41. The demand equation is linear, and we know that the line passes through the points $(1000, 9)$ and $(6000, 4)$. Therefore, the slope of the line is given by $m = \dfrac{4-9}{6000-1000} = -\dfrac{5}{5000} = -0.001$. Since the equation of

the line has the form $p = ax + b$, $9 = -0.001(1000) + b$, so $b = 10$. Therefore, an equation of the line is $p = -0.001x + 10$. If $p = 7.50$, we have $7.50 = -0.001x + 10$, so $0.001x = 2.50$ and $x = 2500$. Thus, the quantity demanded when the unit price is \$7.50 is 2500 units.

42. $p = -0.025x + 50$, so when $p = 0$, $x = 2000$ and when $x = 0$, $p = 50$. The highest price anyone would pay for the watch is \$50 (when $x = 0$).

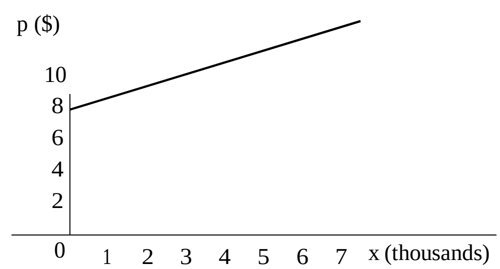


43. a. $3x - 4p + 24 = 0$. Setting $x = 0$, we obtain $3(0) - 4p + 24 = 0$, so $-4p = -24$ and $p = 6$. Setting $p = 0$, we obtain $3x - 4(0) + 24 = 0$, so $3x = -24$ and $x = -8$.

b. When $p = 8$, $3x - 4(8) + 24 = 0$, $3x = 32 - 24 = 8$,
and

3

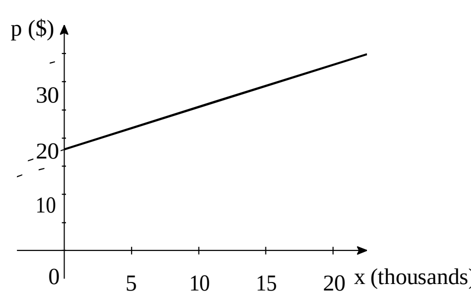
supplied at a unit price of \$8. (Here again x is measured in units of 1000.)



44. a. $\frac{1}{2}x - \frac{2}{3}p + 12 = 0$. When $p = 0$, $x = -24$, and when $x = 0$, $p = 18$.

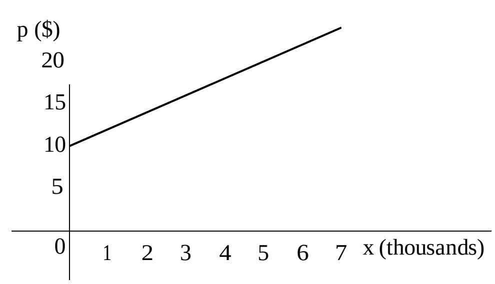
2 3 2

8000 units.



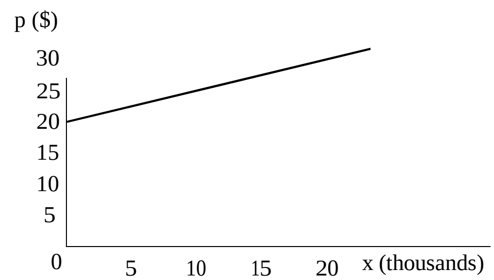
45. a. $p = 2x + 10$, so when $x = 0$, $p = 10$, and when $p = 0$, $x = -5$.

Therefore, when $p = 14$ the supplier will make 2000 units of the commodity available.

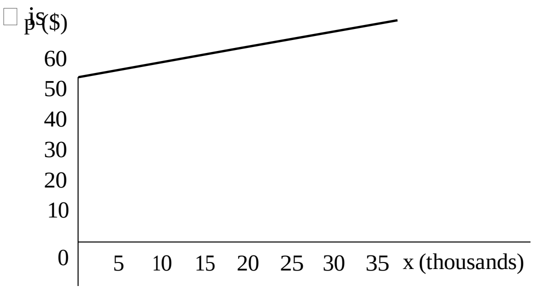


46. a. $p = \frac{1}{2}x + 20$, so when $x = 0$, $p = 20$ and when $p = 0$, $\frac{1}{2}x = -20$ and $x = -40$.

b. When $p = 28$, $28 = \frac{1}{2}x + 20$, so $\frac{1}{2}x = 8$ and $x = 16$. Therefore, 16,000 units will be supplied at a unit price of \$28.



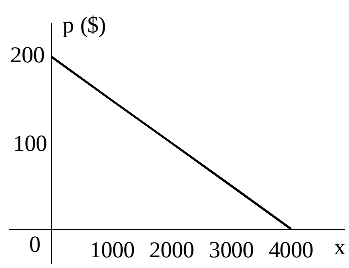
47. When $x = 10{,}000$, $p = 45$ and when $x = 20{,}000$, $p = 50$. The slope of the line passing through $(10000, 45)$ and $(20000, 50)$ is $m = \dfrac{50 - 45}{20{,}000 - 10{,}000} = \dfrac{5}{10{,}000} = 0.0005$, so using the point-slope form of an equation of a line with the point $(10000, 45)$, we have $p - 45 = 0.0005(x - 10{,}000)$, $p = 0.0005x - 5 + 45$, and $p = 0.0005x + 40$.



If $p = 70$, then $70 = 0.0005x + 40$ and $0.0005x = 30$, so $x = \dfrac{30}{0.0005} = 60{,}000$. (If x is expressed in units of a thousand, then the equation may be written in the form $p = \frac{1}{2}x + 40$.)

48. When $x = 2000$, $p = 330$, and when $x = 6000$, $p = 390$. Therefore, the graph of the linear equation passes through $(2000, 330)$ and $(6000, 390)$. The slope of the line is $\dfrac{390 - 330}{6000 - 2000} = \dfrac{3}{200}$. Using the point-slope form of an equation of a line with the point $(2000, 330)$, we obtain $p - 330 = \frac{3}{200}(x - 2000)$, or $p = \frac{3}{200}x + 300$, as the required supply equation. When $p = 450$, we have $450 = \frac{3}{200}x + 300$, $\frac{3}{200}x = 150$ or $x = 10{,}000$, and the number of refrigerators marketed at this price is 10,000. When $x = 0$, $p = 300$, and the lowest price at which a refrigerator will be marketed is \$300.

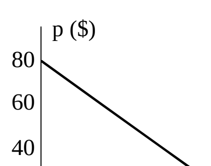
49.



b. The highest price is \$200 per unit.

c. To find the quantity demanded when $p = 100$, we solve $-0.005x + 200 = 100$, obtaining $x = \dfrac{-100}{-0.05} = 2000$, or 2000 units per month.

50.



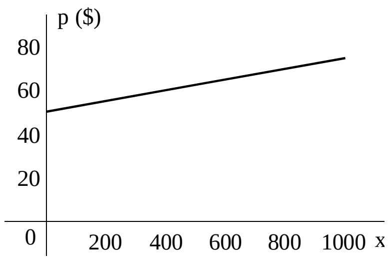
b. The highest price is \$80 per unit.

c. We solve the equation $-0.02x + 80 = 20$, obtaining

60

$$x \;\square\; \frac{\square}{\square 0 \square 02} \;\square\; 3000,$$ or 3000 units per month.

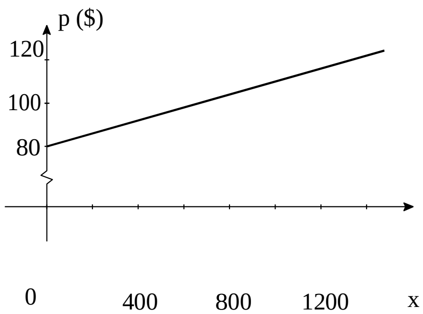
51.



b. The lowest price is $50 per unit.

c. We solve the equation $0.025x + 50 = 100$, obtaining $x = \dfrac{100 - 50}{0.025} = 2000$, or 2000 units per month.

52.



b. The lowest price is $80 per unit.

c. We solve the equation $0.03x + 80 = 110$, obtaining $x = \dfrac{110 - 80}{0.03} = 1000$, or 1000 units per month.

53. False. $P(x) = R(x) - C(x) = sx - (cx + F) = (s - c)x - F$. Therefore, the firm is making a profit if

$$P(x) = (s - c)x - F > 0, \text{ or } x > \frac{F}{s - c}.$$

54. True.

Technology Exercises page 43

1. 2.2875 **2.** 3.0125 **3.** 2.880952381 **4.** 0.7875

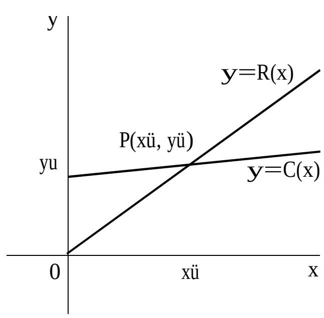
5. 7.2851648352 **6.** −26.82928836 **7.** 2.4680851064 **8.** 1.24375

1.4 Intersection of Straight Lines

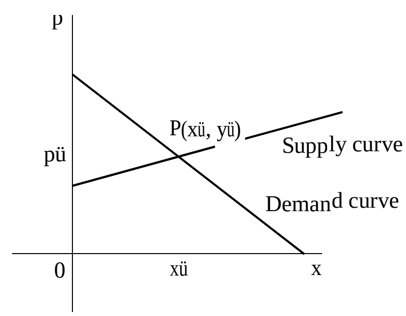
Concept Questions page 49

1. The intersection must lie in the first quadrant because only the parts of the demand and supply curves in the first quadrant are of interest.

2.



3.



Exercises page 50

1. We solve the system $y = 3x + 4$, $y = -2x + 14$. Substituting the first equation into the second yields $3x + 4 = -2x + 14$, $5x = 10$, and $x = 2$. Substituting this value of x into the first equation yields $y = 3(2) + 4$, so $y = 10$. Thus, the point of intersection is $(2, 10)$.

2. We solve the system $y = -4x - 7$, $-y = 5x + 10$. Substituting the first equation into the second yields $-(-4x - 7) = 5x + 10$, $4x + 7 = 5x + 10$, and $x = -3$. Substituting this value of x into the first equation, we obtain $y = -4(-3) - 7 = 12 - 7 = 5$. Therefore, the point of intersection is $(-3, 5)$.

3. We solve the system $2x - 3y = 6$, $3x + 6y = 16$. Solving the first equation for y, we obtain $3y = 2x - 6$, so $y = \frac{2}{3}x - 2$. Substituting this value of y into the second equation, we obtain $3x + 6\left(\frac{2}{3}x - 2\right) = 16$, $3x + 4x - 12 = 16$, $7x = 28$, and $x = 4$. Then $y = \frac{2}{3}(4) - 2 = \frac{2}{3}$, so the point of intersection is $\left(4, \frac{2}{3}\right)$.

4. We solve the system $2x + 4y = 11$, $-5x + 3y = 5$. Solving the first equation for x, we find $x = -2y + \frac{11}{2}$. Substituting this value into the second equation of the system, we have $-5\left(-2y + \frac{11}{2}\right) + 3y = 5$, so $10y - \frac{55}{2} + 3y = 5$, $20y - 55 + 6y = 10$, $26y = 65$, and $y = \frac{5}{2}$. Substituting this value of y into the first equation, we have $2x + 4\left(\frac{5}{2}\right) = 11$, so $2x = 1$ and $x = \frac{1}{2}$. Thus, the point of intersection is $\left(\frac{1}{2}, \frac{5}{2}\right)$.

5. We solve the system $y = \frac{1}{4}x - 5$, $2x - \frac{3}{2}y = 1$. Substituting the value of y given in the first equation into the second equation, we obtain $2x - \frac{3}{2}\left(\frac{1}{4}x - 5\right) = 1$, so $2x - \frac{3}{8}x + \frac{15}{2} = 1$, $16x - 3x + 60 = 8$, $13x = -52$, and $x = -4$. Substituting this value of x into the first equation, we have $y = \frac{1}{4}(-4) - 5 = -1 - 5$, so $y = -6$. Therefore, the point of intersection is $(-4, -6)$.

6. We solve the system $y = \frac{2}{3}x - 4$, $x + 3y + 3 = 0$. Substituting the first equation into the second equation, we obtain $x + 3\left(\frac{2}{3}x - 4\right) + 3 = 0$, so $x + 2x - 12 + 3 = 0$, $3x = 9$, and $x = 3$. Substituting this value of x into the first equation, we have $y = \frac{2}{3}(3) - 4 = -2$. Therefore, the point of intersection is $(3, -2)$.

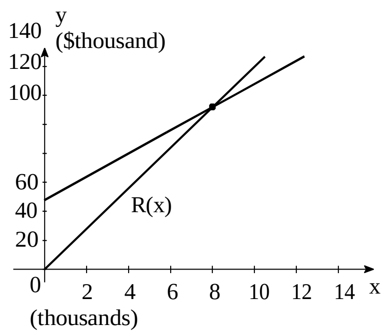
7. We solve the equation $R(x) = C(x)$, or $15x = 5x + 10{,}000$, obtaining $10x = 10{,}000$, or $x = 1000$. Substituting this value of x into the equation $R(x) = 15x$, we find $R(1000) = 15{,}000$. Therefore, the break-even point is $(1000, 15000)$.

8. We solve the equation $R(x) = C(x)$, or $21x = 15x + 12{,}000$, obtaining $6x = 12{,}000$, or $x = 2000$. Substituting this value of x into the equation $R(x) = 21x$, we find $R(2000) = 42{,}000$. Therefore, the break-even point is $(2000, 42000)$.

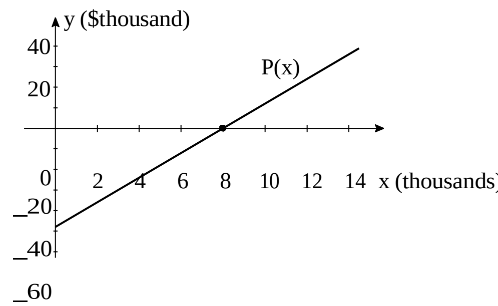
9. We solve the equation $R(x) = C(x)$, or $0.4x = 0.2x + 120$, obtaining $0.2x = 120$, or $x = 600$. Substituting this value of x into the equation $R(x) = 0.4x$, we find $R(600) = 240$. Therefore, the break-even point is $(600, 240)$.

10. We solve the equation $R(x) = C(x)$ or $270x = 150x + 20{,}000$, obtaining $120x = 20{,}000$ or $x = \frac{500}{3} \approx 167$. Substituting this value of x into the equation $R(x) = 270x$, we find $R(167) = 45{,}090$. Therefore, the break-even point is $(167, 45090)$.

11. a.



c.



b. We solve the equation $R(x) = C(x)$ or $14x = 8x + 48{,}000$, obtaining $6x = 48{,}000$, so $x = 8000$. Substituting this value of x into the equation $R(x) = 14x$, we find $R(8000) = 14(8000) = 112{,}000$. Therefore, the break-even point is $(8000, 112000)$.

d. $P(x) = R(x) - C(x) = 14x - 8x - 48{,}000 = 6x - 48{,}000$. The graph of the profit function crosses the x-axis when $P(x) = 0$, or $6x = 48{,}000$ and $x = 8000$. This means that the revenue is equal to the cost when 8000 units are produced and consequently the company breaks even at this point.

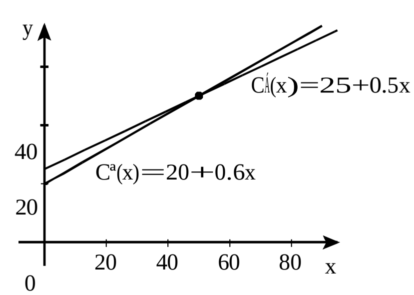
12. a. $R(x) = 8x$ and $C(x) = 25{,}000 + 3x$, so $P(x) = R(x) - C(x) = 5x - 25{,}000$. The break-even point occurs when $P(x) = 0$, that is, $5x - 25{,}000 = 0$, or $x = 5000$. Then $R(5000) = 40{,}000$, so the break-even point is
$(5000, 40000)$.

b. If the division realizes a 15% profit over the cost of making the income tax apps, then $P(x) = 0.15\,C(x)$, so
$5x - 25{,}000 = 0.15(25{,}000 + 3x)$, $4.55x = 28{,}750$, and $x \approx 6318.68$, or approximately 6319 income tax apps.

13. Let x denote the number of units sold. Then, the revenue function R is given by $R(x) = 9x$. Since the variable cost is 40% of the selling price and the monthly fixed costs are $50,000, the cost function C is given by $C(x) = 0.4(9x) + 50{,}000 = 3.6x + 50{,}000$. To find the break-even point, we set $R(x) = C(x)$, obtaining
$9x = 3.6x + 50{,}000$, $5.4x = 50{,}000$, and $x \approx 9259$, or 9259 units. Substituting this value of x into the equation $R(x) = 9x$ gives $R(9259) = 9(9259) = 83{,}331$. Thus, for a break-even operation, the firm should manufacture
9259 bicycle pumps, resulting in a break-even revenue of $83,331.

14. a. The cost function associated with renting a truck from the Ace Truck Leasing Company is $C_1(x) = 25 + 0.5x$. The cost function associated with renting a truck from the Acme Truck Leasing Company is $C_2(x) = 20 + 0.6x$.

b.



c. The cost of renting a truck from the Ace Truck Leasing Company for one day and driving 30 miles is
$C_1(30) = 25 + 0.5(30) = 40$, or $40.

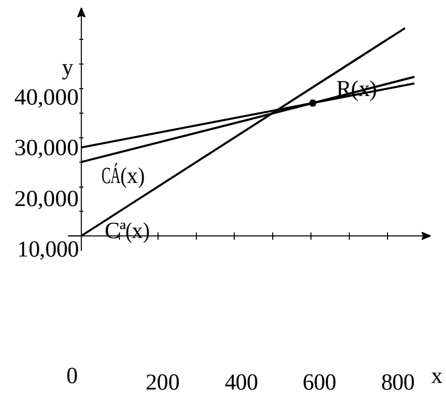
The cost of renting a truck from the Acme Truck Leasing Company for one day and driving it 30 miles is

$C_2(30) = 20 + 0.60(30) = 38$, or \$38. Thus, the customer should rent the truck from Acme Truck Leasing Company. This answer may also be obtained by inspecting the graph of the two functions and noting that the graph of $C_2(x)$ lies below that of $C_1(x)$ for $x < 50$.

d. $C_1(60) = 25 + 0.5(60) = 55$, or \$55. $C_2(60) = 20 + 0.6(60) = 56$, or \$56. Because $C_1(60) < C_2(60)$, the customer should rent the truck from Ace Trucking Company in this case.

15. **a.** The cost function associated with using machine I is $C_1(x) = 18{,}000 + 15x$. The cost function associated with using machine II is $C_2(x) = 15{,}000 + 20x$.

b.



c. Comparing the cost of producing 450 units on each machine, we find $C_1(450) = 18{,}000 + 15(450) = 24{,}750$ or $24,750

$24,000 on machine II. Therefore, machine II should be used in this case. Next, comparing the costs of producing 550 units on each machine, we find $C_1(550) = 18{,}000 + 15(550) = 26{,}250$ or $26,250 on machine I, and $C_2(550) = 15{,}000 + 20(550) = 26{,}000$, or $26,000 on machine II. Therefore, machine II should be used in this instance. Once again, we compare the cost of producing 650 units on each machine and find that $C_1(650) = 18{,}000 + 15(650) = 27{,}750$, or $27,750 on machine I and $C_2(650) = 15{,}000 + 20(650) = 28{,}000$, or $28,000 on machine II. Therefore, machine I should be used in this case.

d. We use the equation $P(x) = R(x) - C(x)$ and find $P(450) = 50(450) - 24{,}000 = -1500$, indicating a loss of $1500 when machine II is used to produce 450 units. Similarly, $P(550) = 50(550) - 26{,}000 = 1500$, indicating a profit of $1500 when machine II is used to produce 550 units. Finally, $P(650) = 50(650) - 27{,}750 = 4750$, for a profit of $4750 when machine I is used to produce 650 units.

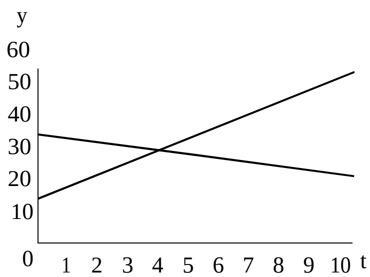
16. First, we find the point of intersection of the two straight lines. (This gives the time when the sales of both companies are the same). Substituting the first equation into the second gives $2.3 + 0.4t = 1.2 + 0.6t$, so $1.1 = 0.2t$ and $t = \frac{1.1}{0.2} = 5.5$. From the observation that the sales of Cambridge Pharmacy are increasing at a faster rate than that of the Crimson Pharmacy (its trend line has the greater slope), we conclude that the sales of the Cambridge Pharmacy will surpass the annual sales of the Crimson Pharmacy in $5\frac{1}{2}$ years.

17. We solve the two equations simultaneously, obtaining $18t + 13.4 = -12t + 88$, $30t = 74.6$, and $t = 2.486$, or approximately 2.5 years. So shipments of LCDs will first overtake shipments of CRTs just before mid-2003.

18. **a.** The number of digital cameras sold in 2001 is given by $f(0) = 3.05(0) + 6.85 = 6.85$, or 6.85 million. The number of film cameras sold in 2001 is given by $g(0) = -1.85(0) + 16.58 = 16.58$, or 16.58 million. Therefore, more film cameras than digital cameras were sold in 2001.

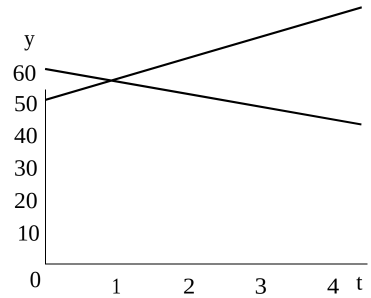
b. The sales are equal when $3.05t + 6.85 = -1.85t + 16.58$, $4.9t = 9.73$, or $t = \frac{9.73}{4.9} \approx 1.986$, approximately 2 years. Therefore, digital camera sales surpassed film camera sales near the end of 2003.

19. a.



b. We solve the two equations simultaneously, obtaining $\frac{11}{3}t + 23 = -\frac{11}{9}t + 43$, $\frac{44}{9}t = 20$, and $t \approx 4.09$. Thus, electronic transactions first exceeded check transactions in early 2005.

20. a.



b. $6.5t + 33 = -3.9t + 42.5$, $10.4t = 9.5$, $t \approx 0.91$, and so $f(0.91) \approx g(0.91) \approx 38.9$. The number of U.S. broadband Internet households was the same as the number of dial-up Internet households (39 million each) around November of 2004. Since then, the former has exceeded the latter.

21. We solve the system $4x + 3p = 59$, $5x - 6p = -14$. Solving the first equation for p, we find $p = -\frac{4}{3}x + \frac{59}{3}$. Substituting this value of p into the second equation, we have $5x - 6\left(-\frac{4}{3}x + \frac{59}{3}\right) = -14$, $5x + 8x - 118 = -14$, $13x = 104$, and $x = 8$. Substituting this value of x into the equation $p = -\frac{4}{3}x + \frac{59}{3}$, we have $p = -\frac{4}{3}(8) + \frac{59}{3} = \frac{27}{3} = 9$. Thus, the equilibrium quantity is 8000 units and the equilibrium price is \$9.

22. We solve the system $2x + 7p = 56$, $3x - 11p = -45$. Solving the first equation for x, we obtain $2x = -7p + 56$, or $x = -\frac{7}{2}p + 28$. Substituting this value of x into the second equation, we obtain $3\left(-\frac{7}{2}p + 28\right) - 11p = -45$, $-\frac{21}{2}p + 84 - 11p = -45$, $-43p = -258$, and $p = 6$. Then $x = -\frac{7}{2}(6) + 28 = -21 + 28 = 7$. Therefore, the equilibrium quantity is 7000 units and the equilibrium price is \$6.

23. We solve the system $p = -2x + 22$, $p = 3x + 12$. Substituting the first equation into the second, we find $-2x + 22 = 3x + 12$, so $5x = 10$ and $x = 2$. Substituting this value of x into the first equation, we obtain $p = -2(2) + 22 = 18$. Thus, the equilibrium quantity is 2000 units and the equilibrium price is \$18.

24. We solve the system $p = -0.3x + 6$, $p = 0.15x + 1.5$. Equating the right-hand sides, we have $-0.3x + 6 = 0.15x + 1.5$, so $-0.45x = -4.5$, or $x = 10$. Substituting this value of x into the first equation gives $p = -0.3(10) + 6$ and $p = 3$. Thus, the equilibrium quantity is 10,000 units and the equilibrium price is \$3.

25. Let x denote the number of DVD players produced per week, and p denote the price of each DVD player.

a. The slope of the demand curve is given by $\frac{\Delta p}{\Delta x} = -\frac{20}{250} = -\frac{2}{25}$. Using the point-slope form of the equation of a line with the point $(3000, 485)$, we have $p - 485 = -\frac{2}{25}(x - 3000)$, so $p = -\frac{2}{25}x + 240 + 485$ or $p = -0.08x + 725$.

b. From the given information, we know that the graph of the supply equation passes through the points $(0, 300)$ and $(2500, 525)$. Therefore, the slope of the supply curve is $m = \dfrac{525 - 300}{2500 - 0} = \dfrac{225}{2500} = 0.09$. Using the point-slope form of the equation of a line with the point $(0, 300)$, we find that $p - 300 = 0.09x$, so $p = 0.09x + 300$.

c. Equating the supply and demand equations, we have $-0.08x + 725 = 0.09x + 300$, so $0.17x = 425$ and $x = 2500$. Then $p = -0.08(2500) + 725 = 525$. We conclude that the equilibrium quantity is 2500 units and the equilibrium price is \$525.

26. We solve the system $x + 4p = 800$, $x - 20p = -1000$. Solving the first equation for x, we obtain $x = -4p + 800$. Substituting this value of x into the second equation, we obtain $-4p + 800 - 20p = -1000$, $-24p = -1800$, and $p = 75$. Substituting this value of p into the first equation, we obtain $x = -4(75) + 800$, or $x = 500$. Thus, the equilibrium quantity is 500 and the equilibrium price is \$75.

27. We solve the system $3x + p = 1500$, $2x - 3p = -1200$. Solving the first equation for p, we obtain $p = 1500 - 3x$. Substituting this value of p into the second equation, we obtain $2x - 3(1500 - 3x) = -1200$, so $11x = 3300$ and $x = 300$. Next, $p = 1500 - 3(300) = 600$. Thus, the equilibrium quantity is 300 and the equilibrium price is \$600.

28. Let x denote the number of espresso makers to be produced per month and p the unit price of the espresso makers.

a. The slope of the demand curve is given by $\frac{\Delta p}{\Delta x} = \frac{110 - 140}{1000 - 250} = -\frac{1}{25}$. Using the point-slope form of the equation of a line with the point $(250, 140)$, we have $p - 140 = -\frac{1}{25}(x - 250)$, so $p = -\frac{1}{25}x + 10 + 140 = -\frac{1}{25}x + 150$.

b. The slope of the supply curve is given by $\frac{\Delta p}{\Delta x} = \frac{80 - 60}{2250 - 750} = \frac{20}{1500} = \frac{1}{75}$. Using the point-slope form of the equation of a line with the point $(750, 60)$, we have $p - 60 = \frac{1}{75}(x - 750)$, so $p = \frac{1}{75}x - 10 + 60$ and $p = \frac{1}{75}x + 50$.

c. Equating the right-hand sides of the demand equation and the supply equation, we have $-\frac{1}{25}x + 150 = \frac{1}{75}x + 50$, so $-\frac{4}{75}x = -100$ and $x = 1875$. Next, $p = \frac{1}{75}(1875) + 50 = 75$. Thus, the equilibrium quantity is 1875 espresso makers and the equilibrium price is \$75.

29. We solve the system of equations $p = -0.05x + 200$, $p = 0.025x + 50$, obtaining $0.025x + 50 = -0.05x + 200$, $0.075x = 150$, and so $x = 2000$. Thus, $p = -0.05(2000) + 200 = 100$, and so the equilibrium quantity is 2000 per month and the equilibrium price is \$100 per unit.

30. We solve the system of equations $p = -0.02x + 80$, $p = 0.03x + 20$, obtaining $0.03x + 20 = -0.02x + 80$, $0.05x = 60$, and so $x = 1200$. Thus, $p = 0.03(1200) + 20 = 56$, and so the equilibrium quantity is 1200 per month and the equilibrium price is \$56 per unit.

31. a. We solve the system of equations $p = cx + d$, $p = ax + b$. Substituting the first into the second gives $cx + d = ax + b$, so $(c - a)x = b - d$ or $x = \frac{b - d}{c - a}$. Since $a < 0$ and $c > 0$, and $b > d > 0$, and $c - a \neq 0$, x is well-defined. Substituting this value of x into the second equation, we obtain

$$p = a\left(\frac{b - d}{c - a}\right) + b = \frac{ab - ad + bc - ab}{c - a} = \frac{bc - ad}{c - a} \quad (1).$$

Therefore, the equilibrium quantity is $\frac{b - d}{c - a}$ and the equilibrium price is $\frac{bc - ad}{c - a}$.

b. If c is increased, the denominator in the expression for x increases and so x gets smaller. At the same time, the first term in equation (1) for p decreases (because a is negative) and so p gets larger. This analysis shows that if the unit price for producing the product is increased then the equilibrium quantity decreases while the equilibrium price increases.

c. If b is decreased, the numerator of the expression for x decreases while the denominator stays the same. Therefore x decreases. The expression for p also shows that p decreases. This analysis shows that if the (theoretical) upper bound for the unit price of a commodity is lowered, then both the equilibrium quantity and the equilibrium price drop.

32. The break-even quantity is found by solving the equation $C(x) = R(x)$, or $cx + F = sx$; that is, $x = \frac{F}{s - c}$ $(s \neq c)$. Substituting this value of x into $R(x) = sx$ gives the break-even revenue as $R(x) = s\left(\frac{F}{s - c}\right) = \frac{sF}{s - c}$.

Our analysis shows that for a break-even operation, the break-even quantity must be equal to the ratio of the fixed cost times the unit selling price and the difference between the unit selling price and unit cost of production.

33. True. $P(x) = R(x) - C(x) = sx - (cx + F) = (s - c)x - F$. Therefore, the firm is making a profit if

$P(x) = (s - c)x - F > 0$; that is, if $x > \frac{F}{s - c}$ $(s \neq c)$.

34. True. In the typical linear demand curve, p drops as x increases, that is, the straight line has negative slope.

35. Solving the two equations simultaneously to find the point(s) of intersection of L_1 and L_2, we obtain $m_1x + b_1 = m_2x + b_2$, so $(m_1 - m_2)x = b_2 - b_1$ (1).

a. If $m_1 = m_2$ and $b_2 \neq b_1$, then there is no solution for (1) and in this case L_1 and L_2 do not intersect.

b. If $m_1 \neq m_2$, then (1) can be solved (uniquely) for x, and this shows that L_1 and L_2 intersect at precisely one point.

c. If $m_1 = m_2$ and $b_1 = b_2$, then (1) is satisfied for all values of x, and this shows that L_1 and L_2 intersect at infinitely many points.

36. a. Rewrite the equations in the form $y = -\frac{a_1}{b_1}x + \frac{c_1}{b_1}$ and $y = -\frac{a_2}{b_2}x + \frac{c_2}{b_2}$, and think of these equations as the equations of the lines L_1 and L_2, respectively. Using the results of Exercise 33, we see that the system has no solution if and only if $-\frac{a_1}{b_1} = -\frac{a_2}{b_2}$, or $a_1b_2 - a_2b_1 = 0$, and $\frac{c_1}{b_1} \neq \frac{c_2}{b_2}$.

b. The system has a unique solution if and only if $a_1b_2 - a_2b_1 \neq 0$.

c. The system has a infinitely many solutions if and only if $a_1b_2 - a_2b_1 = 0$ and $\frac{c_1}{b_1} = \frac{c_2}{b_2}$, or $c_1b_2 - b_1c_2 = 0$.

Technology Exercises page 54

1. □0□6□ 6□2□

2. □0□5273□ 6□8327□

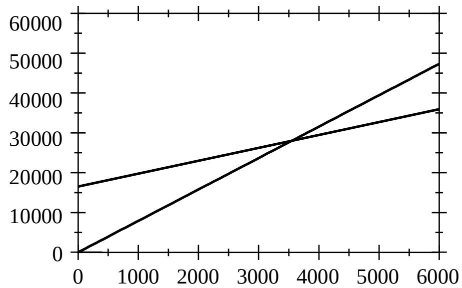
3. □3□8261□
0□1304□

4. □4□2256□
□0□4007□

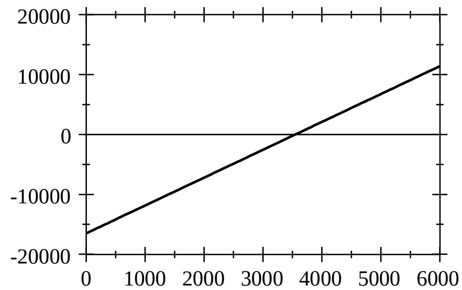
5. □386□9091□ 145□3939□

6. □ □1□5125□

7. a.



b.



From the graph, we see that the break-even point is approximately □3548□ 27997□

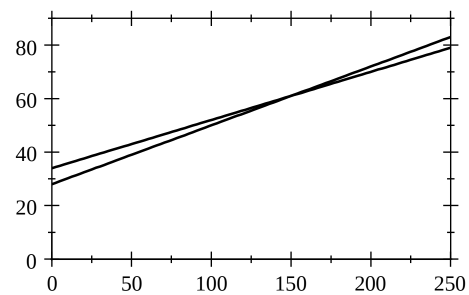
c. The x-intercept is approximately 3548.

8. a. □2492□ 610518□

b. 3438 units

9. a. C_1 □x□ □ 34 □ 0□18x and C_2 □x□ □ 28 □ 0□22x.

b.

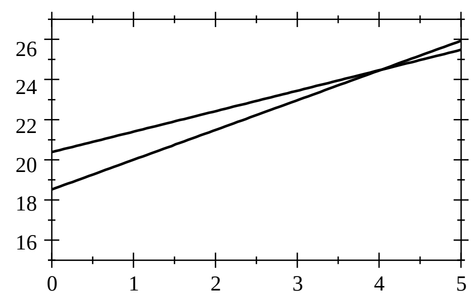


c. □150□ 61□

d. If the distance driven is less than or equal to 150 mi, rent from Acme Truck Leasing; if the distance driven is more than 150 mi, rent from Ace Truck Leasing.

10. a. Randolph Bank: D_1 □t□ □ 20□384 □ 1□019t; Madison Bank: D_2 □t□ □ 18□521 □ 1□482t.

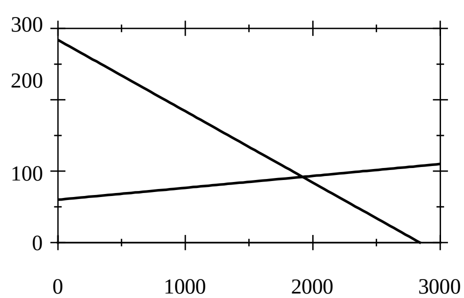
b.



c. Yes; 4 years from now.

11. a. p □ □$\frac{1}{10}x$ □ 284; p □ $\frac{1}{60}x$ □ 60

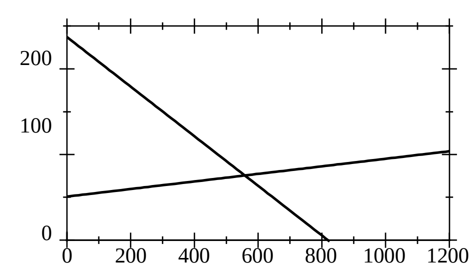
b.



The graphs intersect at roughly □1920□ 92□.

c. 1920□wk; \$92□radio.

12. a.



b. 558 units; \$75□51

1.5 The Method of Least Squares

Concept Questions page 60

1. a. A scatter diagram is a graph showing the data points that describe the relationship between the two variables x and y.

b. The least squares line is the straight line that best fits a set of data points when the points are scattered about a straight line.

2. See page 55 of the text.

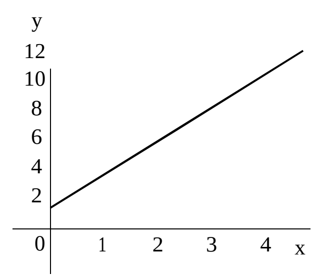
Exercises page 60

1. a. We first summarize the data.

	x	y	x^2	xy
	1	4	1	4
	2	6	4	12
	3	8	9	24
	4	11	16	44
Sum	10	29	30	84

The normal equations are $4b + 10m = 29$ and $10b + 30m = 84$. Solving this system of equations, we obtain $m = 2.3$ and $b = 1.5$, so an equation is $y = 2.3x + 1.5$.

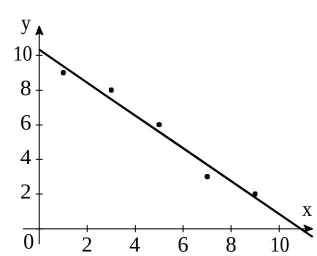
b.



2. a. We first summarize the data.

	x	y	x^2	xy
	1	9	1	9
	3	8	9	24
	5	6	25	30
	7	3	49	21
	9	2	81	18
Sum	25	28	165	102

b.



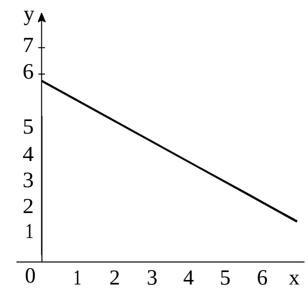
The normal equations are $165m + 25b = 102$ and $25m + 5b = 28$. Solving, we find $m = -0.95$ and $b = 10.35$, so the required equation is $y = -0.95x + 10.35$.

3. a. We first summarize the data.

	x	y	x^2	xy
	1	4.5	1	4.5
	2	5	4	10
	3	3	9	9
	4	2	16	8
	4	3.5	16	14
	6	1	36	6
Sum	20	19	82	51.5

The normal equations are $6b + 20m = 19$ and $20b + 82m = 51.5$. The solutions are $m = -0.7717$ and $b = 5.7391$, so the required equation is $y = -0.772x + 5.739$.

b.

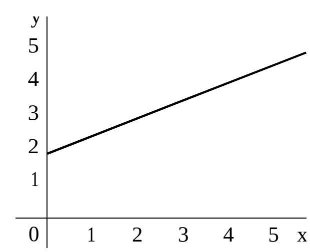


4. a. We first summarize the data:

	x	y	x^2	xy
	1	2	1	2
	1	3	1	3
	2	3	4	6
	3	3.5	9	10.5
	4	3.5	16	14
	4	4	16	16
	5	5	25	25
Sum	20	24	72	76.5

The normal equations are $72m + 20b = 76.5$ and $20m + 7b = 24$. Solving, we find $m = 0.53$ and $b = 1.91$. The required equation is $y = 0.53x + 1.91$.

b.

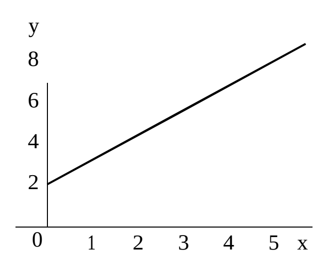


5. a. We first summarize the data:

	2	5	4	10
	3	5	9	15
	4	7	16	28
	5	8	25	40
Sum	15	28	55	96

The normal equations are $55m + 15b = 96$ and $15m + 5b = 28$. Solving, we find $m = 1.2$ and $b = 2$, so the required equation is $y = 1.2x + 2$.

b.

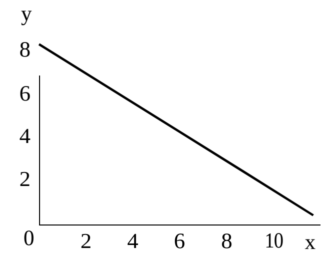


6. a. We first summarize the data:

	x	y	x^2	xy
	7	4	49	28
	10	1	100	10
Sum	25	25	179	88

The normal equations are $5b + 25m = 25$ and $25b + 179m = 88$. The solutions are $m = -0.68519$ and $b = 8.4259$, so the required equation is $y = -0.685x + 8.426$.

b.

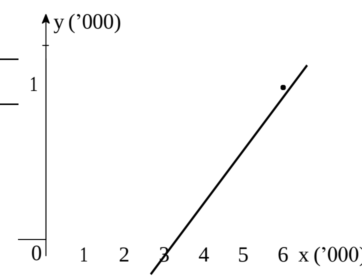


7. a. We first summarize the data:

	x	y	x^2	xy
	4	0.5	16	2
	4.5	0.6	20.25	2.7
	5	0.8	25	4
	5.5	0.9	30.25	4.95
	6	1.2	36	7.2
Sum	25	4	127.5	20.85

The normal equations are $5b + 25m = 4$ and $25b + 127.5m = 20.85$. The solutions are $m = 0.34$ and $b = -0.9$, so the required equation is $y = 0.34x - 0.9$.

b.



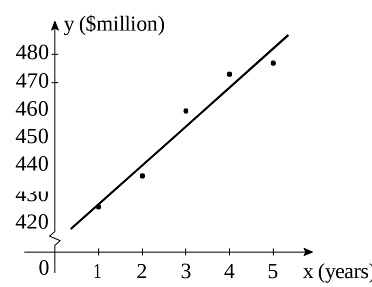
c. $x = 6.4$, then $y = 0.34(6.4) - 0.9 = 1.276$, and so 1276 completed applications can be expected.

8. a. We first summarize the data:

	x	y	x^2	xy
	1	426	1	426
	2	437	4	874
	3	460	9	1380
	4	473	16	1892
	5	477	25	2385
Sum	15	2273	55	6957

The normal equations are $55m + 15b = 6957$ and $15m + 5b = 2273$. Solving, we find $m = 13.8$ and $b = 413.2$, so the required equation is $y = 13.8x + 413.2$.

b.



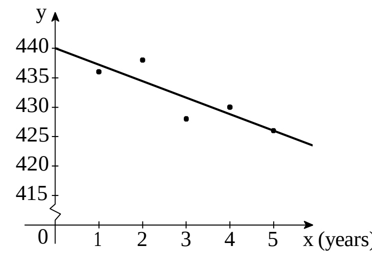
c. When $x = 6$, $y = 13.8(6) + 413.2 = 496$, so the predicted net sales for the upcoming year are $496 million.

9. a. We first summarize the data:

	x	y	x^2	xy
	1	436	1	436
	2	438	4	876
	3	428	9	1284
	4	430	16	1720
	5	426	25	2138
Sum	15	2158	55	6446

The normal equations are 5b □ 15m □ 2158 and 15b □ 55m □ 6446. Solving this system, we find m □ □2□8 and b □ 440. Thus, the equation of the least-squares line is y □ □2□8x □ 440.

b.



c. Two years from now, the average SAT verbal score in that area will be y □ □2□8 □7□ □ 440 □ 420□4, or approximately 420.

10. a. We first summarize the data:

	x	y	x^2	xy
	1	2□1	1	2□1
	2	2□4	4	4□8
	3	2□7	9	8□1
Sum	6	7□2	14	15□0

The normal equations are 3b □ 6m □ 7□2 and 6b □ 14m □ 15. Solving the system, we find m □ 0□3 and b □ 1□8. Thus, the equation of the least-squares line is y □ 0□3x □ 1□8.

b. The amount of money that Hollywood is projected to spend in 2015 is approximately 0□3 □5□ □ 1□8 □ 3□3, or $3□3 billion.

11. a.

	x	y	x^2	xy
	0	154□5	0	0
	1	381□8	1	381□8
	2	654□5	4	1309
	3	845	9	2535
Sum	6	2035□8	14	4225□

The normal equations are 4b □ 6m □ 2035□8 and 6b □ 14m □ 4225□8. The solutions are m □ 234□42 and b □ 157□32, so the required equation is y □ 234□4x □ 157□3.

b. The projected number of Facebook users is f □7□ □ 234□4 □7□ □ 157□3 □ 1798□1, or approximately 1798□1 million.

12. a.

	x	y	x^2	xy
	0	25□3	0	0
	1	33□	1	33□4
	2	39□	4	79
	3	50	9	150
	4	59□	16	238□4
Sum	10	207□	30	500□8

The normal equations are 5b □ 10m □ 207□8 and 10b □ 30m □ 500□8. The solutions are m □ 8□52 and b □ 24□52, so the required equation is y □ 8□52x □ 24□52.

b. The average rate of growth of the number of e-book readers between 2011 and 2015 is projected to be approximately

8□52 million per year.

13. a.

	x	y	x^2	xy
	1	20	1	20
	2	24	4	48
	3	26	9	78
	4	28	16	112
	5	32	25	160
Sum	15	130	55	418

The normal equations are $5b + 15m = 130$ and $15b + 55m = 418$. The solutions are $m = 2.8$ and $b = 17.6$, and so an equation of the line is $y = 2.8x + 17.6$.

b. When $x = 8$, $y = 2.8(8) + 17.6 = 40$. Hence, the state subsidy is expected to be $40 million for the eighth year.

14. a.

	x	y	x^2	xy
	0	26.2	0	0
	1	26.8	1	26.8
	2	27.5	4	55.0
	3	28.3	9	84.9
	4	28.7	16	114.8
Sum	10	137.5	30	281.5

The normal equations are $5b + 10m = 137.5$ and $10b + 30m = 281.5$. Solving this system, we find $m = 0.65$ and $b = 26.2$. Thus, an equation of the least-squares line is $y = 0.65x + 26.2$.

b. The percentage of the population enrolled in college in 2014 is projected to be $0.65(7) + 26.2 = 30.75$, or 30.75

15. a.

	x	y	x^2	xy
	1	26.1	1	26.1
	2	27.2	4	54.4
	3	28.9	9	86.7
	4	31.1	16	124.4
	5	32.6	25	163.0
Sum	15	145.9	55	454.6

The normal equations are $5b + 15m = 145.9$ and $15b + 55m = 454.6$. Solving this system, we find $m =$ and $b = 24.11$. Thus, the required equation is $y = f(x) = 1.69x + 24.11$.

b. The predicted global sales for 2014 are given by $f(8) = 1.69(8) + 24.11 = 37.63$, or 37.6 billion.

16. a.

	x	y	x^2	xy
	0	34.4	0	0
	1	34.1	1	34.1
	2	33.4	4	66.8
	3	33.1	9	99.3
	4	32.7	16	130.8
Sum	10	167.7	30	331.0

The normal equations are $5b + 10m = 167.7$ and $10b + 30m = 331$. Solving this system, we find $m = -0.44$ and $b = 34.42$. Thus, an equation of the least-squares line $y = -0.44x + 34.42$.

b. The percentage of households in which someone is under 18 years old in 2013 is projected to be $-0.44(6) + 34.42 = 31.78$, or 31.78%.

17.

	x	y	x^2	xy
	0	82□0	0	0
	1	84□7	1	84□7
	2	86□8	4	173□6
	3	89□7	9	269□1
	4	91□8	16	367□2
Sum	10	435	30	894□6

The normal equations are 5*b* □ 10*m* □ 435 and 10*b* □ 30*m* □ 894□6. The solutions are *m* □ 2□46 and *b* □ 82□08, so the required equation is *y* □ 2□46*x* □ 82□1.

b. The estimated number of credit union members in 2013 is *f* □5□ □ 2□46 □5□ □ 82□1 □ 94□4, or approximately 94□4 million.

18. a.

	x	y	x^2	xy
	1	95□9	1	95□9
	2	91□7	4	183□4
	3	83□8	9	251□4
	4	78□2	16	312□8
	5	73□5	25	367□5
Sum	15	423□1	55	1211□0

The normal equations are 5*b* □ 15*m* □ 423□1 and 15*b* □ 55*m* □ 1211. Solving this system, we find *m* □ □5□83 and *b* □ 102□11. Thus, an equation of the least-squares line is *y* □ □5□83*x* □ 102□11.

b. The volume of first-class mail in 2014 is projected to be □5□83 □8□ □ 102□11 □ 55□47, or approximately 55□47 billion pieces.

19. a.

	x	y	x^2	xy
	0	29□4	0	0
	1	32□2	1	32□2
	2	34□8	4	69□6
	3	37□7	9	113□1
	4	40□4	16	161□6
Sum	10	174□5	30	376□5

The normal equations are 5*b* □ 10*m* □ 174□5 and 10*b* □ 30*m* □ 376□5. The solutions are *m* □ 2□75 and *b* □ 29□4, so *y* □ 2□75*x* □ 29□4.

b. The average rate of growth of the number of subscribers from 2006 through 2010 was 2□75 million per year.

20. a.

	x	y	x^2	xy
	0	2□0	0	0
	1	3□1	1	3□1
	2	4□5	4	9□0
	3	6□3	9	18□9
	4	7□8	16	31□2
	5	9□3	25	46□5
Sum	15	33□0	55	108□7

The normal equations are 6*b* □ 15*m* □ 33 and 15*b* □ 55*m* □ 108□7. Solving this system, we find *m* □ 1□50 and *b* □ 1□76, so an equation of the least-squares line is *y* □ 1□5*x* □ 1□76.

b. The rate of growth of video advertising spending between 2011 and 2016 is approximated by the slope of the least-squares line, that is $1□5 billion□yr.

21. a.

	x	y	x^2	xy
	0	6.4	0	0
	1	6.8	1	6.8
	2	7.1	4	14.2
	3	7.4	9	22.2
	4	7.6	16	30.4
Sum	10	35.3	30	73.6

The normal equations are $5b + 10m = 35.3$ and $10b + 30m = 73.6$. The solutions are $m = 0.3$ and $b = 6.46$, so the required equation is $y = 0.3x + 6.46$.

b. The rate of change is given by the slope of the least-squares line, that is, approximately \$0.3 billion/yr.

22. a.

	x	y	x^2	xy
	0	12.9	0	0
	1	13.9	1	13.9
	2	14.65	4	29.3
	3	15.25	9	45.75
	4	15.85	16	63.4
Sum	10	72.55	30	152.35

The normal equations are $5b + 10m = 72.55$ and $10b + 30m = 152.35$. The solutions are $m = 0.725$ and $b = 13.06$, so the required equation is $y = 0.725x + 13.06$.

b. $y = 0.725(5) + 13.06 = 16.685$, or approximately \$16.685 million.

23. a. We summarize the data at right. The normal equations are $6b + 39m = 195.5$ and $39b + 271 = 1309$. The solutions are $b = 18.38$ and $m = 2.19$, so the required least-squares line is given by $y = 2.19x + 18.38$.

	x	y	x^2	xy
	4	27.5	16	110
	5	29	25	145
	6	31	36	186
	7	34	49	238
	8	36	64	288
	9	38	81	342
Sum	39	195.5	271	1309

b. The average rate of increase is given by the slope of the least-squares line, namely \$2.19 billion/yr.

c. The revenue from overdraft fees in 2011 is $y = 2.19(11) + 18.38 = 42.47$, or approximately \$42.47 billion.

24. a.

	x	y	x^2	xy
	0	15.9	0	0
	10	16.8	100	168
	20	17.6	400	352
	30	18.5	900	555
	40	19.3	1600	772
	50	20.3	2500	1015
Sum	150	108.4	5500	2862

The normal equations are $6b + 150m = 108.4$ and $150b + 5500m = 2862$. The solutions are $b = 15.90$ $m = 0.09$, so $y = 0.09x + 15.9$.

b. The life expectancy at 65 of a male in 2040 is $y = 0.09(40) + 15.9 = 19.5$, or 19.5 years.

c. The life expectancy at 65 of a male in 2030 is $y = 0.09(30) + 15.9 = 18.6$, or 18.6 years.

21. a. The normal equations are $5b + 10m = 35.3$ and

25. a. The normal equations are $7b + 42m = 726$ and $42b + 364m = 5168$. The solutions are $m = 7.25$ and $b = 60.21$, so the required equation is $y = 7.25x + 60.21$.

	x	y	x^2	xy
	0	60	0	0
	2	74	4	148
	4	90	16	360
	6	106	36	636
	8	118	64	944
	10	128	100	1280
	12	150	144	1800
Sum	42	726	364	5168

b. $y = 7.25(11) + 60.21 = 139.96$, or \$139.96 billion.

c. \$7.25 billion/yr.

26. a. The normal equations are $8b + 28m = 12.67$ and $28b + 140m = 46.79$. The solutions are $m = 0.058$ and $b = 138$, so the required equation is $y = 0.058t + 138$.

	t	y	t^2	ty
	0	1.38	0	0
	1	1.44	1	1.44
	2	1.49	4	2.98
	3	1.56	9	4.68
	4	1.61	16	6.44
	5	1.67	25	8.35
	6	1.74	36	10.44
	7	1.78	49	12.46
Sum	28	12.67	140	46.79

b. The rate of change is given by the slope of the least-squares line, that is, approximately \$0.058 trillion/yr, or \$58 billion/yr.

c. $y = 0.058(10) + 1.38 = 1.96$, or \$1.96 trillion.

27. False. See Example 1 on page 56 of the text.

28. True. The error involves the sum of the squares of the form $\left[f(x_i) - y_i\right]^2$, where f is the least-squares function and y_i is a data point. Thus, the error is zero if and only if $f(x_i) = y_i$ for each $1 \le i \le n$.

29. True.

30. True.

Technology Exercises page 67

1. $y = 2.3596x + 3.8639$

2. $y = 1.4068x + 2.1241$

3. $y = -1.1948x + 3.5525$

4. $y = -2.07715x + 5.23847$

5. a. $22.3x + 143.5$ **b.** \$22.3 billion/yr **c.** \$366.5 billion

6. a. $0.305x + 0.19$ **b.** \$0.305 billion/yr **c.** \$3.24 billion

21. a. The normal equations are 5*b* □ 10*m* □ 35□3 and

7. a. *y* □ 1□5857*t* □ 6□6857 **b.** $19□4 billion

8. a. *y* □ 5□5*x* □ 15□7 **b.** 70□7%

9. a. *y* □ 1□751*x* □ 7□9143 **b.** $22 billion

10. a. *y* □ 46□6*x* □ 495 **b.** $46□60□buyer□yr

CHAPTER 1 Concept Review Questions page 68

1. ordered, abscissa (x-coordinate), ordinate (y-coordinate)

2. a. x-, y- **b.** third

3. $\sqrt{(c-a)^2+(d-b)^2}$

4. $(x-a)^2+(y-b)^2=r^2$

5. a. $\dfrac{y_2-y_1}{x_2-x_1}$ **b.** undefined **c.** zero **d.** positive

6. $m_1=m_2$, $m_1=-\dfrac{1}{m_2}$

7. a. $y-y_1=m(x-x_1)$, point-slope **b.** $y=mx+b$, slope-intercept

8. a. $Ax+By+C=0$, where A and B are not both zero **b.** $-a/b$

9. $mx+b$

10. a. price, demanded, demand **b.** price, supplied, supply

11. break-even

12. demand, supply

CHAPTER 1 Review Exercises page 69

1. The distance is $d=\sqrt{(6-2)^2+(4-1)^2}=\sqrt{4^2+3^2}=\sqrt{25}=5$.

2. The distance is $d=\sqrt{(2-6)^2+(6-9)^2}=\sqrt{(-4)^2+(-3)^2}=\sqrt{25}=5$.

3. The distance is $d=\sqrt{[1-(-2)]^2+[-7-(-3)]^2}=\sqrt{3^2+(-4)^2}=\sqrt{9+16}=\sqrt{25}=5$.

4. The distance is $d=\sqrt{\left(-\frac{1}{2}-\frac{1}{2}\right)^2+\left(2\sqrt{3}-\sqrt{3}\right)^2}=\sqrt{1+3}=\sqrt{4}=2$.

5. Substituting $x=-1$ and $y=\frac{5}{4}$ into the left-hand side of the equation gives $6(-1)+8\left(\frac{5}{4}\right)-16=-12$. The equation is not satisfied, and so we conclude that the point $\left(-1,\frac{5}{4}\right)$ does not lie on the line $6x+8y-16=0$.

6. An equation is $x=-2$.

7. An equation is $y=4$.

8. The slope of L is $m=\dfrac{\frac{7}{2}-4}{3-(-2)}=\dfrac{\frac{7-8}{2}}{5}=-\dfrac{1}{10}$ and an equation of L is $y-4=-\frac{1}{10}[x-(-2)]=-\frac{1}{10}x-\frac{1}{5}$, or

3 □ □□2□ 5 10 10 10 5

y □ □$\frac{1}{10}x$ □ $\frac{19}{5}$. The general form of this equation is x □ $10y$ □ 38 □ 0.

9. The line passes through the points $(-2, 4)$ and $(3, 0)$, so its slope is $m = \frac{4-0}{-2-3} = -\frac{4}{5}$. An equation is

$y - 0 = -\frac{4}{5}(x - 3)$, or $y = -\frac{4}{5}x + \frac{12}{5}$.

10. Writing the given equation in the form $y = \frac{5}{2}x - 3$, we see that the slope of the given line is $\frac{5}{2}$. Thus, an equation is $y - 4 = \frac{5}{2}(x + 2)$, or $y = \frac{5}{2}x + 9$. The general form of this equation is $5x - 2y + 18 = 0$.

11. Writing the given equation in the form $y = -\frac{4}{3}x + 2$, we see that the slope of the given line is $-\frac{4}{3}$. Therefore, the slope of the required line is $\frac{3}{4}$ and an equation of the line is $y - 4 = \frac{3}{4}(x + 2)$, or $y = \frac{3}{4}x + \frac{11}{2}$.

12. Using the slope-intercept form of the equation of a line, we have $y = -\frac{1}{2}x - 3$.

13. Rewriting the given equation in slope-intercept form, we have $-5y = -3x + 6$, or $y = \frac{3}{5}x - \frac{6}{5}$. From this equation, we see that the slope of the line is $\frac{3}{5}$ and its y-intercept is $-\frac{6}{5}$.

14. Rewriting the given equation in slope-intercept form, we have $4y = -3x + 8$, or $y = -\frac{3}{4}x + 2$, and we conclude that the slope of the required line is $-\frac{3}{4}$. Using the point-slope form of the equation of a line with the point $(2, 3)$ and slope $-\frac{3}{4}$, we obtain $y - 3 = -\frac{3}{4}(x - 2)$, so $y = -\frac{3}{4}x + \frac{6}{4} + 3 = -\frac{3}{4}x + \frac{9}{2}$. The general form of this equation is $3x + 4y - 18 = 0$.

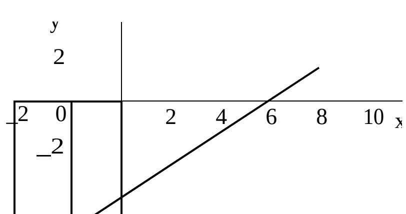
15. The slope of the line joining the points $(-3, 4)$ and $(2, 1)$ is $m = \frac{1-4}{2-(-3)} = -\frac{3}{5}$. Using the point-slope form of the equation of a line with the point $(-1, 3)$ and slope $-\frac{3}{5}$, we have $y - 3 = -\frac{3}{5}[x - (-1)]$, so $y = -\frac{3}{5}(x + 1) + 3 = -\frac{3}{5}x + \frac{12}{5}$.

16. Rewriting the given equation in the slope-intercept form $y = \frac{2}{3}x - 8$, we see that the slope of the line with this equation is $\frac{2}{3}$. The slope of the required line is $-\frac{3}{2}$. Using the point-slope form of the equation of a line with the point $(-2, -4)$ and slope $-\frac{3}{2}$, we have $y - (-4) = -\frac{3}{2}[x - (-2)]$, or $y = -\frac{3}{2}x - 7$. The general form of this equation is $3x + 2y + 14 = 0$.

17. Substituting $x = 2$ and $y = -4$ into the equation, we obtain $2(2) + k(-4) = -8$, so $-4k = -12$ and $k = 3$.

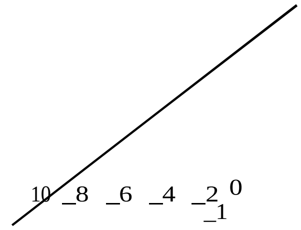
18. $f(1) = m(1) + b = 3$ and $f(3) = m(3) + b = -2$. The first equation gives $b = 3 - m$. Substituting this into the second equation gives $3m + (3 - m) = -2$, so $2m = -5$ and $m = -\frac{5}{2}$. Thus, $b = 3 - \left(-\frac{5}{2}\right) = \frac{11}{2}$.

19. Setting $x = 0$ gives $y = -6$ as the y-intercept. Setting $y = 0$ gives $x = 8$ as the x-intercept.



20. Setting $x = 0$ gives $5y = 15$, or $y = 3$. Setting $y = 0$ gives $-2x = 15$, or $x = -\frac{15}{2}$.

_6



21. In 2015 (when $x = 5$), we have $S(5) = 6000(5) + 30{,}000 = 60{,}000$.

22. Let x denote the time in years. Since the function is linear, we know that it has the form $f(x) = mx + b$.

a. The slope of the line passing through $(0, 2.4)$ and $(5, 7.4)$ is $m = \dfrac{7.4 - 2.4}{5} = 1$. Since the line passes through $(0, 2.4)$, we know that the y-intercept is 2.4. Therefore, the required function is $f(x) = x + 2.4$.

b. In 2013 (when $x = 3$), the sales were $f(3) = 3 + 2.4 = 5.4$, or \$5.4 million.

23. The slope of the line segment joining A and B is given by $m_1 = \dfrac{3 - 1}{5 - 1} = \dfrac{2}{4} = \dfrac{1}{2}$. The slope of the line segment joining B and C is $m_2 = \dfrac{5 - 3}{4 - 5} = \dfrac{2}{-1} = -2$. Since $m_1 = -1/m_2$, $\triangle ABC$ is a right triangle.

24. a. $D(t) = \dfrac{a}{150} t$. The given equation can be expressed in the form $y = mx + b$, where $m = \dfrac{a}{150}$ and $b = 0$.

b. If $a = 500$ and $t = 35$, $D(35) = \frac{500}{150}(35) = 116\frac{2}{3}$, or approximately 117 mg.

25. Let V denote the value of the building after t years.

a. The rate of depreciation is $-\dfrac{\Delta V}{\Delta t} = \dfrac{6{,}000{,}000}{30} = 200{,}000$, or \$200,000/year.

b. From part a, we know that the slope of the line is $-200{,}000$. Using the point-slope form of the equation of a line, we have $V - 0 = -200{,}000(t - 30)$, or $V = -200{,}000t + 6{,}000{,}000$. In the year 2020 (when $t = 10$), we have $V = -200{,}000(10) + 6{,}000{,}000 = 4{,}000{,}000$, or \$4,000,000.

26. Let V denote the value of the machine after t years.

a. The rate of depreciation is $-\dfrac{\Delta V}{\Delta t} = \dfrac{300{,}000 - 30{,}000}{12} = \dfrac{270{,}000}{12} = 22{,}500$, or \$22,500/year.

b. Using the point-slope form of the equation of a line with the point $(0, 300000)$ and $m = -22{,}500$, we have $V - 300{,}000 = -22{,}500(t - 0)$, or $V = -22{,}500t + 300{,}000$.

27. Let x denote the number of units produced and sold.

a. The cost function is $C(x) = 6x + 30{,}000$.

b. The revenue function is $R(x) = 10x$.

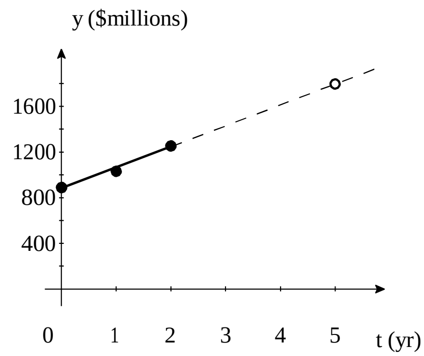
c. The profit function is $P(x) = R(x) - C(x) = 10x - (30{,}000 + 6x) = 4x - 30{,}000$.

d. $P(6000) = 4(6000) - 30{,}000 = -6{,}000$, a loss of \$6000; $P(8000) = 4(8000) - 30{,}000 = 2{,}000$, a profit of \$2000; and $P(12{,}000) = 4(12{,}000) - 30{,}000 = 18{,}000$, a profit of \$18,000.

28. a. The graph of f is the line L passing through $(0, 23.4)$ and $(4, 25.2)$. The slope of L is $\dfrac{25.2 - 23.4}{4 - 0} = 0.45$, so an equation of L is $y - 23.4 = 0.45(t - 0)$, or $y = 0.45t + 23.4$. Thus, $f(t) = 0.45t + 23.4$.

b. The percentage is $f(6) = 0.45(6) + 23.4 = 26.1$, or 26.1%.

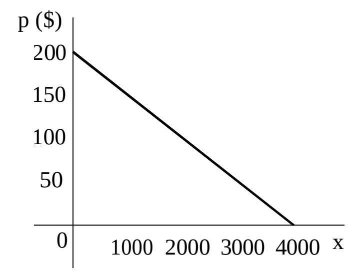
29. a, b.



c. The slope of L is $\frac{}{2 - 0} = 182$, so an equation of L is $y - 887 = 182(t - 0)$ or $y = 182t + 887$.

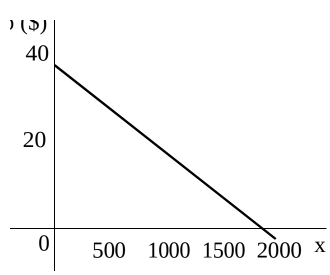
d. The amount consumers are projected to spend on Cyber Monday, 2014 ($t = 5$) is $182(5) + 887$, or $1.797 billion.

30. The slope of the demand curve is $\frac{\Delta p}{\Delta x} = -\frac{10}{200} = -0.05$. Using the point-slope form of the equation of a line with the point $(0, 200)$, we have $p - 200 = -0.05(x)$, or $p = -0.05x + 200$.



31. The slope of the supply curve is $\frac{\Delta p}{\Delta x} = \frac{100 - 50}{2000 - 200} = \frac{50}{1800} = \frac{1}{36}$. Using the point-slope form of the equation of a line with the point $(200, 50)$, we have $p - 50 = \frac{1}{36}(x - 200)$, so $p = \frac{1}{36}x - \frac{200}{36} + 50 = \frac{1}{36}x + \frac{1600}{36} = \frac{1}{36}x + \frac{400}{9}$, or $36p - x - 1600 = 0$.

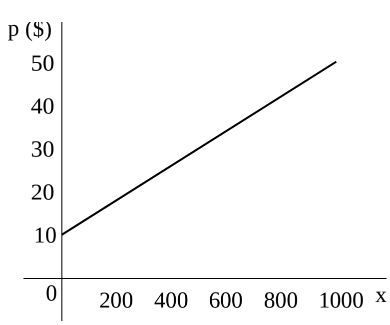
32. a.



b. The highest price is $40 per unit.

c. We solve the equation $-0.02x + 40 = 20$, obtaining $x = 1000$. Thus, the quantity demanded per week is 1000 units.

33. a.



b. The lowest price is $10 per unit.

c. We solve the equation $0.04x + 10 = 20$, obtaining $x = 250$. Thus, the supplier will make 250 headphones available per week.

34. We solve the system $3x + 4y = -6$, $2x + 5y = -11$. Solving the first equation for x, we have $3x = -4y - 6$ and $x = -\frac{4}{3}y - 2$. Substituting this value of x into the second equation yields $2\left(-\frac{4}{3}y - 2\right) + 5y = -11$, so

8 7 4

 3y 4 5y   11, 3y   7, and y   3. Thus, x   3 3 2 4 2 2, so the point of intersection is
2 3.

35. We solve the system $y = \frac{3}{4}x + 6$, $3x - 2y = -3$. Substituting the first equation into the second equation, we have $3x - 2\left(\frac{3}{4}x + 6\right) = -3$, $3x - \frac{3}{2}x - 12 = -3$, $\frac{3}{2}x = 9$, and $x = 6$. Substituting this value of x into the first equation, we have $y = \frac{3}{4}(6) + 6 = \frac{21}{2}$. Therefore, the point of intersection is $\left(6, \frac{21}{2}\right)$.

36. Setting $C(x) = R(x)$, we have $12x + 20{,}000 = 20x$, $8x = 20{,}000$, and $x = 2500$. Next, $R(2500) = 20(2500) = 50{,}000$, and we conclude that the break-even point is $(2500, 50000)$.

37. We solve the system $3x + p = 40$, $2x - p = -10$. Solving the first equation for p, we obtain $p = 40 - 3x$. Substituting this value of p into the second equation, we obtain $2x - (40 - 3x) = -10$, $5x - 40 = -10$, $5x = 30$, and $x = 6$. Next, $p = 40 - 3(6) = 40 - 18 = 22$. Thus, the equilibrium quantity is 6000 units and the equilibrium price is $22.

38. a. The slope of the line is $m = \frac{1.0 - 0.5}{4 - 2} = 0.25$. Using the point-slope form of an equation of a line, we have $y - 1 = 0.25(x - 4)$, or $y = 0.25x$.

b. $y = 0.25(6.4) = 1.6$, or 1600 applications.

39. We solve the system of equations $2x + 7p - 1760 = 0$, $3x - 56p + 2680 = 0$. Solving the first equation for x yields $x = -\frac{7}{2}p + 880$, which when substituted into the second equation gives $3\left(-\frac{7}{2}p + 880\right) - 56p + 2680 = 0$, $-\frac{21}{2}p + 2640 - 56p = -2680$, $-21p + 5280 - 112p = -5360$, $-133p = -10{,}640$, and $p = \frac{10{,}640}{133} = 80$. Substituting this value of p into the expression for x, we find $x = -\frac{7}{2}(80) + 880 = 600$. Thus, the equilibrium quantity is 600 refrigerators and the equilibrium price is $80.

40. a.

	x	y	x^2	xy
	1	87.9	1	87.9
	2	90	4	180
	3	94.2	9	282.6
	4	97.5	16	390
	5	102.6	25	513
	6	106.8	36	640.8
Sum	21	579	91	2094.3

The normal equations are $6b + 21m = 579$ and $21b + 91m = 2094.3$. The solutions are $m = 3.87$ and $b = 82.94$, so the required equation is $y = 3.87x + 82.94$.

b. The FICA wage base for the year 2012 is given by $y = 3.17(9) + 82.94 = 117.77$, or $117,770.

41. We solve the system $p = -0.02x + 40$, $p = 0.04x + 10$, obtaining $0.04x + 10 = -0.02x + 40$, $0.06x = 30$, $x = 500$, and $p = -0.02(500) + 40 = 30$. Thus, the equilibrium quantity is 500 units per week and the equilibrium price is $30 per unit.

42. a.

	x	y	x^2	xy
	0	19.5	0	0
	10	20	100	200
	20	20.6	400	412
	30	21.2	900	636
	40	21.8	1600	872
	50	22.4	2500	1120
Sum	150	125.5	5500	3240

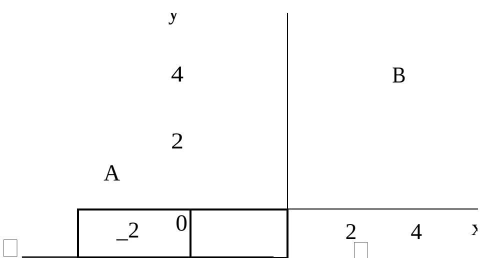
The normal equations are $6b + 150m = 125.5$ and $150b + 5500m = 3240$. Solving, we obtain $b = 19.45$, $m = 0.0586$. Therefore, $y = 0.059x + 19.5$.

b. The life expectancy at 65 of a female in 2040 is $y = 0.059(40) + 19.5 = 21.86$, or 21.9 years.

c. The life expectancy at 65 of a female in 2030 is $y = 0.059(30) + 19.5 = 21.27$, or 21.3 years. The ... gives a life expectancy of 21.2 years.

CHAPTER 1 Before Moving On... page 71

1.



$d = \sqrt{[3 - (-2)]^2 + (4 - 1)^2} = \sqrt{5^2 + 3^2} = \sqrt{34}$.

2. Solving the equation $3x - y - 4 = 0$ gives $y = 3x - 4$, and this tells us that the slope of the second line is 3. Therefore, the slope of the required line is $m = 3$. It equation is $y - 1 = 3(x - 3)$, or $y = 3x - 8$.

3. The slope of the line passing through $(1, 2)$ and $(3, 5)$ is $m_1 = \frac{5-2}{3-1} = \frac{3}{2}$. Solving $2x + 3y = 10$ gives $y = -\frac{2}{3}x + \frac{10}{3}$, and the slope of the line with this equation is $m_2 = -\frac{2}{3} = -\frac{1}{m_1}$. Thus, the two lines are perpendicular.

4. a. The unit cost is given by the coefficient of x in $C(x)$; that is, \$15.

b. The monthly fixed cost is given by the constant term of $C(x)$; that is, \$22,000.

c. The selling price is given by the coefficient of x in $R(x)$; that is, \$18.

5. Solving $2x - 3y = -2$ for x gives $x = \frac{3}{2}y - 1$. Substituting into the second equation gives $9\left(\frac{3}{2}y - 1\right) + 12y = 25$, so $\frac{27}{2}y - 9 + 12y = 25$, $27y - 18 + 24y = 50$, $51y = 68$, and $y = \frac{68}{51} = \frac{4}{3}$. Therefore, $x = \frac{3}{2}\left(\frac{4}{3}\right) - 1 = 1$, and so the point of intersection is $\left(1, \frac{4}{3}\right)$.

6. We solve the equation $S_1 = S_2$: $4.2 + 0.4t = 2.2 + 0.8t$, so $2 = 0.4t$ and $t = \frac{2}{0.4} = 5$. So Lowe's sales will surpass Best's in 5 years.

CHAPTER 1 Explore & Discuss

Page 4

1. Let $P_1 = (2, 6)$ and $P_2 = (-4, 3)$. Then we have $x_1 = 2$, $y_1 = 6$, $x_2 = -4$, and $y_2 = 3$. Using Formula (1), we have $d = \sqrt{[(-4) - 2]^2 + (3 - 6)^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$, as obtained in Example 1.

2. Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be any two points in the plane. Then the result follows from the equality $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Page 6

1. a. All points on and inside the circle with center (h, k) and radius r.

b. All points inside the circle with center (h, k) and radius r.

c. All points on and outside the circle with center (h, k) and radius r.

d. All points outside the circle with center (h, k) and radius r.

2. a. $y^2 = 4 - x^2$, and so $y = \pm\sqrt{4 - x^2}$.

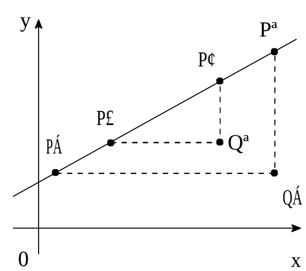
b. i. The upper semicircle with center at the origin and radius 2.

ii. The lower semicircle with center at the origin and radius 2.

Page 12

Refer to the accompanying figure. Observe that triangles $\triangle P_1 Q_1 P_2$ and $\triangle P_3 Q_2 P_4$ are similar. From this we conclude that

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_4 - y_3}{x_4 - x_3}$. Because P_3 and P_4 are arbitrary, the conclusion follows.



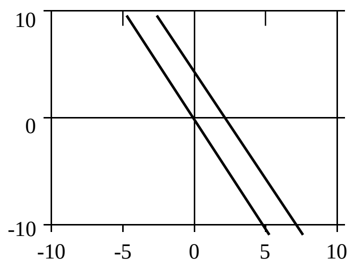
Page 17

In Example 11, we are told that the object is expected to appreciate in value at a given rate for the next five years, and the equation obtained in that example is based on this fact. Thus, the equation may not be used to predict the value of the object much beyond five years from the date of purchase.

CHAPTER 1 Exploring with Technology

Page 15

1.

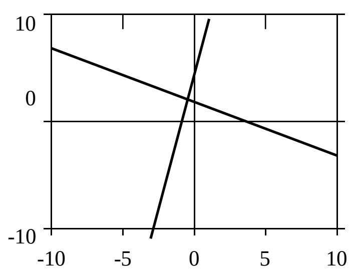


The straight lines L_1 and L_2 are shown in the figure.

a. L_1 and L_2 seem to be parallel.

b. Writing each equation in the slope-intercept form gives $y = -2x + 5$ and $y = -\frac{41}{20}x + \frac{11}{20}$, from which we see that the slopes of L_1 and L_2 are -2 and $-\frac{41}{20} = -2.05$, respectively. This shows that L_1 and L_2 are not parallel.

2.



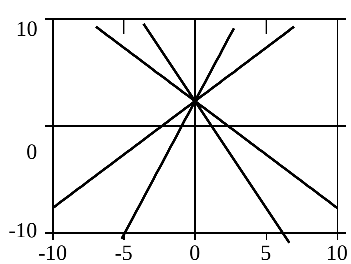
The straight lines L_1 and L_2 are shown in the figure.

a. L_1 and L_2 seem to be perpendicular.

b. The slopes of L_1 and L_2 are $m_1 = -\frac{1}{2}$ and $m_2 = 5$, respectively. Because $m_1 = -\dfrac{1}{2} \neq -\dfrac{1}{5} = -\dfrac{1}{m_2}$, we see that L_1 and L_2 are not perpendicular.

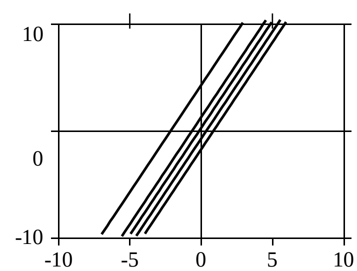
Page 16

1.



The straight lines with the given equations are shown in the figure. Changing the value of m in the equation $y = mx + b$ changes the slope of the line and thus rotates it.

2.

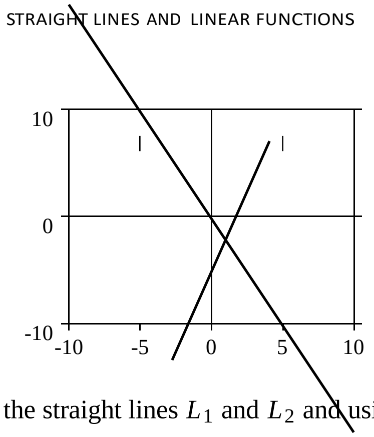


The straight lines of interest are shown in the figure. Changing the value of b in the equation $y = mx + b$ changes the y-intercept of the line and thus translates it (upward if $b > 0$ and downward if $b < 0$).

3. Changing both m and b in the equation $y = mx + b$ both rotates and translates the line.

Page 44

1.



Plotting the straight lines L_1 and L_2 and using TRACE and ZOOM repeatedly, you will see that the iterations approach the answer □1□ 1□. Using the intersection feature of the graphing utility gives the result x □ 1 and y □ 1 immediately.

2. Substituting the first equation into the second yields 3x □ 2 □ □2x □ 3, so 5x □ 5 and x □ 1. Substituting this value of x into either equation gives y □ 1.

3. The iterations obtained using TRACE and ZOOM converge to the solution □1□ 1□. The use of the intersection feature is clearly superior to the first method. The algebraic method also yields the desired result easily.

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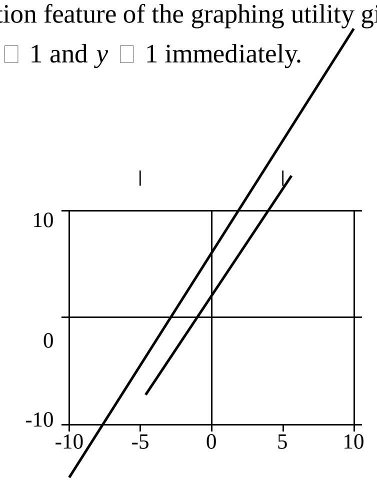
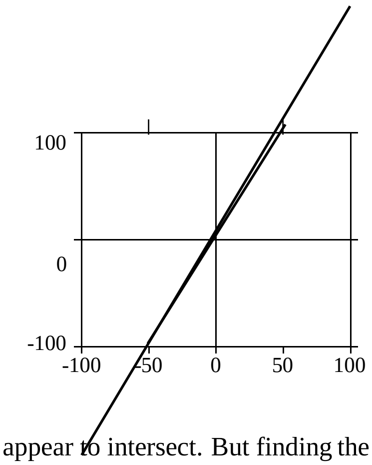
1.



The lines seem to be parallel to each other and do not appear to intersect.

2.



They appear to intersect. But finding the point of intersection using TRACE and ZOOM with any degree of accuracy seems to be an impossible task. Using the intersection feature of the graphing utility yields the point of intersection □□40□ □81□ immediately.

3. Substituting the first equation into the second gives 2x □ 1 □ 2□1x □ 3, □4 □ 0□1x, and thus x □ □40. The corresponding y-value is □81.

4. Using TRACE and ZOOM is not effective. The intersection feature gives the desired result immediately. The algebraic method also yields the answer with little effort and without the use of a graphing utility.

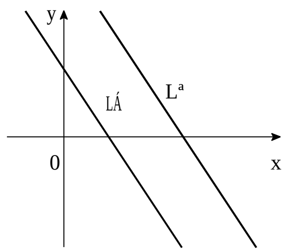
2 SYSTEMS OF LINEAR EQUATIONS AND MATRICES

2.1 Systems of Linear Equations: An Introduction

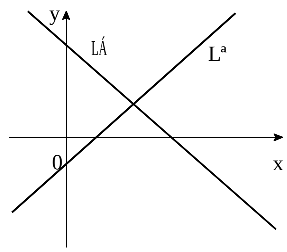
Concept Questions page 79

1. **a.** There may be no solution, a unique solution, or infinitely many solutions.

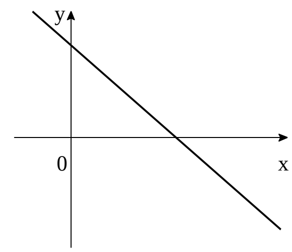
   b. There is no solution if the two lines represented by the given system of linear equations are parallel and distinct; there is a unique solution if the two lines intersect at precisely one point; there are infinitely many solutions if the two lines are parallel and coincident.



No solution



A unique solution

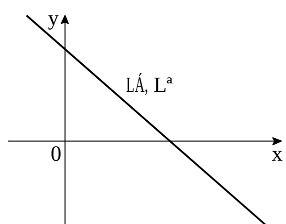


Infinitely many solutions

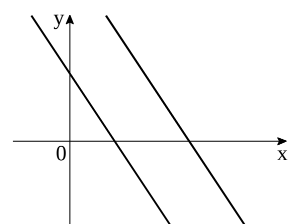
2. **a.** **i.** The system is dependent if the two equations in the system describe the same line.

   ii. The system is inconsistent if the two equations in the system describe two lines that are parallel and distinct.

   b.



Two (coincident) lines in a dependent system



Two lines in an inconsistent system

Exercises page 79

1. Solving the first equation for x, we find $x = 3y - 1$. Substituting this value of x into the second equation yields $4(3y - 1) + 3y = 11$, so $12y - 4 + 3y = 11$ and $y = 1$. Substituting this value of y into the first equation gives $x = 3(1) - 1 = 2$. Therefore, the unique solution of the system is $(2, 1)$.

2. Solving the first equation for x, we have $2x - 4y = -10$, so $x = 2y - 5$. Substituting this value of x into the second equation, we have $3(2y - 5) + 2y = 1$, $6y - 15 + 2y = 1$, $8y = 16$, and $y = 2$. Then $x = 2(2) - 5 = -1$. Therefore, the solution is $(-1, 2)$.