Solution Manual for Mathematical Proofs A Transition to Advanced Mathematics 3rd Edition Chartrand Polimeni Zhang 0321797094 9780321797094

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Exercises for Chapter 2

Exercises for Section 2.1: Statements

2.1 (a) A false statement.

(b) A true statement.

(c) Not a statement.

(d) Not a statement (an open sentence).

(e) Not a statement.

(f) Not a statement (an open sentence).

(g) Not a statement.

2.2 (a) A true statement since $A = \{3n - 2 : n \in N\}$ and so $3 \cdot 9 - 2 = 25 \in A$.

(b) A false statement. Starting with the 3rd term in D, each element is the sum of the two preceding terms. Therefore, all terms following 21 exceed 33 and so $33 \notin D$.

(c) A false statement since $3 \cdot 8 - 2 = 22 \in A$.

(d) A true statement since every prime except 2 is odd.

(e) A false statement since B and D consist only of integers.

(f) A false statement since 53 is prime.

2.3 (a) False. $\emptyset$ has no elements.

(b) True.

(c) True.

(d) False. $\{\emptyset\}$ has $\emptyset$ as its only element.

(e) True.

(f) False. 1 is not a set.

2.4 (a) $x = -2$ and $x = 3$.

(b) All $x \in \mathbb{R}$ such that $x = -2$ and $x = 3$.

2.5 (a) $\{x \in Z : x > 2\}$

(b) $\{x \in Z : x \leq 2\}$

2.6 (a) A can be any of the sets $\emptyset, \{1\}, \{2\}, \{1, 2\}$, that is, A is any subset of $\{1, 2, 4\}$ that does not contain 4.

(b) A can be any of the sets $\{1, 4\}, \{2, 4\}, \{1, 2, 4\}, \{4\}$, that is, A is any subset of $\{1, 2, 4\}$ that contains 4.

(c) $A = \emptyset$ and $A = \{4\}$.

2.7 3, 5, 11, 17, 41, 59.

2.8 (a) $S_1 = \{1, 2, 5\}$ (b) $S_2 = \{0, 3, 4\}$.

2.9 $P(n)$: $\frac{n-1}{2}$ is even. $P(n)$ is true only for $n = 5$ and $n = 9$.

2.10 $P(n)$: $\frac{n}{2}$ is odd. $Q(n)$: $\frac{n^2-2n}{8}$ is even. or $Q(n)$: $n^2 + 9$ is a prime.

Exercises for Section 2.2: The Negation of a Statement

2.11 (a) $\sqrt{2}$ is not a rational number.

(b) 0 is a negative integer.

(c) 111 is not a prime number.

2.12 See Figure 1.

P	Q	∼ P	∼ Q
T	T	F	F
T	F	F	T
F	T	T	F
F	F	T	T

Figure 1: Answer for Exercise 2.12

2.13 (a) The real number r is greater than $\sqrt{2}$.

(b) The absolute value of the real number a is at least 3.

(c) At most one angle of the triangle is 45^o.

(d) The area of the circle is less than 9π.

(e) The sides of the triangle have different lengths.

(f) The point P lies on or within the circle C.

2.14 (a) At most one of my library books is overdue.

(b) My two friends did not misplace their homework assignments.

(c) Someone expected this to happen.

(d) My instructor often teaches that course.

(e) It's not surprising that two students received the same exam score.

Exercises for Section 2.3: The Disjunction and Conjunction of Statements

2.15 See Figure 2.

2.16 (a) True. (b) False. (c) False. (d) True. (e) True.

2.17 (a) $P \vee Q$: 15 is odd or 21 is prime. (True)

P	Q	$\sim Q$	$P \wedge (\sim Q)$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

Figure 2: Answer for Exercise 2.15

(b) $P \wedge Q$: 15 is odd and 21 is prime. (False)

(c) $(\sim P) \vee Q$: 15 is not odd or 21 is prime. (False)

(d) $P \wedge (\sim Q)$: 15 is odd and 21 is not prime. (True)

2.18 (a) All nonempty subsets of $\{1, 3, 5\}$.

(b) All subsets of $\{1, 3, 5\}$.

(c) There are no subsets A of S for which $(\sim P(A)) \wedge (\sim Q(A))$ is true.

Exercises for Section 2.4: The Implication

2.19 (a) $\sim P$: 17 is not even (or 17 is odd). (True)

(b) $P \vee Q$: 17 is even or 19 is prime. (True)

(c) $P \wedge Q$: 17 is even and 19 is prime. (False)

(d) $P \Rightarrow Q$: If 17 is even, then 19 is prime. (True)

2.20 See Figure 3.

P	Q	$\sim P$	$P \Rightarrow Q$	$(P \Rightarrow Q) \Rightarrow (\sim P)$
T	T	F	T	F
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Figure 3: Answer for Exercise 2.20

2.21 (a) $P \Rightarrow Q$: If $\sqrt{2}$ is rational, then 22/7 is rational. (True)

(b) $Q \Rightarrow P$: If 22/7 is rational, then $\sqrt{2}$ is rational. (False)

(c) $(\sim P) \Rightarrow (\sim Q)$: If $\sqrt{2}$ is not rational, then 22/7 is not rational. (False)

(d) $(\sim Q) \Rightarrow (\sim P)$: If 22/7 is not rational, then $\sqrt{2}$ is not rational. (True)

2.22 (a) $(P \wedge Q) \Rightarrow R$: If $\sqrt{2}$ is rational and $\frac{2}{3}$ is rational, then $\sqrt{3}$ is rational. (True)

(b) $(P \wedge Q) \Rightarrow (\sim R)$: If $\sqrt{\ }$ is rational and 2 is rational, then $\sqrt{\ }$ is not rational. (True)

 16

(c) $((\sim P) \wedge Q) \Rightarrow R$: If $\frac{\sqrt{2}}{2}$ is not rational and $\frac{2}{3}$ is rational, then $\frac{\sqrt{3}}{3}$ is rational. (False)

(d) $(P \vee Q) \Rightarrow (\sim R)$: If $\frac{\sqrt{2}}{2}$ is rational or $\frac{2}{3}$ is rational, then $\frac{\sqrt{3}}{3}$ is not rational. (True)

2.23 (a), (c), (d) are true.

2.24 (b), (d), (f) are true.

2.25 (a) true. (b) false. (c) true. (d) true. (e) true.

2.26 (a) false. (b) true. (c) true. (d) false.

2.27 Cindy and Don attended the talk.

2.28 (b), (d), (f), (g) are true.

2.29 Only (c) implies that $P \vee Q$ is false.

Exercises for Section 2.5: More On Implications

2.30 (a) $P(n) \Rightarrow Q(n)$: If $5n + 3$ is prime, then $7n + 1$ is prime.

(b) $P(2) \Rightarrow Q(2)$: If 13 is prime, then 15 is prime. (False)

(c) $P(6) \Rightarrow Q(6)$: If 33 is prime, then 43 is prime. (True)

2.31 (a) $P(x) \Rightarrow Q(x)$: If $|x| = 4$, then $x = 4$.

$P(-4) \Rightarrow Q(-4)$ is false.

$P(-3) \Rightarrow Q(-3)$ is true.

$P(1) \Rightarrow Q(1)$ is true.

$P(4) \Rightarrow Q(4)$ is true.

$P(5) \Rightarrow Q(5)$ is true.

(b) $P(x) \Rightarrow Q(x)$: If $x^2 = 16$, then $|x| = 4$. True for all $x \in S$.

(c) $P(x) \Rightarrow Q(x)$: If $x > 3$, then $4x - 1 > 12$. True for all $x \in S$.

2.32 (a) All $x \in S$ for which $x = 7$.

(b) All $x \in S$ for which $x > -1$.

(c) All $x \in S$.

(d) All $x \in S$.

2.33 (a) True for $(x, y) = (3, 4)$ and $(x, y) = (5, 5)$ and false for $(x, y) = (1, -1)$.

(b) True for $(x, y) = (1, 2)$ and $(x, y) = (6, 6)$ and false for $(x, y) = (2, -2)$.

(c) True for $(x, y) \in \{(1, -1), (-3, 4), (1, 0)\}$ and false for $(x, y) = (0, -1)$.

2.34 (a) If the x-coordinate of a point on the straight line with equation $2y + x - 3 = 0$ is an integer, then its y-coordinate is also an integer. Or: If $-2n + 3 \in Z$, then $n \in Z$.

(b) If n is an odd integer, then n^2 is an odd integer.

(c) Let $n \in Z$. If $3n + 7$ is even, the n is odd.

(d) If $f(x) = \cos x$, then $f'(x) = -\sin x$.

(e) If a circle has circumference 4π, then its area is also 4π.

(f) Let $n \in Z$. If n^3 is even, then n is even.

Exercises for Section 2.6: The Biconditional

2.35 $P \Leftrightarrow Q$: 18 is odd if and only if 25 is even. (True)

2.36 The integer x is odd if and only if x^2 is odd.

That the integer x is odd is a necessary and sufficient condition for x^2 to be odd

2.37 The inequality $|x-3| < 1$ is satisfied if and only if $x \in (2, 4)$.

That $|x-3| < 1$ is necessary and sufficient for $x \in (2, 4)$.

2.38 (a) $\sim P(x)$: $x = -2$. True if $x = 0, 2$.

(b) $P(x) \vee Q(x)$: $x = -2$ or $x^2 = 4$. True if $x = -2, 2$.

(c) $P(x) \wedge Q(x)$: $x = -2$ and $x^2 = 4$. True if $x = -2$.

(d) $P(x) \Rightarrow Q(x)$: If $x = -2$, then $x^2 = 4$. True for all x.

(e) $Q(x) \Rightarrow P(x)$: If $x^2 = 4$, then $x = -2$ True if $x = 0, -2$.

(f) $P(x) \Leftrightarrow Q(x)$: $x = -2$ if and only if $x^2 = 4$. True if $x = 0, -2$.

2.39 (a) True for all $x \in S - \{-4\}$.

(b) True for $x \in S - \{3\}$.

(c) True for $x \in S - \{-4, 0\}$.

2.40 (a) True for $(x, y) \in \{(3, 4), (5, 5)\}$.

(b) True for $(x, y) \in \{(1, 2), (6, 6)\}$.

(c) True for $(x, y) \in \{(1, -1), (1, 0)\}$.

2.41 True if $n = 3$.

2.42 True if $n = 3$.

2.43 $P(1) \Rightarrow Q(1)$ is false (since $P(1)$ is true and $Q(1)$ is false).

$Q(3) \Rightarrow P(3)$ is false (since $Q(3)$ is true and $P(3)$ is false).

$P(2) \Leftrightarrow Q(2)$ is true (since $P(2)$ and $Q(2)$ are both true).

2.44 (i) $P(1) \Rightarrow Q(1)$ is false;

(ii) $Q(4) \Rightarrow P(4)$ is true;

(iii) $P(2) \Leftrightarrow R(2)$ is true;

(iv) $Q(3) \Leftrightarrow R(3)$ is false.

2.45 True for all $n \in S - \{11\}$.

Exercises for Section 2.7: Tautologies and Contradictions

2.46 The compound statement $P \Rightarrow (P \vee Q)$ is a tautology since it is true for all combinations of truth values for the component statements P and Q. See the truth table below.

P	Q	$P \vee Q$	$P \Rightarrow (P \vee Q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

2.47 The compound statement $(P \wedge (\sim Q)) \wedge (P \wedge Q)$ is a contradiction since it is false for all combinations of truth values for the component statements P and Q. See the truth table below.

P	Q	$\sim Q$	$P \wedge Q$	$P \wedge (\sim Q)$	$(P \wedge (\sim Q)) \wedge (P \wedge Q)$
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	F	F	F
F	F	T	F	F	F

2.48 The compound statement $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$ is a tautology since it is true for all combinations of truth values for the component statements P and Q. See the truth table below.

P	Q	$P \Rightarrow Q$	$P \wedge (P \Rightarrow Q)$	$(P \wedge (P \Rightarrow Q)) \Rightarrow Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$(P \wedge (P \Rightarrow Q)) \Rightarrow Q$: If P and P implies Q, then Q.

2.49 The compound statement $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$ is a tautology since it is true for all combinations of truth values for the component statements P, Q and R. See the truth table below.

P	Q	R	$P \Rightarrow Q$	$Q \Rightarrow R$	$(P \Rightarrow Q) \wedge (Q \Rightarrow R)$	$P \Rightarrow R$	$((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$
T	T	T	T	T	T	T	T
T	F	T	F	T	F	T	T
F	T	T	T	T	T	T	T
F	F	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	F	F	T	F	F	T
F	T	F	T	F	F	T	T
F	F	F	T	T	T	T	T

$((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$: If P implies Q and Q implies R, then P implies R.

2.50 (a) $R \vee S$ is a tautology. (b) $R \wedge S$ is a contradiction.

(c) $R \Rightarrow S$ is a contradiction. (d) $S \Rightarrow R$ is a tautology.

17

Exercises for Section 2.8: Logical Equivalence

2.51 (a) See the truth table below.

P	Q	$\sim P$	$\sim Q$	$P \Rightarrow Q$	$(\sim P) \Rightarrow (\sim Q)$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Since $P \Rightarrow Q$ and $(\sim P) \Rightarrow (\sim Q)$ do not have the same truth values for all combinations of truth values for the component statements P and Q, the compound statements $P \Rightarrow Q$ and $(\sim P) \Rightarrow (\sim Q)$ are not logically equivalent. Note that the last two columns in the truth table are not the same.

(b) The implication $Q \Rightarrow P$ is logically equivalent to $(\sim P) \Rightarrow (\sim Q)$.

2.52 (a) See the truth table below.

P	Q	$\sim P$	$\sim Q$	$P \vee Q$	$\sim (P \vee Q)$	$(\sim P) \vee (\sim Q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

Since $\sim (P \vee Q)$ and $(\sim P) \vee (\sim Q)$ do not have the same truth values for all combinations of truth values for the component statements P and Q, the compound statements $\sim (P \vee Q)$ and $(\sim P) \vee (\sim Q)$ are not logically equivalent.

(b) The biconditional $\sim (P \vee Q) \Leftrightarrow ((\sim P) \vee (\sim Q))$ is not a tautology as there are instances when this biconditional is false.

2.53 (a) The statements $P \Rightarrow Q$ and $(P \wedge Q) \Leftrightarrow P$ are logically equivalent since they have the same truth values for all combinations of truth values for the component statements P and Q. See the truth table.

P	Q	$P \Rightarrow Q$	$P \wedge Q$	$(P \wedge Q) \Leftrightarrow P$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

(b) The statements $P \Rightarrow (Q \vee R)$ and $(\sim Q) \Rightarrow ((\sim P) \vee R)$ are logically equivalent since they have the same truth values for all combinations of truth values for the component statements P, Q and R. See the truth table.

P	Q	R	$\sim P$	$\sim Q$	$Q \vee R$	$P \Rightarrow (Q \vee R)$	$(\sim P) \vee R$	$(\sim Q) \Rightarrow ((\sim P) \vee R)$
T	T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T	T
F	T	T	T	F	T	T	T	T
F	F	T	T	T	T	T	T	T
T	T	F	F	F	T	T	F	T
T	F	F	F	T	F	F	F	F
F	T	F	T	F	T	T	T	T
F	F	F	T	T	F	T	T	T

2.54 The statements Q and $(\sim Q) \Rightarrow (P \wedge (\sim P))$ are logically equivalent since they have the same truth values for all combinations of truth values for the component statements P and Q. See the truth table below.

20

P	Q	$\sim$ P	$\sim$ Q	P $\wedge$ ($\sim$ P)	($\sim$ Q) $\Rightarrow$ (P $\wedge$ ($\sim$ P))
T	T	F	F	F	T
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	F	F

2.55 The statements (P $\vee$ Q) $\Rightarrow$ R and (P $\Rightarrow$ R) $\wedge$ (Q $\Rightarrow$ R) are logically equivalent since they have the same truth values for all combinations of truth values for the component statements P , Q and R. See the truth table.

P	Q	R	P $\vee$ Q	(P $\vee$ Q) $\Rightarrow$ R	P $\Rightarrow$ R	Q $\Rightarrow$ R	(P $\Rightarrow$ R) $\wedge$ (Q $\Rightarrow$ R)
T	T	T	T	T	T	T	T
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	F	T	T	T	T
T	T	F	T	F	F	F	F
T	F	F	T	F	F	T	F
F	T	F	T	F	T	F	F
F	F	F	F	T	T	T	T

2.56 If S and T are not logically equivalent, there is some combination of truth values of the component statements P, Q and R for which S and T have different truth values.

2.57 Since there are only four different combinations of truth values of P and Q for the second and third rows of the statements S_1, S_2, S_3, S_4 and S_5, at least two of these must have identical truth tables and so are logically equivalent.

Exercises for Section 2.9: Some Fundamental Properties of Logical Equivalence

2.58 (a) The statement P $\vee$ (Q $\wedge$ R) is logically equivalent to (P $\vee$ Q) $\wedge$ (P $\vee$ R) since the last two columns in the truth table in Figure 4 are the same.

P	Q	R	P $\vee$ Q	P $\vee$ R	Q $\wedge$ R	P $\vee$ (Q $\wedge$ R)	(P $\vee$ Q) $\wedge$ (P $\vee$ R)
T	T	T	T	T	T	T	T
T	F	T	T	T	F	T	T
F	T	T	T	T	T	T	T
F	F	T	F	T	F	F	F
T	T	F	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	F	T	F	F	F	F
F	F	F	F	F	F	F	F

Figure 4: Answer for Exercise 2.58(a)

(b) The statement $\sim$ (P $\vee$ Q) is logically equivalent to ($\sim$ P) $\wedge$ ($\sim$ Q) since the last two columns in the truth table in Figure 5 are the same.

21

2.59 (a) Both $x = 0$ and $y = 0$.

(b) Either the integer a is odd or the integer b is odd.

P	Q	$\sim P$	$\sim Q$	$P \vee Q$	$\sim (P \vee Q)$	$(\sim P) \wedge (\sim Q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Figure 5: Answer for Exercise 2.58(b)

2.60 (a) x and y are even only if xy is even.

(b) If xy is even, then x and y are even.

(c) Either at least one of x and y is odd or xy is even.

(d) x and y are even and xy is odd.

2.61 Either $x^2 = 2$ and $x = \sqrt{2}$ or $x = \sqrt{2}$ and $x^2 = 2$.

2.62 The statement $[(P \vee Q) \wedge \sim (P \wedge Q)]$ is logically equivalent to $\sim (P \Leftrightarrow Q)$ since the last two columns in the truth table below are the same.

P	Q	$P \vee Q$	$P \wedge Q$	$\sim (P \wedge Q)$	$P \Leftrightarrow Q$	$(P \vee Q) \wedge \sim (P \wedge Q)$	$\sim (P \Leftrightarrow Q)$
T	T	T	T	F	T	F	F
T	F	T	F	T	F	T	T
F	T	T	F	T	F	T	T
F	F	F	F	T	T	F	F

2.63 If $3n + 4$ is odd, then $5n - 6$ is odd.

2.64 n^3 is odd if and only if $7n + 2$ is even.

Exercises for Section 2.10: Quantified Statements

2.65 $\forall x \in S, P(x)$: For every odd integer x, the integer $x^2 + 1$ is even.

$\exists x \in S, Q(x)$: There exists an odd integer x such that x^2 is even.

2.66 Let $R(x)$: $x^2 + x + 1$ is even. and let $S = \{x \in Z : x \text{ is odd}\}$.

$\forall x \in S, R(x)$: For every odd integer x, the integer $x^2 + x + 1$ is even.

$\exists x \in S, R(x)$: There exists an odd integer x such that $x^2 + x + 1$ is even.

2.67 (a) There exists a set A such that $A \cap \overline{A} = \emptyset$.

(b) For every set A, we have $\overline{A}$ 6⊆ A.

2.68 (a) There exists a rational number r such that $1/r$ is not rational.

(b) For every rational number r, $r^2 = 2$.

2.69 (a) False, since $P(1)$ is false.

(b) True, for example, $P(3)$ is true.

23

2.70 (a) T (b) T (c) F (d) T (e) T (f) F (g) T (h) F

2.71 (a) $\exists a, b \in Z,\ ab < 0$ and $a + b > 0$.

(b) $\forall x, y \in R,\ x = y$ implies that $x^2 + y^2 > 0$.

(c) For all integers a and b, either $ab \geq 0$ or $a + b \leq 0$.

There exist real numbers x and y such that $x = y$ and $x^2 + y^2 \leq 0$.

(d) $\forall a, b \in Z,\ ab \geq 0$ or $a + b \leq 0$.

$\exists x, y \in R,\ x = y$ and $x^2 + y^2 \leq 0$.

2.72 (d) implies that $(\sim P(x)) \Rightarrow Q(x)$ is false for some $x \in S$ (in fact, for all $x \in S$).

2.73 (b) and (c) imply that $P(x) \Rightarrow Q(x)$ is true for all $x \in T$.

2.74 (a) For all real numbers x, y and z, $(x-1)^2 + (y-2)^2 + (z-2)^2 > 0$.

(b) False, since $P(1, 2, 2)$ is false.

(c) $\exists x, y, z \in R,\ (x-1)^2 + (y-2)^2 + (z-2)^2 \leq 0$. ($\exists x, y, z \in R, \sim P(x, y, z)$.)

(d) There exist real numbers x, y and z such that $(x-1)^2 + (y-2)^2 + (z-2)^2 \leq 0$.

(e) True, since $(1-1)^2 + (2-2)^2 + (2-2)^2 = 0$.

2.75 Let $S = \{3, 5, 11\}$ and $P(s, t) : st - 2$ is prime.

(a) $\forall s, t \in S,\ P(s, t)$.

(b) False since $P(11, 11)$ is false.

(c) $\exists s, t \in S, \sim P(s, t)$.

(d) There exist $s, t \in S$ such that $st - 2$ is not prime.

(e) True since the statement in (a) is false.

2.76 (a) For every circle C_1 with center $(0, 0)$, there exists a circle C_2 with center $(1, 1)$ such that C_1 and C_2 have exactly two points in common.

(b) $\exists C_1 \in A, \forall C_2 \in B, \sim P(C_1, C_2)$.

(c) There exists a circle C_1 with center $(0, 0)$ such that for every circle C_2 with center $(1, 1)$, C_1 and C_2 do not have exactly two points in common.

2.77 (a) There exists a triangle T_1 such that for every triangle T_2, $r(T_2) \geq r(T_1)$.

(b) $\forall T_1 \in A, \exists T_2 \in B, \sim P(T_1, T_2)$.

(c) For every triangle T_1, there exists a triangle T_2 such that $r(T_2) < r(T_1)$.

2.78 (a) For every $a \in A$, there exists $b \in B$ such that $a/b < 1$.

(b) For $a = 2$, let $b = 4$. Then $a/b = 1/2 < 1$.

For $a = 3$, let $b = 4$. Then $a/b = 3/4 < 1$.

For $a = 5$, let $b = 6$. Then $a/b = 5/6 < 1$.

21

2.79 (a) There exists $b \in B$ such that for every $a \in A$, $a - b < 0$.

 22

(b) Let $b = 10$. Then $3 - 10 = -7 < 0$, $5 - 10 = -5 < 0$ and $8 - 10 = -2 < 0$.

Exercises for Section 2.11: Characterizations of Statements

2.80 (a) Two lines in the plane are defined to be perpendicular if they intersect at right angles.

Two lines in the plane are perpendicular if and only if (1) one line is vertical and the other is horizontal or (2) the product of the slopes of the two lines is -1.

(b) A rational number is a real number that can be expressed as a/b, where $a, b \in \mathbf{Z}$ and $b = 0$.

A real number is rational if and only if it has a repeating decimal expansion.

2.81 An integer n is odd if and only if n^2 is odd.

2.82 Only (f) is a characterization; (a), (c) and (e) are implications only; (b) is a definition; and (d) is false.

2.83 (a)-(d) are characterizations. (d) is the Pythagorean theorem. (e) is not a characterization. (Every positive number is the area of some rectangle.)

2.84 (a) and (b) are characterizations.

Additional Exercises for Chapter 2

2.85 See the truth table below.

P	Q	$\sim P$	$Q \Rightarrow (\sim P)$	$P \wedge (Q \Rightarrow (\sim P))$
T	T	F	F	F
T	F	F	T	T
F	T	T	T	F
F	F	T	T	F

2.86 Statements R and P are both true.

2.87 $P \vee (\sim Q)$

2.88 (a) T (b) T (c) F (d) F (e) T (f) F

2.89 (a) (1) A function f is differentiable only if f is continuous.

(2) That a function f is differentiable is sufficient for f to be continuous.

(b) (1) The number $x = -5$ only if $x^2 = 25$.

(2) That $x = -5$ is sufficient for $x^2 = 25$.

2.90 (a) For $S = \{1, 2, 3, 4\}$, $\forall n \in S, P(n)$ is true, $\exists n \in S, \sim P(n)$ is false

(b) For $S = \{1, 2, 3, 4, 5\}$, $\forall n \in S, P(n)$ is false, $\exists n \in S, \sim P(n)$ is true.

(c) The truth value of $\forall n \in S, P(n)$ (or $\exists n \in S, \sim P(n)$) depends on the domain S as well as the open sentence $P(n)$.

 24

2.91 (a) See the truth table below. (b) can be similarly verified.

P	Q	R	$\sim Q$	$\sim R$	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$	$P \wedge (\sim R)$	$(P \wedge (\sim R)) \Rightarrow (\sim Q)$
T	T	T	F	F	T	T	F	T
T	F	T	T	F	F	T	F	T
F	T	T	F	F	F	T	F	T
F	F	T	T	F	F	T	F	T
T	T	F	F	T	T	F	T	F
T	F	F	T	T	F	T	T	T
F	T	F	F	T	F	T	F	T
F	F	F	T	T	F	T	F	T

2.92 If n is a prime and n is even, then $n \leq 2$.

If $n > 2$ and n is even, then n is not a prime.

2.93 If m is even and $m + n$ is even, then n is even.

If n is odd and $m + n$ is even, then m is odd.

2.94 If $f'(x) = 3x^2 - 2x$ and $f(x) = x^3 - x^2 + 4$, then $f(0) = 4$.

If $f(0) = 4$ and $f(x) = x^3 - x^2 + 4$, then $f'(x) = 3x^2 - 2x$.

2.95 Consider the open sentences

$$P(n) : \frac{n^2+3n}{2} \text{ is odd.} \qquad Q(n) : (n-2)^2 > 0. \qquad R(n) : (n+1)^{n-1} \text{ is odd.}$$

The statement $P(n)$ is true for $n = 2, 3$; $Q(n)$ is true for $n = 1, 3$; and $R(n)$ is true for $n = 1, 2$. Thus the implications $P(1) \Rightarrow Q(1)$, $Q(2) \Rightarrow R(2)$ and $R(3) \Rightarrow P(3)$ are true and their respective converses are false.

2.96 No. Since $Q(a) \Rightarrow P(a)$, $R(b) \Rightarrow Q(b)$ and $P(c) \Rightarrow R(c)$ are false, it follows that

$$P(a),\ Q(b) \text{ and } R(c) \text{ are false and } Q(a),\ R(b) \text{ and } P(c) \text{ are true.}$$

At least two of the three elements a, b and c are the same. If $a = b$, then $Q(a)$ and $Q(b)$ are both true and false. This is impossible for a statement. If $a = c$, then $P(c)$ and $P(a)$ are both true and false, again impossible. If $b = c$, then $R(b)$ and $R(c)$ are both true and false, which is impossible.

2.97 Observe that

(1) $P(x)$ is true for $x = 1, 3, 5$ and false for $x = 2, 4, 6$, (2)

$Q(y)$ is true for $y = 2, 4, 6$ and false for $y = 1, 3, 5, 7$, (3)

$P(x) \Rightarrow Q(y)$ is false if $P(x)$ is true and $Q(y)$ is false.

Thus $|S| = 3 \cdot 4 = 12$.

2.98 (a) For every $x \in A$ and $y \in B$, there exists $z \in C$ such that $P(x, y, z)$.

(b) For every $x \in A$ and $y \in B$, there exists $z \in C$ such that $x = yz$.

(c) For $x = 4$ and $y = 2$, let $z = 2$. Then $x = yz$.
For $x = 4$ and $y = 4$, let $z = 1$. Then $x = yz$.
For $x = 8$ and $y = 2$, let $z = 4$. Then $x = yz$.
For $x = 8$ and $y = 4$, let $z = 2$. Then $x = yz$.
Therefore, the quantified statement in (b) is true.

2.99 (a) $\exists x \in A, \exists y \in B, \forall z \in C, \sim P(x, y, z)$.

(b) There exist $x \in A$ and $y \in B$ such that for all $z \in C$, $\sim P(x, y, z)$.

(c) For $x = 1$ and $y = 3$, let $z = 2$. Then $x + z = y$.
For $x = 1$ and $y = 5$, let $z = 4$. Then $x + z = y$.
For $x = 1$ and $y = 7$, let $z = 6$. Then $x + z = y$.
For $x = 3$ and $y = 3$, let $z = 0$. Then $x + z = y$.
For $x = 3$ and $y = 5$, let $z = 2$. Then $x + z = y$.
For $x = 3$ and $y = 7$, let $z = 4$. Then $x + z = y$.
Therefore, there is no pair x, y of elements with $x \in A$ and $y \in B$ such that $x + z = y$ for every $z \in C$. Thus the statement in (b) is false.

2.100 (a) If a triangle has two equal angles, then it is isosceles.

(b) If a circle C has diameter $\sqrt{2/\pi}$, then the area of C is $1/2$.

(c) If n is an odd integer, then n^4 is odd.

(d) If the slope of a line ℓ is 2, then the equation of ℓ is $y = 2x + b$ for some $b \in \mathbf{R}$.

(e) If a and b are nonzero rational numbers, then a/b is a nonzero rational number.

(f) If a, b and c are three integers, then at least one of $a + b, a + c$ and $b + c$ is even.

(g) If the sum of two of the angles of a triangle T is 90°, then T is a right triangle.

(h) If $r = \sqrt{3}$, then r is irrational.