# Solution Manual for Fitzgerald and Kingsleys Electric Machinery 7th Edition by Umans ISBN 0073380466 9780073380469 

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## Chapter 1

Part (a): $\quad \begin{aligned} \mathrm{R} & =I_{\mathrm{c}}=I_{\mathrm{c}}=0 \quad \mathrm{~A} / \mathrm{Wb} \\ \mathrm{c} & \mu A_{\mathrm{c}}{ }_{\mathrm{r} 0 \mathrm{c}}^{\mu A_{\mathrm{c}}} \\ \mathrm{R}_{\mathrm{g}} & =\frac{g}{\mu_{0} A_{\mathrm{c}}}=5.457 \times 10^{6} \quad \mathrm{~A} / \mathrm{Wb}\end{aligned}$

Part (b):

$$
\Phi=\frac{N I}{\mathrm{R}_{\mathrm{c}}+\mathrm{R}_{\mathrm{g}}}=2.437 \times 10^{-5} \mathrm{~Wb}
$$

Part (c):

$$
\lambda=N \Phi=2.315 \times 10^{-3} \quad \mathrm{~Wb}
$$

Part (d):

$$
L=\frac{\lambda}{I}=1.654 \mathrm{mH}
$$

Problem 1-2

Part (a):

$$
\mathrm{R}_{\mathrm{c}}=\frac{l_{\mathrm{c}}}{\mu A}=\mathrm{l}_{\mathrm{c}} \underset{\mathrm{c} 0 \mathrm{c}}{\mu \mu} \quad=2.419 \times 10^{5} \quad \mathrm{~A} / \mathrm{Wb}
$$

$$
\mathrm{R}_{\mathrm{g}}=\mu A
$$

$=5.457 \times 10^{6} \quad \mathrm{~A} / \mathrm{Wb}$

Part (b):

$$
\Phi=\frac{N I}{\mathrm{R}_{\mathrm{c}}+\mathrm{R}_{\mathrm{g}}}=2.334 \times 10^{-5} \quad \mathrm{~Wb}
$$

Part (c):

$$
\lambda=N \Phi=2.217 \times 10^{-3} \quad \mathrm{~Wb}
$$

Part (d):

$$
L=\frac{\lambda}{I}=1.584 \mathrm{mH}
$$

Problem 1-3

Part (a):

$$
N=\frac{s}{\frac{L g}{\mu_{0} A_{\mathrm{c}}}}=287 \text { turns }
$$

Part (b):

$$
I=\frac{B_{\text {core }}}{\mu_{0} N / g}=7.68 \mathrm{~A}
$$

Problem 1-4

Part (a):

$$
N=\frac{\boldsymbol{s}^{\frac{L\left(g+l_{\mathrm{c}} \mu_{0} / \mu\right)}{}} \mu_{0} A_{\mathrm{c}}}{\boldsymbol{s}^{\frac{L\left(g+l_{\mathrm{c}} \mu_{0} /\left(\mu_{\mathrm{r}} \mu_{0}\right)\right)}{}} \mu_{0} A_{\mathrm{c}}}=129 \text { turns }
$$

## Part (b):

$$
I=\frac{B_{\text {core }}}{\mu_{0} N /\left(g+l_{c} \mu_{0} / \mu\right)}=20.78
$$

Problem 1-5
Part (a):


Part (b):

$$
B_{\mathrm{g}}=B_{\mathrm{m}}=2.1 \mathrm{~T}
$$

For $B_{\mathrm{m}}=2.1 \mathrm{~T}, \mu_{\mathrm{r}}=37.88$ and thus

$$
I=\frac{B_{\mathrm{m}}}{\mu_{0} N} g+\frac{l_{\mathrm{c}}}{\mu_{\mathrm{r}}}=158 A
$$

## Part (c):



Problem 1-6

## Part (a):

$$
\mu_{0} N I
$$

## B <br> g <br> $=$ <br> 2

$$
B_{\mathrm{c}}=B_{\mathrm{g}} \frac{A_{\mathrm{g}}}{-\frac{\mu_{0} N I}{A_{\mathrm{c}}}} 1-\frac{x}{2 g}
$$

Part (b): Will assume $l_{c}$ is "large" and $l_{\mathrm{p}}$ is relatively "small". Thus,

$$
B_{\mathrm{g}} A_{\mathrm{g}}=B_{\mathrm{p}} A_{\mathrm{g}}=B_{\mathrm{c}} A_{\mathrm{c}}
$$

We can also write

$$
2 g H_{\mathrm{g}}+H_{\mathrm{p}} l_{\mathrm{p}}+H_{\mathrm{c}} l_{\mathrm{c}}=N I
$$

and

$$
B_{\mathrm{g}}=\mu_{0} H_{\mathrm{g}} ; \quad B_{\mathrm{p}}=\mu H_{\mathrm{p}} \quad B_{\mathrm{c}}=\mu H_{\mathrm{c}}
$$

These equations can be combined to give
and

$$
B_{\mathrm{c}}=1-\frac{x}{X_{0}} B_{\mathrm{g}}
$$

## Problem 1-7

From Problem 1-6, the inductance can be found as

$$
L=\frac{N A_{\mathrm{c}} B_{\mathrm{c}}}{I}=\frac{\mu_{0} N^{2} A_{\mathrm{c}}}{2 g+\mu_{\mu}^{\mu_{0}}\left(l_{\mathrm{p}}+(1-x / X) d\right)}
$$

from which we can solve for $\mu_{\mathrm{r}}$

$$
\mu_{\mathrm{r}}=\frac{\mu}{\mu_{0}}=\frac{L l_{\mathrm{p}}+\left(1-x / X_{0}\right) l_{\mathrm{c}}}{\mu_{0} N^{2} A_{\mathrm{c}}-2 g L}=88.5
$$

Problem 1-8

Part (a):

$$
L=\frac{\mu_{0}(2 N)^{2} A_{\mathrm{c}}}{2 g}
$$

and thus

$$
N=0.5 \frac{\boldsymbol{s}^{\frac{2 g L}{}}}{\frac{A_{\mathrm{c}}}{}}=38.8
$$

which rounds to $N=39$ turns for which $L=12.33 \mathrm{mH}$.

Part (b): $\quad g=0.121 \mathrm{~cm}$

Part(c):

$$
B_{\mathrm{c}}=B_{\mathrm{g}}=\frac{2 \mu_{0} N I}{2 g}
$$

and thus

$$
I=\frac{B_{\mathrm{c}} g}{\mu_{0} N}=37.1 \mathrm{~A}
$$

Problem 1-9

Part (a):

$$
\begin{array}{ll}
{ }_{2}^{L} A_{\mathrm{c}}= & \\
& \mu_{0} N \\
& 2 \\
g
\end{array}
$$

and thus

$$
N=\frac{\boldsymbol{s}^{\overline{2 g L}}}{\frac{A_{\mathrm{c}}}{}}=77.6
$$

which rounds to $N=78$ turns for which $L=12.33 \mathrm{mH}$.

Part (b):

$$
g=0.121 \mathrm{~cm}
$$

Part(c):

$$
B_{\mathrm{c}}=B_{\mathrm{g}}=\frac{\mu_{0}(2 N)(I / 2)}{2 g}
$$

and thus

$$
I=\frac{2 B_{\mathrm{c}} g}{\mu_{0} N}=37.1 \mathrm{~A}
$$

## Problem 1-10

Part (a):

$$
L=\begin{gathered}
\mu_{0}(2 N)^{2} A_{\mathrm{c}} \\
2\left(g+\left(\frac{\mu 0}{\mu}\right) l{ }_{\mathrm{c}}\right)
\end{gathered}
$$

and thus
which rounds to $N=39$ turns for which $L=12.33 \mathrm{mH}$.

Part (b): $\quad g=0.121 \mathrm{~cm}$

Part(c):

$$
B_{\mathrm{c}}=B_{\mathrm{g}}=\frac{2 \mu_{0} N I}{2\left(g+\frac{\left.\mu_{0} l 0\right)}{\mu \mathrm{c}}\right.}
$$

and thus

$$
I=\frac{B_{\mathrm{c}}\left(g+\frac{\mu_{0}}{\mu} l_{\mathrm{c}}\right)}{\mu_{0} N}=40.9 \mathrm{~A}
$$

Problem 1-11

Part (a): From the solution to Problem 1-6 with $x=0$

$$
I=\frac{B_{\mathrm{g}} 2 g+2 \frac{\mu_{0}\left(l_{\mathrm{p}}+l_{\mathrm{c}}\right)}{\mu}}{\mu_{0} N}=1.44 \mathrm{~A}
$$

Part (b): For $B_{\mathrm{m}}=1.25 \mathrm{~T}, \mu_{\mathrm{r}}=941$ and thus $I=2.43 \mathrm{~A}$
Part (c):


Problem 1-12

$$
\begin{aligned}
& g=\underline{\mu_{0} N^{2} A_{\mathrm{c}}}-\mu^{\mu_{0}^{-!}} l \quad 10^{-4} \mathrm{~m} \\
& L \quad \mu \quad \mathrm{c}=7.8 \times
\end{aligned}
$$

Problem 1-13

Part (a):

$$
I_{\mathrm{c}}=2 \pi \quad \frac{R_{\mathrm{i}}+R_{\mathrm{o}}}{2}-g=22.8 \mathrm{~cm}
$$

$$
A_{\mathrm{c}}=h\left(R_{\mathrm{o}}-R_{\mathrm{i}}\right)=1.62 \mathrm{~cm}^{2}
$$

Part (b):

$$
\begin{gathered}
\mathrm{R}_{\mathrm{c}}=\frac{l_{\mathrm{c}}}{\mu A_{\mathrm{c}}}=0 \\
\mathrm{R}_{\mathrm{g}}=\frac{g}{\mu_{0} A_{\mathrm{c}}}=7.37 \times 10^{6} \mathrm{H}^{-1}
\end{gathered}
$$

Part (c):

$$
L=\frac{N^{2}}{\mathrm{R}_{\mathrm{c}}+\mathrm{R}_{\mathrm{g}}}=7.04 \times 10^{-4} \mathrm{H}
$$

Part (d):

$$
I=\frac{B_{\mathrm{g}} \underline{A}\left(\mathrm{R}_{\underline{c}}+\mathrm{R}_{\mathrm{g}}\right)}{N}=20.7 \mathrm{~A}
$$

Part (e):

$$
\lambda=L I=1.46 \times 10^{-2} \mathrm{~Wb}
$$

Problem 1-14

## See solution to Problem 1-13

## Part (a):

$$
\begin{aligned}
I_{\mathrm{c}} & =22.8 \mathrm{~cm} \\
A_{\mathrm{c}} & =1.62 \mathrm{~cm}^{2}
\end{aligned}
$$

Part (b):

$$
\begin{aligned}
& R_{c}=1.37 \times 10^{6} \mathrm{H}^{-1} \\
& \mathrm{R}_{\mathrm{g}}=7.37 \times 10^{6} \mathrm{H}^{-1}
\end{aligned}
$$

Part (c):

$$
L=5.94 \times 10^{-4} \mathrm{H}
$$

Part (d):

$$
I=24.6 \mathrm{~A}
$$

Part (e):

$$
\lambda=1.46 \times 10^{-2} \mathrm{~Wb}
$$

Problem 1-15

$\mu_{\mathrm{r}}$ must be greater than 2886.
Problem 1-16

$$
L=\frac{\mu_{0} N^{2} A_{\mathrm{c}}}{g+l_{\mathrm{c}} / \mu_{\mathrm{r}}}
$$

Problem 1-17

Part (a):

$$
L=\frac{\mu_{0} N^{2} A_{\mathrm{c}}}{g+l_{\mathrm{c}} / \mu_{\mathrm{r}}}=36.6 \mathrm{mH}
$$

Part (b):

$$
\begin{aligned}
& B=\frac{\mu_{0} N^{2}}{g+l_{\mathrm{c}} / \mu_{\mathrm{r}}} I=0.77 \mathrm{~T} \\
& \lambda=L I=4.40 \times 10^{-2} \mathrm{~Wb}
\end{aligned}
$$

Problem 1-18

Part (a): With $\omega=120 \pi$

Part (b): Using $L$ from the solution to Problem 1-17

$$
\begin{gathered}
I_{\text {peak }}=\frac{{ }^{\sqrt{2}} V_{\text {rms }}}{\omega L}=1.66 \mathrm{~A} \\
W_{\text {peak }}=\frac{L I_{\text {peak }}^{2}}{2}=9.13 \times 10^{-2} \mathrm{~J}
\end{gathered}
$$

Problem 1-19

$$
B=0.81 \mathrm{~T} \text { and } \lambda=46.5 \mathrm{mWb}
$$

Problem 1-20

Part (a):

$$
R_{3}={ }^{\mathrm{q}}\left(R_{1}^{2}+R_{2}^{2}\right)=4.49 \mathrm{~cm}
$$

Part (b): For

$$
I_{c}=4 l+R_{2}+R_{3}-2 h
$$

and

$$
\begin{gathered}
A_{\mathrm{g}}=\pi R_{1}^{2} \\
L=\frac{\mu_{0} A_{\mathrm{g}} N^{2}}{g+\left(\mu_{0} / \mu\right) l_{\mathrm{c}}}=61.8 \mathrm{mH}
\end{gathered}
$$

Part (c): For $B_{\text {peak }}=0.6 \mathrm{~T}$ and $\omega=2 \pi 60$

$$
\begin{gathered}
\lambda_{\text {peak }}=A_{\mathrm{g}} N B_{\text {peak }} \\
V_{\text {rms }}={\underset{\sim}{2}}_{\omega \lambda_{\text {peak }}}^{V_{2}}=23.2 \mathrm{~V} \\
I_{\text {rms }}=\frac{V_{\text {rms }}}{\omega L}=0.99 \mathrm{~A} \\
1 \\
\left.W={ }_{-}=\frac{V}{2} \quad \begin{array}{l}
\text { peak } \\
\text { peak } \\
2
\end{array} \quad \mathrm{rms}\right)^{2}=61.0 \mathrm{~mJ}
\end{gathered}
$$

Part (d): For $\omega=2 \pi 50$

$$
\begin{gathered}
V_{\mathrm{rms}}=19.3 \mathrm{~V} \\
I_{\mathrm{rms}}=0.99 \mathrm{~A} \\
W_{\text {peak }}=61.0 \mathrm{~mJ}
\end{gathered}
$$

Problem 1-21
Part (a);


Part (b):

$$
E_{\max }=4 f N A_{\mathrm{c}} B_{\text {peak }}=118 \mathrm{~V}
$$

part (c): For $\mu=1000 \mu_{0}$

$$
I_{\text {peak }}=\frac{l_{c} B_{\text {peak }}}{\mu N}=0.46 \mathrm{~A}
$$

## Problem 1-22

## Part (a);



Part (b): $I_{\text {peak }}=0.6 \mathrm{~A}$


Part (c): $I_{\text {peak }}=4.0 \mathrm{~A}$

Problem 1-23

For part (b), $I_{\text {peak }}=11.9 \mathrm{~A}$. For part (c), $I_{\text {peak }}=27.2 \mathrm{~A}$.


Problem 1-24

$$
\begin{aligned}
& L=\frac{\mu_{0} A_{\mathrm{c}} N^{2}}{g+\left(\mu_{0} / \mu\right) l_{\mathrm{c}}} \\
& B_{\mathrm{c}}=\frac{\mu_{0} N I}{g+\left(\mu_{0} / \mu\right){ }_{\mathrm{c}}}
\end{aligned}
$$

Part (a): For $I=10 \mathrm{~A}, L=23 \mathrm{mH}$ and $B_{\mathrm{c}}=1.7 \mathrm{~T}$

$$
\begin{gathered}
N=\frac{L I}{A_{\mathrm{c}} B_{\mathrm{c}}}=225 \mathrm{turns} \\
g=\frac{\mu_{0} N I}{M_{0}} \frac{\mu_{0} l_{\mathrm{c}}}{}=1.56 \mathrm{~mm} \\
B_{\mathrm{c}} \quad \mu
\end{gathered}
$$

Part (b): For $I=10 \mathrm{~A}$ and $B_{\mathrm{c}}=B_{\mathrm{g}}=1.7 \mathrm{~T}$, from Eq. 3.21

$$
W_{\text {core }}=\frac{E^{2}}{2 \mu} V_{\mathrm{g}}=0.072 \mathrm{~J}
$$

based upon

$$
V_{\text {core }}=A_{\mathrm{c}} l_{\mathrm{c}} \quad V_{\mathrm{g}}=A_{\mathrm{c}} g
$$

Part (c):

$$
W_{\text {tot }}=W_{\mathrm{g}}+W_{\text {core }}=1.15 \mathrm{~J}={ }_{\frac{1}{2}}^{1} L I^{2}
$$

Problem 1-25

$$
L_{\min }=3.6 \mathrm{mH} \quad L_{\max }=86.0 \mathrm{mH}
$$

## Problem 1-26

Part (a):

$$
\begin{gathered}
N=\frac{L I}{B A_{\mathrm{c}}}=167 \\
g=\frac{\mu_{0} N I}{2 B A_{\mathrm{c}}}=0.52 \mathrm{~mm}
\end{gathered}
$$

Part (b):

$$
N=\frac{L I}{2 B A_{\mathrm{c}}}=84
$$

$$
g=\frac{\mu_{0} N I}{B A_{\mathrm{c}}}=0.52 \mathrm{~mm}
$$

Problem 1-27

Part (a):

$$
N=\frac{L I}{B A_{\mathrm{c}}}=167
$$

$$
g=\frac{\mu_{0} N I}{2 B A_{\mathrm{c}}}-\underset{0}{(\mu / \mu) l}=0.39 \mathrm{~mm}
$$

Part (b):

$$
\begin{gathered}
N=\frac{L I}{2 B A_{\mathrm{c}}}=84 \\
g=\frac{\mu_{0} N I}{B A_{\mathrm{c}}}-\underset{0}{(\mu / \mu) l}=0.39 \mathrm{~mm}
\end{gathered}
$$

Problem 1-28

Part (a): $N=450$ and $g=2.2 \mathrm{~mm}$

Part (b): $N=225$ and $g=2.2 \mathrm{~mm}$

Problem 1-29

Part (a):

$$
L=\frac{\mu_{0} N^{2} A}{l}=11.3 \mathrm{H}
$$

where

$$
A=\pi a^{2} \quad l=2 \pi r
$$

Part (b):

$$
W=\frac{B^{2}}{2 \mu_{0}} \times \text { Volume }=6.93 \times 10 \mathrm{~J}
$$

where

$$
\text { Volume }=\left(\pi a^{2}\right)(2 \pi r)
$$

Part (c): For a flux density of 1.80 T,

$$
I=\frac{l B}{\mu_{0} N}=\frac{2 \pi r B}{\mu_{0} N}=6.75 \mathrm{kA}
$$

and

$$
V=L \frac{\underline{\Delta I}}{\Delta t}=113 \times 10 \frac{-36.75 \times 10^{3}}{40}=1.90 \mathrm{kV}
$$

Problem 1-30

Part (a):

$$
\begin{gathered}
\text { Copper cross - sectional area } \equiv A_{\mathrm{cu}}=f_{\mathrm{w}} a b \\
\text { Copper volume }=\mathrm{Vol}_{\mathrm{cu}}=f_{\mathrm{w}} b\left(a+\frac{w}{2}\right)\left(h+\frac{w}{2}\right)-w h
\end{gathered}
$$

Part (b):

$$
B=\frac{\mu_{\mathrm{o}} J_{\mathrm{cu}} A_{\mathrm{cu}}}{g}
$$

Part (c):

## $J_{\text {cu }}=$ <br> NI

A c
u

Part (d):

$$
P_{\mathrm{diss}}=\rho J_{\mathrm{cu}}^{2} \mathrm{Vol}_{\mathrm{cu}}
$$

Part (e):

$$
W_{\text {stored }}=\frac{B^{2}}{2 \mu_{0}} \times \text { gap volume }=\frac{\mu_{0} J^{2} A^{2} w h}{\frac{\mathrm{cu} \mathrm{cu}}{2 g}}
$$

Part (f):

$$
\frac{W_{\text {store }}}{P_{\text {diss }}}=\frac{\frac{1}{2} I^{2} L}{I^{2} R}
$$

and thus

$$
\begin{gathered}
L \\
R^{-} W_{\text {stored }} \\
P_{\text {diss }} \\
=\begin{array}{c}
\mu_{0} A^{2} w h \\
g \rho \frac{\mathrm{cu}}{\mathrm{Vol} \mathrm{l}_{\mathrm{cu}}}
\end{array}
\end{gathered}
$$

Problem 1-31

$$
\begin{gathered}
P_{\text {diss }}=6.20 \mathrm{~W} \quad I=155 \mathrm{~mA} \quad N=12,019 \text { turns } \\
R=258 \Omega \quad L=32 \mathrm{H} \quad \tau=126 \mathrm{msec} \quad \text { Wire size }=34 \mathrm{AWG}
\end{gathered}
$$

Problem 1-32

Part (a) (i):

$$
B_{\mathbf{g} 1}=\frac{\mu_{0} N_{1}}{g_{1}} I_{1} \quad B_{\mathrm{g} 2}=\frac{\mu_{0} N_{1}}{g_{2}} I_{1}
$$

$$
\lambda_{1}=N_{1}\left(A_{1} B_{\mathrm{g} 1}+A_{2} B_{\mathrm{g} 2}\right)=\mu_{0} N^{2} \frac{A_{1}}{{ }_{1}} \frac{A_{2}}{g_{1}} I_{1}
$$

$$
\lambda=N \underset{22 \mathrm{~g} 2}{A_{2}}=\mu_{0} N_{1} N_{2} \frac{A_{2}}{g_{2}} I_{1}
$$

Part (b) (i):

$$
B_{\mathrm{g} 1}=0 \quad B_{\mathrm{g} 2}=\frac{\mu_{0} N_{2}}{g_{2}} I_{2}
$$

(ii)

$$
\begin{aligned}
& \lambda=N A B=\mu N^{2} \underline{A_{2}} I \\
& 2 \quad 22 \mathrm{~g} 2 \quad 0 \quad{ }^{2} g_{2}{ }^{2}
\end{aligned}
$$

Part (c) (i):

$$
B_{\mathrm{g} 1}=\frac{\mu_{0} N_{1}}{g_{1}} I_{1} \quad B_{\mathrm{g} 2}=\frac{\mu_{0} N_{1}}{g_{2}} I_{1}+\frac{\mu_{0} N_{2}}{g_{2}} I_{2}
$$

(ii)

$$
\begin{aligned}
& \lambda_{1}=N_{1}\left(A_{1} B_{\mathrm{g} 1}+A_{2} B_{\mathrm{g} 2}\right)=\mu_{0} N^{2} A_{1}+\underline{A_{2}} I_{1}+\mu_{0} N_{1} N_{2} \underline{A_{2}} I_{2} \\
& { }^{1} \overline{g_{1}} \quad g_{2} \\
& g_{2} \\
& \lambda=N A B=\mu N N \underline{A_{2}} I+\mu N^{2} \underline{A_{2}} I \\
& 2 \quad 22 \mathrm{~g} 2 \quad 0 \quad 1 \quad 2 g_{2}{ }^{1} \quad 0 \quad{ }^{2} g_{2}{ }^{2}
\end{aligned}
$$

Part (d):

$$
L_{1}=\mu_{0} N_{1}^{2} \frac{A_{1}}{g_{1}}+\frac{A_{2}}{g_{2}} \quad \begin{gathered}
L \mu N{ }^{2} A_{2} \\
20 \quad{ }^{2} g_{2}
\end{gathered}
$$

$$
L \underset{12}{=\mu N N} \begin{array}{ll}
N_{012} & A_{2} \\
g_{2}
\end{array}
$$

Problem 1-33

$$
\begin{array}{ll}
\mathrm{R}_{\mathrm{g}}=\frac{g}{\mu_{0} A_{\mathrm{c}}} & \mathrm{R}_{1}=\frac{l_{1}}{\mu A_{\mathrm{c}}} \\
\mathrm{R}_{2}=\frac{l_{2}}{\mu A_{\mathrm{c}}} & \mathrm{R}_{\mathrm{A}}=\frac{l_{\mathrm{A}}}{\mu A_{\mathrm{c}}}
\end{array}
$$

Part (a):

$$
\begin{gathered}
L_{1}=\frac{N \frac{2}{1}}{\mathrm{R}_{\mathrm{g}}+\mathrm{R}_{1}+\mathrm{R}_{2}+\mathrm{R}_{\mathrm{A}} / 2} \\
L_{\mathrm{A}}=L_{\mathrm{B}}=\frac{N^{2}}{\mathrm{R}}
\end{gathered}
$$

where

$$
R=R_{A}+\frac{R_{A}\left(R_{g}+R_{1}+R_{2}\right)}{R_{A}+R_{g}+R_{1}+R_{2}}
$$

Part (b):

$$
\begin{gathered}
L=-L=\frac{N_{1} N}{{ }_{1 \mathrm{~B}} \quad 1 \mathrm{~A} \quad 2\left(\mathrm{R}_{\mathrm{g}}+\mathrm{R}_{1}+\mathrm{R}_{2}+\mathrm{R}_{\mathrm{A}} / 2\right)} \\
N^{2}\left(\mathrm{R}_{\mathrm{g}}+\mathrm{R}_{1}+\mathrm{R}_{2}\right)
\end{gathered}
$$

$$
L_{12}=\overline{2 R^{\mathrm{A}}\left(R^{g}+R^{1}+R\right)^{2}+R^{2 A}}
$$

Part (c):

$$
v_{1}(t)=L_{1 \mathrm{~A}} \frac{d i_{\mathrm{A}}}{d t}+L_{1 \mathrm{~B}} \frac{d i_{\mathrm{B}}}{d t}=L_{1 \mathrm{~A}} \frac{d\left(i_{\mathrm{A}}-i_{\mathrm{B}}\right)}{d t}
$$

Problem 1-34

Part (a):

$$
L_{12}=\frac{\mu_{0} N_{1} N_{2} D(w-x)}{2 g}
$$

Part (b):

$$
v(t)=-\omega I \quad \frac{\mu_{0} N_{1} N_{2} D w}{\cos \omega t} \cos
$$

Problem 1-35

Part (a):

$$
H=\frac{2 N_{1} i_{1}}{\left(R_{o}+R_{i}\right)}
$$

Part (b):

$$
v_{2}(t)=N_{2} w(n \Delta) \frac{d B(t)}{d t}
$$

Part (c):

$$
v_{0}(t)=G N_{2} w(n \Delta) B(t)
$$

Problem 1-36

Must have

$$
\frac{\mu_{0}}{\mu} l_{\mathrm{c}}<0.05 g+\frac{\mu_{0}}{\mu} l_{\mathrm{c}} \Rightarrow \mu={ }_{\bar{H}}^{B}>19 \mu_{0}{ }^{l_{\mathrm{c}}}
$$

For $g=0.05 \mathrm{~cm}$ and $l_{\mathrm{c}}=30 \mathrm{~cm}$, must have $\mu>0.014$. This is satisfied over the approximate range $0.65 \mathrm{~T} \leq B \leq 1.65 \mathrm{~T}$.

Problem 1-37

Part (a): See Problem 1-35. For the given dimensions, $V_{\text {peak }}=20 \mathrm{~V}, B_{\text {peak }}=1 \mathrm{~T}$ and $\omega=100 \pi \mathrm{rad} / \mathrm{sec}$

$$
N_{1}=\frac{V_{\text {peak }}}{\omega\left(R_{\mathrm{o}}-R_{\mathrm{i}}\right)(n \Delta)}=79 \text { turns }
$$

Part (b): (i)

$$
B=\frac{V_{0, \text { peak }}}{G N_{2}\left(R_{0}-R_{\mathrm{i}}\right)(n \Delta)}=0.83 \mathrm{~T}
$$

(ii)

$$
V_{\text {peak }}=\omega N_{1}\left(R_{\mathrm{o}}-R_{\mathrm{i}}\right)(n \Delta) B_{\text {peak }}=9.26 \mathrm{~V}
$$

Problem 1-38

Part (a): From the M-5 dc-magnetization characteristic, $H_{c}=19$ A-turns $/ \mathrm{m}$ at $B_{\mathrm{c}}=$ $B_{\mathrm{g}}=1.3 \mathrm{~T}$. For $H_{\mathrm{g}}=1.3 \mathrm{~T} / \mu_{0}=1.03 \times 10^{6}$ A-turns $/ \mathrm{m}$

$$
I=\frac{H_{\mathrm{c}}\left(l_{\mathrm{A}}+l_{\mathrm{C}}-g\right)+H_{\mathrm{g}} g}{N_{1}}=30.2 \mathrm{~A}
$$

Part(b):

$$
W_{\text {gap }}=g A_{\mathrm{C}} \frac{\stackrel{B}{8}^{2}}{2 \mu_{0}}=3.77 \mathrm{~J}
$$

For $\mu=B_{\mathrm{c}} / H_{\mathrm{c}}=0.0684 \mathrm{H} / \mathrm{m}$

$$
\begin{gathered}
W_{\mathrm{c}}=\left(l_{\mathrm{A}} A_{\mathrm{A}}+l_{\mathrm{B}} A_{\mathrm{B}}+\left(l_{\mathrm{C}}-g\right) A_{\mathrm{C}}\right) \underset{\mathrm{c}}{\mathrm{~B}^{2}} \underset{\mathrm{c}}{\mu^{2}}=4.37 \times 10^{-3} \mathrm{~J} \\
L=\frac{2\left(W_{\text {gap }}+W_{\mathrm{c}}\right)}{I^{2}}=8.26 \mathrm{mH}
\end{gathered}
$$

Part (c):

$$
L=\frac{2 W_{\text {gap }}}{I^{2}}=8.25 \mathrm{mH}
$$

## Problem 1-39

Part (a):


Part (b): Loop area $=191 \mathrm{~J} / \mathrm{m}^{3}$

Part (c):

## Core

loss $=f$
$\times$ Loop
area
$\rho$

For $f=60 \mathrm{~Hz}, \rho=7.65 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$, Core loss $=1.50 \mathrm{~W} / \mathrm{kg}$

Problem 1-40
$B_{\mathrm{rms}}=1.1 \mathrm{~T}$ and $f=60 \mathrm{~Hz}$,

$$
V_{\mathrm{rms}}=\omega N A_{\mathrm{c}} B_{\mathrm{rms}}=86.7 \quad \mathrm{~V}
$$

Core volume $=A_{\mathrm{c}} l_{\mathrm{c}}=1.54 \times 10^{-3} \mathrm{~m}^{3}$. Mass density $=7.65 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$. Thus, the core mass $=\left(1.54 \times 10^{-3}\right)\left(7.65 \times 10^{3}\right)=11.8 \mathrm{~kg}$.

At $B=1.1 \mathrm{Trms}=1.56 \mathrm{~T}$ peak, core loss density $=1.3 \mathrm{~W} / \mathrm{kg}$ and rms VA density is 2.0 VA/kg. Thus, the core loss $=1.3 \times 11.8=15.3 \mathrm{~W}$. The total $/$ exciting VA for the core is $2.0 \times 11.8=23.6 \mathrm{VA}$. Thus, its reactive component is given by $23.6^{2}-15.3^{2}=17.9 \mathrm{VAR}$.

The rms energy storage in the air gap is

$$
W_{\text {gap }}=-\frac{g A_{\mathrm{c}} B^{2}}{\mu_{0}} \frac{\stackrel{\mathrm{rms}}{ }=5.08 \mathrm{~J}}{}
$$

corresponding to an rms reactive power of

$$
\mathrm{VAR}_{\text {gap }}=\omega W_{\text {gap }}=1917 \quad \text { VAR }
$$

Thus, the total rms exciting VA for the magnetic circuit is

$$
\mathrm{VA}_{\mathrm{rms}}=\frac{\mathrm{q}_{\overline{1}}}{15.3^{2}+(1917+17.9)^{2}}=1935 \quad \mathrm{VA}
$$

and the rms current is $I_{\mathrm{rms}}=\mathrm{VA}_{\mathrm{rms}} / V_{\mathrm{rms}}=22.3 \mathrm{~A}$.

## Problem 1-41

Part(a): Area increases by a factor of 4 . Thus the voltage increases by a factor of 4 to $e=1096 \cos (377 t)$.

Part (b): $l_{\mathrm{c}}$ doubles therefore so does the current. Thus $I=0.26 \mathrm{~A}$.

Part (c): Volume increases by a factor of 8 and voltage increases by a factor of 4 . There $I_{\varphi, \text { rms }}$ doubles to 0.20 A .

Part (d): Volume increases by a factor of 8 as does the core loss. Thus $P_{\mathrm{c}}=128 \mathrm{~W}$.

Problem 1-42

From Fig. 1.19, the maximum energy product for samarium-cobalt occurs at (approximately) $B=0.47 \mathrm{~T}$ and $H=-360 \mathrm{kA} / \mathrm{m}$. Thus the maximum energy product is $1.6810^{5}$ $\mathrm{J} / \mathrm{m}^{3}$.

Thus,

$$
A_{\mathrm{m}}=\frac{0.8}{0.47} 2 \mathrm{~cm}^{2}=3.40 \mathrm{~cm}^{2}
$$

and

$$
\begin{aligned}
& l=-0.2 \mathrm{~cm} \quad 0.8 \quad \stackrel{!}{ }=0.35 \mathrm{~cm} \\
& \mathrm{~m} \quad \mu_{0}\left(-3.60 \times 10^{5}\right)
\end{aligned}
$$

Thus the volume is $3.40 \times 0.35=1.20 \mathrm{~cm}^{3}$, which is a reduction by a factor of $5.09 / 1.21$ $=4.9$.

Problem 1-43

From Fig. 1.19, the maximum energy product for neodymium-iron-boron occurs at (approximately) $B=0.63 \mathrm{~T}$ and $H=-470 \mathrm{kA} / \mathrm{m}$. Thus the maximum energy product is 2.90 $\times 10^{5} \mathrm{~J} / \mathrm{m}^{3}$.

Thus,

$$
A_{\mathrm{m}}=\frac{0.8}{0.63} 2 \mathrm{~cm}^{2}=2.54 \mathrm{~cm}^{2}
$$

and

$$
I_{\mathrm{m}}=-0.2 \mathrm{~cm}{\frac{0.8}{\mu_{0}\left(-4.70 \times 10^{5}\right)}}^{\mathbf{!}}=0.27 \mathrm{~cm}
$$

Thus the volume is $2.54 \times 0.25=0.688 \mathrm{~cm}^{3}$, which is a reduction by a factor of $5.09 / 0.688$ $=7.4$.

Problem 1-44

From Fig. 1.19, the maximum energy product for samarium-cobalt occurs at (approximately) $B=0.47 \mathrm{~T}$ and $H=-360 \mathrm{kA} / \mathrm{m}$. Thus the maximum energy product is $1.69 \times 10^{5}$ $\mathrm{J} / \mathrm{m}^{3}$. Thus, we want $B_{\mathrm{g}}=1.3 \mathrm{~T}, B_{\mathrm{m}}=0.47 \mathrm{~T}$ and $H_{\mathrm{m}}=-360 \mathrm{kA} / \mathrm{m}$.

$$
\begin{aligned}
& \mathrm{m} \quad 0 \quad \mathrm{~m} \\
& A_{\mathrm{m}}=A_{\mathrm{g}} \quad \frac{B_{\mathrm{g}}}{B_{\mathrm{m}}}=2 \pi R h \quad \begin{array}{c}
B_{\mathrm{g}} \\
B_{\mathrm{m}}
\end{array}=45.1 \quad \mathrm{~cm}^{2} \\
& R_{\mathrm{m}}=\frac{\mathrm{s}^{\underline{A_{\mathrm{m}}}}}{\pi}=3.66 \mathrm{~cm}
\end{aligned}
$$

Problem 1-45

From Fig. 1.19, the maximum energy product for samarium-cobalt occurs at (approximately) $B=0.63 \mathrm{~T}$ and $H=-482 \mathrm{kA} / \mathrm{m}$. Thus the maximum energy product is $3.03 \times 10^{5}$ $\mathrm{J} / \mathrm{m}^{3}$. Thus, we want $B_{\mathrm{g}}=1.3 \mathrm{~T}, B_{\mathrm{m}}=0.47 \mathrm{~T}$ and $H_{\mathrm{m}}=-360 \mathrm{kA} / \mathrm{m}$.

$$
h_{\mathrm{m}}=-g \quad \begin{gathered}
H_{\mathrm{g}} \\
H_{\mathrm{m}}
\end{gathered}=-g \quad \begin{gathered}
B_{\mathrm{g}} \\
\mu_{0} H_{\mathrm{m}}
\end{gathered}=2.15 \mathrm{~mm}
$$

$$
A_{\mathrm{m}}=A_{\mathrm{g}} \quad \frac{B_{\mathrm{g}}}{B_{\mathrm{m}}}=2 \pi R h \quad \frac{B_{\mathrm{g}}}{B_{\mathrm{m}}}=31.3 \mathrm{~cm}^{2}
$$

$$
R_{\mathrm{m}}=\frac{\mathrm{S}^{\underline{A_{\mathrm{m}}}}}{\pi}=3.16 \mathrm{~cm}
$$

Problem 1-46

For $B_{\mathrm{m}}=\mu_{R}\left(H_{\mathrm{m}}-H_{\mathrm{c}}\right)$, the maximum value of the product $-B_{\mathrm{m}} H_{\mathrm{m}}$ occurs at $H_{\mathrm{m}}=H_{\mathrm{c}} / 2$ and the value is $B^{2} k 4 \mu_{\mathrm{R}}$.

|  |  | Corresponding |  |
| :---: | :---: | :---: | :---: |
| $T[\mathrm{C}]$ | $-\left(B_{\mathrm{m}} H_{\mathrm{m}}\right)_{\max }[\mathrm{kJ} / \mathrm{m}]^{2}$ | $H_{\mathrm{m}}[\mathrm{kA} / \mathrm{m}]$ | $B_{\mathrm{m}}[T]$ |
| 20 | 253.0 | -440.0 | 0.57 |
| 60 | 235.7 | -424.7 | 0.56 |
| 80 | 223.1 | -413.2 | 0.54 |
| 150 | 187.5 | -378.8 | 0.50 |
| 180 | 169.0 | -359.6 | 0.47 |
| 210 | 151.5 | -340.5 | 0.45 |

Problem 1-47

From Fig. 1.19, the maximum energy product for neodymium-iron-boron occurs at (approximately) $B_{\mathrm{m}}=0.63 \mathrm{~T}$ and $H_{\mathrm{m}}=-470 \mathrm{kA} / \mathrm{m}$. The magnetization curve for neodymium-iron-boron can be represented as

$$
B_{\mathrm{m}}=\mu_{\mathrm{R}} H_{\mathrm{m}}+B_{\mathrm{r}}
$$

where $B_{\mathrm{r}}=1.26 \mathrm{~T}$ and $\mu_{R}=1.067 \mu_{0}$. The magnetic circuit must satisfy

$$
H_{\mathrm{m}} d+H_{\mathrm{g}} g=N i ; \quad B_{\mathrm{m}} A_{\mathrm{m}}=B_{\mathrm{g}} A_{\mathrm{g}}
$$

part (a): For $i=0$ and $B_{g}=0.6 \mathrm{~T}$, the minimum magnet volume will occur when the magnet is operating at the maximum energy point.

$$
A_{\mathrm{m}}=\frac{B_{\mathrm{g}}}{B_{\mathrm{m}}} A_{\mathrm{g}}=6.67 \mathrm{~cm}^{2}
$$

$$
\begin{array}{ll}
\mathrm{H}_{\mathrm{g}} & d=- \\
\hline
\end{array}
$$

$$
g=3.47 \mathrm{~mm}
$$

part (b): Want $B_{\mathrm{g}}=0.8 \mathrm{~T}$ when $i=I_{\text {peak }}$

$$
I_{\text {peak }}=\frac{\mu_{\mathrm{g}} \frac{\mu_{\mathrm{g}} A_{\mathrm{m}}}{B_{\mathrm{g}}+\frac{g}{\mu_{0}}-\frac{B_{\mathrm{r}} d^{\mathbf{i}}}{\mu_{\mathrm{R}}}}}{N}=6.37 \mathrm{~A}
$$

Because the neodymium-iron-boron magnet is essentially linear over the operating range of this problem, the system is linear and hence a sinusoidal flux variation will correspond to a sinusoidal current variation.

Problem 1-48

Part(a): Fromthesolution to Problem 1-46, themaximumenergyproductforneodymium-iron-boron at 180 C occurs at (approximately) $B_{\mathrm{m}}=0.47 \mathrm{~T}$ and $H_{\mathrm{m}}=-360 \mathrm{kA} / \mathrm{m}$. The magnetization curve for neodymium-iron-boron can be represented as

$$
B_{\mathrm{m}}=\mu_{\mathrm{R}} H_{\mathrm{m}}+B_{\mathrm{r}}
$$

where $B_{\mathrm{r}}=0.94 \mathrm{~T}$ and $\mu_{R}=1.04 \mu_{0}$. The magnetic circuit must satisfy

$$
H_{\mathrm{m}} d+H_{\mathrm{g}} g=0 ; \quad B_{\mathrm{m}} A_{\mathrm{m}}=B_{\mathrm{g}} A_{\mathrm{g}}
$$

For $B_{\mathrm{g}}=0.8 \mathrm{~T}$, the minimum magnet volume will occur when the magnet is operating at the maximum energy point.

$$
\begin{gathered}
A_{\mathrm{m}}=\underset{B_{\mathrm{m}}}{B_{\mathrm{g}}} A_{\mathrm{g}}=15.3 \mathrm{~cm}^{2} \\
d=-\frac{H_{\mathrm{g}}}{H_{\mathrm{m}}} g=5.66 \mathrm{~mm}
\end{gathered}
$$

Part(b): At $60 \mathrm{C}, B_{\mathrm{r}}=1.12$ T. Combining

$$
B_{\mathrm{m}}=\mu_{\mathrm{R}} H_{\mathrm{m}}+B_{\mathrm{r}}
$$

$$
H_{\mathrm{m}} d+H_{\mathrm{g}} g=0
$$

$$
A_{\mathrm{m}} B_{\mathrm{m}}=A_{\mathrm{g}} A_{\mathrm{g}}=\mu_{0} H_{\mathrm{g}} A_{\mathrm{g}}
$$

gives

$$
B_{\mathrm{g}}=\frac{\mu_{0} d A_{\mathrm{m}}}{\mu_{0} d A_{\mathrm{g}}+\mu_{\mathrm{R}} g A_{\mathrm{m}}} B_{\mathrm{r}}=0.95 \mathrm{~T}
$$

## PROBLEM SOLUTIONS: Chapter 2

Problem 2-1

At $60 \mathrm{~Hz}, \omega=120 п$.

$$
\text { primary: } \quad\left(V_{\mathrm{rms}}\right)_{\max }=N_{1} \omega A_{\mathrm{c}}\left(B_{\mathrm{rms}}\right)_{\max }=3520 \quad \mathrm{~V}, \mathrm{rms}
$$

$$
\text { secondary: } \quad\left(V_{\mathrm{rms}}\right)_{\max }=N_{2} \omega A_{c}\left(B_{\mathrm{rms}}\right)_{\max }=245 \quad \mathrm{~V}, \mathrm{rms}
$$

At $50 \mathrm{~Hz}, \omega=100 \pi$. Primary voltage is 2934 V , rms and secondary voltage is 204 V , rms.

Problem 2-2

$$
N=\frac{\sqrt{ }^{z V_{\text {rms }}}}{\omega A_{c} B_{\text {peak }}}=147 \text { turns }
$$

Problem 2-3

$$
N=\frac{s_{\overline{300}}}{75}=2 \text { turns }
$$

Problem 2-4

Part (a):

$$
R 1={\frac{N_{1}}{N_{2}}}^{2} R_{2}=9.38 \Omega \quad I_{1}=\frac{\frac{V_{1}}{I_{1}}=1.28 \mathrm{~A} \text { } \quad \text {. }}{}
$$

$$
V_{2}=\frac{N_{2}}{N_{1}} \quad V_{1}=48 \mathrm{~V} \quad P_{2}=\frac{V_{2}^{2}}{R_{2}}=14.6 \mathrm{~W}
$$

Part (b): For $\omega=2 \pi f=6.28 \times 10^{3} \Omega$

$$
\begin{gathered}
X_{1}=\omega L=2.14 \Omega \quad I_{1}=\frac{V_{1}}{R_{1}+j X_{1}}=1.25 \mathrm{~A} \\
I_{2}=\frac{N_{1}}{N_{2}} I_{1}=0.312 \mathrm{~A} \quad V_{2}=I_{2} R_{2}=46.8 \mathrm{~V} \\
P_{2}=V_{2} I_{2}=14.6 \mathrm{~W}
\end{gathered}
$$

Problem 2-5

For $\omega=100] \Pi, Z_{\mathrm{L}}=R_{\mathrm{L}}+j \omega L=5.0+j 0.79 \Omega$

$$
V_{\mathrm{L}}=110 \frac{20}{120}=18.3 \mathrm{~V} \quad I_{\mathrm{L}}=\frac{V_{\mathrm{L}}}{I_{\mathrm{L}}} 3.6 \mathrm{~A}
$$

and

$$
I_{\mathrm{H}}=I_{\mathrm{L}} \frac{120}{20}=604 \mathrm{~mA}
$$

## Problem 2-6

The maximum power will be supplied to the load resistor when its impedance, as reflected to the primary of the ideal transformer, equals that of the source $(1.5 \mathrm{k} \Omega)$. Thus the transformer turns ratio $N$ to give maximum power must be

$$
N=\frac{\mathrm{S}_{\underline{R_{\mathrm{s}}}}}{R_{\text {load }}}=4.47
$$

Under these conditions, the source voltage will see a total resistance of $R_{\mathrm{tot}}=3 \mathrm{k} \Omega$ and the source current will thus equal $I=V_{s} / R_{\text {tot }}=4 \mathrm{~mA}$. Thus, the power delivered to the load will equal

$$
P_{\text {load }}=I^{2}\left(N^{2} R_{\text {load }}\right)=24 \mathrm{~mW}
$$

Here is the desired MATLAB plot:


Problem 2-7

The maximum power will be supplied to the load resistor when its impedance, as reflected to the primary of the ideal transformer, equals that of the source $(1.5 \mathrm{k} \Omega)$. Thus the transformer turns ratio $N$ to give maximum power must be

$$
N=\frac{\boldsymbol{s}}{\underline{X_{\mathrm{s}}}}=4.47
$$

Under these conditions, $\sqrt{ }$ the source voltage will see a total impedance of $Z_{\text {tot }}=\sqrt{ } 1.5+j 1.5 \mathrm{k} \Omega$ whose magnitude is $1.52 \mathrm{k} \Omega$. The current will thus equal $I=V_{\mathrm{s}} /\left|Z_{\text {tot }}\right|=5.72 \mathrm{~mA}$. Thus, the power delivered to the load will equal

$$
P_{\text {load }}=I^{2}\left(N^{2} R_{\text {load }}\right)=48 \quad \mathrm{~mW}
$$

Here is the desired MATLAB plot:


Problem 2-8

$$
V_{2}=V_{1} \quad X_{+1}^{X_{m}} \quad \stackrel{!}{!}=V_{1} \quad L_{+m}^{L_{q}} \quad \stackrel{!}{=}=119.86 \quad \mathrm{~V}
$$

$$
\mathrm{l}_{1} \quad \mathrm{~m} \quad \mathrm{l}_{1} \quad \mathrm{~m}
$$

Problem 2-9

Part (a): Referred to the secondary

$$
L_{\mathrm{m}, 2}=\frac{L_{\mathrm{m}, 1}}{N^{2}}=37.9 \mathrm{mH}
$$

Part(b): Referred to the secondary, $X_{\mathrm{m}}=\omega L_{\mathrm{m}, 2}=14.3 \Omega, X_{2}=16.6 \mathrm{~m} \Omega$ and $X_{\mathrm{l}}=$ $16.5 \mathrm{~m} \Omega$. Thus,

$$
\text { (i) } \quad V_{1}=N \frac{X_{m}}{X_{m}+X_{2}} \quad V_{2}=7961 \quad \mathrm{~V}
$$

and

$$
\text { (ii) } I=\underline{V_{2}}=\frac{V_{2}}{}=3629 \mathrm{~A}
$$

$$
\text { sc } \quad X_{\text {sc }} \quad X_{2}+X_{m} \| X_{1}
$$

Problem 2-10

Part (a):

$$
\begin{gathered}
I=\frac{V_{1}}{I=}=4.98 \quad \mathrm{~A} ; \quad V=N V \quad \frac{X_{\mathrm{m}}}{}=6596 \mathrm{~V} \\
\\
\\
\\
X_{\mathrm{l}_{1}}+X_{\mathrm{m}}
\end{gathered}
$$

Part (b): Let $X_{1_{2}}^{0}=X_{1_{2}} / N^{2}$ and $X_{\text {sc }}=X_{1}+X_{\mathrm{m}} \|\left(X_{m}+X_{1_{2}}^{0}\right)$. For $I_{\text {rated }}=45 \mathrm{kVA} / 230 \mathrm{~V}=$

196 A

$$
\begin{gathered}
V_{1}=I_{\text {rated }} X_{\mathrm{sc}}=11.4 \mathrm{~V} \\
I_{2}=1 \\
N \underset{\mathrm{~m}}{+X_{\mathrm{l}_{2}}}
\end{gathered}
$$

Problem 2-11

Part (a): At 60 Hz , all reactances increase by a factor of 1.2 over their $50-\mathrm{Hz}$ values. Thus

$$
X_{\mathrm{m}}=55.4 \Omega \quad X_{\mathrm{l}, 1}=33.4 \mathrm{~m} \Omega \quad X_{\mathrm{l}, 2}=30.4 \Omega
$$

Part (b): For $V_{1}=240 \mathrm{~V}$

$$
I \quad \begin{gathered}
\frac{V 1}{} \\
{ }_{1}=X_{1}+X_{\mathrm{m}} \\
X_{1} \\
=4.33 \mathrm{~A} \\
V_{2}=N V_{1} \\
\overline{X_{\mathrm{m}}+X_{1,1}}
\end{gathered}
$$

Problem 2-12

The load voltage as referred to the high-voltage side is $V 0_{\mathrm{L}}=447 \times 2400 / 460=2332 \mathrm{~V}$. Thus the load current as referred to the high voltage side is

$$
I_{\mathrm{L}}^{0}=\underline{P_{\mathrm{L}}} V_{\mathrm{L}}{ }^{0}=18.0 \mathrm{~A}
$$

and the voltage at the high voltage terminals is

$$
V_{\mathrm{H}}=\left|V_{\mathrm{L}} \mathrm{~L}+j X_{\mathrm{l}, 1 \mathrm{~L}}\right|=2347 \mathrm{~V}
$$

and the power factor is

$$
\mathrm{pf}=\frac{P_{\mathrm{L}}}{V_{\mathrm{H}} I_{\mathrm{L}}{ }^{0}}=0.957 \text { lagging }
$$

here we know that it is lagging because the transformer is inductive.

Problem 2-13

At $50 \mathrm{~Hz}, X_{1}=39.3 \times(5 / 6)=32.8 \Omega$. The load voltage as referred to the high-voltage side is $V_{\mathrm{L}}{ }^{0}=362 \times 2400 / 460=1889 \mathrm{~V}$. Thus the load current as referred to the high voltage side is

$$
I_{\mathrm{L}}^{0}=\frac{P_{\mathrm{L}}}{V_{\mathrm{L}}{ }^{0}}=18.3 \mathrm{~A}
$$

and the voltage at the high voltage terminals is

$$
V_{\mathrm{H}}=\left|V_{\mathrm{L}}^{0}+j X_{\mathrm{i}, 1} I \mathrm{~L}\right|=1981 \mathrm{~V}
$$

Problem 2-14

Part (a):

(ii)

Part (b):

$$
\hat{I}_{\text {load }}=\frac{40 \mathrm{~kW}}{240 \mathrm{~V}} e^{j \varphi}=166.7 e^{j \varphi} \mathrm{~A}
$$

where $\varphi$ is the power-factor angle. Referred to the high voltage side, $\hat{I}_{\mathrm{H}}=5.02 \mathrm{e}^{j \varphi} \mathrm{~A}$.

$$
\hat{V}_{\mathrm{H}}=Z_{\mathrm{H}} \hat{I}_{\mathrm{H}}
$$

Thus, (i) for a power factor of 0.87 lagging, $V_{\mathrm{H}}=7820 \mathrm{~V}$ and (ii) for a power factor of 0.87 leading, $V_{H}=7392 \mathrm{~V}$.
part (c):


Problem 2-15

## Part (a):


(ii)

Part (b):

$$
\hat{I}_{\mathrm{load}}=\frac{40 \mathrm{~kW}}{240 \mathrm{~V}} e^{j \varphi}=326 \cdot 1 e^{j \varphi} \mathrm{~A}
$$

where $\varphi$ is the power-factor angle. Referred to the high voltage side, $\hat{I}_{\mathrm{H}}=19.7 \mathrm{e}^{j \varphi} \mathrm{~A}$.

$$
\hat{V}_{\mathrm{H}}=Z_{\mathrm{H}} \hat{I}_{\mathrm{H}}
$$

Thus, (i) for a power factor of 0.87 lagging, $V_{\mathrm{H}}=3758 \mathrm{~V}$ and (ii) for a power factor of 0.87 leading, $V_{\mathrm{H}}=3570 \mathrm{~V}$.
part (c):


Problem 2-16
$\operatorname{Part}(\mathrm{a}): \hat{\mathrm{I}}_{\text {load }}=(178 / .78) \mathrm{kVA} / 2385 \mathrm{~V}=72.6 \mathrm{~A}$ at $\underline{6}=\cos ^{-1}(0.78)=38.7^{\circ}$

$$
\hat{V}_{\mathrm{t}, \mathrm{H}}=N\left(\hat{V}_{\mathrm{L}}+Z_{\mathrm{t}} \hat{I}_{\text {load }}\right)=34.1 \mathrm{kV}
$$

Part (b):

$$
\hat{V}_{\text {send }}=N\left(\hat{V}_{\mathrm{L}}+\left(Z_{\mathrm{t}}+Z_{\mathrm{f}}\right) \hat{I}_{\text {load }}\right)=33.5 \mathrm{kV}
$$

Part (c): $\hat{I}_{\text {send }}=\hat{I}_{\text {load }} / N$ and

$$
S_{\text {send }}=P_{\text {send }}+j Q_{\text {send }}=\hat{V}_{\text {send }} \hat{I}_{\text {send }}^{*}=138 \mathrm{~kW}-j 93.4 \mathrm{kVAR}
$$

Thus $P_{\text {send }}=138 \mathrm{~kW}$ and $Q_{\text {send }}=-93.4 \mathrm{kVAR}$.

Problem 2-17

## Part (a):

$\mathrm{pf}=0.78$ leading:

$$
\operatorname{part}(\mathrm{a}): \quad V_{\mathrm{t}, \mathrm{H}}=34.1 \mathrm{kV}
$$

part (b): $V_{\text {send }}=33.5 \mathrm{kV}$
part (c): $P_{\text {send }}^{\text {se }}=138.3 \mathrm{~kW}, Q_{\text {send }}=-93.4 \mathrm{kVA}$
$\mathrm{pf}=$ unity:
part (a): $V_{\mathrm{t}, \mathrm{H}}=35.0 \mathrm{kV}$
part (b): $V_{\text {send }}=35.4 \mathrm{kV}$
part (c): $P_{\text {send }}=137.0 \mathrm{~kW}, Q_{\text {send }}=9.1 \mathrm{kVA}$
$\mathrm{pf}=0.78$ lagging:

$$
\begin{aligned}
& \operatorname{part}(\mathrm{a}): V_{\mathrm{t}, \mathrm{H}}=35.8 \mathrm{kV} \\
& \text { part (b): } V_{\text {send }}=37.2 \mathrm{kV} \\
& \operatorname{part}(\mathrm{c}): P_{\text {send }}=138.335 \mathrm{~kW}, Q_{\text {send }}=123.236 \mathrm{kVA}
\end{aligned}
$$

Part (b):


Problem 2-18

Following the methodology of Example 2.6, efficiency $=98.4$ percent and regulation $=$ 1.25 percent.

Problem 2-19

Part (a): The core cross-sectional area increases by a factor of two thus the primary voltage must double to 22 kV to produce the same core flux density.

Part (b): The core volume increases by a factor of $2 \quad-\quad$ and thus the excitation kVA must increase by the same factor which means that the current must increase by a factor of $\overline{2}$ to 0.47 A and the power must increase by a factor of 22 to 7.64 kW .

## Problem 2-20

Part (a):

$$
\begin{gathered}
\left|Z_{\mathrm{eq}, \mathrm{H}}\right|=\frac{V_{\mathrm{sc}, \mathrm{H}}}{I_{\mathrm{sc}, \mathrm{H}}}=14.1 \Omega \\
R_{\mathrm{eq}, \mathrm{H}}=\frac{P_{\mathrm{sc}, \mathrm{H}}}{I_{\mathrm{sc}, \mathrm{H}}^{2}}=752 \mathrm{~m} \Omega \\
X_{\mathrm{eq}, \mathrm{H}}=\frac{\mathrm{q}}{\left|Z_{\mathrm{eq}, \mathrm{H}}\right|^{2}-R_{\mathrm{e}, \mathrm{H}}^{2}}=14.1 \Omega
\end{gathered}
$$

and thus

$$
Z_{\mathrm{eq}, \mathrm{H}}=0.75+j 14.1 \Omega
$$

Part (b): With $N=78 / 8=9.75$

$$
R_{\mathrm{eq}, \mathrm{~L}}=N^{2}=7.91 \mathrm{~m} \Omega
$$

$$
X_{\mathrm{eq}, \mathrm{~L}}=\frac{X_{\mathrm{eq}, \mathrm{H}}}{N^{2}}=148 \mathrm{~m} \Omega
$$

and thus

$$
Z_{\text {eq }, \mathrm{L}}=7.9+j 148 \mathrm{~m} \Omega
$$

Part (c): From the open-circuit test, the core-loss resistance and the magnetizing reactance as referred to the low-voltage side can be found:

$$
\begin{gathered}
R_{\mathrm{c}, \mathrm{~L}}=\frac{V_{\mathrm{oc}, \mathrm{~L}}^{2}}{P_{\mathrm{oc}, \mathrm{~L}}}=742 \Omega \\
S_{\mathrm{oc}, \mathrm{~L}}=V_{\mathrm{oc}, \mathrm{~L}} I_{\mathrm{oc}, \mathrm{~L}}=317 \mathrm{kVA} ; \quad Q_{\mathrm{oc}, \mathrm{~L}}=\mathrm{a}_{\overline{S_{\mathrm{oc}, \mathrm{~L}}^{2}-P_{\mathrm{oc}, \mathrm{~L}}^{2}}} 305 \mathrm{kVAR}
\end{gathered}
$$

and thus

$$
X_{\mathrm{m}, \mathrm{~L}}=\frac{V_{\mathrm{oc}, \mathrm{~L}}^{2}}{Q_{\mathrm{oc}, \mathrm{~L}}}=210 \Omega
$$

The equivalent-T circuit for the transformer from the low-voltage side is thus:


Problem 2-21

Part (a):

$$
\begin{gathered}
\left|Z_{\mathrm{eq}, \mathrm{H}}\right|=\frac{V_{\mathrm{sc}, \mathrm{H}}}{I_{\mathrm{sc}, \mathrm{H}}}=14.1 \Omega \\
R_{\mathrm{eq}, \mathrm{H}}=\frac{P_{\mathrm{sc}, \mathrm{H}}}{I_{\mathrm{sc}, \mathrm{H}}^{2}}=752 \mathrm{~m} \Omega \\
X_{\mathrm{eq}, \mathrm{H}}=\frac{\mathrm{q}}{\left|Z_{\mathrm{eq}, \mathrm{H}}\right|^{2}-R_{\mathrm{R}^{2}, \mathrm{H}}}=14.1 \Omega
\end{gathered}
$$

and thus

$$
Z_{\mathrm{eq}, \mathrm{H}}=0.75+j 14.1 \Omega
$$

Part (b): With $N=78 / 8=9.75$

$$
\begin{aligned}
& R_{\mathrm{eq}, \mathrm{~L}}=\frac{R_{\mathrm{eq}, \mathrm{H}}}{N^{2}}=7.91 \mathrm{~m} \Omega \\
& X_{\mathrm{eq}, \mathrm{~L}}=\frac{X_{\mathrm{eq}, \mathrm{H}}}{N_{2}}=148 \mathrm{~m} \Omega
\end{aligned}
$$

and thus

$$
Z_{\mathrm{eq}, \mathrm{~L}}=7.9+j 148 \mathrm{~m} \Omega
$$

Part (c): From the open-circuit test, the core-loss resistance and the magnetizing reactance as referred to the low-voltage side can be found:

$$
R_{\mathrm{c}, \mathrm{~L}}=\frac{V_{\mathrm{c}, \mathrm{~L}}}{P_{\mathrm{oc}, \mathrm{~L}}}=742 \Omega
$$

$$
S_{\mathrm{oc}, \mathrm{~L}}=V_{\mathrm{oc}, \mathrm{~L}} I_{\mathrm{oc}, \mathrm{~L}}=317 \mathrm{kVA} ; \quad Q_{\mathrm{oc}, \mathrm{~L}}=\mathrm{q}_{\overline{S_{\mathrm{oc}, \mathrm{~L}}^{2}-P_{\mathrm{oc}, \mathrm{~L}}^{2}}}=305 \mathrm{kVAR}
$$

and thus

$$
X_{\mathrm{m}, \mathrm{~L}}=\frac{V_{\mathrm{oc}, \mathrm{~L}}^{2}}{Q_{\mathrm{oc}, \mathrm{~L}}}=210 \Omega
$$

The equivalent-T circuit for the transformer from the low-voltage side is thus:


Problem 2-22

Parts (a) \& (b): For $V_{o c, \mathrm{~L}}=7.96 \mathrm{kV}, I_{\mathrm{oc}, \mathrm{L}}=17.3 \mathrm{~A}$ and $P_{\mathrm{oc}, \mathrm{L}}=48 \mathrm{~kW}$

$$
\begin{gathered}
R_{\mathrm{c}, \mathrm{~L}}=\frac{V_{\mathrm{oc}, \mathrm{~L}}^{2}}{P_{\mathrm{oc}, \mathrm{~L}}}=1.32 \mathrm{k} \Omega \quad R_{\mathrm{c}, \mathrm{H}}=N^{2} R_{\mathrm{c}, \mathrm{~L}}=33.0 \mathrm{k} \Omega \\
Q_{\mathrm{oc}, \mathrm{~L}}=\mathrm{a}_{\overline{S_{\mathrm{oc}, \mathrm{~L}}^{2}-P_{\mathrm{oc}, \mathrm{~L}}^{2}}=}=\left(V_{\mathrm{oc}, \mathrm{~L}} I_{\mathrm{oc}, \mathrm{~L}}-P \mathrm{c}_{\mathrm{c}, \mathrm{~L}}\right)^{2}=129 \mathrm{kVAR} \\
X_{\mathrm{m}, \mathrm{~L}}= \\
\frac{V_{\mathrm{oc} \mathrm{~L}}^{2}}{Q_{\mathrm{oc}, \mathrm{~L}}}=491 \Omega \quad X_{\mathrm{m}, \mathrm{H}}=N^{2} X_{\mathrm{m}, \mathrm{~L}}=12.3 \mathrm{k} \Omega
\end{gathered}
$$

For $V_{\mathrm{sc}, \mathrm{H}}=1.92 \mathrm{kV}, I_{\mathrm{oc}, \mathrm{L}}=252 \mathrm{~A}$ and $P_{\mathrm{oc}, \mathrm{L}}=60.3 \mathrm{~kW}$

$$
\underline{P s c}, \mathrm{H}
$$

$$
R_{\mathrm{H}}=I_{\mathrm{sc}, \mathrm{H}}^{2}=950 \mathrm{~m} \Omega
$$

$$
\begin{aligned}
& \text { q } \\
& \text { q } \\
& X_{\mathrm{H}}=Z^{\mathrm{A}}-R^{\mathrm{R}}=\left(V_{\mathrm{sc}, \mathrm{H}} / I_{\mathrm{sc}, \mathrm{H}}\right)^{2}-R_{\mathrm{H}}^{2}=7.56 \Omega \\
& R_{\mathrm{L}}=\frac{R_{\mathrm{H}}}{\overline{N^{2}}}=38.0 \mathrm{~m} \Omega \quad X_{\mathrm{L}}={ }_{X_{\mathrm{H}}}^{\overline{N^{2}}}=302 \mathrm{~m} \Omega
\end{aligned}
$$

## Part (c):

$$
P_{\mathrm{diss}}=P_{\mathrm{oc}, \mathrm{~L}}+P_{\mathrm{sc}, \mathrm{H}}=108 \mathrm{~kW}
$$

Problem 2-23

Parts (a) \& (b): For $V_{o c, L}=3.81 \mathrm{kV}, I_{\mathrm{oc}, \mathrm{L}}=9.86 \mathrm{~A}$ and $P_{\mathrm{oc}, \mathrm{L}}=8.14 \mathrm{~kW}$

$$
\begin{aligned}
R_{\mathrm{c}, \mathrm{~L}} & =\frac{V_{\mathrm{oc}, \mathrm{~L}}^{2}}{P_{\mathrm{oc}, \mathrm{~L}}}=1.78 \mathrm{k} \Omega \quad R_{\mathrm{c}, \mathrm{H}}=N^{2} R_{\mathrm{c}, \mathrm{~L}}=44.8 \mathrm{k} \Omega \\
Q_{\mathrm{oc}, \mathrm{~L}} & =\frac{\mathrm{a}_{\overline{\mathrm{oc}, \mathrm{~L}}}-P_{\mathrm{oc}, \mathrm{~L}}^{2}}{}=\frac{\mathrm{a}^{2}}{\left(V_{\mathrm{oc}, \mathrm{~L}} I_{\mathrm{oc}, \mathrm{~L}}-P_{\mathrm{cc}, \mathrm{~L})^{2}}^{2}\right.}=36.7 \mathrm{kVAR} \\
X_{\mathrm{m}, \mathrm{~L}} & =\frac{V_{\mathrm{oc}, \mathrm{~L}}^{2}}{Q_{\mathrm{oc}, \mathrm{~L}}}=395 \Omega \quad X_{\mathrm{m}, \mathrm{H}}=N^{2} X_{\mathrm{m}, \mathrm{~L}}=9.95 \mathrm{k} \Omega
\end{aligned}
$$

For $V_{\mathrm{sc}, \mathrm{H}}=920 \mathrm{~V}, I_{\mathrm{oc}, \mathrm{L}}=141 \mathrm{~A}$ and $P_{\mathrm{oc}, \mathrm{L}}=10.3 \mathrm{~kW}$

$$
\begin{gathered}
R_{\mathrm{H}}=\frac{P_{\mathrm{sc}, \mathrm{H}}}{I_{\mathrm{sc}, \mathrm{H}}^{2}}=518 \mathrm{~m} \Omega \\
X_{\mathrm{H}}=\mathrm{a}_{\overline{Z \mathrm{~A}}-R^{2}=}^{\mathrm{a}_{\overline{\mathrm{I}}}} \frac{\left(V_{\mathrm{sc}, \mathrm{H}} / I_{\mathrm{sc}, \mathrm{H}}\right)^{2}-R_{\mathrm{H}}^{2}=6.50 \Omega}{}
\end{gathered}
$$

$$
R_{\mathrm{L}}=\frac{R_{\mathrm{H}}}{N^{2}}=20.6 \mathrm{~m} \Omega \quad X_{\mathrm{L}}=\frac{X_{\mathrm{H}}}{N^{2}}=259 \mathrm{~m} \Omega
$$

Part (c):

$$
P_{\mathrm{diss}}=P_{\mathrm{oc}, \mathrm{~L}}+P_{\mathrm{sc}, \mathrm{H}}=18.4 \mathrm{~kW}
$$

Problem 2-24

Solution the same as Problem 2-22

Problem 2-25

Part (a): $7.69 \mathrm{kV}: 79.6 \mathrm{kV}, 10 \mathrm{MVA}$

Part (b): $17.3 \mathrm{~A}, 48.0 \mathrm{~kW}$

Part (c): Since the number of turns on the high-voltage side have doubled, this will occur at a voltage equal to twice that of the original transformer, i.e. 3.84 kV .

Part (d): The equivalent-circuit parameters referred to the low-voltage side will be unchanged from those of Problem 2-22. Those referred to the high-voltage side will have 4 times the values of Problem 2-22.

$$
\begin{aligned}
& R_{\mathrm{c}, \mathrm{~L}}=1.32 \mathrm{k} \Omega \quad R_{\mathrm{c}, \mathrm{H}}=132 \mathrm{k} \Omega \\
& X_{\mathrm{m}, \mathrm{~L}}=491 \Omega \quad X_{\mathrm{m}, \mathrm{H}}=49.1 \mathrm{k} \Omega \\
& R_{\mathrm{L}}=38.0 \mathrm{~m} \Omega R_{\mathrm{H}}=3.80 \Omega \\
& X_{\mathrm{L}}=302 \mathrm{~m} \Omega \quad X_{\mathrm{H}}=30.2 \Omega
\end{aligned}
$$

Problem 2-26

Part (a): Under this condition, the total transformer power dissipation is 163.7 kW . Thus the efficiency is

$$
\eta=100 \times \frac{25 \mathrm{MW}}{25 \mathrm{MW}+163.7 \mathrm{~kW}}=99.4 \%
$$

From Problem 2-20, the transformer equivalent series impedance from the low voltage side is $Z_{\text {eq, } \mathrm{L}}=7.91+j 148 \mathrm{~m} \Omega$. The transformer rated current is $I_{\mathrm{rated}}=3125 \mathrm{~A}$ and thus under load the transformer high-side voltage (neglecting the effects of magnetizing current) referred to the primary is

$$
\left|V_{\mathrm{H}}^{0}\right|=\left|V_{\mathrm{L}}-I_{\mathrm{rated}} Z_{\text {eq }, \mathrm{L}}\right|=7.989 \mathrm{kV}
$$

and thus the voltage regulation is $100 \times(7.989-8.00) / 7.989=0.14 \%$.

Part (b): Same methodology as part (a) except that the load is 22.5 MW and the current is $\hat{I}=I_{\text {rated }}{ }^{6} \varphi$ where $\varphi=\cos ^{-1}(0.9)=25.8^{\circ}$. In this case, the efficiency is $99.3 \%$ and the regulation is $1.94 \%$.

Problem 2-27

Part (a):


Part (b);


## Problem 2-28

Part (a): The transformer loss will be equal to the sum of the open-circuit and short- circuit losses, i.e. 313 W . With a load of $0.85 \times 25=21.25 \mathrm{~kW}$, the efficiency is equalto

$$
\eta=\frac{21.25}{21.25+0.313}=0.9855=98.55 \%
$$

Part (b): The transformer equivalent-circuit parameters are found as is shown in the solution to Problem 2-23.

$$
\begin{aligned}
& R_{\mathrm{c}, \mathrm{~L}}=414 \Omega R_{\mathrm{c}, \mathrm{H}}=41.4 \mathrm{k} \Omega \\
& X_{\mathrm{m}, \mathrm{~L}}=193 \Omega \quad X_{\mathrm{m}, \mathrm{H}}=19.3 \mathrm{k} \Omega \\
& R_{\mathrm{L}}=17.1 \mathrm{~m} \Omega R_{\mathrm{H}}=1.71 \Omega \\
& X_{\mathrm{L}}=64.9 \mathrm{~m} \Omega X_{\mathrm{H}}=6.49 \Omega
\end{aligned}
$$

The desired solution is 0.963 leading power factor, based upon a MATLAB search for the load power factor that corresponds to rated voltage at both the low- and high-voltage terminals.

Problem 2-29

Efficiency $=98.4 \%$ and regulation $=2.38 \%$.

Problem 2-30

The voltage rating is $280 \mathrm{~V}: 400 \mathrm{~V}$. The rated current of the high voltage terminal is equal to that of the $120-\mathrm{V}$ winding, $I_{\text {rated }}=45 \times 10^{3} / 120=375 \mathrm{~A}$. Hence the kVA rating of the transformer is $400 \times 375=150 \mathrm{kVA}$.

Problem 2-31

Part (a):


Part (b): The rated current of the high voltage terminal is equal to that of the $120-\mathrm{V}$ winding, $I_{\text {rated }}=10^{4} / 120=83.3$ A. Hence the kVArating of the transformer is $600 \times 83.3=$ 50 kVA .

Part (c): The full load loss is equal to that of the transformer in the conventional connection, $P_{\text {loss }}=(1-0.979) 10 \mathrm{~kW}=180 \mathrm{~W}$. Hence as an autotransformer operating with a load at 0.93 power factor ( $\left.P_{\text {load }}=0.93 \times 50 \mathrm{~kW}=46.5 \mathrm{~kW}\right)$, the efficiency will be

$$
\eta=\frac{46.5 \mathrm{~kW}}{46.78 \mathrm{~kW}}=0.996=99.6 \text { percent }
$$

Problem 2-32

Part (a): The voltage rating is $78 \mathrm{kV}: 86 \mathrm{kV}$. The rated current of the high voltage terminal is equal to that of the $8-\mathrm{kV}$ winding, $I_{\text {rated }}=50 \times 106 / 8000=3.125 \mathrm{kA}$. Hence the
kVA rating of the transformer is $86 \mathrm{kV} \times 3.125 \mathrm{kA}=268.8 \mathrm{MVA}$.

Part (b): The loss at rated voltage and current is equal to 164 kW and hence the efficiency will be

$$
\eta=\frac{268.8 \mathrm{MW}}{268.96 \mathrm{MW}}=0.9994=99.94 \text { percent }
$$

Problem 2-33

MATLAB script should reproduce the answers to Problem 2-32.

Problem 2-34

Part (a): $7.97 \mathrm{kV}: 2.3 \mathrm{kV} ; \quad 188 \mathrm{~A}: 652 \mathrm{~A} ; \quad 1500 \mathrm{kVA}$

Part (b): $13.8 \mathrm{kV}: 1.33 \mathrm{kV} ; \quad 109 \mathrm{~A}: 1130 \mathrm{~A} ; \quad 1500 \mathrm{kVA}$

Part (c): $7.97 \mathrm{kV}: 1.33 \mathrm{kV} ; \quad 188 \mathrm{~A}: 1130 \mathrm{~A} ; \quad 1500 \mathrm{kVA}$
part (d): $13.8 \mathrm{kV}: 2.3 \mathrm{kV} ; \quad 109 \mathrm{~A}: 652 \mathrm{~A} ; \quad 1500 \mathrm{kVA}$

Problem 2-35

Part (a):
(i) $68.9 \mathrm{kV}: 230 \mathrm{kV}, 225 \mathrm{MVA}$
(ii) $Z_{\text {eq }}=0.087+j 1.01 \Omega$
(iii) $Z_{\text {eq }}=0.97+j 11.3 \Omega$

Part (b):
(i) $68.9 \mathrm{kV}: 133 \mathrm{kV}, 225 \mathrm{MVA}$
(ii) $Z_{\text {eq }}=0.087+j 1.01 \Omega$
(iii) $Z_{\text {eq }}=0.32+j 3.77 \Omega$

Problem 2-36

Part (a):
(i) $480 \mathrm{~V}: 13.8 \mathrm{kV}, 675 \mathrm{kVA}$
(ii) $Z_{\text {eq }}=0.0031+j 0.0215 \Omega$
(iii) $Z_{\text {eq }}=2.57+j 17.8 \Omega$

Part (b):
(i) $480 \mathrm{~V}: 7.97 \mathrm{kV}, 675 \mathrm{MVA}$
(ii) $Z_{\text {eq }}=0.0031+j 0.0215 \Omega$
(iii) $Z_{\text {eq }}=0.86+j 5.93 \Omega$

Problem 2-37

Following the methodology of Example 2.8, $V_{\text {load }}=236 \mathrm{~V}$, line-to-line.

Problem 2-38

Part (a): The rated current on the high-voltage side of the transformer is

$$
I_{\text {rated }, \mathrm{H}}=\frac{25 \mathrm{MVA}}{3 \times 68 \mathrm{kV}}=209 \mathrm{~A}
$$

The equivalent series impedance reflected to the high-voltage side is

$$
Z_{\mathrm{eq}, \mathrm{H}}=N^{2} Z_{\mathrm{eq}, \mathrm{~L}}=1.55+j 9.70 \Omega
$$

and the corresponding line-neutral voltage magnitude is

$$
V_{\mathrm{H}}=I_{\text {rated }, \mathrm{H}}\left|Z_{\text {eq }, \mathrm{H}}\right|=2.05 \mathrm{kV}
$$

corresponding to a line-line voltage of 3.56 kV .

Part (b): The apparent power at the high-voltage winding is $S=18 / .75=24 \mathrm{MVA}$ and the corresponding current is

$$
I_{\text {load }}=\frac{24 \mathrm{MVA}}{\overline{3} \times 68 \mathrm{kV}}=209 \mathrm{~A}
$$

The power factor angle $\theta=-\cos ^{-1}(0.75)=-41.4^{\circ}$ and thus

$$
\hat{I}_{\text {load }}=2096-41.4^{\circ}
$$

With a high-side line-neutral voltage $V_{\mathrm{H}}=69 \mathrm{kV} / 3 \equiv 39.8 \mathrm{kV}$, referred to the highvoltage side, the line-neutral load voltage referred to the high-voltage side is thus

$$
V_{\text {load }}^{0}=\left|V_{\mathrm{H}}-\hat{I}_{\text {load }} Z_{\text {eq }, \mathrm{H}}\right|=38.7 \mathrm{kV}
$$

Referred to the low-voltage winding, the line-neutral load voltage is

$$
V_{\text {load }}=\frac{13.8}{69} V_{\text {load }}^{0}=7.68 \mathrm{kV}
$$

corresponding to a line-line voltage of 13.3 kV .

Problem 2-39
Part (a): The line-neutral load voltage $V_{\text {load }}=24 \mathrm{kV} /{ }^{\sqrt{ }} 3=13.85 \mathrm{kV}$ and the load current is

$$
\hat{I}_{\text {load }}=\frac{\sqrt{-}}{\frac{\sqrt{ }}{} \mathrm{MVA}^{!}} \mathrm{e}^{j 24 \mathrm{kV}} e^{j \varphi}=9.02 e^{j \varphi} \mathrm{kA}
$$

where $\varphi=\cos ^{-1} 0.89=27.1^{\circ}$.
The transformer turns ratio $N=9.37$ and thus referred to the high voltage side, $V^{0}{ }_{\text {load }}=$ $N V_{\text {load }}=129.9 \mathrm{kV}$ and ${ }^{\hat{l}_{0}}$ load $=\hat{I}_{\text {load }} / N=962 e^{j \varphi}$ A. Thus, the transformer high-side lineneutral terminal voltage is

$$
V_{\mathrm{H}}=\left|V_{\mathrm{L}}^{0}+j X_{\mathrm{t}} \hat{I}_{\text {load }}^{0}\right|=127.3 \mathrm{kV}
$$

corresponding to a line-line voltage of 220.6 kV .

Part (b): In a similar fashion, the line-neutral voltage at the source end of the feeder is given by

$$
V_{s}=\left|V_{\mathrm{L}}^{0}+\left(Z_{\mathrm{f}}+j X_{\mathrm{t}}\right) \hat{I}_{\text {load }}^{0}\right|=126.6 \mathrm{kV}
$$

corresponding to a line-line voltage of 219.3 kV .

Problem 2-40


Problem 2-41

Part (a): For a single transformer

$$
R_{\mathrm{eq}, \mathrm{H}}=\frac{P_{\mathrm{sc}}}{I_{\mathrm{sc}}^{2}}=342 \mathrm{~m} \Omega
$$

$$
\begin{gathered}
S_{\mathrm{sc}}=V_{\mathrm{sc}} I_{\mathrm{sc}}=8.188 \mathrm{kVA} \\
Q_{\mathrm{sc}}=\frac{\mathrm{q}}{S_{\mathrm{s} 2 \mathrm{c}}-P_{\mathrm{s}}^{2 \mathrm{c}}}=8.079 \mathrm{kVAR}
\end{gathered}
$$

and thus

$$
X_{\mathrm{eq}, \mathrm{H}}=\frac{Q_{\mathrm{sc}}}{I^{2}}=2.07 \Omega
$$

For the there-phase bank with the high-voltage side connected in $\Delta$, the transformer series impedance reflected to the high-voltage side will be $1 / 3$ of this value. Thus

Part (b): Referred to the high voltage side, the line-neutral load voltage is $V_{\text {load }}=$ $2400 /$ sqrt $3=1386 \mathrm{~V}$ and the $450-\mathrm{kW}$ load current will be

$$
I_{\text {load }}=\frac{P_{\text {load }}}{3 V_{\text {load }}}=108 \mathrm{~A}
$$

Thus the line-neutral source voltage is

$$
V_{\mathrm{s}}=\left|V_{\text {load }}+\left(Z_{\mathrm{t}, \mathrm{H}}+Z_{\mathrm{f}}\right) I_{\text {load }}\right|=1.40 \mathrm{kV}
$$

corresponding to a line-line voltage of 2.43 kV .
Problem 2-42

Part (a): The transformer turns ratio is $N=13800 / 120=115$. The secondary voltage will thus be

$$
\begin{gathered}
V_{2}=\frac{V_{1}}{j X_{\mathrm{m}}} \\
N R_{1}+j X_{1}+j X_{m}
\end{gathered}
$$

Part (b): Defining $R_{\mathrm{L}}=N^{2} R_{\mathrm{L}}=9.92 \mathrm{M} \Omega$ and

$$
\begin{gathered}
Z_{\mathrm{eq}}=j X_{\mathrm{m}} \|\left(R_{2}^{\ell}+R_{\mathrm{L}}^{\mathrm{L}}+j X_{\mathrm{Z}}\right) \\
\nabla_{2}=\frac{V_{1}}{} \frac{Z_{\mathrm{eq}}}{\underline{!}}=119.80 \underline{6} 0.012^{\circ} \\
R_{1}+j X_{1}+Z_{\mathrm{eq}}
\end{gathered}
$$

Part (c): Defining $X \mathrm{~L}=N^{2} X_{\mathrm{L}}=9.92 \mathrm{M} \Omega$ and

$$
Z_{\mathrm{eq}}=j X_{\mathrm{m}} \|\left(R+j X_{\mathrm{L}}+j X_{\mathrm{t}}\right)
$$

$$
\begin{aligned}
\nabla_{2}= & \frac{V_{1}}{N} \frac{Z_{\mathrm{eq}}}{\underline{!}}=119.78 \underline{6} 0.083^{\circ} \\
& R_{1}+j X_{1}+Z_{\mathrm{eq}}
\end{aligned}
$$

## Problem 2-43

Following the methodology of Part (c) of Problem 2-42 and varying $X_{\mathrm{L}}$ one finds that the minimum reactance is $80.9 \Omega$.

## Problem 2-44

This solution uses the methodology of Problem 2-42.

## Part (a):



Part (b):


Problem 2-45

Part (a): For $I_{1}=150 \mathrm{~A}$ and turns ratio $N=150 / 5=30$

$$
\hat{I}_{2}=\frac{I_{1}}{N} \frac{j X_{\mathrm{m}}}{R_{2}^{0}+j\left(X_{\mathrm{m}}+X^{2}\right)}=4.995{ }_{-}^{6} 0.01^{\circ} \mathrm{A}
$$

Part (b): With $R_{\mathrm{b}}=0.1 \mathrm{~m} \Omega$ and $R_{\mathrm{b}}=N^{2} R_{\mathrm{b}}=90 \mathrm{~m} \Omega$

$$
\hat{I}_{2}=\frac{I_{1}}{N} \frac{j X_{\mathrm{m}}}{R_{\mathfrak{Z}}+R^{b}+j\left(X_{\mathrm{m}}+X^{\mathbb{Q}}\right)}=4.988^{6} 2.99^{\circ} \mathrm{A}
$$

Problem 2-46

This solution uses the methodology of Problem 2-45.

Part (a):


Part (b):


Problem 2-47

The base impedance on the high-voltage side of the transformer is

$$
Z_{\text {base }, \mathrm{H}}=\frac{V_{\text {ated }, \mathrm{H}}^{2}}{P_{\text {rated }}}=136.1 \Omega
$$

Thus, in Ohms referred to the high-voltage side, the primary and secondary impedances are

$$
Z=(0.0029+j 0.023) Z_{\text {base }, \mathrm{H}}=0.29+j 23.0 \mathrm{~m} \Omega
$$

and the magnetizing reactance is similarly found to be $X_{\mathrm{m}}=172 \Omega$.

## Problem 2-48

From the solution to Problem 2-20, as referred to the low voltage side, the total series impedance of the transformer is $7.92+j 148.2 \mathrm{~m} \Omega$, the magnetizing reactance is $210 \Omega$ and the core-loss resistance is $742 \Omega$. The low-voltage base impedance of this transformer is

$$
Z_{\text {base }, \mathrm{L}}=\frac{\left(8 \times 10^{3}\right)^{2}}{25 \times 10^{6}}=2.56 \Omega
$$

and thus the per-unit series impedance is $0.0031+j 0.0579$, the per-unit magnetizing reactance is 82.0 and the per-unit core-loss resistance is 289.8 .

Problem 2-49

From the solution to Problem 2-23, as referred to the low voltage side, the total series impedance of the transformer is $20.6+j 259 \mathrm{~m} \Omega$, the magnetizing reactance is $395 \Omega$ and the core-loss resistance is $1780 \Omega$. The low-voltage base impedance of this transformer is

$$
Z_{\text {base, } \mathrm{L}}=\frac{\left(3.81 \times 10^{3}\right)^{2}}{2.5 \times 10^{6}}=5.81 \Omega
$$

and thus the per-unit series impedance is $0.0035+j 0.0446$, the per-unit magnetizing reactance is 68.0 and the per-unit core-loss resistance is 306.6 .

Problem 2-50

Part (a): (i) The high-voltage base impedance of the transformer is

$$
Z_{\text {base }, \mathrm{H}}=\frac{\left(7.97 \times 10^{3}\right)^{2}}{2.5 \times 10^{3}}=2.54 \mathrm{k} \Omega
$$

and thus the series reactance referred high-voltage terminal is

$$
X_{\mathrm{H}}=0.075 Z_{\text {base }, \mathrm{H}}=191 \Omega
$$

(ii) The low-voltage base impedance is $2.83 \Omega$ and thus the series reactance referred to the low-voltage terminal is $212 \mathrm{~m} \Omega$.

## Part (b):

(i) Power rating: $3 \times 25 \mathrm{kVA}=75 \mathrm{k} \underline{\mathrm{VA}}$

Voltage rating: $3 \times 7.97 \mathrm{kV}: 3 \times 266 \mathrm{~V}=13.8 \mathrm{kV}: 460 \mathrm{~V}$
(ii) The per-unit impedance remains 0.075 per-unit
(iii) Referred to the high-voltage terminal, $X_{\mathrm{H}}=191 \Omega$
(iv) Referred to the low-voltage terminal, $X_{\mathrm{L}}=212 \mathrm{~m} \Omega$

## Part (c):

(i) Power rating: $3 \times 25 \mathrm{kVA}=75 \mathrm{kVA}$

Voltage rating: $3 \times 7.97 \mathrm{kV}: 266 \mathrm{~V}=13.8 \mathrm{kV}: 266 \mathrm{~V}$
(ii) The per-unit impedance remains 0.075 per-unit
(iii) Referred to the high-voltage terminal, $X_{\mathrm{H}}=191 \Omega$
(iv) Referred to the low-voltage terminal, the base impedance is now $Z_{\text {base, } \mathrm{L}}=$ $266^{2} /\left(75 \times 10^{3}\right)=0.943 \Omega$ and thus $X_{\mathrm{L}}=0.943 \times 0.075=70.8 \mathrm{~m} \Omega$

## Problem 2-51

Part (a): 500 V at the high-voltage terminals is equal to $500 / 13.8 \times 10^{3}=0.0362$ per unit. Thus the per-unit short-circuit current will be

$$
I_{\mathrm{sc}}=\frac{0.0363}{0.075}=0.48 \text { perunit }
$$

(i) The base current on the high-voltage side is

$$
I_{\text {base }, \mathrm{L}}=\frac{\sqrt{ }{ }^{75} \times^{10^{3}}}{-}=3.14 \mathrm{~A}
$$

$3 \times 13.8 \times 10$
and thus the short-circuit current at the high-voltage terminals will equal

$$
I_{\mathrm{sc}, \mathrm{H}}=0.48 \times 3.14=1.51 \mathrm{~A}
$$

(ii) The base current on the low-voltage side is

$$
I_{\text {base }, \mathrm{L}}=\frac{75 \times 10^{3}}{\frac{-}{3} \times 460}=94.1 \mathrm{~A}
$$

and thus the short-circuit current at the low-voltage terminals will equal

$$
I_{\mathrm{sc}, \mathrm{~L}}=0.48 \times 94.1=45.4 \mathrm{~A}
$$

Part (b): The per-unit short-circuit current as well as the short-circuit current at the high-voltage terminals remains the same as for Part (a). The base current on the low-voltage side is now

$$
I_{\text {base }, \mathrm{L}}=\frac{75 \times 10^{3}}{\sqrt{3} \times 266}=163 \mathrm{~A}
$$

and thus the short-circuit current at the low-voltage terminals will equal

$$
I_{\mathrm{sc}, \mathrm{~L}}=0.48 \times 163=78.6 \mathrm{~A}
$$

Problem 2-52

Part (a): On the transformer $26-\mathrm{kV}$ base, the transformer base impedance is

$$
Z_{\text {base }, \mathrm{t}}=\frac{26^{2}}{850}=0.795 \Omega
$$

and on the same voltage base, the generator base impedance is

$$
Z_{\text {base }, \mathrm{g}}=\frac{26^{2}}{800}=0.845 \Omega
$$

Thus, on the transformer base, the per-unit generator reactance is

$$
X_{\mathrm{g}}=1.28 \frac{Z_{\text {base }, \mathrm{g}}}{Z_{\text {base,t }}}=1.36 \text { perunit }
$$

Part (b):


Part (c): In per-unit on the transformer base,

$$
V_{\mathrm{t}, \mathrm{H}}=1.0 \text { per unit } P=\frac{750}{\overline{850}}=0.882 \text { per unit } \quad S=\frac{P}{\overline{0.9}}=0.980 \text { per unit }
$$

and thus

$$
\hat{I}=0.98 e^{j \varphi}
$$

where $\varphi=\cos ^{-1}(0.9)=25.8^{\circ}$
Thus, the per-unit generator terminal voltage on the transformer voltage base is

$$
\hat{V}_{\mathrm{g}}=V_{\mathrm{t}, \mathrm{H}}+\left(R_{\mathrm{t}}+j X_{\mathrm{t}}\right) \hat{I}=0.979 \quad 63.01^{\circ} \text { per }- \text { unit }
$$

which corresponds to a terminal voltage of $0.979 \times 26 \mathrm{kV}=25.4 \mathrm{kV}$.
The per-unit generator internal terminal voltage on the transformer voltage base is

$$
\hat{E}_{\mathrm{af}}=V_{\mathrm{t}, \mathrm{H}}+\left(R_{\mathrm{t}}+j X_{\mathrm{t}}+j X_{\mathrm{g}}\right) \hat{I}=1.31 \quad \underline{6} \quad 72.4^{\circ} \text { per }- \text { unit }
$$

which corresponds to a terminal voltage of $1.31 \times 26 \mathrm{kV}=34.1 \mathrm{kV}$.
In per unit, the generator complex output power is

$$
S=\hat{V}_{\mathrm{g}} \hat{I}^{*}=0.884-j 0.373 \text { per unit }
$$

and thus the generator output power is $P_{\text {gen }}=0.884 \times 850=751.4$ MW. The generator power factor is

$$
\mathrm{pf}=\frac{P}{|S|}=0.92
$$

and it is leading.

