

**Solution Manual for Fundamentals of Electromagnetics with Engineering Applications 1st Edition
by Wentworth ISBN 9780470105757**

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SOLUTIONS MANUAL

FOR

Fundamentals of Electromagnetics
with Engineering Applications

1st Edition
By Wentworth
ISBN13-9780470105757

Solutions for Chapter 2 Problems

1. Vectors in the Cartesian Coordinate System

P2.1: Given $P(4,2,1)$ and $\mathbf{A}_{PQ}=2\mathbf{a}_x+4\mathbf{a}_y+6\mathbf{a}_z$, find the point Q .

$$\mathbf{A}_{PQ} = 2 \mathbf{a}_x + 4 \mathbf{a}_y + 6 \mathbf{a}_z = (Q_x - P_x)\mathbf{a}_x + (Q_y - P_y)\mathbf{a}_y + (Q_z - P_z)\mathbf{a}_z$$

$$Q_x - P_x = Q_x - 4 = 2; \quad Q_x = 6$$

$$Q_y - P_y = Q_y - 2 = 4; \quad Q_y = 6$$

$$Q_z - P_z = Q_z - 1 = 6; \quad Q_z = 7$$

Ans: $Q(6,6,7)$

P2.2: Given the points $P(4,1,0)\text{m}$ and $Q(1,3,0)\text{m}$, fill in the table and make a sketch of the vectors found in (a) through (f).

	Vector	Mag	Unit Vector
a. Find the vector \mathbf{A} from the origin to P	$\mathbf{A}_{OP} = 4 \mathbf{a}_x + 1 \mathbf{a}_y$	4.12	$\mathbf{a}_{OP} = 0.97 \mathbf{a}_x + 0.24 \mathbf{a}_y$
b. Find the vector \mathbf{B} from the origin to Q	$\mathbf{B}_{OQ} = 1 \mathbf{a}_x + 3 \mathbf{a}_y$	3.16	$\mathbf{a}_{OQ} = 0.32 \mathbf{a}_x + 0.95 \mathbf{a}_y$
c. Find the vector \mathbf{C} from P to Q	$\mathbf{C}_{PQ} = -3 \mathbf{a}_x + 2 \mathbf{a}_y$	3.61	$\mathbf{a}_{PQ} = -0.83 \mathbf{a}_x + 0.55 \mathbf{a}_y$
d. Find $\mathbf{A} + \mathbf{B}$	$\mathbf{A} + \mathbf{B} = 5 \mathbf{a}_x + 4 \mathbf{a}_y$	6.4	$\mathbf{a} = 0.78 \mathbf{a}_x + 0.62 \mathbf{a}_y$
e. Find $\mathbf{C} - \mathbf{A}$	$\mathbf{C} - \mathbf{A} = -7 \mathbf{a}_x + 1 \mathbf{a}_y$	7.07	$\mathbf{a} = -0.99 \mathbf{a}_x + 0.14 \mathbf{a}_y$
f. Find $\mathbf{B} - \mathbf{A}$	$\mathbf{B} - \mathbf{A} = -3 \mathbf{a}_x + 2 \mathbf{a}_y$	3.6	$\mathbf{a} = -0.83 \mathbf{a}_x + 0.55 \mathbf{a}_y$

a. $\mathbf{A}_{OP} = (4-0)\mathbf{a}_x + (1-0)\mathbf{a}_y + (0-0)\mathbf{a}_z = 4 \mathbf{a}_x + 1 \mathbf{a}_y$.

$$|\mathbf{A}_{OP}| = \sqrt{4^2 + 1^2} = \sqrt{17} = 4.12$$

$$\mathbf{a}_{OP} = \frac{4}{\sqrt{17}}\mathbf{a}_x + \frac{1}{\sqrt{17}}\mathbf{a}_y = 0.97\mathbf{a}_x + 0.24\mathbf{a}_y$$

(see Figure P2.2ab)

b. $\mathbf{B}_{OQ} = (1-0)\mathbf{a}_x + (3-0)\mathbf{a}_y + (0-0)\mathbf{a}_z = 1 \mathbf{a}_x + 3 \mathbf{a}_y$.

$$|\mathbf{B}_{OQ}| = \sqrt{1^2 + 3^2} = \sqrt{10} = 3.16$$

$$\mathbf{a}_{OQ} = \frac{1}{\sqrt{10}}\mathbf{a}_x + \frac{3}{\sqrt{10}}\mathbf{a}_y = 0.32\mathbf{a}_x + 0.95\mathbf{a}_y$$

(see Figure P2.2ab)

c. $\mathbf{C}_{PQ} = (1-4)\mathbf{a}_x + (3-1)\mathbf{a}_y + (0-0)\mathbf{a}_z = -3 \mathbf{a}_x + 2 \mathbf{a}_y$.

$$|\mathbf{C}_{PQ}| = \sqrt{3^2 + 2^2} = \sqrt{13} = 3.61$$

$$\mathbf{a}_{PQ} = \frac{-3}{\sqrt{13}}\mathbf{a}_x + \frac{2}{\sqrt{13}}\mathbf{a}_y = -0.83\mathbf{a}_x + 0.55\mathbf{a}_y$$

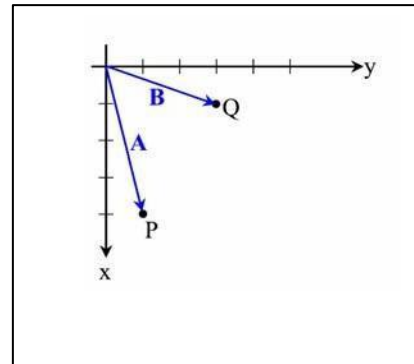


Fig. P2.2ab

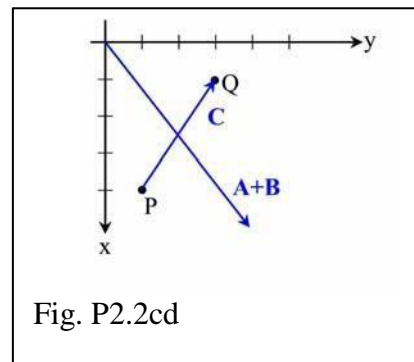


Fig. P2.2cd

(see Figure P2.2cd)

$$d. \mathbf{A} + \mathbf{B} = (4+1)\mathbf{a}_x + (1+3)\mathbf{a}_y + (0-0)\mathbf{a}_z = 5 \mathbf{a}_x + 4 \mathbf{a}_y.$$

$$|\mathbf{A} + \mathbf{B}| = \sqrt{5^2 + 4^2} = \sqrt{41} = 6.4$$

$$\mathbf{a} = \frac{5}{\sqrt{41}}\mathbf{a}_x + \frac{4}{\sqrt{41}}\mathbf{a}_y = 0.78\mathbf{a}_x + 0.62\mathbf{a}_y$$

(see Figure P2.2cd)

$$e. \mathbf{C} - \mathbf{A} = (-3-4)\mathbf{a}_x + (2-1)\mathbf{a}_y + (0-0)\mathbf{a}_z = -7 \mathbf{a}_x + 1 \mathbf{a}_y.$$

$$|\mathbf{C} - \mathbf{A}| = \sqrt{7^2 + 1^2} = \sqrt{50} = 7.07$$

$$\mathbf{a} = \frac{-7}{\sqrt{50}}\mathbf{a}_x + \frac{1}{\sqrt{50}}\mathbf{a}_y = -0.99\mathbf{a}_x + 0.14\mathbf{a}_y$$

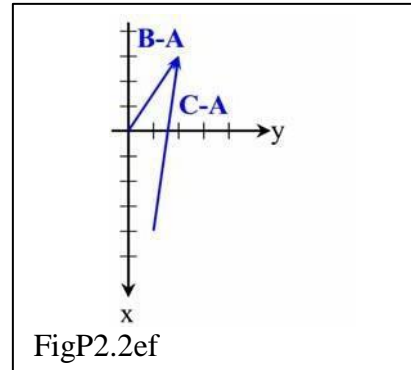
(see Figure P2.2ef)

$$f. \mathbf{B} - \mathbf{A} = (1-4)\mathbf{a}_x + (3-1)\mathbf{a}_y + (0-0)\mathbf{a}_z = -3 \mathbf{a}_x + 2 \mathbf{a}_y.$$

$$|\mathbf{B} - \mathbf{A}| = \sqrt{3^2 + 2^2} = \sqrt{13} = 3.6$$

$$\mathbf{a} = \frac{-3}{\sqrt{13}}\mathbf{a}_x + \frac{2}{\sqrt{13}}\mathbf{a}_y = -0.83\mathbf{a}_x + 0.55\mathbf{a}_y$$

(see Figure P2.2ef)



P2.3: MATLAB: Write a program that will find the vector between a pair of arbitrary points in the Cartesian Coordinate System.

A program or function for this task is really overkill, as it is so easy to perform the task. Enter points P and Q (for example, P=[1 2 3]; Q=[6 5 4]). Then, the vector from P to Q is simply given by Q-P.

As a function we could have:

```
function PQ=vector(P,Q)
% Given a pair of Cartesian points
% P and Q, the program determines the
% vector from P to Q.
PQ=Q-P;
```

Running this function we have:

```
>> P=[1 2 3];
>> Q=[6 5 4];
>> PQ=vector(P,Q)
```

```
PQ =
    5    3    1
```

Alternatively, we could simply perform the math in the command line window:

>> PQ=Q-P
 PQ =
 5 3 1
 >>

2. Coulomb's Law, Electric Field Intensity, and Field Lines

P2.4: Suppose $Q_1(0.0, -3.0\text{m}, 0.0) = 4.0\text{nC}$, $Q_2(0.0, 3.0\text{m}, 0.0) = 4.0\text{nC}$, and $Q_3(4.0\text{m}, 0.0, 0.0) = 1.0\text{nC}$. (a) Find the total force acting on the charge Q_3 . (b) Repeat the problem after changing the charge of Q_2 to -4.0nC . (c) Find the electric field intensity for parts (a) and (b).

$$(a) \mathbf{F}_{13} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{13}^2} \mathbf{a}_{13}, \text{ where } \mathbf{R}_{13} = 4 \mathbf{a}_x + 3 \mathbf{a}_y, R_{13} = 5\text{m}, \mathbf{a}_{13} = 0.8 \mathbf{a}_x + 0.6 \mathbf{a}_y.$$

so

$$\mathbf{F}_{13} = \frac{(4 \times 10^{-9} \text{C})(1 \times 10^{-9} \text{C})(4 \mathbf{a}_x + 3 \mathbf{a}_y) / 5}{4\pi(10^{-9} \text{F}/36\pi \text{m})(5\text{m})^2} \frac{\text{FV}}{\text{C}} \frac{\text{NM}}{\text{VC}}$$

$$= 1.15 \times 10^{-9} \mathbf{a}_x + 0.86 \times 10^{-9} \mathbf{a}_y \text{ N.}$$

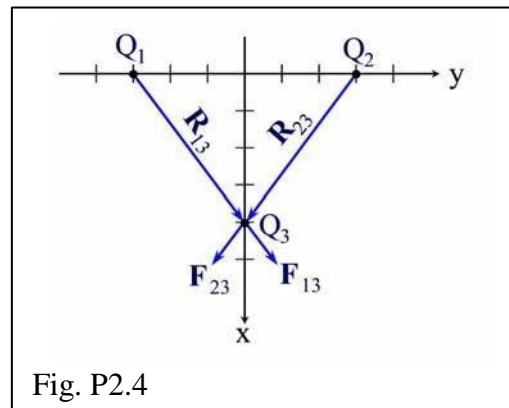
Similarly, $\mathbf{F}_{23} = 1.15 \times 10^{-9} \mathbf{a}_x - 0.86 \times 10^{-9} \mathbf{a}_y \text{ N}$, so $\mathbf{F}_{\text{TOT}} = 2.3 \mathbf{a}_x \text{ nN}$

(b) with $Q_2 = -4 \text{ nC}$, \mathbf{F}_{13} is unchanged but $\mathbf{F}_{23} = -1.15 \times 10^{-9} \mathbf{a}_x + 0.86 \times 10^{-9} \mathbf{a}_y \text{ N}$, so

$$\mathbf{F}_{\text{TOT}} = 1.7 \mathbf{a}_y \text{ nN}.$$

$$(c) \mathbf{E}_a = \frac{\mathbf{F}_{\text{TOT}}}{Q_3} = \frac{(2.3 \times 10^{-9} \mathbf{a}_x \text{ N}) \text{ VC}}{(1 \times 10^{-9} \text{ C}) \text{ Nm}} = 2.3 \mathbf{a}_x \frac{\text{V}}{\text{m}}$$

Likewise, $\mathbf{E}_b = 1.7 \mathbf{a}_y \frac{\text{V}}{\text{m}}$.



P2.5: Find the force exerted by $Q_1(3.0\text{m}, 3.0\text{m}, 3.0\text{m}) = 1.0 \mu\text{C}$ on $Q_2(6.0\text{m}, 9.0\text{m}, 3.0\text{m}) = 10. \text{nC}$.

$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \mathbf{a}_{12}, \text{ where}$$

$$\mathbf{R}_{12} = (6-3)\mathbf{a}_x + (9-3)\mathbf{a}_y + (3-3)\mathbf{a}_z = 3 \mathbf{a}_x + 6 \mathbf{a}_y \text{ m}$$

$$\sqrt{R_{12}^2} = \sqrt{3^2 + 6^2}$$

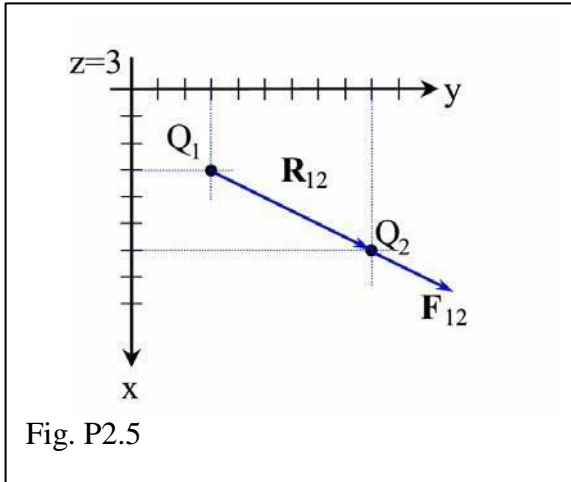
$$= 45m, \mathbf{a}_{12} =$$

$$\frac{3\mathbf{a}_x}{\sqrt{45}} + \frac{6\mathbf{a}_y}{\sqrt{45}},$$

$$\text{and } \frac{4}{5}$$

$$\sqrt{45}$$

$$\mathbf{F}_{12} = \frac{(1 \times 10^{-6} \text{ C})(10 \times 10^{-6} \text{ C})}{4\pi(10^{-9} \text{ F}/36\pi \text{ m})(45 \text{ m}^2)} \frac{3\mathbf{a}_x + 6\mathbf{a}_y}{\sqrt{45}} \frac{\text{FV}}{\text{C VC}} \text{ NM}, \text{ so } \boxed{\mathbf{F}_{12} = 0.89\mathbf{a}_x + 1.8\mathbf{a}_y \mu \text{ N}}$$



P2.6: Suppose 10.0 nC point charges are located on the corners of a square of side 10.0 cm. Locating the square in the x-y plane (at $z = 0.00$) with one corner at the origin and one corner at $P(10.0, 10.0, 0.00)$ cm, find the total force acting at point P.

We arbitrarily label the charges as shown in Figure P2.6. Then

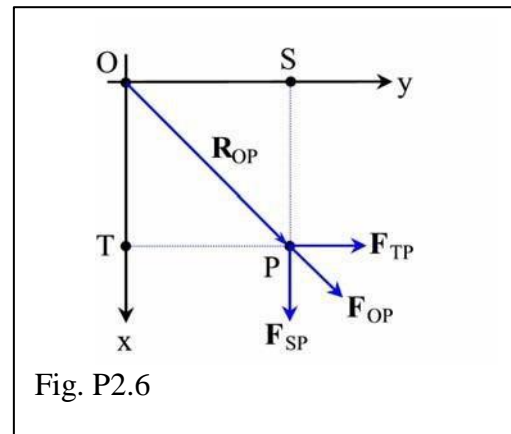
$$\mathbf{R}_{OP} = 0.1 \mathbf{a}_x + 0.1 \mathbf{a}_y$$

$$R_{OP} = 0.141 \text{ m}$$

$$\mathbf{a}_{OP} = 0.707 \mathbf{a}_x + 0.707 \mathbf{a}_y.$$

$$\begin{aligned} \mathbf{F}_{OP} &= \frac{(10 \text{ nC})(10 \text{ nC})(0.707)(\mathbf{a}_x + \mathbf{a}_y)}{(4\pi)(10^{-9} \text{ F}/36\pi \text{ m})(0.141 \text{ m})^2} \\ &= 32(\mathbf{a}_x + \mathbf{a}_y) \mu \text{ N} / \\ &\quad (10 \text{ nC})(10 \text{ nC}) \mathbf{a}_y \\ \mathbf{F}_{TP} &= \frac{(10 \text{ nC})(10 \text{ nC}) \mathbf{a}_y}{(4\pi)(10^{-9} \text{ F}/36\pi \text{ m})(0.1 \text{ m})^2} = 90 \mathbf{a}_y \mu \text{ N} \end{aligned}$$

$$\mathbf{F}_{SP} = \frac{(10 \text{ nC})(10 \text{ nC}) \mathbf{a}_x}{(4\pi)(10^{-9} \text{ F}/36\pi \text{ m})(0.1 \text{ m})^2} = 90 \mathbf{a}_x \mu \text{ N}$$



and then the total (adjusting to 2 significant digits) is:

$$\mathbf{F}_{TOT} = 120(\mathbf{a}_x + \mathbf{a}_y) \mu \text{ N}.$$

P2.7: 1.00 nC point charges are located at (0.00, -2.00, 0.00)m, (0.00, 2.00, 0.00)m, (0.00, 0.00, -2.00)m and (0.00, 0.00, +2.00)m. Find the total force acting on a 1.00 nC charge located at (2.00, 0.00, 0.00)m.

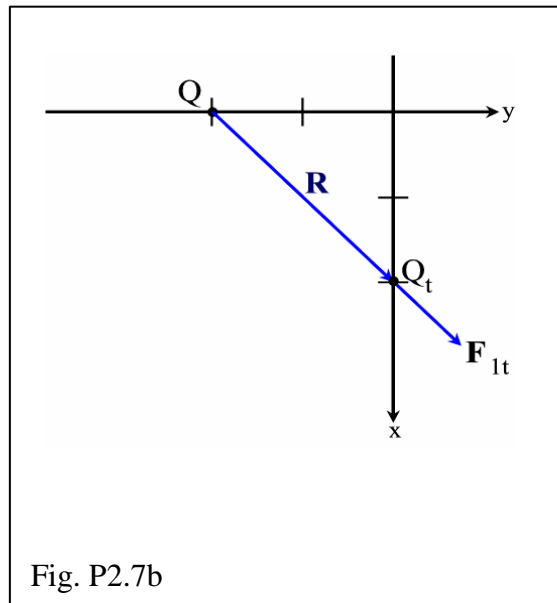
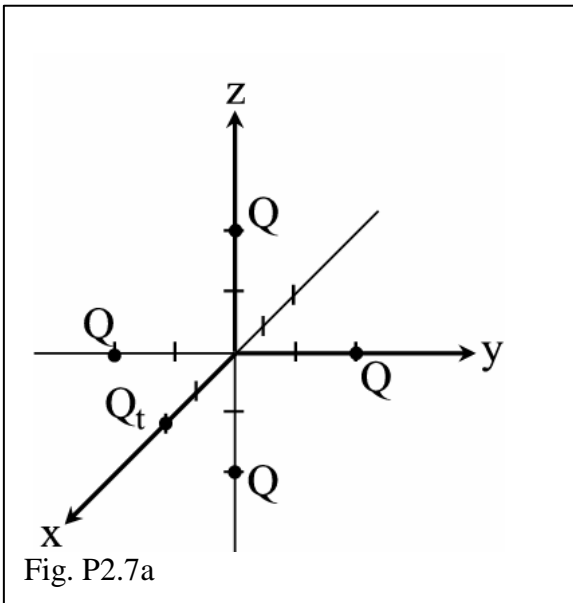
Figure P2.7a shows the situation, but we need only find the x-directed force from one of the charges on Q_t (Figure P2.7b) and multiply this result by 4. Because of the problem's symmetry, the rest of the components cancel.

$$\mathbf{F} = \frac{Q_t Q}{4\pi\epsilon_0 R^2} \mathbf{a}_R, \quad \mathbf{R} = 2\mathbf{a}_x + 2\mathbf{a}_y, \quad R = \sqrt{8} \text{ m}, \quad \mathbf{a}_R = \frac{2\mathbf{a}_x + 2\mathbf{a}_y}{\sqrt{8}}$$

$$\text{so } \mathbf{F} = \frac{(1 \times 10^{-9} \text{ C})(1 \times 10^{-9} \text{ C})}{4\pi(10^{-9} \text{ F/36}\pi \text{ m})(8 \text{ m}^2)} \frac{2\mathbf{a}_x + 2\mathbf{a}_y}{\sqrt{8}} = 796 \times 10^{-12} (\mathbf{a}_x + \mathbf{a}_y) \text{ N}$$

The force from all charges is then

$$\mathbf{F}_{TOT} = (4)(796 \times 10^{-12} \text{ N}) = 3.2 \text{ nN}$$



P2.8: A 20.0 nC point charge exists at P(0.00,0.00,-3.00m). Where must a 10.0 nC charge be located such that the total field is zero at the origin?

For zero field at the origin, we must cancel the $+\mathbf{a}_z$ directed field from Q_P by placing Q at the point $Q(0,0,z)$ (see Figure P2.8). Then we have $\mathbf{E}_{tot} = \mathbf{E}_P + \mathbf{E}_Q = 0$.

$$\frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a}_z = \frac{(20 \times 10^{-9} \text{ C})}{4\pi\epsilon_0 R^2} \mathbf{a}_z$$

$$\text{So, } \mathbf{E}_P = \frac{Q_P}{4\pi\epsilon_0 R^2} \mathbf{a}_z = \frac{20 \times 10^{-9} \text{ C}}{4\pi\epsilon_0 R^2} \mathbf{a}_z$$

$$4\pi \left(10^{-9} \right. \\ \left. F_{36\pi m} \right) (3m) \\ m$$

and

$$\mathbf{E}_Q = \frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a}_R$$

$$= \frac{(10 \times 10^{-9} \text{ C})(-\mathbf{a}_z)}{z^2} = \frac{-90}{z^2} \mathbf{a}_z$$

$$4\pi(10^{-9} / 36\pi \text{ m})(z(\text{m}))^2$$

So then

$$20\mathbf{a}_z - \frac{90}{z^2} \mathbf{a}_z = 0,$$

$$z^2 = \frac{90}{20}, \quad z = 2.12.$$

Thus, Q(0,0,2.12m).

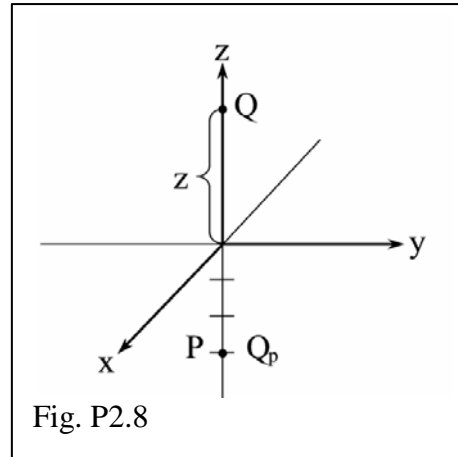


Fig. P2.8

3. The Spherical Coordinate System

P2.9: Convert the following points from Cartesian to Spherical coordinates:

- P(6.0, 2.0, 6.0)
- P(0.0, -4.0, 3.0)
- P(-5.0, -1.0, -4.0)

$$(a) \quad r = \sqrt{6^2 + 2^2 + 6^2} = 8.7, \theta = \cos^{-1} \frac{6}{8.7} = 47^\circ, \phi = \tan^{-1} \frac{2}{6} = 18^\circ$$

$$(b) \quad r = \sqrt{0^2 + 4^2 + 3^2} = 5, \theta = \cos^{-1} \frac{3}{5} = 53^\circ, \phi = \tan^{-1} \frac{-4}{0} = -90^\circ$$

$$(c) \quad r = \sqrt{5^2 + 1^2 + 4^2} = 6.5, \theta = \cos^{-1} \frac{-4.0}{6.5} = 130^\circ, \phi = \tan^{-1} \frac{-1}{-5} = 190^\circ$$

P2.10: Convert the following points from Spherical to Cartesian coordinates:

- P(3.0, 30°, 45°)
- P(5.0, π/4, 3π/2)
- P(10., 135°, 180°)

(a)

$$x = r \sin\theta \cos\phi = 3 \sin 30^\circ \cos 45^\circ = 1.06$$

$$y = r \sin\theta \sin\phi = 3 \sin 30^\circ \sin 45^\circ = 1.06$$

$$z = r \cos\theta = 3 \cos 30^\circ = 2.6$$

so P(1.1, 1.1, 2.6).

(b)

$$x = r \sin\theta \cos\phi = 5 \sin 45^\circ \cos 270^\circ = 0$$

$$y = r \sin\theta \sin\phi = 5 \sin 45^\circ \sin 270^\circ = -3.5$$

$$z = r \cos\theta = 5 \cos 45^\circ = 3.5$$

so $P(0, -3.5, 3.5)$.

(c)

$$x = r \sin\theta \cos\phi = 10 \sin 135^\circ \cos 180^\circ = -7.1$$

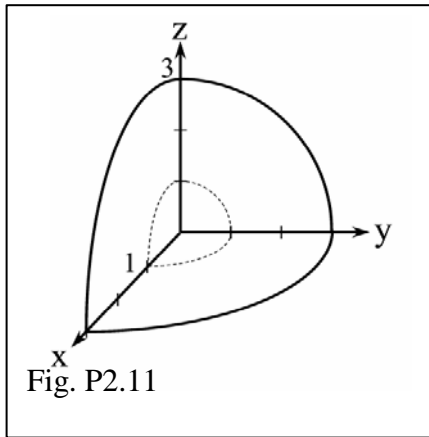
$$y = r \sin\theta \sin\phi = 10 \sin 135^\circ \sin 180^\circ = 0$$

$$z = r \cos\theta = 10 \cos 135^\circ = -7.1$$

so $P(-7.1, 0, -7.1)$.

P2.11: Given a volume defined by $1.0\text{m} \leq r \leq 3.0\text{m}$, $0 \leq \theta \leq \pi/2$, $90^\circ \leq \phi \leq 270^\circ$, (a) sketch the volume, (b) perform the integration to find the volume, and (c) perform the necessary integrations to find the total surface area.

(a)



(b)

$$V = \iiint r^2 \sin\theta \, dr \, d\theta \, d\phi = \int_1^3 r^2 \, dr \int_0^{\pi/2} \sin\theta \, d\theta \int_{\pi/2}^{3\pi/2} d\phi = \frac{13\pi}{3} = 13.6 \text{ m}^3$$

So volume $V = 14 \text{ m}^3$.

(c) There are 5 surfaces: an inner, an outer, and 3 identical sides.

$$S_{side} = \iint r \, dr \, d\phi = \int_1^3 r \, dr \int_{\pi/2}^{3\pi/2} d\phi = 2\pi \text{ m}^2; \quad S_{sides} = 6\pi \text{ m}^2$$

$$S_{outer} = \iint r^2 \sin\theta \, d\theta \, d\phi = 3^2 \int_0^{\pi/2} \sin\theta \, d\theta \int_{\pi/2}^{3\pi/2} d\phi = \frac{9\pi}{2} \text{ m}^2$$

$$S_{inner} = \frac{\pi}{2} \text{ m}^2; \quad S_{TOT} = 11\pi \text{ m}^2 = 34.6 \text{ m}^2$$

So $S_{total} = 35 \text{ m}^2$.

4. Line Charges and the Cylindrical Coordinate System

P2.12: Convert the following points from Cartesian to cylindrical coordinates:

a. $P(0.0, 4.0, 3.0)$

b. $P(-2.0, 3.0, 2.0)$

c. $P(4.0, -3.0, -4.0)$

(a) $\rho = \sqrt{0^2 + 4^2} = 4, \phi = \tan^{-1} \frac{4}{0} = 90^\circ, z = 3, \text{ so } P(4.0, 90^\circ, 3.0)$

(b) $\rho = \sqrt{2^2 + 3^2} = 3.6, \phi = \tan^{-1} \frac{3}{2} = 124^\circ, z = 2, \text{ so } P(3.6, 120^\circ, 2.0)$

(c) $\rho = \sqrt{4^2 + 3^2} = 5, \phi = \tan^{-1} \frac{-3}{4} = -37^\circ, z = -4, \text{ so } P(5.0, -37^\circ, -4.0)$

P2.13: Convert the following points from cylindrical to Cartesian coordinates:

a. $P(2.83, 45.0^\circ, 2.00)$

b. $P(6.00, 120.^\circ, -3.00)$

c. $P(10.0, -90.0^\circ, 6.00)$

(a)

$$x = \rho \cos \phi = 2.83 \cos 45^\circ = 2.00$$

$$y = \rho \sin \phi = 2.83 \sin 45^\circ = 2.00$$

$$z = z = 2.00$$

$$\text{so } P(2.00, 2.00, 2.00).$$

(b)

$$x = \rho \cos \phi = 6.00 \cos 120^\circ = -3.00$$

$$y = \rho \sin \phi = 6.00 \sin 120^\circ = 5.20$$

$$z = z = -3.00$$

$$\text{so } P(-3.00, 5.20, -3.00).$$

(c)

$$x = \rho \cos \phi = 10.0 \cos(-90.0^\circ) = 0$$

$$y = \rho \sin \phi = 10.0 \sin(-90.0^\circ) = -10.0$$

$$z = z = 6.00$$

$$\text{so } P(0, -10.0, 6.00).$$

P2.14: A 20.0 cm long section of copper pipe has a 1.00 cm thick wall and outer diameter of 6.00 cm.

- Sketch the pipe conveniently overlaying the cylindrical coordinate system, lining up the length direction with the z-axis
- Determine the total surface area (this could actually be useful if, say, you needed to do an electroplating step on this piece of pipe)
- Determine the weight of the pipe given the density of copper is 8.96 g/cm^3

(a) See Figure P2.14

(b) The top area, S_{top} , is equal to the bottom area. We must also find the inner area, S_{inner} , and the outer area, S_{outer} .

$$S_{top} = \iint \rho d\rho d\phi = \int_2^3 \rho d\rho \int_0^{2\pi} d\phi = 5\pi \text{ cm}^2.$$

$$S_{bottom} = S_{top}.$$

$$S_{outer} = \iint \rho d\phi dz = 3 \int_0^{2\pi} d\phi \int_0^{20} dz = 120\pi \text{ cm}^2$$

$$S_{inner} = \iint \rho d\phi dz = 2 \int_0^{2\pi} d\phi \int_0^{20} dz = 80\pi \text{ cm}^2$$

The total area, then, is $210\pi \text{ cm}^2$, or $S_{tot} = 660 \text{ cm}^2$.

(c) Determining the weight of the pipe requires the volume:

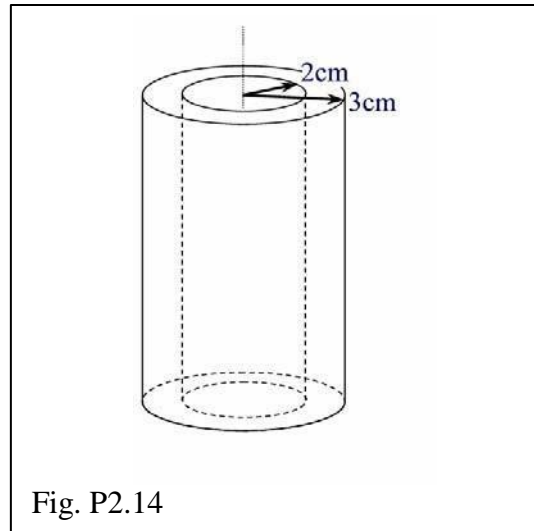
$$V = \iiint \rho d\rho d\phi dz$$

$$= \int_2^3 \rho d\rho \int_0^{2\pi} d\phi \int_0^{20} dz = 100\pi \text{ cm}^3.$$

$$M_{pipe} = \left[\frac{8.96 \text{ g}}{\text{cm}^3} \right] (100\pi \text{ cm}^3)$$

$$= 2815 \text{ g}.$$

$$\text{So } M_{pipe} = 2820 \text{ g}.$$



P2.15: A line charge with charge density 2.00 nC/m exists at $y = -2.00 \text{ m}$, $x = 0.00$. (a) A charge $Q = 8.00 \text{ nC}$ exists somewhere along the y -axis. Where must you locate Q so that the total electric field is zero at the origin? (b) Suppose instead of the 8.00 nC charge of part (a) that you locate a charge Q at $(0.00, 6.00\text{m}, 0.00)$. What value of Q will result in a total electric field intensity of zero at the origin?

(a) The contributions to \mathbf{E} from the line and point charge must cancel, or $\mathbf{E} = \mathbf{E}_L + \mathbf{E}_Q$.

$$\text{For the line: } \mathbf{E}_L = \frac{\rho_L}{2\pi\epsilon_0 \rho} \mathbf{a}_\rho = \frac{(2 \text{ nC/m})}{2\pi(10^{-9} \text{ F/36}\pi\text{m})(2\text{m})} \mathbf{a}_y = 18 \frac{\text{V}}{\text{m}} \mathbf{a}_y$$

and for the point charge, where the point is located a distance y along the y -axis, we

$$\text{have: } \mathbf{E}_Q = \frac{Q}{4\pi\epsilon_0 y^2} (-\mathbf{a}_y) = \frac{(8nC)(-\mathbf{a}_y)}{4\pi(10^{-9} F/36\pi m) y^2} = \frac{72}{y^2} (-\mathbf{a}_y)$$

Therefore:

$$\frac{72}{y^2} = 18, \text{ or } y = \sqrt{\frac{72}{18}} = 2m.$$

So $\boxed{Q(0,2.0m,0)}$

(b)

$$\frac{Q}{4\pi\epsilon_0 (6)^2} = 18,$$

$$Q = \frac{(18)(36)}{9} = 72nC.$$

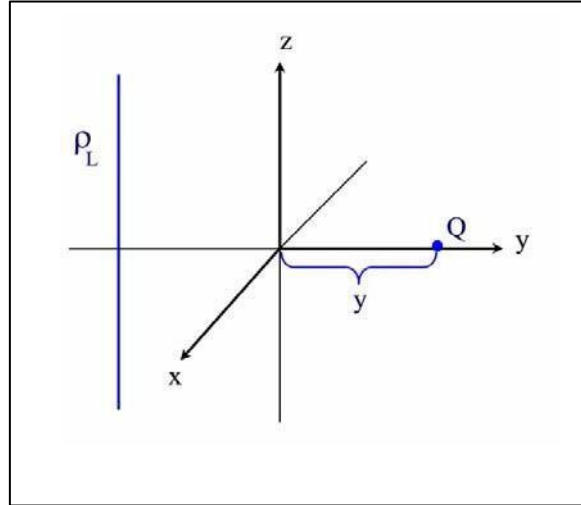


Fig. P2.15

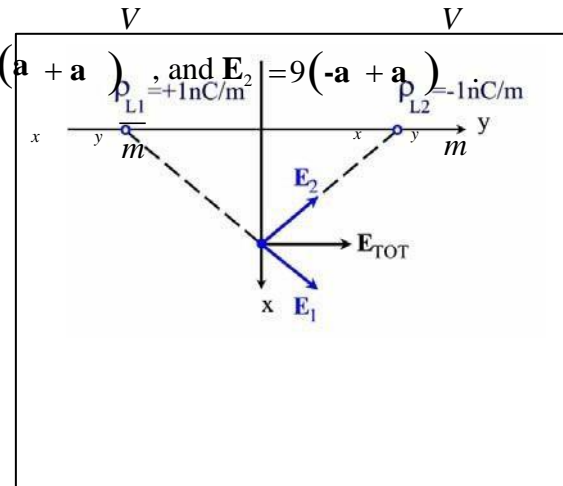
P2.16: You are given two z -directed line charges of charge density $+1 \text{ nC/m}$ at $x = 0, y = -1.0 \text{ m}$, and charge density -1.0 nC/m at $x = 0, y = 1.0 \text{ m}$. Find \mathbf{E} at $P(1.0m,0,0)$.

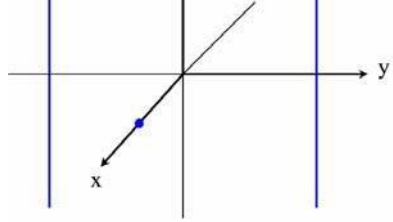
The situation is represented by Figure P2.16a. A better 2-dimensional view in Figure P2.16b is useful for solving the problem.

$$\mathbf{E}_1 = \frac{\rho_{L1}}{2\pi\epsilon} \mathbf{a}_\rho, \text{ and } \rho_{L2} = \frac{\rho_{L1}}{\sqrt{2}}$$

$$\mathbf{E}_1 = \frac{(1 \times 10^{-9} \text{ C})}{2\pi(10^{-9} F/36\pi m)(\sqrt{2}m)} \frac{(\mathbf{a}_x + \mathbf{a}_y)}{\sqrt{2}} = 9 \frac{(\mathbf{a}_x + \mathbf{a}_y)}{m}$$

So $\mathbf{E}_{TOT} = 18 \mathbf{a}_y \text{ V/m}$.





2-14

Fig. P2.16a

Fig. P2.16b

P2.17: MATLAB: Suppose you have a segment of line charge of length $2L$ centered on the z -axis and having a charge distribution ρ_L . Compare the electric field intensity at a point on the y -axis a distance d from the origin with the electric field at that point assuming the line charge is of infinite length. The ratio of E for the segment to E for the infinite line is to be plotted versus the ratio L/d using MATLAB.

This is similar to MATLAB 2.3. We have for the ideal case

$$\mathbf{E}_{ideal} = \frac{\rho_L}{2\pi\epsilon_0} \mathbf{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0 d} \mathbf{a}_\rho.$$

For the actual $2L$ case, we have an integration to perform (Equation (2.35) with different limits):

$$\mathbf{E}_{actual} = \frac{\rho_L \mathbf{a}_\rho}{4\pi\epsilon_0} \int_{-L}^{+L} \frac{dz}{(\rho^2 + z^2)^{3/2}} = \frac{\rho_L \mathbf{a}_\rho}{4\pi\epsilon_0} \left[\frac{z}{d^2 \sqrt{z^2 + d^2}} \right]_{-L}^{+L}$$

$$\mathbf{E}_{actual} = \frac{\rho_L \mathbf{a}_y}{2\pi\epsilon_0 d} \frac{L}{\sqrt{L^2 + d^2}}.$$

Now we manipulate these expressions to get the following ratio:

$$\frac{\mathbf{E}_{actual}}{\mathbf{E}_{ideal}} = \frac{L/d}{\sqrt{1 + (L/d)^2}}.$$

In the program, the actual to ideal field ratio is termed “Eratio” and the charged line half-length L ratioed to the distance d is termed “Lod”.

```
% M-File: MLP0217
%
% This program is similar to ML0203.
% It compares the E-field from a finite length
% segment of charge (from -L to +L on the z-axis)
% to the E-field from an infinite length line
% of charge. The ratio (E from segment to E from
% infinite length line) is plotted versus the ratio
% Lod=L/d, where d is the distance along the y axis.
%
% Wentworth, 12/19/02
%
% Variables:
%     Lod     the ratio L/d
%     Eratio  ratio of E from segment to E from line

clc      %clears the command window
clear   %clears variables

% Initialize Lod array and calculate Eratio
Lod=0.1:0.01:100;
```



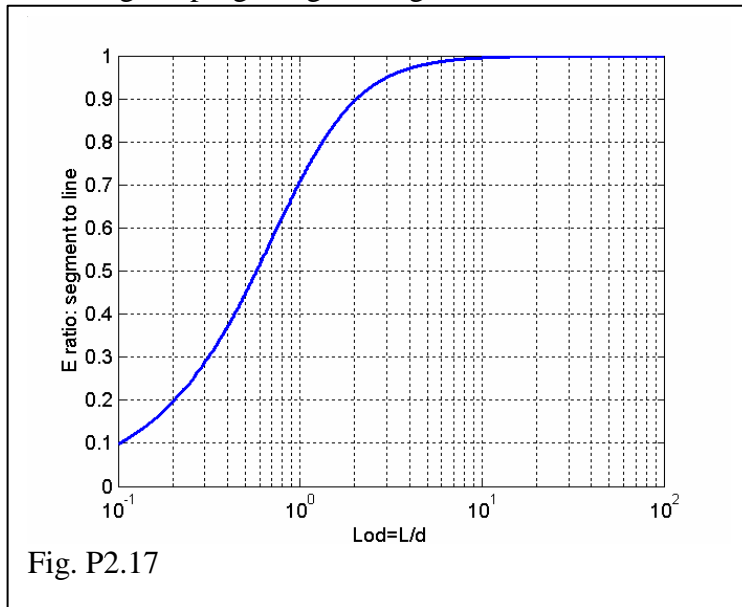
```

Eratio=Lod./(sqrt(1+Lod.^2));

% Plot Eratio versus Lod
semilogx(Lod,Eratio)
grid on
xlabel('Lod=L/d')
ylabel('E ratio: segment to line')

```

Executing the program gives Figure P2.17.



So we see that the field from a line segment of charge appears equivalent to the field from an infinite length line if the test point is close to the line.

P2.18: A segment of line charge $\rho_L = 10 \text{ nC/m}$ exists on the y-axis from the origin to $y = +3.0 \text{ m}$. Determine \mathbf{E} at the point $(3.0, 0, 0) \text{ m}$.

It is clear from a sketch of the problem in Figure P2.18a that the resultant field will be directed in the x-y plane. The situation is redrawn in a temporary coordinate system in Figure P2.18b.

We have from Eqn (2.34) $\mathbf{E} = \int \frac{\rho_L dz}{4\pi\epsilon_0 (\rho^2 + z^2)^{3/2}} (\rho \mathbf{a}_\rho - z \mathbf{a}_z) = E_\rho \mathbf{a}_\rho + E_z \mathbf{a}_z$.

For E_ρ we have:

$$E_\rho = \frac{\rho_L \rho}{4\pi\epsilon_0} \int_0^3 \frac{dz}{(\rho^2 + z^2)^{3/2}} = \frac{\rho_L \rho}{4\pi\epsilon_0} \left[\frac{z}{\rho^2 \sqrt{z^2 + \rho^2}} \right]_0^3$$

With $\rho = 3$, we then have $E_\rho = 21.2 \text{ V/m}$.

For E_z :

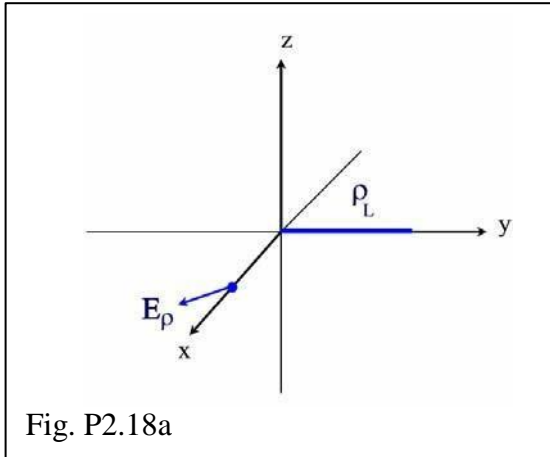


Fig. P2.18a

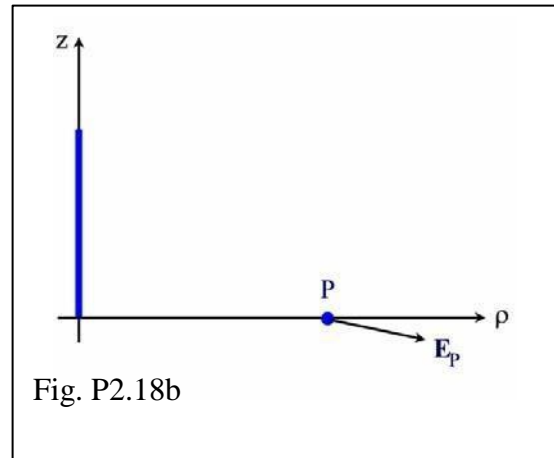


Fig. P2.18b

$$E_z = \frac{-\rho}{4\pi\epsilon} \int \frac{zdz}{(\rho^2 + z^2)^{3/2}} = \frac{\rho}{4\pi\epsilon} \left[\frac{1}{\sqrt{\rho^2 + z^2}} \right]_{z=0}^{\infty} = -8.79 \frac{\rho}{m}$$

Thus we have $\mathbf{E}_{TOT} = 21 \mathbf{a}_\rho - 8.8 \mathbf{a}_z$ V/m.

Converting back to the original coordinates, we have $\mathbf{E}_{TOT} = 21 \mathbf{a}_x - 8.8 \mathbf{a}_y$ V/m.

5. Surface and Volume Charge

P2.19: In free space, there is a point charge $Q = 8.0$ nC at $(-2.0, 0, 0)$ m, a line charge $\rho_L = 10$ nC/m at $y = -9.0$ m, $x = 0$ m, and a sheet charge $\rho_s = 12$ nC/m² at $z = -2.0$ m. Determine \mathbf{E} at the origin.

The situation is represented by Figure P2.19, and the total field is $\mathbf{E}_{TOT} = \mathbf{E}_Q + \mathbf{E}_L + \mathbf{E}_S$.

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a} = \frac{(8 \times 10^{-9} \text{ C})}{4\pi(10^{-9} \text{ F}/36\pi \text{ m})(2 \text{ m})^2} \mathbf{a}_x$$

$$= 18 \mathbf{a}_x \frac{\text{V}}{\text{m}}$$

$$\mathbf{E}_L = \frac{\rho_L}{2\pi\epsilon_0 \rho} \mathbf{a}_\rho = \frac{(10 \times 10^{-9} \text{ C/m})}{2\pi(10^{-9} \text{ F}/36\pi \text{ m})(9 \text{ m})} \mathbf{a}_y$$

$$= 20 \mathbf{a}_y \frac{\text{V}}{\text{m}}$$

$$\mathbf{E}_S = \frac{\rho_s}{2\epsilon_0} \mathbf{a}_N = \frac{(12 \times 10^{-9} \text{ C/m}^2)}{2(10^{-9} \text{ F}/36\pi \text{ m})} \mathbf{a}_z$$

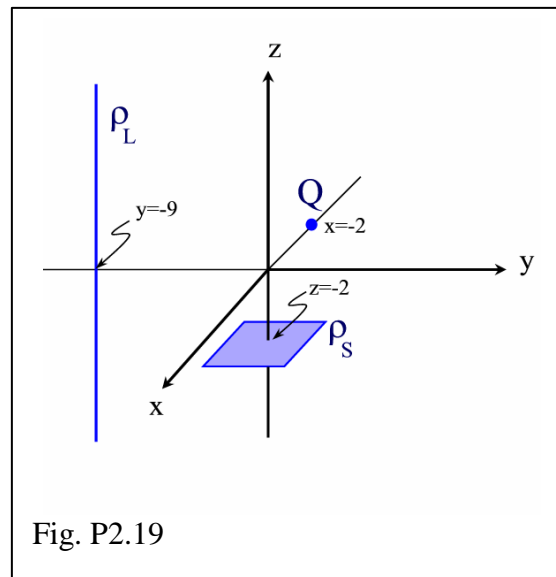


Fig. P2.19

$$= 679 \mathbf{a}_z \frac{\text{V}}{\text{m}}$$

So: $\mathbf{E}_{\text{tot}} = 18 \mathbf{a}_x + 20 \mathbf{a}_y + 680 \mathbf{a}_z \text{ V/m.}$

P2.20: An infinitely long line charge ($\rho_L = 21\pi \text{ nC/m}$) lies along the z-axis. An infinite area sheet charge ($\rho_s = 3 \text{ nC/m}^2$) lies in the x-z plane at $y = 10 \text{ m}$. Find a point on the y-axis where the electric field intensity is zero.

We have $\mathbf{E}_{\text{TOT}} = \mathbf{E}_L + \mathbf{E}_S$.

$$\mathbf{E}_L = \frac{\rho_L \mathbf{a}_\rho}{2\pi\epsilon_0 \rho} = \frac{\rho_L \mathbf{a}_y}{2\pi\epsilon_0 y}$$

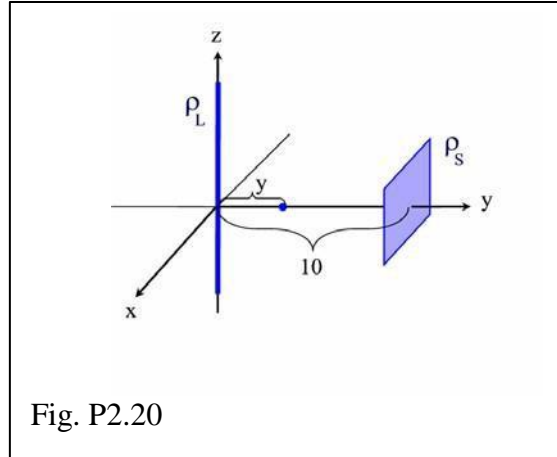
$$= \frac{(21\pi \times 10^{-9} \text{ C/m})}{2\pi \left(10^{-9} \text{ F/36}\pi\text{m}\right) y} \mathbf{a}_y = \frac{378\pi}{y} \mathbf{a}_y$$

$$\mathbf{E}_S = \frac{\rho_s \mathbf{a}_N}{2\epsilon_0} = \frac{(3 \times 10^{-9} \text{ C/m}^2)(-\mathbf{a}_y)}{2 \left(10^{-9} \text{ F/36}\pi\text{m}\right)}$$

$= -54\pi \mathbf{a}_y$

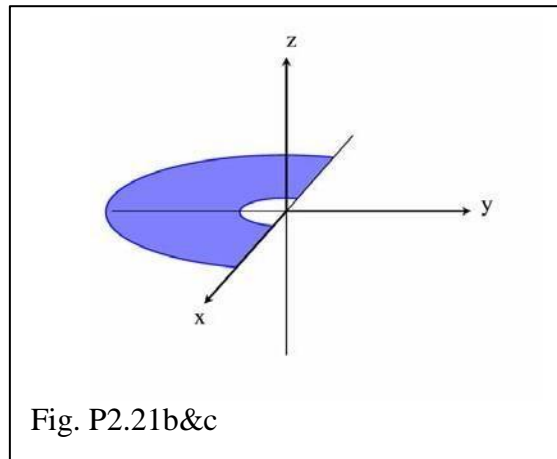
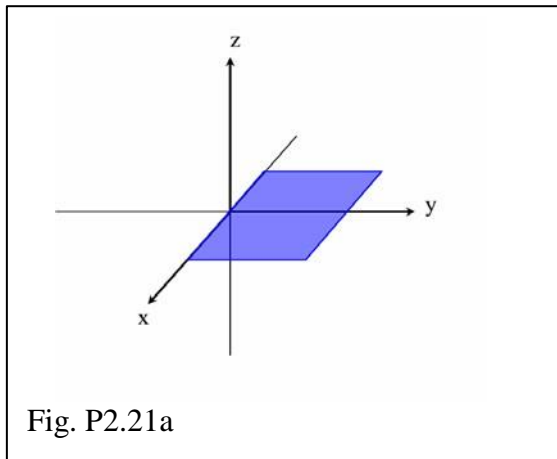
so
 $\frac{378\pi}{y} - 54\pi = 0$, or $y = 7$.

Therefore, $P(0, 7\text{m}, 0)$.



P2.21: Sketch the following surfaces and find the total charge on each surface given a surface charge density of $\rho_s = 1 \text{ nC/m}^2$. Units (other than degrees) are meters.

- (a) $-3 \leq x \leq 3, 0 \leq y \leq 4, z = 0$
- (b) $1 \leq r \leq 4, 180^\circ \leq \phi \leq 360^\circ, \theta = \pi/2$
- (c) $1 \leq \rho \leq 4, 180^\circ \leq \phi \leq 360^\circ, z = 0$



(a) $Q = \int \rho_s dS = \rho_s \int_{-3}^3 dx \int_0^4 dy = 24\rho_s = 24 \text{ nC}$

$$(b) Q = \rho_s \int_1^4 r dr \int_{\pi}^{2\pi} d\phi = \frac{15\pi}{2} \rho_s = 24nC$$

$$(c) Q = \rho_s \int_1^4 \rho d\rho \int_{\pi}^{2\pi} d\phi = 24nC$$

P2.22: Consider a circular disk in the x-y plane of radius 5.0 cm. Suppose the charge density is a function of radius such that $\rho_s = 12\rho \text{ nC/cm}^2$ (when ρ is in cm). Find the electric field intensity a point 20.0 cm above the origin on the z-axis.

From section 4 for a ring of charge of radius a , $\mathbf{E} = \frac{\rho_L a h \mathbf{a}_z}{2\epsilon_o (a^2 + h^2)^{3/2}}$. Now we have

$$\rho_L = \rho_s d\rho \text{ and } d\mathbf{E} = \frac{A\rho d\rho h \mathbf{a}_z}{2\epsilon_o (\rho^2 + h^2)^{3/2}}, \text{ where } \rho_s = A\rho \text{ nC/cm}^2. \text{ Now the total field is given}$$

by the integral:

$$\mathbf{E} = \frac{A h \mathbf{a}_z}{2\epsilon_o} \int \frac{\rho^2 d\rho}{(\rho^2 + h^2)^{3/2}}$$

This can be solved using integration by parts, where $u = \rho$, $du = d\rho$,

$$v = \frac{-1}{\sqrt{\rho^2 + h^2}}, \text{ and } dv = \frac{\rho d\rho}{\sqrt{\rho^2 + h^2}}$$

$$\mathbf{E} = \frac{A h \mathbf{a}_z}{2\epsilon_o} \left[\frac{\rho}{\sqrt{\rho^2 + h^2}} + \ln \frac{\rho + \sqrt{\rho^2 + h^2}}{h} \right]_{\rho=0}^{\rho=5}$$

Plugging in the appropriate values we arrive at $\mathbf{E} = 6.7 \text{ kV/cm } \mathbf{a}_z$.

P2.23: Suppose a ribbon of charge with density ρ_s exists in the y-z plane of infinite length in the z direction and extending from $-a$ to $+a$ in the y direction. Find a general expression for the electric field intensity at a point d along the x-axis.

The problem is represented by Figure P2.23a. A better representation for solving the problem is shown in Figure P2.23b.

We have $d\mathbf{E} = \frac{\rho_L}{2\epsilon_o} \frac{d\mathbf{a}_x - y\mathbf{a}_y}{\sqrt{d^2 + y^2}}$, where $\rho_L = \rho_s dy$. Then, since

$$\rho_L = \rho_s dy$$

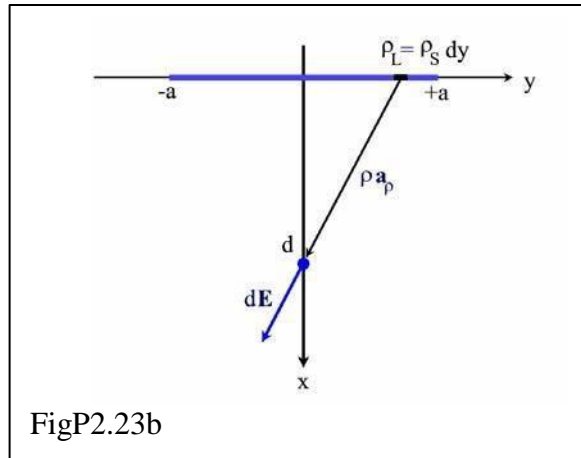
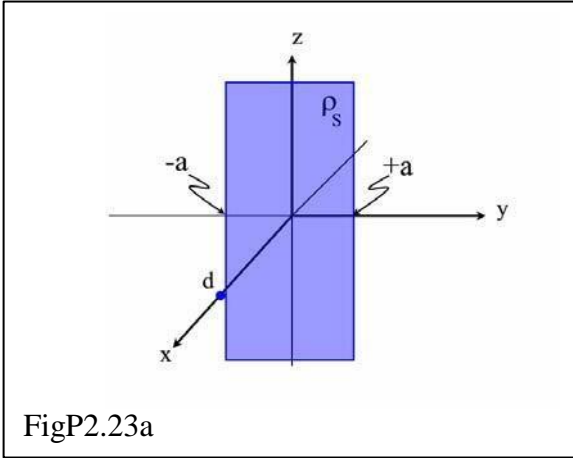
the integral becomes

$$\mathbf{E} = \frac{\rho_s d}{2\epsilon_o} \frac{d\mathbf{a}_x - y\mathbf{a}_y}{\sqrt{d^2 + y^2}}$$

$$\int 2\pi\epsilon_0 y \sqrt{d^2 + y^2} \sqrt{d^2 + y^2}$$

It may be noted that the \mathbf{a}_y component will cancel by symmetry. The \mathbf{a}_x integral is found from the appendix and we have

$$\mathbf{E} = \frac{\rho_s}{\pi\epsilon_0} \tan^{-1} \frac{a}{d} \mathbf{a}_x$$



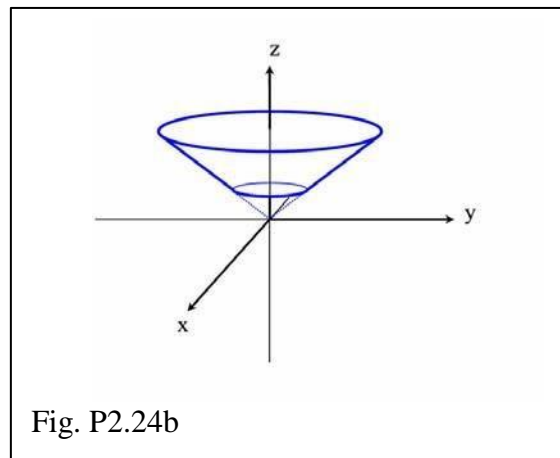
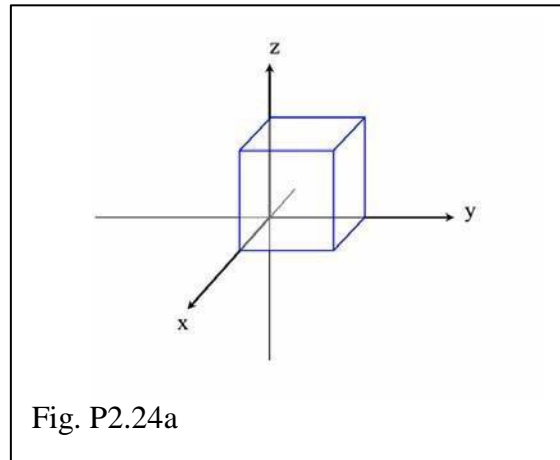
P2.24: Sketch the following volumes and find the total charge for each given a volume charge density of $\rho_v = 1\text{nC/m}^3$. Units (other than degrees) are meters.

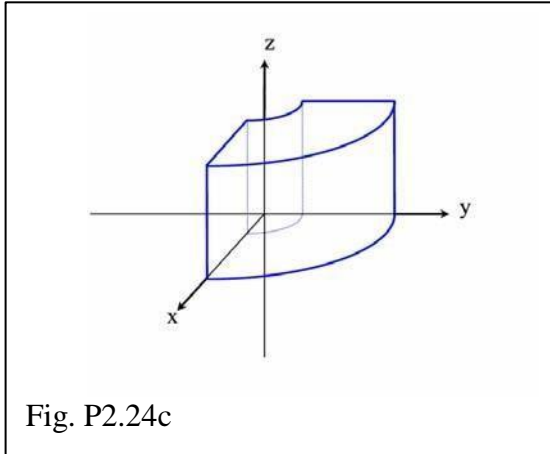
- (a) $0 \leq x \leq 4, 0 \leq y \leq 5, 0 \leq z \leq 6$
- (b) $1 \leq r \leq 5, 0 \leq \theta \leq 60^\circ$
- (c) $1 \leq \rho \leq 5, 0^\circ \leq \phi \leq 90^\circ, 0 \leq z \leq 5$

$$(a) Q = \int \rho_v dv = \rho_v \int_0^4 dx \int_0^5 dy \int_0^6 dz = 120\text{nC}$$

$$(b) Q = \int \rho_v dv = \rho_v \int_1^5 r^2 dr \int_0^{60^\circ} \sin\theta d\theta \int_0^{2\pi} d\phi = 130\text{nC}$$

$$(c) Q = \int \rho_v dv = \rho_v \int_1^5 \rho d\rho \int_0^{\pi/2} d\phi \int_0^5 dz = 94\text{nC}$$





P2.25: You have a cylinder of 4.00 inch diameter and 5.00 inch length (imagine a can of tomatoes) that has a charge distribution that varies with radius as $\rho_v = (6\rho) \text{ nC/in}^3$ where ρ is in inches. (It may help you with the units to think of this as $\rho_v (\text{nC/in}^3) = 6 (\text{nC/in}^4) \rho(\text{in})$). Find the total charge contained in this cylinder.

$$Q = \int_V \rho \, dv = \int_0^L \int_0^{2\pi} \int_0^R (6\rho) \rho \, d\rho \, d\phi \, dz = 6 \int_0^L \rho^2 \, d\rho \int_0^{2\pi} d\phi \int_0^L dz = 160\pi \text{ nC} = 503 \text{ nC}$$

P2.26: MATLAB: Consider a rectangular volume with $0.00 \leq x \leq 4.00 \text{ m}$, $0.00 \leq y \leq 5.00 \text{ m}$ and $-6.00 \text{ m} \leq z \leq 0.00$ with charge density $\rho_v = 40.0 \text{ nC/m}^3$. Find the electric field intensity at the point $P(0.00, 0.00, 20.0 \text{ m})$.

```
% MLP0226
% calculate E from a rectangular volume of charge

% variables
% xstart,xstop      limits on x for vol charge (m)
% ystart,ystop
% zstart,zstop
% xt,yt,zt         test point (m)
% rhov             vol charge density, nC/m^3
% Nx,Ny,Nz         discretization points
% dx,dy,dz         differential lengths
% dQ               differential charge, nC
% eo               free space permittivity (F/m)
% dEi              differential field vector
% dEix,dEiy,dEiz   x,y and z components of dEi
% dEjx,dEjy,dEjz   of dEj
% dEkx,dEky,dEkz   of dEk
% Etot            total field vector, V/m
```



```

clc
clear

% initialize variables
xstart=0;xstop=4;
ystart=0;ystop=5;
zstart=-6;zstop=0;
xt=0;yt=0;zt=20;
rhov=40e-9;
Nx=10;Ny=10;Nz=10;
eo=8.854e-12;

dx=(xstop-xstart)/Nx;
dy=(ystop-ystart)/Ny;
dz=(zstop-zstart)/Nz;
dQ=rhov*dx*dy*dz;

for k=1:Nz
    for j=1:Ny
        for i=1:Nx
            xv=xstart+(i-0.5)*dx;
            yv=ystart+(j-0.5)*dy;
            zv=zstart+(k-0.5)*dz; R=[xt-xv
            yt-yv zt-zv];
            magR=magvector(R);
            uvR=unitvector(R);
            dEi=(dQ/(4*pi*eo*magR^2))*uvR;
            dEix(i)=dEi(1);
            dEiy(i)=dEi(2);
            dEiz(i)=dEi(3);
        end
        dEjx(j)=sum(dEix);
        dEjy(j)=sum(dEiy);
        dEjz(j)=sum(dEiz);
    end

    dEkx(k)=sum(dEjx);
    dEky(k)=sum(dEjy);
    dEkz(k)=sum(dEjz);
end

Etotx=sum(dEkx);
Etoty=sum(dEky);
Etotz=sum(dEkz);
Etot=[Etotx Etoty Etotz]

```

Now to run the program:

Etot =

-6.9983 -8.7104 79.7668

>>

So $\mathbf{E} = -7.0 \mathbf{a}_x - 8.7 \mathbf{a}_y + 80. \mathbf{a}_z$ V/m

P2.27: MATLAB: Consider a sphere with charge density $\rho_v = 120 \text{ nC/m}^3$ centered at the origin with a radius of 2.00 m. Now, remove the top half of the sphere, leaving a hemisphere below the x-y plane. Find the electric field intensity at the point P(8.00m,0.00,0.00). (*Hint: see MATLAB 2.4, and consider that your answer will now have two field components.*)

```
% M-File: MLP0227
%
% This program modifies ML0204 to find the field
% at point P(8m,0,0) from a hemispherical
% distribution of charge given by
% rho_v=120 nC/m^3 from 0 < r < 2m and
% pi/2 < theta < pi.
%
% Wentworth, 12/23/02
%
% Variables:
%   d           y axis distance to test point (m)
%   a           sphere radius (m)
%   dV          differential charge volume where
%               dV=delta_r*delta_theta*delta_phi
%   eo          free space permittivity (F/m)
%   r,theta,phi spherical coordinate location of
%               center of a differential charge element
%   x,y,z       cartesian coord location of charge %
%               element
%   R           vector from charge element to P
%   Rmag        magnitude of R
%   aR          unit vector of R
%   dr,dtheta,dphi differential spherical elements
%   dEi,dEj,dEk partial field values
%   Etot        total field at P resulting from charge

clc      %clears the command window
clear    %clears variables

% Initialize variables
eo=8.854e-12;
d=8;a=2;
```

```

delta_r=40;delta_theta=72;delta_phi=144;

% Perform calculation
for k=(1:delta_phi)
    for j=(1:delta_theta)
        for i=(1:delta_r)
            r=i*a/delta_r;
            theta=(pi/2)+j*pi/(2*delta_theta);
            phi=k*2*pi/delta_phi;
            x=r*sin(theta)*cos(phi);
            y=r*sin(theta)*sin(phi);
            z=r*cos(theta);
            R=[d-x,-y,-z];
            Rmag=magvector(R);
            aR=R/Rmag;
            dr=a/delta_r;
            dtheta=pi/delta_theta;
            dphi=2*pi/delta_phi;
            dV=r^2*sin(theta)*dr*dtheta*dphi;
            dQ=120e-9*dV;
            dEi=dQ*aR/(4*pi*eo*Rmag^2);
            dEix(i)=dEi(1);
            dEiy(i)=dEi(2);
            dEiz(i)=dEi(3);
        end
        dEjx(j)=sum(dEix);
        dEjy(j)=sum(dEiy);
        dEjz(j)=sum(dEiz);
    end

    dEkx(k)=sum(dEjx);
    dEky(k)=sum(dEjy);
    dEkz(k)=sum(dEjz);
end

Etotx=sum(dEkx);
Etoty=sum(dEky);
Etotz=sum(dEkz);

Etot=[Etotx Etoty Etotz]

```

Now to run the program:

```

Etot =
    579.4623    0.0000    56.5317

```

So $\mathbf{E} = 580 \mathbf{a}_x + 57 \mathbf{a}_z$ V/m.

6. Electric Flux Density

P2.28: Use the definition of dot product to find the three interior angles for the triangle bounded by the points $P(-3.00, -4.00, 5.00)$, $Q(2.00, 0.00, -4.00)$, and $R(5.00, -1.00, 0.00)$.

Here we use $\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos \theta_{AB}$.

$$\mathbf{PR} = (5 - (-3))\mathbf{a}_x + (-1 - (-4))\mathbf{a}_y + (0 - 5)\mathbf{a}_z$$

$$\mathbf{PR} = 8\mathbf{a}_x + 3\mathbf{a}_y - 5\mathbf{a}_z, |\mathbf{PR}| = 9.9$$

$$\mathbf{PQ} = 5\mathbf{a}_x + 4\mathbf{a}_y - 9\mathbf{a}_z, |\mathbf{PQ}| = 11.0$$

$$\mathbf{PR} \cdot \mathbf{PQ} = (8)(5) + (3)(4) + (-5)(-9) = 97$$

$$= 9.9 \cdot 11.0 \cos \theta_P$$

$$\theta_P = \cos^{-1} \left(\frac{97}{99} \right) = 27^\circ$$

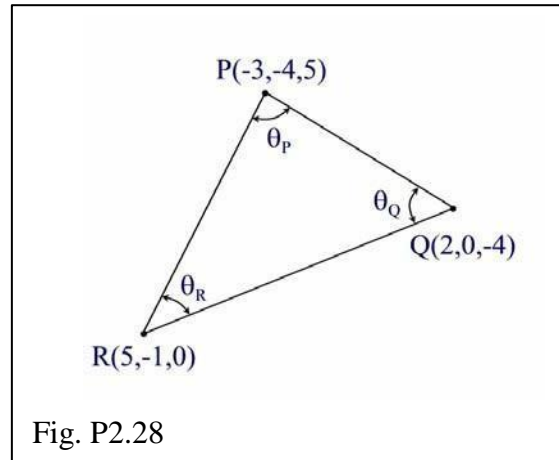
$$|\mathbf{RQ}| = \sqrt{(9)^2 + (1)^2 + (4)^2} = 11.0$$

$$\mathbf{RQ} = -3\mathbf{a}_x + 1\mathbf{a}_y - 4\mathbf{a}_z, |\mathbf{RQ}| = 5.1$$

$$\mathbf{RP} = -8\mathbf{a}_x - 3\mathbf{a}_y + 5\mathbf{a}_z, |\mathbf{RP}| = 9.9$$

$$\mathbf{RQ} \cdot \mathbf{RP} = 1 = (5.1)(9.9) \cos \theta_R, \theta_R = 89^\circ$$

$$\theta_Q = 180^\circ - 27^\circ - 89^\circ = 64^\circ$$



P2.29: Given $\mathbf{D} = 2\rho \mathbf{a}_\rho + \sin \phi \mathbf{a}_z$ C/m², find the electric flux passing through the surface defined by $2.0 \leq \rho \leq 4.0$ m, $90.0^\circ \leq \phi \leq 180^\circ$, and $z = 4.0$ m.

$$\Psi = \int \mathbf{E} \cdot d\mathbf{S}, d\mathbf{S} = \rho d\rho d\phi \mathbf{a}_z$$

$$\Psi = \int (2\rho \mathbf{a}_\rho + \sin \phi \mathbf{a}_z) \cdot \rho d\rho d\phi \mathbf{a}_z = \int_2^4 \rho d\rho \int_{\pi/2}^{\pi} \sin \phi d\phi = 6C$$

P2.30: Suppose the electric flux density is given by $\mathbf{D} = 3r \mathbf{a}_r - \cos \phi \mathbf{a}_\theta + \sin^2 \theta \mathbf{a}_\phi$ C/m². Find the electric flux through both surfaces of a hemisphere of radius 2.00 m and $0.00^\circ \leq \theta \leq 90.0^\circ$.

$$\Psi_1 = \int \mathbf{D} \cdot d\mathbf{S}, d\mathbf{S} = r \sin \theta d\theta d\phi \mathbf{a}_\theta$$

$$\Psi = \int_0^{\pi/2} \int_0^{2\pi} (3r \mathbf{a}_r - \cos \phi \mathbf{a}_\theta + \sin^2 \theta \mathbf{a}_\phi) \cdot (r^2 \sin \theta d\theta d\phi \mathbf{a}_\theta)$$

$$= 3r^3 \int_0^{\pi/2} \sin \theta d\theta \int_0^{2\pi} d\phi = 48\pi C$$

$$d\mathbf{S}_2 = r dr d\phi \mathbf{a}_\theta$$

$$\Psi_2 = \int -\cos\phi \mathbf{a}_\theta \cdot \mathbf{r} dr d\phi \mathbf{a}_\theta$$

$$= -\int_0^{2\pi} \int_0^{\frac{\pi}{2}} r^2 \sin\phi \, d\phi \, d\theta = 0$$

$$\therefore \Psi = 48\pi C$$

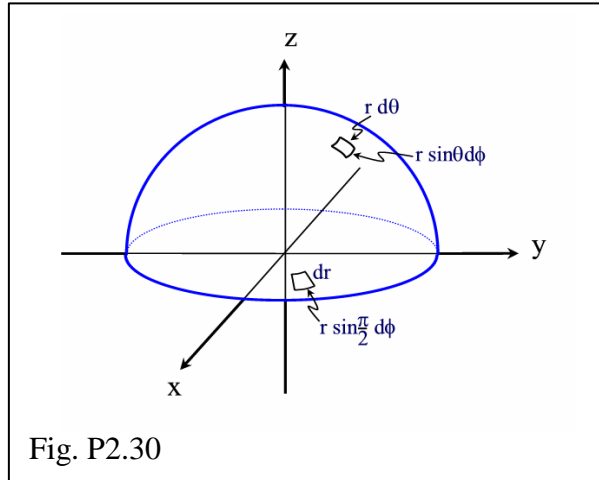


Fig. P2.30

7. Gauss's Law and Applications

P2.31: Given a 3.00 mm radius solid wire centered on the z-axis with an evenly distributed 2.00 coulombs of charge per meter length of wire, plot the electric flux density D_ρ versus radial distance from the z-axis over the range $0 \leq \rho \leq 9$ mm.

For a 1 m length, $\rho_v = \frac{2(C/m)}{\pi a^2} = 70.7 \times 10^3 \frac{C}{m^3}$, ($a = 3\text{mm} = .003\text{m}$)

$Q_{enc} = \oint \mathbf{D} \cdot d\mathbf{S} = \int D_\rho \mathbf{a}_\rho \cdot \rho d\phi dz \mathbf{a}_\rho = 2\pi\rho L D_\rho$, where L is the length of the Gaussian surface. Note that this expression for Q_{enc} is valid for both Gaussian surfaces.

GS1 ($\rho < a$):

$$Q_{enc} = \int \rho_v dv = \rho_v \int_0^\rho \rho d\rho \int_0^{2\pi} d\phi \int_0^L dz = \rho_v \rho^2 \pi L$$

$$\text{so } D_\rho = \frac{\rho_v \rho^2 \pi L}{2\pi\rho L} = \frac{\rho_v}{2} \rho \quad \text{for } \rho \leq a.$$

GS2 ($\rho > a$):

$$Q_{enc} = \rho_v a^2 \pi L, \quad D_\rho = \frac{\rho_v a^2}{2\rho} \quad \text{for } \rho \geq a.$$

This is plotted with the following Matlab routine:

```
% M-File: MLP0231
%
% Gauss's Law Problem
%   solid cylinder with even charge
%
% Variables
% rhov   charge density (C/m^3)
% a      radius of cylinder (m)
% rho    radial distance from z-axis
% rhomm  rho in mm
% D      electric flux density (C/m^3)
```

```

% N      number of data points
% maxrad max radius for plot (m)

clc;clear;

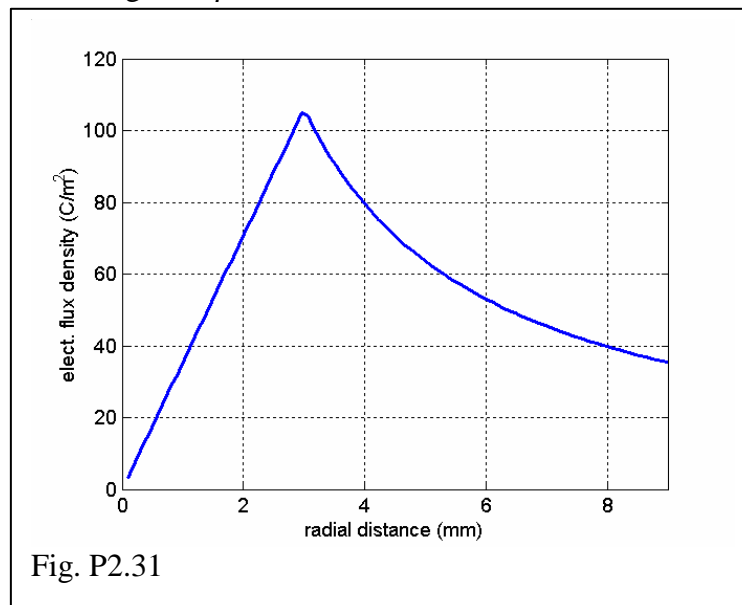
% initialize variables
rhov=70.7e3;
a=0.003;
maxrad=.009;
N=100;
bndy=round(N*a/maxrad);

for i=1:bndy
    rho(i)=i*maxrad/N;
    rhomm(i)=rho(i)*1000;
    D(i)=rhov*rho(i)/2;
end

for i=bndy+1:N
    rho(i)=i*maxrad/N;
    rhomm(i)=rho(i)*1000;
    D(i)=(rhov*a^2)/(2*rho(i));
end
plot(rhomm,D)
xlabel('radial distance (mm)')
ylabel('elect. flux density (C/m^2)')
grid on

```

P2.32: Given a 2.00 cm radius solid wire centered on the z-axis with a charge density $\rho_v = 6\rho \text{ C/cm}^3$ (when ρ is in cm), plot the electric flux density D_ρ versus radial distance from the z-axis over the range $0 \leq \rho \leq 8 \text{ cm}$.



Choose Gaussian surface length L , and as usual we have

$$Q_{enc} = \oint \mathbf{D} \cdot d\mathbf{S} = \int D_\rho \mathbf{a}_\rho \cdot \rho d\phi dz \mathbf{a}_\rho = 2\pi L \rho D_\rho, \text{ valid for both Gaussian surfaces.}$$

$$\text{In GS1 } (\rho < a): Q_{enc} = \int \rho_v dv = \int_0^{\rho} \int_0^{2\pi} \int_0^L \rho \, d\phi dz = 4\pi L \rho^3,$$

$$\text{so } D_\rho = \frac{4\pi L \rho}{2\pi L \rho} = 2\rho \text{ for } \rho \leq a.$$

$$\text{For GS2 } (\rho > a): Q_{enc} = 4\pi L a^3, \quad D_\rho = \frac{2a^3}{\rho} \text{ for } \rho \geq a.$$

This is plotted for the problem values in the following Matlab routine.

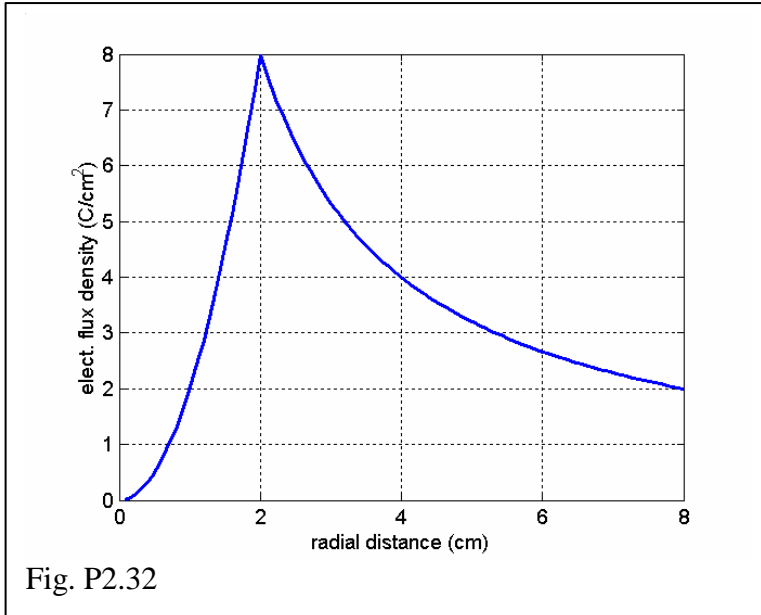
```
% M-File: MLP0232
%
% Gauss's Law Problem
%   solid cylinder with radially-dependent charge
%
% Variables
% a      radius of cylinder (cm)
% rho    radial distance from z-axis
% D      electric flux density (C/cm^3)
% N      number of data points
% maxrad max radius for plot (cm)

clc;clear;

% initialize variables
a=2;
maxrad=8; N=100;
bndy=round(N*a/maxrad);

for i=1:bndy
    rho(i)=i*maxrad/N;
    D(i)=2*rho(i)^2;
end

for i=bndy+1:N
    rho(i)=i*maxrad/N;
    D(i)=(2*a^3)/rho(i);
end
plot(rho,D)
xlabel('radial distance (cm)')
ylabel('elect. flux density (C/cm^2)')
grid on
```



P2.33: A cylindrical pipe with a 1.00 cm wall thickness and an inner radius of 4.00 cm is centered on the z-axis and has an evenly distributed 3.00 C of charge per meter length of pipe. Plot D_ρ as a function of radial distance from the z-axis over the range $0 \leq \rho \leq 10$ cm.

$$Q_{enc} = \oint \mathbf{D} \cdot d\mathbf{S} = \int D_\rho \mathbf{a}_\rho \cdot \rho d\phi dz \mathbf{a}_\rho = 2\pi h \rho D_\rho; \text{ this is true for all the Gaussian surfaces.}$$

GS1 ($\rho < a$): since $Q_{enc} = 0$, $D_\rho = 0$.

GS2 ($a < \rho < b$):

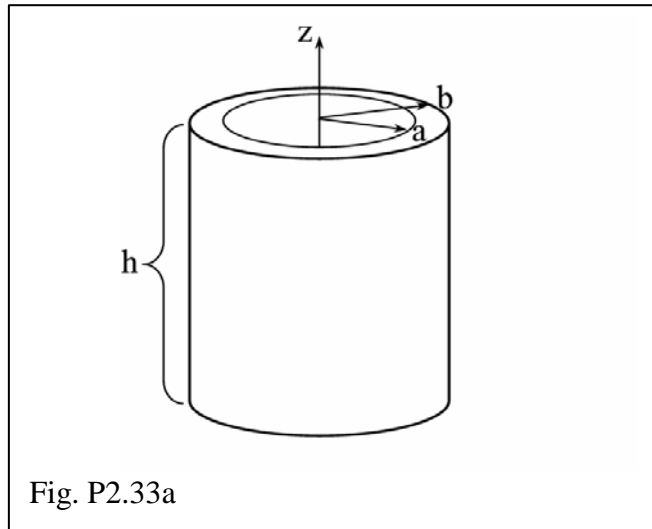
$$\rho_v = \frac{3h}{\iiint \rho d\rho d\phi dz} = \frac{3}{\pi(b^2 - a^2)}$$

$$\begin{aligned} Q_{enc} &= \int \rho_v dv \\ &= \frac{3}{\pi(b^2 - a^2)} \int_a^\rho \rho d\rho \int_0^{2\pi} d\phi \int_0^h dz \\ &= 3h \frac{(\rho^2 - a^2)}{(b^2 - a^2)} \end{aligned}$$

So,

$$D_\rho = \frac{3h(\rho^2 - a^2)}{2\pi h \rho (b^2 - a^2)} = \frac{3(\rho^2 - a^2)}{2\pi \rho (b^2 - a^2)} \text{ for } a \leq \rho \leq b.$$

GS3 ($\rho > b$):



$$Q_{enc} = 3h, D_p = \frac{3h}{2\pi\rho} \quad \text{for } \rho > b.$$

A plot with the appropriate values is generated by the following Matlab routine:

```
% M-File: MLP0233
% Gauss's Law Problem
%     cylindrical pipe with even charge distribution
%
% Variables
% a     inner radius of pipe (m)
% b     outer radius of pipe (m)
% rho   radial distance from z-axis (m)
% rhocm radial distance in cm
% D     electric flux density (C/cm^3)
% N     number of data points
% maxrad max radius for plot (m)
clc;clear;

% initialize variables
a=.04;b=.05;maxrad=0.10;N=100;
bndya=round(N*a/maxrad);
bndyb=round(N*b/maxrad);

for i=1:bndya
    rho(i)=i*maxrad/N;
    rhocm(i)=rho(i)*100;
    D(i)=0;
end

for i=bndya+1:bndyb
    rho(i)=i*maxrad/N;
    rhocm(i)=rho(i)*100;
    D(i)=(3/(2*pi*rho(i)))*((rho(i)^2-a^2)/(b^2-a^2));
end

for i=bndyb+1:N
    rho(i)=i*maxrad/N;
    rhocm(i)=rho(i)*100;
    D(i)=3/(2*pi*rho(i));
end

plot(rhocm,D)
xlabel('radial distance (cm)')
ylabel('elect. flux density (C/m^2)')
grid on
```

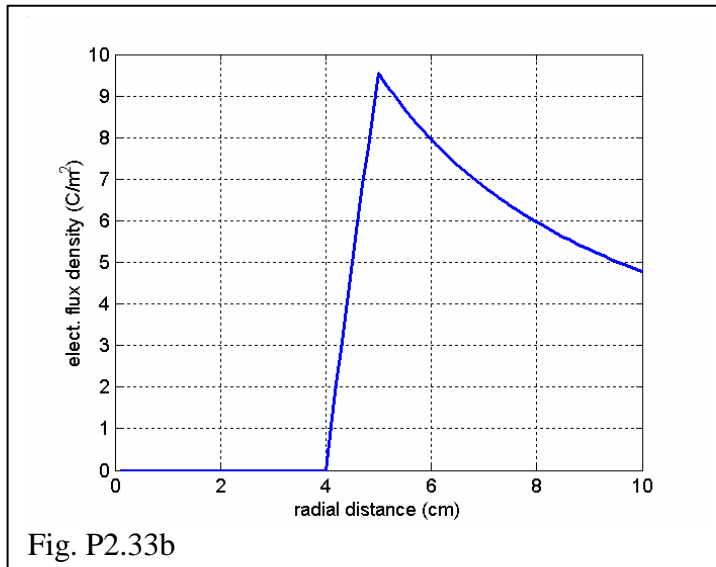


Fig. P2.33b

P2.34: An infinitesimally thin metallic cylindrical shell of radius 4.00 cm is centered on the z-axis and has an evenly distributed charge of 100. nC per meter length of shell. (a) Determine the value of the surface charge density on the conductive shell and (b) plot D_ρ as a function of radial distance from the z-axis over the range $0 \leq \rho \leq 12$ cm.

$$\rho = \frac{Q}{S} = \frac{100 \text{ nC}}{2\pi(0.04 \text{ m})(1 \text{ m})} = 398 \frac{\text{nC}}{\text{m}^2}$$

$$S = \int \rho d\phi dz = (0.04 \text{ m})(2\pi)(1 \text{ m}) \quad \text{m}^2$$

For all Gaussian surfaces, of height h and radius ρ , we have:

$$Q_{enc} = \oint \mathbf{D} \cdot d\mathbf{S}, \text{ where } d\mathbf{S} = \rho d\phi dz \mathbf{a}_\rho,$$

$$\therefore Q_{enc} = 2\pi\rho h D_\rho.$$

GS1 ($\rho < a$): $Q_{enc} = 0$ so $D_\rho = 0$

GS2 ($\rho > a$):

$$Q_{enc} = \int \rho_s dS = \rho_s \int \rho d\phi dz = 2\pi a h \rho_s,$$

$$D = \rho_s \frac{a}{\rho} \quad \text{for } \rho \geq a.$$

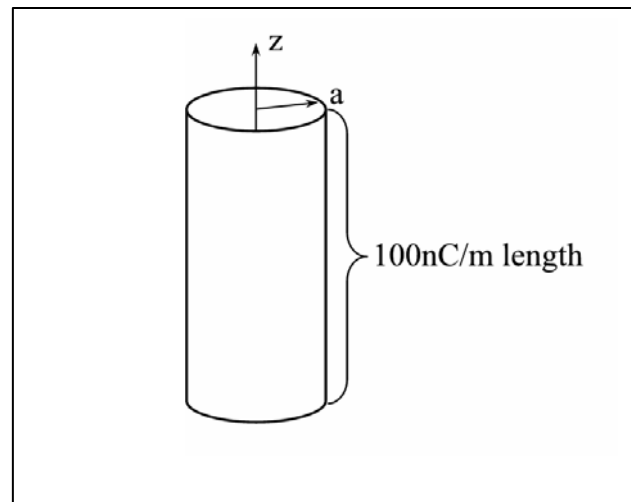


Fig. P2.34a

```
% M-File: MLP0234
```

```
%
```

```
% Gauss's Law Problem
```

```
% cylindrical shell of charge
```

```
%
```

```
% Variables
```

```
% a radius of cylinder (m)
```

```
% Qs surface charge density (nC/m^2)
```

```
% rho radial distance from z-axis (m)
```

```

% rhocm    radial distance in cm
% D        electric flux density (nC/cm^3)
% N        number of data points
% maxrad   max radius for plot (cm)

clc;clear;

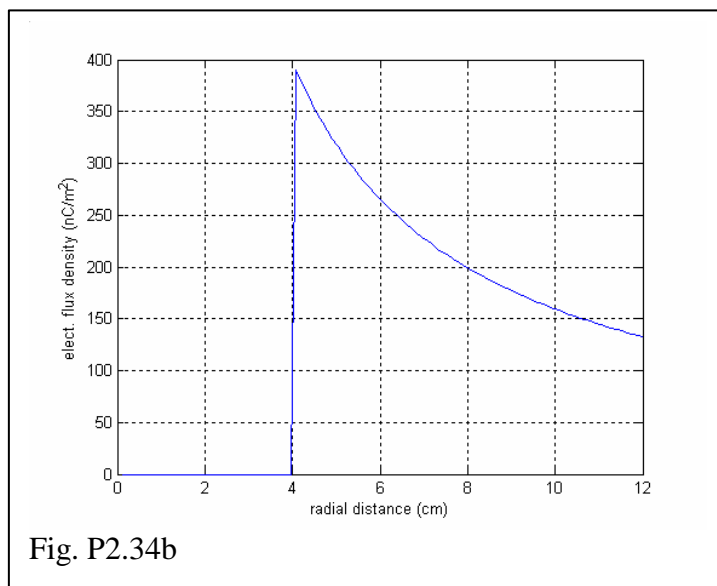
% initialize variables
a=.04;Qs=398;maxrad=0.12;N=100;
bndy=round(N*a/maxrad);

for i=1:bndy
    rho(i)=i*maxrad/N;
    rhocm(i)=rho(i)*100;
    D(i)=0;
end

for i=bndy+1:N
    rho(i)=i*maxrad/N;
    rhocm(i)=rho(i)*100;
    D(i)=Qs*a/rho(i);
end

plot(rhocm,D)
xlabel('radial distance (cm)')
ylabel('elect. flux density (nC/m^2)')
grid on

```



P2.35: A spherical charge density is given by $\rho_v = \rho_o r/a$ for $0 \leq r \leq a$, and $\rho_v = 0$ for $r > a$. Derive equations for the electric flux density for all r .

$Q_{enc} = \oint \mathbf{D} \cdot d\mathbf{S} = \int_{r=r} \mathbf{D} \cdot \mathbf{a}_r r^2 \sin\theta d\theta d\phi = 4\pi r^2 D_r$. This is valid for each Gaussian surface.

$$\text{GS1 } (r < a): Q_{enc} = \int \rho_v dv = \int_0^r \int_0^\pi \int_0^{2\pi} \frac{\rho_o}{a} r^3 \sin\theta d\theta d\phi dr = \frac{\pi \rho_o r^4}{a}.$$

$$\text{So } D_r = \frac{\pi \rho_o r^4}{4\pi r^2 a} = \frac{\rho_o r^2}{4a} \text{ for } r \leq a.$$

$$\text{GS2 } (r > a): Q_{enc} = \pi \rho_o a^3, \quad D_r = \frac{\rho_o a^3}{4r^2} \text{ for } r \geq a.$$

P2.36: A thick-walled spherical shell, with inner radius 2.00 cm and outer radius 4.00 cm, has an evenly distributed 12.0 nC charge. Plot D_r as a function of radial distance from the origin over the range $0 \leq r \leq 10$ cm.

Here we'll let a = inner radius and b = outer radius. Then

$Q_{enc} = \oint \mathbf{D} \cdot d\mathbf{S} = \int_{r=r} D_r \mathbf{a}_r r^2 \sin\theta d\theta d\phi = 4\pi r^2 D_r$; This is true for each Gaussian surface.

The volume containing charge is

$$v = \int_a^b \int_0^\pi \int_0^{2\pi} r^2 \sin\theta d\theta d\phi dr = \frac{4}{3} \pi (b^3 - a^3).$$

$$\text{So } \rho_v = \frac{Q}{v} = \frac{3Q}{4\pi (b^3 - a^3)}.$$

Now we can evaluate Q_{enc} for each Gaussian surface.

GS1 ($r < a$): $Q_{enc} = 0$ so $D_r = 0$.

GS2 ($a < r < b$): $Q_{enc} = \int_a^r \int_0^\pi \int_0^{2\pi} \rho_v r^2 \sin\theta d\theta d\phi dr = \frac{\rho_v}{4\pi} (r^3 - a^3)$.

$$Q_{enc} = \int_a^r \int_0^\pi \int_0^{2\pi} \rho_v r^2 \sin\theta d\theta d\phi dr = \frac{\rho_v}{4\pi} (r^3 - a^3)$$

Inserting our value for ρ_v , we find

$$D_r = \frac{Q (r^3 - a^3)}{4\pi r^2 (b^3 - a^3)} \text{ for } a \leq r \leq b.$$

GS3 ($r > b$): $Q_{enc} = Q$, $D_r = \frac{Q}{4\pi r^2}$, for $r \geq b$.

This is plotted for appropriate values using the following Matlab routine:

```
% M-File: MLP0236
% Gauss's Law Problem
% thick spherical shell with even charge
%
% Variables
```

```

% a      inner radius of sphere (m)
% b      outer radius of sphere (m)
% r      radial distance from origin (m)
% rcm    radial distance in cm
% D      electric flux density (nC/cm^3)
% N      number of data points
% maxr   max radius for plot (m)
% Q      charge (nC)

```

```
clc;clear;
```

```

% initialize variables
a=.02;b=.04;
Q=12;
maxrad=0.10;
N=100;
bndya=round(N*a/maxrad);
bndyb=round(N*b/maxrad);

```

```

for i=1:bndya
    r(i)=i*maxrad/N;
    rcm(i)=r(i)*100;
    D(i)=0;
end

```

```

for i=bndya+1:bndyb
    r(i)=i*maxrad/N;
    rcm(i)=r(i)*100;

```

```

    D(i)=(Q/(4*pi*r(i)^2))*(r(i)^3-a^3)/(b^3-a^3);
end

```

```

for i=bndyb+1:N
    r(i)=i*maxrad/N;
    rcm(i)=r(i)*100;
    D(i)=Q/(4*pi*r(i)^2);
end

```

```

plot(rcm,D)
xlabel('radial distance (cm)')
ylabel('elect. flux density (nC/m^2)')
grid on

```

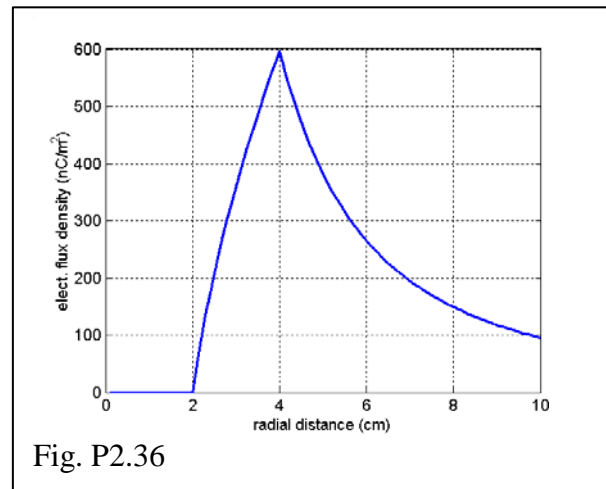


Fig. P2.36

P2.37: Given a coaxial cable with solid inner conductor of radius a , an outer conductor that goes from radius b to c , (so $c > b > a$), a charge $+Q$ that is evenly distributed

throughout a meter length of the inner conductor and a charge $-Q$ that is evenly distributed throughout a meter length of the outer conductor, derive equations for the electric flux density for all ρ . You may orient the cable in any way you wish.

We conveniently center the cable on the z -axis. Then, for a Gaussian surface of length L ,

$$Q_{enc} = \oint \mathbf{D} \cdot d\mathbf{S} = 2\pi\rho LD_\rho; \text{ valid for all Gaussian surfaces.}$$

$$\text{GS1: } (\rho < a): \rho_v = \frac{Q}{(1\text{m})(\pi a^2)};$$

$$Q_1 = \int \rho_v dv = \rho_v \int_0^\rho \rho d\rho \int_0^{2\pi} d\phi \int_0^L dz = \frac{QL}{a^2} \rho^2;$$

$$D_\rho = \frac{QL\rho^2}{a^2 2\pi\rho L} = \frac{Q}{2\pi a^2} \rho \quad \text{for } \rho \leq a$$

$$\text{GS2 } (a < \rho < b): Q_2 = QL; D_\rho = \frac{QL}{2\pi\rho L} = \frac{Q}{2\pi\rho} \quad \text{for } a \leq \rho \leq b.$$

$$\text{GS3 } (b < \rho < c): Q_3 = Q + \int \rho_{vo} dv, \text{ where } \rho_{vo} = \frac{-Q}{(1\text{m})\pi(c^2 - b^2)}$$

$$Q_3 = Q + \frac{-Q}{\pi(c^2 - b^2)} \int_b^\rho \rho d\rho \int_0^{2\pi} d\phi \int_0^L dz = Q \frac{(c^2 - \rho^2)}{(c^2 - b^2)}$$

$$\text{so } D = \frac{Q}{2\pi\rho} \left(\frac{c^2 - \rho^2}{c^2 - b^2} \right) \quad \text{for } b \leq \rho \leq c.$$

$$\text{GS4 } (\rho > c): Q_{enc} = 0, D_\rho = 0.$$

8. Divergence and the Point Form of Gauss's Law

P2.38: Determine the charge density at the point $P(3.0\text{m}, 4.0\text{m}, 0.0)$ if the electric flux density is given as $\mathbf{D} = xyz \mathbf{a}_z \text{ C/m}^2$.

$$\nabla \cdot \mathbf{D} = \frac{\partial D_z}{\partial z} = \frac{\partial (xyz)}{\partial z} = xy = \rho_v$$

$$\rho_v(3,4,0) = (3)(4) = 12 \text{ C/m}^3.$$

P2.39: Given $\mathbf{D} = 3\mathbf{a}_x + 2xy\mathbf{a}_y + 8x^2y^3\mathbf{a}_z \text{ C/m}^2$, (a) determine the charge density at the point $P(1,1,1)$. Find the total flux through the surface of a cube with $0.0 \leq x \leq 2.0\text{m}$, $0.0 \leq y \leq 2.0\text{m}$ and $0.0 \leq z \leq 2.0\text{m}$ by evaluating (b) the left side of the divergence theorem and (c) the right side of the divergence theorem.

$$(a) \nabla \cdot \mathbf{D} = \frac{\partial}{\partial y} (2xy) = 2x, \quad \rho_v(1,1,1) = 2 \frac{C}{m^3}.$$

$$(b) \oint \mathbf{D} \cdot d\mathbf{S} = \int_{\text{top}} + \int_{\text{bottom}} + \int_{\text{left}} + \int_{\text{right}} + \int_{\text{front}} + \int_{\text{back}}$$

$$\int_{\text{top}} = \int_0^2 \int_0^2 8x^2 y^3 \mathbf{a}_z \cdot d\mathbf{a}_z = 8 \int_0^2 x^2 dx \int_0^2 y^3 dy = 85.3C$$

$$\int_{\text{bottom}} = \int_0^2 \int_0^2 8x^2 y^3 \mathbf{a}_z \cdot (-d\mathbf{a}_z) = -85.3C$$

$$\int_{\text{left}} = \int_0^2 2x y|_{y=0} \mathbf{a}_y \cdot (-d\mathbf{a}_y) = 0$$

$$\int_{\text{right}} = \int_0^2 2x y|_{y=2} \mathbf{a}_y \cdot d\mathbf{a}_y = 16C$$

$$\int_{\text{front}} = \int 3\mathbf{a}_x \cdot d\mathbf{a}_x = 12C$$

$$\int_{\text{back}} = \int 3\mathbf{a}_x \cdot (-d\mathbf{a}_x) = -12C$$

$$\therefore Q_{\text{enc}} = \oint \mathbf{D} \cdot d\mathbf{S} = 16C.$$

$$\partial \quad \quad \quad 2 \quad 2 \quad 2$$

$$(c) \nabla \cdot \mathbf{D} = \frac{\partial}{\partial y} (2xy) = 2x; \quad \int \nabla \cdot \mathbf{D} dv = 2 \int_0^2 x dx \int_0^2 dy \int_0^2 dz = 16C.$$

P2.40: Suppose $\mathbf{D} = 6\rho \cos\phi \mathbf{a}_\phi$ C/m². (a) Determine the charge density at the point (3m, 90°, -2m). Find the total flux through the surface of a quartered-cylinder defined by $0 \leq \rho \leq 4m$, $0 \leq \phi \leq 90^\circ$, and $-4m \leq z \leq 0$ by evaluating (b) the left side of the divergence theorem and (c) the right side of the divergence theorem.

$$(a) (\nabla \cdot \mathbf{D})_{\text{cylinder}} = \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} = \frac{1}{\rho} \frac{\partial (6\rho \cos\phi)}{\partial \phi} = -6 \sin\phi.$$

$$\rho_v(3, 90^\circ, -2) = -6 \frac{C}{m^3}$$

$$(b) \oint \mathbf{D} \cdot d\mathbf{S} = \int_{\phi=0^\circ} + \int_{\phi=90^\circ} + \int_{\text{top}} + \int_{\text{bottom}} + \int_{\text{outside}},$$

note that the top, bottom and outside integrals yield zero since there is no component of \mathbf{D} in these $d\mathbf{S}$ directions.

$$\int_{\phi=0^\circ} = \int 6\rho \cos\phi \Big|_{\phi=0^\circ} \mathbf{a}_\phi \mathbf{i} (-d\rho dz \mathbf{a}_\phi) = -192C$$

$$\int_{\phi=90^\circ} = \int 6\rho \cos\phi \Big|_{\phi=90^\circ} \mathbf{a}_\phi \mathbf{i} (d\rho dz \mathbf{a}_\phi) = 0$$

So, $\oint \mathbf{D} \cdot d\mathbf{S} = -192C$.

(c)(c)

$$\nabla \cdot \mathbf{D} = -6 \sin\phi, \quad dv = \rho d\rho d\phi dz$$

$$\int \nabla \cdot \mathbf{D} dv = -6 \int_0^{90^\circ} \sin\phi d\phi \int_0^4 \rho d\rho \int_{-4}^4 dz = -192C.$$

P2.41: Suppose $\mathbf{D} = r^2 \sin\theta \mathbf{a}_r + \sin\theta \cos\phi \mathbf{a}_\phi$ C/m². (a) Determine the charge density at the point (1.0m, 45°, 90°). Find the total flux through the surface of a volume defined by $0.0 \leq r \leq 2.0$ m, $0.0^\circ \leq \theta \leq 90.0^\circ$, and $0.0 \leq \phi \leq 180^\circ$ by evaluating (b) the left side of the divergence theorem and (c) the right side of the divergence theorem.

The volume is that of a quartered-sphere, as indicated in Figure P2.41.

(a)

$$\nabla \cdot \mathbf{D} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r \sin\theta} \frac{\partial D_\theta}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial D_\phi}{\partial \phi} = 4r \sin\theta - \frac{\sin\phi}{r} = \rho_v$$

$$\rho_v(1, 45^\circ, 90^\circ) = 1.83 \frac{C}{m^3}$$

(b) $\oint \mathbf{D} \cdot d\mathbf{S} = \int_{\phi=0^\circ} + \int_{\phi=180^\circ} + \int_{\theta=90^\circ} + \int_{r=2}$; note that $\int_{\theta=90^\circ} = 0$ since $D_\theta = 0$.

$$\int_{\phi=0^\circ} = \int \sin\theta \cos\phi \Big|_{\phi=0^\circ} \mathbf{a}_\phi \mathbf{i} (-r dr d\theta \mathbf{a}_\phi) = -2C$$

$$\int_{\phi=180^\circ} = \int \sin\theta \cos\phi \Big|_{\phi=180^\circ} \mathbf{a}_\phi \mathbf{i} r dr d\theta \mathbf{a}_\phi = -2C$$

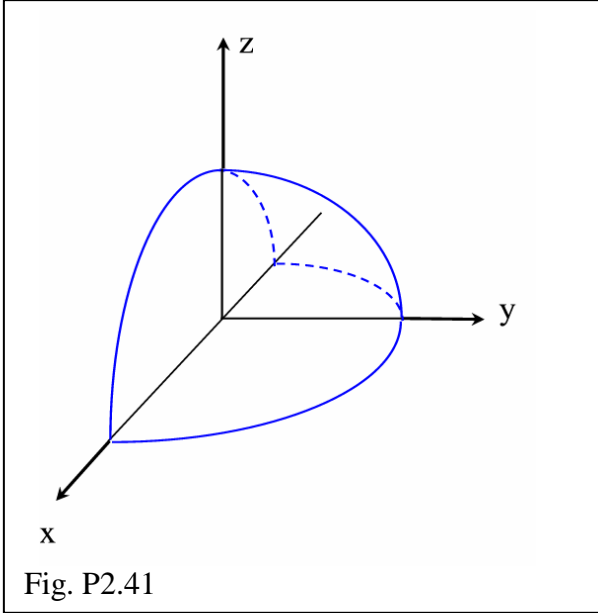
$$\int_{r=2} = \int_0^{90^\circ} \int_0^{180^\circ} r^2 \sin\theta \mathbf{a}_r \mathbf{i} r^2 \sin\theta d\theta d\phi \mathbf{a}_r = r^4 \int_0^{90^\circ} \sin^2\theta d\theta \int_0^{180^\circ} d\phi = 8\pi \int_0^{90^\circ} (1 - \cos 2\theta) d\theta = 4\pi^2 C$$

Summing these terms we have $Q = 4(\pi^2 - 1)C = 35.5C$.

(c)

$$\int \nabla \cdot \mathbf{D} dv = \int_0^2 \int_0^{\pi/2} \int_0^{\pi} 4r \sin\theta - \frac{\sin\phi}{r} r^2 \sin\theta dr d\theta d\phi$$

$$= 4 \int_0^2 r^3 dr \int_0^{\pi/2} \sin^2 \theta d\theta \int_0^{\pi} d\phi - \int_0^2 r dr \int_0^{\pi/2} \sin\theta d\theta \int_0^{\pi} \sin\phi d\phi = 4\pi^2 - 4 = 35.5C.$$



9. Electric Potential

P2.42: A sheet of charge density $\rho_s = 100 \text{ nC/m}^2$ occupies the x-z plane at $y = 0$. (a) Find the work required to move a 2.0 nC charge from P(-5.0m, 10.m, 2.0m) to M(2.0m, 3.0m, 0.0). (b) Find V_{MP} .

(a) $W = -Q \int \mathbf{E} \cdot d\mathbf{L}$; so we need \mathbf{E} for the sheet charge.

$$\mathbf{E} = \frac{\rho_s}{2\epsilon_0} \mathbf{a}_y = \frac{(100 \times 10^{-9} \text{ C})}{2(8.854 \times 10^{-12} \text{ F/m})} \frac{\text{FV}}{\text{C}} \mathbf{a}_y = 5.65 \times 10^3 \frac{\text{V}}{\text{m}} \mathbf{a}_y$$

Notice that we are only concerned with movement in the y-direction. We then have:

$$W = -2 \times 10^{-9} \text{ C} \int_{y=10}^{y=3} 5.65 \times 10^3 \frac{\text{V}}{\text{m}} \mathbf{a}_y \cdot d\mathbf{y} \mathbf{a}_y = -79 \mu\text{J}$$

(b) $V_{MP} = \frac{W}{Q} = \frac{(79 \mu\text{J}) \text{ CV}}{(2 \times 10^{-9} \text{ C}) \text{ J}} = 39.5 \text{ kV}$; so $V_{MP} = 40 \text{ kV}$.

P2.43: A surface is defined by the function $2x + 4y^2 - \ln z = 12$. Use the gradient equation to find a unit vector normal to the plane at the point (3.00m, 2.00m, 1.00m).

Let $F = 2x + 4y^2 - \ln z = 12$, then

$$\mathbf{a} = \frac{\nabla F}{|\nabla F|}; \quad \nabla F = 2\mathbf{a}_x + 8y\mathbf{a}_y - \frac{1}{z}\mathbf{a}_z,$$

At (3,2,1),

$$|\nabla F| = \sqrt{2^2 + 16^2 + 1^2} = 16.16,$$

$$\mathbf{a}_N = 0.124\mathbf{a}_x + 0.990\mathbf{a}_y - 0.062\mathbf{a}_z$$

P2.44: For the following potential distributions, use the gradient equation to find \mathbf{E} .

(a) $V = x + y^2z$ (V)

(b) $V = \rho^2 \sin\phi$ (V)

(c) $V = r \sin\theta \cos\phi$ (V).

(a) $\mathbf{E} = -\nabla V = -\mathbf{a}_x - 2yz\mathbf{a}_y - y^2\mathbf{a}_z$

(b) $\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial \rho}\mathbf{a}_\rho - \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi - \frac{\partial V}{\partial z}\mathbf{a}_z = -2\rho\sin\phi\mathbf{a}_\rho - \rho\cos\phi\mathbf{a}_\phi - \mathbf{a}_z$

(c)

$\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial r}\mathbf{a}_r - \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta - \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi = -\sin\theta\cos\phi\mathbf{a}_r - \cos\theta\cos\phi\mathbf{a}_\theta - \sin\phi\mathbf{a}_\phi$

P2.45: A 100 nC point charge is located at the origin. (a) Determine the potential difference V_{BA} between the point A(0.0,0.0,-6.0)m and point B(0.0,2.0,0.0)m. (b) How much work would be done to move a 1.0 nC charge from point A to point B against the electric field generated by the 100 nC point charge?

(a) $V_{BA} = -\int_A^B \mathbf{E} \cdot d\mathbf{L}$

The potential difference is only a function of radial distance from the origin. Letting $r_a = 6\text{m}$ and $r_b = 2\text{m}$, we then have

$$V_{BA} = -\int_{r_a}^{r_b} \frac{Q}{4\pi\epsilon_0 r^2} \mathbf{a}_r \cdot d\mathbf{r} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a} \right) = 300\text{V}.$$

(b) $W = Q_{BA} V_{BA} = (10^{-9}\text{C})(300\text{V}) = 300\text{nJ}$

P2.46: MATLAB: Suppose you have a pair of charges $Q_1(0.0, -5.0\text{m}, 0.0) = 1.0\text{ nC}$ and $Q_2(0.0, 5.0\text{m}, 0.0) = 2.0\text{ nC}$. Write a MATLAB routine to calculate the potential V_{RO} moving from the origin to the point R(5.0m, 0.0, 0.0). Your numerical integration will involve choosing a step size ΔL and finding the field at the center of the step. You should try several different step sizes to see how much this affects the solution.

```

% M-File: MLP0246
%
% Modify ML0207 to calculate the potential
% difference going from the origin (O) to the point
% R(5,0,0) given a pair of point charges
% Q1(0,-5,0)=1nC and Q2(0,5,0)=2nC.
%
% The approach will be to break up the distance
% from O to R into k sections. The total field E will
% be found at the center of each section (located
% at point P) and then dot(Ep,dLv) will give the
% potential drop across the kth section. Total
% potential is found by summing the potential drops.
%
% Wentworth, 1/7/03
%
% Variables:
%   Q1,Q2   the point charges, in nC
%   k       number of numerical integration steps
%   dL      magnitude of one step
%   dLv     vector for a step
%   x(n)    x location at center of section at P
%
%   R1,R2   vector from Q1,Q2 to P
%   E1,E2   electric fields from Q1 & Q2 at P
%   Etot    total electric field at P
%   V(n)    portion of dot(Etot,dL) at P

clc      %clears the command window
clear    %clears variables

% Initialize variables
k=64;
Q1=1;
Q2=2;
dL=5/k;
dLv=dL*[1 0 0];

% Perform calculation
for n=1:k
    x(n)=(n-1)*dL+dL/2;
    R1=[x(n) 5 0];
    R2=[x(n) -5 0];
    Rmag1=magvector(R1);
    Rmag2=magvector(R2);
    E1=9*Q1*R1/Rmag1^3;

```

```

E2=9*Q2*R2/Rmag2^3;
Etot=E1+E2;
V(n)=dot(Etot,dLv);
end

```

Vtot=sum(-V)

Now running the program:

```

Vtot =
    -1.5817

```

So $V_{RO} = -1.6 \text{ V}$.

P2.47: For an infinite length line of charge density $\rho_L = 20 \text{ nC/m}$ on the z -axis, find the potential difference V_{BA} between point B(0, 2m, 0) and point A(0, 1m, 0).

$$V_{BA} = -\int_A^B \mathbf{E} \cdot d\mathbf{L}; \quad \mathbf{E} = -\frac{\rho_L}{\rho} \mathbf{a}_\rho, \quad d\mathbf{L} = d\rho \mathbf{a}_\rho,$$

$$\text{so } V_{BA} = -\int_A^B \frac{\rho_L}{2\pi\epsilon_0\rho} \mathbf{a}_\rho \cdot d\rho \mathbf{a}_\rho = -\frac{\rho_L}{2\pi\epsilon_0} \ln(2) = -250 \text{ V}$$

P2.48: Find the electric field at point P(0.0,0.0,8.0m) resulting from a surface charge density $\rho_s = 5.0 \text{ nC/m}^2$ existing on the $z = 0$ plane from $\rho = 2.0 \text{ m}$ to $\rho = 6.0 \text{ m}$. Assume $V = 0$ at a point an infinite distance from the origin.

(Method 1)

For a ring of charge it was previously found that

$$\mathbf{E} = \frac{\rho_L a h \mathbf{a}_z}{2\epsilon_0 (a^2 + h^2)^{3/2}}.$$

We can then break up our disk into differential rings (see Figure P2.48), each contributing $d\mathbf{E}$ as:

$$d\mathbf{E} = \frac{\rho_s h \rho d\rho}{2\epsilon_0 (\rho^2 + h^2)^{3/2}} \mathbf{a}_z, \quad \text{where we've used } \rho = \rho d\rho_s$$

So we then have

$$\mathbf{E} = \frac{\rho_s h \mathbf{a}_z}{2\epsilon_0} \int \frac{\rho d\rho}{(\rho^2 + h^2)^{3/2}}.$$

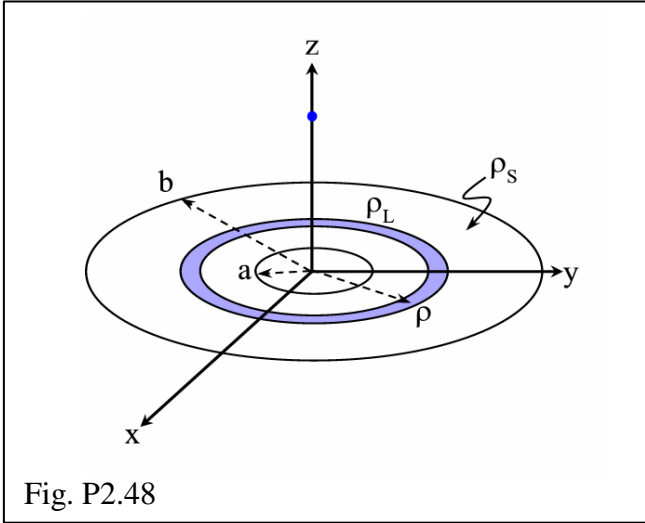
This is easy to integrate if we let $u = \rho^2 + h^2$, then $du = 2\rho d\rho$, and we have

$$\mathbf{E} = \frac{\rho_s h \mathbf{a}_z}{4\epsilon_0} \int_a^b \frac{du}{\sqrt{u}} = \frac{\rho_s h \mathbf{a}_z}{2\epsilon_0} \left. \frac{1}{\sqrt{\rho^2 + h^2}} \right|_a^b$$

Solving, we arrive at

$$\mathbf{E} = \frac{-\rho_s h}{2\epsilon_0} \frac{1}{\sqrt{b^2 + h^2}} - \frac{1}{\sqrt{a^2 + h^2}} \mathbf{a}_z.$$

Upon inserting the appropriate values we find $\mathbf{E} = 48 \text{ V/m } \mathbf{a}_z$.



(method 2)

Find an expression for potential and then evaluate the gradient at the point.

$$V = \int \frac{dQ}{R}, \quad R = \sqrt{\rho^2 + h^2}, \quad dQ = \rho_s \rho d\rho d\phi, \quad \text{so } V = \int \frac{\rho_s \rho d\rho d\phi}{\sqrt{\rho^2 + h^2}}$$

$$V = \int_a^b \frac{4\pi\epsilon_0 \rho_s \rho d\rho}{2\epsilon_0 \sqrt{\rho^2 + h^2}} = \frac{2\pi\epsilon_0 \rho_s}{\epsilon_0} \left[\sqrt{\rho^2 + h^2} \right]_a^b = \frac{2\pi\epsilon_0 \rho_s}{\epsilon_0} \left(\sqrt{b^2 + h^2} - \sqrt{a^2 + h^2} \right)$$

Now we let $h = z$ and $\mathbf{E} = -\nabla V$;

$$\mathbf{E} = -\frac{\rho_s}{2\epsilon_0} \left[\frac{\partial}{\partial z} \left(\sqrt{b^2 + z^2} \right) - \frac{\partial}{\partial z} \left(\sqrt{a^2 + z^2} \right) \right] \mathbf{a}_z$$

$$= -\frac{\rho_s}{2\epsilon_0} \left[\frac{z}{\sqrt{b^2 + z^2}} - \frac{z}{\sqrt{a^2 + z^2}} \right] \mathbf{a}_z = -\frac{\rho_s}{2\epsilon_0} \left[\frac{z}{\sqrt{b^2 + z^2}} - \frac{z}{\sqrt{a^2 + z^2}} \right] \mathbf{a}_z$$

Plugging in the values we find $\mathbf{E} = 48 \text{ V/m } \mathbf{a}_z$.

P2.49: Suppose a 6.0 m diameter ring with charge density 5.0 nC/m lies in the x-y plane with the origin at its center. Determine the potential difference V_{ho} between the point $h(0,0,0,4.0)\text{m}$ and the origin. (Hint: first find an expression for \mathbf{E} on the z-axis as a general function of z .)

For the ring of charge, replacing h with z , we have

$$\mathbf{E} = \frac{\rho_L a \mathbf{a}_z}{\epsilon_0} \frac{z}{\sqrt{a^2 + z^2}}$$

$$2\varepsilon (a^2 + z^2)^2$$

$$V_{ho} = -\int_0^h \mathbf{E} \cdot d\mathbf{L} = -\frac{\rho_L a}{2\epsilon_o} \int_0^h \frac{z dz}{(a^2 + z^2)^{3/2}}$$

Letting $u = a^2 + z^2$, $du = 2z dz$, we have

$$V_{ho} = -\frac{\rho_L a}{4\epsilon_o} \int_{a^2}^{a^2+h^2} u^{-3/2} du = \frac{\rho_L a}{2\epsilon_o} \frac{1}{\sqrt{u}}$$

Replacing u and evaluating from 0 to h ,

$$V_{ho} = \frac{\rho_L a}{2\epsilon_o} \left[\frac{1}{\sqrt{a^2+h^2}} - \frac{1}{a} \right]$$

$$= -36\pi V = -113V$$

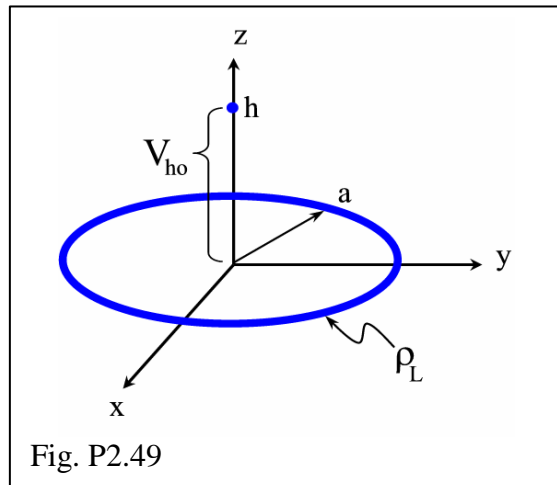


Fig. P2.49

10. Conductivity and Current

P2.50: A columnar beam of electrons from $0 \leq \rho \leq 1$ mm has a charge density $\rho_v = -0.1 \cos(\pi\rho/2)$ nC/mm³ (where ρ is in mm) and a velocity of 6×10^6 m/sec in the $+\mathbf{a}_z$ direction. Find the current.

Let's let $\rho_v = \rho \cos \frac{\pi\rho}{2}$ where $\rho = -0.1$ nC/mm³. Then we'll let $\mathbf{u} = u \mathbf{a}_z$, where $u =$

6×10^9 mm/s. Notice we convert the units to mm. Now,

$$\mathbf{J} = \rho_v \mathbf{u} = \rho u \cos \frac{\pi\rho}{2} \mathbf{a}_z$$

and with $d\mathbf{S} = \rho d\rho d\phi \mathbf{a}_z$, we then have

$$I = \int \mathbf{J} \cdot d\mathbf{S} = \rho u \int_0^1 \int_0^{2\pi} \rho \cos \frac{\pi\rho}{2} d\rho \int_0^{2\pi} d\phi$$

This becomes

$$I = A \int_0^1 \rho \cos \frac{\pi\rho}{2} d\rho, \text{ where } A = 2\pi\rho_o u_o$$

Now we can integrate by parts, or $\int u dv = uv - \int v du$, where $u = A\rho$, $du = A d\rho$, $v = \frac{2}{\pi} \sin \frac{\pi\rho}{2}$, and $dv = \cos \frac{\pi\rho}{2} d\rho$

We then have

$$I = \frac{2Aa}{\pi} \sin \frac{\pi a}{2} - \frac{4A}{\pi^2} \cos \frac{\pi a}{2}$$

To evaluate, we first find $A = 2\pi(-0.1 \times 10^{-9})(6 \times 10^9) = -3.77$, and then $I = 2.40 - 1.53 = 0.87A$.

$$I = 0.87A$$

P2.51: Two spherical conductive shells of radius a and b ($b > a$) are separated by a material with conductivity σ . Find an expression for the resistance between the two spheres.

First find \mathbf{E} for $a < r < b$, assuming $+Q$ at $r = a$ and $-Q$ at $r = b$. From Gauss's law:

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2} \mathbf{a}_r$$

Now find V_{ab} :

$$\begin{aligned} V_{ab} &= -\int_b^a \mathbf{E} \cdot d\mathbf{L} = -\int_b^a \frac{Q}{4\pi\epsilon_0 r^2} \mathbf{a}_r \cdot dr \mathbf{a}_r \\ &= \frac{Q}{4\pi\epsilon_0} \int_b^a \frac{1}{r^2} dr = \frac{Q}{4\pi\epsilon_0} \left[-\frac{1}{r} \right]_b^a \\ &= \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) \end{aligned}$$

Now can find I :

$$\begin{aligned} I &= \int \mathbf{J} \cdot d\mathbf{S} = \int \sigma \mathbf{E} \cdot d\mathbf{S} = \sigma \int \frac{Q}{4\pi\epsilon_0 r^2} \mathbf{a}_r \cdot r^2 \sin\theta d\theta d\phi \mathbf{a}_r \\ &= \frac{\sigma Q}{4\pi\epsilon_0} \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi = \frac{\sigma Q}{\epsilon_0} \end{aligned}$$

Finally, $R = \frac{V_{ab}}{I} = \frac{1}{4\pi\sigma} \left(\frac{1}{a} - \frac{1}{b} \right)$

P2.52: The typical length of each piece of jumper wire on a student's protoboard is 5.0 cm. Assuming AWG-20 (wire diameter 0.812 mm) copper wire, (a) determine the resistance for this length of wire. (b) Determine the power dissipated in the wire for 10. mA of current.

$$(a) R = \frac{1}{\sigma} \frac{L}{\pi a^2} = \frac{1}{5.8 \times 10^7 (S/m)} \frac{0.05m}{\pi (0.406 \times 10^{-3} m)^2} = 1.67 m\Omega$$

so $R = 1.7 m\Omega$

$$(b) P = I^2 R = (10 \times 10^{-3} A)^2 (1.7 \times 10^{-3} \Omega) = 170 nW$$

P2.53: A densely wrapped coil of AWG-22 (0.644 mm diameter) copper magnet wire is 150 m long. The wire has a very thin insulative sheath. Determine the resistance for this length of wire.

$$R = \frac{1}{\sigma} \frac{L}{\pi a^2} = \frac{1}{5.8 \times 10^7 S/m} \frac{150m}{\pi (0.322 \times 10^{-3} m)^2} = 7.94 \Omega$$

so $R = 7.9 \Omega$

P2.54: Determine an expression for the power dissipated per unit length in coaxial cable of inner radius a , outer radius b , and conductivity between the conductors σ if a potential difference V_{ab} is applied.

$$\text{From Eqn(2.84) we have } R = \frac{1}{2\pi\sigma L} \ln \frac{b}{a}$$

Now for a given potential difference V_{ab} we have

$$P = \frac{V_{ab}^2}{R} = \frac{V_{ab}^2}{\frac{1}{2\pi\sigma L} \ln \frac{b}{a}} = 2\pi\sigma L V_{ab}^2 \ln \frac{b}{a}$$

P2.55: Find the resistance per unit length of a stainless steel pipe of inner radius 2.5 cm and outer radius 3.0 cm.

$$R = \frac{L}{\sigma \pi (b^2 - a^2)}$$

so we have $R = \frac{1}{L} \frac{L}{\sigma \pi (b^2 - a^2)} = \frac{1}{\sigma \pi (b^2 - a^2)}$

$$= \frac{1}{1.1 \times 10^6 \text{ S/m} \pi (.030^2 - .025^2) \text{ m}^2} = 1.05 \frac{\text{m}\Omega}{\text{m}}$$

so $R/L = 1.0 \text{ m}\Omega/\text{m}$

P2.56: A nickel wire of diameter 5.0 mm is surrounded by a 0.50 mm thick layer of silver. What is the resistance per unit length for this wire? Assuming 1.0 m of this wire carries 1.0 A of current, determine the power dissipated in the nickel portion and in the silver portion of the wire.

We can treat this wire as two resistors in parallel. We have

$$\frac{R_{Ni}}{L} = \frac{1}{1.5 \times 10^7 \pi (2.5 \times 10^{-3})^2} = 3.4 \times 10^{-3} \frac{\Omega}{\text{m}}$$

$$\frac{R_{Ag}}{L} = \frac{1}{6.2 \times 10^7 \pi (3 \times 10^{-3})^2 - (2.5 \times 10^{-3})^2} = 1.87 \times 10^{-3} \frac{\Omega}{\text{m}}$$

$$\frac{R_{total}}{L} = \frac{R_{Ni}}{L} \parallel \frac{R_{Ag}}{L} = 1.2 \frac{\text{m}\Omega}{\text{m}}$$

To find the power dissipated, we first find the potential difference:

$$V = IR_{total} = 1.2 \text{ mV}$$

then

$$P = \frac{V^2}{R} = 0.42 \text{ mW}, P = \frac{V^2}{R} = 0.77 \text{ mW}$$

$$\begin{matrix} Ni & R \\ & Ni \end{matrix}$$
$$\begin{matrix} Ag & R \\ & Ag \end{matrix}$$

11. Dielectrics

P2.57: A material has 12.0 V/m \mathbf{a}_x field intensity with permittivity 194.5 pF/m. Determine the electric flux density.

$$\mathbf{D} = \epsilon \mathbf{E} = \left(194.5 \times 10^{-12} \frac{\text{F}}{\text{m}}\right) \left(12 \frac{\text{V}}{\text{m}}\right) \frac{\text{C}}{\text{FV}} = 2.3 \frac{\text{nC}}{\text{m}^2} \mathbf{a}_x$$

P2.58: MATLAB: A 20 nC point charge at the origin is embedded in Teflon ($\epsilon_r = 2.1$). Find and plot the magnitudes of the polarization vector, the electric field intensity and the electric flux density at a radial distance from 0.1 cm out to 10 cm.

We use the following equations:

$$E = \frac{Q}{4\pi\epsilon_r \epsilon_o r^2}, P = \chi_e \epsilon_o E, D = \epsilon_r \epsilon_o E$$

```
% M-File: MLP0258
%
% Plot E, P and D vs distance r from a point
% charge Q at the origin with a dielectric.
%
% Variables
% Q      charge (C)
% eo     free space permittivity (F/m)
% r      radial distance (m)
% Chi    electric susceptibility
% E      electric field intensity (V/m)
% D      electric flux density (C/m^2)
% P      polarization vector (C/m^2)

% initialize variables
Q=20e-9;
er=2.1;
eo=8.854e-12;
Chi=er-1;

% perform calculations
r=0.001:.001:0.100;
rcm=r.*100;
E=Q./(4*pi*r.^2);
P=Chi*eo*E;
D=er*eo*E;

% plot data
```

```

subplot(2,1,1)
loglog(rcm,P,'--k',rcm,D,'-k')
legend('P','D')
ylabel('C/m^2')
grid on
subplot(2,1,2)
loglog(rcm,E)
ylabel('V/m')
xlabel('radial distance (cm)')
grid on

```

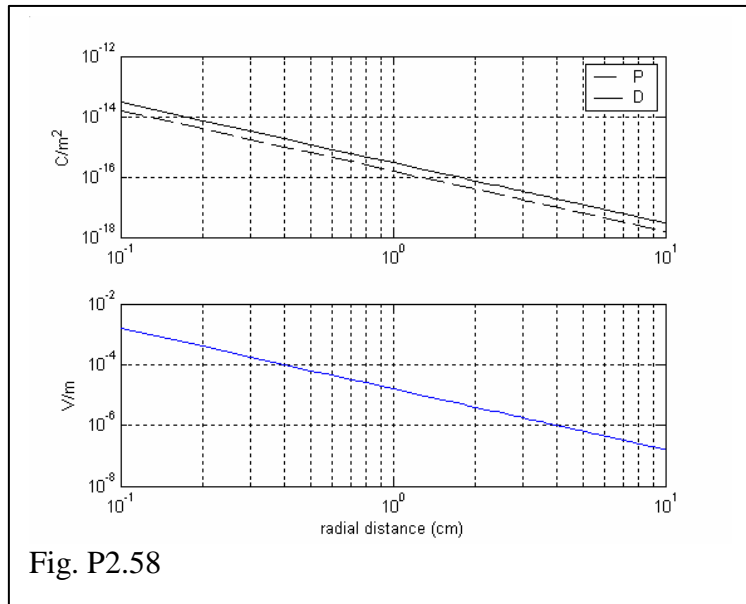


Fig. P2.58

P2.59: Suppose the force is very carefully measured between a pair of point charges separated by a dielectric material and is found to be 20 nN. The dielectric material is removed without changing the position of the point charges, and the force has increased to 100 nN. What is the relative permittivity of the dielectric?

$$F_1 = \frac{Q_1 Q_2}{4\pi\epsilon_r \epsilon_o R^2}, \quad F_2 = \frac{Q_1 Q_2}{4\pi\epsilon_o R^2}, \quad \frac{F_2}{F_1} = \epsilon_r = \frac{100}{20} = 5$$

P2.60: The potential field in a material with $\epsilon_r = 10.2$ is $V = 12xy^2$ (V). Find **E**, **P** and **D**.

$$\mathbf{E} = -\nabla V = -\left(\frac{\partial(12xy^2)}{\partial x} \mathbf{a}_x - \frac{\partial(12xy^2)}{\partial y} \mathbf{a}_y \right) = -12y^2 \mathbf{a}_x - 24xy \mathbf{a}_y \quad \frac{V}{m}$$

$$\mathbf{D} = \epsilon_r \epsilon_o \mathbf{E} = -1.1y^2 \mathbf{a}_x - 2.2xy \mathbf{a}_y \quad \frac{nC}{m^2}$$

$$\chi_e = \epsilon_r - 1 = 9.2$$

$$\mathbf{P} = \chi_e \epsilon_o \mathbf{E} = (9.2)(8.854 \times 10^{-12}) \mathbf{E} = -9.8y^2 \mathbf{a}_x - 2.00xy \mathbf{a}_y \quad \frac{nC}{m^2}$$

P2.61: In a mineral oil dielectric, with breakdown voltage of 15 MV/m, the potential function is $V = x^3 - 6x^2 - 3Ix$ (MV). Is the dielectric likely to breakdown, and if so, where?

$$\mathbf{E} = -\nabla V = (-3x^2 + 12x + 3.1)\mathbf{a}_x \frac{MV}{m}$$

$$\frac{d\mathbf{E}}{dx} = 6x + 12, \quad \frac{d^2\mathbf{E}}{dx^2} = -6, \quad \text{so from } 6x - 12 = 0 \text{ we find the maximum electric field}$$

occurs at $x = 2\text{m}$.

At $x = 2\text{m}$, we have $\mathbf{E} = -12 + 24 + 3.1 = 15.1 \text{ MV/m}$, exceeding the breakdown voltage.

12. Boundary Conditions

P2.62: For $y < 0$, $\epsilon_{r1} = 4.0$ and $\mathbf{E}_1 = 3\mathbf{a}_x + 6\pi\mathbf{a}_y + 4\mathbf{a}_z \text{ V/m}$. At $y = 0$, $\rho_s = 0.25 \text{ nC/m}^2$. If $\epsilon_{r2} = 5.0$ for $y > 0$, find \mathbf{E}_2 .

$\mathbf{E}_1 = 3\mathbf{a}_x + 6\pi\mathbf{a}_y + 4\mathbf{a}_z \text{ V/m}$

(g) $\mathbf{E}_2 = 3\mathbf{a}_x + 20.7\mathbf{a}_y + 4\mathbf{a}_z \text{ V/m}$

(a) $\mathbf{E}_{N1} = 6\pi\mathbf{a}_y$

(f) $\mathbf{E}_{N2} = \mathbf{D}_{N2}/5\epsilon_0 = 20.7\mathbf{a}_y$

(b) $\mathbf{E}_{T1} = 3\mathbf{a}_x + 4\mathbf{a}_z$

(c) $\mathbf{E}_{T2} = \mathbf{E}_{T1} = 3\mathbf{a}_x + 4\mathbf{a}_z$

(d) $\mathbf{D}_{N1} = \epsilon_{r1}\epsilon_0\mathbf{E}_{N1} = 24\pi\epsilon_0 \mathbf{a}_y$

(e) $\mathbf{D}_{N2} = 0.92 \mathbf{a}_y$

(e) $\mathbf{a}_{21} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s, \quad -\mathbf{a}_y \cdot (\mathbf{D}_{N1} - \mathbf{D}_{N2})\mathbf{a}_y = D_{N2} - D_{N1} = \rho_s$

∩

$$D_{N2} = \rho_s + D_{N1} = 0.25 \frac{nC}{m^2} + 24 \frac{10^{-9} F}{36\pi m} \pi \frac{nC}{m^2} = 0.92 \frac{nC}{m^2}$$

P2.63: For $z \leq 0$, $\epsilon_{r1} = 9.0$ and for $z > 0$, $\epsilon_{r2} = 4.0$. If \mathbf{E}_1 makes a 30° angle with a normal to the surface, what angle does \mathbf{E}_2 make with a normal to the surface?

Refer to Figure P2.63.

$E_{T1} = E_1 \sin\theta_1, E_{T2} = E_2 \sin\theta_2,$ and $E_{T1} = E_{T2}$

also

$D_{N1} = \epsilon_{r1}\epsilon_0 E_1 \cos\theta_1, \quad D_{N2} = \epsilon_{r2}\epsilon_0 E_2 \cos\theta_2,$ and $D_{N1} = D_{N2}$ (since $\rho_s = 0$)

Therefore

$$E_{T1} = E_{T2}, \text{ and after routine math we find } \theta_2 = \tan^{-1} \left(\frac{\epsilon_{r2}}{\epsilon_{r1}} \tan\theta_1 \right)$$

Using this formula we obtain for this problem $\theta_2 = 14^\circ$.

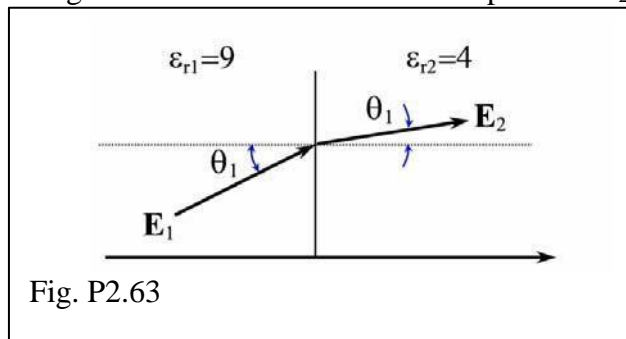


Fig. P2.63

P2.64: A plane defined by $3x + 2y + z = 6$ separates two dielectrics. The first dielectric, on the side of the plane containing the origin, has $\epsilon_{r1} = 3.0$ and $\mathbf{E}_1 = 4.0\mathbf{a}_z$ V/m. The other dielectric has $\epsilon_{r2} = 6.0$. Find \mathbf{E}_2 .

We first use gradient to find a normal to the planar surface.

Let $F = 3x + 2y + z - 6 = 0$.

$$\nabla F = 3\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z, \quad \text{and} \quad |\nabla F| = \sqrt{14},$$

$$\text{so } \mathbf{a}_N = \frac{\nabla F}{|\nabla F|} = 0.802\mathbf{a}_x + 0.534\mathbf{a}_y + 0.267\mathbf{a}_z.$$

$$\mathbf{a}_N = \frac{\nabla F}{|\nabla F|} = 0.802\mathbf{a}_x + 0.534\mathbf{a}_y + 0.267\mathbf{a}_z.$$

Now we can work the boundary condition problem.

$$\mathbf{E}_1 = 4\mathbf{a}_z, \quad \mathbf{E}_{N1} = (\mathbf{E}_1 \cdot \mathbf{a}_N)\mathbf{a}_N = 0.857\mathbf{a}_x + 0.570\mathbf{a}_y + 0.285\mathbf{a}_z.$$

$$\mathbf{E}_{T1} = \mathbf{E}_1 - \mathbf{E}_{N1} = -0.857\mathbf{a}_x - 0.570\mathbf{a}_y + 3.715\mathbf{a}_z, \quad \mathbf{E}_{T2} = \mathbf{E}_{T1}$$

$$\mathbf{D}_{N1} = \epsilon_{r1}\epsilon_0\mathbf{E}_{N1} = \epsilon_0 \{2.571\mathbf{a}_x + 1.710\mathbf{a}_y + 0.855\mathbf{a}_z\} \quad \text{and} \quad \mathbf{D}_{N2} = \mathbf{D}_{N1}$$

$$\mathbf{E}_{N2} = \frac{\mathbf{D}_{N2}}{\epsilon_0\epsilon_{r2}} = \frac{\mathbf{D}_{N1}}{6\epsilon_0} = 0.429\mathbf{a}_x + 0.285\mathbf{a}_y + 0.143\mathbf{a}_z$$

$$\text{Finally we have } \mathbf{E}_2 = \mathbf{E}_{T2} + \mathbf{E}_{N2} = -0.43\mathbf{a}_x - 0.29\mathbf{a}_y + 3.8\mathbf{a}_z \frac{\text{V}}{\text{m}}.$$

P2.65: MATLAB: Consider a dielectric-dielectric charge free boundary at the plane $z = 0$. Construct a program that will allow the user to enter ϵ_{r1} (for $z < 0$), ϵ_{r2} , and \mathbf{E}_1 , and will then calculate \mathbf{E}_2 . (Just for fun, you may want to have the program calculate the angles that \mathbf{E}_1 and \mathbf{E}_2 make with a normal to the surface).

```
% M-File: MLP0265
%
% Given E1 at boundary between a pair of
% dielectrics with no charge at boundary,
% calculate E2. Also calculates angles.
% clc
clear
% enter variables
disp('enter vector quantities in brackets,')
disp('for example: [1 2 3]')
er1=input('relative permittivity in material 1: ');
er2=input('relative permittivity in material 2: ');
a12=input('unit vector from mtrl 1 to mtrl 2: ');
E1=input('electric field intensity vector in mtrl 1: ');

% perform calculations
```

```

En1=dot(E1,a12)*a12;
Et1=E1-En1;
Et2=Et1;
Dn1=er1*En1;      %ignores eo since it will factor out
Dn2=Dn1;
En2=Dn2/er2;
E2=Et2+En2

% calculate the angles
th1=atan(magvector(Et1)/magvector(En1));
th2=atan(magvector(Et2)/magvector(En2));
th1r=th1*180/pi
th2r=th2*180/pi

```

Now run the program:

```

enter vector quantities in brackets,
for example: [1 2 3]
relative permittivity in material 1: 2
relative permittivity in material 2: 5
unit vector from mtrl 1 to mtrl 2: [0 0 1]
electric field intensity vector in mtrl 1: [3 4 5]

```

E2 =

3 4 2

th1r =

45

th2r =

68.1986

P2.66: A 1.0 cm diameter conductor is sheathed with a 0.50 cm thickness of Teflon and then a 2.0 cm (inner) diameter outer conductor. (a) Use Laplace's equations to find an expression for the potential as a function of ρ in the dielectric. (b) Find \mathbf{E} as a function of ρ . (c) What is the maximum potential difference that can be applied across this coaxial cable without breaking down the dielectric?

(a) Since V is only a function of ρ ,

$$\nabla^2 V_{cyl} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) = 0$$

$$\text{so } \rho \frac{\partial V}{\partial \rho} = A$$

$$\text{or } V = A \ln \rho + B$$

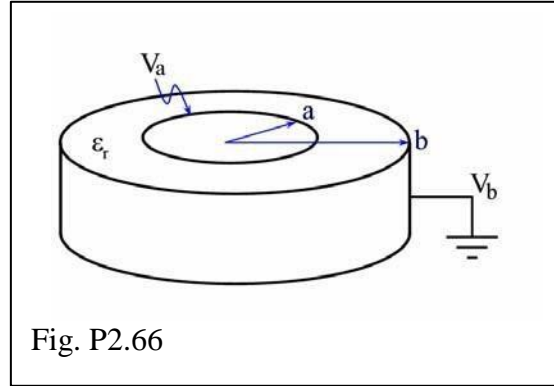
where A and B are constants.

Now we apply boundary conditions.

BC1:

$$0 = A \ln b + B, B = -A \ln b,$$

$$\therefore V = A \ln \frac{\rho}{b}$$



$$\text{BC2: } V = A \ln \frac{a}{b} \quad A = \frac{V_a}{\ln \left(\frac{a}{b} \right)}, \quad V = V_a \frac{\ln \left(\frac{\rho}{b} \right)}{\ln \left(\frac{a}{b} \right)}$$

or

$$V = -1.443 V_a \ln (100\rho).$$

$$(b) \mathbf{E} = -\nabla V = -\frac{\partial V}{\partial \rho} \mathbf{a}_\rho = \frac{1.443 V_a}{\rho} \mathbf{a}_\rho$$

$$(c) \left(\begin{array}{l} c \\) \end{array} = 1.443 V_a = 288.5 V = E = 60 \times 10^6,$$

$$E_{\max} = \frac{60 \times 10^6}{0.005} \quad a \quad br$$

$$\text{so } V_a = \frac{208 kV}{288.5}, \quad \therefore (V_{ab})_{\max} = 210 kV$$

P2.67: A 1.0 m long carbon pipe of inner diameter 3.0 cm and outer diameter 5.0 cm is cut in half lengthwise. Determine the resistance between the inner surface and the outer surface of one of the half sections of pipe.

One approach is to consider the resistance for the half-section of pipe is twice the resistance for a complete cylindrical section, given by Eqn. (2.84). But we'll use the Laplace equation approach instead.

$$\text{Laplace: } \nabla^2 V_{cyl} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) = 0; \text{ here we see } V \text{ only depends on } \rho$$

$= A;$

□
□

$$V = A \ln \rho + B;$$

where A and B are constants.

Now apply boundary conditions.

BC1:

$$V_b = 0 = A \ln b + B; \quad B = -A \ln b;$$

$$V = A \ln \frac{\rho}{b}$$

BC2:

$$V = A \ln \frac{a}{b}; \quad A = \frac{V_a}{\ln(a/b)}$$

$$\ln(\rho/b)$$

$$V = V_a \ln \left(\frac{b}{a} \right)$$

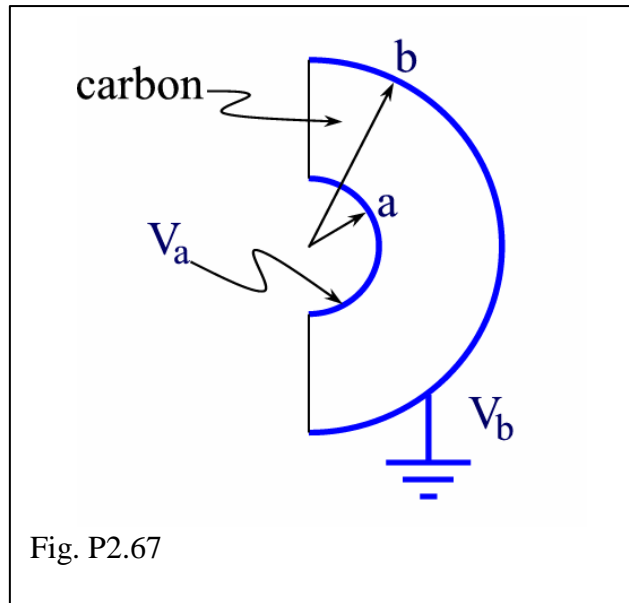
$$\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial \rho} \mathbf{a}_\rho = -\frac{V_a}{\ln(a/b)} \frac{1}{\rho} \mathbf{a}_\rho$$

$$\mathbf{J} = \sigma \mathbf{E}$$

$$I = \int \mathbf{J} \cdot d\mathbf{S} = -\frac{\sigma V_a}{\ln(a/b)} \int_0^L \frac{1}{\rho} \rho d\phi \int_0^{2\pi} dz = \frac{\pi L \sigma V_a}{\ln(b/a)}$$

$$R = \frac{V_a}{I} = \frac{\ln(b/a)}{\pi \sigma L}$$

$$I = \frac{V_a}{R} = 5.4 \mu\Omega$$



P2.68: For a coaxial cable of inner conductor radius a and outer conductor radius b and a dielectric ϵ_r in-between, assume a charge density $\rho_v = \rho_o/\rho$ is added in the dielectric region. Use Poisson's equation to derive an expression for V and \mathbf{E} . Calculate ρ_s on each plate.

$$\nabla^2 V = -\frac{\rho_v}{\epsilon} = -\frac{1}{\epsilon} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) = -\frac{\rho_o}{\epsilon \rho^2}$$

$$\frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) = \frac{\rho_o}{\epsilon}; \quad \int d \left(\rho \frac{\partial V}{\partial \rho} \right) = \int \frac{\rho_o}{\epsilon} d\rho; \quad \rho \frac{\partial V}{\partial \rho} = \frac{\rho_o}{\epsilon} \rho + A, \text{ where } A \text{ is a constant.}$$

$$\frac{\partial V}{\partial \rho} = \frac{\rho_o}{\epsilon} + \frac{A}{\rho}; \quad dV = \int \left(\frac{\rho_o}{\epsilon} + \frac{A}{\rho} \right) d\rho; \quad V = \frac{\rho_o}{2\epsilon} \rho^2 + A \ln \rho + B, \text{ where } B \text{ is a constant.}$$

Now apply boundary conditions: $V = V_a$ at $\rho = a$

Applying the second one gives us:

and $V = 0$ at $\rho = b$

$$V = \frac{\rho_o}{\epsilon} (\rho - b) + A \ln\left(\frac{\rho}{b}\right).$$

Applying the first one:

$$V = \frac{\rho_o}{\epsilon} (a-b) + A \ln\left(\frac{a}{b}\right); \quad A = \frac{V + \frac{\rho_o}{\epsilon}(b-a)}{\ln\left(\frac{a}{b}\right)}$$

Therefore,

$$V(\rho) = \frac{V + \frac{\rho_o}{\epsilon}(b-a)}{\ln\left(\frac{a}{b}\right)} \ln\left(\frac{\rho}{b}\right) + \frac{\rho_o}{\epsilon}(\rho - b)$$

$$\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial \rho} \mathbf{a}_\rho = -\left[\frac{K}{\rho} + \frac{\rho_o}{\epsilon} \right] \mathbf{a}_\rho$$

$$\mathbf{E} = -\left[\frac{K}{\rho} + \frac{\rho_o}{\epsilon} \right] \mathbf{a}_\rho$$

where

$$K = \frac{V + \frac{\rho_o}{\epsilon}(b-a)}{\ln\left(\frac{a}{b}\right)}$$

so $\mathbf{E} = -\left[\frac{K}{\rho} + \frac{\rho_o}{\epsilon} \right] \mathbf{a}_\rho$

$$D_N = \rho_s; \quad D_{Na} = \epsilon \mathbf{E}|_{\rho=a} = -\epsilon \left[\frac{K}{a} + \frac{\rho_o}{\epsilon} \right] \mathbf{a}_\rho = \rho_{sa} \mathbf{a}_\rho$$

$$D_{Nb} = \epsilon \mathbf{E}|_{\rho=b} = -\epsilon \left[\frac{K}{b} + \frac{\rho_o}{\epsilon} \right] \mathbf{a}_\rho = \rho_{sb} \mathbf{a}_\rho$$

$$\rho_v = \rho_o \sin \frac{\pi z}{2a}$$

is added between the plates. Use Poisson's equation to derive a new expression for V and \mathbf{E} . Calculate ρ_s on each plate.

$$\frac{\partial^2 V(z)}{\partial z^2} = -\rho = \frac{-\rho_o \sin(\frac{\pi z}{2d})}{\epsilon}$$

$$\frac{\partial V(z)}{\partial z} = \frac{2\rho_o d}{\pi^2 \epsilon} \cos(\frac{\pi z}{2d}) + A$$

$$V(z) = \frac{2\rho_o d^2}{\pi^2 \epsilon} \sin(\frac{\pi z}{2d}) + Az + B$$

$$\int \frac{2\rho_o d}{\pi^2 \epsilon} \cos(\frac{\pi z}{2d}) dz + \int A dz = \frac{2\rho_o d^2}{\pi^2 \epsilon} \sin(\frac{\pi z}{2d}) + Az + B$$

Now apply the boundary conditions:

$$V = 0 = B; \quad V = \frac{2\rho_o d^2}{\pi^2 \epsilon} \sin(\frac{\pi d}{2d}) + Ad; \quad A = \frac{2\rho_o d^2}{d} - \frac{\pi^2 \epsilon V_d}{d}$$

$$V(z) = \frac{2\rho_o d^2}{\pi^2 \epsilon} \sin(\frac{\pi z}{2d}) + \frac{V_d}{d} z - \frac{2\rho_o d^2}{\pi^2 \epsilon} z$$

$$E = -\nabla V = -\frac{\partial V}{\partial z} \mathbf{a}_z = \frac{2\rho_o d}{\pi^2 \epsilon} \cos(\frac{\pi z}{2d}) - \frac{V_d}{d} + \frac{2\rho_o d}{\pi^2 \epsilon} z \mathbf{a}_z$$

$$E = \frac{2\rho_o d}{\pi^2 \epsilon} \cos(\frac{\pi z}{2d}) - \frac{V_d}{d} + \frac{2\rho_o d}{\pi^2 \epsilon} z \mathbf{a}_z$$

$$E = \frac{2\rho_o d}{\pi^2 \epsilon} \cos(\frac{\pi z}{2d}) - \frac{V_d}{d} + \frac{2\rho_o d}{\pi^2 \epsilon} z \mathbf{a}_z$$

$$\text{at } z = 0, D_N = \epsilon E, \text{ so } \rho_s|_{z=0} = -\frac{\rho_o d}{\pi} - \frac{\epsilon V_d}{d} + \frac{2\rho_o d}{\pi^2} = \rho_s$$

$$\text{at } z = d, D_N = \epsilon E, \text{ so } \rho_s|_{z=d} = -\frac{\epsilon V_d}{d} + \frac{2\rho_o d}{\pi^2} = \rho_s$$

13. Capacitors

P2.70: A parallel plate capacitor is constructed such that the dielectric can be easily removed. With the dielectric in place, the capacitance is 48 nF. With the dielectric removed, the capacitance drops to 12 nF. Determine the relative permittivity of the dielectric.

$$C = \frac{\epsilon_r \epsilon_o A}{d}; \quad C = \frac{\epsilon_o A}{d}; \quad \frac{C_1}{C_2} = \epsilon_r = \frac{48}{12} = 4.0$$

P2.71: A parallel plate capacitor with a 1.0 m² surface area for each plate, a 2.0 mm plate separation, and a dielectric with relative permittivity of 1200 has a 12. V potential difference across the plates. (a) What is the minimum allowed dielectric strength for this

capacitor? Calculate (b) the capacitance, and (c) the magnitude of the charge density on one of the plates.

$$(a) E = \frac{12V}{0.002m} = 6 \frac{kV}{m}; \quad (a)E = 6 \frac{kV}{m}$$

$$(b) C = \frac{\epsilon_r \epsilon_0 A}{d} = \frac{(1200)(8.854 \times 10^{-12} \text{ F/m})(1 \text{ m}^2)}{0.002 \text{ m}} = 5.3 \mu\text{F}$$

$$(c) C = \frac{Q}{V}; \quad Q = CV = (5.3 \times 10^{-6} \text{ F})(12 \text{ V}) = 64 \mu\text{C}$$

P2.72: A conical section of material extends from $2.0 \text{ cm} \leq r \leq 9.0 \text{ cm}$ for $0 \leq \theta \leq 30^\circ$ with $\epsilon_r = 9.0$ and $\sigma = 0.020 \text{ S/m}$. Conductive plates are placed at each radial end of the section. Determine the resistance and capacitance of the section.

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) = 0 \quad \frac{\partial V}{\partial \theta} = 0; \quad V = -\frac{A}{r} + B, \text{ where } A \text{ and } B \text{ are constants.}$$

Boundary conditions: $r = a, V = 0$ and $r = b, V = V_b$

$$V = V \left(\frac{1}{a} - \frac{1}{r} \right)$$

$$\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial r} \mathbf{a}_r$$

$$\mathbf{E} = \frac{-V_b \mathbf{a}_r}{\left(\frac{1}{a} - \frac{1}{r} \right)}, \quad \mathbf{D} = \frac{-\epsilon_r \epsilon_0 V_b \mathbf{a}_r}{\left(\frac{1}{a} - \frac{1}{r} \right)^2}$$

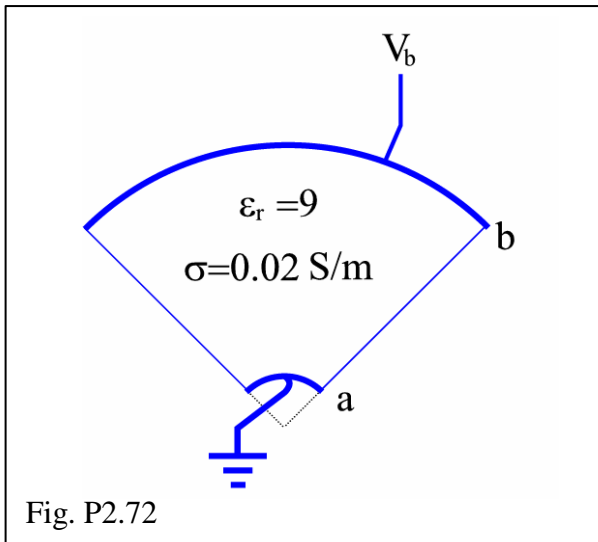


Fig. P2.72

$$\rho_{sb} = \frac{-\epsilon_r \epsilon_0 V_b}{\left(\frac{1}{a} - \frac{1}{b} \right) b^2};$$

$$Q = \int \rho_s dS; \quad dS = r^2 \sin\theta \, d\theta \, d\phi$$

$$Q_b = \frac{\epsilon \epsilon_0 V}{2} \int_0^{30^\circ} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$= 1.73 \times 10^{-12} \text{ V}$$

$$C = \frac{Q_b}{V_b} = 1.7 \text{ pF}; \quad RC = \frac{\epsilon}{\sigma}; \quad R = \frac{1}{\sigma} = 2.3 \text{ k}\Omega$$

$$V_b \quad \sigma \quad C \sigma$$

P2.73: An inhomogeneous dielectric fills a parallel plate capacitor of surface area 50 cm² and thickness 1.0 cm. You are given $\epsilon_r = 3(1 + z)$, where z is measured from the bottom plate in cm. Determine the capacitance.

Place $+Q$ at $z = d$ and $-Q$ at $z = 0$

$$\rho_s = \frac{Q}{S_d} \mathbf{a}_d, \quad \mathbf{D} = -\frac{Q}{S_d} \mathbf{a}_z, \quad \mathbf{E} = -\frac{Q}{\sum_r \sum_o S^z} \mathbf{a}_z$$

$$V_{do} = -\int \mathbf{E} \cdot d\mathbf{L} = -\int \frac{Q}{\epsilon \epsilon_0 S} \mathbf{a}_z \cdot dz \mathbf{a}_z = -\frac{Q}{S} \int \frac{dz}{\epsilon}$$

evaluating the integral:

$$\int \frac{dz}{\epsilon} = \int \frac{dz}{\epsilon_0 3(1+z)} = \frac{1}{3\epsilon_0} \ln(1+z) \Big|_0^1 = \frac{1}{3\epsilon_0} \ln 2 \text{ cm}$$

$$C = \frac{Q}{V_{do}} = \frac{3\epsilon_0 S}{\ln 2} = \frac{3(8.854 \times 10^{-12} \text{ F/m})(50 \text{ cm}^2)}{(\ln(2) \text{ cm})} \cdot \frac{1}{100 \text{ cm}} = 19$$

P2.74: Given $\mathbf{E} = 5xy\mathbf{a}_x + 3z\mathbf{a}_z$ V/m, find the electrostatic potential energy stored in a volume defined by $0 \leq x \leq 2$ m, $0 \leq y \leq 1$ m, and $0 \leq z \leq 1$ m. Assume $\epsilon = \epsilon_0$.

$$W = \frac{1}{2} \epsilon \int \mathbf{E} \cdot \mathbf{E} \, dv = \frac{1}{2} \epsilon \int (25x^2 y^2 + 9z^2) \, dx \, dy \, dz$$

$$\int_0^2 \int_0^1 \int_0^1 (25x^2 y^2 + 9z^2) \, dx \, dy \, dz$$

$$W = \frac{1}{2} \epsilon_0 \int_0^2 \int_0^1 \int_0^1 (25x^2 y^2 + 9z^2) \, dx \, dy \, dz = 125 \text{ pJ}$$

P2.75: Suppose a coaxial capacitor with inner radius 1.0 cm, outer radius 2.0 cm and length 1.0 m is constructed with 2 different dielectrics. When oriented along the z-axis, ϵ_r for $0^\circ \leq \phi \leq 180^\circ$ is 9.0, and for $180^\circ \leq \phi \leq 360^\circ$ is 4.0. (a) Calculate the capacitance. (b) If 9.0 V is applied across the conductors, determine the electrostatic potential energy stored in each dielectric for this capacitor.

(a) a coaxial line,

$$C = \frac{2\pi L \epsilon_r \epsilon_o}{\ln(b/a)}$$

But for only half the line,

$$C = \frac{\pi L \epsilon_r \epsilon_o}{\ln(b/a)}$$

So

$$C = \frac{\pi L \epsilon_{r1} \epsilon_o}{\ln(b/a)} = 361 \text{ pF}$$

and

$$C = \frac{\pi L \epsilon_{r2} \epsilon_o}{\ln(b/a)} = 161 \text{ pF}$$

So

$$C_{TOT} = C_1 + C_2 = 522 \text{ pF}$$

$$(b) W_{E1} = \frac{1}{2} C_1 V^2 = 14.6 \text{ nJ}; \quad W_{E2} = \frac{1}{2} C_2 V^2 = 6.5 \text{ nJ}$$

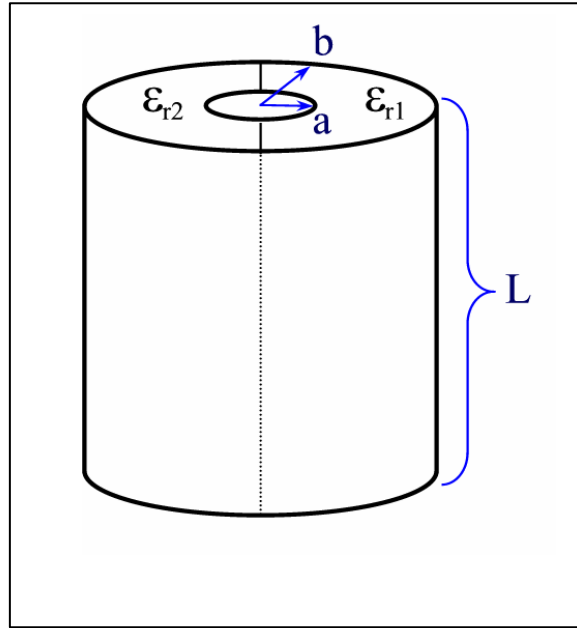


Fig. P2.75