1.1 Toe force, F, of the wind blowing against a building is given by $F = C_D \rho V^2 A/2$, where V is the wind speed, ρ the density of the air, A the cross-sectional area of the building, and C_D is a constant termed the drag coefficient. Detennine the dimensions of the drag coefficient.

$$F = C_D \rho V^2 A/2$$

or

$C_D = 2F/\rho V^2 A$, where $F \doteq MLT^{-2}$, $\rho \doteq ML^{-3}$, $V \doteq LT^{-1}$, $A \doteq L^2$

Thus,

$$C_D \doteq (MLT^{-2}) / [(ML^{-3})(LT^{-1})^2(L^2)] \doteq M^0L^0T^0$$

Hence, C_D is dimensionless.

1.2 Determine the dimensions, in both the *FLT* system and the *MLT* system, for (a) the product of mass times velocity, (b) the product of force times volume, and (c) kinetic energy divided by area.

(a) mass × velocity $\doteq (M)(LT^{-1}) \doteq \underline{\underline{MLT^{-1}}}$

Since $F \doteq MLT^{-2}$

mass × velocity $\doteq (FL^{-1}T^{2})(LT^{-1}) \doteq \underline{\underline{FT}}$

(b) force × volume $\doteq \underline{\underline{FL^{3}}}$

$\doteq (MLT^{-2})(L^{3}) \doteq \underline{\underline{ML^{4}T^{-2}}}$

(c) $\dfrac{\text{kinetic energy}}{\text{area}} \doteq \dfrac{FL}{L^{2}} \doteq \underline{\underline{FL^{-1}}}$

$\doteq \dfrac{(MLT^{-2})L}{L^{2}} \doteq \underline{\underline{MT^{-2}}}$

1.3

1.3 Verify the dimensions, in both the *FLT* and *MLT* systems, of the following quantities which appear in Table 1.1: (a) vol• ume, (b) acceleration, (c) mass, (d) moment of inertia (area), and (e) work.

(a) Volume ≐ ●●

(6) acceleration = lime rle of chang o+ vuloci+

≐ L7′ / T = Lr?

(c) mass ≐ M

or wt F ±MLT? / T

OnAs3± EL ʼ?

(d) moment uf tier±r˜ (rec) = second moment of arc

= (O) - L′

(e) work = force × distance

≐ F L

M

er with -: LT?

Work5 M / LR T-?

1.4 Determine the dimensions, in both the FLT system and the MLT system, for **(a)** the product of force times acceleration, **(b)** the product of force times velocity divided by area, and (c) momentum divided by volume.

(a) Force × acceleration $\doteq (F)(LT^{-2}) \doteq FLT^{-2}$

Since $F \doteq MLT^{-2}$,

force × acceleration $\doteq$ (nr(tr)$\doteq$ L_T'

(b) $\dfrac{\text{force} \times \text{velocity}}{\text{area}} \doteq \dfrac{(F)(LT^{-1})}{L^2} \doteq$ pt "7'

$\doteq \dfrac{(L73lur^{-1})}{L} \doteq MT^3$

(c) $\dfrac{\text{momentum}}{\text{volume}} = \dfrac{\text{mass} \times \text{velocity}}{\text{volume}}$

$\doteq \dfrac{(rr^{-1})(r^{-1})}{L^3} \doteq FL?r$

$\doteq \dfrac{()(Lr)}{L^3} \doteq ML'r'$

1.5 Verify the dimensions, in both the FLT and MLT systems, of the following quantities which appear in Table 1.1: **(a)** angular velocity, **(b)** energy, (c) moment of inertia (area), **(d)** power, and (e) pressure.

(a) angular velocity $= \dfrac{\text{angular displacement}}{\text{time}} \doteq \underline{\underline{T^{-1}}}$

(b) energy ~ capacity of body to do work.
Since work = force × distance,

$$\text{energy} \doteq \underline{\underline{FL}}$$

or with $F \doteq MLT^{-2}$,

$$\text{energy} \doteq (MLT^{-2})(L) \doteq \underline{\underline{ML^{2}T^{-2}}}$$

(c) moment of inertia (area) = second moment of area

$$\doteq (L^{2})(L^{2}) \doteq \underline{L^{4}}$$

(d) power = rate of doing work $\doteq \dfrac{FL}{T} \doteq \underline{\underline{FLT^{-1}}}$

$$\doteq (MLT^{-2})(L)(T^{-1}) \doteq \underline{\underline{ML^{2}T^{-3}}}$$

(e) pressure $= \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^{2}} \doteq \underline{\underline{FL^{-2}}}$

$$\doteq (MLT^{-2})(L^{-2}) \doteq \underline{\underline{ML^{-1}T^{-2}}}$$

I.6

1.6 Verify the dimensions. in both the *FLT* system and the *MLT* system, of the following quantities which-appear in Table 1.1: (a) frequenay, (b) stress, (c) strain, (d) torque, and (e) work.

() frequency = cycles / +in€ – —⁄

(6) shres5 = 4ore / re = F / LZ = F7?

Smc¢ F ± L T2,

≠ess = LT? / L = L"'7'2

cc) strain = ChAnne in lenth / /en4th ≐ kt (aerate)

(d) torque = force × distance ≐ FL

$\doteq (MLT^{-2})(L) \doteq \underline{ML^{2}T^{-2}}$

(&) erk = force s dishane ≐ FL

= (ur)/L)=- Mgr?

1.7

1.7 If u is a velocity, x a length, and t a time, what are the dimensions (in the MLT system) of (a) $\partial u/\partial t$, **(b)** $\partial^2 u/\partial x \partial t$, and (c) $\int (\partial u/\partial t)\, dx$?

(a) $$\frac{\partial u}{\partial t} \doteq \frac{LT^{-1}}{T} \doteq LT^{-2}$$

(b) $$\frac{\partial^2 u}{\partial x \partial t} \doteq \frac{LT^{-1}}{(L)(T)} \doteq T^{-2}$$

(c) $$\int \frac{\partial u}{\partial t}\, dx \doteq \frac{(LT^{-1})}{(T)}(L) \doteq L^2T^{-2}$$

1.8

1.8 Verify the dimensions, in both the *FLT* system and the *MLT* system, of the following quantities which appear in Table 1.1: (a) acceleration, **(b)** stress, (c) moment of a force, (d) volume, and **(e)** work.

(a) acceleration $= \dfrac{\text{velocity}}{\text{time}} \doteq \dfrac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$

(b) stress $= \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

$\text{stress} \doteq \dfrac{MLT^{-2}}{L^2} = \underline{\underline{ML^{-1}T^{-2}}}$

(c) moment of a force $=$ force $\times$ distance $\doteq \underline{\underline{FL}}$

$\doteq (MLT^{-2})L \doteq \underline{\underline{ML^2T^{-2}}}$

(d) volume $= (\text{length})^3 \doteq \underline{\underline{L^3}}$

(e) work $=$ force $\times$ distance $\doteq \underline{\underline{FL}}$

$\doteq (MLT^{-2})L \doteq \underline{\underline{ML^2T^{-2}}}$

1.9 If p is a pressure, V a velocity, and ρ a fluid density, what are the dimensions (on the MLT system) of (a) p/ρ, (b) $pV\rho$, and (c) $p/\rho V^2$?

(a) $$\frac{p}{\rho} \doteq \frac{ML^{-1}T^{-2}}{ML^{-3}} \doteq \underline{\underline{L^2T^{-2}}}$$

(b) $$pV\rho \doteq (ML^{-1}T^{-2})(LT^{-1})(ML^{-3}) \doteq \underline{\underline{M^2L^{-3}T^{-3}}}$$

(c) $$\frac{p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0 \ (\underline{\underline{\text{dimensionless}}})$$

1.10 If P is a force and x a length, what are the dimensions (in the FLT system) of (a) dP/dx, (b) d^3P/dx^3, and (c) $\int P\,dx$?

(a) $\frac{dP}{dx} \doteq \frac{F}{L} = FL^{-1}$

(b) $\frac{d^3P}{dx^3} \doteq \frac{F}{L^3} = FL^{-3}$

(c) $\int P\,dx \doteq FL$

1.14 If V is a velocity, ℓ a length, and ν a fluid property (the kinematic viscosity) having dimensions of ET', which of the following combinations are dimensionless: **(a)** VE, (b) V&/v, (c) Vy, **(d)** V/6v?

(a) $V\ell\nu$ = [illegible] = L^4T^{-4} (not dimensionless)

(b) $\frac{V\ell}{\nu}$ = [illegible] = L^0T^0 (dimensionless)

(c) $V^2\nu$ = [illegible] = L^4T^{-3} (not dimensionless)

(d) $\frac{V}{\ell\nu}$ = [illegible] = L^{-2} (not dimensionless)

I. f2

1.12 If V is a velocity, determine the dimensions of Z, *a*, and *G*, which appear in the dimensionally homogeneous equation

$$V = Z(a - 1) + G$$

[r= 7 (¢−1) + G

[IN«−)+ (6J

Shce ca«ch ~err » The <4alien mus# have the 5ame dimensions, it flos the!

$Z \doteq \underline{LT'}$

Gd. ≐ p?[97° (dimensionless amce obied uoih « number)

$G \doteq \underline{LT^{-1}}$

r~ h·

1.13 The volume rate of flow, Q, through a pipe containing a slowly moving liquid is given by the equation

$$Q = \frac{\pi R^4 \Delta p}{8 \mu \ell}$$

where R is the pipe radius, Δp the pressure drop along the pipe, μ a fluid property called viscosity $(FL^{-2}T)$, and ℓ the length of pipe. What are the dimensions of the constant $\pi/8$? Would you classify this equation as a general homogeneous equation? Explain.

$$[L^3 T^{-1}] \doteq \left[\frac{\pi}{8}\right] \frac{[L^4][FL^{-2}]}{[FL^{-2}T][L]}$$

$$[L^3 T^{-1}] \doteq \left[\frac{\pi}{8}\right] [L^3 T^{-1}]$$

The constant $\pi/8$ is dimensionless and the equation is a general homogeneous equation that is valid in any consistent system of units. Yes

1.14

1.14 According to information found in an old hydraulics book, the energy loss per unit weight of fluid flowing through a nozzle connected to a hose can be estimated by the formula

$$h = (0.04 \text{ to } 0.09)(D/d)^4 V^2/2g$$

where h is the energy loss per unit weight, D the hose diameter, d the nozzle tip diameter, V the fluid velocity in the hose, and g the acceleration of gravity. Do you think this equation is valid in any system of units? Explain.

$$h = (0.04 \text{ to } 0.09)\left(\frac{D}{d}\right)^4 \frac{V^2}{2g}$$

$$[L] \doteq [0.04 \text{ to } 0.09]\left[\frac{L}{L}\right]^4 \frac{[LT^{-1}]^2}{[LT^{-2}]}$$

$$[L] \doteq [0.04 \text{ to } 0.09][L]$$

Since each term in the equation must have the same dimensions, the constant term (0.04 to 0.09) must be dimensionless. Thus, the equation is a general homogeneous equation that is valid in any system of units. Yes.

1.15

1.15 The pressure difference, Δp, across a partial blockage in an artery (called a *stenosis*) is approximated by the equation

$$\Delta p = K_v \frac{\mu V}{D} + K_u\left(\frac{A_0}{A_1} - 1\right)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity $(FL^{-2}T)$, ρ the blood density (ML^{-3}), D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of the stenosis. Determine the dimensions of the constants K_v and K_u. Would this equation be valid in any system of units?

$$\Delta p = K_v \frac{\mu V}{D} + K_u\left[\frac{A_0}{A_1} - 1\right]^2 \rho V^2$$

$$[FL^{-2}] \doteq [K_v]\left[\left(\frac{FT}{L^2}\right)\left(\frac{L}{T}\right)\left(\frac{1}{L}\right)\right] + [K_u]\left[\frac{(L^2)}{(L^2)} - 1\right]^2\left[\frac{FT^2}{L^4}\right]\left[\frac{L}{T}\right]^2$$

$$[FL^{-2}] \doteq [K_v][FL^{-2}] + [K_u][FL^{-2}]$$

Since each term must have the same dimensions, K_v and K_u are ~~dimensionless~~. Thus, the equation is a general homogeneous equation that would be valid in any consistent system of units. ~~Yes~~.

1.16

1.16 Assume that the speed of sound, c, in a fluid depends on an elastic modulus, E_v, with dimensions FL^{-2}, and the fluid density, ρ, in the form $c = (E_v)^a(\rho)^b$. If this is to be a dimensionally homogeneous equation, what are the values for a and b? Is your result consistent with the standard formula for the speed of sound? (See Eq. 1.19.)

$$c = (E_v)^a(\rho)^b$$

Since $c \doteq LT^{-1}$ $\quad E_v \doteq FL^{-2}$ $\quad \rho \doteq FL^{-4}T^{2}$

$$\left[\frac{L}{T}\right] \doteq \left[\frac{F^a}{L^{-2a}}\right]\left[\frac{F^b T^{2b}}{L^{-4b}}\right] \qquad (1)$$

For a dimensionally homogeneous equation each term in the equation must have the same dimensions. Thus the right hand side of Eq. (1) must have the dimensions of LT^{-1}. Therefore,

$a + b = 0$ (to eliminate F)

$2b = -1$ (to satisfy condition on T)

$2a + 4b = -1$ (to satisfy condition on L)

It follows that $a = \frac{1}{2}$ and $b = -\frac{1}{2}$

So

This result is consistent with the standard formula for the speed of sound. Yes.

a h

1.17 A formula to estimate the volume rate of flow, Q, flowing over a dam of length, B, is given by the equation

$$Q = 3.09BH^{3/2}$$

where H is the depth of the water above the top of the dam (called the head). This formula gives Q in ft/s when B and H are in feet. Is the constant, 3.09, dimensionless? Would this equation be valid if units other than feet and seconds were used?

= 30 6 H ³⁄₂]

[er)= O0C]tr³

[r]=[el]D"

jnce each erm in the equation mus# huve the same dimensions the tons#an+ 3O mus} he dimensions of /'7' and is thcreve not dimensionless No.

Ice The cstar# hus dimensions i#s vaIe till channe ith a change in nis. Io,

hi I:

1.18

1.18 The force, *P*, that is exerted on a spherical particle moving slowly through a liquid is given by the equation

$$P = 3DV$$

where p is a fluid property (viscosity) having dimensions of *FLT*, *D* is the particle diameter, and *V* is the particle velocity. What are the dimensions of the constant, 3n? Would you classify this equation as a general homogeneous equation?

P= nADY

[r] + [3r(=e r @4r?

[r- [»[r

∴ 37r s dimensionless, qnd the equa#on is a general homogeneous equation. Yes.

420

1.20 Make use of Table 1.3 to express the following quantities in SI units: **(a)** 10.2 in./min, **(b)** 4.81 slugs, **(c)** 3.02 lb, **(d)** 73.1 ft/s^2, **(e)** 0.0234 lb·s/ft^2.

(a) [illegible]

$= 4.32 \times 10^{-3}\ \frac{m}{s} = 4.32$ [illegible]

(b) [illegible]

(c) 3.02 lb = [illegible] N

(d) $73.1\ \frac{ft}{s^2} = \left(73.1\ \frac{ft}{s^2}\right)\left(3.048 \times 10^{-1}\ \frac{\frac{m}{s^2}}{\frac{ft}{s^2}}\right) = 22.3\ \frac{m}{s^2}$

(e) 0.0234 $\frac{lb \cdot s}{ft^2}$ = [illegible]

$= 1.12\ \frac{N \cdot s}{m^2}$

1.21 Make use of Table 1.4 to express the following quantities in BG units: **(a)** 14.2 km, **(b)** 8.14 N/m³, (c) 1.61 kg/m³, **(d)** 0.0320 N·m/s, **(e)** 5.67 mm/hr.

(a) /42 % = (ZW>>) (Z5±)• 44% r' 4t

(b) &/ N/3 • (e +)zce'#)• 5,18>>? !l £e3

(c) $1.61\ \frac{kg}{m^3} = \left(1.61\ \frac{kg}{m^3}\right)\left(1.940\times10^{-3}\ \frac{\frac{slugs}{ft^3}}{\frac{kg}{m^3}}\right) =$ 53/2 ¥/0⁻³ slugs 443

(d) $0.0320\ \frac{N\cdot m}{s} = \left(0.0320\ \frac{N\cdot m}{s}\right)\left(7.376\times10^{-1}\ \frac{\frac{ft\cdot lb}{s}}{\frac{N\cdot m}{s}}\right)$

$= 2.36\times10^{-2}\ \frac{ft\cdot lb}{s}$

(e) $5.67\ \frac{mm}{hr} = \left(5.67\times10^{-3}\ \frac{m}{hr}\right)\left(3.281\ \frac{ft}{m}\right)\left(\frac{1\ hr}{3600\ s}\right)$

= 5,/7 x10⁻% 44

1.22 Express the following quantities in SI units: **(a)** 160 acre, **(b)** 15 gallons (U.S.), (e) 240 miles, **(d)** 79.1 hp, (e) 60.3 F.

(a) $160 \text{ acre} = (160 \text{ acre})\left(4.356 \times 10^4 \frac{ft^2}{acre}\right)\left(9.290 \times 10^{-2} \frac{m^2}{ft^2}\right)$

$= \underline{\underline{6.47 \times 10^5 m^2}}$

(b) $15 \text{ gallons} = (15 \text{ gallons})\left(3.785 \frac{liters}{gallon}\right)\left(10^{-3} \frac{m^3}{liter}\right) = \underline{\underline{56.8 \times 10^{-2} m^3}}$

<) 2% - =(e 2)(sf;)(vwi'[)= 51S»

(d) hp El, s

and [- \W ss ha±

79.1 hp = 5.90. Z0 4 W

ce) T•5(%r-3) =1s.7°c

er .3

- 15.7C +273 = 28 <