Solution Manual for Mathematics for Elementary Teachers 5th Edition Beckmann 0134392795 9780134392790

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Chapter 2
Fractions and Problem Solving

2.2 Defining and Reasoning About Fractions

1. Answers will vary. Michael needs to learn that the whole or unit amount can be comprised of a collection of objects. The collection of dark marbles is indeed less than the entire collection of marbles, and in that sense they can represent a number that is less than 1. As Michael is able to recognize that since there are three rows, we can group them together and call each row $\frac{1}{3}$ of the total number of marbles.

2. The 3 means that the whole or the unit amount has been partitioned into three parts that are equal in some way. By itself, $\frac{2}{3}$ is implied to mean $\frac{2}{3}$ of a unit or 1. Saying 3 is the whole can be misleading because one may assume then that you are taking $\frac{2}{3}$ of 3. This could be especially confusing in situations where the whole is defined instead of the implied unit. For example, if we were taking $\frac{2}{3}$ of 8 pounds of apples, saying "3 is the whole" confuses the situation greatly.

3. This problem is similar to Practice Exercise 1. The given array of *x*'s represents four parts, each part a column of x's. Since each part is $\frac{1}{5}$ of the original rectangle, then we

want to show five parts or columns, so we add 1 more column of 5 x's as shown below.

x	*x*	*x*	*x*	*x*
x	*x*	*x*	*x*	*x*
x	*x*	*x*	*x*	*x*
x	*x*	*x*	*x*	*x*
x	*x*	*x*	*x*	*x*

4. This problem is very similar to Practice Exercise 2. The given array of *x*'s represents six parts, each part being a row of x's. Since each part or row is $\frac{1}{5}$ of the original rectangle, then we want to only show five parts or rows, so we remove 1 row of 5 x's, as shown.

x	*x*	*x*	*x*	*x*
x	*x*	*x*	*x*	*x*
x	*x*	*x*	*x*	*x*
x	*x*	*x*	*x*	*x*
x	*x*	*x*	*x*	*x*

5. a. Cameron got $\frac{1}{6}$ of the candy bar. See Figure 2.1. Arianna got of the candy bar.
1

Arianna then partitioned that portion into three equal pieces. So Cameron got $\frac{1}{3}$ of one half of the candy bar. Partitioning the other half of the candy bar into three equal pieces shows that each piece is $\frac{1}{6}$ of the candy bar.

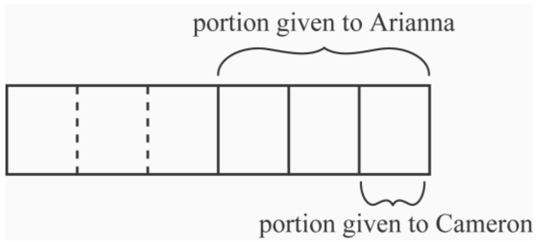


Figure 2.1: Kaitlyn's candy bar

b. The entire candy bar is one of the unit amounts (as in Arianna got $\frac{1}{2}$ of the candy bar). Another unit amount is just $\frac{1}{2}$ of the candy bar, because Cameron got $\frac{1}{3}$ of one half of the candy bar. Depending on which unit amount you use, Cameron's piece could be viewed as $\frac{1}{3}$ or $\frac{1}{6}$.

6. a. $\frac{1}{2}$ of a recipe. See Figure 2.2. Partitioning a cup into 3 parts, we see that two of these parts represents the amount (or portion) needed for a full recipe. Then we look at $\frac{1}{3}$ of a cup of cocoa (or the amount we actually have on hand) and compare this to amount needed to make a full recipe. We see that $\frac{1}{2}$ of the amount needed to make a full recipe is the amount we actually have on hand. So we can only make $\frac{1}{2}$ of the recipe.

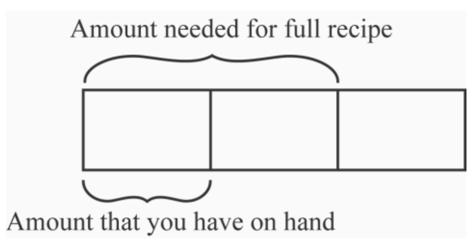


Figure 2.2: Fractions of a cup of cocoa

b. An entire cup of cocoa is one unit amount (as in you need $\frac{2}{3}$ of a cup of cocoa for one batch). The amount needed for the full recipe (or $\frac{2}{3}$ of a cup of cocoa) is another unit amount. The amount that you have on hand is $\frac{1}{2}$ of the amount needed for the full recipe and also it is $\frac{1}{3}$ of a cup of cocoa.

7. a. $\frac{3}{5}$ of the butter she should have used. See Figure 2.3. The recipe calls for $\frac{5}{4}$ of a cup, and Susan uses $\frac{3}{4}$ of a cup. As the math drawing shows, Susan used three parts out of the five called for by the recipe. So she used $\frac{3}{5}$ of the amount called

for by the recipe.

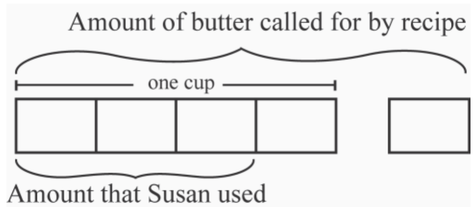


Figure 2.3: Fractions of a cup of butter

b. A cup of butter is one unit amount ($\frac{5}{4}$ of a cup). The amount of butter called for by the recipe is another unit amount ($\frac{3}{5}$ of the amount called for by the recipe). The amount that Susan used could be thought of as $\frac{3}{5}$ of the amount called for by the recipe or as $\frac{3}{4}$ of a cup of butter.

8. a. $\frac{4}{3}$ of the daily value. See Figure 2.4. The full daily value of calcium is contained in $\frac{3}{4}$ of a cup of snack food. So $\frac{1}{4}$ of a cup of snack food contains $\frac{1}{3}$ of the daily value. One cup of snack food is $\frac{4}{4}$ of a cup of snack food, which gives $\frac{4}{3}$ of the full daily value.

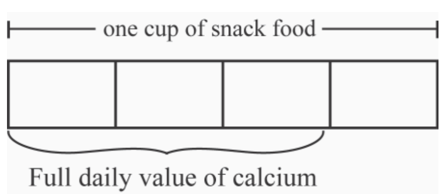


Figure 2.4: Daily value of calcium

b. A cup of snack food is one unit amount ($\frac{3}{4}$ of a cup). The full daily value of calcium is another unit amount ($\frac{1}{3}$ of the daily value.). The amount of daily value of calcium in a cup could be thought of as $\frac{4}{3}$ of the full daily value or as 12 4 24 of a cup of snack food.

9. Answers will vary widely. For example: $\frac{2}{3}$ of the sixth-grade class was on a field trip to

the county courtroom. The students were divided into two equal groups for separate tours. What fraction of the class was in each group? ($\frac{1}{3}$) What fraction of the students on the trip was in each group? ($\frac{1}{2}$)

10. a. Ben has the correct answer, but incorrect reasoning. If Marla ate $\frac{1}{12}$ of each cake, then she ate $\frac{1}{12}$ of the cakes combined (see part c). While it is true that $\frac{1}{\ } = \frac{2}{\ }$, the denominator in the phrase " $\frac{2}{24}$ of the cake", does not represent the 24 pieces in the math drawing but rather $\frac{2}{24}$ of the amount of cake . If the cakes were of the same size, then there would be 24 equal-sized pieces of cake and Ben's reasoning would be correct.

b. Seyong seems to believe that the equation $\frac{1}{12}+\frac{1}{12}=\frac{2}{12}$ applies in this situation. But each twelfth is part of a different whole or unit amount, so this is not a meaningful computation. However, if the pieces were of cakes of the same size, say one pound, then Seyong would be right in a sense because the fractions would be parts of the same size whole. Seyong could say that Marla ate $\frac{2}{12}$ of a pound of cake.

However, Seyong is not correct to say that Marla ate $\frac{2}{12}$ of the cake in either case.

Unless the total amount of cake was split up into 12 pieces, then the piece sizes are not one-twelth of the cake. If the cakes were exactly the same sizes then the piece sizes would be $\frac{1}{24}$ of the cake since there are 24 equal pieces that make up the entire amount of cake.

c. The cakes can be shared equally among 12 people by giving each person a serving like Marla's. Since there are 12 servings for the total, each serving is $\frac{1}{12}$ of the cake. See Practice Exercise 5.

11. Yes, Harry's method is valid. See the solution to Practice Exercise 6.

12. See Figure 2.5. As in Practice Exercise 5, if you shade $\frac{3}{8}$ of each individual pie, then the shaded parts make $\frac{3}{8}$ of the 3 pies combined. Imagine preparing to serve 8 people each getting one piece from each pie. Person 1 gets all the pieces marked with 1, Person 2 gets all the pieces marked with 2, etc. All together, each person's serving size is $\frac{1}{8}$ of the 3 pies combined. If you have only served the shaded portions (or persons 1,2 and 3), then you have only served $\frac{3}{8}$ of the 3 pies combined.

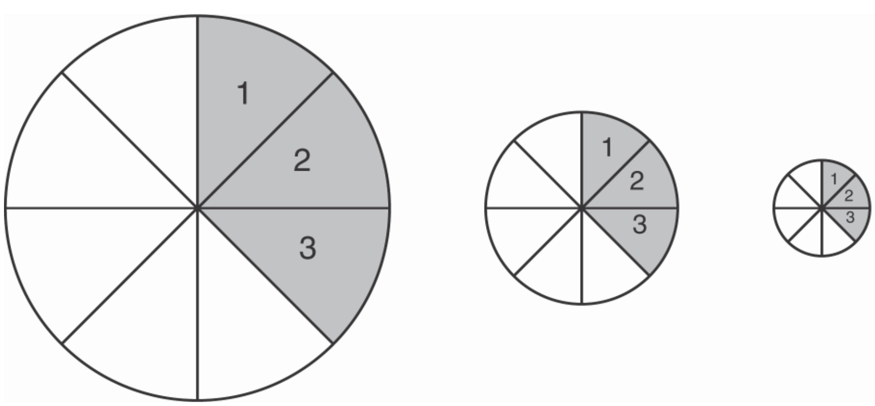


Figure 2.5: Shading three-eighths

13. We need at least three objects to show $\frac{7}{3}$, but that can get confusing because the unit amount could be viewed as one object, the entire collection of objects, or individual pieces of an object. A number line can be a useful way to show improper fractions.

14. In Figure 2.6, the 1-unit has been partitioned into 3 equal-length parts, so the distance between tick marks is $\frac{1}{3}$ of the unit. Then $\frac{1}{3}$ is the first tick mark to the right of zero and $\frac{2}{3}$ is the second tick mark to the right of zero.

Figure 2.6: Number line with $\frac{1}{3}$ and $\frac{2}{3}$ marked.

15. In Figure 2.7, each unit has been partitioned into 3 equal-length parts, so the distance between tick marks is $\frac{1}{3}$ of the unit. Then $\frac{5}{3}$ is the fifth tick mark to the right of zero.

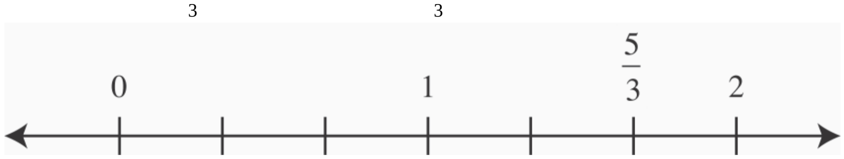


Figure 2.7: Number line with $\frac{5}{3}$ marked.

16. See Figure 2.8.

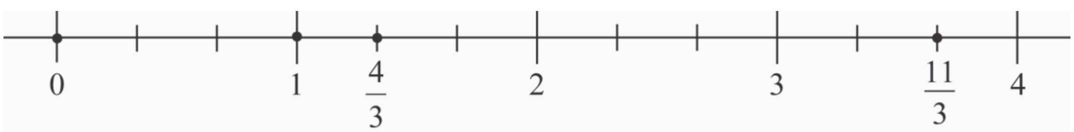


Figure 2.8: Plotting 0, 1, $\frac{4}{3}$ and $\frac{11}{3}$

17. The shaded region could be described as $\frac{7}{8}$ of the total region or as $1\frac{3}{4}$ rectangles. An unambiguous question would be: *If the two rectangles combined is considered to be the unit amount, what fraction names the shaded region?* This question is unambiguous because it clearly states what the unit amount will be.

18. $\frac{9}{12}$ is correct if the entire figure (three rectangles) is meant to represent the unit amount. $\frac{9}{4}$ is correct if one rectangle (partitioned into four pieces) is meant to represent the unit amount.

An unambiguous question might be: *If the three rectangles together make the unit amount, what fraction of this unit amount is shaded?* Any unambiguous question about the shaded region must clearly state what the unit amount is.

19. a. No! The hexagons take up $\frac{2}{3}$ of the area. We do not know how many shapes were in NanHe's design, but based on the fractions in the problem, we can know that the design was made using 11 shapes or a number of shapes that is a multiple

of eleven. Thus, the shapes in each design can be organized into one or more groups of the same eleven shapes: 4 hexagons, 5 rhombi, and 2 triangles. In each group of 11 shapes, if you were to use triangles to make the hexagons, you would need 24 triangles. To do the same with the rhombi would require 10 triangles. So each group in the design could be recreated using 36 triangles, which has the effect of breaking any of these groups of 11 shapes up into equal parts. Thus, in each group of 11 shapes, the hexagons use 24 of the triangles and $\frac{24}{\ } = \frac{2}{\ }$. If the area taken up by the hexagons is $\frac{2}{3}$ of each group of 11 shapes, then $\frac{2}{3}$ is also the fraction of the entire design taken up by hexagons.

b. If we view the hexagons, triangles, and rhombi as all just shapes, then they are equal and this allows us to view each as $\frac{1}{11}$ of the total number of shapes in each group and overall. But if we are talking about area, then the hexagon no longer
equal to a triangle. Instead we see that the hexagon is the same as 6 triangles and we think of each shape in terms of how many triangles make up the shape, giving us equal parts again.

20. a. If School A has 600 students, then $\frac{800}{1500}=\frac{8}{15}$ of the students attend the after school program. If School A has 900 students, then $\frac{700}{1500}=\frac{7}{15}$ of the students attend the after school program. In the first situation, $\frac{1}{3}$ of 600 students is 200 students, and $\frac{2}{3}$ of 900 students is 600 students. So 800 out of the 1500 students are in the after school program.

b. Answers will vary. For example: if School A has 300 students and School B has 1200 students, then the fraction of students in the after school program is
$$\frac{100+800}{1500}=\frac{900}{1500}=\frac{3}{5}.$$
Or:

If the each school has 750 students, then the fraction of students in the after school program is $\frac{250+500}{1500}=\frac{750}{1500}=\frac{1}{2}$.

c. Jamie is correct only when the same number of students attend each school. Jamie is wrong for any other situation because the unit amounts will be different.

2.3 Reasoning About Equivalent Fractions

1. This problem is very similar to Practice Exercise 1. If you have a rod that is partitioned into four equal parts and 3 are shown shaded, and then you partition each of the 4 parts into 3 smaller parts, the shaded portion will then consist of 3×3 parts and the unit amount will consist of 4×3 parts. So the shaded portion of the unit amount can be described both as $\frac{3}{4}$ of the rod and as $\frac{3\times3}{4\times3}=\frac{9}{12}$ of the rod. See Figure 2.9.

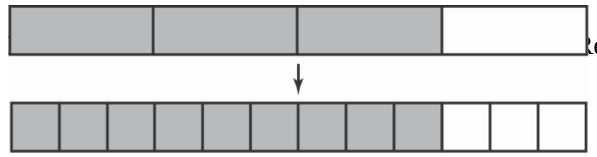


Figure 2.9: Equivalent fractions $\frac{3}{4} = \frac{9}{12}$

2. If you have a unit amount that is partitioned into three parts and two are shaded, and then you partition each of the three parts into four smaller parts, the shaded amount will then consist of 2×4 parts and the unit amount will consist of 3×4 parts. So the shaded portion of the unit amount can be described both as $\frac{2}{3}$ of the unit amount and as

$$\frac{2\times 4}{3\times 4}=\frac{8}{12} \text{ of a unit amount.}$$

Alternatively, consider multiplying $\frac{2}{3}$ by one as shown here:

$$\frac{2}{3}\times 1=\frac{2}{3}\times\frac{4}{4}=\frac{2\times 4}{3\times 4}=\frac{8}{12}.$$

3. To give two fractions common denominators, we use a common multiple of the two numbers. For each fraction, we multiply both the numerator and denominator by a factor that will change the denominator to this common multiple. For the fractions given, this common multiple would be 12. Specifically, we multiply as shown below.

$$\frac{2}{3}=\frac{2\times 4}{3\times 4}=\frac{8}{12}$$
$$\frac{3}{4}=\frac{3\times 3}{4\times 3}=\frac{9}{12}$$

In giving the fractions common denominators, we are partitioning the parts of each fraction into equivalent sized smaller parts. The total number of pieces and the size of the pieces change, while the amount of the unit amount that would be represented by the shaded pieces remains the same.

4. See Figure 2.10. We join together small pieces of the rod to make larger rod pieces. Here we joining 2 smaller pieces to make a larger piece because both 6 and 8 can by divided by 2 (thus 8 pieces or 6 pieces could both be put into groups of 2). When we group the 6 shaded smaller pieces into groups of 2, this yields a total of $6\div 2$ larger

shaded rod pieces. Similarly, when we group the 8 total smaller pieces into groups of 2, this yields a total of $8\div 2$ rod pieces .
Put in math drawing that looks something like this

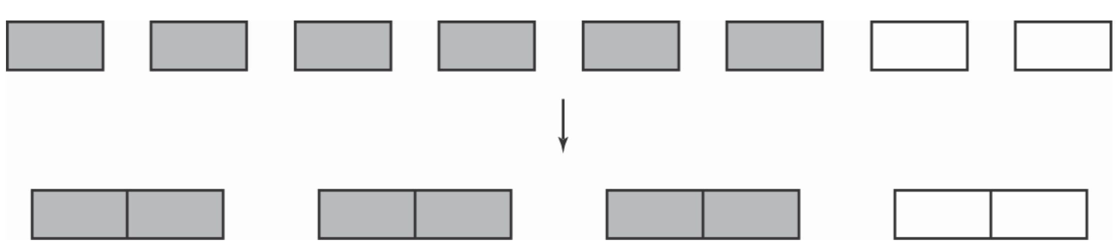


Figure 2.10: Equivalent fractions $\frac{6}{8}=\frac{3}{4}$

Division is typically thought of as getting smaller. Here the pieces are getting larger. But the number of pieces is getting smaller since you had 8 total pieces (and 6 shaded) and now via division you have 4 total rod pieces with 3 shaded rod pieces. We are seeing 3 groups of 2 shaded (6) out of 4 groups of 2 in the unit amount. Arithmetically, we have

$$\frac{6}{8} \underset{\div 2}{\overset{\div 2}{=}} \frac{3}{4}$$

5. To simplify a fraction you look for common factors of the numerator and denominator. This makes sense because we are trying to regroup the smaller pieces of the fraction into larger pieces so that the unit amount is partitioned into fewer parts. Specifically, the six shaded pieces representing the numerator in $\frac{6}{9}$ could be grouped into two pieces, with

three smaller pieces in each group. Likewise, the nine pieces representing the denominator can be grouped into three pieces, again with three smaller pieces in each group. See Figure 2.11.

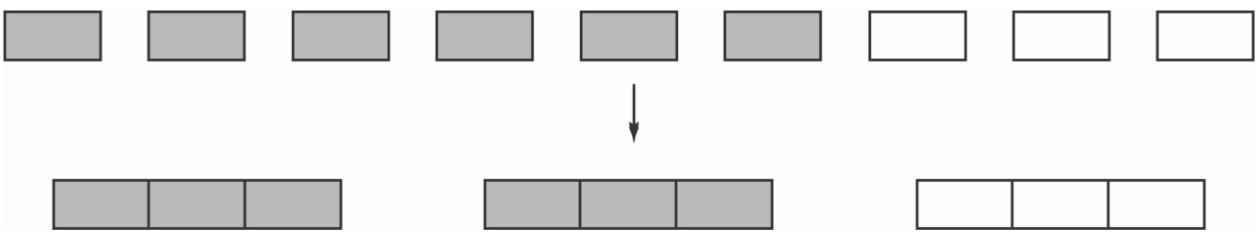


Figure 2.11 Equivalent Fractions $\frac{6}{9} = \frac{2}{3}$

We then end up with two shaded pieces out of three pieces overall, that is, $\frac{2}{3}$. Arithmetically, we have

$$\frac{6}{9} \underset{\div 3}{\overset{\div 3}{=}} \frac{2}{3}$$

The total number of pieces and the size of the pieces change, while the amount of the unit amount that would be represented by the shade pieces remains the same.

6. See Figure 2.12. Since $\frac{3}{8} = \frac{9}{24}$ and $\frac{1}{3} = \frac{8}{24}$, let each tick mark represent $\frac{1}{24}$ of the unit amount.

Figure 2.12: Plotting 0, $\frac{1}{3}$ and $\frac{3}{8}$

7. See Figure 2.13. Since $\frac{1}{\ } = \frac{20}{\ }$, $\frac{3}{\ } = \frac{24}{\ }$, and $\frac{5}{\ } = \frac{25}{\ }$, let each tick mark represent $\frac{1}{\ }$ of the

2 40 5 40 8 40 40

unit amount.

Figure 2.13: Plotting $\frac{1}{2}$, $\frac{3}{5}$ and $\frac{5}{8}$

8. See Figure 2.14. Note that $0.3 = \frac{3}{10}$ and that $\frac{3}{5} = \frac{6}{10}$.

Figure 2.14: Plotting 0, 0.3 and $\frac{3}{5}$

9. See Figure 2.15. Since $\frac{5}{6} = \frac{35}{42}$ and $\frac{8}{7} = \frac{48}{42}$, let each tick mark represent $\frac{1}{42}$.

Figure 2.15: Plotting 1, $\frac{5}{6}$ and $\frac{8}{7}$

10. See Figure 2.16. To have sevenths and thirds both falling on tick marks, we must partition the number line into twenty-firsts. I accomplished this by first dividing the interval from 0 to $\frac{2}{7}$ in half to locate $\frac{1}{7}$. I then partitioned the interval from 0 to $\frac{1}{7}$ into thirds to locate $\frac{1}{21}$, $\frac{2}{21}$, $\frac{3}{21} = \frac{1}{7}$. Then $\frac{1}{3}$ is the same as $\frac{7}{21}$.

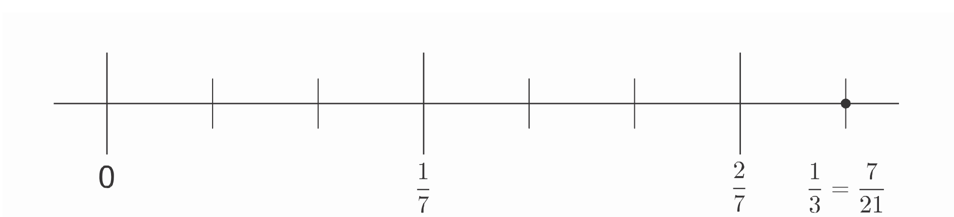


Figure 2.16: Plotting $\frac{1}{3}$

11. See Figure 2.17. To place fifths and halves on the same number line, we must use a common denominator of tenths.

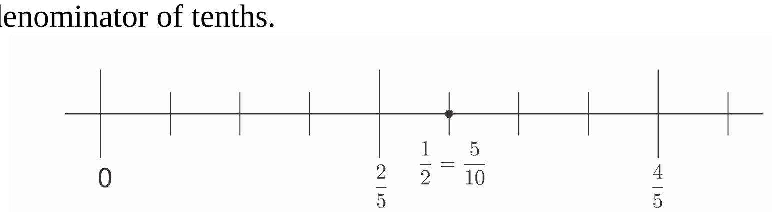


Figure 2.17: Plotting $\frac{1}{2}$

12. See Figure 2.18. To place eighths and twelfths on the same number line, we must use a common denominator of twenty-fourths.

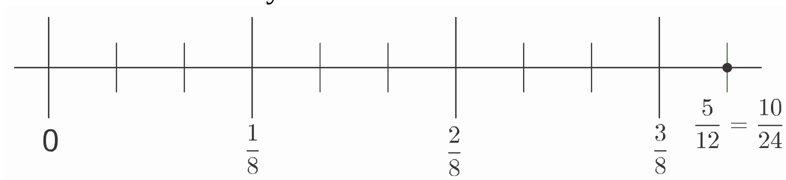


Figure 2.18: Plotting $\frac{5}{12}$

13. See Figure 2.19. $\frac{47}{6} = 7\frac{5}{6}$.

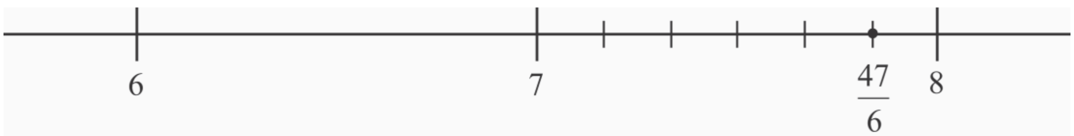


Figure 2.19: Plotting $\frac{47}{6}$

14. Erin is wrong; the tick mark should be labeled $2\frac{2}{8} = 2\frac{1}{4}$. Since the interval from 2 to 3 is partitioned into 8 pieces, each tick mark represents $\frac{1}{8}$ rather than $\frac{1}{10}$.

15. a. $\frac{5}{12}$ of a cup. See Figure 2.20. Representing the two fractions as twelfths allows us to remove the $\frac{4}{12}$ (or $\frac{1}{3}$) of a cup from the $\frac{9}{12}$ (or $\frac{3}{4}$) of a cup and see that there are 5 parts left, each part being $\frac{1}{12}$ of a cup of water.

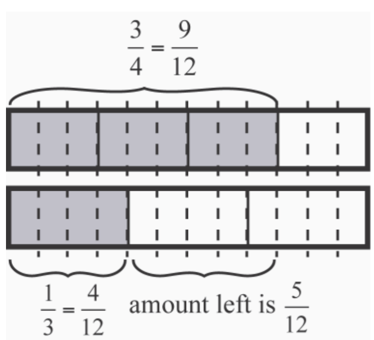


Figure 2.20: Removing some of the water

b. $\frac{3}{4}$ appears as $\frac{9}{12}$. $\frac{1}{3}$ appears as $\frac{4}{12}$.

16. a. $\frac{3}{8}$ of a cup. See Figure 2.21. Expressing $\frac{3}{4}$ of a cup of water as $\frac{6}{8}$ of a cup of water allows us to remove half, or $\frac{3}{8}$ of a cup of water, leaving $\frac{3}{8}$ of a cup in the bowl.

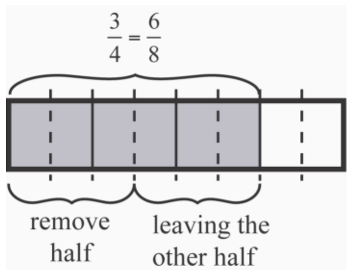


Figure 2.21: Removing half of the water

b. $\frac{3}{4}$ appears as $\frac{6}{8}$.

17. a. $\frac{3}{4}$ of a recipe. See Figure 2.22. Expressing $\frac{2}{3}$ of a cup of flour and $\frac{1}{2}$ of a cup of flour in terms of sixths of a cup of flour allows us to see that of four parts ($\frac{1}{6}$ ths of a cup of flour) needed for a full recipe, we have three of these parts, so we can make $\frac{3}{4}$ of a recipe.

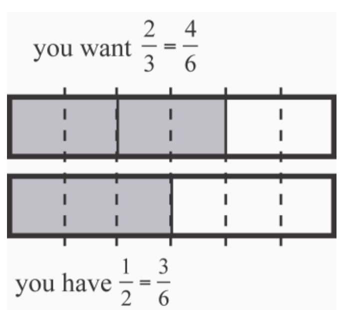


Figure 2.22: What fraction of the recipe?

b. $\frac{2}{3}$ and $\frac{1}{2}$ appear respectively as $\frac{4}{6}$ and $\frac{3}{6}$.

18. a. $\frac{8}{9}$ of his order. See Figure 2.23. Expressing the fractions as twelfths of a ton of gravel shows that Ken got eight out of nine parts ordered.

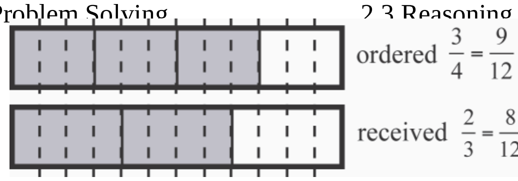


Figure 2.23: Received 8 out of 9 parts

b. $\frac{3}{4}$ and $\frac{2}{3}$ also appear as $\frac{9}{12}$ and $\frac{8}{12}$.

3

19. a. 9 of a cup. See Figure 2.24. Representing of a cup as of a cup shows that 1
2 2 6

9

of this amount is taking two parts out of the six, which represent 9 of a cup.
2 3

(Note that since measuring cups rarely come in ninths, Here,a person could argue that $\frac{1}{4}$ of a cup would be a more practical answer.)

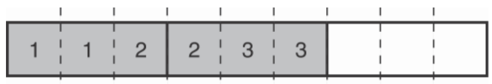


Figure 2.24: One-third of two-thirds

b. 3 also appears as $\frac{6}{9}$.
2

20. See Figure 2.25. Since 2 of a mile corresponds to 5 of Benny's total running distance
1 4

then thinking of Benny's distance ran so far as being made up of 4 parts, I split the number line region from 0 to $\frac{1}{2}$ into 4 parts, each corresponding to one of the 4 parts

Benny has run so far. The bold dark arrow shows how far he has run so far. Extending out the next is $\frac{1}{2}$ of a mile to make the full unit mile, we see that these parts we made are

is $\frac{1}{8}$ of a mile, while at the same time being 1 of Benny's total running distance. So

Benny needs to go one more $\frac{1}{8}$ of a mile to finish for a total of 5 parts run, each of size $\frac{1}{8}$
5

of a mile. So Benny will have run a total of $\frac{5}{8}$ of a mile. This is show in the dotted arrow

below in Figure 2.25.

5

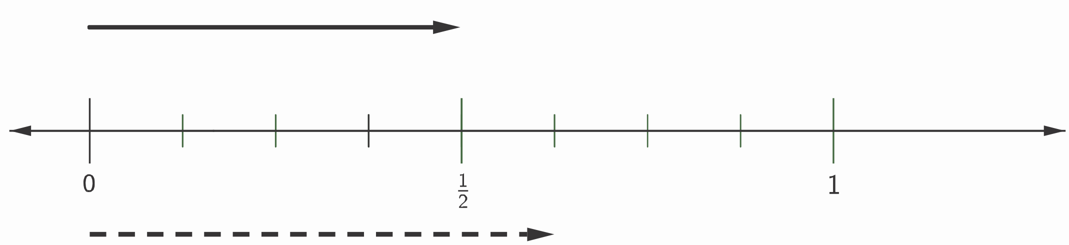


Figure 2.25: Four-fifths of the running distance is one-half of a mile

21. See Figure 2.26. since $\frac{3}{2}$ of a mile is equivalent to 4 of mile, I can split how far has been
6

4 run into 2 equal parts, each being 3

of a mile while at the same time each part is also $\frac{1}{}$ of the total running distance. Then she needs to run 5 of those parts so she will have run $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=\frac{15}{4}$ of a mile.

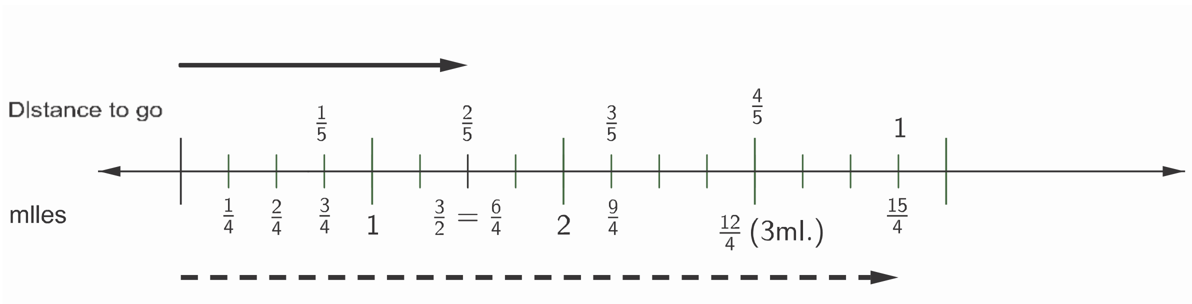


Figure 2.26: Two-fifths of the total running distance is three-halves of a mile

22. a. It would take $3\frac{1}{2}$ copies of strip A to make strip B exactly. Since strip B and strip A are made of equal sized parts, we can use these parts to solve this problem. Each little part is $\frac{1}{2}$ of strip A. You will need 7 of these parts to make strip B. So you have 7 parts, each part is $\frac{1}{2}$ of strip A. So if you make $\frac{7}{2}$ of strip A you will get strip B. Since 2 parts would make an entire copy of A, then you can also say you have $3\frac{1}{2}$ copies of strip A.

When you consider the parts in relation to strip B, you can see that each part is $\frac{1}{7}$ of strip B. Since strip A only uses 2 of these parts, then you can make strip A with only $\frac{2}{7}$ of strip B.

b. It would take $1\frac{1}{4}$ copies of strip C to make strip D exactly. Since strip D and strip C are made of equal sized parts, we can use these parts to solve this problem. Each little part is $\frac{1}{4}$ of strip C. You will need 5 of these parts to make strip D. So you have 5 parts, each part is $\frac{1}{4}$ of strip C. So if you make $\frac{5}{4}$ of strip C you will get strip D. Since 4 parts would make an entire copy of C, then you can also say you have $1\frac{1}{4}$ copies of strip C. On the other hand, you could also think of the candies as the way you are comparing the strips. Each candy strip is made up of the same size and type of candy pieces. For strip C you've got 24 pieces of candy so each piece of candy represents $\frac{1}{24}$ of strip C. Strip D needs 30 of the candy pieces. If you made $\frac{30}{24}$ of strip C you would get strip D. $\frac{5}{4}$ and $\frac{30}{24}$ are equivalent fractions.

When you consider the parts in relation to strip D, you can see that each part is $\frac{1}{5}$ of strip D. Since strip C only uses 4 of these parts, then you can make strip C with only $\frac{4}{5}$ of strip D. In terms of candy pieces, you see that each candy piece is $\frac{1}{30}$ of strip D but you only need 24 to make strip C, so if you made $\frac{24}{30}$ of strip D you would get strip C. $\frac{4}{5}$ and $\frac{24}{30}$ are equivalent fractions.

c. It would take $1\frac{1}{4}$ copies of strip E to make strip F exactly. Since strip F and strip E are made of equal sized parts, we can use these parts to solve this problem. Each little part is $\frac{1}{4}$ of strip E. You will need 5 of these parts to make strip F. So you have 5 parts, each part is $\frac{1}{4}$ of strip E. So you if you make $\frac{5}{4}$ of strip E you will get strip F. Since 4 parts would make an entire copy of E, then you can also say you have $1\frac{1}{4}$ copies of strip E. On the other hand, you could also think of the candies as the way you are comparing the strips. Each candy strip is made up of the same size and type of candy pieces. For strip E you've got 20 pieces of candy so each piece of candy represents $\frac{1}{20}$ of strip E. Strip F needs 25 of the candy pieces. If you made $\frac{25}{20}$ of strip E you would get strip F. $\frac{5}{4}$ and $\frac{25}{20}$ are equivalent fractions.

When you consider the parts in relation to strip F, you can see that each part is $\frac{1}{5}$ of strip F. Since strip E only uses 4 of these parts, then you can make strip E with only $\frac{4}{5}$ of strip F. In terms of candy pieces, you see that each candy piece is $\frac{1}{25}$ of strip F but you only need 20 to make strip E, so if you made $\frac{20}{25}$ of strip F you would get strip E. $\frac{4}{5}$ and $\frac{20}{25}$ are equivalent fractions.

d. See the answers to part b and c. In both answers our reasons are similar to the reasoning given in part a and we get the same answers. See the first explanation given in the answers to parts b and c. In those explanations, we focused on the fact that Strip C (in part b) and Strip E (in part c) were made of 4 equal size parts, while the other strip (Strip D in part b and strip F in part c) were made of 5 of those same size parts. So in both problems, we determined that $1\frac{1}{4}$ copies of the shorter strip (Strip C and Strip E) were needed to make the longer strip (Strip D and Strip F), and $\frac{4}{5}$ of the longer strip was needed to make the shorter strip."

23. a. Rachel has read $\frac{3}{4}$ as many books as Leah has. Because the parts of the strip diagrams are of equal size, we can see that Leah has read four portions of books while Rachel has read 3 portions. Since we are comparing to Leah's amount, then Leah's amount is the whole or unit amount, which tells us that each portion is $\frac{1}{4}$ of the amount that Leah has read. Rachel has 3 of these $\frac{1}{4}$ portions so she has read $\frac{3}{4}$ as many books as Leah has.

b. Leah has read $\frac{4}{3}$ as many books as Rachel has. Since we are comparing to Rachel's amount, then Rachel's amount is the whole or unit amount, which tells us that each portion is $\frac{1}{3}$ of the amount that Rachel has read. Leah has 4 of these $\frac{1}{3}$ portions so she has read $\frac{4}{3}$ as many books as Rachel has.

24. a. See Figure 2.27. Starting with Joseph's strip, dividing it into six equal pieces, then you make Adam's strip equal in length to five of those pieces.

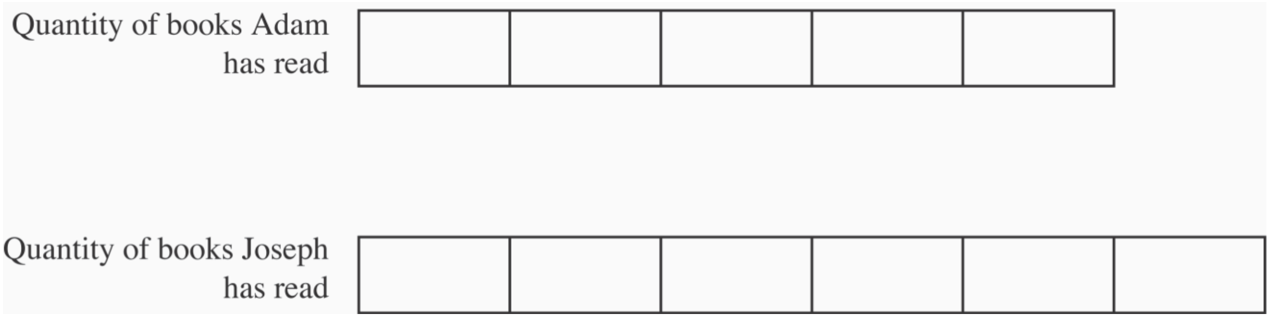


Figure 2.27: Adam and Joseph's books

b. Joseph has read $\frac{5}{6}$ as many books as Adam this year.

25. Becky should pay $\frac{1}{6}$ of the July/August bill, and her two friends should pay $\frac{5}{12}$ of the bill each. See Figure 2.28. Dividing the first month's cost between the two friends and the second month's cost among the three girls yields pieces of different sizes. Expressing everything as twelfths of the total July/August usage makes it much easier to express each person's share.

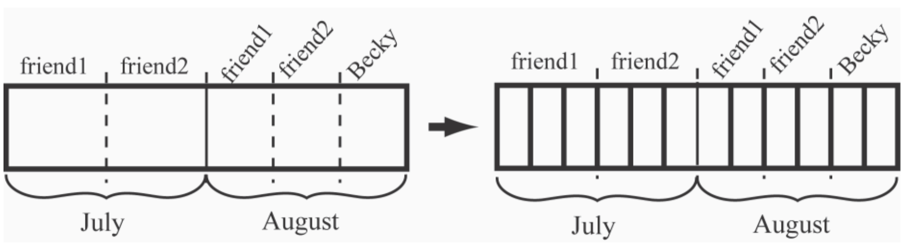


Figure 2.28: Sharing the electricity bill

The cases in parts a. and b. result in the same division of cost. In part a. the bulk of the electricity is shared, so the cost should be shared equally. In part b. the bulk of the electricity is used individually, but the individuals are doing similar things, and thus using roughly equal amounts of electricity, so sharing the cost equally still makes sense.

2.4 Reasoning to Compare Fractions

1. Since the whole or unit is not the same, comparing areas of the two shaded regions is not a valid way to compare the sizes of these fractions. When you compare two fractions like $\frac{3}{4}$ and $\frac{5}{8}$ you are assuming that the unit amount is the same.

2. See the text.

3. See the text.

4. A smaller denominator means the same sized unit it partitioned into a smaller number of pieces. This means each piece must be larger. If numerators are the same, the fraction involving larger pieces is larger overall.

5. $\frac{36}{104}$ is greater. First, notice that the numerator of $\frac{36}{104}$ is greater, so it has more pieces than $\frac{35}{109}$. Also notice that partitioning a unit into 104 pieces yields larger pieces than partitioning it into 109 pieces. So $\frac{36}{104}$ not only has more pieces than $\frac{35}{109}$, it also has a larger number of larger pieces, so $\frac{36}{104}$ is greater.

6. ${}_{47}$ is greater. Both fractions in this problem are very close to one half. In fact they are both less than one half. To make one half when a whole is partitioned into 31 pieces, we would need $15\frac{1}{2}$ pieces. So $\frac{15}{31}$ is $\frac{1}{2}$ of a piece less than one half of the unit amount, where

the pieces are $\frac{1}{31}$ of the unit amount in size. To make one half when a unit amount is partitioned into 47 pieces, we would need $23\frac{1}{2}$ pieces. So $\frac{23}{47}$ is $\frac{1}{2}$ of a piece less than one half of the unit amount, where the pieces are $\frac{1}{47}$ of the unit amount in size. Since these $\frac{1}{47}$ pieces are *smaller* than the $\frac{1}{31}$ pieces, then half of each piece size remains similar, half of a $\frac{1}{47}$ piece is smaller than half of a $\frac{1}{31}$ piece. So $\frac{23}{47}$ is closer to one half of the unit amount.

7. $\frac{\ }{28}$ is larger because it is closer to 1. It is only $\frac{1}{28}$ away from 1 whereas the other fraction is $\frac{1}{20}$ away, and since partitioning a unit into 28 pieces yields smaller pieces than partitioning it into 20 pieces, $\frac{1}{28}$ is the smaller amount, so $\frac{27}{28}$ has less to go to reach 1 than $\frac{19}{20}$ so $\frac{27}{28}$ is larger.

8. The order is $-1.1,\ -1,\ -0.98,\ \frac{11}{10},\ \frac{9}{8},\ 1.2$. One explanation that might be problematic is explaining why $\frac{11}{10}$ is smaller than $\frac{9}{8}$. Since one-tenth is smaller than one-eighth, $\frac{11}{10}$ is closer to 1 (farther to the left) than $\frac{9}{8}$. Another explanation that might be problematic is explaining why $\frac{9}{8}$ is smaller than 1.2. Writing $\frac{9}{8}$ as $1\frac{1}{8}$ and 1.2 as $1\frac{2}{10}=1\frac{1}{5}$ helps explain this ordering.

9. $\frac{23}{84}\approx 0.2738$ and $\frac{29}{98}\approx 0.2959$. The number picked will vary. For example: $\frac{7}{25}=0.28$. See Figure 2.29.

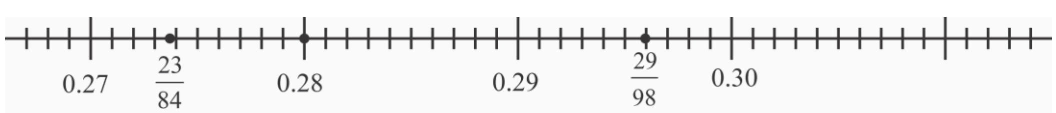


Figure 2.29: A number line

10. $\frac{78}{134}\approx 0.58209$ and $\frac{124}{213}\approx 0.58216$. The number picked will vary. For example: $\frac{5821}{10000}=0.5821$. See Figure 2.30.

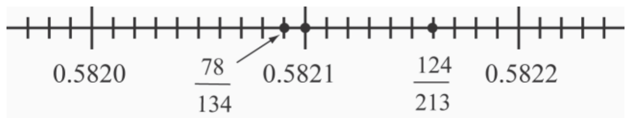


Figure 2.30: A number line

11. Many answers are possible. Since $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \square$ and $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \square$ then we can find as many fractions between $\frac{2}{5}$ and $\frac{3}{5}$ as we want to. For instance, both $\frac{7}{15}$ and $\frac{8}{15}$ work.

12. a. $\frac{3\times 9}{5\times 9} = \frac{27}{45}$ and $\frac{2\times 15}{3\times 15} = \frac{30}{45}$ so choose, for example, $\frac{28}{45}$ and $\frac{29}{45}$.

b. $\frac{3}{5} = 0.6$ and $\frac{2}{3} = 0.6666\ldots$ so choose, for example 0.61 and 0.63 which become $\frac{61}{100}$ and $\frac{63}{100}$.

13. a. $\frac{5}{7} = \frac{35}{49}$ and $\frac{6}{7} = \frac{42}{49}$ so choose, for example, $\frac{37}{49}$ and $\frac{41}{49}$.

b. $\frac{5}{7} \approx 0.7143$ and $\frac{6}{7} \approx 0.8571$ so choose, for example 0.75 and 0.80 which become $\frac{75}{100} = \frac{3}{4}$ and $\frac{8}{10} = \frac{4}{5}$.

14. See Figure 2.31. As shown in this figure, $\frac{1}{5}$ is not halfway between $\frac{1}{4}$ and $\frac{1}{6}$. Instead, $\frac{5}{24}$ is halfway between $\frac{1}{4}$ and $\frac{1}{6}$. This fact is most easily illustrated by finding a common denominator between the two fractions: $\frac{1}{4} = \frac{6}{24}$ and $\frac{1}{6} = \frac{4}{24}$. Just as 5 is halfway between 4 and 6, $\frac{5}{24}$ is halfway between $\frac{4}{24}$ and $\frac{6}{24}$. But, $\frac{1}{5} = \frac{5}{25}$ which is slightly less than $\frac{5}{24}$.

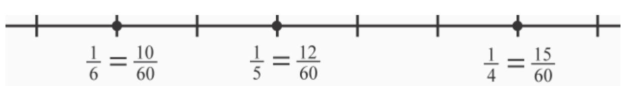


Figure 2.31: Showing $\frac{1}{5}$ is closer to $\frac{1}{6}$

15. Sam is not accounting for the numerators. While a fraction with a larger denominator may have smaller pieces, if it has enough of those smaller pieces (the numerator) then it will represent a larger amount. For example $\frac{9}{10}$ is bigger than $\frac{1}{2}$ even though the halves are bigger than the tenths. But having 9 of these tenths makes up for the fact that they are small and is bigger than a single half.

16. Answers will vary. Here are two examples:

a. Which fraction is greater: $\frac{1}{3}$ or $\frac{2}{9}$? Since $\frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$, $\frac{1}{3}$ is greater even though its numerator and denominator are both smaller than in $\frac{2}{9}$.

b. Which fraction is greater: $\frac{2}{4}$ or $\frac{5}{10}$? Neither, because they both are equivalent to $\frac{1}{2}$

$\frac{1}{2}$. So even though the numerator and denominator of $\frac{5}{10}$ are greater than in $\frac{2}{4}$, the fractions are equal.

17. Malcom's reasoning is not correct. The problem lies in the fact that although $\frac{8}{11}$ does have more pieces of the unit amount represented than $\frac{7}{10}$ does, the pieces in $\frac{8}{11}$ are smaller this complicates the situation. For example, : $\frac{5}{11} < \frac{3}{5}$ and but $5 > 3$ and $11 > 5$. Many counterexamples exist, another is $\frac{8}{13} < \frac{7}{10}$.

18. a. Yes it is valid: $\frac{30}{70} = \frac{3 \times \cancel{10}}{7 \times \cancel{10}} = \frac{3}{7}$ and $\frac{20}{50} = \frac{2 \times \cancel{10}}{5 \times \cancel{10}} = \frac{2}{5}$. "Canceling" zeros comes from canceling the factor of ten from the numerator and denominator.

b. No it is not valid: $\frac{15}{25} = \frac{3\times\cancel{5}}{5\times\cancel{5}} = \frac{3}{5} \neq \frac{1\cancel{5}}{2\cancel{5}}$ and $\frac{105}{205} = \frac{21\times\cancel{5}}{41\times\cancel{5}} = \frac{21}{41} \neq \frac{10\cancel{5}}{20\cancel{5}}$.

c. Canceling happens when the same *factor* appears in the numerator and denominator when they are written as a product of factors. Cancelling is not valid by simply cancelling the same final digit.

19. a. The numerators and denominators are both following the Fibonacci sequence. The next numerator (or denominator) is found by adding the last two numerators (or denominators).

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \frac{89}{55}, \square$$

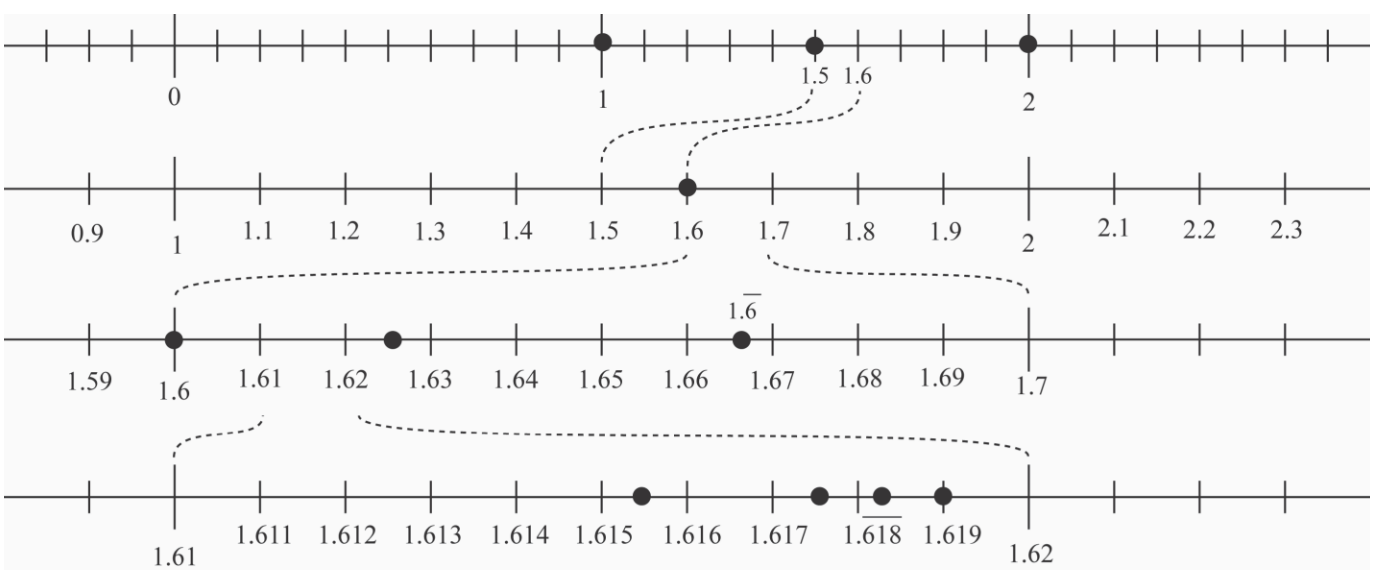
b. The fractions are increasing in size. When comparing the first and third fractions, the first fraction is one-half less than the third. The third fraction is one-tenth less than the fifth fraction. The fifth fraction is one-sixty-fifth less than the seventh fraction, etc.

c. An analogous pattern occurs in the even numbered fractions except that the fractions are decreasing. The fourth fraction is one-third *less* than the second. The sixth fraction is one-twenty-fourth *less* than the fourth, etc.

d. $1,\ 2,\ 1.5,\ 1.\overline{6},\ 1.6,\ 1.625,\ 1.6154,\ 1.6190,\ 1.6176,\ 1.6\overline{18}$. See Figure 2.32.

e. The numbers do appear to be getting closer and closer to a particular number. In fact, the sequence approaches the golden ratio, with a decimal approximation around 1.61803399.

Figure 2.32: Fractions on the number line



20. a. For example: $\frac{2}{3} < \frac{3}{4}$, $\frac{3}{5} < \frac{4}{6}$, $\frac{1}{2} < \frac{2}{3}$, $\frac{3}{7} < \frac{4}{8}$, $\frac{23}{30} < \frac{24}{31}$.

b. Frank is correct, but his reasoning is flawed. For example, $\frac{3}{5}$ has more parts than $\frac{2}{3}$, but $\frac{2}{3}$ is larger. Looking only at the number of parts does not account for the sizes of the parts.

c. The claim is that $\frac{A+1}{B+1} > \frac{A}{B}$. This is only true when $(A+1)\times B > A\times(B+1)$, which can be simplified to $A\times B + B > A\times B + A$ or $B > A$. In other words, $\frac{A+1}{B+1} > \frac{A}{B}$ is true if $B > A$. Since $\frac{A}{B}$ is a proper fraction then we know $B > A$ is true so that $\frac{A+1}{B+1} > \frac{A}{B}$ is also true. Another way to look at is that the fraction $\frac{A}{B}$ is $\frac{B-A}{B}$ less than 1. $\frac{A+1}{B+1}$ is $\frac{(B+1)-(A+1)}{B+1} = \frac{B-A}{B+1}$ less than 1. Since B is always less than $B + 1$ in these situations, and since $B > A$ in these situations, $\frac{B-A}{B} > \frac{B-A}{B+1}$ which means $\frac{A+1}{B+1}$ is closer to 1 than $\frac{A}{B}$, which means $\frac{A}{B} < \frac{A+1}{B+1}$.

2.5 Reasoning About Percent

1. 285 grams. See Figure 2.33. We can find the value by building up the value as follows: $95\% = 50\% + 25\% + 5\% + 5\% + 5\% + 5\%$; but once we know 5% we can simply deduct that from the total: $100\% - 5\% = 95\%$. So 300 – 15 = 285 grams. Numerically, we have $\frac{95}{100} \cdot 300 = 0.95 \cdot 300 = 285$ grams.

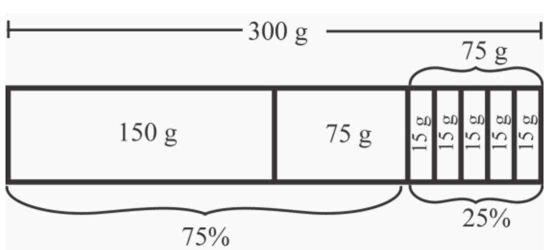


Figure 2.33: Daily value of carbohydrates

We can also solve it with a percent table. See Table 2.1.

Table 2.1: A percent table used to calculate 95% of 300 is ?

100%	$\rightarrow$	300
10%	$\rightarrow$	30
5%	$\rightarrow$	15
$100\% - 5\% = 95\%$	$\rightarrow$	$300 - 15 = 285$

2. 60%. See Figure 2.34. The bar representing 100% of the order has five pieces, with each piece representing one-half ton. Each piece also represents 20% of the order, since five of

them make up the total. The three shaded pieces represent the delivered gravel. Since each piece is 20%, the crew has received $20 \cdot 3 = 60\%$ of the order.

Numerically, $P \cdot \frac{5}{2} = \frac{3}{2} \Rightarrow P = \frac{3}{2} \cdot \frac{2}{5} = \frac{6}{10} = 60\%$.

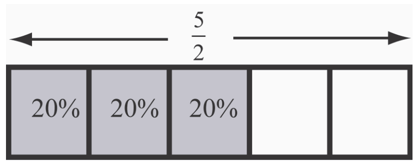


Figure 2.34: What percent of the gravel?

We can also solve it with a percent table. See Table 2.2.

Table 2.2: A percent table used to calculate ?% of $\frac{5}{2}$ is $\frac{3}{2}$

100%	→	$\frac{5}{2}$
$100\% \div 5$	→	$\frac{5}{2} \div 5$
20%	→	$\frac{1}{2}$
$20\% \cdot 3$	→	$\frac{1}{2} \cdot 3$
60%	→	$\frac{3}{2}$

3. 20 grams. See Figure 2.35. Looking for 80% is like looking for 20% since 80% + 20% = 100%. Also, $20\% \cdot 5 = 100\%$ so we can partition the unit amount into five pieces representing 20% each. Numerically, $\frac{80}{100} \cdot 25 = (0.8) \times (25) = 20$ grams.

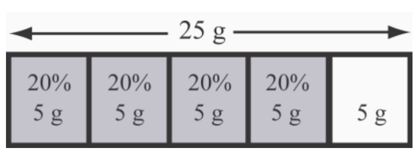


Figure 2.35: 80% of your daily fiber

We can also solve the problem with a percent table. See Table 2.3.

Table 2.3: A percent table used to calculate 80% of 25 is ?

100%	$\rightarrow$	25
$100\% \div 5$	$\rightarrow$	$25 \div 5$
20%	$\rightarrow$	5
$20\% \times 4$	$\rightarrow$	5×4
80%	$\rightarrow$	20

4. 240,000. See Figure 2.36. The math drawing shows building up 100% by putting together 6 groups of 15% each and then $\frac{2}{3}$ of 15%. So

$100\% = 36,000 \cdot 6 + \frac{2}{3} \cdot 36,000 = 216,000 + 24,000 = 240,000$. Numerically,

$15\% \cdot T = 36,000 \Rightarrow T = \dfrac{36,000}{0.15} = 240,000$.

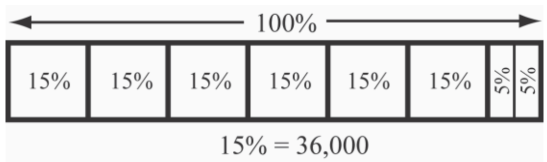


Figure 2.36: Building up 100%

5. 3600 mg. See the Table 2.4 below. $5\% \cdot 20 = 100\%$ so $180 \cdot 20 = 3600$ mg.

Numerically, $0.05 \cdot T = 180 \Rightarrow T = \frac{180}{0.05} = 3600$ mg.

Table 2.4: A percent table used to calculate 5% of ? = 180

5%	$\rightarrow$	180
5% $\cdot$ 20	$\rightarrow$	180 $\cdot$ 20
100%	$\rightarrow$	3600

6. 70%. See Figure 2.37. Each part represents $\frac{1}{2}$-inch of rain. The whole or unit amount represents the average July rainfall. The shaded portion represents this year's July rainfall. Seven parts out of ten are shaded, which is 70%. Numerically,

$P \cdot 5 = 3.5 \Rightarrow P = \frac{3.5}{5} = \frac{35}{50} = \frac{70}{100} = 70\%$.

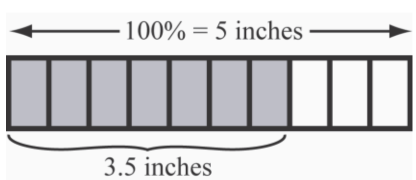


Figure 2.37: July Rainfall

7. $66\frac{2}{3}\%$. See Figure 2.38. Half a cup of cereal can be seen as two parts out of three, or

$\frac{2}{3} = \frac{66\frac{2}{3}}{100}$. Numerically, $P \cdot \frac{3}{4} = \frac{1}{2} \Rightarrow P = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$.

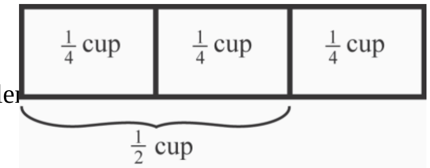


Figure 2.38: Part of a serving of cereal

8. a. See Table 2.5.

Table 2.5: A percent table used to calculate ?% of 250 = 225		
250	$\rightarrow$	100%
25	$\rightarrow$	10%
$250-25=225$	$\rightarrow$	$100\%-10\%=90\%$

25 is 10% of 250. 225 is 25 less than 250, so it is 10% less than 100%. So 225 is 90% of 250.

b. $\frac{P}{100}=\frac{225}{250}=\frac{225\div 25}{250\div 25}=\frac{9}{10}=\frac{90}{100}$. So P = 90%.

c. 50% of 1600 is 800. 10% of 1600 is 160. Since 960 = 800 + 160, then 960 is 60% of 1600. See Table 2.6.

Table 2.6: A percent table used to calculate ?% of 1600 = 960		
1600	$\rightarrow$	100%
800	$\rightarrow$	50%
160	$\rightarrow$	10%
$800+160=960$	$\rightarrow$	$10\%+50\%=60\%$

d. $\frac{P}{100}=\frac{960}{1600}=\frac{960\div 16}{1600\div 16}=\frac{60}{100}$. So P = 60%.

9. a. 3000. See Table 2.7.

Table 2.7: A percent table used to calculate 690 is 23% of ?		
23%	$\rightarrow$	690
1%	$\rightarrow$	$690\div 23=30$
100%	$\rightarrow$	$30\times 100=3000$

b. 7.5%. See Table 2.8.

Table 2.8: A percent table used to calculate ?% of 40 is 3		
40	$\rightarrow$	100%

1	$\rightarrow$	$100\% \div 40 = 2.5\%$
$1 \times 3 = 3$	$\rightarrow$	$2.5\% \times 3 = 7.5\%$

c. 7000. See Table 2.9.

Table 2.9: A percent table used to calculate 630 is9% of ?		
9%	$\rightarrow$	630
$9\% \div 9 = 1\%$	$\rightarrow$	$630 \div 9 = 70$
$1\% \cdot 100 = 100\%$	$\rightarrow$	$70 \cdot 100 = 7000$

d. 4000. See Table 2.10.

Table 2.10: A percent table used to calculate 12% of ? is 480		
12%	$\rightarrow$	480
$12\% \div 12 = 1\%$	$\rightarrow$	$480 \div 12 = 40$
$1\% \cdot 100 = 100\%$	$\rightarrow$	$40 \cdot 100 = 4000$

10. 75%. See Figure 2.39. The bar represents 100% of your full daily value of folic acid, or $\frac{2}{3}$ cup, as $\frac{4}{6}$ cup. Then $\frac{1}{2}$ cup is three parts out of four, or 75% of your full daily value of folic acid. Numerically, $P \cdot \frac{2}{3} = \frac{1}{2} \Rightarrow P = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4} = 75\%$.

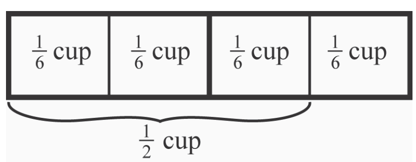


Figure 2.39: Part of a serving of cereal

11. 4,370. This problem is very similar to Problem 1. See Table 2.11. 5% of 100% is $\frac{1}{20}$, so 5% of 4,600 is 4,600 ÷ 20 = 230. So Biggo Corporation hopes all but 230 employees, or 4,600 – 230 = 4,370 employees will participate. Numerically, $\frac{95}{100} \cdot 4,600 = 0.95 \cdot 4,600 = 4,370$.

Table 2.11: A percent table used to calculate 95% of 4600 is ?		
100%	$\rightarrow$	4600
$100\% \div 20 = 5\%$	$\rightarrow$	$4600 \div 20 = 230$
$100\% - 5\% = 95\%$	$\rightarrow$	$4600 - 230 = 4370$

12. 85%. See Table 2.12.

Table 2.12: A percent table used to calculate ?% of 4 is 3.4

3	$\rightarrow$	75%
0.4	$\rightarrow$	10%
$3+0.4=3.4$	$\rightarrow$	$75\%+10\%=85\%$

Split the 3.4 acres and consider 3 acres + 0.4 acres. 3 acres is 75% of 4 acres (think $\frac{3}{4}$), and 0.4 acres is 10% of 4 acres. So with 3.4 acres, the company has 75% + 10%. Or solve the equation: $4 \cdot P = 3.4$.

13. \$4.8 million. See Table 2.13.

Table 2.13: A percent table used to calculate 75% of ? is 3.6 million

75%	$\rightarrow$	$3{,}600{,}000$
$75\% \div 3 = 25\%$	$\rightarrow$	$3{,}600{,}000 \div 3 = 1{,}200{,}000$
$25\% \cdot 4 = 100\%$	$\rightarrow$	$1{,}200{,}000 \cdot 4 = 4{,}800{,}000$

Recognizing that 75% and 100% are both multiples of 25%, I first divided 75% by 3 and 3,600,00 by 3 to show that 1,200,000 matched with 25%. Then to get to 100%, I multiplied 25% by 4 and also 1,200,000 by 4 . Or solve the equation: $0.75 \cdot N = 3.6$.

14. 50,400. See Table 2.14

Table 2.14: A percent table used to calculate 60% of 84,000 is ?

100%	$\rightarrow$	$84{,}000$
50%	$\rightarrow$	$42{,}000$
10%	$\rightarrow$	$8{,}400$
$50\%+10\%=60\%$	$\rightarrow$	$42{,}000+8{,}400=50{,}400$

The problem asks "what is 60% of 84,000?" 50% is 42,000, and 10% is 8,400. So 60% is 42,000 + 8,400 = 50,400. Or solve the equation: $0.60 \cdot (84{,}000) = N$.

15. 125%. See Figure 2.40. Each square represents 20% of Denise's distance but 25% of Frank's distance. The math drawing shows that Frank ran $80\% = \frac{80}{100} = \frac{4}{5}$ of Denise's distance. It also shows that Denise ran $\frac{5}{4}$ of Frank's distance. $\frac{5}{4} = 1\frac{1}{4} = 125\%$.

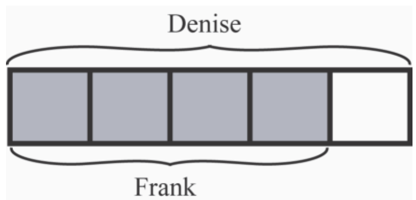


Figure 2.40: Denise's distance as a percentage of Frank's

16. 87%. 115% of BigMart's soda sales is the same as GrandMart's soda sales. So $1.15 \times B = G \Rightarrow B = \frac{G}{1.15} = \frac{1}{1.15} \times G \approx 0.87 \times G$, which means BigMart sells about 87% as much soda as GrandMart.

17. a. No. Benton spent $133\frac{1}{3}\%$ of Connie's ticket price. Figure 2.41 shows that Benton spent $\frac{4}{3}$ of Connie's ticket price. Each square is 25% of Benton's ticket price, but $33\frac{1}{3}\%$ of Connie's ticket price.

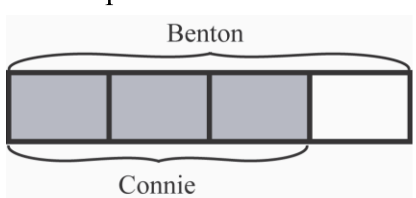


Figure 2.41: Connie spent 75% as much as Benton

b. No. Connie spent 80% of Benton's ticket price. Figure 2.42 shows that Connie spent $\frac{4}{5}$ of Benton's ticket price. Each square is 20% of Benton's ticket price and 25% of Connie's ticket price.

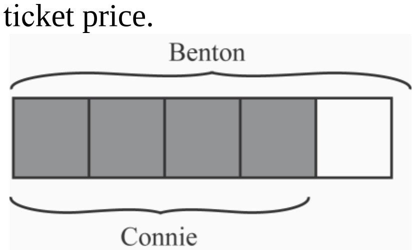


Figure 2.42: Benton spent 125% as much as Connie

18. No. 25% of the items are not newspapers. If these items cost much more than the newspapers or much less than the newspapers, then the income from these items would not equal 25%. The income from newspapers would be 75% of the total only if all the items are the same price.

19. 200. 30% of 500 is 150 style A handbags. 70% of 500 is 350 style B handbags. To bring the balance to 50% style A handbags, we need to increase the number of style A bags sold from 150 to 350.

20 a. We actually cannot determine the percent of students in the county that speak Spanish at home because we don’t know the populations of the two schools. Furthermore, by Tom’s logic we would have to say that 150% of the students *do not* speak Spanish at home because 70% do not at one school and 80% do not at the other. This cannot be true.

b. i. No, the correct percent is 28%. In this scenario, 30% of School A is 120 students, and 20% of School B is 20 students. So 140 students in the county speak Spanish at home out of a total population is 500 students. So the percent is $\frac{140}{500} = \frac{28}{100}$.

ii. No, the correct percent is 22%. In this scenario, 30% of School A is 30 students, and 20% of School B is 80 students. So 110 students in the county speak Spanish at home out of 500 students altogether. So the percent is $\frac{110}{500} = \frac{22}{100}$.

iii. If the two schools have equal populations, then the correct percent is the average of the two percents. For example, if both schools have 100 students, then 30 students in School A and 20 students in School B speak Spanish at home. This gives us 50 students out of 200, which is 25%.

21. a. Think of the can of soup as being 20 parts solid and 80 parts water. Mixing a can of water adds another 100 parts of water. So the mixture has 20 parts solid and 180 parts water, to make 200 parts total. With 180 parts water out of 200 parts total is equivalent to 90 parts water out of 100 parts total, which is 90%.

b. $66\frac{2}{3}\%$. Instead of 20 parts solid and 80 parts water, the concentrated soup is 20 parts solid and 40 parts water. So it is 40 parts water out of 60 parts total, which is equivalent to $\frac{2}{3}$ which is $66\frac{2}{3}\%$.

c. 75%. Continuing with the line of the previous two parts of the solution we assume the 20 parts solid remains constant. To make this 50% of the contents of the can, we need solids to be 20 parts out of 40. So the concentrated soup must be 20 parts water. The soup started with 80 parts water. Removing 60 parts of the water means removing 60 out of 80, which is equivalent to $\frac{6}{8} = \frac{3}{4} = 75\%$.

22. 60 cans. 60% of 300 cans is 180 cans of brand A beans. 40% of 300 cans is 120 cans of brand B beans. After removing some cans of brand B, the 180 cans of brand A now

represent 75% of the total. This means that the 180 cans are $\frac{3}{4}$ of the total because there are 3 parts each being $\frac{1}{4}$ of the total . Since 180 ÷ 3 = 60, then we know that 60 cans are

$\frac{1}{4}$ of the total. So we will have 180 cans of brand A and 60 cans of brand B, which means 60 cans of brand B were removed.

23. 100 pounds. The fresh cucumbers are 99% water, and 1% plant material. This means the plant material weighs $0.01\times 200 = 2$ pounds. Later, the cucumbers are 98% water. The weight of the actual plant material does not change, only the weight from water. So now, the 2 pounds of plant material constitute 2% of the weight. That is:
$\frac{2}{100}W = 2 \Rightarrow W = \frac{100}{2} = 100$ pounds.
2

2.1 Solving Problems and Explaining Solutions

Go to MyMathLab for a video for this section:

Using a math drawing. This video demonstrates how to use a math drawing (specifically, a strip diagram) to solve a word problem. It discusses how math drawings should be as simple as possible and show relationships among quantities. Common Core State Standards Standards for Mathematical Practice 1, 3 are directly related.

You might like to assign this section as reading and weave a discussion of this section into the next section. Although the norms of mathematical reasoning and explanation are certainly based in our natural capacity for logical thought, these norms are culturally transmitted, and students must learn what qualifies as valid mathematical reasoning. This takes many examples as well as guidance from instructors. You could show or assign the video Using a math drawing to help students think about how to use drawings effectively in math.

If you want to assign some problems for students to practice their problem-solving skills, you might consider some of the following:

- The cube problem in Section 4.3;
- the locker problem in Section 8.1;
- the high-five problem of Class Activity 9B (ask about a large number of people);
- the sum of odd numbers in part 4 of Class Activity 9D;
- some of the later problems in Section 9.5;
- some of the later problems in Section 11.4.

2.2 Defining and Reasoning About Fractions

Go to MyMathLab for a video for this section:

The Common Core definition of fraction. This video discusses the Common Core definition of fraction, why this definition is useful, and how it is different from another way people often describe fractions. Common Core State Standard 3.NF.1 is directly related.

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60CHAPTER 2

Go to MyMathLab for additional activities for this section:

The Whole Associated With a Fraction

Is the Meaning of Equal Parts Always Clear?

Improper Fractions

The definition of fractions given in the book is essentially the same as the definition in 3.NF.1 of the CCSS. Research has identified a number of different “personalities” or ways of defining or thinking about fractions, including viewing fractions as division or as ratios. In chapters 6 and 7 we will connect these other ways of thinking about fractions to the definition of fractions given in this chapter.

Class Activity 2A: Getting Familiar with our Definition of Fractions

IMAP Video opportunity

You could show or assign the video Common Core definition of fraction after part 1 or at the end of the activity.

I like to have students do part 1 before I even give our definition of fraction. Once they see the limitation of this “A out of B” definition, they are interested in learning a different definition.

1. It doesn’t make sense to talk about 4 parts out of 3 equal parts because there are only 3 parts making the whole.

   Ask students how they would plot fractions on a number line if they think of them as “A out of B.” For example, how would they plot the fraction $\frac{3}{4}$ if they think of it as “3 out of 4”?

   Also, this “A out of B” language seems to cause some students to focus on the remaining part of the whole when the fractional part is “taken out of the whole,” i.e., removed from the whole. This misconception is discussed in IMAP Video: Jacky compares $\frac{1}{9}$ and $\frac{2}{7}$ (number 12 on the DVD). In particular, the discussion at the end explains why the “A out of B” language may cause problems for students.

2. To explain why an amount is $\frac{1}{3}$ of the whole (or unit amount), students can explain that 3 copies of the amount makes the whole, or that the amount was obtained by dividing the whole into 3 equal parts.

Emphasize that students should describe $\frac{2}{3}$ as 2 parts, each of which is $\frac{1}{3}$ of the whole (or unit amount). Students should use similar wording for the other non-unit fractions.

3. (a) One cup of butter should be represented by a strip that is 5 times as long.
   (b) Students should view the $\frac{3}{5}$ liter bar as 3 parts, each of size $\frac{1}{5}$ of a liter. Therefore they should partition the given bar into 3 parts and draw 5 parts of that size to represent 1 whole liter.
   (c) Students should view the $\frac{4}{9}$ pound bar as 4 parts, each of size $\frac{1}{9}$ of a pound. Therefore they could partition the given bar into 4 parts and draw 9 parts of that size to represent 1 whole pound. They could also draw 2 copies of the original bar and add another piece the size of $\frac{1}{4}$ of the original bar.
   (d) Students should view the $\frac{6}{5}$ kilogram bar as 6 parts, each of size $\frac{1}{5}$ of a pound. Therefore they could partition the given bar into 6 parts and draw 5 parts of that size to represent 1 whole kilogram.

Class Activity 2B: Using our Fraction Definition to Solve Problems

This activity is an opportunity not only for problem-solving, thereby addressing Standard for Mathematical Practice 1, but also for attention to precision in using the definition of fraction, thereby attending to Standard for Mathematical Practice 6.

During the discussion of these problems, be sure students see that an amount can be described with different fractions depending on what the unit amount (whole) is taken to be. This issue is explored further in the activity after this one.

1. Show $\frac{3}{4}$ of the piece of paper because you want to show 3 parts, given that the piece of paper is 4 parts.
2. Benton should use $\frac{1}{3}$ of his dough, which he can get by cutting the dough into 3 equal parts either horizontally or vertically. He needs $\frac{1}{3}$ of the dough because the $\frac{3}{4}$ cup of butter consists of 3 parts (each of which is $\frac{1}{4}$ cup) but he only wants 1 of those parts.
3. Because $\frac{2}{3}$ is 2 parts, each of which is $\frac{1}{3}$ of the other whole design, the hexagon should be thought of as 2 parts. Two trapezoids make the hexagon, so a trapezoid is $\frac{1}{3}$ of the other design. Therefore the other design's area can be made from

3 trapezoids (which can also be made from various combinations of triangles, rhombuses, and trapezoids).

If the hexagon is $\frac{3}{2}$ of the area of another design, then it is 3 parts, each of which is $\frac{1}{2}$ of another design. Three rhombuses make the hexagon, so the other design's area can be made from 2 rhombuses (or 4 triangles, rhombuses or a trapezoid and a triangle).

If the hexagon is $\frac{6}{10}$ of the area of another design, then it is 6 parts, each of which is $\frac{1}{10}$ of another design. Six green triangles make the hexagon, so the other design can be made from 10 triangles.

4. One method is to make 3 copies of the 3 hexagon (making $\frac{6}{7}$ of the whole area) and then adding half of 3 hexagons (e.g., a hexagon and a red trapezoid). Another method is to take half of 3 hexagons and repeat that amount 7 times.

   As an extension, ask students to make a design, then make a fraction of the design and give it to another group as a challenge to make their original design.

Class Activity 2C: Why Are Fractions Numbers? A Measurement Perspective

This measurement perspective on fractions becomes especially important later on for understanding fraction multiplication, in particular for understanding a fraction as a multiplier.

Reciprocals (parts 4 – 7) will be important in fraction division.

1. This first part gives students the opportunity to think about this important question and begin to develop their ideas about how to answer it.
2. 3; $\frac{8}{3}$ or $2\frac{2}{3}$; $\frac{4}{3}$ or $1\frac{1}{3}$, $\frac{2}{3}$.
3. Students may begin to appreciate that measurement is one way that both whole numbers and fractions arise. The text at the top of the next page expresses that idea.
4. Part a: 5. Part b: $\frac{1}{5}$.
5. Part a: $\frac{7}{3}$ because Strip D is 7 parts, each $\frac{1}{3}$ of Strip C. Students may also say $2\frac{1}{3}$, but if they do not produce $\frac{7}{3}$ ask if there is another way to express the answer. Part b: $\frac{3}{7}$.

In conjunction with the mixed number answer $2\frac{1}{3}$ ask students why the fractional part is $\frac{1}{3}$ and not $\frac{1}{7}$. This will help to emphasize the different unit amounts being used.

6. Part a: $\frac{11}{4}$ or $2\frac{3}{4}$. Part b: $\frac{4}{11}$.

7. One is flipped upside down compared to the other. In other words, one fraction is the reciprocal or multiplicative inverse of the other. This previews the relationship $\frac{A}{B} \cdot \frac{B}{A} = 1$.

Class Activity 2D: Relating Fractions to Wholes

This activity is an opportunity not only for problem-solving, thereby addressing Standard for Mathematical Practice 1, but also for attention to precision in using the definition of fraction, thereby attending to Standard for Mathematical Practice 6.

Note that identifying the whole that a fraction is "of" will be especially important later in analyzing fraction multiplication and division (in chapters 5 and 6) because in fraction multiplication and division word problems, different fractions in a problem can refer to different wholes. So during the discussion of this activity you might want to tell students that they will be encountering the "what's the whole?" question later on as well.

We have to specify the whole in order to interpret unambiguously math drawings that represent fractional amounts. Students will see this point in this activity; it is especially important when it comes to improper fractions and mixed numbers. In your class discussions, you might encourage students to draw a separate picture of the whole and to label the whole as such.

Encourage students to talk about fractions as "of a whole" rather than just as "parts out of parts." For example, encourage them to say "$\frac{1}{12}$ of the park" (in addition to saying "1 out of 12 equal parts").

You may wish to use the term "unit amount" in addition to "whole" to help students with the idea that there can be different wholes within one situation. Note that the Common Core State Standards use the term "whole" in relation to fractions.

1. The swing area consists of $\frac{1}{12}$ of the park because it is 1 out of 12 equal parts (after suitably subdividing the park). Note that the whole for this $\frac{1}{12}$ and for the $\frac{1}{3}$ is the area of the entire neighborhood park. The whole for $\frac{1}{4}$ is the area of the playground.

For part (b), the swings' area can be described both as $\frac{1}{4}$ of the area of the playground and as $\frac{1}{12}$ of the area of the park.

You don't need to bring this up now, but some students might observe that the swing problem is a word problem for $\frac{1}{4} \cdot \frac{1}{3}$.

2. Ben should use $\frac{1}{2}$ of the oil in the bottle. Note that the whole for this $\frac{1}{2}$ is the amount of oil in the bottle. But the whole for the $\frac{1}{3}$ and the $\frac{2}{3}$ is a cup of oil.

   For part (b), the $\frac{1}{3}$ of a cup of oil can also be described as $\frac{1}{2}$ of the oil in the bottle.

   You don't need to bring this up now, but some students might observe that the oil problem is a word problem for $\frac{1}{3} \div \frac{2}{3}$.

Class Activity 2E: Critiquing Fraction Arguments

IMAP Video opportunity

2. Mariah's method is not correct because the 10 parts are not equal in area. If the two plots did have the same area, then her reasoning would be correct.

3. Yes, Aysah's picture can be used. It shows that the shaded portion is $\frac{1}{5}$ of Peter's garden because 5 copies of the shaded portion can be joined to make the whole garden. Or in other words, picture shows that the garden is divided into 5 parts and one of those parts is shaded (note that a "part" can consist of several distinct pieces).

4. Matt's reasoning is not valid because the two $\frac{1}{5}$ each refer to a different whole and the question is about Peter's entire garden.

   If the two plots were each 1 acre, then the shaded pieces would in fact be two parts, each of which is $\frac{1}{5}$ of an acre, and therefore the two parts together would be $\frac{2}{5}$ of an acre. So yes, Matt's reasoning can be used to make a correct statement. As always, attending to the whole is crucial when working with fractions!

   At this point, you might like to show IMAP video: Felicia interprets the whole of a fraction (number 14 on the DVD), in which Felisha divides 2 cookies equally among 5 people. There is some confusion about how to describe each person's share. Is it $\frac{2}{10}$ or $\frac{2}{5}$? The answer depends on what we take the whole to be.

(This video is an even better fit for the discussion of the relationship between division and fractions in chapter 6.)

Class Activity 2F: Critique Fraction Locations on Number Lines

Another point to bring up during this activity is that an "A out of B" view of fractions is not helpful for locating fractions on number lines. For example, how could one plot the fraction $\frac{3}{4}$ if one thinks of it as "3 out of 4"?

1. (a) Eric may understand that making fourths involves dividing a unit into 4 pieces, but he may think he can accomplish that by inserting 4 tick marks between 0 and 1 instead of dividing the interval from 0 to 1 into 4 equal pieces. Eric has actually divided the interval from 0 to 1 into 5 equal pieces.

   (b) Kristin may simply be counting tick marks. She may not understand that the interval between 0 and 1 must be divided into 4 equal pieces. Her number line doesn't show 0. Although not incorrect, it probably means she is not attending to the distance from 0.

2. It looks like Tyler has the idea of making 4 equal intervals to plot $\frac{3}{4}$, but he may not realize that he needs to take the interval between 0 and 1 as the unit amount for the fraction.

3. Although it looks like Amy attended to the number of intervals between 0 and 1 (and not the number of tick marks, for example), Amy probably ignored the fact that the tick marks are not equally spaced, and that some intervals are longer than others. She could ignore the first and last tick marks and realize that the box is at $\frac{1}{4}$ or she could put in more tick marks and recognize that the location of the box can also be described as $\frac{2}{8}$.

4. See text for the idea of circling intervals. Students may think of other ways of highlighting the intervals.

Class Activity 2G: Fractions on Number Lines

1. Be sure students talk about dividing the interval between 0 and 1 into 4 equal-length segments, and in particular, focus on length. It will be worth noting that dividing the interval from 0 to 1 does not involve making 4 new tick marks. See text for full discussion of where to locate fractions on number lines and see the answer to the practice problem in this section. You might encourage students to put a strip labeled "1 whole" above the interval from 0 to 1.

2. As the tick marks should be $\frac{1}{4}$ apart, draw 3 equally spaced tick marks between the tick marks for 0 and 1 (so as to divide the interval between 0 and 1 into 4 equal pieces) and then continue on.

3. As the tick marks should be $\frac{1}{2}$ apart, draw 2 equally spaced tick marks between the tick marks for 0 and $\frac{3}{2}$, so as to subdivide the interval between 0 and $\frac{3}{2}$ into 3 equal pieces, and then continue on. Note that students must recognize that $\frac{3}{2}$ consists of 3 pieces, each of length $\frac{1}{2}$.

4. As the tick marks should be $\frac{1}{4}$ apart, draw 2 equally spaced tick marks between the tick marks for 0 and $\frac{3}{4}$ and then continue on.

5. As the tick marks should be $\frac{1}{5}$ apart, draw 3 equally spaced tick marks between the tick marks for 0 and $\frac{4}{5}$ and then continue on.

Class Activity 2H: Improper Fraction Problem Solving with Pattern Tiles

In this activity, students swap out a given tile for other tiles, so you can use this activity to lead into the idea of equivalent fractions.

1. Because $\frac{5}{4}$ means 5 parts, each of which is one fourth (of the original design),

   we need to find how to break Design A into 5 equal parts. We can do this by making the design with 5 green triangles. Then 4 of those green triangles will have the area of the original design (because 4 fourths make the whole). This area can also be made with a trapezoid and 1 triangle or with 2 blue rhombuses.
2. Because $\frac{5}{3}$ means 5 parts, each of which is one third (of the original design),

   we need to find how to break Design B into 5 equal parts. We can do this by making the area of the design with 5 trapezoids (even though we can't make the design itself that way). A design whose area is the same as the area of 3 trapezoids is what we are looking for (because 3 thirds make the whole).

   Another possibility is to make the area of the design with 15 triangles, which can be considered as 5 groups of 3 triangles each. Then make a design using 3 groups of 3 triangles each. Implicitly, this uses the equivalence of the fractions $\frac{5}{3}$ and $\frac{15}{9}$, so could be a springboard to the topic of equivalent fractions.

3. Because $\frac{9}{7}$ means 9 parts, each of which is one seventh (of the original design),

   we need to find how to break Design C into 9 equal parts. We can do this by making the area of the design with 9 blue rhombuses (even though we can't make the actual design that way). Then a design whose area is the same as the area of 7 blue rhombuses is what we are looking for (because 7 sevenths make the whole).

2.3 Reasoning About Equivalent Fractions

Go to MyMathLab for a video for this section:

Equivalent fractions. This video explains in two ways why we can multiply the numerator and denominator of a fraction by the same number to obtain a new fraction that is equal to the original fraction. One explanation involves multiplication by 1. Another explanation involves reasoning about math drawings. Common Core State Standards 4.NF.1, 5.NF.5b are directly related.

Go to MyMathLab for another activity for this section:

Simplifying Fractions.

In discussing this section, you might want to adopt slightly different terminology: you could use the words "partition" and "iterate" when referring to what we do with strips, drawings, or other quantities (real or imagined) and reserve the words "divide" and "multiply" for operations with numbers.

Before you have students work on the next Class Activity, you might have them work with fraction strips first. Give each student 3 long strips of paper (e.g., from a piece of paper cut into strips lengthwise). Ask students how they could fold the paper to show fourths of the strip (fold in half and then in half again while still folded). Have the students fold each strip to make fourths and then unfold and shade 3 of the fourths to show $\frac{3}{4}$ of each strip.

Next have students fold the second strip back into fourths and then fold in half again. Ask students to predict the size of the parts (eighths) and ask them to predict how they will be able to describe the shaded part in terms of those new parts (as 6 eighths).

Have students fold the third strip back up and have them fold and third strips back into fourths, and then fold in half again, and then again. Ask them to predict the size of the parts (sixteenths) and how many parts will be shaded (12 sixteenths). Have students write equations that correspond with what they have found and ask them how the numerators and denominators are related to the numerator and denominator of $\frac{3}{4}$.

Class Activity 2I: Explaining Equivalent Fractions

You could show or assign the video Equivalent fractions after students complete the activity.

Before you do the activity, you might ask students if they can think of ways to explain why the two fractions are equal. Some students will think of the idea of multiplying by 1 in the form of $\frac{4}{4}$. Although this is a nice way to explain fraction equivalence, it is only suitable for students who have already studied fraction multiplication. In the Common Core State Standards, students study equivalent fractions (in Grade 4) before they complete their study of fraction multiplication. Also, standard 4.NF.1 asks students to explain fraction equivalence with visual fraction models (i.e., as in the activity). Note that a brief comment is made about explaining fraction equivalence by multiplying by 1 in the text.

At the conclusion of the activity you could remark that there wasn't anything special about the specific numbers used in the explanations and that the very same reasoning will apply no matter what counting numbers are involved.

1. See text for an explanation using different numbers. You may wish to present an explanation to students first using different numbers. Then have them explain this example to each other. Press students to explain the multiplication of the numerator and denominator and not just to say that $\frac{2}{3} = \frac{8}{12}$.

2. Students will sometimes give an explanation that involves repeating a drawing of the fraction 3 times. However, with this approach, it's hard to explain why the resulting fraction is equal to the original fraction. Such an explanation would be more of an "equivalent ratio" explanation than an equivalent fraction explanation. By subdividing parts, we can argue that we have the same overall amount shaded and in all, therefore the two fractions express the same quantity.

3. The issue addressed in this part may already come up in your discussion of part 1. Note that it makes sense that when we divide each piece into 4 equal pieces there will be 4 times as many pieces (in all, and shaded). So the pieces become smaller and to compensate, there are more of them. The pieces themselves are divided, but the number of pieces is multiplied. In the picture we are working

with the pieces themselves, but in the numerical work we are working with the number of pieces.

Class Activity 2J: Critique Fraction Equivalence Reasoning

1. Even though Anna did "do the same thing to the top and bottom of the fraction," she doesn't get equivalent fractions this way, which we can see immediately by drawing pictures. When we teach about equivalent fractions, we should be more specific about how to get equivalent fractions than just saying "do the same thing to the top and the bottom."

2. Both fractions are "one part away from a whole" but 17ths and 12ths are different size parts.

3. Both fractions are equal to 1. Peter may not realize that when the whole is divided into more parts, each part becomes smaller.

   A similar issue comes up again in the activity "Can We Reason this Way?" in the section on comparing fractions.

Class Activity 2K: Interpreting and Using Common Denomi- nators

1. The two smallest common denominators are 12 and 24. When giving these fractions common denominators we are dividing the strips and number lines into like parts.
2. Let the distance between adjacent tick marks equal $\frac{1}{6}$ and let the leftmost tick mark represent $\frac{2}{3} = \frac{4}{6}$.
3. Let the distance between adjacent tick marks equal $\frac{1}{12}$ and let the leftmost tick mark represent $\frac{33}{4} = \frac{99}{12}$.
4. Let the distance between adjacent tick marks equal $\frac{1}{20} = 0.05$ and let the leftmost tick mark represent $0.7 = \frac{14}{20}$.

Class Activity 2L: Solving Problems by Using Equivalent Frac- tions

1. $\frac{1}{6}$ of the larger (imagined) piece of paper is formed by $\frac{1}{4}$ of the given piece of paper. Since $\frac{2}{3} = \frac{4}{6}$, and since we want 1 sixth and we have 4 sixths, we must show 1 out of 4 equal parts of the given piece of paper. In solving this problem, $\frac{2}{3}$ also appears as $\frac{4}{6}$. We can make 4 equal parts horizontally or vertically, which look different.
2. Using a math drawing, we see that Jean needs $\frac{1}{2} = \frac{3}{6}$ cups of butter but only has $\frac{1}{3} = \frac{2}{6}$ cups of butter, which means she has 2 out of the 3 parts she needs, so she has $\frac{2}{3}$ of what she needs. In solving the problem the fractions appear as equivalent fractions with denominator 6.

   You could also ask students to discuss the different wholes that are used in the problem. The whole associated with $\frac{1}{2}$ and $\frac{1}{3}$ is a cup of butter, but the whole associated with $\frac{2}{3}$ is the amount of butter Jean needs, namely the $\frac{1}{2}$ cup of butter.
3. Using a math drawing, we see that Joey should eat $\frac{3}{8}$ of a cup of cereal. In the drawing, we turn $\frac{3}{4}$ into $\frac{6}{8}$ in order to solve the problem.

   You could also ask students to discuss the different wholes that are used in the problem. The whole associated with the $\frac{3}{4}$ in the problem is a cup of cereal. The whole associated with the answer, $\frac{3}{8}$, is also a cup of cereal. But the whole associated with $\frac{1}{2}$ is a serving of cereal.

Class Activity 2M: Problem Solving with Fractions on Num- ber Lines

In this activity, students will need to give fractions either common denominators (the first 3 parts) or common numerators (the last two parts) to solve the problems and they will need to focus on the meaning of fractions in terms of number lines.

If students are stuck, give them the hint that they could try to make equivalent fractions to help them solve the problems. These are challenging problems!

1. Using the common denominator 12, $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$, so break the interval

between 0 and $\frac{2}{3}$ into 8 equal parts by drawing seven equally spaced tick marks between the tick marks for 0 and $\frac{2}{3}$. The distance between adjacent tick marks

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equals $\frac{1}{12}$. If we extend these tick marks past $\frac{2}{3}$ the tick mark for $\frac{3}{4}$ is the first one past $\frac{2}{3}$.

2. Use the common denominator 10. Draw five equally spaced tick marks between the tick marks for 0 and $\frac{1}{2}$. The distance between adjacent tick marks equals $\frac{1}{10}$. If we extend these tick marks past $\frac{1}{2}$ the tick mark for $\frac{3}{5}$ is the first one past $\frac{1}{2}$.

3. Use the common denominator 24.

4. Since $\frac{2}{3}$ is 2 parts, each of size $\frac{1}{3}$, we need to break the interval from 0 to $\frac{1}{4}$ into 2 equal parts. Therefore turn $\frac{1}{4}$ into the equivalent fraction $\frac{2}{8}$ (in essence we are giving $\frac{1}{4}$ and $\frac{2}{3}$ common numerators). Then place tick marks at $\frac{1}{8}$, $\frac{2}{8}$, and $\frac{3}{8}$. The answer is $\frac{3}{8}$ of a mile.

5. Since $\frac{3}{4}$ is 3 parts, each of size $\frac{1}{4}$, we need to break the interval from 0 to $\frac{2}{3}$ into 3 equal parts. Therefore turn $\frac{2}{3}$ into the equivalent fraction $\frac{6}{9}$ and make a tick mark spaced $\frac{2}{9}$ apart. The fourth tick mark will be at $\frac{8}{9}$ of a mile, which is the solution. Notice that in essence, we are giving $\frac{2}{3}$ and $\frac{3}{4}$ common numerators to make it possible to partition an interval of size $\frac{2}{3}$ into 3 equal parts.

Class Activity 2N: Measuring one Quantity with Another

This activity is designed to help students to distinguish an additive relationship from a multiplicative one and to see that they can focus on numbers of sections or numbers of buttons to describe a relationship. Being able to view an amount in two ways, as 1 group and also as some number of units, will be important for understanding fraction multiplication, and in particular, for understanding a fraction as a multiplier.

1. The strips can be compared by noting how many more sections or buttons one strip has than the other but they can also be compared by taking one strip as a unit amount and considering the other strip in relation to that unit amount.

2. It takes $\frac{8}{3}$ of Strip A to make Strip B because Strip B is make of 8 parts, each of which is the size of $\frac{1}{3}$ of Strip A. Students might also describe this as $2\frac{2}{3}$. If so, ask them if they can also use a fraction.

   Thinking in terms of buttons, it takes $\frac{32}{12}$ of Strip A to make Strip B or $2\frac{8}{12}$.

It takes $\frac{3}{8}$ of Strip B to make Strip A. Thinking in terms of buttons, it takes $\frac{12}{32}$ of Strip B to make Strip A.

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It takes $\frac{8}{3}$ of Button Strip C to make Button Strip D.

Thinking in terms of buttons it takes $\frac{48}{18}$ of Strip C to make Strip D.

It takes $\frac{3}{8}$ of Button Strip D to make Button Strip C.

Thinking in terms of buttons it takes $\frac{18}{48}$ of Strip D to make Strip C.

2.4 Reasoning to Compare Fractions

Go to MyMathLab for another activity for this section:

Can We Compare Fractions This Way?.

I like to begin this section by putting two fractions on the board (such as $\frac{5}{8}$ and $\frac{7}{13}$ or $\frac{2}{3}$ and $\frac{3}{5}$) and asking students what methods they know for determining which of those two fractions is greater. We then discuss the general methods and why they work.

Class Activity 2O: What is another way to Compare these Fractions ?

Use this activity to discuss the method of comparing fractions that have the same numerator by considering the denominators. At the end of this activity, you could mention that even if two fractions don't have the same numerator to start with, we can give the fractions common numerators to compare them. In some cases this is easier than giving fractions common denominators.

1. The second fraction is bigger because when you divide an object into 39 equal pieces, each piece is slightly bigger than if you divided the object into 49 equal pieces.

2. Use the same reasoning.

Class Activity 2P: Comparing Fractions by Reasoning

$\frac{27}{43} > \frac{26}{45}$ because $27 > 26$ and forty-thirds are greater than forty-fifths.

$\frac{13}{25} > \frac{34}{70}$ because the first fraction is greater than $\frac{1}{2}$ but the second fraction is less than $\frac{1}{2}$.

$\frac{17}{18} < \frac{19}{20}$ because each fraction is one piece less than a whole but 20ths are smaller than 19ths, so the second fraction is closer to a whole than the first.

$\frac{51}{53} < \frac{65}{67}$ because both fractions are 2 pieces less than a whole but sixty-sevenths are smaller than fifty-thirds, so the second fraction is closer to a whole than the first.

$\frac{9}{40} < \frac{12}{44}$ because $\frac{9}{40} < \frac{1}{4} < \frac{12}{44}$.

$\frac{13}{25} < \frac{5}{8}$ because both fractions are just over $\frac{1}{2}$ but $\frac{5}{8}$ is one eighth more than $\frac{1}{2}$ whereas $\frac{13}{25}$ is half of a twenty-fifth more than $\frac{1}{2}$.

$\frac{37}{35} < \frac{27}{25}$ because both fractions are "2 parts" over 1, but thirty-fifths are less than twenty-fifths.

Class Activity 2Q: Can We Reason this Way?

IMAP Video opportunity

As an extension of this activity, you could go to MyMathLab to watch four children discuss the sizes of pieces in relation to the number of pieces in the whole.

- Ally, compare and convert fractions;
- Jace, $\frac{1}{5}$ versus $\frac{1}{8}$;
- Jacky, compare fractions;
- Jacky, $\frac{2}{7}$ versus $\frac{1}{7}$.

In this case, Claire reaches the correct conclusion but her reasoning is not valid because even though 4 pieces is more than 3 pieces, 9ths are smaller than 8ths. So the numerators and denominators are actually working against each other in this case. Notice that it's important for both fractions to refer to the same size whole to know that 9ths are smaller than 8ths.

Ask students to think about other examples and whether Claire will always get the right answer using her approach. If they can't find any examples where Claire will get the wrong answer, give them the example of comparing $\frac{4}{11}$ and $\frac{3}{8}$ and ask what Claire would predict and whether her prediction would be correct. Claire would most likely predict that

$$\frac{4}{11} > \frac{3}{8}$$

which is false.

You may wish to show or assign IMAP Video: Ally compares and converts fractions (number 11 on the DVD) in which Ally talks about changing digits in order to determine which fraction is larger. It's not necessarily the same reasoning that Claire and Conrad are using in this activity, but is in the same style. You could also show or assign Video 12 where prospective teachers discuss a misconception in comparing fractions that many noticed when they interviewed students.

2.5 Reasoning About Percent

Go to MyMathLab for another activity for this section:

Equivalent Fractions and Percent.

You might introduce this section by asking students to jot down what they think "percent" means and why they think we have percents. You could then discuss the material at the beginning of the section. You might then ask students why we use 100 as the denominator for percentages instead of 10, say, or 1000. The denominator of 100 allows percentages to be expressed with 2 digits, which is short enough to be quickly grasped yet provides enough information to distinguish and separate many cases.

Notice that the methods shown in the tables in the text apply to different kinds of situations: Table 2.2 involves finding the portion, Table 2.3 involves finding the whole amount, and Table 2.4 involves finding the percent.

Class Activity 2R: Math Drawings, Percentages, and Fractions

This activity gives students a chance to see various percentages as arising from combining other common "benchmark" percentages. This will be useful in mental percent calculations.

1. Diagram 1: 95%

   Diagram 2: 80% (viewed as 75% plus $\frac{1}{5}$ of 25%)

Diagram 3: 45%

Diagram 4: 12.5% (half of 25%)

Diagram 5: 87.5% (75% plus half of 25%)

Class Activity 2S: Reasoning About Percent Tables to Solve "Portion Unknown" Percent Problems

Simple percent drawings and percent tables are thinking tools that can help us reason about how quantities are related. Therefore this could be a good opportunity to discuss Standards for Mathematical Practice 2 and 5 of the Common Core State Standards.

Most students find the "percent tables" to be really helpful for developing a feel for percentages. When we use percent tables, we engage in proportional reasoning.

1. $\frac{1}{10}$ of 80,000 is 8000. So half of 8000, namely 4000, is 5% of 80,000. If we take 4,000 away from 80,000 that will be 95% of 80,000. So the answer is 76,000. Some students may also calculate 9 times 8000 and then add 4000 to that, so it's a good idea to ask for their reasoning, not just their percent tables. Have students recognize that this is another valid way to compute 95% of 80,000.

   Percent diagram:

$$\begin{array}{rcr} 100\% & \longrightarrow & 80,000 \\ 10\% & \longrightarrow & 8000 \\ 5\% & \longrightarrow & 4000 \\ 95\% & \longrightarrow & 76,000 \end{array}$$

2. $\frac{1}{10}$ of 6500 is 650. So half of that, namely 325 is 5% of 6500. Since $10\% + 5\% = 15\%$, we have that $650 + 325 = 975$ is 15% of 6500.

3. 1% of \$25 is \$.25. Hence 7% of \$25 is \$1.75.

4. $\frac{1}{2}$ of 810 is 405. 10% of 810 is 81. So 60% of 810 is $405 + 81 = 486$.

5. 20% of 810 is 162. 40% of 810 is $162 + 162 = 324$. Hence 60% of 162 is $324 + 162 = 486$.

6. $\frac{1}{2}$ of 180 is 90. 10% of that is 9, which is also 5% of 180 (or you can find 5% of 180 by taking 10% of 180 first and then take half of that). So 55% of 180 is $90 + 9 = 99$.

Class Activity 2T: Reasoning about Percent Tables

Notice that the methods shown in the tables in the text apply to different kinds of situations: Table 2.2 involves finding the portion, Table 2.3 involves finding the whole amount, and Table 2.4 involves finding the percent.

1. See Figure 2.1 for a picture.

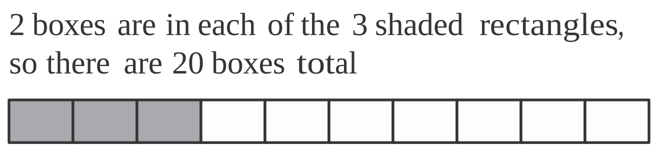

Figure 2.1: Boxes of Paper

Using a percent table:

30% → 6

10% → 2

100% → 20

You could also ask students to reason about equivalent fractions (without cross multiplying) to solve the problem. To do so, they should solve

$$\frac{30}{100} = \frac{6}{?}$$

which they can do this way: Since $\frac{30}{100} = \frac{5\cdot 6}{5\cdot 20} = \frac{6}{20}$ his order must have been 20.

2. 7% → \$2.10

1% → \$.30

100% → \$30

3. As seen in Figure 2.2, last year, 3 out of the 5 parts of normal rainfall fell, so $\frac{3}{5}$ = 60% of normal rainfall fell.

Using a percent table, students can reason this way:

100% → 5 eighths

20% → 1 eighth

60% → 3 eighths

Figure 2.2: Rainfall

Notice that this is another version of "going through 1" because it is going through 1 eighth.

4. Since $\frac{1}{4}$ cup gives 100% of your full daily value of clacium, $\frac{1}{12}$ gives you $33\frac{1}{3}\%$ of your daily value. Hence $\frac{1}{3} = \frac{4}{12}$ of a cup gives you $133\frac{1}{3}\%$ of your daily value.

Class Activity 2U: Percent Problem Solving

1. With a percent table:

$$\begin{array}{rcl} 40\% & \longrightarrow & 30 \\ 20\% & \longrightarrow & 15 \\ 100\% & \longrightarrow & 75 \end{array}$$

With equivalent fractions: we want to solve

$$\frac{40}{100} = \frac{30}{?}$$

$$\frac{40}{100} = \frac{4}{10} = \frac{2}{5} = \frac{30}{75}$$

2. Draw two strips to represent the running distances, making Marcie's 10 pieces long and Andrew's 4 pieces long. From the comparison, we see that Marcie's distance is 250% as long as Andrew's.

3. The 8 extra female bugs are as much as 10% of the male bugs. So there are $10 \cdot 8 = 80$ male bugs and 88 female bugs. This makes 168 bugs all together. Using a percent table we might say

$$\begin{array}{rcl} 10\% & \longrightarrow & 8 \\ 100\% & \longrightarrow & 80 \end{array}$$

but this 100% refers to 100% of the male bugs. It would be easy to think incorrectly that this 100% refers to all the bugs.

4. Draw two strips, one for the dogs, one for the cats. Make the dog strip 25% longer than the cat strip by making the cat strip 4 pieces long and the dog strip one piece longer, so 5 pieces long. Then the cats and dogs together are “made of” a total of 9 pieces and the cats are 4 of those 9 pieces. So the percentage of cats is $\frac{4}{9}$, which is about 44%.