Solution Manual for Mechanics of Materials 7th Edition
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CHAPTER 2

PROBLEM 2.1

A nylon thread is subjected to a 8.5-N tension force. Knowing that $E = 3.3\text{ GPa}$ and that the length of the thread increases by 1.1%, determine (*a*) the diameter of the thread, (*b*) the stress in the thread.

SOLUTION

(*a*) Strain: $\varepsilon = \dfrac{\delta}{L} = \dfrac{1.1}{100} = 0.011$

Stress: $\sigma = E\varepsilon = (3.3\times 10^9)(0.011) = 36.3\times 10^6\text{ Pa}$

$$\sigma = \frac{P}{A}$$

Area: $A = \dfrac{P}{\sigma} = \dfrac{8.5}{36.3\times 10^6} = 234.16\times 10^{-9}\text{ m}^2$

Diameter: $d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(234.16\times 10^{-9})}{\pi}} = 546\times 10^{-6}\text{ m}$ $\qquad d = 0.546\text{ mm}$ ◀

(*b*) Stress: $\qquad \sigma = 36.3\text{ MPa}$ ◀

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PROBLEM 2.2

A 4.8-ft-long steel wire of $\frac{1}{4}$-in.-diameter is subjected to a 750-lb tensile load. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the elongation of the wire, (*b*) the corresponding normal stress.

SOLUTION

(*a*) Deformation: $\delta = \dfrac{PL}{AE}; \qquad A = \dfrac{\pi d^2}{4}$

Area: $A = \dfrac{\pi(0.25 \text{ in.})^2}{4} = 4.9087 \times 10^{-2} \text{ in}^2$

$$\delta = \frac{(750 \text{ lb})(4.8 \text{ ft} \times 12 \text{ in./ft})}{(4.9087 \times 10^{-2} \text{ in}^2)(29 \times 10^6 \text{ psi})}$$

$\delta = 3.0347 \times 10^{-2}$ in.

$\delta = 0.0303$ in. ◀

(*b*) Stress: $\sigma = \dfrac{P}{A}$

Area: $\sigma = \dfrac{(750 \text{ lb})}{(4.9087 \times 10^{-2} \text{ in}^2)}$

$\sigma = 1.52790 \times 10^4$ psi

$\sigma = 15.28$ ksi ◀

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PROBLEM 2.3

An 18-m-long steel wire of 5-mm diameter is to be used in the manufacture of a prestressed concrete beam. It is observed that the wire stretches 45 mm when a tensile force **P** is applied. Knowing that $E = 200$ GPa, determine (*a*) the magnitude of the force **P**, (*b*) the corresponding normal stress in the wire.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}$, or $P = \dfrac{\delta AE}{L}$

with $A = \dfrac{1}{4}\pi d^2 = \dfrac{1}{4}\pi(0.005)^2 = 19.6350 \times 10^{-6}\ \text{m}^2$

$$P = \frac{(0.045\ \text{m})(19.6350\times10^{-6}\ \text{m}^2)(200\times10^9\ \text{N/m}^2)}{18\ \text{m}} = 9817.5\ \text{N}$$

$P = 9.82$ kN ◀

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{9817.5\ \text{N}}{19.6350\times10^{-6}\ \text{m}^2} = 500\times10^6\ \text{Pa}$

$\sigma = 500$ MPa ◀

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PROBLEM 2.4

Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod with E = 73 GPa and an ultimate strength of 140 MPa. Knowing that the distance between the gage marks is 250.28 mm after a load is applied, determine (*a*) the stress in the rod, (*b*) the factor of safety.

SOLUTION

(*a*)

$$\delta = L - L_0$$

$$= 250.28 \text{ mm} - 250 \text{ mm}$$

$$= 0.28 \text{ mm}$$

$$\varepsilon = \frac{\delta}{L_0}$$

$$= \frac{0.28 \text{ mm}}{250 \text{ mm}}$$

$$= 1.11643 \times 10^{-4}$$

$$\sigma = E\varepsilon$$

$$= (73 \times 10^9 \text{ Pa})(1.11643 \times 10^{-4})$$

$$= 8.1760 \times 10^7 \text{ Pa}$$

$$\sigma = 81.8 \text{ MPa} \blacktriangleleft$$

(*b*)

$$\text{F.S.} = \frac{\sigma_u}{\sigma}$$

$$= \frac{140 \text{ MPa}}{81.760 \text{ MPa}}$$

$$= 1.71233$$

$$\text{F.S.} = 1.712 \blacktriangleleft$$

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PROBLEM 2.5

An aluminum pipe must not stretch more than 0.05 in. when it is subjected to a tensile load. Knowing that $E = 10.1 \times 10^6$ psi and that the maximum allowable normal stress is 14 ksi, determine (*a*) the maximum allowable length of the pipe, (*b*) the required area of the pipe if the tensile load is 127.5 kips.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}$

Thus, $L = \dfrac{EA\delta}{P} = \dfrac{E\delta}{\sigma} = \dfrac{(10.1 \times 10^6)(0.05)}{14 \times 10^3}$

$L = 36.1 \text{ in.}$ ◀

(*b*) $\sigma = \dfrac{P}{A}$

Thus, $A = \dfrac{P}{\sigma} = \dfrac{127.5 \times 10^3}{14 \times 10^3}$ $A = 9.11 \text{ in}^2$ ◀

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PROBLEM 2.6

A control rod made of yellow brass must not stretch more than 3 mm when the tension in the wire is 4 kN. Knowing that E = 105 GPa and that the maximum allowable normal stress is 180 MPa, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding maximum length of the rod.

SOLUTION

(*a*) $\sigma = \dfrac{P}{A}; \qquad A = \dfrac{\pi d^2}{4}$

Substituting, we have

$$\sigma = \frac{P}{\left(\dfrac{\pi d^2}{4}\right)} \quad \Rightarrow \quad d = \sqrt{\frac{4P}{\sigma\pi}}$$

$$d = \sqrt{\frac{4(4 \times 10^3 \text{ N})}{(180 \times 10^6 \text{ Pa})\pi}}$$

$$d = 5.3192 \times 10^{-3} \text{ m}$$

$d = 5.32$ mm ◀

(*b*) $\sigma = E\varepsilon; \qquad \varepsilon = \dfrac{\delta}{L}$

Substituting, we have

$$\sigma = E\frac{\delta}{L} \quad \Rightarrow \quad L = \frac{E\partial}{\sigma}$$

$$L = \frac{(105 \times 10^9 \text{ Pa})(3 \times 10^{-3} \text{ m})}{(180 \times 10^6 \text{ Pa})}$$

$L = 1.750$ m ◀

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PROBLEM 2.7

A steel control rod is 5.5 ft long and must not stretch more than 0.04 in. when a 2-kip tensile load is applied to it. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding normal stress caused by the load.

SOLUTION

(*a*) $\delta = \frac{PL}{AE}: \quad 0.04 \text{ in.} = \frac{(2000 \text{ lb})(5.5 \times 12 \text{ in.})}{A(29 \times 10^6 \text{ psi})}$

$$A = \frac{1}{4}\pi d^2 = 0.113793 \text{ in}^2$$

$$d = 0.38063 \text{ in.}$$

$d = 0.381$ in. ◀

(*b*) $\sigma = \frac{P}{A} = \frac{2000 \text{ lb}}{0.113793 \text{ in}^2} = 17575.8 \text{ psi}$

$\sigma = 17.58$ ksi ◀

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PROBLEM 2.8

A cast-iron tube is used to support a compressive load. Knowing that $E = 10 \times 10^6$ psi and that the maximum allowable change in length is 0.025%, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 1600 lb if the outside diameter of the tube is 2.0 in.

SOLUTION

(*a*) $$\frac{\delta}{L} = \frac{\delta}{100} = 0.00025$$

$$\sigma = E\varepsilon; \qquad \varepsilon = \frac{\delta}{L}$$

$$\therefore \qquad \sigma = E\frac{\delta}{L}$$

$$\sigma = (10 \times 10^6 \text{ psi})(0.00025)$$

$$\sigma = 2.50 \times 10^3 \text{ psi}$$

$$\sigma = 2.50 \text{ ksi} \blacktriangleleft$$

(*b*) $$\sigma = \frac{P}{A}; \qquad \therefore \qquad A = \frac{P}{\sigma} = \frac{1600 \text{ lb}}{2.50 \times 10^3 \text{ psi}} = 0.64 \text{ in}^2$$

$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right)$$

$$d_i^2 = d_o^2 - \frac{4A}{\pi}$$

$$d_i^2 = (2.0 \text{ in.})^2 - \frac{4(0.64 \text{ in}^2)}{\pi} = 3.1851 \text{ in}^2$$

$$\therefore \qquad d_i = 1.78469 \text{ in.}$$

$$t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(2.0 \text{ in.} - 1.78469 \text{ in.})$$

$$t = 0.107655 \text{ in.}$$

$$t = 0.1077 \blacktriangleleft$$

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PROBLEM 2.9

A 4-m-long steel rod must not stretch more than 3 mm and the normal stress must not exceed 150 MPa when the rod is subjected to a 10-kN axial load. Knowing that $E = 200\text{ GPa}$, determine the required diameter of the rod.

SOLUTION

$L = 4\text{ m}$

$\delta = 3 \times 10^{-3}\text{ m}, \quad \sigma = 150 \times 10^{6}\text{ Pa}$

$E = 200 \times 10^{9}\text{ Pa}, \quad P = 10 \times 10^{3}\text{ N}$

Stress: $$\sigma = \frac{P}{A}$$

$$A = \frac{P}{\sigma} = \frac{10 \times 10^{3}\text{ N}}{150 \times 10^{6}\text{ Pa}} = 66.667 \times 10^{-6}\text{m}^2 = 66.667\text{ mm}^2$$

Deformation: $$\delta = \frac{PL}{AE}$$

$$A = \frac{PL}{E\delta} = \frac{(10 \times 10^{3})(4)}{(200 \times 10^{9})(3 \times 10^{-3})} = 66.667 \times 10^{-6}\text{m}^2 = 66.667\text{ mm}^2$$

The larger value of A governs: $A = 66.667\text{ mm}^2$

$$A = \frac{\pi}{4} d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(66.667)}{\pi}}$$

$d = 9.21\text{ mm}$ ◀

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PROBLEM 2.10

A nylon thread is to be subjected to a 10-N tension. Knowing that $E = 3.2$ GPa, that the maximum allowable normal stress is 40 MPa, and that the length of the thread must not increase by more than 1%, determine the required diameter of the thread.

SOLUTION

Stress criterion:

$$\sigma = 40 \text{ MPa} = 40 \times 10^6 \text{ Pa} \qquad P = 10 \text{ N}$$

$$\sigma = \frac{P}{A}: \quad A = \frac{P}{\sigma} = \frac{10 \text{ N}}{40 \times 10^6 \text{ Pa}} = 250 \times 10^{-9} \text{m}^2$$

$$A = \frac{\pi}{4} d^2: \quad d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{250 \times 10^{-9}}{\pi}} = 564.19 \times 10^{-6} \text{m}$$

$$d = 0.564 \text{ mm}$$

Elongation criterion:

$$\frac{\delta}{L} = 1\% = 0.01$$

$$\delta = \frac{PL}{AE}:$$

$$A = \frac{P/E}{\delta/L} = \frac{10 \text{ N}/3.2 \times 10^9 \text{ Pa}}{0.01} = 312.5 \times 10^{-9} \text{ m}^2$$

$$d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{312.5 \times 10^{-9}}{\pi}} = 630.78 \times 10^{-6} \text{m}^2$$

$$d = 0.631 \text{ mm}$$

The required diameter is the larger value: $d = 0.631$ mm ◀

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PROBLEM 2.11

A block of 10-in. length and 1.8 × 1.6-in. cross section is to support a centric compressive load **P**. The material to be used is a bronze for which $E = 14 \times 10^6$ psi. Determine the largest load that can be applied, knowing that the normal stress must not exceed 18 ksi and that the decrease in length of the block should be at most 0.12% of its original length.

SOLUTION

Considering allowable stress, $\sigma = 18$ ksi or 18×10^3 psi

Cross-sectional area: $A = (1.8 \text{ in.})(1.6 \text{ in.}) = 2.880 \text{ in}^2$

$$\sigma = \frac{P}{A} \quad \Rightarrow \quad P = \sigma A$$

$$= (18 \times 10^3 \text{psi})(2.880 \text{ in}^2)$$

$$= 5.1840 \times 10^4 \text{ lb}$$

or 51.840 kips

Considering allowable deformation, $\frac{\delta}{L} = 0.12\%$ or 0.0012 in.

$$\delta = \frac{PL}{AE} \quad \Rightarrow \quad P = AE\left(\frac{\delta}{L}\right)$$

$$P = (2.880 \text{ in}^2)(14 \times 10^6 \text{ psi})(0.0012 \text{ in.})$$

$$= 4.8384 \times 10^4 \text{ lb}$$

or 48.384 kips

The smaller value for **P** resulting from the required deformation criteria governs.

48.4 kips ◀

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PROBLEM 2.12

A square yellow-brass bar must not stretch more than 2.5 mm when it is subjected to a tensile load. Knowing that $E = 105$ GPa and that the allowable tensile strength is 180 MPa, determine (*a*) the maximum allowable length of the bar, (*b*) the required dimensions of the cross section if the tensile load is 40 kN.

SOLUTION

$\sigma = 180\times10^6$ Pa $\quad P = 40\times10^3$ N

$E = 105\times10^9$ Pa $\quad \delta = 2.5\times10^{-3}$ m

(*a*) $$\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

$$L = \frac{E\delta}{\sigma} = \frac{(105\times10^9)(2.5\times10^{-3})}{180\times10^6} = 1.45833 \text{ m}$$

$L = 1.458$ m ◀

(*b*) $$\sigma = \frac{P}{A}$$

$$A = \frac{P}{\sigma} = \frac{40\times10^3}{180\times10^6} = 222.22\times10^{-6}\text{ m}^2 = 222.22 \text{ mm}^2$$

$$A = a^2 \quad a = \sqrt{A} = \sqrt{222.22}$$

$a = 14.91$ mm ◀

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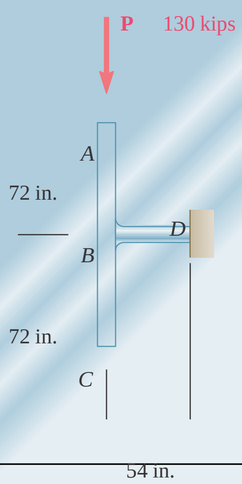
PROBLEM 2.13



Rod BD is made of steel $(E = 29\times10^6 \text{ psi})$ and is used to brace the axially compressed member ABC. The maximum force that can be developed in member BD is $0.02P$. If the stress must not exceed 18 ksi and the maximum change in length of BD must not exceed 0.001 times the length of ABC, determine the smallest-diameter rod that can be used for member BD.

SOLUTION

$$F_{BD} = 0.02P = (0.02)(130) = 2.6 \text{ kips} = 2.6\times10^3 \text{ lb}$$

Considering stress, $\sigma = 18 \text{ ksi} = 18\times10^3 \text{ psi}$

$$\sigma = \frac{F_{BD}}{A} \qquad \therefore \quad A = \frac{F_{BD}}{\sigma} = \frac{2.6}{18} = 0.14444 \text{ in}^2$$

Considering deformation, $\delta = (0.001)(144) = 0.144 \text{ in.}$

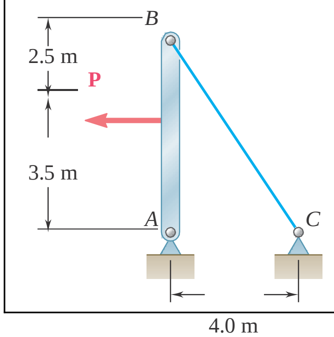
$$\delta = \frac{F_{BD}L_{BD}}{AE} \qquad \therefore \quad A = \frac{F_{BD}L_{BD}}{E\delta} = \frac{(2.6\times10^3)(54)}{(29\times10^6)(0.144)} = 0.03362 \text{ in}^2$$

Larger area governs. $A = 0.14444 \text{ in}^2$

$$A = \frac{\pi}{4}d^2 \qquad \therefore \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.14444)}{\pi}}$$

$d = 0.429$ in. ◀

PROBLEM 2.14

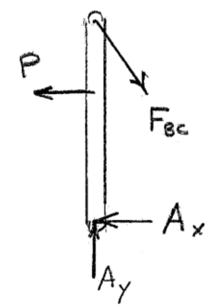


The 4-mm-diameter cable *BC* is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.

SOLUTION

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

Use bar *AB* as a free body.



$$+\circlearrowleft \Sigma M_A = 0: \quad 3.5P - (6)\left(\frac{4}{7.2111}F_{BC}\right) = 0$$

$$P = 0.9509 F_{BC}$$

Considering allowable stress, $\sigma = 190 \times 10^6$ Pa

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_{BC}}{A} \quad \therefore \quad F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$$

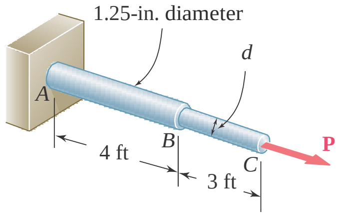
Considering allowable elongation, $\delta = 6 \times 10^{-3}$ m

$$\delta = \frac{F_{BC} L_{BC}}{AE} \quad \therefore \quad F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$$

Smaller value governs. $F_{BC} = 2.091 \times 10^3$ N

$$P = 0.9509 F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N}$$

$P = 1.988$ kN ◀



PROBLEM 2.15

A single axial load of magnitude $P = 15$ kips is applied at end C of the steel rod ABC. Knowing that $E = 30 \times 10^6$ psi, determine the diameter d of portion BC for which the deflection of point C will be 0.05 in.

SOLUTION

$$\delta_C = \sum \frac{PL_i}{A_i E_i} = \left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC}$$

$$L_{AB} = 4 \text{ ft} = 48 \text{ in.}; \qquad L_{BC} = 3 \text{ ft} = 36 \text{ in.}$$

$$A_{AB} = \frac{\pi d^2}{4} = \frac{\pi(1.25 \text{ in.})^2}{4} = 1.22718 \text{ in}^2$$

Substituting, we have

$$0.05 \text{ in.} = \left(\frac{15 \times 10^3 \text{ lb}}{30 \times 10^6 \text{psi}}\right)\left(\frac{48 \text{ in.}}{1.22718 \text{ in}^2} + \frac{36 \text{ in.}}{A_{BC}}\right)$$

$$A_{BC} = 0.59127 \text{ in}^2$$

$$A_{BC} = \frac{\pi d^2}{4}$$

or $$d = \sqrt{\frac{4A_{BC}}{\pi}}$$

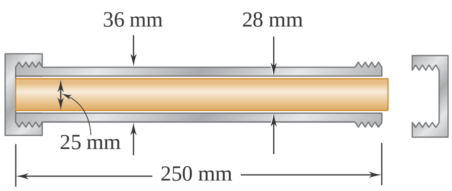
$$d = \sqrt{\frac{4(0.59127 \text{ in}^2)}{\pi}}$$

$$d = 0.86766 \text{ in.}$$

$d = 0.868$ in. ◀

PROBLEM 2.16

A 250-mm-long aluminum tube $(E = 70\text{ GPa})$ of 36-mm outer diameter and 28-mm inner diameter can be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod $(E = 105\text{ GPa})$ of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.



SOLUTION

$$A_{\text{tube}} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12\text{ mm}^2 = 402.12\times10^{-6}\text{ m}^2$$

$$A_{\text{rod}} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(25)^2 = 490.87\text{ mm}^2 = 490.87\times10^{-6}\text{ m}^2$$

$$\delta_{\text{tube}} = \frac{PL}{E_{\text{tube}}A_{\text{tube}}} = \frac{P(0.250)}{(70\times10^9)(402.12\times10^{-6})} = 8.8815\times10^{-9}P$$

$$\delta_{\text{rod}} = -\frac{PL}{E_{\text{rod}}A_{\text{rod}}} = \frac{P(0.250)}{(105\times10^6)(490.87\times10^{-6})} = -4.8505\times10^{-9}P$$

$$\delta^* = \left(\frac{1}{4}\text{ turn}\right)\times1.5\text{ mm} = 0.375\text{ mm} = 375\times10^{-6}\text{ m}$$

$$\delta_{\text{tube}} = \delta^* + \delta_{\text{rod}} \quad\text{or}\quad \delta_{\text{tube}} - \delta_{\text{rod}} = \delta^*$$

$$8.8815\times10^{-9}P + 4.8505\times10^{-9}P = 375\times10^{-6}$$

$$P = \frac{0.375\times10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308\times10^3\text{ N}$$

(*a*) $\sigma_{\text{tube}} = \dfrac{P}{A_{\text{tube}}} = \dfrac{27.308\times10^3}{402.12\times10^{-6}} = 67.9\times10^6\text{ Pa}$ $\qquad \sigma_{\text{tube}} = 67.9\text{ MPa}$ ◀

$\sigma_{\text{rod}} = -\dfrac{P}{A_{\text{rod}}} = -\dfrac{27.308\times10^3}{490.87\times10^{-6}} = -55.6\times10^6\text{ Pa}$ $\qquad \sigma_{\text{rod}} = -55.6\text{ MPa}$ ◀

(*b*)

$\delta_{\text{tube}} = (8.8815\times10^{-9})(27.308\times10^3) = 242.5\times10^{-6}\text{ m}$ $\qquad \delta_{\text{tube}} = 0.243\text{ mm}$ ◀

$$\delta_{\text{rod}} = -(4.8505\times10^{-9})(27.308\times10^{3}) = -132.5\times10^{-6}\ \text{m} \qquad \delta_{\text{rod}} = -0.1325\ \text{mm} \blacktriangleleft$$

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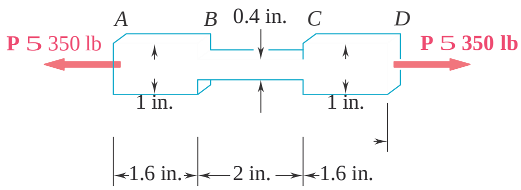

PROBLEM 2.17

The specimen shown has been cut from a $\frac{1}{4}$-in.-thick sheet of vinyl (E = 0.45 × 10^6 psi) and is subjected to a 350-lb tensile load. Determine (*a*) the total deformation of the specimen, (*b*) the deformation of its central portion *BC*.

SOLUTION

$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(350\text{ lb})(1.6\text{ in.})}{(0.45\times 10^6\text{ psi})(1\text{ in.})(0.25\text{ in.})} = 4.9778\times 10^{-3}\text{ in.}$$

$$\delta_{BC} = \frac{PL_{BC}}{EA_{BC}} = \frac{(350\text{ lb})(2\text{ in.})}{(0.45\times 10^6\text{ psi})(0.4\text{ in.})(0.25\text{ in.})} = 15.5556\times 10^{-3}\text{ in.}$$

$$\delta_{CD} = \delta_{AB} = 4.9778\times 10^{-3}\text{ in.}$$

(*a*) Total deformation:

$$\delta = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$\delta = 25.511\times 10^{-3}\text{ in.}$$

$$\delta = 25.5\times 10^{-3}\text{ in.} \blacktriangleleft$$

(*b*) Deformation of portion *BC* :

$$\delta_{BC} = 15.56\times 10^{-3}\text{ in.} \blacktriangleleft$$

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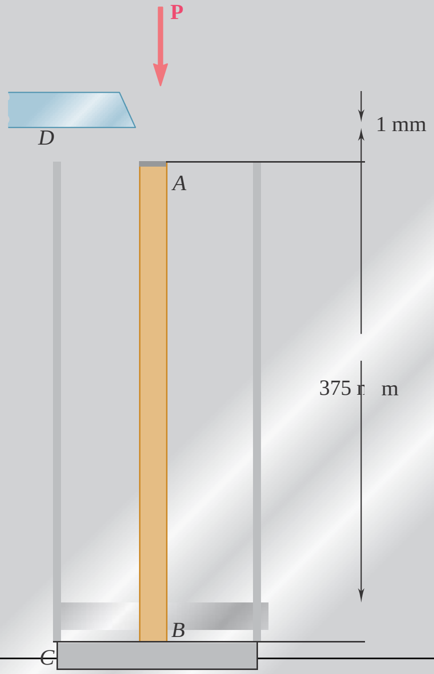
PROBLEM 2.18



The brass tube AB ($E = 105$ GPa) has a cross-sectional area of 140 mm^2 and is fitted with a plug at A. The tube is attached at B to a rigid plate that is itself attached at C to the bottom of an aluminum cylinder ($E = 72$ GPa) with a cross-sectional area of 250 mm^2. The cylinder is then hung from a support at D. In order to close the cylinder, the plug must move down through 1 mm. Determine the force **P** that must be applied to the cylinder.

SOLUTION

Shortening of brass tube AB:

$$L_{AB} = 375 + 1 = 376 \text{ mm} = 0.376 \text{ m} \quad A_{AB} = 140 \text{ mm}^2 = 140 \times 10^{-6} \text{ m}^2$$

$$E_{AB} = 105 \times 10^9 \text{ Pa}$$

$$\delta_{AB} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{P(0.376)}{(105 \times 10^9)(140 \times 10^{-6})} = 25.578 \times 10^{-9} P$$

Lengthening of aluminum cylinder CD:

$$L_{CD} = 0.375 \text{ m} \quad A_{CD} = 250 \text{ mm}^2 = 250 \times 10^{-6} \text{ m}^2 \quad E_{CD} = 72 \times 10^9 \text{ Pa}$$

$$\delta_{CD} = \frac{PL_{CD}}{E_{CD}A_{CD}} = \frac{P(0.375)}{(72 \times 10^9)(250 \times 10^{-6})} = 20.833 \times 10^{-9} P$$

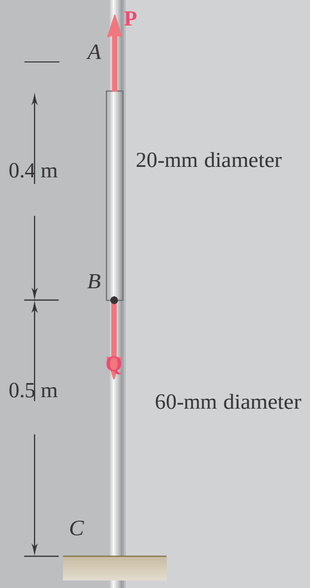
Total deflection: $\delta_A = \delta_{AB} + \delta_{CD}$ where $\delta_A = 0.001$ m

$$0.001 = (25.578 \times 10^{-9} + 20.833 \times 10^{-9})P$$

$$P = 21.547 \times 10^3 \text{ N}$$

$P = 21.5$ kN ◀

PROBLEM 2.19



Both portions of the rod *ABC* are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.

SOLUTION

(*a*) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\,\text{m}^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\,\text{m}^2$

Force in member *AB* is *P* tension.

Elongation:

$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(4\times 10^3)(0.4)}{(70\times 10^9)(314.16\times 10^{-6})} = 72.756\times 10^{-6}\,\text{m}$$

Force in member *BC* is $Q - P$ compression.

Shortening:

$$\delta_{BC} = \frac{(Q-P)L_{BC}}{EA_{BC}} = \frac{(Q-P)(0.5)}{(70\times 10^9)(2.8274\times 10^{-3})} = 2.5263\times 10^{-9}(Q-P)$$

For zero deflection at *A*, $\delta_{BC} = \delta_{AB}$

$$2.5263\times 10^{-9}(Q-P) = 72.756\times 10^{-6} \quad \therefore \quad Q - P = 28.8\times 10^3\ \text{N}$$

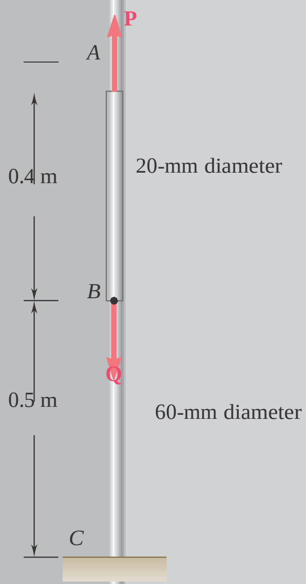
$$Q = 28.3\times 10^3 + 4\times 10^3 = 32.8\times 10^3\ \text{N}$$

$Q = 32.8$ kN ◀

(*b*) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756\times 10^{-6}\,\text{m}$

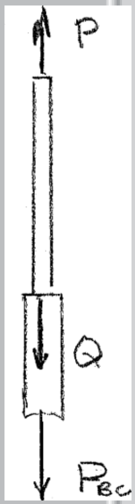
$\delta_{AB} = 0.0728$ mm ↓ ◀

PROBLEM 2.20



The rod ABC is made of an aluminum for which $E = 70$ GPa. Knowing that $\mathbf{P} = 6$ kN and $\mathbf{Q} = 42$ kN, determine the deflection of (a) point A, (b) point B.

SOLUTION



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\ \text{m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\ \text{m}^2$$

$$P_{AB} = P = 6 \times 10^3\ \text{N}$$

$$P_{BC} = P - Q = 6 \times 10^3 - 42 \times 10^3 = -36 \times 10^3\ \text{N}$$

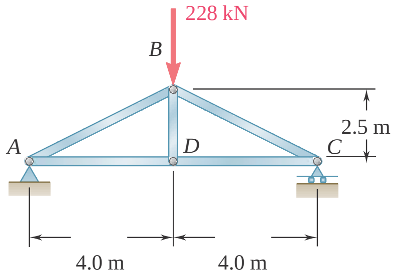
$$L_{AB} = 0.4\ \text{m} \quad L_{BC} = 0.5\ \text{m}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E_A} = \frac{(6 \times 10^3)(0.4)}{(314.16 \times 10^{-6})(70 \times 10^9)} = 109.135 \times 10^{-6}\ \text{m}$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36 \times 10^3)(0.5)}{(2.8274 \times 10^{-3})(70 \times 10^9)} = -90.947 \times 10^{-6}\ \text{m}$$

(a) $\delta_A = \delta_{AB} + \delta_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6}\ \text{m} = 18.19 \times 10^{-6}\ \text{m}$ $\quad\quad \delta_A = 0.01819\ \text{mm} \uparrow$ ◀

(b) $\delta_B = \delta_{BC} = -90.9 \times 10^{-6}\ \text{m} = -0.0909\ \text{mm}$ $\quad$ or $\quad \delta_B = 0.0909\ \text{mm} \downarrow$ ◀



PROBLEM 2.21

For the steel truss $(E = 200\text{ GPa})$ and loading shown, determine the deformations of the members AB and AD, knowing that their cross-sectional areas are 2400 mm^2 and 1800 mm^2, respectively.

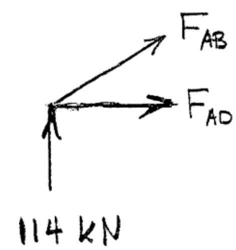
SOLUTION

Statics: Reactions are 114 kN upward at A and C.

Member BD is a zero force member.

$$L_{AB} = \sqrt{4.0^2 + 2.5^2} = 4.717\text{ m}$$

Use joint A as a free body.



$$+\uparrow \Sigma F_y = 0: 114 + \frac{2.5}{4.717} F_{AB} = 0$$

$$F_{AB} = -215.10\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0: F_{AD} + \frac{4}{4.717} F_{AB} = 0$$

$$F_{AD} = -\frac{(4)(-215.10)}{4.717} = 182.4\text{ kN}$$

Member AB:

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{EA_{AB}} = \frac{(-215.10\times 10^3)(4.717)}{(200\times 10^9)(2400\times 10^{-6})}$$

$$= -2.11\times 10^{-3}\text{ m}$$

$\delta_{AB} = -2.11\text{ mm}$ ◀

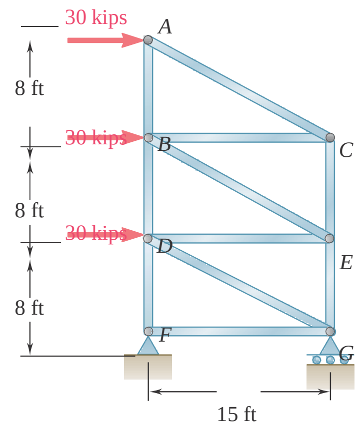
Member AD:

$$\delta_{AD} = \frac{F_{AD} L_{AD}}{EA_{AD}} = \frac{(182.4\times 10^3)(4.0)}{(200\times 10^9)(1800\times 10^{-6})}$$

$$= 2.03\times 10^{-3}\text{ m}$$

$\delta_{AD} = 2.03\text{ mm}$ ◀

114

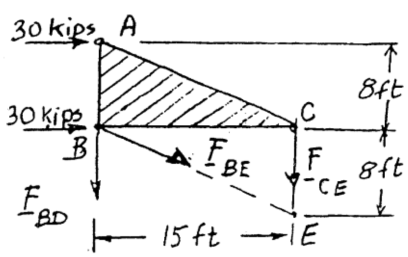


PROBLEM 2.22

For the steel truss $(E = 29\times10^{6}\,\text{psi})$ and loading shown, determine the deformations of the members *BD* and *DE*, knowing that their cross-sectional areas are 2 in^2 and 3 in^2, respectively.

SOLUTION

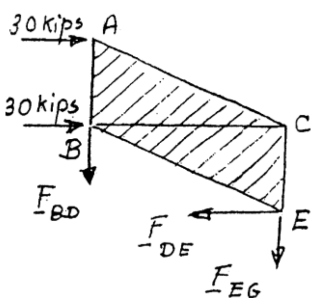
Free body: Portion *ABC* of truss



$$\circlearrowleft\!+\ \Sigma M_E = 0:\ F_{BD}(15\text{ ft}) - (30\text{ kips})(8\text{ ft}) - (30\text{ kips})(16\text{ ft}) = 0$$

$$F_{BD} = +48.0\text{ kips}$$

Free body: Portion *ABEC* of truss



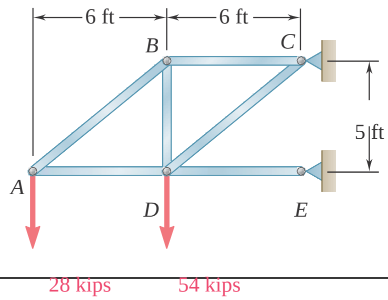
$$\xrightarrow{+}\ \Sigma F_x = 0:\ 30\text{ kips} + 30\text{ kips} - F_{DE} = 0$$

$$F_{DE} = +60.0\text{ kips}$$

$$\delta_{BD} = \frac{PL}{AE} = \frac{(+48.0\times10^{3}\text{ lb})(8\times12\text{ in.})}{(2\text{ in}^2)(29\times10^{6}\text{ psi})} \qquad \delta_{BD} = +79.4\times10^{-3}\text{ in.} \blacktriangleleft$$

$$\delta_{DE} = \frac{PL}{AE} = \frac{(+60.0\times10^{3}\text{ lb})(15\times12\text{ in.})}{(3\text{ in}^2)(29\times10^{6}\text{ psi})} \qquad \delta_{DE} = +124.1\times10^{-3}\text{ in.} \blacktriangleleft$$

 115



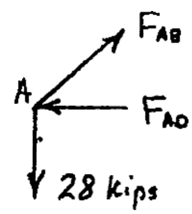
PROBLEM 2.23

Members *AB* and *BC* are made of steel $(E = 29 \times 10^6 \text{psi})$ with cross-sectional areas of 0.80 in^2 and 0.64 in^2, respectively. For the loading shown, determine the elongation of (*a*) member *AB*, (*b*) member *BC*.

SOLUTION

(*a*) $L_{AB} = \sqrt{6^2 + 5^2} = 7.810 \text{ ft} = 93.72 \text{ in.}$

Use joint *A* as a free body.



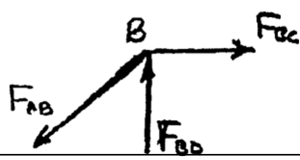
$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{7.810} F_{AB} - 28 = 0$$

$$F_{AB} = 43.74 \text{ kip} = 43.74 \times 10^3 \text{lb}$$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{EA_{AB}} = \frac{(43.74 \times 10^3)(93.72)}{(29 \times 10^6)(0.80)}$$

$\delta_{AB} = 0.1767$ in. ◀

(*b*) Use joint *B* as a free body.



$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad F_{BC} - \frac{6}{7.810} F_{AB} = 0$$

$$F_{BC} = \frac{(6)(43.74)}{7.810} = 33.60 \text{ kip} = 33.60 \times 10^3 \text{ lb}$$

$$\delta_{BC} = \frac{F_{BC} L_{BC}}{EA_{BC}} = \frac{(33.60 \times 10^3)(72)}{(29 \times 10^6)(0.64)}$$

$\delta_{BC} = 0.1304$ in. ◀

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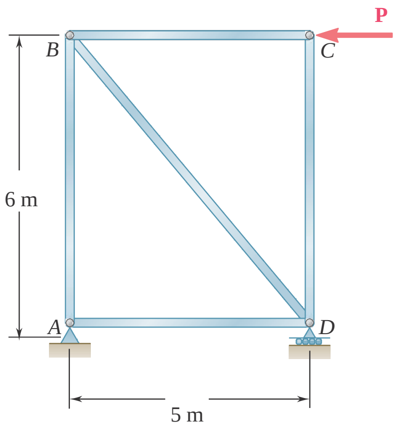


PROBLEM 2.24

The steel frame $(E = 200\text{ GPa})$ shown has a diagonal brace *BD* with an area of 1920 mm^2. Determine the largest allowable load **P** if the change in length of member *BD* is not to exceed 1.6 mm.

SOLUTION

$$\delta_{BD} = 1.6 \times 10^{-3}\text{ m}, \quad A_{BD} = 1920\text{ mm}^2 = 1920 \times 10^{-6}\text{ m}^2$$

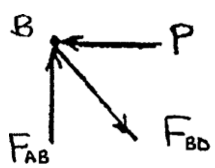
$$L_{BD} = \sqrt{5^2 + 6^2} = 7.810\text{ m}, \quad E_{BD} = 200 \times 10^9\text{ Pa}$$

$$\delta_{BD} = \frac{F_{BD} L_{BD}}{E_{BD} A_{BD}}$$

$$F_{BD} = \frac{E_{BD} A_{BD} \delta_{BD}}{L_{BD}} = \frac{(200 \times 10^9)(1920 \times 10^{-6})(1.6 \times 10^{-3})}{7.81}$$

$$= 78.67 \times 10^3\text{ N}$$

Use joint *B* as a free body. $\xrightarrow{+} \Sigma F_x = 0$:



$$\frac{5}{7.810} F_{BD} - P = 0$$

$$P = \frac{5}{7.810} F_{BD} = \frac{(5)(78.67 \times 10^3)}{7.810}$$

$$= 50.4 \times 10^3\text{ N}$$

$P = 50.4\text{ kN}$ ◀

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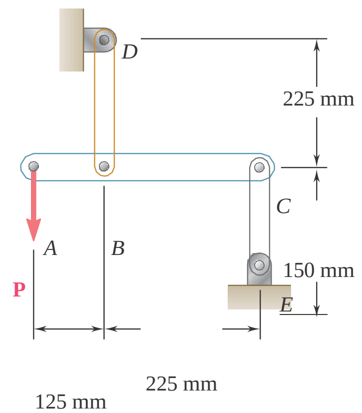
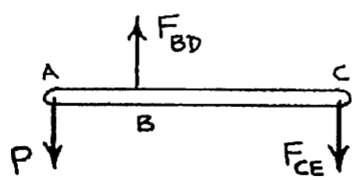


PROBLEM 2.25

Link BD is made of brass $(E = 105 \text{ GPa})$ and has a cross-sectional area of 240 mm^2. Link CE is made of aluminum $(E = 72 \text{ GPa})$ and has a cross-sectional area of 300 mm^2. Knowing that they support rigid member ABC, determine the maximum force **P** that can be applied vertically at point A if the deflection of A is not to exceed 0.35 mm.

SOLUTION

Free body member AC:



$$+\circlearrowleft \Sigma M_C = 0: \; 0.350P - 0.225F_{BD} = 0$$

$$F_{BD} = 1.55556P$$

$$+\circlearrowleft \Sigma M_B = 0: \; 0.125P - 0.225F_{CE} = 0$$

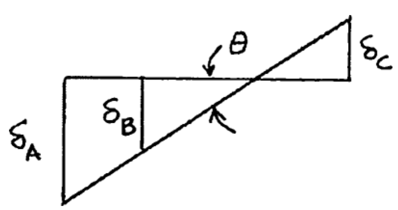
$$F_{CE} = 0.55556P$$

$$\delta_B = \delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}} = \frac{(1.55556P)(0.225)}{(105\times 10^9)(240\times 10^{-6})} = 13.8889\times 10^{-9}P$$

$$\delta_C = \delta_{CE} = \frac{F_{CE}L_{CE}}{E_{CE}A_{CE}} = \frac{(0.55556P)(0.150)}{(72\times 10^9)(300\times 10^{-6})} = 3.8581\times 10^{-9}P$$

Deformation Diagram:

From the deformation diagram,



Slope:
$$\theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{17.7470\times 10^{-9}P}{0.225} = 78.876\times 10^{-9}P$$

$$\delta_A = \delta_B + L_{AB}\theta$$

$$= 13.8889\times 10^{-9}P + (0.125)(78.876\times 10^{-9}P)$$

$$= 23.748\times 10^{-9}P$$

Apply displacement limit. $\delta_A = 0.35\times 10^{-3} \text{ m} = 23.748\times 10^{-9}P$

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$$P = 14.7381 \times 10^3 \text{ N}$$

$P = 14.74$ kN ◀

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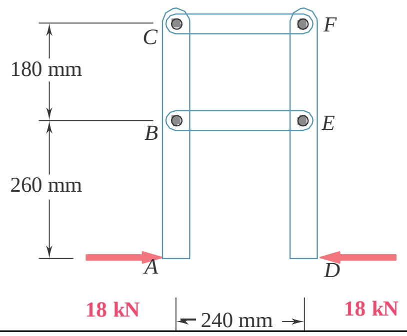
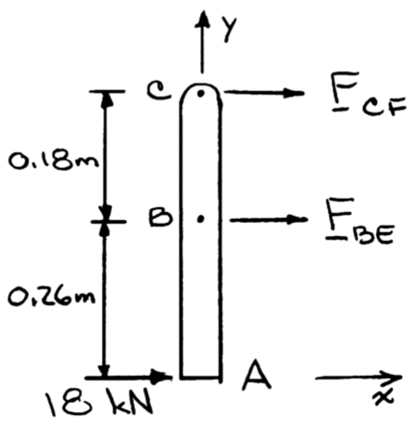


PROBLEM 2.26

Members *ABC* and *DEF* are joined with steel links (E = 200 GPa). Each of the links is made of a pair of 25 × 35-mm plates. Determine the change in length of (*a*) member *BE*, (*b*) member *CF*.

SOLUTION

Free body diagram of Member *ABC*:



$$+\circlearrowleft \Sigma M_B = 0:$$

$$(0.26\text{ m})(18\text{ kN}) - (0.18\text{ m})F_{CF} = 0$$

$$F_{CF} = 26.0\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0:$$

$$18\text{ kN} + F_{BE} + 26.0\text{ kN} = 0$$

$$F_{BE} = -44.0\text{ kN}$$

Area for link made of two plates:

$$A = 2(0.025\text{ m})(0.035\text{ m}) = 1.750 \times 10^{-3}\text{ m}^2$$

(*a*) $$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-44.0 \times 10^3\text{ N})(0.240\text{ m})}{(200 \times 10^9\text{ Pa})(1.75 \times 10^{-3}\text{ m}^2)}$$

$$= -30.171 \times 10^{-6}\text{ m}$$

$$\delta_{BE} = -0.0302\text{ mm} \blacktriangleleft$$

(*b*) $$\delta_{CF} = \frac{F_{BF}L}{EA} = \frac{(26.0 \times 10^3\text{ N})(0.240\text{ m})}{(200 \times 10^9\text{ Pa})(1.75 \times 10^{-3}\text{ m}^2)}$$

$$= 17.8286 \times 10^{-6}\text{ m}$$

$$\delta_{CF} = 0.01783\text{ mm} \blacktriangleleft$$

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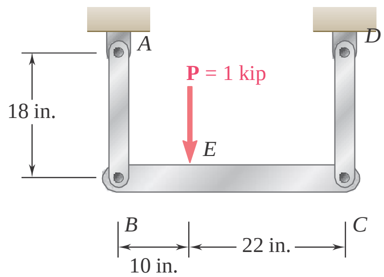
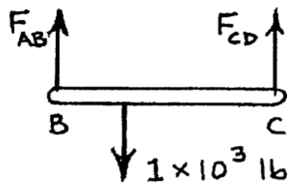


PROBLEM 2.27

Each of the links AB and CD is made of aluminum ($E = 10.9 \times 10^6$ psi) and has a cross-sectional area of 0.2 in^2. Knowing that they support the rigid member BC, determine the deflection of point E.

SOLUTION

Free body BC:



$$+\circlearrowleft \Sigma M_C = 0: \ -(32)F_{AB} + (22)(1\times 10^3) = 0$$

$$F_{AB} = 687.5 \text{ lb}$$

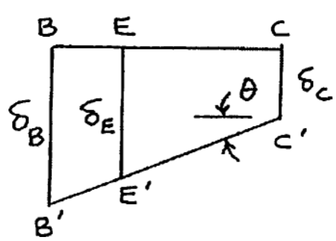
$$+\uparrow \Sigma F_y = 0: \ 687.5 - 1\times 10^3 + F_{CD} = 0$$

$$F_{CD} = 312.5 \text{ lb}$$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{EA} = \frac{(687.5)(18)}{(10.9\times 10^6)(0.2)} = 5.6766\times 10^{-3} \text{ in.} = \delta_B$$

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA} = \frac{(312.5)(18)}{(10.9\times 10^6)(0.2)} = 2.5803\times 10^{-3} \text{ in.} = \delta_C$$

Deformation diagram:



$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{L_{BC}} = \frac{3.0963\times 10^{-3}}{32}$$

$$= 96.759\times 10^{-6} \text{ rad}$$

$$\delta_E = \delta_C + L_{EC}\theta$$

$$= 2.5803\times 10^{-3} + (22)(96.759\times 10^{-6})$$

$$= 4.7090\times 10^{-3} \text{ in.}$$

$$\delta_E = 4.71\times 10^{-3} \text{ in.} \downarrow \blacktriangleleft$$

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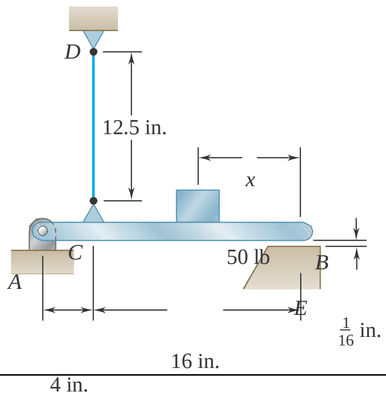


PROBLEM 2.28

The length of the $\frac{3}{32}$-in.-diameter steel wire *CD* has been adjusted so that with no load applied, a gap of $\frac{1}{16}$ in. exists between the end *B* of the rigid beam *ACB* and a contact point *E*. Knowing that $E = 29 \times 10^6$ psi, determine where a 50-lb block should be placed on the beam in order to cause contact between *B* and *E*.

SOLUTION

Rigid beam *ACB* rotates through angle θ to close gap.

$$\theta = \frac{1/16}{20} = 3.125 \times 10^{-3} \text{ rad}$$

Point *C* moves downward.

$$\delta_C = 4\theta = 4(3.125 \times 10^{-3}) = 12.5 \times 10^{-3} \text{ in.}$$

$$\delta_{CD} = \delta_C = 12.5 \times 10^{-3} \text{ in.}$$

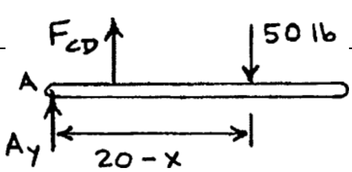
$$A_{CD} = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{3}{32}\right)^2 = 6.9029 \times 10^{-3} \text{ in}^2$$

$$\delta_{CD} = \frac{F_{CD} L_{CD}}{E A_{CD}}$$

$$F_{CD} = \frac{E A_{CD} \delta_{CD}}{L_{CD}} = \frac{(29 \times 10^6)(6.9029 \times 10^{-3})(12.5 \times 10^{-3})}{12.5}$$

$$= 200.18 \text{ lb}$$

Free body *ACB*:



$$+\circlearrowleft \Sigma M_A = 0: \quad 4F_{CD} - (50)(20 - x) = 0$$

$$20 - x = \frac{(4)(200.18)}{50} = 16.0144$$

$$x = 3.9856 \text{ in.}$$

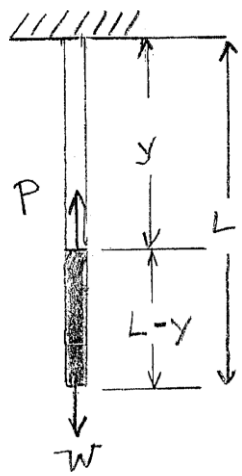
For contact, $x < 3.99$ in. ◀

PROBLEM 2.29

A homogeneous cable of length L and uniform cross section is suspended from one end. (*a*) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (*b*) Show that the same elongation would be obtained if the cable were horizontal and if a force equal to half of its weight were applied at each end.

SOLUTION

(*a*) For element at point identified by coordinate y,



P = weight of portion below the point

$$= \rho g A(L - y)$$

$$d\delta = \frac{P\,dy}{EA} = \frac{\rho g A(L-y)\,dy}{EA} = \frac{\rho g(L-y)}{E}\,dy$$

$$\delta = \int_0^L \frac{\rho g(L-y)}{E}\,dy = \frac{\rho g}{E}\left(Ly - \frac{1}{2}y^2\right)\Bigg|_0^L$$

$$= \frac{\rho g}{E}\left(L^2 - \frac{L^2}{2}\right)$$

$$\delta = \frac{1}{2}\frac{\rho g L^2}{E} \blacktriangleleft$$

(*b*) Total weight: $W = \rho g A L$

$$F = \frac{EA\delta}{L} = \frac{EA}{L}\cdot\frac{1}{2}\frac{\rho g L^2}{E} = \frac{1}{2}\rho g A L$$

$$F = \frac{1}{2}W \blacktriangleleft$$

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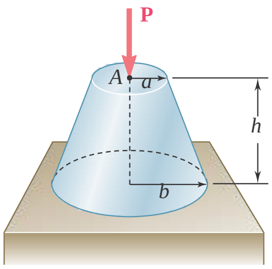
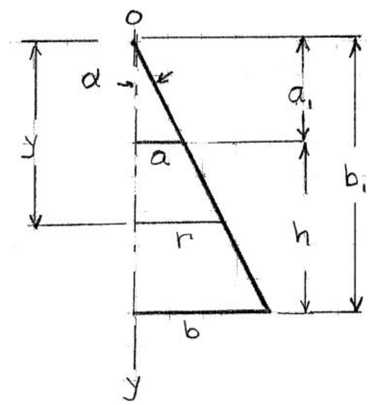


PROBLEM 2.30

A vertical load **P** is applied at the center *A* of the upper section of a homogeneous frustum of a circular cone of height *h*, minimum radius *a*, and maximum radius *b*. Denoting by *E* the modulus of elasticity of the material and neglecting the effect of its weight, determine the deflection of point *A*.

SOLUTION

Extend the slant sides of the cone to meet at a point *O* and place the origin of the coordinate system there.



From geometry, $\tan\alpha = \dfrac{b-a}{h}$

$$a_1 = \frac{a}{\tan\alpha}, \quad b_1 = \frac{b}{\tan\alpha}, \quad r = y\tan\alpha$$

At coordinate point y, $A = \pi r^2$

Deformation of element of height dy: $d\delta = \dfrac{Pdy}{AE}$

$$d\delta = \frac{P}{E\pi}\frac{dy}{r^2} = \frac{P}{\pi E\tan^2\alpha}\frac{dy}{y^2}$$

Total deformation:

$$\delta_A = \frac{P}{\pi E\tan^2\alpha}\int_{a_1}^{b_1}\frac{dy}{y^2} = \frac{P}{\pi E\tan^2\alpha}\left(-\frac{1}{y}\right)\Bigg|_{a_1}^{b_1} = \frac{P}{\pi E\tan^2\alpha}\left(\frac{1}{a_1}-\frac{1}{b_1}\right)$$

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$$= \frac{P}{\pi E \tan^2 \alpha} \frac{b_1 - a_1}{a_1 b_1} = \frac{P(b_1 - a_1)}{\pi E a b}$$

$$\delta_A = \frac{Ph}{\pi E a b} \downarrow \blacktriangleleft$$

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PROBLEM 2.31

Denoting by ε the "engineering strain" in a tensile specimen, show that the true strain is $\varepsilon_t = \ln(1+\varepsilon)$.

SOLUTION

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\frac{L_0+\delta}{L_0} = \ln\left(1+\frac{\delta}{L_0}\right) = \ln(1+\varepsilon)$$

Thus, $\varepsilon_t = \ln(1+\varepsilon)$ ◀

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PROBLEM 2.32

The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is d_1, show that when the diameter is d, the true strain is $\varepsilon_t = 2\ln(d_1/d)$.

SOLUTION

If the volume is constant, $\frac{\pi}{4}d^2L = \frac{\pi}{4}d_1^2L_0$

$$\frac{L}{L_0} = \frac{d_1^2}{d^2} = \left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = 2\ln\frac{d_1}{d} \quad ◀$$

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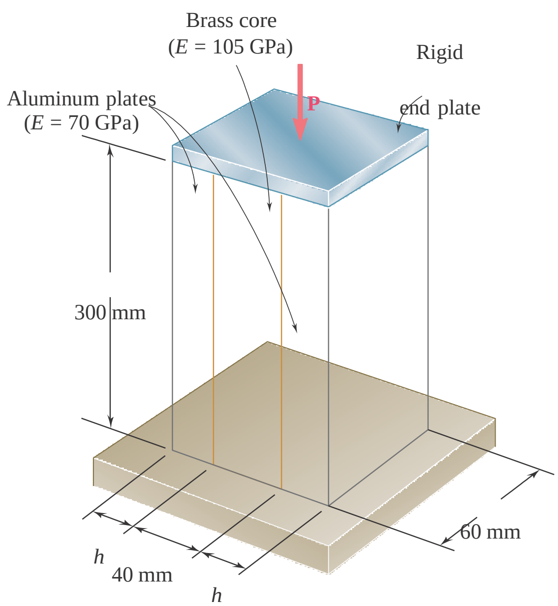

PROBLEM 2.33

An axial centric force of magnitude $P = 450$ kN is applied to the composite block shown by means of a rigid end plate. Knowing that $h = 10$ mm, determine the normal stress in (*a*) the brass core, (*b*) the aluminum plates.

SOLUTION

$$\delta_A = \delta_B = \delta; \qquad P = P_A + P_B$$

$$\delta = \frac{P_A L}{E_A A_A} \quad \text{and} \quad \delta = \frac{P_B L}{E_B A_B}$$

Therefore,

$$P_A = (E_A A_A)\left(\frac{\delta}{L}\right); \qquad P_B = (E_B A_B)\left(\frac{\delta}{L}\right)$$

Substituting,

$$P_A = \left(E_A A_A + E_B A_B\right)\left(\frac{\delta}{L}\right)$$

$$\in = \frac{\delta}{L} = \frac{P}{\left(E_A A_A + E_B A_B\right)}$$

$$\in = \frac{(450 \times 10^3 \text{ N})}{(70 \times 10^9 \text{ Pa})(2)(0.06 \text{ m})(0.01 \text{ m}) + (105 \times 10^9 \text{ Pa})(0.06 \text{ m})(0.04 \text{ m})}$$

$$\in = 1.33929 \times 10^{-3}$$

Now,

$$\sigma = E\in$$

(*a*) Brass-core:

$$\sigma_B = (105 \times 10^9 \text{ Pa})(1.33929 \times 10^{-3})$$

$$= 1.40625 \times 10^8 \text{ Pa}$$

(*b*) Aluminum:

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$$\sigma_A = (70 \times 10^9 \text{ Pa})(1.33929 \times 10^{-3}) = 9.3750 \times 10^7 \text{ Pa}$$

$$\sigma_B = 140.6 \text{ MPa} \blacktriangleleft$$

$$\sigma_A = 93.8 \text{ MPa} \blacktriangleleft$$

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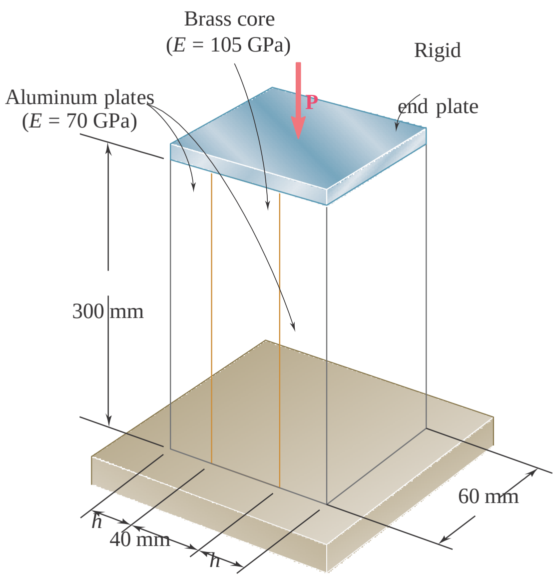

PROBLEM 2.34

For the composite block shown in Prob. 2.33, determine (*a*) the value of *h* if the portion of the load carried by the aluminum plates is half the portion of the load carried by the brass core, (*b*) the total load if the stress in the brass is 80 MPa.

PROBLEM 2.33. An axial centric force of magnitude *P* = 450 kN is applied to the composite block shown by means of a rigid end plate. Knowing that *h* = 10 mm, determine the normal stress in (*a*) the brass core, (*b*) the aluminum plates.

SOLUTION

$$\delta = \delta_a = \delta_b; \qquad P = P_a + P_b$$

$$\delta = \frac{P_a L}{E_a A_a} \quad \text{and} \quad \delta = \frac{P_b L}{E_b A_b}$$

Therefore,

$$P_a = (E_a A_a)\frac{\delta}{L}; \qquad P_b = (E_b A_b)\left(\frac{\delta}{L}\right)$$

(a)

$$P_a = \frac{1}{2} P_b$$

$$(E_a A_a)\left(\frac{\delta}{L}\right) = \frac{1}{2}(E_b A_b)\left(\frac{\delta}{L}\right)$$

$$A_a = \frac{1}{2}\left(\frac{E_b}{E_a}\right) A_b$$

$$A_a = \frac{1}{2}\left(\frac{105 \text{ GPa}}{70 \text{ GPa}}\right)(40 \text{ mm})(60 \text{ mm})$$

$$A_a = 1800 \text{ mm}^2$$

$$1800 \text{ mm}^2 = 2(60 \text{ mm})(h)$$

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$h = 15.00 \text{ mm}$ ◀

(*b*) $\sigma_b = \dfrac{P_b}{A_b} \Rightarrow P_b = \sigma_b A_b$ and $P_a = \dfrac{1}{2} P_b$

$P = P_a + P_b$

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PROBLEM 2.34 *(Continued)*

$$P = \frac{1}{2}(\sigma_b A_b) + \sigma_b A_b$$

$$P = (\sigma_b A_b)1.5$$

$$P = (80 \times 10^6 \text{ Pa})(0.04 \text{ m})(0.06 \text{ m})(1.5)$$

$$P = 2.880 \times 10^5 \text{ N}$$

$P = 288$ kN ◀

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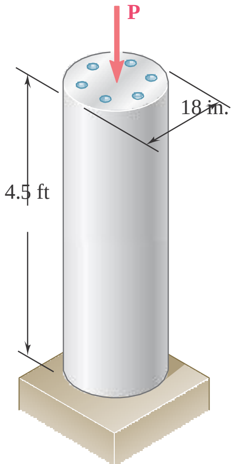

PROBLEM 2.35

The 4.5-ft concrete post is reinforced with six steel bars, each with a $1\frac{1}{8}$-in. diameter. Knowing that $E_s = 29 \times 10^6$ psi and $E_c = 4.2 \times 10^6$ psi, determine the normal stresses in the steel and in the concrete when a 350-kip axial centric force **P** is applied to the post.

SOLUTION

Let P_c = portion of axial force carried by concrete.

P_s = portion carried by the six steel rods.

$$\delta = \frac{P_c L}{E_c A_c} \qquad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{-P}{E_c A_c + E_s A_s}$$

$$A_s = 6\frac{\pi}{4} d_s^2 = \frac{6\pi}{4}(1.125\text{ in.})^2 = 5.9641\text{ in}^2$$

$$A_c = \frac{\pi}{4} d^2 - A_s = \frac{\pi}{4}(18\text{ in.})^2 - 5.9641\text{ in}^2$$

$$= 248.51\text{ in}^2$$

$$L = 4.5\text{ ft} = 54\text{ in.}$$

$$\varepsilon = \frac{-350 \times 10^3\text{ lb}}{(4.2 \times 10^6\text{ psi})(248.51\text{ in}^2) + (29 \times 10^6\text{ psi})(5.9641\text{ in}^2)} = -2.8767 \times 10^{-4}$$

$$\sigma_s = E_s \varepsilon = (29 \times 10^6\text{ psi})(-2.8767 \times 10^{-4}) = -8.3424 \times 10^3\text{ psi} \qquad \sigma_s = -8.34\text{ ksi} \blacktriangleleft$$

$$\sigma_c = E_c \varepsilon = (4.2 \times 10^6\text{ psi})(-2.8767 \times 10^{-4}) = 1.20821 \times 10^3\text{ psi} \qquad \sigma_c = -1.208\text{ ksi} \blacktriangleleft$$

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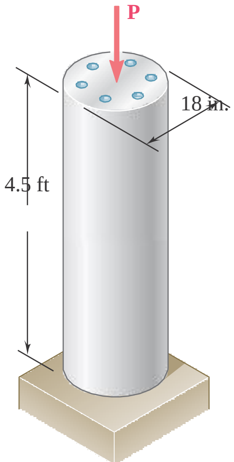

PROBLEM 2.36

For the post of Prob. 2.35, determine the maximum centric force that can be applied if the allowable normal stress is 20 ksi in the steel and 2.4 ksi in the concrete.

PROBLEM 2.35 The 4.5-ft concrete post is reinforced with six steel bars, each with a $1\frac{1}{8}$-in. diameter. Knowing that $E_s = 29 \times 10^6$ psi and $E_c = 4.2 \times 10^6$ psi, determine the normal stresses in the steel and in the concrete when a 350-kip axial centric force **P** is applied to the post.

SOLUTION

Allowable strain in each material:

Steel: $$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{20\times10^3 \text{ psi}}{29\times10^6 \text{ psi}} = 6.8966\times10^{-4}$$

Concrete: $$\varepsilon_c = \frac{\sigma_c}{E_c} = \frac{2.4\times10^3 \text{ psi}}{4.2\times10^6 \text{ psi}} = 5.7143\times10^{-4}$$

Smaller value governs. $$\varepsilon = \frac{\delta}{L} = 5.7143\times10^{-4}$$

Let P_c = Portion of load carried by concrete.

P_s = Portion of load carried by 6 steel rods.

$$\delta = \frac{P_c L}{E_c A_c} \qquad \therefore \qquad P_c = E_c A_c\left(\frac{\delta}{L}\right) = E_c A_c \in$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad \therefore \qquad P_s = E_s A_s\left(\frac{\delta}{L}\right) = E_s A_s \in$$

$$A_s = 6\left(\frac{\pi}{4}\right)d_s^2 = \frac{6\pi}{4}(1.125 \text{ in.})^2 = 5.9641 \text{ in}^2$$

$$A_c = \left(\frac{\pi}{4}\right)d_c^2 - A_s = \frac{\pi}{4}(18 \text{ in.})^2 - 5.9641 \text{ in}^2 = 2.4851\times10^2 \text{ in}^2$$

$$P = P_c + P_s = E_c A_c \in + E_s A_s \in$$

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$$P = [(4.2\times 10^6 \text{ psi})(2.4851\times 10^2 \text{ in}^2) + (29\times 10^6 \text{ psi})(5.9641 \text{ in}^2)](5.7143\times 10^{-4})$$

$$P = 6.9526\times 10^5 \text{ lb}$$

$$P = 695 \text{ kips} \blacktriangleleft$$

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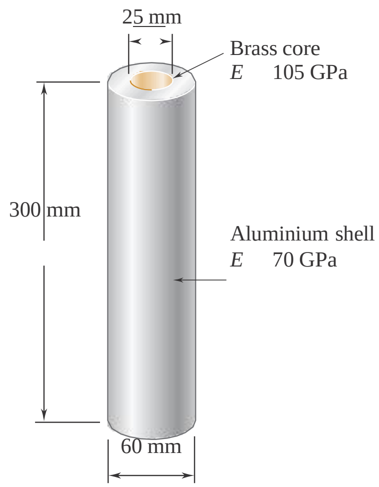

PROBLEM 2.37

An axial force of 200 kN is applied to the assembly shown by means of rigid end plates. Determine (*a*) the normal stress in the aluminum shell, (*b*) the corresponding deformation of the assembly.

SOLUTION

Let P_a = Portion of axial force carried by shell.

P_b = Portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a}, \quad \text{or} \quad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_b L}{E_b A_b}, \quad \text{or} \quad P_b = \frac{E_b A_b}{L}\delta$$

Thus,
$$P = P_a + P_b = (E_a A_a + E_b A_b)\frac{\delta}{L}$$

with
$$A_a = \frac{\pi}{4}[(0.060)^2 - (0.025)^2] = 2.3366 \times 10^{-3}\ \text{m}^2$$

$$A_b = \frac{\pi}{4}(0.025)^2 = 0.49087 \times 10^{-3}\ \text{m}^2$$

$$P = [(70 \times 10^9)(2.3366 \times 10^{-3}) + (105 \times 10^9)(0.49087 \times 10^{-3})]\frac{\delta}{L}$$

$$P = 215.10 \times 10^6 \frac{\delta}{L}$$

Strain:
$$\varepsilon = \frac{\delta}{L} = \frac{P}{215.10 \times 10^6} = \frac{200 \times 10^3}{215.10 \times 10^6} = 0.92980 \times 10^{-3}$$

(*a*) $\sigma_a = E_a\varepsilon = (70 \times 10^9)(0.92980 \times 10^{-3}) = 65.1 \times 10^6\ \text{Pa}$ $\qquad \sigma_a = 65.1\ \text{MPa}$ ◀

(*b*) $\delta = \varepsilon L = (0.92980 \times 10^{-3})(300\ \text{mm})$ $\qquad \delta = 0.279\ \text{mm}$ ◀

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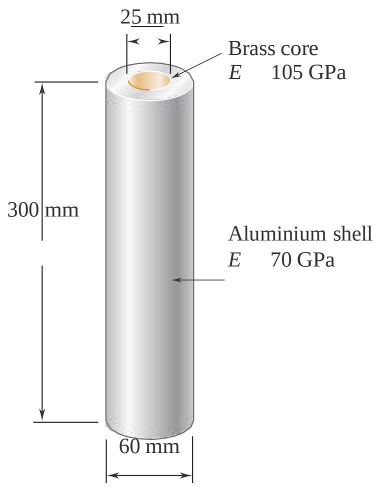

PROBLEM 2.38

The length of the assembly shown decreases by 0.40 mm when an axial force is applied by means of rigid end plates. Determine (*a*) the magnitude of the applied force, (*b*) the corresponding stress in the brass core.

SOLUTION

Let P_a = Portion of axial force carried by shell and P_b = Portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a}, \quad \text{or} \quad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_b L}{E_b A_b}, \quad \text{or} \quad P_b = \frac{E_b A_b}{L}\delta$$

Thus, $$P = P_a + P_b = (E_a A_a + E_b A_b)\frac{\delta}{L}$$

with $$A_a = \frac{\pi}{4}[(0.060)^2 - (0.025)^2] = 2.3366 \times 10^{-3}\ \text{m}^2$$

$$A_b = \frac{\pi}{4}(0.025)^2 = 0.49087 \times 10^{-3}\ \text{m}^2$$

$$P = [(70 \times 10^9)(2.3366 \times 10^{-3}) + (105 \times 10^9)(0.49087 \times 10^{-3})]\frac{\delta}{L} = 215.10 \times 10^6 \frac{\delta}{L}$$

with $\delta = 0.40$ mm, $L = 300$ mm

(*a*) $$P = (215.10 \times 10^6)\frac{0.40}{300} = 286.8 \times 10^3\ \text{N}$$

$P = 287$ kN ◀

(*b*) $$\sigma_b = \frac{P_b}{A_b} = \frac{E_b \delta}{L} = \frac{(105 \times 10^9)(0.40 \times 10^{-3})}{300 \times 10^{-3}} = 140 \times 10^6\ \text{Pa}$$

$\sigma_b = 140.0$ MPa ◀

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A

1.25 in.

25 in. 6 kips 6 kips

B

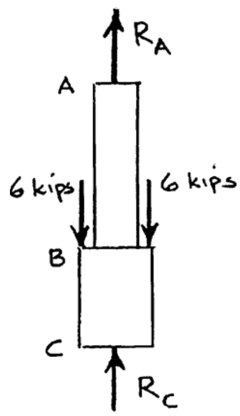
15 in. 2 in.

C

A polystyrene rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends and supports two 6-kip loads as shown. Knowing that $E = 0.45 \times 10^6$ psi, determine (*a*) the reactions at *A* and *C*, (*b*) the normal stress in each portion of

SOLUTION

(*a*) We express that the elongation of the rod is zero.



$$\delta = \frac{P_{AB}L_{AB}}{\frac{\pi}{4}d_{AB}^2 E} + \frac{P_{BC}L_{BC}}{\frac{\pi}{4}d_{BC}^2 E} = 0$$

But $\qquad P_{AB} = +R_A \qquad P_{BC} = -R_C$

Substituting and simplifying,

$$\frac{R_A L_{AB}}{d_{AB}^2} - \frac{R_C L_{BC}}{d_{BC}^2} = 0$$

$$R_C = \frac{L_{AB}}{L_{BC}}\left(\frac{d_{BC}}{d_{AB}}\right)^2 R_A = \frac{25}{15}\left(\frac{2}{1.25}\right)^2 R_A$$

$$R_C = 4.2667 R_A \qquad (1)$$

From the free body diagram, $\qquad R_A + R_C = 12$ kips $\qquad (2)$

Substituting (1) into (2), $\qquad 5.2667 R_A = 12$

$R_A = 2.2785$ kips $\qquad R_A = 2.28$ kips ↑ ◀

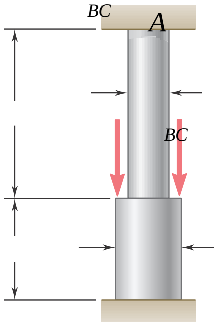
From (1), $\qquad R_C = 4.2667(2.2785) = 9.7217$ kips

$R_C = 9.72$ kips ↑ ◀

(*b*) $$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{+R_A}{A_{AB}} = \frac{2.2785}{\frac{\pi}{4}(1.25)^2}$$

$\sigma_{AB} = +1.857$ ksi ◀

$$\sigma = \frac{P_{BC}}{} =$$



$$\sigma_{BC} = \frac{-R_C}{A_{BC}} = \frac{-9.7217}{\frac{\pi}{4}(2)^2}$$

$$\sigma_{BC} = -3.09 \text{ ksi} \blacktriangleleft$$

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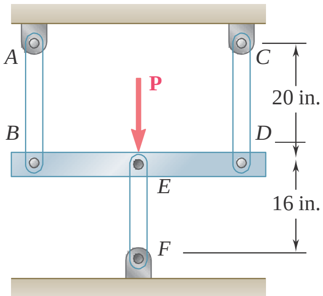
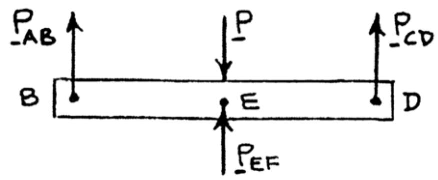


PROBLEM 2.40

Three steel rods ($E = 29 \times 10^6$ psi) support an 8.5-kip load **P**. Each of the rods *AB* and *CD* has a 0.32-in^2 cross-sectional area and rod *EF* has a 1-in^2 cross-sectional area. Neglecting the deformation of bar *BED*, determine (*a*) the change in length of rod *EF*, (*b*) the stress in each rod.

SOLUTION

Use member *BED* as a free body.



By symmetry, or by $\sum M_E = 0$:

$$P_{CD} = P_{AB}$$

$$+\uparrow \sum F_y = 0: \; P_{AB} + P_{CD} + P_{EF} - P = 0$$

$$P = 2P_{AB} + P_{EF}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{EA_{AB}} \qquad \delta_{CD} = \frac{P_{CD} L_{CD}}{EA_{CD}} \qquad \delta_{EF} = \frac{P_{EF} L_{EF}}{EA_{EF}}$$

Since $\quad L_{AB} = L_{CD}$ and $A_{AB} = A_{CD}, \; \delta_{AB} = \delta_{CD}$

Since points *A*, *C*, and *F* are fixed, $\delta_B = \delta_{AB}, \delta_D = \delta_{CD}, \; \delta_E = \delta_{EF}$

Since member *BED* is rigid, $\delta_E = \delta_B = \delta_C$

$$\frac{P_{AB} L_{AB}}{EA_{AB}} = \frac{P_{EF} L_{EF}}{EA_{EF}} \quad \therefore \quad P_{AB} = \frac{A_{AB}}{A_{EF}} \cdot \frac{L_{EF}}{L_{AB}} P_{EF} = \frac{0.32}{1} \cdot \frac{16}{20} P_{EF} = 0.256 P_{EF}$$

$$P = 2P_{AB} + P_{EF} = 2(0.256 P_{EF}) + P_{EF} = 1.512 P_{EF}$$

$$P_{EF} = \frac{P}{1.512} = \frac{8.5}{1.512} = 5.6217 \text{ kips}$$

$$P_{AB} = P_{CD} = 0.256(5.6217) = 1.43916 \text{ kips}$$

(*a*) $\quad \delta_{EF} = \dfrac{P_{EF} L_{EF}}{EA_{EF}} = \dfrac{(5.6217)(16)}{(29 \times 10^3)(1)} = 0.0031016 \text{ in.} \qquad \delta_{EF} = 0.00310 \text{ in.}$ ◀

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(*b*) $\sigma_{AB} = \sigma_{CD} = \frac{P_{AB}}{A_{AB}} = \frac{1.43916}{0.32} = 4.4974\text{ ksi}$ $\sigma_{AB} = \sigma_{CD} = 4.50\text{ ksi}$ ◀

$\sigma_{EF} = \frac{P_{EF}}{A_{EF}} = \frac{5.6217}{1} = 5.6217\text{ ksi}$ $\sigma_{EF} = 5.62\text{ ksi}$ ◀

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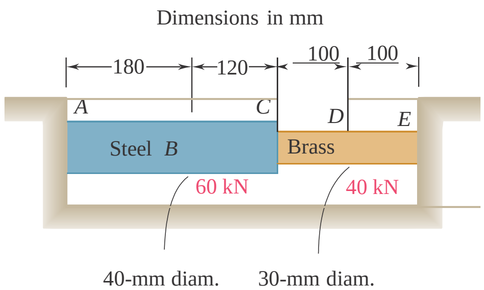


PROBLEM 2.41

C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine

(*a*) the reactions at A and E, (*b*) the deflection of point C.

SOLUTION

<u>*A* to *C*</u>: $E = 200 \times 10^9\,\text{Pa}$

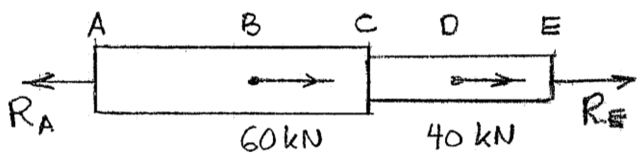
$$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3\,\text{mm}^2 = 1.25664 \times 10^{-3}\,\text{m}^2$$

$$EA = 251.327 \times 10^6\,\text{N}$$

<u>*C* to *E*</u>: $E = 105 \times 10^9\,\text{Pa}$

$$A = \frac{\pi}{4}(30)^2 = 706.86\,\text{mm}^2 = 706.86 \times 10^{-6}\,\text{m}^2$$

$$EA = 74.220 \times 10^6\,\text{N}$$



<u>*A* to *B*</u>: $P = R_A$

$$L = 180\,\text{mm} = 0.180\,\text{m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327 \times 10^6}$$

$$= 716.20 \times 10^{-12} R_A$$

<u>*B* to *C*</u>: $P = R_A - 60 \times 10^3$

$$L = 120\,\text{mm} = 0.120\,\text{m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{251.327 \times 10^6}$$

$$= 447.47 \times 10^{-12} R_A - 26.848 \times 10^{-6}$$

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PROBLEM 2.41 *(Continued)*

<u>*C* to *D*</u>: $P = R_A - 60\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9} R_A - 80.841\times10^{-6}$$

<u>*D* to *E*</u>: $P = R_A - 100\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9} R_A - 134.735\times10^{-6}$$

<u>*A* to *E*</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.85837\times10^{-9} R_A - 242.424\times10^{-6}$$

Since point *E* cannot move relative to *A*, $\delta_{AE} = 0$

(*a*) $3.85837\times10^{-9} R_A - 242.424\times10^{-6} = 0 \quad R_A = 62.831\times10^3 \text{ N}$ $R_A = 62.8 \text{ kN} \leftarrow$ ◀

$R_E = R_A - 100\times10^3 = 62.8\times10^3 - 100\times10^3 = -37.2\times10^3 \text{ N}$ $R_E = 37.2 \text{ kN} \leftarrow$ ◀

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367\times10^{-9} R_A - 26.848\times10^{-6}$

$$= (1.16369\times10^{-9})(62.831\times10^3) - 26.848\times10^{-6}$$

$$= 46.3\times10^{-6} \text{ m}$$

$\delta_C = 46.3 \ \mu\text{m} \rightarrow$ ◀

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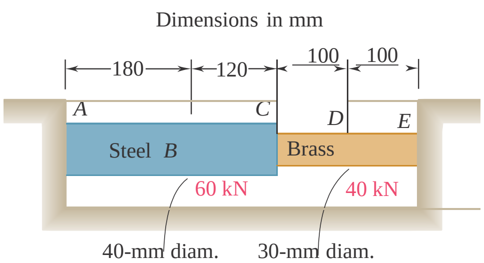


PROBLEM 2.42

rod *CE* is made of steel.

PROBLEM 2.41 Two cylindrical rods, one of steel and the other of brass, are joined at *C* and restrained by rigid supports at *A* and *E*. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at *A* and *E*, (*b*) the deflection of point *C*.

SOLUTION

<u>*A* to *C*</u>: $E = 105 \times 10^9\ \text{Pa}$

$$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3\ \text{mm}^2 = 1.25664 \times 10^{-3}\ \text{m}^2$$

$$EA = 131.947 \times 10^6\ \text{N}$$

<u>*C* to *E*</u>: $E = 200 \times 10^9\ \text{Pa}$

$$A = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86 \times 10^{-6}\ \text{m}^2$$

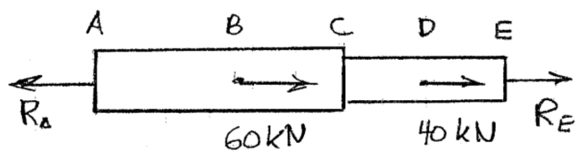
$$EA = 141.372 \times 10^6\ \text{N}$$

<u>*A* to *B*</u>: $P = R_A$

$$L = 180\ \text{mm} = 0.180\ \text{m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{131.947 \times 10^6}$$

$$= 1.36418 \times 10^{-9} R_A$$



<u>*B* to *C*</u>: $P = R_A - 60 \times 10^3$

$$L = 120\ \text{mm} = 0.120\ \text{m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{131.947 \times 10^6}$$

$$= 909.456 \times 10^{-12} R_A - 54.567 \times 10^{-6}$$

<u>*C* to *D*</u>: $P = R_A - 60 \times 10^3$

$$L = 100\ \text{mm} = 0.100\ \text{m}$$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{141.372 \times 10^6}$$

$$= 707.354 \times 10^{-12} R_A - 42.441 \times 10^{-6}$$

PROBLEM 2.42 *(Continued)*

$\underline{D \text{ to } E}$:

$$P = R_A - 100\times10^3$$

$$L = 100 \text{ mm} = 0.100 \text{ m}$$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^3)(0.100)}{141.372\times10^6}$$

$$= 707.354\times10^{-12}R_A - 70.735\times10^{-6}$$

$\underline{A \text{ to } E}$:

$$\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$$

$$= 3.68834\times10^{-9}R_A - 167.743\times10^{-6}$$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(*a*) $3.68834\times10^{-9}R_A - 167.743\times10^{-6} = 0 \quad R_A = 45.479\times10^3 \text{ N}$ $\quad R_A = 45.5 \text{ kN} \leftarrow$ ◀

$R_E = R_A - 100\times10^3 = 45.479\times10^3 - 100\times10^3 = -54.521\times10^3$ $\quad R_E = 54.5 \text{ kN} \leftarrow$ ◀

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 2.27364\times10^{-9}R_A - 54.567\times10^{-6}$

$$= (2.27364\times10^{-9})(45.479\times10^3) - 54.567\times10^{-6}$$

$$= 48.8\times10^{-6} \text{ m}$$

$\delta_C = 48.8\,\mu\text{m} \rightarrow$ ◀

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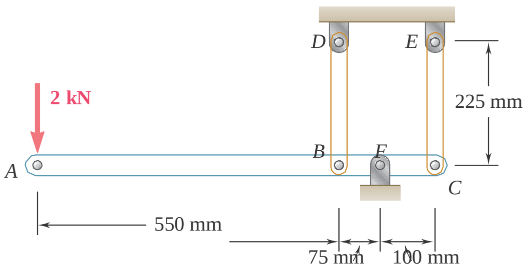
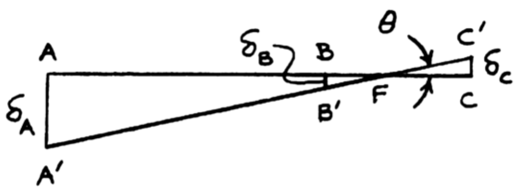


PROBLEM 2.43

Each of the rods *BD* and *CE* is made of brass ($E = 105$ GPa) and has a cross-sectional area of 200 mm^2. Determine the deflection of end *A* of the rigid member *ABC* caused by the 2-kN load.

SOLUTION

Let θ be the rotation of member *ABC* as shown.



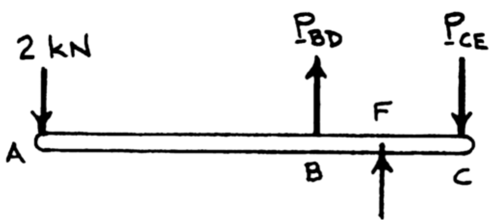
Then $\delta_A = 0.625\theta_1 \qquad \delta_B = 0.075\theta_1 \qquad \delta_C = 0.1\theta$

But $\delta_B = \dfrac{P_{BD} L_{BD}}{AE}$

$$P_{BD} = \frac{EA\delta_B}{L_{BD}} = \frac{(105\times 10^9)(200\times 10^{-6})(0.075\theta)}{0.225}$$

$$= 7\times 10^6\ \theta$$

Free body *ABC*:



$$\delta_C = \frac{P_{CE} L_{CE}}{AE}$$

$$P_{CE} = \frac{EA\delta_C}{L_{CE}} = \frac{(105\times 10^9)(200\times 10^{-6})(0.1\theta)}{0.225}$$

$$= 9.3333\times 10^6\ \theta$$

From free body of member *ABC*:

$$+\circlearrowleft \Sigma \mathbf{M}_F = 0: \quad (0.625)(2000) - 0.075P_{BD} - 0.1P_{CE} = 0$$

or $\quad (0.625)(2000) - 0.075(7\times 10^6\ \theta) - 0.1(9.3333\times 10^6\ \theta) = 0$

$$\theta = 0.85714\times 10^{-3} \text{ rad}$$

and $\quad \delta_A = 0.625\theta = 0.625(0.85714\times 10^{-3}) = 0.53571\times 10^{-3}$ m

$$\delta_A = 0.536 \text{ mm} \downarrow \blacktriangleleft$$

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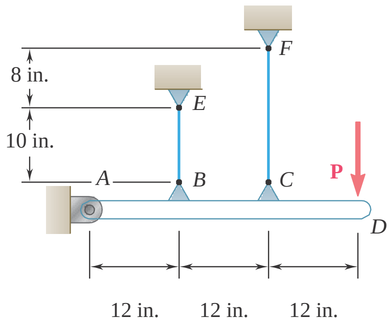


PROBLEM 2.44

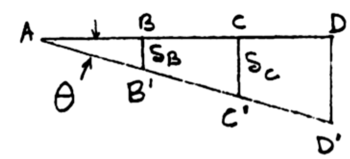
The rigid bar *AD* is supported by two steel wires of $\frac{1}{16}$-in. diameter ($E = 29 \times 10^6$ psi) and a pin and bracket at *A*. Knowing that the wires were initially taut, determine (*a*) the additional tension in each wire when a 220-lb load **P** is applied at *D*, (*b*) the corresponding deflection of point *D*.

SOLUTION

Let θ be the notation of bar *ABCD*.

Then $\delta_B = 12\theta$

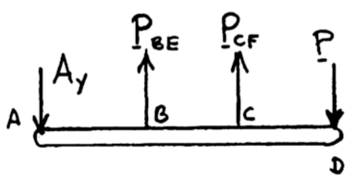
$\delta_C = 24\theta$



$$\delta_B = \frac{P_{BE}L_{BE}}{AE}$$

$$P_{BE} = \frac{EA\delta_{BE}}{L_{BE}} = \frac{(29\times 10^6)\frac{\pi}{4}\left(\frac{1}{6}\right)^2(12\theta)}{10}$$

$$= 106.765 \times 10^3\,\theta$$



$$\delta_C = \frac{P_{CF}L_{CF}}{EA}$$

$$P_{CF} = \frac{EA\delta_{CE}}{L_{CF}} = \frac{(29\times 10^6)\frac{\pi}{4}\left(\frac{1}{16}\right)^2(24\theta)}{18}$$

$$= 118.628 \times 10^3\,\theta$$

Using free body *ABCD*,

$+\circlearrowleft \Sigma \mathbf{M}_A = 0: \qquad 12P_{BE} + 24P_{CF} - 36P = 0$

( ) (*b*)

a $\qquad P_{BE}$

$(12)(106.765\times10^{3}\theta)+(24)(118.628\times10^{6}\theta)-(36)(220)=0$

4.12

$$83\times10^{6}\,\theta=(36)(220)$$

$$\theta=1.91847\times10^{-3}\ \text{rad}$$

$$=(106.765\times10^{3})(1.91847\times10^{-3})=204.83\ \text{lb}$$

P_{CF} $$=(118.628\times10^{3})(1.91847\times10^{-3})=227.58\ \text{lb}$$

δ_D $$=36\,\theta=(36)(1.91847\times10^{-3})=69.1\times10^{-3}\ \text{in.}$$

$P_{BE}=205$ lb ◀

$P_{CF}=228$ lb ◀

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0.0691 in. ◀

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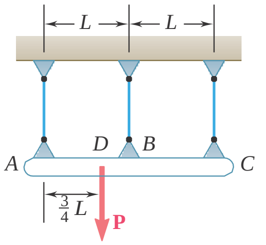
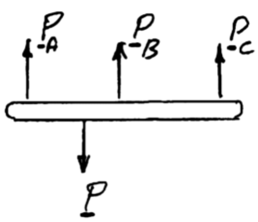
PROBLEM 2.45



The rigid bar *ABC* is suspended from three wires of the same material. The cross-sectional area of the wire at *B* is equal to half of the cross-sectional area of the wires at *A* and *C*. Determine the tension in each wire caused by the load **P** shown.

SOLUTION



$$+\circlearrowleft \Sigma M_A = 0: \quad 2LP_C + LP_B - \frac{3}{4}LP = 0$$

$$P_C = \frac{3}{8}P - \frac{1}{2}P_B$$

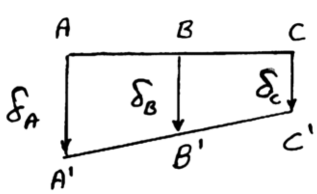
$$+\circlearrowleft \Sigma M_C = 0: \quad -2LP_A - LP_B + \frac{5}{4}LP = 0$$

$$P_A = \frac{5}{8}P - \frac{1}{2}P_B$$

Let l be the length of the wires.

$$\delta_A = \frac{P_A l}{EA} = \frac{l}{EA}\left(\frac{5}{8}P - \frac{1}{2}P_B\right)$$

$$\delta_B = \frac{P_B l}{E(A/2)} = \frac{2l}{EA}P_B$$



$$\delta_C = \frac{P_C l}{EA} = \frac{l}{EA}\left(\frac{3}{8}P - \frac{1}{2}P_B\right)$$

From the deformation diagram,

$$\delta_A - \delta_B = \delta_B - \delta_C$$

or $$\delta_B = \frac{1}{2}(\delta_A + \delta_c)$$

$$\frac{l}{E(A/2)}P_B = \frac{1}{2}\frac{l}{EA}\left(\frac{5}{8}P - \frac{1}{2}P_B + \frac{3}{8}P - \frac{1}{2}P_B\right)$$

$$\frac{5}{2}P_B = \frac{1}{2}P; \qquad P_B = \frac{1}{5}P \qquad\qquad P_B = 0.200P \blacktriangleleft$$

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PROBLEM 2.45 *(Continued)*

$$P_A = \frac{5}{8}P - \frac{1}{2}\left(\frac{P}{5}\right) = \frac{21}{40}P \qquad P_A = 0.525P \blacktriangleleft$$

$$P_C = \frac{3}{8}P - \frac{1}{2}\left(\frac{P}{5}\right) = \frac{11}{40}P \qquad P_C = 0.275P \blacktriangleleft$$

Check:

$$P_A + P_B + P_C = 1.000P \qquad \text{Ok}$$

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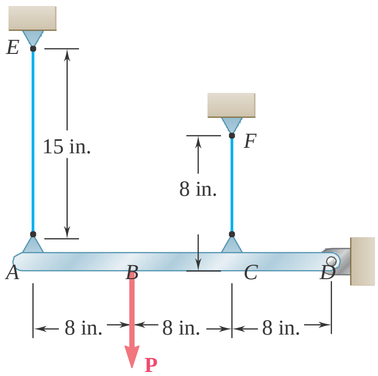


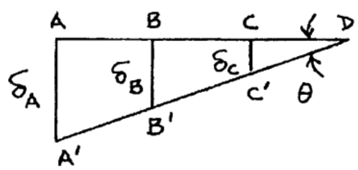
PROBLEM 2.46

The rigid bar AD is supported by two steel wires of $\frac{1}{16}$-in. diameter $(E = 29 \times 10^6 \text{ psi})$ and a pin and bracket at D. Knowing that the wires were initially taut, determine (*a*) the additional tension in each wire when a 120-lb load **P** is applied at B, (*b*) the corresponding deflection of point B.

SOLUTION

Let θ be the rotation of bar $ABCD$.

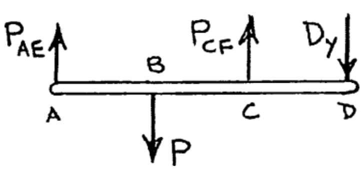
Then $\delta_A = 24\theta \qquad \delta_C = 8\theta$



$$\delta_A = \frac{P_{AE} L_{AE}}{AE}$$

$$P_{AE} = \frac{EA\delta_A}{L_{AE}} = \frac{(29\times10^6)\frac{\pi}{4}\left(\frac{1}{16}\right)^2(24\theta)}{15}$$

$$= 142.353\times10^3\theta$$



$$\delta_C = \frac{P_{CF} L_{CF}}{AE}$$

$$P_{CF} = \frac{EA\delta_C}{L_{CF}} = \frac{(29\times10^6)\frac{\pi}{4}\left(\frac{1}{16}\right)^2(8\theta)}{8}$$

$$= 88.971\times10^3\theta$$

Using free body $ABCD$,

$$+\circlearrowleft \Sigma M_D = 0: \qquad -24P_{AE} + 16P - 8P_{CF} = 0$$

$$-24(142.353\times10^3\theta) + 16(120) - 8(88.971\times10^3\theta) = 0$$

$$\theta = 0.46510\times10^{-3} \text{ rad}$$

(*a*) $P_{AE} = (142.353\times10^3)(0.46510\times10^{-3})$ $\qquad P_{AE} = 66.2 \text{ lb}$ ◀

$P_{CF} = (88.971\times10^3)(0.46510\times10^{-3})$ $\qquad P_{CF} = 41.4 \text{ lb}$ ◀

(*b*) $\delta_B = 16\theta = 16(0.46510\times10^{-3})$ $\qquad \delta_B = 7.44\times10^{-3} \text{ in.}\downarrow$ ◀

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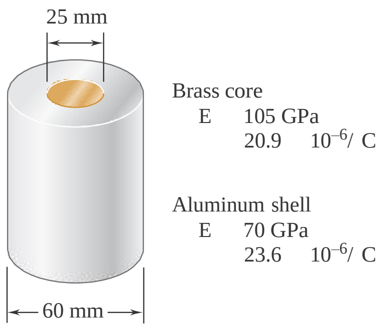


PROBLEM 2.47

The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C.

SOLUTION

Brass core:

$$E = 105 \text{ GPa}$$

$$\alpha = 20.9 \times 10^{-6}/°\text{C}$$

Aluminum shell:

$$E = 70 \text{ GPa}$$

$$\alpha = 23.6 \times 10^{-6}/°\text{C}$$

Let L be the length of the assembly.

Free thermal expansion:

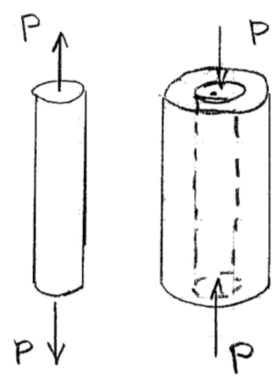
$$\Delta T = 195 - 15 = 180\,°\text{C}$$

Brass core: $(\delta_T)_b = L\alpha_b(\Delta T)$

Aluminum shell: $(\delta_T)_a = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core: $\delta = L(\alpha_a - \alpha_b)(\Delta T)$

Let P be the tensile force in the core and the compressive force in the shell.



Brass core:

$$E_b = 105 \times 10^9 \text{ Pa}$$

$$A_b = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2$$

$$= 490.87 \times 10^{-6} \text{m}^2$$

$$(\delta_P)_b = \frac{PL}{E_b A_b}$$

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PROBLEM 2.47 *(Continued)*

Aluminum shell:
$$(\delta_p)_a = \frac{PL}{E_a A_a}$$

$$E_a = 70\times 10^9\ \text{Pa}$$

$$A_a = \frac{\pi}{4}(60^2 - 25^2)$$

$$= 2.3366\times 10^3\ \text{mm}^2$$

$$= 2.3366\times 10^{-3}\ \text{m}^2$$

$$\delta = (\delta_P)_b + (\delta_P)_a$$

$$L(\alpha_b - \alpha_a)(\Delta T) = \frac{PL}{E_b A_b} + \frac{PL}{E_a A_a} = KPL$$

where

$$K = \frac{1}{E_b A_b} + \frac{1}{E_a A_a}$$

$$= \frac{1}{(105\times 10^9)(490.87\times 10^{-6})} + \frac{1}{(70\times 10^9)(2.3366\times 10^{-3})}$$

$$= 25.516\times 10^{-9}\ \text{N}^{-1}$$

Then

$$P = \frac{(\alpha_b - \alpha_a)(\Delta T)}{K}$$

$$= \frac{(23.6\times 10^{-6} - 20.9\times 10^{-6})(180)}{25.516\times 10^{-9}}$$

$$= 19.047\times 10^3\ \text{N}$$

Stress in aluminum:
$$\sigma_a = -\frac{P}{A_a} = -\frac{19.047\times 10^3}{2.3366\times 10^{-3}} = -8.15\times 10^6\ \text{Pa} \qquad \sigma_a = -8.15\ \text{MPa} \blacktriangleleft$$

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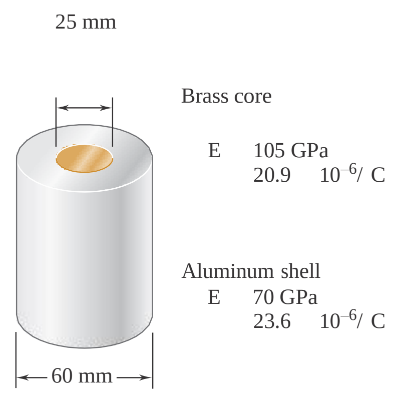


PROBLEM 2.48

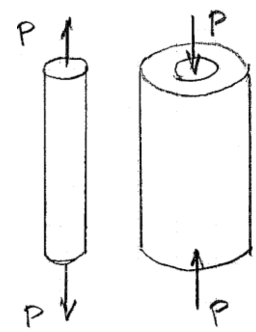
Solve Prob. 2.47, assuming that the core is made of steel $(E_s = 200\ \text{GPa},\ \alpha_s = 11.7 \times 10^{-6}/°\text{C})$ instead of brass.

PROBLEM 2.47 The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C.

SOLUTION

Aluminum shell: $E = 70\ \text{GPa}\quad \alpha = 23.6 \times 10^{-6}/°\text{C}$

Let L be the length of the assembly.



Free thermal expansion: $\Delta T = 195 - 15 = 180\ °\text{C}$

Steel core: $(\delta_T)_s = L\alpha_s(\Delta T)$

Aluminum shell: $(\delta_T)_a = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core: $\delta = L(\alpha_a - \alpha_s)(\Delta T)$

Let P be the tensile force in the core and the compressive force in the shell.

Steel core: $E_s = 200 \times 10^9\ \text{Pa},\quad A_s = \frac{\pi}{4}(25)^2 = 490.87\ \text{mm}^2 = 490.87 \times 10^{-6}\ \text{m}^2$

$$(\delta_P)_s = \frac{PL}{E_s A_s}$$

Aluminum shell: $E_a = 70 \times 10^9\ \text{Pa}$

$$(\delta_P)_a = \frac{PL}{E_a A_a}$$

$$A_a = \frac{\pi}{4}(60^2 - 25)^2 = 2.3366 \times 10^3\ \text{mm}^2 = 2.3366 \times 10^{-3}\ \text{m}^2$$

$$\delta = (\delta_P)_s + (\delta_P)_a$$

$$L(\alpha_a - \alpha_s)(\Delta T) = \frac{PL}{E_s A_s} + \frac{PL}{E_a A_a} = KPL$$

where

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$$K = \frac{1}{E_s A_s} + \frac{1}{E_a A_a}$$

$$= \frac{1}{(200\times10^9)(490.87\times10^{-6})} + \frac{1}{(70\times10^9)(2.3366\times10^{-3})}$$

$$= 16.2999\times10^{-9}\ \text{N}^{-1}$$

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PROBLEM 2.48 *(Continued)*

Then

$$P = \frac{(\alpha_a - \alpha_s)(\Delta T)}{K} = \frac{(23.6 \times 10^{-6} - 11.7 \times 10^{-6})(180)}{16.2999 \times 10^{-9}} = 131.412 \times 10^{3}\ \text{N}$$

Stress in aluminum: $\sigma_a = -\frac{P}{A_a} = -\frac{131.412 \times 10^3}{2.3366 \times 10^{-3}} = -56.241 \times 10^6\ \text{Pa}$ $\qquad \sigma_a = -56.2\ \text{MPa}$ ◀

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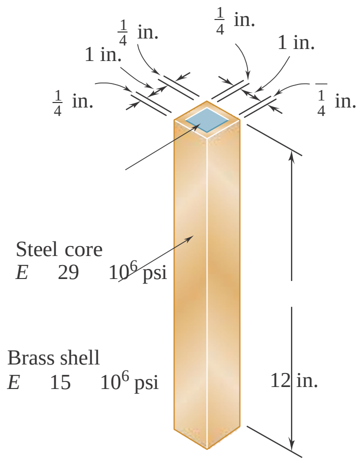

PROBLEM 2.49

The brass shell $(\alpha_b = 11.6 \times 10^{-6}/°\text{F})$ is fully bonded to the steel core $(\alpha_s = 6.5 \times 10^{-6}/°\text{F})$. Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 8 ksi.

SOLUTION

Let P_s = axial force developed in the steel core.

For equilibrium with zero total force, the compressive force in the brass shell is P_s.

Strains:

$$\varepsilon_s = \frac{P_s}{E_sA_s} + \alpha_s(\Delta T)$$

$$\varepsilon_b = -\frac{P_s}{E_bA_b} + \alpha_b(\Delta T)$$

Matching:

$$\varepsilon_s = \varepsilon_b$$

$$\frac{P_s}{E_sA_s} + \alpha_s(\Delta T) = -\frac{P_s}{E_bA_b} + \alpha_b(\Delta T)$$

$$\left(\frac{1}{E_sA_s} + \frac{1}{E_bA_b}\right)P_s = (\alpha_b - \alpha_s)(\Delta T) \qquad (1)$$

$$A_b = (1.5)(1.5) - (1.0)(1.0) = 1.25 \text{ in}^2$$

$$A_s = (1.0)(1.0) = 1.0 \text{ in}^2$$

$$\alpha_b - \alpha_s = 5.1 \times 10^{-6}/°\text{F}$$

$$P_s = \sigma_s A_s = (8 \times 10^3)(1.0) = 8 \times 10^3 \text{ lb}$$

$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(29\times 10^6)(1.0)} + \frac{1}{(15\times 10^6)(1.25)} = 87.816\times 10^{-9}\ \text{lb}^{-1}$$

From (1),
$$(87.816\times 10^{-9})(8\times 10^3) = (5.1\times 10^{-6})(\Delta T)$$

$$\Delta T = 137.8°\text{F} \blacktriangleleft$$

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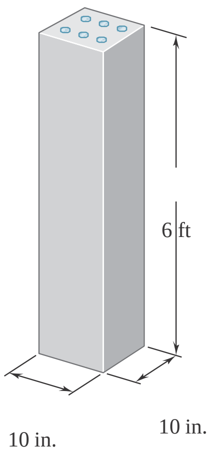

PROBLEM 2.50

The concrete post ($E_c = 3.6\times10^6$ psi and $\alpha_c = 5.5\times10^{-6}/°\text{F}$) is reinforced with six steel bars, each of $\frac{7}{8}$-in. diameter ($E_s = 29\times10^6$ psi and $\alpha_s = 6.5\times10^{-6}/°\text{F}$). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 65°F.

SOLUTION

$$A_s = 6\frac{\pi}{4}d^2 = 6\frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 3.6079 \text{ in}^2$$

$$A_c = 10^2 - A_s = 10^2 - 3.6079 = 96.392 \text{ in}^2$$

Let $P_c =$ tensile force developed in the concrete.

For equilibrium with zero total force, the compressive force in the six steel rods equals P_c.

Strains: $\varepsilon_s = -\frac{P_c}{E_sA_s} + \alpha_s(\Delta T) \qquad \varepsilon_c = \frac{P_c}{E_cA_c} + \alpha_c(\Delta T)$

Matching: $\varepsilon_c = \varepsilon_s \qquad \frac{P_c}{E_cA_c} + \alpha_c(\Delta T) = -\frac{P_c}{E_sA_s} + \alpha_s(\Delta T)$

$$\left(\frac{1}{E_cA_c} + \frac{1}{E_sA_s}\right)P_c = (\alpha_s - \alpha_c)(\Delta T)$$

$$\left[\frac{1}{(3.6\times10^6)(96.392)} + \frac{1}{(29\times10^6)(3.6079)}\right]P_c = (1.0\times10^{-6})(65)$$

$$P_c = 5.2254\times10^3 \text{ lb}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{5.2254\times10^3}{96.392} = 54.210 \text{ psi} \qquad \sigma_c = 54.2 \text{ psi}$$

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$$A_c \qquad 96.392$$

$$\sigma_s = -\frac{P_c}{A_s} = -\frac{5.2254\times 10^3}{3.6079} = -1448.32 \text{ psi} \qquad \sigma_s = -1.448 \text{ ksi}$$

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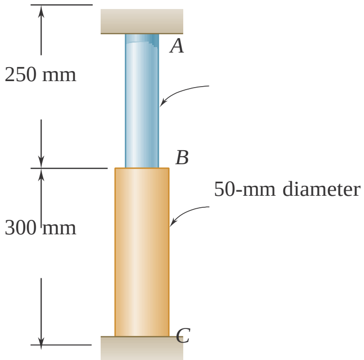

ends. Portion *AB* is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7\times10^{-6}/°C$) and

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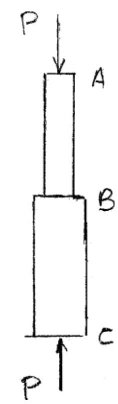
that the rod is initially unstressed, determine the compressive force induced in *ABC* when there is a temperature rise of 50°C.

SOLUTION

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86\times10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(50)^2 = 1.9635\times10^{3} \text{ mm}^2 = 1.9635\times10^{-3} \text{ m}^2$$



Free thermal expansion:

$$\begin{aligned}\delta_T &= L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)\\ &= (0.250)(11.7\times10^{-6})(50) + (0.300)(20.9\times10^{-6})(50)\\ &= 459.75\times10^{-6} \text{ m}\end{aligned}$$

Shortening due to induced compressive force *P*:

$$\begin{aligned}\delta_P &= \frac{PL}{E_s A_{AB}} + \frac{PL}{E_b A_{BC}}\\ &= \frac{0.250P}{(200\times10^{9})(706.86\times10^{-6})} + \frac{0.300P}{(105\times10^{9})(1.9635\times10^{-3})}\\ &= 3.2235\times10^{-9}P\end{aligned}$$

For zero net deflection, $\delta_P = \delta_T$

$$3.2235\times10^{-9}P = 459.75\times10^{-6}$$

$$P = 142.624\times10^{3} \text{ N}$$

$P = 142.6$ kN ◀

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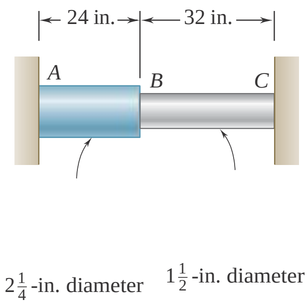
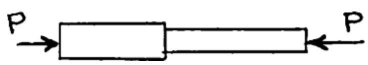


PROBLEM 2.52

A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel $(E_s = 29 \times 10^6 \text{ psi}, \ \alpha_s = 6.5 \times 10^{-6}/°\text{F})$ and portion BC is made of aluminum $(E_a = 10.4 \times 10^6 \text{ psi}, \ \alpha_a = 13.3 \times 10^{-6}/°\text{F})$. Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 70°F, (*b*) the corresponding deflection of point B.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2 \qquad A_{BC} = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$



Free thermal expansion. $\Delta T = 70°\text{F}$

$$(\delta_T)_{AB} = L_{AB}\alpha_s(\Delta T) = (24)(6.5\times 10^{-6})(70) = 10.92\times 10^{-3} \text{ in.}$$

$$(\delta_T)_{BC} = L_{BC}\alpha_a(\Delta T) = (32)(13.3\times 10^{-6})(70) = 29.792\times 10^{-3} \text{ in.}$$

Total: $$\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 40.712\times 10^{-3} \text{ in.}$$

Shortening due to induced compressive force P.

$$(\delta_P)_{AB} = \frac{PL_{AB}}{E_s A_{AB}} = \frac{24P}{(29\times 10^6)(3.9761)} = 208.14\times 10^{-9} P$$

$$(\delta_P)_{BC} = \frac{PL_{BC}}{E_a A_{BC}} = \frac{32P}{(10.4\times 10^6)(1.76715)} = 1741.18\times 10^{-9} P$$

Total: $$\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 1949.32\times 10^{-9} P$$

For zero net deflection, $\delta_P = \delta_T$ $\qquad 1949.32\times 10^{-9} P = 40.712\times 10^{-3}$ $\qquad P = 20.885\times 10^3 \text{ lb}$

(*a*) $$\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{20.885\times 10^3}{3.9761} = -5.25\times 10^3 \text{ psi}$$ $\sigma_{AB} = -5.25$ ksi ◀

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{20.885\times 10^3}{1.76715} = -11.82\times 10^3 \text{ psi}$$ $\sigma_{BC} = -11.82$ ksi ◀

(*b*)

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$$(\delta_P)_{AB} = (208.14\times10^{-9})(20.885\times10^3) = 4.3470\times10^{-3}\text{ in.}$$

$$\delta_B = (\delta_T)_{AB} \rightarrow + (\delta_P)_{AB} \leftarrow = 10.92\times10^{-3} \rightarrow + 4.3470\times10^{-3} \leftarrow \qquad \delta_B = 6.57\times10^{-3}\text{ in.} \rightarrow \blacktriangleleft$$

or

$$(\delta_P)_{BC} = (1741.18\times10^{-9})(20.885\times10^3) = 36.365\times10^{-3}\text{ in.}$$

$$\delta_B = (\delta_T)_{BC} \leftarrow + (\delta_P)_{BC} \rightarrow = 29.792\times10^{-3} \leftarrow + 36.365\times10^{-3} \rightarrow = 6.57\times10^{-3}\text{ in.} \rightarrow \qquad \text{(checks)}$$

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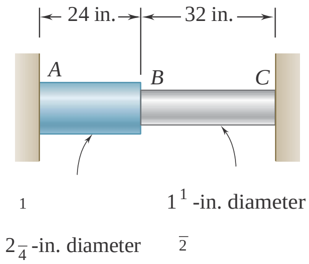
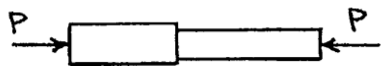
PROBLEM 2.53



Solve Prob. 2.52, assuming that portion *AB* of the composite rod is made of aluminum and portion *BC* is made of steel.

PROBLEM 2.52 A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$) and portion *BC* is made of aluminum ($E_a = 10.4 \times 10^6$ psi, $\alpha_a = 13.3 \times 10^{-6}/°\text{F}$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 70°F, (*b*) the corresponding deflection of point *B*.

SOLUTION



$$A_{AB} = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2 \qquad A_{BC} = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

Free thermal expansion. $\Delta T = 70°\text{F}$

$$(\delta_T)_{AB} = L_{AB}\alpha_a(\Delta T) = (24)(13.3\times 10^{-6})(70) = 22.344\times 10^{-3} \text{in.}$$

$$(\delta_T)_{BC} = L_{BC}\alpha_s(\Delta T) = (32)(6.5\times 10^{-6})(70) = 14.56\times 10^{-3} \text{in.}$$

Total: $$\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 36.904\times 10^{-3} \text{in.}$$

Shortening due to induced compressive force *P*.

$$(\delta_P)_{AB} = \frac{PL_{AB}}{E_a A_{AB}} = \frac{24P}{(10.4\times 10^6)(3.9761)} = 580.39\times 10^{-9} P$$

$$(\delta_P)_{BC} = \frac{PL_{BC}}{E_s A_{BC}} = \frac{32P}{(29\times 10^6)(1.76715)} = 624.42\times 10^{-9} P$$

Total: $$\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 1204.81\times 10^{-9} P$$

For zero net deflection, $\delta_P = \delta_T$ $\quad 1204.81\times 10^{-9} P = 36.904\times 10^{-3}$ $\quad P = 30.631\times 10^3$ lb

(*a*) $$\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{30.631\times 10^3}{3.9761} = -7.70\times 10^3 \text{ psi}$$ $\sigma_{AB} = -7.70$ ksi ◀

$$_{AB} \quad A_{AB} \quad 3.9761 \qquad\qquad _{AB}$$

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{30.631\times10^3}{1.76715} = -17.33\times10^3 \text{ psi}$$

$\sigma_{BC} = -17.33$ ksi ◀

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PROBLEM 2.53 *(Continued)*

(*b*) $(\delta_P)_{AB} = (580.39\times10^{-9})(30.631\times10^{3}) = 17.7779\times10^{-3}$ in.

$$\delta_B = (\delta_T)_{AB} \rightarrow + (\delta_P)_{AB} \leftarrow = 22.344\times10^{-3} \rightarrow + 17.7779\times10^{-3} \leftarrow \qquad \delta_B = 4.57\times10^{-3} \text{ in.} \rightarrow \blacktriangleleft$$

or $(\delta_P)_{BC} = (624.42\times10^{-9})(30.631\times10^{3}) = 19.1266\times10^{-3}$ in.

$$\delta_B = (\delta_T)_{BC} \leftarrow + (\delta_P)_{BC} \rightarrow = 14.56\times10^{-3} \leftarrow + 19.1266\times10^{-3} \rightarrow = 4.57\times10^{-3} \text{ in.} \rightarrow \qquad \text{(checks)}$$

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PROBLEM 2.54

The steel rails of a railroad track (E_s = 200 GPa, α_s = 11.7 × 102^{-6}/°C) were laid at a temperature of 6°C. Determine the normal stress in the rails when the temperature reaches 48°C, assuming that the rails (*a*) are welded to form a continuous track, (*b*) are 10 m long with 3-mm gaps between them.

SOLUTION

(*a*) $\delta_T = \alpha(\Delta T)L = (11.7 \times 10^{-6})(48 - 6)(10) = 4.914 \times 10^{-3}$ m

$$\delta_P = \frac{PL}{AE} = \frac{L\sigma}{E} = \frac{(10)\sigma}{200 \times 10^9} = 50 \times 10^{-12}\sigma$$

$$\delta = \delta_T + \delta_P = 4.914 \times 10^{-3} + 50 \times 10^{-12}\sigma = 0$$

$$\sigma = -98.3 \times 10^6 \text{ Pa}$$

$\sigma = -98.3$ MPa

(*b*) $\delta = \delta_T + \delta_P = 4.914 \times 10^{-3} + 50 \times 10^{-12}\sigma = 3 \times 10^{-3}$

$$\sigma = \frac{3 \times 10^{-3} - 4.914 \times 10^{-3}}{50 \times 10^{-12}}$$

$$= -38.3 \times 10^6 \text{ Pa}$$

$\sigma = -38.3$ MPa

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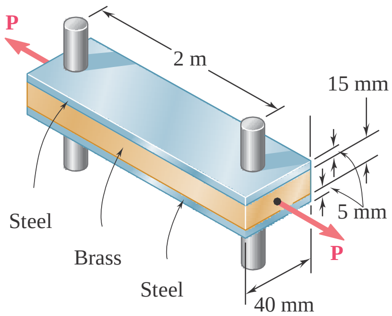

PROBLEM 2.55

Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°\text{C}$) that is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

SOLUTION

(*a*) Required temperature change for fabrication:

$$\delta_T = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

Temperature change required to expand steel bar by this amount:

$$\delta_T = L\alpha_s \Delta T, \quad 0.5 \times 10^{-3} = (2.00)(11.7 \times 10^{-6})(\Delta T),$$

$$\Delta T = 0.5 \times 10^{-3} = (2)(11.7 \times 10^{-6})(\Delta T)$$

$$\Delta T = 21.368\,°\text{C} \qquad 21.4\,°\text{C} \blacktriangleleft$$

(*b*) Once assembled, a tensile force P^* develops in the steel, and a compressive force P^* develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$

$$(\delta_P)_s = \frac{F^* L}{A_s E_s} = \frac{P^*(2.00)}{(400 \times 10^{-6})(200 \times 10^9)} = 25 \times 10^{-9} P^*$$

Contraction of brass: $A_b = (40)(15) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$

$$(\delta_P)_b = \frac{P^* L}{A_b E_b} = \frac{P^*(2.00)}{(600 \times 10^{-6})(105 \times 10^9)} = 31.746 \times 10^{9}\ P^*$$

But $(\delta_P)_s + (\delta_P)_b$ is equal to the initial amount of misfit:

$$(\delta_P)_s + (\delta_P)_b = 0.5 \times 10^{-3}, \quad 56.746 \times 10^{-9} P^* = 0.5 \times 10^{-3}$$

$$P^* = 8.8112 \times 10^3 \text{ N}$$

Stresses due to fabrication:

Steel:

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$$\sigma_s^* = \frac{P^*}{A_s} = \frac{8.8112 \times 10^3}{400 \times 10^{-6}} = 22.028 \times 10^6 \text{ Pa} = 22.028 \text{ MPa}$$

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PROBLEM 2.55 *(Continued)*

Brass: $\sigma_b^* = -\dfrac{P^*}{A_b} = -\dfrac{8.8112\times10^3}{600\times10^{-6}} = -14.6853\times10^6 \text{ Pa} = -14.685 \text{ MPa}$

To these stresses must be added the stresses due to the 25-kN load.

For the added load, the additional deformation is the same for both the steel and the brass. Let δ' be the additional displacement. Also, let P_s and P_b be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_s = \frac{A_s E_s}{L}\delta' = \frac{(400\times10^{-6})(200\times10^9)}{2.00}\delta' = 40\times10^6\delta'$$

$$P_b = \frac{A_b E_b}{L}\delta' = \frac{(600\times10^{-6})(105\times10^9)}{2.00}\delta' = 31.5\times10^6\delta'$$

Total: $P = P_s + P_b = 25\times10^3 \text{ N}$

$$40\times10^6\delta' + 31.5\times10^6\delta' = 25\times10^3 \qquad \delta' = 349.65\times10^{-6} \text{ m}$$

$$P_s = (40\times10^6)(349.65\times10^{-6}) = 13.9860\times10^3 \text{ N}$$

$$P_b = (31.5\times10^6)(349.65\times10^{-6}) = 11.0140\times10^3 \text{ N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.9860\times10^3}{400\times10^{-6}} = 34.965\times10^6 \text{ Pa}$$

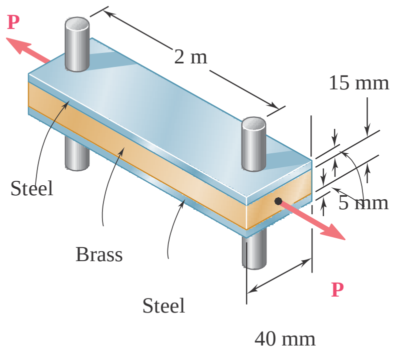
$$\sigma_b = \frac{P_b}{A_b} = \frac{11.0140\times10^3}{600\times10^{-6}} = 18.3566\times10^6 \text{ Pa}$$

Add stress due to fabrication.

Total stresses:

$$\sigma_s = 34.965\times10^6 + 22.028\times10^6 = 56.991\times10^6 \text{ Pa} \qquad \sigma_s = 57.0 \text{ MPa}$$

$$\sigma_b = 18.3566\times10^6 - 14.6853\times10^6 = 3.6713\times10^6 \text{ Pa} \qquad \sigma_b = 3.67 \text{ MPa} \blacktriangleleft$$



PROBLEM 2.56

Determine the maximum load P that may be applied to the brass bar of Prob. 2.55 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

PROBLEM 2.55 Two steel bars $(E_s = 200 \text{ GPa and } \alpha_s = 11.7 \times 10^{-6}/°\text{C})$ are used to reinforce a brass bar $(E_b = 105 \text{ GPa}, \alpha_b = 20.9 \times 10^{-6}/°\text{C})$ that is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

SOLUTION

See solution to Problem 2.55 to obtain the fabrication stresses.

$$\sigma_s^* = 22.028 \text{ MPa}$$

$$\sigma_b^* = 14.6853 \text{ MPa}$$

Allowable stresses: $\sigma_{s,\text{all}} = 30 \text{ MPa}, \sigma_{b,\text{all}} = 25 \text{ MPa}$

Available stress increase from load.

$$\sigma_s = 30 - 22.028 = 7.9720 \text{ MPa}$$

$$\sigma_b = 25 + 14.6853 = 39.685 \text{ MPa}$$

Corresponding available strains.

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{7.9720 \times 10^6}{200 \times 10^9} = 39.860 \times 10^{-6}$$

$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{39.685 \times 10^6}{105 \times 10^9} = 377.95 \times 10^{-6}$$

Smaller value governs $\therefore \quad \varepsilon = 39.860 \times 10^{-6}$

Areas: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{m}^2$

$A_b = (15)(40) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{m}^2$

Forces $P_s = E_s A_s \varepsilon = (200 \times 10^9)(400 \times 10^{-6})(39.860 \times 10^{-6}) = 3.1888 \times 10^3 \text{N}$

$P_b = E_b A_b \varepsilon = (105 \times 10^9)(600 \times 10^{-6})(39.860 \times 10^{-6}) = 2.5112 \times 10^{-3} \text{N}$

Total allowable additional force:

$$P = P_s + P_b = 3.1888 \times 10^3 + 2.5112 \times 10^3 = 5.70 \times 10^3 \text{N}$$

$P = 5.70$ kN ◀

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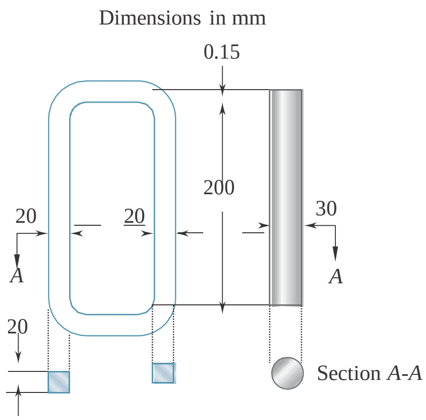

PROBLEM 2.57

An aluminum rod (E_a = 70 GPa, α_a = 23.6 × 10^{-6}/°C) and a steel link (E_s × 200 GPa, α_a = 11.7 × 10^{-6}/°C) have the dimensions shown at a temperature of 20°C. The steel link is heated until the aluminum rod can be fitted freely into the link. The temperature of the whole assembly is then raised to 150°C. Determine the final normal stress (*a*) in the rod, (*b*) in the link.

SOLUTION

$$\Delta T = T_f - T_i = 150°\text{C} - 20°\text{C} = 130°\text{C}$$

Unrestrained thermal expansion of each part:

Aluminum rod: $(\delta_T)_a = L\alpha_a(\Delta T)$

$$(\delta_T)_a = (0.200 \text{ m})(23.6\times10^{-6}/°\text{C})(130°\text{C})$$

$$= 6.1360\times10^{-4} \text{ m}$$

Steel link: $(\delta_T)_s = L\alpha_s(\Delta T)$

$$(\delta_T)_s = (0.200 \text{ m})(11.7\times10^{-6}/°\text{C})(130°\text{C})$$

$$= 3.0420\times10^{-4} \text{ m}$$

Let *P* be the compressive force developed in the aluminum rod. It is also the tensile force in the steel link.

Aluminum rod: $(\delta_P)_a = \dfrac{PL}{E_a A_a}$

$$= \frac{P(0.200 \text{ m})}{(70\times10^9 \text{ Pa})(\pi/4)(0.03 \text{ m})^2}$$

$$= 4.0420\times10^{-9}P$$

Steel link: $(\delta_P)_s = \dfrac{PL}{E_s A_s}$

$$= \frac{P(0.200)}{(200\times10^9 \text{ Pa})(2)(0.02 \text{ m})^2}$$

$$= 1.250\times10^{-9}P$$

Setting the total deformed lengths in the link and rod equal gives

$$(0.200) + (\delta\ )\ + (\delta\ )\ = (0.200) + (0.15\times10^{-3}) + (\delta\ )\ - (\delta\ )$$

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T_s P_s T_a P_a

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PROBLEM 2.57 *(Continued)*

$$(\delta_P)_s + (\delta_P)_a = 0.15\times10^{-3} + (\delta_T)_a - (\delta_T)_s$$

$$1.25\times10^{-9}P + 4.0420\times10^{-9}P = 0.15\times10^{-3} + 6.1360\times10^{-4} - 3.0420\times10^{-4}$$

$$P = 8.6810\times10^{4}\text{ N}$$

(*a*) Stress in rod:

$$\sigma = \frac{P}{A}$$

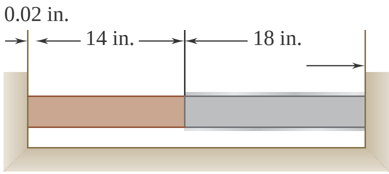
$$\sigma_R = \frac{-8.6810\times10^{4}\text{ N}}{(\pi/4)(0.030\text{ m})^2} = -1.22811\times10^{8}\text{ Pa}$$

$$\sigma_R = -122.8\text{ MPa} \blacktriangleleft$$

(b) Stress in link:

$$\sigma_L = \frac{8.6810\times10^{4}\text{ N}}{(2)(0.020\text{ m})^2} = 1.08513\times10^{8}\text{ Pa}$$

$$\sigma_L = 108.5\text{ MPa} \blacktriangleleft$$



Bronze			Aluminum		
A	2.4 in^2		A	2.8 in^2	
E	15	10^6 psi	E	10.6	10^6 psi
	12	10^{-6}/ F		12.9	10^{-6}/ F

PROBLEM 2.58

Knowing that a 0.02-in. gap exists when the temperature is 75°F, determine (*a*) the temperature at which the normal stress in the aluminum bar will be equal to −11 ksi, (*b*) the corresponding exact length of the aluminum bar.

SOLUTION

$$\sigma_a = -11 \text{ ksi} = -11\times 10^3 \text{ psi}$$

$$P = -\sigma_a A_a = (11\times 10^3)(2.8) = 30.8\times 10^3 \text{ lb}$$

Shortening due to *P*:

$$\delta_P = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a}$$

$$= \frac{(30.8\times 10^3)(14)}{(15\times 10^6)(2.4)} + \frac{(30.8\times 10^3)(18)}{(10.6\times 10^6)(2.8)}$$

$$= 30.657\times 10^{-3} \text{ in.}$$

Available elongation for thermal expansion:

$$\delta_T = 0.02 + 30.657\times 10^{-3} = 50.657\times 10^{-3} \text{ in.}$$

But $\delta_T = L_b\alpha_b(\Delta T) + L_a\alpha_a(\Delta T)$

$$= (14)(12\times 10^{-6})(\Delta T) + (18)(12.9\times 10^{-6})(\Delta T) = (400.2\times 10^{-6})\Delta T$$

Equating, $(400.2\times 10^{-6})\Delta T = 50.657\times 10^{-3}$ $\quad \Delta T = 126.6°\text{F}$

(*a*) $T_{\text{hot}} = T_{\text{cold}} + \Delta T = 75 + 126.6 = 201.6°\text{F}$ $\quad T_{\text{hot}} = 201.6°\text{F}$ ◀

(*b*) $\delta_a = L_a\alpha_a(\Delta T) - \dfrac{PL_a}{E_a A_a}$

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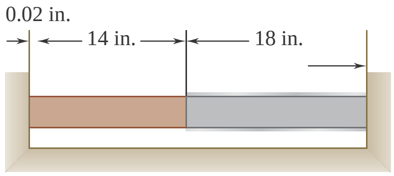
$$= (18)(12.9\times 10^{-6})(26.6) - \frac{(30.8\times 10\ )(18)}{(10.6\times 10^6)(2.8)}$$

$$= 10.712 \times 10^{-3} \text{ in.}$$

$$L_{\text{exact}} = 18 + 10.712 \times 10^{-3} = 18.0107 \text{ in.}$$

$L = 18.0107$ in. ◀



Bronze			Aluminum		
A	2.4 in^2		A	2.8 in^2	
E	15	10^6 psi	E	10.6	10^6 psi
	12	10^{-6}/ F		12.9	10^{-6}/ F

PROBLEM 2.59

Determine (*a*) the compressive force in the bars shown after a temperature rise of 180°F, (*b*) the corresponding change in length of the bronze bar.

SOLUTION

Thermal expansion if free of constraint:

$$\delta_T = L_b\alpha_b(\Delta T) + L_a\alpha_a(\Delta T)$$
$$= (14)(12\times10^{-6})(180) + (18)(12.9\times10^{-6})(180)$$
$$= 72.036\times10^{-3} \text{ in.}$$

Constrained expansion: $\delta = 0.02$ in.

Shortening due to induced compressive force *P*:

$$\delta_P = 72.036\times10^{-3} - 0.02 = 52.036\times10^{-3} \text{in.}$$

But $$\delta_P = \frac{PL_b}{E_bA_b} + \frac{PL_a}{E_aA_a} = \left(\frac{L_b}{E_bA_b} + \frac{L_a}{E_aA_a}\right)P$$

$$= \left(\frac{14}{(15\times10^6)(2.4)} + \frac{18}{(10.6\times10^6)(2.8)}\right)P = 995.36\times10^{-9}P$$

Equating, $$995.36\times10^{-9}P = 52.036\times10^{-3}$$
$$P = 52.279\times10^3 \text{ lb}$$

(*a*) $P = 52.3$ kips ◀

(*b*) $$\delta_b = L_b\alpha_b(\Delta T) - \frac{PL_b}{E_bA_b}$$

$= (14)(12\times10^{-6}$

)
(

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$$180) - \frac{(52.279\times 10\ \ )(14)}{(15\times 10^{6})(2.4)} = 9.91\times 10^{-3}\text{ in.}$$

$$\delta_b = 9.91\times 10^{-3}\text{ in.} \blacktriangleleft$$

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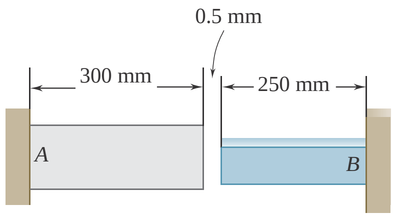

Aluminum
A 5 2000 mm^2
E 5 75 GPa
a 5 23 3 10^{-6}/8C

Stainless steel
A 5 800 mm^2
E 5 190 GPa
a 5 17.3 3 10^{-6}/8C

PROBLEM 2.60

At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 140°C, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.

SOLUTION

$$\Delta T = 140 - 20 = 120°\text{C}$$

<u>Free thermal expansion</u>:

$$\begin{aligned}\delta_T &= L_a\alpha_a(\Delta T) + L_s\alpha_s(\Delta T)\\ &= (0.300)(23\times 10^{-6})(120) + (0.250)(17.3\times 10^{-6})(120)\\ &= 1.347\times 10^{-3}\ \text{m}\end{aligned}$$

<u>Shortening due to *P* to meet constraint</u>:

$$\delta_P = 1.347\times 10^{-3} - 0.5\times 10^{-3} = 0.847\times 10^{-3}\ \text{m}$$

$$\begin{aligned}\delta_P &= \frac{PL_a}{E_aA_a} + \frac{PL_s}{E_sA_s} = \left(\frac{L_a}{E_aA_a} + \frac{L_s}{E_sA_s}\right)P\\ &= \left(\frac{0.300}{(75\times 10^9)(2000\times 10^{-6})} + \frac{0.250}{(190\times 10^9)(800\times 10^{-6})}\right)P\\ &= 3.6447\times 10^{-9}P\end{aligned}$$

Equating,
$$3.6447\times 10^{-9}P = 0.847\times 10^{-3}$$
$$P = 232.39\times 10^3\ \text{N}$$

(*a*) $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{232.39\times 10^3}{2000\times 10^{-6}} = -116.2\times 10^6\ \text{Pa}$ $\qquad \sigma_a = -116.2\ \text{MPa}$ ◀

(*b*) $\delta_a = L_a\alpha_a(\Delta T) - \dfrac{PL_a}{E_aA_a}$

$$= (0.300)(23\times 10^{-6})(120) - \frac{(232.39\times 10^3)(0.300)}{(75\times 10^9)(2000\times 10^{-6})} = 363\times 10^{-6}\ \text{m}$$ $\qquad \delta_a = 0.363\ \text{mm}$ ◀

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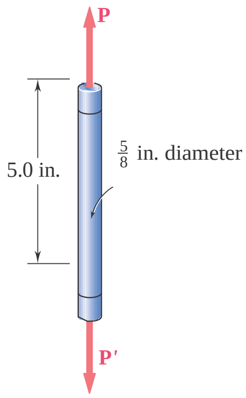

PROBLEM 2.61

A standard tension test is used to determine the properties of an experimental plastic. The test specimen is a $\frac{5}{8}$-in.-diameter rod and it is subjected to an 800-lb tensile force. Knowing that an elongation of 0.45 in. and a decrease in diameter of 0.025 in. are observed in a 5-in. gage length, determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio for the material.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.306796 \text{ in}^2$$

$$P = 800 \text{ lb}$$

$$\sigma_y = \frac{P}{A} = \frac{800}{0.306796} = 2.6076 \times 10^3 \text{ psi}$$

$$\varepsilon_y = \frac{\delta_y}{L} = \frac{0.45}{5.0} = 0.090$$

$$\varepsilon_x = \frac{\delta_x}{d} = \frac{-0.025}{0.625} = -0.040$$

$$E = \frac{\delta_y}{\varepsilon_y} = \frac{2.6076 \times 10^3}{0.090} = 28.973 \times 10^3 \text{ psi}$$

$E = 29.0 \times 10^3$ psi ◀

$$v = -\frac{\varepsilon_x}{\varepsilon_y} = \frac{-0.040}{0.090} = 0.44444$$

$v = 0.444$ ◀

$$\sigma = \frac{E}{2(1+v)} = \frac{28.973 \times 10^3}{(2)(1+0.44444)} = 10.0291 \times 10^3 \text{ psi}$$

$\sigma = 10.03 \times 10^3$ psi ◀

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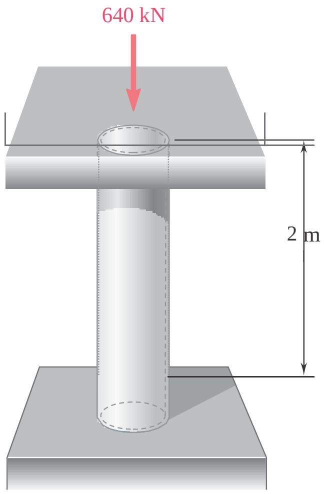

PROBLEM 2.62

A 2-m length of an aluminum pipe of 240-mm outer diameter and 10-mm wall thickness is used as a short column to carry a 640-kN centric axial load. Knowing that $E = 73$ GPa and $\nu = 0.33$, determine (*a*) the change in length of the pipe, (*b*) the change in its outer diameter, (*c*) the change in its wall thickness.

SOLUTION

$$d_o = 0.240 \qquad t = 0.010 \qquad L = 2.0$$

$$d_i = d_o - 2t = 0.240 - 2(0.010) = 0.220 \text{ m} \quad P = 640 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4}(0.240 - 0.220) = 7.2257 \times 10^{-3} \text{ m}^2$$

(*a*) $$\delta = -\frac{PL}{EA} = -\frac{(640 \times 10^3)(2.0)}{(73 \times 10^9)(7.2257 \times 10^{-3})}$$

$$= -2.4267 \times 10^{-3} \text{ m}$$

$\delta = -2.43$ mm ◀

$$\varepsilon = \frac{\delta}{L} = \frac{-2.4267}{2.0} = -1.21335 \times 10^{-3}$$

$$\varepsilon_{LAT} = -\nu\varepsilon = -(0.33)(-1.21335 \times 10^{-3})$$

$$= 4.0041 \times 10^{-4}$$

(b) $$\Delta d_o = d_o \varepsilon_{LAT} = (240 \text{ mm})(4.0041 \times 10^{-4}) = 9.6098 \times 10^{-2} \text{ mm}$$

$\Delta d_o = 0.0961$ mm ◀

$$\Delta t = t\varepsilon_{LAT} = (10 \text{ mm})(4.0041 \times 10^{-4}) = 4.0041 \times 10^{-3} \text{ mm}$$

$\Delta t = 0.00400$ mm ◀

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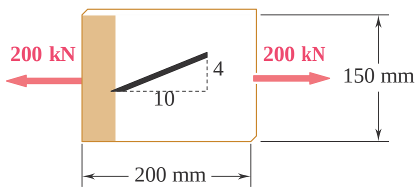


PROBLEM 2.63

A line of slope 4:10 has been scribed on a cold-rolled yellow-brass plate, 150 mm wide and 6 mm thick. Knowing that $E = 105$ GPa and $v = 0.34$, determine the slope of the line when the plate is subjected to a 200-kN centric axial load as shown.

SOLUTION

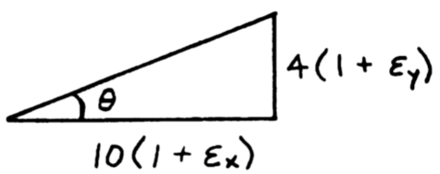
$$A = (0.150)(0.006) = 0.9 \times 10^{-3}\ \text{m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{200 \times 10^3}{0.9 \times 10^{-3}} = 222.22 \times 10^6\ \text{Pa}$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{222.22 \times 10^6}{105 \times 10^9} = 2.1164 \times 10^{-3}$$

$$\varepsilon_y = -\upsilon \varepsilon_x = -(0.34)(2.1164 \times 10^{-3})$$

$$= -0.71958 \times 10^{-3}$$



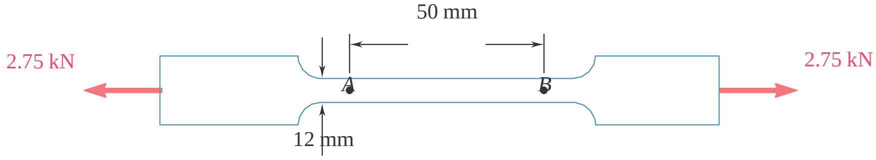
$$\tan\theta = \frac{4(1+\varepsilon_y)}{10(1+\varepsilon_x)}$$

$$= \frac{4(1 - 0.71958 \times 10^{-3})}{10(1 + 2.1164 \times 10^{-3})}$$

$$= 0.39887$$

$$\tan\theta = 0.399 \blacktriangleleft$$

PROBLEM 2.64



A 2.75-kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate (E = 200 GPa, *v* = 0.30). Determine the resulting change (*a*) in the 50-mm gage length, (*b*) in the width of portion *AB* of the test coupon, (*c*) in the thickness of portion *AB*, (*d*) in the cross-sectional area of portion *AB*.

SOLUTION

$$A = (1.6)(12) = 19.20\ \text{mm}^2$$

$$= 19.20 \times 10^{-6}\ \text{m}^2$$

$$P = 2.75 \times 10^3\ \text{N}$$

$$\sigma_x = \frac{P}{A} = \frac{2.75 \times 10^3}{19.20 \times 10^{-6}}$$

$$= 143.229 \times 10^6\ \text{Pa}$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{143.229 \times 10^6}{200 \times 10^9} = 716.15 \times 10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -\upsilon \varepsilon_x = -(0.30)(716.15 \times 10^{-6}) = -214.84 \times 10^{-6}$$

(*a*) $L = 0.050\ \text{m}$ $\qquad \delta_x = L\varepsilon_x = (0.50)(716.15 \times 10^{-6}) = 35.808 \times 10^{-6}\ \text{m}$

0.0358 mm ◀

(*b*) $w = 0.012\ \text{m}$ $\qquad \delta_y = w\varepsilon_y = (0.012)(-214.84 \times 10^{-6}) = -2.5781 \times 10^{-6}\ \text{m}$

–0.00258 mm ◀

(*c*) $t = 0.0016\ \text{m}$ $\qquad \delta_z = t\varepsilon_z = (0.0016)(-214.84 \times 10^{-6}) = -343.74 \times 10^{-9}\ \text{m}$

–0.000344 mm ◀

(*d*) $A = w_0(1+\varepsilon_y)t_0(1+\varepsilon_z) = w_0 t_0(1+\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z)$ $\qquad A_0 = w_0 t_0$

$$\Delta A = A - A_0 = w_0 t_0(\varepsilon_y + \varepsilon_z + \text{negligible term}) = 2w_0 t_0 \varepsilon_y$$

$$= (2)(0.012)(0.0016)(-214.84 \times 10^{-6}) = -8.25 \times 10^{-9}\ \text{m}^2$$

–0.00825 mm^2 ◀

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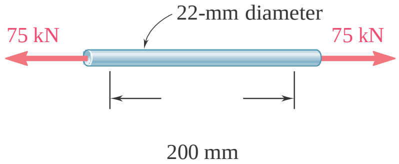
PROBLEM 2.65



In a standard tensile test, a steel rod of 22-mm diameter is subjected to a tension force of 75 kN. Knowing that $v = 0.3$ and $E = 200$ GPa, determine (*a*) the elongation of the rod in a 200-mm gage length, (*b*) the change in diameter of the rod.

SOLUTION

$$P = 75 \text{ kN} = 75 \times 10^3 \text{ N} \qquad A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.022)^2 = 380.13 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{75 \times 10^3}{380.13 \times 10^{-6}} = 197.301 \times 10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{197.301 \times 10^6}{200 \times 10^9} = 986.51 \times 10^{-6}$$

$$\delta_x = L\varepsilon_x = (200 \text{ mm})(986.51 \times 10^{-6})$$

(*a*) $\delta_x = 0.1973 \text{ mm}$ ◀

$$\varepsilon_y = -v\varepsilon_x = -(0.3)(986.51 \times 10^{-6}) = -295.95 \times 10^{-6}$$

$$\delta_y = d\varepsilon_y = (22 \text{ mm})(-295.95 \times 10^{-6})$$

(*b*) $\delta_y = -0.00651 \text{ mm}$ ◀

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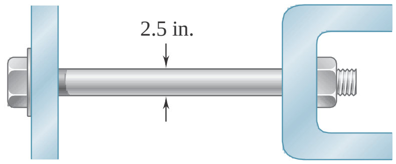

PROBLEM 2.66

The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the internal force in the bolt if the diameter is observed to decrease by 0.5×10^{-3} in.

SOLUTION

$$\delta_y = -0.5 \times 10^{-3} \text{ in.} \qquad d = 2.5 \text{ in.}$$

$$\varepsilon_y = \frac{\varepsilon_y}{d} = -\frac{0.5 \times 10^{-3}}{2.5} = -0.2 \times 10^{-3}$$

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}: \qquad \varepsilon_x = \frac{-\varepsilon_y}{\nu} = \frac{0.2 \times 10^{-3}}{0.3} = 0.66667 \times 10^{-3}$$

$$\sigma_x = E\varepsilon_x = (29 \times 10^6)(0.66667 \times 10^{-3}) = 19.3334 \times 10^3 \text{ psi}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(2.5)^2 = 4.9087 \text{ in}^2$$

$$F = \sigma_x A = (19.3334 \times 10^3)(4.9087) = 94.902 \times 10^3 \text{ lb}$$

$F = 94.9$ kips ◀

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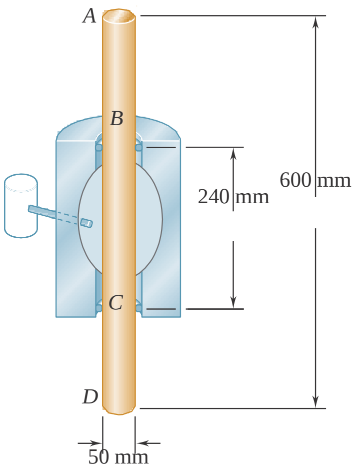

PROBLEM 2.67

The brass rod *AD* is fitted with a jacket that is used to apply a hydrostatic pressure of 48 MPa to the 240-mm portion *BC* of the rod. Knowing that $E = 105$ GPa and $v = 0.33$, determine (*a*) the change in the total length *AD*, (*b*) the change in diameter at the middle of the rod.

SOLUTION

$$\sigma_x = \sigma_z = -p = -48 \times 10^6 \text{ Pa}, \qquad \sigma_y = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \upsilon\sigma_y - \upsilon\sigma_z)$$

$$= \frac{1}{105 \times 10^9}\left[-48 \times 10^6 - (0.33)(0) - (0.33)(-48 \times 10^6)\right]$$

$$= 306.29 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(-\upsilon\sigma_x - \sigma_y - \upsilon\sigma_z)$$

$$= \frac{1}{105 \times 10^9}\left[-(0.33)(-48 \times 10^6) + 0 - (0.33)(-48 \times 10^6)\right]$$

$$= 301.71 \times 10^{-6}$$

(*a*) Change in length: only portion *BC* is strained. $L = 240$ mm

$$\delta_y = L\varepsilon_y = (240)(-301.71 \times 10^{-6}) = -0.0724 \text{ mm} \quad ◀$$

(*b*) Change in diameter: $d = 50$ mm

$$\delta_x = \delta_z = d\varepsilon_x = (50)(-306.29 \times 10^{-6}) = -0.01531 \text{ mm} \quad ◀$$

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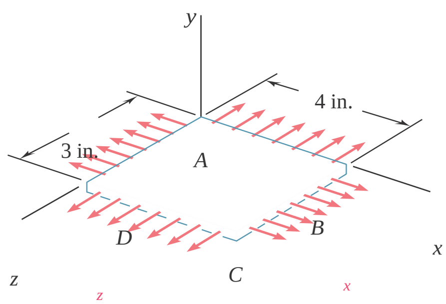
PROBLEM 2.68



A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 18$ ksi and $\sigma_z = 24$ ksi. Knowing that the properties of the fabric can be approximated as $E = 12.6 \times 10^6$ psi and $v = 0.34$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

$\sigma_x = 18$ ksi $\qquad \sigma_y = 0 \qquad \sigma_z = 24$ ksi

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \upsilon\sigma_y - \upsilon\sigma_z) = \frac{1}{12.6\times 10^6}[18{,}000 - (0.34)(24{,}000)] = 780.95\times 10^{-6}$$

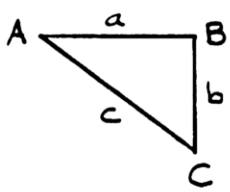
$$\varepsilon_z = \frac{1}{E}(-\upsilon\sigma_x - \upsilon\sigma_y + \sigma_z) = \frac{1}{12.6\times 10^6}[-(0.34)(18{,}000) + 24{,}000] = 1.41905\times 10^{-3}$$

(*a*) $\delta_{AB} = (AB)\varepsilon_x = (4\text{ in.})(780.95\times 10^{-6}) = 0.0031238$ in.

0.00312 in. ◀

(*b*) $\delta_{BC} = (BC)\varepsilon_z = (3\text{ in.})(1.41905\times 10^{-3}) = 0.0042572$ in.

0.00426 in. ◀



Label sides of right triangle *ABC* as *a*, *b*, *c*.

Then $\qquad c^2 = a^2 + b^2$

Obtain differentials by calculus.

$$2cdc = 2ada + 2bdb$$

$$dc = \frac{a}{c}da + \frac{b}{c}db$$

But $a = 4$ in. $\qquad b = 3$ in. $\qquad c = \sqrt{4^2 + 3^2} = 5$ in.

$da = \delta_{AB} = 0.0031238$ in. $\qquad db = \delta_{BC} = 0.0042572$ in.

(*c*) $\delta_{AC} = dc = \frac{4}{5}(0.0031238) + \frac{3}{5}(0.0042572)$

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$_{AC}$ 5 5

0.00505 in. ◀

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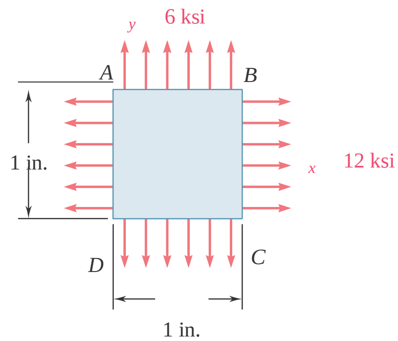


PROBLEM 2.69

A 1-in. square was scribed on the side of a large steel pressure vessel. After pressurization the biaxial stress condition at the square is as shown. Knowing that $E = 29 \times 10^6$ psi and $v = 0.30$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \upsilon\delta_y) = \frac{1}{29\times10^6}\left[12\times10^3 - (0.30)(6\times10^3)\right)$$

$$= 351.72\times10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \upsilon\delta_x) = \frac{1}{29\times10^6}\left[6\times10^3 - (0.30)(12\times10^3)\right)$$

$$= 82.759\times10^{-6}$$

(*a*) $\delta_{AB} = (AB)_0\varepsilon_x = (1.00)(351.72\times10^{-6}) = 352\times10^{-6}$ in. ◀

(*b*) $\delta_{BC} = (BC)_0\varepsilon_y = (1.00)(82.759\times10^{-6}) = 82.8\times10^{-6}$ in. ◀

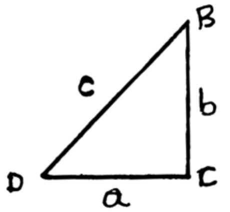
(*c*) $(AC) = \sqrt{(AB)^2 + (BC)^2} = \sqrt{(AB_0 + \delta_x)^2 + (BC_0 + \delta_y)^2}$

$$= \sqrt{(1 + 351.72\times10^{-6})^2 + (1 + 82.759\times10^{-6})^2}$$

$$= 1.41452$$

$(AC)_0 = \sqrt{2}$ $\qquad AC - (AC)_0 = 307\times10^{-6}$ ◀

or use calculus as follows:



Label sides using *a*, *b*, and *c* as shown.

$c^2 = a^2 + b^2$

Obtain differentials. $\qquad 2c\,dc = 2a\,da + 2b\,dc$

from which $\qquad dc = \frac{a}{c}da + \frac{b}{c}dc$

But $a = 100$ in., $b = 1.00$ in., $c = \sqrt{2}$ in.

$da = \delta \quad = 351.72\times10^{-6}$ in., $db = \delta \quad = 82.8\times10^{-6}$ in.

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$$\delta_{AC} = dc = \overset{AB}{\frac{1.00}{\sqrt{2}}}(351.7\times 10^{-6}) + \overset{BC}{\frac{1.00}{\sqrt{2}}}(82.8\times 10^{-6})$$

$$= 307\times 10^{-6}\text{ in.}$$

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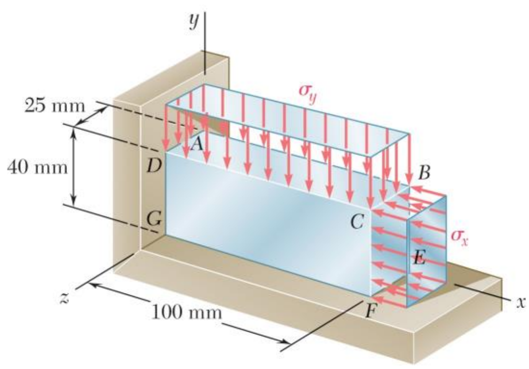
PROBLEM 2.70



The block shown is made of a magnesium alloy, for which $E = 45$ GPa and $\nu = 0.35$. Knowing that $\sigma_x = -180$ MPa, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.

SOLUTION

(*a*) $\delta_y = 0 \qquad \varepsilon_y = 0 \qquad \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$\sigma_y = \nu\sigma_x = (0.35)(-180\times 10^6)$$

$$= -63\times 10^6\ \text{Pa} \qquad \sigma_y = -63.0\ \text{MPa} \blacktriangleleft$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{(0.35)(-243\times 10^6)}{45\times 10^9} = +1.890\times 10^{-3}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x - \nu\sigma_y}{E} = -\frac{157.95\times 10^6}{45\times 10^9} = -3.510\times 10^{-3}$$

(*b*) $A_0 = L_x L_z$

$$A = L_x(1+\varepsilon_x)L_z(1+\varepsilon_z) = L_x L_z(1+\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z)$$

$$\Delta A = A - A_0 = L_x L_z(\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z) \approx L_x L_z(\varepsilon_x+\varepsilon_z)$$

$$\Delta A = (100\ \text{mm})(25\ \text{mm})(-3.510\times 10^{-3} + 1.890\times 10^{-3})$$

(*c*) $V_0 = L_x L_y L_z$

$$V = L_x(1+\varepsilon_x)L_y(1+\varepsilon_y)L_z(1+\varepsilon_z)$$

$$= L_x L_y L_z(1+\varepsilon_x+\varepsilon_y+\varepsilon_z+\varepsilon_x\varepsilon_y+\varepsilon_y\varepsilon_z+\varepsilon_z\varepsilon_x+\varepsilon_x\varepsilon_y\varepsilon_z)$$

$$\Delta V = V - V_0 = L_x L_y L_z(\varepsilon_x+\varepsilon_y+\varepsilon_z+\text{small terms})$$

$$\Delta V = (100)(40)(25)(-3.510\times 10^{-3} + 0 + 1.890\times 10^{-3})$$

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$\Delta A = -4.05\ \text{mm}^2$ ◀

$\Delta V = -162.0\ \text{mm}^3$ ◀

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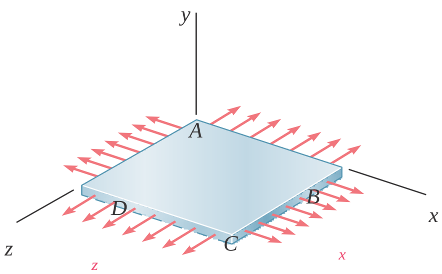


PROBLEM 2.71

The homogeneous plate *ABCD* is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the *x* direction must be zero, that is, $\varepsilon_x = 0$. Denoting by *E* the modulus of elasticity and by *v* Poisson's ratio, determine (*a*) the required magnitude of σ_x, (*b*) the ratio σ_0/ε_z.

SOLUTION

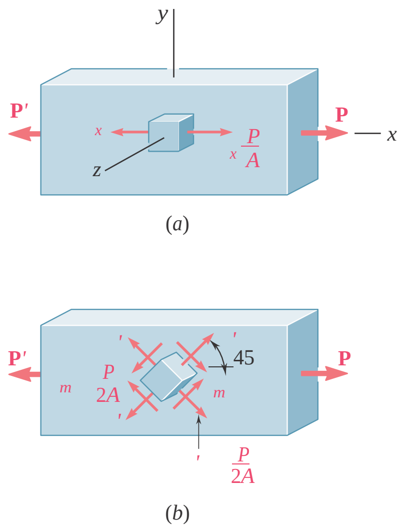
$$\sigma_z = \sigma_0, \quad \sigma_y = 0, \quad \varepsilon_x = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) = \frac{1}{E}(\sigma_x - v\sigma_0)$$

(*a*) $\sigma_x = v\sigma_0$ ◀

(*b*) $\varepsilon_z = \frac{1}{E}(-v\sigma_x - v\sigma_y + \sigma_z) = \frac{1}{E}(-v^2\sigma_0 - 0 + \sigma_0) = \frac{1-v^2}{E}\sigma_0$

$$\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-v^2}$$ ◀



PROBLEM 2.72

For a member under axial loading, express the normal strain ε' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ε_x by (*a*) comparing the hypotenuses of the triangles shown in Fig. 2.43, which represent, respectively, an element before and after deformation, (*b*) using the values of the corresponding stresses of σ' and σ_x shown in Fig. 1.38, and the generalized Hooke's law.

SOLUTION

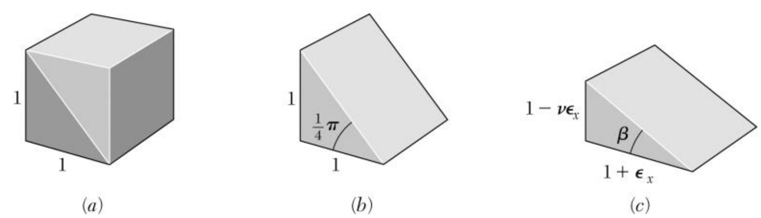


Figure 2.49

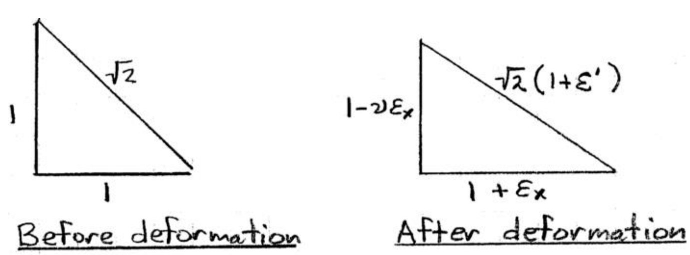
(*a*) $$[\sqrt{2}(1+\varepsilon')]^2 = (1+\varepsilon_x)^2 + (1-\nu\varepsilon_x)^2$$

$$2(1+2\varepsilon'+\varepsilon'^2) = 1+2\varepsilon_x+\varepsilon_x^2+1-2\nu\varepsilon_x+\nu^2\varepsilon_x^2$$

$$4\varepsilon'+2\varepsilon'^2 = 2\varepsilon_x+\varepsilon_x^2-2\nu\varepsilon_x+\nu^2\varepsilon_x^2$$

Neglect squares as small. $4\varepsilon' = 2\varepsilon_x - 2\nu\varepsilon_x$

$$\varepsilon' = \frac{1-\nu}{2}\varepsilon_x \quad ◀$$



(A) **(B)**

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PROBLEM 2.72 *(Continued)*

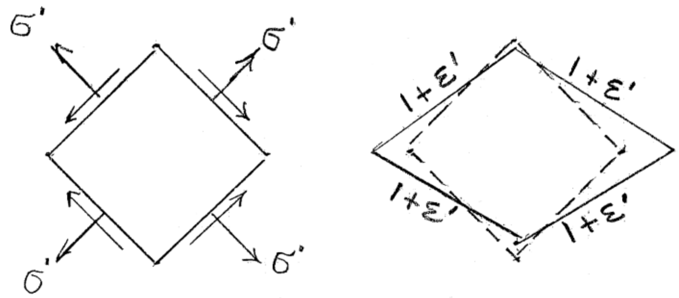
(*b*)

$$\varepsilon' = \frac{\sigma'}{E} - \frac{\nu\sigma'}{E}$$

$$= \frac{1-\nu}{E} \cdot \frac{P}{2A}$$

$$= \frac{1-\nu}{2E}\sigma_x$$

$$= \frac{1-\nu}{2}\varepsilon_x \quad ◀$$



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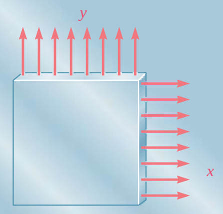


PROBLEM 2.73

In many situations, it is known that the normal stress in a given direction is zero. For example, $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress*, show that if the strains ε_x and ε_y have been determined experimentally, we can express σ_x, σ_y, and ε_z as follows:

$$\sigma_x = E\frac{\varepsilon_x + v\varepsilon_y}{1-v^2} \qquad \sigma_y = E\frac{\varepsilon_y + v\varepsilon_x}{1-v^2} \qquad \varepsilon_z = -\frac{v}{1-v}(\varepsilon_x + \varepsilon_y)$$

SOLUTION

$$\sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y) \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}(-v\sigma_x + \sigma_y) \qquad (2)$$

Multiplying (2) by v and adding to (1),

$$\varepsilon_x + v\varepsilon_y = \frac{1-v^2}{E}\sigma_x \quad \text{or} \quad \sigma_x = \frac{E}{1-v^2}(\varepsilon_x + v\varepsilon_y) \qquad ◀$$

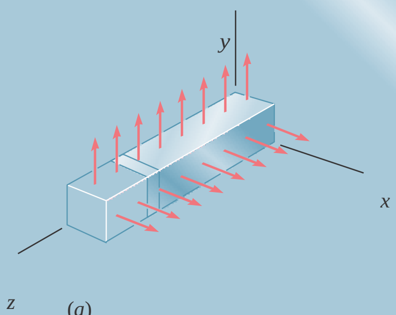
Multiplying (1) by v and adding to (2),

$$\varepsilon_y + v\varepsilon_x = \frac{1-v^2}{E}\sigma_y \quad \text{or} \quad \sigma_y = \frac{E}{1-v^2}(\varepsilon_y + v\varepsilon_x) \qquad ◀$$

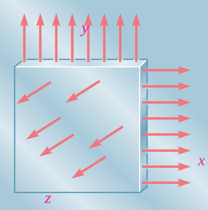
$$\varepsilon_z = \frac{1}{E}(-v\sigma_x - v\sigma_y) = -\frac{v}{E}\cdot\frac{E}{1-v^2}(\varepsilon_x + v\varepsilon_y + \varepsilon_y + v\varepsilon_x)$$

$$= -\frac{v(1+v)}{1-v^2}(\varepsilon_x + \varepsilon_y) = -\frac{v}{1-v}(\varepsilon_x + \varepsilon_y) \qquad ◀$$

PROBLEM 2.74



(*a*)



(*b*)

In many situations, physical constraints prevent strain from occurring in a given direction. For example, $\varepsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation, which is known as *plane strain*, we can express σ_z, ε_x, and ε_y as follows:

$$\sigma_z = v(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}[(1-v^2)\sigma_x - v(1+v)\sigma_y]$$

$$\varepsilon_y = \frac{1}{E}[(1-v^2)\sigma_y - v(1+v)\sigma_x]$$

SOLUTION

$$\varepsilon_z = 0 = \frac{1}{E}(-v\sigma_x - v\sigma_y + \sigma_z) \quad \text{or} \quad \sigma_z = v(\sigma_x + \sigma_y) \quad ◀$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z)$$

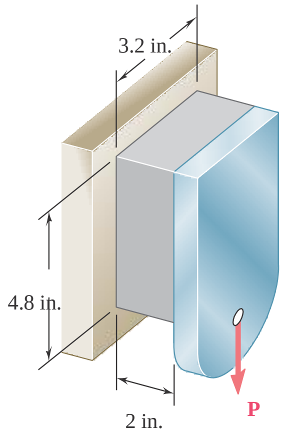
$$= \frac{1}{E}[\sigma_x - v\sigma_y - v^2(\sigma_x + \sigma_y)]$$

$$= \frac{1}{E}[(1-v^2)\sigma_x - v(1+v)\sigma_y] \quad ◀$$

$$\varepsilon_y = \frac{1}{E}(-v\sigma_x + \sigma_y - v\sigma_z)$$

$$= \frac{1}{E}[-v\sigma_x + \sigma_y - v^2(\sigma_x + \sigma_y)]$$

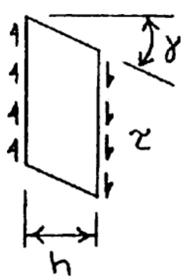
$$= \frac{1}{E}[(1-v^2)\sigma_y - v(1+v)\sigma_x] \quad ◀$$



PROBLEM 2.75

The plastic block shown is bonded to a rigid support and to a vertical plate to which a 55-kip load **P** is applied. Knowing that for the plastic used $G = 150$ ksi, determine the deflection of the plate.

SOLUTION



$$A = (3.2)(4.8) = 15.36 \text{ in}^2$$

$$P = 55 \times 10^3 \text{ lb}$$

$$\tau = \frac{P}{A} = \frac{55 \times 10^3}{15.36} = 3580.7 \text{ psi}$$

$$G = 150 \times 10^3 \text{ psi}$$

$$\gamma = \frac{\tau}{G} = \frac{3580.7}{150 \times 10^3} = 23.871 \times 10^{-3}$$

$$h = 2 \text{ in.}$$

$$\delta = h\gamma = (2)(23.871 \times 10^{-3})$$

$$= 47.7 \times 10^{-3} \text{ in.}$$

$$\delta = 0.0477 \text{ in.} \downarrow \blacktriangleleft$$

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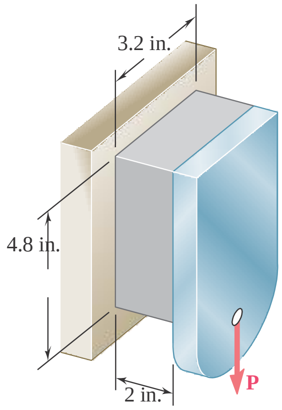
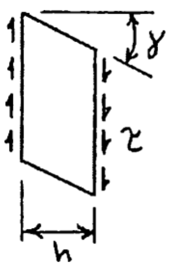
PROBLEM 2.76



What load **P** should be applied to the plate of Prob. 2.75 to produce a $\frac{1}{16}$-in. deflection?

PROBLEM 2.75 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 55-kip load **P** is applied. Knowing that for the plastic used $G = 150$ ksi, determine the deflection of the plate.

SOLUTION



$$\delta = \frac{1}{16} \text{ in.} = 0.0625 \text{ in.}$$

$$h = 2 \text{ in.}$$

$$\gamma = \frac{\delta}{h} = \frac{0.0625}{2} = 0.03125$$

$$G = 150 \times 10^3 \text{ psi}$$

$$\tau = G\gamma = (150 \times 10^3)(0.03125)$$

$$= 4687.5 \text{ psi}$$

$$A = (3.2)(4.8) = 15.36 \text{ in}^2$$

$$P = \tau A = (4687.5)(15.36)$$

$$= 72.0 \times 10^3 \text{ lb}$$

72.0 kips ◀

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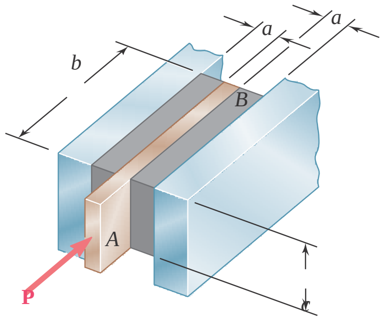
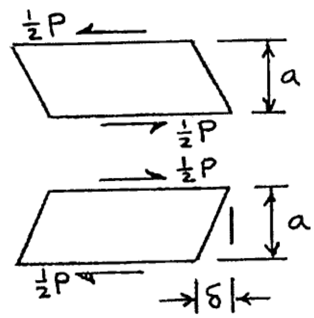


PROBLEM 2.77

Two blocks of rubber with a modulus of rigidity $G = 12$ MPa are bonded to rigid supports and to a plate *AB*. Knowing that $c = 100$ mm and $P = 45$ kN, determine the smallest allowable dimensions *a* and *b* of the blocks if the shearing stress in the rubber is not to exceed 1.4 MPa and the deflection of the plate is to be at least 5 mm.

SOLUTION



Shearing strain: $\gamma = \dfrac{\delta}{a} = \dfrac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(12\times 10^6 \text{ Pa})(0.005 \text{ m})}{1.4\times 10^6 \text{ Pa}} = 0.0429 \text{ m} \qquad a = 42.9 \text{ mm} \blacktriangleleft$$

Shearing stress: $\tau = \dfrac{\frac{1}{2}P}{A} = \dfrac{P}{2bc}$

$$b = \frac{P}{2c\tau} = \frac{45\times 10^3 \text{ N}}{2(0.1 \text{ m})(1.4\times 10^6 \text{ Pa})} = 0.1607 \text{ m} \qquad b = 160.7 \text{ mm} \blacktriangleleft$$

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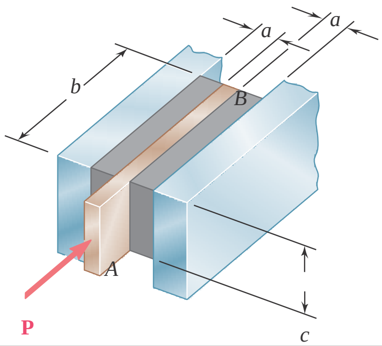
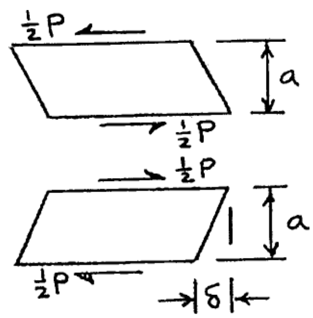


PROBLEM 2.78

Two blocks of rubber with a modulus of rigidity $G = 10$ MPa are bonded to rigid supports and to a plate AB. Knowing that $b = 200$ mm and $c = 125$ mm, determine the largest allowable load P and the smallest allowable thickness a of the blocks if the shearing stress in the rubber is not to exceed 1.5 MPa and the deflection of the plate is to be at least 6 mm.

SOLUTION



Shearing stress: $\tau = \dfrac{\frac{1}{2}P}{A} = \dfrac{P}{2bc}$

$P = 2bc\tau = 2(0.2\text{ m})(0.125\text{ m})(1.5\times 10^3\text{ kPa})$ $\qquad P = 75.0\text{ kN}$ ◀

Shearing strain: $\gamma = \dfrac{\delta}{a} = \dfrac{\tau}{G}$

$a = \dfrac{G\delta}{\tau} = \dfrac{(10\times 10^6\text{ Pa})(0.006\text{ m})}{1.5\times 10^6\text{ Pa}} = 0.04\text{ m}$ $\qquad a = 40.0\text{ mm}$ ◀

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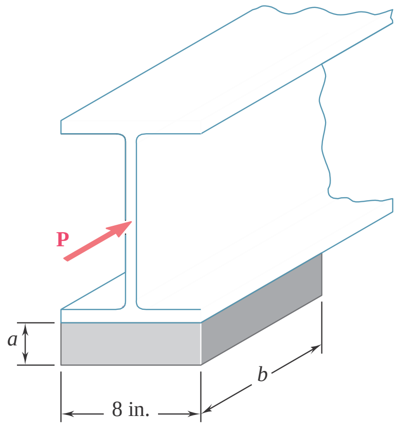
PROBLEM 2.79



An elastomeric bearing ($G = 130$ psi) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than $\frac{3}{8}$ in. when a 5-kip lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 60 psi, determine (*a*) the smallest allowable dimension *b*, (*b*) the smallest required thickness *a*.

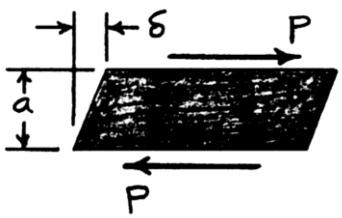
SOLUTION

Shearing force: $P = 5 \text{ kips} = 5000 \text{ lb}$

Shearing stress: $\tau = 60 \text{ psi}$

$$\tau = \frac{P}{A}, \quad \text{or} \quad A = \frac{P}{\tau} = \frac{5000}{60} = 83.333 \text{ in}^2$$

and $A = (8 \text{ in.})(b)$



(*a*) $b = \frac{A}{8} = \frac{83.333}{8} = 10.4166 \text{ in.}$ $\qquad b = 10.42 \text{ in.}$ ◀

$$\gamma = \frac{\tau}{G} = \frac{60}{130} = 461.54 \times 10^{-3} \text{ rad}$$

(*b*) But $\gamma = \frac{\delta}{a}$, or $a = \frac{\delta}{\gamma} = \frac{0.375 \text{ in.}}{461.54 \times 10^{-3}}$ $\qquad a = 0.813 \text{ in.}$ ◀

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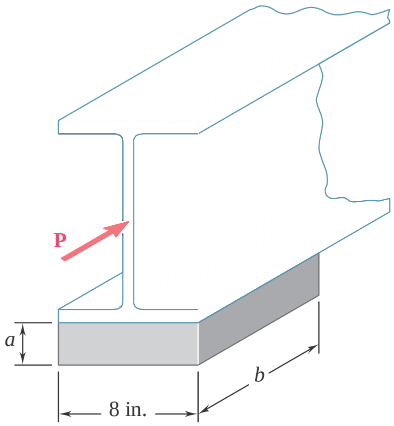
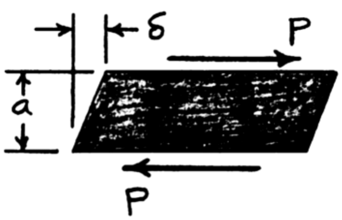


PROBLEM 2.80

For the elastomeric bearing in Prob. 2.79 with $b = 10$ in. and $a = 1$ in., determine the shearing modulus G and the shear stress τ for a maximum lateral load $P = 5$ kips and a maximum displacement $\delta = 0.4$ in.

SOLUTION



Shearing force: $P = 5 \text{ kips} = 5000 \text{ lb}$

Area: $A = (8 \text{ in.})(10 \text{ in.}) = 80 \text{ in}^2$

Shearing stress:

$$\tau = \frac{P}{A} = \frac{5000}{80} \qquad \tau = 62.5 \text{ psi} \blacktriangleleft$$

Shearing strain: $$\gamma = \frac{\delta}{a} = \frac{0.4 \text{ in.}}{1 \text{ in.}} = 0.400 \text{ rad}$$

Shearing modulus: $$G = \frac{\tau}{\gamma} = \frac{62.5}{0.400} \qquad G = 156.3 \text{ psi} \blacktriangleleft$$

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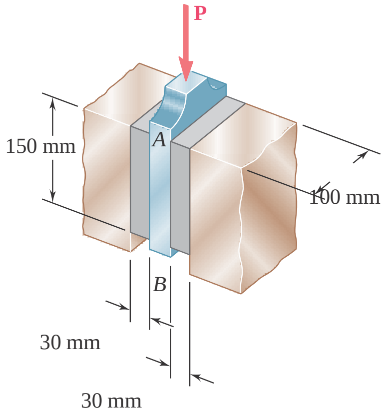
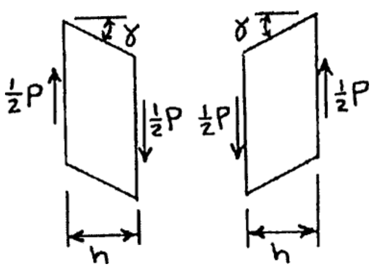


PROBLEM 2.81

A vibration isolation unit consists of two blocks of hard rubber bonded to a plate *AB* and to rigid supports as shown. Knowing that a force of magnitude $P = 25\text{ kN}$ causes a deflection $\delta = 1.5\text{ mm}$ of plate *AB*,

determine the modulus of rigidity of the rubber used.

SOLUTION



$$F = \frac{1}{2}P = \frac{1}{2}(25 \times 10^3\text{ N}) = 12.5 \times 10^3\text{ N}$$

$$\tau = \frac{F}{A} = \frac{(12.5 \times 10^3\text{ N})}{(0.15\text{ m})(0.1\text{ m})} = 833.33 \times 10^3\text{ Pa}$$

$$\delta = 1.5 \times 10^{-3}\text{ m} \qquad h = 0.03\text{ m}$$

$$\gamma = \frac{\delta}{h} = \frac{1.5 \times 10^{-3}}{0.03} = 0.05$$

$$G = \frac{\tau}{\gamma} = \frac{833.33 \times 10^3}{0.05} = 16.67 \times 10^6\text{ Pa}$$

$G = 16.67\text{ MPa}$ ◀

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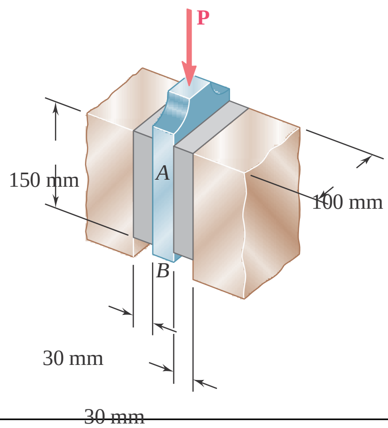
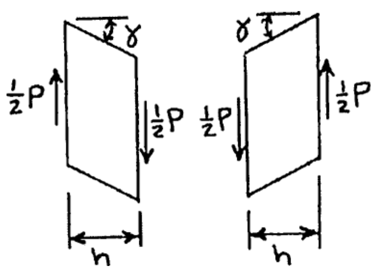


PROBLEM 2.82

A vibration isolation unit consists of two blocks of hard rubber with a modulus of rigidity $G = 19$ MPa bonded to a plate AB and to rigid supports as shown. Denoting by P the magnitude of the force applied to the plate and by δ the corresponding deflection, determine the effective spring constant, $k = P/\delta$, of the system.

SOLUTION



Shearing strain: $\gamma = \dfrac{\delta}{h}$

Shearing stress: $\tau = G\gamma = \dfrac{G\delta}{h}$

Force: $\dfrac{1}{2}P = A\tau = \dfrac{GA\delta}{h}$ or $P = \dfrac{2GA\delta}{h}$

Effective spring constant: $k = \dfrac{P}{\delta} = \dfrac{2GA}{h}$

with $A = (0.15)(0.1) = 0.015\text{ m}^2 \quad h = 0.03\text{ m}$

$$k = \frac{2(19\times 10^6\text{ Pa})(0.015\text{ m}^2)}{0.03\text{ m}} = 19.00\times 10^6\text{ N/m}$$

$$k = 19.00\times 10^3\text{ kN/m} \blacktriangleleft$$

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PROBLEM 2.83*

A 6-in.-diameter solid steel sphere is lowered into the ocean to a point where the pressure is 7.1 ksi (about 3 miles below the surface). Knowing that $E = 29\times10^6$ psi and $v = 0.30$, determine (*a*) the decrease in diameter of the sphere, (*b*) the decrease in volume of the sphere, (*c*) the percent increase in the density of the sphere.

SOLUTION

For a solid sphere,
$$V_0 = \frac{\pi}{6} d_0^3 = \frac{\pi}{6}(6.00)^3 = 113.097 \text{ in}^3$$

$$\sigma_x = \sigma_y = \sigma_z = -p = -7.1\times10^3 \text{ psi}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) = -\frac{(1-2v)p}{E} = -\frac{(0.4)(7.1\times10^3)}{29\times10^6} = -97.93\times10^{-6}$$

Likewise,
$$\varepsilon_y = \varepsilon_z = -97.93\times10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -293.79\times10^{-6}$$

(*a*) $-\Delta d = -d_0\varepsilon_x = -(6.00)(-97.93\times10^{-6}) = 588\times10^{-6}$ in. $\qquad -\Delta d = 588\times10^{-6}$ in. ◀

(*b*) $-\Delta V = -V_0 e = -(113.097)(-293.79\times10^{-6}) = 33.2\times10^{-3}$ in^3 $\qquad -\Delta V = 33.2\times10^{-3}$ in^3 ◀

(*c*) Let m = mass of sphere. $\qquad m$ = constant.

$$m = \rho_0 V_0 = \rho V = \rho V_0(1+e)$$

$$\frac{\rho-\rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = \frac{m}{V_0(1+e)}\times\frac{V_0}{m} - 1 = \frac{1}{1+e} - 1$$

$$= (1 - e + e^2 - e^3 + \cdots) - 1 = -e + e^2 - e^3 + \cdots$$
$$\approx -e = 293.79\times10^{-6}$$

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$$\frac{\rho_0}{\rho_0} \times 100\% = (293.79 \times 10^{-6})(100\%)$$

0.0294% ◀

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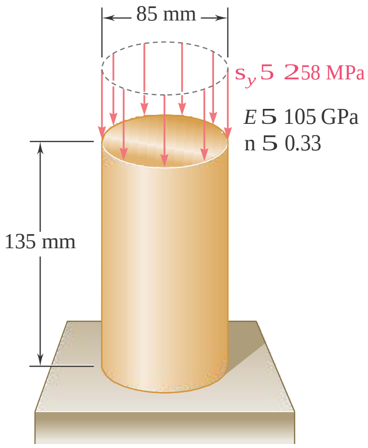

PROBLEM 2.84*

(*a*) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (*b*) Solve part *a*, assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70$ MPa.

SOLUTION

$$h_0 = 135 \text{ mm} = 0.135 \text{ m}$$

$$A_0 = \frac{\pi}{4} d_0^2 = \frac{\pi}{4}(85)^2 = 5.6745 \times 10^3 \text{ mm}^2 = 5.6745 \times 10^{-3} \text{ m}^2$$

$$V_0 = A_0 h_0 = 766.06 \times 10^3 \text{ mm}^3 = 766.06 \times 10^{-6} \text{ m}^3$$

(*a*) $\sigma_x = 0, \quad \sigma_y = -58 \times 10^6 \text{ Pa}, \quad \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{\sigma_y}{E} = -\frac{58 \times 10^6}{105 \times 10^9} = -552.38 \times 10^{-6}$$

$$\Delta h = h_0 \varepsilon_y = (135 \text{ mm})(-552.38 \times 10^{-6})$$

$\Delta h = -0.0746$ mm ◀

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(1-2\nu)\sigma_y}{E} = \frac{(0.34)(-58 \times 10^6)}{105 \times 10^9} = -187.81 \times 10^{-6}$$

$$\Delta V = V_0 e = (766.06 \times 10^3 \text{ mm}^3)(-187.81 \times 10^{-6})$$

$\Delta V = -143.9 \text{ mm}^3$ ◀

(*b*) $\sigma_x = \sigma_y = \sigma_z = -70 \times 10^6 \text{ Pa} \qquad \sigma_x + \sigma_y + \sigma_z = -210 \times 10^6 \text{ Pa}$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1-2\nu}{E}\sigma_y = \frac{(0.34)(-70 \times 10^6)}{105 \times 10^9} = -226.67 \times 10^{-6}$$

$$\Delta h = h_0 \varepsilon_y = (135 \text{ mm})(-226.67 \times 10^{-6})$$

$\Delta h = -0.0306$ mm ◀

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$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(0.34)(-210\times 10^6)}{105\times 10^9} = -680\times 10^{-6}$$

$$\Delta V = V_0 e = (766.06\times 10^3 \text{ mm}^3)(-680\times 10^{-6}) \qquad \Delta V = -521 \text{ mm}^3 \blacktriangleleft$$

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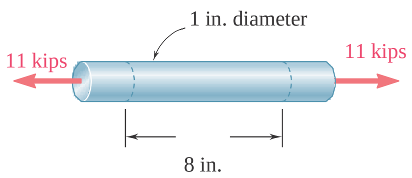

PROBLEM 2.85*

Determine the dilatation e and the change in volume of the 8-in. length of the rod shown if (a) the rod is made of steel with $E = 29 \times 10^6$ psi and $\nu = 0.30$, (b) the rod is made of aluminum with $E = 10.6 \times 10^6$ psi and $\nu = 0.35$.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1)^2 = 0.78540 \text{ in}^2$$

$$P = 11 \times 10^3 \text{ lb}$$

Stresses:
$$\sigma_x = \frac{P}{A} = \frac{11 \times 10^3}{0.78540} = 14.0056 \times 10^3 \text{ psi}$$

$$\sigma_y = \sigma_z = 0$$

(a) Steel. $E = 29 \times 10^6$ psi $\quad \nu = 0.30$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x}{E} = \frac{14.0056 \times 10^3}{29 \times 10^6} = 482.95 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = -\frac{\nu\sigma_x}{E} = -\nu\varepsilon_x = -(0.30)(482.95 \times 10^{-6})$$

$$= -144.885 \times 10^{-6}$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = -\frac{\nu\sigma_x}{E} = \varepsilon_y = -144.885 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 193.2 \times 10^{-6} \quad ◀$$

$$\Delta v = ve = \Delta Le = (0.78540)(8)(193.2 \times 10^{-6}) = 1.214 \times 10^{-3} \text{ in}^3 \quad ◀$$

(b) Aluminum. $E = 10.6 \times 10^6$ psi $\quad \nu = 0.35$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{14.0056 \times 10^3}{10.6 \times 10^6} = 1.32128 \times 10^{-3}$$

$$\varepsilon_y = -\nu\varepsilon_x = -(0.35)(1.32128 \times 10^{-3}) = -462.45 \times 10^{-6}$$

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$$\varepsilon_z = \varepsilon_y = -462.45 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 396 \times 10^{-6} \quad \blacktriangleleft$$

$$\Delta v = ve = \Delta L e = (0.78540)(8)(396 \times 10^{-6}) = 2.49 \times 10^{-3} \text{ in}^3 \quad \blacktriangleleft$$

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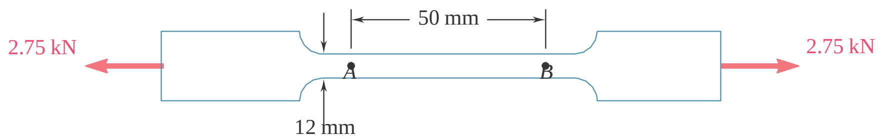
PROBLEM 2.86

Determine the change in volume of the 50-mm gage length segment *AB* in Prob. 2.64 (*a*) by computing the dilatation of the material, (*b*) by subtracting the original volume of portion *AB* from its final volume.

PROBLEM 2.64 A 2.75-kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate (E = 200 GPa, ν = 0.30). Determine the resulting change (*a*) in the 50-mm gage length, (*b*) in the width of portion *AB* of the test coupon, (*c*) in the thickness of portion *AB*, (*d*) in the cross-sectional area of portion *AB*.



SOLUTION

(*a*) $A_0 = (12)(1.6) = 19.2\ \text{mm}^2 = 19.2 \times 10^{-6}\ \text{m}^2$

Volume $V_0 = L_0 A_0 = (50)(19.2) = 960\ \text{mm}^3$

$$\sigma_x = \frac{P}{A_0} = \frac{2.75 \times 10^3}{19.2 \times 10^{-6}} = 143.229 \times 10^6\ \text{Pa} \qquad \sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \upsilon\sigma_y - \upsilon\sigma_z) = \frac{\sigma_x}{E} = \frac{143.229 \times 10^6}{200 \times 10^9} = 716.15 \times 10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -\upsilon\varepsilon_x = -(0.30)(716.15 \times 10^{-3}) = -214.84 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 286.46 \times 10^{-6}$$

$$\Delta v = v_0 e = (960)(286.46 \times 10^{-6}) = 0.275\ \text{mm}^3 \quad ◀$$

(*b*) From the solution to problem 2.64,

$$\delta_x = 0.035808\ \text{mm} \qquad \delta_y = -0.0025781 \qquad \delta_z = -0.00034374\ \text{mm}$$

The dimensions when under the 2.75-kN load are

Length: $L = L_0 + \delta_x = 50 + 0.035808 = 50.035808\ \text{mm}$

Width: $w = w_0 + \delta_y = 12 - 0.0025781 = 11.997422\ \text{mm}$

Thickness: $t = t_0 + \delta_z = 1.6 - 0.00034374 = 1.599656\ \text{mm}$

Volume: $V = Lwt = (50.03581)(11.997422)(1.599656) = 960.275\ \text{mm}^3$

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$$\Delta V = V - V_0 = 960.275 - 960 = 0.275 \text{ mm}^3 \quad \blacktriangleleft$$

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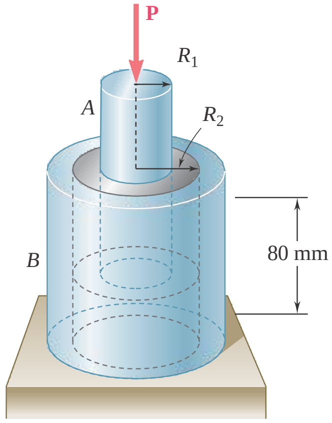
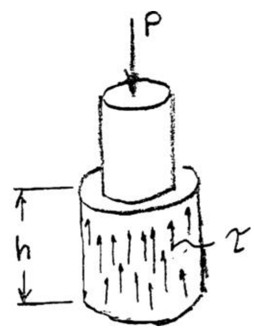


PROBLEM 2.87*

A vibration isolation support consists of a rod A of radius $R_1 = 10$ mm and a tube B of inner radius $R_2 = 25$ mm bonded to an 80-mm-long hollow rubber cylinder with a modulus of rigidity G = 12 MPa. Determine the largest allowable force **P** that can be applied to rod A if its deflection is not to exceed 2.50 mm.

SOLUTION

Let r be a radial coordinate. Over the hollow rubber cylinder, $R_1 \le r \le R_2$.



Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi rh}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi Ghr}$$

Shearing deformation over radial length dr:

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi Gh}\frac{dr}{r}$$

Total deformation.

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi Gh}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi Gh}\ln r\Big|_{R_1}^{R_2} = \frac{P}{2\pi Gh}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi Gh}\ln\frac{R_2}{R_1} \quad \text{or} \quad P = \frac{2\pi Gh\delta}{\ln(R_2/R_1)}$$

Data: $R_1 = 10 \text{ mm} = 0.010 \text{ m},$

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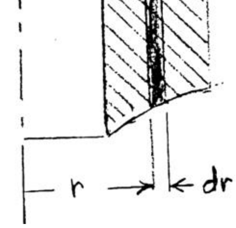

$R_2 = 25$ mm $= 0.025$ m, $h = 80$ mm $= 0.080$ m

$$G = 12\times10^{6}\ \text{Pa} \qquad \delta = 2.50\times10^{-3}\ \text{m}$$

$$P = \frac{(2\pi)(12\times10^{6})(0.080)(2.50\times10^{-3})}{\ln(0.025/0.010)} = 16.46\times10^{3}\ \text{N}$$

16.46 kN ◀

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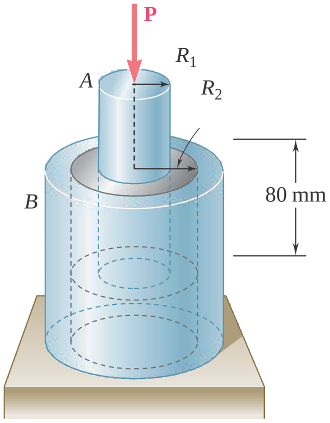
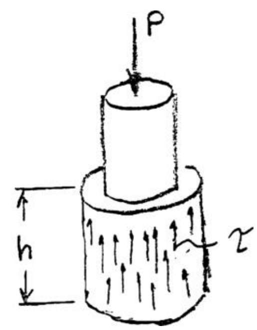


PROBLEM 2.88

A vibration isolation support consists of a rod A of radius R_1 and a tube B of inner radius R_2 bonded to a 80-mm-long hollow rubber cylinder with a modulus of rigidity G = 10.93 MPa. Determine the required value of the ratio R_2/R_1 if a 10-kN force **P** is to cause a 2-mm deflection of rod A.

SOLUTION

Let r be a radial coordinate. Over the hollow rubber cylinder, $R_1 \le r \le R_2$.



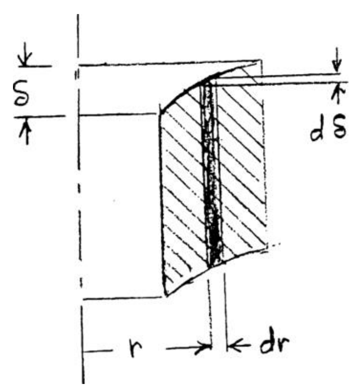
Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi rh}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi Ghr}$$

Shearing deformation over radial length dr:



$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr$$

$$dr\delta = \frac{P}{2\pi Gh}\frac{dr}{r}$$

Total deformation.

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi Gh}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi Gh}\ln r\Big|_{R_1}^{R_2} = \frac{P}{2\pi Gh}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi Gh}\ln\frac{R_2}{R_1}$$

$$\ln\frac{R_2}{R_1} = \frac{2\pi Gh\delta}{P} = \frac{(2\pi)(10.93\times 10^6)(0.080)(0.002)}{10.10^3} = 1.0988$$

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$$\frac{R_2}{R_1} = \exp(1.0988) = 3.00$$

$R_2/R_1 = 3.00$ ◀

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PROBLEM 2.89*

The material constants E, G, k, and v are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that (a) $k = GE/(9G - 3E)$ and (b) $v = (3k - 2G)/(6k + 2G)$.

SOLUTION

$$k = \frac{E}{3(1-2v)} \quad \text{and} \quad G = \frac{E}{2(1+v)}$$

(a) $1 + v = \frac{E}{2G}$ or $v = \frac{E}{2G} - 1$

$$k = \frac{E}{3\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]} = \frac{2EG}{3[2G - 2E + 4G]} = \frac{2EG}{18G - 6E}$$

$$k = \frac{EG}{9G - 6E} \blacktriangleleft$$

(b) $\frac{k}{G} = \frac{2(1+v)}{3(1-2v)}$

$$3k - 6kv = 2G + 2Gv$$

$$3k - 2G = 2G + 6k$$

$$v = \frac{3k - 2G}{6k + 2G} \blacktriangleleft$$

PROBLEM 2.90*

Show that for any given material, the ratio G/E of the modulus of rigidity over the modulus of elasticity is always less than $\frac{1}{2}$ but more than $\frac{1}{3}$. [*Hint:* Refer to Eq. (2.43) and to Sec. 2.13.]

SOLUTION

$$G = \frac{E}{2(1+v)} \qquad \text{or} \qquad \frac{E}{G} = 2(1+v)$$

Assume $v > 0$ for almost all materials, and $v < \frac{1}{2}$ for a positive bulk modulus.

Applying the bounds,
$$2 \le \frac{E}{G} < 2\left(1 + \frac{1}{2}\right) = 3$$

Taking the reciprocals,
$$\frac{1}{2} > \frac{G}{E} > \frac{1}{3}$$

or
$$\frac{1}{3} < \frac{G}{E} < \frac{1}{2} \quad ◀$$

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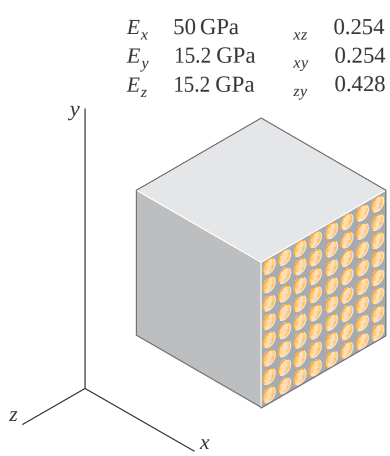

PROBLEM 2.91*

A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the x direction. The cube is constrained against deformations in the y and z directions and is subjected to a tensile load of 65 kN in the x direction. Determine (a) the change in the length of the cube in the x direction, (b) the stresses σ_x, σ_y, and σ_z.

$$E_x = 50 \text{ GPa} \quad \nu_{xz} = 0.254$$
$$E_y = 15.2 \text{ GPa} \quad \nu_{xy} = 0.254$$
$$E_z = 15.2 \text{ GPa} \quad \nu_{zy} = 0.428$$

SOLUTION

Stress-to-strain equations are

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \tag{1}$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \tag{2}$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \tag{3}$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \tag{4}$$

$$\frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \tag{5}$$

$$\frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \tag{6}$$

The constraint conditions are $\varepsilon_y = 0$ and $\varepsilon_z = 0$.

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y}\sigma_y - \frac{\nu_{zy}}{E_z}\sigma_z = \frac{\nu_{xy}}{E_x}\sigma_x \tag{7}$$

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$$-\frac{\nu_{yz}}{E_y}\sigma_y + \frac{1}{E_z}\sigma_z = \frac{\nu_{xz}}{E_x}\sigma_x \qquad (8)$$

$$\frac{1}{15.2}\sigma_y - \frac{0.428}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad \sigma_y - 0.428\sigma_z = 0.077216\sigma_x$$

$$-\frac{0.428}{15.2}\sigma_y + \frac{1}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad -0.428\sigma_y + \sigma_z = 0.077216\sigma_x$$

PROBLEM 2.91* *(Continued)*

Solving simultaneously, $\sigma_y = \sigma_z = 0.134993\sigma_x$

Using (4) and (5) in (1), $\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{xy}}{E_x}\sigma_y - \frac{\nu_{xz}}{E}\sigma_z$

$$E_x = \frac{1}{E_x}[1-(0.254)(0.134993)-(0.254)(0.134993)]\sigma_x$$

$$= \frac{0.93142\sigma_x}{E_x}$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600\times10^{-6} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{65\times10^3}{1600\times10^{-6}} = 40.625\times10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{(0.93142)(40.625\times10^3)}{50\times10^9} = 756.78\times10^{-6}$$

(*a*) $\delta_x = L_x\varepsilon_x = (40 \text{ mm})(756.78\times10^{-6})$ $\delta_x = 0.0303 \text{ mm}$ ◀

(*b*) $\sigma_x = 40.625\times10^6 \text{ Pa}$ $\sigma_x = 40.6 \text{ MPa}$ ◀

$\sigma_y = \sigma_z = (0.134993)(40.625\times10^6) = 5.48\times10^6 \text{ Pa}$ $\sigma_y = \sigma_z = 5.48 \text{ MPa}$ ◀

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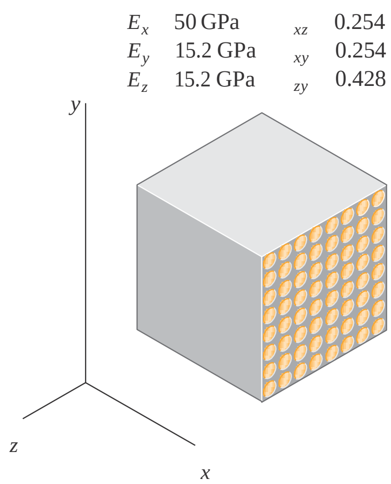

PROBLEM 2.92*

The composite cube of Prob. 2.91 is constrained against deformation in the z direction and elongated in the x direction by 0.035 mm due to a tensile load in the x direction. Determine (a) the stresses σ_x, σ_y, and σ_z and (b) the change in the dimension in the y direction.

$$E_x = 50 \text{ GPa} \quad \nu_{xz} = 0.254$$

$$E_y = 15.2 \text{ GPa} \quad \nu_{xy} = 0.254$$

$$E_z = 15.2 \text{ GPa} \quad \nu_{zy} = 0.428$$

SOLUTION

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \qquad (1)$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \qquad (2)$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \qquad (3)$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \qquad (4)$$

$$\frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \qquad (5)$$

$$\frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \qquad (6)$$

Constraint condition: $\varepsilon_z = 0$

Load condition: $\sigma_y = 0$

From Equation (3),

$$0 = -\frac{\nu_{xz}}{E_x}\sigma_x + \frac{1}{E_z}\sigma_z$$

$$\sigma_z = \frac{\nu_{xz}E_z}{E_x}\sigma_x = \frac{(0.254)(15.2)}{50} = 0.077216\sigma_x$$

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PROBLEM 2.92* *(Continued)*

From Equation (1) with $\sigma_y = 0$,

$$\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{zx}}{E_z}\sigma_z = \frac{1}{E_x}\sigma_x - \frac{\nu_{xz}}{E_x}\sigma_z$$

$$= \frac{1}{E_x}[\sigma_x - 0.254\sigma_z] = \frac{1}{E_x}[1 - (0.254)(0.077216)]\sigma_x$$

$$= \frac{0.98039}{E_x}\sigma_x$$

$$\sigma_x = \frac{E_x \varepsilon_x}{0.98039}$$

But $\quad \varepsilon_x = \dfrac{\delta_x}{L_x} = \dfrac{0.035 \text{ mm}}{40 \text{ mm}} = 875 \times 10^{-6}$

(*a*) $\quad \sigma_x = \dfrac{(50 \times 10^9)(875 \times 10^{-6})}{0.98039} = 44.625 \times 10^3 \text{ Pa}$ $\qquad \sigma_x = 44.6 \text{ MPa}$ ◀

$\sigma_y = 0$ ◀

$\sigma_z = (0.077216)(44.625 \times 10^6) = 3.446 \times 10^6 \text{ Pa}$ $\qquad \sigma_z = 3.45 \text{ MPa}$ ◀

From (2), $\quad \varepsilon_y = \dfrac{\nu_{xy}}{E_x}\sigma_x + \dfrac{1}{E_y}\sigma_y - \dfrac{\nu_{zy}}{E_z}\sigma_z$

$$= -\frac{(0.254)(44.625 \times 10^6)}{50 \times 10^9} + 0 - \frac{(0.428)(3.446 \times 10^6)}{15.2 \times 10^9}$$

$$= -323.73 \times 10^{-6}$$

(*b*) $\quad \delta_y = L_y \varepsilon_y = (40 \text{ mm})(-323.73 \times 10^{-6})$ $\qquad \delta_y = -0.0129 \text{ mm}$ ◀

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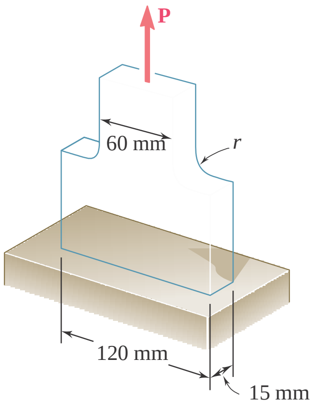

PROBLEM 2.93

Knowing that, for the plate shown, the allowable stress is 125 MPa, determine the maximum allowable value of P when (a) $r = 12$ mm, (b) $r = 18$ mm.

SOLUTION

$$A = (60)(15) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\frac{D}{d} = \frac{120 \text{ mm}}{60 \text{ mm}} = 2.00$$

(a) $r = 12$ mm $\qquad \dfrac{r}{d} = \dfrac{12 \text{ mm}}{60 \text{ mm}} = 0.2$

From Fig. 2.60b, $\quad K = 1.92 \quad \sigma_{max} = K\dfrac{P}{A}$

$$P = \frac{A\sigma_{max}}{K} = \frac{(900 \times 10^{-6})(125 \times 10^{6})}{1.92} = 58.6 \times 10^3 \text{ N}$$

$= 58.3$ kN ◀

(b) $r = 18$ mm, $\qquad \dfrac{r}{d} = \dfrac{18 \text{ mm}}{60 \text{ mm}} = 0.30 \qquad$ From Fig 2.60b, $\quad K = 1.75$

$$P = \frac{A\sigma_{max}}{K} = \frac{(900 \times 10^{-6})(125 \times 10^{6})}{1.75} = 64.3 \times 10^3 \text{ N}$$

$= 64.3$ kN ◀

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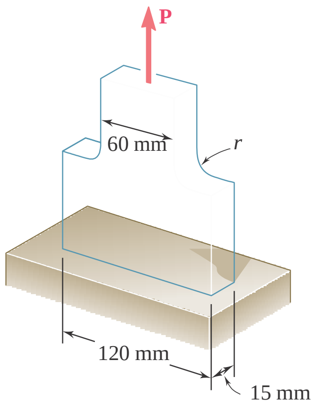
PROBLEM 2.94



Knowing that $P = 38$ kN, determine the maximum stress when (*a*) $r = 10$ mm, (*b*) $r = 16$ mm, (*c*) $r = 18$ mm.

SOLUTION

$$A = (60)(15) = 900 \text{ mm}^2 = 900\times10^{-6} \text{ m}^2$$

$$\frac{D}{d} = \frac{10 \text{ mm}}{60 \text{ mm}} = 2.00$$

(*a*) $r = 10$ mm $\qquad \dfrac{r}{d} = \dfrac{10 \text{ mm}}{60 \text{ mm}} = 0.1667$

From Fig. 2.60b, $\quad K = 2.06 \quad \sigma_{max} = \dfrac{KP}{A}$

$$\sigma_{max} = \frac{(2.06)(38\times10^3)}{900\times10^{-6}} = 87.0\times10^6 \text{ Pa} \qquad = 87.0 \text{ MPa} \blacktriangleleft$$

(*b*) $r = 16$ mm $\qquad \dfrac{r}{d} = \dfrac{16 \text{ mm}}{60 \text{ mm}} = 0.2667$

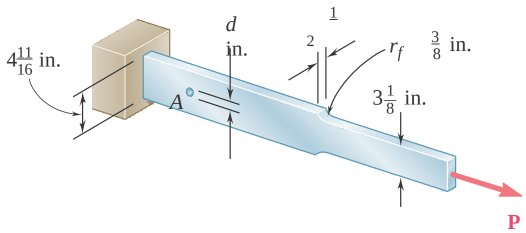
From Fig. 2.60b, $\quad K = 1.78$

$$\sigma_{max} = \frac{(1.78)(38\times10^3)}{900\times10^{-6}} = 75.2\times10^6 \text{ Pa} \qquad = 75.2 \text{ MPa} \blacktriangleleft$$

(*c*) $r = 18$ mm, $\qquad \dfrac{r}{d} = \dfrac{18 \text{ mm}}{60 \text{ mm}} = 0.30$

From Fig 2.60b, $\quad K = 1.75$

$$\sigma_{max} = \frac{(1.75)(38\times10^3)}{900\times10^{-6}} = 73.9\times10^6 \text{ Pa} \qquad = 73.9 \text{ MPa} \blacktriangleleft$$



PROBLEM 2.95

A hole is to be drilled in the plate at A. The diameters of the bits available to drill the hole range from $\frac{1}{2}$ to $1\frac{1}{2}$ in. in $\frac{1}{4}$-in. increments. If the allowable stress in the plate is 21 ksi, determine (*a*) the diameter d of the largest bit that can be used if the allowable load **P** at the hole is to exceed that at the fillets, (*b*) the corresponding allowable load **P**.

SOLUTION

At the fillets: $\frac{D}{d} = \frac{4.6875}{3.125} = 1.5 \qquad \frac{r}{d} = \frac{0.375}{3.125} = 0.12$

From Fig. 2.60b, $K = 2.10$

$$A_{min} = (3.125)(0.5) = 1.5625\text{ in}^2$$

$$\sigma_{max} = K\frac{P_{all}}{A_{min}} = \sigma_{all}$$

$$P_{all} = \frac{A_{min}\sigma_{all}}{K} = \frac{(1.5625)(21)}{2.10} = 15.625\text{ kips}$$

At the hole: $A_{net} = (D - 2r)t,\ K$ from Fig. 2.60a

$$\sigma_{max} = K\frac{P}{A_{net}} = \sigma_{all} \qquad \therefore \qquad P_{all} = \frac{A_{net}\sigma_{all}}{K}$$

with $D = 4.6875\text{ in.} \qquad t = 0.5\text{ in.} \qquad \sigma_{all} = 21\text{ ksi}$

Hole diam.	r	$d = D - 2r$	$2r/D$	K	A_{net}	P_{all}
0.5 in.	0.25 in.	4.1875 in.	0.107	2.68	2.0938 in^2	16.41 kips
0.75 in.	0.375 in.	3.9375 in.	0.16	2.58	1.96875 in^2	16.02 kips
1 in.	0.5 in.	3.6875 in.	0.213	2.49	1.84375 in^2	15.55 kips
1.25 in.	0.625 in.	3.4375 in.	0.267	2.41	1.71875 in^2	14.98 kips
1.5 in.	0.75 in.	3.1875 in.	0.32	2.34	1.59375 in^2	14.30 kips

(*a*) Largest hole with $P_{all} > 15.625\text{ kips}$ is the $\frac{3}{4}$-in.-diameter hole. ◀

(*b*) Allowable load $P_{all} = 15.63$ kips ◀

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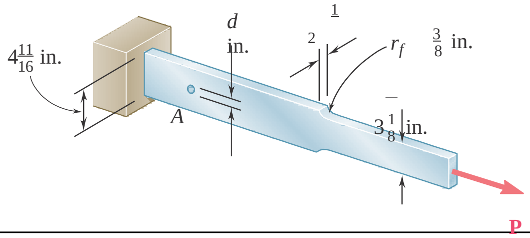

PROBLEM 2.96

(*a*) For $P = 13$ kips and $d = \frac{1}{2}$ in., determine the maximum stress in the plate shown. (*b*) Solve part *a*, assuming that the hole at *A* is not drilled.

SOLUTION

Maximum stress at hole:

Use Fig. 2.60a for values of *K*.

$$\frac{2r}{D} = \frac{0.5}{4.6875} = 0.017, \qquad K = 2.68$$

$$A_{net} = (0.5)(4.6875 - 0.5) = 2.0938 \text{ in}^2$$

$$\sigma_{max} = K\frac{P}{A_{net}} = \frac{(2.68)(13)}{2.0938} = 16.64 \text{ ksi}$$

Maximum stress at fillets:

Use Fig. 2.60b for values of *K*.

$$\frac{r}{d} = \frac{0.375}{3.125} = 0.12 \qquad \frac{D}{d} = \frac{4.6875}{3.125} = 1.5 \qquad K = 2.10$$

$$A_{min} = (0.5)(3.125) = 1.5625 \text{ in}^2$$

$$\sigma_{max} = K\frac{P}{A_{min}} = \frac{(2.10)(13)}{1.5625} = 17.47 \text{ ksi}$$

(*a*) With hole and fillets: 17.47 ksi ◀

(*b*) Without hole: 17.47 ksi ◀

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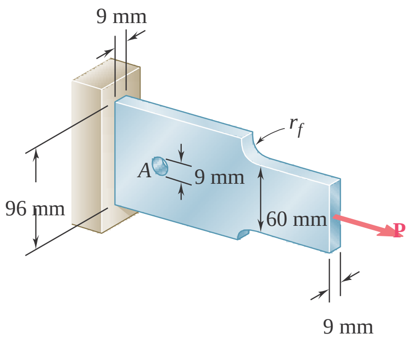


PROBLEM 2.97

Knowing that the hole has a diameter of 9 mm, determine (*a*) the radius r_f of the fillets for which the same maximum stress occurs at the hole *A* and at the fillets, (*b*) the corresponding maximum allowable load **P** if the allowable stress is 100 MPa.

SOLUTION

For the circular hole, $r = \left(\frac{1}{2}\right)(9) = 4.5 \text{ mm}$

$$d = 96 - 9 = 87 \text{ mm} \qquad \frac{2r}{D} = \frac{2(4.5)}{96} = 0.09375$$

$$A_{net} = dt = (0.087 \text{ m})(0.009 \text{ m}) = 783 \times 10^{-6} \text{ m}^2$$

From Fig. 2.60a, $K_{hole} = 2.72$

$$\sigma_{max} = \frac{K_{hole}P}{A_{net}}$$

$$P = \frac{A_{net}\sigma_{max}}{K_{hole}} = \frac{(783 \times 10^{-6})(100 \times 10^6)}{2.72} = 28.787 \times 10^3 \text{ N}$$

(*a*) For fillet, $D = 96 \text{ mm}, \ d = 60 \text{ mm}$

$$\frac{D}{d} = \frac{96}{60} = 1.60$$

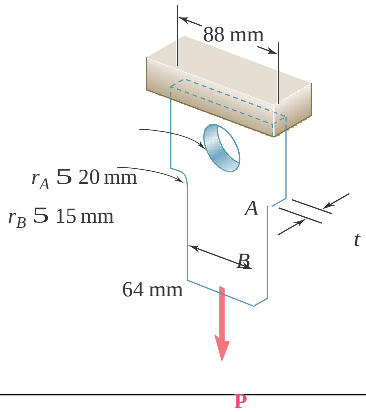
$$A_{min} = dt = (0.060 \text{ m})(0.009 \text{ m}) = 540 \times 10^{-6} \text{ m}^2$$

$$\sigma_{max} = \frac{K_{fillet}P}{A_{min}} \quad \therefore \quad K_{fillet} = \frac{A_{min}\sigma_{max}}{P} = \frac{(5.40 \times 10^{-6})(100 \times 10^6)}{28.787 \times 10^3}$$

$$= 1.876$$

From Fig. 2.60b, $\frac{r_f}{d} \approx 0.19 \quad \therefore \quad r_f \approx 0.19d = 0.19(60) \qquad r_f = 11.4 \text{ mm}$ ◀

(*b*) $P = 28.8 \text{ kN}$ ◀



PROBLEM 2.98

For $P = 100$ kN, determine the minimum plate thickness t required if the allowable stress is 125 MPa.

SOLUTION

At the hole: $r_A = 20\text{ mm} \qquad d_A = 88 - 40 = 48\text{ mm}$

$$\frac{2r_A}{D_A} = \frac{2(20)}{88} = 0.455$$

From Fig. 2.60a, $K = 2.20$

$$\sigma_{max} = \frac{KP}{A_{net}} = \frac{KP}{d_A t} \quad \therefore \quad t = \frac{KP}{d_A \sigma_{max}}$$

$$t = \frac{(2.20)(100 \times 10^3\text{ N})}{(0.048\text{ m})(125 \times 10^6\text{ Pa})} = 36.7 \times 10^{-3}\text{ m} = 36.7\text{ mm}$$

At the fillet: $D = 88\text{ mm}, \quad d_B = 64\text{ mm} \qquad \frac{D}{d_B} = \frac{88}{64} = 1.375$

$$r_B = 15\text{ mm} \qquad \frac{r_B}{d_B} = \frac{15}{64} = 0.2344$$

From Fig. 2.60b, $K = 1.70$

$$\sigma_{max} = \frac{KP}{A_{min}} = \frac{KP}{d_B t}$$

$$t = \frac{KP}{d_B \sigma_{max}} = \frac{(1.70)(100 \times 10^3\text{ N})}{(0.064\text{ m})(125 \times 10^6\text{ Pa})} = 21.25 \times 10^{-3}\text{ m} = 21.25\text{ mm}$$

The larger value is the required minimum plate thickness.

$t = 36.7\text{ mm}$ ◀

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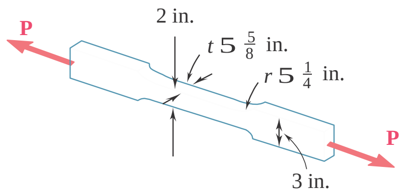

PROBLEM 2.99

(*a*) Knowing that the allowable stress is 20 ksi, determine the maximum allowable magnitude of the centric load **P**. (*b*) Determine the percent change in the maximum allowable magnitude of **P** if the raised portions are removed at the ends of the specimen.

SOLUTION

$$\frac{D}{d} = \frac{3}{2} = 1.50 \qquad \frac{r}{d} = \frac{0.250}{2} = 0.125$$

From Fig. 2.60b, $K = 2.08$

$$A_{min} = td = (0.625)(2) = 1.25\ \text{in}^2$$

(*a*) $\sigma_{max} = \dfrac{KP}{A_{min}} \qquad \therefore \qquad P = \dfrac{A_{min}\sigma_{max}}{K} = \dfrac{(1.25)(20)}{2.08} = 12.0192$ kips $\qquad P = 12.02$ kips ◀

(*b*) Without raised section, $K = 1.00$

$$P = A_{min}\sigma_{max} = (1.25)(20) = 25\ \text{kips}$$

$$\%\ \text{change} = \left(\frac{25 - 12.02}{12.02}\right) \times 100\% \qquad = 108.0\%$$ ◀

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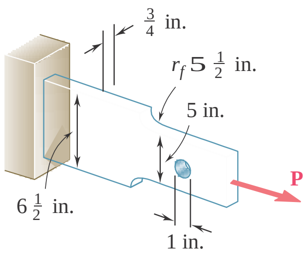

PROBLEM 2.100

A centric axial force is applied to the steel bar shown. Knowing that $\sigma_{all} = 20$ ksi, determine the maximum allowable load **P**.

SOLUTION

At the hole: $r = 0.5$ in. $\quad d = 5 - 1 = 4$ in.

$$\frac{2r}{d} = \frac{2(0.5)}{5} = 0.2 \qquad \text{From Fig. 2.60a,} \qquad K = 2.51$$

$$A_{net} = td = (0.75)(4) = 3\ \text{in}^2$$

$$\sigma_{max} = \frac{KP}{A_{net}}$$

$$P = \frac{A_{net}\sigma_{max}}{K} = \frac{(3)(20)}{2.51} = 23.9\ \text{kips}$$

At the fillet: $D = 6.5$ in., $\quad d = 5$ in., $\quad \frac{D}{d} = \frac{6.5}{5} = 1.3$

$$r = 0.5\ \text{in.} \qquad \frac{r}{d} = \frac{0.5}{5} = 0.1$$

From Fig. 2.60b, $\quad K = 2.04$

$$A_{min} = td = (0.75)(5) = 3.75\ \text{in}^2$$

$$\sigma_{max} = \frac{KP}{A_{min}}$$

$$P = \frac{A_{min}\sigma_{max}}{K} = \frac{(3.75)(20)}{2.04} = 36.8\ \text{kips}$$

Smaller value for P controls. $\qquad P = 23.9$ kips ◀

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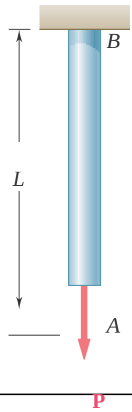
PROBLEM 2.101



The cylindrical rod *AB* has a length L = 5 ft and a 0.75-in. diameter; it is made of a mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 36$ ksi. A force **P** is applied to the bar and then removed to give it a permanent set δ_P. Determine the maximum value of the force **P** and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_P is (*a*) 0.1 in., (*b*) 0.2 in.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2 \qquad L = 5 \text{ ft} = 60 \text{ in.}$$

$$\delta_y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(60)(36 \times 10^3)}{29 \times 10^3} = 0.074483 \text{ in.}$$

When δ_m exceeds δ_Y, thus causing permanent stretch δ_p, the maximum force is

$$P_m = A\sigma_Y = (0.44179)(36 \times 10^3) = 15.9043 \times 10^3 \text{ lb}$$

$P = 15.90$ kips ◀

$$\delta_p = \delta_m - \delta' = \delta_m - \delta_Y \quad \text{so that} \quad \delta_m = \delta_p + \delta_Y$$

(*a*) $\delta_p = 0.1$ in. $\quad \delta_m = 0.1 + 0.074483 = 0.1745$ in. ◀

(*b*) $\delta_p = 0.2$ in. $\quad \delta_m = 0.2 + 0.074483 = 0.274$ in. ◀

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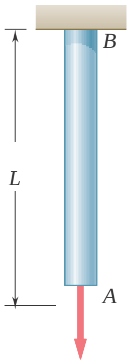


PROBLEM 2.102

The cylindrical rod *AB* has a length $L = 6$ ft and a 1.25-in. diameter; it is made of a mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 36$ ksi. A force **P** is applied to the bar until end *A* has moved down by an amount δ_m. Determine the maximum value of the force **P** and the permanent set of the bar after the force has been removed, knowing (*a*) $\delta_m = 0.125$ in., (*b*) $\delta_m = 0.250$ in.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1.25)^2 = 1.22718 \text{ in}^2 \qquad L = 6 \text{ ft} = 72 \text{ in.}$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(72)(36\times 10^3)}{29\times 10^3} = 0.089379 \text{ in.}$$

If $\delta_m \geq \delta_Y$, $\quad P_m = A\sigma_Y = (1.22718)(36\times 10^3)$

$$= 44.179\times 10^3 \text{ lb} = 44.2 \text{ kips}$$

(*a*) $\quad \delta_m = 0.125 \text{ in.} > \delta_Y \quad$ so that $\quad P_m = 44.2$ kips ◀

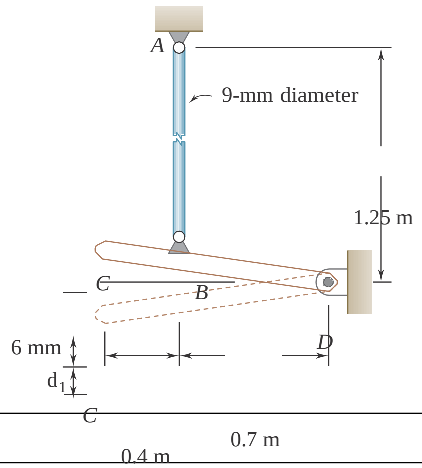
$$\delta' = \frac{P_m L}{AE} = \frac{\sigma_Y L}{E} = \delta_Y = 0.089379$$

$$\delta_p = \delta_m - \delta' = 0.125 - 0.089379 = 0.356 \text{ in.}$$ ◀

(*b*) $\quad \delta_m = 0.250 \text{ in.} > \delta_Y \quad$ so that $\quad P_m = 44.2$ kips ◀

$$\delta' = \delta_Y$$

$$\delta_p = \delta_m - \delta' = 0.250 - 0.089379 = 0.1606 \text{ in.}$$ ◀



PROBLEM 2.103

Rod *AB* is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is 6 mm too high. A vertical force **Q** is then applied at *C* until this point has moved to position C'. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

Since rod *AB* is to be stretched permanently,

$$(F_{AB})_{\max} = A_{AB}\sigma_Y = (63.617 \times 10^{-6})(345 \times 10^6)$$

$$= 21.948 \times 10^3 \text{ N}$$

$$+\circlearrowleft \sum M_D = 0: \quad 1.1Q - 0.7F_{AB} = 0$$

$$Q_{\max} = \frac{0.7}{1.1}(21.948 \times 10^3) = 13.9669 \times 10^{-3} \text{ N}$$

13.97 kN ◀

$$\delta'_{AB} = \frac{(F_{AB})_{\max} L_{AB}}{EA_{AB}} = \frac{(21.948 \times 10^3)(1.25)}{(200 \times 10^9)(63.617 \times 10^{-6})} = 2.15625 \times 10^{-3} \text{ m}$$

$$\theta' = \frac{\delta_{AB'}}{0.7} = 3.0804 \times 10^{-3} \text{ rad}$$

$$\delta_1 = 1.1\theta' = 3.39 \times 10^{-3} \text{ m}$$

3.39 mm ◀

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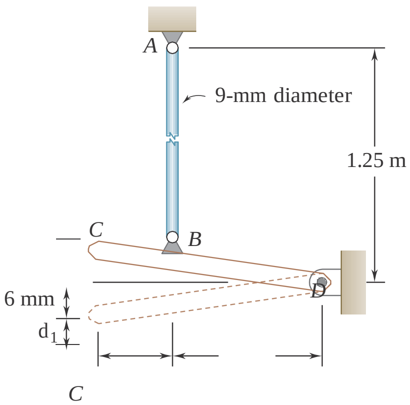

PROBLEM 2.104

Solve Prob. 2.103, assuming that the yield point of the mild steel is 250 MPa.

PROBLEM 2.103 Rod *AB* is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is 6 mm too high. A vertical force **Q** is then applied at *C* until this point has moved to position C'. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

Since rod *AB* is to be stretched permanently,

$$(F_{AB})_{\max} = A_{AB}\sigma_Y = (63.617 \times 10^{-6})(250 \times 10^6)$$

$$= 15.9043 \times 10^3 \text{ N}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad 1.1Q - 0.7F_{AB} = 0$$

$$Q_{\max} = \frac{0.7}{1.1}(15.9043 \times 10^3) = 10.12 \times 10^3 \text{ N}$$

10.12 kN ◀

$$\delta'_{AB} = \frac{(F_{AB})_{\max} L_{AB}}{EA_{AE}} = \frac{(15.9043 \times 10^3)(1.25)}{(200 \times 10^9)(63.617 \times 10^{-6})} = 1.5625 \times 10^{-3} \text{ m}$$

$$\theta' = \frac{\delta'_{AB}}{0.7} = 2.2321 \times 10^{-3} \text{ rad}$$

$$\delta_1 = 1.1\theta' = 2.46 \times 10^{-3} \text{ m}$$

2.46 mm ◀

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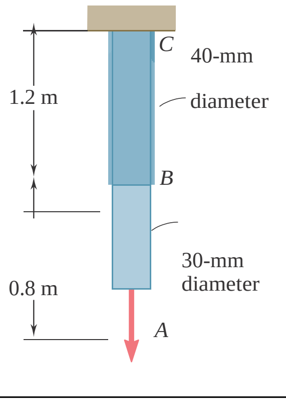
PROBLEM 2.105



Rod ABC consists of two cylindrical portions AB and BC; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod and then removed to give it a permanent set $\delta_p = 2$ mm. Determine the maximum value of the force **P** and the maximum amount δ_m by which the rod should be stretched to give it the desired permanent set.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

$$P_{max} = A_{min}\sigma_Y = (706.86 \times 10^{-6})(250 \times 10^6) = 176.715 \times 10^3 \text{ N}$$

$$P_{max} = 176.7 \text{ kN} \blacktriangleleft$$

$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715 \times 10^3)(0.8)}{(200 \times 10^9)(706.86 \times 10^{-6})} + \frac{(176.715 \times 10^3)(1.2)}{(200 \times 10^9)(1.25664 \times 10^{-3})}$$

$$= 1.84375 \times 10^{-3} \text{ m} = 1.84375 \text{ mm}$$

$$\delta_p = \delta_m - \delta' \text{ or } \delta_m = \delta_p + \delta' = 2 + 1.84375 \qquad \delta_m = 3.84 \text{ mm} \blacktriangleleft$$

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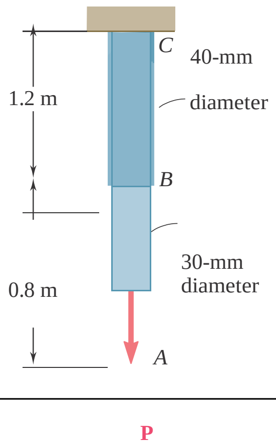


PROBLEM 2.106

Rod *ABC* consists of two cylindrical portions *AB* and *BC*; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod until its end *A* has moved down by an amount $\delta_m = 5$ mm. Determine the maximum value of the force **P** and the permanent set of the rod after the force has been removed.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25644 \times 10^{-3} \text{ m}^2$$

$$P_{max} = A_{min}\,\sigma_Y = (706.86 \times 10^{-6})(250 \times 10^6) = 176.715 \times 10^3 \text{ N}$$

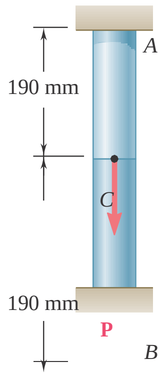
$$P_{max} = 176.7 \text{ kN} \blacktriangleleft$$

$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715 \times 10^3)(0.8)}{(200 \times 10^9)(706.68 \times 10^{-6})} + \frac{(176.715 \times 10^3)(1.2)}{(200 \times 10^9)(1.25664 \times 10^{-3})}$$

$$= 1.84375 \times 10^{-3} \text{ m} = 1.84375 \text{ mm}$$

$$\delta_p = \delta_m - \delta' = 5 - 1.84375 = 3.16 \text{ mm} \qquad \delta_p = 3.16 \text{ mm} \blacktriangleleft$$

PROBLEM 2.107



Rod *AB* consists of two cylindrical portions *AC* and *BC*, each with a cross-sectional area of 1750 mm^2. Portion *AC* is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion *CB* is made of a high-strength steel with $E = 200$ GPa

elastoplastic, determine (*a*) the maximum deflection of *C* if *P* is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of *C*.

SOLUTION

Displacement at *C* to cause yielding of *AC*.

$$\delta_{C,Y} = L_{AC}\varepsilon_{Y,AC} = \frac{L_{AC}\sigma_{Y,AC}}{E} = \frac{(0.190)(250\times10^6)}{200\times10^9} = 0.2375\times10^{-3}\text{ m}$$

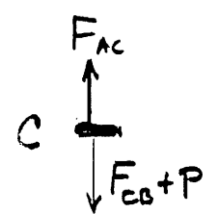
Corresponding force.

$$F_{AC} = A\sigma_{Y,AC} = (1750\times10^{-6})(250\times10^6) = 437.5\times10^3\text{ N}$$

$$F_{CB} = -\frac{EA\delta_C}{L_{CB}} = -\frac{(200\times10^9)(1750\times10^{-6})(0.2375\times10^{-3})}{0.190} = -437.5\times10^3\text{ N}$$

For equilibrium of element at *C*,

$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 875\times10^3\text{ N}$$



Since applied load $P = 975\times10^3\text{ N} > 875\times10^3\text{ N}$, portion *AC* yields.

$$F_{CB} = F_{AC} - P = 437.5\times10^3 - 975\times10^3\text{ N} = -537.5\times10^3\text{ N}$$

(*a*) $$\delta_C = -\frac{F_{CB}L_{CD}}{EA} = \frac{(537.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.29179\times10^{-3}\text{ m}$$

0.292 mm ◀

(*b*) Maximum stresses: $\sigma_{AC} = \sigma_{Y,AC} = 250$ MPa   250 MPa ◀

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{537.5\times10^3}{1750\times10^{-6}} = -307.14\times10^6\text{ Pa} = -307\text{ MPa}$$

−307 MPa ◀

(*c*) Deflection and forces for unloading.

$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad\therefore\quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P'_{AC}$$

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$$P' = 975 \times 10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \qquad P'_{AC} = 487.5 \times 10^{-3}\ \text{N}$$

$$\delta' = \frac{(487.5 \times 10^3)(0.190)}{(200 \times 10^9)(1750 \times 10^{-6})} = 0.26464 \times 10^3\ \text{m}$$

$$\delta_p = \delta_m - \delta' = 0.29179 \times 10^{-3} - 0.26464 \times 10^{-3}$$

$$= 0.02715 \times 10^{-3}\ \text{m}$$

0.0272 mm ◀

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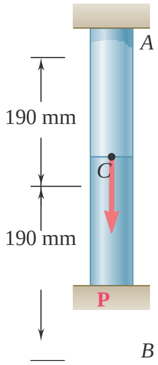
PROBLEM 2.108



For the composite rod of Prob. 2.107, if P is gradually increased from zero until the deflection of point C reaches a maximum value of $\delta_m = 0.3 \text{ mm}$ and then decreased back to zero, determine (a) the maximum value of P, (b) the maximum stress in each portion of the rod, (c) the permanent deflection of C after the load is removed.

PROBLEM 2.107 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm . Portion AC is made of a mild steel with $E = 200 \text{ GPa}$ and $\sigma_Y = 250 \text{ MPa}$, and portion CB is made of a high-strength steel with $E = 200 \text{ GPa}$ and $\sigma_Y = 345 \text{ MPa}$. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (a) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (b) the maximum stress in each portion of the rod, (c) the permanent deflection of C.

SOLUTION

Displacement at C is $\delta_m = 0.30 \text{ mm}$. The corresponding strains are

$$\varepsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.30 \text{ mm}}{190 \text{ mm}} = 1.5789 \times 10^{-3}$$

$$\varepsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.30 \text{ mm}}{190 \text{ mm}} = -1.5789 \times 10^{-3}$$

Strains at initial yielding:

$$\varepsilon_{Y,\,AC} = \frac{\sigma_{Y,\,AC}}{E} = \frac{250 \times 10^6}{200 \times 10^9} = 1.25 \times 10^{-3} \qquad \text{(yielding)}$$

$$\varepsilon_{Y,\,CB} = \frac{\sigma_{Y,\,BC}}{E} = -\frac{345 \times 10^6}{200 \times 10^9} = -1.725 \times 10^{-3} \qquad \text{(elastic)}$$

(a) Forces: $F_{AC} = A\sigma_Y = (1750 \times 10^{-6})(250 \times 10^6) = 437.5 \times 10^{-3} \text{ N}$

$$F_{CB} = EA\varepsilon_{CB} = (200 \times 10^9)(1750 \times 10^{-6})(-1.5789 \times 10^{-3}) = -552.6 \times 10^{-3} \text{ N}$$

For equilibrium of element at C, $F_{AC} - F_{CB} - P = 0$

$$P = F_{AC} - F_{CD} = 437.5 \times 10^3 + 552.6 \times 10^3 = 990.1 \times 10^3 \text{N}$$

990 kN ◀

(b) Stresses: AC: $\sigma_{AC} = \sigma_{Y,\,AC}$ 250 MPa ◀

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$$CB: \quad \sigma_{CB} = \frac{F_{CB}}{A} = -\frac{552.6\times 10^3}{1750\times 10^{-6}} = -316\times 10^6 \text{ Pa} \qquad -316 \text{ MPa} \blacktriangleleft$$

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PROBLEM 2.108 *(Continued)*

(*c*) <u>Deflection and forces for unloading</u>.

$$\delta' = \frac{P'_{AC} L_{AC}}{EA} = -\frac{P'_{CB} L_{CB}}{EA} \quad \therefore \; P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P_{AC}$$

$$P' = P'_{AC} - P'_{CB} = 2P'_{AC} = 990.1\times 10^3 \text{ N} \quad \therefore \; P'_{AC} = 495.05\times 10^3 \text{ N}$$

$$\delta' = \frac{(495.05\times 10^3)(0.190)}{(200\times 10^9)(1750\times 10^{-6})} = 0.26874\times 10^{-3} \text{ m} = 0.26874 \text{ mm}$$

$$\delta_p = \delta_m - \delta' = 0.30 \text{ mm} - 0.26874 \text{ mm}$$

0.0313 mm ◀

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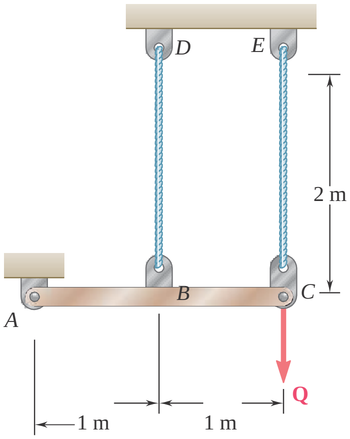


PROBLEM 2.109

Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force

Q is applied at *C* to the rigid bar *ABC* and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable *BD*, (*b*) the maximum deflection of point *C*, (*c*) the final displacement of point *C*. (*Hint:* In part *c*, cable *CE* is not taut.)

SOLUTION

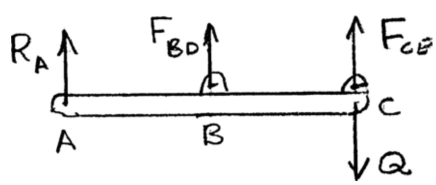
Elongation constraints for taut cables.

Let $\theta =$ rotation angle of rigid bar *ABC*.

$$\theta = \frac{\delta_{BD}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\delta_{CE} = \frac{1}{2}\delta_{CE} \qquad (1)$$

Equilibrium of bar *ABC*.



$$+\circlearrowleft M_A = 0: \; L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \qquad (2)$$

Assume cable *CE* is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}\,\text{N}$

From (2), $F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^{3} - 34.5\times10^{3}) = 31.0\times10^{3}\,\text{N}$

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BD CE

Since $F_{BD} < A\sigma_Y = 34.5\times10^3\,\text{N},$ cable BD is elastic when $Q = 50$ kN.

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PROBLEM 2.109 *(Continued)*

(*a*) Maximum stresses. $\sigma_{CE} = \sigma_Y = 345$ MPa

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times10^3}{100\times10^{-6}} = 310\times10^6 \text{ Pa} \qquad \sigma_{BD} = 310 \text{ MPa} \blacktriangleleft$$

(*b*) Maximum of deflection of point *C*.

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = \frac{(31.0\times10^3)(2)}{(200\times10^9)(100\times10^{-6})} = 3.1\times10^{-3} \text{ m}$$

From (1), $\delta_C = \delta_{CE} = 2\delta_{BD} = 6.2\times10^{-3}$ m $\qquad$ 6.20 mm $\downarrow$ ◀

Permanent elongation of cable *CE*: $(\delta_{CE})_p = (\delta_{CE}) - \dfrac{\sigma_Y L_{CE}}{E}$

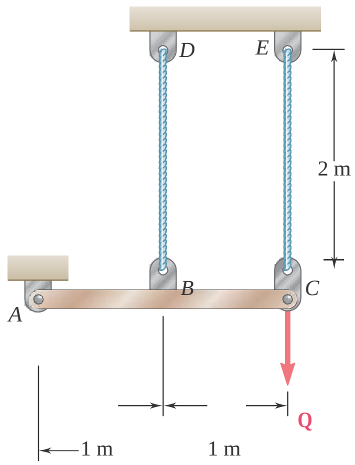
$$(\delta_{CE})_P = (\delta_{CE})_{\max} - \frac{\sigma_Y L_{CE}}{E}$$

$$= 6.20\times10^{-3} - \frac{(345\times10^6)(2)}{200\times10^9} = 2.75\times10^{-3} \text{ m}$$

(*c*) Unloading. Cable *CE* is slack $(F_{CE} = 0)$ at $Q = 0$.

From (2), $F_{BD} = 2(Q - F_{CE}) = 2(0-0) = 0$

Since cable *BD* remained elastic, $\delta_{BD} = \dfrac{F_{BD}L_{BD}}{EA} = 0.$ $\qquad$ 0 ◀



PROBLEM 2.110

Solve Prob. 2.109, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.

PROBLEM 2.109 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at *C* to the rigid bar *ABC* and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable *BD*, (*b*) the maximum deflection of point *C*, (*c*) the final displacement of point *C*. (*Hint:* In part *c*, cable *CE* is not taut.)

SOLUTION

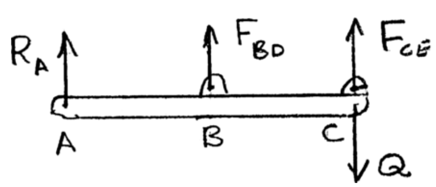
<u>Elongation constraints</u>.

Let $\theta =$ rotation angle of rigid bar ABC.

$$\theta = \frac{\delta_{BC}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\delta_{CE} = \frac{1}{2}\delta_{CE} \qquad (1)$$

<u>Equilibrium of bar *ABC*</u>.



$$+\circlearrowleft M_A = 0: \quad L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \qquad (2)$$

Assume cable *CE* is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}$ N

From (2),

$$F_{CE} = 2(Q - F_{BD}) = (2)(50\times10^3 - 34.5\times10^3) = 31.0\times10^3 \text{ N}$$

Since $F_{BD} < A\sigma_Y = 34.5\times10^3$ N, cable *BD* is elastic when $Q = 50$ kN.

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(*a*) Maximum stresses. $\sigma_{CE} = \sigma_Y = 345 \text{ MPa}$

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times10^3}{100\times10^{-6}} = 310\times10^6 \text{ Pa} \qquad \sigma_{BD} = 310 \text{ MPa} ◀$$

(*b*) Maximum of deflection of point *C*.

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = \frac{(31.0\times10^3)(2)}{(200\times10^9)(100\times10^{-6})} = 3.1\times10^{-3} \text{ m}$$

From (1), $\delta_C = \delta_{CE} = 2\delta_{BD} = 6.2\times10^{-3}$ m $\qquad$ 6.20 mm ↓ ◀

Unloading. $Q' = 50\times10^3$ N, $\delta'_{CE} = \delta'_C$

From (1), $\delta'_{BD} = \frac{1}{2}\delta'_C$

$$\text{Elastic } F''_{BD} = \frac{EA\delta'_{BD}}{L_{BD}} = \frac{(200\times10^9)(100\times10^{-6})(\frac{1}{2}\delta'_C)}{2} = 5\times10^6\delta'_C$$

$$F'_{CE} = \frac{EA\delta'_{CE}}{L_{CE}} = \frac{(200\times10^9)(100\times10^{-6})(\delta'_C)}{2} = 10\times10^6\delta'_C$$

From (2), $Q' = F'_{CE} + \frac{1}{2}F'_{BD} = 12.5\times10^6\delta'_C$

Equating expressions for Q', $12.5\times10^6\delta'_C = 50\times10^3$

$$\delta'_C = 4\times10^{-3} \text{ m}$$

(*c*) Final displacement. $\delta_C = (\delta_C)_m - \delta'_C = 6.2\times10^{-3} - 4\times10^{-3} = 2.2\times10^{-3}$ m $\qquad$ 2.20 mm ↓ ◀

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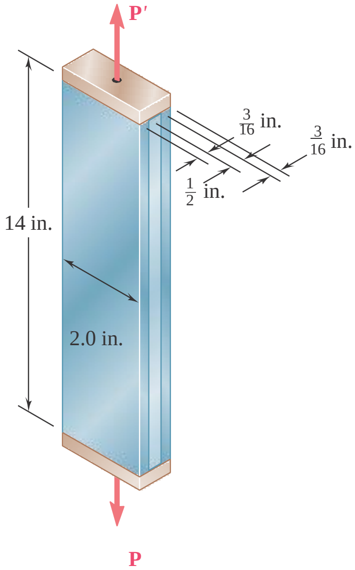

PROBLEM 2.111

Two tempered-steel bars, each $\frac{3}{16}$ in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

For the mild steel, $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$ $\qquad \delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(14)(50\times 10^3)}{29\times 10^6} = 0.024138 \text{ in.}$

For the tempered steel, $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$ $\qquad \delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(14)(100\times 10^3)}{29\times 10^3} = 0.048276 \text{ in.}$

Total area: $A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$. The mild steel yields. Tempered steel is elastic.

(*a*) Forces: $P_1 = A_1\sigma_{Y1} = (1.00)(50\times 10^3) = 50\times 10^3 \text{ lb}$

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times 10^3)(0.75)(0.04)}{14} = 62.14\times 10^3 \text{ lb}$$

$$P = P_1 + P_2 = 112.14\times 10^3 \text{ lb} = 112.1 \text{ kips}$$

$P = 112.1$ kips ◀

(*b*) Stresses: $\sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50\times 10^3 \text{ psi} = 50 \text{ ksi}$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14\times 10^3}{0.75} = 82.86\times 10^3 \text{ psi} = 82.86 \text{ ksi}$$

82.86 ksi ◀

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Unloading: $\delta' = \dfrac{PL}{EA} = \dfrac{(112.14\times10^3)(14)}{(29\times10^6)(1.75)} = 0.03094$ in.

(*c*) Permanent set: $\delta_p = \delta_m - \delta' = 0.04 - 0.03094 = 0.00906$ in. 0.00906 in. ◀

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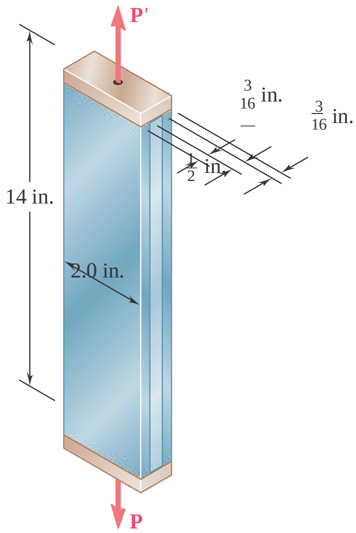

PROBLEM 2.112

For the composite bar of Prob. 2.111, if P is gradually increased from zero to 98 kips and then decreased back to zero, determine (*a*) the maximum deformation of the bar, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$ in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

Areas: Mild steel: $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$

Tempered steel: $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$

Total: $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel:

$$\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50\times 10^3) = 87.50\times 10^3 \text{ lb}$$

$P > P_Y$, therefore, mild steel yields.

Let P_1 = force carried by mild steel.

P_2 = force carried by tempered steel.

$$P_1 = A_1\sigma_1 = (1.00)(50\times 10^3) = 50\times 10^3 \text{ lb}$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98\times 10^3 - 50\times 10^3 = 48\times 10^3 \text{ lb}$$

(*a*) $\delta_m = \frac{P_2 L}{EA_2} = \frac{(48\times 10^3)(14)}{(29\times 10^6)(0.75)}$ 0.0309 in. ◀

(*b*) $\sigma_2 = \frac{P_2}{\ } = \frac{48\times 10^3}{\ } = 64\times 10^3 \text{ psi}$ 64.0 ksi ◀

A_2 0.75

Unloading: $\delta' = \dfrac{PL}{EA} = \dfrac{(98\times 10^3)(14)}{(29\times 10^6)(1.75)} = 0.02703 \text{ in.}$

(*c*) $\delta_P = \delta_m - \delta' = 0.03090 - 0.02703 = 0.003870 \text{ in.}$ 0.00387 in. ◀

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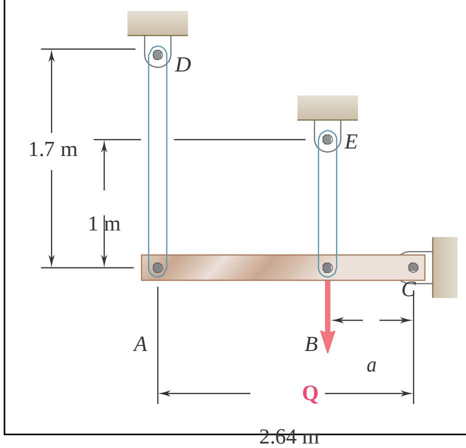


PROBLEM 2.113

The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa.

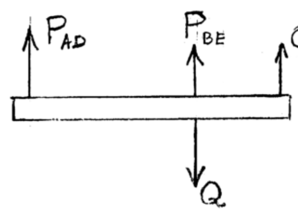
The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

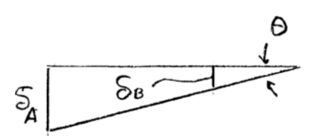
SOLUTION

Statics: $\Sigma M_C = 0: \; 0.640(Q - P_{BE}) - 2.64 P_{AD} = 0$

Deformation: $\delta_A = 2.64\theta, \; \delta_B = a\theta = 0.640\theta$

Elastic analysis:





$$A = (37.5)(6) = 225 \text{ mm}^2 = 225 \times 10^{-6} \text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}} \delta_A = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.7} \delta_A = 26.47 \times 10^6 \delta_A$$

$$= (26.47 \times 10^6)(2.64\theta) = 69.88 \times 10^6 \theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6 \times 10^9 \theta$$

$$P_{BE} = \frac{EA}{L_{BE}} \delta_B = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.0} \delta_B = 45 \times 10^6 \delta_B$$

$$= (45 \times 10^6)(0.640\theta) = 28.80 \times 10^6 \theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 128 \times 10^9 \theta$$

From statics, $Q = P_{BE} + \dfrac{2.64}{0.640} P_{AD} = P_{BE} + 4.125 P_{AD}$

$$= [28.80 \times 10^6 + (4.125)(69.88 \times 10^6)]\theta = 317.06 \times 10^6 \theta$$

θ_Y at yielding of link *AD*: $\quad \sigma_{AD} = \sigma_Y = 250 \times 10^6 = 310.6 \times 10^9 \theta$

$$\theta_Y = 804.89 \times 10^{-6}$$

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$$Q_Y = (317.06 \times 10^6)(804.89 \times 10^{-6}) = 255.2 \times 10^3 \text{ N}$$

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PROBLEM 2.113 *(Continued)*

(*a*) Since $Q = 260\times10^3 > Q_Y$, link *AD* yields. $\sigma_{AD} = 250$ MPa ◀

$$P_{AD} = A\sigma_Y = (225\times10^{-6})(250\times10^{-6}) = 56.25\times10^3 \text{ N}$$

From statics, $P_{BE} = Q - 4.125P_{AD} = 260\times10^3 - (4.125)(56.25\times10^3)$

$$P_{BE} = 27.97\times10^3 \text{ N}$$

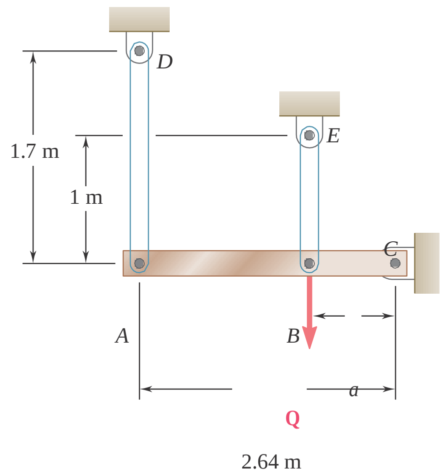
$$\sigma_{BE} = \frac{P_{BE}}{A} = \frac{27.97\times10^3}{225\times10^{-6}} = 124.3\times10^6 \text{ Pa}$$

$\sigma_{BE} = 124.3$ MPa ◀

(*b*)

$$\delta_B = \frac{P_{BE}L_{BE}}{EA} = \frac{(27.97\times10^3)(1.0)}{(200\times10^9)(225\times10^{-6})} = 621.53\times10^{-6} \text{ m}$$

$\delta_B = 0.622$ mm↓ ◀



PROBLEM 2.114

Solve Prob. 2.113, knowing that $a = 1.76$ m and that the magnitude of the force **Q** applied at *B* is gradually increased from zero to 135 kN.

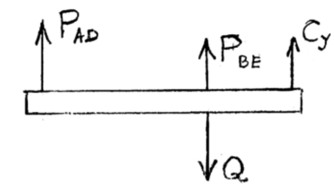
PROBLEM 2.113 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform $37.5 \times 6\text{-mm}$ rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

SOLUTION

Statics: $\Sigma M_C = 0: \; 1.76(Q - P_{BE}) - 2.64 P_{AD} = 0$

Deformation: $\delta_A = 2.64\theta, \; \delta_B = 1.76\theta$

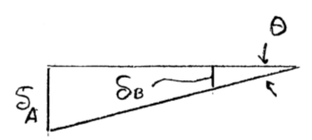
Elastic Analysis:



$$A = (37.5)(6) = 225 \text{ mm}^2 = 225 \times 10^{-6} \text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.7}\delta_A = 26.47 \times 10^6 \delta_A$$

$$= (26.47 \times 10^6)(2.64\theta) = 69.88 \times 10^6 \theta$$



$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6 \times 10^9 \theta$$

$$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.0}\delta_B = 45 \times 10^6 \delta_B$$

$$= (45 \times 10^6)(1.76\theta) = 79.2 \times 10^6 \theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 352 \times 10^9 \theta$$

From statics, $Q = P_{BE} + \frac{2.64}{1.76} P_{AD} = P_{BE} + 1.500 P_{AD}$

$$= [73.8 \times 10^6 + (1.500)(69.88 \times 10^6)]\theta = 178.62 \times 10^6 \theta$$

θ_Y at yielding of link *BE*: $\quad \sigma_{BE} = \sigma_Y = 250 \times 10^6 = 352 \times 10^9 \theta_Y$

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$$\theta_Y = 710.23 \times 10^{-6}$$

$$Q_Y = (178.62 \times 10^6)(710.23 \times 10^{-6}) = 126.86 \times 10^3 \text{ N}$$

(*a*) Since $Q = 135 \times 10^3 \text{ N} > Q_Y$, link *BE* yields. $\sigma_{BE} = \sigma_Y = 250 \text{ MPa}$ ◀

$$P_{BE} = A\sigma_Y = (225 \times 10^{-6})(250 \times 10^6) = 56.25 \times 10^3 \text{ N}$$

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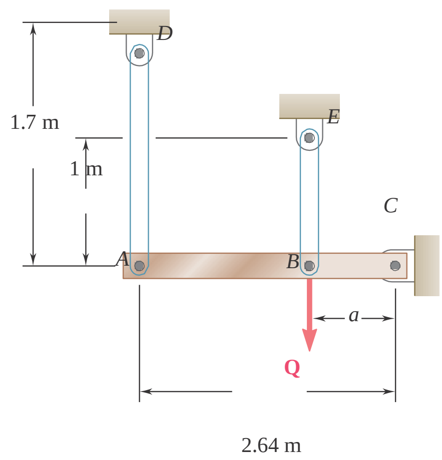
PROBLEM 2.114 *(Continued)*

From statics, $P_{AD} = \frac{1}{1.500}(Q - P_{BE}) = 52.5\times10^3$ N

$$\sigma_{AD} = \frac{P_{AD}}{A} = \frac{52.5\times10^3}{225\times10^{-6}} = 233.3\times10^6 \qquad \sigma_{AD} = 233 \text{ MPa} ◀$$

From elastic analysis of *AD*, $\theta = \frac{P_{AD}}{69.88\times10^6} = 751.29\times10^{-3}$ rad

(*b*) $\delta_B = 1.760\theta = 1.322\times10^{-3}$ m $\qquad \delta_B = 1.322$ mm ↓ ◀



PROBLEM 2.115*

Solve Prob. 2.113, assuming that the magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (*a*) the residual stress in each link, (*b*) the final deflection of point *B*. Assume that the links are braced so that they can carry compressive forces without buckling.

PROBLEM 2.113 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

SOLUTION

See solution to Problem 2.113 for the normal stresses in each link and the deflection of Point *B* after loading.

$$\sigma_{AD} = 250 \times 10^6 \text{ Pa}$$
$$\sigma_{BE} = 124.3 \times 10^6 \text{ Pa}$$
$$\delta_B = 621.53 \times 10^{-6} \text{ m}$$

The elastic analysis given in the solution to Problem 2.113 applies to the unloading.

$$Q' = 317.06 \times 10^6 \theta'$$
$$Q' = \frac{Q}{317.06 \times 10^6} = \frac{260 \times 10^3}{317.06 \times 10^6} = 820.03 \times 10^{-6}$$

$$\sigma'_{AD} = 310.6 \times 10^9 \theta = (310.6 \times 10^9)(820.03 \times 10^{-6}) = 254.70 \times 10^6 \text{ Pa}$$
$$\sigma'_{BE} = 128 \times 10^9 \theta = (128 \times 10^9)(820.03 \times 10^{-6}) = 104.96 \times 10^6 \text{ Pa}$$
$$\delta'_B = 0.640\theta' = 524.82 \times 10^{-6} \text{ m}$$

(*a*) Residual stresses.

$$\sigma_{AD,\text{ res}} = \sigma_{AD} - \sigma'_{AD} = 250 \times 10^6 - 254.70 \times 10^{-6} = -4.70 \times 10^6 \text{ Pa} \qquad = -4.70 \text{ MPa} \blacktriangleleft$$

$$\sigma_{BE,\text{ res}} = \sigma_{BE} - \sigma'_{BE} = 124.3 \times 10^6 - 104.96 \times 10^6 = 19.34 \times 10^6 \text{ Pa} \qquad = 19.34 \text{ MPa} \blacktriangleleft$$

(*b*) −6 −6 −6

$$\delta_{B,P} = \delta_B - \delta'_B = 621.53\times 10 \quad - 524.82\times 10 \quad = 96.71\times 10 \quad \text{m} \qquad = 0.0967 \text{ mm}\downarrow \blacktriangleleft$$

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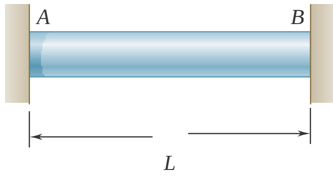
PROBLEM 2.116



A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of 45°F. The steel is assumed to be elastoplastic with $\sigma_Y = 36$ ksi and $E = 29\times 10^6$ psi. Knowing that $\alpha = 6.5\times 10^{-6}$/°F, determine the stress in the bar (*a*) when the temperature is raised to 320°F, (*b*) after the temperature has returned to 45°F.

SOLUTION

Let P be the compressive force in the rod.

Determine temperature change to cause yielding.

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_Y L}{E} + L\alpha(\Delta T)_Y = 0$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E\alpha} = \frac{36\times 10^3}{(29\times 10^6)(6.5\times 10^{-6})} = 190.98°\text{F}$$

But $\Delta T = 320 - 45 = 275°\text{F} > (\Delta T_Y)$

(*a*) Yielding occurs. $\sigma = -\sigma_Y = -36.0$ ksi ◀

Cooling: $(\Delta \text{T})' = 275°\text{F}$

$$\delta' = \delta'_P = \delta'_T = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)'$$

$$= -(29\times 10^6)(6.5\times 10^{-6})(275) = -51.8375\times 10^3 \text{ psi}$$

(*b*) Residual stress:

$$\sigma_{\text{res}} = -\sigma_Y - \sigma' = -36\times 10^3 + 51.8375\times 10^3 = 15.84\times 10 \text{ psi}$$

15.84 ksi ◀

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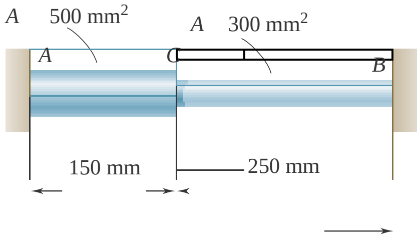
PROBLEM 2.117



The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25°C. The steel is assumed elastoplastic, with E = 200 GPa and $\sigma_y = 250$ MPa. The temperature of both portions of the rod is then raised to 150°C. Knowing that $\alpha = 11.7\times10^{-6}/°\text{C}$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.

SOLUTION

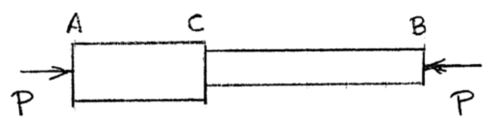
$$A_{AC} = 500\times10^{-6}\,\text{m}^2 \qquad L_{AC} = 0.150\text{ m}$$

$$A_{CB} = 300\times10^{-6}\,\text{m}^2 \qquad L_{CB} = 0.250\text{ m}$$

Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion *CB*.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\alpha(\Delta T)$$



$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding, $P = P_Y = A_{CB}\sigma_Y = (300\times10^{-6})(2.50\times10^{6}) = 75\times10^{3}\text{ N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

$$= \frac{75\times10^{3}}{(0.400)(200\times10^{9})(11.7\times10^{-6})}\left(\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}\right) = 90.812°\text{C}$$

Actual ΔT: $150°\text{C} - 25°\text{C} = 125°\text{C} > (\Delta T)_Y$

Yielding occurs. For $\Delta T > (\Delta T)_Y$, $\quad P = P_Y = 75\times10^{3}\text{ N}$

(*a*) $\sigma_{AC} = -\dfrac{P_Y}{A_{AC}} = -\dfrac{75\times10^{3}}{500\times10^{-6}} = -150\times10^{-6}\text{ Pa}$ $\qquad \sigma_{AC} = -150.0\text{ MPa}$ ◀

$\sigma_{CB} = -\dfrac{P_Y}{A_{CB}} = -\sigma_Y$

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$$\sigma_{CB} = -250 \text{ MPa} \blacktriangleleft$$

(*b*) For $\Delta T > (\Delta T)_Y$, portion *AC* remains elastic.

$$\delta_{C/A} = -\frac{P_Y L_{AC}}{EA_{AC}} + L_{AC}\alpha(\Delta T)$$

$$= -\frac{(75\times 10^3)(0.150)}{(200\times 10^9)(500\times 10^{-6})} + (0.150)(11.7\times 10^{-6})(125) = 106.9\times 10^{-6} \text{ m}$$

Since Point *A* is stationary, $\delta_C = \delta_{C/A} = 106.9\times 10^{-6}$ m $\qquad \delta_C = 0.1069 \text{ mm} \rightarrow \blacktriangleleft$

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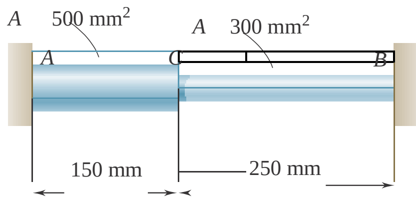
PROBLEM 2.118*



Solve Prob. 2.117, assuming that the temperature of the rod is raised to 150°C and then returned to 25°C.

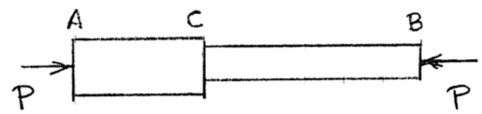
PROBLEM 2.117 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25°C. The steel is assumed elastoplastic, with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The temperature of both portions of the rod is then raised to 150°C. Knowing that $\alpha = 11.7\times10^{-6}/°\mathrm{C}$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.

SOLUTION

$$A_{AC} = 500\times10^{-6}\,\mathrm{m}^2 \qquad L_{AC} = 0.150\ \mathrm{m} \qquad A_{CB} = 300\times10^{-6}\,\mathrm{m}^2 \qquad L_{CB} = 0.250\ \mathrm{m}$$

Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion *CB*.



$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\alpha(\Delta T)$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

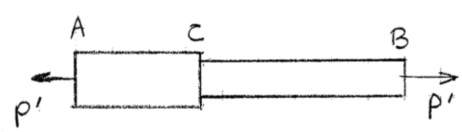
At yielding, $P = P_Y = A_{CB}\sigma_Y = (300\times10^{-6})(250\times10^{6}) = 75\times10^{3}\,\mathrm{N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) = \frac{75\times10^{3}}{(0.400)(200\times10^{9})(11.7\times10^{-6})}\left(\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}\right)$$

$$= 90.812\ °\mathrm{C}$$

Actual ΔT: $150°\mathrm{C} - 25°\mathrm{C} = 125°\mathrm{C} > (\Delta T)_Y$

Yielding occurs. For $\Delta T > (\Delta T)_Y$, $\quad P = P_Y = 75\times10^{3}\ \mathrm{N}$



Cooling:

$$(\Delta T)' = 125°\mathrm{C} \quad P' = \frac{EL_{AB}\alpha(\Delta T)'}{\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)} = \frac{(200\times10^{9})(0.400)(11.7\times10^{-6})(125)}{\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}} = 103.235\times10^{3}\,\mathrm{N}$$

Residual force: P_{res}

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$= P' - P_Y$ $= 103.235 \times 10^3 - 75 \times 10^3 = 28.235 \times 10^3$ N (tension)

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PROBLEM 2.118* *(Continued)*

(*a*) Residual stresses.

$$\sigma_{AC} = \frac{P_{res}}{A_{AC}} = \frac{28.235\times 10^3}{500\times 10^{-6}} \qquad \sigma_{AC} = 56.5 \text{ MPa} \blacktriangleleft$$

$$\sigma_{CB} = \frac{P_{res}}{A_{CB}} = \frac{28.235\times 10^3}{300\times 10^{-6}} \qquad \sigma_{CB} = 9.41 \text{ MPa} \blacktriangleleft$$

(*b*) Permanent deflection of point *C*.

$$\delta_C = \frac{P_{res} L_{AC}}{EA_{AC}} \qquad \delta_C = 0.0424 \text{ mm} \rightarrow \blacktriangleleft$$

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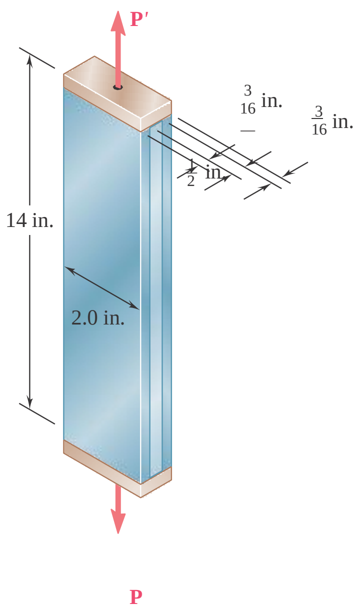

PROBLEM 2.119*

For the composite bar of Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero to 98 kips and then decreased back to zero.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$ in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

Areas. Mild steel: $A_1 = \left(\frac{1}{2}\right)(2) = 1.00\ \text{in}^2$

Tempered steel: $A_2 = (2)\left(\frac{3}{16}\right)(2) = 0.75\ \text{in}^2$

Total: $A = A_1 + A_2 = 1.75\ \text{in}^2$

Total force to yield the mild steel: $\sigma_{Y1} = \frac{P_Y}{A}$ $\therefore$ $P_Y = A\sigma_{Y1} = (1.75)(50\times10^3) = 87.50\times10^3\ \text{lb}$

$P > P_Y$; therefore, mild steel yields.

Let P_1 = force carried by mild steel

P_2 = force carried by tempered steel

$P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3\ \text{lb}$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98\times10^3 - 50\times10^3 = 48\times10^3\ \text{lb}$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{48\times10^3}{0.75} = 64\times10^3\ \text{psi}$$

Unloading. $\sigma' = \frac{P}{A} = 98\times10^3 = 56\times10^3\ \text{psi}$

Mild steel:

Residual stresses.

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1.75

$\sigma_{1,\text{res}} = \sigma_1 - \sigma' = 50 \times 10^3 - 56 \times 10^3 = -6 \times 10^{-3}\ \text{psi} = -6\ \text{ksi}$

Tempered steel: $\sigma_{2,\text{res}} = \sigma_2 - \sigma_1 = 64 \times 10^3 - 56 \times 10^3 = 8 \times 10^3\ \text{psi}$ 8.00 ksi ◀

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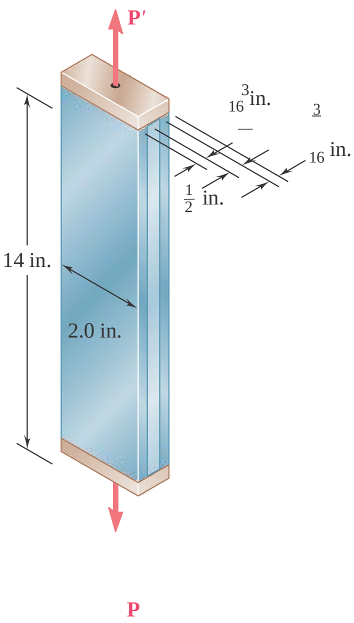

PROBLEM 2.120*

For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and is then decreased back to zero.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$ in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

For the mild steel, $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2 \quad \delta_{Y1} = \frac{L\delta_{Y1}}{E} = \frac{(14)(50 \times 10^3)}{29 \times 10^6} = 0.024138 \text{ in.}$

For the tempered steel, $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2 \quad \delta_{Y2} = \frac{L\delta_{Y2}}{E} = \frac{(14)(100 \times 10^3)}{29 \times 10^6} = 0.048276 \text{ in.}$

Total area: $A = A_1 + A_2 = 1.75 \text{ in}^2$

$$\delta_{Y1} < \delta_m < \delta_{Y2}$$

The mild steel yields. Tempered steel is elastic.

Forces:

$$P_1 = A_1\delta_{Y1} = (1.00)(50 \times 10^3) = 50 \times 10^3 \text{ lb}$$

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29 \times 10^6)(0.75)(0.04)}{14} = 62.14 \times 10^3 \text{ lb}$$

Stresses:

$$\sigma_1 = \frac{P_1}{A_1} = \delta_{Y1} = 50 \times 10^3 \text{ psi} \qquad \sigma_2 = \frac{P_2}{A_2} = \frac{62.14 \times 10^3}{0.75} = 82.86 \times 10^3 \text{ psi}$$

Unloading:

$$\sigma' = \frac{P}{A} = \frac{112.14}{1.75} = 64.08 \times 10^3 \text{ psi}$$

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Residual stresses. $\sigma_{1,\text{res}} = \sigma_1 - \sigma' = 50\times10^3 - 64.08\times10^3 = -14.08\times10^3 \text{ psi} = -14.08 \text{ ksi}$

$$\sigma_{2,\text{res}} = \sigma_2 - \sigma' = 82.86\times10^3 - 64.08\times10^3 = 18.78\times10^3 \text{ psi} = 18.78 \text{ ksi} \quad ◀$$

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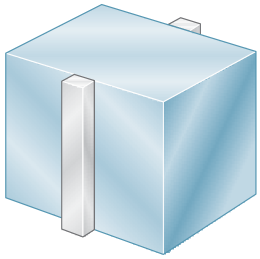
PROBLEM 2.121*



Narrow bars of aluminum are bonded to the two sides of a thick steel plate as shown. Initially, at $T_1 = 70°\text{F}$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1, determine (*a*) the highest temperature T_2 that does *not* result in residual stresses, (*b*) the temperature T_2 that will result in a residual stress in the aluminum equal to 58 ksi. Assume $\alpha_a = 12.8 \times 10^{-6}/°\text{F}$ for the aluminum and $\alpha^s = 6.5 \times 10^{-6}/°\text{F}$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 10.9 \times 10^6$ psi and $\sigma_Y = 58$ ksi. (*Hint:* Neglect the small stresses in the plate.)

SOLUTION

Determine <u>temperature change to cause yielding</u>.

$$\delta = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{58 \times 10^3}{(10.9 \times 10^6)(12.8 - 6.5)(10^{-6})} = 844.62°\text{F}$$

(*a*) $T_{2Y} = T_1 + (\Delta T)_Y = 70 + 844.62 = 915°\text{F}$ 915°F ◀

After yielding,

$$\delta = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

<u>Cooling</u>:

$$\delta' = \frac{P'L}{AE} + L\alpha_a(\Delta T)' = L\alpha_s(\Delta T)'$$

The <u>residual stress</u> is

$$\sigma_{\text{res}} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

Set $\sigma_{\text{res}} = -\sigma_Y$

$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$2\sigma \qquad (2)(58 \times 10^3)$$

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$$\Delta T = \frac{Y}{E(\alpha_a - \alpha_s)} = \frac{}{(10.9\times 10^6)(12.8-6.5)(10^{-6})} = 1689°\text{F}$$

(*b*) $T_2 = T_1 + \Delta T = 70 + 1689 = 1759°\text{F}$ 1759°F ◀

If $T_2 > 1759°\text{F}$, the aluminum bar will most likely yield in compression.

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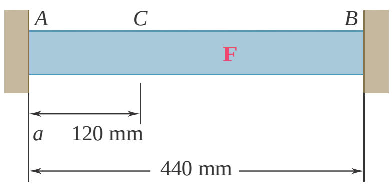


PROBLEM 2.122*

Bar AB has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point C, (*b*) the residual stress in the bar.

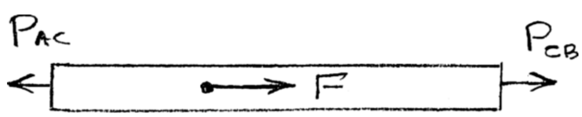
SOLUTION

$$A = 1200\ \text{mm}^2 = 1200 \times 10^{-6}\ \text{m}^2$$

<u>Force to yield portion *AC*</u>:

$$P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6)$$

$$= 300 \times 10^3\ \text{N}$$



For equilibrium, $F + P_{CB} - P_{AC} = 0.$

$$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3$$

$$= -220 \times 10^3\ \text{N}$$

$$\delta_C = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.120)}{(200 \times 10^9)(1200 \times 10^{-6})}$$

$$= 0.29333 \times 10^{-3}\ \text{m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220 \times 10^3}{1200 \times 10^{-6}}$$

$$= -183.333 \times 10^6\ \text{Pa}$$

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PROBLEM 2.122* *(Continued)*

Unloading:

$$\delta'_C = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} = \frac{(F - P'_{AC})L_{CB}}{EA}$$

$$P'_{AC}\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{FL_{CB}}{EA}$$

$$P'_{AC} = \frac{FL_{CB}}{L_{AC} + L_{CB}} = \frac{(520\times10^3)(0.440 - 0.120)}{0.440} = 378.18\times10^3\,\text{N}$$

$$P'_{CB} = P'_{AC} - F = 378.18\times10^3 - 520\times10^3 = -141.820\times10^3\,\text{N}$$

$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{378.18\times10^3}{1200\times10^{-6}} = 315.150\times10^6\,\text{Pa}$$

$$\sigma'_{BC} = \frac{P'_{BC}}{A} = -\frac{141.820\times10^3}{1200\times10^{-6}} = -118.183\times10^6\,\text{Pa}$$

$$\delta'_C = \frac{(378.18\times10^3)(0.120)}{(200\times10^9)(1200\times10^{-6})} = 0.189090\times10^{-3}\,\text{m}$$

(*a*)

$$\delta_{C,p} = \delta_C - \delta'_C = 0.29333\times10^{-3} - 0.189090\times10^{-3} = 0.104240\times10^{-3}\,\text{m} \qquad = 0.1042\text{ mm} \blacktriangleleft$$

(*b*) $$\sigma_{AC,\text{res}} = \sigma_Y - \sigma'_{AC} = 250\times10^6 - 315.150\times10^6 = -65.150\times10^6\,\text{Pa} \qquad = -65.2\text{ MPa} \blacktriangleleft$$

$$\sigma_{CB,\text{res}} = \sigma_{CB} - \sigma'_{CB} = -183.333\times10^6 + 118.183\times10^6 = -65.150\times10^6\,\text{Pa} \qquad = -65.2\text{ MPa} \blacktriangleleft$$

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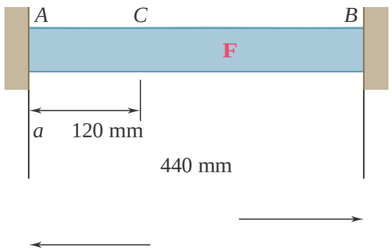
PROBLEM 2.123*



Solve Prob. 2.122, assuming that $a = 180$ mm.

PROBLEM 2.122 Bar *AB* has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point *C*, (*b*) the residual stress in the bar.

SOLUTION

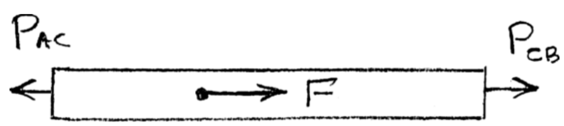
$$A = 1200\ \text{mm}^2 = 1200 \times 10^{-6}\ \text{m}^2$$

Force to yield portion *AC*:

$$P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6)$$

$$= 300 \times 10^3\ \text{N}$$

For equilibrium, $F + P_{CB} - P_{AC} = 0.$



$$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3$$

$$= -220 \times 10^3\ \text{N}$$

$$\delta_C = -\frac{P_{CB}L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.180)}{(200 \times 10^9)(1200 \times 10^{-6})}$$

$$= 0.23833 \times 10^{-3}\ \text{m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = -\frac{220 \times 10^3}{1200 \times 10^{-6}}$$

$$= -183.333 \times 10^6\ \text{Pa}$$

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PROBLEM 2.123* *(Continued)*

Unloading:

$$\delta'_C = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} = \frac{(F - P'_{AC})L_{CB}}{EA}$$

$$= P'_{AC}\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{FL_{CB}}{EA}$$

$$P'_{AC} = \frac{FL_{CB}}{L_{AC} + L_{CB}} = \frac{(520\times 10^3)(0.440 - 0.180)}{0.440} = 307.27\times 10^3 \text{ N}$$

$$P'_{CB} = P'_{AC} - F = 307.27\times 10^3 - 520\times 10^3 = -212.73\times 10^3 \text{ N}$$

$$\delta'_C = \frac{(307.27\times 10^3)(0.180)}{(200\times 10^9)(1200\times 10^{-6})} = 0.23045\times 10^{-3} \text{ m}$$

$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{307.27\times 10^3}{1200\times 10^{-6}} = 256.058\times 10^6 \text{ Pa}$$

$$\sigma'_{CB} = \frac{P'_{CB}}{A} = \frac{-212.73\times 10^3}{1200\times 10^{-6}} = -177.275\times 10^6 \text{ Pa}$$

(*a*)

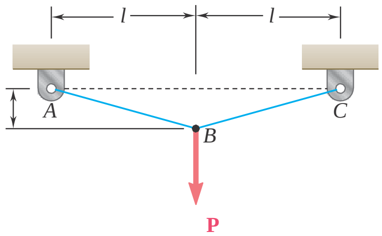
$$\delta_{C,p} = \delta_C - \delta'_C = 0.23833\times 10^{-3} - 0.23045\times 10^{-3} = 0.00788\times 10^{-3} \text{ m} \qquad = 0.00788 \text{ mm} \blacktriangleleft$$

(*b*)

$$\sigma_{AC,\text{res}} = \sigma_{AC} - \sigma'_{AC} = 250\times 10^6 - 256.058\times 10^6 = -6.0580\times 10^6 \text{ Pa} \qquad = -6.06 \text{ MPa} \blacktriangleleft$$

$$\sigma_{CB,\text{res}} = \sigma_{CB} - \sigma'_{CB} = -183.333\times 10^6 + 177.275\times 10^6 = -6.0580\times 10^6 \text{ Pa} \qquad = -6.06 \text{ MPa} \blacktriangleleft$$

PROBLEM 2.124



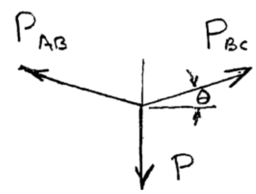
The uniform wire ABC, of unstretched length $2l$, is attached to the supports shown and a vertical load **P** is applied at the midpoint B. Denoting by A the cross-sectional area of the wire and by E the modulus of elasticity, show that, for $\delta \ll l$, the deflection at the midpoint B is

$$\delta = l\sqrt[3]{\frac{P}{AE}}$$

SOLUTION

Use approximation.

$$\sin\theta \approx \tan\theta \approx \frac{\delta}{l}$$



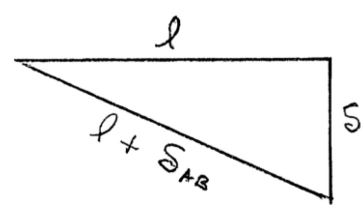
Statics: $+\uparrow \Sigma F_Y = 0: \quad 2P_{AB}\sin\theta - P = 0$

$$P_{AB} = \frac{P}{2\sin\theta} \approx \frac{Pl}{2\delta}$$

Elongation: $\delta_{AB} = \dfrac{P_{AB}\,l}{AE} = \dfrac{Pl^2}{2AE\delta}$

Deflection:

From the right triangle,



$$(l+\delta_{AB})^2 = l^2 + \delta^2$$

$$\delta^2 = l^2 + 2l\delta_{AB} + \delta_{AB}^2 - l^2$$

$$= 2l\delta_{AB}\left(1 + \frac{1}{2}\frac{\delta_{AB}}{l}\right) \approx 2l\delta_{AB}$$

$$\approx \frac{Pl^3}{AE\delta}$$

$$\delta^3 \approx \frac{Pl^3}{AE} \quad \therefore \quad \delta \approx l\sqrt[3]{\frac{P}{AE}} \quad ◀$$

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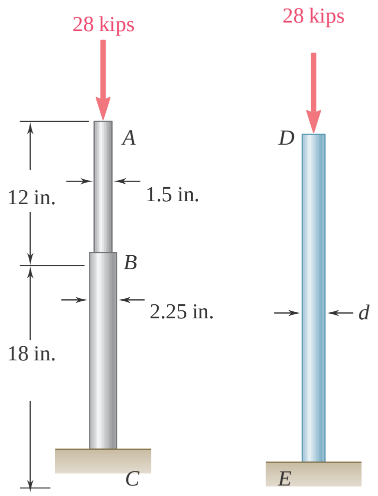

PROBLEM 2.125

The aluminum rod ABC ($E = 10.1 \times 10^6$ psi), which consists of two cylindrical portions AB and BC, is to be replaced with a cylindrical steel

minimum required diameter d of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 24 ksi.

SOLUTION

Deformation of aluminum rod.

$$\delta_A = \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E}$$

$$= \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$= \frac{28 \times 10^3}{10.1 \times 10^6}\left(\frac{12}{\frac{\pi}{4}(1.5)^2} + \frac{18}{\frac{\pi}{4}(2.25)^2}\right)$$

$$= 0.031376 \text{ in.}$$

Steel rod. $\delta = 0.031376$ in.

$$\delta = \frac{PL}{EA} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(28 \times 10^3)(30)}{(29 \times 10^6)(0.031376)} = 0.92317 \text{ in}^2$$

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{28 \times 10^3}{24 \times 10^3} = 1.16667 \text{ in}^2$$

Required area is the larger value. $A = 1.16667 \text{ in}^2$

Diameter:

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(1.16667)}{\pi}}$$

$d = 1.219$ in. ◀

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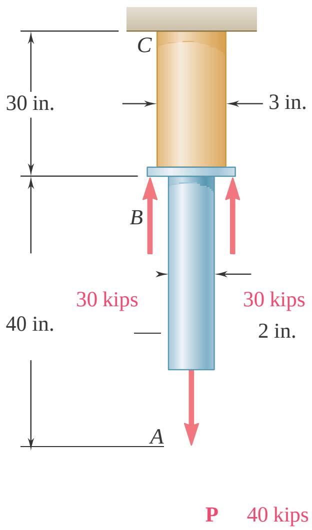

PROBLEM 2.126

Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel $(E = 29 \times 10^6 \text{ psi})$, and rod BC of brass $(E = 15 \times 10^6 \text{ psi})$. Determine (*a*) the total deformation of the composite rod ABC, (*b*) the deflection of point B.

SOLUTION

<u>Portion *AB*</u>:

$$P_{AB} = 40 \times 10^3 \text{ lb}$$

$$L_{AB} = 40 \text{ in.}$$

$$d = 2 \text{ in.}$$

$$A_{AB} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(2)^2 = 3.1416 \text{ in}^2$$

$$E_{AB} = 29 \times 10^6 \text{ psi}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} = \frac{(40 \times 10^3)(40)}{(29 \times 10^6)(3.1416)} = 17.5619 \times 10^{-3} \text{ in.}$$

<u>Portion *BC*</u>:

$$P_{BC} = -20 \times 10^3 \text{ lb}$$

$$L_{BC} = 30 \text{ in.}$$

$$d = 3 \text{ in.}$$

$$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(3)^2 = 7.0686 \text{ in}^2$$

$$E_{BC} = 15 \times 10^6 \text{ psi}$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{E_{BC} A_{BC}} = \frac{(-20 \times 10^3)(30)}{(15 \times 10^6)(7.0686)} = -5.6588 \times 10^{-3} \text{ in.}$$

(*a*) $\delta = \delta_{AB} + \delta_{BC} = 17.5619 \times 10^{-6} - 5.6588 \times 10^{-6}$ $\qquad \delta = 11.90 \times 10^{-3} \text{ in.} \downarrow$ ◀

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(*b*) $\delta_B = -\delta_{BC}$ $\delta_B = 5.66 \times 10^{-3}\,\text{in.}\uparrow$ ◀

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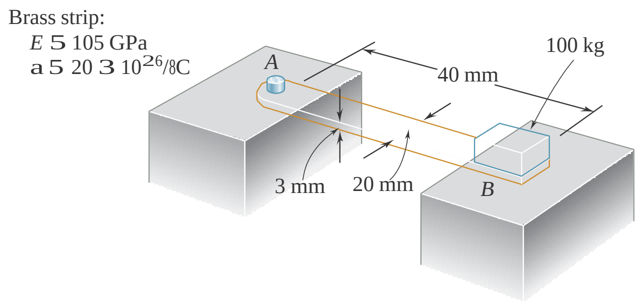


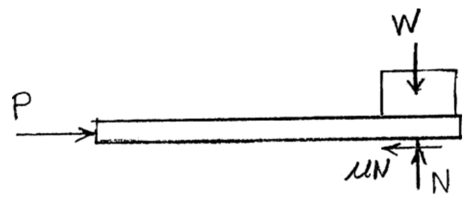
PROBLEM 2.127

The brass strip *AB* has been attached to a fixed support at *A* and rests on a rough support at *B*. Knowing that the coefficient of friction is 0.60 between the strip and the support at *B*, determine the decrease in temperature for which slipping will impend.

SOLUTION

Brass strip:

$$E = 105 \text{ GPa}$$
$$\alpha = 20 \times 10^{-6}/°\text{C}$$



$$+\uparrow \Sigma F_y = 0: \quad N - W = 0 \qquad N = W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - \mu N = 0 \qquad P = \mu W = \mu mg$$

$$\delta = -\frac{PL}{EA} + L\alpha(\Delta T) = 0 \qquad \Delta T = \frac{P}{EA\alpha} = \frac{\mu mg}{EA\alpha}$$

Data: $\mu = 0.60$

$$A = (20)(3) = 60 \text{ mm}^2 = 60 \times 10^{-6} \text{ m}^2$$
$$m = 100 \text{ kg}$$
$$g = 9.81 \text{ m/s}^2$$
$$E = 105 \times 10^9 \text{ Pa}$$

$$\Delta T = \frac{(0.60)(100)(9.81)}{(105 \times 10^9)(60 \times 10^{-6})(20 \times 10^{6})}$$

$\Delta T = 4.67°\text{C}$ ◀

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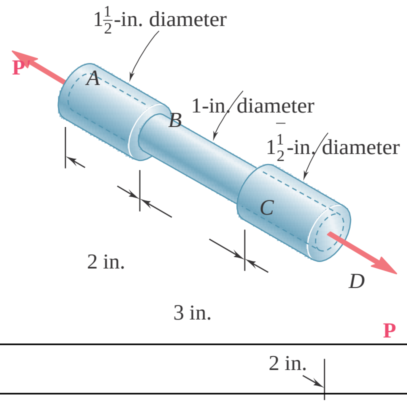

PROBLEM 2.128

The specimen shown is made from a 1-in.-diameter cylindrical steel rod with two 1.5-in.-outer-diameter sleeves bonded to the rod as shown. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the load **P** so that the total deformation is 0.002 in., (*b*) the corresponding deformation of the central portion *BC*.

SOLUTION

(*a*) $\delta = \Sigma \dfrac{P_i L_i}{A_i E_i} = \dfrac{P}{E} \Sigma \dfrac{L_i}{A_i}$

$$P = E\delta \left(\Sigma \frac{L_i}{A_i} \right)^{-1} \qquad A_i = \frac{\pi}{4} d_i^2$$

	L, in.	d, in.	A, in^2	L/A, in^{-1}
AB	2	1.5	1.7671	1.1318
BC	3	1.0	0.7854	3.8197
CD	2	1.5	1.7671	1.1318
				6.083 ← sum

$P = (29 \times 10^6)(0.002)(6.083)^{-1} = 9.353 \times 10^3$ lb

$P = 9.53$ kips ◀

(*b*) $\delta_{BC} = \dfrac{PL_{BC}}{A_{BC}E} = \dfrac{P}{E}\dfrac{L_{BC}}{A_{BC}} = \dfrac{9.535 \times 10^3}{29 \times 10^6}(3.8197)$

$\delta = 1.254 \times 10^{-3}$ in. ◀

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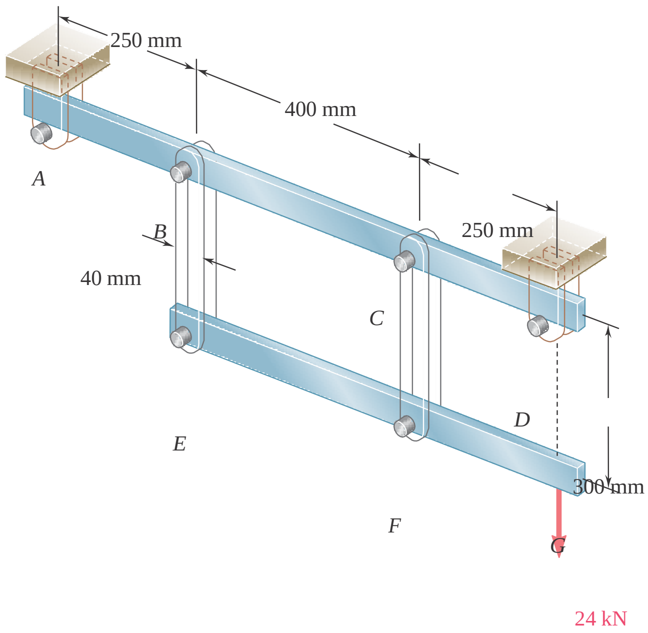
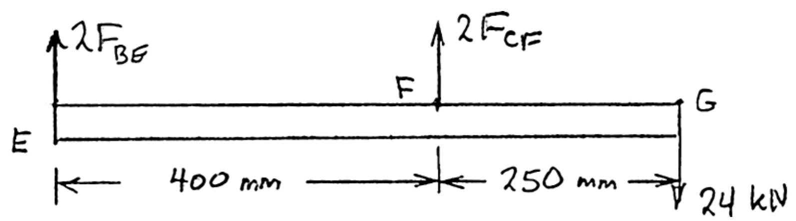
PROBLEM 2.129

Each of the four vertical links connecting the two rigid horizontal members is made of aluminum $(E = 70 \text{ GPa})$ and has a uniform rectangular cross section of 10×40 mm. For the loading shown, determine the deflection of (*a*) point *E*, (*b*) point *F*, (c) point *G*.



SOLUTION

<u>Statics</u>. Free body *EFG*:



$$\circlearrowleft^{+} \Sigma M_F = 0: \; -(400)(2F_{BE}) - (250)(24) = 0$$

$$F_{BE} = -7.5 \text{ kN} = -7.5 \times 10^3 \text{ N}$$

$$\circlearrowleft^{+} \Sigma M_E = 0: \; (400)(2F_{CF}) - (650)(24) = 0$$

$$F_{CF} = 19.5 \text{ kN} = 19.5 \times 10^3 \text{ N}$$

Area of one link:

$$A = (10)(40) = 400 \text{ mm}^2$$
$$= 400 \times 10^{-6} \text{ m}^2$$

Length: $L = 300 \text{ mm} = 0.300 \text{ m}$

<u>Deformations</u>.

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$$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-7.5\times10^3)(0.300)}{(70\times10^9)(400\times10^{-6})} = -80.357\times10^{-6}\,\text{m}$$

$$\delta_{CF} = \frac{F_{CF}L}{EA} = \frac{(19.5\times10^3)(0.300)}{(70\times10^9)(400\times10^{-6})} = 208.93\times10^{-6}\,\text{m}$$

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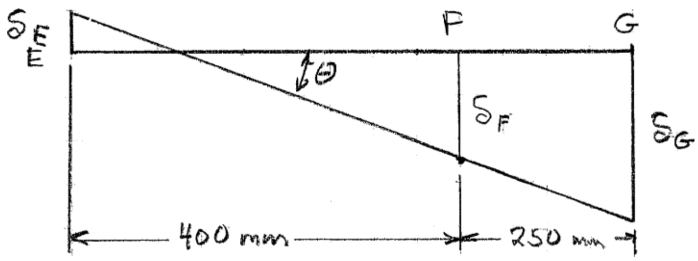
PROBLEM 2.129 *(Continued)*

(*a*) Deflection of Point *E*. $\delta_E = |\delta_{BF}|$ $\delta_E = 80.4\ \mu\text{m}\uparrow$ ◀

(*b*) Deflection of Point *F*. $\delta_F = \delta_{CF}$ $\delta_F = 209\ \mu\text{m}\downarrow$ ◀

Geometry change.



Let θ be the small change in slope angle.

$$\theta = \frac{\delta_E + \delta_F}{L_{EF}} = \frac{80.357\times10^{-6} + 208.93\times10^{-6}}{0.400} = 723.22\times10^{-6}\text{ radians}$$

(*c*) Deflection of Point *G*. $\delta_G = \delta_F + L_{FG}\,\theta$

$$\delta_G = \delta_F + L_{FG}\,\theta = 208.93\times10^{-6} + (0.250)(723.22\times10^{-6})$$

$$= 389.73\times10^{-6}\text{ m}$$ $\delta_G = 390\ \mu\text{m}\downarrow$ ◀

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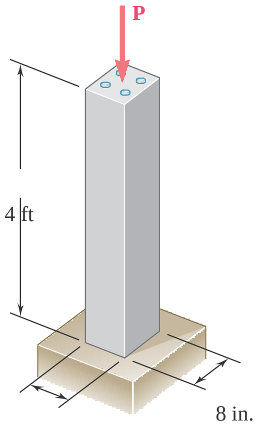
PROBLEM 2.130



A 4-ft concrete post is reinforced with four steel bars, each with a $\frac{3}{4}$-in. diameter. Knowing that $E_s = 29\times10^6$ psi and $E_c = 3.6\times10^6$ psi, determine the normal stresses in the steel and in the concrete when a 150-kip axial centric force **P** is applied to the post.

SOLUTION

$$A_s = 4\left[\frac{\pi}{4}\left(\frac{3}{4}\right)^2\right] = 1.76715\ \text{in}^2$$

$$A_c = 8^2 - A_s = 62.233\ \text{in}^2$$

$$\delta_s = \frac{P_s L}{A_s E_s} = \frac{P_s(48)}{(1.76715)(29\times10^6)} = 0.93663\times10^{-6} P_s$$

$$\delta_c = \frac{P_c L}{A_c E_c} = \frac{P_c(48)}{(62.233)(3.6\times10^6)} = 0.21425\times10^{-6} P_c$$

But $\delta_s = \delta_c$: $0.93663\times10^{-6} P_s = 0.21425\times10^{-6} P_c$

$$P_s = 0.22875 P_c \qquad (1)$$

Also, $$P_s + P_c = P = 150\ \text{kips} \qquad (2)$$

Substituting (1) into (2), $$1.22875 P_c = 150\ \text{kips}$$

$$P_c = 122.075\ \text{kips}$$

From (1), $$P_s = 0.22875(122.075) = 27.925\ \text{kips}$$

$$\sigma_s = -\frac{P_s}{A_s} = -\frac{27.925}{1.76715} \qquad \sigma_c = -\frac{P_c}{A_c}$$

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$$= -\frac{122.075}{62.233}$$

$\sigma_s =$
-15.80 ksi ◀

$\sigma_c =$
-1.962 ksi ◀

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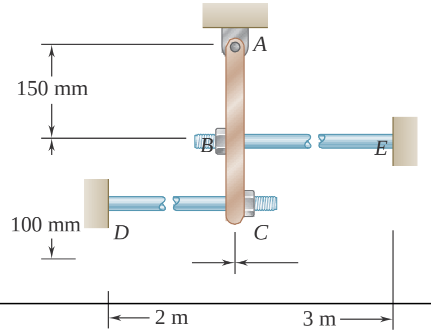
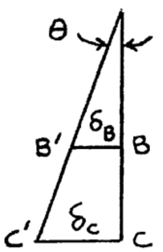


PROBLEM 2.131

The steel rods *BE* and *CD* each have a 16-mm diameter ($E = 200$ GPa); the ends of the rods are single-threaded with a pitch of 2.5 mm. Knowing that after being snugly fitted, the nut at *C* is tightened one full turn, determine (*a*) the tension in rod *CD*, (*b*) the deflection of point *C* of the rigid member *ABC*.

SOLUTION

Let θ be the rotation of bar *ABC* as shown.



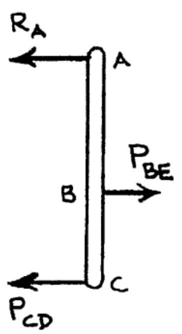
Then $\qquad \delta_B = 0.15\theta \qquad \delta_C = 0.25\theta$

But $\qquad \delta_C = \delta_{\text{turn}} - \dfrac{P_{CD}L_{CD}}{E_{CD}A_{CD}}$

$$P_{CD} = \frac{E_{CD}A_{CD}}{L_{CD}}(\delta_{\text{turn}} - \delta_C)$$

$$= \frac{(200\times10^9\,\text{Pa})\frac{\pi}{4}(0.016\text{ m})^2}{2\text{ m}}(0.0025\text{ m} - 0.25\theta)$$

$$= 50.265\times10^3 - 5.0265\times10^6\theta$$



$$\delta_B = \frac{P_{BE}L_{BE}}{E_{BE}A_{BE}} \qquad \text{or} \qquad P_{BE} = \frac{E_{BE}A_{BE}}{L_{BE}}\delta_B$$

$$P_{BE} = \frac{(200\times10^9\text{ Pa})\frac{\pi}{4}(0.016\text{ m})^2}{3\text{ m}}(0.15\theta)$$

$$= 2.0106\times10^6\theta$$

From free body of member *ABC*:

$$+\circlearrowleft\ \Sigma M_A = 0: \quad 0.15P_{BE} - 0.25P_{CD} = 0$$

$$0.15(2.0106\times10^6\theta) - 0.25(50.265\times10^3 - 5.0265\times10^6\theta) = 0$$

$$\theta = 8.0645\times10^{-3}\text{ rad}$$

(*a*) $\quad P_{CD} = 50.265\times10^3 - 5.0265\times10^6(8.0645\times10^{-3})$

$\qquad = 9.7288\times10^3$ N

(*b*) $\quad \delta_C = 0.25\theta = 0.25(8.0645\times10^{-3})$

$\qquad = 2.0161\times10^{-3}$ m

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$P_{CD} = 9.73 \text{ kN}$ ◀

$\delta_C = 2.02 \text{ mm} \leftarrow$ ◀

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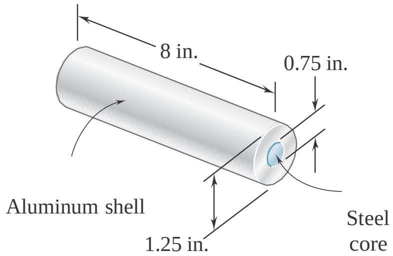
PROBLEM 2.132



The assembly shown consists of an aluminum shell ($E_a = 10.6 \times 10^6$ psi, $\alpha_a = 12.9 \times 10^{-6}/°\text{F}$) fully bonded to a steel core ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$) and is unstressed. Determine (*a*) the largest allowable change in temperature if the stress in the aluminum shell is not to exceed 6 ksi, (*b*) the corresponding change in length of the assembly.

SOLUTION

Since $\alpha_a > \alpha_s$, the shell is in compression for a positive temperature rise.

Let $$\sigma_a = -6 \text{ ksi} = -6 \times 10^3 \text{ psi}$$

$$A_a = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4}(1.25^2 - 0.75^2) = 0.78540 \text{ in}^2$$

$$A_s = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2$$

$$P = -\sigma_a A_a = \sigma_s A_s$$

where P is the tensile force in the steel core.

$$\sigma_s = -\frac{\sigma_a A_a}{A_s} = \frac{(6 \times 10^3)(0.78540)}{0.44179} = 10.667 \times 10^3 \text{ psi}$$

$$\varepsilon = \frac{\sigma_s}{E_s} + \alpha_s(\Delta T) = \frac{\sigma_a}{E_a} + \alpha_a(\Delta T)$$

$$(\alpha_a - \alpha_s)(\Delta T) = \frac{\sigma_s}{E_s} - \frac{\sigma_a}{E_a}$$

$$(6.4 \times 10^{-6})(\Delta T) = \frac{10.667 \times 10^3}{29 \times 10^6} + \frac{6 \times 10^3}{10.6 \times 10^6} = 0.93385 \times 10^{-3}$$

(*a*) $\Delta T = 145.91°\text{F}$ $\qquad\qquad$ $\Delta T = 145.9°\text{F}$ ◀

(*b*) $$\varepsilon = \frac{10.667 \times 10^3}{29 \times 10^6} + (6.5 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$$

or

$$\varepsilon = \frac{-6 \times 10^3}{10.6 \times 10^6} + (12.9 \times 10^{6})(145.91) = 1.3163 \times 10^{3}$$

$$\delta = L\varepsilon = (8.0)(1.3163 \times 10^{-3}) = 0.01053 \text{ in.}$$ $\qquad\qquad$ $\delta = 0.01053$ in. ◀

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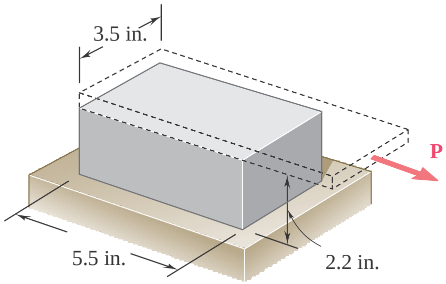


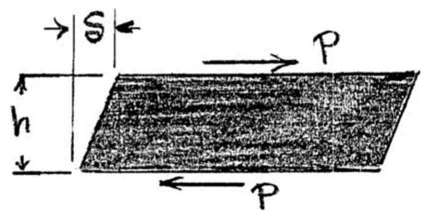
PROBLEM 2.133

The plastic block shown is bonded to a fixed base and to a horizontal rigid plate to which a force **P** is applied. Knowing that for the plastic used $G = 55$ ksi, determine the deflection of the plate when $P = 9$ kips.

SOLUTION

Consider the plastic block. The shearing force carried is $P = 9 \times 10^3$ lb

The area is $A = (3.5)(5.5) = 19.25\ \text{in}^2$



Shearing stress: $\tau = \dfrac{P}{A} = \dfrac{9 \times 10^3}{19.25} = 467.52$ psi

Shearing strain: $\gamma = \dfrac{\tau}{G} = \dfrac{467.52}{55 \times 10^3} = 0.0085006$

But $\gamma = \dfrac{\delta}{h} \quad \therefore \quad \delta = h\gamma = (2.2)(0.0085006)$ $\qquad \delta = 0.01870$ in. ◀

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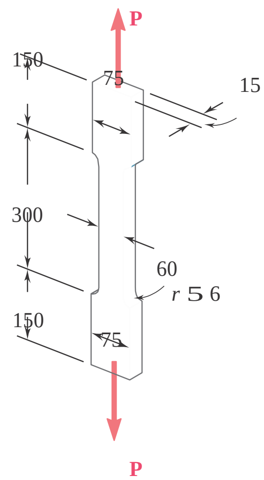
PROBLEM 2.134



Dimensions in mm

The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude *P*. (*a*) Knowing that $E = 70$ GPa and $\sigma_{all} = 200$ MPa, determine the maximum allowable value of *P* and the corresponding total elongation of the specimen. (*b*) Solve part *a*, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60 × 15-mm rectangular cross section.

SOLUTION

$$\sigma_{all} = 200\times10^{6}\text{ Pa} \quad E = 70\times10^{9}\text{ Pa}$$

$$A_{min} = (60\text{ mm})(15\text{ mm}) = 900\text{ mm}^2 = 900\times10^{-6}\text{ m}^2$$

(*a*) Test specimen. $D = 75$ mm, $d = 60$ mm, $r = 6$ mm

$$\frac{D}{d} = \frac{75}{60} = 1.25 \qquad \frac{r}{d} = \frac{6}{60} = 0.10$$

From Fig. 2.60b, $K = 1.95$ $\qquad \sigma_{max} = K\frac{P}{A}$

$$P = \frac{A\sigma_{max}}{K} = \frac{(900\times10^{-6})(200\times10^{6})}{1.95} = 92.308\times10^{3}\text{ N}$$

$P = 92.3$ kN ◀

Wide area $A^* = (75\text{ mm})(15\text{ mm}) = 1125\text{ mm}^2 = 1.125\times10^{-3}\text{ m}^2$

$$\delta = \Sigma\frac{P_iL_i}{A_iE_i} = \frac{P}{E}\Sigma\frac{L_i}{A_i} = \frac{92.308\times10^{3}}{70\times10^{9}}\left[\frac{0.150}{1.125\times10^{-3}} + \frac{0.300}{900\times10^{-6}} + \frac{0.150}{1.125\times10^{-3}}\right]$$

$$= 7.91\times10^{-6}\text{ m}$$

$\delta = 0.791$ mm ◀

(*b*) Uniform bar.

$$P = A\sigma_{all} = (900\times10^{-6})(200\times10^{6}) = 180\times10^{3}\text{ N}$$

$P = 180.0$ kN ◀

$$\delta = PL = (180\times10^{3})(0.600) = 1.714\times10^{-3}\text{m}$$

$\delta = 1.714$ mm ◀

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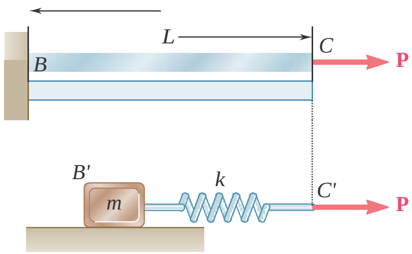
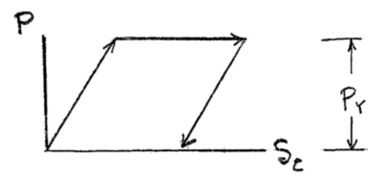
$AE \quad (900 \times 10^{-6})(70 \times 10^{9})$

PROBLEM 2.135



The uniform rod *BC* has a cross-sectional area *A* and is made of a mild steel that can be assumed to be elastoplastic with a modulus of elasticity *E* and a yield strength σ_Y. Using the block-and-spring system shown, it is desired to simulate the deflection of end *C* of the rod as the axial force **P** is gradually applied and removed, that is, the deflection of points *C* and *C′* should be the same for all values of *P*. Denoting by μ the coefficient of friction between the block and the horizontal surface, derive an expression for (*a*) the required mass *m* of the block, (*b*) the required constant *k* of the spring.

SOLUTION

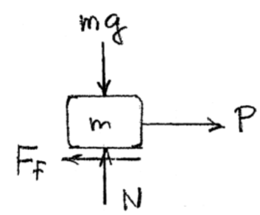


Force-deflection diagram for Point *C* or rod *BC*.

For $P < P_Y = A\sigma_Y$

$$\delta_C = \frac{PL}{EA} \qquad P = \frac{EA}{L}\delta_C$$

$$P_{\max} = P_Y = A\sigma_Y$$



Force-deflection diagram for Point *C′* of block-and-spring system.

$$+\uparrow \Sigma F_y = 0: \quad N - mg = 0 \qquad N = mg$$

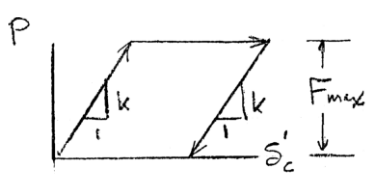
$$\xrightarrow{+} \Sigma F_x = 0: \quad P - F_f = 0 \qquad P = F_f$$

If block does not move, i.e., $F_f < \mu N = \mu mg$ or $P < \mu mg$,

then $\delta_c' = \dfrac{P}{K}$ or $P = k\delta_c'$

If $P = \mu mg$, then slip at $P = F_m = \mu mg$ occurs.

If the force *P* is the removed, the spring returns to its initial length.



(*a*) Equating P_Y and $F_{\max}$, $\quad A\sigma_Y = \mu mg \qquad m = \dfrac{A\sigma_Y}{\mu g}$ ◀

(*b*) Equating slopes, $\quad k = \dfrac{EA}{L}$ ◀

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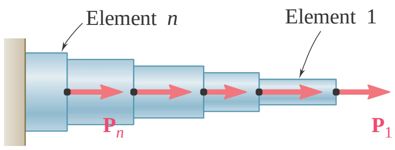

PROBLEM 2.C1

A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (*a*) Write a computer program that can be used to determine the average normal stress in each element, the deformation of each element, and the total deformation of the rod. (*b*) Use this program to solve Probs. 2.20 and 2.126.

SOLUTION

For each element, enter

$$L_i, \quad A_i, \quad E_i$$

<u>Compute deformation</u>

Update axial load $\qquad P = P + P_i$

Compute for each element

$$\sigma_i = P/A_i$$
$$\delta_i = PL_i/A_iE_i$$

Total deformation:

Update through n elements

$$\delta = \delta + \delta_i$$

<u>Program Outputs</u>

<u>Problem 2.20</u>

Element	Stress (MPa)	Deformation (mm)
1	19.0986	0.1091
2	–12.7324	–0.0909
Total Deformation =		0.0182 mm

<u>Problem 2.126</u>

Element	Stress (ksi)	Deformation (in.)
1	12.7324	0.0176

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2	–2.8294	–0.0057
Total Deformation =		0.01190 in.

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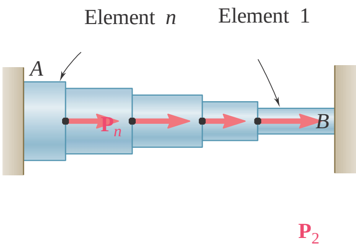
PROBLEM 2.C2



Rod *AB* is horizontal with both ends fixed; it consists of *n* elements, each of which is homogeneous and of uniform cross section, and is subjected to the loading shown. The length of element *i* is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (Note that $P_1 = 0$.) (*a*) Write a computer program which can be used to determine the reactions at *A* and *B*, the average normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs. 2.41 and 2.42.

SOLUTION

We Consider the reaction at *B* redundant and release the rod at *B*

Compute δ_B with $R_B = 0$

For each element, enter

$$L_i, \quad A_i, \quad E_i$$

Update axial load

$$P = P + P_i$$

Compute for each element

$$\sigma_i = P/A_i$$
$$\delta_i = PL_i/A_iE_i$$

Update total deformation

$$\delta_B = \delta_B + \delta_i$$

Compute δ_B due to unit load at *B*

$$\text{Unit} \quad \sigma_i = 1/A_i$$
$$\text{Unit} \quad \delta_i = L_i/A_iE_i$$

Update total unit deformation

$$\text{Unit} \quad \delta_B = \text{Unit } \delta_B + \text{Unit } \delta_i$$

Superposition

For total displacement at $B = 0$

$$\delta_B + R_B \quad \text{Unit} \quad \delta_B = 0$$

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Solving:

$$R_B = -\delta_B/\text{Unit } \delta_B$$

Then: $$R_A = \Sigma P_i + R_B$$

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PROBLEM 2.C2 *(Continued)*

For each element

$$\sigma = \sigma_i + R_B \quad \text{Unit } \sigma_i$$
$$\delta = \delta_i + R_B \quad \text{Unit } \delta_i$$

Program Outputs

Problem 2.41

RA = –62.809 kN
RB = –37.191 kN

Element	Stress (MPa)	Deformation (mm)
1	–52.615	–0.05011
2	3.974	0.00378
3	2.235	0.00134
4	49.982	0.04498

Problem 2.42

RA = –45.479 kN
RB = –54.521 kN

Element	Stress (MPa)	Deformation (mm)
1	–77.131	–0.03857
2	–20.542	–0.01027
3	–11.555	–0.01321
4	36.191	0.06204

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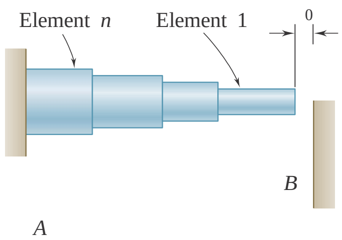

PROBLEM 2.C3

Rod *AB* consists of *n* elements, each of which is homogeneous and of uniform cross section. End *A* is fixed, while initially there is a gap δ_0 between end *B* and the fixed vertical surface on the right. The length of element *i* is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and its coefficient of thermal expansion by α_i. After the temperature of the rod has been increased by ΔT, the gap at *B* is closed and the vertical surfaces exert equal and opposite forces on the rod. (*a*) Write a computer program which can be used to determine the magnitude of the reactions at *A* and *B*, the normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs. 2.59 and 2.60.

SOLUTION

We compute the displacements at *B*.

Assuming there is no support at *B*,

enter $$L_i,\quad A_i,\quad E_i,\quad \alpha_i$$

Enter temperature change *T*. Compute for each element.

$$\delta_i = \alpha_i L_i T$$

Update total deformation.

$$\delta_B = \delta_B + \delta_i$$

Compute δ_B due to unit load at *B*.

$$\text{Unit } \delta_i = L_i / A_i E_i$$

Update total unit deformation.

$$\text{Unit } \delta_B = \text{Unit } \delta_B + \text{Unit } \delta_i$$

Compute reactions.

From superposition,

$$R_B = (\delta_B - \delta_0)/\text{Unit } \delta_B$$

Then

$$R_A = -R_B$$

For each element,

$$\sigma_i = -R_B / A_i$$
$$\delta_i = \alpha_i L_i T + R_B L_i / A_i E_i$$

PROBLEM 2.C3 *(Continued)*

Program Outputs

Problem 2.59.

$R = 52.279$ kips

Element	Stress (ksi)	Deformation (10 * –3 in.)
1	–21.783	9.909
2	–18.671	10.091

Problem 2.60.

$R = 232.390$ kN

Element	Stress (MPa)	Deformation (microm)
1	–116.195	363.220
2	–290.487	136.780

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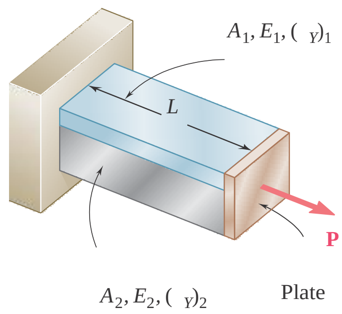
PROBLEM 2.C4



Bar *AB* has a length *L* and is made of two different materials of given cross-sectional area, modulus of elasticity, and yield strength. The bar is subjected as shown to a load **P** that is gradually increased from zero until the deformation of the bar has reached a maximum value δ_m and then decreased back to zero. (*a*) Write a computer program that, for each of 25 values of δ_m equally spaced over a range extending from 0 to a value equal to 120% of the deformation causing both materials to yield, can be used to determine the maximum value P_m of the load, the maximum normal stress in each material, the permanent deformation δ_p of the bar, and the residual stress in each material. (*b*) Use this program to solve Probs. 2.111 and 2.112.

SOLUTION

Note: The following assumes $(\sigma_Y)_1 < (\sigma_Y)_2$

Displacement increment

$$\delta_m = 0.05(\sigma_Y)_2 L/E_2$$

Displacements at yielding

$$\delta_A = (\sigma_Y)_1 L/E_1 \qquad \delta_B = (\sigma_Y)_2 L/E_2$$

For each displacement

If $\delta_m < \delta_A$:

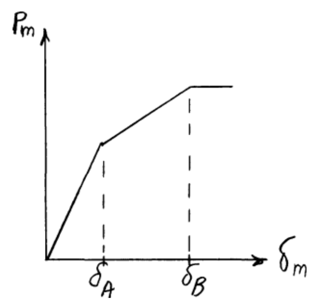
$$\sigma_1 = \delta_m E_1/L$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = (\delta_m/L)(A_1E_1 + A_2E_2)$$

If $\delta_A < \delta_m < \delta_B$:

$$\sigma_1 = (\sigma_Y)_1$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = A_1\sigma_1 + (\delta_m/L)A_2E_2$$

If $\delta_m > \delta_B$:

$$\sigma_1 = (\sigma_Y)_1$$
$$\sigma_2 = (\sigma_Y)_2$$
$$P_m = A_1\sigma_1 + A_2\sigma_2$$



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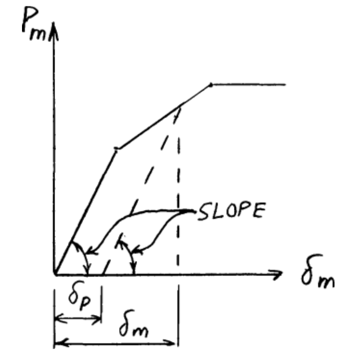
PROBLEM 2.C4 *(Continued)*



Permanent deformations, residual stresses

Slope of first (elastic) segment

$$\text{Slope} = (A_1E_1 + A_2E_2)/L$$

$$\delta_P = \delta_m - (P_m/\text{Slope})$$

$$(\sigma_1)_{\text{res}} = \sigma_1 - (E_1P_m/(L\ \text{Slope}))$$

$$(\sigma_2)_{\text{res}} = \sigma_2 - (E_2P_m/(L\ \text{Slope}))$$

Program Outputs

Problems 2.111 and 2.112

DM 10** – 3 in.	PM kips	SIGM (1) ksi	SIGM (2) ksi	DP 10** – 3 in.	SIGR (1) ksi	SIG (2) ksi	
0.000	0.000	0.000	0.000	0.000	0.000	0.000	
2.414	8.750	5.000	5.000	0.000	0.000	0.000	
4.828	17.500	10.000	10.000	0.000	0.000	0.000	
7.241	26.250	15.000	15.000	0.000	0.000	0.000	
9.655	35.000	20.000	20.000	0.000	0.000	0.000	
12.069	43.750	25.000	25.000	0.000	0.000	0.000	
14.483	52.500	30.000	30.000	0.000	0.000	0.000	
16.897	61.250	35.000	35.000	0.000	0.000	0.000	
19.310	70.000	40.000	40.000	0.000	0.000	0.000	
21.724	78.750	45.000	45.000	0.000	0.000	0.000	
24.138	87.500	50.000	50.000	0.000	0.000	0.000	
26.552	91.250	50.000	55.000	1.379	–2.143	2.857	
28.966	95.000	50.000	60.000	2.759	–4.286	5.714	
31.379	98.750	50.000	65.000	4.138	–6.429	8.571	2.112 ◀
33.793	102.500	50.000	70.000	5.517	–8.571	11.429	
36.207	106.250	50.000	75.000	6.897	–10.714	14.286	
38.621	110.000	50.000	80.000	8.276	–12.857	17.143	
41.034	113.750	50.000	85.000	9.655	–15.000	20.000	2.111 ◀
43.448	117.500	50.000	90.000	11.034	–17.143	22.857	
45.862	121.250	50.000	95.000	12.414	–19.286	25.714	
48.276	125.000	50.000	100.000	13.793	–21.429	28.571	
50.690	125.000	50.000	100.000	16.207	–21.429	28.571	
53.103	125.000	50.000	100.000	18.621	–21.429	28.571	
55.517	125.000	50.000	100.000	21.034	–21.429	28.571	
57.931	125.000	50.000	100.000	23.448	–21.429	28.571	

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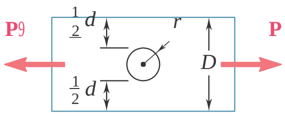
PROBLEM 2.C5



The plate has a hole centered across the width. The stress concentration factor for a flat bar under axial loading with a centric hole is

$$K = 3.00 - 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 - 1.53\left(\frac{2r}{D}\right)^3$$

where r is the radius of the hole and D is the width of the bar. Write a computer program to determine the allowable load **P** for the given values of r, D, the thickness t of the bar, and the allowable stress σ_{all} of the material. Knowing that $t = \frac{1}{4}$ in., $D = 3.0$ in., and $\sigma_{all} = 16$ ksi, determine the allowable load **P** for values of r from 0.125 in. to 0.75 in., using 0.125 in. increments.

SOLUTION

Enter

$$r,\quad D,\quad t,\quad \sigma_{\text{all}}$$

Compute K

$$RD = 2.0r/D$$

$$K = 3.00 - 3.13RD + 3.66RD^2 - 1.53RD^3$$

Compute average stress

$$\sigma_{\text{ave}} = \sigma_{\text{all}}/K$$

Allowable load

$$P_{\text{all}} = \sigma_{\text{ave}}(D - 2.0r)t$$

Program Output

Radius (in.)	Allowable Load (kips)
0.1250	3.9802
0.2500	3.8866
0.3750	3.7154
0.5000	3.4682
0.6250	3.1523
0.7500	2.7794

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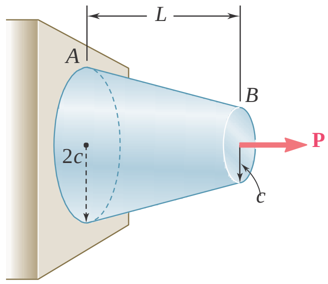


PROBLEM 2.C6

A solid truncated cone is subjected to an axial force **P** as shown. The exact elongation is $(PL)/(2\pi c^2 E)$. By replacing the cone by n circular cylinders of equal thickness, write a computer program that can be used to calculate the elongation of the truncated cone. What is the percentage error in the answer obtained from the program using (*a*) $n = 6$, (*b*) $n = 12$, (*c*) $n = 60$?

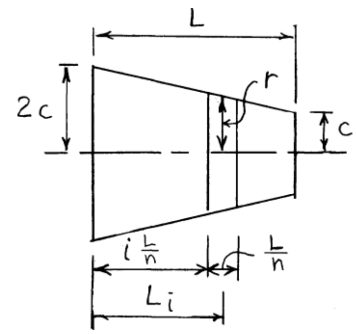
SOLUTION

For

$$i = 1 \text{ to } n:$$

$$L_i = (i + 0.5)(L/n)$$

$$r_i = 2c - c(L_i/L)$$



Area:

$$A = \pi r_i^2$$

Displacement:

$$\delta = \delta + P(L/n)/(AE)$$

Exact displacement:

$$\delta_{exact} = PL/(2.0\pi c^2 E)$$

Percentage error:

$$\text{Percent} = 100(\delta - \delta_{exact})/\delta_{exact}$$

Program Output

n	Approximate	Exact	Percent
6	0.15852	0.15915	–0.40083
12	0.15899	0.15915	–0.10100
60	0.15915	0.15915	–0.00405

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