Solution Manual for Mechanics of Materials 8th Edition Gere Goodno 1111577730 9781111577735

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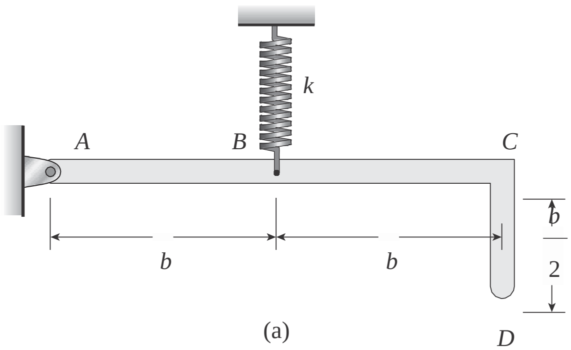
2

Axially Loaded Members

Changes in Lengths of Axially Loaded Members

Problem 2.2-1 The L-shaped arm *ABCD* shown in the figure lies in a vertical plane and pivots about a horizontal pin at *A*. The arm has constant cross-sectional area and total weight *W*.
A vertical spring of stiffness *k* supports the arm at point *B*.

(a) Obtain a formula for the elongation of the spring due to the weight of the arm.
(b) Repeat part (a) if the pin support at *A* is moved to *D*.



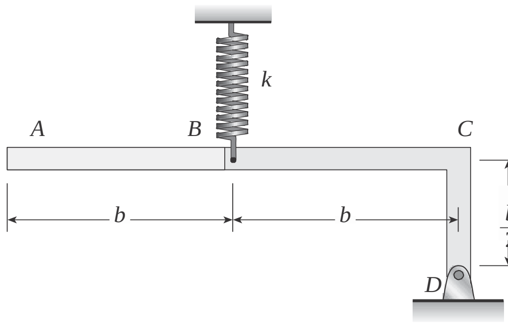
(a)

Solution 2.2-1

(a) Sum moments about *A*

$$©M_A \quad 0 \quad \frac{2b}{\frac{5}{2}b}Wb + \frac{\frac{b}{2}}{\frac{5}{2}b}W(2b)kdb$$

$$\boxed{d \quad \frac{\frac{2b}{\frac{5}{2}b}Wb + \frac{\frac{b}{2}}{\frac{5}{2}b}W(2b)}{kb} \quad \frac{6W}{5k}}$$

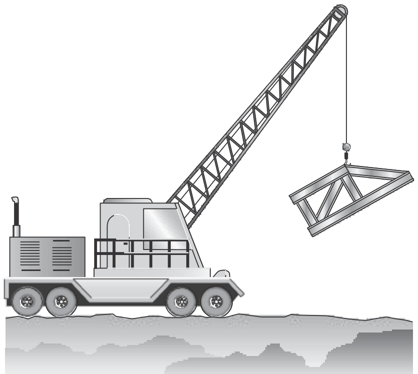


(b)

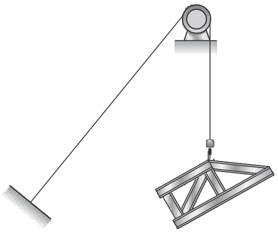
(b) $©M_D \quad 0 \quad kbd \quad \frac{2b}{\frac{5}{2}b}Wb \quad \frac{4Wb}{5}$ so $\boxed{d \quad \frac{\frac{2b}{\frac{5}{2}b}Wb}{kb} \quad \frac{4W}{5k}}$

Problem 2.2-2 A steel cable with nominal diameter 25 mm (see Table 2-1) is used in a construction yard to lift a bridge section weighing 38 kN, as shown in the figure. The cable has an effective modulus of elasticity E 140 GPa.

(a) If the cable is 14 m long, how much will it stretch when the load is picked up?
(b) If the cable is rated for a maximum load of 70 kN, what is the factor of safety with respect to failure of the cable?



Solution 2.2-2 Bridge section lifted by a cable



A 304 mm^2 (from Table 2-1)

W 38 kN

E 140 GPa

L 14 m

(b) FACTOR OF SAFETY

P_{ULT} 406 kN (from Table 2-1)

P_{max} 70 kN

$$n \quad \frac{P_{ULT}}{P_{max}} \quad \frac{406 \text{ kN}}{70 \text{ kN}} \quad 5.8$$

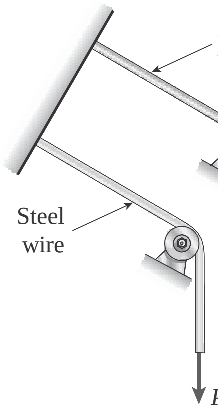
(a) STRETCH OF CABLE

$$d \quad \frac{WL}{EA} \quad \frac{(38 \text{ kN})(14 \text{ m})}{(140 \text{ GPa})(304 \text{ mm}^2)}$$

$$12.5 \text{ mm}$$

Problem 2.2-3 A steel wire and an aluminum alloy wire have equal lengths and support equal loads P (see figure). The moduli of elasticity for the steel and aluminum alloy are E_s 30,000 ksi and E_a 11,000 ksi, respectively.

(a) If the wires have the same diameters, what is the ratio of the elongation of the aluminum alloy wire to the elongation of the steel wire?
(b) If the wires stretch the same amount, what is the ratio of the diameter of the aluminum alloy wire to the diameter of the steel wire?
(c) If the wires have the same diameters and same load P, what is the ratio of the initial length of the aluminum alloy wire to that of the steel wire if the aluminum alloy wire stretches 1.5 times that of the steel wire?
(d) If the wires have the same diameters, same initial length, and same load P, what is the material of the upper wire if it elongates 1.7 times that of the steel wire?



Solution 2.2-3

(a) $$\frac{\delta_a}{\delta_s} = \frac{\frac{PL}{E_aA}}{\left(\frac{PL}{E_sA}\right)} = \frac{E_s}{E_a}$$

$E_s = 30{,}000$ ksi $\quad E_a = 11{,}000$ ksi

$$\boxed{\frac{E_s}{E_a} = 2.727} \quad \frac{30}{11} = 2.727$$

(b) $\delta_a = \delta_s$ so $\dfrac{PL}{E_aA_a} = \dfrac{PL}{E_sA_s}$ so $\dfrac{A_a}{A_s} = \dfrac{E_s}{E_a}$ and $\boxed{\dfrac{d_a}{d_s} = \sqrt{\dfrac{E_s}{E_a}} = 1.651}$

(c) Same diameter, same load, find ratio of length of aluminum to steel wire if elongation of aluminum is 1.5 times that of steel wire

$$\frac{\delta_a}{\delta_s} = \frac{\frac{PL_a}{E_aA}}{\left(\frac{PL_s}{E_sA}\right)} = \frac{\frac{PL_a}{E_aA}}{\left(\frac{PL_s}{E_sA}\right)} = 1.5 \qquad \boxed{\frac{L_a}{L_s} = 1.5\frac{E_a}{E_s} = 0.55}$$

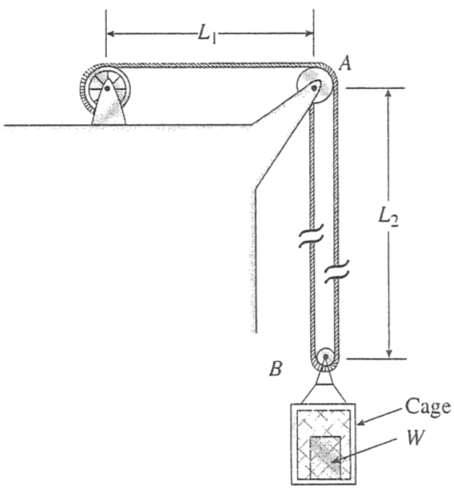
(d) Same diameter, same length, same load—but wire 1 elongation 1.7 times the steel wire what is wire 1 material?

$$\frac{\delta_1}{\delta_s} = \frac{\frac{PL}{E_1A}}{\left(\frac{PL}{E_sA}\right)} = \frac{\frac{PL}{E_1A}}{\left(\frac{PL}{E_sA}\right)} = 1.7 \qquad E_1 = \frac{E_s}{1.7} = 17{,}647 \text{ ksi}$$

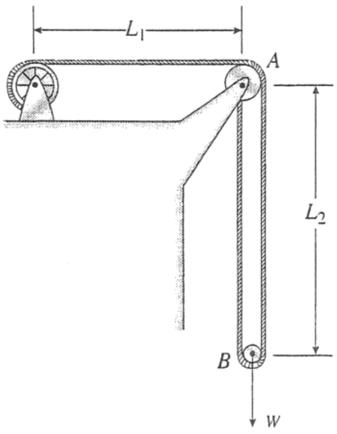
$\boxed{\text{cast iron or copper alloy (see App. I)}}$

Problem 2.2-4 By what distance h does the cage shown in the figure move downward when the weight W is placed inside it?

Consider only the effects of the stretching of the cable, which has axial rigidity $EA = 10{,}700$ kN. The pulley at A has diameter $d_A = 300$ mm and the pulley at B has diameter $d_B = 150$ mm. Also, the distance $L_1 = 4.6$ m, the distance $L_2 = 10.5$ m, and the weight $W = 22$ kN. (*Note*: When calculating the length of the cable, include the parts of the cable that go around the pulleys at A and B.)



Solution 2.2-4 Cage supported by a cable



d_A 300 mm

d_B 150 mm

L_1 4.6 m

L_2 10.5 m

EA 10,700 kN

W 22 kN

TENSILE FORCE IN CABLE

$$T \quad \frac{W}{2} \quad 11 \text{ kN}$$

LENGTH OF CABLE

$$L \quad L_1 + 2L_2 + \frac{1}{4}1pd_A2 + \frac{1}{2}(pd_B)$$

$$4{,}600 \text{ mm} + 21{,}000 \text{ mm} + 236 \text{ mm} + 236 \text{ mm}$$

$$26{,}072 \text{ mm}$$

ELONGATION OF CABLE

$$d \quad \frac{TL}{EA} \quad \frac{(11 \text{ kN})(26{,}072 \text{ mm})}{(10{,}700 \text{ kN})} \quad 26.8 \text{ mm}$$

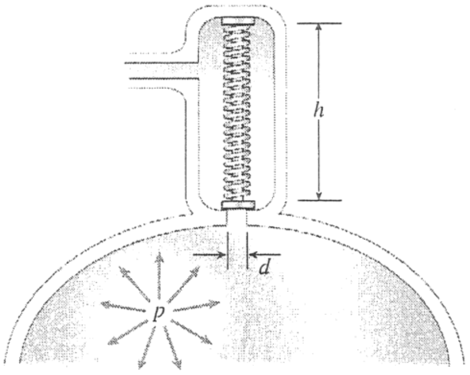
LOWERING OF THE CAGE

h distance the cage moves downward

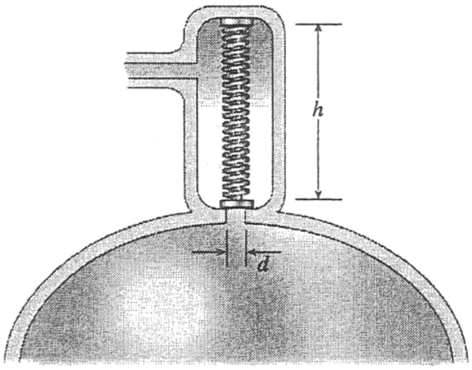
$$h \quad \frac{1}{2}d \quad 13.4 \text{ mm}$$

Problem 2.2-5 A safety valve on the top of a tank containing steam under pressure p has a discharge hole of diameter d (see figure). The valve is designed to release the steam when the pressure reaches the value $p_{\max}$.

If the natural length of the spring is L and its stiffness is k, what should be the dimension h of the valve? (Express your result as a formula for h.)



Solution 2.2-5 Safety valve



h = height of valve (compressed length of the spring)

d = diameter of discharge hole

p = pressure in tank

p_{max} = pressure when valve opens

L = natural length of spring ($L > h$)

k = stiffness of spring

FORCE IN COMPRESSED SPRING

$F = k(L - h)$ (From Eq. 2-1a)

PRESSURE FORCE ON SPRING

$$P = p_{max}\left(\frac{\pi d^2}{4}\right)$$

EQUATE FORCES AND SOLVE FOR h:

$$F = P \quad k(L - h) = \frac{\pi p_{max} d^2}{4}$$

$$h = L - \frac{\pi p_{max} d^2}{4k} \quad \leftarrow$$

Problem 2.2-6 The device shown in the figure consists of a prismatic rigid pointer ABC supported by a uniform translational spring of stiffness $k = 950$ N/m. The spring is positioned at distance $b = 165$ mm from the pinned end A of the pointer. The device is adjusted so that when there is no load P, the pointer reads zero on the angular scale.

(a) If the load $P = 11$ N, at what distance x should the load be placed so that the pointer will read $\theta = 2.5°$ on the scale (see figure part a)?
(b) Repeat part (a) if a rotational spring $k_r = kb^2$ is added at A (see figure part b).
(c) Let $x = 7b/8$. What is P_{max} (N) if θ cannot exceed 2°? Include spring k_r in your analysis.
(d) Now, if the weight of the pointer ABC is known to be $W_p = 3$ N and the weight of the spring is $W_s = 2.75$ N, what initial angular position (i.e., θ in degrees) of the pointer will result in a zero reading on the angular scale once the pointer is released from rest? Assume $P = k_r = 0$.
(e) If the pointer is rotated to a vertical position (see figure part c), find the required load P, applied at mid-height of the pointer, that will result in a pointer reading of $\theta = 2.5°$ on the scale. Consider the weight of the pointer W_p in your analysis.

Solution 2.2-6

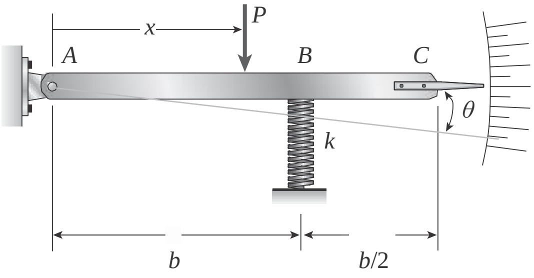
NUMERICAL DATA k 950 N/m b 165 mm P 11 N u 2.5 u_{max} 2

W_p 3 N W_s 2.75 N

(a) If the load P 11 N, at what distance x should the load be placed so that the pointer will read u 2.5° on the scale (see Fig. a)?

Sum moments about A, then solve for x:

$$x \quad \frac{k\,u\,b^2}{P} \quad 102.6\text{ mm} \quad \boxed{x \quad 102.6\text{ mm}}$$



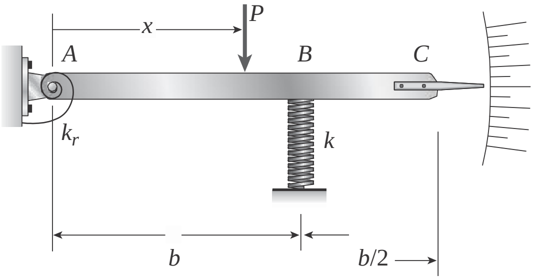
(a)

(b) Repeat (a) if a rotational spring k_r kb^2 is added at A (see Fig. b).

k_r $k\,b^2$ 25864 N·mm

Sum moments about A, then solve for x:

$$x \quad \frac{k\,u\,b^2 + k_r\,u}{P} \quad 205\text{ mm} \quad \frac{x}{b} \quad 1.244 \quad \boxed{x \quad 205\text{ mm}}$$



(b)

(c) Now if x $7b/8$, what is P_{max} (N) if u cannot exceed 2 ? x $\frac{7}{8}b$ 144.375 mm

Sum moments about A, then solve for P: $$P_{max} \quad \frac{k\,u_{max}\,b^2 + k_r\,u_{max}}{\frac{7}{8}b} \quad 12.51\text{ N} \quad \boxed{P_{max} \quad 12.51\text{ N}}$$

(d) Now, if the weight of the pointer ABC is known to be W_p 3 N and the weight of the spring is W_s 2.75 N, what initial angular position (i.e., u in degrees) of the pointer will result in a zero reading on the angular scale once the pointer is released from rest? Assume P k_r 0.

Deflection at spring due to W_p: Deflection at B due to self weight of spring:

$$d_{Bp} \quad \frac{W_p\,a\,\frac{3}{4}b\,b}{k\,b} \quad 2.368\text{ mm} \qquad d_{Bk} \quad \frac{W_s}{2\,k} \quad 1.447\text{ mm}$$

$$d_B \quad d_{Bp} + d_{Bk} \quad 3.816\text{ mm} \qquad u_{init} \quad \frac{d_B}{b} \quad 1.325$$

$$\text{OR} \quad u_{init} \quad \arctan\,a\,\frac{d_B}{b}\,b \quad 1.325 \quad \boxed{u_{init} \quad 1.325}$$

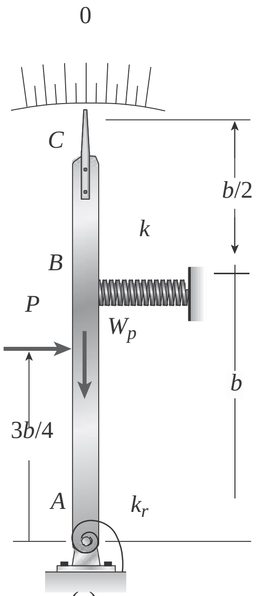
(e) If the pointer is rotated to a vertical position (figure part c), find the required load P, applied at mid-height of the pointer that will result in a pointer reading of $\theta = 2.5^\circ$ on the scale. Consider the weight of the pointer, W_p, in your analysis.

$$k = 950\ \text{N/m} \quad b = 165\ \text{mm} \quad W_p = 3\ \text{N}$$

$$k_r = kb^2 = 25.864\ \text{N}\cdot\text{m} \quad \theta = 2.5^\circ$$

Sum moments about A to get P:

$$P = \frac{\theta}{\left(\frac{3b}{4}\right)}\left[k_r + k\left(\frac{5}{4}b\right)^2 - W_p\left(\frac{3b}{4}\right)\right] = 20.388\ \text{N} \qquad \boxed{P = 20.4\ \text{N}}$$

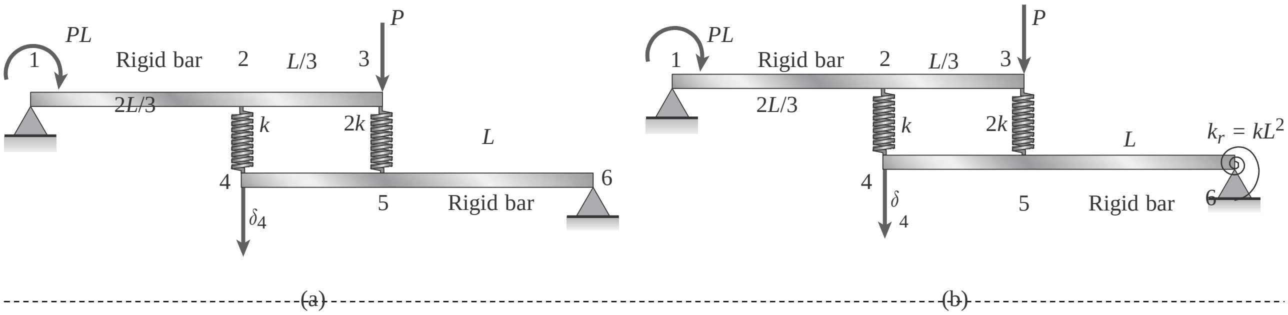


(c)

Problem 2.2-7 Two rigid bars are connected to each other by two linearly elastic springs. Before loads are applied, the lengths of the springs are such that the bars are parallel and the springs are without stress.

(a) Derive a formula for the displacement δ_4 at point 4 when the load P is applied at joint 3 and moment PL is applied at joint 1, as shown in the figure part a. (Assume that the bars rotate through very small angles under the action of the load P.)

(b) Repeat part (a) if a rotational spring, $k_r = kL^2$, is now added at joint 6. What is the ratio of the deflection δ_4 in the figure part a to that in the figure part b?



Solution 2.2-7

(a) Derive a formula for the displacement δ_4 at point 4 when the load P is applied at joint 3 and moment PL is applied at joint 1, as shown.

Cut horizontally through both springs to create upper and lower FBD's. Sum moments about joint 1 for upper FBD and also sum moments about joint 6 for lower FBD to get two equations of equilibrium; assume both springs are in tension.

Note that $\delta_2 = \frac{2}{3}\delta_3$ and $\delta_5 = \frac{3}{4}\delta_4$

Force in left spring: $k\left(\delta_4 \; \frac{2}{3}\delta_3\right)$

Force in right spring: $2k\left(\frac{3}{4}\delta_4 \; \delta_3\right)$

Summing moments about joint 1 (upper FBD) and about joint 6 (lower FBD) then dividing through by k gives

$$\begin{bmatrix} \frac{22}{9} & \frac{13}{6} \\ \frac{26}{9} & \frac{17}{6} \end{bmatrix} \begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \frac{2P}{k} \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{bmatrix} \frac{22}{9} & \frac{13}{6} \\ \frac{26}{9} & \frac{17}{6} \end{bmatrix}^{-1} \begin{pmatrix} \frac{2P}{k} \\ 0 \end{pmatrix} = \begin{bmatrix} \frac{17P}{2k} \\ \frac{26P}{3k} \end{bmatrix} \quad \frac{17}{2} = 8.5 \quad \frac{26}{3} = 8.667 \qquad \boxed{\delta_4 = \frac{26P}{3k}}$$

^ deltas are **positive downward**

(b) Repeat part (a) if a rotational spring $k_r = kL^2$ is now added at joint 6. What is the ratio of the deflection $\delta 4$ in part (a) to that in (b)?

Upper FBD—sum moments about joint 1:

$$k\left(\delta_4 \; \frac{2}{3}\delta_3\right)\frac{2L}{3} + 2k\left(\frac{3}{4}\delta_4 \; \delta_3\right)L = 2PL \quad \text{OR} \quad \left(\frac{22Lk}{9}\right)\delta_3 + \frac{13Lk}{6}\delta_4 = 2PL$$

Lower FBD—sum moments about joint 6:

$$k\left(\delta_4 \; \frac{2}{3}\delta_3\right)\frac{4L}{3} + 2k\left(\frac{3}{4}\delta_4 \; \delta_3\right)L \; k_r u_6 = 0$$

$$\left[k\left(\delta_4 \; \frac{2}{3}\delta_3\right)\frac{4L}{3} + 2k\left(\frac{3}{4}\delta_4 \; \delta_3\right)L\right] + (kL^2)\left(\frac{\delta_4 \; \frac{4}{3}\delta_3}{L}\right) = 0 \quad \text{OR} \quad \left(\frac{26Lk}{9}\right)\delta_3 + \frac{43Lk}{12}\delta_4 = 0$$

Divide matrix equilibrium equations through by k to get the following displacement equations:

$$\begin{bmatrix} \frac{22}{9} & \frac{13}{6} \\ \frac{26}{9} & \frac{43}{6} \end{bmatrix} \begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \frac{2P}{k} \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{bmatrix} \frac{22}{9} & \frac{13}{6} \\ \frac{26}{9} & \frac{43}{12} \end{bmatrix}^{-1} \begin{pmatrix} \frac{2P}{k} \\ 0 \end{pmatrix} = \begin{bmatrix} \frac{43P}{15k} \\ \frac{104P}{45k} \end{bmatrix} \quad \frac{43}{15} = 2.867 \quad \frac{104}{45} = 2.311 \qquad \boxed{\delta_4 = \frac{104P}{45k}}$$

^ deltas are positive downward

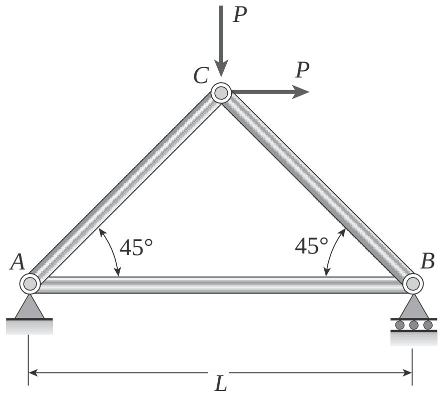
Ratio of the deflection δ_4 in part (a) to that in (b): $\dfrac{\frac{26}{3}}{\frac{104}{45}} = \frac{15}{4} \qquad \boxed{\frac{15}{4} = \text{Ratio}}$

4 3.75

Problem 2.2-8 The three-bar truss ABC shown in figure part a has a span L 3 m and is constructed of steel pipes having cross-sectional area A 3900 mm^2 and modulus of elasticity E 200 GPa. Identical loads P act both vertically and horizontally at joint C, as shown.

(a) If P 475 kN, what is the horizontal displacement of joint B?
(b) What is the maximum permissible load value P_{max} if the displacement of joint B is limited to 1.5 mm?
(c) Repeat parts (a) and (b) if the plane truss is replaced by a space truss (see figure part b).

Solution 2.2-8



(a)

Numerical data

$A \quad 3900\ \text{mm}^2 \qquad E \quad 200\ \text{GPa}$

$P \quad 475\ \text{kN} \qquad L \quad 3000\ \text{mm}$

$d_{Bmax} \quad 1.5\ \text{mm}$

(a) Find horizontal displacement of joint B
Statics To find support reactions and then member forces:

$$\text{g}M_A \quad 0 \qquad B_y \quad \frac{1}{L}\ \text{a}\,2P\,\frac{L}{2}\ \text{b}$$

$$B_y \quad P$$

$$\text{g}F_H \quad 0 \qquad A_x \qquad P$$

$$\text{g}F_V \quad 0 \qquad A_y \quad P \quad B_y \qquad A_y \quad 0$$

Method of Joints: $AC_V \quad A_Y \quad AC_V \quad 0$ Force in AC 0

$AB \quad A_X$

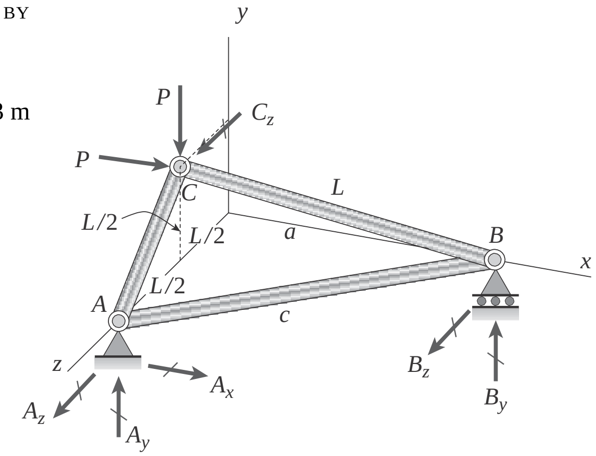
Force in AB is P (tension) so elongation of AB is the horizontal displacement of joint B.

$$d_B \ \frac{F_{AB}L}{EA} \qquad d_B \ \frac{PL}{EA} \qquad d_B \quad 1.82692\ \text{mm} \qquad \boxed{d_B \quad 1.827\ \text{mm}}$$

(b) Find P_{max} if displacement of joint B $\quad d_{Bmax} \quad 1.5\ \text{mm} \qquad P_{max} \quad \frac{EA}{L}\, d_{Bmax} \quad \boxed{P_{max} \quad 390\ \text{kN}}$

(c) Repeat parts (a) and (b) if the plane truss is replaced by a space truss (see figure part b).

Find missing dimensions a and c: $\quad P \quad 475\ \text{kN} \quad L \quad 3\ \text{m}$



(b)

$$a = \sqrt{L^2 - 2\left(\frac{L}{2}\right)^2} = 2.12132\ \text{m} \qquad \frac{a}{L} = 0.707 \qquad a = \frac{L}{\sqrt{2}} = 2.12132\ \text{m}$$

$$c = \sqrt{L^2 + a^2} = 3.67423\ \text{m} \qquad c = \sqrt{L^2 + \left(\frac{L}{\sqrt{2}}\right)^2} = 3.67423\ \text{m} \qquad c = L\sqrt{\frac{3}{2}} = 3.67423\ \text{m}$$

(1) Sum moments about a line thru A which is parallel to the y-axis

$$B_z = P\,\frac{L}{a} = 671.751\ \text{kN}$$

(2) Sum moments about the z-axis

$$B_y = \frac{P\left(\frac{L}{2}\right)}{a} = 335.876\ \text{kN} \qquad \text{SO} \qquad A_y = P - B_y = 139.124\ \text{kN}$$

(3) Sum moments about the x-axis

$$C_z = \frac{A_y L - P\frac{L}{2}}{\frac{L}{2}} = 196.751\ \text{kN}$$

(4) Sum forces in the x- and z-directions $\quad A_x = P = 475$ kN $\quad A_z = C_z \quad B_z = 868.503$ kN

(5) Use method of joints to find member forces

Sum forces in x-direction at joint A: $\quad \frac{a}{c}F_{AB} + A_x = 0 \qquad F_{AB} = \frac{c}{a}A_x = 823$ kN

Sum forces in y-direction at joint A: $\quad \dfrac{\frac{L}{2}}{\sqrt{2}\,\frac{L}{2}}F_{AC} + A_y = 0 \qquad F_{AC} = \sqrt{2}\,A_y = 196.8$ kN

Sum forces in y-direction at joint B: $\quad \dfrac{\frac{L}{2}}{L}F_{BC} + B_y = 0 \qquad F_{BC} = 2B_y = 672$ kN

(6) Find displacement along x-axis at joint B

Find change in length of member AB then find its projection along x axis:

$$d_{AB} = \frac{F_{AB}\,c}{EA} = 3.875\ \text{mm} \qquad \beta = \arctan\left(\frac{L}{a}\right) = 54.736 \qquad d_{Bx} = \frac{d_{AB}}{\cos(\beta)} = 6.713\ \text{mm} \qquad \boxed{d_{Bx} = 6.71\ \text{mm}}$$

(7) Find P_{max} for space truss if δ_{Bx} must be limited to 1.5 mm

Displacements are linearly related to the loads for this linear elastic small displacement problem, so reduce load variable P from 475 kN to

$$\frac{1.5}{6.71254}\,475 = 106.145\ \text{kN} \qquad \boxed{P_{max} = 106.1\ \text{kN}}$$

Repeat space truss analysis using vector operations $\quad a = 2.121$ m $\quad L = 3$ m $\quad P = 475$ kN

POSITION AND UNIT VECTORS:

$$r_{AB} \quad \begin{pmatrix} a \\ 0 \\ L \end{pmatrix} \quad e_{AB} \quad \frac{r_{AB}}{r_{AB}} \quad \begin{pmatrix} 0.577 \\ 0 \\ 0.816 \end{pmatrix} \qquad r_{AC} \quad \begin{pmatrix} 0 \\ \frac{L}{2} \\ \frac{L}{2} \end{pmatrix} \quad e_{AC} \quad \frac{r_{AC}}{r_{AC}} \quad \begin{pmatrix} 0 \\ 0.707 \\ 0.707 \end{pmatrix}$$

FIND MOMENT AT A:

$$M_A \quad r_{AB} * R_B + r_{AC} * R_C$$

$$0 \qquad 2.P \qquad 3.0\text{ m } RB_y + 1.5\text{ m } RC_z \quad 712.5\text{ kN}\cdot\text{m}$$

$$M_A \quad r_{AB} * \begin{pmatrix} RB_y \\ RB_z \end{pmatrix} + r_{AC} * \begin{pmatrix} P \\ RC_z \end{pmatrix} \quad \begin{pmatrix} 2.1213\text{ m } RB_Z \quad 1425.0\text{ kN}\cdot\text{m} \\ 2.1213\text{ m } RB_y \quad 1425.0\text{ kN}\cdot\text{m} \end{pmatrix}$$

FIND MOMENTS ABOUT LINES OR AXES:

$$M_A e_{AB} \quad 1.732\text{ m } RB_y + 1.7321\text{ m } RB_y + 0.86603\text{ m } RC_z + 752.15\text{ kN}\cdot\text{m}$$

$$RC_z \ \frac{244.12}{0.72169} \quad 338.262 \quad C_z \quad 196.751\text{ kN}$$

$$M_A e_{AC} \quad 1.5\text{ m } RB_y + \quad 1.5\text{ m } RB_z \quad \text{So} \quad RB_y \quad RB_z$$

$$M_A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad 2.1213\text{ m } RB_z + \quad 1425.0\text{ kN}\cdot\text{m} \quad \text{So} \quad RB_z \ \frac{462.5}{1.7678} \quad 261.625 \quad B_z \quad 671.75\text{ kN}$$

$$0$$

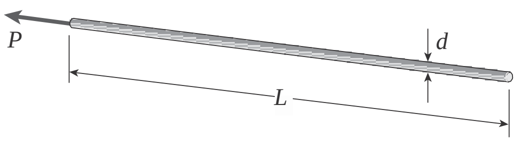
$$M_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad 2.1213\text{ m } RB_y + \quad 1425.0\text{ kN}\cdot\text{m} \quad \text{So} \quad RB_y \quad RB_z \quad 261.625 \quad B_y \quad 335.876\text{ kN}$$

$$\Sigma F_y \quad 0 \quad A_y \quad P \quad B_y \quad 139.124\text{ kN}$$

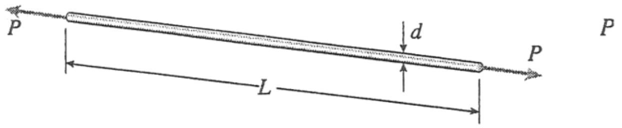
Reactions obtained using vector operations agree with those based on scalar operations.

Problem 2.2-9 An aluminum wire having a diameter d 1/10 in. and length L 12 ft is subjected to a tensile load P (see figure). The aluminum has modulus of elasticity E 10,600 ksi

If the maximum permissible elongation of the wire is 1/8 in. and the allowable stress in tension is 10 ksi, what is the allowable load P_{max}?



Solution 2.2-9



$d \quad \frac{1}{10}$ in. $\quad L \quad 12(12)$ in. $\quad E \quad 10{,}600 \quad (10^3)$ psi

$\delta_a \quad \frac{1}{8}$ in. $\quad \sigma_a \quad 10 * (10^3)$ psi

$A \quad \frac{\pi d^2}{4} \quad A \quad 7.854 \quad 10^{\ 3}$ in.^{2}

$EA \quad 8.325 \quad 10^4$ lb

Maximum load based on elongation:

$P_{max1} \quad \frac{EA}{L} \delta_a \quad P_{max1} \quad 72.3$ lb $\quad$ controls

Maximum load based on stress:

$P_{max2} \quad \sigma_a A \quad P_{max2} \quad 78.5$ lb

Problem 2.2-10 A uniform bar AB of weight $W \quad 25$ N is supported by two springs, as shown in the figure. The spring on the left has stiffness $k_1 \quad 300$ N/m and natural length $L_1 \quad 250$ mm. The corresponding quantities for the spring on the right are $k_2 \quad 400$ N/m and $L_2 \quad 200$ mm. The distance between the springs is $L \quad 350$ mm, and the spring on the right is suspended from a support that is distance $h \quad 80$ mm below the point

of support for the spring on the left. Neglect the weight of the springs.

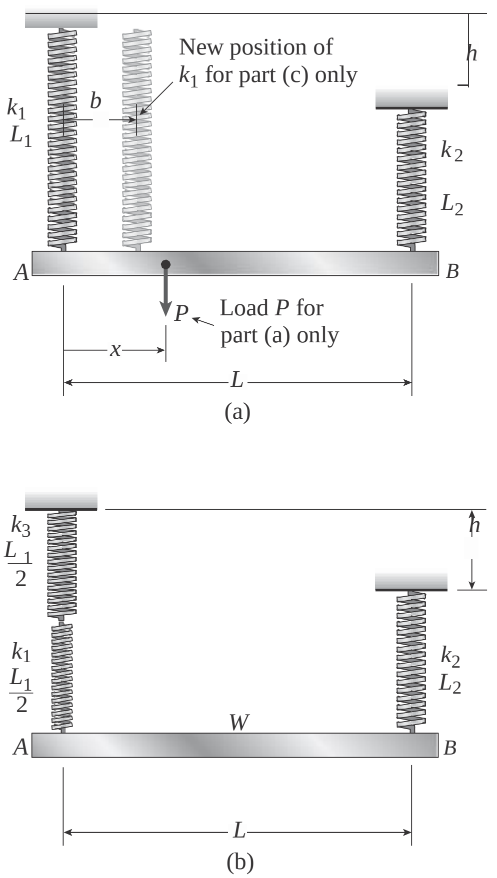
should a load $P \quad 18$ N be placed in order to bring the bar to a horizontal position?

(b) If P is now removed, what new value of k_1 is required so that the

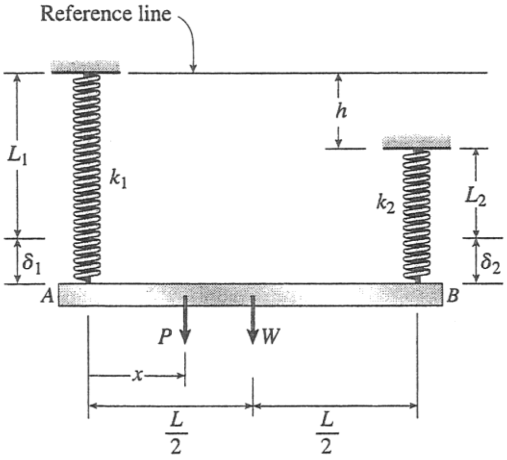
weight W?

(c) If P is removed and $k_1 \quad 300$ N/m, what distance b should spring k_1 be moved to the right so that the bar (figure part a) will hang in a horizontal position under weight W?

(d) If the spring on the left is now replaced by two springs in series ($k_1 \quad 300$N/m, k_3) with overall natural length $L_1 \quad 250$ mm (see figure part b), what value of k_3 is required so that the bar will hang in a horizontal position under weight W?



Solution 2.2-10



NUMERICAL DATA

$W \quad 25\text{ N} \quad k_1 \quad 0.300\text{ N/mm} \quad L_1 \quad 250\text{ mm}$

$k_2 \quad 0.400\text{ N/mm} \quad L_2 \quad 200\text{ mm}$

$L \quad 350\text{ mm} \quad h \quad 80\text{ mm} \quad P \quad 18\text{ N}$

(a) LOCATION OF LOAD P TO BRING BAR TO HORIZONTAL POSITION

Use statics to get forces in both springs:

$$\mathbf{a}\, M_A \quad 0 \qquad F_2 \quad \frac{1}{L} \mathrm{a} W \frac{L}{2} + P x \mathrm{b}$$

$$F_2 \quad \frac{W}{2} + P\frac{x}{L}$$

$$\mathbf{a}\, F_V \quad 0 \qquad F_1 \quad W + P \quad F_2$$

$$F_1 \quad \frac{W}{2} + P \mathrm{a} 1 \quad \frac{x}{L} \mathrm{b}$$

Use constraint equation to define horizontal position, then solve for location x:

$$L_1 + \frac{F_1}{k_1} \quad L_2 + h + \frac{F_2}{k_2}$$

Substitute expressions for F_1 and F_2 above into constraint equilibrium and solve for x:

$$x \quad \frac{2L_1 L k_1 k_2 \quad k_2 W L \quad 2k_2 P L + 2L_2 L k_1 k_2 + 2 h L k_1 k_2 + k_1 W L}{2P1k_1 + k_2 2}$$

$$x \quad 134.7\text{ mm}$$

(b) NEXT REMOVE P AND FIND NEW VALUE OF SPRING CONSTANT k_1 SO THAT BAR IS HORIZONTAL UNDER WEIGHT W

Now, $F_1 \quad \frac{W}{2} \qquad F_2 \quad \frac{W}{2}$ since $P \quad 0$

Same constraint equation as above but now $P \quad 0$:

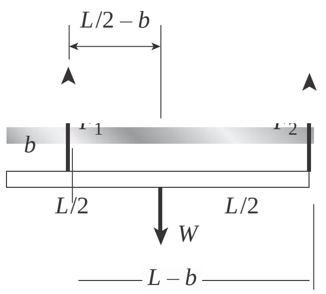
$$L_1 + \frac{\frac{W}{2}}{k_1} \quad 1L_2 + h2 \quad \frac{a\frac{W}{2}b}{k_2} \quad 0$$

Solve for k_1:

$$k_1 \quad \frac{Wk_2}{[2k_2[L_1 \quad (L_2 + h)]] \quad W}$$

$k_1 \quad 0.204$ N/mm ;

(c) USE $k_1 \quad 0.300$ N/mm BUT RELOCATE SPRING k_1 $(x \quad b)$ SO THAT BAR ENDS UP IN HORIZONTAL POSITION UNDER WEIGHT W



FBD

$$b \quad \frac{2L_1k_1k_2L + WLk_2 \quad 2L_2k_1k_2L \quad 2hk_1k_2L \quad Wk_1L}{(2L_1k_1k_2) \quad 2L_2k_1k_2 \quad 2hk_1k_2 \quad 2Wk_1}$$

PART (C)—CONTINUED

STATICS

$$a\, M_{k_1} \quad 0 \quad F_2 \quad \frac{w a\frac{L}{2} \quad b b}{L \quad b}$$

$$a\, F_V \quad 0$$

$$F_1 \quad W \quad F_2$$

$$F_1 \quad W \quad \frac{W a\frac{L}{2} \quad b b}{L \quad b}$$

$$F_1 \quad \frac{WL}{2(L \quad b)}$$

Constraint equation—substitute above expressions for F_1 and F_2 and solve for b:

$$L_1 + \frac{F_1}{k_1} \quad (L_2 + h) \quad \frac{F_2}{k_2} \quad 0$$

Use the following data:

$k_1 \quad 0.300$ N/mm $\quad k_2 \quad 0.4$ N/mm $\quad L_1 \quad 250$ mm

$L_2 \quad 200$ mm $\quad L \quad 350$ mm

$b \quad 74.1$ mm ;

(d) REPLACE SPRING k_1 WITH SPRINGS IN SERIES:

$k_1 \quad 0.3$ N/mm, $L_1/2$, AND k_3, $L_1/2$. FIND k_3 SO THAT BAR HANGS IN HORIZONTAL POSITION

STATICS $\quad F_1 \quad \frac{W}{2} \quad F_2 \quad \frac{W}{2}$

$$k_3 \quad \frac{Wk_1k_2}{2L_1k_1k_2 \quad Wk_2 + 2L_2k_1k_2 + 2hk_1k_2 + Wk_1}$$

New constraint equation; solve for k_3:

$$L_1 + \frac{F_1}{k_1} + \frac{F_1}{k_3} \quad (L_2 + h) \quad \frac{F_2}{k_2} \quad 0$$

$$L_1 + \frac{W/2}{k_1} + \frac{W/2}{k_3} \quad (L_2 + h) \quad \frac{W/2}{k_2} \quad 0$$

$k_3 \quad 0.638$ N/mm ;

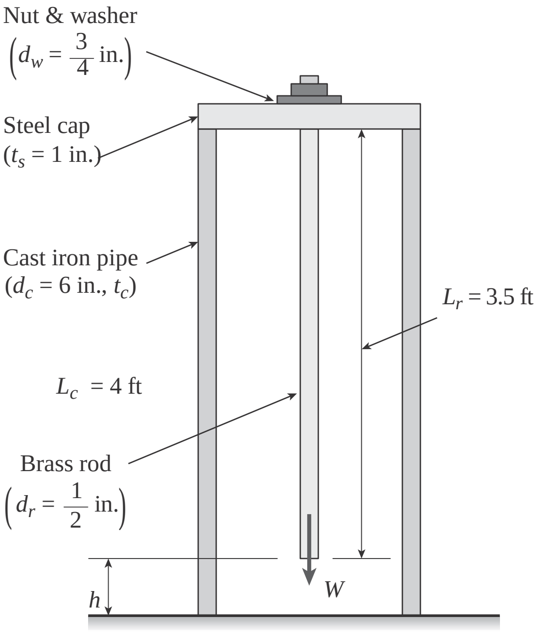
NOTE—equivalent spring constant for series springs:

$k_1k_3 \quad k_e \quad 0.204$ N/mm ; checks—same as (b) above

$$k_e \quad k_1 + k_3$$

Problem 2.2-11 A hollow, circular, cast-iron pipe (E_c 12,000 ksi) supports a brass rod (E_b 14,000 ksi) and weight W 2 kips, as shown. The outside diameter of the pipe is d_c 6 in.

(a) If the allowable compressive stress in the pipe is 5000 psi and the allowable shortening of the pipe is 0.02 in., what is the minimum required wall thickness $t_{c,\min}$? (Include the weights of the rod and steel cap in your calculations.)
(b) What is the elongation of the brass rod d_r due to both load W and its own weight?
(c) What is the minimum required clearance h?



Solution 2.2-11

The figure shows a section cut through the pipe, cap, and rod.

NUMERICAL DATA

E_c 12000 ksi E_b 14,000 ksi

W 2 k d_c 6 in. d_r $\frac{1}{2}$ in.

σ_a 5 ksi δ_a 0.02 in.

Unit weights (see Table I-1): γ_s 2.836 * 10 ^{4} k/in.^{3}

γ_b 3.009 * 10 ^{4} k/in.^{3}

L_c 48 in. L_r 42 in.

t_s 1 in.

(a) MINIMUM REQUIRED WALL THICKNESS OF CAST IRON PIPE, $t_{c\min}$

First check allowable stress then allowable shortening:

$$W_{\text{cap}} \quad \gamma_s \left(\frac{\pi}{4} d_c^2 t_s\right)$$

$$W_{\text{cap}} \quad 8.018 \quad 10^{3} \text{ k}$$

$$W_{\text{rod}} \quad \gamma_b \left(\frac{\pi}{4} d_r^2 L_r\right)$$

$$W_{\text{rod}} \quad 2.482 \quad 10^{3} \text{ k}$$

$$W_t \quad W \quad W_{\text{cap}} \quad W_{\text{rod}} \qquad W_t \quad 2.01 \text{ k}$$

$$A_{\min} \quad \frac{W_t}{\sigma_a} \qquad A_{\min} \quad 0.402 \text{ in.}^2$$

$$A_{\text{pipe}} \quad \frac{\pi}{4}[d_c^2 \quad (d_c \quad 2t_c)^2]$$

$A_{pipe} \quad p t_c(d_c \quad t_c)$

$t_c(d_c \quad t_c) \quad \frac{W_t}{p s_a}$

Let $a \quad \frac{W_t}{p s_a} \quad a \quad 0.128$:

$t_c^2 \quad d_c t_c \quad a \quad 0$

$t_c \quad \frac{d_c \quad 2\, d_c^2 \quad 4a}{2} \qquad t_c \quad 0.021$ in.

^ minimum based on s_a

Now check allowable shortening requirement:

$d_{pipe} \quad \frac{W_t L_c}{E_c A_{min}} \qquad A_{min} \quad \frac{W_t L_c}{E_c d_a}$

$A_{min} \quad 0.447\ \text{in.}^2$ larger than value based on s_a above

$p t_c(d_c \quad t_c) \quad \frac{W_t L_c}{E_c d_a}$

$t_c^2 \quad d_c t_c \quad b \quad 0 \qquad b \quad \frac{W_t L_c}{p E_c d_a}$

$b \quad 0.142$

$t_c \quad \frac{d_c \quad 2\, d_c^2 \quad 4b}{2}$

$t_c \quad 0.021$ in. ; minimum based on d_a and s_a controls

(b) ELONGATION OF ROD DUE TO SELF WEIGHT AND ALSO WEIGHT W

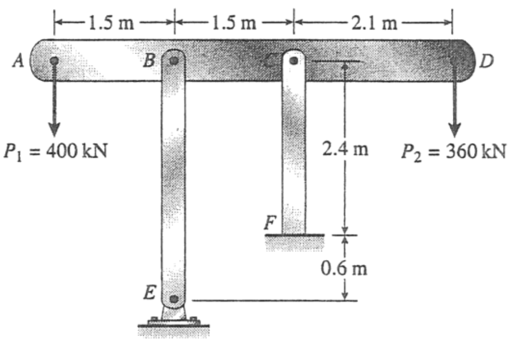
$$d_r \quad \frac{\left(W + \frac{W_{rod}}{2}\right) L_r}{E_b \left(\frac{p}{4} d_r^2\right)} \qquad d_r \quad 0.031\ \text{in.}$$

(c) MINIMUM CLEARANCE h

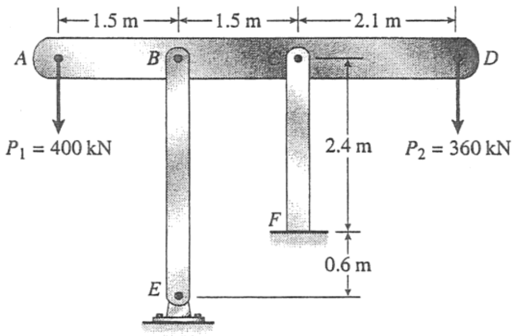
$h_{min} \quad d_a \quad d_r \qquad h_{min} \quad 0.051$ in.

Problem 2.2-12 The horizontal rigid beam *ABCD* is supported by vertical bars *BE* and *CF* and is loaded by vertical forces P_1 400 kN and P_2 360 kN acting at points *A* and *D*, respectively (see figure). Bars *BE* and *CF* are made of steel (E 200 GPa) and have cross-sectional areas A_{BE} 11,100 mm^2 and A_{CF} 9,280 mm^2. The distances between various points on the bars are shown in the figure.

Determine the vertical displacements d_A and d_D of points *A* and *D*, respectively.



Solution 2.2-12 Rigid beam supported by vertical bars



A_{BE} 11,100 mm^2

A_{CF} 9,280 mm^2

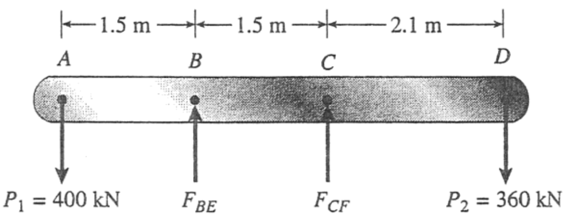
E 200 GPa

L_{BE} 3.0 m

L_{CF} 2.4 m

P_1 400 kN; P_2 360 kN

Free-body diagram of bar *ABCD*



$M_B \quad 0$

$(400 \text{ kN})(1.5 \text{ m}) \quad F_{CF}(1.5 \text{ m}) \quad (360 \text{ kN})(3.6 \text{ m}) \quad 0$

$F_{CF} \quad 464 \text{ kN}$

$M_C \quad 0$

$(400 \text{ kN})(3.0 \text{ m}) \quad F_{BE}(1.5 \text{ m}) \quad (360 \text{ kN})(2.1 \text{ m}) \quad 0$

$F_{BE} \quad 296 \text{ kN}$

Shortening of bar *BE*

$$d_{BE} \quad \frac{F_{BE}L_{BE}}{EA_{BE}} \quad \frac{(296 \text{ kN})(3.0 \text{ m})}{(200 \text{ GPa})(11{,}100 \text{ mm}^2)}$$

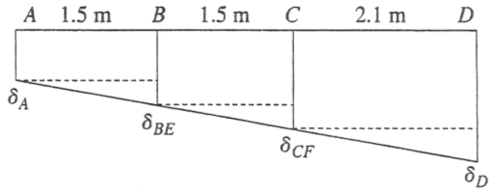
$$0.400 \text{ mm}$$

Shortening of bar *CF*

$$d_{CF} \quad \frac{F_{CF}L_{CF}}{EA_{CF}} \quad \frac{(464 \text{ kN})(2.4 \text{ m})}{(200 \text{ GPa})(9{,}280 \text{ mm}^2)}$$

$$0.600 \text{ mm}$$

Displacement diagram



$d_{BE} \quad d_A \quad d_{CF} \quad d_{BE}$ or $d_A \quad 2d_{BE} \quad d_{CF}$

$d_A \quad 2(0.400 \text{ mm}) \quad 0.600 \text{ m}$

0.200 mm

(Downward)

$$d_D \quad d_{CF} \quad \frac{2.1}{1.5}(d_{CF} \quad d_{BE})$$

or $$d_D \quad \frac{12}{5}d_{CF} \quad \frac{7}{5}d_{BE}$$

$$\frac{12}{5}(0.600 \text{ mm}) \quad \frac{7}{5}(0.400 \text{ mm})$$

0.880 mm

(Downward)

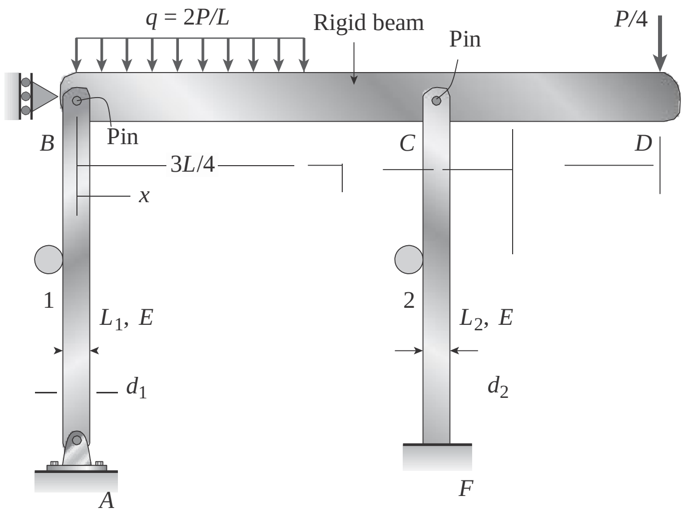
Problem 2.2-13 Two pipe columns (AB, FC) are pin-connected to a *rigid* beam (BCD) as shown in the figure. Each pipe column has modulus E, but heights (L_1 or L_2) and outer diameters (d_1 or d_2) are different for each column. Assume the inner diameter of each column is ¾ of outer diameter. Uniformly distributed downward load $q = 2P/L$ is applied over a distance of $3L/4$ along BC, and concentrated load $P/4$ is applied downward at D.

(a) Derive a formula for the displacement δ_D at point D *in terms of P and column flexibilities* f_1 and f_2.

BCD displaces downward to a horizontal position *under the load system in (a).*

(c) If $L_1 = 2L_2$, find the d_1/d_2 ratio so that beam BCD displaces downward to a horizontal position *under the load system in (a).*

(d) If $d_1 = (9/8)\,d_2$ and $L_1/L_2 = 1.5$, at what horizontal distance x from B should load $P/4$ be placed so that beam BCD displaces downward to a horizontal position under the load system in part (a)?



Solution 2.2-13

(a) Displacement δ_D

Use FBD of beam BCD $\quad \Sigma M_B = 0 \quad R_C = \frac{1}{L}\left[\left(2\frac{P}{L}\right)\left(\frac{3}{4}L\right)\left(\frac{3}{8}L\right) + \frac{P}{4}\left(L + \frac{3}{4}L\right)\right] = P \Leftarrow$ compression force in column CF

$$\Sigma F_V = 0 \quad R_B = \left(2\frac{P}{L}\right)\left(\frac{3}{4}L\right) + \frac{P}{4} - R_C = \frac{3P}{4} \qquad \text{compression force in column } BA$$

Downward displacements at B and C: $\quad \delta_B = R_B f_1 = \dfrac{3Pf_1}{4} \qquad \delta_C = R_C f_2 = P f_2$

Geometry: $\quad \delta_D = \delta_B + (\delta_C - \delta_B)\left(\dfrac{L + \frac{3}{4}L}{L}\right) = \dfrac{7Pf_2}{4} - \dfrac{9Pf_1}{16} \qquad \delta_D = \dfrac{7Pf_2}{4} - \dfrac{9Pf_1}{16} \qquad \boxed{\dfrac{P}{16}(28f_2 - 9f_1)}$

(b) Displacement to horizontal position, so $\delta_C = \delta_B$ and $\dfrac{3Pf_1}{4} = Pf_2$ or $\dfrac{f_1}{f_2} = \dfrac{4}{3}$

$$\frac{\frac{L_1}{EA_1}}{\frac{L_2}{EA_2}} = \frac{4}{3} \quad \text{or} \quad \frac{L_1}{L_2} = \frac{4}{3}\left(\frac{A_1}{A_2}\right) \quad \frac{L_1}{L_2} = \frac{4}{3}\left[\frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2}\right] = \frac{4d_1^2}{3d_2^2} \quad \frac{L_1}{L_2} = \frac{4}{3}\left(\frac{d_1}{d_2}\right)^2 \quad \text{with} \quad \frac{d_1}{d_2} = \frac{9}{8}$$

$$\frac{L_1}{L_2} = \frac{4}{3}\left(\frac{9}{8}\right)^2 = \frac{27}{16} \qquad \boxed{\frac{L_1}{L_2} = \frac{27}{16}}$$

(c) IF $L_1 = 2L_2$, FIND THE d_1/d_2 RATIO SO THAT BEAM BCD DISPLACES DOWNWARD TO A HORIZONTAL POSITION

$\frac{L_1}{L_2} = 2$ and $\delta_C = \delta_B$ from part (b). $\left(\frac{d_1}{d_2}\right)^2 = \frac{3}{4}\left(\frac{L_1}{L_2}\right)$ so $\boxed{\frac{d_1}{d_2} = \sqrt{\frac{3}{4}(2)} = 1.225}$

(d) IF $d_1 = (9/8)\,d_2$ AND $L_1/L_2 = 1.5$, AT WHAT HORIZONTAL DISTANCE x FROM B SHOULD LOAD $P/4$ AT D BE PLACED?

Given $\frac{d_1}{d_2} = \frac{9}{8}$ and $\frac{L_1}{L_2} = 1.5$ or $\frac{f_1}{f_2} = \frac{L_1}{L_2}\left(\frac{A_2}{A_1}\right)$ $\frac{f_1}{f_2} = \frac{L_1}{L_2}\left(\frac{d_2}{d_1}\right)^2 = \frac{3}{2}\left(\frac{8}{9}\right)^2 = \frac{32}{27}$

Recompute column forces R_B and R_C but now with load $P/4$ positioned at distance x from B.

Use FBD of beam BCD: $\Sigma M_B = 0 \quad R_C = \frac{1}{L}\left[\left(2\frac{P}{L}\right)\left(\frac{3}{4}L\right)\left(\frac{3}{8}L\right) + \frac{P}{4}(x)\right] = \frac{\frac{9LP}{16} + \frac{Px}{4}}{L}$

$$\Sigma F_V = 0 \quad R_B = \left(2\frac{P}{L}\right)\left(\frac{3}{4}L\right) + \frac{P}{4} - R_C = \frac{7P}{4} - \frac{\frac{9LP}{16} + \frac{Px}{4}}{L}$$

Horizontal displaced position under load q and load $P/4$ so $\delta_C = \delta_B$ or $R_C f_2 = R_B f_1$.

$$\left(\frac{\frac{9LP}{16} + \frac{Px}{4}}{L}\right) f_2 = \left(\frac{7P}{4} - \frac{\frac{9LP}{16} + \frac{Px}{4}}{L}\right) f_1 \quad \text{solve, } x = \frac{-9Lf_2 + 19Lf_1}{4f_1 + 4f_2} = \frac{-L(9f_2 - 19f_1)}{4(f_1 + f_2)}$$

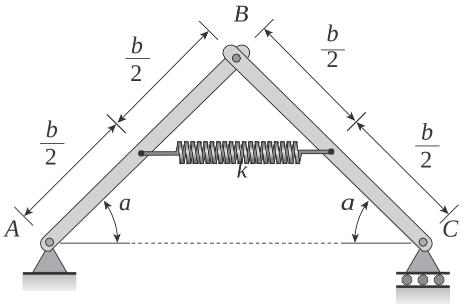
$$x = \frac{-L(9f_2 - 19f_1)}{4(f_1 + f_2)} \quad \text{or} \quad x = L\frac{19\frac{f_1}{f_2} - 9}{4\left(\frac{f_1}{f_2} + 1\right)}$$

Now substitute f_1/f_2 ratio from above: $\boxed{x = L\frac{19\left(\frac{32}{27}\right) - 9}{4\left(\frac{32}{27} + 1\right)} = \frac{365L}{236}}$ $\frac{365}{236} = 1.547$

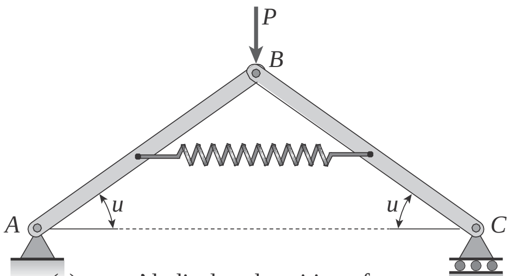
Problem 2.2-14 A framework *ABC* consists of two rigid bars *AB* and *BC*, each having a length *b* (see the first part of the figure part a). The bars have pin connections at *A*,*B*, and *C* and are joined by a spring of stiffness *k*. The spring is attached at the midpoints of the bars. The framework has a pin support at *A* and a roller support a *C*, and the bars are at an angle *a* to the horizontal.

When a vertical load *P* is applied at joint *B* (see the second part of the figure part a) the roller support *C* moves to the right, the spring is stretched, and the angle of the bars decreases from *a* to the angle *u*.

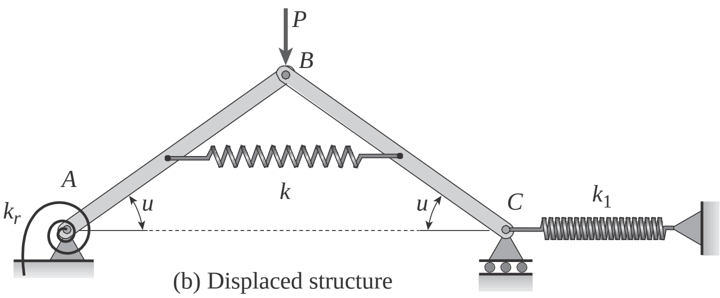
(a) Determine the angle *u* and the increase *d* in the distance between points *A* and *C*. Also find reactions at *A* and *C*. (Use the following data: *b* 200 mm, *k* 3.2 kN/m, *a* 45 , and *P* 50 N.)
(b) Repeat part (a) if a translational spring k_1 $k/2$ is added at *C* and a rotational spring k_r $kb^2/2$ is added at *A* (see figure part b).



(a) Initial position of structure



(a) - cont'd: displaced position of structure



(b) Displaced structure

Solution 2.2-14

Apply the laws of statics to the structure in its displaced position; also use FBD's of the left and right bars alone (referred to as LHFB and RHFB below).

OVERALL FBD: $\Sigma F_H \quad 0 \quad H_A \quad k_1 d \quad 0 \quad \text{so} \quad H_A \quad k_1 d$

$$\Sigma F_V \quad 0 \quad R_A + R_C \quad P$$

$$\Sigma M_A \quad 0 \quad k_r(a \quad u) \quad P\frac{L_2}{2} + R_C L_2 \quad 0 \quad R_C \quad \frac{1}{L_2}\left[P\frac{L_2}{2} \quad k_r(a \quad u)\right]$$

LHFB: $\Sigma M_B \quad 0 \quad H_A h + k\frac{d}{2}\left(\frac{h}{2}\right) \quad R_A\left(\frac{L_2}{2}\right) + k_r(a \quad u) \quad 0$

$$R_A \quad \frac{2}{L_2}\left[k_1 d h + k\frac{d}{2}\left(\frac{h}{2}\right) + k_r(a \quad u)\right]$$

RHFB: $\Sigma M_B \quad 0 \quad k\frac{d}{2}\left(\frac{h}{2}\right) \quad k_1 d h + R_C\frac{L_2}{2} \quad 0 \quad R_C \quad \frac{2}{L_2}\left[k\frac{d}{2}\left(\frac{h}{2}\right) + k_1 d h\right]$

Equate the two expressions for R_C then substitute expressions for L_2, k_r, k_1, h and d

$$\frac{1}{L_2} c P \frac{L_2}{2} \quad k_r(a \quad u) d \qquad \frac{2}{L_2} c k \frac{d}{2} a \frac{h}{2} b + k_1 d h d \quad \text{OR}$$

$$\frac{1}{L_2} c P \frac{L_2}{2} \quad k_r(a \quad u) d \quad c \frac{2}{L_2} c k \frac{2b1\cos1u2 \quad \cos1a22}{2} \frac{b\sin1u2}{2} + k_1[2b1\cos1u2 \quad \cos1a22]1b\sin1u22 d d \quad 0$$

(a) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE u AND DISTANCE INCREASE d

$b \quad 200\text{ mm} \quad k \quad 3.2\text{ kN/m} \quad a \quad 45 \quad P \quad 50\text{ N} \quad k_1 \quad 0 \quad k_r \quad 0$

$L_2 \quad 2b\cos1u2 \quad L_1 \quad 2b\cos1a2 \quad d \quad L_2 \quad L_1 \quad d \quad 2b1\cos1u2 \quad \cos1a22 \quad h \quad b\sin1u2$

$$\frac{1}{L_2} c P \frac{L_2}{2} \quad k_r 1a \quad u2 d \quad c \frac{1}{L_2} c k \frac{2b1\cos1u2 \quad \cos1a22}{2} \frac{b\sin1u2}{2} + k_1[2b1\cos1u2 \quad \cos1a22]1b\sin1u22 d d \quad 0$$

Solving above equation numerically gives $\boxed{u \quad 35.1}$ $d \quad 44.6\text{ mm}$

COMPUTE REACTIONS

$$R_C \quad \frac{2}{L_2} c k \frac{d}{2} a \frac{h}{2} b + k_1 d h d \quad 25\text{ N} \qquad R_C \quad \frac{1}{L} c P \frac{L_2}{2} \quad k_r 1a \quad u2 d \quad 25\text{ N}$$

$$R_A \quad \frac{2}{L_2} c k_1 d h + k \frac{d}{2} a \frac{h}{2} b + k_r 1a \quad u2 d \quad 25\text{ N} \qquad M_A \quad k_r 1a \quad u2 \quad 0$$

$R_A + R_C \quad 50\text{ N} \quad 6\text{ check} \qquad R_A \quad 25\text{ N} \qquad R_C \quad 25\text{ N}$

(b) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE u AND DISTANCE INCREASE d

$b \quad 200\text{ mm} \quad k \quad 3.2\text{ kN/m} \quad a \quad 45 \quad P \quad 50\text{ N} \quad k_1 \quad \frac{k}{2} \quad k_r \quad \frac{k}{2} b^2$

$L_2 \quad 2b\cos1u2 \quad L_1 \quad 2b\cos1a2 \quad d \quad L_2 \quad L_1 \quad d \quad 2b1\cos1u2 \quad \cos1a22 \quad h \quad b\sin1u2$

$$\frac{1}{L_2} c P \frac{L_2}{2} \quad k_r 1a \quad u2 d \quad c \frac{2}{L_2} c k \frac{2b1\cos1u2 \quad \cos1a22}{2} \frac{b\sin1u2}{2} + k_1[2b1\cos1u2 \quad \cos1a22]1b\sin1u22 d d \quad 0$$

Solving above equation numerically gives $\boxed{u \quad 43.3}$ $d \quad 8.19\text{ mm}$

COMPUTE REACTIONS

$$R_C \quad \frac{2}{L^2} c k \frac{d}{2} a \frac{h}{2} b + k_1 d h d \quad 18.5\text{ N} \qquad R_2 \quad \frac{1}{L_2} c P \frac{L_2}{2} \quad k_r 1a \quad u2 d \quad 18.5\text{ N}$$

$$R_A \quad \frac{2}{L_2} c k_1 d h + k \frac{d}{2} a \frac{h}{2} b + k_r 1a \quad u2 d \quad 31.5\text{ N} \qquad M_A \quad k_r 1a \quad u2 \quad 1.882\text{ N·m}$$

$R_A + R_C \quad 50\text{ N} \quad 6\text{ check} \qquad \boxed{R_A \quad 31.5\text{ N}} \qquad \boxed{R_C \quad 18.5\text{ N}} \qquad M_A \quad 1.882\text{ N·m}$

Problem 2.2-15 Solve the preceding problem for the following data:

$b = 8.0$ in., $k = 16$ lb/in., $\alpha = 45°$, and $P = 10$ lb.

Solution 2.2-15

Apply the laws of statics to the structure in its displaced position; also use FBD's of the left and right bars alone (referred to as LHFB and RHFB below)

OVERALL FBD

$$\Sigma F_H = 0 \quad H_A - k_1 d = 0 \quad \text{so} \quad H_A = k_1 d$$

$$\Sigma F_V = 0 \quad R_A + R_C = P$$

$$\Sigma M_A = 0 \quad k_r(\alpha - \theta) - P\frac{L_2}{2} + R_C L_2 = 0 \qquad R_C = \frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right]$$

LHFB

$$\Sigma M_B = 0 \quad H_A h + k\frac{d}{2}\left(\frac{h}{2}\right) - R_A\frac{L_2}{2} + k_r(\alpha - \theta) = 0$$

$$R_A = \frac{2}{L_2}\left[k_1 d h + k\frac{d}{2}\left(\frac{h}{2}\right) + k_r(\alpha - \theta)\right]$$

RHFB

$$\Sigma M_B = 0 \quad -k\frac{d}{2}\left(\frac{h}{2}\right) - k_1 d h + R_C\frac{L_2}{2} = 0 \qquad R_C = \frac{2}{L}\left[k\frac{d}{2}\left(\frac{h}{2}\right) + k_1 d h\right]$$

Equate the two expressions above for R_C then substitute expressions for L_2, k_r, k_1, h, and d

$$\frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] = \frac{2}{L_2}\left[k\frac{d}{2}\left(\frac{h}{2}\right) + k_1 d h\right] \qquad \text{OR}$$

$$\frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] - \left[\frac{2}{L_2}\left[k\frac{2b(\cos(\theta) - \cos(\alpha))}{2}\,\frac{b\sin(\theta)}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right]\right] = 0$$

(a) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE θ AND DISTANCE INCREASE d

$b = 8$ in. $\quad k = 16$ lb/in. $\quad \alpha = 45° \quad P = 10$ lb $\quad k_1 = 0 \quad k_r = 0$

$L_2 = 2b\cos(\theta) \quad L_1 = 2b\cos(\alpha) \quad d = L_2 - L_1 \quad d = 2b(\cos(\theta) - \cos(\alpha)) \quad h = b\sin(\theta)$

$$\frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] - \left[\frac{2}{L_2}\left[k\frac{2b(\cos(\theta) - \cos(\alpha))\,b\sin(\theta)}{2 \cdot 2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right]\right] = 0$$

Solving above equation numerically gives $\boxed{\theta = 35.1°}$ $\boxed{d = 1.782 \text{ in.}}$

COMPUTE REACTIONS

$$R_C = \frac{2}{L_2}\left[k\frac{d}{2}\left(\frac{h}{2}\right) + k_1 d h\right] = 5\text{ lb} \qquad R_C = \frac{1}{L_C}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] = 5\text{ lb}$$

$$R_A = \frac{2}{L_2}\left[k_1 d h + k\frac{d}{2}\left(\frac{h}{2}\right) + k_1(\alpha - \theta)\right] = 5\text{ lb} \qquad M_A = k_r(\alpha - \theta) = 0$$

$R_A + R_C \quad 10 \text{ lb} \qquad 6 \text{ check} \qquad \boxed{R_A \quad 5 \text{ lb}}\ \boxed{R_C \quad 5 \text{ lb}}$

(b) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE u AND DISTANCE INCREASE d

$$b \quad 8\text{ in.} \quad k \quad 16\text{ lb/in.} \quad a \quad 45 \quad P \quad 101b \quad k_1 \quad \frac{k}{2} \quad k_r \quad \frac{k}{2}b^2$$

$$L_2 \quad 2b\cos1u2 \quad L_1 \quad 2b\cos1a2 \quad d \quad L_2 \quad L_1 \quad d \quad 2b1\cos1u2 \quad \cos1a22 \quad h \quad b\sin1u2$$

$$\frac{1}{L_2}\,c\,P\frac{L_2}{2} \quad k_r1a \quad u2d \quad c\frac{2}{L_2}\,ck\frac{2b1\cos1u2 \quad \cos1a22}{2}\frac{b\sin1u2}{2} + k_1[2b1\cos1u2 \quad \cos1a22]1b\sin1u22dd \quad 0$$

Solving above equation numerically gives $\boxed{u \quad 43.3}$ $\quad d \quad 0.327$ in.

COMPUTE REACTIONS

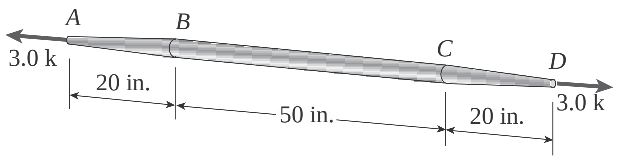
$$R_C \quad \frac{2}{L_2}\,ck\frac{d}{2}a\frac{h}{2}b + k_1dhd \quad 3.71\text{lb} \qquad R_C \quad \frac{1}{L}\,cP\frac{L_2}{2} \quad k_r1a \quad u2d \quad 3.71\text{ lb}$$

$$R_A \quad \frac{2}{L_2}\,ck_1dh + k\frac{d}{2}a\frac{h}{2}b + k_r1a \quad u2d \quad 6.3\text{ lb} \qquad M_A \quad k_r1a \quad u2 \quad 1.252\text{ ft}\cdot\text{lb}$$

$$R_A + R_C \quad 10.01\text{lb} \quad 6 \text{ check} \qquad \boxed{R_A \quad 6.3\text{lb}} \quad \boxed{R_C \quad 3.71\text{lb}} \quad \boxed{M_A \quad 1.252\text{ lb}\cdot\text{ft}}$$

Changes in Lengths under Nonuniform Conditions

Problem 2.3-1



(a) Calculate the elongation of a copper bar of solid circular cross section with tapered ends when it is stretched by axial loads of magnitude 3.0 k (see figure).

(The length of the end segments is 20 in. and the length of the prismatic middle segment is 50 in. Also, the diameters at cross sections *A*, *B*, *C*, and *D* are 0.5, 1.0, 1.0, and 0.5 in., respectively, and the modulus of elasticity is 18,000 ksi. (*Hint*: Use the result of Example 2-4.)

(b) If the total elongation of the bar cannot exceed 0.025 in., what are the required diameters at *B* and *C*? Assume that diameters at *A* and *D* remain at 0.5 in.

Solution 2.3-1

NUMERICAL DATA

$$P \quad 3\text{ k} \quad L_1 \quad 20\text{ in.} \quad L_2 \quad 50\text{ in.} \quad d_A \quad 0.5\text{ in.} \quad d_B \quad 1\text{ in.} \quad E \quad 18000\text{ ksi}$$

(a) TOTAL ELONGATION

$$d_1 \quad \frac{4PL_1}{pEd_Ad_B} \quad 0.00849\text{ in.} \qquad d_2 \quad \frac{PL_2}{E\frac{p}{4}d_B^{\,2}} \quad 0.01061\text{ in.}$$

$$d \quad 2d_1 + d_2 \quad 0.0276\text{ in.} \qquad \boxed{d \quad 0.0276\text{ in.}}$$

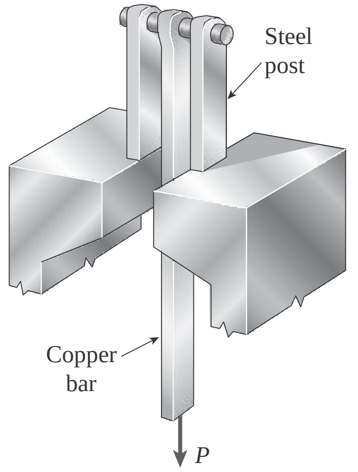
(b) FIND NEW DIAMETERS AT B AND C IF TOTAL ELONGATION CANNOT EXCEED 0.025 in.

$$\frac{4PL_1}{\quad} \qquad \frac{PL_2}{\quad}$$

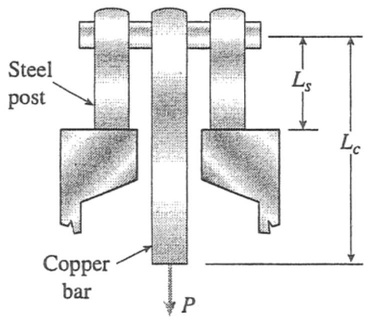
$2\left(p E d_A d_B \right) + E P d_B^2 \; 4$ 0.025 in. Solving for d_B: $\boxed{d_B \quad 1.074 \text{ in.}}$

Problem 2.3-2 A long, rectangular copper bar under a tensile load P hangs from a pin that is supported by two steel posts (see figure). The copper bar has a length of 2.0 m, a cross-sectional area of 4800 mm^2, and a modulus of elasticity $E_c = 120$ GPa. Each steel post has a height of 0.5 m, a cross-sectional area of 4500 mm^2, and a modulus of elasticity $E_s = 200$ GPa.

(a) Determine the downward displacement d of the lower end of the copper bar due to a load $P = 180$ kN.
(b) What is the maximum permissible load P_{max} if the displacement d is limited to 1.0 mm?



Solution 2.3-2 Copper bar with a tensile load



$L_c = 2.0$ m

$A_c = 4800$ mm^2

$E_c = 120$ GPa

$L_s = 0.5$ m

$A_s = 4500$ mm^2

$E_s = 200$ GPa

(a) DOWNWARD DISPLACEMENT d ($P = 180$ kN)

$$d_c = \frac{PL_c}{E_c A_c} = \frac{(180 \text{ kN})(2.0 \text{ m})}{(120 \text{ GPa})(4800 \text{ mm}^2)} = 0.625 \text{ mm}$$

$$d_s = \frac{(P/2)L_s}{E_s A_s} = \frac{(90 \text{ kN})(0.5 \text{ m})}{(200 \text{ GPa})(4500 \text{ mm}^2)} = 0.050 \text{ mm}$$

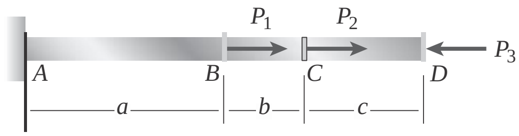
$$d = d_c + d_s = 0.625 \text{ mm} + 0.050 \text{ mm} = 0.675 \text{ mm}$$

(b) MAXIMUM LOAD P_{max} ($d_{max} = 1.0$ mm)

$$\frac{P_{max}}{P} = \frac{d_{max}}{d} \qquad P_{max} = P\left(\frac{d_{max}}{d}\right)$$

$$P_{max} = (180 \text{ kN})\left(\frac{1.0 \text{ mm}}{0.675 \text{ mm}}\right) = 267 \text{ kN}$$

Problem 2.3-3 An aluminum bar AD (see figure) has a cross-sectional area of 0.40 in.^{2} and is loaded by forces $P_1 = 1700$ lb, $P_2 = 1200$ lb, and $P_3 = 1300$ lb. The lengths of the segments of the bar are $a = 60$ in., $b = 24$ in., and $c = 36$ in.



(a) Assuming that the modulus of elasticity $E = 10.4 \times 10^6$ psi, calculate the change in length of the bar. Does the bar elongate or shorten?

(b) By what amount P should the load P_3 be increased so that the bar does not change in length when the three loads are applied?

(c) If P_3 remains at 1300 lb, what revised cross-sectional area for segment AB will result in no change of length when all three loads are applied?

Solution 2.3-3

NUMERICAL DATA

$A = 0.40 \text{ in.}^2 \qquad P_1 = 1700 \text{ lb}$

$P_2 = 1200 \text{ lb} \qquad P_3 = 1300 \text{ lb}$

$E = 10.4(10^6) \text{ psi}$

$a = 60 \text{ in.} \qquad b = 24 \text{ in.} \qquad c = 36 \text{ in.}$

(a) TOTAL ELONGATION

$$\delta = \frac{1}{EA}\left[(P_1 + P_2 - P_3)a + (P_2 - P_3)b + (-P_3)c\right] = 0.01125 \text{ in.} \qquad \boxed{\delta = 0.01125 \text{ in.}} \quad \text{(elongation)}$$

(b) INCREASE P_3 SO THAT BAR DOES NOT CHANGE LENGTH

$$\frac{1}{EA}\left[(P_1 + P_2 - P_3)a + (P_2 - P_3)b + (-P_3)c\right] = 0 \quad \text{solve, } P_3 = 1690 \text{ lb}$$

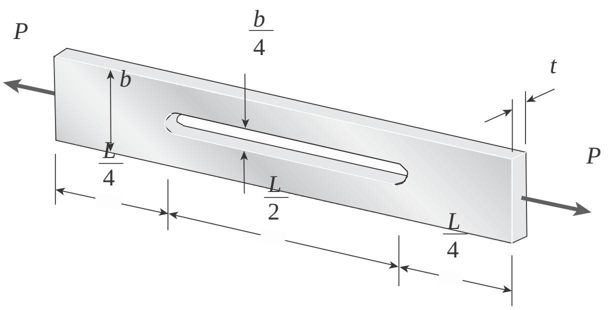
$\boxed{\text{So new value of } P_3 \text{ is 1690 lb, an increase of 390 lb.}}$

(c) NOW CHANGE CROSS-SECTIONAL AREA OF AB SO THAT BAR DOES NOT CHANGE LENGTH $\quad P_3 = 1300$ lb

$$\frac{1}{E}\left[(P_1 + P_2 - P_3)\frac{a}{A_{AB}} + (P_2 - P_3)\frac{b}{A} + (-P_3)\frac{c}{A}\right] = 0$$

Solving for A_{AB}: $\boxed{A_{AB} = 0.78 \text{ in.}^2} \qquad \frac{A_{AB}}{A} = 1.951$

Problem 2.3-4 A rectangular bar of length L has a slot in the middle half of its length (see figure). The bar has width b, thickness t, and modulus of elasticity E. The slot has width $b/4$.



(a) Obtain a formula for the elongation δ of the bar due to the axial loads P.
(b) Calculate the elongation of the bar if the material is high-strength steel, the axial stress in the middle region is 160 MPa, the length is 750 mm, and the modulus of elasticity is 210 GPa.
(c) If the total elongation of the bar is limited to $\delta_{max} = 0.475$ mm, what is the maximum length of the slotted region? Assume that the axial stress in the middle region remains at 160 MPa.

Solution 2.3-4

(a) $$\delta = \frac{P}{E}\left(\frac{2\frac{L}{4}}{bt} + \frac{\frac{L}{2}}{\frac{3}{4}bt}\right) = \frac{7LP}{6Ebt} \qquad \boxed{\delta = \frac{7PL}{6Ebt}}$$

(b) Numerical data $E = 210$ GPa $\quad L = 750$ mm $\quad \sigma_{mid} = 160$ MPa

so $\sigma_{mid} = \dfrac{P}{\frac{3}{4}bt}$ and $\dfrac{P}{bt} = \dfrac{3}{4}\sigma_{mid}$

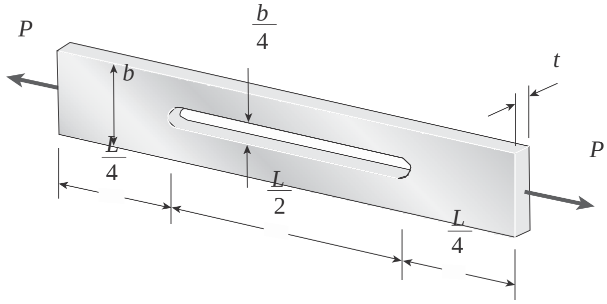
$$\delta = \frac{7LP}{6Ebt} \quad \text{or} \quad \delta = \frac{7L}{6E}\left(\frac{3}{4}\sigma_{mid}\right) = 0.5\text{ mm} \qquad \boxed{\delta = 0.5\text{ mm}}$$

(c) $$\delta_{max} = \frac{P}{E}\left(\frac{L - L_{slot}}{bt} + \frac{L_{slot}}{\frac{3}{4}bt}\right) \quad \text{or} \quad \delta_{max} = \left(\frac{P}{bt}\right)\left(\frac{1}{E}\right)\left(L - L_{slot} + \frac{4}{3}L_{slot}\right)$$

or $$\delta_{max} = \left(\frac{3}{4}\sigma_{mid}\right)\left(\frac{1}{E}\right)\left(L + \frac{L_{slot}}{3}\right)$$ Solving for L_{slot} with $\delta_{max} = 0.475$ mm

$$L_{slot} = \frac{4E\delta_{max} - 3L\sigma_{mid}}{\sigma_{mid}} = 244\text{ mm} \qquad \boxed{L_{slot} = 244\text{ mm}} \qquad \frac{L_{slot}}{L} = 0.325$$

Problem 2.3-5 Solve the preceding problem if the axial stress in the middle region is 24,000 psi, the length is 30 in., and the modulus of elasticity is 30 * 10^6 psi. In part (c), assume that d_{max} 0.02 in.



Solution 2.3-5

(a) $$\boxed{d \quad \frac{P}{E}\left(\frac{2\frac{L}{4}}{bt} + \frac{\frac{L}{2}}{\frac{3}{4}bt}\right) \quad \frac{7LP}{6Ebt}}$$

(b) E 30,000 ksi L 30 in. s_{mid} 24 ksi

So s_{mid} $\frac{P}{\frac{3}{4}bt}$ and $\frac{P}{bt}$ $\frac{3}{4}s_{mid}$

$$d \quad \frac{7LP}{6Ebt} \quad \text{or} \quad d \quad \frac{7L}{6E}\left(\frac{3}{4}s_{mid}\right) \quad 0.021\text{ in.} \qquad \boxed{d \quad 0.021\text{ in.}}$$

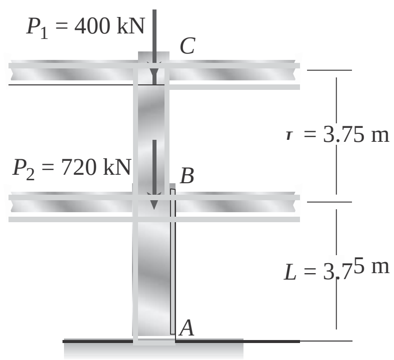
(c) $$d_{max} \quad \frac{P}{E}\left(\frac{L \quad L_{slot}}{bt} + \frac{L_{slot}}{\frac{3}{4}bt}\right) \quad \text{or} \quad d_{max} \quad \left(\frac{P}{bt}\right)\left(\frac{1}{E}\right)\left(L \quad L_{slot} + \frac{4}{3}L_{slot}\right)$$

or $$d_{max} \quad \left(\frac{3}{4}s_{mid}\right)\left(\frac{1}{E}\right)\left(L + \frac{L_{slot}}{3}\right)$$ Solving for L_{slot} with d_{max} 0.02 in.:

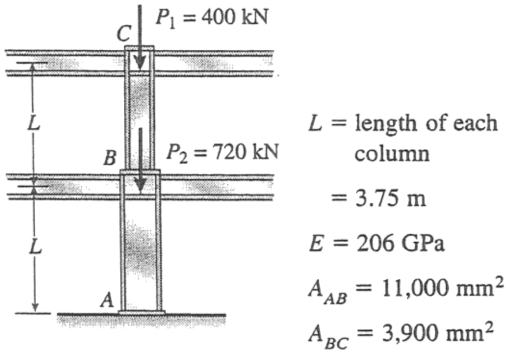
$$L_{slot} \quad \frac{4Ed_{max} \quad 3Ls_{mid}}{s_{mid}} \quad 10\text{ in.} \qquad \boxed{L_{slot} \quad 10\text{ in.}} \qquad \frac{L_{slot}}{L} \quad 0.333$$

Problem 2.3-6 A two-story building has steel columns AB in the first floor and BC in the second floor, as shown in the figure. The roof load P_1 equals 400 kN and the second-floor load P_2 equals 720 kN. Each column has length L 3.75 m. The cross-sectional areas of the first- and second-floor columns are 11,000 mm^2 and 3,900 mm^2, respectively.

(a) Assuming that E 206 GPa, determine the total shortening d_{AC} of the two columns due to the combined action of the loads P_1 and P_2.
(b) How much additional load P_0 can be placed at the top of the column (point C) if the total shortening d_{AC} is not to exceed 4.0 mm?



Solution 2.3-6 Steel columns in a building



(a) Shortening δ_{AC} of the two columns

$$\delta_{AC} = \sum \frac{N_i L_i}{E_i A_i} = \frac{N_{AB} L}{E A_{AB}} + \frac{N_{BC} L}{E A_{BC}}$$

$$= \frac{(1120 \text{ kN})(3.75 \text{ m})}{(206 \text{ GPa})(11{,}000 \text{ mm}^2)} + \frac{(400 \text{ kN})(3.75 \text{ m})}{(206 \text{ GPa})(3{,}900 \text{ mm}^2)}$$

$$= 1.8535 \text{ mm} + 1.8671 \text{ mm} = 3.7206 \text{ mm}$$

$$\delta_{AC} = 3.72 \text{ mm} \;\longleftarrow$$

(b) Additional load P_0 at point C

$(\delta_{AC})_{max} = 4.0$ mm

δ_0 = additional shortening of the two columns due to the load P_0

$$\delta_0 = (\delta_{AC})_{max} - \delta_{AC} = 4.0 \text{ mm} - 3.7206 \text{ mm} = 0.2794 \text{ mm}$$

$$\text{Also, } \delta_0 = \frac{P_0 L}{E A_{AB}} + \frac{P_0 L}{E A_{BC}} = \frac{P_0 L}{E}\left(\frac{1}{A_{AB}} + \frac{1}{A_{BC}}\right)$$

Solve for P_0:

$$P_0 = \frac{E \delta_0}{L}\left(\frac{A_{AB} A_{BC}}{A_{AB} + A_{BC}}\right)$$

Substitute numerical values:

$E = 206 \times 10^9$ N/m² $\quad \delta_0 = 0.2794 \times 10^{-3}$ m

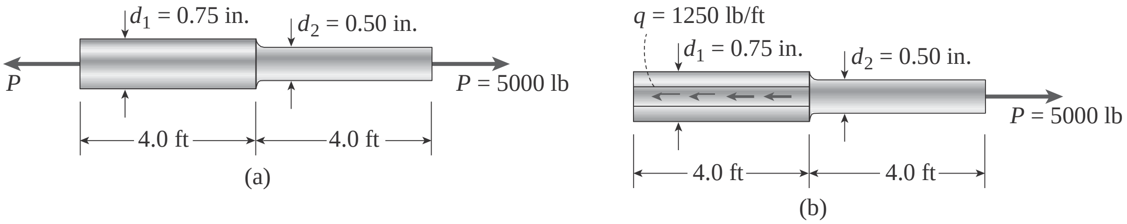
$L = 3.75$ m $\quad A_{AB} = 11{,}000 \times 10^{-6}$ m²

$A_{BC} = 3{,}900 \times 10^{-6}$ m²

$P_0 = 44{,}200$ N $= 44.2$ kN $\longleftarrow$

Problem 2.3-7 A steel bar 8.0 ft long has a circular cross section of diameter $d_1 = 0.75$ in. over one-half of its length and diameter $d_2 = 0.5$ in. over the other half (see figure part a). The modulus of elasticity $E = 30 \times 10^6$ psi.

(a) How much will the bar elongate under a tensile load $P = 5000$ lb?
(b) If the same volume of material is made into a bar of constant diameter d and length 8.0 ft, what will be the elongation under the same load P?
(c) If the uniform axial centroidal load $q = 1250$ lb/ft is applied to the left over segment 1 (see figure part b), find the ratio of the total elongation of the bar to that in parts (a) and (b).



Solution 2.3-7

NUMERICAL DATA $E = 30(10^6)$ psi $P = 5000$ lb $L = 4$ ft $d_1 = 0.75$ in. $d_2 = 0.5$ in.

(a) $$\delta_a = \frac{PL}{E}\left(\frac{1}{\frac{\pi}{4}d_1^2} + \frac{1}{\frac{\pi}{4}d_2^2}\right) = 0.0589\ \text{in.} \qquad \boxed{\delta_a = 0.0589\ \text{in.}}$$

(b) $$V_a = \left(\frac{\pi}{4}d_1^2 + \frac{\pi}{4}d_2^2\right)L = 30.631\ \text{in.}^3 \qquad d = \sqrt{\frac{V_a}{\frac{\pi}{4}(12L)}} = 0.637\ \text{in.} \qquad A = \frac{\pi}{4}d^2 = 0.31907\ \text{in.}^2$$

$$\delta_b = \frac{P(12L)}{EA} = 0.0501\ \text{in.} \qquad \boxed{\delta_b = 0.0501\ \text{in.}}$$

(c) $q = 1250$ lb/ft $L = 4$ ft

$$\delta_c = \frac{qL^2}{2E\left(\frac{\pi}{4}d_1^2\right)} + \frac{PL}{EA} = 0.0341\ \text{in.} \qquad \boxed{\frac{\delta_c}{\delta_a} = 0.58} \qquad \boxed{\frac{\delta_c}{\delta_b} = 0.681}$$

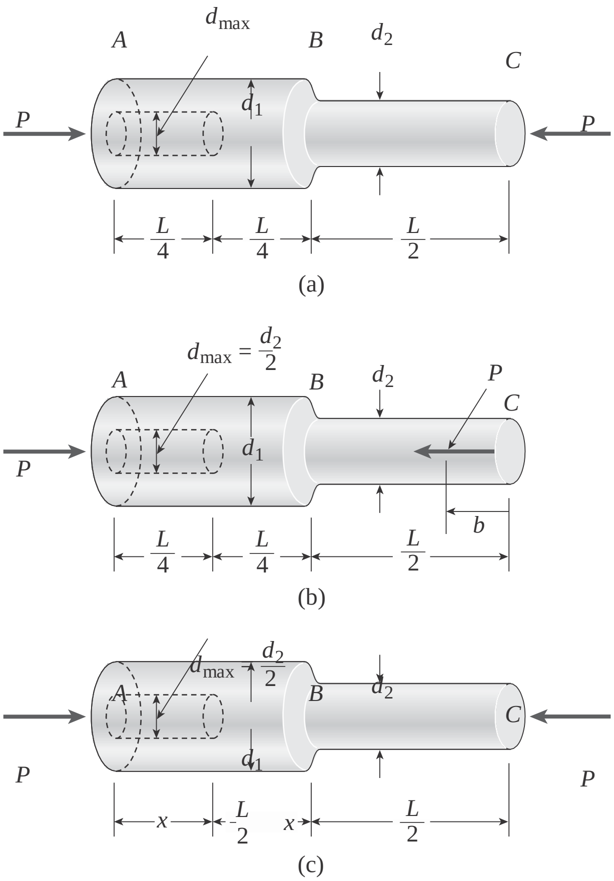
Problem 2.3-8 A bar ABC of length L consists of two parts of equal lengths but different diameters. Segment AB has diameter $d_1 = 100$ mm, and segment BC has diameter $d_2 = 60$ mm. Both segments have length $L/2 = 0.6$ m. A longitudinal hole of diameter d is drilled through segment AB for one-half of its length (distance $L/4 = 0.3$ m). The bar is made of plastic having modulus of elasticity $E = 4.0$ GPa.

Compressive loads $P = 110$ kN act at the ends of the bar.

(a) If the shortening of the bar is limited to 8.0 mm, what is the maximum allowable diameter d_{max} of the hole? (See figure part a.)

(b) Now, if d_{max} is instead set at $d_2/2$, at what distance b from end C should load P be applied to limit the bar shortening to 8.0 mm? (See figure part b.)

(c) Finally, if loads P are applied at the ends and $d_{max} = d_2/2$, what is the permissible length x of the hole if shortening is to be limited to 8.0 mm? (See figure part c.)



Solution 2.3-8

NUMERICAL DATA

$d_1 = 100\text{ mm}\quad d_2 = 60\text{ mm}$

$L = 1200\text{ mm}\quad E = 4.0\text{ GPa}\quad P = 110\text{ kN}$

$\delta_a = 8.0\text{ mm}$

(a) FIND d_{max} IF SHORTENING IS LIMITED TO δ_a

$$A_1 = \frac{\pi}{4}d_1^{\,2}\qquad A_2 = \frac{\pi}{4}d_2^{\,2}$$

$$\delta = \frac{P}{E}\left[\frac{\frac{L}{4}}{\frac{\pi}{4}\left(d_1^{\,2} - d_{max}^{\,2}\right)} + \frac{\frac{L}{4}}{A_1} + \frac{\frac{L}{2}}{A_2}\right]$$

Set δ to δ_a, and solve for d_{max}:

$$d_{max} = d_1\sqrt{\frac{E\delta_a\pi d_1^{\,2}d_2^{\,2} - 2PLd_2^{\,2} - 2PLd_1^{\,2}}{E\delta_a\pi d_1^{\,2}d_2^{\,2} - PLd_2^{\,2} - 2PLd_1^{\,2}}}$$

$d_{max} = 23.9\text{ mm}$ ←

(b) NOW, IF d_{max} IS INSTEAD SET AT $d_2/2$, AT WHAT DISTANCE b FROM END C SHOULD LOAD P BE APPLIED TO LIMIT THE BAR SHORTENING TO $\delta_a = 8.0$ mm?

$$A_0 = \frac{\pi}{4}\left[d_1^2 - \left(\frac{d_2}{2}\right)^2\right]$$

$$A_1 = \frac{\pi}{4}d_1^{\,2}\qquad A_2 = \frac{\pi}{4}d_2^{\,2}$$

$$\delta = \frac{P}{E}\left[\frac{L}{4A_0} + \frac{L}{4A_1} + \frac{\left(\frac{L}{2} - b\right)}{A_2}\right]$$

No axial force in segment at end of length b; set $\delta = \delta_a$ and solve for b:

$$b = \left[\frac{L}{2} - A_2\left[\frac{E\delta_a}{P} - \left(\frac{L}{4A_0} + \frac{L}{4A_1}\right)\right]\right]$$

$b = 4.16\text{ mm}$ ←

(c) FINALLY IF LOADS P ARE APPLIED AT THE ENDS AND $d_{max} = d_2/2$, WHAT IS THE PERMISSIBLE LENGTH x OF THE HOLE IF SHORTENING IS TO BE LIMITED TO $\delta_a = 8.0$ mm?

$$\delta = \frac{P}{E}\left[\frac{x}{A_0} + \frac{\left(\frac{L}{2} - x\right)}{A_1} + \frac{\left(\frac{L}{2}\right)}{A_2}\right]$$

Set $\delta = \delta_a$ and solve for x:

$$x = \frac{\left[A_0A_1\left(\frac{E\delta_a}{P} - \frac{L}{2A_2}\right)\right] - \frac{1}{2}A_0L}{A_1 - A_0}$$

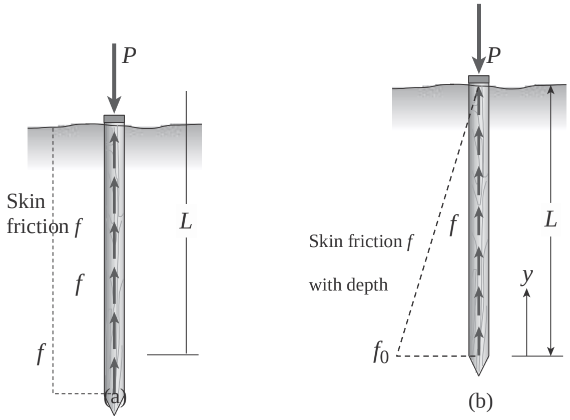
$x = 183.3\text{ mm}$ ←

Problem 2.3-9 A wood pile, driven into the earth, supports a load P entirely by friction along its sides (see figure part a). The friction force f per unit length of pile is assumed to be uniformly distributed over the surface of the pile. The pile has length L, cross-sectional area A, and modulus of elasticity E.

of P, L, E, and A.

(b) Draw a diagram showing how the compressive stress s_c

(c) Repeat parts (a) and (b) if skin friction f varies linearly with depth (see figure part b).



Solution 2.3-9

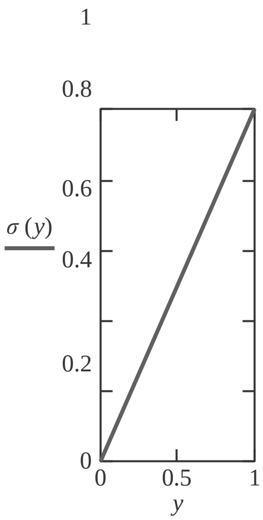
AFD LINEAR

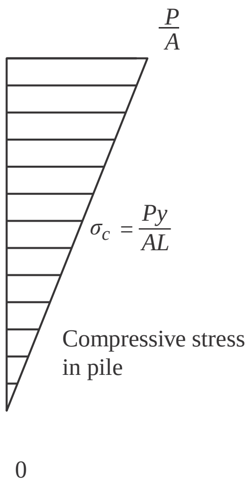
(a) $N(y) \quad fy \qquad d \quad \int_0^L \frac{(fy)}{EA}\,dy \quad \frac{L^2 f}{2AE} \qquad \boxed{d \quad \frac{PL}{2EA}}$

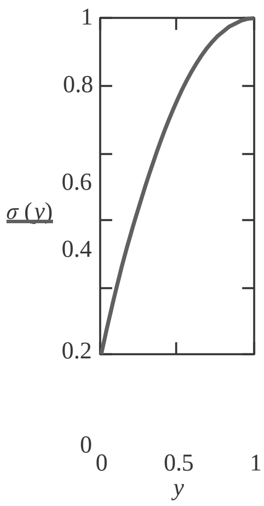
(b) $s(y) \quad \frac{N(y)}{A} \qquad s(y) \quad \frac{fy}{A} \qquad s(L) \quad \frac{fL}{A} \quad \frac{P}{A}$

$s(0) \quad 0$ So linear variation, zero at bottom, P/A at top (i.e., at ground surface)

$N(L) \quad f \qquad \boxed{s(y) \quad \frac{P}{A} \text{ a } \frac{y}{L} \text{ b}}$







$f(y)$ is constant and AFD is linear

$f(y)$ is linear and AFD quadratic

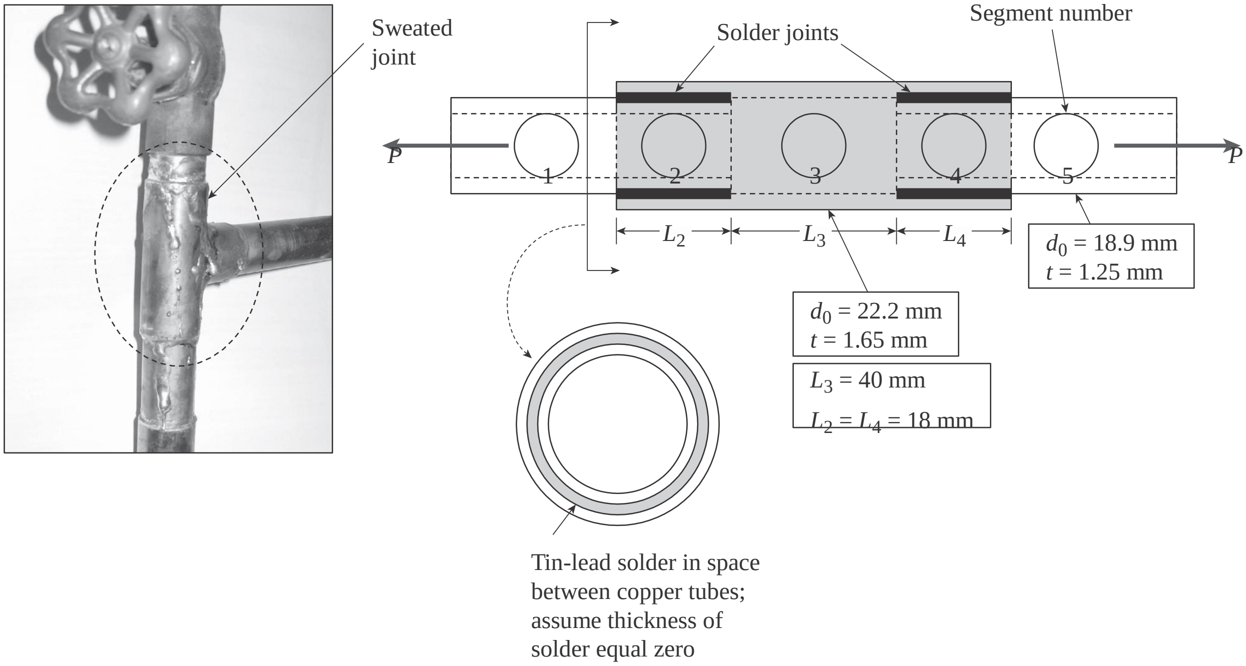
(c) $N(y) = -\int f(y)\,dy$

$$N(y) = -\int_0^y f_0\left(1 - \frac{y}{L}\right)dy = \frac{f_0\,y(y-2)}{2} \qquad N(L) = \frac{f_0}{2} \qquad N(0) = 0$$

$$\delta = \frac{\left(\frac{f_0 L}{2}\right)}{\frac{3}{2}EA} \qquad P = \frac{1}{2}f_0 L \qquad \boxed{\delta = \frac{PL}{EA}\left(\frac{2}{3}\right)} \qquad \boxed{\sigma(y) = \frac{P}{A}\left[\frac{y}{L}\left(2 - \frac{y}{L}\right)\right]} \qquad \sigma(0) = 0 \qquad \sigma(L) = \frac{f_0}{2} = P/A$$

Problem 2.3-10 Consider the copper tubes joined below using a "sweated" joint. Use the properties and dimensions given.

(a) Find the total elongation of segment 2-3-4 ($\delta_{2\text{-}4}$) for an applied tensile force of $P = 5$ kN. Use $E_c = 120$ GPa.
(b) If the yield strength in shear of the tin-lead solder is $\tau_y = 30$ MPa and the tensile yield strength of the copper is $\sigma_y = 200$ MPa, what is the maximum load P_{max} that can be applied to the joint if the desired factor of safety in shear is $FS_\tau = 2$ and in tension is $FS_\sigma = 1.7$?
(c) Find the value of L_2 at which tube and solder capacities are equal.



Solution 2.3-10

NUMERICAL DATA

$P \quad 5\text{ kN} \qquad E_c \quad 120\text{ GPa}$

$L_2 \quad 18\text{ mm} \qquad L_4 \quad L_2$

$L_3 \quad 40\text{ mm}$

$d_{o3} \quad 22.2\text{ mm} \qquad t_3 \quad 1.65\text{ mm}$

$d_{o5} \quad 18.9\text{ mm} \qquad t_5 \quad 1.25\text{ mm}$

$\tau_Y \quad 30\text{ MPa} \qquad \sigma_Y \quad 200\text{ MPa}$

$\text{FS}_\tau \quad 2 \qquad \text{FS}_\sigma \quad 1.7$

$$\tau_a \quad \frac{\tau_Y}{\text{FS}_\tau} \qquad \tau_a \quad 15\text{ MPa}$$

$$\sigma_a \quad \frac{\sigma_Y}{\text{FS}_\sigma} \qquad \sigma_a \quad 117.6\text{ MPa}$$

(a) ELONGATION OF SEGMENT 2-3-4

$$A_2 \quad \frac{\pi}{4}[d_{o3}^2 \quad (d_{o5} \quad 2t_5)^2]$$

$$A_3 \quad \frac{\pi}{4}[d_{o3}^2 \quad 1d_{o3} \quad 2t_3 2^2]$$

$A_2 \quad 175.835\text{ mm}^2 \qquad A_3 \quad 106.524\text{ mm}^2$

$$d_{24} \quad \frac{P}{E_c}\left(\frac{L_2+L_4}{A_2}+\frac{L_3}{A_3}\right)$$

$d_{24} \quad 0.024\text{ mm}$

(b) MAXIMUM LOAD $P_{\max}$ THAT CAN BE APPLIED TO THE JOINT

First check normal stress:

$$A_1 \quad \frac{\pi}{4}[d_{o5}^2 \quad 1d_{o5} \quad 2t_5 2^2]$$

$A_1 \quad 69.311\text{ mm}^2 \qquad$ smallest cross-sectional area controls normal stress

$P_{\max\sigma} \quad \sigma_a A_1 \qquad P_{\max\sigma} \quad 8.15\text{ kN}$ smaller than $P_{\max}$ based on shear below so normal stress controls

Next check shear stress in solder joint:

$A_{sh} \quad \pi d_{o5} L_2 \qquad A_{sh} \quad 1.069 \quad 10^3\text{ mm}^2$

$P_{\max\tau} \quad \tau_a A_{sh} \qquad P_{\max\tau} \quad 16.03\text{ kN}$

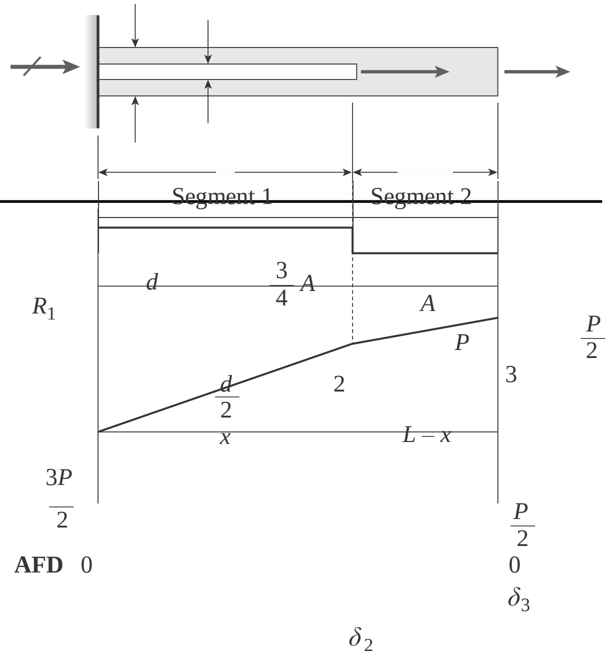
(c) FIND THE VALUE OF L_2 AT WHICH TUBE AND SOLDER *CAPACITIES* ARE EQUAL

Set $P_{\max}$ based on shear strength equal to $P_{\max}$ based on tensile strength and solve for L_2:

$$L_2 \quad \frac{\sigma_a A_1}{\tau_a 1\pi d_{o5} 2} \qquad L_2 \quad 9.16\text{ mm}$$

Problem 2.3-11 The nonprismatic cantilever circular bar shown has an internal cylindrical hole of diameter $d/2$ from 0 to x, so the net area of the cross section for Segment 1 is $(3/4)A$. Load P is applied at x, and load $P/2$ is applied at $x \quad L$. Assume that E is constant.

(a) Find reaction force R_1.
(b) Find internal axial forces N_i in segments 1 and 2.
(c) Find x required to obtain axial displacement at joint 3 of $\delta_3 \quad PL/EA$.
(d) In (c), what is the displacement at joint 2, δ_2?
(e) If P acts at $x \quad 2L/3$ and $P/2$ at joint 3 is replaced by βP, find β so that $\delta_3 \quad PL/EA$.
(f) Draw the *axial force* (AFD: $N(x)$, $0 \quad x \quad L$) and *axial displacement* (ADD: $\delta(x)$, $0 \quad x \quad L$) *diagrams* using results from (b) through (d) above.



ADD	0	0

Solution 2.3-11

(a) STATICS $\sum F_H = 0 \quad R_1 = -P + \frac{P}{2}$

$R_1 = \frac{-3}{2}P \quad \Leftarrow$

(b) DRAW FBD'S CUTTING THROUGH SEGMENT 1 AND AGAIN THROUGH SEGMENT 2

$N_1 = \frac{3P}{2} \quad \Leftarrow$ tension $\qquad N_2 = \frac{P}{2} \quad \Leftarrow$ tension

(c) FIND x REQUIRED TO OBTAIN AXIAL DISPLACEMENT AT JOINT 3 OF $d_3 = PL/EA$

Add axial deformations of segments 1 and 2, then set to d_3; solve for x:

$$\frac{N_1 x}{E\frac{3}{4}A} + \frac{N_2(L - x)}{EA} = \frac{PL}{EA}$$

$$\frac{\frac{3P}{2}x}{E\frac{3}{4}A} + \frac{\frac{P}{2}(L - x)}{EA} = \frac{PL}{EA}$$

$$\frac{3}{2}x = \frac{L}{2} \qquad x = \frac{L}{3} \quad \Leftarrow$$

(d) WHAT IS THE DISPLACEMENT AT JOINT 2, d_2?

$$d_2 = \frac{N_1 x}{E\frac{3}{4}A} \qquad d_2 = \frac{\left(\frac{3P}{2}\right)\left(\frac{L}{3}\right)}{E\frac{3}{4}A}$$

$$d_2 = \frac{2}{3}\frac{PL}{EA}$$

(e) IF $x = 2L/3$ AND $P/2$ AT JOINT 3 IS REPLACED BY bP, FIND b SO THAT $d_3 = PL/EA$

$N_1 = (1 + b)P \quad N_2 = bP \quad x = \frac{2L}{3}$

substitute in axial deformation expression above and solve for b

$$\frac{[(1 + b)P]\frac{2L}{3}}{E\frac{3}{4}A} + \frac{bP\left(L - \frac{2L}{3}\right)}{EA} = \frac{PL}{EA}$$

$$\frac{1}{9}PL\frac{8 + 11b}{EA} = \frac{PL}{EA}$$

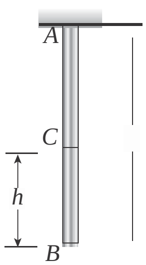
$$(8 + 11b) = 9$$

$$b = \frac{1}{11} \quad \Leftarrow$$

$$b = 0.091$$

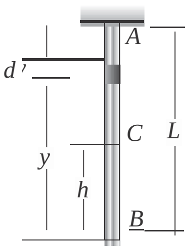
(f) Draw AFD, ADD—see plots for $x = \frac{L}{3}$

Problem 2.3-12 A prismatic bar AB of length L, cross-sectional area A, modulus of elasticity E, and weight W hangs vertically under its own weight (see figure).



(a) Derive a formula for the downward displacement δ_C of point C, located at distance h from the lower end of the bar.
(b) What is the elongation δ_B of the entire bar?
(c) What is the ratio b of the elongation of the upper half of the bar to the elongation of the lower half of the bar?
(d) If bar AB is a riser pipe hanging from a drill rig at sea, what is the total elongation of the pipe? Let $L = 1500$ m, $A = 0.0157$ m^2, $E = 210$ GPa. See Appendix I for weight densities of steel and sea water. (See Problems 1.4-2 and 1.7-11 for additional figures).

Solution 2.3-12 Prismatic bar hanging vertically



W Weight of bar

(a) DOWNWARD DISPLACEMENT d_C

Consider an element at distance y from the lower end.

$$N(y) \quad \frac{Wy}{L} \qquad dd \quad \frac{N(y)\,dy}{EA} \quad \frac{Wydy}{EAL}$$

$$d_C \quad 1_h^L dd \quad 1_h^L \frac{Wydy}{EAL} \quad \frac{W}{2EAL}(L^2 \quad h^2)$$

$$d_C \quad \frac{W}{2EAL}(L^2 \quad h^2)$$

(b) ELONGATION OF BAR (h 0)

$$d_B \quad \frac{WL}{2EA}$$

(c) RATIO OF ELONGATIONS

Elongation of upper half of bar $a h \quad \frac{L}{2} b$:

$$d_{\text{upper}} \quad \frac{3WL}{8EA}$$

Elongation of lower half of bar:

$$d_{\text{lower}} \quad d_B \quad d_{\text{upper}} \quad \frac{WL}{2EA} \quad \frac{3WL}{8EA} \quad \frac{WL}{8EA}$$

$$b \quad \frac{d_{\text{upper}}}{d_{\text{lower}}} \quad \frac{3/8}{1/8} \quad 3$$

(d) NUMERICAL DATA

g_s 77 kN/m^3 $\quad g_w$ 10 kN/m^3 $\quad$ L 1500 m $\quad A$ 0.0157 m^2 $\quad E$ 210 GPa

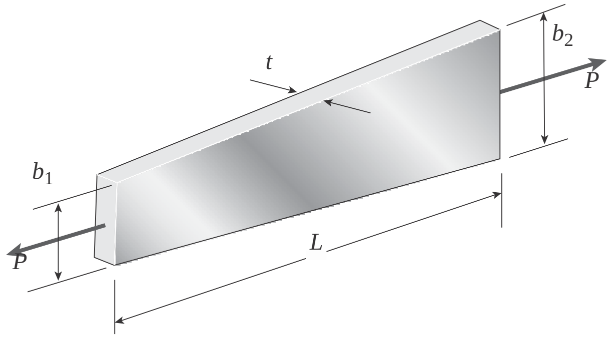
In sea water:

$$W \quad (g_s \quad g_w)AL \quad 1577.85\,\text{kN} \qquad \boxed{d \quad \frac{WL}{2EA} \quad 359 \text{ mm}} \qquad \frac{d}{L} \quad 2.393 * 10^{4}$$

In air:

$$W \quad (g_s)AL \quad 1813.35\,\text{kN} \qquad \boxed{d \quad \frac{WL}{2EA} \quad 412 \text{ mm}} \qquad \frac{d}{L} \quad 2.75 * 10^{4}$$

Problem 2.3-13 A flat bar of rectangular cross section, length L, and constant thickness t is subjected to tension by forces P (see figure). The width of the bar varies linearly from b_1 at the smaller end to b_2 at the larger end. Assume that the angle of taper is small.

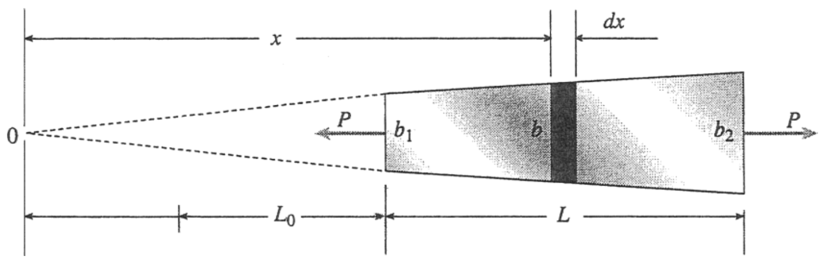


(a) Derive the following formula for the elongation of the bar:

$$\delta = \frac{PL}{Et(b_2 - b_1)} \ln \frac{b_2}{b_1}$$

(b) Calculate the elongation, assuming $L = 5$ ft, $t = 1.0$ in., $P = 25$ k, $b_1 = 4.0$ in., $b_2 = 6.0$ in., and $E = 30 \times 10^6$ psi.

Solution 2.3-13 Tapered bar (rectangular cross section)



$t =$ thickness (constant)

$$b = b_1\left(\frac{x}{L_0}\right) \quad b_2 = b_1\left(\frac{L_0 + L}{L_0}\right) \qquad \text{(Eq. 1)}$$

$$A(x) = bt = b_1 t\left(\frac{x}{L_0}\right)$$

(a) Elongation of the bar

$$d\delta = \frac{P\,dx}{EA(x)} = \frac{PL_0\,dx}{Eb_1 tx}$$

$$\delta = \int_{L_0}^{L_0+L} d\delta = \frac{PL_0}{Eb_1 t}\int_{L_0}^{L_0+L} \frac{dx}{x}$$

$$= \frac{PL_0}{Eb_1 t} \ln x \Big|_{L_0}^{L_0+L} = \frac{PL_0}{Eb_1 t} \ln \frac{L_0 + L}{L_0} \qquad \text{(Eq. 2)}$$

From Eq. (1): $\dfrac{L_0 + L}{L_0} = \dfrac{b_2}{b_1}$ (Eq. 3)

Solve Eq. (3) for L_0: $L_0 = L\left(\dfrac{b_1}{b_2 - b_1}\right)$ (Eq. 4)

Substitute Eqs. (3) and (4) into Eq. (2):

$$\delta = \frac{PL}{Et(b_2 - b_1)} \ln \frac{b_2}{b_1} \qquad \text{(Eq. 5)}$$

(b) Substitute numerical values:

$L = 5$ ft $= 60$ in. $\quad t = 1.0$ in.

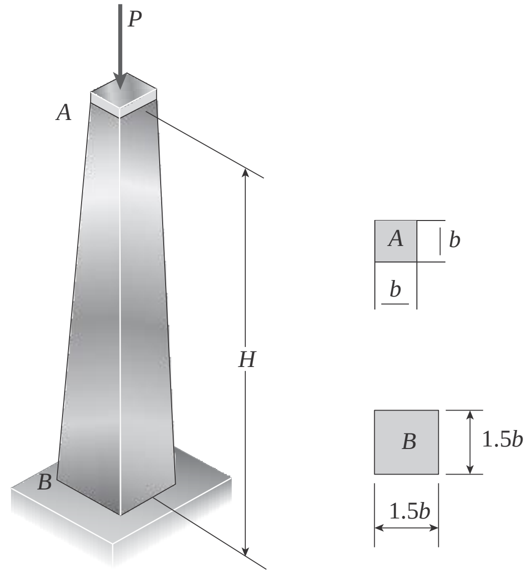
$P = 25$ k $\quad b_1 = 4.0$ in.

$b_2 = 6.0$ in. $\quad E = 30 \times 10^6$ psi

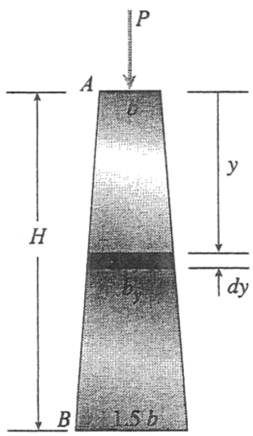
From Eq. (5): $\delta = 0.010$ in. ←

Problem 2.3-14 A post AB supporting equipment in a laboratory is tapered uniformly throughout its height H (see figure). The cross sections of the post are square, with dimensions $b \times b$ at the top and $1.5b \times 1.5b$ at the base.

Derive a formula for the shortening δ of the post due to the compressive load P acting at the top. (Assume that the angle of taper is small and disregard the weight of the post itself.)



Solution 2.3-14 Tapered post



Square cross sections:

b = width at A

$1.5b$ = width at B

b_y = width at distance y

$$= b + (1.5b - b)\frac{y}{H}$$

$$= \frac{b}{H}(H + 0.5y)$$

A_y = cross-sectional area at distance y

$$= (b_y)^2 = \frac{b^2}{H^2}(H + 0.5y)^2$$

Shortening of element dy

$$d\delta = \frac{P\,dy}{EA_y} = \frac{P\,dy}{E\left(\frac{b^2}{H^2}\right)(H + 0.5y)^2}$$

Shortening of entire post

$$\delta = \int d\delta = \frac{PH^2}{Eb^2}\int_0^H \frac{dy}{(H + 0.5y)^2}$$

From Appendix D: $\displaystyle\int \frac{dx}{(a + bx)^2} = -\frac{1}{b(a + bx)}$

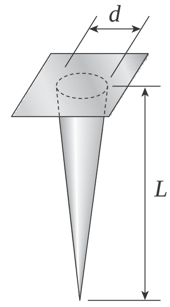
$$\delta = \frac{PH^2}{Eb^2}\left[-\frac{1}{(0.5)(H + 0.5y)}\right]_0^H$$

$$= \frac{PH^2}{Eb^2}\left[-\frac{1}{(0.5)(1.5H)} + \frac{1}{0.5H}\right]$$

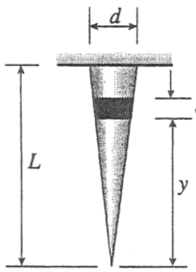
$$= \frac{2PH}{3Eb^2} \quad \Longleftarrow$$

Problem 2.3-15 A long, slender bar in the shape of a right circular cone with length L and base diameter d hangs vertically under the action of its own weight (see figure). The weight of the cone is W and the modulus of elasticity of the material is E.

Derive a formula for the increase d in the length of the bar due to its own weight. (Assume that the angle of taper of the cone is small.)



Solution 2.3-15 Conical bar hanging vertically



TERMINOLOGY

N_y axial force acting on element dy

A_y cross-sectional area at element dy

A_B cross-sectional area at base of cone

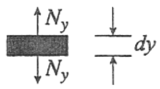
$\frac{pd^2}{4}$ V volume of cone

$\frac{1}{3}A_B L$ V_y volume of cone below element dy

$\frac{1}{3}A_y y$ W_y weight of cone below element dy

$\frac{V_y}{V}(W)$ $\frac{A_y yW}{A_B L}$ N_y W_y

ELEMENT OF BAR



W weight of cone

ELONGATION OF ELEMENT dy

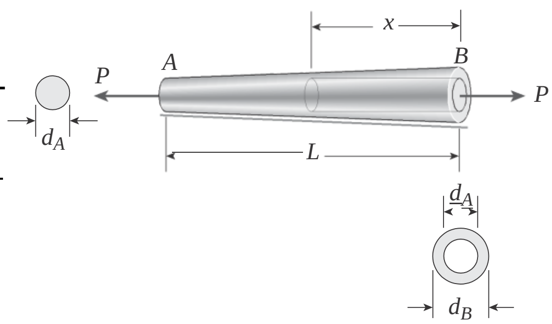
$$dd \quad \frac{N_y\,dy}{E A_y} \quad \frac{Wy\,dy}{E A_B L} \quad \frac{4W}{pd^2 EL} y\,dy$$

ELONGATION OF CONICAL BAR

$$d \quad \int dd \quad \frac{4W}{pd^2 EL}\int_0^L y\,dy \quad \frac{2WL}{pd^2 E}$$

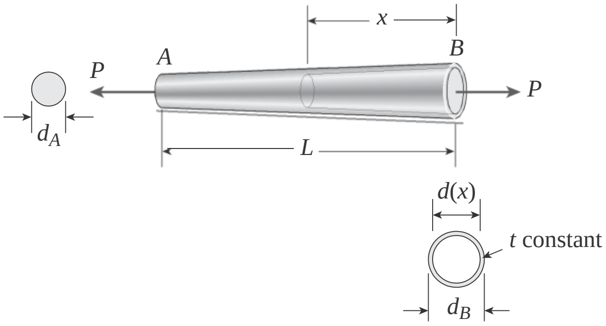
Problem 2.3-16 A uniformly tapered plastic tube AB of circular cross section and length L is shown in the figure. The average diameters at the ends are d_A and d_B $2d_A$. Assume E is constant. Find the elongation d of the tube when it is subjected to loads P acting at the ends. Use the following numerial data: d_A 35 mm, L 300 mm, E 2.1 GPa, P 25 kN. Consider two cases as follows:

(a) A hole of *constant* diameter d_A is drilled from B toward A to form a hollow section of length x $L/2$ (see figure part a).



(a)

(b) A hole of *variable* diameter $d(x)$ is drilled from B toward A to form a hollow section of length $x = L/2$ and constant thickness t (see figure part b). (Assume that $t = d_A/20$.)



(b)

Solution 2.3-16

(a) ELONGATION δ FOR CASE OF CONSTANT DIAMETER HOLE

$d(\xi) = d_A\left(1 + \frac{\xi}{L}\right)$ $\quad A(\xi) = \frac{\pi}{4}d(\xi)^2$ $\quad$ solid portion of length $L - x$

$A(\xi) = \frac{\pi}{4}\left(d(\xi)^2 - d_A^2\right)$ $\quad$ hollow portion of length x

$$\delta = \frac{P}{E}\left(\int_0^L \frac{1}{A(\xi)}\,d\xi\right) \qquad \delta = \frac{P}{E}\left[\int_0^{L-x}\frac{4}{\pi d(\xi)^2}\,d\xi + \int_{L-x}^{L}\frac{4}{\pi\left[d(\xi)^2 - d_A^2\right]}\,d\xi\right]$$

$$\delta = \frac{P}{E}\left[\int_0^{L-x}\frac{1}{\frac{\pi}{4}\left[d_A\left(1+\frac{\xi}{L}\right)\right]^2}\,d\xi + \int_{L-x}^{L}\frac{1}{\frac{\pi}{4}\left[\left[d_A\left(1+\frac{\xi}{L}\right)\right]^2 - d_A^2\right]}\,d\xi\right]$$

$$\delta = \frac{P}{E}\left[\left[-4\frac{L^2}{(2L-x)\pi d_A^2} + 4\frac{L}{\pi d_A^2}\right] + \int_{L-x}^{L}\frac{1}{\frac{\pi}{4}\left[\left[d_A\left(1+\frac{\xi}{L}\right)\right]^2 - d_A^2\right]}\,d\xi\right]$$

$$\delta = \frac{P}{E}\left[-4\frac{L^2}{(2L-x)\pi d_A^2} + \left(4\frac{L}{\pi d_A^2} - 2L\frac{\ln(3)}{\pi d_A^2} + 2L\frac{\ln(L-x) + \ln(3L-x)}{\pi d_A^2}\right)\right]$$

$$\text{if } x = L/2 \qquad \delta = \frac{P}{E}\left[\frac{4}{3}\frac{L}{\pi d_A^2} - 2L\frac{\ln(3)}{\pi d_A^2} - 2L\frac{\ln\left(\frac{1}{2}L\right) + \ln\left(\frac{5}{2}L\right)}{\pi d_A^2}\right]$$

Substitute numerical data:

$\delta = 2.18\text{ mm}$ $\leftarrow$

(b) ELONGATION δ FOR CASE OF VARIABLE DIAMETER HOLE BUT CONSTANT WALL THICKNESS $t = d_A/20$ OVER SEGMENT x

$d(\xi) = d_A\left(1 + \frac{\xi}{L}\right)$ $\quad A(\xi) = \frac{\pi}{4}d(\xi)^2$ solid portion of length $L - x$

$A(\xi) = \frac{\pi}{4}\left[d(\xi)^2 - \left(d(\xi) - 2\frac{d_A}{20}\right)^2\right]$ hollow portion of length x

$$\delta = \frac{P}{E}\left(\int_L \frac{1}{A(\xi)}\,d\xi\right) \qquad \delta = \frac{P}{E}\left[\int_0^{L-x}\frac{4}{\pi d(\xi)^2}\,d\xi + \int_{L-x}^{L}\frac{4}{\pi\left[d(\xi)^2 - \left(d(\xi) - 2\frac{d_A}{20}\right)^2\right]}\,d\xi\right]$$

$$\delta = \frac{P}{E}\left[\int_0^{L-x}\frac{4}{\pi\left[d_A\left(1+\frac{\xi}{L}\right)\right]^2}\,d\xi + \int_{L-x}^{L}\frac{4}{\pi\left[\left[d_A\left(1+\frac{\xi}{L}\right)\right]^2 - \left[d_A\left(1+\frac{\xi}{L}\right) - 2\frac{d_A}{20}\right]^2\right]}\,d\xi\right]$$

$$\delta = \frac{P}{E}\left[4\frac{L^2}{(-2L + x)\pi d_A^2} + 4\frac{L}{\pi d_A^2} + 20L\frac{\ln(3) + \ln(13) + 2\ln(d_A) + \ln(L)}{\pi d_A^2} - 20L\frac{2\ln(d_A) + \ln(39L - 20x)}{\pi d_A^2}\right]$$

if $x = L/2$

$$\delta = \frac{P}{E}\left(\frac{4}{3}\frac{L}{\pi d_A^2} + 20L\frac{\ln(3) + \ln(13) + 2\ln(d_A) + \ln(L)}{\pi d_A^2} - 20L\frac{2\ln(d_A) + \ln(29L)}{\pi d_A^2}\right)$$

Substitute numerical data:

$\delta = 6.74\text{ mm}$ $\leftarrow$

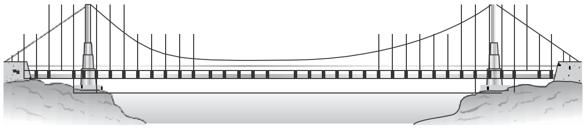
Problem 2.3-17 The main cables of a suspension bridge [see part (a) of the figure] follow a curve that is nearly parabolic because the primary load on the cables is the weight of the bridge deck, which is uniform in intensity along the horizontal. Therefore, let us represent the central region *AOB* of one of the main cables [see part (b) of the figure] as a parabolic cable supported at points *A* and *B* and carrying a uniform load of intensity q along the horizontal. The span of the cable is L, the sag is h,

midspan.

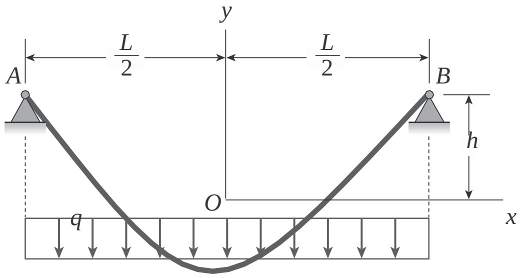
(a) Derive the following formula for the elongation of cable *AOB* shown in part (b) of the figure:

$$d \quad \frac{qL^3}{8hEA}\left(1 + \frac{16h^2}{3L^2}\right)$$

(b) Calculate the elongation d of the central span of one of the main cables of the Golden Gate Bridge, for which the dimensions and properties are L 4200 ft, h 470 ft, q 12,700 lb/ft, and E 28,800,000 psi. The cable consists of 27,572 parallel wires of diameter 0.196 in.



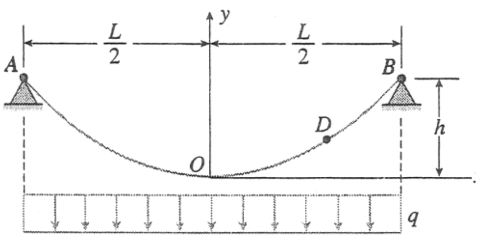
(a)

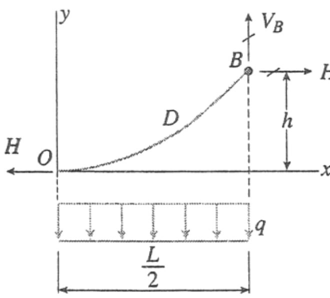


(b)

Hint: Determine the tensile force T at any point in the cable from a free-body diagram of part of the cable; then determine the elongation of an element of the cable of length ds; finally, integrate along the curve of the cable to obtain an equation for the elongation d.

Solution 2.3-17 Cable of a suspension bridge





Equation of parabolic curve:

$$y \quad \frac{4hx^2}{L^2}$$

$$\frac{dy}{dx} \quad \frac{8hx}{L^2}$$

FREE-BODY DIAGRAM OF HALF OF CABLE

$$M_B \quad 0$$

$$Hh + \frac{qL}{2}\left(\frac{L}{4}\right) \quad 0$$

$$H \quad \frac{qL^2}{8h}$$

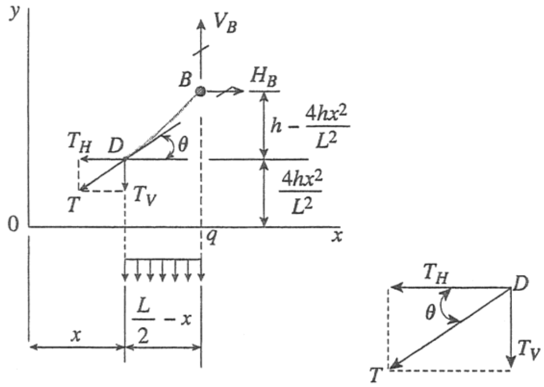
$$F_{\text{horizontal}} \quad 0$$

$$H_B \quad H \quad \frac{qL^2}{8h} \qquad \text{(Eq. 1)}$$

$$F_{\text{vertical}} \quad 0$$

$$V_B \quad \frac{qL}{2} \qquad \text{(Eq. 2)}$$

FREE-BODY DIAGRAM OF SEGMENT DB OF CABLE



$$\Sigma F_{horiz} = 0 \qquad T_H = H_B = \frac{qL^2}{8h} \qquad \text{(Eq. 3)}$$

$$\Sigma F_{vert} = 0 \qquad V_B - T_v - q\left(\frac{L}{2} - x\right) = 0$$

$$T_v = V_B - q\left(\frac{L}{2} - x\right) = \frac{qL}{2} - \frac{qL}{2} + qx$$

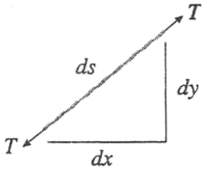
$$= qx \qquad \text{(Eq. 4)}$$

TENSILE FORCE T IN CABLE

$$T = \sqrt{T_H^2 + T_v^2} = \sqrt{\left(\frac{qL^2}{8h}\right)^2 + (qx)^2}$$

$$= \frac{qL^2}{8h}\sqrt{1 + \frac{64h^2x^2}{L^4}} \qquad \text{(Eq. 5)}$$

ELONGATION $d\delta$ OF AN ELEMENT OF LENGTH ds



$$d\delta = \frac{T\,ds}{EA}$$

$$ds = \sqrt{(dx)^2 + (dy)^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$= dx\sqrt{1 + \left(\frac{8hx}{L^2}\right)^2}$$

$$= dx\sqrt{1 + \frac{64h^2x^2}{L^4}} \qquad \text{(Eq. 6)}$$

(a) ELONGATION δ OF CABLE AOB

$$\delta = \int d\delta = \int \frac{T\,ds}{EA}$$

Substitute for T from Eq. (5) and for ds from Eq. (6):

$$\delta = \frac{1}{EA}\int \frac{qL^2}{8h}\left(1 + \frac{64h^2x^2}{L^4}\right)dx$$

For both halves of cable:

$$\delta = \frac{2}{EA}\int_0^{L/2} \frac{qL^2}{8h}\left(1 + \frac{64h^2x^2}{L^4}\right)dx$$

$$\delta = \frac{qL^3}{8hEA}\left(1 + \frac{16h^2}{3L^4}\right) \qquad \text{(Eq. 7)}$$

(b) GOLDEN GATE BRIDGE CABLE

$L = 4200$ ft $\quad h = 470$ ft

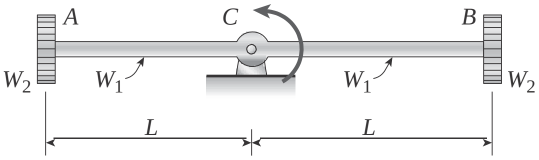
$q = 12{,}700$ lb/ft $\quad E = 28{,}800{,}000$ psi

27,572 wires of diameter $d = 0.196$ in.

$$A = (27{,}572)\left(\frac{\pi}{4}\right)(0.196 \text{ in.})^2 = 831.90 \text{ in.}^2$$

Substitute into Eq. (7):

$\delta = 133.7$ in $= 11.14$ ft



vertical axis at the midpoint C (see figure). The bar, which has length

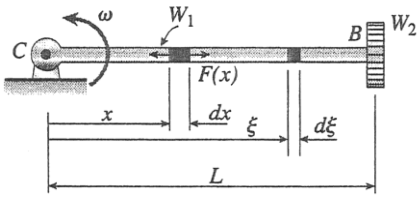
Each half of the bar (AC and BC) has weight W_1 and supports a weight W_2 at its end.

Derive the following formula for the elongation of one-half of the bar (that is, the elongation of either AC or BC):

$$d \quad \frac{L^2 \quad ^2}{3gEA}(W_1 + 3W_2)$$

in which E is the modulus of elasticity of the material of the bar and g is the acceleration of gravity.

Solution 2.3-18 Rotating bar



v angular speed

A cross-sectional area

E modulus of elasticity

g acceleration of gravity

$F(x)$ axial force in bar at distance x from point C

Consider an element of length dx at distance x from point C.

To find the force $F(x)$ acting on this element, we must find the inertia force of the part of the bar from distance x to distance L, plus the inertia force of the weight W_2.

Since the inertia force varies with distance from point C, we now must consider an element of length $d\boldsymbol{j}$ at distance $\boldsymbol{j}$, where $\boldsymbol{j}$ varies from x to L.

Mass of element $d \quad \frac{d}{L}\left(\frac{W_1}{g}\right)$

Acceleration of element $\quad \boldsymbol{j}v^2$

Centrifugal force produced by element

(mass)(acceleration) $\quad \frac{W_1 \quad ^2}{gL} \; d$

Centrifugal force produced by weight W_2

$$\left(\frac{W_2}{g}\right)(L \quad ^2)$$

AXIAL FORCE $F(x)$

$$F(x) \quad \int_x^L \frac{W_1 \quad ^2}{gL} \; d \; + \frac{W_2 L \quad ^2}{g}$$

$$\frac{W_1 \quad ^2}{2gL}(L^2 \quad x^2) + \frac{W_2 L \quad ^2}{g}$$

ELONGATION OF BAR BC

$$d \quad \int_0^L \frac{F(x)\,dx}{EA}$$

$$\int_0^L \frac{W_1 \quad ^2}{2gL}(L^2 \quad x^2)dx + \int_0^L \frac{W_2 L \quad ^2 dx}{gEA}$$

$$\frac{W1L \quad ^2}{2gLEA}\left(\int_0^L L^2 dx \quad \int_0^L x^2\,dx\right) + \frac{W_2 L \quad ^2 dx}{gEA}\int_0^L dx$$

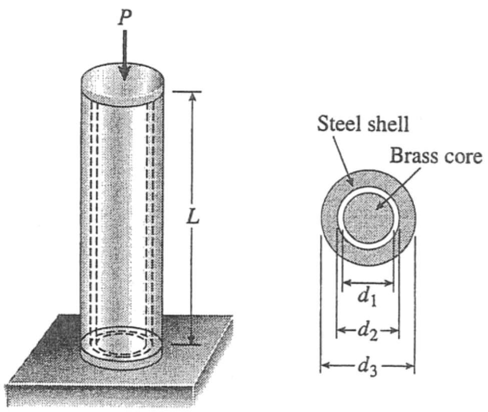
$$\frac{W_1 L^2 \quad ^2}{3gEA} + \frac{W_2 L^2 \quad ^2}{gEA}$$

$$\frac{L^2 \quad ^2}{3gEA} + (W_1 + 3W_2)$$

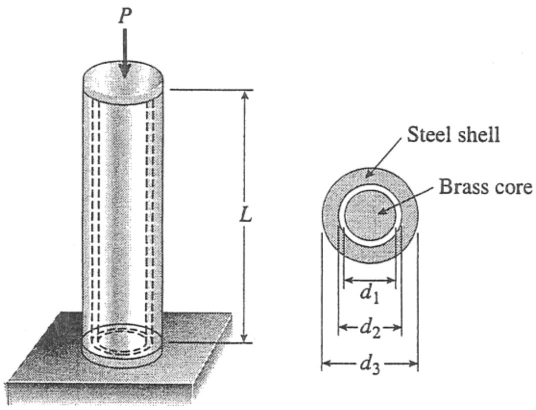
Statically Indeterminate Structures

Problem 2.4-1 The assembly shown in the figure consists of a brass core (diameter d_1 0.25 in.) surrounded by a steel shell (inner diameter d_2 0.28 in., outer diameter d_3 0.35 in.). A load P compresses the core and shell, which have length L 4.0 in. The moduli of elasticity of the brass and steel are E_b 15 10^6 psi and E_s 30 10^6 psi, respectively.

(a) What load P will compress the assembly by 0.003 in.?
(b) If the allowable stress in the steel is 22 ksi and the allowable stress in the brass is 16 ksi, what is the allowable compressive load P_{allow}? (*Suggestion:* Use the equations derived in Example 2-6.)



Solution 2.4-1 Cylindrical assembly in compression



d_1 0.25 in. E_b 15 10^6 psi

d_2 0.28 in. E_s 30 10^6 psi

d_3 0.35 in. A_s $\frac{p}{4}(d_3^2 \quad d_2^2)$ 0.03464 in.^{2}

L 4.0 in. A_b $\frac{p}{4} d_1^2$ 0.04909 in.^{2}

(a) Decrease in length (d 0.003 in.)

Use Eq. (2-18) of Example 2-6.

$$d \quad \frac{PL}{E_s A_s + E_b A_b} \quad \text{or}$$

$$P \quad (E_s A_s + E_s A_b)\text{a}\frac{d}{L}\text{b}$$

Substitute numerical values:

$$E_s A_s + E_b A_b \quad (30 * 10^6 \text{ psi})(0.03464 \text{ in.}^2)$$
$$+ (15 * 10^6 \text{ psi})(0.04909 \text{ in.}^2)$$
$$1.776 * 10^6 \text{ lb}$$

$$P \quad (1.776 * 10^6 \text{ lb})\text{a}\frac{0.003 \text{ in.}}{4.0 \text{ in.}}\text{b}$$
$$1330 \text{ lb}$$

(b) Allowable load

s_s 22 ksi s_b 16 ksi

Use Eqs. (2-17a and b) of Example 2-6.

For steel:

$$s_s \quad \frac{PE_s}{E_s A_s + E_b A_b} \quad P_s \quad (E_s A_s + E_b A_b)\frac{s_s}{E_s}$$

$$P_s \quad (1.776 * 10^6 \text{ lb})\text{a}\frac{22 \text{ ksi}}{30 * 10^6 \text{ psi}}\text{b} \quad 1300 \text{ lb}$$

For brass:

$$s_b \quad \frac{PE_b}{E_s A_s + E_b A_b} \quad P_s \quad (E_s A_s + E_b A_b)\frac{s_b}{E_b}$$

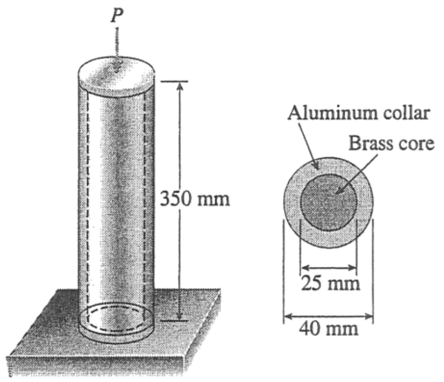
$$P_s \quad (1.776 * 10^6 \text{ lb})\text{a}\frac{16 \text{ ksi}}{15 * 10^6 \text{ psi}}\text{b} \quad 1890 \text{ lb}$$

Steel governs. P_{allow} 1300 lb

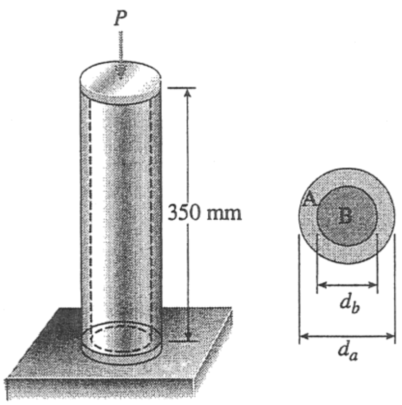
Problem 2.4-2 A cylindrical assembly consisting of a brass core and an aluminum collar is compressed by a load P (see figure). The length of the aluminum collar and brass core is 350 mm, the diameter of the core is 25 mm, and the outside diameter of the collar is 40 mm. Also, the moduli of elasticity of the aluminum and brass are 72 GPa and 100 GPa, respectively.

(a) If the length of the assembly decreases by 0.1% when the load P is applied, what is the magnitude of the load?

(b) What is the maximum permissible load P_{max} if the allowable stresses in the aluminum and brass are 80 MPa and 120 MPa, respectively? (*Suggestion:* Use the equations derived in Example 2-6.)



Solution 2.4-2 Cylindrical assembly in compression



A aluminum

B brass

L 350 mm

d_a 40 mm

d_b 25 mm

$$A_a \quad \frac{\pi}{4}(d_a^2 \quad d_b^2)$$

$$765.8 \text{ mm}^2$$

E_a 72 GPa E_b 100 GPa A_b $\frac{\pi}{4}d_b^2$

490.9 mm^2

(a) DECREASE IN LENGTH

(δ 0.1% of L 0.350 mm)

Use Eq. (2-18) of Example 2-6.

$$\delta \quad \frac{PL}{E_aA_a + E_bA_b} \quad \text{or}$$

$$P \quad (E_aA_a + E_bA_b)\left(\frac{\delta}{L}\right)$$

Substitute numerical values:

$$E_aA_a + E_bA_b \quad (72 \text{ GPa})(765.8 \text{ mm}^2)$$
$$(100 \text{ GPa})(490.9 \text{ mm}^2)$$
$$55.135 \text{ MN} + 49.090 \text{ MN}$$
$$104.23 \text{ MN}$$

$$P \quad (104.23 \text{ MN})\left(\frac{0.350 \text{ mm}}{350 \text{ mm}}\right)$$

$$104.2 \text{ kN}$$

(b) ALLOWABLE LOAD

σ_a 80 MPa σ_b 120 MPa

Use Eqs. (2-17a and b) of Example 2-6.

For aluminum:

$$\sigma_a \quad \frac{PE_a}{E_aA_a + E_bA_b} \quad P_a \quad (E_aA_a + E_bA_b)\left(\frac{\sigma_a}{E_a}\right)$$

$$P_a \quad (104.23 \text{ MN})\left(\frac{80 \text{ MPa}}{72 \text{ GPa}}\right) \quad 115.8 \text{ kN}$$

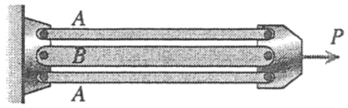
For brass:

$$\sigma_b \quad \frac{PE_b}{E_aA_a + E_bA_b} \quad P_b \quad (E_aA_a + E_bA_b)\left(\frac{\sigma_b}{E_b}\right)$$

$$P_b \quad (104.23 \text{ MN})\left(\frac{120 \text{ MPa}}{100 \text{ GPa}}\right) \quad 125.1 \text{ kN}$$

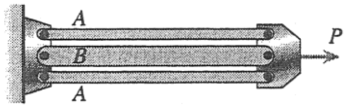
Aluminum governs. P_{max} 116 kN

Problem 2.4-3 Three prismatic bars, two of material *A* and one of material *B*, transmit a tensile load *P* (see figure). The two outer bars (material *A*) are identical. The cross-sectional area of the middle bar (material *B*) is 50% larger than the cross-sectional area of one of the outer bars. Also, the modulus of elasticity of material *A* is twice that of material *B*.

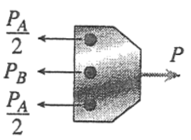


(a) What fraction of the load *P* is transmitted by the middle bar?
(b) What is the ratio of the stress in the middle bar to the stress in the outer bars?
(c) What is the ratio of the strain in the middle bar to the strain in the outer bars?

Solution 2.4-3 Prismatic bars in tension



FREE-BODY DIAGRAM OF END PLATE



EQUATION OF EQUILIBRIUM

$$F_{\text{horiz}} \quad 0 \quad P_A \quad P_B \quad P \quad 0 \qquad (1)$$

EQUATION OF COMPATIBILITY

$$d_A \quad d_B \qquad (2)$$

FORCE-DISPLACEMENT RELATIONS

A_A total area of both outer bars

$$d_A \quad \frac{P_A L}{E_A A_k} \quad d_B \quad \frac{P_B L}{E_B A_B} \qquad (3)$$

Substitute into Eq. (2):

$$\frac{P_A L}{E_A A_A} \quad \frac{P_B L}{E_B A_B} \qquad (4)$$

SOLUTION OF THE EQUATIONS

Solve simultaneously Eqs. (1) and (4):

$$P_A \quad \frac{E_A A_A P}{E_A A_A + E_B A_B} \quad P_B \quad \frac{E_B A_B P}{E_A A_A + E_B A_B} \qquad (5)$$

Substitute into Eq. (3):

$$d \quad d_A \quad d_B \quad \frac{PL}{E_A A_A}$$

STRESSES:

$$s_A \quad \frac{P_A}{A_A} \quad \frac{E_A P}{E_A A_A + E_B A_B}$$

$$s_B \quad \frac{P_B}{A_B} \quad \frac{E_B P}{E_A A_A + E_B A_B} \qquad (7)$$

(a) LOAD IN MIDDLE BAR

$$\frac{P_B}{P} \quad \frac{E_B A_B}{E_A A_A + E_B A_B} \quad \frac{1}{\frac{E_A A_A}{E_B A_B} + 1}$$

$$\text{Given: } \frac{E_A}{E_B} \quad 2 \quad \frac{A_A}{A_B} \quad \frac{1 + 1}{1.5} \quad \frac{4}{3}$$

$$< \frac{P_B}{P} \quad \frac{1}{a\frac{E_A}{E_B} b a \frac{A_A}{A_B} b + 1} \quad \frac{1}{\frac{8}{3} + 1} \quad \frac{3}{11}$$

(b) RATIO OF STRESSES

$$\frac{s_B}{s_A} \quad \frac{E_B}{E_A} \quad \frac{1}{2}$$

(c) RATIO OF STRAINS

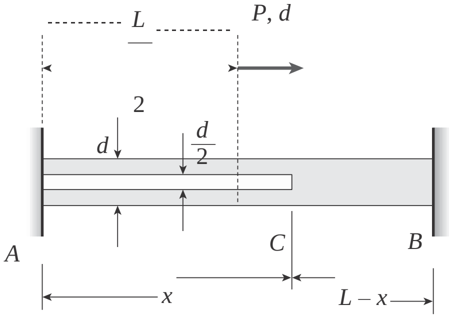
All bars have the same strain

Ratio 1

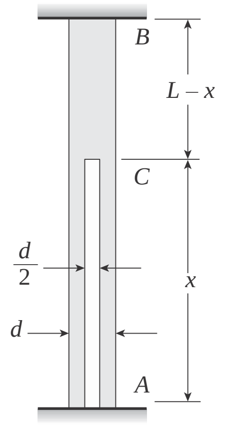
$$+ \ E_B A_B \qquad (6)$$

Problem 2.4-4 A circular bar *ACB* of diameter *d* having a cylindrical hole of length *x* and diameter *d*/2 from *A* to *C* is held between rigid supports at *A* and *B*. A load *P* acts at *L*/2 from ends *A* and *B*. Assume *E* is constant.

(a) Obtain formulas for the reactions R_A and R_B at supports *A* and *B*, respectively, due to the load *P* (see figure part a).
(b) Obtain a formula for the displacement *d* at the point of load application (see figure part a).
(c) For what value of *x* is R_B (6/5) R_A? (See figure part a.)
(d) Repeat part (a) if the bar is now rotated to a vertical position, load *P* is removed, and the bar is hanging under its own weight (assume mass density *r*). (See figure part b.) Assume that *x* *L*/2.



(a)



(b)

Solution 2.4-4

(a) Reactions at *A* and *B* due to load *P* at *L*/2

$A_{AC} \quad \frac{p}{4} c\, d^2 \quad a\frac{d}{2}b^2 d \qquad A_{AC} \quad \frac{3}{16}pd^2$

$A_{CB} \quad \frac{p}{4} d^2$

Select R_B as the redundant; use superposition and a compatibility equation at *B*:

if *x* *L*/2 $\quad d_{B1a} \quad \frac{Px}{EA_{AC}} + \frac{P a\frac{L}{2} \quad x b}{EA_{CB}} \qquad d_{B1a} \quad \frac{P}{E} \pm \frac{x}{\frac{3}{16}pd^2} + \frac{\frac{L}{2} \quad x}{\frac{p}{4}d^2} \leq$

$d_{B1a} \quad \frac{2}{3}P \frac{2x + 3L}{Epd^2}$

if *x* *L*/2 $\quad d_{B1b} \quad \frac{P\frac{L}{2}}{EA_{AC}} \qquad d_{B1b} \quad \frac{P\frac{L}{2}}{E a\frac{3}{16}pd^2 b} \qquad d_{B1b} \quad \frac{8}{3}\frac{PL}{Epd^2}$

The following expression for δ_{B2} is good for all x:

$$\delta_{B2} = \frac{R_B}{E}\left(\frac{x}{A_{AC}} + \frac{L-x}{A_{CB}}\right) \qquad \delta_{B2} = \frac{R_B}{E}\left[\frac{x}{\frac{3}{16}\pi d^2} + \frac{L-x}{\frac{\pi}{4}d^2}\right]$$

$$\delta_{B2} = \frac{R_B}{E}\left(\frac{16}{3}\frac{x}{\pi d^2} + 4\frac{L-x}{\pi d^2}\right)$$

Solve for R_B and R_A assuming that $x \le L/2$:

$$\text{Compatibility:} \quad \delta_{B1a} - \delta_{B2} = 0 \qquad R_{Ba} = \frac{\left(\frac{2}{3}P\frac{2x+3L}{\pi d^2}\right)}{\left(\frac{16}{3}\frac{x}{\pi d^2} + 4\frac{L-x}{\pi d^2}\right)} \qquad R_{Ba} = \frac{1}{2}P\frac{2x+3L}{x+3L} \quad \leftarrow$$

$\Leftarrow$ check—if $x = 0$, $R_B = P/2$

$$\text{Statics:} \quad R_{Aa} = P - R_{Ba} \qquad R_{Aa} = P - \frac{1}{2}P\frac{2x+3L}{x+3L} \qquad R_{Aa} = \frac{3}{2}P\frac{L}{x+3L} \quad \leftarrow$$

$\Leftarrow$ check—if $x = 0$, $R_{A_a} = P/2$

Solve for R_B and R_A assuming that $x \ge L/2$:

$$\text{Compatibility:} \quad \delta_{B1b} - \delta_{B2} = 0 \qquad R_{Bb} = \frac{\frac{8}{3}\frac{PL}{\pi d^2}}{\left(\frac{16}{3}\frac{x}{\pi d^2} + 4\frac{L-x}{\pi d^2}\right)} \qquad R_{Bb} = \frac{2PL}{x+3L} \quad \leftarrow$$

$\Leftarrow$ check—if $x = L$, $R_B = P/2$

$$\text{Statics:} \quad R_{Ab} = P - R_{Bb} \qquad R_{Ab} = P - \left(\frac{2PL}{x+3L}\right) \qquad R_{Ab} = P\frac{x+L}{x+3L} \quad \leftarrow$$

(b) Find δ at point of load application; axial force for segment 0 to $L/2 = R_A$ and $\delta =$ elongation of this segment

Assume that $x \le L/2$:

$$\delta_a = \frac{R_{Aa}}{E}\left[\frac{x}{A_{AC}} + \frac{\frac{L}{2} - x}{A_{CB}}\right] \qquad \delta_a = \frac{\left(\frac{3}{2}P\frac{L}{x+3L}\right)}{E}\left[\frac{x}{\frac{3}{16}\pi d^2} + \frac{\frac{L}{2} - x}{\frac{\pi}{4}d^2}\right]$$

δ_a PL

$$\frac{2x + 3L}{\qquad} \quad (x + 3L)E\pi d^2$$

For $x \quad L/2, \qquad \delta_a \quad \frac{8}{7} L \frac{P}{E\pi d^2}$

ASSUME THAT x Ú $L/2$:

$$\delta_b = \frac{\frac{1}{2} R_{Ab} \frac{L}{2}}{E A_{AC}} \qquad \delta_b = \frac{\left(P \frac{x + L}{x + 3L}\right)\frac{L}{2}}{E\left(\frac{3}{16}\pi d^2\right)} \qquad \delta_b = \frac{8}{3} P \left(\frac{x + L}{x + 3L}\right) \frac{L}{E\pi d^2}$$

for $x = L/2$ $\quad \delta_b = \frac{8}{7} P \frac{L}{E\pi d^2}$ same as δ_a above (OK)

(c) FOR WHAT VALUE OF x IS $R_B = (6/5)\, R_A$?

Guess that $x \leq L/2$ here and use R_{Ba} expression above to find x:

$$\frac{1}{2} P \frac{2x + 3L}{x + 3L} - \frac{6}{5}\left(\frac{3}{2} P \frac{L}{x + 3L}\right) = 0 \qquad \frac{1}{10} P \frac{10x - 3L}{x + 3L} = 0 \qquad x = \frac{3L}{10}$$

Now try $R_{Bb} = (6/5)R_{Ab}$, assuming that $x \geq L/2$

$$\frac{2PL}{x + 3L} - \frac{6}{5}\left(P \frac{x + L}{x + 3L}\right) = 0 \qquad \frac{2}{5} P \frac{2L - 3x}{x + 3L} = 0 \qquad x = \frac{2}{3} L$$

So, there are two solutions for x.

(d) FIND REACTIONS IF THE BAR IS NOW ROTATED TO A VERTICAL POSITION, LOAD P IS REMOVED, AND THE BAR IS HANGING UNDER ITS OWN WEIGHT (ASSUME MASS DENSITY $= \rho$). ASSUME THAT $x = L/2$.

$$A_{AC} = \frac{3}{16}\pi d^2 \qquad A_{CB} = \frac{\pi}{4} d^2$$

Select R_B as the redundant; use superposition and a compatibility equation at B from (a) above. compatibility: $\delta_{B1} + \delta_{B2} = 0$

$$\delta_{B2} = \frac{R_B}{E}\left(\frac{x}{A_{AC}} + \frac{L - x}{A_{CB}}\right) \qquad \text{For } x = L/2,\ \delta_{B2} = \frac{R_B}{E}\left(\frac{14}{3}\frac{L}{\pi d^2}\right)$$

$$\delta_{B1} = \int_0^{\frac{L}{2}} \frac{N_{AC}}{EA_{AC}}\, d\zeta + \int_{\frac{L}{2}}^{L} \frac{N_{CB}}{EA_{CB}}\, d\zeta$$

Where axial forces in bar due to self weight are $W_{AC} = \rho g A_{AC} \frac{L}{2}$ $\quad W_{CB} = \rho g A_{CB} \frac{L}{2}$ (assume z is measured upward from A):

$$N_{AC} = -\left[\rho g A_{CB} \frac{L}{2} + \rho g A_{AC}\left(\frac{L}{2} - \zeta\right)\right] \qquad A_{AC} = \frac{3}{16}\pi d^2 \qquad A_{CB} = \frac{\pi}{4} d^2$$

$$N_{CB} = -[\rho\, g A_{CB}(L - \zeta)]$$

$$\frac{1}{\ }\ _2 \quad \frac{3}{\ }\ _2 \quad 1 \qquad 1 \quad _2$$

N_{AC} 8 $rgp\ d$ L 16 $rgp\ d$ 2 L N_{CB} 4 $rgp\ d\ (L\ )$

$$d_{B1} \int_{0}^{\frac{L}{2}} \frac{\frac{1}{8} rgpd^2L \quad \frac{3}{16} rgpd^2 \,a\frac{1}{2}L \quad b}{E\,a\frac{3}{16}pd^2 b} d + \int_{\frac{L}{2}}^{L} \frac{c\frac{1}{4} rgpd^2 (L \quad) d}{E\,a\frac{p}{4}d^2 b} d$$

$$d_{B1} \quad a\frac{11}{24} rg\frac{L^2}{E} + \frac{1}{8} rg\frac{L^2}{E} b \qquad d_{B1} \quad \frac{7}{12} rg\frac{L^2}{E} \qquad \frac{7}{12} \quad 0.583$$

Compatibility: $d_{B1} \quad d_{B2} \quad 0$

$$R_B \frac{a\frac{7}{12} rg\frac{L^2}{E} b}{a\frac{14}{3}\frac{L}{Epd^2} b} \qquad R_B \quad \frac{1}{8} rgpd^2 L$$

Statics: $R_A \quad (W_{AC} \quad W_{CB}) \quad R_B$

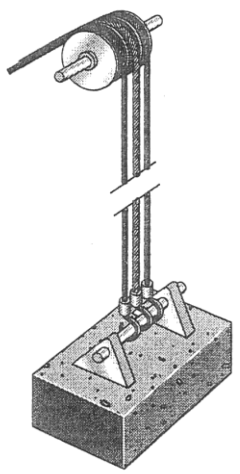
$$R_A \quad c\,c\,rg\,a\frac{3}{16}pd^2 b\frac{L}{2} + rg\,a\frac{p}{4}d^2 b\frac{L}{2} d \quad \frac{1}{8} rgpd^2 L d$$

$$R_A \frac{3}{32} rgp\, d^2 L$$

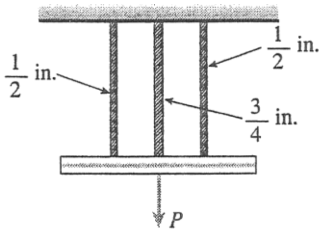
Problem 2.4-5 Three steel cables jointly support a load of 12 k (see figure). The diameter of the middle cable is ¾ in. and the diameter of each outer cable is ½ in. The tensions in the cables are adjusted so that each cable carries one-third of the load (i.e., 4 k). Later, the load is increased by 9 k to a total load of 21 k.

(a) What percent of the total load is now carried by the middle cable?

(b) What are the stresses s_M and s_O in the middle and outer cables, respectively? (**NOTE:** See Table 2-1 in Section 2.2 for properties of cables.)



Solution 2.4-5 Three cables in tension



AREAS OF CABLES (from Table 2-1)

Middle cable: $A_M \quad 0.268 \text{ in.}^2$

Outer cables: $A_O \quad 0.119 \text{ in.}^2$

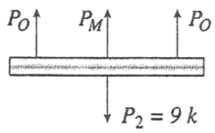
(for each cable)

FIRST LOADING

$P_1 \quad 12 \text{ k} \left(\text{Each cable carries } \frac{P_1}{3} \text{ or } 4 \text{ k.}\right)$

SECOND LOADING

$P_2 = 9$ k (additional load)



EQUATION OF EQUILIBRIUM

$$F_{\text{vert}} \quad 0 \qquad 2P_O \quad P_M \quad P_2 \quad 0 \tag{1}$$

EQUATION OF COMPATIBILITY

$$d_M \quad d_O \tag{2}$$

FORCE-DISPLACEMENT RELATIONS

$$d_M \quad \frac{P_M L}{EA_M} \quad d_O \quad \frac{P_o L}{EA_o} \tag{3, 4}$$

SUBSTITUTE INTO COMPATIBILITY EQUATION:

$$\frac{P_M L}{EA_M} \quad \frac{P_O L}{EA_O} \quad \frac{P_M}{A_M} \quad \frac{P_O}{A_O} \tag{5}$$

SOLVE SIMULTANEOUSLY EQS. (1) AND (5):

$$P_M \quad P_2\left(\frac{A_M}{A_M + 2A_O}\right) \quad (9 \text{ k})\left(\frac{0.268 \text{ in.}^2}{0.506 \text{ in.}^2}\right)$$

$$4.767 \text{ k}$$

$$P_o \quad P_2\left(\frac{A_o}{A^M + 2A_O}\right) \quad (9 \text{ k})\left(\frac{0.119 \text{ in.}^2}{0.506 \text{ in.}^2}\right)$$

$$2.117 \text{ k}$$

FORCES IN CABLES

Middle cable: Force 4 k 4.767 k 8.767 k

Outer cables: Force 4 k 2.117 k 6.117 k

(for each cable)

(a) PERCENT OF TOTAL LOAD CARRIED BY MIDDLE CABLE

$$\text{Percent} \ \frac{8.767 \text{ k}}{21 \text{ k}}(100\%) \quad 41.7\%$$

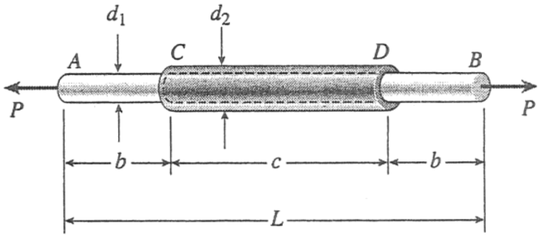
(b) STRESSES IN CABLES ($s \quad P/A$)

Middle cable: $s_M \quad \frac{8.767 \text{ k}}{0.268 \text{ in.}^2} \quad 32.7$ ksi

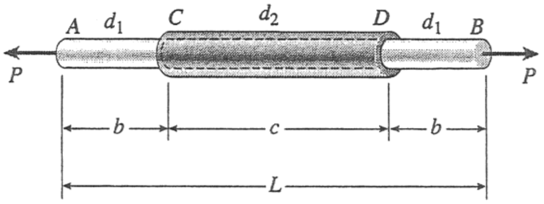
Outer cables: $s_O \quad \frac{6.117 \text{ k}}{0.119 \text{ in.}^2} \quad 51.4$ ksi

Problem 2.4-6 A plastic rod *AB* of length $L = 0.5$ m has a diameter $d_1 = 30$ mm (see figure). A plastic sleeve *CD* of length $c = 0.3$ m and outer diameter $d_2 = 45$ mm is securely bonded to the rod so that no slippage can occur between the rod and the sleeve. The rod is made of an acrylic with modulus of elasticity $E_1 = 3.1$ GPa and the sleeve is made of a polyamide with $E_2 = 2.5$ GPa.

(a) Calculate the elongation δ of the rod when it is pulled by axial forces $P = 12$ kN.
(b) If the sleeve is extended for the full length of the rod, what is the elongation?
(c) If the sleeve is removed, what is the elongation?



Solution 2.4-6 Plastic rod with sleeve



$P = 12$ kN $\quad d_1 = 30$ mm $\quad b = 100$ mm

$L = 500$ mm $\quad d_2 = 45$ mm $\quad c = 300$ mm

Rod: $E_1 = 3.1$ GPa

Sleeve: $E_2 = 2.5$ GPa

Rod: $A_1 = \dfrac{\pi d_1^2}{4} = 706.86 \text{ mm}^2$

Sleeve: $A_2 = \dfrac{\pi}{4}(d_2^2 - d_1^2) = 883.57 \text{ mm}^2$

$E_1A_1 + E_2A_2 = 4.400$ MN

(a) ELONGATION OF ROD

Part *AC*: $\delta_{AC} = \dfrac{Pb}{E_1A_1} = 0.5476$ mm

Part *CD*: $\delta_{CD} = \dfrac{Pc}{E_1A_1 + E_2A_2}$

$= 0.81815$ mm

(From Eq. 2-18 of Example 2-6)

$\delta = 2\delta_{AC} + \delta_{CD} = 1.91$ mm ◄

(b) SLEEVE AT FULL LENGTH

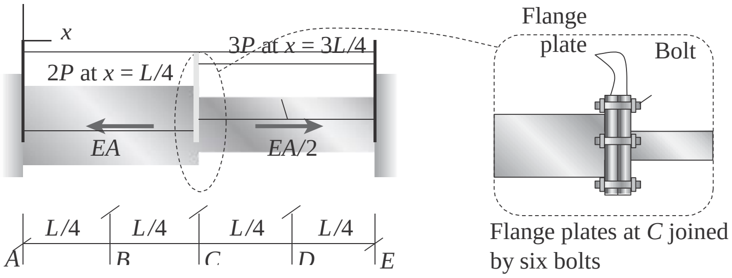
$\delta = \delta_{CD}\left(\dfrac{L}{c}\right) = (0.81815 \text{ mm})\left(\dfrac{500 \text{ mm}}{300 \text{ mm}}\right)$

$= 1.36$ mm ◄

(c) SLEEVE REMOVED

$\delta = \dfrac{PL}{E_1A_1} = 2.74$ mm ◄

Problem 2.4-7 A tube structure is acted on by loads at B and D, as shown in the figure. The tubes are joined using two flange plates at C, which are bolted together using six 0.5 in. diameter bolts.



(a) Derive formulas for the reactions R_A and

R_E at the ends of the bar.

(b) Determine the axial displacement $_B$, $_C$, and $_D$ at points B, C, and D, respectively.

(c) Draw an axial-displacement diagram

(ADD) in which the abscissa is the distance x from support A to any point on the bar and the ordinate is the horizontal displacement d at that point.

(d) Find the maximum value of the load variable P if allowable normal stress in the bolts is 14 ksi.

Solution 2.4-7

NUMERICAL DATA

$n \quad 6 \qquad d_b \quad 0.5\text{ in.} \qquad s_a \quad 14\text{ ksi} \qquad A_b \quad \frac{p}{4} d_b^{\,2} \quad 0.196\text{ in.}^2$

(a) FORMULAS FOR REACTIONS F

Segment ABC flexibility: $f_1 \quad \dfrac{2 a \frac{L}{4} b}{EA} \quad \dfrac{L}{2EA}$

Segment CDE flexibility: $f_2 \quad \dfrac{2 a \frac{L}{4} b}{\frac{1}{2} EA} \quad \dfrac{L}{EA}$

Loads at points B and D:

$P_B \quad 2P \qquad P_D \quad 3P$

(1) Select R_E as the redundant; find axial displacement d_1 displacement at E due to loads P_B and P_D:

$$d_1 \quad \frac{1P_B + P_D 2\frac{L}{4}}{EA} + \frac{P_D \frac{L}{4}}{EA} + \frac{P_D \frac{L}{4}}{\frac{1}{2} EA} \quad \frac{5LP}{2EA}$$

(2) Next apply redundant RE and find axial displacement d_2 displacement at E due to redundant R_E:

$$d_2 \quad R_E 1 f_1 + f_2 2 \quad \frac{3LR_E}{2EA}$$

(3) Use compatibility equation to find redundant R_E then use statics to find R_A:

$$d_1 + d_2 \quad 0 \text{ solve, } R_E \quad \frac{5P}{3} \qquad R_E \quad \frac{5}{3}P$$

$$R_A \quad R_E \quad P_B \quad P_D \quad \frac{2P}{3} \qquad R_A \quad \frac{2P}{3} \qquad \boxed{R_A \quad \frac{2P}{3}} \qquad \boxed{R_E \quad \frac{5P}{3}}$$

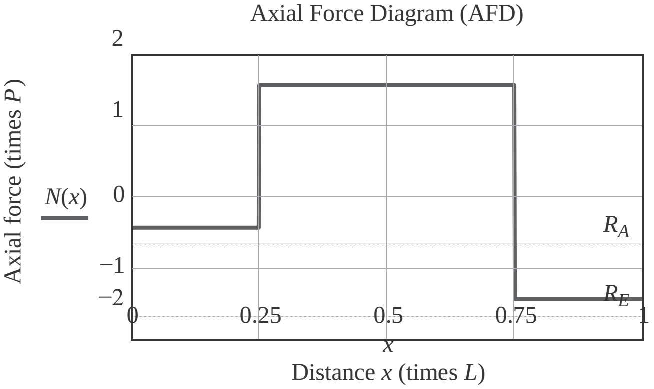
(b) Determine the axial displacements d_B, d_C, and d_D at points B, C, and D, respectively.

$$d_B = \frac{-\left(\frac{2P}{3}\right)\left(\frac{L}{4}\right)}{EA} = \frac{-LP}{6EA} \quad \text{leftward} \qquad d_C = d_B + \frac{\left(\frac{2P}{3}\right)\left(\frac{L}{4}\right)}{EA} = \frac{LP}{6EA} \quad \text{to the right} \qquad d_D = \frac{\left(\frac{5P}{3}\right)\left(\frac{L}{4}\right)}{\frac{EA}{2}} = \frac{5LP}{6EA} \quad \text{to the right}$$

(c) Draw an axial-displacement diagram (ADD) in which the abscissa is the distance x from support A to any point on the bar and the ordinate is the horizontal displacement d at that point.

AFD for use below in Part (d)

AFD is composed of 4 constant segments, so ADD is linear with zero displacements at supports A and E.

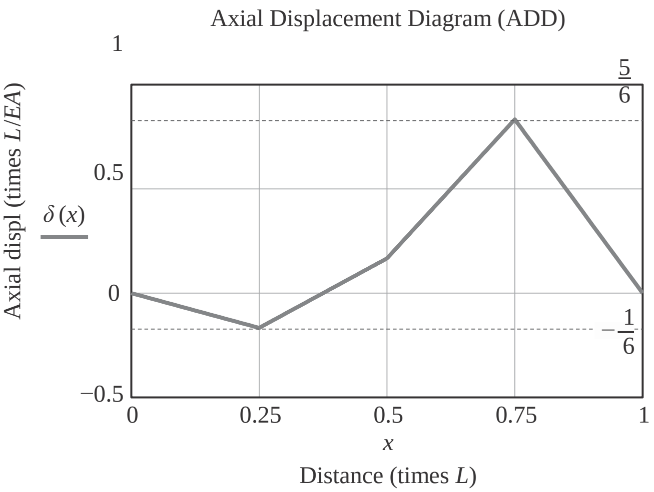


Plot displacements d_B, d_C, and d_D from part (b) above, then connect points using straight lines showing linear variation of axial displacement Between points

$$d_{max} = d_D \qquad d_{max} = \frac{5LP}{6EA} \quad \text{to the right}$$

Boundary conditions at supports:

$$d_A = d_E = 0$$



(d) Maximum permissible value of load variable P based on allowable normal stress in flange bolts

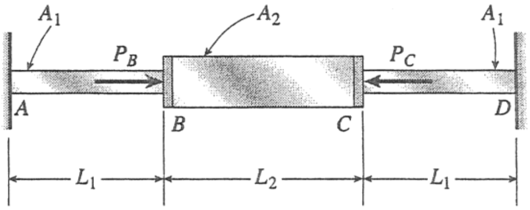
From AFD, force at $L/2$:

$$F_{max} = \frac{4}{3}P \qquad \text{and} \qquad F_{max} = n\sigma_a A_b = 16.493 \text{ k}$$

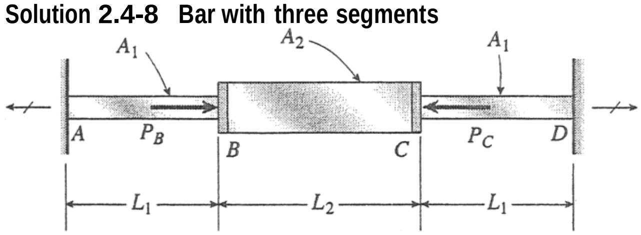
$$P_{max} = \frac{3}{4}F_{max} = 12.37 \text{ k} \qquad \boxed{P_{max} = 12.37 \text{ k}}$$

Problem 2.4-8 The fixed-end bar *ABCD* consists of three prismatic segments, as shown in the figure. The end segments have cross-sectional area A_1 840 mm^2 and length L_1 200 mm. The middle segment has cross-sectional area A_2 1260 mm^2 and length L_2 250 mm. Loads P_B and P_C are equal to 25.5 kN and 17.0 kN, respectively.

(a) Determine the reactions R_A and R_D at the fixed supports.
(b) Determine the compressive axial force F_{BC} in the middle segment of the bar.



Solution 2.4-8 Bar with three segments



P_B 25.5 kN $\quad P_C$ 17.0 kN

L_1 200 mm $\quad L_2$ 250 mm

A_1 840 mm^2 $\quad A_2$ 1260 mm^2

m meter

FREE-BODY DIAGRAM

EQUATION OF EQUILIBRIUM

F_{horiz} 0

$P_B \quad R_D \quad P_C \quad R_A \quad 0$ or

$R_A \quad R_D \quad P_B \quad P_C \quad 8.5$ kN (Eq. 1)

EQUATION OF COMPATIBILITY

d_{AD} elongation of entire bar

$d_{AD} \quad d_{AB} \quad d_{BC} \quad d_{CD} \quad 0$ (Eq. 2)

FORCE-DISPLACEMENT RELATIONS

$$d_{AB} \quad \frac{R_A L_1}{EA_1} \quad \frac{R_A}{E}\left(238.05\,\frac{1}{\mathrm{m}}\right) \qquad \text{(Eq. 3)}$$

$$d_{BC} \quad \frac{(R_A \quad P_B)L_2}{EA_2}$$

$$\frac{R_A}{E}\left(198.413\,\frac{1}{\mathrm{m}}\right) \quad \frac{P_B}{E}\left(198.413\,\frac{1}{\mathrm{m}}\right) \qquad \text{(Eq. 4)}$$

$$d_{CD} \quad \frac{R_D L_1}{EA_1} \quad \frac{R_D}{E}\left(238.095\,\frac{1}{\mathrm{m}}\right) \qquad \text{(Eq. 5)}$$

SOLUTION OF EQUATIONS

Substitute Eqs. (3), (4), and (5) into Eq. (2):

$$\frac{R_A}{E}\left(238.095\,\frac{1}{\mathrm{m}}\right) + \frac{R_A}{E}\left(198.413\,\frac{1}{\mathrm{m}}\right)$$

$$\frac{P_B}{E}\left(198.413\,\frac{1}{\mathrm{m}}\right) + \frac{R_D}{E}\left(238.095\,\frac{1}{\mathrm{m}}\right) \quad 0$$

Simplify and substitute P_B 25.5 kN:

$$R_A\left(436.508\,\frac{1}{\mathrm{m}}\right) + R_D\left(238.095\,\frac{1}{\mathrm{m}}\right)$$

$$5{,}059.53 \text{ kN/m} \qquad \text{(Eq. 6)}$$

(a) REACTIONS R_A AND R_D

Solve simultaneously Eqs. (1) and (6).

From (1): $R_D \quad R_A \quad 8.5$ kN

Substitute into (6) and solve for R_A:

$$R_A\left(674.603\,\frac{1}{\mathrm{m}}\right) \quad 7083.34 \text{ kN/m}$$

R_A 10.5 kN

$R_D \quad R_A \quad 8.5$ kN $\quad 2.0$ kN

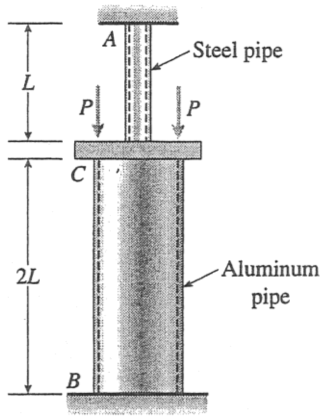
(b) COMPRESSIVE AXIAL FORCE F_{BC}

$F_{BC} \quad P_B \quad R_A \quad P_C \quad R_D \quad 15.0$ kN

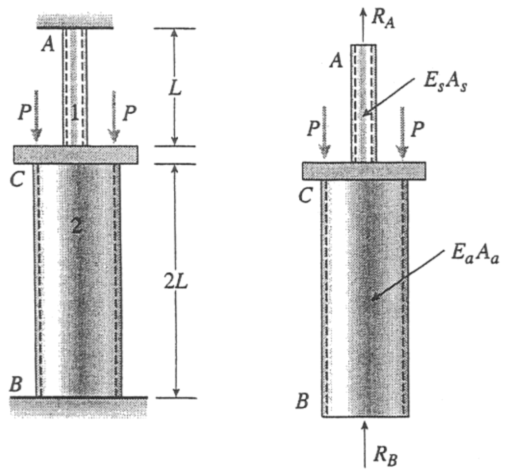
Problem 2.4-9 The aluminum and steel pipes shown in the figure are fastened to rigid supports at ends A and B and to a rigid plate C at their junction. The aluminum pipe is twice as long as the steel pipe. Two equal and symmetrically placed loads P act on the plate at C.

(a) Obtain formulas for the axial stresses s_a and s_s in the aluminum and steel pipes, respectively.

(b) Calculate the stresses for the following data: P 12 k, cross-sectional area of aluminum pipe A_a 8.92 in.^{2}, cross-sectional area of steel pipe A_s 1.03 in.^{2}, modulus of elasticity of aluminum E_a 10 10^6 psi, and modulus of elasticity of steel E_s 29 10^6 psi.



Solution 2.4-9 Pipes with intermediate loads



Pipe 1 is steel.

Pipe 2 is aluminum.

Equation of equilibrium

F_{vert} 0 R_A R_B $2P$ (Eq. 1)

Equation of compatibility

d_{AB} d_{AC} d_{CB} 0 (Eq. 2)

(A positive value of d means elongation.)

Force-displacement relations

$$d_{AC} \quad \frac{R_A L}{E_s A_s} \qquad d_{BC} \quad \frac{R_B(2L)}{E_a A_a} \qquad \text{(Eqs. 3, 4)}$$

Solution of equations

Substitute Eqs. (3) and (4) into Eq. (2):

$$\frac{R_A L}{E_s A_s} \quad \frac{R_B(2L)}{E_a A_a} \quad 0 \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$R_A \quad \frac{4E_s A_s P}{E_a A_a + 2E_s A_s} \qquad R_B \quad \frac{2E_a A_a P}{E_a A_a + 2E_s A_s} \qquad \text{(Eqs. 6, 7)}$$

(a) Axial stresses

$$\text{Aluminum: } s_a \quad \frac{R_B}{A_a} \quad \frac{2E_a P}{E_a A_a + 2E_s A_s}$$

(compression) (Eq. 8)

$$\text{Steel: } s_s \quad \frac{R_A}{A_s} \quad \frac{4E_s P}{E_a A_a + 2E_s A_s} \qquad \text{(Eq. 9)}$$

(tension)

(b) Numerical results

P 12 k A_a 8.92 in.^{2} A_s 1.03 in.^{2}

E_a 10 10^6 psi E_s 29 10^6 psi

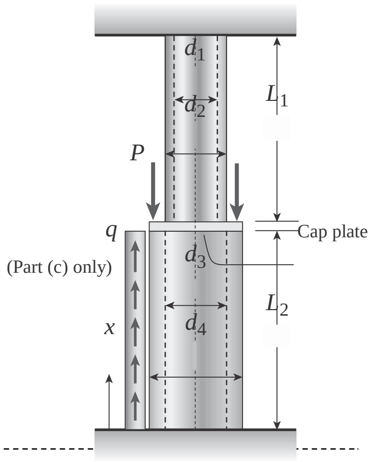
Substitute into Eqs. (8) and (9):

s_a 1,610 psi (compression)

s_s 9,350 psi (tension)

Problem 2.4-10 A hollow circular pipe (see figure) supports a load P which is uniformly distributed around a cap plate at the top of the lower pipe. The inner and outer diameters of the upper and lower parts of the pipe are d_1 50 mm, d_2 60 mm, d_3 57 mm, and d_4 64 mm, respectively. Pipe lengths are L_1 2m and L_2 3 m. Neglect the self-weight of the pipes. Assume that cap plate thickness is small compared to L_1 and L_2. Let E 110 MPa.

(a) If the tensile stress in the upper part is s_1 10.5 MPa, what is load P? Also, what are reactions R_1 at the upper support and R_2 at the lower support. What is the stress s_2 MPa in the lower part?
(b) Find displacement (mm) at the cap plate. Plot the Axial Force Diagram, *AFD* [$N(x)$] and Axial Displacement Diagram, *ADD* [(x)].
(c) Add the uniformly distributed load q along the centroidal axis of pipe segment 2. Find q(kN/m) so that R_2 0. Assume that load P from part (a) is also applied.



Solution 2.4-10

(a) Stresses and reactions: Select R_1 as redundant and do superposition analysis (here q 0; deflection position upward)

d_1 50 mm d_2 60 mm d_3 57 mm d_4 64 mm $A_1 \quad \frac{p}{4}1d_2{}^2 \quad d_1{}^2 2 \quad 863.938\ \text{mm}^2$

E 110 MPa

$A_2 \quad \frac{p}{4}1d_4{}^2 \quad d_3{}^2 2 \quad 665.232\ \text{mm}^2$

Segment flexibilities L_1 2 m L_2 3 m

$f_1 \quad \frac{L_1}{EA_1} \quad 0.02105\ \text{mm/N} \qquad f_2 \quad \frac{L_2}{EA_2} \quad 0.041\ \text{mm/N} \qquad \frac{f_1}{f_2} \quad 0.513$

Tensile stress (s_1) is known in upper segment so $R_1 \quad s_1 * A_1 \qquad s_1 \quad 10.5\ \text{MPa} \qquad R_1 \quad s_1 A_1 \quad 9.07\ \text{kN}$

$d_{1a} \quad P f_2 \qquad d_{1b} \quad R_1 1 f_1 + f_2 2$ Compatibility: $d_{1a} + d_{1b} \quad 0$

Solve for P: $P \quad R_1\, a \frac{f_1 + f_2}{f_2} b \quad 13.73\ \text{kN}$

Finally, use statics to find R_2: $R_2 \quad P \quad R_1 \quad 4.66\ \text{kN} \qquad s_2 \quad \frac{R_2}{A_2} \quad 7\ \text{MPa}$ **compressive** since R_2 is positive (upward)

$\boxed{P \quad 13.73\ \text{kN}}$ $\boxed{R_1 \quad 9.07\ \text{kN}}$ $\boxed{R_2 \quad 4.66\ \text{kN}}$ $\boxed{s_2 \quad 7\ \text{MPa}}$

(b) Displacement at cap plate

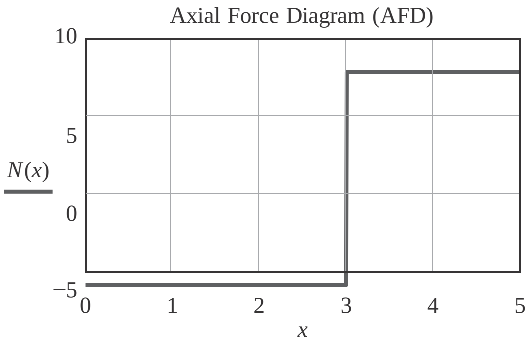
$d_c \quad R_1 f_1 \quad 190.909\ \text{mm}$ 6 downward OR $d_c \quad 1R_2 2 f_2 \quad 190.909\ \text{mm}$ downward (neg. x-direction)

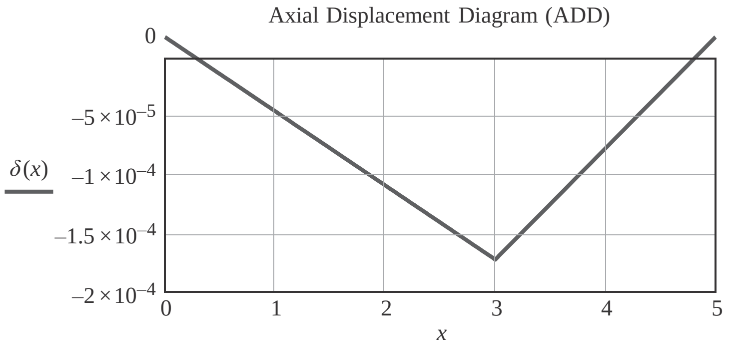
$d_{cap} \quad d_c \quad 0.191\ \text{m} \qquad \boxed{d_{cap} \quad 190.9\ \text{mm}}$

AFD and ADD: R_1 9.071 R_2 4.657 L_1 2 A_1 863.938 A_2 665.232 E 110

L_2 3

NOTE: x is measured up from lower support.





(c) UNIFORM LOAD q ON SEGMENT 2 SUCH THAT R_2 0

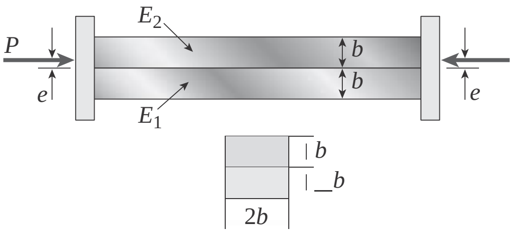
P 13.728 kN R_1 $s_1 A_1$ 9.071 kN L_2 3 m

Equilibrium: $R_1 + R_2$ P qL_2 6 set R_2 0, solve for req'd q q $\dfrac{P \quad R_1}{L_2}$ 1.552 kN/m

$\boxed{q \quad 1.552 \text{ kN/m}}$

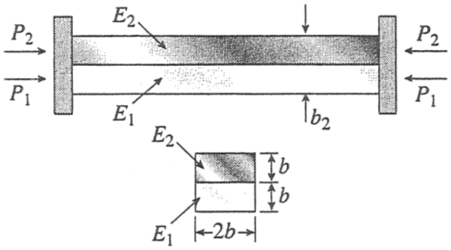
Problem 2.4-11 A *bimetallic* bar (or composite bar) of square cross section with dimensions 2*b*   2*b* is constructed of two different metals

bar have the same cross-sectional dimensions. The bar is compressed by forces *P* acting through rigid end plates. The line of action of the loads has an eccentricity *e* of such magnitude that each part of the bar is stressed uniformly in compression.



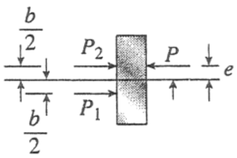
(a) Determine the axial forces P_1 and P_2 in the two parts of the bar.
(b) Determine the eccentricity *e* of the loads.
(c) Determine the ratio s_1/s_2 of the stresses in the two parts of the bar.

Solution 2.4-11 Bimetallic bar in compression



FREE-BODY DIAGRAM

(Plate at right-hand end)



EQUATIONS OF EQUILIBRIUM

$F \quad 0 \quad P_1 \quad P_2 \quad P$ (Eq. 1)

$M \quad 0 \qquad Pe + P_1 a \frac{b}{2} b \quad P_2 a \frac{b}{2} b \quad 0$ (Eq. 2)

EQUATION OF COMPATIBILITY

$d_2 \quad d_1$

$\frac{P_2 L}{E_2 A} \quad \frac{P_1 L}{E_1 A}$ or $\frac{P_2}{E_2} \quad \frac{P_1}{E_1}$ (Eq. 3)

(a) AXIAL FORCES

Solve simultaneously Eqs. (1) and (3):

$$P_1 \quad \frac{PE_1}{E_1 + E_2} \qquad P_2 \quad \frac{PE_2}{E_1 + E_2}$$

(b ECCENTRICITY OF LOAD P

Substitute P_1 and P_2 into Eq. (2) and solve for e:

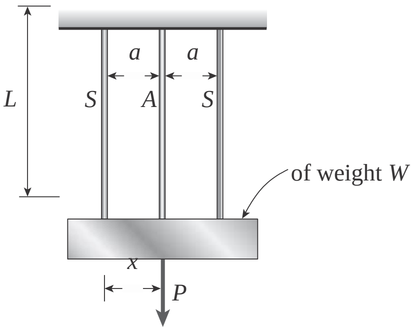
$$e \quad \frac{b(E_2 \quad E_1)}{2(E_2 \quad E_1)}$$

(c) RATIO OF STRESSES

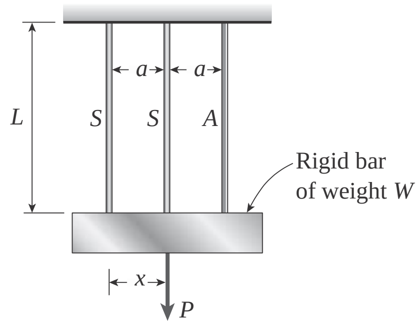
$$s_1 \quad \frac{P_1}{A} \qquad s_2 \quad \frac{P_2}{A} \qquad \frac{s_1}{s_2} \quad \frac{P_1}{P_2} \quad \frac{E_1}{E_2}$$

Problem 2.4-12 A rigid bar of weight W 800 N hangs from three equally spaced vertical wires (length L 150 mm, spacing a 50 mm): two of steel and one of aluminum. The wires also support a load P acting on the bar. The diameter of the steel wires is d_s 2 mm, and the diameter of the aluminum wire is d_a 4 mm. Assume E_s 210 GPa and E_a 70 GPa.

(a) What load P_{allow} can be supported *at the midpoint of the bar* (x a) if the allowable stress in the steel wires is 220 MPa and in the aluminum wire is 80 MPa? (See figure part a.)
(b) What is P_{allow} if the load is positioned at x $a/2$? (See figure part a.)
(c) Repeat (b) above if the second and third wires are *switched* as shown in figure part b.



(a)



(b)

Solution 2.4-12

Numerical data:

W 800 N $\qquad L$ 150 mm

a 50 mm $\qquad d_S$ 2 mm

d_A 4 mm $\qquad E_S$ 210 GPa

E_A 70 GPa

s_{Sa} 220 MPa $\qquad s_{Aa}$ 80 MPa

$A_A \ \frac{p}{4} d_A{}^2 \qquad A_S \ \frac{p}{4} d_S{}^2$

A_A 13 mm^2 $\qquad A_S$ 3 mm^2

(a) P_{allow} AT CENTER OF BAR

One-degree statically indeterminate - use reaction (R_A) at top of aluminum bar as the redundant

compatibility: $d_1 \ \ d_2 \ \ 0$ $\qquad$ Statics: $2R_S \ \ R_A \ \ P \ \ W$

$d_1 \ \frac{P+W}{2} \ \text{a} \frac{L}{E_S A_S} \text{b}$ $\qquad$ downward displacement due to elongation of each steel wire under P W if aluminum wire is cut at top

$d_2 \quad R_A \left(\dfrac{L}{2E_S A_S} + \dfrac{L}{E_A A_A} \right)$ upward displ. due to shortening of steel wires and elongation of aluminum wire under redundant R_A

Enforce compatibility and then solve for R_A:

$$d_1 \quad d_2 \quad \text{so} \quad R_A \quad \frac{\dfrac{P+W}{2} \left(\dfrac{L}{E_S A_S} \right)}{\dfrac{L}{2E_S A_S} + \dfrac{L}{E_A A_A}} \qquad R_A \quad (P + W) \frac{E_A A_A}{E_A A_A + 2E_S A_S} \quad \text{and} \quad s_{Aa} \quad \frac{R_A}{A_A}$$

Now use statics to find R_S:

$$R_S \quad \frac{P + W \quad R_A}{2} \qquad R_S \quad \frac{P + W \quad (P + W) \dfrac{E_A A_A}{E_A A_A + 2E_S A_S}}{2} \qquad R_S \quad (P + W) \frac{E_S A_S}{E_A A_A + 2E_S A_S}$$

$$\text{and} \quad s_{Sa} \quad \frac{R_S}{A_S}$$

Compute stresses and apply allowable stress values:

$$s_{Aa} \quad (P + W) \frac{E_A}{E_A A_A + 2E_S A_S} \qquad s_{Sa} \quad (P + W) \frac{E_S}{E_A A_A + 2E_S A_S}$$

Solve for allowable load P:

$$P_{Aa} \quad s_{Aa} \left(\frac{E_A A_A + 2E_S A_S}{E_A} \right) \quad W \qquad P_{Sa} \quad s_{Sa} \left(\frac{E_A A_A + 2E_S A_S}{E_S} \right) \quad W \quad \text{(lower value of } P \text{ controls)}$$

P_{Aa} 1713 N $\qquad$ P_{Sa} 1504 N $\quad$; P_{allow} is controlled by steel wires

(b) P_{allow} IF LOAD P AT $x \quad a/2$

Again, cut aluminum wire at top, then compute elongations of left and right steel wires:

$$d_{1L} \quad \left(\frac{3P}{4} + \frac{W}{2} \right) \left(\frac{L}{E_S A_S} \right) \qquad d_{1R} \quad \left(\frac{P}{4} + \frac{W}{2} \right) \left(\frac{L}{E_S A_S} \right)$$

$$d_1 \quad \frac{d_{1L} + d_{1R}}{2} \qquad d_1 \quad \frac{P + W}{2} \left(\frac{L}{E_S A_S} \right) \text{ where } d_1 \quad \text{displacement at } x \quad a$$

Use d_2 from part (a):

$$d_2 \quad R_A \left(\frac{L}{2E_S A_S} + \frac{L}{E_A A_A} \right)$$

So equating d_1 and d_2, solve for R_A: $\quad R_A \quad (P + W) \dfrac{E_A A_A}{E_A A_A + 2E_S A_S}$

^ same as in part (a)

$R_{SL} \quad \dfrac{3P}{4} + \dfrac{W}{2} \quad \dfrac{R_A}{2}$ $\qquad$ stress in left steel wire exceeds that in right steel wire

$$\frac{E_A A_A}{\quad}$$

$$R_{SL} \quad \frac{3P}{4} + \frac{W}{2} \quad \frac{(P + W) \quad E_A A_A + 2E_S A_S}{2}$$

$$R_{SL} \quad \frac{PE_AA_A + 6PE_SA_S + 4WE_SA_S}{4E_AA_A + 8E_SA_S} \qquad \sigma_{Sa} \quad \frac{PE_AA_A + 6PE_SA_S + 4WE_SA_S}{4E_AA_A + 8E_SA_S}\left(\frac{1}{A_S}\right)$$

Solve for P_{allow} based on allowable stresses in steel and aluminum:

$$P_{Sa} \quad \frac{\sigma_{Sa}(4A_SE_AA_A + 8E_SA_S^2) \quad (4WE_SA_S)}{E_AA_A + 6E_SA_S} \qquad P_{Aa} \quad 1713\text{ N} \qquad \text{same as in part(a)}$$

P_{Sa} 820 N steel controls

(c) P_{allow} IF WIRES ARE SWITCHED AS SHOWN AND x $a/2$

Select R_A as the redundant; statics on the two released structures:

(1) Cut aluminum wire—apply P and W, compute forces in left and right steel wires, then compute displacements at each steel wire:

$$R_{SL} \quad \frac{P}{2} \qquad R_{SR} \quad \frac{P}{2} + W$$

$$d_{1L} \quad \frac{P}{2}\left(\frac{L}{E_SA_S}\right) \qquad d_{1R} \quad \left(\frac{P}{2} + W\right)\left(\frac{L}{E_SA_S}\right)$$

By geometry, d at aluminum wire location at far right is $d_1 \quad \left(\frac{P}{2} + 2W\right)\left(\frac{L}{E_SA_S}\right)$

(2) Next apply redundant R_A at right wire, compute wire force and displacement at aluminum wire:

$$R_{SL} \quad R_A \qquad R_{SR} \quad 2R_A \qquad d_2 \quad R_A\left(\frac{5L}{E_SA_S} + \frac{L}{E_AA_A}\right)$$

(3) Compatibility equate d_1, d_2 and solve for R_A, then P_{allow} for aluminum wire:

$$R_A \quad \frac{\left(\frac{P}{2} + 2W\right)\left(\frac{L}{E_SA_S}\right)}{\frac{5L}{E_SA_S} + \frac{L}{E_AA_A}} \qquad R_A \quad \frac{E_AA_AP + 4E_AA_AW}{10E_AA_A + 2E_SA_S} \qquad \sigma_{Aa} \quad \frac{R_A}{A_A}$$

$$\sigma_{Aa} \quad \frac{E_AP + 4E_AW}{10E_AA_A + 2E_SA_S}$$

$$P_{Aa} \quad \frac{\sigma_{Aa}(10E_AA_A + 2E_SA_S) \quad 4E_AW}{E_A} \qquad P_{Aa} \quad 1713\text{ N}$$

(4) Statics or superposition—find forces in steel wires, then P_{allow} for steel wires:

$$R_{SL} \quad \frac{P}{2} + R_A \qquad R_{SL} \quad \frac{P}{2} + \frac{E_AA_AP + 4E_AA_AW}{10E_AA_A + 2E_SA_S}$$

$$R_{SL} \quad \frac{6E_AA_AP + PE_SA_S + 4E_AA_AW}{10E_AA_A + 2E_SA_S}$$

larger than R_{SR}, so use in allowable stress calculations

$$R_{SR} \quad \frac{P}{2} + W \quad 2R_A \qquad R_{SR} \quad \frac{P}{2} + W \quad \frac{E_A A_A P + 4E_A A_A W}{5E_A A_A + E_S A_S}$$

$$R_{SR} \quad \frac{3E_A A_A P + PE_S A_S + 2E_A A_A W + 2WE_S A_S}{10E_A A_A + 2E_S A_S}$$

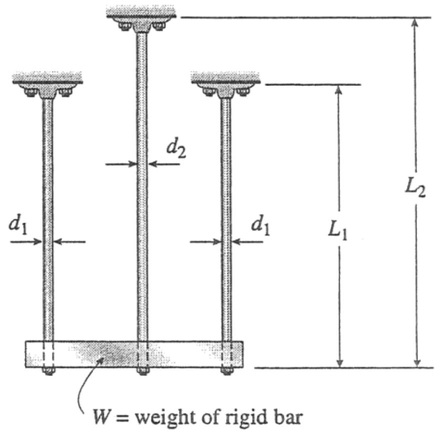
$$s_{Sa} \quad \frac{R_{SL}}{A_S} \qquad P_{Sa} \quad s_{Sa} A_S \left(\frac{10E_A A_A + 2E_S A_S}{6E_A A_A + E_S A_S} \right) \quad \frac{4E_A A_A W}{6E_A A_A + E_S A_S}$$

$$P_{Sa} \quad \frac{10s_{Sa} A_S E_A A_A + 2s_{Sa} A_S^2 E_S \quad 4E_A A_A W}{6E_A A_A + E_S A_S} \qquad P_{Sa} \quad 703 \text{ N}$$

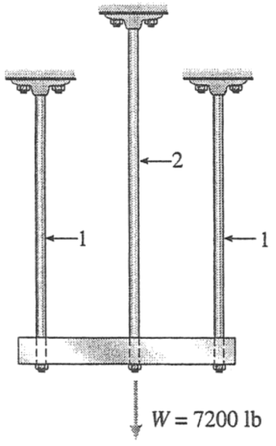
^ steel controls

Problem 2.4-13 A horizontal rigid bar of weight W 7200 lb is supported by three slender circular rods that are equally spaced (see figure). The two outer rods are made of aluminum (E_1 10 10^6 psi) with diameter d_1 0.4 in. and length L_1 40 in. The inner rod is magnesium (E_2 6.5 10^6 psi) with diameter d_2 and length L_2. The allowable stresses in the aluminum and magnesium are 24,000 psi and 13,000 psi, respectively.

If it is desired to have all three rods loaded to their maximum allowable values, what should be the diameter d_2 and length L_2 of the middle rod?



Solution 2.4-13 **Bar supported by three rods**



BAR 1 ALUMINUM

E_1 10 10^6 psi

d_1 0.4 in.

L_1 40 in.

s_1 24,000 psi

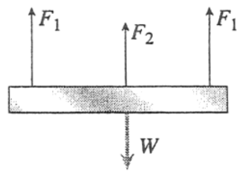
BAR 2 MAGNESIUM

E_2 6.5 10^6 psi

d_2 ? L_2 ?

s_2 13,000 psi

FREE-BODY DIAGRAM OF RIGID BAR
EQUATION OF EQUILIBRIUM



F_{vert} 0

$2F_1 \quad F_2 \quad W \quad 0$ (Eq. 1)

FULLY STRESSED RODS

$F_1 \quad s_1 A_1 \qquad F_2 \quad s_2 A_2$

$$A_1 \quad \frac{pd_1^2}{4} \qquad A_2 \quad \frac{pd_2^2}{4}$$

Substitute into Eq. (1):

$$2s_1 \left(\frac{pd_1^2}{4} \right) + s_2 \left(\frac{pd_2^2}{4} \right) \quad W$$

Diameter d_1 is known; solve for d_2:

$$d_2^2 = \frac{4W}{\pi\sigma_2} - \frac{2\sigma_1 d_1^2}{\sigma_2} \quad \text{(Eq. 2)}$$

SUBSTITUTE NUMERICAL VALUES:

$$d_2^2 = \frac{4(7200 \text{ lb})}{\pi(13{,}000 \text{ psi})} - \frac{2(24{,}000 \text{ psi})(0.4 \text{ in.})^2}{13{,}000 \text{ psi}}$$

$$= 0.70518 \text{ in.}^2 - 0.59077 \text{ in.}^2 = 0.11441 \text{ in.}^2$$

$$d_2 = 0.338 \text{ in.}$$

EQUATION OF COMPATIBILITY

$$\delta_1 = \delta_2 \quad \text{(Eq. 3)}$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_1 = \frac{F_1 L_1}{E_1 A_1} = \sigma_1\left(\frac{L_1}{E_1}\right) \quad \text{(Eq. 4)}$$

$$\delta_2 = \frac{F_2 L_2}{E_2 A_2} = \sigma_2\left(\frac{L_2}{E_2}\right) \quad \text{(Eq. 5)}$$

Substitute (4) and (5) into Eq. (3):

$$\sigma_1\left(\frac{L_1}{E_1}\right) = \sigma_2\left(\frac{L_2}{E_2}\right)$$

Length L_1 is known; solve for L_2:

$$L_2 = L_1\left(\frac{\sigma_1 E_2}{\sigma_2 E_1}\right) \quad \text{(Eq. 6)}$$

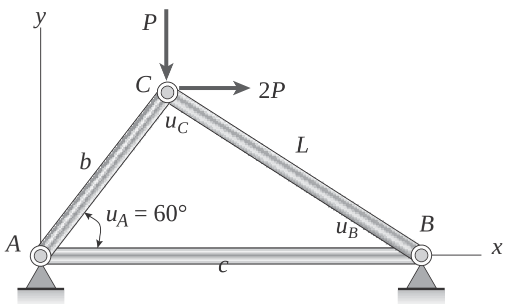
SUBSTITUTE NUMERICAL VALUES:

$$L_2 = (40 \text{ in.})\left(\frac{24{,}000 \text{ psi}}{13{,}000 \text{ psi}}\right)\left(\frac{6.5 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}}\right)$$

$$= 48.0 \text{ in.}$$

Problem 2.4-14 Three-bar truss ABC (see figure) is constructed of steel pipes having a cross-sectional area $A = 3500 \text{ mm}^2$ and a modulus of elasticity $E = 210$ GPa. Member BC is of length $L = 2.5$ m, and the angle between members AC and AB is known to be 60°. Member AC length is $b = 0.71L$. Loads $P = 185$ kN and $2P = 370$ kN act vertically and horizontally at joint C, as shown. Joints A and B are pinned supports. (Use the law of sines and law of cosines to find missing dimensions and angles in the figure.)

(a) Find the support reactions at joints A and B. Use horizontal reaction B_x as the redundant.
(b) What is the maximum permissible value of load variable P if the allowable normal stress in each truss member is 150 MPa?



Solution 2.4-14

NUMERICAL DATA

$L = 2.5$ m $\quad b = 0.71L = 1.775$ m $\quad E = 210$ GPa $\quad A = 3500 \text{ mm}^2 \quad P = 185$ kN $\quad \theta_A = 60°$

$\sigma_a = 150$ MPa

FIND MISSING DIMENSIONS AND ANGLES IN PLANE TRUSS FIGURE

$x_c = b\cos(\theta_A) = 0.8875 \text{ m} \quad y_c = b\sin(\theta_A) = 1.5372 \text{ m}$

$$\frac{b}{\sin(\theta_B)} = \frac{L}{\sin(\theta_A)} \quad \text{so} \quad \theta_B = \operatorname{asin}\left(\frac{b\sin(\theta_A)}{L}\right) = 37.94306°$$

$$u_C \quad 180 \quad (u_A + u_B) \quad 82.05694$$

$$c \quad \frac{L}{\sin(u_A)}\sin(u_C) \quad 2.85906\text{ m} \qquad \text{or} \qquad c \quad 3b^2 + L^2 \quad 2\,b\,L\cos(u_C) \quad 2.85906\text{ m}$$

(a) SELECT B_x AS THE REDUNDANT; PERFORM SUPERPOSITION ANALYSIS TO FIND B_x THEN USE STATICS TO FIND REMAINING REACTIONS. FINALLY USE METHOD OF JOINTS TO FIND MEMBER FORCES (SEE EXAMPLE 1-1)

d_{Bx1} displacement in x-direction in released structure acted upon by loads P and $2P$ at joint C:

d_{Bx1} 1.2789911 mm this displacement equals force in AB divided by flexibility of AB

d_{Bx2} displacement in x-direction in released structure acted upon by redundant B_x: $d_{BX2} \quad B_x \dfrac{c}{EA}$

COMPATIBILITY EQUATION: $d_{BX1} + d_{BX2} \quad 0$ so $B_X \quad \dfrac{EA}{c} d_{BX1} \quad 328.8\text{ kN}$

STATICS: $\Sigma F_X \quad 0 \quad A_X \quad B_X \quad 2P \quad 41.2\text{ kN}$

$\Sigma M_A \quad 0 \quad B_y \quad \dfrac{1}{c}[2P(b\sin(u_A)) + P(b\cos(u_A))] \quad 256.361\text{ kN}$

$\Sigma F_y \quad 0 \quad A_y \quad P \quad B_y \quad 71.361\text{ kN}$

REACTIONS:

$\boxed{A_x \quad 41.2\text{ kN}}$ $\boxed{A_y \quad 71.4\text{ kN}}$ $\boxed{B_x \quad 329\text{ kN}}$ $\boxed{B_y \quad 256\text{ kN}}$

(b) FIND MAXIMUM PERMISSIBLE VALUE OF LOAD VARIABLE P IF ALLOWABLE NORMAL STRESS IS 150 MPA

(1) Use reactions and Method of Joints to find member forces in each member for above loading.

Results: $F_{AB} \quad 0 \quad F_{BC} \quad 416.929\text{ kN} \quad F_{AC} = 82.40\text{ kN}$

(2) Compute member stresses:

$s_{AB} \quad 0 \quad s_{BC} \quad \dfrac{416.93\text{ kN}}{A} \quad 119.123\text{ MPa} \quad s_{AC} \quad \dfrac{82.4\text{ kN}}{A} \quad 23.543\text{ MPa}$

(3) Maximum stress occurs in member BC. For linear analysis, the stress is proportional to the load so

$\boxed{P_{max} \quad \dfrac{s_a}{s_{BC}} P \quad 233\text{ kN}}$

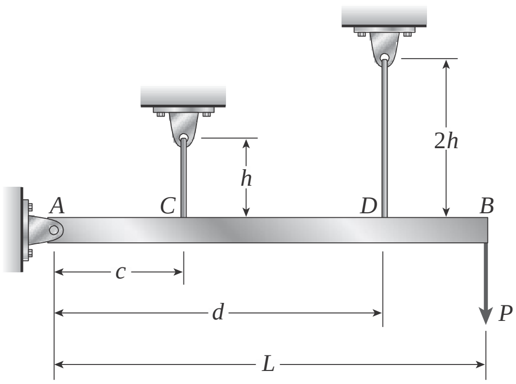
So when downward load P 233 kN is applied at C and horizontal load $2P$ 466 kN is applied to the right at C, the stress in BC is 150 MPa

---

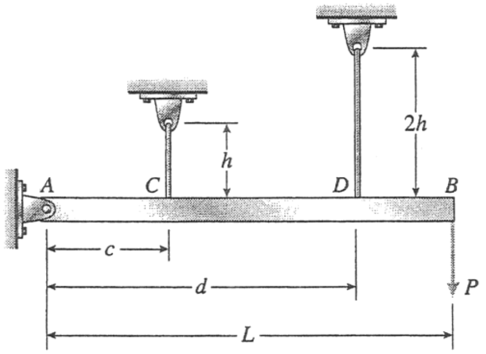
Problem 2.4-15 A rigid bar AB of length L 66 in. is hinged to a support at A and supported by two vertical wires attached at points C and D (see figure). Both wires have the same cross-sectional area (A 0.0272 in.^{2}) and are made of the same material (modulus E 30 10^6 psi). The wire at C has length h 18 in. and the wire at D has length twice that amount. The horizontal distances are c 20 in. and d 50 in.

(a) Determine the tensile stresses s_C and s_D in the wires due to the load P 340 lb acting at end B of the bar.

(b) Find the downward displacement d_B at end B of the bar.



Solution 2.4-15 Bar supported by two wires



$h \quad 18$ in.

$2h \quad 36$ in.

$c \quad 20$ in.

$d \quad 50$ in.

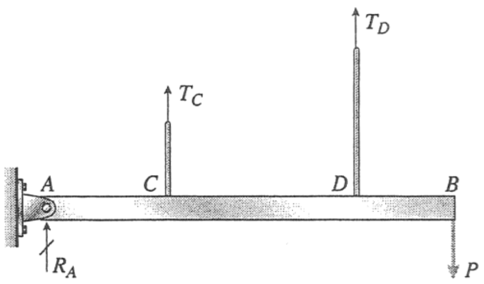
$L \quad 66$ in.

$E \quad 30 \quad 10^6$ psi

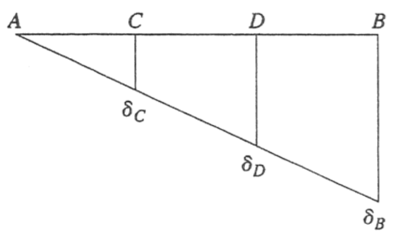
$A \quad 0.0272$ in.^{2}

$P \quad 340$ lb

FREE-BODY DIAGRAM



DISPLACEMENT DIAGRAM



EQUATION OF COMPATIBILITY

$$\frac{\delta_c}{c} \quad \frac{\delta_D}{d} \qquad \text{(Eq. 2)}$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_C \quad \frac{T_C h}{EA} \qquad \delta_D \quad \frac{T_D(2h)}{EA} \qquad \text{(Eqs. 3, 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{T_C h}{cEA} \quad \frac{T_D(2h)}{dEA} \quad \text{or} \quad \frac{T_C}{c} \quad \frac{2T_D}{d} \qquad \text{(Eq. 5)}$$

TENSILE FORCES IN THE WIRES

Solve simultaneously Eqs. (1) and (5):

$$T_C \quad \frac{2cPL}{2c^2 + d^2} \qquad T_D \quad \frac{dPL}{2c^2 + d^2} \qquad \text{(Eqs. 6, 7)}$$

TENSILE STRESSES IN THE WIRES

$$\sigma_C \quad \frac{T_C}{A} \quad \frac{2cPL}{A(2c^2 + d^2)} \qquad \text{(Eq. 8)}$$

$$\sigma_D \quad \frac{T_D}{A} \quad \frac{dPL}{A(2c^2 + d^2)} \qquad \text{(Eq. 9)}$$

DISPLACEMENT AT END OF BAR

$$\delta_B \quad \delta_D\left(\frac{L}{d}\right) \quad \frac{2hT_D}{EA}\left(\frac{L}{d}\right) \quad \frac{2hPL^2}{EA(2c^2 + d^2)} \qquad \text{(Eq. 10)}$$

SUBSTITUTE NUMERICAL VALUES

$$2c^2 \quad d^2 \quad 2(20\text{ in.})^2 \quad (50\text{ in.})^2 \quad 3300\text{ in.}^2$$

$$\text{(a)} \quad \sigma_C \quad \frac{2cPL}{A(2c^2 + d^2)} \quad \frac{2(20\text{ in.})(340\text{ lb})(66\text{ in.})}{(0.0272\text{ in.}^2)(3300\text{ in.}^2)}$$

10,000 psi

$$\sigma_D \quad \frac{dPL}{A(2c^2 + d^2)} \quad \frac{(50\text{ in.})(340\text{ lb})(66\text{ in.})}{(0.0272\text{ in.}^2)(3300\text{ in.}^2)}$$

12,500 psi

$2hPL^2$

EQUATION OF EQUILIBRIUM

(b) d_B

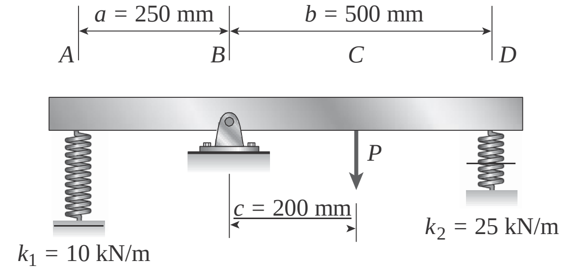
$$EA(2c^2 + d^2)$$

$$\frac{2(18\text{ in.})(3400\text{ lb})(66\text{ in.})^2}{(30 * 10^6\text{ psi})(0.0272\text{ in.}^2)(3300\text{ in.}^2)} = 0.0198\text{ in.}$$

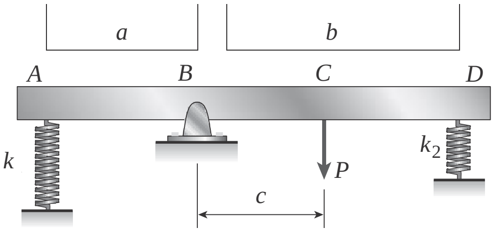
$M_A \quad 0 \quad T_C(c) \quad T_D(d) \quad PL$ (Eq. 1)

Problem 2.4-16 A rigid bar *ABCD* is pinned at point *B* and supported by springs at *A* and *D* (see figure). The springs at *A* and *D* have stiffnesses k_1 10 kN/m and k_2 25 kN/m, respectively, and the dimensions *a*, *b*, and *c* are 250 mm, 500 mm, and 200 mm, respectively. A load *P* acts at point *C*.

If the angle of rotation of the bar due to the action of the load *P* is limited to 3°, what is the maximum permissible load P_{max}?



Solution 2.4-16 Rigid bar supported by springs



NUMERICAL DATA

a 250 mm

b 500 mm

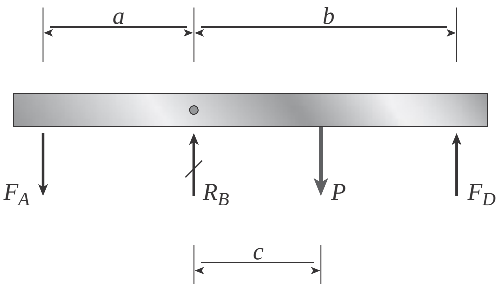
c 200 mm

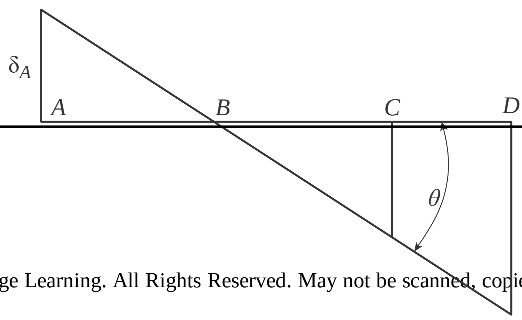
k_1 10 kN/m

k_2 25 kN/m

u_{max} 3 $\frac{p}{60}$ rad

FREE-BODY DIAGRAM AND DISPLACEMENT DIAGRAM





EQUATION OF EQUILIBRIUM

M_B 0 $F_A(a)$ $P(c)$ $F_D(b)$ 0 (Eq. 1)

EQUATION OF COMPATIBILITY

$\frac{d_A}{a}$ $\frac{d_D}{b}$ (Eq. 2)

FORCE-DISPLACEMENT RELATIONS

d_A $\frac{F_A}{k_1}$ d_D $\frac{F_D}{k_2}$ (Eqs. 3, 4)

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$\frac{F_A}{ak_1}$ $\frac{F_D}{bk_2}$ (Eq. 5)

SOLVE SIMULTANEOUSLY EQS. (1) AND (5):

F_A $\frac{ack_1P}{a^2k_1 + b^2k_2}$ F_D $\frac{bck_2P}{a^2k_1 + b^2k_2}$

ANGLE OF ROTATION

d_D $\frac{F_D}{k_2}$ $\frac{bcP}{a^2k_1 + b^2k_2}$ u $\frac{d_D}{b}$ $\frac{cP}{a^2k_1 + b^2k_2}$

MAXIMUM LOAD

P $\frac{u}{}(a^2k_1 + b^2k_2)$

P_{max} $\frac{u_{max}}{c}(a^2k_1 + b^2k_2)$

SUBSTITUTE NUMERICAL VALUES:

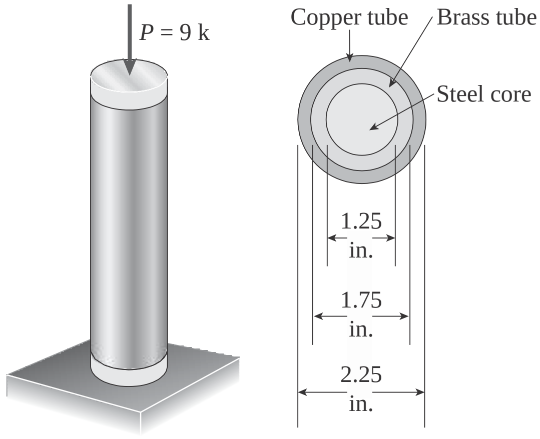
P_{max} $\frac{p/60 \text{ rad}}{200 \text{ mm}}$[(250 mm)^2(10 kN/m) + (500 mm)^2(25 kN/m)]

1800 N

δ_C

Problem 2.4-17 A trimetallic bar is uniformly compressed by an axial force P 9 kips applied through a rigid end plate (see figure). The bar consists of a circular steel core surrounded by brass and copper tubes. The steel core has diameter 1.25 in., the brass tube has outer diameter 1.75 in., and the copper tube has outer diameter 2.25 in. The corresponding moduli of elasticity are E_s 30,000 ksi, E_b 16,000 ksi, and E_c 18,000 ksi.

Calculate the compressive stresses s_s, s_b, and s_c in the steel, brass, and copper, respectively, due to the force P.



Solution 2.4-17

Numerical properties (kips, inches):

d_c 2.25 in. d_b 1.75 in. d_s 1.25 in.

$A_s \quad \frac{p}{4} d_s^2$

E_c 18,000 ksi E_b 16,000 ksi

$A_b \quad \frac{p}{4} 1 d_b^2 \quad d_s^2 2$

E_s 30000 ksi

$A_c \quad \frac{p}{4} 1 d_c^2 \quad d_b^2 2$

P 9 k

EQUATION OF EQUILIBRIUM

$F_{\text{vert}} \quad 0 \quad P_s \quad P_b \quad P_c \quad P$ (Eq. 1)

EQUATIONS OF COMPATIBILITY

$d_s \quad d_b \quad d_c \quad d_s$ (Eqs. 2)

FORCE-DISPLACEMENT RELATIONS

$d_s \quad \frac{P_s L}{E_s A_s} \quad d_b \quad \frac{P_b L}{E_b A_b} \quad d_c \quad \frac{P_c L}{E_c A_c}$ (Eqs. 3, 4, 5)

SOLUTION OF EQUATIONS

Substitute (3), (4), and (5) into Eqs. (2):

$P_b \quad P_s \frac{E_b A_b}{E_s A_s} \quad P_c \quad P_s \frac{E_c A_c}{E_s A_s}$ (Eqs. 6, 7)

SOLVE SIMULTANEOUSLY EQS. (1), (6), AND (7):

$$P_s \quad P\frac{E_s A_s}{E_s A_s + E_b A_b + E_c A_c} \quad 3.95 \text{ k}$$

$$P_b \quad P\frac{E_b A_b}{E_s A_s + E_b A_b + E_c A_c} \quad 2.02 \text{ k}$$

$$P_c \quad P\frac{E_c A_c}{E_s A_s + E_b A_b + E_c A_c} \quad 3.03 \text{ k}$$

$P_s \quad P_b \quad P_c \quad 9$ statics check

COMPRESSIVE STRESSES

Let $EA \quad E_s A_s \quad E_b A_b \quad E_c A_c$

$$s_s \quad \frac{P_s}{A_s} \quad \frac{PE_s}{\copyright EA} \qquad s_s \quad 3.22 \text{ ksi}$$

$$s_b \quad \frac{P_b}{A_b} \quad \frac{PE_b}{\copyright EA} \qquad s_b \quad 1.716 \text{ ksi}$$

$$s_c \quad \frac{P_c}{A_c} \quad \frac{PE_c}{\copyright EA} \qquad s_c \quad 1.93 \text{ ksi}$$

Thermal Effects

Problem 2.5-1 The rails of a railroad track are welded together at their ends (to form continuous rails and thus eliminate the clacking sound of the wheels) when the temperature is 60°F.

What compressive stress s is produced in the rails when they are heated by the sun to 120°F if the coefficient of thermal expansion a 6.5 10 ^{6}/°F and the modulus of elasticity E 30 10^6 psi?

Solution 2.5-1 Expansion of railroad rails

The rails are prevented from expanding because of their great length and lack of expansion joints.

Therefore, each rail is in the same condition as a bar with fixed ends (see Example 2-7).

The compressive stress in the rails may be calculated as follows.

T 120°F 60°F 60°F

s Ea(T)

(30 10^6 psi)(6.5 10 ^{6}/°F)(60°F)

s 11,700 psi

Problem 2.5-2 An aluminum pipe has a length of 60 m at a temperature of 10°C. An adjacent steel pipe at the same temperature is 5 mm longer than the aluminum pipe.

At what temperature (degrees Celsius) will the aluminum pipe be 15 mm longer than the steel pipe? (Assume that the coefficients of thermal expansion of aluminum and steel are a_a 23 10 ^{6}/°C and a_s 12 10 ^{6}/°C, respectively.)

Solution 2.5-2 Aluminum and steel pipes

INITIAL CONDITIONS

L_a 60 m T_0 10°C

L_s 60.005 m T_0 10°C

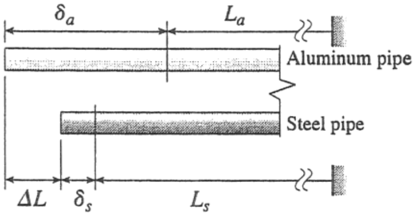
a_a 23 10 ^{6}/°C a_s 12 10 ^{6}/°C

FINAL CONDITIONS

Aluminum pipe is longer than the steel pipe by the amount L 15 mm.

T increase in temperature

d_a a_a(T)L_a d_s a_s(T)L_s



From the figure above:

d_a L_a L d_s L_s

or, a_a(T)L_a L_a L a_s(T)L_s L_s

Solve for T:

$$¢T \quad \frac{¢L + (L_s \quad L_a)}{a_aL_a \quad a_sL_s}$$

Substitute numerical values:

a_aL_a a_sL_s 659.9 10 ^{6} m/°C

$$¢T \quad \frac{15 \text{ mm} + 5 \text{ mm}}{659.9 * 10^{6} \text{ m/ C}} \quad 30.31 \text{ C}$$

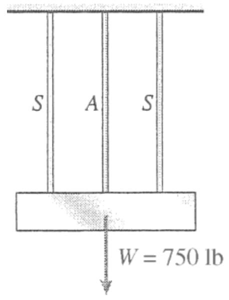
T T_0 + ¢T 10 C + 30.31 C

40.3 C

Problem 2.5-3 A rigid bar of weight W 750 lb hangs from three equally spaced wires, two of steel and one of aluminum (see figure). The diameter of the wires is $^1/_8$ in . Before they were loaded, all three wires had the same length.

What temperature increase T in all three wires will result in the entire load being carried by the steel wires? (Assume E_s 30 10^6 psi, a_s 6.5 10 ^{6}/°F, and a_a 12 10 ^{6}/°F.)

Solution 2.5-3 Bar supported by three wires



S steel A aluminum

W 750 lb

d $\frac{1}{8}$ in.

$A_s \frac{pd^2}{4}$ 0.012272 in.^{2}

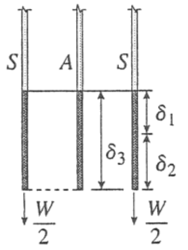
E_s 30 10^6 psi

E_sA_s 368,155 lb

a_s 6.5 10 ^{6}/°F

a_a 12 10 ^{6}/°F

L Initial length of wires



d_1 increase in length of a steel wire due to temperature increase T

a_s ($T)L$

d_2 increase in length of a steel wire due to load $W/2$

$\frac{WL}{2E_sA_s}$

d_3 increase in length of aluminum wire due to temperature increase T

a_a($T)L$

For no load in the aluminum wire:

d_1 d_2 d_3

$$a_s(¢T)L + \frac{WL}{2E_sA_s} \quad a_a(¢T)L$$

or

$$¢T \quad \frac{W}{2E_sA_s(a_a \quad a_s)}$$

Substitute numerical values:

$$¢T \quad \frac{750 \text{ lb}}{(2)(368{,}155 \text{ lb})(5.5 * 10^{6}/\text{ F})}$$

185 F

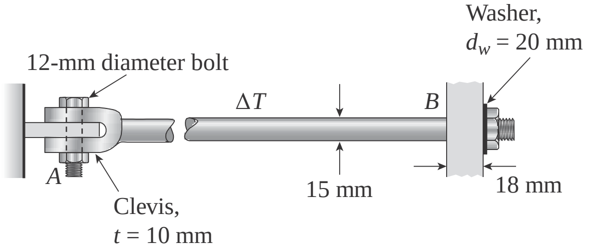
NOTE: If the temperature increase is larger than T, the aluminum wire would be in compression, which is not possible. Therefore, the steel wires continue to carry all of the load. If the temperature increase is less than T, the aluminum wire will be in tension and carry part of the load.

Problem 2.5-4 A steel rod of 15-mm diameter is held snugly (but without any initial stresses) between rigid walls by the arrangement shown in figure part a. (For the steel rod, use $\alpha = 12 \times 10^{-6}/°C$ and $E = 200$ GPa.)

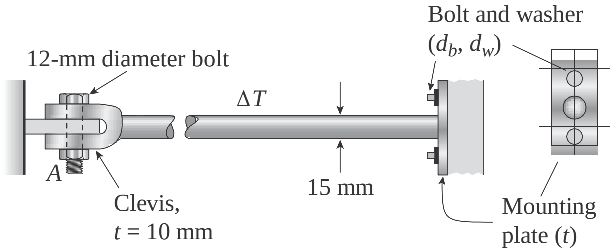
(a) Calculate the temperature drop ΔT (degrees Celsius) at which the average shear stress in the 12-mm diameter bolt becomes 45 MPa. Also, what is the normal stress in the rod?

(b) What are the average bearing stresses in the bolt and clevis at A and between the washer ($d_w = 20$ mm) and wall ($t = 18$ mm) at B?

(c) If the connection to the wall at B is changed to an end plate with two bolts (see figure part b), what is the required diameter d_b of each bolt if the temperature drop is $\Delta T = 38$ °C and the allowable bolt stress is 90 MPa?



(a)



(b)

Solution 2.5-4

Numerical properties

$d_r = 15$ mm $\quad d_b = 12$ mm $\quad d_w = 20$ mm $\quad t_c = 10$ mm $\quad t_{wall} = 18$ mm

$\tau_b = 45$ MPa $\quad \alpha = 12(10^{-6})$ $\quad E = 200$ GPa

(a) Temperature drop resulting in bolt shear stress $\quad \alpha\Delta T \quad \sigma = E\alpha\Delta T$

Rod force $P = (E\alpha\Delta T)\frac{\pi}{4}d_r^2$ and bolt in double shear with shear stress $\tau = \frac{\frac{P}{2}}{A_s}$ $\quad \tau = \frac{P}{2\frac{\pi}{4}d_b^2}$

$$\tau_b = \frac{2}{\pi d_b^2}\left[(E\alpha\Delta T)\frac{\pi}{4}d_r^2\right] \qquad \tau_b = \frac{E\alpha\Delta T}{2}\left(\frac{d_r}{d_b}\right)^2$$

$\tau_b = 45$ MPa

$$\Delta T = \frac{2\tau_b}{E(1000)\alpha}\left(\frac{d_b}{d_r}\right)^2 \qquad \Delta T = 24\ °C \qquad P = (E\alpha\Delta T)\frac{\pi}{4}d_r^2 \qquad P = 10\ \text{kN}$$

$$\sigma_{rod} = \frac{P\,1000}{\frac{\pi}{4}d_r^2} \qquad \boxed{\sigma_{rod} = 57.6\ \text{MPa}}$$

(b) Bearing stresses

Bolt and clevis $\sigma_{bc} = \dfrac{\frac{P}{2}}{d_b t_c}$ $\boxed{\sigma_{bc} = 42.4 \text{ MPa}}$

Washer at wall $\sigma_{bw} = \dfrac{P}{\frac{\pi}{4}\left(d_w^2 - d_r^2\right)}$ $\boxed{\sigma_{bw} = 74.1 \text{ MPa}}$

(c) If the connection to the wall at B is changed to an end plate with two bolts (see Fig. b), what is the required diameter d_b of each bolt if temperature drop $\Delta T = 38°C$ and the allowable bolt stress is 90 MPa?

Find force in rod due to temperature drop.

$\Delta T = 38°C$ $P = (E\alpha\,\Delta T)\dfrac{\pi}{4}d_r^2$

$P = 200\ GPa\,\dfrac{\pi}{4}(15\text{ mm})^2\left[12\left(10^{-6}\right)\right](38) = 16116\text{ N}$ $P = 16.12\text{ kN}$

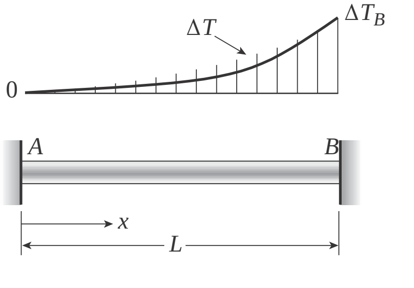
Each bolt carries one half of the force P:

$$d_b = \sqrt{\dfrac{\frac{16.12\text{ kN}}{2}}{\frac{\pi}{4}(90\text{ MPa})}} = 10.68\text{ mm})$$ $\boxed{d_b = 10.68\text{ mm}}$

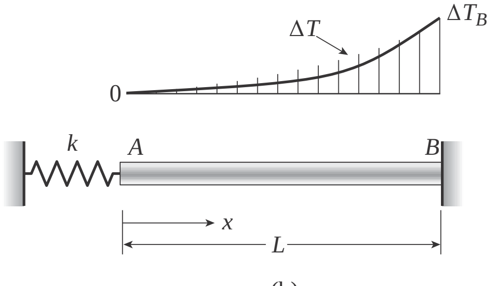
Problem 2.5-5 A bar AB of length L is held between rigid supports and heated nonuniformly in such a manner that the temperature increase ΔT at distance x from end A is given by the expression $\Delta T = \Delta T_B x^3/L^3$, where ΔT_B is the increase in temperature at end B of the bar (see figure part a).

(a) Derive a formula for the compressive stress σ_c in the bar. (Assume that the material has modulus of elasticity E and coefficient of thermal expansion α).
(b) Now modify the formula in (a) if the rigid support at A is replaced by an elastic support at A having a spring constant k (see figure part b).

Assume that only bar AB is subject to the temperature increase.



(a)

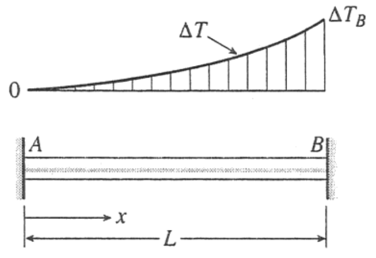


(b)

Solution 2.5-5

(a) One degree statically indeterminate—use superposition select reaction R_B as the redundant; follow procedure

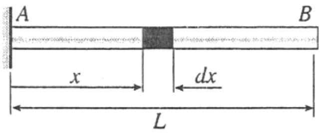
Bar with nonuniform temperature change.



At distance x:

$$\Delta T = \Delta T_B \left(\frac{x^3}{L^3}\right)$$

Remove the support at the end B of the bar:



Consider an element dx at a distance x from end A.

$d\delta$ = Elongation of element dx

$$d\delta = \alpha(\Delta T)dx = \alpha(\Delta T_B)\left(\frac{x^3}{L^3}\right)dx$$

$d\delta$ = elongation of bar

$$\delta = \int_0^L d\delta = \int_0^L \alpha(\Delta T_B)\left(\frac{x^3}{L^3}\right)dx = \frac{1}{4}\alpha(\Delta T_B)L$$

Compressive force P required to shorten the bar by the amount δ

$$P = \frac{EA\delta}{L} = \frac{1}{4}EA\alpha(\Delta T_B)$$

Compressive stress in the bar

$$\sigma_c = \frac{P}{A} = \frac{E\alpha(\Delta T_B)}{4} \leftarrow$$

(b) One degree statically indeterminate—use superposition.

Select reaction R_B as the redundant then compute bar elongations due to ΔT and due to R_B

$$\delta_{B1} = \alpha\Delta T_B\frac{L}{4}$$ due to temperature from above

$$\delta_{B2} = R_B\left(\frac{1}{k} + \frac{L}{EA}\right)$$

Compatibility: solve for R_B: $\delta_{B1} - \delta_{B2} = 0$

$$R_B = \frac{\left(\alpha\Delta T_B\frac{L}{4}\right)}{\left(\frac{1}{k} + \frac{L}{EA}\right)}$$

$$R_B = \alpha\Delta T_B \frac{EA}{\left[4\left(\frac{EA}{kL} + 1\right)\right]}$$

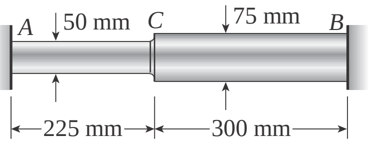
So compressive stress in bar is

$$\sigma_c = \frac{R_B}{A} \qquad \sigma_c = \frac{E\alpha(\Delta T_B)}{4\left(\frac{EA}{kL} + 1\right)} \leftarrow$$

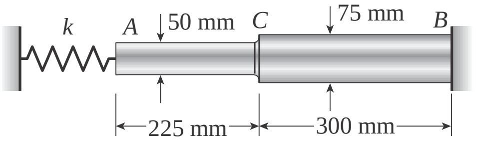
NOTE: σ_c in part (b) is the same as in part (a) if spring constant k goes to infinity.

Problem 2.5-6 A plastic bar *ACB* having two different solid circular cross sections is held between rigid supports as shown in the figure. The diameters in the left- and right-hand parts are 50 mm and 75 mm, respectively. The corresponding lengths are 225 mm and 300 mm. Also, the modulus of elasticity *E* is 6.0 GPa, and the coefficient of thermal expansion a is 100 10 ^{6}/°C. The bar is subjected to a uniform temperature increase of 30°C.

(a) Calculate the following quantities: (1) the compressive force *N* in the bar; (2) the maximum compressive stress s_c; and (3) the displacement d_C of point *C*.
(b) Repeat (a) if the rigid support at *A* is replaced by an elastic support having spring constant k 50 MN/m (see figure part b; assume that only the bar *ACB* is subject to the temperature increase).



(a)



(b)

Solution

NUMERICAL DATA

d_1 50 mm d_2 75 mm

L_1 225 mm L_2 300 mm

E 6.0 GPa a 100 10 ^{6}/°C

T 30°C k 50 MN/m

(a) COMPRESSIVE FORCE N, MAXIMUM COMPRESSIVE STRESS AND DISPLACEMENT OF PT. C

$$A_1 \quad \frac{p}{4} d_1^{\,2} \qquad A_2 \quad \frac{p}{4} d_2^{\,2}$$

One-degree statically indeterminate—use R_B as redundant

$$d_{B1} \quad a \quad T(L_1 \quad L_2)$$

$$d_{B2} \quad R_B \mathrm{a} \frac{L_1}{EA_1} + \frac{L_2}{EA_2} \mathrm{b}$$

Compatibility: d_{B1} d_{B2}, solve for R_B

$$R_B \quad \frac{a¢T(L_1 + L_2)}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2}} \qquad N \quad R_B$$

N 51.8 kN

Maximum compressive stress in *AC* since it has the smaller area (A_1 A_2):

$$s_{c\max} \quad \frac{N}{A_1} \qquad s_{c\max} \quad 26.4 \text{ MPa}$$

$$d_C \quad a¢T(L_1) \quad R_B \frac{L_1}{EA_1}$$

d_C 0.314 mm ( ) sign means joint *C* moves left

(b) COMPRESSIVE FORCE N, MAXIMUM COMPRESSIVE STRESS AND DISPLACEMENT OF PART C FOR ELASTIC SUPPORT CASE

Use R_B as redundant as in part (a):

$$d_{B1} \quad a \quad T(L_1 \quad L_2)$$

$$d_{B_2} \quad R_B \mathrm{a} \frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k} \mathrm{b}$$

Now add effect of elastic support; equate d_{B1} and d_{B2} then solve for R_B:

$$R_B \quad \frac{a¢T1L_1 + L_22}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k}} \qquad N \quad R_B$$

N 31.2 kN

$$s_{c\max} \quad \frac{N}{A_1} \qquad s_{c\max} \quad 15.91 \text{ MPa}$$

Superposition:

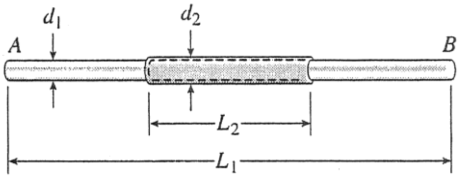
$$d_C \quad a¢T(L_1) \quad R_B \mathrm{a} \frac{L_1}{EA_1} + \frac{1}{k} \mathrm{b}$$

Displacement d_C of point C superposition of displacements in two released structures at C:

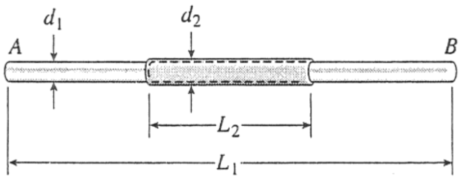
d_C 0.546 mm ( ) sign means joint C moves left

Problem 2.5-7 A circular steel rod *AB* (diameter d_1 1.0 in., length L_1 3.0 ft) has a bronze sleeve (outer diameter d_2 1.25 in., length L_2 1.0 ft) shrunk onto it so that the two parts are securely bonded (see figure).

Calculate the total elongation *d* of the steel bar due to a temperature rise T 500°F. (Material properties are as follows: for steel, E_s 30 10^6 psi and a_s 6.5 10^{6}/°F; for bronze, E_b 15 10^6 psi, and a_b 11 10^{6}/°F.)



Solution 2.5-7 Steel rod with bronze sleeve



L_1 36 in. L_2 12 in.

ELONGATION OF THE TWO OUTER PARTS OF THE BAR

$d_1 \quad a_s(\quad T)(L_1 \quad L_2)$

$(6.5 \quad 10^{6}/°F)(500°F)(36 \text{ in.} \quad 12 \text{ in.})$

0.07800 in.

ELONGATION OF THE MIDDLE PART OF THE BAR

The steel rod and bronze sleeve lengthen the same amount, so they are in the same condition as the bolt and sleeve of Example 2-8. Thus, we can calculate the elongation from Eq. (2-23):

$$d_2 \quad \frac{(a_s E_s A_s + a_b E_b A_b)(¢T)L_2}{E_s A_s + E_b A_b}$$

SUBSTITUTE NUMERICAL VALUES

a_s 6.5 10^{6}/°F a_b 11 10^{6}/°F

E_s 30 10^6 psi E_b 15 10^6 psi

d_1 1.0 in.

$A_s \quad \frac{p}{4} d_1^2 \quad 0.78540 \text{ in.}^2$

d_2 1.25 in.

$A_b \quad \frac{p}{4}(d_2^2 \quad d_1^2) \quad 0.44179 \text{ in.}^2$

T 500°F L_2 12.0 in.

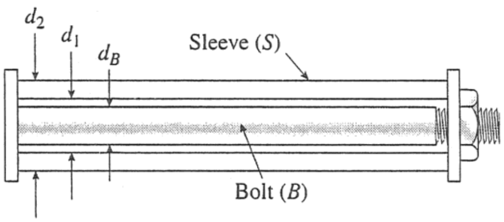
d_2 0.04493 in.

TOTAL ELONGATION

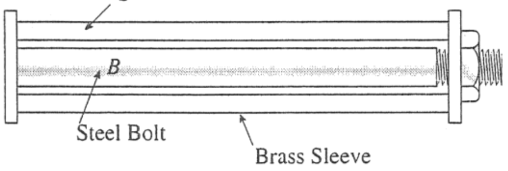
$d \quad d_1 \quad d_2 \quad 0.123$ in.

Problem 2.5-8 A brass sleeve *S* is fitted over a steel bolt *B* (see figure), and the nut is tightened until it is just snug. The bolt has a diameter d_B 25 mm, and the sleeve has inside and outside diameters d_1 26 mm and d_2 36 mm, respectively.

Calculate the temperature rise T that is required to produce a compressive stress of 25 MPa in the sleeve. (Use material properties as follows: for the sleeve, a_S 21 10^{6}/°C and E_S 100 GPa; for the bolt, a_B 10 10^{6}/°C and E_B 200 GPa.)
(*Suggestion*: Use the results of Example 2-8.)



Solution 2.5-8 Brass sleeve fitted over a Steel bolt



Subscript S means "sleeve".

Subscript B means "bolt".

Use the results of Example 2-8.

σ_S = compressive force in sleeve

EQUATION (2-22a):

$$\sigma_S = \frac{(\alpha_S - \alpha_B)(\Delta T)E_S E_B A_B}{E_S A_S + E_B A_B} \quad \text{(Compression)}$$

SOLVE FOR ΔT:

$$\Delta T = \frac{\sigma_S(E_S A_S + E_B A_B)}{(\alpha_S - \alpha_B)E_S E_B A_B}$$

or

$$\Delta T = \frac{\sigma_S}{E_S(\alpha_S - \alpha_B)}\left(1 + \frac{E_S A_S}{E_B A_B}\right) \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$\sigma_S = 25$ MPa

$d_2 = 36$ mm $\quad d_1 = 26$ mm $\quad d_B = 25$ mm

$E_S = 100$ GPa $\quad E_B = 200$ GPa

$\alpha_S = 21 \times 10^{-6}$/°C $\quad \alpha_B = 10 \times 10^{-6}$/°C

$$A_S = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}(620\text{ mm}^2)$$

$$A_B = \frac{\pi}{4}(d_B)^2 = \frac{\pi}{4}(625\text{ mm}^2) \quad 1 + \frac{E_S A_S}{E_B A_B} = 1.496$$

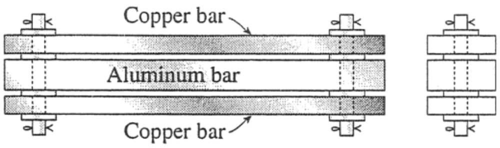
$$\Delta T = \frac{25\text{ MPa }(1.496)}{(100\text{ GPa})(11 \times 10^{-6}/{}^\circ\text{C})}$$

$$\Delta T = 34^\circ\text{C} \longleftarrow$$

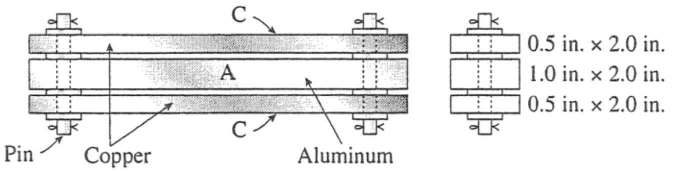
(Increase in temperature)

Problem 2.5-9 Rectangular bars of copper and aluminum are held by pins at their ends, as shown in the figure. Thin spacers provide a separation between the bars. The copper bars have cross-sectional dimensions 0.5 in. × 2.0 in., and the aluminum bar has dimensions 1.0 in. × 2.0 in.

Determine the shear stress in the 7/16 in. diameter pins if the temperature is raised by 100°F. (For copper, $E_c = 18{,}000$ ksi and $\alpha_c = 9.5 \times 10^{-6}$/°F; for aluminum, $E_a = 10{,}000$ ksi, and $\alpha_a = 13 \times 10^{-6}$/°F.) *Suggestion*: Use the results of Example 2-8.



Solution 2.5-9 Rectangular bars held by pins



Diameter of pin: $d_P = \frac{7}{16}$ in. $= 0.4375$ in.

Area of pin: $A_P = \frac{\pi}{4} d_P^2 = 0.15033$ in.^{2}

Area of two copper bars: $A_c = 2.0$ in.^{2}

Area of aluminum bar: $A_a = 2.0$ in.^{2}

$\Delta T = 100$°F

Copper: E_c 18,000 ksi a_c 9.5 10 ^{6}/°F

Aluminum: E_a 10,000 ksi

a_a 13 10 ^{6}/°F

Use the results of Example 2-8.

Find the forces P_a and P_c in the aluminum bar and copper bar, respectively, from Eq. (2-21).

Replace the subscript "*S*" in that equation by "*a*" (for aluminum) and replace the subscript "*B*" by "*c*" (for copper), because *a* for aluminum is larger than *a* for copper.

$$P_a \quad P_c \quad \frac{(a_a \quad a_c)(¢T)E_a A_a E_c A_c}{E_a A_a + E_c A_c}$$

Note that P_a is the compressive force in the aluminum bar and P_c is the combined tensile force in the two copper bars.

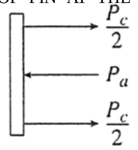
$$P_a \quad P_c \quad \frac{(a_a \quad a_c)(¢T)E_c A_c}{1 + \frac{E_c A_c}{E_a A_a}}$$

SUBSTITUTE NUMERICAL VALUES:

$$P_a \quad P_c \quad \frac{(3.5 * 10 \ ^{6}/ \text{ F})(100 \text{ F})(18{,}000 \text{ ksi})(2 \text{ in.}^2)}{1 + \text{a}\frac{18}{10}\text{b a}\frac{2.0}{2.0}\text{b}}$$

4,500 lb

FREE-BODY DIAGRAM OF PIN AT THE LEFT END



V shear force in pin

$P_c/2$

2,250 lb

t average shear stress on cross section of pin

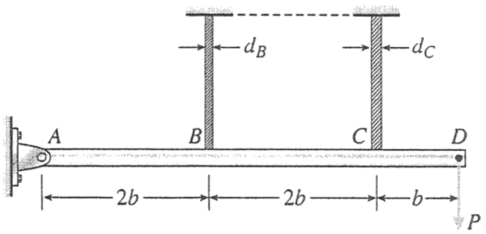
$$t \quad \frac{V}{A_P} \quad \frac{2{,}250 \text{ lb}}{0.15033 \text{ in.}^2}$$

t 15.0 ksi

Problem 2.5-10 A rigid bar *ABCD* is pinned at end *A* and supported by two cables at points *B* and *C* (see figure). The cable at *B* has nominal diameter d_B 12 mm and the cable at *C* has nominal diameter d_C 20 mm. A load *P* acts at end *D* of the bar.

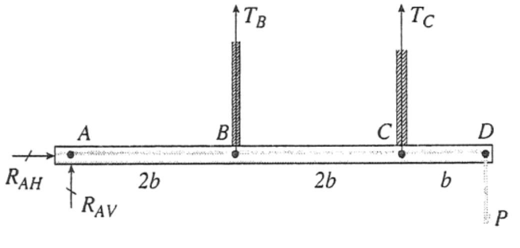
What is the allowable load *P* if the temperature rises by 60°C and each cable is required to have a factor of safety of at least 5 against its ultimate load?

(*Note:* The cables have effective modulus of elasticity *E* 140 GPa and coefficient of thermal expansion *a* 12 10 ^{6}/°C. Other properties of the cables can be found in Table 2-1, Section 2.2.)



Solution 2.5-10 Rigid bar supported by two cables

FREE-BODY DIAGRAM OF BAR *ABCD*



T_B force in cable *B* T_C force in cable *C*

d_B 12 mm d_C 20 mm

From Table 2-1:

A_B 76.7 mm^2 E 140 GPa

T 60°C A_C 173 mm^2

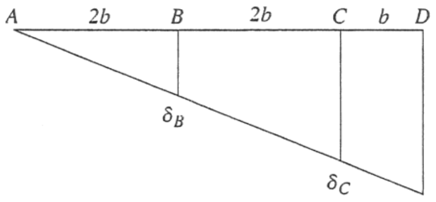
a 12 10 ^{6}/°C

EQUATION OF EQUILIBRIUM

M_A 0 $T_B(2b)$ $T_C(4b)$ $P(5b)$ 0

or $2T_B$ $4T_C$ $5P$ (Eq. 1)

DISPLACEMENT DIAGRAM



COMPATIBILITY:

$d_C \quad 2d_B$ (Eq. 2)

FORCE-DISPLACEMENT AND TEMPERATURE-DISPLACEMENT RELATIONS

$$d_B \quad \frac{T_B L}{EA_B} + a(¢T)L \qquad \text{(Eq. 3)}$$

$$d_C \quad \frac{T_C L}{EA_C} + a(¢T)L \qquad \text{(Eq. 4)}$$

SUBSTITUTE EQS. (3) AND (4) INTO EQ. (2):

$$\frac{T_C L}{EA_C} + a(¢T)L \quad \frac{2T_B L}{EA_B} + 2a(¢T)L$$

or

$2T_B A_C \quad T_C A_B \quad Ea(\;\; T)A_B\, A_C$ (Eq. 5)

SUBSTITUTE NUMERICAL VALUES INTO EQ. (5):

$T_B(346) \quad T_C(76.7) \quad 1{,}338{,}000$ (Eq. 6)

in which T_B and T_C have units of newtons.

SOLVE SIMULTANEOUSLY EQS. (1) AND (6):

$T_B \quad 0.2494\,P \quad 3{,}480$ (Eq. 7)

$T_C \quad 1.1253\,P \quad 1{,}740$ (Eq. 8)

in which P has units of newtons.

SOLVE EQS. (7) AND (8) FOR THE LOAD P:

$P_B \quad 4.0096\,T_B \quad 13{,}953$ (Eq. 9)

$P_C \quad 0.8887\,T_C \quad 1{,}546$ (Eq. 10)

ALLOWABLE LOADS

From Table 2-1:

$(T_B)_{\text{ULT}} \quad 102{,}000$ N $\quad (T_C)_{\text{ULT}} \quad 231{,}000$ N

Factor of safety 5

$(T_B)_{\text{allow}} \quad 20{,}400$ N $\quad (T_C)_{\text{allow}} \quad 46{,}200$ N

From Eq. (9): P_B (4.0096)(20,400 N) 13,953 N
95,700 N

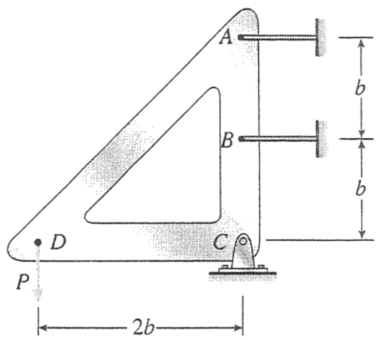
From Eq. (10): P_C (0.8887)(46,200 N) 1546 N
39,500 N

Cable C governs.

$P_{\text{allow}} \quad 39.5$ kN

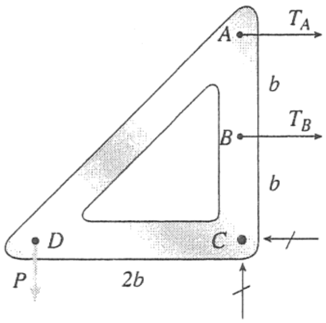
Problem 2.5-11 A rigid triangular frame is pivoted at C and held by two identical horizontal wires at points A and B (see figure). Each wire has axial rigidity EA 120 k and coefficient of thermal expansion a 12.5 10 ^{6}/°F.

(a) If a vertical load P 500 lb acts at point D, what are the tensile forces T_A and T_B in the wires at A and B, respectively?
(b) If, while the load P is acting, both wires have their temperatures raised by 180°F, what are the forces T_A and T_B?
(c) What further increase in temperature will cause the wire at B to become slack?



Solution 2.5-11 Triangular frame held by two wires

FREE-BODY DIAGRAM OF FRAME

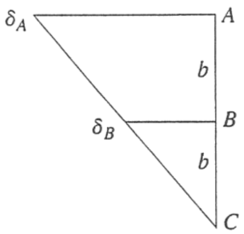


EQUATION OF EQUILIBRIUM

$M_C \quad 0$

$P(2b) \quad T_A(2b) \quad T_B(b) \quad 0$ or $2T_A \quad T_B \quad 2P$ (Eq. 1)

DISPLACEMENT DIAGRAM



EQUATION OF COMPATIBILITY

$d_A \quad 2d_B$ (Eq. 2)

(a) LOAD P ONLY

Force-displacement relations:

$$d_A \quad \frac{T_A L}{EA} \quad d_B \quad \frac{T_B L}{EA} \qquad \text{(Eq. 3, 4)}$$

(L length of wires at A and B.)
Substitute (3) and (4) into Eq. (2):

$$\frac{T_A L}{EA} \quad \frac{2T_B L}{EA}$$

or $T_A \quad 2T_B$ (Eq. 5)

Solve simultaneously Eqs. (1) and (5):

$$T_A \frac{4P}{5} \quad T_B \quad \frac{2P}{5} \qquad \text{(Eqs. 6, 7)}$$

Numerical values:

$P \quad 500$ lb

(b) LOAD P AND TEMPERATURE INCREASE T

Force-displacement and temperature-displacement relations:

$$d_A \quad \frac{T_A L}{EA} + a(¢T)L \qquad \text{(Eq. 8)}$$

$$d_B \quad \frac{T_B L}{EA} + a(¢T)L \qquad \text{(Eq. 9)}$$

Substitute (8) and (9) into Eq. (2):

$$\frac{T_A L}{EA} + a(¢T)L \; \frac{2T_B L}{EA} + 2a(¢T)L$$

or $T_A \quad 2T_B \quad EAa(\quad T)$ (Eq. 10)

Solve simultaneously Eqs. (1) and (10):

$$T_A \quad \frac{1}{5}[4P + EAa(¢T)] \qquad \text{(Eq. 11)}$$

$$T_B \quad \frac{2}{5}[P \quad EAa(¢T)] \qquad \text{(Eq. 12)}$$

Substitute numerical values:

$P \quad 500$ lb $\quad EA \quad 120{,}000$ lb

$T \quad 180°F$

$a \quad 12.5 \quad 10^{\ 6}/°F$

$T_A \quad \frac{1}{5}(2000 \text{ lb} + 270 \text{ lb}) \quad 454$ lb

$T_B \quad \frac{2}{5}(500 \text{ lb} \quad 270 \text{ lb}) \quad 92$ lb

(c) WIRE B BECOMES SLACK

Set $T_B \quad 0$ in Eq. (12):

$P \quad EAa(\quad T)$

or

$$¢T \quad \frac{P}{EAa} \quad \frac{500 \text{ lb}}{(120{,}000 \text{ lb})(12.5 * 10^{\ 6}/ \text{F})}$$

$333.3°F$

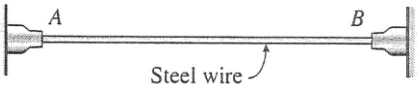
Further increase in temperature:

$T \quad 333.3°F \quad 180°F$

$153°F$

$T_A \quad 400 \text{ lb} \quad T_B \quad 200 \text{ lb}$

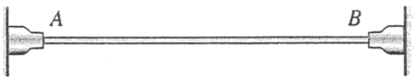
Misfits and Prestrains



Problem 2.5-12 A steel wire *AB* is stretched between rigid supports (see figure). The initial prestress in the wire is 42 MPa when the temperature is 20°C.

(a) What is the stress s in the wire when the temperature drops to 0°C?
(b) At what temperature *T* will the stress in the wire become zero?
(Assume a 14 10 ^{6}/°C and E 200 GPa.)

Solution 2.5-12 Steel wire with initial prestress



Initial prestress: s_1 42 MPa

Initial temperature: T_1 20°C

E 200 GPa

a 14 10 ^{6}/°C

(a) Stress s when temperature drops to 0°C

T_2 0°C T 20°C

NOTE: *Positive* T means a *decrease* in temperature and an *increase* in the stress in the wire.

Negative T means an *increase* in temperature and a *decrease* in the stress.

Stress s equals the initial stress s_1 plus the additional stress s_2 due to the temperature drop.

s_2 $Ea(\ T)$

s s_1 s_2 s_1 $Ea(\ T)$

42 MPa (200 GPa)(14 10 ^{6}/°C)(20°C)

42 MPa 56 MPa 98 MPa

(b) Temperature when stress equals zero

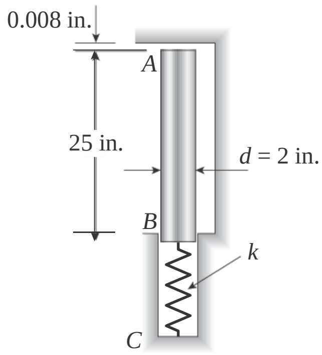
s s_1 s_2 0 s_1 $Ea(\ T)$ 0

$$¢T \quad \frac{s_1}{Ea}$$

(Negative means increase in temp.)

$$¢T \quad \frac{42 \text{ MPa}}{(200 \text{ GPa})(14 * 10^{6}/ \text{C}} \quad 15 \text{ C}$$

T 20°C 15°C 35°C



room temperature with a gap of 0.008 in. between end *A* and a rigid restraint (see figure). The bar is supported at end *B* by an elastic spring with spring constant k 1.2 10^6 lb/in.

(a) Calculate the axial compressive stress s_c in the bar if the temperature rises 50°F. (For copper, use a 9.6 10 ^{6}/°F and E 16 10^6 psi.)

(c) Repeat (a) if k .

Solution 2.5-13

Numerical data:

L 25 in. d 2 in. d 0.008 in.

k 1.2 (10^6) lb/in. E 16 (10^6) psi

a 9.6 (10^{6})/°F T 50°F

$A \quad \frac{p}{4} d^2 \quad A \quad 3.14159 \text{ in.}^2$

(a) ONE-DEGREE STATICALLY INDETERMINATE IF GAP CLOSES

$a \ TL$ 0.012 in. exceeds gap

Select R_A as redundant and do superposition analysis:

$$d_{A1} \qquad d_{A2} \quad R_A a \frac{L}{EA} + \frac{1}{k} b$$

Compatibility: $d_{A1} \quad d_{A2} \quad d \quad d_{A2} \quad d \quad d_{A1}$

$$R_A \quad \frac{d \quad ¢}{\frac{L}{EA} + \frac{1}{k}} \qquad R_A \quad 3006 \text{ lb}$$

Compressive stress in bar:

$$s \quad \frac{R_A}{A} \quad s \quad 957 \text{ psi}$$

(b) FORCE IN SPRING F_k R_C STATICS

$R_A \quad R_C \quad 0$

$R_C \quad R_A$

$R_C \quad 3006 \text{ lb}$

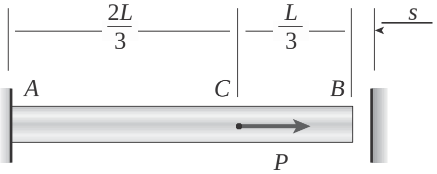
(c) FIND COMPRESSIVE STRESS IN BAR IF k GOES TO INFINITY FROM EXPRESSION FOR R_A ABOVE, $1/k$ GOES TO ZERO

$$R_A \quad \frac{d \quad ¢}{\frac{L}{EA}} \quad R_A \quad 8042 \text{ lb} \quad s \quad \frac{R_A}{A}$$

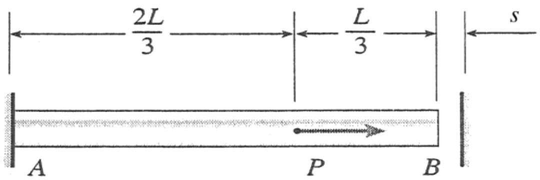
$s \quad 2560$ psi

Problem 2.5-14 A bar AB having length L and axial rigidity EA is fixed at end A (see figure). At the other end a small gap of dimension s exists between the end of the bar and a rigid surface. A load P acts on the bar at point C, which is two-thirds of the length from the fixed end.

If the support reactions produced by the load P are to be equal in magnitude, what should be the size s of the gap?



Solution 2.5-14 Bar with a gap (load P)



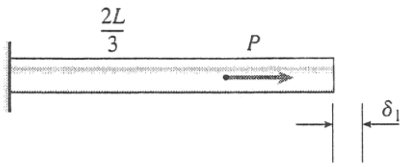
L length of bar

s size of gap

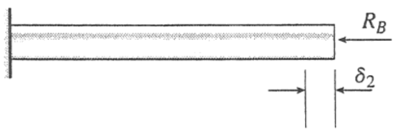
EA axial rigidity

Reactions must be equal; find s.

FORCE-DISPLACEMENT RELATIONS



$$d_1 \quad \frac{P\left(\frac{2L}{3}\right)}{EA}$$



$$d_2 \quad \frac{R_B L}{EA}$$

COMPATIBILITY EQUATION

$d_1 \quad d_2 \quad s$ or

$$\frac{2PL}{3EA} \quad \frac{R_B L}{EA} \quad s \qquad \text{(Eq. 1)}$$

EQUILIBRIUM EQUATION

R_A reaction at end A (to the left)

R_B reaction at end B (to the left)

$P \quad R_A \quad R_B$

Reactions must be equal.

$$R_A \quad R_B \quad P \quad 2R_B \quad R_B \quad \frac{P}{2}$$

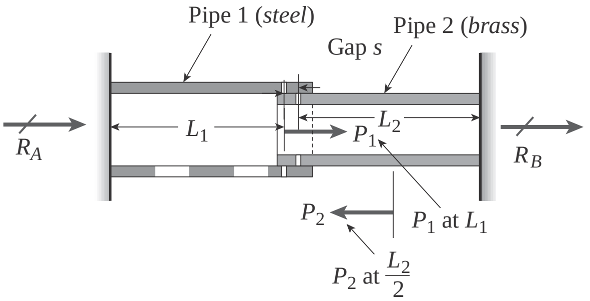
Substitute for R_B in Eq. (1):

$$\frac{2PL}{3EA} \quad \frac{PL}{2EA} \quad s \quad \text{or} \quad s \quad \frac{PL}{6EA}$$

NOTE: The gap closes when the load reaches the value $P/4$. When the load reaches the value P, equal to $6EAs/L$, the reactions are equal ($R_A \quad R_B \quad P/2$). When the load is between $P/4$ and P, R_A is greater than R_B. If the load exceeds P, R_B is greater than R_A.

Problem 2.5-15 Pipe 2 has been inserted snugly into Pipe 1, but the holes for a connecting pin do not line up: there is a gap s. The user decides to apply *either* force P_1 to Pipe 1 *or* force P_2 to Pipe 2, whichever is smaller. Determine the following using the numerical properties in the box.

(a) If only P_1 is applied, find P_1 (kips) required to close gap s; if a pin is then inserted and P_1 removed, what are reaction forces R_A and R_B for this load case?
(b) If only P_2 is applied, find P_2 (kips) required to close gap s; if a pin is inserted and P_2 removed, what are reaction forces R_A and R_B for this load case?
(c) What is the maximum *shear* stress in the pipes, for the loads in parts (a) and (b)?
(d) If a temperature increase T is to be applied to the entire structure to close gap s (*instead of applying forces P_1* and P_2), find the T required to close the gap. If a pin is inserted after the gap has closed, what are reaction forces R_A and R_B for this case?
(e) Finally, if the structure (with pin inserted) then cools to the *original* ambient temperature, what are reaction forces R_A and R_B?



Numerical properties
$E_1 = 30{,}000$ ksi, $E_2 = 14{,}000$ ksi
$a_1 = 6.5 \quad 10^{-6}/°F$, $a_2 = 11 \quad 10^{-6}/°F$
Gap $s = 0.05$ in.
$L_1 = 56$ in., $d_1 = 6$ in., $t_1 = 0.5$ in., $A_1 = 8.64$ in.^{2}
$L_2 = 36$ in., $d_2 = 5$ in., $t_2 = 0.25$ in., $A_2 = 3.73$ in.^{2}

Solution 2.5-15

(a) FIND REACTIONS AT A AND B FOR APPLIED FORCE P_1

First compute P_1, required to close gap:

$$P_1 \quad \frac{E_1 A_1}{L_1} s \quad P_1 \quad 231.4 \text{ k}$$

Statically indeterminate analysis with R_B as the redundant:

$$d_{B1} \quad s \quad d_{B2} \quad R_B \left(\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} \right)$$

Compatibility: $d_{B1} \quad d_{B2} \quad 0$

$$R_B \quad \frac{s}{\left(\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} \right)} \quad R_B \quad 55.2 \text{ k}$$

$R_A \quad R_B$

(b) FIND REACTIONS AT A AND B FOR APPLIED FORCE P_2

$$P_2 \quad \frac{E_2 A_2}{\frac{L_2}{2}} s \quad P_2 \quad 145.1 \text{ k}$$

Analysis after removing P_2 is same as in part (a), so reaction forces are the same

(c) MAXIMUM SHEAR STRESS IN PIPE 1 OR 2 WHEN EITHER P_1 OR P_2 IS APPLIED

$$\tau_{maxa} \quad \frac{\frac{P_1}{A_1}}{2} \quad \tau_{maxa} \quad 13.39 \text{ ksi}$$

$$\tau_{maxb} \quad \frac{\frac{P_2}{A_2}}{2} \quad \tau_{maxb} \quad 19.44 \text{ ksi}$$

(d) REQUIRED ¢T AND REACTIONS AT A AND B

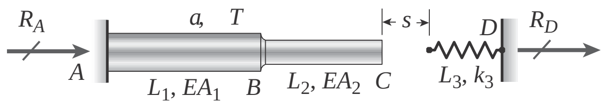
$$¢T_{reqd} \quad \frac{s}{\alpha_1 L_1 + \alpha_2 L_2} \quad T_{reqd} \quad 65.8°\text{F}$$

If pin is inserted but temperature remains at T above ambient temperature, reactions are zero.

(e) IF TEMPERATURE RETURNS TO ORIGINAL AMBIENT TEMPERATURE, FIND REACTIONS AT A AND B

statically indeterminate analysis with R_B as the redundant Compatibility: $\delta_{B1} \quad \delta_{B2} \quad 0$
Analysis is the same as in parts (a) and (b) above since gap s is the same, so reactions are the same.

Problem 2.5-16 A nonprismatic bar ABC made up of segments AB (length L_1, cross-sectional area A_1) and BC (length L_2, cross-sectional area A_2) is fixed at end A and free at end C (see figure). The modulus of elasticity of the bar is E. A small gap of dimension s exists between the end of the bar and an elastic spring of length L_3 and spring constant k_3. If bar ABC only (*not the spring*) is subjected to temperature increase T determine the following.



(a) Write an expression for reaction forces R_A and R_D if the elongation of ABC exceeds gap length s.
(b) Find expressions for the displacements of points B and C if the elongation of ABC exceeds gap length s.

Solution 2.5-16

With gap s closed due to T, structure is one-degree statically-indeterminate; select internal force (Q) at juncture of bar and spring as the redundant. Use superposition of two released structures in the solution.

δ_{rel1} relative displacement between end of bar at C and end of spring due to T

$$\delta_{rel1} \quad \alpha \quad T(L_1 \quad L_2)$$

δ_{rel1} is greater than gap length s

δ_{rel2} relative displacement between ends of bar and spring due to pair of forces Q, one on end of bar at C and the other on end of spring

$$\delta_{rel2} \quad Q\left(\frac{L_1}{EA_1} + \frac{L_2}{EA_2}\right) + \frac{Q}{k_3}$$

$$\delta_{rel2} \quad Q\left(\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}\right)$$

Compatibility: $\delta_{rel1} \quad \delta_{rel2} \quad s \quad \delta_{rel2} \quad s \quad \delta_{rel1}$

$$\delta_{rel2} \quad s \quad \alpha \quad T(L_1 \quad L_2)$$

$$Q \quad \frac{s \quad \alpha ¢T(L_1 + L_2)}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}}$$

$$Q \quad \frac{EA_1A_2k_3}{L_1A_2k_3 + L_2A_1k_3 + EA_1A_2}[s \quad \alpha ¢T(L_1 + L_2)]$$

(a) REACTIONS AT A AND D

Statics: $R_A \quad Q \quad R_D \quad Q$

$$R_A \quad \frac{s + \alpha ¢T(L_1 + L_2)}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}}$$

$$R_D \quad R_A$$

(b) DISPLACEMENTS AT B AND C

Use superposition of displacements in the two released structures:

$$d_B \quad a¢T1\, L_1 2 \quad R_A a \frac{L_1}{EA_1} b$$

$$d_B \quad a¢T1\, L_1 2$$

$$\frac{[\quad s + a¢T1\, L_1 + L_2 2]}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}} \quad a\frac{L_1}{EA_1} b$$

$$d_C \quad a¢T1\, L_1 + L_2 2$$

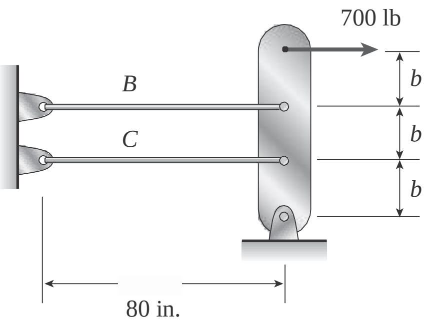
$$R_A a \frac{L_1}{EA_1} + \frac{L_2}{EA_2} b$$

$$d_C \quad a¢T1\, L_1 + L_2 2$$

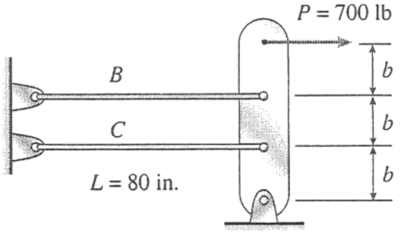
$$\frac{[\quad s + a¢T1\, L_1 + L_2 2]}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}} \quad a\frac{L_1}{EA_1} + \frac{L_2}{EA_2} b$$

Problem 2.5–17 Wires B and C are attached to a support at the left-hand end and to a pin-supported rigid bar at the right-hand end (see figure). Each wire has cross-sectional area A 0.03 in.^{2} and modulus of elasticity E 30 10^6 psi. When the bar is in a vertical position, the length of each wire is L 80 in. However, before being attached to the bar, the length of wire B was 79.98 in. and of wire C was 79.95 in.

Find the tensile forces T_B and T_C in the wires under the action of a force P 700 lb acting at the upper end of the bar.



Solution 2.5–17 Wires B and C attached to a bar



P 700 lb

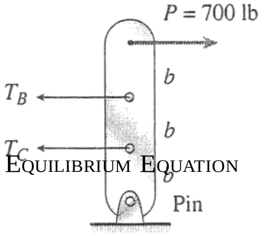
A 0.03 in.^{2}

E 30 10^6 psi

L_B 79.98 in.

L_C 79.95 in.

EQUILIBRIUM EQUATION



M_{pin} 0

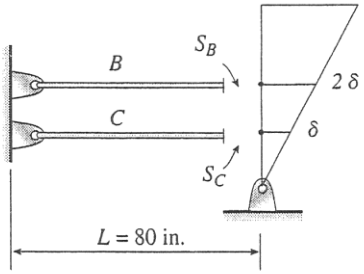
$T_C(b)$ $T_B(2b)$ $P(3b)$

$2T_B$ T_C $3P$ (Eq. 1)

DISPLACEMENT DIAGRAM

$S_B \quad 80 \text{ in.} \quad L_B \quad 0.02 \text{ in.}$

$S_C \quad 80 \text{ in.} \quad L_C \quad 0.05 \text{ in.}$



Elongation of wires:

$d_B \quad S_B \quad 2d$ (Eq. 2)

$d_C \quad S_C \quad d$ (Eq. 3)

FORCE-DISPLACEMENT RELATIONS

$d_B \quad \frac{T_B L}{EA} \quad d_C \quad \frac{T_C L}{EA}$ (Eqs. 4, 5)

SOLUTION OF EQUATIONS

Combine Eqs. (2) and (4):

$\frac{T_B L}{EA} \quad S_B + 2d$ (Eq. 6)

Combine Eqs. (3) and (5):

$\frac{T_C L}{EA} \quad S_C + d$ (Eq. 7)

Eliminate between Eqs. (6) and (7):

$T_B \quad 2T_C \quad \frac{EAS_B}{L} \quad \frac{2EAS_C}{L}$ (Eq. 8)

Solve simultaneously Eqs. (1) and (8):

$T_B \quad \frac{6P}{5} + \frac{EAS_B}{5L} \quad \frac{2EAS_C}{5L}$

$T_C \quad \frac{3P}{5} \quad \frac{2EAS_B}{5L} + \frac{4EAS_C}{5L}$

SUBSTITUTE NUMERICAL VALUES:

$\frac{EA}{5L} \quad 2250 \text{ lb/in.}$

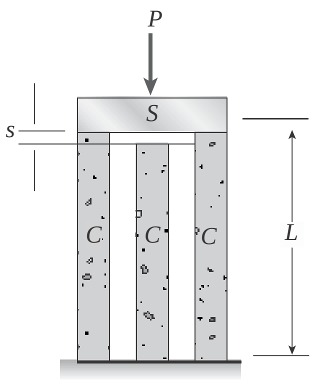
$T_B \quad 840 \text{ lb} \quad 45 \text{ lb} \quad 225 \text{ lb} \quad 660 \text{ lb}$

$T_C \quad 420 \text{ lb} \quad 90 \text{ lb} \quad 450 \text{ lb} \quad 780 \text{ lb}$

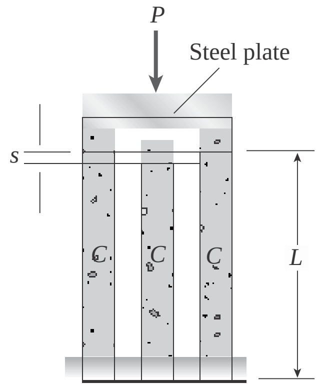
(Both forces are positive, which means tension, as required for wires.)

Problem 2.5-18 A rigid steel plate is supported by three posts of high-strength concrete each having an effective cross-sectional area $A \quad 40{,}000 \text{ mm}^2$ and length $L \quad 2$ m (see figure). Before the load P is applied, the middle post is shorter than the others by an amount $s \quad 1.0$ mm.

Determine the maximum allowable load P_{allow} if the allowable compressive stress in the concrete is $s_{allow} \quad 20$ MPa. (Use $E \quad 30$ GPa for concrete.)



Solution 2.5-18 Plate supported by three posts



s size of gap 1.0 mm

L length of posts 2.0 m

A 40,000 mm^2

$_{\text{allow}}$ 20 MPa

E 30 GPa

C concrete post

DOES THE GAP CLOSE?

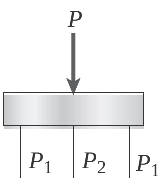
Stress in the two outer posts when the gap is just closed:

$$s \quad E \quad E\left(\frac{s}{L}\right) \quad (30 \text{ GPa})\left(\frac{1.0 \text{ mm}}{2.0 \text{ m}}\right)$$

$$15 \text{ MPa}$$

Since this stress is less than the allowable stress, the allowable force P will close the gap.

EQUILIBRIUM EQUATION



$$2P_1 \quad P_2 \quad P \qquad \text{(Eq. 1)}$$

COMPATIBILITY EQUATION

$_1$ shortening of outer posts

$_2$ shortening of inner post

$$_1 \quad _2 \quad s \qquad \text{(Eq. 2)}$$

FORCE-DISPLACEMENT RELATIONS

$$d_1 \; \frac{P_1 L}{EA} \quad d_2 \; \frac{P_2 L}{EA} \qquad \text{(Eqs. 3, 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_1 L}{EA} \quad \frac{P_2 L}{EA} + s \quad \text{or} \quad P_1 \quad P_2 \quad \frac{EAs}{L} \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$P \quad 3P_1 \quad \frac{EAs}{L}$$

By inspection, we know that P_1 is larger than P_2. Therefore, P_1 will control and will be equal to $_{\text{allow}} A$.

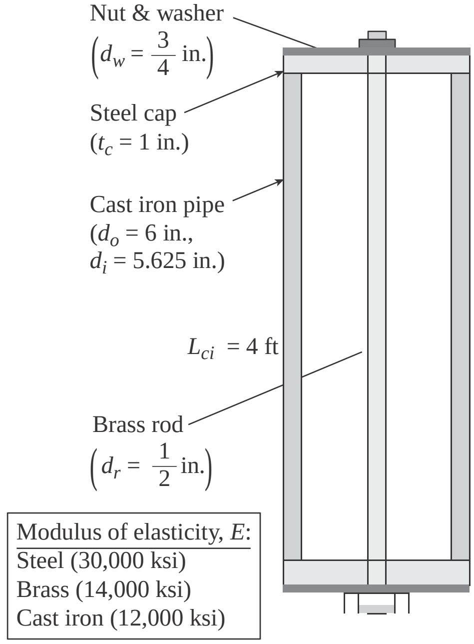
$$P_{\text{allow}} \quad 3s_{\text{allow}} A \quad \frac{EAs}{L}$$

2400 kN 600 kN 1800 kN

1.8 MN

Problem 2.5-19 A capped cast-iron pipe is compressed by a brass rod, as shown. The nut is turned until it is just snug, then add an additional quarter turn to pre-compress the CI pipe. The pitch of the threads of the bolt is p 52 mils (a mil is one-thousandth of an inch). Use the numerical properties provided.

(a) What stresses s_p and s_r will be produced in the cast-iron pipe and brass rod, respectively, by the additional quarter turn of the nut?
(b) Find the bearing stress s_b beneath the washer and the shear stress t_c in the steel cap.



Solution 2.5-19

The figure shows a section through the pipe, cap and rod

NUMERICAL PROPERTIES

L_{ci} 48 in. E_s 30000 ksi E_b 14,000 ksi

E_c 12,000 ksi t_c 1 in. p 52 (10^{-3}) in. n $\frac{1}{4}$

d_w $\frac{3}{4}$ in. d_r $\frac{1}{2}$ in. d_o 6 in. d_i 5.625 in.

(a) FORCES AND STRESSES IN PIPE AND ROD

One degree statically indeterminate—cut rod at cap and use force in rod (Q) as the redundant:

d_{rel1} relative displacement between cut ends of rod due to 1/4 turn of nut

d_{rel1} np Ends of rod move apart, not together, so this is ( ).

d_{rel2} relative displacement between cut ends of rod due pair of forces Q

$$d_{rel2} \quad Q\left(\frac{L_{ci} + 2t_c}{E_b A_{rod}} + \frac{L_{ci}}{E_c A_{pipe}}\right)$$

$$A_{rod} \quad \frac{p}{4} d_r^2 \qquad A_{pipe} \quad \frac{p}{4}(d_o^2 \quad d_i^2)$$

A_{rod} 0.196 in.^{2} A_{pipe} 3.424 in.^{2}

Compatibility equation: d_{rel1} d_{rel2} 0

$$Q \quad \frac{np}{\frac{L_{ci} + 2t_c}{E_b A_{rod}} + \frac{L_{ci}}{E_c A_{pipe}}}$$

Q 0.672 k F_{rod} Q

Statics: F_{pipe} Q

Stresses: s_c $\frac{F_{pipe}}{A_{pipe}}$ s_c 0.196 ksi

s_b $\frac{F_{rod}}{A_{rod}}$ s_b 3.42 ksi

(b) BEARING AND SHEAR STRESSES IN STEEL CAP

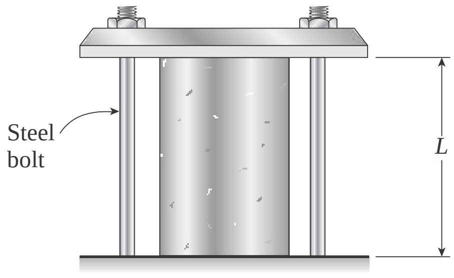
$$s_b \quad \frac{F_{rod}}{\frac{p}{4}(d_w^2 \quad d_r^2)} \qquad s_b \quad 2.74 \text{ ksi}$$

$$t_c \quad \frac{F_{rod}}{p d_w t_c} \qquad t_c \quad 0.285 \text{ ksi}$$

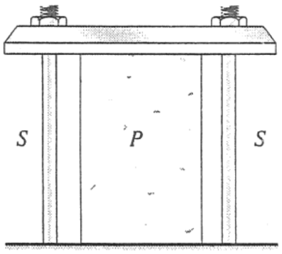
Problem 2.5-20 A plastic cylinder is held snugly between a rigid plate and a foundation by two steel bolts (see figure).

Determine the compressive stress s_p in the plastic when the nuts on the steel bolts are tightened by one complete turn.

Data for the assembly are as follows: length L 200 mm, pitch of the bolt threads p 1.0 mm, modulus of elasticity for steel E_s 200 GPa, modulus of elasticity for the plastic E_p 7.5 GPa, cross-sectional area of one bolt A_s 36.0 mm^2, and cross-sectional area of the plastic cylinder A_p 960 mm^2.



Solution 2.5-20 Plastic cylinder and two steel bolts



L 200 mm

P 1.0 mm

E_s 200 GPa

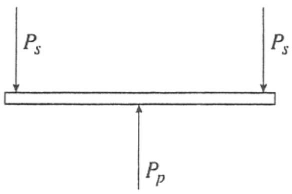
A_s 36.0 mm^2 (for one bolt)

E_p 7.5 GPa

A_p 960 mm^2

n 1 (See Eq. 2-24)

EQUILIBRIUM EQUATION

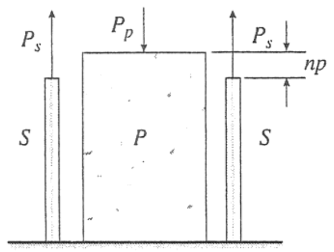


P_s tensile force in one steel bolt

P_p compressive force in plastic cylinder

$P_p \quad 2P_s$ (Eq. 1)

COMPATIBILITY EQUATION



d_s elongation of steel bolt

d_p shortening of plastic cylinder

$d_s \quad d_p \quad np$ (Eq. 2)

FORCE-DISPLACEMENT RELATIONS

$$d_s \quad \frac{P_s L}{E_s A_s} \qquad d_p \quad \frac{P_p L}{E_p A_p}$$ (Eq. 3, Eq. 4)

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_s L}{E_s A_s} + \frac{P_p L}{E_p A_p} \quad np$$ (Eq. 5)

Solve simultaneously Eqs. (1) and (5):

$$P_p \quad \frac{2npE_sA_sE_pA_p}{L(E_pA_p + 2E_sA_s)}$$

STRESS IN THE PLASTIC CYLINDER

$$s_p \quad \frac{P_p}{A_p} \quad \frac{2np\, E_s\, A_s\, E_p}{L(E_p A_p + 2E_s A_s)}$$

SUBSTITUTE NUMERICAL VALUES:

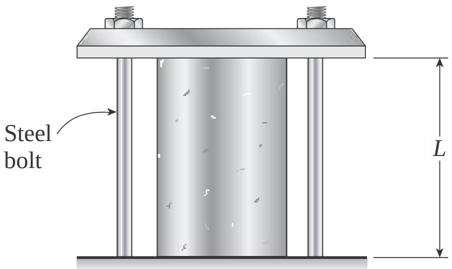
$N \quad E_s\, A_s\, E_p \quad 54.0 \quad 10^{15}\ \mathrm{N^2/m^2}$

$D \quad E_p A_p \quad 2E_s A_s \quad 21.6 \quad 10^6$ N

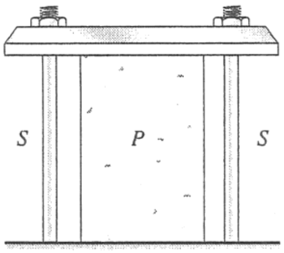
$$s_p \quad \frac{2np}{L} \text{a} \frac{N}{D} \text{b} \quad \frac{2(1)(1.0 \text{ mm})}{200 \text{ mm}} \text{a} \frac{N}{D} \text{b}$$

25.0 MPa

Problem 2.5-21 Solve the preceding problem if the data for the assembly are as follows: length L 10 in., pitch of the bolt threads p 0.058 in., modulus of elasticity for steel E_s 30 10^6 psi, modulus of elasticity for the plastic E_p 500 ksi, cross-sectional area of one bolt A_s 0.06 in.^{2}, and cross-sectional area of the plastic cylinder A_p 1.5 in.^{2}



Solution 2.5-21 Plastic cylinder and two steel bolts



L 10 in.

p 0.058 in.

E_s 30 10^6 psi

A_s 0.06 in.^{2} (for one bolt)

E_p 500 ksi

A_p 1.5 in.^{2}

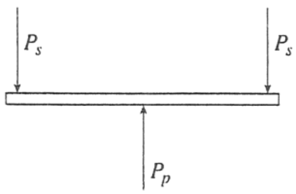
n 1 (see Eq. 2-24)

EQUILIBRIUM EQUATION

P_s tensile force in one steel bolt

P_p compressive force in plastic cylinder

$P_p \quad 2P_s$ (Eq. 1)

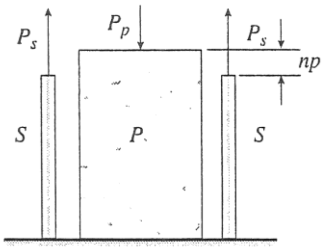


COMPATIBILITY EQUATION

d_s elongation of steel bolt

d_p shortening of plastic cylinder

$d_s \quad d_p \quad np$ (Eq. 2)



FORCE-DISPLACEMENT RELATIONS

$$d_s \quad \frac{P_s L}{E_s A_s} \qquad d_p \quad \frac{P_p L}{E_p A_p} \qquad \text{(Eq. 3, Eq. 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_s L}{E_s A_s} + \frac{P_p L}{E_p A_p} \quad np \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$P_p \quad \frac{2\,np\,E_s\,A_s\,E_p\,A_p}{L(E_p\,A_p + 2E_s\,A_s)}$$

Stress in the plastic cylinder

$$\sigma_p \quad \frac{P_p}{A_p} \quad \frac{2\,np\,E_s\,A_s\,E_p}{L(E_p\,A_p + 2E_s\,A_s)}$$

Substitute numerical values:

$$N \quad E_s A_s E_p \quad 900 \quad 10^9\ \text{lb}^2/\text{in.}^2$$

$$D \quad E_p A_p \quad 2E_s A_s \quad 4350 \quad 10^3\ \text{lb}$$

$$\sigma_p \quad \frac{2np}{L}\left(\frac{N}{D}\right) \quad \frac{2(1)(0.058\ \text{in.})}{10\ \text{in.}}\left(\frac{N}{D}\right)$$

$$2400\ \text{psi}$$

---

solder over distance s. The sleeve has brass caps at both ends, which are held in place by a steel bolt and washer with the nut turned just snug at the outset. Then, two

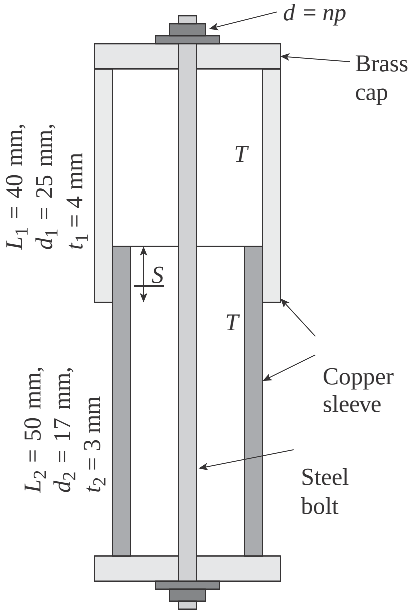
temperature is raised by T 30°C.

(a) Find the forces in the sleeve and bolt, P_s and P_B, due to both the prestress in the bolt and the temperature increase. For copper, use E_c 120 GPa and

The pitch of the bolt threads is p 1.0 mm. Assume s 26 mm and bolt diameter d_b 5 mm.

(b) Find the required length of the solder joint, s, if shear stress in the sweated joint cannot exceed the allowable shear stress τ_{aj} 18.5 MPa.

(c) What is the final elongation of the entire assemblage due to both temperature change T and the initial prestress in the bolt?



Solution 2.5-22

The figure shows a section through the sleeve, cap, and bolt.

Numerical properties

$n \quad \frac{1}{2} \qquad p \quad 1.0\ \text{mm} \qquad T \quad 30°\text{C}$

$E_c \quad 120\ \text{GPa} \qquad \alpha_c \quad 17 \quad (10^{-6})/°\text{C}$

$E_s \quad 200\ \text{GPa} \qquad \alpha_s \quad 12 \quad (10^{-6})/°\text{C}$

$\tau_{aj} \quad 18.5\ \text{MPa} \qquad s \quad 26\ \text{mm} \qquad d_b \quad 5\ \text{mm}$

$L_1 \quad 40\ \text{mm} \quad t_1 \quad 4\ \text{mm} \quad L_2 \quad 50\ \text{mm} \quad t_2 \quad 3\ \text{mm}$

$d_1 \quad 25\ \text{mm} \quad d_1 \quad 2t_1 \quad 17\ \text{mm} \qquad d_2 \quad 17\ \text{mm}$

$$A_b \quad \frac{\pi}{4} d_b^2 \qquad A_1 \quad \frac{\pi}{4}[d_1^2 \quad 1\,d_1 \quad 2t_1\,2^2]$$

$$A_b \quad 19.635\ \text{mm}^2 \qquad A_1 \quad 263.894\ \text{mm}^2$$

$$A_2 \quad \frac{\pi}{4}[\,d_2^2 \quad 1\,d_2 \quad 2t_2 2^2] \qquad A_2 \quad 131.947\ \text{mm}^2$$

(a) Forces in sleeve and bolt

One-degree statically indeterminate—cut bolt and use force in bolt (P_B) as redundant (see sketches):

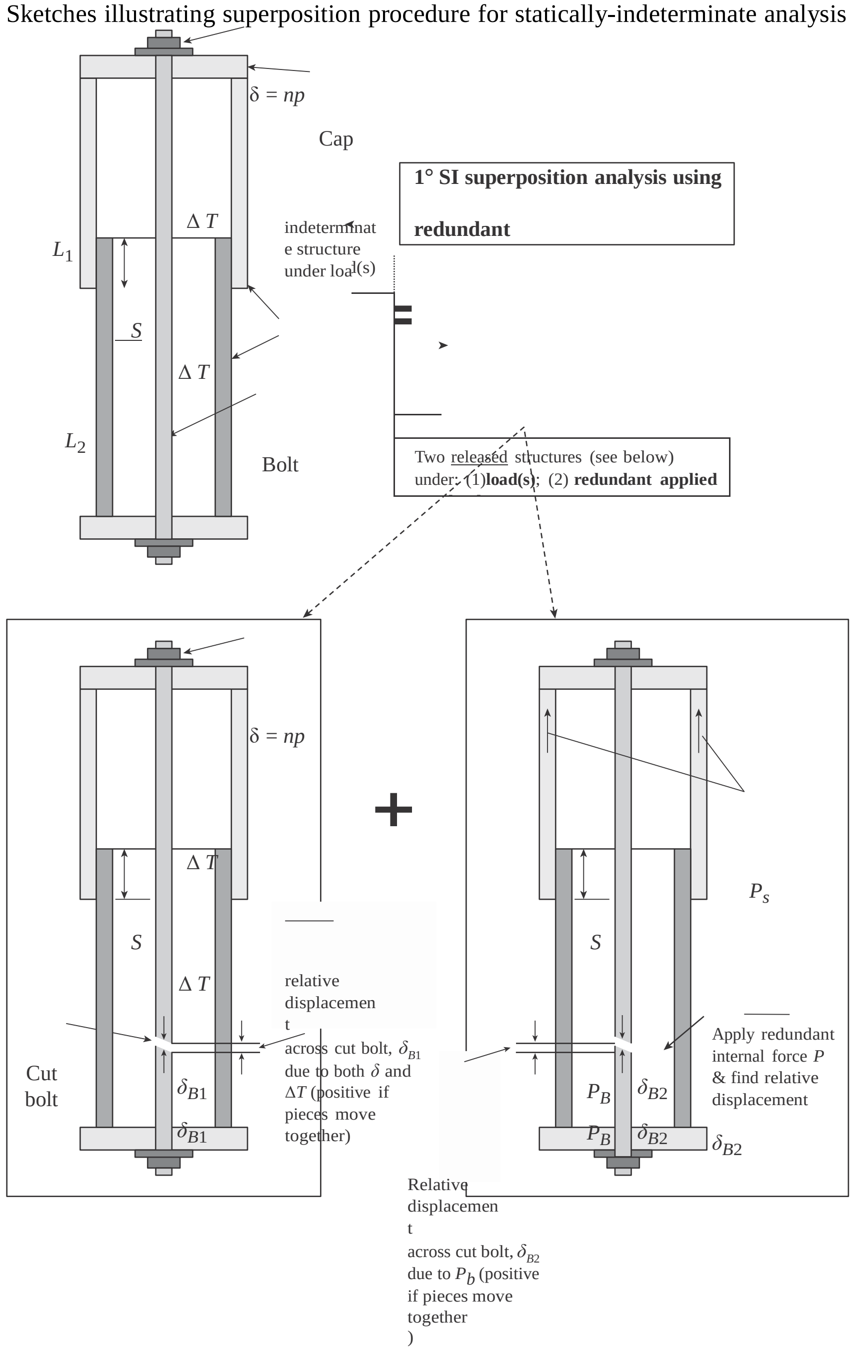
$$\delta_{B1} \quad np \quad \alpha_s \; T(L_1 \quad L_2 \quad s)$$

$$d_{B2} \quad P_B \left(\frac{L_1 + L_2 \quad s}{E_s A_b} + \frac{L_1 \quad s}{E_c A_1} + \frac{L_2 \quad s}{E_c A_2} + \frac{s}{E_c (A_1 + A_2)} \right)$$

Compatibility: $d_{B1} \quad d_{B2} \quad 0$

$$P_B \quad \frac{[\quad np + \alpha_s \Delta T(L_1 + L_2 \quad s)]}{\left(\frac{L_1 + L_2 \quad s}{E_s A_b} + \frac{L_1 \quad s}{E_c A_1} + \frac{L_2 \quad s}{E_c A_2} + \frac{s}{E_c (A_1 + A_2)} \right)} \qquad P_B \quad 25.4 \text{ kN} \qquad P_s \quad P_B$$

Sketches illustrating superposition procedure for statically-indeterminate analysis



(b) REQUIRED LENGTH OF SOLDER JOINT≈

$$t \quad \frac{P}{A_s} \qquad A_s \quad pd_2 s$$

$$s_{\text{reqd}} \quad \frac{P_B}{p d_2 t_{aj}} \qquad s_{\text{reqd}} \quad 25.7 \text{ mm}$$

(c) FINAL ELONGATION

d_f net of elongation of bolt (d_b) and shortening of sleeve (d_s)

$$d_b \quad P_B \left(\frac{L_1 + L_2 \;\; s}{E_s A_b} \right) \qquad d_b \quad 0.413 \text{ mm}$$

$$d_s \quad P_s \left(\frac{L_1 \;\; s}{E_c A_1} + \frac{L_2 \;\; s}{E_c A_2} + \frac{s}{E_c (A_1 + A_2)} \right)$$

$$d_s \quad 0.064 \text{ mm}$$

$$d_f \quad d_b \quad d_s \qquad d_f \quad 0.35 \text{ mm}$$

---

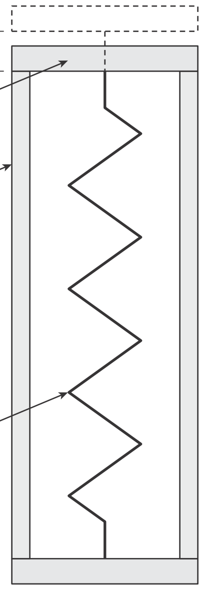
compresses a spring (with undeformed length L_1 L) by amount d (L_1 L).

the redundant. Use numerical properties in the boxes given.

(a) What is the resulting force in the spring, F_k?

(c) What is the final length of the tube, L_f?

(d) What temperature change T inside the tube will result in zero force in the spring?



Modulus of elasticity
Polyethylene tube (E_t = 100 ksi)

Coefficients of thermal expansion
$a_t = 80 \quad 10^{-6}$/°F, $a_k = 6.5 \quad 10^{-6}$/°F

Properties and dimensions
$d_0 = 6$ in. $t = \frac{1}{8}$ in.

$L_1 = 12.125$ in. > $L = 12$ in. $k = 1.5$ k/in.

Solution 2.5-23

The figure shows a section through the tube, cap, and spring.

Properties and dimensions:

d_o 6 in. t $\frac{1}{8}$ in. E_t 100 ksi

A_t $\frac{p}{4}[d_o^2 \quad (d_o \quad 2t)^2]$ A_t 2.307 in.^{2}

L_1 12.125 in. L 12 in. k 1.5 k/in.

Spring is 1/8 in. longer than tube

d L_1 L d 0.125 in.

a_k 6.5(10 ^{6})/ F a_t 80 (10 ^{6})/ F

T 0 note that Q result below is for zero temperature (until part(d))

(a) FORCE IN SPRING F_K REDUNDANT Q

Flexibilities: f $\frac{1}{k}$ f_t $\frac{L}{E_t A_t}$

d_2 relative displacement across cut spring due to redundant $Q(f \quad f_t)$

d_1 relative displacement across cut spring due to precompression and T d a_k TL_1 a_t TL

Compatibility: d_1 d_2 0

Solve for redundant Q:

$$Q \quad \frac{d + ¢T(a_k L_1 + a_t L)}{f + f_t} \quad F_k$$

F_k 0.174 k compressive force in spring (F_k) and also tensile force in tube

(b) F_t force in tube Q

NOTE: If tube is rigid, F_k k 0.1875 k

(c) FINAL LENGTH OF TUBE

L_f L d_{c1} d_{c2} i.e., add displacements for the two released structures to initial tube length L

L_f L Qf_t $a_t(\quad T)L$ L_f 12.01 in.

(d) SET Q 0 TO FIND T REQUIRED TO REDUCE SPRING FORCE TO ZERO

$$¢T_{reqd} \quad \frac{d}{(\quad a_k L_1 + a_t L)}$$

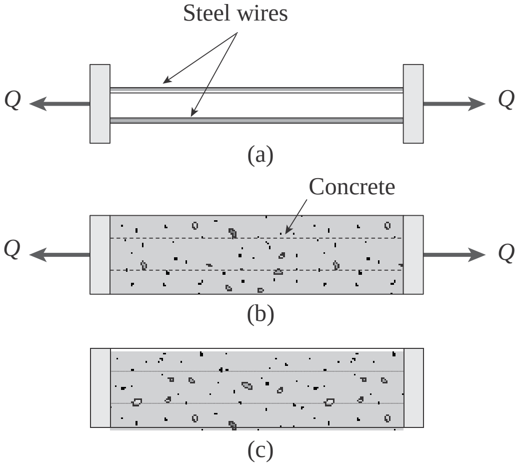
T_{reqd} 141.9 F

Since a_t a_k, a temp. increase is req'd to expand tube so that spring force goes to zero.

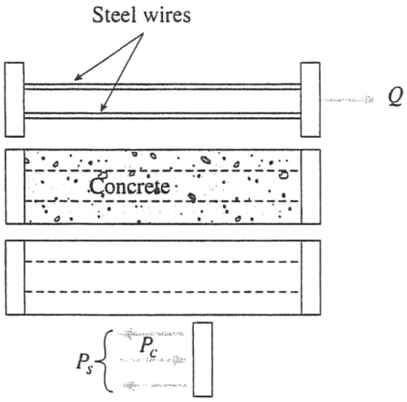
Problem 2.5-24 Prestressed concrete beams are sometimes manufactured in the following manner. High-strength steel wires are stretched by a jacking mechanism that applies a force Q, as represented schematically in part (a) of the figure. Concrete is then poured around the wires to form a beam, as shown in part (b).

After the concrete sets properly, the jacks are released and the force Q is removed [see part (c) of the figure]. Thus, the beam is left in compression.

Let us assume that the prestressing force Q produces in the steel wires an initial stress s_0 620 MPa. If the moduli of elasticity of the steel and concrete are in the ratio 12:1 and the cross-sectional areas are in the ratio 1:50, what are the final stresses s_s and s_c in the two materials?



Solution 2.5-24 Prestressed concrete beam



L length

s_0 initial stress in wires

$\frac{Q}{A_s}$ 620 MPa

A_s total area of steel wires

A_c area of concrete

50 A_s

E_s 12 E_c

P_s final tensile force in steel wires

P_c final compressive force in concrete

EQUILIBRIUM EQUATION

$P_s \quad P_c$ (Eq. 1)

COMPATIBILITY EQUATION AND FORCE-DISPLACEMENT RELATIONS

d_1 initial elongation of steel wires

$$\frac{QL}{E_s A_s} \quad \frac{s_0 L}{E_s}$$

d_2 final elongation of steel wires

$$\frac{P_s L}{E_s A_s}$$

d_3 shortening of concrete

$$\frac{P_c L}{E_c A_c}$$

$d_1 \quad d_2 \quad d_3$ or

$$\frac{s_0 L}{E_s} \quad \frac{P_s L}{E_s A_s} \quad \frac{P_c L}{E_c A_c} \qquad \text{(Eq. 2, Eq. 3)}$$

Solve simultaneously Eqs. (1) and (3):

$$P_s \quad P_c \quad \frac{s_0 A_s}{1 + \frac{E_s A_s}{E_c A_c}}$$

STRESSES

$$s_s \quad \frac{P_s}{A_s} \quad \frac{s_0}{1 + \frac{E_s A_s}{E_c A_c}}$$

$$s_c \quad \frac{P_c}{A_c} \quad \frac{s_0}{\frac{A_c}{A_s} + \frac{E_s}{E_c}}$$

SUBSTITUTE NUMERICAL VALUES:

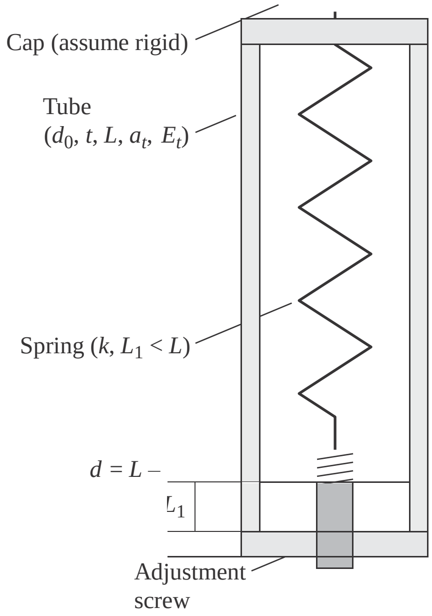
$$s_0 \quad 620 \text{ MPa} \qquad \frac{E_s}{E_c} \quad 12 \qquad \frac{A_s}{A_c} \quad \frac{1}{50}$$

$$s_s \quad \frac{620 \text{ MPa}}{1 + \frac{12}{50}} \quad 500 \text{ MPa (Tension)}$$

$$s_c \quad \frac{620 \text{ MPa}}{50 + 12} \quad 10 \text{ MPa (Compression)}$$

Problem 2.5-25 A polyethylene tube (length L) has a cap which is held in place by a spring (with undeformed length L_1 L). After installing the cap, the spring is post-tensioned by turning an adjustment screw by amount d. Ignore deformations of the cap and base. Use the force at the base of the spring as the redundant. Use numerical properties in the boxes below.

(a) What is the resulting force in the spring, F_k?
(b) What is the resulting force in the tube, F_t?
(c) What is the final length of the tube, L_f?
(d) What temperature change T inside the tube will result in zero force in the spring?



Modulus of elasticity
Polyethylene tube (E_t = 100 ksi)

Coefficients of thermal expansion
$a_t = 80 \quad 10^{-6}$/°F, $a_k = 6.5 \quad 10^{-6}$/°F

Properties and dimensions
$d_0 = 6$ in. $t = \frac{1}{8}$ in.
$L = 12$ in. $L_1 = 11.875$ in. $k = 1.5$ k/in.

Solution 2.5-25

The figure shows a section through the tube, cap, and spring.

Properties and dimensions:

$$d_o \quad 6 \text{ in.} \qquad t \quad \frac{1}{8} \text{ in.} \qquad E_t \quad 100 \text{ ksi}$$

$$L \quad 12 \text{ in.} \qquad L_1 \quad 11.875 \text{ in.} \qquad k \quad 1.5 \text{ k/in.}$$

$$a_k \quad 6.5(10 \quad ^6) \qquad a_t \quad 80 \quad (10 \quad ^6)$$

$$A_t \quad \frac{p}{4}[\,d_o^2 \quad 1\,d_o \quad 2t2^2\,]$$

$$A_t \quad 2.307 \text{ in.}^2$$

Pretension and temperature:
Spring is 1/8 in. shorter than tube.

$d \quad L \quad L_1 \qquad d \quad 0.125$ in. $\qquad T \quad 0$
Note that Q result below is for zero temperature (until part (d)).

Flexibilities: $\quad f \quad \frac{1}{k} \qquad f_t \quad \frac{L}{E_t A_t}$

(a) Force in spring (F_k) redundant (Q)

Follow solution procedure outlined in Prob. 2.5-23 solution:

$$Q \quad \frac{d + ¢T1 \quad a_k L_1 + a_t L2}{f + f_t} \qquad F_k$$

F_k 0.174 k ⟸ also the compressive force in the tube

(b) FORCE IN TUBE F_t Q 0.174 k ⟸

(c) FINAL LENGTH OF TUBE AND SPRING L_f L d_{c1} d_{c2}

L_f L Qf_t $a_t(T)L$ L_f 11.99 in. ⟸

(d) SET Q 0 TO FIND T REQUIRED TO REDUCE SPRING FORCE TO ZERO

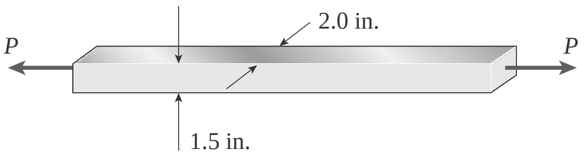
$$\Delta T_{\text{reqd}} \quad \frac{d}{1 \quad a_k L_1 + a_t L_2}$$

T_{reqd} 141.6 F

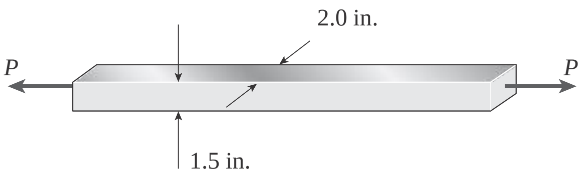
Since a_t a_k, a temperature drop is required to shrink tube so that spring force goes to zero.

Stresses on Inclined Sections

Problem 2.6-1 A steel bar of rectangular cross section (1.5 in. 2.0 in.) carries a tensile load P (see figure). The allowable stresses in tension and shear are 14,500 psi and 7,100 psi, respectively. Determine the maximum permissible load P_{max}.



Solution 2.6-1



NUMERICAL DATA

A 3 in.^{2} s_a 14500 psi

t_a 7100 psi

MAXIMUM LOAD—TENSION

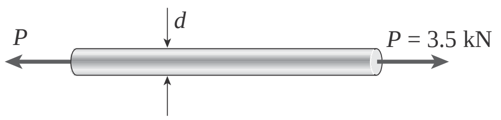
P_{max1} $s_a A$ P_{max1} 43500 lbs

MAXIMUM LOAD—SHEAR

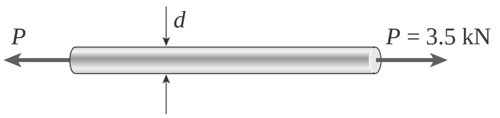
P_{max2} $2t_a A$ P_{max2} 42,600 lbs

Because t_{allow} is less than one-half of s_{allow}, the shear stress governs.

Problem 2.6-2 A circular steel rod of diameter d is subjected to a tensile force P 3.5 kN (see figure). The allowable stresses in tension and shear are 118 MPa and 48 MPa, respectively. What is the minimum permissible diameter d_{min} of the rod?



Solution 2.6-2



NUMERICAL DATA P 3.5 kN s_a 118 MPa

t_a 48 MPa

Find P_{max} then rod diameter.

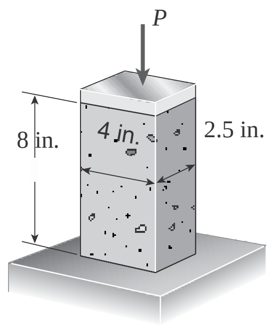
since t_a is less than 1/2 of s_a, shear governs.

$$P_{max} \quad 2t_a \left(\frac{p}{4} d_{min}^2\right)$$

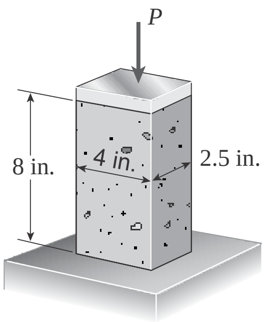
$$d_{min} \quad \sqrt{\frac{2}{p t_a} P}$$

$$d_{min} \quad 6.81 \text{ mm}$$

Problem 2.6-3 A standard brick (dimensions 8 in. 4 in. 2.5 in.) is compressed lengthwise by a force P, as shown in the figure. If the ultimate shear stress for brick is 1200 psi and the ultimate compressive stress is 3600 psi, what force P_{max} is required to break the brick?



Solution 2.6-3 Standard brick in compression



A 2.5 in. 4.0 in. 10.0 in.^{2}

Maximum normal stress:

$$s_x \quad \frac{P}{A}$$

Maximum shear stress:

$$t_{max} \quad \frac{s_x}{2} \quad \frac{P}{2A}$$

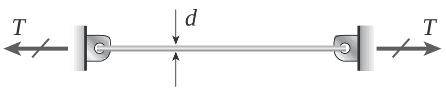
s_{ult} 3600 psi t_{ult} 1200 psi

Because t_{ult} is less than one-half of s_{ult}, the shear stress governs.

$$t_{max} \quad \frac{P}{2A} \quad \text{or} \quad P_{max} \quad 2At_{ult}$$

$$P_{max} \quad 2(10.0 \text{ in.}^2)(1200 \text{ psi}) \quad 24{,}000 \text{ lb}$$

Problem 2.6-4 A brass wire of diameter d 2.42 mm is stretched tightly between rigid supports so that the tensile force is T 98 N (see figure). The coefficient of thermal expansion for the wire is 19.5 * 10 ^{6}/ C and the modulus of elasticity is E 110 GPa.



(a) What is the maximum permissible temperature drop T if the allowable shear stress in the wire is 60 MPa?
(b) At what temperature changes does the wire go slack?

Solution 2.6-4 Brass wire in tension

NUMERICAL DATA

d 2.42 mm T 98 N

a 19.5 (10 ^{6})/°C E 110 GPa

(a) $¢T_{max}$ (DROP IN TEMPERATURE)

$$s \quad \frac{T}{A} \quad (E a\ ¢T) \qquad t_{max} \quad \frac{s}{2}$$

$$t_a \quad \frac{T}{2A} \quad \frac{E a\ ¢T}{2}$$

t_a 60 MPa A $\frac{p}{4} d^2$

$$¢T_{max} \quad \frac{\frac{T}{A} \quad 2\, t_a}{E a}$$

$¢T_{max}$ 46 C (drop)

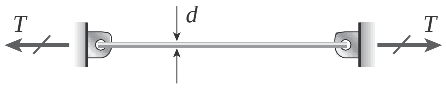
(b) $¢T$ AT WHICH WIRE GOES SLACK

Increase $¢T$ until s 0:

$$¢T \quad \frac{T}{E aA}$$

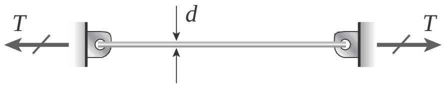
$¢T$ 9.93 C (increase)

Problem 2.6-5 A brass wire of diameter d 1/16 in. is stretched between rigid supports with an initial tension T of 37 lb (see figure). Assume that the coefficient of thermal expansion is 10.6 10 ^{6}/°F and the modulus of elasticity is 15 10^{6} psi.)



(a) If the temperature is lowered by 60°F, what is the maximum shear stress t_{max} in the wire?
(b) If the allowable shear stress is 10,000 psi, what is the maximum permissible temperature drop?
(c) At what temperature change T does the wire go slack?

Solution 2.6-5



NUMERICAL DATA

$d \quad \frac{1}{16}$ in. T 37 lb a 10.6 (10 ^{6})/ F

E 15 (10^{6}) psi T 60 F

$A \quad \frac{p}{4} d^2$

(a) t_{max} (DUE TO DROP IN TEMPERATURE)

$$t_{max} \quad \frac{s_x}{2} \qquad t_{max} \quad \frac{\frac{T}{A} \quad (E a\, ¢T)}{2}$$

t_{max} 10,800 psi

(b) $¢T_{max}$ FOR ALLOWABLE SHEAR STRESS

t_a 10000 psi

$$¢T_{max} \quad \frac{\frac{T}{A} \quad 2t_a}{E a}$$

T_{max} 49.9 F

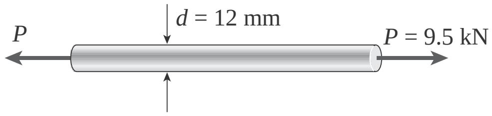
(c) T AT WHICH WIRE GOES SLACK

Increase T until s 0:

$$¢T \quad \frac{T}{E aA}$$

T 75.9 F (increase)

Problem 2.6-6 A steel bar with diameter $d = 12$ mm is subjected to a tensile load $P = 9.5$ kN (see figure).



(a) What is the maximum normal stress σ_{max} in the bar?
(b) What is the maximum shear stress τ_{max}?
(c) Draw a stress element oriented at 45° to the axis of the bar and show all stresses acting on the faces of this element.

(d) Repeat part (c) for a stress element oriented at 22.5° to the axis of the bar.

Solution 2.6-6

(a) $d = 12$ mm $P = 9.5$ kN $A = \frac{\pi}{4}d^2 = 1.131 \times 10^{-4}$ m^2

$\boxed{\sigma_x = \frac{P}{A} = 84 \text{ MPa}}$

(b) $\boxed{\tau_{max} = \frac{\sigma_x}{2} = 42 \text{ MPa}}$ On plane stress element rotated 45°

(c) ROTATED STRESS ELEMENT (45°) HAS NORMAL TENSILE STRESS $\sigma_x/2$ ON ALL FACES, T_{max} (CW) ON x-FACE, AND T_{max} (CCW) ON y-FACE

$\tau_{xy1y1} = \tau_{max}$ $\sigma_{x1} = \frac{\sigma_x}{2}$ $\sigma_{y1} = \sigma_{x1}$

On rotated x-face: $\boxed{\sigma_{x1} = 42 \text{ MPa}}$ $\boxed{\tau_{x1y1} = 42 \text{ MPa}}$

On rotated y-face: $\boxed{\sigma_{y1} = 42 \text{ MPa}}$

(d) $\theta = 22.5°$ CCW ROTATION OF ELEMENT

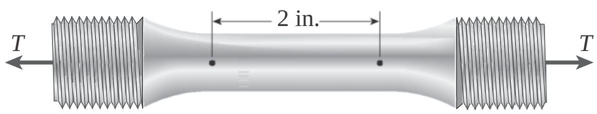
$\sigma_\theta = \sigma_x \cos(\theta)^2 = 71.7$ MPa on rotated x face $\sigma_y = \sigma_x \cos\left(\theta + \frac{\pi}{2}\right)^2 = 12.3$ MPa on rotated y face

Eq. 2-31b $\tau_\theta = \frac{-\sigma_x}{2}\sin(2\theta) = -29.7$ MPa CW on rotated x-face

On rotated x-face: $\boxed{\sigma_{x1} = 71.7 \text{ MPa}}$ $\boxed{\tau_{x1y1} = -29.7 \text{ MPa}}$

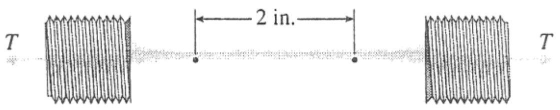
On rotated y-face: $\boxed{\sigma_{y1} = 12.3 \text{ MPa}}$

Problem 2.6-7 During a tension test of a mild-steel specimen (see figure), the extensometer shows an elongation of 0.00120 in. with a gage length of 2 in. Assume that the steel is stressed below the proportional limit and that the modulus of elasticity $E = 30 \times 10^6$ psi.



(a) What is the maximum normal stress σ_{max} in the specimen?
(b) What is the maximum shear stress τ_{max}?
(c) Draw a stress element oriented at an angle of 45° to the axis of the bar and show all stresses acting on the faces of this element.

Solution 2.6-7 Tension test



Elongation: $\delta \quad 0.00120$ in.
(2 in. gage length)

Strain: $\quad \frac{\delta}{L} \quad \frac{0.00120\text{ in.}}{2\text{ in.}} \quad 0.00060$

Hooke's law: $\sigma_x \quad E \quad (30 \quad 10^6\text{ psi})(0.00060)$

$\quad 18{,}000\text{ psi}$

(a) Maximum normal stress

σ_x is the maximum normal stress.

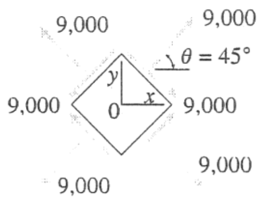
$\sigma_{\max} \quad 18{,}000\text{ psi}$

(b) Maximum shear stress

The maximum shear stress is on a 45° plane and equals $\sigma_x/2$.

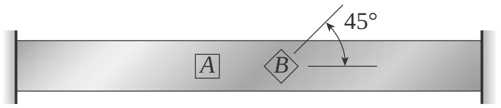
$\tau_{\max} \quad \frac{\sigma_x}{2} \quad 9{,}000\text{ psi}$

(c) Stress element at $\theta \quad 45°$



NOTE: All stresses have units of psi.

Problem 2.6-8 A copper bar with a rectangular cross section is held without stress between rigid supports (see figure). Subsequently, the temperature of the bar is raised 50 C.



(a) Determine the stresses on all faces of the elements A and B, and show these stresses on sketches of the elements. (Assume $\alpha \quad 17.5 * 10^{-6}$/ C and $E \quad 120$ GPa.)

(b) If the shear stress at B is known to be 48 MPa at some inclination θ, find angle θ and show the stresses on a sketch of a properly oriented element.

Solution 2.6-8

(a) $\alpha \quad 17.5 (10^{-6}) \quad \Delta T \quad 50 \quad E \quad 120\text{ GPa}$

$\sigma_x \quad E\alpha\Delta T \quad 105\text{ MPa} \qquad \tau_{\max} \quad \frac{\sigma_x}{2} \quad 52.5\text{ MPa} \qquad \text{at } \theta \quad 45$

(compression)

Element A: $\sigma_x \quad 105$ MPa (compression);   Element B: $\tau_{\max} \quad 52.5$ MPa

(b) $\tau_\theta \quad 48\text{ MPa}$

Eq. 2-31b $\qquad \tau_\theta \quad \frac{-\sigma_x}{2}\sin(2\theta)$

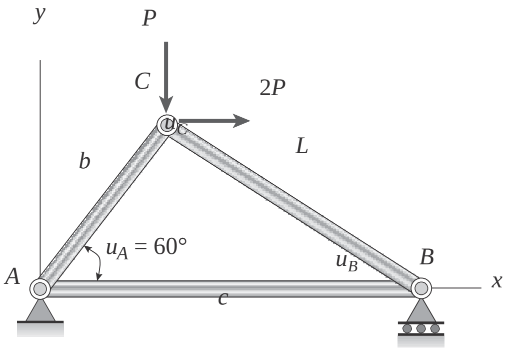
so $\qquad \theta \quad \frac{1}{2}\operatorname{asin}\left(\frac{2\tau_\theta}{\sigma_x}\right) \quad 33.1 \qquad$ CCW rotation of element $\quad \boxed{\theta \quad 33.1}$

$\sigma_\theta \quad \theta_x\cos(\theta)^2 \quad 73.8\text{ MPa} \qquad$ on rotated x face

$\sigma_y \quad \sigma_x\cos\left(\theta + \frac{\pi}{2}\right)^2 \quad 31.2\text{ MPa} \qquad$ on rotated y face

Problem 2.6-9 The plane truss below is assembled from steel C10 * 20 shapes (see Table 3(a) in Appendix F). Assume that L 10 ft and b 0.71 L.

(a) If load variable P 49 k, what is the maximum shear stress t_{max} in each truss member?
(b) What is the maximum permissible value of load variable P if the allowable normal stress is 14 ksi and the allowable shear stress is 7.5 ksi?



Solution 2.6-9

NUMERICAL DATA

L 10 ft $\quad b$ 0.71 $L \quad P$ 49 k $\quad s_a$ 14 ksi $\quad t_a$ 7.5 ksi $\quad A$ 5.87 in.^{2}

(a) FOR LINEAR ANALYSIS, MEMBER FORCES ARE PROPORTIONAL TO LOADING

FROM EXAMPLE 1-1: (solution for P 35 k)

$$F_{AC} \quad \frac{P}{35}15.59 \quad 21.826\text{ k} \qquad F_{AB} \quad \frac{P}{35}62.2 \quad 87.08\text{ k}$$

$$F_{BC} \quad \frac{P}{35}(\ \ 78.9) \qquad F_{BC} \quad 110.46\text{ k}$$

Normal stresses in each member:

$$s_{AC} \quad \frac{F_{AC}}{A} \quad 3.718\text{ ksi} \qquad s_{AB} \quad \frac{F_{AB}}{A} \quad 14.835\text{ ksi}$$

From Eq. 2-33:

$$s_{BC} \quad \frac{F_{BC}}{A} \quad 18.818\text{ ksi}$$

$$\boxed{t_{maxAC} \quad \frac{s_{AC}}{2} \quad 1.859\text{ ksi}} \qquad \boxed{t_{maxAB} \quad \frac{s_{AB}}{2} \quad 7.42\text{ ksi}}$$

$$\boxed{t_{maxBC} \quad \frac{s_{BC}}{2} \quad 9.41\text{ ksi}}$$

(b) s_a 6 2 * T_a so normal stress will control; lowest value governs here

MEMBER AC: $\quad P_{maxs} \quad \frac{P}{F_{AC}}(s_a A) \quad 184.496\text{ k} \qquad P_{maxt} \quad \frac{P}{F_{AC}}(2\,t_a A) \quad 197.675\text{ k}$

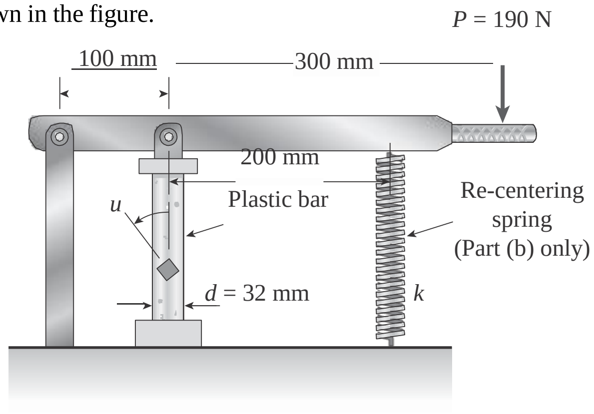
MEMBER AB: $\quad P_{maxs} \quad \frac{P}{F_{AB}}(s_a A) \quad 46.243\text{ k} \qquad P_{maxt} \quad \frac{P}{F_{AB}}(2\,t_a A) \quad 49.546\text{ k}$

MEMBER BC: $\quad \boxed{P_{maxs} \quad \frac{P}{F_{BC}} 1 s_a A2 \quad 36.5\text{ k}} \qquad P_{maxt} \quad \frac{P}{F_{BC}} 12\,t_a A2 \quad 39.059\text{ k}$

Problem 2.6-10 A plastic bar of diameter d 32 mm is compressed in a testing device by a force P 190 N applied as shown in the figure.

(a) Determine the normal and shear stresses acting on all faces of stress elements oriented at (1) an angle u 0°, (2) an angle u 22.5°, and (3) an angle u 45°. In each case, show the stresses on a sketch of a properly oriented element. What are s_{max} and t_{max}?

(b) Find s_{max} and t_{max} in the plastic bar if a re-centering spring of stiffness k is inserted into the testing device, as shown in the figure. The spring stiffness is 1/6 of the axial stiffness of the plastic bar.



Solution

NUMERICAL DATA

d 32 mm A $\frac{p}{4}d^2$

P 190 N A 804.25 mm^2

a 100 mm

b 300 mm

(a) STATICS—FIND COMPRESSIVE FORCE F AND STRESSES IN PLASTIC BAR

F $\frac{P(a+b)}{a}$ F 760 N

s_x $\frac{F}{A}$ s_x 0.945 MPa or s_x 945 kPa

From (1), (2), and (3) below:

s_{max} s_x s_{max} 945 kPa

t_{max} 472 kPa $\frac{s_x}{2}$ 472 kPa

(1) u 0 s_x 945 kPa

(2) u 22.50

On x-face:

s_u $s_x\cos(u)^2$

s_u 807 kPa

t_u $s_x\sin(u)\cos(u)$

t_u 334 kPa

On y-face: u $u + \frac{p}{2}$

s_u $s_x\cos(u)^2$

s_u 138.39 kPa

t_u $s_x\sin(u)\cos(u)$

t_u 334.1 kPa

(3) u 45

On x-face:

s_u $s_x\cos(u)^2$

s_u 472 kPa

t_u $s_x\sin(u)\cos(u)$

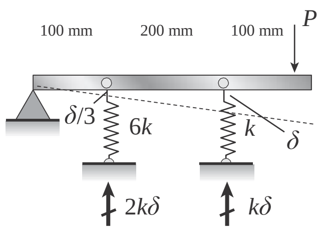
t_u 472 kPa

On y-face: u $u + \frac{p}{2}$

s_u $s_x\cos(u)^2$ s_u 472.49 kPa

t_u $s_x\sin(u)\cos(u)$ t_u 472.49 kPa

(b) ADD SPRING—FIND MAXIMUM NORMAL AND SHEAR STRESSES IN PLASTIC BAR



$a M_{\text{pin}} \quad 0$

$P(400) \quad [2kd(100) \quad kd(300)]$

$d \quad \dfrac{4}{5}\dfrac{P}{k}$

Force in plastic bar: $\quad F \quad (2k)\,a\dfrac{4}{5}\dfrac{P}{k}b$

$F \quad \dfrac{8}{5}P \qquad F \quad 304\ \text{N}$

Normal and shear stresses in plastic bar:

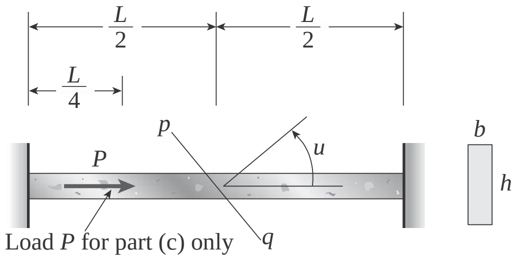
$s_x \quad \dfrac{F}{A} \qquad s_x \quad 0.38$

$s_{\max} \quad 378\ \text{kPa}$

$t_{\max} \quad \dfrac{s_x}{2} \qquad t_{\max} \quad 189\ \text{kPa}$

Problem 2.6-11 A plastic bar of rectangular cross section (b 1.5 in. and h 3 in.) fits snugly between rigid supports at room temperature (68°F) but with no initial stress (see figure). When the temperature of the bar is raised to 160°F, the compressive stress on an inclined plane pq at midspan becomes 1700 psi.

(a) What is the shear stress on plane pq? (Assume a 60 10^{6}/°F and E 450 10^3 psi.)
(b) Draw a stress element oriented to plane pq and show the stresses acting on all faces of this element.
(c) If the allowable normal stress is 3400 psi and the allowable shear stress is 1650 psi, what is the maximum load P (*in x direction*) which can be added at the quarter point (in addition to thermal effects above) without exceeding allowable stress values in the bar?



Solution 2.6-11

NUMERICAL DATA

$b \quad 1.5\ \text{in.} \quad h \quad 3\ \text{in.} \quad A \quad bh \quad T \quad (160 \quad 68)\ \text{F}$

$T \quad 92\ \text{F}$

$A \quad 4.5\ \text{in.}^2 \qquad s_{pq} \quad 1700\ \text{psi}$

$a \quad 60 \quad (10^{6})/\ \text{F}$

$E \quad 450 \quad (10^3)\ \text{psi}$

(a) SHEAR STRESS ON PLANE PQ

Statically indeterminate analysis gives, for reaction at right support:

$R \quad EAa\ T \qquad R \quad 11178\ \text{lb}$

$s_x \quad \dfrac{R}{A} \qquad s_x \quad 2484\ \text{psi}$

Using $s_u \quad s_x\cos(u)^2$: $\quad \cos 1u2^2 \quad \dfrac{s_{pq}}{s_x}$

$u \quad a\cos a\sqrt{\dfrac{s_{pq}}{A\ s_x}}\,b \qquad u \quad 34.2°$

Now with u, can find shear stress on plane pq:

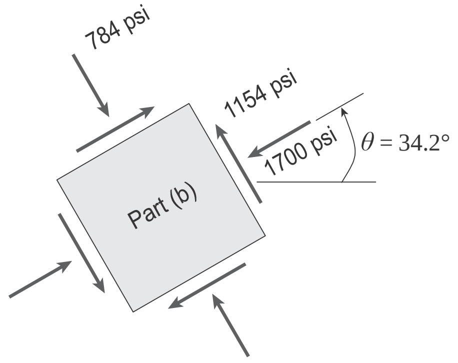
$t_{pq} \quad s_x\sin(u)\cos(u) \qquad t_{pq} \quad 1154\ \text{psi}$

$s_{pq} \quad s_x\cos(u)^2 \qquad s_{pq} \quad 1700\ \text{psi}$

Stresses at $u \quad p/2$ (y-face):

$s_y \quad s_x\cos a u + \dfrac{p}{2}b^2 \qquad s_y \quad 784\ \text{psi}$

(b) STRESS ELEMENT FOR PLANE *pq*



(c) MAXIMUM LOAD AT QUARTER POINT s_a 3400 psi

t_a 1650 psi $2t_a$ 3300 less than s_a, so shear controls

Statically indeterminate analysis for P at $L/4$ gives

for reactions:

$$R_{R2} \quad \frac{P}{4} \qquad R_{L2} \quad \frac{3}{4} P$$

(tension for 0 to $L/4$ and compression for rest of bar)

From part (a) (for temperature increase T):

R_{R1} EAa T R_{L1} EAa T

Stresses in bar (0 to $L/4$):

$$s_x \quad Ea¢T + \frac{3P}{4A} \qquad t_{max} \quad \frac{s_x}{2}$$

Set t_{max} t_a and solve for P_{max1}:

$$t_a \quad \frac{Ea¢T}{2} + \frac{3P}{8A}$$

$$P_{max1} \quad \frac{4A}{3} 12t_a + Ea¢T2$$

P_{max1} 34,704 lb

$$t_{max} \quad \frac{Ea¢T}{2} + \frac{3P_{max1}}{8A}$$

t_{max} 1650 psi check

$$s_x \quad Ea¢T + \frac{3P_{max1}}{4A}$$

s_x 3300 psi less than s_a

Stresses in bar ($L/4$ to L):

$$s_x \quad Ea¢T \quad \frac{P}{4A} \qquad t_{max} \quad \frac{s_x}{2}$$

Set t_{max} t_a and solve for P_{max2}:

P_{max2} $4A($ $2t_a$ Ea $T)$

P_{max2} 14,688 lb shear in segment ($L/4$ to L) controls

$$t_{max} \quad \frac{Ea¢T}{2} \quad \frac{P_{max2}}{8A} \qquad t_{max} \quad 1650 \text{ psi}$$

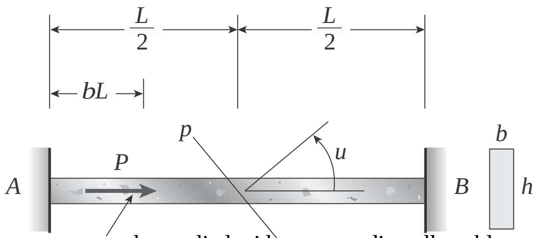
$$s_x \quad Ea¢T \quad \frac{P_{max2}}{4A} \qquad s_x \quad 3300 \text{ psi}$$

Problem 2.6-12 A copper bar of rectangular cross section (b 18 mm and h 40 mm) is held snugly (but without any initial stress) between rigid supports (see figure). The allowable stresses on the inclined plane pq at midspan, for which 55°, are specified as 60 MPa in compression and 30 MPa in shear.

(a) What is the maximum permissible temperature rise T if the allowable stresses on plane pq are not to be exceeded? (Assume a 17 10 ^{6}/°C and E 120 GPa.)

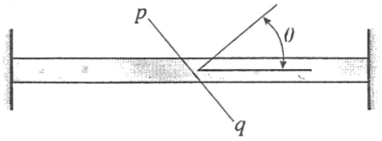
(b) If the temperature increases by the maximum permissible amount, what are the stresses on plane pq?

(c) If the temperature rise T 28°C, how far to the right of end A (distance bL, expressed as a fraction of length L) can load P 15 kN be applied without exceeding allowable stress values in the bar? Assume that s_a 75 MPa and t_a 35 MPa.



Load for part (c) only q

Solution 2.6-12



NUMERICAL DATA

$u \quad 55 a\frac{p}{180}b$ rad

$b \quad 18$ mm $\qquad h \quad 40$ mm

$A \quad bh \qquad A \quad 720\ \text{mm}^2$

$s_{pqa} \quad 60$ MPa $\qquad t_{pqa} \quad 30$ Mpa

$a \quad 17 \quad (10\ ^{6})/\ \text{C} \qquad E \quad 120$ GPa

$T \quad 20$ C $\qquad P \quad 15$ kN

(a) FIND T_{max} BASED ON ALLOWABLE NORMAL AND SHEAR STRESS VALUES ON PLANE pq

$$s_x \quad Ea \ \ T_{max} \qquad ¢T_{max} \quad \frac{s_x}{Ea}$$

$$s_{pq} \quad s_x\cos(u)^2 \qquad t_{pq} \quad s_x\sin(u)\cos(u)$$

Set each equal to corresponding allowable and solve for s_x:

$$s_{x1} \quad \frac{s_{pqa}}{\cos1u2^2} \qquad s_{x1} \quad 182.38\ \text{MPa}$$

$$s_{x2} \quad \frac{t_{pqa}}{\sin1u2\cos1u2} \qquad s_{x2} \quad 63.85\ \text{MPa}$$

Lesser value controls, so allowable shear stress governs.

$$¢T_{max} \quad \frac{s_{x2}}{Ea} \qquad T_{max} \quad 31.3\ \text{C}$$

(b) STRESSES ON PLANE PQ FOR MAXIMUM TEMPERATURE

$$s_x \quad Ea \ \ T_{max} \qquad s_x \quad 63.85\ \text{MPa}$$

$$s_{pq} \quad s_x\cos(u)^2 \qquad s_{pq} \quad 21.0\ \text{MPa}$$

$$t_{pq} \quad s_x\sin(u)\cos(u) \qquad t_{pq} \quad 30\ \text{MPa}$$

(c) ADD LOAD P IN x-DIRECTION TO TEMPERATURE CHANGE AND FIND LOCATION OF LOAD

$T \quad 28$ C

$P \quad 15$ kN from one-degree statically indeterminate analysis, reactions R_A and R_B due to load P:

$R_A \quad (1 \quad b)P \qquad R_B \quad bP$

Now add normal stresses due to P to thermal stresses due to T (tension in segment 0 to bL, compression in segment bL to L).

Stresses in bar (0 to bL):

$$s_x \quad Ea¢T + \frac{R_A}{A} \qquad t_{max} \quad \frac{s_x}{2}$$

Shear controls so set $t_{max} \quad t_a$ and solve for b:

$$2t_a \quad Ea¢T + \frac{(1 \quad b)P}{A}$$

$$b \quad 1 \quad \frac{A}{P}[2t_a + Ea¢T]$$

$$b \quad 5.1$$

Impossible so evaluate segment (bL to L):

Stresses in bar (bL to L):

$$s_x \quad Ea¢T \quad \frac{R_B}{A} \qquad t_{max} \quad \frac{s_x}{2}$$

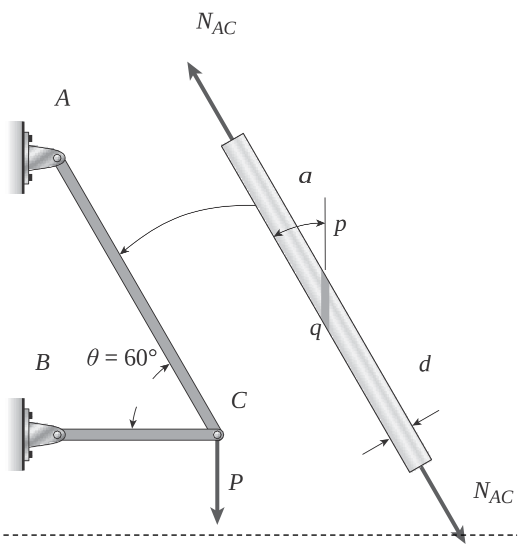
set $t_{max} \quad t_a$ and solve for P_{max2}

$$2t_a \quad Ea¢T \quad \frac{bP}{A}$$

$$b \quad \frac{A}{P}[\ 2t_a + Ea¢T]$$

$$b \quad 0.62$$

Problem 2.6-13 A circular brass bar of diameter d is member AC in truss ABC which has load $P = 5000$ lb applied at joint C. Bar AC is composed of two segments brazed together on a plane pq making an angle $\alpha = 36°$ with the axis of the bar (see figure). The allowable stresses in the brass are 13,500 psi in tension and 6500 psi in shear. On the brazed joint, the allowable stresses are 6000 psi in tension and 3000 psi in shear. What is the tensile force N_{AC} in bar AC? What is the minimum required diameter d_{min} of bar AC?



Solution 2.6-13

Numerical data

$P = 5$ k $\quad \alpha = 36° \quad \sigma_a = 13.5$ ksi

$\tau_a = 6.5$ ksi

$\theta = \frac{\pi}{2} - \alpha \quad \theta = 54°$

$\sigma_{ja} = 6.0$ ksi

$\tau_{ja} = 3.0$ ksi

Tensile force N_{AC} using Method of Joints at C:

$$N_{AC} = \frac{P}{\sin(60°)} \quad \text{(tension)}$$

$$N_{AC} = 5.77 \text{ k} \leftarrow$$

Minimum required diameter of bar AC:

(1) Check tension and shear in bars; $\tau_a < \sigma_a/2$ so shear controls $\tau_{max} = \frac{\sigma_x}{2}$:

$$2\tau_a = \frac{N_{AC}}{A} \quad \sigma_x = 2\tau_a = 13 \text{ ksi}$$

$$A_{reqd} = \frac{N_{AC}}{2\tau_a} \quad A_{reqd} = 0.44 \text{ in.}^2$$

$$d_{min} = \sqrt{\frac{4}{\pi} A_{reqd}} \quad d_{min} = 0.75 \text{ in.}$$

(2) Check tension and shear on brazed joint:

$$\sigma_x = \frac{N_{AC}}{A} \quad \sigma_x = \frac{N_{AC}}{\frac{\pi}{4} d^2} \quad d_{reqd} = \sqrt{\frac{4}{\pi} \frac{N_{AC}}{\sigma_x}}$$

Tension on brazed joint:

$$\sigma_\theta = \sigma_x \cos(\theta)^2$$

Set equal to σ_{ja} and solve for σ_x, then d_{reqd}:

$$\sigma_x = \frac{\sigma_{ja}}{\cos(\theta)^2} \quad \sigma_x = 17.37 \text{ ksi}$$

$$d_{reqd} = \sqrt{\frac{4}{\pi} \frac{N_{AC}}{\sigma_x}} \quad d_{reqd} = 0.65 \text{ in.}$$

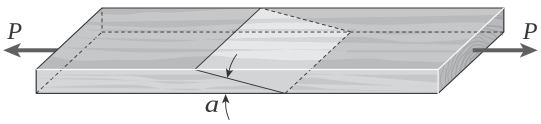
Shear on brazed joint:

$$\tau_\theta = -\sigma_x \sin(\theta) \cos(\theta)$$

$$\sigma_x = \frac{\tau_{ja}}{(\sin(\theta)\cos(\theta))} \quad \sigma_x = 6.31 \text{ ksi}$$

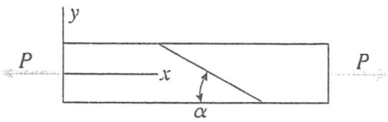
$$d_{reqd} = \sqrt{\frac{4}{\pi} \frac{N_{AC}}{\sigma_x}} \quad d_{reqd} = 1.08 \text{ in.} \leftarrow$$

Problem 2.6-14 Two boards are joined by gluing along a scarf joint, as shown in the figure. For purposes of cutting and gluing, the angle *a* between the plane of the joint and the faces of the boards must be between 10° and 40°. Under a tensile load *P*, the normal stress in the boards is 4.9 MPa.



(a) What are the normal and shear stresses acting on the glued joint if *a* 20°?
(b) If the allowable shear stress on the joint is 2.25 MPa, what is the largest permissible value of the angle *a*?
(c) For what angle *a* will the shear stress on the glued joint be numerically equal to twice the normal stress on the joint?

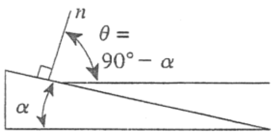
Solution 2.6-14 Two boards joined by a scarf joint



10° *a* 40°

Due to load *P*: s_x 4.9 MPa

(a) STRESSES ON JOINT WHEN *a* 20°



u 90° *a* 70°

s_u $s_x \cos^2 u$ (4.9 MPa)(cos 70°)^2

0.57 MPa

t_u s_x sin *u* cos *u*

(4.9 MPa)(sin 70°)(cos 70°)

1.58 MPa

(b) LARGEST ANGLE *a* IF t_{allow} 2.25 MPa

t_{allow} s_x sin *u* cos *u*

The shear stress on the joint has a negative sign. Its numerical value cannot exceed t_{allow} 2.25 MPa. Therefore,

2.25 MPa (4.9 MPa)(sin *u*)(cos *u*) or sin *u* cos

u 0.4592

From trigonometry: $\sin u \cos u \quad \frac{1}{2} \sin 2u$

Therefore: sin 2*u* 2(0.4592) 0.9184

Solving: 2*u* 66.69° or 113.31°

u 33.34° or 56.66°

a 90° *u* *a* 56.66° or 33.34°

Since *a* must be between 10° and 40°, we select

a 33.3°

NOTE: If *a* is between 10° and 33.3°,

| t_u | 2.25 MPa.

If *a* is between 33.3° and 40°,

| t_u | 2.25 MPa.

(c) WHAT IS *a* if t_u $2s_u$?

Numerical values only:

| t_u | s_x sin *u* cos *u* | s_u | $s_x \cos^2 u$

$$\frac{t_0}{s_0} \quad 2$$

s_x sin *u* cos *u* $2s_x \cos^2 u$

sin *u* 2 cos *u* or tan *u* 2

u 63.43° *a* 90° *u*

a 26.6°

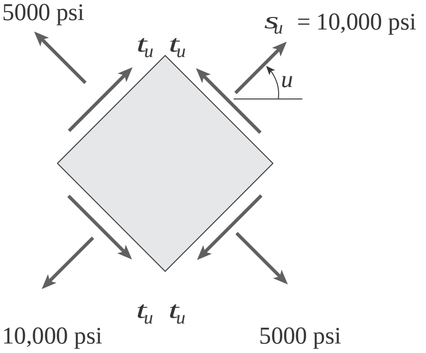
NOTE: For *a* 26.6° and *u* 63.4°, we find s_u 0.98 MPa and t_u 1.96 MPa.

Thus, $\frac{t_0}{s_0}$ 2 as required.

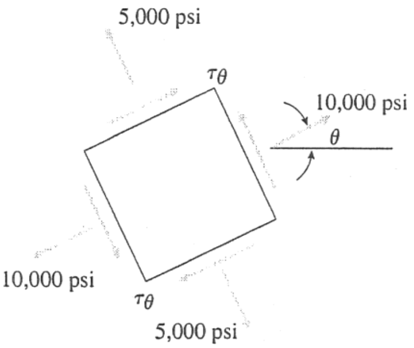
Problem 2.6-15 Acting on the sides of a stress element cut from a bar in uniaxial stress are tensile stresses of 10,000 psi and 5,000 psi, as shown in the figure.

(a) Determine the angle u and the shear stress t_u and show all stresses on a sketch of the element.

(b) Determine the maximum normal stress $s_{\max}$ and the maximum shear stress $t_{\max}$ in the material.



Solution 2.6-15 **Bar in uniaxial stress**



(a) ANGLE u AND SHEAR STRESS t_u

$s_u \quad s_x \cos^2 u$

$s_u \quad 10{,}000$ psi

$$s_x \quad \frac{s_0}{\cos^2 u} \quad \frac{10{,}000 \text{ psi}}{\cos^2 u} \qquad (1)$$

PLANE AT ANGLE $u \quad 90°$

$s_{u \quad 90°} \quad s_x[\cos(u \quad 90°)]^2 \quad s_x[\quad \sin u]^2$

$\quad s_x \sin^2 u$

$s_{u \quad 90°} \quad 5{,}000$ psi

$$s_x \quad \frac{s_{0 \quad 90}}{\sin^2 u} \quad \frac{5{,}000 \text{ psi}}{\sin^2 u} \qquad (2)$$

Equate (1) and (2):

$$\frac{10{,}000 \text{ psi}}{\cos^2 u} \quad \frac{5{,}000 \text{ psi}}{\sin^2 u}$$

$$\tan^2 u \quad \frac{1}{2} \quad \tan u \quad \frac{1}{\mathbf{1}2} \quad u \quad 35.26$$

From Eq. (1) or (2):

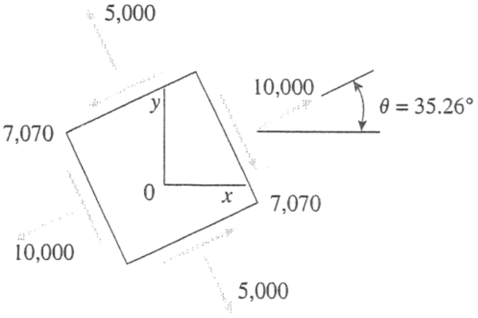
$s_x \quad 15{,}000$ psi

$t_u \quad \quad s_x \sin u \cos u$

$\quad (\quad 15{,}000 \text{ psi})(\sin 35.26°)(\cos 35.26°)$

$\quad 7{,}070$ psi

Minus sign means that t_u acts clockwise on the plane for which $u \quad 35.26°$.



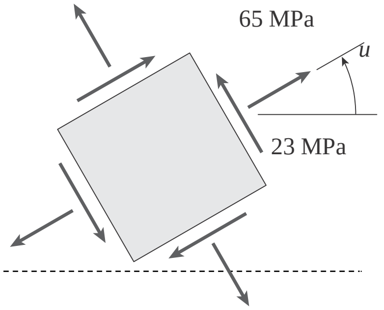
NOTE: All stresses have units of psi.

(b) MAXIMUM NORMAL AND SHEAR STRESSES

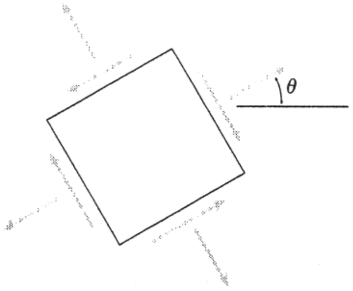
$s_{\max} \quad s_x \quad 15{,}000$ psi

$t_{\max} \quad \frac{s_x}{2} \quad 7{,}500$ psi

Problem 2.6-16 A prismatic bar is subjected to an axial force that produces a tensile stress $\sigma_\theta = 65$ MPa and a shear stress $\tau_\theta = 23$ MPa on a certain inclined plane (see figure). Determine the stresses acting on all faces of a stress element oriented at 30° and show the stresses on a sketch of the element.



Solution 2.6-16



Find θ and σ_x for stress state shown in figure.

$$\sigma_\theta = \sigma_x \cos(\theta)^2 \qquad \cos(\theta) = \sqrt{\frac{\sigma_\theta}{\sigma_x}}$$

$$\text{so} \qquad \sin(\theta) = \sqrt{1 - \frac{\sigma_\theta}{\sigma_x}}$$

$$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta)$$

$$\frac{\tau_\theta}{\sigma_x} = -\sqrt{1 - \frac{\sigma_\theta}{\sigma_x}}\sqrt{\frac{\sigma_\theta}{\sigma_x}}$$

$$\left(\frac{\tau_\theta}{\sigma_x}\right)^2 = \frac{\sigma_\theta}{\sigma_x} - \left(\frac{\sigma_\theta}{\sigma_x}\right)^2$$

$$\left(\frac{23}{\sigma_x}\right)^2 = \frac{65}{\sigma_x} - \left(\frac{65}{\sigma_x}\right)^2$$

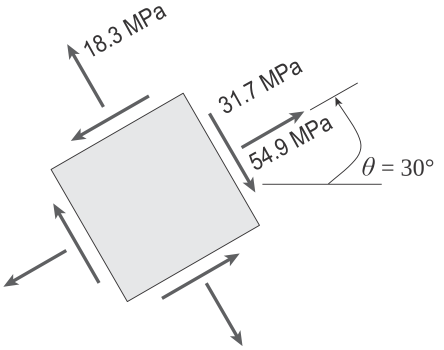
$$\left(\frac{65}{\sigma_x}\right)^2 - \frac{65}{\sigma_x} + \left(\frac{23}{\sigma_x}\right)^2 = 0$$

$$\frac{(4754 + 65\sigma_x)}{\sigma_x^2} = 0$$

$$\sigma_x = \frac{4754}{65}$$

$$\sigma_x = 73.1 \text{ MPa} \qquad \sigma_\theta = 65 \text{ MPa}$$

$$\theta = \operatorname{acos}\left(\sqrt{\frac{\sigma_\theta}{\sigma_x}}\right) \qquad \theta = 19.5$$



Now find σ_θ and τ_θ for $\theta = 30°$:

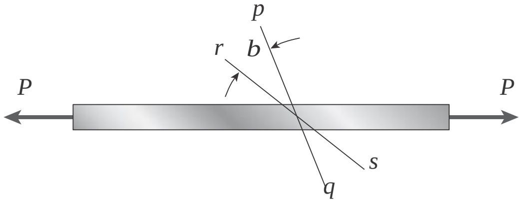
$$\sigma_{\theta 1} = \sigma_x \cos(\theta)^2 \qquad \sigma_{\theta 1} = 54.9 \text{ MPa}$$

$$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta) \qquad \tau_\theta = 31.7 \text{ MPa}$$

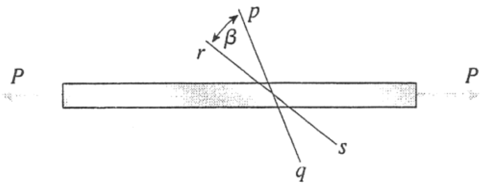
$$\sigma_{\theta 2} = \sigma_x \cos\left(\theta + \frac{\pi}{2}\right)^2 \qquad \sigma_{\theta 2} = 18.3 \text{ MPa}$$

Problem 2.6-17 The normal stress on plane *pq* of a prismatic bar in tension (see figure) is found to be 7500 psi. On plane *rs*, which makes an angle *b* 30° with plane *pq*, the stress is found to be 2500 psi.

Determine the maximum normal stress s_{max} and maximum shear stress t_{max} in the bar.



Solution 2.6-17 Bar in tension



Eq. (2-31a):

$s_u \quad s_x \cos^2 u$

$b \quad 30°$

PLANE *pq*: $s_1 \quad s_x \cos^2 u_1 \qquad s_1 \quad 7500$ psi

PLANE *rs*: $s_2 \quad s_x \cos^2(u_1 \quad b) \qquad s_2 \quad 2500$ psi

Equate s_x from s_1 and s_2:

$$s_x \quad \frac{s_1}{\cos^2 u_1} \quad \frac{s_2}{\cos^2(u_1 + b)} \qquad \text{(Eq. 1)}$$

or

$$\frac{\cos^2 u_1}{\cos^2(u_1 + b)} \quad \frac{s_1}{s_2} \quad \frac{\cos u_1}{\cos(u_1 + b)} \quad A\,\frac{s_1}{s_2} \qquad \text{(Eq. 2)}$$

SUBSTITUTE NUMERICAL VALUES INTO EQ. (2):

$$\frac{\cos u_1}{\cos(u_1 + 30)} \quad A\,\frac{7500 \text{ psi}}{2500 \text{ psi}} \quad 2\underline{3} \quad 1.7321$$

Solve by iteration or a computer program:

$u_1 \quad 30°$

MAXIMUM NORMAL STRESS (FROM EQ. 1)

$$s_{max} \quad s_x \quad \frac{s_1}{\cos^2 u_1} \quad \frac{7500 \text{ psi}}{\cos^2 30}$$

$$10{,}000 \text{ psi}$$

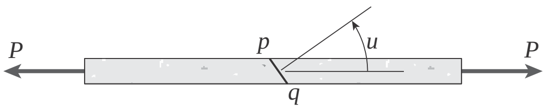
MAXIMUM SHEAR STRESS

$$t_{max} \quad \frac{s_x}{2} \quad 5{,}000 \text{ psi}$$

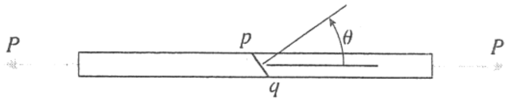
Problem 2.6-18 A tension member is to be constructed of two pieces of plastic glued along plane *pq* (see figure). For purposes of cutting and gluing, the angle *u* must be between 25° and 45°. The allowable stresses on the glued joint in tension and shear are 5.0 MPa and 3.0 MPa, respectively.

(a) Determine the angle *u* so that the bar will carry the largest load *P*. (Assume that the strength of the glued joint controls the design.)

(b) Determine the maximum allowable load P_{max} if the cross-sectional area of the bar is 225 mm^2.



Solution 2.6-18 Bar in tension with glued joint



25° u 45°

A 225 mm^2

On glued joint: s_{allow} 5.0 MPa

t_{allow} 3.0 MPa

ALLOWABLE STRESS s_x IN TENSION

$$s_u \quad s_x\cos^2 u \qquad s_x \quad \frac{s_u}{\cos^2 u} \quad \frac{5.0\text{ MPa}}{\cos^2 u} \qquad (1)$$

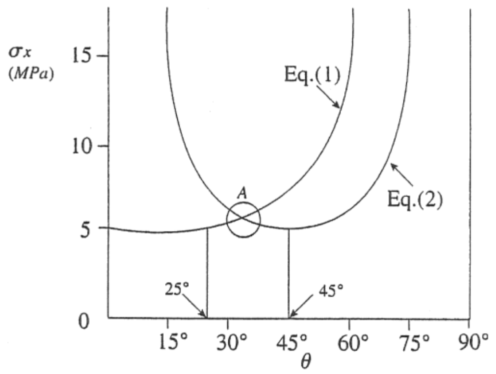
$$t_u \qquad s_x \sin u \cos u$$

Since the direction of t_u is immaterial, we can write:
$t_u|$ $s_x\sin u \cos u$

or

$$s_x \quad \frac{|t_u|}{\sin u \cos u} \quad \frac{3.0\text{ MPa}}{\sin u \cos u} \qquad (2)$$

GRAPH OF EQS. (1) AND (2)



(a) DETERMINE ANGLE Q FOR LARGEST LOAD

Point A gives the largest value of s_x and hence the largest load. To determine the angle u correspon-ding to point A, we equate Eqs. (1) and (2).

$$\frac{5.0\text{ MPa}}{\cos^2 u} \qquad \frac{3.0\text{ MPa}}{\sin u \cos u}$$

$$\tan u \quad \frac{3.0}{5.0} \qquad u \quad 30.96$$

(b) DETERMINE THE MAXIMUM LOAD

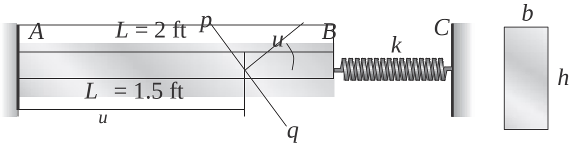
From Eq. (1) or Eq. (2):

$$s_x \quad \frac{5.0\text{ MPa}}{\cos^2 u} \quad \frac{3.0\text{ MPa}}{\sin u \cos u} \quad 6.80\text{ MPa}$$

$$P_{\max} \quad s_x A \quad (6.80\text{ MPa})(225\text{ mm}^2)$$

$$1.53\text{ kN}$$

Problem 2.6-19 Plastic bar *AB* of rectangular cross section (*b* 0.75 in. and *h* 1.5 in.) and length *L* 2 ft. is fixed at *A* and has a spring support (*k* 18 k/in.) at *C* (see figure). Initially, the bar and spring have no stress. When the temperature of the bar is *raised* by 100 F, the *compressive* stress on an inclined plane *pq* at L_u 1.5 ft becomes 950 psi. Assume the spring is massless and is unaffected by the temperature change. Let *a* 55 * 10 ^{6}/ F and *E* 400 ksi.



(a) What is the shear stress t_u on plane *pq*? What is angle *u*?

(b) Draw a stress element oriented to plane *pq*, and show the stresses acting on all faces of this element.
(c) If the allowable normal stress is ; 1000 psi and the allowable shear stress is ; 560 psi, what is the maximum permissible value of spring constant *k* if allowable stress values in the bar are not to be exceeded?
(d) What is the maximum permissible length *L* of the bar if allowable stress values in the bar are not to be exceeded? (Assume *k* 18 k/in.)
(e) What is the maximum permissible temperature *increase* (¢*T*) in the bar if allowable stress values in the bar are not to be exceeded? (Assume *L* 2 ft and *k* 18 k/in.)

Solution 2.6-19

NUMERICAL DATA

$a \quad 55\,110^{\ 6}2 \qquad E \quad 400\text{ ksi} \qquad L \quad 2\text{ ft} \qquad ¢T \quad 100 \qquad k \quad 18\text{ k/in.} \qquad b \quad 0.75\text{ in.} \qquad h \quad 1.5\text{ in.}$

$s_u \quad 950\text{ psi} \qquad s_a \quad 1000\text{ psi} \qquad t_a \quad 560\text{ psi} \qquad L_u \quad 1.5\text{ ft} \qquad A \quad bh \qquad f \quad \frac{1}{k} \quad 5.556 * 10^{\ 5}\text{in./lb}$

(a) FIND *u* AND T_u

$$R_2 \quad \text{redundant} \qquad R_2 \quad \frac{a\,¢TL}{a\frac{L}{EA}b + f} \quad 1.212 * 10^3\text{ lb} \qquad s_x \quad \frac{R_2}{A} \quad 1077.551\text{ psi} \qquad \sqrt{\frac{s_u}{s_x}} \quad 0.939$$

$$u \quad a\cos a\sqrt{\frac{s_u}{s_x}}b \quad 0.351 \qquad \cos(2u) \quad 0.763 \qquad u \quad 20.124$$

$$s_x\cos(u)^2 \quad 950\text{ psi} \qquad \text{or} \qquad \frac{s_x}{2}(1 + \cos(2u)) \quad 950\text{ psi} \qquad s_y \quad s_x\cos a u + \frac{p}{2}b^2 \quad 127.551\text{ psi}$$

$$u \quad 0.351 \qquad u \quad 20.124 \qquad s_x \quad 1077.551\text{ psi} \qquad 2u \quad 0.702$$

$$t_u \quad s_x\sin(u)\cos(u) \quad 348.1\text{ psi} \qquad \text{or} \qquad t_u \quad \frac{s_x}{2}\sin(2u) \quad 348.1\text{ psi}$$

$\boxed{t_u \quad 348\text{ psi}}$ $\boxed{u \quad 20.1}$

(b) FIND s_{x1} AND s_{y1}

$$s_{x1} \quad s_x\cos(u)^2 \qquad s_{y1} \quad s_x\cos a u + \frac{p}{2}b^2$$

$\boxed{s_{x1} \quad 950\text{ psi}}$ $\boxed{s_{y1} \quad 127.6\text{ psi}}$

(c) GIVEN $L = 2$ ft, FIND k_{max}

$$k_{max1} = \frac{\sigma_a A}{\alpha \Delta T L - \sigma_a A \left(\frac{L}{EA}\right)} = 15625 \text{ lb/in.} \Leftarrow \text{ controls (based on } \sigma_{allow})$$

$$\text{or} \quad k_{max2} = \frac{2\tau_a A}{\alpha \Delta T L - 2\tau_a A \left(\frac{L}{EA}\right)} = 19444.444 \text{ lb/in.} \quad \text{based on allowable shear stress}$$

$\boxed{k_{max} = 15625 \text{ lb/in.}}$

(d) GIVEN ALLOWABLE NORMAL AND SHEAR STRESSES, FIND L_{max}

$k = 18000$ lb/in.

$$\sigma_x = \frac{R_2}{A} \qquad \sigma_a A = \frac{\alpha \Delta T L}{\left(\frac{L}{EA}\right) + f} \qquad L_{max1} = \frac{\sigma_a A(f)}{\left(\alpha \Delta T + \frac{\sigma_a}{E}\right)} = 1.736 \text{ ft} \Leftarrow \text{ controls (based on } \sigma_{allow})$$

$$\text{or} \quad L_{max2} = \frac{2\tau_a A(f)}{\left(\alpha \Delta T + \frac{2\tau_a}{E}\right)} = 2.16 \text{ ft} \Leftarrow \text{ based on } \tau_{allow}$$

$\boxed{L_{max} = 1.736 \text{ ft}}$

(e) FIND T_{max} GIVEN L, k, AND ALLOWABLE STRESSES $k = 18000$ lb/in. $L = 2$ ft $\sigma_a = 1000$ psi

$\tau_a = 560$ psi

$$\Delta T_{max1} = \frac{\left(\frac{L}{EA} + f\right)\sigma_a A}{\alpha L} = 92.803\ ^\circ\text{F} \Leftarrow \text{ based on } \sigma_{allow} \qquad \Delta T = 100$$

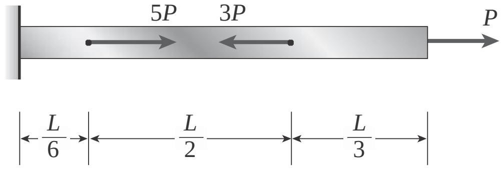
$$\Delta T_{max2} = \frac{\left(\frac{L}{EA} + f\right) 2\tau_a A}{\alpha L} = 103.939\ ^\circ\text{F} \Leftarrow \text{ based on } \tau_{allow}$$

$\boxed{\Delta T_{max} = 92.8\ ^\circ\text{F}}$

Strain Energy

When solving the problems for Section 2.7, assume that the material behaves linearly elastically.

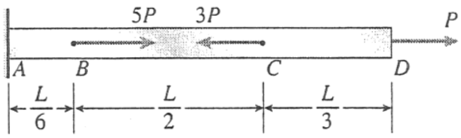
Problem 2.7-1 A prismatic bar *AD* of length *L*, cross-sectional area *A*, and modulus of elasticity *E* is subjected to loads 5*P*, 3*P*, and *P* acting at points *B*, *C*, and *D*, respectively (see figure).



respectively.

(a) Obtain a formula for the strain energy *U* of the bar.
(b) Calculate the strain energy if P 6 k, L 52 in., A 2.76 in.^{2}, and the material is aluminum with E 10.4 10^6 psi.

Solution 2.7-1 Bar with three loads



P 6 k

L 52 in.

E 10.4 10^6 psi

A 2.76 in.^{2}

INTERNAL AXIAL FORCES

N_{AB} $3P$ $\quad N_{BC}$ $2P$ $\quad N_{CD}$ P

LENGTHS

$L_{AB} \quad \frac{L}{6} \qquad L_{BC} \quad \frac{L}{2} \qquad L_{CD} \quad \frac{L}{3}$

(a) STRAIN ENERGY OF THE BAR (EQ. 2-40)

$$U \quad \sum \frac{N_i^2 L_i}{2E_i A_i}$$

$$\frac{1}{2EA}\left[(3P)^2\left(\frac{L}{6}\right) + (\;\;2P)^2\left(\frac{L}{2}\right) + (P)^2\left(\frac{L}{3}\right)\right]$$

$$\frac{P^2L}{2EA}\left(\frac{23}{6}\right) \quad \frac{23P^2L}{12EA}$$

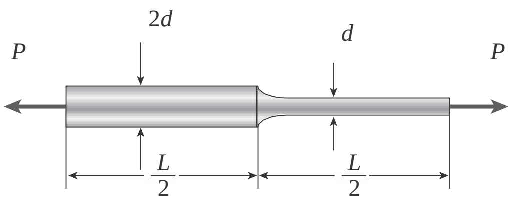
(b) SUBSTITUTE NUMERICAL VALUES:

$$U \quad \frac{23(6\text{ k})^2(52\text{ in.})}{12(10.4 * 10^6\text{ psi})(2.76\text{ in.}^2)}$$

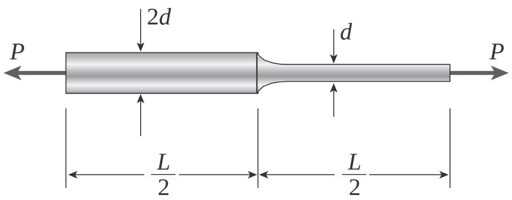
$$125\text{ in.-lb}$$

Problem 2.7-2 A bar of circular cross section having two different diameters d and $2d$ is shown in the figure. The length of each segment of the bar is $L/2$ and the modulus of elasticity of the material is E.

(a) Obtain a formula for the strain energy U of the bar due to the load P.
(b) Calculate the strain energy if the load $P = 27$ kN, the length $L = 600$ mm, the diameter $d = 40$ mm, and the material is brass with $E = 105$ GPa.



Solution 2.7-2 Bar with two segments



(a) Strain energy of the bar

Add the strain energies of the two segments of the bar (see Eq. 2-42).

$$U = \sum_{i=1}^{2} \frac{N_i^2 L_i}{2 E_i A_i} = \frac{P^2(L/2)}{2E}\left[\frac{1}{\frac{\pi}{4}(2d)^2} + \frac{1}{\frac{\pi}{4}(d^2)}\right]$$

$$= \frac{P^2 L}{\pi E}\left(\frac{1}{4d^2} + \frac{1}{d^2}\right) = \frac{5P^2L}{4\pi E d^2}$$

(b) Substitute numerical values:

$P = 27$ kN $\quad L = 600$ mm

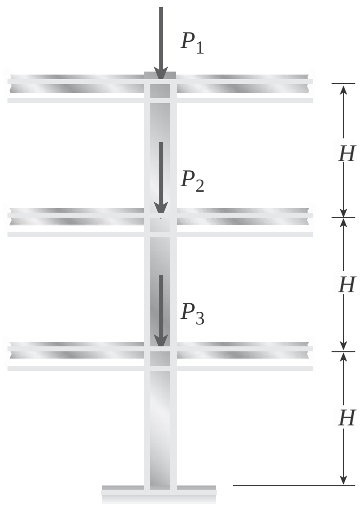
$d = 40$ mm $\quad E = 105$ GPa

$$U = \frac{5(27\text{ kN}^2)(600\text{ mm})}{4\pi(105\text{ GPa})(40\text{ mm})^2}$$

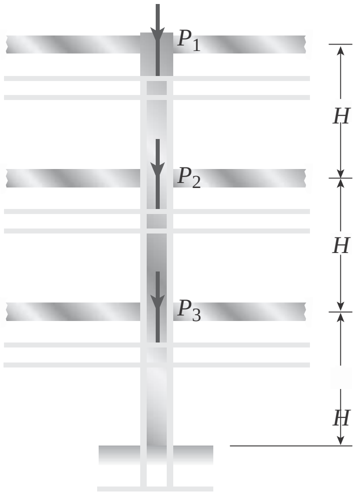
$$= 1.036\ \text{N}\cdot\text{m} = 1.036\ \text{J}$$

Problem 2.7-3 A three-story steel column in a building supports roof and floor loads as shown in the figure. The story height H is 10.5 ft, the cross-sectional area A of the column is 15.5 in.^{2}, and the modulus of elasticity E of the steel is 30×10^6 psi.

Calculate the strain energy U of the column assuming $P_1 = 40$ k and $P_2 = P_3 = 60$ k.



Solution 2.7-3 Three-story column



$H \quad 10.5 \text{ ft} \qquad E \quad 30 \quad 10^6 \text{ psi}$

$A \quad 15.5 \text{ in.}^2 \qquad P_1 \quad 40 \text{ k}$

$P_2 \quad P_3 \quad 60 \text{ k}$

To find the strain energy of the column, add the strain energies of the three segments (see Eq. 2-42).

Upper segment: $N_1 \quad P_1$

Middle segment: $N_2 \quad (P_1 \quad P_2)$

Lower segment: $N_3 \quad (P_1 \quad P_2 \quad P_3)$

STRAIN ENERGY

$$U \quad g\frac{N_i^2 L_i}{2E_i A_i}$$

$$\frac{H}{2EA}[P_1^2 + (P_1 + P_2)^2 + (P_1 + P_2 + P_3)^2]$$

$$\frac{H}{2EA}[Q]$$

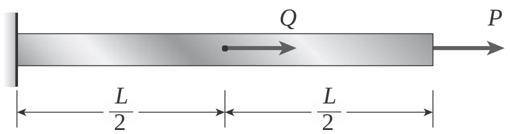
$$[Q] \quad (40 \text{ k})^2 + (100 \text{ k})^2 + (160 \text{ k})^2 \quad 37{,}200 \text{ k}^2$$

$$2EA \quad 2(30 * 10^6 \text{ psi})(15.5 \text{ in.}^2) \quad 930 * 10^6 \text{ lb}$$

$$U \quad \frac{(10.5 \text{ ft})(12 \text{ in./ft})}{930 * 10^6 \text{ lb}}[37{,}200 \text{ k}^2]$$

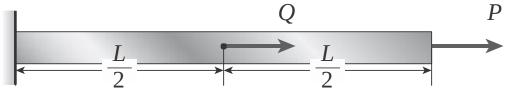
$$5040 \text{ in.-lb}$$

Problem 2.7-4 The bar ABC shown in the figure is loaded by a force P acting at end C and by a force Q acting at the midpoint B. The



(a) Determine the strain energy U_1 of the bar when the force P acts alone ($Q \quad 0$).
(b) Determine the strain energy U_2 when the force Q acts alone ($P \quad 0$).
(c) Determine the strain energy U_3 when the forces P and Q act simultaneously upon the bar.

Solution 2.7-4 Bar with two loads



(a) FORCE P ACTS ALONE (Q 0)

$$U_1 \quad \frac{P^2L}{2EA}$$

(b) FORCE Q ACTS ALONE (P 0)

$$U_2 \quad \frac{Q^2(L/2)}{2EA} \quad \frac{Q^2L}{4EA}$$

(c) FORCES P AND Q ACT SIMULTANEOUSLY

Segment BC: U_{BC} $\quad 2EA \quad 4EA$

Segment AB: U_{AB} $\dfrac{(P+Q)^2(L/2)}{2EA}$

$$\frac{P^2L}{4EA} + \frac{PQL}{2EA} + \frac{Q^2L}{4EA}$$

$$U_3 \quad U_{BC} + U_{AB} \quad \frac{P^2L}{2EA} + \frac{PQL}{2EA} + \frac{Q^2L}{4EA}$$

(Note that U_3 is *not* equal to U_1 U_2. In this case, U_3 U_1 U_2. However, if Q is reversed in direction, U_3 U_1 U_2. Thus, U_3 may be larger or smaller than U_1 U_2.)

Problem 2.7-5 Determine the strain energy per unit volume (units of psi) and the strain energy per unit weight (units of in.) that can be stored in each of the materials listed in the accompanying table, assuming that the material is stressed to the proportional limit.

DATA FOR PROBLEM 2.7-5

Material	Weight density (lb/in.^{3})	Modulus of elasticity (ksi)	Proportional limit (psi)
Mild steel	0.284	30,000	36,000
Tool steel	0.284	30,000	75,000
Aluminum	0.0984	10,500	60,000
Rubber (soft)	0.0405	0.300	300

Solution 2.7-5 Strain-energy density

DATA:

Material	Weight density (lb/in.^{3})	Modulus of elasticity (ksi)	Proportional limit (psi)
Mild steel	0.284	30,000	36,000
Tool steel	0.284	30,000	75,000
Aluminum	0.0984	10,500	60,000
Rubber (soft)	0.0405	0.300	300

STRAIN ENERGY PER UNIT VOLUME

$$U \quad \frac{P^2L}{2EA} \qquad \text{Volume } V \quad AL$$

$$\text{Stress } \sigma \quad \frac{P}{A}$$

$$u \quad \frac{U}{V} \quad \frac{\sigma_{PL}^2}{2E}$$

At the proportional limit:

$u \quad u_R$ modulus of resistance

$$u_R \quad \frac{s_{PL}^2}{2E} \qquad \text{(Eq. 1)}$$

STRAIN ENERGY PER UNIT WEIGHT

$$U \quad \frac{P^2L}{2EA} \qquad \text{Weight } W \quad gAL$$

g weight density

$$u_W \quad \frac{U}{W} \quad \frac{s^2}{2gE}$$

At the proportional limit:

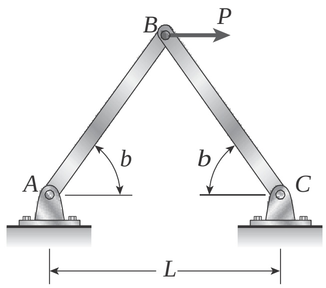
$$u_W \quad \frac{s_{PL}^2}{2gE} \qquad \text{(Eq. 2)}$$

RESULTS

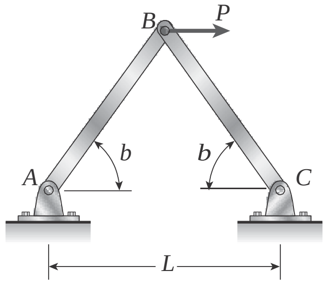
	u_R (psi)	u_w (in.)
Mild steel	22	76
Tool steel	94	330
Aluminum	171	1740
Rubber (soft)	150	3700

Problem 2.7-6 The truss *ABC* shown in the figure is subjected to a horizontal load *P* at joint *B*. The two bars are identical with cross-sectional area *A* and modulus of elasticity *E*.

(a) Determine the strain energy *U* of the truss if the angle $b \quad 60°$.
(b) Determine the horizontal displacement d_B of joint *B* by equating the strain energy of the truss to the work done by the load.



Solution 2.7-6 Truss subjected to a load P



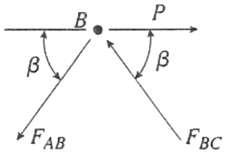
$b \quad 60°$

$L_{AB} \quad L_{BC} \quad L$

$\sin b \quad \sqrt{3}/2$

$\cos b \quad 1/2$

FREE-BODY DIAGRAM OF JOINT *B*



$F_{\text{vert}} \quad 0 \quad \downarrow \uparrow$

$F_{AB} \sin b \quad F_{BC} \sin b \quad 0$

$$F_{AB} \quad F_{BC} \qquad \text{(Eq. 1)}$$

$F_{\text{horiz}} \quad 0 \quad \leftarrow$

$F_{AB} \cos b \quad F_{BC} \cos b \quad P \quad 0$

$$F_{AB} \quad F_{BC} \quad \frac{P}{2\cos b} \quad \frac{P}{2(1/2)} \quad P \qquad \text{(Eq. 2)}$$

Axial forces: N_{AB} $\quad P$ (tension)

N_{BC} $\quad P$ (compression)

(a) STRAIN ENERGY OF TRUSS (EQ. 2-42)

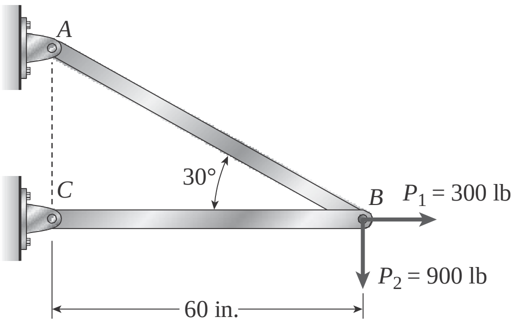
$$U \quad g\frac{N_i^2 L_i}{2E_iA_i} \quad \frac{(N_{AB})^2 L}{2EA} + \frac{(N_{BC})^2 L}{2EA} \quad \frac{P^2 L}{EA}$$

(b) HORIZONTAL DISPLACEMENT OF JOINT B (EQ. 2-44)

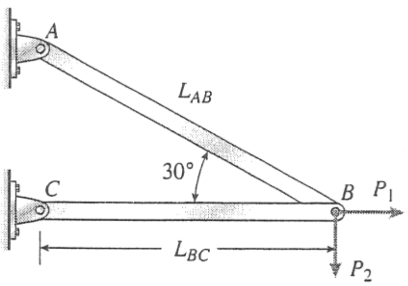
$$d_B \quad \frac{2U}{P} \quad \frac{2}{P}a\frac{P^2L}{EA}b \quad \frac{2PL}{EA}$$

Problem 2.7-7 The truss ABC shown in the figure supports a horizontal load P_1 $\quad$ 300 lb and a vertical load P_2 $\quad$ 900 lb. Both bars have cross-sectional area A $\quad$ 2.4 in.^{2} and are made of steel with E $\quad$ 30 $\quad 10^6$ psi.

(a) Determine the strain energy U_1 of the truss when the load P_1 acts alone (P_2 $\quad$ 0).
(b) Determine the strain energy U_2 when the load P_2 acts alone (P_1 $\quad$ 0).
(c) Determine the strain energy U_3 when both loads act simultaneously.



Solution 2.7-7 Truss with two loads



P_1 $\quad$ 300 lb

P_2 $\quad$ 900 lb A

2.4 in.^{2}

E $\quad$ 30 $\quad 10^6$ psi

L_{BC} $\quad$ 60 in.

b $\quad$ 30°

$$\sin b \quad \sin 30 \quad \frac{1}{2}$$

$$\cos b \quad \cos 30 \quad \frac{\sqrt{3}}{2}$$

$$L_{AB} \quad \frac{L_{BC}}{\cos 30} \quad \frac{120}{\sqrt{3}} \text{ in.} \quad 69.282 \text{ in.}$$

$2EA$ $\quad$ 2(30 $\quad 10^6$ psi)(2.4 in.^{2}) $\quad$ 144 $\quad 10^6$ lb

FORCES F_{AB} AND F_{BC} IN THE BARS

From equilibrium of joint B:

F_{AB} $\quad 2P_2$ $\quad$ 1800 lb

F_{BC} $\quad P_1$ $\quad P_2\sqrt{3}$ $\quad$ 300 lb $\quad$ 1558.8 lb

Force	P_1 alone	P_2 alone	P_1 and P_2
F_{AB}	0	1800 lb	1800 lb
F_{BC}	300 lb	1558.8 lb	1258.8 lb

(a) LOAD P_1 ACTS ALONE

$$U_1 \quad \frac{(F_{BC})^2 L_{BC}}{2EA} \quad \frac{(300 \text{ lb})^2(60 \text{ in.})}{144 * 10^6 \text{ lb}}$$

$$0.0375 \text{ in.-lb}$$

(b) LOAD P_2 ACTS ALONE

$$U_2 \quad \frac{1}{2EA}\left[(F_{AB})^2 L_{AB} + (F_{BC})^2 L_{BC}\right]$$

$$\frac{1}{2EA}\left[(1800\text{ lb})^2(69.282\text{ in.})\right.$$

$$\left. + (\quad 1558.8\text{ lb})^2(60\text{ in.})\right]$$

$$\frac{370.265 * 10^6\text{ lb}^2\text{-in.}}{144 * 10^6\text{ lb}} \quad 2.57\text{ in.-lb} \quad \longleftarrow$$

(c) LOADS P_1 AND P_2 ACT SIMULTANEOUSLY

$$U_3 \quad \frac{1}{2EA}\left[(F_{AB})^2 L_{AB} + (F_{BC})^2 L_{BC}\right]$$

$$\frac{1}{2EA}\left[(1800\text{ lb})^2(69.282\text{ in.})\right.$$

$$\left. + (\quad 1258.8\text{ lb})^2(60\text{ in.})\right]$$

$$\frac{319.548 * 10^6\text{ lb}^2\text{-in.}}{144 * 10^6\text{ lb}}$$

$$2.22\text{ in.-lb} \quad \longleftarrow$$

NOTE: The strain energy U_3 is *not* equal to $U_1 \quad U_2$.

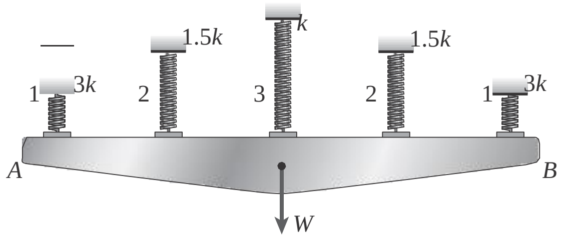
Problem 2.7-8 The statically indeterminate structure shown in the figure consists of a horizontal rigid bar *AB* supported by five equally spaced springs. Springs 1, 2, and 3 have stiffnesses 3*k*, 1.5*k*, and *k*, respectively. When unstressed, the lower ends of all five springs lie along a horizontal line. Bar *AB*, which has weight *W*, causes the springs to elongate by an amount *d*.

(a) Obtain a formula for the total strain energy *U* of the springs in terms of the downward displacement *d* of the bar.

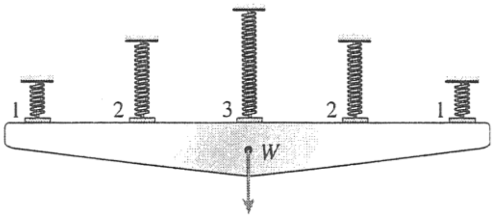
(b) Obtain a formula for the displacement *d* by equating the strain energy of the springs to the work done by the weight *W*.

(c) Determine the forces F_1, F_2, and F_3 in the springs.

(d) Evaluate the strain energy *U*, the displacement *d*, and the forces in the springs if $W \quad 600$ N and $k \quad 7.5$ N/mm.



Solution 2.7-8 Rigid bar supported by springs



$k_1 = 3k$

$k_2 = 1.5k$

$k_3 = k$

δ = downward displacement of rigid bar

For a spring: $U = \frac{k\delta^2}{2}$ Eq. (2-40b)

(a) Strain energy U of all springs

$$U = 2\left(\frac{3k\delta^2}{2}\right) + 2\left(\frac{1.5k\delta^2}{2}\right) + \frac{k\delta^2}{2} = 5k\delta^2$$

(b) Displacement δ

Work done by the weight W equals $\frac{W\delta}{2}$

Strain energy of the springs equals $5k\delta^2$

$$\therefore \frac{W\delta}{2} = 5k\delta^2 \quad \text{and} \quad \delta = \frac{W}{10k}$$

(c) Forces in the springs

$$F_1 = 3k\delta = \frac{3W}{10} \qquad F_2 = 1.5k\delta = \frac{3W}{20}$$

$$F_3 = k\delta = \frac{W}{10}$$

(d) Numerical values

$W = 600$ N $\quad k = 7.5$ N/mm $= 7500$ N/mm

$$U = 5k\delta^2 = 5k\left(\frac{W}{10k}\right)^2 = \frac{W^2}{20k}$$

$$= 2.4 \text{ N·m} = 2.4 \text{ J}$$

$$\delta = \frac{W}{10k} = 8.0 \text{ mm}$$

$$F_1 = \frac{3W}{10} = 180 \text{ N}$$

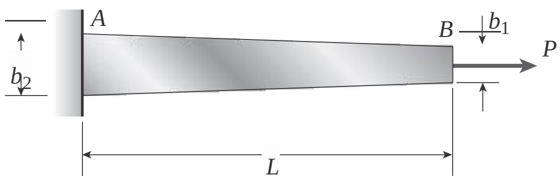
$$F_2 = \frac{3W}{20} = 90 \text{ N}$$

$$F_3 = \frac{W}{10} = 60 \text{ N}$$

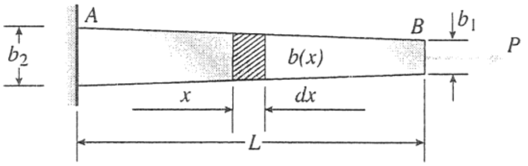
NOTE: $W = 2F_1 + 2F_2 + F_3 = 600$ N (Check)

Problem 2.7-9 A slightly tapered bar AB of rectangular cross section and length L is acted upon by a force P (see figure). The width of the bar varies uniformly from b_2 at end A to b_1 at end B. The thickness t is constant.

(a) Determine the strain energy U of the bar.
(b) Determine the elongation δ of the bar by equating the strain energy to the work done by the force P.



Solution 2.7-9 Tapered bar of rectangular cross section



$$b(x) = b_2 - \frac{(b_2 - b_1)x}{L}$$

$$A(x) = tb(x)$$

$$= t\left[b_2 - \frac{(b_2 - b_1)x}{L}\right]$$

(a) Strain energy of the bar

$$U = \int \frac{[N(x)]^2 dx}{2EA(x)} \quad \text{(Eq. 2-43)}$$

$$= \int_0^L \frac{P^2 dx}{2Etb(x)} = \frac{P^2}{2Et}\int_0^L \frac{dx}{b_2 - (b_2 - b_1)\frac{x}{L}} \qquad (1)$$

From Appendix C: $\displaystyle\int \frac{dx}{a + bx} = \frac{1}{b}\ln(a + bx)$

Apply this integration formula to Eq. (1):

$$U = \frac{P^2}{2Et}\left[\frac{1}{-(b_2 - b_1)\left(\frac{1}{L}\right)}\ln\left[b_2 - \frac{(b_2 - b_1)x}{L}\right]\right]_0^L$$

$$= \frac{P^2}{2Et}\left[\frac{-L}{(b_2 - b_1)}\ln b_1 - \frac{-L}{(b_2 - b_1)}\ln b_2\right]$$

$$U = \frac{P^2 L}{2Et(b_2 - b_1)}\ln\frac{b_2}{b_1} \quad ◄$$

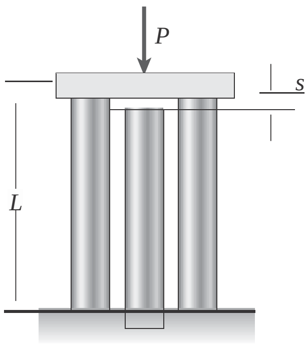
(b) Elongation of the bar (Eq. 2-44)

$$\delta = \frac{2U}{P} = \frac{PL}{Et(b_2 - b_1)}\ln\frac{b_2}{b_1} \quad ◄$$

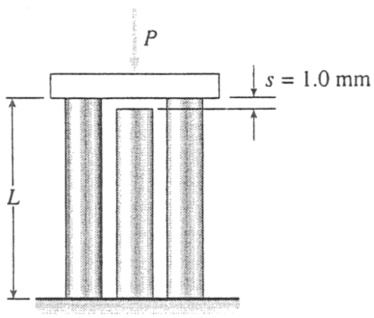
NOTE: This result agrees with the formula derived in Prob. 2.3-13.

Problem 2.7-10 A compressive load P is transmitted through a rigid plate to three magnesium-alloy bars that are identical except that initially the middle bar is slightly shorter than the other bars (see figure). The dimensions and properties of the assembly are as follows: length $L = 1.0$ m, cross-sectional area of each bar $A = 3000$ mm^2, modulus of elasticity $E = 45$ GPa, and the gap $s = 1.0$ mm.

(a) Calculate the load P_1 required to close the gap.
(b) Calculate the downward displacement δ of the rigid plate when $P = 400$ kN.
(c) Calculate the total strain energy U of the three bars when $P = 400$ kN.
(d) Explain why the strain energy U is *not* equal to $P\delta/2$. (*Hint*: Draw a load-displacement diagram.)



Solution 2.7-10 Three bars in compression



s 1.0 mm

L 1.0 m

For each bar:

A 3000 mm^2

E 45 GPa

$\frac{EA}{L}$ $135 * 10^6$ N/m

(a) Load P_1 required to close the gap

In general, $d \quad \frac{PL}{EA}$ and $P \quad \frac{EAd}{L}$

For two bars, we obtain:

$$P_1 \quad 2\left(\frac{EAs}{L}\right) \quad 2(135 * 10^6 \text{ N/m})(1.0 \text{ mm})$$

P_1 270 kN

(b) Displacement d for P 400 kN

Since P P_1, all three bars are compressed. The force P equals P_1 plus the additional force required to compress all three bars by the amount d s.

$$P \quad P_1 + 3\left(\frac{EA}{L}\right)(d \quad s)$$

or 400 kN 270 kN 3(135 10^6 N/m) (d 0.001 m)

Solving, we get d 1.321 mm

(c) Strain energy U for P 400 kN

$$U \quad \sum \frac{EAd^2}{2L}$$

Outer bars: d 1.321 mm

Middle bar: d 1.321 mm s

0.321 mm

$$U \quad \frac{EA}{2L}[2(1.321 \text{ mm})^2 + (0.321 \text{ mm})^2]$$

$$\frac{1}{2}(135 * 10^6 \text{ N/m})(3.593 \text{ mm}^2)$$

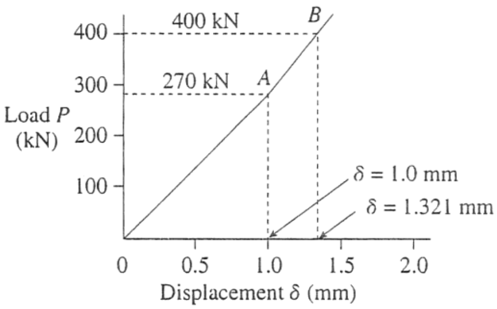
243 N·m 243 J

(d) Load-displacement diagram

U 243 J 243 N·m

$$\frac{Pd}{2} \quad \frac{1}{2}(400 \text{ kN})(1.321 \text{ mm}) \quad 264 \text{ N·m}$$

The strain energy U is *not* equal to $\frac{Pd}{2}$ because the load-displacement relation is not linear.

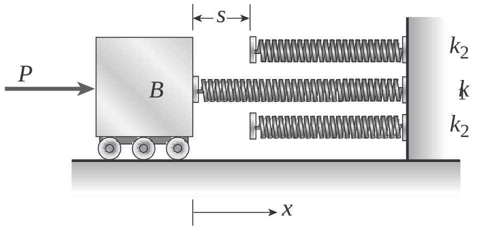


U area under line OAB.

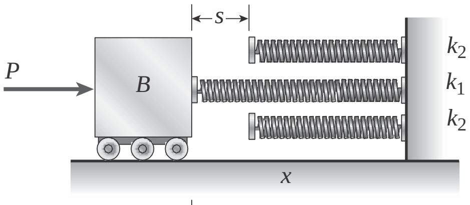
$\frac{Pd}{2}$ area under a straight line from O to B, which is larger than U.

Problem 2.7-11 A block B is pushed against three springs by a force P (see figure). The middle spring has stiffness k_1 and the outer springs each have stiffness k_2. Initially, the springs are unstressed and the middle spring is longer than the outer springs (the difference in length is denoted s).

(a) Draw a force-displacement diagram with the force P as ordinate and the displacement x of the block as abscissa.
(b) From the diagram, determine the strain energy U_1 of the springs when $x \quad 2s$.
(c) Explain why the strain energy U_1 is not equal to $P\delta/2$, where $\delta \quad 2s$.



Solution 2.7-11 Block pushed against three springs



Force P_0 required to close the gap:

$$P_0 \quad k_1 s \qquad (1)$$

FORCE-DISPLACEMENT RELATION BEFORE GAP IS CLOSED

$$P \quad k_1 x \qquad (0 \quad x \quad s)(0 \quad P \quad P_0) \qquad (2)$$

FORCE-DISPLACEMENT RELATION AFTER GAP IS CLOSED

All three springs are compressed. Total stiffness equals $k_1 \quad 2k_2$. Additional displacement equals $x \quad s$. Force P equals P_0 plus the force required to compress all three springs by the amount $x \quad s$.

$$P \quad P_0 \quad (k_1 \quad 2k_2)(x \quad s)$$

$$\quad k_1 s \quad (k_1 \quad 2k_2)x \quad k_1 s \quad 2k_2 s$$

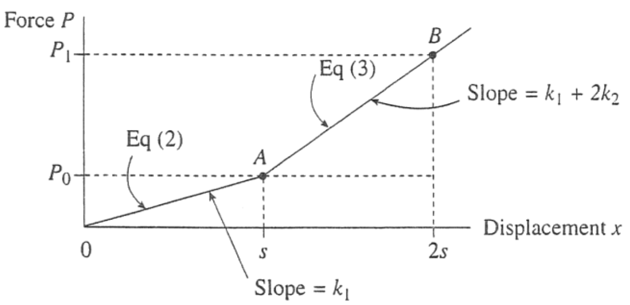
$$P \quad (k_1 \quad 2k_2)x \quad 2k_2 s \qquad (x \quad s); (P \quad P_0) \qquad (3)$$

P_1 force P when $x \quad 2s$

Substitute $x \quad 2s$ into Eq. (3):

$$P_1 \quad 2(k_1 \quad k_2)s \qquad (4)$$

(a) FORCE-DISPLACEMENT DIAGRAM



(b) STRAIN ENERGY U_1 WHEN $x \quad 2s$

U_1 Area below force-displacement curve

$$\frac{1}{2}P_0 s + P_0 s + \frac{1}{2}(P_1 \quad P_0)s \quad P_0 s + \frac{1}{2}P_1 s$$

$$k_1 s^2 + (k_1 + k_2)s^2$$

$$U_1 \quad (2k_1 \quad k_2)s^2 \qquad (5)$$

(c) STRAIN ENERGY U_1 IS NOT EQUAL TO $\frac{Pd}{2}$

For $d \quad 2s$: $\frac{Pd}{2} \quad \frac{1}{2}P_1(2s) \quad P_1 s \quad 2(k_1 + k_2)s^2$

(This quantity is greater than U_1.)

U_1 area under line OAB.

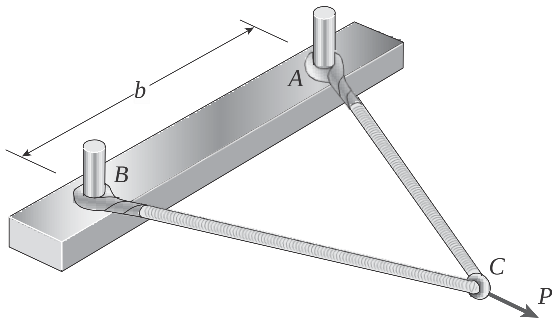
$\frac{Pd}{2}$ area under a straight line from O to B, which is larger than U_1.

Thus, $\frac{Pd}{2}$ is *not* equal to the strain energy because the force-displacement relation is not linear.

Problem 2.7-12 A bungee cord that behaves linearly elastically has an unstressed length $L_0 \quad 760$ mm and a stiffness $k \quad 140$ N/m. The cord is attached to two pegs, distance $b \quad 380$ mm apart, and pulled at its midpoint by a force $P \quad 80$ N (see figure).

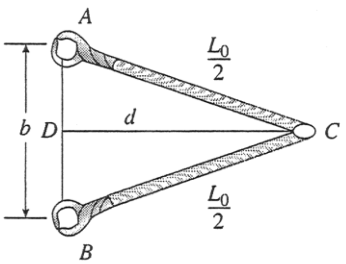
(a) How much strain energy U is stored in the cord?
(b) What is the displacement d_C of the point where the load is applied?
(c) Compare the strain energy U with the quantity $Pd_C/2$.

(*Note*: The elongation of the cord is *not* small compared to its original length.)



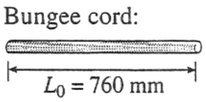
Solution 2.7-12 Bungee cord subjected to a load P.

DIMENSIONS BEFORE THE LOAD P IS APPLIED



$L_0 \quad 760$ mm $\quad \frac{L_0}{2} \quad 380$ mm

$b \quad 380$ mm

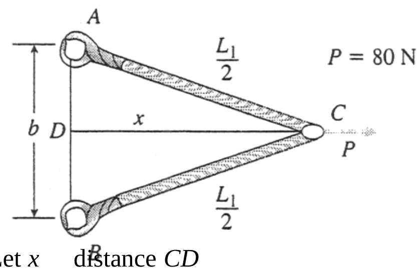


$k \quad 140$ N/m

From triangle ACD:

$$d \quad \frac{1}{2}\sqrt{L_0^2 \quad b^2} \quad 329.09 \text{ mm} \qquad (1)$$

DIMENSIONS AFTER THE LOAD P IS APPLIED



Let x distance CD

Let L_1 stretched length of bungee cord

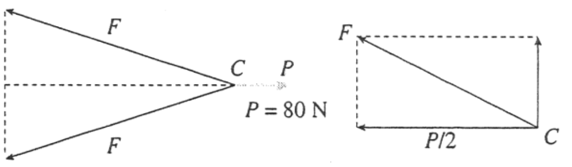
From triangle *ACD*:

$$\frac{L_1}{2} = \sqrt{\left(\frac{b}{2}\right)^2 + x^2} \qquad (2)$$

$$L_1 = \sqrt{b^2 + 4x^2} \qquad (3)$$

EQUILIBRIUM AT POINT C

Let F = tensile force in bungee cord



$$\frac{F}{P/2} = \frac{L_1/2}{x} \qquad F = \left(\frac{P}{2}\right)\left(\frac{L_1}{2}\right)\left(\frac{1}{x}\right)$$

$$= \frac{P}{2}\sqrt{1 + \left(\frac{b}{2x}\right)^2} \qquad (4)$$

ELONGATION OF BUNGEE CORD

Let d = elongation of the entire bungee cord

$$d = \frac{F}{k} = \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} \qquad (5)$$

Final length of bungee cord = original length + d

$$L_1 = L_0 + d = L_0 + \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} \qquad (6)$$

SOLUTION OF EQUATIONS

Combine Eqs. (6) and (3):

$$L_1 = L_0 + \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} = \sqrt{b^2 + 4x^2}$$

$$\text{or} \quad L_1 = L_0 + \frac{P}{4kx}\sqrt{b^2 + 4x^2} = \sqrt{b^2 + 4x^2}$$

$$L_0 = \left(1 - \frac{P}{4kx}\right)\sqrt{b^2 + 4x^2} \qquad (7)$$

This equation can be solved for x.

SUBSTITUTE NUMERICAL VALUES INTO EQ. (7):

$$760 \text{ mm} = \left[1 - \frac{(80 \text{ N})(1000 \text{ mm/m})}{4(140 \text{ N/m})x}\right]$$

$$* \sqrt{(380 \text{ mm})^2 + 4x^2} \qquad (8)$$

$$760 = \left(1 - \frac{142.857}{x}\right)\sqrt{144{,}400 + 4x^2} \qquad (9)$$

Units: x is in millimeters

Solve for x (Use trial-and-error or a computer program):

$x = 497.88$ mm

(a) STRAIN ENERGY U OF THE BUNGEE CORD

$$U = \frac{kd^2}{2} \qquad k = 140 \text{ N/m} \qquad P = 80 \text{ N}$$

From Eq. (5):

$$d = \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} = 305.81 \text{ mm}$$

$$U = \frac{1}{2}(140 \text{ N/m})(305.81 \text{ mm})^2 = 6.55 \text{ N}\cdot\text{m}$$

$U = 6.55$ J ←

(b) DISPLACEMENT d_C OF POINT C

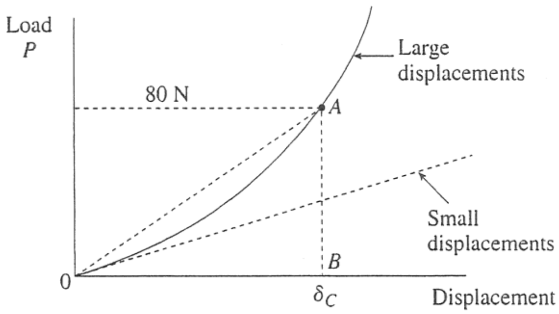
$$d_C = x - d = 497.88 \text{ mm} - 329.09 \text{ mm}$$

$$= 168.8 \text{ mm} \quad ←$$

(c) COMPARISON OF STRAIN ENERGY U WITH THE QUANTITY $Pd_C/2$

$$U \quad 6.55 \text{ J}$$

$$\frac{Pd_C}{2} \quad \frac{1}{2}(80 \text{ N})(168.8 \text{ mm}) \quad 6.75 \text{ J}$$



The two quantities are not the same. The work done by the load P is *not* equal to $Pd_C/2$ because the load-displacement relation (see below) is non-linear when the displacements are large. (The *work* done by the load P is equal to the strain energy because the bungee cord behaves elastically and there are no energy losses.)

U area *OAB* under the curve *OA*.

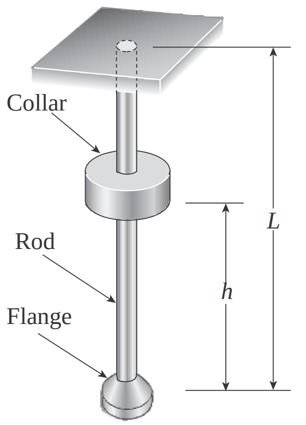
$\frac{Pd_C}{2}$ area of triangle *OAB*, which is greater than U.

Impact Loading

The problems for Section 2.8 are to be solved on the basis of the assumptions and idealizations described in the text. In particular, assume that the material behaves linearly elastically and no energy is lost during the impact.

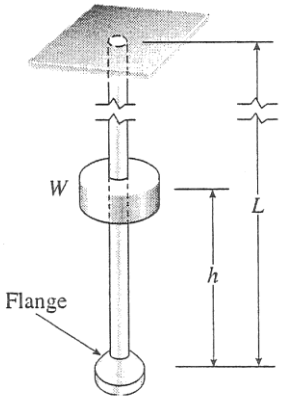
Problem 2.8-1 A sliding collar of weight W 150 lb falls from a height h 2.0 in. onto a flange at the bottom of a slender vertical rod (see figure). The rod has length L 4.0 ft, cross-sectional area A 0.75 in.^{2}, and modulus of elasticity E 30 10^6 psi.

Calculate the following quantities: (a) the maximum downward displacement of the flange, (b) the maximum tensile stress in the rod, and (c) the impact factor.



Probs. 2.8-1, 2.8-2, 2.8-3

Solution 2.8-1 Collar falling onto a flange



W 150 lb

h 2.0 in. L 4.0 ft 48 in.

E 30 10^6 psi A 0.75 in.^{2}

(a) DOWNWARD DISPLACEMENT OF FLANGE

$$d_{st} \quad \frac{WL}{EA} \quad 0.00032 \text{ in.}$$

Eq. (2-55):

$$d_{max} \quad d_{st}\left[1 + \left(1 + \frac{2h}{d_{st}}\right)^{1/2}\right]$$

$$0.0361 \text{ in.}$$

(b) MAXIMUM TENSILE STRESS (EQ. 2-57)

$$s_{max} \quad \frac{Ed_{max}}{L} \quad 22{,}600 \text{ psi}$$

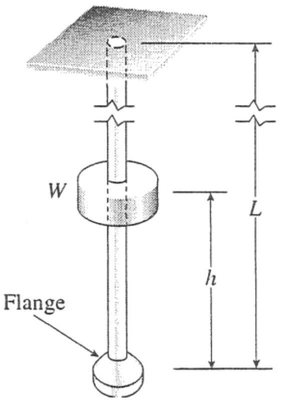
(c) IMPACT FACTOR (EQ. 2-63)

$$\text{Impact factor} \quad \frac{d_{max}}{d_{st}} \quad \frac{0.0361 \text{ in.}}{0.00032 \text{ in.}}$$

$$113$$

Problem 2.8-2 Solve the preceding problem if the collar has mass $M \quad 80$ kg, the height $h \quad 0.5$ m, the length $L \quad 3.0$ m, the cross-sectional area $A \quad 350\ \text{mm}^2$, and the modulus of elasticity $E \quad 170$ GPa.

Solution 2.8-2 Collar falling onto a flange



$M \quad 80$ kg

$W \quad Mg \quad (80\ \text{kg})(9.81\ \text{m/s}^2)$

$\quad 784.8$ N

$h \quad 0.5$ m $\qquad L \quad 3.0$ m

$E \quad 170$ GPa $\qquad A \quad 350\ \text{mm}^2$

(a) Downward displacement of flange

$$\delta_{st} \quad \frac{WL}{EA} \quad 0.03957\ \text{mm}$$

Eq. (2-53): $\delta_{max} \quad \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$

$\quad 6.33$ mm ◀

(b) Maximum tensile stress (Eq. 2-57)

$$\sigma_{max} \quad \frac{E\delta_{max}}{L} \quad 359\ \text{MPa} \;◀$$

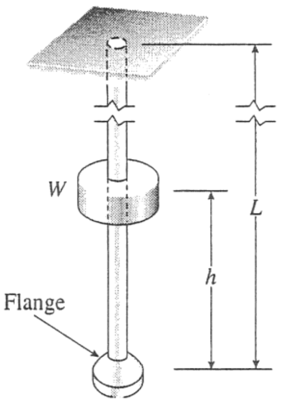
(c) Impact factor (Eq. 2-63)

$$\text{Impact factor} \quad \frac{\delta_{max}}{\delta_{st}} \quad \frac{6.33\ \text{mm}}{0.03957\ \text{mm}}$$

$\quad 160$ ◀

Problem 2.8-3 Solve Problem 2.8-1 if the collar has weight $W \quad 50$ lb, the height $h \quad 2.0$ in., the length $L \quad 3.0$ ft, the cross-sectional area $A \quad 0.25\ \text{in.}^2$, and the modulus of elasticity $E \quad 30{,}000$ ksi.

Solution 2.8-3 Collar falling onto a flange



$W \quad 50$ lb $\qquad h \quad 2.0$ in.

$L \quad 3.0$ ft $\quad 36$ in.

$E \quad 30{,}000$ psi $\qquad A \quad 0.25\ \text{in.}^2$

(a) Downward displacement of flange

$$\delta_{st} \quad \frac{WL}{EA} \quad 0.00024\ \text{in.}$$

Eq. (2-55): $\delta_{max} \quad \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$

$\quad 0.0312$ in. ◀

(b) Maximum tensile stress (Eq. 2-57)

$$\sigma_{max} = \frac{E\delta_{max}}{L} = 26{,}000 \text{ psi}$$

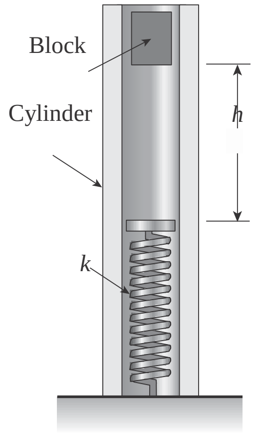
(c) Impact factor (Eq. 2-63)

$$\text{Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{0.0312 \text{ in.}}{0.00024 \text{ in.}}$$

$$= 130$$

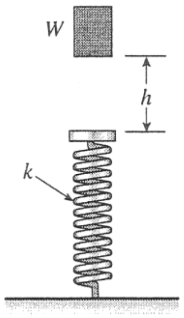
Problem 2.8-4 A block weighing $W = 5.0$ N drops inside a cylinder from a height $h = 200$ mm onto a spring having stiffness $k = 90$ N/m (see figure).

(a) Determine the maximum shortening of the spring due to the impact and (b) determine the impact factor.



Prob. 2.8-4 and 2.8-5

Solution 2.8-4 Block dropping onto a spring



$W = 5.0$ N $\quad h = 200$ mm $\quad k = 90$ N/m

(a) Maximum shortening of the spring

$$\delta_{st} = \frac{W}{k} = \frac{5.0 \text{ N}}{90 \text{ N/m}} = 55.56 \text{ mm}$$

Eq. (2-55): $$\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$= 215 \text{ mm}$$

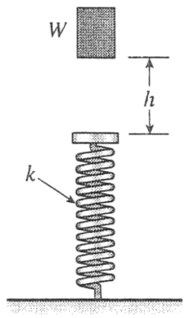
(b) Impact factor (Eq. 2-63)

$$\text{Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{215 \text{ mm}}{55.56 \text{ mm}}$$

$$= 3.9$$

Problem 2.8-5 Solve the preceding problem if the block weighs $W\quad 1.0$ lb, $h\quad 12$ in., and $k\quad 0.5$ lb/in.

Solution 2.8-5 Block dropping onto a spring



$W\quad 1.0$ lb $\qquad h\quad 12$ in. $\qquad k\quad 0.5$ lb/in.

(a) MAXIMUM SHORTENING OF THE SPRING

$$d_{st}\ \frac{W}{k}\quad \frac{1.0 \text{ lb}}{0.5 \text{ lb/in.}}\quad 2.0 \text{ in.}$$

Eq. (2-55): $$d_{max}\quad d_{st}\left[1+\left(1+\frac{2h}{d_{st}}\right)^{1/2}\right]$$

$$9.21 \text{ in.}$$

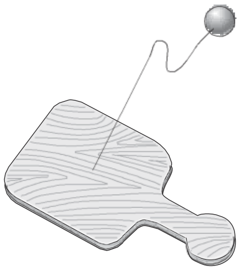
(b) IMPACT FACTOR (EQ. 2-63)

$$\text{Impact factor}\ \frac{d_{max}}{d_{st}}\quad \frac{9.21 \text{ in.}}{2.0 \text{ in.}}$$

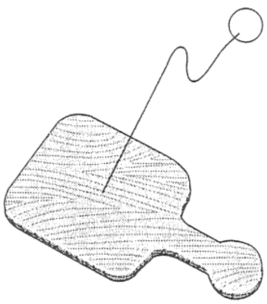
$$4.6$$

Problem 2.8-6 A small rubber ball (weight $W\quad 450$ mN) is attached by a rubber cord to a wood paddle (see figure). The natural length of the cord is $L_0\quad 200$ mm, its cross-sectional area is $A\quad 1.6 \text{ mm}^2$, and its modulus of elasticity is $E\quad 2.0$ MPa. After being struck by the paddle, the ball stretches the cord to a total length $L_1\quad 900$ mm.

What was the velocity v of the ball when it left the paddle? (Assume linearly elastic behavior of the rubber cord, and disregard the potential energy due to any change in elevation of the ball.)



Solution 2.8-6 Rubber ball attached to a paddle



$g\quad 9.81 \text{ m/s}^2 \qquad E\quad 2.0$ MPa

$A\quad 1.6 \text{ mm}^2 \qquad L_0\quad 200$ mm L_1 900 mm $\qquad W\quad 450$ mN

WHEN THE BALL LEAVES THE PADDLE

$$KE\ \frac{Wv^2}{2g}$$

WHEN THE RUBBER CORD IS FULLY STRETCHED:

$$U\ \frac{EA\delta^2}{2L_0}\quad \frac{EA}{2L_0}(L_1\quad L_0)^2$$

CONSERVATION OF ENERGY

$$KE\quad U\quad \frac{Wv^2}{2g}\quad \frac{EA}{2L_0}(L_1\quad L_0)^2$$

$$v^2\quad \frac{gEA}{WL_0}(L_1\quad L_0)^2$$

$$v\quad (L_1\quad L_0)\sqrt{\frac{gEA}{WL_0}}$$

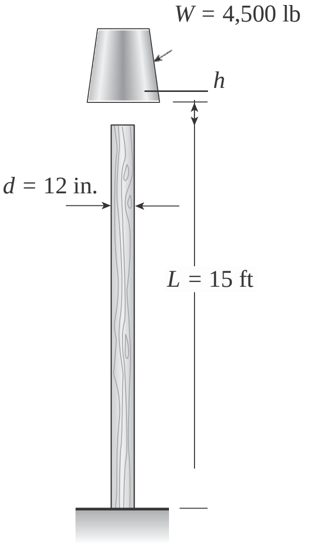
SUBSTITUTE NUMERICAL VALUES:

$$v\quad (700 \text{ mm})\sqrt{\frac{(9.81 \text{ m/s}^2)(2.0 \text{ MPa})(1.6 \text{ mm}^2)}{(450 \text{ mN})(200 \text{ mm})}}$$

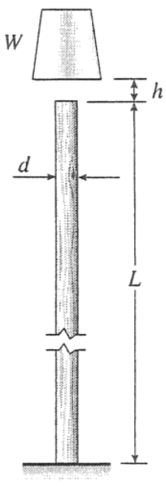
$$13.1 \text{ m/s}$$

Problem 2.8-7 A weight W 4500 lb falls from a height h onto a vertical wood pole having length L 15 ft, diameter d 12 in., and modulus of elasticity E 1.6 10^6 psi (see figure).

If the allowable stress in the wood under an impact load is 2500 psi, what is the maximum permissible height h?



Solution 2.8-7 Weight falling on a wood pole



W 4500 lb d 12 in.

L 15 ft 180 in.

$$A \quad \frac{\pi d^2}{4} \quad 113.10 \text{ in.}^2$$

E 1.6 10^6 psi

σ_{allow} 2500 psi (σ_{max})

Find h_{max}

STATIC STRESS

$$\sigma_{st} \quad \frac{W}{A} \quad \frac{4500 \text{ lb}}{113.10 \text{ in.}^2} \quad 39.79 \text{ psi}$$

MAXIMUM HEIGHT h_{max}

Eq. (2-61): $$\sigma_{max} \quad \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$$

or

$$\frac{\sigma_{max}}{\sigma_{st}} \quad 1 \quad \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for h:

$$h \quad h_{max} \frac{L\sigma_{max}}{2E}\left(\frac{\sigma_{max}}{\sigma_{st}} \quad 2\right) \quad ◀$$

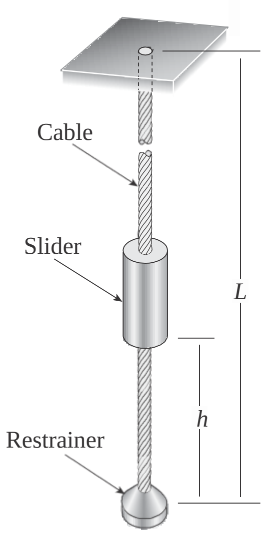
SUBSTITUTE NUMERICAL VALUES:

$$h_{max} \quad \frac{(180 \text{ in.})(2500 \text{ psi})}{2(1.6 * 10^6 \text{ psi})}\left(\frac{2500 \text{ psi}}{39.79 \text{ psi}} \quad 2\right)$$

$$8.55 \text{ in.} \quad ◀$$

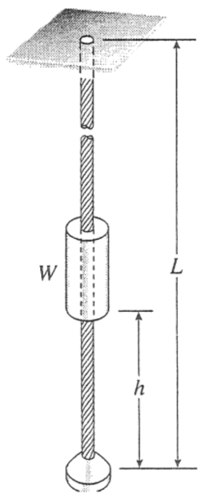
Problem 2.8-8 A cable with a restrainer at the bottom hangs vertically from its upper end (see figure). The cable has an effective cross-sectional area $A = 40\ \text{mm}^2$ and an effective modulus of elasticity $E = 130$ GPa. A slider of mass $M = 35$ kg drops from a height $h = 1.0$ m onto the restrainer.

If the allowable stress in the cable under an impact load is 500 MPa, what is the minimum permissible length L of the cable?



Probs. 2.8-8, 2.8-2, 2.8-9

Solution 2.8-8 Slider on a cable



$W = Mg = (35\ \text{kg})(9.81\ \text{m/s}^2) = 343.4\ \text{N}$

$A = 40\ \text{mm}^2 \qquad E = 130\ \text{GPa}$

$h = 1.0\ \text{m} \qquad \sigma_{allow} = \sigma_{max} = 500\ \text{MPa}$

Find minimum length L_{min}.

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{343.4\ \text{N}}{40\ \text{mm}^2} = 8.585\ \text{MPa}$$

MINIMUM LENGTH L_{min}

Eq. (2-61): $\sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for L:

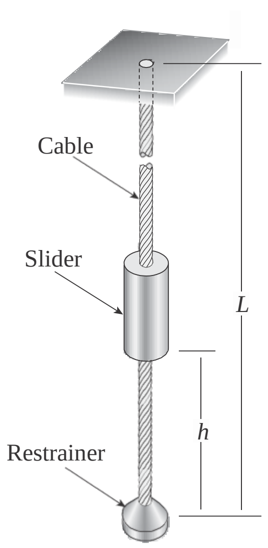
$$L = L_{min} = \frac{2Eh\sigma_{st}}{\sigma_{max}(\sigma_{max} - 2\sigma_{st})}$$

SUBSTITUTE NUMERICAL VALUES:

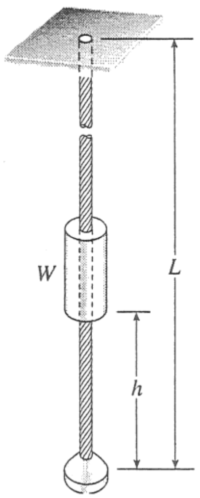
$$L_{min} = \frac{2(130\ \text{GPa})(1.0\ \text{m})(8.585\ \text{MPa})}{(500\ \text{MPa})[500\ \text{MPa} - 2(8.585\ \text{MPa})]}$$

$$= 9.25\ \text{mm}$$

Problem 2.8-9 Solve the preceding problem if the slider has weight $W = 100$ lb, $h = 45$ in., $A = 0.080$ in.^{2}, $E = 21 \times 10^6$ psi, and the allowable stress is 70 ksi.



Solution 2.8-9 Slider on a cable



$W = 100$ lb

$A = 0.080$ in.^{2} $E = 21 \times 10^6$ psi

$h = 45$ in $\sigma_{allow} = \sigma_{max} = 70$ ksi

Find minimum length L_{min}.

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{100 \text{ lb}}{0.080 \text{ in.}^2} = 1250 \text{ psi}$$

MINIMUM LENGTH L_{min}

$$\text{Eq. (2-61):} \quad \sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for L:

$$L = L_{min} = \frac{2Eh\sigma_{st}}{\sigma_{max}(\sigma_{max} - 2\sigma_{st})} \quad \leftarrow$$

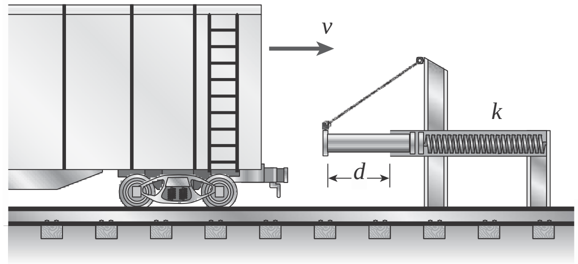
SUBSTITUTE NUMERICAL VALUES:

$$L_{min} = \frac{2(21 \times 10^6 \text{ psi})(45 \text{ in.})(1250 \text{ psi})}{(70{,}000 \text{ psi})[70{,}000 \text{ psi} - 2(1250 \text{ psi})]}$$

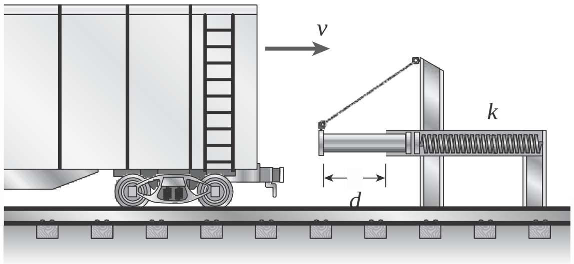
$$= 500 \text{ in.} \quad \leftarrow$$

Problem 2.8-10 A bumping post at the end of a track in a railway yard has a spring constant k 8.0 MN/m (see figure). The maximum possible displacement d of the end of the striking plate is 450 mm.

What is the maximum velocity n_{max} that a railway car of weight W 545 kN can have without damaging the bumping post when it strikes it?



Solution 2.8-10 Bumping post for a railway car



k 8.0 MN/m $\quad W$ 545 kN

d maximum displacement of spring

$d \quad d_{max}$ 450 mm

Find n_{max}.

KINETIC ENERGY BEFORE IMPACT

$$KE \quad \frac{Mv^2}{2} \quad \frac{Wv^2}{2g}$$

STRAIN ENERGY WHEN SPRING IS COMPRESSED TO THE MAXIMUM ALLOWABLE AMOUNT

$$U \quad \frac{kd_{max}^2}{2} \quad \frac{kd^2}{2}$$

CONSERVATION OF ENERGY

$$KE \quad U \quad \frac{Wv^2}{2g} \quad \frac{kd^2}{2} \quad v^2 \quad \frac{kd^2}{W/g}$$

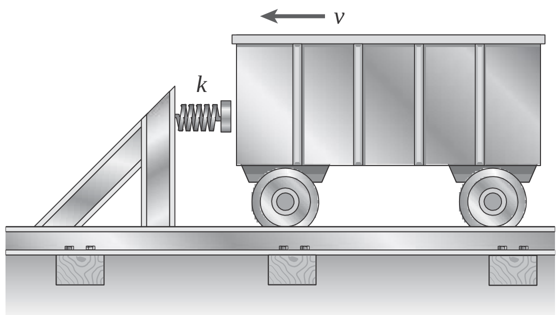
$$v \quad v_{max} \quad d\sqrt{\frac{k}{W/g}}$$

SUBSTITUTE NUMERICAL VALUES:

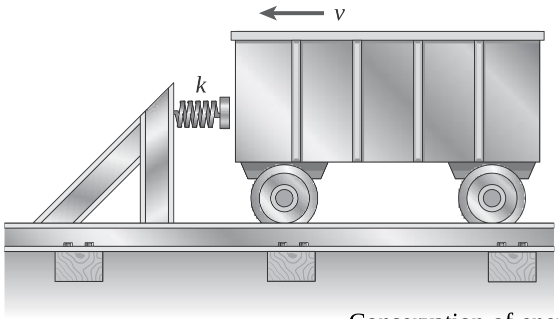
$$v_{max} \quad (450\text{ mm})\sqrt{\frac{8.0\text{ MN/m}}{(545\text{ kN})/(9.81\text{ m/s}^2)}}$$

$$5400\text{ mm/s} \quad 5.4\text{ m/s}$$

Problem 2.8-11 A bumper for a mine car is constructed with a spring of stiffness k 1120 lb/in. (see figure). If a car weighing 3450 lb is traveling at velocity n 7 mph when it strikes the spring, what is the maximum shortening of the spring?



Solution 2.8-11 Bumper for a mine car



k 1120 lb/in. W 3450 lb

n 7 mph 123.2 in./sec

g 32.2 ft/sec^2 386.4 in./sec^2

Find the shortening d_{max} of the spring.

KINETIC ENERGY JUST BEFORE IMPACT

$$KE \quad \frac{Mv^2}{2} \quad \frac{Wv^2}{2g}$$

STRAIN ENERGY WHEN SPRING IS FULLY COMPRESSED

$$U \quad \frac{kd^2_{max}}{2}$$

Conservation of energy

$$KE \quad U \quad \frac{Wv^2}{2g} \quad \frac{kd^2_{max}}{2}$$

Solve for d_{max}: $d_{max} \quad \sqrt{\frac{Wv^2}{gk}}$

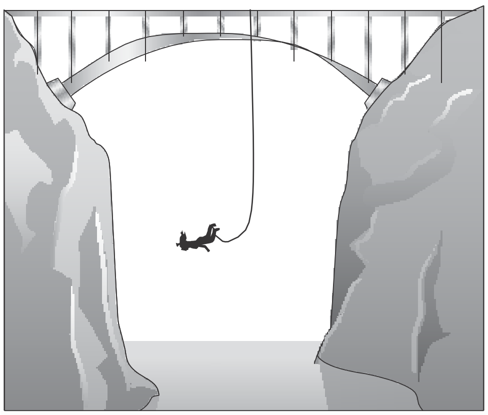
SUBSTITUTE NUMERICAL VALUES:

$$d_{max} \quad \sqrt{\frac{(3450 \text{ lb})(123.2 \text{ in./sec})^2}{(386.4 \text{ in./sec}^2)(1120 \text{ lb/in.})}}$$

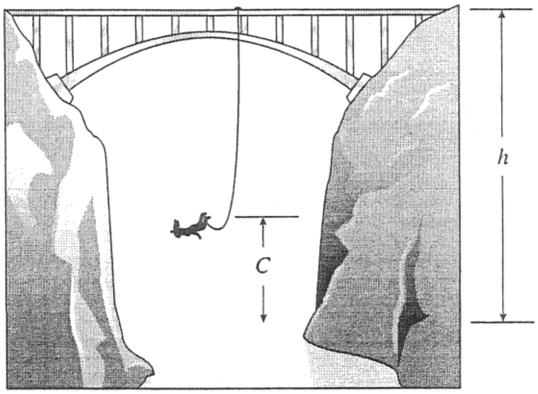
$$11.0 \text{ in.}$$

Problem 2.8-12 A bungee jumper having a mass of 55 kg leaps from a bridge, braking her fall with a long elastic shock cord having axial rigidity EA 2.3 kN (see figure).

If the jumpoff point is 60 m above the water, and if it is desired to maintain a clearance of 10 m between the jumper and the water, what length L of cord should be used?



Solution 2.8-12 Bungee jumper



$W = Mg = (55 \text{ kg})(9.81 \text{ m/s}^2)$

$= 539.55 \text{ N}$

$EA = 2.3 \text{ kN}$

Height: $h = 60 \text{ m}$

Clearance: $C = 10 \text{ m}$

Find length L of the bungee cord.

P.E. Potential energy of the jumper at the top of bridge (with respect to lowest position)

$= W(L + d_{max})$

$U =$ strain energy of cord at lowest position

$$= \frac{EAd_{max}^2}{2L}$$

CONSERVATION OF ENERGY

$$P.E. = U \qquad W(L + d_{max}) = \frac{EAd_{max}^2}{2L}$$

$$\text{or} \qquad d_{max}^2 - \frac{2WL}{EA} d_{max} - \frac{2WL^2}{EA} = 0$$

SOLVE QUADRATIC EQUATION FOR d_{max}:

$$d_{max} = \frac{WL}{EA} + \left[\left(\frac{WL}{EA}\right)^2 + 2L\left(\frac{WL}{EA}\right)\right]^{1/2}$$

$$= \frac{WL}{EA}\left[1 + \left(1 + \frac{2EA}{W}\right)^{1/2}\right]$$

VERTICAL HEIGHT

$$h = C + L + d_{max}$$

$$h - C = L + \frac{WL}{EA}\left[1 + \left(1 + \frac{2EA}{W}\right)^{1/2}\right]$$

SOLVE FOR L:

$$L = \frac{h - C}{1 + \frac{W}{EA}\left[1 + \left(1 + \frac{2EA}{W}\right)^{1/2}\right]} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\frac{W}{EA} = \frac{539.55 \text{ N}}{2.3 \text{ kN}} = 0.234587$$

Numerator $= h - C = 60 \text{ m} - 10 \text{ m} = 50 \text{ m}$

Denominator $= 1 + (0.234587)$

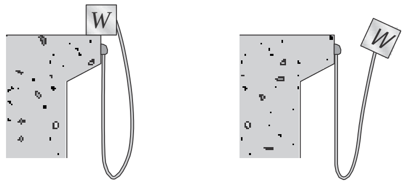
$$\times \left[1 + \left(1 + \frac{2}{0.234587}\right)^{1/2}\right]$$

$= 1.9586$

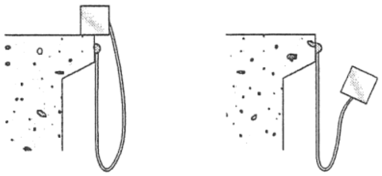
$$L = \frac{50 \text{ m}}{1.9586} = 25.5 \text{ m} \quad \leftarrow$$

Problem 2.8-13 A weight W rests on top of a wall and is attached to one end of a very flexible cord having cross-sectional area A and modulus of elasticity E (see figure). The other end of the cord is attached securely to the wall. The weight is then pushed off the wall and falls freely the full length of the cord.

(a) Derive a formula for the impact factor.
(b) Evaluate the impact factor if the weight, when hanging statically, elongates the band by 2.5% of its original length.



Solution 2.8-13 Weight falling off a wall



$W =$ Weight

Properties of elastic cord:

$E =$ modulus of elasticity

$A =$ cross-sectional area

$L =$ original length

$d_{max} =$ elongation of elastic cord

$P.E. =$ potential energy of weight before fall (with respect to lowest position)

$P.E. = W(L + d_{max})$

Let $U =$ strain energy of cord at lowest position.

$$U = \frac{EAd_{max}^2}{2L}$$

Conservation of energy

$$P.E. = U \qquad W(L + d_{max}) = \frac{EAd_{max}^2}{2L}$$

$$\text{or} \qquad d_{max}^2 - \frac{2WL}{EA}d_{max} - \frac{2WL^2}{EA} = 0$$

Solve quadratic equation for d_{max}:

$$d_{max} = \frac{WL}{EA} + \left[\left(\frac{WL}{EA}\right)^2 + 2L\left(\frac{WL}{EA}\right)\right]^{1/2}$$

Static elongation

$$d_{st} = \frac{WL}{EA}$$

Impact factor

$$\frac{d_{max}}{d_{st}} = 1 + \left[1 + \frac{2EA}{W}\right]^{1/2}$$

Numerical values

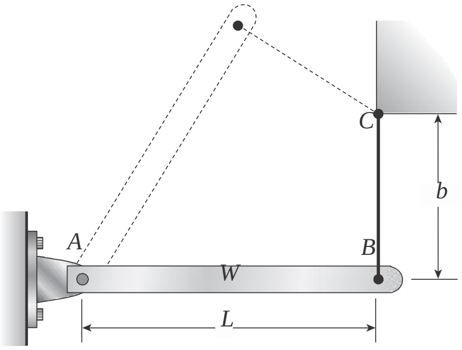
$$d_{st} = (2.5\%)(L) = 0.025L$$

$$d_{st} = \frac{WL}{EA} \qquad \frac{W}{EA} = 0.025 \qquad \frac{EA}{W} = 40$$

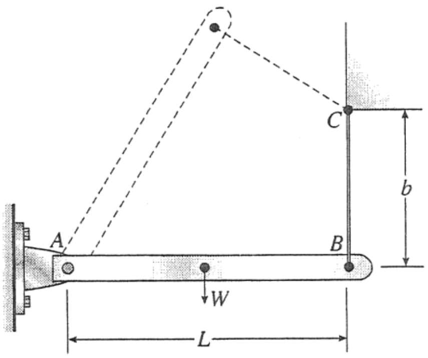
$$\text{Impact factor} = 1 + [1 + 2(40)]^{1/2} = 10$$

Problem 2.8-14 A rigid bar *AB* having mass M 1.0 kg and length L 0.5 m is hinged at end *A* and supported at end *B* by a nylon cord *BC* (see figure). The cord has cross-sectional area A 30 mm^2, length b 0.25 m, and modulus of elasticity E 2.1 GPa.

If the bar is raised to its maximum height and then released, what is the maximum stress in the cord?



Solution 2.8-14 Falling bar AB



RIGID BAR:

W Mg (1.0 kg)(9.81 m/s^2)

9.81 N L

0.5 m NYLON

CORD: A 30

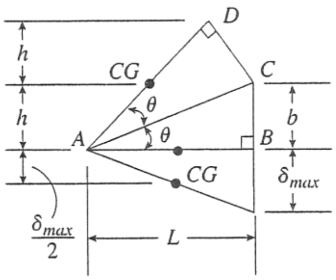
mm^2

b 0.25 m

E 2.1 GPa

Find maximum stress s_{max} in cord *BC*.

GEOMETRY OF BAR *AB* AND CORD *BC*



$\overline{CD}$ $\overline{CB}$ b

$\overline{AD}$ $\overline{AB}$ L

h height of center of gravity of raised bar *AD*

d_{max} elongation of cord

From triangle *ABC*: $\sin u \quad \frac{b}{2b^2 + L^2}$

$\cos u \quad \frac{L}{2b^2 + L^2}$

From line *AD*: $\sin 2u \quad \frac{2h}{AD} \quad \frac{2h}{L}$

From Appendix D: $\sin 2u \quad 2 \sin u \cos u$

$< \quad \frac{2h}{L} \quad 2\,a\frac{b}{2b^2 + L^2}b\,a\frac{L}{2b^2 + L^2}b \quad \frac{2bL}{b^2 + L^2}$

and $h \quad \frac{bL^2}{b^2 + L^2}$ (Eq. 1)

CONSERVATION OF ENERGY

$P.E.$ potential energy of raised bar AD

$$W\, a h + \frac{d_{max}}{2} b$$

U strain energy of stretched cord $\dfrac{EAd^2_{max}}{2b}$

$$P.E. \quad U \quad W\, a h + \frac{d_{max}}{2} b \quad \frac{EAd^2_{max}}{2b} \qquad \text{(Eq. 2)}$$

For the cord: $d_{max} \quad \dfrac{s_{max} b}{E}$

Substitute into Eq. (2) and rearrange:

$$s^2_{max} \quad \frac{W}{A} s_{max} \quad \frac{2WhE}{bA} \quad 0 \qquad \text{(Eq. 3)}$$

Substitute from Eq. (1) into Eq. (3):

$$s^2_{max} \quad \frac{W}{A} s_{max} \quad \frac{2WL^2E}{A(b^2 + L^2)} \quad 0 \qquad \text{(Eq. 4)}$$

SOLVE FOR s_{max}:

$$s_{max} \quad \frac{W}{2A} c 1 + A 1 + \frac{8L^2 EA}{W(b^2 + L^2)} d$$

SUBSTITUTE NUMERICAL VALUES:

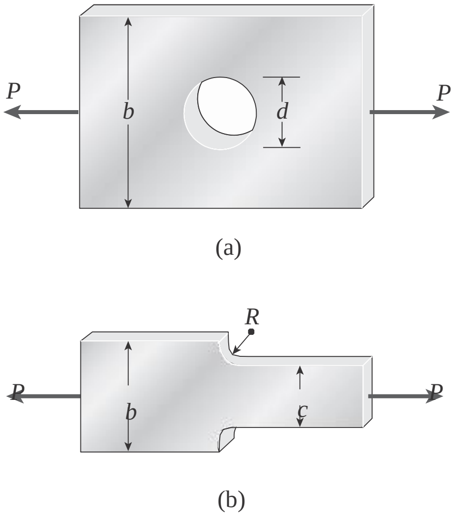
s_{max} 33.3 MPa

Stress Concentrations

The problems for Section 2.10 are to be solved by considering the stress-concentration factors and assuming linearly elastic behavior.

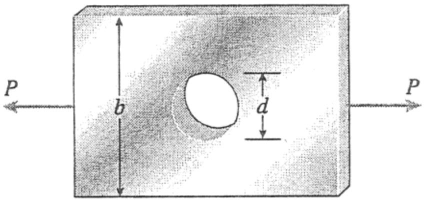
Problem 2.10-1 The flat bars shown in parts (a) and (b) of the figure are subjected to tensile forces P 3.0 k. Each bar has thickness t 0.25 in.

(a) For the bar with a circular hole, determine the maximum stresses for hole diameters d 1 in. and d 2 in. if the width b 6.0 in.
(b) For the stepped bar with shoulder fillets, determine the maximum stresses for fillet radii R 0.25 in. and R 0.5 in. if the bar widths are b 4.0 in. and c 2.5 in.

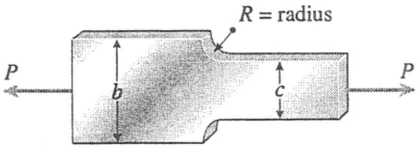


Probs. 2.10-1 and 2.10-2

Solution 2.10-1 Flat bars in tension



(a)



(b)

P 3.0 k t 0.25 in.

(a) BAR WITH CIRCULAR HOLE (b 6 in.)

Obtain K from Fig. 2-63

FOR d 1 in.: c b d 5 in.

$$s_{\text{nom}} \quad \frac{P}{ct} \quad \frac{3.0\text{ k}}{(5\text{ in.})(0.25\text{ in.})} \quad 2.40\text{ ksi}$$

$$d/b\ \frac{1}{6} \quad K \text{ L } 2.60$$

$$s_{\max} \quad ks_{\text{nom}} \quad 6.2\text{ ksi}$$

FOR d 2 in.: c b d 4 in.

$$s_{\text{nom}} \quad \frac{P}{ct} \quad \frac{3.0\text{ k}}{(4\text{ in.})(0.25\text{ in.})} \quad 3.00\text{ ksi}$$

$$d/b\ \frac{1}{3} \quad K \text{ L } 2.31$$

$$s_{\max} \quad Ks_{\text{nom}} \quad 6.9\text{ ksi}$$

(b) STEPPED BAR WITH SHOULDER FILLETS

b 4.0 in. c 2.5 in.; Obtain k from Fig. 2-65

$$s_{\text{nom}} \quad \frac{P}{ct} \quad \frac{3.0\text{ k}}{(2.5\text{ in.})(0.25\text{ in.})} \quad 4.80\text{ ksi}$$

FOR R 0.25 in.: R/c 0.1 b/c 1.60

k 2.30 $s_{\max}$ Ks_{nom} 11.0 ksi

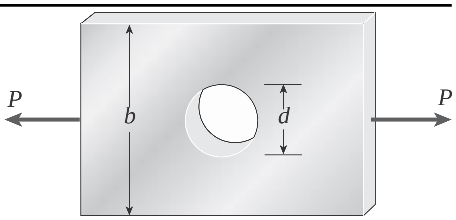
FOR R 0.5 in.: R/c 0.2 b/c 1.60

K 1.87 $s_{\max}$ Ks_{nom} 9.0 ksi

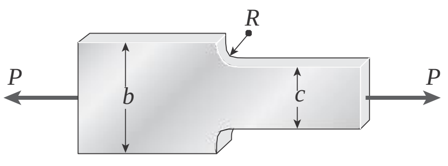
Problem 2.10-2 The flat bars shown in parts (a) and (b) of the figure are subjected to tensile forces P 2.5 kN. Each bar has thickness t 5.0 mm.

(a) For the bar with a circular hole, determine the maximum stresses for hole diameters d 12 mm and d 20 mm if the width b 60 mm.

(b) For the stepped bar with shoulder fillets, determine the maximum stresses for fillet radii R 6 mm and R 10 mm if the bar widths are b 60 mm and c 40 mm.

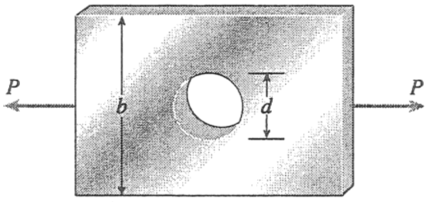


(a)

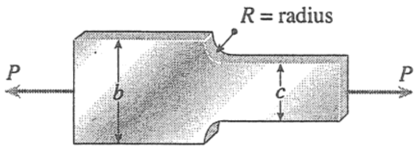


(b)

Solution 2.10-2 Flat bars in tension



(a)



(b)

$P = 2.5$ kN $\quad t = 5.0$ mm

(a) Bar with circular hole ($b = 60$ mm)

Obtain K from Fig. 2-63

For $d = 12$ mm: $c = b - d = 48$ mm

$$\sigma_{nom} = \frac{P}{ct} = \frac{2.5 \text{ kN}}{(48 \text{ mm})(5 \text{ mm})} = 10.42 \text{ MPa}$$

$$d/b = \frac{1}{5} \quad K \approx 2.51$$

$$\sigma_{max} = K\sigma_{nom} = 26 \text{ MPa}$$

For $d = 20$ mm: $c = b - d = 40$ mm

$$\sigma_{nom} = \frac{P}{ct} = \frac{2.5 \text{ kN}}{(40 \text{ mm})(5 \text{ mm})} = 12.50 \text{ MPa}$$

$$d/b = \frac{1}{3} \quad K \approx 2.31$$

$$\sigma_{max} = K\sigma_{nom} = 29 \text{ MPa}$$

(b) Stepped bar with shoulder fillets

$b = 60$ mm $\quad c = 40$ mm;

Obtain K from Fig. 2-65

$$\sigma_{nom} = \frac{P}{ct} = \frac{2.5 \text{ kN}}{(40 \text{ mm})(5 \text{ mm})} = 12.50 \text{ MPa}$$

For $R = 6$ mm: $R/c = 0.15 \quad b/c = 1.5$

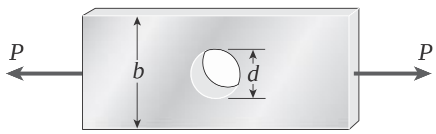
$K = 2.00 \quad \sigma_{max} = K\sigma_{nom} = 25$ MPa

For $R = 10$ mm: $R/c = 0.25 \quad b/c = 1.5$

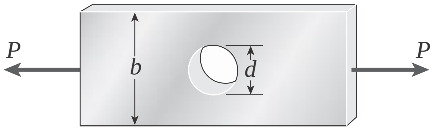
$K = 1.75 \quad \sigma_{max} = K\sigma_{nom} = 22$ MPa

Problem 2.10-3 A flat bar of width b and thickness t has a hole of diameter d drilled through it (see figure). The hole may have any diameter that will fit within the bar.

What is the maximum permissible tensile load P_{max} if the allowable tensile stress in the material is σ_t?



Solution 2.10-3 Flat bar in tension



t = thickness

σ_t = allowable tensile stress

Find P_{max}

Find K from Fig. 2-63

$$P_{max} = \sigma_{nom}ct = \frac{\sigma_{max}}{K}ct = \frac{\sigma_t}{K}(b - d)t$$

$$= \frac{\sigma_t}{K}bt\left(1 - \frac{d}{b}\right)$$

Because σ_t, b, and t are constants, we write:

$$P^* = \frac{P_{max}}{\sigma_t bt} = \frac{1}{K}\left(1 - \frac{d}{b}\right)$$

$\frac{d}{b}$	K	P^*
0	3.00	0.333
0.1	2.73	0.330
0.2	2.50	0.320
0.3	2.35	0.298
0.4	2.24	0.268

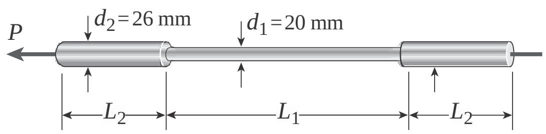
We observe that P_{max} decreases as d/b increases. Therefore, the maximum load occurs when the hole becomes very small.

$$\left(\frac{d}{b} \to 0 \quad \text{and} \quad K \to 3\right)$$

$$P_{max} = \frac{\sigma_t bt}{3} \leftarrow$$

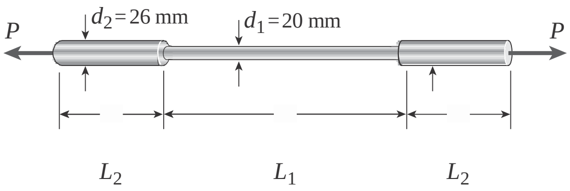
Problem 2.10-4 A round brass bar of diameter $d_1 = 20$ mm has upset ends of diameter $d_2 = 26$ mm (see figure). The lengths of the segments of the bar are $L_1 = 0.3$ m and $L_2 = 0.1$ m. Quarter-circular fillets are used at the shoulders of the bar, and the modulus of elasticity of the brass is $E = 100$ GPa.

If the bar lengthens by 0.12 mm under a tensile load P, what is the maximum stress σ_{max} in the bar?



Probs. 2.10-4 and 2.10-5

Solution 2.10-4 Round brass bar with upset ends



E 100 GPa

d 0.12 mm

L_2 0.1 m

L_1 0.3 m

R radius of fillets $\dfrac{26 \text{ mm} \quad 20 \text{ mm}}{2}$ 3 mm

$$d \quad 2\left(\frac{PL_2}{EA_2}\right) + \frac{PL_1}{EA_1}$$

Solve for P: $\quad P \quad \dfrac{dEA_1A_2}{2L_2A_1 + L_1A_2}$

Use Fig. 2-66 for the stress-concentration factor:

$$s_{\text{nom}} \quad \frac{P}{A_1} \quad \frac{dEA_2}{2L_2A_1 + L_1A_2} \quad \frac{dE}{2L_2\left(\frac{A_1}{A_2}\right) + L_1}$$

$$\frac{dE}{2L_2\left(\frac{d_1}{d_2}\right)^2 + L_1}$$

SUBSTITUTE NUMERICAL VALUES:

$$s_{\text{nom}} \quad \frac{(0.12 \text{ mm})(100 \text{ GPa})}{2(0.1 \text{ m})\left(\frac{20}{26}\right)^2 + 0.3 \text{ m}} \quad 28.68 \text{ MPa}$$

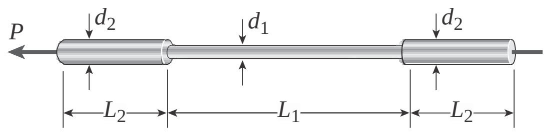
$$\frac{R}{D_1} \quad \frac{3 \text{ mm}}{20 \text{ mm}} \quad 0.15$$

Use the dashed curve in Fig. 2-66. K 1.6

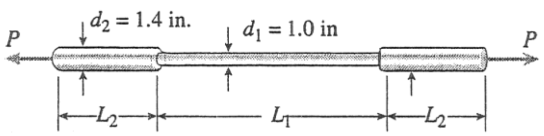
$s_{\max}$ Ks_{nom} (1.6) (28.68 MPa)

46 MPa

Problem 2.10-5 Solve the preceding problem for a bar of monel metal having the following properties: d_1 1.0 in., d_2 1.4 in., L_1 20.0 in., L_2 5.0 in., and E 25 10^6 psi. Also, the bar lengthens by 0.0040 in. when the tensile load is applied.



Solution 2.10-5 Round bar with upset ends



E 25 10^6 psi

d 0.0040 in.

L_1 20 in.

L_2 5 in.

R radius of fillets R

$$d \quad 2\left(\frac{PL_2}{EA_2}\right) + \frac{PL_1}{EA_1}$$

Solve for P: $P \quad \dfrac{dEA_1A_2}{2L_2A_1 + L_1A_2}$

Use Fig. 2-66 for the stress-concentration factor.

$$s_{\text{nom}} \quad \frac{P}{A_1} \quad \frac{dEA_2}{2L_2A_1 + L_1A_2} \quad \frac{dE}{2L_2\left(\frac{A_1}{A_2}\right) + L_1}$$

$$\frac{dE}{}$$

. in.

0 2

$\frac{1.4 \text{ in.} \quad 1.0 \text{ in.}}{2}$ $2L_2$ a $\frac{d_1}{d_2}$ b ^{2} + L_1

SUBSTITUTE NUMERICAL VALUES:

$$\sigma_{nom} \quad \frac{(0.0040 \text{ in.})(25 * 10^6 \text{ psi})}{2(5 \text{ in.})\left(\frac{1.0}{1.4}\right)^2 + 20 \text{ in.}} \quad 3{,}984 \text{ psi}$$

$$\frac{R}{D_1} \quad \frac{0.2 \text{ in.}}{1.0 \text{ in.}} \quad 0.2$$

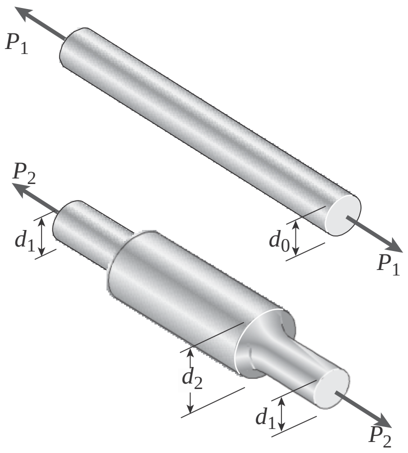
Use the dashed curve in Fig. 2-66. $K \quad 1.53$

$$\sigma_{max} \quad K\sigma_{nom} \quad (1.53)(3984 \text{ psi})$$

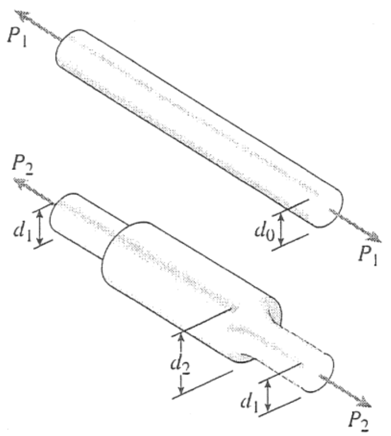
$$6100 \text{ psi} \quad \leftarrow$$

Problem 2.10-6 A prismatic bar of diameter d_0 20 mm is being compared with a stepped bar of the same diameter (d_1 20 mm) that is enlarged in the middle region to a diameter d_2 25 mm (see figure). The radius of the fillets in the stepped bar is 2.0 mm.

(a) Does enlarging the bar in the middle region make it stronger than the prismatic bar? Demonstrate your answer by determining the maximum permissible load P_1 for the prismatic bar and the maximum permissible load P_2 for the enlarged bar, assuming that the allowable stress for the material is 80 MPa.
(b) What should be the diameter d_0 of the prismatic bar if it is to have the same maximum permissible load as does the stepped bar?



Solution 2.10-6 Prismatic bar and stepped bar



$d_0 \quad 20$ mm

$d_1 \quad 20$ mm

$d_2 \quad 25$ mm

Fillet radius: $R \quad 2$ mm

Allowable stress: $\sigma_t \quad 80$ MPa

(a) COMPARISON OF BARS

Prismatic bar: $P_1 \quad \sigma_t A_0 \quad \sigma_t\left(\frac{\pi d_0^2}{4}\right)$

$$(80 \text{ MPa})\left(\frac{\pi}{4}\right)(20\text{mm})^2 \quad 25.1 \text{ kN} \quad \leftarrow$$

Stepped bar: See Fig. 2-66 for the stress-concentration factor.

$R \quad 2.0$ mm $\qquad D_1 \quad 20$ mm $\qquad D_2 \quad 25$ mm

$R/D_1 \quad 0.10 \qquad D_2/D_1 \quad 1.25 \qquad K \quad 1.75$

$$\sigma_{nom} \quad \frac{P_2}{\frac{\pi}{4} d_1^2} \quad \frac{P_2}{A_1} \qquad \sigma_{nom} \quad \frac{\sigma_{max}}{K}$$

$$P_2 = \sigma_{\text{nom}} A_1 = \frac{\sigma_{\max}}{K} A_1 = \frac{\sigma_t}{K} A_1 = \left(\frac{80\text{ MPa}}{1.75}\right)\left(\frac{\pi}{4}\right)(20\text{ mm})^2$$

$$\approx 14.4\text{ kN} \;\leftarrow$$

Enlarging the bar makes it *weaker*, not stronger. The ratio of loads is $P_1/P_2 = K = 1.75$

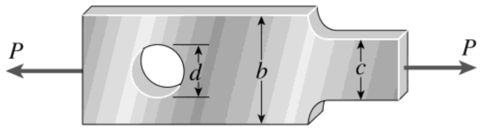
(b) Diameter of prismatic bar for the same allowable load

$$P_1 = P_2 \quad \sigma_t\left(\frac{\pi d_0^2}{4}\right) = \frac{\sigma_t}{K}\left(\frac{\pi d_1^2}{4}\right) \quad d_0^2 = \frac{d_1^2}{K}$$

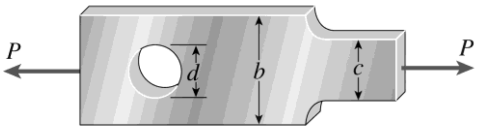
$$d_0 = \frac{d_1}{\sqrt{K}} = \frac{20\text{ mm}}{\sqrt{1.75}} \approx 15.1\text{ mm} \;\leftarrow$$

Problem 2.10-7 A stepped bar with a hole (see figure) has widths $b = 2.4$ in. and $c = 1.6$ in. The fillets have radii equal to 0.2 in.

What is the diameter $d_{\max}$ of the largest hole that can be drilled through the bar without reducing the load-carrying capacity?



Solution 2.10-7 Stepped bar with a hole



$b = 2.4$ in.

$c = 1.6$ in.

Fillet radius: $R = 0.2$ in.

Find $d_{\max}$

Based upon fillets (Use Fig. 2-65)

$b = 2.4$ in. $\quad c = 1.6$ in. $\quad R = 0.2$ in.

$R/c = 0.125 \quad b/c = 1.5 \quad K \approx 2.10$

$$P_{\max} = \sigma_{\text{nom}} ct = \frac{\sigma_{\max}}{K} ct = \frac{\sigma_{\max}}{K}\left(\frac{c}{b}\right)(bt)$$

$$\approx 0.317\, bt\, \sigma_{\max}$$

Based upon hole (Use Fig. 2-63)

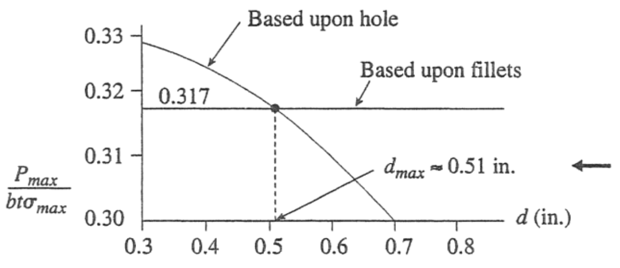
$b = 2.4$ in. $\quad d =$ diameter of the hole (in.)

$c_1 = b - d$

$$P_{\max} = \sigma_{\text{nom}} c_1 t = \frac{\sigma_{\max}}{K}(b - d)t$$

$$= \frac{1}{K}\left(1 - \frac{d}{b}\right) bt\sigma_{\max}$$

d(in.)	d/b	K	$P_{\max}/bt\sigma_{\max}$
0.3	0.125	2.66	0.329
0.4	0.167	2.57	0.324
0.5	0.208	2.49	0.318
0.6	0.250	2.41	0.311
0.7	0.292	2.37	0.299


Based upon hole
Based upon fillets
0.33
0.32
0.317
0.31
0.30
$\frac{P_{max}}{bt\sigma_{max}}$
$d_{max} \approx 0.51$ in.
d (in.)
0.3
0.4
0.5
0.6
0.7
0.8

Nonlinear Behavior (Changes in Lengths of Bars)

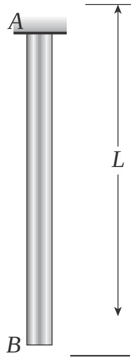
Problem 2.11-1 A bar AB of length L and weight density γ hangs vertically under its own weight (see figure). The stress-strain relation for the material is given by the Ramberg-Osgood equation (Eq. 2-73):

$$\epsilon = \frac{\sigma}{E} + \frac{\sigma_0 \alpha}{E}\left(\frac{\sigma}{\sigma_0}\right)^m$$

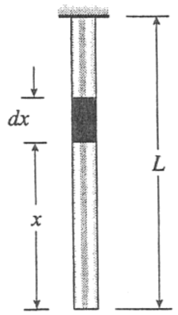
Derive the following formula

$$\delta = \frac{\gamma L^2}{2E} + \frac{\sigma_0 \alpha L}{(m+1)E}\left(\frac{\gamma L}{\sigma_0}\right)^m$$

for the elongation of the bar.



Solution 2.11-1 Bar hanging under its own weight



Let A = cross-sectional area

Let N = axial force at distance x

$N = \gamma A x$

$\sigma = \frac{N}{A} = \gamma x$

STRAIN AT DISTANCE x

$$\epsilon = \frac{\sigma}{E} + \frac{\sigma_0 \alpha}{E}\left(\frac{\sigma}{\sigma_0}\right)^m = \frac{\gamma x}{E} + \frac{\sigma_0}{\alpha E}\left(\frac{\gamma x}{\sigma_0}\right)^m$$

ELONGATION OF BAR

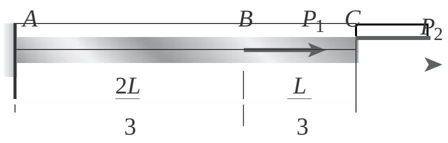
$$\delta = \int_0^L \epsilon\, dx = \int_0^L \frac{\gamma x}{E} dx + \frac{\sigma_0 \alpha}{E}\int_0^L \left(\frac{\gamma x}{\sigma_0}\right)^m dx$$

$$= \frac{\gamma L^2}{2E} + \frac{\sigma_0 \alpha L}{(m+1)E}\left(\frac{\gamma L}{\sigma_0}\right)^m \quad \text{Q.E.D.}$$

Problem 2.11-2 A prismatic bar of length $L = 1.8$ m and cross-sectional area $A = 480$ mm^2 is loaded by forces $P_1 = 30$ kN and $P_2 = 60$ kN (see figure). The bar is constructed of magnesium alloy having a stress-strain curve described by the following Ramberg-Osgood equation:

$$\epsilon = \frac{\sigma}{45{,}000} + \frac{1}{618}\left(\frac{\sigma}{170}\right)^{10} \quad (\sigma = \text{MPa})$$

in which σ has units of megapascals.

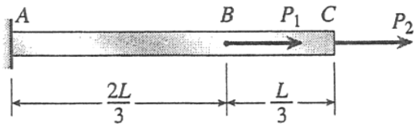


(a) Calculate the displacement δ_C of the end of the bar when the load P_1 acts alone.

(b) Calculate the displacement when the load P_2 acts alone.

(c) Calculate the displacement when both loads act simultaneously.

Solution 2.11-2 Axially loaded bar



L 1.8 m A 480 mm^2

P_1 30 kN P_2 60 kN

Ramberg–Osgood equation:

$$\frac{s}{45{,}000} + \frac{1}{618}a\frac{s}{170}b^{10} \quad (s \quad \text{MPa})$$

Find displacement at end of bar.

(a) P_1 ACTS ALONE

$$AB: s \quad \frac{P_1}{A} \quad \frac{30 \text{ kN}}{480 \text{ mm}^2} \quad 62.5 \text{ MPa}$$

0.001389

$$d_c \quad a\frac{2L}{3}b \quad 1.67 \text{ mm}$$

(b) P_2 ACTS ALONE

$$ABC: s \quad \frac{P_2}{A} \quad \frac{60 \text{ kN}}{480 \text{ mm}^2} \quad 125 \text{ MPa}$$

0.002853

$$d_c \quad L \quad 5.13 \text{ mm}$$

(c) BOTH P_1 AND P_2 ARE ACTING

$$AB: s \quad \frac{P_1 + P_2}{A} \quad \frac{90 \text{ kN}}{480 \text{ mm}^2} \quad 187.5 \text{ MPa}$$

0.008477

$$d_{AB} \quad a\frac{2L}{3}b \quad 10.17 \text{ mm}$$

$$BC: s \quad \frac{P_2}{A} \quad \frac{60 \text{ kN}}{480 \text{ mm}^2} \quad 125 \text{ MPa}$$

0.002853

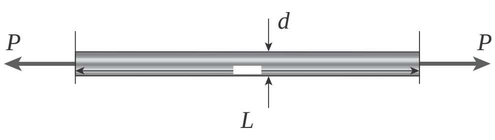
$$d_{BC} \quad a\frac{L}{3}b \quad 1.71 \text{ mm}$$

$$d_C \quad d_{AB} + d_{BC} \quad 11.88 \text{ mm}$$

(Note that the displacement when both loads act simultaneously is *not* equal to the sum of the displacements when the loads act separately.)

Problem 2.11-3 A circular bar of length L 32 in. and diameter d 0.75 in. is subjected to tension by forces P (see figure). The wire is made of a copper alloy having the following *hyperbolic stress-strain relationship*:

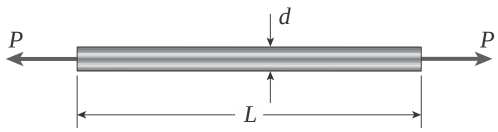
$$s \quad \frac{18{,}000P}{1 + 300P} \quad 0 \ldots P \ldots 0.03 \quad (s \quad \text{ksi})$$



(a) Draw a stress-strain diagram for the material.

(b) If the elongation of the wire is limited to 0.25 in. and the maximum stress is limited to 40 ksi, what is the allowable load P?

Solution 2.11-3 Copper bar in tension

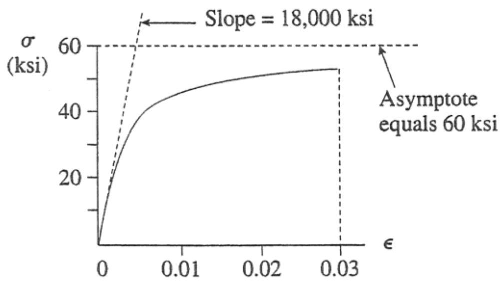


L 32 in. d 0.75 in.

$A \quad \frac{\underline{pd^2}}{4} \quad 0.4418 \text{ in.}^2$

(a) Stress-strain diagram

$s \quad \frac{18{,}000}{1 + 300} \quad 0 \ldots \quad \ldots 0.03 \quad (s \quad \text{ksi})$



(b) Allowable load P

Maximum elongation d_{max} 0.25 in.

Maximum stress s_{max} 40 ksi

Based upon elongation:

$_{max} \quad \frac{d_{max}}{L} \quad \frac{0.25 \text{ in.}}{32 \text{ in.}} \quad 0.007813$

$s_{max} \quad \frac{18{,}000 \quad _{max}}{1 + 300 \quad _{max}} \quad 42.06 \text{ ksi}$

Based upon stress:

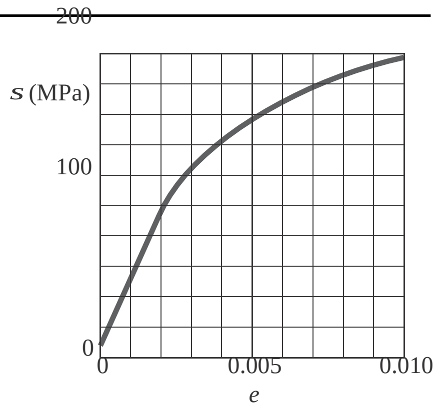
s_{max} 40 ksi

Stress governs. $P \quad s_{max} \, A \quad (40 \text{ ksi})(0.4418 \text{ in.}^2)$

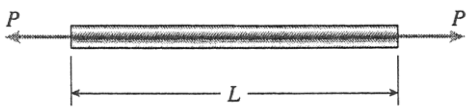
17.7 k

Problem 2.11-4 A prismatic bar in tension has length L 2.0 m and cross-sectional area A 249 mm^2. The material of the bar has the stress-strain curve shown in the figure.

Determine the elongation d of the bar for each of the following axial loads: P 10 kN, 20 kN, 30 kN, 40 kN, and 45 kN. From these results, plot a diagram of load P versus elongation d (load-displacement diagram).



Solution 2.11-4 Bar in tension



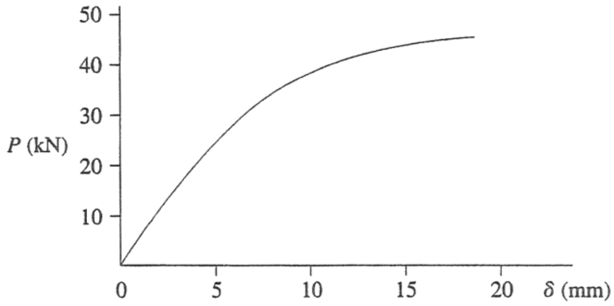
L 2.0 m

A 249 mm^2

STRESS-STRAIN DIAGRAM
(See the problem statement for the diagram)

LOAD-DISPLACEMENT DIAGRAM

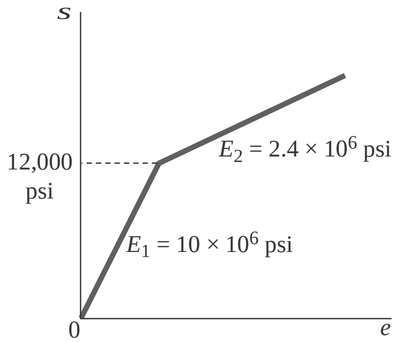
P (kN)	s P/A (MPa)	(from diagram)	d L (mm)
10	40	0.0009	1.8
20	80	0.0018	3.6
30	120	0.0031	6.2
40	161	0.0060	12.0
45	181	0.0081	16.2



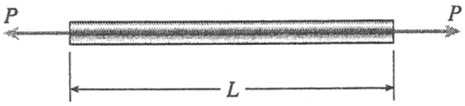
NOTE: The load-displacement curve has the same shape as the stress-strain curve.

Problem 2.11-5 An aluminum bar subjected to tensile forces P has length L 150 in. and cross-sectional area A 2.0 in.^{2} The stress-strain behavior of the aluminum may be represented approximately by the bilinear stress-strain diagram shown in the figure.

Calculate the elongation d of the bar for each of the following axial loads: P 8 k, 16 k, 24 k, 32 k, and 40 k. From these results, plot a diagram of load P versus elongation d (load-displacement diagram).



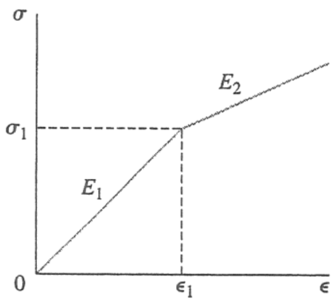
Solution 2.11-5 Aluminum bar in tension



L 150 in.

A 2.0 in.^{2}

STRESS-STRAIN DIAGRAM



E_1 10 10^6 psi

E_2 2.4 10^6 psi

s_1 12,000 psi

$$_1 \quad \frac{s_1}{E_1} \quad \frac{12{,}000 \text{ psi}}{10 * 10^6 \text{ psi}}$$

0.0012

For 0 s s_1:

$$\frac{s}{E_1} \quad \frac{s}{10 * 10^6 \text{psi}} \; (s \quad \text{psi}) \qquad \text{Eq. (1)}$$

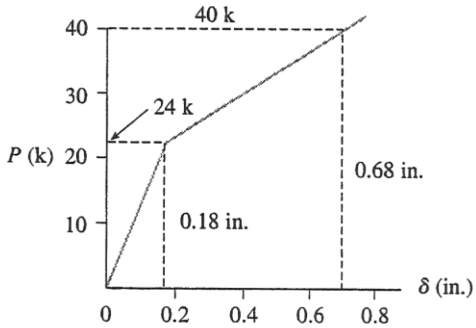
For s s_1:

$$_1 + \frac{s \quad s_1}{E_2} \quad 0.0012 + \frac{s \quad 12{,}000}{2.4 * 10^6}$$

$$\frac{s}{2.4 * 10^6} \quad 0.0038 \quad (s \quad \text{psi}) \quad \text{Eq. (2)}$$

LOAD-DISPLACEMENT DIAGRAM

P (k)	s P/A (psi)	(from Eq. 1 or Eq. 2)	d L (in.)
8	4,000	0.00040	0.060
16	8,000	0.00080	0.120
24	12,000	0.00120	0.180
32	16,000	0.00287	0.430
40	20,000	0.00453	0.680



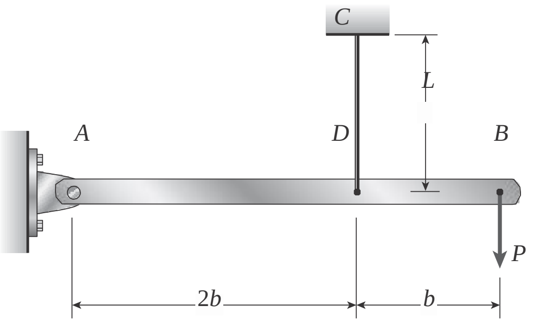
Problem 2.11-6 A rigid bar *AB*, pinned at end *A*, is supported by a wire *CD* and loaded by a force *P* at end *B* (see figure). The wire is made of high-strength steel having modulus of elasticity *E* 210 GPa and yield stress s_Y 820 MPa. The length of the wire is *L* 1.0 m and its diameter is *d* 3 mm. The stress-strain diagram for the steel is defined by the *modified power law*, as follows:

$$s \quad EP \quad 0 \ldots s \ldots s_Y$$

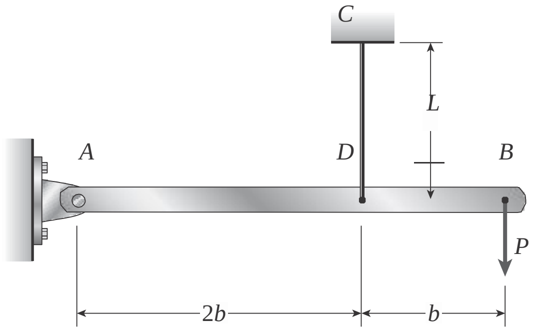
$$s \quad s_Y \text{a} \frac{EP}{s_Y} \text{b}^n \quad s \text{ Ú } s_Y$$

(a) Assuming *n* 0.2, calculate the displacement d_B at the end of the bar due to the load *P*. Take values of *P* from 2.4 kN to 5.6 kN in increments of 0.8 kN.

(b) Plot a load-displacement diagram showing *P* versus d_B.



Solution 2.11-6 Rigid bar supported by a wire



Wire: *E* 210 GPa

s_Y 820 MPa

L 1.0 m

d 3 mm

$$A \quad \frac{pd^2}{4} \quad 7.0686 \text{ mm}^2$$

STRESS-STRAIN DIAGRAM

$$s \quad E \qquad (0 \quad s \quad s_Y) \tag{1}$$

$$s \quad s_Y \text{a} \frac{E}{s_Y} \text{b}^n \qquad (s \quad s_Y) \qquad (n \quad 0.2) \tag{2}$$

(a) DISPLACEMENT d_B AT END OF BAR

$$d \quad \text{elongation of wire } d_B \quad \frac{3}{2}d \quad \frac{3}{2} \quad L \tag{3}$$

Obtain from stress-strain equations:

$$\text{From Eq. (1):} \qquad \frac{sE}{(0 \ldots s \ldots s_Y)} \tag{4}$$

$$\text{From Eq. (2):} \qquad \frac{s_Y}{E} \text{a} \frac{s}{s_Y} \text{b}^{1/n} \tag{5}$$

Axial force in wire: $F \quad \frac{3P}{2}$

$$\text{Stress in wire: } s \quad \frac{F}{A} \quad \frac{3P}{2A} \tag{6}$$

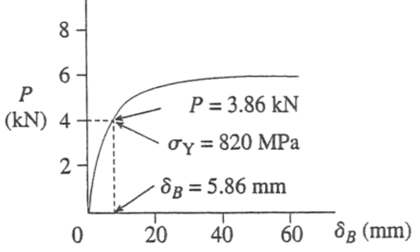
PROCEDURE: Assume a value of *P*
Calculate *s* from Eq. (6)
Calculate from Eq. (4) or (5)
Calculate d_B from Eq. (3)

P (kN)	*s* (MPa) Eq. (6)	Eq. (4) or (5)	d_B (mm) Eq. (3)
2.4	509.3	0.002425	3.64
3.2	679.1	0.003234	4.85
4.0	848.8	0.004640	6.96
4.8	1018.6	0.01155	17.3
5.6	1188.4	0.02497	37.5

For *s* s_Y 820 MPa:

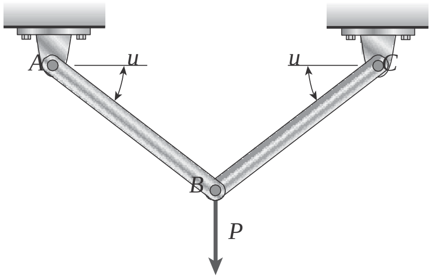
0.0039048 *P* 3.864 kN d_B 5.86 mm

(b) LOAD-DISPLACEMENT DIAGRAM



Elastoplastic Analysis

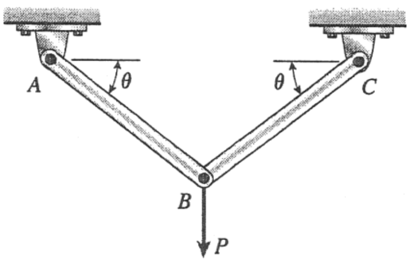
The problems for Section 2.12 are to be solved assuming that the material is elastoplastic with yield stress s_Y, yield strain $_Y$, and modulus of elasticity E in the linearly elastic region (see Fig. 2-72).



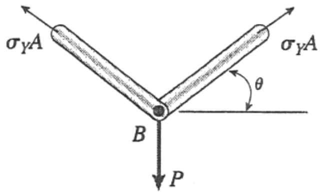
Problem 2.12-1 Two identical bars AB and BC support a vertical load P (see figure). The bars are made of steel having a stress-strain curve that may be idealized as elastoplastic with yield stress s_Y. Each bar has cross-sectional area A.

Determine the yield load P_Y and the plastic load P_P.

Solution 2.12-1 Two bars supporting a load P



Structure is statically determinate. The yield load P_Y and the plastic lead P_P occur at the same time, namely, when both bars reach the yield stress.



JOINT B

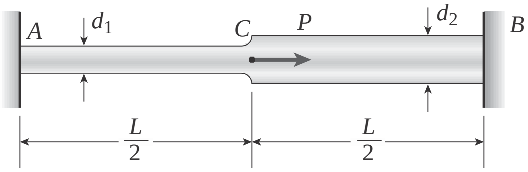
$F_{\text{vert}} \quad 0$

$(2s_YA)\sin u \quad P$

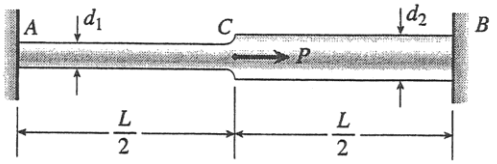
$P_Y \quad P_P \quad 2s_YA\sin u$

Problem 2.12-2 A stepped bar ACB with circular cross sections is held between rigid supports and loaded by an axial force P at midlength (see figure). The diameters for the two parts of the bar are d_1 20 mm and d_2 25 mm, and the material is elastoplastic with yield stress s_Y 250 MPa.

Determine the plastic load P_P.



Solution 2.12-2 Bar between rigid supports

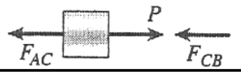


$d_1 \quad 20$ mm $\quad d_2 \quad 25$ mm $\quad \sigma_Y \quad 250$ MPa

DETERMINE THE PLASTIC LOAD P_P:

At the plastic load, all parts of the bar are stressed to the yield stress.

Point C:



$F_{AC} \quad \sigma_Y A_1 \qquad F_{CB} \quad \sigma_Y A_2$

$P \quad F_{AC} \quad F_{CB}$

$P_P \quad \sigma_Y A_1 \quad \sigma_Y A_2 \quad \sigma_Y (A_1 \quad A_2)$

SUBSTITUTE NUMERICAL VALUES:

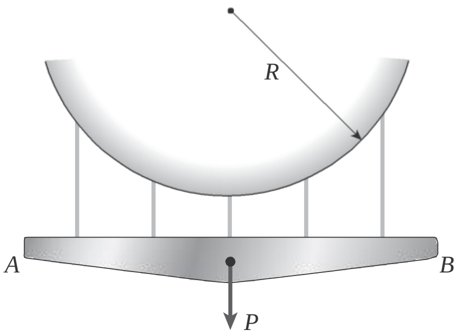
$$P_P \quad (250 \text{ MPa})\left(\frac{\pi}{4}\right)(d_1^2 + d_2^2)$$

$$(250 \text{ MPa})\left(\frac{\pi}{4}\right)[(20 \text{ mm})^2 + (25 \text{ mm})^2]$$

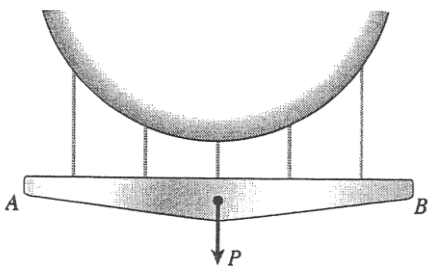
$$201 \text{ kN}$$

Problem 2.12-3 A horizontal rigid bar AB supporting a load P is hung from five symmetrically placed wires, each of cross-sectional area A (see figure). The wires are fastened to a curved surface of radius R.

(a) Determine the plastic load P_P if the material of the wires is elastoplastic with yield stress σ_Y.
(b) How is P_P changed if bar AB is flexible instead of rigid?
(c) How is P_P changed if the radius R is increased?



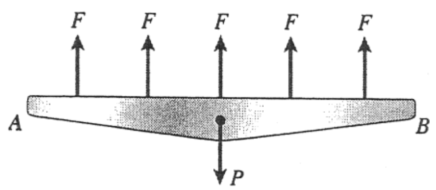
Solution 2.12-3 Rigid bar supported by five wires



(a) PLASTIC LOAD P_P

At the plastic load, each wire is stressed to the yield stress. $\quad P_P \quad 5\sigma_Y A$

$F \quad \sigma_Y A$



(b) BAR AB IS FLEXIBLE

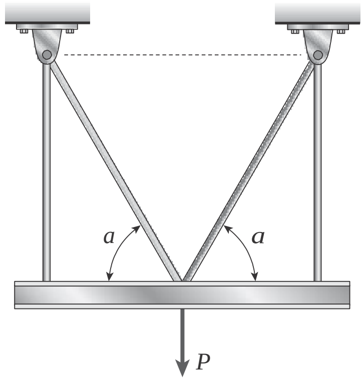
At the plastic load, each wire is stressed to the yield stress, so the plastic load is not changed.

(c) RADIUS R IS INCREASED

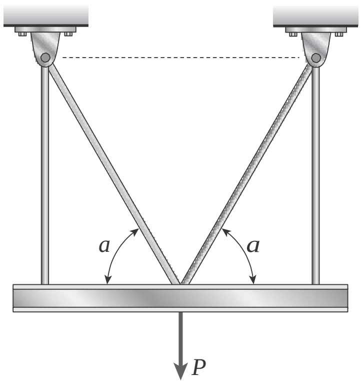
Again, the forces in the wires are not changed, so the plastic load is not changed.

Problem 2.12-4 A load P acts on a horizontal beam that is supported by four rods arranged in the symmetrical pattern shown in the figure. Each rod has cross-sectional area A and the material is elastoplastic with yield stress s_Y.

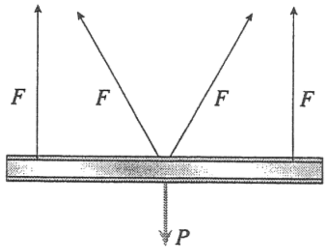
Determine the plastic load P_P.



Solution 2.12-4 Beam supported by four rods



At the plastic load, all four rods are stressed to the yield stress.



$F \quad s_Y A$

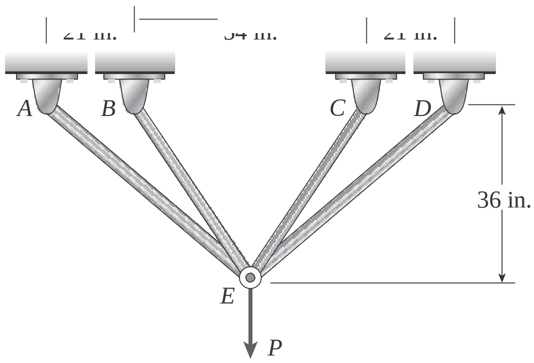
Sum forces in the vertical direction and solve for the

$P_P \quad 2F \quad 2F \sin a$

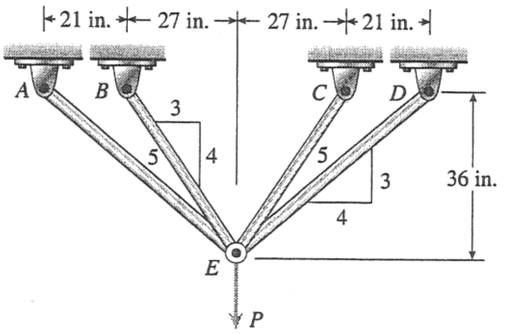
$P_P \quad 2 s_Y A\,(1 \quad \sin a)$

Problem 2.12-5 The symmetric truss $ABCDE$ shown in the figure is constructed of four bars and supports a load P at joint E. Each of the two outer bars has a cross-sectional area of 0.307 in.^{2}, and each of the two inner bars has an area of 0.601 in.^{2} The material is elastoplastic with yield stress $s_Y \quad 36$ ksi.

Determine the plastic load P_P.

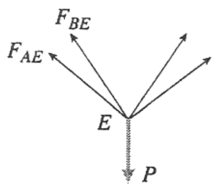


Solution 2.12-5 Truss with four bars



L_{AE} 60 in. L_{BE} 45 in.

Joint E



Equilibrium:

$$2F_{AE}\,a\frac{3}{5}b + 2F_{BE}\,a\frac{4}{5}b \quad P$$

or

$$P \quad \frac{6}{5}F_{AE} + \frac{8}{5}F_{BE}$$

Plastic load P_P

At the plastic load, all bars are stressed to the yield stress.

$$F_{AE} \quad s_Y A_{AE} \qquad F_{BE} \quad s_Y A_{BE}$$

$$P_P \quad \frac{6}{5}s_Y A_{AE} + \frac{8}{5}s_Y A_{BE}$$

Substitute numerical values:

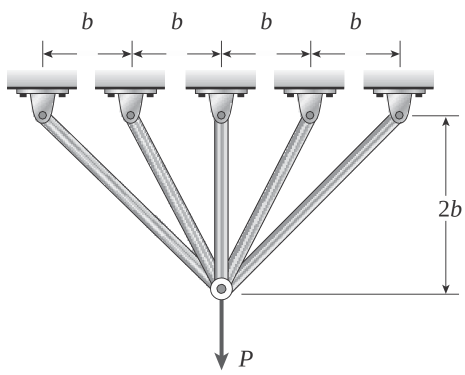
A_{AE} 0.307 in.^{2} A_{BE} 0.601 in.^{2}

s_Y 36 ksi

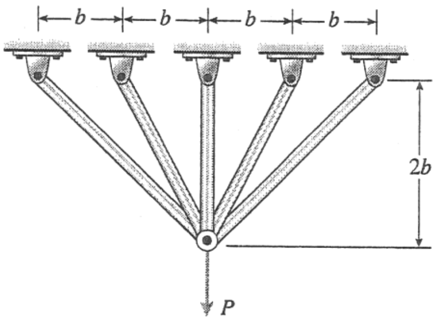
$$P_P \quad \frac{6}{5}(36\text{ ksi})(0.307\text{ in.}^2) + \frac{8}{5}(36\text{ ksi})(0.601\text{ in.}^2)$$

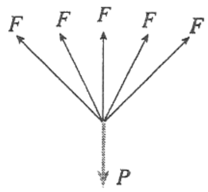
$$13.26\text{ k} + 34.62\text{ k} \quad 47.9\text{ k}$$

Problem 2.12-6 Five bars, each having a diameter of 10 mm, support a load P as shown in the figure. Determine the plastic load P_P if the material is elastoplastic with yield stress s_Y 250 MPa.



Solution 2.12-6 Truss consisting of five bars





At the plastic load, all five bars are stressed to the yield stress

$F = \sigma_Y A$

Sum forces in the vertical direction and solve for the load:

$$P_P = 2F\left(\frac{1}{\sqrt{2}}\right) + 2F\left(\frac{2}{\sqrt{5}}\right) + F$$

$$= \frac{\sigma_Y A}{5}(5\sqrt{2} + 4\sqrt{5} + 5)$$

$$= 4.2031\sigma_Y A$$

$d = 10$ mm

$A = \frac{\pi d^2}{4} = 78.54\ \text{mm}^2$

$\sigma_Y = 250$ MPa

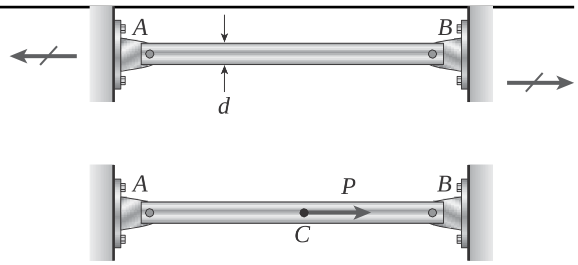
Substitute numerical values:

$P_P = (4.2031)(250\ \text{MPa})(78.54\ \text{mm}^2)$

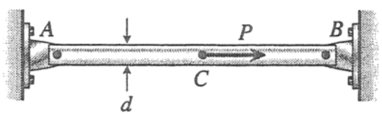
$= 82.5$ kN

Problem 2.12-7 A circular steel rod AB of diameter $d = 0.60$ in. is stretched tightly between two supports so that initially the tensile stress in the rod is 10 ksi (see figure). An axial force P is then applied to the rod at an intermediate location C.

(a) Determine the plastic load P_P if the material is elastoplastic with yield stress $\sigma_Y = 36$ ksi.
(b) How is P_P changed if the initial tensile stress is doubled to 20 ksi?



Solution 2.12-7 Bar held between rigid supports



$d = 0.6$ in.

$\sigma_Y = 36$ ksi

Initial tensile stress $= 10$ ksi

(a) Plastic load P_P

The presence of the initial tensile stress does not affect the plastic load. Both parts of the bar must yield in order to reach the plastic load.

Point C:

$\sigma_Y A \quad P \quad \sigma_Y A$

C

$P_P = 2\sigma_Y A = (2)(36\ \text{ksi})\left(\frac{\pi}{4}\right)(0.60\ \text{in.})^2$

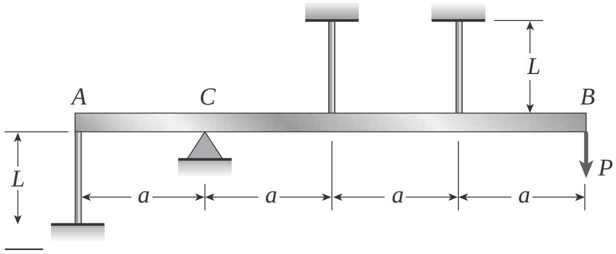
$= 20.4$ k

(b) Initial tensile stress is doubled

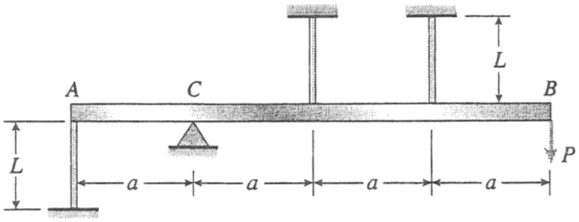
P_P is not changed.

Problem 2.12-8 A rigid bar *ACB* is supported on a fulcrum at *C* and loaded by a force *P* at end *B* (see figure). Three identical wires made of an elastoplastic material (yield stress s_Y and modulus of elasticity *E*) resist the load *P*. Each wire has cross-sectional area *A* and length *L*.

(a) Determine the yield load P_Y and the corresponding yield displacement d_Y at point *B*.
(b) Determine the plastic load P_P and the corresponding displacement d_P at point *B* when the load just reaches the value P_P.
(c) Draw a load-displacement diagram with the load *P* as ordinate and the displacement d_B of point *B* as abscissa.

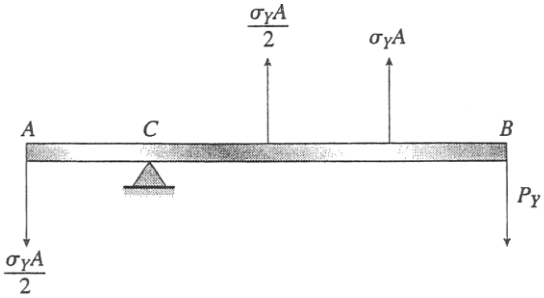


Solution 2.12-8 Rigid bar supported by wires



(a) Yield load P_Y

Yielding occurs when the most highly stressed wire reaches the yield stress s_Y



M_C 0

P_Y $s_Y A$

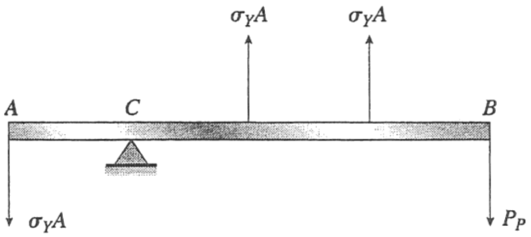
At point *A*:

d_A a $\frac{s_Y A}{2}$ b a $\frac{L}{EA}$ b $\frac{s_Y L}{2E}$

At point *B*:

d_B $3d_A$ d_Y $\frac{3s_Y L}{2E}$

(b) Plastic load P_P



At the plastic load, all wires reach the yield stress.

M_C 0

P_P $\frac{4s_Y A}{3}$

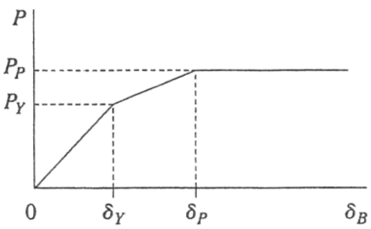
At point *A*:

d_A $(s_Y A)$ a $\frac{L}{EA}$ b $\frac{s_Y L}{E}$

At point *B*:

d_B $3d_A$ d_P $\frac{3s_Y L}{E}$

(c) Load-displacement diagram

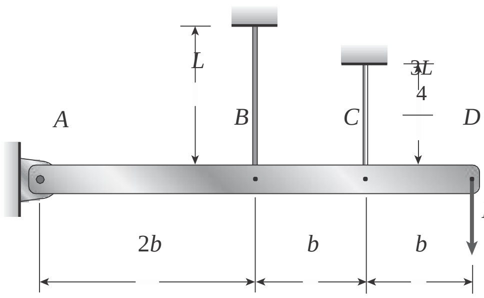


P_P $\frac{4}{3}P_Y$

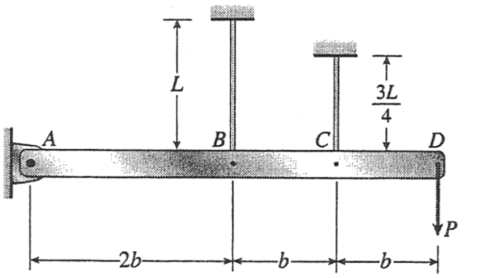
d_P $2d_Y$

Problem 2.12-9 The structure shown in the figure consists of a horizontal rigid bar *ABCD* supported by two steel wires, one of length *L* and the other of length 3*L*/4. Both wires have cross-sectional area *A* and are made of elastoplastic material with yield stress s_Y and modulus of elasticity *E*. A vertical load *P* acts at end *D* of the bar.

(a) Determine the yield load P_Y and the corresponding yield displacement d_Y at point *D*.
(b) Determine the plastic load P_P and the corresponding displacement d_P at point *D* when the load just reaches the value P_P.
(c) Draw a load-displacement diagram with the load *P* as ordinate and the displacement d_D of point *D* as abscissa.



Solution 2.12-9 Rigid bar supported by two wires

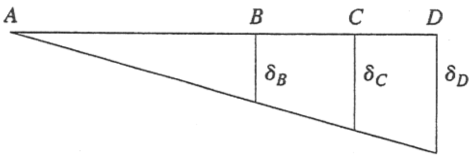


A cross-sectional area

s_Y yield stress

E modulus of elasticity

DISPLACEMENT DIAGRAM

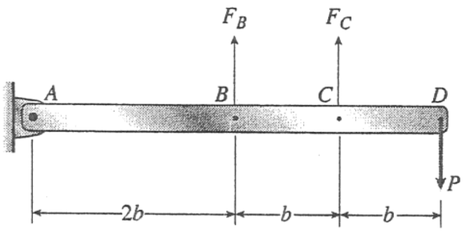


COMPATIBILITY:

$$d_C \quad \frac{3}{2}d_B \qquad (1)$$

$$d_D \quad 2d_B \qquad (2)$$

FREE-BODY DIAGRAM



EQUILIBRIUM:

$$M_A \quad 0 \qquad F_B(2b) \quad F_C(3b) \quad P(4b)$$

$$2F_B \quad 3F_C \quad 4P \qquad (3)$$

FORCE-DISPLACEMENT RELATIONS

$$d_B \; \frac{F_B L}{EA} \quad d_C \; \frac{F_C \mathrm{a}\frac{3}{4}L\mathrm{b}}{EA} \qquad (4, 5)$$

Substitute into Eq. (1):

$$\frac{3F_C L}{4EA} \quad \frac{3F_B L}{2EA}$$

$$F_C \quad 2F_B \qquad (6)$$

STRESSES

$$s_B \quad \frac{F_B}{A} \quad s_C \quad \frac{F_C}{A} \quad s_C \quad 2s_B \qquad (7)$$

Wire C has the larger stress. Therefore, it will yield first.

(a) YIELD LOAD

$$s_C \quad s_Y \qquad s_B \quad \frac{s_C}{2} \quad \frac{s_Y}{2} \qquad \text{(From Eq. 7)}$$

$$F_C \quad s_Y A \qquad F_B \quad \frac{1}{2} s_Y A$$

From Eq. (3):

$$2\text{a}\frac{1}{2}s_Y A\text{b} + 3(s_Y A) \quad 4P$$

$$P \quad P_Y \quad s_Y A \quad ;$$

From Eq. (4):

$$d_B \quad \frac{F_B L}{EA} \quad \frac{s_Y L}{2E}$$

From Eq. (2):

$$d_D \quad d_Y \quad 2d_B \quad \frac{s_Y L}{E} \quad ;$$

(b) PLASTIC LOAD

At the plastic load, both wires yield.

$$s_B \quad s_Y \quad s_C \qquad F_B \quad F_C \quad s_Y A$$

From Eq. (3):

$$2(s_Y A) \quad 3(s_Y A) \quad 4P$$

$$P \quad P_P \quad \frac{5}{4} s_Y A \quad ;$$

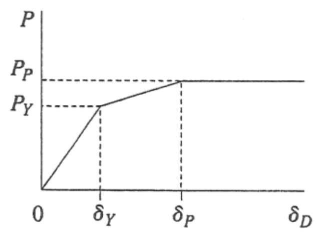
From Eq. (4):

$$d_B \quad \frac{F_B L}{EA} \quad \frac{s_Y L}{E}$$

From Eq. (2):

$$d_D \quad d_P \quad 2d_B \quad \frac{2s_Y L}{E} \quad ;$$

(c) LOAD-DISPLACEMENT DIAGRAM



$$P_P \quad \frac{5}{4} P_Y$$

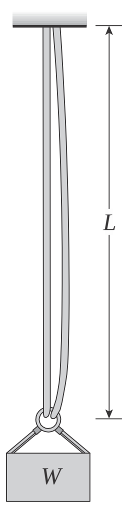
$$d_P \quad 2d_Y$$

Problem 2.12-10 Two cables, each having a length L of approximately 40 m, support a l oaded container of weight W (see figure). The cables, which have effective cross-sectional area A 48.0 mm^2 and effective modulus of elasticity E 160 GPa, are identical except that one cable is longer than the other when they are hanging separately and unloaded. The difference in lengths is d 100 mm. The cables are made of steel having an elastoplastic stress-strain diagram with s_Y 500 MPa. Assume that the weight W is initially zero and is slowly increased by the addition of material to the container.

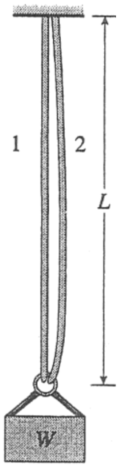
(a) Determine the weight W_Y that first produces yielding of the shorter cable. Also, determine the corresponding elongation d_Y of the shorter cable.

(b) Determine the weight W_P that produces yielding of both cables. Also, determine the elongation d_P of the shorter cable when the weight W just reaches the value W_P.

(c) Construct a load-displacement diagram showing the weight W as ordinate and the elongation d of the shorter cable as abscissa. (*Hint*: The load displacement diagram is not a single straight line in the region 0 W W_Y.)



Solution 2.12-10 Two cables supporting a load



L 40 m A 48.0 mm^2

E 160 GPa

d difference in length 100 mm

s_Y 500 MPa

INITIAL STRETCHING OF CABLE 1

Initially, cable 1 supports all of the load.

Let W_1 load required to stretch cable 1 to the same length as cable 2

$$W_1 \frac{EA}{L} d \quad 19.2 \text{ kN}$$

d_1 100 mm (elongation of cable 1)

$$s_1 \frac{W_1}{A} \quad \frac{Ed}{L} \quad 400 \text{ MPa } (s_1 \; 6 \; s_Y \; < \; 7 \text{ OK})$$

(a) YIELD LOAD W_Y

Cable 1 yields first. F_1 $s_Y A$ 24 kN

d_{1Y} total elongation of cable 1

d_{1Y} total elongation of cable 1

$$d_{1Y} \quad \frac{F_1 L}{EA} \quad \frac{s_Y L}{E} \quad 0.125 \text{ m} \quad 125 \text{ mm}$$

d_Y d_{1Y} 125 mm

d_{2Y} elongation of cable 2

d_{1Y} d 25 mm

$$F_2 \frac{EA}{L} d_{2Y} \quad 4.8 \text{ kN}$$

W_Y $F_1 + F_2$ 24 kN + 4.8 kN

28.8 kN

(b) PLASTIC LOAD W_P

F_1 $s_Y A$ F_2 $s_Y A$ W_P

$2s_Y A$ 48 kN d_{2P}

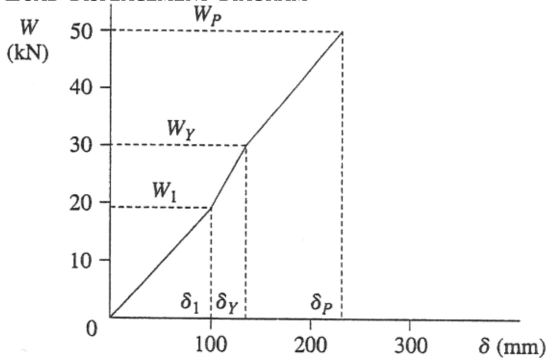
elongation of cable 2

$$F_2 \left(\frac{L}{EA}\right) \frac{s_Y L}{E} \quad 0.125 \text{ mm} \quad 125 \text{ mm}$$

d_{1P} d_{2P} d 225 mm

d_P d_{1P} 225 mm

(c) LOAD-DISPLACEMENT DIAGRAM



$$\frac{W_Y}{W_1} \quad 1.5 \quad \frac{d_Y}{d_1} \quad 1.25$$

$$\frac{W_P}{W_Y} \quad 1.667 \quad \frac{d_P}{d_Y} \quad 1.8$$

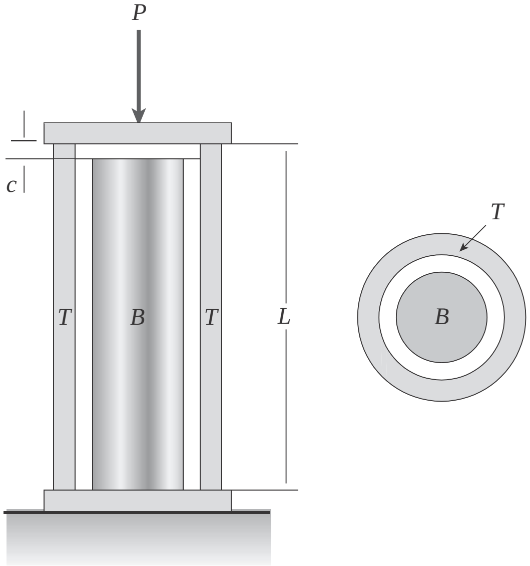
0 W W_1: slope 192,000 N/m

W_1 W W_Y: slope 384,000 N/m

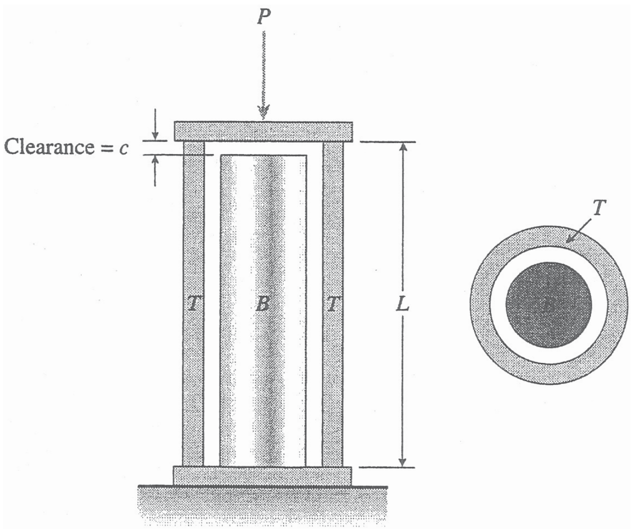
W_Y W W_P: slope 192,000 N/m

Problem 2.12-11 A hollow circular tube T of length L 15 in. is uniformly compressed by a force P acting through a rigid plate (see figure). The outside and inside diameters of the tube are 3.0 and 2.75 in., repectively. A concentric solid circular bar B of 1.5 in. diameter is mounted inside the tube. When no load is present, there is a clearance c 0.010 in. between the bar B and the rigid plate. Both bar and tube are made of steel having an elastoplastic stress-strain diagram with E 29 10^3 ksi and s_Y 36 ksi.

(a) Determine the yield load P_Y and the corresponding shortening d_Y of the tube.
(b) Determine the plastic load P_P and the corresponding shortening d_P of the tube.
(c) Construct a load-displacement diagram showing the load P as ordinate and the shortening d of the tube as abscissa. (*Hint*: The load-displacement diagram is not a single straight line in the region 0 P P_Y.)



Solution 2.12-11 Tube and bar supporting a load



L 15 in.

c 0.010 in.

E 29 10^3 ksi

s_Y 36 ksi

TUBE:

d_2 3.0 in.

d_1 2.75 in.

$A_T \quad \frac{p}{4}(d_2^2 \quad d_1^2) \quad 1.1290 \text{ in.}^2$

BAR

$d \quad 1.5 \text{ in.}$

$$A_B \quad \frac{pd^2}{4} \quad 1.7671 \text{ in.}^2$$

INITIAL SHORTENING OF TUBE T

Initially, the tube supports all of the load.

Let P_1 load required to close the clearance

$$P_1 \quad \frac{EA_T}{L} c \quad 21{,}827 \text{ lb}$$

Let d_1 shortening of tube $d_1 \quad c \quad 0.010$ in.

$$s_1 \quad \frac{P_1}{A_T} \quad 19{,}330 \text{ psi} \qquad (s_1 \quad s_Y \quad \text{OK})$$

(a) YIELD LOAD P_Y

Because the tube and bar are made of the same material, and because the strain in the tube is larger than the strain in the bar, the tube will yield first.

$F_T \quad s_Y A_T \quad 40{,}644$ lb

d_{TY} shortening of tube at the yield stress

$$s_{TY} \quad \frac{F_T L}{EA_T} \quad \frac{s_Y L}{E} \quad 0.018621 \text{ in.}$$

$d_Y \quad d_{TY} \quad 0.018621$ in.

d_{BY} shortening of bar

$d_{TY} \quad c \quad 0.008621$ in.

$$F_B \quad \frac{EA_B}{L} d_{BY} \quad 29{,}453 \text{ lb}$$

$P_Y \quad F_T \quad F_B \quad 40{,}644 \text{ lb} \quad 29{,}453 \text{ lb}$

$70{,}097$ lb

$P_Y \quad 70{,}100$ lb

(b) PLASTIC LOAD P_P

$F_T \quad s_Y A_T \qquad F_B \quad s_Y A_B$

$P_P \quad F_T \quad F_B \quad s_Y(A_T \quad A_B)$

$104{,}300$ lb

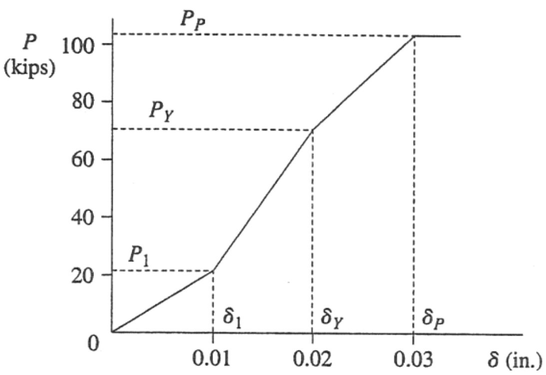
d_{BP} shortening of bar

$$F_B \left(\frac{L}{EA_B}\right) \quad \frac{s_Y L}{E} \quad 0.018621 \text{ in.}$$

$d_{TP} \quad d_{BP} \quad c \quad 0.028621$ in.

$d_P \quad d_{TP} \quad 0.02862$ in.

(c) LOAD-DISPLACEMENT DIAGRAM



$$\frac{P_Y}{P_1} \quad 3.21 \qquad \frac{d_Y}{d_1} \quad 1.86$$

$$\frac{P_P}{P_Y} \quad 1.49 \qquad \frac{d_P}{d_Y} \quad 1.54$$

$0 \quad P \quad P_1$: slope $\quad 2180$ k/in.

$P_1 \quad P \quad P_Y$: slope $\quad 5600$ k/in.

$P_Y \quad P \quad P_P$: slope $\quad 3420$ k/in.