Solution Manual for Mechanisms and Machines Kinematics Dynamics and Synthesis SI Edition 1st Edition Stanisic 1285057562 9781285057569
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Solution Manual:

Chapter 2

Kinematic Analysis Part I: Vector Loop Method - Solutions

For the mechanisms shown in Problems 2.1-2.16,
1.) Draw an appropriate vector loop.
2.) Write out the VLE(s).
3.) Write the X and Y components of the VLE(s) in their simplest form.
4.) Write down all geometric constraints.
5.) Summarize the scalar knowns and the scalar unknowns.
6.) From all the above, deduce the number of degrees of freedom in the systems.
7.) Check your result in 6.) against Gruebler's Criterion.

Problem 2.1

1.) A correct vector loop is drawn below. In all vector loop problems, vectors may be in the opposite direction of what is shown and they can also be in any sequence and numbered in any way. Take origin of coordinate system to be at the pin joint between 1 and 2 (pin joint between 1 and 4 also suitable). Align the X axis with $\bar{r}_1$.
2.) The VLE is,

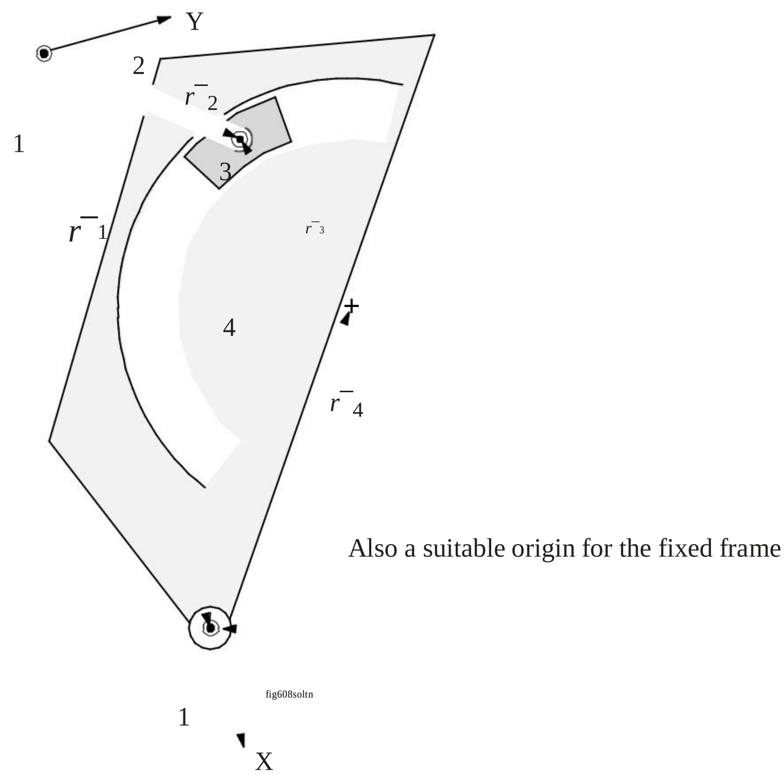
$$\bar{r}_1 + \bar{r}_4 + \bar{r}_3 - \bar{r}_2 = 0$$

3.) The VLE has simplified scalar components (noting $\theta_1 = 0$),

$$r_1 + r_4\cos\theta_4 + r_3\cos\theta_3 - r_2\cos\theta_2 = 0 \quad (1)$$
$$r_4\sin\theta_4 + r_3\sin\theta_3 - r_2\sin\theta_2 = 0 \quad (2)$$

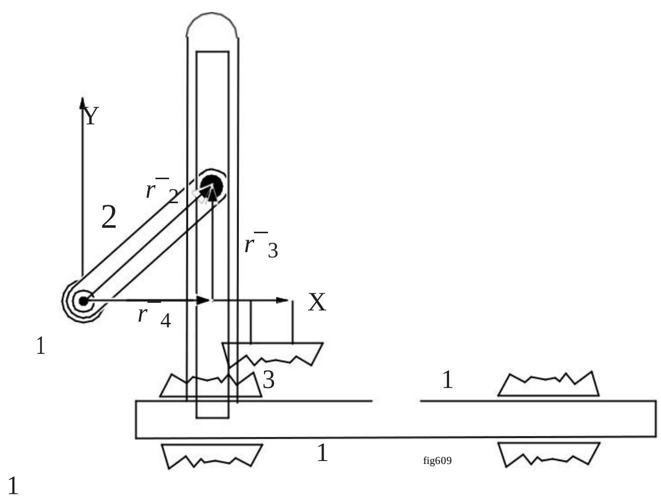
4.) There are no geometric constraints
5.) The two position equations (1) and (2) contain,
scalar knowns: r_1, r_4, r_3, r_2, $\theta_1 = 0$
and
scalar unknowns: θ_4, θ_3 and θ_2.

6.) Two position equations in three scalar unknowns means one scalar unknown must be given so that the remaining two can be calculated from the position equations. So, the system has one degree of freedom, which agrees with Gruebler's Criterion.



Problem 2.2

1.) A correct vector loop is drawn below.



Take the origin of the fixed coordinate system to be at the pin joint between 1 and 2. There is no other fixed point in the vector loop which could serve as the origin. Align the X axis with $\bar{r}_4$.

2.) The VLE is,

$$\bar{r}_4 + \bar{r}_3 - \bar{r}_2 = \bar{0}$$

3.) The VLE has simplified scalar components (noting that $\theta_4 = 0$ and $\theta_3 = \pi/2$),

$$r_4 - r_2\cos\theta_2 = 0 \qquad (1)$$

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$$r_3 - r_2 \sin\theta_2 = 0 \tag{2}$$

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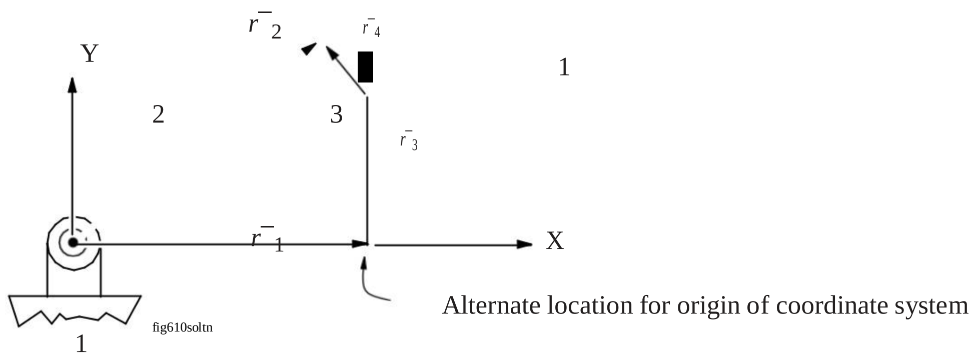
4.) There are no geometric constraints
5.) The two scalar position equations (1) and (2) contain,
scalar knowns: r_2, $\theta_3 = \frac{\pi}{2}$, $\theta_4 = 0$
and
scalar unknowns: θ_2, r_3 and r_4.
6.) Two position equations in three unknowns means one of the scalar unknowns must be given so the remaining two can be calculated. So the system has one degree of freedom, which agrees with Gruebler's Criterion.

Problem 2.3

1.) A correct vector loop is drawn below.



Take the origin of the fixed coordinate system to be at the pin joint between 1 and 2. Could also have been located at the point where $\bar{r}_1$ contact $\bar{r}_4$. Align the X axis with $\bar{r}_1$.
2.) The VLE is,

$$\bar{r}_1 + \bar{r}_3 + \bar{r}_4 - \bar{r}_2 = 0$$

3.) The VLE has simplified scalar components (noting that $\theta_1 = 0$ and $\theta_3 = \pi/2$)

$$r_1 + r_4\cos\theta_4 - r_2\cos\theta_2 = 0 \tag{1}$$

$$r_3 + r_4\sin\theta_4 - r_2\sin\theta_2 = 0. \tag{2}$$

4.) Since $\bar{r}_2$ and $\bar{r}_4$ are always orthogonal, we have a geometric constraint,

$$\theta_2 + \pi/2 = \theta_4 \longrightarrow \theta_2 + \pi/2 - \theta_4 = 0. \tag{3}$$

5.) The three scalar position equations (1) - (3) contain,
scalar knowns: r_1, r_4, $\theta_1 = 0$, $\theta_3 = \frac{\pi}{2}$.

scalar unknowns: r_2, θ_2, θ_4, and r_3.
6.) The three position equations contain four unknowns meaning one of the scalar unknowns must be given so that the remaining three can be calculated from the position equations. This means the system has one degree of freedom which agrees with Gruebler's Criterion.

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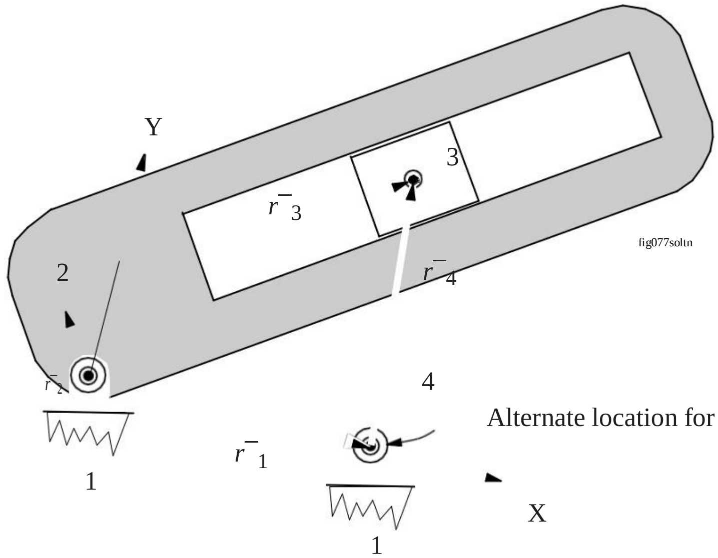
Problem 2.4 Problem 2.4

1.) A correct vector loop is drawn below.



Take the origin of the fixed coordinate system to be at the pin joint between 1 and 2. The origin could also have been located at the pin joint between 1 and 4. Align the X axis with $\bar r_1$.

2.) The VLE is,

$$\bar r_1 + \bar r_4 - \bar r_3 - \bar r_2 = \bar 0$$

3.) The VLE has scalar components (noting that $\theta_1 = 0$),

$$r_1 + r_4\cos\theta_4 - r_3\cos\theta_3 - r_2\cos\theta_2 = 0 \tag{1}$$

$$r_4\sin\theta_4 - r_3\sin\theta_3 - r_2\sin\theta_2 = 0. \tag{2}$$

4.) Vectors $\bar r_2$ and $\bar r_3$ are always orthogonal so,

$$\theta_3 + \pi/2 = \theta_2 \longrightarrow \theta_3 + \pi/2 - \theta_2 = 0. \tag{3}$$

5.) The three position equations (1) - (3) contain,
scalar knowns: r_1, r_2, r_4, $\theta_1 = 0$,
and
scalar unknowns: θ_2, θ_3, r_3 and θ_4.

6.) The three position equations contain four unknowns meaning one of the scalar unknowns must be given so that the remaining three can be calculated from the position equations. This means the system has one degree of freedom, which agrees with Gruebler's Criterion.

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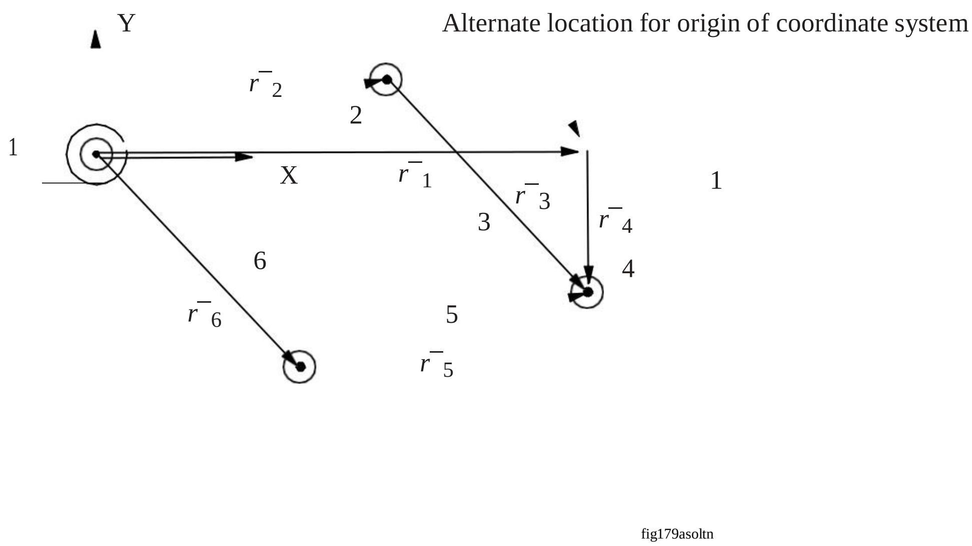
Problem 2.5

1.) Correct vector loops are drawn below.



Take the origin of the fixed coordinate system to be at the pin joint between 1 and 2. The point where $\bar r_4$ and $\bar r_4$ touch is an alternate location for the origin. Align the X axis with $\bar r_1$.

2.) The VLEs are,

$$\bar r_2 + \bar r_3 - \bar r_5 - \bar r_6 = \bar 0$$

$$\bar r_1 + \bar r_4 - \bar r_5 - \bar r_6 = \bar 0$$

3.) The VLE has scalar components (note that $\theta_1 = 0$ and $\theta_4 = -\pi/2$),

$$r_2\cos\theta_2 + r_3\cos\theta_3 - r_5\cos\theta_5 - r_6\cos\theta_6 = 0 \quad (1)$$

$$r_2\sin\theta_2 + r_3\sin\theta_3 - r_5\sin\theta_5 - r_6\sin\theta_6 = 0 \quad (2)$$

$$r_1 - r_5\cos\theta_5 - r_6\cos\theta_6 = 0 \quad (3)$$

$$-r_4 - r_5\sin\theta_5 - r_6\sin\theta_6 = 0 \quad (4)$$

4.) There are no geometric constraints

5.) The four scalar position equations (1) - (4) contain,

<u>scalar knowns</u>: r_1, r_2, r_3, r_5, r_6, $\theta_1 = 0$, $\theta_4 = -\frac{\pi}{2}$

and

<u>scalar unknowns</u>: θ_2, θ_3, r_4, θ_5, θ_6.

6.) The four position equations contain five unknowns meaning one of the scalar unknowns must be given so that the remaining four can be calculated from the position equations. This means the system has one degree of freedom which agrees with Gruebler's Criterion.

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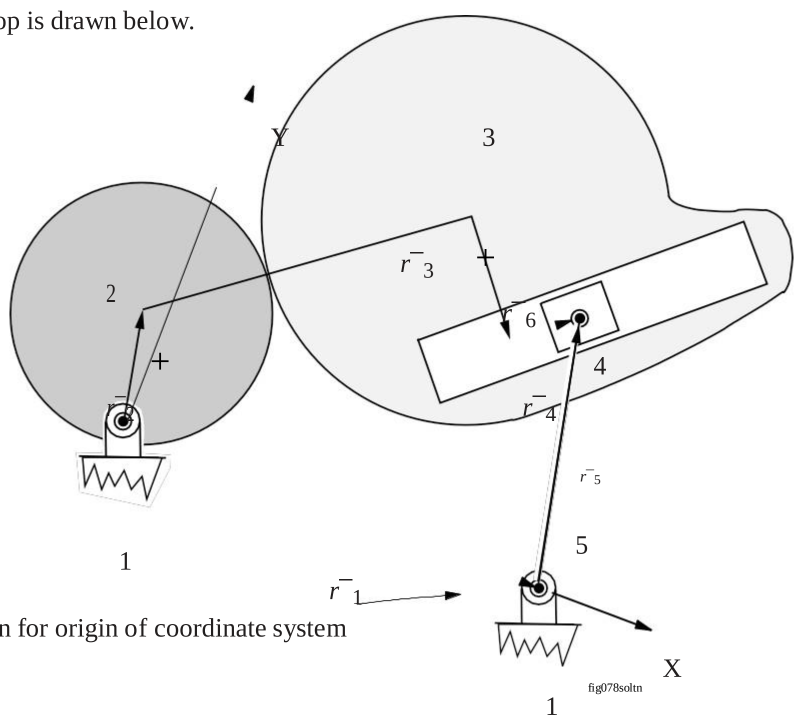
Problem 2.6
Problem 2.6

1.) A correct vector loop is drawn below.



Take the origin of the fixed coordinate system to be at the pin joint between 1 and 2. The pin joint between 1 and 5 is also suitable. Align the X axis with $\bar{r}_1$.

2.) The VLE is,

$$\bar{r}_2 + \bar{r}_3 + \bar{r}_6 + \bar{r}_4 - \bar{r}_5 - \bar{r}_1 = 0$$

3.) The VLE has simplified scalar components (note $\theta_1 = 0$),

$$r_2\cos\theta_2 + r_3\cos\theta_3 + r_6\cos\theta_6 + r_4\cos\theta_4 - r_5\cos\theta_5 - r_1 = 0 \qquad (1)$$

$$r_2\sin\theta_2 + r_3\sin\theta_3 + r_6\sin\theta_6 + r_4\sin\theta_4 - r_5\sin\theta_5 = 0. \qquad (2)$$

4.) Vectors $\bar{r}_6$ and $\bar{r}_4$ are always orthogonal so,

$$\theta_6 + \pi/2 = \theta_4 \longrightarrow \theta_6 + \pi/2 - \theta_4 = 0 \qquad (3)$$

5.) The three position equations (1) - (3) contain,
<u>scalar knowns</u>: r_2, r_3, r_6, r_5, r_1, $\theta_1 = 0$
and
<u>scalar unknowns</u>: θ_2, θ_3 , θ_6, θ_4, θ_5,and r_4.

6.) The three position equations contain six unknowns meaning three of the scalar unknowns must be given so that the remaining three can be calculated from the position equations. This means the system has three degrees of freedom which agrees with Gruebler's Criterion.

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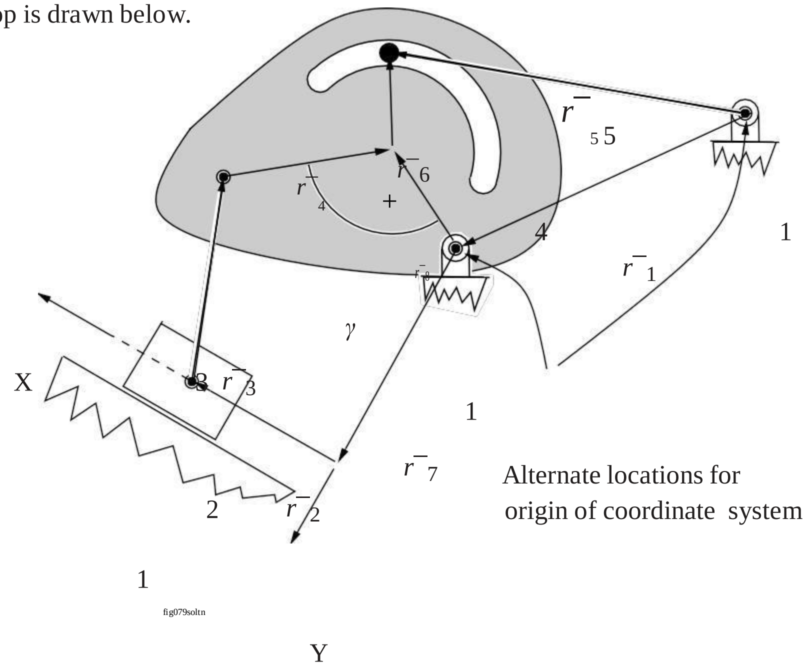
Problem 2.7

1.) A correct vector loop is drawn below.



Take the origin of the fixed coordinate system to be at the point where vectors r_2 and r_7 contact. The origin could also have been located at the pin joint between 1 and 4 or the pin joint between 1 and 5. Align the X axis with $\bar{r}_2$.

2.) The VLEs are,

$$\bar{r}_7 + \bar{r}_2 + \bar{r}_3 + \bar{r}_4 - \bar{r}_8 = \bar{0}$$

$$\bar{r}_8 + \bar{r}_6 - \bar{r}_5 + \bar{r}_1 = \bar{0}$$

3.) The VLE has simplified scalar components (note $\theta_2 = 0$ and $\theta_7 = \pi/2$),

$$r_2 + r_3\cos\theta_3 + r_4\cos\theta_4 - r_8\cos\theta_8 = 0 \quad (1)$$

$$r_7 + r_3\sin\theta_3 + r_4\sin\theta_4 - r_8\sin\theta_8 = 0 \quad (2)$$

$$r_8\cos\theta_8 + r_6\cos\theta_6 - r_5\cos\theta_5 + r_1\cos\theta_1 = 0 \quad (3)$$

$$r_8\sin\theta_8 + r_6\sin\theta_6 - r_5\sin\theta_5 + r_1\sin\theta_1 = 0. \quad (4)$$

4.) Vectors $\bar{r}_8$ and $\bar{r}_4$ both rotate with 4, so the angle between them, γ, is a constant, so

$$\theta_4 + \gamma = \theta_8 \longrightarrow \theta_4 + \gamma - \theta_8 = 0 \quad (5)$$

5.) These five position equations (1) - (5) contain,

<u>scalar knowns</u>: r_1, θ_1, r_2, $\theta_2 = 0$, r_3, r_4, r_5, r_6, r_7, $\theta_7 = \pi/2$, r_8 and

<u>scalar unknowns</u>: r_2, θ_3, θ_4, θ_5, θ_6, θ_8.

6.) The five scalar position equations contain six scalar unknowns meaning one of the scalar unknowns must be given so that the remaining five can be calculated from the position equations. This means the system has one degree of freedom, which agrees with Gruebler’s Criterion.

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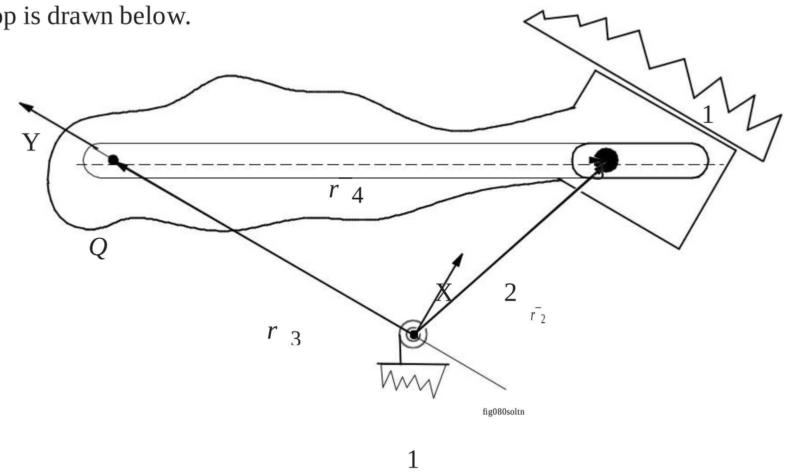
Problem 2.8

1.) A correct vector loop is drawn below.



The simplest possible vector loop uses a reference point Q on a hypothetical extension of 3. You can imagine the slot on 3 extending out to include the point Q on it. Take the origin of the fixed coordinate system to be at the pin joint between 1 and 2. This is the only possible origin. Align the Y axis with $\bar{r}_3$.

2.) The VLE is,

$$\bar{r}_2 - \bar{r}_4 - \bar{r}_3 = 0$$

3.) The VLE has simplified scalar components (note $\theta_3 = \pi/2$),

$$r_2\cos\theta_2 - r_4\cos\theta_4 = 0 \quad (1)$$

$$r_2\sin\theta_2 - r_4\sin\theta_4 - r_3 = 0 \quad (2)$$

4.) There are no geometric constraints.

5.) The position equations (1) and (2) contain,

scalar knowns: r_2, $\theta_3 = \frac{\pi}{2}$, θ_4

scalar unknowns: θ_2, r_3 and r_4.

6.) The two position equations (1) and (2) contain three unknowns meaning one of the scalar unknowns must be given so that the remaining two can be calculated from the position equations. This means the system has one degree of freedom which agrees with Gruebler's Criterion.

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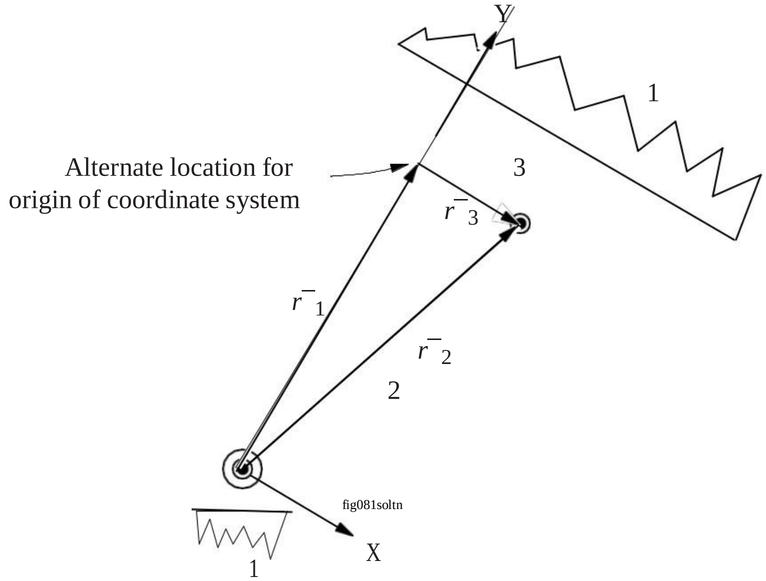
Problem 2.9

Problem 2.9

1.) A correct vector loop is drawn below.



Take the origin of the fixed coordinate system to be at the pin joint between 1 and 2. The origin also can be placed where $\bar{r}_1$ contacts $\bar{r}_3$.

2.) Align the Y axis with $\bar{r}_1$. The VLE is,

$$\bar{r}_1 + \bar{r}_3 - \bar{r}_2 = 0$$

3.) The VLE has simplified scalar components (note $\theta_1 = \pi/2$ and $\theta_3 = 0$),

$$r_3 - r_2\cos\theta_2 = 0 \qquad (1)$$

$$r_1 - r_2\sin\theta_2 = 0 \qquad (2)$$

4.) There are no geometric constraints.

5.) The position equations (1) and (2) contain,

<u>scalar knowns</u>: r_1, $\theta_1 = \pi/2$, r_2, $\theta_3 = 0$

and

<u>scalar unknowns</u>: θ_2 and r_3.

6.) The two position equations contain two unknowns meaning none of the scalar unknowns need be given. This means the system has zero degrees of freedom and is a statically determinate structure, which agrees with Gruebler's Criterion.

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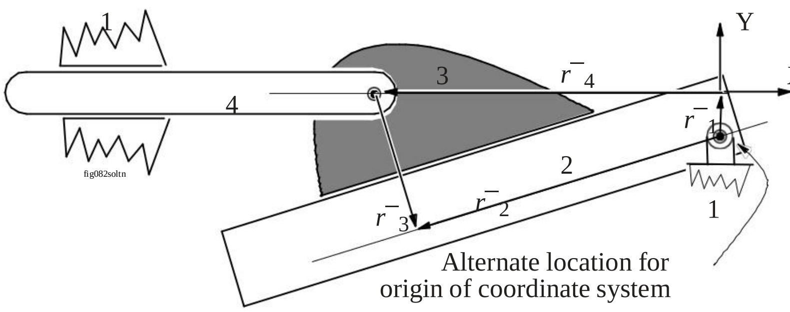
Problem 2.10
Problem 2.10

1.) A correct vector loop is drawn below.



Take the origin of the fixed coordinate system to be at point where $\bar r_1$ touches $\bar r_4$. The pin joint between 1 and 2 is an alternate location for the origin. Align the X axis with $\bar r_4$.

2.) The VLE is,

$$\bar r_1 + \bar r_4 + \bar r_3 - \bar r_2 = 0$$

3.) The VLE has simplified scalar components (note $\theta_1 = \pi/2$ and $\theta_4 = \pi$),

$$-r_4 + r_3\cos\theta_3 - r_2\cos\theta_2 = 0 \qquad (1)$$

$$r_1 + r_3\sin\theta_3 - r_2\sin\theta_2 = 0. \qquad (2)$$

4.) $\bar r_3$ and $\bar r_2$ both rotate with 2 and are always orthogonal, so

$$\theta_2 + \pi/2 = \theta_3 \longrightarrow \theta_2 + \pi/2 - \theta_3 = 0 \qquad (3)$$

5.) The three position equations (1) - (3) contain,
scalar knowns: r_1, $\theta_1 = 0$, $\theta_4 = \pi$, r_3,
and
scalar unknowns: r_4, θ_3, θ_2 and r_2.

6.) The three position equations contain four unknowns meaning one of the scalar unknowns must be given so that the remaining three can be calculated from the position equations. This means the system has one degree of freedom which agrees with Gruebler's Criterion.

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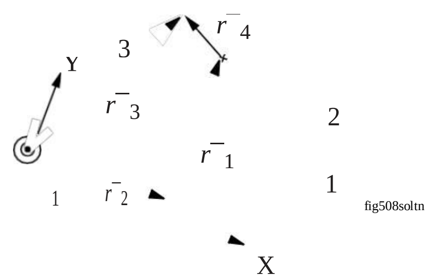
Problem 2.11

1.) A correct vector loop is drawn below.



Take the origin of the fixed coordinate system to be at pin joint between 1 and 3. There are no other possibilities for this origin. Align the X axis with $\bar{r}_2$.

2.) The VLE is,

$$\bar{r}_2 + \bar{r}_1 + \bar{r}_4 - \bar{r}_3 = 0$$

3.) The VLE has simplified scalar components (note $\theta_1 = \pi/2$ and $\theta_2 = 0$),

$$r_2 + r_4\cos\theta_4 - r_3\cos\theta_3 = 0 \tag{1}$$

$$r_1 + r_4\sin\theta_4 - r_3\sin\theta_3 = 0. \tag{2}$$

4.) $\bar{r}_3$ and $\bar{r}_4$ both rotate with 3 and are always orthogonal so,

$$\theta_3 + \pi/2 = \theta_4 \longrightarrow \theta_3 + \pi/2 - \theta_4 = 0. \tag{3}$$

5.) The three position equations (1) - (3) contain,
<u>scalar knowns</u>: $\theta_2 = 0$, r_1, $\theta_1 = \pi/2$, r_4,
and
<u>scalar unknowns</u>: r_2, θ_4, r_3 and θ_3.

6.) The three position equations contain four unknowns meaning one of the scalar unknowns must be given so that the remaining three can be calculated from the position equations. This means the system has one degree of freedom which agrees with Gruebler's Criterion.

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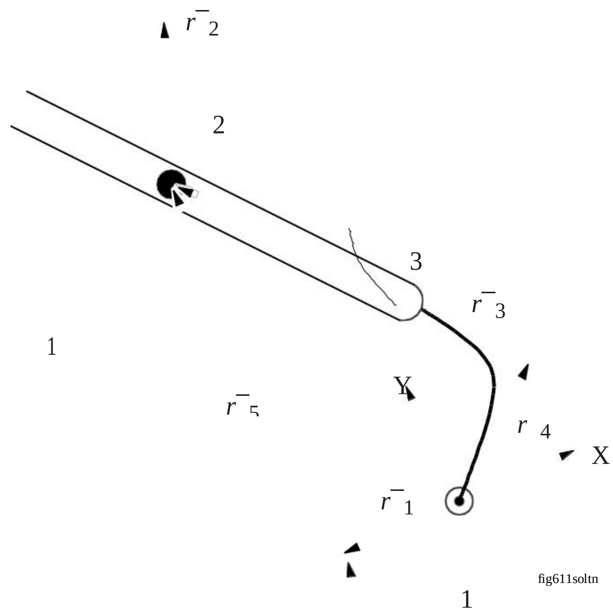
Problem 2.12
Problem 2.12

1.) A correct vector loop is drawn below.



Alternate location for origin of coordinate system

Take the origin of the fixed coordinate system to be at the pin joint between 1 and 3. An alternate location would be where vectors $\bar{r}_1$ and $\bar{r}_2$ touch. Align the X axis with $\bar{r}_1$.
2.) The VLE is,

$$\bar{r}_1 + \bar{r}_2 - \bar{r}_3 - \bar{r}_4 = 0$$

3.) The VLE has simplified scalar components (note $\theta_1 = \pi$ and $\theta_2 = \pi/2$),

$$-r_1 - r_3\cos\theta_3 - r_4\cos\theta_4 = 0 \tag{1}$$

$$r_2 - r_3\sin\theta_3 - r_4\sin\theta_4 = 0. \tag{2}$$

4.) $\bar{r}_3$ and $\bar{r}_4$ both rotate with 3 and are always orthogonal so,

$$\theta_4 + \pi/2 = \theta_3 \longrightarrow \theta_4 + \pi/2 - \theta_3 = 0. \tag{3}$$

5.) The three position equations (1) - (3) contain,
<u>scalar knowns</u>: r_1, $\theta_1 = \pi$, r_2, $\theta_5 = \pi/2$, r_4,
and
<u>scalar unknowns</u>: r_5, θ_2, r_3, θ_3 and θ_4.
6.) The three position equations contain five unknowns meaning two of the scalar unknowns must be given so that the remaining three can be calculated from the position equations. This means the system has two degrees-of-freedom which agrees with Gruebler's Criterion.

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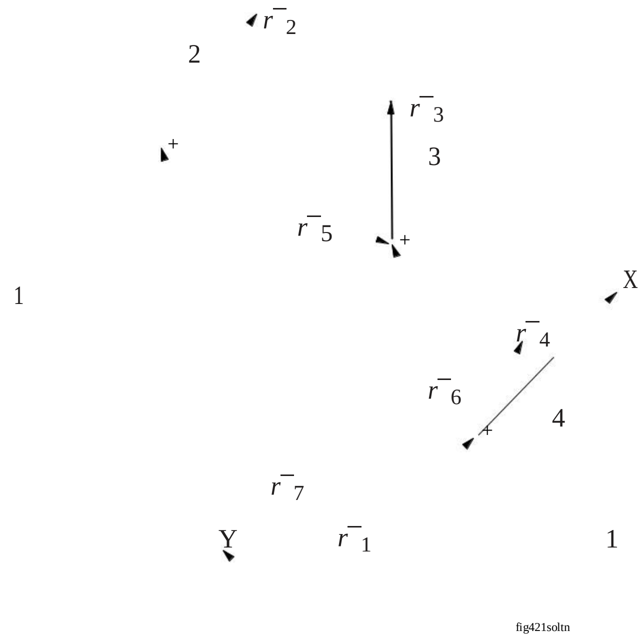
Problem 2.13 Problem 2.13

1.) A correct vector loop is drawn below.



Take the origin of the fixed coordinate system to be at the point where $\bar{r}_1$ and $\bar{r}_7$ touch. There are no other possibilities for this origin. Align the X axis with $\bar{r}_1$.

2.) The VLE is,

$$\bar{r}_1 + \bar{r}_6 - \bar{r}_5 - \bar{r}_7 = \bar{0}$$

but none of these vectors capture the rotation of 2, 3 or 4, so we need to attach vectors to those three bodies. These are the vectors $\bar{r}_2$, $\bar{r}_3$ and $\bar{r}_4$ respectively. Their magnitudes r_2, r_3 and r_4 are arbitrary but known.

3.) The VLE has simplified scalar components (note $\theta_1 = 0$),

$$r_1 + r_6\cos\theta_6 - r_5\cos\theta_5 - r_7\cos\theta_7 = 0 \quad (1)$$

$$r_6\sin\theta_6 - r_5\sin\theta_5 - r_7\sin\theta_7 = 0 \quad (2)$$

4.) There are no geometric constraints.

5.) The position equations (1) and (2)and the additionally needed vectors $\bar{r}_2$, $\bar{r}_3$ and $\bar{r}_2$ contain

<u>scalar knowns</u>: r_2, r_3, r_4, $\theta_1 = 0$, θ_7, r_5, r_6,

and

<u>scalar unknowns</u>: θ_2, θ_3, θ_4, θ_5, θ_6, r_1 and r_7.

6.) The two position equations along with the three additionally required vectors contain seven unknowns meaning five of the unknowns must be given in order to compute the remaining two. This means the system has five degrees of freedom which agrees with Gruebler's Criterion.

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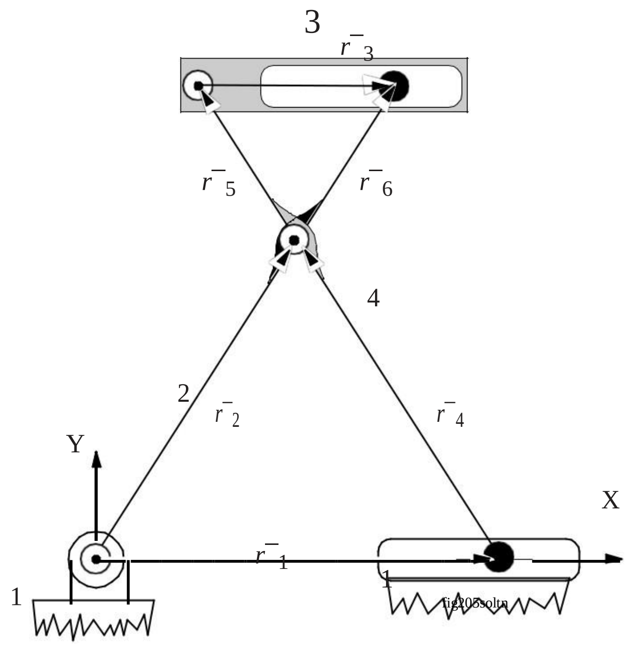
 Problem 2.14

1.) A correct vector loop is drawn below. The shown origin is the only possibility. Align the X axis with $\bar{r}_1$. 2.) The VLEs are,



$$\bar{r}_1 + \bar{r}_4 + \bar{r}_5 + \bar{r}_3 - \bar{r}_6 - \bar{r}_2 = 0 \quad \text{and} \quad \bar{r}_6 - \bar{r}_3 - \bar{r}_5 = 0.$$

3.) The VLEs simplified scalar components are (note $\theta_1 = 0$),

$$r_1 + r_4\cos\theta_4 + r_5\cos\theta_5 + r_3\cos\theta_3 - r_6\cos\theta_6 - r_2\cos\theta_2 = 0 \tag{1}$$
$$r_4\sin\theta_4 + r_5\sin\theta_5 + r_3\sin\theta_3 - r_6\sin\theta_6 - r_2\sin\theta_2 = 0 \tag{2}$$
$$r_6\cos\theta_6 - r_3\cos\theta_3 - r_5\cos\theta_5 = 0 \tag{3}$$
$$r_6\sin\theta_6 - r_3\sin\theta_3 - r_5\sin\theta_5 = 0. \tag{4}$$

4.) $\bar{r}_2$ and $\bar{r}_6$ both rotate with 2 and are always in-line. Likewise $\bar{r}_4$ and $\bar{r}_5$ both rotate with 4 and are always in-line so,

$$\theta_2 = \theta_6 \longrightarrow \theta_2 - \theta_6 = 0 \tag{5}$$
$$\theta_4 = \theta_5 \longrightarrow \theta_4 - \theta_5 = 0. \tag{6}$$

5.) This system of 6 position equations (1) - (6)has
<u>scalar knowns</u>: r_2, r_6, r_4, r_5, $\theta_1 = 0$,
and
<u>scalar unknowns</u>: θ_2, θ_6, r_3, θ_3, θ_5, θ_4, and r_1.

6.) The six position equations along with the seven unknowns means one of the unknowns must be given in order to compute the remaining six. This means the system has one degrees of freedom which agrees with Gruebler's Criterion. Many people are confused by the variable θ_3 being an unknown. They are inclined to say $\theta_3 = 0$. If the exacting conditions of $r_2 = r_4$ and $r_5 = r_6$ are met, along with exacting conditions that $\bar{r}_2$, $\bar{r}_6$ and $\bar{r}_4$, $\bar{r}_5$ in-line , then it is true

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Problem 2.15
Problem 2.15

that θ_3 will be zero. However these precise conditions can never be met as there are tolerances involved with any real system. Consequently, θ_3 is an unknown and as the mechanism articulates it will vary. The variation may be so small as to not be observable, but it exists.

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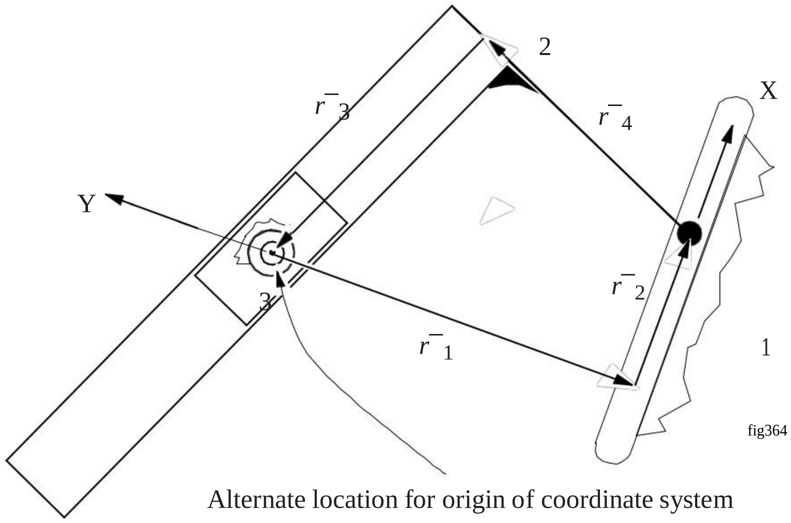
Problem 2.16

1.) A correct vector loop is drawn below. Take the origin at the point where $\bar{r}_1$ touches $\bar{r}_2$. Align the X axis



with $\bar{r}_2$. An alternate origin is the pin joint between 1 and 3.

2.) The VLE is,

$$\bar{r}_1 + \bar{r}_2 + \bar{r}_4 + \bar{r}_3 = \bar{0}$$

3.) The VLE has simplified scalar components (note $\theta_1 = -\pi/2$ and $\theta_2 = 0$),

$$r_2 + r_4\cos\theta_4 + r_3\cos\theta_3 = 0 \tag{1}$$

$$-r_1 + r_4\sin\theta_4 + r_3\sin\theta_3 = 0. \tag{2}$$

4.) $\bar{r}_3$ and $\bar{r}_4$ rotate with 3 and are always orthogonal so,

$$\theta_4 + \pi/2 = \theta_3 \longrightarrow \theta_4 + \pi/2 - \theta_3 = 0 \tag{3}$$

5.) The three position equations (1) and (3) contain,
<u>scalar knowns</u> : r_1, $\theta_1 = -\pi/2$, $\theta_2 = 0$, r_4
and
<u>scalar unknowns</u>: r_2, θ_4, θ_3 and r_3.

6.) The three position equations contain four unknowns meaning one of the scalar unknowns must be given so that the remaining three can be calculated from the position equations. This means the system has one degree of freedom which agrees with Gruebler's Criterion.

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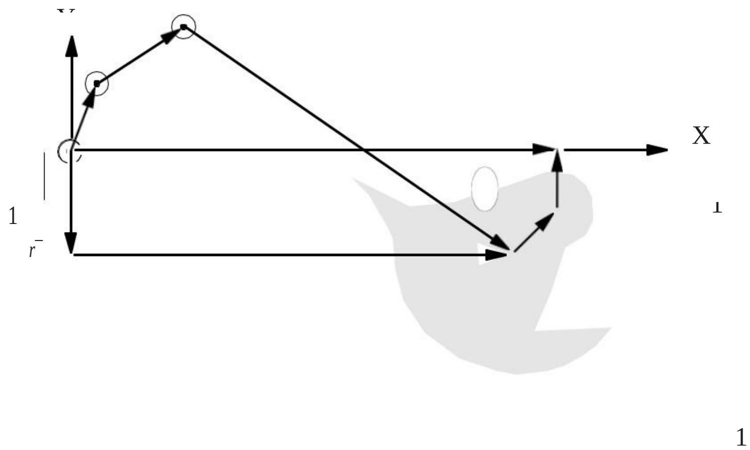
Problem 2.17

Problem 2.17

1.) A correct vector loop is drawn below. Take the origin of the fixed coordinate system to be at the pin joint



between 1 and 2. There is no alternate possibility for the origin. Align the X axis with $\bar{r}_8$ (and $\bar{r}_5$). 2.) The VLEs are,

$$\bar{r}_2 + \bar{r}_3 + \bar{r}_4 - \bar{r}_5 - \bar{r}_1 = \bar{0}$$

$$\bar{r}_2 + \bar{r}_3 + \bar{r}_4 + \bar{r}_6 + \bar{r}_7 - \bar{r}_8 = 0$$

3.) The VLEs have simplified scalar components (note $\theta_1 = -\pi/2$, $\theta_5 = \theta_8 = 0$, $\theta_7 = \pi/2$),

$$r_2\cos\theta_2 + r_3\cos\theta_3 + r_4\cos\theta_4 - r_5 = 0 \tag{1}$$

$$r_2\sin\theta_2 + r_3\sin\theta_3 + r_4\sin\theta_4 + r_1 = 0 \tag{2}$$

$$r_2\cos\theta_2 + r_3\cos\theta_3 + r_4\cos\theta_4 + r_6\cos\theta_2 - r_8 = 0 \tag{3}$$

$$r_2\sin\theta_2 + r_3\sin\theta_3 + r_4\sin\theta_4 + r_6\sin\theta_2 + r_7 = 0. \tag{4}$$

4.) $\bar{r}_4$ and $\bar{r}_6$ both rotate with link 4 so the angle between them (γ as shown) is constant, so

$$\theta_6 + \gamma + \pi = \theta_4 \longrightarrow \theta_6 + \gamma + \pi - \theta_4 = 0 \tag{5}$$

5.) The five position equations (1) and (5) contain,

<u>scalar knowns</u>: r_2, r_3, r_4, $\theta_5 = 0$, r_1, $\theta_1 = -\pi/2$, r_6, $\theta_7 = \pi/2$, r_8, $\theta_8 = 0$, and

<u>scalar unknowns</u>: θ_2, θ_3 and θ_4 , r_5, θ_6, r_7

6.) The five position equations contain six unknowns meaning one of the scalar unknowns must be given so that the remaining five can be calculated from the position equations. This means the system has one degree of freedom which agrees with Gruebler's Criterion.

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Problem 2.17

1.) Take θ_2 as the known input. Outline all equations needed to implement Newton's Method to solve for the remaining five unknowns as was done in Section 2.4, using,

$$\bar{x} = \begin{bmatrix} r_3 \\ r_4 \\ r_5 \\ \theta_3 \\ \theta_4 \end{bmatrix}.$$

Solution to Part 1.)

Define a vector $\bar{f}$ of the homogeneous functions whose roots are to be found.

$$\bar{f}(\bar{x}) = \begin{bmatrix} f_1(\bar{x}) \\ f_2(\bar{x}) \\ f_3(\bar{x}) \\ f_4(\bar{x}) \\ f_5(\bar{x}) \end{bmatrix} = \begin{bmatrix} r_2\cos\theta_2 - r_3\cos\theta_3 + r_1 \\ r_2\sin\theta_2 - r_3\sin\theta_3 + r \\ r_6 - r_4\cos\theta_4 \\ -r_5 - r_4\sin\theta_4 \\ \theta_4 - \theta_3 \end{bmatrix} = \bar{0}$$

Find the Jacobian matrix of partial derivatives.

$$J(\bar{x}) = \begin{bmatrix} \partial\bar{f}/\partial r_3 & \partial\bar{f}/\partial r_4 & \partial\bar{f}/\partial r_5 & \partial\bar{f}/\partial\theta_3 & \partial\bar{f}/\partial\theta_4 \end{bmatrix}$$

$$= \begin{bmatrix} -\cos\theta_3 & 0 & 0 & r_3\sin\theta_3 & 0 \\ -\sin\theta_3 & 0 & 0 & -r_3\cos\theta_3 & 0 \\ 0 & -\cos\theta_4 & 0 & 0 & r_4\sin\theta_4 \\ 0 & -\sin\theta_4 & -1 & 0 & -r_4\cos\theta_4 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

2.) Flowchart how these equations would be used in a computer program to solve for the output variables.

Solution to Part 2.)
Flowchart of Newton's Method

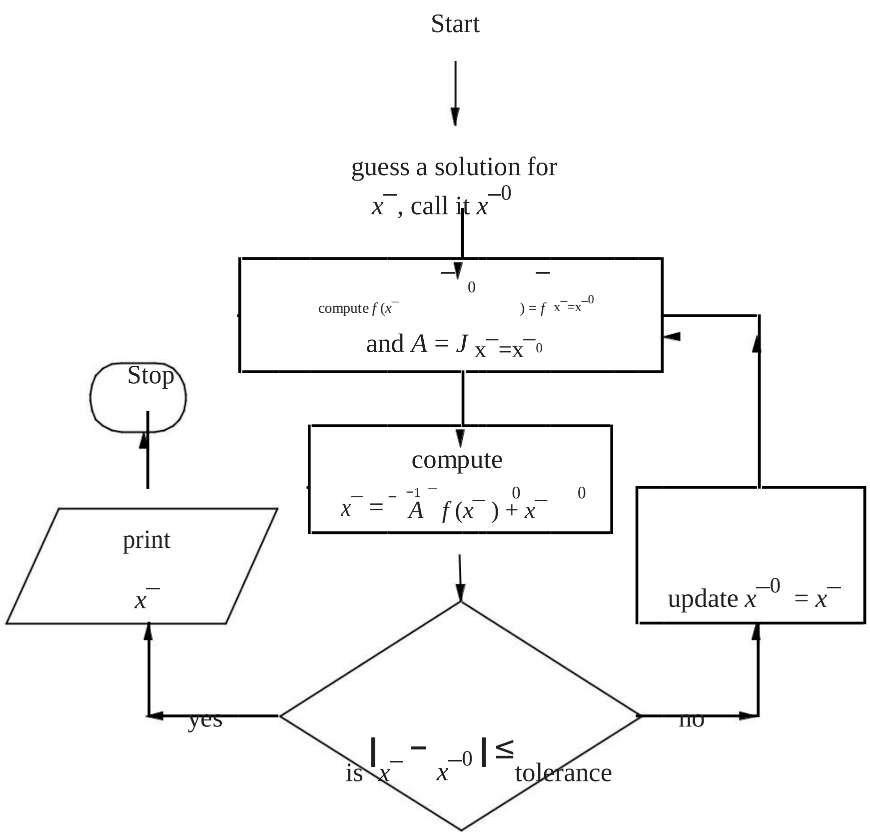

fig 087b

2.1 Programming Problems

For the following problems you may use any programming language or script of your choice.

Programming Problem 1

The Matlab code

```
pi=4.0*atan(1.0);
t2=283.0*pi/180;
r1=4.8;
r2=2.0;
r6=3.65;
r3=3;
r4=11;
r5=4;
t3=300*pi/180;
t4=300*pi/180;
x0=[r3;r4;r5;t3;t4];
t3d=t3*180/pi;
t4d=t4*180/pi;
xd0=[r3;r4;r5;t3d;t4d];
for  i=1:6
ct2=cos(t2);
st2=sin(t2);

ct3=cos(t3);
st3=sin(t3);
ct4=cos(t4);
st4=sin(t4);
f1=r2*ct2-r3*ct3+r1;
f2=r2*st2-r3*st3;
f3=r6-r4*ct4+r1;
f4=-r5-r4*st4;
f5=t4-t3;
f=[f1;f2;f3;f4;f5];
a11=-ct3;
a12=0;
a13=0;
a14=r3*st3;
a15=0;
a21=-st3;
a22=0;
a23=0;
a24=-r3*ct3;
a25=0;
a31=0;
a32=-ct4;
a33=0;
a34=0;
a35=r4*st4;
a41=0;
a42=-st4;
a43=-1;
a44=0;
```

```
a45=-r4*ct4;
a51=0;
a52=0;
a53=0;
a54=-1;
a55=1;
A = [a11 a12 a13 a14 a15;a21 a22 a23 a24 a25;a31 a32 a33 a34 a35; ...
            a41 a42 a43 a44 a45;a51 a52 a53 a54 a55]; x
= x0-inv(A)*f;
x0=x;
x xd=[r3;r4;r5;t3d;t4d]; xd


r3=x(1,1);
r4=x(2,1);
r5=x(3,1);
t3=x(4,1);
t4=x(5,1);
t3d=t3*180/pi;
t4d=t4*180/pi;
end
```

The Matlab Output

Initial $\bar{x}^0$ as given in problem statement

$$\bar{x}^0 = \begin{bmatrix} 3.0000 \\ 11.0000 \\ 4.0000 \\ 5.2360 \\ 5.2360 \end{bmatrix}$$

Computed $\bar{x}$ after 1st iteration

$$\bar{x}^0 = \begin{bmatrix} 4.3126 \\ -5.7864 \\ -1.5601 \\ 6.4267 \\ 6.4267 \end{bmatrix}$$

Computed $\bar{x}$ after 2nd iteration

$$\bar{x}^0 = \begin{bmatrix} 4.9172 \\ 9.0574 \\ -4.8539 \\ 5.8054 \\ 5.8054 \end{bmatrix}$$

Computed $\bar{x}$ after 3rd iteration

$$\bar{x}^0 = \begin{bmatrix} 5.5580 \\ 8.8637 \\ 2.9577 \\ 5.9444 \end{bmatrix}$$

5.9444

Computed $\bar{x}$ after 4th iteration

$$\bar{x}^{0} = \begin{bmatrix} 5.5991 \\ 9.0116 \\ 3.1351 \\ 5.9276 \\ 5.9276 \end{bmatrix}$$

Computed $\bar{x}$ after 5th iteration

$$\bar{x}^{0} = \begin{bmatrix} 5.5999 \\ 9.0134 \\ 3.1366 \\ 5.9278 \\ 5.9278 \end{bmatrix}$$

Programming Problem 2

To capture the angles θ_2 , θ_3 and θ_4 as defined, you must use a vector loop which has the vectors $\bar{r}_2$, $\bar{r}_3$ and $\bar{r}_4$, and the direction of the X axis, as defined in the figure below. The direction of $\bar{r}_1$ is immaterial.

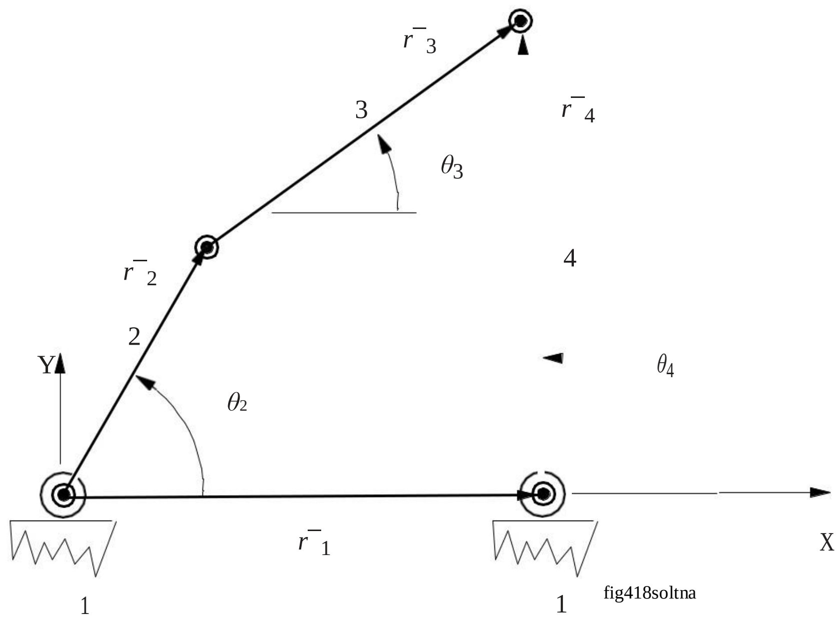

Figure 2.1: Open four bar mechanism

The vector loop equation has scalar components,

$$r_2\cos\theta_2 + r_3\cos\theta_3 - r_4\cos\theta_4 = 0$$
$$r_2\sin\theta_2 + r_3\sin\theta_3 - r_4\sin\theta_4 - r_1 = 0$$

The scalar unknowns are θ_2, θ_3 and θ_4. Applying Grashof's Criterion shows the four bar mechanism is a crank input (2) and a rocker output (4). We write the following Matlab code that increments θ_2 from 0° to 360° and computes the corresponding values of θ_3 and θ_4 using Newton's Method then produces the desired plots.

The Matlab Code

Note that the initial guesses of θ_3 and θ_4 for the crossed four bar mechanism are commented out.

```
clear all;
pi=4.0*atan(1.0);
   values of scalar knowns (dimensions)
r1=10.0;
r2=6.0;
r3=8.0;
r4=10.0;
% set value of input angle theta2 (given joint variable) in radians t2=0;

% guess values of scalar unknowns the (remaining joint variables) the values of these
% guesses dictate whether the solution converges to the open or crossed case. these
% guesses lead to the open case. the guesses below that are not commented out are for
   the open case.
t3=pi/2;
```

```
t4=3*pi/4;
```


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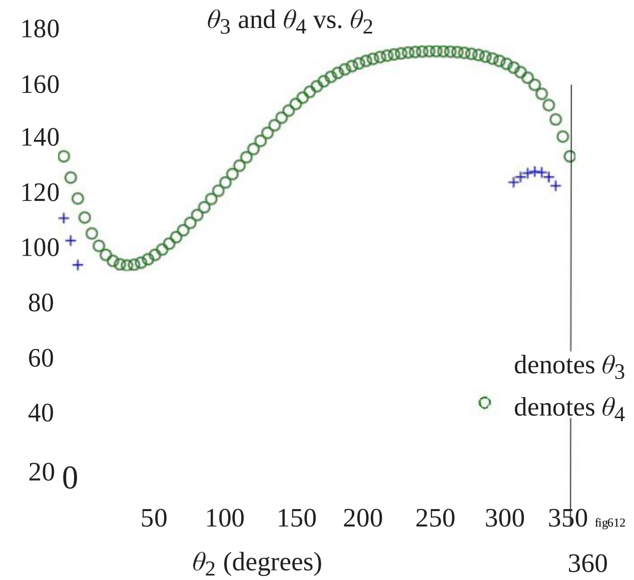
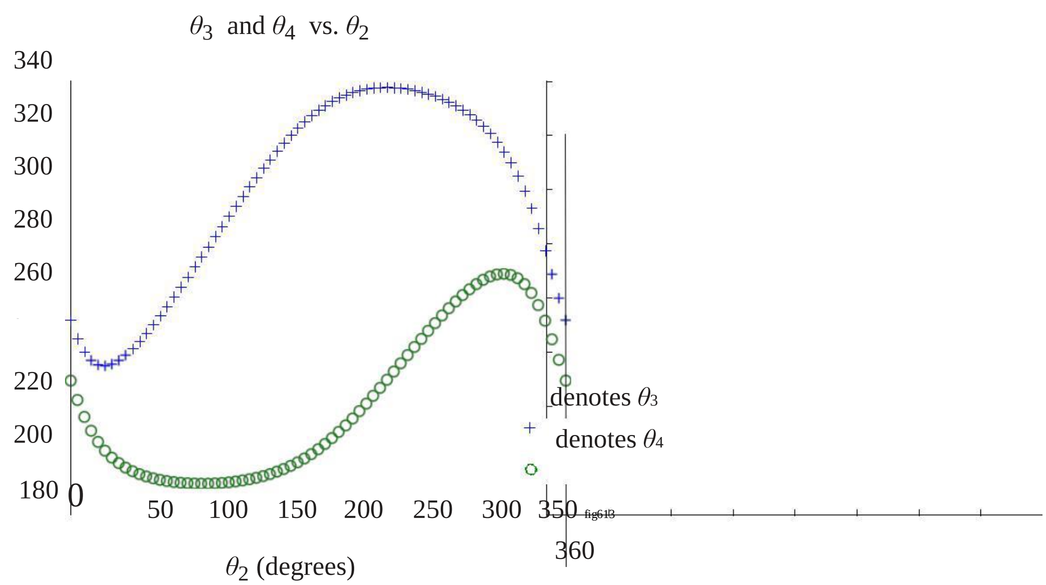
```
% the guesses below that are commented out are for the crossed case.
%t3=3*pi/2;
%t4=5*pi/4;
   define vector of initial guesses
x0=[t3;t4];
   initialize counter i
i=1;
while t2<360*pi/180
% set initial error as large, so as to enter the loop below error=10;

while error>.01
% compute the necessary sines and cosines of angles theta2-theta4 ct2=cos(t2);

st2=sin(t2);
ct3=cos(t3);
st3=sin(t3);
ct4=cos(t4);
st4=sin(t4);
%compute the functions at the guessed value
f1=r2*ct2+r3*ct3-r4*ct4-r1;
f2=r2*st2+r3*st3-r4*st4;
% define vector of functions computed at the guessed solution f=[f1;f2];

% calculate the partials of f w.r.t. each element of x dfdt3=[-
r3*st3;r3*ct3];
dfdt4=[r4*st4;-r4*ct4];
    define the A matrix
A = [dfdt3 dfdt4];
% Compute the solution x
x = x0-inv(A)*f;
error=norm(x-x0);
x0=x;
t3=x(1);
t4=x(2);
end
% building vectors of the angles, in units of degrees
t2d(i)=t2*180/pi;
t3d(i)=t3*180/pi;
t4d(i)=t4*180/pi;
% taking 5 degree step in theta2
t2=t2+5*pi/180;
i=i+1;
end
% plotting theta3 versus theta2 using "+"
% and theta4 versus theta2 using "o'" where all
% angles are in degrees
plot(t2d,t3d,'+',t2d,t4d,'o');
```



Open four bar mechanism



Crossed four bar mechanism

Programming Problem 3

The Matlab Code

It is important that the command “axis equal;” is used after the plot command. If it were not used, then the coupler curve would have been compressed in either the X or Y direction because one inch along each axis would not have been the same distance.

```
clear all;
pi=4.0*atan(1.0);
   values of scalar knowns (dimensions)
r1=10.0;
r2=6.0;
r3=8.0;
$ the variables r3p and phi3 define the location of Q on the coupler as per Figure 2.29
r3p=6;
phi3=68.3*pi/180;
r4=10.0;
% set value of input angle theta2 (given joint variable) in radians t2=60*pi/180;

% guess values of scalar unknowns the (remaining joint variables) the values of these
% guesses dictate whether the solution converges to the open or crossed case. these
% guesses lead to the open case.
%t3=pi/2; %
t4=3*pi/4;
t3=pi/4;
t4=pi/2;
   define vector of initial guesses
x0=[t3;t4];
   initialize counter i
i=1;
while t2<420*pi/180
% set initial error as large, so as to enter the loop below error=10;

while error>.01
% compute the necessary sines and cosines of angles theta2-theta4 ct2=cos(t2);

st2=sin(t2);
ct3=cos(t3);
st3=sin(t3);
ct4=cos(t4);
st4=sin(t4);
%compute the functions at the guessed value
f1=r2*ct2+r3*ct3-r4*ct4-r1;
f2=r2*st2+r3*st3-r4*st4;
% define vector of functions computed at the guessed solution f=[f1;f2];

% calculate the partials of f w.r.t. each element of x dfdt3=[-
r3*st3;r3*ct3];
dfdt4=[r4*st4;-r4*ct4];
    define the A matrix
A = [dfdt3 dfdt4];
% Compute the solution x
x = x0-inv(A)*f;
error=norm(x-x0);
x0=x;
```

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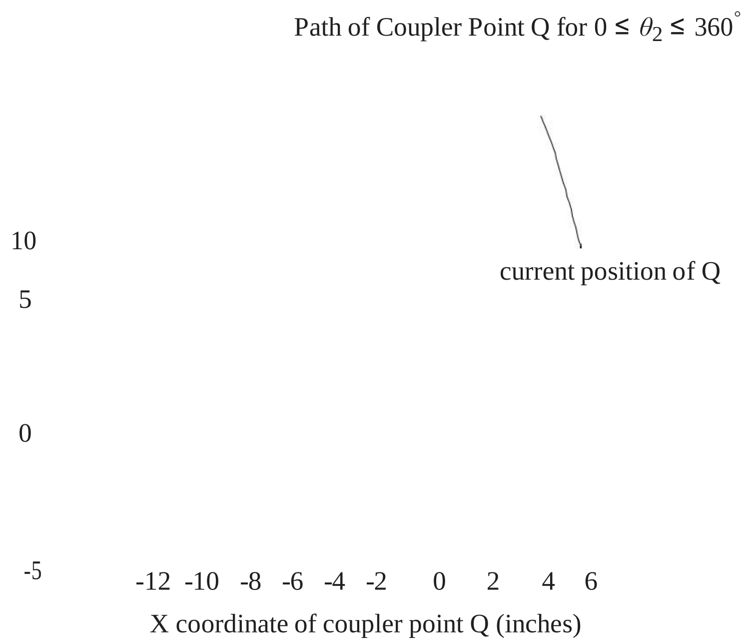
```
t3=x(1);
t4=x(2);
end
% building vectors of the coordinates of Q, inches
X(i)=r2*ct2+r3p*cos(t3+phi3); Y(i)=r2*st2+r3p*sin(t3+phi3);

    taking 5 degree step in theta2
t2=t2+5*pi/180;
i=i+1;
end
plot(X(1),Y(1),'*',X,Y,'+');axis  equal;
```



Coupler curve of point Q