1.1)

Given:

T-38A weighing 10,000 *lbf* and flying at 20,000 *ft*

From Fig. P1.1, the maximum Mach number with afterburner ("Max" thrust curve) is $M \approx 1.075$. From Table 1.2b, the standard day speed of sound at 20,000 *ft* is 1036.94 *ft/s*. Using the definition of the Mach number as $M \equiv V/a$ (the instructor may have to give this definition to the students since it is not found in Chapter 1), the velocity is:

$$V = Ma = (1.075)(1036.94\,ft/s) = 1114.7\ ft/s$$

The maximum lift-to-drag ratio, $(L/D)_{max}$, occurs at the point of minimum drag, $D_{min} = 850\ lbf$. Assuming that the airplane is in steady, level, unaccelerated flight, $L = W = 10{,}000\ lbf$, and:

$$(L/D)_{max} = 10{,}000 lbf / 850 lbf = 11.76$$

The Mach number where this occurs is $M \approx 0.53$. The minimum velocity of the aircraft under the given conditions is $M \approx 0.34$ and is due to the buffet (or stall) limit.

1.2)

Given:

T-38A weighing 10,000 *lbf* and flying at 20,000 *ft* with "Mil" thrust at $M = 0.65$

From Eqn. 1.1, the total energy of the aircraft is:

$$E = 0.5mV^2 + mgh$$

The mass of the airplane is given by Eqn. 1.2:

$$m = W/g = 10{,}000 lbf / 32.174\,ft/s^2 = 310.81\ slugs$$

As in Problem 1.1, the velocity of the airplane can be found as:

$$V = Ma = (0.65)(1036.94\,ft/s) = 674.01\ ft/s$$

which yields a total energy of:

1.2) contd.

The energy height is given by Eqn. 1.3:

$$H_e = E/W = 270.60 \times 10^6\ ft - lbf / 10{,}000 lbf = 27{,}060\ ft$$

The specific excess power is given by Eqn. 1.7:

$$P_s = \frac{(T - D)V}{W}$$

At the given conditions, $T = 2{,}500\ lbf$ and $D = 1{,}000\ lbf$ from Fig. P1.1, and the specific excess power is:

$$P_s = \frac{(T - D)V}{W} = \frac{(2{,}500 lbf - 1{,}000 lbf)(674.01 ft/s)}{10{,}000 lbf} = 101.1\ ft/s$$

1.3)

Given:

T-38A weighing 10,000 *lbf* and flying at 20,000 *ft* with "Mil" thrust at $M = 0.65$

The acceleration possible is given by Eqn. 1.5 as:

$$\frac{(T - D)V}{W} = \frac{V}{g}\frac{dV}{dt}$$

For the conditions of Problem 1.2, the acceleration would be:

$$\frac{dV}{dt} = \frac{(T - D)V}{W}\frac{g}{V} = P_s \frac{g}{V} = (101.1 ft/s)\frac{32.174 ft/s^2}{674.01 ft/s} = 4.826 ft/s^2$$

The rate of climb is given by Eqn. 1.7 as:

$$\frac{dh}{dt} = P_s = 101.1 ft/s = 6{,}066 ft/m$$

1.4)

Given:

T-38A flying at 20,000 *ft* with weight of 8,000, 10,000, and 12,000 *lbf*

As in Problem 1.1, the maximum lift-to-drag ratio, $(L/D)_{max}$, occurs at the minimum drag, D_{min}. Also, assuming that the airplane is in steady, level, unaccelerated flight, $L = W$. For the three weights you can generate a table for finding $(L/D)_{max}$ using values from Fig. P1.1 and Tab. 1.2b:

$W = L$ (*lbf*)	D_{min} (*lbf*)	$(L/D)_{max}$	$M_{(L/D)_{max}}$	$V_{(L/D)_{max}}$ (*ft/s*)
8,000	650	12.31	0.47	487.3
10,000	850	11.76	0.53	549.6
12,000	1050	11.43	0.60	622.2

As can be seen, higher weight requires higher velocities to maintain aerodynamic efficiency, but also results in a reduction of that efficiency.

1.5)

Given:

10,000 *lbf* T-38A flying at 20,000 *ft* at $M = 0.35$, $M_{(L/D)_{max}}$, and 0.70

As in Problem 1.2, the specific excess power is given by Eqn. 1.7:

$$P_s = \frac{(T - D)V}{W}$$

For the three velocities you can generate a table for specific excess power using values from Fig. P1.1:

M	T (*lbf*)	D (*lbf*)	V (*ft/s*)	P_s (*ft/s*)
0.35	2250	1600	362.93	23.6
0.53	2400	850	549.58	85.2
0.70	2600	1100	725.86	108.9

Notice the large increase in specific excess power from $M = 0.35$ to 0.53 as the airplane becomes more aerodynamically efficient. While the specific excess power continues to increase as the speed increases to $M = 0.70$, the increase in P_s is not as dramatic due to the increase in drag.

1.6)

Given:

Air and nitrogen with associated gas and Sutherland's law constants at $p = 150\ psia$ and $T = 350^\circ F$.

For a thermally perfect gas, the equation of state is given by Eqn. 1.10:

$$p = \rho RT$$

The fluid density can be found as:

$$\rho = \frac{p}{RT}$$

where R, the gas constant, is given in the problem statement, and $T = 350 + 459.67 = 809.67^\circ R$. The resulting densities are:

$$\rho_{Air} = \frac{p_{Air}}{R_{Air} T_{Air}} = \frac{(150 lbf/in^2)(144 in^2/ft^2)}{\left(53.34 \frac{ft-lbf}{lbm-^\circ R}\right)(809.67^\circ R)} = 0.50014\ lbm/ft^3 = 0.01554\ slug/ft^3$$

$$\rho_{N} = \frac{p_{N_2}}{R_{N_2} T_{N_2}} = \frac{(150 lbf/in^2)(144 in^2/ft^2)}{\left(55.15 \frac{ft-lbf}{lbm-^\circ R}\right)(809.67^\circ R)} = 0.48373\ lbm/ft^3 = 0.01503\ slug/ft^3$$

These values are very close to each other due to the very similar gas constants. The viscosity is given by Sutherland's law in English units, Eqn. 1.12b:

$$\mu = C_1 \frac{T^{1.5}}{(T + C_2)}$$

where the constants are also given in the problem statement. The resulting viscosities are:

$$\mu_{Air} = 2.27 \times 10^{-8} \frac{lbf-s}{ft^2 -^\circ R^{0.5}} \frac{(809.67^\circ R)^{1.5}}{(809.67^\circ R + 198.6^\circ R)} = 5.18694 \times 10^{-7} \frac{lbf-s}{ft^2}$$

$$\mu_{N_2} = 2.16 \times 10^{-8} \frac{lbf-s}{ft^2 -^\circ R^{0.5}} \frac{(809.67^\circ R)^{1.5}}{(809.67^\circ R + 183.6^\circ R)} = 5.01012 \times 10^{-7} \frac{lbf-s}{ft^2}$$

1.6) contd.

Finally, the kinematic viscosity is given by Eqn. 1.6 as:

$$\nu = \frac{\mu}{\rho}$$

and the resulting kinematic viscosities are:

$$\nu_{Air} = \frac{\mu_{Air}}{\rho_{Air}} = \frac{5.18694 \times 10^{-7} \frac{lbf - s}{ft^2}}{0.01553 \frac{slug}{ft^3}} = 3.33995 \times 10^{-5}\ ft^2/s$$

$$\nu_{N_2} = \frac{\mu_{N_2}}{\rho_{N_2}} = \frac{5.01012 \times 10^{-7} \frac{lbf - s}{ft^2}}{0.01502 \frac{slug}{ft^3}} = 3.33563 \times 10^{-5}\ ft^2/s$$

1.7)

Given:

Air and nitrogen with associated gas and Sutherland's law constants at $p = 586\ N/m^2$ and $T = 54.3K$.

For a thermally perfect gas, the equation of state is given by Eqn. 1.10:

$$p = \rho RT$$

The fluid density can be found as:

$$\rho = \frac{p}{RT}$$

where R, the gas constant, is given in the problem statement. The resulting densities are:

$$\rho_{Air} = \frac{p_{Air}}{R_{Air} T_{Air}} = \frac{58.6 N/m^2}{\left(287.05 \frac{N - m}{kg - K}\right)(54.3K)} = 0.03760\ kg/m^3$$

1.7) contd.

$$\rho_N = \frac{p_{N_2}}{R_{N_2}T_{N_2}} = \frac{(58.6N/m^2)}{\left(297\frac{N-m}{kg-K}\right)(54.3K)} = 0.03634\,kg/m^3$$

These values are very close to each other due to the very similar gas constants. The viscosity is given by Sutherland's law in English units, Eqn. 1.12a:

$$\mu = C_1\frac{T^{1.5}}{(T+C_2)}$$

where the constants are also given in the problem statement. The resulting viscosities are:

$$\mu_{Air} = 1.458\times10^{-6}\frac{kg}{s-m-K^{0.5}}\frac{(54.3K)^{1.5}}{(54.3K+110.4K)} = 5.18694\times10^{-7}\frac{lbf-s}{ft^2}$$

$$\mu_{N_2} = 1.458\times10^{-6}\frac{kg}{s-m-K^{0.5}}\frac{(54.3K)^{1.5}}{(54.3K+102K)} = 5.01012\times10^{-7}\frac{lbf-s}{ft^2}$$

Finally, the kinematic viscosity is given by Eqn. 1.6 as:

$$\nu = \frac{\mu}{\rho}$$

and the resulting kinematic viscosities are:

$$\nu_{Air} = \frac{\mu_{Air}}{\rho_{Air}} = \frac{5.18694\times10^{-7}\frac{lbf-s}{ft^2}}{0.01553\frac{slug}{ft^3}} = 3.33995\times10^{-5}\,ft^2/s$$

$$\nu_{N_2} = \frac{\mu_{N_2}}{\rho_{N_2}} = \frac{5.01012\times10^{-7}\frac{lbf-s}{ft^2}}{0.01502\frac{slug}{ft^3}} = 3.33563\times10^{-5}\,ft^2/s$$

1.8)

Given:

A perfect gas process which doubles the pressure and the density is decreased by three-quarters. Initial temperature $T_1 = 200°F$.

The pressure and density changes during the process may be represented as:

$$\frac{p_2}{p_1} = 2 \text{ and } \frac{\rho_2}{\rho_1} = \frac{3}{4}$$

The perfect gas law is given by Eqn. 1.10:

$$p = \rho RT$$

So each state may be represented by:

$$R_1 = \frac{p_1}{\rho_1 T_1} \text{ and } R_2 = \frac{p_2}{\rho_2 T_2}$$

Assuming that the gas constant does not change significantly due to changes in temperature (i.e. $R_1 = R_2$), the equations can be combined to yield:

$$\frac{T_2}{T_1} = \frac{p_2/p_1}{\rho_2/\rho_1} = \frac{2}{3/4} = \frac{8}{3}$$

Remembering that temperatures in the perfect gas law must be in absolute values, the final temperature can then be found as:

$$T_2 = \frac{8}{3}T_1 = \frac{8}{3}(200 + 459.67)^{\circ} R = 1759^{\circ} R = 1299^{\circ} F = 703.9^{\circ} C$$

1.9)

Given:

Isentropic expansion of perfect air $\frac{p}{\rho^{1.4}}$ = constant and $\frac{p_2}{p_1} = 0.5$

Initial temperature $T_1 = 20°$ C

For a perfect gas $\rho = \frac{p}{RT}$.

1.9) contd.

$$\frac{p_1}{\rho_1^{1.4}} = \frac{p_2}{\rho_2^{1.4}} \text{ so } p_1\left(\frac{RT_1}{p_1}\right)^{1.4} = p_2\left(\frac{RT_2}{p_2}\right)^{1.4}$$

Simplifying yields:

$$p_1^{-0.4}T_1^{1.4} = p_2^{-0.4}T_2^{1.4} \text{ or } \left(\frac{T_2}{T_1}\right)^{1.4} = \left(\frac{p_2}{p_1}\right)^{0.4}$$

Find the temperature ratio as:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{0.4/1.4} = 0.820, \text{ so the temperature decreases by about } 18.0\%.$$

Remember that temperature ratios must always be calculated using absolute temperatures. So use $T_1 = 293.15\,K$.

$$T_2 = 0.820\,T_1 = 240.481\,K = -32.67°C.$$

1.10)

Given:

Pressure and temperature for the standard atmosphere at 20 *km* altitude.

From Table 1.2a:

$$p = \frac{p}{p_{SL}} p_{SL} = \left(5.4570\times10^{-2}\right)\left(1.01325\times10^{5}\,N/m^2\right) = 5.5293\times10^{3}\ N/m^2$$

$$T = 216.65K$$

The density can be found using the perfect gas law, Eqn. 1.10:

$$\rho = \frac{p}{RT} = \frac{5.5293\times10^{3}\,N/m^2}{\left(287.05\,\frac{N-m}{kg-K}\right)(216.65K)} = 0.08891\ kg/m^3$$

1.10) contd.

The value for density from Table 1.2a is:

$$\rho = \frac{\rho}{\rho_{SL}}\rho_{SL} = \left(7.2580\times10^{-2}\right)\left(1.2250kg/m^3\right) = 0.08891\ kg/m^3$$

which is the same as the calculated value from above. The viscosity is given by Eqn. 1.12a:

$$\mu = 1.458\times10^{-6}\frac{T^{1.5}}{(T+110.4K)}$$
$$= 1.458\times10^{-6}\frac{\left(216.65^{\circ}R\right)^{1.5}}{\left(216.65K+110.4K\right)} = 1.422\times10^{-5}\ kg/s\text{-}m$$

The value for viscosity from Table 1.2b is:

$$\mu = \frac{\mu}{\mu_{SL}}\mu_{SL} = (0.79447)\left(1.7894\times10^{-5}kg/s-m\right) = 1.422\times10^{-5}\ kg/s\text{-}m$$

which is the same as the calculated value from above.

1.11)

Given:

Pressure and temperature for the standard atmosphere at 35,000 *ft* altitude.

From Table 1.2b:

$$p = \frac{p}{p_{SL}}p_{SL} = (0.23617)\left(2116.22lbf/ft^2\right) = 499.79\ lbf/ft^2$$

$$T = 394.07^{o}R$$

The density can be found using the perfect gas law, Eqn. 1.10:

$$\rho = \frac{p}{RT} = \frac{499.79lbf/ft^2}{\left(53.34\frac{ft-lbf}{lbm-^{\circ}R}\right)\left(394.07^{\circ}R\right)} = 0.02378\ lbm/ft^3 = 0.000739\ slug/ft^3$$

1.11) contd.

The value for density from Table 1.2b is:

$$\rho = \frac{\rho}{\rho_{SL}} \rho_{SL} = (0.31075)(0.002377 slug/ft^3) = 0.000739 \; slug/ft^3$$

which is the same as the calculated value from above. The viscosity is given by Eqn. 1.12b:

$$\mu = 2.27 \times 10^{-8} \frac{T^{1.5}}{(T + 198.6^\circ R)}$$
$$= 2.27 \times 10^{-8} \frac{(394.07^\circ R)^{1.5}}{(394.07^\circ R + 198.6^\circ R)} = 2.996 \times 10^{-7} \; lbf\text{-}s/ft^2$$

The value for viscosity from Table 1.2b is:

$$\mu = \frac{\mu}{\mu_{SL}} \mu_{SL} = (0.801425)(3.740 \times 10^{-7} lbf - s/ft^2) = 2.997 \times 10^{-7} \; lbf\text{-}s/ft^2$$

which is the same as the calculated value from above.

1.12)

$$p_{t1} = 5.723 \times 10^6 \; N/m^2 \; (= 56.48 \text{ atmospheres})$$
$$T_t = 750K$$
$$\rho_{t1} = \frac{p_{t1}}{R \, T_t} = \frac{5.723 \times 10^6 \frac{N}{m^2}}{(287.05 \frac{N \cdot m}{kg \cdot K})(750K)} = 26.583 \frac{kg}{m^3}$$
$$\mu_t = 1.458 \times 10^{-6} \frac{T_t^{1.5}}{T_t + 110.4} = 3.481 \times 10^{-5} \frac{kg}{s \cdot m}$$