Solution Manual for Algebra for College Students 8th Edition Lial Hornsby McGinnis 032196926X 9780321969262

Chapter 2 Linear Equations, Graphs, and Functions

2.1 Linear Equations in Two Variables

Classroom Examples, Now Try Exercises

1. To complete the ordered pairs, substitute the given value of *x* or *y* in the equation.
For (0, ____), let $x = 0$.

$$3x - 4y = 12$$
$$3(0) - 4y = 12$$
$$-4y = 12$$
$$y = -3$$

The ordered pair is $(0, -3)$.

For (____, 0) let $y = 0$.

$$3x - 4y = 12$$
$$3x - 4(0) = 12$$
$$3x = 12$$
$$x = 4$$

The ordered pair is (4, 0).

For (____, − 2), let $3x -$ $4y = 12$

The completed table follows.

x	y
0	−3
4	0
$\frac{4}{3}$	−2
−6	$-\frac{15}{2}$

N1. To complete the ordered pairs, substitute the given value of *x* or *y* in the equation.
For (0, ____), let $x = 0$.

$$2x - y = 4$$
$$2(0) - y = 4$$
$$-y = 4$$
$$y = -4$$

The ordered pair is $(0, -4)$.

For (____, 0) let $y = 0$.

$$2x - y = 4$$
$$2x - 0 = 4$$

$$\begin{aligned} 3x - 4(-2) &= 12 \\ 3x + 8 &= 12 \\ 3x &= 4 \\ x &= \frac{4}{3} \end{aligned} \qquad y = -2.$$

The ordered pair is $\left(\frac{4}{3}, -2\right)$.

For $(-6, \_\_\_\_)$, let $x = -6$.

$$\begin{aligned} 3x - 4y &= 12 \\ 3(-6) - 4y &= 12 \\ -18 - 4y &= 12 \\ -4y &= 30 \end{aligned}$$

$$y = -\frac{30}{4} = -\frac{15}{2}$$

The ordered pair is $\left(-6, -\frac{15}{2}\right)$.

$$\begin{aligned} 2x &= 4 \\ x &= 2 \end{aligned}$$

The ordered pair is (2, 0).
For (4, ____), let $x = 4$.

$$\begin{aligned} 2x - y &= 4 \\ 2(4) - y &= 4 \\ 8 - y &= 4 \\ -y &= -4 \\ y &= 4 \end{aligned}$$

The ordered pair is (4, 4).

For (____, 2), let $y = 2$.

$$\begin{aligned} 2x - y &= 4 \\ 2x - 2 &= 4 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

The ordered pair is (3, 2).
The completed table follows.

x	y
0	−4
2	0
4	4
3	2

2. To find the x-intercept, let $y = 0$.

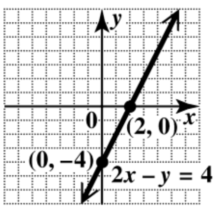
$2x - y = 4$
$2x - 0 = 4$
$2x = 4$
$x = 2$

The x-intercept is (2, 0).
To find the y-intercept, let $x = 0$.

$2x - y = 4$
$2(0) - y = 4$
$-y = 4$
$y = -4$

The y-intercept is $(0, -4)$.

Plot the intercepts, and draw the line through them.



N2. To find the x-intercept, let $y = 0$.

$x - 2y = 4$
$x - 2(0) = 4$
$x = 4$

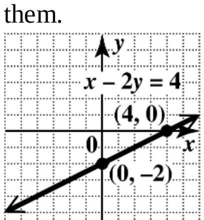
The x-intercept is (4, 0).

To find the y-intercept, let $x = 0$.

$x - 2y = 4$
$0 - 2y = 4$
$-2y = 4$
$y = -2$

The y-intercept is $(0, -2)$.

Plot the intercepts, and draw the line through them.



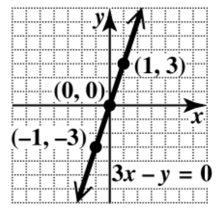
3. To find the x-intercept, let $y = 0$.

$3x - 0 = 0$
$3x = 0$
$x = 0$

Find another point. Let $x = 1$.

$3(1) - y = 0$
$3 - y = 0$
$y = 3$

This gives the ordered pair (1, 3). Plot (1, 3) and (0, 0) and draw the line through them.



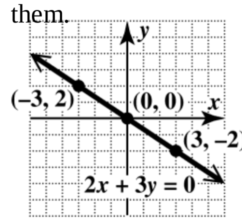
N3. To find the x-intercept, let $y = 0$.

$2x + 3(0) = 0$
$2x = 0$
$x = 0$

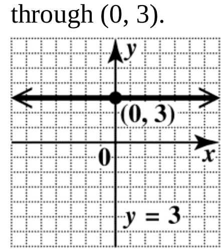
Since the x-intercept is (0, 0), the y-intercept is also (0, 0).
Find another point. Let $x = 3$.

$2(3) + 3y = 0$
$6 + 3y = 0$
$3y = -6$
$y = -2$

This gives the ordered pair $(3, -2)$. Plot $(3, -2)$ and (0, 0) and draw the line through them.

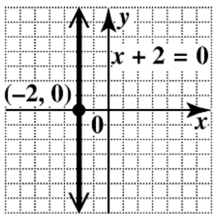


4. **(a)** In standard form, the equation is $0x + y = 3$. Every value of x leads to $y = 3$, so the y-intercept is (0, 3). There is no x-intercept. The graph is the horizontal line through (0, 3).

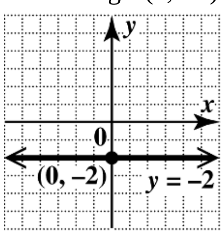


Since the *x*-intercept is (0, 0), the *y*-intercept is also (0, 0).

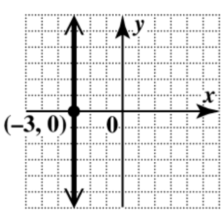
(b) In standard form, the equation is $x + 0y = -2$. Every value of y leads to $x = -2$, so the x-intercept is $(-2, 0)$. There is no y-intercept. The graph is the vertical line through $(-2, 0)$.



N4. **(a)** In standard form, the equation is $0x + y = -2$. Every value of x leads to $y = -2$, so the y-intercept is $(0, -2)$. There is no x-intercept. The graph is the horizontal line through $(0, -2)$.



(b) In standard form, the equation is $x + 0y = -3$. Every value of y leads to $x = -3$, so the x-intercept is $(-3, 0)$. There is no y-intercept. The graph is the vertical line through $(-3, 0)$.



5. By the midpoint formula, the midpoint of the segment with endpoints $(-5, 8)$ and $(2, 4)$ is
$$\left(\frac{-5+2}{2}, \frac{8+4}{2}\right) = \left(\frac{-3}{2}, \frac{12}{2}\right) = (-1.5, 6).$$

N5. By the midpoint formula, the midpoint of the segment with endpoints $(2, -5)$ and $(-4, 7)$ is
$$\left(\frac{2+(-4)}{2}, \frac{-5+7}{2}\right) = \left(\frac{-2}{2}, \frac{2}{2}\right) = (-1, 1).$$

2. For any value of x, the point $(x, 0)$ lies on the <u>x</u>-axis. For any value of y, the point $(0, y)$ lies on the <u>y</u>-axis.

3. The x-intercept is the point where a line crosses the x-axis. To find the x-intercept of a line, we let <u>y</u> equal 0 and solve for <u>x</u>.
The y-intercept is the point where a line crosses the y-axis. To find the y-intercept of a line, we let <u>x</u> equal 0 and solve for <u>y</u>.

4. The equation $y = 4$ has a <u>horizontal</u> line as its graph. The equation $x = 4$ has a <u>vertical</u> line as its graph.

5. To graph a straight line, we must find a minimum of <u>two</u> points. The points $(3, \underline{2})$ and $(\underline{6}, 4)$ lie on the graph of $2x - 3y = 0$.

6. The equation of the x-axis is $y = 0$.

The equation of the y-axis is $x = 0$.

7. **(a)** x represents the year; y represents the personal spending on medical care in billions of dollars.

(b) The dot above the year 2012 appears to be at about 2360, so the spending in 2012 was about \$2360 billion.

(c) The ordered pair is $(x, y) = (2012, 2360)$.

(d) In 2008, personal spending on medical care was about \$2000 billion.

8. **(a)** x represents the year; y represents the percentage of Americans who moved.

(b) The dot above the year 2013 appears to be at about 11, so about 11% of Americans moved in 2013.

(c) The ordered pair is $(x, y) = (2013, 11)$.

(d) In 1960, the percentage of Americans who moved was about 20%.

9. **(a)** The point $(1, 6)$ is located in quadrant I, since the x- and y-coordinates are both positive.

(b) The point $(-4, -2)$ is located in quadrant III, since the x- and y-coordinates

(2 2) (2 2)

Exercises

1. The point with coordinates (0, 0) is the origin of a rectangular coordinate system.

are both negative.

(c) The point $(-3, 6)$ is located in quadrant II, since the x-coordinate is negative and the y-coordinate is positive.

(d) The point $(7, -5)$ is located in quadrant IV, since the *x*-coordinate is positive and the *y*-coordinate is negative.

(e) The point $(-3, 0)$ is located on the *x*-axis, so it does not belong to any quadrant.

(f) The point $(0, -0.5)$ is located on the *y*-axis, so it does not belong to any quadrant.

10. (a) The point $(-2, -10)$ is located in quadrant III, since the *x*- and *y*-coordinates are both negative.

(b) The point (4, 8) is located in quadrant I, since the *x*- and *y*-coordinates are both positive.

(c) The point $(-9, 12)$ is located in quadrant II, since the *x*-coordinate is negative and the *y*-coordinate is positive.

(d) The point $(3, -9)$ is located in quadrant IV, since the *x*-coordinate is positive and the *y*-coordinate is negative.

(e) The point $(0, -8)$ is located on the *y*-axis, so it does not belong to any quadrant.

(f) The point (2.3, 0) is located on the *x*-axis, so it does not belong to any quadrant.

11. (a) If $xy > 0$, then both *x* and *y* have the same sign.
(*x*, *y*) is in quadrant I if *x* and *y* are positive.
(*x*, *y*) is in quadrant III if *x* and *y* are negative.

(b) If $xy < 0$, then *x* and *y* have different signs.

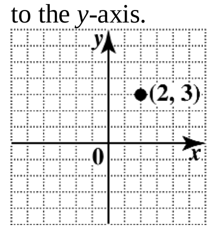
(*x*, *y*) is in quadrant II if $x < 0$ and $y > 0$.
(*x*, *y*) is in quadrant IV if $x > 0$ and $y < 0$.

(c) If $\frac{x}{y} < 0$, then *x* and *y* have different signs.

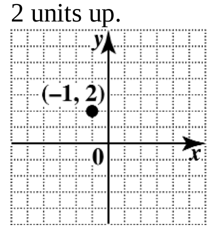
(*x*, *y*) is in either quadrant II or quadrant IV. (See part (b).)

(d) If $\frac{x}{y} > 0$, then *x* and *y* have the same sign.

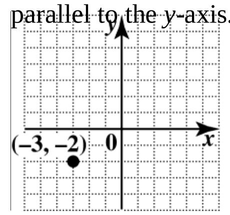
13. To plot $(2, 3)$, go 2 units from zero to the right along the *x*-axis, and then go 3 units up parallel to the *y*-axis.



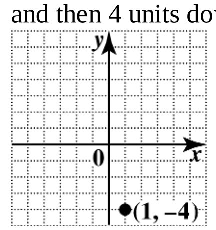
14. To plot $(-1, 2)$, go 1 unit in the negative direction—that is, left—on the *x*-axis and then 2 units up.



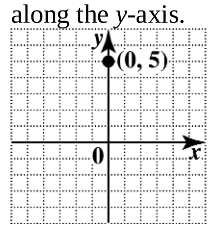
15. To plot $(-3, -2)$, go 3 units from zero to the left along the *x*-axis, and then go 2 units down parallel to the *y*-axis.



16. To plot $(1, -4)$, go 1 unit right on the *x*-axis and then 4 units down.



17. To plot $(0, 5)$, do not move along the *x*-axis at all since the *x*-coordinate is 0. Move 5 units up along the *y*-axis.

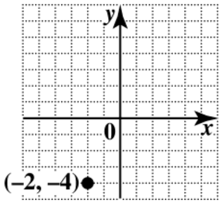


2.1 Linear Equations in Two Variables 129

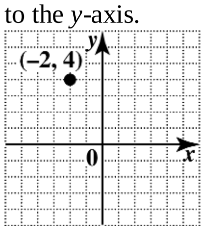
(*x*, *y*) is in either quadrant I or quadrant III. (See part (a).)

12. Any point that lies on an axis must have one coordinate that is 0.

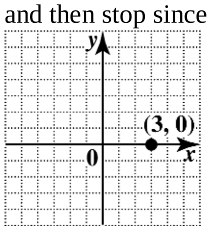
18. To plot $(-2, -4)$, go 2 units left on the x-axis and then 4 units down.



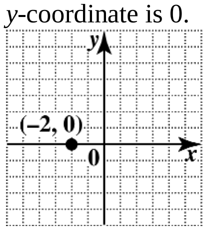
19. To plot $(-2, 4)$, go 2 units from zero to the left along the x-axis, and then go 4 units up parallel to the y-axis.



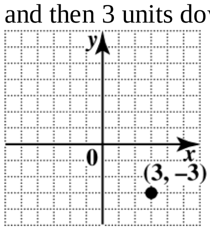
20. To plot $(3, 0)$, go 3 units right on the x-axis and then stop since the y-coordinate is 0.



21. To plot $(-2, 0)$, go 2 units to the left along the x-axis. Do not move up or down since the y-coordinate is 0.



22. To plot $(3, -3)$, go 3 units right on the x-axis and then 3 units down.



23. **(a)** To complete the table, substitute the given values for x and y in the equation.

For $x = 1$: $y = x - 4$
$y = 1 - 4$
$y = -3$ $(1, -3)$

For $x = 2$: $y = x - 4$
$y = 2 - 4$
$y = -2$ $(2, -2)$

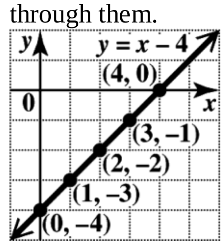
For $x = 3$: $y = x - 4$
$y = 3 - 4$
$y = -1$ $(3, -1)$

For $x = 4$: $y = x - 4$
$y = 4 - 4$
$y = 0$ $(4, 0)$

This is shown in the table below.

x	y
0	-4
1	-3
2	-2
3	-1
4	0

(b) Plot the ordered pairs and draw the line through them.



24. **(a)** To complete the table, substitute the given values for x and y in the equation.

For $x = 0$: $y = x + 3$
$y = 0 + 3$
$y = 3$ $(0, 3)$

For $x = 1$: $y = x + 3$
$y = 1 + 3$
$y = 4$ $(1, 4)$

For $x = 2$: $y = x + 3$
$y = 2 + 3$
$y = 5$ $(2, 5)$

For $x = 0$: $y = x - 4$
$y = 0 - 4$
$y = -4$ (0, −4)

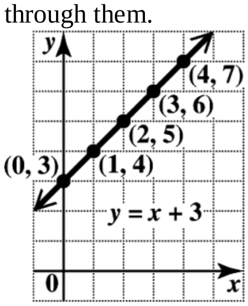
For $x = 3$: $y = x + 3$
$y = 3 + 3$
$y = 6$ (3, 6)

For $x = 4$: $y = x + 3$
$y = 4 + 3$
$y = 7$ (4, 7)

This is shown in the table below.

x	y
0	3
1	4
2	5
3	6
4	7

(b) Plot the ordered pairs and draw the line through them.



25. (a) To complete the table, substitute the given values for x and y in the equation.

For $x = 0$: $x - y = 3$
$0 - y = 3$
$y = -3$ (0, −3)

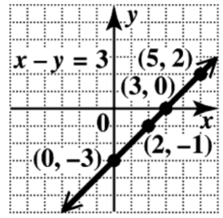
For $y = 0$: $x - y = 3$
$x - 0 = 3$
$x = 3$ (3, 0)

For $x = 5$: $x - y = 3$
$5 - y = 3$
$-y = -2$
$y = 2$ (5, 2)

For $x = 2$: $x - y = 3$
$2 - y = 3$
$-y = 1$
$y = -1$ (2, −1)

x	y
0	−3
3	0
5	2
2	−1

(b) Plot the ordered pairs and draw the line through them.



26. (a) For $x = 0$: $0 - y = 5$
$-y = 5$
$y = -5$ (0, −5)

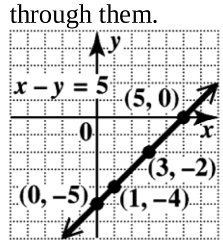
For $y = 0$: $x - 0 = 5$
$x = 5$ (5, 0)

For $x = 1$: $1 - y = 5$
$-y = 4$
$y = -4$ (1, −4)

For $x = 3$: $3 - y = 5$
$-y = 2$
$y = -2$ (3, −2)

x	y
0	−5
5	0
1	−4
3	−2

(b) Plot the ordered pairs and draw the line through them.



27. (a) To complete the table, substitute the given values for x or y in the equation.

For $x = 0$: $x + 2y = 5$
$0 + 2y = 5$
$2y = 5$
$y = \frac{5}{2}$ $\left(0, \frac{5}{2}\right)$

For $y = 0$:

$x + 2y = 5$

$$x + 2(0) = 5$$
$$x + 0 = 5$$
$$x = 5 \quad (5, 0)$$

For $x = 2$: $x + 2y = 5$

$2 + 2y = 5$

$2y = 3$

$y = \frac{3}{2}$ $\left(2, \frac{3}{2}\right)$

For $y = 2$: $x + 2y = 5$

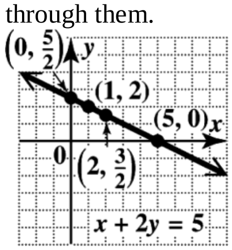
$x + 2(2) = 5$

$x + 4 = 5$

$x = 1$ $(1, 2)$

x	y
0	$\frac{5}{2}$
5	0
2	$\frac{3}{2}$
1	2

(b) Plot the ordered pairs and draw the line through them.



28. **(a)** For $x = 0$: $0 + 3y = -5$

$3y = -5$

$y = -\frac{5}{3}$ $\left(0, -\frac{5}{3}\right)$

For $y = 0$: $x + 3(0) = -5$

$x = -5$ $(-5, 0)$

For $x = 1$: $1 + 3y = -5$

$3y = -6$

$y = -2$ $(1, -2)$

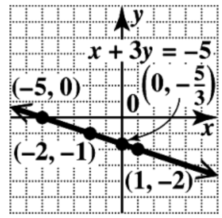
For $y = -1$: $x + 3(-1) = -5$

$x - 3 = -5$

$x = -2$ $(-2, -1)$

x	y
	–

(b) Plot the ordered pairs and draw the line through them.



29. **(a)** For $x = 0$: $4x - 5y = 20$

$4(0) - 5y = 20$

$-5y = 20$

$y = -4$ $(0, -4)$

For $y = 0$: $4x - 5y = 20$

$4x - 5(0) = 20$

$4x = 20$

$x = 5$ $(5, 0)$

For $x = 2$: $4x - 5y = 20$

$4(2) - 5y = 20$

$8 - 5y = 20$

$-5y = 12$

$y = -\frac{12}{5}$ $\left(2, -\frac{12}{5}\right)$

For $y = -3$: $4x - 5y = 20$

$4x - 5(-3) = 20$

$4x + 15 = 20$

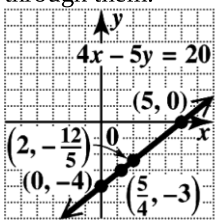
$4x = 5$

$x = \frac{5}{4}$ $\left(\frac{5}{4}, -3\right)$

x	y
0	−4
5	0
2	$-\frac{12}{5}$
$\frac{5}{4}$	−3

0	$-\frac{5}{3}$
−5	0
1	−2
−2	−1

(b) Plot the ordered pairs and draw the line through them.



30. (a) For $x = 0$: $6(0) - 5y = 30$

$$-5y = 30$$
$$y = -6 \quad (0, -6)$$

For $y = 0$: $6x - 5(0) = 30$

$$6x = 30$$
$$x = 5 \quad (5, 0)$$

For $x = 3$: $6(3) - 5y = 30$

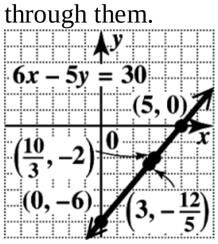
$$18 - 5y = 30$$
$$-5y = 12$$
$$y = -\frac{12}{5} \quad \left(3, -\frac{12}{5}\right)$$

For $y = -2$:

$$6x - 5(-2) = 30$$
$$6x + 10 = 30$$
$$6x = 20$$
$$x = \frac{20}{6} = \frac{10}{3} \quad \left(\frac{10}{3}, -2\right)$$

x	y
0	−6
5	0
3	$-\frac{12}{5}$
$\frac{10}{3}$	−2

(b) Plot the ordered pairs and draw the line through them.



31. (a) For $x = 0$: $y = -2(0) + 3$

$$y = 3 \quad (0, 3)$$

For $x = 1$: $y = -2(1) + 3$

$$y = 1 \quad (1, 1)$$

For $x = 2$: $y = -2(2) + 3$

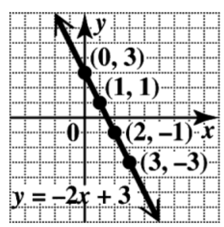
$$y = -1 \quad (2, -1)$$

For $x = 3$: $y = -2(3) + 3$

$$y = -3 \quad (3, -3)$$

x	y
0	3
1	1
2	−1
3	−3

(b) Notice that as the value of x increases by 1, the value of y decreases by 2.



32. (a) For $x = 0$: $y = -3(0) + 1$

$$y = 1 \quad (0, 1)$$

For $x = 1$: $y = -3(1) + 1$

$$y = -2 \quad (1, -2)$$

For $x = 2$: $y = -3(2) + 1$

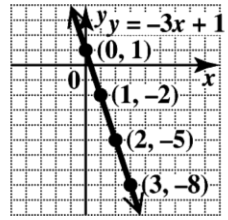
$$y = -5 \quad (2, -5)$$

For $x = 3$: $y = -3(3) + 1$

$$y = -8 \quad (3, -8)$$

0	1
1	−2
2	−5
3	−8

(b) Notice that as the value of x increases by 1, the value of y decreases by 3.



33. **(a)** The y-values corresponding to the x-values for Exercise 23 are $-4, -3, -2,$ and $-1.$ The difference between each is 1 unit. Therefore, for every increase in x by 1 unit, y increases by 1 unit.

(b) The y-values corresponding to the x-values for Exercise 31 are $3, 1, -1,$ and $-3.$ The difference between each is 2 units, and the values are decreasing. Therefore, for every increase in x by 1 unit, y decreases by 2 units.

(c) It appears that the y-value increases (or decreases) by the value of the coefficient of x. So for $y = 2x + 4,$ a conjecture is "for every increase in x by 1 unit, y increases by 2 units."

For $x = 0$: $y = 2(0) + 4$
$y = 4$ $(0, 4)$
For $x = 1$: $y = 2(1) + 4$
$y = 6$ $(1, 6)$
For $x = 2$: $y = 2(2) + 4$
$y = 8$ $(2, 8)$
For $x = 3$: $y = 2(3) + 4$
$y = 10$ $(3, 10)$

x	y
0	4
1	6
2	8
3	10

The difference between each y-value is 2 units, and the values are increasing. Therefore, the conjecture is true.

34. The choices C and D are horizontal lines. The equation $y + 3 = 0$ can be rewritten as $y = -3.$ Because y always equals $-3,$ there is no corresponding value to $y = 0$ and so the graph has no y-intercept. Since the line never crosses the y-axis, it must be vertical. The equation $x + 1 = 5$ can be rewritten as $x = 4.$ Because x always equals 4, there is no corresponding value to $x = 0$ and so the graph has no y-intercept. Since the line never crosses the y-axis, it must also be vertical.

The equation $x + y = 0$ is neither horizontal nor vertical. Neither the x-coordinate nor the y-coordinate is a fixed value, and the line crosses both the x-axis and y-axis.

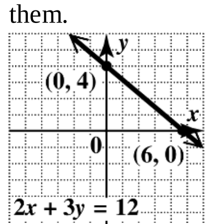
35. To find the x-intercept, let $y = 0.$

$$2x + 3y = 12$$
$$2x + 3(0) = 12$$
$$2x = 12$$
$$x = 6$$

The x-intercept is (6, 0).
To find the y-intercept, let $x = 0.$

$$2x + 3y = 12$$
$$2(0) + 3y = 12$$
$$3y = 12$$
$$y = 4$$

The y-intercept is (0, 4).
Plot the intercepts and draw the line through them.



36. $5x + 2y = 10$
To find the x-intercept, let $y = 0.$

$$5x + 2(0) = 10$$
$$5x = 10$$
$$x = 2$$

The x-intercept is (2, 0).

To find the y-intercept, let $x = 0.$

$$5(0) + 2y = 10$$

has no *x*-intercept. Since the line never crosses the *x*-axis, it must be horizontal. Because *y* always equals -10, there is no corresponding

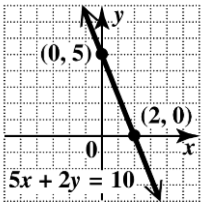
value to $y = 0$ and so the graph has no

x-intercept. Since the line never crosses the *x*-axis, it must also be horizontal.
The choices A and E are vertical lines. The equation $x - 6 = 0$ can be rewritten as $x = 6$. Because *x* always equals 6, there is no corresponding value to $x = 0$ and so the graph

$$2y = 10$$
$$y = 5$$

The *y*-intercept is (0, 5).

Plot the intercepts and draw the line through them.



37. To find the x-intercept, let $y = 0$.

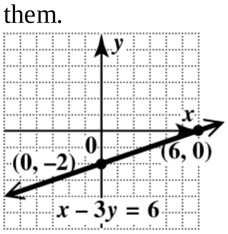
$$x - 3y = 6$$
$$x - 3(0) = 6$$
$$x - 0 = 6$$
$$x = 6$$

The x-intercept is (6, 0).
To find the y-intercept, let $x = 0$.

$$x - 3y = 6$$
$$0 - 3y = 6$$
$$-3y = 6$$
$$y = -2$$

The y-intercept is $(0, -2)$.

Plot the intercepts and draw the line through them.



38. To find the x-intercept, let $y = 0$.

$$x - 2(0) = -4$$
$$x = -4$$

The x-intercept is $(-4, 0)$.
To find the y-intercept, let $x = 0$.

$$0 - 2y = -4$$
$$-2y = -4$$
$$y = 2$$

The y-intercept is (0, 2).
Plot the intercepts and draw the line through them.

39. To find the x-intercept, let $y = 0$.

$$5x + 6(0) = -10$$
$$5x = -10$$
$$x = -2$$

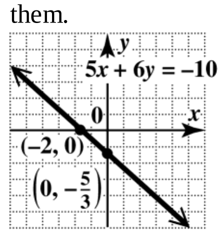
The x-intercept is $(-2, 0)$.

To find the y-intercept, let $x = 0$.

$$5(0) + 6y = -10$$
$$6y = -10$$
$$y = -\frac{10}{6} = -\frac{5}{3}$$

The y-intercept is $\left(0, -\frac{5}{3}\right)$.

Plot the intercepts and draw the line through them.



40. To find the x-intercept, let $y = 0$.

$$3x - 7y = 9$$
$$3x - 7(0) = 9$$
$$3x = 9$$
$$x = 3$$

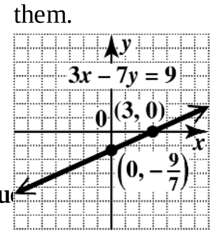
The x-intercept is (3, 0).

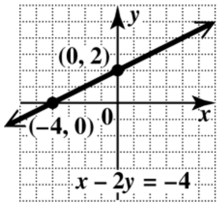
To find the y-intercept, let $x = 0$.

$$3x - 7y = 9$$
$$3(0) - 7y = 9$$
$$-7y = 9$$
$$y = -\frac{9}{7}$$

The y-intercept is $\left(0, -\frac{9}{7}\right)$.

Plot the intercepts and draw the line through them.




y
(0, 2)
(−4, 0)
0
x
x − 2y = −4

41. To find the x-intercept, let $y = 0$.

$$\frac{2}{3}x - 3(0) = 7$$
$$\frac{2}{3}x = 7$$
$$x = \frac{3}{2} \cdot 7 = \frac{21}{2}$$

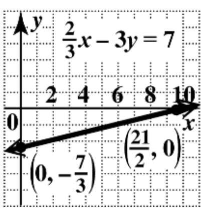
The x-intercept is $\left(\frac{21}{2}, 0\right)$.

To find the y-intercept, let $x = 0$.

$$\frac{2}{3}(0) - 3y = 7$$
$$-3y = 7$$
$$y = -\frac{7}{3}$$

The y-intercept is $\left(0, -\frac{7}{3}\right)$.

Plot the intercepts and draw the line through them.



42. To find the x-intercept, let $y = 0$.

$$\frac{5}{7}x + \frac{6}{7}(0) = -2$$
$$\frac{5}{7}x = -2$$
$$x = \frac{7}{5}(-2) = -\frac{14}{5}$$

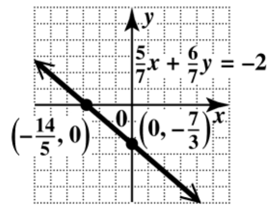
The x-intercept is $\left(-\frac{14}{5}, 0\right)$.

To find the y-intercept, let $x = 0$.

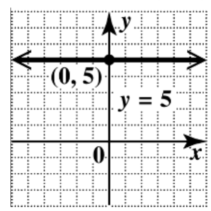
$$\frac{5}{7}(0) + \frac{6}{7}y = -2$$
$$\frac{6}{7}y = -2$$
$$y = \frac{7}{6}(-2) = -\frac{7}{3}$$

The y-intercept is $\left(0, -\frac{7}{3}\right)$.

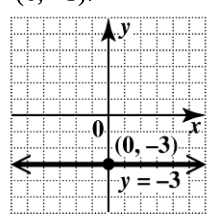
Plot the intercepts and draw the line through them.



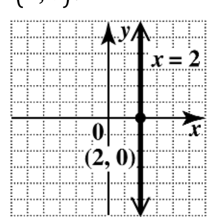
43. This is a horizontal line. Every point has y-coordinate 5, so no point has y-coordinate 0. There is no x-intercept.
Since every point of the line has y-coordinate 5, the y-intercept is $(0, 5)$. Draw the horizontal line through $(0, 5)$.



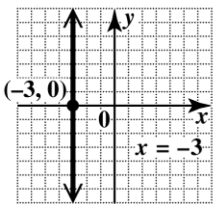
44. This is a horizontal line. Every point has y-coordinate -3, so no point has y-coordinate 0. There is no x-intercept. Since every point of the line has y-coordinate -3, the y-intercept is $(0, -3)$. Draw the horizontal line through $(0, -3)$.



45. This is a vertical line. Every point has x-coordinate 2, so the x-intercept is $(2, 0)$.
Since every point of the line has x-coordinate 2, no point has x-coordinate 0. There is no y-intercept. Draw the vertical line through $(2, 0)$.

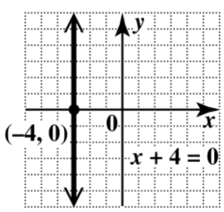


46. This is a vertical line. Every point has x-coordinate -3, so the x-intercept is $(-3, 0)$. Since every point of the line has x-coordinate -3, no point has x-coordinate 0. There is no y-intercept. Draw the vertical line through $(-3, 0)$.

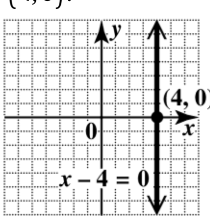


47. This is a vertical line. Every point has x-coordinate -4, so the x-intercept is $(-4, 0)$. Since every point of the line has x-coordinate -4, no point has x-coordinate 0. There is no y-intercept. Draw the vertical line through

$(-4, 0)$.

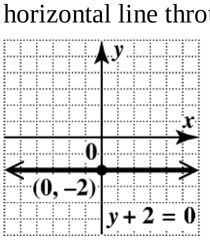


48. This is a vertical line. Every point has x-coordinate 4, so the x-intercept is $(4, 0)$. Since every point of the line has x-coordinate 4, no point has x-coordinate 0. There is no y-intercept. Draw the vertical line through $(4, 0)$.

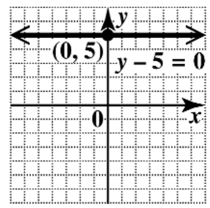


49. This is a horizontal line. Every point has y-coordinate -2, so no point has y-coordinate

0. There is no x-intercept.
Since every point of the line has y-coordinate -2, the y-intercept is $(0, -2)$. Draw the horizontal line through $(0, -2)$.



50. This is a horizontal line. Every point has y-coordinate 5, so no point has y-coordinate 0. There is no x-intercept.
Since every point of the line has y-coordinate 5, the y-intercept is $(0, 5)$. Draw the horizontal line through $(0, 5)$.



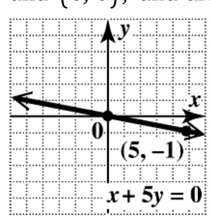
51. To find the x-intercept, let $y = 0$.

$$x + 5y = 0$$
$$x + 5(0) = 0$$
$$x = 0$$

The x-intercept is (0, 0), and since $x = 0$, this is also the y-intercept. Since the intercepts are the same, another point is needed to graph the line. Choose any number for y, say $y = -1$, and solve the equation for x.

$$x + 5y = 0$$
$$x + 5(-1) = 0$$
$$x = 5$$

This gives the ordered pair $(5, -1)$. Plot $(5, -1)$ and $(0, 0)$, and draw the line through them.



52. To find the x-intercept, let $y = 0$.

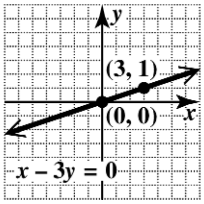
$$x - 3(0) = 0$$
$$x = 0$$

The x-intercept is (0, 0), and since $x = 0$, this is also the y-intercept. Since the intercepts are the same, another point is needed to graph the line.

Choose any number for y, say $y = 1$, and solve the equation for x.

$$x - 3(1) = 0$$
$$x = 3$$

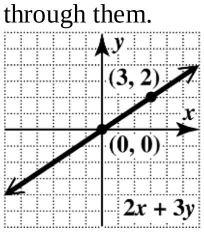
This gives the ordered pair (3, 1). Plot (3, 1) and (0, 0), and draw the line through them.



53. If $x = 0$, then $y = 0$, so the x- and y-intercepts are $(0, 0)$. To get another point, let $x = 3$.

$$2(3) = 3y$$
$$2 = y$$

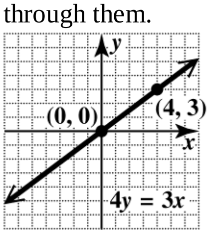
Plot $(3, 2)$ and $(0, 0)$, and draw the line through them.



54. If $x = 0$, then $y = 0$, so the x- and y-intercepts are $(0, 0)$. To get another point, let $x = 4$.

$$4y = 3(4)$$
$$y = 3$$

Plot $(4, 3)$ and $(0, 0)$, and draw the line through them.



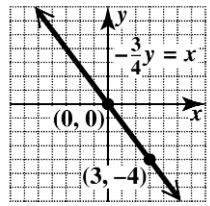
55. If $x = 0$, then $y = 0$, so the x- and y-intercepts are $(0, 0)$. To get another point, let $y = -3$.

$$-\frac{2}{3}(-3) = x$$
$$2 = x$$

56. If $x = 0$, then $y = 0$, so the x- and y-intercepts are $(0, 0)$. To get another point, let $y = -4$.

$$-\frac{3}{4}(-4) = x$$
$$3 = x$$

Plot $(3, -4)$ and $(0, 0)$, and draw the line through them.



57. **(a)** From the table, when $y = 0, x = -2$, so the x-intercept is $(-2, 0)$. When $x = 0, y = 3$, so the y-intercept is $(0, 3)$.

(b) Find the intercepts in each equation and compare them to the table to see which of the choices is correct.
Find the intercepts in equation A.

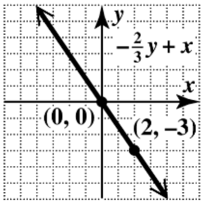
$$3x + 2y = 6$$
$$3x + 2(0) = 6$$
$$3x = 6$$
$$x = 2$$

$$3x + 2y = 6$$
$$3(0) + 2y = 6$$
$$2y = 6$$
$$y = 3$$

The intercepts are $(2, 0)$ and $(0, 3)$. This is not the correct choice.
Find the intercepts in equation B.

$$3x - 2y = -6$$
$$3x - 2(0) = -6$$
$$3x = -6$$
$$x = -2$$
$$3x - 2y = -6$$
$$3(0) - 2y = -6$$

Plot $(2, -3)$ and $(0, 0)$, and draw the line through them.

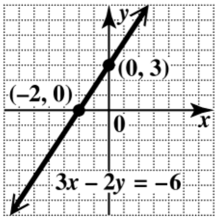


$$-2y = -6$$
$$y = 3$$

The intercepts are $(-2, 0)$ and $(0, 3)$. This

is the correct choice. So equation B corresponds to the given table.
(Note: Equations C and D would be tested similarly if the correct choice had not yet been found.)

(c) Plot the x-intercept and y-intercept. Draw the line through them.



58. (a) From the table, when $y = 0, x = 2,$ so the x-intercept is $(2, 0)$. When $x = 0, y = 4,$ so the y-intercept is $(0, 4)$.

(b) Find the intercepts in each equation and compare them to the table to see which of the choices is correct.
Find the intercepts in equation A.

$$2x - y = 4$$
$$2x - (0) = 4$$
$$2x = 4$$
$$x = 2$$
$$2x - y = 4$$
$$2(0) - y = 4$$
$$-y = 4$$
$$y = -4$$

The intercepts are $(2, 0)$ and $(0, -4)$. This is not the correct choice.
Find the intercepts in equation B.

$$2x + y = -4$$
$$2x + (0) = -4$$
$$2x = -4$$
$$x = -2$$
$$2x + y = -4$$
$$2(0) + y = -4$$
$$y = -4$$

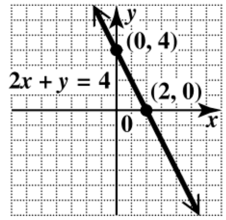
The intercepts are $(-2, 0)$ and $(0, -4)$.

This is not the correct choice.
Find the intercepts in equation C.

$$2x + y = 4$$
$$2x + (0) = 4$$
$$2x = 4$$
$$x = 2$$
$$2x + y = 4$$

The intercepts are $(2, 0)$ and $(0, 4)$. This is not the correct choice.
So equation C corresponds to the given table.
(Note: Equation D would be tested similarly if the correct choice had not yet been found.)

(c) Plot the x-intercept and y-intercept. Draw the line through them.

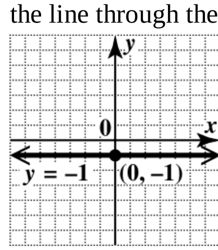


59. (a) From the table, when $x = 0, y = -1,$ so the y-intercept is $(0, -1)$. Note that the y-coordinate of all the points is $-1,$ so the equation is a horizontal line, with no x-intercept.

(b) The equation is a horizontal line through $(0, -1)$. Since the y-coordinate is always $-1,$ the equation is $y = -1.$

So equation A corresponds to the given table.

(c) Plot the x-intercept and y-intercept. Draw the line through them.



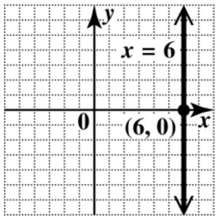
60. (a) From the table, when $y = 0, x = 6,$ so the x-intercept is $(6, 0)$. Note that the

$$2(0) + y = 4$$
$$y = 4$$

x-coordinate of all the points is 6, so the equation is a vertical line, with no *y*-intercept.

(b) The equation is a vertical line through $(6, 0)$. Since the *x*-coordinate is always 6, the equation is $x = 6$. So equation D corresponds to the given table.

(c) Draw the line through the *x*-intercept.



61. Find the intercepts first since they are plotted on the graph. To find the *x*-intercept, let $y = 0$.

$x + 3y = 3$
$x + 3(0) = 3$
$x + 0 = 3$
$x = 3$

The *x*-intercept is $(3, 0)$.
To find the *y*-intercept, let $x = 0$.

$x + 3y = 3$
$0 + 3y = 3$
$3y = 3$
$y = 1$

The *y*-intercept is $(0, 1)$.
Graph C has these intercepts.

62. Find the intercepts first since they are plotted on the graph. To find the *x*-intercept, let $y = 0$.

$x - 3y = -3$
$x - 3(0) = -3$
$x - 0 = -3$
$x = -3$

The *x*-intercept is $(-3, 0)$.
To find the *y*-intercept, let $x = 0$.

$x - 3y = -3$
$0 - 3y = -3$
$-3y = -3$
$y = 1$

The *y*-intercept is $(0, 1)$.
Graph D has these intercepts

63. Find the intercepts first since they are plotted on the graph. To find the *x*-intercept, let $y = 0$.

$x - 3y = 3$

To find the *y*-intercept, let $x = 0$.

$x - 3y = 3$
$0 - 3y = 3$
$-3y = 3$
$y = -1$

The *y*-intercept is $(0, -1)$.
Graph B has these intercepts.

64. Find the intercepts first since they are plotted on the graph. To find the *x*-intercept, let $y = 0$.

$x + 3y = -3$
$x + 3(0) = -3$
$x + 0 = -3$
$x = -3$

The *x*-intercept is $(-3, 0)$.
To find the *y*-intercept, let $x = 0$.

$x + 3y = -3$
$0 + 3y = -3$
$3y = -3$
$y = -1$

The *y*-intercept is $(0, -1)$.
Graph A has these intercepts.

65. By the midpoint formula, the midpoint of the segment with endpoints $(-8, 4)$ and $(-2, -6)$ is

$$\left(\frac{-8+(-2)}{2}, \frac{4+(-6)}{2}\right) = \left(\frac{-10}{2}, \frac{-2}{2}\right) = (-5, -1).$$

66. By the midpoint formula, the midpoint of the segment with endpoints (5, 2) and $(-1, 8)$ is

$$\left(\frac{5+(-1)}{2}, \frac{2+8}{2}\right) = \left(\frac{4}{2}, \frac{10}{2}\right) = (2, 5).$$

67. By the midpoint formula, the midpoint of the segment with endpoints $(3, -6)$ and (6, 3) is

$$\left(\frac{3+6}{2}, \frac{-6+3}{2}\right) = \left(\frac{9}{2}, \frac{-3}{2}\right) = \left(\frac{9}{2}, -\frac{3}{2}\right).$$

68. By the midpoint formula, the midpoint of the segment with endpoints $(-10, 4)$ and (7, 1) is

$$\left(\frac{-10+7}{2}, \frac{4+1}{2}\right) = \left(\frac{-3}{2}, \frac{5}{2}\right) = \left(-\frac{3}{2}, \frac{5}{2}\right).$$

$x - 3(0) = 3$

$x - 0 = 3$

$x = 3$

The x-intercept is $(3, 0)$.

2 2 2 2 2 2

69. By the midpoint formula, the midpoint of the segment with endpoints $(-9, 3)$ and (9, 8) is $\left(\frac{-9+9}{2}, \frac{3+8}{2}\right) = \left(\frac{0}{2}, \frac{11}{2}\right) = \left(0, \frac{11}{2}\right)$.

70. By the midpoint formula, the midpoint of the segment with endpoints $(4,-3)$ and $(-1,3)$ is

$$\left(\frac{4+(-1)}{2}, \frac{-3+3}{2}\right) = \left(\frac{3}{2}, \frac{0}{2}\right) = \left(\frac{3}{2}, 0\right).$$

71. By the midpoint formula, the midpoint of the segment with endpoints (2.5, 3.1) and $(1.7, -1.3)$ is

$$\left(\frac{2.5+1.7}{2}, \frac{3.1+(-1.3)}{2}\right) = \left(\frac{4.2}{2}, \frac{1.8}{2}\right) = (2.1, 0.9).$$

72. By the midpoint formula, the midpoint of the segment with endpoints $(6.2, 5.8)$ and $(1.4, -0.6)$ is

$$\left(\frac{6.2+1.4}{2}, \frac{5.8+(-0.6)}{2}\right) = \left(\frac{7.6}{2}, \frac{5.2}{2}\right) = (3.8, 2.6).$$

73. By the midpoint formula, the midpoint of the segment with endpoints $\left(\frac{1}{2}, \frac{1}{3}\right)$ and $\left(\frac{3}{2}, \frac{5}{3}\right)$ is

$$\left(\frac{\frac{1}{2}+\frac{3}{2}}{2}, \frac{\frac{1}{3}+\frac{5}{3}}{2}\right) = \left(\frac{\frac{4}{2}}{2}, \frac{\frac{6}{3}}{2}\right) = \left(\frac{2}{2}, \frac{2}{2}\right) = (1,1).$$

74. By the midpoint formula, the midpoint of the segment with endpoints $\left(\frac{21}{4}, \frac{2}{5}\right)$ and $\left(\frac{7}{4}, \frac{3}{5}\right)$ is

$$\left(\frac{\frac{21}{4}+\frac{7}{4}}{}, \frac{\frac{2}{5}+\frac{3}{5}}{}\right) \quad \left(\frac{28}{}\quad\frac{5}{}\right)$$

$\left(\frac{1}{2}, -\frac{7}{2}\right)$ is

$$\left(\frac{\frac{3}{5}+\frac{1}{2}}{2}, \frac{-\frac{1}{3}+\left(-\frac{7}{2}\right)}{2}\right) = \left(\frac{\frac{11}{10}}{2}, \frac{-\frac{23}{6}}{2}\right) = \left(\frac{11}{20}, -\frac{23}{12}\right).$$

77. midpoint of $P(5, 8)$ and $Q(x, y) = M(8, 2)$

$$\left(\frac{5+x}{2}, \frac{8+y}{2}\right) = (8, 2)$$

The x- and y-coordinates must be equal.

$$\frac{5+x}{2} = 8 \qquad \frac{8+y}{2} = 2$$
$$5+x = 16 \qquad 8+y = 4$$
$$x = 11 \qquad y = -4$$

Thus, the endpoint Q is $(11, -4)$.

78. midpoint of $P(7, 10)$ and $Q(x, y) = M(5, 3)$

$$\left(\frac{7+x}{2}, \frac{10+y}{2}\right) = (5, 3)$$

The x- and y-coordinates must be equal.

$$\frac{7+x}{2} = 5 \qquad \frac{10+y}{2} = 3$$
$$7+x = 10 \qquad 10+y = 6$$
$$x = 3 \qquad y = -4$$

Thus, the endpoint Q is $(3, -4)$.

79. midpoint of $P(1.5, 1.25)$ and $Q(x, y) = M(3, 1)$

$$\left(\frac{1.5+x}{2}, \frac{1.25+y}{2}\right) = (3, 1)$$

$$\left(\frac{4\quad 4}{2}, \frac{5\quad 5}{2}\right) = \left(\frac{4}{2}, \frac{5}{2}\right) = \left(\frac{7}{2}, \frac{1}{2}\right).$$

75. By the midpoint formula, the midpoint of the segment with endpoints $\left(-\frac{1}{3}, \frac{2}{7}\right)$ and $\left(-\frac{1}{2}, \frac{1}{14}\right)$ is

$$\left(\frac{-\frac{1}{3}+\left(-\frac{1}{2}\right)}{2}, \frac{\frac{2}{7}+\frac{1}{14}}{2}\right) = \left(\frac{-\frac{5}{6}}{2}, \frac{\frac{5}{14}}{2}\right) = \left(-\frac{5}{12}, \frac{5}{28}\right).$$

76. By the midpoint formula, the midpoint of the segment with endpoints $\left(\frac{3}{5}, -\frac{1}{3}\right)$ and

The x- and y-coordinates must be equal.

$$\frac{1.5+x}{2} = 3 \qquad \frac{1.25+y}{2} = 1$$
$$1.5 + x = 6 \qquad 1.25 + y = 2$$
$$x = 4.5 \qquad y = 0.75$$

Thus, the endpoint Q is (4.5, 0.75).

80. midpoint of $P(2.5, 1.75)$ and $Q(x, y) = M(3, 2)$

$$\left(\frac{2.5+x}{2}, \frac{1.75+y}{2}\right) = (3, 2)$$

The x- and y-coordinates must be equal.

$$\frac{2.5+x}{2} = 3 \qquad \frac{1.75+y}{2} = 2$$
$$2.5 + x = 6 \qquad 1.75 + y = 4$$
$$x = 3.5 \qquad y = 2.25$$

Thus, the endpoint Q is (3.5, 2.25).

2.2 The Slope of a Line

Classroom Examples, Now Try Exercises

1. If $(x_1, y_1) = (-6, 9)$ and $(x_2, y_2) = (3, -5)$, then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 9}{3 - (-6)} = \frac{-14}{9} = -\frac{14}{9}$. The slope is $-\frac{14}{9}$.

N1. If $(x_1, y_1) = (2, -6)$ and $(x_2, y_2) = (-3, 5)$, then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-6)}{-3 - 2} = \frac{11}{-5} = -\frac{11}{5}$. The slope is $-\frac{11}{5}$.

2. To find the slope of the line with equation $3x - 4y = 12$, first find the intercepts. The x-intercept is (4, 0), and the y-intercept is $(0, -3)$. The slope is then

$$m = \frac{-3 - 0}{0 - 4} = \frac{-3}{-4} = \frac{3}{4}.$$

N2. To find the slope of the line with equation $3x - 7y = 21$, first find the intercepts. The x-intercept is (7, 0), and the y-intercept is $(0, -3)$. The slope is then

$$m = \frac{-3 - 0}{0 - 7} = \frac{-3}{-7} = \frac{3}{7}.$$

3. **(a)** To find the slope of the line with equation $y + 3 = 0$, select two different points on the line, such as $(0, -3)$ and $(2, -3)$, and use the slope formula.

$$m = \frac{-3 - (-3)}{2 - 0} = \frac{0}{2} = 0$$

N3. **(a)** To find the slope of the line with equation $x = 4$, select two different points on the line, such as (4, 0) and (4, 3), and use the slope formula.

$$m = \frac{3 - 0}{4 - 4} = \frac{3}{0}$$

Since division by zero is undefined, the slope is undefined.

(b) To find the slope of the line with equation $y - 6 = 0$, select two different points on the line, such as (0, 6) and (2, 6), and use the slope formula.

$$m = \frac{6 - 6}{2 - 0} = \frac{0}{2} = 0$$

The slope is 0.

4. Solve the equation for y.

$3x + 4y = 9$

$4y = -3x + 9$ Subtract $3x$.

$y = -\frac{3}{4}x + \frac{9}{4}$ Divide by 4.

The slope is given by the coefficient of x, so the slope is $-\frac{3}{4}$.

N4. Solve the equation for y.

$5x - 4y = 7$

$-4y = -5x + 7$ Subtract $5x$.

$y = \frac{5}{4}x - \frac{7}{4}$ Divide by -4.

The slope is given by the coefficient of x, so the slope is $\frac{5}{4}$.

5. Through $(-3, -2)$; $m = \frac{1}{2}$

The slope is 0.

(b) To find the slope of the line with equation $x = -6$, select two different points on the line, such as $(-6, 0)$ and $(-6, 3)$, and use the slope formula.

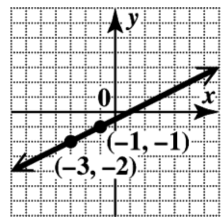
$$m = \frac{3-0}{-6-(-6)} = \frac{3}{0}$$

Since division by zero is undefined, the slope is undefined.

Locate the point $(-3, -2)$ on the graph. Use the slope formula to find a second point on the line.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{1}{2}$$

From $(-3, -2)$, move up 1 unit and then 2 units to the right to $(-1, -1)$. Draw the line through the two points.

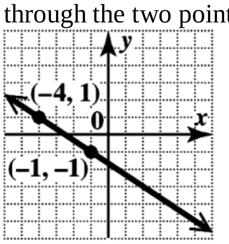


N5. Through $(-4, 1)$; $m = -\frac{2}{3}$

Locate the point $(-4, 1)$ on the graph. Use the slope formula to find a second point on the line.

$m = \frac{\text{change in } y}{\text{change in } x} = \frac{-2}{3}$

From $(-4, 1)$, move down 2 units and then 3 units to the right to $(-1, -1)$. Draw the line through the two points.



6. Find the slope of each line.
The line through $(-1, 2)$ and (3, 5) has slope

$m_1 = \frac{5-2}{3-(-1)} = \frac{3}{4}.$

The line through (4, 7) and (8, 10) has slope

$m_2 = \frac{10-7}{8-4} = \frac{3}{4}.$

The slopes are the same, so the lines are parallel.

N6. Find the slope of each line.
The line through (2, 5) and (4, 8) has slope

$m_1 = \frac{8-5}{4-2} = \frac{3}{2}.$

The line through (2, 0) and $(-1, -2)$ has slope

$m_2 = \frac{-2-0}{-1-2} = \frac{-2}{-3} = \frac{2}{3}.$

The slopes are not the same, so the lines are not parallel.

7. Solve each equation for *y*.

$3x + 5y = 6$ $\quad$ $5x - 3y = 2$

N7. Solve each equation for *y*.

$x + 2y = 7$ $\quad$ $2x = y - 4$

$2y = -x + 7$ $\quad$ $-y = -2x - 4$

$y = -\frac{1}{2}x + \frac{7}{2}$ $\quad$ $y = 2x + 4$

The slope is $m = -\frac{1}{2}$. The slope is $m = 2$.

Since $m_1 m_2 = \left(-\frac{1}{2}\right)(2) = -1$, the lines are perpendicular.

8. Solve each equation for *y*.

$4x - y = 2$ $\quad$ $x - 4y = -8$

$-y = -4x + 2$ $\quad$ $-4y = -x - 8$

$y = 4x - 2$ $\quad$ $y = \frac{1}{4}x + 2$

The slope is $m = 4$. The slope is $m = \frac{1}{4}$.

Since $m_1 \neq m_2$, the lines are not parallel. Since $m_1 m_2 = 4\left(\frac{1}{4}\right) = 1$, the lines are not perpendicular either. Therefore, the answer is neither.

N8. Solve each equation for *y*.

$2x - y = 4$ $\quad$ $2x + y = 6$

$-y = -2x + 4$ $\quad$ $y = -2x + 6$

$y = 2x - 4$

The slope is $m = 2$. The slope is $m = -2$.

Since $m_1 \neq m_2$, the lines are not parallel. Since $m_1 m_2 = 2(-2) = -4$, the lines are not perpendicular either. Therefore, the answer is neither.

9. $(x_1, y_1) = (2010, 45)$ and $(x_2, y_2) = (2012, 47)$.

average rate of change $= \frac{y_2 - y_1}{}$

$$5y = -3x + 6 \qquad -3y = -5x + 2$$

$$y = -\frac{3}{5}x + \frac{6}{5} \qquad y = \frac{5}{3}x - \frac{2}{3}$$

The slope is $m = -\frac{3}{5}$. The slope is $m = \frac{5}{3}$.

Since $m_1 m_2 = \left(-\frac{3}{5}\right)\left(\frac{5}{3}\right) = -1$, the lines are perpendicular.

$$x_2 - x_1$$

$$= \frac{47 - 45}{2012 - 2010}$$

$$= \frac{2}{2} = 1$$

The average rate of change is about 1 million customers per year. This is less than the average rate of change from 2007 to 2012, which is 2 million customers per year.

N9. $(x_1, y_1) = (2008, 40)$ and $(x_2, y_2) = (2012, 47)$.

$$\text{average rate of change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{47 - 40}{2012 - 2008} = \frac{7}{4} = 1.75$$

The average rate of change is about 1.75 million customers per year. This is less than the average rate of change from 2007 to 2012, which is 2 million customers per year.

10. $(x_1, y_1) = (2000, 943)$ and $(x_2, y_2) = (2011, 241)$.

$$\text{average rate of change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{241 - 943}{2011 - 2000} = \frac{-702}{11} \approx -63.8$$

Thus, the average rate of change from 2000 to 2011 was about -64 million CDs per year.

N10. $(x_1, y_1) = (2010, 1150)$ and $(x_2, y_2) = (2013, 137)$.

$$\text{average rate of change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{137 - 1150}{2013 - 2010} = \frac{-1013}{3} \approx -337.7$$

Thus, the average rate of change in sales of digital camcorders in the United States from 2010 to 2013 was about $-\$338$ million per year.

Exercises

1. $\text{slope} = \frac{\text{change in vertical position}}{\text{change in horizontal position}}$

$= \frac{30\text{ feet}}{100\text{ feet}}$

Choices A, 0.3; B, $\frac{3}{x}$; D, $\frac{30}{}$; and F, 30%,

2. $\text{slope} = \frac{\text{change in vertical position}}{\text{change in horizontal position}} = \frac{2\text{ feet}}{24\text{ feet}}$

Choices B, $\frac{2}{24}$; C, $\frac{1}{12}$; and E, 8.3%, are all correct.

3. $\text{slope} = \frac{\text{change in vertical position}}{\text{change in horizontal position}}$

$3 = \frac{15\text{ feet}}{\text{change in horizontal position}}$

$3 \times \text{change} = 15$

$\text{change} = \frac{15}{3} = 5$

So the change in horizontal position is 5 feet.

4. $\text{slope} = \frac{\text{change in vertical position}}{\text{change in horizontal position}}$

$0.05 = \frac{\text{change in vertical position}}{50\text{ feet}}$

So the change in vertical position is $0.05(50\text{ feet}) = 2.5$ feet.

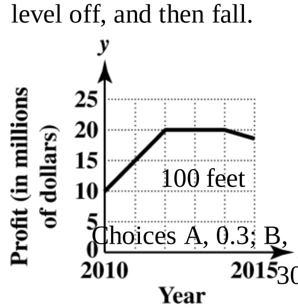
5. **(a)** Graph C indicates that sales leveled off during the second quarter.

(b) Graph A indicates that sales leveled off during the fourth quarter.

(c) Graph D indicates that sales rose sharply during the first quarter and then fell to the original level during the second quarter.

(d) Graph B is the only graph that indicates that sales fell during the first two quarters.

6. Answers will vary, but the graphs will all rise, level off, and then fall.



10 100
are all correct.

7. To get to B from A, we must go up 2 units and move right 1 unit. Thus,

slope of $AB = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2.$

8. slope of $BC = \frac{\text{rise}}{\text{run}} = \frac{0}{-4} = 0$

9. slope of $CD = \frac{\text{rise}}{\text{run}} = \frac{-7}{0}$, which is undefined.

10. slope of $DE = \frac{\text{rise}}{\text{run}} = \frac{-1}{3} = -\frac{1}{3}$

11. slope of $EF = \frac{\text{rise}}{\text{run}} = \frac{3}{3} = 1$

12. slope of $FG = \frac{\text{rise}}{\text{run}} = \frac{-4}{1} = -4$

13. slope of $AF = \frac{\text{rise}}{\text{run}} = \frac{-3}{3} = -1$

14. slope of $BD = \frac{\text{rise}}{\text{run}} = \frac{-7}{-4} = \frac{7}{4}$

15. **(a)** "The line has positive slope" means that the line goes up from left to right. This is line B.

(b) "The line has negative slope" means that the line goes down from left to right. This is line C.

(c) "The line has slope 0" means that there is no vertical change—that is, the line is horizontal. This is line A.

(d) "The line has undefined slope" means that there is no horizontal change—that is, the line is vertical. This is line D.

16. B and D are correct. Choice A is wrong

17.

22. $m = \frac{-2-(-2)}{4-(-3)} = \frac{-2+2}{4+3} = \frac{0}{7} = 0$

23. $m = \frac{3-8}{-2-(-2)} = \frac{-5}{-2+2} = \frac{-5}{0}$, which is undefined.

24. $m = \frac{5-6}{-8-(-8)} = \frac{-1}{-8+8} = \frac{-1}{0}$, which is undefined.

25. **(a)** Let $(x_1, y_1) = (-2, -3)$ and $(x_2, y_2) = (-1, 5)$. Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-3)}{-1-(-2)} = \frac{8}{1} = 8.$
The slope is 8.

(b) The slope is positive, so the line rises.

26. **(a)** Let $(x_1, y_1) = (-4, 1)$ and $(x_2, y_2) = (-3, 4)$. Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{-3-(-4)} = \frac{3}{1} = 3$
The slope is 3.

(b) The slope is positive, so the line rises.

27. **(a)** Let $(x_1, y_1) = (-4, 1)$ and $(x_2, y_2) = (2, 6)$. Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-1}{2-(-4)} = \frac{5}{6}.$
The slope is $\frac{5}{6}$.

because the order of subtraction must be the same in the numerator and denominator. Choice C is wrong because slope is defined as the change in *y* divided by the change in *x*.

$m = \frac{6-2}{5-3} = \frac{4}{2} = 2$

18. $m = \frac{5-7}{-4-2} = \frac{-2}{-6} = \frac{1}{3}$

19. $m = \frac{4-(-1)}{-3-(-5)} = \frac{4+1}{-3+5} = \frac{5}{2}$

20. $m = \frac{-6-0}{0-(-3)} = \frac{-6}{3} = -2$

21. $m = \frac{-5-(-5)}{3-2} = \frac{-5+5}{1} = \frac{0}{1} = 0$

(b) The slope is positive, so the line rises.

28. **(a)** Let $(x_1, y_1) = (-3, -3)$ and $(x_2, y_2) = (5, 6)$. Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-3)}{5-(-3)} = \frac{9}{8}$.

The slope is $\frac{9}{8}$.

(b) The slope is positive, so the line rises.

29. **(a)** Let $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (-4, 4)$.

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-4}{-4-2} = \frac{0}{-6} = 0.$

The slope is 0.

(b) The slope is zero, so the line is horizontal.

30. **(a)** Let $(x_1, y_1) = (-6, 3)$ and $(x_2, y_2) = (2, 3)$.

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-3}{2-(-6)} = \frac{0}{8} = 0.$

The slope is 0.

(b) The slope is zero, so the line is horizontal.

31. **(a)** Let $(x_1, y_1) = (-2, 2)$ and $(x_2, y_2) = (4, -1)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-2}{4-(-2)} = \frac{-3}{6} = -\frac{1}{2}.$$

The slope is $-\frac{1}{2}$.

(b) The slope is negative, so the line falls.

32. **(a)** Let $(x_1, y_1) = (-3, 1)$ and $(x_2, y_2) = (6, -2)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-1}{6-(-3)} = \frac{-3}{9} = -\frac{1}{3}.$$

The slope is $-\frac{1}{3}$.

(b) The slope is negative, so the line falls.

33. **(a)** Let $(x_1, y_1) = (5, -3)$ and $(x_2, y_2) = (5, 2)$.

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-3)}{5-5} = \frac{5}{0}.$

The slope is undefined.

(b) The slope is undefined, so the line is vertical.

34. **(a)** Let $(x_1, y_1) = (4, -1)$ and $(x_2, y_2) = (4, 3)$.

$\frac{y_2 - y_1}{}\quad \frac{3-(-1)}{}\quad \frac{4}{}$

36. **(a)** Let $(x_1, y_1) = (3.4, 4.2)$ and $(x_2, y_2) = (1.4, 10.2)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10.2-4.2}{1.4-3.4} = \frac{6}{-2} = -3.$$

The slope is -3.

(b) The slope is negative, so the line falls.

37. Let $(x_1, y_1) = \left(\frac{1}{6}, \frac{1}{2}\right)$ and $(x_2, y_2) = \left(\frac{5}{6}, \frac{9}{2}\right)$.

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{9}{2} - \frac{1}{2}}{\frac{5}{6} - \frac{1}{6}} = \frac{\frac{8}{2}}{\frac{4}{6}} = \frac{4}{\frac{2}{3}} = 6.$

The slope is 6.

38. Let $(x_1, y_1) = \left(\frac{3}{4}, \frac{1}{3}\right)$ and $(x_2, y_2) = \left(\frac{5}{4}, \frac{10}{3}\right)$.

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{10}{3} - \frac{1}{3}}{\frac{5}{4} - \frac{3}{4}} = \frac{\frac{9}{3}}{\frac{2}{4}} = 3 \cdot 2 = 6.$

The slope is 6.

39. Let $(x_1, y_1) = \left(-\frac{2}{9}, \frac{5}{18}\right)$ and $(x_2, y_2) = \left(\frac{1}{18}, -\frac{5}{9}\right)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-\frac{5}{9} - \frac{5}{18}}{\frac{1}{18} - \left(-\frac{2}{9}\right)} = \frac{-\frac{15}{18}}{\frac{5}{18}}$$

$$= -\frac{15}{18} \cdot \frac{18}{5} = -3.$$

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\quad}{4 - 4} = \frac{\quad}{0}$.

The slope is undefined.

(b) The slope is undefined, so the line is vertical.

35. **(a)** Let $(x_1, y_1) = (1.5, 2.6)$ and $(x_2, y_2) = (0.5, 3.6)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3.6 - 2.6}{0.5 - 1.5} = \frac{1}{-1} = -1.$$

The slope is -1.

(b) The slope is negative, so the line falls.

The slope is -3.

40. Let $(x_1, y_1) = \left(-\frac{4}{5}, \frac{9}{10}\right)$ and $(x_2, y_2) = \left(-\frac{3}{10}, \frac{1}{5}\right)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{1}{5} - \frac{9}{10}}{-\frac{3}{10} - \left(-\frac{4}{5}\right)} = \frac{-\frac{7}{10}}{\frac{5}{10}}$$

$$= -\frac{7}{10} \cdot \frac{10}{5} = -\frac{7}{5}.$$

The slope is $-\frac{7}{5}$.

41. Since the points lie on a line, the slope between any two points will be the same. To find the slope, any two points can be used, but using the x- and y-intercepts will make the calculations simple. Let $(x_1, y_1) = (0, 6)$ and $(x_2, y_2) = (3, 0)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-6}{3-0} = \frac{-6}{3} = -2.$$

The slope is -2.

42. Since the points lie on a line, the slope between any two points will be the same. To find the slope, any two points can be used, but using the x- and y-intercepts will make the calculations simple. Let $(x_1, y_1) = (-1, 0)$ and $(x_2, y_2) = (0, -3)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-0}{0-(-1)} = \frac{-3}{1} = -3.$$

The slope is -3.

43. Since the points lie on a line, the slope between any two points will be the same. To find the slope, any two points can be used, but using the x- and y-intercepts will make the calculations simple. Let $(x_1, y_1) = (-3, 0)$ and $(x_2, y_2) = (0, 4)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{0-(-3)} = \frac{4}{3}.$$

The slope is $\frac{4}{3}$.

44. Since the points lie on a line, the slope between any two points will be the same. To find the slope, any two points can be used, but using the x- and y-intercepts will make the calculations simple. Let $(x_1, y_1) = (0, -2)$ $(x_2, y_2) = (5, 0)$. Then y

46. The points shown on the line are $(1, -1)$ and $(3, 3)$. The slope is $m = \frac{3-(-1)}{3-1} = \frac{4}{2} = 2.$

47. The points shown on the line are $(3, 3)$ and $(3, -3)$. The slope is $m = \frac{-3-3}{3-3} = \frac{-6}{0}$, which is undefined.

48. The points shown on the line are $(2, 2)$ and $(-2, 2)$. The slope is $m = \frac{2-2}{-2-2} = \frac{0}{-4} = 0.$

49. **(a)** Answers will vary. The intercepts are $(4, 0)$ and $(0, -8)$.

Let $(x_1, y_1) = (4, 0)$ and $(x_2, y_2) = (0, -8)$.

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8-0}{0-4} = \frac{-8}{-4} = 2.$

The slope is 2.

(b) $2x - y = 8$

$-y = -2x + 8$

$y = 2x - 8$

From this equation, the slope is also 2.

(c) $2x - 1y = 8$

$A = 2$ and $B = -1$, so $-\frac{A}{B} = -\frac{2}{-1} = 2.$

50. **(a)** Answers will vary. The intercepts are $(-2, 0)$ and $(0, 6)$. Let $(x_1, y_1) = (-2, 0)$ and $(x_2, y_2) = (0, 6)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{0-(-2)} = \frac{6}{2} = 3.$$

$$m = \frac{y_2 \quad 0-(-2)}{\quad} = \frac{2}{\quad} = \quad .$$

and

$x_2 - x_1 \quad 5 - 0 \quad 5$

The slope is $\frac{2}{5}$.

45. The points shown on the line are $(-3, 3)$ and $(-1, -2)$. The slope is

$$m = \frac{-2 - 3}{-1 - (-3)} = \frac{-5}{2} = -\frac{5}{2}.$$

The slope is 3.

(b) $3x - y = -6$

$$-y = -3x - 6$$

$$y = 3x + 6$$

From this equation, the slope is also 3.

(c) $3x - 1y = -6$

$A = 3$ and $B = -1$, so $-\frac{A}{B} = -\frac{3}{-1} = 3.$

51. **(a)** Answers will vary. The intercepts are $(4, 0)$ and $(0, 3)$. Let $(x_1, y_1) = (4, 0)$ and $(x_2, y_2) = (0, 3)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{0 - 4} = \frac{3}{-4} = -\frac{3}{4}.$$

The slope is $-\frac{3}{4}$.

(b) $3x + 4y = 12$

$4y = -3x + 12$

$y = -\frac{3}{4}x + 3$

From this equation, the slope is also $-\frac{3}{4}$.

(c) $3x + 4y = 12$

$A = 3$ and $B = 4$, so $-\frac{A}{B} = -\frac{3}{4}$.

52. **(a)** Answers will vary. The intercepts are $(5, 0)$ and $(0, 6)$. Let $(x_1, y_1) = (5, 0)$ and $(x_2, y_2) = (0, 6)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{0 - 5} = \frac{6}{-5} = -\frac{6}{5}.$$

The slope is $-\frac{6}{5}$.

(b) $6x + 5y = 30$

$5y = -6x + 30$

$y = -\frac{6}{5}x + 6$

From this equation, the slope is also $-\frac{6}{5}$.

(c) $6x + 5y = 30$

$A = 6$ and $B = 5$, so $-\frac{A}{B} = -\frac{6}{5}$.

53. **(a)** Answers will vary. The intercepts are

(c) $1x + 1y = -3$

$A = 1$ and $B = 1$, so $-\frac{A}{B} = -\frac{1}{1} = -1$.

54. **(a)** Answers will vary. The intercepts are $(4, 0)$ and $(0, -4)$. Let $(x_1, y_1) = (4, 0)$ and $(x_2, y_2) = (0, -4)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 0}{0 - 4} = \frac{-4}{-4} = 1.$$

The slope is 1.

(b) $x - y = 4$

$-y = -x + 4$

$y = x - 4$

From this equation, the slope is also 1.

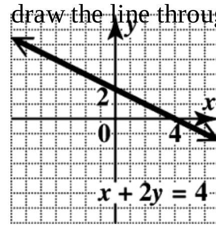
(c) $1x - 1y = 4$

$A = 1$ and $B = -1$, so $-\frac{A}{B} = -\frac{1}{-1} = 1$.

55. To find the slope of $x + 2y = 4$, first find the intercepts. Replace y with 0 to find that the x-intercept is (4, 0); replace x with 0 to find that the y-intercept is (0, 2). The slope is then

$$m = \frac{2 - 0}{0 - 4} = -\frac{2}{4} = -\frac{1}{2}.$$

To sketch the graph, plot the intercepts and draw the line through them.



56. To find the slope of $x + 3y = -6$, first find the intercepts. Replace y with 0 to find that the x-intercept is $(-6, 0)$; replace x with 0 to find that the y-intercept is $(0, -2)$. The slope is then

$-2 - 0 \quad -2 \quad 1$

$(-3, 0)$ and $(0, -3)$. Let $(x_1, y_1) = (-3, 0)$ and $(x_2, y_2) = (0, -3)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{0 - (-3)} = \frac{-3}{3} = -1.$$

The slope is -1.

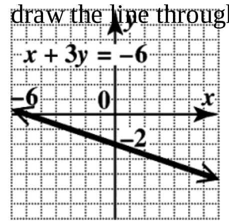
(b) $x + y = -3$

$y = -x - 3$

From this equation, the slope is also -1.

$$m = \frac{}{0 - (-6)} = \frac{}{6} = -\frac{}{3}.$$

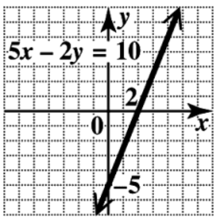
To sketch the graph, plot the intercepts and draw the line through them.



57. To find the slope of $5x - 2y = 10$, first find the intercepts. Replace y with 0 to find that the x-intercept is $(2, 0)$; replace x with 0 to find that the y-intercept is $(0, -5)$. The slope is then

$$m = \frac{-5-0}{0-2} = \frac{-5}{-2} = \frac{5}{2}.$$

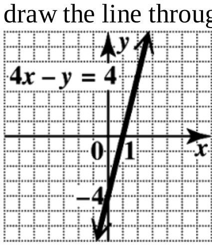
To sketch the graph, plot the intercepts and draw the line through them.



58. To find the slope of $4x - y = 4$, first find the intercepts. Replace y with 0 to find that the x-intercept is $(1, 0)$; replace x with 0 to find that the y-intercept is $(0, -4)$. The slope is then

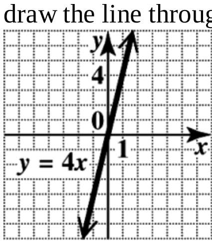
$$m = \frac{-4-0}{0-1} = \frac{-4}{-1} = 4.$$

To sketch the graph, plot the intercepts and draw the line through them.



59. In the equation $y = 4x$, replace x with 0 and then x with 1 to get the ordered pairs $(0, 0)$ and $(1, 4)$, respectively. (There are other possibilities for ordered pairs.) The slope is then $m = \frac{4-0}{1-0} = \frac{4}{1} = 4.$

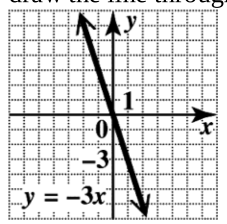
To sketch the graph, plot the two points and draw the line through them.



60. In the equation $y = -3x$, replace x with 0 and then x with 1 to get the ordered pairs $(0, 0)$ and $(1, -3)$, respectively. (There are other possibilities for ordered pairs.) The slope is then

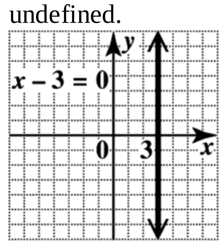
$$m = \frac{-3-0}{1-0} = \frac{-3}{1} = -3.$$

To sketch the graph, plot the two points and draw the line through them.



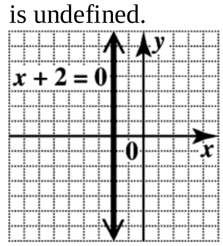
61. $x - 3 = 0 \quad (x = 3)$

The graph of $x = 3$ is the vertical line with x-intercept $(3, 0)$. The slope of a vertical line is undefined.



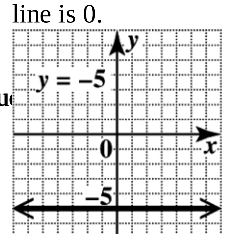
62. $x + 2 = 0 \quad (x = -2)$

The graph of $x = -2$ is the vertical line with x-intercept $(-2, 0)$. The slope of a vertical line is undefined.



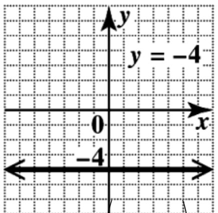
63. $y = -5$

The graph of $y = -5$ is the horizontal line with y-intercept $(0, -5)$. The slope of a horizontal line is 0.



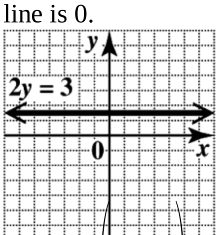
64. $y = -4$

The graph of $y = -4$ is the horizontal line with y-intercept $(0, -4)$. The slope of a horizontal line is 0.



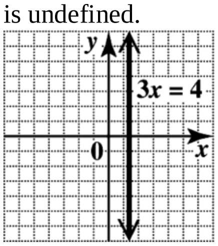
65. $2y = 3 \quad \left(y = \dfrac{3}{2} \right)$

The graph of $y = \dfrac{3}{2}$ is the horizontal line with y-intercept $\left(0, \dfrac{3}{2}\right)$. The slope of a horizontal line is 0.

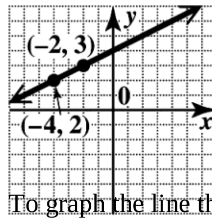


66. $3x = 4 \quad \left(x = \dfrac{4}{3} \right)$

The graph of $x = \dfrac{4}{3}$ is the vertical line with x-intercept $\left(\dfrac{4}{3}, 0\right)$. The slope of a vertical line is undefined.



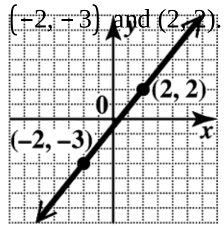
67. To graph the line through $(-4, 2)$ with slope $m = \dfrac{1}{2}$, locate $(-4, 2)$ on the graph. From $(-4, 2)$, go up 1 unit. Then go 2 units to the right to get to $(-2, 3)$. Draw the line through $(-4, 2)$ and $(-2, 3)$.



68. To graph the line through $(-2, -3)$ with slope $m = \dfrac{5}{4}$, locate $(-2, -3)$ on the graph. To find a second point, use the definition of slope.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{5}{4}$$

From $(-2, 3)$, go up 5 units. Then go 4 units to the right to get to (2, 2). Draw the line through $(-2, -3)$ and (2, 2).



69. To graph the line through $(0, -2)$ with slope $m = -\dfrac{2}{3}$, locate the point $(0, -2)$ on the graph.

To find a second point on the line, use the definition of slope, writing $-\dfrac{2}{3}$ as $\dfrac{-2}{3}$.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{-2}{3}$$

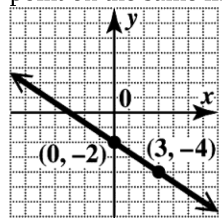
From $(0, -2)$, move 2 units down and then 3 units to the right. Draw a line through this second point and $(0, -2)$. (Note that the slope could also be written as $\dfrac{2}{-3}$. In this case, move

$m = \frac{1}{2}$, locate $(-4, 2)$ on the graph. To find a second point, use the definition of slope.

$m = \frac{\text{change in } y}{\text{change in } x} = \frac{1}{2}$

−3

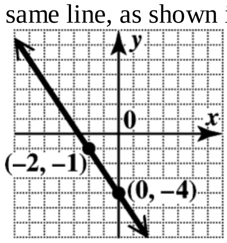
2 units up and 3 units to the left to get another point on the same line.)



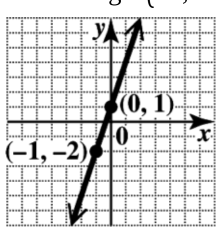
70. To graph the line through $(0, -4)$ with slope $m = -\frac{3}{2}$, locate the point $(0, -4)$ on the graph. To find a second point on the line, use the definition of slope, writing $-\frac{3}{2}$ as $\frac{-3}{2}$.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{-3}{2}$$

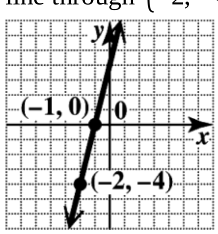
From $(0, -4)$, move 3 units down and then 2 units to the right. Draw a line through this second point and $(0, -4)$. The slope could also be written as $\frac{3}{-2}$. In this case, move 3 units up and 2 units to the left to get another point on the same line, as shown in the figure.



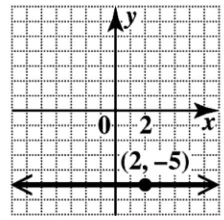
71. Locate $(-1, -2)$. Then use $m = 3 = \frac{3}{1}$ to go 3 units up and 1 unit right to (0, 1). Draw the line through $(-1, -2)$ and (0, 1).



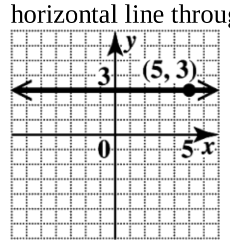
72. Locate $(-2, -4)$. Then use $m = 4 = \frac{4}{1}$ to go 4 units up and 1 unit right to $(-1, 0)$. Draw the line through $(-2, -4)$ and $k = 5$.



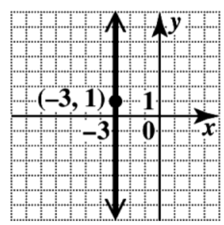
73. Locate $(2, -5)$. A slope of 0 means that the line is horizontal, so $y = -5$ at every point. Draw the horizontal line through $(2, -5)$.



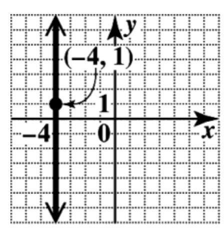
74. Locate (5, 3). A slope of 0 means that the line is horizontal, so $y = 3$ at every point. Draw the horizontal line through (5, 3).



75. Locate $(-3, 1)$. Since the slope is undefined, the line is vertical. The x-value of every point is -3. Draw the vertical line through $(-3, 1)$.



76. Locate $(-4, 1)$. Since the slope is undefined, the line is vertical. The x-value of every point is -4. Draw the vertical line through $(-4, 1)$.



77. If a line has slope $-\frac{4}{9}$, then any line parallel to it has slope $-\frac{4}{9}$

(the slope must be the same),

and any line perpendicular to it has slope $\frac{9}{4}$ (the slope must be the negative reciprocal).

78. If a line has slope 0.2, then any line parallel to it has slope 0.2 (the slope must be the same), and any line perpendicular to it has slope $\frac{-1}{0.2} = -5$ (the slope must be the negative reciprocal).

79. The slope of the line through (15, 9) and $(12, -7)$ is
$$m = \frac{-7-9}{12-15} = \frac{-16}{-3} = \frac{16}{3}.$$
The slope of the line through $(8, -4)$ and $(5, -20)$ is
$$m = \frac{-20-(-4)}{5-8} = \frac{-16}{-3} = \frac{16}{3}.$$
Since the slopes are equal, the two lines are parallel.

80. The slope of the line through (4, 6) and $(-8, 7)$ is
$$m = \frac{7-6}{-8-4} = \frac{1}{-12} = -\frac{1}{12}.$$
The slope of the line through $(-5, 5)$ and $(7, 4)$ is
$$m = \frac{4-5}{7-(-5)} = \frac{-1}{12} = -\frac{1}{12}.$$
Since the slopes are equal, the two lines are parallel.

81. Solve the equations for *y*.
$$x + 4y = 7 \qquad 4x - y = 3$$
$$4y = -x + 7 \qquad -y = -4x + 3$$
$$\frac{1}{}x + \frac{7}{}$$

83. Solve the equations for *y*.
$$4x - 3y = 6 \qquad 3x - 4y = 2$$
$$-3y = -4x + 6 \qquad -4y = -3x + 2$$
$$y = \frac{4}{3}x - 2 \qquad y = \frac{3}{4}x - \frac{1}{2}$$
The slopes are $\frac{4}{3}$ and $\frac{3}{4}$. The lines are neither parallel nor perpendicular.

84. Solve the equations for *y*.
$$2x + y = 6 \qquad x - y = 4$$
$$y = -2x + 6 \qquad -y = -x + 4$$
$$y = x - 4$$
The slopes are -2 and 1. The lines are neither parallel nor perpendicular.

85. The second equation can be simplified as $x = -2$. Both lines are vertical lines, so they are parallel.

86. The slope of the first line is the coefficient of *x*, namely 3. Solve the second equation for *y*.
$$2y - 6x = 5$$
$$2y = 6x + 5$$
$$y = 3x + \frac{5}{2}$$
The slope of the second line is also 3, so the lines are parallel.

87. Solve the equations for *y*.
$$4x + y = 0 \qquad 5x - 8 = 2y$$
$$y = -4x \qquad \frac{5}{2}x - 4 = y$$
The slopes are -4 and $\frac{5}{2}$. The lines are neither

$y = -$

4 4 $y = 4x - 3$

The slopes, $-\frac{1}{4}$ and 4, are negative reciprocals of one another, so the lines are perpendicular.

82. Solve the equations for y.

$2x + 5y = -7$ $\quad$ $5x - 2y = 1$

$5y = -2x - 7$ $\quad$ $-2y = -5x + 1$

$y = -\frac{2}{5}x - \frac{7}{5}$ $\quad$ $y = \frac{5}{2}x - \frac{1}{2}$

The slopes, $-\frac{2}{5}$ and $\frac{5}{2}$, are negative reciprocals of one another, so the lines are perpendicular.

parallel nor perpendicular.

88. Solve the equations for y.

$2x + 5y = -8$ $\quad$ $6 + 2x = 5y$

$5y = -2x - 8$ $\quad$ $5y = 2x + 6$

$y = -\frac{2}{5}x - \frac{8}{5}$ $\quad$ $y = \frac{2}{5}x + \frac{6}{5}$

The slopes are $-\frac{2}{5}$ and $\frac{2}{5}$. The lines are neither parallel nor perpendicular.

89. Solve the equations for y.

$2x = y + 3$ $\quad$ $2y + x = 3$

$2x - 3 = y$ $\quad$ $2y = -x + 3$

$y = -\frac{1}{2}x + \frac{3}{2}$

The slopes, 2 and $-\frac{1}{2}$, are negative reciprocals of one another, so the lines are perpendicular.

90. Solve the equations for y.

$$4x-3y=8 \qquad 4y+3x=12$$
$$-3y=-4x+8 \qquad 4y=-3x+12$$
$$y=\frac{4}{3}x-\frac{8}{3} \qquad y=-\frac{3}{4}x+3$$

The slopes, $\frac{4}{3}$ and $-\frac{3}{4}$, are negative reciprocals of one another, so the lines are perpendicular.

91. Let y be the vertical rise.
Since the slope is the vertical rise divided by the horizontal run, $0.13=\frac{y}{150}$. Solving for y gives $y=0.13(150)=19.5$.

The vertical rise could be a maximum of 19.5 ft.

92. The vertical change is 63 ft, and the horizontal change is $250-160=90$ ft.

The slope is $\frac{63}{90}=\frac{7}{10}$.

93. Use the points (0, 20) and (4, 4).

average rate of change

$$=\frac{\text{change in } y}{\text{change in } x}=\frac{4-20}{4-0}=\frac{-16}{4}=-4$$

The average rate of change is $-\$4000$ per year—that is, the value of the machine is decreasing $4000 each year during these years.

94. Use the points (0, 0) and (4, 200).
average rate of change

$$=\frac{\text{change in } y}{\text{change in } x}=\frac{200-0}{4-0}=\frac{200}{4}=50$$

The average rate of change is $50 per month—that is, the amount saved is increasing $50 each month during these months.

97. **(a)** In 2012, there were 326 million wireless subscriber connections in the United States.

(b) $m=\frac{326-255}{2012-2007}=\frac{71}{5}=14.2$

(c) The number of subscribers increased by an average of 14.2 million per year from 2007 to 2012.

98. **(a)** In 2012, 38% of U.S. households were wireless-only households.

(b) $m=\frac{38-16}{2012-2007}=\frac{22}{5}=4.4$

(c) The percent of wireless-only housleholds increased by an average of 4.4% per year from 2007 to 2012.

99. **(a)** Use $(2005, 402)$ and $(2012, 350)$.

$$m=\frac{350-402}{2012-2005}=\frac{-52}{7}\approx -7.4$$

The average rate of change is about -7 theaters per year.

(b) The negative slope means that the number of drive-in theaters decreased by an average of 7 each year from 2005 to 2012.

100. **(a)** Use $(2000, 15{,}189)$ and $(2011, 11{,}595)$.

$$m=\frac{11{,}595-15{,}189}{2011-2000}=\frac{-3594}{11}\approx -326.7$$

The average rate of change is about -327 thousand travelers per year.

(b) The negative slope means that the number of U.S. travelers to Canada decreased by an average of 327 thousand each year from 2000 to 2011.

101. Use $(1980, 1.22)$ and $(2012, 3.70)$.

95. We can see that there is no change in the percent of pay raise. Thus, the average rate of change is 0% per year—that is, the percent of pay raise is not changing; it is 3% each year during these years.

96. If the graph of a linear equation rises from left to right, then the average rate of change is *positive*. If the graph of a linear equation falls from left to right, then the average rate of change is *negative*.

$$m = \frac{3.70 - 1.22}{2012 - 1980} = \frac{2.48}{32} \approx 0.078$$

The average rate of change is about 7.8 cents per year—that is, the price of a gallon of gasoline increased by an average of \$0.08 per year from 1980 to 2012.

102. Use $(1990, 4.23)$ and $(2012, 7.96)$.

$$m = \frac{7.96 - 4.23}{2012 - 1990} = \frac{3.73}{22} \approx 0.17$$

The average rate of change is about 17 cents per year—that is, the price of a movie ticket increased by an average of \$0.17 per year from 1990 to 2012.

103. Use $(2010, 7246)$ and $(2013, 1670)$.

$$m = \frac{1670 - 7246}{2013 - 2010} = \frac{-5576}{3} \approx -1858.7$$

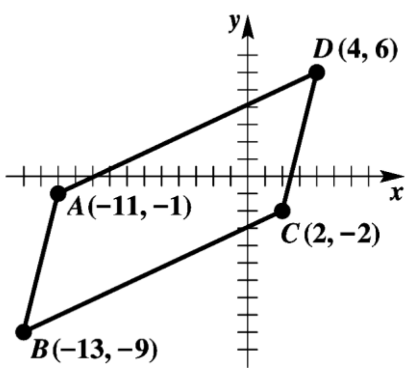
The average rate of change is about -1858.7 digital cameras sold per year—that is, the number of digital cameras sold decreased by an average of 1859 thousand per year from 2010 to 2013.

104. Use $(2010, 7390)$ and $(2013, 6876)$.

$$m = \frac{6876 - 7390}{2013 - 2010} = \frac{-514}{3} \approx -171.3$$

The average rate of change is about -171.3 sales of desktop computers per year—that is, the sales of desktop computers decreased by an average of $171 million per year from 2010 to 2013.

105. Label the points as shown in the figure.



In order to determine whether *ABCD* is a parallelogram, we need to show that the slope of *AB* equals the slope of *CD* and that the slope of $\overline{AD}$ equals the slope of $\overline{BC}$.

Slope of $\overline{AB} = \frac{-9-(-1)}{-13-(-11)} = \frac{-8}{-2} = 4$

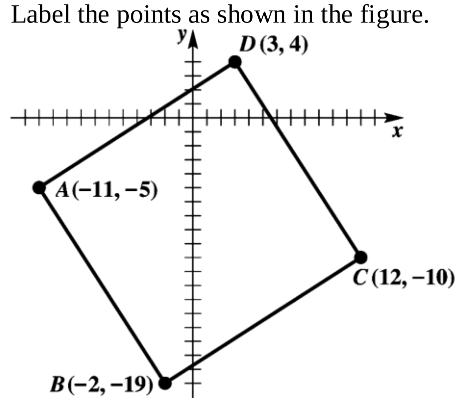
Slope of $\overline{CD} = \frac{6-(-2)}{4-2} = \frac{8}{2} = 4$

Slope of $\overline{AD} = \frac{6-(-1)}{4-(-11)} = \frac{7}{15}$

Slope of $\overline{BC} = \frac{-2-(-9)}{2-(-13)} = \frac{7}{15}$

Thus, the figure is a parallelogram.

106. Label the points as shown in the figure.



In order to determine whether *ABCD* is a parallelogram, we need to show that the slope of *AB* equals the slope of *CD* and that the slope of $\overline{AD}$ equals the slope of $\overline{BC}$.

Slope of $\overline{AB} = \frac{-19-(-5)}{-2-(-11)} = \frac{-14}{9} = -\frac{14}{9}$

Slope of $\overline{CD} = \frac{4-(-10)}{3-12} = \frac{14}{-9} = -\frac{14}{9}$

Slope of $\overline{AD} = \frac{4-(-5)}{3-(-11)} = \frac{9}{14}$

Slope of $\overline{BC} = \frac{-10-(-19)}{12-(-2)} = \frac{9}{14}$

Thus, the figure is a parallelogram. If two adjacent sides form a right angle, the parallelogram is a rectangle. A right angle is formed by perpendicular lines. Notice $\overline{AB}$ is perpendicular to $\overline{BC}$ since $-\frac{14}{9}\left(\frac{9}{14}\right) = -1$. Therefore, the figure is a rectangle.

107. For *A*(3, 1) and *B*(6, 2), the slope of $\overline{AB}$ is

$$m = \frac{2-1}{6-3} = \frac{1}{3}.$$

108. For *B*(6, 2) and *C*(9, 3), the slope of $\overline{BC}$ is

$$m = \frac{3-2}{9-6} = \frac{1}{3}.$$

109. For *A*(3, 1) and *C*(9, 3), the slope of $\overline{AC}$ is

$$m = \frac{3-1}{} = \frac{2}{} = \frac{1}{}.$$

9 − 3 6 3

110. The slope of $\overline{AB}$ = slope of $\overline{BC}$

$$= \text{slope of } \overline{AC} = \frac{1}{3}.$$

111. For $A(1, -2)$ and $B(3, -1)$, the slope of AB is

$$m = \frac{-1-(-2)}{3-1} = \frac{1}{2}.$$

For $B(3, -1)$ and $C(5, 0)$, the slope of $\overline{BC}$ is

$$m = \frac{0-(-1)}{5-3} = \frac{1}{2}.$$

For $A(1, -2)$ and $C(5, 0)$, the slope of AC is

$$m = \frac{0-(-2)}{5-1} = \frac{2}{4} = \frac{1}{2}.$$

Since the three slopes are the same, the three points are collinear.

112. For $A(0, 6)$ and $B(4, -5)$, the slope of $\overline{AB}$ is

$$m = \frac{-5-6}{4-0} = \frac{-11}{4} = -\frac{11}{4}.$$

For $B(4, -5)$ and $C(-2, 12)$, the slope of $\overline{BC}$ is $m = \frac{12-(-5)}{-2-4} = \frac{17}{-6} = -\frac{17}{6}.$

Since these two slopes are not the same, the three points are not collinear.

2.3 Writing Equations of Lines

Classroom Examples, Now Try Exercises

1. Slope 2; y-intercept $(0, -3)$

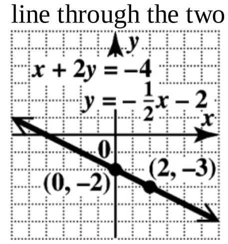
Here $m = 2$ and $b = -3$. Substitute these values into the slope-intercept form.

$$y = mx + b$$
$$y = 2x + (-3)$$
$$y = 2x - 3$$

N1. Slope $\frac{2}{3}$; y-intercept $(0, 1)$

Here $m = \frac{2}{3}$ and $b = 1$. Substitute these values into the slope-intercept form.

Plot the y-intercept $(0, -2)$. The slope can be interpreted as either $\frac{-1}{2}$ or $\frac{1}{-2}$. Using $\frac{-1}{2}$, move from $(0, -2)$ down 1 unit and to the right 2 units to locate the point $(2, -3)$. Draw a line through the two points.

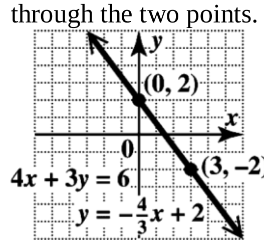


N2. $4x + 3y = 6$

Solve the equation for y.

$$3y = -4x + 6$$
$$y = -\frac{4}{3}x + 2$$

Plot the y-intercept (0, 2). The slope can be interpreted as either $\frac{-4}{3}$ or $\frac{4}{-3}$. Using $\frac{-4}{3}$, move from (0, 2) *down* 4 units and to the *right* 3 units to locate the point $(3, -2)$. Draw a line through the two points.



3. Through $(3, -4)$; slope $= m = \frac{2}{5}$

Use the point-slope form with $(x_1, y_1) = (3, -4)$ and $m = \frac{2}{5}$.

$$y - y_1 = m(x - x_1)$$

$$y = mx + b$$

$y = \frac{2}{3}x + 1$

2. $x + 2y = -4$

Solve the equation for *y*.

$2y = -x - 4$

$y = -\frac{1}{2}x - 2$

$y - (-4) = \frac{2}{5}(x - 3)$

$y + 4 = \frac{2}{5}(x - 3)$

$y + 4 = \frac{2}{5}x - \frac{6}{5}$

$y = \frac{2}{5}x - \frac{6}{5} - \frac{20}{5}$

$y = \frac{2}{5}x - \frac{26}{5}$

N3. Through $(5, -3)$; slope $= m = -\frac{1}{5}$

Use the point-slope form with

$(x_1, y_1) = (5, -3)$ and $m = -\frac{1}{5}$.

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = -\frac{1}{5}(x - 5)$$
$$y + 3 = -\frac{1}{5}(x - 5)$$
$$y + 3 = -\frac{1}{5}x + 1$$
$$y = -\frac{1}{5}x - 2$$

4. Through $(-2, 6)$ and $(1, 4)$

$$m = \frac{4-6}{1-(-2)} = \frac{-2}{3} = -\frac{2}{3}$$

Let $(x_1, y_1) = (1, 4)$.

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -\frac{2}{3}(x - 1)$$
$$3y - 12 = -2x + 2$$
$$2x + 3y = 14 \qquad \text{Standard form}$$

N4. Through $(3, -4)$ and $(-2, -1)$

$$m = \frac{-1-(-4)}{-2-3} = \frac{3}{-5} = -\frac{3}{5}$$

Let $(x_1, y_1) = (3, -4)$.

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = -\frac{3}{5}(x - 3)$$
$$y + 4 = -\frac{3}{5}(x - 3)$$
$$5y + 20 = -3x + 9$$
$$3x + 5y = -11 \qquad \text{Standard form}$$

5. **(a)** Through $(2, -1)$; m undefined

This is a vertical line since the slope is undefined. A vertical line through the

(b) Through $(2, -1)$; $m = 0$

Since the slope is 0, this is a horizontal line. A horizontal line through the point (a, b) has equation $y = b$. Here the y-coordinate is -1, so the equation is $y = -1$.

N5. **(a)** Through $(4, -4)$; m undefined

This is a vertical line since the slope is undefined. A vertical line through the point (a, b) has equation $x = a$. Here the x-coordinate is 4, so the equation is $x = 4$.

(b) Through $(4, -4)$; $m = 0$

Since the slope is 0, this is a horizontal line. A horizontal line through the point *(a, b)* has equation $y = b$. Here the y-coordinate is -4, so the equation is $y = -4$.

6. **(a)** Through $(-8, 3)$; parallel to the line $2x - 3y = 10$

Find the slope of the given line.

$$2x - 3y = 10$$
$$-3y = -2x + 10$$
$$y = \frac{2}{3}x - \frac{10}{3}$$

The slope is $\frac{2}{3}$, so a line parallel to it also has slope $\frac{2}{3}$. Use $m = \frac{2}{3}$ and $(x_1, y_1) = (-8, 3)$ in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{2}{3}[x - (-8)]$$
$$y - 3 = \frac{2}{3}(x + 8)$$
$$y - 3 = \frac{2}{3}x + \frac{16}{3}$$

2 16 9

point (a, b) has equation $x = a$. Here the x-coordinate is 2, so the equation is $x = 2$.

$$y = \frac{}{3}x + \frac{}{3} + 3$$

$$y = \frac{2}{3}x + \frac{25}{3}$$

(b) Through $(-8, 3)$; perpendicular to $2x-3y=10$

The slope of $2x-3y=10$ is $\frac{2}{3}$. The negative reciprocal of $\frac{2}{3}$ is $-\frac{3}{2}$, so the slope of the line through $(-8, 3)$ is $-\frac{3}{2}$.

$$y-y_1=m(x-x_1)$$
$$y-3=-\frac{3}{2}[x-(-8)]$$
$$y-3=-\frac{3}{2}(x+8)$$
$$y-3=-\frac{3}{2}x-12$$
$$y=-\frac{3}{2}x-9$$

N6. **(a)** Through $(6, -1)$; parallel to the line $3x-5y=7$

Find the slope of the given line.

$$3x-5y=7$$
$$-5y=-3x+7$$
$$y=\frac{3}{5}x-\frac{7}{5}$$

The slope is $\frac{3}{5}$, so a line parallel to it also has slope $\frac{3}{5}$. Use $m=\frac{3}{5}$ and $(x_1, y_1)=(6, -1)$ in the point-slope form.

$$y-y_1=m(x-x_1)$$
$$y-(-1)=\frac{3}{5}(x-6)$$
$$y+1=\frac{3}{5}x-\frac{18}{5}$$
$$y=\frac{3}{5}x-\frac{18}{5}-\frac{5}{5}$$
$$y=\frac{3}{5}x-\frac{23}{5}$$

(b) Through $(6, -1)$; perpendicular to $3x-5y=7$

The slope of $3x-5y=7$ is $\frac{3}{5}$. The negative reciprocal of $\frac{3}{5}$ is $-\frac{5}{3}$, so the slope of the line through $(6, -1)$ is $-\frac{5}{3}$.

$$y-y_1=m(x-x_1)$$
$$y-(-1)=-\frac{5}{3}(x-6)$$
$$y+1=-\frac{5}{3}x+10$$
$$y=-\frac{5}{3}x+9$$

7. Since the price you pay is \$0.10 per minute plus a flat rate of \$0.20, an equation that gives the cost y in dollars for a call of x minutes is $y=0.1x+0.2$.

N7. Since the price you pay is \$85 per month plus a flat fee of \$100, an equation that gives the cost y in dollars for x months of service is $y=85x+100$.

8. **(a)** Use (0, 34.3) for 1950 and (60, 87.1) for 2010.

$$m=\frac{87.1-34.3}{60-0}=\frac{52.8}{60}=0.88$$
$$y=mx+b$$
$$y=0.88x+34.3$$

(b) For 2012, $x=2012-1950=62$.

$$y=0.88x+34.3$$
$$y=0.88(62)+34.3 \quad \text{Let } x=62.$$
$$y=54.56+34.3$$
$$y=88.86$$

About 88.9% of the U.S. population 25 yr or older were at least high school graduates in 2012.

N8. **(a)** Use (0, 2787) for 2009 and (3, 3216) for 2012.

$$m=\frac{3216-2787}{3-0}=\frac{429}{3}=143$$
$$y=mx+b$$
$$y=143x+2787$$

(b) For 2011, $x = 2011 - 2009 = 2.$

$y = 143x + 2787$

$y = 143(2) + 2787$ Let $x = 2.$

$y = 286 + 2787$

$y = 3073$

According to the model, average tuition and fees for in-state students at public two-year colleges in 2011 were about $3073.

9. **(a)** Use (9, 255) and (11, 262).

$m = \frac{262 - 255}{11 - 9} = \frac{7}{2} = 3.5$

Use the point-slope form with $(x_1, y_1) = (9, 255).$

$y - y_1 = m(x - x_1)$

$y - 255 = 3.5(x - 9)$

$y - 255 = 3.5x - 31.5$

$y = 3.5x + 223.5$

(b) For 2014, $x = 2014 - 2000 = 14.$

$y = 3.5x + 223.5$

$y = 3.5(14) + 223.5$ Let $x = 14.$

$y = 49 + 223.5$

$y = 272.5$

The estimated retail spending on prescription drugs in 2014 is $272.5 billion.

N9. **(a)** Use (8, 243) and (12, 263).

$m = \frac{263 - 243}{12 - 8} = \frac{20}{4} = 5$

Use the point-slope form with $(x_1, y_1) = (8, 243).$

$y - y_1 = m(x - x_1)$

$y - 243 = 5(x - 8)$

$y - 243 = 5x - 40$

$y = 5x + 203$

(b) For 2015, $x = 2015 - 2000 = 15.$

$y = 5x + 203$

$y = 5(15) + 203$ Let $x = 15.$

$= 75 + 203$

$= 278$

The estimated retail spending on prescription drugs in 2015 is $278 billion.

2. Choice C, $y - 3 = 2(x - 1),$ is in the form

$y - y_1 = m(x - x_1).$

3. Choice A, $y = 6x + 2,$ is in the form $y = mx + b.$

4. $y + 2 = -3(x - 4)$

$y + 2 = -3x + 12$

$y = -3x + 10$

5. $y + 2 = -3(x - 4)$

$y + 2 = -3x + 12$

$3x + y = 10$ Standard form

Exercises

1. Choice A, $3x - 2y = 5,$ is in the form

6. Solve $10x - 7y = 70$ for y.

$-7y = -10x + 70$

$y = \frac{10}{7}x - 10$

7. This line is in slope-intercept form with slope $m = 2$ and y-intercept $(0, b) = (0, 3)$. The only graph with positive slope and with a positive y-coordinate of its y-intercept is A.

8. This line is in slope-intercept form with slope $m = -2$ and y-intercept $(0, b) = (0, 3)$. The only graph with negative slope and with a positive y-coordinate of its y-intercept is D.

9. This line is in slope-intercept form with slope $m = -2$ and y-intercept $(0, b) = (0, -3)$. The only graph with negative slope and with a negative y-coordinate of its y-intercept is C.

$Ax + By = C$ with $A \geq 0$ and integers A, B, and C having no common factor (except 1).

10. This line has slope $m = 2$ and y-intercept $(0, b) = (0, -3)$. The only graph with positive slope and with a negative y-coordinate of its y-intercept is F.

11. This line has slope $m = 2$ and y-intercept $(0, b) = (0, 0)$. The only graph with positive slope and with y-intercept (0, 0) is H.

12. This line has slope $m = -2$ and y-intercept $(0, b) = (0, 0)$. The only graph with negative slope and with y-intercept (0, 0) is G.

13. This line is a horizontal line with y-intercept (0, 3). Its y-coordinate is positive. The only graph that has these characteristics is B.

14. This line is a horizontal line with y-intercept $(0, -3)$. Its y-coordinate is negative. The only graph that has these characteristics is E.

15. $m = 5$ and $b = 15$
Substitute these values in the slope-intercept form.

$y = mx + b$
$y = 5x + 15$

16. $m = 2$ and $b = 12$

Substitute these values in the slope-intercept form.
$y = mx + b$

$y = 2x + 12$

17. $m = -\frac{2}{3}$ and $b = \frac{4}{5}$

Substitute these values in the slope-intercept form.
$y = mx + b$

$y = -\frac{2}{3}x + \frac{4}{5}$

18. $m = -\frac{5}{8}$ and $b = -\frac{1}{3}$
Substitute these values in the slope-intercept form.
$y = mx + b$

$y = -\frac{5}{8}x - \frac{1}{3}$

19. Here, $m = 1$ and $b = -1$. Substitute these values in the slope-intercept form.
$y = mx + b$
$y = 1x - 1$, or $y = x - 1$

20. Here, $m = -1$ and $b = -3$. Substitute these values in the slope-intercept form.
$y = mx + b$
$y = -1x - 3$, or $y = -x - 3$

21. Here, $m = \frac{2}{}$

23. To get to the point (3, 3) from the y-intercept (0, 1), we must go up 2 units and to the right 3 units, so the slope is $\frac{2}{3}$. The slope-intercept form is $y = \frac{2}{3}x + 1$.

24. To get to the point (2, 2) from the y-intercept $(0, -3)$, we must go up 5 units and to the right 2 units, so the slope is $\frac{5}{2}$. The slope-intercept form is $y = \frac{5}{2}x - 3$.

25. To get to the point $(-3, 1)$ from the y-intercept $(0, -2)$, we must go up 3 units and to the left 3 units, so the slope is $\frac{3}{-3} = -1$. The slope-intercept form is $y = -1x - 2$, or $y = -x - 2$.

26. To get to the point $(3, -1)$ from the y-intercept (0, 2), we must go down 3 units and to the right 3 units, so the slope is $\frac{-3}{3} = -1$. The slope-intercept form is $y = -1x + 2$, or $y = -x + 2$.

27. Use the points $(0, -4)$ and $(1, -2)$ to find the slope of the line.

$$m = \frac{-2-(-4)}{1-0} = \frac{-2+4}{1} = 2$$

The slope is 2. The y-intercept is $(0, -4)$. The equation in slope-intercept form is $y = 2x - 4$.

28. Use the points $(0, 3)$ and $(2, 9)$ to find the slope of the line.
$m = \frac{9-3}{} = \frac{6}{} = 3$
and $b = 5$. Substitute these values

2 − 0 2 The slope is 3. The *y*-intercept is (0, 3).

in the slope-intercept form.

$y = mx + b$

$y = \frac{2}{5}x + 5$

22. Here, $m = -\frac{3}{4}$ and $b = 7$. Substitute these values in the slope-intercept form.

$y = mx + b$

$y = -\frac{3}{4}x + 7$

The equation in slope-intercept form is $y = 3x + 3$.

29. Use the points (0, 3) and (5, 0) to find the slope of the line.

$m = \frac{0-3}{5-0} = \frac{-3}{5} = -\frac{3}{5}$

The slope is $-\frac{3}{5}$. The y-intercept is $(0, 3)$.
The equation in slope-intercept form is
$y = -\frac{3}{5}x + 3.$

30. Use the points $(0, -5)$ and $(2, -10)$ to find the slope of the line.

$$m = \frac{-10 - (-5)}{2 - 0} = \frac{-10 + 5}{2} = \frac{-5}{2} = -\frac{5}{2}$$

The slope is $-\frac{5}{2}$. The y-intercept is $(0, -5)$.
The equation in slope-intercept form is
$y = -\frac{5}{2}x - 5.$

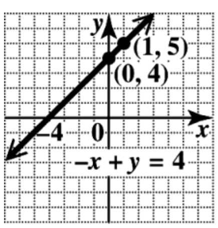
31. **(a)** Solve for y to get the equation in slope-intercept form.

$$-x + y = 4$$
$$y = x + 4$$

(b) The slope is the coefficient of x, 1.

(c) The y-intercept is the point $(0, b)$, or $(0, 4)$.

(d)



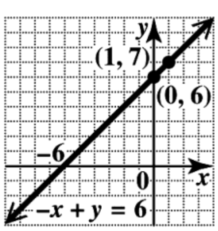
32. **(a)** Solve for y to get the equation in slope-intercept form.

$$-x + y = 6$$
$$y = x + 6$$

(b) The slope is the coefficient of x, 1.

(c) The y-intercept is the point $(0, b)$, or $(0, 6)$.

(d)



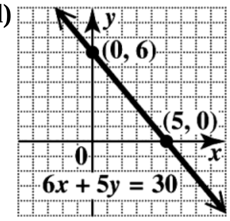
33. **(a)** Solve for y to get the equation in slope-intercept form.

$$6x + 5y = 30$$
$$5y = -6x + 30$$

(b) The slope is the coefficient of x, $-\frac{6}{5}$.

(c) The y-intercept is the point $(0, b)$, or $(0, 6)$.

(d)

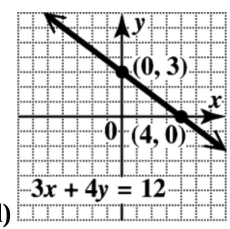


34. **(a)** Solve for y to get the equation in slope-intercept form.

$$3x + 4y = 12$$
$$4y = -3x + 12$$
$$y = -\frac{3}{4}x + 3$$

(b) The slope is the coefficient of x, $-\frac{3}{4}$.

(c) The y-intercept is the point $(0, b)$, or $(0, 3)$.



(d)

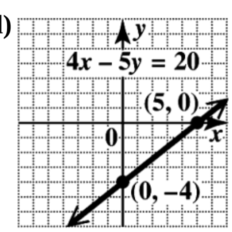
35. **(a)** Solve for y to get the equation in slope-intercept form.

$$4x - 5y = 20$$
$$-5y = -4x + 20$$
$$y = \frac{4}{5}x - 4$$

(b) The slope is the coefficient of x, $\frac{4}{5}$.

(c) The y-intercept is the point $(0, b)$, or $(0, -4)$.

(d)



$$y = -\frac{6}{5}x + 6$$

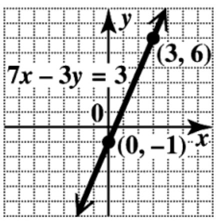
36. (a) Solve for y to get the equation in slope-intercept form.

$$7x - 3y = 3$$
$$-3y = -7x + 3$$
$$y = \frac{7}{3}x - 1$$

(b) The slope is the coefficient of x, $\frac{7}{3}$.

(c) The y-intercept is the point $(0, b)$, or $(0, -1)$.

(d)



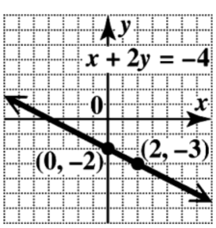
37. (a) Solve for y to get the equation in slope-intercept form.

$$x + 2y = -4$$
$$2y = -x - 4$$
$$y = -\frac{1}{2}x - 2$$

(b) The slope is the coefficient of x, $-\frac{1}{2}$.

(c) The y-intercept is the point $(0, b)$, or $(0, -2)$.

(d)



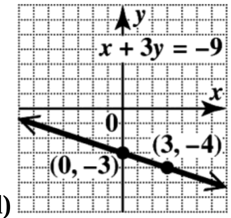
38. (a) Solve for y to get the equation in slope-intercept form.

$$x + 3y = -9$$
$$3y = -x - 9$$
$$y = -\frac{1}{3}x - 3$$

(b) The slope is the coefficient of x, $-\frac{1}{3}$.

(c) The y-intercept is the point $(0, b)$, or $(0, -3)$.

(d)



39. (a) Use the slope-intercept formula with the given slope and point.

$$8 = -2(5) + b$$
$$8 = -10 + b$$
$$18 = b$$
$$y = -2x + 18$$

(b) Use the equation in part (a) and rewrite it in standard form.

$$y = -2x + 18$$
$$2x + y = 18$$

40. (a) Use the slope-intercept formula with the given slope and point.

$$10 = 1(12) + b$$
$$10 = 12 + b$$
$$-2 = b$$
$$y = x - 2$$

(b) Use the equation in part (a) and rewrite it in standard form.

$$y = x - 2$$
$$-x + y = -2$$
$$x - y = 2$$

41. (a) Use the slope-intercept formula with the given slope and point.

$$4 = -\frac{3}{4}(-2) + b$$
$$4 = \frac{3}{2} + b$$
$$8 = 3 + 2b$$
$$5 = 2b$$
$$\frac{5}{2} = b$$
$$y = -\frac{3}{4}x + \frac{5}{2}$$

(b) Use the equation in part (a) and rewrite it in standard form.

$$y = -\frac{3}{4}x + \frac{5}{2}$$
$$\frac{3}{4}x + y = \frac{5}{2}$$
$$3x + 4y = 10$$

42. **(a)** Use the slope-intercept formula with the given slope and point.

$$6 = -\frac{5}{6}(-1) + b$$
$$6 = \frac{5}{6} + b$$
$$36 = 5 + 6b$$
$$31 = 6b$$
$$\frac{31}{6} = b$$
$$y = -\frac{5}{6}x + \frac{31}{6}$$

(b) Use the equation in part (a) and rewrite it in standard form.

$$y = -\frac{5}{6}x + \frac{31}{6}$$
$$\frac{5}{6}x + y = \frac{31}{6}$$
$$5x + 6y = 31$$

43. **(a)** Use the slope-intercept formula with the given slope and point.

$$4 = \frac{1}{2}(-5) + b$$
$$4 = -\frac{5}{2} + b$$
$$8 = -5 + 2b$$
$$13 = 2b$$
$$\frac{13}{2} = b$$
$$y = \frac{1}{2}x + \frac{13}{2}$$

(b) Use the equation in part (a) and rewrite it in standard form.

$$y = \frac{1}{2}x + \frac{13}{2}$$
$$-\frac{1}{2}x + y = \frac{13}{2}$$

44. **(a)** Use the slope-intercept formula with the given slope and point.

$$-2 = \frac{1}{4}(7) + b$$
$$-2 = \frac{7}{4} + b$$
$$-8 = 7 + 4b$$
$$-15 = 4b$$
$$-\frac{15}{4} = b$$
$$y = \frac{1}{4}x - \frac{15}{4}$$

(b) Use the equation in part (a) and rewrite it in standard form.

$$y = \frac{1}{4}x - \frac{15}{4}$$
$$-\frac{1}{4}x + y = -\frac{15}{4}$$
$$x - 4y = 15$$

45. **(a)** Use the slope-intercept formula with the given slope and point.

$$x - 2y = -13$$

$$0 = 4(3) + b$$
$$0 = 12 + b$$
$$-12 = b$$
$$y = 4x - 12$$

(b) Use the equation in part (a) and rewrite it in standard form.

$$y = 4x - 12$$
$$-4x + y = -12$$
$$4x - y = 12$$

46. **(a)** Use the slope-intercept formula with the given slope and point.

$$0 = -5(-2) + b$$
$$0 = 10 + b$$
$$-10 = b$$
$$y = -5x - 10$$

(b) Use the equation in part (a) and rewrite it in standard form.

$$y = -5x - 10$$
$$5x + y = -10$$

47. **(a)** Use the slope-intercept formula with the given slope and point.

$$6.8 = 1.4(2) + b$$
$$6.8 = 2.8 + b$$
$$4 = b$$
$$y = 1.4x + 4$$

(b) Use the equation in part (a) and rewrite it in standard form.

$$y = 1.4x + 4$$
$$-1.4x + y = 4$$
$$7x - 5y = -20$$

48. **(a)** Use the slope-intercept formula with the given slope and point.

$$-1.2 = 0.8(6) + b$$
$$-1.2 = 4.8 + b$$
$$-6 = b$$
$$y = 0.8x - 6$$

(b) Use the equation in part (a) and rewrite it in standard form.

$$y = 0.8x - 6$$
$$-0.8x + y = -6$$
$$4x - 5y = 30$$

49. Find the slope.

$$m = \frac{8-4}{5-3} = \frac{4}{2} = 2$$

Use the point-slope form with $(x_1, y_1) = (3, 4)$ and $m = 2.$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 2(x - 3)$$
$$y - 4 = 2x - 6$$
$$-2x + y = -2$$
$$2x - y = 2$$

50. Find the slope.

$$m = \frac{14-(-2)}{-3-5} = \frac{16}{-8} = -2$$

Use the point-slope form with

Use the point-slope form with $(x_1, y_1) = (6,1)$ and $m = -\frac{1}{2}.$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -\frac{1}{2}(x - 6)$$
$$2(y - 1) = -1(x - 6)$$
$$2y - 2 = -x + 6$$
$$x + 2y = 8$$

52. Find the slope.

$$m = \frac{1-5}{-8-(-2)} = \frac{-4}{-6} = \frac{2}{3}$$

Let $(x_1, y_1) = (-2, 5).$

$$y - 5 = \frac{2}{3}[x - (-2)]$$
$$3(y - 5) = 2(x + 2)$$
$$3y - 15 = 2x + 4$$
$$-2x + 3y = 19$$
$$2x - 3y = -19$$

53. Find the slope.

$$m = \frac{5-5}{1-2} = \frac{0}{-1} = 0$$

A line with slope 0 is horizontal. A horizontal line through the point (x, k) has equation $y = k,$ so the equation is $y = 5.$

54. Find the slope.

$$m = \frac{2-2}{4-(-2)} = \frac{0}{6} = 0$$

A line with slope 0 is horizontal. A horizontal line through the point (x, k) has equation $y = k,$ so the equation is $y = 2.$

55. Find the slope.

$$m = \frac{-8-6}{7-7} = \frac{-14}{0} \quad \text{Undefined}$$

A line with undefined slope is a vertical line. The equation of a vertical line is $x = k,$ where

$$(x_1, y_1) = (5, -2).$$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -2(x - 5)$$
$$y + 2 = -2x + 10$$
$$2x + y = 8$$

51. Find the slope.

$$m = \frac{5 - 1}{-2 - 6} = \frac{4}{-8} = -\frac{1}{2}$$

k is the common x-value. So the equation is $x = 7$.

56. $m = \frac{-1 - 5}{13 - 13} = \frac{-6}{0}$ Undefined

A line with undefined slope is a vertical line. The equation of a vertical line is $x = k$, where k is the common x-value. So the equation is $x = 13$.

57. Find the slope.

$$m = \frac{-3-(-3)}{-\frac{2}{3}-\frac{1}{2}} = \frac{0}{-\frac{7}{6}} = 0$$

A line with slope 0 is horizontal. A horizontal line through the point (x, k) has equation $y = k$, so the equation is $y = -3$.

58. $$m = \frac{-6-(-6)}{\frac{12}{7}-\left(-\frac{4}{9}\right)} = \frac{0}{\frac{136}{63}} = 0$$

A line with slope 0 is horizontal. A horizontal line through the point (x, k) has equation $y = k$, so the equation is $y = -6$.

59. Find the slope.

$$m = \frac{\frac{2}{3}-\frac{2}{5}}{\frac{4}{3}-\left(-\frac{2}{5}\right)} = \frac{\frac{10-6}{15}}{\frac{20+6}{15}} = \frac{\frac{4}{15}}{\frac{26}{15}} = \frac{4}{26} = \frac{2}{13}$$

Use the point-slope form with $(x_1, y_1) = \left(-\frac{2}{5}, \frac{2}{5}\right)$ and $m = \frac{2}{13}$.

$$y - \frac{2}{5} = \frac{2}{13}\left[x - \left(-\frac{2}{5}\right)\right]$$
$$13\left(y - \frac{2}{5}\right) = 2\left(x + \frac{2}{5}\right)$$
$$13y - \frac{26}{5} = 2x + \frac{4}{5}$$
$$-2x + 13y = \frac{30}{5}$$
$$2x - 13y = -6$$

60. $$m = \frac{\frac{2}{3}-\frac{8}{3}}{\frac{2}{5}-\frac{3}{4}} = \frac{\frac{2-8}{3}}{\frac{8-15}{20}} = \frac{-\frac{6}{3}}{-\frac{7}{20}} = (2)\left(\frac{20}{7}\right) = \frac{40}{7}$$

Let $(x_1, y_1) = \left(\frac{3}{4}, \frac{8}{3}\right)$.

$$y - \frac{8}{3} = \frac{40}{7}\left(x - \frac{3}{4}\right)$$
$$7\left(y - \frac{8}{3}\right) = 40\left(x - \frac{3}{4}\right)$$
$$7y - \frac{56}{3} = 40x - 30$$
$$-40x + 7y = -\frac{34}{3}$$
$$120x - 21y = 34$$

61. A line with slope 0 is a horizontal line. A horizontal line through the point (x, k) has equation $y = k$. Here $k = 5$, so an equation is $y = 5$.

62. An equation of this line is $y = -2$.

63. A vertical line has undefined slope and equation $x = c$. Since the x-value in (9, 10) is 9, the equation is $x = 9$.

64. A line with undefined slope is a vertical line in the form $x = c$. The equation of this line is $x = -2$.

2.3 Writing Equations of Lines 168

65. The equation of this line is $y = -\frac{3}{2}$.

66. The equation of this line is $y = -\frac{9}{2}$.

67. A horizontal line through the point (x, k) has equation $y = k$, so the equation is $y = 8$.

68. A horizontal line through the point (x, k) has equation $y = k$, so the equation is $y = -7$.

69. A vertical line through the point (k, y) has equation $x = k$. Here $k = 0.5$, so the equation is $x = 0.5$.

70. A vertical line through the point (k, y) has equation $x = k$. Here $k = 0.1$, so the equation is $x = 0.1$.

71. **(a)** Find the slope of $3x - y = 8$.

$$-y = -3x + 8$$
$$y = 3x - 8$$

The slope is 3, so a line parallel to it also has slope 3. Use $m = 3$ and $(x_1, y_1) = (7, 2)$ in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 3(x - 7)$$
$$y - 2 = 3x - 21$$
$$y = 3x - 19$$

(b) $$y = 3x - 19$$
$$-3x + y = -19$$
$$3x - y = 19$$

72. **(a)** Find the slope of $2x + 5y = 10$.

$$5y = -2x + 10$$
$$y = -\frac{2}{5}x + 2$$

The slope is $-\frac{2}{5}$. We are to find the equation of a line parallel to this line, so its slope is also $-\frac{2}{5}$. Use $m = -\frac{2}{5}$ and $(x_1, y_1) = (4, 1)$ in the point-slope form.

$$y - 1 = -\frac{2}{5}(x - 4)$$
$$y - 1 = -\frac{2}{5}x + \frac{8}{5}$$
$$y = -\frac{2}{5}x + \frac{13}{5}$$

(b) $$y = -\frac{2}{5}x + \frac{13}{5}$$
$$5y = -2x + 13 \quad \text{Multiply by 5.}$$
$$2x + 5y = 13$$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = \frac{1}{2}[x - (-2)]$$
$$y + 2 = \frac{1}{2}(x + 2)$$
$$y + 2 = \frac{1}{2}x + 1$$
$$y = \frac{1}{2}x - 1$$

(b) $$y = \frac{1}{2}x - 1$$
$$2y = x - 2 \quad \text{Multiply by 2.}$$
$$-x + 2y = -2$$
$$x - 2y = 2$$

74. **(a)** Find the slope of $-x + 3y = 12$.

$$3y = x + 12$$
$$y = \frac{1}{3}x + 4$$

The slope of the required line is the same as the slope of this line, $\frac{1}{3}$. Use $m = \frac{1}{3}$ and $(x_1, y_1) = (-1, 3)$ in the point-slope form.

$$y - 3 = \frac{1}{3}[x - (-1)]$$
$$y - 3 = \frac{1}{3}(x + 1)$$
$$y - 3 = \frac{1}{3}x + \frac{1}{3}$$
$$y = \frac{1}{3}x + \frac{10}{3}$$

(b) $$y = \frac{1}{3}x + \frac{10}{}$$

73. **(a)** Find the slope of $-x + 2y = 10$.

$2y = x + 10$

$y = \frac{1}{2}x + 5$

The slope is $\frac{1}{2}$, so a line parallel to it also

has slope $\frac{1}{2}$. Use $m = \frac{1}{2}$ and

$(x_1, y_1) = (-2, -2)$ in the point-slope form.

3 3

$3y = x + 10$ Multiply by 3.

$-x + 3y = 10$

$x - 3y = -10$

75. **(a)** Find the slope of $2x - y = 7$.

$-y = -2x + 7$

$y = 2x - 7$

The slope of the line is 2. Therefore, the slope of the line perpendicular to it is $-\frac{1}{2}$ since $2\left(-\frac{1}{2}\right) = -1$. Use $m = -\frac{1}{2}$ and $(x_1, y_1) = (8, 5)$ in the point-slope form.

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{2}(x - 8)$

$y - 5 = -\frac{1}{2}x + 4$

$y = -\frac{1}{2}x + 9$

$y = -\frac{1}{2}x + 9$

(b) $2y = -x + 18$ Multiply by 2.

$x + 2y = 18$

76. **(a)** Find the slope of $5x + 2y = 18$.

$2y = -5x + 18$

$y = -\frac{5}{2}x + 9$

$m_1 = -\frac{5}{2}$

We wish to find m_2 such that $m_1 m_2 = -1$, or $m_2 = \frac{-1}{m_1}$.

$m_2 = \frac{-1}{-\frac{5}{2}} = (-1)\left(-\frac{2}{5}\right) = \frac{2}{5}$

Use the point-slope form.

$y - (-7) = \frac{2}{5}(x - 2)$

$y + 7 = \frac{2}{5}x - \frac{4}{5}$

$y = \frac{2}{5}x - \frac{39}{5}$

(b) $y = \frac{2}{5}x - \frac{39}{5}$

$5y = 2x - 39$ Multiply by 5.

$-2x + 5y = -39$

$2x - 5y = 39$

77. **(a)** $x = 9$ is a vertical line, so a line perpendicular to it will be a horizontal line. It goes through $(-2, 7)$, so its equation is $y = 7$.

(b) $y = 7$ is already in standard form.

78. **(a)** $x = -3$ is a vertical line, so a line perpendicular to it will be a horizontal line. It goes through (8, 4), so its equation is $y = 4$.

(b) $y = 4$ is already in standard form.

79. Distance = (rate)(time), so $y = 45x$.

x	$y = 45x$	**Ordered Pair**
0	45(0) = 0	(0, 0)
5	45(5) = 225	(5, 225)
10	45(10) = 450	(10, 450)

80. Total cost = (cost/t-shirt)(number of t-shirts), so $y = 26x$.

x	$\boldsymbol{y = 26x}$	**Ordered Pair**
0	26(0) = 0	(0, 0)
5	26(5) = 130	(5, 130)
10	26(10) = 260	(10, 260)

81. Total cost = (cost/gal)(number of gallons), so $y = 3.75x$.

x	$\boldsymbol{y = 3.75x}$	**Ordered Pair**
0	3.75(0) = 0	(0, 0)
5	3.75(5) = 18.75	(5, 18.75)
10	3.75(10) = 37.50	(10, 37.50)

82. Total cost = (cost/day)(number of days), so
$y = 4.50x$.

x	$y = 4.50x$	Ordered Pair
0	$4.50(0) = 0$	(0, 0)
5	$4.50(5) = 22.50$	(5, 22.50)
10	$4.50(10) = 45.00$	(10, 45.00)

83. Total cost = (cost/credit)(number of credits), so $y = 140x$.

x	$y = 140x$	Ordered Pair
0	140(0) = 0	(0, 0)
5	140(5) = 700	(5, 700)
10	140(10) = 1400	(10, 1400)

84. Total cost = (cost/ticket)(number of tickets), so $y = 125x$.

x	$y = 125x$	Ordered Pair
0	125(0) = 0	(0, 0)
5	125(5) = 625	(5, 625)
10	125(10) = 1250	(10, 1250)

85. (a) The fixed cost is $15, so that is the value of *b*. The variable cost is $149, so

$y = mx + b = 149x + 15.$

(b) If $x = 5$, $y = 149(5) + 15 = 760$. The ordered pair is (5, 760). The cost of five tickets and a parking pass is $760.

(c) If $x = 2$, $y = 149(2) + 15 = 313$. The cost of two tickets and a parking pass is $313.

86. (a) The fixed cost is $20, so that is the value of *b*. The variable cost is $105.90, so $y = mx + b = 105.90x + 20$.

(b) If $x = 5$, $y = 105.90(5) + 20 = 549.50$. The ordered pair is (5, 549.5). The cost of five credit hours and the application fee is $549.50.

(c) If $x = 15$, $y = 105.90(15) + 20 = 1608.50$. The cost of 15 credit hours and the application fee is $1608.50.

87. (a) The fixed cost is $99, so that is the value of *b*. The variable cost is $41, so $y = mx + b = 41x + 99$.

(b) If $x = 5$, $y = 41(5) + 99 = 304$. The ordered pair is (5, 304). The cost for a five-month membership is $304.

88. (a) The fixed cost is $159, so that is the value of *b*. The variable cost is $57, so $y = mx + b = 57x + 159$.

(b) If $x = 5$, $y = 57(5) + 159 = 444$ The ordered pair is (5, 444). The cost of a five-month membership is $444.

(c) For 12 months, $x = 12$, so $y = 57(12) + 159 = 843$. The cost for a one-year membership is $843.

89. (a) The fixed cost is $36, so that is the value of *b*. The variable cost is $95, so $y = mx + b = 95x + 36$.

(b) If $x = 5$, $y = 95(5) + 36 = 511$. The ordered pair is (5, 511). The cost of a plan over a five-month contract is $511.

(c) For a two-year contract, $x = 24$, so $y = 95(24) + 36 = 2316$. The cost of a plan over a two-year contract is $2316.

90. (a) The fixed cost is $\$36 + \$99 = \$135$, so that

is the value of *b*. The variable cost is $110,

(c) If $x = 12$, $y = 41(12) + 99 = 591$. The cost for the first year's membership is $591.

so $y = mx + b = 110x + 135.$

(b) If $x = 5,\ y = 110(5) + 135 = 685.$ The ordered pair is (5, 685). The cost of a plan
over a five-month contract is $685.

(c) For a two-year contract, $x = 24,$ so $y = 110(24) + 135 = 2775.$ The cost of a plan over a two-year contract is $2775.

91. **(a)** The fixed cost is $30, so that is the value of *b*. The variable cost is $6, so $y = mx + b = 6x + 30.$

(b) If $x = 5,\ y = 6(5) + 30 = 60.$ The ordered pair is (5, 60). It costs $60 to rent the saw
for five days.

(c) $138 = 6x + 30$ Let $y = 138.$

$108 = 6x$

$x = \frac{108}{6} = 18$

The saw is rented for 18 days.

92. **(a)** The fixed cost is $50, so that is the value of *b*. The variable cost is $0.45, so $y = mx + b = 0.45x + 50.$

(b) If $x = 5$, $y = 0.45(5) + 50 = 52.25$. The ordered pair is (5, 52.25). The charge for driving 5 miles is $52.25.

(c) $127.85 = 0.45x + 50$ Let $y = 127.85$.

$77.85 = 0.45x$

$x = \frac{77.85}{0.45} = 173$

The car was driven 173 miles.

93. **(a)** Use (0, 7030) and (3, 2959).

$m = \frac{2959 - 7030}{3 - 0} = \frac{-4071}{3} = -1357$

The equation is $y = -1357x + 7030$. The slope tells us that the sales of portable media/MP3 players in the United States decreased by $1357 million per year from 2010 to 2013.

(b) The year 2011 corresponds to $x = 1$, so portable media/MP3 player sales were approximately

$y = -1357(1) + 7030 = \$5673$ million in the United States in 2011.

94. **(a)** Use (0, 17.6) and (3, 37.9).

$m = \frac{37.9 - 17.6}{3 - 0} = \frac{20.3}{3} \approx 6.8$

The equation is $y = 6.8x + 17.6$. The slope tells us that the sales of smartphones in the United States increased by $6.8 billion per year from 2010 to 2013.

(b) The year 2011 corresponds to $x = 1$, so smartphone sales were approximately $y = 6.8(1) + 17.6 = \$24.4$ billion in the United States in 2011.

95. **(a)** Use (8, 62.3) and (12, 77.8).

$m = \frac{77.8 - 62.3}{12 - 8} = \frac{15.5}{4} = 3.875$

Now use the point-slope form.

$y - 62.3 = 3.875(x - 8)$

$y - 62.3 = 3.875x - 31$

96. **(a)** Use (7, 95.9) and (12, 68.7).

$m = \frac{68.7 - 95.9}{12 - 7} = \frac{-27.2}{5} = -5.44$

Now use the point-slope form.

$y - 95.9 = -5.44(x - 7)$

$y - 95.9 = -5.44x + 38.08$

$y = -5.44x + 133.98$

(b) The year 2010 corresponds to $x = 10$, so the number of pieces of mail was approximately

$y \approx -5.44(10) + 133.98 \approx 79.6$ billion in 2010. This value is greater than the actual value.

97. When $C = 0°$, $F = \underline{32°}$.

When $C = 100°$, $F = $ ~~212°~~.

98. The two points of the form (C, F) would be (0, 32) and (100, 212).

99. $m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$

100. Let $m = \frac{9}{5}$ and $(x_1, y_1) = (0, 32)$.

$y - y_1 = m(x - x_1)$

$F - 32 = \frac{9}{\ }(C - 0)$

101. $y = 3.875x + 31.3$

102.

5

$F - 32 = \frac{9}{5}C$

5

$F = \frac{9}{5}C + 32$

$F = {}^{9}C + 32$

(b) The year 2011 corresponds to $x = 11$, so spending on home health care was

approximately
$y \approx 3.875(11) + 31.3 \approx \$73.9$ billion in

2011. This value is very close to the actual value.

5
$F \quad 32 = \frac{9}{5}C$

$\frac{5}{9}(F - 32) = C$

$F = \frac{9}{5}C + 32$

If $C = 30$, $F = \frac{9}{5}(30) + 32$
$= 54 + 32 = 86.$

Thus, when $C = 30°$, $F = 86°$.

103. $C = \frac{5}{9}(F - 32)$

When $F = 50$, $C = \frac{5}{9}(50 - 32)$

$= \frac{5}{9}(18) = 10.$

Thus, when $F = 50°, C = 10°$.

104. Let $F = C$ in the equation obtained in Exercise 100.

$F = \frac{9}{5}C + 32$

$C = \frac{9}{5}C + 32$ Let F be C.

$5C = 5\left(\frac{9}{5}C + 32\right)$ Multiply by 5.

$5C = 9C + 160$

$-4C = 160$ Subtract $9C$.

$C = -40$ Divide by -4.

(The same result may be found by using either form of the equation obtained in Exercise 101.)

The Celsius and Fahrenheit temperatures are equal $(F = C)$ at -40 degrees.

Summary Exercises Finding Slopes and Equations of Lines

1. The slope is $m = \frac{-6 - (-3)}{8 - 3} = \frac{-3}{5} = -\frac{3}{5}$.

2. The slope is $m = \frac{-5 - (-5)}{-1 - 4} = \frac{0}{-5} = 0.$

3. Rewrite the equation to have a coefficient next to the x-variable.
$y = 1x - 5$
Compare this to the slope-intercept form, $y = mx + b$.
The slope can be identified as $m = 1$ by inspection.

5. The graph of $x - 4 = 0$, or $x = 4$, is a vertical line with x-intercept (4, 0). The slope of a vertical line is undefined because the denominator equals zero in the slope formula.

6. Solve the equation for y.
$4x + 7y = 3$

$7y = -4x + 3$

$y = -\frac{4}{7}x + \frac{3}{7}$

The slope is $-\frac{4}{7}$.

7. **(a)** The slope-intercept form of a line, $y = mx + b$, becomes $y = -0.5x - 2$, or

$y = -\frac{1}{2}x - 2$, which is choice B.

(b) $m = \frac{2 - 0}{0 - 4} = \frac{2}{-4} = -\frac{1}{2}$

Using $m = -\frac{1}{2}$ and a y-intercept of (0, 2),

we get $y = -\frac{1}{2}x + 2$. Changing this

equation to the standard form gives us $2y = -x + 4$, or $x + 2y = 4$, which is choice F.

(c) $m = \frac{0 - (-2)}{0 - 4} = \frac{2}{-4} = -\frac{1}{2}$

Using $m = -\frac{1}{2}$ and a y-intercept of (0, 0),

we get $y = -\frac{1}{2}x + 0$, or $y = -\frac{1}{2}x$, which is

4. Solve the equation for y.
$3x - 7y = 21$
$-7y = -3x + 21$

$$y = \frac{3}{7}x - 3$$

The slope is $\frac{3}{7}$.

choice A.

(d) Use the point-slope form with $(x_1, y_1) = (-2, -2)$ and $m = \frac{1}{2}$.

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = \frac{1}{2}[x - (-2)]$$
$$y + 2 = \frac{1}{2}(x + 2)$$
$$2(y + 2) = x + 2$$
$$2y + 4 = x + 2$$
$$2 = x - 2y$$

This is choice C.

(e) Use the point-slope form with

$(x_1, y_1) = (0, 0)$ and $m = \frac{1}{2}$.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{1}{2}(x - 0)$$
$$y = \frac{1}{2}x \text{ or } 2y = x$$

This is choice E.

(f) The slope-intercept form of a line,

$y = mx + b$, becomes $y = 2x + 0$, or

$y = 2x$, which is choice D.

8. The only equation written in standard from with a positive whole number x coefficient and no common factor is C.

A. $y = -4x - 7$
$4x + y = -7$

B. $-3x + 4y = 12$
$3x - 4y = -12$

D. $\frac{1}{2}x + y = 0$
$x + 2y = 0$

E. $6x - 2y = 10$
$3x - y = 5$

F. $3y - 5x = -15$
$5x - 3y = 15$

9. **(a)** The slope is $m = \frac{1-6}{4-(-2)} = \frac{-5}{6} = -\frac{5}{6}$.

$x +$

Use the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 6 = -\frac{5}{6}[x - (-2)]$$
$$y - 6 = -\frac{5}{6}x - \frac{5}{3}$$
$$y = -\frac{5}{6}x - \frac{5}{3} + \frac{18}{3}$$

10. **(a)** A slope of 0 means that the line is a horizontal line of the form $y = k$, where k is the y-coordinate of any point on the line. It goes through $(5, -8)$ so its equation is $y = -8$.

(b) $y = -8$ is already in standard form.

11. **(a)** Use the point-slope form with

$(x_1, y_1) = (4, -2)$ and $m = -3$.

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -3(x - 4)$$
$$y + 2 = -3x + 12$$
$$y = -3x + 10$$

(b) $y = -3x + 10$

$3x + y = 10$

12. **(a)** The slope is $m = \frac{12 - (-8)}{-4 - 4} = \frac{20}{-8} = -\frac{5}{2}$.

Use the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - (-8) = -\frac{5}{2}(x - 4)$$
$$y + 8 = -\frac{5}{2}x + 10$$
$$y = -\frac{5}{2}x + 2$$

(b) $y = -\frac{5}{2}x + 2$

$y = -\frac{5}{6} \quad \frac{13}{3}$

$$2y = -5x + 4 \quad \text{Multiply by 2.}$$
$$5x + 2y = 4$$

13. (a) $x = \frac{2}{3}$ is a vertical line, so a line perpendicular to it will be a horizontal line. It goes through $\left(\frac{3}{4}, -\frac{7}{9}\right)$, so its equation is $y = -\frac{7}{9}$.

(b) $y = -\frac{5}{6}x + \frac{13}{3}$

$$6y = -5x + 26 \quad \text{Multiply by 6.}$$
$$5x + 6y = 26$$

(b) $y = -\frac{7}{9}$

$$9y = -7 \quad \text{Multiply by 9.}$$

14. **(a)** Use the point-slope form with $(x_1, y_1) = (-3, 6)$ and $m = \frac{2}{3}$.

$$y - y_1 = m(x - x_1)$$
$$y - 6 = \frac{2}{3}[x - (-3)]$$
$$y - 6 = \frac{2}{3}x + 2$$
$$y = \frac{2}{3}x + 8$$

(b)
$$y = \frac{2}{3}x + 8$$
$$3y = 2x + 24 \quad \text{Multiply by 3.}$$
$$-2x + 3y = 24$$
$$2x - 3y = -24$$

15. **(a)** Find the slope of $2x - 5y = 6$.

$$-5y = -2x + 6$$
$$y = \frac{2}{5}x - \frac{6}{5}$$

The slope of the line is $\frac{2}{5}$. Therefore, the slope of the line perpendicular to it is $-\frac{5}{2}$ since $\frac{2}{5}\left(-\frac{5}{2}\right) = -1$. Use $m = -\frac{5}{2}$ and $(x_1, y_1) = (0, 0)$ in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -\frac{5}{2}(x - 0)$$
$$y = -\frac{5}{2}x$$

(b)
$$y = -\frac{5}{2}x$$
$$2y = -5x$$
$$5x + 2y = 0$$

16. **(a)** Find the slope of $3x - y = 4$.

$$-y = -3x + 4$$
$$y = 3x - 4$$

The slope is 3, so a line parallel to it also has slope 3. Use $m = 3$ and $(x_1, y_1) = (-2, 5)$ in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 3[x - (-2)]$$
$$y - 5 = 3(x + 2)$$
$$y - 5 = 3x + 6$$
$$y = 3x + 11$$

(b)
$$y = 3x + 11$$
$$-3x + y = 11$$
$$3x - y = -11$$

17. **(a)** The slope of the line through (3, 9) and (6, 11) is $m = \frac{11 - 9}{6 - 3} = \frac{2}{3}$.

Use the point-slope form with $(x_1, y_1) = (-4, 2)$ and $m = \frac{2}{3}$ (since the slope of the desired line must equal the slope of the given line).

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{2}{3}[x - (-4)]$$
$$y - 2 = \frac{2}{3}(x + 4)$$
$$y - 2 = \frac{2}{3}x + \frac{8}{3}$$
$$y = \frac{2}{3}x + \frac{8}{3} + \frac{6}{3}$$
$$y = \frac{2}{3}x + \frac{14}{3}$$

(b)
$$y = \frac{2}{3}x + \frac{14}{3}$$
$$3y = 2x + 14$$
$$-2x + 3y = 14$$
$$2x - 3y = -14$$

18. **(a)** The slope of the line through (3, 7) and (5, 6) is $m = \frac{6-7}{5-3} = \frac{-1}{2} = -\frac{1}{2}.$

The slope of a line perpendicular to the given line is 2 (the negative reciprocal of $-\frac{1}{2}$). Use the point-slope form with $(x_1, y_1) = (4, -2)$ and $m = 2.$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = 2(x - 4)$$
$$y + 2 = 2x - 8$$
$$y = 2x - 10$$

(b)
$$y = 2x - 10$$
$$-2x + y = -10$$
$$2x - y = 10$$

2.4 Linear Inequalities in Two Variables

Classroom Examples, Now Try Exercises

1. $x + y \le 4$

Step 1

Graph the line, $x + y = 4,$ which has intercepts (4, 0) and (0, 4), as a solid line since the inequality involves $\le$.

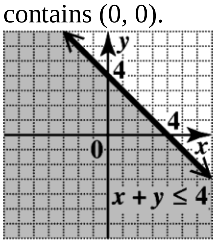
Step 2

Test (0, 0).

$$x + y \le 4$$
$$0 + 0 \overset{?}{\le} 4$$
$$0 \le 4 \quad \text{True}$$

Step 3

Since the result is true, shade the region that contains (0, 0).



N1. $-x + 2y \ge 4$

Step 1

Graph the line, $-x + 2y = 4,$ which has intercepts $(-4, 0)$ and (0, 2), as a solid line since the inequality involves $\le$.

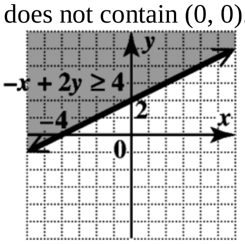
Step 2

Test (0, 0).

$$x + y \ge 4$$
$$0 + 0 \overset{?}{\ge} 4$$
$$0 \ge 4 \quad \text{False}$$

Step 3

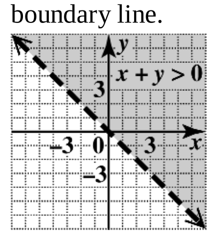
Since the result is false, shade the region that does not contain (0, 0).



2. Solve the inequality for *y*.

$$x + y > 0$$
$$y > -x \quad \text{Subtract } x.$$

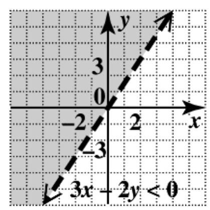
Graph the boundary line, $y = -x$ [which has slope -1 and *y*-intercept (0, 0)], as a dashed line because the inequality symbol is >. Since the inequality is solved for *y* and the inequality symbol is >, we shade the half-plane above the boundary line.



N2. Solve the inequality for *y*.

$$3x - 2y < 0$$
$$-2y < -3x \quad \text{Subtract } 3x.$$
$$y > \frac{3}{2}x \quad \text{Divide by } -2.$$

Graph the boundary line, $y = \frac{3}{2}x$ [which has slope $\frac{3}{2}$ and *y*-intercept (0, 0)], as a dashed line because the inequality symbol is >. Since the inequality is solved for *y* and the inequality symbol is >, we shade the half-plane above the boundary line.



3. Solve the inequality for y.

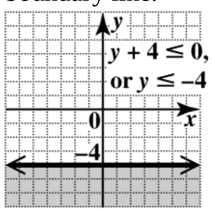
$y + 4 \le 0$

$y \le -4$ Subtract 4.

Graph the boundary line, $y = -4$ [which has

slope 0 and y-intercept $(0, -4)$], as a solid

line because the inequality symbol is $\le$. Since the inequality is solved for y and the inequality symbol is $\le$, we shade the half-plane below the boundary line.

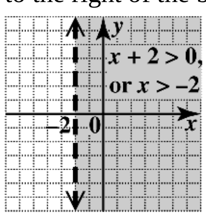


N3. Solve the inequality for x.

$x + 2 > 0$

$x > -2$ Subtract 2.

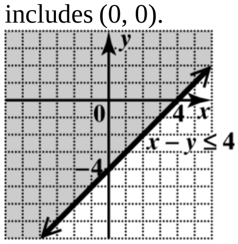
Graph the boundary line, $x = -2$ (which has an undefined slope and no y-intercerpt) as a dashed line because the inequality symbol is >. Since the inequality is solved for y and the inequality symbol is >, we shade the half-plane to the right of the boundary line.



4. Graph $x - y = 4$, which has intercepts (4, 0)

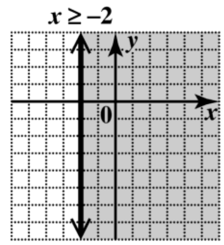
and $(0, -4)$, as a solid line since the

inequality involves $\le$. Test (0, 0), which yields $0 \le 4$, a true statement. Shade the region that includes (0, 0).

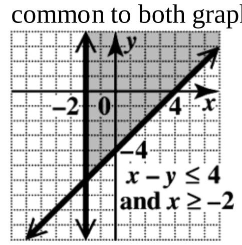


Graph $x = -2$ as a solid vertical line through

$(-2, 0)$. Shade the region to the right of $x = -2$.

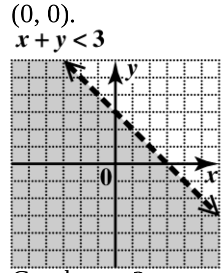


The graph of the intersection is the region common to both graphs.



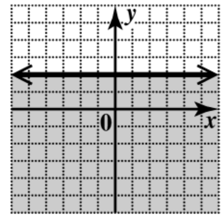
N4. Graph $x + y = 3$, which has intercepts (3, 0)

and (0, 3), as a dashed line since the inequality involves <. Test (0, 0), which yields $0 < 3$, a true statement. Shade the region that includes (0, 0).

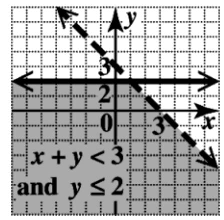


Graph $y = 2$ as a solid horizontal line through

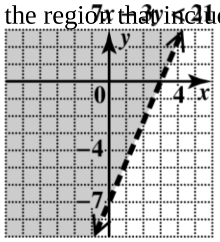
(0, 2). Shade the region below $y = 2$.



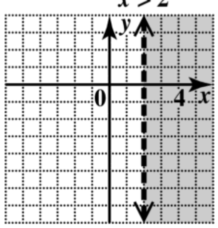
The graph of the intersection is the region common to both graphs.



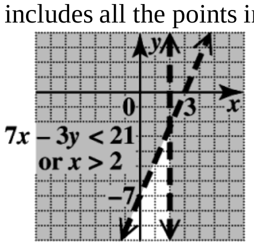
5. Graph $7x - 3y = 21$ as a dashed line through its intercepts (3, 0) and $(0, \ -7)$. Test (0, 0), which yields $0 < 21$, a true statement. Shade the region that includes (0, 0).



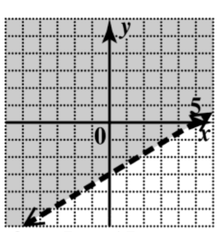
Graph $x = 2$ as a dashed vertical line through (2, 0). Shade the region to the right of $x = 2$.



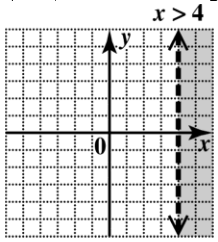
The graph of the union is the region that includes all the points in both graphs.



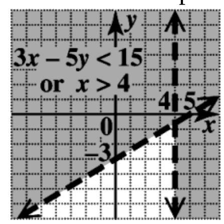
N5. Graph $3x - 5y = 15$ as a dashed line through its intercepts (5, 0) and $(0, \ -3)$. Test (0, 0), which yields $0 < 15$, a true statement. Shade the region that includes (0, 0).



Graph $x = 4$ as a dashed vertical line through (4, 0). Shade the region to the right of $x = 4$.



The graph of the union is the region that includes all the points in both graphs.



Exercises

1. **(a)** $x - 2y \le 4$

$$0 - 2(0) \stackrel{?}{\le} 4$$

$$0 \le 4 \quad \text{True}$$

The ordered pair (0, 0) is a solution.

(b) $x - 2y \le 4$

$$2 - 2(-1) \stackrel{?}{\le} 4$$

$$4 \le 4 \quad \text{True}$$

The ordered pair $(2, \ -1)$ is a solution.

(c) $x - 2y \le 4$

$$7 - 2(1) \stackrel{?}{\le} 4$$

$$5 \le 4 \quad \text{False}$$

The ordered pair (7, 1) is not a solution.

(d) $x - 2y \le 4$

$$0 - 2(2) \stackrel{?}{\le} 4$$

$$-4 \le 4 \quad \text{True}$$

The ordered pair (0, 2) is a solution.

2. **(a)** $x + y > 0$

$$0 + 0 \stackrel{?}{>} 0$$

$$0 > 0 \quad \text{False}$$

The ordered pair (0, 0) is not a solution.

(b) $x + y > 0$

$$-2 + 1 \stackrel{?}{>} 0$$

$$-1 > 0 \quad \text{False}$$

The ordered pair $(-2, \ 1)$ is not a solution.

(c) $x + y > 0$

$$2 - 1 \overset{?}{>} 0$$

$$1 > 0 \quad \text{True}$$

The ordered pair $(2, -1)$ is a solution.

(d) $x + y > 0$

$-4 + 6 \overset{?}{>} 0$

$2 > 0$ True

The ordered pair $(-4, 6)$ is a solution.

3. **(a)** $x - 5 > 0$

$0 - 5 \overset{?}{>} 0$

$-5 > 0$ False

The ordered pair (0, 0) is not a solution.

(b) $x - 5 > 0$

$5 - 5 \overset{?}{>} 0$

$0 > 0$ False

The ordered pair $(5, 0)$ is not a solution.

(c) $x - 5 > 0$

$-1 - 5 \overset{?}{>} 0$

$-6 > 0$ False

The ordered pair $(-1, 3)$ is not a solution.

(d) $x - 5 > 0$

$6 - 5 \overset{?}{>} 0$

$1 > 0$ True

The ordered pair $(6, 2)$ is a solution.

4. **(a)** $y \le 1$

$0 \overset{?}{\le} 1$

$0 \le 1$ True

The ordered pair (0, 0) is a solution.

(b) $y \le 1$

$1 \overset{?}{\le} 1$

$1 \le 1$ True

The ordered pair $(3, 1)$ is a solution.

(c) $y \le 1$

$-1 \overset{?}{\le} 1$

$-1 \le 1$ True

(d) $y \le 1$

$3 \overset{?}{\le} 1$

$3 \le 1$ False

The ordered pair $(-3, 3)$ is not a solution.

5. The boundary of the graph of $y \le -x + 2$ will be a <u>solid</u> line (since the inequality involves $\le$), and the shading will be <u>below</u> the line (since the inequality sign is $\le$ or <).

6. The boundary of the graph of $y < -x + 2$ will be a <u>dashed</u> line (since the inequality involves <), and the shading will be <u>below</u> the line (since the inequality sign is $\le$ or <).

7. The boundary of the graph of $y > -x + 2$ will be a <u>dashed</u> line (since the inequality involves >), and the shading will be <u>above</u> the line (since the inequality sign is $\ge$ or >).

8. The boundary of the graph of $y \ge -x + 2$ will be a <u>solid</u> line (since the inequality involves $\ge$), and the shading will be <u>above</u> the line (since the inequality sign is $\ge$ or >).

9. The graph has a solid line and is shaded to the left of $x = 4$.

$x \le 4$

10. The graph has a solid line and is shaded above $y = -3$.

$y \geq -3$

11. The graph has a dashed line and is shaded above the line $y = 3x - 2$.

$y \geq 3x - 2$

12. The graph has a dashed line and is shaded below the line $y = -x + 3$.

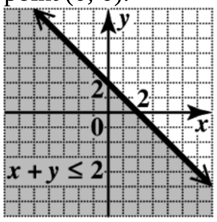
$y \le -x + 3$

13. Graph the line $x + y = 2$ by drawing a solid line (since the inequality involves $\le$) through the intercepts (2, 0) and (0, 2).
The ordered pair $(2, -1)$ is a solution.

Test a point not on this line, such as (0, 0).

$$x + y \le 2$$
$$0 + 0 \overset{?}{\le} 2$$
$$0 \le 2 \quad \text{True}$$

Shade the side of the line containing the test point (0, 0).

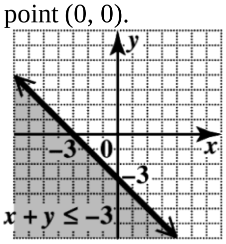


14. Graph the line $x + y = -3$ by drawing a solid line (since the inequality involves $\le$) through the intercepts $(-3,\ 0)$ and $(0,\ -3)$.

Test a point not on this line, such as (0, 0).

$$x + y \le -3$$
$$0 + 0 \overset{?}{\le} -3$$
$$0 \le -3 \quad \text{False}$$

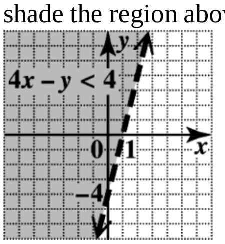
Shade the side of the line not containing the test point (0, 0).



15. Graph the line $4x - y = 4$ by drawing a dashed line (since the inequality involves <) through the intercepts (1, 0) and $(0,\ -4)$. Instead of using a test point, we will solve the inequality for *y*.

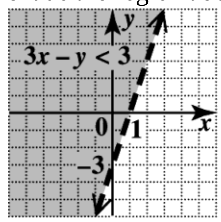
$$-y < -4x + 4$$
$$y > 4x - 4$$

Since we have "$y >$" in the last inequality, shade the region above the boundary line.



16. Graph the line $3x - y = 3$ by drawing a dashed line (since the inequality involves <) through the intercepts (1, 0) and $(0,\ -3)$. Instead of

Since we have "$y >$" in the last inequality, shade the region above the boundary line.

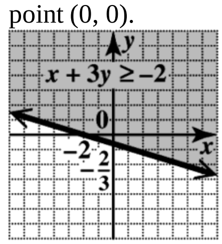


17. Graph the solid line $x + 3y = -2$ (since the inequality involves $\ge$) through the intercepts $(-2,\ 0)$ and $\left(0,\ -\frac{2}{3}\right)$.

Test a point not on this line, such as (0, 0).

$$0 + 3(0) \overset{?}{\ge} -2$$
$$0 \ge -2 \quad \text{True}$$

Shade the side of the line containing the test point (0, 0).

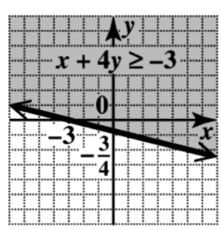


18. Graph the solid line $x + 4y = -3$ (since the inequality involves $\ge$) through the intercepts $(-3,\ 0)$ and $\left(0,\ -\frac{3}{4}\right)$.

Test a point not on this line, such as (0, 0).

$$0 + 4(0) \overset{?}{\ge} -3$$
$$0 \ge -3 \quad \text{True}$$

Shade the side of the line containing the test point (0, 0).



19. Graph the dashed line $y = \frac{1}{2}x + 3$ (since the inequality involves $<$) through the intercepts using a test point, we will solve the inequality for y.

$$-y < -3x + 3$$

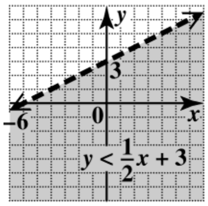
$$y > 3x - 3$$

$(-6, 0)$ and $(0, 3)$. Test a point not on this line, such as (0, 0).

$$0 \overset{?}{<} \frac{1}{2}(0) + 3$$

$$0 < 3 \qquad \text{True}$$

Shade the side of the line containing the test point (0, 0).

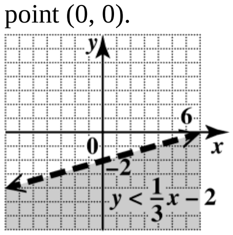


20. Graph the dashed line $y = \frac{1}{3}x - 2$ (since the inequality involves $<$) through the intercepts $(6, 0)$ and $(0,\ -2)$. Test a point not on this line, such as (0, 0).

$0 \overset{?}{<} \frac{1}{3}(0) - 2$

$0 < -2$ False

Shade the side of the line not containing the test point (0, 0).

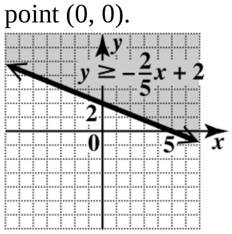


21. Graph the solid line $y = -\frac{2}{5}x + 2$ (since the inequality involves $\geq$) through the intercepts $(5, 0)$ and $(0,\ 2)$. Test a point not on this line, such as (0, 0).

$0 \overset{?}{\geq} -\frac{2}{5}(0) + 2$

$0 \geq 2$ False

Shade the side of the line not containing the test point (0, 0).

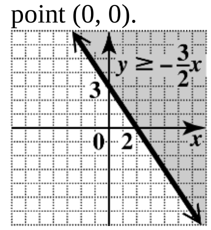


22. Graph the solid line $y = -\frac{3}{2}x + 3$ (since the inequality involves $\geq$) through the intercepts (2, 0) and (0, 3). Test a point not on this line, such as (0, 0).

$0 \overset{?}{\geq} -\frac{3}{2}(0) + 3$

$0 \geq 3$ False

Shade the side of the line not containing the test point (0, 0).

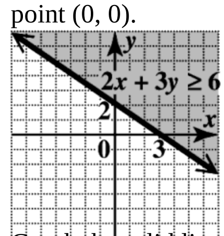


23. Graph the solid line $2x + 3y = 6$ (since the inequality involves $\geq$) through the intercepts (3, 0) and (0, 2). Test a point not on this line, such as (0, 0).

$2(0) + 3(0) \overset{?}{\geq} 6$

$0 \geq 6$ False

Shade the side of the line not containing the test point (0, 0).

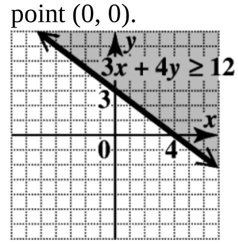


24. Graph the solid line $3x + 4y = 12$ (since the inequality involves $\geq$) through the intercepts (4, 0) and (0, 3). Test a point not on this line, such as (0, 0).

$3(0) + 4(0) \overset{?}{\geq} 12$

$0 \geq 12$ False

Shade the side of the line not containing the test point (0, 0).



25. Graph the dashed line $5x - 3y = 15$ (since the inequality involves $>$) through the intercepts

2

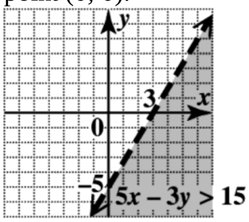
inequality involves ≥) through the intercepts $(2, 0)$ and $(0, 3)$. Test a point not on this line, such as (0, 0).

(3, 0) and $(0, -5)$. Test a point not on this line, such as (0, 0).

$$5(0) - 3(0) \overset{?}{>} 15$$

$$0 > 15 \quad \text{False}$$

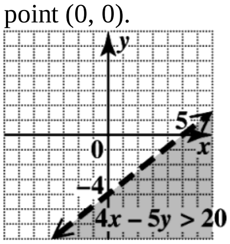
Shade the side of the line not containing the test point (0, 0).



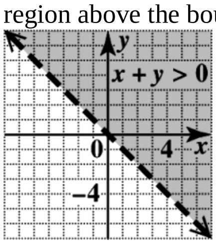
26. Graph the dashed line $4x - 5y = 20$ (since the inequality involves >) through the intercepts (5, 0) and $(0,\ -4)$. Test a point not on this line, such as (0, 0).

$$4(0) - 5(0) \overset{?}{>} 20$$
$$0 > 20 \quad \text{False}$$

Shade the side of the line not containing the test point (0, 0).



27. Graph the line $x + y = 0$, which includes the points (0, 0) and $(2,\ -2)$, as a dashed line (since the inequality involves >). Solving the inequality for *y* gives us $y > -x$. So shade the region above the boundary line.

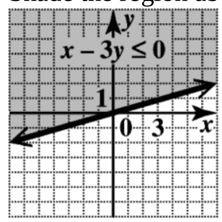


28. Graph the line $x + 2y = 0$, which includes the points (0, 0) and $(-4,\ 2)$, as a dashed line (since the inequality involves >). Solving the inequality for *y* gives us $y > -\frac{1}{2}x$. So shade the

29. Graph the solid line $x - 3y = 0$ through the points (0, 0) and (3, 1). Solve the inequality for *y*.

$$-3y \le -x$$
$$y \ge \frac{1}{3}x$$

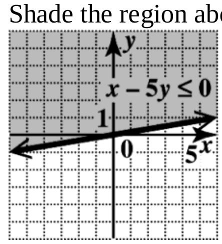
Shade the region above the boundary line.



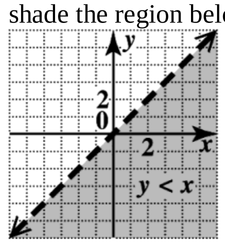
30. Graph the solid line $x - 5y = 0$ through the points (0, 0) and (5, 1). Solve the inequality for *y*.

$$-5y \le -x$$
$$y \ge \frac{1}{5}x$$

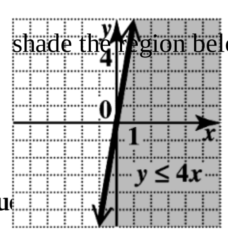
Shade the region above the boundary line.



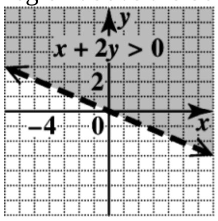
31. Graph the dashed line $y = x$ through (0, 0) and (2, 2). Since we have "$y <$" in the inequality, shade the region below the boundary line.



32. Graph the solid line $y = 4x$ through (0, 0) and (1, 4). Since we have "$y \le$" in the inequality, shade the region below the boundary line.

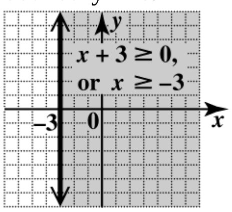


region above the boundary line.

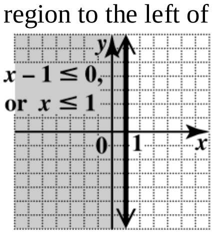


33. The line $x+3\geq 0$ has an intercept at $(-3,\ 0)$ and is a vertical line. Graph the solid line $x=-3$ (since the inequality involves $\geq$).

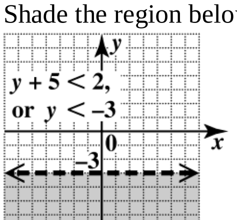
Shade the region to the right of the boundary line.



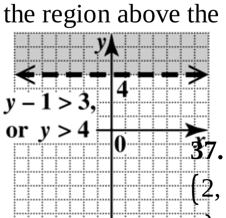
34. The line $x-1\leq 0$ has an intercept at $(1,\ 0)$ and is a vertical line. Graph the solid line $x=1$ (since the inequality involves $\leq$). Shade the region to the left of the boundary line.



35. The line $y+5<2$ has an intercept at $(0,\ -3)$ and is a horizontal line. Graph the dashed line $y=-3$ (since the inequality involves $<$). Shade the region below the boundary line.



36. The line $y-1>3$ has an intercept at $(0,\ 4)$ and is a vertical line. Graph the dashed line $y=4$ (since the inequality involves $>$). Shade the region above the boundary line.



37. $(2,\ 0)$

The boundary line here is *solid*, and the region *above* it is shaded.
The inequality symbol to indicate this is $\geq$.

Inequality for the graph: $\underline{y\geq 2x-4}$

38. $(3,\ 0)$ and $(0,\ 2)$

$$m=\frac{0-(2)}{3-0}=\frac{-2}{3}=-\frac{2}{3}$$

Slope: $-\frac{2}{3}$

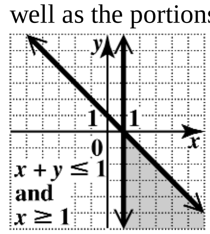
y-intercept: $(0,\ 2)$

Equation: $y=-\frac{2}{3}x+2$

The boundary line here is *dashed*, and the region *below* it is shaded.
The inequality symbol to indicate this is $<$.

Inequality for the graph: $y<-\frac{2}{3}x+2$

39. Graph the solid line $x+y=1$ through (0, 1) and (1, 0). The inequality $x+y\leq 1$ can be written as $y\leq -x+1$, so shade the region below the boundary line.
Graph the solid vertical line $x=1$ through (1, 0) and shade the region to the right. The required graph is the common shaded area as well as the portions of the lines that bound it.



40. Graph $x-y=2$ as a solid line through (2, 0) and $(0,\ -2)$. Test (0, 0).

$0-0\overset{?}{\geq}2$

and $(0,$

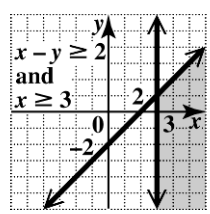
$\underline{0-(-4)}$

$-4)$

4

$0 \geq 2$ False

The graph is the region that does not contain (0, 0). Graph $x = 3$ as a solid vertical line through (3, 0). The graph of the inequality is the region to the right of the line. Shade the region that includes the overlap of the two graphs.



$m = \frac{}{2-0} = \frac{}{2} = 2$

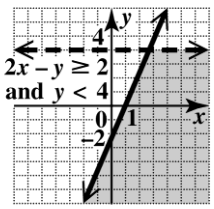
Slope: 2

y-intercept: $(0, \ -4)$

Equation: $y = 2x - 4$

41. Graph the solid line $2x - y = 2$ through the intercepts (1, 0) and $(0, -2)$. Test (0, 0) to get $0 \geq 2$, a false statement. Shade the side of the line not containing (0, 0).

To graph $y < 4$ on the same axes, graph the dashed horizontal line through (0, 4). Test (0, 0) to get $0 < 4$, a true statement. Shade the side of the dashed line containing (0, 0). The word "and" indicates the intersection of the two graphs. The final solution set consists of the region where the two shaded regions overlap.



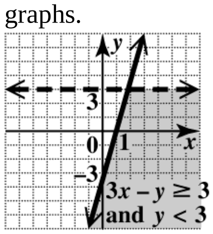
42. Graph $3x - y = 3$ as a solid line through (1, 0) and $(0, -3)$. Test (0, 0).

$$3(0) - 0 \overset{?}{\geq} 3$$

$$0 \geq 3 \quad \text{False}$$

The graph is the region that does not contain (0, 0).

Graph $y = 3$ as a dashed horizontal line through (0, 3). The graph of the inequality is the region below the dashed line. Shade the region that includes the overlap of the two graphs.



43. Graph $x + y = -5$, which has intercepts $(-5, 0)$ and $(0, -5)$, as a dashed line. Test

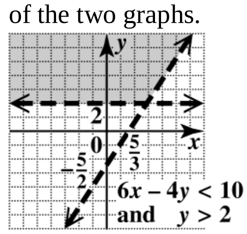
44. Graph the dashed line $6x - 4y = 10$ through $\left(\frac{5}{3}, 0\right)$ and $\left(0, -\frac{5}{2}\right)$. Test (0, 0).

$$6(0) - 4(0) \overset{?}{<} 10$$

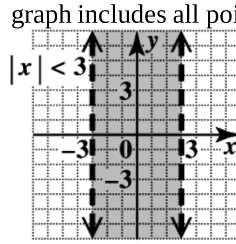
$$0 < 1 \quad \text{True}$$

The graph includes the region that includes (0, 0).

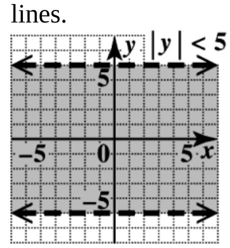
Graph the dashed horizontal line $y = 2$ through (0, 2). The graph includes the region above the line. Shade the region that includes the overlap of the two graphs.



45. $|x| < 3$ can be rewritten as $-3 < x < 3$. The boundaries are the dashed vertical lines $x = -3$ and $x = 3$. Since x is between -3 and 3, the graph includes all points between the lines.



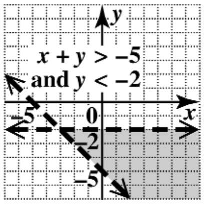
46. $|y| < 5$ can be rewritten as $-5 < y < 5$. The boundaries are the dashed horizontal lines $y = -5$ and $y = 5$. Since y is between -5 and 5, the graph includes all points between the lines.



(0, 0), which yields $0 > -5$, a true statement.
Shade the region that includes (0, 0).
Graph $y = -2$ as a dashed horizontal line.

Shade the region below $y = -2$. The required

graph of the intersection is the region common to both graphs.

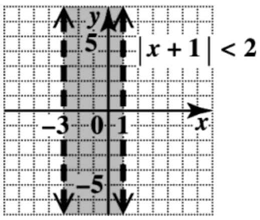


47. $|x+1|<2$ can be rewritten as the following.

$-2 < x+1 < 2$

$-3 < x < 1$

The boundaries are the dashed vertical lines $x=-3$ and $x=1$. Since x is between -3 and 1, the graph includes all points between the lines.

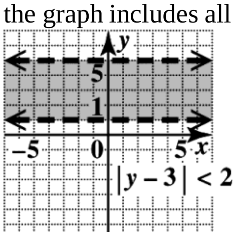


48. $|y-3|<2$ can be rewritten as the following.

$-2 < y-3 < 2$

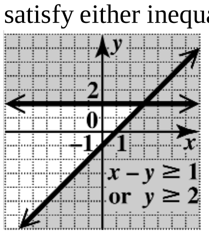
$1 < y < 5$

The boundaries are the dashed horizontal lines $y=1$ and $y=5$. Since y is between 1 and 5, the graph includes all points between the lines.



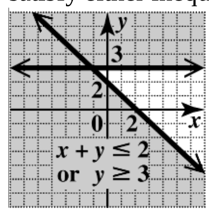
49. Graph the solid line $x-y=1$, which crosses the y-axis at -1 and the x-axis at 1. Use (0, 0) as a test point, which yields $0 \geq 1$, a false statement. Shade the region that does not include (0, 0).

Now graph the solid line $y=2$. Since the inequality is $y \geq 2$, shade above this line. The required graph of the union includes all the shaded regions—that is, all the points that satisfy either inequality.



50. Graph the solid line $x+y=2$ through (2, 0) and (0, 2). Use (0, 0) as a test point, which yields $0 \leq 2$, a true statement. Shade the region that includes (0, 0).

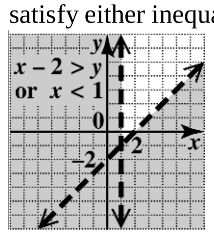
Graph the solid horizontal line $y=3$ through (0, 3). Shade the region above the line. The required graph of the union includes all the shaded regions—that is, all the points that satisfy either inequality.



51. Graph $x-2=y$, which has intercepts (2, 0) and (0, -2), as a dashed line. Test (0, 0), which yields $-2>0$, a false statement. Shade the region that does not include (0, 0).

Graph $x=1$ as a dashed vertical line. Shade the region to the left of $x=1$.

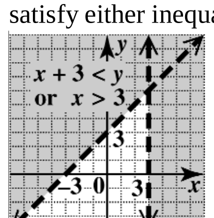
The required graph of the union includes all the shaded regions—that is, all the points that satisfy either inequality.



52. Graph the dashed line $x+3=y$ through $(-3, 0)$ and (0, 3). Use (0, 0) as a test point, which yields $3<0$, a false statement. Shade the region that does not include (0, 0).

Graph the dashed vertical line $x=3$ through (3, 0). Shade the region to the right of the line.

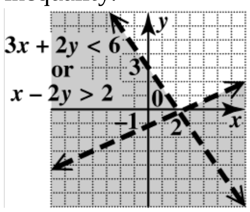
The required graph of the union includes all the shaded regions—that is, all the points that satisfy either inequality.



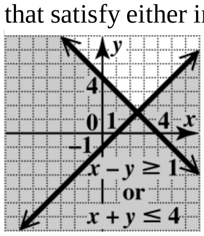
53. Graph $3x + 2y = 6$, which has intercepts (2, 0) and (0, 3), as a dashed line. Test (0, 0), which yields $0 < 6$, a true statement. Shade the region that includes (0, 0).

Graph $x - 2y = 2$, which has intercepts (2, 0)

and $(0, -1)$, as a dashed line. Test (0, 0), which yields $0 > 2$, a false statement. Shade the region that does not include (0, 0). The required graph of the union includes all the shaded regions—that is, all the points that satisfy either inequality.



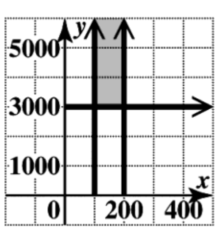
54. Graph the solid line $x - y = 1$ through (1, 0) and $(0, -1)$. Test (0, 0), which yields $0 \geq 1$, a false statement. Shade the region that does not include (0, 0).
Graph the solid line $x + y = 4$ through (4, 0) and (0, 4). Test (0, 0), which yields $0 \leq 4$, a true statement. Shade the region that includes (0, 0). The required graph of the union includes all the shaded regions—that is, all the points that satisfy either inequality.



55. "A factory can have *no more than* 200 workers on a shift but must have *at least* 100" can be translated as $x \leq 200$ and $x \geq 100$. "Must manufacture *at least* 3000 units" can be

translated as $y \geq 3000$.

56.



58. Some examples of points in the shaded region are (150, 4000), (150, 5000), (120, 3500), and (180, 6000). Some examples of points on the boundary are (100, 5000), (150, 3000), and

(200, 4000).
The corner points are (100, 3000) and

(200, 3000).

59.

(x, y)	$\mathbf{50x + 100y = C}$
(150, 4000)	$50(150) + 100(4000) = 407{,}500$
(120, 3500)	$50(120) + 100(3500) = 356{,}000$
(180, 6000)	$50(180) + 100(6000) = 609{,}000$
(100, 5000)	$50(100) + 100(5000) = 505{,}000$
(150, 3000)	$50(150) + 100(3000) = 307{,}500$
(200, 4000)	$50(200) + 100(4000) = 410{,}000$
(100, 3000) (least value)	$50(100) + 100(3000) = 305{,}000$
(150, 5000)	$50(150) + 100(5000) = 507{,}500$
(200, 3000)	$50(200) + 100(3000) = 310{,}000$

60. The company should use 100 workers and manufacture 3000 units to achieve the least possible cost.

2.5 Introduction to Relations and Functions

Classroom Examples, Now Try Exercises

1. The numbers in the table define a relation between *x* and *y*. They can be written as $\{(-4, 0), (0, -2), (3, 1), (5, 1)\}$.

N1. The data in the table defines a relation between

the average gas price per gallon and the year. It can be written as
$\{(2000, 1.56),\ (2005, 2.34),$
$(2010, 2.84),\ (2015, 3.39)\}$.

57. The total daily cost *C* consists of \$50 per worker and \$100 to manufacture one unit, so $C = 50x + 100y$.

2. **(a)** $\{(0,3),(-1,2),(-1,3)\}$
The last two ordered pairs have the *same* *x*-value paired with *two different y*-values (-1 is paired with both 2 and 3), so this relation *is not a function*.

(b) $\{(5, 4), (6, 4), (7, 4)\}$
The relation *is a function* because for each different x-value there is exactly one y-value. It is acceptable to have different x-values paired with the same y-value.

N2. **(a)** $\{(1, 5), (3, 5), (5, 5)\}$
The relation *is a function* because for each different x-value there is exactly one y-value. It is acceptable to have different x-values paired with the same y-value.

(b) $\{(-1, -3), (0, 2), (-1, 6)\}$
The first and last ordered pairs have the *same* x-value paired with *two different* y-values (-1 is paired with both -3 and 6), so this relation *is not a function*.

3. The domain of this relation is the set of all first components—that is, $\{0, 1, 2, 3, 4\}$. The range of this relation is the set of all second components—that is, $\{0, 3.50, 7.00, 10.50, 14.00\}$. This relation *is a function* because for each different first component, there is exactly one second component.

N3. **(a)** $\{(2, 2), (2, 5), (4, 8)\}$
The first two ordered pairs have the *same* x-value paired with *two different* y-values (2 is paired with both 2 and 5), so this relation *is not a function*. The domain is $\{2, 4\}$, and the range is $\{2, 5, 8\}$.

(b) The domain of this relation is the set of all first components—that is, $\{5, 10, 20, 40\}$. The range of this relation is the set of all second components—that is, $\{40, 80, 160, 320\}$. This relation *is a function* because for each different first component, there is exactly one second component.

4. The arrowheads indicate that the graph extends indefinitely left and right, as well as downward. The domain includes all real numbers, written $(-\infty, \infty)$. Because there is a greatest y-value, 4, the range includes all numbers less than or equal to 4, $(-\infty, 4]$.

N4. The arrowheads indicate that the graph extends indefinitely left and right, as well as upward.

N5. A vertical line intersects the graph more than once, so the relation is *not* a function.

6. **(a)** $y = -2x + 7$ is a function because each value of x corresponds to exactly one value of y. Its domain is the set of all real numbers, $(-\infty, \infty)$.

(b) $y = \sqrt{5x - 6}$ is a function because each value of x corresponds to exactly one value of y. Since the quantity under the radical must be nonnegative, the domain is the set of real numbers that satisfy the condition

$$5x - 6 \ge 0$$
$$5x \ge 6$$
$$x \ge \frac{6}{5}.$$

Therefore, the domain is $\left[\frac{6}{5}, \infty\right)$.

(c) $y^4 = x$ is not a function. If $x = 1$, for example, $y^4 = 1$ and $y = 1$ or $y = -1$. Since y^4 must be nonnegative, the domain is the set of nonnegative real numbers, $[0, \infty)$.

(d) $y \ge 4x + 2$ is not a function because if $x = 0$, then $y \ge 2$. Thus, the x-value 0

(e)

corresponds to many *y*-values. Its domain is the set of all real numbers, $(-\infty, \infty)$.

$$y = \frac{6}{5+3x}$$

Given any value of *x* in the domain, we find *y* by multiplying by 3, adding 5, and then dividing the result into 6. This process produces exactly one value of *y* for each value in the domain, so the given equation defines a function. The domain includes all real numbers except those that make the denominator 0. We find those numbers by setting the denominator equal to 0 and solving for *x*.

$$5 + 3x = 0$$

$$3x = -5$$

The domain includes all real numbers, written $(-\infty, \infty)$. Because there is a least y-value, -2, the range includes all numbers greater than or equal to -2, $[-2, \infty)$.

5. Any vertical line would intersect the graph at most once, so the relation is a function.

2.5 Introduction to Relations and Functions 181

$$x = -\frac{5}{3}$$

The domain includes all real numbers *except* $-\frac{5}{3}$, written as $\left(-\infty, -\frac{5}{3}\right) \cup \left(-\frac{5}{3}, \infty\right)$.

N6. **(a)** $y = 4x - 3$ is a function because each value of *x* corresponds to exactly one value of *y*. Its domain is the set of all real numbers,

$(-\infty, \infty)$.

(b) $y = \sqrt{2x - 4}$ is a function because each

value of *x* corresponds to exactly one value of *y*. Since the quantity under the radical must be nonnegative, the domain is the set of real numbers that satisfy the condition

$$2x - 4 \geq 0$$
$$2x \geq 4$$
$$x \geq 2.$$

Therefore, the domain is $[2, \infty)$.

(c) $y = \dfrac{1}{x - 2}$

Given any value of *x* in the domain, we find *y* by subtracting 2 and then dividing the result into 1. This process produces exactly one value of *y* for each value in the domain, so the given equation defines a function. The domain includes all real numbers except those that make the denominator 0. We find those numbers by setting the denominator equal to 0 and solving for *x*.

$$x - 2 = 0$$
$$x = 2$$

The domain includes all real numbers *except* 2, written as $(-\infty, 2) \cup (2, \infty)$.

(d) $y < 3x + 1$ is not a function because if

$x = 0$, then $y < 1$. Thus, the *x*-value 0

corresponds to many *y*-values. Its domain is the set of all real numbers, $(-\infty, \infty)$.

Exercises

1. A relation is any set of ordered pairs $\{(x, y)\}$.

2. A function is a relation in which for each distinct value of the first component of the ordered pairs, there is exactly one value of the second component.

3. In a relation $\{(x, y)\}$, the domain is the set of *x*-values, and the range is the set of *y*-values.

domain, and the set $\{-2, -1, -4, 3\}$ is its range.

5. Consider the function $d = 50t$, where *d* represents distance and *t* represents time. The value of *d* depends on the value of *t*, so the

variable *t* is the independent variable, and the variable *d* is the dependent variable.

6. The vertical line test is used to determine whether a graph is that of a function. It says that any vertical line can intersect the graph of a function in no more than *one* point.

7. The numbers in the table define a relation between *x* and *y*. They can be written as $\{(2, -2), (2, 0), (2, 1)\}$.

8. The numbers in the table define a relation between *x* and *y*. They can be written as $\{(-1, -1), (0, -1), (1, -1)\}$.

9. The data in the table defines a relation between the average movie ticket price and year.
$\{(1960,\ 0.76),\ (1980,\ 2.69),$
$(2000,\ 5.39),\ (2013,\ 8.38)\}$

10. The data in the table defines a relation between the average ACT composite score and the year.
$\{(2010,\ 21.0),\ (2011,\ 21.1),$
$(2012,\ 21.1),\ (2013,\ 20.9)\}$

11. The mapping defines a relation. It can be written as

$\{(\text{A}, 4),\ (\text{B}, 3),\ (\text{C}, 2),\ (\text{D}, 1),\ (\text{F}, 0)\}$.

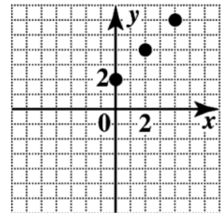
12. The mapping defines a relation. It can be written as
$\{(\text{A}, 1),\ (\text{E}, 5),\ (\text{I}, 9),\ (\text{O}, 15),\ (\text{U}, 21)\}$.

4. The relation

13. We can represent this set of ordered pairs by plotting them on a graph.

$\{(0,\ -2),\ (2,\ -1),\ (2,-4),\ (5,\ 3)\}$ *does not*

define a function. The set $\{0,\ 2,\ 5\}$ is its



14. We can represent this table as a set of ordered pairs: $\{(-1,-3),\ (0,-1),\ (1,1),(3,3)\}$.

15. We can represent the diagram in table form.

x	y
-3	-4
-3	1
2	0

16. No, the same x-value, -3, is paired with two different y-values, -4 and 1.

17. The relation is a function since for each x-value, there is only one y-value.
The domain is the set of x-values: {5, 3, 4, 7}.
The range is the set of y-values: {1, 2, 9, 6}.

18. The relation is a function since for each x-value, there is only one y-value.
The domain is the set of x-values: {8, 5, 9, 3}.
The range is the set of y-values: {0, 4, 3, 8}.

19. The relation is not a function since the x-value 2 has two different y-values associated with it, 4 and 5.
The domain is the set of x-values: {2, 0}.
The range is the set of y-values: {4, 2, 5}.

20. The relation is not a function since the x-value 9 has two different y-values associated with it, -2 and 2.

The domain is the set of x-values: $\{9, -3\}$.

The range is the set of y-values: $\{-2, 5, 2\}$.

21. The relation is a function since for each x-value, there is only one y-value.
The domain is the set of x-values:

$\{-3, 4, -2\}$.

The range is the set of y-values: {1, 7}.

22. The relation is a function since for each x-value, there is only one y-value.
The domain is the set of x-values:

$\{-12, -10, 8\}$.

The range is the set of y-values: {5, 3}.

23. The relation is not a function since the x-value 1 has two different y-values associated with it, 1 and -1. (A similar statement can be made for $x = 2$.)

24. The relation is a function since for each x-value, there is only one y-value.
The domain is the set of x-values: {2, 3, 4, 5}.
The range is the set of y-values: {5, 7, 9, 11}.

25. The relation can be described by the set of ordered pairs {(2, 1), (5, 1), (11, 7), (17, 20), (3, 20)}. The relation is a function since for each x-value, there is only one y-value.
The domain is the set of x-values: {2, 5, 11, 17, 3}.
The range is the set of y-values: {1, 7, 20}.

26. The relation can be described by the set of ordered pairs
$\{(1,10), (2,15), (2,19), (3,19), (5,27)\}$.
The relation is not a function since the x-value 2 has two different y-values associated with it, 15 and 19.
The domain is the set of x-values: {1, 2, 3, 5}.
The range is the set of y-values: {10, 15, 19, 27}.

27. The relation can be described by the set of ordered pairs $\{(1,5), (1,2), (1,-1), (1,-4)\}$.
The relation is not a function since the x-value 1 has four different y-values associated with it, 5, 2, -1, and -4.
The domain is the set of x-values: {1}.
The range is the set of y-values:

$\{5, 2, -1, -4\}$.

28. The relation can be described by the set of ordered pairs
$\{(-4,-4), (-4,0), (-4,4), (-4,8)\}$.
The relation is not a function since the x-value

-4 has four different y-values associated with

it, -4, 0, 4, and 8.
The domain is the set of x-values: $\{-4\}$.
The range is the set of y-values: $\{-4, 0, 4, 8\}$.

29. The relation can be described by the set of

The domain is the set of x-values: {1, 0, 2}. The range is the set of y-values:

ordered pairs
$\{(4, -3), (2, -3), (0, -3), (-2, -3)\}$.
The relation is a function since for each x-value, there is only one y-value.

$\{1, -1, 0, 4, -4\}$.

The domain is the set of x-values: {4, 2, 0,
$\{4, 2, 0 - 2,\}$.

The range is the set of y-values: $\{-3\}$.

30. The relation can be described by the set of ordered pairs $\{(-3,-6),(-1,-6),(1,-6),(3,-6)\}$. The relation is a function since for each x-value, there is only one y-value.

The domain is the set of x-values:

$\{-3,\ -1,\ 1,\ 3\}$.

The range is the set of y-values: $\{-6\}$.

31. The relation can be described by the set of ordered pairs $\{(-2,2),(0,3),(3,2)\}$. The relation is a function since for each x-value, there is only one y-value.
The domain is the set of x-values: $\{-2,\ 0,\ 3\}$.
The range is the set of y-values: $\{2, 3\}$.

32. The relation can be described by the set of ordered pairs $\{(-1,-3),(1,-3),(4,0)\}$. The relation is a function since for each x-value, there is only one y-value.
The domain is the set of x-values: $\{-1,\ 1,\ 4\}$.
The range is the set of y-values: $\{-3,\ 0\}$.

33. Using the vertical line test, we find that any vertical line will intersect the graph at most once. This indicates that the graph represents a function. This graph extends indefinitely to the left $(-\infty)$ and indefinitely to the right (∞).

Therefore, the domain is $(-\infty, \infty)$.
This graph extends indefinitely downward $(-\infty)$ and indefinitely upward (∞). Thus, the range is $(-\infty, \infty)$.

34. Using the vertical line test, we find that any vertical line will intersect the graph at most once. This indicates that the graph represents a function. This graph extends indefinitely to the left $(-\infty)$ and indefinitely to the right (∞).

Therefore, the domain is $(-\infty, \infty)$.
This graph extends indefinitely downward

36. Using the vertical line test, we find that any vertical line will intersect the graph at most once. This indicates that the graph represents a function. This graph extends indefinitely to the left $(-\infty)$ and indefinitely to the right (∞).

Therefore, the domain is $(-\infty, \infty)$. The y-value of the graph is constant, so the range is $\{2\}$.

37. Since a vertical line, such as $x = -4$, intersects the graph in two points, the relation is not a function.
The domain is $[-\infty,\ 0]$, and the range is $(-\infty, \infty)$.

38. The relation is not a function since a vertical line may intersect the graph in more than one point.
The domain is the set of x-values, $[-2,\ 2]$.
The range is the set of y-values, $[-2,\ 2]$.

39. Using the vertical line test, we find that any vertical line will intersect the graph at most once. This indicates that the graph represents a function. This graph extends indefinitely to the left $(-\infty)$ and indefinitely to the right (∞).

Therefore, the domain is $(-\infty, \infty)$.
This graph extends indefinitely downward $(-\infty)$ and reaches a high point at $y = 4$.

Therefore, the range is $(-\infty, 4]$.

40. Since any vertical line that intersects the graph intersects it in no more than one point, the relation represented by the graph is a function.
The domain is $[-2,\ 2]$, and the range is $[0, 4]$.

41. Since a vertical line can intersect the graph of the relation in more than one point, the relation is not a function.

The domain, the x-values of the points on the graph, is $[-4, 4]$.
The range, the y-values of the points on the

$(-\infty)$ and indefinitely upward (∞). Thus, the range is $(-\infty, \infty)$.

35. Using the vertical line test shows that one vertical line intersects the graph and it is at every point. This indicates that the graph does not represent a function.
The domain is $\{2\}$ because the *x*-value does not change, and the range is $(-\infty, \infty)$.

graph, is $[-3, 3]$.

42. Since a vertical line, such as $x = 4$, intersects the graph in two points, the relation is not a function.
The domain is $[3, \infty)$, and the range is $(-\infty, \infty)$.

43. For each x-value, there are multiple y-values associated with it, all of which are 2 or greater. Thus, this relation does not define a function. The domain is $(-\infty, \infty)$, and the range is $[2, \infty)$.

44. For each y-value, there are multiple x-values associated with it, all of which are 3 or less. Thus, this relation does not define a function. The domain is $(-\infty, 3]$, and the range is $(-\infty, \infty)$.

45. Each value of x corresponds to one y-value. For example, if $x = 3$, then $y = -6(3) = -18$. Therefore, $y = -6x$ defines y as a function of x. Since any x-value, positive, negative, or zero, can be multiplied by -6, the domain is $(-\infty, \infty)$.

46. Each value of x corresponds to one y-value. For example, if $x = 3$ then $y = -9(3) = -27$. Therefore, $y = -9x$ defines y as a function of x. Since any x-value, positive, negative, or zero, can be multiplied by -9, the domain is $(-\infty, \infty)$.

47. For any value of x, there is exactly one value of y, so this equation defines a function. The domain is the set of all real numbers, $(-\infty, \infty)$.

48. For any value of x, there is exactly one value of y, so this equation defines a function. The domain is the set of all real numbers, $(-\infty, \infty)$.

49. Each value of x corresponds to one y-value. For example, if $x = 3$, then $y = 3^2 = 9$. Therefore, $y = x^2$ defines y as a function of x. Since any x-value, positive, negative, or zero, can be squared, the domain is $(-\infty, \infty)$.

50. Each value of x corresponds to one y-value. For

52. The ordered pairs $(16, 2)$ and $(16, -2)$ both satisfy the equation. Since one value of x, 16, corresponds to two values of y, 2 and -2, the relation does not define a function. Because x is equal to the fourth power of y, the values of x must always be nonnegative. The domain is $[0, \infty)$.

53. For a particular x-value, more than one y-value can be selected to satisfy $x + y < 4$. Look at the given example.

$x = 2,\ y = 0$

$2 + 0 < 4$ True

Now, if $x = 2$ and $y = 1$, then $2 + 1 < 4$ is a true statement. Therefore, $x + y < 4$ does not define y as a function of x.

The graph of $x + y < 4$ is equivalent to the graph of $y < -x + 4$, which consists of the shaded region below the dashed line $y = -x + 4$, which extends indefinitely from left to right. Therefore, the domain is $(-\infty, \infty)$.

54. Let $x = 1$.

$1 - y < 3$

$-y < 2$

$y > -2$

So for $x = 1$, y may be any number greater than -2. $x - y < 3$ does not define y as a function of x. The x-values may be any number. The domain is $(-\infty, \infty)$.

55. For any value of x, there is exactly one corresponding value for y, so this relation defines a function. Since the radicand must be a nonnegative number, x must always be nonnegative. The domain is $[0, \infty)$.

56. For any value of x, there is exactly one corresponding value for y, so this relation defines a function. Since the radicand must be a
example, if $x = 3$, then $y = 3^3 = 27$. Therefore,

$y = x^3$ defines y as a function of x. Since any

x-value, positive, negative, or zero, can be

cubed, the domain is $(-\infty, \infty)$.

51. The ordered pairs (64, 2) and $(64, -2)$ both satisfy the equation. Since one value of x, 64, corresponds to two values of y, 2 and -2, the relation does not define a function. Because x is equal to the sixth power of y, the values of x must always be nonnegative. The domain is $[0, \infty)$.

nonnegative number, x must always be

nonnegative. The domain is $[0, \infty)$.

57. $y = \sqrt{x-3}$ is a function because each value of

x in the domain corresponds to exactly one value of y. Since the quantity under the radical must be nonnegative, the domain is the set of real numbers that satisfy the following condition.

$$x - 3 \geq 0$$
$$x \geq 3$$

Therefore, the domain is $[3, \infty)$.

58. $y=\sqrt{x-7}$ is a function because each value of x in the domain corresponds to exactly one value of y. Since the quantity under the radical must be nonnegative, the domain is the set of real numbers that satisfy the following condition.

$x-7\geq 0$

$x\geq 7$

Therefore, the domain is $[7, \infty)$.

59. $y=\sqrt{4x+2}$ is a function because each value of x in the domain corresponds to exactly one value of y. Since the quantity under the radical must be nonnegative, the domain is the set of real numbers that satisfy the following condition.

$4x+2\geq 0$

$4x\geq -2$

$x\geq -\frac{1}{2}$

Therefore, the domain is $\left[-\frac{1}{2}, \infty\right)$.

60. $y=\sqrt{2x+9}$ is a function because each value of x in the domain corresponds to exactly one value of y. Since the quantity under the radical must be nonnegative, the domain is the set of real numbers that satisfy the following condition.

$2x+9\geq 0$

$2x\geq -9$

$x\geq -\frac{9}{2}$

Therefore, the domain is $\left[-\frac{9}{2}, \infty\right)$.

61. Given any value of x, y is found by adding 4 and then dividing the result by 5. This process produces exactly one value of y for each x-value in the domain, so the relation represents a function. The denominator is never 0, so the domain is $(-\infty, \infty)$.

process produces exactly one value of y for each x-value in the domain, so the relation represents a function. The domain includes all real numbers except those that make the denominator 0, namely 0. The domain is $(-\infty, 0)\cup(0, \infty)$.

64. Given any value of x, y is found by dividing that value into 6 and negating that result. This process produces exactly one value of y for each x-value in the domain, so the relation represents a function. The domain includes all real numbers except those that make the denominator 0, namely 0. The domain is $(-\infty, 0)\cup(0, \infty)$.

65. Given any value of x, y is found by subtracting 4 and then dividing the result into 2. This process produces exactly one value of y for each x-value in the domain, so the relation represents a function. The domain includes all real numbers except those that make the denominator 0, namely 4. The domain is $(-\infty, 4)\cup(4, \infty)$.

66. Given any value of x, y is found by subtracting 2 and then dividing the result into 7. This process produces exactly one value of y for each x-value in the domain, so the relation represents a function. The domain includes all real numbers except those that make the denominator 0, namely 2. The domain is $(-\infty, 2)\cup(2, \infty)$.

67. Rewrite $xy=1$ as $y=\frac{1}{x}$. Note that x can never equal 0; otherwise the denominator would equal 0. The domain is $(-\infty, 0)\cup(0, \infty)$.

Each nonzero x-value gives exactly one y-value. Therefore, $xy=1$ defines y as a function of x.

68. Rewrite $xy=3$ as $y=\frac{3}{x}$. Note that x can never

2.5 Introduction to Relations and Functions 191

62. Given any value of x, y is found by subtracting 3 and then dividing the result by 2. This process produces exactly one value of y for each x-value in the domain, so the relation represents a function. The denominator is never 0, so the domain is $(-\infty, \infty)$.

63. Given any value of x, y is found by dividing that value into 2 and negating that result. This equal 0; otherwise the denominator would equal 0. The domain is $(-\infty, 0) \cup (0, \infty)$. Each nonzero x-value gives exactly one y-value. Therefore, $xy = 3$ defines y as a function of x.

69. **(a)** Each year corresponds to exactly one percentage, so the table defines a function.

(b) The domain is {2009, 2010, 2011, 2012, 2013}.
The range is {44.0, 43.4, 43.1, 42.9}.

(c) The range element that corresponds to 2012 is 42.9. The domain element that corresponds to 43.4 is 2010.

(d) Answers will vary. Two possible answers are (2010, 43.4) and (2013, 43.1).

70. (a) Each year corresponds to exactly one percentage, so the table defines a function.

(b) The domain is {2008, 2009, 2010, 2011, 2012}.
The range is {4.4, 4.2, 3.7, 2.8, 2.6}.

(c) The range element that corresponds to 2012 is 2.6. The domain element that corresponds to 2.8 is 2011.

(d) Answers will vary. Two possible answers are (2008, 4.4) and (2012, 2.6).

2.6 Function Notation and Linear Functions

Classroom Examples, Now Try Exercises

1. (a) $f(x) = 6x - 2$

$f(-3) = 6(-3) - 2$ Replace x with -3.
$= -18 - 2$ Multiply.
$= -20$ Subtract.

(b) $f(x) = 6x - 2$

$f(0) = 6(0) - 2$ Replace x with 0.
$= 0 - 2$ Multiply.
$= -2$ Subtract.

N1. (a) $f(x) = 4x + 3$
$f(-2) = 4(-2) + 3$ Replace x with -2.
$= -8 + 3$ Multiply.
$= -5$ Add.

(b) $f(x) = 4x + 3$

$f(0) = 4(0) + 3$ Replace x with 0.

(b) $f(x) = -x^2 + 3x + 3$
$f(t) = -(t)^2 + 3(t) + 3$
$f(t) = -t^2 + 3t + 3$

N2. (a) $f(x) = 2x^2 - 4x + 1$

$f(-2) = 2(-2)^2 - 4(-2) + 1$
$= 8 + 8 + 1 = 17$

(b) $f(x) = 2x^2 - 4x + 1$
$f(a) = 2a^2 - 4a + 1$

3. $g(x) = 5x - 1$
$g(t+2) = 5(t+2) - 1$
$= 5t + 10 - 1$
$= 5t + 9$

N3. $g(x) = 8x - 5$
$g(a-2) = 8(a-2) - 5$
$= 8a - 16 - 5$
$= 8a - 21$

4. (a) When $x = 2$, $y = 6$, so $f(2) = 6$.

(b) $f(x) = -x^2$
$f(2) = -(2)^2 = -(4) = -4$

N4. (a) When $x = -1$, $y = 4$, so $f(-1) = 4$.

(b) $f(x) = x^2 - 12$
$f(-1) = (-1)^2 - 12 = 1 - 12 = -11$

5. (a) When $x = 2$, $y = 1$, so $f(2) = 1$.

(b) When $x = -2$, $y = 3$, so $f(-2) = 3$.

(c) $f(x) = 0$ is equivalent to $y = 0$, and $y = 0$ when $x = 4$.

N5. (a) When $x = -1$, $y = 0$, so $f(-1) = 0$.

$= 0 + 3$ Multiply.
$= 3$ Add.

2. **(a)** $f(x) = -x^2 + 3x + 3$

$f(-3) = -(-3)^2 + 3(-3) + 3$

$f(-3) = -9 - 9 + 3$

$f(-3) = -15$

(b) $f(x) = 2$ is equivalent to $y = 2$, and $y = 2$ when $x = 1$.

6. $x^2 - 4y = 3$

Solve for y.

$-4y = -x^2 + 3$

$y = \frac{1}{4}x^2 - \frac{3}{4}$ or $y = \frac{x^2 - 3}{4}$

Replace y with $f(x)$.

$$f(x)=\frac{x^2-3}{4}$$

$$f(1)=\frac{(1)^2-3}{4}=\frac{1-3}{4}=\frac{-2}{4}=-\frac{1}{2}$$

$$f(a)=\frac{a^2-3}{4}$$

N6. $-4x^2+y=5$

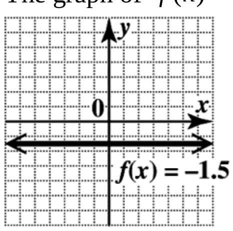
Solving for y gives us $y=4x^2+5$.
Replace y with $f(x)$.

$$f(x)=4x^2+5$$

$$f(-3)=4(-3)^2+5=36+5=41$$

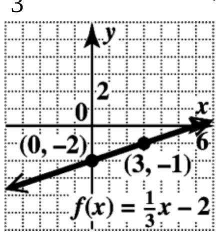
$$f(h)=4h^2+5$$

7. The graph of $f(x)=-1.5$ is a horizontal line.



The domain is $(-\infty,\infty)$ and the range is $\{-1.5\}$.

N7. The graph of $g(x)=\frac{1}{3}x-2$ is a line with slope $\frac{1}{3}$ and y-intercept $(0,\ -2)$.



The domain and range are $(-\infty,\ \infty)$.

Exercises

1. To emphasize that "y is a function of x" for a given function f, we use function notation and

2. For a function f, the notation $f(3)$ means <u>the value of the dependent variable when the independent variable is 3</u>. This is choice B.

3. The equation $2x+y=4$ has a straight <u>line</u> as its graph. One point that lies on the graph is $(3,\ \underline{-2})$. If we solve the equation for y and use function notation, we have a <u>linear</u> function, $f(x)=\underline{-2x+4}$. For this function, $f(3)=\underline{-2}$, meaning that the point $(3,\ \underline{-2})$ lies on the graph of the function.

4. Choice A, $f(x)=\frac{1}{4}x-\frac{5}{4}$, is the only choice that defines y as a linear function of x.

5.
$$\begin{aligned} f(x)&=-3x+4\\ f(0)&=-3(0)+4\\ &=0+4\\ &=4\end{aligned}$$

6.
$$\begin{aligned} g(x)&=-x^2+4x+1\\ g(0)&=-(0)^2+4(0)+1\\ &=0+0+1\\ &=1\end{aligned}$$

7.
$$\begin{aligned} f(x)&=-3x+4\\ f(-3)&=-3(-3)+4\\ &=9+4\\ &=13\end{aligned}$$

8.
$$\begin{aligned} f(x)&=-3x+4\\ f(-5)&=-3(-5)+4\\ &=15+4\\ &=19\end{aligned}$$

9.
$$\begin{aligned} g(x)&=-x^2+4x+1\\ g(-2)&=-(-2)^2+4(-2)+1\\ &=-(4)-8+1\\ &=-11\end{aligned}$$

10. $g(x)=-x^2+4x+1$

write $y = \underline{f(x)}$. Here, *f* is the name of the

function, *x* is a value from the domain, and $f(x)$ is a function value (or *y*-value) that corresponds to x. We read $f(x)$ as "*f* of *x*" (or "*f* at *x*").

$$g(-1) = -(-1)^2 + 4(-1) + 1$$
$$= -(1) - 4 + 1$$
$$= -4$$

11. $g(x) = -x^2 + 4x + 1$
$g(3) = -(3)^2 + 4(3) + 1$
$= -9 + 12 + 1$
$= 4$

12. $g(x) = -x^2 + 4x + 1$
$g(10) = -(10)^2 + 4(10) + 1$
$= -100 + 40 + 1$
$= -59$

13. $f(x) = -3x + 4$
$f(100) = -3(100) + 4$
$= -300 + 4$
$= -296$

14. $f(x) = -3x + 4$
$f(-100) = -3(-100) + 4$
$= 300 + 4$
$= 304$

15. $f(x) = -3x + 4$
$f\left(\frac{1}{3}\right) = -3\left(\frac{1}{3}\right) + 4$
$= -1 + 4$
$= 3$

16. $f(x) = -3x + 4$
$f\left(\frac{7}{3}\right) = -3\left(\frac{7}{3}\right) + 4$
$= -7 + 4$
$= -3$

17. $g(x) = -x^2 + 4x + 1$
$g(0.5) = -(0.5)^2 + 4(0.5) + 1$
$= -0.25 + 2 + 1$
$= 2.75$

18. $g(x) = -x^2 + 4x + 1$
$g(1.5) = -(1.5)^2 + 4(1.5) + 1$
$= -2.25 + 6 + 1$
$= 4.75$

19. $f(x) = -3x + 4$
$f(p) = -3(p) + 4$
$= -3p + 4$

20. $g(x) = -x^2 + 4x + 1$
$g(k) = -k^2 + 4k + 1$

21. $f(x) = -3x + 4$
$f(-x) = -3(-x) + 4$
$= 3x + 4$

22. $g(x) = -x^2 + 4x + 1$
$g(-x) = -(-x)^2 + 4(-x) + 1$
$= -x^2 - 4x + 1$

23. $f(x) = -3x + 4$
$f(x+2) = -3(x+2) + 4$
$= -3x - 6 + 4$
$= -3x - 2$

24. $f(x) = -3x + 4$
$f(x-2) = -3(x-2) + 4$
$= -3x + 6 + 4$
$= -3x + 10$

25. $f(x) = -3x + 4$
$f(2t+1) = -3(2t+1) + 4$
$= -6t - 3 + 4$
$= -6t + 1$

26. $f(x) = -3x + 4$
$f(3t-2) = -3(3t-2) + 4$
$= -9t + 6 + 4$
$= -9t + 10$

27. $g(x) = -x^2 + 4x + 1$
$g(\pi) = -\pi^2 + 4\pi + 1$

28. $g(x) = -x^2 + 4x + 1$
$g(t) = -t^2 + 4t + 1$

29. $f(x) = -3x + 4$
$f(x+h) = -3(x+h) + 4$
$= -3x - 3h + 4$

30. $f(x) = -3x + 4$
$f(a+b) = -3(a+b) + 4$
$= -3a - 3b + 4$

31. $g(x) = -x^2 + 4x + 1$

$$g\left(\frac{p}{3}\right) = -\left(\frac{p}{3}\right)^2 + 4\left(\frac{p}{3}\right) + 1$$

$$g\left(\frac{p}{3}\right) = -\frac{p^2}{9} + \frac{4p}{3} + 1$$

32. $g(x) = -x^2 + 4x + 1$

$$g\left(\frac{1}{x}\right) = -\left(\frac{1}{x}\right)^2 + 4\left(\frac{1}{x}\right) + 1$$

$$g\left(\frac{1}{x}\right) = -\frac{1}{x^2} + \frac{4}{x} + 1$$

33. **(a)** When $x = 2$, $y = -1$, so $f(2) = -1$.

(b) When $x = -1$, $y = -1$, so $f(-1) = -1$.

34. **(a)** When $x = 2$, $y = -5$, so $f(2) = -5$.

(b) When $x = -1$, $y = -5$, so $f(-1) = -5$.

35. **(a)** When $x = 2$, $y = 2$, so $f(2) = 2$.

(b) When $x = -1$, $y = 3$, so $f(-1) = 3$.

36. **(a)** When $x = 2$, $y = 5$, so $f(2) = 5$.

(b) When $x = -1$, $y = 11$, so $f(-1) = 11$.

37. **(a)** When $x = 2$, $y = 15$, so $f(2) = 15$.

(b) When $x = -1$, $y = 10$, so $f(-1) = 10$.

38. **(a)** When $x = 2$, $y = 1$, so $f(2) = 1$.

(b) When $x = -1$, $y = 7$, so $f(-1) = 7$.

39. **(a)** When $x = 2$,

(
b
)

W
h
e

43. **(a)** The point $(2, -3)$ is on the graph of *f*, so

$f(2) = -3$.

(b) The point $(-1, 2)$ is on the graph of *f*, so

$f(-1) = 2$.

44. **(a)** The point $(2, -2)$ is on the graph of *f*, so

$f(2) = -2$.

(b) The point $(-1, 4)$ is on the graph of *f*, so

$f(-1) = 4$.

45. **(a)** $f(x) = 3$: when $y = 3$, $x = 2$

(b) $f(x) = -1$: when $y = -1$, $x = 0$

(c) $f(x) = -3$: when $y = -3$, $x = -1$

46. **(a)** $f(x) = 4$: when $y = 4$, $x = 3$

(b) $f(x) = -2$: when $y = -2$, $x = 0$

(c) $f(x) = 0$: when $y = 0$, $x = 1$

47. **(a)** Solve the equation for *y*.

$$x + 3y = 12$$
$$3y = 12 - x$$
$$y = \frac{12 - x}{3}$$

Since $y = f(x)$, $f(x) = \frac{12 - x}{3} = -\frac{1}{3}x + 4$.

(b) $f(3) = \frac{12 - 3}{3} = \frac{9}{3} = 3$

48. **(a)** Solve the equation for *y*.

n $x = -1$,

$y = 4$, so $f(2) = 4$.

$y = 1$, so $f(-1) = 1$.

$x - 4y = 8$

$-4y = 8 - x$

$y = \frac{8 - x}{-4}$

$f(x) = \frac{8 - x}{-4} = \frac{1}{4}x - 2$

(b) $f(3) = \frac{8 - 3}{-4} = \frac{5}{-4} = -\frac{5}{4}$

40. **(a)** When $x = 2$, $y = 0$, so $f(2) = 0$.

(b) When $x = -1$, $y = -3$, so $f(-1) = -3$.

41. **(a)** The point (2, 3) is on the graph of *f*, so $f(2) = 3$.

(b) The point $(-1, -3)$ is on the graph of *f*, so $f(-1) = -3$.

42. **(a)** The point (2, 2) is on the graph of *f*, so $f(2) = 2$.

(b) The point $(-1, -4)$ is on the graph of *f*, so $f(-1) = -4$.

49. **(a)** Solve the equation for *y*.

$y + 2x^2 = 3$

$y = 3 - 2x^2$

Since $y = f(x)$, $f(x) = 3 - 2x^2$.

(b) $f(3) = 3 - 2(3)^2$
$= 3 - 2(9)$
$= -15$

50. **(a)** Solve the equation for *y*.

$$y - 3x^2 = 2$$
$$y = 2 + 3x^2$$
$$f(x) = 2 + 3x^2$$

(b) $f(3) = 2 + 3(3)^2$
$= 2 + 3(9)$
$= 29$

51. **(a)** Solve the equation for *y*.

$$4x - 3y = 8$$
$$-3y = 8 - 4x$$
$$y = \frac{8 - 4x}{-3}$$

Since $y = f(x)$,

$$f(x) = \frac{8 - 4x}{-3} = \frac{4}{3}x - \frac{8}{3}.$$

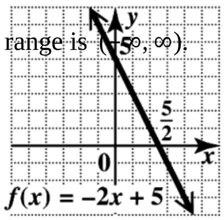
(b) $f(3) = \frac{8 - 4(3)}{-3} = \frac{8 - 12}{-3}$
$= \frac{-4}{-3} = \frac{4}{3}$

52. **(a)** Solve the equation for *y*.

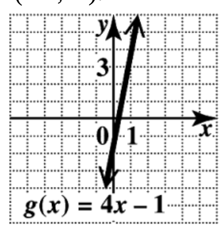
$$-2x + 5y = 9$$
$$5y = 9 + 2x$$
$$y = \frac{9 + 2x}{5}$$
$$f(x) = \frac{9 + 2x}{5} = \frac{2}{5}x + \frac{9}{5}$$

(b) $f(3) = \frac{9 + 2(3)}{5}$
$= \frac{9 + 6}{5} = \frac{15}{5} = 3$

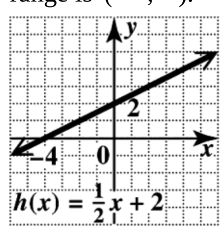
53. The graph will be a line. The intercepts are $(0, 5)$ and $\left(\frac{5}{2}, 0\right)$. The domain is $(-\infty, \infty)$. The range is $(-\infty, \infty)$.



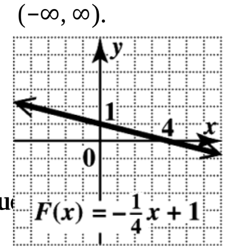
54. Using a *y*-intercept of (0, 1) and a slope of $m = 4 = \frac{4}{1}$, graph the line. From the graph we see that the domain is $(-\infty, \infty)$. The range is $(-\infty, \infty)$.



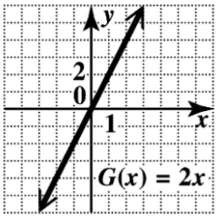
55. The graph will be a line. The intercepts are $(0, 2)$ and $(-4, 0)$. The domain is $(-\infty, \infty)$. The range is $(-\infty, \infty)$.



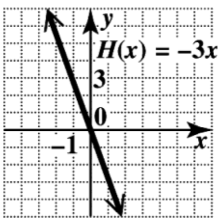
56. Using a *y*-intercept of (0, 1) and a slope of $m = -\frac{1}{4}$, we graph the line. From the graph, we see that the domain is $(-\infty, \infty)$. The range is $(-\infty, \infty)$.



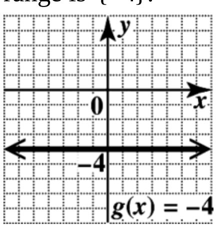
57. This line includes the points (0, 0), (1, 2), and (2, 4). The domain is $(-\infty, \infty)$. The range is $(-\infty, \infty)$.



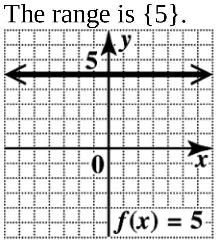
58. Using a *y*-intercept of (0, 0) and a slope of $m = -3 = \frac{-3}{1}$, we graph the line. From the graph we see that the domain is $(-\infty, \infty)$. The range is $(-\infty, \infty)$.



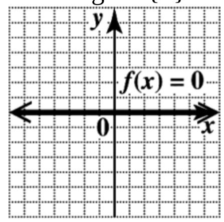
59. Using a *y*-intercept of $(0, -4)$ and a slope of $m = 0$, we graph the horizontal line. From the graph we see that the domain is $(-\infty, \infty)$. The range is $\{-4\}$.



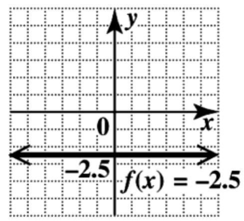
60. Draw the horizontal line through the point (0, 5). On the horizontal line the value of *x* can be any real number, so the domain is $(-\infty, \infty)$. The range is $\{5\}$.



61. Draw the horizontal line through the point (0, 0). On the horizontal line the value of *x* can be any real number, so the domain is $(-\infty, \infty)$. The range is $\{0\}$.



62. Draw the horizontal line through the point $(0, -2.5)$. On the horizontal line the value of *x* can be any real number, so the domain is $(-\infty, \infty)$. The range is $\{-2.5\}$.



63. $f(x) = 0$, or $y = 0$, is the *x*-axis.

64. No, because the equation of a line with an undefined slope is $x = a$. The ordered pairs have the form (a, y), where *a* is a constant and *y* is a variable. Thus, the number *a* corresponds to an infinite number of values of *y*.

65. **(a)** $f(x) = 3.75x$

$$f(3) = 3.75(3)$$
$$= 11.25 \text{ (dollars)}$$

(b) 3 is the value of the independent variable, which represents a package weight of 3 pounds. *f*(3) is the value of the dependent variable representing the cost to mail a 3-lb package.

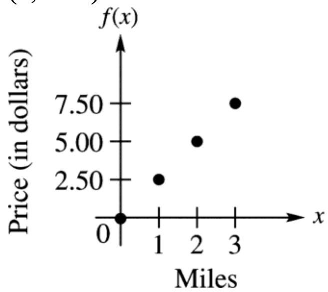
(c) The cost to mail a 5-lb package is $3.75(5) = \$18.75$. Using function notation, we have $f(5) = 18.75$.

66. **(a)**

x	*f*(*x*)
0	0
1	$2.50
2	$5.00
3	$7.50

(b) Since the charge equals the cost per mile, \$2.50, times the number of miles, the linear function that gives a rule for the amount charged is $f(x) = \underline{\$2.50x}$.

(c) To graph $y = f(x)$ for $x \in \{0, 1, 2, 3\}$, plot the points (0, 0), (1, 2.50), (2, 5.00), and (3, 7.50) from the chart.



67. **(a)** $f(x) = 12x + 100$

(b) $f(125) = 12(125) + 100$
$= 1500 + 100 = 1600$

The cost to print 125 t-shirts is $1600.

(c) $f(x) = 1000$
$12x + 100 = 1000$
$12x = 900$ Subtract 100.
$x = 75$ Divide by 12.

In function notation, $f(75) = 1000.$ The cost to print 75 t-shirts is $1000.

68. **(a)** $f(x) = 0.50x + 150$

(b) $f(250) = 0.50(250) + 150$
$= 125 + 150 = 275$

In function notation, $f(250) = 275.$

(c) $f(x) = 400$
$0.50x + 150 = 400$
$0.50x = 250$ Subtract 150.
$x = 500$ Multiply by 2.

It costs $400 to drive a rental car 500 miles.

69. **(a)** $f(2) = 1.1$

(b) y is -2.5 when x is 5. So, if $f(x) = -2.5,$ then $x = 5.$

(c) Let $(x_1, y_1) = (0, 3.5)$ and $(x_2, y_2) = (1, 2.3).$ Then

$$m = \frac{2.3 - 3.5}{} = \frac{-1.2}{} = -1.2.$$

(e) Use the slope-intercept form of the equation of a line and the information found in parts (c) and (d).
$f(x) = mx + b$
$f(x) = -1.2x + 3.5$

70. **(a)** $f(2) = 0.6$

(b) y is 2.1 when x is 3. So, if $f(x) = 2.1,$ then $x = 3.$

(c) Let $(x_1, y_1) = (0, -2.4)$ and $(x_2, y_2) = (1, -0.9).$ Then
$$m = \frac{-0.9 - (-2.4)}{1 - 0} = \frac{1.5}{1} = 1.5.$$
The slope is 1.5.

(d) When $x = 0,$ y is $-2.4,$ so the y-intercept is $(0, -2.4).$

(e) Use the slope-intercept form of the equation of a line and the information found in parts (c) and (d).
$f(x) = mx + b$
$f(x) = 1.5x - 2.4$

71. **(a)** The independent variable is t, the number of hours, and the possible values are in the set [0, 100]. The dependent variable is g, the number of gallons, and the possible values are in the set [0, 3000].

(b) The graph rises for the first 25 hours, so the water level increases for 25 hours. The graph falls for $t = 50$ to $t = 75,$ so the water level decreases for 25 hours.

(c) There are 2000 gallons in the pool when $t = 90.$

(d) $f(0)$ is the number of gallons in the pool at time $t = 0.$ Here, $f(0) = 0,$ which means the pool is empty at time 0.

(e) $f(25) = 3000;$ After 25 hours, there are 3000 gallons of water in the pool.

$1 - 0 \qquad 1$

The slope is -1.2.

(d) When $x = 0$, y is 3.5, so the y-intercept is (0, 3.5).

72. (a) For every hour, there is one and only one megawatt reading. Thus, the graph passes the vertical line test, so it is the graph of a function.

(b) We start the day at midnight and end the day at midnight. The domain is [0, 24].

(c) At 8 A.M., it appears that the number of megawatts used is halfway between 1100 and 1300. So 1200 is a good estimate.

(d) The most electricity was used at 18 hours (6 P.M.) and the least electricity was used at 4 hours (4 A.M.).

(e) $f(12) = 1800$; At 12 noon, electricity use is 1800 megawatts.

73. **(a)** Since the length of a man's femur is given, use the formula $h(r) = 69.09 + 2.24r$.

$h(56) = 69.09 + 2.24(56)$ Let $r = 56$.

$= 194.53$

The man is 194.53 cm tall.

(b) Use the formula $h(t) = 81.69 + 2.39t$.

$h(40) = 81.69 + 2.39(40)$ Let $t = 40$.

$= 177.29$

The man is 177.29 cm tall.

(c) Since the length of a woman's femur is given, use the formula $h(r) = 61.41 + 2.32r$.

$h(50) = 61.41 + 2.32(50)$ Let $r = 50$.

$= 177.41$

The woman is 177.41 cm tall.

(d) Use the formula $h(t) = 72.57 + 2.53t$.

$h(36) = 72.57 + 2.53(36)$ Let $t = 36$.

$= 163.65$

The woman is 163.65 cm tall.

74. **(a)** $f(x) = 0.91(3.14)x^2$

$f(0.8) = 0.91(3.14)(0.8)^2$

$= 1.828736$

To the nearest hundredth, the volume of the pool is 1.83 m^3.

(b) $f(x) = 0.91(3.14)x^2$

$f(1.0) = 0.91(3.14)(1.0)^2$

$= 2.8574$

To the nearest hundredth, the volume of the pool is 2.86 m^3.

(c) $f(x) = 0.91(3.14)x^2$

(d) $f(x) = 0.91(3.14)x^2$

$f(1.5) = 0.91(3.14)(1.5)^2$

$= 6.42915$

To the nearest hundredth, the volume of the pool is 6.43 m^3.

75. Because it falls from left to right, the slope is negative.

76. $m = \dfrac{-1-5}{3-(-1)} = \dfrac{-6}{4} = -\dfrac{3}{2}$

The slope is $-\frac{3}{2}$.

77. Parallel lines have the same slope, so their slope will be $-\frac{3}{2}$. Any perpendicular line will have a slope that is the negative reciprocal of $-\frac{3}{2}$.

$$-\frac{1}{\left(-\frac{3}{2}\right)} = \frac{2}{3}$$

The slope of a perpendicular line will be $\frac{2}{3}$.

78.
$$2y = -3x + 7$$
$$2(0) = -3x + 7$$
$$0 = -3x + 7$$
$$-7 = -3x$$
$$\frac{-7}{-3} = x$$
$$\frac{7}{3} = x$$

(7

$f(1.2) = 0.91(3.14)(1.2)^2 = 4.114656$

To the nearest hundredth, the volume of the pool is $4.11\,\text{m}^3$.

79.

80.

The x-intercept is $\left(3 \quad 2 \quad 2, 0\right)$.

$$2y = -3x + 7$$
$$2y = -3(0) + 7$$
$$2y = 0 + 7$$
$$2y = 7$$
$$y = \frac{7}{2}$$

The x-intercept is $\left(0, \frac{7}{2}\right)$.

$$f(x) = -\frac{3}{x} + \frac{7}{}$$

81. $f(x) = -\frac{3}{2}x + \frac{7}{2}$

$f(8) = -\frac{3}{2}(8) + \frac{7}{2}$

$f(8) = -\frac{24}{2} + \frac{7}{2}$

$f(8) = -\frac{17}{2}$

82. $f(x) = -\frac{3}{2}x + \frac{7}{2}$

$-8 = -\frac{3}{2}x + \frac{7}{2}$

$-\frac{23}{2} = -\frac{3}{2}x$

$23 = 3x$

$\frac{23}{3} = x$

Chapter 2 Review Exercises

1. For $x = 0$:

$3(0) + 2y = 10$

$2y = 10$

$y = 5 \quad (0, 5)$

For $y = 0$:

$3x + 2(0) = 10$

$3x = 10$

$x = \frac{10}{3} \quad \left(\frac{10}{3}, 0\right)$

For $x = 2$:

$3(2) + 2y = 10$

$6 + 2y = 10$

$2y = 4$

$y = 2 \quad (2, 2)$

For $y = -2$:

$3x + 2(-2) = 10$ 3

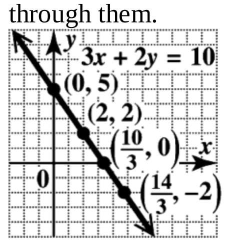
$3x - 4 = 10$

$3x = 14$

$x = \frac{14}{3} \quad \left(\frac{14}{3},\right.$

x	y
0	5
$\frac{10}{3}$	0
2	2
$\frac{14}{3}$	−2

Plot the ordered pairs, and draw the line through them.



2. For $x = 2$:

$2 - y = 8$

$-y = 6$

$y = -6 \quad (2, -6)$

For $y = -3$:

$x - (-3) = 8$

$x + 3 = 8$

$x = 5 \quad (5, -3)$

For $x = 3$:

$3 - y = 8$

$-y = 5$

$y = -5 \quad (3, -5)$

For $y = -2$:

$x - (-2) = 8$

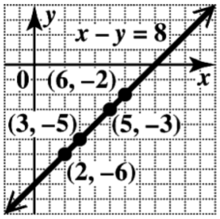
$x + 2 = 8$

$x = 6 \quad (6, -2)$

x	y
2	−6
5	−3
3	−5
6	−2

$\left(\quad -2 \right)$

Plot the ordered pairs, and draw the line through them.



3. To find the x-intercept, let $y = 0$.

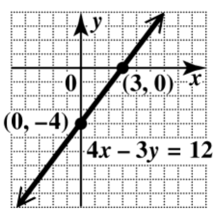
$$4x - 3y = 12$$
$$4x - 3(0) = 12$$
$$4x = 12$$
$$x = 3$$

The x-intercept is (3, 0).

To find the y-intercept, let $x = 0$.

$$4x - 3y = 12$$
$$4(0) - 3y = 12$$
$$-3y = 12$$
$$y = -4$$

The y-intercept is $(0, -4)$.
Plot the intercepts and draw the line through them.



4. To find the x-intercept, let $y = 0$.

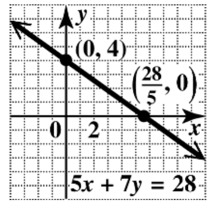
$$5x + 7y = 28$$
$$5x + 7(0) = 28$$
$$5x = 28$$
$$x = \frac{28}{5}$$

The x-intercept is $\left(\frac{28}{5}, 0\right)$.

To find the y-intercept, let $x = 0$.

$$5x + 7y = 28$$
$$5(0) + 7y = 28$$
$$7y = 28$$

Plot the intercepts and draw the line through them.



5. To find the x-intercept, let $y = 0$.

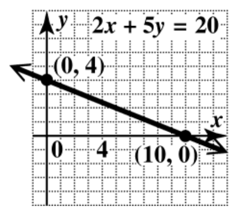
$$2x + 5y = 20$$
$$2x + 5(0) = 20$$
$$2x = 20$$
$$x = 10$$

The x-intercept is (10, 0).

To find the y-intercept, let $x = 0$.

$$2x + 5y = 20$$
$$2(0) + 5y = 20$$
$$5y = 20$$
$$y = 4$$

The y-intercept is (0, 4).
Plot the intercepts and draw the line through them.



6. To find the x-intercept, let $y = 0$.

$$x - 4y = 8$$
$$x - 4(0) = 8$$
$$x = 8$$

The x-intercept is (8, 0).

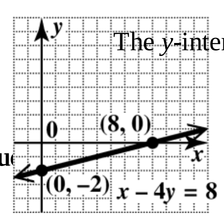
To find the y-intercept, let $x = 0$.

$$0 - 4y = 8$$
$$-4y = 8$$
$$y = -2$$

The y-intercept is $(0, -2)$.

$$y = 4$$

The y-intercept is (0, 4).



Plot the intercepts and draw the line through them.

7. By the midpoint formula, the midpoint of the segment with endpoints $(-8, -12)$ and (8, 16) is $\left(\frac{-8+8}{2}, \frac{-12+16}{2}\right) = \left(\frac{0}{2}, \frac{4}{2}\right) = (0, 2).$

8. By the midpoint formula, the midpoint of the segment with endpoints $(0, -5)$ and $(-9, 8)$ is $\left(\frac{0+(-9)}{2}, \frac{-5+8}{2}\right) = \left(\frac{-9}{2}, \frac{3}{2}\right) = \left(-\frac{9}{2}, \frac{3}{2}\right)$

9. $m = \frac{\text{change in } y}{\text{change in } x} = \frac{-5-2}{4-(-1)} = \frac{-7}{5} = -\frac{7}{5}$

10. $(x_1, y_1) = (0, 3)$ and $(x_2, y_2) = (-2, 4).$
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{-2-0} = \frac{1}{-2} = -\frac{1}{2}$

11. The slope of $y = 2x + 3$ is 2, the coefficient of x.

12. Write the equation in slope-intercept form.
$$3x - 4y = 5$$
$$-4y = -3x + 5$$
$$y = \frac{3}{4}x - \frac{5}{4}$$
The slope is $\frac{3}{4}$.

13. $x = 5$ is a vertical line and has undefined slope.

14. Write the equation in slope-intercept form.
$$3y = 2x + 5$$
$$y = \frac{2}{3}x + \frac{5}{3}$$
The slope of $3y = 2x + 5$ is $\frac{2}{3}$; all lines parallel to it will also have a slope of $\frac{2}{3}$.

15. Solve for y.

17. $m = \frac{\Delta y}{\Delta x} = \frac{1-(-1)}{-3-3} = \frac{2}{-6} = -\frac{1}{3}.$

18. The x-intercept is (2, 0) and the y-intercept is (0, 2). The slope is
$m = \frac{\text{change in } y}{\text{change in } x} = \frac{2-0}{0-2} = \frac{2}{-2} = -1.$

19. The line goes up from left to right, so it has positive slope.

20. The line goes down from left to right, so it has negative slope.

21. The line is vertical, so it has undefined slope.

22. The line is horizontal, so it has 0 slope.

23. To rise 1 foot, we must move 4 feet in the horizontal direction. To rise 3 feet, we must move $3(4) = 12$ feet in the horizontal direction.

24. Let $(x_1, y_1) = (1980, 21{,}000)$ and $(x_2, y_2) = (2012, 51{,}017).$
$$\text{average rate of change} = \frac{51{,}017 - 21{,}000}{2012 - 1980} = \frac{30{,}017}{32} \approx 938$$
The average rate of change is $938 per year (to the nearest dollar).

25. (a) Use the slope-intercept form with $m = -\frac{1}{3}$ and $b = -1$.
$$y = mx + b$$
$$y = -\frac{1}{3}x - 1$$

(b) $y = -\frac{1}{3}x - 1$
$$3x - y = 4$$
$$y = 3x - 4$$
The slope is 3; the slope of a line perpendicular

$$3y = -x - 3$$
$$x + 3y = -3$$

to it is $-\frac{1}{3}$ since $3\left(-\frac{1}{3}\right) = -1$.

16. $m = \frac{\Delta y}{\Delta x} = \frac{-4-5}{-1-(-1)} = \frac{-9}{0}$ Undefined

This is a vertical line; it has undefined slope.

26. (a) Use the slope-intercept form with $m = 0$ and $b = -2$.

$$y = mx + b$$
$$y = (0)x - 2$$
$$y = -2$$

(b) $y = -2$ is already in standard form.

27. **(a)** Use the point-slope form with $m = -\frac{4}{3}$ and $(x_1, y_1) = (2, 7)$.

$$y - y_1 = m(x - x_1)$$
$$y - 7 = -\frac{4}{3}(x - 2)$$
$$y - 7 = -\frac{4}{3}x + \frac{8}{3}$$
$$y = -\frac{4}{3}x + \frac{29}{3}$$

(b)
$$y = -\frac{4}{3}x + \frac{29}{3}$$
$$3y = -4x + 29$$
$$4x + 3y = 29$$

28. **(a)** Use the point-slope form with $m = 3$ and $(x_1, y_1) = (-1, 4)$.

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 3[x - (-1)]$$
$$y - 4 = 3(x + 1)$$
$$y - 4 = 3x + 3$$
$$y = 3x + 7$$

(b)
$$y = 3x + 7$$
$$-3x + y = 7$$
$$3x - y = -7$$

29. **(a)** The equation of any vertical line is in the form $x = k$. Since the line goes through (2, 5), the equation is $x = 2$. (Slope-intercept form is not possible.)

(b) $x = 2$ is already in standard form.

30. **(a)** Find the slope.

$$m = \frac{\Delta y}{\Delta x} = \frac{4 - (-5)}{1 - 2} = \frac{9}{-1} = -9$$

Use the point-slope form with $m = -9$ and $(x_1, y_1) = (2, -5)$.

31. **(a)** Find the slope.

$$m = \frac{\Delta y}{\Delta x} = \frac{6 - (-1)}{2 - (-3)} = \frac{7}{5}$$

Use the point-slope form with $m = \frac{7}{5}$ and $(x_1, y_1) = (2, 6)$.

$$y - y_1 = m(x - x_1)$$
$$y - 6 = \frac{7}{5}(x - 2)$$
$$y - 6 = \frac{7}{5}x - \frac{14}{5}$$
$$y = \frac{7}{5}x + \frac{16}{5}$$

(b)
$$y = \frac{7}{5}x + \frac{16}{5}$$
$$5y = 7x + 16$$
$$-7x + 5y = 16$$
$$7x - 5y = -16$$

32. **(a)** From Exercise 18, we have $m = -1$ and a y-intercept of (0, 2). The slope-intercept form is $y = -1x + 2$, or $y = -x + 2$.

(b)
$$y = -x + 2$$
$$x + y = 2$$

33. **(a)** Parallel to $4x - y = 3$ and through $(7, -1)$

Writing $4x - y = 3$ in slope-intercept form gives us $y = 4x - 3$, which has slope 4.

Lines parallel to it will also have slope 4. The line with slope 4 through $(7, -1)$ is

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = 4(x - 7)$$
$$y + 1 = 4x - 28$$
$$y = 4x - 29.$$

$$y - y_1 = m(x - x_1)$$

(b)

$$y - (-5) = -9(x - 2)$$
$$y + 5 = -9x + 18$$
$$y = -9x + 13$$

(b) $y = -9x + 13$

$9x + y = 13$

$$y = 4x - 29$$
$$-4x + y = -29$$
$$4x - y = 29$$

34. **(a)** Write the equation in slope-intercept form.

$$2x - 5y = 7$$
$$-5y = -2x + 7$$
$$y = \frac{2}{5}x - \frac{7}{5}$$

$y = \frac{2}{5}x - \frac{7}{5}$ has slope $\frac{2}{5}$ and is perpendicular to lines with slope $-\frac{5}{2}$. The line with slope $-\frac{5}{2}$ through (4, 3) is

$$y - y_1 = m(x - x_1)$$
$$y - 3 = -\frac{5}{2}(x - 4)$$
$$y - 3 = -\frac{5}{2}x + 10$$
$$y = -\frac{5}{2}x + 13.$$

(b) $y = -\frac{5}{2}x + 13$

$$2y = -5x + 26$$
$$5x + 2y = 26$$

35. The fixed cost is \$159, so that is the value of *b*. The variable cost is \$47, so $y = mx + b = 47x + 159.$

The cost of a one-year membership can be found by substituting 12 for *x*.

$$y = 47x + 159.$$
$$y = 47(12) + 159$$
$$= 564 + 159 = 723$$

The cost is \$723.

36. **(a)** Use (8, 2476) and (12, 2628).

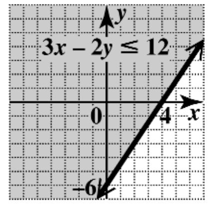
$$m = \frac{\Delta y}{\Delta x} = \frac{2628 - 2476}{12 - 8} = \frac{152}{4} = 38$$

Use the point-slope form of a line.

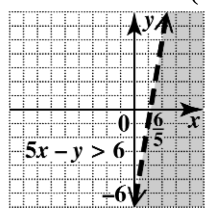
$$y - y_1 = m(x - x_1)$$
$$y - 2476 = 38(x - 8)$$
$$y - 2476 = 38x - 304$$
$$y = 38x + 2172$$

The slope, 38, indicates that the revenue from skiing facilities increased by an average of \$38 million each year from 2008

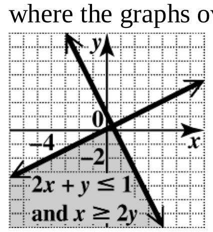
Since (0, 0) satisfies the inequality, shade the region on the side of the line containing (0, 0).



38. Graph $5x - y = 6$ as a dashed line through $(0, -6)$ and $\left(\frac{6}{5}, 0\right)$. Use (0, 0) as a test point. Since (0, 0) does not satisfy the inequality, shade the region on the side of the line that does not contain (0, 0).



39. Graph $2x + y = 1$ as a solid line through $\left(\frac{1}{2}, 0\right)$ and (0, 1), and shade the region on the side containing (0, 0) since it satisfies the inequality. Next, graph $x = 2y$ as a solid line through (0, 0) and (2, 1), and shade the region on the side containing (2, 0) since $2 > 2(0)$, or $2 > 0$, is true. The intersection is the region where the graphs overlap.



40. Graph $x = 2$ as a solid vertical line through (2, 0). Shade the region to the right of $x = 2$. Graph $y = 2$ as a solid horizontal line through (0, 2). Shade the region above $y = 2$. The to 2012.

(b) The year 2010 corresponds to $x = 10$.

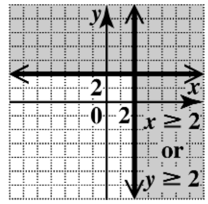
$y = 38(10) + 2172$

$= 380 + 2172 = 2552$

According to the equation from part (a), we estimate the revenue from skiing facilities to be $2552 million (to the nearest million).

37. Graph $3x - 2y = 12$ as a solid line through $(0, -6)$ and (4, 0). Use (0, 0) as a test point.

graph of $x \geq 2$ or $y \geq 2$ includes all the shaded regions.



41. The domain, the set of x-values, is $\{-4, 1\}$. The range, the set of y-values, is $\{2, -2, 5, -5\}$. Since each x-value has more than one y-value, the relation is not a function.

42. The relation can be described by the set of ordered pairs
$\{(9, 32), (11, 47), (4, 47), (17, 69), (25, 14)\}$.
The relation is a function since for each x-value, there is only one y-value.
The domain is the set of x-values:
$\{9, 11, 4, 17, 25\}$.
The range is the set of y-values:
$\{32, 47, 69, 14\}$.

43. The domain, the x-values of the points on the graph, is $[-4, 4]$. The range, the y-values of the points on the graph, is $[0, 2]$. Since a vertical line intersects the graph of the relation in at most one point, the relation is a function.

44. The x-values are negative or zero, so the domain is $(-\infty, 0]$. The y-values can be any real number, so the range is $(-\infty, \infty)$. A vertical line, such as $x = -3$, will intersect the graph twice, so by the vertical line test, the relation is not a function.

45. For any value of x, there is exactly one value of y, so the equation defines a function. The function is a linear function. The domain is the set of all real numbers, $(-\infty, \infty)$.

46. For any value of x, there are many values of y. For example, (1, 0) and (1, 1) are both solutions of the inequality that have the same x-value but different y-values. The inequality does not define a function. The domain is the set of all real numbers, $(-\infty, \infty)$.

47. For any value of x, there is exactly one value of y, so the equation defines a function. The domain is the set of all real numbers, $(-\infty, \infty)$.

48. Given any value of x, y is found by multiplying x by 4, adding 7, and taking the square root of the result. This process produces exactly one value of y for each x-value in the domain, so the equation defines a function. Since the radicand must be nonnegative,
$4x + 7 \ge 0$

49. The ordered pairs (4, 2) and $(4, -2)$ both satisfy the equation. Since one value of x, 4, corresponds to two values of y, 2 and -2, the equation does not define a function. Because x is equal to the square of y, the values of x must always be nonnegative. The domain is $[0, \infty)$.

50. Given any value of x, y is found by subtracting 6 and then dividing the result into 7. This process produces exactly one value of y for each x-value in the domain, so the equation defines a function. The domain includes all real numbers except those that make the denominator 0, namely 6. The domain is $(-\infty, 6) \cup (6, \infty)$.

51. $f(0) = -2(0)^2 + 3(0) - 6 = -6$

52. $f(2.1) = -2(2.1)^2 + 3(2.1) - 6$
$= -8.82 + 6.3 - 6 = -8.52$

53. $f\left(-\frac{1}{2}\right) = -2\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) - 6$
$= -\frac{1}{2} - \frac{3}{2} - 6 = -8$

54. $f(k) = -2k^2 + 3k - 6$

55. $2x^2 - y = 0$
$-y = -2x^2$
$y = 2x^2$
Since $y = 2x^2$, $f(x) = 2x^2$.

$4x \ge -7$
$x \ge \frac{7}{4}$

$f(x) = 2x$

$f(3) = 2(3)^2$

$= 2(9)$

$= 18$

56. Solve for y in terms of x.

$2x - 5y = 7$

$2x - 7 = 5y$

The domain is $\left[-\frac{7}{4}, \infty\right)$.

$\frac{2}{5}x - \frac{7}{5} = y$

Thus, choice C is correct.

57. The graph of a constant function is a horizontal line.

58. **(a)** For each year, there is exactly one life expectancy associated with the year, so the table defines a function.

(b) The domain is the set of years—that is,

{1960, 1970, 1980, 1990, 2000, 2010}.

The range is the set of life expectancies—that is, {69.7, 70.8, 73.7, 75.4, 76.8, 78.7}.

(c) Answers will vary. Two possible answers are (1960, 69.7) and (2010, 78.7).

(d) $f(1980) = 73.7.$ In 1980, life expectancy at birth was 73.7 yr.

(e) Since $f(2000) = 76.8,\ x = 2000.$

Chapter 2 Mixed Review Exercises

1. Determine the slope of both lines.

$$3x + y = 4 \quad\text{and}\quad 3y = x - 6$$

$$y = -3x + 4 \qquad y = \frac{x}{3} - 2$$

$$m = -3 \qquad m = \frac{1}{3}$$

The lines are perpendicular because their slopes are negative reciprocals of each other.

2. Determine the slope of both lines.

$$4x + 3y = 8 \quad\text{and}\quad 6y = 7 - 8x$$

$$3y = -4x + 8 \qquad 6y = -8x + 7$$

$$y = -\frac{4}{3}x + \frac{8}{3} \qquad y = -\frac{4}{3}x + \frac{7}{6}$$

$$m = -\frac{4}{3} \qquad m = -\frac{4}{3}$$

The lines are parallel because their slopes are the same.

3. Use (2003, 46.8) and (2011, 32.7).

$$\text{average rate of change} = \frac{32.7 - 46.8}{2011 - 2003} = \frac{-14.1}{8} \approx -1.8$$

The average rate of change is -1.8 lb per year. From 2003 to 2011 the per capita consumption

of potatoes decreased by an average of 1.8 lb per year.

4. The point (0, 46.8) is the y-intercept, so the

6. Use the two points to determine the slope.

$$m = \frac{3 - 4}{0 - (-2)} = \frac{-1}{2} = -\frac{1}{2}$$

The point (0, 3) is the y-intercept, so the equation is $y = -\frac{1}{2}x + 3.$ Change the equation from slope-intercept form to standard form.

$$y = -\frac{1}{2}x + 3$$

$$\frac{1}{2}x + y = 3$$

$$x + 2y = 6$$

7. $x = 2$ is a vertical line, so a perpendicular line to $x = 2$ would be a horizontal line. The general form of a horizontal line is $y = a.$ Use the y-value from an ordered pair to find the equation. Using the the point $(2,\ -3)$ gives the equation $y = -3.$

8. Choice A gives an equation whose graph has one intercept since it is a vertical line and crosses only the x-axis. Choice B gives an equation whose graph has one intercept since the graph crosses the x-axis and y-axis at the same point. Choice D gives an equation whose graph has one intercept since it is a horizontal line and crosses only the y-axis.

9. In $y < 4x + 3,$ the $<$ symbol indicates that the graph is a dashed boundary line and that the shading is below the line, so the correct choice is D.

10. **(a)** The graph has a value of negative one when *x* has a value of negative two.

$f(-2) = -1$

(b) The graph has a value of negative two when *x* has a value of zero.

equation is $y = -1.8x + 46.8.$

5. $y = mx + b$

$y = 3x + b$

$0 = 3(0) + b$

$0 = b$

$y = 3x$

$f(0) = -2$

(c) When the graph has a *y*-value of negative three, the corresponding *x*-value is two.

$f(2) = -3$

(d) The graph catches all *x*- and *y*-values. Therefore, the domain is $(-\infty, \infty)$, and the

range is $(-\infty, \infty)$.

Chapter 2 Test

1. For $x = 1$:

$$2(1) - 3y = 12$$
$$2 - 3y = 12$$
$$-3y = 10$$
$$y = -\frac{10}{3} \quad \left(1, -\frac{10}{3}\right)$$

For $x = 3$:

$$2(3) - 3y = 12$$
$$6 - 3y = 12$$
$$-3y = 6$$
$$y = -2 \quad (3, -2)$$

For $y = -4$:

$$2x - 3(-4) = 12$$
$$2x + 12 = 12$$
$$2x = 0$$
$$x = 0 \quad (0, -4)$$

x	y
1	$-\frac{10}{3}$
3	-2
0	-4

2. To find the x-intercept, let $y = 0$.

$$3x - 2(0) = 20$$
$$3x = 20$$
$$x = \frac{20}{3}$$

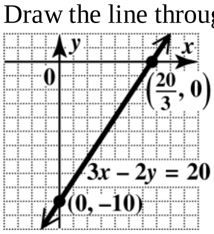
The x-intercept is $\left(\frac{20}{3}, 0\right)$.

To find the y-intercept, let $x = 0$.

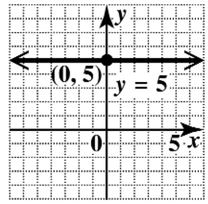
$$3(0) - 2y = 20$$
$$-2y = 20$$
$$y = -10$$

The y-intercept is $(0, -10)$.

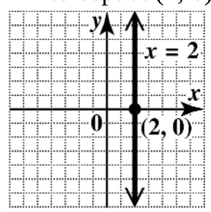
Draw the line through these two points.



3. The graph of $y = 5$ is the horizontal line with slope 0 and y-intercept (0, 5). There is no x-intercept.



4. The graph of $x = 2$ is the vertical line with x-intercept at (2, 0). There is no y-intercept.



5. $m = \dfrac{\Delta y}{\Delta x} = \dfrac{-1-4}{-4-6} = \dfrac{-5}{-10} = \dfrac{1}{2}$

The slope of the line is $\frac{1}{2}$.

6. The graph of a line with undefined slope is the graph of a vertical line.

7. Find the slope of each line.

$$5x - y = 8$$
$$-y = -5x + 8$$
$$y = 5x - 8$$

The slope is 5.

$$5y = -x + 3$$
$$y = -\frac{1}{5}x + \frac{3}{5}$$

The slope is $-\frac{1}{5}$.

Since $5\left(-\frac{1}{5}\right) = -1$, the two slopes are negative reciprocals and the lines are perpendicular.

8. Find the slope of each line.

$$2y = 3x + 12$$
$$y = \frac{3}{2}x + 6$$

$$3y = 2x - 5$$
$$y = \frac{2}{3}x - \frac{5}{3}$$

The slope is $\frac{2}{3}$.

The slope is $\frac{2}{3}$.

The lines are neither parallel nor perpendicular.

9. Use the points (1980, 119,000) and (2012, 92,200).
average rate of change

$$= \frac{\text{change in } y}{\text{change in } x} = \frac{92{,}200 - 119{,}000}{2012 - 1980} = \frac{-26{,}800}{32} \approx -838$$

The average rate of change is about -838 farms per year, that is, the number of farms decreased by about 838 each year from 1980 to 2012.

10. **(a)** Let $m = -5$ and $(x_1, y_1) = (4, -1)$ in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -5(x - 4)$$
$$y + 1 = -5x + 20$$
$$y = -5x + 19$$

(b) $y = -5x + 19$ From part (a)

$5x + y = 19$ Standard form

11. **(a)** A horizontal line has equation $y = k$. Here $k = 14$, so the line has equation $y = 14$.

(b) $y = 14$ is already in standard form.

12. **(a)** First find the slope.

$$m = \frac{\Delta y}{\Delta x} = \frac{-1 - 3}{6 - (-2)} = \frac{-4}{8} = -\frac{1}{2}$$

Use $m = -\frac{1}{2}$ and $(x_1, y_1) = (-2, 3)$ in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = -\frac{1}{2}[x - (-2)]$$

(b) $y = -\frac{1}{2}x + 2$

$$\frac{1}{2}x + y = 2$$
$$2\left(\frac{1}{2}x + y\right) = 2(2)$$
$$x + 2y = 4$$

13. **(a)** The equation of any vertical line is in the form $x = k$. Since the line goes through $(5, -6)$, the equation is $x = 5$. Writing $x = 5$ in slope-intercept form is not possible since there is no y-term.

(b) From part (a), the standard form is $x = 5$.

14. **(a)** To find the slope of $3x + 5y = 6$, write the equation in slope-intercept form by solving for y.

$$3x + 5y = 6$$
$$5y = -3x + 6$$
$$y = -\frac{3}{5}x + \frac{6}{5}$$

The slope is $-\frac{3}{5}$, so a line parallel to it also has slope $-\frac{3}{5}$. Let $m = -\frac{3}{5}$ and $(x_1, y_1) = (-7, 2)$ in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -\frac{3}{5}[x - (-7)]$$
$$y - 2 = -\frac{3}{5}(x + 7)$$
$$y - 2 = -\frac{3}{5}x - \frac{21}{5}$$
$$y = -\frac{3}{5}x - \frac{11}{5}$$

(b) $y = -\frac{3}{5}x - \frac{11}{5}$

$$y - 3 = -\frac{1}{2}(x + 2)$$
$$y - 3 = -\frac{1}{2}x - 1$$
$$y = -\frac{1}{2}x + 2$$

5 5

$$\frac{3}{5}x + y = -\frac{11}{5}$$
$$5\left(\frac{3}{5}x + y\right) = 5\left(-\frac{11}{5}\right)$$
$$3x + 5y = -11$$

15. **(a)** Since $y = 2x$ is in slope-intercept form $(b = 0)$, the slope, m, of $y = 2x$ is 2. A line perpendicular to it has a slope that is the negative reciprocal of 2—that is, $-\frac{1}{2}$. Let $m = -\frac{1}{2}$ and $(x_1, y_1) = (-7, 2)$ in the point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= -\frac{1}{2}(x + 7) \\ y - 2 &= -\frac{1}{2}x - \frac{7}{2} \\ y &= -\frac{1}{2}x - \frac{3}{2} \end{aligned}$$

(b)

$$\begin{aligned} y &= -\frac{1}{2}x - \frac{3}{2} \\ \frac{1}{2}x + y &= -\frac{3}{2} \\ 2\left(\frac{1}{2}x + y\right) &= 2\left(-\frac{3}{2}\right) \\ x + 2y &= -3 \end{aligned}$$

16. Positive slope means that the line goes up from left to right. The only line that has positive slope and a negative y-coordinate for its y-intercept is choice B.

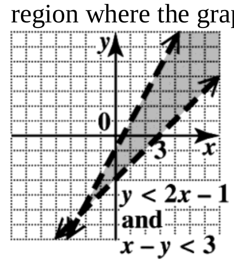
17. **(a)** The fixed cost is $45, so that is the value of b. The variable cost is $142.75, so $y = mx + b = 142.75x + 45.$

(b) $y = 142.75(6) + 45$ Let $x = 6$.
$= 901.5$

The cost for 6 tickets and a parking pass is $901.50.

18. Graph the line $3x - 2y = 6$, which has intercepts (2, 0) and $(0, -3)$, as a dashed line since the inequality involves >. Test (0, 0), which yields $0 > 6$, a false statement. Shade the region that does not include (0, 0).

19. First graph $y = 2x - 1$ as a dashed line through (2, 3) and $(0, -1)$. Test (0, 0), which yields $0 < -1$, a false statement. Shade the side of the line not containing (0, 0).
Next, graph $x - y = 3$ as a dashed line through (3, 0) and $(0, -3)$. Test (0, 0), which yields $0 < 3$, a true statement. Shade the side of the line containing (0, 0). The intersection is the region where the graphs overlap.



20. Choice D is the only graph that passes the vertical line test.

21. Choice D does not define a function, since its domain (input) element 0 is paired with two different range (output) elements, 1 and 2.

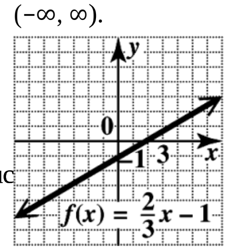
22. The x-values are greater than or equal to zero, so the domain is $[0, \infty)$. Since y can be any value, the range is $(-\infty, \infty)$.

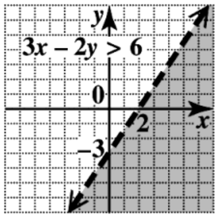
23. The domain is the set of x-values: $\{0, -2, 4\}$. The range is the set of y-values: $\{1, 3, 8\}$.

24. **(a)**
$$\begin{aligned} f(1) &= -(1)^2 + 2(1) - 1 \\ &= -1 + 2 - 1 \\ &= 0 \end{aligned}$$

(b) $f(a) = -a^2 + 2a - 1$

25. This function represents a line with y-intercept $(0, -1)$ and x-intercept $\left(\frac{3}{2}, 0\right)$.
Draw the line through these two points. The domain is $(-\infty, \infty)$, and the range is $(-\infty, \infty)$.




y
3x – 2y > 6
0
2
x
–3

Chapters R–2 Cumulative Review

Exercises

1. The absolute value of a negative number is a positive number, and the additive inverse of the same negative number is the same positive number. For example, suppose the negative number is -5:

$|-5| = -(-5) = 5$ and $-(-5) = 5$

The statement is *always true*.

2. The sum of two negative numbers is another negative number, so the statement is *never true*.

3. The statement is *sometimes true*. For example, $3+(-3)=0,$ but $3+(-1)=2\neq 0.$

4.
$$\begin{aligned}-|-2|-4+|-3|+7 &= -2-4+3+7\\ &= -6+3+7\\ &= -3+7\\ &= 4\end{aligned}$$

5. $(-0.8)^2 = (-0.8)(-0.8) = 0.64$

6. $\sqrt{-64}$ is not a real number.

7.
$$\begin{aligned}-(-4x+3) &= -(-4x)-3\\ &= 4x-3\end{aligned}$$

8.
$$\begin{aligned}&3x^2-4x+4+9x-x^2\\ &= 3x^2-x^2-4x+9x+4\\ &= 2x^2+5x+4\end{aligned}$$

9.
$$\begin{aligned}\frac{(4^2-4)-(-1)7}{4+(-6)} &= \frac{(16-4)-(-7)}{-2}\\ &= \frac{12+7}{-2} = -\frac{19}{2}\end{aligned}$$

10.
$$\begin{aligned}&\sqrt{25}-5(-1)^2\\ &= 5-5(1)\\ &= 5-5\\ &= 0\end{aligned}$$

11. $-3(2q-3p) = -3\left[2\left(\frac{1}{2}\right)-3(-4)\right]$

12.
$$\begin{aligned}\frac{\sqrt{r}}{8p+2r} &= \frac{\sqrt{16}}{8(-4)+2(16)}\\ &= \frac{4}{-32+32}\\ &= \frac{4}{0}, \text{ which is undefined.}\end{aligned}$$

13.
$$\begin{aligned}2z-5+3z &= 4-(z+2)\\ 5z-5 &= 2-z\\ 6z &= 7\\ z &= \frac{7}{6}\end{aligned}$$

The solution set is $\left\{\frac{7}{6}\right\}$.

14. Multiply both sides by the LCD, 10.

$$\frac{3x-1}{5}+\frac{x+2}{2} = -\frac{3}{10}$$

$$10\left(\frac{3x-1}{5}+\frac{x+2}{2}\right) = 10\left(-\frac{3}{10}\right)$$

$$\begin{aligned}2(3x-1)+5(x+2) &= -3\\ 6x-2+5x+10 &= -3\\ 11x+8 &= -3\\ 11x &= -11\\ x &= -1\end{aligned}$$

The solution set is $\{-1\}$.

15. Let x denote the side of the original square and $4x$ the perimeter. Now $x+4$ is the side of the new square and $4(x+4)$ is its perimeter. "The perimeter would be 8 inches less than twice the perimeter of the original square" translates to the following.

$$\begin{aligned}4(x+4) &= 2(4x)-8\\ 4x+16 &= 8x-8\\ 24 &= 4x\\ 6 &= x\end{aligned}$$

The length of a side of the original square is 6 inches.

$$= -3(1+12)$$
$$= -3(13)$$
$$= -39$$

16. Let x = the time it takes for the planes to be 2100 miles apart. Make a table. Use the formula $d = rt$.

	r	*t*	*d*
Eastbound Plane	550	x	$550x$
Westbound Plane	500	x	$500x$

The total distance is 2100 miles.

$$550x + 500x = 2100$$
$$1050x = 2100$$
$$x = 2$$

It will take 2 hr for the planes to be 2100 mi apart.

17. $-4 < 3 - 2k < 9$

$$-7 < -2k < 6$$

Divide by -2, and reverse the inequalities.

$$\frac{7}{2} > k > -3$$

$$-3 < k < \frac{7}{2} \quad \text{Equivalent inequality}$$

The solution set is $\left(-3, \frac{7}{2}\right)$.

18. $-0.3x + 2.1(x - 4) \le -6.6$

$$-3x + 21(x - 4) \le -66 \quad \text{Multiply by 10.}$$
$$-3x + 21x - 84 \le -66$$
$$18x - 84 \le -66$$
$$18x \le 18$$
$$x \le 1$$

The solution set is $(-\infty, 1]$.

19. $\frac{1}{2}x > 3$ and $\frac{1}{3}x < \frac{8}{3}$

$x > 6$ $\qquad$ $x < 8$

This is the intersection. The solution set is

21. $|2k - 7| + 4 = 11$

$$|2k - 7| = 7$$

$2k - 7 = 7$ or $2k - 7 = -7$

$2k = 14$ $\qquad$ $2k = 0$

$k = 7$ or $k = 0$

The solution set is $\{0, 7\}$.

22. $|3x + 6| \ge 0$

The absolute value of an expression is always nonnegative, so the inequality is true for any real number x.

The solution set is $(-\infty, \infty)$.

23. To find the x-intercept, let $y = 0$.

$$3x + 5(0) = 12$$
$$3x = 12$$
$$x = 4$$

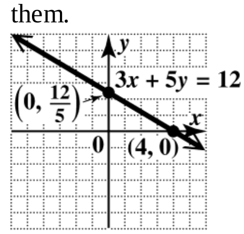
The x-intercept is (4, 0).

To find the y-intercept, let $x = 0$.

$$3(0) + 5y = 12$$
$$5y = 12$$
$$y = \frac{12}{5}$$

The y-intercept is $\left(0, \frac{12}{5}\right)$.

Plot the intercepts and draw the line through them.



24. **(a)** The slope of line AB is

$$m = \frac{\Delta y}{\Delta x} = \frac{-5-1}{3-(-2)} = \frac{-6}{5} = -\frac{6}{5}.$$

$(6, 8)$.

20. $-5x+1 \geq 11$ or $3x+5 > 26$

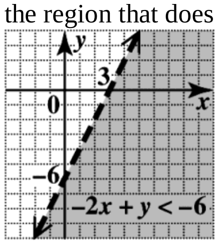
$-5x \geq 10$ $\quad 3x > 21$

$x \leq -2$ or $x > 7$

This is the union. The solution set is $(-\infty, -2] \cup (7, \infty)$.

(b) The slope of a line perpendicular to line *AB* is the negative reciprocal of $-\frac{6}{5}$, which is $\frac{5}{6}$.

25. Graph the line $-2x + y = -6$, which has intercepts (3, 0) and $(0, -6)$, as a dashed line since the inequality involves <. Test (0, 0), which yields $0 < -6$, a false statement. Shade the region that does not include (0, 0).



26. **(a)** To write an equation of this line, let $m = -\frac{3}{4}$ and $b = -1$ in the slope-intercept form.

$$y = mx + b$$
$$y = -\frac{3}{4}x - 1$$

(b)
$$y = -\frac{3}{4}x - 1$$
$$4y = -3x - 4$$
$$3x + 4y = -4$$

27. **(a)** First find the slope of the line.

$$m = \frac{\Delta y}{\Delta x} = \frac{1-(-3)}{1-4} = \frac{4}{-3} = -\frac{4}{3}$$

Now substitute $(x_1, y_1) = (4, -3)$ and $m = -\frac{4}{3}$ in the point-slope form. Then solve for *y*.

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = -\frac{4}{3}(x - 4)$$
$$y + 3 = -\frac{4}{3}x + \frac{16}{3}$$
$$y = -\frac{4}{3}x + \frac{7}{3}$$

(b)
$$y = -\frac{4}{3}x + \frac{7}{3}$$
$$3y = -4x + 7$$
$$4x + 3y = 7$$

28. The domain of the relation consists of the elements in the leftmost figure—that is, {14, 91, 75, 23}.
The range of the relation consists of the elements in the rightmost figure—that is, {9, 70, 56, 5}.
Since the element 75 in the domain is paired with two different values, 70 and 56, in the range, the relation is not a function.