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Chapter 2: One-Dimensional Kinematics

Answers to Even-Numbered Conceptual Questions

An odometer measures the distance traveled by a car. You can tell this by the fact that an odometer has a nonzero reading after a round trip.

No. Their velocities are different because they travel in different directions.

Since the car circles the track its direction of motion must be changing. Therefore, its velocity changes as well. Its speed, however, can be constant.

(a) The time required to stop is doubled. **(b)** The distance required to stop increases by a factor of four.

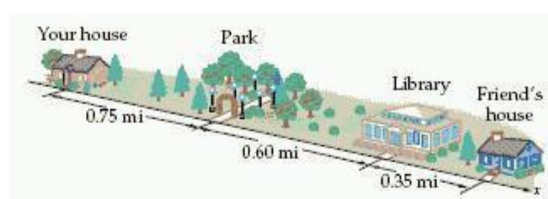
Yes, if it moves with constant velocity.

(a) No. If air resistance can be ignored, the acceleration of the ball is the same at each point on its flight. **(b)** Same answer as part (a).

(a) No. Displacement is the *change* in position, and therefore it is independent of the location chosen for the origin. **(b)** Yes. In order to know whether an object's displacement is positive or negative, we need to know which direction has been chosen to be positive.

Solutions to Problems and Conceptual Exercises

Picture the Problem: You walk in both the positive and negative directions along a straight line.



Strategy: The distance is the total length of travel, and the displacement is the net change in position. We place the origin at the location labeled "Your house."

Solution: 1. (a) Add the lengths: $(0.75 + 0.60 \text{ mi}) + (0.60 \text{ mi}) = \boxed{1.95 \text{ mi}}$

2. (b) Subtract x_i from x_f to find the displacement. $\Delta x = x_f - x_i = 0.75 - 0.00 \text{ mi} = \boxed{0.75 \text{ mi}}$

Insight: The distance traveled is always positive, but the displacement can be negative.

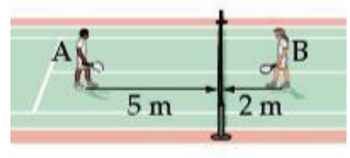
Picture the Problem: You walk in both the positive and negative directions along a straight line.

Strategy: The distance is the total length of travel, and the displacement is the net change in position. We place the origin at the location labeled "Your house."

Solution: 1. (a) Add the lengths: $(0.60 + 0.35 \text{ mi}) + (0.75 + 0.60 + 0.35 \text{ mi}) = \boxed{2.65 \text{ mi}}$

2. (b) Subtract x_i from x_f to find the displacement. $\Delta x = x_f - x_i = 0.00 - 0.75\text{ mi} = \boxed{-0.75\text{ mi}}$

Insight: The distance traveled is always positive, but the displacement can be negative.



Picture the Problem: Player A walks in the positive direction and player B walks in the negative direction.

Strategy: In each case the distance is the total length of travel, and the displacement is the net change in position.

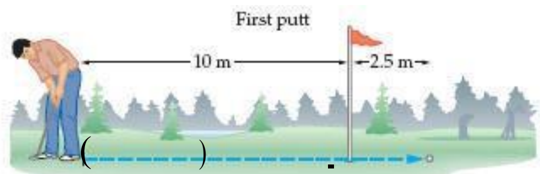
Solution: 1. (a) Note the distance traveled by player A: $\boxed{\ \text{m}}$

The displacement of player A is positive: $x = x_f - x_i = 5\ \text{m} - 0\ \text{m} = \boxed{5\ \text{m}}$

(b) Note the distance traveled by player B: $\boxed{\ \text{m}}$

The displacement of player B is negative. Let the origin be at the initial position of player A. $x = x_f - x_i = 5\ \text{m} - 7\ \text{m} = \boxed{-2\ \text{m}}$

Insight: The distance traveled is always positive, but the displacement can be negative.



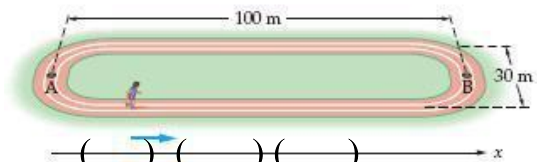
Picture the Problem: The ball is putted in the positive direction and then the negative direction.

Strategy: The distance is the total length of travel, and the displacement is the net change in position.

Solution: 1. (a) Add the lengths: $(10 + 2.5\ \text{m}) + 2.5\ \text{m} = 15\ \text{m}$

2. (b) Subtract x_i from x_f to find the displacement. $\Delta x = x_f - x_i = 10 - 0\ \text{m} = \boxed{10\ \text{m}}$

Insight: The distance traveled is always positive, but the displacement can be negative.



Picture the Problem: The runner moves along the oval track.

Strategy: The distance is the total length of travel, and the displacement is the net change in position.

Solution: 1. (a) Add the lengths: $(15\ \text{m}) + (100\ \text{m}) + (15\ \text{m}) = \boxed{130\ \text{m}}$

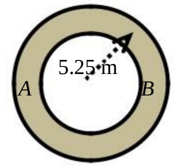
2. Subtract x_i from x_f to find the displacement. $\Delta x = x_f - x_i = 100 - 0\ \text{m} = \boxed{100\ \text{m}}$

3. (b) Add the lengths: $15 + 100 + 30 + 100 + 15\ \text{m} = \boxed{260\ \text{m}}$

4. Subtract x_i from x_f to find the displacement. $\Delta x = x_f - x_i = 0 - 0\ \text{m} = \boxed{0\ \text{m}}$

Insight: The distance traveled is always positive, but the displacement can be negative. The displacement is always zero for a complete circuit, as in this case.

Picture the Problem: The pony walks around the circular track.



Strategy: The distance is the total length of travel, and the displacement is the net change in position.

Solution: (a) 1. The distance traveled is half the circumference: $= \frac{1}{2}(2\pi r) = \pi r = \pi(5.25\text{ m}) = \boxed{16.5\text{ m}}$

2. The displacement is the distance from *A* to *B*: $x = x_f - x_i = 2r = 2(5.25\text{ m}) = \boxed{10.5\text{ m}}$

(b) The distance traveled will increase when the child completes one circuit, because the pony will have taken more steps.

(c) The displacement will decrease when the child completes one circuit, because the displacement is maximum when the child has gone halfway around, and is zero when the child returns to the starting position.

5. (d) The distance traveled equals the circumference: $d = 2\pi r = 2\pi(5.25\text{ m}) = \boxed{33.0\text{ m}}$

The displacement is zero because the child has returned to her starting position.

Insight: The distance traveled is always positive, but the displacement can be negative. The displacement is always zero for a complete circuit, as in this case.

Picture the Problem: You drive your car in a straight line at two different speeds.

Strategy: We could calculate the average speed with the given information by determining the total distance traveled and dividing by the elapsed time. However, we can arrive at a conceptual understanding of the answer by remembering that average speed is an average over time, not an average over the distance traveled.

Solution: 1. (a) The average speed will be less than 20 m/s because you will spend a longer time driving at the lower speed. You will cover the second 10 km distance in less time at the higher speed than you did at the lower speed.

2. (b) The best answer is **I**. More time is spent at 15 m/s than at 25 m/s because the distances traveled at each speed are the same, so that it will take a longer time at the slower speed to cover the same distance. Statement II is true but irrelevant and statement III is false.

Insight: The time elapsed at the lower speed is $(10{,}000\text{ m})/(15\text{ m/s}) = 667\text{ s}$ and the time elapsed at the higher speed is $(10{,}000\text{ m})/(25\text{ m/s}) = 400\text{ s}$, hence the average speed is $(20{,}000\text{ m})/(1067\text{ s}) = 18.7\text{ m/s}$.

Picture the Problem: You drive your car in a straight line at two different speeds.

Strategy: We could calculate the average speed with the given information by determining the total distance traveled and dividing by the elapsed time. However, we can arrive at a conceptual understanding of the answer by remembering that average speed is an average over time, not an average over the distance traveled.

Solution: 1. (a) The average speed will be equal to 20 m/s because you will spend an equal amount of time driving at the lower speed as at the higher speed. The average speed is therefore the mean value of the two speeds.

2. (b) The best answer is **III**. Equal time is spent at 15 m/s and 25 m/s because that fact is stated in the question. Statements I and II are both false.

Insight: The distance traveled at the lower speed would be $(15\text{ m/s})(600\text{ s}) = 9000\text{ m}$ and the distance traveled at the higher speed would be $(25\text{ m/s})(600\text{ s}) = 15{,}000\text{ m}$ so the average speed is $(24{,}000\text{ m})/(1200\text{ s}) = 20.0\text{ m/s}$.

Picture the Problem: A runner sprints in the forward direction.

Strategy: The average speed is the distance divided by elapsed time.

Solution: Divide the distance by the time: $s = \frac{\text{distance}}{\text{time}} = \frac{200.0\text{ m}}{19.19\text{ s}} = \boxed{10.42\text{ m/s}} \times \frac{1\text{ mi}}{1609\text{ m}} \times \frac{3600\text{ s}}{1\text{ h}} = \boxed{23.32\text{ mi/h}}$

Insight: The displacement would be complicated in this case because the 200-m dash usually takes place on a curved track. Fortunately, the average speed depends upon distance traveled, not displacement.

Picture the Problem: A kangaroo hops in the forward direction.

Strategy: The distance is the average speed multiplied by the time elapsed. The time elapsed is the distance divided by the average speed.

Solution: 1. (a) Multiply the average speed by the time elapsed: $d = s\,t = \left(65\frac{\text{km}}{\text{h}}\right)\left(3.2\text{ min} \times \frac{1\text{ h}}{60\text{ min}}\right) = \boxed{3.5\text{ km}}$

(b) Divide the distance by the average speed: $t = \frac{d}{s} = \frac{0.25\text{ km}}{65\text{ km/h}} \times \frac{60\text{ min}}{1\text{ h}} = \boxed{14\text{ s}}$

Insight: The instantaneous speed might vary from 65 km/h, but the time elapsed and the distance traveled depend only upon the average speed during the interval in question.

Picture the Problem: Rubber ducks drift along the ocean surface.

Strategy: The average speed is the distance divided by elapsed time.

Solution: 1. (a) Divide the distance by the time: $s = \frac{d}{t} = \frac{1600\text{ mi}}{10\text{ mo}} \times \frac{1609\text{ m}}{1\text{ mi}} \times \frac{1\text{ mo}}{30.5\text{ d}} \times \frac{1\text{ d}}{8.64\times10^4\text{ s}} = \boxed{0.098\text{ m/s}}$

2. (b) Divide the distance by the time: $s = \frac{d}{t} = \frac{1600\text{ mi}}{10\text{ mo}} \times \frac{1\text{ mo}}{30.5\text{ d}} \times \frac{1\text{ d}}{24\text{ h}} = \boxed{0.22\text{ mi/h}}$

Insight: The instantaneous speed might vary from 0.098 m/s, but we can calculate only average speed from the total distance traveled and time elapsed.

Picture the Problem: Radio waves propagate in a straight line.

Strategy: The time elapsed is the distance divided by the average speed. The distance to the Moon is 2.39×10^5 mi. We must double this distance because the signal travels there and back again.

Solution: Divide the distance by the average speed: $t = \frac{2d}{s} = \frac{2\left(2.39\times10^5\text{ mi}\right)}{1.86\times10^5\text{ mi/s}} = \boxed{2.57\text{ s}}$

Insight: The time is slightly shorter than this because the given distance is from the center of the Earth to the center of the Moon, but presumably any radio communications would occur between the surfaces of the Earth and Moon. When the radii of the two spheres is taken into account, the time decreases to 2.52 s.

Picture the Problem: Sound waves propagate in a straight line from a thunderbolt to your ears.

Strategy: The distance is the average speed multiplied by the time elapsed. We will neglect the time it takes for the light wave to arrive at your eyes because it is vastly smaller than the time it takes the sound wave to travel.

Solution: Multiply the average speed by the time elapsed: $d = s\,t = (340\text{ m/s})(6.5\text{ s}) = 2200\text{ m} = \boxed{2.2\text{ km}}$

Insight: The speed of sound, 340 m/s, works out to approximately one mile every five seconds, a useful rule of thumb for estimating the distance to an approaching thunderstorm!

Picture the Problem: Human nerve impulses propagate at a fixed speed.

Strategy: The time elapsed is the distance divided by the average speed. The distance from your toes to your brain is on the order of two meters.

Solution: Divide the distance by the average speed:

$$t = \frac{d}{s} = \frac{2 \text{ m}}{1\times10^2 \text{ m/s}} = \boxed{0.02 \text{ s}}$$

Insight: This nerve impulse travel time is not the limiting factor for human reaction time, which is about 0.2 s.

Picture the Problem: A finch travels a short distance on the back of the tortoise and a longer distance through the air, with both displacements along the same direction.

Strategy: First find the total distance traveled by the finch and then determine the average speed by dividing by the total time elapsed.

Solution: 1. Determine the total distance traveled:

$$d = s_1\,\Delta t_1 + s_2\,\Delta t_2 = \left[(0.060 \text{ m/s})(1.5 \text{ min}) + (11 \text{ m/s})(1.5 \text{ min})\right] \times 60 \text{ s/min} = 995 \text{ m}$$

2. Divide the distance by the time elapsed:

$$s = \frac{d}{\Delta t} = \frac{995 \text{ m}}{3.0 \text{ min} \times 60 \text{ s/min}} = \boxed{5.5 \text{ m/s}}$$

Insight: Most of the distance traveled by the finch occurred by air. In fact, if we neglect the 5.4 m the finch traveled while on the tortoise's back, we still get an average speed of 5.5 m/s over the 3.0 min time interval! The bird might as well have been at rest during the time it perched on the tortoise's back.

Picture the Problem: You jog for 5.0 km and then travel an additional 13 km by car, with both displacements along the same direction.

Strategy: First find the total time elapsed by dividing the distance traveled by the average speed. Find the time elapsed while jogging, and subtract it from the total time to find the time elapsed while in the car. Finally use the travel-by-car distance and time information to find the average speed with which you must drive the car.

Solution: 1. Use the definition of average speed to determine the total time elapsed.

$$\Delta t = \frac{d}{s_{av}} = \frac{5.0 + 13 \text{ km}}{25 \text{ km/h}} = 0.72 \text{ h}$$

Find the time elapsed while jogging:

$$\Delta t_1 = \frac{d_1}{v_1} = \frac{5.0 \text{ km}}{9.1 \text{ km/h}} = 0.55 \text{ h}$$

Find the time elapsed while in the car:

$$\Delta t_2 = \Delta t - \Delta t_1 = 0.72 \text{ h} - 0.55 \text{ h} = 0.17 \text{ h}$$

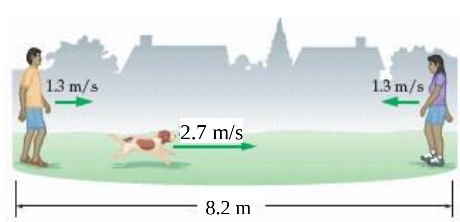
Find the speed of the car:

$$s_2 = \frac{d_2}{\Delta t_2} = \frac{13 \text{ km}}{0.17 \text{ h}} = \boxed{76 \text{ km/h}}$$

Insight: Notice that the average speed is not the average of 9.1 km/h and 76 km/h (which would be 43 km/h) because you spend a much longer time jogging at low speed than you spend driving at high speed.

Picture the Problem: A dog continuously runs back and forth as the owners close the distance between each other.

Strategy: First find the time that will elapse before the owners meet each other. Then determine the distance the dog will cover if it continues running at constant speed over that time interval.



Solution: 1. Find the time it takes each owner to walk half the distance (4.10 m) before meeting each other:

$$\Delta t = \frac{d}{s_{av}} = \frac{4.10 \text{ m}}{1.3 \text{ m/s}} = 3.15 \text{ s}$$

Find the distance the dog runs:

$$d = s\,\Delta t = (2.7 \text{ m/s})(3.15 \text{ s}) = \boxed{8.5 \text{ m}}$$

Insight: The dog will actually run a shorter distance than this, because it is impossible for it to maintain the same 2.7 m/s as it turns around to run to the other owner. It must first slow down to zero speed and then accelerate again.

Picture the Problem: Blood flows at two different speeds through arteries during a specified time interval.

Strategy: Determine the average speed by first calculating the total distance traveled and then dividing it by the total time elapsed.

Solution: 1. (a) Because the time intervals are the same, the blood spends equal times at 1.0 m/s and 0.60 m/s, and its average speed will be equal to 0.80 m/s.

2. (b) Divide the total distance by the time elapsed:

$$s_{av} = \frac{s_1 \Delta t_1 + s_2 \Delta t_2}{\Delta t_1 + \Delta t_2} = \frac{(1.0\text{ m/s})(0.50\text{ s}) + (0.60\text{ m/s})(0.50\text{ s})}{0.50 + 0.50\text{ s}} = \frac{0.80\text{ m}}{1.00\text{ s}}$$

$$= \boxed{0.80\text{ m/s}}$$

Insight: Average speed is a weighted average according to how much *time* the blood spends traveling at each speed.

Picture the Problem: Blood flows at two different speeds through arteries over a specified distance.

Strategy: Determine the average speed by first calculating the total distance traveled and then dividing it by the total time elapsed.

Solution: 1. (a) The distance intervals are the same but the time intervals are different. The blood will spend more time at the lower speed than at the higher speed. Because the average speed is a time-weighted average, it will be less than 0.80 m/s.

2. (b) Divide the total distance by the time elapsed:

$$s_{av} = \frac{d_1 + d_2}{\Delta t_1 + \Delta t_2} = \frac{d_1 + d_2}{\frac{d_1}{s_1} + \frac{d_2}{s_2}} = \frac{1.00\text{ m}}{\left(\frac{0.50\text{ m}}{1.0\text{ m/s}} + \frac{0.50\text{ m}}{0.60\text{ m/s}}\right)}$$

$$s_{av} = \boxed{0.75\text{ m/s}}$$

Insight: The blood spends 0.50 s flowing at 1.0 m/s and 0.83 s flowing at 0.60 m/s.

Picture the Problem: You travel in a straight line at two different speeds during the specified time interval.

Strategy: Determine the distance traveled during each leg of the trip in order to plot the graph.

Solution: 1. (a) Calculate the distance traveled in the first leg:

$$d_1 = s_1 \Delta t_1 = (12\text{ m/s})(1.5\text{ min} \times 60\text{ s/min}) = 1080\text{ m}$$

Calculate the distance traveled in the second leg:

$$d_2 = s_2 \Delta t_2 = (0\text{ m/s})(3.5\text{ min}) = 0\text{ m}$$

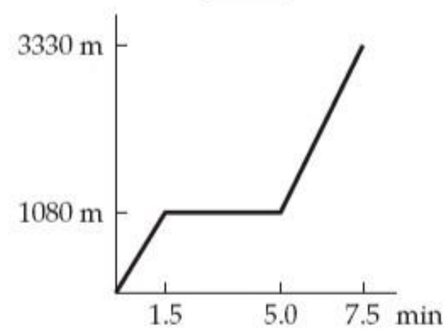
Calculate the distance traveled in the third leg:

$$d_3 = s_3 \Delta t_3 = (15\text{ m/s})(2.5\text{ min} \times 60\text{ s/min}) = 2250\text{ m}$$

Calculate the total distance traveled:

$$= d_1 + d_2 + d_3 = 3330\text{ m}$$

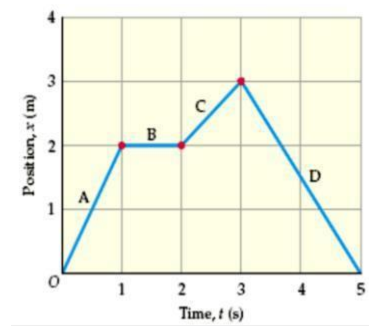
Draw the graph:



(b) Divide the total distance by the time elapsed:

$$s_{av} = \frac{d_1 + d_2 + d_3}{\Delta t_1 + \Delta t_2 + \Delta t_3} = \frac{3330\text{ m}}{7.5\text{ min} \times 60\text{ s/min}} = \boxed{7.4\text{ m/s}}$$

Insight: The average speed is a weighted average according to how much *time* you spend traveling at each speed. Here you spend the most amount of time at rest, so the average speed is less than either 12 m/s or 15 m/s.



Picture the Problem: As specified in the position-*versus*-time graph, the father walks forward, stops, walks forward again, and then walks backward.

Strategy: Determine the direction of the velocity from the slope of the graph along each segment. Then determine the magnitude of the velocity by calculating the slope of the graph at each specified point.

Solution: 1. (a) The slope at A is positive so the velocity is positive.
(b) The velocity at B is zero. **(c)** The velocity at C is positive. **(d)** The velocity at D is negative.

(e) Find the slope of the graph at A: $v_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{2.0\text{ m}}{1.0\text{ s}} = \boxed{2.0\text{ m/s}}$

(f) Find the slope of the graph at B: $v_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{0.0\text{ m}}{1.0\text{ s}} = \boxed{0.0\text{ m/s}}$

(g) Find the slope of the graph at C: $v_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{1.0\text{ m}}{1.0\text{ s}} = \boxed{1.0\text{ m/s}}$

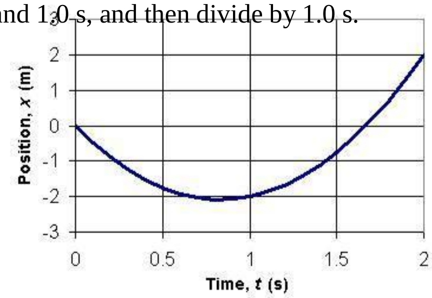
(h) Find the slope of the graph at D: $v_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{-3.0\text{ m}}{2.0\text{ s}} \quad \boxed{-1.5\text{ m/s}}$

Insight: The signs of each answer in (e) through (h) match those predicted in parts (a) through (d). With practice you can form both a qualitative and quantitative "movie" of the motion in your head simply by examining the position-*versus*-time graph.

Picture the Problem: The given position function indicates the particle begins traveling in the negative direction but is accelerating in the positive direction.

Strategy: Create the *x*-*versus*-*t* plot using a spreadsheet, or calculate individual values by hand and sketch the curve using graph paper. Use the known *x* and *t* information to determine the average velocity. To find the average speed, we must find the total distance that the particle travels between 0 and 1.0 s, and then divide by 1.0 s.

Solution: 1. (a) Use a spreadsheet or similar program to create the plot shown at right. Notice that the average velocity over the first second of time is equal to the slope of a straight line drawn from the origin to the value of the curve at $t = 1.0$ s. At that time the position is −2.0 m.



(b) Find the average velocity from $t = 0$ to $t = 1.0$ s:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{\left[(-5\text{ m/s})(1.0\text{ s}) + (3\text{ m/s}^2)(1.0\text{ s})^2\right] - [0.0\text{ m}]}{1.0\text{ s}} = \boxed{-2.0\text{ m/s}}$$

(c) To find the average speed, we must determine the distance traveled. First calculate the time at which $x = 0$:

$$0 = (-5\text{ m/s})t + (3\text{ m/s}^2)t^2$$

$$5\text{ m/s} = (3\text{ m/s}^2)t \Rightarrow t = 5/3\text{ s} = 1.67\text{ s}$$

The time at which the particle turns around is half the time found in step 3. Find *x* at the turnaround time:

$$x = (-5\text{ m/s})(5/6\text{ s}) + (3\text{ m/s})(5/6\text{ s})^2 = -2.083\text{ m}$$

5. At $t = 1$ s, the particle is at $x = -2$ m, so it has traveled an additional 0.083 m after turning around. Find the average speed:

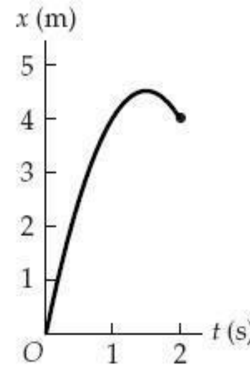
$$s_{av} = \frac{2.083 + 0.083\text{ m}}{1.0\text{ s}} = \boxed{2.2\text{ m/s}}$$

Insight: The instantaneous speed is always the magnitude of the instantaneous velocity, but the average speed is not always the magnitude of the average velocity. For instance, in this problem the particle returns to $x = 0$ after 1.67 s, at which time its average speed is $s_{av} = 4.17\text{ m}/1.67\text{ s} = 2.50$ m/s, but its average velocity is zero because $\Delta x = 0$.

Picture the Problem: The given position function indicates the particle begins traveling in the positive direction but is accelerating in the negative direction.

Strategy: Create the *x-versus-t* plot using a spreadsheet, or calculate individual values by hand and sketch the curve using graph paper. Use the known *x* and *t* information to determine the average speed and velocity.

Solution: 1. (a) Use a spreadsheet to create the plot shown at right:



(b) Find the average velocity from $t = 0$ to $t = 1.0$ s:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{\left[(6\text{ m/s})(1.0\text{ s}) + (-2\text{ m/s}^2)(1.0\text{ s})^2\right] - [0.0\text{ m}]}{1.0\text{ s}}$$

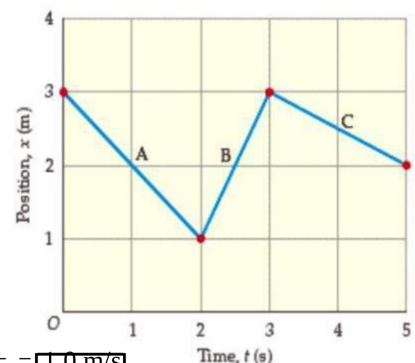
$$v_{av} = \boxed{4.0\text{ m/s}}$$

(c) The average speed is the magnitude of the average velocity:

$$s_{av} = |v_{av}| = \boxed{4.0\text{ m/s}}$$

Insight: Notice that the average velocity over the first second of time is equal to the slope of a straight line drawn from the origin to the curve at $t = 1.0$ s. At that time the position is 4.0 m.

Picture the Problem: Following the motion specified in the position-*versus*-time graph, the tennis player moves left, then right, then left again, if we take left to be in the negative direction.



Strategy: Determine the direction of the velocity from the slope of the graph. The speed will be greatest for the segment of the curve that has the largest slope magnitude.

Solution: 1. (a) The magnitude of the slope at B is larger than A or C so we conclude $\boxed{\text{the speed is greatest at B}}$.

2. (b) Find the slope of the graph at A:

$$s_{av} = \left|\frac{\Delta x}{\Delta t}\right| = \left|\frac{-2.0\text{ m}}{2.0\text{ s}}\right| = \boxed{1.0\text{ m/s}}$$

3. (c) Find the slope of the graph at B:

$$s_{av} = \left|\frac{\Delta x}{\Delta t}\right| = \left|\frac{2.0\text{ m}}{1.0\text{ s}}\right| = \boxed{2.0\text{ m/s}}$$

4. (d) Find the slope of the graph at C:

$$s_{av} = \left|\frac{\Delta x}{\Delta t}\right| = \left|\frac{-1.0\text{ m}}{2.0\text{ s}}\right| = \boxed{0.50\text{ m/s}}$$

Insight: The speed during segment B is larger than the speed during segments A and C, as predicted. Speeds are always positive because they do not involve direction, but velocities can be negative to indicate their direction.

Picture the Problem: You travel in the forward direction along the roads leading to the wedding ceremony, but your average speed is different during the first and second portions of the trip.

Strategy: First find the distance traveled during the first 15 minutes in order to calculate the distance yet to travel. Then determine the speed you need during the second 15 minutes of travel.

Solution: 1. Use the definition of average speed to determine the distance traveled:

$$d_1 = s_1 \Delta t_1 = \left(5.0 \frac{\text{mi}}{\text{h}}\right)\left(15.0 \text{ min} \times \frac{1 \text{ h}}{60 \text{ min}}\right) = 1.25 \text{ mi}$$

Find the remaining distance to travel:

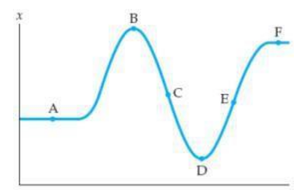
$$d_2 = d_{\text{total}} - d_1 = 10.0 - 1.25 \text{ mi} = 8.8 \text{ mi}$$

Find the required speed for the second part of the trip:

$$s_2 = \frac{d_2}{\Delta t_2} = \frac{8.8 \text{ mi}}{0.250 \text{ h}} = \boxed{35 \text{ mi/h}}$$

Insight: The car needs an average speed of 10 mi/0.5 h = 20 mi/h for the entire trip. However, in order to make it on time it must go seven times faster in the second half (time-wise) of the trip than it did in the first half of the trip.

Picture the Problem: The graph in the problem statement depicts the position of a boat as a function of time.



Strategy: The velocity of the boat is equal to the slope of its position-*versus*-time graph.

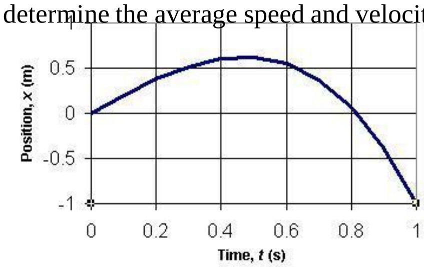
Solution: By examining the graph we can see that the steepest slope in the negative direction (down and to the right) is at point C. Therefore, the boat had its most negative velocity at that time. Points A, B, D, and F all correspond to times of zero velocity because the slope of the graph is zero at those points. Point E has a large positive slope and we conclude the boat had its most positive velocity at that time. Therefore, the ranking is: $\boxed{C < A = B = D = F < E}$.

Insight: The portion of the graph to the left of point B also corresponds to a time of high positive velocity.

Picture the Problem: The given position function indicates the particle begins traveling in the positive direction but is accelerating in the negative direction.

Strategy: Create the *x-versus-t* plot using a spreadsheet, or calculate individual values by hand and sketch the curve using graph paper. Use the known *x* and *t* information to determine the average speed and velocity.

Solution: 1. (a) Use a spreadsheet to create the plot:



(b) Find the average velocity from $t = 0.35$ to $t = 0.45$ s:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{\left[(2\text{ m/s})(0.45\text{ s}) - (3\text{ m/s}^3)(0.45\text{ s})^3\right] - \left[(2\text{ m/s})(0.35\text{ s}) - (3\text{ m/s}^3)(0.35\text{ s})^3\right]}{0.10\text{ s}}$$

$$\boxed{0.55\text{ m/s}}$$

(c) Find the average velocity from $t = 0.39$ to $t = 0.41$ s:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{\left[(2\text{ m/s})(0.41\text{ s}) - (3\text{ m/s}^3)(0.41\text{ s})^3\right] - \left[(2\text{ m/s})(0.39\text{ s}) - (3\text{ m/s}^3)(0.39\text{ s})^3\right]}{0.41 - 0.39\text{ s}}$$

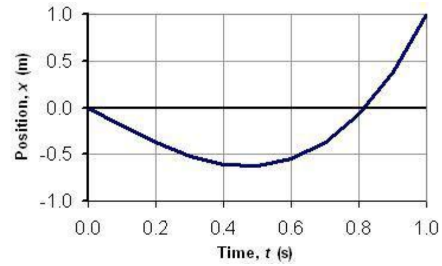
$$\boxed{0.56\text{ m/s}}$$

(d) The instantaneous speed at $t = 0.40$ s will be $\boxed{\text{closer to 0.56 m/s.}}$ As the time interval becomes smaller the average velocity is approaching 0.56 m/s, so we conclude the average speed over an infinitesimally small time interval will be very close to that value.

Insight: Notice that the instantaneous velocity at 0.40 s is equal to the slope of a straight line drawn tangent to the curve at that point. Because it is difficult to accurately draw a tangent line, we often resort to mathematical methods like those illustrated above to determine the instantaneous velocity.

Picture the Problem: The given position function indicates the particle begins traveling in the negative direction but is accelerating in the positive direction.

Strategy: Create the *x-versus-t* plot using a spreadsheet, or calculate individual values by hand and sketch the curve using graph paper. Use the known *x* and *t* information to determine the average speed and velocity.



Solution: 1. (a) Use a spreadsheet to create the plot:

(b) Find the average velocity from $t = 0.150$ to $t = 0.250$ s:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{\left[(-2\text{ m/s})(0.250\text{ s}) + (3\text{ m/s}^3)(0.250\text{ s})^3\right] - \left[(-2\text{ m/s})(0.150\text{ s}) + (3\text{ m/s}^3)(0.150\text{ s})^3\right]}{0.250 - 0.150\text{ s}} = \boxed{-1.63\text{ m/s}}$$

(c) Find the average velocity from $t = 0.190$ to $t = 0.210$ s:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{\left[(-2\text{ m/s})(0.210\text{ s}) + (3\text{ m/s}^3)(0.210\text{ s})^3\right] - \left[(-2\text{ m/s})(0.190\text{ s}) + (3\text{ m/s}^3)(0.190\text{ s})^3\right]}{0.210 - 0.190\text{ s}} = \boxed{-1.64\text{ m/s}}$$

(d) The instantaneous speed at $t = 0.200$ s will be closer to −1.64 m/s. As the time interval becomes smaller the average velocity approaches −1.64 m/s, and we conclude the average speed over an infinitesimally small time interval will be very close to that value.

Insight: Notice that the instantaneous velocity at 0.200 s is equal to the slope of a straight line drawn tangent to the curve at that point. Because it is difficult to accurately draw a tangent line, we often resort to mathematical methods like those illustrated above to determine the instantaneous velocity.

Picture the Problem: You accelerate your car from rest along two on-ramps of different lengths.

Strategy: Use the definitions of average speed and acceleration to compare your motion along the two on-ramps.

Solution: 1. (a) We can reason that because you accelerate between the same initial and final velocities, you must have the same average speed along both on-ramps. If you have the same average speed, then you will accelerate for a shorter period of time along the shorter on-ramp A. Your acceleration must be greater to achieve the same final velocity in a shorter time. We conclude that your acceleration along on-ramp A is greater than your acceleration along on-ramp B.

(b) As discussed above, the best explanation is I. The shorter acceleration distance along ramp A requires a greater acceleration. Statement II is true but is not a complete explanation, and statement III is false.

Insight: We could also set $v_0 = 0$ in the equation, $v^2 = v_0^2 + 2a\,\Delta x$ and solve for a: $a = v^2/2\Delta x$ From this expression we can see that for the same final velocity *v*, you will have a smaller acceleration when you accelerate over the greater distance Δ*x*.

Picture the Problem: An airplane accelerates uniformly along a straight runway.

Strategy: The average acceleration is the change of the velocity divided by the elapsed time.

Solution: Divide the change in velocity by the time:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{156 - 0\text{ mi/h}}{35.2\text{ s}} \times \frac{0.447\text{ m/s}}{\text{mi/h}} = \boxed{1.98\text{ m/s}^2}$$

Insight: The instantaneous acceleration might vary from 1.98 m/s^2, but we can calculate only average acceleration from the net change in velocity and time elapsed.

Picture the Problem: A runner accelerates uniformly along a straight track.

Strategy: The change in velocity is the average acceleration multiplied by the elapsed time.

Solution: 1. (a) Multiply the acceleration by the time: $v = v_0 + a\,t = 0 \text{ m/s} + \left(1.9 \text{ m/s}^2\right)\left(2.0 \text{ s}\right) = \boxed{3.8 \text{ m/s}}$

2. (b) The runner's speed will be the same at the end of the race as it is at $t = 5.2$ s: $v = v_0 + a\,t = 0 \text{ m/s} + \left(1.9 \text{ m/s}^2\right)\left(5.2 \text{ s}\right) = \boxed{9.9 \text{ m/s}}$

Insight: World class sprinters have top speeds of over 10 m/s and can get up to speed in much less than 5.2 s.

Picture the Problem: An airplane slows down uniformly along a straight runway as it travels toward the east.

Strategy: The average acceleration is the change of the velocity divided by the elapsed time. Assume that east is in the positive direction.

Solution: 1. Divide the change in velocity by the time: $a_{av} = \dfrac{v_f - v_i}{\Delta t} = \dfrac{0 - 70.6 \text{ m/s}}{13.0 \text{ s}} = -5.43 \text{ m/s}^2$

We note from the previous step that the acceleration is negative. Because east is the positive direction, negative acceleration must be toward the west. Thus the jet has an acceleration of $\boxed{5.43 \text{ m/s}^2 \text{ toward the west.}}$

Insight: In physics we almost never talk about deceleration. Instead, we call it *negative acceleration.*

Picture the Problem: A car travels in a straight line due north, either speeding up or slowing down, depending upon the direction of the acceleration.

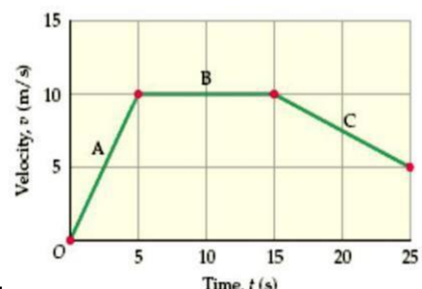
Strategy: Use the definition of acceleration to determine the final velocity over the specified time interval. Let north be the positive direction.

Solution: 1. (a) Calculate the velocity: $v = v_0 + at = 23.6 \text{ m/s} + \left(1.30 \text{ m/s}^2\right)\left(7.10 \text{ s}\right) = \boxed{32.8 \text{ m/s north}}$

2. (b) Calculate the velocity: $v = v_0 + at = 23.6 \text{ m/s} + \left(-1.15 \text{ m/s}^2\right)\left(7.10 \text{ s}\right) = \boxed{15.4 \text{ m/s north}}$

Insight: In physics we almost never talk about deceleration. Instead, we call it *negative acceleration*. In this problem south is considered the negative direction, and in part (b) the car is slowing down or undergoing negative acceleration.

Picture the Problem: Following the motion specified in the velocity-*versus*-time graph, the motorcycle is speeding up, then moving at constant speed, then slowing down.



Strategy: Determine the acceleration from the slope of the graph.

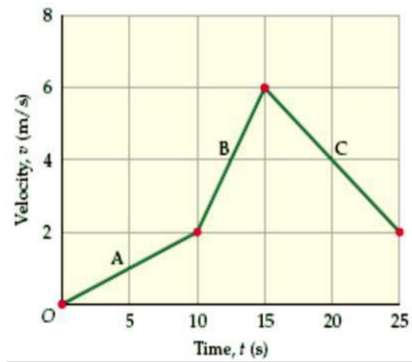
Solution: 1. (a) Find the slope at A: $a_{av} = \dfrac{\Delta v}{\Delta t} = \dfrac{10 \text{ m/s}}{5.0 \text{ s}} = \boxed{2.0 \text{ m/s}^2}$

(b) Find the slope of the graph at B: $a_{av} = \dfrac{\Delta v}{\Delta t} = \dfrac{0 \text{ m/s}}{10.0 \text{ s}} = \boxed{0.0 \text{ m/s}^2}$

(c) Find the slope of the graph at C: $a_{av} = \dfrac{\Delta v}{\Delta t} = \dfrac{-5.0 \text{ m/s}}{10.0 \text{ s}} = \boxed{-0.50 \text{ m/s}^2}$

Insight: The acceleration during segment A is larger than the acceleration during segments B and C because the slope there has the greatest magnitude.

Picture the Problem: Following the motion specified in the velocity-*versus*-time graph, the person on horseback is speeding up, then accelerating at an even greater rate, then slowing down.



Strategy: We could determine the acceleration from the slope of the graph, and then use the acceleration and initial velocity to determine the displacement. Alternatively, we could use the initial and final velocities in each segment to determine the average velocity and the time elapsed to find the displacement during each interval.

Solution: 1. (a) Use the average velocity during interval A to calculate the displacement:

$$x = \tfrac{1}{2}(v_0 + v)t = \tfrac{1}{2}(0 + 2.0\text{ m/s})(10\text{ s}) = \boxed{10\text{ m}}$$

(b) Calculate the displacement during segment B:

$$x = \tfrac{1}{2}(v_0 + v)t = \tfrac{1}{2}(2.0 + 6.0\text{ m/s})(5.0\text{ s}) = \boxed{20\text{ m}}$$

(c) Calculate the displacement during segment C:

$$x = \tfrac{1}{2}(v_0 + v)t = \tfrac{1}{2}(6.0 + 2.0\text{ m/s})(10\text{ s}) = \boxed{40\text{ m}}$$

Insight: There are often several ways to solve motion problems involving constant acceleration, some easier than others.

Picture the Problem: A horse travels in a straight line in the positive direction while accelerating in the negative direction (slowing down).

Strategy: Use the definition of acceleration to determine the time elapsed for the specified change in velocity.

Solution: Calculate the time interval:

$$t = \frac{v - v_0}{a} = \frac{5.5 - 9.2\text{ m/s}}{-1.81\text{ m/s}^2} = \boxed{2.0\text{ s}}$$

Insight: An acceleration of greater magnitude would decrease the horse's velocity in a shorter period of time.

Picture the Problem: Your car travels in a straight line in the positive direction while accelerating in the negative direction (slowing down).

Strategy: Use the constant acceleration equation of motion to determine the time elapsed for the specified change in velocity.

Solution: 1. (a) The time required to come to a stop is the change in velocity divided by the acceleration. In both cases the final velocity is zero, so the change in velocity doubles when you double the initial velocity. Therefore, the stopping time will increase by a factor of two when you double your driving speed.

2. (b) Calculate the stopping time:

$$t = \frac{v - v_0}{a} = \frac{0 - 18\text{ m/s}}{-4.2\text{ m/s}^2} = \boxed{4.3\text{ s}}$$

3. (c) Calculate the stopping time:

$$t = \frac{v - v_0}{a} = \frac{0 - 36\text{ m/s}}{-4.2\text{ m/s}^2} = \boxed{8.6\text{ s}}$$

Insight: Notice that the deceleration is treated as a negative acceleration in this problem and elsewhere in the text.

Picture the Problem: A train travels in a straight line in the positive direction while accelerating in the positive direction (speeding up).

Strategy: First find the acceleration and then determine the final velocity.

Solution: 1. Use the definition of acceleration: $a = \frac{v - v_0}{t} = \frac{4.7 - 0\text{ m/s}}{5.0\text{ s}} = \underline{0.94\text{ m/s}^2}$

Calculate the final speed of the second segment, using the final speed from the first segment as the initial speed: $v = v_0 + a\,t = 4.7\text{ m/s} + \left(0.94\text{ m/s}^2\right)\left(5.0\text{ s}\right)$ $\boxed{v = 9.4\text{ m/s}}$

Insight: Another way to tackle this problem is to set up similar triangles on a velocity-*versus*-time graph. The answer would then be calculated as $v = (4.7\text{ m/s}) \times 10\text{ s} / 5\text{ s} = 9.4\text{ m/s}$. Try it!

Picture the Problem: A particle travels in a straight line in the positive direction while accelerating in the positive direction (speeding up).

Strategy: Use the constant acceleration equation of motion to find the initial velocity.

Solution: Calculate v_0 : $v_0 = v - a\,t = 9.31\text{ m/s} - \left(6.24\text{ m/s}^2\right)\left(0.450\text{ s}\right) = \boxed{6.50\text{ m/s}}$

Insight: As expected, the initial velocity is less than the final velocity because the particle is speeding up.

Picture the Problem: A jet travels in a straight line toward the south while accelerating in the northerly direction (slowing down).

Strategy: Use the relationship between acceleration, velocity, and displacement (Equation 2-12). The acceleration should be negative if we take the direction of the jet's motion (to the south) to be positive.

Solution: Solve for the acceleration: $a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{0^2 - (71.4\text{ m/s})^2}{2\left(949\text{ m}\right)} = -\,2.69\text{ m/s}^2 = \boxed{2.69\text{ m/s}^2\text{ to the north}}$

Insight: The negative acceleration indicates the jet is slowing down during that time interval. Notice that Equation 2-12 is a good choice for problems in which no time information is given or requested.

Picture the Problem: Your car travels in a straight line toward the west while accelerating in the easterly direction (slowing down).

Strategy: The average velocity is simply half the sum of the initial and final velocities because the acceleration is uniform.

Solution: Calculate half the sum of the velocities: $v_{av} = \tfrac{1}{2}\left(v_0 + v\right) = \tfrac{1}{2}\left(18 + 0\text{ m/s}\right) = \boxed{9.0\text{ m/s to the west}}$

Insight: The average velocity of any object that slows down and comes to a stop is just half the initial velocity.

Picture the Problem: A ball rolls down an inclined plane with constant acceleration.

Strategy: The ball starts at a positive value of its position x and must therefore travel in the negative direction in order to reach the location $x = 0$.

Solution: 1. (a) No matter how fast the ball might initially move in the positive direction, away from $x = 0$, a constant negative acceleration will eventually slow it down, bring it briefly to rest, and speed it up back toward $x = 0$. Therefore, in cases 3 and 4, where $a < 0$, the ball will certainly pass $x = 0$.

(b) It is possible for the initial velocity to be so large in the negative direction that a positive acceleration cannot bring it to rest before it passes $x = 0$. Therefore, in case 2 where $v_0 < 0$ and $a > 0$, it is possible that the ball will pass $x = 0$, but we need more information about the relative magnitudes of v_0 and a in order to be certain.

(c) Whenever the initial velocity is opposite in sign to the acceleration, the ball will eventually come to rest briefly and then speed up in the direction of the acceleration. Therefore, in cases 2 and 3 we know that the ball will momentarily come to rest.

Insight: If we suppose that $a = +4.00\text{ m/s}^2$ and that $x_0 = 2.00$ m, we can determine that an initial velocity of $v_0^2 = v^2 - 2a\,\Delta x = 0^2 - 2(4.00\text{ m/s}^2)(-2.00\text{ m}) \Rightarrow v_0 = -\sqrt{8} = -2.83\text{ m/s}$ is the threshold initial velocity for the ball to reach the $x = 0$ position. With that initial velocity the ball will come to rest momentarily at $x = 0$ before speeding up in the positive direction again.

Picture the Problem: A boat travels in a straight line with constant positive acceleration.

Strategy: The average speed is simply half the sum of the initial and final velocities because the acceleration is uniform.

Solution: 1. (a) Calculate half the sum of the velocities: $v_{av} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(0 + 4.82\text{ m/s}) = \boxed{2.41\text{ m/s}}$

2. (b) The distance traveled is the average velocity multiplied by the time elapsed: $d = v_{av}\,t = (2.41\text{ m/s})(4.77\text{ s}) = \boxed{11.5\text{ m}}$

Insight: The average velocity of any object that speeds up from rest is just half the final velocity.

Picture the Problem: The given position function indicates the car begins at a positive position, but is traveling in the negative direction and accelerating in the negative direction.

Strategy: Compare the given position as a function of time with the symbolic expression to determine the initial position, initial velocity, and acceleration of the car. Create the *x-versus-t* plot using a spreadsheet, or calculate individual values by hand and sketch the curve using graph paper. Finally, use the known *x* and *t* information to determine the distance traveled and the average velocity.

Solution: 1. (a) Compare the symbolic formula with the given equation to find the initial position:

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 = (50\text{ m}) + (-5.0\text{ m/s})t + (-10\text{ m/s}^2)t^2$$

$$x_0 = \boxed{50\text{ m}}$$

Compare the symbolic formula with the given equation to find the initial velocity:

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 = (50\text{ m}) + (-5.0\text{ m/s})t + (-10\text{ m/s}^2)t^2$$

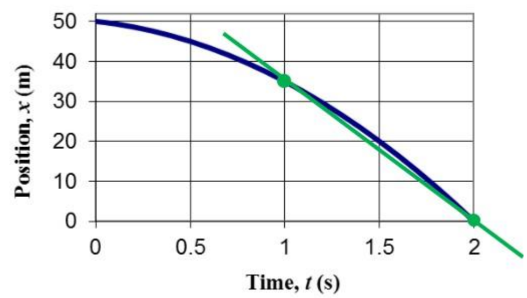
$$v_0 = \boxed{-5.0\text{ m/s}}$$

Compare the symbolic formula with the given equation to find acceleration:

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 = (50\text{ m}) + (-5.0\text{ m/s})t + (-10\text{ m/s}^2)t^2$$

$$\tfrac{1}{2} a = -10\text{ m/s}^2 \quad \Rightarrow a = \boxed{-20\text{ m/s}^2}$$

(b) Use a spreadsheet or similar program to create the blue plot shown at right. The average velocity of the car between 1.0 and 2.0 s is equal to the slope of a straight line drawn from its position at t = 1.0 s and that at t =2.0 s as shown.



(c) Because the car travels in a straight line and does not reverse direction, the distance traveled equals the magnitude of the displacement:

$$x_i = (50\text{ m}) + (-5.0\text{ m/s})(0\text{ s}) + (-10\text{ m/s}^2)(0\text{ s})^2 = 50\text{ m}$$

$$x_f = (50\text{ m}) + (-5.0\text{ m/s})(1.0\text{ s}) + (-10\text{ m/s}^2)(1.0\text{ s})^2 = 35\text{ m}$$

$$\Delta x = x_f - x_i = 35 - 50\text{ m} = -15\text{ m} \quad \Rightarrow \text{distance} = |\Delta x| = \boxed{15\text{ m}}$$

(d) Find the average velocity from t = 1.0 s to t = 2.0 s:

$$x_i = (50\text{ m}) + (-5.0\text{ m/s})(1.0\text{ s}) + (-10\text{ m/s}^2)(1.0\text{ s})^2 = 35\text{ m}$$

$$x_f = (50\text{ m}) + (-5.0\text{ m/s})(2.0\text{ s}) + (-10\text{ m/s}^2)(2.0\text{ s})^2 = 0\text{ m}$$

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{0 - 35\text{ m}}{2.0 - 1.0\text{ s}} = \boxed{-35\text{ m/s}}$$

Insight: Average speed and average velocity are always the same as long as the object continuously travels in the same direction. If it reverses course or travels in two (or three) dimensions, the relationship between the two is more complex, but the distance traveled will always be greater than or equal to the displacement, so the average speed will always be greater than or equal to the average velocity.

Picture the Problem: The given position function indicates the ball begins traveling in the positive direction but is accelerating in the negative direction.

Strategy: Compare the given position as a function of time with the symbolic expression to determine the initial position, initial velocity, and acceleration of the ball. Create the *x-versus-t* plot using a spreadsheet, or calculate individual values by hand and sketch the curve using graph paper. Finally, use the known *x* and *t* information to determine the average velocity and the average speed.

Solution: 1. (a) Compare the symbolic formula with the given equation to find the initial position:

$$= x_0 + v_0 t + \tfrac{1}{2} a t^2 = (0\text{ m}) + (5.0\text{ m/s})t + (-10\text{ m/s}^2)t^2$$

$$x_0 = \boxed{0\text{ m}}$$

Compare the symbolic formula with the given equation to find the initial velocity:

$$= x_0 + v_0 t + \tfrac{1}{2} a t^2 = (0\text{ m}) + (5.0\text{ m/s})t + (-10\text{ m/s}^2)t^2$$

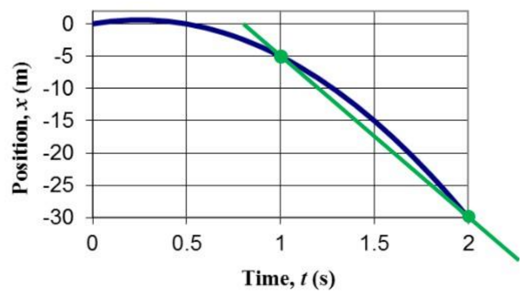
$$v_0 = \boxed{5.0\text{ m/s}}$$

Compare the symbolic formula with the given equation to find acceleration:

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 = (0\text{ m}) + (5.0\text{ m/s})t + (-10\text{ m/s}^2)t^2$$

$$\tfrac{1}{2} a = -10\text{ m/s}^2 \Rightarrow a = \boxed{-20\text{ m/s}^2}$$

(b) Use a spreadsheet or similar program to create the blue plot shown at right. The average speed of the ball between 1.0 and 2.0 s is equal to the slope of a straight line drawn from its position at t = 1.0 s and that at t =2.0 s as shown.



(c) Because the ball reverses direction, the average velocity from t = 0 to t = 1.0 s should be calculated with careful attention to the signs:

$$x_i = (0\text{ m}) + (5.0\text{ m/s})(0\text{ s}) + (-10\text{ m/s}^2)(0\text{ s})^2 = 0\text{ m}$$

$$x_f = (0\text{ m}) + (5.0\text{ m/s})(1.0\text{ s}) + (-10\text{ m/s}^2)(1.0\text{ s})^2 = -5.0\text{ m}$$

$$\Delta x = x_f - x_i = -5.0 - 0\text{ m} = -5\text{ m}$$

$$v_{av} = \Delta x / \Delta t = (-5.0\text{ m})/(1.0\text{ s}) = \boxed{-5.0\text{ m/s}}$$

(d) Because the ball does not reverse direction between t = 1.0 s to t = 2.0 s, the average speed is the magnitude of the average velocity:

$$x_i = (0\text{ m}) + (5.0\text{ m/s})(1.0\text{ s}) + (-10\text{ m/s}^2)(1.0\text{ s})^2 = -5.0\text{ m}$$

$$x_f = (0\text{ m}) + (5.0\text{ m/s})(2.0\text{ s}) + (-10\text{ m/s}^2)(2.0\text{ s})^2 = -30\text{ m}$$

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{-30 - (-5.0)\text{ m}}{2.0 - 1.0\text{ s}} = -25\text{ m/s} \Rightarrow |v_{av}| = \boxed{25\text{ m/s}}$$

Insight: The instantaneous speed is always the magnitude of the instantaneous velocity, but the average speed is not always the magnitude of the average velocity. For instance, in this problem the ball travels to +0.625 m at t = 0.250 s and then to −5.00 m at t = 1.00 s, a total distance of 6.25 m, while its displacement is −5.00 m. Hence its average speed is 6.25 m/s while its average velocity is −5.00 m/s over the time interval between t = 0 and t = 1.0 s

Picture the Problem: A cheetah runs in a straight line with constant positive acceleration.

Strategy: The average velocity is simply half the sum of the initial and final velocities because the acceleration is uniform. The distance traveled is the average velocity multiplied by the time elapsed.

Solution: 1. (a) Calculate half the sum of the velocities: $v_{av} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(0 + 25.0 \text{ m/s}) = \underline{\underline{12.5 \text{ m/s}}}$

2. Use the average velocity to find the distance: $d = v_{av}t = (12.5 \text{ m/s})(6.22 \text{ s}) = \boxed{77.8 \text{ m}}$

(b) For a constant acceleration the velocity varies linearly with time. Therefore we expect the velocity to be equal to $\boxed{5 \text{ m/s}}$ after half the time (3.11 s) has elapsed.

4. (c) Calculate half the sum of the velocities: $v_{av,1} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(0 + 12.5 \text{ m/s}) = \boxed{6.25 \text{ m/s}}$

5. Calculate half the sum of the velocities: $v_{av,2} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(12.5 + 25.0 \text{ m/s}) = \boxed{18.8 \text{ m/s}}$

6. (d) Use the average velocity to find the distance: $d_1 = v_{av,1}t = (6.25 \text{ m/s})(3.11 \text{ s}) = \boxed{19.4 \text{ m}}$

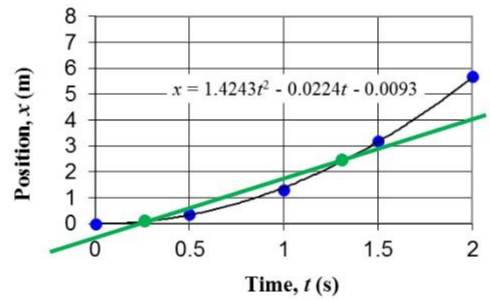
7. Use the average velocity to find the distance: $d_2 = v_{av,2}t = (18.8 \text{ m/s})(3.11 \text{ s}) = \boxed{58.5 \text{ m}}$

Insight: The distance traveled is always the average velocity multiplied by the time. This is a consequence of the definition of average velocity.

Picture the Problem: Measurements taken from a video of a sled traveling down an icy slope can be used to determine the average speed and the acceleration of the sled.

Strategy: Create an *x-versus-t* plot using a spreadsheet, or plot the individual values by hand using graph paper. Use the known *x* and *t* information to determine the average velocity over the specified time interval. Use either the spreadsheet features or the equation $x = x_0 + v_0 t + \frac{1}{2} a t^2$ to determine the average acceleration of the sled.

Solution: 1. (a) Use a spreadsheet or similar program to create the plot shown at right. The average velocity of the sled between 0.25 and 1.3 s is equal to the slope of a straight line drawn from its position at t =0.25 s and that at t =1.3 s as shown.



2. (b) Draw a smooth curve to represent the sled data, and use the smooth curve to determine the approximate average speed:

From the plot, $x_i \approx 0.15$ m and $x_f \approx 2.5$ m

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{2.5 - 0.15 \text{ m}}{1.3 - 0.25 \text{ s}} = \boxed{2.2 \text{ m/s}}$$

3. Check your answer using the least-squares regression from the spreadsheet:

$$x_i = (-0.0093 \text{ m}) + (-0.0224 \text{ m/s})(0.25 \text{ s}) + (1.4243 \text{ m/s}^2)(0.25 \text{ s})^2$$
$$= 0.074 \text{ m}$$

$$x_f = (-0.0093 \text{ m}) + (-0.0224 \text{ m/s})(1.3 \text{ s}) + (1.4243 \text{ m/s}^2)(1.3 \text{ s})^2$$
$$= 2.37 \text{ m}$$

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{2.37 - 0.074 \text{ m}}{1.3 - 0.25 \text{ s}} = 2.19 \text{ m/s} \quad \therefore \text{ confirmed}$$

(c) Calculate the average acceleration of the sled from the equation, $x = x_0 + v_0 t + \frac{1}{2} a t^2$:

$$= x_0 + v_0 t + \tfrac{1}{2} a t^2 = 0 + 0 + \tfrac{1}{2} a t^2$$

$$a = \frac{2x}{t^2} = \frac{2(8.8 \text{ m})}{(2.5 \text{ s})^2} = \boxed{2.8 \text{ m/s}^2}$$

Check your answer using the least-squares regression from the spreadsheet:

$$\tfrac{1}{2} a = 1.4243 \text{ m/s}^2 \quad \Rightarrow a = 2.8486 \text{ m/s}^2 \quad \therefore \text{ confirmed}$$

Insight: Spreadsheet software usually includes powerful tools like regression to analyze data like these.

Picture the Problem: A child slides down the hill in a straight line with constant positive acceleration.

Strategy: Use the known acceleration and times to determine the positions of the child. In each case x_0 and v_0 are zero.

Solution: 1. (a) Calculate her position:

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 = 0 + 0 + \tfrac{1}{2}(1.6 \text{ m/s}^2)(1.0 \text{ s})^2 = \boxed{0.80 \text{ m}}$$

2. (b) Calculate her position:

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 = 0 + 0 + \tfrac{1}{2}(1.6 \text{ m/s}^2)(2.0 \text{ s})^2 = \boxed{3.2 \text{ m}}$$

3. (c) Calculate her position:

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 = 0 + 0 + \tfrac{1}{2}(1.6 \text{ m/s}^2)(3.0 \text{ s})^2 = \boxed{7.2 \text{ m}}$$

Insight: Her position varies with the square of the time because of her constant acceleration.

Picture the Problem: Passengers on the Detonator ride accelerate straight downward.

Strategy: Use the known initial and final velocities and the elapsed time to find the acceleration.

Solution: Calculate the acceleration: $a = \frac{v_f - v_i}{\Delta t} = \frac{(45 - 0 \text{ mi/h})}{2.2 \text{ s}} \times \frac{0.447 \text{ m/s}}{\text{mi/h}} = \boxed{9.1 \text{ m/s}^2}$

Insight: The passenger's acceleration is just less than that for a free-falling object. What a thrill!

Picture the Problem: Jules Verne's *Columbiad* spaceship accelerates from rest down the barrel of the cannon.

Strategy: Employ the relationship between acceleration, displacement, and velocity (Equation 2-12) to find the acceleration.

Solution: Calculate the acceleration: $a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{(12000 \text{ yd/s} \times 3 \text{ ft/yd} \times 0.305 \text{ m/ft})^2 - 0^2}{2(700 \text{ ft} \times 0.305 \text{ m/ft})} = \boxed{2.8\times10^5 \text{ m/s}^2}$

Insight: An acceleration this great would tear the occupants of the spacecraft apart! Notice that the equation $v^2 = v_0^2 + 2a\Delta x$ is a good choice for problems in which no time information is given or requested.

Picture the Problem: An *Escherichia coli* bacterium accelerates from rest in the forward direction.

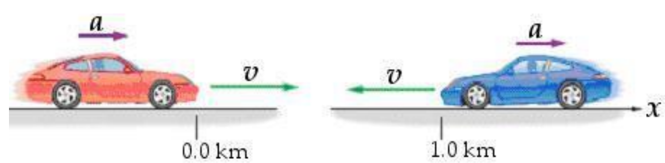
Strategy: Employ the definition of acceleration to find the time elapsed, and the relationship between acceleration, displacement, and velocity (Equation 2-12) to find the distance traveled.

Solution: 1. (a) Calculate the time to accelerate: $t = \frac{v - v_0}{a} = \frac{12 - 0 \text{ µm/s}}{156 \text{ µm/s}^2} = \boxed{0.077 \text{ s}}$

2. (b) Calculate the displacement: $\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{(12 \text{ µm/s})^2 - 0^2}{2(156 \text{ µm/s}^2)} = \boxed{0.46 \text{ µm}}$

Insight: The accelerations are tiny but so are the bacteria! The average speed here is about 3 body lengths per second if each bacterium were 2 μm long. If this were a human that would be 6 m/s or 13 mi/h, much faster than we can swim!

Picture the Problem: Two cars are traveling in opposite directions.



Strategy: Write the equations of motion based upon Equation 2-11, and set them equal to each other to find the time at which the two cars pass each other.

Solution: 1. (a) Write an equation for the position of car 1, which is traveling east and speeding up. Let east be the positive direction:

$$x_1 = x_{0,1} + v_{0,1}t + \tfrac{1}{2}a_1 t^2 = \boxed{x_1 = (20.0 \text{ m/s})t + \tfrac{1}{2}(2.5 \text{ m/s}^2)t^2}$$

Write an equation for the position of car 2, which is traveling west but slowing down, which means it is accelerating toward the east:

$$x_2 = x_{0,2} + v_{0,2}t + \tfrac{1}{2}a_2 t^2 = \boxed{x_2 = 100 \text{ m} - (30.0 \text{ m/s})t + \tfrac{1}{2}(3.2 \text{ m/s}^2)t^2}$$

(b) Set $x_1 = x_2$ and solve for t:

$$(20.0 \text{ m/s})t + (1.25 \text{ m/s}^2)t^2 = 100 \text{ m} - (30.0 \text{ m/s})t + (1.6 \text{ m/s}^2)t^2$$

$$0 = 100 - 50t + 0.35t^2$$

$$t = \frac{50 \pm \sqrt{50^2 - 4(0.35)(100)}}{0.70} = 2.03,\ 141 \text{ s} \Rightarrow \boxed{2.0 \text{ s}}$$

Insight: We chose smaller of the two roots, which corresponds to the first time the cars pass each other. The larger acceleration of car 2 means that it'll come to rest, speed up in the positive direction, and overtake car 1 at $t = 141$ s.

Picture the Problem: A meteorite accelerates from a high speed to rest after impacting the car.

Strategy: Employ the relationship between acceleration, displacement, and velocity (Equation 2-12) to find the acceleration.

Solution: Calculate the acceleration: $|a| = \left|\dfrac{v^2 - v_0^2}{2\Delta x}\right| = \left|\dfrac{0^2 - (130\text{ m/s})^2}{2(0.22\text{ m})}\right| = \boxed{3.8\times10^4 \text{ m/s}^2}$

Insight: The high stiffness of steel is responsible for the tremendous (negative) acceleration of the meteorite.

Picture the Problem: A rocket accelerates straight upward.

Strategy: Employ the relationship between acceleration, displacement, and time (Equation 2-11) to find the acceleration. Because the rocket was at rest before blast off, the initial velocity v_0 is zero, and so is the initial position x_0 . Once the acceleration is known, we can use the constant acceleration equation (Equation 2-7) to find the speed.

Solution: **1. (a)** Write out the position vs. time equation: $x = x_0 + v_0 t + \frac{1}{2} a t^2$

2. Let $x_0 = v_0 = 0$ and solve for acceleration: $a = \dfrac{2x}{t^2} = \dfrac{2(91\text{ m})}{(2.8\text{ s})^2} = \boxed{23 \text{ m/s}^2 \text{ upward}}$

3. (b) Calculate the final speed: $v = 0 + a\,t = (23.2\text{ m/s}^2)(2.8\text{ s}) = \boxed{65\text{ m/s}}$

Insight: The position vs. time equation simplifies considerably if the initial position and the initial velocity are zero.

Picture the Problem: You drive in a straight line and then slow down to a stop.

Strategy: Employ the relationship between acceleration, displacement, and velocity (Equation 2-12) to find the displacement. Equation 2-12 is a good choice for problems in which no time information is given or requested. In this case the acceleration is negative because the car is slowing down.

Solution: 1. (a) Calculate the displacement: $\Delta x = \dfrac{v^2 - v_0^2}{2a} = \dfrac{0^2 - v_0^2}{2a} = -\dfrac{v_0^2}{2a} = -\dfrac{(12.0\text{ m/s})^2}{2(-3.5\text{ m/s}^2)} = \boxed{21\text{ m}}$

(b) Because velocity is proportional to the square root of displacement, cutting the distance in half will reduce the velocity by $\sqrt{2}$, not 2. Therefore the speed will be greater than 6.0 m/s after traveling half the distance.

3. (c) Calculate the speed after half the displacement: $v = \sqrt{v_0^2 + 2a(\tfrac{1}{2}\Delta x)} = \sqrt{v_0^2 + a\left(-\dfrac{v_0^2}{2a}\right)} = \sqrt{\dfrac{v_0^2}{2}} = \dfrac{12.0\text{ m/s}}{\sqrt{2}} = \boxed{8.49\text{ m/s}}$

Insight: For constant acceleration, the velocity changes linearly with time, but nonlinearly with distance.

Picture the Problem: You drive in a straight line and then slow down to a stop.

Strategy: Use the constant acceleration equation of motion (Equation 2-7) to find the time. Once the time is known, we can use the same equation to find the speed. In this case, the acceleration is negative because the car is slowing down.

Solution: 1. (a) Calculate the stopping time: $t = \dfrac{v - v_0}{a} = \dfrac{0 - 16\text{ m/s}}{-3.2\text{ m/s}^2} = \boxed{5.0\text{ s}}$

(b) Because the velocity varies linearly with time for constant acceleration, the velocity will be half the initial velocity when you have braked for half the time. Therefore the speed after braking 2.5 s will be equal to 8.0 m/s.

3. (c) Calculate the speed after half the time: $v = v_0 + a\,t = 16\text{ m/s} + (-3.2\text{ m/s}^2)(2.5\text{ s}) = 8.0\text{ m/s}$

Insight: For constant acceleration, the velocity changes linearly with time, but nonlinearly with distance.

Picture the Problem: A chameleon's tongue accelerates in a straight line until it is extended to its full length.

Strategy: Employ the relationship between acceleration, displacement, and time (Equation 2-11) to find the acceleration. Let the initial velocity v_0 and the initial position x_0 of the tongue each be zero.

Solution: 1. (a) Let $x_0 = v_0 = 0$ and calculate the acceleration:

$$a = \frac{2x}{t^2} = \frac{2(0.16\text{ m})}{(0.10\text{ s})^2} = \boxed{32\text{ m/s}^2}$$

(b) Because the displacement varies with the square of the time for constant acceleration, the displacement will be less than half its final value when half the time has elapsed. Most of the displacement occurs when the tongue's speed is greatest, late in the time interval. Therefore we expect the tongue to have extended $\boxed{\text{less than 8.0 cm}}$ after 0.050 s.

3. (c) Calculate the position of the tongue after half the time:

$$x = \tfrac{1}{2}at^2 = \tfrac{1}{2}(32\text{ m/s}^2)(0.050\text{ s})^2 = \boxed{4.0\text{ cm}}$$

Insight: For constant acceleration, the displacement changes nonlinearly with both time and velocity. Notice that the acceleration of the chameleon's tongue is over three times the acceleration of gravity!

Picture the Problem: David Purley travels in a straight line, slowing down at a uniform rate until coming to rest.

Strategy: Use the time-free relationship between displacement, velocity, and acceleration (Equation 2-12) to find the acceleration.

Solution: Calculate the acceleration:

$$a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{0^2 - \left(173\text{ km/h} \times \frac{0.278\text{ m/s}}{1\text{ km/h}}\right)^2}{2(0.66\text{ m})}$$

$$a = -1800\text{ m/s}^2 \times \frac{1.00g}{9.81\text{ m/s}^2} = -180g \quad \Rightarrow |a| = \boxed{180g}$$

Insight: Mr. Purley was lucky to escape death when experiencing an acceleration this large! We'll learn in Chapter 5 that a large acceleration implies a large force, which in this case must have been applied to his body in just the right way to produce a non-lethal injury.

Picture the Problem: A boat slows down at a uniform rate as it coasts in a straight line.

Strategy: Because the initial and final velocities are known, the time can be determined from the average velocity and the distance traveled. Then use the constant acceleration equation of motion (Equation 2-7) to find the acceleration and the time-free equation (Equation 2-12) to find the velocity after the boat had coasted half the distance.

Solution: 1. (a) Use the displacement and the average velocity to find the time elapsed:

$$t = \frac{\Delta x}{\frac{1}{2}(v + v_0)} = \frac{12\text{ m}}{\frac{1}{2}(1.6 + 2.6\text{ m/s})} = \boxed{5.7\text{ s}}$$

(b) Apply the definition of acceleration:

$$a = \frac{v - v_0}{t} = \frac{1.6 - 2.6\text{ m/s}}{5.7\text{ s}} = \boxed{-0.175\text{ m/s}^2}$$ where the negative sign means opposite the direction of motion.

(c) Calculate the velocity after coasting 6.0 m using the time-free equation of motion:

$$v^2 = v_0^2 + 2a\Delta x$$

$$v = \sqrt{(2.6\text{ m/s})^2 + 2(-0.175\text{ m/s}^2)(6.0\text{ m})} = \boxed{2.2\text{ m/s}}$$

Insight: For constant acceleration, the velocity changes linearly with time but nonlinearly with distance. That is why the 2.2-m/s velocity after coasting 6.0 m is greater than the 2.1-m/s average speed the boat has over the entire 12-m distance it coasted.

Picture the Problem: A model rocket accelerates straight upward at a constant rate.

Strategy: Because the initial and final velocities are known, the time can be determined from the average velocity and the distance traveled. The constant acceleration equation of motion (Equation 2-7) can then be used to find the acceleration. Once that is known, the position of the rocket as a function of time is given by Equation 2-11, and the velocity as a function of time is given by Equation 2-7.

Solution: 1. (a) Use the displacement and the average velocity to find the time elapsed:

$$t = \frac{\Delta x}{\frac{1}{2}(v + v_0)} = \frac{4.2\text{ m}}{\frac{1}{2}(0 + 26.0\text{ m/s})} = 0.323\text{ s} = \boxed{0.32\text{ s}}$$

(b) Apply the definition of acceleration:

$$a = \frac{v - v_0}{t} = \frac{26.0 - 0\text{ m/s}}{0.323\text{ s}} = \boxed{80\text{ m/s}^2}$$

(c) Find the rocket's height, assuming $x_0 = v_0 = 0$:

$$x = \tfrac{1}{2} a t^2 = \tfrac{1}{2}(80\text{ m/s}^2)(0.10\text{ s})^2 = \boxed{0.40\text{ m}}$$

Find the velocity of the rocket, assuming $v_0 = 0$:

$$v = 0 + a t = (80\text{ m/s}^2)(0.10\text{ s}) = \boxed{8.0\text{ m/s}}$$

Insight: Model rockets accelerate at very large rates, but only for a very short time. Still, even inexpensive starter rockets can reach 1500 ft in altitude and can be great fun to build and launch!

Picture the Problem: The infamous chicken dashes toward home plate while playing baseball, and then slides along a straight line and comes to rest.

Strategy: Because the initial and final velocities and the time elapsed are known, the acceleration can be determined from the constant acceleration equation of motion (Equation 2-7). The distance traveled can be found from the average velocity and the time elapsed (Equation 2-10).

Solution: 1. (a) Calculate the acceleration:

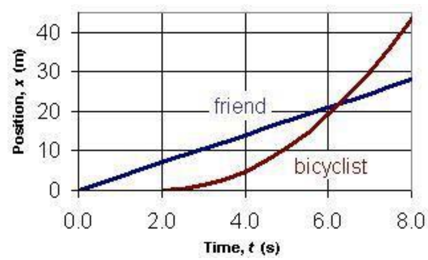
$$a = \frac{v - v_0}{t} = \frac{0 - 5.7\text{ m/s}}{1.2\text{ s}} = \boxed{-4.8\text{ m/s}^2}$$, where the negative sign means opposite the direction of motion, or $\boxed{\text{toward third base.}}$

(b) Use the average velocity and time to find the distance the chicken slides:

$$\Delta x = \tfrac{1}{2}(v + v_0)t = \tfrac{1}{2}(0 + 5.7\text{ m/s})(1.2\text{ s}) = \boxed{3.4\text{ m}}$$

Insight: If the dirt had accelerated the chicken at a lesser rate, the chicken would have had nonzero speed as it crossed home plate. A larger magnitude acceleration would stop the chicken before reaching the plate, and it would be out!

Picture the Problem: The distance-*versus*-time plot at right shows how a bicyclist can overtake his friend by pedaling at constant acceleration.



Strategy: To find the time elapsed when the two bicyclists meet, we must set the constant velocity equation of motion of the friend (Equation 2-8) equal to the constant acceleration equation of motion (Equation 2-11) of the bicyclist. Once the time is known, the displacement and velocity of the bicyclist can be determined from Eqs. 2-10 and 2-7, respectively.

Solution: 1. (a) Set the two equations of motion equal to each other. For the friend, use Equation 2-8 with $x_0 = 0$ and for the bicyclist, use Equation 2-11 with $x_0 = 0$ and $v_0 = 0$:

$$x_{\text{friend}} = x_{\text{bicyclist}}$$

$$v_{\text{friend}}\, t = 0 + 0 + \tfrac{1}{2} a_{\text{bicyclist}} (t-2)^2$$

Solve the two equations for t by rearranging them into a quadratic expression:

$$v_{\text{friend}}\, t = \tfrac{1}{2} a_{\text{bicyclist}} \left(t^2 - 4t + 4\right)$$

$$0 = t^2 - \left[4 + \frac{2v_{\text{friend}}}{a_{\text{bicyclist}}}\right]t + 4 = t^2 - \left[4 + \frac{2\,(3.5\text{ m/s})}{2.4\text{ m/s}^2}\right]t + 4$$

$$0 = t^2 - 6.92t + 4$$

Now use the quadratic formula:

$$t = \frac{+6.92 \pm \sqrt{6.92^2 - 4(1)(4)}}{2} = 6.3,\ 0.64\text{ s}$$

We choose the larger root because the time must be greater than 2.0 s, the time at which the bicyclist began pursuing his friend. The bicyclist will overtake his friend 6.3 s after his friend passes him.

(b) Use the known time to find the position:

$$x_{\text{friend}} = v_0\, t = (3.5\text{ m/s})(6.3\text{ s}) = 22\text{ m}$$

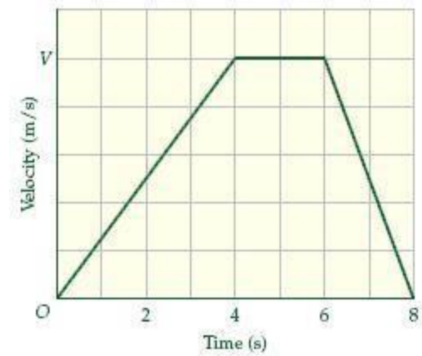
$$x_{\text{bicyclist}} = \tfrac{1}{2} a_{\text{bicyclist}} (t-2)^2 = \tfrac{1}{2}\left(2.4\text{ m/s}^2\right)(4.3\text{ s})^2 = 22\text{ m}$$

(c) Use Equation 2-7 to find $v_{\text{bicyclist}}$. Keep in mind that $v_0 = 0$ and that the bicyclist doesn't begin accelerating until two seconds have elapsed:

$$v = 0 + a_{\text{bicyclist}}(t-2) = \left(2.4\text{ m/s}^2\right)(6.3 - 2.0\text{ s}) = 10\text{ m/s}$$

Insight: Even a smaller acceleration would allow the bicyclist to catch up to the friend, because the speed is always increasing for any nonzero acceleration. Hence the bicyclist's speed would eventually exceed the friend's speed and the two would meet some time after that.

Picture the Problem: The velocity-*versus*-time plot at right indicates the velocity of a car as it accelerates in the forward direction, maintains a constant speed, and then rapidly slows down to a stop.



Strategy: The distance traveled by the car is equal to the area under the velocity-*versus*-time plot. Because the distance traveled is known to be 22 m, we can use that fact to determine the unknown speed *V*. Once we know the velocity as a function of time we can answer any other question about its motion during the time interval.

Solution: 1. (a) Determine the area under the curve by adding the area of the triangle from 0 to 4 s, the rectangle from 4 to 6 s, and the triangle from 6 to 8 s.

$$= \tfrac{1}{2}(4-0\text{ s})V + (6-4\text{ s})V + \tfrac{1}{2}(8-6\text{ s})V = (5\text{ s})V$$

Set x equal to 22 m and solve for V:

$$= (5.0\text{ s})V = 22\text{ m} \Rightarrow V = (22/5.0)\text{ m/s} = \underline{\underline{4.4\text{ m/s}}}$$

Now find the area of the triangle from 0 to 4 s:

$$x_1 = \tfrac{1}{2}(4-0\text{ s})(4.4\text{ m/s}) = \boxed{8.8\text{ m}}$$

(b) Find the area of the triangle from 6 to 8 s:

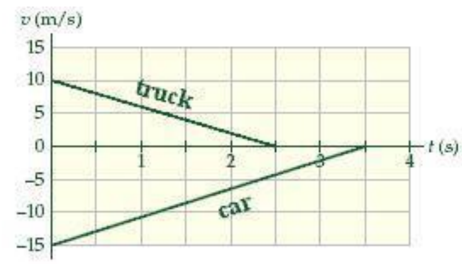
–

(c) We found the unknown speed in step 2:

$$V = \boxed{4.4\text{ m/s}}$$

Insight: The velocity-*versus*-time graph is a rich source of information. Besides velocity and time information, you can determine acceleration from the slope of the graph and distance traveled from the area under the graph.

Picture the Problem: The velocity-*versus*-time plots of the car and the truck are shown at right. The car begins with a positive position and a negative velocity, so it must be represented by the lower line. The truck begins with a negative position and a positive velocity, so it is represented by the upper line.



Strategy: The distances traveled by the car and the truck are equal to the areas under their velocity-*versus*-time plots. We can determine the distances traveled from the plots and use the known initial positions to find the final positions and the final separation.

Solution: 1. Find the final position of the truck. The truck's displacement Δx_{truck} is the area under its v vs. t graph:

$$x_{truck} = x_{0,truck} + \Delta x_{truck} = (-35\text{ m}) + \tfrac{1}{2}(2.5-0\text{ s})(10\text{ m/s}) = \underline{\underline{-22.5\text{ m}}}$$

2. Find the final position of the car. The car's displacement Δx_{car} is the area under its v vs. t graph:

$$x_{car} = x_{0,car} + \Delta x_{car} = (15\text{ m}) + \tfrac{1}{2}(3.5-0\text{ s})(-15\text{ m/s}) = \underline{\underline{-11.25\text{ m}}}$$

3. Now find the separation:

$$x_{car} - x_{truck} = (-11.25\text{ m}) - (-22.5\text{ m}) = \boxed{11.3\text{ m}}$$

Insight: The velocity-*versus*-time graph is a rich source of information. Besides velocity and time information, you can determine acceleration from the slope of the graph, and distance traveled from the area under the graph. In this case, we can see the acceleration of the car (4.29 m/s^2) has a greater magnitude than the acceleration of the truck (−4.00 m/s^2).

Picture the Problem: Penguins slide down three different frictionless ramps, A, B, and C. The distance along each ramp and the average sliding times are recorded.

Strategy: Use the relationship between distance, acceleration, and time (Equation 2-11) to determine the accelerations of the penguins. Then use $a = g \sin\theta$ to determine the angle of inclination θ for each ramp.

Solution: 1. (a) Write an expression for the acceleration, assuming that $x_0 = v_0 = 0$:

$$x = 0 + 0 + \tfrac{1}{2} a t^2 \quad \Rightarrow a = 2x/t^2$$

Calculate the acceleration of penguins that slide along ramp A:

$$a_A = \frac{2x_A}{t_A^2} = \frac{2(4.09\text{ m})}{(2.19\text{ s})^2} = \boxed{1.71\text{ m/s}^2}$$

Calculate the acceleration of penguins that slide along ramp B:

$$a_B = \frac{2x_B}{t_B^2} = \frac{2(1.96\text{ m})}{(1.08\text{ s})^2} = \boxed{3.36\text{ m/s}^2}$$

Calculate the acceleration of penguins that slide along ramp C:

$$a_C = \frac{2x_C}{t_C^2} = \frac{2(1.08\text{ m})}{(0.663\text{ s})^2} = \boxed{4.91\text{ m/s}^2}$$

(b) Write an expression for the angle of incline θ :

$$a = g\sin\theta \quad \Rightarrow \theta = \sin^{-1}\left(\frac{a}{g}\right)$$

Calculate the angle of incline for ramp A:

$$\theta_A = \sin^{-1}\left(\frac{1.71\text{ m/s}^2}{9.81\text{ m/s}^2}\right) = \boxed{10.0°}$$

Calculate the angle of incline for ramp B:

$$\theta_B = \sin^{-1}\left(\frac{3.36\text{ m/s}^2}{9.81\text{ m/s}^2}\right) = \boxed{20.0°}$$

Calculate the angle of incline for ramp C:

$$\theta_C = \sin^{-1}\left(\frac{4.91\text{ m/s}^2}{9.81\text{ m/s}^2}\right) = \boxed{30.0°}$$

Insight: Along a steeper ramp there is a greater component of gravitational force that is parallel to the ramp, resulting in a larger acceleration.

Picture the Problem: Two balls are each thrown with speed v_0 from the same initial height. Ball 1 is thrown straight upward and ball 2 is thrown straight downward.

Strategy: Use the known set of kinematic equations that describe motion with constant acceleration to determine the relative speeds of balls 1 and 2 when they hit the ground.

Solution: 1. Solve Equation 2-12 for v_1, assuming the ball is thrown upward with velocity v_0 :

$$v_1 = \sqrt{v_0^2 + 2(-g)\Delta x} = \sqrt{v_0^2 - 2g\,\Delta x}$$

Solve Equation 2-12 for v_2, assuming the ball is thrown downward with velocity v_0 :

$$v_2 = \sqrt{(-v_0)^2 + 2(-g)\Delta x} = \sqrt{v_0^2 - 2g\,\Delta x}$$

By comparing the two expressions for v above we can conclude that the best answer is **B**. The speed of ball 1 is equal to the speed of ball 2.

Insight: In a later chapter we'll come to the same conclusion from an understanding of the conservation of mechanical energy. The balls have the same speed just before they land because they both have the same downward speed when they are at the level of the roof. Ball 2 simply starts off with the speed v_0 downward. Ball 1 travels upward initially, but when it returns to the level of the roof it is moving downward with the speed v_0, just like ball 2.

Picture the Problem: A cliff diver drops from rest, picking up speed with the acceleration of gravity.

Strategy: Convert the speed in mi/h to m/s, and then solve Equation 2-7 for the time required for the cliff diver to reach that speed when she accelerates at 9.81 m/s^2.

Solution: 1. Convert v to units of m/s^2:

$$60\ \frac{\text{mi}}{\text{h}} \times \frac{1\ \text{m/s}}{2.24\ \text{mi/h}} = 26.8\ \text{m/s}$$

2. Solve Equation 2-7 for the time:

$$v = v_0 + g\,t = 0 + g\,t$$

$$t = \frac{v}{g} = \frac{26.8\ \text{m/s}}{9.81\ \text{m/s}^2} = \boxed{2.7\ \text{s}}$$

Insight: This is significantly less than the 3.5 s required for a powerful car to achieve 60.0 mi/h from rest.

Picture the Problem: A juggler throws a ball straight upward and later catches it at the same height it was thrown.

Strategy: Use the known acceleration of gravity (9.81 m/s^2) and Equation 2-7 to find the initial speed of the ball, assuming by symmetry that the final speed is the same as the initial speed, except the final velocity is downward (negative).

Solution: Solve Equation 2-7 for the initial velocity:

$$-v_0 = v_0 - g\,t \;\Rightarrow g\,t = 2v_0$$

$$v_0 = \tfrac{1}{2}g\,t = \tfrac{1}{2}\left(9.81\ \text{m/s}^2\right)\left(3.2\ \text{s}\right) = \boxed{16\ \text{m/s}}$$

Insight: This speed is equivalent to 35 mi/h, a reasonably easy throw for an accomplished juggler.

Picture the Problem: Snowboarder Shaun White soars straight upward a distance 6.4 m above the rim of a half-pipe.

Strategy: Because the height of the snowboard and rider is known, the time-free equation of motion (velocity in terms of displacement, Equation 2-12) can be used to find the takeoff speed.

Solution: Solve the time-free equation of motion for v_0 :

$$v_0 = \sqrt{v^2 - 2g\,\Delta x} = \sqrt{0^2 - 2\left(-9.81\ \text{m/s}^2\right)\left(6.4\ \text{m}\right)} = \boxed{11\ \text{m/s}}$$

Insight: That speed is about 25 mi/h straight upward! Olympic snowboarders must be very athletic as well as acrobatic to perform the feats we witness during the Games.

Picture the Problem: A gull drops a clam shell, which falls from rest straight down under the influence of gravity.

Strategy: Because the distance of the fall is known, use the time-free equation of motion (velocity in terms of displacement, Equation 2-12) to find the landing speed.

Solution: Solve the time-free equation of motion for v. Let $v_0 = 0$ and let downward be the positive direction.

$$v = \sqrt{v_0^2 + 2\,g\,\Delta x} = \sqrt{0^2 + 2\left(9.81\ \text{m/s}^2\right)\left(17\ \text{m}\right)} = \boxed{18\ \text{m/s}}$$

Insight: That speed (about 41 mi/h) is sufficient to shatter the shell and provide a tasty meal!

Picture the Problem: A volcano launches a lava bomb straight upward. It slows down under the influence of gravity, coming to rest momentarily before falling downward.

Strategy: Because the acceleration of gravity is known, the constant acceleration equation of motion (velocity as a function of time, Equation 2-7) can be used to find the speed and velocity as a function of time. Let upward be the positive direction.

Solution: 1. (a) Apply Equation 2-7 directly with $a = -g$:

$$v = v_0 - g\,t = 28\text{ m/s} - \left(9.81\text{ m/s}^2\right)\left(2.0\text{ s}\right) = \boxed{8.4\text{ m/s}}$$

2. (b) Apply Equation 2-7 directly with $a = -g$:

$$v = v_0 - g\,t = 28\text{ m/s} - \left(9.81\text{ m/s}^2\right)\left(3.0\text{ s}\right) = \boxed{-1.4\text{ m/s}}$$

The positive sign for the velocity in part (a) indicates that the lava bomb is traveling upward, and the negative sign for part (b) means it is traveling downward.

Insight: We can see the lava bomb must have reached its peak between 2.0 and 3.0 seconds. In fact, it reached it at $= \left(v - v_0\right)/a = \left(0 - 28\text{ m/s}\right)/\left(-9.81\text{ m/s}^2\right) = 2.85\text{ s}$.

Picture the Problem: Volcanic material on Io travels straight upward, slowing down under the influence of gravity until it momentarily comes to rest at its maximum altitude.

Strategy: Because the maximum altitude is known, use the time-free equation of motion (velocity in terms of displacement, Equation 2-12) to find the initial velocity. Let upward be the positive direction, so that $a = -1.80\text{ m/s}^2$.

Solution: Solve the time-free equation of motion for v_0 :

$$v_0 = \sqrt{v^2 - 2a\,\Delta x} = \sqrt{0^2 - 2\left(-1.80\text{ m/s}^2\right)\left(3.00\times10^5\text{ m}\right)}$$

$$= 1040\text{ m/s} = \boxed{1.04\text{ km/s}}$$

Insight: On Earth that speed would only hurl the material to an altitude of 55 km, as opposed to 300 km on Io. Still, that's a very impressive initial velocity! It is equivalent to the muzzle velocity of a bullet, and is 2.5 times the speed of sound on Earth.

Picture the Problem: A ruler falls straight down under the influence of gravity.

Strategy: Because the acceleration and initial velocity (zero) of the ruler are known, use the position as a function of time equation of motion (Equation 2-11) to find the time.

Solution: Solve Equation 2-11 for t. Let $v_0 = 0$ and let downward be the positive direction.

$$t = \sqrt{\frac{2\Delta x}{g}} = \sqrt{\frac{2\left(0.052\text{ m}\right)}{9.81\text{ m/s}^2}} = \boxed{0.10\text{ s}}$$

Insight: This is a very good reaction time, about half the average human reaction time of 0.20 s.

Picture the Problem: A hammer drops straight downward and passes by two windows of equal height.

Strategy: Use the definition of acceleration together with the knowledge that a falling hammer undergoes constant acceleration to answer the conceptual question.

Solution: 1. (a) The acceleration of the hammer is a constant throughout its flight (neglecting air friction) so its speed increases by the same amount for each equivalent *time* interval. However, it passes by the second window in a smaller amount of time than it took to pass by the first window because its speed has increased. We conclude that increase in speed of the hammer as it drops past window 1 is greater than the increase in speed as it drops past window 2.

2. (b) The best explanation (see the discussion above) is **III**. The hammer spends more time dropping past window 1. Statement 1 is false because acceleration is independent of speed, and statement II is false because acceleration is rate of change of speed per *time* not distance.

Insight: If the hammer were thrown upward, its speed decrease as it passes window 2 would be less than the decrease in its speed as it passes window 1, again because it is traveling slower as it passes window 1.

Picture the Problem: A hammer drops straight downward and passes by two windows of equal height.

Strategy: The velocity-*versus*-time graph contains two pieces of information: the slope of the graph is the acceleration, and the area under the graph is the distance traveled. Use this knowledge to answer the conceptual question.

Solution: 1. (a) The two windows have the same height, so the hammer travels the same distance as it passes each window. We conclude that the area of the shaded region corresponding to window 1 is equal to the area of the shaded region corresponding to window 2.

(b) The best explanation (see the discussion above) is **II**. The windows are equally tall. Statement I is true, but not relevant, and statement III is true, but not relevant.

Insight: If the hammer were thrown upward, the velocity-*versus*-time graph would have a negative slope, but the shaded areas corresponding to each window would still be equal, with the tall and narrow shaded area for window 2 on the left (because the hammer passes it first) and the short and wide shaded area for window 1 on the right.

Picture the Problem: Two balls are thrown upward with the same initial speed but at different times. The second ball is thrown at the instant the first ball has reached the peak of its flight.

Strategy: The average speed of the ball is smaller at altitudes above $\frac{1}{2}h$, so that it spends a greater fraction of time in that region than it does at altitudes below $\frac{1}{2}h$. Use this insight to answer the conceptual question.

Solution: The second ball will reach $\frac{1}{2}h$ on its way up sooner than the first ball will reach $\frac{1}{2}h$ on its way down because the speed of each ball is greater at low altitudes than at high altitudes. We conclude that the two balls pass at an altitude that is above $\frac{1}{2}h$.

Insight: A careful analysis reveals that the two balls will pass each other at altitude of $\frac{3}{4}h$.

Picture the Problem: Several swimmers fall straight down from a bridge into the Snohomish River.

Strategy: The initial velocities of the swimmers are zero because they step off the bridge rather than jump up or dive downward. Use the equation of motion for position as a function of time and acceleration, realizing that the acceleration in each case is 9.81 m/s^2. Set $x_0 = 0$ and let downward be the positive direction for simplicity. The known acceleration can be used to find velocity as a function of time for part (b). Finally, the same equation of motion for part (a) can be solved for time in order to answer part (c).

Solution: 1. (a) Calculate the fall distance:

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 = 0.0\text{ m} + 0 + \tfrac{1}{2}\left(9.81\text{ m/s}^2\right)\left(1.5\text{ s}\right)^2$$

$$x = \boxed{11\text{ m}}$$

2. (b) Calculate the final speed if $v_0 = 0$:

$$v = v_0 + a t = 0 + \left(9.81\text{ m/s}^2\right)\left(1.5\text{ s}\right) = \boxed{15\text{ m/s}}$$

3. (c) Calculate the fall time for twice the distance:

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2\left(11\text{ m}\times 2\right)}{9.81\text{ m/s}^2}} = \boxed{2.1\text{ s}}$$

Insight: The time in part (c) doesn't double because it depends upon the *square root* of the distance the swimmer falls. If you want to double the fall time you must *quadruple* the height of the bridge!

Picture the Problem: Water in the highest fountain is projected with a large upward velocity, rises straight upward, and momentarily comes to rest before falling straight back down again.

Strategy: By analyzing the time-free equation of motion (Equation 2-12) with $v = 0$ (because the water briefly comes to rest at the top of its trajectory), we can see that the initial velocity v_0 increases with the square root of the fountain height. The known fountain height and acceleration of gravity can also be used to determine the time it takes for the water to reach the peak using the position as a function of time (Equation 2-11).

Solution: 1. (a) Calculate v_0 assuming the water comes to rest ($v = 0$) at the top:

$$0^2 = v_0^2 - 2g\,\Delta x$$

$$v_0 = \sqrt{2g\,\Delta x} = \sqrt{2\left(9.81 \text{ m/s}^2\right)\left(560 \text{ ft} \times 0.305 \text{ m/ft}\right)} = \boxed{58 \text{ m/s}}$$

(b) Calculate the time required for the water to reach the top of the fountain:

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2\left(560 \text{ ft} \times 0.305 \text{ m/ft}\right)}{9.81 \text{ m/s}^2}} = \boxed{5.9 \text{ s}}$$

Insight: The speed of 58 m/s corresponds to 130 mi/h. The fountain is produced by a world-class water pump!

Picture the Problem: A basketball bounces straight up, momentarily comes to rest, and then falls straight back down.

Strategy: If air friction is neglected, the time it takes the ball to fall is the same as the time it takes the ball to rise. Therefore, the maximum height of the ball is also the distance a ball will fall for 1.6 s. Use the equation of motion for position as a function of time and acceleration, realizing that the acceleration in each case is 9.81 m/s^2. Set $x_0 = v_0 = 0$ and let downward be the positive direction for simplicity.

Solution: Calculate the maximum height: $x = x_0 + v_0 t + \tfrac{1}{2} a t^2 = 0.0 \text{ m} + 0 + \tfrac{1}{2}\left(9.81 \text{ m/s}^2\right)\left(1.6 \text{ s}\right)^2 = 12.6 \text{ m} = \boxed{13 \text{ m}}$

Insight: The 12.6-m height corresponds to 41 ft. The ball must have rebounded from the floor with a speed of 15.7 m/s or 35 mi/h. The player was pretty angry!

Picture the Problem: A baseball glove rises straight up, momentarily comes to rest, and then falls straight back down.

Strategy: The glove will land with the same speed it was released, neglecting any air friction, so the final velocity = −6.5 m/s. We can use the equation of motion for velocity as a function of time (Eq. 2-7) to find the time of flight.

Solution: 1. (a) Calculate the total time of flight

$$t = \frac{v - v_0}{a} = \frac{(-6.5) - (6.5) \text{ m/s}}{-9.81 \text{ m/s}^2} = \boxed{1.3 \text{ s}}$$

2. (b) Calculate the time to reach maximum height:

$$t = \frac{v - v_0}{a} = \frac{0 - 6.5 \text{ m/s}}{-9.81 \text{ m/s}^2} = \boxed{0.66 \text{ s}}$$

Insight: Throwing the glove upward with twice the speed will double the time of flight, but the maximum height attained by the glove (2.15 m for a 6.5 m/s initial speed) will increase by only a factor of $\sqrt{2}$.

Picture the Problem: Two balls fall straight down under the influence of gravity. The first ball falls from rest but the second ball is given an initial downward velocity.

Strategy: Because the fall distance is known in each case, use the velocity in terms of displacement equation of motion (Equation 2-12) to predict the final velocity. Let downward be the positive direction for simplicity.

Solution: 1. (a) The speed increases linearly with time but nonlinearly with distance. Because the first ball has a lower initial velocity and hence a lower average velocity, it spends more time in the air. The first (dropped) ball will therefore experience a larger increase in speed.

(b) First ball: Solve Eq. 2-12 for v, setting $v_0 = 0$:

$$= \sqrt{0^2 + 2g\,\Delta x} = \sqrt{2(9.81\ \text{m/s}^2)(30.5\ \text{m})} = 24.5\ \text{m/s}$$

Second ball: Solve Eq. 2-12 for v:

$$= \sqrt{v_0^2 + 2g\,\Delta x} = \sqrt{(11.2\ \text{m/s})^2 + 2(9.81\ \text{m/s}^2)(30.5\ \text{m})} = 26.9\ \text{m/s}$$

Compare the Δv values: $\Delta v_1 = 24.5 - 0$ m/s = 24.5 m/s for the first ball and $\Delta v_2 = 26.9 - 11.2$ m/s = 15.7 m/s for the second ball.

Insight: The second ball is certainly going faster, but its *change* in speed is less than the first ball.

Picture the Problem: An arrow rises straight upward, slowing down due to the acceleration of gravity.

Strategy: Because the position, time, and acceleration are all known, we can use the equation of motion for position as a function of time (Equation 2-11) to find the initial velocity v_0 . The same equation could be used to find the time required to rise to a height of 15.0 m above its launch point. Let the launch position be $x_0 = 0$ and let upward be the positive direction.

Solution: 1. (a) Calculate v_0 from a rearrangement of Equation 2-11:

$$v_0 = \frac{x - \frac{1}{2}a t^2}{t} = \frac{30.0\ \text{m} - \frac{1}{2}(-9.81\ \text{m/s}^2)(2.00\ \text{s})^2}{2.00\ \text{s}} = \boxed{24.8\ \text{m/s}}$$

2. (b) Solve Equation 2-11 with x = 15.0 m:

$$15.0\ \text{m} = (24.8\ \text{m/s})t - \tfrac{1}{2}(9.81\ \text{m/s}^2)t^2$$
$$0 = (-4.905\ \text{m/s}^2)t^2 + (24.8\ \text{m/s})t - 15.0\ \text{m}$$

3. Now use the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-24.8 \pm \sqrt{(24.8)^2 - 4(-4.905)(-15.0)}}{-9.81}$$

$$t = \boxed{0.702\ \text{s}},\ 4.36\ \text{s}$$

Insight: The second root of the solution to part (b) corresponds to the time when the arrow, after rising to its maximum height, falls back to a position 15.0 m above the launch point.

Picture the Problem: A book accelerates straight downward and hits the floor of an elevator that is descending at constant speed.

Strategy: The constant speed motion of the elevator does not affect the acceleration of the book. From the perspective of an observer outside the elevator, both the book and the floor have an initial downward velocity of 3.0 m/s. Therefore, from your perspective the motion of the book is no different than if the elevator were at rest. Solve the position as a function of time and acceleration equation (Equation 2-11) for t, setting $v_0 = 0$ and letting downward be the positive direction. Then use velocity as a function of time (Equation 2-7) to find the speed of the book when it lands.

Solution: 1. (a) Solve Equation 2-11 for t, setting $x_0 = v_0 = 0$:

$$t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2\,(1.2\text{ m})}{9.81\text{ m/s}^2}} = \boxed{0.49\text{ s}}$$

2. (b) Apply Equation 2-7 to find v:

$$v = v_0 + g\,t = 0 + \left(9.81\text{ m/s}^2\right)\left(0.49\text{ s}\right) = \boxed{4.8\text{ m/s}}$$

Insight: The speed in part (b) is relative to you. Relative to the ground the velocity of the book is 4.8 + 3.0 = 7.8 m/s in the downward direction.

Picture the Problem: A camera has an initial downward velocity of 2.3 m/s when it is dropped from a hot-air balloon. The camera accelerates straight downward before striking the ground.

Strategy: One way to solve this problem is to use the quadratic formula to find t from the position as a function of time and acceleration equation (Equation 2-11). Then the definition of acceleration can be used to find the final velocity. Here's another way: Find the final velocity from the time-free equation of motion (Equation 2-12) and use the relationship between average velocity, position, and time (Equation 2-10) to find the time. We'll therefore be solving this problem backwards, finding the answer to (b) first and then (a). Let upward be the positive direction, so that $v_0 = -2.3\text{ m/s}$ and $\Delta x = x - x_0 = 0 - 41\text{ m} = -41\text{ m}$.

Solution: 1. (a) Solve Equation 2-12 for v:

$$v = \sqrt{v_0^2 + 2g\,\Delta x} = \sqrt{(-2.3\text{ m/s})^2 + 2\left(-9.81\text{ m/s}^2\right)(-41\text{ m})} = \underline{-28\text{ m/s}}$$

2. Solve Equation 2-10 for t:

$$t = \frac{\Delta x}{\frac{1}{2}(v + v_0)} = \frac{-41\text{ m}}{\frac{1}{2}(-28 - 2.3\text{ m/s})} = \boxed{2.7\text{ s}}$$

3. (b) We found v in step 1:

$$v = \boxed{-28\text{ m/s}}$$

Insight: There is often more than one way to approach constant acceleration problems, some easier than others. In this case our strategy allowed us to avoid using the quadratic formula to find t.

Picture the Problem: A model rocket rises straight upward, accelerating over a distance of 29 m and then slowing down and coming to rest at some altitude higher than 29 m.

Strategy: Use the given acceleration and distance and the time-free equation of motion (Equation 2-12) to find the velocity of the rocket at the end of its acceleration phase, when its altitude is 29 m. Use that as the initial velocity of the free-fall stage in order to find the maximum altitude (Equation 2-12 again). Then apply Equation 2-12 a third time to find the velocity of the rocket when it returns to the ground. The given and calculated positions at various stages of the flight can then be used to find the elapsed time in each stage and the total time of flight.

Solution: 1. (a) Find the velocity at the end of the boost phase using Equation 2-12:

$$v_{\text{boost}} = \sqrt{v_0^2 + 2g\,\Delta x} = \sqrt{0^2 + 2\left(12\text{ m/s}^2\right)(29\text{ m})} = \underline{26.4\text{ m/s}}$$

Find the height change during the boost phase using Equation 2-12 and a final speed of zero:

$$0^2 = v_{\text{boost}}^2 - 2g\,\Delta x_{\text{boost}} \Rightarrow \Delta x_{\text{boost}} = \frac{v_{\text{boost}}^2}{2g}$$

Now find the overall maximum height:

$$h_{\max} = h_0 + \Delta x_{\text{boost}} = 29\text{ m} + \frac{v_{\text{boost}}^2}{2g}$$

$$29\text{ m} + \frac{(26.4\text{ m/s})^2}{2\left(9.81\text{ m/s}^2\right)} = 29 + 36\text{ m} = \boxed{65\text{ m}}$$

(b) Apply Equation 2-12 once again between the end of the boost phase and the point where it hits the ground:

$$v^2 = v_{\text{boost}}^2 - 2g\,\Delta x$$

$$v = \sqrt{v_{\text{boost}}^2 - 2g\,\Delta x} = \sqrt{(26.4\text{ m/s})^2 - 2\left(9.81\text{ m/s}^2\right)(-29\text{ m})}$$

$$\boxed{36\text{ m/s}}$$

(c) First find the duration of the boost phase. Use the known positions and Equation 2-10:

$$t_{\text{boost}} = \frac{\Delta x_{\text{boost}}}{\frac{1}{2}\left(v_0 + v_{\text{boost}}\right)} = \frac{29\text{ m}}{\frac{1}{2}(0 + 26.4\text{ m/s})} = \underline{2.2\text{ s}}$$

Now find the time for the rocket to reach its maximum altitude from the end of the boost phase:

$$t_{\text{up}} = \frac{\Delta x_{\text{boost}}}{\frac{1}{2}\left(v_{\text{boost}} + v_{\text{top}}\right)} = \frac{36\text{ m}}{\frac{1}{2}(26.4 + 0\text{ m/s})} = \underline{2.7\text{ s}}$$

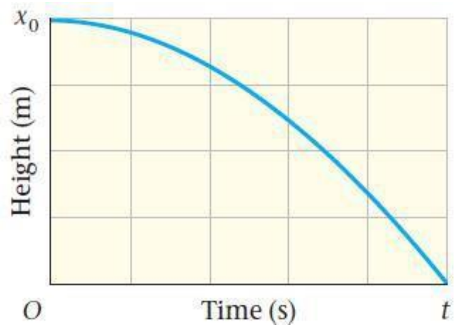
Now find the time for the rocket to fall back to the ground:

$$t_{\text{down}} = \frac{\Delta x_{\text{down}}}{\frac{1}{2}\left(v_{\text{top}} + v_{\text{ground}}\right)} = \frac{65\text{ m}}{\frac{1}{2}(0 + 36\text{ m/s})} = \underline{3.6\text{ s}}$$

Sum the times to find the time of flight:

$$t_{\text{total}} = t_{\text{boost}} + t_{\text{up}} + t_{\text{down}} = 2.2 + 2.7 + 3.6\text{ s} = \boxed{8.5\text{ s}}$$

Insight: Notice how knowledge of the initial and final velocities in each stage, and the distance traveled in each stage, allowed the calculation of the elapsed times using the relatively simple Equation 2-10, as opposed to the quadratic Equation 2-11. Learning to recognize the easiest route to the answer is an important skill to obtain.



Picture the Problem: The vertical position-*versus*-time plot of a flying squirrel is shown at right. The squirrel starts from rest and drops a distance of x_0 = 4.0 m in t = 1.10 s.

Strategy: Because the squirrel starts from rest and lands at x = 0, the equation of motion for position as a function of time (Equation 2-11) can be solved to find the acceleration a. We expect the acceleration to be negative because the squirrel begins from a positive height and ends at a zero height. The negative slope of the plot also indicates the velocity of the squirrel is downward and increasing in magnitude.

Solution: 1. Solve the position as a function of time equation for the acceleration a:

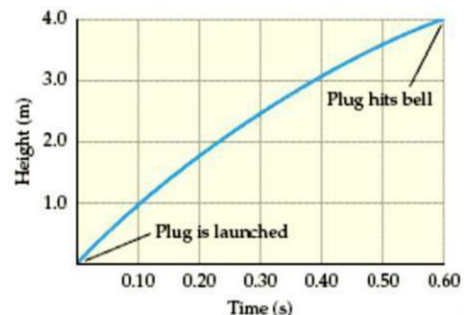
$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$0 = x_0 + 0 + \tfrac{1}{2} a t^2 \Rightarrow \frac{-2x_0}{t^2} = a$$

Calculate the squirrel's acceleration:

$$a = -\frac{2x_0}{t^2} = -\frac{2(4.0\text{ m})}{(1.10\text{ s})^2} = \boxed{-6.6\text{ m/s}^2}$$

Insight: If the squirrel did not have a *patagium* to slow its descent, its acceleration would be close to −9.81 m/s^2.



Picture the Problem: The height-*versus*-time plot of a "high striker" plug is shown at right. The plug starts with a high velocity and begins to slow down when it hits the bell after 0.60 s.

Strategy: The average velocity is the distance traveled by the plug divided by the time (Equation 2-10). Assuming there is no friction, the time and free fall acceleration (−9.81 m/s^2) can be used to find the change in velocity (Equation 2-7). The initial velocity can then be determined from the change in velocity and average velocities by combining Equations 2-7 and 2-9.

Solution: 1. (a) Find the average velocity using Equation 2-10:

$$v_{av} = \frac{x - x_0}{t} = \frac{4.0 - 0\text{ m}}{0.60\text{ s}} = \boxed{6.7\text{ m/s}}$$

(b) Find the change in velocity using Eq. 2-7:

$$\Delta v = v - v_0 = at = (-9.81\text{ m/s}^2)(0.60\text{ s}) = \boxed{-5.9\text{ m/s}}$$

(c) Solve Equation 2-7 for v_0:

$$v = v_0 + at \Rightarrow v_0 = v - at$$

Solve Equation 2-9 for v:

$$v_{av} = \tfrac{1}{2}(v_0 + v) \Rightarrow v = 2v_{av} - v_0$$

Substitute the expression for v into the equation for v_0:

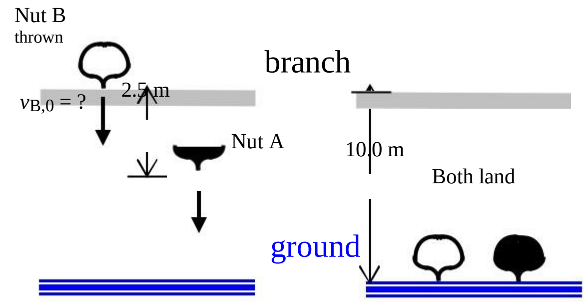
$$v_0 = (2v_{av} - v_0) - a\,t$$

6. Now solve that expression for v_0:

$$v_0 = \tfrac{1}{2}(2v_{av} - a\,t) = \tfrac{1}{2}\left[2(6.7\text{ m/s}) - (-9.81\text{ m/s}^2)(0.60\text{ s})\right] \boxed{v_0 = 9.6\text{ m/s}}$$

Insight: There are several other ways of finding these speeds, including graphical analysis. Try measuring the slope of the graph at the launch point and the point at which the plug hits the bell to find the initial and final speeds.

88. **Picture the Problem**: Chestnut A is dropped from rest. When it has fallen 2.5 m, chestnut B is thrown downward with an initial speed $v_{B,0}$. Both nuts land at the same time after falling 10.0 m.



Strategy: First find the time it takes for nut A to fall 2.5 m using the equation of motion for position as a function of time and acceleration (Equation 2-11). Also find the time required for nut A to fall the entire 10.0 m. Subtract the first time from the second to find the time interval over which nut B must reach the ground in order to land at the same instant as nut A. Then use Equation 2-11 again to find the initial velocity $v_{B,0}$ required in order for nut B to reach the ground in that time.

Solution: 1. Find the time it takes for nut A to fall 2.5 m by solving Equation 2-11 for t and setting $v_{A,0}$ = 0.

$$t_{A,1} = \sqrt{\frac{2\Delta x}{g}} = \sqrt{\frac{2(2.5\text{ m})}{9.81\text{ m/s}^2}} = 0.714\text{ s}$$

Find the time it takes for nut A to fall the entire 0 m:

$$t_{A,\text{total}} = \sqrt{\frac{2\Delta x}{g}} = \sqrt{\frac{2(10.0\text{ m})}{9.81\text{ m/s}^2}} = 1.428\text{ s}$$

Subtract the times to find the time over which nut B must reach the ground:

$$t_{B,\text{total}} = t_{A,\text{total}} - t_{A,1} = 1.428 - 0.714\text{ s} = 0.714\text{ s}$$

Solve Equation 2-11 for $v_{B,0}$:

$$v_{B,0} = \frac{\Delta x - \frac{1}{2} g t_{B,\text{total}}^2}{t_{B,\text{total}}} = \frac{10.0\text{ m} - \frac{1}{2}(9.81\text{ m/s}^2)(0.714\text{ s})^2}{0.714\text{ s}}$$

$$v_{B,0} = 10.5\text{ m/s} \Rightarrow \boxed{11\text{ m/s}}$$

Insight: In this problem we kept an additional significant figure than is warranted in steps 1, 2, and 3 in an attempt to get a more accurate answer in step 4. However, if you choose not to do so, differences in rounding will lead to an answer of 10 m/s. The specified 2.5 m drop distance for nut A limits the answer to two significant digits, and because the answer is right between 10 and 11 m/s, it could correctly go either way.

Picture the Problem: A rock accelerates from rest straight downward and lands on the surface of the Moon.

Strategy: Employ the relationship between acceleration, displacement, and velocity (Equation 2-12) to find the final velocity.

Solution: Solve Equation 2-12 for velocity v:

$$v = \sqrt{v_0^2 + 2a\,\Delta x} = \sqrt{0^2 + 2(1.62\text{ m/s}^2)(1.25\text{ m})} = \boxed{2.01\text{ m/s}}$$

Insight: On Earth the rock would be traveling 4.95 m/s, but the weaker gravity on the Moon accelerates the rock only about one-sixth as much as would the Earth's gravity.

Picture the Problem: An elevator in the Taipei 101 skyscraper accelerates to its maximum speed.

Strategy: Because time information is neither given nor requested, the time-free equation for velocity in terms of displacement (Equation 2-12) is the best choice for finding the displacement.

Solution: Solve Equation 2-12 for displacement:

$$\Delta x = v^2 - v_0^2 = (16\text{ m/s})^2 - 0^2 = \boxed{116\text{ m}}$$

$$2a \qquad 2\left(1.1\ \text{m/s}^2\right)$$

Insight: The observatory elevators in Taipei 101 were the world's fastest when installed, whisking passengers from the fifth floor to the 89th-floor observatory, a distance of 369 m, in only 37 seconds. The 16.83 m/s maximum speed is equivalent to 37.7 mi/h. The passengers are treated to a memorable ride!

Picture the Problem: Water pouring through an ancient Strait of Gibraltar accelerates downward and impacts the water surface below.

Strategy: Employ the relationship between acceleration, displacement, and velocity (Equation 2-12) to find the height from which the water must fall so that its final velocity just before landing is 340 m/s.

Solution: Solve Equation 2-12 for velocity Δx: $\Delta x = \dfrac{v^2 - v_0^2}{2g} = \dfrac{(340\text{ m/s})^2 - 0^2}{2\left(9.81\text{ m/s}^2\right)} = 5900\text{ m} = \boxed{5.9\text{ km}}$

Insight: This height corresponds to 3.7 miles or over 19,000 feet! With air resistance, however, an even higher altitude would be required to obtain speeds this great.

Picture the Problem: A juggler throws a ball straight upward, it briefly comes to rest, and falls downward, returning to the juggler's hand.

Strategy: By symmetry the total time of flight is exactly twice the amount of time elapsed as the ball falls from rest from its maximum height. Use this observation, together with the equation for position as a function of time (Equation 2-11) to find the maximum height of the ball above the juggler's hand.

Solution: 1. Solve Equation 2-11 for x_0, assuming that the final position $x = 0$ and initial speed $v_0 = 0$:

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$0 = x_0 + 0 + \tfrac{1}{2}(-g)t^2 \Rightarrow x_0 = \tfrac{1}{2} g t^2$$

2. Substitute values to find the maximum height: $x_0 = \tfrac{1}{2}\left(9.81\text{ m/s}^2\right)\left(\tfrac{1}{2}\times 0.98\text{ s}\right)^2 = \boxed{1.2\text{ m}}$

Insight: You can show that the ball left the juggler's hand with an upward velocity of 4.8 m/s, or about 11 mi/h.

Picture the Problem: Ball A is dropped from rest at the edge of a roof, and at the same instant ball B is thrown upward from the ground with an initial velocity v_0 sufficient to reach the original location of ball A.

Strategy: Use an understanding of velocity and acceleration to answer the conceptual question. Let upward be the positive direction.

Solution: 1. The velocity of ball A is $\boxed{\text{negative}}$ because it is falling downward.

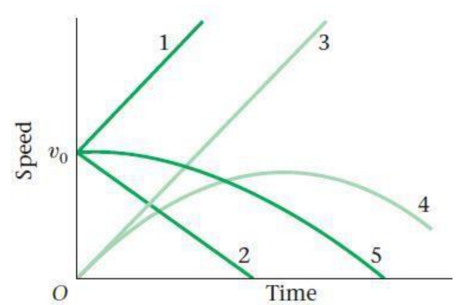
The acceleration of ball A is $\boxed{\text{negative}}$ because gravity acts in the downward direction.

The velocity of ball B is $\boxed{\text{positive}}$ because it is traveling upward.

The acceleration of ball B is $\boxed{\text{negative}}$ because gravity acts in the downward direction.

Insight: Acceleration is the rate of change of velocity, so acceleration can be zero when the velocity has a large magnitude (for example, a car traveling along a highway at constant speed), and the velocity can be zero when the acceleration has a large magnitude (for example, a ball at the top of its vertical flight). The acceleration of ball B is always downward, even when its velocity is upward.

Picture the Problem: Two balls are released simultaneously. Ball A is dropped from rest but ball B is thrown upward with an initial velocity v_0.

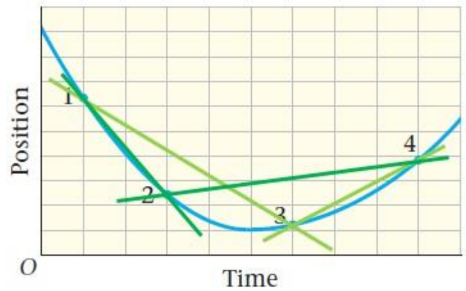


Strategy: Use a correct interpretation of motion graphs to answer the conceptual questions. Recall that the slope of a velocity-*versus*-time graph is the acceleration.

Solution: 1. (a) The speed of ball A starts at zero and then increases linearly with a slope of 9.81 m/s^2. The graph that corresponds to that description is $\boxed{\text{plot 3}}$.

(b) The speed of ball B starts at v_0 and then decreases linearly with a slope of −9.81 m/s^2, equal in magnitude but opposite in direction to the slope of ball A's plot. The graph that corresponds to that description is $\boxed{\text{plot 2.}}$

Insight: Even if ball B were fired upward at an extremely high speed, its velocity-*versus*-time graph would still be linear with a slope of −9.81 m/s^2, but the line would begin very high on the speed axis of the graph.



Picture the Problem: A plot of position vs. time yields information about the average velocity of an object.

Strategy: Use a correct interpretation of position-time plots to answer the conceptual questions. Recall that the slope of a position-*versus*-time graph is the velocity.

Solution: 1. (a) The average speed of the object is the magnitude of the slope of a line between the two points. The line between points 1 and 2 (dark solid line) has a steeper slope than the line between points 1 and 3 (light solid line). Therefore, the average speed for the time interval between points 1 and 2 is greater than the average speed for the time interval between points 1 and 3.

2. (b) The average velocity of the object is the slope of a line between the two points. The line between points 2 and 4 (dark dashed line) has a smaller slope than the line between points 3 and 4 (light dashed line). Therefore, the average velocity for the time interval between points 2 and 4 is less than the average velocity for the time interval between points 3 and 4.

Insight: The negative slopes of the two solid lines indicate the velocity of the object is negative for those time intervals. However, the question in part (a) asked about the speed, not the velocity, hence only the magnitude of the slopes was considered.

Picture the Problem: A package falls straight downward, accelerating for 2.2 seconds before impacting air bags.

Strategy: Find the distance the package will fall from rest in 2.2 seconds by using Equation 2-11. Use the known acceleration and time to find the velocity of the package just before impact by using Equation 2-7. Finally, use the known initial and final velocities, together with the distance over which the package comes to rest when in contact with the air bags, to find the stopping acceleration using Equation 2-12.

Solution: 1. (a) Find the distance the package falls from rest in 2.2 s using Equation 2-11:

$$x = v_0 t + \tfrac{1}{2} g t^2 = 0 + \tfrac{1}{2}(9.81 \text{ m/s}^2)(2.2 \text{ s})^2 = \boxed{24 \text{ m}}$$

(b) Find the velocity just before impact using Equation 2-7:

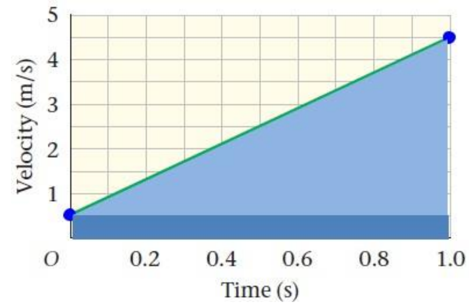
$$v_{\text{land}} = v_0 + g t = 0 + (9.81 \text{ m/s}^2)(2.2 \text{ s}) = \boxed{22 \text{ m/s}} = 48 \text{ mi/h!}$$

(c) Solve Equation 2-12 for a:

$$a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{0^2 - (22 \text{ m/s})^2}{2(0.75 \text{ m})} = \boxed{-320 \text{ m/s}^2} = -33g$$

Insight: Increasing the stopping distance will decrease the stopping acceleration. We will return to this idea when we discuss impulse and momentum in Chapter 9.

97. **Picture the Problem**: A plot of velocity vs. time yields information about the acceleration of an object.



Strategy: Use a correct interpretation of motion graphs to answer the conceptual questions. Recall that the slope of a velocity-*versus*-time graph is the acceleration.

Solution: 1. (a) The acceleration of the object is the slope of the velocity-*versus*-time graph: $a = \dfrac{v - v_0}{t - t_0} = \dfrac{4.5 - 0.5\ \text{m/s}}{1.0 - 0\ \text{s}} = \boxed{4.0\ \text{m/s}^2}$

(b) The displacement is the area under the velocity-versus-time curve:

x = area of bottom rectangle + area of triangle

$(1.0\ \text{s})(0.50\ \text{m/s}) + \frac{1}{2}(1.0\ \text{s})(4.0\ \text{m/s}) = 2.5\ \text{m}$

The final position is the initial position plus the displacement:

$x = x_0 + \Delta x = 12.0\ \text{m} + 2.5\ \text{m} = \boxed{14.5\ \text{m}}$

(c) Use the known acceleration, initial velocity, and initial position to find the final position at $t = 5.00$ s:

$x = x_0 + v_0 t + \frac{1}{2} a t^2$

$= (12.0\ \text{m}) + (0.50\ \text{m/s})(5.00\ \text{s}) + \frac{1}{2}(4.0\ \text{m/s}^2)(5.00\ \text{s})^2$

$= \boxed{64.5\ \text{m}}$

Insight: Equation 2-11 can also be used to find the final position in part (b), instead of determining the area under the velocity-*versus*-time graph: $x = x_0 + v_0 t + \frac{1}{2} a t^2 = (12.0\ \text{m}) + (0.50\ \text{m/s})(1.00\ \text{s}) + \frac{1}{2}(4.0\ \text{m/s}^2)(1.00\ \text{s})^2 = 14.5\ \text{m}.$

Picture the Problem: A golf ball rolls in a straight line, decreasing its speed at a constant rate until it comes to rest.

Strategy: You could find the (negative) acceleration by using Equation 2-12 and the known initial and final velocities and the distance traveled. Then employ Equation 2-12 again using the same acceleration, but solving for the v_0 required to go the longer distance. Instead, we'll present a way to calculate the same answer using a ratio, which will also be useful to calculate the initial speed needed to make the putt over the new 6.00-ft distance.

Solution: 1. (a) Calculate the ratio of initial velocities based upon Equation 2-12:

$$\frac{v_{\text{make},0}}{v_{\text{miss},0}} = \frac{\sqrt{v^2 - 2a\,\Delta x_{\text{make}}}}{\sqrt{v^2 - 2a\,\Delta x_{\text{miss}}}} = \frac{\sqrt{0^2 - 2a\,\Delta x_{\text{make}}}}{\sqrt{0^2 - 2a\,\Delta x_{\text{miss}}}} = \sqrt{\frac{\Delta x_{\text{make}}}{\Delta x_{\text{miss}}}}$$

Now solve for $v_{\text{make},0}$, the initial speed needed to make the 23.5-ft putt:

$$v_{\text{make},0} = v_{\text{miss},0}\sqrt{\frac{\Delta x_{\text{make}}}{\Delta x_{\text{miss}}}} = (1.54\ \text{m/s})\sqrt{\frac{23.5\ \text{ft}}{23.5 - 6.00\ \text{ft}}} = \boxed{1.77\ \text{m/s}}$$

(b) Employ the same ratio to find the initial speed for the new 6.00-ft putt:

$$v_{\text{new},0} = v_{\text{miss},0}\sqrt{\frac{\Delta x_{\text{new}}}{\Delta x_{\text{miss}}}} = (1.54\ \text{m/s})\sqrt{\frac{6.00\ \text{ft}}{23.5 - 6.00\ \text{ft}}} = \boxed{0.902\ \text{m/s}}$$

Insight: Calculating ratios can often be a convenient and simple way to solve a problem. In this case a three-step solution became two steps when we calculated the ratio, and furthermore we never needed to convert feet to meters because the units cancel out in the ratio. Learning to calculate ratios in this manner is a valuable skill in physics.

Picture the Problem: After its release by a glaucous-winged gull, a shell rises straight upward, slows down, and momentarily comes to rest before falling straight downward again.

Strategy: Find the extra altitude attained by the shell due to its upward initial velocity upon release, and add that value to 12.5 m to find the maximum height it reaches above ground. The time-free equation for velocity in terms of displacement (Equation 2-12) can be employed for this purpose. The time the shell spends going up and the time it spends going down can each be found from the known heights and speeds (Equations 2-7 and 2-11). Then the speed upon landing can be determined from the known time it spends falling (Equation 2-7). Let upward be the positive direction throughout the solution to this problem.

Solution: 1. (a) The motion of the shell is influenced only by gravity once it has been released by the gull. Therefore its acceleration will be $\boxed{9.81\text{ m/s}^2\text{ downward}}$ from the moment it is released, even though it is moving upward at the release.

(b) Use Equation 2-12, setting the final speed $v = 0$, to find the extra altitude gained by the shell due to its initial upward speed, and add it to the 12.5 m:

$$x_{max} = 12.5\text{ m} + \frac{v^2 - v_0^2}{-2g} = 12.5\text{ m} + \frac{0^2 - (5.20\text{ m/s})^2}{-2\left(9.81\text{ m/s}^2\right)}$$

$$x_{max} = 12.5\text{ m} + 1.38\text{ m} = \boxed{13.9\text{ m}}$$

(c) The time the shell travels upward is the time it takes gravity to bring the speed to zero (Equation 2-7):

$$t = \frac{v - v_0}{-g} = \frac{0 - 5.2\text{ m/s}}{-9.81\text{ m/s}^2} = 0.53\text{ s}$$

The time the shell travels down is governed by the distance and the acceleration (Equation 2-11):

$$x = x_0 + v_0 t - \tfrac{1}{2}gt^2 \Rightarrow 0 = x_0 + 0 - \tfrac{1}{2}gt^2$$

$$t = \sqrt{\frac{2x_0}{g}} = \sqrt{\frac{2(13.9\text{ m})}{9.81\text{ m/s}^2}} = 1.68\text{ s}$$

The total time of flight is the sum:

$$t_{total} = t_{up} + t_{down} = 0.53 + 1.68\text{ s} = \boxed{2.21\text{ s}}$$

(d) The speed of the shell upon impact is given by the acceleration of gravity and the fall time (Equation 2-7):

$$v = v_0 - g\,t = 0 - \left(9.81\text{ m/s}^2\right)(1.68\text{ s}) = -16.5\text{ m/s}$$

$$|v| = \boxed{16.5\text{ m/s}}$$

Insight: There are a variety of other ways to solve this problem. For instance, it is possible to find the final velocity of 16.5 m/s in part (d) by using Equation 2-12 with $v_0 = 5.2$ m/s and $\Delta x = -12.5$ m without using any time information. Try it for yourself!

Picture the Problem: Liquid from a syringe squirts straight upward, slows down, and momentarily comes to rest before falling straight downward again.

Strategy: Find the time of flight by exploiting the symmetry of the situation. If it takes time t for gravity to slow the liquid drops down from their initial speed v_0 to zero, it will take the same amount of time to accelerate them back to the same speed. They therefore return to the needle tip at the same speed v_0 with which they were squirted. Use this fact together with Equation 2-7 to find the time of flight. The maximum height the drops achieve is related to the square of v_0, as indicated by Equation 2-12.

Solution: 1. (a) Calculate he time of flight for $v_0 = 1.5$ m/s , using Equation 2-7:

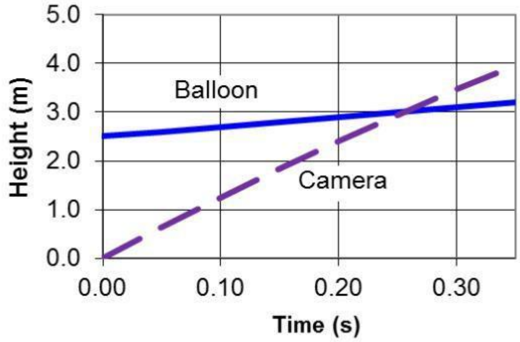
$$t = \frac{v - v_0}{-g} = \frac{(-v_0) - v_0}{-g} = \frac{2v_0}{g} = \frac{2(1.5\text{ m/s})}{9.81\text{ m/s}^2} = \boxed{0.31\text{ s}}$$

(b) Calculate the maximum height for $v_0 = 1.5$ m/s , using Equation 2-12:

$$\Delta x = \frac{v^2 - v_0^2}{-2g} = \frac{0^2 - v_0^2}{-2g} = \frac{v_0^2}{2g} = \frac{(1.5\text{ m/s})^2}{2\left(9.81\text{ m/s}^2\right)} = \boxed{0.11\text{ m}}$$

Insight: The symmetry of the motion of a freely falling object can often be a useful tool for solving problems quickly.

Picture the Problem: The trajectories of a hot-air balloon and a camera are shown at right. The balloon rises at a steady rate while the camera's speed is continually slowing down under the influence of gravity. The camera is caught when the two trajectories meet.



Strategy: The equation of motion for position as a function of time (Equation 2-10) can be used to describe the balloon, while the equation for position as a function of time and acceleration (Equation 2-11) can be used to describe the camera's motion. Set these two equations equal to each other to find the time at which the camera is caught. Then find the height of the balloon at the instant the camera is caught.

Solution: 1. Write Equation 2-10 for the balloon: $x_b = x_{b,0} + v_b t$

Write Equation 2-11 for the camera: $x_c = 0 + v_{c,0}\, t - \frac{1}{2} g t^2$

Set $x_b = x_c$ and solve for t:

$$x_{b,0} + v_b t = v_{c,0}\, t - \tfrac{1}{2} g t^2$$
$$= -x_{b,0} + (v_{c,0} - v_b) t - \tfrac{1}{2} g t^2$$

Multiply by −1 and substitute the numerical values:

$$= 2.5 \text{ m} - (13 - 2.0 \text{ m/s})t + \tfrac{1}{2}(9.81 \text{ m/s}^2)t^2$$
$$= 2.5 - 11t + 4.9\,t^2$$

Apply the quadratic formula and solve for t. The larger root corresponds to the time when the camera would pass the balloon a second time, on its way down back to the ground.

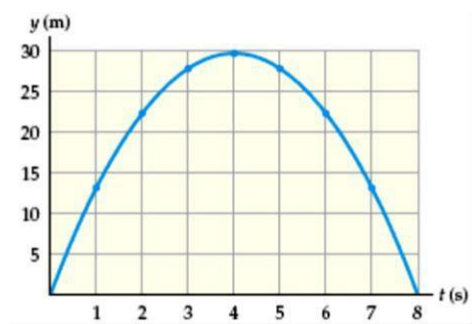
$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+11 \pm \sqrt{11^2 - 4(4.9)(2.5)}}{9.8}$$
$$t = \underline{\underline{0.26}} \text{ or } 2.0 \text{ s}$$

Find the height of the balloon at that time:

$$x_b = x_{b,0} + v_b t = 2.5 \text{ m} + (2.0 \text{ m/s})(0.26 \text{ s}) = \boxed{3.0 \text{ m}}$$

Insight: If the passenger misses the camera the first time, she has another shot at it after 2.0 s (from the time it is thrown) when the camera is on its way back toward the ground. That is the meaning of the second solution for t.

Picture the Problem: The height-*versus*-time plot of a rock on a distant planet is shown at right. The rock starts with a high velocity upward, slows down and momentarily comes to rest after about 4.0 seconds of flight, and then falls straight down and lands at about 8.0 seconds.



Strategy: The equation of motion for position as a function of time and acceleration (Equation 2-11) can be used to find the acceleration from the second half of the trajectory, where the rock falls 30 m from rest and lands 4.0 seconds later. Once acceleration is known, the initial velocity can be determined from Equation 2-7. Let upward be the positive direction.

Solution: 1. (a) Solve Equation 2-11 for acceleration, assuming $v_0 = 0$ at the peak of its flight and the rock falls 30 m in 4.0 s:

$$a = \frac{2\Delta x}{t^2} = \frac{2(-30 \text{ m})}{(4.0 \text{ s})^2} = -3.8 \text{ m/s}^2 \quad \Rightarrow |a| = \boxed{3.8 \text{ m/s}^2}$$

(b) Find the initial velocity using Eq. 2-7, concentrating on the first half of the flight that ends with $v = 0$ at the peak:

$$v = v_0 + a t$$
$$v_0 = v - a t = 0 - (-3.8 \text{ m/s}^2)(4.0 \text{ s}) = \boxed{15 \text{ m/s}}$$

Insight: There are several other ways of finding the answers, including graphical analysis. Try measuring the slope of the graph at the launch point and the point at which the rock lands to find the initial and final velocities. Those values (about ±15 m/s) can then be used to find the acceleration.

Picture the Problem: A squid emits a jet of water, propelling itself forward with constant acceleration, then coasts to rest with constant (negative) acceleration.

Strategy: The magnitude of the acceleration can be determined from the position as a function of time equation (Equation 2-11) and the given information. During the first part of the squid's motion the initial velocity is zero, and during the second part the final velocity is zero.

Solution: 1. (a) Find the acceleration during the first part of the squid's motion, noting that $x_0 = v_0 = 0$:

$$x_1 = 0 + 0 + \tfrac{1}{2} a_1 t_1^2$$

$$a_1 = \frac{2x_1}{t_1^2} = \frac{2(0.179\text{ m})}{(0.170\text{ s})^2} = \boxed{12.4\text{ m/s}^2}$$

(b) Find the squid's velocity at the end of the first part of its motion:

$$v_1 = v_0 + a_1 t_1 = 0 + (12.4\text{ m/s}^2)(0.170\text{ s}) = 2.11\text{ m/s}$$

The time elapsed during the second part of the squid's motion is found by subtraction:

$$t_2 = t_{\text{total}} - t_1 = 0.400 - 0.170\text{ s} = 0.230\text{ s}$$

The distance traveled during the second part of the squid's motion is found by subtraction:

$$x_2 = x_{\text{total}} - x_1 = 0.421 - 0.179\text{ m} = 0.242\text{ m}$$

Calculate the squid's acceleration during the second part of its motion:

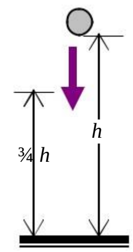
$$x_2 = 0 + v_1 t_2 + \tfrac{1}{2} a_2 t_2^2$$

$$a_2 = \frac{2(x_2 - v_1 t_2)}{t_2^2} = \frac{2\left[(0.242\text{ m}) - (2.11\text{ m/s})(0.230\text{ s})\right]}{(0.230\text{ s})^2}$$

$$a_2 = \boxed{-9.20\text{ m/s}^2}$$

Insight: Notice that the answer to part (b) can also be determined without finding the squid's velocity. Instead, work backward and pretend the squid starts from rest and covers 0.242 m in 0.230 s. Then $a_2 = 2x_2/t_2^2 = 2(0.242\text{ m})/(0.230\text{ s})^2 = 9.15\text{ m/s}^2$, which is the same result to within rounding error, although you must recognize that the acceleration is negative, not positive.

104. **Picture the Problem**: A ball falls straight downward from rest at an initial height *h*.



Strategy: The problem requires that the time to fall the final 3/4 *h* from rest is 1.00 s. Find the velocity v_1 at ¾ *h* above the ground using Equation 2-12. Use Equation 2-11 along with that initial velocity and the time elapsed to determine *h*. Then the total time of fall can be found using Equation 2-11 again, this time with an initial velocity of zero.

Solution: 1. (a) Find the velocity v_1 of the ball after falling a distance ¼ *h*:

$$v_1^2 = 0^2 + 2g\,\Delta x = 2g\left(\tfrac{1}{4}h\right) \Rightarrow v_1 = \sqrt{\tfrac{1}{2}gh}$$

Now insert that velocity as the initial velocity for the remaining portion of the fall into Equation 2-11:

$$\Delta x = v_1 t + \tfrac{1}{2} g t^2$$

$$\tfrac{3}{4}h = \left(\sqrt{\tfrac{1}{2}gh}\right)t + \tfrac{1}{2} g t^2$$

The time *t* is 1.00 s as given in the problem statement. Rearrange the above equation and square both sides to get a quadratic equation:

$$\tfrac{3}{4}h - \tfrac{1}{2} g t^2 = \left(\sqrt{\tfrac{1}{2}gh}\right)t$$

$$\tfrac{9}{16}h^2 - 2\left(\tfrac{1}{2}g t^2\right)\left(\tfrac{3}{4}h\right) + \tfrac{1}{4}g^2 t^4 = \tfrac{1}{2} g h t^2$$

$$h^2 - \left(\tfrac{20}{9} g t^2\right)h + \tfrac{4}{9} g^2 t^4 = 0$$

$$h^2 - \tfrac{20}{9}\left(9.81\text{ m/s}^2\right)\left(1.00\text{ s}\right)^2 h + \tfrac{4}{9}\left(9.81\text{ m/s}^2\right)^2\left(1.00\text{ s}\right)^4 = 0$$

$$h^2 - 21.8h + 42.8 = 0$$

Now apply the quadratic formula for *h*:

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{21.8 \pm \sqrt{(21.8)^2 - 4(1)(42.8)}}{2(1)} = 2.18,\ \boxed{19.6\text{ m}}$$

(b) Use Equation 2-11 again to find the total time of fall:

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(19.6\text{ m})}{9.81\text{ m/s}^2}} = \boxed{2.00\text{ s}}$$

Insight: The first root in step 4 (2.18 m) is thrown out because the total fall time from that height would be less than 1.00 s, but the ball is supposed to be in the air for longer than 1.00 s. Notice it takes half the total flight time to fall the first quarter of the fall distance, and half to fall the final three quarters.

Picture the Problem: A ski glove falls straight downward from rest, accelerates to a maximum speed under the influence of gravity, then decelerates due to its interaction with the snow before coming to rest at a depth *d* below the surface of the snow.

Strategy: We can find the maximum speed of the glove from its initial height and the acceleration of gravity by using Equation 2-12. The same equation can be applied again, this time with a zero final speed instead of zero initial speed, to find the acceleration caused by the snow. Let downward be the positive direction.

Solution: 1. (a) Solve Equation 2-12 for *v*, assuming $v_0 = 0$:

$$v = \sqrt{0^2 + 2gh} = \boxed{\sqrt{2gh}}$$

2. (b) Use Equation 2-12 to find the acceleration caused by the snow:

$$0^2 = v_0^2 + 2ad \Rightarrow -2ad = \left(\sqrt{2gh}\right)^2 \Rightarrow a = \boxed{-\frac{h}{d}g}$$

The negative sign on the acceleration means the glove is accelerated upward during its interaction with the snow.

Insight: In Chapter 5 we will analyze the motion of objects like this glove in terms of force vectors. This motion can also be explained in terms of energy using the tools introduced in Chapters 7 and 8.

Picture the Problem: A ball rises straight upward, passes a power line, momentarily comes to rest, and falls back to Earth again, passing the power line a second time on its way down.

Strategy: The ball will reach the peak of its flight at a time directly between the times it passes the power line. The time to reach the peak of flight can be used to find the initial velocity using Equation 2-7, and the initial velocity can then be used to find the height of the power lines using Equation 2-11.

Solution: 1. Find the time at which the ball reaches its maximum altitude:

$$t_{peak} = t_{line\ up} + \tfrac{1}{2}\left(t_{line\ down} - t_{line\ up}\right) = 0.75\ \text{s} + \tfrac{1}{2}(1.5 - 0.75\ \text{s})$$

$$t_{peak} = \underline{\underline{1.1\ \text{s}}}$$

Find the initial velocity using Equation 2-7:

$$0 = v_0 - g\,t_{peak} \Rightarrow v_0 = \left(9.81\ \text{m/s}^2\right)(1.1\ \text{s}) = \boxed{11\ \text{m/s}}$$

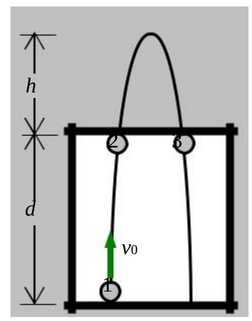
Find the height of the power line using Equation 2-11:

$$x = 0 + v_0\,t_{line\ up} - \tfrac{1}{2} g\,t_{line\ up}^2$$

$$x = (11\ \text{m/s})(0.75\ \text{s}) - \tfrac{1}{2}\left(9.81\ \text{m/s}^2\right)(0.75\ \text{s})^2 = \boxed{5.5\ \text{m}}$$

Insight: As is often the case, there are several other ways to solve this problem. Try setting the heights at 0.75 s and 1.5 s equal to each other and solving for v_0. Can you think of yet another way?

Picture the Problem: A ball appears at the bottom edge of the window, rising straight upward with initial speed v_0. It travels upward, disappearing beyond the top edge of the window, comes to rest momentarily, and then falls straight downward, reappearing some time later at the top edge of the window. In the drawing at right the motion of the ball is offset horizontally for clarity.



Strategy: Let $t = 0$ correspond to the instant the ball first appears at the bottom edge of the window with speed v_0. Write the equation of position as a function of time and accelera-tion (Equation 2-11) for when the ball is at the top edge (position 2) in order to find v_0. Use v_0 to find the time to go from position 1 to the peak of the flight (Equation 2-7). Subtract 0.25 s from that time to find the time to go from position 2 to the peak of the flight. The time elapsed between positions 2 and 3 is twice the time to go from position 2 to the peak of the flight. The time from position 2 to the peak can be used to find h from Equation 2-11.

Solution: 1. (a) Write Equation 2-11 for positions 1 and 2, and solve for v_0:

$$d = v_0\,t_2 - \tfrac{1}{2} g\,t_2^2$$

$$v_0 = \frac{d + \tfrac{1}{2} g\,t_2^2}{t_2} = \frac{1.05\ \text{m} + \tfrac{1}{2}\left(9.81\ \text{m/s}^2\right)(0.25\ \text{s})^2}{0.25\ \text{s}} = \underline{\underline{5.4\ \text{m/s}}}$$

Find the time to go from position 1 to the peak of the flight using Equation 2-7:

$$\Delta t_{1,p} = \frac{0 - v_0}{-g} = \frac{5.4\ \text{m/s}}{9.81\ \text{m/s}^2} = \underline{\underline{0.55\ \text{s}}}$$

Subtract 0.25 s to find the time to go from position 2 to the peak of the flight:

$$\Delta t_{2,p} = \Delta t_{1,p} - \Delta t_{1,2} = 0.55 - 0.25\ \text{s} = \underline{\underline{0.30\ \text{s}}}$$

The time to reappear is twice this time:

$$\Delta t_{2,3} = 2\Delta t_{2,p} = 2(0.30\ \text{s}) = \boxed{0.60\ \text{s}}$$

(b) The height h can be found from $\Delta t_{2,p}$ and Equation 2-11, by considering the ball dropping from rest at the peak to position 3:

$$0 = h + 0 - \tfrac{1}{2} g\,\Delta t_{2,p}^2$$

$$h = \tfrac{1}{2}\left(9.81\ \text{m/s}^2\right)(0.30\ \text{s})^2 = \boxed{0.44\ \text{m}}$$

Insight: As usual there are other ways to solve this problem. Try finding the velocity at position 2 and use it together with the acceleration of gravity and the average velocity from position 2 to the peak to find $\Delta t_{2,3}$ and h.

Picture the Problem: The lunar lander falls straight downward, accelerating over a distance of 4.30 ft before impacting the lunar surface.

Strategy: Use the given acceleration and distance and the time-free equation of motion (Equation 2-12) to find the velocity of the lander just before impact. Use the known initial and final velocities, together with the distance of the fall, to find the time elapsed using Equation 2-10.

Solution: 1. Find the velocity just before impact using Equation 2-12:

$$v_{\text{land}} = \sqrt{v_0^2 + 2a\,\Delta x}$$
$$= \sqrt{(0.500\text{ ft/s})^2 + 2\left(1.62\text{ m/s}^2 \times 3.28\text{ ft/m}\right)(4.30\text{ ft})} = 6.78\text{ ft/s}$$

Solve Equation 2-10 for t_{fall}:

$$t_{\text{fall}} = \frac{\Delta x_{\text{fall}}}{\frac{1}{2}(v_0 + v_{\text{land}})} = \frac{4.30\text{ ft}}{\frac{1}{2}(0.500 + 6.78\text{ ft/s})} = \boxed{1.18\text{ s}}$$

Insight: An alternative strategy would be to solve Equation 2-11 as a quadratic equation in t. Assuming the lander feet had little in the way of shock absorbers, the lander came to rest in a distance given by the amount the lunar dust compacted underneath the feet. Supposing it was about 2 cm, the astronauts experienced a brief deceleration of $106\text{ m/s}^2 = 11g$! Bam!

Picture the Problem: The lunar lander falls straight downward, accelerating over a distance of 4.30 ft before impacting the lunar surface.

Strategy: Use the given acceleration and distance and the time-free equation of motion (Equation 2-12) to find the velocity of the lander just before impact.

Solution: Find the velocity just before impact using Equation 2-12:

$$v_{\text{land}} = \sqrt{v_0^2 + 2a\,\Delta x}$$
$$= \sqrt{(0.500\text{ ft/s})^2 + 2\left(1.62\text{ m/s}^2 \times 3.28\text{ ft/m}\right)(4.30\text{ ft})} = \boxed{6.78\text{ ft/s}}$$

Insight: The initial speed made little difference; if you set $v_0 = 0$ you'll note that $v_{\text{land}} = 6.76$ ft/s.

Picture the Problem: The lunar lander falls straight downward, accelerating over a distance of 4.30 ft before impacting the lunar surface.

Strategy: The lander has an initial downward velocity and accelerates downward at a constant rate. Use the knowledge that the velocity-*versus*-time graph is a straight line for constant acceleration to determine which graph is the appropriate one.

Solution: Graph B is the only one that depicts the speed increasing linearly with time.

Insight: Graph D would be an appropriate depiction of the altitude *versus* time graph.

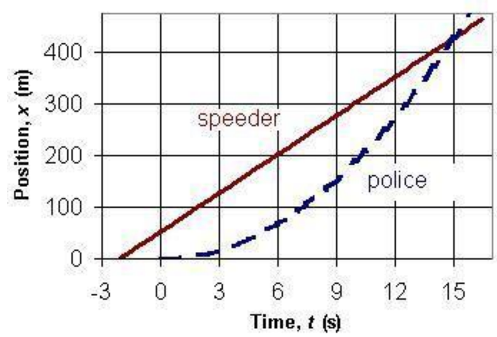
Picture the Problem: We imagine that the astronauts increase the upward thrust, giving the lunar lander a small upward acceleration.

Strategy: The lander has an initial downward velocity and accelerates upward at a constant rate. This means the lander's speed would decrease at a constant rate. Use the knowledge that the velocity-*versus*-time graph is a straight line for constant acceleration to determine which graph is the appropriate one.

Solution: Plot C is the only one that depicts the speed decreasing linearly with time.

Insight: The altitude-*versus*-time graph in this case would curve upward much like plot A but would have an initially negative slope like plot D.

Picture the Problem: The trajectories of the speeder and police car are shown at right. The speeder moves at a constant velocity while the police car has a constant acceleration, except the police car is delayed in time from when the speeder passes it at $x = 0$.



Strategy: The equation of motion for position as a function of time and velocity (Equation 2-10) can be used to describe the speeder, while the equation for position as a function of time and acceleration (Equation 2-11) can be used to describe the police car's motion. Set these two equations equal to each other and solve the resulting equation to find the speeder's head-start x_{shs} .

Solution: 1. Write Equation 2-10 for the speeder, with $t = 0$ corresponding $x_s = x_{shs} + v_s t$ to the instant it passes the police car:

2. Write Equation 2-11 for the police car: $x_p = 0 + 0 + \frac{1}{2} a_p t^2$

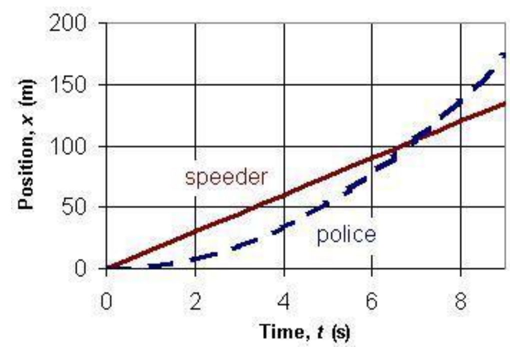
3. Set $x_p = x_s$ and solve for x_{shs}: $\frac{1}{2} a_p t^2 = x_{shs} + v_s t$

$$x_{shs} = \frac{1}{2} a_p t^2 - v_s t = \frac{1}{2}\left(3.8\text{ m/s}^2\right)\left(15\text{ s}\right)^2 - \left(25\text{ m/s}\right)\left(15\text{ s}\right)$$

$$x_{shs} = 53\text{ m}$$

Insight: This head start corresponds to about 2.10 seconds (verify for yourself, and/or examine the plot) so the police officer has to be ready to start the chase very soon after the speeder passes by!

Picture the Problem: The trajectories of the speeder and police car are shown at right. The speeder moves at a constant velocity while the police car has a constant acceleration.



Strategy: The equation of motion for position as a function of time and velocity (Equation 2-10) can be used to describe the speeder, while the equation for position as a function of time and acceleration (Equation 2-11) can be used to describe the police car's motion. Set these two equations equal to each other and solve the resulting equation for the acceleration of the police car.

Solution: 1. Write Equation 2-10 for the speeder, with $t = 0$ corresponding to the instant it passes the police car: $x_s = 0 + v_s t$

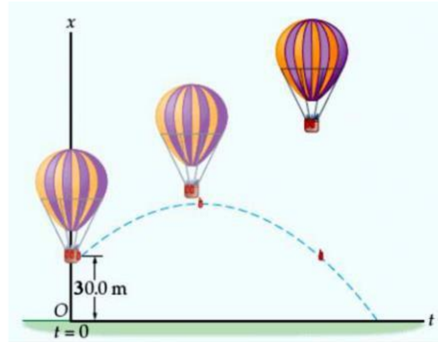
2. Write Equation 2-11 for the police car: $x_p = 0 + 0 + \frac{1}{2} a_p t^2$

3. Set $x_p = x_s$ and solve for a_p : $\frac{1}{2} a_p t^2 = v_s t$

$$a_p = \frac{2v_s}{t} = \frac{2\left(15\text{ m/s}\right)}{7.0\text{ s}} = \boxed{4.3\text{ m/s}^2}$$

Insight: A faster acceleration of the police car would allow it to catch the speeder in less than 7.0 s.

Picture the Problem: The trajectory of a bag of sand is shown at right. After release from the balloon it rises straight up and comes momentarily to rest before accelerating straight downward and impacting the ground.



Strategy: Because the initial velocity, acceleration, and altitude are known, we need only use Equation 2-12 to find the final velocity.

Solution: 1. (a) Because the upward speed of the sandbag is the same, it will gain the same additional 2 m in altitude as it did in the original Example 2-12. Therefore the maximum height will be equal to 32 m.

2. (b) Apply Equation 2-12 to find the final velocity:

$$v^2 = v_0^2 + 2a\,\Delta x$$

$$v = \sqrt{(6.5\text{ m/s})^2 + 2\left(-9.81\text{ m/s}^2\right)(-30.0\text{ m})} = \boxed{25\text{ m/s}}$$

Insight: Another way to find the final velocity just before impact is to allow the sandbag to fall from rest a distance of 32 m. Try it!

Picture the Problem: A bag of sand has an initial downward velocity when it breaks free from the balloon, and is accelerated by gravity until it hits the ground.

Strategy: Because the initial velocity, acceleration, and altitude are known, we need only use Equation 2-12 to find the final velocity. The time can then be found from the average velocity and the distance.

Solution: 1. (a) Apply Equation 2-12 to find the final v:

$$v^2 = v_0^2 + 2a\,\Delta x$$

$$v = \sqrt{(4.2\text{ m/s})^2 + 2\left(-9.81\text{ m/s}^2\right)(-35.0\text{ m})} = \underline{26.5\text{ m/s}}$$

2. Use Equation 2-10 to find the time:

$$t = \frac{x - x_0}{\frac{1}{2}(v_0 + v)} = \frac{0 - 35\text{ m}}{\frac{1}{2}(-4.5 - 26.5\text{ m/s})} = \boxed{2.3\text{ s}}$$

3. (b) Apply Equation 2-12 again to find v at x = 15 m:

$$v^2 = v_0^2 + 2a\,\Delta x$$

$$v = \sqrt{(4.2\text{ m/s})^2 + 2\left(-9.81\text{ m/s}^2\right)(15 - 35\text{ m})} = \boxed{20\text{ m/s}}$$

Insight: Another way to find the descent time of the bag of sand is to solve Equation 2-11 using the quadratic formula. Try it!

Chapter 2: One-Dimensional Kinematics

Answers to Conceptual Questions

Can you drive your car in such a way that the distance it covers is (a) greater than, (b) equal to, or (c) less than the magnitude of its displacement? In each case, give an example if your answer is yes, explain why not if your answer is no.

Yes. If you drive in a complete circle your distance is the circumference of the circle, but your displacement is zero. Yes. The distance and the magnitude of the displacement are equal if you drive in a straight line. (c) No. Any deviation from a straight line results in a distance that is greater than the magnitude of the displacement.

CE **Predict/Explain** You drive your car in a straight line at 15 m/s for 10 minutes, then at 25 m/s for another 10 minutes. **(a)** Is your average speed for the entire trip more than, less than, or equal to 20 m/s? **(b)** Choose the *best* explanation from among the following:

More time is required to drive at 15 m/s than at 25 m/s.
II. Less distance is covered at 25 m/s than at 15 m/s.
III. Equal time is spent at 15 m/s and 25 m/s.

Yes. For example, your friends might have backed out of a parking place at some point in the trip, giving a negative velocity for a short time.

In 1992 Zhuang Yong of China set a women's Olympic record in the 100-meter freestyle swim with a time of 54.64 seconds. What was her average speed in m/s and mi/h?

No. If you throw a ball upward, for example, you might choose the release point to be $y = 0$. This doesn't change the fact that the initial upward speed is nonzero.

A finch rides on the back of a Galapagos tortoise, which walks at the stately pace of 0.060 m/s. After 1.2 minutes the finch tires of the tortoise's slow pace, and takes flight in the same direction for another 1.2 minutes at 12 m/s. What was the average speed of the finch for this 2.4-minute interval?

Ignoring air resistance, the two gloves have the same acceleration.

Solutions to Problems and Conceptual Exercises

In 1992 Zhuang Yong of China set a women's Olympic record in the 100-meter freestyle swim with a time of 54.64 seconds. What was her average speed in m/s and mi/h?

Picture the Problem: The swimmer swims in the forward direction.

Strategy: The average speed is the distance divided by elapsed time.

Solution: Divide the distance by the time:
$$s = \frac{\text{distance}}{\text{time}} = \frac{100.0\text{ m}}{54.64\text{ s}} = \boxed{1.830\text{ m/s}} \times \frac{1\text{ mi}}{1609\text{ m}} \times \frac{3600\text{ s}}{1\text{ h}} = \boxed{4.095\text{ mi/h}}$$

Insight: The displacement would be zero in this case because the swimmer swims either two lengths of a 50-m pool or four lengths of a 25-m pool, returning to the starting point each time. However, the average speed depends upon distance traveled, not displacement.

Estimate how fast your hair grows in miles per hour.

Picture the Problem: Your hair grows at a fixed speed.

Strategy: The growth rate is the length gained divided by the time elapsed. Hair grows at a rate of about half an inch a month, or about 1 cm or 0.01 m per month.

Solution: Divide the length gained by the elapsed time: $s = \dfrac{d}{t} = \dfrac{0.010\text{ m}}{1\text{ mo}} \times \dfrac{1\text{ mi}}{1609\text{ m}} \times \dfrac{1\text{ mo}}{30.5\text{ d}} \times \dfrac{1\text{ d}}{24\text{ h}} = \boxed{8.5\times10^{-9}\text{ mi/h}}$

Insight: Try converting this growth rate to a more appropriate unit such as μm/h. (Answer: 14 μm/h.) Choosing an appropriate unit can help you communicate a number more effectively.

IP You drive in a straight line at 20.0 m/s for 10.0 minutes, then at 30.0 m/s for another 10.0 minutes. **(a)** Is your average speed 25.0 m/s, more than 25.0 m/s, or less than 25.0 m/s? Explain. **(b)** Verify your answer to part (a) by calculating the average speed.

Picture the Problem: You travel in a straight line at two different speeds during the specified time interval.

Strategy: Determine the average speed by first calculating the total distance traveled and then dividing it by the total time elapsed.

Solution: 1. (a) Because the time intervals are the same, you spend equal times at 20 m/s and 30 m/s, and your average speed will be $\boxed{\text{equal to 25.0 m/s}}$.

2. (b) Divide the total distance by the time elapsed: $s_{av} = \dfrac{s_1 t_1 + s_2 t_2}{t_1 + t_2} = \dfrac{(20.0\text{ m/s})(10.0\text{ min}\times 60\text{ s}) + (30.0\text{ m/s})(600\text{ s})}{600 + 600\text{ s}}$

$s_{av} = \boxed{25.0\text{ m/s}}$

Insight: The average speed is a weighted average according to how much *time* you spend traveling at each speed.

IP You drive in a straight line at 20.0 m/s for 10.0 miles, then at 30.0 m/s for another 10.0 miles. **(a)** Is your average speed 25.0 m/s, more than 25.0 m/s, or less than 25.0 m/s? Explain. **(b)** Verify your answer to part (a) by calculating the average speed.

Picture the Problem: You travel in a straight line at two different speeds during the specified time interval.

Strategy: Determine the average speed by first calculating the total distance traveled and then dividing it by the total time elapsed.

Solution: 1. (a) The distance intervals are the same but the time intervals are different. You will spend more time at the lower speed than at the higher speed. Because the average speed is a time weighted average, it will be $\boxed{\text{less than 25.0 m/s}}$.

2. (b) Divide the total distance by the time elapsed: $s_{av} = \dfrac{d_1 + d_2}{t_1 + t_2} = \dfrac{d_1 + d_2}{\dfrac{d_1}{s_1} + \dfrac{d_2}{s_2}} = \dfrac{20.0\text{ mi}}{\left(\dfrac{10.0\text{ mi}}{20.0\text{ m/s}} + \dfrac{10.0\text{ mi}}{30.0\text{ m/s}}\right)}$

$s_{av} = \boxed{24.0\text{ m/s}}$

Insight: Notice that in this case it is not necessary to convert miles to meters in both the numerator and denominator because the units cancel out and leave m/s in the numerator.

CE Predict/Explain Two bows shoot identical arrows with the same launch speed. To accomplish this, the string in bow 1 must be pulled back farther when shooting its arrow than the string in bow 2. **(a)** Is the acceleration of the arrow shot by bow 1 greater than, less than, or equal to the acceleration of the arrow shot by bow 2? **(b)** Choose the *best explanation* from among the following:

The arrow in bow 2 accelerates for a greater time.

II. Both arrows start from rest.

The arrow in bow 1 accelerates for a greater time.

Picture the Problem: Two arrows are launched by two different bows.

Strategy: Use the definitions of average speed and acceleration to compare the motions of the two arrows.

Solution: 1. (a) We can reason that because both arrows undergo uniform acceleration between the same initial and final velocities, both arrows must have the same average speed. If they have the same average speed, then arrow 1, which must travel a longer distance, will be accelerated for a longer period of time. We conclude that the acceleration of the arrow shot by bow 1 is less than the acceleration of the arrow shot by bow 2.

(b) As discussed above, the best explanation is **III**. The arrow in bow 1 accelerates over a greater time. Statement I is false and statement II is true but is not a complete explanation.

Insight: We could also set $v_0 = 0$ in the equation, $v^2 = v_0^2 + 2a\,x$ and solve for a: $a = v^2/2x$ From this expression we can see that for the same final velocity v, the arrow that is accelerated over the greater distance x will have the smaller acceleration.

IP In the previous problem, **(a)** does the distance needed to stop increase by a factor of two or a factor of four? Explain. Verify your answer to part (a) by calculating the stopping distances for initial speeds of **(b)** 16 m/s and **(c)** 32 m/s.

Picture the Problem: The car travels in a straight line in the positive direction while accelerating in the negative direction (slowing down).

Strategy: Use the average velocity and the time elapsed to determine the distance traveled for the specified change in velocity.

Solution: 1. (a) Because the distance traveled is proportional to the square of the time (Equation 2-11), or alternatively, because both the time elapsed and the average velocity change by a factor of two, the stopping distance will increase by a factor of four when you double your driving speed.

2. (b) Evaluate Equation 2-10 directly: $x = \frac{1}{2}(v_0 + v)\,t = \frac{1}{2}(16 + 0\text{ m/s})(3.8) = \boxed{30\text{ m}} = 0.030\text{ km}$

3. (c) Evaluate Equation 2-10 directly: $x = \frac{1}{2}(v_0 + v)\,t = \frac{1}{2}(32 + 0\text{ m/s})(7.6) = \boxed{120\text{ m}} = 0.12\text{ km}$

Insight: Doubling your speed will quadruple the stopping distance for a constant acceleration. We will learn in chapter 7 that this can be explained in terms of energy; that is, doubling your speed quadruples your kinetic energy.

Suppose the car in Problem 44 comes to rest in 35 m. How much time does this take?

Picture the Problem: The car travels in a straight line toward the west while accelerating in the easterly direction (slowing down).

Strategy: The average velocity is simply half the sum of the initial and final velocities because the acceleration is uniform. Use the average velocity together with Equation 2-10 to find the time.

Solution: Solve Equation 2-10 for time: $t = \dfrac{x}{\frac{1}{2}(v_0 + v)} = \dfrac{35\text{ m}}{\frac{1}{2}(12 + 0\text{ m/s})} = 5.8\text{ s}$

Insight: The distance traveled is always the average velocity multiplied by the time. This stems from the definition of average velocity.

Air Bags Air bags are designed to deploy in 10 ms. Estimate the acceleration of the front surface of the bag as it expands. Express your answer in terms of the acceleration of gravity *g*.

Picture the Problem: An air bag expands outward with constant positive acceleration.

Strategy: Assume the air bag has a thickness of 1 ft or about 0.3 m. It must expand that distance within the given time of 10 ms. Employ the relationship between acceleration, displacement, and time (Equation 2-11) to find the acceleration.

Solution: Solve Equation 2-11 for a:

$$a = \frac{2x}{t^2} = \frac{2(0.3\text{ m})}{(10\text{ ms} \times 0.001\text{ s/ms})^2} = 6000\text{ m/s}^2 \times \frac{1\,g}{9.81\text{ m/s}^2} = \boxed{600g}$$

Insight: The very large acceleration of an expanding airbag can cause severe injury to a small child whose head is too close to the bag when it deploys. Children are safest in the back seat!

IP Coasting due west on your bicycle at 8.4 m/s, you encounter a sandy patch of road 7.2 m across. When you leave the sandy patch your speed has been reduced by 2.0 m/s to 6.4 m/s. **(a)** Assuming the sand causes a constant acceleration, what was the bicycle's acceleration in the sandy patch? Give both magnitude and direction. **(b)** How long did it take to cross the sandy patch? **(c)** Suppose you enter the sandy patch with a speed of only 5.4 m/s. Is your final speed in this case 3.4 m/s, more than 3.4 m/s, or less than 3.4 m/s? Explain.

Picture the Problem: A bicycle travels in a straight line, slowing down at a uniform rate as it crosses the sandy patch.

Strategy: Use the time-free relationship between displacement, velocity, and acceleration (Equation 2-12) to find the acceleration. The time can then be determined from the average velocity and the distance across the sandy patch.

Solution: 1. (a) Calculate the acceleration:

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(6.4\text{ m/s})^2 - (8.4\text{ m/s})^2}{2(7.2\text{ m})} = -2.1\text{ m/s}^2$$

where the negative sign means $\boxed{2.1\text{ m/s}^2 \text{ to the east}}$.

2. (b) Solve Equation 2-10 for t:

$$t = \frac{x}{\frac{1}{2}(v + v_0)} = \frac{7.2\text{ m}}{\frac{1}{2}(8.4 + 6.4\text{ m/s})} = \boxed{0.97\text{ s}}$$

(c) Examining $v^2 = v_0^2 + 2a\,x$ (Equation 2-12) in detail, we note that the acceleration is negative, and that the final velocity is the square root of the difference between v_0^2 and $|2a\,x|$. Because $|2a\,x|$ is constant because the sandy patch doesn't change, it now represents a larger fraction of the smaller v_0^2, and the final velocity v will be more than 2.0 m/s different than v_0. We therefore expect a final speed of less $\boxed{\text{than 3.4 m/s.}}$

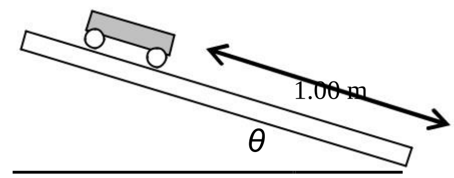
Insight: In fact, if you try to calculate v in part (c) with Equation 2-12 you end up with the square root of a negative number, because the bicycle will come to rest in a distance $x = \frac{0^2 - v_0^2}{2a} = \frac{-(5.4\text{ m/s})^2}{2(-2.1\text{ m/s}^2)} = 6.9\text{ m}$, less than the 7.2 m length of the sandy patch.

129. In a physics lab, students measure the time it takes a small cart to slide a distance of 1.00 m on a smooth track inclined at an angle above the horizontal. Their results are given in the following table.

θ	**10.0°**	**20.0°**	**30.0°**
time, s	1.08	0.770	0.640

(a) Find the magnitude of the cart's acceleration for each angle.
(b) Show that your results for part (a) are in close agreement with the formula, $a = g \sin\theta$. (We will derive this formula in Chapter 5.)

Picture the Problem: The cart slides down the inclined track, each time traveling a distance of 1.00 m along the track.



Strategy: The distance traveled by the cart is given by the constant-acceleration equation of motion for position as a function of time (Equation 2-11), where $x_0 = v_0 = 0$. The magnitude of the acceleration can thus be determined from the given distance traveled and the time elapsed in each case. We can then make the comparison with $a = g \sin\theta$.

Solution: 1. Find the acceleration from Equation 2-11: $x = 0 + 0 + \frac{1}{2}at^2 \Rightarrow a = \frac{2x}{t^2}$ $\quad a = g\sin\theta$

Now find the values for $\theta = 10.0°$: $a = \frac{2.00\text{ m}}{(1.08\text{ s})^2} = \boxed{1.71\text{ m/s}^2}$ $\quad a = (9.81\text{ m/s}^2)\sin 10.0 = \boxed{1.70\text{ m/s}^2}$

Now find the values for $\theta = 20.0°$: $a = \frac{2.00\text{ m}}{(0.770\text{ s})^2} = \boxed{3.37\text{ m/s}^2}$ $\quad a = (9.81\text{ m/s}^2)\sin 20.0 = \boxed{3.35\text{ m/s}^2}$

Now find the values for $\theta = 30.0°$: $a = \frac{2.00\text{ m}}{(0.640\text{ s})^2} = \boxed{4.88\text{ m/s}^2}$ $\quad a = (9.81\text{ m/s}^2)\sin 10.0 = \boxed{4.91\text{ m/s}^2}$

Insight: We see very good agreement between the formula $a = g\sin\theta$ and the measured acceleration. The experimental accuracy gets more and more difficult to control as the angle gets bigger because the elapsed times become very small and more difficult to measure accurately. For this reason Galileo's experimental approach (rolling balls down an incline with a small angle) gave him an opportunity to make accurate observations about free fall without fancy electronic equipment.

Legend has it that Isaac Newton was hit on the head by a falling apple, thus triggering his thoughts on gravity. Assuming the story to be true, estimate the speed of the apple when it struck Newton.

Picture the Problem: An apple falls straight downward under the influence of gravity.

Strategy: The distance of the fall is estimated to be about 3.0 m (about 10 ft). Then use the time-free equation of motion (Equation 2-12) to estimate the speed of the apple.

Solution: 1. Solve Equation 2-12 for v, assuming the apple drops from rest ($v_0 = 0$): $v = \sqrt{0 + 2a\,x}$

2. Let $a = g$ and calculate v: $v = \sqrt{2(9.81\text{ m/s}^2)(3.0\text{ m})} = \boxed{7.7\text{ m/s}} = 17\text{ mi/h}$

Insight: Newton supposedly then reasoned that the same force that made the apple fall also keeps the Moon in orbit around the Earth, leading to his universal law of gravity (Chapter 12). One lesson we might learn here is—wear a helmet when sitting under an apple tree!

Jordan's Jump Michael Jordan's vertical leap is reported to be 48 inches. What is his takeoff speed? Give your answer in meters per second.

Picture the Problem: Michael Jordan jumps vertically, the acceleration of gravity slowing him down and bringing him momentarily to rest at the peak of his flight.

Strategy: Because the height of the leap is known, use the time-free equation of motion (Equation 2-12) to find the takeoff speed.

Solution: Solve Eq. 2-12 for v_0:

$$v_0 = \sqrt{v^2 - 2g\,x} = \sqrt{0^2 - 2\left(-9.81\text{ m/s}^2\right)\left(48\text{ in}\times 0.0254\text{ m/in}\right)} = \boxed{4.9\text{ m/s}}$$

Insight: That speed is about half of what champion sprinters achieve in the horizontal direction, but is very good among athletes for a vertical leap. High jumpers can jump even higher, but use the running start to their advantage.

Bill steps off a 3.0-m-high diving board and drops to the water below. At the same time, Ted jumps upward with a speed of 4.2 m/s from a 1.0-m-high diving board. Choosing the origin to be at the water's surface, and upward to be the positive *x* direction, write *x*-versus-*t* equations of motion for both Bill and Ted.

Picture the Problem: Two divers move vertically under the influence of gravity.

Strategy: In both cases we wish to write the equation of motion for position as a function of time and acceleration (Equation 2-11). In Bill's case, the initial height $x_0 = 3.0$ m , but the initial velocity is zero because he steps off the diving board. In Ted's case the initial height $x_0 = 1.0$ m and the initial velocity is +4.2 m/s. In both cases the acceleration is -9.81 m/s^2.

Solution: 1. Equation 2-11 for Bill:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = 3.0\text{ m} + 0 + \tfrac{1}{2}\left(-9.81\text{ m/s}^2\right)t^2$$

$$\boxed{x = \left(3.0\text{ m}\right) - \left(4.9\text{ m/s}^2\right)t^2}$$

2. Equation 2-11 for Ted:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = 1.0\text{ m} + \left(4.2\text{ m/s}\right)t + \tfrac{1}{2}\left(-9.81\text{ m/s}^2\right)t^2$$

$$\boxed{x = \left(1.0\text{ m}\right) + \left(4.2\text{ m/s}\right)t - \left(4.9\text{ m/s}^2\right)t^2}$$

Insight: The different initial velocities result in significantly different trajectories for Bill and Ted.

Repeat the previous problem, this time with the origin 3.0 m above the water, and with downward as the positive *x* direction.

Picture the Problem: Two divers move vertically under the influence of gravity.

Strategy: In both cases we wish to write the equation of motion for position as a function of time and acceleration (Equation 2-11). Here we'll take the origin to be at the level of Bill's board above the water, Ted's diving board to be at +2.0 m, and the water surface at +3.0 m. Downward is the positive direction so that the acceleration is 9.81 m/s^2. In Bill's case, the initial height $x_0 = 0.0$ m and his initial velocity is zero because he steps off the diving board. In Ted's case the initial height is $x_0 = +2.0$ m and the initial velocity is -4.2 m/s (upward).

Solution: 1. Equation 2-11 for Bill:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = 0.0\text{ m} + 0 + \tfrac{1}{2}\left(9.81\text{ m/s}^2\right)t^2$$

$$\boxed{x = \left(4.9\text{ m/s}^2\right)t^2}$$

2. Equation 2-11 for Ted:

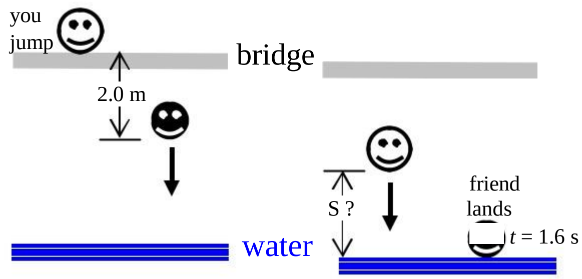
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = 2.0\text{ m} + \left(-4.2\text{ m/s}\right)t + \tfrac{1}{2}\left(9.81\text{ m/s}^2\right)t^2$$

$$\boxed{x = \left(2.0\text{ m}\right) + \left(-4.2\text{ m/s}\right)t + \left(4.9\text{ m/s}^2\right)t^2}$$

Insight: The different initial velocities result in significantly different trajectories for Bill and Ted.

IP Standing side by side, you and a friend step off a bridge at different times and fall for 1.6 s to the water below. Your friend goes first, and you follow after she has dropped a distance of 2.0 m. **(a)** When your friend hits the water, is the separation between the two of you 2.0 m, less than 2.0 m, or more than 2.0 m? **(b)** Verify your answer to part (a) with a calculation.

Picture the Problem: You and your friend both accelerate from rest straight downward, but at different times. You step off the bridge when your friend has fallen 2.0 m, and your friend hits the water while you are still in the air.



Strategy: First find the time it takes for your friend to fall 2.0 m using the equation of motion for position as a function of time and acceleration (Equation 2-11). Subtract that time from 1.6 s to find the time elapsed between when you jump and when your friend hits the water. Use Equation 2-11 and the times found above to find the positions of you and your friend at the time your friend lands. Then determine the separation between the known positions.

Solution: 1. (a) Because your friend has a greater average speed than you do during the time between when you jump and your friend lands, the separation between the two of you will increase to a value $\boxed{\text{more than 2.0 m.}}$

(b) Find the time it takes to fall 2.0 m from Equation 2-11 with $v_0 = 0$:

$$t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(2.0\text{ m})}{9.81\text{ m/s}^2}} = 0.64\text{ s}$$

Find the distance your friend fell in 1.6 s:

$$x_{\text{friend}} = \tfrac{1}{2}gt^2 = \tfrac{1}{2}(9.81\text{ m/s}^2)(1.6\text{ s})^2 = 13\text{ m}$$

Find the distance you fell in the shorter time:

$$x_{\text{you}} = \tfrac{1}{2}g(t - t_{2.0\text{ m}})^2 = \tfrac{1}{2}(9.81\text{ m/s}^2)(1.6 - 0.64\text{ s})^2 = 4.5\text{ m}$$

Find the difference in your positions:

$$S = x_{\text{friend}} - x_{\text{you}} = 13 - 4.5\text{ m} = \boxed{8\text{ m}}$$

Insight: Because of her head start, your friend will always have a higher average velocity than you, and the separation between you and her will continue to increase the longer you both fall.

In a well-known Jules Verne novel, Phileas Fogg travels around the world in 80 days. What was Mr. Fogg's approximate average speed during his adventure?

Picture the Problem: Phileas Fogg travels in a straight line all the way around the world.

Strategy: The average speed is the distance divided by elapsed time. We will estimate that Mr. Fogg travels a distance equal to the equatorial circumference of the Earth. This is an approximation, because his path was most likely much more complicated than that, but we were asked only for the approximate speed.

Solution: Find the circumference of the Earth:

$$d = 2\pi r = 2\pi(6370\times 10^3\text{ m}) = 4.0\times 10^7\text{ m}$$

Divide the distance by the time:

$$s = \frac{\text{distance}}{\text{time}} = \frac{4.0\times 10^7\text{ m}}{80\text{ d}\times 24\text{ h/d}\times 3600\text{ s/h}} = \boxed{5.8\text{ m/s}}$$

Insight: This speed corresponds to about 13 mi/h and is faster than humans can walk. Giving time for sleeping, eating, and other delays, Mr. Fogg needs a relatively fast means of travel.

You jump from the top of a boulder to the ground 1.5 m below. Estimate your deceleration on landing.

Picture the Problem: You jump off a boulder, accelerate from rest straight downward and land, bending your knees so that your center of mass comes to rest over a short vertical distance.

Strategy: Employ the relationship between acceleration, displacement, and velocity (Equation 2-12) to find your final velocity just before landing. Then estimate the distance your center of mass will move after your feet contact the ground, and use that distance to estimate your deceleration rate.

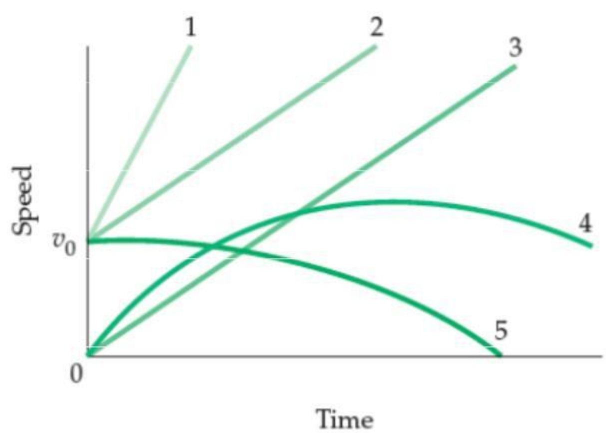
Solution: 1. Solve Equation 2-12 for velocity v: $v = \sqrt{v_0^2 + 2a\,x} = \sqrt{0^2 + 2\left(9.81\text{ m/s}^2\right)\left(1.5\text{ m}\right)} = \underline{5.4\text{ m/s}}$

Estimate your center of mass moves downward about 0.5 m after your feet contact the ground and you bend your knees into a crouching position.

Solve Equation 2-12 for acceleration: $a = \dfrac{v^2 - v_0^2}{2\,y} = \dfrac{0^2 - (5.4\text{ m/s})^2}{2\left(0.50\text{ m}\right)} = \boxed{-29\text{ m/s}^2} = -3.0g$

Insight: When a gymnast lands from an even higher altitude, she might try to bend her knees even less in order to impress the judges. If she lands from an altitude of 3.0 m and bends her knees so her center of mass moves only 0.2 m, her acceleration is $-15g$!

CE At the edge of a roof you drop ball A from rest, and then throw ball B downward with an initial velocity of v_0 . Is the increase in speed just before the balls land more for ball A, more for ball B, or the same for each ball?



Picture the Problem: Two balls are released from the edge of a roof. Ball A is dropped from rest but ball B is thrown downward with an initial velocity v_0 .

Strategy: Use the definition of acceleration to answer the conceptual question, keeping in mind the average speed of ball B is greater than the average speed of ball A.

Solution: The two balls fall the same distance but ball B has the greater average speed and falls for a shorter length of time. Because each ball accelerates at the same rate of 9.81 m/s^2, ball A accelerates for a longer time and the increase in speed is $\boxed{\text{more for ball A}}$ than it is for ball B.

Insight: If ball B were fired downward at an extremely high speed, it would reach the ground within a very short interval of time and its speed would hardly change at all.

IP A youngster bounces straight up and down on a trampoline. Suppose she doubles her initial speed from 2.0 m/s to 4.0 m/s. **(a)** By what factor does her time in the air increase? **(b)** By what factor does her maximum height increase? **(c)** Verify your answers to parts (a) and (b) with an explicit calculation.

Picture the Problem: A youngster bounces straight up and down on a trampoline. The child rises straight upward, slows down, and momentarily comes to rest before falling straight downward again.

Strategy: Find the time of flight by exploiting the symmetry of the situation. If it takes time t for gravity to slow the child down from her initial speed v_0 to zero, it will take the same amount of time to accelerate her back to the same speed. She therefore lands at the same speed v_0 with which she took off. Use this fact together with Equation 2-7 to find the time of flight. The maximum height she achieves is related to the square of v_0, as indicated by Equation 2-12.

Solution: 1. (a) Because the time of flight depends linearly upon the initial velocity, doubling v_0 will increase her time of flight by a factor of 2.

(b) Because the time of flight depends upon the square of the initial velocity, doubling v_0 will increase her maximum altitude by a factor of 4.

3. (c) The time of flight for $v_0 = 2.0$ m/s, using Eq. 2-7: $t = \dfrac{v - v_0}{-g} = \dfrac{(-v_0) - v_0}{-g} = \dfrac{2v_0}{g} = \dfrac{2\,(2.0 \text{ m/s})}{9.81 \text{ m/s}^2} = \boxed{0.41 \text{ s}}$

4. The time of flight for $v_0 = 4.0$ m/s: $t = \dfrac{2v_0}{g} = \dfrac{2\,(4.0 \text{ m/s})}{9.81 \text{ m/s}^2} = \boxed{0.82 \text{ s}}$

5. The maximum height for $v_0 = 2.0$ m/s, using Eq. 2-12: $x = \dfrac{v^2 - v_0^2}{-2g} = \dfrac{0^2 - v_0^2}{-2g} = \dfrac{v_0^2}{2g} = \dfrac{(2.0 \text{ m/s})^2}{2\left(9.81 \text{ m/s}^2\right)} = \boxed{0.20 \text{ m}}$

6. The maximum height for $v_0 = 4.0$ m/s: $x = \dfrac{v_0^2}{2g} = \dfrac{(4.0 \text{ m/s})^2}{2\left(9.81 \text{ m/s}^2\right)} = \boxed{0.82 \text{ m}}$

Insight: The reason the answer in step 6 is not exactly four times larger than the answer in step 5 is due to the rounding required by the fact that there are only two significant digits. If you recalculate using 2.00 m/s and 4.00 m/s, the answers are 0.204 and 0.816 m, respectively.

IP A popular entertainment at some carnivals is the blanket toss (see photo, p. 39). **(a)** If a person is thrown to a maximum height of 28.0 ft above the blanket, how long does she spend in the air? **(b)** Is the amount of time the person is above a height of 14.0 ft more than, less than, or equal to the amount of time the person is below a height of 14.0 ft? Explain. **(c)** Verify your answer to part (b) with a calculation.

Picture the Problem: The person is thrown straight upward, slows down, and momentarily comes to rest before falling straight downward again.

Strategy: Find the time of flight by exploiting the symmetry of the situation. If it takes time t for gravity to slow the person down from her initial speed v_0 to zero, it will take the same amount of time to accelerate her back to the same speed. It therefore takes the same amount of time for her to rise to the peak of her flight than it does for her to return to the blanket. Use this fact together with Equation 2-11 with $v_0 = 0$ (corresponding to the second half of her flight, from the peak back down to the blanket) to find the time of flight. The time above and below 14.0 ft can be found using the same equation.

Solution: 1. (a) The time of flight can be found from Equation 2-11: $t = 2 \times t_{\text{down}} = 2\sqrt{\dfrac{2x}{g}} = 2\sqrt{\dfrac{2(28.0 \text{ ft} \times 0.305 \text{ m/ft})}{9.81 \text{ m/s}^2}} = \boxed{2.64 \text{ s}}$

(b) The person's average speed is less during the upper half of her trajectory, so the time she spends in that portion of her flight is more than the time she spends in the lower half of her flight.

3. (c) The time she spends above 14.0 ft is the same time of her flight if her maximum height were 14.0 ft:

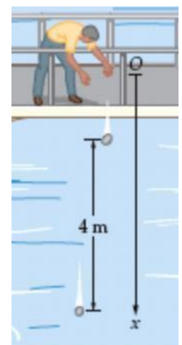
$$t_{\text{above}} = 2\times\sqrt{\frac{2x}{g}} = 2\sqrt{\frac{2(14.0\text{ ft}\times 0.305\text{ m/ft})}{9.81\text{ m/s}^2}} = \boxed{1.87\text{ s}}$$

4. The time spent below 14.0 ft is the remaining portion of the total time of flight:

$$t_{\text{below}} = t_{\text{total}} - t_{\text{above}} = 2.64 - 1.87\text{ s} = \boxed{0.77\text{ s}}$$

Insight: The symmetry of the motion of a freely falling object can often be a useful tool for solving problems quickly.

Referring to Conceptual Checkpoint 2–5, find the separation between the rocks at **(a)** $t = 1.0$ s, **(b)** $t = 2.0$ s, and $t = 3.0$ s, where time is measured from the instant the second rock is dropped. **(d)** Verify that the separation increases linearly with time.



Picture the Problem: The two rocks fall straight downward along a similar path except at different times.

Strategy: First find the time elapsed between the release of the two rocks by finding the time required for the first rock to fall 4.00 m, using the equation of motion for position as a function of time and acceleration (Equation 2-11). The positions as a function of time for each rock can then be compared to find a separation distance as a function of time.

Solution: 1. (a) Find the time required for rock A to fall 4.00 m:

$$t_4 = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00\text{ m})}{9.81\text{ m/s}^2}} = \underline{0.903\text{ s}}$$

Let t represent the time elapsed from the instant rock B is dropped. The position of rock A (Equation 2-11) is thus:

$$x_A = 0 + \tfrac{1}{2}g\left(t + t_4\right)^2 = \tfrac{1}{2}gt^2 + g\,t\,t_4 + \tfrac{1}{2}gt_4^2$$

The position of rock B (Equation 2-11) is:

$$x_B = 0 + \tfrac{1}{2}gt^2 = \tfrac{1}{2}gt^2$$

Find the separation between the rocks:

$$x = x_A - x_B = \left(\tfrac{1}{2}gt^2 + g\,t\,t_4 + \tfrac{1}{2}gt_4^2\right) - \tfrac{1}{2}gt^2$$

$$= g\,t\,t_4 + \tfrac{1}{2}gt_4^2 = \left(9.81\text{ m/s}^2\right)t\left(0.903\text{ s}\right) + \tfrac{1}{2}\left(9.81\text{ m/s}^2\right)\left(0.903\text{ s}\right)^2$$

$$= \left(8.86\text{ m/s}\right)t + 4.00\text{ m}$$

Find x for $t = 1.0$ s:

$$= \left(8.86\text{ m/s}\right)\left(1.0\text{ s}\right) + 4.00\text{ m} = \boxed{12.9}\text{ m}$$

(b) Find x for $t = 2.0$ s:

$$x = \left(8.86\text{ m/s}\right)\left(2.0\text{ s}\right) + 4.00\text{ m} = \boxed{22\text{ m}}$$

8. (c) Find x for $t = 1.0$ s:

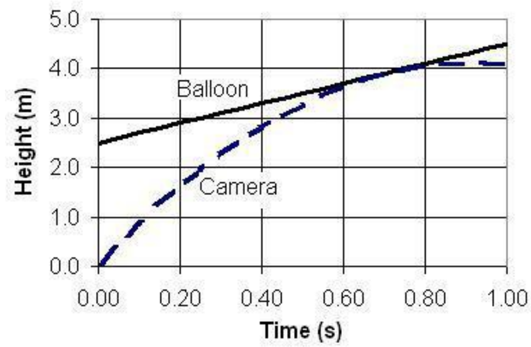
$$x = \left(8.86\text{ m/s}\right)\left(3.0\text{ s}\right) + 4.00\text{ m} = \boxed{31\text{ m}}$$

(d) The linear dependence of x upon t can be verified by examining the equation derived in step 5.

Insight: The only way for rock B to catch up to rock A would be for rock B to be thrown downward with a large initial speed. In that case the separation becomes $x = \left(8.86\text{ m/s} - v_{B,0}\right)t + 4.00\text{ m}$, which decreases to zero as long as $v_{B,0}$ is greater than 8.86 m/s.

In the previous problem, what is the minimum initial speed of the camera if it is to just reach the passenger? (*Hint:* When the camera is thrown with its minimum speed, its speed on reaching the passenger is the same as the speed of the passenger.)

Picture the Problem: The trajectories of the balloon and camera are shown at right. The balloon rises at a steady rate while the camera's speed is continually slowing down under the influence of gravity. The camera is caught when the two trajectories meet.



Strategy: The camera meets the balloon when the positions are equal, so that is our starting point. For the case when the camera just barely meets the balloon, the velocity of the camera must match the velocity of the balloon (2.0 m/s). We use this fact to find the time the two must meet, and substitute that into the position equation. We can then solve for the initial velocity of the camera.

Solution: 1. Write Equation 2-10 for the balloon: $x_b = x_{b,0} + v_b t$

Write Equation 2-12 for the camera: $x_c = \dfrac{v_c^2 - v_{c,0}^2}{-2g}$

Set $x_b = x_c$ and solve for $v_{c,0}$: $x_{b,0} + v_b t = \dfrac{v_c^2 - v_{c,0}^2}{-2g} \Rightarrow v_{c,0}^2 = v_c^2 \quad 2g\left(x_{b,0} + v_b t\right)$

As indicated above, the camera will be caught not only when it's at the same position as the balloon, but when its velocity is the same as well, so set $v_c = v_b$: $v_{c,0}^2 = v_b^2 + 2g x_{b,0} + 2g v_b t$

The two will meet at a time when their velocities are equal. Write Equation 2-7 for the camera and set its final velocity equal to the balloon's velocity, and find the time. $v_c = v_{c,0} - gt = v_b$ $= \dfrac{v_{c,0} - v_b}{g}$

Substitute the time into the equation in step 4: $v_{c,0}^2 = v_b^2 + 2g x_{b,0} + 2v_b\left(v_{c,0} - v_b\right)$

$v_{c,0}^2 - 2v_b v_{c,0} + v_b^2 - 2g x_{b,0} = 0$

$v_{c,0}^2 - 2(2.0\text{ m/s})\, v_{c,0} + (2.0\text{ m/s})^2 \quad 2(9.81\text{ m/s}^2)(2.5\text{ m}) = 0$

$_{,0} - 4.0v \quad - 45\text{ m}^2/\text{s}^2 = 0$

You can get the roots using the quadratic formula, but you might recognize the simple factors here. Only the positive root corresponds to the camera going *upward*: $v_c + 5)(v_c - 9) = 0 \quad v_c = -5.0, \boxed{9.0\text{ m/s}}$

Insight: This is a complicated problem that always ends with a quadratic solution. It required the kind of strategy that must usually be mapped out after trying a few things; don't feel bad if you didn't intuitively choose this strategy. There are other strategies that work, but they are equally complicated.

Old Faithful Watching Old Faithful erupt, you notice that it takes a time t for water to emerge from the base of the geyser and reach its maximum height. **(a)** What is the height of the geyser, and **(b)** what is the initial speed of the water? Evaluate your expressions for **(c)** the height and **(d)** the initial speed for a measured time of 1.65 s.

Picture the Problem: The water shoots straight upward, slows down, and momentarily comes to rest before falling straight downward again.

Strategy: Find the height of the geyser by exploiting the symmetry of the situation. If it takes time t for gravity to slow the water down from its initial speed v_0 to zero, it will take the same amount of time to accelerate it back to the same speed. The height of the geyser is therefore determined by the distance the water will fall from rest in time t (Equation 2-11). Gravity will slow the water down from its initial velocity to zero in time t at a known rate (-9.81 m/s^2), so that fact can be used to find the initial velocity (Equation 2-7).

Solution: 1. (a) Solve Equation 2-11 for x_0, setting $x = 0$ and $v_0 = 0$ for the case when the water falls from rest in time t:

$$0 = x_0 + 0 - \tfrac{1}{2} g t^2$$

(b) Use Equation 2-7 to find the initial velocity if the final velocity is zero (upward portion of the flight):

$$v = v_0 - g t = 0$$

$$v_0 = \boxed{g t}$$

(c) Substitute $t = 1.65$ s into the equation from step 1:

$$x_{max} = \tfrac{1}{2}\left(9.81\text{ m/s}^2\right)(1.65\text{ s})^2 = \boxed{13.4}$$

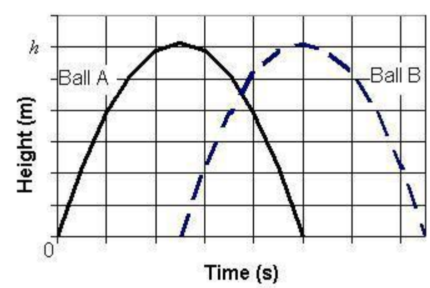
(d) Substitute $t = 1.65$ s into the equation from step 2:

$$\text{m } v_0 = \left(9.81\text{ m/s}^2\right)(1.65\text{ s}) = 16.2\text{ m/s}$$

Insight: If you round off $g = 10\text{ m/s}^2$, you can impress your friends by memorizing these simple formulae and doing the quick calculations in your head!

IP A ball is thrown upward with an initial speed v_0 . When it reaches the top of its flight, at a height h, a second ball is thrown upward with the same initial velocity. **(a)** Sketch an x-versus-t plot for each ball. **(b)** From your graph, decide whether the balls cross paths at $h/2$, above $h/2$, or below $h/2$. **(c)** Find the height where the paths cross.

Picture the Problem: The trajectories of the two balls are shown at right. Remember that in each case the balls are traveling straight up and straight down; the graphs look parabolic because time is the x axis. Ball B is tossed upward at the instant ball A reaches the peak of its flight. Ball A has begun its descent when it is passed by ball B, which is still on its way up toward its peak.



Strategy: The positions are equal to each other when the balls cross paths. The launch times are offset by the time it takes the ball to reach the peak of its flight. That time is given by the time it takes gravity to slow the ball from v_0 down to zero (Equation 2-7). The time the balls cross is directly between the time ball B is launched and ball A lands. Once we have the time figured out we can find the position of ball A in terms of its maximum height h.

Solution: 1. The plot of *x-versus-t* for the two balls is shown above. **2.** Judging from the plot the balls will cross paths $\boxed{\text{above } h/2.}$

3. Find the time it takes ball A to reach its peak:

$$t = \frac{v - v_0}{-g} = \frac{0 - v_0}{-g} = \frac{v_0}{g}$$

Because ball B is launched at time v_0/g and ball A lands at time $2v_0/g$, the two balls will cross at a time midway between these, or at time $t_{cross} = 3v_0/2g$.

$$2 \quad \left(\frac{3v}{}\right) \quad 1 \quad \left(\frac{3v}{}\right)^2 \quad \frac{3v^2}{}$$

5. Find the position of ball A at time t_{cross} using Eq. 2-11:

$$x_A = v_0 t_{\text{cross}} - \tfrac{1}{2} g t_{\text{cross}}^2 = v_0\left(\frac{\;_0}{2g}\right) - \;g\left(\frac{\;_0}{2g}\right)^2 = \frac{\;_0}{8g}$$

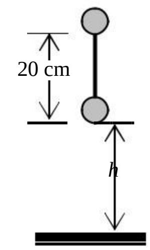
6. Find the maximum height h using Equation 2-12: $0^2 = v_0^2 - 2gh \Rightarrow h = \dfrac{v_0^2}{2g}$

7. Now write x_A in terms of h: $\dfrac{x_A}{h} = \dfrac{3v_0^2/8g}{v_0^2/2g} = \dfrac{3}{4} \Rightarrow x_A = \boxed{\tfrac{3}{4}}\ h$

Insight: The balls do not cross right at h / 2 because they spend more time above h / 2 than they do below, because their average speeds are smaller during the top half of their flight.

Weights are tied to each end of a 20.0-cm string. You hold one weight in your hand and let the other hang vertically a height h above the floor. When you release the weight in your hand, the two weights strike the ground one after the other with audible thuds. Find the value of h for which the time between release and the first thud is equal to the time between the first thud and the second thud.

Picture the Problem: The two weights fall straight downward from rest along a similar path except at different times.



Strategy: The problem requires that the time to fall a distance h from rest (the time between release and the first thud) is the time to fall a distance h + 20 cm (second thud) minus the time to fall a distance h (first thud). We can set these times equal to each other, use Equation 2-11 to write the times in terms of heights, and then solve for h.

Solution: 1. Set the time intervals equal to each other: $t_h = t_{h+20} - t_h \Rightarrow 2t_h = t_{h+20}$

2. Now use Equation 2-11 to write the times in terms of the heights: $2\sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2(h+20.0\text{ cm})}{g}}$

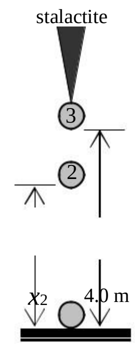
3. Square both sides and multiply by g / 2: $4h = h + 20.0\text{ cm}$

$h = \dfrac{20.0}{3}\text{ cm} = \boxed{6.67\text{ cm}}$

Insight: The tension in the string will be zero during the descent because each ball accelerates at the same rate. Therefore the string will have no effect upon the motion of the balls.

A stalactite on the roof of a cave drips water at a steady rate to a pool 4.0 m below. As one drop of water hits the pool, a second drop is in the air, and a third is just detaching from the stalactite. **(a)** What are the position and velocity of the second drop when the first drop hits the pool? **(b)** How many drops per minute fall into the pool?

Picture the Problem: The three drops are positioned as depicted at right. They all fall straight downward from an initial height of 4.0 m.



Strategy: The time interval between drops is half the time it takes a drop to fall the entire 4.0 m. Use this fact to find the position and velocity of drop 2 when drop 1 hits the pool (equations 2- 11 and 2-7). Then the time interval between drops can be used to find the number of drops per minute.

Solution: 1. (a) Find the time interval between drops, using Equation 2-11 to find the fall time: $\Delta t = \tfrac{1}{2}t_{\text{fall}} = \dfrac{1}{2}\sqrt{\dfrac{2x}{g}} = \dfrac{1}{2}\sqrt{\dfrac{2(4.0\text{ m})}{9.81\text{ m/s}^2}} = \underline{\underline{0.45\text{ s}}}$

Now use Equation 2-11 to find the position of drop 2: $x_2 = 0 + \tfrac{1}{2}g(\Delta t)^2 = \tfrac{1}{2}(9.81\text{ m/s}^2)(0.45\text{ s})^2$

$x_2 = 0.99$ m below the stalactite or

$4.0 - 0.99\text{ m} = \boxed{3.0\text{ m}}$ above the pool

Use Equation 2-7 to find the speed of drop 2: $v = 0 + g\,\Delta t = (9.81\text{ m/s}^2)(0.45\text{ s}) = \boxed{4.4\text{ m/s}}$

(b) Find the drop rate from the time interval: $D = \dfrac{1\text{ drop}}{} \times \dfrac{60\text{ s}}{} = \boxed{130\text{ drops/min}}$

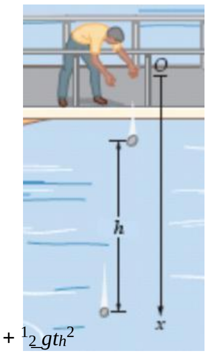
0.45 s 1 min

Insight: Note that it takes half the drop time to fall the first quarter of the drop distance, and half the time to fall the final three quarters of the distance.

Suppose the first rock in Conceptual Checkpoint 2–5 drops through a height h before the second rock is released from rest. Show that the separation between the rocks, S, is given by the following expression:

$$S = h + (\sqrt{2gh})t$$

In this result, the time t is measured from the time the second rock is dropped.



Picture the Problem: The two rocks fall straight downward along a similar path except at different times.

Strategy: First find the time elapsed between the release of the two rocks by finding the time required for the first rock to fall a distance h, using the equation of motion for position as a function of time and acceleration (Equation 2-11). The positions as a function of time for each rock can then be compared to find a separation distance as a function of time.

Solution: 1. (a) Find the time required for rock A to fall a distance h:

$$t_h = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2h}{g}}$$

Let t represent the time elapsed from the instant rock B is dropped. The position of rock A (equation 2-11) is thus:

$$x_A = 0 + \tfrac{1}{2}g(t + t_h)^2 = \tfrac{1}{2}gt^2 + g\,t\,t_h + \tfrac{1}{2}gt_h^2$$

3. The position of rock B (Equation 2-11) is:

$$x_B = 0 + \tfrac{1}{2}g\,t^2 = \tfrac{1}{2}gt^2$$

4. Find the separation between the rocks:

$$S = x_A - x_B = \left(\tfrac{1}{2}gt^2 + g\,t\,t_h + \tfrac{1}{2}gt_h^2\right) - \tfrac{1}{2}gt^2$$

$$S = g\,t\,t_h + \tfrac{1}{2}gt_h^2 = gt\sqrt{\frac{2h}{g}} + \tfrac{1}{2}g\,\frac{2h}{g}$$

$$S = t\sqrt{2gh} + h = \boxed{h + \left(\sqrt{2gh}\right)t}$$

Insight: The separation between the two rocks increases linearly with time t.

An arrow is fired with a speed of 20.0 m/s at a block of Styrofoam resting on a smooth surface. The arrow penetrates a certain distance into the block before coming to rest relative to it. During this process the arrow's deceleration has a magnitude of 1550 m/s^2 and the block's acceleration has a magnitude of 450 m/s^2. **(a)** How long does it take for the arrow to stop moving with respect to the block? **(b)** What is the common speed of the arrow and block when this happens? **(c)** How far into the block does the arrow penetrate?

Picture the Problem: An arrow travels horizontally at 20.0 m/s and impacts the Styrofoam. It continues to travel in the positive direction, but more slowly due to its collision with the Styrofoam. The arrow and the Styrofoam then move together at the same speed in the positive direction.

Strategy: Find the final velocity of the block in terms of the collision time t by using Equation 2-7. Because this is also the final velocity of the arrow, the collision time t can be determined by using the known accelerations and the initial velocity of the arrow. The final velocity and penetration depth traveled can then be found from applying equations 2-7 and 2-11.

Solution: 1. (a) Set the final velocities of the arrow and the block equal to each other and apply Equation 2-7 to find t :

$$v_a = v_b$$

$$v_{a,0} + a_a\,t = 0 + a_b\,t$$

$$t = \frac{-v_{a,0}}{a_a - a_b} = \frac{v_{a,0}}{a_b - a_a} = \frac{20.0\text{ m/s}}{450 - (-1550)\text{ m/s}^2}$$

$$= 0.0100\text{ s} = \boxed{10.0\text{ ms}}$$

2. (b) Now apply Equation 2-7 to find v_b :

$$v_b = a_b t = (450\text{ m/s}^2)(0.0100\text{ s}) = \boxed{4.50\text{ m/s}}$$

(c) The penetration distance is a bit tricky because both the arrow and the block move while they are colliding. The penetration distance is the difference between how far the arrow moves and how far the block moves during the collision time interval.

$$d = x_{\text{arrow}} - x_{\text{block}}$$

$$= \left(v_{a,0}t + \tfrac{1}{2}a_a t^2\right) - \left(\tfrac{1}{2}a_b t^2\right)$$

$$= \left[(20.0\text{ m/s})(0.0100\text{ s}) + \tfrac{1}{2}(-1550\text{ m/s}^2)(0.0100\text{ s})^2\right] - \left[\tfrac{1}{2}(450\text{ m/s}^2)(0.0100\text{ s})^2\right]$$

$$d = 0.1225\text{ m} - 0.0225\text{ m} = 0.100\text{ m} = \boxed{10.0\text{ cm}}$$

Insight: We could also analyze this collision using the concept of momentum conservation (Chapter 9) and work and energy (Chapter 7).

Sitting in a second-story apartment, a physicist notices a ball moving straight upward just outside her window. The ball is visible for 0.25 s as it moves a distance of 1.05 m from the bottom to the top of the window. **(a)** How long does it take before the ball reappears? **(b)** What is the greatest height of the ball above the top of the window?

Picture the Problem: This exercise considers a generic object traveling in a straight line with constant acceleration.

Strategy: Manipulate the suggested equations with algebra to derive the desired results.

Solution: 1. (a) Begin with Equation 2-12:

$$v^2 = v_0^2 + 2a(x - x_0)$$

2. Set $x = 0$ and solve for v:

$$v = \boxed{\pm\sqrt{v_0^2 - 2ax_0}}$$

(b) First write Equation 2-7 and substitute for v. Then solve for t:

$$v = v_0 + at$$

$$\sqrt{v_0^2 - 2ax_0} = v_0 + at$$

$$\boxed{\frac{-v_0 \pm \sqrt{v_0^2 - 2ax_0}}{a}} = t$$

(c) Write Equation 2-11 as given and apply the quadratic formula to solve for t:

$$0 = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\tfrac{1}{2}a\right)(x_0)}}{2\left(\tfrac{1}{2}a\right)}$$

$$t = \boxed{\frac{-v_0 \pm \sqrt{v_0^2 - 2ax_0}}{a}}$$

Insight: When an object undergoes uniform acceleration its position is a quadratic function of time. The quadratic formula is therefore an appropriate one to describe the motion of the object.