Solution Manual for Power System Analysis and Design SI Edition 6th Edition Glover Overbye Sarma 130563618X 9781305636187

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Chapter 2
Fundamentals

ANSWERS TO MULTIPLE-CHOICE TYPE QUESTIONS

2.1	b		**2.19**	a
2.2	a		**2.20**	A. c
2.3	c			B. a
2.4	a			C. b
2.5	b		**2.21**	a
2.6	c		**2.22**	a
2.7	a		**2.23**	b
2.8	c		**2.24**	a
2.9	a		**2.25**	a
2.10	c		**2.26**	b
2.11	a		**2.27**	a
2.12	b		**2.28**	b
2.13	b		**2.29**	a
2.14	c		**2.30**	(i) c
2.15	a			(ii) b
2.16	b			(iii) a
2.17	A. a			(iv) d
	B. b		**2.31**	a
	C. a		**2.32**	a
2.18	c			

2.1 (a) $\bar{A}_1 = 6\angle 30° = 6[\cos 30° + j\sin 30°] = 5.20 + j3$

(b) $\bar{A}_2 = -4 + j5 = \sqrt{16 + 25}\ \angle \tan^{-1}\dfrac{5}{-4} = 6.40\angle 128.66° = 6.40e^{j128.66°}$

(c) $\bar{A}_3 = (5.20 + j3) + (-4 + j5) = 1.20 + j8 = 8.01\angle 81.50°$

(d) $\bar{A}_4 = (6\angle 30°)(6.40\angle 128.66°) = 38.414\angle 158.658° = -35.78 + j13.98$

(e) $\bar{A}_5 = (6\angle 30°)/(6.40\angle -128.66°) = 0.94\angle 158.66° = 0.94e^{j158.66°}$

2.2 (a) $\bar{I} = 500\angle -30° = 433.01 - j250$

(b) $i(t) = 4\sin(\omega t + 30°) = 4\cos(\omega t + 30° - 90°) = 4\cos(\omega t - 60°)$

$\bar{I} = (4)\angle -60° = 2.83\angle -60° = 1.42 - j2.45$

(c) $\bar{I} = (5/\sqrt{2})\angle -15° + 4\angle -60° = (3.42 - j0.92) + (2 - j3.46)$

$= 5.42 - j4.38 = 6.964\angle -38.94°$

2.3 (a) $V_{\max} = 400\,\text{V}; I_{\max} = 100\,\text{A}$

(b) $V = 400/\sqrt{2} = 282.84\,\text{V}; I = 100/\sqrt{2} = 70.71\,\text{A}$

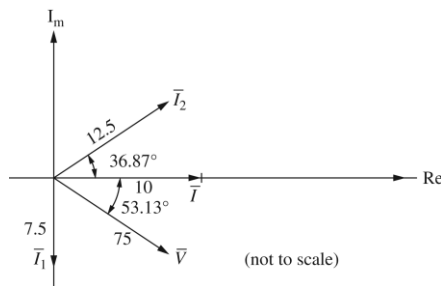(c) $\bar{V} = 282.84\angle 30°\,\text{V}; \bar{I} = 70.71\angle -80°\,\text{A}$

2.4 (a) $\bar{I}_1 = 10\angle 0°\dfrac{-j6}{8 + j6 - j6} = 10\dfrac{6\angle -90°}{8} = 7.5\angle -90°\,\text{A}$

$\bar{I}_2 = \bar{I} - \bar{I}_1 = 10\angle 0° - 7.3\angle -90° = 10 + j7.5 = 12.5\angle 36.87°\,\text{A}$

$\bar{V} = \bar{I}_2(-j6) = (12.5\angle 36.87°)(6\angle -90°) = 75\angle -53.13°\,\text{V}$

(b)

(not to scale)

2.5 (a) $\upsilon(t) = 277\sqrt{2}\cos(\omega t + 30°) = 391.7\cos(\omega t + 30°)\,\text{V}$

(b) (c)

$$\overline{I} = \overline{V} /$$

$$20 = 13.85\angle 30° \,\text{A}$$

$$i(t) = 19.58\cos(\omega t + 30°)\,\text{A}$$

$$\overline{Z} = j\omega L = j(2\pi 60)(10 \times 10^{-3}) = 3.771\angle 90° \,\Omega$$

$$\overline{I} = \overline{V}\,\overline{Z} \quad \Big/ = \frac{(277\angle 30°)}{(3.771\angle 90°)} = 73.46\angle -60° \,\text{A}$$

$$i(t) = 73.46\sqrt{2}\cos(\omega t - 60°) = 103.9\cos(\omega t - 60°)\,\text{A}$$

(d) $\mathbb{Z} = -j\,25\,\Omega$

$$\overline{I} = \overline{V}/\overline{Z} = \left(277\angle 30°\right)/\left(25\angle -90°\right) = 11.08\angle 120°\,\text{A}$$

$$i(t) = 11.08\sqrt{2}\cos\left(\omega t + 120°\right) = 15.67\cos\left(\omega t + 120°\right)\text{A}$$

2.6　(a) $\overline{V} = \left(75/\sqrt{2}\right)\angle -15° = 53.03\angle -15°$; ω does not appear in the answer.

(b) $\upsilon(t) = 50\sqrt{2}\cos\left(\omega t + 10°\right)$; with $\omega = 377$,

$$\upsilon(t) = 70.71\cos\left(377t + 10°\right)$$

(c) $\overline{A} = A\angle\alpha$; $\overline{B} = B\angle\beta$; $\overline{C} = \overline{A} + \overline{B}$

$$c(t) = a(t) + b(t) = \sqrt{2}\,\text{Re}\left[\overline{C}e^{j\omega t}\right]$$

The resultant has the same frequency ω.

2.7　(a) The circuit diagram is shown below:

(b) $\overline{Z} = 3 + j8 - j4 = 3 + j4 = 5\angle 53.1°\,\Omega$

(c) $\overline{I} = \left(100\angle 0°\right)/\left(5\angle 53.1°\right) = 20\angle -53.1°\,\text{A}$

The current lags the source voltage by $53.1°$
Power Factor $= \cos 53.1° = 0.6$ Lagging

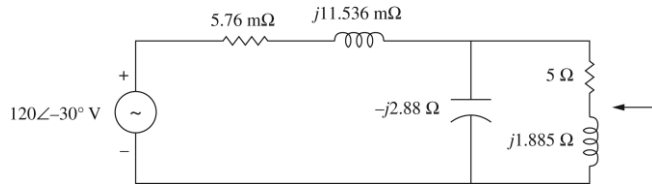**2.8**　$\overline{Z}_{LT} = j\left(377\right)\left(30.6 \times 10^{-6}\right) = j11.536\,\text{m}\Omega$

$\overline{Z}_{LL} = j\left(377\right)\left(5 \times 10^{-3}\right) = j1.885\,\Omega$

$$\overline{Z}_C = -j\,\frac{1}{\left(377\right)\left(921 \times 10^{-6}\right)} = -j2.88\,\Omega$$

$$\overline{V} = \frac{120\sqrt{2}}{\sqrt{2}}\angle -30°\,\text{V}$$

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The circuit transformed to phasor domain is shown below:

2.9 $KVL: \ 120\angle 0° = (60\angle 0°)(0.1 + j0.5) + V_{LOAD}$

$$\therefore \ \bar{V}_{LOAD} = 120\angle 0° - (60\angle 0°)(0.1 + j0.5)$$
$$= 114.1 - j30.0 = 117.9\angle -14.7° \text{ V} \ \leftarrow$$

2.10   (a) $p(t) = \upsilon(t)i(t) = \left[400\cos(\omega t + 30°)\right]\left[100\cos(\omega t - 80°)\right]$

$$= \frac{1}{2}(400)(100)\left[\cos 110° + \cos(2\omega t - 50°)\right]$$
$$= -6840.4 + 2\times 10^4 \cos(2\omega t - 50°)\text{ W}$$

(b) $P = VI\cos(\delta - \beta) = (282.84)(70.71)\cos(30° + 80°)$

$$= -6840\,\text{W} \quad \text{Absorbed}$$
$$= +6840 \text{ W} \quad \text{Delivered}$$

(c) $Q = VI\sin(\delta - \beta) = (282.84)(70.71)\sin 110°$

$$= 18.79 \text{ kVAR Absorbed}$$

(d) The phasor current $(-\bar{I}) = 70.71\angle -80° + 180° = 70.71\angle 100°$ A leaves the positive terminal of the generator.

The generator power factor is then $\cos(30° - 100°) = 0.3420$ leading

2.11   (a) $p(t) = \upsilon(t)i(t) = 391.7 \times 19.58\cos^2(\omega t + 30°)$

$$= 0.7669\times 10^4 \left(\frac{1}{2}\right)\left[1 + \cos(2\omega t + 60°)\right]$$
$$= 3.834\times 10^3 + 3.834\times 10^3 \cos(2\omega t + 60°)\text{ W}$$
$$P = VI\cos(\delta - \beta) = 277\times 13.85\cos 0° = 3.836\,\text{kW}$$
$$Q = VI\sin(\delta - \beta) = 0\,\text{VAR}$$

Source Power Factor $= \cos(\delta - \beta) = \cos(30° - 30°) = 1.0$

(b) $p(t) = \upsilon(t)i(t) = 391.7\times 103.9\cos(\omega t + 30°)\cos(\omega t - 60°)$

$$= 4.07\times 10^4 \left(\frac{1}{2}\right)\left[\cos 90° + \cos(2\omega t - 30°)\right]$$
$$= 2.035\times 10^4 \cos(2\omega t - 30°)\text{ W}$$
$$P = VI\cos(\delta - \beta) = 277\times 73.46\cos(30° + 60°) = 0\,\text{W } Q$$
$$= VI\sin(\delta - \beta) = 277\times 73.46\sin 90° = 20.35\,\text{kVAR } pf$$
$$= \cos(\delta - \beta) = 0 \text{ Lagging}$$

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(c) $p(t) = \upsilon(t)i(t) = 391.7 \times 15.67 \cos(\omega t + 30°)\cos(\omega t + 120°)$

$$= 6.138 \times 10^3 \left(\frac{1}{2}\right)\left[\cos(-90°) + \cos(2\omega t + 150°)\right] = 3.069 \times 10^3 \cos(2\omega t + 150°)\,\text{W}$$

$$P = VI\cos(\delta - \beta) = 277 \times 11.08\cos(30° - 120°) = 0\,\text{W}$$

$$Q = VI\sin(\delta - \beta) = 277 \times 11.08\sin(-90°)$$

$$= -3.069\,\text{kVAR Absorbed} = +3.069\,\text{kVAR Delivered}$$

$$pf = \cos(\delta - \beta) = \cos(-90°) = 0\,\text{Leading}$$

2.12 (a) $p_R(t) = (359.3\cos\omega t)(35.93\cos\omega t)$

$$= 6455 + 6455\cos 2\omega t\,\text{W}$$

(b) $p_x(t) = (359.3\cos\omega t)\left[14.37\cos(\omega t + 90°)\right]$

$$= 2582\cos(2\cot + 90°)$$

$$= -2582\sin 2\omega t\,\text{W}$$

(c) $P = V^2/R = \left(359.3/\sqrt{2}\right)^2 \big/ 10 = 6455\,\text{W Absorbed}$

(d) $Q = V^2/X = \left(359.3/\sqrt{2}\right)^2 \big/ 25 = 2582\,\text{VARS Delivered}$

(e) $(\beta - \delta) = \tan^{-1}(Q/P) = \tan^{-1}(2582/6455) = 21.8°$

Power factor $= \cos(\delta - \beta) = \cos(21.8°) = 0.9285$ Leading

2.13 $\underline{Z} = R - jx_c = 10 - j25 = 26.93\angle - 68.2°\,\Omega$

$i(t) = (359.3/26.93)\cos(\omega t + 68.2°)$

$$= 13.34\cos(\omega t + 68.2°)\,\text{A}$$

(a) $p_R(t) = \left[13.34\cos(\omega t + 68.2°)\right]\left[133.4\cos(\omega t + 68.2°)\right]$

$$= 889.8 + 889.8\cos\left[2(\omega t + 68.2°)\right]\,\text{W}$$

(b) $p_x(t) = \left[13.34\cos(\omega t + 68.2°)\right]\left[333.5\cos(\omega t + 68.2° - 90°)\right]$

$$= 2224\sin\left[2(\omega t + 68.2°)\right]\,\text{W}$$

(c) $P = I^2 R = \left(13.34/\sqrt{2}\right)^2 10 = 889.8\,\text{W}$

(d) $Q = I^2 X = \left(13.34/\sqrt{2}\right)^2 25 = 2224\,\text{VARS}$

(e) $pf = \cos\left[\tan^{-1}(Q/P)\right] = \cos\left[\tan^{-1}(2224/889.8)\right]$

$$= 0.3714\,\text{Leading}$$

2.14 (a) $\bar{I} = 2\angle 0° \, \text{kA}$

$$\bar{V} = \bar{Z}\,\bar{I} = (3\angle - 45°)(2\angle 0°) = 6\angle - 45° \, \text{kV}$$

$$\upsilon(t) = 6\sqrt{2}\cos(\omega t - 45°)\,\text{kV}$$
$$p(t) = \upsilon(t)i(t) = \left[6\sqrt{2}\cos(\omega t - 45°)\right]\left[2\sqrt{2}\cos\omega t\right]$$
$$= 24\left(\frac{1}{2}\right)\left[\cos(-45°) + \cos(2\omega t - 45°)\right]$$
$$= 8.49 + 12\cos(2\omega t - 45°)\,\text{MW}$$

(b) $P = VI\cos(\delta - \beta) = 6 \times 2\cos(-45° - 0°) = 8.49\,\text{MW}$ Delivered

(c) $Q = VI\sin(\delta - \beta) = 6 \times 2\sin(-45° - 0°)$
$$= -8.49\,\text{MVAR Delivered} = +8.49\,\text{MVAR Absorbed}$$

(d) $pf = \cos(\delta - \beta) = \cos(-45° - 0°) = 0.707$ Leading

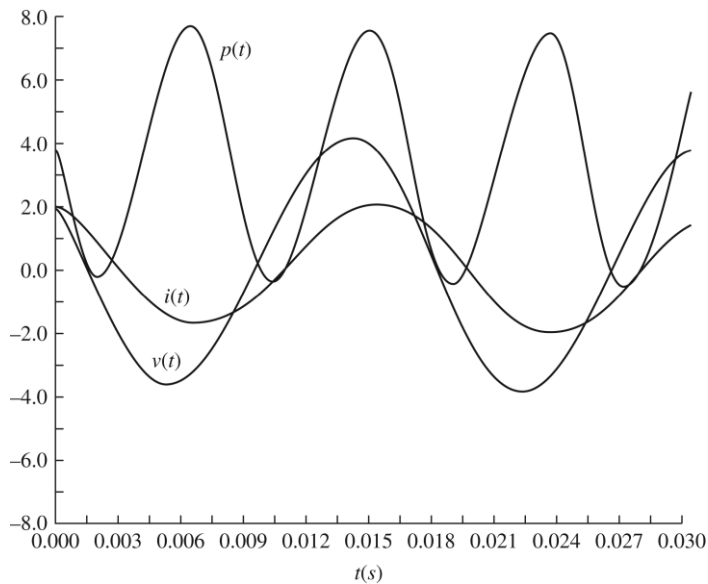**2.15** (a) $\bar{I} = \left[\left(4/\sqrt{2}\right)\angle 60°\right]\big/(2\angle 30°) = \sqrt{2}\,\angle 30° \, \text{A}$

$$i(t) = 2\cos(\omega t + 30°)\,\text{A} \text{ with } \omega = 377\,\text{rad/s}$$
$$p(t) = \upsilon(t)i(t) = 4\left[\cos 30° + \cos(2\omega t + 90°)\right]$$
$$= 3.46 + 4\cos(2\omega t + 90°)\,\text{W}$$

(b) $\upsilon(t)$, $i(t)$, and $p(t)$ are plotted below.

(c) The instantaneous power has an average value of 3.46 W, and the frequency is <u>twice</u> that of the voltage or current.

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2.16 (a) $\mathcal{Z} = 10 + j120\pi \times 0.04 = 10 + j15.1 = 18.1\angle 56.4° \, \Omega$

$pf = \cos 56.4° = 0.553$ Lagging

(b) $\bar{V} = 120 \angle 0° \, V$

The current supplied by the source is

$\bar{I} = (120 \angle 0°)/(18.1 \angle 56.4°) = 6.63 \angle -56.4° \, A$

The real power absorbed by the load is

$P = 120 \times 6.63 \times \cos 56.4° = 440 \, W$

which can be checked by $I^2 R = (6.63)^2 \, 10 = 440 \, W$

The reactive power absorbed by the load is

$Q = 120 \times 6.63 \times \sin 36.4° = 663 \, VAR$

(c) Peak Magnetic Energy $= W = LI^2 = 0.04(6.63)^2 = 1.76 \, J$

$\qquad\qquad Q = \omega W = 377 \times 1.76 = 663 \, VAR$ is satisfied.

2.17 (a) $\bar{S} = \bar{V} \bar{I}^* = \mathcal{Z} \bar{I} \bar{I}^* = \mathcal{Z} |\bar{I}|^2 = j\omega L I^2$

$Q = \text{Im}[\bar{S}] = \omega L I^2 \quad \leftarrow$

(b) $\upsilon(t) = L\dfrac{di}{dt} = -\sqrt{2}\,\omega L I \sin(\omega t + \theta)$

$p(t) = \upsilon(t) \cdot i(t) = -2\omega L I^2 \sin(\omega t + \theta)\cos(\omega t + \theta)$

$\qquad\qquad = -\omega L I^2 \sin 2(\omega t + \theta) \quad \leftarrow$

$\qquad\qquad = -Q \sin 2(\omega t + \theta) \quad \leftarrow$

Average real power P supplied to the inductor $= 0 \quad \leftarrow$

Instantaneous power supplied (to sustain the changing energy in the magnetic field) has a maximum value of Q. $\quad \leftarrow$

2.18 (a) $\bar{S} = \bar{V}\bar{I}^* = \mathcal{Z}\underline{I}\,\underline{I}^* = \text{Re}\;\mathcal{Z}\,I^2 + j\,\text{Im}\;\mathcal{Z}\,I^2$

$\qquad\qquad = P + jQ$

$\therefore P = \mathcal{Z}\,I^2 \cos\angle\mathcal{Z}\,; Q = \mathcal{Z}\,I^2 \sin\angle\mathcal{Z} \quad \leftarrow$

(b) Choosing $i(t) = \sqrt{2}\,I\cos\omega t,$

Then $\upsilon(t) = \sqrt{2}\,\mathcal{Z}\,I\cos(\omega t + \angle\mathcal{Z})$

$\therefore p(t) = \upsilon(t) \cdot i(t) = \mathcal{Z}\,I^2 \cos(\omega t + \angle\mathcal{Z}) \cdot \cos\omega t$

$\qquad\qquad = \mathcal{Z}\,I^2 \left[\cos\angle\mathcal{Z} + \cos(2\omega t + \angle\mathcal{Z})\right]$

$\qquad\qquad = \mathcal{Z}\,I^2 \left[\cos\angle\mathcal{Z} + \cos 2\omega t \cos\angle\mathcal{Z} - \sin 2\omega t \sin\angle\mathcal{Z}\right]$

$\qquad\qquad = P(1 + \cos 2\omega t) - Q\sin 2\omega t \quad \leftarrow$

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(c) $\overline{Z} = R + j\omega L + \dfrac{1}{j\omega C}$

From part (a), $P = RI^2$ and $Q = Q_L + Q_C$

where $Q_L = \omega L I^2$ and $Q_C = -\dfrac{1}{\omega C} I^2$

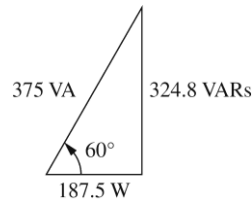which are the reactive powers into L and C, respectively.

Thus $p(t) = P(1 + \cos 2\omega t) - Q_L \sin 2\omega t - Q_C \sin 2\omega t$ ←

If $\omega^2 LC = 1$, $Q_L + Q_C = Q = 0$
Then $p(t) = P(1 + \cos 2\omega t)$ ←

2.19 (a) $\overline{S} = VI^* = \left(\dfrac{150}{\sqrt{2}} \angle 10°\right)\left(\dfrac{5}{\sqrt{2}} \angle -50°\right)^* = 375 \angle 60°$

$= 187.5 + j324.8$

$P = \mathrm{Re}\,\overline{S} = 187.5\,\text{W Absorbed}$

$Q = \mathrm{Im}\,\overline{S} = 324.8\,\text{VARS Absorbed}$

375 VA   324.8 VARs   60°   187.5 W

(b) $pf = \cos(60°) = 0.5\,\text{Lagging}$

(c) $Q_S = P \tan Q_S = 187.5 \tan\left[\cos^{-1} 0.9\right] = 90.81\,\text{VARS}$

$Q_C = Q_L - Q_S = 324.8 - 90.81 = 234\,\text{VARS}$

2.20 $\overline{Y}_1 = \dfrac{1}{\overline{Z}_1} = \dfrac{1}{20\angle 30°} = 0.05 \angle -30° = (0.0433 - j0.025)\text{S} = G_1 - jB_1$

$\overline{Y}_2 = \dfrac{1}{\overline{Z}_2} = \dfrac{1}{25\angle 60°} = 0.04 \angle -60° = \left(0.02 - j0.03464\right)\text{S} = G_2 + jB_2$

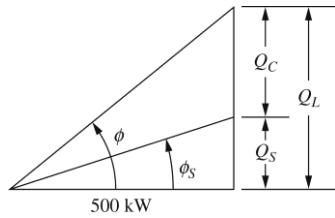$P_1 = V^2 G_1 = (100)^2\, 0.0433 = 433\,\text{W Absorbed}$

$Q_1 = V^2 B_1 = (100)^2\, 0.025 = 250\,\text{VARS Absorbed}$

$P_2 = V^2 G_2 = (100)^2\, 0.02 = 200\,\text{W Absorbed}$

$Q_2 = V^2 B_2 = (100)^2\, 0.03464 = 346.4\,\text{VARS Absorbed}$

500 VA $Q_1 = 250$ VARS

$P_1 = 433$ W

400 VA $Q_2 = 346.4$ VARS

$P_2 = 200$ W

869.7 VA $Q_S = Q_1 + Q_2$ $= 596.4$ VARS

$P_S = P_1 + P_2 = 633$ W

2.21 (a)

$$\phi_L = \cos^{-1} 0.6 = 53.13°$$
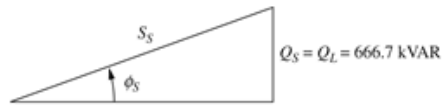$$Q_L = P\tan\phi_L = 500\tan 53.13° = 666.7 \text{ kVAR}$$
$$\phi_S = \cos^{-1} 0.9 = 25.84°$$
$$Q_S = P\tan\phi_S = 500\tan 25.84° = 242.2 \text{ kVAR}$$
$$Q_C = Q_L - Q_S = 666.7 - 242.2 = 424.5 \text{ kVAR}$$
$$S_C = Q_C = 424.5 \text{ kVA}$$

(b) The synchronous motor absorbs $P_m = \dfrac{375}{0.9} = 416.7 \text{ kW}$ and $Q_m = 0 \text{ kVAR}$

$P_S = P + P_m = 500 + 416.7 = 916.7 \text{ kW}$

Source $\text{PF} = \cos\left[\tan^{-1}\left(666.7/916.7\right)\right] = 0.809$ Lagging

2.22 (a) $\bar{Y}_1 = \dfrac{1}{\mathbb{Z}_1} = \dfrac{1}{(4+j5)} = \dfrac{1}{6.4\angle 51.34°} = 0.16\angle -51.34°$

$$= (0.1 - j0.12)\text{S}$$

$$\bar{Y}_2 = \dfrac{1}{\mathbb{Z}_2} = \dfrac{1}{10} = 0.1\text{S}$$

$$P = V^2\left(G_1 + G_2\right) \Rightarrow V = \sqrt{\dfrac{P}{G_1 + G_2}} = \sqrt{\dfrac{1000}{(0.1+0.1)}} = 70.71 \text{ V}$$

$$P_1 = V^2 G_1 = (70.71)^2\, 0.1 = 500 \text{ W}$$

$$P_2 = V^2 G_2 = (70.71)^2\, 0.1 = 500 \text{ W}$$

(b) $\bar{Y}_{eq} = \bar{Y}_1 + \bar{Y}_2 = (0.1 - j0.12) + 0.1 = 0.2 - j0.12$

$$= 0.233\angle -30.96° \text{ S}$$

$$I_S = V\, Y_{eq} = 70.71(0.233) = 16.48 \text{ A}$$

2.23 $\bar{S} = \bar{V}\bar{I}^* = (120\angle 0°)(15\angle - 30°) = 1800\angle - 30°$

$$= 1558.85 - j900$$

$P = \text{Re}\,\bar{S} = 1558.85\,\text{W Delivered}$

$Q = \text{Im}\,\bar{S} = -900\,\text{VAR S Delivered} = +900\,\text{VAR S Absorbed}$

2.24 $\underline{S}_1 = P_1 + jQ_1 = 10 + j0;\ \underline{S}_2 = 10\angle\cos^{-1}0.9 = 9 + j4.359$

$\bar{S}_3 = \dfrac{7.5}{0.85\times 0.95}\angle - \cos^{-1}0.95 = 9.288\angle -18.19° = 8.824 - j2.899$

$\bar{S}_S = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 27.82 + j1.460 = 27.86\angle 3.00°$

$P_S = \text{Re}(\bar{S}_S) = 27.82\,\text{kW}$

$Q_S = \text{Im}(\bar{S}_S) = 1.460\,\text{kVAR}$
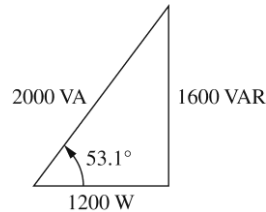
$S_S = |\bar{S}_S| = 27.86\,\text{kVA}$

2.25 $\bar{S}_R = \bar{V}_R\bar{I}^* = R\bar{I}\,\bar{I}^* = I^2R = (20)^2\,3 = 1200 + j0$

$\bar{S}_L = \bar{V}_L\bar{I}^* = (jX_L\bar{I})\bar{I}^* = jX_LI^2 = j8(20)^2 = 0 + j3200$

$\bar{S}_C = \bar{V}_C\bar{I}^* = (-j\bar{I}X_C)\bar{I}^* = -jX_CI^2 = -j4(20)^2 = 0 - j1600$

Complex power absorbed by the total load $\bar{S}_{LOAD} = \bar{S}_R + \bar{S}_L + \bar{S}_C = 2000\angle 53.1°$

Power Triangle:

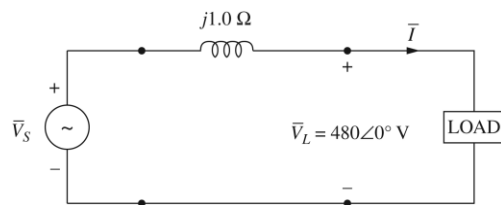Complex power delivered by the source is

$\bar{S}_{SOURCE} = \bar{V}\bar{I}^* = (100\angle 0°)(20\angle -53.1°)^* = 2000\angle 53.1°$

The complex power delivered by the source is equal to the total complex power absorbed by the load.

2.26 (a) The problem is modeled as shown in figure below:
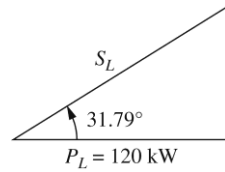

$P_L = 120\,\text{kW}$

$pf_L = 0.85\,\text{Lagging}$

$\theta_L = \cos^{-1}0.85 = 31.79°$

Power triangle for the load:

$$\overline{S}_L = P_L + jQ_L = 141.18\angle31.79°\,\text{kVA}$$
$$I = S_L/V = 141{,}180/480 = 294.13\,\text{A}$$

$$Q_L = P_L \tan(31.79°)$$
$$= 74.364\,\text{kVAR}$$

Real power loss in the line is zero.

Reactive power loss in the line is $Q_{LINE} = I^2 X_{LINE} = (294.13)^2\,1$

$$= 86.512\,\text{kVAR}$$

$$\therefore \overline{S}_S = P_S + jQ_S = 120 + j(74.364 + 86.512) = 200.7\angle53.28°\,\text{kVA}$$

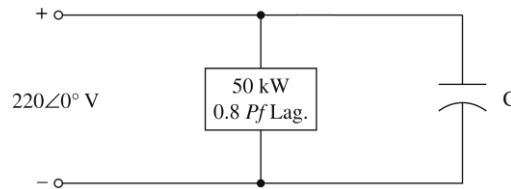The input voltage is given by $V_S = S_S/I = 682.4\,\text{V (rms)}$

The power factor at the input is $\cos 53.28° = 0.6$ Lagging

(b) Applying KVL, $\overline{V}_S = 480\angle0° + j1.0(294.13\angle-31.79°)$

$$= 635 + j250 = 682.4\angle21.5°\,\text{V (rms)}$$

$$(pf)_S = \cos(21.5° + 31.79°) = 0.6 \text{ Lagging}$$

2.27 The circuit diagram is shown below:

$$P_{old} = 50\,\text{kW}; \cos^{-1}0.8 = 36.87°; \theta_{OLD} = 36.87°; Q_{old} = P_{old}\tan(\theta_{old})$$

$$= 37.5\,\text{kVAR}$$

$$\therefore \overline{S}_{old} = 50{,}000 + j37{,}500$$

$$\theta_{new} = \cos^{-1}0.95 = 18.19°; \overline{S}_{new} = 50{,}000 + j50{,}000\tan(18.19°)$$

$$= 50{,}000 + j16{,}430$$

Hence $\overline{S}_{cap} = \overline{S}_{new} - \overline{S}_{old} = -j21{,}070\,\text{VA}$

$$\therefore C = \frac{21{,}070}{(377)(220)^2} = 1155\,\mu\text{F} \quad \leftarrow$$

2.28 $\underline{S}_1 = 15 + j6.667$

$\overline{S}_2 = 3(0.96) - j3\left[\sin\left(\cos^{-1} 0.96\right)\right] = 2.88 - j0.84$

$\overline{S}_3 = 15 + j0$

$\overline{S}_{TOTAL} = \overline{S}_1 + \overline{S}_2 + \overline{S}_3 = (32.88 + j5.827)\,\text{kVA}$

(i) Let $\overline{Z}$ be the impedance of a series combination of R and X

Since $S = V I^* = V \left|\dfrac{\overline{V}}{-\overline{Z}}\right)^* = \dfrac{V^2}{Z^*}$, it follows that

$Z^* = \dfrac{V^2}{S} = \dfrac{(240)^2}{(32.88 + j5.827)10^3} = (1.698 - j0.301)\,\Omega$

$\therefore \overline{Z} = (1.698 + j0.301)\,\Omega \quad \leftarrow$

(ii) Let $\overline{Z}$ be the impedance of a parallel combination of R and X

Then $R = \dfrac{(240)^2}{(32.88)10^3} = 1.7518\,\Omega$

$X = \dfrac{(240)^2}{(5.827)10^3} = 9.885\,\Omega$

$\therefore \overline{Z} = \left(1.7518 \| j9.885\right)\Omega \quad \leftarrow$

2.29 Since complex powers satisfy KCL at each bus, it follows that

$\overline{S}_{13} = (1 + j1) - (1 - j1) - (0.4 + j0.2) = -0.4 + j1.8 \quad \leftarrow$

$\overline{S}_{31} = -\overline{S}_{13}^* = 0.4 + j1.8 \quad \leftarrow$

Similarly, $\overline{S}_{23} = (0.5 + j0.5) - (1 + j1) - (-0.4 + j0.2) = -0.1 - j0.7 \quad \leftarrow$

$\overline{S}_{32} = -\overline{S}_{23}^* = 0.1 - j0.7 \quad \leftarrow$

At Bus 3, $\overline{S}_{G3} = \overline{S}_{31} + \overline{S}_{32} = (0.4 + j1.8) + (0.1 - j0.7) = 0.5 + j1.1 \quad \leftarrow$

2.30   (a) For load 1: $\theta_1 = \cos^{-1}(0.28) = 73.74°$ Lagging

$\overline{S}_1 = 125\angle 73.74° = 35 + j120$

$\overline{S}_2 = 10 - j40$

$\overline{S}_3 = 15 + j0$

$\overline{S}_{TOTAL} = \overline{S}_1 + \overline{S}_2 + \overline{S}_3 = 60 + j80 = 100\angle 53.13°\,\text{kVA} = P + jQ$

$\therefore P_{TOTAL} = 60\,\text{kW}; Q_{TOTAL} = 80\,\text{kVAR}; \text{kVA}_{TOTAL} = S_{TOTAL} = 100\,\text{kVA}. \quad \leftarrow$

Supply $pf = \cos(53.13°) = 0.6$ Lagging $\quad \leftarrow$

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(b) $\bar{I}_{TOTAL} = \dfrac{S^*}{\overline{V}^*} = \dfrac{100 \times 10^3 \angle -53.13°}{1000 \angle 0°} = 100 \angle -53.13° \, A$

At the new *pf* of 0.8 lagging, P_{TOTAL} of 60kW results in the new reactive power Q', such that

$$\theta' = \cos^{-1}(0.8) = 36.87°$$

and $Q' = 60\tan(36.87°) = 45\,\text{kVAR}$

∴ The required capacitor's kVAR is $Q_C = 80 - 45 = 35\,\text{kVAR}$ ←

It follows then $X_C = \dfrac{V^2}{S_C^*} = \dfrac{(1000)^2}{j35000} = -j28.57\,\Omega$

and $\qquad C = \dfrac{10^6}{2\pi(60)(28.57)} = 92.85\,\mu\text{F}$ ←

The new current is $I' = \dfrac{\overline{S}'^*}{\overline{V}^*} = \dfrac{60{,}000 - j45{,}000}{1000\angle 0°} = 60 - j45 = 75\angle -36.87°\,\text{A}$

The supply current, in magnitude, is reduced from 100A to 75A ←

2.31 (a) $\overline{I}_{12} = \dfrac{V_1\angle\delta_1 - V_2\angle\delta_2}{X\angle 90°} = \left(\dfrac{V_1}{X}\angle\delta_1 - 90°\right) - \dfrac{V_2}{X}\angle\delta_2 - 90°$

Complex power $\overline{S}_{12} = \overline{V}_1\overline{I}_{12}^* = V_1\angle\delta_1\left[\dfrac{V_1}{X}\angle 90° - \delta_1 - \dfrac{V_2}{X}\angle 90° - \delta_2\right]$

$\qquad\qquad = \dfrac{V_1^2}{X}\angle 90° - \dfrac{V_1V_2}{X}\angle 90° + \delta_1 - \delta_2$

∴ The real and reactive power at the sending end are

$$P_{12} = \dfrac{V_1^2}{X}\cos 90° - \dfrac{V_1V_2}{X}\cos(90° + \delta_1 - \delta_2)$$

$$= \dfrac{V_1V_2}{X}\sin(\delta_1 - \delta_2) \leftarrow$$

$$Q_{12} = \dfrac{V_1^2}{X}\sin 90° - \dfrac{V_1V_2}{X}\sin(90° + \delta_1 - \delta_2)$$

$$= \dfrac{V_1}{X}\left[V_1 - V_2\cos(\delta_1 - \delta_2)\right] \leftarrow$$

Note: If $\overline{V}_1$ leads $\overline{V}_2$, $\delta = \delta_1 - \delta_2$ is positive and the real power flows from node 1 to node 2. If $\overline{V}_1$ Lags $\overline{V}_2$, δ is negative and power flows from node 2 to node 1.

(b) Maximum power transfer occurs when $\delta = 90° = \delta_1 - \delta_2$ ←

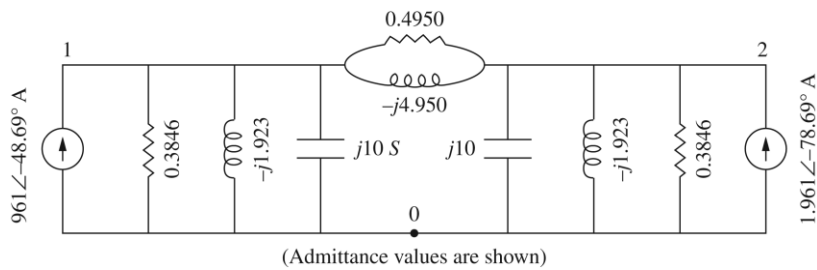$$P_{\text{MAX}} = \dfrac{V_1V_2}{X} \leftarrow$$

2.32 4 Mvar minimizes the real power line losses, while 4.5 Mvar minimizes the MVA power flow into the feeder.

2.33

Qcap	MW Losses	Mvar Losses
0	0.42	0.84
0.5	0.4	0.8
1	0.383	0.766
1.5	0.369	0.738
2	0.357	0.714
2.5	0.348	0.696
3	0.341	0.682
3.5	0.337	0.675
4	0.336	0.672
4.5	0.337	0.675
5	0.341	0.682
5.5	0.348	0.696
6	0.357	0.714
6.5	0.369	0.738
7	0.383	0.766
7.5	0.4	0.801
8	0.42	0.84
8.5	0.442	0.885
9	0.467	0.934
9.5	0.495	0.99
10	0.525	1.05

2.34 7.5 Mvars

2.35

(Admittance values are shown)

$$\begin{bmatrix} (.3846+.4950)+j(10-1.923-4.950) & -(.4950-j4.950) \\ -(.4950-j4.950) & (.3846+.4950)+j(10-1.923-4.95) \end{bmatrix}\begin{bmatrix} \bar{V}_{10} \\ \bar{V}_{20} \end{bmatrix}$$

$$=\begin{bmatrix} 1.961\angle -48.69° \\ 1.961\angle -78.69° \end{bmatrix}$$

$$\begin{bmatrix} 0.8796+j3.127 & -0.4950+j4.950 \\ -0.4950+j4.950 & -0.8796+j3.127 \end{bmatrix}\begin{bmatrix} \bar{V}_{10} \\ V_{20} \end{bmatrix}=\begin{bmatrix} 1.961\angle -48.69° \\ 1.961\angle -78.69° \end{bmatrix}$$

2.36 Note that there are two buses plus the reference bus and one line for this problem. After converting the voltage sources in Fig. 2.29 to current sources, the equivalent source impedances are:

$$\bar{Z}_{S1}=\bar{Z}_{S2}=(0.1+j0.5)//(-j0.1)=\frac{(0.1+j0.5)(-j0.1)}{0.1+j0.5-j0.1}$$

$$=\frac{(0.5099\angle 78.69°)(0.1\angle -90°)}{0.4123\angle 75.96°}=0.1237\angle -87.27°$$
$$=0.005882-j0.1235\,\Omega$$

The rest is left as an exercise to the student.

2.37 After converting impedance values in Figure 2.30 to admittance values, the bus admittance matrix is:

$$\bar{Y}_{bus}=\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-j1\right) & -\left(\frac{1}{3}-j1\right) & 1\left(\_\right) \\ 0 & -\left(\frac{1}{3}-j1\right) & \left(\frac{1}{3}-j1+j\frac{1}{4}+j\frac{1}{2}\right) & -j\left(\frac{1}{4}\right) \\ 0 & -\left(\frac{1}{4}\right) & -\left(j\frac{1}{4}\right) & \left(\frac{1}{4}+j\frac{1}{4}-j\frac{1}{3}\right) \end{bmatrix}$$
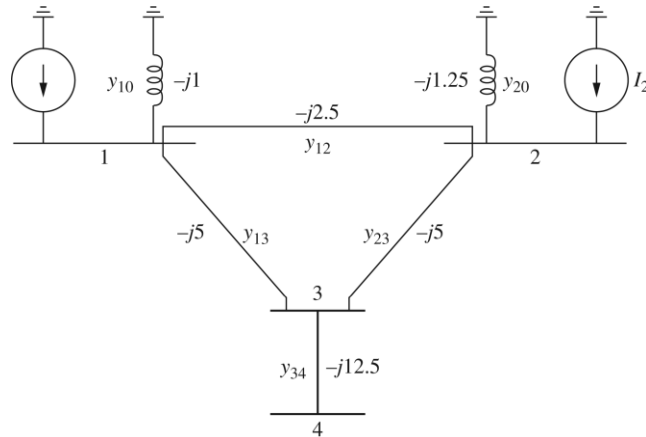
Writing nodal equations by inspection:

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & (2.083-j1) & (-0.3333+j1) & ^-0.25 \\ 0 & (-0.3333+j1) & (0.3333-j0.25) & -j0.25 \\ 0 & (-0.25) & -j0.25 & (0.25-j0.08333) \end{bmatrix}\begin{bmatrix} \bar{V}_{10} \\ \bar{V}_{20} \\ V_{30} \\ V \end{bmatrix}=\begin{bmatrix} 1\angle 0° \\ 0 \\ 0 \\ \end{bmatrix}$$

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$\left| 2\angle 30° \right|$

$\left\lfloor {}_{40} \right\rfloor \quad \left\lfloor \right\rfloor$

2.38 The admittance diagram for the system is shown below:

$$\bar{Y}_{BUS} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \bar{Y}_{13} & \bar{Y}_{14} \\ \bar{Y}_{21} & \bar{Y}_{22} & \bar{Y}_{23} & \bar{Y}_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ \bar{Y}_{41} & \bar{Y}_{42} & \bar{Y}_{43} & \bar{Y}_{44} \end{bmatrix} = j \begin{bmatrix} -8.5 & 2.5 & 5.0 & 0 \\ 2.5 & -8.75 & 5.0 & 0 \\ 5.0 & 5.0 & -22.5 & 12.5 \\ 0 & 0 & 12.5 & -12.5 \end{bmatrix} S$$

where $\bar{Y}_{11} = y_{10} + y_{12} + y_{13}$; $Y_{22} = y_{20} + y_{12} + y_{23}$; $Y_{23} = y_{13} + y_{23} + y_{34}$

$\bar{Y}_{44} = y_{34}$; $\bar{Y}_{12} = \bar{Y}_{21} = -\bar{y}_{12}$; $\bar{Y}_{13} = \bar{Y}_{31} = -\bar{y}_{13}$; $\bar{Y}_{23} = \bar{Y}_{32} = -\bar{y}_{23}$

and $\bar{Y}_{34} = \bar{Y}_{43} = -\bar{y}_{34}$

2.39 (a)

$$\begin{bmatrix} \bar{Y}_c + \bar{Y}_d + \bar{Y}_f & -\bar{Y}_d & -\bar{Y}_c & -\bar{Y}_f \\ -\bar{Y}_d & \bar{Y}_b + \bar{Y}_d + \bar{Y}_e & -\bar{Y}_b & -\bar{Y}_e \\ -\bar{Y}_c & -Y_b & Y_a + Y_b + Y_c & 0 \\ -\bar{Y}_f & -\bar{Y}_e & 0 & \bar{Y}_e + \bar{Y}_f + \bar{Y}_g \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} \bar{I}_1 = 0 \\ \bar{I}_2 = 0 \\ I_3 \\ I_4 \end{bmatrix}$$

(b) $j \begin{bmatrix} -14.5 & 8 & 4 & 2.5 \\ 8 & -17 & 4 & 5 \\ 4 & 4 & -8.8 & 0 \\ 2.5 & 5 & 0 & -8.3 \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ V_3 \\ \bar{V}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1\angle -90° \\ 0.62\angle -135° \end{bmatrix}$

$\bar{Y} \quad \bar{V} = \bar{I}; \bar{Y}^{-1} \bar{Y} \quad \bar{V} = \bar{Y}^{-1} \bar{I}$

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BUS   *BUS*   *BUS*   *BUS*

where $\bar{Y}^{-1} = \bar{Z} = j$
$$\begin{bmatrix} 0.7187 & 0.6688 & 0.6307 & 0.6194 \\ 0.6688 & 0.7045 & 0.6242 & 0.6258 \\ 0.6307 & 0.7045 & 0.6840 & 0.5660 \\ 0.6194 & 0.6258 & 0.5660 & 0.6840 \end{bmatrix} \Omega$$

$$\bar{V} = \bar{Y}_{BUS}^{-1} \, \bar{I}$$

where $\bar{V} = \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \end{bmatrix}$ and $\bar{I} = \begin{bmatrix} 0 \\ 0 \\ 1\angle -90° \\ 0.62\angle -135° \end{bmatrix}$

Then solve for $\bar{V}_1$, $\bar{V}_2$, $\bar{V}_3$, and $\bar{V}_4$.

2.40 (a) $\bar{V}_{AN} = \dfrac{240}{\sqrt{3}} \angle 0° = 138.56 \angle 0° \, \text{V}$ (Assumed as Reference)

$\bar{V}_{AB} = 240 \angle 30° \, \text{V}; \bar{V}_{BC} = 240 \angle -90° \, \text{V}; \bar{I}_A = 15 \angle -90° \, \text{A}$

$\bar{Z}_Y = \dfrac{\bar{V}_{AN}}{\bar{I}_A} = \dfrac{138.56 \angle 0°}{15 \angle -90°} = 9.24 \angle 90° = (0 + j9.24) \, \Omega$

(b) $\bar{I}_{AB} = \dfrac{\bar{I}_A}{\sqrt{3}} \angle 30° = \dfrac{15}{\sqrt{3}} \angle -90° + 30° = 8.66 \angle -60° \text{A}$

$\bar{Z}_\Delta = \dfrac{\bar{V}_{AB}}{\bar{I}_{AB}} = \dfrac{240 \angle 30°}{8.66 \angle -60°} = 27.71 \angle 90° = (0 + j27.71) \, \Omega$
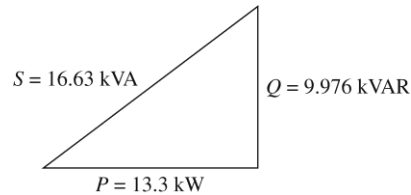
Note: $\bar{Z}_Y = \bar{Z}_\Delta / 3$

2.41 $\bar{S}_{3\phi} = \sqrt{3} V_{LL} \, I_L \angle \cos^{-1}(pf)$

$= \sqrt{3} \, (480)(20) \angle \cos^{-1} 0.8$

$= 16.627 \times 10^3 \angle 36.87°$

$= (13.3 \times 10^3) + j(9.976 \times 10^3)$

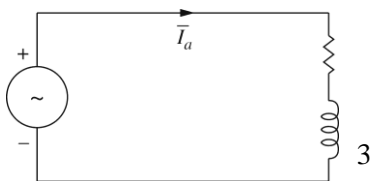S = 16.63 kVA   Q = 9.976 kVAR   P = 13.3 kW

$P_{3\phi} = \text{Re } \bar{S}_{3\phi} = 13.3 \, \text{kW}$ Delivered

$Q_{3\phi} = I_m \bar{S}_{3\phi} = 9.976 \, \text{kVAR}$ Delivered

2.42 (a) With $\bar{V}_{ab}$ as reference

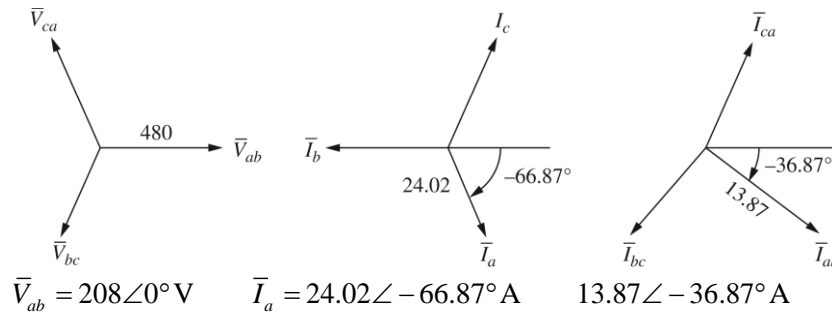$\bar{V}_{an} = \dfrac{208}{\sqrt{}} \angle -30°$ $\bar{Z}_\Delta = 4 + j3 = 5 \angle 36.87° \, \Omega$

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$$3 \qquad\qquad\qquad\qquad 3$$

$$\overline{I}_a = \frac{\overline{V}_{an}}{\left(\mathbb{Z}_\Delta/3\right)} = \frac{120.1\angle-30°}{5\angle36.87°} = 24.02\angle-66.87°\,\text{A}$$

$$\overline{I}_a = \frac{\overline{V}_{an}}{\left(\mathbb{Z}_\Delta/3\right)} = \frac{120.1\angle-30°}{5\angle36.87°} = 24.02\angle-66.87°\,\text{A}$$

$$\overline{S}_{3\phi} = 3\overline{V}_{an}\overline{I}_a^* = 3(120.1\angle - 30°)(24.02\angle + 66.87°)$$
$$= 8654\angle 36.87° = 6923 + j5192$$

$P_{3\phi} = 6923\,\text{W}; Q_{3\phi} = 5192\,\text{VAR};$ both absorbed by the load

$pf = \cos(36.87°) = 0.8$ Lagging; $S_{3\phi} = |\overline{S}_{3\phi}| = 8654\,\text{VA}$

(b)

$\overline{V}_{ab} = 208\angle 0°\,\text{V}$ $\overline{I}_a = 24.02\angle - 66.87°\,\text{A}$ $13.87\angle - 36.87°\,\text{A}$
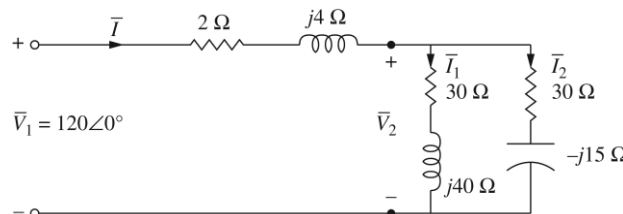
2.43 (a) Transforming the Δ-connected load into an equivalent Y, the impedance per phase of the equivalent Y is

$$\overline{Z}_2 = \frac{60 - j45}{3} = (20 - j15)\,\Omega$$

With the phase voltage $V_1 = \frac{120\sqrt{3}}{\sqrt{3}} = 120\,\text{V}$ taken as a reference, the per-phase equivalent

circuit is shown below:

Total impedance viewed from the input terminals is

$$\overline{Z} = 2 + j4 + \frac{(30 + j40)(20 - j15)}{(30 + j40) + (20 - j15)} = 2 + j4 + 22 - j4 = 24\,\Omega$$

$$\overline{I} = \frac{\overline{V}_1}{\overline{Z}} = \frac{120\angle 0°}{24} = 5\angle 0°\,\text{A}$$

The three-phase complex power supplied $= \overline{S} = 3\overline{V}_1\,\overline{I}^* = 1800\,\text{W}$

$P = 1800\,\text{W}$ and $Q = 0\,\text{VAR}$ delivered by the sending-end source

(b) Phase voltage at load terminals $\overline{V}_2 = 120\angle 0° - (2 + j4)(5\angle 0°)$

$$= 110 - j20 = 111.8\angle - 10.3° \, V$$

The line voltage magnitude at the load terminal is

$$(V_{\text{LOAD}})_{L\text{-}L} = \sqrt{3} \, 111.8 = 193.64 \, V$$

(c) The current per phase in the Y-connected load and in the equiv. Y of the Δ-load:

$$\overline{I}_1 = \frac{\overline{V}_2}{\mathbf{Z}_1} = 1 - j2 = 2.236\angle - 63.4° \, A$$

$$\overline{I}_2 = \frac{\overline{V}_2}{\mathbf{Z}_2} = 4 + j2 = 4.472\angle 26.56° \, A$$

The phase current magnitude in the original Δ-connected load

$$(I_{ph})_\Delta = \frac{I_2}{\sqrt{3}} = \frac{4.472}{\sqrt{3}} = 2.582 \, A$$

(d) The three-phase complex power absorbed by each load is

$$\overline{S}_1 = 3\overline{V}_2\overline{I}_1^* = 430 \, W + j600 \, VAR$$
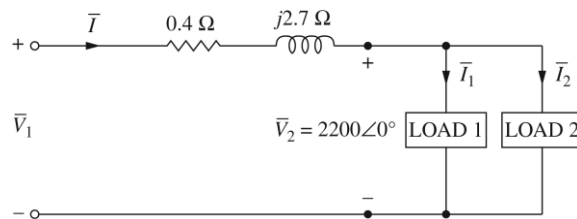$$\overline{S}_2 = 3\overline{V}_2\overline{I}_2^* = 1200 \, W - j900 \, VAR$$

The three-phase complex power absorbed by the line is

$$\overline{S}_L = 3(R_L + jX_L)I^2 = 3(2 + j4)(5)^2 = 150 \, W + j300 \, VAR$$

The sum of load powers and line losses is equal to the power delivered from the supply:

$$\overline{S}_1 + \overline{S}_2 + \overline{S}_L = (450 + j600) + (1200 - j900) + (150 + j300)$$
$$= 1800 \, W + j0 \, VAR$$

2.44 (a) The per-phase equivalent circuit for the problem is shown below:

Phase voltage at the load terminals is $V_2 = \dfrac{2200\sqrt{3}}{\sqrt{3}} = 2200 \, V$ taken as Ref.

Total complex power at the load end or receiving end is

$$\overline{S}_{R(3\phi)} = 560.1(0.707 + j0.707) + 132 = 528 + j396 = 660\angle 36.87° \, kVA$$

With phase voltage $\overline{V}_2$ as reference,

$$\overline{I} = \frac{S^*_{R(3\phi)}}{3V^*_2} = \frac{660{,}000\angle -36.87°}{3(2200\angle 0°)} = 100\angle -36.87° \,\text{A}$$

Phase voltage at sending end is given by

$$\overline{V}_1 = 2200\angle 0° + (0.4 + j2.7)(100\angle -36.87°) = 2401.7\angle 4.58° \,\text{V}$$

The magnitude of the line to line voltage at the sending end of the line is

$$(V_1)_{L\text{-}L} = \sqrt{3}V_1 = \sqrt{3}\,(2401.7) = 4160 \,\text{V}$$

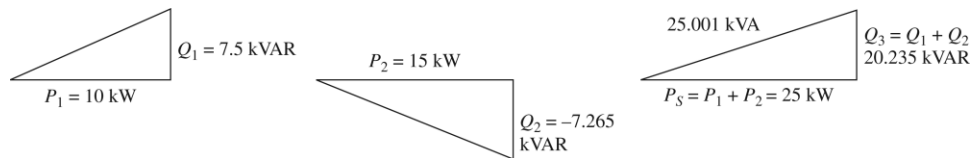(b) The three-phase complex-power loss in the line is given by

$$\overline{S}_{L(3\phi)} = 3RI^2 + j3 \times I^2 = 3(0.4)\left(100^2\right) + j3(2.7)\left(100^2\right)$$

$$= 12\,\text{kW} + j81\,\text{kVAR}$$

(c) The three-phase sending power is

$$\overline{S}_{S(3\phi)} = 3\overline{V}_1\overline{I}^* = 3(2401.7\angle 4.58°)(100\angle 36.87°)$$

$$= 540\,\text{kW} + j477\,\text{kVAR}$$

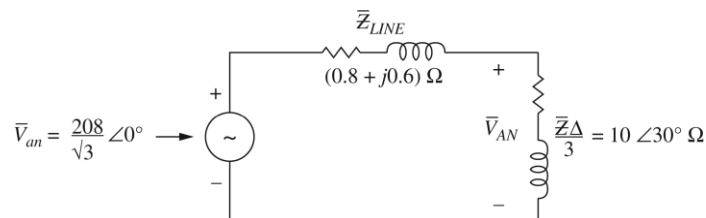Note that $\overline{S}_{S(3\phi)} = \overline{S}_{R(3\phi)} + \overline{S}_{L(3\phi)}$

2.45 (a)

$$I_S = \frac{S}{\sqrt{3}V_{LL}} = \frac{25.001\times 10^3}{\sqrt{3}\,(480)} = 30.07 \,\text{A}$$

(b) The ammeter reads zero, because in a balanced three-phase system, there is no neutral current.
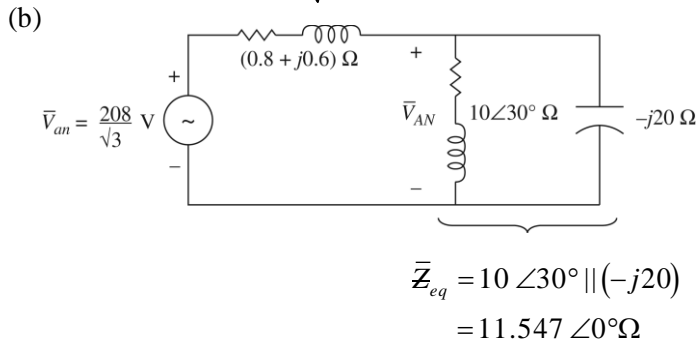
2.46 (a)

Using voltage division: $\bar{V}_{AN} = \bar{V}_{an} \dfrac{\underline{Z}_\Delta/3V}{(\underline{Z}_\Delta/3)+\underline{Z}_{LINE}}$

$$= \frac{208}{\sqrt{3}}\angle 0° \frac{10\angle 30°}{10\angle 30° + (0.8+j0.6)}$$

$$= \frac{(120.09)(10\angle 30°)}{9.46+j5.6} = \frac{1200.9\angle 30°}{10.99\angle 30.62°}$$

$$= 109.3\angle -0.62° \text{ V}$$

Load voltage $= V_{AB} = \sqrt{3}\,(109.3) = 189.3$ V Line-to-Line

(b)

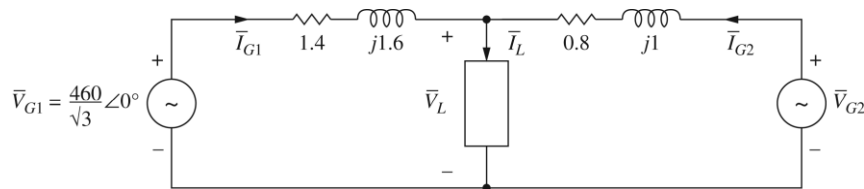$$\bar{Z}_{eq} = 10\angle 30° \,||\, (-j20)$$
$$= 11.547\angle 0°\Omega$$

$$\bar{V}_{AN} = \bar{V}_{an}\, \frac{\bar{Z}_{eq}}{\underline{Z}_{eq}+\underline{Z}_{LINE}}$$

$$= (208/\sqrt{3}) \frac{11.547}{(11.547+0.8+j0.6)}$$

$$= \frac{1386.7}{12.362\angle 2.78°} = 112.2\angle -2.78° \text{ V}$$

Load voltage Line-to-Line $V_{AB} = \sqrt{3}\,(112.2) = 194.3$ V

2.47

(a) $\bar{I}_{G1} = \dfrac{15\times 10^3}{\sqrt{8}\,(460)(0.8)}\angle -\cos^{-1}0.8 = 23.53\angle -36.87° \text{ A}$

$\bar{V}_L = \bar{V}_{G1} - \bar{Z}_{LINE1}\,\bar{I}_{G1} = \dfrac{460}{\sqrt{3}}\angle 0° - (1.4+j1.6)(23.53\angle -36.87°)$

$$= 216.9\angle -2.73° \text{ V Line to Neutral}$$

Load Voltage $V_L = \sqrt{3}\,216.9 = 375.7$ V Line to line

(b) $\overline{I}_L = \dfrac{30 \times 10^3}{\sqrt{3}(375.7)(0.8)} \angle 2.73° \cos^{-1} 0.8 \quad 57.63 \angle 39.6°$ A

$$\overline{I}_{G2} = \overline{I}_L - \overline{I}_{G1} = 57.63\angle -39.6° - 23.53\angle -36.87°$$
$$= 34.14\angle -41.49° \, \text{A}$$
$$\overline{V}_{G2} = \overline{V}_L + \overline{Z}_{LINE\,2}\,\overline{I}_{G2} = 216.9\angle -2.73° + (0.8 + j1)(34.14\angle -41.49°)$$
$$= 259.7\angle -0.63° \, \text{V}$$

Generator 2 line-to-line voltage $V_{G2} = \sqrt{3}(259.7)$

$$= 449.8 \, \text{V}$$

(c) $S_{G2} = 3\overline{V}_{G2}\,\overline{I}_{G_2}^* = 3\left(259.7\angle -0.63°\right)\left(34.14\angle 41.49°\right)$

$$= 20.12 \times 10^3 + j17.4 \times 10^3$$

$P_{G2} = 20.12\,\text{kW}; Q_{G2} = 17.4\,\text{kVAR};$ Both delivered

2.48 (a)

(b) $pf = \cos 31.32° = 0.854$ Lagging

(c) $I_L = \dfrac{S}{\sqrt{3}V_{LL}} = \dfrac{26.93 \times 10^3}{\sqrt{3}(480)} = 32.39\,\text{A}$

(d) $Q_C = Q_L = 14 \times 10^3 \, \text{VAR} = 3\left(V_{LL}\right)^2 / X_\Delta$

$$X_\Delta = \dfrac{3(480)^2}{14 \times 10^3} = 49.37\,\Omega$$

(e) $I_C = V_{LL} / X_\Delta = 480 / 49.37 = 9.72\,\text{A}$

$$I_{LINE} = \dfrac{P}{\sqrt{3}\,V_{LL}} = \dfrac{23 \times 10^3}{\sqrt{3}\,480} = 27.66\,\text{A}$$

2.49 (a) Let $\bar{Z}_Y = \bar{Z}_A = \bar{Z}_B = \bar{Z}_C$ for a balanced Y-load

$$\bar{Z}_\Delta = \bar{Z}_{AB} = \bar{Z}_{BC} = \bar{Z}_{CA}$$

Using equations in Fig. 2.27

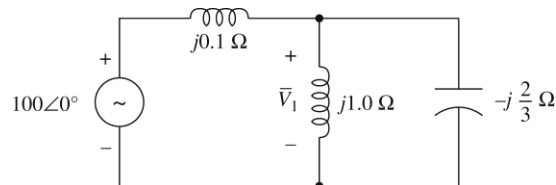$$\bar{Z}_\Delta = \frac{\bar{Z}_{Yq}^2 + \bar{Z}_{qY}^2 + \bar{Z}_Y^2}{\bar{Z}_Y} = 3\bar{Z}_Y$$

and

$$\bar{Z}_Y = \frac{\bar{Z}_{\Delta}^2}{\bar{Z}_\Delta + \bar{Z}_\Delta + \bar{Z}_\Delta} = \frac{\bar{Z}_\Delta}{3}$$

(b) $\bar{Z}_A = \dfrac{(j10)(-j25)}{j10 + j20 - j25} = -j50 \ \Omega$

$\bar{Z}_B = \dfrac{(j10)(j20)}{j5} = j40 \ \Omega; \bar{Z}_C = \dfrac{(j20)(-j25)}{j5} = -j100 \ \Omega$

2.50 Replace delta by the equivalent WYE: $\bar{Z}_Y = -j\dfrac{2}{3} \ \Omega$

Per-phase equivalent circuit is shown below:

Noting that $\left(j1.0 \| -j\dfrac{2}{3} \right) = -j2$, by voltage-divider law,

$$\bar{V}_1 = \frac{-j2}{-j2 + j0.1}(100\angle 0°) = 105\angle 0°$$

$$\therefore \upsilon_1(t) = 105\sqrt{2}\cos(\omega t + 0°) = 148.5\cos\omega t \ \text{V} \ \leftarrow$$

In order to find $i_2(t)$ in the original circuit, let us calculate $\bar{V}_{A'B'}$

$$\bar{V}_{A'B'} = \bar{V}_{A'N'} - \bar{V}_{B'N'} = \sqrt{3} \, e^{j30°}\bar{V}_{A'N'} = 173.2\angle 30°$$

Then

$$\bar{I}_{A'B'} = \frac{173.2\angle 30°}{-j2} = 86.6\angle 120°$$

$$\therefore i_2(t) = 86.6\sqrt{2}\cos(\omega t + 120°)$$

$$= 122.5\cos(\omega t + 120°)\text{A} \ \leftarrow$$

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2.51 On a per-phase basis $\bar{S}_1 = \frac{1}{3}(150 + j120) = (50 + j40)\,\text{kVA}$

$$\therefore \bar{I}_1 = \frac{(50 - j40)10^3}{2000} = (25 - j20)\,\text{A}$$

Note: PF Lagging

Load 2: Convert Δ into an equivalent Y

$$\bar{Z}_{2Y} = \frac{1}{3}(150 - j48) = (50 - j16)\,\Omega$$

$$\therefore \bar{I}_2 = \frac{2000\angle 0°}{50 - j16} = 38.1\angle 17.74°$$

$$= (36.29 + j11.61)\,\text{A}$$

Note: PF Leading

$\bar{S}_3$ per phase $= \frac{1}{3}\lceil (120 \times 0.6) - j120\sin(\cos^{-1}0.6) = (24 - j32)\,\text{kVA}$

$$\therefore \bar{I}_3 = \frac{(24 + j32)10^3}{2000} = (12 + j16)\,\text{A}$$

Note: PF Leading

Total current drawn by the three parallel loads $\bar{I}_T = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$

$$\bar{I}_{TOTAL} = (73.29 + j7.61)\,\text{A}$$

Note: PF Leading

Voltage at the sending end: $\bar{V}_{AN} = 2000\angle 0° + (73.29 + j7.61)(0.2 + j1.0)$

$$= 2007.05 + j74.81 = 2008.44\angle 2.13°\,\text{V}$$

Line-to-line voltage magnitude at the sending end $= \sqrt{3}(2008.44) = 3478.62\,\text{V} \quad \leftarrow$

2.52 (a) Let $\bar{V}_{AN}$ be the reference: $\overline{V}_{AN} = \frac{2160}{\sqrt{3}}\angle 0° \simeq 2400\angle 0°\,\text{V}$

Total impedance per phase $\bar{Z} = (4.7 + j9) + (0.3 + j1) = (5 + j10)\,\Omega$

$$\therefore \text{Line Current} = \frac{2400\angle 0°}{5 + j10} = 214.7\angle -63.4°\,\text{A} = \bar{I}_A \quad \leftarrow$$

With positive A-B-C phase sequence,

$$\bar{I}_B = 214.7\angle -183.4°\,\text{A}; \bar{I}_C = 214.7\angle -303.4° = 214.7\angle 56.6°\,\text{A} \quad \leftarrow$$

(b) $\left(\bar{V}_{A'N}\right)_{LOAD} = 2400\angle 0° - \left[\left(214.7\angle -63.4°\right)\left(0.3+j1\right)\right]$

$= 2400\angle 0° - 224.15\angle 9.9° = 2179.2 - j38.54$

$= 2179.5\angle -1.01° \, \text{V} \; \leftarrow$

$\left(\bar{V}_{B'N}\right)_{LOAD} = 2179.5\angle -121.01°\,\text{V} \, \Box ; \left(V_{C'N}\right)_{LOAD} = 2179.5\angle -241.01°\,\text{V} \, \Box$

(c) $S/\text{Phase} = \left(V_{A'N}\right)_{LOAD} I_A = \left(2179.5\right)\left(214.7\right) = 467.94\,\text{kVA} \; \leftarrow$

Total apparent power dissipated in all three phases in the load

$$\left[S_{3\phi \;\; LOAD} = 3\left(467.94\right) = 1403.82\,\text{kVA} \; \leftarrow\right.$$

Active power dissipated per phase in load $= \left(P_{1\phi}\right)_{LOAD}$

$$= \left(2179.5\right)\left(214.7\right)\cos\left(62.39°\right) = 216.87\,\text{kW} \; \leftarrow$$

$$\therefore \left[P_{3\phi \;\; LOAD} = 3\left(216.87\right) = 650.61\,\text{kW} \; \leftarrow\right.$$

Reactive power dissipated per phase in load $= \left(Q_{1\phi}\right)_{LOAD}$

$$= \left(2179.5\right)\left(214.7\right)\sin\left(62.39°\right) = 414.65\,\text{kVAR} \; \leftarrow$$

$$\therefore \left[Q_{3\phi \;\; LOAD} = 3\left(414.65\right) = 1243.95\,\text{kVAR} \; \leftarrow\right.$$

(d) Line losses per phase $\left(P_{1\phi}\right)_{LOSS} = \left(214.7\right)^2 0.3 = 13.83\,\text{kW} \; \leftarrow$

Total line loss $\left(P_{3\phi}\right)_{LOSS} = 13.83 \times 3 = 41.49\,\text{kW} \; \leftarrow$

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