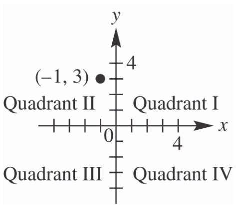
Solution Manual for Precalculus 6th Edition Lial Hornsby Schneider Daniels 013421742X 9780134217420

Chapter 2
GRAPHS AND FUNCTIONS

Section 2.1 Rectangular Coordinates and Graphs

1. The point (–1, 3) lies in quadrant II in the rectangular coordinate system.



2. The point (4, 6) lies on the graph of the equation $y = 3x - 6$. Find the y-value by letting $x = 4$ and solving for y.
$$y = 3(4) - 6 = 12 - 6 = 6$$

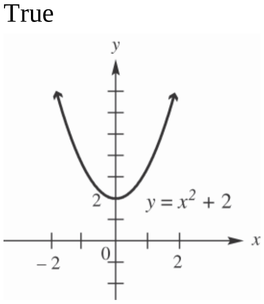
3. Any point that lies on the x-axis has y-coordinate equal to 0.

4. The y-intercept of the graph of $y = -2x + 6$ is (0, 6).

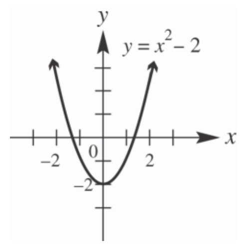
5. The x-intercept of the graph of $2x + 5y = 10$ is (5, 0). Find the x-intercept by letting $y = 0$ and solving for x.
$$2x + 5(0) = 10 \Rightarrow 2x = 10 \Rightarrow x = 5$$

6. The distance from the origin to the point (–3, 4) is 5. Using the distance formula, we have
$$d(P, Q) = \sqrt{(-3-0)^2 + (4-0)^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

7. True



8. True



9. False. The midpoint of the segment joining (0, 0) and (4, 4) is
$$\left(\frac{4+0}{2}, \frac{4+0}{2}\right) = \left(\frac{4}{2}, \frac{4}{2}\right) = (2, 2).$$

10. False. The distance between the point (0, 0) and (4, 4) is
$$d(P, Q) = \sqrt{(4-0)^2 + (4-0)^2} = \sqrt{4^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

11. Any three of the following:
$(2,-5), (-1,7), (3,-9), (5,-17), (6,-21)$

12. Any three of the following:
$(3,3), (-5,-21), (8,18), (4,6), (0,-6)$

13. Any three of the following: (1999, 35), (2001, 29), (2003, 22), (2005, 23), (2007, 20), (2009, 20)

14. Any three of the following:
$(2002, 86.8), (2004, 89.8), (2006, 90.7), (2008, 97.4), (2010, 106.5), (2012, 111.4), (2014, 111.5)$

15. $P(-5, -6)$, $Q(7, -1)$

(a) $$d(P, Q) = \sqrt{[7-(-5)]^2 + [-1-(-6)]^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$$

(b) The midpoint M of the segment joining points P and Q has coordinates
$$\left(\frac{-5+7}{2}, \frac{-6+(-1)}{2}\right) = \left(\frac{2}{2}, -\frac{7}{2}\right) = \left(1, -\frac{7}{2}\right).$$

16. $P(-4, 3)$, $Q(2, -5)$

(a) $d(P, Q) = \sqrt{[2-(-4)]^2 + (-5-3)^2}$
$= \sqrt{6^2 + (-8)^2} = \sqrt{100} = 10$

(b) The midpoint M of the segment joining points P and Q has coordinates
$\left(\frac{-4+2}{2}, \frac{3+(-5)}{2}\right) = \left(\frac{-2}{2}, \frac{-2}{2}\right)$
$= (-1, -1).$

17. $P(8, 2)$, $Q(3, 5)$

(a) $d(P, Q) = \sqrt{(3-8)^2 + (5-2)^2}$
$= \sqrt{(-5)^2 + 3^2}$
$= \sqrt{25+9} = \sqrt{34}$

(b) The midpoint M of the segment joining points P and Q has coordinates

$\left(\frac{8+3}{2}, \frac{2+5}{2}\right) = \left(\frac{11}{2}, \frac{7}{2}\right).$

18. $P(-8, 4)$, $Q(3, -5)$

(a) $d(P, Q) = \sqrt{[3-(-8)]^2 + (-5-4)^2}$
$= \sqrt{11^2 + (-9)^2} = \sqrt{121+81}$
$= \sqrt{202}$

(b) The midpoint M of the segment joining

points P and Q has coordinates

$\left(\frac{-8+3}{2}, \frac{4+(-5)}{2}\right) = \left(-\frac{5}{2}, -\frac{1}{2}\right).$

19. $P(-6, -5)$, $Q(6, 10)$

(a) $d(P, Q) = \sqrt{[6-(-6)]^2 + [10-(-5)]^2}$
$= \sqrt{12^2 + 15^2} = \sqrt{144+225}$

21. $P\left(3\sqrt{2}, 4\sqrt{5}\right)$, $Q\left(\sqrt{2}, -\sqrt{5}\right)$

(a) $d(P, Q)$
$= \sqrt{\left(\sqrt{2} - 3\sqrt{2}\right)^2 + \left(-\sqrt{5} - 4\sqrt{5}\right)^2}$
$= \sqrt{\left(-2\sqrt{2}\right)^2 + \left(-5\sqrt{5}\right)^2}$
$= \sqrt{8+125} = \sqrt{133}$

(b) The midpoint M of the segment joining points P and Q has coordinates
$\left(\frac{3\sqrt{2}+\sqrt{2}}{2}, \frac{4\sqrt{5}+(-\sqrt{5})}{2}\right)$
$= \left(\frac{4\sqrt{2}}{2}, \frac{3\sqrt{5}}{2}\right) = \left(2\sqrt{2}, \frac{3\sqrt{5}}{2}\right).$

22. $P\left(-\sqrt{7}, 8\sqrt{3}\right), Q\left(5\sqrt{7}, -\sqrt{3}\right)$

(a) $d(P, Q)$
$= \sqrt{[5\sqrt{7} - (-\sqrt{7})]^2 + (-\sqrt{3} - 8\sqrt{3})^2}$
$= \sqrt{(6\sqrt{7})^2 + (-9\sqrt{3})^2} = \sqrt{252+243}$
$= \sqrt{495} = 3\sqrt{55}$

(b) The midpoint M of the segment joining

points P and Q has coordinates
$\left(\frac{-\sqrt{7}+5\sqrt{7}}{2}, \frac{8\sqrt{3}+(-\sqrt{3})}{2}\right)$
$= \left(\frac{4\sqrt{7}}{2}, \frac{7\sqrt{3}}{2}\right) = \left(2\sqrt{7}, \frac{7\sqrt{3}}{2}\right).$

23. Label the points $A(-6, -4)$, $B(0, -2)$, and $C(-10, 8)$. Use the distance formula to find the length of each side of the triangle.

$d(A, B) = \sqrt{[0-(-6)]^2 + [-2-(-4)]^2}$

$= \sqrt{369} = 3\sqrt{41}$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{-6+6}{2}, \frac{-5+10}{2}\right) = \left(\frac{0}{2}, \frac{5}{2}\right) = \left(0, \frac{5}{2}\right).$

20. $P(6, -2)$, $Q(4, 6)$

(a) $d(P, Q) = \sqrt{(4-6)^2 + [6-(-2)]^2}$

$= \sqrt{(-2)^2 + 8^2}$

$= \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{6+4}{2}, \frac{-2+6}{2}\right) = \left(\frac{10}{2}, \frac{4}{2}\right) = (5, 2)$

$= \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40}$

$d(B, C) = \sqrt{(-10-0)^2 + [8-(-2)]^2}$

$= \sqrt{(-10)^2 + 10^2} = \sqrt{100+100}$

$= \sqrt{200}$

$d(A, C) = \sqrt{[-10-(-6)]^2 + [8-(-4)]^2}$

$= \sqrt{(-4)^2 + 12^2} = \sqrt{16+144} = \sqrt{160}$

Because $\left(\sqrt{40}\right)^2 + \left(\sqrt{160}\right)^2 = \left(\sqrt{200}\right)^2$,

triangle ABC is a right triangle.

24. Label the points $A(-2, -8)$, $B(0, -4)$, and $C(-4, -7)$. Use the distance formula to find the length of each side of the triangle.

$$d(A, B) = \sqrt{[0-(-2)]^2 + [-4-(-8)]^2} = \sqrt{2^2 + 4^2} = \sqrt{4+16} = \sqrt{20}$$

$$d(B, C) = \sqrt{(-4-0)^2 + [-7-(-4)]^2} = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$d(A, C) = \sqrt{[-4-(-2)]^2 + [-7-(-8)]^2} = \sqrt{(-2)^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$$

Because $(\sqrt{5})^2 + (\sqrt{20})^2 = 5 + 20 = 25 = 5^2$, triangle ABC is a right triangle.

25. Label the points $A(-4, 1)$, $B(1, 4)$, and $C(-6, -1)$.

$$d(A, B) = \sqrt{[1-(-4)]^2 + (4-1)^2} = \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}$$

$$d(B, C) = \sqrt{(-6-1)^2 + (-1-4)^2} = \sqrt{(-7)^2 + (-5)^2} = \sqrt{49+25} = \sqrt{74}$$

$$d(A, C) = \sqrt{[-6-(-4)]^2 + (-1-1)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$$

Because $(\sqrt{8})^2 + (\sqrt{34})^2 \neq (\sqrt{74})^2$ because $8 + 34 = 42 \neq 74$, triangle ABC is not a right triangle.

26. Label the points $A(-2, -5)$, $B(1, 7)$, and $C(3, 15)$.

$$d(A, B) = \sqrt{[1-(-2)]^2 + [7-(-5)]^2} = \sqrt{3^2 + 12^2} = \sqrt{9+144} = \sqrt{153}$$

$$d(B, C) = \sqrt{(3-1)^2 + (15-7)^2}$$

$$d(B, C) = \sqrt{(-1-2)^2 + (-6-5)^2} = \sqrt{(-3)^2 + (-11)^2} = \sqrt{9+121} = \sqrt{130}$$

$$d(A, C) = \sqrt{[-1-(-4)]^2 + (-6-3)^2} = \sqrt{3^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90}$$

Because $\left(\sqrt{40}\right)^2 + \left(\sqrt{90}\right)^2 = \left(\sqrt{130}\right)^2$, triangle ABC is a right triangle.

28. Label the points $A(-7, 4)$, $B(6, -2)$, and $C(0, -15)$.

$$d(A, B) = \sqrt{[6-(-7)]^2 + (-2-4)^2} = \sqrt{13^2 + (-6)^2} = \sqrt{169+36} = \sqrt{205}$$

$$d(B, C) = \sqrt{(0-6)^2 + [-15-(-2)]^2} = \sqrt{(-6)^2 + (-13)^2} = \sqrt{36+169} = \sqrt{205}$$

$$d(A, C) = \sqrt{[0-(-7)]^2 + (-15-4)^2} = \sqrt{7^2 + (-19)^2} = \sqrt{49+361} = \sqrt{410}$$

Because $\left(\sqrt{205}\right)^2 + \left(\sqrt{205}\right)^2 = \left(\sqrt{410}\right)^2$, triangle ABC is a right triangle.

29. Label the given points $A(0, -7)$, $B(-3, 5)$, and $C(2, -15)$. Find the distance between each pair of points.

$$d(A, B) = \sqrt{(-3-0)^2 + [5-(-7)]^2} = \sqrt{(-3)^2 + 12^2} = \sqrt{9+144} = \sqrt{153} = 3\sqrt{17}$$

$$= \sqrt{2^2 + 8^2} = \sqrt{4 + 64} = \sqrt{68}$$

$$d(A, C) = \sqrt{[3 - (-2)]^2 + [15 - (-5)]^2}$$
$$= \sqrt{5^2 + 20^2} = \sqrt{25 + 400} = \sqrt{425}$$

Because $(\sqrt{68})^2 + (\sqrt{153})^2 \neq \left(\sqrt{425}\right)^2$ because

$68 + 153 = 221 \neq 425$, triangle ABC is not a

right triangle.

27. Label the points $A(-4, 3)$, $B(2, 5)$, and $C(-1, -6)$.

$$d(A, B) = \sqrt{\left[2 - (-4)\right]^2 + (5 - 3)^2}$$
$$= \sqrt{6^2 + 2^2} = \sqrt{36 + 4} = \sqrt{40}$$

$$d(B, C) = \sqrt{\left[2 - (-3)\right]^2 + (-15 - 5)^2}$$
$$= \sqrt{5^2 + (-20)^2} = \sqrt{25 + 400}$$
$$= \sqrt{425} = 5\sqrt{17}$$

$$d(A, C) = \sqrt{(2 - 0)^2 + \left[-15 - (-7)\right]^2}$$
$$= \sqrt{2^2 + (-8)^2} = \sqrt{68} = 2\sqrt{17}$$

Because $d(A, B) + d(A, C) = d(B, C)$ or $3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$, the points are collinear.

30. Label the points $A(-1, 4)$, $B(-2, -1)$, and $C(1, 14)$. Apply the distance formula to each pair of points.

$$d(A,\ B) = \sqrt{\left[-2-(-1)\right]^2 + (-1-4)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{26}$$

$$d(B,\ C) = \sqrt{\left[1-(-2)\right]^2 + \left[14-(-1)\right]^2} = \sqrt{3^2 + 15^2} = \sqrt{234} = 3\sqrt{26}$$

$$d(A,\ C) = \sqrt{\left[1-(-1)\right]^2 + (14-4)^2} = \sqrt{2^2 + 10^2} = \sqrt{104} = 2\sqrt{26}$$

Because $\sqrt{26} + 2\sqrt{26} = 3\sqrt{26}$, the points are collinear.

31. Label the points $A(0, 9)$, $B(-3, -7)$, and $C(2, 19)$.

$$d(A,\ B) = \sqrt{(-3-0)^2 + (-7-9)^2} = \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265} \approx 16.279$$

$$d(B,\ C) = \sqrt{\left[2-(-3)\right]^2 + \left[19-(-7)\right]^2} = \sqrt{5^2 + 26^2} = \sqrt{25+676} = \sqrt{701} \approx 26.476$$

$$d(A,\ C) = \sqrt{(2-0)^2 + (19-9)^2} = \sqrt{2^2 + 10^2} = \sqrt{4+100} = \sqrt{104} \approx 10.198$$

Because $d(A,\ B) + d(A,\ C) \neq d(B,\ C)$

or $\sqrt{265} + \sqrt{104} \neq \sqrt{701}$

$$16.279 + 10.198 \neq 26.476,$$
$$26.477 \neq 26.476,$$

the three given points are not collinear. (Note, however, that these points are very close to

$$d(A,\ C) = \sqrt{\left[1-(-1)\right]^2 + \left[-11-(-3)\right]^2} = \sqrt{2^2 + (-8)^2} = \sqrt{4+64} = \sqrt{68} \approx 8.2462$$

Because $d(A, B) + d(A, C) \neq d(B, C)$

or $\sqrt{241} + \sqrt{68} \neq \sqrt{565}$

$$15.5242 + 8.2462 \neq 23.7697$$
$$23.7704 \neq 23.7697,$$

the three given points are not collinear. (Note, however, that these points are very close to lying on a straight line and may appear to lie on a straight line when graphed.)

33. Label the points $A(-7, 4)$, $B(6, -2)$, and $C(-1, 1)$.

$$d(A,\ B) = \sqrt{\left[6-(-7)\right]^2 + (-2-4)^2} = \sqrt{13^2 + (-6)^2} = \sqrt{169+36} = \sqrt{205} \approx 14.3178$$

$$d(B,\ C) = \sqrt{(-1-6)^2 + \left[1-(-2)\right]^2} = \sqrt{(-7)^2 + 3^2} = \sqrt{49+9} = \sqrt{58} \approx 7.6158$$

$$d(A,\ C) = \sqrt{\left[-1-(-7)\right]^2 + (1-4)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45} \approx 6.7082$$

Because $d(B, C) + d(A, C) \neq d(A, B)$ or

$$\sqrt{58} + \sqrt{45} \neq \sqrt{205}$$

$$7.6158 + 6.7082 \neq 14.3178$$
$$14.3240 \neq 14.3178,$$

lying on a straight line and may appear to lie on a straight line when graphed.)

32. Label the points $A(-1, -3)$, $B(-5, 12)$, and $C(1, -11)$.

the three given points are not collinear. (Note, however, that these points are very close to lying on a straight line and may appear to lie on a straight line when graphed.)

$$d(A, B) = \sqrt{[-5-(-1)]^2 + [12-(-3)]^2}$$
$$= \sqrt{(-4)^2 + 15^2} = \sqrt{16+225}$$
$$= \sqrt{241} \approx 15.5242$$

$$d(B, C) = \sqrt{[1-(-5)]^2 + (-11-12)^2}$$
$$= \sqrt{6^2 + (-23)^2} = \sqrt{36+529}$$
$$= \sqrt{565} \approx 23.7697$$

34. Label the given points $A(-4, 3)$, $B(2, 5)$, and $C(-1, 4)$. Find the distance between each pair of points.

$$d(A, B) = \sqrt{[2-(-4)]^2 + (5-3)^2} = \sqrt{6^2 + 2^2}$$
$$= \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$$

$$d(B, C) = \sqrt{(-1-2)^2 + (4-5)^2}$$
$$= \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$d(A, C) = \sqrt{[-1-(-4)]^2 + (4-3)^2}$$
$$= \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

Because $d(B, C) + d(A, C) = d(A, B)$ or $\sqrt{10} + \sqrt{10} = 2\sqrt{10}$, the points are collinear.

35. Midpoint (5, 8), endpoint (13, 10)

$$\frac{13+x}{2}=5 \quad\text{and}\quad \frac{10+y}{2}=8$$
$$13+x=10 \quad\text{and}\quad 10+y=16$$
$$x=-3 \quad\text{and}\quad y=6.$$

The other endpoint has coordinates (–3, 6).

36. Midpoint (–7, 6), endpoint (–9, 9)

$$\frac{-9+x}{2}=-7 \quad\text{and}\quad \frac{9+y}{2}=6$$
$$-9+x=-14 \quad\text{and}\quad 9+y=12$$
$$x=-5 \quad\text{and}\quad y=3.$$

The other endpoint has coordinates (–5, 3).

37. Midpoint (12, 6), endpoint (19, 16)

$$\frac{19+x}{2}=12 \quad\text{and}\quad \frac{16+y}{2}=6$$
$$19+x=24 \quad\text{and}\quad 16+y=12$$
$$x=5 \quad\text{and}\quad y=-4.$$

The other endpoint has coordinates (5, –4).

38. Midpoint (–9, 8), endpoint (–16, 9)

$$\frac{-16+x}{2}=-9 \quad\text{and}\quad \frac{9+y}{2}=8$$
$$-16+x=-18 \quad\text{and}\quad 9+y=16$$
$$x=-2 \quad\text{and}\quad y=7$$

The other endpoint has coordinates (–2, 7).

39. Midpoint (a, b), endpoint (p, q)

$$\frac{p+x}{2}=a \quad\text{and}\quad \frac{q+y}{2}=b$$
$$p+x=2a \quad\text{and}\quad q+y=2b$$
$$x=2a-p \quad\text{and}\quad y=2b-q$$

The other endpoint has coordinates $(2a-p, 2b-q)$.

40. Midpoint $(6a, 6b)$, endpoint $(3a, 5b)$

42. The endpoints are (2006, 7505) and (2012, 3335)

$$M=\left(\frac{2006+2012}{2}, \frac{7505+3335}{2}\right)$$
$$=(2009,\ 5420)$$

According to the model, the average national advertising revenue in 2009 was \$5420 million. This is higher than the actual value of \$4424 million.

43. The points to use are (2011, 23021) and (2013, 23834). Their midpoint is

$$\left(\frac{2011+2013}{2}, \frac{23,021+23,834}{2}\right) = (2012,\ 23427.5).$$

In 2012, the poverty level cutoff was approximately \$23,428.

44. **(a)** To estimate the enrollment for 2003, use the points (2000, 11,753) and (2006, 13,180)

$$M=\left(\frac{2000+2006}{2}, \frac{11,753+13,180}{2}\right)$$
$$=(2003,\ 12466.5)$$

The enrollment for 2003 was about 12,466.5 thousand.

(b) To estimate the enrollment for 2009, use the points (2006, 13,180) and (2012, 14,880)

$$M=\left(\frac{2006+2012}{2}, \frac{13,180+14,880}{2}\right)$$
$$=(2009,\ 14030)$$

The enrollment for 2009 was about 14,030 thousand.

45. The midpoint M has coordinates

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right).$$

$$\frac{3a+x}{2} = 6a \quad \text{and} \quad \frac{5b+y}{2} = 6b$$

$$3a + x = 12a \quad \text{and} \quad 5b + y = 12b$$

$$x = 9a \quad \text{and} \quad y = 7b$$

The other endpoint has coordinates $(9a, 7b)$.

41. The endpoints of the segment are $(1990, 21.3)$ and $(2012, 30.1)$.

$$M = \left(\frac{1990+2012}{2}, \frac{21.3+30.9}{2}\right)$$

$$= (2001,\ 26.1)$$

The estimate is 26.1%. This is very close to the actual figure of 26.2%.

$d(P, M)$

$$= \sqrt{\left(\frac{x_1+x_2}{2} - x_1\right)^2 + \left(\frac{y_1+y_2}{2} - y_1\right)^2}$$

$$= \sqrt{\left(\frac{x_1+x_2}{2} - \frac{2x_1}{2}\right)^2 + \left(\frac{y_1+y_2}{2} - \frac{2y_1}{2}\right)^2}$$

$$= \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2}$$

$$= \sqrt{\frac{(x_2-x_1)^2}{4} + \frac{(y_2-y_1)^2}{4}}$$

$$= \sqrt{\frac{(x_2-x_1)^2 + (y_2-y_1)^2}{4}}$$

$$= \frac{1}{2}\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

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$d(M,Q)$

$$= \sqrt{\left(x_2 - \frac{x_1+x_2}{2}\right)^2 + \left(y_2 - \frac{y_1+y_2}{2}\right)^2}$$

$$= \sqrt{\left(\frac{2x_2}{2} - \frac{x_1+x_2}{2}\right)^2 + \left(\frac{2y_2}{2} - \frac{y_1+y_2}{2}\right)^2}$$

$$= \sqrt{\frac{(x_2-x_1)^2}{4} + \frac{(y_2-y_1)^2}{4}}$$

$$= \sqrt{\frac{(x_2-x_1)^2+(y_2-y_1)^2}{4}}$$

$$= \frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d(P,Q) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Because $\frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$+\frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2},$

this shows $d(P,M)+d(M,Q)=d(P,Q)$ and $d(P,M)=d(M,Q)$.

46. The distance formula,

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, can be written

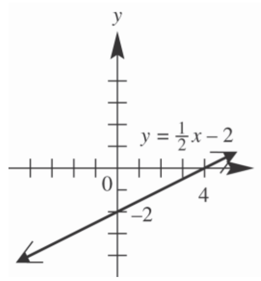
as $d=[(x_2-x_1)^2+(y_2-y_1)^2]^{1/2}$.

In exercises 47−58, other ordered pairs are possible.

47. (a)

x	y	
0	−2	y-intercept: $x=0 \Rightarrow$ $y=\frac{1}{2}(0)-2=-2$
4	0	x-intercept: $y=0\Rightarrow$ $0=\frac{1}{2}x-2\Rightarrow$ $2=\frac{1}{2}x\Rightarrow 4=x$
2	−1	additional point

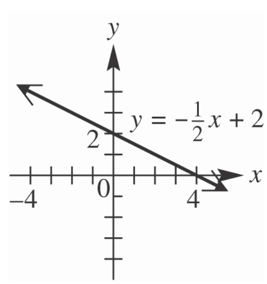
(b)



48. (a)

x	y	
0	2	y-intercept: $x=0\Rightarrow$ $y=-\frac{1}{2}(0)+2=2$
4	0	x-intercept: $y=0\Rightarrow$ $0=-\frac{1}{2}x+2\Rightarrow$
2	1	additional point

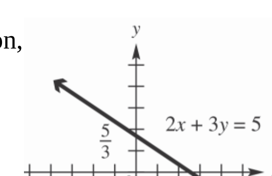
(b)



49. (a)

x	y	
0	$\frac{5}{3}$	y-intercept: $x=0\Rightarrow$ $2(0)+3y=5\Rightarrow$ $3y=5\Rightarrow y=\frac{5}{3}$
$\frac{5}{2}$	0	x-intercept: $y=0\Rightarrow$ $2x+3(0)=5\Rightarrow$ $2x=5\Rightarrow x=\frac{5}{2}$
4	−1	additional point

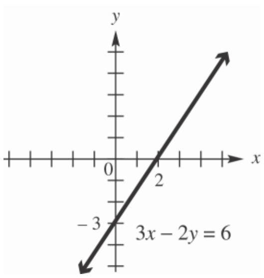
(b)



50. (a)

x	y	
0	−3	y-intercept: $x = 0 \Rightarrow$ $3(0) - 2y = 6 \Rightarrow$ $-2y = 6 \Rightarrow y = -3$
2	0	x-intercept: $y = 0 \Rightarrow$ $3x - 2(0) = 6 \Rightarrow$ $3x = 6 \Rightarrow x = 2$
4	3	additional point

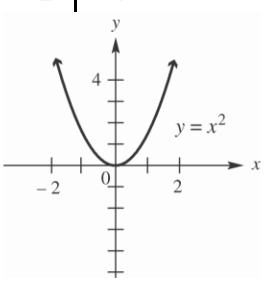
(b)



51. (a)

x	y	
0	0	x- and y-intercept: $0 = 0^2$
1	1	additional point
−2	4	additional point

(b)



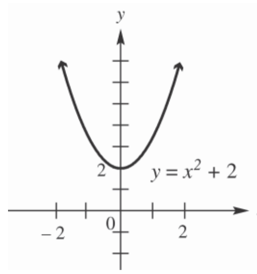
52. (a)

x	y	
0	2	y-intercept: $x = 0 \Rightarrow$ $y = 0^2 + 2 \Rightarrow$ $y = 0 + 2 \Rightarrow y = 2$
−1	3	additional point
2	6	additional point

no x-intercept:

$y = 0 \Rightarrow 0 = x^2 + 2 \Rightarrow$
$-2 = x^2 \Rightarrow \pm\sqrt{-2} = x$

(b)

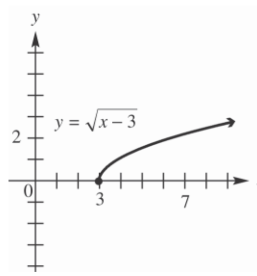


53. (a)

x	y	
3	0	x-intercept: $y = 0 \Rightarrow$ $0 = \sqrt{x-3} \Rightarrow$ $0 = x - 3 \Rightarrow 3 = x$
4	1	additional point
7	2	additional point

no y-intercept:
$x = 0 \Rightarrow y = \sqrt{0-3} \Rightarrow y = \sqrt{-3}$

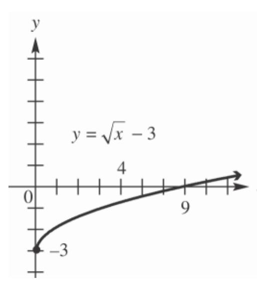
(b)



54. (a)

x	y	
0	−3	y-intercept: $x = 0 \Rightarrow$ $y = \sqrt{0} - 3 \Rightarrow$ $y = 0 - 3 \Rightarrow y = -3$
4	−1	additional point
9	0	x-intercept: $y = 0 \Rightarrow$ $0 = \sqrt{x} - 3 \Rightarrow$ $3 = \sqrt{x} \Rightarrow 9 = x$

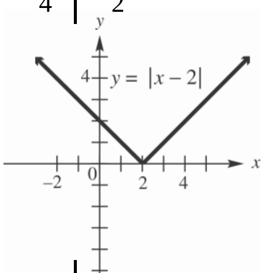
(b)



55. (a)

x	y	
0	2	y-intercept: $x = 0 \Rightarrow$ $y = \lvert 0-2 \rvert \Rightarrow$ $y = \lvert -2 \rvert \Rightarrow y = 2$
2	0	x-intercept: $y = 0 \Rightarrow$ $0 = \lvert x-2 \rvert \Rightarrow$ $0 = x-2 \Rightarrow 2 = x$
−2	4	additional point
4	2	additional point

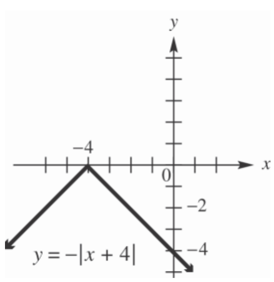
(b)



56. (a)

x	y	
−2	−2	additional point
−4	0	x-intercept: $y = 0 \Rightarrow$ $0 = -\lvert x+4 \rvert \Rightarrow$ $0 = \lvert x+4 \rvert \Rightarrow$ $0 = x+4 \Rightarrow -4 = x$
0	−4	y-intercept: $x = 0 \Rightarrow$ $y = -\lvert 0+4 \rvert \Rightarrow$ $y = -\lvert 4 \rvert \Rightarrow y = -4$

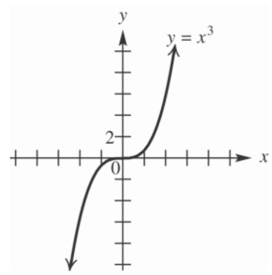
(b)



57. (a)

x	y	
0	0	x- and y-intercept: $0 = 0^3$
−1	−1	additional point
2	8	additional point

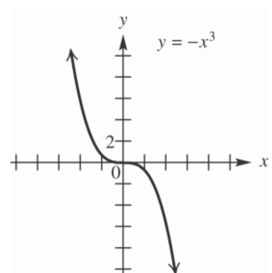
(b)



58. (a)

x	y	
0	0	x- and y-intercept: $0 = -0^3$
1	−1	additional point
2	−8	additional point

(b)

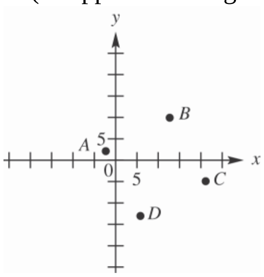


59. Points on the x-axis have y-coordinates equal to 0. The point on the x-axis will have the same x-coordinate as point (4, 3). Therefore, the line will intersect the x-axis at (4, 0).

60. Points on the y-axis have x-coordinates equal to 0. The point on the y-axis will have the same y-coordinate as point (4, 3). Therefore, the line will intersect the y-axis at (0, 3).

61. Because (a, b) is in the second quadrant, a is negative and b is positive. Therefore, $(a, -b)$ will have a negative x–coordinate and a negative y-coordinate and will lie in quadrant III.
$(-a, b)$ will have a positive x-coordinate and a positive y-coordinateand will lie in quadrant I.
$(-a, -b)$ will have a positive x-coordinate and a negative y-coordinate and will lie in quadrant IV.
(b, a) will have a positive x-coordinate and a negative y-coordinate and will lie in quadrant IV.

62. Label the points $A(-2,2)$, $B(13,10)$, $C(21,-5)$, and $D(6,-13)$. To determine which points form sides of the quadrilateral (as opposed to diagonals), plot the points.



Use the distance formula to find the length of each side.

$$d(A,B)=\sqrt{\left[13-(-2)\right]^2+(10-2)^2}=\sqrt{15^2+8^2}=\sqrt{225+64}=\sqrt{289}=17$$

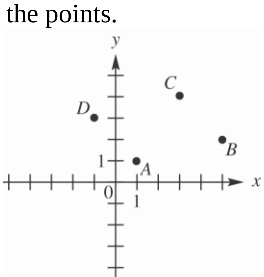
$$d(B,C)=\sqrt{(21-13)^2+(-5-10)^2}=\sqrt{8^2+(-15)^2}=\sqrt{64+225}=\sqrt{289}=17$$

$$d(C,D)=\sqrt{(6-21)^2+\left[-13-(-5)\right]^2}=\sqrt{(-15)^2+(-8)^2}=\sqrt{225+64}=\sqrt{289}=17$$

$$d(D,A)=\sqrt{(-2-6)^2+\left[2-(-13)\right]^2}=\sqrt{(-8)^2+15^2}=\sqrt{64+225}=\sqrt{289}=17$$

Because all sides have equal length, the four points form a rhombus.

63. To determine which points form sides of the quadrilateral (as opposed to diagonals), plot the points.



$$d(B,C)=\sqrt{(3-5)^2+(4-2)^2}=\sqrt{(-2)^2+2^2}=\sqrt{4+4}=\sqrt{8}$$

$$d(C,D)=\sqrt{(-1-3)^2+(3-4)^2}=\sqrt{(-4)^2+(-1)^2}=\sqrt{16+1}=\sqrt{17}$$

$$d(D,A)=\sqrt{\left[1-(-1)\right]^2+(1-3)^2}=\sqrt{2^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}$$

Because $d(A, B) = d(C, D)$ and $d(B, C) = d(D, A)$, the points are the vertices of a parallelogram. Because $d(A, B) \neq d(B, C)$, the points are not the vertices of a rhombus.

64. For the points $A(4, 5)$ and $D(10, 14)$, the difference of the x-coordinates is $10 - 4 = 6$ and the difference of the y-coordinates is $14 - 5 = 9$. Dividing these differences by 3, we obtain 2 and 3, respectively. Adding 2 and 3 to the x and y coordinates of point A, respectively, we obtain $B(4 + 2, 5 + 3)$ or $B(6, 8)$.
Adding 2 and 3 to the x- and y- coordinates of point B, respectively, we obtain $C(6 + 2, 8 + 3)$ or $C(8, 11)$. The desired points are $B(6, 8)$ and $C(8, 11)$.

We check these by showing that $d(A, B) = d(B, C) = d(C, D)$ and that $d(A, D) = d(A, B) + d(B, C) + d(C, D)$.

$$d(A,B)=\sqrt{(6-4)^2+(8-5)^2}=\sqrt{2^2+3^2}=\sqrt{4+9}=\sqrt{13}$$

$$d(B,C)=\sqrt{(8-6)^2+(11-8)^2}=\sqrt{2^2+3^2}=\sqrt{4+9}=\sqrt{13}$$

$$d(C,D)=\sqrt{(10-8)^2+(14-11)^2}=\sqrt{2^2+3^2}=\sqrt{4+9}=\sqrt{13}$$

$$d(A,D)=\sqrt{(10-4)^2+(14-5)^2}$$

Use the distance formula to find the length of each side.

$$d(A,B) = \sqrt{(5-1)^2 + (2-1)^2}$$
$$= \sqrt{4^2 + 1^2} = \sqrt{16+1} = \sqrt{17}$$

$$= \sqrt{6^2 + 9^2} = \sqrt{36+81}$$
$$= \sqrt{117} = \sqrt{9(13)} = 3\sqrt{13}$$

$d(A, B)$, $d(B, C)$, and $d(C, D)$ all have the same measure and $d(A, D) = d(A, B) + d(B, C) + d(C, D)$ Because $3\sqrt{13} = \sqrt{13} + \sqrt{13} + \sqrt{13}$.

Section 2.2 Circles

1. The circle with equation $x^2 + y^2 = 49$ has center with coordinates (0, 0) and radius equal to 7.

2. The circle with center (3, 6) and radius 4 has equation $(x-3)^2 + (y-6) = 16$.

3. The graph of $(x-4)^2 + (y+7)^2 = 9$ has center with coordinates (4, –7).

4. The graph of $x^2 + (y-5)^2 = 9$ has center with coordinates (0, 5).

5. This circle has center (3, 2) and radius 5. This is graph B.

6. This circle has center (3, –2) and radius 5. This is graph C.

7. This circle has center (–3, 2) and radius 5. This is graph D.

8. This circle has center (–3, –2) and radius 5. This is graph A.

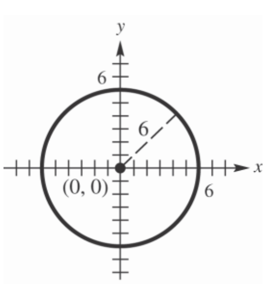
9. The graph of $x^2 + y^2 = 0$ has center (0, 0) and radius 0. This is the point (0, 0). Therefore, there is one point on the graph.

10. $\sqrt{-100}$ is not a real number, so there are no points on the graph of $x^2 + y^2 = -100$.

11. (a) Center (0, 0), radius 6

$$\sqrt{(x-0)^2 + (y-0)^2} = 6$$
$$(x-0)^2 + (y-0)^2 = 6^2 \Rightarrow x^2 + y^2 = 36$$

(b)

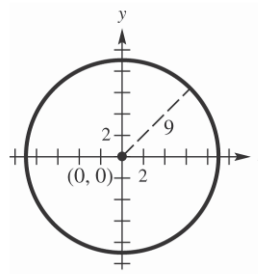


$x^2 + y^2 = 36$

12. (a) Center (0, 0), radius 9

$$\sqrt{(x-0)^2 + (y-0)^2} = 9$$
$$(x-0)^2 + (y-0)^2 = 9^2 \Rightarrow x^2 + y^2 = 81$$

(b)

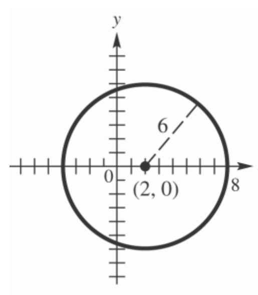


$x^2 + y^2 = 81$

13. (a) Center (2, 0), radius 6

$$\sqrt{(x-2)^2 + (y-0)^2} = 6$$
$$(x-2)^2 + (y-0)^2 = 6^2$$
$$(x-2)^2 + y^2 = 36$$

(b)

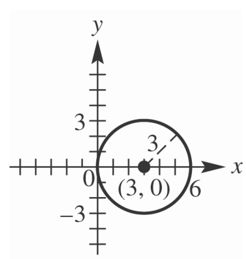


$(x-2)^2 + y^2 = 36$

14. (a) Center (3, 0), radius 3

$$\sqrt{(x-3)^2 + (y-0)^2} = 3$$
$$(x-3)^2 + y^2 = 9$$

(b)

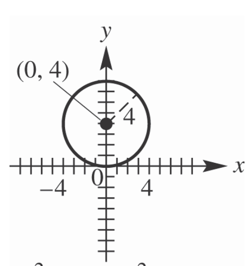


$(x-3)^2 + y^2 = 9$

15. (a) Center (0, 4), radius 4

$$\sqrt{(x-0)^2 + (y-4)^2} = 4$$
$$x^2 + (y-4)^2 = 16$$

(b)

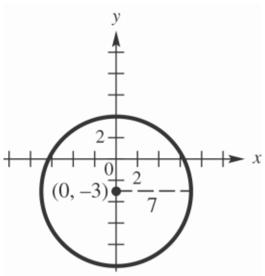


$x^2 + (y-4)^2 = 16$

16. **(a)** Center (0, –3), radius 7

$$\sqrt{(x-0)^2+[y-(-3)]^2}=7$$
$$(x-0)^2+[y-(-3)]^2=7^2$$
$$x^2+(y+3)^2=49$$

(b)

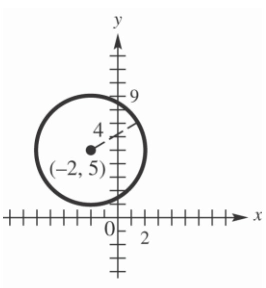


$x^2+(y+3)^2=49$

17. **(a)** Center (–2, 5), radius 4

$$\sqrt{[x-(-2)]^2+(y-5)^2}=4$$
$$[x-(-2)]^2+(y-5)^2=4^2$$
$$(x+2)^2+(y-5)^2=16$$

(b)

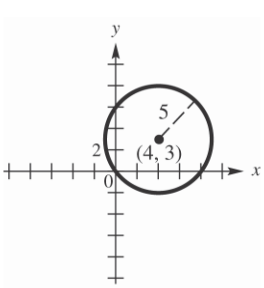


$(x+2)^2+(y-5)^2=16$

18. **(a)** Center (4, 3), radius 5

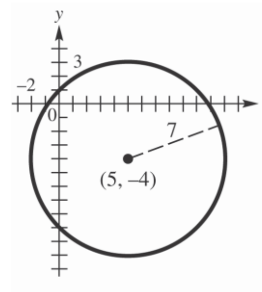
$$\sqrt{(x-4)^2+(y-3)^2}=5$$
$$(x-4)^2+(y-3)^2=5^2$$
$$(x-4)^2+(y-3)^2=25$$

(b)



$(x-4)^2+(y-3)^2=25$

(b)

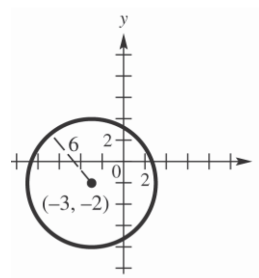


$(x-5)^2+(y+4)^2=49$

20. **(a)** Center (–3, –2), radius 6

$$\sqrt{[x-(-3)]^2+[y-(-2)]^2}=6$$
$$[x-(-3)]^2+[y-(-2)]^2=6^2$$
$$(x+3)^2+(y+2)^2=36$$

(b)

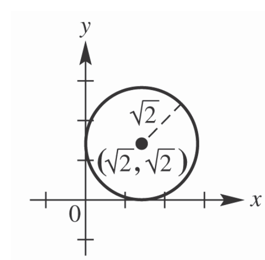


$(x+3)^2+(y+2)^2=36$

21. **(a)** Center $(\sqrt{2},\sqrt{2})$, radius $\sqrt{2}$

$$\sqrt{(x-\sqrt{2})^2+(y-\sqrt{2})^2}=\sqrt{2}$$
$$(x-\sqrt{2})^2+(y-\sqrt{2})^2=2$$

(b)



$(x-\sqrt{2})^2+(y-\sqrt{2})^2=2$

22. **(a)** Center $(-\sqrt{3},-\sqrt{3})$, radius $\sqrt{3}$

$$\sqrt{[x-(-\sqrt{3})]^2+[y-(-\sqrt{3})]^2}=\sqrt{3}$$
$$[x-(-\sqrt{3})]^2+[y-(-\sqrt{3})]^2=(\sqrt{3})^2$$

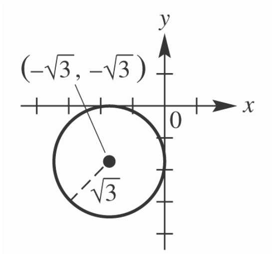
$$\left(x+\;3\right)^2+\left(y+\sqrt{3}\right)^2=3$$

19. **(a)** Center (5, –4), radius 7

$$\sqrt{(x-5)^2+\left[y-(-4)\right]^2}=7$$

$$(x-5)^2+[y-(-4)]^2=7^2$$
$$(x-5)^2+(y+4)^2=49$$

(b)



$(x+\sqrt{3})^2+(y+\sqrt{3})^2=3$

23. **(a)** The center of the circle is located at the midpoint of the diameter determined by the points (1, 1) and (5, 1). Using the midpoint formula, we have

$C=\left(\frac{1+5}{2},\frac{1+1}{2}\right)=(3,1)$. The radius is one-half the length of the diameter:

$$r=\frac{1}{2}\sqrt{(5-1)^2+(1-1)^2}=2$$

The equation of the circle is

$$(x-3)^2+(y-1)^2=4$$

(b) Expand $(x-3)^2+(y-1)^2=4$ to find the equation of the circle in general form:

$$(x-3)^2+(y-1)^2=4$$

$$x^2-6x+9+y^2-2y+1=4$$
$$x^2+y^2-6x-2y+6=0$$

24. **(a)** The center of the circle is located at the midpoint of the diameter determined by the points (−1, 1) and (−1, −5). Using the midpoint formula, we have

$$C=\left(\frac{-1+(-1)}{2},\frac{1+(-5)}{2}\right)=(-1,-2).$$

The radius is one-half the length of the diameter:

$$r=\frac{1}{2}\sqrt{\left[-1-(-1)\right]^2+(-5-1)^2}=3$$

The equation of the circle is

$$(x+1)^2+(y+2)^2=9$$

(b) Expand $(x+1)^2+(y+2)^2=9$ to find the equation of the circle in general form:

25. **(a)** The center of the circle is located at the midpoint of the diameter determined by the points (−2, 4) and (−2, 0). Using the midpoint formula, we have

$$C=\left(\frac{-2+(-2)}{2},\frac{4+0}{2}\right)=(-2,2).$$

The radius is one-half the length of the diameter:

$$r=\frac{1}{2}\sqrt{\left[-2-(-2)\right]^2+(4-0)^2}=2$$

The equation of the circle is

$$(x+2)^2+(y-2)^2=4$$

(b) Expand $(x+2)^2+(y-2)^2=4$ to find the equation of the circle in general form:

$$(x+2)^2+(y-2)^2=4$$
$$x^2+4x+4+y^2-4y+4=4$$
$$x^2+y^2+4x-4y+4=0$$

26. **(a)** The center of the circle is located at the midpoint of the diameter determined by the points (0, −3) and (6, −3). Using the midpoint formula, we have

$$C=\left(\frac{0+6}{2},\frac{-3+(-3)}{2}\right)=(3,-3).$$

The radius is one-half the length of the diameter:

$$r=\frac{1}{2}\sqrt{(6-0)^2+\left[-3-(-3)\right]^2}=3$$

The equation of the circle is

$$(x-3)^2+(y+3)^2=9$$

(b) Expand $(x-3)^2+(y+3)^2=9$ to find the equation of the circle in general form:

$$(x-3)^2+(y+3)^2=9$$

$$x^2-6x+9+y^2+6y+9=9$$
$$x^2+y^2-6x+6y+9=0$$

$$(x+1)^2+(y+2)^2=9$$

27. $x^2 + y^2 + 6x + 8y + 9 = 0$

Complete the square on x and y separately.

$(x^2 + 6x) + (y^2 + 8y) = -9$

$(x^2 + 6x + 9) + (y^2 + 8y + 16) = -9 + 9 + 16$

$$x^2 + 2x + 1 + y^2 + 4y + 4 = 9$$

$$x^2 + y^2 + 2x + 4y - 4 = 0$$

$$(x+3) \;\; + (y+4) \;\; = 16$$

Yes, it is a circle. The circle has its center at (–3, –4) and radius 4.

28. $x^2 + y^2 + 8x - 6y + 16 = 0$

Complete the square on x and y separately.

$$\left(x^2 + 8x\right) + \left(y^2 - 6y\right) = -16$$

$$\left(x^2 + 8x + 16\right) + \left(y^2 - 6y + 9\right) = -16 + 16 + 9$$

$$(x + 4)^2 + (y - 3)^2 = 9$$

Yes, it is a circle. The circle has its center at (–4, 3) and radius 3.

29. $x^2 + y^2 - 4x + 12y = -4$

Complete the square on x and y separately.

$$\left(x^2 - 4x\right) + \left(y^2 + 12y\right) = -4$$

$$\left(x^2 - 4x + 4\right) + \left(y^2 + 12y + 36\right) = -4 + 4 + 36$$

$$(x - 2)^2 + (y + 6)^2 = 36$$

Yes, it is a circle. The circle has its center at (2, –6) and radius 6.

30. $x^2 + y^2 - 12x + 10y = -25$

Complete the square on x and y separately.

$$\left(x^2 - 12x\right) + \left(y^2 + 10y\right) = -25$$

$$\left(x^2 - 12x + 36\right) + \left(y^2 + 10y + 25\right) = -25 + 36 + 25$$

$$(x - 6)^2 + (y + 5)^2 = 36$$

Yes, it is a circle. The circle has its center at (6, –5) and radius 6.

31. $4x^2 + 4y^2 + 4x - 16y - 19 = 0$

Complete the square on x and y separately.

$$4\left(x^2 + x\right) + 4\left(y^2 - 4y\right) = 19$$

$$4\left(x^2 + x + \tfrac{1}{4}\right) + 4\left(y^2 - 4y + 4\right) = 19 + 4\left(\tfrac{1}{4}\right) + 4(4)$$

$$4\left(x + \tfrac{1}{2}\right)^2 + 4(y - 2)^2 = 36$$

$$\left(x + \tfrac{1}{2}\right)^2 + (y - 2)^2 = 9$$

Yes, it is a circle with center $\left(-\tfrac{1}{2}, 2\right)$ and radius 3.

32. $9x^2 + 9y^2 + 12x - 18y - 23 = 0$

Complete the square on x and y separately.

$$9\left(x + \tfrac{2}{3}\right)^2 + 9(y - 1)^2 = 36$$

$$\left(x + \tfrac{2}{3}\right)^2 + (y - 1)^2 = 4$$

Yes, it is a circle with center $\left(-\tfrac{2}{3}, 1\right)$ and radius 2.

33. $x^2 + y^2 + 2x - 6y + 14 = 0$

Complete the square on x and y separately.

$$\left(x^2 + 2x\right) + \left(y^2 - 6y\right) = -14$$

$$\left(x^2 + 2x + 1\right) + \left(y^2 - 6y + 9\right) = -14 + 1 + 9$$

$$(x + 1)^2 + (y - 3)^2 = -4$$

The graph is nonexistent.

34. $x^2 + y^2 + 4x - 8y + 32 = 0$

Complete the square on x and y separately.

$$\left(x^2 + 4x\right) + \left(y^2 - 8y\right) = -32$$

$$\left(x^2 + 4x + 4\right) + \left(y^2 - 8y + 16\right) = -32 + 4 + 16$$

$$(x + 2)^2 + (y - 4)^2 = -12$$

The graph is nonexistent.

35. $x^2 + y^2 - 6x - 6y + 18 = 0$

Complete the square on x and y separately.

$$\left(x^2 - 6x\right) + \left(y^2 - 6y\right) = -18$$

$$\left(x^2 - 6x + 9\right) + \left(y^2 - 6y + 9\right) = -18 + 9 + 9$$

$$(x - 3)^2 + (y - 3)^2 = 0$$

The graph is the point (3, 3).

36. $x^2 + y^2 + 4x + 4y + 8 = 0$

Complete the square on x and y separately.

$$\left(x^2 + 4x\right) + \left(y^2 + 4y\right) = -8$$

$$\left(x^2 + 4x + 4\right) + \left(y^2 + 4y + 4\right) = -8 + 4 + 4$$

$$(x + 2)^2 + (y + 2)^2 = 0$$

The graph is the point (−2, −2).

37. $9x^2 + 9y^2 - 6x + 6y - 23 = 0$

Complete the square on x and y separately.

$$\left(9x^2 - 6x\right) + \left(9y^2 + 6y\right) = 23$$

$$9\left(x^2+\frac{4}{3}x\right)+9\left(y^2-2y\right)=23$$

$$9\left(x^2+\frac{4}{3}x+\frac{4}{9}\right)+9\left(y^2-2y+1\right)=23+9\left(\frac{4}{9}\right)+9(1)$$

$$9\left(x^2-\frac{2}{3}x\right)+9\left(y^2+\frac{2}{3}y\right)=23$$

$$\left(x^2-\frac{2}{3}x+\frac{1}{9}\right)+\left(y^2+\frac{2}{3}y+\frac{1}{9}\right)^2=\frac{23}{9}+\frac{1}{9}+\frac{1}{9}$$

$$\left(x-\frac{1}{3}\right)^2+\left(y+\frac{1}{3}\right)^2=\frac{25}{9}=\left(\frac{5}{3}\right)^2$$

Yes, it is a circle with center $\left(\frac{1}{3},-\frac{1}{3}\right)$ and radius $\frac{5}{3}$.

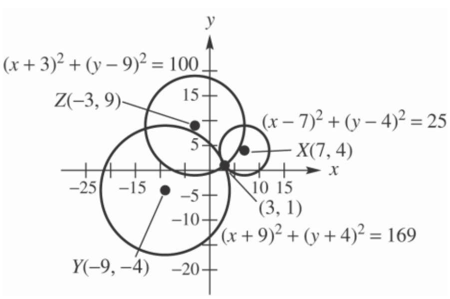
38. $4x^2+4y^2+4x-4y-7=0$

Complete the square on x and y separately.

$$4\left(x^2+x\right)+4\left(y^2-y\right)=7$$
$$4\left(x^2+x+\tfrac{1}{4}\right)+4\left(y^2-y+\tfrac{1}{4}\right)=7+4\left(\tfrac{1}{4}\right)+4\left(\tfrac{1}{4}\right)$$
$$4\left(x+\tfrac{1}{2}\right)^2+4\left(y-\tfrac{1}{2}\right)^2=9$$
$$\left(x+\tfrac{1}{2}\right)^2+\left(y-\tfrac{1}{2}\right)^2=\tfrac{9}{4}$$

Yes, it is a circle with center $\left(-\frac{1}{2},\frac{1}{2}\right)$ and radius $\frac{3}{2}$.

39. The equations of the three circles are $(x-7)^2+(y-4)^2=25$, $(x+9)^2+(y+4)^2=169$, and $(x+3)^2+(y-9)^2=100$. From the graph of the three circles, it appears that the epicenter is located at (3, 1).

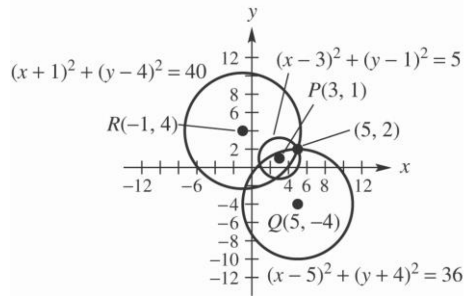


Check algebraically:

$$(x-7)^2+(y-4)^2=25$$
$$(3-7)^2+(1-4)^2=25$$
$$4^2+3^2=25\Rightarrow 25=25$$
$$(x+9)^2+(y+4)^2=169$$
$$(3+9)^2+(1+4)^2=169$$
$$12^2+5^2=169\Rightarrow 169=169$$
$$(x+3)^2+(y-9)^2=100$$
$$(3+3)^2+(1-9)^2=100$$
$$6^2+(-8)^2=100\Rightarrow 100=100$$

(3, 1) satisfies all three equations, so the epicenter is at (3, 1).

40. The three equations are $(x-3)^2+(y-1)^2=5$, $(x-5)^2+(y+4)^2=36$, and $(x+1)^2+(y-4)^2=40$.

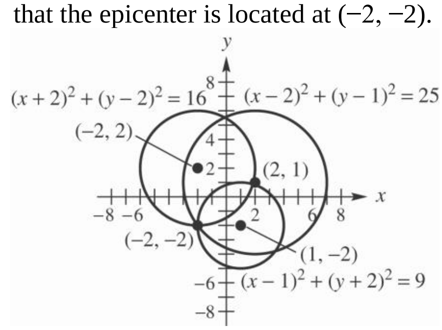


Check algebraically:

$$(x-3)^2+(y-1)^2=5$$
$$(5-3)^2+(2-1)^2=5$$
$$2^2+1^2=5\Rightarrow 5=5$$
$$(x-5)^2+(y+4)^2=36$$
$$(5-5)^2+(2+4)^2=36$$
$$6^2=36\Rightarrow 36=36$$
$$(x+1)^2+(y-4)^2=40$$
$$(5+1)^2+(2-4)^2=40$$
$$6^2+(-2)^2=40\Rightarrow 40=40$$

(5, 2) satisfies all three equations, so the epicenter is at (5, 2).

41. From the graph of the three circles, it appears that the epicenter is located at (−2, −2).



Check algebraically:

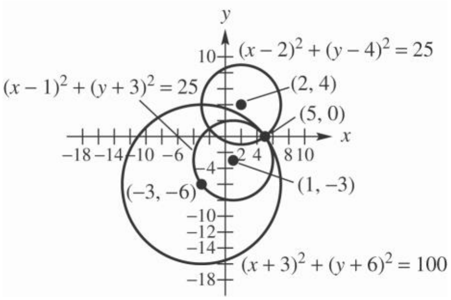
$$(x-2)^2+(y-1)^2=25$$
$$(-2-2)^2+(-2-1)^2=25$$
$$(-4)^2+(-3)^2=25$$
$$25=25$$
$$(x+2)^2+(y-2)^2=16$$
$$(-2+2)^2+(-2-2)^2=16$$
$$0^2+(-4)^2=16$$

$(x+1)^2+(y-4)^2=40$. From the graph of the three circles, it appears that the epicenter is located at (5, 2).

$$
\begin{aligned}
16 &= 16 \\
(x-1)^2+(y+2)^2 &= 9 \\
(-2-1)^2+(-2+2)^2 &= 9 \\
(-3)^2+0^2 &= 9 \\
9 &= 9
\end{aligned}
$$

(−2, −2) satisfies all three equations, so the epicenter is at (−2, −2).

42. From the graph of the three circles, it appears that the epicenter is located at (5, 0).



Check algebraically:

$$(x-2)^2+(y-4)^2=25$$
$$(5-2)^2+(0-4)^2=25$$
$$3^2+(-4)^2=25$$
$$25=25$$
$$(x-1)^2+(y+3)^2=25$$
$$(5-1)^2+(0+3)^2=25$$
$$4^2+3^2=25$$
$$25=25$$
$$(x+3)^2+(y+6)^2=100$$
$$(5+3)^2+(0+6)^2=100$$
$$8^2+6^2=100$$
$$100=100$$

(5, 0) satisfies all three equations, so the epicenter is at (5, 0).

43. The radius of this circle is the distance from the center $C(3, 2)$ to the x-axis. This distance is 2, so $r = 2$.

$$(x-3)^2+(y-2)^2=2^2 \Rightarrow$$
$$(x-3)^2+(y-2)^2=4$$

44. The radius is the distance from the center $C(-4, 3)$ to the point $P(5, 8)$.

$$r=\sqrt{[5-(-4)]^2+(8-3)^2}$$
$$=\sqrt{9^2+5^2}=\sqrt{106}$$

The equation of the circle is

$$[x-(-4)]^2+(y-3)^2=(\sqrt{106})^2 \Rightarrow$$
$$(x+4)^2+(y-3)^3=106$$

$$(1-x)^2+(3-x)^2=16$$
$$1-2x+x^2+9-6x+x^2=16$$
$$2x^2-8x+10=16$$
$$2x^2-8x-6=0$$
$$x^2-4x-3=0$$

To solve this equation, we can use the quadratic formula with $a = 1$, $b = -4$, and $c = -3$.

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-3)}}{2(1)}$$
$$=\frac{4\pm\sqrt{16+12}}{2}=\frac{4\pm\sqrt{28}}{2}$$
$$=\frac{4\pm 2\sqrt{7}}{2}=2\pm\sqrt{7}$$

Because $x = y$, the points are $\left(2+\sqrt{7}, 2+\sqrt{7}\right)$ and $\left(2-\sqrt{7}, 2-\sqrt{7}\right)$.

46. Let $P(-2, 3)$ be a point which is 8 units from $Q(x, y)$. We have

$$d(P, Q)=\sqrt{(-2-x)^2+(3-y)^2}=8 \Rightarrow$$
$$(-2-x)^2+(3-y)^2=64.$$

Because $x + y = 0$, $x = -y$. We can either substitute $-x$ for y or $-y$ for x. Substituting $-x$ for y we solve the following:

$$(-2-x)^2+[3-(-x)]^2=64$$
$$(-2-x)^2+(3+x)^2=64$$
$$4+4x+x^2+9+6x+x^2=64$$
$$2x^2+10x+13=64$$
$$2x^2+10x-51=0$$

To solve this equation, use the quadratic formula with $a = 2$, $b = 10$, and $c = -51$.

$$x=\frac{-10\pm\sqrt{10^2-4(2)(-51)}}{2(2)}$$
$$=\frac{-10\pm\sqrt{100+408}}{4}$$

45. Label the points $P(x, y)$ and $Q(1, 3)$.

If $d(P,Q)=4$, $\sqrt{(1-x)^2+(3-y)^2}=4 \Rightarrow$

$(1-x)^2+(3-y)^2=16.$

If $x = y$, then we can either substitute x for y or y for x. Substituting x for y we solve the following:

$$=\frac{-10\pm\sqrt{508}}{4}=\frac{-10\pm\sqrt{4(127)}}{4}$$

$$=\frac{-10\pm 2\sqrt{127}}{4}=\frac{-5\pm\sqrt{127}}{2}$$

Because $y=-x$ the points are

$\left(\frac{-5-\sqrt{127}}{2}, \frac{5+\sqrt{127}}{2}\right)$ and

$\left(\frac{-5+\sqrt{127}}{2}, \frac{5-\sqrt{127}}{2}\right)$.

47. Let $P(x, y)$ be a point whose distance from $A(1, 0)$ is $\sqrt{10}$ and whose distance from $B(5, 4)$ is $\sqrt{10}$. $d(P, A) = \sqrt{10}$, so

$$\sqrt{(1-x)^2 + (0-y)^2} = \sqrt{10} \Rightarrow$$

$(1-x)^2 + y^2 = 10.$ $d(P, B) = \sqrt{10}$, so

$$\sqrt{(5-x)^2 + (4-y)^2} = \sqrt{10} \Rightarrow$$

$(5-x)^2 + (4-y)^2 = 10.$ Thus,

$$(1-x)^2 + y^2 = (5-x)^2 + (4-y)^2$$

$$1 - 2x + x^2 + y^2 = 25 - 10x + x^2 + 16 - 8y + y^2$$
$$1 - 2x = 41 - 10x - 8y$$
$$8y = 40 - 8x$$
$$y = 5 - x$$

Substitute $5 - x$ for y in the equation $(1-x)^2 + y^2 = 10$ and solve for x.

$$(1-x)^2 + (5-x)^2 = 10 \Rightarrow$$
$$1 - 2x + x^2 + 25 - 10x + x^2 = 10$$
$$2x^2 - 12x + 26 = 10 \Rightarrow 2x^2 - 12x + 16 = 0$$
$$x^2 - 6x + 8 = 0 \Rightarrow (x-2)(x-4) = 0 \Rightarrow$$
$$x - 2 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = 2 \quad \text{or} \quad x = 4$$

To find the corresponding values of y use the equation $y = 5 - x$. If $x = 2$, then $y = 5 - 2 = 3$.

If $x = 4$, then $y = 5 - 4 = 1$. The points satisfying the conditions are (2, 3) and (4, 1).

48. The circle of smallest radius that contains the points $A(1, 4)$ and $B(-3, 2)$ within or on its boundary will be the circle having points A and B as endpoints of a diameter. The center will be M, the midpoint:

$$\left(\frac{1+(-3)}{2}, \frac{4+2}{2}\right) = \left(\frac{-2}{2}, \frac{6}{2}\right) = (-1, 3).$$

The radius will be the distance from M to either A or B:

$$d(M, A) = [1-(-1)]^2 + (4-3)^2$$

49. Label the points $A(3, y)$ and $B(-2, 9)$. If $d(A, B) = 12$, then

$$\sqrt{(-2-3)^2 + (9-y)^2} = 12$$
$$\sqrt{(-5)^2 + (9-y)^2} = 12$$
$$(-5)^2 + (9-y)^2 = 12^2$$
$$25 + 81 - 18y + y^2 = 144$$
$$y^2 - 18y - 38 = 0$$

Solve this equation by using the quadratic formula with $a = 1$, $b = -18$, and $c = -38$:

$$y = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(1)(-38)}}{2(1)}$$
$$= \frac{18 \pm \sqrt{324 + 152}}{2(1)} = \frac{18 \pm \sqrt{476}}{2}$$
$$= \frac{18 \pm \sqrt{4(119)}}{2} = \frac{18 \pm 2\sqrt{119}}{2} = 9 \pm \sqrt{119}$$

The values of y are $9 + \sqrt{119}$ and $9 - \sqrt{119}$.

50. Because the center is in the third quadrant, the radius is $\sqrt{2}$, and the circle is tangent to both axes, the center must be at $(-\sqrt{2}, -\sqrt{2})$.

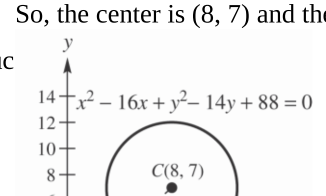
Using the center-radius of the equation of a circle, we have

$$\left[x - (-\sqrt{2})\right]^2 + \left[y - (-\sqrt{2})\right]^2 = (\sqrt{2})^2 \Rightarrow$$
$$(x + \sqrt{2})^2 + (y + \sqrt{2})^2 = 2.$$

51. Let $P(x, y)$ be the point on the circle whose distance from the origin is the shortest. Complete the square on x and y separately to write the equation in center-radius form:

$$x^2 - 16x + y^2 - 14y + 88 = 0$$
$$x^2 - 16x + 64 + y^2 - 14y + 49 = -88 + 64 + 49$$
$$(x-8)^2 + (y-7)^2 = 25$$

So, the center is (8, 7) and the radius is 5.



$= \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$

The equation of the circle is

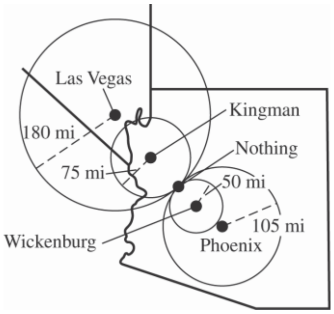
$[x-(-1)]^2 + (y-3)^2 = (\sqrt{5})^2 \Rightarrow$

$(x+1)^2 + (y-3)^2 = 5.$

$d(C, O) = \sqrt{8^2 + 7^2} = \sqrt{113}$. Because the

length of the radius is 5, $d(P, O) = \sqrt{113} - 5$.

52. Using compasses, draw circles centered at Wickenburg, Kingman, Phoenix, and Las Vegas with scaled radii of 50, 75, 105, and 180 miles respectively. The four circles should intersect at the location of Nothing.



53. The midpoint M has coordinates

$$\left(\frac{-1+5}{2}, \frac{3+(-9)}{2}\right) = \left(\frac{4}{2}, \frac{-6}{2}\right) = (2, -3).$$

54. Use points $C(2, -3)$ and $P(-1, 3)$.

$$\begin{aligned} d(C, P) &= \sqrt{(-1-2)^2 + [3-(-3)]^2} \\ &= \sqrt{(-3)^2 + 6^2} = \sqrt{9+36} \\ &= \sqrt{45} = 3\sqrt{5} \end{aligned}$$

The radius is $3\sqrt{5}$.

55. Use points $C(2, -3)$ and $Q(5, -9)$.

$$\begin{aligned} d(C, Q) &= \sqrt{(5-2)^2 + [-9-(-3)]^2} \\ &= \sqrt{3^2 + (-6)^2} = \sqrt{9+36} \\ &= \sqrt{45} = 3\sqrt{5} \end{aligned}$$

The radius is $3\sqrt{5}$.

56. Use the points $P(-1, 3)$ and $Q(5, -9)$.

Because $d(P, Q) = \sqrt{[5-(-1)]^2 + (-9-3)^2}$

$$= \sqrt{6^2 + (-12)^2} = \sqrt{36+144} = \sqrt{180}$$

58. Label the endpoints of the diameter $P(3, -5)$ and $Q(-7, 3)$. The midpoint M of the segment joining P and Q has coordinates

$$\left(\frac{3+(-7)}{2}, \frac{-5+3}{2}\right) = \left(\frac{-4}{2}, \frac{-2}{2}\right) = (-2, -1).$$

The center is $C(-2, -1)$. To find the radius, we can use points $C(-2, -1)$ and $P(3, -5)$

$$\begin{aligned} d(C, P) &= \sqrt{[3-(-2)]^2 + [-5-(-1)]^2} \\ &= \sqrt{5^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41} \end{aligned}$$

We could also use points $C(-2, -1)$.and $Q(-7, 3)$.

$$\begin{aligned} d(C, Q) &= \sqrt{[-7-(-2)]^2 + [3-(-1)]^2} \\ &= \sqrt{(-5)^2 + 4^2} = \sqrt{25+16} = \sqrt{41} \end{aligned}$$

We could also use points $P(3, -5)$ and $Q(-7, 3)$ to find the length of the diameter. The length of the radius is one-half the length of the diameter.

$$\begin{aligned} d(P, Q) &= \sqrt{(-7-3)^2 + [3-(-5)]^2} \\ &= \sqrt{(-10)^2 + 8^2} = \sqrt{100+64} \\ &= \sqrt{164} = 2\sqrt{41} \end{aligned}$$

$$\frac{1}{2} d(P, Q) = \frac{1}{2}\left(2\sqrt{41}\right) = \sqrt{41}$$

The center-radius form of the equation of the circle is

$$[x-(-2)]^2 + [y-(-1)]^2 = (\sqrt{41})^2$$

$$(x+2)^2 + (y+1)^2 = 41$$

59. Label the endpoints of the diameter $P(-1, 2)$ and $Q(11, 7)$. The midpoint M of the segment joining P and Q has coordinates

$$\left(\frac{-1+11}{2}, \frac{2+7}{2}\right) = \left(5, \frac{9}{2}\right).$$

The center is $C\left(5, \frac{9}{2}\right)$. To find the radius, we

$= 6\sqrt{5}$, the radius is $\frac{1}{2}d(P, Q)$. Thus

$$r = \frac{1}{2}\left(6\sqrt{5}\right) = 3\sqrt{5}.$$

57. The center-radius form for this circle is
$(x-2)^2 + (y+3)^2 = (3\sqrt{5})^2 \Rightarrow$

$$(x-2)^2 + (y+3)^2 = 45.$$

can use points $C\left(5, \frac{9}{2}\right)$ and $P(-1, 2)$.

$$d(C, P) = \sqrt{\left[5-(-1)\right]^2 + \left(\frac{9}{2}-2\right)^2} = \sqrt{6^2 + \left(\frac{5}{2}\right)^2} = \sqrt{\frac{169}{4}} = \frac{13}{2}$$

We could also use points $C\left(5, \frac{9}{2}\right)$ and $Q(11, 7)$.

$$d(C, Q) = \sqrt{(5-11)^2 + \left(\frac{9}{2}-7\right)^2} = \sqrt{(-6)^2 + \left(-\frac{5}{2}\right)^2} = \sqrt{\frac{169}{4}} = \frac{13}{2}$$

(*continued on next page*)

(*continued*)

Using the points P and Q to find the length of the diameter, we have

$$d(P,\ Q) = \sqrt{(-1-11)^2 + (2-7)^2} = \sqrt{(-12)^2 + (-5)^2} = \sqrt{169} = 13$$

$$\frac{1}{2}d(P,\ Q) = \frac{1}{2}(13) = \frac{13}{2}$$

The center-radius form of the equation of the circle is

$$(x-5)^2 + \left(y - \tfrac{9}{2}\right)^2 = \left(\tfrac{13}{2}\right)^2$$

$$(x-5)^2 + \left(y - \tfrac{9}{2}\right)^2 = \tfrac{169}{4}$$

60. Label the endpoints of the diameter $P(5, 4)$ and $Q(-3, -2)$. The midpoint M of the segment joining P and Q has coordinates

$$\left(\frac{5+(-3)}{2},\ \frac{4+(-2)}{2}\right) = (1,\ 1).$$

The center is $C(1, 1)$. To find the radius, we can use points $C(1, 1)$ and $P(5, 4)$.

$$d(C,\ P) = \sqrt{(5-1)^2 + (4-1)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

We could also use points $C(1, 1)$ and $Q(-3, -2)$.

$$d(C,\ Q) = \sqrt{[1-(-3)]^2 + [1-(-2)]^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

Using the points P and Q to find the length of the diameter, we have

$$d(P,\ Q) = \sqrt{[5-(-3)]^2 + [4-(-2)]^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

$$\frac{1}{2}d(P,\ Q) = \frac{1}{2}(10) = 5$$

The length of the diameter PQ is

$$\sqrt{(1-5)^2 + (4-1)^2} = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5.$$

The length of the radius is $\frac{1}{2}(5) = \frac{5}{2}$.

The center-radius form of the equation of the circle is

$$(x-3)^2 + \left(y - \tfrac{5}{2}\right)^2 = \left(\tfrac{5}{2}\right)^2$$

$$(x-3)^2 + \left(y - \tfrac{5}{2}\right)^2 = \tfrac{25}{4}$$

62. Label the endpoints of the diameter $P(-3, 10)$ and $Q(5, -5)$. The midpoint M of the segment joining P and Q has coordinates

$$\left(\frac{-3+5}{2},\ \frac{10+(-5)}{2}\right) = \left(1,\ \tfrac{5}{2}\right).$$

The center is $C\left(1,\ \frac{5}{2}\right)$.

The length of the diameter PQ is

$$\sqrt{(-3-5)^2 + [10-(-5)]^2} = \sqrt{(-8)^2 + 15^2} = \sqrt{289} = 17.$$

The length of the radius is $\frac{1}{2}(17) = \frac{17}{2}$.

The center-radius form of the equation of the circle is

$$(x-1)^2 + \left(y - \tfrac{5}{2}\right)^2 = \left(\tfrac{17}{2}\right)^2$$

$$(x-1)^2 + \left(y - \tfrac{5}{2}\right)^2 = \tfrac{289}{4}$$

Section 2.3 Functions

1. The domain of the relation

$$\{(3,5),\ (4,9),\ (10,13)\} \text{ is } \underline{\{3, 4, 10\}}.$$

2. The range of the relation in Exercise 1 is $\underline{\{5, 9, 13\}}$.

3. The equation $y = 4x - 6$ defines a function with

The center-radius form of the equation of the circle is

$$(x-1)^2+(y-1)^2=5^2$$

$$(x-1)^2+(y-1)^2=25$$

61. Label the endpoints of the diameter $P(1, 4)$ and $Q(5, 1)$. The midpoint M of the segment joining P and Q has coordinates

$$\left(\frac{1+5}{2}, \frac{4+1}{2}\right)=\left(3, \frac{5}{2}\right).$$

The center is $C\left(3, \frac{5}{2}\right)$.

independent variable $\underline{x}$ and dependent variable $\underline{y}$.

4. The function in Exercise 3 includes the ordered pair (6, $\underline{18}$).

5. For the function $f(x)=-4x+2$,

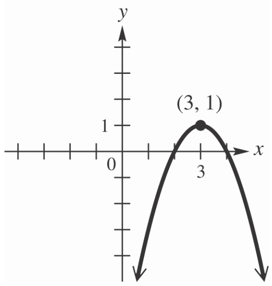
$$f(-2)=-4(-2)+2=8+2=\underline{10}.$$

6. For the function $g(x)=\sqrt{x}$, $g(9)=\sqrt{9}=\underline{3}$.

7. The function in Exercise 6, $g(x)=\sqrt{x}$, has domain $\underline{[0, \infty)}$.

8. The function in Exercise 6, $g(x) = \sqrt{x}$, has range $\underline{[0, \infty)}$.

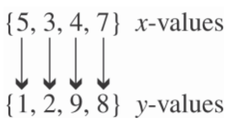
For exercises 9 and 10, use this graph.



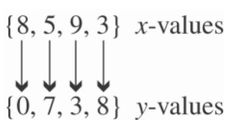
9. The largest open interval over which the function graphed here increases is $\underline{(-\infty, 3)}$.

10. The largest open interval over which the function graphed here decreases is $\underline{(3, \infty)}$.

11. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.



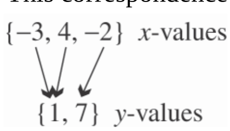
12. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.



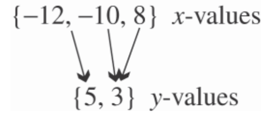
13. Two ordered pairs, namely (2, 4) and (2, 6), have the same x-value paired with different y-values, so the relation is not a function.

14. Two ordered pairs, namely (9, −2) and (9, 1), have the same x-value paired with different y-values, so the relation is not a function.

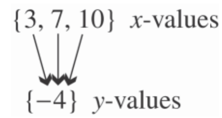
15. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.



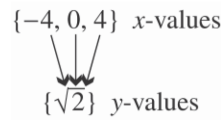
16. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.



17. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.

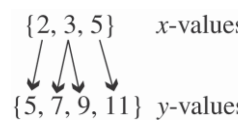


18. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.



19. Two sets of ordered pairs, namely (1, 1) and (1, −1) as well as (2, 4) and (2, −4), have the same x-value paired with different y-values, so the relation is not a function.
domain: {0, 1, 2}; range: {−4, −1, 0, 1, 4}

20. The relation is not a function because the x-value 3 corresponds to two y-values, 7 and 9. This correspondence can be shown as follows.

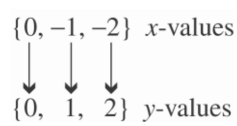


domain: {2, 3, 5}; range: {5, 7, 9, 11}

21. The relation is a function because for each different x-value there is exactly one y-value.
domain: {2, 3, 5, 11, 17}; range: {1, 7, 20}

22. The relation is a function because for each different x-value there is exactly one y-value.
domain: {1, 2, 3, 5}; range: {10, 15, 19, 27}

23. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.



Domain: {0, −1, −2}; range: {0, 1, 2}

24. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.

$\{0, \;\; 1, \;\; 2\}$ x-values
$\downarrow \;\; \downarrow \;\; \downarrow$
$\{0, -1, -2\}$ y-values

Domain: {0, 1, 2}; range: {0, −1, −2}

25. The relation is a function because for each different year, there is exactly one number for visitors.
domain: {2010, 2011, 2012, 2013}
range: {64.9, 63.0, 65.1, 63.5}

26. The relation is a function because for each basketball season, there is only one number for attendance.
domain: {2011, 2012, 2013, 2014}
range: {11,159,999, 11,210,832, 11,339,285, 11,181,735}

27. This graph represents a function. If you pass a vertical line through the graph, one x-value corresponds to only one y-value.
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

28. This graph represents a function. If you pass a vertical line through the graph, one x-value corresponds to only one y-value.
domain: $(-\infty, \infty)$; range: $(-\infty, 4]$

29. This graph does not represent a function. If you pass a vertical line through the graph, there are places where one value of x corresponds to two values of y.

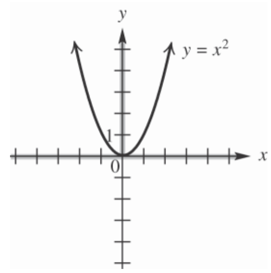
domain: $[3, \infty)$; range: $(-\infty, \infty)$

30. This graph does not represent a function. If you pass a vertical line through the graph, there are places where one value of x corresponds to two values of y.
domain: [−4, 4]; range: [−3, 3]

31. This graph represents a function. If you pass a vertical line through the graph, one x-value corresponds to only one y-value.
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

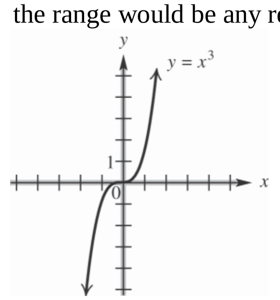
32. This graph represents a function. If you pass a vertical line through the graph, one x-value corresponds to only one y-value.
domain: [−2, 2]; range: [0, 4]

33. $y = x^2$ represents a function because y is always found by squaring x. Thus, each value of x corresponds to just one value of y. x can be any real number. Because the square of any real number is not negative, the range would be zero or greater.



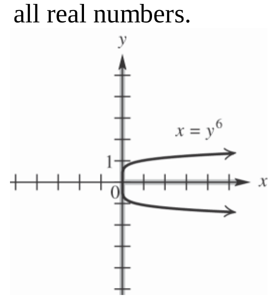
domain: $(-\infty, \infty)$; range: $[0, \infty)$

34. $y = x^3$ represents a function because y is always found by cubing x. Thus, each value of x corresponds to just one value of y. x can be any real number. Because the cube of any real number could be negative, positive, or zero, the range would be any real number.



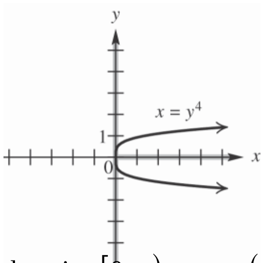
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

35. The ordered pairs (1, 1) and (1, −1) both satisfy $x = y^6$. This equation does not represent a function. Because x is equal to the sixth power of y, the values of x are nonnegative. Any real number can be raised to the sixth power, so the range of the relation is all real numbers.



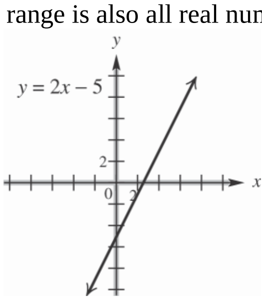
domain: $[0, \infty)$ range: $(-\infty, \infty)$

36. The ordered pairs (1, 1) and (1, −1) both satisfy $x = y^4$. This equation does not represent a function. Because x is equal to the fourth power of y, the values of x are nonnegative. Any real number can be raised to the fourth power, so the range of the relation is all real numbers.



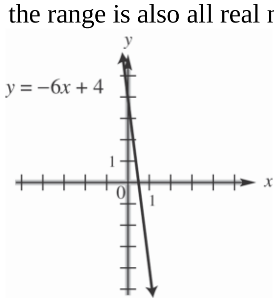
domain: $[0, \infty)$ range: $(-\infty, \infty)$

37. $y = 2x - 5$ represents a function because y is found by multiplying x by 2 and subtracting 5. Each value of x corresponds to just one value of y. x can be any real number, so the domain is all real numbers. Because y is twice x, less 5, y also may be any real number, and so the range is also all real numbers.



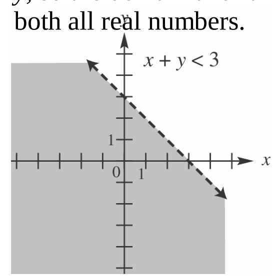
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

38. $y = -6x + 4$ represents a function because y is found by multiplying x by −6 and adding 4. Each value of x corresponds to just one value of y. x can be any real number, so the domain is all real numbers. Because y is −6 times x, plus 4, y also may be any real number, and so the range is also all real numbers.



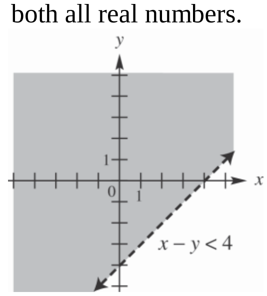
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

39. By definition, y is a function of x if every value of x leads to exactly one value of y. Substituting a particular value of x, say 1, into $x + y < 3$ corresponds to many values of y. The ordered pairs (1, –2), (1, 1), (1, 0), (1, −1), and so on, all satisfy the inequality. Note that the points on the graphed line do not satisfy the inequality and only indicate the boundary of the solution set. This does not represent a function. Any number can be used for x or for y, so the domain and range of this relation are both all real numbers.



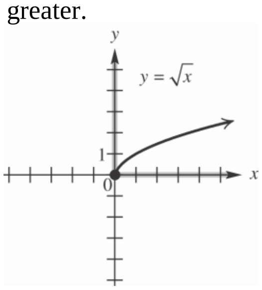
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

40. By definition, y is a function of x if every value of x leads to exactly one value of y. Substituting a particular value of x, say 1, into $x - y < 4$ corresponds to many values of y. The ordered pairs (1, −1), (1, 0), (1, 1), (1, 2), and so on, all satisfy the inequality. Note that the points on the graphed line do not satisfy the inequality and only indicate the boundary of the solution set. This does not represent a function. Any number can be used for x or for y, so the domain and range of this relation are both all real numbers.



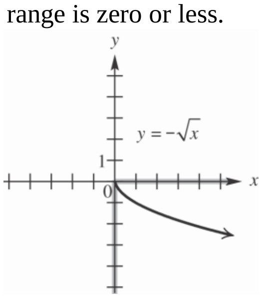
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

41. For any choice of x in the domain of $y = \sqrt{x}$, there is exactly one corresponding value of y, so this equation defines a function. Because the quantity under the square root cannot be negative, we have $x \geq 0$. Because the radical is nonnegative, the range is also zero or greater.



domain: $[0, \infty)$; range: $[0, \infty)$

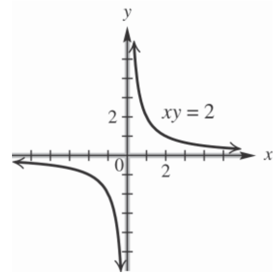
42. For any choice of x in the domain of $y = -\sqrt{x}$, there is exactly one corresponding value of y, so this equation defines a function. Because the quantity under the square root cannot be negative, we have $x \geq 0$. The outcome of the radical is nonnegative, when you change the sign (by multiplying by −1), the range becomes nonpositive. Thus the range is zero or less.



domain: $[0, \infty)$; range: $(-\infty, 0]$

43. Because $xy = 2$ can be rewritten as $y = \frac{2}{x}$,

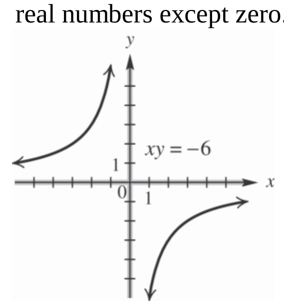
we can see that y can be found by dividing x into 2. This process produces one value of y for each value of x in the domain, so this equation is a function. The domain includes all real numbers except those that make the denominator equal to zero, namely $x = 0$. Values of y can be negative or positive, but never zero. Therefore, the range will be all real numbers except zero.



domain: $(-\infty, 0) \cup (0, \infty)$;
range: $(-\infty, 0) \cup (0, \infty)$

44. Because $xy = -6$ can be rewritten as $y = \frac{-6}{x}$,

we can see that y can be found by dividing x into −6. This process produces one value of y for each value of x in the domain, so this equation is a function. The domain includes all real numbers except those that make the denominator equal to zero, namely $x = 0$. Values of y can be negative or positive, but never zero. Therefore, the range will be all real numbers except zero.

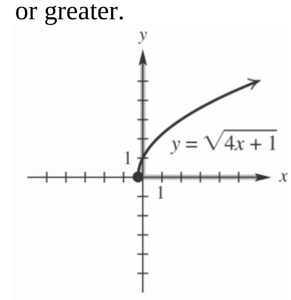


domain: $(-\infty, 0) \cup (0, \infty)$;
range: $(-\infty, 0) \cup (0, \infty)$

45. For any choice of x in the domain of $y = \sqrt{4x+1}$ there is exactly one

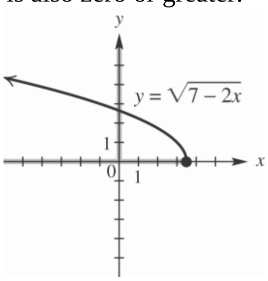
corresponding value of y, so this equation defines a function. Because the quantity under

the square root cannot be negative, we have $4x + 1 \geq 0 \Rightarrow 4x \geq -1 \Rightarrow x \geq -\frac{1}{4}$. Because the radical is nonnegative, the range is also zero or greater.



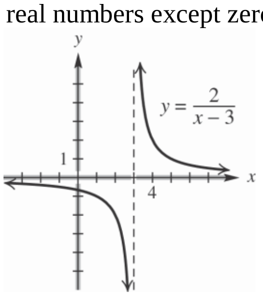
domain: $\left[-\frac{1}{4}, \infty\right)$; range: $[0, \infty)$

46. For any choice of x in the domain of $y = \sqrt{7-2x}$ there is exactly one corresponding value of y, so this equation defines a function. Because the quantity under the square root cannot be negative, we have $7 - 2x \geq 0 \Rightarrow -2x \geq -7 \Rightarrow x \leq \frac{-7}{-2}$ or $x \leq \frac{7}{2}$. Because the radical is nonnegative, the range is also zero or greater.



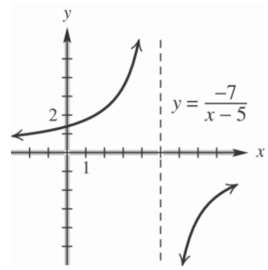
domain: $\left(-\infty, \frac{7}{2}\right]$; range: $[0, \infty)$

47. Given any value in the domain of $y = \frac{2}{x-3}$, we find y by subtracting 3, then dividing into 2. This process produces one value of y for each value of x in the domain, so this equation is a function. The domain includes all real numbers except those that make the denominator equal to zero, namely $x = 3$. Values of y can be negative or positive, but never zero. Therefore, the range will be all real numbers except zero.



domain: $(-\infty, 3) \cup (3, \infty)$;
range: $(-\infty, 0) \cup (0, \infty)$

48. Given any value in the domain of $y = \frac{-7}{x-5}$, we find y by subtracting 5, then dividing into −7. This process produces one value of y for each value of x in the domain, so this equation is a function. The domain includes all real numbers except those that make the denominator equal to zero, namely $x = 5$. Values of y can be negative or positive, but never zero. Therefore, the range will be all real numbers except zero.



domain: $(-\infty, 5) \cup (5, \infty)$;
range: $(-\infty, 0) \cup (0, \infty)$

49. B. The notation $f(3)$ means the value of the dependent variable when the independent variable is 3.

50. Answers will vary. An example is: <u>The cost of gasoline</u> depends on <u>the number of gallons used;</u> so <u>cost</u> is a function of <u>number of</u> gallons.

51. $f(x) = -3x + 4$

$f(0) = -3 \cdot 0 + 4 = 0 + 4 = 4$

52. $f(x) = -3x + 4$
$f(-3) = -3(-3) + 4 = 9 + 4 = 13$

53. $g(x) = -x^2 + 4x + 1$

$g(-2) = -(-2)^2 + 4(-2) + 1$

$= -4 + (-8) + 1 = -11$

54. $g(x) = -x^2 + 4x + 1$
$g(10) = -10^2 + 4 \cdot 10 + 1$
$= -100 + 40 + 1 = -59$

55. $f(x) = -3x + 4$
$f\left(\frac{1}{3}\right) = -3\left(\frac{1}{3}\right) + 4 = -1 + 4 = 3$

56. $f(x) = -3x + 4$
$f\left(-\frac{7}{3}\right) = -3\left(-\frac{7}{3}\right) + 4 = 7 + 4 = 11$

57. $g(x) = -x^2 + 4x + 1$

59. 2 2 2

58.

$$g\left(\tfrac{1}{2}\right) = -\left(\tfrac{1}{2}\right)^2 + 4\left(\tfrac{1}{2}\right) + 1$$
$$= -\tfrac{1}{4} + 2 + 1 = \tfrac{11}{4}$$

$$g(x) = -x^2 + 4x + 1$$
$$g\left(-\tfrac{1}{4}\right) = -\left(-\tfrac{1}{4}\right)^2 + 4\left(-\tfrac{1}{4}\right) + 1$$
$$= -\tfrac{1}{16} - 1 + 1 = -\tfrac{1}{16}$$

$$f(x) = -3x + 4$$
$$f(p) = -3p + 4$$

60. $g(x) = -x^2 + 4x + 1$
$g(k) = -k^2 + 4k + 1$

61. $f(x) = -3x + 4$
$f(-x) = -3(-x) + 4 = 3x + 4$

62. $g(x) = -x^2 + 4x + 1$

$g(-x) = -(-x)^2 + 4(-x) + 1$
$= -x^2 - 4x + 1$

63. $f(x) = -3x + 4$

$f(x+2) = -3(x+2) + 4$
$= -3x - 6 + 4 = -3x - 2$

64. $f(x) = -3x + 4$
$f(a+4) = -3(a+4) + 4$
$= -3a - 12 + 4 = -3a - 8$

65. $f(x) = -3x + 4$

$f(2m-3) = -3(2m-3) + 4$
$= -6m + 9 + 4 = -6m + 13$

66. $f(x) = -3x + 4$
$f(3t-2) = -3(3t-2) + 4$
$= -9t + 6 + 4 = -9t + 10$

67. **(a)** $f(2) = 2$ **(b)** $f(-1) = 3$

68. **(a)** $f(2) = 5$ **(b)** $f(-1) = 11$

69. **(a)** $f(2) = 15$ **(b)** $f(-1) = 10$

70. **(a)** $f(2) = 1$ **(b)** $f(-1) = 7$

71. **(a)** $f(2) = 3$ **(b)** $f(-1) = -3$

72. **(a)** $f(2) = -3$ **(b)** $f(-1) = 2$

73. **(a)** $f(-2) = 0$ **(b)** $f(0) = 4$
(c) $f(1) = 2$ **(d)** $f(4) = 4$

74. **(a)** $f(-2) = 5$ **(b)** $f(0) = 0$
(c) $f(1) = 2$ **(d)** $f(4) = 4$

75. **(a)** $f(-2) = -3$ **(b)** $f(0) = -2$
(c) $f(1) = 0$ **(d)** $f(4) = 2$

77. **(a)** $x + 3y = 12$
$3y = -x + 12$
$y = \frac{-x+12}{3}$
$y = -\frac{1}{3}x + 4 \Rightarrow f(x) = -\frac{1}{3}x + 4$

(b) $f(3) = -\frac{1}{3}(3) + 4 = -1 + 4 = 3$

78. **(a)** $x - 4y = 8$
$x = 8 + 4y$
$x - 8 = 4y$
$\frac{x-8}{4} = y$
$y = \frac{1}{4}x - 2 \Rightarrow f(x) = \frac{1}{4}x - 2$

(b) $f(3) = \frac{1}{4}(3) - 2 = \frac{3}{4} - 2 = \frac{3}{4} - \frac{8}{4} = -\frac{5}{4}$

79. **(a)** $y + 2x^2 = 3 - x$
$y = -2x^2 - x + 3$
$f(x) = -2x^2 - x + 3$

(b) $f(3) = -2(3)^2 - 3 + 3$
$= -2 \cdot 9 - 3 + 3 = -18$

80. **(a)** $y - 3x^2 = 2 + x$
$y = 3x^2 + x + 2$
$f(x) = 3x^2 + x + 2$

(b) $f(3) = 3(3)^2 + 3 + 2$
$= 3 \cdot 9 + 3 + 2 = 32$

81. **(a)** $4x - 3y = 8$
$4x = 3y + 8$
$4x - 8 = 3y$
$\frac{4x-8}{3} = y$
$y = \frac{4}{3}x - \frac{8}{3} \Rightarrow f(x) = \frac{4}{3}x - \frac{8}{3}$

(b) $f(3) = \frac{4}{3}(3) - \frac{8}{3} = \frac{12}{3} - \frac{8}{3} = \frac{4}{3}$

82. **(a)** $-2x + 5y = 9$
$5y = 2x + 9$
$y = \frac{2x+9}{5}$
$y = \frac{2}{5}x + \frac{9}{5} \Rightarrow f(x) = \frac{2}{5}x + \frac{9}{5}$

(b) $f(3) = \frac{2}{5}(3) + \frac{9}{5} = \frac{6}{5} + \frac{9}{5} = \frac{15}{5}$

$f\ 3 = {}_5\ 3 + {}_5 = {}_5 + {}_5 = {}_5 = 3$

76. **(a)** $f(-2) = 3$ **(b)** $f(0) = 3$

(c) $f(1) = 3$ **(d)** $f(4) = 3$

83. $f(3) = 4$

84. Because $f(0.2) = 0.2^2 + 3(0.2) + 1$

= 0.04 + 0.6 + 1 = 1.64, the height of the rectangle is 1.64 units. The base measures 0.3 − 0.2 = 0.1 unit. Because the area of a rectangle is base times height, the area of this rectangle is 0.1(1.64) = 0.164 square unit.

85. $f(3)$ is the y-component of the coordinate, which is −4.

86. $f(-2)$ is the y-component of the coordinate, which is −3.

87. **(a)** $(-2, 0)$ **(b)** $(-\infty, -2)$

(c) $(0, \infty)$

88. **(a)** $(-3, -1)$ **(b)** $(-1, \infty)$

(c) $(-\infty, -3)$

89. **(a)** $(-\infty, -2); (2, \infty)$

(b) $(-2, -2)$ **(c)** none

90. **(a)** $(-3, 3)$ **(b)** $(-\infty, -3); (3, \infty)$

(c) none

91. **(a)** $(-1, 0); (1, \infty)$

(b) $(-\infty, -1); (0, 1)$

(c) none

92. **(a)** $(-\infty, -2); (0, 2)$

(b) $(-2, 0); (2, \infty)$

(c) none

93. **(a)** Yes, it is the graph of a function.

(b) [0, 24]

(c) When $t = 8$, $y = 1200$ from the graph. At 8 A.M., approximately 1200 megawatts is being used.

(d) The most electricity was used at 17 hr or 5 P.M. The least electricity was used at 4 A.M.

(e) $f(12) \approx 1900$

At 12 noon, electricity use is about 1900 megawatts.

(f) increasing from 4 A.M. to 5 P.M.; decreasing from midnight to 4 A.M. and

from 5 P.M. to midnight

94. **(a)** At $t = 2$, $y = 240$ from the graph. Therefore, at 2 seconds, the ball is 240 feet high.

(b) At $y = 192$, $x = 1$ and $x = 5$ from the graph. Therefore, the height will be 192 feet at 1 second and at 5 seconds.

(c) The ball is going up from 0 to 3 seconds and down from 3 to 7 seconds.

(d) The coordinate of the highest point is (3, 256). Therefore, it reaches a maximum height of 256 feet at 3 seconds.

(e) At $x = 7$, $y = 0$. Therefore, at 7 seconds, the ball hits the ground.

95. **(a)** At $t = 12$ and $t = 20$, $y = 55$ from the graph. Therefore, after about 12 noon

until about 8 P.M. the temperature was over 55º.

(b) At $t = 6$ and $t = 22$, $y = 40$ from the graph. Therefore, until about 6 A.M. and after 10 P.M. the temperature was below 40º.

(c) The temperature at noon in Bratenahl, Ohio was 55º. Because the temperature in Greenville is 7º higher, we are looking for the time at which Bratenahl, Ohio was at or above 48º. This occurred at approximately 10 A.M and 8:30 P.M.

(d) The temperature is just below 40° from midnight to 6 A.M., when it begins to rise until it reaches a maximum of just below 65° at 4 P.M. It then begins to fall util it reaches just under 40° again at midnight.

96. **(a)** At t = 8, y = 24 from the graph. Therefore, there are 24 units of the drug in the bloodstream at 8 hours.

(b) The level increases between 0 and 2 hours after the drug is taken and decreases between 2 and 12 hours after the drug is taken.

(c) The coordinates of the highest point are (2, 64). Therefore, at 2 hours, the level of the drug in the bloodstream reaches its

greatest value of 64 units.

(d) After the peak, $y = 16$ at $t = 10$. 10 hours – 2 hours = 8 hours after the peak. 8 additional hours are required for the level to drop to 16 units.

(e) When the drug is administered, the level is 0 units. The level begins to rise quickly for 2 hours until it reaches a maximum of 64 units. The level then begins to decrease gradually until it reaches a level of 12 units, 12 hours after it was administered.

Section 2.4 Linear Functions

1. B; $f(x) = 3x + 6$ is a linear function with y-intercept (0, 6).

2. H; $x = 9$ is a vertical line.

3. C; $f(x) = -8$ is a constant function.

4. G; $2x - y = -4$ or $y = 2x + 4$ is a linear equation with x-intercept (–2, 0) and y-intercept (0, 4).

5. A; $f(x) = 5x$ is a linear function whose graph passes through the origin, (0, 0). $f(0) = 5(0) = 0$.

6. D; $f(x) = x^2$ is a function that is not linear.

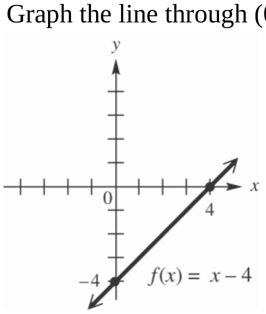
7. $m = -3$ matches graph C because the line falls rapidly as x increases.

8. $m = 0$ matches graph A because horizontal lines have slopes of 0.

9. $m = 3$ matches graph D because the line rises rapidly as x increases.

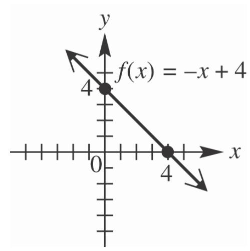
10. m is undefined for graph B because vertical lines have undefined slopes.

11. $f(x) = x - 4$
Use the intercepts.
$f(0) = 0 - 4 = -4:$ y-intercept
$0 = x - 4 \Rightarrow x = 4:$ x-intercept
Graph the line through (0, –4) and (4, 0).



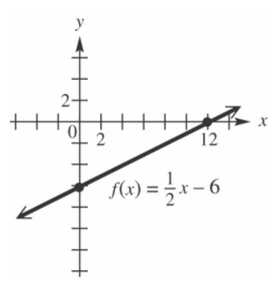
The domain and range are both $(-\infty, \infty)$.

12. $f(x) = -x + 4$
Use the intercepts.
$f(0) = -0 + 4 = 4:$ y-intercept
$0 = -x + 4 \Rightarrow x = 4:$ x-intercept
Graph the line through (0, 4) and (4, 0).



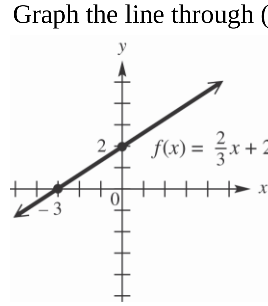
The domain and range are both $(-\infty, \infty)$.

13. $f(x) = \frac{1}{2}x - 6$
Use the intercepts.
$f(0) = \frac{1}{2}(0) - 6 = -6:$ y-intercept
$0 = \frac{1}{2}x - 6 \Rightarrow 6 = \frac{1}{2}x \Rightarrow x = 12:$ x-intercept
Graph the line through (0, –6) and (12, 0).



The domain and range are both $(-\infty, \infty)$.

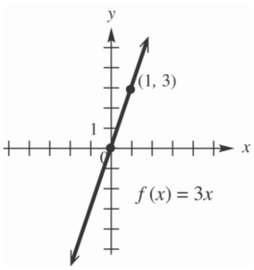
14. $f(x) = \frac{2}{3}x + 2$
Use the intercepts.
$f(0) = \frac{2}{3}(0) + 2 = 2:$ y-intercept
$0 = \frac{2}{3}x + 2 \Rightarrow -2 = \frac{2}{3}x \Rightarrow x = -3:$ x-intercept
Graph the line through (0, 2) and (–3, 0).



The domain and range are both $(-\infty, \infty)$.

15. $f(x) = 3x$

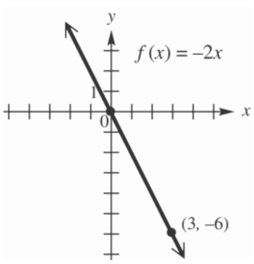
The x-intercept and the y-intercept are both zero. This gives us only one point, (0, 0). If $x = 1$, $y = 3(1) = 3$. Another point is (1, 3). Graph the line through (0, 0) and (1, 3).



The domain and range are both $(-\infty, \infty)$.

16. $f(x) = -2x$

The x-intercept and the y-intercept are both zero. This gives us only one point, (0, 0). If $x = 3$, $y = -2(3) = -6$, so another point is (3, –6). Graph the line through (0, 0) and (3, –6).

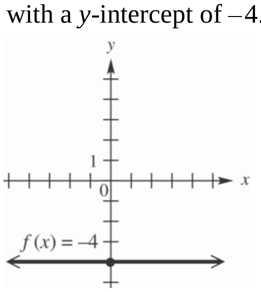


The domain and range are both $(-\infty, \infty)$.

17. $f(x) = -4$ is a constant function.

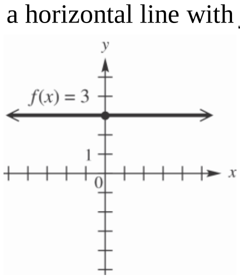
The graph of $f(x) = -4$ is a horizontal line

with a y-intercept of –4.



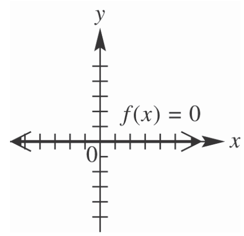
domain: $(-\infty, \infty)$; range: $\{-4\}$

18. $f(x) = 3$ is a constant function whose graph is a horizontal line with y-intercept of 3.



domain: $(-\infty, \infty)$; range: $\{3\}$

19. $f(x) = 0$ is a constant function whose graph is the x-axis.



domain: $(-\infty, \infty)$; range: $\{0\}$

20. $f(x) = 9x$

The domain and range are both $(-\infty, \infty)$.

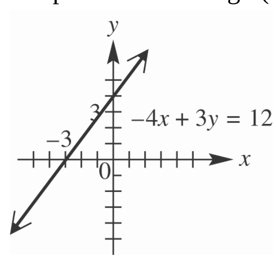
21. $-4x + 3y = 12$

Use the intercepts.

$-4(0) + 3y = 12 \Rightarrow 3y = 12 \Rightarrow$
$y = 4$: y-intercept

$-4x + 3(0) = 12 \Rightarrow -4x = 12 \Rightarrow$
$x = -3$: x-intercept

Graph the line through (0, 4) and (−3, 0).



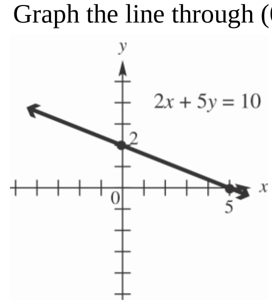
The domain and range are both $(-\infty, \infty)$.

22. $2x + 5y = 10$; Use the intercepts.

$2(0) + 5y = 10 \Rightarrow 5y = 10 \Rightarrow$
$y = 2$: y-intercept

$2x + 5(0) = 10 \Rightarrow 2x = 10 \Rightarrow$
$x = 5$: x-intercept

Graph the line through (0, 2) and (5, 0):



The domain and range are both $(-\infty, \infty)$.

23. $3y - 4x = 0$

Use the intercepts.

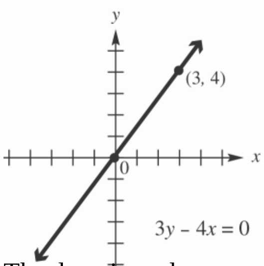
$3y - 4(0) = 0 \Rightarrow 3y = 0 \Rightarrow y = 0$: y-intercept

$3(0) - 4x = 0 \Rightarrow -4x = 0 \Rightarrow x = 0$: x-intercept

The graph has just one intercept. Choose an additional value, say 3, for x.

$3y - 4(3) = 0 \Rightarrow 3y - 12 = 0 \Rightarrow$
$3y = 12 \Rightarrow y = 4$

Graph the line through (0, 0) and (3, 4):



The domain and range are both $(-\infty, \infty)$.

24. $3x + 2y = 0$

Use the intercepts.

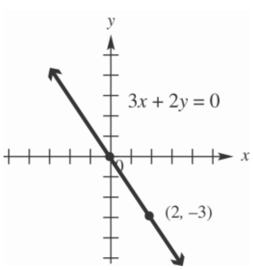
$3(0) + 2y = 0 \Rightarrow 2y = 0 \Rightarrow y = 0$: y-intercept

$3x + 2(0) = 0 \Rightarrow 3x = 0 \Rightarrow x = 0$: x-intercept

The graph has just one intercept. Choose an additional value, say 2, for x.

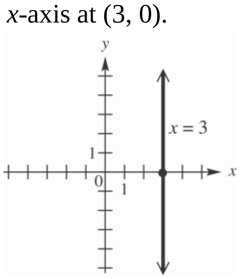
$3(2) + 2y = 0 \Rightarrow 6 + 2y = 0 \Rightarrow$
$2y = -6 \Rightarrow y = -3$

Graph the line through (0, 0) and (2, −3):



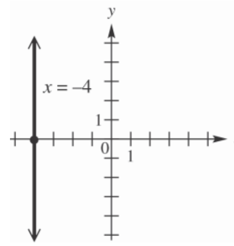
The domain and range are both $(-\infty, \infty)$.

25. $x = 3$ is a vertical line, intersecting the x-axis at (3, 0).



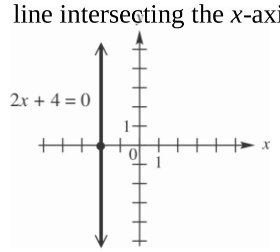
domain: {3}; range: $(-\infty, \infty)$

26. $x = -4$ is a vertical line intersecting the x-axis at (–4, 0).



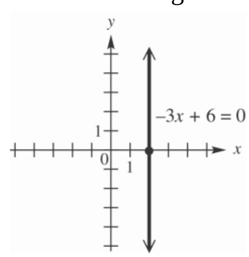
domain: $\{-4\}$; range: $(-\infty, \infty)$

27. $2x + 4 = 0 \Rightarrow 2x = -4 \Rightarrow x = -2$ is a vertical line intersecting the x-axis at (–2, 0).



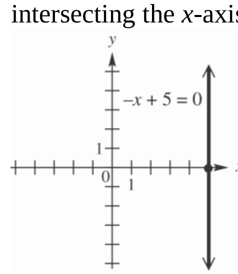
domain: $\{-2\}$; range: $(-\infty, \infty)$

28. $-3x + 6 = 0 \Rightarrow -3x = -6 \Rightarrow x = 2$ is a vertical line intersecting the x-axis at (2, 0).



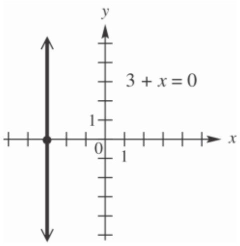
domain: {2}; range: $(-\infty, \infty)$

29. $-x + 5 = 0 \Rightarrow x = 5$ is a vertical line intersecting the x-axis at (5, 0).



domain: {5}; range: $(-\infty, \infty)$

30. $3 + x = 0 \Rightarrow x = -3$ is a vertical line intersecting the x-axis at $(-3, 0)$.



domain: $\{-3\}$; range: $(-\infty, \infty)$

31. $y = 5$ is a horizontal line with y-intercept 5.

Choice A resembles this.

32. $y = -5$ is a horizontal line with y-intercept -5.

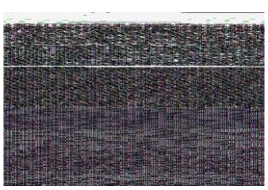
Choice C resembles this.

33. $x = 5$ is a vertical line with x-intercept 5.

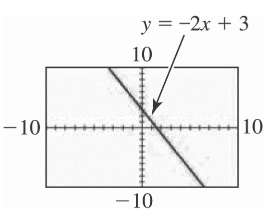
Choice D resembles this.

34. $x = -5$ is a vertical line with x-intercept -5. Choice B resembles this.

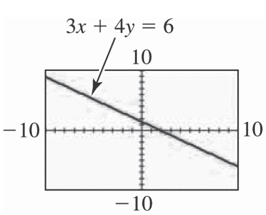
35.



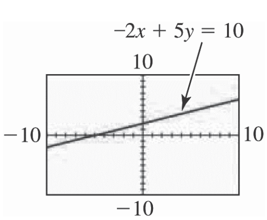
36.



37.



38.



40. The pitch or slope is $\frac{1}{4}$. If the rise is 4 feet then $\frac{1}{4} = \frac{\text{rise}}{\text{run}} = \frac{4}{x}$ or $x = 16$ feet. So 16 feet in the horizontal direction corresponds to a rise of 4 feet.

41. Through (2, –1) and (–3, –3)
Let $x_1 = 2,\ y_1 = -1,\ x_2 = -3,$ and $y_2 = -3.$

Then rise $= \Delta y = -3 - (-1) = -2$ and run $= \Delta x = -3 - 2 = -5.$

The slope is $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{-2}{-5} = \frac{2}{5}.$

42. Through (−3, 4) and (2, −8)

Let $x_1 = -3,\ y_1 = 4,\ x_2 = 2,$ and $y_2 = -8.$

Then rise $= \Delta y = -8 - 4 = -12$ and run $= \Delta x = 2 - (-3) = 5.$

The slope is $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{-12}{5} = -\frac{12}{5}.$

43. Through (5, 8) and (3, 12)

Let $x_1 = 5,\ y_1 = 8,\ x_2 = 3,$ and $y_2 = 12.$

Then rise $= \Delta y = 12 - 8 = 4$ and run $= \Delta x = 3 - 5 = -2.$

The slope is $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{4}{-2} = -2.$

44. Through (5, –3) and (1, –7)
Let $x_1 = 5, y_1 = -3,\ x_2 = 1,$ and $y_2 = -7.$

Then rise $= \Delta y = -7 - (-3) = -4$ and run $= \Delta x = 1 - 5 = -4.$

The slope is $m = \frac{\Delta y}{\Delta x} = \frac{-4}{-4} = 1.$

45. Through (5, 9) and (–2, 9)
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 9}{-2 - 5} = \frac{0}{-7} = 0$$

46. Through (–2, 4) and (6, 4)
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{6 - (-2)} = \frac{0}{8} = 0$$

39. The rise is 2.5 feet while the run is 10 feet so the slope is $\frac{2.5}{10} = 0.25 = 25\% = \frac{1}{4}$. So A = 0.25, C = $\frac{2.5}{10}$, D = 25%, and E = $\frac{1}{4}$ are all expressions of the slope.

47. Horizontal, through (5, 1)
The slope of every horizontal line is zero, so $m = 0$.

48. Horizontal, through (3, 5)
The slope of every horizontal line is zero, so $m = 0$.

49. Vertical, through (4, –7)

The slope of every vertical line is undefined; m is undefined.

50. Vertical, through (–8, 5)
The slope of every vertical line is undefined; *m* is undefined.

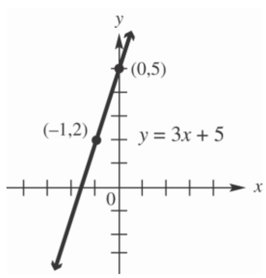
51. (a) $y = 3x + 5$

Find two ordered pairs that are solutions to the equation.
If $x = 0$, then $y = 3(0) + 5 \Rightarrow y = 5$.

If $x = -1$, then
$y = 3(-1) + 5 \Rightarrow y = -3 + 5 \Rightarrow y = 2.$
Thus two ordered pairs are (0, 5) and (−1, 2)

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{-1-0} = \frac{-3}{-1} = 3.$$

(b)



52. $y = 2x - 4$

Find two ordered pairs that are solutions to the equation. If $x = 0$, then $y = 2(0) - 4 \Rightarrow$

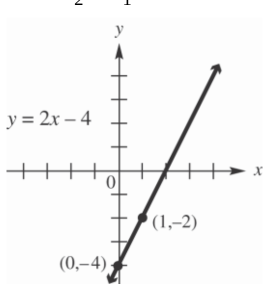
$y = -4$. If $x = 1$, then $y = 2(1) - 4 \Rightarrow$

$y = 2 - 4 \Rightarrow y = -2$. Thus two ordered pairs

are $(0, -4)$ and $(1, -2)$.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-4)}{1-0} = \frac{2}{1} = 2.$$

(b)

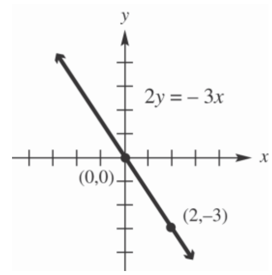


53. $2y = -3x$

Find two ordered pairs that are solutions to the equation. If $x = 0$, then $2y = 0 \Rightarrow y = 0$.
If $y = -3$, then $2(-3) = -3x \Rightarrow -6 = -3x \Rightarrow$

$x = 2.$ $(2, -3).$

(b)



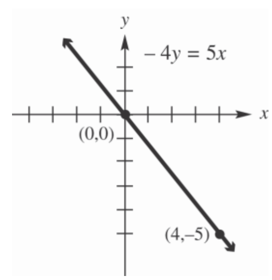
54. $-4y = 5x$

Find two ordered pairs that are solutions to the equation. If $x = 0$, then $-4y = 0 \Rightarrow y = 0$.
If $x = 4$, then $-4y = 5(4) \Rightarrow -4y = 20 \Rightarrow$

$y = -5$. Thus two ordered pairs are $(0, 0)$ and $(4, -5)$.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-0}{4-0} = -\frac{5}{4}.$$

(b)



55. $5x - 2y = 10$

Find two ordered pairs that are solutions to the equation. If $x = 0$, then $5(0) - 2y = 10 \Rightarrow$

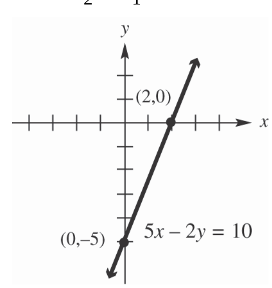
$y = -5$. If $y = 0$, then $5x - 2(0) = 10 \Rightarrow$

$5x = 10 \Rightarrow x = 2$.

Thus two ordered pairs are $(0, -5)$ and $(2, 0)$.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-5)}{2-0} = \frac{5}{2}.$$

(b)



Thus two ordered pairs are $(0, 0)$ and

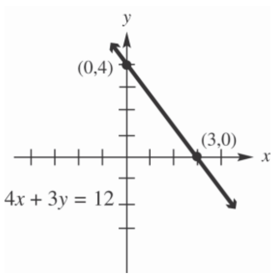
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{2 - 0} = -\frac{3}{2}.$$

56. $4x+3y=12$

Find two ordered pairs that are solutions to the equation. If $x=0$, then $4(0)+3y=12 \Rightarrow$ $3y=12 \Rightarrow y=4$. If $y=0$, then $4x+3(0)=12 \Rightarrow 4x=12 \Rightarrow x=3$. Thus two ordered pairs are $(0,4)$ and $(3,0)$.

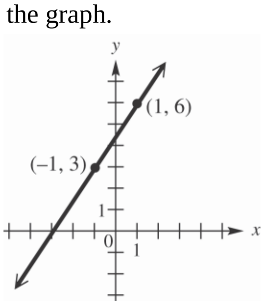
$$m=\frac{\text{rise}}{\text{run}}=\frac{y_2-y_1}{x_2-x_1}=\frac{0-4}{3-0}=-\frac{4}{3}.$$

(b)

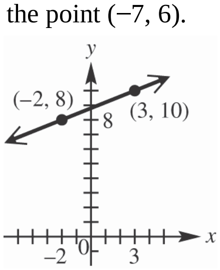


57. Through (–1, 3), $m=\frac{3}{2}$

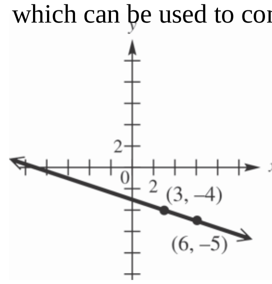
First locate the point (–1, 3). Because the slope is $\frac{3}{2}$, a change of 2 units horizontally (2 units to the right) produces a change of 3 units vertically (3 units up). This gives a second point, (1, 6), which can be used to complete the graph.



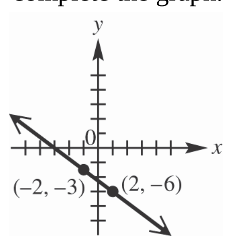
58. Through (–2, 8), $m=\frac{2}{5}$. Because the slope is $\frac{2}{5}$, a change of 5 units horizontally (to the right) produces a change of 2 units vertically (2 units up). This gives a second point (3, 10), which can be used to complete the graph. Alternatively, a change of 5 units to the left produces a change of 2 units down. This gives the point (–7, 6).



59. Through (3, –4), $m=-\frac{1}{3}$. First locate the point (3, –4). Because the slope is $-\frac{1}{3}$, a change of 3 units horizontally (3 units to the right) produces a change of –1 unit vertically (1 unit down). This gives a second point, (6, –5), which can be used to complete the graph.

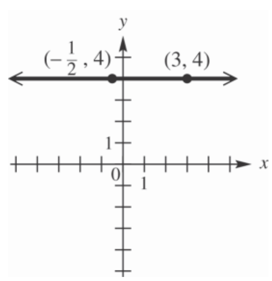


60. Through (–2, –3), $m=-\frac{3}{4}$. Because the slope is $-\frac{3}{4}=\frac{-3}{4}$, a change of 4 units horizontally (4 units to the right) produces a change of –3 units vertically (3 units down). This gives a second point (2, –6), which can be used to complete the graph.

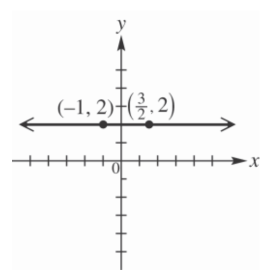


61. Through $\left(-\frac{1}{2}, 4\right)$, $m=0$.

The graph is the horizontal line through $\left(-\frac{1}{2}, 4\right)$.



Exercise 61

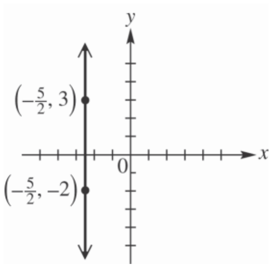


Exercise 62

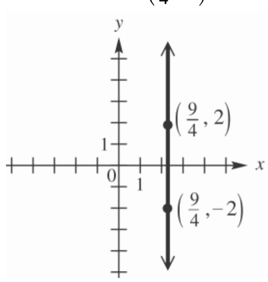
62. Through $\left(\frac{3}{2}, 2\right)$, $m=0$.

The graph is the horizontal line through $\left(\frac{3}{2}, 2\right)$.

63. Through $\left(-\frac{5}{2}, 3\right)$, undefined slope. The slope is undefined, so the line is vertical, intersecting the x-axis at $\left(-\frac{5}{2}, 0\right)$.



64. Through $\left(\frac{9}{4}, 2\right)$, undefined slope. The slope is undefined, so the line is vertical, intersecting the x-axis at $\left(\frac{9}{4}, 0\right)$.



65. The average rate of change is

$$m = \frac{f(b)-f(a)}{b-a}$$

$\frac{20-4}{0-4} = \frac{-16}{4} = -\$4$ (thousand) per year. The value of the machine is decreasing \$4000 each year during these years.

66. The average rate of change is

$$m = \frac{f(b)-f(a)}{b-a}$$

$\frac{200-0}{4-0} = \frac{200}{4} = \$50$ per month. The amount saved is increasing \$50 each month during these months.

67. The graph is a horizontal line, so the average rate of change (slope) is 0. The percent of pay raise is not changing—it is 3% each year.

68. The graph is a horizontal line, so the average rate of change (slope) is 0. That means that the number of named hurricanes remained the same, 10, for the four consecutive years shown.

69. $m = \frac{f(b)-f(a)}{b-a} = \frac{2562-5085}{2012-1980} = \frac{-2523}{32}$

$= -78.8$ thousand per year

The number of high school dropouts decreased by an average of 78.8 thousand per year from 1980 to 2012.

70. $m = \frac{f(b)-f(a)}{b-a} = \frac{1709-5302}{2013-2006}$

$= \frac{-3593}{7} \approx -\$513.29$

Sales of plasma flat-panel TVs decreased by an average of \$513.29 million per year from 2006 to 2013.

71. **(a)** The slope of –0.0167 indicates that the average rate of change of the winning time for the 5000 m run is 0.0167 min less. It is negative because the times are generally decreasing as time progresses.

(b) The Olympics were not held during World Wars I (1914−1919) and II (1939–1945).

(c) $y = -0.0167(2000) + 46.45 = 13.05$ min

The model predicts a winning time of 13.05 minutes. The times differ by 13.35 − 13.05 = 0.30 min.

72. **(a)** From the equation, the slope is 200.02. This means that the number of radio stations increased by an average of 200.02 per year.

(b) The year 2018 is represented by $x = 68$.

$y = 200.02(68) + 2727.7 = 16{,}329.06$

According to the model, there will be about 16,329 radio stations in 2018.

73. $\frac{f(2013)-f(2008)}{2013-2008} = \frac{335{,}652-270{,}334}{2013-2008}$

$= \frac{65{,}318}{5} = 13{,}063.6$

The average annual rate of change from 2008 through 2013 is about 13,064 thousand.

74. $\frac{f(2014)-f(2006)}{2014-2006} = \frac{3.74-4.53}{2014-2006}$

$= -\frac{0.79}{8} \approx -0.099$

The average annual rate of change from 2006 through 2014 is about –0.099.

75. (a) $m = \dfrac{f(b)-f(a)}{b-a} = \dfrac{56.3-138}{2013-2003}$
$= \dfrac{-81.7}{10} = -8.17$

The average rate of change was −8.17 thousand mobile homes per year.

(b) The negative slope means that the number of mobile homes decreased by an average of 8.17 thousand each year from 2003 to 2013.

76. $\dfrac{f(2013)-f(1991)}{2013-1991} = \dfrac{26.6-61.8}{2013-1991}$
$= -\dfrac{35.2}{22} = -1.6$

There was an average decrease of 1.6 births per thousand per year from 1991 through 2013.

77. (a) $C(x) = 10x + 500$

(b) $R(x) = 35x$

(c) $P(x) = R(x) - C(x)$
$= 35x - (10x + 500)$
$= 35x - 10x - 500 = 25x - 500$

(d) $C(x) = R(x)$
$10x + 500 = 35x$
$500 = 25x$
$20 = x$
20 units; do not produce

78. (a) $C(x) = 150x + 2700$

(b) $R(x) = 280x$

(c) $P(x) = R(x) - C(x)$
$= 280x - (150x + 2700)$
$= 280x - 150x - 2700$
$= 130x - 2700$

(d) $C(x) = R(x)$
$150x + 2700 = 280x$
$2700 = 130x$
$20.77 \approx x$ or 21 units

21 units; produce

79. (a) $C(x) = 400x + 1650$

(b) $R(x) = 305x$

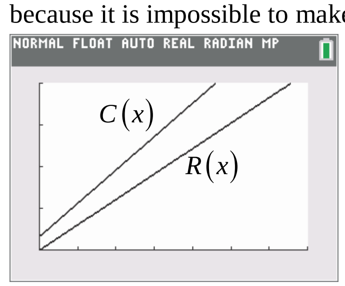
(c) $P(x) = R(x) - C(x)$
$= 305x - (400x + 1650)$
$= 305x - 400x - 1650$
$= -95x - 1650$

(d) $C(x) = R(x)$
$400x + 1650 = 305x$
$95x + 1650 = 0$
$95x = -1650$
$x \approx -17.37$ units

This result indicates a negative "break-even point," but the number of units produced must be a positive number. A calculator graph of the lines

$y_1 = C(x) = 400x + 1650$ and
$y_2 = R(x) = 305x$ in the window

[0, 70] × [0, 20000] or solving the inequality $305x < 400x + 1650$ will show that $R(x) < C(x)$ for all positive values of x (in fact whenever x is greater than −17.4). Do not produce the product because it is impossible to make a profit.



80. (a) $C(x) = 11x + 180$

(b) $R(x) = 20x$

(c) $P(x) = R(x) - C(x)$
$= 20x - (11x + 180)$
$= 20x - 11x - 180 = 9x - 180$

(d) $C(x) = R(x)$
$11x + 180 = 20x$
$180 = 9x$
$20 = x$
20 units; produce

81. $C(x) = R(x) \Rightarrow 200x + 1000 = 240x \Rightarrow$
$1000 = 40x \Rightarrow 25 = x$
The break-even point is 25 units.
$C(25) = 200(25) + 1000 = \$6000$ which is the same as $R(25) = 240(25) = \$6000$

82. $C(x)=R(x)\Rightarrow 220x+1000=240x\Rightarrow$
$1000=20x\Rightarrow 50=x$
The break-even point is 50 units instead of 25 units. The manager is not better off because twice as many units must be sold before beginning to show a profit.

83. The first two points are $A(0,-6)$ and $B(1,-3)$.

$$m=\frac{-3-(-6)}{1-0}=\frac{3}{1}=3$$

84. The second and third points are B(1, –3) and C(2, 0).

$$m=\frac{0-(-3)}{2-1}=\frac{3}{1}=3$$

85. If we use any two points on a line to find its slope, we find that the slope is <u>the same</u> in all cases.

86. The first two points are $A(0,-6)$ and $B(1,-3)$.

$$d(A,\ B)=\sqrt{(1-0)^2+[-3-(-6)]^2}$$
$$=\sqrt{1^2+3^2}=\sqrt{1+9}=\sqrt{10}$$

87. The second and fourth points are $B(1,-3)$ and $D(3, 3)$.

$$d(B,\ D)=\sqrt{(3-1)^2+[3-(-3)]^2}$$
$$=\sqrt{2^2+6^2}=\sqrt{4+36}$$
$$=\sqrt{40}=2\sqrt{10}$$

88. The first and fourth points are $A(0,-6)$ and $D(3, 3)$.

$$d(A,\ D)=\sqrt{(3-0)^2+[3-(-6)]^2}$$
$$=\sqrt{3^2+9^2}=\sqrt{9+81}$$
$$=\sqrt{90}=3\sqrt{10}$$

89. $\sqrt{10}+2\sqrt{10}=3\sqrt{10}$; The sum is $3\sqrt{10}$, which is equal to the answer in Exercise 88.

90. If points *A*, *B*, and *C* lie on a line in that order, then the distance between *A* and *B* added to the distance between <u>*B*</u> and <u>*C*</u> is equal to the distance between <u>*A*</u> and <u>*C*</u>.

91. The midpoint of the segment joining $A(0,-6)$ and $G(6, 12)$ has coordinates

$\left(\frac{0+6}{2},\frac{-6+12}{2}\right)=\left(\frac{6}{2},\frac{6}{2}\right)=(3,3)$. The midpoint is

$M(3, 3)$, which is the same as the middle entry in the table.

Chapter 2 Quiz
(Sections 2.1–2.4)

1. $d(A,B)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$$=\sqrt{(-8-(-4))^2+(-3-2)^2}$$
$$=\sqrt{(-4)^2+(-5)^2}=\sqrt{16+25}=\sqrt{41}$$

2. To find an estimate for 2006, find the midpoint of (2004, 6.55) and (2008, 6.97:

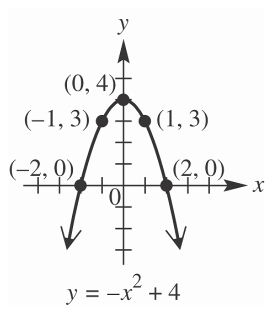
$$M=\left(\frac{2004+2008}{2},\frac{6.55+6.97}{2}\right)$$
$$=(2006,\ 6.76)$$

The estimated enrollment for 2006 was 6.76 million.
To find an estimate for 2010, find the midpoint of (2008, 6.97) and (2012, 7.50):

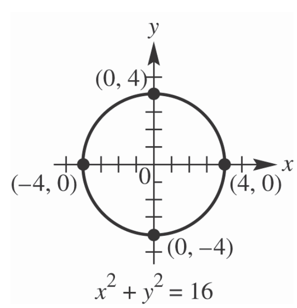
$$M=\left(\frac{2008+2012}{2},\frac{6.97+7.50}{2}\right)$$
$$=(2010,\ 7.235)$$

The estimated enrollment for 2006 was about 7.24 million.

3.



4.



5. $x^2+y^2-4x+8y+3=0$

Complete the square on *x* and *y* separately.
$(x^2-4x+4)+(y^2+8y+16)=-3+4+16\Rightarrow$

$(x-2)^2+(y+4)^2=17$

92. The midpoint of the segment joining $E(4, 6)$ and $F(5, 9)$ has coordinates $\left(\frac{4+5}{2}, \frac{6+9}{2}\right) = \left(\frac{9}{2}, \frac{15}{2}\right) = (4.5, 7.5)$. If the x-value 4.5 were in the table, the corresponding y-value would be 7.5.

The radius is 17 and the midpoint of the circle is $(2, -4)$.

6. From the graph, $f(-1)$ is 2.

7. Domain: $(-\infty, \infty)$; range: $[0, \infty)$

8. (a) The largest open interval over which *f* is decreasing is $(-\infty, -3)$.

(b) The largest open interval over which *f* is increasing is $(-3, \infty)$.

(c) There is no interval over which the function is constant.

9. (a) $m = \dfrac{11-5}{5-1} = \dfrac{6}{4} = \dfrac{3}{2}$

(b) $m = \dfrac{4-4}{-1-(-7)} = \dfrac{0}{6} = 0$

(c) $m = \dfrac{-4-12}{6-6} = \dfrac{-16}{0} \Rightarrow$ the slope is undefined.

10. The points to use are (2009, 10,602) and (2013, 15,884). The average rate of change is

$$\frac{15{,}884 - 10{,}602}{2013 - 2009} = \frac{5282}{4} = 1320.5$$

The average rate of change was 1320.5 thousand cars per year. This means that the number of new motor vehicles sold in the United States increased by an average of 1320.5 thousand per year from 2009 to 2013.

Section 2.5 Equations of Lines and Linear Models

1. The graph of the line $y - 3 = 4(x - 8)$ has slope 4 and passes through the point (8, 3).

2. The graph of the line $y = -2x + 7$ has slope –2 and *y*-intercept (0, 7).

3. The vertical line through the point (–4, 8) has equation $x = -4$.

4. The horizontal line through the point (–4, 8) has equation $y = 8$.

For exercises 5 and 6,

$6x + 7y = 0 \Rightarrow 7y = -6x \Rightarrow y = -\frac{6}{7}x$

5. Any line parallel to the graph of $6x + 7y = 0$ must have slope $-\frac{6}{7}$.

6. Any line perpendicular to the graph of $6x + 7y = 0$ must have slope $\frac{7}{6}$.

8. $4x + 3y = 12$ or $3y = -4x + 12$ or $y = -\frac{4}{3}x + 4$ is graphed in B. The slope is $-\frac{4}{3}$ and the *y*-intercept is (0, 4).

9. $y - (-1) = \frac{3}{2}(x - 1)$ is graphed in C. The slope is $\frac{3}{2}$ and a point on the graph is (1, –1).

10. $y = 4$ is graphed in A. $y = 4$ is a horizontal line with *y*-intercept (0, 4).

11. Through (1, 3), $m = -2$.
Write the equation in point-slope form.
$y - y_1 = m(x - x_1) \Rightarrow y - 3 = -2(x - 1)$
Then, change to standard form.
$y - 3 = -2x + 2 \Rightarrow 2x + y = 5$

12. Through (2, 4), m = –1
Write the equation in point-slope form.
$y - y_1 = m(x - x_1) \Rightarrow y - 4 = -1(x - 2)$
Then, change to standard form.
$y - 4 = -x + 2 \Rightarrow x + y = 6$

13. Through (–5, 4), $m = -\frac{3}{2}$
Write the equation in point-slope form.
$y - 4 = -\frac{3}{2}[x - (-5)]$
Change to standard form.
$2(y - 4) = -3(x + 5)$
$2y - 8 = -3x - 15$
$3x + 2y = -7$

14. Through $(-4, 3)$, $m = \frac{3}{4}$
Write the equation in point-slope form.
$y - 3 = \frac{3}{4}[x - (-4)]$

7. $y = \frac{1}{4}x + 2$ is graphed in D.

Change to standard form.

$$4(y-3)=3(x+4)$$
$$4y-12=3x+12$$
$$-3x+4y=24 \quad \text{or} \quad 3x-4y=-24$$

15. Through (–8, 4), undefined slope
Because undefined slope indicates a vertical line, the equation will have the form $x = a$.

The slope is $\frac{1}{4}$ and the y-intercept is (0, 2).

The equation of the line is $x = -8$.

16. Through (5, 1), undefined slope
This is a vertical line through (5, 1), so the equation is $x = 5$.

17. Through (5, −8), $m = 0$
This is a horizontal line through (5, −8), so the equation is $y = -8$.

18. Through (−3, 12), $m = 0$
This is a horizontal line through (−3, 12), so the equation is $y = 12$.

19. Through (–1, 3) and (3, 4)
First find m.
$$m=\frac{4-3}{3-(-1)}=\frac{1}{4}$$
Use either point and the point-slope form.
$$\begin{aligned} y-4&=\tfrac{1}{4}(x-3)\\ 4y-16&=x-3\\ -x+4y&=13\\ x-4y&=-13 \end{aligned}$$

20. Through (2, 3) and (−1, 2)
First find m.
$$m=\frac{2-3}{-1-2}=\frac{-1}{-3}=\frac{1}{3}$$
Use either point and the point-slope form.
$$\begin{aligned} y-3&=\tfrac{1}{3}(x-2)\\ 3y-9&=x-2\\ -x+3y&=7\\ x-3y&=-7 \end{aligned}$$

21. x-intercept (3, 0), y-intercept (0, –2)
The line passes through (3, 0) and (0, –2). Use these points to find m.
$$m=\frac{-2-0}{0-3}=\frac{2}{3}$$
Using slope-intercept form we have
$y=\frac{2}{3}x-2.$

22. x-intercept (–4, 0), y-intercept (0, 3)
The line passes through the points (–4, 0) and (0, 3). Use these points to find m.
$$m=\frac{3-0}{0-(-4)}=\frac{3}{4}$$
Using slope-intercept form we have
$y=\frac{3}{4}x+3.$

23. Vertical, through (–6, 4)
The equation of a vertical line has an equation of the form $x = a$. Because the line passes through (–6, 4), the equation is
$x = –6$. (Because the slope of a vertical line is undefined, this equation cannot be written in slope-intercept form.)

24. Vertical, through (2, 7)
The equation of a vertical line has an equation of the form $x = a$. Because the line passes through (2, 7), the equation is
$x = 2$. (Because the slope of a vertical line is undefined, this equation cannot be written in slope-intercept form.)

25. Horizontal, through (−7, 4)
The equation of a horizontal line has an equation of the form $y = b$. Because the line passes through (−7, 4), the equation is
$y = 4$.

26. Horizontal, through (−8, −2)
The equation of a horizontal line has an equation of the form $y = b$. Because the line passes through (−8, −2), the equation is
$y = −2$.

27. $m = 5$, $b = 15$
Using slope-intercept form, we have
$y=5x+15.$

28. $m = −2$, $b = 12$
Using slope-intercept form, we have
$y=-2x+12.$

29. Through (−2, 5), slope = −4
$$\begin{aligned} y-5&=-4(x-(-2))\\ y-5&=-4(x+2)\\ y-5&=-4x-8\\ y&=-4x-3 \end{aligned}$$

30. Through (4, −7), slope = −2
$$\begin{aligned} y-(-7)&=-2(x-4)\\ y+7&=-2x+8\\ y&=-2x+1 \end{aligned}$$

31. slope 0, y-intercept $\left(0,\frac{3}{2}\right)$
These represent $m=0$ and $b=\frac{3}{2}$.
Using slope-intercept form, we have
$y=(0)x+\frac{3}{2}\Rightarrow y=\frac{3}{2}.$

32. slope 0, y-intercept $\left(0,-\frac{5}{4}\right)$
These represent $m=0$ and $b=-\frac{5}{4}$.
Using slope-intercept form, we have
$y=(0)x-\frac{5}{4}\Rightarrow y=-\frac{5}{4}.$

33. The line $x + 2 = 0$ has x-intercept <u>(–2, 0)</u>. It <u>does not</u> have a y-intercept. The slope of his line is <u>undefined</u>. The line $4y = 2$ has y-intercept $\underline{\left(0,\frac{1}{2}\right)}$. It <u>does not</u> have an x-intercept. The slope of this line is <u>0</u>.

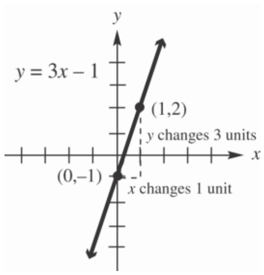
34. **(a)** The graph of $y = 3x + 2$ has a positive slope and a positive y-intercept. These conditions match graph D.

(b) The graph of $y = -3x + 2$ has a negative slope and a positive y-intercept. These conditions match graph B.

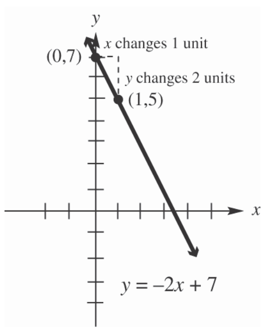
(c) The graph of $y = 3x - 2$ has a positive slope and a negative y-intercept. These conditions match graph A.

(d) The graph of $y = -3x - 2$ has a negative slope and a negative y-intercept. These conditions match graph C.

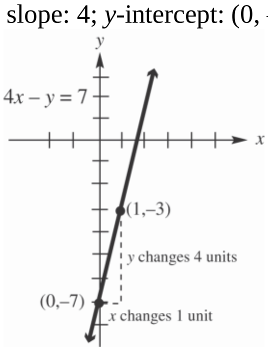
35. $y = 3x - 1$
This equation is in the slope-intercept form, $y = mx + b$.
slope: 3;
y-intercept: (0, –1)



36. $y = -2x + 7$
slope: –2;
y-intercept: (0, 7)



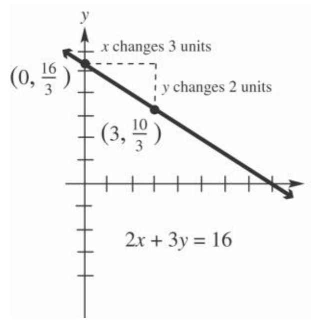
37. $4x - y = 7$
Solve for y to write the equation in slope-intercept form.
$-y = -4x + 7 \Rightarrow y = 4x - 7$
slope: 4; y-intercept: (0, –7)



38. $2x + 3y = 16$
Solve the equation for y to write the equation in slope-intercept form.

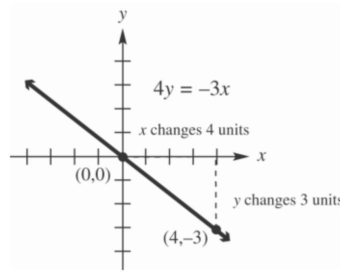
$3y = -2x + 16 \Rightarrow y = -\frac{2}{3}x + \frac{16}{3}$

slope: $-\frac{2}{3}$; y-intercept: $\left(0, \frac{16}{3}\right)$



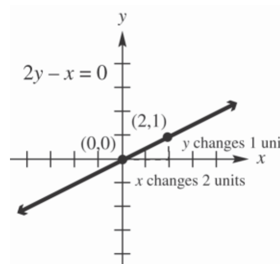
39. $4y = -3x \Rightarrow y = -\frac{3}{4}x$ or $y = -\frac{3}{4}x + 0$

slope: $-\frac{3}{4}$; y-intercept (0, 0)



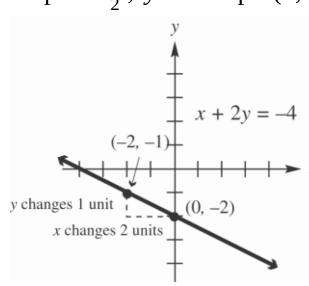
40. $2y = x \Rightarrow y = \frac{1}{2}x$ or $y = \frac{1}{2}x + 0$

slope is $\frac{1}{2}$; y-intercept: (0, 0)



41. $x + 2y = -4$
Solve the equation for y to write the equation in slope-intercept form.
$2y = -x - 4 \Rightarrow y = -\frac{1}{2}x - 2$

slope: $-\frac{1}{2}$; y-intercept: (0, −2)

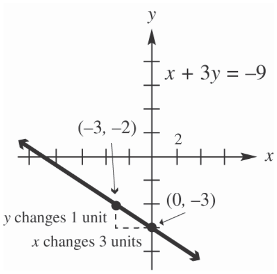


42. $x+3y=-9$

Solve the equation for y to write the equation in slope-intercept form.

$3y=-x-9 \Rightarrow y=-\frac{1}{3}x-3$

slope: $-\frac{1}{3}$; y-intercept: (0,−3)

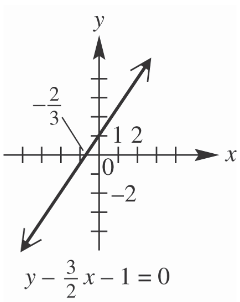


43. $y-\frac{3}{2}x-1=0$

Solve the equation for y to write the equation in slope-intercept form.

$y-\frac{3}{2}x-1=0 \Rightarrow y=\frac{3}{2}x+1$

slope: $\frac{3}{2}$; y-intercept: (0, 1)



44. **(a)** Use the first two points in the table, $A(-2,-11)$ and $B(-1,-8)$.

$$m=\frac{-8-(-11)}{-1-(-2)}=\frac{3}{1}=3$$

(b) When $x=0$, $y=-5$. The y-intercept is (0, –5).

(c) Substitute 3 for m and –5 for b in the slope-intercept form.

$y=mx+b \Rightarrow y=3x-5$

45. **(a)** The line falls 2 units each time the x value increases by 1 unit. Therefore the slope is −2. The graph intersects the y-axis at the point (0, 1) and intersects the x-axis at $\left(\frac{1}{2},0\right)$, so the y-intercept is (0, 1) and the x-intercept is $\left(\frac{1}{2}, 0\right)$.

(b) An equation defining f is $y=-2x+1$.

46. **(a)** The line rises 2 units each time the x value increases by 1 unit. Therefore the slope is 2. The graph intersects the y-axis at the point (0, −1) and intersects the x-axis at $\left(\frac{1}{2},0\right)$, so the y-intercept is (0, −1) and the x-intercept is $\left(\frac{1}{2}, 0\right)$.

(b) An equation defining f is $y=2x-1$.

47. **(a)** The line falls 1 unit each time the x value increases by 3 units. Therefore the slope is $-\frac{1}{3}$. The graph intersects the y-axis at the point (0, 2), so the y-intercept is (0, 2). The graph passes through (3, 1) and will fall 1 unit when the x value increases by 3, so the x-intercept is (6, 0).

(b) An equation defining f is $y=-\frac{1}{3}x+2$.

48. **(a)** The line rises 3 units each time the x value increases by 4 units. Therefore the slope is $\frac{3}{4}$. The graph intersects the y-axis at the point (0, −3) and intersects the x-axis at (4, 0), so the y-intercept is (0, −3) and the x-intercept is 4.

(b) An equation defining f is $y=\frac{3}{4}x-3$.

49. **(a)** The line falls 200 units each time the x value increases by 1 unit. Therefore the slope is −200. The graph intersects the y-axis at the point (0, 300) and intersects the x-axis at $\left(\frac{3}{2},0\right)$, so the y-intercept is (0, 300) and the x-intercept is $\left(\frac{3}{2}, 0\right)$.

(b) An equation defining f is $y=-200x+300$.

50. **(a)** The line rises 100 units each time the x value increases by 5 units. Therefore the slope is 20. The graph intersects the y-axis at the point (0, −50) and intersects the x-axis at $\left(\frac{5}{2},0\right)$, so the y-intercept is (0, −50) and the x-intercept is $\left(\frac{5}{2}, 0\right)$.

(b) An equation defining f is $y=20x-50$.

51. (a) through (–1, 4), parallel to $x + 3y = 5$
Find the slope of the line $x + 3y = 5$ by writing this equation in slope-intercept form.
$x+3y=5 \Rightarrow 3y=-x+5 \Rightarrow$
$y=-\frac{1}{3}x+\frac{5}{3}$
The slope is $-\frac{1}{3}$.

Because the lines are parallel, $-\frac{1}{3}$ is also the slope of the line whose equation is to be found. Substitute $m=-\frac{1}{3}$, $x_1=-1$, and $y_1=4$ into the point-slope form.

$$y-y_1=m(x-x_1)$$
$$y-4=-\frac{1}{3}[x-(-1)]$$
$$y-4=-\frac{1}{3}(x+1)$$
$$3y-12=-x-1 \Rightarrow x+3y=11$$

(b) Solve for y.
$3y=-x+11 \Rightarrow y=-\frac{1}{3}x+\frac{11}{3}$

52. (a) through (3, –2), parallel to $2x - y = 5$
Find the slope of the line $2x - y = 5$ by writing this equation in slope-intercept form.
$2x-y=5 \Rightarrow -y=-2x+5 \Rightarrow$
$y=2x-5$
The slope is 2. Because the lines are parallel, the slope of the line whose equation is to be found is also 2.

Substitute $m = 2$, $x_1=3$, and $y_1=-2$ into the point-slope form.
$y-y_1=m(x-x_1) \Rightarrow$

$y+2=2(x-3) \Rightarrow y+2=2x-6 \Rightarrow$
$-2x+y=-8$ or $2x-y=8$

(b) Solve for y. $y = 2x - 8$

53. (a) through (1, 6), perpendicular to $3x + 5y = 1$
Find the slope of the line $3x + 5y = 1$ by writing this equation in slope-intercept form.
$3x+5y=1 \Rightarrow 5y=-3x+1 \Rightarrow$
$y=-\frac{3}{5}x+\frac{1}{5}$
This line has a slope of $-\frac{3}{5}$. The slope of any line perpendicular to this line is $\frac{5}{3}$, because $-\frac{3}{5}\left(\frac{5}{3}\right)=-1.$ Substitute $m=\frac{5}{3}$,

$$y-6=\frac{5}{3}(x-1)$$
$$3(y-6)=5(x-1)$$
$$3y-18=5x-5$$
$$-13=5x-3y \text{ or } 5x-3y=-13$$

(b) Solve for y.
$3y=5x+13 \Rightarrow y=\frac{5}{3}x+\frac{13}{3}$

54. (a) through (–2, 0), perpendicular to $8x – 3y = 7$
Find the slope of the line $8x – 3y = 7$ by writing the equation in slope-intercept form.
$8x-3y=7 \Rightarrow -3y=-8x+7 \Rightarrow$

$y=\frac{8}{3}x-\frac{7}{3}$
This line has a slope of $\frac{8}{3}$. The slope of any line perpendicular to this line is $-\frac{3}{8}$, because $\frac{8}{3}\left(-\frac{3}{8}\right)=-1.$

Substitute $m=-\frac{8}{3}$, $x_1=-2$, and $y_1=0$ into the point-slope form.
$$y-0=-\frac{3}{8}(x+2)$$
$$8y=-3(x+2)$$
$$8y=-3x-6 \Rightarrow 3x+8y=-6$$

(b) Solve for y.
$8y=-3x-6 \Rightarrow y=-\frac{3}{8}x-\frac{6}{8} \Rightarrow$
$y=-\frac{3}{8}x-\frac{3}{4}$

55. (a) through (4, 1), parallel to $y = −5$
Because $y = −5$ is a horizontal line, any line parallel to this line will be horizontal and have an equation of the form $y = b$. Because the line passes through (4, 1), the equation is $y = 1$.

(b) The slope-intercept form is $y = 1$.

56. (a) through $(-2,-2)$, parallel to $y = 3$.
Because $y = 3$ is a horizontal line, any line parallel to this line will be horizontal and have an equation of the form $y = b$. Because the line passes through $(-2,-2)$, the equation is $y = –2$.

(b) The slope-intercept form is $y = −2$

57. (a) through (–5, 6), perpendicular to $x = –2$.

Because $x = -2$ is a vertical line, any line perpendicular to this line will be

$x_1 = 1$, and $y_1 = 6$ into the point-slope form.

horizontal and have an equation of the form $y = b$. Because the line passes through (–5, 6), the equation is $y = 6$.

(b) The slope-intercept form is $y = 6$.

58. (a) Through $(4, -4)$, perpendicular to $x = 4$

Because $x = 4$ is a vertical line, any line perpendicular to this line will be horizontal and have an equation of the form $y = b$. Because the line passes through $(4, -4)$, the equation is $y = -4$.

(b) The slope-intercept form is $y = -4$.

59. (a) Find the slope of the line $3y + 2x = 6$.

$$3y + 2x = 6 \Rightarrow 3y = -2x + 6 \Rightarrow$$
$$y = -\frac{2}{3}x + 2$$

Thus, $m = -\frac{2}{3}$. A line parallel to $3y + 2x = 6$ also has slope $-\frac{2}{3}$.

Solve for k using the slope formula.

$$\frac{2-(-1)}{k-4} = -\frac{2}{3}$$
$$\frac{3}{k-4} = -\frac{2}{3}$$
$$3(k-4)\left(\frac{3}{k-4}\right) = 3(k-4)\left(-\frac{2}{3}\right)$$
$$9 = -2(k-4)$$
$$9 = -2k + 8$$
$$2k = -1 \Rightarrow k = -\frac{1}{2}$$

(b) Find the slope of the line $2y - 5x = 1$.

$$2y - 5x = 1 \Rightarrow 2y = 5x + 1 \Rightarrow$$
$$y = \frac{5}{2}x + \frac{1}{2}$$

Thus, $m = \frac{5}{2}$. A line perpendicular to $2y - 5x = 1$ will have slope $-\frac{2}{5}$, because

$$\frac{5}{2}\left(-\frac{2}{5}\right) = -1.$$

Solve this equation for k.

$$\frac{3}{k-4} = -\frac{2}{5}$$
$$5(k-4)\left(\frac{3}{k-4}\right) = 5(k-4)\left(-\frac{2}{5}\right)$$
$$15 = -2(k-4)$$
$$15 = -2k + 8$$
$$2k = -7 \Rightarrow k = -\frac{7}{2}$$

60. (a) Find the slope of the line $2x - 3y = 4$.

$$2x - 3y = 4 \Rightarrow -3y = -2x + 4 \Rightarrow$$
$$y = \frac{2}{3}x - \frac{4}{3}$$

Thus, $m = \frac{2}{3}$. A line parallel to $2x - 3y = 4$ also has slope $\frac{2}{3}$. Solve for r using the slope formula.

$$\frac{r-6}{-4-2} = \frac{2}{3} \Rightarrow \frac{r-6}{-6} = \frac{2}{3} \Rightarrow$$
$$-6\left(\frac{r-6}{-6}\right) = -6\left(\frac{2}{3}\right) \Rightarrow$$
$$r - 6 = -4 \Rightarrow r = 2$$

(b) Find the slope of the line $x + 2y = 1$.

$$x + 2y = 1 \Rightarrow 2y = -x + 1 \Rightarrow$$
$$y = -\frac{1}{2}x + \frac{1}{2}$$

Thus, $m = -\frac{1}{2}$. A line perpendicular to the line $x + 2y = 1$ has slope 2, because $-\frac{1}{2}(2) = -1$. Solve for r using the slope formula.

$$\frac{r-6}{-4-2} = 2 \Rightarrow \frac{r-6}{-6} = 2 \Rightarrow$$
$$r - 6 = -12 \Rightarrow r = -6$$

61. (a) First find the slope using the points $(0, 6312)$ and $(3, 7703)$.

$$m = \frac{7703 - 6312}{3 - 0} = \frac{1391}{3} \approx 463.67$$

The y-intercept is $(0, 6312)$, so the equation of the line is $y = 463.67x + 6312$.

(b) The value $x = 4$ corresponds to the year 2013.

$$y = 463.67(4) + 6312 = 8166.68$$

The model predicts that average tuition and fees were \$8166.68 in 2013. This is \$96.68 more than the actual amount.

62. (a) First find the slope using the points $(0, 6312)$ and $(2, 7136)$.

$$m = \frac{7136 - 6312}{2 - 0} = \frac{824}{2} = 412$$

The y-intercept is $(0, 6312)$, so the equation of the line is $y = 412x + 6312$.

(b) The value $x = 4$ corresponds to the year 2013.

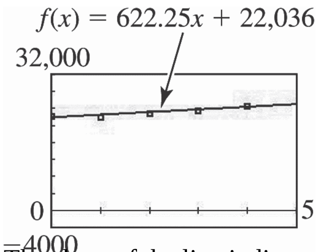
$$y = 412(4) + 6312 = 7960$$

The model predicts that average tuition and fees were \$7960 in 2013. This is \$110 less than the actual amount.

63. (a) First find the slope using the points (0, 22036) and (4, 24525).

$$m = \frac{24525 - 22036}{4 - 0} = \frac{2489}{4} = 622.25$$

The y-intercept is (0, 22036), so the equation of the line is $y = 622.25x + 22,036.$



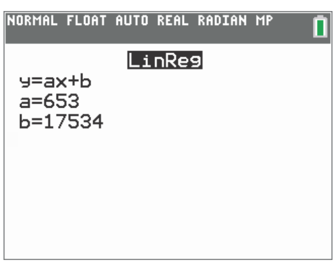
The slope of the line indicates that the average tuition increase is about $622 per year from 2009 through 2013.

(b) The year 2012 corresponds to $x = 3$.

$$y = 622.25(3) + 22,036 = 23,902.75$$

According to the model, average tuition and fees were $23,903 in 2012. This is $443 more than the actual amount $23,460.

(c) Using the linear regression feature, the equation of the line of best fit is $y = 653x + 21,634.$

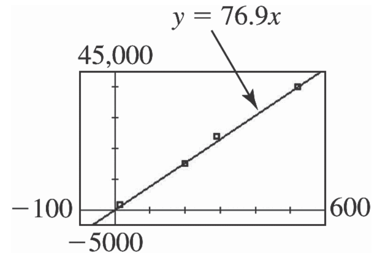


64. (a) See the graph in the answer to part (b).There appears to be a linear relationship between the data. The farther the galaxy is from Earth, the faster it is receding.

(b) Using the points (520, 40,000) and (0, 0), we obtain

$$m = \frac{40,000 - 0}{520 - 0} = \frac{40,000}{520} \approx 76.9.$$

The equation of the line through these two points is $y = 76.9x$.



(c) $76.9x = 60,000$

$$x = \frac{60,000}{76.9} \Rightarrow x \approx 780$$

According to the model, the galaxy Hydra is approximately 780 megaparsecs away.

(d) $A = \dfrac{9.5 \times 10^{11}}{m}$

$$A = \frac{9.5 \times 10^{11}}{76.9} \approx 1.235 \times 10^{10} \approx 12.35 \times 10^{9}$$

Using $m = 76.9$, we estimate that the age of the universe is approximately 12.35 billion years.

(e) $A = \dfrac{9.5 \times 10^{11}}{50} = 1.9 \times 10^{10}$ or 19×10^{9}

$$A = \frac{9.5 \times 10^{11}}{100} = 9.5 \times 10^{9}$$

The range for the age of the universe is between 9.5 billion and 19 billion years.

65. (a) The ordered pairs are (0, 32) and (100, 212).

The slope is $m = \dfrac{212 - 32}{100 - 0} = \dfrac{180}{100} = \dfrac{9}{5}.$

Use $(x_1, y_1) = (0, 32)$ and $m = \frac{9}{5}$ in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 32 = \tfrac{9}{5}(x - 0)$$
$$y - 32 = \tfrac{9}{5}x$$
$$y = \tfrac{9}{5}x + 32 \Rightarrow F = \tfrac{9}{5}C + 32$$

(b)

$$F = \tfrac{9}{5}C + 32$$
$$5F = 9(C + 32)$$
$$5F = 9C + 160 \Rightarrow 9C = 5F - 160 \Rightarrow$$
$$9C = 5(F - 32) \Rightarrow C = \tfrac{5}{9}(F - 32)$$

(c)

$$F = C \Rightarrow F = \tfrac{5}{9}(F - 32) \Rightarrow$$
$$9F = 5(F - 32) \Rightarrow 9F = 5F - 160 \Rightarrow$$
$$4F = -160 \Rightarrow F = -40$$

$F = C$ when F is –40°.

66. (a) The ordered pairs are (0, 1) and (100, 3.92).
The slope is
$$m = \frac{3.92-1}{100-0} = \frac{2.92}{100} = 0.0292 \quad \text{and } b = 1.$$

Using slope-intercept form we have
$y = 0.0292x + 1$ or $p(x) = 0.0292x + 1$.

(b) Let $x = 60$.

$P(60) = 0.0292(60) + 1 = 2.752$
The pressure at 60 feet is approximately 2.75 atmospheres.

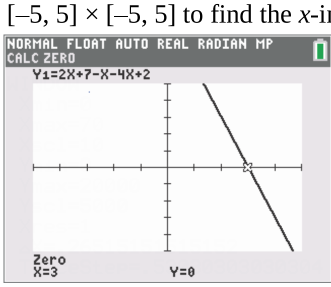
67. (a) Because we want to find C as a function of I, use the points (12026, 10089) and (14167, 11484), where the first component represents the independent variable, I. First find the slope of the line.
$$m = \frac{11484-10089}{14167-12026} = \frac{1395}{2141} \approx 0.6516$$
Now use either point, say (12026, 10089), and the point-slope form to find the equation.
$$C - 10089 = 0.6516(I - 12026)$$
$$C - 10089 \approx 0.6516I - 7836$$
$$C \approx 0.6516I + 2253$$

(b) Because the slope is 0.6516, the marginal

propensity to consume is 0.6516.

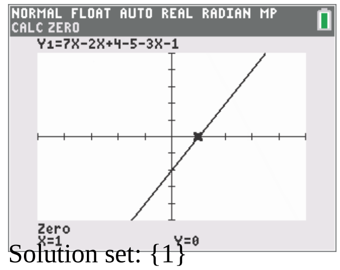
68. D is the only possible answer, because the x-intercept occurs when $y = 0$. We can see from the graph that the value of the x-intercept exceeds 10.

69. Write the equation as an equivalent equation with 0 on one side: $2x + 7 - x = 4x - 2 \Rightarrow$ $2x + 7 - x - 4x + 2 = 0$. Now graph $y = 2x + 7 - x - 4x + 2$ in the window $[-5, 5] \times [-5, 5]$ to find the x-intercept:



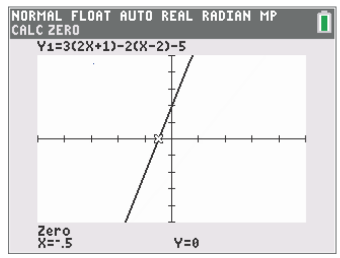
Solution set: {3}

70. Write the equation as an equivalent equation with 0 on one side: $7x - 2x + 4 - 5 = 3x + 1 \Rightarrow$ $7x - 2x + 4 - 5 - 3x - 1 = 0$. Now graph $y = 7x - 2x + 4 - 5 - 3x - 1$ in the window $[-5, 5] \times [-5, 5]$ to find the x-intercept:



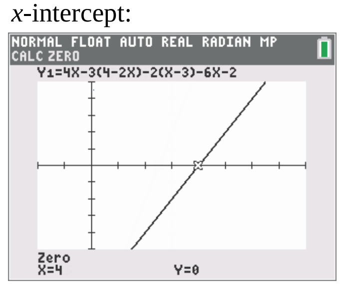
Solution set: {1}

71. Write the equation as an equivalent equation with 0 on one side: $3(2x+1) - 2(x-2) = 5 \Rightarrow$ $3(2x+1) - 2(x-2) - 5 = 0$. Now graph $y = 3(2x+1) - 2(x-2) - 5$ in the window $[-5, 5] \times [-5, 5]$ to find the x-intercept:



Solution set: $\left\{-\frac{1}{2}\right\}$ or $\{-0.5\}$

72. Write the equation as an equivalent equation with 0 on one side:
$4x - 3(4-2x) = 2(x-3) + 6x + 2 \Rightarrow$
$4x - 3(4-2x) - 2(x-3) - 6x - 2 = 0$.
Now graph $y = 4x - 3(4-2x) - 2(x-3) - 6x - 2$ in the window $[-2, 8] \times [-5, 5]$ to find the x-intercept:



Solution set: {4}

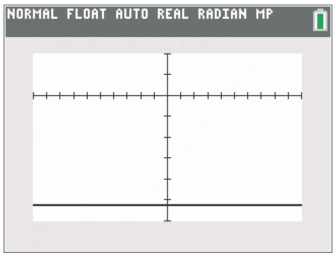
73. (a)
$$-2(x-5) = -x - 2$$
$$-2x + 10 = -x - 2$$
$$10 = x - 2$$
$$12 = x$$
Solution set: {12}

(b) Answers will vary. Sample answer: The solution does not appear in the standard viewing window x-interval [10, –10]. The minimum and maximum values must include 12.

74. Rewrite the equation as an equivalent equation with 0 on one side.

$$-3(2x+6) = -4x+8-2x$$
$$-6x-18-(-4x+8-2x) = 0$$

Now graph $y = -6x - 18 - (-4x + 8 - 2x)$ in the window $[-10, 10] \times [-30, 10]$.



The graph is a horizontal line that does not intersect the x-axis. Therefore, the solution set is $\varnothing$. We can verify this algebraically.

$$-3(2x+6) = -4x+8-2x$$
$$-6x-18 = -6x+8 \Rightarrow 0 = 26$$

Because this is a false statement, the solution set is $\varnothing$.

75. $A(-1, 4)$, $B(-2, -1)$, $C(1, 14)$

For A and B, $m = \dfrac{-1-4}{-2-(-1)} = \dfrac{-5}{-1} = 5$

For B and C, $m = \dfrac{14-(-1)}{1-(-2)} = \dfrac{15}{3} = 5$

For A and C, $m = \dfrac{14-4}{1-(-1)} = \dfrac{10}{2} = 5$

Since all three slopes are the same, the points are collinear.

76. $A(0, -7)$, $B(-3, 5)$, $C(2, -15)$

For A and B, $m = \dfrac{5-(-7)}{-3-0} = \dfrac{12}{-3} = -4$

77. $A(-1, -3)$, $B(-5, 12)$, $C(1, -11)$

For A and B, $m = \dfrac{12-(-3)}{-5-(-1)} = -\dfrac{15}{4}$

For B and C, $m = \dfrac{-11-12}{1-(-5)} = -\dfrac{23}{6}$

For A and C, $m = \dfrac{-11-(-3)}{1-(-1)} = -\dfrac{8}{2} = -4$

Since all three slopes are not the same, the points are not collinear.

78. $A(0, 9)$, $B(-3, -7)$, $C(2, 19)$

For A and B, $m = \dfrac{-7-9}{-3-0} = \dfrac{-16}{-3} = \dfrac{16}{3}$

For B and C, $m = \dfrac{19-(-7)}{2-(-3)} = \dfrac{26}{5}$

For A and C, $m = \dfrac{19-9}{2-0} = \dfrac{10}{2} = 5$

Because all three slopes are not the same, the points are not collinear.

79. $d(O, P) = \sqrt{(x_1-0)^2 + (m_1x_1-0)^2} = \sqrt{x_1^2 + m_1^2x_1^2}$

80. $d(O, Q) = \sqrt{(x_2-0)^2 + (m_2x_2-0)^2} = \sqrt{x_2^2 + m_2^2x_2^2}$

81. $d(P, Q) = \sqrt{(x_2-x_1)^2 + (m_2x_2-m_1x_1)^2}$

82. $[d(O, P)]^2 + [d(O, Q)]^2 = [d(P, Q)]^2$

$$\left[\sqrt{x_1^2 + m_1^2x_1^2}\right]^2 + \left[\sqrt{x_2^2 + m_2^2x_2^2}\right]^2 = \left[\sqrt{(x_2-x_1)^2 + (m_2x_2-m_1x_1)^2}\right]^2$$

$$\left(x_1^2 + m_1^2x_1^2\right) + \left(x_2^2 + m_2^2x_2^2\right) = \left(x_2-x_1\right)^2 + \left(m_2x_2-m_1x_1\right)^2$$

$$x_1^2 + m_1^2x_1^2 + x_2^2 + m_2^2x_2^2$$

For B and C, $m = \dfrac{-15-5}{2-(-3)} = \dfrac{-20}{5} = -4$

For A and C, $m = \dfrac{-15-(-7)}{2-0} = \dfrac{-8}{2} = -4$

Since all three slopes are the same, the points are collinear.

$$= x_2 \; - 2x_2x_1 + x_1 \; + m_2 \; x_2$$

$$-2m_1m_2x_1x_2 + m_1^2x_1^2$$

$$0 = -2x_2x_1 - 2m_1m_2x_1x_2 \Rightarrow$$

$$-2m_1m_2x_1x - 2x_2x_1 = 0$$

83. $-2m_1m_2x_1x_2 - 2x_1x_2 = 0$

$-2x_1x_2(m_1m_2 + 1) = 0$

84. $-2x_1x_2(m_1m_2 + 1) = 0$

Because $x_1 \neq 0$ and $x_2 \neq 0$, we have $m_1m_2 + 1 = 0$ implying that $m_1m_2 = -1$.

85. If two nonvertical lines are perpendicular, then the product of the slopes of these lines is –1.

Summary Exercises on Graphs, Circles, Functions, and Equations

1. $P(3, 5),\ Q(2, -3)$

(a) $d(P, Q) = \sqrt{(2-3)^2 + (-3-5)^2}$
$= \sqrt{(-1)^2 + (-8)^2}$
$= \sqrt{1 + 64} = \sqrt{65}$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{3+2}{2}, \frac{5+(-3)}{2}\right) = \left(\frac{5}{2}, \frac{2}{2}\right) = \left(\frac{5}{2}, 1\right).$$

(c) First find m: $m = \dfrac{-3-5}{2-3} = \dfrac{-8}{-1} = 8$

Use either point and the point-slope form.
$y - 5 = 8(x - 3)$

Change to slope-intercept form.

$y - 5 = 8x - 24 \Rightarrow y = 8x - 19$

2. $P(-1, 0),\ Q(4, -2)$

(a) $d(P, Q) = \sqrt{[4-(-1)]^2 + (-2-0)^2}$
$= \sqrt{5^2 + (-2)^2}$
$= \sqrt{25 + 4} = \sqrt{29}$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{-1+4}{2}, \frac{0+(-2)}{2}\right) = \left(\frac{3}{2}, \frac{-2}{2}\right) = \left(\frac{3}{2}, -1\right).$$

(c) First find m: $m = \dfrac{-2-0}{4-(-1)} = \dfrac{-2}{5} = -\dfrac{2}{5}$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{-2+3}{2}, \frac{2+2}{2}\right) = \left(\frac{1}{2}, \frac{4}{2}\right) = \left(\frac{1}{2}, 2\right).$$

(c) First find m: $m = \dfrac{2-2}{3-(-2)} = \dfrac{0}{5} = 0$

All lines that have a slope of 0 are horizontal lines. The equation of a horizontal line has an equation of the form $y = b$. Because the line passes through (3, 2), the equation is $y = 2$.

4. $P\left(2\sqrt{2}, \sqrt{2}\right),\ Q\left(\sqrt{2}, 3\sqrt{2}\right)$

(a) $d(P, Q) = \sqrt{\left(\sqrt{2} - 2\sqrt{2}\right)^2 + \left(3\sqrt{2} - \sqrt{2}\right)^2}$
$= \sqrt{\left(-\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2}$
$= \sqrt{2 + 8} = \sqrt{10}$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{2\sqrt{2}+\sqrt{2}}{2}, \frac{\sqrt{2}+3\sqrt{2}}{2}\right)$$
$$= \left(\frac{3\sqrt{2}}{2}, \frac{4\sqrt{2}}{2}\right) = \left(\frac{3\sqrt{2}}{2}, 2\sqrt{2}\right).$$

(c) First find m: $m = \dfrac{3\sqrt{2} - \sqrt{2}}{\sqrt{2} - 2\sqrt{2}} = \dfrac{2\sqrt{2}}{-\sqrt{2}} = -2$

Use either point and the point-slope form.

$y - \sqrt{2} = -2\left(x - 2\sqrt{2}\right)$

Change to slope-intercept form.
$y - \sqrt{2} = -2x + 4\sqrt{2} \Rightarrow y = -2x + 5\sqrt{2}$

5. $P(5, -1),\ Q(5, 1)$

(a) $d(P, Q) = \sqrt{(5-5)^2 + [1-(-1)]^2}$
$= \sqrt{0^2 + 2^2} = \sqrt{0+4} = \sqrt{4} = 2$

Use either point and the point-slope form.

$y-0=-\frac{2}{5}\left[x-(-1)\right]$

Change to slope-intercept form.

$$5y=-2(x+1)$$
$$5y=-2x-2$$
$$y=-\tfrac{2}{5}x-\tfrac{2}{5}$$

3. $P(-2, 2)$, $Q(3, 2)$

(a) $d(P, Q)=\sqrt{[3-(-2)]^2+(2-2)^2}$
$=\sqrt{5^2+0^2}=\sqrt{25+0}=\sqrt{25}=5$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{5+5}{2}, \frac{-1+1}{2}\right)=\left(\frac{10}{2}, \frac{0}{2}\right)=(5,0).$$

(c) First find m.

$$m = \frac{1-(-1)}{5-5} = \frac{2}{0} = \text{undefined}$$

All lines that have an undefined slope are vertical lines. The equation of a vertical line has an equation of the form $x = a$. The line passes through (5, 1), so the equation is $x = 5$. (Because the slope of a vertical line is undefined, this equation cannot be written in slope-intercept form.)

6. $P(1, 1),\ Q(-3, -3)$

(a)
$$d(P, Q) = \sqrt{(-3-1)^2 + (-3-1)^2} = \sqrt{(-4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{1+(-3)}{2}, \frac{1+(-3)}{2}\right) = \left(\frac{-2}{2}, \frac{-2}{2}\right) = (-1, -1).$$

(c) First find m: $m = \dfrac{-3-1}{-3-1} = \dfrac{-4}{-4} = 1$

Use either point and the point-slope form.

$$y - 1 = 1(x-1)$$

Change to slope-intercept form.

$$y - 1 = x - 1 \Rightarrow y = x$$

7. $P\left(2\sqrt{3}, 3\sqrt{5}\right),\ Q\left(6\sqrt{3}, 3\sqrt{5}\right)$

(a)
$$d(P, Q) = \sqrt{\left(6\sqrt{3} - 2\sqrt{3}\right)^2 + \left(3\sqrt{5} - 3\sqrt{5}\right)^2} = \sqrt{\left(4\sqrt{3}\right)^2 + 0} = \sqrt{48} = 4\sqrt{3}$$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{2\sqrt{3}+6\sqrt{3}}{2}, \frac{3\sqrt{5}+3\sqrt{5}}{2}\right)$$

(c) First find m: $m = \dfrac{3\sqrt{5} - 3\sqrt{5}}{6\sqrt{3} - 2\sqrt{3}} = \dfrac{0}{4\sqrt{3}} = 0$

All lines that have a slope of 0 are horizontal lines. The equation of a horizontal line has an equation of the form $y = b$. Because the line passes through $\left(2\sqrt{3}, 3\sqrt{5}\right)$, the equation is $y = 3\sqrt{5}$.

8. $P(0, -4),\ Q(3, 1)$

(a)
$$d(P, Q) = \sqrt{(3-0)^2 + [1-(-4)]^2} = \sqrt{3^2 + 5^2} = \sqrt{9+25} = \sqrt{34}$$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{0+3}{2}, \frac{-4+1}{2}\right) = \left(\frac{3}{2}, \frac{-3}{2}\right) = \left(\frac{3}{2}, -\frac{3}{2}\right).$$

(c) First find m: $m = \dfrac{1-(-4)}{3-0} = \dfrac{5}{3}$

Using slope-intercept form we have $y = \frac{5}{3}x - 4$.

9. Through $(-2, 1)$ and $(4, -1)$

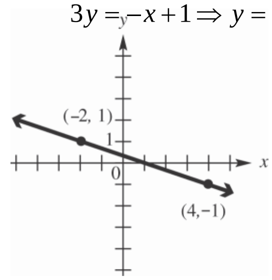
First find m: $m = \dfrac{-1-1}{4-(-2)} = \dfrac{-2}{6} = -\dfrac{1}{3}$

Use either point and the point-slope form.

$$y - (-1) = -\tfrac{1}{3}(x-4)$$

Change to slope-intercept form.

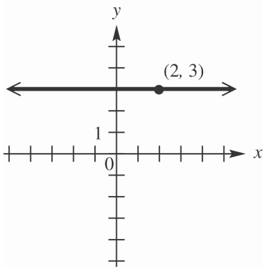
$$3(y+1) = -(x-4) \Rightarrow 3y + 3 = -x + 4 \Rightarrow$$
$$3y = -x + 1 \Rightarrow y = -\tfrac{1}{3}x + \tfrac{1}{3}$$



$$= \left(\frac{8\sqrt{3}}{2}, \frac{6\sqrt{5}}{2} \right) = \left(4\sqrt{3}, 3\sqrt{5} \right).$$

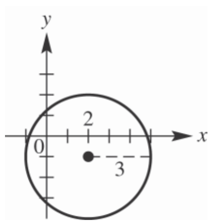
10. the horizontal line through (2, 3)

The equation of a horizontal line has an equation of the form $y = b$. Because the line passes through (2, 3), the equation is $y = 3$.



11. the circle with center (2, –1) and radius 3

$$(x-2)^2 + [y-(-1)]^2 = 3^2$$
$$(x-2)^2 + (y+1)^2 = 9$$

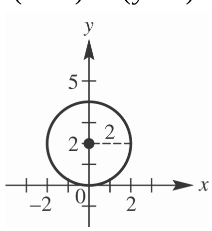


12. the circle with center (0, 2) and tangent to the

x-axis

The distance from the center of the circle to the x-axis is 2, so $r = 2$.

$$(x-0)^2 + (y-2)^2 = 2^2 \Rightarrow x^2 + (y-2)^2 = 4$$



13. the line through $(3, -5)$ with slope $-\frac{5}{6}$

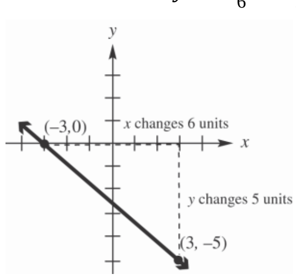
Write the equation in point-slope form.

$$y-(-5) = -\tfrac{5}{6}(x-3)$$

Change to standard form.

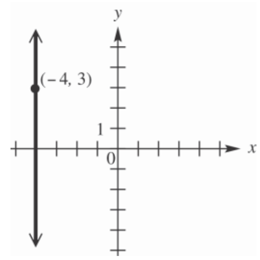
$$6(y+5) = -5(x-3) \Rightarrow 6y + 30 = -5x + 15$$

$$6y = -5x - 15 \Rightarrow y = -\tfrac{5}{6}x - \tfrac{15}{6}$$
$$y = -\tfrac{5}{6}x - \tfrac{5}{2}$$



14. the vertical line through (−4, 3)

The equation of a vertical line has an equation of the form $x = a$. Because the line passes through (−4, 3), the equation is $x = -4$.



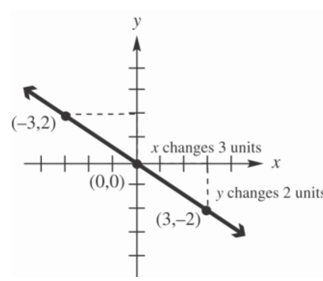
15. a line through (–3, 2) and parallel to the line $2x + 3y = 6$

First, find the slope of the line $2x + 3y = 6$ by writing this equation in slope-intercept form.

$$2x + 3y = 6 \Rightarrow 3y = -2x + 6 \Rightarrow y = -\tfrac{2}{3}x + 2$$

The slope is $-\frac{2}{3}$. Because the lines are parallel, $-\frac{2}{3}$ is also the slope of the line whose equation is to be found. Substitute $m = -\frac{2}{3}$, $x_1 = -3$, and $y_1 = 2$ into the point-slope form.

$$y - y_1 = m(x - x_1) \Rightarrow y - 2 = -\tfrac{2}{3}[x-(-3)] \Rightarrow$$
$$3(y-2) = -2(x+3) \Rightarrow 3y - 6 = -2x - 6 \Rightarrow$$
$$3y = -2x \Rightarrow y = -\tfrac{2}{3}x$$



16. a line through the origin and perpendicular to the line $3x - 4y = 2$

First, find the slope of the line $3x - 4y = 2$ by writing this equation in slope-intercept form.

$$3x - 4y = 2 \Rightarrow -4y = -3x + 2 \Rightarrow$$

$$y = \tfrac{3}{4}x - \tfrac{2}{4} \Rightarrow y = \tfrac{3}{4}x - \tfrac{1}{2}$$

This line has a slope of $\frac{3}{4}$. The slope of any line perpendicular to this line is $-\frac{4}{3}$, because $-\frac{4}{3}\left(\frac{3}{4}\right) = -1$. Using slope-intercept form we have $y = -\frac{4}{3}x + 0$ or $y = -\frac{4}{3}x$.

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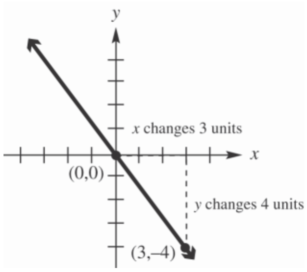

17. $x^2 - 4x + y^2 + 2y = 4$

Complete the square on x and y separately.

$$\left(x^2 - 4x\right) + \left(y^2 + 2y\right) = 4$$

$$\left(x^2 - 4x + 4\right) + \left(y^2 + 2y + 1\right) = 4 + 4 + 1$$

$$(x-2)^2 + (y+1)^2 = 9$$

Yes, it is a circle. The circle has its center at (2, −1) and radius 3.

18. $x^2 + 6x + y^2 + 10y + 36 = 0$

Complete the square on x and y separately.

$$\left(x^2 + 6x\right) + \left(y^2 + 10y\right) = -36$$

$$\left(x^2 + 6x + 9\right) + \left(y^2 + 10y + 25\right) = -36 + 9 + 25$$

$$(x+3)^2 + (y+5)^2 = -2$$

No, it is not a circle.

19. $x^2 - 12x + y^2 + 20 = 0$

Complete the square on x and y separately.

$$\left(x^2 - 12x\right) + y^2 = -20$$

$$\left(x^2 - 12x + 36\right) + y^2 = -20 + 36$$

$$(x-6)^2 + y^2 = 16$$

Yes, it is a circle. The circle has its center at (6, 0) and radius 4.

20. $x^2 + 2x + y^2 + 16y = -61$

Complete the square on x and y separately.

$$\left(x^2 + 2x\right) + \left(y^2 + 16y\right) = -61$$

$$\left(x^2 + 2x + 1\right) + \left(y^2 + 16y + 64\right) = -61 + 1 + 64$$

$$(x+1)^2 + (y+8)^2 = 4$$

21. $x^2 - 2x + y^2 + 10 = 0$

Complete the square on x and y separately.

$$\left(x^2 - 2x\right) + y^2 = -10$$

$$\left(x^2 - 2x + 1\right) + y^2 = -10 + 1$$

$$(x-1)^2 + y^2 = -9$$

No, it is not a circle.

22. $x^2 + y^2 - 8y - 9 = 0$

Complete the square on x and y separately.

$$x^2 + \left(y^2 - 8y\right) = 9$$

$$x^2 + \left(y^2 - 8y + 16\right) = 9 + 16$$

$$x^2 + (y-4)^2 = 25$$

Yes, it is a circle. The circle has its center at (0, 4) and radius 5.

23. The equation of the circle is $(x-4)^2 + (y-5)^2 = 4^2$.

Let $y = 2$ and solve for x:

$$(x-4)^2 + (2-5)^2 = 4^2 \Rightarrow$$

$$(x-4)^2 + (-3)^2 = 4^2 \Rightarrow (x-4)^2 = 7 \Rightarrow$$

$$x - 4 = \pm\sqrt{7} \Rightarrow x = 4 \pm \sqrt{7}$$

The points of intersection are $\left(4+\sqrt{7}, 2\right)$ and $\left(4-\sqrt{7}, 2\right)$

24. Write the equation in center-radius form by completing the square on x and y separately:

$$x^2 + y^2 - 10x - 24y + 144 = 0$$

$$\left(x^2 - 10x + \quad\right) + \left(y^2 - 24y + 144\right) = 0$$

$$(x^2 - 10x + 25) + (y^2 - 24y + 144) = 25$$

$$(x-5)^2 + (y-12)^2 = 25$$

The center of the circle is (5, 12) and the radius is 5.

Now use the distance formula to find the distance from the center (5, 12) to the origin:

$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$$= \sqrt{(5-0)^2 + (12-0)^2} = \sqrt{25 + 144} = 13$$

Yes, it is a circle. The circle has its center at $(-1, -8)$ and radius 2.

The radius is 5, so the shortest distance from the origin to the graph of the circle is $13 - 5 = 8$.

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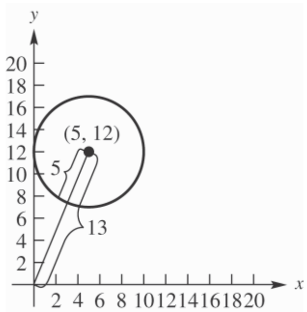


25. **(a)** The equation can be rewritten as

$$-4y = -x - 6 \Rightarrow y = \frac{1}{4}x + \frac{6}{4} \Rightarrow y = \frac{1}{4}x + \frac{3}{2}.$$

x can be any real number, so the domain is all real numbers and the range is also all real numbers.
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

(b) Each value of x corresponds to just one value of y. $x - 4y = -6$ represents a function.

$$y = \frac{1}{4}x + \frac{3}{2} \Rightarrow f(x) = \frac{1}{4}x + \frac{3}{2}$$

$$f(-2) = \frac{1}{4}(-2) + \frac{3}{2} = -\frac{1}{2} + \frac{3}{2} = \frac{2}{2} = 1$$

26. **(a)** The equation can be rewritten as

$y^2 - 5 = x$. y can be any real number.

Because the square of any real number is not negative, y^2 is never negative.

Taking the constant term into consideration, domain would be $[-5, \infty)$.
domain: $[-5, \infty)$; range: $(-\infty, \infty)$

(b) Because $(-4, 1)$ and $(-4, -1)$ both satisfy the relation, $y^2 - x = 5$ does not represent a function.

27. **(a)** $(x+2)^2 + y^2 = 25$ is a circle centered at $(-2, 0)$ with a radius of 5. The domain will start 5 units to the left of -2 and end

28. **(a)** The equation can be rewritten as

$-2y = -x^2 + 3 \Rightarrow y = \frac{1}{2}x^2 - \frac{3}{2}$. x can be any real number. Because the square of any real number is not negative, $\frac{1}{2}x^2$ is never negative. Taking the constant term into consideration, range would be $\left[-\frac{3}{2}, \infty\right)$.
domain: $(-\infty, \infty)$; range: $\left[-\frac{3}{2}, \infty\right)$

(b) Each value of x corresponds to just one value of y. $x^2 - 2y = 3$ represents a function.

$$y = \frac{1}{2}x^2 - \frac{3}{2} \Rightarrow f(x) = \frac{1}{2}x^2 - \frac{3}{2}$$

$$f(-2) = \frac{1}{2}(-2)^2 - \frac{3}{2} = \frac{1}{2}(4) - \frac{3}{2} = \frac{4}{2} - \frac{3}{2} = \frac{1}{2}$$

Section 2.6 Graphs of Basic Functions

1. The equation $f(x) = x^2$ matches graph E.

   The domain is $(-\infty, \infty)$. | |

2. The equation of $f(x) = x$ matches graph G.

   The function is increasing on $(0, \infty)$.

3. The equation $f(x) = x^3$ matches graph A.

   The range is $(-\infty, \infty)$.

4. Graph C is not the graph of a function. Its equation is $x = y^2$.

5. Graph F is the graph of the identity function.

   Its equation is $f(x) = x$.

6. The equation $f(x) = [\![x]\!]$ matches graph B.

   $f[\![1.5]\!] = 1$

7. The equation $f(x) = \sqrt[3]{x}$ matches graph H.

5 units to the right of –2. The domain will be [−2 − 5, 2 + 5] = [−7, 3]. The range will start 5 units below 0 and end 5 units above 0. The range will be [0 − 5, 0 + 5] = [−5, 5].

(b) Because (−2, 5) and (−2, −5) both satisfy the relation, $(x+2)^2 + y^2 = 25$ does not represent a function.

No, there is no interval over which the function is decreasing.

8. The equation of $f(x) = \sqrt{x}$ matches graph D.

The domain is $[0, \infty)$.

9. The graph in B is discontinuous at many points. Assuming the graph continues, the range would be $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

10. The graphs in E and G decrease over part of the domain and increase over part of the domain. They both increase over $(0,\infty)$ and decrease over $(-\infty,0)$.

11. The function is continuous over the entire domain of real numbers $(-\infty,\infty)$.

12. The function is continuous over the entire domain of real numbers $(-\infty,\infty)$.

13. The function is continuous over the interval $[0,\infty)$.

14. The function is continuous over the interval $(-\infty,0]$.

15. The function has a point of discontinuity at (3, 1). It is continuous over the interval $(-\infty,3)$ and the interval $(3,\infty)$.

16. The function has a point of discontinuity at $x = 1$. It is continuous over the interval $(-\infty,1)$ and the interval $(1,\infty)$.

17. $f(x)=\begin{cases} 2x & \text{if } x \le -1 \\ x-1 & \text{if } x > -1 \end{cases}$

(a) $f(-5) = 2(-5) = -10$

(b) $f(-1) = 2(-1) = -2$

(c) $f(0) = 0 - 1 = -1$

(d) $f(3) = 3 - 1 = 2$

18. $f(x)=\begin{cases} x-2 & \text{if } x < 3 \\ 5-x & \text{if } x \ge 3 \end{cases}$

(a) $f(-5) = -5 - 2 = -7$

(b) $f(-1) = -1 - 2 = -3$

(c) $f(0) = 0 - 2 = -2$

(d) $f(3) = 5 - 3 = 2$

19. $f(x)=\begin{cases} 2+x & \text{if } x < -4 \\ -x & \\ 3x & \end{cases}$

20. $f(x)=\begin{cases} -2x & \text{if } x < -3 \\ 3x-1 & \text{if } -3 \le x \le 2 \\ -4x & \text{if } x > 2 \end{cases}$

(a) $f(-5) = -2(-5) = 10$

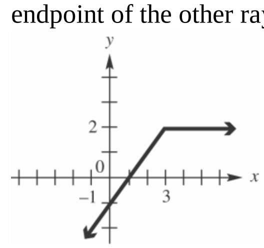
(b) $f(-1) = 3(-1) - 1 = -3 - 1 = -4$

(c) $f(0) = 3(0) - 1 = 0 - 1 = -1$

(d) $f(3) = -4(3) = -12$

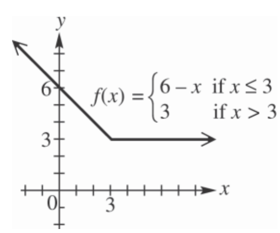
21. $f(x)=\begin{cases} x-1 & \text{if } x \le 3 \\ 2 & \text{if } x > 3 \end{cases}$

Draw the graph of $y = x - 1$ to the left of $x = 3$, including the endpoint at $x = 3$. Draw the graph of $y = 2$ to the right of $x = 3$, and note that the endpoint at $x = 3$ coincides with the endpoint of the other ray.



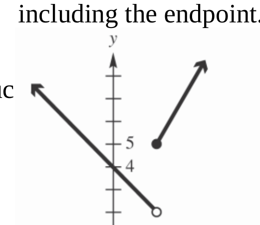
22. $f(x)=\begin{cases} 6-x & \text{if } x \le 3 \\ 3 & \text{if } x > 3 \end{cases}$

Graph the line $y = 6 - x$ to the left of $x = 3$, including the endpoint. Draw $y = 3$ to the right of $x = 3$. Note that the endpoint at $x = 3$ coincides with the endpoint of the other ray.



23. $f(x)=\begin{cases} 4-x & \text{if } x < 2 \\ 1+2x & \text{if } x \ge 2 \end{cases}$

Draw the graph of $y = 4 - x$ to the left of $x = 2$, but do not include the endpoint. Draw the graph of $y = 1 + 2x$ to the right of $x = 2$, including the endpoint.



i $-4 \le x \le 2$ if
f $x > 2$

(a) $f(-5) = 2 + (-5) = -3$

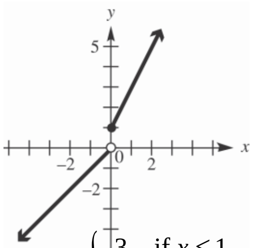
(b) $f(-1) = -(-1) = 1$

(c) $f(0) = -0 = 0$

(d) $f(3) = 3 \cdot 3 = 9$

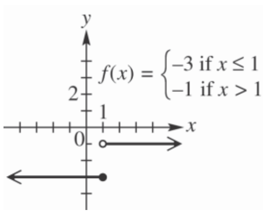
24. $f(x)=\begin{cases}2x+1 & \text{if } x\ge 0\\ x & \text{if } x<0\end{cases}$

Graph the line $y = 2x + 1$ to the right of $x = 0$, including the endpoint. Draw $y = x$ to the left of $x = 0$, but do not include the endpoint.



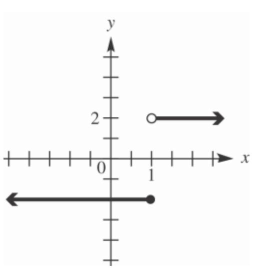
25. $f(x)=\begin{cases}-3 & \text{if } x\le 1\\ -1 & \text{if } x>1\end{cases}$

Graph the line $y = -3$ to the left of $x = 1$, including the endpoint. Draw $y = -1$ to the right of $x = 1$, but do not include the endpoint.

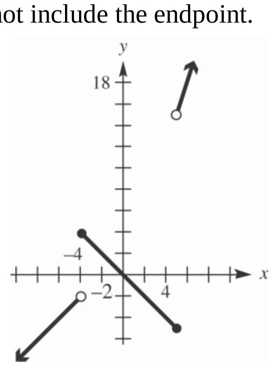


26. $f(x)=\begin{cases}-2 & \text{if } x\le 1\\ 2 & \text{if } x>1\end{cases}$

Graph the line $y = -2$ to the left of $x = 1$, including the endpoint. Draw $y = 2$ to the right of $x = 1$, but do not include the endpoint.



Exercise 26



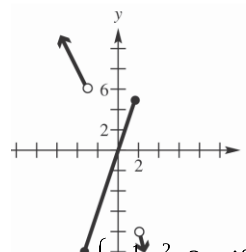
Exercise 27

27. $f(x)=\begin{cases}2+x & \text{if } x<-4\\ -x & \text{if } -4\le x\le 5\\ 3x & \text{if } x>5\end{cases}$

Draw the graph of $y = 2 + x$ to the left of –4, but do not include the endpoint at $x = 4$. Draw the graph of $y = -x$ between –4 and 5, including both endpoints. Draw the graph of

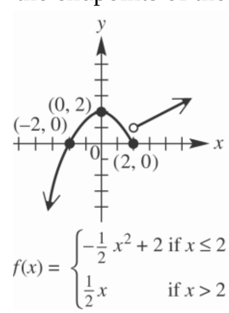
28. $f(x)=\begin{cases}-2x & \text{if } x<-3\\ 3x-1 & \text{if } -3\le x\le 2\\ -4x & \text{if } x>2\end{cases}$

Graph the line $y = -2x$ to the left of $x = -3$, but do not include the endpoint. Draw $y = 3x - 1$ between $x = -3$ and $x = 2$, and include both endpoints. Draw $y = -4x$ to the right of $x = 2$, but do not include the endpoint. Notice that the endpoints of the pieces do not coincide.



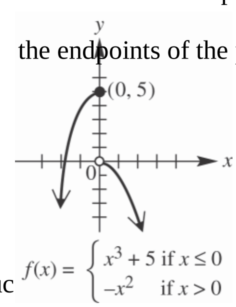
29. $f(x)=\begin{cases}-\frac{1}{2}x^2+2 & \text{if } x\le 2\\ \frac{1}{2}x & \text{if } x>2\end{cases}$

Graph the curve $y=-\frac{1}{2}x^2+2$ to the left of $x = 2$, including the endpoint at (2, 0). Graph the line $y=\frac{1}{2}x$ to the right of $x = 2$, but do not include the endpoint at (2, 1). Notice that the endpoints of the pieces do not coincide.



30. $f(x)=\begin{cases}x^3+5 & \text{if } x\le 0\\ -x^2 & \text{if } x>0\end{cases}$

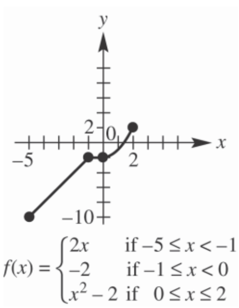
Graph the curve $y=x^3+5$ to the left of $x = 0$, including the endpoint at (0, 5). Graph the line $y=-x^2$ to the right of $x = 0$, but do not include the endpoint at (0, 0). Notice that the endpoints of the pieces do not coincide.



$y = 3x$ to the right of 5, but do not include the endpoint at $x = 5$.

31. $f(x)=\begin{cases} 2x & \text{if } -5\le x<-1 \\ -2 & \text{if } -1\le x<0 \\ x^2-2 & \text{if } 0\le x\le 2 \end{cases}$

Graph the line $y = 2x$ between $x = -5$ and $x = -1$, including the left endpoint at $(-5, -10)$, but not including the right endpoint at $(-1, -2)$. Graph the line $y = -2$ between $x = -1$ and $x = 0$, including the left endpoint at $(-1, -2)$ and not including the right endpoint at $(0, -2)$. Note that $(-1, -2)$ coincides with the first two sections, so it is included. Graph the curve $y = x^2 - 2$ from $x = 0$ to $x = 2$, including the endpoints at $(0, -2)$ and $(2, 2)$. Note that $(0, -2)$ coincides with the second two sections, so it is included. The graph ends at $x = -5$ and $x = 2$.

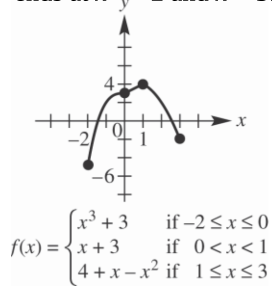


32. $f(x)=\begin{cases} 0.5x^2 & \text{if } -4\le x\le -2 \\ x & \text{if } -2<x<2 \\ x^2-4 & \text{if } 2\le x\le 4 \end{cases}$

Graph the curve $y = 0.5x^2$ between $x = -4$ and $x = -2$, including the endpoints at $(-4, 8)$ and $(-2, 2)$. Graph the line $y = x$ between $x = -2$ and $x = 2$, but do not include the endpoints at $(-2, -2)$ and $(2, 2)$. Graph the curve $y = x^2 - 4$ from $x = 2$ to $x = 4$, including the endpoints at $(2, 0)$ and $(4, 12)$. The graph ends at $x = -4$ and $x = 4$.

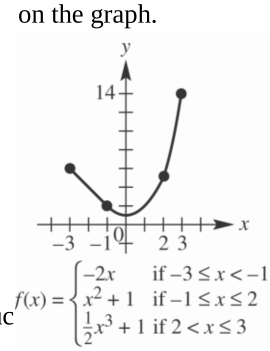
33. $f(x)=\begin{cases} x^3+3 & \text{if } -2\le x\le 0 \\ x+3 & \text{if } 0<x<1 \\ 4+x-x^2 & \text{if } 1\le x\le 3 \end{cases}$

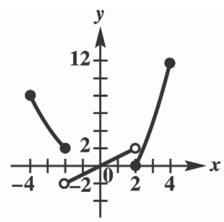
Graph the curve $y = x^3 + 3$ between $x = -2$ and $x = 0$, including the endpoints at $(-2, -5)$ and $(0, 3)$. Graph the line $y = x + 3$ between $x = 0$ and $x = 1$, but do not include the endpoints at $(0, 3)$ and $(1, 4)$. Graph the curve $y = 4 + x - x^2$ from $x = 1$ to $x = 3$, including the endpoints at $(1, 4)$ and $(3, -2)$. The graph ends at $x = -2$ and $x = 3$.



34. $f(x)=\begin{cases} -2x & \text{if } -3\le x<-1 \\ x^2+1 & \text{if } -1\le x\le 2 \\ \frac{1}{2}x^3+1 & \text{if } 2<x\le 3 \end{cases}$

Graph the curve $y = -2x$ to from $x = -3$ to $x = -1$, including the endpoint $(-3, 6)$, but not including the endpoint $(-1, 2)$. Graph the curve $y = x^2 + 1$ from $x = -1$ to $x = 2$, including the endpoints $(-1, 2)$ and $(2, 5)$. Graph the curve $y = \frac{1}{2}x^3 + 1$ from $x = 2$ to $x = 3$, including the endpoint $(3, 14.5)$ but not including the endpoint $(2, 5)$. Because the endpoints that are not included coincide with endpoints that are included, we use closed dots on the graph.





$$f(x) = \begin{cases} 0.5x^2 & \text{if } -4 \le x \le -2 \\ x & \text{if } -2 < x < 2 \\ x^2 - 4 & \text{if } 2 \le x \le 4 \end{cases}$$

35. The solid circle on the graph shows that the endpoint (0, –1) is part of the graph, while the open circle shows that the endpoint (0, 1) is not part of the graph. The graph is made up of parts of two horizontal lines. The function which fits this graph is

$$f(x) = \begin{cases} -1 & \text{if } x \le 0 \\ 1 & \text{if } x > 0. \end{cases}$$

domain: $(-\infty, \infty)$; range: {–1, 1}

36. We see that $y = 1$ for every value of x except $x = 0$, and that when $x = 0$, $y = 0$. We can write the function as

$$f(x) = \begin{cases} 1 & \text{if } x \ne 0 \\ 0 & \text{if } x = 0. \end{cases}$$

domain: $(-\infty, \infty)$; range: {0, 1}

37. The graph is made up of parts of two horizontal lines. The solid circle shows that the endpoint (0, 2) of the one on the left belongs to the graph, while the open circle shows that the endpoint (0, –1) of the one on the right does not belong to the graph. The function that fits this graph is

$$f(x) = \begin{cases} 2 & \text{if } x \le 0 \\ -1 & \text{if } x > 1. \end{cases}$$

domain: $(-\infty, 0] \cup (1, \infty)$; range: {–1, 2}

38. We see that $y = 1$ when $x \le -1$ and that $y = -1$ when $x > 2$. We can write the function as

$$f(x) = \begin{cases} 1 & \text{if } x \le -1 \\ -1 & \text{if } x > 2. \end{cases}$$

domain: $(-\infty, -1] \cup (2, \infty)$; range: {–1, 1}

39. For $x \le 0$, that piece of the graph goes through the points (−1, −1) and (0, 0). The slope is 1, so the equation of this piece is $y = x$. For $x > 0$, that piece of the graph is a horizontal line passing through (2, 2), so its equation is $y = 2$. We can write the function as

$$f(x) = \begin{cases} x & \text{if } x \le 0 \\ 2 & \text{if } x > 0 \end{cases}.$$

domain: $(-\infty, \infty)$ range: $(-\infty, 0] \cup \{2\}$

40. For $x < 0$, that piece of the graph is a horizontal line passing though (−3, −3), so the equation of this piece is $y = -3$. For $x \ge 0$, the curve passes through (1, 1) and (4, 2), so the equation of this piece is $y = \sqrt{x}$. We can

41. For $x < 1$, that piece of the graph is a curve passes through (−8, −2), (−1, −1) and (1, 1), so the equation of this piece is $y = \sqrt[3]{x}$. The right piece of the graph passes through (1, 2) and (2, 3). $m = \dfrac{2-3}{1-2} = 1$, and the equation of the line is $y - 2 = x - 1 \Rightarrow y = x + 1$. We can write the function as

$$f(x) = \begin{cases} \sqrt[3]{x} & \text{if } x < 1 \\ x + 1 & \text{if } x \ge 1 \end{cases}$$

domain: $(-\infty, \infty)$ range: $(-\infty, 1) \cup [2, \infty)$

42. For all values except $x = 2$, the graph is a line. It passes through (0, −3) and (1, −1). The slope is 2, so the equation is $y = 2x - 3$. At $x = 2$, the graph is the point (2, 3). We can write the function as

$$f(x) = \begin{cases} 3 & \text{if } x = 2 \\ 2x - 3 & \text{if } x \ne 2 \end{cases}.$$

domain: $(-\infty, \infty)$ range: $(-\infty, 1) \cup (1, \infty)$

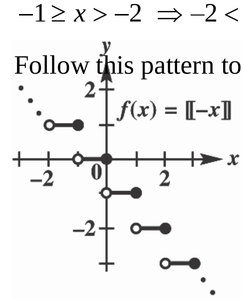
43. $f(x) = [\![-x]\!]$

Plot points.

x	$-x$	$f(x) = [\![-x]\!]$
–2	2	2
–1.5	1.5	1
–1	1	1
–0.5	0.5	0
0	0	0
0.5	–0.5	–1
1	–1	–1
1.5	–1.5	–2
2	–2	–2

More generally, to get $y = 0$, we need $0 \le -x < 1 \Rightarrow 0 \ge x > -1 \Rightarrow -1 < x \le 0$. To get $y = 1$, we need $1 \le -x < 2 \Rightarrow -1 \ge x > -2 \Rightarrow -2 < x \le -1$.

Follow this pattern to graph the step function.



write the function as $f(x)=\begin{cases} -3 & \text{if } x<0 \\ \sqrt{x} & \text{if } x\ge 0 \end{cases}$.

domain: $(-\infty,\infty)$; range: $\{\ldots,-2,-1,0,1,2,\ldots\}$

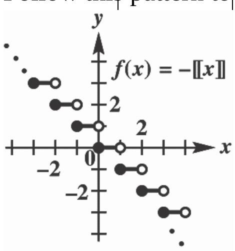
domain: $(-\infty,\infty)$ range: $\{-3\}\cup[0,\infty)$

44. $f(x) = -[\![x]\!]$

Plot points.

x	$[\![x]\!]$	$f(x) = -[\![x]\!]$
–2	–2	2
–1.5	–2	2
–1	–1	1
–0.5	–1	1
0	0	0
0.5	0	0
1	1	–1
1.5	1	–1
2	2	–2

Follow this pattern to graph the step function.



domain: $(-\infty, \infty)$; range: {...,–2,–1,0,1,2,...}

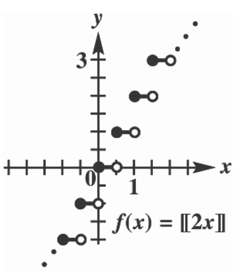
45. $f(x) = [\![2x]\!]$

To get $y = 0$, we need $0 \le 2x < 1 \Rightarrow 0 \le x < \frac{1}{2}$.

To get $y = 1$, we need $1 \le 2x < 2 \Rightarrow \frac{1}{2} \le x < 1$.

To get $y = 2$, we need $2 \le 2x < 3 \Rightarrow 1 \le x < \frac{3}{2}$.

Follow this pattern to graph the step function.



domain: $(-\infty, \infty)$; range: {...,–2,–1,0,1,2,...}

46. $g(x) = [\![2x - 1]\!]$

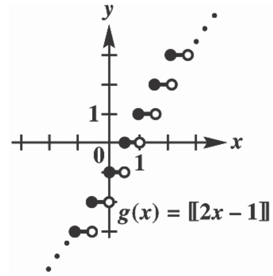
To get $y = 0$, we need

$0 \le 2x - 1 < 1 \Rightarrow 1 \le 2x < 2 \Rightarrow \frac{1}{2} \le x < 1.$

To get $y = 1$, we need

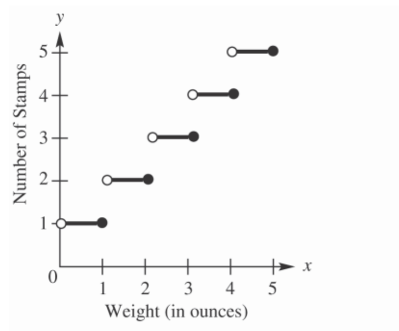
$1 \le 2x - 1 < 2 \Rightarrow 2 \le 2x < 3 \Rightarrow 1 \le x < \frac{3}{2}.$

Follow this pattern to graph the step function.

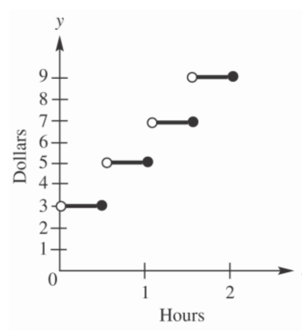


domain: $(-\infty, \infty)$; range: {..., 2,–1,0,1,2,...}

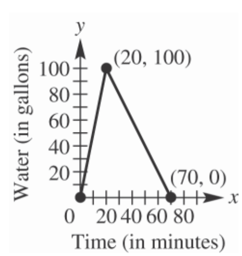
47. The cost of mailing a letter that weighs more than 1 ounce and less than 2 ounces is the same as the cost of a 2-ounce letter, and the cost of mailing a letter that weighs more than 2 ounces and less than 3 ounces is the same as the cost of a 3-ounce letter, etc.



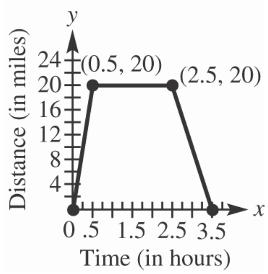
48. The cost is the same for all cars parking between $\frac{1}{2}$ hour and 1-hour, between 1 hour and $1\frac{1}{2}$ hours, etc.



49.



50.



51. **(a)** For $0 \le x \le 8$, $m = \dfrac{49.8 - 34.2}{8 - 0} = 1.95$,

so $y = 1.95x + 34.2$. For $8 < x \le 13$,

$m = \dfrac{52.2 - 49.8}{13 - 8} = 0.48$, so the equation is $y - 52.2 = 0.48(x - 13) \Rightarrow$
$y = 0.48x + 45.96$

(b) $f(x) = \begin{cases} 1.95x + 34.2 & \text{if } 0 \le x \le 8 \\ 0.48x + 45.96 & \text{if } 8 < x \le 13 \end{cases}$

52. When $0 \le x \le 3$, the slope is 5, which means that the inlet pipe is open, and the outlet pipe is closed. When $3 < x \le 5$, the slope is 2, which means that both pipes are open. When $5 < x \le 8$, the slope is 0, which means that both pipes are closed. When $8 < x \le 10$, the slope is −3, which means that the inlet pipe is closed, and the outlet pipe is open.

53. **(a)** The initial amount is 50,000 gallons. The final amount is 30,000 gallons.

(b) The amount of water in the pool remained constant during the first and fourth days.

(c) $f(2) \approx 45{,}000$; $f(4) = 40{,}000$

(d) The slope of the segment between (1, 50000) and (3, 40000) is −5000, so the water was being drained at 5000 gallons per day.

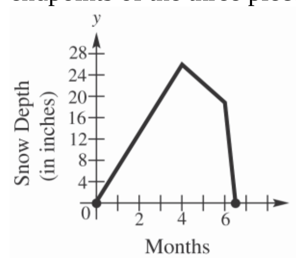
54. **(a)** There were 20 gallons of gas in the tank at $x = 3$.

(b) The slope is steepest between $t = 1$ and $t \approx 2.9$, so that is when the car burned gasoline at the fastest rate.

55. **(a)** There is no charge for additional length, so we use the greatest integer function.

56. **(a)** $f(x) = \begin{cases} 6.5x & \text{if } 0 \le x \le 4 \\ -5.5x + 48 & \text{if } 4 < x \le 6 \\ -30x + 195 & \text{if } 6 < x \le 6.5 \end{cases}$

Draw a graph of $y = 6.5x$ between 0 and 4, including the endpoints. Draw the graph of $y = -5.5x + 48$ between 4 and 6, including the endpoint at 6 but not the one at 4. Draw the graph of $y = -30x + 195$, including the endpoint at 6.5 but not the one at 6. Notice that the endpoints of the three pieces coincide.



(b) From the graph, observe that the snow depth, y, reaches its deepest level (26 in.) when $x = 4$, $x = 4$ represents 4 months after the beginning of October, which is the beginning of February.

(c) From the graph, the snow depth y is nonzero when x is between 0 and 6.5. Snow begins at the beginning of October and ends 6.5 months later, in the middle of April.

Section 2.7 Graphing Techniques

1. To graph the function $f(x) = x^2 - 3$, shift the graph of $y = x^2$ down <u>3</u> units.

2. To graph the function $f(x) = x^2 + 5$, shift the graph of $y = x^2$ up <u>5</u> units.

3. The graph of $f(x) = (x + 4)^2$ is obtained by shifting the graph of $y = x^2$ to the <u>left</u> 4 units.

4. The graph of $f(x) = (x - 7)^2$ is obtained by shifting the graph of $y = x^2$ to the <u>right</u> 7 units.

The cost is based on multiples of two feet, so $f(x) = 0.8\left\lfloor \frac{x}{2} \right\rfloor$ if $6 \le x \le 18$.

(b)

$$f(8.5) = 0.8\left\lfloor \frac{8.5}{2} \right\rfloor = 0.8(4) = \$3.20$$

$$f(15.2) = 0.8\left\lfloor \frac{15.2}{2} \right\rfloor = 0.8(7) = \$5.60$$

5. The graph of $f(x) = -\lfloor x \rfloor$ is a reflection of the graph of $f(x) = \lfloor x \rfloor$ across the x-axis.

6. The graph of $f(x) = \lfloor -x \rfloor$ is a reflection of the graph of $f(x) = \lfloor x \rfloor$ across the y-axis.

7. To obtain the graph of $f(x)=(x+2)^3-3$, shift the graph of $y=x^3$ 2 units to the left and 3 units down.

8. To obtain the graph of $f(x)=(x-3)^3+6$, shift the graph of $y=x^3$ 3 units to the right and 6 units up.

9. The graph of $f(x)=|-x|$ is the same as the graph of $y=|x|$ because reflecting it across the y-axis yields the same ordered pairs.

10. The graph of $x=y^2$ is the same as the graph of $x=(-y)^2$ because reflecting it across the x-axis yields the same ordered pairs.

11. **(a)** B; $y=(x-7)^2$ is a shift of $y=x^2$, 7 units to the right.

(b) D; $y=x^2-7$ is a shift of $y=x^2$, 7 units downward.

(c) E; $y=7x^2$ is a vertical stretch of $y=x^2$, by a factor of 7.

(d) A; $y=(x+7)^2$ is a shift of $y=x^2$, 7 units to the left.

(e) C; $y=x^2+7$ is a shift of $y=x^2$, 7 units upward.

12. **(a)** E; $y=4\sqrt[3]{x}$ is a vertical stretch of $y=\sqrt[3]{x}$, by a factor of 4.

(b) C; $y=-\sqrt[3]{x}$ is a reflection of $y=\sqrt[3]{x}$,

(c) G; $y=(x+2)^2$ is a shift of $y=x^2$, 2 units to the left.

(d) C; $y=(x-2)^2$ is a shift of $y=x^2$, 2 units to the right.

(e) F; $y=2x^2$ is a vertical stretch of $y=x^2$, by a factor of 2.

(f) D; $y=-x^2$ is a reflection of $y=x^2$, across the x-axis.

(g) H; $y=(x-2)^2+1$ is a shift of $y=x^2$, 2 units to the right and 1 unit upward.

(h) E; $y=(x+2)^2+1$ is a shift of $y=x^2$, 2 units to the left and 1 unit upward.

(i) I; $y=(x+2)^2-1$ is a shift of $y=x^2$, 2 units to the left and 1 unit down.

14. **(a)** G; $y=\sqrt{x+3}$ is a shift of $y=\sqrt{x}$, 3 units to the left.

(b) D; $y=\sqrt{x}-3$ is a shift of $y=\sqrt{x}$, 3 units downward.

(c) E; $y=\sqrt{x}+3$ is a shift of $y=\sqrt{x}$, 3 units upward.

(d) B; $y=3\sqrt{x}$ is a vertical stretch of $y=\sqrt{x}$, by a factor of 3.

(e) C; $y=-\sqrt{x}$ is a reflection of $y=\sqrt{x}$ across the x-axis.

(f) A; $y=\sqrt{x-3}$ is a shift of $y=\sqrt{x}$, 3 units to the right.

over the x-axis.

(c) D; $y = \sqrt[3]{-x}$ is a reflection of $y = \sqrt[3]{x}$, over the y-axis.

(d) A; $y = \sqrt[3]{x-4}$ is a shift of $y = \sqrt[3]{x}$, 4 units to the right.

(e) B; $y = \sqrt[3]{x} - 4$ is a shift of $y = \sqrt[3]{x}$, 4 units down.

13. (a) B; $y = x^2 + 2$ is a shift of $y = x^2$, 2 units upward.

(b) A; $y = x^2 - 2$ is a shift of $y = x^2$, 2 units downward.

(g) H; $y = \sqrt{x-3} + 2$ is a shift of $y = \sqrt{x}$, 3 units to the right and 2 units upward.

(h) F; $y = \sqrt{x+3} + 2$ is a shift of $y = \sqrt{x}$, 3 units to the left and 2 units upward.

(i) I; $y = \sqrt{x-3} - 2$ is a shift of $y = \sqrt{x}$, 3 units to the right and 2 units downward.

15. (a) F; $y = |x-2|$ is a shift of $y = |x|$ 2 units to the right.

(b) C; $y = |x| - 2$ is a shift of $y = |x|$ 2 units downward.

(c) H; $y = |x| + 2$ is a shift of $y = |x|$ 2 units upward.

(d) D; $y = 2|x|$ is a vertical stretch of $y = |x|$ by a factor of 2.

(e) G; $y = -|x|$ is a reflection of $y = |x|$ across the x-axis.

(f) A; $y = |-x|$ is a reflection of $y = |x|$ across the y-axis.

(g) E; $y = -2|x|$ is a reflection of $y = 2|x|$ across the x-axis. $y = 2|x|$ is a vertical stretch of $y = |x|$ by a factor of 2.

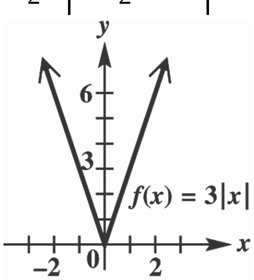
(h) I; $y = |x - 2| + 2$ is a shift of $y = |x|$ 2 units to the right and 2 units upward.

(i) B; $y = |x + 2| - 2$ is a shift of $y = |x|$ 2 units to the left and 2 units downward.

16. The graph of $f(x) = 2(x+1)^3 - 6$ is the graph of $f(x) = x^3$ stretched vertically by a factor of 2, shifted left 1 unit and down 6 units.

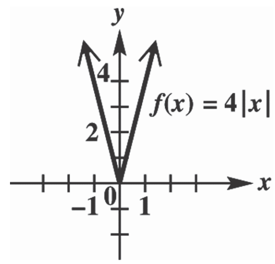
17. $f(x) = 3|x|$

x	$h(x) = \|x\|$	$f(x) = 3\|x\|$
−2	2	6
−1	1	3
0	0	0
1	1	3
2	2	6



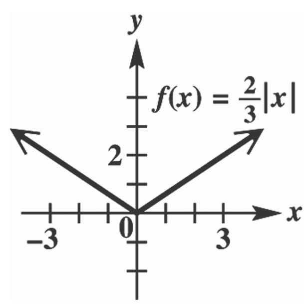
18. $f(x) = 4|x|$

x	$h(x) = \|x\|$	$f(x) = 4\|x\|$
−2	2	8
−1	1	4
0	0	0
1	1	4
2	2	8



19. $f(x) = \frac{2}{3}|x|$

x	$h(x) = \|x\|$	$f(x) = \frac{2}{3}\|x\|$
−3	3	2
−2	2	$\frac{4}{3}$
−1	1	$\frac{2}{3}$
0	0	0
1	1	$\frac{2}{3}$
2	2	$\frac{4}{3}$
3	3	2



20. $f(x) = \frac{3}{4}|x|$

x	$h(x) = \|x\|$	$f(x) = \frac{3}{4}\|x\|$
−4	4	3
−3	3	$\frac{9}{4}$
−2	2	$\frac{3}{2}$
−1	1	$\frac{3}{4}$
0	0	0
1	1	$\frac{3}{4}$
2	2	$\frac{3}{2}$
3	3	$\frac{9}{4}$
4	4	3

(*continued on next page*)

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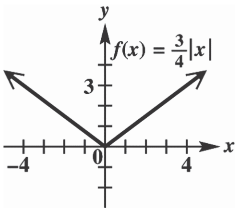
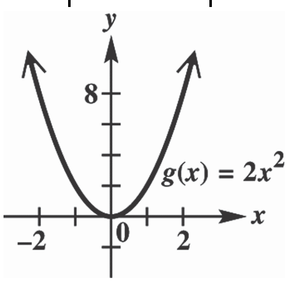


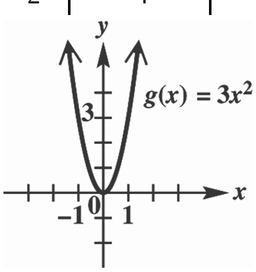
21. $f(x) = 2x^2$

x	$h(x) = x^2$	$f(x) = 2x^2$
−2	4	8
−1	1	2
0	0	0
1	1	2
2	4	8



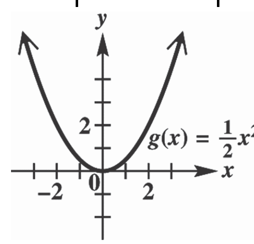
22. $f(x) = 3x^2$

x	$h(x) = x^2$	$f(x) = 3x^2$
−2	4	12
−1	1	3
0	0	0
1	1	3
2	4	12



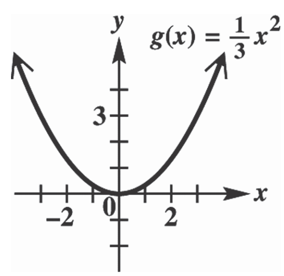
23. $f(x) = \frac{1}{2}x^2$

x	$h(x) = x^2$	$f(x) = \frac{1}{2}x^2$
−2	4	2
−1	1	$\frac{1}{2}$
0	0	0
1	1	$\frac{1}{2}$
2	4	2



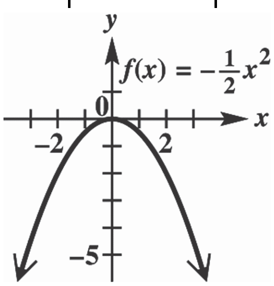
24. $f(x) = \frac{1}{3}x^2$

x	$h(x) = x^2$	$f(x) = \frac{1}{3}x^2$
−3	9	3
−2	4	$\frac{4}{3}$
−1	1	$\frac{1}{3}$
0	0	0
1	1	$\frac{1}{3}$
2	4	$\frac{4}{3}$
3	9	3



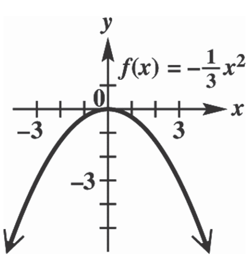
25. $f(x) = -\frac{1}{2}x^2$

x	$h(x) = x^2$	$f(x) = -\frac{1}{2}x^2$
−3	9	$-\frac{9}{2}$
−2	4	−2
−1	1	$-\frac{1}{2}$
0	0	0
1	1	$-\frac{1}{2}$
2	4	−2
3	9	$-\frac{9}{2}$



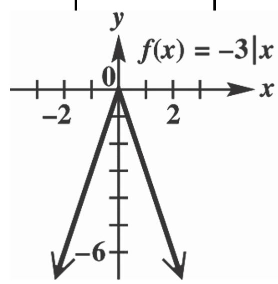
26. $f(x) = -\frac{1}{3}x^2$

x	$h(x) = x^2$	$f(x) = -\frac{1}{3}x^2$
−3	9	−3
−2	4	$-\frac{4}{3}$
−1	1	$-\frac{1}{3}$
0	0	0
1	1	$-\frac{1}{3}$
2	4	$-\frac{4}{3}$
3	9	−3



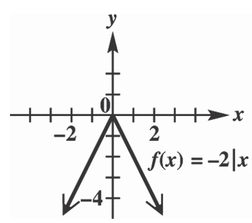
27. $f(x) = -3|x|$

x	$h(x) = \lvert x\rvert$	$f(x) = -3\lvert x\rvert$
−2	2	−6
−1	1	−3
0	0	0
1	1	−3
2	2	−6



28. $f(x) = -2|x|$

x	$h(x) = \lvert x\rvert$	$f(x) = -2\lvert x\rvert$
−2	2	−4
−1	1	−2
0	0	0
1	1	−2
2	2	−4



29. $h(x) = \left|-\frac{1}{2}x\right|$

x	$f(x) = \lvert x\rvert$	$h(x) = \left\lvert-\frac{1}{2}x\right\rvert = \left\lvert-\frac{1}{2}\right\rvert\lvert x\rvert = \frac{1}{2}\lvert x\rvert$
−4	4	2
−3	3	$\frac{3}{2}$
−2	2	1
−1	1	$\frac{1}{2}$
0	0	0

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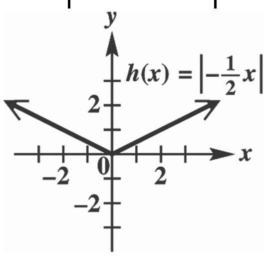
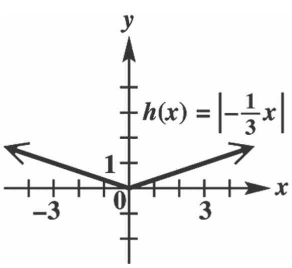
(*continued*)

x	$f(x)=\lvert x\rvert$	$h(x)=\left\lvert -\frac{1}{2}x\right\rvert = \left\lvert -\frac{1}{2}\right\rvert\lvert x\rvert = \frac{1}{2}\lvert x\rvert$
1	1	$\frac{1}{2}$
2	2	1
3	3	$\frac{3}{2}$
4	4	2



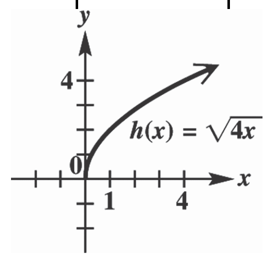
30. $h(x)=\left\lvert -\frac{1}{3}x\right\rvert$

x	$f(x)=\left\lvert -\frac{1}{3}x\right\rvert$	$h(x)=\left\lvert -\frac{1}{3}x\right\rvert = \left\lvert -\frac{1}{3}\right\rvert\lvert x\rvert = \frac{1}{3}\lvert x\rvert$
−3	3	1
−2	2	$\frac{2}{3}$
−1	1	$\frac{1}{3}$
0	0	0
1	1	$\frac{1}{3}$
2	2	$\frac{2}{3}$
3	3	1



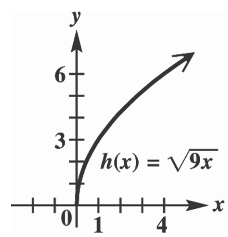
31. $h(x)=\sqrt{4x}$

x	$f(x)=\sqrt{x}$	$h(x)=\sqrt{4x}=2\sqrt{x}$
0	0	0
1	$\sqrt{1}$	2
2	2	$2\sqrt{2}$
3	$\sqrt{3}$	$2\sqrt{3}$
4	2	4



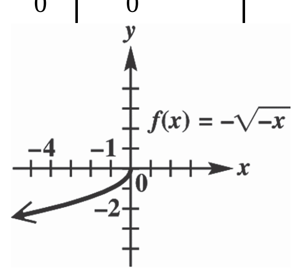
32. $h(x)=\sqrt{9x}$

x	$f(x)=\sqrt{x}$	$h(x)=\sqrt{9x}=3\sqrt{x}$
0	0	0
1	1	3
2	$\sqrt{2}$	$3\sqrt{2}$
3	$\sqrt{3}$	$3\sqrt{3}$
4	2	6



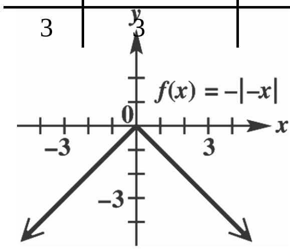
33. $f(x)=-\sqrt{-x}$

x	$h(x)=\sqrt{-x}$	$f(x)=-\sqrt{-x}$
−4	2	−2
−3	$\sqrt{3}$	$-\sqrt{3}$
−2	$\sqrt{2}$	$-\sqrt{2}$
−1	1	−1
0	0	0



34. $f(x) = -|-x|$

x	$h(x) = \|-x\|$	$f(x) = -\|-x\|$
−3	3	−3
−2	2	−2
−1	1	−1
0	0	0
1	1	−1
2	2	−2
3	3	−3



35. (a) $y = f(x+4)$ is a horizontal translation of *f*, 4 units to the left. The point that corresponds to (8, 12) on this translated function would be $(8-4, 12) = (4, 12)$.

(b) $y = f(x)+4$ is a vertical translation of *f*, 4 units up. The point that corresponds to (8, 12) on this translated function would be $(8, 12+4) = (8, 16)$.

36. (a) $y = \frac{1}{4} f(x)$ is a vertical shrinking of *f*, by a factor of $\frac{1}{4}$. The point that corresponds to (8, 12) on this translated function would be $\left(8, \frac{1}{4}\cdot 12\right) = (8, 3)$.

(b) $y = 4f(x)$ is a vertical stretching of *f*, by a factor of 4. The point that corresponds to (8, 12) on this translated function would be $(8, 4\cdot 12) = (8, 48)$.

37. (a) $y = f(4x)$ is a horizontal shrinking of *f*,

by a factor of 4. The point that corresponds to (8, 12) on this translated function is $\left(8\cdot\frac{1}{4}, 12\right) = (2, 12)$.

(b) $y = f\left(\frac{1}{4}x\right)$ is a horizontal stretching of *f*, by a factor of 4. The point that corresponds to (8, 12) on this translated function is $(8\cdot 4, 12) = (32, 12)$.

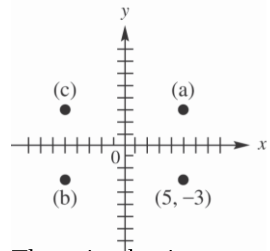
38. (a) The point that corresponds to (8, 12) when reflected across the x-axis would be (8, −12).

(b) The point that corresponds to (8, 12) when reflected across the y-axis would be (−8, 12).

39. (a) The point that is symmetric to (5, –3) with respect to the *x*-axis is (5, 3).

(b) The point that is symmetric to (5, –3) with respect to the *y*-axis is (–5, –3).

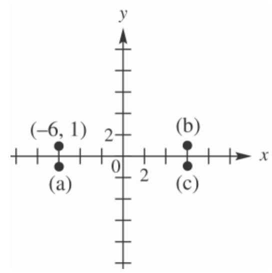
(c) The point that is symmetric to (5, –3) with respect to the origin is (–5, 3).



40. (a) The point that is symmetric to (–6, 1) with respect to the *x*-axis is (–6, –1).

(b) The point that is symmetric to (–6, 1) with respect to the *y*-axis is (6, 1).

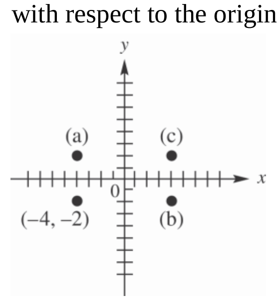
(c) The point that is symmetric to (–6, 1) with respect to the origin is (6, –1).



41. (a) The point that is symmetric to (–4, –2) with respect to the *x*-axis is (–4, 2).

(b) The point that is symmetric to (–4, –2) with respect to the *y*-axis is (4, –2).

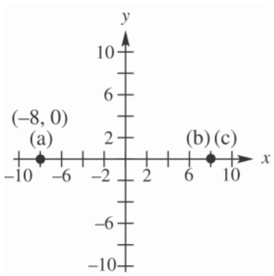
(c) The point that is symmetric to (–4, –2) with respect to the origin is (4, 2).



42. **(a)** The point that is symmetric to (–8, 0) with respect to the x-axis is (–8, 0) because this point lies on the x-axis.

(b) The point that is symmetric to the point (–8, 0) with respect to the y-axis is (8, 0).

(c) The point that is symmetric to the point (–8, 0) with respect to the origin is (8, 0).



43. The graph of $y = |x - 2|$ is symmetric with respect to the line $x = 2$.

44. The graph of $y = -|x + 1|$ is symmetric with respect to the line $x = -1$.

45. $y = x^2 + 5$

Replace x with $-x$ to obtain $y = (-x)^2 + 5 = x^2 + 5$. The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Because y is a function of x, the graph cannot be symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain

$-y = (-x)^2 + 2 \Rightarrow -y = x^2 + 2 \Rightarrow y = -x^2 - 2.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the y-axis only.

46. $y = 2x^4 - 3$

Replace x with $-x$ to obtain

$y = 2(-x)^4 - 3 = 2x^4 - 3$

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Because y is a function of x, the graph cannot be symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain $-y = 2(-x)^4 - 3 \Rightarrow -y = 2x^4 - 3 \Rightarrow y = -2x^4 + 3$. The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the y-axis only.

47. $x^2 + y^2 = 12$

Replace x with $-x$ to obtain

$(-x)^2 + y^2 = 12 \Rightarrow x^2 + y^2 = 12$.

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$x^2 + (-y)^2 = 12 \Rightarrow x^2 + y^2 = 12$

The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Because the graph is symmetric with respect to the x-axis and y-axis, it is also symmetric with respect to the origin.

48. $y^2 - x^2 = 6$

Replace x with $-x$ to obtain

$y^2 - (-x)^2 = 6 \Rightarrow y^2 - x^2 = 6$

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$(-y)^2 - x^2 = 6 \Rightarrow y^2 - x^2 = 6$

The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Because the graph is symmetric with respect to the x-axis and y-axis, it is also symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the x-axis, the y-axis, and the origin.

49. $y = -4x^3 + x$

Replace x with $-x$ to obtain

$y = -4(-x)^3 + (-x) \Rightarrow y = -4(-x^3) - x \Rightarrow$
$y = 4x^3 - x.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain $-y = -4x^3 + x \Rightarrow y = 4x^3 - x.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain

$-y = -4(-x)^3 + (-x) \Rightarrow -y = -4(-x^3) - x \Rightarrow$
$-y = 4x^3 - x \Rightarrow y = -4x^3 + x.$

The result is the same as the original equation, so the graph is symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the origin only.

50. $y = x^3 - x$
Replace x with $-x$ to obtain

$y = (-x)^3 - (-x) \Rightarrow y = -x^3 + x.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain $-y = x^3 - x \Rightarrow y = -x^3 + x.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain $-y = (-x)^3 - (-x) \Rightarrow -y = -x^3 + x \Rightarrow$ $y = x^3 - x.$ The result is the same as the original equation, so the graph is symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the origin only.

51. $y = x^2 - x + 8$
Replace x with $-x$ to obtain
$y = (-x)^2 - (-x) + 8 \Rightarrow y = x^2 + x + 8.$
The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Because y is a function of x, the graph cannot be symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain $-y = (-x)^2 - (-x) + 8 \Rightarrow$
$-y = x^2 + x + 8 \Rightarrow y = -x^2 - x - 8.$
The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph has none of the listed symmetries.

52. $y = x + 15$
Replace x with $-x$ to obtain
$y = (-x) + 15 \Rightarrow y = -x + 15.$
The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Because y is a function of x, the graph cannot be symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain $-y = (-x) + 15 \Rightarrow y = x - 15.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph has none of the listed symmetries.

53. $f(x) = -x^3 + 2x$

$f(-x) = -(-x)^3 + 2(-x)$

$= x^3 - 2x = -\left(-x^3 + 2x\right) = -f(x)$

The function is odd.

54. $f(x) = x^5 - 2x^3$

$f(-x) = (-x)^5 - 2(-x)^3$

$= -x^5 + 2x^3 = -\left(x^5 - 2x^3\right) = -f(x)$

The function is odd.

55. $f(x) = 0.5x^4 - 2x^2 + 6$

$f(-x) = 0.5(-x)^4 - 2(-x)^2 + 6$
$= 0.5x^4 - 2x^2 + 6 = f(x)$
The function is even.

56. $f(x) = 0.75x^2 + |x| + 4$

$f(-x) = 0.75(-x)^2 + |-x| + 4$
$= 0.75x^2 + |x| + 4 = f(x)$
The function is even.

57. $f(x) = x^3 - x + 9$

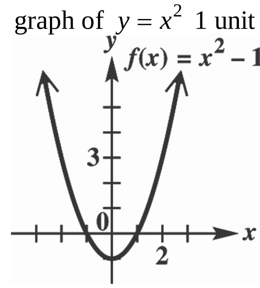
$f(x) = (-x)^3 - (-x) + 9$
$= -x^3 + x + 9 = -\left(x^3 - x - 9\right) \neq -f(x)$

The function is neither.

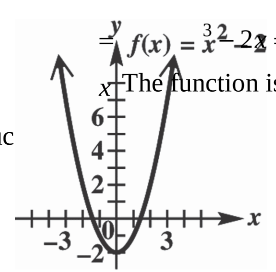
58. $f(x) = x^4 - 5x + 8$

$f(-x) = (-x)^4 - 5(-x) + 8$
$= x^4 + 5x + 8 \neq f(x)$
The function is neither.

59. $f(x) = x^2 - 1$
This graph may be obtained by translating the graph of $y = x^2$ 1 unit downward.



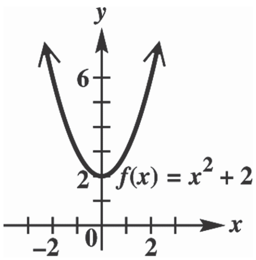
60. $f(x) = x^2 - 2$
This graph may be obtained by translating the



graph of $y = x^2$ 2 units downward.

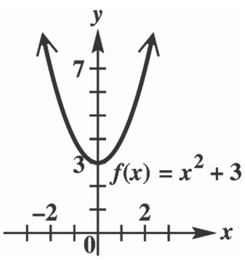
61. $f(x) = x^2 + 2$

This graph may be obtained by translating the graph of $y = x^2$ 2 units upward.



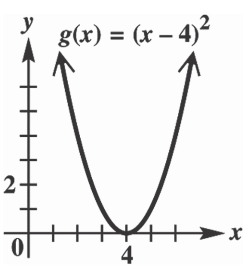
62. $f(x) = x^2 + 3$

This graph may be obtained by translating the graph of $y = x^2$ 3 units upward.



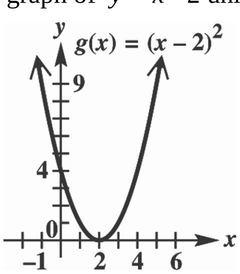
63. $g(x) = (x-4)^2$

This graph may be obtained by translating the graph of $y = x^2$ 4 units to the right.



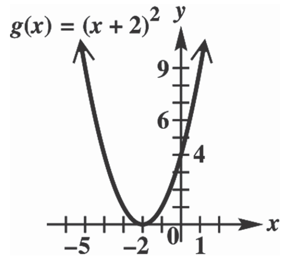
64. $g(x) = (x-2)^2$

This graph may be obtained by translating the graph of $y = x^2$ 2 units to the right.



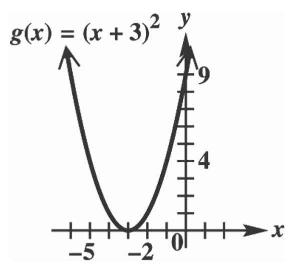
65. $g(x) = (x+2)^2$

This graph may be obtained by translating the graph of $y = x^2$ 2 units to the left.



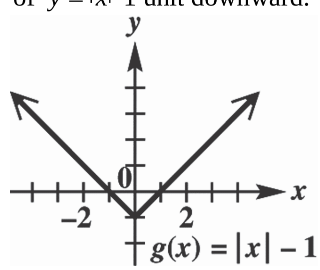
66. $g(x) = (x+3)^2$

This graph may be obtained by translating the graph of $y = x^2$ 3 units to the left.



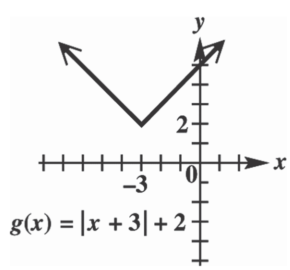
67. $g(x) = |x| - 1$

The graph is obtained by translating the graph of $y = |x|$ 1 unit downward.



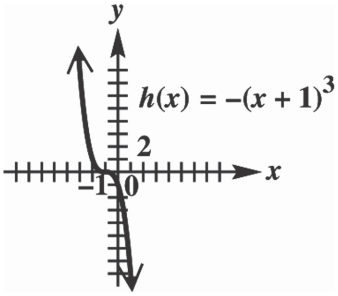
68. $g(x) = |x + 3 + 2$

This graph may be obtained by translating the graph of $y = |x|$ 3 units to the left and 2 units upward.



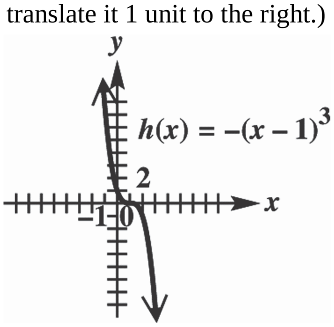
69. $h(x) = -(x+1)^3$

This graph may be obtained by translating the graph of $y = x^3$ 1 unit to the left. It is then reflected across the *x*-axis.



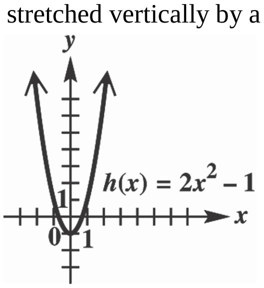
70. $h(x) = -(x-1)^3$

This graph can be obtained by translating the graph of $y = x^3$ 1 unit to the right. It is then reflected across the *x*-axis. (We may also reflect the graph about the *x*-axis first and then translate it 1 unit to the right.)



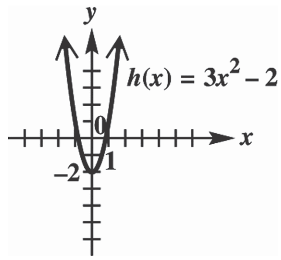
71. $h(x) = 2x^2 - 1$

This graph may be obtained by translating the graph of $y = x^2$ 1 unit down. It is then stretched vertically by a factor of 2.



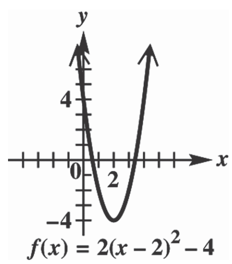
72. $h(x) = 3x^2 - 2$

This graph may be obtained by stretching the graph of $y = x^2$ vertically by a factor of 3, then shifting the resulting graph down 2 units.



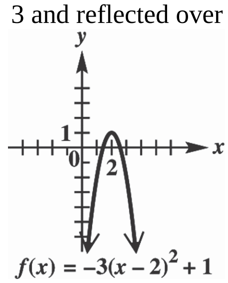
73. $f(x) = 2(x-2)^2 - 4$

This graph may be obtained by translating the graph of $y = x^2$ 2 units to the right and 4 units down. It is then stretched vertically by a factor of 2.



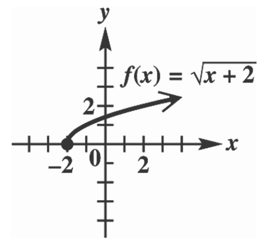
74. $f(x) = -3(x-2)^2 + 1$

This graph may be obtained by translating the graph of $y = x^2$ 2 units to the right and 1 unit up. It is then stretched vertically by a factor of 3 and reflected over the *x*-axis.



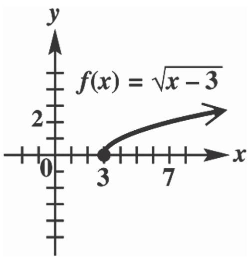
75. $f(x) = \sqrt{x+2}$

This graph may be obtained by translating the graph of $y = \sqrt{x}$ two units to the left.


y
f(x) = √x + 2
2
x
−2
0
2

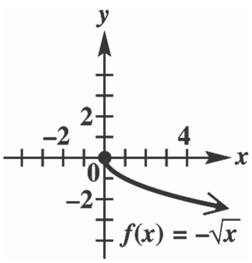
76. $f(x) = \sqrt{x-3}$

This graph may be obtained by translating the graph of $y = \sqrt{x}$ three units to the right.



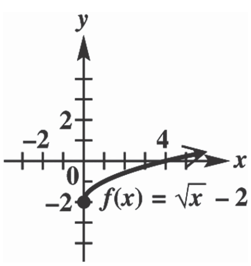
77. $f(x) = -\sqrt{x}$

This graph may be obtained by reflecting the graph of $y = \sqrt{x}$ across the x-axis.



78. $f(x) = \sqrt{x} - 2$

This graph may be obtained by translating the graph of $y = \sqrt{x}$ two units down.

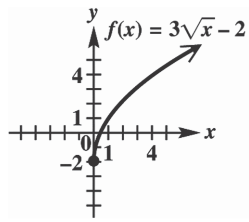


79. $f(x) = 2\sqrt{x} + 1$

This graph may be obtained by stretching the graph of $y = \sqrt{x}$ vertically by a factor of two and then translating the resulting graph one unit up.

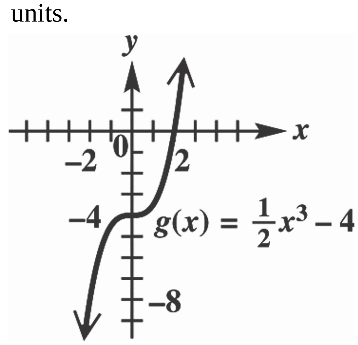
80. $f(x) = 3\sqrt{x} - 2$

This graph may be obtained by stretching the graph of $y = \sqrt{x}$ vertically by a factor of three and then translating the resulting graph two units down.



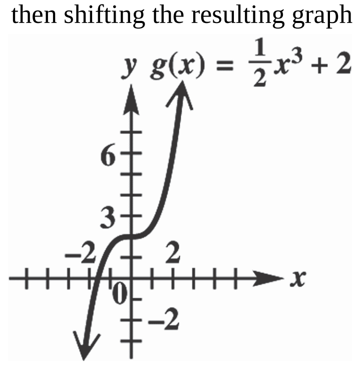
81. $g(x) = \frac{1}{2}x^3 - 4$

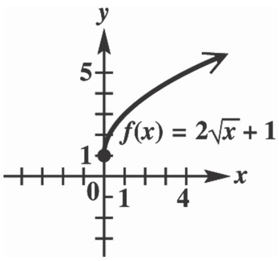
This graph may be obtained by stretching the graph of $y = x^3$ vertically by a factor of $\frac{1}{2}$, then shifting the resulting graph down four units.



82. $g(x) = \frac{1}{2}x^3 + 2$

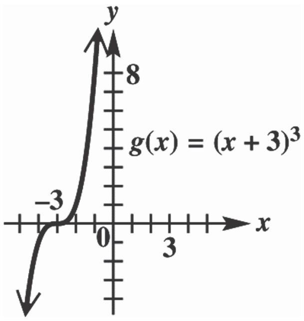
This graph may be obtained by stretching the graph of $y = x^3$ vertically by a factor of $\frac{1}{2}$, then shifting the resulting graph up two units.




y
5
1
0
1
4
x
f(x) = 2√x + 1

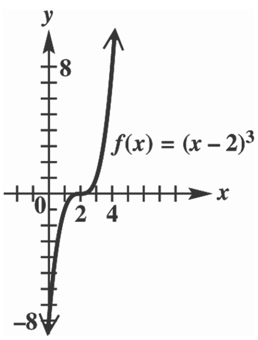
83. $g(x)=(x+3)^3$

This graph may be obtained by shifting the graph of $y=x^3$ three units left.



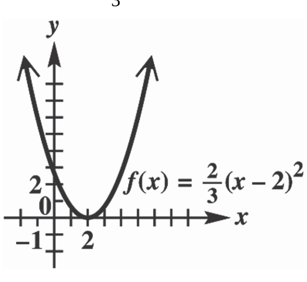
84. $f(x)=(x-2)^3$

This graph may be obtained by shifting the graph of $y=x^3$ two units right.



85. $f(x)=\frac{2}{3}(x-2)^2$

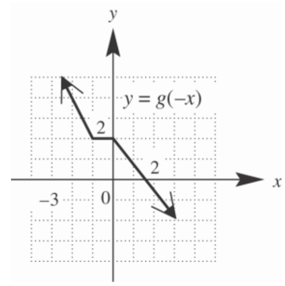
This graph may be obtained by translating the graph of $y=x^2$ two units to the right, then stretching the resulting graph vertically by a factor of $\frac{2}{3}$.



86. Because $g(x)=|-x|=|x|=f(x),$ the graphs are the same.

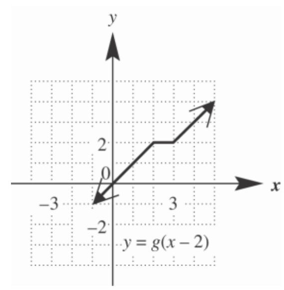
87. **(a)** $y=g(-x)$

The graph of $g(x)$ is reflected across the y-axis.



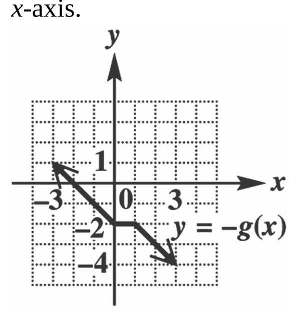
(b) $y=g(x-2)$

The graph of $g(x)$ is translated to the right 2 units.



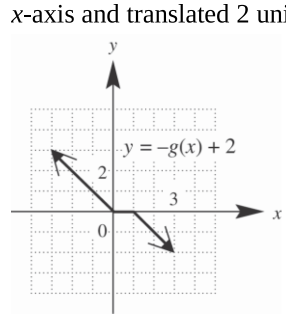
(c) $y=-g(x)$

The graph of $g(x)$ is reflected across the x-axis.



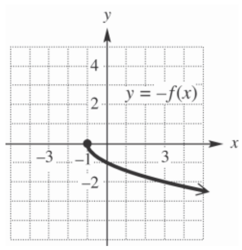
(d) $y=-g(x)+2$

The graph of $g(x)$ is reflected across the x-axis and translated 2 units up.



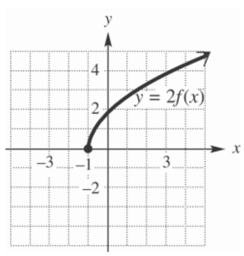
88. **(a)** $y = -f(x)$

The graph of $f(x)$ is reflected across the x-axis.



(b) $y = 2f(x)$

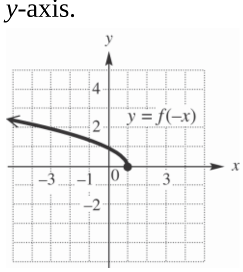
The graph of $f(x)$ is stretched vertically by a factor of 2.



(c) $y = f(-x)$

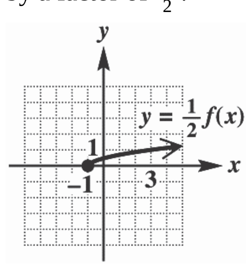
The graph of $f(x)$ is reflected across the

y-axis.



(d) $y = \frac{1}{2}f(x)$

The graph of $f(x)$ is compressed vertically by a factor of $\frac{1}{2}$.



90. It is the graph of $g(x) = \sqrt{x}$ translated 4 units to the left, reflected across the x-axis, and translated two units up. The equation is

$y = -\sqrt{x+4} + 2.$

91. It is the graph of $f(x) = \sqrt{x}$ translated one unit right and then three units down. The equation is $y = \sqrt{x-1} - 3.$

92. It is the graph of $f(x) = |x|$ translated 2 units to the right, shrunken vertically by a factor of $\frac{1}{2}$, and translated one unit down. The equation is $y = \frac{1}{2}|x-2| - 1.$

93. It is the graph of $g(x) = \sqrt{x}$ translated 4 units to the left, stretched vertically by a factor of 2, and translated four units down. The equation is $y = 2\sqrt{x+4} - 4.$

94. It is the graph of $f(x) = |x|$ reflected across the x-axis and then shifted two units down. The equation is $y = -|x| - 2$.

95. Because $f(3) = 6$, the point (3, 6) is on the graph. Because the graph is symmetric with respect to the origin, the point (–3, –6) is on the graph. Therefore, $f(-3) = -6$.

96. Because $f(3) = 6$, (3, 6) is a point on the graph. The graph is symmetric with respect to the y-axis, so (–3, 6) is on the graph. Therefore, $f(-3) = 6$.

97. Because $f(3) = 6$, the point (3, 6) is on the graph. The graph is symmetric with respect to the line $x = 6$ and the point (3, 6) is 3 units to the left of the line $x = 6$, so the image point of (3, 6), 3 units to the right of the line $x = 6$ is (9, 6). Therefore, $f(9) = 6$.

98. Because $f(3) = 6$ and $f(-x) = f(x)$, $f(-3) = f(3)$. Therefore, $f(-3) = 6$.

99. Because (3, 6) is on the graph, (–3, –6) must also be on the graph. Therefore, $f(-3) = -6$.

100. If f is an odd function, $f(-x) = -f(x)$. Because $f(3) = 6$ and $f(-x) = -f(x)$, $f(-3) = -f(3)$. Therefore, $f(-3) = -6$.

89. It is the graph of $f(x)=|x|$ translated 1 unit to the left, reflected across the *x*-axis, and translated 3 units up. The equation is $y=-|x+1|+3.$

101. $f(x) = 2x + 5$

Translate the graph of $f(x)$ up 2 units to obtain the graph of

$t(x) = (2x+5)+2 = 2x+7.$

Now translate the graph of $t(x) = 2x + 7$ left 3 units to obtain the graph of

$g(x) = 2(x+3)+7 = 2x+6+7 = 2x+13.$

(Note that if the original graph is first translated to the left 3 units and then up 2 units, the final result will be the same.)

102. $f(x) = 3 - x$

Translate the graph of $f(x)$ down 2 units to

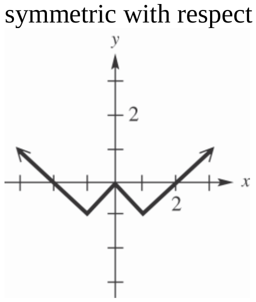
obtain the graph of $t(x) = (3-x)-2 = -x+1.$

Now translate the graph of $t(x) = -x+1$ right 3 units to obtain the graph of

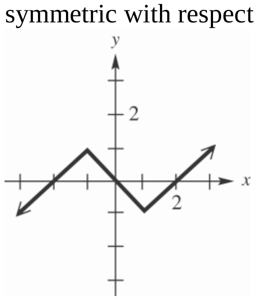
$g(x) = -(x-3)+1 = -x+3+1 = -x+4.$

(Note that if the original graph is first translated to the right 3 units and then down 2 units, the final result will be the same.)

103. **(a)** Because $f(-x) = f(x)$, the graph is symmetric with respect to the y-axis.



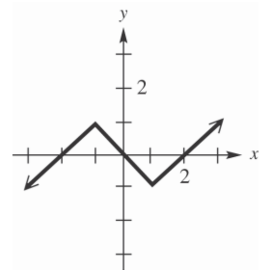
(b) Because $f(-x) = -f(x)$, the graph is symmetric with respect to the origin.



104. **(a)** $f(x)$ is odd. An odd function has a graph

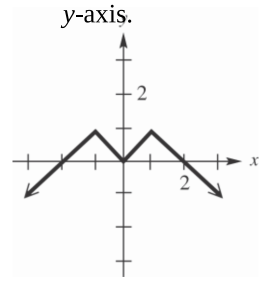
symmetric with respect to the origin. Reflect the left half of the graph in the

origin.



(b) $f(x)$ is even. An even function has a graph symmetric with respect to the y-axis. Reflect the left half of the graph in the

y-axis.



Chapter 2 Quiz
(Sections 2.5−2.7)

1. **(a)** First, find the slope: $m = \dfrac{9-5}{-1-(-3)} = 2$

Choose either point, say, (−3, 5), to find the equation of the line:

$y-5 = 2(x-(-3)) \Rightarrow y = 2(x+3)+5 \Rightarrow$

$y = 2x+11$.

(b) To find the x-intercept, let $y = 0$ and solve for x: $0 = 2x+11 \Rightarrow x = -\frac{11}{2}$. The x-intercept is $\left(-\frac{11}{2}, 0\right)$.

2. Write $3x - 2y = 6$ in slope-intercept form to find its slope: $3x-2y=6 \Rightarrow y = \frac{3}{2}x-3.$

Then, the slope of the line perpendicular to this graph is $-\frac{2}{3}$. $y-4 = -\frac{2}{3}(x-(-6)) \Rightarrow$

$y = -\frac{2}{3}(x+6))+4 \Rightarrow y = -\frac{2}{3}x$

3. **(a)** $x = -8$ **(b)** $y = 5$

4. **(a)** Cubing function; domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$; increasing over $(-\infty, \infty)$.

(b) Absolute value function; domain: $(-\infty, \infty)$; range: $[0, \infty)$; decreasing over

$(-\infty, 0)$;
increasin
g over $(0,$
$\infty)$

(c) Cube root function: domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$; increasing over $(-\infty, \infty)$.

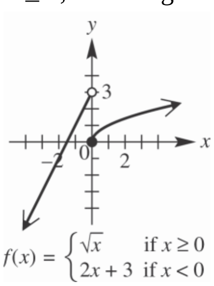
5. $f(x) = 0.40\lbrack\!\lbrack x \rbrack\!\rbrack + 0.75$

$$f(5.5) = 0.40\lbrack\!\lbrack 5.5 \rbrack\!\rbrack + 0.75$$
$$= 0.40(5) + 0.75 = 2.75$$

A 5.5-minute call costs \$2.75.

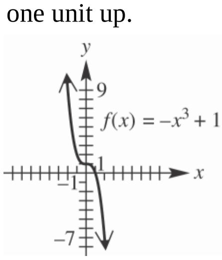
6. $f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ 2x + 3 & \text{if } x < 0 \end{cases}$

For values of $x < 0$, the graph is the line $y = 2x + 3$. Do not include the right endpoint (0, 3). Graph the line $y = \sqrt{x}$ for values of $x \geq 0$, including the left endpoint (0, 0).



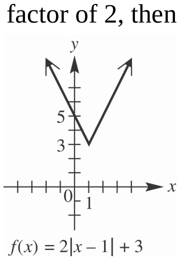
7. $f(x) = -x^3 + 1$

Reflect the graph of $f(x) = x^3$ across the x-axis, and then translate the resulting graph one unit up.



8. $f(x) = 2|x - 1| + 3$

Shift the graph of $f(x) = |x|$ one unit right, stretch the resulting graph vertically by a factor of 2, then shift this graph three units up.



9. This is the graph of $g(x) = \sqrt{x}$, translated four units to the left, reflected across the x-axis, and then translated two units down. The equation is $y = -\sqrt{x + 4} - 2$.

10. (a) $f(x) = x^2 - 7$

Replace x with $-x$ to obtain

$$f(-x) = (-x)^2 - 7 \Rightarrow$$
$$f(-x) = x^2 - 7 = f(x)$$

The result is the same as the original function, so the function is even.

(b) $f(x) = x^3 - x - 1$

Replace x with $-x$ to obtain

$$f(-x) = (-x)^3 - (-x) - 1$$
$$= -x^3 + x - 1 \neq f(x)$$

The result is not the same as the original equation, so the function is not even. Because $f(-x) \neq -f(x)$, the function is not odd. Therefore, the function is neither even nor odd.

(c) $f(x) = x^{101} - x^{99}$

Replace x with $-x$ to obtain

$$f(-x) = (-x)^{101} - (-x)^{99}$$
$$= -x^{101} - (-x^{99})$$
$$= -(x^{101} - x^{99})$$
$$= -f(x)$$

Because $f(-x) = -f(x)$, the function is odd.

Section 2.8 Function Operations and Composition

In exercises 1–10, $f(x) = x + 1$ and $g(x) = x^2$.

1. $(f + g)(2) = f(2) + g(2)$
$$= (2 + 1) + 2^2 = 7$$

2. $(f - g)(2) = f(2) - g(2)$
$$= (2 + 1) - 2^2 = -1$$

3. $(fg)(2) = f(2) \cdot g(2)$
$$= (2 + 1) \cdot 2^2 = 12$$

4. $\left(\dfrac{f}{g}\right)(2)=\dfrac{f(2)}{g(2)}=\dfrac{2+1}{2^2}=\dfrac{3}{4}$

5. $(f\circ g)(2)=f\left(g(2)\right)=f\left(2^2\right)=2^2+1=5$

6. $(g \circ f)(2) = g(f(2)) = g(2+1) = (2+1)^2 = 9$

7. f is defined for all real numbers, so its domain is $(-\infty, \infty)$.

8. g is defined for all real numbers, so its domain is $(-\infty, \infty)$.

9. $f + g$ is defined for all real numbers, so its domain is $(-\infty, \infty)$.

10. $\frac{f}{g}$ is defined for all real numbers except those values that make $g(x) = 0$, so its domain is $(-\infty, 0) \cup (0, \infty)$.

In Exercises 11–18, $f(x) = x^2 + 3$ and $g(x) = -2x + 6$.

11. $$\begin{aligned}(f+g)(3) &= f(3) + g(3) \\ &= \left[(3)^2 + 3\right] + \left[-2(3) + 6\right] \\ &= 12 + 0 = 12\end{aligned}$$

12. $$\begin{aligned}(f+g)(-5) &= f(-5) + g(-5) \\ &= [(-5)^2 + 3] + [-2(-5) + 6] \\ &= 28 + 16 = 44\end{aligned}$$

13. $$\begin{aligned}(f-g)(-1) &= f(-1) - g(-1) \\ &= [(-1)^2 + 3] - [-2(-1) + 6] \\ &= 4 - 8 = -4\end{aligned}$$

14. $$\begin{aligned}(f-g)(4) &= f(4) - g(4) \\ &= [(4)^2 + 3] - [-2(4) + 6] \\ &= 19 - (-2) = 21\end{aligned}$$

15. $$\begin{aligned}(fg)(4) &= f(4) \cdot g(4) \\ &= [4^2 + 3] \cdot [-2(4) + 6] \\ &= 19 \cdot (-2) = -38\end{aligned}$$

16. $$\begin{aligned}(fg)(-3) &= f(-3) \cdot g(-3) \\ &= [(-3)^2 + 3] \cdot [-2(-3) + 6] \\ &= 12 \cdot 12 = 144\end{aligned}$$

17. $$\left(\frac{f}{g}\right)(-1) = \frac{f(-1)}{g(-1)} = \frac{(-1)^2 + 3}{-2(-1) + 6} = \frac{4}{8} = \frac{1}{2}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x) \\ &= (3x+4) - (2x-5) = x + 9\end{aligned}$$

$$\begin{aligned}(fg)(x) &= f(x) \cdot g(x) = (3x+4)(2x-5) \\ &= 6x^2 - 15x + 8x - 20 \\ &= 6x^2 - 7x - 20\end{aligned}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x+4}{2x-5}$$

The domains of both f and g are the set of all real numbers, so the domains of $f + g$, $f - g$, and fg are all $(-\infty, \infty)$. The domain of $\frac{f}{g}$ is the set of all real numbers for which $g(x) \neq 0$. This is the set of all real numbers except $\frac{5}{2}$, which is written in interval notation as $\left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$.

20. $f(x) = 6 - 3x$, $g(x) = -4x + 1$

$$\begin{aligned}(f+g)(x) &= f(x) + g(x) \\ &= (6-3x) + (-4x+1) \\ &= -7x + 7\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x) \\ &= (6-3x) - (-4x+1) = x + 5\end{aligned}$$

$$\begin{aligned}(fg)(x) &= f(x) \cdot g(x) = (6-3x)(-4x+1) \\ &= -24x + 6 + 12x^2 - 3x \\ &= 12x^2 - 27x + 6\end{aligned}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{6-3x}{-4x+1}$$

The domains of both f and g are the set of all real numbers, so the domains of $f + g$, $f - g$, and fg are all $(-\infty, \infty)$. The domain of $\frac{f}{g}$ is the set of all real numbers for which $g(x) \neq 0$. This is the set of all real numbers except $\frac{1}{4}$, which is written in interval notation as $\left(-\infty, \frac{1}{4}\right) \cup \left(\frac{1}{4}, \infty\right)$.

21. $f(x) = 2x^2 - 3x$, $g(x) = x^2 - x + 3$

$$\begin{aligned}(f+g)(x) &= f(x) + g(x) \\ &= (2x^2 - 3x) + (x^2 - x + 3) \\ &= 3x^2 - 4x + 3\end{aligned}$$

18. $\left(\dfrac{f}{g}\right)(5) = \dfrac{f(5)}{g(5)} = \dfrac{(5)^2 + 3}{-2(5) + 6} = \dfrac{28}{-4} = -7$

19. $f(x) = 3x + 4,\ g(x) = 2x - 5$

$$(f + g)(x) = f(x) + g(x) = (3x + 4) + (2x - 5) = 5x - 1$$

$$(f - g)(x) = f(x) - g(x) = (2x^2 - 3x) - (x^2 - x + 3) = 2x^2 - 3x - x^2 + x - 3 = x^2 - 2x - 3$$

(*continued on next page*)

(*continued*)

$$(fg)(x) = f(x)\cdot g(x)$$
$$= (2x^2-3x)(x^2-x+3)$$
$$= 2x^4-2x^3+6x^2-3x^3+3x^2-9x$$
$$= 2x^4-5x^3+9x^2-9x$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x^2-3x}{x^2-x+3}$$

The domains of both *f* and *g* are the set of all real numbers, so the domains of $f+g$, $f-g$, and fg are all $(-\infty,\infty)$. The domain of $\frac{f}{g}$ is the set of all real numbers for which $g(x)\neq 0$. If $x^2-x+3=0$, then by the quadratic formula $x=\frac{1\pm i\sqrt{11}}{2}$. The equation has no real solutions. There are no real numbers which make the denominator zero. Thus, the domain of $\frac{f}{g}$ is also $(-\infty,\infty)$.

22. $f(x)=4x^2+2x,\ g(x)=x^2-3x+2$

$$(f+g)(x) = f(x)+g(x)$$
$$= (4x^2+2x)+(x^2-3x+2)$$
$$= 5x^2-x+2$$
$$(f-g)(x) = f(x)-g(x)$$
$$= (4x^2+2x)-(x^2-3x+2)$$
$$= 4x^2+2x-x^2+3x-2$$
$$= 3x^2+5x-2$$
$$(fg)(x) = f(x)\cdot g(x)$$
$$= (4x^2+2x)(x^2-3x+2)$$
$$= 4x^4-12x^3+8x^2+2x^3-6x^2+4x$$
$$= 4x^4-10x^3+2x^2+4x$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{4x^2+2x}{x^2-3x+2}$$

The domains of both *f* and *g* are the set of all real numbers, so the domains of $f+g$, $f-g$,

23. $f(x)=\sqrt{4x-1},\ g(x)=\frac{1}{x}$

$$(f+g)(x) = f(x)+g(x) = \sqrt{4x-1}+\frac{1}{x}$$
$$(f-g)(x) = f(x)-g(x) = \sqrt{4x-1}-\frac{1}{x}$$
$$(fg)(x) = f(x)\cdot g(x)$$
$$= \sqrt{4x-1}\left(\frac{1}{x}\right) = \frac{\sqrt{4x-1}}{x}$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{4x-1}}{\frac{1}{x}} = x\sqrt{4x-1}$$

Because $4x-1\geq 0 \Rightarrow 4x\geq 1 \Rightarrow x\geq \frac{1}{4}$, the domain of *f* is $\left[\frac{1}{4},\infty\right)$. The domain of *g* is $(-\infty,0)\cup(0,\infty)$. Considering the intersection of the domains of *f* and *g*, the domains of $f+g$, $f-g$, and fg are all $\left[\frac{1}{4},\infty\right)$. Because $\frac{1}{x}\neq 0$ for any value of *x*, the domain of $\frac{f}{g}$ is also $\left[\frac{1}{4},\infty\right)$.

24. $f(x)=\sqrt{5x-4},\ g(x)=-\frac{1}{x}$

$$(f+g)(x) = f(x)+g(x)$$
$$= \sqrt{5x-4}+\left(-\frac{1}{x}\right) = \sqrt{5x-4}-\frac{1}{x}$$
$$(f-g)(x) = f(x)-g(x)$$
$$= \sqrt{5x-4}-\left(-\frac{1}{x}\right) = \sqrt{5x-4}+\frac{1}{x}$$
$$(fg)(x) = f(x)\cdot g(x)$$
$$= \left(\sqrt{5x-4}\right)\left(-\frac{1}{x}\right) = -\frac{\sqrt{5x-4}}{x}$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{5x-4}}{-\frac{1}{x}} = -x\sqrt{5x-4}$$

and fg are all $(-\infty, \infty)$. The domain of $\frac{f}{g}$ is the set of all real numbers x such that $x^2 - 3x + 2 \neq 0$. Because $x^2 - 3x + 2 = (x-1)(x-2)$, the numbers which give this denominator a value of 0 are $x = 1$ and $x = 2$. Therefore, the domain of $\frac{f}{g}$ is the set of all real numbers except 1 and 2, which is written in interval notation as

$(-\infty, 1) \cup (1, 2) \cup (2, \infty)$.

$g \quad g(x) \quad \sqrt{-\frac{1}{x}}$

Because $5x - 4 \geq 0 \Rightarrow 5x \geq 4 \Rightarrow x \geq \frac{4}{5}$, the domain of f is $\left[\frac{4}{5}, \infty\right)$. The domain of g is $(-\infty, 0) \cup (0, \infty)$. Considering the intersection of the domains of f and g, the domains of $f + g$, $f - g$, and fg are all $\left[\frac{4}{5}, \infty\right)$. $-\frac{1}{x} \neq 0$ for any value of x, so the domain of $\frac{f}{g}$ is also $\left[\frac{4}{5}, \infty\right)$.

25. $M(2008) \approx 280$ and $F(2008) \approx 470$, thus
$$T(2008) = M(2008) + F(2008) = 280 + 470 = 750 \text{ (thousand).}$$

26. $M(2012) \approx 390$ and $F(2012) \approx 630$, thus
$$T(2012) = M(2012) + F(2012) = 390 + 630 = 1020 \text{ (thousand).}$$

27. Looking at the graphs of the functions, the slopes of the line segments for the period 2008−2012 are much steeper than the slopes of the corresponding line segments for the period 2004−2008. Thus, the number of associate's degrees increased more rapidly during the period 2008−2012.

28. If $2004 \le k \le 2012$, $T(k) = r$, and $F(k) = s$, then $M(k) = \underline{r - s}$.

29. $(T - S)(2000) = T(2000) - S(2000) = 19 - 13 = 6$
It represents the dollars in billions spent for general science in 2000.

30. $(T - G)(2010) = T(2010) - G(2010) \approx 29 - 11 = 18$
It represents the dollars in billions spent on space and other technologies in 2010.

31. Spending for space and other technologies spending decreased in the years 1995−2000 and 2010–2015.

32. Total spending increased the most during the years 2005−2010.

33. **(a)** $(f + g)(2) = f(2) + g(2) = 4 + (-2) = 2$

(b) $(f - g)(1) = f(1) - g(1) = 1 - (-3) = 4$

(c) $(fg)(0) = f(0) \cdot g(0) = 0(-4) = 0$

(d) $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{1}{-3} = -\frac{1}{3}$

34. **(a)** $(f + g)(0) = f(0) + g(0) = 0 + 2 = 2$

(b) $(f - g)(-1) = f(-1) - g(-1) = -2 - 1 = -3$

(c) $(fg)(1) = f(1) \cdot g(1) = 2 \cdot 1 = 2$

(d) $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{4}{-2} = -2$

35. **(a)** $(f + g)(-1) = f(-1) + g(-1) = 0 + 3 = 3$

(b) $(f - g)(-2) = f(-2) - g(-2) = -1 - 4 = -5$

(c) $(fg)(0) = f(0) \cdot g(0) = 1 \cdot 2 = 2$

(d) $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{3}{0} =$ undefined

36. **(a)** $(f + g)(1) = f(1) + g(1) = -3 + 1 = -2$

(b) $(f - g)(0) = f(0) - g(0) = -2 - 0 = -2$

(c) $(fg)(-1) = f(-1) \cdot g(-1) = -3(-1) = 3$

(d) $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{-3}{1} = -3$

37. **(a)** $(f + g)(2) = f(2) + g(2) = 7 + (-2) = 5$

(b) $(f - g)(4) = f(4) - g(4) = 10 - 5 = 5$

(c) $(fg)(-2) = f(-2) \cdot g(-2) = 0 \cdot 6 = 0$

(d) $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{5}{0} =$ undefined

38. **(a)** $(f + g)(2) = f(2) + g(2) = 5 + 4 = 9$

(b) $(f - g)(4) = f(4) - g(4) = 0 - 0 = 0$

(c) $(fg)(-2) = f(-2) \cdot g(-2) = -4 \cdot 2 = -8$

(d) $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{8}{-1} = -8$

39.

x	$f(x)$	$g(x)$	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
-2	0	6	$0+6=6$	$0-6=-6$	$0\cdot 6=0$	$\frac{0}{6}=0$
0	5	0	$5+0=5$	$5-0=5$	$5\cdot 0=0$	$\frac{5}{0}=$ undefined
2	7	-2	$7+(-2)=5$	$7-(-2)=9$	$7(-2)=-14$	$\frac{7}{-2}=-3.5$
4	10	5	$10+5=15$	$10-5=5$	$10\cdot 5=50$	$\frac{10}{5}=2$

40.

x	$f(x)$	$g(x)$	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
-2	-4	2	$-4+2=-2$	$-4-2=-6$	$-4\cdot 2=-8$	$\frac{-4}{2}=-2$
0	8	-1	$8+(-1)=7$	$8-(-1)=9$	$8(-1)=-8$	$\frac{8}{-1}=-8$
2	5	4	$5+4=9$	$5-4=1$	$5\cdot 4=20$	$\frac{5}{4}=1.25$
4	0	0	$0+0=0$	$0-0=0$	$0\cdot 0=0$	$\frac{0}{0}=$ undefined

41. Answers may vary. Sample answer: Both the slope formula and the difference quotient

represent the ratio of the vertical change to the horizontal change. The slope formula is stated for a line while the difference quotient is stated for a function *f*.

42. Answers may vary. Sample answer: As *h* approaches 0, the slope of the secant line *PQ* approaches the slope of the line tangent of the curve at *P*.

43. $f(x)=2-x$

(a) $f(x+h)=2-(x+h)=2-x-h$

(b) $f(x+h)-f(x)=(2-x-h)-(2-x)$
$=2-x-h-2+x=-h$

(c) $\dfrac{f(x+h)-f(x)}{h}=\dfrac{-h}{h}=-1$

44. $f(x)=1-x$

(a) $f(x+h)=1-(x+h)=1-x-h$

(b) $f(x+h)-f(x)=(1-x-h)-(1-x)$
$=1-x-h-1+x=-h$

(c) $\dfrac{f(x+h)-f(x)}{h}=\dfrac{-h}{h}=-1$

45. $f(x)=6x+2$

(a) $f(x+h)=6(x+h)+2=6x+6h+2$

(b) $f(x+h)-f(x)$
$=(6x+6h+2)-(6x+2)$
$=6x+6h+2-6x-2=6h$

(c) $\dfrac{f(x+h)-f(x)}{h}=\dfrac{6h}{h}=6$

46. $f(x)=4x+11$

(a) $f(x+h)=4(x+h)+11=4x+4h+11$

(b) $f(x+h)-f(x)$
$=(4x+4h+11)-(4x+11)$
$=4x+4h+11-4x-11=4h$

(c) $\dfrac{f(x+h)-f(x)}{h}=\dfrac{4h}{h}=4$

47. $f(x)=-2x+5$

(a) $f(x+h)=-2(x+h)+5$
$=-2x-2h+5$

(b) $f(x+h)-f(x)$
$=(-2x-2h+5)-(-2x+5)$
$=-2x-2h+5+2x-5=-2h$

(c) $\dfrac{f(x+h)-f(x)}{h}=\dfrac{-2h}{h}=-2$

48. $f(x) = -4x + 2$

(a) $f(x+h) = -4(x+h) + 2 = -4x - 4h + 2$

(b) $f(x+h) - f(x)$

$= -4x - 4h + 2 - (-4x + 2)$
$= -4x - 4h + 2 + 4x - 2$
$= -4h$

(c) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-4h}{h} = -4$

49. $f(x) = \dfrac{1}{x}$

(a) $f(x+h) = \dfrac{1}{x+h}$

(b) $f(x+h) - f(x)$

$= \dfrac{1}{x+h} - \dfrac{1}{x} = \dfrac{x - (x+h)}{x(x+h)}$

$= \dfrac{-h}{x(x+h)}$

(c) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\frac{-h}{x(x+h)}}{h} = \dfrac{-h}{hx(x+h)}$

$= -\dfrac{1}{x(x+h)}$

50. $f(x) = \dfrac{1}{x^2}$

(a) $f(x+h) = \dfrac{1}{(x+h)^2}$

(b) $f(x+h) - f(x)$

$= \dfrac{1}{(x+h)^2} - \dfrac{1}{x^2} = \dfrac{x^2 - (x+h)^2}{x^2(x+h)^2}$

$= \dfrac{x^2 - (x^2 + 2xh + h^2)}{x^2(x+h)^2} = \dfrac{-2xh - h^2}{x^2(x+h)^2}$

(c) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\frac{-2xh - h^2}{x^2(x+h)^2}}{h} = \dfrac{-2xh - h^2}{hx^2(x+h)^2}$

$= \dfrac{h(-2x-h)}{}$

51. $f(x) = x^2$

(a) $f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$

(b) $f(x+h) - f(x) = x^2 + 2xh + h^2 - x^2$

$= 2xh + h^2$

(c) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2xh + h^2}{h}$

$= \dfrac{h(2x+h)}{h}$

$= 2x + h$

52. $f(x) = -x^2$

(a) $f(x+h) = -(x+h)^2$
$= -(x^2 + 2xh + h^2)$
$= -x^2 - 2xh - h^2$

(b) $f(x+h) - f(x) = -x^2 - 2xh - h^2 - (-x^2)$

$= -x^2 - 2xh - h^2 + x^2$
$= -2xh - h^2$

(c) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-2xh - h^2}{h}$

$= \dfrac{-h(2x+h)}{h}$

$= -2x - h$

53. $f(x) = 1 - x^2$

(a) $f(x+h) = 1 - (x+h)^2$
$= 1 - (x^2 + 2xh + h^2)$
$= 1 - x^2 - 2xh - h^2$

(b) $f(x+h) - f(x)$
$= (1 - x^2 - 2xh - h^2) - (1 - x^2)$
$= 1 - x^2 - 2xh - h^2 - 1 + x^2$
$= -2xh - h^2$

(c) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-2xh - h^2}{h}$

$= \dfrac{h(-2x-h)}{h}$

$= -2x - h$

54. $f(x) = 1 + 2x^2$

$$hx^2\left(x+h\right)^2$$

$$= \frac{-2x-h}{x^2\left(x+h\right)^2}$$

(a) $f(x+h) = 1 + 2(x+h)^2$

$$= 1 + 2(x^2 + 2xh + h^2)$$

$$= 1 + 2x^2 + 4xh + 2h^2$$

(b) $f(x+h)-f(x)$
$$=\left(1+2x^2+4xh+2h^2\right)-\left(1+2x^2\right)$$
$$=1+2x^2+4xh+2h^2-1-2x^2$$
$$=4xh+2h^2$$

(c) $$\frac{f(x+h)-f(x)}{h}=\frac{4xh+2h^2}{h}$$
$$=\frac{h(4x+2h)}{h}$$
$$=4x+2h$$

55. $f(x)=x^2+3x+1$

(a) $f(x+h)=(x+h)^2+3(x+h)+1$
$$=x^2+2xh+h^2+3x+3h+1$$

(b) $f(x+h)-f(x)$
$$=\left(x^2+2xh+h^2+3x+3h+1\right)-\left(x^2+3x+1\right)$$
$$=x^2+2xh+h^2+3x+3h+1-x^2-3x-1$$
$$=2xh+h^2+3h$$

(c) $$\frac{f(x+h)-f(x)}{h}=\frac{2xh+h^2+3h}{h}$$
$$=\frac{h(2x+h+3)}{h}$$
$$=2x+h+3$$

56. $f(x)=x^2-4x+2$

(a) $f(x+h)=(x+h)^2-4(x+h)+2$
$$=x^2+2xh+h^2-4x-4h+2$$

(b) $f(x+h)-f(x)$
$$=\left(x^2+2xh+h^2-4x-4h+2\right)-\left(x^2-4x+2\right)$$
$$=x^2+2xh+h^2-4x-4h+2-x^2+4x-2$$
$$=2xh+h^2-4h$$

(c) $$\frac{f(x+h)-f(x)}{h}=\frac{2xh+h^2-4h}{h}$$
$$=\frac{h(2x+h-4)}{h}$$
$$=2x+h-4$$

58. $g(x)=-x+3\Rightarrow g(2)=-2+3=1$
$$(f\circ g)(2)=f[g(2)]=f(1)$$
$$=2(1)-3=2-3=-1$$

59. $g(x)=-x+3\Rightarrow g(-2)=-(-2)+3=5$
$$(f\circ g)(-2)=f[g(-2)\ \ =f(5)$$
$$=2(5)-3=10-3=7$$

60. $f(x)=2x-3\Rightarrow f(3)=2(3)-3=6-3=3$
$$(g\circ f)(3)=g\ \ f(3)\ \ =g(3)=-3+3=0$$

61. $f(x)=2x-3\Rightarrow f(0)=2(0)-3=0-3=-3$
$$(g\circ f)(0)=g\ \ f(0)\ \ =g(-3)$$
$$=-(-3)+3=3+3=6$$

62. $f(x)=2x-3\Rightarrow f(-2)=2(-2)-3=-7$
$$(g\circ f)(-2)=g[f(-2)]=g(-7)$$
$$=-(-7)+3=7+3=10$$

63. $f(x)=2x-3\Rightarrow f(2)=2(2)-3=4-3=1$
$$(f\circ f)(2)=f\ \ f(2)\ \ =f(1)=2(1)-3=-1$$

64. $g(x)=-x+3\Rightarrow g(-2)=-(-2)+3=5$
$$(g\circ g)(-2)=g[g(-2)\ \ =g(5)=-5+3=-2$$

65. $(f\circ g)(2)=f[g(2)]=f(3)=1$

66. $(f\circ g)(7)=f[g(7)]=f(6)=9$

67. $(g\circ f)(3)=g[f(3)]=g(1)=9$

68. $(g\circ f)(6)=g[f(6)]=g(9)=12$

69. $(f\circ f)(4)=f[f(4)]=f(3)=1$

70. $(g\circ g)(1)=g[g(1)]=g(9)=12$

71. $(f\circ g)(1)=f[g(1)]=f(9)$
However, $f(9)$ cannot be determined from the table given.

72. $(g\circ(f\circ g))(7)=g(f(g(7)))$
$$=g(f(6))=g(9)=12$$

73. **(a)** $(f\circ g)(x)=f(g(x))=f(5x+7)$

57. $g(x) = -x + 3 \Rightarrow g(4) = -4 + 3 = -1$

$$(f \circ g)(4) = f\left[g(4)\right] = f(-1)$$

$$= 2(-1) - 3 = -2 - 3 = -5$$

$$= -6(5x + 7) + 9$$

$$= -30x - 42 + 9 = -30x - 33$$

The domain and range of both *f* and *g* are $(-\infty, \infty)$, so the domain of $f \circ g$ is $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g(-6x+9)$

$$= 5(-6x+9)+7$$
$$= -30x+45+7 = -30x+52$$

The domain of $g \circ f$ is $(-\infty, \infty)$.

74. (a) $(f \circ g)(x) = f(g(x)) = f(3x-1)$
$$= 8(3x-1)+12$$
$$= 24x-8+12 = 24x+4$$

The domain and range of both *f* and *g* are $(-\infty, \infty)$, so the domain of $f \circ g$ is $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g(8x+12)$
$$= 3(8x+12)-1$$
$$= 24x+36-1 = 24x+35$$

The domain of $g \circ f$ is $(-\infty, \infty)$.

75. (a) $(f \circ g)(x) = f(g(x)) = f(x+3) = \sqrt{x+3}$

The domain and range of *g* are $(-\infty, \infty)$, however, the domain and range of *f* are $[0, \infty)$. So, $x+3 \geq 0 \Rightarrow x \geq -3$.

Therefore, the domain of $f \circ g$ is

$[-3, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \sqrt{x}+3$

The domain and range of *g* are $(-\infty, \infty)$, however, the domain and range of *f* are $[0, \infty)$ .Therefore, the domain of $g \circ f$ is $[0, \infty)$.

76. (a) $(f \circ g)(x) = f(g(x)) = f(x-1) = \sqrt{x-1}$

The domain and range of *g* are $(-\infty, \infty)$, however, the domain and range of *f* are $[0, \infty)$. So, $x-1 \geq 0 \Rightarrow x \geq 1$. Therefore,

the domain of $f \circ g$ is $[1, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \sqrt{x}-1$

The domain and range of *g* are $(-\infty, \infty)$,

(b) $(g \circ f)(x) = g(f(x)) = g(x^3)$

$$= \left(x^3\right)^2 + 3\left(x^3\right) - 1$$

$$= x^6 + 3x^3 - 1$$

The domain and range of *f* and *g* are $(-\infty, \infty)$, so the domain of $g \circ f$ is $(-\infty, \infty)$.

78. (a) $(f \circ g)(x) = f(g(x)) = f(x^4+x^2-4)$
$$= x^4+x^2-4+2$$
$$= x^4+x^2-2$$

The domain of *f* and *g* is $(-\infty, \infty)$, while the range of *f* is $(-\infty, \infty)$ and the range of *g* is $[-4, \infty)$, so the domain of $f \circ g$ is $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g(x+2)$

$$= (x+2)^4 + (x+2)^2 - 4$$

The domain of *f* and *g* is $(-\infty, \infty)$, while the range of *f* is $(-\infty, \infty)$ and the range of *g* is $[-4, \infty)$, so the domain of $g \circ f$ is $(-\infty, \infty)$.

79. (a) $(f \circ g)(x) = f(g(x)) = f(3x) = \sqrt{3x-1}$

The domain and range of *g* are $(-\infty, \infty)$, however, the domain of *f* is $[1, \infty)$, while the range of *f* is $[0, \infty)$. So, $3x-1 \geq 0 \Rightarrow x \geq \frac{1}{3}$. Therefore, the domain of $f \circ g$ is $\left[\frac{1}{3}, \infty\right)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x-1}\right)$
$$= 3\sqrt{x-1}$$

The domain and range of *g* are $(-\infty, \infty)$, however, the range of *f* is $[0, \infty)$. So

$x-1 \geq 0 \Rightarrow x \geq 1$. Therefore, the domain however, the domain and range of *f* are $[0,\infty)$. Therefore, the domain of $g\circ f$ is

$[0,\infty)$.

77. (a) $(f\circ g)(x) = f(g(x)) = f(x^2+3x-1)$
$= (x^2+3x-1)^3$

The domain and range of *f* and *g* are $(-\infty,\infty)$, so the domain of $f\circ g$ is $(-\infty,\infty)$.

of $g\circ f$ is $[1,\infty)$.

80. (a) $(f\circ g)(x) = f(g(x)) = f(2x) = \sqrt{2x-2}$

The domain and range of *g* are $(-\infty,\infty)$, however, the domain of *f* is $[2,\infty)$. So, $2x-2\ge 0 \Rightarrow x\ge 1$. Therefore, the domain of $f\circ g$ is $[1,\infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x-2}\right)$

$= 2\sqrt{x-2}$

The domain and range of *g* are $(-\infty, \infty)$, however, the range of *f* is $[0, \infty)$. So

$x - 2 \ge 0 \Rightarrow x \ge 2$. Therefore, the domain

of $g \circ f$ is $[2, \infty)$.

81. **(a)** $(f \circ g)(x) = f(g(x)) = f(x+1) = \frac{2}{x+1}$

The domain and range of *g* are $(-\infty, \infty)$, however, the domain of *f* is $(-\infty, 0) \cup (0, \infty)$. So, $x + 1 \ne 0 \Rightarrow x \ne -1$.

Therefore, the domain of $f \circ g$ is

$(-\infty, -1) \cup (-1, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\frac{2}{x}\right) = \frac{2}{x} + 1$

The domain and range of *f* is $(-\infty, 0) \cup (0, \infty)$, however, the domain and range of *g* are $(-\infty, \infty)$. So $x \ne 0$.

Therefore, the domain of $g \circ f$ is $(-\infty, 0) \cup (0, \infty)$.

82. **(a)** $(f \circ g)(x) = f(g(x)) = f(x+4) = \frac{4}{x+4}$

The domain and range of *g* are $(-\infty, \infty)$,

however, the domain of *f* is $(-\infty, 0) \cup (0, \infty)$. So, $x + 4 \ne 0 \Rightarrow x \ne -4$. Therefore, the domain of $f \circ g$ is $(-\infty, -4) \cup (-4, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\frac{4}{x}\right) = \frac{4}{x} + 4$

The domain and range of *f* is $(-\infty, 0) \cup (0, \infty)$, however, the domain and range of *g* are $(-\infty, \infty)$. So $x \ne 0$.

Therefore, the domain of $g \circ f$ is $(-\infty, 0) \cup (0, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x+2}\right) = -\frac{1}{\sqrt{x+2}}$

The domain of *f* is $[-2, \infty)$ and its range is $[0, \infty)$. The domain and range of *g* are $(-\infty, 0) \cup (0, \infty)$. So $x + 2 > 0 \Rightarrow x > -2$.

Therefore, the domain of $g \circ f$ is $(-2, \infty)$.

84. **(a)** $(f \circ g)(x) = f(g(x)) = f\left(-\frac{2}{x}\right) = \sqrt{-\frac{2}{x} + 4}$

The domain and range of *g* are $(-\infty, 0) \cup (0, \infty)$, however, the domain of *f* is $[-4, \infty)$. So, $-\frac{2}{x} + 4 \ge 0 \Rightarrow$

$x < 0$ or $x \ge \frac{1}{2}$ (using test intervals).

Therefore, the domain of $f \circ g$ is

$(-\infty, 0) \cup \left[\frac{1}{2}, \infty\right)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x+4}\right) = -\frac{2}{\sqrt{x+4}}$

The domain of *f* is $[-4, \infty)$ and its range is $[0, \infty)$. The domain and range of *g* are $(-\infty, 0) \cup (0, \infty)$. So $x + 4 > 0 \Rightarrow x > -4$.

Therefore, the domain of $g \circ f$ is $(-4, \infty)$.

85. **(a)** $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x+5}\right) = \sqrt{\frac{1}{x+5}}$

The domain of *g* is $(-\infty, -5) \cup (-5, \infty)$, and the range of *g* is $(-\infty, 0) \cup (0, \infty)$. The domain of *f* is $[0, \infty)$. Therefore, the domain of $f \circ g$ is $(-5, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \frac{1}{\sqrt{x}+5}$

The domain and range of *f* is $[0, \infty)$. The domain of *g* is $(-\infty, -5) \cup (-5, \infty)$.

Therefore, the domain of $g \circ f$ is $[0, \infty)$.

86. **(a)** $\left(\frac{3}{x}\right) = \sqrt{\frac{3}{x}}$

83. **(a)** $(f \circ g)(x) = f(g(x)) = f\left(-\frac{1}{x}\right) = -\frac{1}{x} + 2$

The domain and range of g are $(-\infty, 0) \cup (0, \infty)$, however, the domain of f is $[-2, \infty)$. So, $-\frac{1}{x} + 2 \geq 0 \Rightarrow$

$x < 0$ or $x \geq \frac{1}{2}$ (using test intervals).

Therefore, the domain of $f \circ g$ is

$(-\infty, 0) \cup \left[\frac{1}{2}, \infty\right)$.

$(f \circ g)(x) = f(g(x)) = f\left(x+6\right) = \sqrt{x+6}$

The domain of g is $(-\infty, -6) \cup (-6, \infty)$, and the range of g is $(-\infty, 0) \cup (0, \infty)$. The domain of f is $[0, \infty)$. Therefore, the domain of $f \circ g$ is $(-6, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \frac{3}{\sqrt{x} + 6}$

The domain and range of f is $[0, \infty)$. The domain of g is $(-\infty, -6) \cup (-6, \infty)$.

Therefore, the domain of $g \circ f$ is $[0, \infty)$.

87. (a) $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x - 2} = \frac{x}{1-2x}$

The domain and range of g are $(-\infty, 0) \cup (0, \infty)$. The domain of f is $(-\infty, -2) \cup (-2, \infty)$, and the range of f is $(-\infty, 0) \cup (0, \infty)$. So, $\frac{x}{1-2x} < 0 \Rightarrow x < 0$ or $0 < x < \frac{1}{2}$ or $x > \frac{1}{2}$ (using test intervals).

Thus, $x \neq 0$ and $x \neq \frac{1}{2}$. Therefore, the domain of $f \circ g$ is

$$\left(-\infty, 0\right) \cup \left(0, \tfrac{1}{2}\right) \cup \left(\tfrac{1}{2}, \infty\right).$$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x-2}\right) = \frac{1}{1/(x-2)}$
$= x - 2$

The domain and range of g are $(-\infty, 0) \cup (0, \infty)$. The domain of f is $(-\infty, 2) \cup (2, \infty)$, and the range of f is $(-\infty, 0) \cup (0, \infty)$. Therefore, the domain of $g \circ f$ is $(-\infty, 2) \cup (2, \infty)$.

88. (a) $(f \circ g)(x) = f(g(x)) = f\left(-\frac{1}{x}\right) = \frac{1}{-1/x + 4}$

$= \frac{x}{-1+4x}$

The domain and range of g are $(-\infty, 0) \cup (0, \infty)$. The domain of f is $(-\infty, -4) \cup (-4, \infty)$, and the range of f is $(-\infty, 0) \cup (0, \infty)$. So, $\frac{x}{-1+4x} < 0 \Rightarrow x < 0$ or $0 < x < \frac{1}{4}$ or $-1 + 4x < 0 \Rightarrow x > \frac{1}{4}$ (using test intervals). Thus, $x \neq 0$ and $x \neq \frac{1}{4}$. Therefore, the domain of $f \circ g$ is

$$\left(-\infty, 0\right) \cup \left(0, \tfrac{1}{4}\right) \cup \left(\tfrac{1}{4}, \infty\right).$$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x+4}\right) = -\frac{1}{1/(x+4)}$

$= -x - 4$

The domain and range of g are

90. $f(x)$ is odd, so $f(-1) = -f(1) = -(-2) = 2.$

Because $g(x)$ is even, $g(1) = g(-1) = 2$ and $g(2) = g(-2) = 0.$ $(f \circ g)(-1) = 1,$ so $f[g(-1)] = 1$ and $f(2) = 1.$ $f(x)$ is odd, so $f(-2) = -f(2) = -1.$ Thus,
$(f \circ g)(-2) = f[g(-2)] = f(0) = 0$ and
$(f \circ g)(1) = f[g(1)] = f(2) = 1$ and
$(f \circ g)(2) = f[g(2)] = f(0) = 0.$

x	–2	–1	0	1	2
$f(x)$	**–1**	**2**	0	–2	**1**
$g(x)$	0	2	1	**2**	**0**
$(f \circ g)(x)$	**0**	1	–2	**1**	**0**

91. Answers will vary. In general, composition of functions is not commutative. Sample answer:

$$(f \circ g)(x) = f(2x-3) = 3(2x-3) - 2 = 6x - 9 - 2 = 6x - 11$$
$$(g \circ f)(x) = g(3x-2) = 2(3x-2) - 3 = 6x - 4 - 3 = 6x - 7$$

Thus, $(f \circ g)(x)$ is not equivalent to $(g \circ f)(x)$.

92.
$$(f \circ g)(x) = f[g(x)] = f\left(\sqrt[3]{x-7}\right) = \left(\sqrt[3]{x-7}\right)^3 + 7 = (x-7) + 7 = x$$
$$(g \circ f)(x) = g(f(x)) = g\left(x^3+7\right) = \sqrt[3]{\left(x^3+7\right) - 7} = \sqrt[3]{x^3} = x$$

93. $(f \circ g)(x) = f[g(x)] = 4\left[\frac{1}{4}(x-2)\right] + 2$

$(-\infty, 0) \cup (0, \infty)$. The domain of f is $(-\infty, -4) \cup (-4, \infty)$, and the range of f is

$= \left(4 \cdot \frac{1}{4}\right)(x-2)+2$

$(-\infty, 0) \cup (0, \infty)$. Therefore, the domain of

$g \circ f$ is $(-\infty, -4) \cup (-4, \infty)$.

89. $g\left[f(2)\right] = g(1) = 2$ and $g\left[f(3)\right] = g(2) = 5$
Since $g\left[f(1)\right] = 7$ and $f(1) = 3$, $g(3) = 7$.

x	$f(x)$	$g(x)$	$g\left[f(x)\right]$
1	3	2	7
2	1	5	**2**
3	2	**7**	**5**

$$= (x-2)+2 = x-2+2 = x$$

$$(g \circ f)(x) = g\left[f(x)\right] = \tfrac{1}{4}\left[(4x+2)-2\right]$$
$$= \tfrac{1}{4}(4x+2-2) = \tfrac{1}{4}(4x) = x$$

94. $(f \circ g)(x) = f\left[g(x)\right] = -3\left(-\frac{1}{3}x\right)$
$$= \left[-3\left(-\tfrac{1}{3}\right)\right]x = x$$

$$(g \circ f)(x) = g\left[f(x)\right] = -\tfrac{1}{3}(-3x)$$
$$= \left[-\tfrac{1}{3}(-3)\right]x = x$$

95. $(f \circ g)(x) = f\left[g(x)\right] = \sqrt[3]{5\left(\frac{1}{5}x^3 - \frac{4}{5}\right) + 4}$
$= \sqrt[3]{x^3 - 4 + 4} = \sqrt[3]{x^3} = x$

$(g \circ f)(x) = g\left[f(x)\right] = \frac{1}{5}\left(\sqrt[3]{5x+4}\right)^3 - \frac{4}{5}$
$= \frac{1}{5}(5x+4) - \frac{4}{5} = \frac{5x}{5} + \frac{4}{5} - \frac{4}{5}$
$= \frac{5x}{5} = x$

96. $(f \circ g)(x) = f\left[g(x)\right] = \sqrt[3]{(x^3 - 1) + 1}$
$= \sqrt[3]{x^3 - 1 + 1} = \sqrt[3]{x^3} = x$

$(g \circ f)(x) = g\left[f(x)\right] = \left(\sqrt[3]{x+1}\right)^3 - 1$
$= x + 1 - 1 = x$

In Exercises 97−102, we give only one of many possible answers.

97. $h(x) = (6x - 2)^2$

Let $g(x) = 6x - 2$ and $f(x) = x^2$.
$(f \circ g)(x) = f(6x - 2) = (6x - 2)^2 = h(x)$

98. $h(x) = (11x^2 + 12x)^2$

Let $g(x) = 11x^2 + 12x$ and $f(x) = x^2$.

$(f \circ g)(x) = f(11x^2 + 12x)$
$= (11x^2 + 12x)^2 = h(x)$

99. $h(x) = \sqrt{x^2 - 1}$

Let $g(x) = x^2 - 1$ and $f(x) = \sqrt{x}$.

$(f \circ g)(x) = f(x^2 - 1) = \sqrt{x^2 - 1} = h(x)$.

100. $h(x) = (2x - 3)^3$

Let $g(x) = 2x - 3$ and $f(x) = x^3$.
$(f \circ g)(x) = f(2x - 3) = (2x - 3)^3 = h(x)$

101. $h(x) = \sqrt{6x} + 12$

104. $f(x) = 3x,\ g(x) = 1760x$

$(f \circ g)(x) = f\left(g(x)\right) = f(1760x)$

$= 3(1760x) = 5280x$

$(f \circ g)(x)$ compute the number of feet in x miles.

105. $\mathcal{A}(x) = \frac{\sqrt{3}}{4}x^2$

(a) $\mathcal{A}(2x) = \frac{\sqrt{3}}{4}(2x)^2 = \frac{\sqrt{3}}{4}(4x^2) = \sqrt{3}x^2$

(b) $\mathcal{A}(16) = A(2 \cdot 8) = \sqrt{3}(8)^2$
$= 64\sqrt{3}$ square units

106. (a) $x = 4s \Rightarrow \frac{x}{4} = s \Rightarrow s = \frac{x}{4}$

(b) $y = s^2 = \left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$

(c) $y = \frac{6^2}{16} = \frac{36}{16} = 2.25$ square units

107. (a) $r(t) = 4t$ and $\mathcal{A}(r) = \pi r^2$
$(\mathcal{A} \circ r)(t) = \mathcal{A}[r(t)]$
$= \mathcal{A}(4t) = \pi(4t)^2 = 16\pi t^2$

(b) $(\mathcal{A} \circ r)(t)$ defines the area of the leak in terms of the time t, in minutes.

(c) $\mathcal{A}(3) = 16\pi(3)^2 = 144\pi\ \text{ft}^2$

108. (a) $(\mathcal{A} \circ r)(t) = \mathcal{A}[r(t)]$
$= \mathcal{A}(2t) = \pi(2t)^2 = 4\pi t^2$

(b) It defines the area of the circular layer in terms of the time t, in hours.

(c) $(\mathcal{A} \circ r)(4) = 4\pi(4)^2 = 64\pi\ \text{mi}^2$

Let $g(x) = 6x$ and $f(x) = \sqrt{x} + 12$.

$(f \circ g)(x) = f(6x) = \sqrt{6x} + 12 = h(x)$

102. $h(x) = \sqrt[3]{2x+3} - 4$

Let $g(x) = 2x + 3$ and $f(x) = \sqrt[3]{x} - 4$.

$(f \circ g)(x) = f(2x+3) = \sqrt[3]{2x+3} - 4 = h(x)$

103. $f(x) = 12x$, $g(x) = 5280x$

$$\begin{aligned}(f \circ g)(x) &= f[g(x)] = f(5280x)\\ &= 12(5280x) = 63,360x\end{aligned}$$

The function $f \circ g$ computes the number of inches in x miles.

109. Let x = the number of people less than 100 people that attend.

(a) x people fewer than 100 attend, so $100 - x$ people do attend $N(x) = 100 - x$

(b) The cost per person starts at \$20 and

increases by \$5 for each of the x people that do not attend. The total increase is \$5x, and the cost per person increases to \$20 + \$5x. Thus, $G(x) = 20 + 5x$.

(c) $C(x) = N(x) \cdot G(x) = (100 - x)(20 + 5x)$

(d) If 80 people attend, $x = 100 - 80 = 20$.

$$C(20) = (100-20)[20+5(20)]$$
$$= (80)(20+100)$$
$$= (80)(120) = \$9600$$

110. (a) $y_1 = 0.02x$

(b) $y_2 = 0.015(x+500)$

(c) $y_1 + y_2$ represents the total annual interest.

(d) $(y_1 + y_2)(250)$

$$= y_1(250) + y_2(250)$$
$$= 0.02(250) + 0.015(250+500)$$
$$= 5 + 0.015(750) = 15 + 11.25$$
$$= \$16.25$$

111. (a) $g(x) = \frac{1}{2}x$

(b) $f(x) = x+1$

(c) $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{2}x\right) = \frac{1}{2}x + 1$

(d) $(f \circ g)(60) = \frac{1}{2}(60) + 1 = \$31$

112. If the area of a square is x^2 square inches, each side must have a length of x inches. If 3 inches is added to one dimension and 1 inch is subtracted from the other, the new dimensions will be $x + 3$ and $x - 1$. Thus, the area of the resulting rectangle is □$(x) = (x + 3)(x - 1)$.

Chapter 2 Review Exercises

1. $P(3, -1)$, $Q(-4, 5)$

$$d(P, Q) = \sqrt{(-4-3)^2 + [5-(-1)]^2}$$
$$= \sqrt{(-7)^2 + 6^2} = \sqrt{49+36} = \sqrt{85}$$

Midpoint:

$$\left(\frac{3+(-4)}{2}, \frac{-1+5}{2}\right) = \left(\frac{-1}{2}, \frac{4}{2}\right) = \left(-\frac{1}{2}, 2\right)$$

3. $A(-6, 3)$, $B(-6, 8)$

$$d(A, B) = \sqrt{[-6-(-6)]^2 + (8-3)^2}$$
$$= \sqrt{0+5^2} = \sqrt{25} = 5$$

Midpoint:

$$\left(\frac{-6+(-6)}{2}, \frac{3+8}{2}\right) = \left(\frac{-12}{2}, \frac{11}{2}\right) = \left(-6, \frac{11}{2}\right)$$

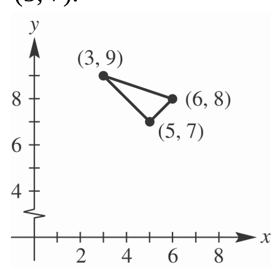
4. Label the points $A(5, 7)$, $B(3, 9)$, and $C(6, 8)$.

$$d(A, B) = \sqrt{(3-5)^2 + (9-7)^2}$$
$$= \sqrt{(-2)^2 + 2^2} = \sqrt{4+4} = \sqrt{8}$$

$$d(A, C) = \sqrt{(6-5)^2 + (8-7)^2}$$
$$= \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

$$d(B, C) = \sqrt{(6-3)^2 + (8-9)^2}$$
$$= \sqrt{3^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Because $\left(\sqrt{8}\right)^2 + \left(\sqrt{2}\right)^2 = \left(\sqrt{10}\right)^2$, triangle ABC is a right triangle with right angle at $(5, 7)$.



5. Let B have coordinates (x, y). Using the midpoint formula, we have

$$(8, 2) = \left(\frac{-6+x}{2}, \frac{10+y}{2}\right) \Rightarrow$$

$$\frac{-6+x}{2} = 8 \qquad \frac{10+y}{2} = 2$$
$$-6 + x = 16 \qquad 10 + y = 4$$
$$x = 22 \qquad y = -6$$

The coordinates of B are (22, −6).

2 2 2 2 2

2. $M(-8, 2)$, $N(3, -7)$

$$d(M, N) = \sqrt{[3-(-8)]^2 + (-7-2)^2} = \sqrt{11^2 + (-9)^2} = \sqrt{121+81} = \sqrt{202}$$

Midpoint: $\left(\frac{-8+3}{2}, \frac{2+(-7)}{2}\right) = \left(-\frac{5}{2}, -\frac{5}{2}\right)$

6. $P(-2, -5)$, $Q(1, 7)$, $R(3, 15)$

$$d(P, Q) = \sqrt{(1-(-2))^2 + (7-(-5))^2} = \sqrt{(3)^2 + (12)^2} = \sqrt{9+144} = \sqrt{153} = 3\sqrt{17}$$

$$d(Q, R) = \sqrt{(3-1)^2 + (15-7)^2} = \sqrt{2^2 + 8^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$$

$$d(P, R) = \sqrt{(3-(-2))^2 + (15-(-5))^2} = \sqrt{(5)^2 + (20)^2} = \sqrt{25+400} = 5\sqrt{17}$$

(*continued on next page*)

(*continued*)

$d(P, Q) + d(Q, R) = 3\sqrt{17} + 2\sqrt{17}$
$= 5\sqrt{17} = d(P, R)$, so these three points are collinear.

7. Center (–2, 3), radius 15

$$(x-h)^2 + (y-k)^2 = r^2$$
$$[x-(-2)]^2 + (y-3)^2 = 15^2$$
$$(x+2)^2 + (y-3)^2 = 225$$

8. Center $(\sqrt{5}, -\sqrt{7})$, radius $\sqrt{3}$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$\left(x-\sqrt{5}\right)^2 + \left[y-\left(-\sqrt{7}\right)\right]^2 = \left(\sqrt{3}\right)^2$$
$$\left(x-\sqrt{5}\right)^2 + \left(y+\sqrt{7}\right)^2 = 3$$

9. Center (–8, 1), passing through (0, 16)
The radius is the distance from the center to any point on the circle. The distance between (–8, 1) and (0, 16) is

$$r = \sqrt{(0-(-8))^2 + (16-1)^2} = \sqrt{8^2 + 15^2}$$
$$= \sqrt{64+225} = \sqrt{289} = 17.$$

The equation of the circle is

$$[x-(-8)]^2 + (y-1)^2 = 17^2$$
$$(x+8)^2 + (y-1)^2 = 289$$

10. Center (3, –6), tangent to the x-axis

The point (3, –6) is 6 units directly below the x-axis. Any segment joining a circle's center to a point on the circle must be a radius, so in this case the length of the radius is 6 units.

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-3)^2 + [y-(-6)]^2 = 6^2$$
$$(x-3)^2 + (y+6)^2 = 36$$

14. The center of the circle is (5, 6). Use the distance formula to find the radius:
$r^2 = (4-5)^2 + (9-6)^2 = 1 + 9 = 10$
The equation is $(x-5)^2 + (y-6)^2 = 10$.

15. $x^2 - 4x + y^2 + 6y + 12 = 0$
Complete the square on x and y to put the equation in center-radius form.

$$\left(x^2 - 4x\right) + \left(y^2 + 6y\right) = -12$$
$$\left(x^2 - 4x + 4\right) + \left(y^2 + 6y + 9\right) = -12 + 4 + 9$$
$$\left(x-2\right)^2 + \left(y+3\right)^2 = 1$$

The circle has center (2, –3) and radius 1.

16. $x^2 - 6x + y^2 - 10y + 30 = 0$

Complete the square on x and y to put the equation in center-radius form.

$$(x^2 - 6x + 9) + (y^2 - 10y + 25) = -30 + 9 + 25$$
$$(x-3)^2 + (y-5)^2 = 4$$

The circle has center (3, 5) and radius 2.

17.
$$2x^2 + 14x + 2y^2 + 6y + 2 = 0$$
$$x^2 + 7x + y^2 + 3y + 1 = 0$$
$$\left(x^2 + 7x\right) + \left(y^2 + 3y\right) = -1$$
$$\left(x^2 + 7x + \tfrac{49}{4}\right) + \left(y^2 + 3y + \tfrac{9}{4}\right) = -1 + \tfrac{49}{4} + \tfrac{9}{4}$$
$$\left(x + \tfrac{7}{2}\right)^2 + \left(y + \tfrac{3}{2}\right)^2 = -\tfrac{4}{4} + \tfrac{49}{4} + \tfrac{9}{4}$$
$$\left(x + \tfrac{7}{2}\right)^2 + \left(y + \tfrac{3}{2}\right)^2 = \tfrac{54}{4}$$

The circle has center $\left(-\frac{7}{2}, -\frac{3}{2}\right)$ and radius

$$\sqrt{\tfrac{54}{4}} = \tfrac{\sqrt{54}}{\sqrt{4}} = \tfrac{\sqrt{9 \cdot 6}}{\sqrt{4}} = \tfrac{3\sqrt{6}}{2}.$$

18.
$$3x^2 + 33x + 3y^2 - 15y = 0$$

11. The center of the circle is (0, 0). Use the distance formula to find the radius:

$$r^2 = (3-0)^2 + (5-0)^2 = 9 + 25 = 34$$

The equation is $x^2 + y^2 = 34$.

12. The center of the circle is (0, 0). Use the distance formula to find the radius:

$$r^2 = (-2-0)^2 + (3-0)^2 = 4 + 9 = 13$$

The equation is $x^2 + y^2 = 13$.

13. The center of the circle is (0, 3). Use the distance formula to find the radius:

$$r^2 = (-2-0)^2 + (6-3)^2 = 4 + 9 = 13$$

The equation is $x^2 + (y-3)^2 = 13$.

$$x^2 + 11x + y^2 - 5y = 0$$
$$\left(x^2 + 11x\right) + \left(y^2 - 5y\right) = 0$$
$$\left(x^2 + 11x + \tfrac{121}{4}\right) + \left(y^2 - 5y + \tfrac{25}{4}\right) = 0 + \tfrac{121}{4} + \tfrac{25}{4}$$
$$\left(x + \tfrac{11}{2}\right)^2 + \left(y - \tfrac{5}{2}\right)^2 = \tfrac{146}{4}$$

The circle has center $\left(-\frac{11}{2}, \frac{5}{2}\right)$ and radius $\frac{\sqrt{146}}{2}$.

19. This is not the graph of a function because a vertical line can intersect it in two points. domain: [−6, 6]; range: [−6, 6]

20. This is not the graph of a function because a vertical line can intersect it in two points. domain: $(-\infty, \infty)$; range: $[0, \infty)$

21. This is not the graph of a function because a vertical line can intersect it in two points.
domain: $(-\infty, \infty)$; range: $(-\infty, -1] \cup [1, \infty)$

22. This is the graph of a function. No vertical line will intersect the graph in more than one point.
domain: $(-\infty, \infty)$; range: $[0, \infty)$

23. This is not the graph of a function because a vertical line can intersect it in two points.
domain: $[0, \infty)$; range: $(-\infty, \infty)$

24. This is the graph of a function. No vertical line will intersect the graph in more than one point.
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

25. $y = 6 - x^2$
Each value of *x* corresponds to exactly one value of *y*, so this equation defines a function.

26. The equation $x = \frac{1}{3}y^2$ does not define *y* as a function of *x*. For some values of *x*, there will be more than one value of *y*. For example, ordered pairs (3, 3) and (3, –3) satisfy the relation. Thus, the relation would not be a function.

27. The equation $y = \pm\sqrt{x-2}$ does not define *y* as a function of *x*. For some values of *x*, there will be more than one value of *y*. For example, ordered pairs (3, 1) and (3, −1) satisfy the relation.

28. The equation $y = -\dfrac{4}{x}$ defines *y* as a function of *x* because for every *x* in the domain, which is $(-\infty, 0) \cup (0, \infty)$, there will be exactly one value of *y*.

29. In the function $f(x) = -4 + |x|$, we may use any real number for *x*. The domain is $(-\infty, \infty)$.

30. $f(x) = \dfrac{8+x}{8-x}$
x can be any real number except 8 because this will give a denominator of zero. Thus, the domain is $(-\infty, 8) \cup (8, \infty)$.

31. $f(x) = \sqrt{6-3x}$

32. **(a)** As *x* is getting larger on the interval $(2, \infty)$, the value of *y* is increasing.

(b) As *x* is getting larger on the interval $(-\infty, -2)$, the value of *y* is decreasing.

(c) *f*(*x*) is constant on (−2, 2).

In exercises 33–36, $f(x) = -2x^2 + 3x - 6$.

33. $$\begin{aligned} f(3) &= -2 \cdot 3^2 + 3 \cdot 3 - 6 \\ &= -2 \cdot 9 + 3 \cdot 3 - 6 \\ &= -18 + 9 - 6 = -15 \end{aligned}$$

34. $$\begin{aligned} f(-0.5) &= -2(-0.5)^2 + 3(-0.5) - 6 \\ &= -2(0.25) + 3(-0.5) - 6 \\ &= -0.5 - 1.5 - 6 = -8 \end{aligned}$$

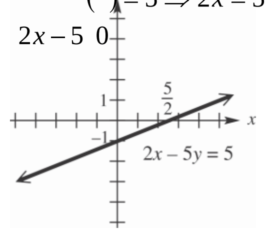
35. $f(0) = -2(0)^2 + 3(0) - 6 = -6$

36. $f(k) = -2k^2 + 3k - 6$

37. $2x - 5y = 5 \Rightarrow -5y = -2x + 5 \Rightarrow y = \frac{2}{5}x - 1$

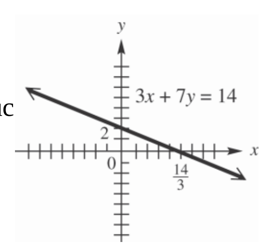
The graph is the line with slope $\frac{5}{2}$ and *y*-intercept (0, –)1. It may also be graphed using intercepts. To do this, locate the *x*-intercept: $y = 0$

$2x - 5(0) = 5 \Rightarrow 2x = 5 \Rightarrow x = \frac{5}{2}$



38. $3x + 7y = 14 \Rightarrow 7y = -3x + 14 \Rightarrow y = -\frac{3}{7}x + 2$

The graph is the line with slope of $-\frac{3}{7}$ and *y*-intercept (0, 2). It may also be graphed using intercepts. To do this, locate the *x*-intercept by setting *y* = 0:
$3x + 7(0) = 14 \Rightarrow 3x = 14 \Rightarrow x = \frac{14}{3}$



In the function $y = \sqrt{6-3x}$, we must have $6-3x \geq 0$.

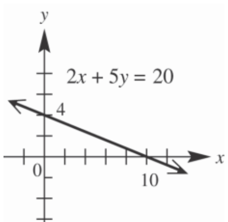
$6-3x \geq 0 \Rightarrow 6 \geq 3x \Rightarrow 2 \geq x \Rightarrow x \leq 2$

Thus, the domain is $(-\infty, 2]$.

39. $2x + 5y = 20 \Rightarrow 5y = -2x + 20 \Rightarrow y = -\frac{2}{5}x + 4$

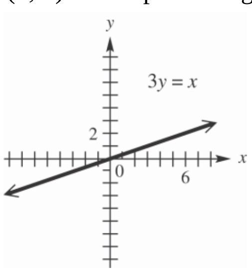
The graph is the line with slope of $-\frac{2}{5}$ and y-intercept (0, 4). It may also be graphed using intercepts. To do this, locate the x-intercept:
x-intercept: $y = 0$

$2x + 5(0) = 20 \Rightarrow 2x = 20 \Rightarrow x = 10$

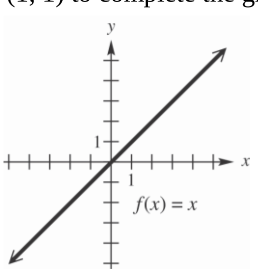


40. $3y = x \Rightarrow y = \frac{1}{3}x$

The graph is the line with slope $\frac{1}{3}$ and y-intercept (0, 0), which means that it passes through the origin. Use another point such as (6, 2) to complete the graph.

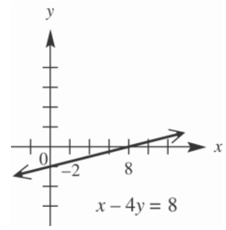


41. $f(x) = x$
The graph is the line with slope 1 and y-intercept (0, 0), which means that it passes through the origin. Use another point such as (1, 1) to complete the graph.

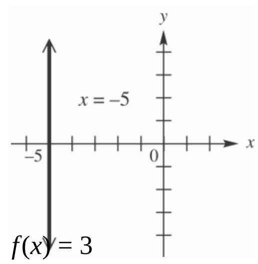


42. $x - 4y = 8$
$-4y = -x + 8$
$y = \frac{1}{4}x - 2$

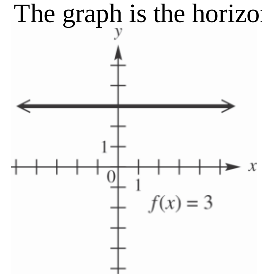
The graph is the line with slope $\frac{1}{4}$ and y-intercept (0, –2). It may also be graphed using intercepts. To do this, locate the x-intercept:
$y = 0 \Rightarrow x - 4(0) = 8 \Rightarrow x = 8$



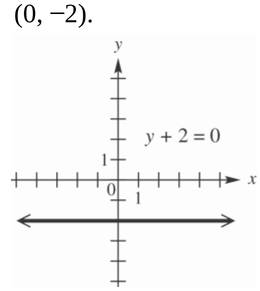
43. $x = -5$
The graph is the vertical line through (–5, 0).



44. $f(x) = 3$
The graph is the horizontal line through (0, 3).



45. $y + 2 = 0 \Rightarrow y = -2$
The graph is the horizontal line through (0, −2).



46. The equation of the line that lies along the x-axis is $\underline{y = 0}$.

47. Line through (0, 5), $m = -\frac{2}{3}$

Note that $m = -\frac{2}{3} = \frac{-2}{3}$.

Begin by locating the point (0, 5). Because the slope is $\frac{-2}{3}$, a change of 3 units horizontally (3 units to the right) produces a change of –2 units vertically (2 units down). This gives a second point, (3, 3), which can be used to complete the graph.

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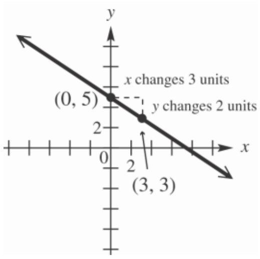
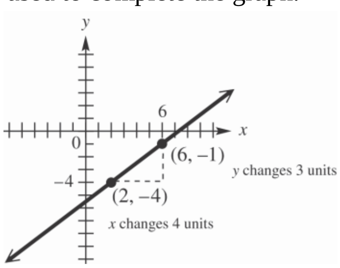


48. Line through (2, –4), $m = \frac{3}{4}$

First locate the point (2, –4).

Because the slope is $\frac{3}{4}$, a change of 4 units horizontally (4 units to the right) produces a change of 3 units vertically (3 units up). This gives a second point, (6, –1), which can be used to complete the graph.



49. through (2, –2) and (3, –4)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-(-2)}{3-2} = \frac{-2}{1} = -2$$

50. through (8, 7) and $\left(\frac{1}{2}, -2\right)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-7}{\frac{1}{2}-8} = \frac{-9}{-\frac{15}{2}}$$

$$= -9\left(-\frac{2}{15}\right) = \frac{18}{15} = \frac{6}{5}$$

51. through (0, –7) and (3, –7)

$$m = \frac{-7-(-7)}{3-0} = \frac{0}{3} = 0$$

52. through (5, 6) and (5, –2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-6}{5-5} = \frac{-8}{0}$$

The slope is undefined.

53. $11x + 2y = 3$

Solve for *y* to put the equation in slope-intercept form.

$2y = -11x + 3 \Rightarrow y = -\frac{11}{2}x + \frac{3}{2}$

Thus, the slope is $-\frac{11}{2}$.

54. $9x - 4y = 2$.

Solve for *y* to put the equation in slope-intercept form.

$-4y = -9x + 2 \Rightarrow y = \frac{9}{4}x - \frac{1}{2}$

Thus, the slope is $\frac{9}{4}$.

55. $x - 2 = 0 \Rightarrow x = 2$

The graph is a vertical line, through (2, 0). The slope is undefined.

56. $x - 5y = 0$.

Solve for *y* to put the equation in slope-intercept form.

$-5y = -x \Rightarrow y = \frac{1}{5}x$

Thus, the slope is $\frac{1}{5}$.

57. Initially, the car is at home. After traveling for 30 mph for 1 hr, the car is 30 mi away from home. During the second hour the car travels 20 mph until it is 50 mi away. During the third hour the car travels toward home at 30 mph until it is 20 mi away. During the fourth hour the car travels away from home at 40 mph until it is 60 mi away from home. During the last hour, the car travels 60 mi at 60 mph until it arrived home.

58. **(a)** This is the graph of a function because no vertical line intersects the graph in more than one point.

(b) The lowest point on the graph occurs in December, so the most jobs lost occurred in December. The highest point on the graph occurs in January, so the most jobs gained occurred in January.

(c) The number of jobs lost in December is approximately 6000. The number of jobs gained in January is approximately 2000.

(d) It shows a slight downward trend.

59. **(a)** We need to first find the slope of a line that passes between points (0, 30.7) and (12, 82.9)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{82.9 - 30.7}{12 - 0} = \frac{52.2}{12} = 4.35$$

Now use the point-intercept form with $b = 30.7$ and $m = 4.35$.
$y = 4.35x + 30.7$
The slope, 4.35, indicates that the number of e-filing taxpayers increased by 4.35% each year from 2001 to 2013.

(b) For 2009, we evaluate the function for $x = 8$. $y = 4.35(8) + 30.7 = 65.5$
65.5% of the tax returns are predicted to have been filed electronically.

60. We need to find the slope of a line that passes between points (1980, 21000) and (2013, 63800)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{63{,}800 - 21{,}000}{2013 - 1980}$$
$$= \frac{42{,}800}{33} \approx \$1297 \text{ per year}$$

The average rate of change was about \$1297 per year.

61. **(a)** through (3, –5) with slope –2

Use the point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-5) &= -2(x - 3) \\ y + 5 &= -2(x - 3) \\ y + 5 &= -2x + 6 \\ y &= -2x + 1 \end{aligned}$$

(b) Standard form: $y = -2x + 1 \Rightarrow 2x + y = 1$

62. **(a)** through (–2, 4) and (1, 3)

First find the slope.

$$m = \frac{3 - 4}{1 - (-2)} = \frac{-1}{3}$$

Now use the point-slope form with $(x_1, y_1) = (1, 3)$ and $m = -\frac{1}{3}$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= -\tfrac{1}{3}(x - 1) \\ 3(y - 3) &= -1(x - 1) \\ 3y - 9 &= -x + 1 \\ 3y &= -x + 10 \Rightarrow y = -\tfrac{1}{3}x + \tfrac{10}{3} \end{aligned}$$

(b) Standard form:

$$y = -\tfrac{1}{3}x + \tfrac{10}{3} \Rightarrow 3y = -x + 10 \Rightarrow$$
$$x + 3y = 10$$

63. **(a)** through (2, –1) parallel to $3x - y = 1$

Find the slope of $3x - y = 1$.

$$3x - y = 1 \Rightarrow -y = -3x + 1 \Rightarrow y = 3x - 1$$

The slope of this line is 3. Because parallel lines have the same slope, 3 is also the slope of the line whose equation is to be found. Now use the point-slope form with $(x_1, y_1) = (2, -1)$ and $m = 3$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-1) &= 3(x - 2) \\ y + 1 &= 3x - 6 \Rightarrow y = 3x - 7 \end{aligned}$$

(b) Standard form:

$$y = 3x - 7 \Rightarrow -3x + y = -7 \Rightarrow 3x - y = 7$$

64. **(a)** x-intercept (–3, 0), y-intercept (0, 5)

Two points of the line are (–3, 0) and (0, 5). First, find the slope.

$$m = \frac{5 - 0}{0 + 3} = \frac{5}{3}$$

The slope is $\frac{5}{3}$ and the y-intercept is (0, 5). Write the equation in slope-intercept form: $y = \frac{5}{3}x + 5$

(b) Standard form:

$$y = \tfrac{5}{3}x + 5 \Rightarrow 3y = 5x + 15 \Rightarrow$$
$$-5x + 3y = 15 \Rightarrow 5x - 3y = -15$$

65. **(a)** through (2, –10), perpendicular to a line with an undefined slope

A line with an undefined slope is a vertical line. Any line perpendicular to a vertical line is a horizontal line, with an equation of the form $y = b$. The line passes through (2, –10), so the equation of the line is $y = -10$.

(b) Standard form: $y = -10$

66. **(a)** through (0, 5), perpendicular to $8x + 5y = 3$

Find the slope of $8x + 5y = 3$.

$$8x + 5y = 3 \Rightarrow 5y = -8x + 3 \Rightarrow$$
$$y = -\tfrac{8}{5}x + \tfrac{3}{5}$$

The slope of this line is $-\frac{8}{5}$. The slope of any line perpendicular to this line is $\frac{5}{8}$, because $-\frac{8}{5}\left(\frac{5}{8}\right) = -1$.

The equation in slope-intercept form with slope $\frac{5}{8}$ and y-intercept (0, 5) is $y = \frac{5}{8}x + 5$.

(b) Standard form:

$$y = \tfrac{5}{8}x + 5 \Rightarrow 8y = 5x + 40 \Rightarrow$$
$$-5x + 8y = 40 \Rightarrow 5x - 8y = -40$$

67. **(a)** through (–7, 4), perpendicular to $y = 8$

The line $y = 8$ is a horizontal line, so any line perpendicular to it will be a vertical line. Because x has the same value at all points on the line, the equation is $x = -7$. It is not possible to write this in slope-intercept form.

(b) Standard form: $x = -7$

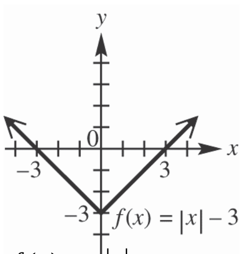
68. **(a)** through (3, –5), parallel to $y = 4$

This will be a horizontal line through (3, –5). Because y has the same value at all points on the line, the equation is $y = –5$.

(b) Standard form: $y = -5$

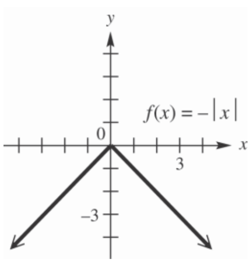
69. $f(x) = |x| - 3$

The graph is the same as that of $y = |x|$, except that it is translated 3 units downward.



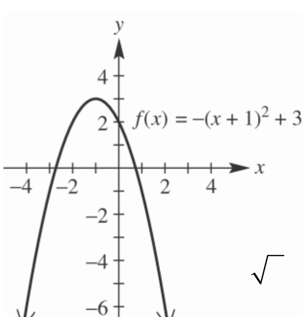
70. $f(x) = -|x|$

The graph of $f(x) = -|x|$ is the reflection of the graph of $y = |x|$ about the x-axis.



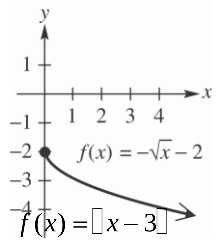
71. $f(x) = -(x+1)^2 + 3$

The graph of $f(x) = -(x+1)^2 + 3$ is a translation of the graph of $y = x^2$ to the left 1 unit, reflected over the x-axis and translated up 3 units.



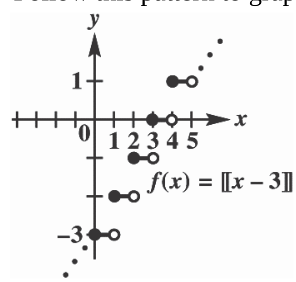
72. $f(x) = -\sqrt{x} - 2$

The graph of $f(x) = -\sqrt{x} - 2$ is the reflection of the graph of $y = \sqrt{x}$ about the x-axis, translated down 2 units.



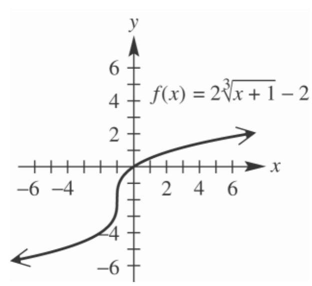
73. $f(x) = [\![x - 3]\!]$

To get $y = 0$, we need $0 \le x - 3 < 1 \Rightarrow 3 \le x < 4.$ To get $y = 1$, we need $1 \le x - 3 < 2 \Rightarrow 4 \le x < 5.$
Follow this pattern to graph the step function.



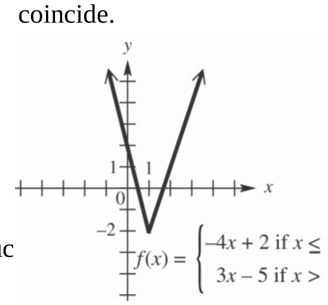
74. $f(x) = 2\sqrt[3]{x+1} - 2$

The graph of $f(x) = 2\sqrt[3]{x+1} - 2$ is a translation of the graph of $y = \sqrt[3]{x}$ to the left 1 unit, stretched vertically by a factor of 2, and translated down 2 units.



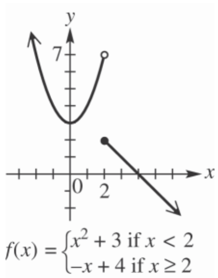
75. $f(x) = \begin{cases} -4x + 2 & \text{if } x \le 1 \\ 3x - 5 & \text{if } x > 1 \end{cases}$

Draw the graph of $y = -4x + 2$ to the left of $x = 1$, including the endpoint at $x = 1$. Draw the graph of $y = 3x - 5$ to the right of $x = 1$, but do not include the endpoint at $x = 1$. Observe that the endpoints of the two pieces coincide.



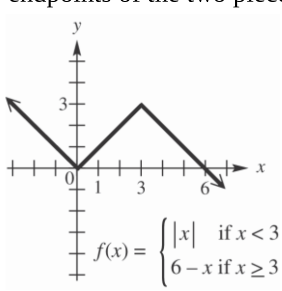
76. $f(x) = \begin{cases} x^2+3 & \text{if } x<2 \\ -x+4 & \text{if } x \geq 2 \end{cases}$

Graph the curve $y = x^2+3$ to the left of $x = 2$, and graph the line $y = -x + 4$ to the right of $x = 2$. The graph has an open circle at (2, 7) and a closed circle at (2, 2).



77. $f(x) = \begin{cases} |x| & \text{if } x<3 \\ 6-x & \text{if } x \geq 3 \end{cases}$

Draw the graph of $y = |x|$ to the left of $x = 3$, but do not include the endpoint. Draw the graph of $y = 6 - x$ to the right of $x = 3$, including the endpoint. Observe that the endpoints of the two pieces coincide.



78. Because x represents an integer, $[\![x]\!] = x$. Therefore, $[\![x]\!] + x = x + x = 2x$.

79. True. The graph of an even function is symmetric with respect to the y-axis.

80. True. The graph of a nonzero function cannot be symmetric with respect to the x-axis. Such a graph would fail the vertical line test

81. False. For example, $f(x) = x^2$ is even and (2, 4) is on the graph but (2, –4) is not.

82. True. The graph of an odd function is symmetric with respect to the origin.

83. True. The constant function $f(x) = 0$ is both

84. False. For example, $f(x) = x^3$ is odd, and (2, 8) is on the graph but (–2, 8) is not.

85. $x + y^2 = 10$

Replace x with $-x$ to obtain $(-x) + y^2 = 10.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain $x + (-y)^2 = 10 \Rightarrow x + y^2 = 10.$ The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain $(-x) + (-y)^2 = 10 \Rightarrow (-x) + y^2 = 10.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. The graph is symmetric with respect to the x-axis only.

86. $5y^2 + 5x^2 = 30$

Replace x with $-x$ to obtain $5y^2 + 5(-x)^2 = 30 \Rightarrow 5y^2 + 5x^2 = 30.$ The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain $5(-y)^2 + 5x^2 = 30 \Rightarrow 5y^2 + 5x^2 = 30.$ The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. The graph is symmetric with respect to the y-axis and x-axis, so it must also be symmetric with respect to the origin. Note that this equation is the same as $y^2 + x^2 = 6$, which is a circle centered at the origin.

87. $x^2 = y^3$

Replace x with $-x$ to obtain $(-x)^2 = y^3 \Rightarrow x^2 = y^3.$ The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain $x^2 = (-y)^3 \Rightarrow x^2 = -y^3.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain $(-x)^2 = (-y)^3 \Rightarrow x^2 = -y^3.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the y-axis only.

even and odd. Because $f(-x) = 0 = f(x)$,

the function is even. Also $f(-x) = 0 = -0 = -f(x)$, so the function is odd.

88. $y^3 = x + 4$

Replace x with $-x$ to obtain $y^3 = -x + 4$.

The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$(-y)^3 = x + 4 \Rightarrow -y^3 = x + 4 \Rightarrow y^3 = -x - 4$

The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain

$(-y)^3 = (-x) + 4 \Rightarrow -y^3 = -x + 4 \Rightarrow y^3 = x - 4.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph has none of the listed symmetries.

89. $6x + y = 4$

Replace x with $-x$ to obtain $6(-x) + y = 4 \Rightarrow -6x + y = 4$. The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain $6x + (-y) = 4 \Rightarrow 6x - y = 4$. The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain $6(-x) + (-y) = 4 \Rightarrow -6x - y = 4$. This equation is not equivalent to the original one, so the graph is not symmetric with respect to the origin. Therefore, the graph has none of the listed symmetries.

90. $|y| = -x$

Replace x with $-x$ to obtain

$|y| = -(-x) \Rightarrow |y| = x.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$|-y| = -x \Rightarrow |y| = -x.$ The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain

$|-y| = -(-x) \Rightarrow |y| = x.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph is symmetric with respect

92. $|x| = |y|$

Replace x with $-x$ to obtain

$|-x| = |y| \Rightarrow |x| = |y|$.

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$|x| = |-y| \Rightarrow |x| = |y|$. The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Because the graph is symmetric with respect to the x-axis and with respect to the y-axis, it must also by symmetric with respect to the origin.

93. $x^2 - y^2 = 0$

Replace x with $-x$ to obtain

$(-x)^2 - y^2 = 0 \Rightarrow x^2 - y^2 = 0.$ The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$x^2 - (-y)^2 = 0 \Rightarrow x^2 - y^2 = 0.$ The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Because the graph is symmetric with respect to the x-axis and with respect to the y-axis, it must also by symmetric with respect to the origin.

94. $x^2 + (y-2)^2 = 4$

Replace x with $-x$ to obtain

$(-x)^2 + (y-2)^2 = 4 \Rightarrow x^2 + (y-2)^2 = 4.$

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$x^2 + (-y-2)^2 = 4.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain

$(-x)^2 + (-y-2)^2 = 4 \Rightarrow x^2 + (-y-2)^2 = 4,$

which is not equivalent to the original equation. Therefore, the graph is not symmetric with respect to the origin.

95. To obtain the graph of $g(x) = -|x|$, reflect the graph of $f(x) = |x|$ across the x-axis.

to the x-axis only.

91. $y = 1$
This is the graph of a horizontal line through (0, 1). It is symmetric with respect to the y-axis, but not symmetric with respect to the x-axis and the origin.

96. To obtain the graph of $h(x) = x - 2$, translate the graph of $f(x) = |x|$ down 2 units.

97. To obtain the graph of $k(x) = 2|x-4|$, translate the graph of $f(x) = |x|$ to the right 4 units and stretch vertically by a factor of 2.

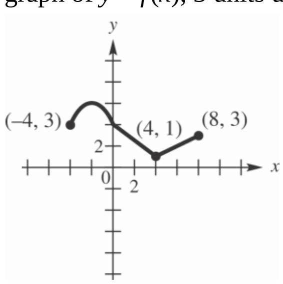
98. If the graph of $f(x) = 3x - 4$ is reflected about the *x*-axis, we obtain a graph whose equation is $y = -(3x-4) = -3x+4$.

99. If the graph of $f(x) = 3x - 4$ is reflected about the *y*-axis, we obtain a graph whose equation is $y = f(-x) = 3(-x) - 4 = -3x - 4$.

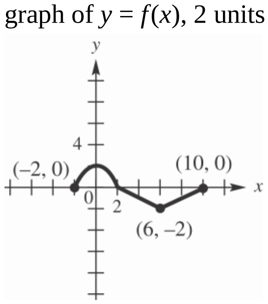
100. If the graph of $f(x) = 3x - 4$ is reflected about the origin, every point (x, y) will be replaced by the point $(-x, -y)$. The equation for the graph will change from $y = 3x - 4$ to

$-y = 3(-x) - 4 \Rightarrow -y = -3x - 4 \Rightarrow$
$y = 3x + 4$.

101. **(a)** To graph $y = f(x) + 3$, translate the graph of $y = f(x)$, 3 units up.

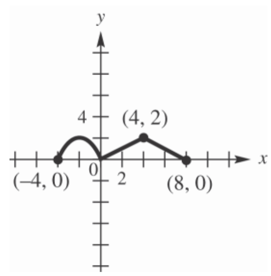


(b) To graph $y = f(x-2)$, translate the graph of $y = f(x)$, 2 units to the right.



(c) To graph $y = f(x+3) - 2$, translate the

(d) To graph $y = |f(x)|$, keep the graph of $y = f(x)$ as it is where $y \geq 0$ and reflect the graph about the *x*-axis where $y < 0$.



102. No. For example suppose $f(x) = \sqrt{x-2}$ and $g(x) = 2x$. Then

$(f \circ g)(x) = f(g(x)) = f(2x) = \sqrt{2x-2}$

The domain and range of *g* are $(-\infty, \infty)$, however, the domain of *f* is $[2, \infty)$. So, $2x - 2 \geq 0 \Rightarrow x \geq 1$. Therefore, the domain of $f \circ g$ is $[1, \infty)$. The domain of g, $(-\infty, \infty)$, is not a subset of the domain of $f \circ g$, $[1, \infty)$.

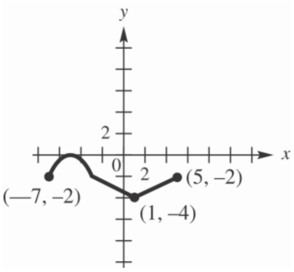
For Exercises 103–110, $f(x) = 3x^2 - 4$ and $g(x) = x^2 - 3x - 4$.

103.
$$\begin{aligned}(fg)(x) &= f(x) \cdot g(x) \\ &= (3x^2 - 4)(x^2 - 3x - 4) \\ &= 3x^4 - 9x^3 - 12x^2 - 4x^2 + 12x + 16 \\ &= 3x^4 - 9x^3 - 16x^2 + 12x + 16\end{aligned}$$

104.
$$\begin{aligned}(f-g)(4) &= f(4) - g(4) \\ &= (3 \cdot 4^2 - 4) - (4^2 - 3 \cdot 4 - 4) \\ &= (3 \cdot 16 - 4) - (16 - 3 \cdot 4 - 4) \\ &= (48 - 4) - (16 - 12 - 4) \\ &= 44 - 0 = 44\end{aligned}$$

105.
$$\begin{aligned}(f+g)(-4) &= f(-4) + g(-4) \\ &= [3(-4)^2 - 4] + [(-4)^2 - 3(-4) - 4] \\ &= [3(16) - 4] + [16 - 3(-4) - 4] \\ &= [48 - 4] + [16 + 12 - 4] \\ &= 44 + 24 = 68\end{aligned}$$

graph of $y = f(x)$, 3 units to the left and 2 units down.



106. $$\begin{aligned}(f+g)(2k) &= f(2k)+g(2k)\\ &= [3(2k)^2-4]+[(2k)^2-3(2k)-4]\\ &= [3(4)k^2-4]+[4k^2-3(2k)-4]\\ &= (12k^2-4)+(4k^2-6k-4)\\ &= 16k^2-6k-8\end{aligned}$$

107. $\left(\frac{f}{g}\right)(3) = \frac{f(3)}{g(3)} = \frac{3\cdot 3^2 - 4}{3^2 - 3\cdot 3 - 4} = \frac{3\cdot 9 - 4}{9 - 3\cdot 3 - 4}$
$= \frac{27-4}{9-9-4} = \frac{23}{-4} = -\frac{23}{4}$

108. $\left(\frac{f}{g}\right)(-1) = \frac{3(-1)^2 - 4}{(-1)^2 - 3(-1) - 4} = \frac{3(1) - 4}{1 - 3(-1) - 4}$
$= \frac{3-4}{1+3-4} = \frac{-1}{0} =$ undefined

109. The domain of $(fg)(x)$ is the intersection of the domain of $f(x)$ and the domain of $g(x)$. Both have domain $(-\infty, \infty)$, so the domain of $(fg)(x)$ is $(-\infty, \infty)$.

110. $\left(\frac{f}{g}\right)(x) = \frac{3x^2 - 4}{x^2 - 3x - 4} = \frac{3x^2 - 4}{(x+1)(x-4)}$

Because both $f(x)$ and $g(x)$ have domain $(-\infty, \infty)$, we are concerned about values of x that make $g(x) = 0$. Thus, the expression is undefined if $(x + 1)(x - 4) = 0$, that is, if $x = -1$ or $x = 4$. Thus, the domain is the set of all real numbers except $x = -1$ and $x = 4$, or $(-\infty, -1) \cup (-1, 4) \cup (4, \infty)$.

111. $f(x) = 2x + 9$

$f(x+h) = 2(x+h) + 9 = 2x + 2h + 9$
$f(x+h) - f(x) = (2x + 2h + 9) - (2x + 9)$
$= 2x + 2h + 9 - 2x - 9 = 2h$

Thus, $\frac{f(x+h) - f(x)}{h} = \frac{2h}{h} = 2.$

112. $f(x) = x^2 - 5x + 3$

$f(x+h) = (x+h)^2 - 5(x+h) + 3$
$= x^2 + 2xh + h^2 - 5x - 5h + 3$
$f(x+h) - f(x)$
$= (x^2 + 2xh + h^2 - 5x - 5h + 3) - (x^2 - 5x + 3)$
$= x^2 + 2xh + h^2 - 5x - 5h + 3 - x^2 + 5x - 3$
$= 2xh + h^2 - 5h$
$\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 - 5h}{h}$

For Exercises 113–118,

$f(x) = \sqrt{x-2}$ and $g(x) = x^2$.

113. $(g \circ f)(x) = g[f(x)] = g\left(\sqrt{x-2}\right)$
$= \left(\sqrt{x-2}\right)^2 = x - 2$

114. $(f \circ g)(x) = f[g(x)] = f(x^2) = \sqrt{x^2 - 2}$

115. $f(x) = \sqrt{x-2}$, so $f(3) = \sqrt{3-2} = \sqrt{1} = 1.$

Therefore,
$(g \circ f)(3) = g\left[f(3)\right] = g(1) = 1^2 = 1.$

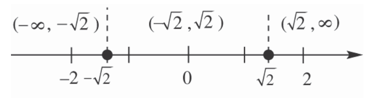
116. $g(x) = x^2$, so $g(-6) = (-6)^2 = 36.$
Therefore, $(f \circ g)(-6) = f\left[g(-6)\right] = f(36)$
$= \sqrt{36-2} = \sqrt{34}$.

117. $(g \circ f)(-1) = g\left(f(-1)\right) = g\left(\sqrt{-1-2}\right) = g\left(\sqrt{-3}\right)$

Because $\sqrt{-3}$ is not a real number, $(g \circ f)(-1)$ is not defined.

118. To find the domain of $f \circ g$, we must consider the domain of g ~~as well~~ as the composed function, $f \circ g$. Because $(f \circ g)(x) = f\left[g(x)\right] = \sqrt{x^2 - 2}$ we need to determine when $x^2 - 2 \geq 0$.
Step 1: Find the values of x that satisfy $x^2 - 2 = 0$.
$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

Step 2: The two numbers divide a number line into three regions.



Step 3 Choose a test value to see if it satisfies the inequality, $x^2 - 2 \geq 0$.

Interval	Test Value	Is $x^2 - 2 \geq 0$ true or false?
$\left(-\infty, -\sqrt{2}\right)$	-2	$(-2)^2 - 2 \geq 0$? $2 \geq 0$ True
$\left(-\sqrt{2}, \sqrt{2}\right)$	0	$0^2 - 2 \geq 0$? $-2 \geq 0$ False
$\left(\sqrt{2}, \infty\right)$	2	$2^2 - 2 \geq 0$? $2 \geq 0$ True

The domain of $f \circ g$ is $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$.

119. $(f+g)(1) = f(1) + g(1) = 7 + 1 = 8$

120. $(f-g)(3) = f(3) - g(3) = 9 - 9 = 0$

121. $(fg)(-1) = f(-1)\cdot g(-1) = 3(-2) = -6$

122. $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{5}{0} =$ undefined

123. $(g \circ f)(-2) = g[f(-2)] = g(1) = 2$

124. $(f \circ g)(3) = f[g(3)] = f(-2) = 1$

125. $(f \circ g)(2) = f[g(2)] = f(2) = 1$

126. $(g \circ f)(3) = g[f(3)] = g(4) = 8$

127. Let x = number of yards.

$f(x) = 36x$, where $f(x)$ is the number of inches. $g(x) = 1760x$, where $g(x)$ is the number of yards. Then

$$(g \circ f)(x) = g\left[f(x)\right] = 1760(36x) = 63,360x.$$

There are $63{,}360x$ inches in x miles

128. Use the definition for the perimeter of a rectangle.

P = length + width + length + width

$P(x) = 2x + x + 2x + x = 6x$

This is a linear function.

129. If $V(r) = \frac{4}{3}\pi r^3$ and if the radius is increased by 3 inches, then the amount of volume gained is given by

$$V_g(r) = V(r+3) - V(r) = \frac{4}{3}\pi(r+3)^3 - \frac{4}{3}\pi r^3.$$

130. **(a)** $V = \pi r^2 h$

If d is the diameter of its top, then $h = d$ and $r = \frac{d}{2}$. So,

$$V(d) = \pi\left(\frac{d}{2}\right)^2(d) = \pi\left(\frac{d^2}{4}\right)(d) = \frac{\pi d^3}{4}.$$

(b) $S = 2\pi r^2 + 2\pi rh \Rightarrow$

(b) The range of $f(x) = \sqrt{x-3}$ is all real numbers greater than or equal to 0. In interval notation, this correlates to the interval in D, $[0,\infty)$.

(c) The domain of $f(x) = x^2 - 3$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty,\infty)$.

(d) The range of $f(x) = x^2 + 3$ is all real numbers greater than or equal to 3. In interval notation, this correlates to the interval in B, $[3,\infty)$.

(e) The domain of $f(x) = \sqrt[3]{x-3}$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty,\infty)$.

(f) The range of $f(x) = \sqrt[3]{x} + 3$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty,\infty)$.

(g) The domain of $f(x) = x - 3$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty,\infty)$.

(h) The range of $f(x) = |x+3|$ is all real numbers greater than or equal to 0. In interval notation, this correlates to the interval in D, $[0,\infty)$.

(i) The domain of $x = y^2$ is $x \geq 0$ because when you square any value of y, the outcome will be nonnegative. In interval notation, this correlates to the interval in D, $[0,\infty)$.

(j) The range of $x = y^2$ is all real numbers.

$$S(d) = 2\pi\left(\frac{d}{2}\right)^2 + 2\pi\left(\frac{d}{2}\right)(d) = \frac{\pi d^2}{2} + \pi d^2$$
$$= \frac{\pi d^2}{2} + \frac{2\pi d^2}{2} = \frac{3\pi d^2}{2}$$

Chapter 2 Test

1. (a) The domain of $f(x) = \sqrt{x} + 3$ occurs when $x \ge 0$. In interval notation, this correlates to the interval in D, $[0, \infty)$.

In interval notation, this correlates to the interval in C, $(-\infty, \infty)$.

2. Consider the points $(-2,1)$ and $(3,4)$.

$$m = \frac{4-1}{3-(-2)} = \frac{3}{5}$$

3. We label the points $A(-2,1)$ and $B(3,4)$.

$$d(A, B) = \sqrt{[3-(-2)]^2 + (4-1)^2}$$
$$= \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}$$

4. The midpoint has coordinates
$$\left(\frac{-2+3}{2}, \frac{1+4}{2}\right) = \left(\frac{1}{2}, \frac{5}{2}\right).$$

5. Use the point-slope form with $(x_1, y_1) = (-2,1)$ and $m = \frac{3}{5}$.
$$y - y_1 = m(x - x_1)$$
$$y - 1 = \tfrac{3}{5}[x - (-2)]$$
$$y - 1 = \tfrac{3}{5}(x+2) \Rightarrow 5(y-1) = 3(x+2) \Rightarrow$$
$$5y - 5 = 3x + 6 \Rightarrow 5y = 3x + 11 \Rightarrow$$
$$-3x + 5y = 11 \Rightarrow 3x - 5y = -11$$

6. Solve $3x - 5y = -11$ for y.
$$3x - 5y = -11$$
$$-5y = -3x - 11$$
$$y = \tfrac{3}{5}x + \tfrac{11}{5}$$

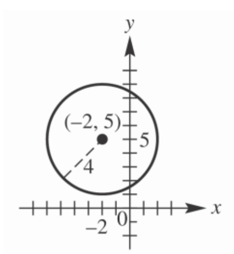
Therefore, the linear function is
$$f(x) = \tfrac{3}{5}x + \tfrac{11}{5}.$$

7. **(a)** The center is at (0, 0) and the radius is 2, so the equation of the circle is $x^2 + y^2 = 4$.

(b) The center is at (1, 4) and the radius is 1, so the equation of the circle is $(x-1)^2 + (y-4)^2 = 1$

8. $x^2 + y^2 + 4x - 10y + 13 = 0$

Complete the square on x and y to write the equation in standard form:
$$x^2 + y^2 + 4x - 10y + 13 = 0$$
$$\left(x^2 + 4x + \quad\right) + \left(y^2 - 10y + \quad\right) = -13$$
$$\left(x^2 + 4x + 4\right) + \left(y^2 - 10y + 25\right) = -13 + 4 + 25$$
$$(x+2)^2 + (y-5)^2 = 16$$
The circle has center (−2, 5) and radius 4.



$x^2 + y^2 + 4x - 10y + 13 = 0$

(b) This is the graph of a function because no vertical line intersects the graph in more than one point. The domain of the function is $(-\infty, -1) \cup (-1, \infty)$. The range is $(-\infty, 0) \cup (0, \infty)$. As x is getting larger on the intervals $(-\infty, -1)$ and $(-1, \infty)$, the value of y is decreasing, so the function is decreasing on these intervals. (The function is never increasing or constant.)

10. Point A has coordinates (5, –3).

(a) The equation of a vertical line through A is $x = 5$.

(b) The equation of a horizontal line through A is $y = -3$.

11. The slope of the graph of $y = -3x + 2$ is –3.

(a) A line parallel to the graph of $y = -3x + 2$ has a slope of –3. Use the point-slope form with $(x_1, y_1) = (2, 3)$ and $m = -3$.
$$y - y_1 = m(x - x_1)$$
$$y - 3 = -3(x - 2)$$
$$y - 3 = -3x + 6 \Rightarrow y = -3x + 9$$

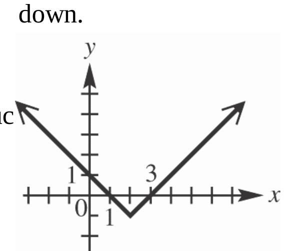
(b) A line perpendicular to the graph of $y = -3x + 2$ has a slope of $\frac{1}{3}$ because $-3\left(\frac{1}{3}\right) = -1.$
$$y - 3 = \tfrac{1}{3}(x - 2)$$
$$3(y-3) = x - 2 \Rightarrow 3y - 9 = x - 2 \Rightarrow$$
$$3y = x + 7 \Rightarrow y = \tfrac{1}{3}x + \tfrac{7}{3}$$

12. **(a)** $(2, \infty)$ **(b)** $(0, 2)$

(c) $(-\infty, 0)$ **(d)** $(-\infty, \infty)$

(e) $(-\infty, \infty)$ **(f)** $[-1, \infty)$

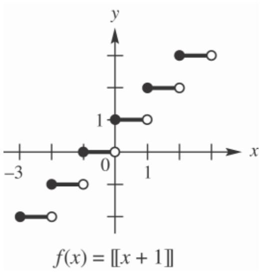
13. To graph $f(x) = |x-2| - 1$, we translate the graph of $y = |x|$, 2 units to the right and 1 unit down.



9. (a) This is not the graph of a function because some vertical lines intersect it in more than one point. The domain of the relation is [0, 4]. The range is [–4, 4].

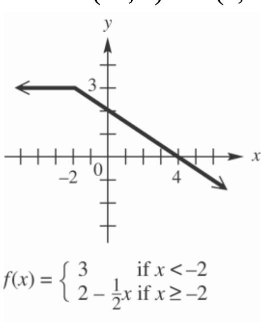
14. $f(x) = [\![x+1]\!]$

To get $y = 0$, we need $0 \le x+1 < 1 \Rightarrow -1 \le x < 0$. To get $y = 1$, we need $1 \le x+1 < 2 \Rightarrow 0 \le x < 1$. Follow this pattern to graph the step function.

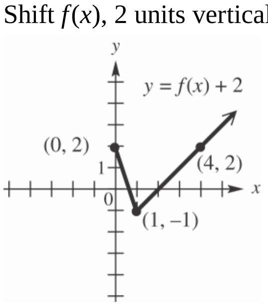


15. $f(x) = \begin{cases} 3 & \text{if } x < -2 \\ 2 - \frac{1}{2}x & \text{if } x \ge -2 \end{cases}$

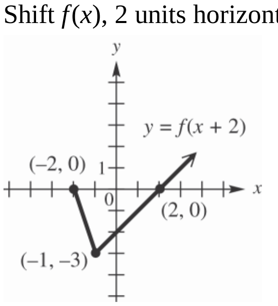
For values of x with $x < -2$, we graph the horizontal line $y = 3$. For values of x with $x \ge -2$, we graph the line with a slope of $-\frac{1}{2}$ and a y-intercept of (0, 2). Two points on this line are (–2, 3) and (0, 2).



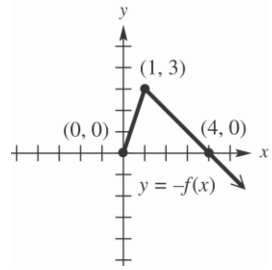
16. **(a)** Shift $f(x)$, 2 units vertically upward.



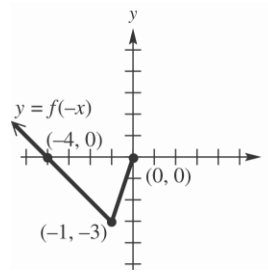
(b) Shift $f(x)$, 2 units horizontally to the left.



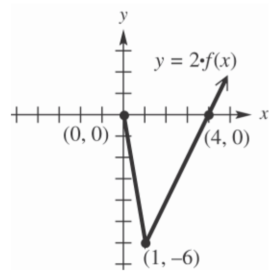
(c) Reflect $f(x)$, across the x-axis.



(d) Reflect $f(x)$, across the y-axis.



(e) Stretch $f(x)$, vertically by a factor of 2.



17. Starting with $y = \sqrt{x}$, we shift it to the left 2 units and stretch it vertically by a factor of 2. The graph is then reflected over the x-axis and then shifted down 3 units.

18. $3x^2 - 2y^2 = 3$

(a) Replace y with $-y$ to obtain $3x^2 - 2(-y)^2 = 3 \Rightarrow 3x^2 - 2y^2 = 3.$ The result is the same as the original equation, so the graph is symmetric with respect to the x-axis.

(b) Replace x with $-x$ to obtain $3(-x)^2 - 2y^2 = 3 \Rightarrow 3x^2 - 2y^2 = 3.$ The result is the same as the original equation, so the graph is symmetric with respect to the y-axis.

(c) The graph is symmetric with respect to the x-axis and with respect to the y-axis, so it must also be symmetric with respect to the origin.

19. $f(x) = 2x^2 - 3x + 2,\ g(x) = -2x + 1$

(a) $(f - g)(x) = f(x) - g(x)$
$= \left(2x^2 - 3x + 2\right) - \left(-2x + 1\right)$
$= 2x^2 - 3x + 2 + 2x - 1$
$= 2x^2 - x + 1$

(b) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x^2 - 3x + 2}{-2x + 1}$

(c) We must determine which values solve the equation $-2x + 1 = 0$.

$-2x + 1 = 0 \Rightarrow -2x = -1 \Rightarrow x = \frac{1}{2}$

Thus, $\frac{1}{2}$ is excluded from the domain, and the domain is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

(d) $f(x) = 2x^2 - 3x + 2$

$f\left(x + h\right) = 2\left(x + h\right)^2 - 3\left(x + h\right) + 2$
$= 2\left(x^2 + 2xh + h^2\right) - 3x - 3h + 2$
$= 2x^2 + 4xh + 2h^2 - 3x - 3h + 2$

$f(x + h) - f(x)$
$= (2x^2 + 4xh + 2h^2 - 3x - 3h + 2) - (2x^2 - 3x + 2)$
$= 2x^2 + 4xh + 2h^2 - 3x - 3h + 2 - 2x^2 + 3x - 2$
$= 4xh + 2h^2 - 3h$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{4xh + 2h^2 - 3h}{h}$
$= \dfrac{h(4x + 2h - 3)}{h}$
$= 4x + 2h - 3$

(e) $(f + g)(1) = f(1) + g(1)$
$= (2 \cdot 1^2 - 3 \cdot 1 + 2) + (-2 \cdot 1 + 1)$
$= (2 \cdot 1 - 3 \cdot 1 + 2) + (-2 \cdot 1 + 1)$
$= (2 - 3 + 2) + (-2 + 1)$
$= 1 + (-1) = 0$

(f) $(fg)(2) = f(2) \cdot g(2)$
$= (2 \cdot 2^2 - 3 \cdot 2 + 2) \cdot (-2 \cdot 2 + 1)$
$= (2 \cdot 4 - 3 \cdot 2 + 2) \cdot (-2 \cdot 2 + 1)$
$= (8 - 6 + 2) \cdot (-4 + 1)$
$= 4(-3) = -12$

(g) $g\left(x\right) = -2x + 1 \Rightarrow g\left(0\right) = -2\left(0\right) + 1$
$= 0 + 1 = 1.$ Therefore,
$\left(f \circ g\right)\left(0\right) = f\left[g\left(0\right)\right]$
$= f\left(1\right) = 2 \cdot 1^2 - 3 \cdot 1 + 2$
$= 2 \cdot 1 - 3 \cdot 1 + 2$
$= 2 - 3 + 2 = 1$

For exercises 20 and 21, $f\left(x\right) = \sqrt{x + 1}$ and $g\left(x\right) = 2x - 7.$

20. $\left(f \circ g\right) = f\left(g\left(x\right)\right) = f\left(2x - 7\right)$
$= \sqrt{(2x - 7) + 1} = \sqrt{2x - 6}$

The domain and range of g are $(-\infty, \infty)$, while the domain of f is $[0, \infty)$. We need to find the values of x which fit the domain of f: $2x - 6 \geq 0 \Rightarrow x \geq 3$. So, the domain of $f \circ g$ is $[3, \infty)$.

21. $\left(g \circ f\right) = g\left(f\left(x\right)\right) = g\left(\sqrt{x + 1}\right)$
$= 2\sqrt{x + 1} - 7$

The domain and range of g are $(-\infty, \infty)$, while the domain of f is $[0, \infty)$. We need to find the values of x which fit the domain of f: $x + 1 \geq 0 \Rightarrow x \geq -1$. So, the domain of $g \circ f$ is $[-1, \infty)$.

22. **(a)** $C(x) = 3300 + 4.50x$

(b) $R(x) = 10.50x$

(c) $P(x) = R(x) - C(x)$
$= 10.50x - (3300 + 4.50x)$
$= 6.00x - 3300$

(d) $P(x) > 0$
$6.00x - 3300 > 0$
$6.00x > 3300$
$x > 550$

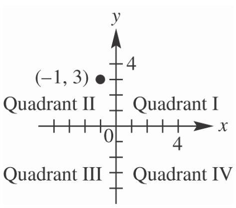
She must produce and sell 551 items before she earns a profit.

Chapter 2

GRAPHS AND FUNCTIONS

Section 2.1 Rectangular Coordinates and Graphs

1. The point (–1, 3) lies in quadrant <u>II</u> in the rectangular coordinate system.



2. The point (4, <u>6</u>) lies on the graph of the equation $y = 3x - 6$. Find the y-value by letting $x = 4$ and solving for y.

$$y = 3(4) - 6 = 12 - 6 = 6$$

3. Any point that lies on the x-axis has y-coordinate equal to <u>0</u>.

4. The y-intercept of the graph of $y = -2x + 6$ is <u>(0, 6)</u>.

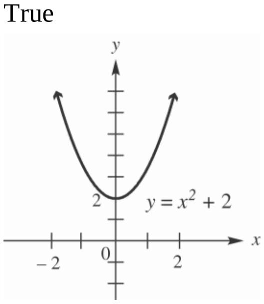
5. The x-intercept of the graph of $2x + 5y = 10$ is <u>(5, 0)</u>. Find the x-intercept by letting $y = 0$ and solving for x.

$$2x + 5(0) = 10 \Rightarrow 2x = 10 \Rightarrow x = 5$$

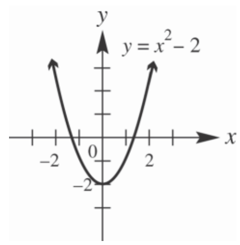
6. The distance from the origin to the point (–3, 4) is <u>5</u>. Using the distance formula, we have

$$d(P, Q) = \sqrt{(-3-0)^2 + (4-0)^2}$$
$$= \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

7. True



8. True



9. False. The midpoint of the segment joining (0, 0) and (4, 4) is

$$\left(\frac{4+0}{2}, \frac{4+0}{2}\right) = \left(\frac{4}{2}, \frac{4}{2}\right) = (2, 2).$$

10. False. The distance between the point (0, 0) and (4, 4) is

$$d(P, Q) = \sqrt{(4-0)^2 + (4-0)^2} = \sqrt{4^2 + 4^2}$$
$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

11. Any three of the following:
$(2,-5), (-1,7), (3,-9), (5,-17), (6,-21)$

12. Any three of the following:
$(3,3), (-5,-21), (8,18), (4,6), (0,-6)$

13. Any three of the following: (1999, 35), (2001, 29), (2003, 22), (2005, 23), (2007, 20), (2009, 20)

14. Any three of the following:

$(2002, 86.8), (2004, 89.8), (2006, 90.7),$

$(2008, 97.4), (2010, 106.5), (2012, 111.4),$
$(2014, 111.5)$

15. $P(-5, -6)$, $Q(7, -1)$

(a) $$d(P, Q) = \sqrt{[7-(-5)]^2 + [-1-(-6)]^2}$$
$$= \sqrt{12^2 + 5^2} = \sqrt{169} = 13$$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{-5+7}{2}, \frac{-6+(-1)}{2}\right) = \left(\frac{2}{2}, -\frac{7}{2}\right)$$
$$= \left(1, -\frac{7}{2}\right).$$

16. $P(-4, 3), Q(2, -5)$

(a) $d(P, Q) = \sqrt{[2-(-4)]^2 + (-5-3)^2}$
$= \sqrt{6^2 + (-8)^2} = \sqrt{100} = 10$

(b) The midpoint M of the segment joining points P and Q has coordinates
$\left(\frac{-4+2}{2}, \frac{3+(-5)}{2}\right) = \left(\frac{-2}{2}, \frac{-2}{2}\right) = (-1, -1).$

17. $P(8, 2), Q(3, 5)$

(a) $d(P, Q) = \sqrt{(3-8)^2 + (5-2)^2}$
$= \sqrt{(-5)^2 + 3^2}$
$= \sqrt{25+9} = \sqrt{34}$

(b) The midpoint M of the segment joining points P and Q has coordinates

$\left(\frac{8+3}{2}, \frac{2+5}{2}\right) = \left(\frac{11}{2}, \frac{7}{2}\right).$

18. $P(-8, 4), Q(3, -5)$

(a) $d(P, Q) = \sqrt{[3-(-8)]^2 + (-5-4)^2}$
$= \sqrt{11^2 + (-9)^2} = \sqrt{121+81}$
$= \sqrt{202}$

(b) The midpoint M of the segment joining points P and Q has coordinates

$\left(\frac{-8+3}{2}, \frac{4+(-5)}{2}\right) = \left(-\frac{5}{2}, -\frac{1}{2}\right).$

19. $P(-6, -5), Q(6, 10)$

(a) $d(P, Q) = \sqrt{[6-(-6)]^2 + [10-(-5)]^2}$
$= \sqrt{12^2 + 15^2} = \sqrt{144+225}$

21. $P\left(3\sqrt{2}, 4\sqrt{5}\right), Q\left(\sqrt{2}, -\sqrt{5}\right)$

(a) $d(P, Q)$
$= \sqrt{\left(\sqrt{2} - 3\sqrt{2}\right)^2 + \left(-\sqrt{5} - 4\sqrt{5}\right)^2}$
$= \sqrt{\left(-2\sqrt{2}\right)^2 + \left(-5\sqrt{5}\right)^2}$
$= \sqrt{8+125} = \sqrt{133}$

(b) The midpoint M of the segment joining points P and Q has coordinates
$\left(\frac{3\sqrt{2}+\sqrt{2}}{2}, \frac{4\sqrt{5}+(-\sqrt{5})}{2}\right)$
$= \left(\frac{4\sqrt{2}}{2}, \frac{3\sqrt{5}}{2}\right) = \left(2\sqrt{2}, \frac{3\sqrt{5}}{2}\right).$

22. $P\left(-\sqrt{7}, 8\sqrt{3}\right), Q\left(5\sqrt{7}, -\sqrt{3}\right)$

(a) $d(P, Q)$
$= \sqrt{[5\sqrt{7}-(-\sqrt{7})]^2 + (-\sqrt{3}-8\sqrt{3})^2}$
$= \sqrt{(6\sqrt{7})^2 + (-9\sqrt{3})^2} = \sqrt{252+243}$
$= \sqrt{495} = 3\sqrt{55}$

(b) The midpoint M of the segment joining points P and Q has coordinates
$\left(\frac{-\sqrt{7}+5\sqrt{7}}{2}, \frac{8\sqrt{3}+(-\sqrt{3})}{2}\right)$
$= \left(\frac{4\sqrt{7}}{2}, \frac{7\sqrt{3}}{2}\right) = \left(2\sqrt{7}, \frac{7\sqrt{3}}{2}\right).$

23. Label the points $A(-6, -4)$, $B(0, -2)$, and $C(-10, 8)$. Use the distance formula to find the length of each side of the triangle.

$d(A, B) = \sqrt{[0-(-6)]^2 + [-2-(-4)]^2}$

$$= \sqrt{369} = 3\sqrt{41}$$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{-6+6}{2}, \frac{-5+10}{2}\right) = \left(\frac{0}{2}, \frac{5}{2}\right) = \left(0, \frac{5}{2}\right)$.

20. $P(6, -2)$, $Q(4, 6)$

(a) $$d(P, Q) = \sqrt{(4-6)^2 + [6-(-2)]^2} = \sqrt{(-2)^2 + 8^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$$

(b) The midpoint M of the segment joining points P and Q has coordinates $\left(\frac{6+4}{2}, \frac{-2+6}{2}\right) = \left(\frac{10}{2}, \frac{4}{2}\right) = (5, 2)$

$$= \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40}$$

$$d(B, C) = \sqrt{(-10-0)^2 + [8-(-2)]^2} = \sqrt{(-10)^2 + 10^2} = \sqrt{100+100} = \sqrt{200}$$

$$d(A, C) = \sqrt{[-10-(-6)]^2 + [8-(-4)]^2} = \sqrt{(-4)^2 + 12^2} = \sqrt{16+144} = \sqrt{160}$$

Because $\left(\sqrt{40}\right)^2 + \left(\sqrt{160}\right)^2 = \left(\sqrt{200}\right)^2$, triangle ABC is a right triangle.

24. Label the points $A(-2, -8)$, $B(0, -4)$, and $C(-4, -7)$. Use the distance formula to find the length of each side of the triangle.

$$d(A, B) = \sqrt{[0-(-2)]^2 + [-4-(-8)]^2}$$
$$= \sqrt{2^2 + 4^2} = \sqrt{4+16} = \sqrt{20}$$

$$d(B, C) = \sqrt{(-4-0)^2 + [-7-(-4)]^2}$$
$$= \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9}$$
$$= \sqrt{25} = 5$$

$$d(A, C) = \sqrt{[-4-(-2)]^2 + [-7-(-8)]^2}$$
$$= \sqrt{(-2)^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$$

Because $(\sqrt{5})^2 + (\sqrt{20})^2 = 5 + 20 = 25 = 5^2$, triangle ABC is a right triangle.

25. Label the points $A(-4, 1)$, $B(1, 4)$, and $C(-6, -1)$.

$$d(A, B) = \sqrt{[1-(-4)]^2 + (4-1)^2}$$
$$= \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}$$

$$d(B, C) = \sqrt{(-6-1)^2 + (-1-4)^2}$$
$$= \sqrt{(-7)^2 + (-5)^2} = \sqrt{49+25} = \sqrt{74}$$

$$d(A, C) = \sqrt{[-6-(-4)]^2 + (-1-1)^2}$$
$$= \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$$

Because $(\sqrt{8})^2 + (\sqrt{34})^2 \neq (\sqrt{74})^2$ because $8 + 34 = 42 \neq 74$, triangle ABC is not a right triangle.

26. Label the points $A(-2, -5)$, $B(1, 7)$, and $C(3, 15)$.

$$d(A, B) = \sqrt{[1-(-2)]^2 + [7-(-5)]^2}$$
$$= \sqrt{3^2 + 12^2} = \sqrt{9+144} = \sqrt{153}$$

$$d(B, C) = \sqrt{(3-1)^2 + (15-7)^2}$$

$$d(B, C) = \sqrt{(-1-2)^2 + (-6-5)^2}$$
$$= \sqrt{(-3)^2 + (-11)^2}$$
$$= \sqrt{9+121} = \sqrt{130}$$

$$d(A, C) = \sqrt{[-1-(-4)]^2 + (-6-3)^2}$$
$$= \sqrt{3^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90}$$

Because $(\sqrt{40})^2 + (\sqrt{90})^2 = (\sqrt{130})^2$, triangle ABC is a right triangle.

28. Label the points $A(-7, 4)$, $B(6, -2)$, and $C(0, -15)$.

$$d(A, B) = \sqrt{[6-(-7)]^2 + (-2-4)^2}$$
$$= \sqrt{13^2 + (-6)^2}$$
$$= \sqrt{169+36} = \sqrt{205}$$

$$d(B, C) = \sqrt{(0-6)^2 + [-15-(-2)]^2}$$
$$= \sqrt{(-6)^2 + (-13)^2}$$
$$= \sqrt{36+169} = \sqrt{205}$$

$$d(A, C) = \sqrt{[0-(-7)]^2 + (-15-4)^2}$$
$$= \sqrt{7^2 + (-19)^2} = \sqrt{49+361} = \sqrt{410}$$

Because $(\sqrt{205})^2 + (\sqrt{205})^2 = (\sqrt{410})^2$, triangle ABC is a right triangle.

29. Label the given points $A(0, -7)$, $B(-3, 5)$, and $C(2, -15)$. Find the distance between each pair of points.

$$d(A, B) = \sqrt{(-3-0)^2 + [5-(-7)]^2}$$
$$= \sqrt{(-3)^2 + 12^2} = \sqrt{9+144}$$
$$= \sqrt{153} = 3\sqrt{17}$$

$$= \sqrt{2^2 + 8^2} = \sqrt{4 + 64} = \sqrt{68}$$

$$d(A,\ C) = \sqrt{[3 - (-2)]^2 + [15 - (-5)]^2}$$
$$= \sqrt{5^2 + 20^2} = \sqrt{25 + 400} = \sqrt{425}$$

Because $(\sqrt{68})^2 + (\sqrt{153})^2 \neq \left(\sqrt{425}\right)^2$ because $68 + 153 = 221 \neq 425$, triangle ABC is not a right triangle.

27. Label the points $A(-4, 3)$, $B(2, 5)$, and $C(-1, -6)$.

$$d(A,\ B) = \sqrt{\left[2 - (-4)\right]^2 + (5 - 3)^2}$$
$$= \sqrt{6^2 + 2^2} = \sqrt{36 + 4} = \sqrt{40}$$

$$d(B,\ C) = \sqrt{\left[2 - (-3)\right]^2 + (-15 - 5)^2}$$
$$= \sqrt{5^2 + (-20)^2} = \sqrt{25 + 400}$$
$$= \sqrt{425} = 5\sqrt{17}$$

$$d(A,\ C) = \sqrt{(2 - 0)^2 + \left[-15 - (-7)\right]^2}$$
$$= \sqrt{2^2 + (-8)^2} = \sqrt{68} = 2\sqrt{17}$$

Because $d(A,\ B) + d(A,\ C) = d(B,\ C)$ or $3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$, the points are collinear.

30. Label the points $A(-1, 4)$, $B(-2, -1)$, and $C(1, 14)$. Apply the distance formula to each pair of points.

$$d(A, B) = \sqrt{[-2-(-1)]^2 + (-1-4)^2}$$
$$= \sqrt{(-1)^2 + (-5)^2} = \sqrt{26}$$

$$d(B, C) = \sqrt{[1-(-2)]^2 + [14-(-1)]^2}$$
$$= \sqrt{3^2 + 15^2} = \sqrt{234} = 3\sqrt{26}$$

$$d(A, C) = \sqrt{[1-(-1)]^2 + (14-4)^2}$$
$$= \sqrt{2^2 + 10^2} = \sqrt{104} = 2\sqrt{26}$$

Because $\sqrt{26} + 2\sqrt{26} = 3\sqrt{26}$, the points are collinear.

31. Label the points $A(0, 9)$, $B(-3, -7)$, and $C(2, 19)$.

$$d(A, B) = \sqrt{(-3-0)^2 + (-7-9)^2}$$
$$= \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256}$$
$$= \sqrt{265} \approx 16.279$$

$$d(B, C) = \sqrt{[2-(-3)]^2 + [19-(-7)]^2}$$
$$= \sqrt{5^2 + 26^2} = \sqrt{25+676}$$
$$= \sqrt{701} \approx 26.476$$

$$d(A, C) = \sqrt{(2-0)^2 + (19-9)^2}$$
$$= \sqrt{2^2 + 10^2} = \sqrt{4+100}$$
$$= \sqrt{104} \approx 10.198$$

Because $d(A, B) + d(A, C) \neq d(B, C)$

or $\sqrt{265} + \sqrt{104} \neq \sqrt{701}$

$$16.279 + 10.198 \neq 26.476,$$
$$26.477 \neq 26.476,$$

the three given points are not collinear. (Note, however, that these points are very close to

$$d(A, C) = \sqrt{[1-(-1)]^2 + [-11-(-3)]^2}$$
$$= \sqrt{2^2 + (-8)^2} = \sqrt{4+64}$$
$$= \sqrt{68} \approx 8.2462$$

Because $d(A, B) + d(A, C) \neq d(B, C)$

or $\sqrt{241} + \sqrt{68} \neq \sqrt{565}$

$$15.5242 + 8.2462 \neq 23.7697$$
$$23.7704 \neq 23.7697,$$

the three given points are not collinear. (Note, however, that these points are very close to lying on a straight line and may appear to lie on a straight line when graphed.)

33. Label the points $A(-7, 4)$, $B(6,-2)$, and $C(-1,1)$.

$$d(A, B) = \sqrt{[6-(-7)]^2 + (-2-4)^2}$$
$$= \sqrt{13^2 + (-6)^2} = \sqrt{169+36}$$
$$= \sqrt{205} \approx 14.3178$$

$$d(B, C) = \sqrt{(-1-6)^2 + [1-(-2)]^2}$$
$$= \sqrt{(-7)^2 + 3^2} = \sqrt{49+9}$$
$$= \sqrt{58} \approx 7.6158$$

$$d(A, C) = \sqrt{[-1-(-7)]^2 + (1-4)^2}$$
$$= \sqrt{6^2 + (-3)^2} = \sqrt{36+9}$$
$$= \sqrt{45} \approx 6.7082$$

Because $d(B, C) + d(A, C) \neq d(A, B)$ or

$$\sqrt{58} + \sqrt{45} \neq \sqrt{205}$$

$$7.6158 + 6.7082 \neq 14.3178$$
$$14.3240 \neq 14.3178,$$

lying on a straight line and may appear to lie on a straight line when graphed.)

32. Label the points $A(-1, -3)$, $B(-5, 12)$, and $C(1, -11)$.

the three given points are not collinear. (Note, however, that these points are very close to lying on a straight line and may appear to lie on a straight line when graphed.)

$$d(A, B) = \sqrt{[-5-(-1)]^2 + [12-(-3)]^2}$$
$$= \sqrt{(-4)^2 + 15^2} = \sqrt{16+225}$$
$$= \sqrt{241} \approx 15.5242$$

$$d(B, C) = \sqrt{[1-(-5)]^2 + (-11-12)^2}$$
$$= \sqrt{6^2 + (-23)^2} = \sqrt{36+529}$$
$$= \sqrt{565} \approx 23.7697$$

34. Label the given points $A(-4, 3)$, $B(2, 5)$, and $C(-1, 4)$. Find the distance between each pair of points.

$$d(A, B) = \sqrt{[2-(-4)]^2 + (5-3)^2} = \sqrt{6^2 + 2^2}$$
$$= \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$$

$$d(B, C) = \sqrt{(-1-2)^2 + (4-5)^2}$$
$$= \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$d(A, C) = \sqrt{[-1-(-4)]^2 + (4-3)^2}$$
$$= \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

Because $d(B, C) + d(A, C) = d(A, B)$ or $\sqrt{10} + \sqrt{10} = 2\sqrt{10}$, the points are collinear.

35. Midpoint (5, 8), endpoint (13, 10)

$$\frac{13+x}{2}=5 \quad\text{and}\quad \frac{10+y}{2}=8$$
$$13+x=10 \quad\text{and}\quad 10+y=16$$
$$x=-3 \quad\text{and}\quad y=6.$$

The other endpoint has coordinates (–3, 6).

36. Midpoint (–7, 6), endpoint (–9, 9)

$$\frac{-9+x}{2}=-7 \quad\text{and}\quad \frac{9+y}{2}=6$$
$$-9+x=-14 \quad\text{and}\quad 9+y=12$$
$$x=-5 \quad\text{and}\quad y=3.$$

The other endpoint has coordinates (–5, 3).

37. Midpoint (12, 6), endpoint (19, 16)

$$\frac{19+x}{2}=12 \quad\text{and}\quad \frac{16+y}{2}=6$$
$$19+x=24 \quad\text{and}\quad 16+y=12$$
$$x=5 \quad\text{and}\quad y=-4.$$

The other endpoint has coordinates (5, –4).

38. Midpoint (–9, 8), endpoint (–16, 9)

$$\frac{-16+x}{2}=-9 \quad\text{and}\quad \frac{9+y}{2}=8$$
$$-16+x=-18 \quad\text{and}\quad 9+y=16$$
$$x=-2 \quad\text{and}\quad y=7$$

The other endpoint has coordinates (–2, 7).

39. Midpoint (a, b), endpoint (p, q)

$$\frac{p+x}{2}=a \quad\text{and}\quad \frac{q+y}{2}=b$$
$$p+x=2a \quad\text{and}\quad q+y=2b$$
$$x=2a-p \quad\text{and}\quad y=2b-q$$

The other endpoint has coordinates $(2a-p, 2b-q)$.

40. Midpoint $(6a, 6b)$, endpoint $(3a, 5b)$

42. The endpoints are (2006, 7505) and (2012, 3335)

$$M=\left(\frac{2006+2012}{2}, \frac{7505+3335}{2}\right)$$
$$=(2009, 5420)$$

According to the model, the average national advertising revenue in 2009 was \$5420 million. This is higher than the actual value of \$4424 million.

43. The points to use are (2011, 23021) and (2013, 23834). Their midpoint is

$$\left(\frac{2011+2013}{2}, \frac{23,021+23,834}{2}\right)$$
$$=(2012, 23427.5).$$

In 2012, the poverty level cutoff was approximately \$23,428.

44. **(a)** To estimate the enrollment for 2003, use the points (2000, 11,753) and (2006, 13,180)

$$M=\left(\frac{2000+2006}{2}, \frac{11,753+13,180}{2}\right)$$
$$=(2003, 12466.5)$$

The enrollment for 2003 was about 12,466.5 thousand.

(b) To estimate the enrollment for 2009, use the points (2006, 13,180) and (2012, 14,880)

$$M=\left(\frac{2006+2012}{2}, \frac{13,180+14,880}{2}\right)$$
$$=(2009, 14030)$$

The enrollment for 2009 was about 14,030 thousand.

45. The midpoint M has coordinates

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right).$$

$$\frac{3a+x}{2} = 6a \quad \text{and} \quad \frac{5b+y}{2} = 6b$$

$$3a + x = 12a \quad \text{and} \quad 5b + y = 12b$$

$$x = 9a \quad \text{and} \quad y = 7b$$

The other endpoint has coordinates $(9a, 7b)$.

41. The endpoints of the segment are $(1990, 21.3)$ and $(2012, 30.1)$.

$$M = \left(\frac{1990+2012}{2}, \frac{21.3+30.9}{2}\right)$$

$$= (2001,\ 26.1)$$

The estimate is 26.1%. This is very close to the actual figure of 26.2%.

$d(P, M)$

$$= \sqrt{\left(\frac{x_1+x_2}{2} - x_1\right)^2 + \left(\frac{y_1+y_2}{2} - y_1\right)^2}$$

$$= \sqrt{\left(\frac{x_1+x_2}{2} - \frac{2x_1}{2}\right)^2 + \left(\frac{y_1+y_2}{2} - \frac{2y_1}{2}\right)^2}$$

$$= \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2}$$

$$= \sqrt{\frac{(x_2-x_1)^2}{4} + \frac{(y_2-y_1)^2}{4}}$$

$$= \sqrt{\frac{(x_2-x_1)^2 + (y_2-y_1)^2}{4}}$$

$$= \frac{1}{2}\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

(*continued on next page*)

(*continued*)

$d(M,Q)$

$$= \sqrt{\left(x_2 - \frac{x_1+x_2}{2}\right)^2 + \left(y_2 - \frac{y_1+y_2}{2}\right)^2}$$

$$= \sqrt{\left(\frac{2x_2}{2} - \frac{x_1+x_2}{2}\right)^2 + \left(\frac{2y_2}{2} - \frac{y_1+y_2}{2}\right)^2}$$

$$= \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2}$$

$$= \sqrt{\frac{(x_2-x_1)^2}{4} + \frac{(y_2-y_1)^2}{4}}$$

$$= \sqrt{\frac{(x_2-x_1)^2+(y_2-y_1)^2}{4}}$$

$$= \frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d(P,Q) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Because $\frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} + \frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$,

this shows $d(P,M)+d(M,Q)=d(P,Q)$ and $d(P,M)=d(M,Q)$.

46. The distance formula,

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, can be written

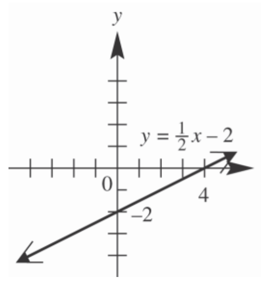
as $d=[(x_2-x_1)^2+(y_2-y_1)^2]^{1/2}$.

In exercises 47−58, other ordered pairs are possible.

47. (a)

x	***y***	
0	−2	*y*-intercept: $x=0\Rightarrow$ $y=\frac{1}{2}(0)-2=-2$
4	0	*x*-intercept: $y=0\Rightarrow$ $0=\frac{1}{2}x-2\Rightarrow$ $2=\frac{1}{2}x\Rightarrow 4=x$
2	−1	additional point

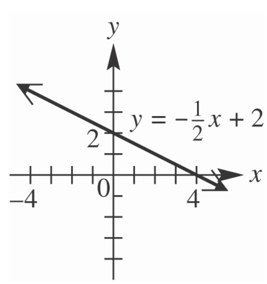
(b)



48. (a)

x	***y***	
0	2	*y*-intercept: $x=0\Rightarrow$ $y=-\frac{1}{2}(0)+2=2$
4	0	*x*-intercept: $y=0\Rightarrow$ $0=-\frac{1}{2}x+2\Rightarrow$ $\frac{1}{2}x=2\Rightarrow 4=x$
2	1	additional point

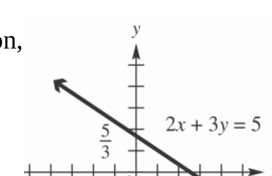
(b)



49. (a)

x	***y***	
0	$\frac{5}{3}$	*y*-intercept: $x=0\Rightarrow$ $2(0)+3y=5\Rightarrow$ $3y=5\Rightarrow y=\frac{5}{3}$
$\frac{5}{2}$	0	*x*-intercept: $y=0\Rightarrow$ $2x+3(0)=5\Rightarrow$ $2x=5\Rightarrow x=\frac{5}{2}$
4	−1	additional point

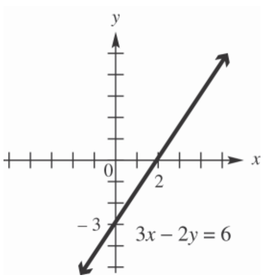
(b)



50. **(a)**

x	y	
0	-3	y-intercept: $x = 0 \Rightarrow$ $3(0) - 2y = 6 \Rightarrow$ $-2y = 6 \Rightarrow y = -3$
2	0	x-intercept: $y = 0 \Rightarrow$ $3x - 2(0) = 6 \Rightarrow$ $3x = 6 \Rightarrow x = 2$
4	3	additional point

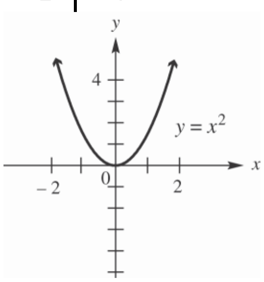
(b)



51. **(a)**

x	y	
0	0	x- and y-intercept: $0 = 0^2$
1	1	additional point
-2	4	additional point

(b)



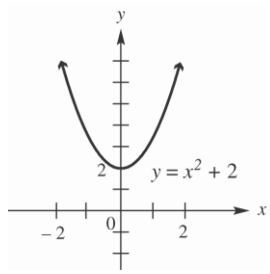
52. **(a)**

x	y	
0	2	y-intercept: $x = 0 \Rightarrow$ $y = 0^2 + 2 \Rightarrow$ $y = 0 + 2 \Rightarrow y = 2$
-1	3	additional point
2	6	additional point

no x-intercept:

$y = 0 \Rightarrow 0 = x^2 + 2 \Rightarrow$
$-2 = x^2 \Rightarrow \pm\sqrt{-2} = x$

(b)

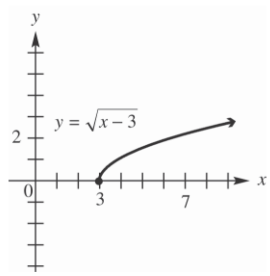


53. **(a)**

x	y	
3	0	x-intercept: $y = 0 \Rightarrow$ $0 = \sqrt{x-3} \Rightarrow$ $0 = x - 3 \Rightarrow 3 = x$
4	1	additional point
7	2	additional point

no y-intercept:
$x = 0 \Rightarrow y = \sqrt{0-3} \Rightarrow y = \sqrt{-3}$

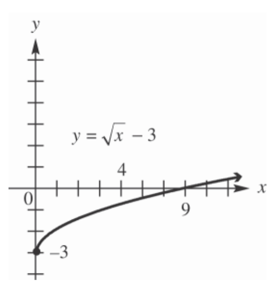
(b)



54. **(a)**

x	y	
0	-3	y-intercept: $x = 0 \Rightarrow$ $y = \sqrt{0} - 3 \Rightarrow$ $y = 0 - 3 \Rightarrow y = -3$
4	-1	additional point
9	0	x-intercept: $y = 0 \Rightarrow$ $0 = \sqrt{x} - 3 \Rightarrow$ $3 = \sqrt{x} \Rightarrow 9 = x$

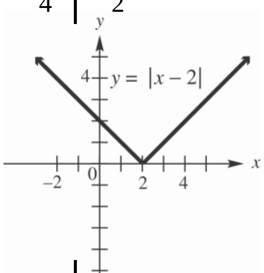
(b)



55. **(a)**

x	y	
0	2	y-intercept: $x = 0 \Rightarrow$ $y = \lvert 0-2 \rvert \Rightarrow$ $y = \lvert -2 \rvert \Rightarrow y = 2$
2	0	x-intercept: $y = 0 \Rightarrow$ $0 = \lvert x-2 \rvert \Rightarrow$ $0 = x-2 \Rightarrow 2 = x$
−2	4	additional point
4	2	additional point

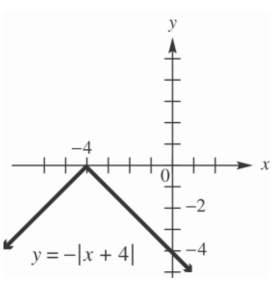
(b)



56. **(a)**

x	y	
−2	−2	additional point
−4	0	x-intercept: $y = 0 \Rightarrow$ $0 = -\lvert x+4 \rvert \Rightarrow$ $0 = \lvert x+4 \rvert \Rightarrow$ $0 = x+4 \Rightarrow -4 = x$
0	−4	y-intercept: $x = 0 \Rightarrow$ $y = -\lvert 0+4 \rvert \Rightarrow$ $y = -\lvert 4 \rvert \Rightarrow y = -4$

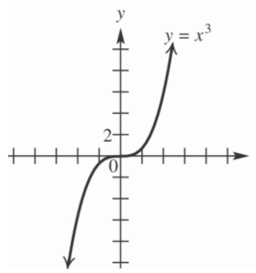
(b)



57. **(a)**

x	y	
0	0	x- and y-intercept: $0 = 0^3$
−1	−1	additional point
2	8	additional point

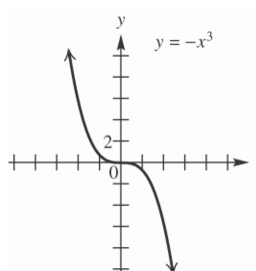
(b)



58. **(a)**

x	y	
0	0	x- and y-intercept: $0 = -0^3$
1	−1	additional point
2	−8	additional point

(b)

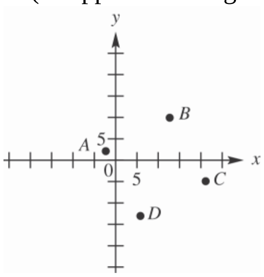


59. Points on the x-axis have y-coordinates equal to 0. The point on the x-axis will have the same x-coordinate as point (4, 3). Therefore, the line will intersect the x-axis at (4, 0).

60. Points on the y-axis have x-coordinates equal to 0. The point on the y-axis will have the same y-coordinate as point (4, 3). Therefore, the line will intersect the y-axis at (0, 3).

61. Because (a, b) is in the second quadrant, a is negative and b is positive. Therefore, $(a, -b)$ will have a negative x–coordinate and a negative y-coordinate and will lie in quadrant III.
$(-a, b)$ will have a positive x-coordinate and a positive y-coordinateand will lie in quadrant I.
$(-a, -b)$ will have a positive x-coordinate and a negative y-coordinate and will lie in quadrant IV.
(b, a) will have a positive x-coordinate and a negative y-coordinate and will lie in quadrant IV.

62. Label the points $A(-2,2)$, $B(13,10)$, $C(21,-5)$, and $D(6,-13)$. To determine which points form sides of the quadrilateral (as opposed to diagonals), plot the points.



Use the distance formula to find the length of each side.

$$d(A,B)=\sqrt{[13-(-2)]^2+(10-2)^2}=\sqrt{15^2+8^2}=\sqrt{225+64}=\sqrt{289}=17$$

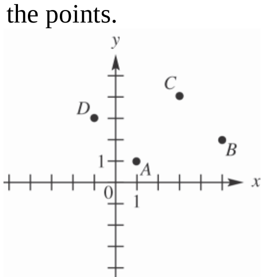
$$d(B,C)=\sqrt{(21-13)^2+(-5-10)^2}=\sqrt{8^2+(-15)^2}=\sqrt{64+225}=\sqrt{289}=17$$

$$d(C,D)=\sqrt{(6-21)^2+[-13-(-5)]^2}=\sqrt{(-15)^2+(-8)^2}=\sqrt{225+64}=\sqrt{289}=17$$

$$d(D,A)=\sqrt{(-2-6)^2+[2-(-13)]^2}=\sqrt{(-8)^2+15^2}=\sqrt{64+225}=\sqrt{289}=17$$

Because all sides have equal length, the four points form a rhombus.

63. To determine which points form sides of the quadrilateral (as opposed to diagonals), plot the points.



$$d(B,C)=\sqrt{(3-5)^2+(4-2)^2}=\sqrt{(-2)^2+2^2}=\sqrt{4+4}=\sqrt{8}$$

$$d(C,D)=\sqrt{(-1-3)^2+(3-4)^2}=\sqrt{(-4)^2+(-1)^2}=\sqrt{16+1}=\sqrt{17}$$

$$d(D,A)=\sqrt{[1-(-1)]^2+(1-3)^2}=\sqrt{2^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}$$

Because $d(A, B) = d(C, D)$ and $d(B, C) = d(D, A)$, the points are the vertices of a parallelogram. Because $d(A, B) \neq d(B, C)$, the points are not the vertices of a rhombus.

64. For the points $A(4, 5)$ and $D(10, 14)$, the difference of the x-coordinates is $10 – 4 = 6$ and the difference of the y-coordinates is $14 – 5 = 9$. Dividing these differences by 3, we obtain 2 and 3, respectively. Adding 2 and 3 to the x and y coordinates of point A, respectively, we obtain $B(4 + 2, 5 + 3)$ or $B(6, 8)$.
Adding 2 and 3 to the x- and y- coordinates of point B, respectively, we obtain $C(6 + 2, 8 + 3)$ or $C(8, 11)$. The desired points are $B(6, 8)$ and $C(8, 11)$.

We check these by showing that $d(A, B) = d(B, C) = d(C, D)$ and that $d(A, D) = d(A, B) + d(B, C) + d(C, D)$.

$$d(A,B)=\sqrt{(6-4)^2+(8-5)^2}=\sqrt{2^2+3^2}=\sqrt{4+9}=\sqrt{13}$$

$$d(B,C)=\sqrt{(8-6)^2+(11-8)^2}=\sqrt{2^2+3^2}=\sqrt{4+9}=\sqrt{13}$$

$$d(C,D)=\sqrt{(10-8)^2+(14-11)^2}=\sqrt{2^2+3^2}=\sqrt{4+9}=\sqrt{13}$$

$$d(A,D)=\sqrt{(10-4)^2+(14-5)^2}$$

Use the distance formula to find the length of each side.

$$d(A,B) = \sqrt{(5-1)^2 + (2-1)^2}$$
$$= \sqrt{4^2 + 1^2} = \sqrt{16+1} = \sqrt{17}$$

$$= \sqrt{6^2 + 9^2} = \sqrt{36+81}$$
$$= \sqrt{117} = \sqrt{9(13)} = 3\sqrt{13}$$

$d(A, B)$, $d(B, C)$, and $d(C, D)$ all have the same measure and $d(A, D) = d(A, B) + d(B, C) + d(C, D)$ Because $3\sqrt{13} = \sqrt{13} + \sqrt{13} + \sqrt{13}$.

Section 2.2 Circles

1. The circle with equation $x^2 + y^2 = 49$ has center with coordinates <u>(0, 0)</u> and radius equal to <u>7</u>.

2. The circle with center (3, 6) and radius 4 has equation $\underline{(x-3)^2 + (y-6) = 16}$.

3. The graph of $(x-4)^2 + (y+7)^2 = 9$ has center with coordinates <u>(4, –7)</u>.

4. The graph of $x^2 + (y-5)^2 = 9$ has center with coordinates <u>(0, 5)</u>.

5. This circle has center (3, 2) and radius 5. This is graph B.

6. This circle has center (3, –2) and radius 5. This is graph C.

7. This circle has center (–3, 2) and radius 5. This is graph D.

8. This circle has center (–3, –2) and radius 5. This is graph A.

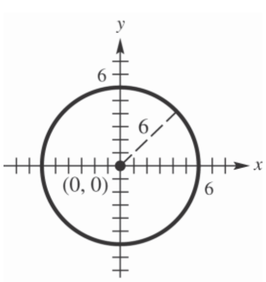
9. The graph of $x^2 + y^2 = 0$ has center (0, 0) and radius 0. This is the point (0, 0). Therefore, there is one point on the graph.

10. $\sqrt{-100}$ is not a real number, so there are no points on the graph of $x^2 + y^2 = -100$.

11. **(a)** Center (0, 0), radius 6

$$\sqrt{(x-0)^2 + (y-0)^2} = 6$$
$$(x-0)^2 + (y-0)^2 = 6^2 \Rightarrow x^2 + y^2 = 36$$

(b)

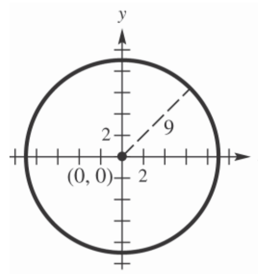


$x^2 + y^2 = 36$

12. **(a)** Center (0, 0), radius 9

$$\sqrt{(x-0)^2 + (y-0)^2} = 9$$
$$(x-0)^2 + (y-0)^2 = 9^2 \Rightarrow x^2 + y^2 = 81$$

(b)

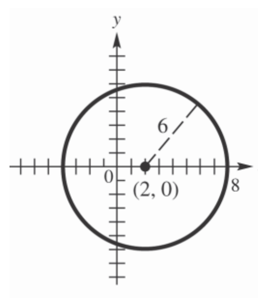


$x^2 + y^2 = 81$

13. **(a)** Center (2, 0), radius 6

$$\sqrt{(x-2)^2 + (y-0)^2} = 6$$
$$(x-2)^2 + (y-0)^2 = 6^2$$
$$(x-2)^2 + y^2 = 36$$

(b)

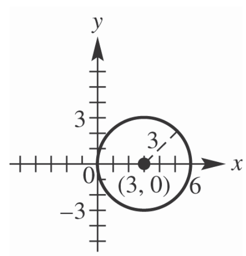


$(x-2)^2 + y^2 = 36$

14. **(a)** Center (3, 0), radius 3

$$\sqrt{(x-3)^2 + (y-0)^2} = 3$$
$$(x-3)^2 + y^2 = 9$$

(b)

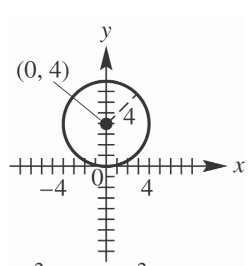


$(x-3)^2 + y^2 = 9$

15. **(a)** Center (0, 4), radius 4

$$\sqrt{(x-0)^2 + (y-4)^2} = 4$$
$$x^2 + (y-4)^2 = 16$$

(b)

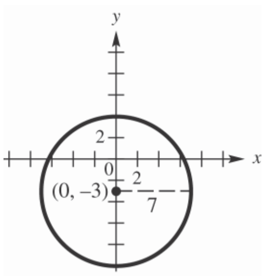


$x^2 + (y-4)^2 = 16$

16. **(a)** Center (0, –3), radius 7

$$\sqrt{(x-0)^2+[y-(-3)]^2}=7$$
$$(x-0)^2+[y-(-3)]^2=7^2$$
$$x^2+(y+3)^2=49$$

(b)

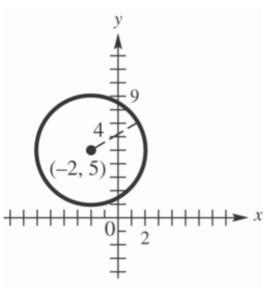


$x^2+(y+3)^2=49$

17. **(a)** Center (–2, 5), radius 4

$$\sqrt{[x-(-2)]^2+(y-5)^2}=4$$
$$[x-(-2)]^2+(y-5)^2=4^2$$
$$(x+2)^2+(y-5)^2=16$$

(b)

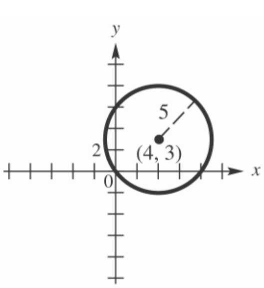


$(x+2)^2+(y-5)^2=16$

18. **(a)** Center (4, 3), radius 5

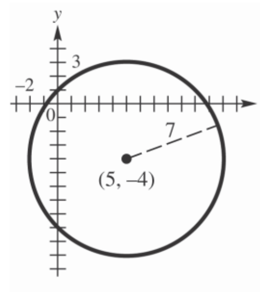
$$\sqrt{(x-4)^2+(y-3)^2}=5$$
$$(x-4)^2+(y-3)^2=5^2$$
$$(x-4)^2+(y-3)^2=25$$

(b)



$(x-4)^2+(y-3)^2=25$

(b)

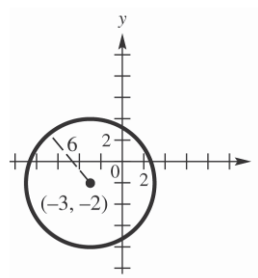


$(x-5)^2+(y+4)^2=49$

20. **(a)** Center (–3, –2), radius 6

$$\sqrt{[x-(-3)]^2+[y-(-2)]^2}=6$$
$$[x-(-3)]^2+[y-(-2)]^2=6^2$$
$$(x+3)^2+(y+2)^2=36$$

(b)

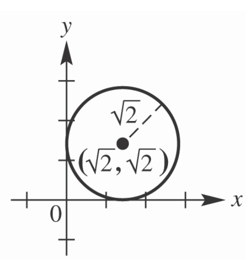


$(x+3)^2+(y+2)^2=36$

21. **(a)** Center $(\sqrt{2},\sqrt{2})$, radius $\sqrt{2}$

$$\sqrt{(x-\sqrt{2})^2+(y-\sqrt{2})^2}=\sqrt{2}$$
$$(x-\sqrt{2})^2+(y-\sqrt{2})^2=2$$

(b)



$(x-\sqrt{2})^2+(y-\sqrt{2})^2=2$

22. **(a)** Center $(-\sqrt{3},-\sqrt{3})$, radius $\sqrt{3}$

$$\sqrt{\left[x-(-\sqrt{3})\right]^2+\left[y-(-\sqrt{3})\right]^2}=\sqrt{3}$$
$$\left[x-(-\sqrt{3})\right]^2+\left[y-(-\sqrt{3})\right]^2=(\sqrt{3})^2$$

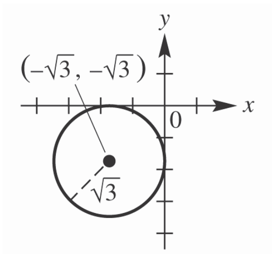
$$\left(x+\sqrt{3}\right)^2+\left(y+\sqrt{3}\right)^2=3$$

19. **(a)** Center (5, –4), radius 7

$$\sqrt{(x-5)^2+[y-(-4)]^2}=7$$

$$(x-5)^2+[y-(-4)]^2=7^2$$
$$(x-5)^2+(y+4)^2=49$$

(b)



$(x+\sqrt{3})^2 + (y+\sqrt{3})^2 = 3$

23. (a) The center of the circle is located at the midpoint of the diameter determined by the points (1, 1) and (5, 1). Using the midpoint formula, we have

$C = \left(\frac{1+5}{2}, \frac{1+1}{2}\right) = (3,1)$. The radius is one-half the length of the diameter:

$$r = \frac{1}{2}\sqrt{(5-1)^2 + (1-1)^2} = 2$$

The equation of the circle is

$(x-3)^2 + (y-1)^2 = 4$

(b) Expand $(x-3)^2 + (y-1)^2 = 4$ to find the equation of the circle in general form:

$$(x-3)^2 + (y-1)^2 = 4$$

$$x^2 - 6x + 9 + y^2 - 2y + 1 = 4$$
$$x^2 + y^2 - 6x - 2y + 6 = 0$$

24. (a) The center of the circle is located at the midpoint of the diameter determined by the points (−1, 1) and (−1, −5). Using the midpoint formula, we have

$C = \left(\frac{-1+(-1)}{2}, \frac{1+(-5)}{2}\right) = (-1,-2)$.

The radius is one-half the length of the diameter:

$$r = \frac{1}{2}\sqrt{\left[-1-(-1)\right]^2 + (-5-1)^2} = 3$$

The equation of the circle is

$(x+1)^2 + (y+2)^2 = 9$

(b) Expand $(x+1)^2 + (y+2)^2 = 9$ to find the equation of the circle in general form:

25. (a) The center of the circle is located at the midpoint of the diameter determined by the points (−2, 4) and (−2, 0). Using the midpoint formula, we have

$C = \left(\frac{-2+(-2)}{2}, \frac{4+0}{2}\right) = (-2,2)$.

The radius is one-half the length of the diameter:

$$r = \frac{1}{2}\sqrt{\left[-2-(-2)\right]^2 + (4-0)^2} = 2$$

The equation of the circle is

$(x+2)^2 + (y-2)^2 = 4$

(b) Expand $(x+2)^2 + (y-2)^2 = 4$ to find the equation of the circle in general form:

$$(x+2)^2 + (y-2)^2 = 4$$
$$x^2 + 4x + 4 + y^2 - 4y + 4 = 4$$
$$x^2 + y^2 + 4x - 4y + 4 = 0$$

26. (a) The center of the circle is located at the midpoint of the diameter determined by the points (0, −3) and (6, −3). Using the midpoint formula, we have

$C = \left(\frac{0+6}{2}, \frac{-3+(-3)}{2}\right) = (3,-3)$.

The radius is one-half the length of the diameter:

$$r = \frac{1}{2}\sqrt{(6-0)^2 + \left[-3-(-3)\right]^2} = 3$$

The equation of the circle is

$(x-3)^2 + (y+3)^2 = 9$

(b) Expand $(x-3)^2 + (y+3)^2 = 9$ to find the equation of the circle in general form:

$$(x-3)^2 + (y+3)^2 = 9$$

$$x^2 - 6x + 9 + y^2 + 6y + 9 = 9$$
$$x^2 + y^2 - 6x + 6y + 9 = 0$$

$$(x+1)^2 + (y+2)^2 = 9$$

27. $x^2 + y^2 + 6x + 8y + 9 = 0$

Complete the square on x and y separately.

$$\left(x^2 + 6x\right) + \left(y^2 + 8y\right) = -9$$

$$\left(x^2 + 6x + 9\right) + \left(y^2 + 8y + 16\right) = -9 + 9 + 16$$

2 2

$$x^2 + 2x + 1 + y^2 + 4y + 4 = 9$$

$$x^2 + y^2 + 2x + 4y - 4 = 0$$

$$(x+3)\ \ +(y+4)\ \ =16$$

Yes, it is a circle. The circle has its center at (–3, –4) and radius 4.

28. $x^2 + y^2 + 8x - 6y + 16 = 0$
Complete the square on x and y separately.

$$\left(x^2 + 8x\right) + \left(y^2 - 6y\right) = -16$$

$$\left(x^2 + 8x + 16\right) + \left(y^2 - 6y + 9\right) = -16 + 16 + 9$$

$$(x + 4)^2 + (y - 3)^2 = 9$$

Yes, it is a circle. The circle has its center at (–4, 3) and radius 3.

29. $x^2 + y^2 - 4x + 12y = -4$
Complete the square on x and y separately.

$$\left(x^2 - 4x\right) + \left(y^2 + 12y\right) = -4$$

$$\left(x^2 - 4x + 4\right) + \left(y^2 + 12y + 36\right) = -4 + 4 + 36$$

$$(x - 2)^2 + (y + 6)^2 = 36$$

Yes, it is a circle. The circle has its center at (2, –6) and radius 6.

30. $x^2 + y^2 - 12x + 10y = -25$
Complete the square on x and y separately.

$$\left(x^2 - 12x\right) + \left(y^2 + 10y\right) = -25$$

$$\left(x^2 - 12x + 36\right) + \left(y^2 + 10y + 25\right) = -25 + 36 + 25$$

$$(x - 6)^2 + (y + 5)^2 = 36$$

Yes, it is a circle. The circle has its center at (6, –5) and radius 6.

31. $4x^2 + 4y^2 + 4x - 16y - 19 = 0$

Complete the square on x and y separately.

$$4\left(x^2 + x\right) + 4\left(y^2 - 4y\right) = 19$$

$$4\left(x^2 + x + \tfrac{1}{4}\right) + 4\left(y^2 - 4y + 4\right) = 19 + 4\left(\tfrac{1}{4}\right) + 4(4)$$

$$4\left(x + \tfrac{1}{2}\right)^2 + 4(y - 2)^2 = 36$$

$$\left(x + \tfrac{1}{2}\right)^2 + (y - 2)^2 = 9$$

Yes, it is a circle with center $\left(-\tfrac{1}{2}, 2\right)$ and radius 3.

32. $9x^2 + 9y^2 + 12x - 18y - 23 = 0$
Complete the square on x and y separately.

$$9\left(x + \tfrac{2}{3}\right)^2 + 9(y - 1)^2 = 36$$

$$\left(x + \tfrac{2}{3}\right)^2 + (y - 1)^2 = 4$$

Yes, it is a circle with center $\left(-\tfrac{2}{3}, 1\right)$ and radius 2.

33. $x^2 + y^2 + 2x - 6y + 14 = 0$
Complete the square on x and y separately.

$$\left(x^2 + 2x\right) + \left(y^2 - 6y\right) = -14$$

$$\left(x^2 + 2x + 1\right) + \left(y^2 - 6y + 9\right) = -14 + 1 + 9$$

$$(x + 1)^2 + (y - 3)^2 = -4$$

The graph is nonexistent.

34. $x^2 + y^2 + 4x - 8y + 32 = 0$
Complete the square on x and y separately.

$$\left(x^2 + 4x\right) + \left(y^2 - 8y\right) = -32$$

$$\left(x^2 + 4x + 4\right) + \left(y^2 - 8y + 16\right) = -32 + 4 + 16$$

$$(x + 2)^2 + (y - 4)^2 = -12$$

The graph is nonexistent.

35. $x^2 + y^2 - 6x - 6y + 18 = 0$
Complete the square on x and y separately.

$$\left(x^2 - 6x\right) + \left(y^2 - 6y\right) = -18$$

$$\left(x^2 - 6x + 9\right) + \left(y^2 - 6y + 9\right) = -18 + 9 + 9$$

$$(x - 3)^2 + (y - 3)^2 = 0$$

The graph is the point (3, 3).

36. $x^2 + y^2 + 4x + 4y + 8 = 0$

Complete the square on x and y separately.

$$\left(x^2 + 4x\right) + \left(y^2 + 4y\right) = -8$$

$$\left(x^2 + 4x + 4\right) + \left(y^2 + 4y + 4\right) = -8 + 4 + 4$$

$$(x + 2)^2 + (y + 2)^2 = 0$$

The graph is the point (−2, −2).

37. $9x^2 + 9y^2 - 6x + 6y - 23 = 0$
Complete the square on x and y separately.

$$\left(9x^2 - 6x\right) + \left(9y^2 + 6y\right) = 23$$

$$9\left(x^2+\frac{4}{3}x\right)+9\left(y^2-2y\right)=23$$

$$9\left(x^2+\frac{4}{3}x+\frac{4}{9}\right)+9\left(y^2-2y+1\right)=$$

$$23+9\left(\frac{4}{9}\right)+9(1)$$

$$9\left(x^2-\frac{2}{3}x\right)+9\left(y^2+\frac{2}{3}y\right)=23$$

$$\left(x^2-\frac{2}{3}x+\frac{1}{9}\right)+\left(y^2+\frac{2}{3}y+\frac{1}{9}\right)^2=\frac{23}{9}+\frac{1}{9}+\frac{1}{9}$$

$$\left(x-\frac{1}{3}\right)^2+\left(y+\frac{1}{3}\right)^2=\frac{25}{9}=\left(\frac{5}{3}\right)^2$$

Yes, it is a circle with center $\left(\frac{1}{3},-\frac{1}{3}\right)$ and radius $\frac{5}{3}$.

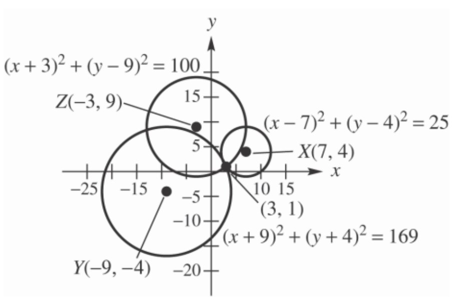
38. $4x^2 + 4y^2 + 4x - 4y - 7 = 0$

Complete the square on x and y separately.

$$4\left(x^2 + x\right) + 4\left(y^2 - y\right) = 7$$
$$4\left(x^2 + x + \tfrac{1}{4}\right) + 4\left(y^2 - y + \tfrac{1}{4}\right) = 7 + 4\left(\tfrac{1}{4}\right) + 4\left(\tfrac{1}{4}\right)$$
$$4\left(x + \tfrac{1}{2}\right)^2 + 4\left(y - \tfrac{1}{2}\right)^2 = 9$$
$$\left(x + \tfrac{1}{2}\right)^2 + \left(y - \tfrac{1}{2}\right)^2 = \tfrac{9}{4}$$

Yes, it is a circle with center $\left(-\tfrac{1}{2}, \tfrac{1}{2}\right)$ and radius $\tfrac{3}{2}$.

39. The equations of the three circles are $(x-7)^2 + (y-4)^2 = 25$, $(x+9)^2 + (y+4)^2 = 169$, and $(x+3)^2 + (y-9)^2 = 100$. From the graph of the three circles, it appears that the epicenter is located at (3, 1).

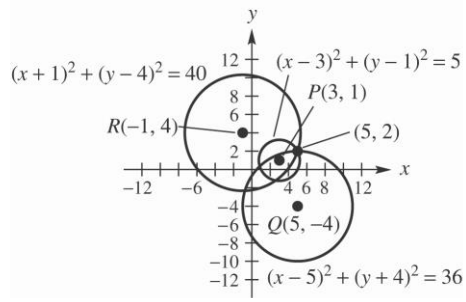


Check algebraically:

$$(x-7)^2 + (y-4)^2 = 25$$
$$(3-7)^2 + (1-4)^2 = 25$$
$$4^2 + 3^2 = 25 \Rightarrow 25 = 25$$
$$(x+9)^2 + (y+4)^2 = 169$$
$$(3+9)^2 + (1+4)^2 = 169$$
$$12^2 + 5^2 = 169 \Rightarrow 169 = 169$$
$$(x+3)^2 + (y-9)^2 = 100$$
$$(3+3)^2 + (1-9)^2 = 100$$
$$6^2 + (-8)^2 = 100 \Rightarrow 100 = 100$$

(3, 1) satisfies all three equations, so the epicenter is at (3, 1).

40. The three equations are $(x-3)^2 + (y-1)^2 = 5$, $(x-5)^2 + (y+4)^2 = 36$, and

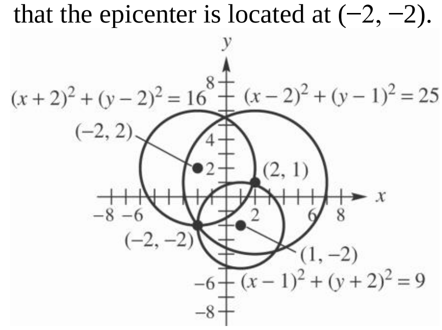


Check algebraically:

$$(x-3)^2 + (y-1)^2 = 5$$
$$(5-3)^2 + (2-1)^2 = 5$$
$$2^2 + 1^2 = 5 \Rightarrow 5 = 5$$
$$(x-5)^2 + (y+4)^2 = 36$$
$$(5-5)^2 + (2+4)^2 = 36$$
$$6^2 = 36 \Rightarrow 36 = 36$$
$$(x+1)^2 + (y-4)^2 = 40$$
$$(5+1)^2 + (2-4)^2 = 40$$
$$6^2 + (-2)^2 = 40 \Rightarrow 40 = 40$$

(5, 2) satisfies all three equations, so the epicenter is at (5, 2).

41. From the graph of the three circles, it appears that the epicenter is located at (−2, −2).



Check algebraically:

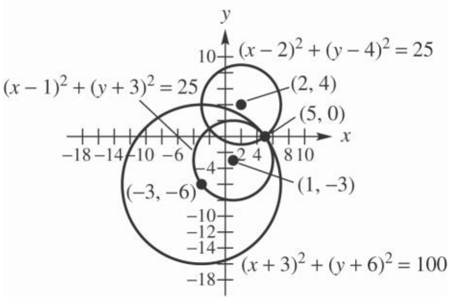
$$(x-2)^2 + (y-1)^2 = 25$$
$$(-2-2)^2 + (-2-1)^2 = 25$$
$$(-4)^2 + (-3)^2 = 25$$
$$25 = 25$$
$$(x+2)^2 + (y-2)^2 = 16$$
$$(-2+2)^2 + (-2-2)^2 = 16$$
$$0^2 + (-4)^2 = 16$$

$(x+1)^2 + (y-4)^2 = 40$. From the graph of the three circles, it appears that the epicenter is located at (5, 2).

$$16 = 16$$
$$(x-1)^2 + (y+2)^2 = 9$$
$$(-2-1)^2 + (-2+2)^2 = 9$$
$$(-3)^2 + 0^2 = 9$$
$$9 = 9$$

(−2, −2) satisfies all three equations, so the epicenter is at (−2, −2).

42. From the graph of the three circles, it appears that the epicenter is located at (5, 0).



Check algebraically:

$$(x-2)^2+(y-4)^2=25$$
$$(5-2)^2+(0-4)^2=25$$
$$3^2+(-4)^2=25$$
$$25=25$$

$$(x-1)^2+(y+3)^2=25$$
$$(5-1)^2+(0+3)^2=25$$
$$4^2+3^2=25$$
$$25=25$$

$$(x+3)^2+(y+6)^2=100$$
$$(5+3)^2+(0+6)^2=100$$
$$8^2+6^2=100$$
$$100=100$$

(5, 0) satisfies all three equations, so the epicenter is at (5, 0).

43. The radius of this circle is the distance from the center $C(3, 2)$ to the x-axis. This distance is 2, so $r = 2$.

$$(x-3)^2+(y-2)^2=2^2 \Rightarrow$$
$$(x-3)^2+(y-2)^2=4$$

44. The radius is the distance from the center $C(-4, 3)$ to the point $P(5, 8)$.

$$r=\sqrt{[5-(-4)]^2+(8-3)^2}$$
$$=\sqrt{9^2+5^2}=\sqrt{106}$$

The equation of the circle is

$$[x-(-4)]^2+(y-3)^2=(\sqrt{106})^2 \Rightarrow$$
$$(x+4)^2+(y-3)^3=106$$

$$(1-x)^2+(3-x)^2=16$$
$$1-2x+x^2+9-6x+x^2=16$$
$$2x^2-8x+10=16$$
$$2x^2-8x-6=0$$
$$x^2-4x-3=0$$

To solve this equation, we can use the quadratic formula with $a = 1$, $b = -4$, and $c = -3$.

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-3)}}{2(1)}$$
$$=\frac{4\pm\sqrt{16+12}}{2}=\frac{4\pm\sqrt{28}}{2}$$
$$=\frac{4\pm 2\sqrt{7}}{2}=2\pm\sqrt{7}$$

Because $x = y$, the points are $\left(2+\sqrt{7}, 2+\sqrt{7}\right)$ and $\left(2-\sqrt{7}, 2-\sqrt{7}\right)$.

46. Let $P(-2, 3)$ be a point which is 8 units from $Q(x, y)$. We have

$$d(P,\ Q)=\sqrt{(-2-x)^2+(3-y)^2}=8 \Rightarrow$$
$$(-2-x)^2+(3-y)^2=64.$$

Because $x + y = 0$, $x = -y$. We can either substitute $-x$ for y or $-y$ for x. Substituting $-x$ for y we solve the following:

$$(-2-x)^2+[3-(-x)]^2=64$$
$$(-2-x)^2+(3+x)^2=64$$
$$4+4x+x^2+9+6x+x^2=64$$
$$2x^2+10x+13=64$$
$$2x^2+10x-51=0$$

To solve this equation, use the quadratic formula with $a = 2$, $b = 10$, and $c = -51$.

$$x=\frac{-10\pm\sqrt{10^2-4(2)(-51)}}{2(2)}$$
$$=\frac{-10\pm\sqrt{100+408}}{4}$$

45. Label the points $P(x, y)$ and $Q(1, 3)$.

If $d(P, Q) = 4$, $\sqrt{(1-x)^2 + (3-y)^2} = 4 \Rightarrow$

$(1-x)^2 + (3-y)^2 = 16.$

If $x = y$, then we can either substitute x for y or y for x. Substituting x for y we solve the following:

$$= \frac{-10 \pm \sqrt{508}}{4} = \frac{-10 \pm \sqrt{4(127)}}{4}$$

$$= \frac{-10 \pm 2\sqrt{127}}{4} = \frac{-5 \pm \sqrt{127}}{2}$$

Because $y = -x$ the points are

$\left(\frac{-5-\sqrt{127}}{2}, \frac{5+\sqrt{127}}{2}\right)$ and

$\left(\frac{-5+\sqrt{127}}{2}, \frac{5-\sqrt{127}}{2}\right).$

47. Let $P(x, y)$ be a point whose distance from $A(1, 0)$ is $\sqrt{10}$ and whose distance from $B(5, 4)$ is $\sqrt{10}$. $d(P, A) = \sqrt{10}$, so

$$\sqrt{(1-x)^2 + (0-y)^2} = \sqrt{10} \Rightarrow$$

$$(1-x)^2 + y^2 = 10. \ d(P,\ B) = \sqrt{10}, \text{so}$$

$$\sqrt{(5-x)^2 + (4-y)^2} = \sqrt{10} \Rightarrow$$

$(5-x)^2 + (4-y)^2 = 10$. Thus,

$$(1-x)^2 + y^2 = (5-x)^2 + (4-y)^2$$

$$1-2x+x^2+y^2 = 25-10x+x^2+16-8y+y^2$$
$$1-2x = 41-10x-8y$$
$$8y = 40-8x$$
$$y = 5-x$$

Substitute $5 - x$ for y in the equation $(1-x)^2 + y^2 = 10$ and solve for x.

$$(1-x)^2 + (5-x)^2 = 10 \Rightarrow$$
$$1-2x+x^2+25-10x+x^2 = 10$$

$$2x^2 - 12x + 26 = 10 \Rightarrow 2x^2 - 12x + 16 = 0$$
$$x^2 - 6x + 8 = 0 \Rightarrow (x-2)(x-4) = 0 \Rightarrow$$

$$x - 2 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = 2 \quad \text{or} \quad x = 4$$

To find the corresponding values of y use the equation $y = 5 - x$. If $x = 2$, then $y = 5 - 2 = 3$.

If $x = 4$, then $y = 5 - 4 = 1$. The points satisfying the conditions are $(2, 3)$ and $(4, 1)$.

48. The circle of smallest radius that contains the points $A(1, 4)$ and $B(-3, 2)$ within or on its boundary will be the circle having points A and B as endpoints of a diameter. The center will be M, the midpoint:

$$\left(\frac{1+(-3)}{2}, \frac{4+2}{2}\right) = \left(\frac{-2}{2}, \frac{6}{2}\right) = (-1, 3).$$

The radius will be the distance from M to either A or B:

$$d(M,\ A) = \sqrt{[1-(-1)]^2 + (4-3)^2}$$

49. Label the points $A(3, y)$ and $B(-2, 9)$. If $d(A, B) = 12$, then

$$\sqrt{(-2-3)^2 + (9-y)^2} = 12$$
$$\sqrt{(-5)^2 + (9-y)^2} = 12$$
$$(-5)^2 + (9-y)^2 = 12^2$$
$$25 + 81 - 18y + y^2 = 144$$
$$y^2 - 18y - 38 = 0$$

Solve this equation by using the quadratic formula with $a = 1$, $b = -18$, and $c = -38$:

$$y = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(1)(-38)}}{2(1)}$$

$$= \frac{18 \pm \sqrt{324 + 152}}{2(1)} = \frac{18 \pm \sqrt{476}}{2}$$

$$= \frac{18 \pm \sqrt{4(119)}}{2} = \frac{18 \pm 2\sqrt{119}}{2} = 9 \pm \sqrt{119}$$

The values of y are $9 + \sqrt{119}$ and $9 - \sqrt{119}$.

50. Because the center is in the third quadrant, the radius is $\sqrt{2}$, and the circle is tangent to both axes, the center must be at $(-\sqrt{2}, -\sqrt{2})$.

Using the center-radius of the equation of a circle, we have

$$\left[x - (-\sqrt{2})\right]^2 + \left[y - (-\sqrt{2})\right]^2 = (\sqrt{2})^2 \Rightarrow$$

$$(x + \sqrt{2})^2 + (y + \sqrt{2})^2 = 2.$$

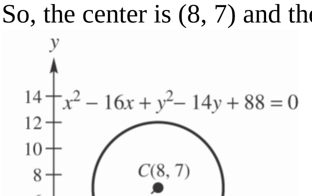
51. Let $P(x, y)$ be the point on the circle whose distance from the origin is the shortest. Complete the square on x and y separately to write the equation in center-radius form:

$$x^2 - 16x + y^2 - 14y + 88 = 0$$

$$x^2 - 16x + 64 + y^2 - 14y + 49 = -88 + 64 + 49$$

$$(x-8)^2 + (y-7)^2 = 25$$

So, the center is $(8, 7)$ and the radius is 5.



$$= \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$$

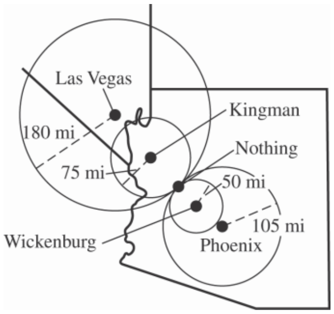
The equation of the circle is

$$\left[x - (-1)\right]^2 + (y-3)^2 = \left(\sqrt{5}\right)^2 \Rightarrow$$

$$(x+1)^2 + (y-3)^2 = 5.$$

$d(C, O) = \sqrt{8^2 + 7^2} = \sqrt{113}$. Because the length of the radius is 5, $d(P, O) = \sqrt{113} - 5$.

52. Using compasses, draw circles centered at Wickenburg, Kingman, Phoenix, and Las Vegas with scaled radii of 50, 75, 105, and 180 miles respectively. The four circles should intersect at the location of Nothing.



53. The midpoint M has coordinates

$$\left(\frac{-1+5}{2}, \frac{3+(-9)}{2}\right) = \left(\frac{4}{2}, \frac{-6}{2}\right) = (2, -3).$$

54. Use points $C(2, -3)$ and $P(-1, 3)$.

$$d(C, P) = \sqrt{(-1-2)^2 + [3-(-3)]^2} = \sqrt{(-3)^2 + 6^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

The radius is $3\sqrt{5}$.

55. Use points $C(2, -3)$ and $Q(5, -9)$.

$$d(C, Q) = \sqrt{(5-2)^2 + [-9-(-3)]^2} = \sqrt{3^2 + (-6)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

The radius is $3\sqrt{5}$.

56. Use the points $P(-1, 3)$ and $Q(5, -9)$.

Because $d(P, Q) = \sqrt{[5-(-1)]^2 + (-9-3)^2}$

$$= \sqrt{6^2 + (-12)^2} = \sqrt{36+144} = \sqrt{180}$$

58. Label the endpoints of the diameter $P(3, -5)$ and $Q(-7, 3)$. The midpoint M of the segment joining P and Q has coordinates

$$\left(\frac{3+(-7)}{2}, \frac{-5+3}{2}\right) = \left(\frac{-4}{2}, \frac{-2}{2}\right) = (-2, -1).$$

The center is $C(-2, -1)$. To find the radius, we can use points $C(-2, -1)$ and $P(3, -5)$

$$d(C, P) = \sqrt{[3-(-2)]^2 + [-5-(-1)]^2} = \sqrt{5^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$$

We could also use points $C(-2, -1)$.and $Q(-7, 3)$.

$$d(C, Q) = \sqrt{[-7-(-2)]^2 + [3-(-1)]^2} = \sqrt{(-5)^2 + 4^2} = \sqrt{25+16} = \sqrt{41}$$

We could also use points $P(3, -5)$ and $Q(-7, 3)$ to find the length of the diameter. The length of the radius is one-half the length of the diameter.

$$d(P, Q) = \sqrt{(-7-3)^2 + [3-(-5)]^2} = \sqrt{(-10)^2 + 8^2} = \sqrt{100+64} = \sqrt{164} = 2\sqrt{41}$$

$$\frac{1}{2} d(P, Q) = \frac{1}{2}\left(2\sqrt{41}\right) = \sqrt{41}$$

The center-radius form of the equation of the circle is

$$[x-(-2)]^2 + [y-(-1)]^2 = (\sqrt{41})^2$$

$$(x+2)^2 + (y+1)^2 = 41$$

59. Label the endpoints of the diameter $P(-1, 2)$ and $Q(11, 7)$. The midpoint M of the segment joining P and Q has coordinates

$$\left(\frac{-1+11}{2}, \frac{2+7}{2}\right) = \left(5, \frac{9}{2}\right).$$

The center is $C\left(5, \frac{9}{2}\right)$. To find the radius, we

$= 6\sqrt{5}$, the radius is $\frac{1}{2}d(P, Q)$. Thus

$$r = \frac{1}{2}\left(6\sqrt{5}\right) = 3\sqrt{5}.$$

57. The center-radius form for this circle is $(x-2)^2 + (y+3)^2 = (3\sqrt{5})^2 \Rightarrow$

$$(x-2)^2 + (y+3)^2 = 45.$$

can use points $C\left(5, \frac{9}{2}\right)$ and $P(-1, 2)$.

$$d(C, P) = \sqrt{\left[5-(-1)\right]^2 + \left(\frac{9}{2}-2\right)^2} = \sqrt{6^2 + \left(\frac{5}{2}\right)^2} = \sqrt{\frac{169}{4}} = \frac{13}{2}$$

We could also use points $C\left(5, \frac{9}{2}\right)$ and $Q(11, 7)$.

$$d(C, Q) = \sqrt{(5-11)^2 + \left(\frac{9}{2}-7\right)^2} = \sqrt{(-6)^2 + \left(-\frac{5}{2}\right)^2} = \sqrt{\frac{169}{4}} = \frac{13}{2}$$

(*continued on next page*)

(*continued*)

Using the points P and Q to find the length of the diameter, we have

$$d(P,\,Q) = \sqrt{(-1-11)^2 + (2-7)^2}$$
$$= \sqrt{(-12)^2 + (-5)^2}$$
$$= \sqrt{169} = 13$$

$$\frac{1}{2}d(P,\,Q) = \frac{1}{2}(13) = \frac{13}{2}$$

The center-radius form of the equation of the circle is

$$(x-5)^2 + \left(y-\tfrac{9}{2}\right)^2 = \left(\tfrac{13}{2}\right)^2$$
$$(x-5)^2 + \left(y-\tfrac{9}{2}\right)^2 = \tfrac{169}{4}$$

60. Label the endpoints of the diameter $P(5, 4)$ and $Q(-3, -2)$. The midpoint M of the segment joining P and Q has coordinates

$$\left(\frac{5+(-3)}{2}, \frac{4+(-2)}{2}\right) = (1, 1).$$

The center is $C(1, 1)$. To find the radius, we can use points $C(1, 1)$ and $P(5, 4)$.

$$d(C,\,P) = \sqrt{(5-1)^2 + (4-1)^2}$$
$$= \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

We could also use points $C(1, 1)$ and $Q(-3, -2)$.

$$d(C,\,Q) = \sqrt{[1-(-3)]^2 + [1-(-2)]^2}$$
$$= \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

Using the points P and Q to find the length of the diameter, we have

$$d(P,\,Q) = \sqrt{[5-(-3)]^2 + [4-(-2)]^2}$$
$$= \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

$$\frac{1}{2}d(P,\,Q) = \frac{1}{2}(10) = 5$$

The length of the diameter PQ is

$$\sqrt{(1-5)^2 + (4-1)^2} = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5.$$

The length of the radius is $\frac{1}{2}(5) = \frac{5}{2}$.

The center-radius form of the equation of the circle is

$$(x-3)^2 + \left(y-\tfrac{5}{2}\right)^2 = \left(\tfrac{5}{2}\right)^2$$
$$(x-3)^2 + \left(y-\tfrac{5}{2}\right)^2 = \tfrac{25}{4}$$

62. Label the endpoints of the diameter $P(-3, 10)$ and $Q(5, -5)$. The midpoint M of the segment joining P and Q has coordinates

$$\left(\frac{-3+5}{2}, \frac{10+(-5)}{2}\right) = \left(1, \tfrac{5}{2}\right).$$

The center is $C\left(1, \tfrac{5}{2}\right)$.

The length of the diameter PQ is

$$\sqrt{(-3-5)^2 + [10-(-5)]^2} = \sqrt{(-8)^2 + 15^2}$$
$$= \sqrt{289} = 17.$$

The length of the radius is $\frac{1}{2}(17) = \frac{17}{2}$.

The center-radius form of the equation of the circle is

$$(x-1)^2 + \left(y-\tfrac{5}{2}\right)^2 = \left(\tfrac{17}{2}\right)^2$$
$$(x-1)^2 + \left(y-\tfrac{5}{2}\right)^2 = \tfrac{289}{4}$$

Section 2.3 Functions

1. The domain of the relation

$$\{(3,5),\,(4,9),\,(10,13)\} \text{ is } \{3,4,10\}.$$

2. The range of the relation in Exercise 1 is $\{5,9,13\}$.

3. The equation $y = 4x - 6$ defines a function with

The center-radius form of the equation of the circle is

$$(x-1)^2+(y-1)^2=5^2$$

$$(x-1)^2+(y-1)^2=25$$

61. Label the endpoints of the diameter $P(1, 4)$ and $Q(5, 1)$. The midpoint M of the segment joining P and Q has coordinates

$$\left(\frac{1+5}{2}, \frac{4+1}{2}\right)=\left(3, \frac{5}{2}\right).$$

The center is $C\left(3, \frac{5}{2}\right)$.

independent variable $\underline{x}$ and dependent variable $\underline{y}$.

4. The function in Exercise 3 includes the ordered pair (6, $\underline{18}$).

5. For the function $f(x)=-4x+2$,

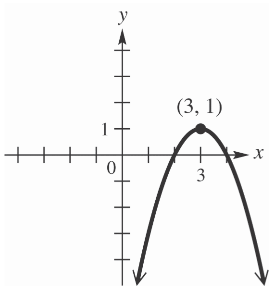
$$f(-2)=-4(-2)+2=8+2=\underline{10}.$$

6. For the function $g(x)=\sqrt{x}$, $g(9)=\sqrt{9}=\underline{3}$.

7. The function in Exercise 6, $g(x)=\sqrt{x}$, has domain $\underline{[0, \infty)}$.

8. The function in Exercise 6, $g(x) = \sqrt{x}$, has range $[0, \infty)$.

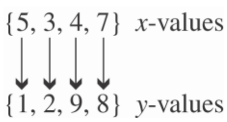
For exercises 9 and 10, use this graph.



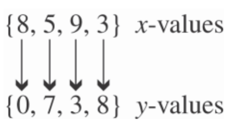
9. The largest open interval over which the function graphed here increases is $(-\infty, 3)$.

10. The largest open interval over which the function graphed here decreases is $(3, \infty)$.

11. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.



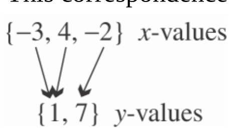
12. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.



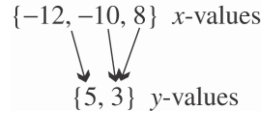
13. Two ordered pairs, namely (2, 4) and (2, 6), have the same x-value paired with different y-values, so the relation is not a function.

14. Two ordered pairs, namely (9, −2) and (9, 1), have the same x-value paired with different y-values, so the relation is not a function.

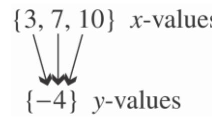
15. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.



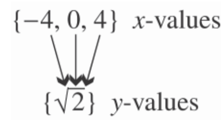
16. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.



17. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.

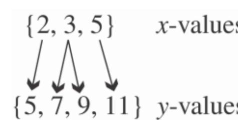


18. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.



19. Two sets of ordered pairs, namely (1, 1) and (1, −1) as well as (2, 4) and (2, −4), have the same x-value paired with different y-values, so the relation is not a function.
domain: {0, 1, 2}; range: {−4, −1, 0, 1, 4}

20. The relation is not a function because the x-value 3 corresponds to two y-values, 7 and 9. This correspondence can be shown as follows.

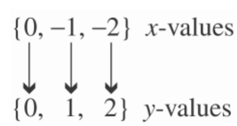


domain: {2, 3, 5}; range: {5, 7, 9, 11}

21. The relation is a function because for each different x-value there is exactly one y-value.
domain: {2, 3, 5, 11, 17}; range: {1, 7, 20}

22. The relation is a function because for each different x-value there is exactly one y-value.
domain: {1, 2, 3, 5}; range: {10, 15, 19, 27}

23. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.



Domain: {0, −1, −2}; range: {0, 1, 2}

24. The relation is a function because for each different x-value there is exactly one y-value. This correspondence can be shown as follows.

$$\begin{array}{ccc} \{0, & 1, & 2\} \\ \downarrow & \downarrow & \downarrow \\ \{0, & -1, & -2\} \end{array} \quad \begin{array}{l} x\text{-values} \\ \\ y\text{-values} \end{array}$$

Domain: $\{0, 1, 2\}$; range: $\{0, -1, -2\}$

25. The relation is a function because for each different year, there is exactly one number for visitors.
domain: {2010, 2011, 2012, 2013}
range: {64.9, 63.0, 65.1, 63.5}

26. The relation is a function because for each basketball season, there is only one number for attendance.
domain: {2011, 2012, 2013, 2014}
range: {11,159,999, 11,210,832, 11,339,285, 11,181,735}

27. This graph represents a function. If you pass a vertical line through the graph, one x-value corresponds to only one y-value.
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

28. This graph represents a function. If you pass a vertical line through the graph, one x-value corresponds to only one y-value.
domain: $(-\infty, \infty)$; range: $(-\infty, 4]$

29. This graph does not represent a function. If you pass a vertical line through the graph, there are places where one value of x corresponds to two values of y.

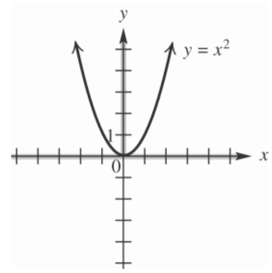
domain: $[3, \infty)$; range: $(-\infty, \infty)$

30. This graph does not represent a function. If you pass a vertical line through the graph, there are places where one value of x corresponds to two values of y.
domain: $[-4, 4]$; range: $[-3, 3]$

31. This graph represents a function. If you pass a vertical line through the graph, one x-value corresponds to only one y-value.
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

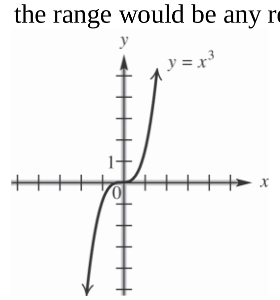
32. This graph represents a function. If you pass a vertical line through the graph, one x-value corresponds to only one y-value.
domain: $[-2, 2]$; range: $[0, 4]$

33. $y = x^2$ represents a function because y is always found by squaring x. Thus, each value of x corresponds to just one value of y. x can be any real number. Because the square of any real number is not negative, the range would be zero or greater.



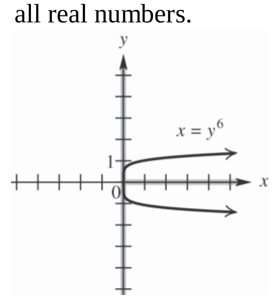
domain: $(-\infty, \infty)$; range: $[0, \infty)$

34. $y = x^3$ represents a function because y is always found by cubing x. Thus, each value of x corresponds to just one value of y. x can be any real number. Because the cube of any real number could be negative, positive, or zero, the range would be any real number.



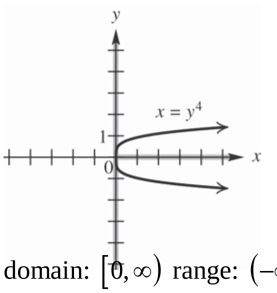
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

35. The ordered pairs (1, 1) and (1, −1) both satisfy $x = y^6$. This equation does not represent a function. Because x is equal to the sixth power of y, the values of x are nonnegative. Any real number can be raised to the sixth power, so the range of the relation is all real numbers.



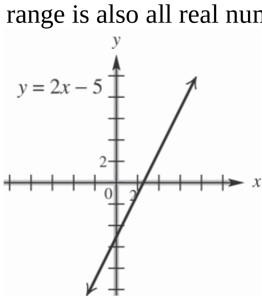
domain: $[0, \infty)$ range: $(-\infty, \infty)$

36. The ordered pairs (1, 1) and (1, −1) both satisfy $x = y^4$. This equation does not represent a function. Because *x* is equal to the fourth power of *y*, the values of *x* are nonnegative. Any real number can be raised to the fourth power, so the range of the relation is all real numbers.



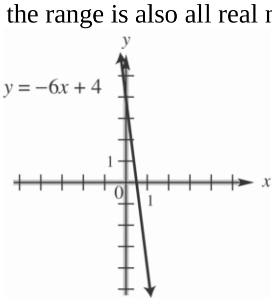
domain: $[0, \infty)$ range: $(-\infty, \infty)$

37. $y = 2x - 5$ represents a function because *y* is found by multiplying *x* by 2 and subtracting 5. Each value of *x* corresponds to just one value of *y*. *x* can be any real number, so the domain is all real numbers. Because *y* is twice *x*, less 5, y also may be any real number, and so the range is also all real numbers.



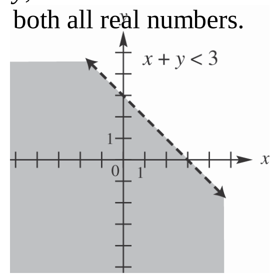
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

38. $y = -6x + 4$ represents a function because *y* is found by multiplying *x* by −6 and adding 4. Each value of *x* corresponds to just one value of *y*. *x* can be any real number, so the domain is all real numbers. Because *y* is −6 times *x*, plus 4, y also may be any real number, and so the range is also all real numbers.



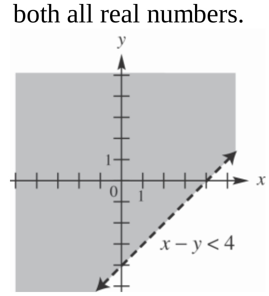
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

39. By definition, *y* is a function of *x* if every value of *x* leads to exactly one value of *y*. Substituting a particular value of *x*, say 1, into $x + y < 3$ corresponds to many values of *y*. The ordered pairs (1, –2), (1, 1), (1, 0), (1, −1), and so on, all satisfy the inequality. Note that the points on the graphed line do not satisfy the inequality and only indicate the boundary of the solution set. This does not represent a function. Any number can be used for *x* or for *y*, so the domain and range of this relation are both all real numbers.



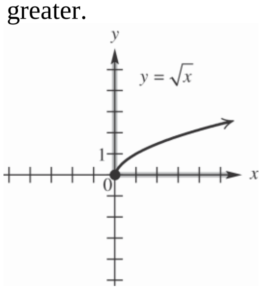
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

40. By definition, *y* is a function of *x* if every value of *x* leads to exactly one value of *y*. Substituting a particular value of *x*, say 1, into $x - y < 4$ corresponds to many values of *y*. The ordered pairs (1, −1), (1, 0), (1, 1), (1, 2), and so on, all satisfy the inequality. Note that the points on the graphed line do not satisfy the inequality and only indicate the boundary of the solution set. This does not represent a function. Any number can be used for *x* or for *y*, so the domain and range of this relation are both all real numbers.



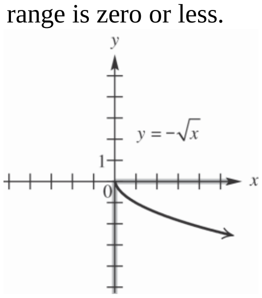
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

41. For any choice of x in the domain of $y = \sqrt{x}$, there is exactly one corresponding value of y, so this equation defines a function. Because the quantity under the square root cannot be negative, we have $x \ge 0$. Because the radical is nonnegative, the range is also zero or greater.



domain: $[0, \infty)$; range: $[0, \infty)$

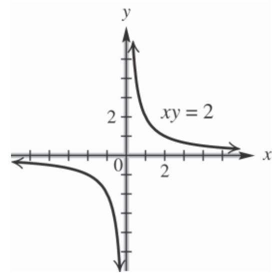
42. For any choice of x in the domain of $y = -\sqrt{x}$, there is exactly one corresponding value of y, so this equation defines a function. Because the quantity under the square root cannot be negative, we have $x \ge 0$. The outcome of the radical is nonnegative, when you change the sign (by multiplying by -1), the range becomes nonpositive. Thus the range is zero or less.



domain: $[0, \infty)$; range: $(-\infty, 0]$

43. Because $xy = 2$ can be rewritten as $y = \frac{2}{x}$,

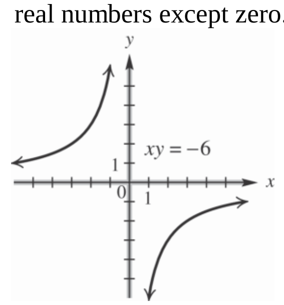
we can see that y can be found by dividing x into 2. This process produces one value of y for each value of x in the domain, so this equation is a function. The domain includes all real numbers except those that make the denominator equal to zero, namely $x = 0$. Values of y can be negative or positive, but never zero. Therefore, the range will be all real numbers except zero.



domain: $(-\infty, 0) \cup (0, \infty)$;
range: $(-\infty, 0) \cup (0, \infty)$

44. Because $xy = -6$ can be rewritten as $y = \frac{-6}{x}$,

we can see that y can be found by dividing x into -6. This process produces one value of y for each value of x in the domain, so this equation is a function. The domain includes all real numbers except those that make the denominator equal to zero, namely $x = 0$. Values of y can be negative or positive, but never zero. Therefore, the range will be all real numbers except zero.

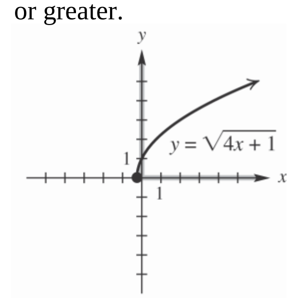


domain: $(-\infty, 0) \cup (0, \infty)$;
range: $(-\infty, 0) \cup (0, \infty)$

45. For any choice of x in the domain of $y = \sqrt{4x+1}$ there is exactly one

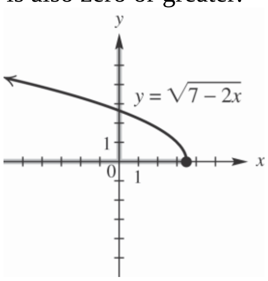
corresponding value of y, so this equation defines a function. Because the quantity under

the square root cannot be negative, we have $4x + 1 \ge 0 \Rightarrow 4x \ge -1 \Rightarrow x \ge -\frac{1}{4}$. Because the radical is nonnegative, the range is also zero or greater.



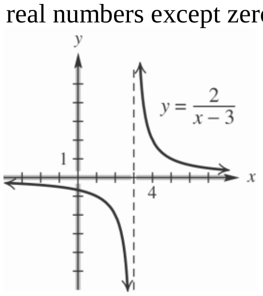
domain: $\left[-\frac{1}{4}, \infty\right)$; range: $[0, \infty)$

46. For any choice of x in the domain of $y=\sqrt{7-2x}$ there is exactly one corresponding value of y, so this equation defines a function. Because the quantity under the square root cannot be negative, we have $7-2x \ge 0 \Rightarrow -2x \ge -7 \Rightarrow x \le \frac{-7}{-2}$ or $x \le \frac{7}{2}$. Because the radical is nonnegative, the range is also zero or greater.



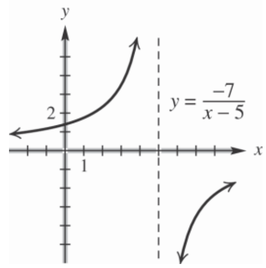
domain: $\left(-\infty, \frac{7}{2}\right]$; range: $[0,\infty)$

47. Given any value in the domain of $y=\frac{2}{x-3}$, we find y by subtracting 3, then dividing into 2. This process produces one value of y for each value of x in the domain, so this equation is a function. The domain includes all real numbers except those that make the denominator equal to zero, namely $x = 3$. Values of y can be negative or positive, but never zero. Therefore, the range will be all real numbers except zero.



domain: $(-\infty,3)\cup(3,\infty)$;
range: $(-\infty,0)\cup(0,\infty)$

48. Given any value in the domain of $y=\frac{-7}{x-5}$, we find y by subtracting 5, then dividing into −7. This process produces one value of y for each value of x in the domain, so this equation is a function. The domain includes all real numbers except those that make the denominator equal to zero, namely $x = 5$. Values of y can be negative or positive, but never zero. Therefore, the range will be all real numbers except zero.



domain: $(-\infty,5)\cup(5,\infty)$;
range: $(-\infty,0)\cup(0,\infty)$

49. B. The notation $f(3)$ means the value of the dependent variable when the independent variable is 3.

50. Answers will vary. An example is: <u>The cost of gasoline</u> depends on <u>the number of gallons used;</u> so <u>cost</u> is a function of <u>number of</u> gallons.

51. $f(x)=-3x+4$
$$f(0)=-3\cdot 0+4=0+4=4$$

52. $f(x)=-3x+4$
$$f(-3)=-3(-3)+4=9+4=13$$

53. $g(x)=-x^2+4x+1$
$$g(-2)=-(-2)^2+4(-2)+1$$
$$=-4+(-8)+1=-11$$

54. $g(x)=-x^2+4x+1$
$$g(10)=-10^2+4\cdot 10+1$$
$$=-100+40+1=-59$$

55. $f(x)=-3x+4$
$$f\left(\tfrac{1}{3}\right)=-3\left(\tfrac{1}{3}\right)+4=-1+4=3$$

56. $f(x)=-3x+4$
$$f\left(-\tfrac{7}{3}\right)=-3\left(-\tfrac{7}{3}\right)+4=7+4=11$$

57. $g(x)=-x^2+4x+1$

59. 2 2 2

58.

$g\left(\frac{1}{}\right) = -\left(\frac{1}{}\right)^2 + 4\left(\frac{1}{}\right) + 1$

$= -\frac{1}{} + 2 + 1 = \frac{11}{}$

4 4

$g(x) = -x^2 + 4x + 1$

$g\left(-\frac{1}{}\right) = -\left(-\frac{1}{}\right)^2 + 4\left(-\frac{1}{}\right) + 1$

$= -\frac{1}{} - 1 + 1 = -\frac{1}{}$

4 4 4

16 16

$f(x) = -3x + 4$

$f(p) = -3p + 4$

60. $g(x) = -x^2 + 4x + 1$
$g(k) = -k^2 + 4k + 1$

61. $f(x) = -3x + 4$
$f(-x) = -3(-x) + 4 = 3x + 4$

62. $g(x) = -x^2 + 4x + 1$

$$\begin{aligned} g(-x) &= -(-x)^2 + 4(-x) + 1 \\ &= -x^2 - 4x + 1 \end{aligned}$$

63. $f(x) = -3x + 4$

$$\begin{aligned} f(x+2) &= -3(x+2) + 4 \\ &= -3x - 6 + 4 = -3x - 2 \end{aligned}$$

64. $f(x) = -3x + 4$

$$\begin{aligned} f(a+4) &= -3(a+4) + 4 \\ &= -3a - 12 + 4 = -3a - 8 \end{aligned}$$

65. $f(x) = -3x + 4$

$$\begin{aligned} f(2m-3) &= -3(2m-3) + 4 \\ &= -6m + 9 + 4 = -6m + 13 \end{aligned}$$

66. $f(x) = -3x + 4$

$$\begin{aligned} f(3t-2) &= -3(3t-2) + 4 \\ &= -9t + 6 + 4 = -9t + 10 \end{aligned}$$

67. **(a)** $f(2) = 2$ **(b)** $f(-1) = 3$

68. **(a)** $f(2) = 5$ **(b)** $f(-1) = 11$

69. **(a)** $f(2) = 15$ **(b)** $f(-1) = 10$

70. **(a)** $f(2) = 1$ **(b)** $f(-1) = 7$

71. **(a)** $f(2) = 3$ **(b)** $f(-1) = -3$

72. **(a)** $f(2) = -3$ **(b)** $f(-1) = 2$

73. **(a)** $f(-2) = 0$ **(b)** $f(0) = 4$
(c) $f(1) = 2$ **(d)** $f(4) = 4$

74. **(a)** $f(-2) = 5$ **(b)** $f(0) = 0$
(c) $f(1) = 2$ **(d)** $f(4) = 4$

75. **(a)** $f(-2) = -3$ **(b)** $f(0) = -2$
(c) $f(1) = 0$ **(d)** $f(4) = 2$

77. **(a)**
$$\begin{aligned} x + 3y &= 12 \\ 3y &= -x + 12 \\ y &= \frac{-x+12}{3} \\ y &= -\tfrac{1}{3}x + 4 \Rightarrow f(x) = -\tfrac{1}{3}x + 4 \end{aligned}$$

(b) $f(3) = -\frac{1}{3}(3) + 4 = -1 + 4 = 3$

78. **(a)**
$$\begin{aligned} x - 4y &= 8 \\ x &= 8 + 4y \\ x - 8 &= 4y \\ \frac{x-8}{4} &= y \\ y &= \tfrac{1}{4}x - 2 \Rightarrow f(x) = \tfrac{1}{4}x - 2 \end{aligned}$$

(b) $f(3) = \frac{1}{4}(3) - 2 = \frac{3}{4} - 2 = \frac{3}{4} - \frac{8}{4} = -\frac{5}{4}$

79. **(a)**
$$\begin{aligned} y + 2x^2 &= 3 - x \\ y &= -2x^2 - x + 3 \\ f(x) &= -2x^2 - x + 3 \end{aligned}$$

(b)
$$\begin{aligned} f(3) &= -2(3)^2 - 3 + 3 \\ &= -2 \cdot 9 - 3 + 3 = -18 \end{aligned}$$

80. **(a)**
$$\begin{aligned} y - 3x^2 &= 2 + x \\ y &= 3x^2 + x + 2 \\ f(x) &= 3x^2 + x + 2 \end{aligned}$$

(b)
$$\begin{aligned} f(3) &= 3(3)^2 + 3 + 2 \\ &= 3 \cdot 9 + 3 + 2 = 32 \end{aligned}$$

81. **(a)**
$$\begin{aligned} 4x - 3y &= 8 \\ 4x &= 3y + 8 \\ 4x - 8 &= 3y \\ \frac{4x-8}{3} &= y \\ y &= \tfrac{4}{3}x - \tfrac{8}{3} \Rightarrow f(x) = \tfrac{4}{3}x - \tfrac{8}{3} \end{aligned}$$

(b) $f(3) = \frac{4}{3}(3) - \frac{8}{3} = \frac{12}{3} - \frac{8}{3} = \frac{4}{3}$

82. **(a)**
$$\begin{aligned} -2x + 5y &= 9 \\ 5y &= 2x + 9 \\ y &= \frac{2x+9}{5} \\ y &= \tfrac{2}{5}x + \tfrac{9}{5} \Rightarrow f(x) = \tfrac{2}{5}x + \tfrac{9}{5} \end{aligned}$$

(b) $f(3) = \frac{2}{5}(3) + \frac{9}{5} = \frac{6}{5} + \frac{9}{5} = \frac{15}{5}$

$$f\ 3 = {}_5\ 3 + {}_5 = {}_5 + {}_5 = {}_5 = 3$$

76. **(a)** $f(-2) = 3$ **(b)** $f(0) = 3$

(c) $f(1) = 3$ **(d)** $f(4) = 3$

83. $f(3) = 4$

84. Because $f(0.2) = 0.2^2 + 3(0.2) + 1$

$= 0.04 + 0.6 + 1 = 1.64$, the height of the rectangle is 1.64 units. The base measures $0.3 - 0.2 = 0.1$ unit. Because the area of a rectangle is base times height, the area of this rectangle is $0.1(1.64) = 0.164$ square unit.

85. $f(3)$ is the y-component of the coordinate, which is −4.

86. $f(-2)$ is the y-component of the coordinate, which is −3.

87. **(a)** $(-2, 0)$ **(b)** $(-\infty, -2)$

(c) $(0, \infty)$

88. **(a)** $(-3, -1)$ **(b)** $(-1, \infty)$

(c) $(-\infty, -3)$

89. **(a)** $(-\infty, -2)$; $(2, \infty)$

(b) $(-2, -2)$ **(c)** none

90. **(a)** $(-3, 3)$ **(b)** $(-\infty, -3)$; $(3, \infty)$

(c) none

91. **(a)** $(-1, 0)$; $(1, \infty)$

(b) $(-\infty, -1)$; $(0, 1)$

(c) none

92. **(a)** $(-\infty, -2)$; $(0, 2)$

(b) $(-2, 0)$; $(2, \infty)$

(c) none

93. **(a)** Yes, it is the graph of a function.

(b) [0, 24]

(c) When $t = 8$, $y = 1200$ from the graph. At 8 A.M., approximately 1200 megawatts is being used.

(d) The most electricity was used at 17 hr or 5 P.M. The least electricity was used at 4 A.M.

(e) $f(12) \approx 1900$

At 12 noon, electricity use is about 1900 megawatts.

(f) increasing from 4 A.M. to 5 P.M.; decreasing from midnight to 4 A.M. and

from 5 P.M. to midnight

94. **(a)** At $t = 2$, $y = 240$ from the graph. Therefore, at 2 seconds, the ball is 240 feet high.

(b) At $y = 192$, $x = 1$ and $x = 5$ from the graph. Therefore, the height will be 192 feet at 1 second and at 5 seconds.

(c) The ball is going up from 0 to 3 seconds and down from 3 to 7 seconds.

(d) The coordinate of the highest point is (3, 256). Therefore, it reaches a maximum height of 256 feet at 3 seconds.

(e) At $x = 7$, $y = 0$. Therefore, at 7 seconds, the ball hits the ground.

95. **(a)** At $t = 12$ and $t = 20$, $y = 55$ from the graph. Therefore, after about 12 noon

until about 8 P.M. the temperature was over 55º.

(b) At $t = 6$ and $t = 22$, $y = 40$ from the graph. Therefore, until about 6 A.M. and after 10 P.M. the temperature was below 40º.

(c) The temperature at noon in Bratenahl, Ohio was 55º. Because the temperature in Greenville is 7º higher, we are looking for the time at which Bratenahl, Ohio was at or above 48º. This occurred at approximately 10 A.M and 8:30 P.M.

(d) The temperature is just below 40° from midnight to 6 A.M., when it begins to rise until it reaches a maximum of just below 65° at 4 P.M. It then begins to fall util it reaches just under 40° again at midnight.

96. **(a)** At t = 8, y = 24 from the graph. Therefore, there are 24 units of the drug in the bloodstream at 8 hours.

(b) The level increases between 0 and 2 hours after the drug is taken and decreases between 2 and 12 hours after the drug is taken.

(c) The coordinates of the highest point are (2, 64). Therefore, at 2 hours, the level of the drug in the bloodstream reaches its

greatest value of 64 units.

(d) After the peak, $y = 16$ at $t = 10$.
10 hours – 2 hours = 8 hours after the peak. 8 additional hours are required for the level to drop to 16 units.

(e) When the drug is administered, the level is 0 units. The level begins to rise quickly for 2 hours until it reaches a maximum of 64 units. The level then begins to decrease gradually until it reaches a level of 12 units, 12 hours after it was administered.

Section 2.4 Linear Functions

1. B; $f(x) = 3x + 6$ is a linear function with y-intercept (0, 6).

2. H; $x = 9$ is a vertical line.

3. C; $f(x) = -8$ is a constant function.

4. G; $2x - y = -4$ or $y = 2x + 4$ is a linear equation with x-intercept (–2, 0) and y-intercept (0, 4).

5. A; $f(x) = 5x$ is a linear function whose graph passes through the origin, (0, 0). $f(0) = 5(0) = 0$.

6. D; $f(x) = x^2$ is a function that is not linear.

7. $m = -3$ matches graph C because the line falls rapidly as x increases.

8. $m = 0$ matches graph A because horizontal lines have slopes of 0.

9. $m = 3$ matches graph D because the line rises rapidly as x increases.

10. m is undefined for graph B because vertical lines have undefined slopes.

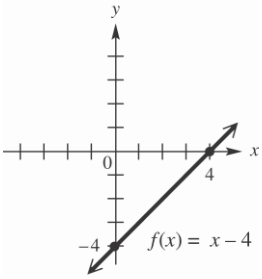
11. $f(x) = x - 4$

Use the intercepts.

$f(0) = 0 - 4 = -4:$ y-intercept

$0 = x - 4 \Rightarrow x = 4:$ x-intercept

Graph the line through (0, –4) and (4, 0).



The domain and range are both $(-\infty, \infty)$.

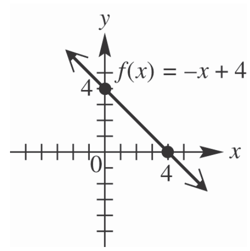
12. $f(x) = -x + 4$

Use the intercepts.

$f(0) = -0 + 4 = 4:$ y-intercept

$0 = -x + 4 \Rightarrow x = 4:$ x-intercept

Graph the line through (0, 4) and (4, 0).



The domain and range are both $(-\infty, \infty)$.

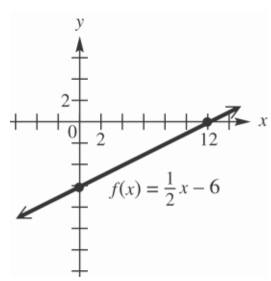
13. $f(x) = \frac{1}{2}x - 6$

Use the intercepts.

$f(0) = \frac{1}{2}(0) - 6 = -6:$ y-intercept

$0 = \frac{1}{2}x - 6 \Rightarrow 6 = \frac{1}{2}x \Rightarrow x = 12:$ x-intercept

Graph the line through (0, –6) and (12, 0).



The domain and range are both $(-\infty, \infty)$.

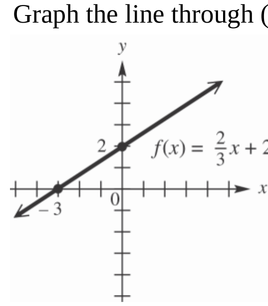
14. $f(x) = \frac{2}{3}x + 2$

Use the intercepts.

$f(0) = \frac{2}{3}(0) + 2 = 2:$ y-intercept

$0 = \frac{2}{3}x + 2 \Rightarrow -2 = \frac{2}{3}x \Rightarrow x = -3:$ x-intercept

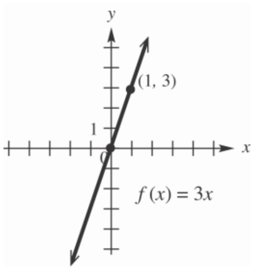
Graph the line through (0, 2) and (–3, 0).



The domain and range are both $(-\infty, \infty)$.

15. $f(x) = 3x$

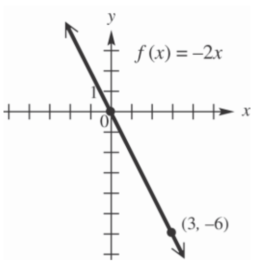
The x-intercept and the y-intercept are both zero. This gives us only one point, (0, 0). If $x = 1$, $y = 3(1) = 3$. Another point is (1, 3). Graph the line through (0, 0) and (1, 3).



The domain and range are both $(-\infty, \infty)$.

16. $f(x) = -2x$

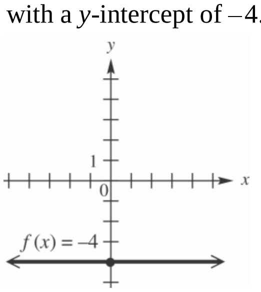
The x-intercept and the y-intercept are both zero. This gives us only one point, (0, 0). If $x = 3$, $y = -2(3) = -6$, so another point is (3, −6). Graph the line through (0, 0) and (3, −6).



The domain and range are both $(-\infty, \infty)$.

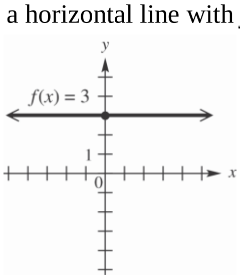
17. $f(x) = -4$ is a constant function.

The graph of $f(x) = -4$ is a horizontal line with a y-intercept of −4.



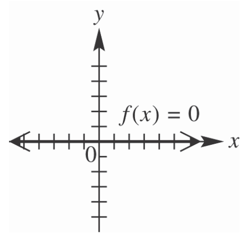
domain: $(-\infty, \infty)$; range: $\{-4\}$

18. $f(x) = 3$ is a constant function whose graph is a horizontal line with y-intercept of 3.



domain: $(-\infty, \infty)$; range: $\{3\}$

19. $f(x) = 0$ is a constant function whose graph is the x-axis.



domain: $(-\infty, \infty)$; range: $\{0\}$

20. $f(x) = 9x$

The domain and range are both $(-\infty, \infty)$.

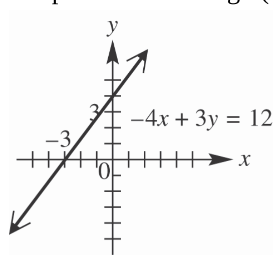
21. $-4x + 3y = 12$

Use the intercepts.

$-4(0) + 3y = 12 \Rightarrow 3y = 12 \Rightarrow$
$y = 4$: y-intercept

$-4x + 3(0) = 12 \Rightarrow -4x = 12 \Rightarrow$
$x = -3$: x-intercept

Graph the line through (0, 4) and (−3, 0).



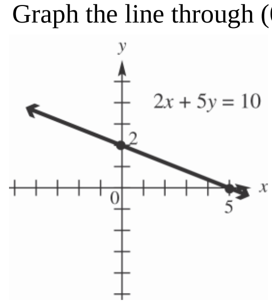
The domain and range are both $(-\infty, \infty)$.

22. $2x + 5y = 10$; Use the intercepts.

$2(0) + 5y = 10 \Rightarrow 5y = 10 \Rightarrow$
$y = 2$: y-intercept

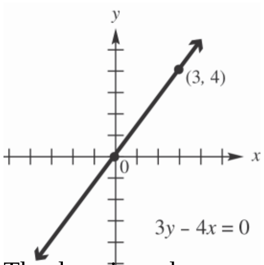
$2x + 5(0) = 10 \Rightarrow 2x = 10 \Rightarrow$
$x = 5$: x-intercept

Graph the line through (0, 2) and (5, 0):



The domain and range are both $(-\infty, \infty)$.

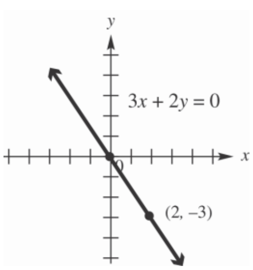
23. $3y - 4x = 0$
Use the intercepts.
$3y - 4(0) = 0 \Rightarrow 3y = 0 \Rightarrow y = 0$: y-intercept
$3(0) - 4x = 0 \Rightarrow -4x = 0 \Rightarrow x = 0$: x-intercept
The graph has just one intercept. Choose an additional value, say 3, for x.
$3y - 4(3) = 0 \Rightarrow 3y - 12 = 0 \Rightarrow$
$3y = 12 \Rightarrow y = 4$
Graph the line through (0, 0) and (3, 4):



The domain and range are both $(-\infty, \infty)$.

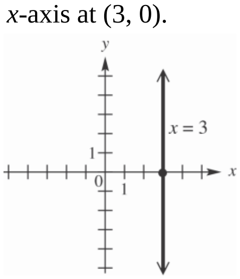
24. $3x + 2y = 0$
Use the intercepts.
$3(0) + 2y = 0 \Rightarrow 2y = 0 \Rightarrow y = 0$: y-intercept

$3x + 2(0) = 0 \Rightarrow 3x = 0 \Rightarrow x = 0$: x-intercept
The graph has just one intercept. Choose an additional value, say 2, for x.
$3(2) + 2y = 0 \Rightarrow 6 + 2y = 0 \Rightarrow$
$2y = -6 \Rightarrow y = -3$
Graph the line through (0, 0) and (2, −3):



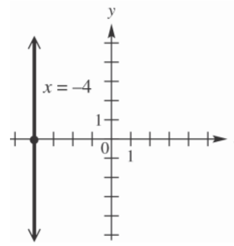
The domain and range are both $(-\infty, \infty)$.

25. $x = 3$ is a vertical line, intersecting the x-axis at (3, 0).



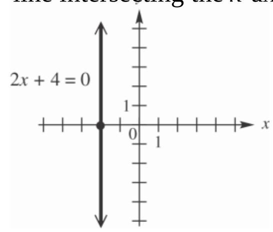
domain: {3}; range: $(-\infty, \infty)$

26. $x = -4$ is a vertical line intersecting the x-axis at (–4, 0).



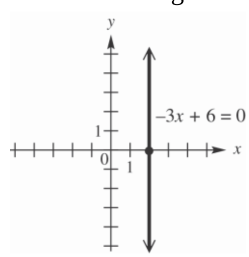
domain: $\{-4\}$; range: $(-\infty, \infty)$

27. $2x + 4 = 0 \Rightarrow 2x = -4 \Rightarrow x = -2$ is a vertical line intersecting the x-axis at (–2, 0).



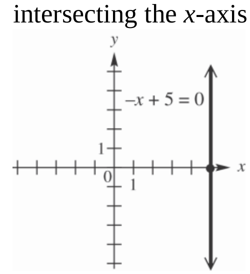
domain: $\{-2\}$; range: $(-\infty, \infty)$

28. $-3x + 6 = 0 \Rightarrow -3x = -6 \Rightarrow x = 2$ is a vertical line intersecting the x-axis at (2, 0).



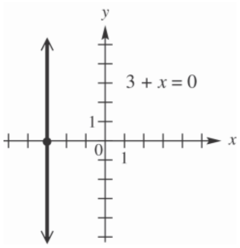
domain: {2}; range: $(-\infty, \infty)$

29. $-x + 5 = 0 \Rightarrow x = 5$ is a vertical line intersecting the x-axis at (5, 0).



domain: {5}; range: $(-\infty, \infty)$

30. $3 + x = 0 \Rightarrow x = -3$ is a vertical line intersecting the x-axis at $(-3, 0)$.



domain: $\{-3\}$; range: $(-\infty, \infty)$

31. $y = 5$ is a horizontal line with y-intercept 5.

Choice A resembles this.

32. $y = -5$ is a horizontal line with y-intercept -5.

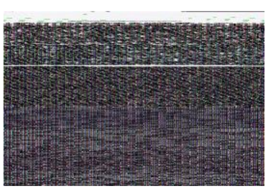
Choice C resembles this.

33. $x = 5$ is a vertical line with x-intercept 5.

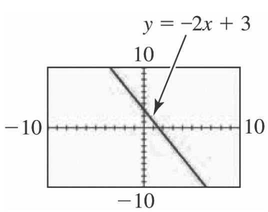
Choice D resembles this.

34. $x = -5$ is a vertical line with x-intercept -5. Choice B resembles this.

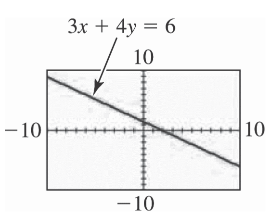
35.



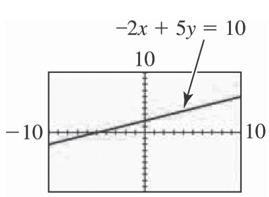
36.



37.



38.



40. The pitch or slope is $\frac{1}{4}$. If the rise is 4 feet then $\frac{1}{4} = \frac{\text{rise}}{\text{run}} = \frac{4}{x}$ or $x = 16$ feet. So 16 feet in the horizontal direction corresponds to a rise of 4 feet.

41. Through (2, –1) and (–3, –3)
Let $x_1 = 2,\ y_1 = -1,\ x_2 = -3,$ and $y_2 = -3.$

Then rise $= \Delta y = -3 - (-1) = -2$ and run $= \Delta x = -3 - 2 = -5.$

The slope is $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{-2}{-5} = \frac{2}{5}.$

42. Through (−3, 4) and (2, −8)

Let $x_1 = -3,\ y_1 = 4,\ x_2 = 2,$ and $y_2 = -8.$

Then rise $= \Delta y = -8 - 4 = -12$ and run $= \Delta x = 2 - (-3) = 5.$

The slope is $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{-12}{5} = -\frac{12}{5}.$

43. Through (5, 8) and (3, 12)

Let $x_1 = 5,\ y_1 = 8,\ x_2 = 3,$ and $y_2 = 12.$

Then rise $= \Delta y = 12 - 8 = 4$ and run $= \Delta x = 3 - 5 = -2.$

The slope is $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{4}{-2} = -2.$

44. Through (5, –3) and (1, –7)
Let $x_1 = 5,\ y_1 = -3,\ x_2 = 1,$ and $y_2 = -7.$

Then rise $= \Delta y = -7 - (-3) = -4$ and run $= \Delta x = 1 - 5 = -4.$

The slope is $m = \frac{\Delta y}{\Delta x} = \frac{-4}{-4} = 1.$

45. Through (5, 9) and (–2, 9)
$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-9}{-2-5} = \frac{0}{-7} = 0$

46. Through (–2, 4) and (6, 4)
$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-4}{6-(-2)} = \frac{0}{8} = 0$

39. The rise is 2.5 feet while the run is 10 feet so the slope is $\frac{2.5}{10} = 0.25 = 25\% = \frac{1}{4}$. So A = 0.25, C = $\frac{2.5}{10}$, D = 25%, and E = $\frac{1}{4}$ are all expressions of the slope.

47. Horizontal, through (5, 1)
The slope of every horizontal line is zero, so $m = 0$.

48. Horizontal, through (3, 5)
The slope of every horizontal line is zero, so $m = 0$.

49. Vertical, through (4, –7)

The slope of every vertical line is undefined; m is undefined.

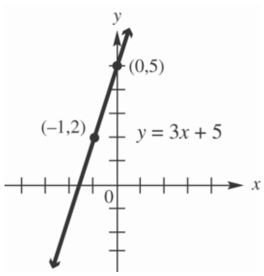
50. Vertical, through (–8, 5)
The slope of every vertical line is undefined; m is undefined.

51. **(a)** $y = 3x + 5$
Find two ordered pairs that are solutions to the equation.
If $x = 0$, then $y = 3(0) + 5 \Rightarrow y = 5$.

If $x = -1$, then
$y = 3(-1) + 5 \Rightarrow y = -3 + 5 \Rightarrow y = 2.$
Thus two ordered pairs are (0, 5) and (−1, 2)

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{-1-0} = \frac{-3}{-1} = 3.$$

(b)



52. $y = 2x - 4$
Find two ordered pairs that are solutions to the equation. If $x = 0$, then $y = 2(0) - 4 \Rightarrow$

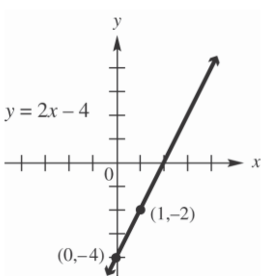
$y = -4$. If $x = 1$, then $y = 2(1) - 4 \Rightarrow$

$y = 2 - 4 \Rightarrow y = -2$. Thus two ordered pairs

are $(0, -4)$ and $(1, -2)$.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-4)}{1-0} = \frac{2}{1} = 2.$$

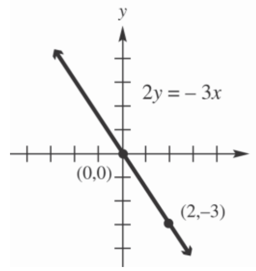
(b)



53. $2y = -3x$
Find two ordered pairs that are solutions to the equation. If $x = 0$, then $2y = 0 \Rightarrow y = 0$.
If $y = -3$, then $2(-3) = -3x \Rightarrow -6 = -3x \Rightarrow$

$x = 2$. $(2, -3)$.

(b)

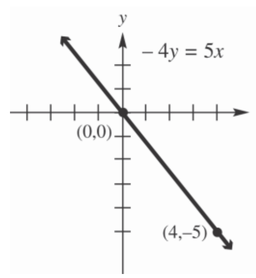


54. $-4y = 5x$
Find two ordered pairs that are solutions to the equation. If $x = 0$, then $-4y = 0 \Rightarrow y = 0$.
If $x = 4$, then $-4y = 5(4) \Rightarrow -4y = 20 \Rightarrow$

$y = -5$. Thus two ordered pairs are $(0, 0)$ and $(4, -5)$.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-0}{4-0} = -\frac{5}{4}.$$

(b)



55. $5x - 2y = 10$

Find two ordered pairs that are solutions to the equation. If $x = 0$, then $5(0) - 2y = 10 \Rightarrow$

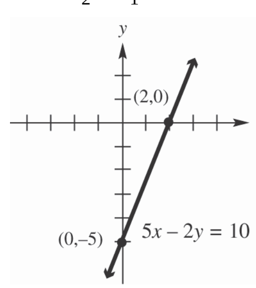
$y = -5$. If $y = 0$, then $5x - 2(0) = 10 \Rightarrow$

$5x = 10 \Rightarrow x = 2$.

Thus two ordered pairs are $(0, -5)$ and $(2, 0)$.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-5)}{2-0} = \frac{5}{2}.$$

(b)



Thus two ordered pairs are $(0,0)$ and

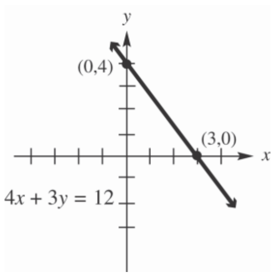
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-0}{2-0} = -\frac{3}{2}.$$

56. $4x + 3y = 12$

Find two ordered pairs that are solutions to the equation. If $x = 0$, then $4(0) + 3y = 12 \Rightarrow$ $3y = 12 \Rightarrow y = 4$. If $y = 0$, then $4x + 3(0) = 12 \Rightarrow 4x = 12 \Rightarrow x = 3$. Thus two ordered pairs are $(0,4)$ and $(3,0)$.

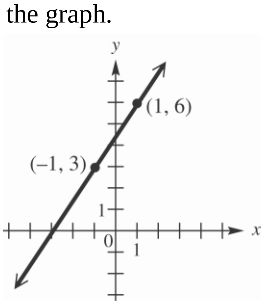
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{3-0} = -\frac{4}{3}.$$

(b)

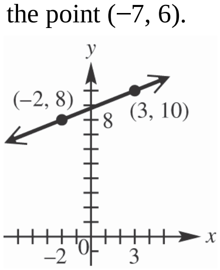


57. Through (–1, 3), $m = \frac{3}{2}$

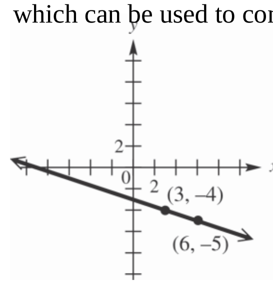
First locate the point (–1, 3). Because the slope is $\frac{3}{2}$, a change of 2 units horizontally (2 units to the right) produces a change of 3 units vertically (3 units up). This gives a second point, (1, 6), which can be used to complete the graph.



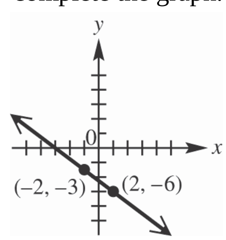
58. Through (–2, 8), $m = \frac{2}{5}$. Because the slope is $\frac{2}{5}$, a change of 5 units horizontally (to the right) produces a change of 2 units vertically (2 units up). This gives a second point (3, 10), which can be used to complete the graph. Alternatively, a change of 5 units to the left produces a change of 2 units down. This gives the point (–7, 6).



59. Through (3, –4), $m = -\frac{1}{3}$. First locate the point (3, –4). Because the slope is $-\frac{1}{3}$, a change of 3 units horizontally (3 units to the right) produces a change of –1 unit vertically (1 unit down). This gives a second point, (6, –5), which can be used to complete the graph.

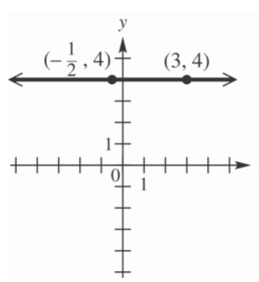


60. Through (–2, –3), $m = -\frac{3}{4}$. Because the slope is $-\frac{3}{4} = \frac{-3}{4}$, a change of 4 units horizontally (4 units to the right) produces a change of –3 units vertically (3 units down). This gives a second point (2, –6), which can be used to complete the graph.

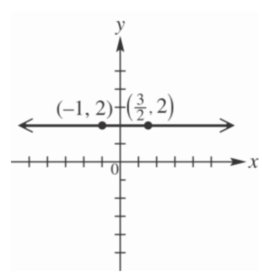


61. Through $\left(-\frac{1}{2}, 4\right)$, $m = 0$.

The graph is the horizontal line through $\left(-\frac{1}{2}, 4\right)$.



Exercise 61

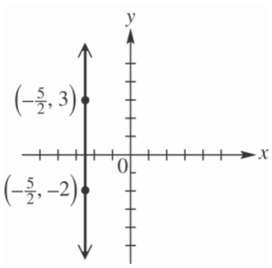


Exercise 62

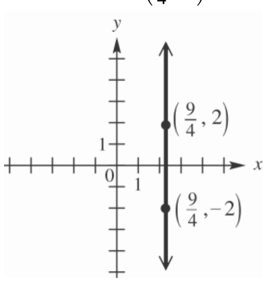
62. Through $\left(\frac{3}{2}, 2\right)$, $m = 0$.

The graph is the horizontal line through $\left(\frac{3}{2}, 2\right)$.

63. Through $\left(-\frac{5}{2}, 3\right)$, undefined slope. The slope is undefined, so the line is vertical, intersecting the x-axis at $\left(-\frac{5}{2}, 0\right)$.



64. Through $\left(\frac{9}{4}, 2\right)$, undefined slope. The slope is undefined, so the line is vertical, intersecting the x-axis at $\left(\frac{9}{4}, 0\right)$.



65. The average rate of change is

$$m = \frac{f(b) - f(a)}{b - a}$$

$\frac{20 - 4}{0 - 4} = \frac{-16}{4} = -\$4$ (thousand) per year. The value of the machine is decreasing \$4000 each year during these years.

66. The average rate of change is

$$m = \frac{f(b) - f(a)}{b - a}$$

$\frac{200 - 0}{4 - 0} = \frac{200}{4} = \$50$ per month. The amount saved is increasing \$50 each month during these months.

67. The graph is a horizontal line, so the average rate of change (slope) is 0. The percent of pay raise is not changing—it is 3% each year.

68. The graph is a horizontal line, so the average rate of change (slope) is 0. That means that the number of named hurricanes remained the same, 10, for the four consecutive years shown.

69. $m = \frac{f(b) - f(a)}{b - a} = \frac{2562 - 5085}{2012 - 1980} = \frac{-2523}{32}$

$= -78.8$ thousand per year

The number of high school dropouts decreased by an average of 78.8 thousand per year from 1980 to 2012.

70. $m = \frac{f(b) - f(a)}{b - a} = \frac{1709 - 5302}{2013 - 2006}$

$= \frac{-3593}{7} \approx -\$513.29$

Sales of plasma flat-panel TVs decreased by an average of \$513.29 million per year from 2006 to 2013.

71. **(a)** The slope of –0.0167 indicates that the average rate of change of the winning time for the 5000 m run is 0.0167 min less. It is negative because the times are generally decreasing as time progresses.

(b) The Olympics were not held during World Wars I (1914−1919) and II (1939–1945).

(c) $y = -0.0167(2000) + 46.45 = 13.05$ min

The model predicts a winning time of 13.05 minutes. The times differ by 13.35 − 13.05 = 0.30 min.

72. **(a)** From the equation, the slope is 200.02. This means that the number of radio stations increased by an average of 200.02 per year.

(b) The year 2018 is represented by $x = 68$.

$y = 200.02(68) + 2727.7 = 16{,}329.06$

According to the model, there will be about 16,329 radio stations in 2018.

73. $\frac{f(2013) - f(2008)}{2013 - 2008} = \frac{335{,}652 - 270{,}334}{2013 - 2008}$

$= \frac{65{,}318}{5} = 13{,}063.6$

The average annual rate of change from 2008 through 2013 is about 13,064 thousand.

74. $\frac{f(2014) - f(2006)}{2014 - 2006} = \frac{3.74 - 4.53}{2014 - 2006}$

$= -\frac{0.79}{8} \approx -0.099$

The average annual rate of change from 2006 through 2014 is about –0.099.

75. **(a)** $m = \frac{f(b)-f(a)}{b-a} = \frac{56.3-138}{2013-2003}$

$= \frac{-81.7}{10} = -8.17$

The average rate of change was −8.17 thousand mobile homes per year.

(b) The negative slope means that the number of mobile homes decreased by an average of 8.17 thousand each year from 2003 to 2013.

76. $\frac{f(2013)-f(1991)}{2013-1991} = \frac{26.6-61.8}{2013-1991}$

$= -\frac{35.2}{22} = -1.6$

There was an average decrease of 1.6 births per thousand per year from 1991 through 2013.

77. **(a)** $C(x) = 10x + 500$

(b) $R(x) = 35x$

(c) $P(x) = R(x) - C(x)$

$= 35x - (10x + 500)$

$= 35x - 10x - 500 = 25x - 500$

(d) $C(x) = R(x)$

$10x + 500 = 35x$

$500 = 25x$

$20 = x$

20 units; do not produce

78. **(a)** $C(x) = 150x + 2700$

(b) $R(x) = 280x$

(c) $P(x) = R(x) - C(x)$

$= 280x - (150x + 2700)$

$= 280x - 150x - 2700$

$= 130x - 2700$

(d) $C(x) = R(x)$

$150x + 2700 = 280x$

$2700 = 130x$

$20.77 \approx x$ or 21 units

21 units; produce

79. **(a)** $C(x) = 400x + 1650$

(b) $R(x) = 305x$

(c) $P(x) = R(x) - C(x)$

$= 305x - (400x + 1650)$

$= 305x - 400x - 1650$

$= -95x - 1650$

(d) $C(x) = R(x)$

$400x + 1650 = 305x$

$95x + 1650 = 0$

$95x = -1650$

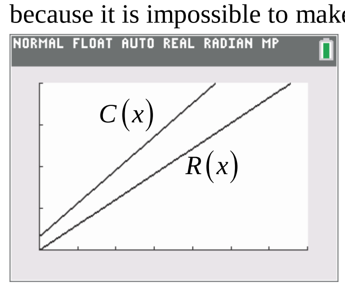
$x \approx -17.37$ units

This result indicates a negative "break-even point," but the number of units produced must be a positive number. A calculator graph of the lines

$y_1 = C(x) = 400x + 1650$ and

$y_2 = R(x) = 305x$ in the window

[0, 70] × [0, 20000] or solving the inequality $305x < 400x + 1650$ will show that $R(x) < C(x)$ for all positive values of x (in fact whenever x is greater than −17.4). Do not produce the product because it is impossible to make a profit.



80. **(a)** $C(x) = 11x + 180$

(b) $R(x) = 20x$

(c) $P(x) = R(x) - C(x)$

$= 20x - (11x + 180)$

$= 20x - 11x - 180 = 9x - 180$

(d) $C(x) = R(x)$

$11x + 180 = 20x$

$180 = 9x$

$20 = x$

20 units; produce

81. $C(x) = R(x) \Rightarrow 200x + 1000 = 240x \Rightarrow$
$1000 = 40x \Rightarrow 25 = x$

The break-even point is 25 units.

$C(25) = 200(25) + 1000 = \$6000$ which is the same as $R(25) = 240(25) = \$6000$

82. $C(x) = R(x) \Rightarrow 220x + 1000 = 240x \Rightarrow$
$1000 = 20x \Rightarrow 50 = x$
The break-even point is 50 units instead of 25 units. The manager is not better off because twice as many units must be sold before beginning to show a profit.

83. The first two points are $A(0, -6)$ and $B(1, -3)$.

$$m = \frac{-3-(-6)}{1-0} = \frac{3}{1} = 3$$

84. The second and third points are B(1, –3) and C(2, 0).

$$m = \frac{0-(-3)}{2-1} = \frac{3}{1} = 3$$

85. If we use any two points on a line to find its slope, we find that the slope is <u>the same</u> in all cases.

86. The first two points are $A(0, -6)$ and $B(1, -3)$.

$$d(A,\ B) = \sqrt{(1-0)^2 + [-3-(-6)]^2} = \sqrt{1^2+3^2} = \sqrt{1+9} = \sqrt{10}$$

87. The second and fourth points are $B(1, -3)$ and $D(3, 3)$.

$$d(B,\ D) = \sqrt{(3-1)^2 + [3-(-3)]^2} = \sqrt{2^2+6^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$$

88. The first and fourth points are $A(0, -6)$ and $D(3, 3)$.

$$d(A,\ D) = \sqrt{(3-0)^2 + [3-(-6)]^2} = \sqrt{3^2+9^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$$

89. $\sqrt{10} + 2\sqrt{10} = 3\sqrt{10}$; The sum is $3\sqrt{10}$, which is equal to the answer in Exercise 88.

90. If points A, B, and C lie on a line in that order, then the distance between A and B added to the distance between <u>B</u> and <u>C</u> is equal to the distance between <u>A</u> and <u>C</u>.

91. The midpoint of the segment joining $A(0, -6)$ and $G(6, 12)$ has coordinates

$\left(\frac{0+6}{2}, \frac{-6+12}{2}\right) = \left(\frac{6}{2}, \frac{6}{2}\right) = (3, 3)$. The midpoint is $M(3, 3)$, which is the same as the middle entry in the table.

Chapter 2 Quiz
(Sections 2.1–2.4)

1. $d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-8-(-4))^2 + (-3-2)^2}$
$= \sqrt{(-4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$

2. To find an estimate for 2006, find the midpoint of (2004, 6.55) and (2008, 6.97:

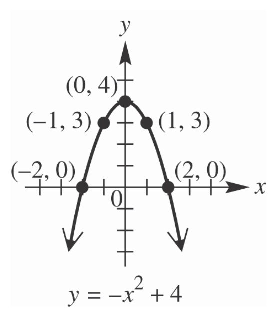
$$M = \left(\frac{2004+2008}{2}, \frac{6.55+6.97}{2}\right) = (2006,\ 6.76)$$

The estimated enrollment for 2006 was 6.76 million.
To find an estimate for 2010, find the midpoint of (2008, 6.97) and (2012, 7.50):

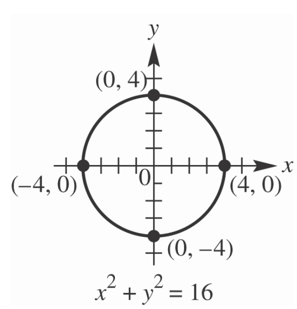
$$M = \left(\frac{2008+2012}{2}, \frac{6.97+7.50}{2}\right) = (2010,\ 7.235)$$

The estimated enrollment for 2006 was about 7.24 million.

3.



4.



5. $x^2 + y^2 - 4x + 8y + 3 = 0$

Complete the square on x and y separately.
$(x^2 - 4x + 4) + (y^2 + 8y + 16) = -3 + 4 + 16 \Rightarrow$

$(x-2)^2 + (y+4)^2 = 17$

92. The midpoint of the segment joining $E(4, 6)$ and $F(5, 9)$ has coordinates $\left(\frac{4+5}{2}, \frac{6+9}{2}\right) = \left(\frac{9}{2}, \frac{15}{2}\right) = (4.5, 7.5)$. If the x-value 4.5 were in the table, the corresponding y-value would be 7.5.

The radius is 17 and the midpoint of the circle is $(2, -4)$.

6. From the graph, $f(-1)$ is 2.

7. Domain: $(-\infty, \infty)$; range: $[0, \infty)$

8. **(a)** The largest open interval over which *f* is decreasing is $(-\infty, -3)$.

(b) The largest open interval over which *f* is increasing is $(-3, \infty)$.

(c) There is no interval over which the function is constant.

9. **(a)** $m = \frac{11-5}{5-1} = \frac{6}{4} = \frac{3}{2}$

(b) $m = \frac{4-4}{-1-(-7)} = \frac{0}{6} = 0$

(c) $m = \frac{-4-12}{6-6} = \frac{-16}{0} \Rightarrow$ the slope is undefined.

10. The points to use are (2009, 10,602) and (2013, 15,884). The average rate of change is
$\frac{15,884-10,602}{2013-2009} = \frac{5282}{4} = 1320.5$
The average rate of change was 1320.5 thousand cars per year. This means that the number of new motor vehicles sold in the United States increased by an average of 1320.5 thousand per year from 2009 to 2013.

Section 2.5 Equations of Lines and Linear Models

1. The graph of the line $y-3=4(x-8)$ has slope 4 and passes through the point (8, 3).

2. The graph of the line $y=-2x+7$ has slope –2 and *y*-intercept (0, 7).

3. The vertical line through the point (–4, 8) has equation *x* = –4.

4. The horizontal line through the point (–4, 8) has equation *y* = 8.

For exercises 5 and 6,
$6x+7y=0 \Rightarrow 7y=-6x \Rightarrow y=-\frac{6}{7}x$

5. Any line parallel to the graph of $6x+7y=0$ must have slope $-\frac{6}{7}$.

6. Any line perpendicular to the graph of $6x+7y=0$ must have slope $\frac{7}{6}$.

8. $4x + 3y = 12$ or $3y = -4x + 12$ or $y = -\frac{4}{3}x + 4$ is graphed in B. The slope is $-\frac{4}{3}$ and the *y*-intercept is (0, 4).

9. $y-(-1)=\frac{3}{2}(x-1)$ is graphed in C. The slope is $\frac{3}{2}$ and a point on the graph is (1, –1).

10. *y* = 4 is graphed in A. *y* = 4 is a horizontal line with *y*-intercept (0, 4).

11. Through (1, 3), *m* = –2.
Write the equation in point-slope form.
$y-y_1=m(x-x_1) \Rightarrow y-3=-2(x-1)$
Then, change to standard form.
$y-3=-2x+2 \Rightarrow 2x+y=5$

12. Through (2, 4), m = –1
Write the equation in point-slope form.
$y-y_1=m(x-x_1) \Rightarrow y-4=-1(x-2)$
Then, change to standard form.
$y-4=-x+2 \Rightarrow x+y=6$

13. Through (–5, 4), $m=-\frac{3}{2}$
Write the equation in point-slope form.
$y-4=-\frac{3}{2}[x-(-5)]$
Change to standard form.
$$2(y-4)=-3(x+5)$$
$$2y-8=-3x-15$$
$$3x+2y=-7$$

14. Through $(-4,3)$, $m=\frac{3}{4}$
Write the equation in point-slope form.
$y-3=\frac{3}{4}[x-(-4)]$

7. $y=\frac{1}{4}x+2$ is graphed in D.

Change to standard form.

$$4(y-3)=3(x+4)$$
$$4y-12=3x+12$$
$$-3x+4y=24 \text{ or } 3x-4y=-24$$

15. Through (–8, 4), undefined slope
Because undefined slope indicates a vertical line, the equation will have the form $x = a$.

The slope is $\frac{1}{4}$ and the y-intercept is (0, 2).

The equation of the line is $x = –8$.

16. Through (5, 1), undefined slope
This is a vertical line through (5, 1), so the equation is $x = 5$.

17. Through (5, −8), $m = 0$
This is a horizontal line through (5, −8), so the equation is $y = −8$.

18. Through (−3, 12), $m = 0$
This is a horizontal line through (−3, 12), so the equation is $y = 12$.

19. Through (–1, 3) and (3, 4)
First find m.
$$m = \frac{4-3}{3-(-1)} = \frac{1}{4}$$
Use either point and the point-slope form.
$$\begin{aligned} y-4 &= \tfrac{1}{4}(x-3) \\ 4y-16 &= x-3 \\ -x+4y &= 13 \\ x-4y &= -13 \end{aligned}$$

20. Through (2, 3) and (−1, 2)
First find m.
$$m = \frac{2-3}{-1-2} = \frac{-1}{-3} = \frac{1}{3}$$
Use either point and the point-slope form.
$$\begin{aligned} y-3 &= \tfrac{1}{3}(x-2) \\ 3y-9 &= x-2 \\ -x+3y &= 7 \\ x-3y &= -7 \end{aligned}$$

21. x-intercept (3, 0), y-intercept (0, –2)
The line passes through (3, 0) and (0, –2). Use these points to find m.
$$m = \frac{-2-0}{0-3} = \frac{2}{3}$$
Using slope-intercept form we have
$y = \frac{2}{3}x - 2.$

22. x-intercept (–4, 0), y-intercept (0, 3)
The line passes through the points (–4, 0) and (0, 3). Use these points to find m.
$$m = \frac{3-0}{0-(-4)} = \frac{3}{4}$$
Using slope-intercept form we have
$y = \frac{3}{4}x + 3.$

23. Vertical, through (–6, 4)
The equation of a vertical line has an equation of the form $x = a$. Because the line passes through (–6, 4), the equation is
$x = -6$. (Because the slope of a vertical line is undefined, this equation cannot be written in slope-intercept form.)

24. Vertical, through (2, 7)
The equation of a vertical line has an equation of the form $x = a$. Because the line passes through (2, 7), the equation is
$x = 2$. (Because the slope of a vertical line is undefined, this equation cannot be written in slope-intercept form.)

25. Horizontal, through (−7, 4)
The equation of a horizontal line has an equation of the form $y = b$. Because the line passes through (−7, 4), the equation is
$y = 4$.

26. Horizontal, through (−8, −2)
The equation of a horizontal line has an equation of the form $y = b$. Because the line passes through (−8, −2), the equation is
$y = -2$.

27. $m = 5$, $b = 15$
Using slope-intercept form, we have
$y = 5x + 15.$

28. $m = -2$, $b = 12$
Using slope-intercept form, we have
$y = -2x + 12.$

29. Through (−2, 5), slope = −4
$$\begin{aligned} y-5 &= -4(x-(-2)) \\ y-5 &= -4(x+2) \\ y-5 &= -4x-8 \\ y &= -4x-3 \end{aligned}$$

30. Through (4, −7), slope = −2
$$\begin{aligned} y-(-7) &= -2(x-4) \\ y+7 &= -2x+8 \\ y &= -2x+1 \end{aligned}$$

31. slope 0, y-intercept $\left(0, \frac{3}{2}\right)$
These represent $m = 0$ and $b = \frac{3}{2}$.
Using slope-intercept form, we have
$y = (0)x + \frac{3}{2} \Rightarrow y = \frac{3}{2}.$

32. slope 0, y-intercept $\left(0, -\frac{5}{4}\right)$
These represent $m = 0$ and $b = -\frac{5}{4}$.
Using slope-intercept form, we have
$y = (0)x - \frac{5}{4} \Rightarrow y = -\frac{5}{4}.$

33. The line $x + 2 = 0$ has x-intercept <u>(–2, 0)</u>. It <u>does not</u> have a y-intercept. The slope of his line is <u>undefined</u>. The line $4y = 2$ has y-intercept $\underline{\left(0, \frac{1}{2}\right)}$. It <u>does not</u> have an x-intercept. The slope of this line is <u>0</u>.

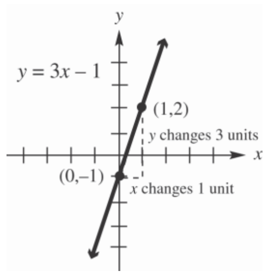
34. **(a)** The graph of $y = 3x + 2$ has a positive slope and a positive y-intercept. These conditions match graph D.

(b) The graph of $y = -3x + 2$ has a negative slope and a positive y-intercept. These conditions match graph B.

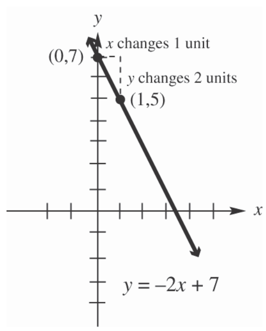
(c) The graph of $y = 3x - 2$ has a positive slope and a negative y-intercept. These conditions match graph A.

(d) The graph of $y = -3x - 2$ has a negative slope and a negative y-intercept. These conditions match graph C.

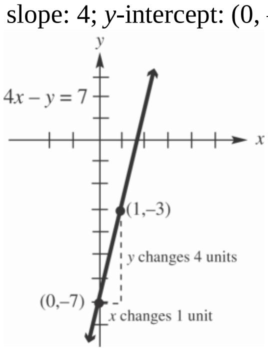
35. $y = 3x - 1$
This equation is in the slope-intercept form, $y = mx + b$.
slope: 3;
y-intercept: $(0, -1)$



36. $y = -2x + 7$
slope: -2;
y-intercept: $(0, 7)$



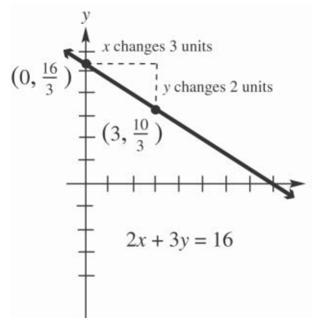
37. $4x - y = 7$
Solve for y to write the equation in slope-intercept form.
$-y = -4x + 7 \Rightarrow y = 4x - 7$
slope: 4; y-intercept: $(0, -7)$



38. $2x + 3y = 16$
Solve the equation for y to write the equation in slope-intercept form.

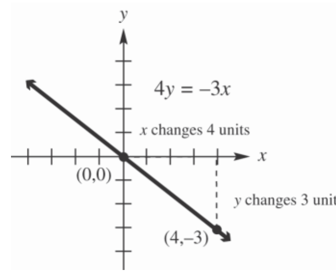
$3y = -2x + 16 \Rightarrow y = -\frac{2}{3}x + \frac{16}{3}$

slope: $-\frac{2}{3}$; y-intercept: $\left(0, \frac{16}{3}\right)$



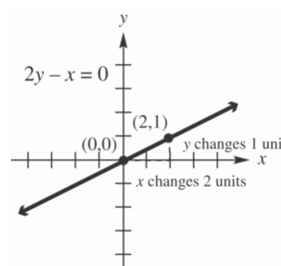
39. $4y = -3x \Rightarrow y = -\frac{3}{4}x$ or $y = -\frac{3}{4}x + 0$

slope: $-\frac{3}{4}$; y-intercept $(0, 0)$



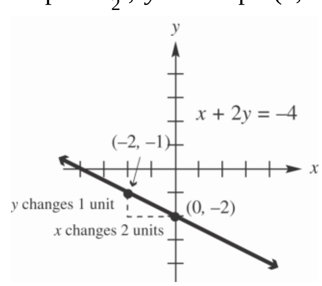
40. $2y = x \Rightarrow y = \frac{1}{2}x$ or $y = \frac{1}{2}x + 0$

slope is $\frac{1}{2}$; y-intercept: $(0, 0)$



41. $x + 2y = -4$
Solve the equation for y to write the equation in slope-intercept form.
$2y = -x - 4 \Rightarrow y = -\frac{1}{2}x - 2$

slope: $-\frac{1}{2}$; y-intercept: $(0, -2)$

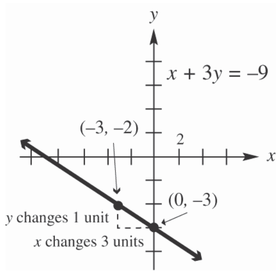


42. $x+3y=-9$

Solve the equation for y to write the equation in slope-intercept form.

$3y=-x-9 \Rightarrow y=-\frac{1}{3}x-3$

slope: $-\frac{1}{3}$; y-intercept: $(0,-3)$

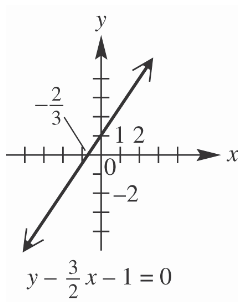


43. $y-\frac{3}{2}x-1=0$

Solve the equation for y to write the equation in slope-intercept form.

$y-\frac{3}{2}x-1=0 \Rightarrow y=\frac{3}{2}x+1$

slope: $\frac{3}{2}$; y-intercept: $(0, 1)$



44. **(a)** Use the first two points in the table, $A(-2, -11)$ and $B(-1, -8)$.

$$m=\frac{-8-(-11)}{-1-(-2)}=\frac{3}{1}=3$$

(b) When $x = 0$, $y = -5$. The y-intercept is $(0, -5)$.

(c) Substitute 3 for m and -5 for b in the slope-intercept form.

$y=mx+b \Rightarrow y=3x-5$

45. **(a)** The line falls 2 units each time the x value increases by 1 unit. Therefore the slope is -2. The graph intersects the y-axis at the point $(0, 1)$ and intersects the x-axis at $\left(\frac{1}{2}, 0\right)$, so the y-intercept is $(0, 1)$ and the x-intercept is $\left(\frac{1}{2}, 0\right)$.

(b) An equation defining f is $y = -2x + 1$.

46. **(a)** The line rises 2 units each time the x value increases by 1 unit. Therefore the slope is 2. The graph intersects the y-axis at the point $(0, -1)$ and intersects the x-axis at $\left(\frac{1}{2}, 0\right)$, so the y-intercept is $(0, -1)$ and the x-intercept is $\left(\frac{1}{2}, 0\right)$.

(b) An equation defining f is $y = 2x - 1$.

47. **(a)** The line falls 1 unit each time the x value increases by 3 units. Therefore the slope is $-\frac{1}{3}$. The graph intersects the y-axis at the point $(0, 2)$, so the y-intercept is $(0, 2)$. The graph passes through $(3, 1)$ and will fall 1 unit when the x value increases by 3, so the x-intercept is $(6, 0)$.

(b) An equation defining f is $y=-\frac{1}{3}x+2$.

48. **(a)** The line rises 3 units each time the x value increases by 4 units. Therefore the slope is $\frac{3}{4}$. The graph intersects the y-axis at the point $(0, -3)$ and intersects the x-axis at $(4, 0)$, so the y-intercept is $(0, -3)$ and the x-intercept is 4.

(b) An equation defining f is $y=\frac{3}{4}x-3$.

49. **(a)** The line falls 200 units each time the x value increases by 1 unit. Therefore the slope is -200. The graph intersects the y-axis at the point $(0, 300)$ and intersects the x-axis at $\left(\frac{3}{2}, 0\right)$, so the y-intercept is $(0, 300)$ and the x-intercept is $\left(\frac{3}{2}, 0\right)$.

(b) An equation defining f is $y = -200x + 300$.

50. **(a)** The line rises 100 units each time the x value increases by 5 units. Therefore the slope is 20. The graph intersects the y-axis at the point $(0, -50)$ and intersects the x-axis at $\left(\frac{5}{2}, 0\right)$, so the y-intercept is $(0, -50)$ and the x-intercept is $\left(\frac{5}{2}, 0\right)$.

(b) An equation defining f is $y = 20x - 50$.

51. (a) through (–1, 4), parallel to $x + 3y = 5$
Find the slope of the line $x + 3y = 5$ by writing this equation in slope-intercept form.

$$x+3y=5 \Rightarrow 3y=-x+5 \Rightarrow$$
$$y=-\tfrac{1}{3}x+\tfrac{5}{3}$$

The slope is $-\frac{1}{3}$.

Because the lines are parallel, $-\frac{1}{3}$ is also the slope of the line whose equation is to be found. Substitute $m=-\frac{1}{3}$, $x_1=-1$, and $y_1=4$ into the point-slope form.

$$y-y_1=m\left(x-x_1\right)$$
$$y-4=-\tfrac{1}{3}\left[x-(-1)\right]$$
$$y-4=-\tfrac{1}{3}\left(x+1\right)$$
$$3y-12=-x-1 \Rightarrow x+3y=11$$

(b) Solve for y.

$$3y=-x+11 \Rightarrow y=-\tfrac{1}{3}x+\tfrac{11}{3}$$

52. (a) through (3, –2), parallel to $2x – y = 5$
Find the slope of the line $2x – y = 5$ by writing this equation in slope-intercept form.

$$2x-y=5 \Rightarrow -y=-2x+5 \Rightarrow$$
$$y=2x-5$$

The slope is 2. Because the lines are parallel, the slope of the line whose equation is to be found is also 2.

Substitute $m = 2$, $x_1=3$, and $y_1=-2$ into the point-slope form.

$$y-y_1=m\left(x-x_1\right) \Rightarrow$$
$$y+2=2\left(x-3\right) \Rightarrow y+2=2x-6 \Rightarrow$$
$$-2x+y=-8 \text{ or } 2x-y=8$$

(b) Solve for y. $y = 2x – 8$

53. (a) through (1, 6), perpendicular to $3x + 5y = 1$
Find the slope of the line $3x + 5y = 1$ by writing this equation in slope-intercept form.

$$3x+5y=1 \Rightarrow 5y=-3x+1 \Rightarrow$$
$$y=-\tfrac{3}{5}x+\tfrac{1}{5}$$

This line has a slope of $-\frac{3}{5}$. The slope of any line perpendicular to this line is $\frac{5}{3}$, because $-\frac{3}{5}\left(\frac{5}{3}\right)=-1$. Substitute $m=\frac{5}{3}$,

$$y-6=\tfrac{5}{3}(x-1)$$
$$3(y-6)=5(x-1)$$
$$3y-18=5x-5$$
$$-13=5x-3y \text{ or } 5x-3y=-13$$

(b) Solve for y.

$$3y=5x+13 \Rightarrow y=\tfrac{5}{3}x+\tfrac{13}{3}$$

54. (a) through (–2, 0), perpendicular to $8x – 3y = 7$
Find the slope of the line $8x – 3y = 7$ by writing the equation in slope-intercept form.

$$8x-3y=7 \Rightarrow -3y=-8x+7 \Rightarrow$$
$$y=\tfrac{8}{3}x-\tfrac{7}{3}$$

This line has a slope of $\frac{8}{3}$. The slope of any line perpendicular to this line is $-\frac{3}{8}$, because $\frac{8}{3}\left(-\frac{3}{8}\right)=-1$.

Substitute $m=-\frac{3}{8}$, $x_1=-2$, and $y_1=0$ into the point-slope form.

$$y-0=-\tfrac{3}{8}(x+2)$$
$$8y=-3(x+2)$$
$$8y=-3x-6 \Rightarrow 3x+8y=-6$$

(b) Solve for y.

$$8y=-3x-6 \Rightarrow y=-\tfrac{3}{8}x-\tfrac{6}{8} \Rightarrow$$
$$y=-\tfrac{3}{8}x-\tfrac{3}{4}$$

55. (a) through (4, 1), parallel to $y = −5$
Because $y = −5$ is a horizontal line, any line parallel to this line will be horizontal and have an equation of the form $y = b$. Because the line passes through (4, 1), the equation is $y = 1$.

(b) The slope-intercept form is $y = 1$.

56. (a) through $(-2,-2)$, parallel to $y = 3$.
Because $y = 3$ is a horizontal line, any line parallel to this line will be horizontal and have an equation of the form $y = b$. Because the line passes through $(-2,-2)$, the equation is $y = –2$.

(b) The slope-intercept form is $y = −2$

57. (a) through (–5, 6), perpendicular to $x = –2$.

Because $x = –2$ is a vertical line, any line perpendicular to this line will be horizontal and have an equation of the form $y = b$. Because the line passes through (–5, 6), the equation is $y = 6$.

$x_1 = 1$, and $y_1 = 6$ into the point-slope form.

(b) The slope-intercept form is $y = 6$.

58. (a) Through $(4, -4)$, perpendicular to $x = 4$

Because $x = 4$ is a vertical line, any line perpendicular to this line will be horizontal and have an equation of the form $y = b$. Because the line passes through $(4, -4)$, the equation is $y = -4$.

(b) The slope-intercept form is $y = -4$.

59. (a) Find the slope of the line $3y + 2x = 6$.

$$3y + 2x = 6 \Rightarrow 3y = -2x + 6 \Rightarrow$$
$$y = -\frac{2}{3}x + 2$$

Thus, $m = -\frac{2}{3}$. A line parallel to $3y + 2x = 6$ also has slope $-\frac{2}{3}$. Solve for k using the slope formula.

$$\frac{2-(-1)}{k-4} = -\frac{2}{3}$$
$$\frac{3}{k-4} = -\frac{2}{3}$$
$$3(k-4)\left(\frac{3}{k-4}\right) = 3(k-4)\left(-\frac{2}{3}\right)$$
$$9 = -2(k-4)$$
$$9 = -2k + 8$$
$$2k = -1 \Rightarrow k = -\frac{1}{2}$$

(b) Find the slope of the line $2y - 5x = 1$.

$$2y - 5x = 1 \Rightarrow 2y = 5x + 1 \Rightarrow$$
$$y = \frac{5}{2}x + \frac{1}{2}$$

Thus, $m = \frac{5}{2}$. A line perpendicular to $2y - 5x = 1$ will have slope $-\frac{2}{5}$, because

$$\frac{5}{2}\left(-\frac{2}{5}\right) = -1.$$

Solve this equation for k.

$$\frac{3}{k-4} = -\frac{2}{5}$$
$$5(k-4)\left(\frac{3}{k-4}\right) = 5(k-4)\left(-\frac{2}{5}\right)$$
$$15 = -2(k-4)$$
$$15 = -2k + 8$$
$$2k = -7 \Rightarrow k = -\frac{7}{2}$$

60. (a) Find the slope of the line $2x - 3y = 4$.

$$2x - 3y = 4 \Rightarrow -3y = -2x + 4 \Rightarrow$$
$$y = \frac{2}{3}x - \frac{4}{3}$$

Thus, $m = \frac{2}{3}$. A line parallel to $2x - 3y = 4$ also has slope $\frac{2}{3}$. Solve for r using the slope formula.

$$\frac{r-6}{-4-2} = \frac{2}{3} \Rightarrow \frac{r-6}{-6} = \frac{2}{3} \Rightarrow$$
$$-6\left(\frac{r-6}{-6}\right) = -6\left(\frac{2}{3}\right) \Rightarrow$$
$$r - 6 = -4 \Rightarrow r = 2$$

(b) Find the slope of the line $x + 2y = 1$.

$$x + 2y = 1 \Rightarrow 2y = -x + 1 \Rightarrow$$
$$y = -\frac{1}{2}x + \frac{1}{2}$$

Thus, $m = -\frac{1}{2}$. A line perpendicular to the line $x + 2y = 1$ has slope 2, because $-\frac{1}{2}(2) = -1$. Solve for r using the slope formula.

$$\frac{r-6}{-4-2} = 2 \Rightarrow \frac{r-6}{-6} = 2 \Rightarrow$$
$$r - 6 = -12 \Rightarrow r = -6$$

61. (a) First find the slope using the points $(0, 6312)$ and $(3, 7703)$.

$$m = \frac{7703 - 6312}{3 - 0} = \frac{1391}{3} \approx 463.67$$

The y-intercept is $(0, 6312)$, so the equation of the line is $y = 463.67x + 6312$.

(b) The value $x = 4$ corresponds to the year 2013.

$$y = 463.67(4) + 6312 = 8166.68$$

The model predicts that average tuition and fees were \$8166.68 in 2013. This is \$96.68 more than the actual amount.

62. (a) First find the slope using the points $(0, 6312)$ and $(2, 7136)$.

$$m = \frac{7136 - 6312}{2 - 0} = \frac{824}{2} = 412$$

The y-intercept is $(0, 6312)$, so the equation of the line is $y = 412x + 6312$.

(b) The value $x = 4$ corresponds to the year 2013.

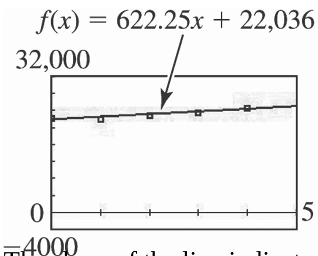
$$y = 412(4) + 6312 = 7960$$

The model predicts that average tuition and fees were \$7960 in 2013. This is \$110 less than the actual amount.

63. (a) First find the slope using the points (0, 22036) and (4, 24525).

$$m = \frac{24525 - 22036}{4 - 0} = \frac{2489}{4} = 622.25$$

The y-intercept is (0, 22036), so the equation of the line is $y = 622.25x + 22,036.$



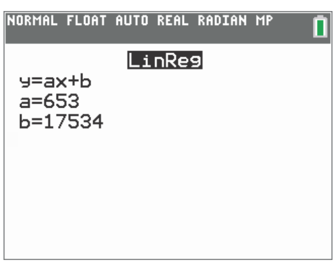
The slope of the line indicates that the average tuition increase is about \$622 per year from 2009 through 2013.

(b) The year 2012 corresponds to $x = 3$.

$$y = 622.25(3) + 22,036 = 23,902.75$$

According to the model, average tuition and fees were \$23,903 in 2012. This is \$443 more than the actual amount \$23,460.

(c) Using the linear regression feature, the equation of the line of best fit is $y = 653x + 21,634.$

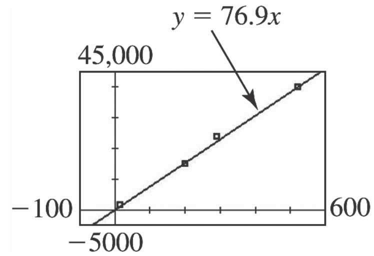


64. (a) See the graph in the answer to part (b).There appears to be a linear relationship between the data. The farther the galaxy is from Earth, the faster it is receding.

(b) Using the points (520, 40,000) and (0, 0), we obtain

$$m = \frac{40,000 - 0}{520 - 0} = \frac{40,000}{520} \approx 76.9.$$

The equation of the line through these two points is $y = 76.9x$.



(c)

$$76.9x = 60,000$$

$$x = \frac{60,000}{76.9} \Rightarrow x \approx 780$$

According to the model, the galaxy Hydra is approximately 780 megaparsecs away.

(d)

$$A = \frac{9.5 \times 10^{11}}{m}$$

$$A = \frac{9.5 \times 10^{11}}{76.9} \approx 1.235 \times 10^{10} \approx 12.35 \times 10^{9}$$

Using $m = 76.9$, we estimate that the age of the universe is approximately 12.35 billion years.

(e)

$$A = \frac{9.5 \times 10^{11}}{50} = 1.9 \times 10^{10} \text{ or } 19 \times 10^{9}$$

$$A = \frac{9.5 \times 10^{11}}{100} = 9.5 \times 10^{9}$$

The range for the age of the universe is between 9.5 billion and 19 billion years.

65. (a) The ordered pairs are (0, 32) and (100, 212).

The slope is $m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}.$

Use $(x_1, y_1) = (0, 32)$ and $m = \frac{9}{5}$ in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 32 = \tfrac{9}{5}(x - 0)$$
$$y - 32 = \tfrac{9}{5}x$$
$$y = \tfrac{9}{5}x + 32 \Rightarrow F = \tfrac{9}{5}C + 32$$

(b)

$$F = \tfrac{9}{5}C + 32$$
$$5F = 9(C + 32)$$
$$5F = 9C + 160 \Rightarrow 9C = 5F - 160 \Rightarrow$$
$$9C = 5(F - 32) \Rightarrow C = \tfrac{5}{9}(F - 32)$$

(c)

$$F = C \Rightarrow F = \tfrac{5}{9}(F - 32) \Rightarrow$$
$$9F = 5(F - 32) \Rightarrow 9F = 5F - 160 \Rightarrow$$
$$4F = -160 \Rightarrow F = -40$$

$F = C$ when F is $-40°$.

66. **(a)** The ordered pairs are (0, 1) and (100, 3.92). The slope is

$$m = \frac{3.92-1}{100-0} = \frac{2.92}{100} = 0.0292 \quad \text{and } b = 1.$$

Using slope-intercept form we have $y = 0.0292x + 1$ or $p(x) = 0.0292x + 1.$

(b) Let $x = 60$.

$P(60) = 0.0292(60) + 1 = 2.752$
The pressure at 60 feet is approximately 2.75 atmospheres.

67. **(a)** Because we want to find C as a function of I, use the points (12026, 10089) and (14167, 11484), where the first component represents the independent variable, I. First find the slope of the line.

$$m = \frac{11484 - 10089}{14167 - 12026} = \frac{1395}{2141} \approx 0.6516$$

Now use either point, say (12026, 10089), and the point-slope form to find the equation.

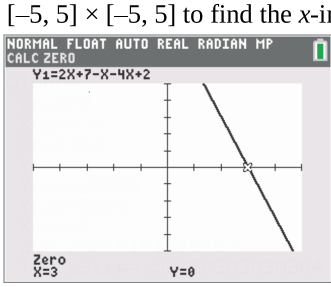
$$C - 10089 = 0.6516(I - 12026)$$
$$C - 10089 \approx 0.6516I - 7836$$
$$C \approx 0.6516I + 2253$$

(b) Because the slope is 0.6516, the marginal

propensity to consume is 0.6516.

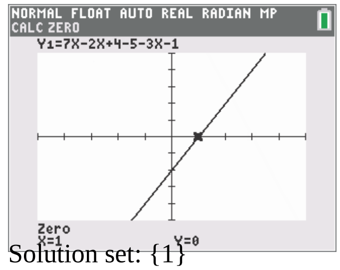
68. D is the only possible answer, because the x-intercept occurs when $y = 0$. We can see from the graph that the value of the x-intercept exceeds 10.

69. Write the equation as an equivalent equation with 0 on one side: $2x + 7 - x = 4x - 2 \Rightarrow$ $2x + 7 - x - 4x + 2 = 0$. Now graph $y = 2x + 7 - x - 4x + 2$ in the window $[-5, 5] \times [-5, 5]$ to find the x-intercept:



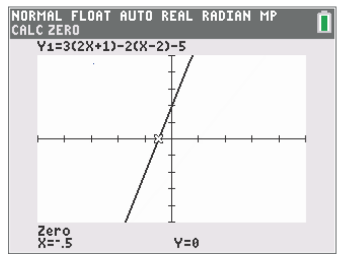
Solution set: {3}

70. Write the equation as an equivalent equation with 0 on one side: $7x - 2x + 4 - 5 = 3x + 1 \Rightarrow$ $7x - 2x + 4 - 5 - 3x - 1 = 0$. Now graph $y = 7x - 2x + 4 - 5 - 3x - 1$ in the window $[-5, 5] \times [-5, 5]$ to find the x-intercept:



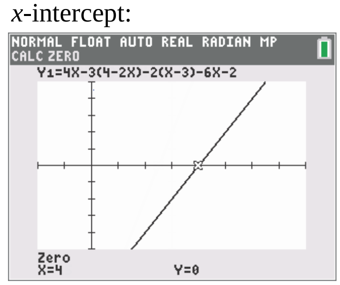
Solution set: {1}

71. Write the equation as an equivalent equation with 0 on one side: $3(2x+1) - 2(x-2) = 5 \Rightarrow$ $3(2x+1) - 2(x-2) - 5 = 0$. Now graph $y = 3(2x+1) - 2(x-2) - 5$ in the window $[-5, 5] \times [-5, 5]$ to find the x-intercept:



Solution set: $\left\{-\frac{1}{2}\right\}$ or $\{-0.5\}$

72. Write the equation as an equivalent equation with 0 on one side:
$4x - 3(4 - 2x) = 2(x - 3) + 6x + 2 \Rightarrow$
$4x - 3(4 - 2x) - 2(x - 3) - 6x - 2 = 0$.
Now graph $y = 4x - 3(4 - 2x) - 2(x - 3) - 6x - 2$ in the window $[-2, 8] \times [-5, 5]$ to find the x-intercept:



Solution set: {4}

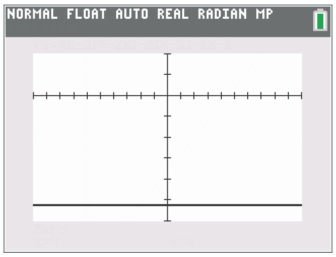
73. **(a)**
$$-2(x-5) = -x - 2$$
$$-2x + 10 = -x - 2$$
$$10 = x - 2$$
$$12 = x$$
Solution set: {12}

(b) Answers will vary. Sample answer: The solution does not appear in the standard viewing window x-interval [10, –10]. The minimum and maximum values must include 12.

74. Rewrite the equation as an equivalent equation with 0 on one side.

$$-3(2x+6) = -4x+8-2x$$
$$-6x-18-(-4x+8-2x)=0$$

Now graph $y = -6x - 18 - (-4x + 8 - 2x)$ in the window $[-10, 10] \times [-30, 10]$.



The graph is a horizontal line that does not intersect the x-axis. Therefore, the solution set is $\varnothing$. We can verify this algebraically.

$$-3(2x+6) = -4x+8-2x$$
$$-6x-18 = -6x+8 \Rightarrow 0 = 26$$

Because this is a false statement, the solution set is $\varnothing$.

75. $A(-1, 4), B(-2, -1), C(1, 14)$

For A and B, $m = \dfrac{-1-4}{-2-(-1)} = \dfrac{-5}{-1} = 5$

For B and C, $m = \dfrac{14-(-1)}{1-(-2)} = \dfrac{15}{3} = 5$

For A and C, $m = \dfrac{14-4}{1-(-1)} = \dfrac{10}{2} = 5$

Since all three slopes are the same, the points are collinear.

76. $A(0, -7), B(-3, 5), C(2, -15)$

For A and B, $m = \dfrac{5-(-7)}{-3-0} = \dfrac{12}{-3} = -4$

77. $A(-1, -3), B(-5, 12), C(1, -11)$

For A and B, $m = \dfrac{12-(-3)}{-5-(-1)} = -\dfrac{15}{4}$

For B and C, $m = \dfrac{-11-12}{1-(-5)} = -\dfrac{23}{6}$

For A and C, $m = \dfrac{-11-(-3)}{1-(-1)} = -\dfrac{8}{2} = -4$

Since all three slopes are not the same, the points are not collinear.

78. $A(0, 9), B(-3, -7), C(2, 19)$

For A and B, $m = \dfrac{-7-9}{-3-0} = \dfrac{-16}{-3} = \dfrac{16}{3}$

For B and C, $m = \dfrac{19-(-7)}{2-(-3)} = \dfrac{26}{5}$

For A and C, $m = \dfrac{19-9}{2-0} = \dfrac{10}{2} = 5$

Because all three slopes are not the same, the points are not collinear.

79. $d(O, P) = \sqrt{(x_1-0)^2 + (m_1x_1-0)^2}$
$= \sqrt{x_1^2 + m_1^2x_1^2}$

80. $d(O, Q) = \sqrt{(x_2-0)^2 + (m_2x_2-0)^2}$
$= \sqrt{x_2^2 + m_2^2x_2^2}$

81. $d(P, Q) = \sqrt{(x_2-x_1)^2 + (m_2x_2-m_1x_1)^2}$

82. $[d(O, P)]^2 + [d(O, Q)]^2 = [d(P, Q)]^2$

$$\left[\sqrt{x_1^2+m_1^2x_1^2}\right]^2 + \left[\sqrt{x_2^2+m_2^2x_2^2}\right]^2 = \left[\sqrt{(x_2-x_1)^2+(m_2x_2-m_1x_1)^2}\right]^2$$

$$\left(x_1^2+m_1^2x_1^2\right)+\left(x_2^2+m_2^2x_2^2\right) = \left(x_2-x_1\right)^2+\left(m_2x_2-m_1x_1\right)^2$$

$$x_1^2+m_1^2x_1^2+x_2^2+m_2^2x_2^2$$

For B and C, $m = \frac{-15-5}{2-(-3)} = \frac{-20}{5} = -4$

For A and C, $m = \frac{-15-(-7)}{2-0} = \frac{-8}{2} = -4$

Since all three slopes are the same, the points are collinear.

$$= x_2 - 2x_2x_1 + x_1 + m_2 x_2 - 2m_1m_2x_1x_2 + m_1^2x_1^2$$

$$0 = -2x_2x_1 - 2m_1m_2x_1x_2 \Rightarrow$$

$$-2m_1m_2x_1x - 2x_2x_1 = 0$$

83. $-2m_1m_2x_1x_2 - 2x_1x_2 = 0$

$-2x_1x_2(m_1m_2 + 1) = 0$

84. $-2x_1x_2(m_1m_2 + 1) = 0$

Because $x_1 \neq 0$ and $x_2 \neq 0$, we have $m_1m_2 + 1 = 0$ implying that $m_1m_2 = -1$.

85. If two nonvertical lines are perpendicular, then the product of the slopes of these lines is –1.

Summary Exercises on Graphs, Circles, Functions, and Equations

1. $P(3, 5),\ Q(2, -3)$

(a) $$d(P, Q) = \sqrt{(2-3)^2 + (-3-5)^2} = \sqrt{(-1)^2 + (-8)^2} = \sqrt{1+64} = \sqrt{65}$$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{3+2}{2}, \frac{5+(-3)}{2}\right) = \left(\frac{5}{2}, \frac{2}{2}\right) = \left(\frac{5}{2}, 1\right).$$

(c) First find m: $m = \dfrac{-3-5}{2-3} = \dfrac{-8}{-1} = 8$

Use either point and the point-slope form.

$y - 5 = 8(x-3)$

Change to slope-intercept form.

$y - 5 = 8x - 24 \Rightarrow y = 8x - 19$

2. $P(-1, 0),\ Q(4, -2)$

(a) $$d(P, Q) = \sqrt{[4-(-1)]^2 + (-2-0)^2} = \sqrt{5^2 + (-2)^2} = \sqrt{25+4} = \sqrt{29}$$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{-1+4}{2}, \frac{0+(-2)}{2}\right) = \left(\frac{3}{2}, \frac{-2}{2}\right) = \left(\frac{3}{2}, -1\right).$$

(c) First find m: $m = \dfrac{-2-0}{4-(-1)} = \dfrac{-2}{5} = -\dfrac{2}{5}$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{-2+3}{2}, \frac{2+2}{2}\right) = \left(\frac{1}{2}, \frac{4}{2}\right) = \left(\frac{1}{2}, 2\right).$$

(c) First find m: $m = \dfrac{2-2}{3-(-2)} = \dfrac{0}{5} = 0$

All lines that have a slope of 0 are horizontal lines. The equation of a horizontal line has an equation of the form $y = b$. Because the line passes through $(3, 2)$, the equation is $y = 2$.

4. $P\left(2\sqrt{2}, \sqrt{2}\right),\ Q\left(\sqrt{2}, 3\sqrt{2}\right)$

(a) $$d(P, Q) = \sqrt{\left(\sqrt{2} - 2\sqrt{2}\right)^2 + \left(3\sqrt{2} - \sqrt{2}\right)^2} = \sqrt{\left(-\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2} = \sqrt{2+8} = \sqrt{10}$$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{2\sqrt{2}+\sqrt{2}}{2}, \frac{\sqrt{2}+3\sqrt{2}}{2}\right) = \left(\frac{3\sqrt{2}}{2}, \frac{4\sqrt{2}}{2}\right) = \left(\frac{3\sqrt{2}}{2}, 2\sqrt{2}\right).$$

(c) First find m: $m = \dfrac{3\sqrt{2}-\sqrt{2}}{\sqrt{2}-2\sqrt{2}} = \dfrac{2\sqrt{2}}{-\sqrt{2}} = -2$

Use either point and the point-slope form.

$y - \sqrt{2} = -2\left(x - 2\sqrt{2}\right)$

Change to slope-intercept form.

$y - \sqrt{2} = -2x + 4\sqrt{2} \Rightarrow y = -2x + 5\sqrt{2}$

5. $P(5, -1),\ Q(5, 1)$

(a) $$d(P, Q) = \sqrt{(5-5)^2 + [1-(-1)]^2} = \sqrt{0^2 + 2^2} = \sqrt{0+4} = \sqrt{4} = 2$$

Use either point and the point-slope form.

$y - 0 = -\frac{2}{5}\left[x - (-1)\right]$

Change to slope-intercept form.

$$5y = -2(x+1)$$
$$5y = -2x - 2$$
$$y = -\tfrac{2}{5}x - \tfrac{2}{5}$$

3. $P(-2, 2)$, $Q(3, 2)$

(a) $d(P, Q) = \sqrt{[3-(-2)]^2 + (2-2)^2}$
$= \sqrt{5^2 + 0^2} = \sqrt{25+0} = \sqrt{25} = 5$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{5+5}{2}, \frac{-1+1}{2}\right) = \left(\frac{10}{2}, \frac{0}{2}\right) = (5, 0).$$

(c) First find m.

$$m = \frac{1-(-1)}{5-5} = \frac{2}{0} = \text{undefined}$$

All lines that have an undefined slope are vertical lines. The equation of a vertical line has an equation of the form $x = a$. The line passes through (5, 1), so the equation is $x = 5$. (Because the slope of a vertical line is undefined, this equation cannot be written in slope-intercept form.)

6. $P(1, 1),\ Q(-3, -3)$

(a)
$$d(P, Q) = \sqrt{(-3-1)^2 + (-3-1)^2} = \sqrt{(-4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{1+(-3)}{2}, \frac{1+(-3)}{2}\right) = \left(\frac{-2}{2}, \frac{-2}{2}\right) = (-1, -1).$$

(c) First find m: $m = \frac{-3-1}{-3-1} = \frac{-4}{-4} = 1$

Use either point and the point-slope form.

$$y - 1 = 1(x - 1)$$

Change to slope-intercept form.

$$y - 1 = x - 1 \Rightarrow y = x$$

7. $P\left(2\sqrt{3}, 3\sqrt{5}\right),\ Q\left(6\sqrt{3}, 3\sqrt{5}\right)$

(a)
$$d(P, Q) = \sqrt{\left(6\sqrt{3} - 2\sqrt{3}\right)^2 + \left(3\sqrt{5} - 3\sqrt{5}\right)^2} = \sqrt{\left(4\sqrt{3}\right)^2 + 0} = \sqrt{48} = 4\sqrt{3}$$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{2\sqrt{3} + 6\sqrt{3}}{2}, \frac{3\sqrt{5} + 3\sqrt{5}}{2}\right)$$

(c) First find m: $m = \frac{3\sqrt{5} - 3\sqrt{5}}{6\sqrt{3} - 2\sqrt{3}} = \frac{0}{4\sqrt{3}} = 0$

All lines that have a slope of 0 are horizontal lines. The equation of a horizontal line has an equation of the form $y = b$. Because the line passes through $\left(2\sqrt{3}, 3\sqrt{5}\right)$, the equation is $y = 3\sqrt{5}$.

8. $P(0, -4),\ Q(3, 1)$

(a)
$$d(P, Q) = \sqrt{(3-0)^2 + [1-(-4)]^2} = \sqrt{3^2 + 5^2} = \sqrt{9+25} = \sqrt{34}$$

(b) The midpoint M of the segment joining points P and Q has coordinates

$$\left(\frac{0+3}{2}, \frac{-4+1}{2}\right) = \left(\frac{3}{2}, \frac{-3}{2}\right) = \left(\frac{3}{2}, -\frac{3}{2}\right).$$

(c) First find m: $m = \frac{1-(-4)}{3-0} = \frac{5}{3}$

Using slope-intercept form we have $y = \frac{5}{3}x - 4$.

9. Through $(-2, 1)$ and $(4, -1)$

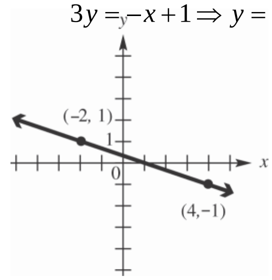
First find m: $m = \frac{-1-1}{4-(-2)} = \frac{-2}{6} = -\frac{1}{3}$

Use either point and the point-slope form.

$$y - (-1) = -\frac{1}{3}(x - 4)$$

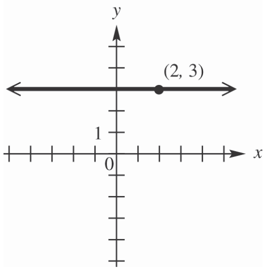
Change to slope-intercept form.

$$3(y+1) = -(x-4) \Rightarrow 3y + 3 = -x + 4 \Rightarrow$$
$$3y = -x + 1 \Rightarrow y = -\frac{1}{3}x + \frac{1}{3}$$



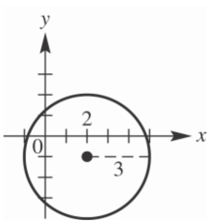
$$= \left(\frac{8\sqrt{3}}{2}, \frac{6\sqrt{5}}{2} \right) = \left(4\sqrt{3}, 3\sqrt{5} \right).$$

10. the horizontal line through (2, 3)
The equation of a horizontal line has an equation of the form $y = b$. Because the line passes through (2, 3), the equation is $y = 3$.



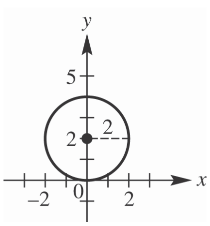
11. the circle with center (2, –1) and radius 3

$$(x-2)^2+[y-(-1)]^2=3^2$$
$$(x-2)^2+(y+1)^2=9$$



12. the circle with center (0, 2) and tangent to the *x*-axis
The distance from the center of the circle to the *x*-axis is 2, so $r = 2$.

$$(x-0)^2+(y-2)^2=2^2 \Rightarrow x^2+(y-2)^2=4$$

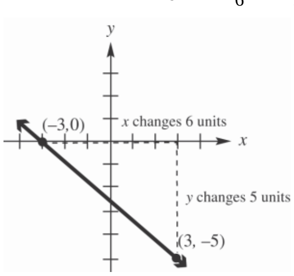


13. the line through $(3,-5)$ with slope $-\frac{5}{6}$
Write the equation in point-slope form.

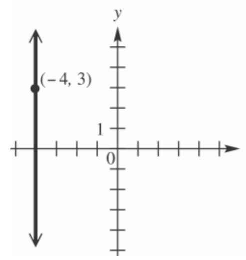
$$y-(-5)=-\tfrac{5}{6}(x-3)$$

Change to standard form.

$$6(y+5)=-5(x-3)\Rightarrow 6y+30=-5x+15$$
$$6y=-5x-15\Rightarrow y=-\tfrac{5}{6}x-\tfrac{15}{6}$$
$$y=-\tfrac{5}{6}x-\tfrac{5}{2}$$



14. the vertical line through (−4, 3)
The equation of a vertical line has an equation of the form $x = a$. Because the line passes through (−4, 3), the equation is $x = -4$.

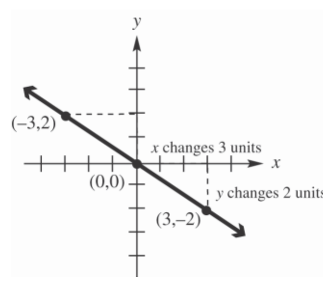


15. a line through (–3, 2) and parallel to the line $2x + 3y = 6$
First, find the slope of the line $2x + 3y = 6$ by writing this equation in slope-intercept form.

$$2x+3y=6\Rightarrow 3y=-2x+6\Rightarrow y=-\tfrac{2}{3}x+2$$

The slope is $-\frac{2}{3}$. Because the lines are parallel, $-\frac{2}{3}$ is also the slope of the line whose equation is to be found. Substitute $m=-\frac{2}{3}$, $x_1=-3$, and $y_1=2$ into the point-slope form.

$$y-y_1=m(x-x_1)\Rightarrow y-2=-\tfrac{2}{3}\left[x-(-3)\right]\Rightarrow$$
$$3(y-2)=-2(x+3)\Rightarrow 3y-6=-2x-6\Rightarrow$$
$$3y=-2x\Rightarrow y=-\tfrac{2}{3}x$$



16. a line through the origin and perpendicular to the line $3x-4y=2$
First, find the slope of the line $3x-4y=2$ by writing this equation in slope-intercept form.

$$3x-4y=2\Rightarrow -4y=-3x+2\Rightarrow$$
$$y=\tfrac{3}{4}x-\tfrac{2}{4}\Rightarrow y=\tfrac{3}{4}x-\tfrac{1}{2}$$

This line has a slope of $\frac{3}{4}$. The slope of any line perpendicular to this line is $-\frac{4}{3}$, because $-\frac{4}{3}\left(\frac{3}{4}\right)=-1$. Using slope-intercept form we have $y=-\frac{4}{3}x+0$ or $y=-\frac{4}{3}x$.

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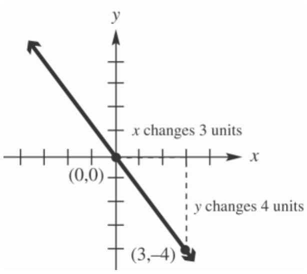

17. $x^2 - 4x + y^2 + 2y = 4$

Complete the square on x and y separately.

$$\left(x^2 - 4x\right) + \left(y^2 + 2y\right) = 4$$

$$\left(x^2 - 4x + 4\right) + \left(y^2 + 2y + 1\right) = 4 + 4 + 1$$

$$(x-2)^2 + (y+1)^2 = 9$$

Yes, it is a circle. The circle has its center at (2, −1) and radius 3.

18. $x^2 + 6x + y^2 + 10y + 36 = 0$

Complete the square on x and y separately.

$$\left(x^2 + 6x\right) + \left(y^2 + 10y\right) = -36$$

$$\left(x^2 + 6x + 9\right) + \left(y^2 + 10y + 25\right) = -36 + 9 + 25$$

$$(x+3)^2 + (y+5)^2 = -2$$

No, it is not a circle.

19. $x^2 - 12x + y^2 + 20 = 0$

Complete the square on x and y separately.

$$\left(x^2 - 12x\right) + y^2 = -20$$

$$\left(x^2 - 12x + 36\right) + y^2 = -20 + 36$$

$$(x-6)^2 + y^2 = 16$$

Yes, it is a circle. The circle has its center at (6, 0) and radius 4.

20. $x^2 + 2x + y^2 + 16y = -61$

Complete the square on x and y separately.

$$\left(x^2 + 2x\right) + \left(y^2 + 16y\right) = -61$$

$$\left(x^2 + 2x + 1\right) + \left(y^2 + 16y + 64\right) = -61 + 1 + 64$$

$$(x+1)^2 + (y+8)^2 = 4$$

21. $x^2 - 2x + y^2 + 10 = 0$

Complete the square on x and y separately.

$$\left(x^2 - 2x\right) + y^2 = -10$$

$$\left(x^2 - 2x + 1\right) + y^2 = -10 + 1$$

$$(x-1)^2 + y^2 = -9$$

No, it is not a circle.

22. $x^2 + y^2 - 8y - 9 = 0$

Complete the square on x and y separately.

$$x^2 + \left(y^2 - 8y\right) = 9$$

$$x^2 + \left(y^2 - 8y + 16\right) = 9 + 16$$

$$x^2 + (y-4)^2 = 25$$

Yes, it is a circle. The circle has its center at (0, 4) and radius 5.

23. The equation of the circle is

$(x-4)^2 + (y-5)^2 = 4^2$.

Let $y = 2$ and solve for x:

$(x-4)^2 + (2-5)^2 = 4^2 \Rightarrow$

$(x-4)^2 + (-3)^2 = 4^2 \Rightarrow (x-4)^2 = 7 \Rightarrow$

$x - 4 = \pm\sqrt{7} \Rightarrow x = 4 \pm \sqrt{7}$

The points of intersection are $\left(4 + \sqrt{7}, 2\right)$ and $\left(4 - \sqrt{7}, 2\right)$

24. Write the equation in center-radius form by completing the square on x and y separately:

$$x^2 + y^2 - 10x - 24y + 144 = 0$$

$$\left(x^2 - 10x + \quad\right) + \left(y^2 - 24y + 144\right) = 0$$

$$(x^2 - 10x + 25) + (y^2 - 24y + 144) = 25$$

$$(x-5)^2 + (y-12)^2 = 25$$

The center of the circle is (5, 12) and the radius is 5.

Now use the distance formula to find the distance from the center (5, 12) to the origin:

$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$$= \sqrt{(5-0)^2 + (12-0)^2} = \sqrt{25 + 144} = 13$$

Yes, it is a circle. The circle has its center at $(-1, -8)$ and radius 2.

The radius is 5, so the shortest distance from the origin to the graph of the circle is $13 - 5 = 8$.

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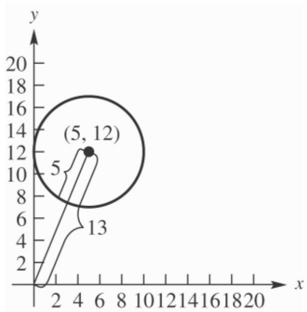


25. **(a)** The equation can be rewritten as

$-4y = -x - 6 \Rightarrow y = \frac{1}{4}x + \frac{6}{4} \Rightarrow y = \frac{1}{4}x + \frac{3}{2}.$
x can be any real number, so the domain is all real numbers and the range is also all real numbers.
domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

(b) Each value of x corresponds to just one value of y. $x - 4y = -6$ represents a function.

$y = \frac{1}{4}x + \frac{3}{2} \Rightarrow f(x) = \frac{1}{4}x + \frac{3}{2}$

$f(-2) = \frac{1}{4}(-2) + \frac{3}{2} = -\frac{1}{2} + \frac{3}{2} = \frac{2}{2} = 1$

26. **(a)** The equation can be rewritten as

$y^2 - 5 = x$. y can be any real number.

Because the square of any real number is not negative, y^2 is never negative.

Taking the constant term into consideration, domain would be $[-5, \infty)$.
domain: $[-5, \infty)$; range: $(-\infty, \infty)$

(b) Because $(-4, 1)$ and $(-4, -1)$ both satisfy the relation, $y^2 - x = 5$ does not represent a function.

27. **(a)** $(x+2)^2 + y^2 = 25$ is a circle centered at $(-2, 0)$ with a radius of 5. The domain will start 5 units to the left of -2 and end

28. **(a)** The equation can be rewritten as

$-2y = -x^2 + 3 \Rightarrow y = \frac{1}{2}x^2 - \frac{3}{2}$. x can be any real number. Because the square of any real number is not negative, $\frac{1}{2}x^2$ is never negative. Taking the constant term into consideration, range would be $\left[-\frac{3}{2}, \infty\right)$.
domain: $(-\infty, \infty)$; range: $\left[-\frac{3}{2}, \infty\right)$

(b) Each value of x corresponds to just one value of y. $x^2 - 2y = 3$ represents a function.
$y = \frac{1}{2}x^2 - \frac{3}{2} \Rightarrow f(x) = \frac{1}{2}x^2 - \frac{3}{2}$
$f(-2) = \frac{1}{2}(-2)^2 - \frac{3}{2} = \frac{1}{2}(4) - \frac{3}{2} = \frac{4}{2} - \frac{3}{2} = \frac{1}{2}$

Section 2.6 Graphs of Basic Functions

1. The equation $f(x) = x^2$ matches graph E.

The domain is $(-\infty, \infty)$.

2. The equation of $f(x) = x$ matches graph G.

The function is increasing on $(0, \infty)$.

3. The equation $f(x) = x^3$ matches graph A.

The range is $(-\infty, \infty)$.

4. Graph C is not the graph of a function. Its equation is $x = y^2$.

5. Graph F is the graph of the identity function.

Its equation is $f(x) = x$.

6. The equation $f(x) = [\![x]\!]$ matches graph B.

$f[\![1.5]\!] = 1$

7. The equation $f(x) = \sqrt[3]{x}$ matches graph H.

5 units to the right of –2. The domain will be [−2 − 5, 2 + 5] = [−7, 3]. The range

will start 5 units below 0 and end 5 units

above 0. The range will be [0 − 5, 0 + 5] = [−5, 5].

(b) Because (−2, 5) and (−2, −5) both satisfy the relation, $(x+2)^2 + y^2 = 25$ does not represent a function.

No, there is no interval over which the function is decreasing.

8. The equation of $f(x) = \sqrt{x}$ matches graph D.

The domain is $[0, \infty)$.

9. The graph in B is discontinuous at many points. Assuming the graph continues, the range would be $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

10. The graphs in E and G decrease over part of the domain and increase over part of the domain. They both increase over $(0, \infty)$ and decrease over $(-\infty, 0)$.

11. The function is continuous over the entire domain of real numbers $(-\infty, \infty)$.

12. The function is continuous over the entire domain of real numbers $(-\infty, \infty)$.

13. The function is continuous over the interval $[0, \infty)$.

14. The function is continuous over the interval $(-\infty, 0]$.

15. The function has a point of discontinuity at (3, 1). It is continuous over the interval $(-\infty, 3)$ and the interval $(3, \infty)$.

16. The function has a point of discontinuity at $x = 1$. It is continuous over the interval $(-\infty, 1)$ and the interval $(1, \infty)$.

17. $f(x) = \begin{cases} 2x & \text{if } x \le -1 \\ x - 1 & \text{if } x > -1 \end{cases}$

(a) $f(-5) = 2(-5) = -10$

(b) $f(-1) = 2(-1) = -2$

(c) $f(0) = 0 - 1 = -1$

(d) $f(3) = 3 - 1 = 2$

18. $f(x) = \begin{cases} x - 2 & \text{if } x < 3 \\ 5 - x & \text{if } x \ge 3 \end{cases}$

(a) $f(-5) = -5 - 2 = -7$

(b) $f(-1) = -1 - 2 = -3$

(c) $f(0) = 0 - 2 = -2$

(d) $f(3) = 5 - 3 = 2$

19. $f(x) = \begin{cases} 2 + x & \text{if } x < -4 \\ -x \\ 3x \end{cases}$

20. $f(x) = \begin{cases} -2x & \text{if } x < -3 \\ 3x - 1 & \text{if } -3 \le x \le 2 \\ -4x & \text{if } x > 2 \end{cases}$

(a) $f(-5) = -2(-5) = 10$

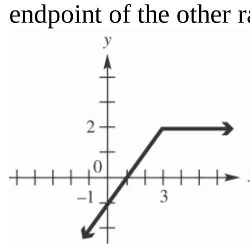
(b) $f(-1) = 3(-1) - 1 = -3 - 1 = -4$

(c) $f(0) = 3(0) - 1 = 0 - 1 = -1$

(d) $f(3) = -4(3) = -12$

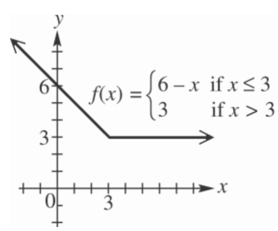
21. $f(x) = \begin{cases} x - 1 & \text{if } x \le 3 \\ 2 & \text{if } x > 3 \end{cases}$

Draw the graph of $y = x - 1$ to the left of $x = 3$, including the endpoint at $x = 3$. Draw the graph of $y = 2$ to the right of $x = 3$, and note that the endpoint at $x = 3$ coincides with the endpoint of the other ray.



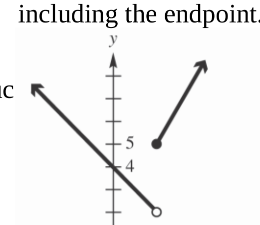
22. $f(x) = \begin{cases} 6 - x & \text{if } x \le 3 \\ 3 & \text{if } x > 3 \end{cases}$

Graph the line $y = 6 - x$ to the left of $x = 3$, including the endpoint. Draw $y = 3$ to the right of $x = 3$. Note that the endpoint at $x = 3$ coincides with the endpoint of the other ray.



23. $f(x) = \begin{cases} 4 - x & \text{if } x < 2 \\ 1 + 2x & \text{if } x \ge 2 \end{cases}$

Draw the graph of $y = 4 - x$ to the left of $x = 2$, but do not include the endpoint. Draw the graph of $y = 1 + 2x$ to the right of $x = 2$, including the endpoint.



i $-4 \le x \le 2$ if
f $x > 2$

(a) $f(-5) = 2 + (-5) = -3$

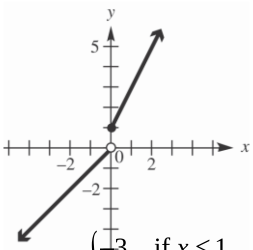
(b) $f(-1) = -(-1) = 1$

(c) $f(0) = -0 = 0$

(d) $f(3) = 3 \cdot 3 = 9$

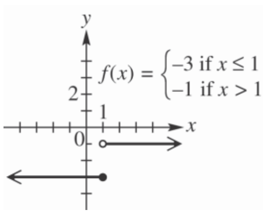
24. $f(x)=\begin{cases}2x+1 & \text{if } x\ge 0\\ x & \text{if } x<0\end{cases}$

Graph the line $y = 2x + 1$ to the right of $x = 0$, including the endpoint. Draw $y = x$ to the left of $x = 0$, but do not include the endpoint.



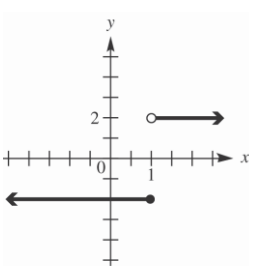
25. $f(x)=\begin{cases}-3 & \text{if } x\le 1\\ -1 & \text{if } x>1\end{cases}$

Graph the line $y = -3$ to the left of $x = 1$, including the endpoint. Draw $y = -1$ to the right of $x = 1$, but do not include the endpoint.

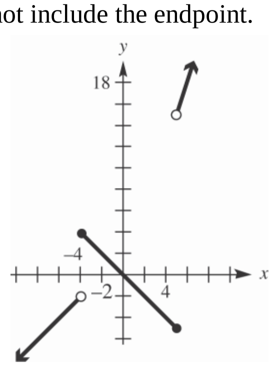


26. $f(x)=\begin{cases}-2 & \text{if } x\le 1\\ 2 & \text{if } x>1\end{cases}$

Graph the line $y = -2$ to the left of $x = 1$, including the endpoint. Draw $y = 2$ to the right of $x = 1$, but do not include the endpoint.



Exercise 26



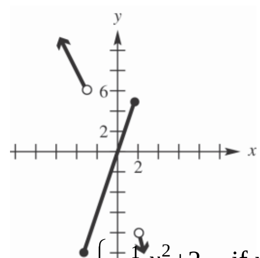
Exercise 27

27. $f(x)=\begin{cases}2+x & \text{if } x<-4\\ -x & \text{if } -4\le x\le 5\\ 3x & \text{if } x>5\end{cases}$

Draw the graph of $y = 2 + x$ to the left of –4, but do not include the endpoint at $x = 4$. Draw the graph of $y = -x$ between –4 and 5, including both endpoints. Draw the graph of

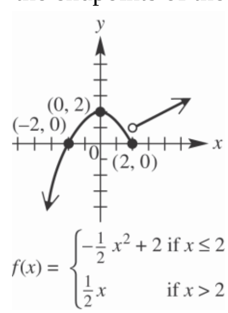
28. $f(x)=\begin{cases}-2x & \text{if } x<-3\\ 3x-1 & \text{if } -3\le x\le 2\\ -4x & \text{if } x>2\end{cases}$

Graph the line $y = -2x$ to the left of $x = -3$, but do not include the endpoint. Draw $y = 3x - 1$ between $x = -3$ and $x = 2$, and include both endpoints. Draw $y = -4x$ to the right of $x = 2$, but do not include the endpoint. Notice that the endpoints of the pieces do not coincide.



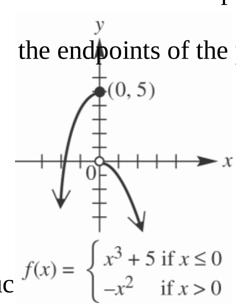
29. $f(x)=\begin{cases}-\frac{1}{2}x^2+2 & \text{if } x\le 2\\ \frac{1}{2}x & \text{if } x>2\end{cases}$

Graph the curve $y=-\frac{1}{2}x^2+2$ to the left of $x = 2$, including the endpoint at (2, 0). Graph the line $y=\frac{1}{2}x$ to the right of $x = 2$, but do not include the endpoint at (2, 1). Notice that the endpoints of the pieces do not coincide.



30. $f(x)=\begin{cases}x^3+5 & \text{if } x\le 0\\ -x^2 & \text{if } x>0\end{cases}$

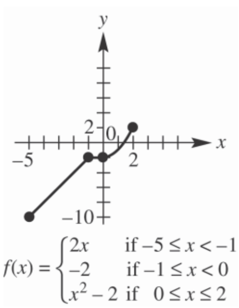
Graph the curve $y=x^3+5$ to the left of $x = 0$, including the endpoint at (0, 5). Graph the line $y=-x^2$ to the right of $x = 0$, but do not include the endpoint at (0, 0). Notice that the endpoints of the pieces do not coincide.



$y = 3x$ to the right of 5, but do not include the endpoint at $x = 5$.

31. $f(x) = \begin{cases} 2x & \text{if } -5 \le x < -1 \\ -2 & \text{if } -1 \le x < 0 \\ x^2 - 2 & \text{if } 0 \le x \le 2 \end{cases}$

Graph the line $y = 2x$ between $x = -5$ and $x = -1$, including the left endpoint at $(-5, -10)$, but not including the right endpoint at $(-1, -2)$. Graph the line $y = -2$ between $x = -1$ and $x = 0$, including the left endpoint at $(-1, -2)$ and not including the right endpoint at $(0, -2)$. Note that $(-1, -2)$ coincides with the first two sections, so it is included. Graph the curve $y = x^2 - 2$ from $x = 0$ to $x = 2$, including the endpoints at $(0, -2)$ and $(2, 2)$. Note that $(0, -2)$ coincides with the second two sections, so it is included. The graph ends at $x = -5$ and $x = 2$.

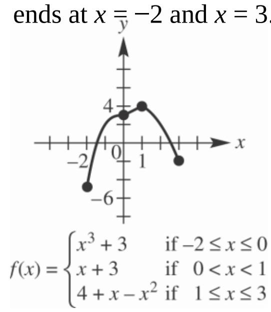


32. $f(x) = \begin{cases} 0.5x^2 & \text{if } -4 \le x \le -2 \\ x & \text{if } -2 < x < 2 \\ x^2 - 4 & \text{if } 2 \le x \le 4 \end{cases}$

Graph the curve $y = 0.5x^2$ between $x = -4$ and $x = -2$, including the endpoints at $(-4, 8)$ and $(-2, 2)$. Graph the line $y = x$ between $x = -2$ and $x = 2$, but do not include the endpoints at $(-2, -2)$ and $(2, 2)$. Graph the curve $y = x^2 - 4$ from $x = 2$ to $x = 4$, including the endpoints at $(2, 0)$ and $(4, 12)$. The graph ends at $x = -4$ and $x = 4$.

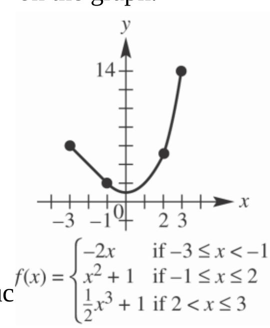
33. $f(x) = \begin{cases} x^3 + 3 & \text{if } -2 \le x \le 0 \\ x + 3 & \text{if } 0 < x < 1 \\ 4 + x - x^2 & \text{if } 1 \le x \le 3 \end{cases}$

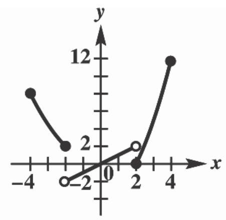
Graph the curve $y = x^3 + 3$ between $x = -2$ and $x = 0$, including the endpoints at $(-2, -5)$ and $(0, 3)$. Graph the line $y = x + 3$ between $x = 0$ and $x = 1$, but do not include the endpoints at $(0, 3)$ and $(1, 4)$. Graph the curve $y = 4 + x - x^2$ from $x = 1$ to $x = 3$, including the endpoints at $(1, 4)$ and $(3, -2)$. The graph ends at $x = -2$ and $x = 3$.



34. $f(x) = \begin{cases} -2x & \text{if } -3 \le x < -1 \\ x^2 + 1 & \text{if } -1 \le x \le 2 \\ \frac{1}{2}x^3 + 1 & \text{if } 2 < x \le 3 \end{cases}$

Graph the curve $y = -2x$ to from $x = -3$ to $x = -1$, including the endpoint $(-3, 6)$, but not including the endpoint $(-1, 2)$. Graph the curve $y = x^2 + 1$ from $x = -1$ to $x = 2$, including the endpoints $(-1, 2)$ and $(2, 5)$. Graph the curve $y = \frac{1}{2}x^3 + 1$ from $x = 2$ to $x = 3$, including the endpoint $(3, 14.5)$ but not including the endpoint $(2, 5)$. Because the endpoints that are not included coincide with endpoints that are included, we use closed dots on the graph.





$$f(x) = \begin{cases} 0.5x^2 & \text{if } -4 \le x \le -2 \\ x & \text{if } -2 < x < 2 \\ x^2 - 4 & \text{if } 2 \le x \le 4 \end{cases}$$

35. The solid circle on the graph shows that the endpoint (0, –1) is part of the graph, while the open circle shows that the endpoint (0, 1) is not part of the graph. The graph is made up of parts of two horizontal lines. The function which fits this graph is

$$f(x) = \begin{cases} -1 & \text{if } x \le 0 \\ 1 & \text{if } x > 0. \end{cases}$$

domain: $(-\infty, \infty)$; range: $\{-1, 1\}$

36. We see that $y = 1$ for every value of x except $x = 0$, and that when $x = 0$, $y = 0$. We can write the function as

$$f(x) = \begin{cases} 1 & \text{if } x \ne 0 \\ 0 & \text{if } x = 0. \end{cases}$$

domain: $(-\infty, \infty)$; range: $\{0, 1\}$

37. The graph is made up of parts of two horizontal lines. The solid circle shows that the endpoint (0, 2) of the one on the left belongs to the graph, while the open circle shows that the endpoint (0, –1) of the one on the right does not belong to the graph. The function that fits this graph is

$$f(x) = \begin{cases} 2 & \text{if } x \le 0 \\ -1 & \text{if } x > 1. \end{cases}$$

domain: $(-\infty, 0] \cup (1, \infty)$; range: $\{-1, 2\}$

38. We see that $y = 1$ when $x \le -1$ and that $y = -1$ when $x > 2$. We can write the function as

$$f(x) = \begin{cases} 1 & \text{if } x \le -1 \\ -1 & \text{if } x > 2. \end{cases}$$

domain: $(-\infty, -1] \cup (2, \infty)$; range: $\{-1, 1\}$

39. For $x \le 0$, that piece of the graph goes through the points (−1, −1) and (0, 0). The slope is 1, so the equation of this piece is $y = x$. For $x > 0$, that piece of the graph is a horizontal line passing through (2, 2), so its equation is $y = 2$. We can write the function as

$$f(x) = \begin{cases} x & \text{if } x \le 0 \\ 2 & \text{if } x > 0 \end{cases}.$$

domain: $(-\infty, \infty)$ range: $(-\infty, 0] \cup \{2\}$

40. For $x < 0$, that piece of the graph is a horizontal line passing though (−3, −3), so the equation of this piece is $y = -3$. For $x \ge 0$, the curve passes through (1, 1) and (4, 2), so the equation of this piece is $y = \sqrt{x}$. We can

41. For $x < 1$, that piece of the graph is a curve passes through (−8, −2), (−1, −1) and (1, 1), so the equation of this piece is $y = \sqrt[3]{x}$. The right piece of the graph passes through (1, 2) and (2, 3). $m = \dfrac{2-3}{1-2} = 1$, and the equation of the line is $y - 2 = x - 1 \Rightarrow y = x + 1$. We can write the function as $f(x) = \begin{cases} \sqrt[3]{x} & \text{if } x < 1 \\ x+1 & \text{if } x \ge 1 \end{cases}$

domain: $(-\infty, \infty)$ range: $(-\infty, 1) \cup [2, \infty)$

42. For all values except $x = 2$, the graph is a line. It passes through (0, −3) and (1, −1). The slope is 2, so the equation is $y = 2x - 3$. At $x = 2$, the graph is the point (2, 3). We can write the function as $f(x) = \begin{cases} 3 & \text{if } x = 2 \\ 2x-3 & \text{if } x \ne 2 \end{cases}$.

domain: $(-\infty, \infty)$ range: $(-\infty, 1) \cup (1, \infty)$

43. $f(x) = [\![-x]\!]$

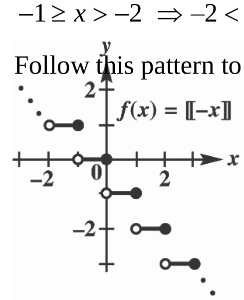
Plot points.

x	$-x$	$f(x) = [\![-x]\!]$
–2	2	2
–1.5	1.5	1
–1	1	1
–0.5	0.5	0
0	0	0
0.5	–0.5	–1
1	–1	–1
1.5	–1.5	–2
2	–2	–2

More generally, to get $y = 0$, we need $0 \le -x < 1 \Rightarrow 0 \ge x > -1 \Rightarrow -1 < x \le 0$. To get $y = 1$, we need $1 \le -x < 2 \Rightarrow$

$-1 \ge x > -2 \Rightarrow -2 < x \le -1$.

Follow this pattern to graph the step function.



write the function as $f(x)=\begin{cases} -3 & \text{if } x<0 \\ \sqrt{x} & \text{if } x\geq 0 \end{cases}$.

domain: $(-\infty,\infty)$; range: $\{\ldots,-2,-1,0,1,2,\ldots\}$

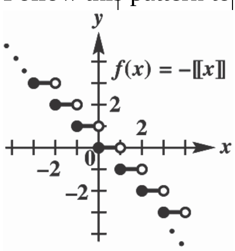
domain: $(-\infty,\infty)$ range: $\{-3\}\cup[0,\infty)$

44. $f(x) = -[\![x]\!]$

Plot points.

x	$[\![x]\!]$	$f(x) = -[\![x]\!]$
–2	–2	2
–1.5	–2	2
–1	–1	1
–0.5	–1	1
0	0	0
0.5	0	0
1	1	–1
1.5	1	–1
2	2	–2

Follow this pattern to graph the step function.



domain: $(-\infty, \infty)$; range: {...,–2,–1,0,1,2,...}

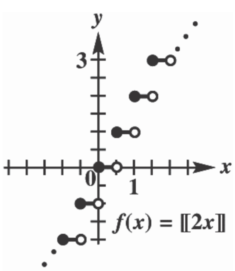
45. $f(x) = [\![2x]\!]$

To get $y = 0$, we need $0 \le 2x < 1 \Rightarrow 0 \le x < \frac{1}{2}$.

To get $y = 1$, we need $1 \le 2x < 2 \Rightarrow \frac{1}{2} \le x < 1$.

To get $y = 2$, we need $2 \le 2x < 3 \Rightarrow 1 \le x < \frac{3}{2}$.

Follow this pattern to graph the step function.



domain: $(-\infty, \infty)$; range: {...,–2,–1,0,1,2,...}

46. $g(x) = [\![2x-1]\!]$

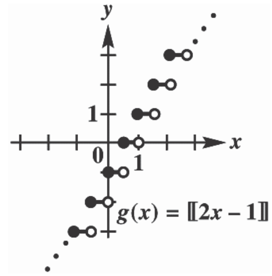
To get $y = 0$, we need

$0 \le 2x - 1 < 1 \Rightarrow 1 \le 2x < 2 \Rightarrow \frac{1}{2} \le x < 1$.

To get $y = 1$, we need

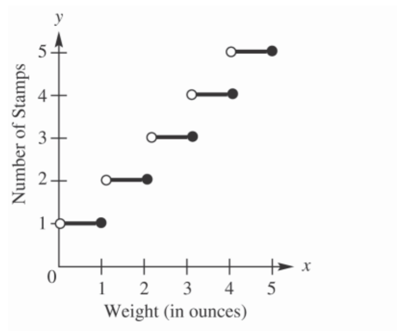
$1 \le 2x - 1 < 2 \Rightarrow 2 \le 2x < 3 \Rightarrow 1 \le x < \frac{3}{2}$.

Follow this pattern to graph the step function.

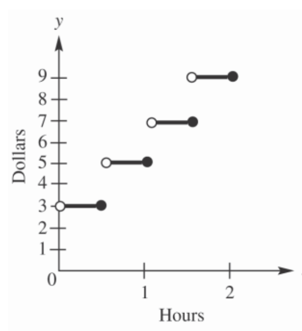


domain: $(-\infty, \infty)$; range: {..., 2,–1,0,1,2,...}

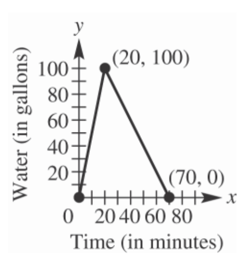
47. The cost of mailing a letter that weighs more than 1 ounce and less than 2 ounces is the same as the cost of a 2-ounce letter, and the cost of mailing a letter that weighs more than 2 ounces and less than 3 ounces is the same as the cost of a 3-ounce letter, etc.



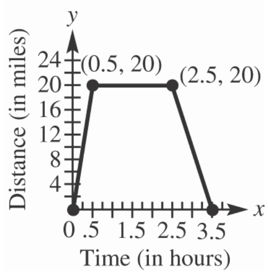
48. The cost is the same for all cars parking between $\frac{1}{2}$ hour and 1-hour, between 1 hour and $1\frac{1}{2}$ hours, etc.



49.



50.



51. **(a)** For $0 \le x \le 8$, $m = \dfrac{49.8 - 34.2}{8 - 0} = 1.95$,

so $y = 1.95x + 34.2$. For $8 < x \le 13$,
$m = \dfrac{52.2 - 49.8}{13 - 8} = 0.48$, so the equation
is $y - 52.2 = 0.48(x - 13) \Rightarrow$
$y = 0.48x + 45.96$

(b) $f(x) = \begin{cases} 1.95x + 34.2 & \text{if } 0 \le x \le 8 \\ 0.48x + 45.96 & \text{if } 8 < x \le 13 \end{cases}$

52. When $0 \le x \le 3$, the slope is 5, which means that the inlet pipe is open, and the outlet pipe is closed. When $3 < x \le 5$, the slope is 2, which means that both pipes are open. When $5 < x \le 8$, the slope is 0, which means that both pipes are closed. When $8 < x \le 10$, the slope is −3, which means that the inlet pipe is closed, and the outlet pipe is open.

53. **(a)** The initial amount is 50,000 gallons. The final amount is 30,000 gallons.

(b) The amount of water in the pool remained constant during the first and fourth days.

(c) $f(2) \approx 45{,}000$; $f(4) = 40{,}000$

(d) The slope of the segment between (1, 50000) and (3, 40000) is −5000, so the water was being drained at 5000 gallons per day.

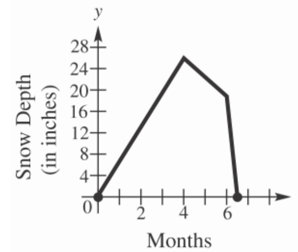
54. **(a)** There were 20 gallons of gas in the tank at $x = 3$.

(b) The slope is steepest between $t = 1$ and $t \approx 2.9$, so that is when the car burned gasoline at the fastest rate.

55. **(a)** There is no charge for additional length, so we use the greatest integer function.

56. **(a)** $f(x) = \begin{cases} 6.5x & \text{if } 0 \le x \le 4 \\ -5.5x + 48 & \text{if } 4 < x \le 6 \\ -30x + 195 & \text{if } 6 < x \le 6.5 \end{cases}$

Draw a graph of $y = 6.5x$ between 0 and 4, including the endpoints. Draw the graph of $y = -5.5x + 48$ between 4 and 6, including the endpoint at 6 but not the one at 4. Draw the graph of $y = -30x + 195$, including the endpoint at 6.5 but not the one at 6. Notice that the endpoints of the three pieces coincide.



(b) From the graph, observe that the snow depth, y, reaches its deepest level (26 in.) when $x = 4$, $x = 4$ represents 4 months after the beginning of October, which is the beginning of February.

(c) From the graph, the snow depth y is nonzero when x is between 0 and 6.5. Snow begins at the beginning of October and ends 6.5 months later, in the middle of April.

Section 2.7 Graphing Techniques

1. To graph the function $f(x) = x^2 - 3$, shift the graph of $y = x^2$ down <u>3</u> units.

2. To graph the function $f(x) = x^2 + 5$, shift the graph of $y = x^2$ up <u>5</u> units.

3. The graph of $f(x) = (x + 4)^2$ is obtained by shifting the graph of $y = x^2$ to the <u>left</u> 4 units.

4. The graph of $f(x) = (x - 7)^2$ is obtained by shifting the graph of $y = x^2$ to the <u>right</u> 7 units.

The cost is based on multiples of two feet, so $f(x) = 0.8\left\lfloor \frac{x}{2} \right\rfloor$ if $6 \le x \le 18$.

(b) $f(8.5) = 0.8\left\lfloor \frac{8.5}{2} \right\rfloor = 0.8(4) = \$3.20$

$f(15.2) = 0.8\left\lfloor \frac{15.2}{2} \right\rfloor = 0.8(7) = \$5.60$

5. The graph of $f(x) = -\sqrt{x}$ is a reflection of the graph of $f(x) = \sqrt{x}$ across the x-axis.

6. The graph of $f(x) = \sqrt{-x}$ is a reflection of the graph of $f(x) = \sqrt{x}$ across the y-axis.

7. To obtain the graph of $f(x)=(x+2)^3-3$, shift the graph of $y=x^3$ 2 units to the left and 3 units down.

8. To obtain the graph of $f(x)=(x-3)^3+6$, shift the graph of $y=x^3$ 3 units to the right and 6 units up.

9. The graph of $f(x)=|-x|$ is the same as the graph of $y=|x|$ because reflecting it across the y-axis yields the same ordered pairs.

10. The graph of $x=y^2$ is the same as the graph of $x=(-y)^2$ because reflecting it across the x-axis yields the same ordered pairs.

11. (a) B; $y=(x-7)^2$ is a shift of $y=x^2$, 7 units to the right.

(b) D; $y=x^2-7$ is a shift of $y=x^2$, 7 units downward.

(c) E; $y=7x^2$ is a vertical stretch of $y=x^2$, by a factor of 7.

(d) A; $y=(x+7)^2$ is a shift of $y=x^2$, 7 units to the left.

(e) C; $y=x^2+7$ is a shift of $y=x^2$, 7 units upward.

12. (a) E; $y=4\sqrt[3]{x}$ is a vertical stretch of $y=\sqrt[3]{x}$, by a factor of 4.

(b) C; $y=-\sqrt[3]{x}$ is a reflection of $y=\sqrt[3]{x}$,

(c) G; $y=(x+2)^2$ is a shift of $y=x^2$, 2 units to the left.

(d) C; $y=(x-2)^2$ is a shift of $y=x^2$, 2 units to the right.

(e) F; $y=2x^2$ is a vertical stretch of $y=x^2$, by a factor of 2.

(f) D; $y=-x^2$ is a reflection of $y=x^2$, across the x-axis.

(g) H; $y=(x-2)^2+1$ is a shift of $y=x^2$, 2 units to the right and 1 unit upward.

(h) E; $y=(x+2)^2+1$ is a shift of $y=x^2$, 2 units to the left and 1 unit upward.

(i) I; $y=(x+2)^2-1$ is a shift of $y=x^2$, 2 units to the left and 1 unit down.

14. (a) G; $y=\sqrt{x+3}$ is a shift of $y=\sqrt{x}$, 3 units to the left.

(b) D; $y=\sqrt{x}-3$ is a shift of $y=\sqrt{x}$, 3 units downward.

(c) E; $y=\sqrt{x}+3$ is a shift of $y=\sqrt{x}$, 3 units upward.

(d) B; $y=3\sqrt{x}$ is a vertical stretch of $y=\sqrt{x}$, by a factor of 3.

(e) C; $y=-\sqrt{x}$ is a reflection of $y=\sqrt{x}$ across the x-axis.

(f) A; $y=\sqrt{x-3}$ is a shift of $y=\sqrt{x}$, 3 units to the right.

over the x-axis.

(c) D; $y = \sqrt[3]{-x}$ is a reflection of $y = \sqrt[3]{x}$, over the y-axis.

(d) A; $y = \sqrt[3]{x-4}$ is a shift of $y = \sqrt[3]{x}$, 4 units to the right.

(e) B; $y = \sqrt[3]{x} - 4$ is a shift of $y = \sqrt[3]{x}$, 4 units down.

13. **(a)** B; $y = x^2 + 2$ is a shift of $y = x^2$, 2 units upward.

(b) A; $y = x^2 - 2$ is a shift of $y = x^2$, 2 units downward.

(g) H; $y = \sqrt{x-3} + 2$ is a shift of $y = \sqrt{x}$, 3 units to the right and 2 units upward.

(h) F; $y = \sqrt{x+3} + 2$ is a shift of $y = \sqrt{x}$, 3 units to the left and 2 units upward.

(i) I; $y = \sqrt{x-3} - 2$ is a shift of $y = \sqrt{x}$, 3 units to the right and 2 units downward.

15. **(a)** F; $y = |x-2|$ is a shift of $y = |x|$ 2 units to the right.

(b) C; $y = |x| - 2$ is a shift of $y = |x|$ 2 units downward.

(c) H; $y = |x| + 2$ is a shift of $y = |x|$ 2 units upward.

(d) D; $y = 2|x|$ is a vertical stretch of $y = |x|$ by a factor of 2.

(e) G; $y = -|x|$ is a reflection of $y = |x|$ across the x-axis.

(f) A; $y = |-x|$ is a reflection of $y = |x|$ across the y-axis.

(g) E; $y = -2|x|$ is a reflection of $y = 2|x|$ across the x-axis. $y = 2|x|$ is a vertical stretch of $y = |x|$ by a factor of 2.

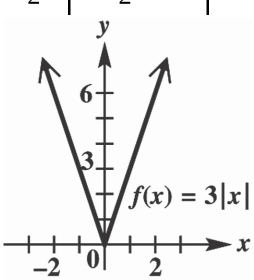
(h) I; $y = |x-2| + 2$ is a shift of $y = |x|$ 2 units to the right and 2 units upward.

(i) B; $y = |x+2| - 2$ is a shift of $y = |x|$ 2 units to the left and 2 units downward.

16. The graph of $f(x) = 2(x+1)^3 - 6$ is the graph of $f(x) = x^3$ stretched vertically by a factor of 2, shifted left 1 unit and down 6 units.

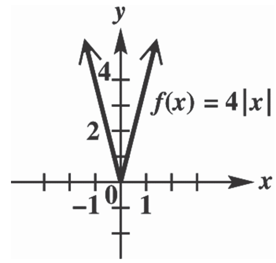
17. $f(x) = 3|x|$

x	$h(x) = \|x\|$	$f(x) = 3\|x\|$
−2	2	6
−1	1	3
0	0	0
1	1	3
2	2	6



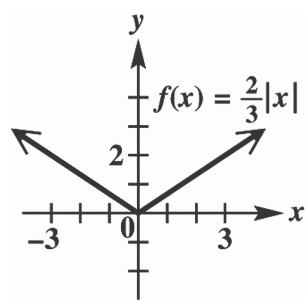
18. $f(x) = 4|x|$

x	$h(x) = \|x\|$	$f(x) = 4\|x\|$
−2	2	8
−1	1	4
0	0	0
1	1	4
2	2	8



19. $f(x) = \frac{2}{3}|x|$

x	$h(x) = \|x\|$	$f(x) = \frac{2}{3}\|x\|$
−3	3	2
−2	2	$\frac{4}{3}$
−1	1	$\frac{2}{3}$
0	0	0
1	1	$\frac{2}{3}$
2	2	$\frac{4}{3}$
3	3	2



20. $f(x) = \frac{3}{4}|x|$

x	$h(x) = \|x\|$	$f(x) = \frac{3}{4}\|x\|$
−4	4	3
−3	3	$\frac{9}{4}$
−2	2	$\frac{3}{2}$
−1	1	$\frac{3}{4}$
0	0	0
1	1	$\frac{3}{4}$
2	2	$\frac{3}{2}$
3	3	$\frac{9}{4}$
4	4	3

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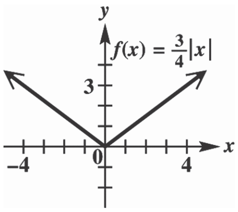
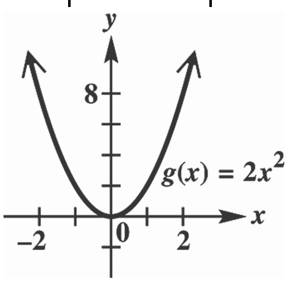


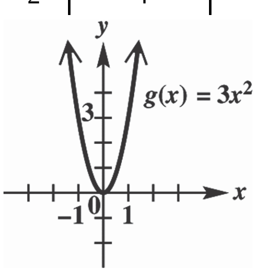
21. $f(x) = 2x^2$

x	$h(x) = x^2$	$f(x) = 2x^2$
−2	4	8
−1	1	2
0	0	0
1	1	2
2	4	8



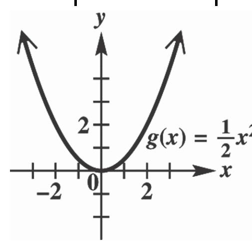
22. $f(x) = 3x^2$

x	$h(x) = x^2$	$f(x) = 3x^2$
−2	4	12
−1	1	3
0	0	0
1	1	3
2	4	12



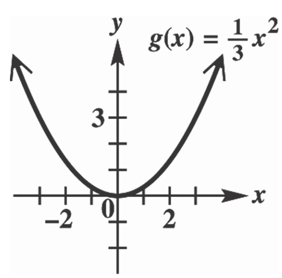
23. $f(x) = \frac{1}{2}x^2$

x	$h(x) = x^2$	$f(x) = \frac{1}{2}x^2$
−2	4	2
−1	1	$\frac{1}{2}$
0	0	0
1	1	$\frac{1}{2}$
2	4	2



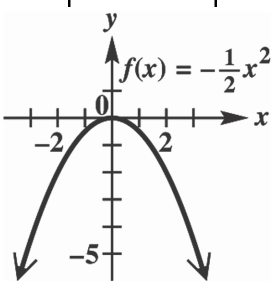
24. $f(x) = \frac{1}{3}x^2$

x	$h(x) = x^2$	$f(x) = \frac{1}{3}x^2$
−3	9	3
−2	4	$\frac{4}{3}$
−1	1	$\frac{1}{3}$
0	0	0
1	1	$\frac{1}{3}$
2	4	$\frac{4}{3}$
3	9	3



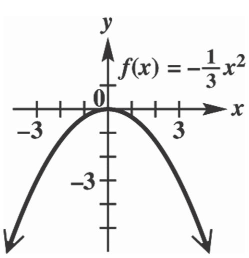
25. $f(x) = -\frac{1}{2}x^2$

x	$h(x) = x^2$	$f(x) = -\frac{1}{2}x^2$
−3	9	$-\frac{9}{2}$
−2	4	−2
−1	1	$-\frac{1}{2}$
0	0	0
1	1	$-\frac{1}{2}$
2	4	−2
3	9	$-\frac{9}{2}$



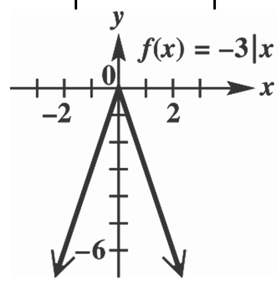
26. $f(x) = -\frac{1}{3}x^2$

x	$h(x) = x^2$	$f(x) = -\frac{1}{3}x^2$
−3	9	−3
−2	4	$-\frac{4}{3}$
−1	1	$-\frac{1}{3}$
0	0	0
1	1	$-\frac{1}{3}$
2	4	$-\frac{4}{3}$
3	9	−3



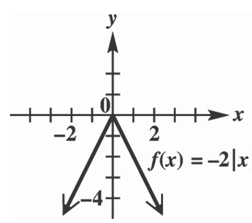
27. $f(x) = -3|x|$

x	$h(x) = \lvert x\rvert$	$f(x) = -3\lvert x\rvert$
−2	2	−6
−1	1	−3
0	0	0
1	1	−3
2	2	−6



28. $f(x) = -2|x|$

x	$h(x) = \lvert x\rvert$	$f(x) = -2\lvert x\rvert$
−2	2	−4
−1	1	−2
0	0	0
1	1	−2
2	2	−4



29. $h(x) = \left|-\frac{1}{2}x\right|$

x	$f(x) = \lvert x\rvert$	$h(x) = \left\lvert -\frac{1}{2}x\right\rvert = \left\lvert -\frac{1}{2}\right\rvert \lvert x\rvert = \frac{1}{2}\lvert x\rvert$
−4	4	2
−3	3	$\frac{3}{2}$
−2	2	1
−1	1	$\frac{1}{2}$
0	0	0

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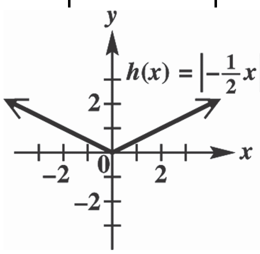
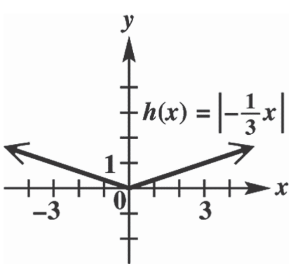
x	$f(x)=\lvert x\rvert$	$h(x)=\left\lvert-\frac{1}{2}x\right\rvert = \left\lvert-\frac{1}{2}\right\rvert\lvert x\rvert=\frac{1}{2}\lvert x\rvert$
1	1	$\frac{1}{2}$
2	2	1
3	3	$\frac{3}{2}$
4	4	2



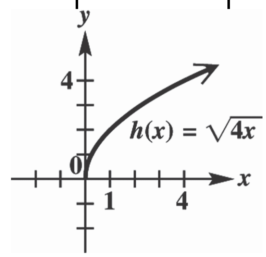
30. $h(x)=\left\lvert-\frac{1}{3}x\right\rvert$

x	$f(x)=\left\lvert-\frac{1}{3}x\right\rvert$	$h(x)=\left\lvert-\frac{1}{3}x\right\rvert =\left\lvert-\frac{1}{3}\right\rvert\lvert x\rvert=\frac{1}{3}\lvert x\rvert$
−3	3	1
−2	2	$\frac{2}{3}$
−1	1	$\frac{1}{3}$
0	0	0
1	1	$\frac{1}{3}$
2	2	$\frac{2}{3}$
3	3	1



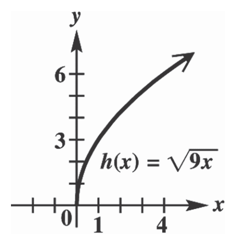
31. $h(x)=\sqrt{4x}$

x	$f(x)=\sqrt{x}$	$h(x)=\sqrt{4x}=2\sqrt{x}$
0	0	0
1	$\sqrt{1}$	2
2	2	$2\sqrt{2}$
3	$\sqrt{3}$	$2\sqrt{3}$
4	2	4



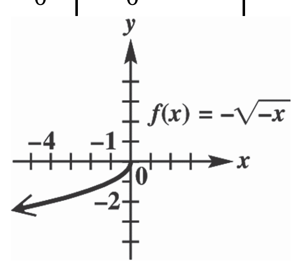
32. $h(x)=\sqrt{9x}$

x	$f(x)=\sqrt{x}$	$h(x)=\sqrt{9x}=3\sqrt{x}$
0	0	0
1	1	3
2	$\sqrt{2}$	$3\sqrt{2}$
3	$\sqrt{3}$	$3\sqrt{3}$
4	2	6



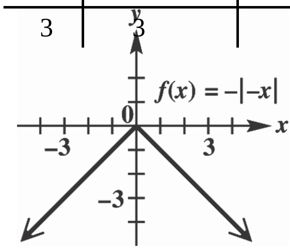
33. $f(x)=-\sqrt{-x}$

x	$h(x)=\sqrt{-x}$	$f(x)=-\sqrt{-x}$
−4	2	−2
−3	$\sqrt{3}$	$-\sqrt{3}$
−2	$\sqrt{2}$	$-\sqrt{2}$
−1	1	−1
0	0	0



34. $f(x) = -|-x|$

x	$h(x) = \|-x\|$	$f(x) = -\|-x\|$
−3	3	−3
−2	2	−2
−1	1	−1
0	0	0
1	1	−1
2	2	−2
3	3	−3



35. **(a)** $y = f(x+4)$ is a horizontal translation of *f*, 4 units to the left. The point that corresponds to (8, 12) on this translated function would be $(8-4, 12) = (4, 12)$.

(b) $y = f(x) + 4$ is a vertical translation of *f*, 4 units up. The point that corresponds to (8, 12) on this translated function would be $(8, 12+4) = (8, 16)$.

36. **(a)** $y = \frac{1}{4} f(x)$ is a vertical shrinking of *f*, by a factor of $\frac{1}{4}$. The point that corresponds to (8, 12) on this translated function would be $\left(8, \frac{1}{4} \cdot 12\right) = (8, 3)$.

(b) $y = 4f(x)$ is a vertical stretching of *f*, by a factor of 4. The point that corresponds to (8, 12) on this translated function would be $(8, 4 \cdot 12) = (8, 48)$.

37. **(a)** $y = f(4x)$ is a horizontal shrinking of *f*,

by a factor of 4. The point that corresponds to (8, 12) on this translated function is $\left(8 \cdot \frac{1}{4}, 12\right) = (2, 12)$.

(b) $y = f\left(\frac{1}{4}x\right)$ is a horizontal stretching of *f*, by a factor of 4. The point that corresponds to (8, 12) on this translated function is $(8 \cdot 4, 12) = (32, 12)$.

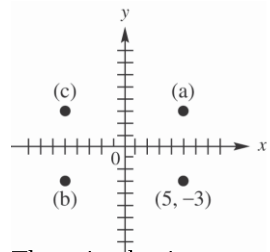
38. **(a)** The point that corresponds to (8, 12) when reflected across the x-axis would be (8, −12).

(b) The point that corresponds to (8, 12) when reflected across the y-axis would be (−8, 12).

39. **(a)** The point that is symmetric to (5, –3) with respect to the *x*-axis is (5, 3).

(b) The point that is symmetric to (5, –3) with respect to the *y*-axis is (–5, –3).

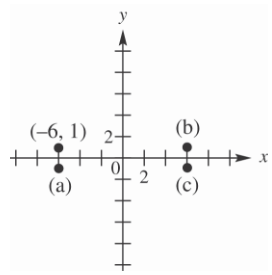
(c) The point that is symmetric to (5, –3) with respect to the origin is (–5, 3).



40. **(a)** The point that is symmetric to (–6, 1) with respect to the *x*-axis is (–6, –1).

(b) The point that is symmetric to (–6, 1) with respect to the *y*-axis is (6, 1).

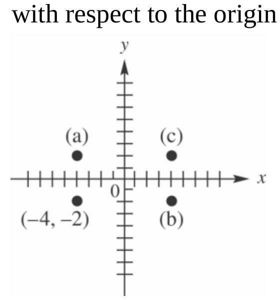
(c) The point that is symmetric to (–6, 1) with respect to the origin is (6, –1).



41. **(a)** The point that is symmetric to (–4, –2) with respect to the *x*-axis is (–4, 2).

(b) The point that is symmetric to (–4, –2) with respect to the *y*-axis is (4, –2).

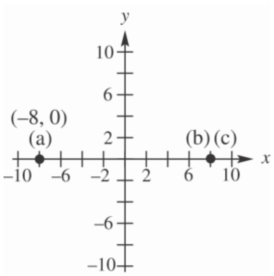
(c) The point that is symmetric to (–4, –2) with respect to the origin is (4, 2).



42. (a) The point that is symmetric to (–8, 0) with respect to the x-axis is (–8, 0) because this point lies on the x-axis.

(b) The point that is symmetric to the point (–8, 0) with respect to the y-axis is (8, 0).

(c) The point that is symmetric to the point (–8, 0) with respect to the origin is (8, 0).



43. The graph of $y = |x - 2|$ is symmetric with respect to the line $x = 2$.

44. The graph of $y = -|x + 1|$ is symmetric with respect to the line $x = -1$.

45. $y = x^2 + 5$

Replace x with $–x$ to obtain $y = (-x)^2 + 5 = x^2 + 5$. The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Because y is a function of x, the graph cannot be symmetric with respect to the x-axis. Replace x with $–x$ and y with $–y$ to obtain

$-y = (-x)^2 + 2 \Rightarrow -y = x^2 + 2 \Rightarrow y = -x^2 - 2.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the y-axis only.

46. $y = 2x^4 - 3$

Replace x with $–x$ to obtain

$y = 2(-x)^4 - 3 = 2x^4 - 3$

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Because y is a function of x, the graph cannot be symmetric with respect to the x-axis. Replace x with $–x$ and y with $–y$ to obtain $-y = 2(-x)^4 - 3 \Rightarrow -y = 2x^4 - 3 \Rightarrow y = -2x^4 + 3$. The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the y-axis only.

47. $x^2 + y^2 = 12$

Replace x with $–x$ to obtain

$(-x)^2 + y^2 = 12 \Rightarrow x^2 + y^2 = 12$.

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $–y$ to obtain

$x^2 + (-y)^2 = 12 \Rightarrow x^2 + y^2 = 12$

The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Because the graph is symmetric with respect to the x-axis and y-axis, it is also symmetric with respect to the origin.

48. $y^2 - x^2 = 6$

Replace x with $–x$ to obtain

$y^2 - (-x)^2 = 6 \Rightarrow y^2 - x^2 = 6$

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $–y$ to obtain

$(-y)^2 - x^2 = 6 \Rightarrow y^2 - x^2 = 6$

The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Because the graph is symmetric with respect to the x-axis and y-axis, it is also symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the x-axis, the y-axis, and the origin.

49. $y = -4x^3 + x$

Replace x with $–x$ to obtain

$y = -4(-x)^3 + (-x) \Rightarrow y = -4(-x^3) - x \Rightarrow$
$y = 4x^3 - x.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $–y$ to obtain $-y = -4x^3 + x \Rightarrow y = 4x^3 - x.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Replace x with $–x$ and y with $–y$ to obtain

$-y = -4(-x)^3 + (-x) \Rightarrow -y = -4(-x^3) - x \Rightarrow$
$-y = 4x^3 - x \Rightarrow y = -4x^3 + x.$

The result is the same as the original equation, so the graph is symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the origin only.

50. $y = x^3 - x$
Replace x with $-x$ to obtain

$y = (-x)^3 - (-x) \Rightarrow y = -x^3 + x.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain $-y = x^3 - x \Rightarrow y = -x^3 + x.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain $-y = (-x)^3 - (-x) \Rightarrow -y = -x^3 + x \Rightarrow$ $y = x^3 - x.$ The result is the same as the original equation, so the graph is symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the origin only.

51. $y = x^2 - x + 8$
Replace x with $-x$ to obtain
$y = (-x)^2 - (-x) + 8 \Rightarrow y = x^2 + x + 8.$
The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Because y is a function of x, the graph cannot be symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain $-y = (-x)^2 - (-x) + 8 \Rightarrow$
$-y = x^2 + x + 8 \Rightarrow y = -x^2 - x - 8.$
The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph has none of the listed symmetries.

52. $y = x + 15$
Replace x with $-x$ to obtain
$y = (-x) + 15 \Rightarrow y = -x + 15.$
The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Because y is a function of x, the graph cannot be symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain $-y = (-x) + 15 \Rightarrow y = x - 15.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph has none of the listed symmetries.

53. $f(x) = -x^3 + 2x$

$f(-x) = -(-x)^3 + 2(-x)$
$= x^3 - 2x = -(-x^3 + 2x) = -f(x)$
The function is odd.

54. $f(x) = x^5 - 2x^3$

$f(-x) = (-x)^5 - 2(-x)^3$
$= -x^5 + 2x^3 = -(x^5 - 2x^3) = -f(x)$

The function is odd.

55. $f(x) = 0.5x^4 - 2x^2 + 6$

$f(-x) = 0.5(-x)^4 - 2(-x)^2 + 6$
$= 0.5x^4 - 2x^2 + 6 = f(x)$
The function is even.

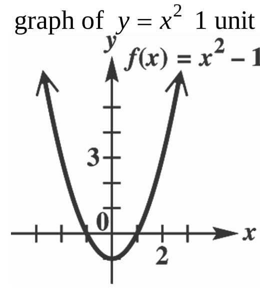
56. $f(x) = 0.75x^2 + |x| + 4$
$f(-x) = 0.75(-x)^2 + |-x| + 4$
$= 0.75x^2 + |x| + 4 = f(x)$
The function is even.

57. $f(x) = x^3 - x + 9$
$f(x) = (-x)^3 - (-x) + 9$
$= -x^3 + x + 9 = -(x^3 - x - 9) \neq -f(x)$

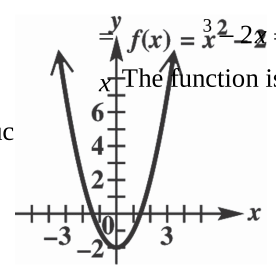
The function is neither.

58. $f(x) = x^4 - 5x + 8$
$f(-x) = (-x)^4 - 5(-x) + 8$
$= x^4 + 5x + 8 \neq f(x)$
The function is neither.

59. $f(x) = x^2 - 1$
This graph may be obtained by translating the graph of $y = x^2$ 1 unit downward.



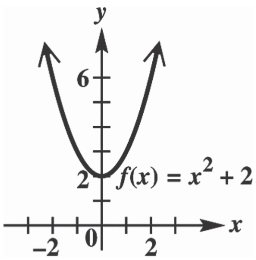
60. $f(x) = x^2 - 2$
This graph may be obtained by translating the



graph of $y = x^2$ 2 units downward.

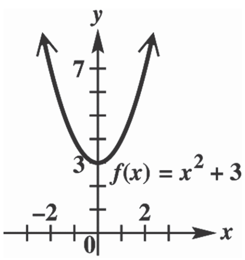
61. $f(x) = x^2 + 2$

This graph may be obtained by translating the graph of $y = x^2$ 2 units upward.



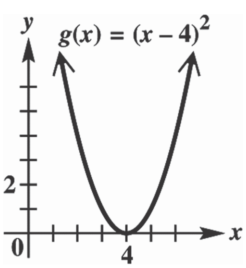
62. $f(x) = x^2 + 3$

This graph may be obtained by translating the graph of $y = x^2$ 3 units upward.



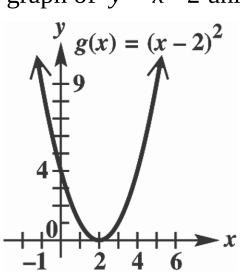
63. $g(x) = (x-4)^2$

This graph may be obtained by translating the graph of $y = x^2$ 4 units to the right.



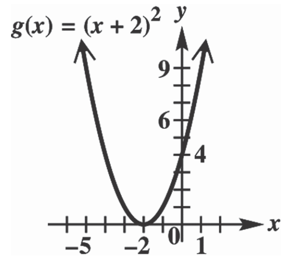
64. $g(x) = (x-2)^2$

This graph may be obtained by translating the graph of $y = x^2$ 2 units to the right.



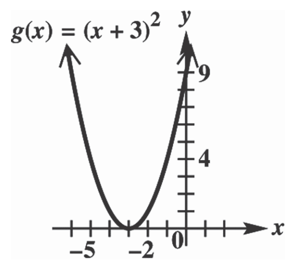
65. $g(x) = (x+2)^2$

This graph may be obtained by translating the graph of $y = x^2$ 2 units to the left.



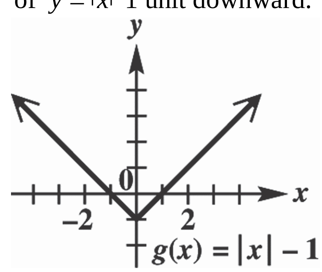
66. $g(x) = (x+3)^2$

This graph may be obtained by translating the graph of $y = x^2$ 3 units to the left.



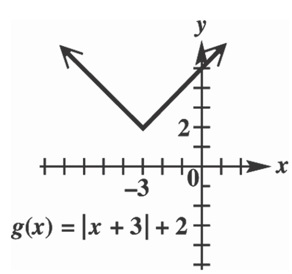
67. $g(x) = |x| - 1$

The graph is obtained by translating the graph of $y = |x|$ 1 unit downward.



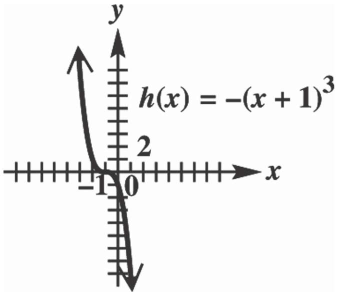
68. $g(x) = |x + 3 + 2$

This graph may be obtained by translating the graph of $y = |x|$ 3 units to the left and 2 units upward.



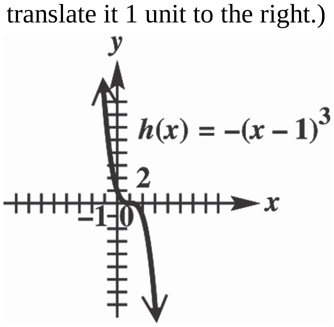
69. $h(x) = -(x+1)^3$

This graph may be obtained by translating the graph of $y = x^3$ 1 unit to the left. It is then reflected across the *x*-axis.



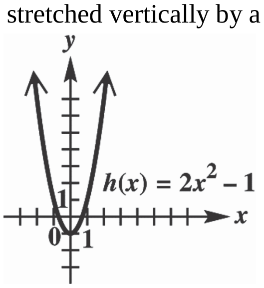
70. $h(x) = -(x-1)^3$

This graph can be obtained by translating the graph of $y = x^3$ 1 unit to the right. It is then reflected across the *x*-axis. (We may also reflect the graph about the *x*-axis first and then translate it 1 unit to the right.)



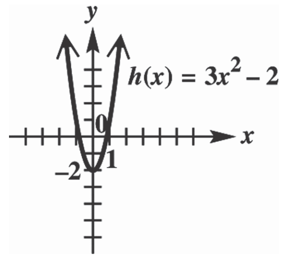
71. $h(x) = 2x^2 - 1$

This graph may be obtained by translating the graph of $y = x^2$ 1 unit down. It is then stretched vertically by a factor of 2.



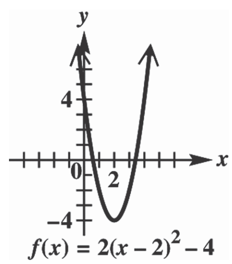
72. $h(x) = 3x^2 - 2$

This graph may be obtained by stretching the graph of $y = x^2$ vertically by a factor of 3, then shifting the resulting graph down 2 units.



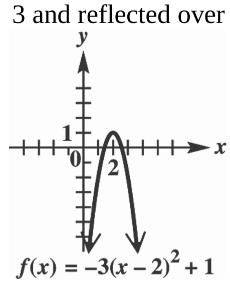
73. $f(x) = 2(x-2)^2 - 4$

This graph may be obtained by translating the graph of $y = x^2$ 2 units to the right and 4 units down. It is then stretched vertically by a factor of 2.



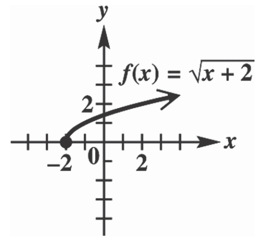
74. $f(x) = -3(x-2)^2 + 1$

This graph may be obtained by translating the graph of $y = x^2$ 2 units to the right and 1 unit up. It is then stretched vertically by a factor of 3 and reflected over the *x*-axis.



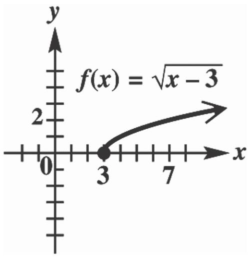
75. $f(x) = \sqrt{x+2}$

This graph may be obtained by translating the graph of $y = \sqrt{x}$ two units to the left.


y
$f(x) = \sqrt{x+2}$
2
−2
0
2
x

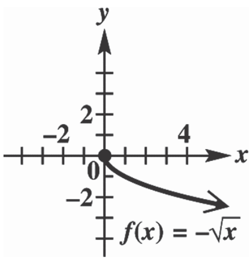
76. $f(x) = \sqrt{x - 3}$

This graph may be obtained by translating the graph of $y = \sqrt{x}$ three units to the right.



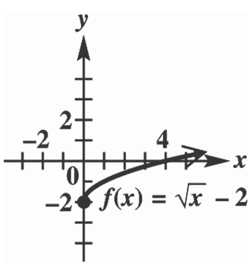
77. $f(x) = -\sqrt{x}$

This graph may be obtained by reflecting the graph of $y = \sqrt{x}$ across the x-axis.



78. $f(x) = \sqrt{x} - 2$

This graph may be obtained by translating the graph of $y = \sqrt{x}$ two units down.

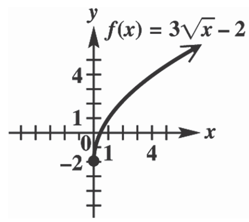


79. $f(x) = 2\sqrt{x} + 1$

This graph may be obtained by stretching the graph of $y = \sqrt{x}$ vertically by a factor of two and then translating the resulting graph one unit up.

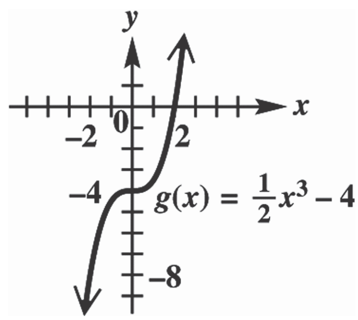
80. $f(x) = 3\sqrt{x} - 2$

This graph may be obtained by stretching the graph of $y = \sqrt{x}$ vertically by a factor of three and then translating the resulting graph two units down.



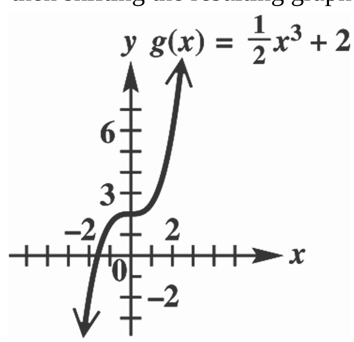
81. $g(x) = \frac{1}{2}x^3 - 4$

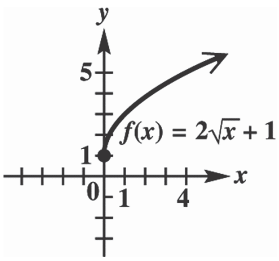
This graph may be obtained by stretching the graph of $y = x^3$ vertically by a factor of $\frac{1}{2}$, then shifting the resulting graph down four units.



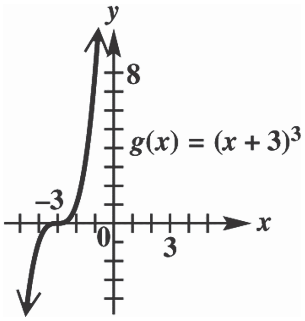
82. $g(x) = \frac{1}{2}x^3 + 2$

This graph may be obtained by stretching the graph of $y = x^3$ vertically by a factor of $\frac{1}{2}$, then shifting the resulting graph up two units.

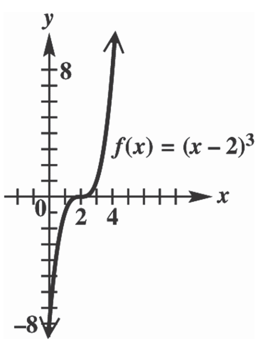



y
5
1
0
1
4
x
f(x) = 2√x + 1

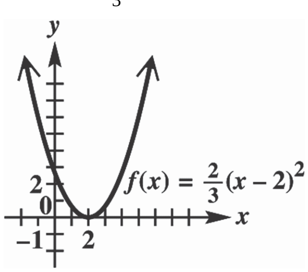
83. $g(x)=(x+3)^3$
This graph may be obtained by shifting the graph of $y=x^3$ three units left.



84. $f(x)=(x-2)^3$
This graph may be obtained by shifting the graph of $y=x^3$ two units right.

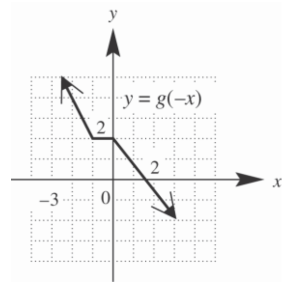


85. $f(x)=\frac{2}{3}(x-2)^2$
This graph may be obtained by translating the graph of $y=x^2$ two units to the right, then stretching the resulting graph vertically by a factor of $\frac{2}{3}$.

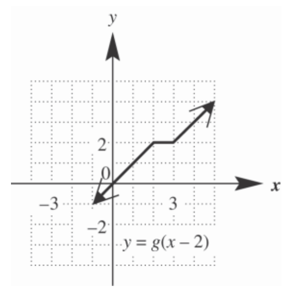


86. Because $g(x)=|-x|=|x|=f(x)$, the graphs are the same.

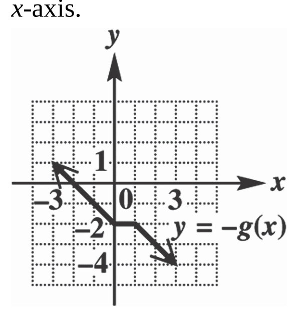
87. **(a)** $y=g(-x)$
The graph of $g(x)$ is reflected across the y-axis.



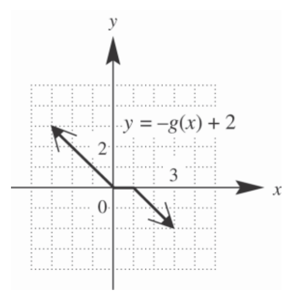
(b) $y=g(x-2)$
The graph of $g(x)$ is translated to the right 2 units.



(c) $y=-g(x)$
The graph of $g(x)$ is reflected across the x-axis.

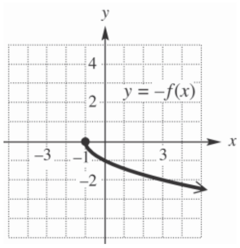


(d) $y=-g(x)+2$
The graph of $g(x)$ is reflected across the x-axis and translated 2 units up.



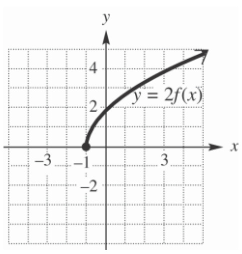
88. **(a)** $y = -f(x)$

The graph of $f(x)$ is reflected across the x-axis.



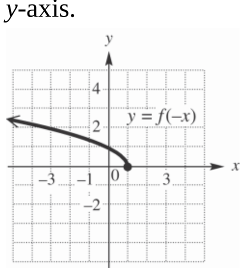
(b) $y = 2f(x)$

The graph of $f(x)$ is stretched vertically by a factor of 2.



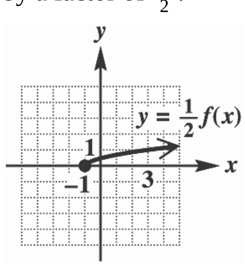
(c) $y = f(-x)$

The graph of $f(x)$ is reflected across the y-axis.



(d) $y = \frac{1}{2} f(x)$

The graph of $f(x)$ is compressed vertically by a factor of $\frac{1}{2}$.



90. It is the graph of $g(x) = \sqrt{x}$ translated 4 units to the left, reflected across the x-axis, and translated two units up. The equation is

$y = -\sqrt{x+4} + 2.$

91. It is the graph of $f(x) = \sqrt{x}$ translated one unit right and then three units down. The equation is $y = \sqrt{x-1} - 3.$

92. It is the graph of $f(x) = |x|$ translated 2 units to the right, shrunken vertically by a factor of $\frac{1}{2}$, and translated one unit down. The equation is $y = \frac{1}{2}|x-2| - 1.$

93. It is the graph of $g(x) = \sqrt{x}$ translated 4 units to the left, stretched vertically by a factor of 2, and translated four units down. The equation is $y = 2\sqrt{x+4} - 4.$

94. It is the graph of $f(x) = |x|$ reflected across the x-axis and then shifted two units down. The equation is $y = -|x| - 2$.

95. Because $f(3) = 6$, the point (3, 6) is on the graph. Because the graph is symmetric with respect to the origin, the point (–3, –6) is on the graph. Therefore, $f(-3) = -6$.

96. Because $f(3) = 6$, (3, 6) is a point on the graph. The graph is symmetric with respect to the y-axis, so (–3, 6) is on the graph. Therefore, $f(-3) = 6$.

97. Because $f(3) = 6$, the point (3, 6) is on the graph. The graph is symmetric with respect to the line $x = 6$ and the point (3, 6) is 3 units to the left of the line $x = 6$, so the image point of (3, 6), 3 units to the right of the line $x = 6$ is (9, 6). Therefore, $f(9) = 6$.

98. Because $f(3) = 6$ and $f(-x) = f(x)$, $f(-3) = f(3)$. Therefore, $f(-3) = 6$.

99. Because (3, 6) is on the graph, (–3, –6) must also be on the graph. Therefore, $f(-3) = -6$.

100. If f is an odd function, $f(-x) = -f(x)$. Because $f(3) = 6$ and $f(-x) = -f(x)$, $f(-3) = -f(3)$. Therefore, $f(-3) = -6$.

89. It is the graph of $f(x) = |x|$ translated 1 unit to the left, reflected across the *x*-axis, and translated 3 units up. The equation is $y = -|x+1| + 3.$

101. $f(x) = 2x + 5$

Translate the graph of $f(x)$ up 2 units to obtain the graph of

$t(x) = (2x+5)+2 = 2x+7.$

Now translate the graph of $t(x) = 2x + 7$ left 3 units to obtain the graph of

$g(x) = 2(x+3)+7 = 2x+6+7 = 2x+13.$

(Note that if the original graph is first translated to the left 3 units and then up 2 units, the final result will be the same.)

102. $f(x) = 3 - x$

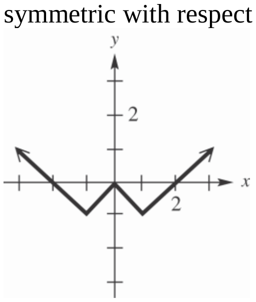
Translate the graph of $f(x)$ down 2 units to obtain the graph of $t(x) = (3-x)-2 = -x+1.$

Now translate the graph of $t(x) = -x+1$ right 3 units to obtain the graph of

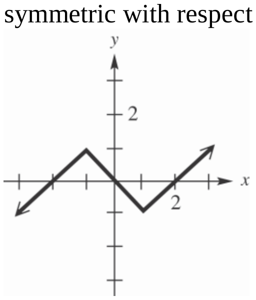
$g(x) = -(x-3)+1 = -x+3+1 = -x+4.$

(Note that if the original graph is first translated to the right 3 units and then down 2 units, the final result will be the same.)

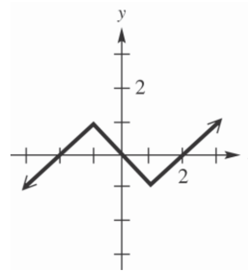
103. **(a)** Because $f(-x) = f(x)$, the graph is symmetric with respect to the y-axis.



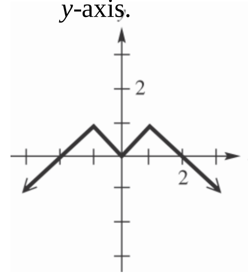
(b) Because $f(-x) = -f(x)$, the graph is symmetric with respect to the origin.



104. **(a)** $f(x)$ is odd. An odd function has a graph symmetric with respect to the origin. Reflect the left half of the graph in the origin.



(b) $f(x)$ is even. An even function has a graph symmetric with respect to the y-axis. Reflect the left half of the graph in the y-axis.



Chapter 2 Quiz
(Sections 2.5−2.7)

1. **(a)** First, find the slope: $m = \dfrac{9-5}{-1-(-3)} = 2$

Choose either point, say, (−3, 5), to find the equation of the line:

$y-5 = 2(x-(-3)) \Rightarrow y = 2(x+3)+5 \Rightarrow$

$y = 2x+11$.

(b) To find the x-intercept, let $y = 0$ and solve for x: $0 = 2x+11 \Rightarrow x = -\frac{11}{2}$. The x-intercept is $\left(-\frac{11}{2},\, 0\right)$.

2. Write $3x - 2y = 6$ in slope-intercept form to find its slope: $3x-2y=6 \Rightarrow y = \frac{3}{2}x-3.$

Then, the slope of the line perpendicular to this graph is $-\frac{2}{3}$. $y-4 = -\frac{2}{3}(x-(-6)) \Rightarrow$

$y = -\frac{2}{3}(x+6))+4 \Rightarrow y = -\frac{2}{3}x$

3. **(a)** $x = -8$ **(b)** $y = 5$

4. **(a)** Cubing function; domain: $(-\infty,\infty)$; range: $(-\infty,\infty)$; increasing over $(-\infty,\infty)$.

(b) Absolute value function; domain: $(-\infty,\infty)$; range: $[0,\infty)$; decreasing over

$(-\infty, 0)$;
increasin
g over $(0,$
$\infty)$

(c) Cube root function: domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$; increasing over $(-\infty, \infty)$.

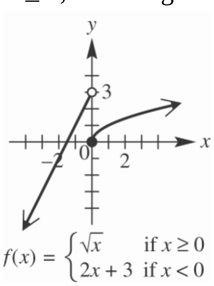
5. $f(x) = 0.40[\![x]\!] + 0.75$

$$f(5.5) = 0.40[\![5.5]\!] + 0.75 = 0.40(5) + 0.75 = 2.75$$

A 5.5-minute call costs $2.75.

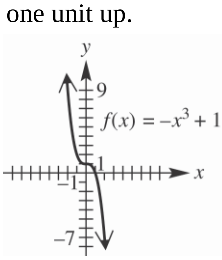
6. $f(x) = \begin{cases} \sqrt{x} & \text{if } x \ge 0 \\ 2x+3 & \text{if } x < 0 \end{cases}$

For values of $x < 0$, the graph is the line $y = 2x + 3$. Do not include the right endpoint (0, 3). Graph the line $y = \sqrt{x}$ for values of $x \ge 0$, including the left endpoint (0, 0).



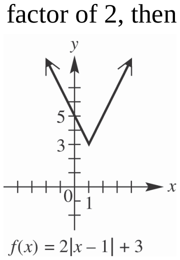
7. $f(x) = -x^3 + 1$

Reflect the graph of $f(x) = x^3$ across the x-axis, and then translate the resulting graph one unit up.



8. $f(x) = 2|x-1| + 3$

Shift the graph of $f(x) = |x|$ one unit right, stretch the resulting graph vertically by a factor of 2, then shift this graph three units up.



9. This is the graph of $g(x) = \sqrt{x}$, translated four units to the left, reflected across the x-axis, and then translated two units down. The equation is $y = -\sqrt{x+4} - 2$.

10. (a) $f(x) = x^2 - 7$

Replace x with $-x$ to obtain

$$f(-x) = (-x)^2 - 7 \Rightarrow$$
$$f(-x) = x^2 - 7 = f(x)$$

The result is the same as the original function, so the function is even.

(b) $f(x) = x^3 - x - 1$

Replace x with $-x$ to obtain

$$f(-x) = (-x)^3 - (-x) - 1 = -x^3 + x - 1 \ne f(x)$$

The result is not the same as the original equation, so the function is not even. Because $f(-x) \ne -f(x)$, the function is not odd. Therefore, the function is neither even nor odd.

(c) $f(x) = x^{101} - x^{99}$

Replace x with $-x$ to obtain

$$\begin{aligned} f(-x) &= (-x)^{101} - (-x)^{99} \\ &= -x^{101} - (-x^{99}) \\ &= -(x^{101} - x^{99}) \\ &= -f(x) \end{aligned}$$

Because $f(-x) = -f(x)$, the function is odd.

Section 2.8 Function Operations and Composition

In exercises 1–10, $f(x) = x + 1$ and $g(x) = x^2$.

1. $(f+g)(2) = f(2) + g(2) = (2+1) + 2^2 = 7$

2. $(f-g)(2) = f(2) - g(2) = (2+1) - 2^2 = -1$

3. $(fg)(2) = f(2) \cdot g(2) = (2+1) \cdot 2^2 = 12$

4. $\left(\frac{f}{g}\right)(2)=\frac{f(2)}{g(2)}=\frac{2+1}{2^2}=\frac{3}{4}$

5. $(f\circ g)(2)=f(g(2))=f\left(2^2\right)=2^2+1=5$

6. $(g \circ f)(2) = g(f(2)) = g(2+1) = (2+1)^2 = 9$

7. f is defined for all real numbers, so its domain is $(-\infty, \infty)$.

8. g is defined for all real numbers, so its domain is $(-\infty, \infty)$.

9. $f + g$ is defined for all real numbers, so its domain is $(-\infty, \infty)$.

10. $\frac{f}{g}$ is defined for all real numbers except those values that make $g(x) = 0$, so its domain is $(-\infty, 0) \cup (0, \infty)$.

In Exercises 11–18, $f(x) = x^2 + 3$ and $g(x) = -2x + 6$.

11. $(f+g)(3) = f(3) + g(3)$
$= \left[(3)^2 + 3\right] + \left[-2(3) + 6\right]$
$= 12 + 0 = 12$

12. $(f+g)(-5) = f(-5) + g(-5)$
$= [(-5)^2 + 3] + [-2(-5) + 6]$
$= 28 + 16 = 44$

13. $(f-g)(-1) = f(-1) - g(-1)$
$= [(-1)^2 + 3] - [-2(-1) + 6]$
$= 4 - 8 = -4$

14. $(f-g)(4) = f(4) - g(4)$
$= [(4)^2 + 3] - [-2(4) + 6]$
$= 19 - (-2) = 21$

15. $(fg)(4) = f(4) \cdot g(4)$
$= [4^2 + 3] \cdot [-2(4) + 6]$
$= 19 \cdot (-2) = -38$

16. $(fg)(-3) = f(-3) \cdot g(-3)$
$= [(-3)^2 + 3] \cdot [-2(-3) + 6]$
$= 12 \cdot 12 = 144$

17. $\left(\frac{f}{g}\right)(-1) = \frac{f(-1)}{g(-1)} = \frac{(-1)^2 + 3}{-2(-1) + 6} = \frac{4}{8} = \frac{1}{2}$

$\left(\frac{f}{}\right)$

$(f-g)(x) = f(x) - g(x)$
$= (3x+4) - (2x-5) = x + 9$

$(fg)(x) = f(x) \cdot g(x) = (3x+4)(2x-5)$
$= 6x^2 - 15x + 8x - 20$
$= 6x^2 - 7x - 20$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x+4}{2x-5}$

The domains of both f and g are the set of all real numbers, so the domains of $f + g$, $f - g$, and fg are all $(-\infty, \infty)$. The domain of $\frac{f}{g}$ is the set of all real numbers for which $g(x) \neq 0$. This is the set of all real numbers except $\frac{5}{2}$, which is written in interval notation as $\left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$.

20. $f(x) = 6 - 3x$, $g(x) = -4x + 1$

$(f+g)(x) = f(x) + g(x)$
$= (6-3x) + (-4x+1)$
$= -7x + 7$

$(f-g)(x) = f(x) - g(x)$
$= (6-3x) - (-4x+1) = x + 5$

$(fg)(x) = f(x) \cdot g(x) = (6-3x)(-4x+1)$
$= -24x + 6 + 12x^2 - 3x$
$= 12x^2 - 27x + 6$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{6-3x}{-4x+1}$

The domains of both f and g are the set of all real numbers, so the domains of $f + g$, $f - g$, and fg are all $(-\infty, \infty)$. The domain of $\frac{f}{g}$ is the set of all real numbers for which $g(x) \neq 0$. This is the set of all real numbers except $\frac{1}{4}$, which is written in interval notation as $\left(-\infty, \frac{1}{4}\right) \cup \left(\frac{1}{4}, \infty\right)$.

21. $f(x) = 2x^2 - 3x$, $g(x) = x^2 - x + 3$

$(f+g)(x) = f(x) + g(x)$
$= (2x^2 - 3x) + (x^2 - x + 3)$
$= 3x^2 - 4x + 3$

$\frac{f(5)}{+3}$ $\frac{(5)^2}{28}$

18. $\left(\frac{f}{g}\right)(5) = \frac{}{g(5)} = \frac{}{-2(5)+6} = \frac{}{-4} = -7$

19. $f(x) = 3x + 4,\ g(x) = 2x - 5$

$(f + g)(x) = f(x) + g(x)$
$= (3x + 4) + (2x - 5) = 5x - 1$

$(f - g)(x) = f(x) - g(x)$
$= (2x^2 - 3x) - (x^2 - x + 3)$
$= 2x^2 - 3x - x^2 + x - 3$
$= x^2 - 2x - 3$

(*continued on next page*)

(*continued*)

$$(fg)(x) = f(x)\cdot g(x)$$
$$= (2x^2 - 3x)(x^2 - x + 3)$$
$$= 2x^4 - 2x^3 + 6x^2 - 3x^3 + 3x^2 - 9x$$
$$= 2x^4 - 5x^3 + 9x^2 - 9x$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x^2 - 3x}{x^2 - x + 3}$$

The domains of both *f* and *g* are the set of all real numbers, so the domains of $f + g$, $f - g$, and fg are all $(-\infty, \infty)$. The domain of $\frac{f}{g}$ is the set of all real numbers for which $g(x) \neq 0$. If $x^2 - x + 3 = 0$, then by the quadratic formula $x = \frac{1 \pm i\sqrt{11}}{2}$. The equation has no real solutions. There are no real numbers which make the denominator zero. Thus, the domain of $\frac{f}{g}$ is also $(-\infty, \infty)$.

22. $f(x) = 4x^2 + 2x,\ g(x) = x^2 - 3x + 2$

$$(f+g)(x) = f(x) + g(x)$$
$$= (4x^2 + 2x) + (x^2 - 3x + 2)$$
$$= 5x^2 - x + 2$$
$$(f-g)(x) = f(x) - g(x)$$
$$= (4x^2 + 2x) - (x^2 - 3x + 2)$$
$$= 4x^2 + 2x - x^2 + 3x - 2$$
$$= 3x^2 + 5x - 2$$
$$(fg)(x) = f(x)\cdot g(x)$$
$$= (4x^2 + 2x)(x^2 - 3x + 2)$$
$$= 4x^4 - 12x^3 + 8x^2 + 2x^3 - 6x^2 + 4x$$
$$= 4x^4 - 10x^3 + 2x^2 + 4x$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{4x^2 + 2x}{x^2 - 3x + 2}$$

The domains of both *f* and *g* are the set of all real numbers, so the domains of $f + g$, $f - g$,

23. $f(x) = \sqrt{4x-1},\ g(x) = \frac{1}{x}$

$$(f+g)(x) = f(x) + g(x) = \sqrt{4x-1} + \frac{1}{x}$$
$$(f-g)(x) = f(x) - g(x) = \sqrt{4x-1} - \frac{1}{x}$$
$$(fg)(x) = f(x)\cdot g(x)$$
$$= \sqrt{4x-1}\left(\frac{1}{x}\right) = \frac{\sqrt{4x-1}}{x}$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{4x-1}}{\frac{1}{x}} = x\sqrt{4x-1}$$

Because $4x - 1 \geq 0 \Rightarrow 4x \geq 1 \Rightarrow x \geq \frac{1}{4}$, the domain of *f* is $\left[\frac{1}{4}, \infty\right)$. The domain of *g* is $(-\infty, 0) \cup (0, \infty)$. Considering the intersection of the domains of *f* and *g*, the domains of $f + g$, $f - g$, and fg are all $\left[\frac{1}{4}, \infty\right)$. Because $\frac{1}{x} \neq 0$ for any value of *x*, the domain of $\frac{f}{g}$ is also $\left[\frac{1}{4}, \infty\right)$.

24. $f(x) = \sqrt{5x-4},\ g(x) = -\frac{1}{x}$

$$(f+g)(x) = f(x) + g(x)$$
$$= \sqrt{5x-4} + \left(-\frac{1}{x}\right) = \sqrt{5x-4} - \frac{1}{x}$$
$$(f-g)(x) = f(x) - g(x)$$
$$= \sqrt{5x-4} - \left(-\frac{1}{x}\right) = \sqrt{5x-4} + \frac{1}{x}$$
$$(fg)(x) = f(x)\cdot g(x)$$
$$= \left(\sqrt{5x-4}\right)\left(-\frac{1}{x}\right) = -\frac{\sqrt{5x-4}}{x}$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{5x-4}}{-\frac{1}{x}} = -x\sqrt{5x-4}$$

and fg are all $(-\infty, \infty)$. The domain of $\frac{f}{g}$ is the set of all real numbers x such that $x^2 - 3x + 2 \neq 0$. Because $x^2 - 3x + 2 = (x-1)(x-2)$, the numbers which give this denominator a value of 0 are $x = 1$ and $x = 2$. Therefore, the domain of $\frac{f}{g}$ is the set of all real numbers except 1 and 2, which is written in interval notation as

$(-\infty, 1) \cup (1, 2) \cup (2, \infty)$.

$g \qquad g(x) \qquad \sqrt{\quad} - \frac{1}{x}$

Because $5x - 4 \geq 0 \Rightarrow 5x \geq 4 \Rightarrow x \geq \frac{4}{5}$, the domain of f is $\left[\frac{4}{5}, \infty\right)$. The domain of g is $(-\infty, 0) \cup (0, \infty)$. Considering the intersection of the domains of f and g, the domains of $f + g$, $f - g$, and fg are all $\left[\frac{4}{5}, \infty\right)$. $-\frac{1}{x} \neq 0$ for any value of x, so the domain of $\frac{f}{g}$ is also

$\left[\frac{4}{5}, \infty\right)$.

25. $M(2008) \approx 280$ and $F(2008) \approx 470$, thus
$$T(2008) = M(2008) + F(2008) = 280 + 470 = 750 \text{ (thousand).}$$

26. $M(2012) \approx 390$ and $F(2012) \approx 630$, thus
$$T(2012) = M(2012) + F(2012) = 390 + 630 = 1020 \text{ (thousand).}$$

27. Looking at the graphs of the functions, the slopes of the line segments for the period 2008−2012 are much steeper than the slopes of the corresponding line segments for the period 2004−2008. Thus, the number of associate's degrees increased more rapidly during the period 2008−2012.

28. If $2004 \le k \le 2012$, $T(k) = r$, and $F(k) = s$, then $M(k) = \underline{r - s}$.

29. $$(T - S)(2000) = T(2000) - S(2000) = 19 - 13 = 6$$
It represents the dollars in billions spent for general science in 2000.

30. $$(T - G)(2010) = T(2010) - G(2010) \approx 29 - 11 = 18$$
It represents the dollars in billions spent on space and other technologies in 2010.

31. Spending for space and other technologies spending decreased in the years 1995−2000 and 2010–2015.

32. Total spending increased the most during the years 2005−2010.

33. **(a)** $(f + g)(2) = f(2) + g(2) = 4 + (-2) = 2$

(b) $(f - g)(1) = f(1) - g(1) = 1 - (-3) = 4$

(c) $(fg)(0) = f(0) \cdot g(0) = 0(-4) = 0$

(d) $\left(\dfrac{f}{g}\right)(1) = \dfrac{f(1)}{g(1)} = \dfrac{1}{-3} = -\dfrac{1}{3}$

34. **(a)** $(f + g)(0) = f(0) + g(0) = 0 + 2 = 2$

(b) $(f - g)(-1) = f(-1) - g(-1) = -2 - 1 = -3$

(c) $(fg)(1) = f(1) \cdot g(1) = 2 \cdot 1 = 2$

(d) $\left(\dfrac{f}{g}\right)(2) = \dfrac{f(2)}{g(2)} = \dfrac{4}{-2} = -2$

35. **(a)** $(f + g)(-1) = f(-1) + g(-1) = 0 + 3 = 3$

(b) $(f - g)(-2) = f(-2) - g(-2) = -1 - 4 = -5$

(c) $(fg)(0) = f(0) \cdot g(0) = 1 \cdot 2 = 2$

(d) $\left(\dfrac{f}{g}\right)(2) = \dfrac{f(2)}{g(2)} = \dfrac{3}{0} = \text{undefined}$

36. **(a)** $(f + g)(1) = f(1) + g(1) = -3 + 1 = -2$

(b) $(f - g)(0) = f(0) - g(0) = -2 - 0 = -2$

(c) $(fg)(-1) = f(-1) \cdot g(-1) = -3(-1) = 3$

(d) $\left(\dfrac{f}{g}\right)(1) = \dfrac{f(1)}{g(1)} = \dfrac{-3}{1} = -3$

37. **(a)** $(f + g)(2) = f(2) + g(2) = 7 + (-2) = 5$

(b) $(f - g)(4) = f(4) - g(4) = 10 - 5 = 5$

(c) $(fg)(-2) = f(-2) \cdot g(-2) = 0 \cdot 6 = 0$

(d) $\left(\dfrac{f}{g}\right)(0) = \dfrac{f(0)}{g(0)} = \dfrac{5}{0} = \text{undefined}$

38. **(a)** $(f + g)(2) = f(2) + g(2) = 5 + 4 = 9$

(b) $(f - g)(4) = f(4) - g(4) = 0 - 0 = 0$

(c) $(fg)(-2) = f(-2) \cdot g(-2) = -4 \cdot 2 = -8$

(d) $\left(\dfrac{f}{g}\right)(0) = \dfrac{f(0)}{g(0)} = \dfrac{8}{-1} = -8$

39.

x	$f(x)$	$g(x)$	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
-2	0	6	$0+6=6$	$0-6=-6$	$0\cdot 6=0$	$\frac{0}{6}=0$
0	5	0	$5+0=5$	$5-0=5$	$5\cdot 0=0$	$\frac{5}{0}=$ undefined
2	7	-2	$7+(-2)=5$	$7-(-2)=9$	$7(-2)=-14$	$\frac{7}{-2}=-3.5$
4	10	5	$10+5=15$	$10-5=5$	$10\cdot 5=50$	$\frac{10}{5}=2$

40.

x	$f(x)$	$g(x)$	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
-2	-4	2	$-4+2=-2$	$-4-2=-6$	$-4\cdot 2=-8$	$\frac{-4}{2}=-2$
0	8	-1	$8+(-1)=7$	$8-(-1)=9$	$8(-1)=-8$	$\frac{8}{-1}=-8$
2	5	4	$5+4=9$	$5-4=1$	$5\cdot 4=20$	$\frac{5}{4}=1.25$
4	0	0	$0+0=0$	$0-0=0$	$0\cdot 0=0$	$\frac{0}{0}=$ undefined

41. Answers may vary. Sample answer: Both the slope formula and the difference quotient

represent the ratio of the vertical change to the horizontal change. The slope formula is stated for a line while the difference quotient is stated for a function *f*.

42. Answers may vary. Sample answer: As *h* approaches 0, the slope of the secant line *PQ* approaches the slope of the line tangent of the curve at *P*.

43. $f(x)=2-x$

(a) $f(x+h)=2-(x+h)=2-x-h$

(b) $f(x+h)-f(x)=(2-x-h)-(2-x)$
$=2-x-h-2+x=-h$

(c) $\dfrac{f(x+h)-f(x)}{h}=\dfrac{-h}{h}=-1$

44. $f(x)=1-x$

(a) $f(x+h)=1-(x+h)=1-x-h$

(b) $f(x+h)-f(x)=(1-x-h)-(1-x)$
$=1-x-h-1+x=-h$

(c) $\dfrac{f(x+h)-f(x)}{h}=\dfrac{-h}{h}=-1$

45. $f(x)=6x+2$

(a) $f(x+h)=6(x+h)+2=6x+6h+2$

(b) $f(x+h)-f(x)$
$=(6x+6h+2)-(6x+2)$
$=6x+6h+2-6x-2=6h$

(c) $\dfrac{f(x+h)-f(x)}{h}=\dfrac{6h}{h}=6$

46. $f(x)=4x+11$

(a) $f(x+h)=4(x+h)+11=4x+4h+11$

(b) $f(x+h)-f(x)$
$=(4x+4h+11)-(4x+11)$
$=4x+4h+11-4x-11=4h$

(c) $\dfrac{f(x+h)-f(x)}{h}=\dfrac{4h}{h}=4$

47. $f(x)=-2x+5$

(a) $f(x+h)=-2(x+h)+5$
$=-2x-2h+5$

(b) $f(x+h)-f(x)$
$=(-2x-2h+5)-(-2x+5)$
$=-2x-2h+5+2x-5=-2h$

(c) $\dfrac{f(x+h)-f(x)}{h}=\dfrac{-2h}{h}=-2$

48. $f(x) = -4x + 2$

(a) $f(x+h) = -4(x+h)+2$
$= -4x - 4h + 2$

(b) $f(x+h) - f(x)$
$= -4x - 4h + 2 - (-4x+2)$
$= -4x - 4h + 2 + 4x - 2$
$= -4h$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-4h}{h} = -4$

49. $f(x) = \dfrac{1}{x}$

(a) $f(x+h) = \dfrac{1}{x+h}$

(b) $f(x+h) - f(x)$
$= \dfrac{1}{x+h} - \dfrac{1}{x} = \dfrac{x-(x+h)}{x(x+h)}$
$= \dfrac{-h}{x(x+h)}$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{-h}{x(x+h)}}{h} = \dfrac{-h}{hx(x+h)}$
$= -\dfrac{1}{x(x+h)}$

50. $f(x) = \dfrac{1}{x^2}$

(a) $f(x+h) = \dfrac{1}{(x+h)^2}$

(b) $f(x+h) - f(x)$
$= \dfrac{1}{(x+h)^2} - \dfrac{1}{x^2} = \dfrac{x^2-(x+h)^2}{x^2(x+h)^2}$
$= \dfrac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2} = \dfrac{-2xh-h^2}{x^2(x+h)^2}$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{-2xh-h^2}{x^2(x+h)^2}}{h} = \dfrac{-2xh-h^2}{hx^2(x+h)^2}$
$= \dfrac{h(-2x-h)}{}$

51. $f(x) = x^2$

(a) $f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$

(b) $f(x+h) - f(x) = x^2 + 2xh + h^2 - x^2$
$= 2xh + h^2$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{2xh+h^2}{h}$
$= \dfrac{h(2x+h)}{h}$
$= 2x + h$

52. $f(x) = -x^2$

(a) $f(x+h) = -(x+h)^2$
$= -(x^2 + 2xh + h^2)$
$= -x^2 - 2xh - h^2$

(b) $f(x+h) - f(x) = -x^2 - 2xh - h^2 - (-x^2)$
$= -x^2 - 2xh - h^2 + x^2$
$= -2xh - h^2$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-2xh-h^2}{h}$
$= \dfrac{-h(2x+h)}{h}$
$= -2x - h$

53. $f(x) = 1 - x^2$

(a) $f(x+h) = 1-(x+h)^2$
$= 1-(x^2+2xh+h^2)$
$= 1 - x^2 - 2xh - h^2$

(b) $f(x+h) - f(x)$
$= (1 - x^2 - 2xh - h^2) - (1 - x^2)$
$= 1 - x^2 - 2xh - h^2 - 1 + x^2$
$= -2xh - h^2$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-2xh-h^2}{h}$
$= \dfrac{h(-2x-h)}{h}$
$= -2x - h$

54. $f(x) = 1 + 2x^2$

$$hx^2\left(x+h\right)^2$$

$$= \frac{-2x-h}{x^2\left(x+h\right)^2}$$

(a) $f(x+h) = 1+2(x+h)^2$

$= 1+2(x^2+2xh+h^2)$

$= 1+2x^2+4xh+2h^2$

(b) $f(x+h)-f(x)$
$= \left(1+2x^2+4xh+2h^2\right)-\left(1+2x^2\right)$
$= 1+2x^2+4xh+2h^2-1-2x^2$
$= 4xh+2h^2$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{4xh+2h^2}{h}$
$= \dfrac{h(4x+2h)}{h}$
$= 4x+2h$

55. $f(x)=x^2+3x+1$

(a) $f(x+h)=(x+h)^2+3(x+h)+1$
$= x^2+2xh+h^2+3x+3h+1$

(b) $f(x+h)-f(x)$
$= \left(x^2+2xh+h^2+3x+3h+1\right)$
$-\left(x^2+3x+1\right)$
$= x^2+2xh+h^2+3x+3h+1-x^2-3x-1$
$= 2xh+h^2+3h$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{2xh+h^2+3h}{h}$
$= \dfrac{h(2x+h+3)}{h}$
$= 2x+h+3$

56. $f(x)=x^2-4x+2$

(a) $f(x+h)=(x+h)^2-4(x+h)+2$
$= x^2+2xh+h^2-4x-4h+2$

(b) $f(x+h)-f(x)$
$= \left(x^2+2xh+h^2-4x-4h+2\right)$
$-\left(x^2-4x+2\right)$
$= x^2+2xh+h^2-4x-4h+2-x^2+4x-2$
$= 2xh+h^2-4h$

(c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{2xh+h^2-4h}{h}$
$= \dfrac{h(2x+h-4)}{h}$
$= 2x+h-4$

58. $g(x)=-x+3 \Rightarrow g(2)=-2+3=1$
$(f\circ g)(2)=f[g(2)]=f(1)$
$= 2(1)-3=2-3=-1$

59. $g(x)=-x+3 \Rightarrow g(-2)=-(-2)+3=5$
$(f\circ g)(-2)=f[g(-2) \;\; = f(5)$
$= 2(5)-3=10-3=7$

60. $f(x)=2x-3 \Rightarrow f(3)=2(3)-3=6-3=3$
$(g\circ f)(3)=g \;\; f(3) \;\; = g(3)=-3+3=0$

61. $f(x)=2x-3 \Rightarrow f(0)=2(0)-3=0-3=-3$
$(g\circ f)(0)=g \;\; f(0) \;\; = g(-3)$
$= -(-3)+3=3+3=6$

62. $f(x)=2x-3 \Rightarrow f(-2)=2(-2)-3=-7$
$(g\circ f)(-2)=g[f(-2)]=g(-7)$
$= -(-7)+3=7+3=10$

63. $f(x)=2x-3 \Rightarrow f(2)=2(2)-3=4-3=1$
$(f\circ f)(2)=f \;\; f(2) \;\; = f(1)=2(1)-3=-1$

64. $g(x)=-x+3 \Rightarrow g(-2)=-(-2)+3=5$
$(g\circ g)(-2)=g[g(-2) \;\; = g(5)=-5+3=-2$

65. $(f\circ g)(2)=f[g(2)]=f(3)=1$

66. $(f\circ g)(7)=f[g(7)]=f(6)=9$

67. $(g\circ f)(3)=g[f(3)]=g(1)=9$

68. $(g\circ f)(6)=g[f(6)]=g(9)=12$

69. $(f\circ f)(4)=f[f(4)]=f(3)=1$

70. $(g\circ g)(1)=g[g(1)]=g(9)=12$

71. $(f\circ g)(1)=f[g(1)]=f(9)$
However, $f(9)$ cannot be determined from the table given.

72. $(g\circ(f\circ g))(7)=g(f(g(7)))$
$= g(f(6))=g(9)=12$

73. **(a)** $(f\circ g)(x)=f(g(x))=f(5x+7)$

57. $g(x) = -x + 3 \Rightarrow g(4) = -4 + 3 = -1$

$(f \circ g)(4) = f[g(4)] = f(-1)$

$= 2(-1) - 3 = -2 - 3 = -5$

$= -6(5x + 7) + 9$

$= -30x - 42 + 9 = -30x - 33$

The domain and range of both *f* and *g* are $(-\infty, \infty)$, so the domain of $f \circ g$ is $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g(-6x+9)$

$$= 5(-6x+9)+7$$
$$= -30x+45+7 = -30x+52$$

The domain of $g \circ f$ is $(-\infty, \infty)$.

74. (a) $(f \circ g)(x) = f(g(x)) = f(3x-1)$
$$= 8(3x-1)+12$$
$$= 24x-8+12 = 24x+4$$

The domain and range of both *f* and *g* are $(-\infty, \infty)$, so the domain of $f \circ g$ is $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g(8x+12)$
$$= 3(8x+12)-1$$
$$= 24x+36-1 = 24x+35$$

The domain of $g \circ f$ is $(-\infty, \infty)$.

75. (a) $(f \circ g)(x) = f(g(x)) = f(x+3) = \sqrt{x+3}$

The domain and range of *g* are $(-\infty, \infty)$, however, the domain and range of *f* are $[0, \infty)$. So, $x+3 \ge 0 \Rightarrow x \ge -3$.

Therefore, the domain of $f \circ g$ is

$[-3, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \sqrt{x}+3$

The domain and range of *g* are $(-\infty, \infty)$, however, the domain and range of *f* are $[0, \infty)$.Therefore, the domain of $g \circ f$ is $[0, \infty)$.

76. (a) $(f \circ g)(x) = f(g(x)) = f(x-1) = \sqrt{x-1}$

The domain and range of *g* are $(-\infty, \infty)$, however, the domain and range of *f* are $[0, \infty)$. So, $x-1 \ge 0 \Rightarrow x \ge 1$. Therefore,

the domain of $f \circ g$ is $[1, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \sqrt{x}-1$

The domain and range of *g* are $(-\infty, \infty)$,

(b) $(g \circ f)(x) = g(f(x)) = g(x^3)$

$$= \left(x^3\right)^2 + 3\left(x^3\right) - 1$$

$$= x^6 + 3x^3 - 1$$

The domain and range of *f* and *g* are $(-\infty, \infty)$, so the domain of $g \circ f$ is $(-\infty, \infty)$.

78. (a) $(f \circ g)(x) = f(g(x)) = f(x^4+x^2-4)$
$$= x^4+x^2-4+2$$
$$= x^4+x^2-2$$

The domain of *f* and *g* is $(-\infty, \infty)$, while the range of *f* is $(-\infty, \infty)$ and the range of *g* is $[-4, \infty)$, so the domain of $f \circ g$ is $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g(x+2)$
$$= (x+2)^4 + (x+2)^2 - 4$$

The domain of *f* and *g* is $(-\infty, \infty)$, while the range of *f* is $(-\infty, \infty)$ and the range of *g* is $[-4, \infty)$, so the domain of $g \circ f$ is $(-\infty, \infty)$.

79. (a) $(f \circ g)(x) = f(g(x)) = f(3x) = \sqrt{3x-1}$

The domain and range of *g* are $(-\infty, \infty)$, however, the domain of *f* is $[1, \infty)$, while the range of *f* is $[0, \infty)$. So, $3x-1 \ge 0 \Rightarrow x \ge \frac{1}{3}$. Therefore, the domain of $f \circ g$ is $\left[\frac{1}{3}, \infty\right)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x-1}\right)$
$$= 3\sqrt{x-1}$$

The domain and range of *g* are $(-\infty, \infty)$, however, the range of *f* is $[0, \infty)$. So

$x-1 \ge 0 \Rightarrow x \ge 1$. Therefore, the domain however, the domain and range of *f* are $[0,\infty)$. Therefore, the domain of $g\circ f$ is

$[0,\infty)$.

77. (a) $(f\circ g)(x) = f(g(x)) = f(x^2+3x-1)$
$= (x^2+3x-1)^3$

The domain and range of *f* and *g* are $(-\infty,\infty)$, so the domain of $f\circ g$ is $(-\infty,\infty)$.

of $g\circ f$ is $[1,\infty)$.

80. (a) $(f\circ g)(x) = f(g(x)) = f(2x) = \sqrt{2x-2}$

The domain and range of *g* are $(-\infty,\infty)$, however, the domain of *f* is $[2,\infty)$. So, $2x-2\ge 0 \Rightarrow x\ge 1$. Therefore, the domain of $f\circ g$ is $[1,\infty)$.

(b) $(g\circ f)(x) = g(f(x)) = g\left(\sqrt{x-2}\right)$

$= 2\sqrt{x-2}$

The domain and range of g are $(-\infty,\infty)$, however, the range of f is $[0,\infty)$. So

$x-2\ge 0 \Rightarrow x\ge 2$. Therefore, the domain of $g\circ f$ is $[2,\infty)$.

81. (a) $(f\circ g)(x) = f(g(x)) = f(x+1) = \frac{2}{x+1}$

The domain and range of g are $(-\infty,\infty)$, however, the domain of f is $(-\infty,0)\cup(0,\infty)$. So, $x+1\ne 0 \Rightarrow x\ne -1$.

Therefore, the domain of $f\circ g$ is

$(-\infty,-1)\cup(-1,\infty)$.

(b) $(g\circ f)(x) = g(f(x)) = g\left(\frac{2}{x}\right) = \frac{2}{x}+1$

The domain and range of f is $(-\infty,0)\cup(0,\infty)$, however, the domain and range of g are $(-\infty,\infty)$. So $x\ne 0$.

Therefore, the domain of $g\circ f$ is $(-\infty,0)\cup(0,\infty)$.

82. (a) $(f\circ g)(x) = f(g(x)) = f(x+4) = \frac{4}{x+4}$

The domain and range of g are $(-\infty,\infty)$,

however, the domain of f is $(-\infty,0)\cup(0,\infty)$. So, $x+4\ne 0 \Rightarrow x\ne -4$. Therefore, the domain of $f\circ g$ is $(-\infty,-4)\cup(-4,\infty)$.

(b) $(g\circ f)(x) = g(f(x)) = g\left(\frac{4}{x}\right) = \frac{4}{x}+4$

The domain and range of f is $(-\infty,0)\cup(0,\infty)$, however, the domain and range of g are $(-\infty,\infty)$. So $x\ne 0$.

Therefore, the domain of $g\circ f$ is $(-\infty,0)\cup(0,\infty)$.

(b) $(g\circ f)(x) = g(f(x)) = g\left(\sqrt{x+2}\right) = -\frac{1}{\sqrt{x+2}}$

The domain of f is $[-2,\infty)$ and its range is $[0,\infty)$. The domain and range of g are $(-\infty,0)\cup(0,\infty)$. So $x+2>0 \Rightarrow x>-2$.

Therefore, the domain of $g\circ f$ is $(-2,\infty)$.

84. (a) $(f\circ g)(x) = f(g(x)) = f\left(-\frac{2}{x}\right) = \sqrt{-\frac{2}{x}+4}$

The domain and range of g are $(-\infty,0)\cup(0,\infty)$, however, the domain of f is $[-4,\infty)$. So, $-\frac{2}{x}+4\ge 0 \Rightarrow$

$x<0$ or $x\ge \frac{1}{2}$ (using test intervals).

Therefore, the domain of $f\circ g$ is

$(-\infty,0)\cup\left[\frac{1}{2},\infty\right)$.

(b) $(g\circ f)(x) = g(f(x)) = g\left(\sqrt{x+4}\right) = -\frac{2}{\sqrt{x+4}}$

The domain of f is $[-4,\infty)$ and its range is $[0,\infty)$. The domain and range of g are $(-\infty,0)\cup(0,\infty)$. So $x+4>0 \Rightarrow x>-4$.

Therefore, the domain of $g\circ f$ is $(-4,\infty)$.

85. (a) $(f\circ g)(x) = f(g(x)) = f\left(\frac{1}{x+5}\right) = \sqrt{\frac{1}{x+5}}$

The domain of g is $(-\infty,-5)\cup(-5,\infty)$, and the range of g is $(-\infty,0)\cup(0,\infty)$. The domain of f is $[0,\infty)$. Therefore, the domain of $f\circ g$ is $(-5,\infty)$.

(b) $(g\circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \frac{1}{\sqrt{x}+5}$

The domain and range of f is $[0,\infty)$. The domain of g is $(-\infty,-5)\cup(-5,\infty)$.

Therefore, the domain of $g\circ f$ is $[0,\infty)$.

86. (a) $\left(-\frac{3}{x}\right)$ $\sqrt{\frac{3}{x}}$

83. (a) $(f\circ g)(x) = f(g(x)) = f\left(-\frac{1}{x}\right) = -\frac{1}{x}+2$

The domain and range of g are $(-\infty,0)\cup(0,\infty)$, however, the domain of f is $[-2,\infty)$. So, $-\frac{1}{x}+2\ge 0 \Rightarrow$

$x<0$ or $x\ge\frac{1}{2}$ (using test intervals).

Therefore, the domain of $f\circ g$ is

$(-\infty,0)\cup\left[\frac{1}{2},\infty\right)$.

$(f\circ g)(x) = f(g(x)) = f\left(\frac{}{x+6}\right) = \sqrt{\frac{}{x+6}}$

The domain of g is $(-\infty,-6)\cup(-6,\infty)$, and the range of g is $(-\infty,0)\cup(0,\infty)$. The domain of f is $[0,\infty)$. Therefore, the domain of $f\circ g$ is $(-6,\infty)$.

(b) $(g\circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \frac{3}{\sqrt{x}+6}$

The domain and range of f is $[0,\infty)$. The domain of g is $(-\infty,-6)\cup(-6,\infty)$.

Therefore, the domain of $g\circ f$ is $[0,\infty)$.

87. **(a)** $(f\circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x-2} = \frac{x}{1-2x}$

The domain and range of g are $(-\infty,0)\cup(0,\infty)$. The domain of f is $(-\infty,-2)\cup(-2,\infty)$, and the range of f is $(-\infty,0)\cup(0,\infty)$. So, $\frac{x}{1-2x}<0 \Rightarrow x<0$ or $0<x<\frac{1}{2}$ or $x>\frac{1}{2}$ (using test intervals).

Thus, $x\neq 0$ and $x\neq\frac{1}{2}$. Therefore, the domain of $f\circ g$ is

$(-\infty,0)\cup\left(0,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$.

(b) $(g\circ f)(x) = g(f(x)) = g\left(\frac{1}{x-2}\right) = \frac{1}{1/(x-2)}$

$= x-2$

The domain and range of g are $(-\infty,0)\cup(0,\infty)$. The domain of f is $(-\infty,2)\cup(2,\infty)$, and the range of f is $(-\infty,0)\cup(0,\infty)$. Therefore, the domain of $g\circ f$ is $(-\infty,2)\cup(2,\infty)$.

88. **(a)** $(f\circ g)(x) = f(g(x)) = f\left(-\frac{1}{x}\right) = \frac{1}{-1/x+4}$

$= \frac{x}{-1+4x}$

The domain and range of g are $(-\infty,0)\cup(0,\infty)$. The domain of f is $(-\infty,-4)\cup(-4,\infty)$, and the range of f is $(-\infty,0)\cup(0,\infty)$. So, $\frac{x}{-1+4x}<0 \Rightarrow x<0$ or $0<x<\frac{1}{4}$ or $-1+4x<0 \Rightarrow x>\frac{1}{4}$ (using test intervals). Thus, $x\neq 0$ and $x\neq\frac{1}{4}$. Therefore, the domain of $f\circ g$ is

$(-\infty,0)\cup\left(0,\frac{1}{4}\right)\cup\left(\frac{1}{4},\infty\right)$.

(b) $(g\circ f)(x) = g(f(x)) = g\left(\frac{1}{x+4}\right) = -\frac{1}{1/(x+4)}$

$= -x-4$

The domain and range of g are

90. $f(x)$ is odd, so $f(-1) = -f(1) = -(-2) = 2.$

Because $g(x)$ is even, $g(1) = g(-1) = 2$ and $g(2) = g(-2) = 0.$ $(f\circ g)(-1) = 1,$ so $f[g(-1)] = 1$ and $f(2) = 1.$ $f(x)$ is odd, so $f(-2) = -f(2) = -1.$ Thus, $(f\circ g)(-2) = f[g(-2)] = f(0) = 0$ and $(f\circ g)(1) = f[g(1)] = f(2) = 1$ and $(f\circ g)(2) = f[g(2)] = f(0) = 0.$

x	-2	-1	0	1	2
$f(x)$	**-1**	**2**	0	-2	**1**
$g(x)$	0	2	1	**2**	**0**
$(f\circ g)(x)$	**0**	1	-2	**1**	**0**

91. Answers will vary. In general, composition of functions is not commutative. Sample answer:

$(f\circ g)(x) = f(2x-3) = 3(2x-3)-2$
$= 6x-9-2 = 6x-11$

$(g\circ f)(x) = g(3x-2) = 2(3x-2)-3$
$= 6x-4-3 = 6x-7$

Thus, $(f\circ g)(x)$ is not equivalent to $(g\circ f)(x)$.

92. $(f\circ g)(x) = f[g(x)] = f\left(\sqrt[3]{x-7}\right)$
$= \left(\sqrt[3]{x-7}\right)^3 + 7$
$= (x-7)+7 = x$

$(g\circ f)(x) = g(f(x)) = g(x^3+7)$
$= \sqrt[3]{(x^3+7)-7} = \sqrt[3]{x^3} = x$

93. $(f\circ g)(x) = f[g(x)] = 4\left[\frac{1}{4}(x-2)\right]+2$

$(-\infty,0)\cup(0,\infty)$. The domain of f is $(-\infty,-4)\cup(-4,\infty)$, and the range of f is

$=\left(4\cdot\frac{1}{4}\right)(x-2)+2$

$(-\infty,0)\cup(0,\infty)$. Therefore, the domain of

$g\circ f$ is $(-\infty,-4)\cup(-4,\infty)$.

89. $g\left[f(2)\right]=g(1)=2$ and $g\left[f(3)\right]=g(2)=5$
Since $g\left[f(1)\right]=7$ and $f(1)=3$, $g(3)=7$.

x	$f(x)$	$g(x)$	$g\left[f(x)\right]$
1	3	2	7
2	1	5	**2**
3	2	**7**	**5**

$$=(x-2)+2=x-2+2=x$$

$$(g\circ f)(x)=g\left[f(x)\right]=\frac{1}{4}\left[(4x+2)-2\right]$$
$$=\frac{1}{4}(4x+2-2)=\frac{1}{4}(4x)=x$$

94. $(f\circ g)(x)=f\left[g(x)\right]=-3\left(-\frac{1}{3}x\right)$

$$=\left[-3\left(-\frac{1}{3}\right)\right]x=x$$

$$(g\circ f)(x)=g\left[f(x)\right]=-\frac{1}{3}(-3x)$$

$$=\left[-\frac{1}{3}(-3)\right]x=x$$

95. $(f\circ g)(x) = f\left[g(x)\right] = \sqrt[3]{5\left(\frac{1}{5}x^3 - \frac{4}{5}\right)+4}$
$= \sqrt[3]{x^3 - 4 + 4} = \sqrt[3]{x^3} = x$

$(g\circ f)(x) = g\left[f(x)\right] = \frac{1}{5}\left(\sqrt[3]{5x+4}\right)^3 - \frac{4}{5}$

$= \frac{1}{5}(5x+4) - \frac{4}{5} = \frac{5x}{5} + \frac{4}{5} - \frac{4}{5}$
$= \frac{5x}{5} = x$

96. $(f\circ g)(x) = f\left[g(x)\right] = \sqrt[3]{\left(x^3-1\right)+1}$

$= \sqrt[3]{x^3 - 1 + 1} = \sqrt[3]{x^3} = x$

$(g\circ f)(x) = g\left[f(x)\right] = \left(\sqrt[3]{x+1}\right)^3 - 1$

$= x + 1 - 1 = x$

In Exercises 97−102, we give only one of many possible answers.

97. $h(x) = (6x-2)^2$

Let $g(x) = 6x - 2$ and $f(x) = x^2$.
$(f\circ g)(x) = f(6x-2) = (6x-2)^2 = h(x)$

98. $h(x) = (11x^2 + 12x)^2$

Let $g(x) = 11x^2 + 12x$ and $f(x) = x^2$.

$(f\circ g)(x) = f(11x^2 + 12x)$
$= (11x^2 + 12x)^2 = h(x)$

99. $h(x) = \sqrt{x^2 - 1}$

Let $g(x) = x^2 - 1$ and $f(x) = \sqrt{x}$.

$(f\circ g)(x) = f(x^2 - 1) = \sqrt{x^2 - 1} = h(x)$.

100. $h(x) = (2x-3)^3$

Let $g(x) = 2x - 3$ and $f(x) = x^3$.
$(f\circ g)(x) = f(2x-3) = (2x-3)^3 = h(x)$

101. $h(x) = \sqrt{6x} + 12$

104. $f(x) = 3x,\ g(x) = 1760x$

$(f\circ g)(x) = f\left(g(x)\right) = f(1760x)$

$= 3(1760x) = 5280x$
$(f\circ g)(x)$ compute the number of feet in x miles.

105. $\square(x) = \frac{\sqrt{3}}{4}x^2$

(a) $\square(2x) = \frac{\sqrt{3}}{4}(2x)^2 = \frac{\sqrt{3}}{4}(4x^2) = \sqrt{3}x^2$

(b) $\square(16) = A(2\cdot 8) = \sqrt{3}(8)^2$
$= 64\sqrt{3}$ square units

106. **(a)** $x = 4s \Rightarrow \frac{x}{4} = s \Rightarrow s = \frac{x}{4}$

(b) $y = s^2 = \left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$

(c) $y = \frac{6^2}{16} = \frac{36}{16} = 2.25$ square units

107. **(a)** $r(t) = 4t$ and $\square(r) = \pi r^2$
$(\square\circ r)(t) = \square[r(t)]$

$= \square(4t) = \pi(4t)^2 = 16\pi t^2$

(b) $(\square\circ r)(t)$ defines the area of the leak in terms of the time t, in minutes.

(c) $\square(3) = 16\pi(3)^2 = 144\pi\ \text{ft}^2$

108. **(a)** $(\square\circ r)(t) = \square[r(t)]$
$= \square(2t) = \pi(2t)^2 = 4\pi t^2$

(b) It defines the area of the circular layer in terms of the time t, in hours.

(c) $(\square\circ r)(4) = 4\pi(4)^2 = 64\pi\ \text{mi}^2$

Let $g(x) = 6x$ and $f(x) = \sqrt{x} + 12$.

$(f \circ g)(x) = f(6x) = \sqrt{6x} + 12 = h(x)$

102. $h(x) = \sqrt[3]{2x+3} - 4$

Let $g(x) = 2x + 3$ and $f(x) = \sqrt[3]{x} - 4$.

$(f \circ g)(x) = f(2x+3) = \sqrt[3]{2x+3} - 4 = h(x)$

103. $f(x) = 12x$, $g(x) = 5280x$

$$\begin{aligned}(f \circ g)(x) &= f[g(x)] = f(5280x)\\ &= 12(5280x) = 63{,}360x\end{aligned}$$

The function $f \circ g$ computes the number of inches in x miles.

109. Let x = the number of people less than 100 people that attend.

(a) x people fewer than 100 attend, so $100 - x$ people do attend $N(x) = 100 - x$

(b) The cost per person starts at \$20 and

increases by \$5 for each of the x people that do not attend. The total increase is \$5x, and the cost per person increases to \$20 + \$5x. Thus, $G(x) = 20 + 5x$.

(c) $C(x) = N(x) \cdot G(x) = (100 - x)(20 + 5x)$

(d) If 80 people attend, $x = 100 - 80 = 20.$

$$C(20) = (100-20)[20+5(20)]$$
$$= (80)(20+100)$$
$$= (80)(120) = \$9600$$

110. **(a)** $y_1 = 0.02x$

(b) $y_2 = 0.015(x+500)$

(c) $y_1 + y_2$ represents the total annual interest.

(d) $(y_1 + y_2)(250)$

$$= y_1(250) + y_2(250)$$
$$= 0.02(250) + 0.015(250+500)$$
$$= 5 + 0.015(750) = 15 + 11.25$$
$$= \$16.25$$

111. **(a)** $g(x) = \frac{1}{2}x$

(b) $f(x) = x + 1$

(c) $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{2}x\right) = \frac{1}{2}x + 1$

(d) $(f \circ g)(60) = \frac{1}{2}(60) + 1 = \$31$

112. If the area of a square is x^2 square inches, each side must have a length of x inches. If 3 inches is added to one dimension and 1 inch is subtracted from the other, the new dimensions will be $x + 3$ and $x - 1$. Thus, the area of the resulting rectangle is □$(x) = (x + 3)(x - 1)$.

Chapter 2 Review Exercises

1. $P(3, -1), Q(-4, 5)$

$$d(P, Q) = \sqrt{(-4-3)^2 + [5-(-1)]^2}$$
$$= \sqrt{(-7)^2 + 6^2} = \sqrt{49+36} = \sqrt{85}$$

Midpoint:

$$\left(\frac{3+(-4)}{2}, \frac{-1+5}{2}\right) = \left(\frac{-1}{2}, \frac{4}{2}\right) = \left(-\frac{1}{2}, 2\right)$$

3. $A(-6, 3), B(-6, 8)$

$$d(A, B) = \sqrt{[-6-(-6)]^2 + (8-3)^2}$$
$$= \sqrt{0+5^2} = \sqrt{25} = 5$$

Midpoint:

$$\left(\frac{-6+(-6)}{2}, \frac{3+8}{2}\right) = \left(\frac{-12}{2}, \frac{11}{2}\right) = \left(-6, \frac{11}{2}\right)$$

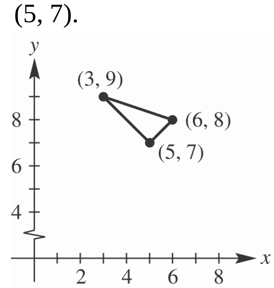
4. Label the points $A(5, 7)$, $B(3, 9)$, and $C(6, 8)$.

$$d(A, B) = \sqrt{(3-5)^2 + (9-7)^2}$$
$$= \sqrt{(-2)^2 + 2^2} = \sqrt{4+4} = \sqrt{8}$$

$$d(A, C) = \sqrt{(6-5)^2 + (8-7)^2}$$
$$= \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

$$d(B, C) = \sqrt{(6-3)^2 + (8-9)^2}$$
$$= \sqrt{3^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Because $\left(\sqrt{8}\right)^2 + \left(\sqrt{2}\right)^2 = \left(\sqrt{10}\right)^2$, triangle *ABC* is a right triangle with right angle at (5, 7).



5. Let *B* have coordinates (x, y). Using the midpoint formula, we have

$$(8, 2) = \left(\frac{-6+x}{2}, \frac{10+y}{2}\right) \Rightarrow$$

$$\frac{-6+x}{2} = 8 \qquad \frac{10+y}{2} = 2$$
$$-6 + x = 16 \qquad 10 + y = 4$$
$$x = 22 \qquad y = -6$$

The coordinates of B are $(22, -6)$.

2. $M(-8, 2)$, $N(3, -7)$

$$d(M, N) = \sqrt{[3-(-8)]^2 + (-7-2)^2} = \sqrt{11^2 + (-9)^2} = \sqrt{121+81} = \sqrt{202}$$

Midpoint: $\left(\frac{-8+3}{2}, \frac{2+(-7)}{2}\right) = \left(-\frac{5}{2}, -\frac{5}{2}\right)$

6. $P(-2, -5)$, $Q(1, 7)$, $R(3, 15)$

$$d(P, Q) = \sqrt{(1-(-2))^2 + (7-(-5))^2} = \sqrt{(3)^2 + (12)^2} = \sqrt{9+144} = \sqrt{153} = 3\sqrt{17}$$

$$d(Q, R) = \sqrt{(3-1)^2 + (15-7)^2} = \sqrt{2^2 + 8^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$$

$$d(P, R) = \sqrt{(3-(-2))^2 + (15-(-5))^2} = \sqrt{(5)^2 + (20)^2} = \sqrt{25+400} = 5\sqrt{17}$$

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(*continued*)

$d(P, Q) + d(Q, R) = 3\sqrt{17} + 2\sqrt{17}$
$= 5\sqrt{17} = d(P, R),$ so these three points are collinear.

7. Center (–2, 3), radius 15

$$(x-h)^2 + (y-k)^2 = r^2$$
$$[x-(-2)]^2 + (y-3)^2 = 15^2$$
$$(x+2)^2 + (y-3)^2 = 225$$

8. Center $(\sqrt{5}, -\sqrt{7})$, radius $\sqrt{3}$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$\left(x-\sqrt{5}\right)^2 + \left[y-\left(-\sqrt{7}\right)\right]^2 = \left(\sqrt{3}\right)^2$$
$$\left(x-\sqrt{5}\right)^2 + \left(y+\sqrt{7}\right)^2 = 3$$

9. Center (–8, 1), passing through (0, 16)
The radius is the distance from the center to any point on the circle. The distance between (–8, 1) and (0, 16) is

$$r = \sqrt{(0-(-8))^2 + (16-1)^2} = \sqrt{8^2 + 15^2}$$
$$= \sqrt{64 + 225} = \sqrt{289} = 17.$$

The equation of the circle is

$$[x-(-8)]^2 + (y-1)^2 = 17^2$$
$$(x+8)^2 + (y-1)^2 = 289$$

10. Center (3, –6), tangent to the x-axis

The point (3, –6) is 6 units directly below the x-axis. Any segment joining a circle's center to a point on the circle must be a radius, so in this case the length of the radius is 6 units.

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-3)^2 + [y-(-6)]^2 = 6^2$$
$$(x-3)^2 + (y+6)^2 = 36$$

14. The center of the circle is (5, 6). Use the distance formula to find the radius:

$$r^2 = (4-5)^2 + (9-6)^2 = 1 + 9 = 10$$

The equation is $(x-5)^2 + (y-6)^2 = 10$.

15. $x^2 - 4x + y^2 + 6y + 12 = 0$
Complete the square on x and y to put the equation in center-radius form.

$$\left(x^2 - 4x\right) + \left(y^2 + 6y\right) = -12$$
$$\left(x^2 - 4x + 4\right) + \left(y^2 + 6y + 9\right) = -12 + 4 + 9$$
$$\left(x-2\right)^2 + \left(y+3\right)^2 = 1$$

The circle has center (2, –3) and radius 1.

16. $x^2 - 6x + y^2 - 10y + 30 = 0$

Complete the square on x and y to put the equation in center-radius form.

$$(x^2 - 6x + 9) + (y^2 - 10y + 25) = -30 + 9 + 25$$
$$(x-3)^2 + (y-5)^2 = 4$$

The circle has center (3, 5) and radius 2.

17.

$$2x^2 + 14x + 2y^2 + 6y + 2 = 0$$
$$x^2 + 7x + y^2 + 3y + 1 = 0$$
$$\left(x^2 + 7x\right) + \left(y^2 + 3y\right) = -1$$
$$\left(x^2 + 7x + \tfrac{49}{4}\right) + \left(y^2 + 3y + \tfrac{9}{4}\right) = -1 + \tfrac{49}{4} + \tfrac{9}{4}$$
$$\left(x + \tfrac{7}{2}\right)^2 + \left(y + \tfrac{3}{2}\right)^2 = -\tfrac{4}{4} + \tfrac{49}{4} + \tfrac{9}{4}$$
$$\left(x + \tfrac{7}{2}\right)^2 + \left(y + \tfrac{3}{2}\right)^2 = \tfrac{54}{4}$$

The circle has center $\left(-\frac{7}{2}, -\frac{3}{2}\right)$ and radius

$$\sqrt{\tfrac{54}{4}} = \tfrac{\sqrt{54}}{\sqrt{4}} = \tfrac{\sqrt{9\cdot 6}}{\sqrt{4}} = \tfrac{3\sqrt{6}}{2}.$$

18.

$$3x^2 + 33x + 3y^2 - 15y = 0$$

11. The center of the circle is (0, 0). Use the distance formula to find the radius:

$$r^2 = (3-0)^2 + (5-0)^2 = 9 + 25 = 34$$

The equation is $x^2 + y^2 = 34$.

12. The center of the circle is (0, 0). Use the distance formula to find the radius:

$$r^2 = (-2-0)^2 + (3-0)^2 = 4 + 9 = 13$$

The equation is $x^2 + y^2 = 13$.

13. The center of the circle is (0, 3). Use the distance formula to find the radius:

$$r^2 = (-2-0)^2 + (6-3)^2 = 4 + 9 = 13$$

The equation is $x^2 + (y-3)^2 = 13$.

$$x^2 + 11x + y^2 - 5y = 0$$
$$\left(x^2 + 11x\right) + \left(y^2 - 5y\right) = 0$$
$$\left(x^2 + 11x + \tfrac{121}{4}\right) + \left(y^2 - 5y + \tfrac{25}{4}\right) = 0 + \tfrac{121}{4} + \tfrac{25}{4}$$
$$\left(x + \tfrac{11}{2}\right)^2 + \left(y - \tfrac{5}{2}\right)^2 = \tfrac{146}{4}$$

The circle has center $\left(-\frac{11}{2}, \frac{5}{2}\right)$ and radius $\frac{\sqrt{146}}{2}$.

19. This is not the graph of a function because a vertical line can intersect it in two points.
domain: [−6, 6]; range: [−6, 6]

20. This is not the graph of a function because a vertical line can intersect it in two points.
domain: $(-\infty, \infty)$; range: $[0, \infty)$

21. This is not the graph of a function because a vertical line can intersect it in two points.
domain: $(-\infty,\infty)$; range: $(-\infty, -1]\cup[1, \infty)$

22. This is the graph of a function. No vertical line will intersect the graph in more than one point.
domain: $(-\infty,\infty)$; range: $[0,\infty)$

23. This is not the graph of a function because a vertical line can intersect it in two points.
domain: $[0,\infty)$; range: $(-\infty,\infty)$

24. This is the graph of a function. No vertical line will intersect the graph in more than one point.
domain: $(-\infty,\infty)$; range: $(-\infty,\infty)$

25. $y=6-x^2$
Each value of *x* corresponds to exactly one value of *y*, so this equation defines a function.

26. The equation $x=\frac{1}{3}y^2$ does not define *y* as a function of *x*. For some values of *x*, there will be more than one value of *y*. For example, ordered pairs (3, 3) and (3, –3) satisfy the relation. Thus, the relation would not be a function.

27. The equation $y=\pm\sqrt{x-2}$ does not define *y* as a function of *x*. For some values of *x*, there will be more than one value of *y*. For example, ordered pairs (3, 1) and (3, −1) satisfy the relation.

28. The equation $y=-\frac{4}{x}$ defines *y* as a function of *x* because for every *x* in the domain, which is $(-\infty, 0)\cup(0, \infty)$, there will be exactly one value of *y*.

29. In the function $f(x)=-4+|x|$, we may use any real number for *x*. The domain is $(-\infty,\infty)$.

30. $f(x)=\dfrac{8+x}{8-x}$
x can be any real number except 8 because this will give a denominator of zero. Thus, the domain is $(-\infty, 8)\cup(8, \infty)$.

31. $f(x)=\sqrt{6-3x}$

32. **(a)** As *x* is getting larger on the interval $(2,\infty)$, the value of *y* is increasing.

(b) As *x* is getting larger on the interval $(-\infty,-2)$, the value of *y* is decreasing.

(c) *f*(*x*) is constant on (−2, 2).

In exercises 33–36, $f(x)=-2x^2+3x-6$.

33. $f(3)=-2\cdot 3^2+3\cdot 3-6$
$=-2\cdot 9+3\cdot 3-6$
$=-18+9-6=-15$

34. $f(-0.5)=-2(-0.5)^2+3(-0.5)-6$
$=-2(0.25)+3(-0.5)-6$
$=-0.5-1.5-6=-8$

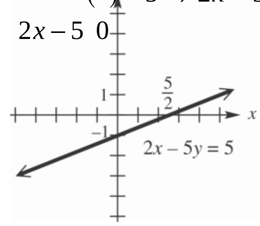
35. $f(0)=-2(0)^2+3(0)-6=-6$

36. $f(k)=-2k^2+3k-6$

37. $2x-5y=5\Rightarrow -5y=-2x+5\Rightarrow y=\frac{2}{5}x-1$

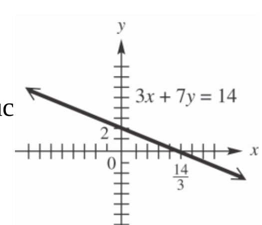
The graph is the line with slope $\frac{5}{2}$ and *y*-intercept (0, –)1. It may also be graphed using intercepts. To do this, locate the *x*-intercept: $y=0$

$2x-5(0)=5\Rightarrow 2x=5\Rightarrow x=\frac{5}{2}$



38. $3x+7y=14\Rightarrow 7y=-3x+14\Rightarrow y=-\frac{3}{7}x+2$

The graph is the line with slope of $-\frac{3}{7}$ and *y*-intercept (0, 2). It may also be graphed using intercepts. To do this, locate the *x*-intercept by setting *y* = 0:
$3x+7(0)=14\Rightarrow 3x=14\Rightarrow x=\frac{14}{3}$



In the function $y = \sqrt{6-3x}$, we must have $6-3x \ge 0$.

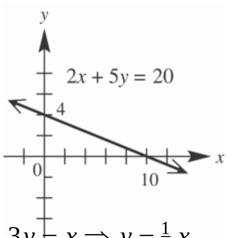
$6-3x \ge 0 \Rightarrow 6 \ge 3x \Rightarrow 2 \ge x \Rightarrow x \le 2$

Thus, the domain is $(-\infty, 2]$.

39. $2x+5y=20 \Rightarrow 5y=-2x+20 \Rightarrow y=-\frac{2}{5}x+4$

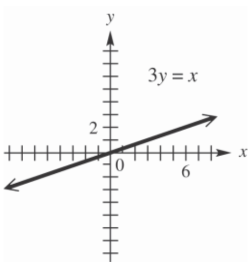
The graph is the line with slope of $-\frac{2}{5}$ and y-intercept (0, 4). It may also be graphed using intercepts. To do this, locate the x-intercept:
x-intercept: $y=0$

$2x+5(0)=20 \Rightarrow 2x=20 \Rightarrow x=10$

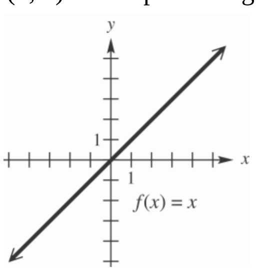


40. $3y=x \Rightarrow y=\frac{1}{3}x$

The graph is the line with slope $\frac{1}{3}$ and y-intercept (0, 0), which means that it passes through the origin. Use another point such as (6, 2) to complete the graph.

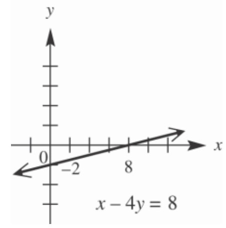


41. $f(x)=x$
The graph is the line with slope 1 and y-intercept (0, 0), which means that it passes through the origin. Use another point such as (1, 1) to complete the graph.

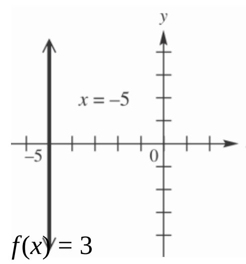


42.
$$\begin{aligned} x-4y&=8 \\ -4y&=-x+8 \\ y&=\tfrac{1}{4}x-2 \end{aligned}$$

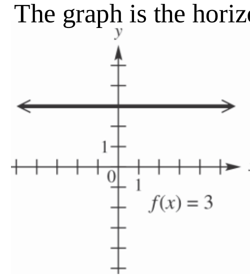
The graph is the line with slope $\frac{1}{4}$ and y-intercept (0, –2). It may also be graphed using intercepts. To do this, locate the x-intercept:
$y=0 \Rightarrow x-4(0)=8 \Rightarrow x=8$



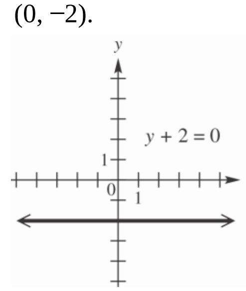
43. $x=-5$
The graph is the vertical line through (–5, 0).



44. $f(x)=3$
The graph is the horizontal line through (0, 3).



45. $y+2=0 \Rightarrow y=-2$
The graph is the horizontal line through (0, −2).



46. The equation of the line that lies along the x-axis is $\underline{y=0}$.

47. Line through (0, 5), $m=-\frac{2}{3}$

Note that $m=-\frac{2}{3}=\frac{-2}{3}$.

Begin by locating the point (0, 5). Because the slope is $\frac{-2}{3}$, a change of 3 units horizontally (3 units to the right) produces a change of –2 units vertically (2 units down). This gives a second point, (3, 3), which can be used to complete the graph.

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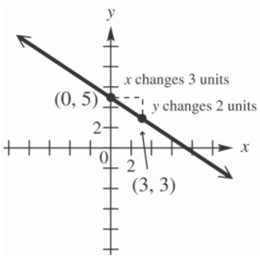
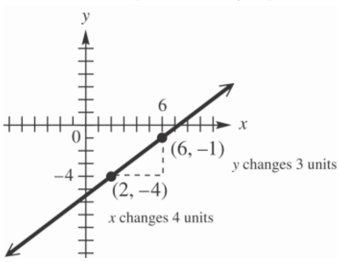


48. Line through (2, –4), $m = \frac{3}{4}$
First locate the point (2, –4).
Because the slope is $\frac{3}{4}$, a change of 4 units horizontally (4 units to the right) produces a change of 3 units vertically (3 units up). This gives a second point, (6, –1), which can be used to complete the graph.



49. through (2, –2) and (3, –4)
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-(-2)}{3-2} = \frac{-2}{1} = -2$$

50. through (8, 7) and $\left(\frac{1}{2}, -2\right)$
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-7}{\frac{1}{2}-8} = \frac{-9}{-\frac{15}{2}}$$
$$= -9\left(-\frac{2}{15}\right) = \frac{18}{15} = \frac{6}{5}$$

51. through (0, –7) and (3, –7)
$$m = \frac{-7-(-7)}{3-0} = \frac{0}{3} = 0$$

52. through (5, 6) and (5, –2)
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-6}{5-5} = \frac{-8}{0}$$
The slope is undefined.

53. $11x + 2y = 3$
Solve for *y* to put the equation in slope-intercept form.
$2y = -11x + 3 \Rightarrow y = -\frac{11}{2}x + \frac{3}{2}$
Thus, the slope is $-\frac{11}{2}$.

54. $9x - 4y = 2$.
Solve for *y* to put the equation in slope-intercept form.
$-4y = -9x + 2 \Rightarrow y = \frac{9}{4}x - \frac{1}{2}$
Thus, the slope is $\frac{9}{4}$.

55. $x - 2 = 0 \Rightarrow x = 2$
The graph is a vertical line, through (2, 0). The slope is undefined.

56. $x - 5y = 0$.
Solve for *y* to put the equation in slope-intercept form.
$-5y = -x \Rightarrow y = \frac{1}{5}x$
Thus, the slope is $\frac{1}{5}$.

57. Initially, the car is at home. After traveling for 30 mph for 1 hr, the car is 30 mi away from home. During the second hour the car travels 20 mph until it is 50 mi away. During the third hour the car travels toward home at 30 mph until it is 20 mi away. During the fourth hour the car travels away from home at 40 mph until it is 60 mi away from home. During the last hour, the car travels 60 mi at 60 mph until it arrived home.

58. **(a)** This is the graph of a function because no vertical line intersects the graph in more than one point.

(b) The lowest point on the graph occurs in December, so the most jobs lost occurred in December. The highest point on the graph occurs in January, so the most jobs gained occurred in January.

(c) The number of jobs lost in December is approximately 6000. The number of jobs gained in January is approximately 2000.

(d) It shows a slight downward trend.

59. **(a)** We need to first find the slope of a line that passes between points (0, 30.7) and (12, 82.9)
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{82.9 - 30.7}{12 - 0} = \frac{52.2}{12} = 4.35$$
Now use the point-intercept form with $b = 30.7$ and $m = 4.35$.
$y = 4.35x + 30.7$
The slope, 4.35, indicates that the number of e-filing taxpayers increased by 4.35% each year from 2001 to 2013.

(b) For 2009, we evaluate the function for $x = 8$. $y = 4.35(8) + 30.7 = 65.5$
65.5% of the tax returns are predicted to have been filed electronically.

60. We need to find the slope of a line that passes between points (1980, 21000) and (2013, 63800)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{63{,}800 - 21{,}000}{2013 - 1980}$$

$$= \frac{42{,}800}{33} \approx \$1297 \text{ per year}$$

The average rate of change was about $1297 per year.

61. **(a)** through (3, –5) with slope –2

Use the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - (-5) = -2(x - 3)$$
$$y + 5 = -2(x - 3)$$
$$y + 5 = -2x + 6$$
$$y = -2x + 1$$

(b) Standard form: $y = -2x + 1 \Rightarrow 2x + y = 1$

62. **(a)** through (–2, 4) and (1, 3)

First find the slope.

$$m = \frac{3 - 4}{1 - (-2)} = \frac{-1}{3}$$

Now use the point-slope form with $(x_1, y_1) = (1, 3)$ and $m = -\frac{1}{3}$.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = -\tfrac{1}{3}(x - 1)$$
$$3(y - 3) = -1(x - 1)$$
$$3y - 9 = -x + 1$$
$$3y = -x + 10 \Rightarrow y = -\tfrac{1}{3}x + \tfrac{10}{3}$$

(b) Standard form:

$$y = -\tfrac{1}{3}x + \tfrac{10}{3} \Rightarrow 3y = -x + 10 \Rightarrow$$
$$x + 3y = 10$$

63. **(a)** through (2, –1) parallel to $3x - y = 1$

Find the slope of $3x - y = 1$.

$$3x - y = 1 \Rightarrow -y = -3x + 1 \Rightarrow y = 3x - 1$$

The slope of this line is 3. Because parallel lines have the same slope, 3 is also the slope of the line whose equation is to be found. Now use the point-slope form with $(x_1, y_1) = (2, -1)$ and $m = 3$.

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = 3(x - 2)$$
$$y + 1 = 3x - 6 \Rightarrow y = 3x - 7$$

(b) Standard form:

$$y = 3x - 7 \Rightarrow -3x + y = -7 \Rightarrow 3x - y = 7$$

64. **(a)** x-intercept (–3, 0), y-intercept (0, 5)

Two points of the line are (–3, 0) and (0, 5). First, find the slope.

$$m = \frac{5 - 0}{0 + 3} = \frac{5}{3}$$

The slope is $\frac{5}{3}$ and the y-intercept is (0, 5). Write the equation in slope-intercept form: $y = \frac{5}{3}x + 5$

(b) Standard form:

$$y = \tfrac{5}{3}x + 5 \Rightarrow 3y = 5x + 15 \Rightarrow$$
$$-5x + 3y = 15 \Rightarrow 5x - 3y = -15$$

65. **(a)** through (2, –10), perpendicular to a line with an undefined slope

A line with an undefined slope is a vertical line. Any line perpendicular to a vertical line is a horizontal line, with an equation of the form $y = b$. The line passes through (2, –10), so the equation of the line is $y = -10$.

(b) Standard form: $y = -10$

66. **(a)** through (0, 5), perpendicular to $8x + 5y = 3$

Find the slope of $8x + 5y = 3$.

$$8x + 5y = 3 \Rightarrow 5y = -8x + 3 \Rightarrow$$
$$y = -\tfrac{8}{5}x + \tfrac{3}{5}$$

The slope of this line is $-\frac{8}{5}$. The slope of any line perpendicular to this line is $\frac{5}{8}$, because $-\frac{8}{5}\left(\frac{5}{8}\right) = -1$.

The equation in slope-intercept form with slope $\frac{5}{8}$ and y-intercept (0, 5) is $y = \frac{5}{8}x + 5$.

(b) Standard form:

$$y = \tfrac{5}{8}x + 5 \Rightarrow 8y = 5x + 40 \Rightarrow$$
$$-5x + 8y = 40 \Rightarrow 5x - 8y = -40$$

67. **(a)** through (–7, 4), perpendicular to $y = 8$

The line $y = 8$ is a horizontal line, so any line perpendicular to it will be a vertical line. Because x has the same value at all points on the line, the equation is $x = -7$. It is not possible to write this in slope-intercept form.

(b) Standard form: $x = -7$

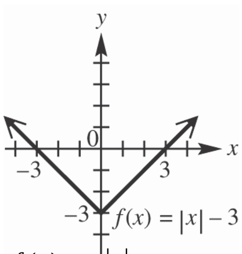
68. **(a)** through (3, –5), parallel to $y = 4$

This will be a horizontal line through (3, –5). Because y has the same value at all points on the line, the equation is $y = -5$.

(b) Standard form: $y = -5$

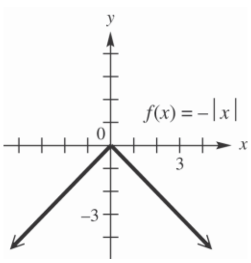
69. $f(x) = |x| - 3$

The graph is the same as that of $y = |x|$, except that it is translated 3 units downward.



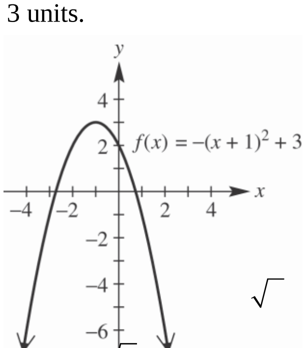
70. $f(x) = -|x|$

The graph of $f(x) = -|x|$ is the reflection of the graph of $y = |x|$ about the x-axis.



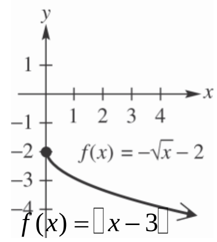
71. $f(x) = -(x+1)^2 + 3$

The graph of $f(x) = -(x+1)^2 + 3$ is a translation of the graph of $y = x^2$ to the left 1 unit, reflected over the x-axis and translated up 3 units.



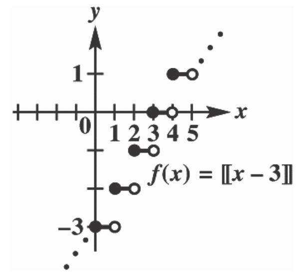
72. $f(x) = -\sqrt{x} - 2$

The graph of $f(x) = -\sqrt{x} - 2$ is the reflection of the graph of $y = \sqrt{x}$ about the x-axis, translated down 2 units.



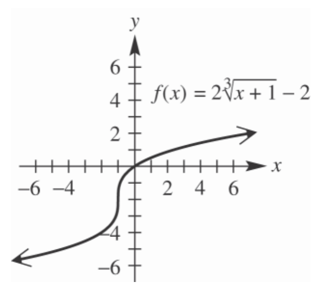
73. $f(x) = [\![x - 3]\!]$

To get $y = 0$, we need $0 \le x - 3 < 1 \Rightarrow 3 \le x < 4.$ To get $y = 1$, we need $1 \le x - 3 < 2 \Rightarrow 4 \le x < 5.$
Follow this pattern to graph the step function.



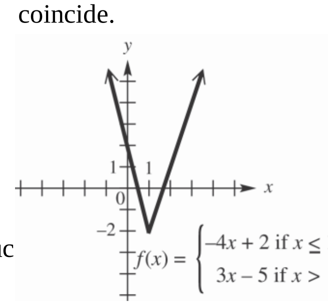
74. $f(x) = 2\sqrt[3]{x+1} - 2$

The graph of $f(x) = 2\sqrt[3]{x+1} - 2$ is a translation of the graph of $y = \sqrt[3]{x}$ to the left 1 unit, stretched vertically by a factor of 2, and translated down 2 units.



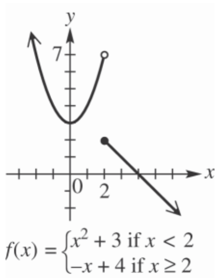
75. $f(x) = \begin{cases} -4x + 2 & \text{if } x \le 1 \\ 3x - 5 & \text{if } x > 1 \end{cases}$

Draw the graph of $y = -4x + 2$ to the left of $x = 1$, including the endpoint at $x = 1$. Draw the graph of $y = 3x - 5$ to the right of $x = 1$, but do not include the endpoint at $x = 1$. Observe that the endpoints of the two pieces coincide.



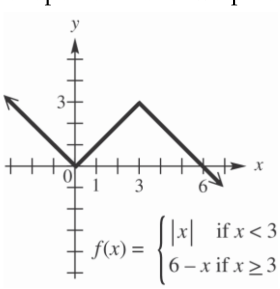
76. $f(x) = \begin{cases} x^2 + 3 & \text{if } x < 2 \\ -x + 4 & \text{if } x \ge 2 \end{cases}$

Graph the curve $y = x^2 + 3$ to the left of $x = 2$, and graph the line $y = -x + 4$ to the right of $x = 2$. The graph has an open circle at (2, 7) and a closed circle at (2, 2).



77. $f(x) = \begin{cases} |x| & \text{if } x < 3 \\ 6 - x & \text{if } x \ge 3 \end{cases}$

Draw the graph of $y = |x|$ to the left of $x = 3$, but do not include the endpoint. Draw the graph of $y = 6 - x$ to the right of $x = 3$, including the endpoint. Observe that the endpoints of the two pieces coincide.



78. Because x represents an integer, $[\![x]\!] = x$. Therefore, $[\![x]\!] + x = x + x = 2x$.

79. True. The graph of an even function is symmetric with respect to the y-axis.

80. True. The graph of a nonzero function cannot be symmetric with respect to the x-axis. Such a graph would fail the vertical line test

81. False. For example, $f(x) = x^2$ is even and (2, 4) is on the graph but (2, –4) is not.

82. True. The graph of an odd function is symmetric with respect to the origin.

83. True. The constant function $f(x) = 0$ is both

84. False. For example, $f(x) = x^3$ is odd, and (2, 8) is on the graph but (–2, 8) is not.

85. $x + y^2 = 10$

Replace x with $-x$ to obtain $(-x) + y^2 = 10$. The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain $x + (-y)^2 = 10 \Rightarrow x + y^2 = 10$. The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain $(-x) + (-y)^2 = 10 \Rightarrow (-x) + y^2 = 10$. The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. The graph is symmetric with respect to the x-axis only.

86. $5y^2 + 5x^2 = 30$

Replace x with $-x$ to obtain $5y^2 + 5(-x)^2 = 30 \Rightarrow 5y^2 + 5x^2 = 30$. The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain $5(-y)^2 + 5x^2 = 30 \Rightarrow 5y^2 + 5x^2 = 30$. The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. The graph is symmetric with respect to the y-axis and x-axis, so it must also be symmetric with respect to the origin. Note that this equation is the same as $y^2 + x^2 = 6$, which is a circle centered at the origin.

87. $x^2 = y^3$

Replace x with $-x$ to obtain $(-x)^2 = y^3 \Rightarrow x^2 = y^3$. The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain $x^2 = (-y)^3 \Rightarrow x^2 = -y^3$. The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain $(-x)^2 = (-y)^3 \Rightarrow x^2 = -y^3$. The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph is symmetric with respect to the y-axis only.

even and odd. Because $f(-x) = 0 = f(x)$,

the function is even. Also
$f(-x) = 0 = -0 = -f(x)$, so the function is odd.

88. $y^3 = x + 4$

Replace x with $-x$ to obtain $y^3 = -x + 4$.

The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$(-y)^3 = x + 4 \Rightarrow -y^3 = x + 4 \Rightarrow y^3 = -x - 4$

The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain

$(-y)^3 = (-x) + 4 \Rightarrow -y^3 = -x + 4 \Rightarrow y^3 = x - 4.$

The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph has none of the listed symmetries.

89. $6x + y = 4$

Replace x with $-x$ to obtain $6(-x) + y = 4 \Rightarrow$ $-6x + y = 4$. The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$6x + (-y) = 4 \Rightarrow 6x - y = 4$. The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain

$6(-x) + (-y) = 4 \Rightarrow -6x - y = 4$. This equation is not equivalent to the original one, so the graph is not symmetric with respect to the origin. Therefore, the graph has none of the listed symmetries.

90. $|y| = -x$

Replace x with $-x$ to obtain

$|y| = -(-x) \Rightarrow |y| = x.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$|-y| = -x \Rightarrow |y| = -x.$ The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain

$|-y| = -(-x) \Rightarrow |y| = x.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the origin. Therefore, the graph is symmetric with respect

92. $|x| = |y|$

Replace x with $-x$ to obtain

$|-x| = |y| \Rightarrow |x| = |y|$.

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$|x| = |-y| \Rightarrow |x| = |y|$. The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Because the graph is symmetric with respect to the x-axis and with respect to the y-axis, it must also by symmetric with respect to the origin.

93. $x^2 - y^2 = 0$

Replace x with $-x$ to obtain

$(-x)^2 - y^2 = 0 \Rightarrow x^2 - y^2 = 0.$ The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$x^2 - (-y)^2 = 0 \Rightarrow x^2 - y^2 = 0.$ The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Because the graph is symmetric with respect to the x-axis and with respect to the y-axis, it must also by symmetric with respect to the origin.

94. $x^2 + (y-2)^2 = 4$

Replace x with $-x$ to obtain

$(-x)^2 + (y-2)^2 = 4 \Rightarrow x^2 + (y-2)^2 = 4.$

The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain

$x^2 + (-y-2)^2 = 4.$ The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Replace x with $-x$ and y with $-y$ to obtain

$(-x)^2 + (-y-2)^2 = 4 \Rightarrow x^2 + (-y-2)^2 = 4,$

which is not equivalent to the original equation. Therefore, the graph is not symmetric with respect to the origin.

95. To obtain the graph of $g(x) = -|x|$, reflect the graph of $f(x) = |x|$ across the x-axis.

to the x-axis only.

91. $y = 1$
This is the graph of a horizontal line through (0, 1). It is symmetric with respect to the y-axis, but not symmetric with respect to the x-axis and the origin.

96. To obtain the graph of $h(x) = \ x - 2$, translate the graph of $f(x) = |x|$ down 2 units.

97. To obtain the graph of $k(x) = 2|x-4|$, translate the graph of $f(x) = |x|$ to the right 4 units and stretch vertically by a factor of 2.

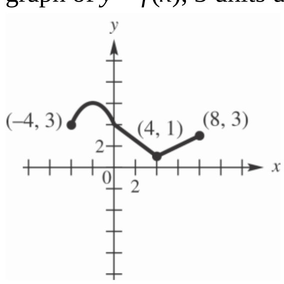
98. If the graph of $f(x) = 3x - 4$ is reflected about the *x*-axis, we obtain a graph whose equation is $y = -(3x-4) = -3x + 4.$

99. If the graph of $f(x) = 3x - 4$ is reflected about the *y*-axis, we obtain a graph whose equation is $y = f(-x) = 3(-x) - 4 = -3x - 4.$

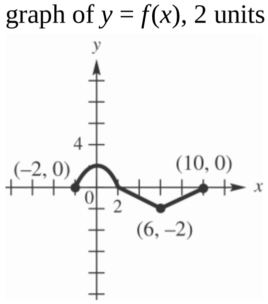
100. If the graph of $f(x) = 3x - 4$ is reflected about the origin, every point (x, y) will be replaced by the point $(-x, -y)$. The equation for the graph will change from $y = 3x - 4$ to

$-y = 3(-x) - 4 \Rightarrow -y = -3x - 4 \Rightarrow$
$y = 3x + 4.$

101. **(a)** To graph $y = f(x) + 3$, translate the graph of $y = f(x)$, 3 units up.

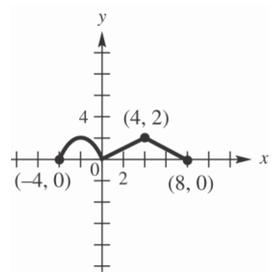


(b) To graph $y = f(x-2)$, translate the graph of $y = f(x)$, 2 units to the right.



(c) To graph $y = f(x+3) - 2$, translate the

(d) To graph $y = |f(x)|$, keep the graph of $y = f(x)$ as it is where $y \geq 0$ and reflect the graph about the *x*-axis where $y < 0$.



102. No. For example suppose $f(x) = \sqrt{x-2}$ and $g(x) = 2x$. Then

$(f \circ g)(x) = f(g(x)) = f(2x) = \sqrt{2x-2}$

The domain and range of g are $(-\infty, \infty)$, however, the domain of f is $[2, \infty)$. So, $2x - 2 \geq 0 \Rightarrow x \geq 1$. Therefore, the domain of $f \circ g$ is $[1, \infty)$. The domain of g, $(-\infty, \infty)$, is not a subset of the domain of $f \circ g$, $[1, \infty)$.

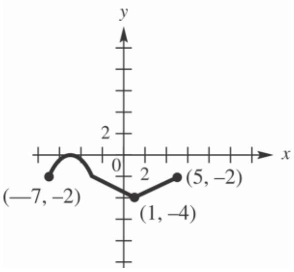
For Exercises 103–110, $f(x) = 3x^2 - 4$ and $g(x) = x^2 - 3x - 4$.

103.
$$\begin{aligned}(fg)(x) &= f(x) \cdot g(x)\\ &= (3x^2-4)(x^2-3x-4)\\ &= 3x^4 - 9x^3 - 12x^2 - 4x^2 + 12x + 16\\ &= 3x^4 - 9x^3 - 16x^2 + 12x + 16\end{aligned}$$

104.
$$\begin{aligned}(f-g)(4) &= f(4) - g(4)\\ &= (3 \cdot 4^2 - 4) - (4^2 - 3 \cdot 4 - 4)\\ &= (3 \cdot 16 - 4) - (16 - 3 \cdot 4 - 4)\\ &= (48 - 4) - (16 - 12 - 4)\\ &= 44 - 0 = 44\end{aligned}$$

105.
$$\begin{aligned}(f+g)(-4) &= f(-4) + g(-4)\\ &= [3(-4)^2 - 4] + [(-4)^2 - 3(-4) - 4]\\ &= [3(16) - 4] + [16 - 3(-4) - 4]\\ &= [48 - 4] + [16 + 12 - 4]\\ &= 44 + 24 = 68\end{aligned}$$

graph of $y = f(x)$, 3 units to the left and 2 units down.



106. $$\begin{aligned}(f+g)(2k) &= f(2k)+g(2k)\\ &= [3(2k)^2-4]+[(2k)^2-3(2k)-4]\\ &= [3(4)k^2-4]+[4k^2-3(2k)-4]\\ &= (12k^2-4)+(4k^2-6k-4)\\ &= 16k^2-6k-8\end{aligned}$$

107. $\left(\dfrac{f}{g}\right)(3) = \dfrac{f(3)}{g(3)} = \dfrac{3\cdot 3^2-4}{3^2-3\cdot 3-4} = \dfrac{3\cdot 9-4}{9-3\cdot 3-4}$

$= \dfrac{27-4}{9-9-4} = \dfrac{23}{-4} = -\dfrac{23}{4}$

108. $\left(\dfrac{f}{g}\right)(-1) = \dfrac{3(-1)^2-4}{(-1)^2-3(-1)-4} = \dfrac{3(1)-4}{1-3(-1)-4}$

$= \dfrac{3-4}{1+3-4} = \dfrac{-1}{0} =$ undefined

109. The domain of $(fg)(x)$ is the intersection of the domain of $f(x)$ and the domain of $g(x)$. Both have domain $(-\infty,\infty)$, so the domain of $(fg)(x)$ is $(-\infty,\infty)$.

110. $\left(\dfrac{f}{g}\right)(x) = \dfrac{3x^2-4}{x^2-3x-4} = \dfrac{3x^2-4}{(x+1)(x-4)}$

Because both $f(x)$ and $g(x)$ have domain $(-\infty,\infty)$, we are concerned about values of x that make $g(x)=0$. Thus, the expression is undefined if $(x+1)(x-4)=0$, that is, if $x=-1$ or $x=4$. Thus, the domain is the set of all real numbers except $x=-1$ and $x=4$, or $(-\infty,-1)\cup(-1,4)\cup(4,\infty)$.

111. $f(x) = 2x+9$

$$f(x+h) = 2(x+h)+9 = 2x+2h+9$$
$$f(x+h)-f(x) = (2x+2h+9)-(2x+9) = 2x+2h+9-2x-9 = 2h$$

Thus, $\dfrac{f(x+h)-f(x)}{h} = \dfrac{2h}{h} = 2.$

112. $f(x) = x^2-5x+3$

$$f(x+h) = (x+h)^2-5(x+h)+3 = x^2+2xh+h^2-5x-5h+3$$
$$f(x+h)-f(x)$$
$$= (x^2+2xh+h^2-5x-5h+3)-(x^2-5x+3)$$
$$= x^2+2xh+h^2-5x-5h+3-x^2+5x-3$$
$$= 2xh+h^2-5h$$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{2xh+h^2-5h}{h}$

For Exercises 113–118,

$f(x) = \sqrt{x-2}$ and $g(x) = x^2$.

113. $(g\circ f)(x) = g[f(x)] = g\left(\sqrt{x-2}\right)$

$= \left(\sqrt{x-2}\right)^2 = x-2$

114. $(f\circ g)(x) = f[g(x)] = f(x^2) = \sqrt{x^2-2}$

115. $f(x) = \sqrt{x-2}$, so $f(3) = \sqrt{3-2} = \sqrt{1} = 1$.

Therefore, $(g\circ f)(3) = g\left[f(3)\right] = g(1) = 1^2 = 1.$

116. $g(x) = x^2$, so $g(-6) = (-6)^2 = 36.$

Therefore, $(f\circ g)(-6) = f\left[g(-6)\right] = f(36)$

$= \sqrt{36-2} = \sqrt{34}$.

117. $(g\circ f)(-1) = g\left(f(-1)\right) = g\left(\sqrt{-1-2}\right) = g\left(\sqrt{-3}\right)$

Because $\sqrt{-3}$ is not a real number, $(g\circ f)(-1)$ is not defined.

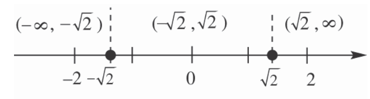
118. To find the domain of $f\circ g$, we must consider the domain of g ~~as well~~ as the composed function, $f\circ g$. Because

$(f\circ g)(x) = f\left[g(x)\right] = \sqrt{x^2-2}$ we need to determine when $x^2-2\ge 0$.

Step 1: Find the values of x that satisfy $x^2-2=0$.

$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

Step 2: The two numbers divide a number line into three regions.



Step 3 Choose a test value to see if it satisfies the inequality, $x^2 - 2 \geq 0$.

Interval	Test Value	Is $x^2 - 2 \geq 0$ true or false?
$\left(-\infty, -\sqrt{2}\right)$	-2	$(-2)^2 - 2 \geq 0$? $2 \geq 0$ True
$\left(-\sqrt{2}, \sqrt{2}\right)$	0	$0^2 - 2 \geq 0$? $-2 \geq 0$ False
$\left(\sqrt{2}, \infty\right)$	2	$2^2 - 2 \geq 0$? $2 \geq 0$ True

The domain of $f \circ g$ is $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$.

119. $(f+g)(1) = f(1) + g(1) = 7 + 1 = 8$

120. $(f-g)(3) = f(3) - g(3) = 9 - 9 = 0$

121. $(fg)(-1) = f(-1) \cdot g(-1) = 3(-2) = -6$

122. $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{5}{0} = \text{undefined}$

123. $(g \circ f)(-2) = g[f(-2)] = g(1) = 2$

124. $(f \circ g)(3) = f[g(3)] = f(-2) = 1$

125. $(f \circ g)(2) = f[g(2)] = f(2) = 1$

126. $(g \circ f)(3) = g[f(3)] = g(4) = 8$

127. Let x = number of yards.

$f(x) = 36x$, where $f(x)$ is the number of inches. $g(x) = 1760x$, where $g(x)$ is the number of yards. Then

$$(g \circ f)(x) = g\left[f(x)\right] = 1760(36x) = 63{,}360x.$$

There are 63,360x inches in x miles

128. Use the definition for the perimeter of a rectangle.

P = length + width + length + width

$P(x) = 2x + x + 2x + x = 6x$
This is a linear function.

129. If $V(r) = \frac{4}{3}\pi r^3$ and if the radius is increased by 3 inches, then the amount of volume gained is given by

$$V_g(r) = V(r+3) - V(r) = \frac{4}{3}\pi(r+3)^3 - \frac{4}{3}\pi r^3.$$

130. **(a)** $V = \pi r^2 h$

If d is the diameter of its top, then $h = d$ and $r = \frac{d}{2}$. So,

$$V(d) = \pi\left(\frac{d}{2}\right)^2(d) = \pi\left(\frac{d^2}{4}\right)(d) = \frac{\pi d^3}{4}.$$

(b) $S = 2\pi r^2 + 2\pi rh \Rightarrow$

(b) The range of $f(x) = \sqrt{x-3}$ is all real numbers greater than ~~or equal~~ to 0. In interval notation, this correlates to the interval in D, $[0, \infty)$.

(c) The domain of $f(x) = x^2 - 3$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty, \infty)$.

(d) The range of $f(x) = x^2 + 3$ is all real numbers greater than or equal to 3. In interval notation, this correlates to the interval in B, $[3, \infty)$.

(e) The domain of $f(x) = \sqrt[3]{x-3}$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty, \infty)$.

(f) The range of $f(x) = \sqrt[3]{x} + 3$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty, \infty)$.

(g) The domain of $f(x) = |x| - 3$ is all real numbers. In interval notation, this correlates to the interval in C, $(-\infty, \infty)$.

(h) The range of $f(x) = |x+3|$ is all real numbers greater than or equal to 0. In interval notation, this correlates to the interval in D, $[0, \infty)$.

(i) The domain of $x = y^2$ is $x \geq 0$ because when you square any value of y, the outcome will be nonnegative. In interval notation, this correlates to the interval in D, $[0, \infty)$.

(j) The range of $x = y^2$ is all real numbers.

$$S(d) = 2\pi\left(\frac{d}{2}\right)^2 + 2\pi\left(\frac{d}{2}\right)(d) = \frac{\pi d^2}{2} + \pi d^2$$
$$= \frac{\pi d^2}{2} + \frac{2\pi d^2}{2} = \frac{3\pi d^2}{2}$$

Chapter 2 Test

1. **(a)** The domain of $f(x) = \sqrt{x} + 3$ occurs when $x \geq 0$. In interval notation, this correlates to the interval in D, $[0, \infty)$.

In interval notation, this correlates to the interval in C, $(-\infty, \infty)$.

2. Consider the points $(-2, 1)$ and $(3, 4)$.

$$m = \frac{4-1}{3-(-2)} = \frac{3}{5}$$

3. We label the points $A(-2, 1)$ and $B(3, 4)$.

$$d(A, B) = \sqrt{[3-(-2)]^2 + (4-1)^2}$$
$$= \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}$$

4. The midpoint has coordinates

$$\left(\frac{-2+3}{2}, \frac{1+4}{2}\right) = \left(\frac{1}{2}, \frac{5}{2}\right).$$

5. Use the point-slope form with $(x_1, y_1) = (-2,1)$ and $m = \frac{3}{5}$.

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \tfrac{3}{5}[x - (-2)]$$
$$y - 1 = \tfrac{3}{5}(x+2) \Rightarrow 5(y-1) = 3(x+2) \Rightarrow$$
$$5y - 5 = 3x + 6 \Rightarrow 5y = 3x + 11 \Rightarrow$$
$$-3x + 5y = 11 \Rightarrow 3x - 5y = -11$$

6. Solve $3x - 5y = -11$ for y.

$$3x - 5y = -11$$
$$-5y = -3x - 11$$
$$y = \tfrac{3}{5}x + \tfrac{11}{5}$$

Therefore, the linear function is

$$f(x) = \frac{3}{5}x + \frac{11}{5}.$$

7. **(a)** The center is at (0, 0) and the radius is 2, so the equation of the circle is $x^2 + y^2 = 4$.

(b) The center is at (1, 4) and the radius is 1, so the equation of the circle is $(x-1)^2 + (y-4)^2 = 1$

8. $x^2 + y^2 + 4x - 10y + 13 = 0$

Complete the square on x and y to write the equation in standard form:

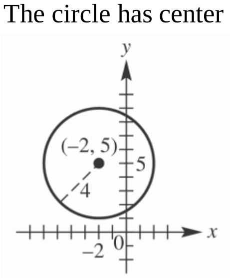
$$x^2 + y^2 + 4x - 10y + 13 = 0$$

$$\left(x^2 + 4x + \quad\right) + \left(y^2 - 10y + \quad\right) = -13$$

$$\left(x^2 + 4x + 4\right) + \left(y^2 - 10y + 25\right) = -13 + 4 + 25$$

$$(x+2)^2 + (y-5)^2 = 16$$

The circle has center (−2, 5) and radius 4.



$x^2 + y^2 + 4x - 10y + 13 = 0$

(b) This is the graph of a function because no vertical line intersects the graph in more than one point. The domain of the function is $(-\infty, -1) \cup (-1, \infty)$. The range is $(-\infty, 0) \cup (0, \infty)$. As x is getting larger on the intervals $(-\infty, -1)$ and $(-1, \infty)$, the value of y is decreasing, so the function is decreasing on these intervals. (The function is never increasing or constant.)

10. Point A has coordinates (5, –3).

(a) The equation of a vertical line through A is $x = 5$.

(b) The equation of a horizontal line through A is $y = -3$.

11. The slope of the graph of $y = -3x + 2$ is –3.

(a) A line parallel to the graph of $y = -3x + 2$ has a slope of –3. Use the point-slope form with $(x_1, y_1) = (2, 3)$ and $m = -3$.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = -3(x - 2)$$
$$y - 3 = -3x + 6 \Rightarrow y = -3x + 9$$

(b) A line perpendicular to the graph of $y = -3x + 2$ has a slope of $\frac{1}{3}$ because $-3\left(\frac{1}{3}\right) = -1$.

$$y - 3 = \tfrac{1}{3}(x - 2)$$

$$3(y-3) = x - 2 \Rightarrow 3y - 9 = x - 2 \Rightarrow$$

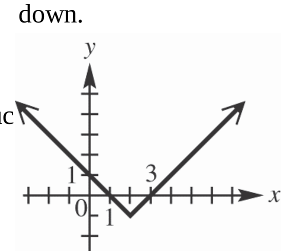
$$3y = x + 7 \Rightarrow y = \tfrac{1}{3}x + \tfrac{7}{3}$$

12. **(a)** $(2, \infty)$ **(b)** $(0, 2)$

(c) $(-\infty, 0)$ **(d)** $(-\infty, \infty)$

(e) $(-\infty, \infty)$ **(f)** $[-1, \infty)$

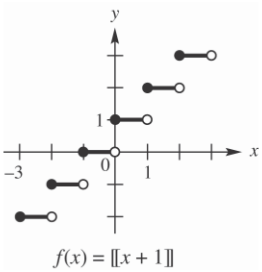
13. To graph $f(x) = |x - 2| - 1$, we translate the graph of $y = |x|$, 2 units to the right and 1 unit down.



9. **(a)** This is not the graph of a function because some vertical lines intersect it in more than one point. The domain of the relation is [0, 4]. The range is [–4, 4].

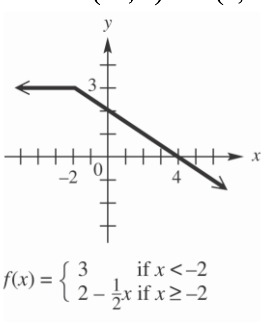
14. $f(x) = [\![x+1]\!]$

To get $y = 0$, we need $0 \le x+1 < 1 \Rightarrow -1 \le x < 0$. To get $y = 1$, we need $1 \le x+1 < 2 \Rightarrow 0 \le x < 1$. Follow this pattern to graph the step function.

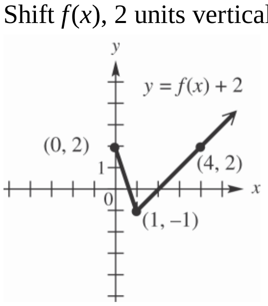


15. $f(x) = \begin{cases} 3 & \text{if } x < -2 \\ 2 - \frac{1}{2}x & \text{if } x \ge -2 \end{cases}$

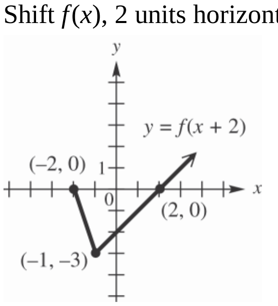
For values of x with $x < -2$, we graph the horizontal line $y = 3$. For values of x with $x \ge -2$, we graph the line with a slope of $-\frac{1}{2}$ and a y-intercept of (0, 2). Two points on this line are (–2, 3) and (0, 2).



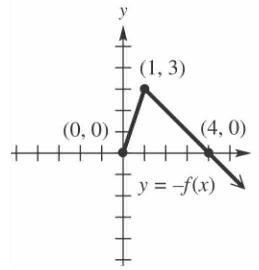
16. **(a)** Shift $f(x)$, 2 units vertically upward.



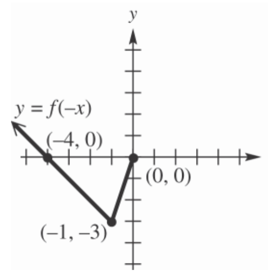
(b) Shift $f(x)$, 2 units horizontally to the left.



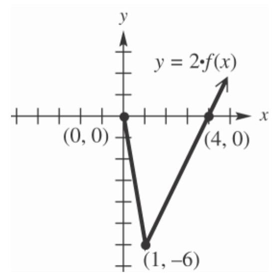
(c) Reflect $f(x)$, across the x-axis.



(d) Reflect $f(x)$, across the y-axis.



(e) Stretch $f(x)$, vertically by a factor of 2.



17. Starting with $y = \sqrt{x}$, we shift it to the left 2 units and stretch it vertically by a factor of 2. The graph is then reflected over the x-axis and then shifted down 3 units.

18. $3x^2 - 2y^2 = 3$

(a) Replace y with $-y$ to obtain $3x^2 - 2(-y)^2 = 3 \Rightarrow 3x^2 - 2y^2 = 3.$ The result is the same as the original equation, so the graph is symmetric with respect to the x-axis.

(b) Replace x with $-x$ to obtain $3(-x)^2 - 2y^2 = 3 \Rightarrow 3x^2 - 2y^2 = 3.$ The result is the same as the original equation, so the graph is symmetric with respect to the y-axis.

(c) The graph is symmetric with respect to the x-axis and with respect to the y-axis, so it must also be symmetric with respect to the origin.

19. $f(x) = 2x^2 - 3x + 2,\ g(x) = -2x + 1$

(a) $$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= \left(2x^2 - 3x + 2\right) - \left(-2x + 1\right)\\ &= 2x^2 - 3x + 2 + 2x - 1\\ &= 2x^2 - x + 1\end{aligned}$$

(b) $$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x^2 - 3x + 2}{-2x + 1}$$

(c) We must determine which values solve the equation $-2x + 1 = 0.$

$$-2x + 1 = 0 \Rightarrow -2x = -1 \Rightarrow x = \frac{1}{2}$$

Thus, $\frac{1}{2}$ is excluded from the domain, and the domain is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

(d) $f(x) = 2x^2 - 3x + 2$

$$\begin{aligned}f(x+h) &= 2(x+h)^2 - 3(x+h) + 2\\ &= 2\left(x^2 + 2xh + h^2\right) - 3x - 3h + 2\\ &= 2x^2 + 4xh + 2h^2 - 3x - 3h + 2\end{aligned}$$

$$\begin{aligned}&f(x+h) - f(x)\\ &= (2x^2 + 4xh + 2h^2 - 3x - 3h + 2)\\ &\qquad - (2x^2 - 3x + 2)\\ &= 2x^2 + 4xh + 2h^2 - 3x\\ &\qquad -3h + 2 - 2x^2 + 3x - 2\\ &= 4xh + 2h^2 - 3h\end{aligned}$$

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{4xh + 2h^2 - 3h}{h}\\ &= \frac{h(4x + 2h - 3)}{h}\\ &= 4x + 2h - 3\end{aligned}$$

(e) $$\begin{aligned}(f+g)(1) &= f(1) + g(1)\\ &= (2\cdot 1^2 - 3\cdot 1 + 2) + (-2\cdot 1 + 1)\\ &= (2\cdot 1 - 3\cdot 1 + 2) + (-2\cdot 1 + 1)\\ &= (2 - 3 + 2) + (-2 + 1)\\ &= 1 + (-1) = 0\end{aligned}$$

(f) $$\begin{aligned}(fg)(2) &= f(2)\cdot g(2)\\ &= (2\cdot 2^2 - 3\cdot 2 + 2)\cdot(-2\cdot 2 + 1)\\ &= (2\cdot 4 - 3\cdot 2 + 2)\cdot(-2\cdot 2 + 1)\\ &= (8 - 6 + 2)\cdot(-4 + 1)\\ &= 4(-3) = -12\end{aligned}$$

(g) $g(x) = -2x + 1 \Rightarrow g(0) = -2(0) + 1$ $= 0 + 1 = 1.$ Therefore,

$$\begin{aligned}(f\circ g)(0) &= f\left[g(0)\right]\\ &= f(1) = 2\cdot 1^2 - 3\cdot 1 + 2\\ &= 2\cdot 1 - 3\cdot 1 + 2\\ &= 2 - 3 + 2 = 1\end{aligned}$$

For exercises 20 and 21, $f(x) = \sqrt{x+1}$ and $g(x) = 2x - 7.$

20. $$\begin{aligned}(f\circ g) &= f\left(g(x)\right) = f(2x - 7)\\ &= \sqrt{(2x-7)+1} = \sqrt{2x - 6}\end{aligned}$$

The domain and range of g are $(-\infty, \infty)$, while the domain of f is $[0, \infty)$. We need to find the values of x which fit the domain of f: $2x - 6 \ge 0 \Rightarrow x \ge 3$. So, the domain of $f\circ g$ is $[3, \infty)$.

21. $$\begin{aligned}(g\circ f) &= g\left(f(x)\right) = g\left(\sqrt{x+1}\right)\\ &= 2\sqrt{x+1} - 7\end{aligned}$$

The domain and range of g are $(-\infty, \infty)$, while the domain of f is $[0, \infty)$. We need to find the values of x which fit the domain of f: $x + 1 \ge 0 \Rightarrow x \ge -1$. So, the domain of $g\circ f$ is $[-1, \infty)$.

22. **(a)** $C(x) = 3300 + 4.50x$

(b) $R(x) = 10.50x$

(c) $$\begin{aligned}P(x) &= R(x) - C(x)\\ &= 10.50x - (3300 + 4.50x)\\ &= 6.00x - 3300\end{aligned}$$

(d) $$\begin{aligned}P(x) &> 0\\ 6.00x - 3300 &> 0\\ 6.00x &> 3300\\ x &> 550\end{aligned}$$

She must produce and sell 551 items before she earns a profit.