Solution Manual for Precalculus A Right Triangle Approach 5th Edition Beecher Penna Bittinger 0321969553 9780321969552

Chapter 2

More on Functions

Exercise Set 2.1

1. a) For x-values from -5 to 1, the y-values increase from -3 to 3. Thus the function is increasing on the interval $(-5, 1)$.

   b) For x-values from 3 to 5, the y-values decrease from 3 to 1. Thus the function is decreasing on the interval $(3, 5)$.

   c) For x-values from 1 to 3, y is 3. Thus the function is constant on $(1, 3)$.

2. a) For x-values from 1 to 3, the y-values increase from 1 to 2. Thus, the function is increasing on the $(1, 3)$.

   b) For x-values from -5 to 1, the y-values decrease from 4 to 1. Thus the function is decreasing on the interval $(-5, 1)$.

   c) For x-values from 3 to 5, y is 2. Thus the function is constant on $(3, 5)$.

3. a) For x-values from -3 to -1, the y-values increase from -4 to 4. Also, for x-values from 3 to 5, the y-values increase from 2 to 6. Thus the function is increasing on $(-3, -1)$ and on $(3, 5)$.

   b) For x-values from 1 to 3, the y-values decrease from 3 to 2. Thus the function is decreasing on the interval $(1, 3)$.

   c) For x-values from -5 to -3, y is 1. Thus the function is constant on $(-5, -3)$.

4. a) For x-values from 1 to 2, the y-values increase from 1 to 2. Thus the function is increasing on the interval $(1, 2)$.

   b) For x-values from -5 to -2, the y-values decrease from 3 to 1. For x-values from -2 to 1, the y-values decrease from 3 to 1. And for x-values from 3 to 5, the y-values decrease from 2 to 1. Thus the function is decreasing on $(-5, -2)$, on $(-2, 1)$, and on $(3, 5)$.

   c) For x-values from 2 to 3, y is 2. Thus the function is constant on $(2, 3)$.

5. a) For x-values from $-\infty$ to -8, the y-values increase from $-\infty$ to 2. Also, for x-values from -3 to -2, the y-values increase from -2 to 3. Thus the function is increasing on $(-\infty, -8)$ and on $(-3, -2)$.

   b) For x-values from -8 to -6, the y-values decrease from 2 to -2. Thus the function is decreasing on the interval $(-8, -6)$.

   c) For x-values from -6 to -3, y is -2. Also, for x-values from -2 to ∞, y is 3. Thus the function is constant on $(-6, -3)$ and on $(-2, \infty)$.

6. a) For x-values from 1 to 4, the y-values increase from 2 to 11. Thus the function is increasing on the interval $(1, 4)$.

   b) For x-values from -1 to 1, the y-values decrease from 6 to 2. Also, for x-values from 4 to ∞, the y-values decrease from 11 to $-\infty$. Thus the function is decreasing on $(-1, 1)$ and on $(4, \infty)$.

   c) For x-values from $-\infty$ to -1, y is 3. Thus the function is constant on $(-\infty, -1)$.

7. The x-values extend from -5 to 5, so the domain is $[-5, 5]$. The y-values extend from -3 to 3, so the range is

8. Domain: $[-5, 5]$; range: $[1, 4]$

9. The x-values extend from -5 to -1 and from 1 to 5, so the domain is $[-5, -1] \cup [1, 5]$.

   The y-values extend from -4 to 6, so the range is $[-4, 6]$

10. Domain: $[-5, 5]$; range: $[1, 3]$

11. The x-values extend from $-\infty$ to ∞, so the domain is

$(-\infty,$
$\infty)$.

The y-values extend from $-\infty$ to 3, so the range is $(-\infty, 3]$.

12. Domain: $(-\infty, \infty)$; range: $(-\infty, 11]$

13. From the graph we see that a relative maximum value of the function is 3.25. It occurs at $x = 2.5$. There is no relative minimum value.

    The graph starts rising, or increasing, from the left and stops increasing at the relative maximum. From this point, the graph decreases. Thus the function is increasing on $(-\infty, 2.5)$ and is decreasing on $(2.5, \infty)$.

14. From the graph we see that a relative minimum value of 2 occurs at $x = 1$. There is no relative maximum value.

    The graph starts falling, or decreasing, from the left and stops decreasing at the relative minimum. From this point, the graph increases. Thus the function is increasing on $(1, \infty)$ and is decreasing on $(-\infty, 1)$.

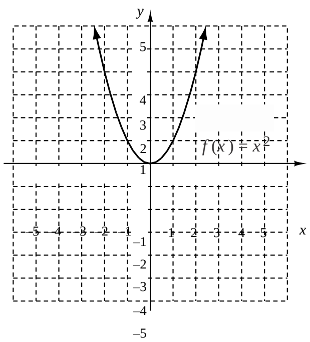
15. From the graph we see that a relative maximum value of the function is 2.370. It occurs at $x = -0.667$. We also see that a relative minimum value of 0 occurs at $x = 2$.

    The graph starts rising, or increasing, from the left and stops increasing at the relative maximum. From this point it decreases to the relative minimum and then increases again. Thus the function is increasing on $(-\infty, -0.667)$ and on $(2, \infty)$. It is decreasing on $(-0.667, 2)$.

16. From the graph we see that a relative maximum value of
    2.921 occurs at $x = 3.601$. Arelative minimum value of
    0.995 occurs at $x =$
    0.103.

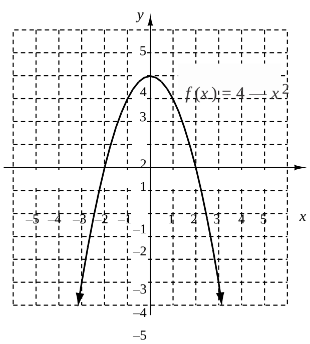
The graph starts decreasing from the left and stops decreasing at the relative minimum. From this point it increases to the relative maximum and then decreases again. Thus the function is increasing on $(0.103, 3.601)$ and is decreasing on $(-\infty, 0.103)$ and on $(3.601, \infty)$.

17.



The function is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$. We estimate that the minimum is 0 at $x = 0$. There are no maxima.

18.



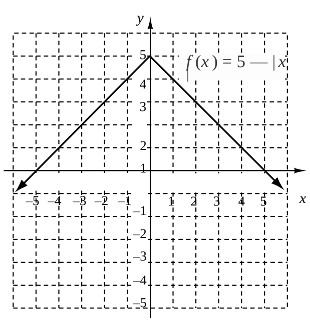
Increasing: $(-\infty, 0)$

Decreasing: $(0, \infty)$

Maximum: 4 at $x = 0$

Minima: none

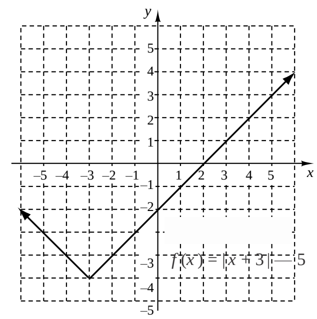
19.



The function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. We estimate that the maximum is 5 at $x = 0$.

There are no minima.

20.



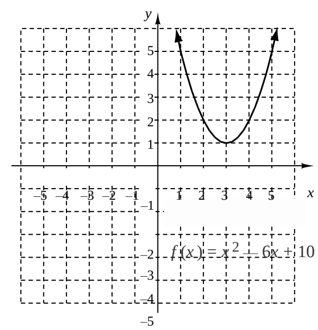
Increasing: $(-3, \infty)$

Decreasing: $(-\infty,$

$-3)$ Maxima: none

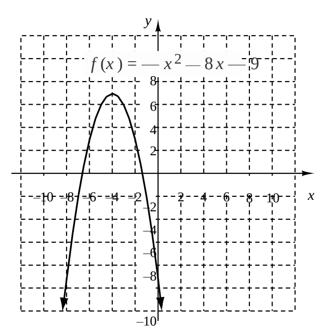
Minimum: -5 at $x = -3$

21.



The function is decreasing on $(-\infty, 3)$ and increasing on $(3, \infty)$. We estimate that the minimum is 1 at $x = 3$. There are no maxima.

22.



Increasing: $(-\infty, -4)$

Decreasing: $(-4, \infty)$

Maximum: 7 at $x =$

-4

Minima: none

23. If $x =$ the length of the rectangle, in meters, then the width is $\frac{480 - 2x}{2}$, or $240 - x$. We use the formula Area = length $\times$ width:

$$A(x) = x(240 - x), \text{ or}$$

$$A(x) = 240x - x^2$$

24. Let h = the height of the scarf, in inches. Then the length of the base $= 2h - 7$.

$$A(h) = \frac{1}{2}(2h - 7)(h)$$

$$A(h) = h^2 - \frac{7}{2}h$$

25. We use the Pythagorean theorem.

$$[h(d)]^2 + 3500^2 = d^2$$
$$[h(d)]^2 = d^2 - 3500^2$$
$$h(d) = \sqrt{d^2 - 3500^2}$$

We considered only the positive square root since distance must be nonnegative.

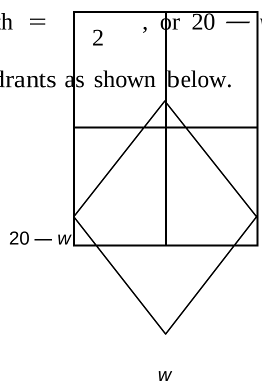
26. After t minutes, the balloon has risen $120t$ ft. We use the Pythagorean theorem.

$$[d(t)]^2 = (120t)^2 + 400^2$$

$$d(t) = \sqrt{(120t)^2 + 400^2}$$

We considered only the positive square root since distance must be nonnegative.

27. Let w = the width of the rectangle. Then the length $= \frac{40 - 2w}{2}$, or $20 - w$. Divide the rectangle into quadrants as shown below.



In each quadrant there are two congruent triangles. One triangle is part of the rhombus and both are part of the rectangle. Thus, in each quadrant the area of the rhombus is one-half the area of the rectangle. Then, in total, the area of the rhombus is one-half the area of the rectangle.

$$A(w) = \frac{1}{2}(20 - w)(w)$$

$$A(w) = 10w - \frac{w^2}{2}$$

28. Let w = the width, in feet. Then the length $= \frac{46 - 2w}{2}$, or $23 - w$.

$$A(w) = (23 - w)w$$
$$A(w) = 23w - w^2$$

$$\frac{3d}{7} = \frac{\frac{1}{2}}{\frac{S}{12}}$$

$$\frac{S}{12} \cdot 3d = 7 \cdot \frac{1}{2}$$

$$\frac{Sd}{4} = \frac{7}{2}$$

$$d = \frac{4}{S} \cdot \frac{7}{2}, \text{ so}$$

$$d(S) = \frac{14}{S}.$$

30. The volume of the tank is the sum of the volume of a sphere with radius r and a right circular cylinder with radius r and height 6 ft.

$$V(r) = \frac{4}{3}\pi r^3 + 6\pi r^2$$

31. a) After 4 pieces of float line, each of length X ft, are used for the sides perpendicular to the beach, there remains $(240 - 4X)$ ft of float line for the side parallel to the beach. Thus we have a rectangle with length $240 - 4X$ and width X. Then the total area of the three swimming areas is

$$A(X) = (240 - 4X)X, \text{ or } 240X - 4X^2.$$

b) The length of the sides labeled X must be positive and their total length must be less than 240 ft, so $4X < 240$, or $X < 60$. Thus the domain is $\{X|0 < X < 60\}$, or $(0, 60)$.

c) We see from the graph that the maximum value of the area function on the interval $(0, 60)$ appears to be 3600 when $X = 30$. Thus the dimensions that yield the maximum area are 30 ft by $240 - 4 \cdot 30$, or $240 - 120$, or 120 ft.

32. a) If the length $= X$ feet, then the width $= 24 - X$ feet.

$$A(X) = X(24 - X)$$
$$A(X) = 24X - X^2$$

b) The length of the rectangle must be positive and less than 24 ft, so the domain of the function is $\{X|0 < X < 24\}$, or $(0, 24)$.

c) We see from the graph that the maximum value of the area function on the interval $(0, 24)$ appears to be 144 when $X = 12$. Then the dimensions that yield the maximum area are length = 12 ft and width = $24 - 12$, or 12 ft.

33. a) When a square with sides of length X is cut from each corner, the length of each of the remaining sides of the piece of cardboard is $12 - 2X$. Then the di-

mensions of the box are X by $12 - 2X$ by $12 - 2X$. We use the formula Volume = length × width × height to find the volume of the box:

$$V(X) = (12 - 2X)(12 - 2X)(X)$$
$$V(X) = (144 - 48X + 4X^2)(X)$$
$$V(X) = 144X - 48X^2 + 4X^3$$

This can also be expressed as $V(X) = 4X(X - 6)^2$, or $V(X) = 4X(6 - X)^2$.

29. We will use similar triangles, expressing all distances in feet. $6 \text{ in.} = \frac{1}{2}$ ft, S in. $= \frac{S}{12}$ ft, and d yd $= 3d$ ft We have

b) The length of the sides of the square corners that are cut out must be positive and less than half the length of a side of the piece of cardboard. Thus, the domain of the function is $\{z|0 < z < 6\}$, or $(0, 6)$.

c) We see from the graph that the maximum value of the area function on the interval $(0, 6)$ appears to be 128 when $z = 2$. When $z = 2$, then $12 - 2z = 12 - 2 \cdot 2 = 8$, so the dimensions that yield the maximum volume are 8 cm by 8 cm by 2 cm.

34. a) $V(z) = 8z(14 - 2z)$, or $112z - 16z^2$

b) The domain is $\left\{z \,\middle|\, 0 < z < \frac{14}{2}\right\}$, or $\{z|0 < z < 7\}$, or $(0, 7)$.

c) The maximum occurs when $z = 3.5$, so the file should be 3.5 in. tall.

35. $g(z) = \begin{cases} z + 4, & \text{for } z < 1, \\ 8 - z, & \text{for } z > 1 \end{cases}$

Since $-4 < 1$, $g(-4) = -4 + 4 = 0$.

Since $3 > 1$, $g(3) = 8 - 3 = 5$.

$\begin{cases} 3, & \text{for } z < -2, \\ \frac{1}{2}z + 6, & \text{for } z > -2 \end{cases}$

$f(-5) = 3$

$f(-2) = 3$

$f(0) = \frac{1}{2} \cdot 0 + 6 = 6$

$f(2) = \frac{1}{2} \cdot 2 + 6 = 7$

37. $h(z) = 1$, for $-5 < z < 1$,

Since -5 is in the interval $[-5, 1)$, $h(-5) = 1$.

Since $1 > 1$, $h(1) = 1 + 2 = 3$.

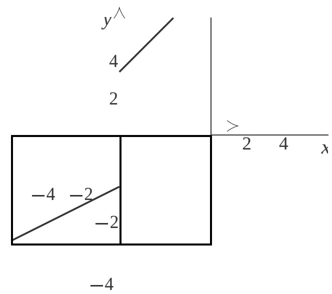
Since $4 > 1$, $h(4) = 4 + 2 = 6$.

38. $f(z) = \begin{cases} -5z - 8, & \text{for } z < -2, \\ \frac{1}{2}z + 5, & \text{for } -2 < z < 4, \\ 10 - 2z, & \text{for } z > 4 \end{cases}$

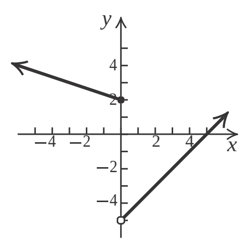
Since $-4 < -2$, $f(-4) = -5(-4) - 8 = 12$.

39. $f(z) = \begin{cases} \frac{1}{2}z, & \text{for } z < 0, \\ z + 3, & \text{for } z > 0 \end{cases}$

We create the graph in two parts. Graph $f(z) = \frac{1}{2}z$ for inputs z less than 0. Then graph $f(z) = z + 3$ for inputs z greater than or equal to 0.

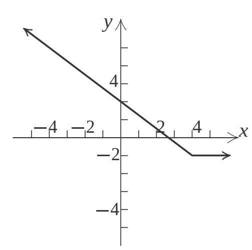


40. $f(z) = \begin{cases} -\frac{1}{3}z + 2, & \text{for } z < 0, \\ z - 5, & \text{for } z > 0 \end{cases}$

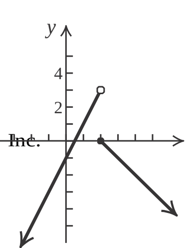


41. $f(z) = \begin{cases} -\frac{3}{4}z + 2, & \text{for } z < 4, \\ -1, & \text{for } z > 4 \end{cases}$

We create the graph in two parts. Graph $f(z) = -\frac{3}{4}z + 2$ for inputs z less than 4. Then graph $f(z) = -1$ for inputs z greater than or equal to 4.



42. $h(z) = \begin{cases} 2z - 1, & \text{for } z < 2 \\ 2 - z, & \text{for } z > 2 \end{cases}$



1

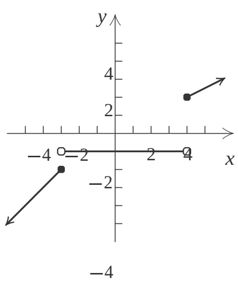
−4 −2 2 4 x
−2
−4

Since 4 is in the interval $[-2, 4]$, $f(4) = \frac{1}{2} \cdot 4 + 5 = 7$.

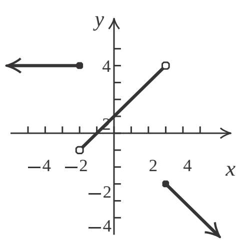
Since $6 > 4$, $f(6) = 10 - 2 \cdot 6 = -2$.

43. $f(a) = \begin{cases} a+1, & \text{for } a < -3, \\ -1, & \text{for } -3 < a < 4 \\ \frac{1}{2}a, & \text{for } a > 4 \end{cases}$

We create the graph in three parts. Graph $f(a) = a + 1$ for inputs a less than or equal to -3. Graph $f(a) = -1$ for inputs greater than -3 and less than 4. Then graph $f(a) = \frac{1}{2}a$ for inputs greater than or equal to 4.

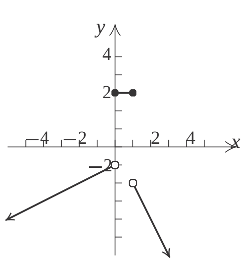


44. $f(a) = \begin{cases} 4, & \text{for } a < -2, \\ a+1, & \text{for } -2 < a < 3 \\ -a, & \text{for } a > 3 \end{cases}$

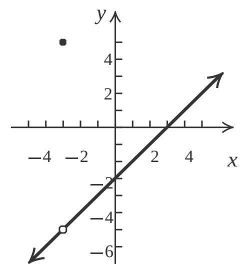


45. $g(a) = \begin{cases} \frac{1}{2}a - 1, & \text{for } a < 0, \\ 3, & \text{for } 0 < a < 1 \\ -2a, & \text{for } a > 1 \end{cases}$

We create the graph in three parts. Graph $g(a) = \frac{1}{2}a - 1$ for inputs less than 0. Graph $g(a) = 3$ for inputs greater than or equal to 0 and less than or equal to 1. Then graph $g(a) = -2a$ for inputs greater than 1.



46. $f(a) = \begin{cases} \frac{a^2 - 9}{a+3}, & \text{for } a = -3, \\ 5, & \text{for } a = -3 \end{cases}$



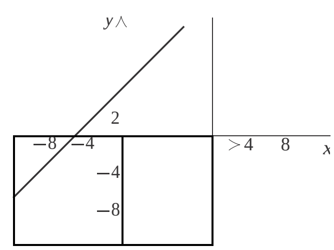
47. $f(a) = \begin{cases} 2, & \text{for } a = 5, \\ \frac{a^2 - 25}{a-5}, & \text{for } a = 5 \end{cases}$

When $a = 5$, the denominator of $(a^2 - 25)/(a - 5)$ is nonzero so we can simplify:

$$\frac{a^2 - 25}{a-5} = \frac{(a+5)(a-5)}{a-5} = a + 5.$$

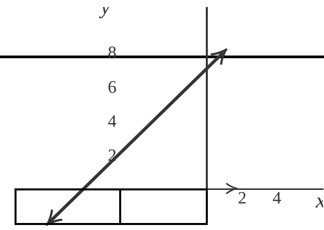
Thus, $f(a) = a + 5$, for $a = 5$.

The graph of this part of the function consists of a line with a "hole" at the point $(5, 10)$, indicated by an open dot. At $a = 5$, we have $f(5) = 2$, so the point $(5, 2)$ is



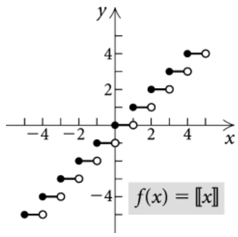
48. $f(a) = \begin{cases} \frac{a^2 + 3a + 2}{a+1}, & \text{for } a = -1, \\ 7, & \text{for } a = -1 \end{cases}$

−4


y
8
6
4
2
−4 −2
2 4
x

49. $f(O) = [[O]]$

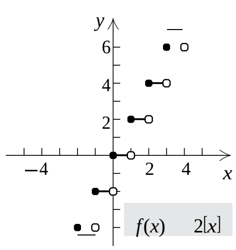
See Example 9.



50. $f(O) = 2[[O]]$

This function can be defined by a piecewise function with an infinite number of statements:

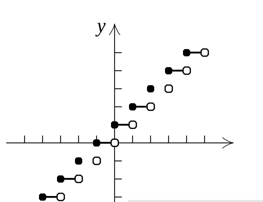
$$f(O) = \begin{cases} \vdots \\ -4, & \text{for } -2 < O < -1, \\ -2, & \text{for } -1 < O < 0, \\ 0, & \text{for } 0 < O < 1, \\ 2, & \text{for } 1 < O < 2, \\ \vdots \end{cases}$$



51. $f(O) = 1 + [[O]]$

This function can be defined by a piecewise function with an infinite number of statements:

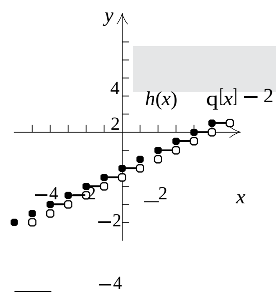
$$f(O) = \begin{cases} \vdots \\ -1, & \text{for } -2 < O < -1, \\ 0, & \text{for } -1 < O < 0, \\ 1, & \text{for } 0 < O < 1, \\ 2, & \text{for } 1 < O < 2, \\ \vdots \end{cases}$$



52. $f(O) = \frac{1}{2}[[O]] - 2$

This function can be defined by a piecewise function with an infinite number of statements:

$$f(O) = \begin{cases} \vdots \\ -2\frac{1}{2}, & \text{for } -1 < O < 0, \\ -2, & \text{for } 0 < O < 1, \\ -1\frac{1}{2}, & \text{for } 1 < O < 2, \\ -1, & \text{for } 2 < O < 3, \\ \vdots \end{cases}$$



53. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $(-\infty, 0) \cup [3, \infty)$.

54. Domain: $(-\infty, \infty)$; range: $(-5, \infty)$

55. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $[-1, \infty)$.

56. Domain: (∞, ∞); range: $(-\infty, 3)$

57. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $\{y \mid y < -2 \text{ or } y = -1 \text{ or } y > 2\}$.

58. Domain: $(-\infty, \infty)$; range: $(-\infty, -3] \cup (-1, 4]$

59. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $\{-5, -2, 4\}$. An equation for the function is:

$$f(O) = \begin{cases} -2, & \text{for } O < 2, \\ -5, & \text{for } O = 2, \\ 4, & \text{for } O > 2 \end{cases}$$

60. Domain: $(-\infty, \infty)$; range: $\{y \mid y = -3 \text{ or } y > 0\}$

$$g(O) = \begin{cases} -3, & \text{for } O < 0, \\ 0, & \text{for } O > 0 \end{cases}$$

61. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $(-\infty, -1] \cup [2, \infty)$. Finding the slope of each segment and using the slope-intercept or point-slope formula, we find that an equation for the function is:

$0, \quad \text{for } O < -1,$

4 —
2

−4 _ 2 4 x
−2

−4 $g(x)$ 1 x

$$g(O) = \begin{cases} 2, & \text{for } -1 < 0 < 2, \\ 0, & \text{for } 0 > 2 \end{cases}$$

This can also be expressed as follows:

$$g(O) = \begin{cases} 0, & \text{for } O < -1, \\ 2, & \text{for } -1 < 0 < 2, \\ 0, & \text{for } O > 2 \end{cases}$$

62. Domain: $(-\infty,\infty)$; range: $\{′]′ = -2$ *or* $′ > 0\}$. An equation for the function is:

$$h(c) = \begin{cases}]c], & \text{for } c < 3, \\ -2, & \text{for } c > 3 \end{cases}$$

This can also be expressed as follows:

$$h(c) = \begin{cases} -c, & \text{for } c < 0, \\ c, & \text{for } 0 < c < 3, \\ -2, & \text{for } c > 3 \end{cases}$$

It can also be expressed as follows:

$$h(c) = \begin{cases} -c, & \text{for } c < 0, \\ c, & \text{for } 0 < c < 3, \\ -2, & \text{for } c > 3 \end{cases}$$

63. From the graph we see that the domain is $[-5,3]$ and the range is $(-3,5)$. Finding the slope of each segment and using the slope-intercept or point-slope formula, we find that an equation for the function is:

$$h(c) = \begin{cases} c\,4\,8, & \text{for } -5 < c < -3, \\ 3, & \text{for } -3 < c < 1, \\ 3c - 6, & \text{for } 1 < c < 3 \end{cases}$$

64. Domain: $[-4,\infty)$; range: $[-2,4]$

$$f(c) = \begin{cases} -2c - 4, & \text{for } -4 < c < -1, \\ c - 1, & \text{for } -1 < c < 2, \\ 2, & \text{for } c > 2 \end{cases}$$

This can also be expressed as:

$$f(c) = \begin{cases} -2c - 4, & \text{for } -4 < c < -1, \\ c - 1, & \text{for } -1 < c < 2, \\ 2, & \text{for } c > 2 \end{cases}$$

65. $f(c) = 5c^2 - 7$

a) $f(-3) = 5(-3)^2 - 7 = 5 \cdot 9 - 7 = 45 - 7 = 38$

b) $f(3) = 5 \cdot 3^2 - 7 = 5 \cdot 9 - 7 = 45 - 7 = 38$

c) $f(a) = 5a^2 - 7$

d) $f(-a) = 5(-a)^2 - 7 = 5a^2 - 7$

66. $f(c) = 4c^3 - 5c$

a) $f(2) = 4 \cdot 2^3 - 5 \cdot 2 = 4 \cdot 8 - 5 \cdot 2 = 32 - 10 = 22$

b) $f(-2) = 4(-2)^3 - 5(-2) = 4(-8) - 5(-2) = -32\,4\,10 = -22$

c) $f(a) = 4a^3 - 5a$

d) $f(-a) = 4(-a)^3 - 5(-a) = 4(-a^3) - 5(-a) = -4a^3\,4\,5a$

67. First find the slope of the given line.

$$8c - ′ = 10$$
$$8c = ′\,4\,10$$
$$8c - 10 = ′$$

The slope of the given line is 8. The slope of a line per- pendicular to this line is the opposite of the reciprocal of 8, or $-\frac{1}{8}$.

$$′ - ′_1 = m(c - c_1)$$
$$′ - 1 = -\frac{1}{8}[c - (-1)]$$
$$′ - 1 = -\frac{1}{8}(c\,4\,1)$$
$$′ - 1 = -\frac{1}{8}c - \frac{1}{8}$$
$$′ = -\frac{1}{8}c\,4\,\frac{7}{8}$$

68. $2c - 9′\,4\,1 = 0$

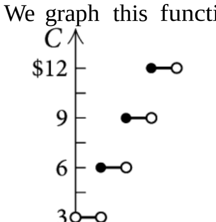
$$2c\,4\,1 = 9′$$
$$\frac{2}{9}c\,4\,\frac{1}{9} = ′$$

Slope: $\frac{2}{9}$; $′$-intercept: $\left(0, \frac{1}{9}\right)$

69. a) The function $C(t)$ can be defined piecewise.

$$C(t) = \begin{cases} 3, & \text{for } 0 < t < 1, \\ 6, & \text{for } 1 < t < 2, \\ 9, & \text{for } 2 < t < 3, \\ \vdots \end{cases}$$

We graph this function.



b) From the definition of the function in part (a), we see that it can be written as

$$C(t) = 3[[t]] \, 4 \, 1, \ t > 0.$$

70. If $[[c \, 4 \, 2]] = -3$, then $-3 < c \, 4 \, 2 < -2$, or $-5 < c < -4$. The possible inputs for c are $\{c] - 5 < c < -4\}$.

71. If $[[c]]^2 = 25$, then $[[c]] = -5$ or $[[c]] = 5$. For $-5 < c < -4$, $[[c]] = -5$. For $5 < c < 6$, $[[c]] = 5$. Thus, the possible inputs for c are $\{c] - 5 < c < -4 \text{ or } 5 < c < 6\}$.

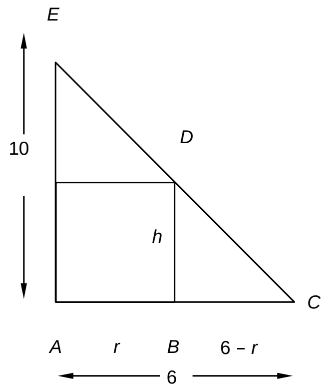
72. a) The distance from A to S is $4 - c$.

Using the Pythagorean theorem, we find that the distance from S to C is $\sqrt{1 \, 4 \, c^2}$.

Then $C(c) = 3000(4 - c) \, 4 \, 5000\sqrt{1 \, 4 \, c^2}$, or $12{,}000 - 3000c \, 4 \, 5000\sqrt{1 \, 4 \, c^2}$.

b) Use a graphing calculator to graph $' = 12{,}000 - 3000c \, 4 \, 5000\sqrt{1 \, 4 \, c^2}$ in a window such as $[0, 5, 10{,}000, 20{,}000]$, Xscl = 1, Yscl = 1000. Using the MINIMUM feature, we find that cost is mini- mized when $c = 0{\bullet}75$, so the line should come to shore 0.75 mi from B.

73. a) We add labels to the drawing in the text.



We write a proportion involving the lengths of the sides of the similar triangles BCD and ACE. Then we solve it for h.

$$\frac{h}{6-r} = \frac{10}{6}$$

$$h = \frac{10}{6}(6-r) = \frac{5}{3}(6-r)$$

$$h = \frac{30-5r}{3}$$

Thus, $h(r) = \dfrac{30-5r}{3}$.

b) $V = \pi r^2 h$

$V(r) = \pi r^2 \left(\dfrac{30-5r}{3}\right)$ Substituting for h

c) We first express r in terms of h.

$$h = \frac{30-5r}{3}$$

$$3h = 30 - 5r$$

$$5r = 30 - 3h$$

$$r = \frac{30-3h}{5}$$

$$V = \pi r^2 h$$

$V(h) = \pi \left(\dfrac{30-3h}{5}\right)^2 h$

Substituting for r

We can also write $V(h) = \pi h \left(\dfrac{30-3h}{5}\right)^2$.

Exercise Set 2.2

1. $(f 4 g)(5) = f(5) 4 g(5)$
$= (5^2 - 3) 4 (2 \cdot 5 4 1)$

3. $(f - g)(-1) = f(-1) - g(-1)$
$= ((-1)^2 - 3) - (2(-1) 4 1)$
$= -2 - (-1) = -2 4 1$
$= -1$

4. $(fg)(2) = f(2) \cdot g(2)$
$= (2^2 - 3)(2 \cdot 2 4 1)$
$= 1 \cdot 5 = 5$

5. $(f/g)\left(-\dfrac{1}{2}\right) = \dfrac{f\left(-\frac{1}{2}\right)}{g\left(-\frac{1}{2}\right)}$
$= \dfrac{\left(-\frac{1}{2}\right)^2 - 3}{2\left(-\frac{1}{2}\right) 4 1}$
$= \dfrac{\frac{1}{4} - 3}{-1 4 1}$
$= \dfrac{-\frac{11}{4}}{0}$

Since division by 0 is not defined, $(f/g)\left(-\dfrac{1}{2}\right)$ does not exist.

6. $(f - g)(0) = f(0) - g(0)$
$= (0^2 - 3) - (2 \cdot 0 4 1)$
$= -3 - 1 = -4$

7. $(fg)\left(-\dfrac{1}{2}\right) = f\left(-\dfrac{1}{2}\right) \cdot g\left(-\dfrac{1}{2}\right)$
$= \left[\left(-\dfrac{1}{2}\right)^2 - 3\right]\left[2\left(-\dfrac{1}{2}\right) 4 1\right]$
$= -\dfrac{11}{4} \cdot 0 = 0$

8. $(f/g)(-\sqrt{3}) = \dfrac{f(-\sqrt{3})}{g(-\sqrt{3})}$
$= \dfrac{(-\sqrt{3})^2 - 3}{2(-\sqrt{3}) 4 1}$
$= \dfrac{0}{-2\sqrt{3} 4 1} = 0$

9. $(g-f)(-1) = g(-1) - f(-1)$

$= 25 - 3\ 4\ 10\ 4\ 1$

$= 33$

2. $(fg)(0) = f(0) \cdot g(0)$

$= (0^2 - 3)(2 \cdot 0\ 4\ 1)$

$= -3(1) = -3$

$= [2(-1)\ 4\ 1] - [(-1)^2 - 3]$

$= (-2\ 4\ 1) - (1-3)$

$= -1 - (-2)$

$= -1\ 4\ 2$

$= 1$

10. $(g/f)\left(-\frac{1}{2}\right) = \dfrac{g\left(-\frac{1}{2}\right)}{f\left(-\frac{1}{2}\right)}$

$= \dfrac{2\left(-\frac{1}{2}\right)+1}{\left(-\frac{1}{2}\right)^2 - 3}$

$= \dfrac{0}{-\frac{11}{4}}$

$= 0$

11. $(h-g)(-4) = h(-4) - g(-4)$

$= (-4+4) - \sqrt{-4-1}$

$= 0 - \sqrt{-5}$

Since $\sqrt{-5}$ is not a real number, $(h-g)(-4)$ does not exist.

12. $(gh)(10) = g(10)\cdot h(10)$

$= \sqrt{10-1}(10+4)$

$= \sqrt{9}(14)$

$= 3\cdot 14 = 42$

13. $(g/h)(1) = \dfrac{g(1)}{h(1)}$

$= \dfrac{\sqrt{1-1}}{1+4}$

$= \dfrac{\sqrt{0}}{5}$

$= \dfrac{0}{5} = 0$

14. $(h/g)(1) = \dfrac{h(1)}{g(1)}$

$= \dfrac{1+4}{\sqrt{1-1}}$

$= \dfrac{5}{0}$

Since division by 0 is not defined, $(h/g)(1)$ does not exist.

15. $(g+h)(1) = g(1) + h(1)$

$= \sqrt{1-1} + (1+4)$

$= \sqrt{0} + 5$

$= 0 + 5 = 5$

16. $(hg)(3) = h(3)\cdot g(3)$

$= (3+4)\sqrt{3-1}$

$= 7\sqrt{2}$

$-\frac{3}{2}$ since $f\left(-\frac{3}{2}\right) = 0$. The domain of g/f is $\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, \infty\right)$.

b) $(f+g)(c) = f(c) + g(c) = (2c+3) + (3-5c) = -3c+6$

$(f-g)(c) = f(c) - g(c) = (2c+3) - (3-5c) = 2c+3-3+5c = 7c$

$(fg)(c) = f(c)\cdot g(c) = (2c+3)(3-5c) = 6c - 10c^2 + 9 - 15c = -10c^2 - 9c + 9$

$(ff)(c) = f(c)\cdot f(c) = (2c+3)(2c+3) = 4c^2 + 12c + 9$

$(f/g)(c) = \dfrac{f(c)}{g(c)} = \dfrac{2c+3}{3-5c}$

$(g/f)(c) = \dfrac{g(c)}{f(c)} = \dfrac{3-5c}{2c+3}$

18. $f(c) = -c+1$, $g(c) = 4c-2$

a) The domain of f, g, $f+g$, $f-g$, fg, and ff is $(-\infty, \infty)$. Since $g\left(\frac{1}{2}\right) = 0$, the domain of f/g is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$. Since $f(1) = 0$, the domain of g/f is $(-\infty, 1) \cup (1, \infty)$.

b) $(f+g)(c) = (-c+1) + (4c-2) = 3c - 1$

$(f-g)(c) = (-c+1) - (4c-2) = -c+1-4c+2 = -5c+3$

$(fg)(c) = (-c+1)(4c-2) = -4c^2 + 6c - 2$

$(ff)(c) = (-c+1)(-c+1) = c^2 - 2c + 1$

$(f/g)(c) = \dfrac{-c+1}{4c-2}$

$(g/f)(c) = \dfrac{4c-2}{-c+1}$

19. $f(c) = c - 3$, $g(c) = \sqrt{c+4}$

a) Any number can be an input in f, so the domain of f is the set of all real numbers, or $(-\infty, \infty)$.

The domain of g consists of all values of c for which $c+4$ is nonnegative, so we have $c+4 \geq 0$, or $c \geq -4$. Thus, the domain of g is $[-4, \infty)$.

17. $f(c) = 2c + 3$, $g(c) = 3 - 5c$

a) The domain of f and of g is the set of all real numbers, or $(-\infty, \infty)$. Then the domain of $f + g$, $f - g$, ff, and fg is also $(-\infty, \infty)$. For f/g we must exclude $\frac{3}{5}$ since $g\left(\frac{3}{5}\right) = 0$. Then the domain of f/g is

$\left(-\infty, \frac{3}{5}\right) \cup \left(\frac{3}{5}, \infty\right)$. For g/f we must exclude

The domain of $f + g$, $f - g$, and fg is the set of all numbers in the domains of both f and g. This is $[-4, \infty)$.

The domain of ff is the domain of f, or $(-\infty, \infty)$.

The domain of f/g is the set of all numbers in the domains of f and g, excluding those for which $g(c) = 0$. Since $g(-4) = 0$, the domain of f/g is $(-4, \infty)$.

The domain of g/f is the set of all numbers in the domains of g and f, excluding those for which $f(c) = 0$. Since $f(3) = 0$, the domain of g/f is $[-4, 3) \cup (3, \infty)$.

b) $(f \, 4 \, g)(c) = f(c) \, 4 \, g(c) = c - 3 \, 4 \sqrt{c \, 4 \, 4}$

$(f - g)(c) = f(c) - g(c) = c - 3 - \sqrt{c \, 4 \, 4}$

$(fg)(c) = f(c) \cdot g(c) = (c - 3)\sqrt{c \, 4 \, 4}$

$(ff)(c) = [f(c)]^2 = (c - 3)^2 = c^2 - 6c \, 4 \, 9$

$(f/g)(c) = \frac{f(c)}{g(c)} = \frac{c - 3}{\sqrt{c \, 4 \, 4}}$

$(g/f)(c) = \frac{g(c)}{f(c)} = \frac{\sqrt{c \, 4 \, 4}}{c - 3}$

20. $f(c) = c \, 4 \, 2$, $g(c) = \sqrt{c - 1}$

a) The domain of f is (−∞» ∞). The domain of g consists of all the values of c for which $c - 1$ is nonnegative, or [1» ∞). Then the domain of $f \, 4 \, g$, $f - g$, and fg is [1» ∞). The domain of ff is (−∞» ∞). Since $g(1) = 0$, the domain of f/g is (1» ∞). Since $f(-2) = 0$ and -2 is not in the domain of g, the domain of g/f is [1» ∞).

b) $(f \, 4 \, g)(c) = c \, 4 \, 2 \, 4 \sqrt{c - 1}$

$(f - g)(c) = c \, 4 \, 2 - \sqrt{c - 1}$

$(fg)(c) = (c \, 4 \, 2)\sqrt{c - 1}$

$(ff)(c) = (c \, 4 \, 2)(c \, 4 \, 2) = c^2 \, 4 \, 4c \, 4 \, 4$

$(f/g)(c) = \frac{c \, 4 \, 2}{\sqrt{c - 1}}$

$(g/f)(c) = \frac{\sqrt{c - 1}}{c \, 4 \, 2}$

21. $f(c) = 2c - 1$, $g(c) = -2c^2$

a) The domain of f and of g is (−∞» ∞). Then the domain of $f \, 4 \, g$, $f - g$, fg, and ff is (−∞» ∞). For f/g, we must exclude 0 since $g(0) = 0$. The domain of f/g is (−∞» 0) ∪ (0» ∞). For g/f, we must exclude $\frac{1}{2}$ since $f\left(\frac{1}{2}\right) = 0$. The domain of g/f is $\left(-\infty» \frac{1}{2}\right) \cup \left(\frac{1}{2}» \infty\right)$.

b) $(f \, 4 \, g)(c) = f(c) \, 4 \, g(c) = (2c - 1) \, 4 \, (-2c^2) = -2c^2 \, 4 \, 2c - 1$

$(f - g)(c) = f(c) - g(c) = (2c - 1) - (-2c^2) = 2c^2 \, 4 \, 2c - 1$

$(fg)(c) = f(c) \cdot g(c) = (2c - 1)(-2c^2) = -4c^3 \, 4 \, 2c^2$

$(ff)(c) = f(c) \cdot f(c) = (2c - 1)(2c - 1) = 4c^2 - 4c \, 4 \, 1$

b) $(f \, 4 \, g)(c) = c^2 - 1 \, 4 \, 2c \, 4 \, 5 = c^2 \, 4 \, 2c \, 4 \, 4$

$(f - g)(c) = c^2 - 1 - (2c \, 4 \, 5) = c^2 - 2c - 6$

$(fg)(c) = (c^2 - 1)(2c \, 4 \, 5) = 2c^3 \, 4 \, 5c^2 - 2c - 5$

$(ff)(c) = (c^2 - 1)^2 = c^4 - 2c^2 \, 4 \, 1$

$(f/g)(c) = \frac{c^2 - 1}{2c \, 4 \, 5}$

$(g/f)(c) = \frac{2c \, 4 \, 5}{c^2 - 1}$

23. $f(c) = \sqrt{c - 3}$, $g(c) = \sqrt{c \, 4 \, 3}$

a) Since $f(c)$ is nonnegative for values of c in [3» ∞), this is the domain of f. Since $g(c)$ is nonnegative for values of c in [−3» ∞), this is the domain of g. The domain of $f \, 4 \, g$, $f - g$, and fg is the intersection of the domains of f and g, or [3» ∞). The domain of ff is the same as the domain of f, or [3» ∞). For f/g, we must exclude −3 since $g(-3) = 0$. This is not in [3» ∞), so the domain of f/g is [3» ∞). For g/f, we must exclude 3 since $f(3) = 0$. The domain of g/f is (3» ∞).

b) $(f \, 4 \, g)(c) = f(c) \, 4 \, g(c) = \sqrt{c - 3} \, 4 \sqrt{c \, 4 \, 3}$

$(f - g)(c) = f(c) - g(c) = \sqrt{c - 3} - \sqrt{c \, 4 \, 3}$

$(fg)(c) = f(c) \cdot g(c) = \sqrt{c - 3} \cdot \sqrt{c \, 4 \, 3} = \sqrt{c^2 - 9}$

$(ff)(c) = f(c) \cdot f(c) = \sqrt{c - 3} \cdot \sqrt{c - 3} =]c - 3]$

$(f/g)(c) = \frac{\sqrt{c - 3}}{\sqrt{c \, 4 \, 3}}$

$(g/f)(c) = \frac{\sqrt{c \, 4 \, 3}}{\sqrt{c - 3}}$

24. $f(c) = \sqrt{c}$, $g(c) = \sqrt{2 - c}$

a) The domain of f is [0» ∞). The domain of g is (−∞» 2]. Then the domain of $f \, 4 \, g$, $f - g$, and fg is [0» 2]. The domain of ff is the same as the domain of f, [0» ∞). Since $g(2) = 0$, the domain of f/g is [0» 2). Since $f(0) = 0$, the domain of g/f is (0» 2].

b) $(f \, 4 \, g)(c) = \sqrt{c} \, 4 \sqrt{2 - c}$

$(f - g)(c) = \sqrt{c} - \sqrt{2 - c}$

$(fg)(c) = \sqrt{c} \cdot \sqrt{2 - c} = \sqrt{2c - c^2}$

$(ff)(c) = \sqrt{c} \cdot \sqrt{c} = \sqrt{c^2} =]c]$

$$(f/g)(c) = \frac{f(c)}{g(c)} = \frac{2c-1}{-2c^2}$$

$$(g/f)(c) = \frac{g(c)}{f(c)} = \frac{-2c^2}{2c-1}$$

22. $f(c) = c^2 - 1$, $g(c) = 2c + 5$

a) The domain of f and of g is the set of all real numbers, or $(-\infty, \infty)$. Then the domain of $f + g$, $f - g$, fg and ff is $(-\infty, \infty)$. Since $g\left(-\frac{5}{2}\right) = 0$, the domain of f/g is $\left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, \infty\right)$. Since $f(1) = 0$ and $f(-1) = 0$, the domain of g/f is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

$$(f/g)(c) = \frac{\sqrt{c}}{\sqrt{2-c}}$$

$$(g/f)(c) = \frac{\sqrt{2-c}}{\sqrt{c}}$$

25. $f(c) = c + 1$, $g(c) = |c|$

a) The domain of f and of g is $(-\infty, \infty)$. Then the domain of $f + g$, $f - g$, fg, and ff is $(-\infty, \infty)$.

For f/g, we must exclude 0 since $g(0) = 0$. The domain of f/g is $(-\infty, 0) \cup (0, \infty)$. For g/f, we must exclude -1 since $f(-1) = 0$. The domain of g/f is $(-\infty, -1) \cup (-1, \infty)$.

b) $(f\,4\,g)(c) = f(c)\,4\,g(c) = c\,4\,1\,4\;]c]$
$(f-g)(c) = f(c) - g(c) = c\,4\,1 -]c]$
$(fg)(c) = f(c)\cdot g(c) = (c\,4\,1)]c]$
$(ff)(c) = f(c)\cdot f(c) = (c41)(c41) = c^2\,4\,2c\,4\,1$

$(f/g)(c) = \dfrac{c\,4\,1}{]c]}$

$(g/f)(c) = \dfrac{]c]}{c\,4\,1}$

26. $f(c) = 4]c],\; g(c) = 1 - c$

a) The domain of f and of g is $(-\infty\text{»}\,\infty)$. Then the domain of $f4g,\, f-g,\, fg$, and ff is $(-\infty\text{»}\,\infty)$. Since $g(1) = 0$, the domain of f/g is $(-\infty\text{»}\,1)\cup(1\text{»}\,\infty)$. Since $f(0) = 0$, the domain of g/f is $(-\infty\text{»}\,0)\cup(0\text{»}\,\infty)$.

b) $(f\,4\,g)(c) = 4]c]\,4\,1 - c$
$(f-g)(c) = 4]c] - (1 - c) = 4]c] - 1\,4\,c$
$(fg)(c) = 4]c](1 - c) = 4]c] - 4c]c]$
$(ff)(c) = 4]c]\cdot 4]c] = 16c^2$

$(f/g)(c) = \dfrac{4]c]}{1 \quad c}$
$-$

$(g/f)(c) = \dfrac{1-\;c}{4\,c}$
]]

27. $f(c) = c^3,\; g(c) = 2c^2\,4\,5c - 3$

a) Since any number can be an input for either f or g, the domain of $f,\, g,\, f\,4\,g,\, f-g,\, fg$, and ff is the set of all real numbers, or $(-\infty\text{»}\,\infty)$.

Since $g(-3) = 0$ and $g\left(\frac{1}{2}\right) = 0$, the domain of f/g is $(-\infty\text{»}\,-3)\cup\left(-3\text{»}\,\frac{1}{2}\right)\cup\left(\frac{1}{2}\text{»}\,\infty\right)$.

Since $f(0) = 0$, the domain of g/f is $(-\infty\text{»}\,0)\cup(0\text{»}\,\infty)$.

b) $(f\,4\,g)(c) = f(c)\,4\,g(c) = c^3\,4\,2c^2\,4\,5c - 3$

$(f-g)(c) = f(c) - g(c) = c^3 - (2c^2 4 5c - 3) =$
$c^3 - 2c^2 - 5c\,4\,3$
$(fg)(c) = f(c)\cdot g(c) = c^3(2c^2\,4\,5c - 3) =$
$2c^5\,4\,5c^4 - 3c^3$
$(ff)(c) = f(c)\cdot f(c) = c^3\cdot c^3 = c^6$

b) $(f\,4\,g)(c) = c^2 - 4\,4\,c^3$» or $c^3\,4\,c^2 - 4$
$(f-g)(c) = c^2 - 4 - c^3$» or $-c^3\,4\,c^2 - 4$
$(fg)(c) = (c^2 - 4)(c^3) = c^5 - 4c^3$
$(ff)(c) = (c^2 - 4)(c^2 - 4) = c^4 - 8c^2\,4\,16$

$(f/g)(c) = \dfrac{{}^2 - 4}{c^3}$

$(g/f)(c) = \dfrac{c^{3}}{c^2 - 4}$

29. $f(c) = \dfrac{4}{c\,4\,1},\; g(c) = \dfrac{1}{6 - c}$

a) Since $c\,4\,1 = 0$ when $c = -1$, we must exclude -1 from the domain of f. It is $(-\infty\text{»}\,-1)\cup(-1\text{»}\,\infty)$. Since $6 - c = 0$ when $c = 6$, we must exclude 6 from the domain of g. It is $(-\infty\text{»}\,6)\cup(6\text{»}\,\infty)$. The domain of $f\,4\,g$, $f - g$, and fg is the intersection of the domains of f and g, or $(-\infty\text{»}\,-1)\cup(-1\text{»}\,6)\cup(6\text{»}\,\infty)$. The domain of ff is the same as the domain of f, or $(-\infty\text{»}\,-1)\cup(-1\text{»}\,\infty)$. Since there are no values of c for which $g(c) = 0$ or $f(c) = 0$, the domain of f/g and g/f is $(-\infty\text{»}\,-1)\cup(-1\text{»}\,6)\cup(6\text{»}\,\infty)$.

b) $(f\,4\,g)(c) = f(c)\,4\,g(c) = \dfrac{4}{c\,4\,1}\;4\;\dfrac{1}{6 - c}$

$(f-g)(c) = f(c) - g(c) = \dfrac{4}{c\,4\,1} - \dfrac{1}{6 - c}$

$(fg)(c) = f(c)\cdot g(c) = \dfrac{4}{c41}\cdot\dfrac{1}{6-c} = \dfrac{4}{(c41)(6-c)}$

$(ff)(c) = f(c)\;f(c) = \dfrac{4}{c\,4\,1}\cdot\dfrac{4}{c\,4\,1} = \dfrac{16}{(c\,4\,1)^2}$» or

$\dfrac{16}{c^2\,4\,2c\,4\,1}$

$(f/g)(c) = \dfrac{\frac{4}{c\,4\,1}}{\frac{1}{6 - c}} = \dfrac{4}{c\,4\,1}\cdot\dfrac{6-\;c}{1} = \dfrac{4(6-\;c)}{c\,4\,1}$

$(g/f)(c) = \dfrac{\frac{1}{6-\;c}}{\frac{4}{c\,4\,1}} = \dfrac{1}{6 - c}\cdot\dfrac{c\,4\,1}{4} = \dfrac{c\,4\,1}{4(6 - c)}$

2 2

3

30. $f(c) = 2c$, $g(c) =$

$$(f/g)(c) = \frac{f(c)}{g(c)} = \frac{c}{2c^2 \, 4 \, 5c - 3}$$

$$(g/f)(c) = \frac{g(c)}{f(c)} = \frac{2c^2 \;\; 45c - 3}{c^3}$$

28. $f(c) = c^2 - 4$, $g(c) = c^3$

a) The domain of f and of g is $(-\infty, \infty)$. Then the domain of $f 4 g$, $f - g$, fg, and ff is $(-\infty, \infty)$. Since $g(0) = 0$, the domain of f/g is $(-\infty, 0) \cup (0, \infty)$. Since $f(-2) = 0$ and $f(2) = 0$, the domain of g/f is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

$c - 5$

a) The domain of f is $(-\infty, \infty)$. Since $c - 5 = 0$ when $c = 5$, the domain of g is $(-\infty, 5) \cup (5, \infty)$. Then the domain of $f 4 g$, $f - g$, and fg is $(-\infty, 5) \cup (5, \infty)$. The domain of ff is $(-\infty, \infty)$. Since there are no values of c for which $g(c) = 0$, the domain of f/g is $(-\infty, 5) \cup (5, \infty)$. Since $f(0) = 0$, the domain of g/f is $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$.

b) $(f 4 g)(c) = 2c \;\; 4^2 \frac{2}{c - 5}$

$$(f - g)(c) = 2c^2 - \frac{2}{c - 5}$$

$$(fg)(c) = 2c^2 \cdot \frac{2}{c - 5} = \frac{4c^2}{c - 5}$$

$$(ff)(c) = 2c^2 \cdot 2c^2 = 4c^4$$

$$(f/g)(c) = \frac{2c^2}{\frac{2}{c-5}} = 2c^2 \cdot \frac{c-5}{2} = c^2(c-5) = c^3 - 5c^2$$

$$(g/f)(c) = \frac{\frac{2}{c-5}}{2c^2} = \frac{2}{c-5} \cdot \frac{1}{2c^2} = \frac{1}{c^2(c-5)} = \frac{1}{c^3 - 5c^2}$$

31. $f(c) = \frac{1}{c}$, $g(c) = c - 3$

a) Since $f(0)$ is not defined, the domain of f is $(-\infty, 0) \cup (0, \infty)$. The domain of g is $(-\infty, \infty)$. Then the domain of $f\,4\,g$, $f - g$, fg, and ff is $(-\infty, 0) \cup (0, \infty)$. Since $g(3) = 0$, the domain of f/g is $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$. There are no values of c for which $f(c) = 0$, so the domain of g/f is $(-\infty, 0) \cup (0, \infty)$.

b) $(f\,4\,g)(c) = f(c)\,4\,g(c) = \frac{1}{c}\,4\,c - 3$

$(f-g)(c) = f(c) - g(c) = \frac{1}{c} - (c-3) = \frac{1}{c} - c\,4\,3$

$(fg)(c) = f(c) \cdot g(c) = \frac{1}{c} \cdot (c-3) = \frac{c-3}{c}$, or $1 - \frac{3}{c}$

$(ff)(c) = f(c) \cdot f(c) = \frac{1}{c} \cdot \frac{1}{c} = \frac{1}{c^2}$

$(f/g)(c) = \frac{f(c)}{g(c)} = \frac{\frac{1}{c}}{c-3} = \frac{1}{c} \cdot \frac{1}{c-3} = \frac{1}{c(c-3)}$

$(g/f)(c) = \frac{g(c)}{f(c)} = \frac{c-3}{\frac{1}{c}} = (c-3) \cdot \frac{c}{1} = c(c-3)$, or $c^2 - 3c$

32. $f(c) = \sqrt{c\,4\,6}$, $g(c) = \frac{1}{c}$

a) The domain of $f(c)$ is $[-6, \infty)$. The domain of $g(c)$ is $(-\infty, 0) \cup (0, \infty)$. Then the domain of $f\,4\,g$, $f - g$, and fg is $[-6, 0) \cup (0, \infty)$. The domain of ff is $[-6, \infty)$. Since there are no values of c for which $g(c) = 0$, the domain of f/g is $[-6, 0) \cup (0, \infty)$. Since $f(-6) = 0$, the domain of g/f is $(-6, 0) \cup (0, \infty)$.

33. $f(c) = \frac{3}{c-2}$, $g(c) = \sqrt{c-1}$

a) Since $f(2)$ is not defined, the domain of f is $(-\infty, 2) \cup (2, \infty)$. Since $g(c)$ is nonnegative for values of c in $[1, \infty)$, this is the domain of g. The domain of $f\,4\,g$, $f - g$, and fg is the intersection of the domains of f and g, or $[1, 2) \cup (2, \infty)$. The domain of ff is the same as the domain of f, or $(-\infty, 2) \cup (2, \infty)$. For f/g, we must exclude 1 since $g(1) = 0$, so the domain of f/g is $(1, 2) \cup (2, \infty)$. There are no values of c for which $f(c) = 0$, so the domain of g/f is $[1, 2) \cup (2, \infty)$.

b) $(f\,4\,g)(c) = f(c)\,4\,g(c) = \frac{3}{c-2}\,4\,\sqrt{c-1}$

$(f-g)(c) = f(c) - g(c) = \frac{3}{c-2} - \sqrt{c-1}$

$(fg)(c) = f(c) \cdot g(c) = \frac{3}{c-2}(\sqrt{c-1})$, or $\frac{3\sqrt{c-1}}{c-2}$

$(ff)(c) = f(c) \cdot f(c) = \frac{3}{c-2} \cdot \frac{3}{c-2} = \frac{9}{(c-2)^2}$

$(f/g)(c) = \frac{f(c)}{g(c)} = \frac{\frac{3}{c-2}}{\sqrt{c-1}} = \frac{3}{(c-2)\sqrt{c-1}}$

$(g/f)(c) = \frac{g(c)}{f(c)} = \frac{\sqrt{c-1}}{\frac{3}{c-2}} = \frac{(c-2)\sqrt{c-1}}{3}$

34. $f(c) = \frac{2}{4-c}$, $g(c) = \frac{5}{c-1}$

a) The domain of f is $(-\infty, 4) \cup (4, \infty)$. The domain of g is $(-\infty, 1) \cup (1, \infty)$. The domain of $f\,4\,g$, $f - g$, and fg is $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$. The domain of ff is $(-\infty, 4) \cup (4, \infty)$. The domain of f/g and of g/f is $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$.

b) $(f\,4\,g)(c) = \frac{2}{4-c}\,4\,\frac{5}{c-1}$

$(f-g)(c) = \frac{2}{4-c} - \frac{5}{c-1}$

b) $(f+g)(c) = \sqrt{c+6} + \frac{1}{c}$

$(f-g)(c) = \sqrt{c+6} - \frac{1}{c}$

$(fg)(c) = \sqrt{c+6} \cdot \frac{1}{c} = \frac{\sqrt{c+6}}{c}$

$(ff)(c) = \sqrt{c+6} \cdot \sqrt{c+6} = c+6$

$(f/g)(c) = \frac{\sqrt{c+6}}{\frac{1}{c}} = \sqrt{c+6} \cdot \frac{c}{1} = c\sqrt{c+6}$

$(g/f)(c) = \frac{\frac{1}{c}}{\sqrt{c+6}} = \frac{1}{c} \cdot \frac{1}{\sqrt{c+6}} = \frac{1}{c\sqrt{c+6}}$

$(fg)(c) = \frac{2}{4-c} \cdot \frac{5}{c-1} = \frac{10}{(4-c)(c-1)}$

$(ff)(c) = \frac{2}{4-c} \cdot \frac{2}{4-c} = \frac{4}{(4-c)^2}$

$(f/g)(c) = \frac{\frac{2}{4-c}}{\frac{5}{c-1}} = \frac{2(c-1)}{5(4-c)}$

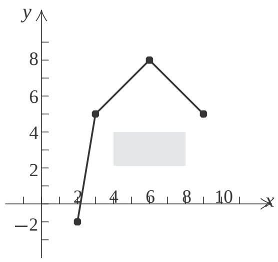
$(g/f)(c) = \frac{\frac{5}{c-1}}{\frac{2}{4-c}} = \frac{5(4-c)}{2(c-1)}$

35. From the graph we see that the domain of F is [2, 11] and the domain of G is [1, 9]. The domain of $F+G$ is the set of numbers in the domains of both F and G. This is [2, 9].

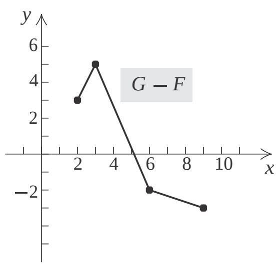
36. The domain of $F-G$ and FG is the set of numbers in the domains of both F and G. (See Exercise 35.) This is [2, 9].

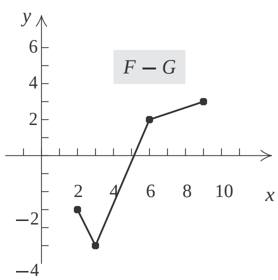
The domain of F/G is the set of numbers in the domains of both F and G, excluding those for which $G = 0$. Since $G > 0$ for all values of ¢ in its domain, the domain of F/G is $[2, 9]$.

37. The domain of G/F is the set of numbers in the domains of both F and G (See Exercise 35.), excluding those for which $F = 0$. Since $F(3) = 0$, the domain of G/F is $[2, 3) \cup (3, 9]$.



39.





41. From the graph, we see that the domain of F is $[0, 9]$ and the domain of G is $[3, 10]$. The domain of $F + G$ is the set of numbers in the domains of both F and G. This is $[3, 9]$.

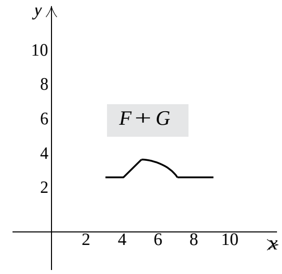
42. The domain of $F - G$ and FG is the set of numbers in the domains of both F and G. (See Exercise 41.) This is $[3, 9]$.

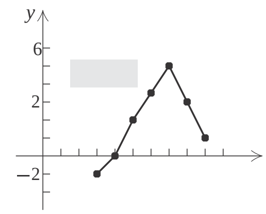
The domain of F/G is the set of numbers in the domains of both F and G, excluding those for which $G = 0$. Since $G > 0$ for all values of ¢ in its domain, the domain of F/G is $[3, 9]$.

43. The domain of G/F is the set of numbers in the domains

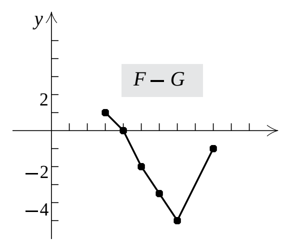
of both F and G (See Exercise 41.), excluding those for which $F = 0$. Since $F(6) = 0$ and $F(8) = 0$, the domain of G/F is $[3, 6) \cup (6, 8) \cup (8, 9]$.

44. $(F + G)(¢) = F(¢) + G(¢)$





46.



47. a) $P(¢) = R(¢) — C(¢) = 60¢ — 0.4¢^2 — (3¢ + 13) =$

$6000 — 4000 = 2000$

$C(100) = 3 \cdot 100 + 13 = 300 + 13 = 313$

$P(100) = R(100) — C(100) = 2000 — 313 = 1687$

48. a) $P(¢) = 200¢ — ¢^2 — (5000 + 8¢) =$
$200¢ — ¢^2 — 5000 — 8¢ = —¢^2 + 192¢ — 5000$

b) $R(175) = 200(175) — 175^2 =$
4375

$C(175) = 5000 + 8 \cdot 175 = 6400$

$P(175) = R(175) — C(175) = 4375 — 6400 =$
$—2025$ (We could also use the function found in part (a) to find $P(175)$.)

49. $f(¢) = 3¢ — 5$

$f(¢ + h) = 3(¢ + h) — 5 = 3¢ + 3h —$
5

$\frac{f(¢ + h) — f(¢)}{h} = \frac{3¢ + 3h — 5 — (3¢ — 5)}{h}$

$= \frac{3¢ + 3h — 5 — 3¢ + 5}{}$

h

$$= \frac{3h}{h} = 3$$

50. $f(¢) = 4¢ - 1$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{4(¢+h) - 1 - (4¢ - 1)}{h} = \frac{4¢ + 4h - 1 - 4¢ + 1}{h} = \frac{4h}{h} = 4$$

51. $f(¢) = 6¢ + 2$

$f(¢ + h) = 6(¢ + h) + 2 = 6¢ + 6h + 2$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{6¢ + 6h + 2 - (6¢ + 2)}{h} = \frac{6¢ + 6h + 2 - 6¢ - 2}{h} = \frac{6h}{h} = 6$$

52. $f(¢) = 5¢ + 3$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{5(¢+h) + 3 - (5¢ + 3)}{h} = \frac{5¢ + 5h + 3 - 5¢ - 3}{h} = \frac{5h}{h} = 5$$

53. $f(¢) = \frac{1}{3}¢ + 1$

$f(¢ + h) = \frac{1}{3}(¢ + h) + 1 = \frac{1}{3}¢ + \frac{1}{3}h + 1$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{\frac{1}{3}¢ + \frac{1}{3}h + 1 - \left(\frac{1}{3}¢ + 1\right)}{h} = \frac{\frac{1}{3}¢ + \frac{1}{3}h + 1 - \frac{1}{3}¢ - 1}{h} = \frac{\frac{1}{3}h}{h} = \frac{1}{3}$$

54. $f(¢) = -\frac{1}{2}¢ + 7$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{-\frac{1}{2}(¢ + h) + 7 - \left(-\frac{1}{2}¢ + 7\right)}{h} =$$

55. $f(¢) = \frac{1}{3¢}$

$f(¢ + h) = \frac{1}{3(¢ + h)}$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{\frac{1}{3(¢+h)} - \frac{1}{3¢}}{h} = \frac{\frac{1}{3(¢+h)} \cdot \frac{¢}{¢} - \frac{1}{3¢} \cdot \frac{¢+h}{¢+h}}{h}$$

$$= \frac{\frac{¢}{3¢(¢+h)} - \frac{¢+h}{3¢(¢+h)}}{h} = \frac{\frac{¢ - (¢ + h)}{3¢(¢+h)}}{h} = \frac{\frac{¢ - ¢ - h}{3¢(¢+h)}}{h}$$

$$= \frac{\frac{-h}{3¢(¢+h)}}{h} = \frac{-h}{3¢(¢+h)} \cdot \frac{1}{h} = \frac{-h}{3¢(¢+h) \cdot h} = \frac{-1 \cdot h}{3¢(¢+h) \cdot h}$$

$$= \frac{-1}{3¢(¢+h)}, \text{ or } -\frac{1}{3¢(¢+h)}$$

56. $f(¢) = \frac{1}{2¢}$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{\frac{1}{2(¢+h)} - \frac{1}{2¢}}{h} = \frac{\frac{1}{2(¢+h)} \cdot \frac{¢}{¢} - \frac{1}{2¢} \cdot \frac{¢+h}{¢+h}}{h} =$$

$$\frac{\frac{¢}{2¢(¢+h)} - \frac{¢+h}{2¢(¢+h)}}{h} = \frac{\frac{¢ - ¢ - h}{2¢(¢+h)}}{h} = \frac{\frac{-h}{2¢(¢+h)}}{h} =$$

$$\frac{-h}{2¢(¢+h)} \cdot \frac{1}{h} = \frac{-1}{2¢(¢+h)}, \text{ or } -\frac{1}{2¢(¢+h)}$$

$$\frac{\frac{1}{2} \quad - \quad 2}{h} =$$

57. $f(¢) = -\frac{1}{4¢}$

$$\frac{-\frac{1}{2}¢ - \frac{1}{2}h + 7 + \frac{1}{2}¢ - 7}{h} = \frac{-\frac{1}{2}h}{h} = -\frac{1}{2}$$

$$f(¢+h) = -\frac{1}{4(¢+h)}$$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{-\frac{1}{4(¢+h)} - \left(-\frac{1}{4¢}\right)}{h}$$

$$= \frac{-\frac{1}{4(¢+h)}\cdot\frac{¢}{¢} - \left(-\frac{1}{4¢}\right)\cdot\frac{¢+h}{¢+h}}{h}$$

$$= \frac{-\frac{¢}{4¢(¢+h)} + \frac{¢+h}{4¢(¢+h)}}{h}$$

$$= \frac{\frac{-¢+¢+h}{4¢(¢+h)}}{h} = \frac{\frac{h}{4¢(¢+h)}}{h}$$

$$= \frac{h}{4¢(¢+h)}\cdot\frac{1}{h} = \frac{h\cdot 1}{4¢(¢+h)\cdot h} = \frac{1}{4¢(¢+h)}$$

58. $f(¢) = \frac{1}{¢}$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{\frac{1}{¢+h} - \frac{1}{¢}}{h} =$$

$$\frac{\frac{1}{¢+h}\cdot\frac{¢}{¢} - \frac{1}{¢}\cdot\frac{¢+h}{¢+h}}{h} = \frac{\frac{¢}{¢(¢+h)} - \frac{¢+h}{¢(¢+h)}}{h} =$$

$$\frac{\frac{¢ - ¢ - h}{¢(¢+h)}}{h} = \frac{\frac{-h}{¢(¢+h)}}{h} = \frac{-h}{¢(¢+h)}\cdot\frac{1}{h} = \frac{-1}{¢(¢+h)}$$

59. $f(¢) = ¢^2 + 1$

$f(¢+h) = (¢+h)^2 + 1 = ¢^2 + 2¢h + h^2 + 1$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{¢^2 + 2¢h + h^2 + 1 - (¢^2+1)}{h}$$

$$= \frac{¢^2 + 2¢h + h^2 + 1 - ¢^2 - 1}{h}$$

$$= \frac{2¢h + h^2}{h}$$

$$= \frac{h(2¢+h)}{h}$$

$$= \frac{h}{h}\cdot\frac{2¢+h}{1}$$

$= 2¢ + h$

60. $f(¢) = ¢^2 - 3$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{(¢+h)^2 - 3 - (¢^2 - 3)}{h} =$$

$$\frac{¢^2 + 2¢h + h^2 - 3 - ¢^2 + 3}{h} = \frac{2¢h + h^2}{h} = \frac{h(2¢+h)}{h} =$$

$2¢ + h$

61. $f(¢) = 4 - ¢^2$

63. $f(¢) = 3¢^2 - 2¢ + 1$

$f(¢+h) = 3(¢+h)^2 - 2(¢+h) + 1 =$

$3(¢^2 + 2¢h + h^2) - 2(¢+h) + 1 =$

$3¢^2 + 6¢h + 3h^2 - 2¢ - 2h + 1$

$f(¢) = 3¢^2 - 2¢ + 1$

$$\frac{f(¢+h) - f(¢)}{h} =$$

$$\frac{(3¢^2 + 6¢h + 3h^2 - 2¢ - 2h + 1) - (3¢^2 - 2¢ + 1)}{h}$$

$$\frac{3¢^2 + 6¢h + 3h^2 - 2¢ - 2h + 1 - 3¢^2 + 2¢ - 1}{h} =$$

$$\frac{6¢h + 3h^2 - 2h}{h} = \frac{h(6¢ + 3h - 2)}{h\cdot 1} =$$

$$\frac{h}{h}\cdot\frac{6¢ + 3h - 2}{1} = 6¢ + 3h - 2$$

64. $f(¢) = 5¢^2 + 4¢$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{(5¢^2 + 10¢h + 5h^2 + 4¢ + 4h) - (5¢^2 + 4¢)}{h} =$$

$$\frac{10¢h + 5h^2 + 4h}{h} = 10¢ + 5h + 4$$

65. $f(¢) = 4 + 5/¢/$

$f(¢+h) = 4 + 5/¢+h/$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{4 + 5/¢+h/ - (4 + 5/¢/)}{h}$$

$$= \frac{4 + 5/¢+h/ - 4 - 5/¢/}{h}$$

$$\frac{5/¢+h/ - 5/¢/}{h}$$

66. $f(¢) = 2/¢/ + 3¢$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{(2/¢+h/ + 3¢ + 3h) - (2/¢/ + 3¢)}{h} =$$

$f(¢+h) = 4 - (¢+h)^2 = 4 - (¢^2 + 2¢h + h^2) =$

$4 - ¢^2 - 2¢h - h^2$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{4 - ¢^2 - 2¢h - h^2 - (4 - ¢^2)}{h}$$

$$= \frac{4 - ¢^2 - 2¢h - h^2 - 4 + ¢^2}{h}$$

$$= \frac{-2¢h - h^2}{h} = \frac{h(-2¢ - h)}{h}$$

$$= -2¢ - h$$

62. $f(¢) = 2 - ¢^2$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{2 - (¢+h)^2 - (2 - ¢^2)}{h} =$$

$$\frac{2 - ¢^2 - 2¢h - h^2 - 2 + ¢^2}{h} = \frac{2¢h - h^2}{h} =$$

$$\frac{(-2¢ - h)}{h} = -2¢ - h$$

$$\frac{2|¢+h| - 2|¢| + 3h}{h}$$

67. $f(¢) = ¢^3$

$f(¢+h) = (¢+h)^3 = ¢^3 + 3¢^2h + 3¢h^2 + h^3$

$f(¢) = ¢^3$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{¢^3 + 3¢^2h + 3¢h^2 + h^3 - ¢^3}{h} =$$

$$\frac{3¢^2h + 3¢h^2 + h^3}{h} = \frac{h(3¢^2 + 3¢h + h^2)}{h \cdot 1} =$$

$$\frac{h}{h} \cdot \frac{3¢^2 + 3¢h + h^2}{1} = 3¢^2 + 3¢h + h^2$$

68. $f(¢) = ¢^3 - 2¢$

$$\frac{f(¢+h) - f(¢)}{h} = \frac{(¢+h)^3 - 2(¢+h) - (¢^3 - 2¢)}{h} =$$

$$\frac{¢^3 + 3¢^2h + 3¢h^2 + h^3 - 2¢ - 2h - ¢^3 + 2¢}{h} =$$

$$\frac{3¢^2h + 3¢h^2 + h^3 - 2h}{h} = \frac{(3¢^2 + 3¢h + h^2 - 2)}{h} =$$

$$3¢^2 + 3¢h + h^2 - 2$$

69. $f(v) = \frac{-\ 4}{v+3}$

$\frac{+h\ -}{4}$ $\frac{-}{4}$

$\frac{f(v+h)-f(v)}{h} = \frac{v+h+3 \ - \ v+3}{h} =$

$\frac{+h\ -}{4}$ $\frac{-\ 4}{}$

$\frac{v+h+3 - v+3}{h} \cdot \frac{(v+h+3)(v+3)}{(v+h+3)(v+3)} =$

$= \frac{(v+h-4)(v+3)-(v-4)(v+h+3)}{h(v+h+3)(v+3)}$

$\frac{^2+hv-4v+3v+3h-12-(v^2+hv+3v-4v-4h-12)}{h(v+h+3)(v+3)} =$

$\frac{^2+hv-v+3h-12-v^2-hv+v+4h+12}{h(v+h+3)(v+3)} =$

$\frac{7h}{h(v+h+3)(v+3)} = \frac{h}{h} \cdot \frac{7}{(v+h+3)(v+3)} =$

$\frac{7}{(v+h+3)(v+3)}$

70. $f(v) = \frac{v}{2-v}$

$\frac{f(v+h)-f(v)}{h} = \frac{\frac{v+h}{2-(v+h)} - \frac{v}{2-v}}{h} =$

$= \frac{\frac{(v+h)(2-v)-v(2-v-h)}{(2-v-h)(2-v)}}{h}$

$\frac{\frac{2h}{(2-v-h)(2-v)}}{h} =$

$\frac{2h}{(2-v-h)(2-v)} \cdot \frac{1}{h} = \frac{2}{(2-v-h)(2-v)}$

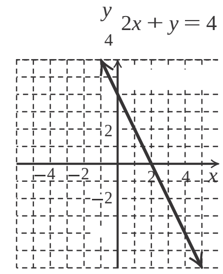
71. Graph $g = 3v - 1$.

We find some ordered pairs that are solutions of the equation, plot these points, and draw the graph.

When $v = -1$, $g = 3(-1) - 1 = -3 - 1 = -4$.

When $v = 0$, $g = 3 \cdot 0 - 1 = 0 - 1 = -1$.

72.



73. Graph $v - 3g = 3$.

First we find the v- and g-intercepts.

$v - 3 \cdot 0 = 3$

$v = 3$

The v-intercept is (3, 0).

$0 - 3g = 3$

$-3g = 3$

$g = -1$

The g-intercept is $(0, -1)$.

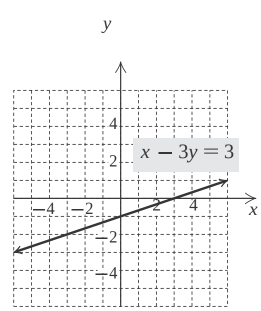
We find a third point as a check. We let $v = -3$ and solve for g.

$-3 - 3g = 3$

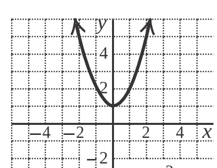
$-3g = 6$

$g = -2$

Another point on the graph is (3, 2). We plot the points

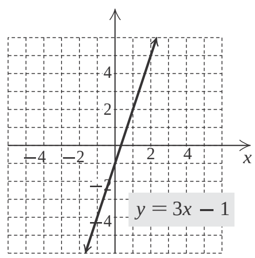


74.



y

v	g
-1	-4
0	-1
2	5



75. Answers may vary; $f(v) = \frac{1}{v+7}$, $g(v) = \frac{1}{v-3}$

76. The domain of $h + f$, $h - f$, and hf consists of all numbers that are in the domain of both h and f, or $\{-4, 0, 3\}$.

The domain of h/f consists of all numbers that are in the

77. The domain of $h(v)$ is $v\ v = \frac{7}{3}]$, and the domain of $g(v)$ is $\{v|v = 3\}$, so $\frac{7}{3}$ and 3 are not in the domain of $(h/g)(v)$. We must also exclude the value of v for which $g(v) = 0$.

$\frac{a^4-1}{5a-15}=0$

$a^4-1=0$ Multiplying by $5a-15$

$a^4=1$

$a=\pm 1$

Then the domain of $(h/g)(a)$ is $\left\{a \mid a \neq \frac{7}{3} \text{ and } a \neq 3 \text{ and } a \neq -1 \text{ and } a \neq 1\right\}$, or

$(-\infty,-1) \cup (-1,1) \cup \left(1,\frac{7}{3}\right) \cup \left(\frac{7}{3},3\right) \cup (3,\infty)$.

Exercise Set 2.3

1. $(f \circ g)(-1)=f(g(-1))=f((-1)^2-2(-1)-6)=$
$f(1+2-6)=f(-3)=3(-3)+1=-9+1=-8$

2. $(g \circ f)(-2)=g(f(-2))=g(3(-2)+1)=g(-5)=$
$(-5)^2-2(-5)-6=25+10-6=29$

3. $(h \circ f)(1)=h(f(1))=h(3\cdot 1+1)=h(3+1)=$
$h(4)=4^3=64$

4. $(g \circ h)\left(\frac{1}{2}\right)=g\left(h\left(\frac{1}{2}\right)\right)=g\left(\left(\frac{1}{2}\right)^3\right)=g\left(\frac{1}{8}\right)=$
$\left(\frac{1}{8}\right)^2-2\left(\frac{1}{8}\right)-6=\frac{1}{64}-\frac{1}{4}-6=-\frac{399}{64}$

5. $(g \circ f)(5)=g(f(5))=g(3\cdot 5+1)=g(15+1)=$
$g(16)=16^2-2\cdot 16-6=218$

6. $(f \circ g)\left(\frac{1}{3}\right)=f\left(g\left(\frac{1}{3}\right)\right)=f\left(\left(\frac{1}{3}\right)^2-2\left(\frac{1}{3}\right)-6\right)=$
$f\left(\frac{1}{9}-\frac{2}{3}-6\right)=f\left(-\frac{59}{9}\right)=3\left(-\frac{59}{9}\right)+1=-\frac{56}{3}$

7. $(f \circ h)(-3)=f(h(-3))=f((-3)^3)=f(-27)=$
$3(-27)+1=-81+1=-80$

8. $(h \circ g)(3)=h(g(3))=h(3^2-2\cdot 3-6)=$
$h(9-6-6)=h(-3)=(-3)^3=-27$

9. $(g \circ g)(-2)=g(g(-2))=g((-2)^2-2(-2)-6)=$
$g(4+4-6)=g(2)=2^2-2\cdot 2-6=4-4-6=-6$

17. $(f \circ g)(a)=f(g(a))=f(a-3)=a-3+3=a$
$(g \circ f)(a)=g(f(a))=g(a+3)=a+3-3=a$
The domain of f and of g is $(-\infty,\infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty,\infty)$.

18. $(f \circ g)(a)=f\left(\frac{4}{5}a\right)=\frac{5}{4}\cdot\frac{4}{5}a=a$
$(g \circ f)(a)=g\left(\frac{5}{4}a\right)=\frac{4}{5}\cdot\frac{5}{4}a=a$
The domain of f and of g is $(-\infty,\infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty,\infty)$.

19. $(f \circ g)(a)=f(g(a))=f(3a^2-2a-1)=3a^2-2a-1+1=$
$3a^2-2a$
$(g \circ f)(a)=g(f(a))=g(a+1)=3(a+1)^2-2(a+1)-1=$
$3(a^2+2a+1)-2(a+1)-1=3a^2+6a+3-2a-2-1=$
$3a^2+4a$
The domain of f and of g is $(-\infty,\infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty,\infty)$.

20. $(f \circ g)(a)=f(a^2+5)=3(a^2+5)-2=3a^2+15-2=$
$3a^2+13$
$(g \circ f)(a)=g(3a-2)=(3a-2)^2+5=9a^2-12a+4+5=$
$9a^2-12a+9$
The domain of f and of g is $(-\infty,\infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty,\infty)$.

21. $(f \circ g)(a)=f(g(a))=f(4a-3)=(4a-3)^2-3=$
$16a^2-24a+9-3=16a^2-24a+6$
$(g \circ f)(a)=g(f(a))=g(a^2-3)=4(a^2-3)-3=$
$4a^2-12-3=4a^2-15$
The domain of f and of g is $(-\infty,\infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty,\infty)$.

22. $(f \circ g)(a)=f(2a-7)=4(2a-7)^2-(2a-7)+10=$
$4(4a^2-28a+49)-(2a-7)+10=$
$16a^2-112a+196-2a+7+10=16a^2-114a+213$
$(g \circ f)(a)=g(4a^2-a+10)=2(4a^2-a+10)-7=$
$8a^2-2a+20-7=8a^2-2a+13$
The domain of f and of g is $(-\infty,\infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty,\infty)$.

23. $(f \circ g)(a)=f(g(a))=f\left(\frac{1}{a}\right)=\frac{4}{1-5\cdot\frac{1}{a}}=\frac{4}{1-\frac{5}{a}}=$

10. $(g \circ g)(3) = g(g(3)) = g(3^2 - 2 \cdot 3 - 6) = g(9 - 6 - 6) =$
$g(-3) = (-3)^2 - 2(-3) - 6 = 9 + 6 - 6 = 9$

11. $(h \circ h)(2) = h(h(2)) = h(2^3) = h(8) = 8^3 = 512$

12. $(h \circ h)(-1) = h(h(-1)) = h((-1)^3) = h(-1) = (-1)^3 = -1$

13. $(f \circ f)(-4) = f(f(-4)) = f(3(-4) + 1) = f(-12 + 1) =$
$f(-11) = 3(-11) + 1 = -33 + 1 = -32$

14. $(f \circ f)(1) = f(f(1)) = f(3 \cdot 1 + 1) = f(3 + 1) = f(4) =$
$3 \cdot 4 + 1 = 12 + 1 = 13$

15. $(h \circ h)(a) = h(h(a)) = h(a^3) = (a^3)^3 = a^9$

16. $(f \circ f)(a) = f(f(a)) = f(3a + 1) = 3(3a + 1) + 1 =$
$9a + 3 + 1 = 9a + 4$

$a \qquad a$

$$\frac{4}{\frac{-5}{a}} = 4 \cdot \frac{a}{a - 5} = \frac{4a}{a - 5}$$

$$(g \circ f)(a) = g(f(a)) = g\left(\frac{4}{1 - 5a}\right) = \frac{1}{\frac{4}{1 - 5a}} =$$

$$1 \cdot \frac{1 - 5a}{4} = \frac{1 - 5a}{4}$$

The domain of f is $\left\{a \,\middle|\, a \neq \frac{1}{5}\right\}$ and the domain of g is $\{a|a \neq 0\}$. Consider the domain of $f \circ g$. Since 0 is not in the domain of g, 0 is not in the domain of $f \circ g$. Since $\frac{1}{5}$ is not in the domain of f, we know that $g(a)$ cannot be $\frac{1}{5}$. We find the value(s) of a for which $g(a) = \frac{1}{5}$.

$\frac{1}{x} = \frac{1}{5}$

$5 = x$ Multiplying by $5x$

Thus 5 is also not in the domain of $f \circ g$. Then the domain of $f \circ g$ is $\{x|x \neq 0 \text{ and } x \neq 5\}$, or $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$.

Now consider the domain of $g \circ f$. Recall that $\frac{1}{5}$ is not in the domain of f, so it is not in the domain of $g \circ f$. Now 0 is not in the domain of g but $f(x)$ is never 0, so the domain of $g \circ f$ is $\left\{x \middle| x \neq \frac{1}{5}\right\}$, or $\left(-\infty, \frac{1}{5}\right) \cup \left(\frac{1}{5}, \infty\right)$.

24. $(f \circ g)(x) = f\left(\frac{1}{2x+1}\right) = \frac{6}{\frac{1}{2x+1}} = 6 \cdot \frac{2x+1}{1} = 6(2x+1)$, or $12x + 6$

$(g \circ f)(x) = g\left(\frac{6}{x}\right) = \frac{1}{2 \cdot \frac{6}{x} + 1} = \frac{1}{\frac{12}{x} + 1} = \frac{1}{\frac{12+x}{x}} = 1 \cdot \frac{x}{12+x} = \frac{x}{12+x}$

The domain of f is $\{x|x \neq 0\}$ and the domain of g is $\left\{x \middle| x \neq -\frac{1}{2}\right\}$. Consider the domain of $f \circ g$. Since $-\frac{1}{2}$ is not in the domain of g, $-\frac{1}{2}$ is not in the domain of $f \circ g$. Now 0 is not in the domain of f but $g(x)$ is never 0, so the domain of $f \circ g$ is $\left\{x \middle| x \neq -\frac{1}{2}\right\}$, or $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

Now consider the domain of $g \circ f$. Since 0 is not in the domain of f, then 0 is not in the domain of $g \circ f$. Also, since $-\frac{1}{2}$ is not in the domain of g, we find the value(s) of x for which $f(x) = -\frac{1}{2}$.

$\frac{6}{x} = -\frac{1}{2}$

$-12 = x$

Then the domain of $g \circ f$ is $\{x|x \neq -12 \text{ and } x \neq 0\}$, or $(-\infty, -12) \cup (-12, 0) \cup (0, \infty)$.

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

27. $(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = 2\sqrt{x} + 1$

$(g \circ f)(x) = g(f(x)) = g(2x+1) = \sqrt{2x+1}$

The domain of f is $(-\infty, \infty)$ and the domain of g is $\{x|x \geq 0\}$. Thus the domain of $f \circ g$ is $\{x|x \geq 0\}$, or $[0, \infty)$.

Now consider the domain of $g \circ f$. There are no restrictions on the domain of f, but the domain of g is $\{x|x \geq 0\}$. Since $f(x) \geq 0$ for $x \geq -\frac{1}{2}$, the domain of $g \circ f$ is $\left\{x \middle| x \geq -\frac{1}{2}\right\}$, or $\left[-\frac{1}{2}, \infty\right)$.

28. $(f \circ g)(x) = f(2 - 3x) = \sqrt{2 - 3x}$

$(g \circ f)(x) = g(\sqrt{x}) = 2 - 3\sqrt{x}$

The domain of f is $\{x|x \geq 0\}$ and the domain of g is $(-\infty, \infty)$. Since $g(x) \geq 0$ when $x \leq \frac{2}{3}$, the domain of $f \circ g$ is $\left(-\infty, \frac{2}{3}\right]$.

Now consider the domain of $g \circ f$. Since the domain of f is $\{x|x \geq 0\}$ and the domain of g is $(-\infty, \infty)$, the domain of $g \circ f$ is $\{x|x \geq 0\}$, or $[0, \infty)$.

29. $(f \circ g)(x) = f(g(x)) = f(0.05) = 20$

$(g \circ f)(x) = g(f(x)) = g(20) = 0.05$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

30. $(f \circ g)(x) = (\sqrt[4]{x})^4 = x$

$(g \circ f)(x) = \sqrt[4]{x^4} = |x|$

The domain of f is $(-\infty, \infty)$ and the domain of g is $\{x|x \geq 0\}$, so the domain of $f \circ g$ is $\{x|x \geq 0\}$, or $[0, \infty)$. Now consider the domain of $g \circ f$. There are no restrictions on the domain of f and $f(x) \geq 0$ for all values of x, so the domain is $(-\infty, \infty)$.

31. $(f \circ g)(x) = f(g(x)) = f(x^2 - 5) = \sqrt{x^2 - 5 + 5} = \sqrt{x^2} = |x|$

25. $(f \circ g)(X) = f(g(X)) = f\left(\frac{}{3}\right) =$

$$3\left(\frac{47}{3}\right) - 7 = X\,4\,7 - 7 = X$$

$$(g \circ f)(X) = g(f(X)) = g(3X - 7) = \frac{(3X - 7)\,4\,7}{3} =$$

$$\frac{3X}{3} = X$$

The domain of f and of g is $(-O, O)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-O, O)$.

26. $(f \circ g)(X) = f(1.5X\,4\,1.2) = \frac{2}{3}(1.5X\,4\,1.2) - \frac{4}{5}$

$$X\,4\,0.8 - \frac{4}{5} = X$$

$$(g \circ f)(X) = g\left(\frac{2}{3}X - \frac{4}{5}\right) = 1.5\left(\frac{2}{3}X - \frac{4}{5}\right)4\,1.2 =$$

$$X - 1.2\,4\,1.2 = X$$

$(g \circ f)(X) = g(f(X)) = g(\sqrt{X\,4\,5}) =$

$$(\sqrt{X\,4\,5})^2 - 5 = X\,4\,5 - 5 = X$$

The domain of f is $\{X \mid X \geq -5\}$ and the domain of g is $(-O, O)$. Since $X^2 \geq 0$ for all values of X, then $X^2 - 5 \geq -5$ for all values of X and the domain of $g \circ f$ is $(-O, O)$.

Now consider the domain of $f \circ g$. There are no restrictions on the domain of g, so the domain of $f \circ g$ is the same as the domain of f, $\{X \mid X \geq -5\}$, or $[-5, O)$.

32. $(f \circ g)(X) = (\sqrt[5]{X\,4\,2})^5 - 2 = X\,4\,2 - 2 = X$

$(g \circ f)(X) = \sqrt[5]{X^5 - 2\,4\,2} = \sqrt[5]{X^5} = X$

The domain of f and of g is $(-O, O)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-O, O)$.

33. $(f \circ g)(0) = f(g(0)) = f(\sqrt{3-0}) = (\sqrt{3-0})^2 + 2 =$

$3 - 0 + 2 = 5 - 0$

$(g \circ f)(0) = g(f(0)) = g(0^2 + 2) = \sqrt{3 - (0^2 + 2)} =$

$\sqrt{3 - 0^2 - 2} = \sqrt{1 - 0^2}$

The domain of f is $(-O, O)$ and the domain of g is $\{0|0 \le 3\}$, so the domain of $f \circ g$ is $\{0|0 \le 3\}$, or $(-O, 3]$.

Now consider the domain of $g \circ f$. There are no restrictions on the domain of f and the domain of g is $\{0|0 \le 3\}$, so we find the values of 0 for which $f(0) \le 3$. We see that

$0^2 + 2 \le 3$ for $-1 \le 0 \le 1$, so the domain of $g \circ f$ is $\{0| -1 \le 0 \le 1\}$, or $[-1, 1]$.

34. $(f \circ g)(0) = f(\sqrt{0^2 - 25}) = 1 - (\sqrt{0^2 - 25})^2 =$

$1 - (0^2 - 25) = 1 - 0^2 + 25 = 26 - 0^2$

$(g \circ f)(0) = g(1 - 0^2) = \sqrt{(1 - 0^2)^2 - 25} =$

$\sqrt{1 - 20^2 + 0^4 - 25} = \sqrt{0^4 - 20^2 - 24}$

The domain of f is $(-O, O)$ and the domain of g is

$\{0|0 \le -5 \text{ or } 0 > 5\}$, so the domain of $f \circ g$ is $\{0|0 \le -5 \text{ or } 0 > 5\}$, or $(-O, -5] \cup [5, O)$.

Now consider the domain of $g \circ f$. There are no restrictions on the domain of f and the domain of g is $\{0|0 \le -5 \text{ or } 0 > 5\}$, so we find the values of 0 for which $f(0) \le -5$ or $f(0) > 5$. We see that $1 - 0^2 \le -5$ when $0 \le -\sqrt{6}$ or $0 > \sqrt{6}$ and $1 - 0^2 \ge 5$ has no solution, so the domain of $g \circ f$ is $\{0|0 \le -\sqrt{6} \text{ or } 0 > \sqrt{6}\}$, or $(-O, -\sqrt{6}] \cup [\sqrt{6}, O)$.

35. $(f \circ g)(0) = f(g(0)) = f\left(\frac{1}{1+0}\right) =$

$\dfrac{1 - \frac{1}{1+0}}{\frac{1}{1+0}} = \dfrac{\frac{1+0-1}{1+0}}{\frac{1}{1+0}} =$

$\dfrac{0}{1+0} \cdot \dfrac{1+0}{1} = 0$

$(g \circ f)(0) = g(f(0)) = g\left(\frac{1-0}{0}\right) =$

$\dfrac{1}{1 + \frac{1-0}{0}} = \dfrac{1}{\frac{0+1-0}{0}} = \dfrac{1}{\frac{1}{0}} = 0$

$\dfrac{1-0}{0} = -1$

$1 - 0 = -0$ Multiplying by 0

$1 = 0$ False equation

We see that there are no values of 0 for which $f(0) = -1$, so the domain of $g \circ f$ is $\{0|0 = 0\}$, or $(-O, 0) \cup (0, O)$.

36. $(f \circ g)(0) = f\left(\frac{1}{0} + 2\right) = \dfrac{1}{\frac{1}{0} + 2 - 2}$

$= \dfrac{1}{\frac{1 + 20}{0} - \frac{20}{0}} = \dfrac{1}{\frac{-0+2}{0}}$, $= 1 \cdot \dfrac{0}{-0+2} = \dfrac{0}{-0+2}$, or $\dfrac{0}{2-0}$

$(g \circ f)(0) = g\left(\frac{1}{0-2}\right) = \dfrac{1}{\frac{1}{0-2}} + 2 = \dfrac{\frac{1 + 20 - 4}{0-2}}{\frac{1}{0-2}} = \dfrac{\frac{20-3}{0-2}}{\frac{1}{0-2}}$

$= \dfrac{20-3}{0-2} \cdot \dfrac{0-2}{1} = 20 - 3$

The domain of f is $\{0|0 = 2\}$ and the domain of g is $\{0|0 = 0\}$, so 0 is not in the domain of $f \circ g$. We find the value of 0 for which $g(0) = 2$.

$\dfrac{0+2}{0} = 2$

$0 + 2 = 20$

$2 = 0$

Then the domain of $f \circ g$ is $(-O, 0) \cup (0, 2) \cup (2, O)$.

Now consider the domain of $g \circ f$. Since the domain of f is $\{0|0 = 2\}$, we know that 2 is not in the domain of $g \circ f$. Since the domain of g is $\{0|0 = 0\}$, we find the value of 0 for which $f(0) = 0$.

$\dfrac{1}{0-2} = 0$

$1 = 0$

We get a false equation, so there are no such values. Then

$$\frac{1}{\frac{1}{0}} = 1 \circ \frac{0}{1} = 0$$

The domain of f is $\{0 l 0 = 0\}$ and the domain of g is

$\{0 l 0 = -1\}$, so we know that -1 is not in the domain of $f \ o \ g$. Since 0 is not in the domain of f, values of 0 for which $g(0) = 0$ are not in the domain of $f \ o \ g$. But $g(0)$ is never 0, so the domain of $f \ o \ g$ is $\{0 l 0 = -1\}$, or $(-O, -1)\ U\ (-1, O)$.

Now consider the domain of $g \ o \ f$. Recall that 0 is not in the domain of f. Since -1 is not in the domain of g, we know that $g(0)$ cannot be -1. We find the value(s) of 0 for which $f(0) = -1$.

the domain of $g \ o \ f$ is $(-O, 2)\ U\ (2, O)$.

37. $(f \ o \ g)(0) = f(g(0)) = f(0 + 1) =$
$(0 + 1)^3 - 5(0 + 1)^2 + 3(0 + 1) + 7 =$

$0^3 + 30^2 + 30 + 1 - 50^2 - 100 - 5 + 30 + 3 + 7 =$

$0^3 - 20^2 - 40 + 6$

$(g \ o \ f)(0) = g(f(0)) = g(0^3 - 50^2 + 30 + 7) =$
$0^3 - 50^2 + 30 + 7 + 1 = 0^3 - 50^2 + 30 + 8$

The domain of f and of g is $(-O, O)$, so the domain of $f \ o \ g$ and of $g \ o \ f$ is $(-O, O)$.

38. $(g \circ f)(0) = 0^3 + 20^2 - 30 - 9 - 1 =$
$0^3 + 20^2 - 30 - 10$
$(g \circ f)(0) = (0 - 1)^3 + 2(0 - 1)^2 - 3(0 - 1) - 9 =$
$0^3 - 30^2 + 30 - 1 + 20^2 - 40 + 2 - 30 + 3 - 9 =$

$0^3 - 0^2 - 40 - 5$

The domain of f and of g is $(-O, O)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-O, O)$.

39. $h(0) = (4 + 30)^5$

This is $4 + 30$ to the 5th power. The most obvious answer is $f(0) = 0^5$ and $g(0) = 4 + 30$.

40. $f(0) = \sqrt[3]{0}$, $g(0) = 0^2 - 8$

41. $h(0) = \dfrac{1}{(0 \quad 2)^4}$

This is 1 divided by $(0 - 2)$ to the 4th power. One obvious answer is $f(0) = \dfrac{1}{0^4}$ and $g(0) = 0 - 2$. Another possibility is $f(0) = \dfrac{1}{0}$ and $g(0) = (0 - 2)^4$.

42. $f(0) = \dfrac{1}{\sqrt{0}}$, $g(0) = 30 + 7$

43. $f(0) = \dfrac{0 - 1}{0 + 1}$, $g(0) = 0^3$

44. $f(0) = |0|$, $g(0) = 90^2 - 4$

45. $f(0) = 0^6$, $g(0) = \dfrac{2 + 0^3}{2 - 0^3}$

46. $f(0) = 0^4$, $g(0) = \sqrt{0} - 3$

47. $f(0) = \sqrt{0}$, $g(0) = \dfrac{0 - 5}{0 + 2}$

48. $f(0) = \sqrt{1 + 0}$, $g(0) = \sqrt{1 + 0}$

49. $f(0) = 0^3 - 50^2 + 30 - 1$, $g(0) = 0 + 2$

50. $f(0) = 20^{5/3} + 50^{2/3}$, $g(0) = 0 - 1$, or

$f(0) = 20^5 + 50^2$, $g(0) = (0 - 1)^{1/3}$

51. a) Use the distance formula, distance = rate $\times$ time. Substitute 3 for the rate and t for time.
$r(t) = 3t$

b) Use the formula for the area of a circle.
$A(r) = \pi r^2$

c) $(A \circ r)(t) = A(r(t)) = A(3t) = \pi(3t)^2 = 9\pi t^2$
This function gives the area of the ripple in terms of time t.

52. a) $h = 2r$
$S(r) = 2\pi r(2r) + 2\pi r^2$

b) $r = \dfrac{h}{2}$

$S(h) = 2\pi \left(\dfrac{h}{2}\right) h + 2\pi \left(\dfrac{h}{2}\right)^2$

$S(h) = \pi h^2 + \dfrac{\pi h^2}{2}$

$S(h) = \dfrac{3}{2}\pi h^2$

53. $f(0) = (t \circ s)(0) = t(s(0)) = t(0 - 3) = 0 - 3 + 4 = 0 + 1$
We have $f(0) = 0 + 1$.

54. The manufacturer charges $m + 6$ per drill. The chain store sells each drill for $150\%(m + 6)$, or $1.5(m + 6)$, or $1.5m + 9$. Thus, we have $P(m) = 1.5m + 9$.

55. Equations (a) – (f) are in the form $g = m0 + b$, so we can read the g-intercepts directly from the equations. Equations (g) and (h) can be written in this form as $g = \dfrac{2}{3}0 - 2$ and $g = -20 + 3$, respectively. We see that only equation (c) has g-intercept (0, 1).

56. None (See Exercise 55.)

57. If a line slopes down from left to right, its slope is negative. The equations $g = m0 + b$ for which m is negative are (b), (d), (f), and (h). (See Exercise 55.)

58. The equation for which $|m|$ is greatest is the equation with the steepest slant. This is equation (b). (See Exercise 55.)

59. The only equation that has (0, 0) as a solution is (a).

60. Equations (c) and (g) have the same slope. (See Exercise 55.)

61. Only equations (c) and (g) have the same slope and different g-intercepts. They represent parallel lines.

62. The only equations for which the product of the slopes is -1 are (a) and (f).

63. Only the composition $(c \circ p)(a)$ makes sense. It represents

$S(r) = 4\pi r^2 + 2\pi r^2$
$S(r) = 6\pi r^2$

the cost of the grass seed required to seed a lawn with area
a.

64. Answers may vary. One example is $f(x) = 2x + 5$ and
$g(x) = \frac{x - 5}{2}$. Other examples are found in Exercises 17,
18, 25, 26, 32 and 35.

Chapter 2 Mid-Chapter Mixed Review

1. The statement is true. See page 100 in the text.

2. The statement is false. See page 113 in the text.

3. The statement is true. See Example 2 in Section 2.3 in the text, for instance.

4. a) For x-values from 2 to 4, the y-values increase from 2 to 4. Thus the function is increasing on the interval (2, 4).

b) For 0-values from -5 to -3, the g-values decrease from 5 to 1. Also, for 0-values from 4 to 5, the g-values decrease from 4 to -3. Thus the function is decreasing on $(-5, -3)$ and on $(4, 5)$.

c) For 0-values from -3 to -1, g is 3. Thus the function is constant on $(-3, -1)$.

5. From the graph we see that a relative maximum value of 6.30 occurs at $0 = -1.29$. We also see that a relative minimum value of -2.30 occurs at $0 = 1.29$.

The graph starts rising, or increasing, from the left and stops increasing at the relative maximum. From this point it decreases to the relative minimum and then increases again. Thus the function is increasing on $(-O, -1.29)$ and on $(1.29, O)$. It is decreasing on $(-1.29, 1.29)$.

6. The 0-values extend from -5 to -1 and from 2 to 5, so the domain is $[-5, -1]\ U\ [2, 5]$. The g-values extend from -3 to 5, so the range is $[-3, 5]$.

7. $A(h) = \frac{1}{2}(h + 4)h$

$A(h) = \frac{1}{2}h^2 + 2h$, or $\frac{h^2}{2} + 2h$

8. $f(0) = \begin{cases} 0 - 5, & \text{for } 0 \leq -3, \\ 20 + 3, & \text{for } -3 < 0 \leq 0, \\ \frac{1}{2}0, & \text{for } 0 > 0, \end{cases}$

Since $-5 \leq -3$, $f(-5) = -5 - 5 = -10$.

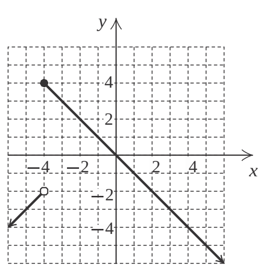
Since $-3 \leq -3$, $f(-3) = -3 - 5 = -8$.

Since $-3 < -1 \leq 0$, $f(-1) = 2(-1) + 3 = -2 + 3 = 1$.

Since $6 > 0$, $f(6) = \frac{1}{2} \circ 6 = 3$.

9. $g(0) = \begin{cases} 0 + 2, & \text{for } 0 < -4, \\ -0, & \text{for } 0 > -4 \end{cases}$

We create the graph in two parts. Graph $g(0) = 0 + 2$ for inputs less than -4. Then graph $g(0) = -0$ for inputs



10. $(f + g)(-1) = f(-1) + g(-1)$

12. $(g - f)(3) = g(3) - f(3)$

$= (3^2 + 4) - (3 \circ 3 - 1)$

$= 9 + 4 - (9 - 1)$

$= 9 + 4 - 9 + 1$

$= 5$

13. $(g/f)\left(\frac{1}{3}\right) = \frac{g\left(\frac{1}{3}\right)}{f\left(\frac{1}{3}\right)}$

$= \frac{\left(\frac{1}{3}\right)^2}{3 \cdot \frac{1}{3} - 1}$

$\frac{1}{}$

$= \frac{9 + 4}{1 - 1}$

$\underline{37}$

$= \frac{9}{0}$

Since division by 0 is not defined, $(g/f)\left(\frac{1}{3}\right)$ does not exist.

14. $f(0) = 20 + 5$, $g(0) = -0 - 4$

a) The domain of f and of g is the set of all real numbers, or $(-O, O)$. Then the domain of $f + g$, $f - g$, fg, and ff is also $(-O, O)$.

For f/g we must exclude -4 since $g(-4) = 0$. Then the domain of f/g is $(-O, -4)\ U\ (-4, O)$.

For g/f we must exclude $-\frac{5}{2}$ since $f\left(-\frac{5}{2}\right) = 0$. Then the domain of g/f is

$\left(-O, -\frac{5}{2}\right) U \left(-\frac{5}{2}, O\right)$.

b) $(f+g)(0) = f(0)+g(0) = (20+5)+(-0-4) = 0+1$

$20 + 5 + 0 + 4 = 30 + 9$

$-20^2 - 80 - 50 - 20 = -20^2 - 130 - 20$

$(ff)(0) = f(0) \circ f(0) = (20 + 5) \circ (20 + 5) =$

$(f/g)(0) = \frac{f(0)}{} = \frac{20 + 5}{}$

$(g/f)(0) = \frac{g(0)}{} \qquad f(0)$

$=$

$\underline{0 - 4}$

$$= [3(-1) - 1] + [(-1)^2 + 4]$$
$$= -3 - 1 + 1 + 4$$
$$= 1$$

11. $(fg)(0) = f(0) \circ g(0)$
$$= (3 \circ 0 - 1) \circ (0^2 + 4)$$
$$= -1 \circ 4$$
$$= -4$$

15. $f(0) = 0 - 1$, $g(0) = \sqrt{\frac{20 + 5}{0 + 2}}$

a) Any number can be an input for f, so the domain of f is the set of all real numbers, or $(-O, O)$.

The domain of g consists of all values for which $0+2$ is nonnegative, so we have $0 + 2 > 0$, or $0 > -2$, or $[-2, O)$. Then the domain of $f + g$, $f - g$, and fg is $[-2, O)$.

The domain of ff is $(-O, O)$.

Since $g(-2) = 0$, the domain of f/g is $(-2, O)$.

Since $f(1) = 0$, the domain of g/f is $[-2, 1) U (1, O)$.

b) $(f + g)(0) = f(0) + g(0) = 0 - 1 + \sqrt{0+2}$

$(f - g)(0) = f(0) - g(0) = 0 - 1 - \sqrt{0+2}$

$(fg)(0) = f(0) \circ g(0) = (0 - 1)\sqrt{0+2}$

$(ff)(0) = f(0) \circ f(0) = (0 - 1)(0 - 1) =$
$0^2 - 0 - 0 + 1 = 0^2 - 20 + 1$

$$(f/g)(0) = \frac{f(0)}{g(0)} = \frac{0 - 1}{\sqrt{0+2}}$$

$$(g/f)(0) = \frac{g(0)}{f(0)} = \frac{\sqrt{0+2}}{0 - 1}$$

16. $f(0) = 40 - 3$

$$\frac{f(0+h) - f(0)}{h} = \frac{4(0+h) - 3 - (40 - 3)}{h} =$$

$$\frac{40 + 4h - 3 - 40 + 3}{h} = \frac{4h}{h} = 4$$

17. $f(0) = 6 - 0^2$

$$\frac{f(0+h) - f(0)}{h} = \frac{6 - (0+h)^2 - (6 - 0^2)}{h} =$$

$$\frac{6 - (0^2 + 20h + h^2) - 6 + 0^2}{h} = \frac{6 - 0^2 - 20h - h^2 - 6 + 0^2}{h} =$$

$$\frac{-20h - h^2}{h} = \frac{h(-20 - h)}{h \circ 1} = -20 - h$$

18. $(f \circ g)(1) = f(g(1)) = f(1^3 + 1) = f(1 + 1) = f(2) = 5 \circ 2 - 4 = 10 - 4 = 6$

19. $(g \circ h)(2) = g(h(2)) = g(2^2 - 2 \circ 2 + 3) = g(4 - 4 + 3) = g(3) = 3^3 + 1 = 27 + 1 = 28$

20. $(f \circ f)(0) = f(f(0)) = f(5 \circ 0 - 4) = f(-4) = 5(-4) - 4 = -20 - 4 = -24$

21. $(h \circ f)(-1) = h(f(-1)) = h(5(-1) - 4) = h(-5 - 4) = h(-9) = (-9)^2 - 2(-9) + 3 = 81 + 18 + 3 = 102$

22. $(f \circ g)(0) = f(g(0)) = f(60 + 4) = \frac{1}{2}(60 + 4) = 30 + 2$

$$(g \circ f)(0) = g(f(0)) = g\left(\frac{1}{2}0\right) = 6 \circ \frac{1}{2}0 + 4 = 30 + 4$$

The domain of f and g is $(-O, O)$, so the domain of $f \circ g$ and $g \circ f$ is $(-O, O)$.

23. $(f \circ g)(0) = f(g(0)) = f(\sqrt{0}) = 3\sqrt{0} + 2$
$(g \circ f)(0) = g(f(0)) = g(30 + 2) = \sqrt{30 + 2}$

The domain of f is $(-O, O)$ and the domain of g is $[0, O)$. Consider the domain of $f \circ g$. Since any number can be an input for f, the domain of $f \circ g$ is the same as the domain of g, $[0, O)$.

24. The graph of $Y = (h - g)(0)$ will be the same as the graph of $Y = h(0)$ moved down b units.

25. Under the given conditions, $(f + g)(0)$ and $(f/g)(0)$ have different domains if $g(0) = 0$ for one or more real numbers 0.

26. If f and g are linear functions, then any real number can be an input for each function. Thus, the domain of $f \circ g =$ the domain of $g \circ f = (-O, O)$.

27. This approach is not valid. Consider Exercise 23 in Exercise Set 2.3 in the text, for example. Since $(f \circ g)(0) = \frac{40}{0 - 5}$, an examination of only this composed function would lead to the incorrect conclusion that the domain of $f \circ g$ is $(-O, 5) U (5, O)$. However, we must also exclude from the domain of $f \circ g$ those values of 0 that are not in the domain of g. Thus, the domain of $f \circ g$ is $(-O, 0) U (0, 5) U (5, O)$.

Exercise Set 2.4

1. If the graph were folded on the 0-axis, the parts above and below the 0-axis would not coincide, so the graph is not symmetric with respect to the 0-axis.

If the graph were folded on the Y-axis, the parts to the left and right of the Y-axis would coincide, so the graph is symmetric with respect to the Y-axis.

If the graph were rotated 180°, the resulting graph would not coincide with the original graph, so it is not symmetric with respect to the origin.

2. If the graph were folded on the 0-axis, the parts above and below the 0-axis would not coincide, so the graph is not symmetric with respect to the 0-axis.

If the graph were folded on the Y-axis, the parts to the left and right of the Y-axis would coincide, so the graph is symmetric with respect to the Y-axis.

If the graph were rotated 180°, the resulting graph would not coincide with the original graph, so it is not symmetric with respect to the origin.

3. If the graph were folded on the 0-axis, the parts above and below the 0-axis would coincide, so the graph is symmetric

Now consider the domain of $g \circ f$. Since the inputs of g must be nonnegative, we must have $3x+2 \geq 0$, or $x \geq -\frac{2}{3}$.

Thus the domain of $g \circ f$ is $\left[-\frac{2}{3}, \infty\right)$.

with respect to the x-axis.

If the graph were folded on the Y-axis, the parts to the left and right of the Y-axis would not coincide, so the graph is not symmetric with respect to the Y-axis.

If the graph were rotated 180°, the resulting graph would not coincide with the original graph, so it is not symmetric with respect to the origin.

4. If the graph were folded on the x-axis, the parts above and below the x-axis would not coincide, so the graph is not symmetric with respect to the x-axis.

   If the graph were folded on the Y-axis, the parts to the left and right of the Y-axis would not coincide, so the graph is not symmetric with respect to the Y-axis.

If the graph were rotated 180°, the resulting graph would coincide with the original graph, so it is symmetric with respect to the origin.

5. If the graph were folded on the 0-axis, the parts above and below the 0-axis would not coincide, so the graph is not symmetric with respect to the 0-axis.

If the graph were folded on the Y-axis, the parts to the left and right of the Y-axis would not coincide, so the graph is

not symmetric with respect to the Y-axis.

If the graph were rotated 180°, the resulting graph would coincide with the original graph, so it is symmetric with respect to the origin.

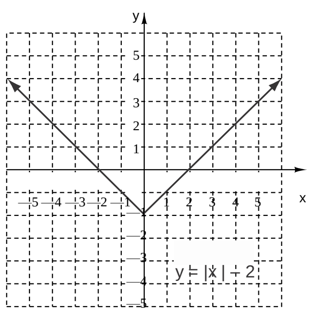
6. If the graph were folded on the 0-axis, the parts above and

below the 0-axis would coincide, so the graph is symmetric with respect to the 0-axis.

If the graph were folded on the Y-axis, the parts to the left and right of the Y-axis would coincide, so the graph is symmetric with respect to the Y-axis.

If the graph were rotated 180°, the resulting graph would coincide with the original graph, so it is symmetric with respect to the origin.

7.



The graph is symmetric with respect to the Y-axis. It is not symmetric with respect to the 0-axis or the origin.

$-Y = l0I - 2$ Replacing Y by $-Y$

$Y = -I0l + 2$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the 0-axis.

Test algebraically for symmetry with respect to the Y-axis:

$Y = I - OI - 2$ Replacing 0 by -0

$Y = l0I - 2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the Y-axis.

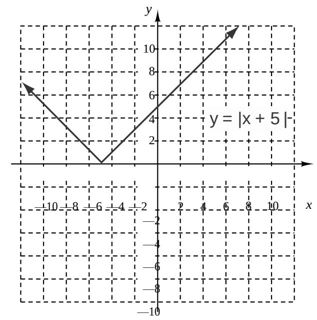
Test algebraically for symmetry with respect to the origin:

$Y = l0I - 2$ Original equation

$-Y = I - OI - 2$ Replacing 0 by -0 and Y by $-Y$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

8.



The graph is not symmetric with respect to the 0-axis, the Y-axis, or the origin.

Test algebraically for symmetry with respect to the 0-axis:

$Y = I0 + 5I$ Original equation

$-Y = I0 + 5I$ Replacing Y by $-Y$

$Y = -I0 + 5I$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the 0-axis.

Test algebraically for symmetry with respect to the Y-axis:

$Y = I0 + 5I$ Original equation

$Y = I - 0 + 5I$ Replacing 0 by -0

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the Y-axis.

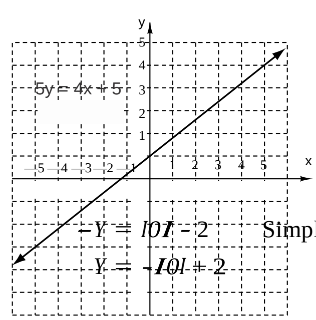
Test algebraically for symmetry with respect to the origin:

$Y = I0 + 5I$ Original equation

$-Y = I - 0 + 5I$ Replacing 0 by -0 and Y by $-Y$

$Y = -I - 0 + 5I$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.



$-Y = l0I - 2$ Simplifying

$Y = -I0l + 2$

—1
—2
—3
—4
—5

The graph is not symmetric with respect to the 0-axis, the Y-axis, or the origin.

Test algebraically for symmetry with respect to the 0-axis:

$5Y = 40 + 5$ Original equation

$5(-Y) = 40 + 5$ Replacing Y by $-Y$

$-5Y = 40 + 5$ Simplifying

$5Y = -40 - 5$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the 0-axis.

$5Y = 4(-0) + 5$ Replacing 0 by -0

$5Y = -40 + 5$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the Y-axis.

Test algebraically for symmetry with respect to the origin:

$5Y = 40 + 5$ Original equation

$5(-Y) = 4(-0) + 5$ Replacing 0 by -0 and

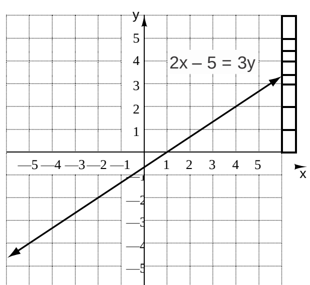
Y by $-Y$

$-5Y = -40 + 5$ Simplifying

$5Y = 40 - 5$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

10.



The graph is not symmetric with respect to the 0-axis, the Y-axis, or the origin.

Test algebraically for symmetry with respect to the 0-axis:

$20 - 5 = 3Y$ Original equation

$20 - 5 = 3(-Y)$ Replacing Y by $-Y$

$-20 + 5 = 3Y$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the 0-axis.

Test algebraically for symmetry with respect to the Y-axis:

$20 - 5 = 3Y$ Original equation

$2(-0) - 5 = 3Y$ Replacing 0 by -0

$-20 - 5 = 3Y$ Simplifying

The last equation is not equivalent to the original equation,

so the graph is not symmetric with respect to the Y-axis.

Test algebraically for symmetry with respect to the origin:

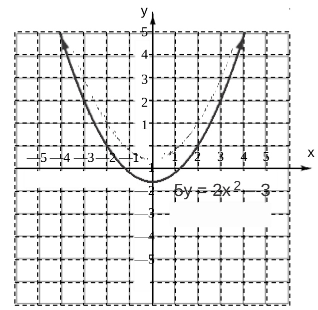
$20 - 5 = 3Y$ Original equation

$2(-0) - 5 = 3(-Y)$ Replacing 0 by -0 and Y by $-Y$

$-20 - 5 = -3Y$ Simplifying

$20 + 5 = 3Y$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.



The graph is symmetric with respect to the Y-axis. It is not symmetric with respect to the 0-axis or the origin.

Test algebraically for symmetry with respect to the 0-axis:

$5Y = 20^2 - 3$ Original equation

$5(-Y) = 20^2 - 3$ Replacing Y by $-Y$

$-5Y = 20 - 3$ Simplifying

$5Y = -20^2 + 3$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the 0-axis.

Test algebraically for symmetry with respect to the Y-axis:

$5Y = 20^2 - 3$ Original equation

$5Y = 2(-0)^2 - 3$ Replacing 0 by -0

$5Y = 20^2 - 3$

The last equation is equivalent to the original equation, so Test algebraically for symmetry with respect to the origin:

$5Y = 20^2 - 3$ Original equation

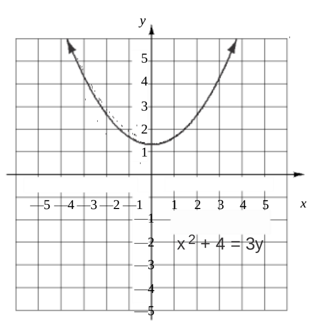
$5(-Y) = 2(-0)^2 - 3$ Replacing 0 by -0 and Y by $-Y$

$-5Y = 20^2 - 3$ Simplifying

$5Y = -20^2 + 3$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

12.



The graph is symmetric with respect to the Y-axis. It is not symmetric with respect to the 0-axis or the origin.

Test algebraically for symmetry with respect to the 0-axis:

$0^2 + 4 = 3Y$ Original equation

$0^2 + 4 = 3(-Y)$ Replacing Y by $-Y$

$-0^2 - 4 = 3Y$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the 0-axis.

Test algebraically for symmetry with respect to the Y-axis:

$$0^2 + 4 = 3Y \quad \text{Original equation}$$
$$(-0)^2 + 4 = 3Y \quad \text{Replacing } 0 \text{ by } -0$$
$$0^2 + 4 = 3Y$$

The last equation is equivalent to the original equation, so

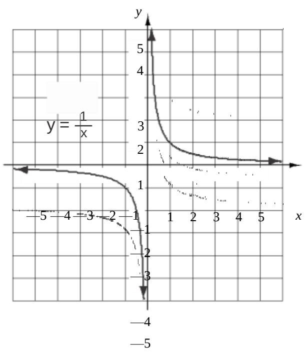
the graph is symmetric with respect to the Y-axis.

Test algebraically for symmetry with respect to the origin:

$$0^2 + 4 = 3Y \quad \text{Original equation}$$
$$(-0)^2 + 4 = 3(-Y) \quad \text{Replacing } 0 \text{ by } -0 \text{ and } Y \text{ by } -Y$$
$$0^2 + 4 = -3Y \quad \text{Simplifying}$$
$$-0^2 - 4 = 3Y$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

13.



The graph is not symmetric with respect to the 0-axis or the Y-axis. It is symmetric with respect to the origin.

Test algebraically for symmetry with respect to the 0-axis:

$$Y = \frac{1}{0} \quad \text{Original equation}$$
$$-Y = \frac{1}{0} \quad \text{Replacing } Y \text{ by } -Y$$
$$Y = -\frac{1}{0} \quad \text{Simplifying}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the 0-axis.

Test algebraically for symmetry with respect to the Y-axis:

$$Y = \frac{1}{0} \quad \text{Original equation}$$
$$Y = \frac{1}{-0} \quad \text{Replacing } 0 \text{ by } -0$$
$$Y = -\frac{1}{0} \quad \text{Simplifying}$$

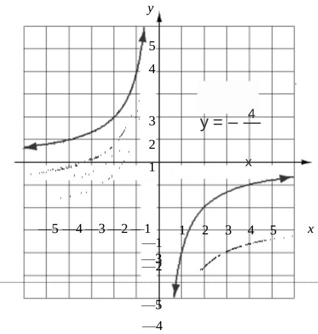
The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the Y-axis.

Test algebraically for symmetry with respect to the origin:

$$Y = \frac{1}{0}$$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

14.



The graph is not symmetric with respect to the 0-axis or the Y-axis. It is symmetric with respect to the origin.

Test algebraically for symmetry with respect to the 0-axis:

$$Y = -\frac{4}{0} \quad \text{Original equation}$$
$$-Y = -\frac{4}{0} \quad \text{Replacing } Y \text{ by } -Y$$
$$Y = \frac{4}{0} \quad \text{Simplifying}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the 0-axis.

Test algebraically for symmetry with respect to the Y-axis:

$$Y = -\frac{4}{0} \quad \text{Original equation}$$
$$Y = -\frac{4}{-0} \quad \text{Replacing } 0 \text{ by } -0$$
$$Y = \frac{4}{0} \quad \text{Simplifying}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the Y-axis.

Test algebraically for symmetry with respect to the origin:

$$Y = -\frac{4}{0} \quad \text{Original equation}$$
$$-Y = -\frac{4}{-0} \quad \text{Replacing } 0 \text{ by } -0 \text{ and } Y \text{ by } -Y$$
$$Y = -\frac{4}{0} \quad \text{Simplifying}$$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

15. Test for symmetry with respect to the 0-axis:

$$50 - 5Y = 0 \quad \text{Original equation}$$
$$50 - 5(-Y) = 0 \quad \text{Replacing } Y \text{ by } -Y$$
$$50 + 5Y = 0 \quad \text{Simplifying}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the 0-axis.

Test for symmetry with respect to the Y-axis:

$-Y =$ -0

$Y = \frac{1}{0}$ Original equation

Replacing 0 by -0 and Y by $-Y$

Simplifying

$50 - 5Y = 0$ Original equation

$5(-0) - 5Y = 0$ Replacing 0 by -0

$-50 - 5Y = 0$ Simplifying

$50 + 5Y = 0$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$5x - 5y = 0$ Original equation

$5(-x) - 5(-y) = 0$ Replacing x by $-x$ and y by $-y$

$-5x + 5y = 0$ Simplifying

$5x - 5y = 0$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

16. Test for symmetry with respect to the x-axis:

$6x + 7y = 0$ Original equation

$6x + 7(-y) = 0$ Replacing y by $-y$

$6x - 7y = 0$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$6x + 7y = 0$ Original equation

$6(-x) + 7y = 0$ Replacing x by $-x$

$6x - 7y = 0$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$6x + 7y = 0$ Original equation

$6(-x) + 7(-y) = 0$ Replacing x by $-x$ and y by $-y$

$6x + 7y = 0$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

17. Test for symmetry with respect to the x-axis:

$3x^2 - 2y^2 = 3$ Original equation

$3x^2 - 2(-y)^2 = 3$ Replacing y by $-y$

$3x^2 - 2y^2 = 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$3x^2 - 2y^2 = 3$ Original equation

$3(-x)^2 - 2y^2 = 3$ Replacing x by $-x$

$3x^2 - 2y^2 = 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$3x^2 - 2y^2 = 3$ Original equation

$3(-x)^2 - 2(-y)^2 = 3$ Replacing x by $-x$

18. Test for symmetry with respect to the x-axis:

$5y = 7x^2 - 2x$ Original equation

$5(-y) = 7x^2 - 2x$ Replacing y by $-y$

$5y = -7x^2 + 2x$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$5y = 7x^2 - 2x$ Original equation

$5y = 7(-x)^2 - 2(-x)$ Replacing x by $-x$

$5y = 7x^2 + 2x$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$5y = 7x^2 - 2x$ Original equation

$5(-y) = 7(-x)^2 - 2(-x)$ Replacing x by $-x$ and y by $-y$

$-5y = 7x^2 + 2x$ Simplifying

$5y = -7x^2 - 2x$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

19. Test for symmetry with respect to the x-axis:

$y = |2x|$ Original equation

$-y = |2x|$ Replacing y by $-y$

$y = -|2x|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$y = |2x|$ Original equation

$y = |2(-x)|$ Replacing x by $-x$

$y = |-2x|$ Simplifying

$y = |2x|$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$y = |2x|$ Original equation

$-y = |2(-x)|$ Replacing x by $-x$ and y by $-y$

$-y = |-2x|$ Simplifying

$-y = |2x|$

$y = -|2x|$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

20. Test for symmetry with respect to the x-axis:

$y^3 = 2x^2$ Original equation

$(-y)^3 = 2x^2$ Replacing y by $-y$

and y by $-y$

$3x^2 - 2y^2 = 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

— —

$-y^3 = 2x^2$ Simplifying

$y^3 = -2x^2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$y^3 = 2x^2$ Original equation

$y^3 = 2(-x)^2$ Replacing x by $-x$

$y^3 = 2x^2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$y^3 = 2x^2$ Original equation

$(-y)^3 = 2(-x)^2$ Replacing x by $-x$ and y by $-y$

$-y^3 = 2x^2$ Simplifying

$y^3 = -2x^2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

21. Test for symmetry with respect to the x-axis:

$2x^4 + 3 = y^2$ Original equation

$2x^4 + 3 = (-y)^2$ Replacing y by $-y$

$2x^4 + 3 = y^2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$2x^4 + 3 = y^2$ Original equation

$2(-x)^4 + 3 = y^2$ Replacing x by $-x$

$2x^4 + 3 = y^2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$2x^4 + 3 = y^2$ Original equation

$2(-x)^4 + 3 = (-y)^2$ Replacing x by $-x$ and y by $-y$

$2x^4 + 3 = y^2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

22. Test for symmetry with respect to the x-axis:

$2y^2 = 5x^2 + 12$ Original equation

$2(-y)^2 = 5x^2 + 12$ Replacing y by $-y$

$2y^2 = 5x^2 + 12$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$2y^2 = 5x^2 + 12$ Original equation

$2y^2 = 5(-x)^2 + 12$ Replacing x by $-x$

$2y^2 = 5x^2 + 12$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

23. Test for symmetry with respect to the x-axis:

$3y^3 = 4x^3 + 2$ Original equation

$3(-y)^3 = 4x^3 + 2$ Replacing y by $-y$

$-3y^3 = 4x^3 + 2$ Simplifying

$3y^3 = -4x^3 - 2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$3y^3 = 4x^3 + 2$ Original equation

$3y^3 = 4(-x)^3 + 2$ Replacing x by $-x$

$3y^3 = -4x^3 + 2$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$3y^3 = 4X^3 + 2$ Original equation

$3(-y)^3 = 4(-x)^3 + 2$ Replacing x by $-x$ and y by $-y$

$-3y^3 = -4x^3 + 2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$2y^2 = 5x^2 + 12$ Original equation

$2(-y)^2 = 5(-x)^2 + 12$ Replacing x by $-x$ and y by $-y$

$2y^2 = 5x^2 + 12$ Simplifying

$3y^3 = 4x^3 - 2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

24. Test for symmetry with respect to the x-axis:

$3x = |y|$ Original equation

$3x = |-y|$ Replacing y by $-y$

$3x = |y|$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$3x = |y|$ Original equation

$3(-x) = |y|$ Replacing x by $-x$

$-3x = |y|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$3x = |y|$ Original equation

$3(-x) = |-y|$ Replacing x by $-x$ and y by $-y$

$-3x = |y|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

25. Test for symmetry with respect to the x-axis:

$xy = 12$ Original equation

$x(-y) = 12$ Replacing y by $-y$

$-xy = 12$ Simplifying

$xy = -12$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$vy = 12$ Original equation

$-vy = 12$ Replacing v by $-v$

$vy = -12$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$vy = 12$ Original equation

$-v(-y) = 12$ Replacing v by $-v$ and y by $-y$

$vy = 12$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

26. Test for symmetry with respect to the v-axis:

$vy - v^2 = 3$ Original equation

$v(-y) - v^2 = 3$ Replacing y by $-y$

$vy + v^2 = -3$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the v-axis.

Test for symmetry with respect to the y-axis:

$vy - v^2 = 3$ Original equation

$-vy - (-v)^2 = 3$ Replacing v by $-v$

$vy + v^2 = -3$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$vy - v^2 = 3$ Original equation

$-v(-y) - (-v)^2 = 3$ Replacing v by $-v$ and y by $-y$

$vy - v^2 = 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

27. v-axis: Replace y with $-y$; $(-5, -6)$

y-axis: Replace v with $-v$; $(5, 6)$

Origin: Replace v with $-v$ and y with $-y$; $(5, -6)$

28. v-axis: Replace y with $-y$; $\left(\frac{7}{2}, 0\right)$

y-axis: Replace v with $-v$; $\left(-\frac{7}{2}, 0\right)$

Origin: Replace v with $-v$ and y with $-y$; $\left(-\frac{7}{2}, 0\right)$

29. v-axis: Replace y with $-y$; $(-10, 7)$

y-axis: Replace v with $-v$; $(10, -7)$

Origin: Replace v with $-v$ and y with $-y$; $(10, 7)$

30. v-axis: Replace y with $-y$; y-axis: Replace v with $-v$;

31. v-axis: Replace y with $-y$; $(0, 4)$

y-axis: Replace v with $-v$; $(0, -4)$

Origin: Replace v with $-v$ and y with $-y$; $(0, 4)$

32. v-axis: Replace y with $-y$; $(8, 3)$

y-axis: Replace v with $-v$; $(-8, -3)$

Origin: Replace v with $-v$ and y with $-y$; $(-8, 3)$

33. The graph is symmetric with respect to the y-axis, so the function is even.

34. The graph is symmetric with respect to the y-axis, so the function is even.

35. The graph is symmetric with respect to the origin, so the function is odd.

36. The graph is not symmetric with respect to either the y-axis or the origin, so the function is neither even nor odd.

37. The graph is not symmetric with respect to either the y-axis or the origin, so the function is neither even nor odd.

38. The graph is not symmetric with respect to either the y-axis or the origin, so the function is neither even nor odd.

39. $f(v) = -3v^3 + 2v$

$f(-v) = -3(-v)^3 + 2(-v) = 3v^3 - 2v$

$-f(v) = -(-3v^3 + 2v) = 3v^3 - 2v$

$f(-v) = -f(v)$, so f is odd.

40. $f(v) = 7v^3 + 4v - 2$

$f(-v) = 7(-v)^3 + 4(-v) - 2 = -7v^3 - 4v - 2$

$-f(v) = -(7v^3 + 4v - 2) = -7v^3 - 4v + 2$

$f(v) \neq f(-v)$, so f is not even.

$f(-v) \neq -f(v)$, so f is not odd.

Thus, $f(v) = 7v^3 + 4v - 2$ is neither even nor odd.

41. $f(v) = 5v^2 + 2v^4 - 1$

$f(-v) = 5(-v)^2 + 2(-v)^4 - 1 = 5v^2 + 2v^4 - 1$

$f(v) = f(-v)$, so f is even.

42. $f(v) = v + \frac{1}{v}$

$f(-v) = -v + \frac{1}{-v} = -v - \frac{1}{v}$

$-f(v) = -\left(v + \frac{1}{v}\right) = -v - \frac{1}{v}$

$f(-v) = -f(v)$, so f is odd.

43. $f(v) = v^{17}$

$f(-v) = (-v)^{17} = -v^{17}$

$-f(v) = -v^{17}$

$\left(1, -\frac{3}{8}\right)$

$\left(-1, \frac{3}{8}\right)$

$f(-v) = -f(v)$, so f is odd.

$\left(\quad 3 \right)$

44. $f(v) = \sqrt[3]{v}$

$f(-v) = \sqrt[3]{-v} = -\sqrt[3]{v}$

$-f(v) = -\sqrt[3]{v}$

Origin: Replace v with $-v$ and y with $-y$; $-1, -\frac{}{8}$

$f(-v) = -f(v)$, so f is odd.

45. $f(v) = v - |v|$

$f(-v) = (-v) - |(-v)| = -v - |v|$

$-f(v) = -(v - |v|) = -v + |v|$

$f(v) \neq f(-v)$, so f is not even.

$f(-v) \neq -f(v)$, so f is not odd.

Thus, $f(v) = v - |v|$ is neither even nor odd.

46. $f(v) = \frac{1}{v^2}$

$f(-v) = \frac{1}{(-v)^2} = \frac{1}{v^2}$

$f(v) = f(-v)$, so f is even.

47. $f(v) = 8$

$f(-v) = 8$

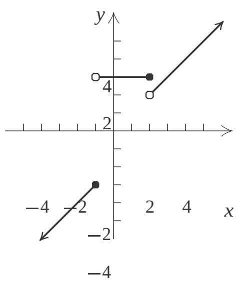
$f(v) = f(-v)$, so f is even.

48. $f(v) = \sqrt{v^2 + 1}$

$f(-v) = \sqrt{(-v)^2 + 1} = \sqrt{v^2 + 1}$

$f(v) = f(-v)$, so f is even.

49.



50. Let v = the number of volunteers from the University of Wisconsin - Madison. Then $v + 464$ = the number of volunteers from the University of California - Berkeley.

Solve: $v + (v + 464) = 6688$

$v = 3112$, so there were 3112 volunteers from the University of Wisconsin - Madison and $3112 + 464$, or 3576 volunteers from the University of California - Berkeley.

51. $f(v) = v\sqrt{10 - v^2}$

$f(-v) = -v\sqrt{10 - (-v)^2} = -v\sqrt{10 - v^2}$

$-f(v) = -v\sqrt{10 - v^2}$

Since $f(-v) = -f(v)$, f is odd.

52. $f(v) = \frac{v^2 + 1}{v^3 + 1}$

$\frac{(-v)^2 + 1}{} \quad \frac{v^2 + 1}{}$

53. If the graph were folded on the v-axis, the parts above and below the v-axis would coincide, so the graph is symmetric with respect to the v-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would not coincide, so the graph is not symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would not coincide with the original graph, so it is not symmetric with respect to the origin.

54. If the graph were folded on the v-axis, the parts above and below the v-axis would not coincide, so the graph is not symmetric with respect to the v-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would not coincide, so the graph is not symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would coincide with the original graph, so it is symmetric with respect to the origin.

55. See the answer section in the text.

56. $O(-v) = \frac{f(-v) - f(-(-v))}{2} = \frac{f(-v) - f(v)}{2}$,

$-O(v) = -\frac{f(v) - f(-v)}{2} = \frac{f(-v) - f(v)}{2}$. Thus,

$O(-v) = -O(v)$ and O is odd.

57. a), b) See the answer section in the text.

58. Let $f(v) = g(v) = v$. Now f and g are odd functions, but $(fg)(v) = v^2 = (fg)(-v)$. Thus, the product is even, so the statement is false.

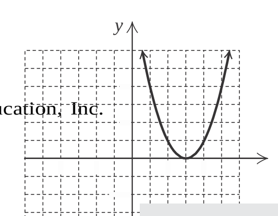
59. Let $f(v)$ and $g(v)$ be even functions. Then by definition, $f(v) = f(-v)$ and $g(v) = g(-v)$. Thus, $(f + g)(v) = f(v) + g(v) = f(-v) + g(-v) = (f + g)(-v)$ and $f + g$ is even. The statement is true.

60. Let $f(v)$ be an even function, and let $g(v)$ be an odd function. By definition $f(v) = f(-v)$ and $g(-v) = -g(v)$, or $g(v) = -g(-v)$. Then $fg(v) = f(v) \cdot g(v) = f(-v) \cdot [-g(-v)] = -f(-v) \cdot g(-v) = -fg(-v)$, and fg is odd.

The statement is true.

Exercise Set 2.5

1. Shift the graph of $f(v) = v^2$ right 3 units.



$f(-v) = \frac{2}{(-v)^3}$ =

$-f(v) = -\frac{v^2+1}{v^3+1}$

Since $f(v) = f(-v)$, f is not even.

Since $f(-v) = -f(v)$, f is not odd.

2

$v^3 + 1$

4

2

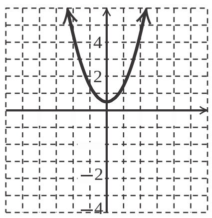
−4 −2 2 4 x

−2

−4

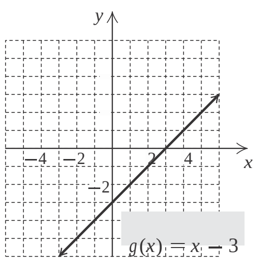
$f(x) = (x - 3)^2$

2. Shift the graph of $g(v) = v^2$ up $\frac{1}{2}$ unit.

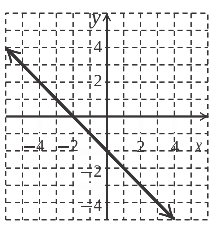


$g(x) = x^2 + \frac{1}{2}$

3. Shift the graph of $g(v) = v$ down 3 units.

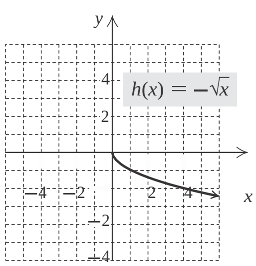


4. Reflect the graph of $g(v) = v$ across the v-axis and then shift it down 2 units.

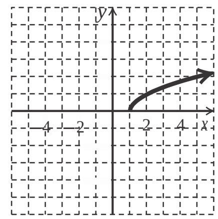


$g(x) = -x - 2$

5. Reflect the graph of $h(v) = \sqrt{v}$ across the v-axis.

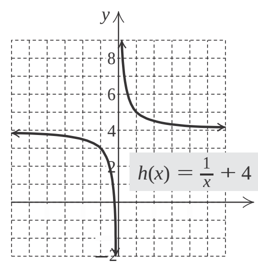


6. Shift the graph of $g(v) = \sqrt{v}$ right 1 unit.

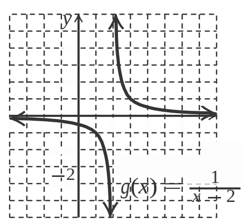


$g(x) = \sqrt{x - 1}$

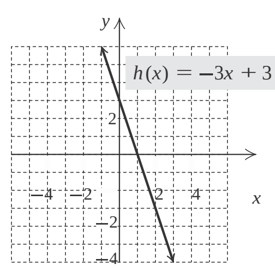
7. Shift the graph of $h(v) = \frac{1}{v}$ up 4 units.



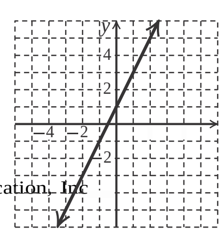
8. Shift the graph of $g(v) = \frac{1}{v}$ right 2 units.



9. First stretch the graph of $h(v) = v$ vertically by multiplying each y-coordinate by 3. Then reflect it across the v-axis and shift it up 3 units.



10. First stretch the graph of $f(v) = v$ vertically by multiplying each y-coordinate by 2. Then shift it up 1 unit.

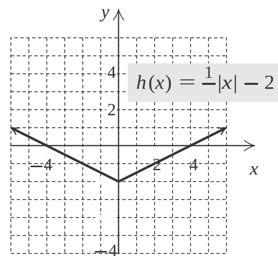


−4

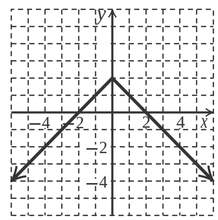
2 4 x

$f(x) = 2x + 1$

11. First shrink the graph of $h(v) = |v|$ vertically by multiply- $\frac{1}{2}$

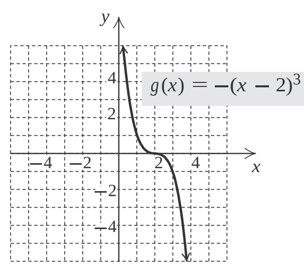


it up 2 units.

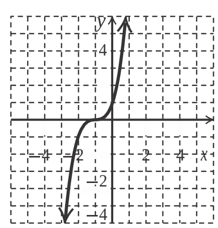


$g(x) = -|x| + 2$

13. Shift the graph of $g(v) = v^3$ right 2 units and reflect it

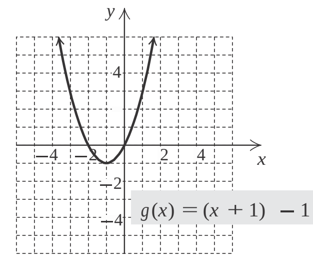


14. Shift the graph of $f(v) = v^3$ left 1 unit.

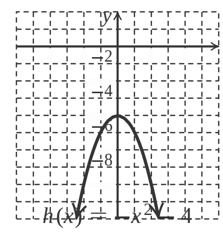


$f(x) = (x + 1)^3$

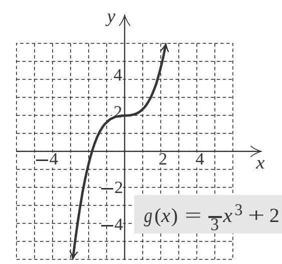
15. Shift the graph of $g(v) = v^2$ left 1 unit and down 1 unit.



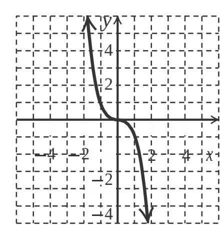
16. Reflect the graph of $h(v) = v^2$ across the v-axis and down 4 units.



17. First shrink the graph of $g(v) = v^3$ vertically by multiplying each y-coordinate by $\frac{1}{3}$. Then shift it up 2 units.

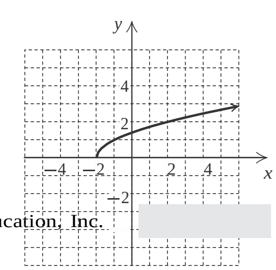


18. Reflect the graph of $h(v) = v^3$ across the y-axis.



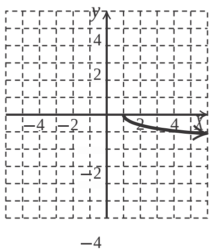
$h(x) = (-x)^3$

19. Shift the graph of $f(v) = \sqrt{v}$ left 2 units.



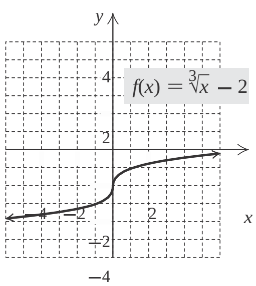
-4 $f(x) = \sqrt{x+2}$

20. First shift the graph of $f(¢) = \sqrt{¢}$ right 1 unit. Shrink it vertically by multiplying each y-coordinate by $\frac{1}{2}$ and then reflect it across the ¢-axis.

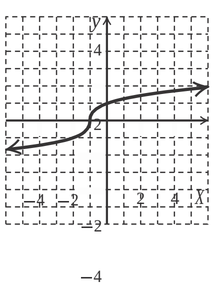


$f(x) = -\frac{1}{2}\sqrt{x-1}$

21. Shift the graph of $f(¢) = \sqrt[3]{¢}$ down 2 units.



22. Shift the graph of $h(¢) = \sqrt[3]{¢}$ left 1 unit.



$h(x) = \sqrt[3]{x+1}$

23. Think of the graph of $f(¢) = |¢|$. Since $g(¢) = f(3¢)$, the graph of $g(¢) = |3¢|$ is the graph of $f(¢) = |¢|$ shrunk horizontally by dividing each ¢-coordinate by 3 or multiplying each ¢-coordinate by $\frac{1}{3}$.

24. Think of the graph of $g(¢) = \sqrt[3]{¢}$. Since $f(¢) = \frac{1}{2}g(¢)$, the

28. Think of the graph of $g(¢) = \frac{1}{¢}$. Since $f(¢) = 5 - g(¢)$, or $f(¢) = -g(¢) + 5$, the graph of $f(¢) = 5 - \frac{1}{¢}$ is the graph of $g(¢) = \frac{1}{¢}$ reflected across the ¢-axis and then shifted up 5 units.

29. Think of the graph of $f(¢) = |¢|$. Since $g(¢) = f\left(\frac{1}{3}¢\right) - 4$, the graph of $g(¢) = \left|\frac{1}{3}¢\right| - 4$ is the graph of $f(¢) = |¢|$ stretched horizontally by multiplying each ¢-coordinate by 3 and then shifted down 4 units.

30. Think of the graph of $g(¢) = ¢^3$. Since $f(¢) = \frac{2}{3}$ ... $\frac{2}{3}$ graph of $g(¢) = ¢^3$ shrunk vertically by multiplying each $\frac{2}{-}$

31. Think of the graph of $g(¢) = ¢^2$. Since $f(¢) = -\frac{1}{4}g(¢-5)$, the graph of $f(¢) = -\frac{1}{4}(¢-5)^2$ is the graph of $g(¢) = ¢^2$ y-coordinate by $\frac{1}{4}$, and reflected across the ¢-axis.

32. Think of the graph of $g(¢) = ¢^3$. Since $f(¢) = g(-¢) - 5$, reflected across the y-axis and shifted down 5 units.

33. Think of the graph of $g(¢) = \frac{1}{¢}$. Since $f(¢) = g(¢+3) + 2$, the graph of $f(¢) = \frac{1}{¢+3} + 2$ is the graph of $g(¢) = \frac{1}{¢}$ shifted left 3 units and up 2 units.

34. Think of the graph of $f(¢) = \sqrt{¢}$. Since $g(¢) = f(-¢) + 5$, the graph of $g(¢) = \sqrt{-¢} + 5$ is the graph of $f(¢) = \sqrt{¢}$ reflected across the y-axis and shifted up 5 units.

35. Think of the graph of $f(¢) = ¢^2$. Since $h(¢) = -f(¢-3) + 5$, the graph of $h(¢) = -(¢-3)^2 + 5$ is the graph of $f(¢) = ¢^2$ shifted right 3 units, reflected across the ¢-axis, and shifted up 5 units.

36. Think of the graph of $g(¢) = ¢^2$. Since $f(¢) = 3g(¢+4) -$

graph of $f(¢) = \frac{1}{2}\sqrt[3]{¢}$ is the graph of $g(¢) = \sqrt[3]{¢}$ shrunk vertically by multiplying each y-coordinate by $\frac{1}{2}$.

25. Think of the graph of $f(¢) = \frac{1}{¢}$. Since $h(¢) = 2f(¢)$, the graph of $h(¢) = \frac{2}{¢}$ is the graph of $f(¢) = \frac{1}{¢}$ stretched vertically by multiplying each y-coordinate by 2.

26. Think of the graph of $g(¢) = |¢|$. Since $f(¢) = g(¢ - 3) - 4$, the graph of $f(¢) = |¢ - 3| - 4$ is the graph of $g(¢) = |¢|$ shifted right 3 units and down 4 units.

27. Think of the graph of $g(¢) = \sqrt{¢}$. Since $f(¢) = 3g(¢) - 5$, the graph of $f(¢) = 3\sqrt{¢} - 5$ is the graph of $g(¢) = \sqrt{¢}$ stretched vertically by multiplying each y-coordinate by 3 and then shifted down 5 units.

3, the graph of $f(¢) = 3(¢+4)^2 - 3$ is the graph of $g(¢) = ¢^2$ shifted left 4 units, stretched vertically by multiplying each y-coordinate by 3, and then shifted down 3 units.

37. The graph of $y = g(¢)$ is the graph of $y = f(¢)$ shrunk vertically by a factor of $\frac{1}{2}$. Multiply the y-coordinate by $\frac{1}{2}$: $(-12, 2)$.

38. The graph of $y = g(¢)$ is the graph of $y = f(¢)$ shifted right 2 units. Add 2 to the $¢$-coordinate: $(-10, 4)$.

39. The graph of $y = g(¢)$ is the graph of $y = f(¢)$ reflected across the y-axis, so we reflect the point across the y-axis: $(12, 4)$.

40. The graph of $Y = 9(v)$ is the graph of $Y = f(v)$ shrunk horizontally. The v-coordinates of $Y = 9(v)$ are $\frac{1}{4}$ the corresponding v-coordinates of $Y = f(v)$, so we divide the v-coordinate by 4 or multiply it by $\frac{1}{4}$: $(-3, 4)$.

41. The graph of $Y = 9(v)$ is the graph of $Y = f(v)$ shifted down 2 units. Subtract 2 from the Y-coordinate: $(-12, 2)$.

42. The graph of $Y = 9(v)$ is the graph of $Y = f(v)$ stretched horizontally. The v-coordinates of $Y = 9(v)$ are twice the corresponding v-coordinates of $Y = f(v)$, so we multiply the v-coordinate by 2 or divide it by $\frac{1}{2}$: $(-24, 4)$.

43. The graph of $Y = 9(v)$ is the graph of $Y = f(v)$ stretched vertically by a factor of 4. Multiply the Y-coordinate by 4: $(-12, 16)$.

44. The graph of $Y = 9(v)$ is the graph $Y = f(v)$ reflected across the v-axis. Reflect the point across the v-axis: $(-12, -4)$.

45. $9(v) = v^2 + 4$ is the function $f(v) = v^2 + 3$ shifted up 1 unit, so $9(v) = f(v) + 1$. Answer B is correct.

46. If we substitute $3v$ for v in f, we get $9v^2 + 3$, so $9(v) = f(3v)$. Answer D is correct.

47. If we substitute $v - 2$ for v in f, we get $(v - 2)^3 + 3$, so $9(v) = f(v - 2)$. Answer A is correct.

48. If we multiply $v^2 + 3$ by 2, we get $2v^2 + 6$, so $9(v) = 2f(v)$. Answer C is correct.

49. Shape: $h(v) = v^2$

Turn $h(v)$ upside-down (that is, reflect it across the v-axis): $9(v) = -h(v) = -v^2$

Shift $9(v)$ right 8 units: $f(v) = 9(v - 8) = -(v - 8)^2$

50. Shape: $h(v) = \sqrt{v}$

Shift $h(v)$ left 6 units: $9(v) = h(v + 6) = \sqrt{v + 6}$

Shift $9(v)$ down 5 units: $f(v) = 9(v) - 5 = \sqrt{v + 6} - 5$

51. Shape: $h(v) = [v]$

Shift $h(v)$ left 7 units: $9(v) = h(v + 7) = [v + 7]$

Shift $9(v)$ up 2 units: $f(v) = 9(v) + 2 = [v + 7] + 2$

52. Shape: $h(v) = v^3$

Turn $h(v)$ upside-down (that is, reflect it across the v-axis): $9(v) = -h(v) = -v^3$

Shift $9(v)$ right 5 units: $f(v) = 9(v - 5) = -(v - 5)^3$

53. Shape: $h(v) = \frac{1}{v}$

Shrink $h(v)$ vertically by a factor of $\frac{1}{2}$ that is, multiply each function value by $\frac{1}{2}$:

54. Shape: $h(v) = v^2$

Shift $h(v)$ right 6 units: $9(v) = h(v - 6) = (v - 6)^2$

Shift $9(v)$ up 2 units: $f(v) = 9(v) + 2 = (v - 6)^2 + 2$

55. Shape: $m(v) = v^2$

Turn $m(v)$ upside-down (that is, reflect it across the v-axis): $h(v) = -m(v) = -v^2$

Shift $h(v)$ right 3 units: $9(v) = h(v - 3) = -(v - 3)^2$

Shift $9(v)$ up 4 units: $f(v) = 9(v) + 4 = -(v - 3)^2 + 4$

56. Shape: $h(v) = [v]$

Stretch $h(v)$ horizontally by a factor of 2 that is, multiply each v-value by $\frac{1}{2}$: $9(v) = h\left(\frac{1}{2}v\right) = \left[\frac{1}{2}v\right]$

Shift $9(v)$ down 5 units: $f(v) = 9(v) - 5 = \left[\frac{1}{2}v\right] - 5$

57. Shape: $m(v) = \sqrt{v}$

Reflect $m(v)$ across the Y-axis: $h(v) = m(-v) = \sqrt{-v}$

Shift $h(v)$ left 2 units: $9(v) = h(v + 2) = \sqrt{-(v + 2)}$

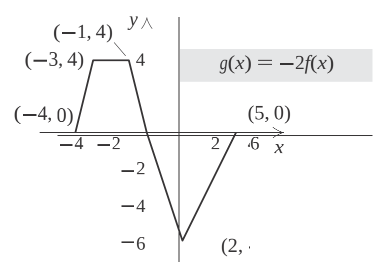
Shift $9(v)$ down 1 unit: $f(v) = 9(v) - 1 = \sqrt{-(v + 2)} - 1$

58. Shape: $h(v) = \frac{1}{v}$

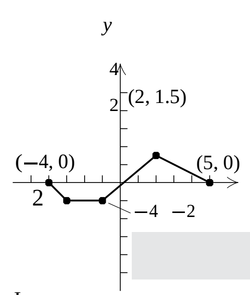
Reflect $h(v)$ across the v-axis: $9(v) = -h(v) = -\frac{1}{v}$

Shift $9(v)$ up 1 unit: $f(v) = 9(v) + 1 = -\frac{1}{v} + 1$

59. Each Y-coordinate is multiplied by -2. We plot and connect $(-4, 0)$, $(-3, 4)$, $(-1, 4)$, $(2, -6)$, and $(5, 0)$.



60. Each Y-coordinate is multiplied by $\frac{1}{2}$. We plot and connect $(-4, 0)$, $(-3, -1)$, $(-1, -1)$, $(2, 1.5)$, and $(5, 0)$.



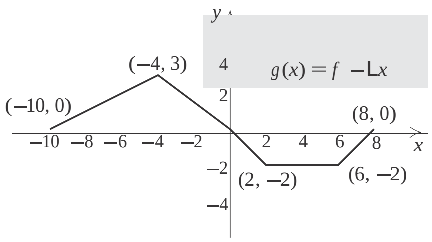
2 4 x
$(-1, -1)$

$g(x) =$
$qf(x)$

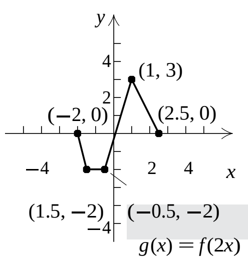
$g(v) = \frac{1}{2}h(v) = \frac{1}{2}\cdot\frac{1}{v}$, or $\frac{1}{2v}$

Shift $g(v)$ down 3 units: $f(v) = g(v) - 3 = \frac{1}{2v} - 3$

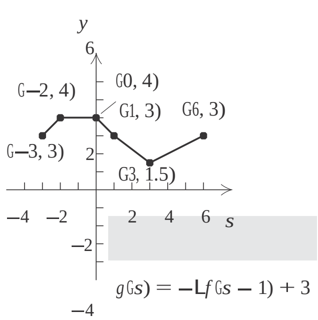
61. The graph is reflected across the Y-axis and stretched horizontally by a factor of 2. That is, each v-coordinate is multiplied by -2 or divided by $-\frac{1}{2}$. We plot and connect $(8, 0)$, $(6, -2)$, $(2, -2)$, $(-4, 3)$, and $(-10, 0)$.



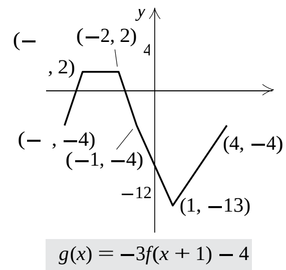
62. The graph is shrunk horizontally by a factor of 2. That is, each v-coordinate is divided by 2 or multiplied by $\frac{1}{2}$. We plot and connect $(-2, 0)$, $(-1.5, -2)$, $(-0.5, -2)$, $(1, 3)$, and $(2.5, 0)$.



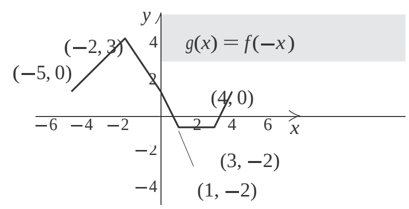
63. The graph is shifted right 1 unit so each v-coordinate is increased by 1. The graph is also reflected across the v-axis, shrunk vertically by a factor of 2, and shifted up 3 units. Thus, each Y-coordinate is multiplied by $-\frac{1}{2}$ and then increased by 3. We plot and connect $(-3, 3)$, $(-2, 4)$, $(0, 4)$, $(3, 1.5)$, and $(6, 3)$.



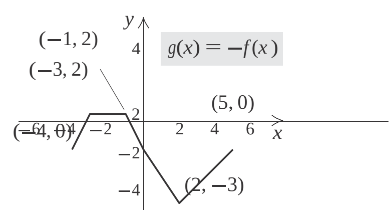
64. The graph is shifted left 1 unit so each v-coordinate is decreased by 1. The graph is also reflected across the v-axis, stretched vertically by a factor of 3, and shifted down 4 units. Thus, each Y-coordinate is multiplied by -3 and then decreased by 4. We plot and connect $(-5, -4)$, $(-4, 2)$, $(-2, 2)$, $(1, -13)$, and $(4, -4)$.



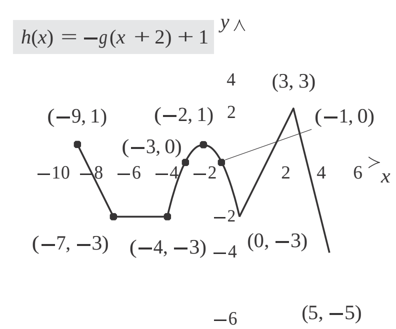
65. The graph is reflected across the Y-axis so each v-coordinate is replaced by its opposite.



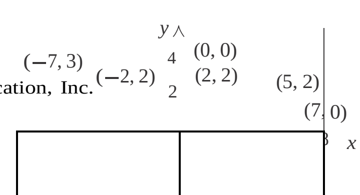
66. The graph is reflected across the v-axis so each Y-coordinate is replaced by its opposite.

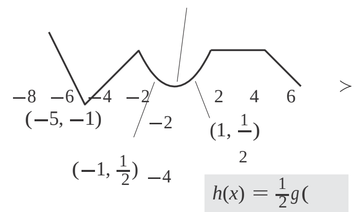


67. The graph is shifted left 2 units so each v-coordinate is decreased by 2. It is also reflected across the v-axis so each Y-coordinate is replaced with its opposite. In addition, the graph is shifted up 1 unit, so each Y-coordinate is then increased by 1.

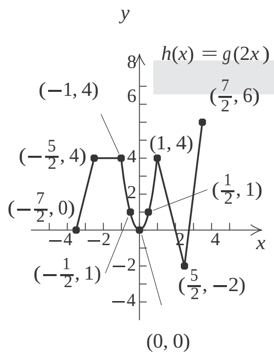


68. The graph is reflected across the Y-axis so each v-coordinate is replaced with its opposite. It is also shrunk vertically by a factor of $\frac{1}{2}$, so each Y-coordinate is multiplied by $\frac{1}{2}$(or divided by 2).

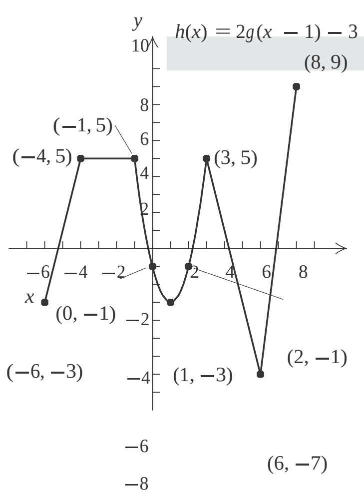



−8 −6 −4 −2
2 4 6
(−5, −1)
−2
(1, 1/2)
2
(−1, 1/2)
−4
h(x) = 1/2 g(

69. The graph is shrunk horizontally. The 0-coordinates of $Y = h(0)$ are one-half the corresponding 0-coordinates of $Y = 9(0)$.



70. The graph is shifted right 1 unit, so each 0-coordinate is increased by 1. It is also stretched vertically by a factor of 2, so each Y-coordinate is multiplied by 2 7 or divided by $\frac{1}{2}$. In addition, the graph is shifted down 3 units, so each Y-coordinate is decreased by 3.



71. $9(0) = f(-0)\ 4\ 3$

The graph of $9(0)$ is the graph of $f(0)$ reflected across the Y-axis and shifted up 3 units. This is graph (f).

72. $9(0) = f(0)\ 4\ 3$

The graph of $9(0)$ is the graph of $f(0)$ shifted up 3 units. This is graph (h).

73. $9(0) = -f(0)\ 4\ 3$

The graph of $9(0)$ is the graph of $f(0)$ reflected across the 0-axis and shifted up 3 units. This is graph (f).

74. $9(0) = -f(-0)$

The graph of $9(0)$ is the graph of $f(0)$ reflected across the 0-axis and the Y-axis. This is graph (a).

75. $9(0) = \frac{1}{3}f(0 - 2)$

The graph of $9(0)$ is the graph of $f(0)$ shrunk vertically by a factor of 3 that is, each Y-coordinate is multiplied 1

76. $9(0) = \frac{1}{3}f(0) - 3$

The graph of $9(0)$ is the graph of $f(0)$ shrunk vertically by a factor of 3 that is, each Y-coordinate is multiplied by $\frac{1}{3}$ and then shifted down 3 units. This is graph (e).

77. $9(0) = \frac{1}{3}f(0\ 4\ 2)$

The graph of $9(0)$ is the graph of $f(0)$ shrunk vertically by a factor of 3 that is, each Y-coordinate is multiplied by $\frac{1}{3}$ and then shifted left 2 units. This is graph (c).

78. $9(0) = -f(0\ 4\ 2)$

The graph of $9(0)$ is the graph $f(0)$ reflected across the 0-axis and shifted left 2 units. This is graph (b).

79. $f(-0) = 2(-0)^4 - 35(-0)^3\ 4\ 3(-0) - 5 =$

$20^4\ 4\ 350^3 - 30 - 5 = 9(0)$

80. $f(-0) = \frac{1}{4}(-0)^4\ 4\ \frac{1}{5}(-0)^3 - 81(-0)^2 - 17 =$

$\frac{1}{4}0^4 - \frac{1}{5}0^3 - 810^2 - 17 = 9(0)$

81. The graph of $f(0) = 0^3 - 30^2$ is shifted up 2 units. A formula for the transformed function is $9(0) = f(0)\ 4\ 2$, or $9(0) = 0^3 - 30^2\ 4\ 2$.

82. Each Y-coordinate of the graph of $f(0) = 0^3 - 30^2$ is multiplied by $\frac{1}{2}$. A formula for the transformed function is $h(0) = \frac{1}{2}f(0)$, or $h(0) = \frac{1}{2}(0^3 - 30^2)$.

83. The graph of $f(0) = 0^3 - 30^2$ is shifted left 1 unit. A formula for the transformed function is $k(0) = f(0\ 4\ 1)$, or $k(0) = (0\ 4\ 1)^3 - 3(0\ 4\ 1)^2$.

84. The graph of $f(0) = 0^3 - 30^2$ is shifted right 2 units and up 1 unit. A formula for the transformed function 2 is $t(0) = f(0 - 2)\ 4\ 1$, or $t(0) = (0 - 2)^3 - 3(0 - 2)\ \ 4\ 1$.

85. Test for symmetry with respect to the 0-axis.

$Y = 30^4 - 3$ Original equation

$-Y = 30^4 - 3$ Replacing Y by $-Y$

$Y = -30^4 4 3$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the 0-axis.

Test for symmetry with respect to the Y-axis.

$Y = 30^4 - 3$ Original equation

by 3 and then shifted right 2 units. This is graph (d).

$Y = 3(-0)^4 - 3$ Replacing 0 by -0

$Y = 30^4 - 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the Y-axis.

Test for symmetry with respect to the origin:

$Y = 30^4 - 3$

$-Y = 3(-0)^4 - 3$ Replacing 0 by -0 and

Y by $-Y$

$-Y = 30^4 - 3$

$Y = -30^4 4 3$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

86. Test for symmetry with respect to the X-axis.

$y^2 = X$ Original equation

$(-y)^2 = X$ Replacing y by $-y$

$y^2 = X$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the X-axis.

Test for symmetry with respect to the y-axis:

$y^2 = X$ Original equation

$y^2 = -X$ Replacing X by $-X$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$y^2 = X$ Original equation

$(-y)^2 = -X$ Replacing X by $-X$ and y by $-y$

$y^2 = -X$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

87. Test for symmetry with respect to the X-axis:

$2X - 5y = 0$ Original equation

$2X - 5(-y) = 0$ Replacing y by $-y$

$2X 4 5y = 0$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the X-axis.

Test for symmetry with respect to the y-axis:

$2X - 5y = 0$ Original equation

$2(-X) - 5y = 0$ Replacing X by $-X$

$-2X - 5y = 0$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$2X - 5y = 0$ Original equation

$2(-X) - 5(-y) = 0$ Replacing X by $-X$ and y by $-y$

$-2X 4 5y = 0$

$2X - 5y = 0$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

88. Let p = the number of pages of federal tax rules in 1995. Then the number of pages in 2014 was p 4 84.2% of p, or $1.842p$.

Solve: $1.842p = 74,608$

$p = 40,504$ pages

89. *Familiarize*. Let n = the number of guns that were found with airline travelers in 2010.

Translate.

Number of guns found in 2013	was twice	number of guns found in 2010	less 418.
1828	$= 2 \cdot$	n	$-$ 418

Solve.

$1828 = 2n - 418$

$2246 = 2n$

$1123 = n$

Check. $2 \cdot 1123 = 2246$ and $2246 - 418 = 1828$. This is the number of guns found in 2013, so the answer checks.

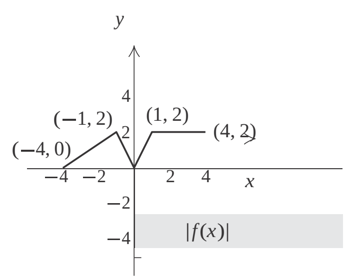
State. In 2010, 1123 guns were found with airline travelers.

90. Let a = the total number of acres of pumpkins harvested in Michigan, Ohio, and Illinois in 2012.

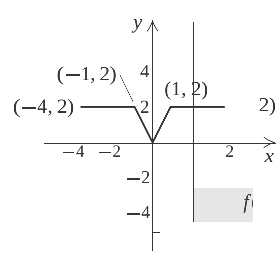
Solve: $0.545a = 16,200$

$a \approx 29,700$ acres

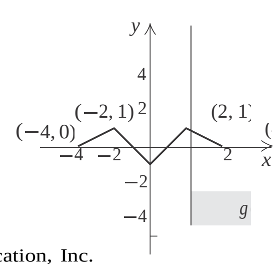
91. Each point for which $f(X) < 0$ is reflected across the X-axis.



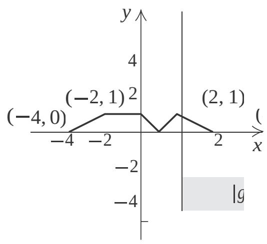
92. The graph of $y = f(JXJ)$ consists of the points of $y = f(X)$ for which $X \geq 0$ along with their reflections across the y-axis.



93. The graph of $y = 9(JXJ)$ consists of the points of $y = 9(X)$ for which $X \geq 0$ along with their reflections across the y-axis.



94. Each point for which $9(X) < 0$ is reflected across the X-axis.



95. $f(2 - 3) = f(-1) = 5$, so $b = 5$.
(The graph of $y = f(X - 3)$ is the graph of $y = f(X)$ shifted right 3 units, so the point $(-1, 5)$ on $y = f(X)$ is transformed to the point $(-1\ 4\ 3, 5)$, or $(2, 5)$ on $y = f(X - 3)$.)

96. Call the transformed function $9(X)$.
Then $9(5) = 4 - f(-3) = 4 - f(5 - 8)$,
$9(8) = 4 - f(0) = 4 - f(8 - 8)$,
and $9(11) = 4 - f(3) = 4 - f(11 - 8)$.
Thus $9(X) = 4 - f(X - 8)$, or $9(X) = 4 -]X - 8]$.

Exercise Set 2.6

1. $y = kX$
$54 = k \cdot 12$
$\frac{54}{12} = k$, or $k = \frac{9}{2}$
The variation constant is $\frac{9}{2}$, or 4.5. The equation of variation is $y = \frac{9}{2}X$, or $y = 4.5X$.

2. $y = kX$
$0.1 = k(0.2)$
$\frac{1}{2} = k$ Variation constant
Equation of variation: $y = \frac{1}{2}X$, or $y = 0.5X$.

3. $y = \frac{k}{X}$
$3 = \frac{k}{12}$
$36 = k$
The variation constant is 36. The equation of variation is $y = \frac{36}{X}$.

4. $y = \frac{k}{X}$
$12 = \frac{k}{5}$
$60 = k$ Variation constant
Equation of variation: $y = \frac{60}{X}$

5. $y = kX$
$1 = k \cdot \frac{1}{4}$
$4 = k$
The variation constant is 4. The equation of variation is $y = 4X$.

6. $y = \frac{k}{X}$
$0.1 = \frac{k}{0.5}$
$0.05 = k$ Variation constant
Equation of variation: $y = \frac{0.05}{X}$

7. $y = \frac{k}{X}$
$32 = \frac{k}{\frac{1}{8}}$
$\frac{1}{8} \cdot 32 = k$
$4 = k$
The variation constant is 4. The equation of variation is $y = \frac{4}{X}$.

8. $y = kX$
$3 = k \cdot 33$
$\frac{1}{11} = k$ Variation constant
Equation of variation: $y = \frac{1}{11}X$

9. $y = kX$
$\frac{3}{4} = k \cdot 2$
$\frac{1}{2} \cdot \frac{3}{4} = k$
$\frac{3}{8} = k$
The variation constant is $\frac{3}{8}$. The equation of variation is $y = \frac{3}{8}X$.

10. $y = \frac{k}{X}$
$\frac{1}{5} = \frac{k}{35}$
$7 = k$ Variation constant
Equation of variation: $y = \frac{7}{X}$

11. $y = \frac{k}{X}$
$1.8 = \frac{k}{0.3}$
$0.54 = k$
The variation constant is 0.54. The equation of variation
0.54

is $y = \frac{}{X}$.

12. $y = kX$

$0.9 = k(0.4)$

$\frac{9}{4} = k$ Variation constant

Equation of variation: $y = \frac{9}{4}X$, or $y = 2.25X$

13. Let W = the weekly allowance and a = the child's age.

$W = ka$

$5.50 = k \cdot 6$

$\frac{11}{12} = k$

$W = \frac{11}{12}X$

$W = \frac{11}{12} \cdot 9$

$W = \$8.25$

14. Let S = the sales tax and p = the purchase price.

$S = kp$ S varies directly as p.

$7.14 = k \cdot 119$ Substituting

$0.06 = k$ Variation constant

$S = 0.06p$ Equation of variation

$S = 0.06(21)$ Substituting

$S \approx 1.26$

The sales tax is \$1.26.

15. $t = \frac{k}{r}$

$5 = \frac{k}{80}$

$400 = r$

$t = \frac{400}{r}$

$t = \frac{400}{70}$

$t = \frac{40}{7}$, or $5\frac{5}{7}$ hr

16. $W = \frac{k}{L}$ W varies inversely as L.

$1200 = \frac{k}{8}$ Substituting

$9600 = k$ Variation constant

$W = \frac{9600}{L}$ Equation of variation

$W = \frac{9600}{14}$ Substituting

$W \approx 686$

A 14-m beam can support about 686 kg.

17. Let F = the number of grams of fat and w = the weight.

$F = kw$ F varies directly as w.

$60 = k \cdot 120$ Substituting

$\frac{60}{120} = k$, or Solving for k

$\frac{1}{2} = k$ Variation constant

$F = \frac{1}{2}w$ Equation of variation

$F = \frac{1}{2} \cdot 180$ Substituting

$F = 90$

The maximum daily fat intake for a person weighing 180 lb is 90 g.

18. $N = kP$

$53 = k \cdot 38,040,000$ Substituting

$\frac{53}{38,040,000} = k$ Variation constant

$N = \frac{53}{38,040,000}P$

$N = \frac{53}{38,040,000} \cdot 26,060,000$ Substituting

$N \approx 36$

Texas has 36 representatives.

19. $T = \frac{k}{P}$ T varies inversely as P.

$5 = \frac{k}{7}$ Substituting

$35 = k$ Variation constant

$T = \frac{35}{P}$ Equation of variation

$T = \frac{35}{10}$ Substituting

$T = 3.5$

It will take 10 bricklayers 3.5 hr to complete the job.

20. $t = \frac{k}{r}$

$45 = \frac{k}{600}$

$27,000 = k$

$t = \frac{27,000}{r}$

$t = \frac{27,000}{1000}$

$t = 27$ min

21. $d = km$ d varies directly as m.

$40 = k \cdot 3$ Substituting

$$\frac{40}{3} = k \quad \text{Variation constant}$$

$d = \frac{40}{3}m$ Equation of variation

$d = \frac{40}{3} \cdot 5 = \frac{200}{3}$ Substituting

$d = 66\frac{2}{3}$

A 5-kg mass will stretch the spring $66\frac{2}{3}$ cm.

22. $f = kF$

$6.3 = k \cdot 150$

$0.042 = k$

$f = 0.042F$

$f = 0.042(80)$

$f = 3.36$

23. $P = \frac{k}{W}$ P varies inversely as W.

$330 = \frac{k}{3.2}$ Substituting

$1056 = k$ Variation constant

$P = \frac{1056}{W}$ Equation of variation

$550 = \frac{1056}{W}$ Substituting

$550W = 1056$ Multiplying by W

$W = \frac{1056}{550}$ Dividing by 550

$W = 1.92$ Simplifying

A tone with a pitch of 550 vibrations per second has a wavelength of 1.92 ft.

24. $M = kE$ M varies directly as E.

$35.9 = k \cdot 95$ Substituting

$\frac{35.9}{95} = k$ Variation constant

$M = \frac{35.9}{95}E$ Equation of variation

$M = \frac{35.9}{95} \cdot 100$ Substituting

$M \approx 37.8$

A 100-lb person would weigh about 37.8 lb on Mars.

25. $y = \frac{k}{X^2}$

$0.15 = \frac{k}{(0.1)^2}$ Substituting

$0.15 = \frac{k}{0.01}$

$0.15(0.01) = k$

$0.0015 = k$

26. $y = \frac{k}{X^2}$

$6 = \frac{k}{3^2}$

$54 = k$

$y = \frac{54}{X^2}$

27. $y = kX^2$

$0.15 = k(0.1)^2$ Substituting

$0.15 = 0.01k$

$\frac{0.15}{0.01} = k$

$15 = k$

The equation of variation is $y = 15X^2$.

28. $y = kX^2$

$6 = k \cdot 3^2$

$\frac{2}{3} = k$

$y = \frac{2}{3}X^2$

29. $y = kXz$

$56 = k \cdot 7 \cdot 8$ Substituting

$56 = 56k$

$1 = k$

The equation of variation is $y = Xz$.

30. $y = \frac{kX}{z}$

$4 = \frac{k \cdot 12}{15}$

$5 = k$

$y = \frac{5X}{z}$

31. $y = kXz^2$

$105 = k \cdot 14 \cdot 5^2$ Substituting

$105 = 350k$

$\frac{105}{350} = k$

$\frac{3}{10} = k$

The equation of variation is $y = \frac{3}{10}Xz^2$.

32. $y = k \cdot \frac{Xz}{w}$

$\frac{3}{2} = k \cdot \frac{2 \cdot 3}{4}$

The equation of variation is $y = \frac{0.0015}{X^2}$.

$1 = k$

$y = \frac{Xz}{w}$

33. $y = k\frac{Xz}{wp}$

$\frac{3}{28} = k\frac{3 \cdot 10}{7 \cdot 8}$ Substituting

$\frac{3}{28} = k \cdot \frac{30}{56}$

$\frac{3}{28} \cdot \frac{56}{30} = k$

$\frac{1}{5} = k$

The equation of variation is $y = \frac{1}{5}\frac{Xz}{wp}$, or $\frac{Xz}{5wp}$.

34. $y = k \cdot \frac{Xz}{w^2}$

$\frac{12}{5} = k \cdot \frac{16 \cdot 3}{5^2}$

$\frac{5}{4} = k$

$y = \frac{5}{4}\frac{Xz}{w^2}$, or $\frac{5Xz}{4w^2}$

35. $I = \frac{k}{d^2}$

$90 = \frac{k}{5^2}$ Substituting

$90 = \frac{k}{25}$

$2250 = k$

The equation of variation is $I = \frac{2250}{d^2}$.

Substitute 40 for I and find d.

$40 = \frac{2250}{d^2}$

$40d^2 = 2250$

$d^2 = 56.25$

$d = 7.5$

The distance from 5 m to 7.5 m is 7.5 − 5, or 2.5 m, so it is 2.5 m further to a point where the intensity is 40 W/m^2.

36. $D = kAv$

$222 = k \cdot 37.8 \cdot 40$

$\frac{37}{252} = k$

$D = \frac{37}{252}Av$

$430 = \frac{37}{252} \cdot 51v$

The equation of variation is $d = \frac{1}{18}r^2$.

Substitute 72 for d and find r.

$72 = \frac{1}{18}r^2$

$1296 = r^2$

$36 = r$

A car can travel 36 mph and still stop in 72 ft.

38. $W = \frac{k}{d^2}$

$220 = \frac{k}{(3978)^2}$

$3,481,386,480 = k$

$W = \frac{3,481,386,480}{d^2}$

$W = \frac{3,481,386,480}{(3978 + 200)^2}$

$W \approx 199$ lb

39. $E = \frac{kR}{I}$

We first find k.

$3.75 = \frac{k \cdot 89}{213.1}$ Substituting

$3.75\left(\frac{213.1}{89}\right) = k$ Multiplying by $\frac{213.1}{89}$

$9 \approx k$

The equation of variation is $E = \frac{9R}{I}$.

Substitute 3.75 for E and 235 for I and solve for R.

$3.75 = \frac{9R}{235}$

$3.75\left(\frac{235}{9}\right) = R$ Multiplying by $\frac{235}{9}$

$98 \approx R$

John Lester would have given up about 98 earned runs if he had pitched 235 innings.

40. $V = \frac{kT}{P}$

$231 = \frac{k \cdot 42}{20}$

$110 = k$

$V = \frac{110T}{P}$

$\frac{110 \cdot 30}{}$

$v \approx 57.4$ mph

37. $d = kr^2$

$200 = k \cdot 60^2$ Substituting

$200 = 3600k$

$\frac{200}{3600} = k$

$\frac{1}{18} = k$

$V =$

15

$V = 220 \text{ cm}^3$

41. parallel

42. zero

43. relative minimum

44. odd function

45. inverse variation

46. a) $7Xy = 14$

$y = \frac{2}{X}$

Inversely

b) $X - 2y = 12$

$y = \frac{X}{2} - 6$

Neither

c) $-2X + 4y = 0$

$y = 2X$

Directly

d) $X = \frac{3}{4}y$

$y = \frac{4}{3}X$

Directly

e) $\frac{X}{y} = 2$

$y = \frac{1}{2}X$

Directly

47. Let V represent the volume and p represent the price of a jar of peanut butter.

$V = kp$		V varies directly as p.
$\pi\left(\frac{3}{2}\right)^2(5) = k(2.89)$		Substituting
$3.89\pi \approx k$		Variation constant
$V = 3.89\pi p$		Equation of variation
$\pi(1.625)^2(5.25) = 3.89\pi p$		Substituting
$3.56 \approx p$		

If cost is directly proportional to volume, the larger jar should cost \$3.56.

Now let W represent the weight and p represent the price of a jar of peanut butter.

$W = kp$	
$18 = k(2.89)$	Substituting
$6.23 \approx k$	Variation constant
$W = 6.23p$	Equation of variation
$22 = 6.23p$	Substituting
$3.53 = p$	

If cost is directly proportional to weight, the larger jar should cost \$3.53.

48. $Q = \frac{kp^2}{q^3}$

Q varies directly as the square of p and inversely as the

49. We are told $A = kd^2$, and we know $A = \pi r^2$ so we have:

$kd^2 = \pi r^2$

$kd^2 = \pi\left(\frac{d}{2}\right)^2 \qquad r = \frac{d}{2}$

$kd^2 = \frac{\pi d^2}{4}$

$k = \frac{\pi}{4}$ Variation constant

Chapter 2 Review Exercises

1. This statement is true by the definition of the greatest integer function.

2. This statement is false. See Example 3 in Section 2.3 in the text.

3. The graph of $y = f(X - d)$ is the graph of $y = f(X)$ shifted right d units, so the statement is true.

4. The graph of $y = -f(X)$ is the reflection of the graph of $y = f(X)$ across the X-axis, so the statement is true.

5. a) For X-values from -4 to -2, the y-values increase from 1 to 4. Thus the function is increasing on the interval $(-4, -2)$.

b) For X-values from 2 to 5, the y-values decrease from 4 to 3. Thus the function is decreasing on the interval $(2, 5)$.

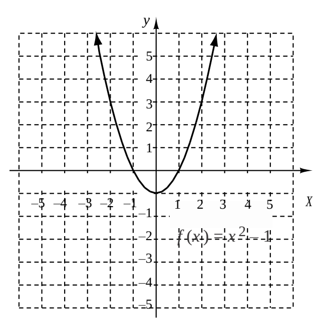
c) For X-values from -2 to 2, y is 4. Thus the function is constant on the interval $(-2, 2)$.

6. a) For X-values from -1 to 0, the y-values increase from 3 to 4. Also, for X-values from 2 to ∞, the y-values increase from 0 to ∞. Thus the function is increasing on the intervals $(-1, 0)$, and $(2, \infty)$.

b) For X-values from 0 to 2, the y-values decrease from 4 to 0. Thus, the function is decreasing on the interval $(0, 2)$.

c) For X-values from $-\infty$ to -1, y is 3. Thus the function is constant on the interval $(-\infty, -1)$.

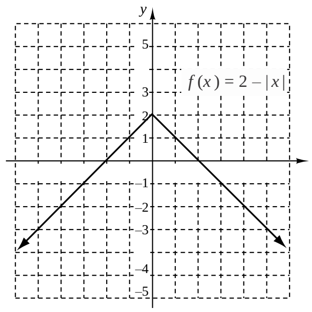
7.



cube of q.

The function is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$. We estimate that the minimum value is -1 at $X = 0$. There are no maxima.

8.



The function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. We estimate that the maximum value is 2 at $X = 0$. There are no minima.

9. If two sides of the patio are each X feet, then the remaining

length × width.

$A(X) = X(48 - 2X)$, or $48X - 2X^2$

10. The length of the rectangle is $2X$. The width is the second coordinate of the point (X, y) on the circle. The circle has

by $A(X) = 2X\sqrt{4 - X^2}$.

108 in^3, we have:

$108 = X \cdot X \cdot h$

$108 = X^2 h$

$\frac{108}{X^2} = h$

Now find the surface area.

$S = X^2 4 4 \cdot X \cdot h$

$S(X) = X^2 4 4 \cdot X \cdot \frac{108}{X^2}$

$S(X) = X^2 4 \frac{432}{X}$

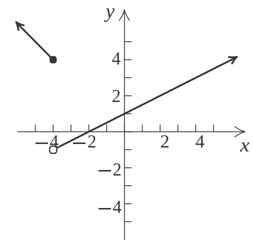
b) X must be positive, so the domain is $(0, \infty)$.

c) From the graph, we see that the minimum value of the function occurs when $X = 6$ in. For this value of X,

$$h = \frac{108}{X^2} = \frac{108}{6^2} = \frac{108}{36} = 3 \text{ in.}$$

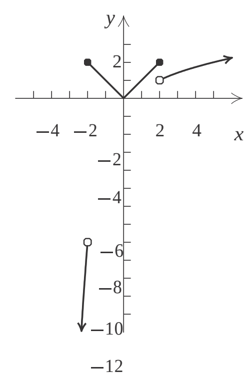
12. $f(X) = \begin{cases} -X, & \text{for } X \leq -4, \\ \frac{1}{2}X 4 1, & \text{for } X > -4 \end{cases}$

We create the graph in two parts. Graph $f(X) = -X$ for inputs less than or equal to -4. Then graph $f(X) = \frac{1}{2}X 4 1$ for inputs greater than -4.



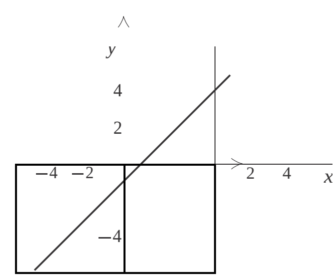
13. $f(X) = \begin{cases} X^3, & \text{for } X < -2, \\ ]X], & \text{for } -2 \leq X \leq 2, \\ \sqrt{X - 1}, & \text{for } X > 2 \end{cases}$

We create the graph in three parts. Graph $f(X) = X^3$ for inputs less than -2. Then graph $f(X) =]X]$ for inputs greater than or equal to -2 and less than or equal to 2. Finally graph $f(X) = \sqrt{X - 1}$ for inputs greater than 2.

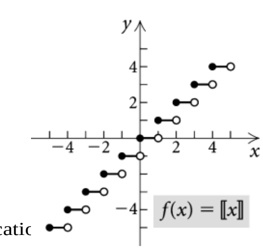


14. $f(X) = \begin{cases} \frac{X^2 - 1}{X 4 1}, & \text{for } X \neq 1, \\ 3, & \text{for } X = -1 \end{cases}$

We create the graph in two parts. Graph $f(X) = \frac{X^2 - 1}{X 4 1}$ for all inputs except -1. Then graph $f(X) = 3$ for $X = -1$.



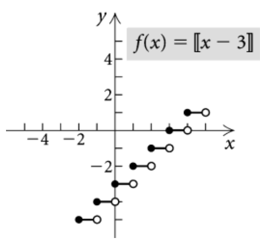
15. $f(X) = [[X]]$. See Example 9 in Section 2.1 of the text.



16. $f(X) = [[X - 3]]$

This function could be defined by a piecewise function with an infinite number of statements.

$$f(x) = \begin{cases} \vdots \\ -4, & \text{for } -1 \le x < 0, \\ -3, & \text{for } 0 \le x < 1, \\ -2, & \text{for } 1 \le x < 2, \\ -1, & \text{for } 2 \le x < 3, \\ \vdots \end{cases}$$



17. $f(x) = \begin{cases} x^3, & \text{for } x < -2, \\ [\![x]\!], & \text{for } -2 \le x \le 2, \\ \sqrt{x-1}, & \text{for } x > 2 \end{cases}$

Since -1 is in the interval $[-2, 2]$, $f(-1) = [\![-1]\!] = 1$.

Since $5 > 2$, $f(5) = \sqrt{5-1} = \sqrt{4} = 2$.

Since -2 is in the interval $[-2, 2]$, $f(-2) = [\![-2]\!] = 2$.

Since $-3 < -2$, $f(-3) = (-3)^3 = -27$.

18. $f(x) = \begin{cases} \dfrac{x^2-1}{x+1}, & \text{for } x \neq -1, \\ 3, & \text{for } x = -1 \end{cases}$

Since $-2 \neq -1$, $f(-2) = \dfrac{(-2)^2-1}{-2+1} = \dfrac{4-1}{-1} = \dfrac{3}{-1} = -3$.

Since $x = -1$, we have $f(-1) = 3$.

Since $0 \neq -1$, $f(0) = \dfrac{0^2-1}{0+1} = \dfrac{-1}{1} = -1$.

Since $4 \neq -1$, $f(4) = \dfrac{4^2-1}{4+1} = \dfrac{16-1}{5} = \dfrac{15}{5} = 3$.

22. $f(x) = \dfrac{4}{x^2}$, $g(x) = 3 - 2x$

a) Division by zero is undefined, so the domain of f is $\{x|x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$. The domain of g is the set of all real numbers, or $(-\infty, \infty)$.

The domain of $f+g$, $f-g$ and fg is $\{x|x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$. Since $g\left(\dfrac{3}{2}\right) = 0$, the domain of f/g is $\left\{x \middle| x \neq 0 \text{ and } x \neq \dfrac{3}{2}\right\}$, or $(-\infty, 0) \cup \left(0, \dfrac{3}{2}\right) \cup \left(\dfrac{3}{2}, \infty\right)$.

b) $(f+g)(x) = \left(\dfrac{4}{x^2}\right) + (3-2x) = \dfrac{4}{x^2} + 3 - 2x$

$(f-g)(x) = \left(\dfrac{4}{x^2}\right) - (3-2x) = \dfrac{4}{x^2} - 3 + 2x$

$(fg)(x) = \left(\dfrac{4}{x^2}\right)(3-2x) = \dfrac{12}{x^2} - \dfrac{8}{x}$

$(f/g)(x) = \dfrac{\left(\dfrac{4}{x^2}\right)}{(3-2x)} = \dfrac{4}{x^2(3-2x)}$

23. $f(x) = 3x^2 + 4x$, $g(x) = 2x - 1$

a) The domain of f, g, $f+g$, $f-g$, and fg is all real numbers, or $(-\infty, \infty)$. Since $g\left(\dfrac{1}{2}\right) = 0$, the domain of f/g is $\left\{x \middle| x \neq \dfrac{1}{2}\right\}$, or $\left(-\infty, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, \infty\right)$.

b) $(f+g)(x) = (3x^2+4x) + (2x-1) = 3x^2 + 6x - 1$

$(f-g)(x) = (3x^2+4x) - (2x-1) = 3x^2 + 2x + 1$

$(fg)(x) = (3x^2+4x)(2x-1) = 6x^3 + 5x^2 - 4x$

$(f/g)(x) = \dfrac{3x^2+4x}{2x-1}$

24. $P(x) = R(x) - C(x)$
$= (120x - 0.5x^2) - (15x + 6)$
$= 120x - 0.5x^2 - 15x - 6$
$= -0.5x^2 + 105x - 6$

25. $f(x) = 2x + 7$

19. $(f - g)(6) = f(6) - g(6)$

$= \sqrt{6-2} - (6^2 - 1)$

$= \sqrt{4} - (36 - 1)$

$= 2 - 35$

$= -33$

20. $(fg)(2) = f(2) \cdot g(2)$

$= \sqrt{2-2} \cdot (2^2 - 1)$

$= 0 \cdot (4 - 1)$

$= 0$

21. $(f + g)(-1) = f(-1) + g(-1)$

$= \sqrt{-1-2} + ((-1)^2 - 1)$

$= \sqrt{-3} + (1 - 1)$

Since $\sqrt{-3}$ is not a real number, $(f + g)(-1)$ does not exist.

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)+7-(2x+7)}{h} = \frac{2x+2h+7-2x-7}{h} = \frac{2h}{h} = 2$$

26. $f(x) = 3 - x^2$

$f(x+h) = 3 - (x+h)^2 = 3 - (x^2 + 2xh + h^2) = 3 - x^2 - 2xh - h^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{3 - x^2 - 2xh - h^2 - (3 - x^2)}{h}$$

$$= \frac{3 - x^2 - 2xh - h^2 - 3 + x^2}{h}$$

$$= \frac{-2xh - h^2}{h} = \frac{h(-2x - h)}{h}$$

$$= \frac{h}{h} \cdot \frac{-2x - h}{1} = -2x - h$$

27. $f(X) = \frac{4}{X}$

$$\frac{f(X4h) - f(X)}{h} = \frac{\frac{4}{X4h} - \frac{4}{X}}{h} = \frac{\frac{4}{X4h}\cdot\frac{X}{X} - \frac{4}{X}\cdot\frac{X4h}{X4h}}{h} =$$

$$\frac{\frac{4X - 4(X4h)}{X(X4h)}}{h} = \frac{\frac{4X - 4X - 4h}{X(X4h)}}{h} = \frac{\frac{-4h}{X(X4h)}}{h} =$$

$$\frac{-4h}{X(X4h)}\cdot\frac{1}{h} = \frac{-4\cdot \not{h}}{X(X4h)\cdot \not{h}} = \frac{-4}{X(X4h)}, \text{ or } -\frac{4}{X(X4h)}$$

28. $(f \circ 9)(1) = f(9(1)) = f(1^2 4 4) = f(1 4 4) = f(5) =$
$2 \cdot 5 - 1 = 10 - 1 = 9$

29. $(9 \circ f)(1) = 9(f(1)) = 9(2 \cdot 1 - 1) = 9(2 - 1) = 9(1) =$
$1^2 4 4 = 1 4 4 = 5$

30. $(h \circ f)(-2) = h(f(-2)) = h(2(-2) - 1) =$

$h(-4 - 1) = h(-5) = 3 - (-5)^3 = 3 - (-125) =$
$3 4 125 = 128$

31. $(9 \circ h)(3) = 9(h(3)) = 9(3 - 3^3) = 9(3 - 27) =$
$9(-24) = (-24)^2 4 4 = 576 4 4 = 580$

32. $(f \circ h)(-1) = f(h(-1)) = f(3 - (-1)^3) =$
$f(3 - (-1)) = f(3 4 1) = f(4) = 2 \cdot 4 - 1 = 8 - 1 = 7$

33. $(h \circ 9)(2) = h(9(2)) = h(2^2 4 4) = h(4 4 4)$
$h(8) = 3 - 8^3 = 3 - 512 = -509$

34. $(f \circ f)(X) = f(f(X)) = f(2X - 1) = 2(2X - 1) - 1 =$
$4X - 2 - 1 = 4X - 3$

35. $(h \circ h)(X) = h(h(X)) = h(3 - X^3) = 3 - (3 - X^3)^3 =$

$3 - (27 - 27X^3 4 9X^6 - X^9) = 3 - 27 4 27X^3 - 9X^6 4 X^9 =$

$-24 4 27X^3 - 9X^6 4 X^9$

36. a) $f \circ 9(X) = f(3 - 2X) = \frac{4}{(3 \quad 2X)^2}$

$$9 \circ f(X) = 9\left(\frac{4}{X^2}\right) = 3 - 2\left(\frac{4}{X^2}\right) = 3 - \frac{8}{X^2}$$

b) The domain of f is $\{X | X = 0\}$ and the domain of 9 is the set of all real numbers. To find the domain of $f \circ 9$, we find the values of X for which $9(X) = 0$. Since $3 - 2X = 0$ when $X = \frac{3}{2}$, the domain of $f \circ 9$ is $\left\{X \middle| X = \frac{3}{2}\right\}$, or $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$. Since any real number can be an input for 9, the domain of $9 \circ f$ is the same as the domain of f, $\{X | X = 0\}$, or $(-\infty, 0) \cup (0, \infty)$.

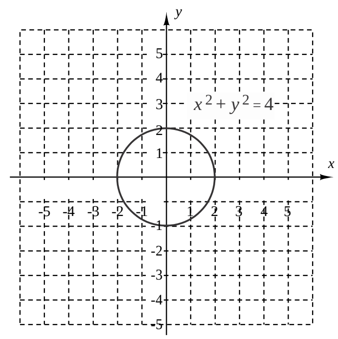
$(9 \circ f)(X) = 9(3X^2 4 4X)$
$= 2(3X^2 4 4X) - 1$
$= 6X^2 4 8X - 1$

b) Domain of f = domain of 9 = all real numbers, so domain of $f \circ 9$ = domain of $9 \circ f$ = all real numbers, or $(-\infty, \infty)$.

38. $f(X) = \sqrt{X}$, $9(X) = 5X 4 2$. Answers may vary.

39. $f(X) = 4X^2 4 9$, $9(X) = 5X - 1$. Answers may vary.

40. $X^2 4 y^2 = 4$



The graph is symmetric with respect to the X-axis, the y-axis, and the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the X-axis.

$X^2 4 (-y)^2 = 4$
$X^2 4 y^2 = 4$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the X-axis.

Replace X with $-X$ to test algebraically for symmetry with respect to the y-axis.

$(-X)^2 4 y^2 = 4$
$X^2 4 y^2 = 4$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace X and $-X$ and y with $-y$ to test for symmetry with respect to the origin.

$(-X)^2 4 (-y)^2 = 4$
$X^2 4 y^2 = 4$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

37. a) $f \circ 9(X) = f(2X - 1)$

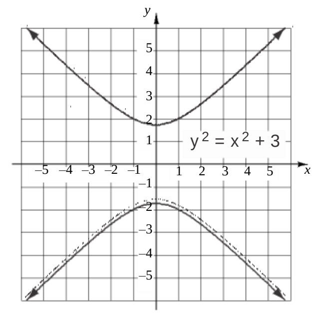
$= 3(2X - 1)^2 + 4(2X - 1)$

$= 3(4X^2 - 4X + 1) + 4(2X - 1)$

$= 12X^2 - 12X + 3 + 8X - 4$

$= 12X^2 - 4X - 1$

41. $y^2 = X^2 + 3$



The graph is symmetric with respect to the X-axis, the y-axis, and the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the X-axis.

$$(-y)^2 = X^2\,4\,3$$
$$y^2 = X^2\,4\,3$$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the X-axis.

Replace X with $-X$ to test algebraically for symmetry with respect to the y-axis.

$$y^2 = (-X)^2\,4\,3$$
$$y^2 = X^2\,4\,3$$

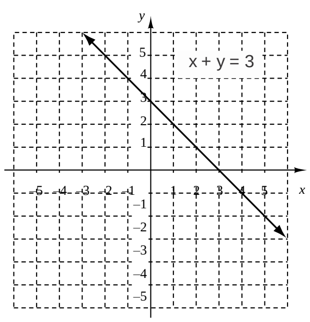
The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace X and $-X$ and y with $-y$ to test for symmetry with respect to the origin.

$$(-y)^2 = (-X)^2\,4\,3$$
$$y^2 = X^2\,4\,3$$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

42. $X\,4\,y = 3$



The graph is not symmetric with respect to the X-axis, the y-axis, or the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the X-axis.

$$X - y = 3$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the X-axis.

Replace X with $-X$ to test algebraically for symmetry with respect to the y-axis.

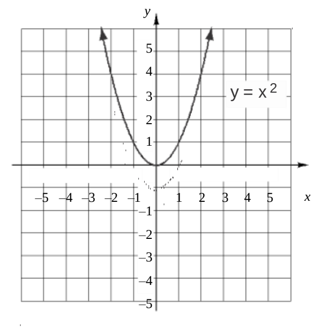
$$-X\,4\,y = 3$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Replace X and $-X$ and y with $-y$ to test for symmetry with respect to the origin.

$$-X - y = 3$$
$$X\,4\,y = -3$$

43. $y = X^2$



The graph is symmetric with respect to the y-axis. It is not symmetric with respect to the X-axis or the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the X-axis.

$$-y = X^2$$
$$y = -X^2$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the X-axis.

Replace X with $-X$ to test algebraically for symmetry with respect to the y-axis.

$$y = (-X)^2$$
$$y = X^2$$

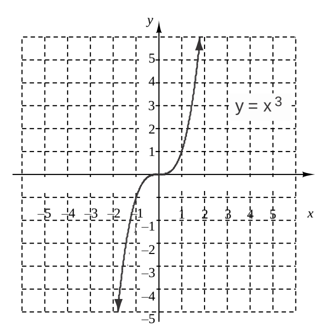
The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace X and $-X$ and y with $-y$ to test for symmetry with respect to the origin.

$$-y = (-X)^2$$
$$-y = X^2$$
$$y = -X^2$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

44. $y = X^3$



The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

The graph is symmetric with respect to the origin. It is not symmetric with respect to the X-axis or the y-axis.

Replace y with $-y$ to test algebraically for symmetry with respect to the X-axis.

$$-y = X^3$$
$$y = -X^3$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the X-axis.

Replace X with $-X$ to test algebraically for symmetry with respect to the y-axis.

$$y = (-X)^3$$
$$y = -X^3$$

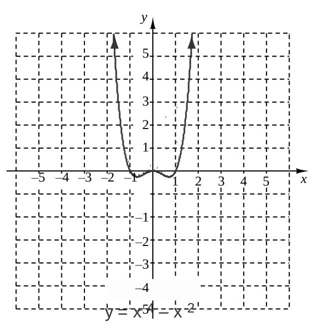
The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Replace X and $-X$ and y with $-y$ to test for symmetry with respect to the origin.

$$-y = (-X)^3$$
$$-y = -X^3$$
$$y = X^3$$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

45. $y = X^4 - X^2$



The graph is symmetric with respect to the y-axis. It is not symmetric with respect to the X-axis or the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the X-axis.

$$-y = X^4 - X^2$$
$$y = -X^4\ 4\ X^2$$

The resulting equation is not equivalent to the original

equation, so the graph is not symmetric with respect to the X-axis.

Replace X with $-X$ to test algebraically for symmetry with respect to the y-axis.

$$y = (-X)^4 - (-X)^2$$
$$y = X^4 - X^2$$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace X and $-X$ and y with $-y$ to test for symmetry

with respect to the origin.

$$-y = (-X)^4 - (-X)^2$$
$$-y = X^4 - X^2$$
$$y = -X^4\ 4\ X^2$$

46. The graph is symmetric with respect to the y-axis, so the function is even.

47. The graph is symmetric with respect to the y-axis, so the function is even.

48. The graph is symmetric with respect to the origin, so the function is odd.

49. The graph is symmetric with respect to the y-axis, so the function is even.

50. $f(X) = 9 - X^2$
$f(-X) = 9 - (-X^2) = 9 - X^2$
$f(X) = f(-X)$, so f is even.

51. $f(X) = X^3 - 2X\ 4\ 4$
$f(-X) = (-X)^3 - 2(-X)\ 4\ 4 = -X^3\ 4\ 2X\ 4\ 4$
$f(X) = f(-X)$, so f is not even.
$-f(X) = -(X^3 - 2X\ 4\ 4) = -X^3\ 4\ 2X - 4$
$f(-X) = -f(X)$, so f is not odd.
Thus, $f(X) = X^3 - 2X\ 4\ 4$ is neither even or odd.

52. $f(X) = X^7 - X^5$
$f(-X) = (-X)^7 - (-X)^5 = -X^7\ 4\ X^5$
$f(X) = f(-X)$, so f is not even.
$-f(X) = -(X^7 - X^5) = -X^7\ 4\ X^5$
$f(-X) = -f(X)$, so f is odd.

53. $f(X) =]X]$
$f(-X) =] - X] =]X]$
$f(X) = f(\_X)$, so f is even.

54. $f(X) = \sqrt{16 - X^2}$
$f(-X) = \sqrt{16 - (-X^2)} = \sqrt{16 - X^2}$
$f(X) = f(-X)$, so f is even.

55. $f(X) = \dfrac{10X}{X^2\ 4\ 1}$
$f(-X) = \dfrac{10(-X)}{(-X)^2\ 4\ 1} = -\dfrac{10X}{X^2\ 4\ 1}$
$f(X) = f(-X)$, so $f(X)$ is not even.
$-f(X) = -\dfrac{10X}{X^2\ 4\ 1}$
$f(-X) = -f(X)$, so f is odd.

56. Shape: $9(X) = X^2$
Shift $9(X)$ left 3 units: $f(X) = 9(X\ 4\ 3) = (X\ 4\ 3)^2$

57. Shape: $t(X) = \sqrt{X}$
Turn $t(X)$ upside down (that is, reflect it across the X-axis):
$h(X) = -t(X) = -\sqrt{X}$.
Shift $h(X)$ right 3 units: $9(X) = h(X - 3) = -\sqrt{X - 3}$.
Shift $9(X)$ up 4 units:

$f(X) = 9(X)$ 4 4 = – X – 3 4 4.

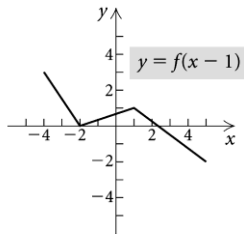
58. Shape: $h(X) =]X]$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

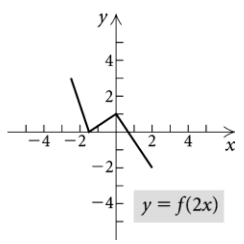
Stretch $h(X)$ vertically by a factor of 2 (that is, multiply each function value by 2): $9(X) = 2h(X) = 2]X]$.

Shift $9(X)$ right 3 units: $f(X) = 9(X - 3) = 2]X - 3]$.

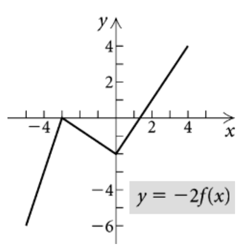
59. The graph is shifted right 1 unit so each X-coordinate is increased by 1. We plot and connect $(-4, 3)$, $(-2, 0)$, $(1, 1)$ and $(5, -2)$.



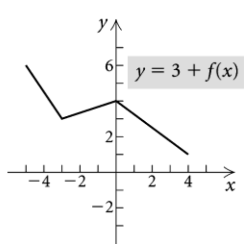
60. The graph is shrunk horizontally by a factor of 2. That is, each X-coordinate is divided by 2. We plot and connect $\left(-\frac{5}{2}, 3\right)$, $\left(-\frac{3}{2}, 0\right)$, $(0, 1)$ and $(2, -2)$.



61. Each y-coordinate is multiplied by -2. We plot and connect $(-5, -6)$, $(-3, 0)$, $(0, -2)$ and $(4, 4)$.



62. Each y-coordinate is increased by 3. We plot and connect $(-5, 6)$, $(-3, 3)$, $(0, 4)$ and $(4, 1)$.



63.
$$y = kX$$
$$100 = 25X$$
$$4 = X$$
Equation of variation: $y = 4X$

64.
$$y = kX$$
$$6 = 9X$$
$$\frac{2}{3} = X \quad \text{Variation constant}$$
Equation of variation: $y = \frac{2}{3}X$

65.
$$y = \frac{k}{X}$$
$$100 = \frac{k}{25}$$
$$2500 = k$$
Equation of variation: $y = \frac{2500}{X}$

66.
$$y = \frac{k}{X}$$
$$6 = \frac{k}{9}$$
$$54 = k \quad \text{Variation constant}$$
Equation of variation: $y = \frac{54}{X}$

67.
$$y = \frac{k}{X^2}$$
$$12 = \frac{k}{2^2}$$
$$48 = k$$
$$y = \frac{48}{X^2}$$

68.
$$y = \frac{kXz^2}{w}$$
$$2 = \frac{k(16)\left(\frac{1}{2}\right)^2}{0.2}$$
$$2 = \frac{k(16)\frac{1}{4}}{0.2}$$
$$2 = \frac{4k}{0.2}$$
$$2 = 20k$$
$$\frac{1}{10} = k$$
$$y = \frac{1}{10}\frac{Xz^2}{w}$$

69.
$$t = \frac{k}{r}$$
$$35 = \frac{k}{800}$$
$$28,000 = k$$
$$t = \frac{28,000}{r}$$
$$t = \frac{28,000}{1400}$$
$$t = 20 \text{ min}$$

70. $N = ka$
$87 = k \cdot 29$
$3 = k$
$N = 3a$
$N = 3 \cdot 25$
$N = 75$

Ellen's score would have been 75 if she had answered 25 questions correctly.

71. $P = kC^2$
$180 = k \cdot 6^2$
$5 = k$ Variation constant
$P = 5C^2$ Variation equation

$P = 5 \cdot 10^2$

$P = 500$ watts

72. $f(X) = X 4 1,\ 9(X) = \overline{\ \ } X$

The domain of f is $(-\infty, \infty)$, and the domain of 9 is $[0, \infty)$. To find the domain of $(9 \circ f)(X)$, we find the values of X for which $f(X) \geq 0$.

$X 4 1 \geq 0$
$X \geq -1$

Thus the domain of $(9 \circ f)(X)$ is $[-1, \infty)$. Answer A is correct.

73. For $b > 0$, the graph of $y = f(X) 4 b$ is the graph of $y = f(X)$ shifted up b units. Answer C is correct.

74. The graph of $9(X) = -\frac{1}{2}f(X) 4 1$ is the graph of $y = f(X)$ shrunk vertically by a factor of $\frac{1}{2}$, then reflected across the X-axis, and shifted up 1 unit. The correct graph is B.

75. Let $f(X)$ and $9(X)$ be odd functions. Then by definition, $f(-X) = -f(X)$, or $f(X) = -f(-X)$, and $9(-X) = -9(X)$, or $9(X) = -9(-X)$. Thus $(f 4 9)(X) = f(X) 4 9(X) = -f(-X) 4 [-9(-X)] = -[f(-X) 4 9(-X)] = -(f 4 9)(-X)$ and $f 4 9$ is odd.

76. Reflect the graph of $y = f(X)$ across the X-axis and then across the y-axis.

77. $f(X) = 4X^3 - 2X 4 7$

a) $f(X) 4 2 = 4X^3 - 2X 4 7 4 2 = 4X^3 - 2X 4 9$

b) $f(X 4 2) = 4(X 4 2)^3 - 2(X 4 2) 4 7$

$= 4(X^3 4 6X^2 4 12X 4 8) - 2(X 4 2) 4 7$

$= 4X^3 4 24X^2 4 48X 4 32 - 2X - 4 4 7$

$= 4X^3 4 24X^2 4 46X 4 35$

c) $f(X) 4 f(2) = 4X^3 - 2X 4 7 4 4 \cdot 2^3 - 2 \cdot 2 4 7$

$= 4X^3 - 2X 4 7 4 32 - 4 4 7$

$= 4X^3 - 2X$
$4 42$

78. In the graph of $y = f(cX)$, the constant c stretches or shrinks the graph of $y = f(X)$ horizontally. The constant c in $y = cf(X)$ stretches or shrinks the graph of $y = f(X)$ vertically. For $y = f(cX)$, the X-coordinates of $y = f(X)$ are divided by c; for $y = cf(X)$, the y-coordinates of $y = f(X)$ are multiplied by c.

79. The graph of $f(X) = 0$ is symmetric with respect to the X-axis, the y-axis, and the origin. This function is both even and odd.

80. If all of the exponents are even numbers, then $f(X)$ is an even function. If $a_0 = 0$ and all of the exponents are odd numbers, then $f(X)$ is an odd function.

81. Let $y(X) = kX^2$. Then $y(2X) = k(2X)^2 = k \cdot 4X^2 = 4 \cdot kX^2 = 4 \cdot y(X)$. Thus, doubling X causes y to be quadrupled.

82. Let $y = k_1 X$ and $X = \frac{k_2}{z}$. Then $y = k_1 \cdot \frac{k_2}{z}$, or $y = \frac{k_1 k_2}{z}$, so y varies inversely as z.

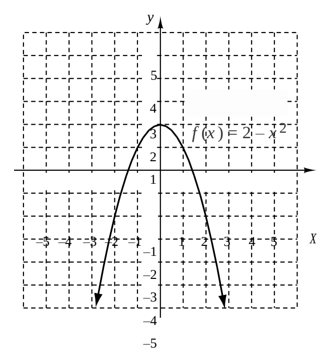
Chapter 2 Test

1. a) For X-values from -5 to -2, the y-values increase from -4 to 3. Thus the function is increasing on the interval $(-5, -2)$.

   b) For X-values from 2 to 5, the y-values decrease from 2 to -1. Thus the function is decreasing on the interval $(2, 5)$.

   c) For X-values from -2 to 2, y is 2. Thus the function is constant on the interval $(-2, 2)$.

2.



The function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. The relative maximum is 2 at $X = 0$. There are no minima.

3. If b = the length of the base, in inches, then the height = $4b - 6$. We use the formula for the area of a triangle,

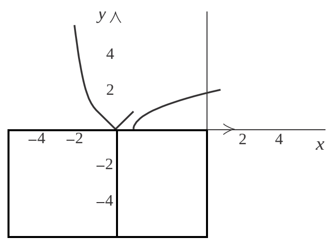
$f(X) 4 2$ adds 2 to each function value; $f(X 4 2)$ adds 2 to each input before the function value is found; $f(X) 4 f(2)$ adds the output for 2 to the output for X.

$$A = \frac{1}{2}bh.$$

$$A(b) = \frac{1}{2}b(4b - 6), \text{ or}$$

$$A(b) = 2b^2 - 3b$$

4. $f(X) = \begin{cases} X^2, & \text{for } X < -1, \\ ]X], & \text{for } -1 \le X \le 1, \\ \sqrt{X-1}, & \text{for } X > 1 \end{cases}$



5. Since $-1 \le -\frac{7}{8} \le 1$, $f\left(-\frac{7}{8}\right) = \left|-\frac{7}{8}\right| = \frac{7}{8}$.

Since $5 > 1$, $f(5) = \sqrt{5-1} = \sqrt{4} = 2$.

Since $-4 < -1$, $f(-4) = (-4)^2 = 16$.

6. $(f \,4\, 9)(-6) = f(-6) \,4\, 9(-6) =$
$(-6)^2 - 4(-6) \,4\, 3 \,4\, \sqrt{3-(-6)} =$

$36 \,4\, 24 \,4\, 3 \,4\, \sqrt{3 \,4\, 6} = 63 \,4\, \sqrt{9} = 63 \,4\, 3 = 66$

7. $(f - 9)(-1) = f(-1) - 9(-1) =$
$(-1)^2 - 4(-1) \,4\, 3 - \sqrt{3-(-1)} =$
$1 \,4\, 4 \,4\, 3 - \sqrt{3 \,4\, 1} = 8 - \sqrt{4} = 8 - 2 = 6$

8. $(f9)(2) = f(2) \cdot 9(2) = (2^2 - 4 \cdot 2 \,4\, 3)(\sqrt{3-2}) =$
$(4 - 8 \,4\, 3)(\sqrt{1}) = -1 \cdot 1 = -1$

9. $(f/9)(1) = \frac{f(1)}{9(1)} = \frac{1^2 - 4 \cdot 1 \,4\, 3}{\sqrt{3-1}} = \frac{1 - 4 \,4\, 3}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0$

10. Any real number can be an input for $f(X) = X^2$, so the domain is the set of real numbers, or $(-\infty, \infty)$.

11. The domain of $9(X) = \sqrt{X-3}$ is the set of real numbers for which $X - 3 \ge 0$, or $X \ge 3$. Thus the domain is $\{X|X \ge 3\}$, or $[3, \infty)$.

12. The domain of $f \,4\, 9$ is the intersection of the domains of f and 9. This is $\{X|X \ge 3\}$, or $[3, \infty)$.

13. The domain of $f - 9$ is the intersection of the domains of f and 9. This is $\{X|X \ge 3\}$, or $[3, \infty)$.

14. The domain of $f9$ is the intersection of the domains of f and 9. This is $\{X|X \ge 3\}$, or $[3, \infty)$.

15. The domain of $f/9$ is the intersection of the domains of f and 9, excluding those X-values for which $9(X) = 0$. Since $X - 3 = 0$ when $X = 3$, the domain is $(3, \infty)$.

20. $f(X) = \frac{1}{2}X \,4\, 4$

$f(X \,4\, h) = \frac{1}{2}(X \,4\, h) \,4\, 4 = \frac{1}{2}X \,4\, \frac{1}{2}h \,4\, 4$

$\frac{f(X \,4\, h) - f(X)}{h} = \frac{\frac{1}{2}X \,4\, \frac{1}{2}h \,4\, 4 - \left(\frac{1}{2}X \,4\, 4\right)}{h}$

$= \frac{\frac{1}{2}X \,4\, \frac{1}{2}h \,4\, 4 - \frac{1}{2}X - 4}{h}$

$= \frac{\frac{1}{2}h}{h} = \frac{1}{2}h \cdot \frac{1}{h} = \frac{1}{2} \cdot \frac{h}{h} = \frac{1}{2}$

21. $f(X) = 2X^2 - X \,4\, 3$

$f(X4h) = 2(X4h)^2 - (X4h)43 = 2(X^2 42Xh4h^2) - X - h43 =$

$2X^2 \,4\, 4Xh \,4\, 2h^2 - X - h \,4\, 3$

$\frac{f(X4h) - f(X)}{h} = \frac{2X^2 44Xh42h^2 - X - h43 - (2X^2 - X43)}{h}$

$= \frac{2X^2 44Xh42h^2 - X - h43 - 2X^2 4X - 3}{h}$

$= \frac{4Xh42h^2 - h}{h}$

$= \frac{h(4X \,4\, 2h - 1)}{h}$

$= 4X \,4\, 2h - 1$

22. $(9 \circ h)(2) = 9(h(2)) = 9(3 \cdot 2^2 \,4\, 2 \cdot 2 \,4\, 4) =$
$9(3 \cdot 4 \,4\, 4 \,4\, 4) = 9(12 \,4\, 4 \,4\, 4) = 9(20) = 4 \cdot 20 \,4\, 3 =$
$80 \,4\, 3 = 83$

23. $(f \circ 9)(-1) = f(9(-1)) = f(4(-1) \,4\, 3) = f(-4 \,4\, 3) =$
$f(-1) = (-1)^2 - 1 = 1 - 1 = 0$

24. $(h \circ f)(1) = h(f(1)) = h(1^2 - 1) = h(1 - 1) = h(0) =$

16. $(f \,4\, 9)(X) = f(X) \,4\, 9(X) = X^2 \,4\, \sqrt{X-3}$

17. $(f - 9)(X) = f(X) - 9(X) = X^2 - \sqrt{X-3}$

18. $(f9)(X) = f(X) \cdot 9(X) = X^2\sqrt{X-3}$

19. $(f/9)(X) = \frac{f(X)}{9(X)} = \frac{X^2}{\sqrt{X-3}}$

$3 \cdot 0^2$ 4 $2 \cdot 0$ 4 4 = 0 4 0 4 4 = 4

25. $(9 \circ 9)(X) = 9(9(X)) = 9(4X$ 4 $3) = 4(4X$ 4 $3)$ 4 3 = 16X 4 12 4 3 = 16X 4 15

26. $(f \circ 9)(X) = f(9(X)) = f(X^2$ 4 $1) = \overline{\ }X^2$ 4 1 − 5 = $X^2 - 4$

$(9 \circ f)(X) = 9(f(X)) = 9(\overline{\ }X - 5) = (\overline{\ }X - 5)^2$ 4 1 = $X - 5$ 4 1 = $X - 4$

$9(X)$ $X - 3$

27. The inputs for $f(X)$ must be such that $X - 5 \geq 0$, or $X \geq 5$. Then for $(f \circ 9)(X)$ we must have $9(X) \geq 5$, or X^2 4 1 ≥ 5, or $X^2 \geq 4$. Then the domain of $(f \circ 9)(X)$ is $(-\infty, -2] \cup [2, \infty)$.

Since we can substitute any real number for X in 9, the domain of $(9 \circ f)(X)$ is the same as the domain of $f(X)$, $[5, \infty)$.

28. Answers may vary. $f(X) = \overline{X^4}$, $9(X) = 2\overline{X - 7}$

29. $y = X^4 - 2X^2$

Replace y with $-y$ to test for symmetry with respect to the X-axis.

$-y = X^4 - 2X^2$

$y = -X^4 4 2X^2$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the X-axis.

Replace X with $-X$ to test for symmetry with respect to the y-axis.

$y = (-X)^4 - 2(-X)^2$

$y = X^4 - 2X^2$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace X with $-X$ and y with $-y$ to test for symmetry with respect to the origin.

$-y = (-X)^4 - 2(-X)^2$

$-y = X^4 - 2X^2$

$y = -X^4 4 2X^2$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

30. $f(X) = \frac{2X}{X^2 4 1}$

$f(-X) = \frac{2(-X)}{(-X)^2 4 1} = -\frac{2X}{X^2 4 1}$

$f(X) = f(-X)$, so f is not even.

$-f(X) = -\frac{2X}{X^2 4 1}$

$f(-X) = -f(X)$, so f is odd.

31. Shape: $h(X) = X^2$

Shift $h(X)$ right 2 units: $9(X) = h(X - 2) = (X - 2)^2$

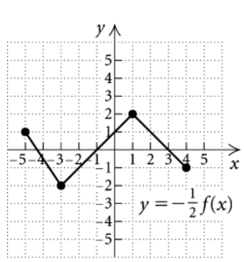
Shift $9(X)$ down 1 unit: $f(X) = (X - 2)^2 - 1$

32. Shape: $h(X) = X^2$

Shift $h(X)$ left 2 units: $9(X) = h(X 4 2) = (X 4 2)^2$

Shift $9(X)$ down 3 units: $f(X) = (X 4 2)^2 - 3$

33. Each y-coordinate is multiplied by $-\frac{1}{2}$. We plot and connect $(-5, 1)$, $(-3, -2)$, $(1, 2)$ and $(4, -1)$.



34. $y = \frac{k}{X}$

$5 = \frac{k}{6}$

$30 = k$ Variation constant

Equation of variation: $y = \frac{30}{X}$

35. $y = kX$

$60 = k \cdot 12$

$5 = k$ Variation constant

Equation of variation: $y = 5X$

36. $y = \frac{kXz^2}{w}$

$100 = \frac{k(0.1)(10)^2}{5}$

$100 = 2k$

$50 = k$ Variation constant

$y = \frac{50Xz^2}{w}$ Equation of variation

37. $d = kr^2$

$200 = k \cdot 60^2$

$\frac{1}{18} = k$ Variation constant

$d = \frac{1}{18}r^2$ Equation of variation

$d = \frac{1}{18} \cdot 30^2$

$d = 50$ ft

38. The graph of $9(X) = 2f(X) - 1$ is the graph of $y = f(X)$ stretched vertically by a factor of 2 and shifted down 1 unit. The correct graph is C.

39. Each X-coordinate on the graph of $y = f(X)$ is divided by 3 on the graph of $y = f(3X)$. Thus the point $\left(\frac{3}{3}, 1\right)$, or $(-1, 1)$ is on the graph of $f(3X)$.