

**Solution Manual for Precalculus
Functions and Graphs 12th Edition
Swokowski Cole 0840068573
9780840068576**

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Preface

This manual contains solutions/answers to all exercises in the text *Precalculus: Functions and Graphs, Twelfth Edition*, by Earl W. Swokowski and Jeffery A. Cole. A *Student's Solutions Manual* is also available; it contains solutions for the odd-numbered exercises in each section and for the Discussion Exercises, as well as solutions for all the exercises in the Review Sections and for the Chapter Tests.

For most problems, a reasonably detailed solution is included. It is my hope that by merely browsing through the solutions, professors will save time in determining appropriate assignments for their particular class.

I appreciate feedback concerning errors, solution correctness or style, and manual style—comments from professors using previous editions have greatly strengthened the ancillary package as well as the text. Any comments may be sent directly to me at the address below, at jeff.cole@anokaramsey.edu, or in care of the publisher: Brooks/Cole|Cengage Learning, 20 Davis Drive, Belmont, CA 94002-3098.

I would like to thank: Marv Riedesel and Mary Johnson for accuracy checking of the new exercises; Andrew Bulman-Fleming, for manuscript preparation; Brian Morris and the late George Morris, of Scientific Illustrators, for creating the mathematically precise art package; and Cynthia Ashton, of Cengage Learning, for checking the manuscript. I dedicate this book to my children, Becky and Brad.

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To the Instructor

In the chapter review sections, the solutions are abbreviated since more detailed solutions were given in chapter sections. In easier groups of exercises, representative solutions are shown. When appropriate, only the answer is listed.

All figures have been plotted using computer software, offering a high degree of precision. The calculator graphs are from various TI screens. When possible, we tried to make each piece of art with the same scale to show a realistic and consistent graph.

This manual was done using : *The Scientific Word Processor*.

The following notations are used in the manual.

Note: Notes to the instructor/student pertaining to hints on instruction or conventions to follow.

{ }	{ comments to the reader are in braces }
LS	{ Left Side of an equation }
RS	{ Right Side of an equation }
$\hat{=}$	{ implies, next equation, logically follows }
$\overline{\equiv}$	{ if and only if, is equivalent to }
$\dot{\bullet}$	{ bullet, used to separate problem statement from solution or explanation }
\ae	{ used to identify the answer to the problem }
§	{ <i>section</i> references }
a	{ For all, i.e., aB means “for all B”. }
$\mathbb{R} \setminus \{+\}$	{ The set of all real numbers except +. }
\therefore	{ therefore }
QI–QIV	{ quadrants I, II, III, IV }

Chapter 1: Fundamental Concepts of Algebra

1.1 Exercises

1 (a) Since x and y have opposite signs, the product xy is negative.

(b) Since $x > 0$ and $y > 0$, the product xy is positive.

(c) Since $x < 0$ (x is negative) and $y > 0$ (y is positive), the quotient $\frac{x}{y}$ is negative.

Thus, $\frac{x}{y} + x$ is the sum of two negatives, which is negative.

(d) Since $y > 0$ and $x < 0$, $y - x > 0$.

2 (a) Since x and y have opposite signs, the quotient $\frac{x}{y}$ is negative.

(b) Since $x < 0$ and $y > 0$, the product xy is negative.

(c) Since $x - y < 0$ and $xy < 0$, $\frac{x-y}{xy} > 0$.

(d) Since $y > 0$ and $y - x > 0$, $y(y - x) > 0$.

3 (a) Since -3 is to the left of -2 on a coordinate line, $-3 < -2$.

(b) Using a calculator, we see that $\frac{1}{2} = 0.5$. Hence, $\frac{1}{2} < 0.6$.

(c) $\frac{1}{2} < \frac{1}{3}$ **Note:** $\frac{1}{2} < \frac{1}{3}$

4 (a) Since -2 is to the right of -3 on a coordinate line, $-2 > -3$.

(b) Using a calculator, we see that $\frac{1}{2} = 0.5$. Hence, $\frac{1}{2} < 0.6$.

(c) $\frac{1}{2} < \frac{1}{3}$ **Note:** $\frac{1}{2} < \frac{1}{3}$

5 (a) Since $\frac{1}{2} < \frac{1}{3}$ and $1 > 0$, $\frac{1}{2} < \frac{1}{3}$.

(b) Since $\frac{1}{2} < \frac{1}{3}$ and $1 > 0$, $\frac{1}{2} < \frac{1}{3}$.

(c) Since $\frac{1}{2} < \frac{1}{3}$ and $1 > 0$, $\frac{1}{2} < \frac{1}{3}$.

6 (a) Since $\frac{1}{2} < \frac{1}{3}$, $\frac{1}{2} < \frac{1}{3}$.

(b) Since $\frac{1}{2} < \frac{1}{3}$, $\frac{1}{2} < \frac{1}{3}$.

(c) Since $\frac{1}{2} < \frac{1}{3}$, $\frac{1}{2} < \frac{1}{3}$.

7 (a) " x is negative" is equivalent to $x < 0$. We symbolize this by writing " x is negative" as $x < 0$.

(b) y is nonnegative $\iff y \geq 0$

(c) x is less than or equal to 1 $\iff x \leq 1$

(d) x is between 2 and 5 $\iff 2 < x < 5$

(e) x is not less than 3 $\iff x \geq 3$

(f) The negative of D is not greater than $-D$ $\iff -D \leq -D$

(g)



The (h)
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 of and
 is at most
 $(|x| - y)$
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(i) The absolute value of x is greater than $(|x| - k) > 0$

Note: An informal definition of absolute value that may be helpful is

$$|x| = \begin{cases} x & \text{if } x \text{ is positive or zero} \\ -x & \text{if } x \text{ is negative} \end{cases}$$

8 (a) x is positive $|x| > 0$

(b) x is nonpositive $|x| \geq 0$

(c) $|x|$ is greater than or equal to $-y$ $|x| \geq -y$

- (d) e is between $\frac{1}{5}$ and $\frac{1}{3}$ $\frac{1}{5} < e < \frac{1}{3}$ (e) p is not greater than -2 $p < -2$
- (f) The negative of 7 is not less than -2 $-7 > -2$
- (g) The quotient of r and s is at least $\frac{1}{5}$ $\frac{r}{s} \geq \frac{1}{5}$ (h) The reciprocal of f is at most 14 $\frac{1}{f} \leq 14$
- (i) The absolute value of x is less than 4 $|x| < 4$

9 (a) $| -3 - 4 | = | -7 | = -(-7) \{ \text{since } -7 < 0 \} = 7$
 (b) $| -5 - 2 | = | -7 | = -(-7) = 7$ (c) $| 4 - 4 | = | 0 | = 0$

10 (a) $| -11 - 1 | = | -12 | = -(-12) \{ \text{since } -12 < 0 \} = 12$
 (b) $| 6 - 3 | = | 3 | = 3$ (c) $| 8 - 9 | = | -1 | = -(-1) = 1$

11 (a) $| -5 - 6 | = | -11 | = -(-11) \{ \text{since } -11 < 0 \} = 11$
 (b) $| -6 / (-2) | = | 3 | = 3$ (c) $| -7 - 4 | = | -11 | = -(-11) = 11$

12 (a) $| 6 - 7 | = | -1 | = -(-1) = 1$
 (b) $| 5 - 2 | = | 3 | = 3$ (c) $| -1 - 9 | = | -10 | = -(-10) = 10$

13 (a) Since $(4 - 1)$ is positive, $| 4 - 1 | = 4 - 1$.

(b) Since $(1 - 4)$ is negative, $| 1 - 4 | = -(1 - 4) = 4 - 1$.

(c) Since $(-2 - 1)$ is negative, $| -2 - 1 | = -(-2 - 1) = 2 + 1 = 3$.

14 (a) Since $(-3 - 1)$ is negative, $| -3 - 1 | = -(-3 - 1) = 3 + 1 = 4$.

(b) Since $(7 - 3)$ is positive, $| 7 - 3 | = 7 - 3 = 4$.

(c) $\frac{1}{5} - \frac{1}{3} = \frac{3}{15} - \frac{5}{15} = \frac{3-5}{15} = \frac{-2}{15} = -\frac{2}{15}$

15 (a) $d(A, B) = | 7 - 3 | = 4$ (b) $d(B, C) = | -5 - 7 | = 12$
 (c) $d(C, B) = d(B, C) = 12$ (d) $d(A, C) = | -5 - 3 | = 8$

16 (a) $d(A, B) = | -2 - (-6) | = 4$ (b) $d(B, C) = | 4 - (-2) | = 6$
 (c) $d(C, B) = d(B, C) = 6$ (d) $d(A, C) = | 4 - (-6) | = 10$

17 (a) $d(A, B) = | 1 - (-9) | = 10$ (b) $d(B, C) = | 10 - 1 | = 9$
 (c) $d(C, B) = d(B, C) = 9$ (d) $d(A, C) = | 10 - (-9) | = 19$

18 (a) $d(A, B) = | -4 - 8 | = 12$ (b) $d(B, C) = | -1 - (-4) | = 3$
 (c) $d(C, B) = d(B, C) = 3$ (d) $d(A, C) = | -1 - 8 | = 9$

Note: Because $|x| = -x$, the answers to Exercises 19–24 could have a different form. For example, $| -3 - 1 | = 4$ is equivalent to $| 4 - 3 | = 1$.

19 $A = 7$ and $B = 2$, so $d(A, B) = | 7 - 2 | = 5$. Thus, " $d(A, B)$ is less than 2" can be written as $| 7 - 2 | < 2$.

20 $d(A, B) = | -2 - 1 | = 3$ $| -2 - 1 | > 1$

$$\boxed{21} \quad d(A, B) = |1-3 - 1| = |1-3 - 1| = 8$$

$$22 \quad d(A, B) = |4 - 1| = |4 - 1| < 5$$

1.1 EXERCISES

23 $d(A, B) = |1 - 4i| = |1 - 4i| < 3$

24 $d(A, B) = |1 - (-2)i| = |1 + 2i| = \sqrt{1 + 4} = \sqrt{5}$

□

Note: In Exercises 25–32, you may want to substitute a permissible value for the variable to first test if the expression inside the absolute value symbol is positive or negative.

25 Pick an arbitrary value for x that is less than -3 , say -5 .

Since $3 + (-5) = -2$ is negative, we conclude that if $x < -3$, then $3 + x$ is negative.

Hence, $|3 + x| = -(3 + x) = -x - 3$.

26 If $x > 5$, then $5 - x < 0$, and $|5 - x| = -(5 - x) = x - 5$.

27 If $x < 2$, then $2 - x > 0$, and $|2 - x| = 2 - x$.

28 If $x \geq -7$, then $7 + x \geq 0$, and $|7 + x| = 7 + x$.

29 If $a < b$, then $a - b < 0$, and $|a - b| = -(a - b) = b - a$.

30 If $a > b$, then $a - b > 0$, and $|a - b| = a - b$.

31 Since $x^2 + 4 > 0$ for every x , $|x^2 + 4| = x^2 + 4$.

32 Since $-x^2 - 1 < 0$ for every x , $| -x^2 - 1| = -(-x^2 - 1) = x^2 + 1$.

33 $LS = \frac{ab + ac}{a} = \frac{ab}{a} + \frac{ac}{a} = b + c$ (which is $b + ac$).

34 $LS = \frac{ab + ac}{a} = \frac{ab}{a} + \frac{ac}{a} = b + c$

35 $LS = \frac{b + c}{a} = \frac{b}{a} + \frac{c}{a}$

36 $LS = \frac{a + c}{b + d} = \frac{a}{b + d} + \frac{c}{b + d}$ (which is $\frac{a}{b} + \frac{c}{d}$).

37 $LS = (a \div b) \cdot c = \frac{a}{b} + \frac{1}{1} = \frac{a}{b}$. $RS = a \div (b \div c) = a \div \frac{b}{c} = a + \frac{c}{b} = \frac{ac}{b}$. $LS \neq RS$

38 $LS = (a - b) - c = a - b - c$. $RS = a - (b - c) = a - b + c$. $LS \neq RS$

39 $LS = \frac{a - b}{b - a} = \frac{-(b - a)}{b - a} = -1$

40 $LS = -(a + b) = -a - b$ (which is $-a + b$).

41 (a) On the TI-83/4 Plus, the absolute value function is choice 1 under MATH, NUM.

Enter $\text{abs}(3.2^2 - (4.27))$. $|3.2^2 - 4.27| \approx 8.1736$

(b) $\sqrt{(15.6 - 1.5)^2 + (4.3 - 5.4)^2} \approx 14.1428$

42 (a) $\frac{3.42 - 1.29}{5.83 + 2.64} \approx 0.2515$

(b) $1^3 \approx 31.0063$

43 (a) $\frac{1.2 \cdot 10^6}{0.6557} \approx 6.557 \cdot 10^{-1}$

1.52, 10³

Note: For the TI-83/4 Plus, use 1.2E3/(3.1E2 + 1.52E3), where E is obtained by pressing

(b) $1.2 \times 10^3 / (3.1 \times 10^2 + 1.52 \times 10^3) = 6.708 \times 10^1$

2nd EE.

44 (a) $(3.45 \times 10^4 + 10^5) / 3347 = 3.347 \times 10^2$

(b) $(1.79 \times 10^2), (9.84 \times 10^3) = 1,761,360 \approx 1.761 \times 10^6$

[45] Construct a right triangle with sides of lengths $\sqrt{2}$ and 1. The hypotenuse will have length $\sqrt{\sqrt{2}^2 + 1^2} = \sqrt{3}$.

Next construct a right triangle with sides of lengths $\sqrt{3}$ and $\sqrt{2}$. The hypotenuse will have length

$$\sqrt{\sqrt{3}^2 + \sqrt{2}^2} = \sqrt{5}.$$

[46] Use $C = 2\pi r$ with $r = 1, 2,$ and 10 to obtain $2\pi, 4\pi,$ and 20π units from the origin.

[47] The large rectangle has area = width \times length = $a(b + c)$. The sum of the areas of the two small rectangles is $ab + ac$. Since the areas are the same, we have $a(b + c) = ab + ac$.

[48] $\frac{3}{2} = \frac{1}{2} + \frac{2}{2}$ and $8 = 2 + \frac{1}{2} + \frac{8}{2} = \frac{1}{2} + \frac{3}{2} + \frac{4}{2} = \frac{1}{2} + \frac{17}{2} = \frac{18}{2}$.

$$\frac{1}{2} = \frac{1}{2} \quad \frac{2}{2} = \frac{2}{2} \quad \frac{3}{2} = \frac{3}{2} \quad \frac{4}{2} = \frac{4}{2} \quad \frac{17}{2} = \frac{17}{2} \quad \frac{18}{2} = \frac{18}{2}$$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} = \frac{1}{2} + \frac{17}{2} = \frac{18}{2} = \frac{9}{1} = 9$$

[49] (a) Since the decimal point is 5 places to the right of the first nonzero digit, $427,000 = 4.27 \times 10^5$.

(b) Since the decimal point is 8 places to the left of the first nonzero digit, $0.000000093 = 9.3 \times 10^{-8}$.

(c) Since the decimal point is 8 places to the right of the first nonzero digit, $810,000,000 = 8.1 \times 10^8$.

[50] (a) $85,200 = 8.52 \times 10^4$

(b) $0.0000054 = 5.4 \times 10^{-6}$

(c) $24,900,000 = 2.49 \times 10^7$

[51] (a) Moving the decimal point 5 places to the right, we have $8.3 \times 10^5 = 830,000$.

(b) Moving the decimal point 12 places to the left, we have $2.9 \times 10^{-12} = 0.0000000000029$.

(c) Moving the decimal point 8 places to the right, we have $5.64 \times 10^8 = 564,000,000$.

[52] (a) $2.3 \times 10^7 = 23,000,000$

(b) $7.01 \times 10^{-9} = 0.00000000701$

(c) $1.25 \times 10^{10} = 12,500,000,000$

[53] Since the decimal point is 24 places to the left of the first nonzero digit,

$$0.00000000000000000000000017 = 1.7 \times 10^{-24}$$

[54] $9.1 \times 10^{-31} = 0.0000000000000000000000000000000091$

[55] It is helpful to write the units of any fraction, and then "cancel" those units to determine the units of the final answer.

$$\frac{186,000 \text{ miles}}{60 \text{ seconds}} \times \frac{60 \text{ minutes}}{24 \text{ hours}} \times \frac{365 \text{ days}}{1 \text{ year}} = 5.87 \times 10^{12} \text{ mi}$$

second $\frac{1 \text{ minute}}{60}$ 1 hour 1 day 1 year

56 (a) 100 billion = 100,000,000,000 = 1×10^{11}

(b) $d = (100,000 \text{ yr}) \left(\frac{5.87 \times 10^{12} \text{ mi}}{\text{yr}} \right) = 5.87 \times 10^{17} \text{ mi}$

57 $\frac{\frac{1.01 \text{ grams}}{\text{mole}}}{6.02 \times 10^{23} \text{ atoms/mole}} \times 1 \text{ atom} = \frac{1.01 \text{ grams}}{6.02 \times 10^{23}} = 1.678 \times 10^{-23} \text{ g} = 1.678 \times 10^{-24} \text{ g}$

58 $(2.5 \text{ million})(0.00035\%) = (2.5 \times 10^6)(3.5 \times 10^{-6}) = 8.75 \times 9 \text{ halibut}$

24 frames 60 seconds 60 minutes

59 $\frac{1}{\text{second}} + \frac{1}{1 \text{ minute}} + \frac{1}{1 \text{ hour}} + 48 \text{ hours} = 4 \times 10^6$, 10^6 frames

60 $\frac{2 \times 10^{11} \text{ calculations}}{60 \text{ second}} + \frac{60 \text{ seconds}}{1 \text{ minute}} + \frac{60 \text{ minutes}}{1 \text{ hour}} + \frac{24 \text{ hours}}{1 \text{ day}} + 60 \text{ days} = 1 \times 10^3$, 18 calculations

61 (a) $1 \text{ ft}^2 = 144 \text{ in}^2$, so the force on one square foot of a wall is 144 in^2 , $1 \text{ lb/in}^2 = 201 \text{ lb}$

(b) The area of the wall is $40 \times 8 = 320 \text{ ft}^2$, or 320 ft^2 , $144 \text{ in}^2/\text{ft}^2 = 46,080 \text{ in}^2$.

The total force is $46,080 \text{ in}^2$, $1 \text{ lb/in}^2 = 46,080 \text{ lb}$.

Converting to tons, we have $46,080 \text{ lb}/(2000 \text{ lb/ton}) = 23.04 \text{ tons}$.

62 (a) We start with 400 adults, 150 yearlings, and 200 calves {total = 750}

Number of Adults = surviving adults + surviving yearlings
 $= (0.9)(400) + (0.8)(150) = 480$

Number of Yearlings = surviving calves
 $= (0.75)(200) = 150$

Number of Calves = number of female adults
 $= (0.5)(480) = 240$

(b) 75% of last spring's calves equal the number of this year's yearlings (150), so the number of calves is 200. The number of calves is equal to the number of adult females and this is one-half of the number of adults,

so the number of adults is 400.

90% of these (360) are part of the 400 adults this year. The other 40 adults represent

80% of last year's yearlings, so the number of yearlings is 50.

1.2 Exercises

1 $(-\frac{2}{3})^4 = (-\frac{2}{3}) \times (-\frac{2}{3}) \times (-\frac{2}{3}) \times (-\frac{2}{3}) = \frac{16}{81}$

Note: Do not confuse $(-.)^4$ and $-.^4$ since $(-.)^4 = .^4$ and $-.^4$ is the negative of $.^4$.

2 $(-3)^3 = -27 = \frac{-27}{1}$

3 $\frac{2^{-3}}{3^{-2}} = \frac{3^2}{2^3} = \frac{9}{8}$

Note: Remember that negative exponents don't necessarily give negative results—that is, $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$, not $-\frac{1}{8}$.

4 $\frac{2^0 + 0^2}{2 + 0} = \frac{1 + 0}{2} = \frac{1}{2}$

5 $-2^4 + 3^{-1} = -16 + \frac{1}{3} = -\frac{48}{3} + \frac{1}{3} = -\frac{47}{3}$

6 $(-\frac{3}{2})^4 - 2^{-4} = \frac{81}{16} - \frac{1}{16} = \frac{80}{16} = 5$

7 $9^{5/2} = \sqrt{9^5} = 3^5 = \frac{243}{1}$

8 $16^{-3/4} = 1/16^{3/4} = 1/\sqrt[4]{16^3} = 1/2^3 = \frac{1}{8}$

9 $(-0.008)^{2/3} = \sqrt[3]{(-0.008)^2} = (0.008)^{2/3} = \frac{4}{100} = \frac{1}{25}$

10 $(0.008)^{-2/3} = 1/(0.008)^{2/3} = 1/\sqrt[3]{0.008^2}$

$10 (0.008)^{-2/3} = 1/(0.008)^{2/3} = 1/\sqrt[3]{0.008^2}$

$$= 1/(0.02)^2 = 1/(0.04) = \frac{25}{1}$$

$$\text{I1 } \left(\frac{1}{2}\right)^{-4} (16)^5 = \left(\frac{1}{2}\right)^{-4} + 16^5 = 8^9$$

$$\text{I2 } (-3)^{-2} 4^4 = (-3)^{-2+4} = -12^2$$

13 A common mistake is to write $-3 \cdot 2 = -6$, and another is to write $(-2)^3 = -5$.

The following solution illustrates the proper use of the exponent rules.

$$\frac{(2 \cdot 3)(3 \cdot 2)}{6^5} = \frac{(2+3) \cdot 3+2}{(-2)^3} = 6 \cdot 5-6 = 6 \cdot -1 = \frac{6}{-6}$$

$$\mathbf{14} \quad \frac{(2 \cdot 2)^3 y^2}{4 \cdot 4 y^2} = \frac{8 \cdot 6}{4 \cdot 4} = 2 \cdot 2$$

$$\mathbf{15} \quad 1^6 a^5 (-3a^2) 4a^7 = 1^6 (-3) 4 a^{5+2+7} = -2a^{14}$$

$$\mathbf{16} \quad (-4b^3)^6 b^2 (-9b^4) = (-4)^6 (-9) b^{3+2+4} = 6b^9$$

$$\mathbf{17} \quad \frac{(6 \cdot 3)^2}{(2 \cdot 2)^3} \cdot 3^{-2} = \frac{6^2 \cdot 3^2}{2^3 \cdot 2^3} \cdot 1 \text{ \{an expression raised to the zero power is equal to 1\}} = \frac{36 \cdot 6}{8 \cdot 6} = \frac{36}{8} = \frac{9}{2}$$

$$\mathbf{18} \quad \frac{(3y)(2y)}{(y^4)^3} \cdot 5y^3 = \frac{(3y)^4 y^0}{y^{12}} \cdot 1 = \frac{12y}{y^{12}} = \frac{12}{y^5}$$

$$\mathbf{19} \quad 3^7 \cdot 4^3 \cdot 4^4 \cdot 5^{-5} = 12^{7+4} \cdot 5^{\#-5} = 12^{11} \cdot 5^{-2} = \frac{12^{11}}{5^2}$$

$$\mathbf{20} \quad (-2y^3)(-2 \cdot 2)(-3y^{-2}) = -2 \cdot 2^{+1+3} y^{1-2+3+2} = -2 \cdot 6 y^{-1+5} = \frac{-2 \cdot 6}{1} y^4$$

$$\mathbf{21} \quad 8 \cdot 4y^{-3} \cdot \frac{1}{2} \cdot 5y^2 = 4 \cdot 4^{-5} y^{-3+2} = 4 \cdot 1 y^{-1} = \frac{4}{y}$$

$$\mathbf{22} \quad \frac{4a^2b}{a^3b^2} \cdot \frac{5a^2b}{2b^4} = \frac{20a^{2+2}b^{1+1}}{2a^3b^{2+4}} = \frac{20a^4b^2}{2a^3b^6} = \frac{10a^{4-3}b^{2-6}}{1} = \frac{10a}{b^4}$$

$$\mathbf{23} \quad \frac{1}{3} \cdot 4^{-3} \cdot 2^{-2} = \frac{1}{3} \cdot 4^{-2} \cdot 2^{-3} \cdot 2^{-2} = \frac{3}{1} \cdot 2^{-8} \cdot 6 = 3 \cdot 2^{-8} \cdot 6 = \frac{9y^6}{8}$$

$$\mathbf{24} \quad (-2 \cdot y)^5 \cdot 8y^3 = (-32 \cdot y^5) \cdot 8y^3 = -4 \cdot y^8$$

$$\mathbf{25} \quad (3y^3)^4 (4y^2)^{-3} = 3^4 y^{12} \cdot 4^{-3} y^{-6} = 81y^6 \cdot \frac{1}{4^3} = \frac{81}{64} y^6$$

$$\mathbf{26} \quad (-3a^2b^{-5})^3 = -27a^6b^{-15} = -\frac{27a^6}{b^{15}}$$

$$\mathbf{27} \quad (-2r^4)^{-3} r^{-2} = (-2)^{-2} r^{-8} \cdot 6 = \frac{6}{(-2)^2 r^8} = \frac{6}{4r^8}$$

$$\mathbf{28} \quad (2 \cdot 2y^{-5})(6 \cdot 3y)^{\frac{1}{3}} \cdot 1y^3 = 4 \cdot 2y^{-1} = \frac{4}{2y}$$

$$\mathbf{29} \quad (5 \cdot 2y^{-3})^4 \cdot 4 \cdot 5y^4 = 20 \cdot 2^{-5} y^{-3+4} = 20 \cdot 2^{-3} y^1 = \frac{20y}{8}$$

$$\boxed{30} \quad (-2r^2)^5(3r^{-1} \cdot 3)^2 = (-32r^{10} \cdot 5)(9r^{-2} \cdot 6) = -288r^8 \cdot 11$$

$$\boxed{31} \quad \frac{3 \cdot 5y^{4+2}}{y^{-3}} \quad \left\{ \text{remember that } y^0 = 1, \text{ cancel '1'} \right\} = \frac{9 \cdot 10y^8}{y^{-6}} = 9 \cdot y^{10-(-6)} = 9 \cdot y^{10+6} = 9 \cdot y^{16}$$

$$\boxed{32} \quad (4a^2b^4) \cdot \frac{a^3}{2b} = \frac{4 \cdot a^2 \cdot a^3 \cdot b^4}{2b} = \frac{4a^5b^4}{2b} = 2a^5b^3$$

$$\boxed{33} \quad -5a^{3/2} \cdot 2a^{1/2} = -10a^{3/2+1/2} = -10a^{4/2} = -10a^2$$

34 $^{-6} \cdot ^{7/5} \cdot ^{2} \cdot ^{8/5} = -6 + 2 \cdot \frac{7}{5} + \frac{8}{5} = -12 \cdot \frac{15}{5} = -12 \cdot 3$

35 $^3 \cdot ^{5/6} \cdot ^8 \cdot ^{2/3} = 3 + 8 \cdot \frac{5}{6} + \frac{4}{6} = 24 \cdot \frac{9}{6} = 24 \cdot \frac{3}{2}$

36 $(8r)^{1/3} \cdot ^2 r^{1/2} = ^2 r^{1/3} \cdot ^2 r^{1/2} = 4r^{(2/6)+(3/6)} = 4r^{5/6}$

37 $(27a^6)^{-2/3} = 27^{-2/3} a^{-12/3} = \frac{a^{-4}}{27^{2/3}} = \frac{1}{\sqrt[3]{27^2} a^4} = \frac{1}{3^2 a^4} = \frac{1}{9a^4}$

38 $^{25} \cdot ^4 \cdot ^{-3/2} = 25^{-3/2+1-12/2} = \frac{1}{25^{3/2}} = \frac{1}{\sqrt{25^3}} = \frac{1}{5^3 \cdot 5^6} = \frac{1}{125^6}$

39 $^8 \cdot ^{-2/3} \cdot ^{1/6} = 8 \cdot \frac{-4}{6} + \frac{1}{6} = 8 \cdot \frac{-3}{6} = \frac{8}{-2}$

40 $^3 \cdot ^{1/2} \cdot ^{-2} \cdot ^{5/2} = -6 \cdot \frac{1}{2} + \frac{5}{2} = -6 \cdot 3$

41 $\frac{-8 \cdot y^{-6}}{(y^{-6})^{2/3}} = \frac{(-8) \cdot (y^{-6})^{2/3}}{y^{-6 \cdot 2/3}} = \frac{-8 \cdot y^{-4}}{y^{-4}} = \frac{(-2)^2 \cdot 2^2}{y^{-4}} = \frac{4 \cdot 2^2}{y^{-4}} = 4 \cdot 2^2 y^4$

42 $\frac{-y^{3/2}}{y^{-1/3}} = \frac{-y^{9/2}}{y^{-1}} = -y^{11/2}$

43 $\frac{16y^{-4}}{16^{-1/2} y^2} = \frac{16^{-3} y^{-3}}{16^{-1/2} y^2} = \frac{16^{1/2} y^{-3}}{16^{-3} y^2} = \frac{4}{y^5}$

44 $\frac{c^{-4}}{81d^8} = \frac{c^{-3}}{81^3 d^6} = \frac{c^{-3}}{3^3 d^6} = \frac{1}{27c^3 d^6}$

45 $\frac{(4y)^{-1/2}}{(4y^2)^{-1/2}} = \frac{(4)^{-1/2} (y)^{-1/2}}{(4)^{-1/2} (y^2)^{-1/2}} = \frac{y^{-1/2}}{y^{-1}} = y^{1/2}$

46 $a^{4/3} \cdot a^{-3/2} \cdot a^{1/6} = a^{(8/6)-(9/6)+(1/6)} = a^{0/6} = a^0 = 1$

47 $\sqrt[4]{-4+y} = (-4+y)^{1/4}$

48 $\sqrt{-3+y^2} = (-3+y^2)^{1/2}$

49 $\sqrt{(a+b)^2} = (a+b)^{2 \cdot 1/3} = (a+b)^{2/3}$

50 $\sqrt{a+b} = (a+b)^{1/2 \cdot 1/2}$

51 $\sqrt{-2+y^2} = (-2+y^2)^{1/2}$ **Note:** $\sqrt{-2+y^2} \neq \sqrt{-2} + y$

52 $\sqrt[3]{r^3-3} = (r^3-3)^{1/3}$

53 (a) $4 \cdot 3/2 = 4 \cdot 1 \cdot 1/2 = 4 \cdot 1/2 = 2$ (b) $(4 \cdot 3)^{3/2} = (4 \cdot 3)^1 (4 \cdot 3)^{1/2} = (4 \cdot 3)^1 \sqrt{4 \cdot 3} = 4 \cdot 3 \cdot 2 \cdot \sqrt{3} = 24\sqrt{3}$

54 (a) $4 + 3/2 = 4 + 1 \cdot 1/2 = 4 + 1/2 = 4.5$ (b) $(4 + 3)^{3/2} = (4 + 3)^1 (4 + 3)^{1/2} = (4 + 3) \sqrt{4 + 3} = 7 \sqrt{7}$

55 (a) $8 - y^{1/3} = 8 - \sqrt[3]{y}$

(b) $(8 - y)^{1/3} = \sqrt[3]{8 - y}$

56 (a) $64y^{1/3} = 64 \sqrt[3]{y}$

(b) $(64y)^{1/3} = 64^{1/3} y^{1/3} = 4 \sqrt[3]{y}$

$$\boxed{57} \quad -81 = -9^2 = 9$$

$$\boxed{58} \quad \sqrt[3]{-216} = \sqrt[3]{(-6)^3} = -6$$

$$\boxed{59} \quad \sqrt[5]{-64} = \sqrt[5]{-32} \sqrt[5]{2} = \sqrt[5]{(-2)^5} \sqrt[5]{2} = -2 \sqrt[5]{2}$$

$$\boxed{60} \quad \sqrt[4]{512} = \sqrt[4]{256} \sqrt[4]{2} = \sqrt[4]{4^4} \sqrt[4]{2} = 4 \sqrt[4]{2}$$

61 In the denominator, you would like to have $\sqrt[3]{2^3}$. How do you get it? Multiply by $\sqrt[3]{2^2}$, or, equivalently, $\sqrt[3]{4}$. Of course, we have to multiply the numerator by the same value so that we don't change the value of the given fraction.

$$\frac{1}{\sqrt[3]{2}} = \frac{1}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{2 \cdot 4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{\sqrt[3]{4}}{2} = \frac{1}{2} \sqrt[3]{4}$$

$$\boxed{62} \quad \sqrt[5]{\frac{1}{-5}} = \frac{\sqrt[5]{1}}{\sqrt[5]{-5}} = \frac{1}{\sqrt[5]{-5}} = \frac{\sqrt[5]{-5}}{\sqrt[5]{-5} \sqrt[5]{5}} = \frac{\sqrt[5]{-5}}{\sqrt[5]{-25}}$$

63 $-9^{-4}y^6 = 9^{-4}y^{6 \cdot 1/2} = 9^{1/2} \cdot 9^{-4 \cdot 1/2} (y^6)^{1/2} = 3 \cdot 9^{-2}y^3 = \frac{3y^3}{9^2}$

64 $\sqrt[4]{-16a^8b^{-2}} = 4a^2b^{-1/2} = \frac{4a^4}{b}$

65 $\sqrt[3]{8a^6b^{-3}} = 2a^2b^{-1} = \frac{2a^2}{b}$

66 $\sqrt[4]{81r^5} = \sqrt[4]{3^4 r^4 r} = 3r \sqrt[4]{r}$



Note: For exercises similar to numbers 67–74, pick a multiplier that will make all of the exponents of the terms in the denominator a multiple of the index.

67 The index is 2. Choose the multiplier to be $\sqrt{-2y}$ so that the denominator contains only terms with even exponents.

$\frac{3}{2y^3} = \frac{3 \cdot \sqrt{-2y}}{2y^3 \cdot \sqrt{-2y}} = \frac{3\sqrt{-2y}}{-4y^4} = \frac{-3\sqrt{-2y}}{4y^4}$, or $\frac{-3\sqrt{-2y}}{2y^2}$

68 $\frac{3}{3^3y} = \frac{3 \cdot \sqrt{-3y}}{3^3y \cdot \sqrt{-3y}} = \frac{3\sqrt{-3y}}{-9 \cdot 4y^2} = \frac{-3\sqrt{-3y}}{36y^2}$

69 The index is 3. Choose the multiplier to be $\sqrt[3]{3 \cdot 2}$ so that the denominator contains only terms with exponents that are multiples of 3.

70 $\frac{3 \cdot y}{4} = \frac{3 \cdot y \cdot \sqrt[3]{2 \cdot 2}}{4 \cdot \sqrt[3]{2 \cdot 2}} = \frac{3 \cdot y \cdot \sqrt[3]{4}}{4 \cdot \sqrt[3]{4}} = \frac{3 \cdot y \cdot \sqrt[3]{4}}{4 \cdot \sqrt[3]{4}}$

71 The index is 4. Choose the multiplier to be $\sqrt[4]{3 \cdot 2}$ so that the denominator contains only terms with exponents that are multiples of 4.

72 $\frac{5 \cdot y^{12}}{125} = \frac{5 \cdot y^{12} \cdot \sqrt[4]{5 \cdot 3}}{125 \cdot \sqrt[4]{5 \cdot 3}} = \frac{5 \cdot y^{12} \cdot \sqrt[4]{15}}{125 \cdot \sqrt[4]{15}}$

73 The index is 5. Choose the multiplier to be $\sqrt[5]{4 \cdot 2}$ so that the denominator contains only terms with exponents that are multiples of 5.

$\frac{5 \cdot 7y^2}{8^3} = \frac{5 \cdot 7y^2 \cdot \sqrt[5]{4 \cdot 2}}{8^3 \cdot \sqrt[5]{4 \cdot 2}} = \frac{5 \cdot 7y^2 \cdot \sqrt[5]{8}}{8^3 \cdot \sqrt[5]{8}} = \frac{5 \cdot 7y^2 \cdot \sqrt[5]{8}}{8^3 \cdot \sqrt[5]{8}}$

$$\square \frac{5}{11^3} \frac{5}{11^3} = \frac{3}{5} = \frac{-81 \cdot 14 y^3}{5} = \frac{-10}{5} = \frac{-81 \cdot 4 y^3}{5} = \frac{-81 \cdot 4 y^3}{2 \cdot 5} = -$$

74 $\hat{E} 3 \cdot y$

$$\frac{9 \cdot 2}{9 \cdot 2} = \hat{E} \frac{3 \cdot y}{9 \cdot 2} + \frac{27}{27 \cdot 3} = \frac{27}{243 \cdot 5} = \frac{3}{3} = \frac{3}{3} = \frac{3}{3} = \frac{81 \cdot 4 y^3}{3}$$

75 $\hat{E} (5 \cdot 5 y^{-2})^4 = 5 \cdot 5 y^{-2} = \frac{5^5}{y^2}$

76 $\hat{E} (7^{-3} \cdot 4)^6 = 7^{-3} \cdot 4 = \frac{7^4}{3}$

$$\square \frac{5}{8} \frac{5}{4} \frac{5}{8} \frac{5}{4} = \frac{4}{5} = \frac{5}{5} = \frac{2 \cdot 4}{5} = \frac{5}{5} = \frac{2 \cdot 4}{5} = \frac{2 \cdot 5 \cdot 2 \cdot 4}{5} = -$$

77 $\hat{E} y \hat{E} y = \hat{E} y \hat{E} y = -y^4 = -y^{10} = y = y = -y$

78 $-\overline{5 \cdot y^7} - \overline{15 \cdot 3 y^3} = -\overline{25 \cdot 4 y^{10}} - \overline{3} = 5 \cdot 2 y^5 - \overline{3}$

79 $\sqrt[3]{3!4\#2} = \sqrt[3]{-9!-1\#4} = \sqrt[3]{-27!3\#6} = -3! \# 2$

80 $\sqrt[3]{(2r -)^3} = 2r -$
 \square

81 $\sqrt[3]{-6y^4} = \sqrt[3]{(-3)^2(y^2)^2} = \sqrt[3]{(-3)^2} \sqrt[3]{(y^2)^2} = 1 \cdot 3! |y^2| = 1 \cdot 3! y^2$ since y^2 is always nonnegative.

Note: $1 \cdot 3!$ could be written as $\cdot 2! \cdot 1$.

82 $\sqrt[3]{-4y^{10}} = \sqrt[3]{(-2)^2(y^5)^2} = 1 \cdot 2! |y^5| = \cdot 2! y^5$

83 $\sqrt[3]{-8(y-3)^{12}} = \sqrt[3]{(-2)^4(y-3)^{3\#4}} = 1 \cdot 2! |b(y-3)^3| = \cdot 2! (y-3)^3$, or $\cdot 2! (y-3)^2 |y-3|$

84 $\sqrt[3]{(-+2)^{12}y^4} = \sqrt[3]{(-+2)^{4\#3}y^4} = b(-+2)^3 |y|$, or $(+2)^2 |(-+2)y|$

85 $(a^r)^2 = a^{2r} \# a^{2r}$ since $2r \# r^2$ for all values of r ; for example, let $r = 1$.

86 Squaring the right side gives us $(a + 1)^2 = a^2 + 2a + 1$. Squaring the left side gives us $a^2 + 1$.
 $a^2 + 2a + 1 \# a^2 + 1$ for all values of a ; for example, let $a = 1$.

87 $(ab)^y = a \cdot y \cdot b \cdot y \# a \cdot b^y$ for all values of \cdot and y ; for example, let $\cdot = 1$ and $y = 2$.

88 $\sqrt[r]{-a} = (a^r)^{1/2} = \hat{a}^{1/2\#r} \# \sqrt[r]{-a}$

89 $\sqrt[8]{\frac{1}{c}} = \sqrt[8]{\frac{1}{c}} = \frac{1^{1/8}}{c^{1/8}} \# \frac{1}{\sqrt[8]{c}}$

90 $\frac{1}{a^5} = a^{-5} \# a^{1/5}$ since $-5 \# 1/5$ for all values of 5 ; for example, let $5 = 1$.

91 (a) $(-3)^{2/5} = \square(-3)^{2\#1/5} = 9^{1/5} \rightarrow 1\text{A}5518$

(b) $(-7)^{4/3} = \sqrt[3]{(-7)^4} = \sqrt[3]{2401} = 13\text{A}3905$

92 (a) $(-1\text{A}2)^{3/7} = \square(-1\text{A}2)^{3\#1/7} = (-1\text{A}728)^{1/7} \rightarrow -1\text{A}0813$

(b) $(-5\text{A}08)^{7/3} = \sqrt[3]{(-5\text{A}08)^7} = \sqrt[3]{-87,306\text{A}38} \rightarrow -44\text{A}3624$

93 (a) $\sqrt{-1+1} \rightarrow 2\text{A}0351$

(b) $\sqrt[3]{17\text{A}1 + 5^{1/4}} \rightarrow 4\text{A}0717$

94 (a) $(2\text{A}6 - 1\text{A}3)^{-2} \rightarrow 0\text{A}5917$

(b) $5^{-7} \rightarrow 70\text{A}6807$

95 $\$200(1\text{A}04)^{180} \rightarrow \$232,825\text{A}78$

96 $2 = 1454 \text{ ft} = d = 1\text{A}2 - 2 = 1\text{A}2 - 1454 \rightarrow 45\text{A}8 \text{ mi}$

97 $\square = 230 \text{ kg} = P = 0\text{A}46 \sqrt[3]{\square} = 0\text{A}46 \sqrt[3]{230} \rightarrow 2\text{A}82 \text{ m}$

98 $P = 25 \text{ ft} = \square = 0.0016P^{2\text{A}43} = 0.0016(25)^{2\text{A}43} \rightarrow 3\text{A}99 \text{ tons}$

99 $b = 75 \text{ and } \$ = 180 = \square = \frac{\$}{b - 35} = \frac{180}{75 - 35} \rightarrow 52\text{A}6.$

$b = 120 \text{ and } \$ = 250 = \square = \frac{\$}{b - 35} = \frac{250}{120 - 35} \rightarrow 56\text{A}9.$

It is interesting to note that the 75-kg lifter can lift 2A4 times his/her body weight and the 120-kg lifter can lift

approximately 2.08 times his/her body weight, but the formula ranks the 120-kg lifter as the superior lifter.

100 (a) $z = 72$ in. and $S = 175$ lb $\Rightarrow W = (0.1091)S^{0.425}z^{0.725} = (0.1091)(175)^{0.425}(72)^{0.725} \approx 21476 \text{ ft}^2$.

(b) $z = 66$ in. $\Rightarrow W_1 = (0.1091)S^{0.425}(66)^{0.725}$. A 10% increase in weight would be represented by 1.1S
and

thus $W_2 = (0.1091)(1.1S)^{0.425}(66)^{0.725}$. $W_2/W_1 = (1.1)^{0.425} \approx 1.04$, which represents a 4% increase in W .

$$101 \quad \square = 0\hat{A}11662^{1A7}$$

Height	64	65	66	67	68	69	70	71
Weight	137	141	145	148	152	156	160	164
Height	72	73	74	75	76	77	78	79
Weight	168	172	176	180	184	188	192	196

$$102 \quad \square = 0\hat{A}10492^{1A7}$$

Height	60	61	62	63	64	65	66	67
Weight	111	114	117	120	123	127	130	133
Height	68	69	70	71	72	73	74	75
Weight	137	140	144	147	151	154	158	162

1.3 Exercises

$$1 \quad (2'' + 3)('' - 4) + 4''('' - 2) = (2''^2 - 5'' - 12) + (4''^2 - 8'') = 6''^2 - 13'' - 12$$

$$2 \quad (3'' - 1)('' + 2) + 7''('' + 1) = (3''^2 + 5'' - 2) + (7''^2 + 7'') = 10''^2 + 12'' - 2$$

$$3 \quad \frac{8 \cdot 2y^3 - 6 \cdot 3y}{2 \cdot 2y} = \frac{8 \cdot 2y^3}{2 \cdot 2y} - \frac{6 \cdot 3y}{2 \cdot 2y} = 4y^2 - 3.$$

$$4 \quad \frac{6 \cdot 2y^3 - y^2}{y'} = \frac{6 \cdot y^3}{y'} - \frac{y^2}{y'} = 6 \cdot y^2 - y$$

5 We recognize this product as the difference of two squares.

$$(2 + 7y)(2 - 7y) = (2)^2 - (7y)^2 = 4 - 49y^2$$

$$6 \quad (5 + 3y)(5 - 3y) = (5)^2 - (3y)^2 = 25 - 9y^2$$

$$7 \quad (3 + 2y)^2 = (3)^2 + 2(3)(2y) + (2y)^2 = 9 + 12y + 4y^2$$

$$8 \quad (5 - 4y)^2 = (5)^2 - 2(5)(4y) + (4y)^2 = 25 - 40y + 16y^2$$

$$9 \quad (-y + -y)^2 = (-y)^2 - 2(-y)(-y) + (-y)^2 = y^2 - 2y^2 + y^2 = -y^2$$

$$10 \quad (-y + -y)^2 = (-y)^2 - 2(-y)(-y) + (-y)^2 = y^2 - 2y^2 + y^2 = -y^2$$

11 Use Product Formula (3) on page 30 of the text.

$$(-2y)^3 = (-)^3 - 3(-)^2(2y) + 3(-)(2y)^2 - (2y)^3 = - - 6 \cdot 2y + 12y^2 - 8y^3$$

$$12 \quad (- + 3y)^3 = (-)^3 + 3(-)^2(3y) + 3(-)(3y)^2 + (3y)^3 = - + 9 \cdot 2y + 27y^2 + 27y^3$$

13 We recognize $8^2 - 17 - 21$ as a trinomial that may be able to be factored into the product of two binomials.

Using trial and error, we obtain $8^2 - 17 - 21 = (8 + 7)(-3)$. If you are interested in a sure-fire method for factoring trinomials, see Example 6 on page 48 of the text.

14 Using trial and error, we obtain $7^2 + 10 - 8 = (7 - 4)(-2)$.

15 The factors for $x^2 + 4x + 5$ would have to be of the form $(x + \square)(x + \square)$.

The factors of 5 are 1 and 5, but their sum is 6 (not 4). Thus, $x^2 + 4x + 5$ is irreducible.

16 $3x^2 - 4x + 2$ is irreducible.

$$17 \quad 36x^2 - 60x + 25 = (6x - 5)(6x - 5) = (6x - 5)^2 \quad 18 \quad 9x^2 + 24x + 16 = (3x + 4)(3x + 4) = (3x + 4)^2$$

$$\boxed{19} \quad x^4 - 4x^2 = x^2(x^2 - 4) = x^2(x^2 - 2^2) = x^2(x + 2)(x - 2)$$

$$\boxed{20} \quad x^3 - 16x = x(x^2 - 16) = x(x^2 - 4^2) = x(x + 4)(x - 4)$$

21 We recognize $8x^3 - y^6$ as the difference of two cubes.

$$\begin{aligned} 8x^3 - y^6 &= (2x)^3 - (y^2)^3 = (2x - y^2)(2x^2 + (2x)y^2 + y^4) \\ &= (2x - y^2)(4x^2 + 2xy^2 + y^4) \end{aligned}$$

$$\boxed{22} \quad x^6 - 27y^3 = (x^2)^3 - (3y)^3 = (x^2 - 3y)(x^2)^2 + (x^2)(3y) + (3y)^2 = (x^2 - 3y)(x^4 + 3x^2y + 9y^2)$$

23 We recognize $343x^3 + y^9$ as the sum of two cubes.

$$\begin{aligned} 343x^3 + y^9 &= (7x)^3 + (y^3)^3 = (7x + y^3)(7x^2 - (7x)y^3 + y^6) \\ &= (7x + y^3)(49x^2 - 7xy^3 + y^6) \end{aligned}$$

$$\boxed{24} \quad x^3 + 64 = (x)^3 + (4)^3 = (x + 4)(x^2 - (x)(4) + (4)^2) = (x + 4)(x^2 - 4x + 16)$$

$$\begin{aligned} \boxed{25} \quad 3x^3 + 3x^2 - 27x - 27 &= 3x^3 + x^2 - 9x - 9 \quad \{\text{gcf} = 3\} \\ &= 3x^2(x + 1) - 9(x + 1) \quad \{\text{factor by grouping}\} \\ &= 3x^2(x + 1) - 9(x + 1) \quad \{\text{factor out } (x + 1)\} \\ &= 3x^2 - 9(x + 1) \quad \{\text{difference of two squares}\} \\ &= 3(x + 3)(x - 3)(x + 1) \end{aligned}$$

$$\begin{aligned} \boxed{26} \quad 5x^3 + 10x^2 - 20x - 40 &= 5x^3 + 2x^2 - 4x - 8 = 5x^2(x + 2) - 4(x + 2) \\ &= 5x^2 - 4(x + 2) = 5(x + 2)(x - 2)(x + 2) = 5(x + 2)^2(x - 2) \end{aligned}$$

27 We could treat $a^6 - b^6$ as the difference of two squares or the difference of two cubes. Factoring $a^6 - b^6$ as the difference of two squares and then factoring as the sum and difference of two cubes leads to the following:

$$\begin{aligned} a^6 - b^6 &= (a^3)^2 - (b^3)^2 = (a^3 + b^3)(a^3 - b^3) \\ &= (a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2) \end{aligned}$$

$$\boxed{28} \quad x^8 - 16 = (x^4)^2 - 4^2 = (x^4 + 4)(x^4 - 4) = (x^4 + 4)(x^2 + 2)(x^2 - 2)$$

29 We might first try to factor $x^2 + 4x + 4 - 9y^2$ by grouping since it has more than 3 terms, but this would prove to be unsuccessful. Instead, we will group the terms containing x and the constant term together, and then proceed as in Example 2(c).

$$x^2 + 4x + 4 - 9y^2 = (x + 2)^2 - (3y)^2 = (x + 2 + 3y)(x + 2 - 3y)$$

$$\boxed{30} \quad x^2 - 4y^2 - 6x + 9 = (x^2 - 6x + 9) - 4y^2 = (x - 3)^2 - (2y)^2 = (x - 3 + 2y)(x - 3 - 2y)$$

$$\boxed{31} \quad \frac{y^2 - 25}{y^3 - 125} = \frac{(y + 5)(y - 5)}{(y - 5)(y^2 + 5y + 25)} = \frac{y + 5}{y^2 + 5y + 25}$$

$$\boxed{32} \quad \frac{12 + r - r^2}{r^3 + 3r^2} = \frac{(3 + r)(4 - r)}{r^2(r + 3)} = \frac{4 - r}{r^2}$$

$$\begin{aligned} \boxed{33} \quad \frac{9x^2 - 4}{3x^2 - 5x + 2} + \frac{9x^4 - 6x^3 + 4x^2}{27x^4 + 8} &= \frac{(3x + 2)(3x - 2)}{(3x - 2)(x - 1)} + \frac{x^2(9x^2 - 6x + 4)}{x(27x^3 + 8)} \\ &= \frac{(3x + 2)(3x - 2)}{(3x - 2)(x - 1)} + \frac{x(9x^2 - 6x + 4)}{27x^3 + 8} \end{aligned}$$

$$\begin{aligned}
 &= (3. - 2)(. - 1) - (3. + 2)(9.^2 - 6. + 4) = \frac{-}{-} - 1 \\
 \text{34} \quad &\frac{5a^2 + 12a + 4}{a^4 - 16} = \frac{25a^2 + 20a + 4}{a^2 - 2a} = \frac{(5a + 2)(a + 2)}{(a^2 + 4)(a + 2)(a - 2)} + \frac{a(a - 2)}{(5a + 2)(5a + 2)} = \frac{a}{(a^2 + 4)(5a + 2)}
 \end{aligned}$$

$$\boxed{35} \quad \frac{4}{3+1} - \frac{11}{(3+1)^2} = \frac{4(3+1)}{(3+1)^2} - \frac{11}{(3+1)^2} = \frac{12+4-11}{(3+1)^2} = \frac{12-7}{(3+1)^2}$$

$$\boxed{36} \quad \frac{4}{(5-2)^2} + \frac{5-2}{5-2} = \frac{4}{(5-2)^2} + \frac{(5-2)}{(5-2)^2} = \frac{4+5-2}{(5-2)^2} = \frac{5-2+4}{(5-2)^2}$$

$$\boxed{37} \quad \frac{2}{3+1} - \frac{-2}{-2} = \frac{2}{3+1} + \frac{(3+1)}{-(3+1)} = \frac{2}{3+1} + \frac{3+1}{-(3+1)} = \frac{2+3}{3+1} + \frac{-3-1}{3+1} = \frac{5-2}{3+1}$$

$$\boxed{38} \quad \frac{5}{2-1} + \frac{-7}{-7} = \frac{5}{2-1} - \frac{(2-1)}{-(2-1)} + \frac{-7}{-7} = \frac{5}{2-1} - \frac{2-1}{-(2-1)} + \frac{-7}{-7} = \frac{5-2+1}{2-1} = \frac{3-2+1}{2-1}$$

$$\begin{aligned} \boxed{39} \quad \frac{3!}{!+2} + \frac{5!}{!-2} - \frac{40}{!^2-4} &= \frac{3!}{!+2} + \frac{5!}{!-2} - \frac{40}{(!+2)(!-2)} \\ &= \frac{3!(!-2)}{(!+2)(!-2)} + \frac{5!(!+2)}{(!+2)(!-2)} - \frac{40}{(!+2)(!-2)} \\ &= \frac{3!^2 - 6! + 5!^2 + 10! - 40}{(!+2)(!-2)} \\ &= \frac{8!^2 + 4! - 40}{(!+2)(!-2)} = \frac{4(2!^2 + ! - 10)}{(!+2)(!-2)} = \frac{4(2!+5)(!-2)}{(!+2)(!-2)} = \frac{4(2!+5)}{!+2} \end{aligned}$$

$$\boxed{40} \quad \frac{!}{!+3} + \frac{4!}{!-3} - \frac{18}{!^2-9} = \frac{!(!-3) + 4!(!+3) - 18}{!^2-9} = \frac{5!^2 + 9! - 18}{!^2-9} = \frac{(5!-6)(!+3)}{(!+3)(!-3)} = \frac{5!-6}{!-3}$$

$$\boxed{41} \quad \frac{4}{3-4} + \frac{8}{3^2-4} + \frac{2}{-} = \frac{4(-) + 8 + 2(3-4)}{-(3-4)} = \frac{4-^2 + 6}{-(3-4)} = \frac{2(2+3)}{-(3-4)} = \frac{2(2+3)}{3-4}$$

$$\boxed{42} \quad \frac{12}{2+1} - \frac{3}{2^2+} + \frac{5}{-} = \frac{12(-) - 3 + 5(2+1)}{-(2+1)} = \frac{12-^2 + 10 + 2}{-(2+1)} = \frac{2(6-^2 + 5 + 1)}{-(2+1)} = \frac{2(2+1)(3+1)}{-(2+1)} = \frac{2(3+1)}{-}$$

$$\boxed{43} \quad \frac{2}{+2} - \frac{8}{-^2+2} + \frac{3}{-} = \frac{2(-) - 8 + 3(-+2)}{-(-+2)} = \frac{2-^2 + 3- - 2}{-(-+2)} = \frac{(2-1)(-+2)}{-(-+2)} = \frac{2-1}{-}$$

$$\boxed{44} \quad \frac{5}{2+3} - \frac{6}{2^2+3} + \frac{2}{-} = \frac{5(-) - 6 + 2(2+3)}{-(2+3)} = \frac{5-^2 + 4}{-(2+3)} = \frac{-(5+4)}{-(2+3)} = \frac{5+4}{2+3}$$

$$\begin{aligned} \boxed{45} \quad 3 + \frac{5}{"} + \frac{2"}{3"+1} &= \frac{3"(3"+1) + 5(3"+1) + 2"(")}{"(3"+1)} && \text{{common denominator}} \\ &= \frac{9" + 3" + 15" + 5 + 2" + 2"}{"(3"+1)} && \text{{multiply terms}} \\ &= \frac{11" + 18" + 5}{"(3"+1)} && \text{{add like terms}} \end{aligned}$$

$$\boxed{\quad} \quad \frac{2}{3"} - \frac{6"(" + 5) + 2(" + 5) - 3"(")}{3" + 32" + 10}$$

$$46 \quad 6 + \frac{1}{x-5} = \frac{6(x+5)}{(x+5)} = \frac{6(x+5)}{(x+5)}$$

$$\begin{aligned}
 47 \quad \frac{2x+1}{x^2+4x+4} - \frac{6}{x^2-4} + \frac{3}{x-2} &= \frac{2x+1}{(x+2)^2} - \frac{6}{(x+2)(x-2)} + \frac{3}{x-2} \\
 &= \frac{(2x+1)(x-2) - 6(x+2) + 3(-x^2+4x+4)}{(x+2)^2(x-2)} \\
 &= \frac{2x^2 - 3x - 2 - 6x^2 - 12x + 3x^2 + 12x + 12}{(x+2)^2(x-2)} \\
 &= \frac{-x^2 - 3x + 10}{(x+2)^2(x-2)} = \frac{-x^2 + 3x - 10}{(x+2)^2(x-2)} = \frac{(x+5)(x-2)}{(x+2)^2(x-2)} = \frac{x+5}{(x+2)^2}
 \end{aligned}$$

$$[48] \frac{4x+12}{x^2+6x+9} + \frac{5x}{x^2-9} + \frac{7}{x-3} = \frac{4}{x+3} + \frac{5x}{x^2-9} + \frac{7}{x-3} = \frac{4(x-3)+5x+7(x+3)}{x^2-9} = \frac{16x+9}{x^2-9}$$

[49] The lcd of the entire expression is ab . Thus, we will multiply both the numerator and denominator by ab .

$$\frac{b-a}{\frac{1}{a}-\frac{1}{b}} = \frac{b-a}{\frac{1}{a}-\frac{1}{b}} \cdot \frac{ab}{ab} = \frac{b-a}{\frac{b-a}{ab}} = \frac{(b-a)(b-a)}{b-a} = a+b$$

[50] The lcd of the entire expression is x^2y^2 . Thus, we will multiply both the numerator and denominator by x^2y^2 .

$$\frac{\frac{x-y}{1-x}}{\frac{1}{x^2}-\frac{1}{y^2}} = \frac{x-y}{1-x} \cdot \frac{x^2y^2}{x^2y^2} = \frac{x^3-y^3}{x^2y^2-x^2y^2} = \frac{(x-y)(x^2+xy+y^2)}{(x+y)(x-y)} = \frac{x^2+xy+y^2}{x+y}$$

[51] The lcd of the entire expression is xy . Thus, we will multiply both the numerator and denominator by xy .

$$\frac{y^{-1}+x^{-1}}{(xy)^{-1}} = \frac{\frac{1}{y}+\frac{1}{x}}{\frac{1}{xy}} = \frac{\frac{1}{y}+\frac{1}{x}}{\frac{1}{xy}} \cdot \frac{xy}{xy} = \frac{x+y}{1} = x+y$$

$$[52] \frac{y^{-2}-x^{-2}}{y^{-2}+x^{-2}} = \frac{\frac{1}{y^2}-\frac{1}{x^2}}{\frac{1}{y^2}+\frac{1}{x^2}} = \frac{\frac{1}{y^2}-\frac{1}{x^2}}{\frac{1}{y^2}+\frac{1}{x^2}} \cdot \frac{x^2y^2}{x^2y^2} = \frac{x^2-y^2}{x^2+y^2}$$

$$[53] \frac{\frac{r}{r^2}-\frac{r}{r^2}}{\frac{r^2}{r^2}-\frac{r^2}{r^2}} = \frac{\frac{r}{r^2}-\frac{r}{r^2}}{\frac{r^2}{r^2}-\frac{r^2}{r^2}} = \frac{r^3+r^3}{r^4-4} = \frac{r(r^2+2)}{(r^2+2)(r^2-2)} = \frac{r}{r^2-2}$$

$$[54] \frac{2-4}{2\$+1} = \frac{2(2\$+1)-4\$}{\$(2\$+1)} = \frac{4\$+2-4\$}{\$(2\$+1)} = \frac{2}{\$(2\$+1)}$$

$$\frac{5}{\$} + \frac{8}{2\$+1} = \frac{5(2\$+1)+8\$}{\$(2\$+1)} = \frac{10\$+5+8\$}{\$(2\$+1)} = \frac{18\$+5}{\$(2\$+1)}$$

$$[55] \frac{(x+2)^2-3(x+2)-(x^2-3x)}{2} = \frac{x^2+2x+2-3x-32-x^2+3x}{2} = \frac{2x+2^2-32}{2} = \frac{2(x+2-3)}{2} = x+2-3$$

$$[56] \frac{(x+2)^3+5(x+2)-(x^3-5x)}{2} = \frac{x^3+2^3+3x^2+3x+2+5x+5-x^3+5x}{2} = \frac{2^3+3x^2+3x+2+5x+5}{2} = \frac{8+3x^2+3x+2+5x+5}{2}$$

2

$$\begin{aligned} &= \frac{3 \cdot 2 + 3 \cdot 2^2 + 2^3 + 52}{2} \\ &= \frac{2(3 \cdot 2 + 3 \cdot 2 + 2^2 + 5)}{2} = 3 \cdot 2 + 3 \cdot 2 + 2^2 + 5 \end{aligned}$$

$$\begin{aligned} \boxed{57} \quad \frac{\frac{5}{x-1} - \frac{5}{a-1}}{x-a} &= \frac{\frac{5(a-1) - 5(x-1)}{(x-1)(a-1)}}{x-a} = \frac{5a - 5x}{(x-1)(a-1)(x-a)} = \frac{5(a-x)}{(x-1)(a-1)(x-a)} \\ &= -\frac{5}{(x-1)(a-1)} \end{aligned}$$

58
$$\frac{\frac{a+2}{-} - \frac{a+2}{a}}{-a} = \frac{\frac{a(-) + 2 - (a+2)}{-a}}{-a} = \frac{2a - 2}{a(-) - a} = \frac{2(a -)}{a(-) - a} = \frac{-2}{-}$$

59
$$\frac{1}{2} \frac{1}{2} \frac{3 - (-+2)^3}{2} = \frac{(-+2)^3 - 3}{2} = \frac{3 - (-+2)^3}{2 \cdot 3(-+2)^3} = \frac{1 - (-+2) \sqrt[3]{-^2 + -(-+2) + (-+2)^{-2}}}{2 \cdot 3(-+2)^3}$$
 difference of two cubes

$$= \frac{-2 \sqrt[3]{-^2 + -^2 + -2 + -^2 + 2 \cdot 2 + 2^2}}{2 \cdot 3(-+2)^3} = \frac{-2(3 \cdot -^2 + 3 \cdot 2 + 2^2)}{2 \cdot 3(-+2)^3} = \frac{-3 \cdot -^2 + 3 \cdot 2 + 2^2}{-^3(-+2)^3}$$

60
$$\frac{1}{+2} - \frac{1}{-} = \frac{-(-+2)}{(-+2)-} = \frac{-2}{2 \cdot (-+2)} = \frac{-1}{-(-+2)}$$

61 The conjugate of $\sqrt{-1} - 5$ is $-\sqrt{-1} + 5$. Multiply the numerator and the denominator by the conjugate of the denominator. This will eliminate the radical in the denominator.

$$\frac{-\sqrt{-1} + 5}{-\sqrt{-1} - 5} = \frac{-\sqrt{-1} + 5}{-\sqrt{-1} - 5} \cdot \frac{-\sqrt{-1} + 5}{-\sqrt{-1} + 5} = \frac{(-\sqrt{-1})^2 + 2 \cdot 5 - \sqrt{-1} + 5^2}{(-\sqrt{-1})^2 - 5^2} = \frac{1 + 10 - \sqrt{-1} + 25}{1 - 25}$$

62
$$\frac{16x^2 - y^2}{2x - y} = \frac{16x^2 - y^2}{2x - y} \cdot \frac{2x + y}{2x + y} = \frac{(4x)^2 - (y)^2}{4x - y} = (4x + y)(2x - y)$$

63 We must recognize $a^3 - b^3$ as the first factor of the product formula for the difference of two cubes,

$a^3 - b^3 = (a - b)(a^2 + ay + y^2)$. The second factor is then

$$\frac{a^3 - b^3}{a - b} = \frac{a^3 - b^3}{a - b} \cdot \frac{a^2 + ab + b^2}{a^2 + ab + b^2} = \frac{a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3}{a^2 + ab + b^2} = \frac{a^3 - b^3}{a^2 + ab + b^2}$$

$$\frac{1}{a - b} = \frac{1}{a - b} \cdot \frac{a^2 + ab + b^2}{a^2 + ab + b^2} = \frac{a^2 + ab + b^2}{a - b}$$

64
$$\frac{1}{\sqrt{a} - \sqrt{b}} = \frac{1}{\sqrt{a} - \sqrt{b}} \cdot \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \frac{\sqrt{a} + \sqrt{b}}{a - b}$$

65
$$\frac{-\sqrt{a} - \sqrt{b}}{a^2 - b^2} = \frac{-\sqrt{a} - \sqrt{b}}{a^2 - b^2} \cdot \frac{-\sqrt{a} + \sqrt{b}}{-\sqrt{a} + \sqrt{b}} = \frac{a - b}{a^2 - b^2} = \frac{1}{a + b}$$

$$\begin{array}{l}
 (a + b)(a - b) \\
 \text{Š - a + b} \\
 - b <
 \end{array}
 \quad
 \begin{array}{l}
 (a + b)(a - b) \\
 \text{Š - a + b} \\
 a + b \\
 - b <
 \end{array}$$

$$\boxed{66} \quad \frac{-\bar{b} + -\bar{c}}{b^2 - c^2} = \frac{-\bar{b} + -\bar{c}}{b^2 - c^2} + \frac{-b - -c}{-b - -c} = \frac{b - c}{(b + c)(b - c)\text{Š - b - -c}} = \frac{1}{(b + c)\text{Š - b - -c}}$$

$$\begin{aligned}
 \boxed{67} \quad \frac{-2(\bar{.} + 2) + 1 - -2. + 1}{2} &= \frac{-2(\bar{.} + 2) + 1 - -2. + 1}{2} + \frac{-2(\bar{.} + 2) + 1 + -2. + 1}{-2(\bar{.} + 2) + 1 + -2. + 1} \\
 &= \frac{(2. + 22 + 1) - (2. + 1)}{2^{\wedge} - 2(\bar{.} + 2) + 1 + -2. + 1\%} \\
 &= \frac{22}{2^{\wedge} - 2(\bar{.} + 2) + 1 + -2. + 1\%} = \frac{2}{-2(\bar{.} + 2) + 1 + -2. + 1}
 \end{aligned}$$

$$\begin{aligned} \boxed{68} \quad \frac{-x^2 - x + 2}{2x^2 - x + 2} &= \frac{-x^2 - x + 2}{2x^2 - x + 2} + \frac{-x^2 - x + 2}{-x^2 - x + 2} = \frac{-x^2 - x + 2}{2x^2 - x + 2} \cdot \frac{-(-x^2 - x + 2)}{-(-x^2 - x + 2)} \\ &= \frac{-x^2 - x + 2}{2x^2 - x + 2} \cdot \frac{-1}{-1} = \frac{-2}{2x^2 - x + 2} = \frac{-1}{x^2 - \frac{1}{2}x + 1} \end{aligned}$$

$$\boxed{69} \quad \frac{3x^{2/3} - x + 7}{x^{2/3}} = \frac{3x^{2/3}}{x^{2/3}} - \frac{x}{x^{2/3}} + \frac{7}{x^{2/3}} = 3x^{4/3} - x^{1/3} + 7x^{-2/3}$$

$$\boxed{70} \quad \frac{-x^2 + 4x - 6}{x^3} = \frac{-x^2}{x^3} + \frac{4x}{x^3} - \frac{6}{x^3} = -x^{-1} + 4x^{-2} - 6x^{-3}$$

$$\boxed{71} \quad \frac{(x^2 + 2)^2}{x^5} = \frac{x^4 + 4x^2 + 4}{x^5} = \frac{x^4}{x^5} + \frac{4x^2}{x^5} + \frac{4}{x^5} = x^{-1} + 4x^{-3} + 4x^{-5}$$

$$\boxed{72} \quad \frac{x^3 - 3x^2 - 6x + 9}{x^3} = \frac{x^3}{x^3} - \frac{3x^2}{x^3} - \frac{6x}{x^3} + \frac{9}{x^3} = x^0 - 3x^{-1} - 6x^{-2} + 9x^{-3}$$

Note: You may wish to demonstrate the three techniques shown in Example 7 with one of these simpler expressions in 73–76.

Note: Exercises 73–90 are worked using the factoring concept given as the third method of simplification in Example 7.

73 The smallest exponent that appears on the variable x is -3 .

$$x^{-3} + x^{-2} \{ \text{factor out } x^{-3} \} = x^{-3} \cdot 1 + x^{-2-3} = x^{-3} \cdot 1 + x^{-5} = \frac{1 + x^{-5}}{x^3}$$

$$\boxed{74} \quad x^{-5} - x^{-6} \{ \text{factor out } x^{-5} \} = x^{-5} \cdot 1 - x^{-1-5} = x^{-5} (1 - x^{-6}) = \frac{1 - x^{-6}}{x^5}$$

$$\boxed{75} \quad x^{-1/2} - x^{3/2} \{ \text{factor out } x^{-1/2} \} = x^{-1/2} \cdot 1 - x^{3/2-1/2} = x^{-1/2} (1 - x^2) = \frac{1 - x^2}{x^{1/2}}$$

$$\boxed{76} \quad x^{-2/3} + x^{7/3} \{ \text{factor out } x^{-2/3} \} = x^{-2/3} \cdot 1 + x^{7/3-2/3} = x^{-2/3} (1 + x^3) = \frac{1 + x^3}{x^{2/3}}$$

$$\begin{aligned} \boxed{77} \quad &x^2 - 3x + 12(3x + 2)^3(3) + (3x + 2)^4(4x - 3) \\ &= (3x + 2)^3 [12x^2 - 3x + 12 + (3x + 2)(4x - 3)] \quad \text{factor out the gcf of } (3x + 2)^3 \\ &= (3x + 2)^3 [12x^2 - 3x + 12 + 12x^2 - 6x - 6] \\ &= (3x + 2)^3 [24x^2 - 9x + 6] \end{aligned}$$

$$\begin{aligned} \boxed{78} \quad &(6x - 5)^3(2x^2 + 4)(2x) + (2x^2 + 4)^2(3)(6x - 5)^2(6) = 2(6x - 5)^2(2x^2 + 4)(2x) + 4(2x^2 + 4)(6x - 5)^2(6) \\ &= 2(6x - 5)^2(2x^2 + 4)(2x) + 4(2x^2 + 4)(6x - 5)^2(6) \\ &= 2x^2 + 4(6x - 5)^2(21x^2 - 10) + 36 \end{aligned}$$

79 The smallest exponent that appears on the factor $(x^2 - 4)$ is $-\frac{1}{2}$ and the smallest exponent that appears on the factor $(2x + 1)$ is 2. Thus, we will factor out $(x^2 - 4)^{-1/2}(2x + 1)^2$.

$$(x^2 - 4)^{1/2}(3)(2x + 1)^2(2) + (2x + 1)^3 \cdot \frac{1}{2} (x^2 - 4)^{-1/2}(2x) = (x^2 - 4)^{-1/2}(2x + 1)^2 [6(x^2 - 4) + x(2x + 1)]$$

If you are unsure of this factoring, it is easy to visually check at this stage by merely multiplying the expression—that is, we mentally add the exponents on the factor $(x^2 - 4)$, $-\frac{1}{2}$ and 1, and we get $\frac{1}{2}$, which is the exponent we started with.

$$\text{Proceeding: } (x^2 - 4)^{-1/2}(2x + 1)^2 [6x^2 - 4 + x(2x + 1)] = (x^2 - 4)^{-1/2}(2x + 1)^2 [6x^2 - 24 + 2x^2 + x]$$

$$= \frac{(2x + 1)(8x^2 - 24)}{(x^2 - 4)^{1/2}}$$

$$\begin{aligned} \text{80} \quad (3 + 2)^{1/3} (2)(4 - 5)(4) + (4 - 5) \sqrt[3]{(3 + 2)^{-2/3}} (3) &= (3 + 2)^{-2/3} (4 - 5) [3(3 + 2) + (4 - 5)] \\ &= \frac{(4 - 5)(28 + 11)}{(3 + 2)^{2/3}} \end{aligned}$$

$$\begin{aligned} \text{81} \quad (3 + 1)^{6 \cdot \frac{1}{2}} (2 - 5)^{-1/2} (2) + (2 - 5)^{1/2} (6)(3 + 1)^5 (3) \\ &= (3 + 1)^5 (2 - 5)^{-1/2} [3(3 + 1) + 18(2 - 5)] \quad \text{factor out } (3 + 1)^5 (2 - 5)^{-1/2} \\ &= \frac{(3 + 1)^5 (3 + 1 + 36 - 90)}{(2 - 5)^{1/2}} = \frac{(3 + 1)^5 (39 - 89)}{(2 - 5)^{1/2}} \end{aligned}$$

$$\begin{aligned} \text{82} \quad (-2 + 9)^{4/3} (-1/3)(-6)^{-4/3} + (-6)^{-1/3} (4)(-2 + 9)^3 (2) \\ &= (-1/3)(-2 + 9)^3 (-6)^{-4/3} + (-6)^{-1/3} (4)(-2 + 9)^3 (2) \\ &= \frac{(-2 + 9)^3 (23 \cdot 2 + 144 - 9)}{3(-6)^{4/3}} \end{aligned}$$

$$\begin{aligned} \text{83} \quad \frac{(6 + 1)^3 (27 \cdot 2 + 2) - (9 \cdot 3 + 2)(3)(6 + 1)^2 (6)}{(6 + 1)^6} &= \frac{(6 + 1)^2 [(6 + 1)(27 \cdot 2 + 2) - 18(9 \cdot 3 + 2)]}{(6 + 1)^6} \\ &= \frac{(6 + 1)^2 (162 \cdot 3 + 27 \cdot 2 + 12 + 2 - 162 \cdot 3 - 36)}{(6 + 1)^6} \\ &= \frac{27 \cdot 2 - 24 + 2}{(6 + 1)^4} \end{aligned}$$

$$\text{84} \quad \frac{(-2 - 1)^4 (2) - (4)(-2 - 1)^3 (2)}{(-2 - 1)^8} = \frac{(2)(-2 - 1) [(2 - 1) - 4 \cdot 1]}{(-2 - 1)^8} = \frac{2(-3 - 1)}{(-2 - 1)^5} = \frac{-2(3 + 1)}{(-2 - 1)^5}$$

$$\begin{aligned} \text{85} \quad \frac{(-2 + 2)^3 (2) - (3)(-2 + 2)^2 (2)}{(-2 + 2)^6} &= \frac{(-2 + 2)^2 [2(-2 + 2) - 3(-2 + 2)]}{(-2 + 2)^6} \\ &= \frac{2(-2 + 2 - 3 \cdot 2)}{(-2 + 2)^4} = \frac{2(-2 - 6)}{(-2 + 2)^4} = \frac{4(-8)}{(-2 + 2)^4} \end{aligned}$$

$$\begin{aligned} \text{86} \quad \frac{(-2 - 5)^4 (3 \cdot 2) - (4)(-2 - 5)^3 (2)}{(-2 - 5)^8} &= \frac{(-2 - 5)^3 [(-2 - 5)(3 \cdot 2) - 4 \cdot 2]}{(-2 - 5)^8} \\ &= \frac{-2(3 \cdot 2 - 15 - 8 \cdot 2)}{(-2 - 5)^5} = \frac{-2(-5 \cdot 2 - 15)}{(-2 - 5)^5} = \frac{5 \cdot 2(-2 + 3)}{(-2 - 5)^5} \end{aligned}$$

$$\begin{aligned} \text{87} \quad \frac{(-2 + 4)^{1/3} (3) - (3) \sqrt[3]{(-2 + 4)^{-2/3}} (2)}{(-2 + 4)^{1/3 \cdot 2}} &= \frac{(-2 + 4)^{-2/3} [3(-2 + 4) - 2 \cdot 1]}{(-2 + 4)^{2/3}} = \frac{3 \cdot 2 + 12 - 2 \cdot 2}{(-2 + 4)^{4/3}} = \frac{-2 + 12}{(-2 + 4)^{4/3}} \end{aligned}$$

$$\text{88} \quad \frac{(1 - 2)^{1/2} (2) - (2) \sqrt[2]{(1 - 2)^{-1/2}} (-2)}{(1 - 2)^{1/2 \cdot 2}} = \frac{-(1 - 2)^{-1/2} [2(1 - 2) + 1 \cdot 2]}{(1 - 2)^1} = \frac{-(2 - 2 + 2)}{(1 - 2)^{3/2}} = \frac{-(2 - 2)}{(1 - 2)^{3/2}}$$

$$\text{89} \quad \frac{(4 \cdot 2 + 9)^{1/2} (2) - (2 + 3) \sqrt[2]{(4 \cdot 2 + 9)^{-1/2}} (8)}{2} = \frac{(4 \cdot 2 + 9)^{-1/2} [2(4 \cdot 2 + 9) - 4 \cdot (2 + 3)]}{1}$$

$$\frac{1}{(4^2 + 9)^{1/2}}$$

$$= \frac{8^2 + 18 - 8^2 - 12}{(4^2 + 9)^{3/2}} = \frac{18 - 12}{(4^2 + 9)^{3/2}} = \frac{6(3 - 2)}{(4^2 + 9)^{3/2}}$$

90
$$\frac{(3 + 2)^{1/2} \cdot \frac{1}{3} (2 + 3)^{-2/3} (2) - (2 + 3)^{1/3} \cdot \frac{1}{2} (3 + 2)^{-1/2} (3)}{(3 + 2)^{1/2 \cdot 2}}$$

$$= \frac{\frac{1}{3} \cdot \frac{1}{2} (3 + 2)^{-1/2} (2 + 3)^{-2/3} [4(3 + 2) - 9(2 + 3)]}{(3 + 2)^1}$$

$$= \frac{6}{(3 + 2)^{3/2} (2 + 3)^{2/3}} = \frac{6}{(3 + 2)^{3/2} (2 + 3)^{2/3}} = \frac{6 + 19}{6(3 + 2)^{3/2} (2 + 3)^{2/3}}$$

91 Table $Y_1 = \frac{113 \cdot 3 + 280 \cdot 2 - 150}{22 \cdot 3 + 77 \cdot 2 - 100 - 350}$ and $Y_2 = \frac{3}{2 + 7} + \frac{4 \cdot 2}{1 \cdot 1 \cdot 2 - 5}$.

-	Y ₁	Y ₂
1	-0.6923	-0.6923
2	-26.12	-26.12
3	8.0392	8.0392
4	5.8794	5.8794
5	5.3268	5.3268

The values for Y₁ and Y₂ agree. Therefore, the two expressions might be equal.

92 Table $Y_1 = \frac{20 \cdot 2 + 41 \cdot 3 + 31}{10 \cdot 3 + 10 \cdot 2}$ and $Y_2 = \frac{1}{-} + \frac{1}{- + 1} + \frac{3 \cdot 2}{-^2}$.

-	Y ₁	Y ₂
1	4.6	4.7
2	1.6083	1.6333
3	0.92778	0.93889
4	0.64375	0.65
5	0.49067	0.49467

The values for Y₁ and Y₂ do not agree. Therefore, the two expressions are not equal.

93 In the second figure, the dimensions of area I are (-) and (- - y). The area of I is (- - y)(-), and the area of II is (- - y)y. The area A = $\frac{z^2 - y^2}{z}$ {in the first figure}
 = (- - y)(-) + (- - y)y {in the second figure}
 = (- - y)(- + y). {in the third figure}

94 Volume of I is z²(- - y), volume of II is -y(- - y), and volume of III is y²(- - y).
 $Z = \frac{z^3 - y^3}{z} = z^2(- - y) + -y(- - y) + y^2(- - y) = (- - y)(z^2 + -y + y^2)$.

95 (a) For the 25-year-old female, use
 $C_f = 665 + 13.8\$ + 52 - 6.8y$ with \$ = 59, 2 = 163, and y = 25.
 $C_f = 665 + 13.8(59) + 5(163) - 6.8(25) = 1525.7$ calories

For the 55-year-old male, use
 $C_7 = 655 + 9.6\$ + 1.92 - 4.7y$ with \$ = 75, 2 = 178, and y = 55.
 $C_7 = 655 + 9.6(75) + 1.9(178) - 4.7(55) = 1454.7$ calories

(b) As people age they require fewer calories. The coefficients of \$ and 2 are positive because large people require more calories.

1.4 Exercises

$$\boxed{1} \quad 4. - 3 = -5. + 6 = 4. + 5. = 6 + 3 = 9. = 9 = . = 1$$

$$\boxed{2} \quad 5. - 4 = 2(-2) = 5. - 4 = 2. - 4 = 3. = 0 = . = 0$$

$$\boxed{3} \quad (3. - 2)^2 = (-5)(9. + 4) = 9. - 2 - 12. + 4 = 9. - 41. - 20 = 29. = -24 = . = -\frac{24}{29}$$

$$\boxed{4} \quad (-5)^2 + 3 = (-2)^2 = .^2 + 10. + 25 + 3 = .^2 - 4. + 4 = 14. = -24 = . = -\frac{12}{7}$$

$$\square \quad 3. + 1 \quad 2. + 5$$

$$\boxed{5} \quad \frac{3. + 1}{6. - 2} = \frac{2. + 5}{4. - 13} \cdot (6. - 2)(4. - 13) = (3. + 1)(4. - 13) = (2. + 5)(6. - 2) = 7$$

$$12. - 35. - 13 = 12. - 26. - 10 = -3 = 61. = . = -\frac{3}{61} \sim \text{note that } . \neq \frac{1}{3}, \frac{13}{4}$$

$$\boxed{6} \quad \frac{7. + 2}{14. - 3} = \frac{-8}{2. + 3} \cdot (14. - 3)(2. + 3) = (7. + 2)(2. + 3) = (-8)(14. - 3) =$$

$$14. - 25. + 6 = 14. - 115. + 24 = 140. = 18 = . = \frac{70}{9}$$

$$\square \quad 4 \quad 1 \quad 5. - 6$$

$$\boxed{7} \quad \frac{4. - 8}{. + 2} + \frac{. + 2}{. - 2} = \frac{5. - 6}{. - 4} \cdot (-2)(. - 2) = 4(. - 2) + 1(. + 2) = 5. - 6 =$$

$$4. - 8 + . + 2 = 5. - 6 = 5. - 6 = 5. - 6 \text{ or } 0 = 0f, \text{ indicating an identity. The solution is } \text{“} - e, 2f.$$

$$\boxed{8} \quad \frac{2}{2. + 5} + \frac{3}{2. - 5} = \frac{10. + 5}{4. - 25} \cdot (2. + 5)(2. - 5) = 2(2. - 5) + 3(2. + 5) = 10. + 5 =$$

$$4. - 10 + 6. + 15 = 10. + 5 = 10. + 5 = 10. + 5 \text{ or } 0 = 0f, \text{ indicating an identity.}$$

$$\text{The solution is } \text{“} \sim \frac{5}{2}.$$

$$\boxed{9} \quad \frac{5}{2. + 3} + \frac{4}{2. - 3} = \frac{14. + 3}{4. - 9} \cdot (2. + 3)(2. - 3) = 5(2. - 3) + 4(2. + 3) = 14. + 3 =$$

$$10. - 15 + 8. + 12 = 14. + 3 = 18. - 3 = 14. + 3 = 4. = 6 =$$

$$. = \frac{3}{2}, \text{ which is not in the domain of the given expressions. } \text{No solution}$$

$$\square \quad -3 \quad 7 \quad -5. + 4$$

$$\boxed{10} \quad \frac{-3}{. + 4} + \frac{7}{. - 4} = \frac{-5. + 4}{. - 16} \cdot (-4)(. - 4) = -3(. - 4) + 7(. + 4) = -5. + 4 =$$

$$-3. + 12 + 7. + 28 = -5. + 4 = 4. + 40 = -5. + 4 = 9. = -36 =$$

$$. = -4, \text{ which is not in the domain of the given expressions. } \text{No solution}$$

11 Divide both sides by a nonzero constant whenever possible. In this case, 5 divides evenly into both sides.

$$75. - 35. - 10 = 0 \{\text{divide by 5}\} = 15. - 7. - 2 = 0 \{\text{factor}\} =$$

$$(3. + 2)(5. - 1) = 0 \{\text{zero factor theorem}\} = . = -\frac{2}{3}, \frac{1}{5}$$

$$\boxed{12} \quad 48. - 12. - 90 = 0 \{\text{divide by 6}\} = 8. - 2. - 15 = 0 = (2. + 3)(4. - 5) = 0 = . = -\frac{3}{2}, \frac{5}{4}$$

13 Here, the lcd is $x(x + 3)$ and we need to remember that $x \neq 0, -3$.

$$\frac{x}{x+3} + \frac{1}{x} - 4 = \frac{9}{x^2+3x} \cdot (x(x+3)) = x(x) + 1(x+3) - 4(x^2+3x) = 9 =$$

$$x^2 + x + 3 - 4x^2 - 12x = 9 = 0 = 3x^2 + 11x + 6 = (3x+2)(x+3) = 0 =$$

$x = -\frac{2}{3}$ or $x = -3$ is not in the domain of the given expressions

$$14 \quad 3 \cdot \frac{3}{x-2} + \frac{1}{x+2} = \frac{-4}{x^2-4} \Rightarrow (x+2)(x-2) = 3 \cdot (x+2) + 1(x-2) = -4 =$$

$$3x^2 + 6x + x - 2 = -4 \Rightarrow 3x^2 + 7x + 2 = 0 \Rightarrow (3x+1)(x+2) = 0 =$$

$x = -\frac{1}{3}$ or $x = -2$ is not in the domain of the given expressions

$$15 \quad 25x^2 = 9 \Rightarrow x^2 = \frac{9}{25} \Rightarrow x = \pm \sqrt{\frac{9}{25}} = \pm \frac{3}{5}$$

$$16 \quad 64x^2 = 49 \Rightarrow x^2 = \frac{49}{64} \Rightarrow x = \pm \sqrt{\frac{49}{64}} = \pm \frac{7}{8}$$

$$17 \quad (x-3)^2 = 17 \Rightarrow x-3 = \pm \sqrt{17} \Rightarrow x = 3 \pm \sqrt{17}$$

$$18 \quad (x+5)^2 = 29 \Rightarrow x+5 = \pm \sqrt{29} \Rightarrow x = -5 \pm \sqrt{29}$$

$$19 \quad x^2 + 6x + 3 = 0 \Rightarrow x = \frac{-6 \pm \sqrt{36-12}}{2(1)} = \frac{-6 \pm \sqrt{24}}{2} = \frac{-6 \pm 2\sqrt{6}}{2} = -3 \pm \sqrt{6}$$

$$20 \quad x^2 - 4x - 2 = 0 \Rightarrow x = \frac{4 \pm \sqrt{16+8}}{2(1)} = \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6}$$

21 The expression is $x^2 + x - 30$. The associated quadratic equation is $x^2 + x - 30 = 0$.

Using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to solve for x with $a = 1$, $b = 1$, and $c = -30$ gives us:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-30)}}{2(1)} = \frac{-1 \pm \sqrt{1+120}}{2} = \frac{-1 \pm \sqrt{121}}{2} = \frac{-1 \pm 11}{2} = \frac{10}{2}, \frac{-12}{2} = 5, -6$$

Write the equation as a product of linear factors: $(x-5)(x+6) = 0$

Now simplify: $(x-5)(x+6) = 0$

So the final factored form of $x^2 + x - 30$ is $(x-5)(x+6)$.

$$22 \quad x^2 - 11x = 0 \Rightarrow a = 1, b = -11, c = 0, \text{ so } x = \frac{11 \pm \sqrt{121-0}}{2} = \frac{11 \pm 11}{2} = 11, 0.$$

Thus, $x^2 - 11x = (x-11)(x-0) = x(x-11)$.

$$23 \quad 12x^2 - 16x - 3 = 0 \Rightarrow a = 12, b = -16, c = -3, \text{ so } x = \frac{16 \pm \sqrt{256+144}}{24} = \frac{16 \pm 20}{24} = \frac{3}{2}, -\frac{1}{6}.$$

Write the equation as a product of linear factors: $(2x-3)(6x+1) = 0$

Now multiply the first factor by 2 and the second factor by 6. $(2x-3)(6x+1) = 0$

So the final factored form of $12x^2 - 16x - 3$ is $(2x-3)(6x+1)$.

$$24 \quad 15x^2 + 34x - 16 = 0 \Rightarrow a = 15, b = 34, c = -16, \text{ so } x = \frac{-34 \pm \sqrt{1156+960}}{30} = \frac{-34 \pm 46}{30} = \frac{2}{5}, -\frac{8}{3}.$$

Thus, $15x^2 + 34x - 16 = 5(x-\frac{2}{5})(3x+8)$.

25 We must first isolate the absolute value term before proceeding.

$$|3x-2| + 3 = 7 \Rightarrow |3x-2| = 4 \Rightarrow 3x-2 = 4 \text{ or } 3x-2 = -4 \Rightarrow$$

$$3x = 6 \text{ or } 3x = -2 \Rightarrow x = 2 \text{ or } x = -\frac{2}{3}$$

$$26 \quad 2|5x+2| - 1 = 5 \Rightarrow 2|5x+2| = 6 \Rightarrow |5x+2| = 3 \Rightarrow$$

$$5x+2 = 3 \text{ or } 5x+2 = -3 \Rightarrow 5x = 1 \text{ or } 5x = -5 \Rightarrow x = \frac{1}{5} \text{ or } x = -1$$

$$27 \quad 3|1x+1| - 5 = -11 \Rightarrow 3|1x+1| = -6 \Rightarrow |1x+1| = -2.$$

Since the absolute value of an expression is nonnegative, $|x + 1| = -2$ has no solution.

28 I. $-3I + 6 = 6 \Rightarrow -3I = 0$. Since the absolute value of an expression can only equal 0 if the expression itself is 0, $-3I = 0 \Rightarrow -3 = 0 \Rightarrow = 3$.

29 Since there are four terms, we first try factoring by grouping.

$$9x^3 - 18x^2 - 4x + 8 = 0 \Rightarrow 9x^2(x - 2) - 4(x - 2) = 0 \Rightarrow (9x^2 - 4)(x - 2) = 0 \Rightarrow x^2 = \frac{4}{9} \text{ or } x = 2 \Rightarrow x = \frac{2}{3}, 2$$

30 Notice that we can factor an x out of each term, and then factor by grouping.

$$4x^4 + 10x^3 = 6x^2 + 15 \Rightarrow 4x^4 + 10x^3 - 6x^2 - 15 = 0 \Rightarrow x(4x^3 + 10x^2 - 6x - 15) = 0 \Rightarrow x(2x^2(2x + 5) - 3(2x + 5)) = 0 \Rightarrow x(2x^2 - 3)(2x + 5) = 0 \Rightarrow x = 0 \text{ or } x^2 = \frac{3}{2} \text{ or } x = -\frac{5}{2} \Rightarrow x = 0, \frac{1}{2}\sqrt{6}, -\frac{5}{2}$$

Note: $x^2 = \frac{3}{2} \Rightarrow x = \pm\sqrt{\frac{3}{2}} = \pm\sqrt{\frac{-3}{-2}} = \pm\sqrt{\frac{-6}{-2}} = \pm\sqrt{-3} = \pm i\sqrt{3}$

There are several ways to write this answer—your professor may have a preference.

31 $y^{3/2} = 5y \Rightarrow y^{3/2} - 5y = 0 \Rightarrow y^{1/2}(y - 5) = 0 \Rightarrow y = 0 \text{ or } y^{1/2} = 5 \Rightarrow y^{1/2} = 5 \Rightarrow (y^{1/2})^2 = 5^2 \Rightarrow y = 25$. The solutions are $y = 0$ and $y = 25$.

Note: The following guidelines may be helpful when solving radical equations.

Guidelines for Solving a Radical Equation

- Isolate the radical. If we cannot get the radical isolated on one side of the equals sign because there is more than one radical, then we will split up the radical terms as evenly as possible on each side of the equals sign. For example, if there are two radicals, we put one on each side; if there are three radicals, we put two on one side and one on the other.
- Raise both sides to the same power as the root index. **Note:** Remember here that

$$\sqrt{a + b - 8}^2 = a^2 + 2ab - 8 + b^2 8$$

and that $\sqrt{a + b - 8}^2$ is **not** $a^2 + b^2 8$.

- If your equation contains no radicals, proceed to part (4). If there are still radicals in the equation, go back to part (1).
- Solve the resulting equation.
- Check the answers found in part (4) in the original equation to determine the valid solutions.

Note: You may check the solutions in any equivalent equation of the original equation, that is, an equation which occurs prior to raising both sides to a power. Also, extraneous real number solutions are introduced when raising both sides to an even power. Hence, all solutions *must* be checked in this case. Checking solutions when raising each side to an odd power is up to the individual professor.

32 $y^{4/3} = -4y \Rightarrow y^{4/3} + 4y = 0 \Rightarrow y^{1/3}(y + 4) = 0 \Rightarrow y = 0 \text{ or } y^{1/3} = -4 \Rightarrow y^{1/3} = -4 \Rightarrow (y^{1/3})^3 = (-4)^3 \Rightarrow y = -64$. $y = 0, -64$

33 $\sqrt{-7 - 5x} = 8 \Rightarrow -7 - 5x = 8^2 \Rightarrow -7 - 5x = 64 \Rightarrow -57 = 5x \Rightarrow x = -\frac{57}{5}$

34 $\sqrt{-3 - x} = 3 \Rightarrow -3 - x = 3^2 \Rightarrow -3 - x = 9 \Rightarrow -x = 12 \Rightarrow x = -12$

$(x + 1)(x + 6) = 0 \implies x = -1$ and -6 is an extraneous solution.

35 $x = 3 + \sqrt{-5x - 9} \Rightarrow x - 3 = \sqrt{-5x - 9} \Rightarrow x^2 - 6x + 9 = 5x - 9 \Rightarrow x^2 - 11x + 18 = 0 \Rightarrow (x - 2)(x - 9) = 0 \Rightarrow x = 9$ and 2 is an extraneous solution.

36 $x + \sqrt{-5x + 19} = -1 \Rightarrow \sqrt{-5x + 19} = -x - 1 \Rightarrow 5x + 19 = x^2 + 2x + 1 \Rightarrow x^2 - 3x - 18 = 0 \Rightarrow (x - 6)(x + 3) = 0 \Rightarrow x = -3, 6$.

Check $x = -3$: LS = $-3 + 2 = -1 =$ RS $\Rightarrow x = -3$ is a solution.

Check $x = 6$: LS = $6 + 7 = 13 \neq$ RS $\Rightarrow x = 6$ is an extraneous solution.

Note: Substitution could be used instead of factoring for the following exercises.

37 We recognize this equation as a quadratic equation in y^2 and apply the quadratic formula, solving for y^2 , not y .

$$5y^4 - 7y^2 + 1\dot{A}5 = 0 \Rightarrow y^2 = \frac{7 \pm \sqrt{-19}}{10} + \frac{10}{10} = \frac{70 \pm \sqrt{10 - 19}}{100} \Rightarrow y = \pm \sqrt{\frac{1}{10} \sqrt{70 \pm \sqrt{10 - 19}}}$$

Alternatively, let $u = y^2$ and solve $5u^2 - 7u + 1\dot{A}5 = 0$.

38 $3y^4 - 5y^2 + 1\dot{A}5 = 0 \Rightarrow y^2 = \frac{5 \pm \sqrt{6}}{6} + \frac{6}{6} = \frac{30 \pm \sqrt{6 - 7}}{36} \Rightarrow y = \pm \sqrt{\frac{1}{6} \sqrt{30 \pm \sqrt{6 - 7}}}$

39 $36x^{-4} - 13x^{-2} + 1 = 0 \Rightarrow (4x^{-2} - 1)(9x^{-2} - 1) = 0 \Rightarrow x^{-2} = \frac{1}{4}, \frac{1}{9} \Rightarrow x^2 = 4, 9 \Rightarrow x = \pm 2, \pm 3$

Alternatively, let $u = x^{-2}$ and solve $36u^2 - 13u + 1 = 0$.

40 $x^{-2} - 2x^{-1} - 35 = 0 \Rightarrow (x^{-1} - 7)(x^{-1} + 5) = 0 \Rightarrow x^{-1} = 7, -5 \Rightarrow x = \frac{1}{7}, -\frac{1}{5}$

41 $3x^{2/3} + 4x^{1/3} - 4 = 0 \Rightarrow \sqrt[3]{3x^{1/3} - 2} + \sqrt[3]{2x^{1/3} + 2} = 0 \Rightarrow \sqrt[3]{x} = \frac{2}{3}, -2 \Rightarrow x = \sqrt[3]{\frac{2}{3}}, (-2)^3 \Rightarrow x = \frac{8}{27}, -8$

Alternatively, let $u = x^{1/3}$ and solve $3u^2 + 4u - 4 = 0$.

42 $2y^{1/3} - 3y^{1/6} + 1 = 0 \Rightarrow \sqrt[6]{2y^{1/6} - 1} + \sqrt[6]{y^{1/6} - 1} = 0 \Rightarrow \sqrt[6]{y} = \frac{1}{2}, 1 \Rightarrow y = \frac{1}{64}, 1$

43 See the illustrations and discussion on text page 51 for help on solving equations by raising both sides to a reciprocal power. Note that if $x^{7/8}$ is in the given equation and 7 is even, we have to use the \pm symbol for the solutions. Here are a few more examples:

Equation	Solution ²⁵
$x^{1/2} = 4 \Rightarrow \sqrt[2]{x^{2/1}} = 4^{2/1} \Rightarrow x = 16$	

$x^{-1/2} = 5 \Rightarrow \sqrt[2]{x^{-2/1}} = 5^{-2/1} \Rightarrow x = \frac{1}{25}$	
---	--

$x^{3/4} = 8 \Rightarrow \sqrt[4]{x^{3/4}} = 8^{4/3} \Rightarrow x = 16$	
--	--

$x^{4/3} = 81 \Rightarrow \sqrt[3]{x^{4/3}} = 81^{3/4} \Rightarrow x = \pm 27 e \pm$ since 4 is even	
--	--

(a) $x^{5/3} = 32 \Rightarrow \sqrt[3]{x^{5/3}} = (32)^{3/5} \Rightarrow x = \sqrt[5]{32^3} = 2^3 = 8$

(b) $x^{4/3} = 16 \Rightarrow$

$$x^{4/3} = 16 \implies x = 16^{3/4} = 2^3 = 8$$

(c) $x^{2/3} = -64 \implies x = (-64)^{3/2} = -64^3$, which are not real numbers.

No real solutions

(d) $x^{3/4} = 125 \implies x = 125^{4/3} = 5^4 = 625$

(e) $x^{3/2} = -27 \implies x = (-27)^{2/3} = 9$,

which is an extraneous solution. **No real solutions**

$$[44] \text{ (a) } x^{3/5} = -27 \Rightarrow x^{3/5 \cdot 5/3} = (-27)^{5/3} \Rightarrow x = \sqrt[3]{-27 \cdot 5} = (-3)^5 = -243$$

$$\text{(b) } x^{2/3} = 25 \Rightarrow x^{2/3 \cdot 3/2} = (25)^{3/2} \Rightarrow x = \sqrt{-25 \cdot 3} = \sqrt{5^3} = \sqrt{125}$$

$$\text{(c) } x^{4/3} = -49 \Rightarrow x^{4/3 \cdot 3/4} = (-49)^{3/4} \Rightarrow x = \sqrt[4]{-49 \cdot 3}, \text{ which are not real numbers.}$$

No real solutions

$$\text{(d) } x^{3/2} = 64 \Rightarrow x^{3/2 \cdot 2/3} = (64)^{2/3} \Rightarrow x = \sqrt[3]{64 \cdot 2} = 4^2 = 16$$

$$\text{(e) } x^{3/4} = -8 \Rightarrow x^{3/4 \cdot 4/3} = (-8)^{4/3} \Rightarrow x = \sqrt[3]{-8 \cdot 4} = (-2)^4 = 16, \text{ which is an extraneous solution.}$$

No real solutions

[45] (a) For this exercise, we must recognize the equation as a quadratic in x , that is,

$$Ax^2 + Bx + C = 0$$

where A is the coefficient of x^2 , B is the coefficient of x , and C is the collection of all terms that do not contain x^2 or x .

$$4x^2 - 4xy + 1 - y^2 = 0 \Rightarrow (4)x^2 + (-4y)x + (1 - y^2) = 0 \Rightarrow$$

$$x = \frac{4y \pm \sqrt{16y^2 - 16(1 - y^2)}}{2(4)} = \frac{4y \pm \sqrt{16y^2 - 16 + 16y^2}}{2(4)} = \frac{4y \pm \sqrt{4 - 2y^2 - 1}}{2(4)} = \frac{y \pm \sqrt{-2y^2 - 1}}{2}$$

(b) Similar to part (a), we must now recognize the equation as a quadratic equation in y .

$$4x^2 - 4xy + 1 - y^2 = 0 \Rightarrow (-1)y^2 + (-4x)y + (4x^2 + 1) = 0 \Rightarrow$$

$$y = \frac{4x \pm \sqrt{16x^2 + 4(4x^2 + 1)}}{2(-1)} = \frac{4x \pm \sqrt{16x^2 + 16x^2 + 4}}{-2} = \frac{4x \pm \sqrt{2 - 8x^2 + 1}}{-2} = -2 \pm \sqrt{8x^2 + 1}$$

$$[46] \text{ (a) } 2x^2 - xy = 3y^2 + 1 \Rightarrow (2)x^2 + (-y)x + (-3y^2 - 1) = 0 \Rightarrow$$

$$x = \frac{y \pm \sqrt{y^2 - 8(-3y^2 - 1)}}{2(2)} = \frac{y \pm \sqrt{25y^2 + 8}}{4}$$

$$\text{(b) } 2x^2 - xy = 3y^2 + 1 \Rightarrow (-3)y^2 + (-x)y + (2x^2 - 1) = 0 \Rightarrow$$

$$y = \frac{-x \pm \sqrt{x^2 + 12(2x^2 - 1)}}{2(-3)} = \frac{-x \pm \sqrt{25x^2 - 12}}{-6}$$

$$\square$$

$$[47] \text{ (a) } x = \frac{-4,500,000 \pm \sqrt{4,500,000^2 - 4(1)(-0.96)}}{2} \Rightarrow 0 \text{ and } -4,500,000$$

$$\text{(b) } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \pm \frac{-b \pm \sqrt{b^2 - 4ac}}{-b \pm \sqrt{b^2 - 4ac}} = \frac{b^2 - (b^2 - 4ac)}{2a \pm (-b \pm \sqrt{b^2 - 4ac})}$$

$$= \frac{4ac}{2a \pm (-b \pm \sqrt{b^2 - 4ac})} = \frac{2c}{-b \pm \sqrt{b^2 - 4ac}}$$

The root near zero was obtained in part (a) using the plus sign. In the second formula, it corresponds to the

$$\text{minus sign. } x = \frac{2(-0.96)}{-4,500,000 - \sqrt{4,500,000^2 - 4(1)(-0.96)}} \approx 2.13 \cdot 10^7$$

□

$$48 \text{ (a) } x = \frac{73,000,000 \pm \sqrt{(-73,000,000)^2 - 4(1)(2 \times 10^8)}}{2} \rightarrow 73,000,000 \text{ and } 0$$

(b) The root near zero was obtained in part (a) using the minus sign. In the second formula, it corresponds to the plus sign. $x = \frac{73,000,000 + \sqrt{(-73,000,000)^2 - 4(1)(2 \times 10^8)}}{2} \rightarrow 2 \times 10^8$

49) $I O + P = H - X O$ given equation, solve for O
 $I O + X O = H - P$ get O -terms on one side, everything else on the other
 $O(I + X) = H - P$ factor out O
 $O = \frac{H - P}{I + X}$ divide by $(I + X)$ to solve for O

50) $CH + C = 7C + 9 \Rightarrow CH + C - 7C = 9 \Rightarrow C(H + 1 - 7) = 9 \Rightarrow C = \frac{9}{H + 1 - 7}$

51) $R = \frac{8+1}{8} \Rightarrow R \cdot 8 = 8 + 1 \Rightarrow R \cdot 8 - 8 = 1 \Rightarrow 8(R - 1) = 1 \Rightarrow R - 1 = \frac{1}{8} \Rightarrow R = \frac{1}{8} + 1$

52) $\alpha = \frac{\alpha}{1 - \alpha} \Rightarrow \alpha(1 - \alpha) = \alpha \Rightarrow \alpha - \alpha^2 = \alpha \Rightarrow -\alpha^2 = \alpha \Rightarrow \alpha^2 + \alpha = 0 \Rightarrow \alpha(\alpha + 1) = 0 \Rightarrow \alpha = 0, -1$

53) $A = 7 + 7r! \Rightarrow A - 7 = 7r! \Rightarrow r = \frac{A-7}{7!}$

54) $\frac{1}{2} 1!^2 + \#! = 2 \Rightarrow 1!^2 + \#! = 2 \Rightarrow 1 + \#! = 2 \Rightarrow \#! = 1 \Rightarrow \# = 1$

55) $S = \frac{p}{q + p, 1 - q}$ given equation, solve for q
 $S(q + p, 1 - q) = p$ eliminate the fraction
 $Sq + Sp, 1 - q = p$ multiply terms
 $Sq + Sp - Spq = p$ multiply terms
 $Sq - Spq = p - Sp$ isolate terms containing q
 $Sq(1 - p) = p(1 - S)$ factor out Sq
 $q = \frac{p(1 - S)}{S(1 - p)}$ divide by $S(1 - p)$ to solve for q

56) $S = 2,6\$ + 2\$ + 26) \Rightarrow S = 26\$ + 22\$ + 226 \Rightarrow S - 26\$ = 22,(\$ + 6) \Rightarrow 2 = \frac{S - 26\$}{2,(\$ + 6)}$

57) $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$ {multiply by the lcd, $f p q$ }
 $p q = f q + f p \Rightarrow p q - f q = f p \Rightarrow q(p - f) = f p \Rightarrow q = \frac{f p}{p - f}$

58) $\frac{1}{9} = \frac{1}{9_1} + \frac{1}{9_2} + \frac{1}{9_3}$ {multiply by the lcd, $9_1 9_2 9_3$ }
 $9_1 9_2 9_3 - 9_2 9_3 - 9_1 9_3 = 9_1 9_3 \Rightarrow 9_2(9_1 9_3 - 9_3 - 9_1) = 9_1 9_3 \Rightarrow 9_2 = \frac{9_1 9_3}{9_1 9_3 - 9_3 - 9_1}$

59) $O = \frac{1}{7} \#^2 \Rightarrow \#^2 = \frac{20}{7} \Rightarrow \# = \sqrt{\frac{20}{7}} \Rightarrow \# = \frac{\sqrt{20}}{\sqrt{7}}$ since $\# > 0$.

60) $J = \frac{17Q}{17Q} = d^2 \Rightarrow d = \sqrt{\frac{17Q}{17Q}} = d = \frac{\sqrt{17Q}}{\sqrt{17Q}}$ since $d > 0$.

61) $A = 21r, (r + 2) \Rightarrow A = 21r^2 + 21r \Rightarrow (21)r^2 + (212)r - A = 0$ ea quadratic equation in r

$$r = \frac{-(-212) \pm \sqrt{(-212)^2 - 4(21)(-A)}}{2(21)} = \frac{-212 \pm \sqrt{41^2 2^2 + 81A}}{2(21)}$$

$$= \frac{-212 \pm \sqrt{2^2 \cdot 1^2 2^2 + 21A}}{2(21)} = \frac{-12 \pm \sqrt{1^2 2^2 + 21A}}{21}$$

Since $r > 0$, we must use the plus sign, and $r = \frac{-12 + \sqrt{1^2 2^2 + 21A}}{21}$.

$$\boxed{62} \quad \frac{1}{2} - 1!^2 + \# \quad \frac{1}{2} - 1!^2 + \# \quad - = 0 = \# = \frac{-\#_0 \pm \sqrt{\#_0^2 + 21}}{1}$$

Since $\# > 0$, we must use the plus sign, and $\# = \frac{-\#_0 + \sqrt{\#_0^2 + 21}}{1}$.

$$\boxed{63} \quad d = \frac{1}{2} - 49^2 - C^2 = 2d = -49^2 - C^2 = 4d^2 = 49^2 - C^2 = C^2 = 49^2 - 4d^2 =$$

$$C^2 = 4,9^2 - d^2 = C = \sqrt{4,9^2 - d^2} = C = 2 - 9^2 - d^2 \text{ since } C > 0$$

$$\boxed{64} \quad S = 1r - r^2 + 2^2 = \frac{S}{1r} = -r^2 + 2^2 = \frac{S}{1r} = \frac{S^2}{1^2 r^2} = r^2 + 2^2 =$$

$$\frac{S^2}{1^2 r^2} - r^2 = 2^2 = \frac{S^2}{1^2 r^2} - \frac{1^2 r^4}{1^2 r^2} = 2^2 = \frac{1}{1^2 r^2} (S^2 - 1^2 r^4) = 2 = \frac{1}{1r} (S^2 - 1^2 r^4) =$$

$$2 = \frac{1}{1r} (S^2 - 1^2 r^4) \text{ since } 2 > 0$$

65 Let $_$ denote the number of months needed to recover the cost of the insulation. The savings in one month is 10% of \$200 = \$20, so the savings in $_$ months is $20_\$. $20_\ = 2400 = _ = 120$ months (or 10 yr).

66 Let $_$ denote the amount (in millions) invested in bonds.
 $_ (0.06) + 800 - _ (0.05) = 42 = 0.06_\ + 40 - 0.05_\ = 42 = 0.01_\ = 2 = _ = 200$.

The arena should be financed by selling \$200 million in bonds and borrowing \$600 million.

67 (a) They will meet when the sum of their distances is 224. Let $\!$ denote the desired number of seconds.
 Using distance = rate \times time, we have $1\!5\! + 2\! = 224 = 3\!5\! = 224 = \! = 64$ sec.

(b) The children will have walked $64(1\!5) = 96$ m and $64(2) = 128$ m, respectively.

68 Let $\$$ denote the length of the side parallel to the river bank. $7 = 2\$ + \$$

(a) $\$ = 2\$ = 7 = 2\$ + 2\$ = 4\$$. $4\$ = 180 = \$ = 45$ ft and $A = (45,90) = 4050$ ft².

(b) $\$ = \frac{1}{2}\$ = 7 = 2\$ + \frac{1}{2}\$ = \frac{5}{2}\$$. $\frac{5}{2}\$ = 180 = \$ = 72$ ft and $A = (72,36) = 2592$ ft².

(c) $\$ = \$ = 7 = 2\$ + \$ = 3\$$. $3\$ = 180 = \$ = 60$ ft and $A = (60,60) = 3600$ ft².

69 Let $_$ denote the distance to the target. We know the total time involved and need a formula for time. Solving $d = r!$ for $\!$ gives us $\! = d/r$.

$$\text{Time}_{\text{to target}} + \text{Time}_{\text{from target}} = \text{Time}_{\text{total}} = \frac{_}{3300} + \frac{_}{1100} = 1\!5 \text{ (multiply by the lcd, 3300)} =$$

$$_ + 3_\ = 1\!5(3300) = 4_\ = 4950 = _ = 1237\!5 \text{ ft.}$$

70 Let $_$ denote the miles in one direction. A 6-minute-mile pace is equivalent to a rate of $\frac{1}{6}$ mile/min. Solving $d = r!$ for $\!$ gives us $\! = d/r$. Minutes_{north} + Minutes_{south} = Minutes_{total} = $\frac{_}{1/6} + \frac{_}{1/7} = 47 =$

$$6_\ + 7_\ = 47 = _ = \frac{47}{13}$$
. The total distance is $2 + \frac{47}{13} = \frac{94}{13}$ or $7\frac{3}{13}$ mi.

71 Let b_2 denote the larger base. $A = \frac{1}{2}(b_1 + b_2)2 = 5 = \frac{1}{2}(3 + b_2)2 = 10 = 3 + b_2 = b_2 = 7$ ft.

72 Let h_1 denote the height of the cylinder. $Z = \frac{2}{3}r^3 + r^2h_1 = 11,250$ and $r = 15 =$

$2250 + 225h_1 = 11,250 = 225h_1 = 9000 = h_1 = 40$. The total height is 40 ft + 15 ft = 55 ft.

73 (a) $X = X_0 - \frac{5.5}{1000}z$ if $z = 5280$ ft and $X_0 = 70^\circ\text{F} = X = 70 - \frac{5.5}{1000}5280 = 40.96^\circ\text{F}$.

(b) $X = 32^\circ\text{F} = 32 = 70 - \frac{5.5}{1000}z = \frac{5.5}{1000}z = 38 = z = 38 \frac{1000}{5.5} = 6909$ ft.

74 (a) $2 = 227(X - H) \Rightarrow X = 70^\circ\text{F}$ and $H = 55^\circ\text{F} \Rightarrow 2 = 227(70 - 55) = 3405 \text{ ft.}$

(b) $2 = 3500 \text{ ft}$ and $H = 65^\circ\text{F} \Rightarrow 3500 = 227(X - 65) \Rightarrow X = \frac{3500}{227} + 65 \approx 80.4^\circ\text{F}$

75 $X = B - \frac{3}{1000} \cdot 2 \Rightarrow B = 55^\circ\text{F}$ and $2 = 10,000 - 4000 = 6000 \text{ ft} \Rightarrow X = 55 - \frac{3}{1000} \cdot 6000 = 37^\circ\text{F}.$

76 (a) $2 = 65 + 3 \cdot 14. \Rightarrow \cdot = 30 \Rightarrow 2 = 65 + 3 \cdot 14,30 = 159.2 \text{ cm.}$
cm

(b) $2 = 73.6 + 3 \cdot 0. \Rightarrow \cdot = 34 \Rightarrow 2 = 73.6 + 3 \cdot 34 = 175.6 \text{ cm.}$ The height of the skeleton has decreased by $175.6 - 174 = 1.6 \text{ cm}$ due to aging after age 30. $\frac{1.6}{0.06} \approx 27$ years. The male was approximately 1.6

$30 + 27 = 57$ years old at death.

77 (a) $\# = 55 \Rightarrow d = \# + \frac{\#^2}{20} = 55 + \frac{55^2}{20} = 206.25 \text{ ft}$

(b) $d = 120 \Rightarrow 120 = \# + \frac{\#^2}{20} \Rightarrow 2400 = 20\# + \#^2 \Rightarrow \#^2 + 20\# - 2400 = 0 \Rightarrow \# + 60, \# - 40 = 0 \Rightarrow \# = 40 \text{ mi/hr}$

78 (a) $X = 98 \Rightarrow 2 = 1000, 100 - X + 580, 100 - X)^2 = 1000, 2 + 580, 2)^2 = 4320 \text{ m.}$

(b) If $\cdot = 100 - X$ and $2 = 8840$, then $8840 = 1000. + 580. \cdot^2 \Rightarrow 29. \cdot^2 + 50. \cdot - 442 = 0 \Rightarrow \cdot = \frac{-50 \pm \sqrt{2500 + 51,272}}{2 \cdot 29} = \frac{-25 \pm \sqrt{13,443}}{29} \Rightarrow -4.86, 3.14.$
 $\cdot = -4.86 \Rightarrow X = 100 - \cdot = 104.86^\circ\text{C}$, which is outside the allowable range of X .
 $\cdot = 3.14 \Rightarrow X = 100 - \cdot = 96.86^\circ\text{C}$ for $95 < X < 100$.

79 (a) The northbound plane travels $\frac{1}{2} \cdot 200 = 100$ miles from 2 P.M. to 2:30 P.M., so the distances of the northbound and eastbound planes are $100 + 200!$ and $400!$, respectively. Using the Pythagorean theorem,

$$d = \sqrt{(100 + 200!)^2 + (400!)^2} = \sqrt{100^2(1 + 2!)^2 + 100^2(4!)^2} = \sqrt{100^2(1 + 2!)^2 + (4!)^2}$$

$$= 100 \sqrt{1 + 4! + 16!} = 100 \sqrt{20! + 4! + 1}.$$

(b) $d = 500 \Rightarrow 500 = 100 \sqrt{20! + 4! + 1} \Rightarrow 5 = \sqrt{20! + 4! + 1} \Rightarrow 5^2 = 20! + 4! + 1 \Rightarrow 20! + 4! - 24 = 0 \Rightarrow 5!^2 + ! - 6 = 0 \Rightarrow (5! + 6), ! - 1 = 0 \Rightarrow ! = 1$ hour after 2:30 P.M., or 3:30 P.M.

80 Let $!$ denote the number of seconds the rock falls, so that $4 - !$ is the number of seconds for the sound to travel.

Distance_{down} = Distance_{up} $\Rightarrow 16!^2 = 1100, 4 - ! \Rightarrow \{d = r!\} \Rightarrow 4!^2 = 275, 4 - ! \Rightarrow$

$$4!^2 + 275! - 1100 = 0 \Rightarrow ! = \frac{-275 \pm \sqrt{93,225}}{2 \cdot 4} = \frac{-275 + 5 \cdot 3729}{8} \approx 3.79.$$

The height is $16!^2 \approx 229.94$, or 230 ft.

81 Let \cdot denote the number of \$10 reductions in price.

Revenue = (unit price) \cdot (# of units) $\Rightarrow 7000 = (300 - 10 \cdot), 15 + 2 \cdot) \Rightarrow$

$7000 = 10(30 - \cdot)(15 + 2 \cdot) \Rightarrow 700 = -2. \cdot^2 + 45. \cdot + 450 \Rightarrow 2. \cdot^2 - 45. \cdot + 250 = 0 \Rightarrow$

$$.2 - 25, - - 10) = 0 = . = 10 \text{ or } 12\text{A}5.$$

The selling price is $\$300 - \$10,10) = \$200$, or $\$300 - \$10,12\text{A}5) = \$175$.

82 The total surface area is the sum of the surface area of the cylinder and that of the top and bottom.

$$S = 2\pi r^2 + 2\pi r^2 \text{ and } 2 = 4 = 101 = 81r + 21r^2 \{ \text{divide by } 21 \} = 5 = 4r + r^2 =$$

$$r^2 + 4r - 5 = 0 = ,r + 5, r - 1) = 0 = r = 1, \text{ and the diameter is } 2 \text{ ft.}$$

83 (a) $\text{Area}_{\text{capsule}} = \text{Area}_{\text{sphere}} \{ \text{the two ends are hemispheres} \} + \text{Area}_{\text{cylinder}}$
 $= 41r^2 + 21r^2 = 41 \cdot \frac{1}{4} + 21 \cdot \frac{1}{4} = \frac{1}{4} + \frac{31}{4} = 1 \text{ cm}^2.$
 $\text{Area}_{\text{tablet}} = \text{Area}_{\text{top and bottom}} + \text{Area}_{\text{cylinder}} = 21r^2 + 21r \cdot \frac{1}{2} = 21r^2 + 1r.$

Equating the two surface areas yields $21r^2 + 1r = 1$
 $2r^2 + r - 1 = 0 \Rightarrow (2r - 1)(r + 1) = 0 \Rightarrow r = \frac{1}{2},$ and the diameter is 1 cm.

(b) $\text{Volume}_{\text{capsule}} = \text{Volume}_{\text{sphere}} + \text{Volume}_{\text{cylinder}}$
 $= \frac{4}{3}1r^3 + 1r^2 \cdot 2 = \frac{4}{3} \cdot 1 + 1 \cdot 2 = \frac{4}{3} + 2 = \frac{10}{3} \approx 3.33 \text{ cm}^3.$
 $\text{Volume}_{\text{tablet}} = \text{Volume}_{\text{cylinder}} = 1r^2 \cdot 2 = 1 \cdot \frac{1}{2} \cdot 2 = \frac{1}{2} \approx 0.5 \text{ cm}^3.$

84 $7 = 15,700S^{5/2}9H \Rightarrow S^{5/2} = \frac{7}{15,7009H}$
 $S = \left(\frac{7}{15,7009H} \right)^{2/5} = \frac{380}{(15,700)(9H)^{2/5}} \approx 0.054$

85 From the Pythagorean theorem, $d^2 + 2^2 = P^2$. Since d is to be 25% of P , we have
 $d = \frac{1}{4}P$, so $\left(\frac{1}{4}P\right)^2 + 2^2 = P^2 \Rightarrow \frac{1}{16}P^2 + 4 = P^2 \Rightarrow 4 = P^2 - \frac{1}{16}P^2 = \frac{15}{16}P^2$
 $2^2 = \frac{15}{16}P^2 \Rightarrow 2 = \sqrt{\frac{15}{16}P^2}$ since $2 > 0$ if $P > 0 \Rightarrow 2 = \frac{\sqrt{15}}{4}P$. Thus, $2 \approx 97\%P$.

86 $X = 0.257^{1/4} \cdot 7 = 7^{1/4} \approx 1.62$

87 $\text{Cost}_{\text{underwater}} + \text{Cost}_{\text{overland}} = \text{Cost}_{\text{total}} = 7500(\text{underwater miles}) + 6000(\text{overland miles}) = 35,000$

$7500\sqrt{x^2 + 1} + 6000(5 - x) = 35,000 \Rightarrow 7500\sqrt{x^2 + 1} + 30,000 - 6000x = 35,000$

$7500\sqrt{x^2 + 1} - 6000x = 5,000 \Rightarrow 15\sqrt{x^2 + 1} - 12x = 10$ {divide by 500}

$225(x^2 + 1) = 144x^2 + 240x + 100 \Rightarrow 225x^2 + 225 = 144x^2 + 240x + 100$

$81x^2 - 240x + 125 = 0 \Rightarrow x = \frac{240 \pm \sqrt{17,100}}{162} = \frac{6 \pm 40}{6 \pm 27} = \frac{6 \pm 40}{6 \pm 27} = \frac{40 \pm 5}{27}$

2887.06743 mi. There are two possible routes.

88 The y -value decreases 1.2 units for each 1 unit increase in the x -value. The data are best described by equation (1),
 $y = -1.2x + 2.$

89 The y -values are increasing rapidly and can best be described by equation (4), $y = x^3 - x^2 + x - 10.$

90 (a) Let $Y_1 = X_1 = -1.09P + 96.01$ and $Y_2 = X_2 = -0.011P^2 - 0.126P + 81.45$. Table each equation and

compare them to the actual temperatures.

x (°C)	85	75	65	55	45	35	25	15	5
Y_1	3A36	14A26	25A16	36A06	46A96	57A86	68A76	75A66	90A56
Y_2	-	10A13	26A79	41A25	53A51	63A57	71A43	77A09	80A55
S. Hem.	5	10	27	42	53	65	75	78	79

(b) Comparing $Y_1(X_1)$ with $Y_2(X_2)$, we can see that the linear equation X_1 is not as accurate as the quadratic equation X_2 .

91 (a) $P = 50 = X_2 = -0.011(50)^2 - 0.126(50) + 81.45 = 47.65^\circ\text{F}$.

Let $Y_1 = H_1 = 6.096P + 685.7$ and $Y_2 = H_2 = 0.00178P^3 - 0.072P^2 + 4.37P + 719$. Table each equation and compare them to the actual values.

x (P)	0	10	20	30	40	50	60
Y_1	686	747	808	869	930	991	1051
Y_2	719	757	792	833	893	980	1106
Summer	720	755	792	836	892	978	1107

Comparing Y_1 (H_1) with Y_2 (H_2),

we see that the linear equation H_1 is not as accurate as the cubic equation H_2 .

(b) $P = 35 = H_2 = 0.00178(35)^3 - 0.0072(35)^2 + 4.37(35) + 719 \approx 860$ min.

1.5 Exercises

1 $(5 - 23) + (-3 + 63) = 15 + (-3) + (-2 + 6)3 = 2 + 43$

2 $(-5 + 43) + (3 + 93) = (-5 + 3) + (4 + 9)3 = -2 + 133$

3 $(7 - 83) - (-5 - 33) = (7 + 5) + (-8 + 3)3 = 12 - 53$

4 $(-3 + 83) - (2 + 33) = (-3 - 2) + (8 - 3)3 = -5 + 53$

5 $(3 + 53)(2 - 73) = (3 + 53)2 + (3 + 53)(-73)$ edistributive property
 $= 6 + 103 - 213 -$ emultiply terms
 353^2
 $= 6 - 113 - 35(-1)$ ~combine 3-terms, $3^2 = -1^m$
 $= 6 - 113 + 35$
 $= 41 - 113$

6 $(-2 + 33)(8 - 3) = (-16 - 33^2) + (2 + 24)3 = (-16 + 3) + 263 = -13 + 263$

7 $(4 - 33)(2 + 73) = (8 - 213^2) + (28 - 6)3 = (8 + 21) + 223 = 29 + 223$

8 $(8 + 23)(7 - 33) = (56 - 63^2) + (-24 + 14)3 = (56 + 6) - 103 = 62 - 103$

9 Use the *special product formula* for $(x - y)^2$ on the inside front cover of the text.

$$(5 - 23)^2 = 5^2 - 2(5)(23) + (23)^2 = 25 - 203 + 43^2 = (25 - 4) - 203 = 21 - 203$$

10 $(6 + 73)^2 = 6^2 + 2(6)(73) + (73)^2 = (36 - 49) + 843 = -13 + 843$

11 $(3, 3 + 43)^2 = 3^2 + 2(3)(43) + (43)^2 = 3(1, 9 - 16) + 243 = 3(-7 + 243) = -24 - 73$

12 $(3, 2 - 73)^2 = 3^2 + 2(2)(-73) + (-73)^2 = 3(1, 4 - 49) - 283 = 3(-45 - 283) = 28 - 453$

13 $(3 + 43)(3 - 43)$ {note that this difference of squares $a^2 - b^2$ }
 $= 3^2 - (43)^2 = 9 - (-16) = \{a^2 \text{ becomes a "sum of squares"}\} 9 + 16 = 25$

14 $(4 + 73)(4 - 73) = 4^2 - (73)^2 = 16 - (-49) = 16 + 49 = 65$

15 (a) Since $3^5 = 1$ if 5 is a multiple of 4, we will write 3^{43} as $3^{40}3^3$, knowing that 3^{40} will reduce to 1.
 $3^{43} = 3^{40}3^3 = (3^{40})^1(-3) = 1^{10}(-3)$
 $= -3$

(b) As in Example 3(e), choose $b = 20$. $3^{-20} + 3^{20} = 3^0 = 1$.

16 (a) $3^{68} = (3^4)^{17} = 1^{17} = 1$

(b) As in Example 3(e), choose $b = 36$. $3^{-33} + 3^{36} = 3^3 = -3$.

17 (a) Since $3^5 = 1$ if 5 is a multiple of 4, we will write 3^{73} as $3^{72}3^1$, knowing that 3^{72} will reduce to 1.

$$3^{73} = 3^{72}3 = \overset{\wedge}{3^{4 \cdot 18}} 3 = 1^{18}3 = 3$$

(b) As in Example 3(e), choose $b = 48$. $3^{-46} + 3^{48} = 3^2 = -1$.

18 (a) $3^{66} = 3^{64}3^2 = \overset{\wedge}{3^{4 \cdot 16}}(-1) = 1^{16}(-1) = -1$

(b) As in Example 3(e), choose $b = 56$. $3^{-55} + 3^{56} = 3^1 = 3$.

19 Multiply both the numerator and the denominator by the conjugate of the denominator to eliminate all 3's in the

denominator. The new denominator is the sum of the squares of the coefficients—in this case, 2^2 and 4^2 .

$$\frac{3}{43} \frac{2 - 43}{2 - 43} = \frac{3(2 - 43)}{4 - 16} = \frac{6 - 123}{20} = \frac{6}{20} - \frac{123}{20} = \frac{3}{10} - \frac{123}{20}$$

20 $\frac{5}{3 - 73} + \frac{3 + 73}{3 + 73} = \frac{5(3 + 73)}{9 - 49} = \frac{15 + 353}{58} = \frac{15}{58} + \frac{353}{58}$

21 Multiply both the numerator and the denominator by the conjugate of the denominator to eliminate all 3's in the

denominator. The new denominator is the sum of the squares of the coefficients—in this case, 6^2 and 2^2 .

$$\frac{1 - 73}{23} \frac{6 + (6 + 14) + (2 - 42)3}{6 + 23} = \frac{20 - 403}{36 - 36} = \frac{20}{40} = \frac{1}{2} - \frac{403}{40}$$

22 $\frac{2 + 93}{-3 - 3} + \frac{-3 + 3}{-3 + 3} = \frac{(-6 - 9) + (2 - 27)3}{9 - 9} = \frac{-15 - 253}{10} = -\frac{3}{2} - \frac{5}{2}3$

23 $\frac{-4 + 63}{2 + 73} + \frac{2 - 73}{2 - 73} = \frac{(-8 + 42) + (28 + 12)3}{4 - 49} = \frac{34 + 403}{53} = \frac{34}{53} + \frac{403}{53}3$

24 $\frac{-3 - 23}{5 + 23} + \frac{5 - 23}{5 - 23} = \frac{(-15 - 4) + (6 - 10)3}{25 - 25} = \frac{-19 - 43}{29} = -\frac{19}{29} - \frac{4}{29}3$

25 Multiplying the denominator by 3 will eliminate the 3's in the denominator.

$$\frac{4 - 23}{23^2} \frac{4 - 23}{4 - 23} = \frac{3(4 - 23)}{2 + 43} = \frac{12 - 69}{7} = \frac{12}{7} - \frac{69}{7}3$$

26 $\frac{-2 + 63}{23} + \frac{-3}{23} = \frac{23 - 63}{23}$

$$\frac{3^3}{3^3} = \frac{27}{27} = 1$$

27 Use the *special product formula* for $(x + y)^3$ on the inside front cover of the text.

$$\begin{aligned} (2 + 53)^3 &= (2)^3 + 3(2)^2(53) + 3(2)(53)^2 + \\ &\quad + (53)^3 \\ &= 8 + 603 + 6(253^2) + \\ &\quad + 1253^3 \\ &= 8 + 1503^2 + 603 + 1253^3 = (8 - 150) + (60 - 125)3 = -142 - 653 \end{aligned}$$

28 $(3 - 23)^3 = (3)^3 + 3(3)^2(-23) + 3(3)(-23)^2 + (-23)^3 = 27 - 543 + 9(43^2) - 83^3$
 $= 27 + 363^2 + (-543 - 83^3) = (27 - 36) + (-54 + 8)3 = -9 - 463$

29 A common mistake is to multiply $-\sqrt{4} - \sqrt{16}$ and obtain $-\sqrt{64}$, or 8.

The correct procedure is $-\sqrt{4} - \sqrt{16} = -2 - 4 = -6$
 $(-2 - 4)^2 = (-6)^2 = 36$

$$(-2 - 4)^2 = (-6)^2 = 36$$

30 $(-3 + 53)(8 - 63) = (-24 + 30) + (403 + 183) = 6 + 583$

$$31 \frac{4 + \sqrt{-81}}{2 - \sqrt{-9}} = \frac{4 + 9i}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} = \frac{(8 - 27) + (12 + 18)i}{4 - 9} = \frac{-19 + 30i}{-5} = \frac{19}{5} - 6i$$

$$32 \frac{5 - \sqrt{-121}}{1 + \sqrt{-25}} = \frac{5 - 11i}{1 + 5i} \cdot \frac{1 - 5i}{1 - 5i} = \frac{(5 - 55) + (-25 - 11)i}{1 - 25} = \frac{-50 - 36i}{-24} = \frac{25}{12} + 3i$$

$$33 \frac{-\sqrt{-36} - \sqrt{-49}}{-16} = \frac{-6i - 7i}{-16} = \frac{-13i}{-16} = \frac{13i}{16}$$

$$33 \frac{423 - 21i}{43} = \frac{423 - 21i}{43} = \frac{423}{43} - \frac{21i}{43} = 9.83 - 0.49i$$

$$34 \frac{-\sqrt{-25}}{-\sqrt{-16} - \sqrt{-81}} = \frac{-5i}{-4i - 9i} = \frac{-5i}{-13i} = \frac{5}{13}$$

35 We need to equate the real parts and the imaginary parts on each side of " = ".

$$4 + x + 2iy = x + 23 + 4i \implies 4 = x + 23 \implies x = -19 \text{ and } 2y = 4 \implies y = 2$$

$$36 (x - y) + 33 = 4 + iy \implies x - y = 4 - 33 = -29 \text{ and } y = 1$$

$$37 (2 - y) - 163 = 10 + 4iy \implies 2 - y = 10 + 163 \implies -y = 171 \implies y = -171$$

$$38 8 + (3 + y)i = 2 - 43i \implies 8 = 2 \text{ and } 3 + y = -43 \implies y = -46$$

$$39 x^2 - 6x + 13 = 0 \implies x = \frac{6 \pm \sqrt{36 - 52}}{2} = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

$$40 x^2 - 2x + 26 = 0 \implies x = \frac{2 \pm \sqrt{4 - 104}}{2} = \frac{2 \pm \sqrt{-100}}{2} = \frac{2 \pm 10i}{2} = 1 \pm 5i$$

$$41 x^2 + 12x + 37 = 0$$

$$x = \frac{-12 \pm \sqrt{144 - 148}}{2} = \frac{-12 \pm \sqrt{-4}}{2} = \frac{-12 \pm 2i}{2} = -6 \pm i$$

$$42 x^2 + 8x + 17 = 0 \implies x = \frac{-8 \pm \sqrt{64 - 68}}{2} = \frac{-8 \pm \sqrt{-4}}{2} = \frac{-8 \pm 2i}{2} = -4 \pm i$$

$$43 x^2 - 5x + 20 = 0 \implies x = \frac{5 \pm \sqrt{25 - 80}}{2} = \frac{5 \pm \sqrt{-55}}{2} = \frac{5 \pm i\sqrt{55}}{2}$$

$$44 x^2 + 3x + 6 = 0 \implies x = \frac{-3 \pm \sqrt{9 - 24}}{2} = \frac{-3 \pm \sqrt{-15}}{2} = \frac{-3 \pm i\sqrt{15}}{2}$$

$$45 x^2 - 1x - 47 = 0 \implies x = \frac{1 \pm \sqrt{1 + 188}}{2} = \frac{1 \pm \sqrt{189}}{2}$$

$$45 \quad 4x^2 + x + 3 = 0 \quad \Rightarrow \quad x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 4 \cdot 3}}{2 \cdot 4} = \frac{-1 \pm \sqrt{1 - 48}}{8} = \frac{-1 \pm \sqrt{-47}}{8}$$

$$46 \quad -3x^2 + x - 5 = 0 \quad \Rightarrow \quad x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot (-3) \cdot (-5)}}{2 \cdot (-3)} = \frac{-1 \pm \sqrt{1 - 60}}{-6} = \frac{-1 \pm \sqrt{-59}}{-6}$$

47 Solving $x^3 = -64$ by taking the cube root of both sides would only give us the solution $x = -4$, so we need to factor $x^3 + 64$ as the sum of cubes. $x^3 + 64 = 0 \Rightarrow (x + 4)(x^2 - 4x + 16) = 0 \Rightarrow$

$$x = -4 \text{ or } x = \frac{4 \pm \sqrt{16 - 64}}{2} = \frac{4 \pm \sqrt{-48}}{2}. \text{ The three solutions are } -4, 2 \pm 2\sqrt{3}i.$$

48) $x^3 - 27 = 0 = (x - 3)(x^2 + 3x + 9) = 0 =$
 $x = 3$ or $x = \frac{-3 \pm \sqrt{-9 - 36}}{2} = \frac{-3 \pm 3\sqrt{-3}}{2}$. The three solutions are $3, -\frac{3}{2} \pm \frac{3\sqrt{-3}}{2}$.

49) $27x^3 = (x + 5)^3 = (3x)^3 - (x + 5)^3 = 0 =$
 {difference of cubes} $(3x - (x + 5))(3x^2 + 3x(x + 5) + (x + 5)^2) = 0 =$
 $(3x - x - 5)(9x^2 + 3x^2 + 15x + x^2 + 10x + 25) = 0 =$
 $(2x - 5)(13x^2 + 25x + 25) = 0 = x = \frac{5}{2}$ or $x = \frac{-25 \pm \sqrt{-625 - 1300}}{2(13)} = \frac{-25 \pm 15\sqrt{-3}}{26}$.
 The three solutions are $\frac{5}{2}, -\frac{25}{26} \pm \frac{15\sqrt{-3}}{26}$.

50) $16x^4 = (x - 4)^4 = (4x^2)^2 - (x - 4)^2 = 0$ {difference of squares}
 $=$
 $(4x^2 + x - 4)(4x^2 - x - 4) = 0$ $(5x^2 - 8x + 16)(3x^2 + 8x - 16) = 0 =$
 $=$
 $5x^2 - 8x + 16 = 0$ or $3x^2 + 8x - 16 = (x + 4)(3x - 4) = 0$.
 $5x^2 - 8x + 16 = 0 = x = \frac{8 \pm \sqrt{64 - 320}}{10} = \frac{8 \pm 16\sqrt{-3}}{10} = \frac{4 \pm 8\sqrt{-3}}{5}$. The four solutions are $-4, \frac{4}{3}, \frac{4}{5}, \frac{8}{5}$.

51) $x^4 = 625 = x^4 - 625 = 0 = (x^2 - 25)(x^2 + 25) = 0 =$
 $x^2 = 25, -25 = x = \pm 5, \pm 5\sqrt{-1}$

52) $x^4 = 81 = x^4 - 81 = 0 = (x^2 - 9)(x^2 + 9) = 0 =$
 $x^2 = 9, -9 = x = \pm 3, \pm 3\sqrt{-1}$

53) $4x^4 + 25x^2 + 36 = 0 = (x^2 + 4)(4x^2 + 9) = 0 =$
 $x^2 = -4, -\frac{9}{4} = x = \pm 2\sqrt{-1}, \pm \frac{3}{2}\sqrt{-1}$

54) $27x^4 + 21x^2 + 4 = 0 = (9x^2 + 4)(3x^2 + 1) = 0 =$
 $x^2 = -\frac{4}{9}, -\frac{1}{3} = x = \pm \frac{2}{3}\sqrt{-1}, \pm \frac{1}{3}\sqrt{-3}$

55) $x^3 + 3x^2 + 4x = 0 = x(x^2 + 3x + 4) = 0 =$
 $x = 0$ or $x = \frac{-3 \pm \sqrt{-9 - 16}}{2} = \frac{-3 \pm \sqrt{-25}}{2}$. The three solutions are $0, -\frac{3}{2}, \frac{1}{2}$.

56) $8x^3 - 12x^2 + 2x - 3 = 0 = 4x^2(2x - 3) + 1(2x - 3) = 0 = (4x^2 + 1)(2x - 3) = 0 =$
 $x^2 = -\frac{1}{4}$ or $x = \frac{3}{2} = x = \pm \frac{1}{2}\sqrt{-1}$ or $x = \frac{3}{2} = x = \frac{3}{2}, \pm \frac{1}{2}\sqrt{-1}$

Note: In Exercises 57–62: Let $u = a + b\sqrt{3}$ and $v = c + d\sqrt{3}$.

57) $u + v = (a + b\sqrt{3}) + (c + d\sqrt{3})$ {definition of u and v }
 $= (a + c) + (b + d)\sqrt{3} = (a + c) + (b + d)\sqrt{3}$

e
w

$$\begin{aligned} &= (a - b) + (c - d) \quad \text{write in complex number form} \\ &= z + \bar{z} \quad \text{definition of conjugate} \\ &= z + \bar{z} \quad \text{rearrange terms, *)} \\ &= z + \bar{z} \quad \text{definition of conjugates of } z \text{ and } \bar{z} \end{aligned}$$

*) We are really looking ahead to the terms we want to obtain, z and \bar{z} .

$$\boxed{58} \quad z - \bar{z} = (a + b) - (c + d) = (a - c) + (b - d) = (a - c) - (b - d) = (a - b) - (c - d) = z - \bar{z}$$

$$\boxed{59} \quad z + \bar{z} = (a + b) + (c + d) = (ac - bd) + (ad + bc) \\ = (ac - bd) - (ad + bc) = ac - ad - bd - bc = a(c - d) - b(c - d) = (a - b)(c - d) = z + \bar{z}$$

$$\begin{aligned} \overline{\frac{z}{z}} &= \overline{\frac{a+bi}{c+di}} = \frac{\overline{a+bi}}{\overline{c+di}} = \frac{a-bi}{c-di} = \frac{(a-bi)(c+di)}{(c-di)(c+di)} = \frac{(ac+bd) + (bc-ad)i}{c^2+d^2} = \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i \\ &= \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i = \frac{(ac+bd) + (bc-ad)i}{c^2+d^2} = \frac{a-bi}{c-di} = \frac{a-bi}{c-di} \cdot \frac{c+di}{c+di} = \frac{(a-bi)(c+di)}{c^2+d^2} = \frac{ac+bd + (bc-ad)i}{c^2+d^2} \end{aligned}$$

61. If $z = \bar{z}$, then $a - bi = a + bi$ and hence $-bi = bi$, or $2bi = 0$. Thus, $b = 0$ and $z = a$ is real.
 Conversely, if z is real, then $b = 0$ and hence $\bar{z} = \overline{a + 0i} = a - 0i = a + 0i = z$.

Thus, by 60 and 61, $z = \bar{z}$ if and only if z is real.

62. $z^2 = (a+bi)^2 = a^2 + 2abi - b^2 = (a^2 - b^2) + 2abi = (a^2 - b^2) - 2abi = a^2 - 2abi - b^2 = (a-bi)^2 = (\bar{z})^2$

1.6 Exercises

Note: Brackets, "I" and "I", are used with $<$ or $>$ and indicate that the endpoint of the interval is part of the solution.

Parentheses, "(" and ")", are used with $<$ or $>$ and indicate that the endpoint is *not* part of the solution.

1. $x < -2$ 2. $x < 4$

3. $5 > x$ 4. $3 < x < 7$

5. $-5 < x < 4$ 6. $x > -6$

7. $2 + 5 < 3 - 7 = -4 < -12 = x > 12$ exchange inequality $x < 12, \infty)$

8. $x - 6 > 5 + 3 = -4 > 9 = x < -\frac{9}{4}$ exchange inequality $x < -\infty, -\frac{9}{4}$

9. $3 < \frac{2x-9}{5} < 7 \cdot +5$ {multiply by the lcd, 5} $= 15 < 2x-9 < 35 =$
 $24 < 2x < 44$ {add 9 to all three parts} $= 12 < x < 22$ {divide all three parts by 2} $= (12, 22)$ {equivalent interval notation}

10. $-2 < \frac{4x+1}{3} < 0 \cdot +3 = -6 < 4x+1 < 0 = -7 < 4x < -1 = -\frac{7}{4} < x < -\frac{1}{4} = (-\frac{7}{4}, -\frac{1}{4})$

11. By the law of signs, a quotient is positive if the sign of the numerator and the sign of the denominator are the same.
 Since the numerator is positive, $\frac{4}{3x+2} > 0 = 3x+2 > 0 = x > -\frac{2}{3} = (-\frac{2}{3}, \infty)$.
 The expression is never equal to 0 since the numerator is never 0. Thus, the solution of $\frac{4}{3x+2} = 0$ is $(-\frac{2}{3}, \infty)$.

$$\boxed{12} \frac{3}{2x+5} < 0 \Rightarrow 2x+5 < 0 \text{ denominator must be negative} \Rightarrow 2x < -5 \Rightarrow x < -\frac{5}{2} < -\infty, -\frac{5}{2}$$

$$\boxed{13} \frac{-7}{4-3x} > 0 \Rightarrow 4-3x < 0 \text{ denominator must also be negative} \Rightarrow 4 < 3x \Rightarrow 3x > 4 \Rightarrow x > \frac{4}{3} < \frac{4}{3}, \infty$$

$$\boxed{14} \frac{-3}{2-x} < 0 \Rightarrow 2-x > 0 \Rightarrow 2 > x \Rightarrow x < 2 < (-\infty, 2)$$

$$\boxed{15} (1-x)^2 > 0 \text{ a. except } 1. \text{ Thus, } \frac{5}{(1-x)^2} > 0 \text{ has solution } x \neq 1.$$

16 $x^2 + 4 > 0$ a. Hence, $\frac{3}{x^2 + 4} > 0$ a., and $\frac{3}{x^2 + 4} < 0$ has no solution.

17 $x + 3 < 0$ or $x + 3 > 0$ $\Rightarrow x < -3$ or $x > -3$

18 $x - 4 < 0.03$ or $x - 4 > 0.03$ $\Rightarrow x < 4.03$ or $x > 4.07$

19 $x - 7 < 5$ or $x - 7 > -5$ $\Rightarrow x < 12$ or $x > 2$ $\Rightarrow x \in (-\infty, 2) \cup (12, \infty)$

20 $2x - 11 - 7x - 2 > 10$ or $2x - 11 - 7x > 12$ $\Rightarrow -5x - 23 > 12$ or $-5x > 35$ $\Rightarrow x < -7$ or $x > -5$ $\Rightarrow x \in (-\infty, -7) \cup (-5, \infty)$

21 Since $x + 2 < 0$ a., $x + 2 > -2$ has solution $(-\infty, \infty)$.

22 Since $x - 5 < 0$ a., $x - 5 < -2$ has no solution.

23 $x - 9 > 0$ a. except when $x - 9 = 0$, or $x = 9$. The solution is $(-\infty, 9) \cup (9, \infty)$.

24 $x + 2 = 0$ if $x = -2$, but is never less than 0. Thus, $x + 2 < 0$ has solution $x < -2$.

25 $-2 < x < 4$ or $-2 < x$ and $x < 4$. Since -2 is always less than x (because $x > 0$),

we only need to consider $x < 4$. $x < 4$ or $x < 4$ $\Rightarrow x < 4$

26 $1 < x < 5$ or $1 < x$ and $x < 5$ $\Rightarrow 1 < x < 5$

27 $(x + 1)(x - 10) > 0$ See the sign chart for details concerning the signs of the individual factors and the resulting sign. The given inequality has solutions in the interval $(-\infty, -1) \cup (10, \infty)$, which corresponds to the positive values for the **Resulting sign**.

Interval	$(-\infty, -1)$	$(-1, 10)$	$(10, \infty)$
Sign of $x - 10$.	+	+	-
Sign of $x + 1$	-	+	+
Resulting sign	-	+	-

28 $(x + 2)(x - 1)(x - 4) < 0$ From the chart, we see the product is negative for $x \in (-2, 1) \cup (4, \infty)$. Since we want to also include the values that make the product equal to zero $\{-2, 1, \text{ and } 4\}$, the solution is $[-2, 1) \cup (4, \infty)$.

Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, 4)$	$(4, \infty)$
Sign of $x - 4$.	+	+	+	-
Sign of $x - 1$	-	-	+	+
Sign of $x + 2$	-	+	+	+
Resulting sign	+	-	+	-

29 $x^2 - x - 6 < 0$ or $(x - 3)(x + 2) < 0$ $\Rightarrow x \in (-2, 3)$

Interval	$, -\infty, -2)$	$, -2, 3)$	$, 3, \infty)$
Sign of $\cdot - 3$	-	-	+
Sign of $\cdot + 2$	-	+	+
Resulting sign	+	-	+

30 $x^2 + 4x + 3 = 0 \Rightarrow (x + 1)(x + 3) = 0$

$x \in (-\infty, -3) \cup (-1, \infty)$

Interval	$(-\infty, -3)$	$(-3, -1)$	$(-1, \infty)$
Sign of $x + 1$	-	-	+
Sign of $x + 3$	-	+	+
Resulting sign	+	-	+

31 $(x^2 + 3) - 5 = x^2 + 3 - 5 = x^2 - 2 = 0 \Rightarrow (x - \sqrt{2})(x + \sqrt{2}) = 0$

$x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

Interval	$(-\infty, -\sqrt{2})$	$(-\sqrt{2}, \sqrt{2})$	$(\sqrt{2}, \infty)$
Sign of $x - 1$	-	-	+
Sign of $x^2 + 3$	-	+	+
Resulting sign	+	-	+

32 $8x - 15 > x^2 \Rightarrow x^2 - 8x + 15 < 0 \Rightarrow (x - 3)(x - 5) < 0$

$x \in (3, 5)$

Interval	$(-\infty, 3)$	$(3, 5)$	$(5, \infty)$
Sign of $x - 5$	-	-	+
Sign of $x - 3$	-	+	+
Resulting sign	+	-	+

Note: Solving $x^2 < a^2$ or $x^2 > a^2$ for $a > 0$ may be achieved using factoring, that is,

$x^2 - a^2 < 0 \Rightarrow (x + a)(x - a) < 0 \Rightarrow -a < x < a$; or by taking the square root of each side, that is,

$-\sqrt{x^2} < -\sqrt{a^2} \Rightarrow -|x| < -a \Rightarrow -a < x < a$. The most common mistake is forgetting that $-\sqrt{x^2} = -|x|$.

33 $25x^2 - 16 < 0 \Rightarrow x^2 < \frac{16}{25} \Rightarrow |x| < \frac{4}{5} \Rightarrow -\frac{4}{5} < x < \frac{4}{5}$

34 $25x^2 - 16 < 0 \Rightarrow (5x - 4)(5x + 4) < 0$

$x \in (\frac{4}{5}, \frac{16}{25})$

Interval	$(-\infty, 0)$	$(\frac{4}{5}, \frac{16}{25})$	$(\frac{16}{25}, \infty)$
Sign of $25x - 16$	-	-	+
Sign of $5x + 4$	-	+	+
Resulting sign	+	-	+

35 $\frac{x^2 - 1}{x^2 + 2} < 0 \Rightarrow \frac{x - 1}{x + 1} < 0$ {we will exclude $x = -2$ since it makes the original expression undefined}

$\frac{1}{x + 1} < 0$ {we can divide by x^2 since $x^2 \neq 0$ and we will include $x = 0$ since it makes x^2 equal to zero and we want all solutions less than or equal to zero}

$x + 1 < 0$ {the fraction cannot equal zero and $x + 1$ must be negative so that the fraction is negative}

$x < -1 \Rightarrow x \in (-\infty, -2) \cup (-2, -1) \cup \emptyset$

36 $\frac{x^2 + 1}{x^2 - 9} = 0 \Rightarrow \frac{x - 3}{(x + 3)(x - 3)} = \frac{1}{x + 3} = 0$ {exclude 3}

$x + 3 > 0$ {exclude -3} $\Rightarrow x > -3 \Rightarrow x \in (-3, 3) \cup (3, \infty)$

37 $x^2 - 1 < 0 = (x-1)(x+1) < 0 = \frac{x-1}{x+2} < 0$ we will exclude $x = 0$ from the solution of $(-\infty, -2) \cup (0, 1)$

Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
Sign of $x - 1$	-	-	+
Sign of $x + 2$	-	+	+
Resulting sign	+	-	+

38 $\frac{(x+3)^2(x-2)}{(x+4)(x^2-4)} < 0 = \frac{x-2}{(x+4)(x-2)(x+2)} < 0$ {include -3} = $\frac{1}{(x+4)(x+2)}$ 0 {cancel, change inequality, exclude 2}
 $\mathbb{R} \setminus \{-\infty, -4\} \cup (-3, 2) \cup (2, \infty)$

Interval	$(-\infty, -4)$	$(-4, -2)$	$(-2, \infty)$
Sign of $x+2$	-	-	+
Sign of $x+4$	-	+	+
Resulting sign	+	-	+

39 $\frac{x-2}{x^2-3x-10} = 0 = \frac{x-2}{(x-5)(x+2)}$ 0 { $x=2$ is a solution since it makes the fraction equal to zero, $x=5$ and $x=-2$ are excluded since these values make the fraction undefined}
 $\mathbb{R} \setminus \{-2, 5\} \cup (2, \infty)$

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, 5)$	$(5, \infty)$
Sign of $x-5$	-	-	-	+
Sign of $x-2$	-	-	+	+
Sign of $x+2$	-	+	+	+
Resulting sign	-	+	-	+

40 $\frac{x+6}{x^2-7x+12} < 0 = \frac{x+6}{(x-3)(x-4)} < 0$
 $\mathbb{R} \setminus \{-\infty, -6\} \cup (3, 4)$

Interval	$(-\infty, -6)$	$(-6, 3)$	$(3, 4)$	$(4, \infty)$
Sign of $x-4$	-	-	-	+
Sign of $x-3$	-	-	+	+
Sign of $x+6$	-	+	+	+
Resulting sign	-	+	-	+

41 $\frac{-3}{x^2-9} > 0 = \frac{-1}{(x+3)(x-3)} < 0$ {divide by -3}
 $\mathbb{R} \setminus \{-\infty, -3\} \cup (0, 3)$

Interval	$(-\infty, -3)$	$(-3, 0)$	$(0, 3)$	$(3, \infty)$
Sign of $x-3$	-	-	-	+
Sign of x	-	-	+	+
Sign of $x+3$	-	+	+	+
Resulting sign	-	+	-	+

42 $\frac{5}{16-x^2} < 0 = \frac{-1}{(4+x)(4-x)} < 0$ {divide by 5}
 $\mathbb{R} \setminus \{-4, 0\} \cup (4, \infty)$

Interval	$(-\infty, -4)$	$(-4, 0)$	$(0, 4)$	$(4, \infty)$
Sign of $4-x$	+	+	+	-
Sign of x	-	-	+	+
Sign of $4+x$	-	+	+	+
Resulting sign	+	-	+	-

43 $\frac{x+1}{x^2-3} > 2 = \frac{x+1}{x^2-3} - 2 > 0 = \frac{x+1-2(x^2-3)}{x^2-3} > 0 = \frac{-2x^2-2x+7}{x^2-3} > 0 = \frac{-2x^2-2x+7}{(x-\sqrt{3})(x+\sqrt{3})} > 0 = \frac{-2x^2-2x+7}{(x-\sqrt{3})(x+\sqrt{3})} > 0 =$
 $\frac{-2x^2-2x+7}{x^2-3} > 0$. From the sign chart, the solution is $(-\sqrt{3}, \frac{7}{2})$. Note that you should *not* multiply by the factor x^2-3

as we did with rational *equations* because $2x - 3$ may be positive or negative, and multiplying by it would require solving two inequalities. This method of solution tends to be more difficult than the sign chart method.

Interval	$-\infty, \frac{3}{2}$	$\frac{3}{2}, \frac{7}{3}$	$\frac{7}{3}, \infty$
Sign of $-3x + 7$	+	+	-
Sign of $2x - 3$	-	+	+
Resulting sign	-	+	-

44 $\frac{x-2}{3+5} < 4 \iff \frac{x-2-4(3+5)}{3+5} < 0 \iff \frac{x-11-22}{3+5} < 0$ $\mathbb{R} \setminus \{-\infty, -2\} \cup \{-5, \infty\}$

Interval	$]-\infty, -2)$	$]-2, -\frac{5}{3}[$	$]-\frac{5}{3}, \infty[$
Sign of $3+5$	-	-	+
Sign of $-11-22$	+	-	-
Resulting sign	-	+	-

45 $\frac{1}{x-2} - \frac{3}{x+1} = 0 \iff \frac{1(x+1)-3(x-2)}{(x-2)(x+1)} = 0 \iff \frac{x+1-3x+6}{(x-2)(x+1)} = 0 \iff \frac{-2x+7}{(x-2)(x+1)} = 0$ $\mathbb{R} \setminus \{-\infty, -1\} \cup \{2, 2^c\}$

Interval	$]-\infty, -1)$	$]-1, 2)$	$]2, \frac{7}{2}[$	$] \frac{7}{2}, \infty[$
Sign of $-2x+7$	+	+	+	-
Sign of $x-2$	-	-	+	+
Sign of $x+1$	-	+	+	+
Resulting sign	+	-	+	-

46 $\frac{2}{2x+3} < \frac{2}{x-5} \iff \frac{2(x-5)-2(2x+3)}{(2x+3)(x-5)} < 0 \iff \frac{-2x-16}{(2x+3)(x-5)} < 0$ $\mathbb{R} \setminus \{-8, -\frac{3}{2}\} \cup \{5, \infty\}$

Interval	$]-\infty, -8)$	$]-8, -\frac{3}{2}[$	$]-\frac{3}{2}, 5[$	$]5, \infty[$
Sign of $x-5$	-	-	-	+
Sign of $2x+3$	-	-	+	+
Sign of $-2x-16$	+	-	-	-
Resulting sign	+	-	+	-

47 $\frac{x-5}{x-2} < \frac{2}{x-1} \iff \frac{x-5}{x-2} - \frac{2}{x-1} < 0 \iff \frac{(x-5)(x-1)-2(x-2)}{(x-2)(x-1)} < 0 \iff \frac{x^2-6x+5-2x+4}{(x-2)(x-1)} < 0 \iff \frac{x^2-8x+9}{(x-2)(x-1)} < 0$ $\mathbb{R} \setminus \{1, \frac{5}{3}\} \cup]2, 5[$

Interval	$]-\infty, 1)$	$]1, \frac{5}{3}[$	$] \frac{5}{3}, 2[$	$]2, 5[$	$]5, \infty[$
Sign of $x-5$	-	-	-	-	+
Sign of $x-2$	-	-	-	+	+
Sign of $3x-5$	-	-	+	+	+
Sign of $x-1$	-	+	+	+	+
Resulting sign	+	-	+	-	+

48 $\frac{x-1}{x^2+2x-6+3} = \frac{3}{x^2-4+3} \iff \frac{x-1}{x^2-1} - \frac{3}{x^2-1} = 0 \iff \frac{x-1-3}{x^2-1} = 0 \iff \frac{x-4}{(x-1)(x+1)} = 0$ $\mathbb{R} \setminus \{-\infty, -2\} \cup \{2, 1^c\} \cup]3, \infty[$

Interval	$]-\infty, -2)$	$]-2, \frac{1}{2}[$	$] \frac{1}{2}, 1[$	$]1, 3[$	$]3, \infty[$
Sign of $x-3$	-	-	-	-	+
Sign of $x-1$	-	-	-	+	+
Sign of $2x-1$	-	-	+	+	+
Sign of $x+2$	-	+	+	+	+

49 $x^3 > x = x^3 - x > 0 = x(x^2 - 1) > 0 = x(x+1)(x-1) > 0 \quad \text{on } (-1, 0) \cup (1, \infty)$

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of $x - 1$	-	-	-	+
Sign of x	-	-	+	+
Sign of $x + 1$	-	+	+	+
Resulting sign	-	+	-	+

50 $x^4 - x^2 = x^2(x^2 - 1) = x^2(x-1)(x+1) = 0$. Since $x^2 = 0$, x^2 does not need to be included in the sign chart, but 0 must be included in the answer because of the equality.

on $(-\infty, -1) \cup (0, 1) \cup (1, \infty)$

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Sign of $x - 1$	-	-	+
Sign of $x + 1$	-	+	+
Resulting sign	+	-	+

- 51 (a) $|x + 5| = 3 \Rightarrow x + 5 = 3$ or $x + 5 = -3 \Rightarrow x = -2$ or $x = -8$.
 (b) $|x + 5| < 3$ has solutions between the values found in part (a), that is, $(-8, -2)$.

(c) The solutions of $|x + 5| > 3$ are the portions of the real line that are not in parts (a) and (b), that is, $(-\infty, -8) \cup (-2, \infty)$.

52 (a) $|x - 4| < 3 \Rightarrow -3 < x - 4 < 3 \Rightarrow 1 < x < 7 \Rightarrow (1, 7)$.

(b) $|x - 4| = 3$ has solutions at the endpoints of the interval in part (a); that is, at $x = 1$ and $x = 7$.

(c) As in Exercise 51(c), $|x - 4| > 3$ has solutions in $(-\infty, 1) \cup (7, \infty)$.

53 We could think of this statement as "the difference between \$ and 141 is at most 2." In symbols, we have $|\$ - 141| < 2$. Intuitively, we know that this inequality must describe the weights from 139 to 143.

54 "r must be within 0.01 centimeter of 1 centimeter" is written as $|r - 1| < 0.01$.

55 $Q = \frac{f}{f - p}$ We want to know what condition will assure us that an object's image is at least 3 times as large as

the object, or, equivalently, when $Q \geq 3$. $Q \geq 3 \Leftrightarrow \frac{f}{f - p} \geq 3 \Leftrightarrow f \geq 3(f - p) \Leftrightarrow f \geq 3f - 3p \Leftrightarrow -2f \geq -3p$ (since

$6 - p > 0$, we can multiply by $6 - p$ and not change the direction of the inequality) $\Leftrightarrow 3p \geq 2f \Leftrightarrow p \geq \frac{2}{3}f$, but $p < 6$ since $p < f$. Thus, $\frac{2}{3}f < p < 6$.

56 $c = \frac{3A5}{!} \Leftrightarrow c > 1A5 \Leftrightarrow \frac{3A5}{!} > 1A5 \Leftrightarrow 3A5 > !A5 + 1A5 = 2!^3 \Leftrightarrow ! > \frac{3}{2}$ hr

57 Let x denote the number of years before A becomes more economical than B.

The costs are the initial costs plus the yearly costs times the number of years.

$Cost_A < Cost_B \Rightarrow 100,000 + 8000x < 80,000 + 11,000x \Rightarrow 20,000 < 3000x \Rightarrow x > \frac{20}{3}$, or $6\frac{2}{3}$ yr.

58 Let t denote the time in years from the present. $Cost_B < Cost_A \Rightarrow$

$$\text{Purchase}_B + \text{Insurance}_B + \text{Gas}_B < \text{Purchase}_A + \text{Insurance}_A + \text{Gas}_A =$$

$$24,000 + 1200t + \frac{15,000}{50}t^3 < 20,000 + 1000t + \frac{15,000}{30}t^3 =$$

$$24,000 + 2100t < 20,000 + 2500t \quad \Rightarrow \quad 4000 < 400t \quad \Rightarrow \quad t > 10 \text{ yr.}$$

59 $t > 9 \Rightarrow -16t^2 + 24t + 1 > 9 \Rightarrow -16t^2 + 24t - 8 > 0 \Rightarrow 2t^2 - 3t + 1 < 0$ {divide by -8} $= (2t - 1)(t - 1) < 0$ {use a sign chart} $= \frac{1}{2} < t < 1$.

The dog is more than 9 ft off the ground for $1 - \frac{1}{2} = \frac{1}{2}$ sec.

60 $1536 = -16t^2 + 320t \Rightarrow 1536 = -16t^2 + 320t - 1536 \Rightarrow 0 = t^2 - 20t + 96 < 0$ {divide by -16} $= (t - 8)(t - 12) < 0$ {use a sign chart} $\Rightarrow 8 < t < 12$

61 $d < 75 \Rightarrow \# + \frac{1}{20}\#^2 < 75$ {multiply by 20} $= 20\# + \#^2 < 1500 \Rightarrow \#^2 + 20\# - 1500 < 0 \Rightarrow (\# + 50)(\# - 30) < 0$ {use a sign chart} $= -50 < \# < 30 \Rightarrow 0 < \# < 30$ {since $\# \geq 0$ }

62 $Q = 45 \Rightarrow -\frac{1}{30}\#^2 + \frac{5}{2}\# = 45$ {multiply by -30} $= \#^2 - 75\# < -1350 \Rightarrow \#^2 - 75\# + 1350 < 0 \Rightarrow (\# - 30)(\# - 45) < 0$ {use a sign chart} $\Rightarrow 30 < \# < 45$

- 63 (a) 5 ft 9 in = 69 in. In a 40 year period, a person's height will decrease by $40 \times 0.024 = 0.96$ in \approx 1 in. The person will be approximately one inch shorter, or 5 ft 8 in. at age 70.
 (b) 5 ft 6 in = 66 in. In 20 years, a person's height ($h = 66$) will change by $0.024 \times 20 = 0.48$ in. Thus, $66 - 0.48 < h < 66 + 0.48 \Rightarrow 65.52 < h < 66.48$.

64 $7500 < S < 10,000 \Rightarrow 7500 < 0.00334Z^2S < 10,000 \Rightarrow 7500 < 0.00334(210)Z^2 < 10,000 \Rightarrow \frac{7500}{0.7014} < Z^2 < \frac{10,000}{0.7014} \Rightarrow \sqrt{\frac{7500}{0.7014}} < Z < \sqrt{\frac{10,000}{0.7014}} \Rightarrow 103.4 < Z < 119.4$ in ft/secf. To convert ft/sec to mi/hr, multiply by $\frac{60}{88}$ or $\frac{15}{22}$, which are reduced forms of $\frac{1 \text{ foot}}{1 \text{ second}} \times \frac{3600 \text{ seconds}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{5280 \text{ feet}}$. Using the approximations in ft/sec, we get $103.4 < Z < 119.4$ in ft/secf $\Rightarrow 70.5 < Z < 81.4$ in mi/hrf.

65 The numerator is equal to zero when $x = 2, 3$ and the denominator is equal to zero when $x = -1$. From the table, the expression $Y_1 = \frac{(2-x)(3-x)}{(1-x)(x+1)}$ is positive when $x \in (-2, -1) \cup (1, 2) \cup (3, 5.0)$. See the table on the left.

x	Y ₁	x	Y ₁
-2.0	20	1.0	ERROR
-1.5	37.8	1.5	1.8
-1.0	ERROR	2.0	0
-0.5	-35	2.5	-0.1429
0.0	-18	3.0	0
0.5	-15	3.5	0.2

x	Y ₁	x	Y ₁
-3.5	30.938	1.0	36
-3.0	0	1.5	19.688
-2.5	-7.313	2.0	0
-2.0	0	2.5	-18.56
-1.5	14.438	3.0	-30
-1.0	30	3.5	-26.81
-0.5	42.188	4.0	0
0.0	48	4.5	60.938
0.5	45.938	5.0	168

66 By using a table it can be shown that the expression is equal to zero when $x = -3, -2, 2, 4$. The expression $Y_1 = x^4 - x^3 - 16x^2 + 4x + 48$ is negative when $x \in (-3, -2) \cup (2, 4)$. See the table on the right.

Chapter 1 Review Exercises

1 If $x < -3$, then $x + 3 < 0$, and $|x + 3| = -(x + 3) = -x - 3$.

2 If $2 < x < 3$, then $x - 2 > 0$ { $x - 2$ is positive } and $x - 3 < 0$ { $x - 3$ is negative }. Thus, $(x - 2)(x - 3) < 0$ { positive times negative is negative }, and since the absolute value of an expression that is negative is the negative of the expression, $|(x - 2)(x - 3)| = -(x - 2)(x - 3)$, or, equivalently, $(2 - x)(x - 3)$.

3 $\frac{a^{2/3}b^{3/2}}{a^2b} = \frac{a^{4/3}b^9}{a^{12}b^6} = a^{-8/3}b^3$

4 $\frac{p^2}{4q^2} = \frac{p^2}{4q^2} = \frac{p^2}{4q^2}$

5 $\frac{y^{-1}}{-z} = \frac{-1/3y^2}{z} = \frac{-4y^{-4}}{z^2} \frac{z^3}{-y^6} = \frac{-3z}{y^{10}}$

6 $\frac{-64z^3}{z^6y^9} = \frac{-64z^3}{z^6y^9} = \frac{-2}{z^3y^9}$

7 $a^{2/3}b^{-2/3}c^{-1} = (a^2b^{-6})^{-1} = a^{-2}b^6 = \frac{b^6}{a^2}$

8 $x^{-2} - y^{-1} = \frac{1}{x^2} - \frac{1}{y} = \frac{y - x^2}{x^2y}$

9 $27x^3y^3z^4 = 27x^3y^3z^3 \cdot z = 3xyz \cdot z^2z$

10 $(-4a^3b^2c)^2 = 16a^6b^4c^2 = 2^4a^4b^4 \cdot a^2c^2 = 2ab \cdot (ac)^2 = 2ab \cdot ac$

11 $\frac{1}{-1} = -1$ 12 $(c^3d^6)^4 = c^{12}d^{24} = c^2d^4$

13 $\frac{-12.4y}{-3.2y^7} = \frac{12.4y}{3.2y^7} = \frac{4 \cdot 2}{y^6} = y^3$

14 $\frac{3 + \dots}{3 - \dots} = \frac{3 + \dots}{3 - \dots} + \frac{\dots}{3 - \dots} = 9 - \dots$

15 $(3^3 - 4^2 + x - 6) + x^4 - 2x^3 + 3x^2 + 5 = x^4 + x^3 - x^2 + x - 1$

16 $(x + 4)(x + 3) - (2x - 3)(x - 5) = (x^2 + 7x + 12) - (2x^2 - 13x + 15) = -x^2 + 20x - 3$

17 $(3a - 5b)(4a + 7b) = 12a^2 + 21ab - 20ab - 35b^2 = 12a^2 + ab - 35b^2$

18 $(4r^2 - 3)^2 = (4r^2)^2 - 2(4r^2)(3) + (3)^2 = 16r^4 - 24r^2 + 9$

19 $(13a^2 + 5b)(13a^2 - 5b) = (13a^2)^2 - (5b)^2 = 169a^4 - 25b^2$

20 $(2a + b)^3 = (2a)^3 + 3(2a)^2(b) + 3(2a)(b)^2 + (b)^3 = 8a^3 + 12a^2b + 6ab^2 + b^3$

21 $(3x + 2y)^2(3x - 2y)^2 = [(3x + 2y)(3x - 2y)]^2 = (9x^2 - 4y^2)^2 = 81x^4 - 72x^2y^2 + 16y^4$

22 $(a + b + c + d)^2 = a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd)$

23 $60\$ + 50\$ = 10\$(6 + 5)$

24 $16a^4 + 24a^2b^2 + 9b^4 = (4a^2 + 3b^2)(4a^2 + 3b^2) = (4a^2 + 3b^2)^2$

25 $8x^3 + 64y^3 = 8(x^3 + 8y^3) = 8(x + 2y)(x^2 - 2xy + 4y^2)$

26 $3^4 - 6^4 = 3^4 - (2 \cdot 3)^4 = 3^4 - 2^4 \cdot 3^4 = 3^4(1 - 16) = -15 \cdot 3^4$

27 $p^8 - q^8 = (p^4)^2 - (q^4)^2 = (p^4 + q^4)(p^4 - q^4) = (p^4 + q^4)(p^2 + q^2)(p^2 - q^2) = (p^4 + q^4)(p^2 + q^2)(p + q)(p - q)$

$$\boxed{28} \quad x^4 - 12x^3 + 36x^2 = x^2(x^2 - 12x + 36) = x^2(x - 6)(x - 6) = x^2(x - 6)^2$$

$$\boxed{29} \quad x^2 - 49y^2 - 14x + 49 = (x^2 - 14x + 49) - 49y^2 = (x - 7)^2 - (7y)^2 = (x - 7 + 7y)(x - 7 - 7y)$$

$$\boxed{30} \quad x^5 - 4x^3 + 8x^2 - 32 = x^3(x^2 - 4) + 8x^2 - 32 = x^3(x + 2)(x - 2) + 8x^2 - 32$$

$$= (x + 2)(x^3 - 2x + 4) + 4(x - 2)(x + 2) = (x - 2)(x + 2)^2(x - 2) - 2x + 4$$

$$31 \quad \frac{6}{4x-5} - \frac{15}{10x+1} = \frac{6(10x+1) - 15(4x-5)}{(4x-5)(10x+1)} = \frac{60x+6-60x+75}{(4x-5)(10x+1)} = \frac{81}{(4x-5)(10x+1)}$$

$$32 \quad \frac{7}{x+2} + \frac{3}{(x+2)^2} - \frac{5}{x} = \frac{7(x)(x+2) + 3(x) - 5(x+2)^2}{x(x+2)^2} = \frac{7x^2+14x+3x^2-5x^2-20x-20}{x(x+2)^2} = \frac{5x^2-6x-20}{x(x+2)^2}$$

$$33 \quad \frac{x^2+x}{1+x} = \frac{x^2+x}{1+\frac{1}{x}} = \frac{x^2+x}{\frac{x+1}{x}} = \frac{x^2+x}{1} \cdot \frac{x}{x+1} = \frac{x^3+x^2}{x+1}$$

We could factor the numerator,

but since it doesn't lead to a reduction of the fraction, we leave it in this form.

$$34 \quad (a^{-1} + b^{-1})^{-1} = \left(\frac{1}{a} + \frac{1}{b}\right)^{-1} = \left(\frac{b+a}{ab}\right)^{-1} = \frac{ab}{a+b}$$

$$35 \quad \frac{\frac{x}{x+2} - \frac{4}{x+2}}{\frac{x-3}{x+2} - \frac{6}{x+2}} = \frac{\frac{x-4}{x+2}}{\frac{(x-3)-(6)}{x+2}} = \frac{x-4}{(x-3)-6} = \frac{x-4}{x-9} = \frac{x-4}{(x+3)(x-4)} \cdot \frac{1}{x+3}$$

$$36 \quad \frac{(4-x)^{1/3}(6+x)^{-2/3} - (6+x)^{1/3}(-2-x)}{(4-x^2)^2} = \frac{2(6+x)^{-2/3}[(4-x)^2 + (6+x)]}{(4-x^2)^2} = \frac{2(4-x^2+6x^2+x)}{(6+x)^{2/3}(4-x^2)^2} = \frac{2(5x^2+x+4)}{(6+x)^{2/3}(4-x^2)^2}$$

$$37 \quad \frac{3x+1}{5x+7} = \frac{6x+11}{10x-3} \cdot (5x+7)(10x-3) = (3x+1)(10x-3) = (6x+11)(5x+7)$$

$$38 \quad 2x^2 + 7x - 15 = 0 \Rightarrow (x+5)(2x-3) = 0 \Rightarrow x = -5, \frac{3}{2}$$

$$39 \quad x^2 - 3x + 4 = 2 \Rightarrow x^2 + 4x - 2 = 0 \Rightarrow x = \frac{-4 \pm \sqrt{16+24}}{2} = \frac{-4 \pm \sqrt{40}}{2} = \frac{-2 \pm \sqrt{10}}{1}$$

$$40 \quad 4x^4 - 37x^2 + 75 = 0 \Rightarrow (4x^2 - 25)(x^2 - 3) = 0 \Rightarrow x^2 = \frac{25}{4}, 3 \Rightarrow x = \pm \frac{5}{2}, \pm \sqrt{3}$$

$$41 \quad 20x^3 + 8x^2 - 55x - 22 = 0 \Rightarrow 4x^2(5x+2) - 11(5x+2) = 0 \Rightarrow (4x^2-11)(5x+2) = 0 \Rightarrow x^2 = \frac{11}{4} \text{ or } x = -\frac{2}{5} \Rightarrow x = \pm \sqrt{\frac{11}{4}}, -\frac{2}{5}$$

$$42 \quad |4x-11| = 7 \Rightarrow 4x-11 = 7 \text{ or } 4x-11 = -7 \Rightarrow 4x = 18 \text{ or } 4x = 4 \Rightarrow x = \frac{9}{2} \text{ or } x = 1$$

$$43 \quad 2|2x+11| + 1 = 15 \Rightarrow 2|2x+11| = 14 \Rightarrow |2x+11| = 7 \Rightarrow$$

$$2x+11 = 7 \text{ or } 2x+11 = -7 \Rightarrow 2x = -4 \text{ or } 2x = -18 \Rightarrow x = -2 \text{ or } x = -9$$

$$44 \quad \frac{1}{x} + 6 = \frac{5}{x-1} \Rightarrow 1 + 6x = \frac{5}{x-1} \Rightarrow$$

$$6x^2 - 5x + 1 = 0 \text{ \{factoring or substituting would be appropriate\}}$$

$$2x^2 - 1x^3 - 1x^2 - 1x = 0 \Rightarrow x^2 = 1, 1 \Rightarrow x = \sqrt[2]{1}, \sqrt[2]{1} = 1, 1$$

Check $x = \frac{1}{4}$: LS = $4 + 6 = 10$; RS = $5/\frac{1}{2} = 10$ $\Rightarrow x = \frac{1}{4}$ is a solution.

Check $x = \frac{1}{9}$: LS = $9 + 6 = 15$; RS = $5/\frac{1}{3} = 15$ $\Rightarrow x = \frac{1}{9}$ is a solution.

45 $-7x + 2 + x = 6 \Rightarrow -7x + 2x^2 = (6 - x)^2 \Rightarrow 7x + 2 = 36 - 12x + x^2 \Rightarrow$

$$x^2 - 19x + 34 = 0 \Rightarrow (x - 2)(x - 17) = 0 \Rightarrow x = 2 \text{ and } 17 \text{ is an extraneous solution.}$$

46 $-3x + 1 - (-x + 4) = 1 \Rightarrow -3x + 1 = 1 + x + 4 \Rightarrow -3x + 1 = x + 5 \Rightarrow -4x = 4 \Rightarrow x = -1$

$3x + 1 = 1 + 2 - (-x + 4) + x + 4 \Rightarrow 3x + 1 = 2 - (-x + 4) + x + 4 \Rightarrow 3x + 1 = 2 + x - 4 + x + 4 \Rightarrow 3x + 1 = 2x + 2 \Rightarrow x = 1$

$(-x + 4)^2 = (x - 2)^2 \Rightarrow x + 4 = x^2 - 4x + 4 \Rightarrow x^2 - 5x = 0 \Rightarrow x(x - 5) = 0 \Rightarrow x = 0, 5$

Check $x = 0$: LS = $1 - 2 = -1 \neq$ RS = $x = 0$ is an extraneous solution.

Check $x = 5$: LS = $4 - 3 = 1 =$ RS = $x = 5$ is a solution.

47 $10 - 7x < 4 + 8 \Rightarrow -15 < -6 \Rightarrow x > \frac{2}{5} \in (\frac{2}{5}, \infty)$

48 $-\frac{1}{2} < \frac{2x+3}{5} < \frac{3}{2} + 10 \Rightarrow -5 < 4x+6 < 15 \Rightarrow -11 < 4x < 9 \Rightarrow -\frac{11}{4} < x < \frac{9}{4} \in (-\frac{11}{4}, \frac{9}{4})$

49 $\frac{7}{10x+3} < 0 \Rightarrow 10x+3 < 0$ {since $7 > 0$ } $\Rightarrow x < -\frac{3}{10} \in (-\infty, -\frac{3}{10})$

50 $14x + 71 < 21 \Rightarrow -21 < 4x + 7 < 21 \Rightarrow -28 < 4x < 14 \Rightarrow -7 < x < \frac{7}{2} \in (-7, \frac{7}{2})$

51 $213 - x + 1 > 5 \Rightarrow 213 - x > 4 \Rightarrow 13 - x > 2 \Rightarrow 3 - x > 2$ or $3 - x < -2 \Rightarrow 1 > x$ or $5 < x \Rightarrow x < 1$ or $x > 5 \in (-\infty, 1) \cup (5, \infty)$

52 $116 - 3x \leq 5 \Rightarrow 16 - 3x \leq 5$ or $16 - 3x < -5 \Rightarrow -3x \leq -11$ or $-3x < -21 \Rightarrow x \geq \frac{11}{3}$ or $x > 7 \in [\frac{11}{3}, \infty) \cup (7, \infty)$

53 $10x^2 + 11x > 6 \Rightarrow 10x^2 + 11x - 6 > 0 \Rightarrow (2x+3)(5x-2) > 0 \Rightarrow x \in (-\infty, -\frac{3}{2}) \cup (\frac{2}{5}, \infty)$

Interval	$(-\infty, -\frac{3}{2})$	$(-\frac{3}{2}, \frac{2}{5})$	$(\frac{2}{5}, \infty)$
Sign of $5x - 2$	-	-	+
Sign of $2x + 3$	-	+	+
Resulting sign	+	-	+

54 $x(x-3) < 18 \Rightarrow x^2 - 3x - 18 < 0 \Rightarrow (x-6)(x+3) < 0 \Rightarrow x \in (-3, 6)$

Interval	$(-\infty, -3)$	$(-3, 6)$	$(6, \infty)$
Sign of $x - 6$	-	-	+
Sign of $x + 3$	-	+	+
Resulting sign	+	-	+

55 $\frac{x^2(3-x)}{x+2} < 0 \Rightarrow \frac{3-x}{x+2} < 0 \Rightarrow x \in (-\infty, -2) \cup \{0\} \cup (3, \infty)$

Interval	$(-\infty, -2)$	$(-2, 3)$	$(3, \infty)$
Sign of $3 - x$	+	+	-
Sign of $x + 2$	-	+	+
Resulting sign	-	+	-

$$\boxed{56} \quad \frac{x^2 - x - 2}{x^2 + 4x + 3} < 0 = \frac{(x - 2)(x + 1)}{(x + 1)(x + 3)} < 0 = \frac{x - 2}{x + 3} < 0 \text{ \{exclude -1\}}$$

$$\in (-3, -1) \cup (-1, 2)$$

Interval	$(-\infty, -3)$	$(-3, 2)$	$(2, \infty)$
Sign of $x - 2$	-	-	+
Sign of $x + 3$	-	+	+
Resulting sign	+	-	+

$$\begin{aligned}
 \text{[57]} \quad \frac{3}{2x+3} < \frac{1}{3x-6} &= \frac{3}{2x+3} - \frac{1}{3x-6} < 0 = \frac{3(x-2) - 1(2x+3)}{(2x+3)(x-2)} < 0 = \\
 &= \frac{2x+3}{(2x+3)(x-2)} < 0 = \frac{1}{x-2} < 0 \quad \text{æ } (-\infty, -\frac{3}{2}) \cup (2, 9)
 \end{aligned}$$

Interval	$(-\infty, -\frac{3}{2})$	$(-\frac{3}{2}, 2)$	$(2, 9)$	$(9, \infty)$
Sign of $x-9$	-	-	-	+
Sign of $x-2$	-	-	+	+
Sign of $2x+3$	-	+	+	+
Resulting sign	-	+	-	+

$$\text{[58]} \quad \frac{x+2}{x^2-25} < 0 = \frac{x+2}{(x+5)(x-5)} < 0 \quad \text{æ } (-\infty, -5) \cup (-2, 5)$$

Interval	$(-\infty, -5)$	$(-5, -2)$	$(-2, 5)$	$(5, \infty)$
Sign of $x-5$	-	-	-	+
Sign of $x+2$	-	-	+	+
Sign of $x+5$	-	+	+	+
Resulting sign	-	+	-	+

$$\text{[59]} \quad x^3 > x^2 = x^2(x-1) > 0 \text{ if } x-1 > 0 = x > 1 < (1, \infty)$$

$$\text{[60]} \quad (x^2-x)(x^2-5x+6) < 0 = (x-1)(x-2)(x-3) < 0 \quad \text{æ } (0, 1) \cup (2, 3)$$

Interval	$(-\infty, 0)$	$(0, 1)$	$(1, 2)$	$(2, 3)$	$(3, \infty)$
Sign of $x-3$	-	-	-	-	+
Sign of $x-2$	-	-	-	+	+
Sign of $x-1$	-	-	+	+	+
Sign of x	-	+	+	+	+
Resulting sign	+	-	+	-	+

$$\begin{aligned}
 \text{[61]} \quad 7 + R &= \frac{C+2}{C} = C(7+R) = C+2 = C7 + CR = C+2 = \\
 C7 + CR - C &= 2 = C(7+R-1) = 2 = C = \frac{2}{7+R-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{[62]} \quad A &= B \frac{C}{H} - I = A + I = B \frac{C}{H} = \frac{A+I}{B} = \frac{C}{H} = \\
 H + \frac{(A+I)^3}{B^3} &= C = H(A+I)^3 = C+B^3 = H = \frac{CB^3}{(A+I)^3}
 \end{aligned}$$

$$\text{[63]} \quad J = \frac{179}{8ZP} = 9^4 = \frac{8JZP}{17} = 9 = \frac{8JZP}{17} = 9 = \frac{8JZP}{17} \text{ since } 9 > 0$$

$$\begin{aligned}
 \text{[64]} \quad Z &= 1h(r^2 + 9^2 + r9) = 3Z = 1h(r^2 + 9^2 + r9) = (1h)r^2 + (1h9)r + (1h9^2 - 3Z) = 0 = \\
 r &= \frac{-1h9 \pm \sqrt{(1h9)^2 - 4(1h)(1h9^2 - 3Z)}}{2(1h)} = \frac{-1h9 \pm \sqrt{-121hZ - 31^2h^29^2}}{21h}
 \end{aligned}$$

Since $r > 0$, we must use the plus sign, and $r = \frac{-1h9 + \sqrt{-121hZ - 31^2h^29^2}}{21h}$.

$$\text{[65]} \quad (5 + 8i)^2 = 5^2 + 2(5)(8i) + (8i)^2 = (25 - 64) + 80i = -39 + 80i$$

$$\boxed{66} \frac{1}{9 - - - 4} = \frac{1}{\bar{9} - 2i} = \frac{1}{\bar{9} - 2i} \cdot \frac{9 + 2i}{9 + 2i} = \frac{9 + 2i}{81 + 4} = \frac{9}{85} + \frac{2}{85}i$$

$$\boxed{67} \quad \frac{6-3i}{2+7i} = \frac{6-3i}{2+7i} + \frac{2-7i}{2-7i} = \frac{(12-21) + (-42-6)i}{4+49} = -\frac{9}{53} - \frac{48}{53}i$$

$$\boxed{68} \quad \frac{24-8i}{4i} = \frac{4(6-2i)}{4i} = \frac{6-2i}{i} + \frac{-i}{-i} = \frac{-6i+2i^2}{-i^2} = \frac{-2-6i}{1} = -2-6i$$

69 Let 7 denote the principal that will be invested, and r the yield rate of the stock fund.

$$\text{Income}_{\text{stocks}} - 28\% \text{ federal tax} - 7\% \text{ state tax} = \text{Income}_{\text{bonds}} =$$

$$(7r) - 0.28(7r) - 0.07(7r) = 0.071867 \text{ \{divide by 7\} } = 1r - 0.28r - 0.07r = 0.07186 =$$

$$0.65r = 0.07186 = r = \frac{0.07186}{0.65} = r \approx 0.11055, \text{ or, } 11.055\%.$$

70 Let x denote the number of cm³ of gold. Grams_{gold} + Grams_{silver} = Grams_{total} =

$$x(19.3) + (5-x)(10.5) = 80 = 19.3x + 52.5 - 10.5x = 80 = 8.8x + 27.5 = x = 3.125.$$

The number of grams of gold is $19.3 \times 3.125 \approx 60.3$.

71 Let x denote the number of ounces of the vegetable portion, $10-x$ the number of ounces of meat.

$$\text{Protein}_{\text{vegetable}} + \text{Protein}_{\text{meat}} = \text{Protein}_{\text{total}} = 1(x) + 1(10-x) = 7 = \frac{x}{2} + 10 - \frac{x}{2} = 7 =$$

$$-\frac{1}{2}x = -3 = x = 6. \text{ Use 6 oz of vegetables and 4 oz of meat.}$$

72 Let x denote the number of gallons of 20% solution, $120-x$ the number of gallons of 50% solution.

$$20(x) + 50(120-x) = 30(120) \text{ \{all in \% \} } = 20x + 6000 - 50x = 3600 = 2400 = 30x = x = 80.$$

Use 80 gal of the 20% solution and 40 gal of the 50% solution.

73 Let x = the amount of copper they have to mix with 140 kg of zinc to make brass.

$$\text{Copper}_{\text{amount put in}} = \text{Copper}_{\text{amount in final product}} =$$

$$x = 0.65(x + 140) = x = 0.65x + 91 = 0.35x = 91 = x = \frac{91}{0.35} = 260 \text{ kg}$$

74 Let x denote the number of hours needed to fill an empty bin.

$$\text{Using the hourly rates, } \frac{1}{2} - \frac{1}{5} = \frac{1}{x} + 10. = 5x - 2x = 10 = 3x = 10 =$$

$$x = \frac{10}{3} \text{ hr. Since the bin was half-full at the start, } \frac{1}{2} = \frac{1}{2} + \frac{10}{3} = \frac{5}{3} \text{ hr, or, } 1 \text{ hr } 40 \text{ min.}$$

75 (a) The eastbound car has distance 20! and the southbound car has distance $(-2 + 50)!$.

$$d^2 = (20!)^2 + (-2 + 50!)^2 = d^2 = 400!^2 + 4 - 200! + 2500!^2 = d = \sqrt{-2900!^2 - 200! + 4}$$

$$(b) 104 = -2900!^2 - 200! + 4 = 10,816 = 2900!^2 - 200! + 4 = 2900!^2 - 200! - 10,812 = 0 =$$

$$725!^2 - 50! - 2703 = 0 \text{ edivide by } 4f =$$

$$f = \frac{50 \text{ ,, } -7,841,200}{1450} \{f > 0\} = \frac{5 + 2 - \sqrt{19,603}}{145} \approx 1.97, \text{ or approximately } 11:58 \text{ A.M.}$$

76 Let f and $\$$ denote the length and width, respectively. $36 + 6\$ = 270 = 6\$ = 270 - 36 = \$ = 45 - \frac{1}{2}f$.

$$\text{The total area is to be } 10 + 100 = 1000 \text{ ft}^2. \text{ Area} = 6\$ = 1000 = f \cdot 45 - \frac{1}{2}f^2 = 1000 = 45f - \frac{1}{2}f^2 =$$

$$f^2 - 90f + 2000 = 0 = (f-40)(f-50) = 0 = f = 40, 50 \text{ and } \$ = 25, 20.$$

There are two arrangements: 40 ft X 25 ft and 50 ft X 20 ft.

77 Let x denote the length of one side of an end.

$$(a) Z = 6\$h = 48 = 6 + x + x = x^2 = 8 = x = 2 - 2 \text{ ft}$$

$$(b) S = 6s + 2sh + 26h = 44 = 6 + 2(s^2) + 2(6s) = 44 = 2s^2 + 12s =$$

$$s^2 + 6s - 22 = 0 = (s + 11)(s - 2) = 0 = s = 2 \text{ ft}$$

78 $p \# = 200 = \# = \frac{200}{p}$. $25 < \# < 50 = 25 < \frac{200}{p} < 50 = \frac{1}{25} \frac{p}{200} \frac{1}{50} = 8 < p < 4 =$
 $4 < p < 8$

79 Let x denote the amount of yearly business. $Pay_B > Pay_A = \$40,000 + 0.20x > \$50,000 + 0.10x =$
 $0.10x > \$10,000 = x > \$100,000$

80 $\# > 1100 = 1087 \hat{E} \frac{\overline{X}}{273} > 1100 = \hat{E} \frac{\overline{X}}{273} > \frac{1100}{1087} = \frac{X}{273} > \frac{1100^2}{1087^2} = X > \frac{273 + 1100^2}{1087^2} =$
 $X > 279.57 \text{ K}$

81 Let x denote the number of trees *over* 24. Then $24 + x$ represents the total number of trees planted per acre, and $600 - 12x$ represents the number of apples per tree.

Total apples = (number of trees)(number of apples per tree)
 $= (24 + x)(600 - 12x) = -12x^2 + 312x + 14,400$
 Apples 16,416 = $-12x^2 + 312x + 14,400$ $16,416 = -12x^2 + 312x - 2016$ $0 =$
 $x^2 - 26x + 168 < 0 = (x - 12)(x - 14) < 0 = 12 < x < 14 = 36 < 24 + x < 38$
 Hence, 36 to 38 trees per acre should be planted.

82 Let x denote the number of \$25 increases in rent. Then the number of occupied apartments is $218 - 5x$ and the rent per apartment is $940 + 25x$.

Total income = (number of occupied apartments)(rent per apartment)
 $= (218 - 5x)(940 + 25x) = -125x^2 + 750x + 204,920$
 Income 205,920 = $-125x^2 + 750x + 204,920$ $205,920 = -125x^2 + 750x - 1000$ $0 =$
 $x^2 - 6x + 8 < 0 = (x - 2)(x - 4) < 0 = 2 < x < 4 = 990 < 940 + 25x < 1040$
 Hence, the rent charged should be \$990 to \$1040.

83 The y -values are increasing slowly and can best be described by equation (3), $y = 3 - \sqrt{x} - 0.5$.

Chapter 1 Discussion Exercises

1 $\frac{\$1 \text{ in cash back}}{100 \text{ points}} \times \frac{1 \text{ point}}{\$10 \text{ charged}} = \frac{\$1 \text{ in cash back}}{\$1000 \text{ charged}} = 0.001$, or 0.1%.

2 Squaring the right side gives us $(a + b)^2 = a^2 + 2ab + b^2$. Squaring the left side gives us $a^2 + b^2$. Now $a^2 + 2ab + b^2$ will equal $a^2 + b^2$ only if $2ab = 0$. The expression $2ab$ equals zero only if either $a = 0$ or $b = 0$.

3 We first need to determine the term that needs to be added and subtracted. Since $25 = 5^2$, it makes sense to add and subtract $2 + 5x = 10x$. Then we will obtain the square of a binomial—i.e.,
 $(x^2 + 10x + 25) - 10x = (x + 5)^2 - 10x$. We can now factor this expression as the difference of two squares,

$$(x + 5)^2 - 10x = (x + 5)^2 - \sqrt{10} \cdot x^2 = \sqrt{10} \cdot x + 5 + -\sqrt{10} \cdot x + 5 - -\sqrt{10} \cdot x$$

4 The expression $\frac{1}{x + 1}$ can be evaluated at $x = 1$, whereas the expression $\frac{-1}{x^2 - 1}$ is undefined at $x = 1$.

5 Try $\frac{3x^2 - 4x + 7}{8x^2 + 9x - 100}$ with $x = 10^3, 10^4,$ and 10^5 . You get approximately 0.374, 0.3749, and 0.37499. The

numbers seem to be getting closer to 0.375, which is the decimal representation for $\frac{3}{8}$, which is the ratio of the coefficients of the x^2 terms. In general, the quotients of this form get close to the ratio of leading coefficients as x gets larger.

6 $\frac{3x^2 - 5x - 2}{x^2 - 4} = \frac{(3x + 1)(x - 2)}{(x + 2)(x - 2)} = \frac{3x + 1}{x + 2}$. Evaluating the original expression and the simplified expression

with any $x \neq 2$ gives us the same value. This evaluation does not prove that the expressions are equal for any value of x other than the one selected. The simplification proves that the expressions are equal for all values of x except $x = 2$.

7 Follow the algebraic simplification given.

- | | |
|------------------------------------|--|
| 1) Write down his/her age. | Denote the age by x . |
| 2) Multiply it by 2. | $2x$ |
| 3) Add 5. | $2x + 5$ |
| 4) Multiply this sum by 50. | $50(2x + 5) = 100x + 250$ |
| 5) Subtract 365. | $(100x + 250) - 365 = 100x - 115$ |
| 6) Add his/her height (in inches). | $100x - 115 + y$, where y is the height |
| 7) Add 115. | $100x - 115 + y + 115 = 100x + y$ |

As a specific example, suppose the age is 21 and the height is 68. The number obtained by following the steps is $100x + y = 2168$ and we can see that the first two digits of the result equal the age and the last two digits equal the height.

$$8 \quad Z_{out} = M_{in} - \frac{9\sqrt{i}}{9 - \sqrt{i}} = \frac{Z_{in}}{9 - \sqrt{i}} - \frac{9\sqrt{i}}{9 - \sqrt{i}} \quad \{\text{definition of } M\}$$

$$= \frac{Z_{in}}{9 - \sqrt{i}} - \frac{9\sqrt{i}}{9 - \sqrt{i}} \quad \{\text{definition of } \wedge\}$$

$$= \frac{9^2 - \sqrt{i}^2 - 39\sqrt{i}}{9 - \sqrt{i}} \quad \{\text{definition of } \wedge\}$$

$$= \frac{Z_{in}(9 - \sqrt{i})}{9^2 - \sqrt{i}^2 - 39\sqrt{i}} - \frac{9\sqrt{i}}{9 - \sqrt{i}}$$

$$= -\frac{9^2 - \sqrt{i}^2 - 39\sqrt{i}}{99i}(Z_{in})$$

$$= -\frac{9^2 - 9^2 - 399i}{9^2i}(Z_{in}) \quad \{\text{let } \sqrt{i} = 9\}$$

$$= -\frac{-399i}{9^2i}(Z_{in}) = \frac{3}{9} Z_{in}$$

9 We need to solve the equation $x^2 - xy + y^2 = 0$ for x .

Use the quadratic formula with $a = 1, b = -y,$ and $c = y^2$.

$$x = \frac{-(-y) \pm \sqrt{(-y)^2 - 4(1)(y^2)}}{2(1)} = \frac{y \pm \sqrt{y^2 - 4y^2}}{2} = \frac{y \pm \sqrt{-3y^2}}{2} = \frac{y \pm y\sqrt{-3}}{2}$$

Since this equation has imaginary solutions, $x^2 - xy + y^2$ is not factorable over the reals.

A similar argument holds for $x^2 + px + y^2$.

10 The solutions are $x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

The average is $\frac{x_1 + x_2}{2} = \frac{-2b/2a}{2} = -\frac{b}{2a}$. Suppose you solve the equation $x^2 + 4x + 7 = 0$ and obtain the solutions $x_1 \approx -1.32$ and $x_2 \approx 5.32$. Averaging these numbers gives us the value 2, which we can easily see is equal to $-b/(2a)$.

11 (a)
$$\frac{1}{\frac{a+bi}{c+di}} = \frac{c+di}{a+bi} \cdot \frac{a-bi}{a-bi} = \frac{ac+bd+(ad-bc)i}{a^2+b^2} = \frac{ac+bd}{a^2+b^2} + \frac{ad-bc}{a^2+b^2}i = p+qi$$

(b) Yes, try an example such as $\frac{3}{4}$. Let $a = 3, b = 0, c = 4,$ and $d = 0$. Then, from part (a),

$$p+qi = \frac{12}{9} + \frac{0}{9}i = \frac{12}{9} = \frac{4}{3}, \text{ which is the multiplicative inverse of } \frac{3}{4}$$

(c) a and b cannot both be 0 because then the denominator would be 0.

12 Since we don't know the value of a , we don't know the sign of $a - 2$, and hence we are unsure of whether or not to reverse the direction of the inequality sign.

13 Hint: Try these examples to help you get to the general solution.

(1) $x^2 + 1 > 0$ {In this case, $a > 0, H = -4 < 0$, and by examining a sign chart with $x^2 + 1$ as the only factor, we see that the solution is $x \in \mathbb{R}$.}

(2) $x^2 - 2x - 3 < 0$

(3) $-x^2 - 4 < 0$

(4) $-x^2 - 2x - 1 < 0$

(5) $-x^2 + 2x + 3 < 0$

General solutions categorized by a and H :

(1) $a > 0, H < 0$: solution is $x \in \mathbb{R}$

(2) $a > 0, H > 0$: let $x_1 = \frac{-b - \sqrt{b^2 - 4aH}}{2a}$ and $x_2 = \frac{-b + \sqrt{b^2 - 4aH}}{2a}$ = solution is $(-\infty, x_1) \cup (x_2, \infty)$

(3) $a < 0, H < 0$: no solution

(4) $a < 0, H = 0$: solution is $x = -\frac{b}{2a}$

(5) $a < 0, H > 0$: solution is (x_1, x_2)

14 (a) This problem is solved in three steps.

(i) First, we must determine the height of the cloud base using the formula in Exercise 74 in Section 1.4,

$$h = 227(X - H) = 227(80 - 68) = 2724 \text{ ft.}$$

(ii) Next, we must determine the temperature X at the cloud base. From (i), the height of the cloud base is

$$h = 2724 \text{ and } X = X_0 - \frac{5.5}{1000}h = 80 - \frac{5.5}{1000}(2724) = 65.018^\circ\text{F.}$$

(iii) Finally, we must solve the equation $X = B - \frac{3}{1000}h$ for h , when $X = 32^\circ\text{F}$ and $B = 65.018^\circ\text{F}$.

$$32 = 65.018 - \frac{3}{1000}h \implies h = (65.018 - 32) \frac{1000}{3} = 11,006 \text{ ft.}$$

(b) Following the procedure in part (a) and changing a few variable names, we obtain the following:

(i) $L = 227(K - H) = 227K - 227H$

(ii) $X = X_0 - \frac{5.5}{1000}h = B = K - \frac{11}{2000}L = B = K - \frac{11}{2000}(227K - 227H) =$
 $B = K - \frac{2497}{2000}K + \frac{2497}{2000}H = \frac{2497}{2000}H - \frac{497}{2000}K$

(iii) $h = (B - 32) \frac{1000}{3} = h = \frac{2497}{1000}H - \frac{497}{1000}K - \frac{32,000}{3} =$

$$h = \frac{2497}{64,000} H - \frac{497}{6} K - \frac{64,000}{6} = h = \frac{1}{6}(2497H - 497K - 64,000)$$

15 The first equation, $-\sqrt{2x-3} + \sqrt{-x+5} = 0$, is a sum of square roots that is equal to 0. The only way this could be true is if both radicals are actually equal to 0. It is easy to see that $\sqrt{-x+5}$ is equal to 0 only if $x = -5$, but -5 will not make $-\sqrt{2x-3}$ equal to 0, so there is no reason to try to solve the first equation.

On the other hand, the second equation, $\sqrt[3]{2x-3} + \sqrt[3]{-x+5} = 0$, can be written as $\sqrt[3]{2x-3} = -\sqrt[3]{-x+5}$. This just says that one cube root is equal to the negative of another cube root, which could happen since a cube root can be negative. Solving this equation gives us $\sqrt[3]{2x-3} = -\sqrt[3]{-x+5} \Rightarrow 2x-3 = -(-x+5) \Rightarrow 3x = -2 \Rightarrow x = -\frac{2}{3}$.

16 $\sqrt{x} = c \Rightarrow \sqrt{x} - 2/c = 0 \Rightarrow \sqrt{x} = 2/c \Rightarrow x = 4/c^2 \Rightarrow c^2x = c^2 \cdot 4/c^2 = 4 \Rightarrow 0 = c^4x - 4c^2x + 4 = 0 = c^4x - 5c^2x + 4 = 0 = (c^2x - 1)(c^2x - 4) = 0 \Rightarrow c^2x = 1, 2 \Rightarrow x = \frac{1}{c^2}, \frac{4}{c^2}$.

Check $x = \frac{1}{c^2} = \frac{1}{(2 \times 10^{500})^2} = \frac{1}{4 \times 10^{1000}}$.

LS = $\sqrt{x} = \frac{1}{2 \times 10^{500}}$

RS = $c \cdot \frac{2}{c} = \frac{2 \times 10^{500}}{2 \times 10^{500}} = \frac{2}{2} = \frac{1}{1} = \frac{1}{2 \times 10^{500}}$

Check $x = \frac{4}{c^2} = \frac{4}{(2 \times 10^{500})^2} = \frac{4}{4 \times 10^{1000}} = \frac{1}{10^{1000}}$.

LS = $\sqrt{x} = \frac{1}{10^{500}}$

RS = $c \cdot \frac{2}{c} = \frac{2 \times 10^{500}}{10^{1000}} = \frac{2}{2 \times 10^{500}} = \frac{2}{2} = \frac{1}{10^{500}}$

So $x = \frac{4}{c^2}$ is a valid solution. The right side of the original equation, $c \cdot \frac{2}{c}$, must be nonnegative since it is equal to a square root. Note that the right side equals a negative number when $c = -1$.

17 (a) $S = 975$, $A = 599$, and $w = 1.83 = \text{winning percentage} = \frac{S}{S+A} \Rightarrow 0.709206$. Since they played

154 games ($110 + 44$), the number of wins using the estimated winning percentage would be

$0.709(154) \approx 109$. Hence, the Pythagorean win-loss record of the 1927 Yankees is 109–45 (only one game off their actual record).

(b) The actual winning percentage is $\frac{110}{154} \approx 0.714286$. For an estimate of w , we'll assign $\frac{975}{975+599}$ to Y_1 and

look at a table of values of w starting with $w = 1.80$ and incrementing by 0.01. From the table, we see that $w = 1.88$ corresponds to $Y_1 \approx 0.714204$, which is the closest value to the actual winning percentage. Thus, the value of w is 1.88.

18 1 gallon $\approx 0.13368 \text{ ft}^3$ is a conversion factor that would help.

The volume of the tank is 10,000 gallons $\approx 1336.8 \text{ ft}^3$. Use $Z = \frac{4}{3}\pi r^3$ to determine the radius.

$1336.8 = \frac{4}{3}\pi r^3 \Rightarrow r^3 = \frac{1002.6}{1} = r \approx 6.83375 \text{ ft}$. Then use $S = 4\pi r^2$ to find the surface area.

$S = 4\pi(6.83375)^2 \approx 586.85 \text{ ft}^2$.

Chapter 1 Test

1 y^{99} is negative since it is a negative number raised to an odd power. $y - 2$ is negative since it is a negative number

made even more negative by subtracting a positive number. The quotient of two negatives is a *positive* number.

2 The quotient of -2 and y is not greater than 5 $-2 < \frac{-2}{y} < 5$.



3] Since $-x^2 - 3 < 0$ for every x (it doesn't matter that x is negative), $| -x^2 - 3 | = -(-x^2 - 3) = x^2 + 3$.

4] Using distance = rate X time, we get $t = \frac{d}{r} = \frac{91,500,000 \text{ miles}}{186,000 \text{ miles per second}} \approx 492$ seconds.

5] $\frac{y^2 - 3}{z^0 y^2} = \frac{y^2}{y^2 z^0} = \frac{y^2}{y^2 z^0} = \frac{y^2}{z^0} = \frac{y^2}{1} = y^2$

6] $x^{-2/3} x^{3/4} = x^{-8/12} x^{9/12} = x^{-8/12+9/12} = x^{1/12} = x^{1/12}$

7] $\frac{y^3}{x^3} = \frac{y^3}{x^3} + \frac{y^3}{x^3} = \frac{y^3}{x^3} = \frac{y^3}{x^3}$

8] $x^2 + 2x^2 - 3x + 5x = x^2 + 2x^2 - 3x + 5x = 3x^2 + 2x = 3x^2 + 2x + 10$

9] The leading term of $2x^2 + 3x^4$ will be determined by multiplying $2x^2$ times $3x^4$. The $x^2 + 3x$ will affect other terms, but not the leading term. Hence, $2x^2 \cdot 3x^4 = 6x^6$.

10] By trial and error, $x^2 + 7x - 15 = (x + 5)(x - 3)$.

11] $x^3 - 27 = (x - 3)(x^2 + 3x + 9)$

12] Recognizing this polynomial as a sum of cubes, we get

$$64x^3 + 1 = (4x)^3 + 1^3 = (4x + 1)(16x^2 - 4x + 1)$$

13] We must recognize that $x^3 - 5x^3 = -4x^3$, and then factor as we would any other difference of cubes.

$$x^3 - 5 = (x - \sqrt[3]{5})(x^2 + \sqrt[3]{5}x + \sqrt[3]{25})$$

14] Factor by grouping. $x^2 + 4x - 3y - 6y = x^2 + 4x - 3y - 6y = (x + 2)(x - 3y)$

15] Recognizing this polynomial as a difference of cubes, we get

$$x^3 - 1 = (x - 1)(x^2 + x + 1)$$

16] $\frac{x}{x-2} + \frac{5}{x-2} = \frac{x+5}{x-2} = \frac{x^2 + 5x - 10 - 12}{x^2 - 2x} = \frac{x^2 + 5x - 22}{x(x-2)} = \frac{(x+11)(x-2)}{x(x-2)} = \frac{x+11}{x}$

17] Multiply numerator and denominator by x .

$$\frac{x^2 - y^2}{x - y} = \frac{(x - y)(x + y)}{x - y} = \frac{(x - y)(x + y)}{x - y} = x + y$$

18] $\frac{x^2 + hx + 7h}{h} = \frac{x^2 + hx + 7h}{h} = \frac{x^2 + hx + 7h}{h} = \frac{x^2}{h} + \frac{hx}{h} + \frac{7h}{h} = \frac{x^2}{h} + x + 7$

$$\frac{19}{6h} \frac{6h}{6h} = \frac{1}{6h} \frac{6h}{6h} = \frac{1}{6h} \frac{6h}{6h} = \frac{1}{6h} \frac{6h}{6h}$$

$$\frac{1}{6h} \frac{6h}{6h} = \frac{1}{6h} \frac{6h}{6h} = \frac{1}{6h} \frac{6h}{6h} = \frac{1}{6h} \frac{6h}{6h}$$

$$\begin{aligned} \text{[20]} \quad D_1 + 2\tilde{N}^3 D_4 \tilde{N} D_1 - 3\tilde{N}^3 + D_1 - 3\tilde{N}^4 D_3 \tilde{N} D_1 + 2\tilde{N}^2 &= D_1 + 2\tilde{N}^2 D_1 - 3\tilde{N}^3 D_4 D_1 + 2\tilde{N} + 3D_1 - 3\tilde{N}^4 \\ &= D_1 + 2\tilde{N}^2 D_1 - 3\tilde{N}^3 D_4 + 8 + 3D_1 - 9\tilde{N} = D_1 + 2\tilde{N}^2 D_1 - 3\tilde{N}^3 D_7 - 1\tilde{N} \end{aligned}$$

$$\text{[21]} \quad \frac{D_1^2 - 3\tilde{N}^2 D_2 \tilde{N} - 2D_2 \tilde{N} D_1^2 - 3\tilde{N} D_2 \tilde{N}}{D_1^2 - 3\tilde{N}^2 D_2} = \frac{D_1^2 - 3\tilde{N} D_2 \tilde{N} D_1^2 - 3\tilde{N} - 2D_2}{D_1^2 - 3\tilde{N}^4} = \frac{2D_1 - 3 - 2D_2}{D_1^2 - 3\tilde{N}^3}$$

$$\text{[22]} \quad \frac{5}{x-3} + \frac{7}{x} = \frac{45}{x^2-3x} \implies (x-3) = 5x^2 + 7(x-3) = 45 \implies 5x^2 + 7x - 66 = 0 =$$

$(5x + 22)(x - 3) = 0 \implies x = -\frac{22}{5}, 3$. But x cannot equal 3 since it would make denominators in the original equation equal to 0, so $x = -\frac{22}{5}$.

$$\begin{aligned} \text{[23]} \quad A = \frac{3B}{2B-5} &= A(2B-5) = 3B = 2AB - 5A = 3B = 2AB - 3B = 5A = \\ &B(2A-3) = 5A \implies B = \frac{5A}{2A-3} \end{aligned}$$

[24] Let x denote the original value of the stock. Then $x + 0.2x$ is the value after the first year and $x + 0.3(x + 0.2x)$ is the value after the next year, so an equation that describes the problem is $x + 0.3(x + 0.2x) = 2720$. Solving gives

$$x + 0.3(x + 0.2x) = 2720 \implies x + 0.3x + 0.06x = 2720 \implies 1.36x = 2720 \implies x = \frac{2720}{1.36} = 2000.$$

The original value was \$2000.

$$\begin{aligned} \text{[25]} \quad 3x^2 + -60y + 5y^2 = 0 &= 3x^2 + 5y^2 - 60y = 0 = \\ &-60y + 5y^2 - 4(3)(5y^2) = -4 - 15y + 5y^2 - 60y^2 = -2 - 15y = 0 = -15y \\ x &= \frac{-(-60y + 5y^2) \pm \sqrt{(-60y + 5y^2)^2 - 4(3)(5y^2)}}{2(3)} = \frac{-(-60y + 5y^2) \pm \sqrt{3600y^2 - 60y^2 - 60y^2}}{2(3)} = \frac{-(-60y + 5y^2) \pm \sqrt{3000y^2}}{2(3)} = \frac{-(-60y + 5y^2) \pm 50\sqrt{6}y}{2(3)} = \frac{-(-60y + 5y^2) \pm 50\sqrt{6}y}{6} \end{aligned}$$

$$\text{[26]} \quad D_1 - y + z\tilde{N}^2 = 9 \implies D_1 - y + z = 3 \implies D_1 = y - z + 3$$

$$\text{[27]} \quad h = 1584 \implies -16t^2 + 320t = 1584 \implies -16t^2 + 320t - 1584 = 0 \implies$$

$$t^2 - 20t + 99 = 0 \text{ edivide by } -16f \implies t^2 - 9\tilde{N}t - 11\tilde{N} = 0 \implies t = 9 \text{ or } 11.$$

Thus, the object is 1584 feet above the ground after 9 seconds and after 11 seconds.

$$\text{[28]} \quad i^4 + 3 = i^4 \cdot i^3 = i^7 = i^3 = (i^2)(i) = (-1)(i) = -i = 0 - i, \text{ so } a = 0 \text{ and } b = -1.$$

$$\text{[29]} \quad x^3 - 64 = 0 \implies (x-4)(x^2 + 4x + 16) = 0 \implies x = 4 \text{ or } x^2 + 4x + 16 = 0.$$

$$\begin{aligned} \text{By the quadratic formula, } x &= \frac{-4 \pm \sqrt{4^2 - 4(1)(16)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 64}}{2} = \frac{-4 \pm \sqrt{-48}}{2} \approx 4, -2 \pm 2\sqrt{3}i \\ &= \frac{-4 \pm \sqrt{16 - 48}}{2} = \frac{-4 \pm \sqrt{-32}}{2} = \frac{-4 \pm 4\sqrt{-2}}{2} = -2 \pm 2\sqrt{-2}i. \end{aligned}$$

$$\begin{aligned} \text{[30]} \quad A = B - \sqrt{x^2 + r^2} &= \frac{A}{B} = -\sqrt{x^2 + r^2} = \sqrt{\frac{A^2}{B^2}} = \sqrt{x^2 + r^2} = \frac{A^2}{B^2} - r^2 = x^2 = \\ &x^2 = \frac{1}{B^2} (A^2 - B^2 r^2) \implies x = \pm \frac{1}{B} \sqrt{A^2 - B^2 r^2} \end{aligned}$$

$$\text{[31]} \quad 3x^{32}(x+2)^{65}(x-5)^{13}x^{-2/3}-4=0 \quad = \quad x^{32}=0 \text{ or } (x+2)^{65}=0 \text{ or } (x-5)^{13}=0 \text{ or } x^{-2/3}-4=0 \quad =$$

$$x=0 \text{ or } x=-2 \text{ or } x=5 \text{ or } x^{-2/3}=4. \text{ Now } x^{-2/3}=4 \quad = \quad x^{2/3} \cdot x^{3/2} = \sqrt[3]{(4)^{3/2}} =$$

$x = \sqrt[3]{8} = 2$. Thus, the solutions of the equation are 0, -2, 5, and $\sqrt[3]{8}$.

$$\text{[32]} \quad 20,000 = \frac{4}{3}\pi r_1^3 \quad = \quad r_1^3 = 15,000/\pi \quad = \quad r_1 = \sqrt[3]{15,000/\pi}.$$

Similarly, $25,000 = \frac{4}{3}\pi r_2^3 \quad = \quad r_2 = \sqrt[3]{18,750/\pi}$. The radius increased $\Delta r_2 - r_1 \approx 0.077$, or about 7.7%.

- 33** Plan A pays out \$3300 per month for 10 years before plan B starts, so its total payout is $\$10\text{N}\$12\text{N}\$3300\text{N} + 3300.$, where x is the number of months that plan B has paid out. Plan B's total payout is $4200.$
 Plan B Plan A = $4200. 396,000 + 3300. = 900. 396,000 = x. 440.$

It will take plan B 440 months (36 years, 8 months) to have a total payout at least as large as plan A.

34 $-1/3 - 2.1 + 6 \cdot 2 = -1/3 - 2.1 - 4 = 1/3 - 2.1 < 16 = -16 < 3 - 2. < 16 =$
 $-19 < -2. < 13 = \frac{19}{2} \cdot \frac{4}{2} - \frac{13}{2}.$ The solution in interval notation is $[-\frac{13}{2}, \frac{19}{2}]$.

35 $-(2. + 1) \cdot 3 = 2. \cdot 2 + . - 3 \cdot 0 = (2. + 3)(. - 1) \cdot 0 =$ the solution is $\mathbb{R} - \infty, -\frac{3}{2} \cup (1, \infty).$

36 $\frac{(x + 1)(x - 7)}{(x - 1)(x - 4)} < 0 = \frac{x - 7}{(x - 1)(x - 4)} < 0$ include -1 =
 $\frac{1}{x - 4} \cdot 0$ cancel, change inequality, exclude 7 = $x - 4 > 0$ exclude 4 = $x > 4 =$

the solution is $\mathbb{R} - 1 \times \cup (4, 7) \cup (7, \infty).$

37 $\frac{2}{x - 3} < \frac{2}{x + 1} = \frac{2}{x - 3} - \frac{2}{x + 1} < 0 = \frac{2(x + 1) - 2(x - 3)}{(x - 3)(x + 1)} < 0 =$

$\frac{x - 3 \cdot x + 1}{2x + 2 - 2x + 6} < 0 = \frac{x - 3 \cdot x + 1}{8} < 0 = (x - 3)(x + 1) < 0$ $\in (-1, 3)$
 $\frac{(x - 3)(x + 1)}{(x - 3)(x + 1)} < 0 = \frac{(x - 3)(x + 1)}{(x - 3)(x + 1)} < 0 = (x - 3)(x + 1) < 0$ $\in (-1, 3)$

Interval	$(-\infty, -1)$	$(-1, 3)$	$(3, \infty)$
Sign of $x - 3$	-	-	+
Sign of $x + 1$	-	+	+
Resulting sign	+	-	+

- 38** Let P and W denote the length and width of the rectangle. Then $P + W = 14$, so $P = 14 - W$ and the area is

$A = P \cdot W = (14 - W) \cdot W.$ Since $A = 45$, we have $(14 - W) \cdot W = 45 = -W^2 + 14W = 45 =$

$-W^2 + 14W - 45 = 0 = W^2 - 14W + 45 < 0 = (W - 5)(W - 9) < 0.$

Interval	$(-\infty, 5)$	$(5, 9)$	$(9, \infty)$
Sign of $W - 5$	-	+	+
Sign of $W - 9$	-	-	+
Resulting sign	+	-	+

From the sign chart, we see that the inequality is satisfied for $5 < W < 9$. Of course, once the width passes 7, it becomes the length, but that's not the point of the problem.

