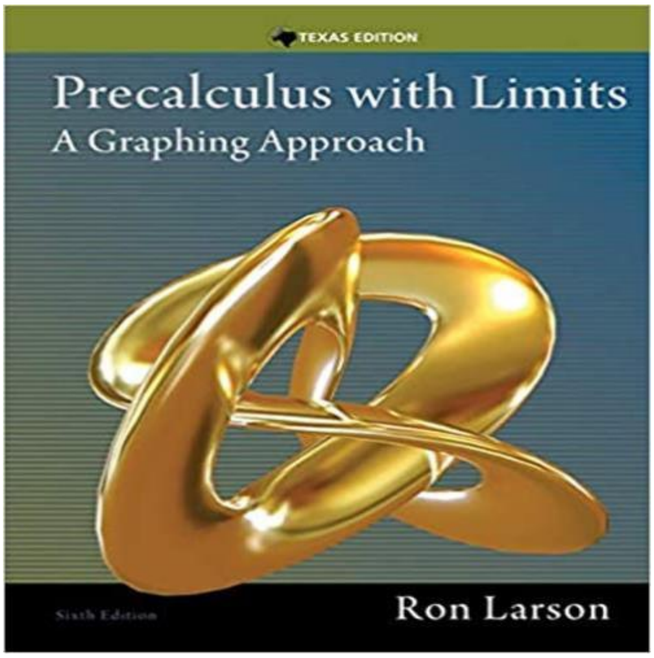
Solution Manual for Precalculus with Limits A Graphing Approach Texas Edition 6th Edition Larson 1285867718 9781285867717



C H A P T E R 2

Section 2.1

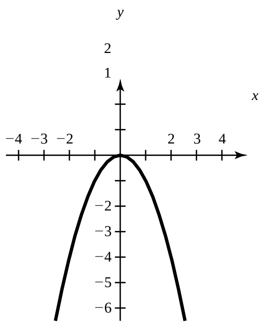
1. nonnegative integer, real

3. Yes, $f(x) = (x-2)^2 + 3$ is in the form $f(x) = a(x-h)^2 + k$. The vertex is $(2, 3)$.

5. $f(x) = (x-2)^2$ opens upward and has vertex $(2, 0)$. Matches graph (c).

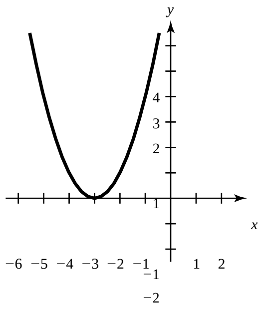
7. $f(x) = x^2 + 3$ opens upward and has vertex $(0, 3)$. Matches graph (b).

9.



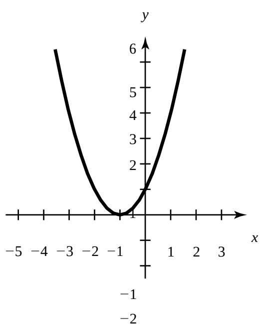
The graph of $y = -x^2$ is a reflection of $y = x^2$ in the x-axis.

11.

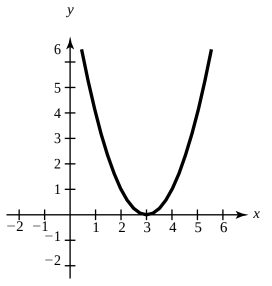


The graph of $y = (x+3)^2$ is a horizontal shift three units to the left of $y = x^2$.

13.



15.



The graph of $y = (x-3)^2$ is a horizontal shift three units to the right of $y = x^2$.

17. $f(x) = 25 - x^2$

$= -x^2 + 25$

A parabola opening downward with vertex $(0, 25)$

19. $f(x) = \frac{1}{2}x^2 - 4$

A parabola opening upward with vertex $(0, -4)$

21. $f(x) = (x+4)^2 - 3$

A parabola opening upward with vertex $(-4, -3)$

23. $h(x) = x^2 - 8x + 16$

$= (x-4)^2$

A parabola opening upward with vertex $(4, 0)$

25. $f(x) = x^2 - x + \frac{5}{4}$

$= (x^2 - x) + \frac{5}{4}$

$= \left(x^2 - x + \frac{1}{4}\right) + \frac{5}{4} - \frac{1}{4}$

$= \left(x - \frac{1}{2}\right)^2 + 1$

A parabola opening upward with vertex $\left(\frac{1}{2}, 1\right)$

27. $f(x) = -x^2 + 2x + 5$

$= -(x^2 - 2x) + 5$

$= -(x^2 - 2x + 1) + 5 + 1$

The graph of $y=(x+1)^2$ is a horizontal shift one unit to the left of $y=x^2$.

$= -(x-1)^2 + 6$

A parabola opening downward with vertex $(1,6)$

29. $h(x) = 4x^2 - 4x + 21$

$= 4(x^2 - x) + 21$

$= 4\left(x^2 - x + \frac{1}{4}\right) + 21 - 4\left(\frac{1}{4}\right)$

$= 4\left(x - \frac{1}{2}\right)^2 + 20$

A parabola opening upward with vertex $\left(\frac{1}{2}, 20\right)$

31. $f(x) = -(x^2 + 2x - 3)$

$= -(x^2 + 2x) + 3$

$= -(x^2 + 2x + 1) + 3 + 1$

$= -(x + 1)^2 + 4$

$-(x^2 + 2x - 3) = 0$

$-(x + 3)(x - 1) = 0$

$x + 3 = 0 \Rightarrow x = -3$

$x - 1 = 0 \Rightarrow x = 1$

A parabola opening downward with vertex $(-1, 4)$ and x-intercepts $(-3, 0)$ and $(1, 0)$.

33. $g(x) = x^2 + 8x + 11$

$= (x^2 + 8x) + 11$

$= (x^2 + 8x + 16) + 11 - 16$

$= (x + 4)^2 - 5$

$x^2 + 8x + 11 = 0$

$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(11)}}{2(1)}$

$= \frac{-8 \pm \sqrt{64 - 44}}{2}$

$= \frac{-8 \pm \sqrt{20}}{2}$

$= \frac{-8 \pm 2\sqrt{5}}{2}$

$= -4 \pm \sqrt{5}$

A parabola opening upward with vertex $(-4, -5)$ and x-intercepts $(-4 \pm \sqrt{5}, 0)$.

35. $f(x) = -2x^2 + 16x - 31$

$= -2(x^2 - 8x) - 31$

$= -2(x^2 - 8x + 16) - 31 + 32$

$= -2(x - 4)^2 + 1$

$-2x^2 + 16x - 31 = 0$

$x = \frac{-16 \pm \sqrt{16^2 - 4(-2)(-31)}}{2(-2)}$

$= \frac{-16 \pm \sqrt{256 - 248}}{-4}$

$= \frac{-16 \pm \sqrt{8}}{-4}$

$= \frac{-16 \pm 2\sqrt{2}}{-4}$

$= 4 \pm \frac{1}{2}\sqrt{2}$

A parabola opening downward with vertex (4, 1) and x-intercepts $\left(4 \pm \frac{1}{2}\sqrt{2}, 0\right)$

37. $(-1, 4)$ is the vertex.

$f(x) = a(x + 1)^2 + 4$

Since the graph passes through the point $(1, 0)$, we have:

$0 = a(1 + 1)^2 + 4$

$0 = 4a + 4$

$-1 = a$

Thus, $f(x) = -(x + 1)^2 + 4$. Note that $(-3, 0)$ is on the parabola.

39. $(-2, 5)$ is the vertex.

$f(x) = a(x + 2)^2 + 5$

Since the graph passes through the point $(0, 9)$, we have:

$9 = a(0 + 2)^2 + 5$

$4 = 4a$

$1 = a$

Thus, $f(x) = (x + 2)^2 + 5$.

41. $(1, -2)$ is the vertex.

$f(x) = a(x-1)^2 - 2$

Since the graph passes through the point $(-1, 14)$, we have:

$$\begin{aligned} 14 &= a(-1-1)^2 - 2 \\ 14 &= 4a - 2 \\ 16 &= 4a \\ 4 &= a \end{aligned}$$

Thus, $f(x) = 4(x-1)^2 - 2$.

43. $\left(\frac{1}{2}, 1\right)$ is the vertex.

$$f(x) = a\left(x - \frac{1}{2}\right)^2 + 1$$

Since the graph passes through the point $\left(-2, -\frac{21}{5}\right)$, we have:

$$-\frac{21}{5} = a\left(-2 - \frac{1}{2}\right)^2 + 1$$

$$-\frac{21}{5} = \frac{25}{4}a + 1$$

$$-\frac{26}{5} = \frac{25}{4}a$$

$$-\frac{104}{125} = a$$

Thus, $f(x) = -\frac{104}{125}\left(x - \frac{1}{2}\right)^2 + 1.$

45. $y = x^2 - 4x - 5$

x-intercepts: $(5, 0)$, $(-1, 0)$

$$0 = x^2 - 4x - 5$$

$$0 = (x - 5)(x + 1)$$

$x = 5$ or $x = -1$

47. $y = x^2 + 8x + 16$

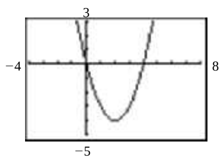
x-intercept: $(-4, 0)$

$$0 = x^2 + 8x + 16$$

$$0 = (x + 4)^2$$

$$x = -4$$

49. $y = x^2 - 4x$



x-intercepts: $(0, 0)$, $(4, 0)$

$$0 = x^2 - 4x$$

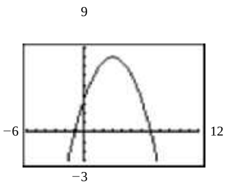
$$0 = x(x - 4)$$

$$0 = 2x^2 - 7x - 30$$

$$0 = (2x + 5)(x - 6)$$

$x = -\frac{5}{2}$ or $x = 6$

53. $y = -\frac{1}{2}(x^2 - 6x - 7)$



x-intercepts: $(-1, 0)$, $(7, 0)$

$$0 = -\frac{1}{2}(x^2 - 6x - 7)$$

$$0 = x^2 - 6x - 7$$

$$0 = (x + 1)(x - 7)$$

$$x = -1, 7$$

55. $f(x) = [x - (-1)](x - 3)$, opens upward

$$= (x + 1)(x - 3)$$
$$= x^2 - 2x - 3$$

$g(x) = -[x - (-1)](x - 3)$, opens downward

$$= -(x + 1)(x - 3)$$
$$= -(x^2 - 2x - 3)$$
$$= -x^2 + 2x + 3$$

Note: $f(x) = a(x + 1)(x - 3)$ has x-intercepts $(-1, 0)$ and $(3, 0)$ for all real numbers $a \neq 0$.

57. $f(x) = [x - (-3)]\left[x - \left(-\frac{1}{2}\right)\right](2)$, opens upward

$$= (x + 3)\left(x + \frac{1}{2}\right)(2)$$
$$= (x + 3)(2x + 1)$$
$$= 2x^2 + 7x + 3$$

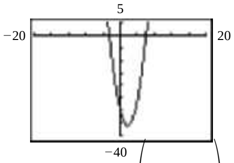
$g(x) = -(2x^2 + 7x + 3)$, opens downward

$$= -2x^2 - 7x - 3$$

Note: $f(x) = a(x + 3)(2x + 1)$ has x-intercepts $(-3, 0)$

$x = 0$ or $x = 4$

51. $y = 2x^2 - 7x - 30$



x-intercepts: $\left(-\frac{5}{2},\ 0\right)$, $(6,\ 0)$

and $\left(-\frac{1}{2},\ 0\right)$ for all real numbers $a \neq 0$.

59. Let $x =$ the first number and $y =$ the second number.

Then the sum is $x + y = 110 \Rightarrow y = 110 - x$.

The product is

$P(x) = xy = x(110 - x) = 110x - x^2$.

$P(x) = -x^2 + 110x$

$= -(x^2 - 110x + 3025 - 3025)$
$= -[(x-55)^2 - 3025]$
$= -(x-55)^2 + 3025$

The maximum value of the product occurs at the vertex of $P(x)$ and is 3025. This happens when $x = y = 55$.

61. Let x be the first number and y be the second number. Then $x + 2y = 24 \Rightarrow x = 24 - 2y$. The product is

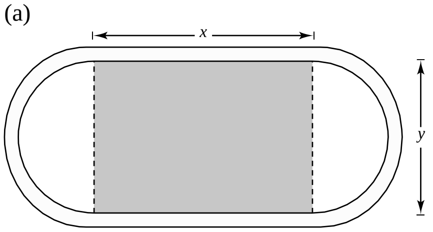
$$P = xy = (24 - 2y)y = 24y - 2y^2.$$

Completing the square,

$$P = -2y^2 + 24y$$
$$= -2(y^2 - 12y + 36) + 72$$
$$= -2(y-6)^2 + 72.$$

The maximum value of the product P occurs at the vertex of the parabola and equals 72. This happens when $y = 6$ and $x = 24 - 2(6) = 12$.

63. (a)



(b) Radius of semicircular ends of track: $r = \frac{1}{2}y$

Distance around two semicircular parts of track:

$$d = 2\pi r = 2\pi\left(\frac{1}{2}y\right) = \pi y$$

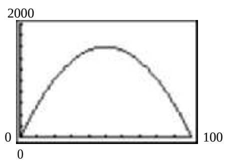
(c) Distance traveled around track in one lap:

$$d = \pi y + 2x = 200$$
$$\pi y = 200 - 2x$$
$$y = \frac{200 - 2x}{\pi}$$

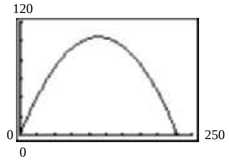
(d) Area of rectangular region: $A = xy = x\left(\frac{200 - 2x}{\pi}\right)$

(e) The area is maximum when $x = 50$ and

$$y = \frac{200 - 2(50)}{\pi} = \frac{100}{\pi}.$$



65. (a)



(b) When $x = 0$, $y = \frac{3}{2}$ feet.

(c) The vertex occurs at

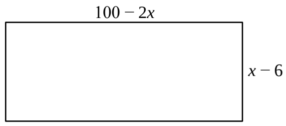
$$x = \frac{-b}{2a} = \frac{-9/5}{2(-16/2025)} = \frac{3645}{32} \approx 113.9.$$

The maximum height is

$$y = \frac{-16}{2025}\left(\frac{3645}{32}\right)^2 + \frac{9}{5}\left(\frac{3645}{32}\right) + \frac{3}{2}$$
$$\approx 104.0 \text{ feet.}$$

(d) Using a graphing utility, the zero of y occurs at $x \approx 228.6$, or 228.6 feet from the punter.

67. (a)



$$A = lw$$
$$A = (100 - 2x)(x - 6)$$
$$A = -2x^2 + 112x - 600$$

(b) $Y_1 = -2x^2 + 112x - 600$

X	Y
25	950
26	960
27	966
28	968
29	966
30	960

The area is maximum when $x = 28$ inches.

69. $R(p) = -10p^2 + 1580p$

(a) When $p = \$50$, $R(50) = \$54{,}000$.
When $p = \$70$, $R(70) = \$61{,}600$.
When $p = \$90$, $R(90) = \$61{,}200$.

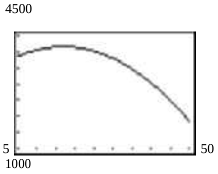
(b) The maximum R occurs at the vertex,

$$p = \frac{-b}{2a}$$
$$p = \frac{-1580}{2(-10)} = \$79$$

(c) When $p = \$79$, $R(79) = \$62{,}410$.

(d) Answers will vary.

71. (a)



(b) Using the graph, during 1966 the maximum average annual consumption of cigarettes appears to have occurred and was 4155 cigarettes per person.

Yes, the warning had an effect because the maximum consumption occurred in 1966 and consumption decreased from then on.

(c) In 2000, $C(50) = 1852$ cigarettes per person.

$\frac{1852}{365} \approx 5$ cigarettes per day

73. True.

$-12x^2 - 1 = 0$

$12x^2 = -1$, impossible

75. The parabola opens downward and the vertex is $(-2, -4)$. Matches (c) and (d).

77. The graph of $f(x) = (x - z)^2$ would be a horizontal shift z units to the right of $g(x) = x^2$.

79. The graph of $f(x) = z(x - 3)^2$ would be a vertical stretch $(z > 1)$ and horizontal shift three units to the right of $g(x) = x^2$. The graph of $f(x) = z(x - 3)^2$ would be a vertical shrink $(0 < z < 1)$ and horizontal shift three units to the right of $g(x) = x^2$.

81. For $a < 0$, $f(x) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$ is a maximum when $x = \frac{-b}{2a}$. In this case, the maximum value is $c - \frac{b^2}{4a}$. Hence,

$$25 = -75 - \frac{b^2}{4(-1)}$$

$$-100 = 300 - b^2$$

$$400 = b^2$$

$$b = \pm 20.$$

$$10 = 26 - \frac{b^2}{4}$$

$$40 = 104 - b^2$$

$$b^2 = 64$$

$$b = \pm 8.$$

85. Let x = first number and y = second number.
Then $x + y = s$ or $y = s - x$.
The product is given by $P = xy$ or $P = x(s - x)$.

$$P = x(s - x)$$

$$P = sx - x^2$$

The maximum P occurs at the vertex when $x = \frac{-b}{2a}$.

$$x = \frac{-s}{2(-1)} = \frac{s}{2}$$

When $x = \frac{s}{2}$, $y = s - \frac{s}{2} = \frac{s}{2}$.

So, the numbers x and y are both $\frac{s}{2}$.

87. $y = ax^2 + bx - 4$

$(1, 0)$ on graph: $0 = a + b - 4$

$(4, 0)$ on graph: $0 = 16a + 4b - 4$

From the first equation, $b = 4 - a$.

Thus, $0 = 16a + 4(4 - a) - 4 = 12a + 12 \Rightarrow a = -1$ and hence $b = 5$, and $y = -x^2 + 5x - 4$.

89. $x + y = 8 \Rightarrow y = 8 - x$

$$-\frac{2}{3}x + 8 - x = 6$$

$$-\frac{5}{3}x + 8 = 6$$

$$-\frac{5}{3}x = -2$$

$$x = 1.2$$

$$y = 8 - 1.2 = 6.8$$

The point of intersection is $(1.2, 6.8)$.

91. $y = x + 3 = 9 - x^2$

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0$$

$$x = -3, \quad x = 2$$

$$y = -3 + 3 = 0$$

$$y = 2 + 3 = 5$$

83. For $a > 0$, $f(x) = a\left(x + \right.$

$$\left(\frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$$ is a minimum

Thus, $(-3,\ 0)$ and $(2,\ 5)$ are the points of intersection.

when $x = \frac{-b}{2a}$. In this case, the minimum value is $c - \frac{b^2}{4a}$. Hence,

93. Answers will vary. (Make a Decision)

Section 2.2

1. continuous

3. (a) solution

(b) $(x - a)$

(c) $(a, 0)$

5. No. If f is an even-degree fourth-degree polynomial function, its left and right end behavior is either that it rises left and right or falls left and right.

7. Because f is a polynomial, it is a continuous on $[x_1, x_2$ and $f(x_1) < 0$ and $f(x_2) > 0$. Then $f(x) = 0$

for some value of x in $[x_1, x_2]$.

9. $f(x) = -2x + 3$ is a line with y-intercept $(0, 3)$. Matches graph (f).

11. $f(x) = -2x^2 - 5x$ is a parabola with x-intercepts $(0, 0)$ and $\left(-\frac{5}{2}, 0\right)$ and opens downward. Matches graph (c).

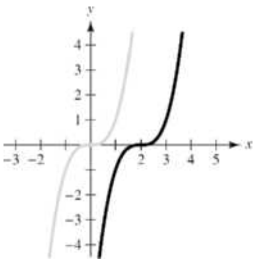
13. $f(x) = -\frac{1}{4}x^4 + 3x^2$ has intercepts $(0, 0)$ and

$(\pm 2\sqrt{3}, 0)$. Matches graph (e).

15. $f(x) = x^4 + 2x^3$ has intercepts $(0, 0)$ and $(-2, 0)$. Matches graph (g).

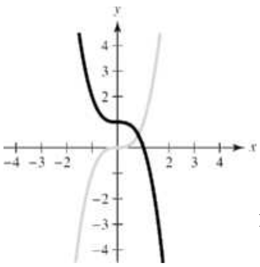
17. The graph of $f(x) = (x - 2)^3$ is a horizontal shift two

units to the right of $y = x^3$.



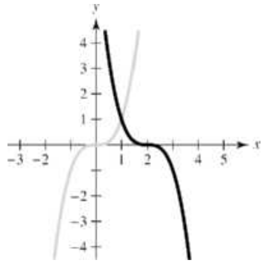
19. The graph of $f(x) = -x^3 + 1$ is a reflection in the x-axis

and a vertical shift one unit upward of $y = x^3$.

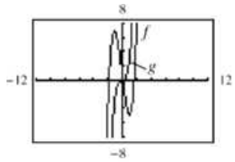


21. The graph of $f(x) = -(x - 2)^3$ is a horizontal shift two

units to the right and a reflection in the x-axis of $y = x^3$.

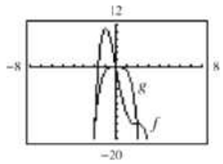


23.



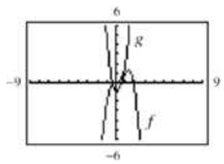
Yes, because both graphs have the same leading coefficient.

25.



Yes, because both graphs have the same leading coefficient.

27.



No, because the graphs have different leading coefficients.

29. $f(x) = 2x^4 - 3x + 1$

Degree: 4

Leading coefficient: 2

The degree is even and the leading coefficient is positive. The graph rises to the left and right.

31. $g(x) = 5 - \frac{7}{2}x - 3x^2$

Degree: 2

Leading coefficient: –3

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

33. $f(x) = \dfrac{6x^5 - 2x^4 + 4x^2 - 5x}{3}$

Degree: 5

Leading coefficient: $\frac{6}{3} = 2$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

35. $h(t) = -\frac{2}{3}\left(t^2 - 5t + 3\right)$

Degree: 2

Leading coefficient: $-\frac{2}{3}$

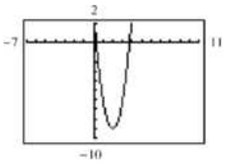
The degree is even and the leading coefficient is negative. The graph falls to the left and right.

37. (a) $f(x) = 3x^2 - 12x + 3$

$= 3\left(x^2 - 4x + 1\right) = 0$

$x = \dfrac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$

(b)



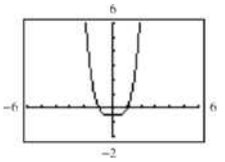
(c) $x \approx 3.732, 0.268$; the answers are approximately the same.

39. (a) $g(t) = \frac{1}{2}t^4 - \frac{1}{2}$

$= \frac{1}{2}\left(t + 1\right)(t - 1)(t^2 + 1) = 0$

$t = \pm 1$

(b)



(c) $t = \pm 1$; the answers are the same.

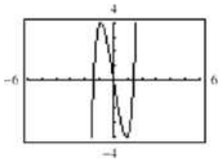
41. (a) $f(x) = x^5 + x^3 - 6x$

$= x\left(x^4 + x^2 - 6\right)$

$= x\left(x^2 + 3\right)(x^2 - 2) = 0$

$x = 0, \ \pm\sqrt{2}$

(b)



(c) $x = 0, 1.414, -1.414$; the answers are approximately the same.

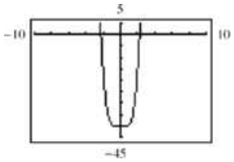
43. (a) $f(x) = 2x^4 - 2x^2 - 40$

$= 2\left(x^4 - x^2 - 20\right)$

$= 2\left(x^2 + 4\right)\left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right) = 0$

$x = \pm\sqrt{5}$

(b)



(c) $x = 2.236, -2.236$; the answers are approximately the same.

45. (a) $f(x) = x^3 - 4x^2 - 25x + 100$

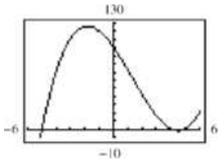
$= x^2(x - 4) - 25(x - 4)$

$= (x^2 - 25)(x - 4)$

$= (x - 5)(x + 5)(x - 4) = 0$

$x = \pm 5, \ 4$

(b)



(c) $x = 4, 5, -5$; the answers are the same.

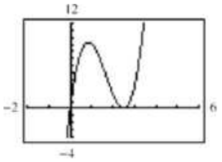
47. (a) $y = 4x^3 - 20x^2 + 25x$

$0 = 4x^3 - 20x^2 + 25x$

$0 = x(2x - 5)^2$

$x = 0, \ \frac{5}{2}$

(b)



(c) $x = 0, \ \frac{5}{2}$; the answers are the same.

49. $f(x) = x^2 - 25$

$= (x + 5)(x - 5)$

$x = \pm 5$ (multiplicity 1)

51. $h(t) = t^2 - 6t + 9$

$= (t - 3)^2$

$t = 3$ (multiplicity 2)

53. $f(x) = x^2 + x - 2$

$= (x + 2)(x - 1)$

$x = -2, 1$ (multiplicity 1)

55. $f(t) = t^3 - 4t^2 + 4t$

$= t(t - 2)^2$

$t = 0$ (multiplicity 1), 2 (multiplicity 2)

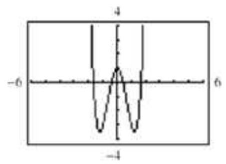
57. $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x - \frac{3}{2}$

$= \frac{1}{2}(x^2 + 5x - 3)$

$x = \frac{-5 \pm \sqrt{25 - 4(-3)}}{2} = -\frac{5}{2} \pm \frac{\sqrt{37}}{2}$

$\approx 0.5414, \ -5.5414$ (multiplicity 1)

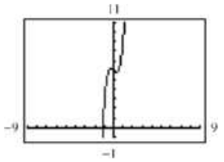
59. $f(x) = 2x^4 - 6x^2 + 1$



Zeros: $x \approx \pm 0.421, \pm 1.680$

Relative maximum: $(0, 1)$

61. $f(x) = x^5 + 3x^3 - x + 6$

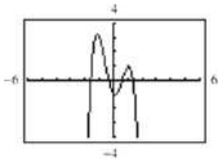


Zero: $x \approx -1.178$

Relative maximum: $(-0.324, \ 6.218)$

Relative minimum: $(0.324, \ 5.782)$

63. $f(x) = -2x^4 + 5x^2 - x - 1$



Zeros: $-1.618, \ -0.366, \ 0.618, 1.366$

Relative minimum: $(0.101, \ -1.050)$

Relative maxima: $(-1.165, \ 3.267), \ (1.064, \ 1.033)$

65. $f(x) = (x - 0)(x - 4) = x^2 - 4x$

Note: $f(x) = a(x - 0)(x - 4) = ax(x - 4)$ has zeros 0 and 4 for all nonzero real numbers a.

67. $f(x) = (x - 0)(x + 2)(x + 3) = x^3 + 5x^2 + 6x$

Note: $f(x) = ax(x + 2)(x + 3)$ has zeros $0, \ -2,$ and -3 for all nonzero real numbers a.

69. $f(x) = (x - 4)(x + 3)(x - 3)(x - 0)$

$= (x - 4)(x^2 - 9)x$

$= x^4 - 4x^3 - 9x^2 + 36x$

Note: $f(x) = a(x^4 - 4x^3 - 9x^2 + 36x)$ has zeros $4, \ -3, \ 3,$ and 0 for all nonzero real numbers a.

71. $f(x) = \left[x - \left(1 + \sqrt{3}\right)\right]\left[x - \left(1 - \sqrt{3}\right)\right]$

$= \left[(x - 1) - \sqrt{3}\right]\left[(x - 1) + \sqrt{3}\right]$

$= (x - 1)^2 - \left(\sqrt{3}\right)^2$

$= x^2 - 2x + 1 - 3$

$= x^2 - 2x - 2$

Note:

Relative minima: $(1.225, -3.5)$, $(-1.225, -3.5)$

$f(x) = a(x^2 - 2x - 2)$ has zeros $1 + \sqrt{3}$ and $1 - \sqrt{3}$ for all nonzero real numbers a.

73. $f(x) = (x - 2)\left[x - \left(4 + \sqrt{5}\right)\right]\left[x - \left(4 - \sqrt{5}\right)\right]$

$= (x - 2)\left[(x - 4) - \sqrt{5}\right]\left[(x - 4) + \sqrt{5}\right]$

$= (x - 2)[(x - 4)^2 - 5]$

$= x^3 - 10x^2 + 27x - 22$

Note: $f(x) = a(x - 2)[(x - 4)^2 - 5]$ has zeros $2,\ 4 + \sqrt{5},$ and $4 - \sqrt{5}$ for all nonzero real numbers a.

75. $f(x) = (x + 2)^2(x + 1) = x^3 + 5x^2 + 8x + 4$

Note: $f(x) = a(x + 2)^2(x + 1)$ has zeros $-2,\ -2,$ and -1 for all nonzero real numbers a.

77. $f(x) = (x + 4)^2(x - 3)^2$

$= x^4 + 2x^3 - 23x^2 - 24x + 144$

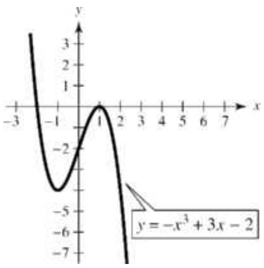
Note: $f(x) = a(x + 4)^2(x - 3)^2$ has zeros $-4,\ -4,\ 3,\ 3$ for all nonzero real numbers a.

79. $f(x) = -(x + 1)^2(x + 2)$

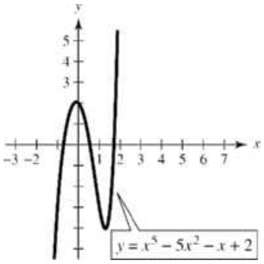
$= -x^3 - 4x^2 - 5x - 2$

Note: $f(x) = a(x + 1)^2(x + 2)^2,\ a < 0,$ has zeros $-1,\ -1,\ -2,$ rises to the left, and falls to the right.

81.



83.

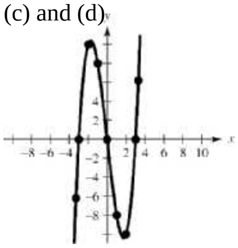


85. (a) The degree of *f* is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $f(x) = x^3 - 9x = x(x^2 - 9) = x(x - 3)(x + 3)$

Zeros: $0, 3, -3$

(c) and (d)

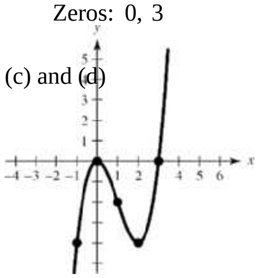


87. (a) The degree of *f* is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $f(x) = x^3 - 3x^2 = x^2(x - 3)$

Zeros: $0,\ 3$

(c) and (d)

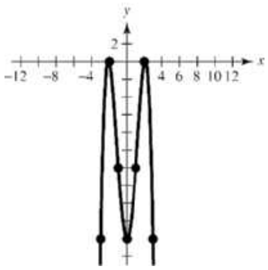


89. (a) The degree of *f* is even and the leading coefficient is -1. The graph falls to the left and falls to the right.

(b) $f(x) = -x^4 + 9x^2 - 20 = -(x^2 - 4)(x^2 - 5)$

Zeros: $\pm 2,\ \pm\sqrt{5}$

(c) and (d)



91. (a) The degree of f is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

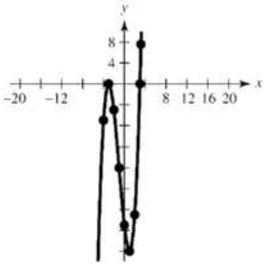
(b) $f(x) = x^3 + 3x^2 - 9x - 27 = x^2(x+3) - 9(x+3)$

$= (x^2 - 9)(x+3)$

$= (x-3)(x+3)^2$

Zeros: $3, \ -3$

(c) and (d)

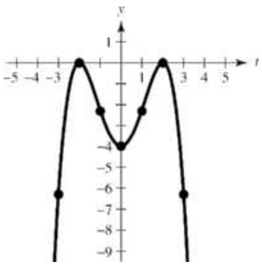


93. (a) The degree of g is even and the leading coefficient is $-\frac{1}{4}$, The graph falls to the left and falls to the right.

(b) $g(t) = -\frac{1}{4}(t^4 - 8t^2 + 16) = -\frac{1}{4}(t^2 - 4)^2$

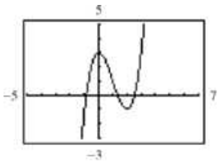
Zeros: $-2, \ -2, 2, 2$

(c) and (d)



95. $f(x) = x^3 - 3x^2 + 3$

(a)



The function has three zeros. They are in the intervals $(-1, 0)$, $(1, 2)$ and $(2, 3)$.

(b) Zeros: $-0.879, 1.347, 2.532$

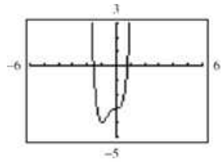
x	y
–0.9	–0.159
–0.89	–0.0813
–0.88	–0.0047
–0.87	0.0708
–0.86	0.14514
–0.85	0.21838
–0.84	0.2905

x	y
1.3	0.127
1.31	0.09979
1.32	0.07277
1.33	0.04594
1.34	0.0193
1.35	–0.0071
1.36	–0.0333

x	y
2.5	–0.125
2.51	–0.087
2.52	–0.0482
2.53	–0.0084
2.54	0.03226
2.55	0.07388
2.56	0.11642

97. $g(x) = 3x^4 + 4x^3 - 3$

(a)



The function has two zeros. They are in the intervals $(-2, -1)$ and $(0, 1)$.

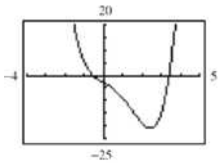
(b) Zeros: $-1.585, 0.779$

x	y_1
–1.6	0.2768
–1.59	0.09515
–1.58	–0.0812
–1.57	–0.2524
–1.56	–0.4184
–1.55	–0.5795
–1.54	–0.7356

x	y_1
0.75	–0.3633
0.76	–0.2432
0.77	–0.1193
0.78	0.00866
0.79	0.14066
0.80	0.2768
0.81	0.41717

99. $f(x) = x^4 - 3x^3 - 4x - 3$

(a)



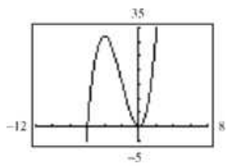
The function has two zeros. They are in the intervals $(-1, 0)$ and $(3, 4)$.

(b) Zeros: $-0.578, 3.418$

x	y_1
−0.61	0.2594
−0.60	0.1776
−0.59	0.09731
−0.58	0.0185
−0.57	−0.0589
−0.56	−0.1348
−0.55	−0.2094

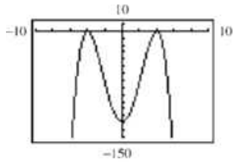
x	y_1
3.39	−1.366
3.40	−0.8784
3.41	−0.3828
3.42	0.12071
3.43	0.63205
3.44	1.1513
3.45	1.6786

101. $f(x) = x^2(x+6)$



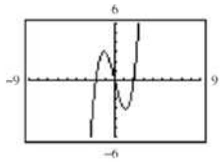
No symmetry
Two x-intercepts

103. $g(t) = -\frac{1}{2}(t-4)^2(t+4)^2$



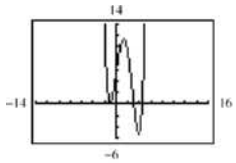
Symmetric with respect to the y-axis
Two x-intercepts

105. $f(x) = x^3 - 4x = x(x+2)(x-2)$



Symmetric with respect to the origin
Three x-intercepts

107. $g(x) = \frac{1}{5}(x+1)^2(x-3)(2x-9)$



No symmetry
Three x-intercepts

109. (a) Volume = length × width × height

Because the box is made from a square, length = width.

Thus: Volume = $(\text{length})^2 \times$ height = $(36-2x)^2 x$

(b) Domain: $0 < 36 - 2x < 36$

$-36 < -2x < 0$

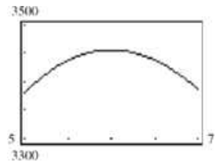
$18 > x > 0$

(c)

Height, x	Length and Width	Volume, V
1	$36 - 2(1)$	$1[36-2(1)]^2 = 1156$
2	$36 - 2(2)$	$2[36-2(2)]^2 = 2048$
3	$36 - 2(3)$	$3[36-2(3)]^2 = 2700$
4	$36 - 2(4)$	$4[36-2(4)]^2 = 3136$
5	$36 - 2(5)$	$5[36-2(5)]^2 = 3380$
6	$36 - 2(6)$	$6[36-2(6)]^2 = 3456$
7	$36 - 2(7)$	$7[36-2(7)]^2 = 3388$

Maximum volume is in the interval $5 < x < 7$.

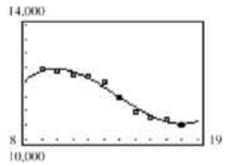
(d)



$x = 6$ when $V(x)$ is maximum.

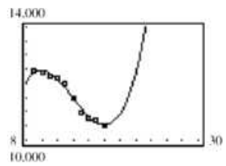
111. The point of diminishing returns (where the graph changes from curving upward to curving downward) occurs when $x = 200$ The point is (200, 160) which corresponds to spending $2,000,000 on advertising to obtain a revenue of $160 million.

113. (a)



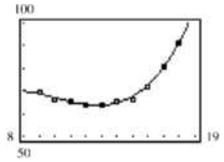
The model fits the data well.

(b)

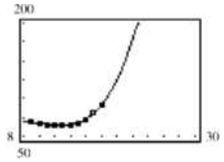


Answers will vary. Sample answer: You could use the model to estimate production in 2010 because the result is somewhat reasonable, but you would not use the model to estimate the 2020 production because the result is unreasonably high.

(c)



The model fits the data well.



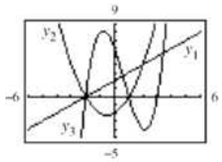
Answers will vary. Sample answer: You could use

the model to estimate production in 2010 because the result is somewhat reasonable, but you would not use the model to estimate the 2020 production because the result is unreasonably high.

115. True. The degree is odd and the leading coefficient is –1.

117. False. The graph crosses the *x*-axis at $x = -3$ and $x = 0$.

119.



The graph of y_3 will fall to the left and rise to the right. It will have another *x*-intercept at $(3, 0)$ of odd multiplicity (crossing the *x*-axis).

121. $(f+g)(-4) = f(-4) + g(-4)$

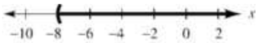
$= -59 + 128 = 69$

123. $(f \circ g)\left(-\frac{4}{7}\right) = f\left(-\frac{4}{7}\right) g\left(-\frac{4}{7}\right) = (-11)\left(\frac{8 \cdot 16}{49}\right)$

$= -\frac{1408}{49} \approx -28.7347$

125. $(f \circ g)(-1) = f(g(-1)) = f(8) = 109$

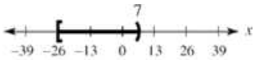
127. $3(x-5) < 4x - 7$



$3x - 15 < 4x - 7$

$-8 < x$

129. $\frac{5x - 2}{x - 7} \leq 4$



$\frac{5x - 2}{x - 7} - 4 \leq 0$

$\frac{5x - 2 - 4(x - 7)}{x - 7} \leq 0$

$\frac{x + 26}{x - 7} \leq 0$

$[x + 26 \geq 0 \text{ and } x - 7 < 0$ or $[x + 26 \leq 0 \text{ and } x - 7 > 0$

$[x \geq -26 \text{ and } x < 7$ or $[x \leq -26 \text{ and } x > 7$

$-26 \leq x < 7$ impossible

Section 2.3

1. $f(x)$ is the dividend, $d(x)$ is the divisor, $q(x)$ is the quotient, and $r(x)$ is the remainder.

3. constant term, leading coefficient

5. upper, lower

7. According to the Remainder Theorem, if you divide

$f(x)$ by $x - 4$ and the remainder is 7, then $f(4) = 7$.

9.

$$\begin{array}{r} 2x + 4 \\ x + 3 \overline{)2x^2 + 10x + 12} \\ \underline{2x^2 + 6x} \quad\quad \\ 4x + 12 \\ \underline{4x + 12} \\ 0 \end{array}$$

$\frac{2x^2 + 10x + 12}{x + 3} = 2x + 4, \ x \neq -3$

11. $x+2\overline{\big)\,x^4+5x^3+6x^2-x-2}$ with quotient x^3+3x^2-1

$$\begin{array}{l}
\underline{x^4+2x^3}\\
3x^3+6x^2\\
\underline{3x^3+6x^2}\\
-x-2\\
\underline{-x-2}\\
0
\end{array}$$

$$\frac{x^4+5x^3+6x^2-x-2}{x+2}=x^3+3x^2-1,\ x\neq-2$$

13. $4x+5\overline{\big)\,4x^3-7x^2-11x+5}$ with quotient x^2-3x+1

$$\begin{array}{l}
\underline{-4x^3+5x^2}\\
-12x^2-11x\\
\underline{-12x^2-15x}\\
4x+5\\
-\underline{4x+5}\\
0
\end{array}$$

$$\frac{4x^3-7x^2-11x+5}{4x+5}=x^2-3x+1,\ x\neq-\frac{5}{4}$$

15. $x+2\overline{\big)\,7x^3+0x^2+0x+3}$ with quotient $7x^2-14x+28$

$$\begin{array}{l}
\underline{7x^3+14x^2}\\
-14x^2\\
\underline{-14x^2-28x}\\
28x+3\\
\underline{28x+56}\\
-53
\end{array}$$

$$\frac{7x^3+3}{x+2}=7x^2-14x+28-\frac{53}{x+2}$$

17. $2x^2+0x+1\overline{\big)\,6x^3+10x^2+x+8}$ with quotient $3x+5$

$$\begin{array}{l}
\underline{6x^3+0x^2+3x}\\
10x^2-2x+8\\
\underline{10x^2+0x+5}\\
-2x+3
\end{array}$$

$$\frac{6x^3+10x^2+x+8}{2x^2+1}=3x+5-\frac{2x-3}{2x^2+1}$$

21. $x^2-2x+1\overline{\big)\,2x^3-4x^2-15x+5}$ with quotient $2x$

$$\begin{array}{l}
\underline{2x^3-4x^2+2x}\\
-17x+5
\end{array}$$

$$\frac{2x^3-4x^2-15x+5}{(x-1)^2}=2x-\frac{17x-5}{(x-1)^2}$$

23.
$$\begin{array}{c|rrrr}
5 & 3 & -17 & 15 & -25\\
 & & 15 & -10 & 25\\
\hline
 & 3 & -2 & 5 & 0
\end{array}$$

$$\frac{3x^3-17x^2+15x-25}{x-5}=3x^2-2x+5,\ x\neq5$$

25.
$$\begin{array}{c|rrrr}
3 & 6 & 7 & -1 & 26\\
 & & 18 & 75 & 222\\
\hline
 & 6 & 25 & 74 & 248
\end{array}$$

$$\frac{6x^3+7x^2-x+26}{x-3}=6x^2+25x+74+\frac{248}{x-3}$$

27.
$$\begin{array}{c|rrrr}
2 & 9 & -18 & -16 & 32\\
 & & 18 & 0 & -32\\
\hline
 & 9 & 0 & -16 & 0
\end{array}$$

$$\frac{9x^3-18x^2-16x+32}{x-2}=9x^2-16,\ x\neq2$$

29.
$$\begin{array}{c|rrrr}
-8 & 1 & 0 & 0 & 512\\
 & & -8 & 64 & -512\\
\hline
 & 1 & -8 & 64 & 0
\end{array}$$

$$\frac{x^3+512}{x+8}=x^2-8x+64,\ x\neq-8$$

31.
$$\begin{array}{c|rrrr}
-\frac{1}{2} & 4 & 16 & -23 & -15\\
 & & -2 & -7 & 15\\
\hline
 & 4 & 14 & -30 & 0
\end{array}$$

$$\frac{4x^3+16x^2-23x-15}{x+\frac{1}{2}}=4x^2+14x-30,\ x\neq-\frac{1}{2}$$

19.
$$\begin{array}{r} x \\ x^2+1\overline{\smash{)}x^3+0x^2+0x-9} \\ \underline{x^3 \qquad\quad +\ x} \\ -x-9 \end{array}$$

$$\frac{x^3-9}{x^2+1} = x - \frac{x+9}{x^2+1}$$

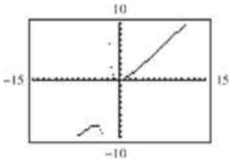
33. $y_2 = x - 2 + \dfrac{4}{x+2}$

$= \dfrac{(x-2)(x+2)+4}{x+2}$

$= \dfrac{x^2 - 4 + 4}{x+2}$

$= \dfrac{x^2}{x+2}$

$= y_1$



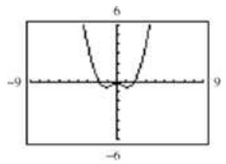
35. $y_2 = x^2 - 8 + \dfrac{39}{x^2+5}$

$= \dfrac{(x^2-8)(x^2+5)+39}{x^2+5}$

$= \dfrac{x^4 - 8x^2 + 5x^2 - 40 + 39}{x^2+5}$

$= \dfrac{x^4 - 3x^2 - 1}{x^2+5}$

$= y_1$



37. $f(x) = x^3 - x^2 - 14x + 11,\ k = 4$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 11 \\ & & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

$f(x) = (x-4)(x^2 + 3x - 2) + 3$

$f(4) = (0)(26) + 3 = 3$

39.

$$\begin{array}{r|rrrr} \sqrt{2} & 1 & 3 & -2 & -14 \\ & & \sqrt{2} & 2+3\sqrt{2} & 6 \\ \hline & 1 & 3+ & & \end{array}$$

43. $f(x) = 2x^3 - 7x + 3$

(a)
$$\begin{array}{r|rrrr} 1 & 2 & 0 & -7 & 3 \\ & & 2 & 2 & -5 \\ \hline & 2 & 2 & -5 & -2 \end{array} = f(1)$$

(b)
$$\begin{array}{r|rrrr} -2 & 2 & 0 & -7 & 3 \\ & & -4 & 8 & -2 \\ \hline & 2 & -4 & 1 & 1 \end{array} = f(-2)$$

(c)
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 0 & -7 & 3 \\ & & 1 & \frac{1}{2} & -\frac{13}{4} \\ \hline & 2 & 1 & -\frac{13}{2} & -\frac{1}{4} \end{array} = f\left(\frac{1}{2}\right)$$

(d)
$$\begin{array}{r|rrrr} 2 & 2 & 0 & -7 & 3 \\ & & 4 & 8 & 2 \\ \hline & 2 & 4 & 1 & 5 \end{array} = f(2)$$

45. $h(x) = x^3 - 5x^2 - 7x + 4$

(a)
$$\begin{array}{r|rrrr} 3 & 1 & -5 & -7 & 4 \\ & & 3 & -6 & -39 \\ \hline & 1 & -2 & -13 & -35 \end{array} = h(3)$$

(b)
$$\begin{array}{r|rrrr} 2 & 1 & -5 & -7 & 4 \\ & & 2 & -6 & -26 \\ \hline & 1 & -3 & -13 & -22 \end{array} = h(2)$$

(c)
$$\begin{array}{r|rrrr} -2 & 1 & -5 & -7 & 4 \\ & & -2 & 14 & -14 \\ \hline & 1 & -7 & 7 & -10 \end{array} = h(-2)$$

(d)
$$\begin{array}{r|rrrr} -5 & 1 & -5 & -7 & 4 \\ & & -5 & 50 & -215 \\ \hline & 1 & -10 & 43 & -211 \end{array} = h(-5)$$

47.
$$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$x^3 - 7x + 6 = (x-2)(x^2 + 2x - 3)$
$= (x-2)(x+3)(x-1)$

Zeros: $2,\ -3,\ 1$

2 3 2 −8

$f(x) = \left(x - \sqrt{2}\right)\left(x^2 + \left(3 + \sqrt{2}\right)x + 3\sqrt{2}\right) - 8$

$f\left(\sqrt{2}\right) = 0\left(4 + 6\sqrt{2}\right) - 8 = -8$

41.

$1-\sqrt{3}$	4	−6	−12	−4
		$4-4\sqrt{3}$	$10-2\sqrt{3}$	4
	4	$-2-4\sqrt{3}$	$-2-2\sqrt{3}$	0

$f(x) = (x - 1 + \sqrt{3})[4x^2 - (2 + 4\sqrt{3})x - (2 + 2\sqrt{3})]$

$f(1 - \sqrt{3}) = 0$

49.

$\frac{1}{2}$	2	−15	27	−10
		1	−7	10
	2	−14	20	0

$2x^3 - 15x^2 + 27x - 10$

$= (x - \frac{1}{2})(2x^2 - 14x + 20)$

$= (2x - 1)(x - 2)(x - 5)$

Zeros: $\frac{1}{2}$, 2, 5

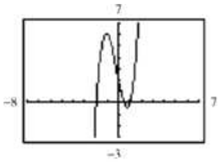
51. (a)

-2	2	1	-5	2
		-4	6	-2
	2	-3	1	0

(b) $2x^2 - 3x + 1 = (2x - 1)(x - 1)$
Remaining factors: $(2x - 1)$, $(x - 1)$

(c) $f(x) = (x + 2)(2x - 1)(x - 1)$

(d) Real zeros: $-2, \frac{1}{2}, 1$



53. (a)

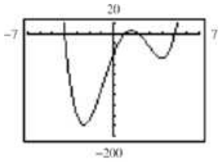
5	1	-4	-15	58	-40
		5	5	-50	40
	1	1	-10	8	0

-4	1	1	-10	8
		-4	12	-8
	1	-3	2	0

(b) $x^2 - 3x + 2 = (x - 2)(x - 1)$
Remaining factors: $(x - 2)$, $(x - 1)$

(c) $f(x) = (x - 5)(x + 4)(x - 2)(x - 1)$

(d) Real zeros: $5, -4, 2, 1$



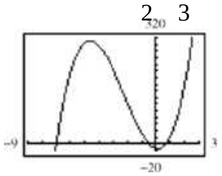
55. (a)

$-\frac{1}{2}$	6	41	-9	-14
		-3	-19	14
	6	38	-28	0

(b) $6x^2 + 38x - 28 = (3x - 2)(2x + 14)$
Remaining factors: $(3x - 2)$, $(x + 7)$

(c) $f(x) = (2x + 1)(3x - 2)(x + 7)$

(d) Real zeros: $-\frac{1}{2}, \frac{2}{3}, -7$



57. $f(x) = x^3 + 3x^2 - x - 3$

p = factor of -3
q = factor of 1

Possible rational zeros: $\pm 1, \pm 3$

$f(x) = x^2(x + 3) - (x + 3) = (x + 3)(x^2 - 1)$

Rational zeros: $\pm 1, -3$

59. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$

p = factor of -45
q = factor of 2

Possible rational zeros: $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45,$

$\pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{9}{2}, \pm \frac{15}{2}, \pm \frac{45}{2}$

Using synthetic division, -1, 3, and 5 are zeros.

$f(x) = (x + 1)(x - 3)(x - 5)(2x - 3)$

Rational zeros: $-1, 3, 5, \frac{3}{2}$

61. $f(x) = 2x^4 - x^3 + 6x^2 - x + 5$

4 variations in sign $\Rightarrow$ 4, 2, or 0 positive real zeros
$f(-x) = 2x^4 + x^3 + 6x^2 + x + 5$
0 variations in sign $\Rightarrow$ 0 negative real zeros

63. $g(x) = 4x^3 - 5x + 8$

2 variations in sign $\Rightarrow$ 2 or 0 positive real zeros
$g(-x) = -4x^3 + 5x + 8$

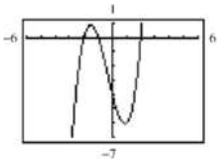
1 variation in sign $\Rightarrow$ 1 negative real zero

65. $f(x) = x^3 + x^2 - 4x - 4$

(a) $f(x)$ has 1 variation in sign $\Rightarrow$ 1 positive real zero.
$f(-x) = -x^3 + x^2 + 4x - 4$ has 2 variations in sign $\Rightarrow$ 2 or 0 negative real zeros.

(b) Possible rational zeros: $\pm 1, \pm 2, \pm 4$

(c)



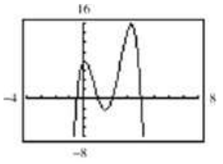
(d) Real zeros: −2, −1, 2

67. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

(a) $f(x)$ has variations in sign $\Rightarrow$ 3 or 1 positive real zeros.
$f(-x) = -2x^4 - 13x^3 - 21x^2 - 2x + 8$ has 1 variation in sign $\Rightarrow$ 1 negative real zero.

(b) Possible rational zeros: $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

(c)



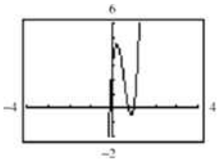
(d) Real zeros: $-\frac{1}{2}, 1, 2, 4$

69. $f(x) = 32x^3 - 52x^2 + 17x + 3$

(a) $f(x)$ has 2 variations in sign $\Rightarrow$ 2 or 0 positive real zeros.
$f(-x) = -32x^3 - 52x^2 - 17x + 3$ has 1 variation in sign $\Rightarrow$ 1 negative real zero.

(b) Possible rational zeros:
$\pm\frac{1}{32}, \pm\frac{1}{16}, \pm\frac{1}{8}, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{32}, \pm\frac{3}{16}, \pm\frac{3}{8}, \pm\frac{3}{4}, \pm\frac{3}{2}, \pm 3$

(c)



(d) Real zeros: $1, \frac{3}{4}, -\frac{1}{8}$

71. $f(x) = x^4 - 4x^3 + 15$

$$\begin{array}{r|rrrrr} 4 & 1 & -4 & 0 & 0 & 15 \\ & & 4 & 0 & 0 & 0 \\ \hline & 1 & 0 & 0 & 0 & 15 \end{array}$$

4 is an upper bound.

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & 0 & 0 & 15 \\ & & -1 & 5 & -5 & 5 \\ \hline & 1 & -5 & 5 & -5 & 20 \end{array}$$

-1 is a lower bound.
Real zeros: 1.937, 3.705

73. $f(x) = x^4 - 4x^3 + 16x - 16$

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & 0 & 16 & -16 \\ & & 25 & 105 & 525 & 2705 \\ \hline & 5 & 21 & 105 & 541 & 2689 \end{array}$$

5 is an upper bound.

$$\begin{array}{r|rrrrr} -3 & 1 & -4 & 0 & 16 & -16 \\ & & -3 & 21 & -63 & 141 \\ \hline & 1 & -7 & 21 & -47 & 125 \end{array}$$

-3 is a lower bound.

Real zeros: $-2, 2$

75. $P(x) = x^4 - \frac{25}{4}x^2 + 9$

$= \frac{1}{4}(4x^4 - 25x^2 + 36)$

$= \frac{1}{4}(4x^2 - 9)(x^2 - 4)$

$= \frac{1}{4}(2x + 3)(2x - 3)(x + 2)(x - 2)$

The rational zeros are $\pm\frac{3}{2}$ and ± 2.

77. $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4}$

$= \frac{1}{4}(4x^3 - x^2 - 4x + 1)$

$= \frac{1}{4}[x^2(4x - 1) - (4x - 1)]$

$= \frac{1}{4}(4x - 1)(x^2 - 1)$

$= \frac{1}{4}(4x - 1)(x + 1)(x - 1)$

The rational zeros are $\frac{1}{4}$ and ± 1.

79. $f(x) = x^3 - 1$

$= (x - 1)(x^2 + x + 1)$

Rational zeros: $1\ (x = 1)$

Irrational zeros: 0

Matches (d).

81. $f(x) = x^3 - x = x(x+1)(x-1)$

Rational zeros: $3\ (x = 0, \pm 1)$

Irrational zeros: 0

Matches (b).

83. $y = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

Using the graph and synthetic division, $-\frac{1}{2}$ is a zero.

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 2 & -9 & 5 & 3 & -1 \\ & & -1 & 5 & -5 & 1 \\ \hline & 2 & -10 & 10 & -2 & 0 \end{array}$$

$y = \left(x + \frac{1}{2}\right)(2x^3 - 10x^2 + 10x - 2)$

$x = 1$ is a zero of the cubic, so
$y = (2x+1)(x-1)(x^2 - 4x + 1)$.

For the quadratic term, use the Quadratic Formula.

$$x = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$$

The real zeros are $-\frac{1}{2}, 1, 2 \pm \sqrt{3}$.

85. $y = -2x^4 + 17x^3 - 3x^2 - 25x - 3$

Using the graph and synthetic division, -1 and $\frac{3}{2}$ are zeros.

$y = -(x+1)(2x-3)(x^2 - 8x - 1)$

For the quadratic term, use the Quadratic Formula.

$$x = \frac{8 \pm \sqrt{64+4}}{2} = 4 \pm \sqrt{17}$$

The real zeros are $-1, \frac{3}{2}, 4 \pm \sqrt{17}$.

87. $3x^4 - 14x^2 - 4x = 0$

$x(3x^3 - 14x - 4) = 0$

$x = 0$ is a real zero.

Possible ratoinal zeros: $\pm 1, \pm 2, \pm 4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

$$\begin{array}{r|rrrr} -2 & 3 & 0 & -14 & -4 \\ & & -6 & 12 & 4 \\ \hline & 3 & -6 & -2 & 0 \end{array}$$

$x = -2$ is a real zero.

Use the Quadratic Formula. $3x^2 - 6x - 2 = 0$

$$x = \frac{3 \pm \sqrt{15}}{3}$$

Real zeros: $x = 0, -2, \frac{3 \pm \sqrt{15}}{3}$

89. $z^4 - z^3 - 2z - 4 = 0$

Possible rational zeros: $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 0 & -2 & -4 \\ & & -1 & 2 & -2 & 4 \\ \hline & 1 & -2 & 2 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 2 & -4 \\ & & 2 & 0 & 4 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

$z = -1$ and $z = 2$ are real zeros.

$z^4 - z^3 - 2z - 4 = (z+1)(z-2)(z^2 + 2) = 0$

The only real zeros are –1 and 2. You can verify this by graphing the function $f(z) = z^4 - z^3 - 2z - 4$.

91. $2y^4 + 7y^3 - 26y^2 + 23y - 6 = 0$

Possible rational zeros:

$\pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & 7 & -26 & 23 & -6 \\ & & 1 & 4 & -11 & 6 \\ \hline & 2 & 8 & -22 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & 8 & -22 & 12 \\ & & 2 & 10 & -12 \\ \hline & 2 & 10 & -12 & 0 \end{array}$$

$$\begin{array}{r|rrr} -6 & 2 & 10 & -12 \\ & & -12 & 12 \\ \hline & 2 & -2 & 0 \end{array}$$

$$\begin{array}{r|rr} 1 & 2 & -2 \\ & & 2 \\ \hline & 2 & 0 \end{array}$$

$x = \frac{1}{2}$, $x = 1$, $x = -6$, and $x = 1$ are real zeros.

$(y + 6)(y - 1)^2(2y - 1) = 0$

Real zeros: $x = -6, 1, \frac{1}{2}$ –

93. $4x^4 - 55x^2 - 45x + 36 = 0$

Possible rational zeros:

$\pm\frac{1}{2}, \pm\frac{3}{4}, \pm 1, \pm\frac{3}{2}, \pm 2, \pm\frac{9}{4}, \pm 3, \pm 4, \pm\frac{9}{2}, \pm 6, \pm 9, \pm 18$

$$\begin{array}{r|rrrrr} 4 & 4 & 0 & -55 & -45 & 36 \\ & & 16 & 64 & 36 & -36 \\ \hline & 4 & 16 & 9 & -9 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 4 & 16 & 9 & -9 \\ & & -12 & -12 & 9 \\ \hline & 4 & 4 & -3 & 0 \end{array}$$

95. $8x^4 + 28x^3 + 9x^2 - 9x = 0$

$x(8x^3 + 28x^2 + 9x - 9) = 0$

$x = 0$ is a real zero.

Possible rational zeros:

$\pm 1, \pm 3, \pm 9, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{9}{8}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}$

$$\begin{array}{r|rrrr} -3 & 8 & 28 & 9 & -9 \\ & & -24 & -12 & 9 \\ \hline & 8 & 4 & -3 & 0 \end{array}$$

$x = -3$ is a real zeros

Use the Quadratic Formula.

$8x^2 + 4x - 3 = 0 \qquad x = \frac{- \pm \sqrt{7}}{4}$

Real zeros: $x = 0, -3, \frac{-1 \pm \sqrt{7}}{4}$

97. $4x^5 + 12x^4 - 11x^3 - 42x^2 + 7x + 30 = 0$

Possible rational zeros:

$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30,$

$\pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{5}{2}, \pm\frac{5}{4}, \pm\frac{15}{2}, \pm\frac{15}{4}$

$$\begin{array}{r|rrrrrr} 1 & 4 & 12 & -11 & -42 & 7 & 30 \\ & & 4 & 16 & 5 & -37 & -30 \\ \hline & 4 & 16 & 5 & -37 & -30 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 4 & 16 & 5 & -37 & -30 \\ & & -4 & -12 & 7 & 30 \\ \hline & 4 & 12 & -7 & -30 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 4 & 12 & -7 & -30 \\ & & -8 & -8 & 30 \\ \hline & 4 & 4 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{3}{2} & 4 & 4 & -15 \end{array}$$

$$\begin{array}{r|rrr} \frac{1}{2} & 4 & 4 & -3 \\ & & 2 & 3 \\ \hline & 4 & 6 & 0 \end{array}$$

$$\begin{array}{r|rr} -\frac{3}{2} & 4 & 6 \\ & & -6 \\ \hline & 4 & 0 \end{array}$$

$x = 4,\ x = -3,\ x = \frac{1}{2},$ and $x = -\frac{3}{2}$ are real zeros.

$(x - 4)(x + 3)(2x - 1)(2x + 3) = 0$

Real zeros: $x = 4, -3, \frac{1}{2}, -\frac{3}{2}$

$$\begin{array}{rrr} & 6 & 15 \\ \hline 4 & 10 & 0 \end{array}$$

$$\begin{array}{r|rr} -\frac{5}{2} & 4 & 10 \\ & & -10 \\ \hline & 4 & 0 \end{array}$$

$x = 1,\ x = -1,\ x = -2,\ x = \frac{3}{2},$ and $x = -\frac{5}{2}$ are real zeros.

$(x - 1)(x + 1)(x + 2)(2x - 3)(2x + 5) = 0$

Real zeros: $x = 1, -1, -2, \frac{3}{2}, -\frac{5}{2}.$

99. $h(t) = t^3 - 2t^2 - 7t + 2$

(a) Zeros: $-2,\ 3.732,\ 0.268$

(b)

-2	1	-2	-7	2
		-2	8	-2
	1	-4	1	0

$t = -2$ is a zero.

(c) $h(t) = (t + 2)(t^2 - 4t + 1)$

$= (t + 2)\left[t - (\sqrt{3} + 2)\right]\left[t + (\sqrt{3} - 2)\right]$

101. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

(a) $x = 0,\ 3,\ 4,\ \pm 1.414$

(b)

3	1	-7	10	14	-24
		3	-12	-6	24
	1	-4	-2	8	0

$x = 3$ is a zero.

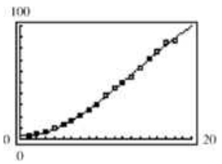
4	1	-4	-2	8
		4	0	-8
	1	0	-2	0

$x = 4$ is a zero.

(c) $h(x) = x(x - 3)(x - 4)(x^2 - 2)$

$= x(x - 3)(x - 4)(x - \sqrt{2})(x + \sqrt{2})$

103. (a)



(b) The model fits the data well.

(c) $S = -0.0135t^3 + 0.545t^2 - 0.71t + 3.6$

25	-0.0135	0.545	-0.71	3.6
		-0.3375	5.1875	111.9375
	-0.0135	0.2075	4.4775	115.5375

In 2015 (t = 25), the model predicts approximately 116 subscriptions per 100 people, obviously not a reasonable prediction because you cannot have more subscriptions than people.

105. (a) Combined length and width:

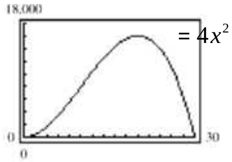
$4x + y = 120 \Rightarrow y = 120 - 4x$

$\text{Volume} = l \cdot w \cdot h = x^2 y$

$= x^2(120 - 4x)$

$= 4x^2(30 - x)$

(b)



Dimension with maximum volume: $20 \times 20 \times 40$

(c) $13{,}500 = 4x^2(30 - x)$

$4x^3 - 120x^2 + 13{,}500 = 0$

$x^3 - 30x^2 + 3375 = 0$

15	1	-30	0	3375
		15	-225	-3375
	1	-15	-225	0

$(x - 15)(x^2 - 15x - 225) = 0$

Using the Quadratic Formula, $x = 15$ or $\dfrac{15 \pm 15\sqrt{5}}{2}$.

The value of $\dfrac{15 - 15\sqrt{5}}{2}$ is not possible because it is negative.

107. False, $-\dfrac{4}{7}$ is a zero of *f*.

109. The zeros are 1, 1, and –2. The graph falls to the right.

$y = a(x - 1)^2(x + 2),\ a < 0$

Since $f(0) = -4,\ a = -2.$

$y = -2(x - 1)^2(x + 2)$

111. $f(x) = -(x + 1)(x - 1)(x + 2)(x - 2)$

113. (a) $\dfrac{x^2-1}{x-1} = x+1,\ x \neq 1$

(b) $\dfrac{x^3-1}{x-1} = x^2+x+1,\ x \neq 1$

(c) $\dfrac{x^4-1}{x-1} = x^3+x^2+x+1,\ x \neq 1$

In general,

$$\frac{x^n-1}{x-1} = x^{n-1} + x^{n-2} + \cdots + x + 1,\ x \neq 1.$$

115. $9x^2 - 25 = 0$

$(3x+5)(3x-5) = 0$

$x = -\dfrac{5}{3}, \dfrac{5}{3}$

117. $2x^2 + 6x + 3 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-6 \pm \sqrt{12}}{4}$$

$$= \frac{-3 \pm \sqrt{3}}{2}$$

$$x = -\frac{3}{2} + \frac{\sqrt{3}}{2}, -\frac{3}{2} - \frac{\sqrt{3}}{2}$$

Section 2.4

1. (a) ii

(b) iii

(c) i

3. $(7+6i) + (8+5i) = (7+8) + (6+5)i$

$= 15 + 11i$

The real part is 15 and the imaginary part is $11i$.

5. The additive inverse of $2-4i$ is $-2+4i$ so that $(2-4i) + (-2+4i) = 0$.

7. $a + bi = -9 + 4i$

$a = -9$

$b = 4$

9. $3a + (b+3)i = 9 + 8i$

$3a = 9 \qquad b + 3 = 8$

$a = 3 \qquad b = 5$

11. $5 + \sqrt{-16} = 5 + \sqrt{16(-1)}$

$= 5 + 4i$

13. $-6 = -6 + 0i$

15. $-5i + i^2 = -5i - 1 = -1 - 5i$

17. $\left(\sqrt{-75}\right)^2 = -75$

19. $\sqrt{-0.09} = \sqrt{0.09}i = 0.3i$

27. $\left(\dfrac{3}{2} + \dfrac{5}{2}i\right) + \left(\dfrac{5}{3} + \dfrac{11}{3}i\right) = \left(\dfrac{3}{2} + \dfrac{5}{3}\right) + \left(\dfrac{5}{2} + \dfrac{11}{3}\right)i$

$= \dfrac{9+10}{6} + \dfrac{15+22}{6}i$

$= \dfrac{19}{6} + \dfrac{37}{6}i$

29. $(1.6 + 3.2i) + (-5.8 + 4.3i) = -4.2 + 7.5i$

31. $4(3+5i) = 12 + 20i$

33. $(1+i)(3-2i) = 3 - 2i + 3i - 2i^2$

$= 3 + i + 2$

$= 5 + i$

35. $4i(8+5i) = 32i + 20i^2$

$= 32i + 20(-1)$

$= -20 + 32i$

37. $\left(\sqrt{14} + \sqrt{10}\,i\right)\left(\sqrt{14} - \sqrt{10}\,i\right) = 14 - 10i^2 = 14 + 10 = 24$

39. $(6+7i)^2 = 36 + 42i + 42i + 49i^2$

$= 36 + 84i - 49$

$= -13 + 84i$

41. $(4+5i)^2 - (4-5i)^2$

$= \left[(4+5i) + (4-5i)\right]\left[(4+5i) - (4-5i)\right] = 8(10i) = 80i$

21. $(4 + i) - (7 - 2i) = (4 - 7) + (1 + 2)i$
$= -3 + 3i$

23. $(-1 + 8i) + (8 - 5i) = (-1 + 8) + (8 - 5)i$
$= 7 + 3i$

25. $13i - (14 - 7i) = 13i - 14 + 7i = -14 + 20i$

43. $4 - 3i$ is the complex conjugate of $4 + 3i$.
$(4 + 3i)(4 - 3i) = 16 + 9 = 25$

45. $-6 + \sqrt{5}i$ is the complex conjugate of $-6 - \sqrt{5}i$.

$\left(-6 - \sqrt{5}i\right)\left(-6 + \sqrt{5}i\right) = 36 + 5 = 41$

47. $-\sqrt{20}i$ is the complex conjugate of $\sqrt{-20} = \sqrt{20}i$.

$$\left(\sqrt{20}i\right)\left(-\sqrt{20}i\right) = 20$$

49. $3+\sqrt{2}i$ is the complex conjugate of

$$3-\sqrt{-2} = 3-\sqrt{2}i$$

$$(3-\sqrt{2}i)(3+\sqrt{2}i) = 9+2 = 11$$

51. $$\frac{6}{i} = \frac{6}{i}\cdot\frac{-i}{-i} = \frac{-6i}{-i^2} = \frac{-6i}{1} = -6i$$

53. $$\frac{2}{4-5i} = \frac{2}{4-5i}\cdot\frac{4+5i}{4+5i} = \frac{8+10i}{16+25} = \frac{8}{41}+\frac{10}{41}i$$

55. $$\frac{2+i}{2-i} = \frac{2+i}{2-i}\cdot\frac{2+i}{2+i} = \frac{4+4i+i^2}{4+1} = \frac{3+4i}{5} = \frac{3}{5}+\frac{4}{5}i$$

57. $$\frac{i}{\left(4-5i\right)^2} = \frac{i}{16-25-40i} = \frac{i}{-9-40i}\cdot\frac{-9+40i}{-9+40i} = \frac{-40-9i}{81+40^2} = -\frac{40}{1681}-\frac{9}{1681}i$$

59. $$\frac{2}{1+i}-\frac{3}{1-i} = \frac{2\left(1-i\right)-3\left(1+i\right)}{\left(1+i\right)\left(1-i\right)} = \frac{2-2i-3-3i}{1+1} = \frac{-1-5i}{2} = -\frac{1}{2}-\frac{5}{2}i$$

61. $$\frac{i}{}+\frac{2i}{} = \frac{3i+8i^2+6i-4i^2}{}$$

63. $$\sqrt{-18}-\sqrt{-54} = 3\sqrt{2}i-3\sqrt{6}i = 3\left(\sqrt{2}-\sqrt{6}\right)i$$

65. $$\left(-3+\sqrt{-24}\right)+\left(7-\sqrt{-44}\right) = \left(-3+2\sqrt{6}i\right)+\left(7-2\sqrt{11}i\right) = 4+\left(2\sqrt{6}-2\sqrt{11}\right)i = 4+2\left(\sqrt{6}-\sqrt{11}\right)i$$

67. $$\sqrt{-6}\cdot\sqrt{-2} = \left(\sqrt{6}i\right)\left(\sqrt{2}i\right) = \sqrt{12}\,i^2 = \left(2\sqrt{3}\right)\left(-1\right) = -2\sqrt{3}$$

69. $$\left(\sqrt{-10}\right)^2 = \left(\sqrt{10}i\right)^2 = 10i^2 = -10$$

71. $$\left(2-\sqrt{-6}\right)^2 = \left(2-\sqrt{6}i\right)\left(2-\sqrt{6}i\right)$$
$$= 4-2\sqrt{6}i-2\sqrt{6}i+6i^2$$
$$= 4-2\sqrt{6}i-2\sqrt{6}i+6(-1)$$
$$= 4-6-4\sqrt{6}i$$
$$= -2-4\sqrt{6}i$$

73. $$x^2+25 = 0$$
$$x^2 = -25$$
$$x = \pm 5i$$

75. $x^2-2x+2 = 0;\ a = 1,\ b = -2,\ c = 2$

$$x = \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)} = \frac{2\pm\sqrt{-4}}{2} = \frac{2\pm 2i}{2} = 1\pm i$$

77. $4x^2+16x+17 = 0;\ a = 4,\ b = 16,\ c = 17$

$3 - 2i \quad 3 + 8i \quad \left(3 - 2i\right)\left(3 + 8i\right)$

$$= \frac{-4 + 9i}{9 + 18i + 16}$$

$$= \frac{-4 + 9i}{25 + 18i} \cdot \frac{25 - 18i}{25 - 18i}$$

$$= \frac{-100 + 72i + 225i + 162}{25^2 + 18^2}$$

$$= \frac{62 + 297i}{949}$$

$$= \frac{62}{949} + \frac{297}{949}i$$

$$x = \frac{-16 \pm \left(16\right)^2 - 4(4)(17)}{2(4)}$$

$$= \frac{-16 \pm \sqrt{-16}}{8}$$

$$= \frac{-16 \pm 4i}{8}$$

$$= -2 \pm \frac{1}{2}i$$

79. $16t^2 - 4t + 3 = 0;\ a = 16,\ b = -4,\ c = 3$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)}$$

$$= \frac{4 \pm \sqrt{-176}}{32}$$

$$= \frac{4 \pm 4\sqrt{11}i}{32}$$

$$= \frac{1}{8} \pm \frac{\sqrt{11}}{8}i$$

81. $\frac{3}{2}x^2 - 6x + 9 = 0$ Multiply both sides by 2.

$3x^2 - 12x + 18 = 0;\ a = 3,\ b = -12,\ c = 18$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(18)}}{2(3)}$$

$$= \frac{12 \pm \sqrt{-72}}{6}$$

$$= \frac{12 \pm 6\sqrt{2}i}{6} = 2 \pm \sqrt{2}i$$

83. $1.4x^2 - 2x - 10 = 0$ Multiply both sides by 5.

$7x^2 - 10x - 50 = 0;\ a = 7,\ b = -10,\ c = -50$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(7)(-50)}}{2(7)}$$

$$= \frac{10 \pm \sqrt{1500}}{14} = \frac{10 \pm 10\sqrt{15}}{14}$$

$$= \frac{5}{7} \pm \frac{5\sqrt{15}}{7}$$

85. $-6i^3 + i^2 = -6i^2 i + i^2 = -6(-1)i + (-1) = 6i - 1 = -1 + 6i$

87. $\left(\sqrt{-75}\right)^3 = \left(5\sqrt{3}i\right)^3 = 5^3\left(\sqrt{3}\right)^3 i^3 = 125\left(3\sqrt{3}\right)(-i)$

$$= -375\sqrt{3}i$$

89. $\frac{1}{i^3} = \frac{1}{i^3} \cdot \frac{i}{i} = \frac{i}{i^4} = \frac{i}{1} = i$

91. (a) $(2)^3 = 8$

(b) $\left(-1 + \sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(\sqrt{3}i\right) + 3(-1)\left(\sqrt{3}i\right)^2 + \left(\sqrt{3}i\right)^3$

$= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^3$

$= -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i$

$= 8$

(c) $\left(-1 - \sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(-\sqrt{3}i\right) + 3(-1)\left(-\sqrt{3}i\right)^2 + \left(-\sqrt{3}i\right)^3$

$= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^3$

$= -1 - 3\sqrt{3}i + 9 + 3\sqrt{3}i$

$= 8$

The three numbers are cube roots of 8.

93. (a) $i^{20} = (i^4)^5 = (1)^5 = 1$

(b) $i^{45} = (i^4)^{11} i = (1)^{11} i = i$

(c) $i^{67} = (i^4)^{16} i^3 = (1)^{16}(-i) = -i$

(d) $i^{114} = (i^4)^{28} i^2 = (1)^{28}(-1) = -1$

95. False. A real number $a + 0i = a$ is equal to its conjugate.

97. False. For example, $(1 + 2i) + (1 - 2i) = 2,$ which is not an imaginary number.

99. True. Let $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i.$ Then

$\overline{z_1 z_2} = \overline{(a_1 + b_1 i)(a_2 + b_2 i)}$

$= \overline{(a_1a_2 - b_1b_2) + (a_1b_2 + b_1a_2)i}$

$= (a_1a_2 - b_1b_2) - (a_1b_2 + b_1a_2)i$

$= (a_1 - b_1 i)(a_2 - b_2 i)$

$= \overline{a_1 + b_1 i}\ \overline{a_2 + b_2 i}$

$= \overline{z_1}\,\overline{z_2}.$

101. $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = 6i^2 = -6$

103. $(4x - 5)(4x + 5) = 16x^2 - 20x + 20x - 25 = 16x^2 - 25$

105. $\left(3x - \frac{1}{2}\right)(x + 4) = 3x^2 - \frac{1}{2}x + 12x - 2 = 3x^2 + \frac{23}{2}x - 2$

Section 2.5

1. Fundamental Theorem, Algebra

3. The Linear Factorization Theorem states that a polynomial function f of degree n, $n > 0$, has exactly n linear factors

$$f(x) = a(x - c_1)(x - c_2)\ldots(x - c_n).$$

5. $f(x) = x^3 + x$ has exactly 3 zeros. Matches (c).

7. $f(x) = x^5 + 9x^3$ has exactly 5 zeros. Matches (d).

9. $f(x) = x^2 + 25$

$$\begin{aligned} x^2 + 25 &= 0 \\ x^2 &= -25 \\ x &= \pm\sqrt{-25} \\ x &= \pm 5i \end{aligned}$$

11. $f(x) = x^3 + 9x$

$$x^3 + 9x = 0$$
$$x(x^2 + 9) = 0$$
$$x = 0 \quad x^2 + 9 = 0$$
$$x^2 = -9$$
$$x = \pm\sqrt{-9}$$
$$x = \pm 3i$$

13. $f(x) = x^3 - 4x^2 + x - 4 = x^2(x-4) + 1(x-4) = (x-4)(x^2+1)$

Zeros: $4, \pm i$

The only real zero of *f(x)* is $x = 4$. This corresponds to the *x*-intercept of $(4, 0)$ on the graph.

15. $f(x) = x^4 + 4x^2 + 4 = (x^2+2)^2$

Zeros: $\pm\sqrt{2}i, \ \pm\sqrt{2}i$

f(*x*) has no real zeros and the graph of *f*(*x*) has no *x*-intercepts.

17. $h(x) = x^2 - 4x + 1$

h has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}.$$

$$h(x) = \left[x - \left(2 + \sqrt{3}\right)\right]\left[x - \left(2 - \sqrt{3}\right)\right]$$
$$= (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$$

19. $f(x) = x^2 - 12x + 26$

f has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(26)}}{2} = 6 \pm \sqrt{10}.$$

$$f(x) = \left[x - (6 + \sqrt{10})\right]\left[x - (6 - \sqrt{10})\right]$$
$$= (x - 6 - \sqrt{10})(x - 6 + \sqrt{10})$$

21. $f(x) = x^2 + 25$

25. $f(z) = z^2 - z + 56$

$$z = \frac{1 \pm \sqrt{1 - 4(56)}}{2}$$
$$= \frac{1 \pm \sqrt{-223}}{2}$$
$$= \frac{1}{2} \pm \frac{\sqrt{223}}{2}i$$

Zeros: $\frac{1}{2} \pm \frac{\sqrt{223}}{2}i$

$$f(z) = \left(z - \frac{1}{2} + \frac{\sqrt{223}i}{2}\right)\left(z - \frac{1}{2} - \frac{\sqrt{223}i}{2}\right)$$

27. $f(x) = x^4 + 10x^2 + 9$

$$= (x^2+1)(x^2+9)$$
$$= (x+i)(x-i)(x+3i)(x-3i)$$

The zeros of *f*(*x*) are $x = \pm i$ and $x = \pm 3i$.

29. $f(x) = 3x^3 - 5x^2 + 48x - 80$

Using synthetic division, $\frac{5}{3}$ is a zero:

$\frac{5}{3}$	3	−5	48	−80
		5	0	80
	3	0	48	0

$$f(x) = \left(x - \frac{5}{3}\right)\left(3x^2 + 48\right)$$
$$= (3x-5)(x^2+16)$$
$$= (3x-5)(x+4i)(x-4i)$$

The zeros are $\frac{5}{3}, 4i, -4i$.

31. $f(t) = t^3 - 3t^2 - 15t + 125$

Possible rational zeros: $\pm1, \pm5, \pm25, \pm125$

−5	1	−3	−15	125
		−5	40	−125

Zeros: $\pm 5i$

$f(x) = (x + 5i)(x - 5i)$

23. $f(x) = 16x^4 - 81$

$= (4x^2 - 9)(4x^2 + 9)$

$= (2x - 3)(2x + 3)(2x + 3i)(2x - 3i)$

Zeros: $\pm \frac{3}{2}, \; \pm \frac{3}{2}i$

1 −8 25 0

By the Quadratic Formula, the zeros of $t^2 - 8t + 25$ are

$t = \frac{8 \pm \sqrt{64 - 100}}{2} = 4 \pm 3i.$

The zeros of $f(t)$ are $t = -5$ and $t = 4 \pm 3i$.

$$f(t) = \left[t - (-5)\right]\left[t - (4 + 3i)\right]\left[t - (4 - 3i)\right]$$
$$= (t + 5)(t - 4 - 3i)(t - 4 + 3i)$$

33. $f(x) = 5x^3 - 9x^2 + 28x + 6$

Possible rational zeros: $\pm 6, \pm \frac{6}{5}, \pm 3, \pm \frac{3}{5}, \pm 2, \pm \frac{2}{5}, \pm 1, \pm \frac{1}{5}$

$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & -9 & 28 & 6 \\ & & -1 & 2 & -6 \\ \hline & 5 & -10 & 30 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $5x^2 - 10x + 30$ are those of $x^2 - 2x + 6$:

$$x = \frac{2 \pm \sqrt{4 - 4(6)}}{2} = 1 \pm \sqrt{5}i$$

Zeros: $-\frac{1}{5}, 1 \pm \sqrt{5}i$

$$f(x) = 5\left(x + \frac{1}{5}\right)\left[x - \left(1 + \sqrt{5}i\right)\right]\left[x - \left(1 - \sqrt{5}i\right)\right]$$

$$= (5x + 1)\left(x - 1 - \sqrt{5}i\right)\left(x - 1 + \sqrt{5}i\right)$$

35. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 8 & -16 & 16 \\ & & 2 & -4 & 8 & -16 \\ \hline 2 & 1 & -2 & 4 & -8 & 0 \\ & & 2 & 0 & 8 & \\ \hline & 1 & 0 & 4 & 0 & \end{array}$$

$g(x) = (x - 2)(x - 2)(x^2 + 4)$

$= (x - 2)^2 (x + 2i)(x - 2i)$

The zeros of g are $2, 2$, and $\pm 2i$.

37. (a) $f(x) = x^2 - 14x + 46.$

By the Quadric Formula,

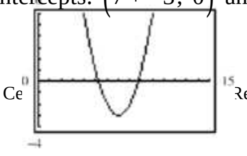
$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(46)}}{2} = 7 \pm \sqrt{3}.$$

The zeros are $7 + \sqrt{3}$ and $7 - \sqrt{3}$.

(b) $f(x) = \left[x - (7 + \sqrt{3})\right]\left[x - (7 - \sqrt{3})\right]$

$= (x - 7 - \sqrt{3})(x - 7 + \sqrt{3})$

(c) x-intercepts: $\left(7 + \sqrt{3}, 0\right)$ and $\left(7 - \sqrt{3}, 0\right)$



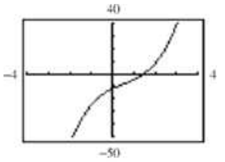
39. (a) $f(x) = 2x^3 - 3x^2 + 8x - 12$

$= (2x - 3)(x^2 + 4)$

The zeros are $\frac{3}{2}$ and $\pm 2i$.

(b) $f(x) = (2x - 3)(x + 2i)(x - 2i)$

(c) x-intercept: $\left(\frac{3}{2}, 0\right)$



41. (a) $f(x) = x^3 - 11x + 150$

$= (x + 6)(x^2 - 6x + 25)$

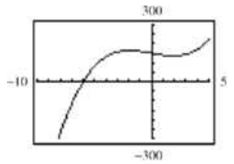
Use the Quadratic Formula to find the zeros of $x^2 - 6x + 25$.

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(25)}}{2} = 3 \pm 4i.$$

The zeros are $-6, 3 + 4i$, and $3 - 4i$.

(b) $f(x) = (x + 6)(x - 3 + 4i)(x - 3 - 4i)$

(c) x-intercept: $(-6, 0)$



43. (a) $f(x) = x^4 + 25x^2 + 144$

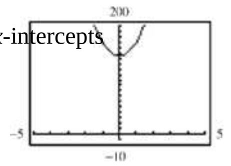
$= (x^2 + 9)(x^2 + 16)$

The zeros are $\pm 3i, \pm 4i$.

(b) $f(x) = (x^2 + 9)(x^2 + 16)$

$= (x + 3i)(x - 3i)(x + 4i)(x - 4i)$

(c) No x-intercepts



45. $f(x) = (x-2)(x-i)(x+i)$

$= (x-2)(x^2+1)$

Note that $f(x) = a(x^3 - 2x^2 + x - 2)$, where a is any nonzero real number, has zeros $2, \ \pm i$.

47. $f(x) = (x-2)^2(x-4-i)(x-4+i)$
$= (x-2)^2(x-8x+16+1)$
$= (x^2-4x+4)(x^2-8x+17)$
$= x^4-12x^3+53x^2-100x+68$

Note that $f(x) = a(x^4-12x^3+53x^2-100x+68)$, where a is any nonzero real number, has zeros $2,\ 2,\ 4\pm i$.

49. Because $1+\sqrt{2}i$ is a zero, so is $1-\sqrt{2}i$.
$f(x) = (x-0)(x+5)(x-1-\sqrt{2}i)(x-1+\sqrt{2}i)$
$= (x^2+5x)(x^2-2x+1+2)$
$= (x^2+5x)(x^2-2x+3)$
$= x^4+3x^3-7x^2-15x$
Note that $f(x) = a(x^4+3x^3-7x^2+15x)$, where a is any nonzero real number, has zeros $0,\ -5,\ 1\pm\sqrt{2}i$.

51. (a) $f(x) = a(x-1)(x+2)(x-2i)(x+2i)$
$= a(x-1)(x+2)(x^2+4)$
$f(1) = 10 = a(-2)(1)(5) \Rightarrow a = -1$
$f(x) = -(x-1)(x+2)(x-2i)(x+2i)$

(b) $f(x) = -(x-1)(x+2)(x^2+4)$
$= -(x^2+x-2)(x^2+4)$
$= -x^4-x^3-2x^2-4x+8$

53. (a) $f(x) = a(x+1)(x-2-\sqrt{5}i)(x-2+\sqrt{5}i)$
$= a(x+1)(x^2-4x+4+5)$
$= a(x+1)(x^2-4x+9)$
$f(-2) = 42 = a(-1)(4+8+9) \Rightarrow a = -2$
$f(x) = -2(x+1)(x-2-\sqrt{5}i)(x-2+\sqrt{5}i)$

(b) $f(x) = -2(x+1)(x^2-4x+9)$
$= -2x^3+6x^2-10x-18$

55. $f(x) = x^4-6x^2-7$

(a) $f(x) = (x^2-7)(x^2+1)$

(b) (c) $f(x) = (x-$

59. $f(x) = 2x^3+3x^2+50x+75$
Since is $5i$ a zero, so is $-5i$.

$5i$	2	3	50	75
		$10i$	$-50+15i$	-75
	2	$3+10i$	$15i$	0

$-5i$	2	$3+10i$	$15i$
		$-10i$	$-15i$
	2	3	0

The zero of $2x+3$ is $x = -\frac{3}{2}$. The zeros of f are $x = -\frac{3}{2}$ and $x = \pm 5i$.

Alternate solution

Since $x = \pm 5i$ are zeros of $f(x)$, $(x+5i)(x-5i) = x^2+25$ is a factor of $f(x)$. By long division we have:

$$\begin{array}{r} 2x+3 \\ x^2+0x+25\ \overline{)\,2x^3+3x^2+50x+75} \\ \underline{2x^3+0x^2+50x} \\ 3x^2+0x+75 \\ \underline{3x^2+0x+75} \\ 0 \end{array}$$

Thus, $f(x) = (x^2+25)(2x+3)$ and the zeros of f are $x = \pm 5i$ and $x = -\frac{3}{2}$.

61. $g(x) = x^3-7x^2-x+87$. Since $5+2i$ is a zero, so is $5-2i$.

$5+2i$	1	-7	-1	87
		$5+2i$	$-14+6i$	-87
	1	$-2+2i$	$-15+6i$	0

$5-2i$	1	$-2+2i$	$-15+6i$
		$5-2i$	$15-6i$
	1	3	0

The zero of $x+3$ is $x = -3$.
The zeros of f are $-3, 5\pm 2i$. .

$f(x) = \left(x \quad 7\right)\left(x + \quad 7\right)\left(x^2\right.$

$- \quad 7\big)\big(x + \sqrt{} + 1\big)$

$7\big)\big(x \quad \sqrt{}$

$+ i\big)\big(x$

$\sqrt{}\, i\big)$

57. $f(x) = x^4 - 2x^3 - 3x^2 + 12x - 18$

(a) $f(x) = \left(x^2 - 6\right)\left(x^2 - 2x + 3\right)$

(b) $f(x) = \left(x + \sqrt{6}\right)\left(x - \sqrt{6}\right)\left(x^2 - 2x + 3\right)$

(c) $f(x) = \left(x + \sqrt{6}\right)\left(x - \sqrt{6}\right)\left(x - 1 - \sqrt{2}i\right)\left(x - 1 + \sqrt{2}i\right)$

63. $h(x) = 3x^3 - 4x^2 + 8x + 8$ Since $1 - \sqrt{3}i$ is a zero, so is $1 + \sqrt{3}i$.

$$\begin{array}{r|rrrr} 1-\sqrt{3}i & 3 & -4 & 8 & 8 \\ & & 3-3\sqrt{3}i & -10-2\sqrt{3}i & -8 \\ \hline & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i & 0 \end{array}$$

$$\begin{array}{r|rrr} 1+\sqrt{3}i & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i \\ & & 3+3\sqrt{3}i & 2+2\sqrt{3}i \\ \hline & 3 & 2 & 0 \end{array}$$

The zero of $3x + 2$ is $x = -\frac{2}{3}$. The zeros of h are $x = -\frac{2}{3}, 1 \pm \sqrt{3}i$.

65. $h(x) = 8x^3 - 14x^2 + 18x - 9$. Since $\frac{1}{2}(1 - \sqrt{5}i)$ is a zero, so is $\frac{1}{2}(1 + \sqrt{5}i)$.

$$\begin{array}{r|rrrr} \frac{1}{2}\left(1-\sqrt{5}i\right) & 8 & -14 & 18 & -9 \\ & & 4-4\sqrt{5}i & -15+3\sqrt{5}i & 9 \\ \hline & 8 & -10-4\sqrt{5}i & 3+3\sqrt{5}i & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{1}{2}\left(1+\sqrt{5}i\right) & 8 & -10-4\sqrt{5}i & 3+3\sqrt{5}i \\ & & 4+4\sqrt{5}i & -3-3\sqrt{5}i \\ \hline & 8 & -6 & 0 \end{array}$$

The zero of $8x - 6$ is $x = \frac{3}{4}$. The zeros of h are $x = \frac{3}{4}, \frac{1}{2}(1 \pm \sqrt{5}i)$.

67. $f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$

(a) The *root* feature yields the real roots 1 and 2, and the complex roots $-3 \pm 1.414i$.

(b) By synthetic division:

$$\begin{array}{r|rrrrr} 1 & 1 & 3 & -5 & -21 & 22 \\ & & 1 & 4 & -1 & -22 \\ \hline & 1 & 4 & -1 & -22 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -1 & -22 \\ & & 2 & 12 & 22 \\ \hline & 1 & 6 & 11 & 0 \end{array}$$

The complex roots of $x^2 + 6x + 11$ are

69. $h(x) = 8x^3 - 14x^2 + 18x - 9$

(a) The *root* feature yields the real root 0.75, and the complex roots $0.5 \pm 1.118i$.

(b) By synthetic division:

$$\begin{array}{r|rrrr} \frac{3}{4} & 8 & -14 & 18 & -9 \\ & & 6 & -6 & 9 \\ \hline & 8 & -8 & 12 & 0 \end{array}$$

The complex roots of $8x^2 - 8x + 12$ are

$$x = \frac{8 \pm \sqrt{64 - 4(8)(12)}}{2(8)} = \frac{1}{2} \pm \frac{\sqrt{5}}{2}i.$$

71. To determine if the football reaches a height of 50 feet, set $h(t) = 50$ and solve for t.

$$-16t^2 + 48t = 50$$

$$-16t^2 + 48t - 50 = 0$$

Using the Quadratic formula:

$$t = \frac{-(48) \pm \sqrt{48^2 - 4(-16)(-50)}}{2(-16)}$$

$$t = \frac{-(48) \pm \sqrt{-896}}{-32}$$

Because the discriminant is negative, the solutions are not real, therefore the football does not reach a height of 50 feet.

73. False, a third-degree polynomial must have at least one real zero.

75. Answers will vary.

77. $f(x) = x^2 - 7x - 8$

$$= \left(x - \frac{7}{2}\right)^2 - \frac{81}{4}$$

A parabola opening upward with vertex $\left(\frac{7}{2}, -\frac{81}{4}\right)$

79. $f(x) = 6x^2 + 5x - 6$

$$= 6\left(x + \frac{5}{12}\right)^2 - \frac{169}{24}$$

A parabola opening upward with vertex $\left(-\frac{5}{\ }, \frac{169}{\ }\right.$

$-$

$\left(\quad 12 \qquad 24 \right)$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(11)}}{2} = -3 \pm \sqrt{2}i.$$

Section 2.6

1. rational functions

3. To determine the vertical asymptote(s) of the graph of $y = \frac{9}{x-3}$, find the real zeros of the denominator of the equation. (Assuming no common factors in the numerator and denominator)

5. $f(x) = \frac{1}{x-1}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	−2
0.9	−10
0.99	−100
0.999	−1000

x	$f(x)$
1.5	2
1.1	10
1.01	100
1.001	1000

x	$f(x)$
5	0.25
10	$0.\overline{1}$
100	0.01
1000	$0.\overline{001}$

x	$f(x)$
−5	$-0.1\overline{6}$
−10	$-0.\overline{09}$
−100	$-0.\overline{0099}$
−1000	$-0.\overline{00099}$

(c) f approaches $-\infty$ from the left of 1 and ∞ from the right of 1.

7. $f(x) = \frac{3x}{|x-1|}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	3
0.9	27
0.99	297
0.999	2997

x	$f(x)$
1.5	9
1.1	33
1.01	303
1.001	3003

x	$f(x)$
5	3.75
10	$3.\overline{3}$
100	$3.\overline{03}$
1000	$3.\overline{003}$

x	$f(x)$
−5	−2.5
−10	−2.727
−100	−2.970
−1000	−2.997

(c) f approaches ∞ from both the left and the right of 1.

9. $f(x) = \frac{3x^2}{x^2 - 1}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	−1
0.9	−12.79
0.99	−147.8
0.999	−1498

x	$f(x)$
1.5	5.4
1.1	17.29
1.01	152.3
1.001	1502.3

x	$f(x)$
5	3.125
10	$3.\overline{03}$
100	3.0003
1000	3

x	$f(x)$
−5	3.125
−10	$3.\overline{03}$
−100	3.0003
−1000	3

(c) f approaches $-\infty$ from the left of 1, and ∞ from the right of 1. f approaches ∞ from the left of –1, and $-\infty$ from the right of –1.

11. $f(x) = \frac{2}{x+2}$

Vertical asymptote: $x = -2$

Horizontal asymptote: $y = 0$

Matches graph (a).

13. $f(x) = \frac{4x+1}{x}$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 4$

Matches graph (c).

15. $f(x) = \frac{x-2}{x-4}$

Vertical asymptote: $x = 4$

Horizontal asymptote: $y = 1$

Matches graph (b).

17. $f(x) = \frac{1}{x^2}$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$ or x-axis

19. $f(x) = \frac{2x^2}{x^2 + x - 6} = \frac{2x^2}{(x+3)(x-2)}$

Vertical asymptotes: $x = -3,\ x = 2$

Horizontal asymptote: $y = 2$

21. $f(x) = \frac{x(2+x)}{2x - x^2} = \frac{\cancel{x}(x+2)}{-\cancel{x}(x-2)} = -\frac{x+2}{x-2},\ x \neq 0$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = -1$

Hole at $x = 0$

23. $f(x) = \dfrac{x^2-25}{x^2+5x} = \dfrac{(x+5)(x-5)}{x(x+5)} = \dfrac{x-5}{x},\ x \neq -5$

Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 1$
Hole at $x = -5$

25. $f(x) = \dfrac{3x^2+x-5}{x^2+1}$

(a) Domain: all real numbers
(b) Continuous
(c) Vertical asymptote: none
Horizontal asymptote: $y = 3$

27. $f(x) = \dfrac{x^2+3x-4}{-x^3+27} = \dfrac{(x+4)(x-1)}{-(x-3)(x^2+3x+9)}$

(a) Domain: all real numbers x except $x = 3$
(b) Not continuous at $x = 3$
(c) Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 0$ or x-axis

29. $f(x) = \dfrac{x^2-16}{x-4} = \dfrac{(x+4)(x-4)}{x-4} = x+4,\ x \neq 4$

$g(x) = x + 4$

(a) Domain of f: all real x except $x = 4$
Domain of g: all real x
(b) Vertical asymptote:

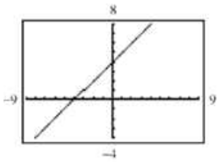
f has none. *g* has none.

Hole: *f* has a hole at $x = 4$; *g* has none.

(c)

x	1	2	3	4	5	6	7
$f(x)$	5	6	7	Undef.	9	10	11
$g(x)$	5	6	7	8	9	10	11

(d)



(e) Graphing utilities are limited in their resolution and therefore may not show a hole in a graph.

31. $f(x) = \dfrac{x^2-1}{x^2-2x-3} = \dfrac{(x+1)(x-1)}{(x-3)(x+1)} = \dfrac{x-1}{x-3},\ x \neq -1$

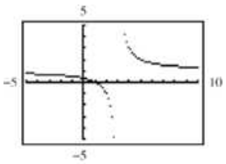
$g(x) = \dfrac{x-1}{x-3}$

(a) Domain of f: all real x except $x = 3$ and $x = -1$
Domain of g: all real x except $x = 3$
(b) Vertical asymptote: f has a vertical asymptote at

(c)

x	-2	-1	0	1	2	3	4
$f(x)$	$\frac{3}{5}$	Undef.	$\frac{1}{3}$	0	-1	Undef.	3
$g(x)$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{1}{3}$	0	-1	Undef.	3

(d)



(e) Graphing utilities are limited in their resolution and therefore may not show a hole in a graph.

33. $f(x) = 4 - \dfrac{1}{x}$

As $x \to \pm\infty$,
As $x \to \infty$,
As $x \to -\infty$,

35. $f(x) = \dfrac{2x-1}{x-3}$

As $x \to \pm\infty$, $f(x) \to 2$.
As $x \to \infty$, $f(x) \to 2$ but is greater than 2.
As $x \to -\infty$, $f(x) \to 2$ but is less than 2.

37. $g(x) = \dfrac{x^2-4}{x+3} = \dfrac{(x-2)(x+2)}{x+3}$

The zeros of g correspond to the zeros of the numerator and are $x = \pm 2$.

39. $f(x) = 1 - \dfrac{2}{x-5} = \dfrac{x-7}{x-5}$

The zero of f corresponds to the zero of the numerator and is $x = 7$.

41. $g(x) = \dfrac{x^2-2x-3}{x^2+1} = \dfrac{(x-3)(x+1)}{x^2+1} = 0$

Zeros: $x = -1, 3$

43. $f(x) = \dfrac{2x^2-5x+2}{2x^2-7x+3} = \dfrac{(2x-1)(x-2)}{(2x-1)(x-3)} = \dfrac{x-2}{x-3},\ x \neq \dfrac{1}{2}$

Zero: $x = 2$ ($x = \dfrac{1}{2}$ is not in the domain.)

$x = 3$.

g has a vertical asymptote at $x = 3$.

Hole: f has a hole at $x = -1$; g has none.

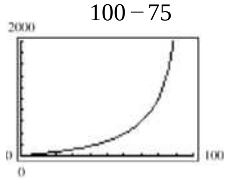
45. $C = \dfrac{255p}{100 - p},\ 0 \le p < 100$

(a) find $C(10) = \dfrac{255(10)}{100 - 10} = \$28.3$ million

$C(40) = \dfrac{255(40)}{100 - 40} = \$170$ million

$C(75) = \dfrac{255(75)}{100 - 75} = \$765$ million

(b)



(c) No. The function is undefined at $p = 100\,\%$.

47.

(a)

M	200	400	600	800	1000	1200	1400	1600	1800	2000
t	0.472	0.596	0.710	0.817	0.916	1.009	1.096	1.178	1.255	1.328

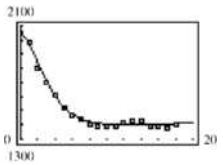
The greater the mass, the more time required per oscillation. The model is a good fit to the actual data.

(b) You can find M corresponding to $t = 1.056$ by finding the point of intersection of

$$t = \frac{38M + 16{,}965}{10(M + 500)} \text{ and } t = 1.056.$$

If you do this, you obtain $M \approx 1306$ grams.

49. (a) The model fits the data well.



(a) $N = \dfrac{77.095t^2 - 216.04t + 2050}{0.052t^2 - 0.8t + 1}$

$2009: N(19) \approx 1{,}412{,}000$

$2010: N(20) \approx 1{,}414{,}000$

$2011: N(11) \approx 1{,}416{,}000$

Answers will vary.

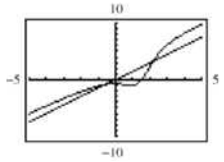
(b) Horizontal asymptote: $N = 1482.6$

(approximately) Answers will vary.

51. False. For example, $f(x) = \dfrac{1}{x^2 + 1}$ has no vertical asymptote.

53. No. If $x = c$ is also a zero of the denominator of *f*, then *f* is undefined at $x = c$, and the graph of *f* may have a hole of vertical asymptote at $x = c$.

55.



The graphs of $y_1 = \dfrac{3x^3 - 5x^2 + 4x - 5}{2x^2 - 6x + 7}$ and $y_2 = \dfrac{3x^3}{2x^2}$

are approximately the same graph as $x \to \infty$ and $x \to -\infty$.

Therefore as $x \to \pm\infty$, the graph of a rational function $y = \dfrac{a_n x^n + \dots a_1 x + a_0}{b_m x^m + \dots b_1 x + b_0}$ appears to be very close to the graph of $y = \dfrac{a_n x^n}{b_m x^m}$.

57. $y - 2 = \dfrac{-1 - 2}{0 - 3}(x - 3) = 1(x - 3)$

$y = x - 1$

$y - x + 1 = 0$

59. $y - 7 = \dfrac{10 - 7}{3 - 2}(x - 2) = 3(x - 2)$

$y = 3x + 1$

$3x - y + 1 = 0$

61.

$$\begin{array}{r|l} & x + 9 \\ x - 4 & x^2 + 5x + 6 \\ & \underline{x^2 - 4x} \\ & 9x + 6 \\ & \underline{9x - 36} \\ & 42 \end{array}$$

$$\frac{x^2 + 5x + 6}{x - 4} = x + 9 + \frac{42}{x - 4}$$

63.

$$\begin{array}{r|l} & 2x^2 - 9 \\ x^2 + 5 & 2x^4 + 0x^3 + x^2 + 0x - 11 \\ & \underline{2x^4 + 10x^2} \\ & -9x^2 - 11 \\ & \underline{-9x^2 - 45} \\ & 34 \end{array}$$

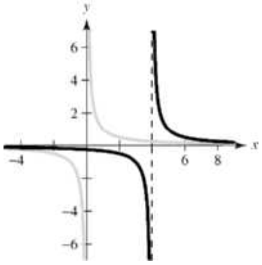
$$\frac{2x^4 + x^2 - 11}{x^2 + 5} = 2x^2 - 9 + \frac{34}{x^2 + 5}$$

Section 2.7

1. slant, asymptote

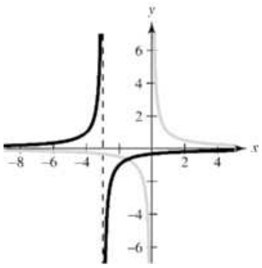
3. Yes. Because the numerator's degree is exactly 1 greater than that of the denominator, the graph of f has a slant asymptote.

5.



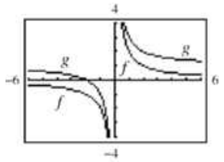
The graph of $g = \dfrac{1}{x-4}$ is a horizontal shift four units to the right of the graph of $f(x) = \dfrac{1}{x}$.

7.



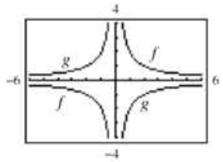
The graph of $g(x) = \dfrac{-1}{x+3}$ is a reflection in the x-axis and a horizontal shift three units to the left of the graph of $f(x) = \dfrac{1}{x}$.

9. $g(x) = \dfrac{2}{x} + 1$



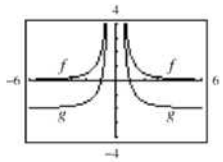
Vertical shift one unit upward

11. $g(x) = -\dfrac{2}{x}$



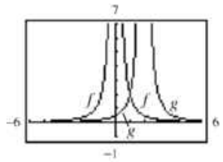
Reflection in the x-axis

13. $g(x) = \dfrac{2}{x^2} - 2$



Vertical shift two units downward

15. $g(x) = \dfrac{2}{(x-2)^2}$

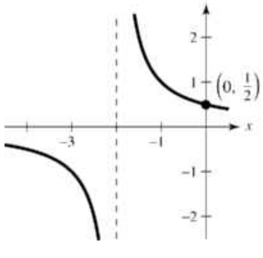


Horizontal shift two units to the right

17. $f(x) = \dfrac{1}{x+2}$

y-intercept: $\left(0, \dfrac{1}{2}\right)$

Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 0$



x	-4	-3	-1	0	1
y	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$

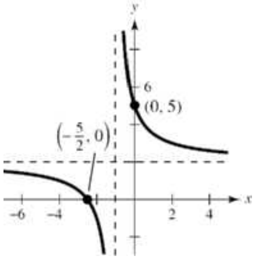
19. $C(x) = \dfrac{5 + 2x}{1 + x} = \dfrac{2x + 5}{x + 1}$

x-intercept: $\left(-\dfrac{5}{2}, 0\right)$

y-intercept: $(0, 5)$

Vertical asymptote: $x = -1$

Horizontal asymptote: $y = 2$



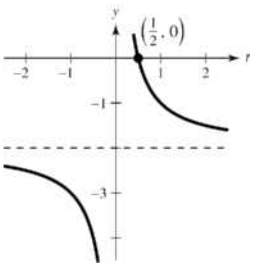
x	-4	-3	-2	0	1	2
$C(x)$	1	$\frac{1}{2}$	-1	5	$\frac{7}{2}$	3

21. $f(t) = \dfrac{1 - 2t}{t} = -\dfrac{2t - 1}{t}$

t-intercept: $\left(\dfrac{1}{2}, 0\right)$

Vertical asymptote: $t = 0$

Horizontal asymptote: $y = -2$



x	-2	-1	$\frac{1}{2}$	1	2
y	$-\frac{5}{2}$	-3	0	-1	$-\frac{3}{2}$

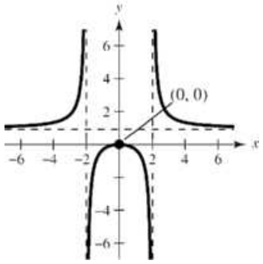
23. $f(x) = \dfrac{x^2}{x^2 - 4}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = 2$, $x = -2$

Horizontal asymptote: $y = 1$

y-axis symmetry



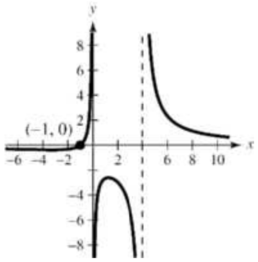
x	-4	-1	0	-1	4
y	$\frac{4}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{4}{3}$

25. $g(x) = \dfrac{4(x + 1)}{x(x - 4)}$

Intercept: $(-1, 0)$

Vertical asymptotes: $x = 0$ and $x = 4$

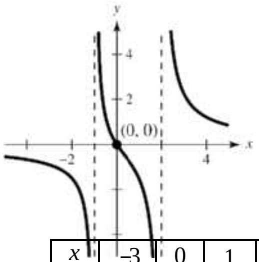
Horizontal asymptote: $y = 0$



x	-2	-1	1	2	3	5	6
y	$-\frac{1}{3}$	0	$-\frac{8}{3}$	-3	$-\frac{16}{3}$	$\frac{24}{5}$	$\frac{7}{3}$

27. $f(x) = \dfrac{3x}{x^2 - x - 2} = \dfrac{3x}{(x+1)(x-2)}$

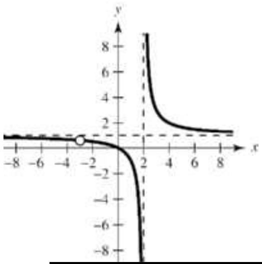
Intercept: $(0, 0)$
Vertical asymptotes: $x = -1, 2$
Horizontal asymptote: $y = 0$



x	-3	0	1	3	4
y	$-\frac{9}{10}$	0	$-\frac{3}{2}$	$\frac{9}{4}$	$\frac{6}{5}$

29. $f(x) = \dfrac{x^2 + 3x}{x^2 + x - 6} = \dfrac{x(x+3)}{(x-2)(x+3)} = \dfrac{x}{x-2},$

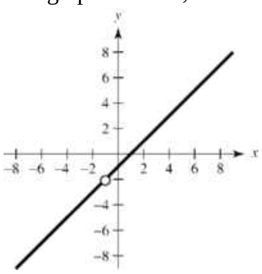
$x \neq -3$
Intercept: $(0, 0)$
Vertical asymptote: $x = 2$
(There is a hole at $x = -3$.)
Horizontal asymptote: $y = 1$



x	-2	-1	0	1	2	3
y	$\frac{1}{2}$	$\frac{1}{3}$	0	-1	Undef.	3

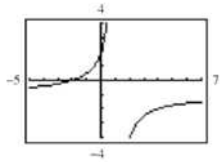
31. $f(x) = \dfrac{x^2 - 1}{x+1} = \dfrac{(x+1)(x-1)}{x+1} = x - 1,$

$x \neq -1$
The graph is a line, with a hole at $x = -1$.



33. $f(x) = \dfrac{2 + x}{1 - x} = -\dfrac{x+2}{x-1}$

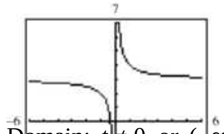
Vertical asymptote: $x = 1$
Horizontal asymptote: $y = -1$



Domain: $x \neq 1$ or $(-\infty, 1) \cup (1, \infty)$

35. $f(t) = \dfrac{3t + 1}{t}$

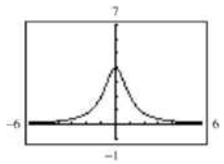
Vertical asymptote: $t = 0$
Horizontal asymptote: $y = 3$



Domain: $t \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

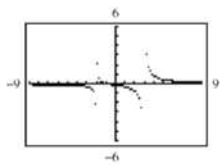
37. $h(t) = \dfrac{4}{t^2 + 1}$

Domain: all real numbers or $(-\infty, \infty)$
Horizontal asymptote: $y = 0$

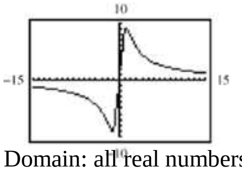


39. $f(x) = \dfrac{x+1}{x^2 - x - 6} = \dfrac{x+1}{(x-3)(x+2)}$

Domain: all real numbers except $x = 3, -2$
Vertical asymptotes: $x = 3, x = -2$
Horizontal asymptote: $y = 0$



41. $f(x) = \dfrac{20x}{x^2 + 1} - \dfrac{1}{x} = \dfrac{19x^2 - 1}{x(x^2 + 1)}$

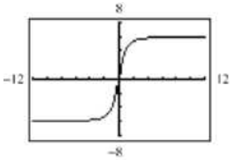


Domain: all real numbers except 0, or $(-\infty, 0) \cup (0, \infty)$

Vertical asymptote: $x = 0$

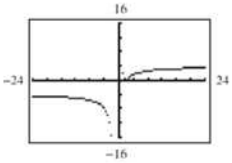
Horizontal asymptote: $y = 0$

43. $h(x) = \dfrac{6x}{\sqrt{x^2 + 1}}$



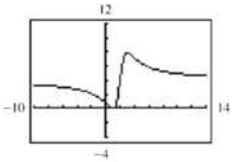
There are two horizontal asymptotes, $y = \pm 6$.

45. $g(x) = \dfrac{4|x - 2|}{x + 1}$



There are two horizontal asymptotes, $y = \pm 4$ and one vertical asymptote, $x = -1$.

47. $f(x) = \dfrac{4(x - 1)^2}{x^2 - 4x + 5}$



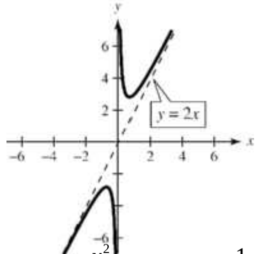
The graph crosses its horizontal asymptote, $y = 4$.

49. $f(x) = \dfrac{2x^2 + 1}{x} = 2x + \dfrac{1}{x}$

Vertical asymptote: $x = 0$

Slant asymptote: $y = 2x$

Origin symmetry

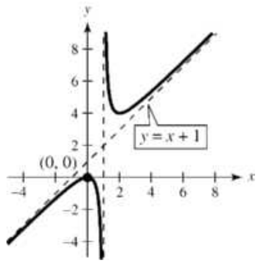


51. $h(x) = \dfrac{x^2}{x - 1} = x + 1 + \dfrac{1}{x - 1}$

Intercept: $(0, 0)$

Vertical asymptote: $x = 1$

Slant asymptote: $y = x + 1$



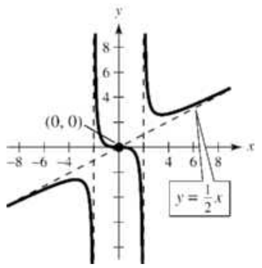
53. $g(x) = \dfrac{x^3}{2x^2 - 8} = \dfrac{1}{2}x + \dfrac{4x}{2x^2 - 8}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = \pm 2$

Slant asymptote: $y = \dfrac{1}{2}x$

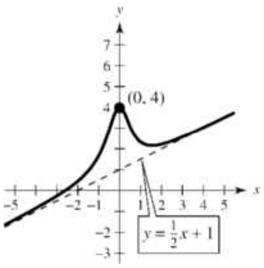
Origin symmetry



55. $f(x) = \dfrac{x^3 + 2x^2 + 4}{2x^2 + 1} = \dfrac{x}{2} + 1 + \dfrac{3 - \frac{x}{2}}{2x^2 + 1}$

Intercepts: $(-2.594, 0), (0, 4)$

Slant asymptote: $y = \dfrac{x}{2} + 1$



57. $y = \dfrac{x+1}{x-3}$

x-intercept: $(-1, 0)$

$0 = \dfrac{x+1}{x-3}$

$0 = x + 1$
$-1 = x$

59. $y = \dfrac{1}{x} - x$

x-intercepts: $(\pm 1, 0)$

$0 = \dfrac{1}{x} - x$

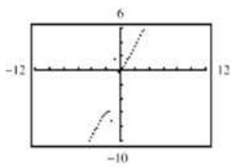
$x = \dfrac{1}{x}$

$x^2 = 1$
$x = \pm 1$

61. $y = \dfrac{2x^2 + x}{x+1} = 2x - 1 + \dfrac{1}{x+1}$

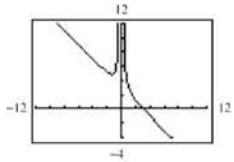
Domain: all real numbers except $x = -1$
Vertical asymptote: $x = -1$

Slant asymptote: $y = 2x - 1$



63. $y = \dfrac{1 + 3x^2 - x^3}{x^2} = \dfrac{1}{x^2} + 3 - x = -x + 3 + \dfrac{1}{x^2}$

Domain: all real numbers except 0
or $(-\infty, 0) \cup (0, \infty)$
Vertical asymptote: $x = 0$
Slant asymptote: $y = -x + 3$



65. $f(x) = \dfrac{x^2 - 5x + 4}{x^2 - 4} = \dfrac{(x-4)(x-1)}{(x-2)(x+2)}$

Vertical asymptotes: $x = 2,\ x = -2$
Horizontal asymptote: $y = 1$
No slant asymptotes, no holes

67. $f(x) = \dfrac{2x^2 - 5x + 2}{2x^2 - x - 6} = \dfrac{(2x-1)(x-2)}{(2x+3)(x-2)} = \dfrac{2x-1}{2x+3},$

$x \neq 2$

Vertical asymptote: $x = -\dfrac{3}{2}$

Horizontal asymptote: $y = 1$
No slant asymptotes
Hole at $x = 2,\ \left(2, \dfrac{3}{7}\right)$

69. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2}$

$= \dfrac{(x-1)(x+1)(2x-1)}{(x+1)(x+2)}$

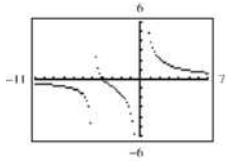
$= \dfrac{(x-1)(2x-1)}{x+2},\ x \neq -1$

Long division gives

$f(x) = \dfrac{2x^2 - 3x + 1}{x+2} = 2x - 7 + \dfrac{15}{x+2}.$

Vertical asymptote: $x = -2$
No horizontal asymptote
Slant asymptote: $y = 2x - 7$
Hole at $x = -1,\ (-1, 6)$

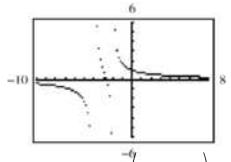
71. $y = \dfrac{1}{x+5} + \dfrac{4}{x}$



x-intercept: $(-4,\ 0)$

$$\begin{aligned} 0 &= \frac{1}{x+5} + \frac{4}{x} \\ -\frac{4}{x} &= \frac{1}{x+5} \\ -4(x+5) &= x \\ -4x - 20 &= x \\ -5x &= 20 \\ x &= -4 \end{aligned}$$

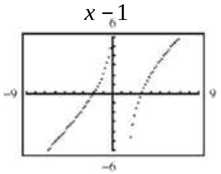
73. $y = \dfrac{1}{x+2} + \dfrac{2}{x+4}$



x-intercept: $\left(-\dfrac{8}{3},\ 0\right)$

$$\begin{aligned} \frac{1}{x+2} + \frac{2}{x+4} &= 0 \\ \frac{1}{x+2} &= \frac{-2}{x+4} \\ x + 4 &= -2x - 4 \\ 3x &= -8 \\ x &= -\frac{8}{3} \end{aligned}$$

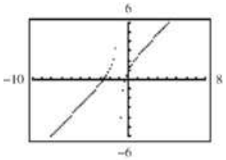
75. $y = x - \dfrac{6}{x-1}$



x-intercepts: $(-2, 0),\ (3, 0)$

$$\begin{aligned} 0 &= x - \frac{6}{x-1} \\ \frac{6}{x-1} &= x \\ 6 &= x(x-1) \\ 0 &= x^2 - x - 6 \\ 0 &= (x+2)(x-3) \\ x &= -2,\ x = 3 \end{aligned}$$

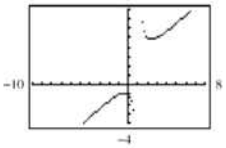
77. $y = x + 2 - \dfrac{1}{x+1}$



x-intercepts: $(-2.618, 0),\ (-0.382, 0)$

$$\begin{aligned} x + 2 &= \frac{1}{x+2} \\ x^2 + 3x + 2 &= 1 \\ x^2 + 3x + 1 &= 0 \\ x &= \frac{-3 \pm \sqrt{9-4}}{2} \\ &= -\frac{3}{2} \pm \frac{\sqrt{5}}{2} \\ &\approx -2.618, -0.382 \end{aligned}$$

79. $y = x + 1 + \dfrac{2}{x-1}$



No x-intercepts

$$\begin{aligned} x + 1 + \frac{2}{x-1} &= 0 \\ \frac{2}{x-1} &= -x - 1 \end{aligned}$$

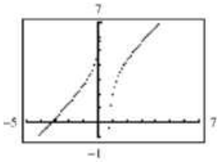
x

$\overline{1}$ $\quad 2 = -x^2 + 1$

$x^2 + 1 = 0$

No real zeros

81. $y = x + 3 - \dfrac{2}{2x - 1}$



x-intercepts: $(0.766, 0)$, $(-3.266, 0)$

$$x + 3 - \frac{2}{2x - 1} = 0$$
$$x + 3 = \frac{2}{2x - 1}$$
$$2x^2 + 5x - 3 = 2$$
$$2x^2 + 5x - 5 = 0$$
$$x = \frac{-5 \pm \sqrt{25 - 4(2)(-5)}}{4}$$
$$= \frac{-5 \pm \sqrt{65}}{4}$$
$$\approx 0.766, -3.266$$

83.

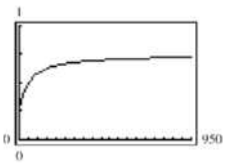
(a) $0.25(50) + 0.75(x) = C(50 + x)$

$$\frac{12.5 + 0.75x}{50 + x} = C$$
$$\frac{50 + 3x}{200 + 4x} = C$$
$$C = \frac{3x + 50}{4(x + 50)}$$

(b) Domain: $x \ge 0$ and $x \le 1000 - 50 = 950$

Thus, $0 \le x \le 950$.

(c)



As the tank fills, the rate that the concentration is increasing slows down. It approaches the horizontal asymptote $C = \dfrac{3}{4} = 0.75$. When the tank is full $(x = 950)$, the concentration is $C = 0.725$.

85.

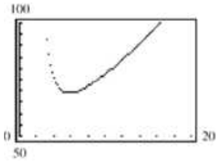
(a) $A = xy$ and

$$(x - 2)(y - 4) = 30$$
$$y - 4 = \frac{30}{x - 2}$$
$$y = 4 + \frac{30}{x - 2} = \frac{4x + 22}{x - 2}$$

Thus, $A = xy = x\left(\dfrac{4x + 22}{x - 2}\right) = \dfrac{2x(2x + 11)}{x - 2}$.

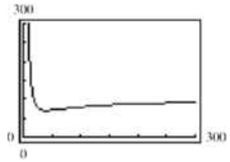
(b) Domain: Since the margins on the left and right are each 1 inch, $x > 2$, or $(2, \infty)$.

(c)



The area is minimum when $x \approx 5.87$ in. and $y \approx 11.75$ in.

87. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x + 30}\right)$, $1 \le x$



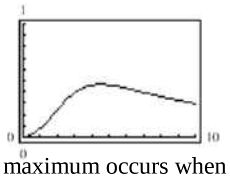
The minimum occurs when $x = 40.4 \approx 40$.

89.

$C = \dfrac{3t^2 + t}{t^3 + 50}$, $0 \le t$

(a) The horizontal asymptote is the t-axis, or $C = 0$. This indicates that the chemical eventually dissipates.

(b)

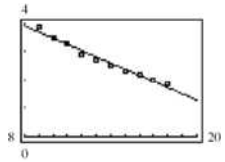


The maximum occurs when $t \approx 4.5$.

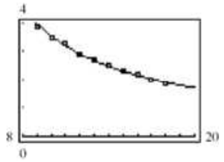
(c) Graph C together with $y = 0.345$. The graphs intersect at $t \approx 2.65$ and $t \approx 8.32$. $C < 0.345$ when $0 \le t \le 2.65$ hours and when $t > 8.32$ hours.

91.

(a) $A = -0.2182t + 5.665$



(b) $A = \dfrac{1}{0.0302t - 0.020}$



(c)

Year	1999	2000	2001	2002
Original data, A	3.9	3.5	3.3	2.9
Model from (a), A	3.7	3.5	3.3	3.0
Model from (b), A	4.0	3.5	3.2	2.9

Year	2003	2004	2005	2006
Original data, A	2.7	2.5	2.3	2.2
Model from (a), A	2.8	2.6	2.4	2.2
Model from (b), A	2.7	2.5	2.3	2.2

Year	2007	2008
Original data, A	2.0	1.9
Model from (a), A	2.0	1.7
Model from (b), A	2.0	1.9

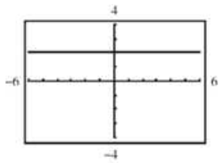
Answers will vary.

93. False. The graph of a rational function is continuous when the polynomial in the denominator has no real zeros.

95. $h(x) = \dfrac{6 - 2x}{3 - x} = \dfrac{2(3 - x)}{3 - x} = 2,\ x \neq 3$

Since *h(x)* is not reduced and $(3 - x)$ is a factor of both the numerator and the denominator, $x = 3$ is not a horizontal asymptote.

There is a hole in the graph at $x = 3$.



97. *Horizontal asymptotes*:

If the degree of the numerator is greater than the degree of the denominator, then there is no horizontal asymptote. If the degree of the numerator is less than the degree of the denominator, then there is a horizontal asymptote at $y = 0$.

If the degree of the numerator is equal to the degree of the denominator, then there is a horizontal asymptote at the line given by the ratio of the leading coefficients.

Vertical asymptotes:

Set the denominator equal to zero and solve.

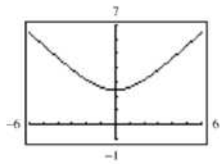
Slant asymptotes:

If there is no horizontal asymptote and the degree of the numerator is exactly one greater than the degree of the denominator, then divide the numerator by the denominator. The slant asymptote is the result, not including the remainder.

99. $\left(\dfrac{x}{8}\right)^{-3} = \left(\dfrac{8}{x}\right)^{3} = \dfrac{512}{x^3}$

101. $\dfrac{3^{7/6}}{3^{1/6}} = 3^{6/6} = 3$

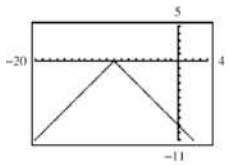
103.



Domain: all x

Range: $y \geq \sqrt{6}$

105.



Domain: all x

Range: $y \leq 0$

107. Answers will vary.

Section 2.8

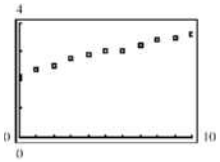
1. quadratic

3. Quadratic

5. Linear

7. Neither

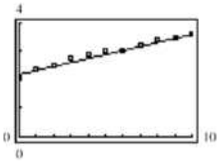
9. (a)



(b) Linear model is better.

(c) $y = 0.14x + 2.2$, linear

(d)

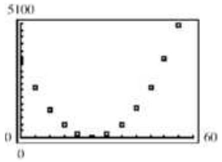


(e)

x	0	1	2	3	4
y	2.1	2.4	2.5	2.8	2.9
Model	2.2	2.4	2.5	2.6	2.8

x	5	6	7	8	9	10
y	3.0	3.0	3.2	3.4	3.5	3.6
Model	2.9	3.0	3.2	3.4	3.5	3.6

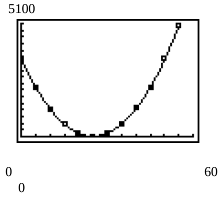
11. (a)



(b) Quadratic model is better.

(c) $y = 5.55x^2 - 277.5x + 3478$

(d)

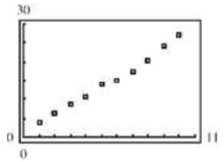


(e)

x	0	5	10	15	20	25
y	3480	2235	1250	565	150	12
Model	3478	2229	1258	564	148	9

x	30	35	40	45	50	55
y	145	575	1275	2225	3500	5010
Model	148	564	1258	2229	3478	5004

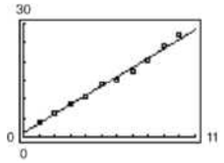
13. (a)



(b) Linear model is better.

(c) $y = 2.48x + 1.1$

(d)

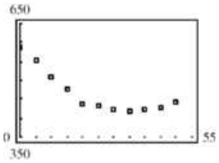


(e)

x	1	2	3	4	5
Actual, y	4.0	6.5	8.8	10.6	13.9
Model, y	3.6	6.1	8.5	11.0	13.5

x	6	7	8	9	10
Actual, y	15.0	17.5	20.1	24.0	27.1
Model, y	16.0	18.5	20.9	23.4	25.9

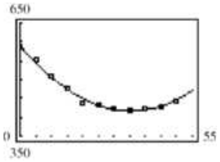
15. (a)



(b) Quadratic is better.

(c) $y = 0.14x^2 - 9.9x + 591$

(d)

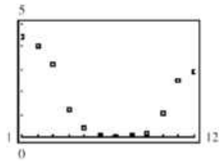


(e)

x	0	5	10	15	20	25
Actual, y	587	551	512	478	436	430
Model, y	591	545	506	474	449	431

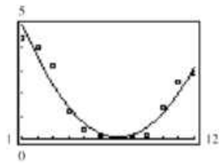
x	30	35	40	45	50
Actual, y	424	420	423	429	444
Model, y	420	416	419	429	446

17. (a)



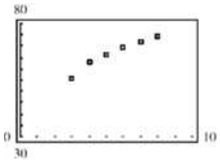
(b) $P = 0.1323t^2 - 1.893t + 6.85$

(c)



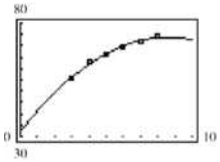
(d) The model's minimum is $H \approx 0.1$ at $t = 7.4$. This corresponds to July.

19. (a)



(b) $P = -0.5638t^2 + 9.690t + 32.17$

(c)



(d) To determine when the percent P of the U.S. population who used the Internet falls below 60%, set P = 60. Using the Quadratic Formula, solve for t.

$$-0.5638t^2 + 9.690t + 32.17 = 60$$
$$-0.5638t^2 + 9.690t - 27.83 = 0$$

$$t = \frac{-(9.690) \pm \sqrt{(9.690)^2 - 4(-0.5638)(-27.83)}}{2(-0.5638)}$$

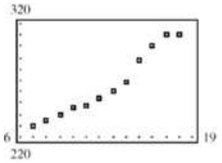
$$t = \frac{-9.690 \pm \sqrt{156.658316}}{-1.1276}$$

$$t \approx 13.54 \text{ or } 2014$$

Therefore after 2014, the percent of the U.S. population who use the Internet will fall below 60%.

This is not a good model for predicting future years. In fact, by 2021, the model gives negative values for the percentage.

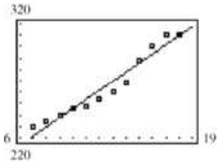
21. (a)



(b) $T = 7.97t + 166.1$

$r^2 \approx 0.9469$

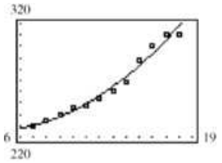
(c)



(d) $T = 0.459t^2 - 3.51t + 232.4$

$r^2 \approx 0.9763$

(e)



(f) The quadratic model is a better fit. Answers will vary.

(g) To determine when the number of televisions, *T*, in homes reaches 350 million, set *T* for the model equal to 350 and solve for *t*.

Linear model:

$$7.97t + 166.1 = 350$$
$$7.97t = 183.9$$
$$t \approx 23.1 \text{ or } 2014$$

Quadratic model:

$$0.459t^2 - 3.51t + 232.4 = 350$$
$$0.459t^2 - 3.51t - 117.6 = 0$$

Using the Quadratic Formula,

$$t = \frac{-(-3.51) \pm \sqrt{(-3.51)^2 - 4(0.459)(-117.6)}}{2(0.459)}$$

$$t = \frac{3.51 \pm \sqrt{228.2337}}{0.918}$$

$t \approx 20.3$ or 2011

23. True. A quadratic model with a positive leading coefficient opens upward. So, its vertex is at its lowest point.

25. The model is above all data points.

27. (a) $(f \circ g)(x) = f(x^2 + 3) = 2(x^2 + 3) - 1 = 2x^2 + 5$

(b) $(g \circ f)(x) = g(2x - 1) = (2x - 1)^2 + 3 = 4x^2 - 4x + 4$

29. (a) $(f \circ g)(x) = f(\sqrt[3]{x + 1}) = x + 1 - 1 = x$

(b) $(g \circ f)(x) = g(x^3 - 1) = \sqrt[3]{x^3 - 1 + 1} = x$

31. *f* is one-to-one.

$$y = 2x + 5$$
$$x = 2y + 5$$
$$2y = x - 5$$
$$y = \frac{(x - 5)}{2} \Rightarrow f^{-1}(x) = \frac{x - 5}{2}$$

33. *f* is one-to-one on $[0, \infty)$.

$$y = x^2 + 5, \ x \ge 0$$
$$x = y^2 + 5, \ y \ge 0$$
$$y^2 = x - 5$$
$$y = \sqrt{x - 5} \Rightarrow f^{-1}(x) = \sqrt{x - 5}, \ x \ge 5$$

35. For $1 - 3i$, the complex conjugate is $1 + 3i$.

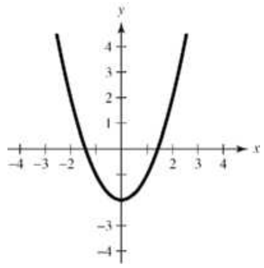
$$(1 - 3i)(1 + 3i) = 1 - 9i^2$$
$$= 1 + 9$$
$$= 10$$

37. For $-5i$, the complex conjugate is $5i$.

$$(-5i)(5i) = -25i^2$$
$$= 25$$

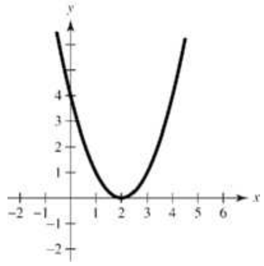
Chapter 2 Review Exercises

1.



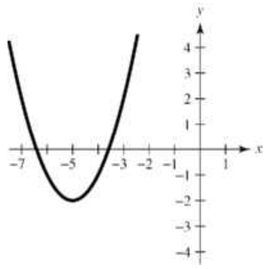
The graph of $y = x^2 - 2$ is a vertical shift two units downward of $y = x^2$.

3.



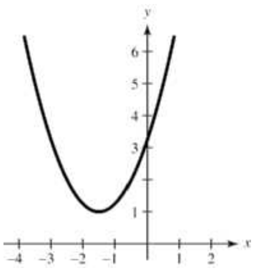
The graph of $y = (x-2)^2$ is a horizontal shift two units to the right of $y = x^2$.

5.



The graph of $y = (x+5)^2 - 2$ is a horizontal shift five units to the left and a vertical shift two units downward of $y = x^2$.

7. The graph of $f(x) = \left(x + 3/2\right)^2 + 1$ is a parabola opening upward with vertex $\left(-\dfrac{3}{2},\ 1\right)$, and no x-intercepts.



9.

$$\begin{aligned} f(x) &= \frac{1}{3}(x^2 + 5x - 4) \\ &= \frac{1}{3}\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4}\right) - \frac{4}{3} \\ &= \frac{1}{3}\left(x + \frac{5}{2}\right)^2 - \frac{41}{12} \end{aligned}$$

The graph of f is a parabola opening upward with vertex $\left(-\dfrac{5}{2},\ -\dfrac{41}{12}\right)$.

x-intercepts: $\dfrac{1}{3}\left(x + \dfrac{5}{2}\right)^2 - \dfrac{41}{12} = 0.$

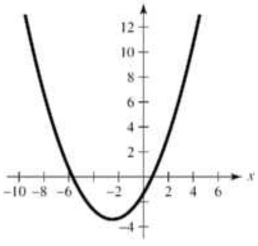
or

$$\frac{1}{3}\left(x^2 + 5x - 4\right) = 0$$
$$x^2 + 5x - 4 = 0$$

Use Quadratic formula.

$$x = \frac{-5 \pm \sqrt{41}}{2}$$

$$\left(\frac{-5 + \sqrt{41}}{2},\ 0\right), \left(\frac{-5 - \sqrt{41}}{2},\ 0\right)$$



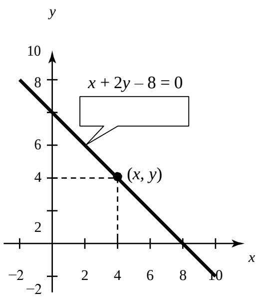
11. Vertex : $(1, -4) \Rightarrow f(x) = a(x-1)^2 - 4$

Point: $(2, -3) \Rightarrow -3 = a(2-1)^2 - 4$

$1 = a$

Thus, $f(x) = (x-1)^2 - 4$.

13.



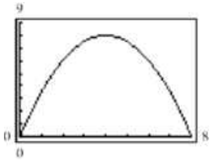
(a) $A = xy$

If $x + 2y - 8 = 0$, then $y = \dfrac{8-x}{2}$.

$A = x\left(\dfrac{8-x}{2}\right)$

$A = 4x - \dfrac{1}{2}x^2, \ 0 < x < 8$

(b)



Using the graph, when $x = 4$, the area is a maximum.

When $x = 4$, $y = \dfrac{8-4}{2} = 2$.

(c) $A = -\dfrac{1}{2}x^2 + 4x$

$= -\dfrac{1}{2}(x^2 - 8x)$

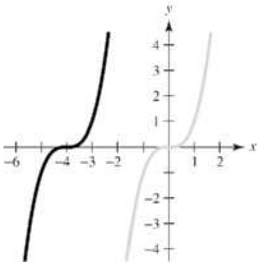
$= -\dfrac{1}{2}(x^2 - 8x + 16 - 16)$

$= -\dfrac{1}{2}(x-4)^2 + 8$

The graph of A is a parabola opening downward, with vertex $(4, 8)$. Therefore, when $x = 4$, the maximum area is 8 square units.

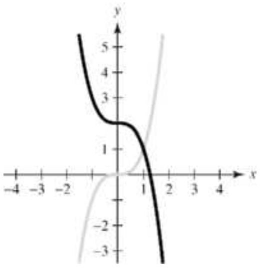
Yes, graphically and algebraically the same dimensions result.

15.



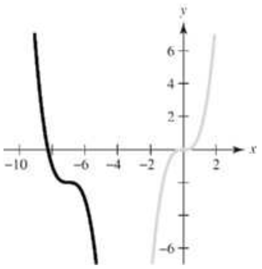
The graph of $f(x) = (x+4)^3$ is a horizontal shift four units to the left of $y = x^3$.

17.



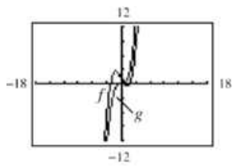
The graph if $f(x) = -x^3 + 2$ is a reflection in the x-axis and a vertical shift two units upward of $y = x^3$.

19.



The graph of $f(x) = -\left(x+7\right)^3 - 2$ is a horizontal shift seven units to the left, a reflection in the x-axis, and a vertical shift two units downward of $y = x^3$.

21. $f(x) = \dfrac{1}{2}x^3 - 2x + 1$; $g(x) = \dfrac{1}{2}x^3$



The graphs have the same end behavior. Both functions are of the same degree and have positive leading coefficients.

23. $f(x) = -x^2 + 6x + 9$

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

25. $f(x) = \frac{3}{4}\left(x^4 + 3x^2 + 2\right)$

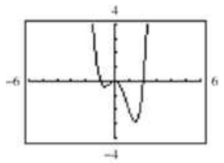
The degree is even and the leading coefficient is positive. The graph rises to the left and right.

27. (a) $x^4 - x^3 - 2x^2 = x^2\left(x^2 - x - 2\right)$

$= x^2\left(x - 2\right)\left(x + 1\right) = 0$

Zeros: $x = -1,\ 0,\ 0,\ 2$

(b)

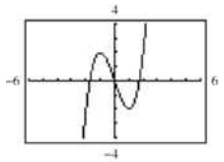


(c) Zeros: $x = -1,\ 0,\ 0,\ 2$; the same

29. (a) $t^3 - 3t = t\left(t^2 - 3\right) = t\left(t + \sqrt{3}\right)\left(t - \sqrt{3}\right) = 0$

Zeros: $t = 0, \pm\sqrt{3}$

(b)

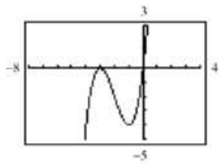


(c) Zeros: $t = 0, \pm 1.732$; the same

31. (a) $x\left(x + 3\right)^2 = 0$

Zeros: $x = 0, -3, -3$

(b)



(c) Zeros: $x = -3,\ -3,\ 0$; the same

33. $f(x) = \left(x + 2\right)\left(x - 1\right)^2\left(x - 5\right)$

$= x^4 - 5x^3 - 3x^2 + 17x - 10$

35. $f(x) = \left(x - 3\right)\left(x - 2 + \sqrt{3}\right)\left(x - 2 - \sqrt{3}\right)$

$= x^3 - 7x^2 + 13x - 3$

37. (a) The degree of f is even and the leading coefficient is 1. The graph rises to the left and rises to the right.

(b) $f(x) = x^4 - 2x^3 - 12x^2 + 18x + 27$

$= (x^4 - 12x^2 + 27) - (2x^3 - 18x)$

$= (x^2 - 9)(x^2 - 3) - 2x(x^2 - 9)$

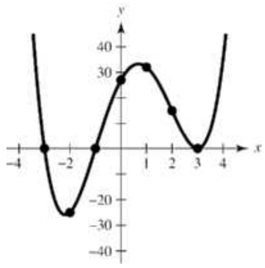
$= (x^2 - 9)(x^2 - 3 - 2x)$

$= (x + 3)(x - 3)(x^2 - 2x - 3)$

$= (x + 3)(x - 3)(x - 3)(x + 1)$

Zeros: $-3,\ 3,\ 3,\ -1$

(c) and (d)

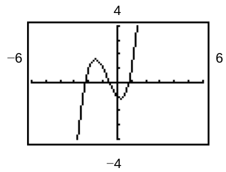


39. $f(x) = x^3 + 2x^2 - x - 1$

(a) $f(-3) < 0,\ f(-2) > 0 \Rightarrow$ zero in $(-3,\ -2)$

$f(-1) > 0,\ f(0) < 0 \Rightarrow$ zero in $(-1,\ 0)$

$f(0) < 0,\ f(1) > 0 \Rightarrow$ zero in $(0,\ 1)$

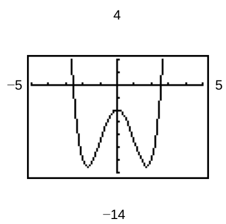


(b) Zeros: $-2.247,\ -0.555,\ 0.802$

41. $f(x) = x^4 - 6x^2 - 4$

(a) $f(-3) > 0, f(-2) < 0 \Rightarrow$ zero in $(-3,\ -2)$

$f(2) < 0, f(3) > 0 \Rightarrow$ zero in $(2,\ 3)$



(b) Zeros: ± 2.570

43.

$$\begin{array}{r} 8x + 5 \\ 3x - 2\,\overline{\big)\,24x^2 - x - 8} \\ \underline{24x^2 - 16x} \end{array}$$

15x − 8

1

5

x 10

2

Thus, $\frac{24x^2 - x - 8}{3x - 2} = 8x + 5 + \frac{2}{3x - 2}$.

45.

$$\begin{array}{r|l} & x^2 - 2 \\ x^2 - 1 & x^4 - 3x^2 + 2 \\ & \underline{x^4 - x^2} \\ & -2x^2 + 2 \\ & \underline{-2x^2 + 2} \\ & 0 \end{array}$$

Thus, $\dfrac{x^4 - 3x^2 + 2}{x^2 - 1} = x^2 - 2,\ (x \neq \pm 1)$.

47.

$$\begin{array}{r|l} & 5x + 2 \\ x^2 - 3x + 1 & 5x^3 - 13x^2 - x + 2 \\ & \underline{5x^3 - 15x^2 + 5x} \\ & 2x^2 - 6x + 2 \\ & \underline{2x^2 - 6x + 2} \\ & 0 \end{array}$$

Thus, $\dfrac{5x^3 - 13x^2 - x + 2}{x^2 - 3x + 1} = 5x + 2,$

$\left(x \neq \dfrac{1}{2}(3 \pm \sqrt{5})\right)$.

49.

$$\begin{array}{r|l} & 3x^2 + 5x + 8 \\ 2x^2 + 0x - 1 & 6x^4 + 10x^3 + 13x^2 - 5x + 2 \\ & \underline{6x^4 + 0x^3 - 3x^2} \\ & 10x^3 + 16x^2 - 5x \\ & \underline{10x^3 + 0x^2 - 5x} \\ & 16x^2 - 0 + 2 \\ & \underline{16x^2 + 0 - 8} \\ & 10 \end{array}$$

Thus,

$$\frac{6x^4 + 10x^3 + 13x^2 - 5x + 2}{2x^2 - 1} = 3x^2 + 5x + 8 + \frac{10}{2x^2 - 1}.$$

51.

$$\begin{array}{r|rrrrr} -2 & 0.25 & -4 & 0 & 0 & 0 \\ & & -\frac{1}{2} & 9 & -18 & 36 \\ \hline & \frac{1}{4} & -\frac{9}{2} & 9 & -18 & 36 \end{array}$$

Thus, $\dfrac{0.25x^4 - 4x^3}{x + 2} = \dfrac{1}{4}x^3 - \dfrac{9}{2}x^2 + 9x - 18 + \dfrac{36}{x + 2}$.

57. (a)

$$\begin{array}{r|rrrrr} -3 & 1 & 10 & -24 & 20 & 44 \\ & & -3 & -21 & 135 & -465 \\ \hline & 1 & 7 & -45 & 155 & -421 = f(-3) \end{array}$$

(b)

$$\begin{array}{r|rrrrr} -2 & 1 & 10 & -24 & 20 & 44 \\ & & -2 & -16 & 80 & -200 \\ \hline & 1 & 8 & -40 & 100 & -156 = f(-2) \end{array}$$

59. $f(x) = x^3 + 4x^2 - 25x - 28$

(a)

$$\begin{array}{r|rrrr} 4 & 1 & 4 & -25 & -28 \\ & & 4 & 32 & 28 \\ \hline & 1 & 8 & 7 & 0 \end{array}$$

$(x - 4)$ is a factor.

(b) $x^2 + 8x + 7 = (x + 1)(x + 7)$

Remaining factors: $(x + 1),\ (x + 7)$

(c) $f(x) = (x - 4)(x + 1)(x + 7)$

(d) Zeros: $4,\ -1,\ -7$

61. $f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$

(a)

$$\begin{array}{r|rrrrr} -2 & 1 & -4 & -7 & 22 & 24 \\ & & -2 & 12 & -10 & -24 \\ \hline & 1 & -6 & 5 & 12 & 0 \end{array}$$

$(x + 2)$ is a factor.

$$\begin{array}{r|rrrr} 3 & 1 & -6 & 5 & 12 \\ & & 3 & -9 & -12 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

$(x - 3)$ is a factor.

(b) $x^2 - 3x - 4 = (x - 4)(x + 1)$

Remaining factors: $(x - 4),\ (x + 1)$

(c) $f(x) = (x + 2)(x - 3)(x - 4)(x + 1)$

(d) Zeros: $-2,\ 3,\ 4,\ -1$

63. $f(x) = 4x^3 - 11x^2 + 10x - 3$

Possible rational zeros: $\pm 1,\ \pm 3,\ \pm \frac{3}{2},\ \pm \frac{3}{4},\ \pm \frac{1}{2},\ \pm \frac{1}{4}$

Zeros: $1,\ 1,\ \dfrac{3}{4}$

65. $g(x) = 5x^3 - 6x + 9$ has two variation in sign $\Rightarrow$ 0 or 2 positive real zeros.

$g(-x) = -5x^3 + 6x + 9$ has one variation in sign

$\Rightarrow$ 1 negative real zero.

53. $\frac{2}{3}$

$\frac{2}{3}$	6	−4	−27	18	0
		4	0	−18	0
	6	0	−27	0	0

Thus, $\dfrac{6x^4 - 4x^3 - 27x^2 + 18x}{x - (2/3)} = 6x^3 - 27x,\ x \neq \dfrac{2}{3}.$

55.

4	3	−10	12	−22
		12	8	80
	3	2	20	58

Thus, $\dfrac{3x^3 - 10x^2 + 12x - 22}{x - 4} = 3x^2 + 2x + 20 + \dfrac{58}{x - 4}.$

67.
$$\begin{array}{r|rrrr} 1 & 4 & -3 & 4 & -3 \\ & & 4 & 1 & 5 \\ \hline & 4 & 1 & 5 & 2 \end{array}$$

All entries positive; $x = 1$ is upper bound.

$$\begin{array}{r|rrrr} -\frac{1}{4} & 4 & -3 & 4 & -3 \\ & & -1 & 1 & -\frac{5}{4} \\ \hline & 4 & -4 & 5 & -\frac{17}{4} \end{array}$$

Alternating signs; $x = -\frac{1}{4}$ is lower bound.

Real zero: $x = \dfrac{3}{4}$

69. $f(x) = 6x^3 + 31x^2 - 18x - 10$

Possible rational zeros:

$\pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}, \pm \frac{1}{6}, \pm \frac{5}{6}$

Using synthetic division and a graph check $x = \frac{5}{6}$.

$$\begin{array}{r|rrrr} \frac{5}{6} & 6 & 31 & -18 & -10 \\ & & 5 & 30 & 10 \\ \hline & 6 & 36 & 12 & 0 \end{array}$$

$x = \frac{5}{6}$ is a real zero.

Rewrite in polynomial form and use the Quadratic Formula:

$$6x^2 + 36x + 12 = 0$$
$$6(x^2 + 6x + 2) = 0$$
$$x^2 + 6x + 2 = 0$$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{28}}{2} = \frac{-6 \pm 2\sqrt{7}}{2} = -3 \pm \sqrt{7}$$

Real zeros: $x = \frac{5}{6}, -3 \pm \sqrt{7}$

71. $f(x) = 6x^4 - 25x^3 + 14x^2 + 27x - 18$

Possible rational zeros:

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{6}$

Use a graphing utility to see that $x = -1$ and $x = 3$ are probably zeros.

$$\begin{array}{r|rrrrr} -1 & 6 & -25 & 14 & 27 & -18 \end{array}$$

73. $6 + \sqrt{-25} = 6 + 5i$

75. $-2i^2 + 7i = 2 + 7i$

77. $(7 + 5i) + (-4 + 2i) = (7 - 4) + (5i + 2i)$
$= 3 + 7i$

79. $5i(13 - 8i) = 65i - 40i^2 = 40 + 65i$

81. $(10 - 8i)(2 - 3i) = 20 - 30i - 16i + 24i^2$
$= -4 - 46i$

83. $(3 + 7i)^2 + (3 - 7i)^2 = (9 + 42i - 49) + (9 - 42i - 49)$
$= -80$

85. $\left(\sqrt{-16} + 3\right)\left(\sqrt{-25} - 2\right) = (4i + 3)(5i - 2)$
$= -20 - 8i + 15i - 6$
$= -26 + 7i$

87. $\sqrt{-9} + 3 + \sqrt{-36} = 3i + 3 + 6i$
$= 3 + 9i$

89. $\dfrac{6 + i}{i} = \dfrac{6 + i}{i} \cdot \dfrac{-i}{-i} = \dfrac{-6i - i^2}{-i^2}$
$= \dfrac{-6i + 1}{1} = 1 - 6i$

91. $\dfrac{3 + 2i}{5 + i} \cdot \dfrac{5 - i}{5 - i} = \dfrac{15 + 10i - 3i + 2}{25 + 1}$
$= \dfrac{17}{26} + \dfrac{7}{26}i$

93. $x^2 + 16 = 0$
$x^2 = -16$
$x = \pm\sqrt{-16}$
$x = \pm 4i$

95. $x^2 + 3x + 6 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{-15}}{2}$$

$$x = -\frac{3}{2} \pm \frac{\sqrt{15}}{2}i$$

	−6	31	−45	18
6	−31	45	−18	0

3	6	−31	45	−18
		18	−39	18
	6	−13	6	0

$$6x^4 - 25x^3 + 14x^2 + 27x - 18 = (x+1)(x-3)(6x^2 - 13x + 6)$$
$$= (x+1)(x-3)(3x-2)(2x-3)$$

Zeros: $x = -1, 3, \frac{2}{3}, \frac{3}{2}$

97. $3x^2 - 5x + 6 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(6)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{-47}}{6}$$

$$x = \frac{5}{6} \pm \frac{\sqrt{47}}{6}i$$

99. $x^2 + 6x + 9 = 0$

$(x + 3)^2 = 0$

$x + 3 = 0$

$x = -3$

101. $x^3 + 16x = 0$

$x(x^2 + 16) = 0$

$x = 0 \quad x^2 + 16 = 0$

$x^2 = -16$

$x = \pm 4i$

103. $f(x) = 3x(x - 2)^2$

Zeros: 0, 2, 2

105. $h(x) = x^3 - 7x^2 + 18x - 24$

4	1	−7	18	−24
		4	−12	24
	1	−3	6	0

$x = 4$ is a zero. Applying the Quadratic Formula on $x^2 - 3x + 6$,

$$x = \frac{3 \pm \sqrt{9 - 4(6)}}{2} = \frac{3}{2} \pm \frac{\sqrt{15}}{2}i.$$

Zeros: $4, \frac{3}{2} + \frac{\sqrt{15}}{2}i, \frac{3}{2} - \frac{\sqrt{15}}{2}i$

$$h(x) = (x - 4)\left(x - \frac{3 + \sqrt{15}i}{2}\right)\left(x - \frac{3 - \sqrt{15}i}{2}\right)$$

107. $f(x) = 2x^4 - 5x^3 + 10x - 12$

2	2	−5	0	10	−12
		4	−2	−4	12
	2	−1	−2	6	0

$x = 2$ is a zero.

109. $f(x) = x^5 + x^4 + 5x^3 + 5x^2$

$= x^2(x^3 + x^2 + 5x + 5)$

$= x^2\left[x^2(x + 1) + 5(x + 1)\right]$

$= x^2(x + 1)(x^2 + 5)$

$= x^2(x + 1)(x + \sqrt{5}i)(x - \sqrt{5}i)$

Zeros: $0, 0, -1, \pm\sqrt{5}i$

111. $f(x) = x^3 - 4x^2 + 6x - 4$

(a) $x^3 - 4x^2 + 6x - 4 = (x - 2)(x^2 - 2x + 2)$

By the Quadratic Formula for $x^2 - 2x + 2$,

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(2)}}{2} = 1 \pm i$$

Zeros: $2, 1 + i, 1 - i$

(b) $f(x) = (x - 2)(x - 1 - i)(x - 1 + i)$

(c) x-intercept: $(2, 0)$

113. (a) $f(x) = -3x^3 - 19x^2 - 4x + 12$

$= -(x + 1)(3x^2 + 16x - 12)$

−1	−3	−19	−4	12
		3	16	−12
	−3	−16	12	0

$3x^2 + 16x - 12 = 0$

$(3x - 2)(x + 6) = 0$

$3x - 2 = 0 \Rightarrow x = \frac{2}{3}$

$x + 6 = 0 \Rightarrow x = -6$

Zeros: $-1, \frac{2}{3}, -6$

(b) $f(x) = -(x + 1)(3x - 2)(x + 6)$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & -1 & -2 & 6 \\ & & -3 & 6 & -6 \\ \hline & 2 & -4 & 4 & 0 \end{array}$$

$x = -\dfrac{3}{2}$ is a zero.

$$f(x) = (x - 2)\left(x + \frac{3}{2}\right)(2x^2 - 4x + 4)$$

$$= (x - 2)(2x + 3)(x^2 - 2x + 2)$$

By the Quadratic Formula, applied to $x^2 - 2x + 2$,

$$x = \frac{2 \pm \sqrt{4 - 4(2)}}{2} = 1 \pm i.$$

Zeros: $2, -\dfrac{3}{2}, 1 \pm i$

$$f(x) = (x - 2)(2x + 3)(x - 1 + i)(x - 1 - i)$$

(c) x-intercepts: $(-1, 0)$, $(-6, 0)$, $\left(\dfrac{2}{3}, 0\right)$

115. $f(x) = x^4 + 34x^2 + 225$

(a) $x^4 + 34x^2 + 225 = (x^2 + 9)(x^2 + 25)$

Zeros: $\pm 3i, \pm 5i$

(b) $(x + 3i)(x - 3i)(x + 5i)(x - 5i)$

(c) No x-intercepts

117. Since $5i$ is a zero, so is $-5i$.

$$f(x) = (x - 4)(x + 2)(x - 5i)(x + 5i)$$

$$= (x^2 - 2x - 8)(x^2 + 25)$$

$$= x^4 - 2x^3 + 17x^2 - 50x - 200$$

119. Since $-3+5i$ is a zero, so is $-3-5i$.

$$f(x) = (x-1)(x+4)(x+3-5i)(x+3+5i)$$
$$= (x^2+3x-4)((x+3)^2+25)$$
$$= (x^2+3x-4)(x^2+6x+34)$$
$$= x^4+9x^3+48x^2+78x-136$$

121. $f(x) = x^4-2x^3+8x^2-18x-9$

(a) $f(x) = (x^2+9)(x^2-2x-1)$

(b) For the quadratic

$x^2-2x-1, x = \dfrac{2 \pm \sqrt{(-2)^2-4(-1)}}{2} = 1 \pm \sqrt{2}.$

$f(x) = \left(x^2+9\right)\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right)$

(c) $f(x) = \left(x+3i\right)\left(x-3i\right)\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right)$

123. Zeros: $-2i$, 2i

$(x+2i)(x-2i) = x^2+4$ is a factor.

$f(x) = (x^2+4)(x+3)$

Zeros: $\pm 2i, -3$

125. $f(x) = \dfrac{2-x}{x+3}$

(a) Domain: all $x \neq -3$
(b) Not continuous
(c) Horizontal asymptote: $y = -1$
Vertical asymptote: $x = -3$

127. $f(x) = \dfrac{2}{x^2-3x-18} = \dfrac{2}{(x-6)(x+3)}$

(a) Domain: all $x \neq 6, -3$
(b) Not continuous
(c) Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = 6$, $x = -3$

129. $f(x) = \dfrac{7+x}{7-x}$

(a) Domain: all $x \neq 7$
(b) Not continuous
(c) Horizontal asymptote: $y = -1$
Vertical asymptote: $x = 7$

131. $f(x) = \dfrac{4x^2}{2x^2-3}$

(a) Domain: all $x \neq \pm\sqrt{\dfrac{3}{2}} = \pm\dfrac{\sqrt{6}}{2}$

(b) Not continuous

133. $f(x) = \dfrac{2x-10}{x^2-2x-15} = \dfrac{2(x-5)}{(x-5)(x+3)} = \dfrac{2}{x+3}, x \neq 5$

(a) Domain: all $x \neq 5, -3$
(b) Not continuous
(c) Vertical asymptote: $x = -3$
(There is a hole at $x = 5.$)
Horizontal asymptote: $y = 0$

135. $f(x) = \dfrac{x-2}{|x|+2}$

(a) Domain: all real numbers
(b) Continuous
(c) No vertical asymptotes
Horizontal asymptotes: $y = 1$, $y = -1$

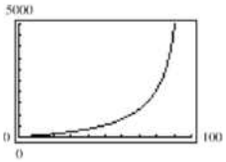
137. $C = \dfrac{528p}{100-p}, \ 0 \le p < 100$

(a) When $p = 25$, $C = \dfrac{528(25)}{100-25} = \$176$ million.

When $p = 50$, C$=\dfrac{528(50)}{100-50} = \$528$ million.

When $p = 75$, C$=\dfrac{528(75)}{100-75} = \$1584$ million.

(b)



Answers will vary.

(c) No. As $p \to 100$, C approaches infinity.

139. $f(x) = \dfrac{x^2-5x+4}{x^2-1}$

$= \dfrac{(x-4)(x-1)}{(x-1)(x+1)}$

$= \dfrac{x-4}{x+1}, x \neq 1$

Vertical asymptote: $x = -1$
Horizontal asymptote: $y = 1$
No slant asymptotes
Hole at $x = 1$

(c) Horizontal asymptote: $y = 2$

Vertical asymptote: $x = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2}$

141. $f(x) = \frac{3x^2 + 5x - 2}{x + 1}$

$= \frac{(3x - 1)(x + 2)}{x + 1}$

Vertical asymptote: $x = -1$

Horizontal asymptote: none

Long division gives:

$$\begin{array}{r} 3x + 2 \\ x + 1 \overline{)3x^2 + 5x - 2} \\ \underline{3x^2 + 3x} \\ 2x - 2 \\ \underline{2x + 2} \\ -4 \end{array}$$

Slant asymptote: $y = 3x + 2$

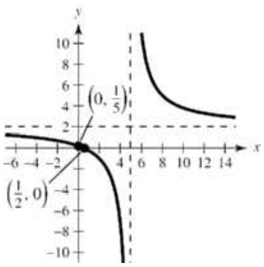
143. $f(x) = \frac{2x - 1}{x - 5}$

Intercepts: $\left(0, \frac{1}{5}\right), \left(\frac{1}{2}, 0\right)$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = 2$

x	-4	-1	0	$\frac{1}{2}$	1	6	8
y	1	$\frac{1}{2}$	$\frac{1}{5}$	0	$-\frac{1}{4}$	11	5



145. $f(x) = \frac{2x^2}{x^2 - 4}$

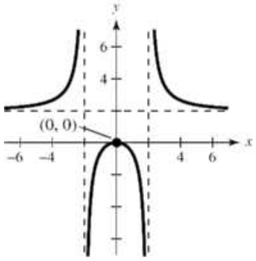
Intercept: $(0, 0)$

y-axis symmetry

Vertical asymptotes: $x = \pm 2$

Horizontal asymptote: $y = 2$

	-6	-4	-1	0	1	4	6
y	$\frac{9}{4}$	$\frac{8}{3}$	$-\frac{2}{3}$	0	$-\frac{2}{3}$	$\frac{8}{3}$	$\frac{9}{4}$



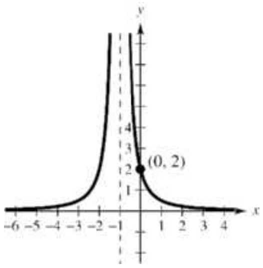
147. $f(x) = \frac{2}{(x + 1)^2}$

Intercept: $(0, 2)$

Horizontal asymptote: $y = 0$

Vertical asymptote: $x = -1$

x	-4	-3	-2	0	1	2
y	$\frac{2}{9}$	$\frac{1}{2}$	2	2	$\frac{1}{2}$	$\frac{2}{9}$



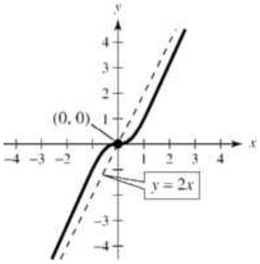
149. $f(x) = \dfrac{2x^3}{x^2+1} = 2x - \dfrac{2x}{x^2+1}$

Intercept: $(0, 0)$

Origin symmetry

Slant asymptote: $y = 2x$

x	-2	-1	0	1	2
y	$-\frac{16}{5}$	-1	0	1	$\frac{16}{5}$



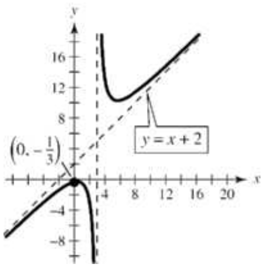
151. $f(x) = \dfrac{x^2 - x + 1}{x - 3} = x + 2 + \dfrac{7}{x-3}$

Intercept: $\left(0, -\dfrac{1}{3}\right)$

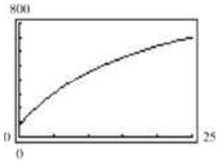
Vertical asymptote: $x = 3$

Slant asymptote: $y = x + 2$

x	-4	0	2	4	5
y	-3	$-\frac{1}{3}$	-3	13	10.5



153. (a)



5 years: $N(5) = \dfrac{20(4+3(5))}{1+0.05(5)} = 304$ thousand fish

10 years: $N(10) = \dfrac{20(4+3(10))}{1+0.05(10)} = 453.\overline{3}$ thousand fish

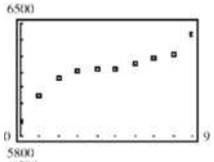
25 years: $N(25) = \dfrac{20(4+3(25))}{1+0.05(25)} = 702.\overline{2}$ thousand fish

(b) The maximum number of fish is $N = 1{,}200{,}000.$ The graph of N has a horizontal asymptote at $N = 1200$ or 1,200,000 fish.

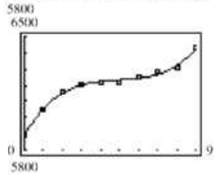
155. Quadratic model

157. Linear model

159. (a)

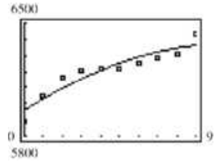


(b)



(c) $S = -3.49t^2 + 76.3t + 5958;\ R^2 \approx 0.8915$

(d)



(e) The cubic model is a better fit because it more closely follows the pattern of the data.

(f) For 2012, let $t = 12.$

$$S(12) = 2.520(12)^3 - 37.51(12)^2 + 192.4(12) + 5895$$
$$= 7157 \text{ stations}$$

161. False. The degree of the numerator is two more than the degree of the denominator.

163. False. $(1+i) + (1-i) = 2,$ a real number

165. Not every rational function has a vertical asymptote. For example,

$$y = \frac{x}{x^2+1}.$$

167. The error is $\sqrt{-4} \neq 4i.$ In fact,

$$-i\left(\sqrt{-4} - 1\right) = -i(2i - 1) = 2 + i.$$

Chapter 2 Test

1. $y = x^2 + 4x + 3 = x^2 + 4x + 4 - 1 = (x+2)^2 - 1$

Vertex: $(-2, -1)$

$x = 0 \Rightarrow y = 3$

$y = 0 \Rightarrow x^2 + 4x + 3 = 0 \Rightarrow (x+3)(x+1) = 0 \Rightarrow x = -1, -3$

Intercepts: $(0, 3)$, $(-1, 0)$, $(-3, 0)$

2. Let $y = a(x-h)^2 + k$. The vertex $(3, -6)$ implies that $y = a(x-3)^2 - 6$. For $(0, 3)$ you obtain

$3 = a(0-3)^2 - 6 = 9a - 6 \Rightarrow a = 1.$

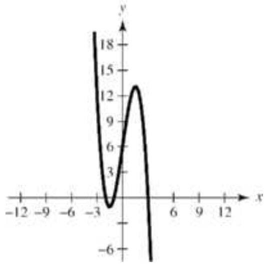
Thus, $y = (x-3)^2 - 6 = x^2 - 6x + 3$.

3. $f(x) = 4x^3 + 4x^2 + x = x(4x^2 + 4x + 1) = x(2x+1)^2$

Zeros: 0 (multiplicity 1)

$-\frac{1}{2}$ (multiplicity 2)

4. $f(x) = -x^3 + 7x + 6$



5.

$$\begin{array}{r} 3x \\ x^2+1 \overline{)\,3x^3 + 0x^2 + 4x - 1} \\ \underline{3x^3 \quad + 3x} \\ x - 1 \end{array}$$

$3x + \frac{x-1}{x^2+1}$

6.

2	2	0	−5	0	−3
		4	8	6	12
	2	4	3	6	9

$2x^3 + 4x^2 + 3x + 6 + \frac{9}{x-2}$

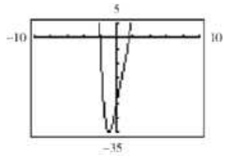
7.

−2	3	0	−6	5	−1
		−6	12	−12	14
	3	−6	6	−7	13

$f(-2) = 13$

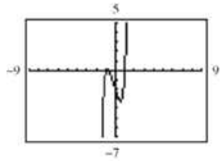
8. Possible rational zeros:

$\pm 24, \pm 12, \pm 8, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1, \pm\frac{3}{2}, \pm\frac{1}{2}$



Rational zeros: $-2, \frac{3}{2}$

9. Possible rational zeros: $\pm 2, \pm 1, \pm\frac{2}{3}, \pm\frac{1}{3}$



Rational zeros: $\pm 1, -\frac{2}{3}$

10. $f(x) = x^3 - 7x^2 + 11x + 19$

$= (x+1)(x^2 - 8x + 19)$

For the quadratic,

$x = \frac{8 \pm \sqrt{64 - 4(19)}}{2} = 4 \pm \sqrt{3}i.$

Zeros: $-1, 4 \pm \sqrt{3}\,i$

$f(x) = (x+1)(x - 4 + \sqrt{3}i)(x - 4 - \sqrt{3}i)$

11. $(-8 - 3i) + (-1 - 15i) = -9 - 18i$

12. $\left(10 + \sqrt{-20}\right) - \left(4 - \sqrt{-14}\right) = 6 + 2\sqrt{5}i + \sqrt{14}i = 6 + \left(2\sqrt{5} + \sqrt{14}\right)i$

13. $(2+i)(6-i) = 12 + 6i - 2i + 1 = 13 + 4i$

14. $(4+3i)^2 - (5+i)^2 = (16 + 24i - 9) - (25 + 10i - 1) = -17 + 14i$

15. $\frac{8+5i}{6-i} \cdot \frac{6+i}{6+i} = \frac{48 + 30i + 8i - 5}{36+1} = \frac{43}{37} + \frac{38}{37}i$

16. $\frac{5i}{2+i} \cdot \frac{2-i}{2-i} = \frac{10i+5}{4+1} = 1 + 2i$

17. $\frac{(2i-1)}{(3i+2)} \cdot \frac{2-3i}{2-3i} = \frac{6 - 2 + 4i + 3i}{4+9}$

$= \frac{4}{13} + \frac{7}{13}i$

18. $x^2 + 75 = 0$

$x^2 = -75$

$x = \pm\sqrt{-75}$

$= \pm 5\sqrt{3}i$

19. $x^2 - 2x + 8 = 0$

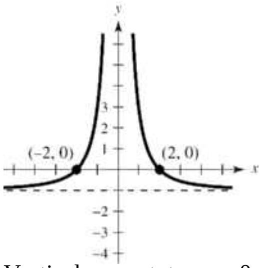
$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(8)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{-28}}{2}$

$x = \dfrac{2 \pm 2\sqrt{7}i}{2}$

$x = 1 \pm \sqrt{7}i$

20.



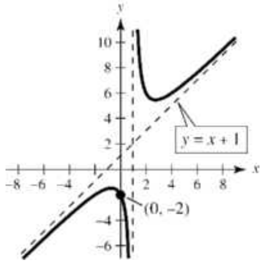
Vertical asymptote: $x = 0$
Intercepts: $(2, 0)$, $(-2, 0)$
Symmetry: y-axis
Horizontal asymptote: $y = -1$

21. $g(x) = \dfrac{x^2 + 2}{x - 1} = x + 1 + \dfrac{3}{x - 1}$

Vertical asymptote: $x = 1$

Intercept: $(0, -2)$

Slant asymptote: $y = x + 1$

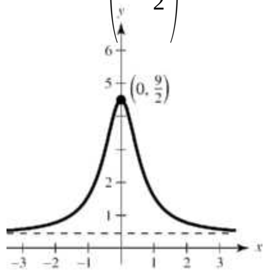


22. $f(x) = \dfrac{2x^2 + 9}{5x^2 + 2}$

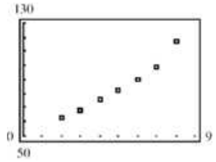
Horizontal asymptote: $y = \dfrac{2}{5}$

y-axis symmetry

Intercept: $\left(0, \dfrac{9}{2}\right)$

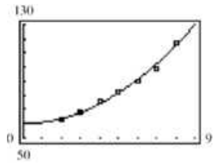


23. (a)



(b) $A = 0.861t^2 + 0.03t + 60.0$

(c)



The model fits the data well.

(d) For 2010, let $t = 10$.

$A(10) = 0.861(10)^2 + 0.03(10) + 60.0$

$= \$146.4$ million

For 2012, let $t = 12$.

$A(12) = 0.861(12)^2 + 0.03(12) + 60.0$

$\approx \$184.3$ billion

(e) Answers will vary.