

**Solution Manual for Probability and Statistical Inference 9th Edition  
Hogg Tanis Zimmerman 0321923278 9780321923271**

**Full link download**

**Solution Manual:**

<https://testbankpack.com/p/solution-manual-for-probability-and-statistical-inference-9th-edition-hogg-tanis-zimmerman-0321923278-9780321923271/>

# INSTRUCTOR'S SOLUTIONS MANUAL

## PROBABILITY AND STATISTICAL INFERENCE

NINTH EDITION

ROBERT V. HOGG

*University of Iowa*

Elliot A. Tanis

*Hope College*

Dale L. Zimmerman

*University of Iowa*

**PEARSON**

Boston Columbus Indianapolis New York San Francisco Upper Saddle River Amsterdam  
Cape Town Dubai London Madrid Milan Munich Paris Montreal Toronto Delhi  
Mexico City São Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo



**This work is protected by United States copyright laws and is provided solely for the use of instructors in teaching their courses and assessing student learning. Dissemination or sale of any part of this work (including on the World Wide Web) will destroy the integrity of the work and is not permitted. The work and materials from it should never be made available to students except by instructors using the accompanying text in their classes. All recipients of this work are expected to abide by these restrictions and to honor the intended pedagogical purposes and the needs of other instructors who rely on these materials.**

The author and publisher of this book have used their best efforts in preparing this book. These efforts include the development, research, and testing of the theories and programs to determine their effectiveness. The author and publisher make no warranty of any kind, expressed or implied, with regard to these programs or the documentation contained in this book. The author and publisher shall not be liable in any event for incidental or consequential damages in connection with, or arising out of, the furnishing, performance, or use of these programs.

Reproduced by Pearson from electronic files supplied by the author.

Copyright © 2015 Pearson Education, Inc.  
Publishing as Pearson, 75 Arlington Street, Boston, MA 02116.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher. Printed in the United States of America.

ISBN-13: 978-0-321-91379-1

ISBN-10: 0-321-91379-5

[www.pearsonhighered.com](http://www.pearsonhighered.com)

**PEARSON**

# Contents

Preface	v
<b>1 Probability</b>	<b>1</b>
1.1 Properties of Probability . . . . .	1
1.2 Methods of Enumeration . . . . .	2
1.3 Conditional Probability . . . . .	3
1.4 Independent Events . . . . .	4
1.5 Bayes' Theorem . . . . .	5
<b>2 Discrete Distributions</b>	<b>7</b>
2.1 Random Variables of the Discrete Type . . . . .	7
2.2 Mathematical Expectation . . . . .	10
2.3 Special Mathematical Expectations . . . . .	11
2.4 The Binomial Distribution . . . . .	14
2.5 The Negative Binomial Distribution . . . . .	16
2.6 The Poisson Distribution . . . . .	17
<b>3 Continuous Distributions</b>	<b>19</b>
3.1 Random Variables of the Continuous Type . . . . .	19
3.2 The Exponential, Gamma, and Chi-Square Distributions . . . . .	25
3.3 The Normal Distribution . . . . .	28
3.4 Additional Models . . . . .	29
<b>4 Bivariate Distributions</b>	<b>33</b>
4.1 Bivariate Distributions of the Discrete Type . . . . .	33
4.2 The Correlation Coefficient . . . . .	34
4.3 Conditional Distributions . . . . .	35
4.4 Bivariate Distributions of the Continuous Type . . . . .	36
4.5 The Bivariate Normal Distribution . . . . .	41
<b>5 Distributions of Functions of Random Variables</b>	<b>43</b>
5.1 Functions of One Random Variable . . . . .	43
5.2 Transformations of Two Random Variables . . . . .	44
5.3 Several Random Variables . . . . .	48
5.4 The Moment-Generating Function Technique . . . . .	50
5.5 Random Functions Associated with Normal Distributions . . . . .	52
5.6 The Central Limit Theorem . . . . .	55
5.7 Approximations for Discrete Distributions . . . . .	56
5.8 Chebyshev's Inequality and Convergence in Probability . . . . .	58
5.9 Limiting Moment-Generating Functions . . . . .	59

	<b>Point Estimation</b>	<b>61</b>
	6.1 Descriptive Statistics . . . . .	61
6	6.2 Exploratory Data Analysis . . . . .	63
	6.3 Order Statistics . . . . .	68
	6.4 Maximum Likelihood Estimation . . . . .	71
	6.5 A Simple Regression Problem . . . . .	74
	6.6 Asymptotic Distributions of Maximum Likelihood Estimators . . . . .	78
	6.7 Sufficient Statistics . . . . .	79
	6.8 Bayesian Estimation . . . . .	81
	6.9 More Bayesian Concepts . . . . .	83
7	<b>Interval Estimation</b>	<b>85</b>
	7.1 Confidence Intervals for Means . . . . .	85
	7.2 Confidence Intervals for the Difference of Two Means . . . . .	86
	7.3 Confidence Intervals For Proportions . . . . .	88
	7.4 Sample Size . . . . .	89
	7.5 Distribution-Free Confidence Intervals for Percentiles . . . . .	90
	7.6 More Regression . . . . .	92
	7.7 Resampling Methods . . . . .	98
8	<b>Tests of Statistical Hypotheses</b>	<b>105</b>
	8.1 Tests About One Mean . . . . .	105
	8.2 Tests of the Equality of Two Means . . . . .	107
	8.3 Tests about Proportions . . . . .	110
	8.4 The Wilcoxon Tests . . . . .	111
	8.5 Power of a Statistical Test . . . . .	115
	8.6 Best Critical Regions . . . . .	119
	8.7 Likelihood Ratio Tests . . . . .	121
9	<b>More Tests</b>	<b>125</b>
	9.1 Chi-Square Goodness-of-Fit Tests . . . . .	125
	9.2 Contingency Tables . . . . .	128
	9.3 One-Factor Analysis of Variance . . . . .	128
	9.4 Two-Way Analysis of Variance . . . . .	132
	9.5 General Factorial and $2^k$ Factorial Designs . . . . .	133
	9.6 Tests Concerning Regression and Correlation . . . . .	134
	9.7 Statistical Quality Control . . . . .	135

---

Copyright © 2015 Pearson Education, Inc.

# Preface

This solutions manual provides answers for the even-numbered exercises in Probability and Statistical Inference, 9th edition, by Robert V. Hogg, Elliot A. Tanis, and Dale L. Zimmerman. Complete solutions are given for most of these exercises. You, the instructor, may decide how many of these solutions and answers you want to make available to your students. Note that the answers for the odd-numbered exercises are given in the textbook.

All of the figures in this manual were generated using Maple, a computer algebra system. Most of the figures were generated and many of the solutions, especially those involving data, were solved using procedures that were written by Zaven Karian from Denison University. We thank him for providing these. These procedures are available free of charge for your use. They are available for download at <http://www.math.hope.edu/tanis/>. Short descriptions of these procedures are provided on the “Maple Card.” Complete descriptions of these procedures are given in Probability and Statistics: Explorations with MAPLE, second edition, 1999, written by Zaven Karian and Elliot Tanis, published by Prentice Hall (ISBN 0-13-021536-8). You can download a copy of this manual at <http://www.math.hope.edu/tanis/MapleManual.pdf>.

Our hope is that this solutions manual will be helpful to each of you in your teaching.

If you find an error or wish to make a suggestion, send these to Elliot Tanis, [tanis@hope.edu](mailto:tanis@hope.edu), and he will post corrections on his web page, <http://www.math.hope.edu/tanis/>.

R.V.H.  
E.A.T.  
D.L.Z.





# Chapter 1

## Probability

### 1.1 Properties of Probability

1.1-2 Sketch a figure and fill in the probabilities of each of the disjoint sets.

Let  $A = \{\text{insure more than one car}\}$ ,  $P(A) =$

0.85. Let  $B = \{\text{insure a sports car}\}$ ,  $P(B) =$

0.23.

Let  $C = \{\text{insure exactly one car}\}$ ,  $P(C) = 0.15$ .

It is also given that  $P(A \cap B) = 0.17$ . Since  $A \cap C = \emptyset$ ,  $P(A \cap C) = 0$ . It follows that

$P(A \cap B \cap C^c) = 0.17$ . Thus  $P(A^c \cap B \cap C^c) = 0.06$  and  $P(A^c \cap B^c \cap C) = 0.09$ .

1.1-4 (a)  $S = \{\text{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTT}\}$ ;

(b) (i)  $5/16$ , (ii)  $0$ , (iii)  $11/16$ , (iv)  $4/16$ , (v)  $4/16$ , (vi)  $9/16$ , (vii)  $4/16$ .

1.1-6 (a)  $P(A \cup B) = 0.4 + 0.5 - 0.3 = 0.6$ ;

(b)  $A = (A \cap B^c) \cup (A \cap B)$   
 $P(A) = P(A \cap B^c) + P(A \cap B)$   
 $0.4 = P(A \cap B^c) + 0.3$   
 $P(A \cap B^c) = 0.1$ ;

(c)  $P(A^c \cap B^c) = P[(A \cap B)^c] = 1 - P(A \cap B) = 1 - 0.3 = 0.7$ .

1.1-8 Let  $A = \{\text{lab work done}\}$ ,  $B = \{\text{referral to a specialist}\}$ ,

$P(A) = 0.41$ ,  $P(B) = 0.53$ ,  $P([A \cup B]^c) = 0.21$ .

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$   
 $0.79 = 0.41 + 0.53 - P(A \cap B)$   
 $P(A \cap B) = 0.41 + 0.53 - 0.79 = 0.15$ .

$$\begin{aligned}
 1.1-10 \quad P(A \cup B \cup C) &= P(A \cup (B \cup C)) \\
 &= P(A) + P(B \cup C) - P[A \cap (B \cup C)] \\
 &= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)] \\
 &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C)
 \end{aligned}$$

1.1-12 (a)  $1/3$ ; (b)  $2/3$ ; (c)  $0$ ; (d)  $1/2$ .

$$1.1-14 \quad P(A) = \frac{2[r-r]}{3/2] - 2r} = 1 - \frac{\sqrt{3}}{2}.$$

1.1-16 Note that the respective probabilities are  $P_0, P_1 = P_0/4, P_2 = P_0/4^2, \dots$

$$\begin{aligned} \sum_{k=0}^{\infty} P_0 4^{-k} &= 1 \\ \frac{P_0}{1 - 1/4} &= 1 \\ P_0 &= \frac{3}{4} \\ 1 - P_0 - P_1 &= 1 - \frac{15}{16} = \frac{1}{16} \end{aligned}$$

## 1.2 Methods of Enumeration

1.2-2 (a)  $(4)(5)(2) = 40$ ; (b)  $(2)(2)(2) = 8$ .

$$1.2-4 \quad (a) \quad {}_4P_3 = 80;$$

$$(b) \quad 4(2^6) = 256;$$

$$(c) \quad \frac{(4 - 1 + 3)!}{(4 - 1)!3!} = 20.$$

1.2-6  $S = \{ DDD, DDFD, DFDD, FDDD, DDFFD, DFDFD, FDDFD, DFFDD, FDFDD, FFDDD, FFF, FFDF, FDFF, DFFF, FFDFF, FDFDF, DFFDF, FDDFF, DFDFD, DDDFF \}$  so there are 20 possibilities.

$$1.2-8 \quad 3 \cdot 3 \cdot 2^{12} = 36,864.$$

$$1.2-10 \quad \sum_{r=1}^{n-1} \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!} = \frac{(n-1)(n-1)! + r(n-1)!}{r!(n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}.$$

$$1.2-12 \quad 0 = (1-1)^n = \sum_{r=0}^n \binom{n}{r} (-1)^r (1)^{n-r} = \sum_{r=0}^n \binom{n}{r} (-1)^r.$$

$$2^n = (1+1)^n = \sum_{r=0}^n \binom{n}{r} (1)^r (1)^{n-r} = \sum_{r=0}^n \binom{n}{r}.$$

$$1.2-14 \quad \sum_{r=0}^{10-1+36} \binom{10-1+36}{r} =$$

$$\frac{3}{6} \frac{45!}{36!9!} = 886,163,135.$$

$$1.2-16 (a) \frac{\binom{52}{3} \binom{52-19}{6}}{\binom{52}{9}} = \frac{102,486}{351,325} = 0.2917;$$

$$(b) \frac{\binom{52}{3} \binom{52-19}{2} \binom{52-19-10}{1} \binom{52-19-10-7}{0} \binom{52-19-10-7-3}{1} \binom{52-19-10-7-3-6}{0} \binom{52-19-10-7-3-6-2}{2}}{\binom{52}{9}} = \frac{7,695}{1,236,664} = 0.00622.$$

### 1.3 Conditional Probability

1.3-2 (a)  $\frac{1041}{1456}$ ;

(b)  $\frac{392}{633}$ ;

(c)  $\frac{649}{823}$ .

(d) The proportion of women who favor a gun law is greater than the proportion of men who favor a gun law.

1.3-4 (a)  $P(HH) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$ ;

(b)  $P(HC) = \frac{13}{52} \cdot \frac{13}{51} = \frac{13}{204}$

(c)  $P(\text{Non-Ace Heart, Ace}) + P(\text{Ace of Hearts, Non-Heart Ace})$

$$= \frac{12}{52} \cdot \frac{4}{51} + \frac{1}{52} \cdot \frac{3}{51} = \frac{51}{52 \cdot 51} = \frac{1}{52}$$

1.3-6 Let  $H = \{\text{died from heart disease}\}$ ;  $P = \{\text{at least one parent had heart disease}\}$ .

$$P(H | P^c) = \frac{N(H \cap P^c)}{N(P^c)} = \frac{110}{648}$$

1.3-8 (a)  $\frac{3}{20} \cdot \frac{2}{19} \cdot \frac{1}{18} = \frac{1}{1140}$ ;

(b)  $\frac{\frac{2}{20} \cdot \frac{1}{17} \cdot \frac{1}{3}}{\frac{1}{380}}$

(c)  $\sum_{k=1}^2 \frac{\frac{2}{20} \cdot \frac{2k-2}{17} \cdot \frac{1}{20-2k}}{\frac{1}{76}} = 0.4605$ .

(d) Draw second. The probability of winning is  $1 - 0.4605 = 0.5395$ .

1.3-10 (a)  $P(A) = \frac{52}{52} \cdot \frac{51}{52} \cdot \frac{50}{52} \cdot \frac{49}{52} \cdot \frac{48}{52} \cdot \frac{47}{52} = \frac{8,808,975}{11,881,376} = 0.741414$ ;

$$(b) P(A^c) = 1 - P(A) = 0.25859.$$

1.3-12 (a) It doesn't matter because  $P(B_1) = \frac{1}{18}$ ,  $P(B_5) = \frac{1}{18}$ ,  $P(B_{18}) = \frac{1}{18}$ ;

$$(b) P(B) = \frac{2}{18} = \frac{1}{9} \text{ on each draw.}$$

1.3-14 (a)  $P(A_1) = 30/100$ ;

$$(b) P(A_3 \cap B_2) = 9/100$$
;

$$(c) P(A_2 \cup B_3) = 41/100 + 28/100 - 9/100 = 60/100$$
;

$$(d) P(A_1 | B_2) = 11/41;$$

$$(e) P(B_1 | A_3) = 13/29.$$

$$1.3-16 \quad \frac{3}{5} \cdot \frac{5}{8} + \frac{2}{5} \cdot \frac{4}{8} = \frac{23}{40}.$$

## 1.4 Independent Events

$$1.4-2 (a) \quad P(A \cap B) = P(A)P(B) = (0.3)(0.6) = 0.18;$$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.3 + 0.6 - 0.18 \\ &= 0.72. \end{aligned}$$

$$(b) \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.18}{0.6} = 0.3$$

$$\begin{aligned} 1.4-4 \text{ Proof of (b): } P(A^1 \cap B) &= P(B)P(A^1|B) \\ &= P(B)[1 - P(A|B)] \\ &= P(B)[1 - P(A)] \\ &= P(B)P(A^1). \end{aligned}$$

$$\begin{aligned} \text{Proof of (c): } P(A^1 \cap B^1) &= P[(A \cup B)^1] \\ &= 1 - P(A \cup B) \\ &= 1 - P(A) - P(B) + P(A \cap B) \\ &= 1 - P(A) - P(B) + P(A)P(B) \\ &= [1 - P(A)][1 - P(B)] \\ &= P(A^1)P(B^1). \end{aligned}$$

$$= P(A^1)P(B)P(C^1)$$

$$= P(A^1)P(B \cap C^1)$$

$$P[A^1 \cap B^1 \cap C^1] = P[(A \cup B \cup C)^c]$$

4 Chapter 1 Probability Section 1.4 Independent Events 6

---


$$= 1 - P(A \cup B \cup C)$$

$$= 1 - P(A) - P(B) - P(C) + P(A)P(B) + P(A)P(C) + P(B)P(C) - P(A)P(B)P(C)$$

$$= [1 - P(A)][1 - P(B)][1 - P(C)]$$

$$= P(A^1)P(B^1)P(C^1).$$

1.4-8  $\frac{1}{6} \cdot \frac{2}{6} \cdot \frac{3}{6} + \frac{1}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} + \frac{5}{6} \cdot \frac{2}{6} \cdot \frac{3}{6} = \frac{2}{9}$ .



1.4-10 (a)  $\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$  ;  
 (b)  $\frac{1}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{2}{4} = \frac{9}{16}$  ;  
 (c)  $\frac{2}{4} \cdot \frac{1}{4} + \frac{2}{4} \cdot \frac{4}{4} = \frac{10}{16}$  .

1.4-12 (a)  $\frac{{}^2P_1 \cdot {}^3P_1 \cdot {}^2P_1 \cdot {}^1P_2}{-}$  ;  
 (b)  $\frac{{}^2P_1 \cdot {}^2P_3 \cdot {}^2P_1 \cdot {}^1P_2}{-}$  ;  
 (c)  $\frac{{}^2P_1 \cdot {}^2P_3 \cdot {}^2P_1 \cdot {}^1P_2}{-}$  ;  
 (d)  $\frac{5!}{3!2!} \cdot \frac{{}^2P_1 \cdot {}^3P_1 \cdot {}^2P_1 \cdot {}^1P_2}{2 \cdot 2}$  .

1.4-14 (a)  $1 - (0.4)^3 = 1 - 0.064 = 0.936$ ;  
 (b)  $1 - (0.4)^8 = 1 - 0.00065536 = 0.99934464$ .

1.4-16 (a)  $\sum_{k=0}^5 \frac{{}^5P_k \cdot {}^4P_{5-k}}{5!} = \frac{5}{5!}$  ;  
 (b)  $\frac{1}{5} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{3}{5}$  .

1.4-18 (a) 7; (b)  $(1/2)^7$ ; (c) 63; (d) No!  $(1/2)^{63} = 1/9,223,372,036,854,775,808$ .

1.4-20 No.

### 1.5 Bayes' Theorem

1.5-2 (a)  $P(G) = P(A \cap G) + P(B \cap G)$   
 $= P(A)P(G|A) + P(B)P(G|B)$   
 $= (0.40)(0.85) + (0.60)(0.75) = 0.79$ ;

(b)  $P(A|G) = \frac{P(A \cap G)}{P(G)}$   
 $= \frac{(0.40)(0.85)}{0.79} = 0.43$ .

1.5-4 Let event B denote an accident and let  $A_1$  be the event that age of the driver is 16–25. Then

$$\begin{aligned} P(A_1 | B) &= \frac{(0.1)(0.05)}{(0.1)(0.05) + (0.55)(0.02) + (0.20)(0.03) + (0.15)(0.04)} \\ &= \frac{50}{50 + 110 + 60 + 60} = \frac{50}{280} = 0.179. \end{aligned}$$

1.5-6 Let B be the event that the policyholder dies. Let  $A_1, A_2, A_3$  be the events that the deceased is standard, preferred and ultra-preferred, respectively. Then

$$\begin{aligned}
 \underline{\underline{P(A_1 | B)}} &= \frac{(0.60)(0.01)}{(0.60)(0.01) + (0.30)(0.008) + (0.10)(0.007)} \\
 &= \frac{60}{60 + 24 + 7} = \frac{60}{91} = 0.659; \\
 P(A_2 | B) &= \frac{24}{91} = 0.264; \\
 P(A_3 | B) &= \frac{7}{91} = 0.077.
 \end{aligned}$$

1.5-8 Let A be the event that the tablet is under warranty.

$$\begin{aligned}
 \underline{\underline{P(B_1 | A)}} &= \frac{(0.40)(0.10)}{(0.40)(0.10) + (0.30)(0.05) + (0.20)(0.03) + (0.10)(0.02)} \\
 &= \frac{40}{40 + 15 + 6 + 2} = \frac{40}{63} = 0.635; \\
 P(B_2 | A) &= \frac{15}{63} = 0.238; \\
 P(B_3 | A) &= \frac{6}{63} = 0.095; \\
 \underline{\underline{P(B_4 | A)}} &= \frac{2}{63} = 0.032.
 \end{aligned}$$

1.5-10 (a)  $P(D^+) = (0.02)(0.92) + (0.98)(0.05) = 0.0184 + 0.0490 = 0.0674;$

(b)  $\underline{\underline{P(A^- | D^+)}} = \frac{0.0490}{0.0674} = 0.727; \quad \underline{\underline{P(A^+ | D^+)}} = \frac{0.0184}{0.0674} = 0.273;$   
 $\underline{\underline{P(A^- | D^-)}} = \frac{(0.98)(0.95)}{16 + 9310} = 0.998;$

(c)  $\underline{\underline{P(A^- | D^-)}} = \frac{(0.02)(0.08) + (0.98)(0.95)}{16 + 9310} = 0.998;$   
 $\underline{\underline{P(A^+ | D^-)}} = 0.002.$

(d) Yes, particularly those in part (b).

1.5-12 Let  $D = \{\text{has the disease}\}$ ,  $DP = \{\text{detects presence of disease}\}$ . Then

$$\begin{aligned}
 P(D | DP) &= \frac{P(D \cap DP)}{P(DP)} \\
 &= \frac{P(D) \cdot P(DP | D)}{P(D) \cdot P(DP | D) + P(D^1) \cdot P(DP | D^1)} \\
 &= \frac{(0.005)(0.90)}{(0.005)(0.90) + (0.995)(0.02)} \\
 &= \frac{0.0045}{0.0045 + 0.0199} = \frac{0.0045}{0.0244} = 0.1844.
 \end{aligned}$$

1.5-14 Let  $D = \{\text{defective roll}\}$ . Then

$$\underline{\underline{P(I | D)}} = \frac{P(I \cap D)}{P(D)} = \underline{\underline{\quad}}$$

$$\begin{aligned} &= P(D) \\ &= \frac{P(I) \cdot P(D|I)}{P(I) \cdot P(D|I) + P(II) \cdot P(D|II)} \\ &= \frac{(0.60)(0.03)}{(0.60)(0.03) + (0.40)(0.01)} \\ &= \frac{0.018}{0.018 + 0.004} = \frac{0.018}{0.022} = 0.818. \end{aligned}$$

## Chapter 2

# Discrete Distributions

### 2.1 Random Variables of the Discrete Type

2.1-2 (a)

$$f(x) = \begin{cases} 0.6, & x = 1, \\ 0.3, & x = 5, \\ 0.1, & x = 10, \end{cases}$$

(b)

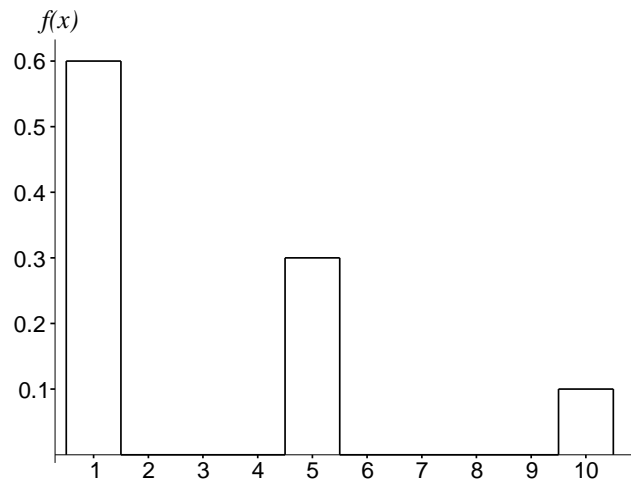


Figure 2.1-2: A probability histogram

2.1-4 (a)  $f(x) = \frac{1}{10}, \quad x = 0, 1, 2, \dots, 9;$

(b)  $N(\{0\})/150 = 11/150 = 0.073; \quad N(\{5\})/150 = 13/150 = 0.087; \quad N(\{1\})/150 = 14/150 = 0.093; \quad N(\{6\})/150 = 22/150 = 0.147; \quad N(\{2\})/150 = 13/150 = 0.087; \quad N(\{7\})/150 = 16/150 = 0.107; \quad N(\{3\})/150 = 12/150 = 0.080; \quad N(\{8\})/150 = 18/150 = 0.120; \quad N(\{4\})/150 = 16/150 = 0.107; \quad N(\{9\})/150 = 15/150 = 0.100.$

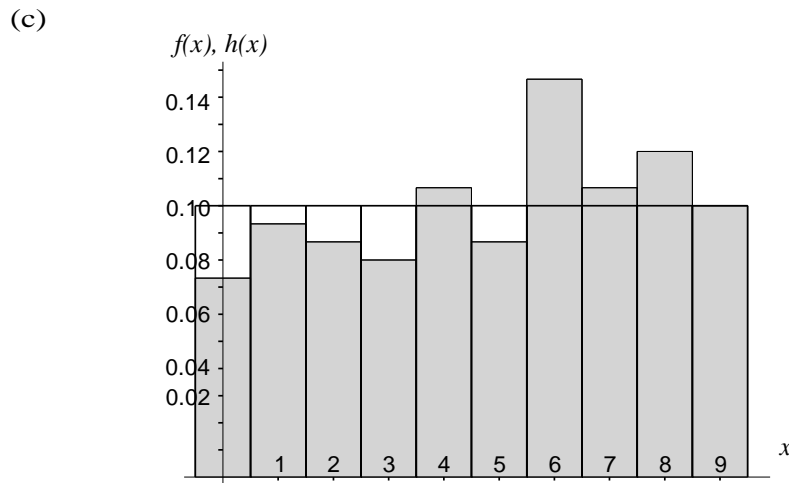


Figure 2.1-4: Michigan daily lottery digits

2.1-6 (a)  $f(x) = \frac{6 - |7 - x|}{36}$ ,  $x = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$ .

(b)

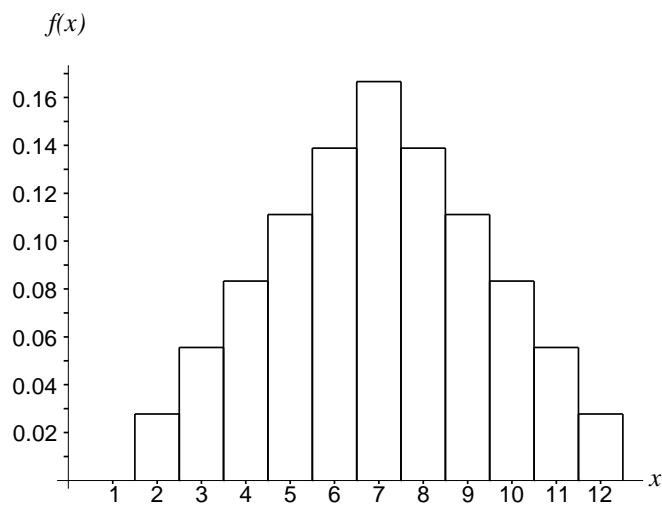


Figure 2.1-6: Probability histogram for the sum of a pair of dice

2.1-8 (a) The space of  $W$  is  $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$ .

$$\begin{aligned}
 P(W = 0) &= P(X = 0, Y = 0) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}, \text{ assuming independence.} \\
 P(W = 1) &= P(X = 0, Y = 1) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \\
 P(W = 2) &= P(X = 1, Y = 0) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \\
 P(W = 3) &= P(X = 1, Y = 1) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \\
 P(W = 4) &= P(X = 2, Y = 0) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \\
 P(W = 5) &= P(X = 2, Y = 1) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \\
 P(W = 6) &= P(X = 3, Y = 0) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \\
 P(W = 7) &= P(X = 3, Y = 1) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}
 \end{aligned}$$

That is,  $f(w) = P(W = w) = \frac{1}{8}, w \in S$ .

(b)

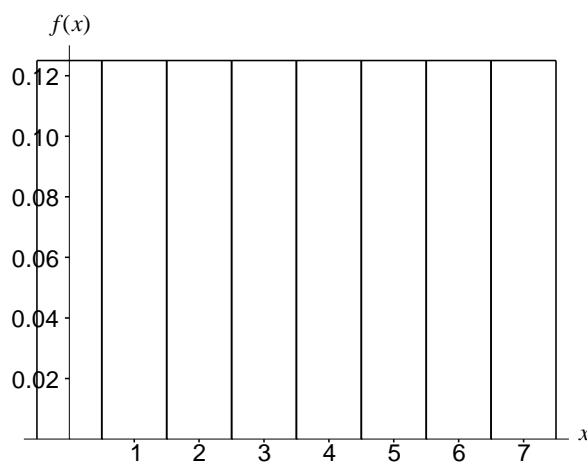


Figure 2.1-8: Probability histogram of sum of two special dice

$$\mu_3 \mu_{47}$$

2.1-10 (a)  $\frac{1}{9} = \frac{39}{39}$ ;

$$\begin{aligned}
 & \sum_{x=0}^{10} x \binom{10}{x} 3^x 47^{10-x} \\
 (b) \quad & \sum_{x=0}^{10} \frac{x(10-x)}{10} \binom{10}{x} 3^x 47^{10-x} = 245.
 \end{aligned}$$



$$\begin{aligned}
 2.1-12 \quad P(X > 4 | X > 1) &= \frac{P(X > 4)}{P(X > 1)} = \frac{1 - P(X \leq 3)}{1 - P(X = 0)} \\
 &= \frac{1 - [1 - 1/2 + 1/2 - 1/3 + 1/3 - 1/4 + 1/4 - 1/5]}{1 - [1 - 1/2]} = \frac{2}{5}.
 \end{aligned}$$

$$2.1-14 \quad P(X > 1) = 1 - P(X = 0) = \frac{\binom{0}{1} \binom{5}{1}}{\binom{20}{1}} = 1 - \frac{91}{228} = \frac{137}{228} = 0.60.$$

2.1-16 (a) P(2, 1, 6, 10) means that 2 is in position 1 so 1 cannot be selected. Thus

$$P(2, 1, 6, 10) = \frac{\binom{0}{1} \binom{1}{1} \binom{5}{8}}{\binom{10}{6}} = \frac{56}{210} = \frac{4}{15};$$

$$(b) \quad P(i, r, k, n) = \frac{\binom{r-1}{i-1} \binom{1}{1} \binom{k-r}{n-i}}{\binom{k}{n}}.$$

## 2.2 Mathematical Expectation

$$2.2-2 \quad E(X) = (-1) \frac{\binom{4}{1}}{\binom{9}{1}} + (0) \frac{\binom{1}{1}}{\binom{9}{1}} + (1) \frac{\binom{4}{1}}{\binom{9}{1}} = 0;$$

$$\begin{aligned}
 E(X^2) &= (-1)^2 \frac{\binom{4}{1}}{\binom{9}{1}} + (0)^2 \frac{\binom{1}{1}}{\binom{9}{1}} + (1)^2 \frac{\binom{4}{1}}{\binom{9}{1}} = \frac{8}{9} \\
 E(3X^2 - 2X + 4) &= 3 \frac{8}{9} - 2(0) + 4 = \frac{20}{3}.
 \end{aligned}$$

$$2.2-4 \quad f(x) = \frac{2}{49} + c \left( \frac{1}{10} + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \right)$$

$$c = \frac{2}{49};$$

$$E(\text{Payment}) = \frac{2}{49} \left( 1 \cdot \frac{1}{10} + 2 \cdot \frac{1}{1} + 3 \cdot \frac{1}{2} + 4 \cdot \frac{1}{3} + 5 \cdot \frac{1}{4} + 6 \cdot \frac{1}{5} \right) \text{ units.}$$

49    2    3    4    5    6    490

2.2-6 Note that  $\sum_{x=1}^{\infty} \frac{6}{\pi^2 x^2} = \frac{6}{\pi^2} \sum_{x=1}^{\infty} \frac{1}{x^2} = 1$ , so this is a pdf

$$E(X) = \sum_{x=1}^{\infty} x \frac{6}{\pi^2 x^2} = \frac{6}{\pi^2} \sum_{x=1}^{\infty} \frac{1}{x} = +\infty$$

and it is well known that the sum of this harmonic series is not finite.

2.2-8  $E(|X - c|) = \sum_{x \in S} \frac{1}{7} |x - c|$ , where  $S = \{1, 2, 3, 5, 15, 25, 50\}$ .

7

When  $c = 5$ ,

$$E(|X - 5|) = \frac{1}{7} [(5 - 1) + (5 - 2) + (5 - 3) + (5 - 5) + (15 - 5) + (25 - 5) + (50 - 5)].$$

If  $c$  is either increased or decreased by 1, this expectation is increased by  $1/7$ . Thus  $c = 5$ , the median, minimizes this expectation while  $b = E(X) = \mu$ , the mean, minimizes  $E[(X - b)^2]$ . You could also let  $h(c) = E(|X - c|)$  and show that  $h'(c) = 0$  when  $c = 5$ .

$$2.2-10 (1) \cdot \frac{15}{36} + (-1) \cdot \frac{21}{36} = \frac{-6}{36} = \frac{-1}{6};$$

$$(1) \cdot \frac{15}{36} + (-1) \cdot \frac{21}{36} = \frac{-6}{36} = \frac{-1}{6};$$

$$(4) \cdot \frac{6}{36} + (-1) \cdot \frac{30}{36} = \frac{-6}{36} = \frac{-1}{6}.$$

2.2-12 (a) The average class size is  $\frac{(16)(25) + (3)(100) + (1)(300)}{20} = 50$ ;

(b)

$$f(Z) = \begin{cases} 0.4, & Z = 25, \\ 0.3, & Z = 100, \\ 0.3, & Z = 300, \end{cases}$$

(c)  $E(X) = 25(0.4) + 100(0.3) + 300(0.3) = 130$ .

### 2.3 Special Mathematical Expectations

2.3-2 (a)  
 $E(X)$

$$\begin{aligned} \mu &= \sum_{x=0}^3 x \binom{3-x}{x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x} \\ &= \sum_{x=1}^3 x \frac{3!}{x!(3-x)!} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x} \\ &= \sum_{k=0}^2 \frac{3!}{k!(2-k)!} \left(\frac{1}{4}\right)^{k+1} \left(\frac{3}{4}\right)^{2-k} \\ &= \frac{3}{4} \left[ \frac{1}{4} + \frac{3}{4} \right] = \frac{3}{4}; \end{aligned}$$

$$\begin{aligned} E[X(X-1)] &= \sum_{x=2}^3 x(x-1) \frac{3!}{x!(3-x)!} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x} \\ &= 2(3) \frac{1}{3} = 2. \end{aligned}$$

$$= \frac{1}{6} \binom{4}{1} \binom{4}{2} \binom{4}{3} \binom{4}{4} = \frac{1}{2} \binom{4}{1} \binom{4}{3} \binom{4}{4} ;$$

$$\begin{aligned} \sigma^2 &= E[X(X - 1)] + E(X) - \mu^2 \\ &= (2) \frac{1}{3} \binom{4}{3} \binom{4}{1} \binom{4}{3} \binom{4}{4} + \frac{4}{3} - \left(\frac{4}{3}\right)^2 \end{aligned}$$

$$= (2) \frac{1}{3} \binom{4}{3} \binom{4}{1} \binom{4}{3} \binom{4}{4} - \frac{4}{3} = \frac{4}{3} \binom{4}{3} \binom{4}{1} \binom{4}{3} \binom{4}{4} - \frac{4}{3} ;$$

$$\frac{4}{3} \binom{4}{3} \binom{4}{1} \binom{4}{3} \binom{4}{4} - \frac{4}{3}$$

$$\begin{aligned}
 \text{(b) } \mu &= E(X) \\
 &= \sum_{x=1}^4 Z \frac{4!}{Z!(4-Z)!} \left(\frac{1}{2}\right)^Z \left(\frac{1}{2}\right)^{4-Z} \\
 &= \sum_{k=0}^3 \frac{4!}{k!(3-k)!} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{3-k} \\
 &= \sum_{k=0}^3 \frac{4!}{k!(3-k)!} \left(\frac{1}{2}\right)^3 = 4 \cdot \frac{1}{2} = 2; \\
 E[X(X-1)] &= \sum_{x=2}^4 Z(Z-1) \frac{4!}{Z!(4-Z)!} \left(\frac{1}{2}\right)^Z \left(\frac{1}{2}\right)^{4-Z} \\
 &= 2(6) \frac{1}{2^4} + (6)(4) \frac{1}{2^4} + (12) \frac{1}{2^4} \\
 &= 48 \frac{1}{2^4} = 12 \frac{1}{2}; \\
 \sigma^2 &= (12) \frac{1}{2} + \frac{4}{2} - \frac{4}{2} = 1.
 \end{aligned}$$

2.3-4  $E[(X - \mu)/\sigma] = (1/\sigma)[E(X) - \mu] = (1/\sigma)(\mu - \mu) = 0;$

$E\{[(X - \mu)/\sigma]^2\} = (1/\sigma^2)E[(X - \mu)^2] = (1/\sigma^2)(\sigma^2) = 1.$

2.3-6  $f(1) = \frac{3}{8}, f(2) = \frac{2}{8}, f(3) = \frac{3}{8}$

$\mu = 1 \cdot \frac{3}{8} + 2 \cdot \frac{2}{8} + 3 \cdot \frac{3}{8} = 2,$   
 $\sigma^2 = 1^2 \cdot \frac{3}{8} + 2^2 \cdot \frac{2}{8} + 3^2 \cdot \frac{3}{8} - 2^2 = \frac{3}{4}.$

2.3-8  $E(X) = \sum_{x=1}^4 Z \frac{2Z-1}{16}$

$$E(X^2) = \sum_{x=1}^4 x^2 \cdot \frac{2x-1}{16} = 3.125;$$

$$\text{Var}(X) = \frac{85}{8} - \left(\frac{25}{8}\right)^2 = \frac{55}{64} = 0.8594;$$

$$\sigma = \frac{\sqrt{55}}{8} = 0.9270.$$

2.3-10 We have  $N = N_1 + N_2$ . Thus

$$\begin{aligned}
 E[X(X - 1)] &= \sum_{x=0}^{\infty} x(x-1)f(x) \\
 &= \sum_{x=2}^{\infty} x(x-1) \frac{N_1!}{x!(N_1 - x)!} \cdot \frac{N_2!}{(n - x)!(N_2 - n + x)!} \\
 &= \sum_{x=2}^{\infty} \frac{(N_1 - 2)!}{(x-2)!(N_1 - x)!} \cdot \frac{N_2!}{(n - x)!(N_2 - n + x)!} \\
 &= N_1(N_1 - 1) \sum_{x=2}^{\infty} \frac{1}{x!} \cdot \frac{1}{n^{n-x}}
 \end{aligned}$$

In the summation, let  $k = x - 2$ , and in the denominator, note that

$$\frac{1}{n^{n-x}} = \frac{1}{n^{n-(k+2)}} = \frac{1}{n^{n-2-k}} = \frac{n(n-1) \dots (n-k)}{n^{n-2}}$$

Thus

$$\begin{aligned}
 E[X(X - 1)] &= \frac{N_1(N_1 - 1)}{N(N - 1)} \sum_{k=0}^{n-2} \frac{k}{n^{n-2-k}} \\
 &= \frac{N_1(N_1 - 1)(n)(n - 1)}{N(N - 1)}
 \end{aligned}$$

2.3-12 (a)  $f(Z) = \frac{1}{365^Z} \cdot \frac{1}{365}$ ,  $Z = 1, 2, 3, \dots$

(b)  $\mu = \sum_{Z=1}^{\infty} Z \cdot \frac{1}{365^Z} = 365$

$\sigma^2 = \sum_{Z=1}^{\infty} Z^2 \cdot \frac{1}{365^Z} - \mu^2 = 132,860$

$\sigma = \sqrt{132,860} \approx 364.500$ ;  $P(X > 400) =$

$$P(X < 300) = 1 - \frac{\binom{364}{400}}{\binom{365}{400}} = 1 - 0.3337 = 0.6663.$$

$$2.3-14 \quad P(X > 100) = P(X > 99) = (0.99)^{99} = 0.3697.$$

$$2.3-16 \quad (a) \quad f(Z) = (1/2)^{Z-1}, \quad Z = 2, 3, 4, \dots;$$



$$\begin{aligned}
 \text{(b) } M(t) &= E[e^{tx}] = \sum_{x=2}^{\infty} e^{tx} (1/2)^{x-1} \\
 &= \sum_{x=2}^{\infty} 2 \cdot (e^t/2)^x \\
 &= \frac{2(e^t/2)^2}{1 - e^t/2} = \frac{e^{2t}}{2 - e^t}, \quad t < \ln 2;
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) } M^1(t) &= \frac{4e^{2t} - e^{3t}}{(2 - e^t)^2} \\
 \mu &= M^1(0) = 3; \\
 M^{11}(t) &= \frac{(2 - e^t)^2(8e^{2t} - 3e^{3t}) - (4e^{2t} - e^{3t})2 \cdot (2 - e^t)(-e^t)}{(2 - e^t)^4} \\
 \sigma^2 &= M^{11}(0) - \mu^2 = 11 - 9 = 2; \\
 \text{(d) (i) } P(X \leq 3) &= 3/4; \text{ (ii) } P(X > 5) = 1/8; \text{ (iii) } P(X = 3) = 1/4.
 \end{aligned}$$

$$\begin{aligned}
 2.3-18 \quad P(X > k + j | X > k) &= \frac{P(X > k + j)}{P(X > k)} \\
 &= \frac{q^{k+j}}{q^k} = q^j = P(X > j).
 \end{aligned}$$

## 2.4 The Binomial Distribution

$$\begin{aligned}
 2.4-2 \quad f(-1) &= \frac{11}{18}, \quad f(1) = \frac{7}{18}; \\
 \mu &= (-1)\frac{11}{18} + (1)\frac{7}{18} = -\frac{4}{18}; \\
 \sigma^2 &= -1 + \frac{4}{18} + \frac{11}{18} + 1 + \frac{4}{18} - \frac{7}{18} = \frac{77}{81}.
 \end{aligned}$$

2.4-4 (a) X is b(7, 0.15);

$$\text{(b) (i) } P(X > 2) = 1 - P(X \leq 1) = 1 - 0.7166 = 0.2834;$$

$$\text{(ii) } P(X = 1) = P(X \leq 1) - P(X \leq 0) = 0.7166 - 0.3060 = 0.4106;$$

$$\text{(iii) } P(X \leq 3) = 0.9879.$$

2.4-6 (a) X is b(15, 0.75); 15 - X is b(15, 0.25);

$$\text{(b) } P(X > 10) = P(15 - X \leq 5) = 0.8516;$$

$$\text{(c) } P(X \leq 10) = P(15 - X > 5) = 1 - P(15 - X \leq 4) = 1 - 0.6865 = 0.3135;$$

$$\begin{aligned}
 \text{(d) } P(X = 10) &= P(X > 10) - P(X > 11) \\
 &= P(15 - X \leq 5) - P(15 - X \leq 4) = 0.8516 - 0.6865 = 0.1651;
 \end{aligned}$$

$$(e) \mu = (15)(0.75) = 11.25, \quad \sigma^2 = (15)(0.75)(0.25) = 2.8125; \quad \sigma = \sqrt{2.8125} = 1.67705.$$

$$2.4-8 (a) 1 - 0.01^4 = 0.99999999; \quad (b) 0.99^4 = 0.960596.$$

2.4-10 (a)  $X$  is  $b(8, 0.90)$ ;

$$(b) \quad (i) \quad P(X = 8) = P(8 - X = 0) = 0.4305;$$

$$(ii) \quad P(X \leq 6) = P(8 - X \geq 2)$$

$$= 1 - P(8 - X \leq 1) = 1 - 0.8131 =$$

$$(iii) \quad P(X > 6) = P(8 - X \leq 2) = 0.9619.$$

2.4-12 (a)

□	125/216,	Z = -
□	75/216,	Z = 1,
=	15/216,	Z = 2,
□	1/216,	Z = 3;

(b)  $\mu = (-1) \cdot \frac{125}{216} + (1) \cdot \frac{75}{216} + (2) \cdot \frac{15}{216} + (3) \cdot \frac{1}{216} = \frac{17}{216};$   
 $\sigma^2 = E(X^2) - \mu^2 = \frac{269}{216} - \left(\frac{17}{216}\right)^2 = 1.2392;$   
 $\sigma = 1.11;$

(c) See Figure 2.4-12.  $f(x)$

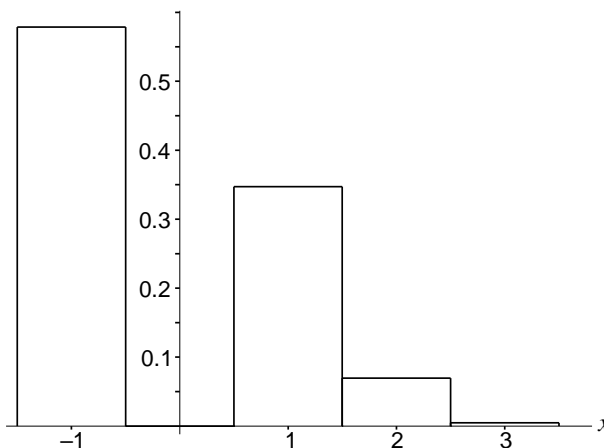


Figure 2.4-12: Losses in chuck-a-luck

2.4-14 Let X equal the number of winning tickets when n tickets are purchased. Then

$$P(X > 1) = 1 - P(X = 0)$$

$$= 1 - \frac{9^n}{10^n}$$

(a)  $1 - (0.9)^n = 0.50$   
 $(0.9)^n = 0.50$   
 $n \ln 0.9 = \ln 0.50 = 6.58$   
 $n = \frac{\ln 0.50}{\ln 0.9}$

so  $n = 7.$

(b)  $1 - (0.9)^n = \frac{\ln 0.05}{\ln 0.09}$   
 $(0.9)^n = \frac{\ln 0.05}{\ln 0.09}$   
 $n = \frac{\ln \left(\frac{\ln 0.05}{\ln 0.09}\right)}{\ln 0.9}$

$$= 28.43$$

so  $n = 29$ .

2.4-16 It is given that  $X$  is  $b(10, 0.10)$ . We are to find  $M$  so that

$P(1000X \leq M) > 0.99$  or  $P(X \leq M/1000) > 0.99$ . From Appendix Table II,  
 $P(X \leq 4) = 0.9984 > 0.99$ . Thus  $M/1000 = 4$  or  $M = 4000$  dollars.

2.4-18  $X$  is  $b(5, 0.05)$ . The expected number of tests is

$$1P(X = 0) + 6P(X > 0) = 1(0.7738) + 6(1 - 0.7738) = 2.131.$$

2.4-20 (a) (i)  $b(5, 0.7)$ ; (ii)  $\mu = 3.5, \sigma^2 = 1.05$ ; (iii) 0.1607;

(b) (i) geometric,  $p = 0.3$ ; (ii)  $\mu = 10/3, \sigma^2 = 70/9$ ; (iii) 0.51;

(c) (i) Bernoulli,  $p = 0.55$ ; (ii)  $\mu = 0.55, \sigma^2 = 0.2475$ ; (iii) 0.55;

(d) (ii)  $\mu = 2.1, \sigma^2 = 0.89$ ; (iii) 0.7;

(e) (i) discrete uniform on  $1, 2, \dots, 10$ ; (ii) 5.5, 8.25; (iii) 0.2.

## 2.5 The Negative Binomial Distribution

$$2.5-2 \quad \binom{10}{5-1} \cdot 1^5 \cdot \frac{1}{2} = \frac{126}{1024} = \frac{63}{512}.$$

2.5-4 Let “being missed” be a success and let  $X$  equal the number of trials until the first success. Then  $p = 0.01$ .

$$P(X \leq 50) = 1 - 0.99^{50} = 1 - 0.605 = 0.395.$$

2.5-6 (a)  $R(t) = \ln(1 - p + pe^t)$ ,

$$R'(t) = \frac{pe^t}{1 - p + pe^t} = p,$$

$$R''(t) = \frac{(1 - p + pe^t)(pe^t) - (pe^t)(pe^t)}{(1 - p + pe^t)^2} = p(1 - p);$$

(b)  $R(t) = n \ln(1 - p + pe^t)$ ,

$$R'(t) = \frac{npe^t}{1 - p + pe^t} = np,$$

$$R''(t) = n \frac{(1 - p + pe^t)(pe^t) - (pe^t)(pe^t)}{(1 - p + pe^t)^2} = np(1 - p);$$

(c)  $R(t) = \ln p + t - \ln[1 - (1 - p)e^t]$ ,

$$R'(t) = 1 + \frac{(1 - p)e^t}{1 - (1 - p)e^t} = 1 + \frac{1 - p}{1 - p e^t} = \frac{1}{p}$$

$$R''(t) = r^2(-1)\{1 - (1 - p)e^{-t}\}^{-2}\{-(-1 - p)e^{-t}\} = \frac{1 - p}{p};$$

$$(d) \quad R(t) = r[\ln p + t - \ln\{1 - (1 - p)e^{-t}\}]$$

$$R'(t) = r \frac{1}{1 - (1 - p)e^{-t}} = \frac{r}{p};$$

$$R''(t) = r^2(-1)\{1 - (1 - p)e^{-t}\}^{-2}\{-(-1 - p)e^{-t}\} = \frac{r(1 - p)}{p^2}.$$

$$2.5-8 \quad (0.7)(0.7)(0.3) = 0.147.$$

2.5-10 (a) Let  $X$  equal the number of boxes that must be purchased. Then

$$E(X) = \sum_{x=1}^{\infty} x \frac{1}{(13-x)/12} = \frac{86021}{2310} \approx 37.2385;$$

$$(b) \frac{100 \cdot E(X)}{365} \approx 10.2.$$

## 2.6 The Poisson Distribution

2.6-2  $\lambda = \mu = \sigma^2 = 3$  so  $P(X = 2) = 0.423 = 0.199 = 0.224$ .

$$2.6-4 \quad 3 \frac{\lambda^1 e^{-\lambda}}{1!} = \frac{\lambda^2 e^{-\lambda}}{2!}$$

$$e^{-\lambda} \lambda (\lambda - 6) = 0$$

$$\lambda = 6$$

Thus  $P(X = 4) = 0.285 = 0.151 = 0.134$ .

2.6-6  $\lambda = (1)(50/100) = 0.5$ , so  $P(X = 0) = e^{-0.5}/0! = 0.607$ .

2.6-8  $np = 1000(0.005) = 5$ ;

(a)  $P(X \leq 1) \approx 0.040$ ;

(b)  $P(X = 4, 5, 6) = P(X \leq 6) - P(X \leq 3) \approx 0.762 - 0.265 = 0.497$ .

2.6-10  $\sigma = \sqrt{9} = 3$ ,

$$P(3 < X < 15) = P(X \leq 14) - P(X \leq 3) = 0.959 - 0.021 = 0.938.$$

2.6-12 Since  $E(X) = 0.2$ , the expected loss is  $(0.02)(\$10,000) = \$2,000$ .





# Chapter 3

## Continuous Distributions

### 3.1 Random Variables of the Continuous Type

3.1-2  $\mu = 0, \sigma^2 = (1 + 1)^2/12 = 1/3.$

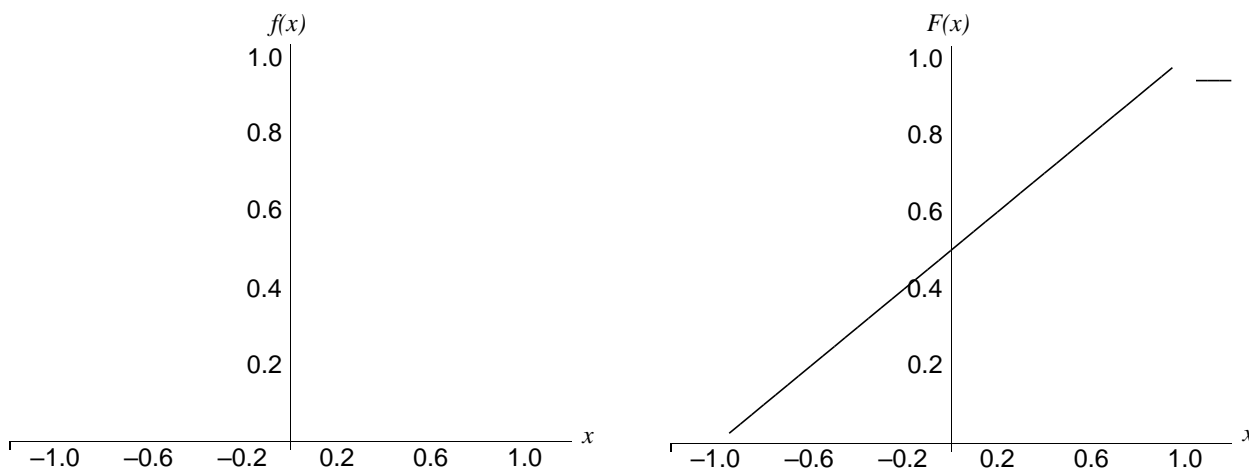


Figure 3.1-2:  $f(Z) = 1/2$  and  $F(Z) = (Z + 1)/2$

3.1-4 X is U(4, 5);

(a)  $\mu = 9/2;$  (b)  $\sigma^2 = 1/12;$  (c) 0.5.

$$\begin{aligned}
 3.1-6 \quad E(\text{profit}) &= \int_0^{200} [Z - 0.5(n - Z)] \frac{1}{200} dZ + \int_n^{200} [n - 5(Z - n)] \frac{1}{200} dZ \\
 &= \frac{1}{200} \int_0^{200} Z^2 + \frac{(n - Z)^2}{2} + \frac{1}{200} \int_n^{200} 6nZ - 5Z^2 \cdot \frac{1}{200} \\
 &= \frac{200}{1} \cdot \frac{2}{3} + \frac{4}{0} \cdot \frac{200}{2} - \frac{2}{n} \cdot \frac{200}{2} \\
 &= \text{der} \quad \text{ive}
 \end{aligned}$$

$$\begin{aligned}
 & 200 && \frac{?}{3.2} \\
 & 1 && \frac{5n^2}{12} \\
 & 200 && + \\
 & && 00n \\
 & && - \\
 & && 100 \\
 & && 000 \\
 & && [- \\
 & && 6.5 \\
 & && n \\
 & && + \\
 & && 12 \\
 & && 00] \\
 & && - \\
 & && 0 \\
 & n & - & \frac{1200}{6.5} \approx 185.
 \end{aligned}$$

$$\begin{aligned}
 3.1-8 \text{ (a) (i)} \quad & \int_0^c Z^3/4 dZ = 1 \\
 & c^4/16 = 1 \\
 & c = 2; \\
 \text{(ii) } F(Z) &= \int_{-\infty}^Z f(t) dt \\
 &= \int_0^Z t^3/4 dt \\
 &= Z^4/16, \\
 & \square 0, \quad -\infty < Z < 0, \\
 F(Z) &= Z^4/16, \quad 0 \leq Z < 2, \\
 & \square 1, \quad 2 \leq Z < \infty.
 \end{aligned}$$

(iii)

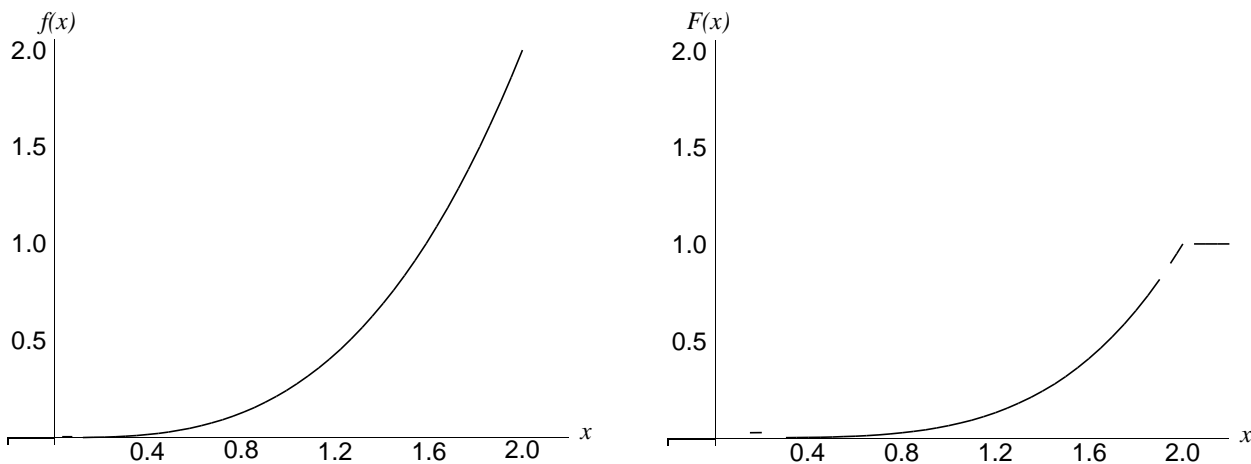


Figure 3.1-8: (a) Continuous distribution pdf and cdf

$$\begin{aligned}
 \text{(iv)} \quad \mu &= \int_0^2 (Z)(Z^3/4) dZ = \frac{8}{5}; \\
 E(X^2) &= \int_0^2 (Z^2)(Z^3/4) dZ = \frac{8}{3}; \\
 \sigma^2 &= \frac{8}{3} - \frac{8^2}{5} = \frac{8}{75}.
 \end{aligned}$$

$$(b) (i) \int_{-c}^{Z_c} (3/16)Z^2 dZ = 1$$

$$c^3/8 = 1$$

$$c = 2;$$

$$(ii) F(Z) = \int_{-\infty}^{Z_x} f(t) dt$$

$$= \int_{-\infty}^{-2} (3/16)t^2 dt$$

$$= \frac{r}{16} t^3 \Big|_{-\infty}^{-2}$$

$$= \frac{Z^3}{16} + \frac{1}{2}$$

$$F(Z) = \begin{cases} 0, & -\infty < Z < -2, \\ \frac{Z^3}{16} + \frac{1}{2}, & -2 \leq Z < 2, \\ 1, & 2 \leq Z < \infty. \end{cases}$$

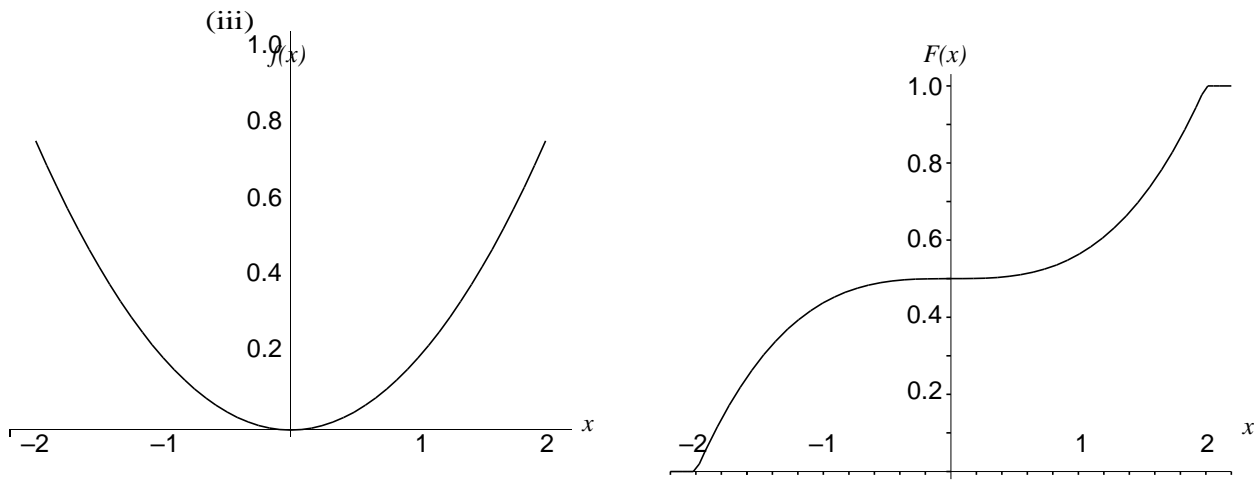


Figure 3.1-8: (b) Continuous distribution pdf and cdf

$$(iv) \mu = \int_{-\infty}^{\infty} Z (3/16)(Z^2) dZ = 0;$$

$$\sigma^2 = \int_{-\infty}^{\infty} (Z^2)(3/16)(Z^2) dZ = \frac{12}{5}$$

$$(c) \quad (i) \quad \int_0^1 \frac{1}{\sqrt{t}} dt = 1$$

$$2c = 1$$

$$c = 1/2.$$

The pdf in part (c) is unbounded.

$$(ii) \quad F(X) = \int_{-\infty}^X f(t) dt$$

$$= \int_0^X \frac{1}{2\sqrt{t}} dt$$

$$= \frac{1}{2} \sqrt{t} \Big|_0^X = \sqrt{X},$$

$$F(X) = \begin{cases} 0, & -\infty < X < 0, \\ \sqrt{X}, & 0 \leq X < 1, \\ 1, & 1 \leq X < \infty. \end{cases}$$

(iii)

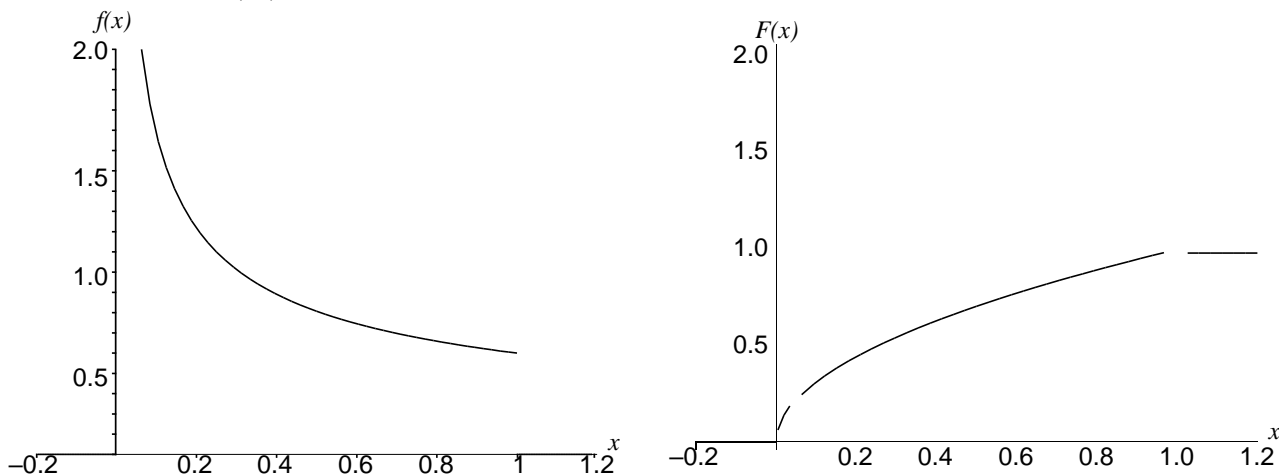


Figure 3.1-8: (c) Continuous distribution pdf and cdf

$$(iv) \quad \mu = \int_0^1 (X)(1/2) \frac{1}{\sqrt{X}} dX = \frac{1}{2} \int_0^1 \sqrt{X} dX = \frac{1}{2} \left[ \frac{2}{3} X^{3/2} \right]_0^1 = \frac{1}{3}$$

$$E(X^2) = \int_0^1 (X^2)(1/2) \frac{1}{\sqrt{X}} dX = \frac{1}{2} \int_0^1 X^{3/2} dX = \frac{1}{2} \left[ \frac{2}{5} X^{5/2} \right]_0^1 = \frac{1}{5}$$

$$\sigma^2 = \frac{1}{5} - \left( \frac{1}{3} \right)^2 = \frac{4}{45}$$

$$3.1-10 (a) \quad \int_1^{\infty} \frac{c}{X^2} dX = 1$$

$$\int_1^{\infty} \frac{c}{X^2} dX = \left[ -\frac{c}{X} \right]_1^{\infty} = 1$$

$$c = 1;$$

(b)  $E(X) = \int_1^{\infty} \frac{x}{x^2} dx = [\ln x]_1^{\infty}$ , which is unbounded.

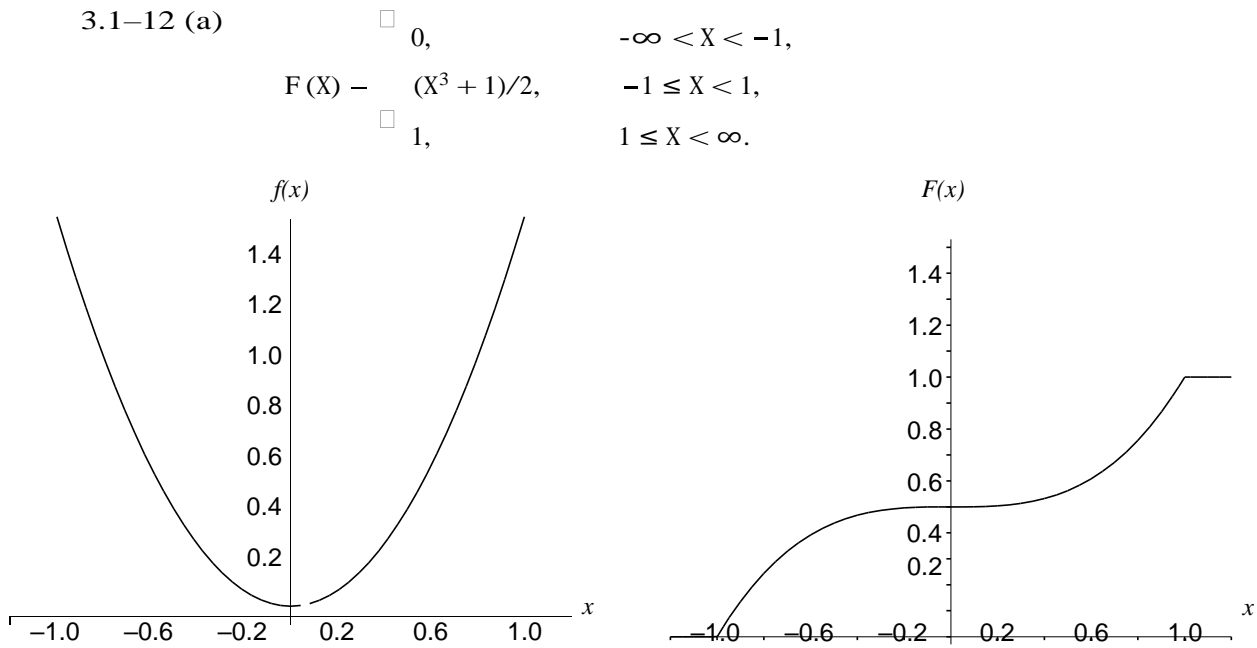


Figure 3.1-12: (a)  $f(X) = (3/2)X^2$  and  $F(X) = (X^3 + 1)/2$

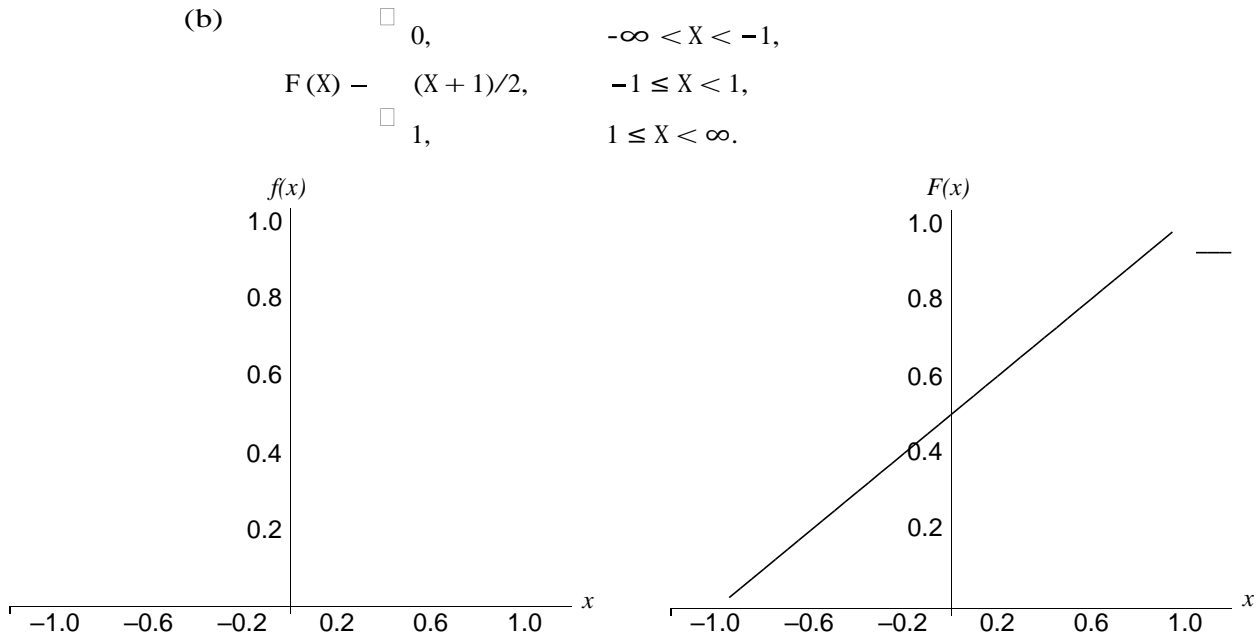


Figure 3.1-12: (b)  $f(X) = 1/2$  and  $F(X) = (X + 1)/2$

(c)

$$f(x) = \begin{cases} 0, & -\infty < X < -1, \\ (X + 1)^2/2, & -1 \leq X < 0, \\ 1 - (1 - X)^2/2, & 0 \leq X < 1, \\ 1, & 1 \leq X < \infty. \end{cases}$$

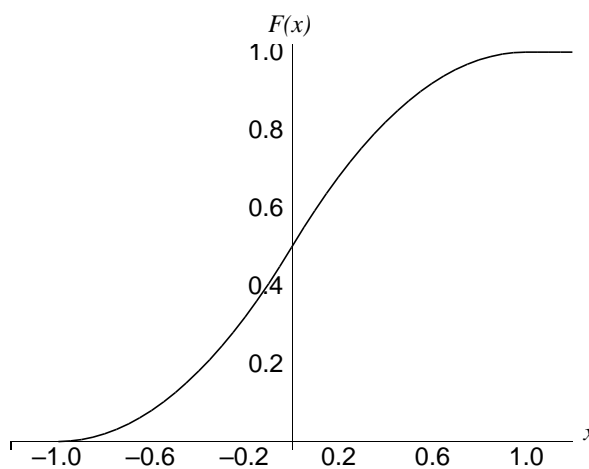
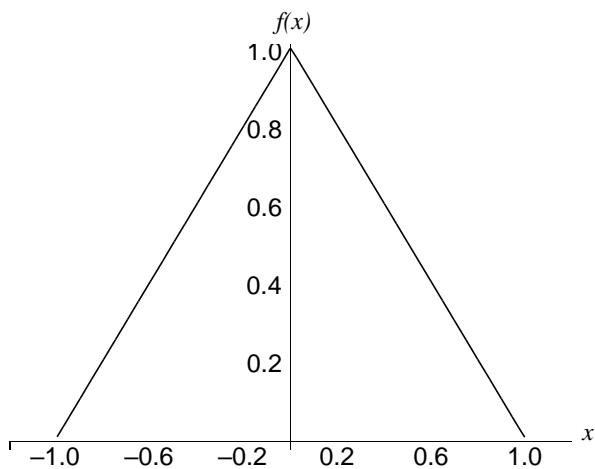
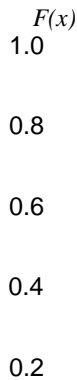
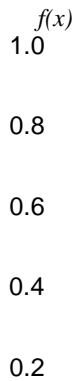


Figure 3.1-12: (c)  $f(x)$  and  $F(x)$  for Exercise 3.1-12(c)

3.1-14 (b)

$$f(x) = \begin{cases} 0, & -\infty < X \leq 0, \\ X, & 0 < X \leq 1, \\ 2 - X, & 1 < X \leq 2, \\ 1, & 2 \leq X < 3, \\ 0, & 3 \leq X < \infty; \end{cases}$$



x





Figure 3.1-14:  $f(X)$  and  $F(X)$  for Exercise 3.1-14(a) and (b)