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CHAPTER 2

Section 2.1

1.

a. S = {1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3124, 3142, 4123, 4132, 3214, 3241, 4213, 4231}.

b. Event A contains the outcomes where 1 is first in the list:
A = {1324, 1342, 1423, 1432}.

c. Event B contains the outcomes where 2 is first or second:
B = {2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231}.

d. The event $A \cup B$ contains the outcomes in A or B or both:
$A \cup B$ = {1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231}.
$A \cap B = \varnothing$, since 1 and 2 can't both get into the championship game.
$A' = S - A$ = {2314, 2341, 2413, 2431, 3124, 3142, 4123, 4132, 3214, 3241, 4213, 4231}.

2.

a. $A = \{RRR, LLL, SSS\}$.

b. $B = \{RLS, RSL, LRS, LSR, SRL, SLR\}$.

c. $C = \{RRL, RRS, RLR, RSR, LRR, SRR\}$.

d. $D = \{RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS\}$

e. Event D' contains outcomes where either all cars go the same direction or they all go different directions:
$D' = \{RRR, LLL, SSS, RLS, RSL, LRS, LSR, SRL, SLR\}$.
Because event D totally encloses event C (see the lists above), the compound event $C \cup D$ is just event D:
$C \cup D = D = \{RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS\}$.
Using similar reasoning, we see that the compound event $C \cap D$ is just event C:
$C \cap D = C = \{RRL, RRS, RLR, RSR, LRR, SRR\}$.

3.

a. $A = \{SSF, SFS, FSS\}$.

b. $B = \{SSS, SSF, SFS, FSS\}$.

c. For event C to occur, the system must have component 1 working (S in the first position), then at least one of the other two components must work (at least one S in the second and third positions): $C = \{SSS, SSF, SFS\}$.

d. $C' = \{SFF, FSS, FSF, FFS, FFF\}$.
$A \cup C = \{SSS, SSF, SFS, FSS\}$.
$A \cap C = \{SSF, SFS\}$.
$B \cup C = \{SSS, SSF, SFS, FSS\}$. Notice that B contains C, so $B \cup C = B$.
$B \cap C = \{SSS\ SSF, SFS\}$. Since B contains C, $B \cap C = C$.

4.

a. The $2^4 = 16$ possible outcomes have been numbered here for later reference.

	Home Mortgage Number			
Outcome	1	2	3	4
1	*F*	*F*	*F*	*F*
2	*F*	*F*	*F*	*V*
3	*F*	*F*	*V*	*F*
4	*F*	*F*	*V*	*V*
5	*F*	*V*	*F*	*F*
6	*F*	*V*	*F*	*V*
7	*F*	*V*	*V*	*F*
8	*F*	*V*	*V*	*V*
9	*V*	*F*	*F*	*F*
10	*V*	*F*	*F*	*V*
11	*V*	*F*	*V*	*F*
12	*V*	*F*	*V*	*V*
13	*V*	*V*	*F*	*F*
14	*V*	*V*	*F*	*V*
15	*V*	*V*	*V*	*F*
16	*V*	*V*	*V*	*V*

b. Outcome numbers 2, 3, 5, 9 above.

c. Outcome numbers 1, 16 above.

d. Outcome numbers 1, 2, 3, 5, 9 above.

e. In words, the union of (c) and (d) is the event that either all of the mortgages are variable, or that at most one of them is variable-rate: outcomes 1, 2, 3, 5, 9, 16. The intersection of (c) and (d) is the event that all of the mortgages are fixed-rate: outcome 1.

f. The union of (b) and (c) is the event that either exactly three are fixed, or that all four are the same: outcomes 1, 2, 3, 5, 9, 16. The intersection of (b) and (c) is the event that exactly three are fixed and all four are the same type. This cannot happen (the events have no outcomes in common), so the intersection of (b) and (c) is $\varnothing$.

5.

a. The $3^3 = 27$ possible outcomes are numbered below for later reference.

Outcome Number	Outcome	Outcome Number	Outcome
1	111	15	223
2	112	16	231
3	113	17	232
4	121	18	233
5	122	19	311
6	123	20	312
7	131	21	313
8	132	22	321
9	133	23	322
10	211	24	323
11	212	25	331
12	213	26	332
13	221	27	333
14	222		

b. Outcome numbers 1, 14, 27 above.

c. Outcome numbers 6, 8, 12, 16, 20, 22 above.

d. Outcome numbers 1, 3, 7, 9, 19, 21, 25, 27 above.

6.

a. S = {123, 124, 125, 213, 214, 215, 13, 14, 15, 23, 24, 25, 3, 4, 5}.

b. A = {3, 4, 5}.

c. B = {125, 215, 15, 25, 5}.

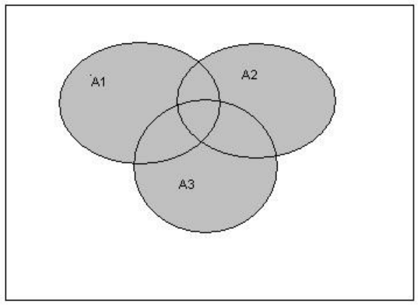
d. C = {23, 24, 25, 3, 4, 5}.

7.

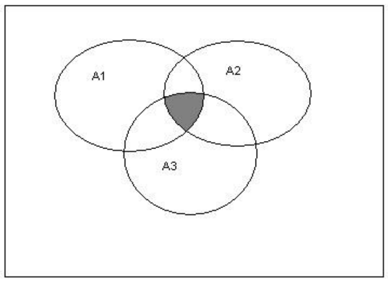
a. S = {*BBBAAAA*, *BBABAAA*, *BBAABAA*, *BBAAABA*, *BBAAAAB*, *BABBAAA*, *BABABAA*, *BABAABA*, *BABAAAB*, *BAABBAA*, *BAABABA*, *BAABAAB*, *BAAABBA*, *BAAABAB*, *BAAAABB*, *ABBBAAA*, *ABBABAA*, *ABBAABA*, *ABBAAAB*, *ABABBAA*, *ABABABA*, *ABABAAB*, *ABAABBA*, *ABAABAB*, *ABAAABB*, *AABBBAA*, *AABBABA*, *AABBAAB*, *AABABBA*, *AABABAB*, *AABAABB*, *AAABBBA*, *AAABBAB*, *AAABABB*, *AAAABBB*}.

b. *AAAABBB*, *AAABABB*, *AAABBAB*, *AABAABB*, *AABABAB*.

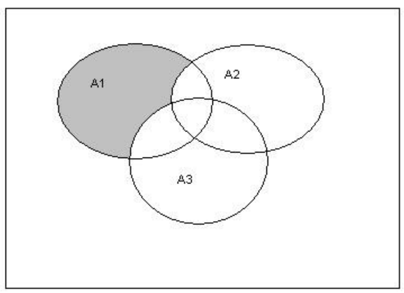
8.



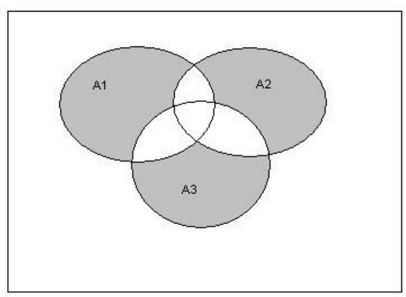
a. $A_1 \cup A_2 \cup A_3$



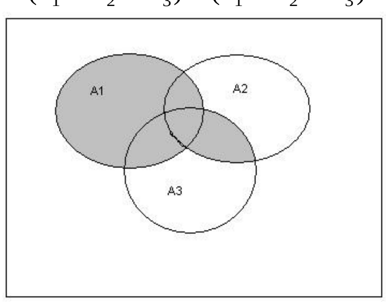
b. $A_1 \cap A_2 \cap A_3$



c. $A_1 \cap A_2' \cap A_3'$



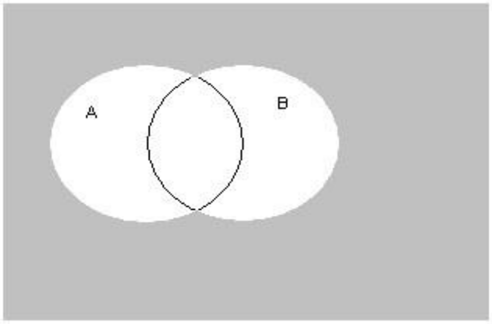
d. $(A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3') \cup (A_1' \cap A_2' \cap A_3)$

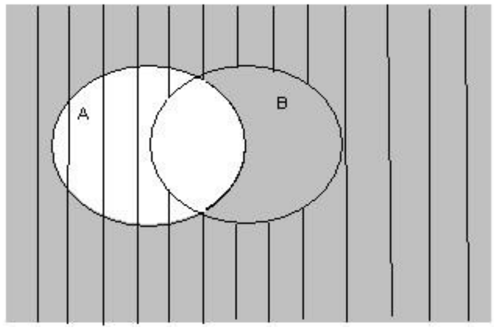


e. $A_1 \cup (A_2 \cap A_3)$

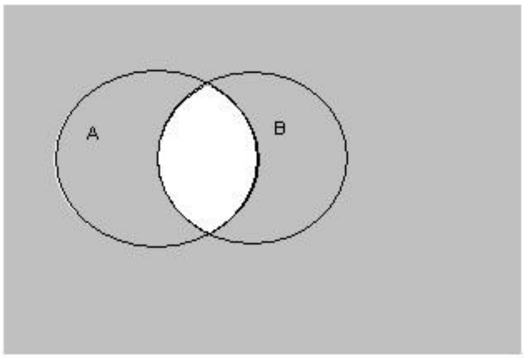
9.

a. In the diagram on the left, the shaded area is $(A \cup B)'$. On the right, the shaded area is A', the striped area is B', and the intersection $A' \cap B'$ occurs where there is both shading <u>and</u> stripes. These two diagrams display the same area.





b. In the diagram below, the shaded area represents $(A \cap B)'$. Using the right-hand diagram from (a), the <u>union</u> of A' and B' is represented by the areas that have either shading <u>or</u> stripes (or both). Both of the diagrams display the same area.



10.

a. Many examples exist; e.g., A = {Chevy, Buick}, B = {Ford, Lincoln}, C = {Toyota} are three mutually exclusive events.

b. No. Let E = {Chevy, Buick}, F = {Buick, Ford}, G = {Toyota}. These events are <u>not</u> mutually exclusive (E and F have an outcome in common), yet there is no outcome common to all three events.

Section 2.2

11.

a. .07.

b. .15 + .10 + .05 = .30.

c. Let A = the selected individual owns shares in a stock fund. Then $P(A) = .18 + .25 = .43$. The desired probability, that a selected customer does not shares in a stock fund, equals $P(A') = 1 - P(A) = 1 - .43 = .57$. This could also be calculated by adding the probabilities for all the funds that are not stocks.

12.

a. No, this is not possible. Since event $A \cap B$ is contained within event B, it must be the case that $P(A \cap B) \le P(B)$. However, .5 > .4.

b. By the addition rule, $P(A \cup B) = .5 + .4 - .3 = .6$.

c. $P(\text{neither } A \text{ nor } B) = P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - .6 = .4$.

d. The event of interest is $A \cap B'$; from a Venn diagram, we see $P(A \cap B') = P(A) - P(A \cap B) = .5 - .3 = .2$.

e. From a Venn diagram, we see that the probability of interest is $P(\text{exactly one}) = P(\text{at least one}) - P(\text{both}) = P(A \cup B) - P(A \cap B) = .6 - .3 = .3$.

13.

a. $A_1 \cup A_2$ = "awarded either #1 or #2 (or both)": from the addition rule,
$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = .22 + .25 - .11 = .36$.

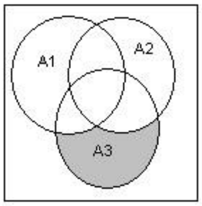
b. $A_1' \cap A_2'$ = "awarded neither #1 or #2": using the hint and part (a),
$P(A_1' \cap A_2') = P((A_1 \cup A_2)') = 1 - P(A_1 \cup A_2) = 1 - .36 = .64$.

c. $A_1 \cup A_2 \cup A_3$ = "awarded at least one of these three projects": using the addition rule for 3 events,
$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) =$

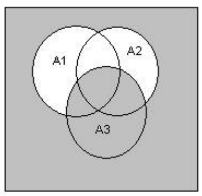
$.22 + .25 + .28 - .11 - .05 - .07 + .01 = .53$.

d. $A_1' \cap A_2' \cap A_3'$ = "awarded none of the three projects":
$P(A_1' \cap A_2' \cap A_3') = 1 - P(\text{awarded at least one}) = 1 - .53 = .47$.

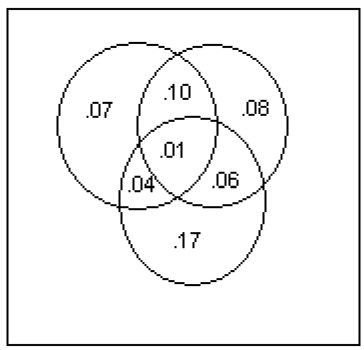
e. $A_1' \cap A_2' \cap A_3$ = "awarded #3 but neither #1 nor #2": from a Venn diagram,
$P(A_1' \cap A_2' \cap A_3) = P(A_3) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) =$
.28 – .05 – .07 + .01 = .17. The last term addresses the "double counting" of the two subtractions.



f. $(A_1' \cap A_2') \cup A_3$ = "awarded neither of #1 and #2, or awarded #3": from a Venn diagram,
$P((A_1' \cap A_2') \cup A_3) = P(\text{none awarded}) + P(A_3)$ = .47 (from **d**) + .28 = 75.



Alternatively, answers to **a-f** can be obtained from probabilities on the accompanying Venn diagram:



14. Let A = an adult consumes coffee and B = an adult consumes carbonated soda. We're told that $P(A) = .55$, $P(B) = .45$, and $P(A \cup B) = .70$.

a. The addition rule says $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, so .70 = .55 + .45 – $P(A \cap B)$ or $P(A \cap B)$ = .55 + .45 – .70 = .30.

b. There are two ways to read this question. We can read "does not (consume at least one)," which means the adult consumes neither beverage. The probability is then $P(\text{neither } A \text{ nor } B) = P(A' \cap B') = 1 - P(A \cup B)$ = 1 – .70 = .30.

The other reading, and this is presumably the intent, is "there is at least one beverage the adult does not consume, i.e. $A' \cup B'$. The probability is $P(A' \cup B') = 1 - P(A \cap B)$ = 1 – .30 from **a** = .70. (It's just a

coincidence this equals $P(A \cup B)$.)

Both of these approaches use *deMorgan's laws*, which say that $P(A' \cap B') = 1 - P(A \cup B)$ and $P(A' \cup B') = 1 - P(A \cap B)$.

15.

a. Let E be the event that at most one purchases an electric dryer. Then E' is the event that at least two purchase electric dryers, and $P(E') = 1 - P(E) = 1 - .428 = .572$.

b. Let A be the event that all five purchase gas, and let B be the event that all five purchase electric. All other possible outcomes are those in which at least one of each type of clothes dryer is purchased. Thus, the desired probability is $1 - [P(A) - P(B)] =$
$1 - [.116 + .005] = .879$.

16.

a. There are six simple events, corresponding to the outcomes *CDP*, *CPD*, *DCP*, *DPC*, *PCD*, and *PDC*. Since the same cola is in every glass, these six outcomes are equally likely to occur, and the probability assigned to each is $\frac{1}{6}$.

b. $P(C \text{ ranked first}) = P(\{CPD, CDP\}) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = .333$.

c. $P(C \text{ ranked first and } D \text{ last}) = P(\{CPD\}) = \frac{1}{6}$.

17.

a. The probabilities do not add to 1 because there are other software packages besides SPSS and SAS for which requests could be made.

b. $P(A') = 1 - P(A) = 1 - .30 = .70$.

c. Since A and B are mutually exclusive events, $P(A \cup B) = P(A) + P(B) = .30 + .50 = .80$.

d. By deMorgan's law, $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - .80 = .20$.
In this example, deMorgan's law says the event "neither A nor B" is the complement of the event "either A or B." (That's true regardless of whether they're mutually exclusive.)

18. The only reason we'd need at least two selections to find a \$10 bill is if the first selection was not a \$10 bill bulb. There are 4 + 6 = 10 non-\$10 bills out of 5 + 4 + 6 = 15 bills in the wallet, so the probability of this event is simply 10/15, or 2/3.

19. Let A be that the selected joint was found defective by inspector A, so $P(A) = \frac{724}{10{,}000}$. Let B be analogous for inspector B, so $P(B) = \frac{751}{10{,}000}$. The event "at least one of the inspectors judged a joint to be defective is $A \cup B$, so $P(A \cup B) = \frac{1159}{10{,}000}$.

a. By deMorgan's law, $P(\text{neither } A \text{ nor } B) = P(A' \cap B') = 1 - P(A \cup B) = 1 - \frac{1159}{10{,}000} = \frac{8841}{10{,}000} = .8841$.

b. The desired event is $B \cap A'$. From a Venn diagram, we see that $P(B \cap A') = P(B) - P(A \cap B)$. From the addition rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ gives $P(A \cap B) = .0724 + .0751 - .1159 = .0316$. Finally, $P(B \cap A') = P(B) - P(A \cap B) = .0751 - .0316 = .0435$.

20.

a. Let S_1, S_2 and S_3 represent day, swing, and night shifts, respectively. Let C_1 and C_2 represent unsafe conditions and unrelated to conditions, respectively. Then the simple events are S_1C_1, S_1C_2, S_2C_1, S_2C_2, S_3C_1, S_3C_2.

b. $P(C_1)$= P({S_1C_1, S_2C_1, S_3C_1})= .10 + .08 + .05 = .23.

c. $P(S_1')$ = 1 – P({S_1C_1, S_1C_2}) = 1 – (.10 + .35) = .55.

21. In what follows, the first letter refers to the auto deductible and the second letter refers to the homeowner's deductible.

a. $P(MH)$ = .10.

b. P(low auto deductible) = P({LN, LL, LM, LH}) = .04 + .06 + .05 + .03 = .18. Following a similar pattern, P(low homeowner's deductible) = .06 + .10 + .03 = .19.

c. P(same deductible for both) = P({LL, MM, HH}) = .06 + .20 + .15 = .41.

d. P(deductibles are different) = 1 – P(same deductible for both) = 1 – .41 = .59.

e. P(at least one low deductible) = P({LN, LL, LM, LH, ML, HL}) = .04 + .06 + .05 + .03 + .10 + .03 = .31.

f. P(neither deductible is low) = 1 – P(at least one low deductible) = 1 – .31 = .69.

22. Let A = motorist must stop at first signal and B = motorist must stop at second signal. We're told that $P(A)$ = .4, $P(B)$ = .5, and $P(A \cup B)$ = .6.

a. From the addition rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, so .6 = .4 + .5 – $P(A \cap B)$, from which $P(A \cap B)$ = .4 + .5 – .6 = .3.

b. From a Venn diagram, $P(A \cap B') = P(A) - P(A \cap B)$ = .4 – .3 = .1.

c. From a Venn diagram, P(stop at exactly one signal) = $P(A \cup B) - P(A \cap B)$ = .6 – .3 = .3. Or, P(stop at exactly one signal) = $P([A \cap B'] \cup [A' \cap B]) = P(A \cap B') + P(A' \cap B) = [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)]$ = [.4 – .3] + [.5 – .3] = .1 + .2 = .3.

23. Assume that the computers are numbered 1-6 as described and that computers 1 and 2 are the two laptops. There are 15 possible outcomes: (1,2) (1,3) (1,4) (1,5) (1,6) (2,3) (2,4) (2,5) (2,6) (3,4) (3,5) (3,6) (4,5) (4,6) and (5,6).

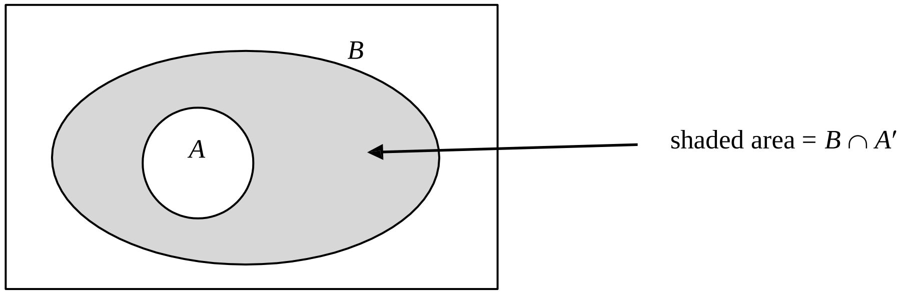
a. P(both are laptops) = P({(1,2)}) = $\frac{1}{15}$ =.067.

b. P(both are desktops) = P({(3,4) (3,5) (3,6) (4,5) (4,6) (5,6)}) = $\frac{6}{15}$ = .40.

c. P(at least one desktop) = 1 – P(no desktops) = 1 – P(both are laptops) = 1 – .067 = .933.

d. P(at least one of each type) = 1 – P(both are the same) = 1 – [P(both are laptops) + P(both are desktops)] = 1 – [.067 + .40] = .533.

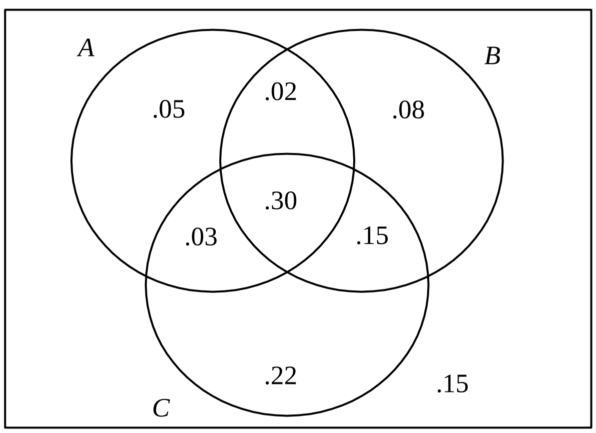
24. Since A is contained in B, we may write $B = A \cup (B \cap A')$, the union of two mutually exclusive events. (See diagram for these two events.) Apply the axioms:
$P(B) = P(A \cup (B \cap A')) = P(A) + P(B \cap A')$ by Axiom 3. Then, since $P(B \cap A') \geq 0$ by Axiom 1, $P(B) = P(A) + P(B \cap A') \geq P(A) + 0 = P(A)$. This proves the statement.



For general events A and B (i.e., not necessarily those in the diagram), it's always the case that $A \cap B$ is contained in A as well as in B, while A and B are both contained in $A \cup B$. Therefore, $P(A \cap B) \leq P(A) \leq P(A \cup B)$ and $P(A \cap B) \leq P(B) \leq P(A \cup B)$.

25. By rearranging the addition rule, $P(A \cap B) = P(A) + P(B) - P(A \cup B) = .40 + .55 - .63 = .32$. By the same method, $P(A \cap C) = .40 + .70 - .77 = .33$ and $P(B \cap C) = .55 + .70 - .80 = .45$. Finally, rearranging the addition rule for 3 events gives
$P(A \cap B \cap C) = P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(A \cap C) + P(B \cap C) = .85 - .40 - .55 - .70 + .32 + .33 + .45 = .30$.

These probabilities are reflected in the Venn diagram below.



a. $P(A \cup B \cup C) = .85$, as given.

b. P(none selected) = 1 – P(at least one selected) = $1 - P(A \cup B \cup C) = 1 - .85 = .15$.

c. From the Venn diagram, P(only automatic transmission selected) = .22.

d. From the Venn diagram, P(exactly one of the three) = .05 + .08 + .22 = .35.

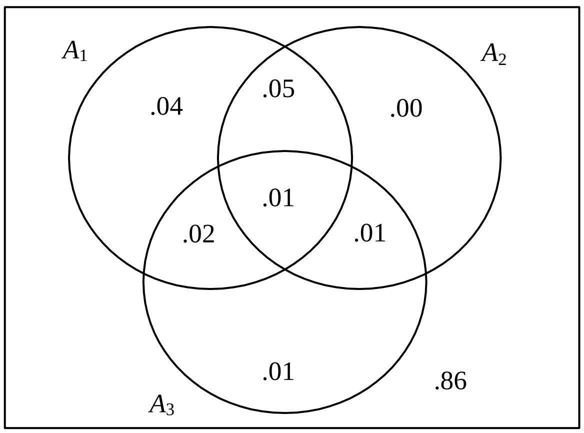
26. These questions can be solved algebraically, or with the Venn diagram below.

a. $P(A_1') = 1 - P(A_1)$ = 1 – .12 = .88.

b. The addition rule says $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Solving for the intersection ("and") probability, you get $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$ = .12 + .07 – .13 = .06.

c. A Venn diagram shows that $P(A \cap B') = P(A) - P(A \cap B)$. Applying that here with $A = A_1 \cap A_2$ and B = A_3, you get $P([A_1 \cap A_2] \cap A_3') = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3)$ = .06 – .01 = .05.

d. The event "at most two defects" is the complement of "all three defects," so the answer is just 1 – $P(A_1 \cap A_2 \cap A_3)$ = 1 – .01 = .99.



27. There are 10 equally likely outcomes: {A, B} {A, Co} {A, Cr} {A,F} {B, Co} {B, Cr} {B, F} {Co, Cr} {Co, F} and {Cr, F}.

a. P({A, B}) = $\frac{1}{10}$ = .1.

b. P(at least one C) = P({A, Co} or {A, Cr} or {B, Co} or {B, Cr} or {Co, Cr} or {Co, F} or {Cr, F}) = $\frac{7}{10}$ = .7.

c. Replacing each person with his/her years of experience, P(at least 15 years) = P({3, 14} or {6, 10} or {6, 14} or {7, 10} or {7, 14} or {10, 14}) = $\frac{6}{10}$ = .6.

28. Recall there are 27 equally likely outcomes.

a. P(all the same station) = P((1,1,1) or (2,2,2) or (3,3,3)) = $\frac{3}{27} = \frac{1}{9}$.

b. P(at most 2 are assigned to the same station) = 1 – P(all 3 are the same) = 1 – $\frac{1}{9} = \frac{8}{9}$.

c. P(all different stations) = P((1,2,3) or (1,3,2) or (2,1,3) or (2,3,1) or (3,1,2) or (3,2,1)) = $\frac{6}{27} = \frac{2}{9}$.

Section 2.3

29.

a. There are 26 letters, so allowing repeats there are $(26)(26) = (26)^2 = 676$ possible 2-letter domain names. Add in the 10 digits, and there are 36 characters available, so allowing repeats there are $(36)(36) = (36)^2 = 1296$ possible 2-character domain names.

b. By the same logic as part **a**, the answers are $(26)^3 = 17{,}576$ and $(36)^3 = 46{,}656$.

c. Continuing, $(26)^4 = 456{,}976$; $(36)^4 = 1{,}679{,}616$.

d. P(4-character sequence is already owned) = 1 – P(4-character sequence still available) = 1 – $97{,}786/(36)^4 = .942$.

30.

a. Because order is important, we'll use $P_{3,8} = (8)(7)(6) = 336$.

b. Order doesn't matter here, so we use $\binom{30}{6} = 593{,}775$.

c. The number of ways to choose 2 zinfandels from the 8 available is $\binom{8}{2}$. Similarly, the number of ways to choose the merlots and cabernets are $\binom{10}{2}$ and $\binom{12}{2}$, respectively. Hence, the total number of options (using the Fundamental Counting Principle) equals $\binom{8}{2}\binom{10}{2}\binom{12}{2} = (28)(45)(66) = 83{,}160$.

d. The numerator comes from part **c** and the denominator from part **b**: $\frac{83{,}160}{593{,}775} = .140$.

e. We use the same denominator as in part **d**. The number of ways to choose all zinfandel is $\binom{8}{6}$, with similar answers for all merlot and all cabernet. Since these are disjoint events, P(all same) = P(all zin) + P(all merlot) + P(all cab) = $\frac{\binom{8}{6}+\binom{10}{6}+\binom{12}{6}}{\binom{30}{6}} = \frac{1162}{593{,}775} = .002$.

31.

a. Use the Fundamental Counting Principle: $(9)(5) = 45$.

b. By the same reasoning, there are $(9)(5)(32) = 1440$ such sequences, so such a policy could be carried out for 1440 successive nights, or almost 4 years, without repeating exactly the same program.

32.

a. Since there are 5 receivers, 4 CD players, 3 speakers, and 4 turntables, the total number of possible selections is (5)(4)(3)(4) = 240.

b. We now only have 1 choice for the receiver and CD player: (1)(1)(3)(4) = 12.

c. Eliminating Sony leaves 4, 3, 3, and 3 choices for the four pieces of equipment, respectively: (4)(3)(3)(3) = 108.

d. From **a**, there are 240 possible configurations. From **c**, 108 of them involve zero Sony products. So, the number of configurations with at least one Sony product is 240 – 108 = 132.

e. Assuming all 240 arrangements are equally likely, $P(\text{at least one Sony}) = \frac{132}{240} = .55$.

Next, P(exactly one component Sony) = P(only the receiver is Sony) + P(only the CD player is Sony) + P(only the turntable is Sony). Counting from the available options gives

$$P(\text{exactly one component Sony}) = \frac{(1)(3)(3)(3) + (4)(1)(3)(3) + (4)(3)(3)(1)}{240} = \frac{99}{240} = .413 .$$

33.

a. Since there are 15 players and 9 positions, and order matters in a line-up (catcher, pitcher, shortstop, etc. are different positions), the number of possibilities is $P_{9,15}$ = (15)(14)…(7) or 15!/(15–9)! = 1,816,214,440.

b. For each of the starting line-ups in part (a), there are 9! possible batting orders. So, multiply the answer from (a) by 9! to get (1,816,214,440)(362,880) = 659,067,881,472,000.

c. Order still matters: There are $P_{3,5}$ = 60 ways to choose three left-handers for the outfield and $P_{6,10}$ = 151,200 ways to choose six right-handers for the other positions. The total number of possibilities is = (60)(151,200) = 9,072,000.

34.

a. Since order doesn't matter, the number of ways to randomly select 5 keyboards from the 25 available is $\binom{25}{5}$ = 53,130.

b. Sample in two stages. First, there are 6 keyboards with an electrical defect, so the number of ways to select exactly 2 of them is $\binom{6}{2}$. Next, the remaining 5 – 2 = 3 keyboards in the sample must have mechanical defects; as there are 19 such keyboards, the number of ways to randomly select 3 is $\binom{19}{3}$. So, the number of ways to achieve both of these in the sample of 5 is the product of these two counting numbers: $\binom{6}{2}\binom{19}{3}$ = (15)(969) = 14,535.

c. Following the analogy from **b**, the number of samples with exactly 4 mechanical defects is $\binom{19}{4}\binom{6}{1}$, and the number with exactly 5 mechanical defects is $\binom{19}{5}\binom{6}{0}$. So, the number of samples with <u>at least</u> 4 mechanical defects is $\binom{19}{4}\binom{6}{1}+\binom{19}{5}\binom{6}{0}$, and the probability of this event is

$$\frac{\binom{19}{4}\binom{6}{1}+\binom{19}{5}\binom{6}{0}}{\binom{25}{5}} = \frac{34{,}884}{53{,}130} = .657.$$ (The denominator comes from **a**.)

35.

a. There are $\binom{10}{5} = 252$ ways to select 5 workers from the day shift. In other words, of all the ways to select 5 workers from among the 24 available, 252 such selections result in 5 day-shift workers. Since the grand total number of possible selections is $\binom{24}{5} = 42504$, the probability of randomly selecting 5 day-shift workers (and, hence, no swing or graveyard workers) is 252/42504 = .00593.

b. Similar to **a**, there are $\binom{8}{5} = 56$ ways to select 5 swing-shift workers and $\binom{6}{5} = 6$ ways to select 5 graveyard-shift workers. So, there are 252 + 56 + 6 = 314 ways to pick 5 workers from the same shift. The probability of this randomly occurring is 314/42504 = .00739.

c. P(at least two shifts represented) = 1 – P(all from same shift) = 1 – .00739 = .99261.

d. There are several ways to approach this question. For example, let A_1 = "day shift is unrepresented," A_2 = "swing shift is unrepresented," and A_3 = "graveyard shift is unrepresented." Then we want $P(A_1 \cup A_2 \cup A_3)$.

$N(A_1)$ = N(day shift unrepresented) = N(all from swing/graveyard) = $\binom{8+6}{5} = 2002$,

since there are 8 + 6 = 14 total employees in the swing and graveyard shifts. Similarly,

$N(A_2) = \binom{10+6}{5} = 4368$ and $N(A_3) = \binom{10+8}{5} = 8568$. Next, $N(A_1 \cap A_2)$ = N(all from graveyard) = 6 from **b**. Similarly, $N(A_1 \cap A_3) = 56$ and $N(A_2 \cap A_3) = 252$. Finally, $N(A_1 \cap A_2 \cap A_3) = 0$, since at least one shift must be represented. Now, apply the addition rule ~~for 3~~ events:

$$P(A_1 \cup A_2 \cup A_3) = \frac{2002+4368+8568-6-56-252+0}{42504} = \frac{14624}{42504} = .3441.$$

(5

36. There are $\binom{\;}{2} = 10$ possible ways to select the positions for *B*'s votes: *BBAAA*, *BABAA*, *BAABA*, *BAAAB*, *ABBAA*, *ABABA*, *ABAAB*, *AABBA*, *AABAB*, and *AAABB*. Only the last two have *A* ahead of *B* throughout the vote count. Since the outcomes are equally likely, the desired probability is 2/10 = .20.

37.

a. By the Fundamental Counting Principle, with $n_1 = 3$, $n_2 = 4$, and $n_3 = 5$, there are (3)(4)(5) = 60 runs.

b. With $n_1 = 1$ (just one temperature), $n_2 = 2$, and $n_3 = 5$, there are (1)(2)(5) = 10 such runs.

c. For each of the 5 specific catalysts, there are (3)(4) = 12 pairings of temperature and pressure. Imagine we separate the 60 possible runs into those 5 sets of 12. The number of ways to select exactly one run from each of these 5 sets of 12 is $\binom{12}{1}^5 = 12^5$. Since there are $\binom{60}{5}$ ways to select the 5 runs overall, the desired probability is $\binom{12}{1}^5 / \binom{60}{5} = 12^5 / \binom{60}{5}$ = .0456.

38.

a. A sonnet has 14 lines, each of which may come from any of the 10 pages. Order matters, and we're sampling with replacement, so the number of possibilities is $10 \times 10 \times \ldots \times 10 = 10^{14}$.

b. Similarly, the number of sonnets you could create avoiding the first and last pages (so, only using lines from the middle 8 sonnets) is 8^{14}. Thus, the probability that a randomly-created sonnet would not use any lines from the first or last page is $8^{14}/10^{14} = .8^{14} = .044$.

39. In **a-c**, the size of the sample space is $N = \binom{5+6+4}{3} = \binom{15}{3} = 455$.

a. There are four 23W bulbs available and 5+6 = 11 non-23W bulbs available. The number of ways to select exactly two of the former (and, thus, exactly one of the latter) is $\binom{4}{2}\binom{11}{1}$ = 6(11) = 66. Hence, the probability is 66/455 = .145.

b. The number of ways to select three 13W bulbs is $\binom{5}{3}$ = 10. Similarly, there are $\binom{6}{3}$ = 20 ways to select three 18W bulbs and $\binom{4}{3}$ = 4 ways to select three 23W bulbs. Put together, there are 10 + 20 + 4 = 34 ways to select three bulbs of the same wattage, and so the probability is 34/455 = .075.

c. The number of ways to obtain one of each type is $\binom{5}{1}\binom{6}{1}\binom{4}{1}$ = (5)(6)(4) = 120, and so the probability is 120/455 = .264.

d. Rather than consider many different options (choose 1, choose 2, etc.), re-frame the problem this way: at least 6 draws are required to get a 23W bulb iff a random sample of <u>five</u> bulbs fails to produce a 23W bulb. Since there are 11 non-23W bulbs, the chance of getting no 23W bulbs in a sample of size 5 is $\binom{11}{5} / \binom{15}{5}$ = 462/3003 = .154.

40.

a. If the *A*'s were distinguishable from one another, and similarly for the *B*'s, *C*'s and *D*'s, then there would be 12! possible chain molecules. Six of these are:

$A_1A_2A_3B_2C_3C_1D_3C_2D_1D_2B_3B_1$ $A_1A_3A_2B_2C_3C_1D_3C_2D_1D_2B_3B_1$
$A_2A_1A_3B_2C_3C_1D_3C_2D_1D_2B_3B_1$ $A_2A_3A_1B_2C_3C_1D_3C_2D_1D_2B_3B_1$
$A_3A_1A_2B_2C_3C_1D_3C_2D_1D_2B_3B_1$ $A_3A_2A_1B_2C_3C_1D_3C_2D_1D_2B_3B_1$

These 6 (=3!) differ only with respect to ordering of the 3 *A*'s. In general, groups of 6 chain molecules can be created such that within each group only the ordering of the *A*'s is different. When the A subscripts are suppressed, each group of 6 "collapses" into a single molecule (*B*'s, *C*'s and *D*'s are still distinguishable).

At this point there are (12!/3!) different molecules. Now suppressing subscripts on the *B*'s, *C*'s, and *D*'s in turn gives $\frac{12!}{(3!)^4} = 369{,}600$ chain molecules.

b. Think of the group of 3 *A*'s as a single entity, and similarly for the *B*'s, *C*'s, and *D*'s. Then there are 4! = 24 ways to order these triplets, and thus 24 molecules in which the *A*'s are contiguous, the *B*'s, *C*'s, and *D*'s also. The desired probability is $\frac{24}{369{,}600} = .00006494$.

41.

a. (10)(10)(10)(10) = 10^4 = 10,000. These are the strings 0000 through 9999.

b. Count the number of prohibited sequences. There are (i) 10 with all digits identical (0000, 1111, …, 9999); (ii) 14 with sequential digits (0123, 1234, 2345, 3456, 4567, 5678, 6789, and 7890, plus these same seven descending); (iii) 100 beginning with 19 (1900 through 1999). That's a total of 10 + 14 + 100 = 124 impermissible sequences, so there are a total of 10,000 – 124 = 9876 permissible sequences. The chance of randomly selecting one is just $\frac{9876}{10{,}000}$ = .9876.

c. All PINs of the form 8xx1 are legitimate, so there are (10)(10) = 100 such PINs. With someone randomly selecting 3 such PINs, the chance of guessing the correct sequence is 3/100 = .03.

d. Of all the PINs of the form 1xx1, eleven is prohibited: 1111, and the ten of the form 19x1. That leaves 89 possibilities, so the chances of correctly guessing the PIN in 3 tries is 3/89 = .0337.

42.

a. If Player X sits out, the number of possible teams is $\binom{3}{1}\binom{4}{2}\binom{4}{2}$ = 108. If Player X plays guard, we need one <u>more</u> guard, and the number of possible teams is $\binom{3}{1}\binom{4}{1}\binom{4}{2}$ = 72. Finally, if Player X plays forward, we need one <u>more</u> forward, and the number of possible teams is $\binom{3}{1}\binom{4}{2}\binom{4}{1}$ = 72. So, the total possible number of teams from this group of 12 players is 108 + 72 + 72 = 252.

b. Using the idea in **a**, consider all possible scenarios. If Players X and Y both sit out, the number of possible teams is $\binom{3}{1}\binom{5}{2}\binom{5}{2}$ = 300. If Player X plays while Player Y sits out, the number of possible teams is $\binom{3}{1}\binom{5}{1}\binom{5}{2}+\binom{3}{1}\binom{5}{2}\binom{5}{1}$ = 150 + 150 = 300. Similarly, there are 300 teams with Player X benched and Player Y in. Finally, there are three cases when X and Y both play: they're both guards, they're both forwards, or they split duties. The number of ways to select the rest of the team under these scenarios is $\binom{3}{1}\binom{5}{0}\binom{5}{2}+\binom{3}{1}\binom{5}{2}\binom{5}{0}+\binom{3}{1}\binom{5}{1}\binom{5}{1}$ = 30 + 30 + 75 = 135.

Since there are $\binom{15}{5}$ = 3003 ways to randomly select 5 players from a 15-person roster, the probability of randomly selecting a legitimate team is $\frac{300+300+135}{3003}=\frac{735}{3003}$ = .245.

43. There are $\binom{52}{5}$ = 2,598,960 five-card hands. The number of 10-high straights is (4)(4)(4)(4)(4) = 4^5 = 1024 (any of four 6s, any of four 7s, etc.). So, P(10 high straight) = $\frac{1024}{2{,}598{,}960}=.000394$. Next, there ten "types of straight: A2345, 23456, …, 910JQK, 10JQKA. So, P(straight) = $10\times\frac{1024}{2{,}598{,}960}=.00394$. Finally, there are only 40 straight flushes: each of the ten sequences above in each of the 4 suits makes (10)(4) = 40. So, P(straight flush) = $\frac{40}{2{,}598{,}960}=.00001539$.

44. $\binom{n}{k}=\frac{n!}{k!(n-k)!}=\frac{n!}{(n-k)!k!}=\binom{n}{n-k}$

The number of subsets of size k equals the number of subsets of size $n-k$, because to each subset of size k there corresponds exactly one subset of size $n-k$: the $n-k$ objects not in the subset of size k. The combinations formula counts the number of ways to split n objects into two subsets: one of size k, and one of size $n-k$.

Section 2.4

45.

a. $P(A) = .106 + .141 + .200 = .447$, $P(C) = .215 + .200 + .065 + .020 = .500$, and $P(A \cap C) = .200$.

b. $P(A|C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{.200}{.500} = .400$. If we know that the individual came from ethnic group 3, the probability that he has Type A blood is .40. $P(C|A) = \dfrac{P(A \cap C)}{P(A)} = \dfrac{.200}{.447} = .447$. If a person has Type A blood, the probability that he is from ethnic group 3 is .447.

c. Define D = "ethnic group 1 selected." We are asked for $P(D|B')$. From the table, $P(D \cap B') = .082 + .106 + .004 = .192$ and $P(B') = 1 - P(B) = 1 - [.008 + .018 + .065] = .909$. So, the desired probability is $P(D|B') = \dfrac{P(D \cap B')}{P(B')} = \dfrac{.192}{.909} = .211$.

46. Let A be that the individual is more than 180 cm tall. Let B be that the individual is a professional basketball player. Then $P(A|B)$ = the probability of the individual being more than 180 cm tall, knowing that the individual is a professional basketball player, while $P(B|A)$ = the probability of the individual being a professional basketball player, knowing that the individual is more than 180 cm tall. $P(A|B)$ will be larger. Most professional basketball players are tall, so the probability of an individual in that reduced sample space being more than 180 cm tall is very large. On the other hand, the number of individuals that are pro basketball players is small in relation to the number of males more than 180 cm tall.

47.

a. Apply the addition rule for three events: $P(A \cup B \cup C) = .6 + .4 + .2 - .3 - .15 - .1 + .08 = .73$.

b. $P(A \cap B \cap C') = P(A \cap B) - P(A \cap B \cap C) = .3 - .08 = .22$.

c. $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.3}{.6} = .50$ and $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.3}{.4} = .75$. Half of students with Visa cards also have a MasterCard, while three-quarters of students with a MasterCard also have a Visa card.

d. $P(A \cap B \mid C) = \dfrac{P([A \cap B] \cap C)}{P(C)} = \dfrac{P(A \cap B \cap C)}{P(C)} = \dfrac{.08}{.2} = .40$.

e. $P(A \cup B \mid C) = \dfrac{P([A \cup B] \cap C)}{P(C)} = \dfrac{P([A \cap C] \cup [B \cap C])}{P(C)}$. Use a distributive law:

$$= \frac{P(A \cap C) + P(B \cap C) - P([A \cap C] \cap [B \cap C])}{P(C)} = \frac{P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)}{P(C)} =$$

$$\frac{.15 + .1 - .08}{.2}$$

= .85.

48.

a. $P(A_2 \mid A_1) = \frac{P(A_2 \cap A_1)}{P(A_1)} = \frac{.06}{.12} = .50$. The numerator comes from Exercise 26.

b. $P(A_1 \cap A_2 \cap A_3 \mid A_1) = \frac{P([A_1 \cap A_2 \cap A_3] \cap A_1)}{P(A_1)} = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{.01}{.12} = .0833$. The numerator simplifies because $A_1 \cap A_2 \cap A_3$ is a subset of A_1, so their intersection is just the smaller event.

c. For this example, you definitely need a Venn diagram. The seven pieces of the partition inside the three circles have probabilities .04, .05, .00, .02, .01, .01, and .01. Those add to .14 (so the chance of no defects is .86).
Let E = "exactly one defect." From the Venn diagram, $P(E)$ = .04 + .00 + .01 = .05. From the addition above, P(at least one defect) = $P(A_1 \cup A_2 \cup A_3) = .14$. Finally, the answer to the question is $P(E \mid A_1 \cup A_2 \cup A_3) = \frac{P(E \cap [A_1 \cup A_2 \cup A_3])}{P(A_1 \cup A_2 \cup A_3)} = \frac{P(E)}{P(A_1 \cup A_2 \cup A_3)} = \frac{.05}{.14} = .3571$. The numerator simplifies because E is a subset of $A_1 \cup A_2 \cup A_3$.

d. $P(A_3' \mid A_1 \cap A_2) = \frac{P(A_3' \cap [A_1 \cap A_2])}{P(A_1 \cap A_2)} = \frac{.05}{.06} = .8333$. The numerator is Exercise 26(c), while the denominator is Exercise 26(b).

49.

a. P(small cup) = .14 + .20 = .34. P(decaf) = .20 + .10 + .10 = .40.

b. P(decaf | small) = $\frac{P(\text{small} \cap \text{decaf})}{P(\text{small})} = \frac{.20}{.34} = .588$. 58.8% of all people who purchase a small cup of coffee choose decaf.

c. P(small | decaf) = $\frac{P(\text{small} \cap \text{decaf})}{P(\text{decaf})} = \frac{.20}{.40} = .50$. 50% of all people who purchase decaf coffee choose the small size.

50.

a. P(**M** ∩ **LS** ∩ **PR**) = .05, directly from the table of probabilities.

b. P(**M** ∩ **Pr**) = P(**M** ∩ **LS** ∩ **PR**) + P(**M** ∩ **SS** ∩ **PR**) = .05 + .07 = .12.

c. P(**SS**) = sum of 9 probabilities in the **SS** table = .56. P(**LS**) = 1 – .56 = .44.

d. From the two tables, P(**M**) = .08 + .07 + .12 + .10 + .05 + .07 = .49. P(**Pr**) = .02 + .07 + .07 + .02 + .05 + .02 = .25.

e. P(**M**|**SS** ∩ **Pl**) = .556.

f. P(**SS**|**M** ∩ **Pl**) =

$$\frac{P(\mathbf{M} \cap \mathbf{SS} \cap \mathbf{Pl})}{P(\mathbf{SS} \cap \mathbf{Pl})} = \frac{.08}{.04 + .08 + .03} = .533 .$$

$$\frac{P(\mathbf{SS} \cap \mathbf{M} \cap \mathbf{Pl})}{P(\mathbf{M} \cap \mathbf{Pl})} = \frac{.08}{.08 + .10} = .444 .$$

$P(\mathbf{LS}|\mathbf{M} \cap \mathbf{Pl}) = 1 - P(\mathbf{SS}|\mathbf{M} \cap \mathbf{Pl}) = 1 - .444 =$

51.

a. Let A = child has a food allergy, and R = child has a history of severe reaction. We are told that $P(A)$ = .08 and $P(R \mid A)$ = .39. By the multiplication rule, $P(A \cap R) = P(A) \times P(R \mid A) = (.08)(.39) = .0312$.

b. Let M = the child is allergic to multiple foods. We are told that $P(M \mid A)$ = .30, and the goal is to find $P(M)$. But notice that M is actually a subset of A: you can't have multiple food allergies without having at least one such allergy! So, apply the multiplication rule again:

$P(M) = P(M \cap A) = P(A) \times P(M \mid A) = (.08)(.30) = .024$.

52. We know that $P(A_1 \cup A_2) = .07$ and $P(A_1 \cap A_2) = .01$, and that $P(A_1) = P(A_2)$ because the pumps are identical. There are two solution methods. The first doesn't require explicit reference to q or r: Let A_1 be the event that #1 fails and A_2 be the event that #2 fails.

Apply the addition rule: $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \Rightarrow .07 = 2P(A_1) - .01 \Rightarrow P(A_1) = .04$.

Otherwise, we assume that $P(A_1) = P(A_2) = q$ and that $P(A_1 \mid A_2) = P(A_2 \mid A_1) = r$ (the goal is to find q). Proceed as follows: $.01 = P(A_1 \cap A_2) = P(A_1)\, P(A_2 \mid A_1) = qr$ and $.07 = P(A_1 \cup A_2) =$
$P(A_1 \cap A_2) + P(A_1' \cap A_2) + P(A_1 \cap A_2') = .01 + q(1-r) + q(1-r) \Rightarrow q(1-r) = .03$.

These two equations give $2q - .01 = .07$, from which $q = .04$ (and $r = .25$).

53. $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{P(B)}{P(A)}$ (since B is contained in A, $A \cap B = B$)

$= \dfrac{.05}{.60} = .0833$

54.

a. $P(A_2 \mid A_1) = \dfrac{P(A_1 \cap A_2)}{P(A_1)} = \dfrac{.11}{.22} = .50$. If the firm is awarded project 1, there is a 50% chance they will also be awarded project 2.

b. $P(A_2 \cap A_3 \mid A_1) = \dfrac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \dfrac{.01}{.22} = .0455$. If the firm is awarded project 1, there is a 4.55% chance they will also be awarded projects 2 and 3.

c. $P(A_2 \cup A_3 \mid A_1) = \dfrac{P[A_1 \cap (A_2 \cup A_3)]}{P(A_1)} = \dfrac{P[(A_1 \cap A_2) \cup (A_1 \cap A_3)]}{P(A_1)}$

$= \dfrac{P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \dfrac{.15}{.22} = .682$. If the firm is awarded project 1, there is a 68.2% chance they will also be awarded at least one of the other two projects.

d. $P(A_1 \cap A_2 \cap A_3 \mid A_1 \cup A_2 \cup A_3) = \dfrac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)} = \dfrac{.01}{.53} = .0189$. If the firm is awarded at least one of the projects, there is a 1.89% chance they will be awarded all three projects.

55. Let A = {carries Lyme disease} and B = {carries HGE}. We are told $P(A) = .16$, $P(B) = .10$, and $P(A \cap B \mid A \cup B) = .10$. From this last statement and the fact that $A \cap B$ is contained in $A \cup B$,

$.10 = \dfrac{P(A \cap B)}{P(A \cup B)} \Rightarrow P(A \cap B) = .10P(A \cup B) = .10[P(A) + P(B) - P(A \cap B)] = .10[.10 + .16 - P(A \cap B)] \Rightarrow$

$1.1P(A \cap B) = .026 \Rightarrow P(A \cap B) = .02364$.

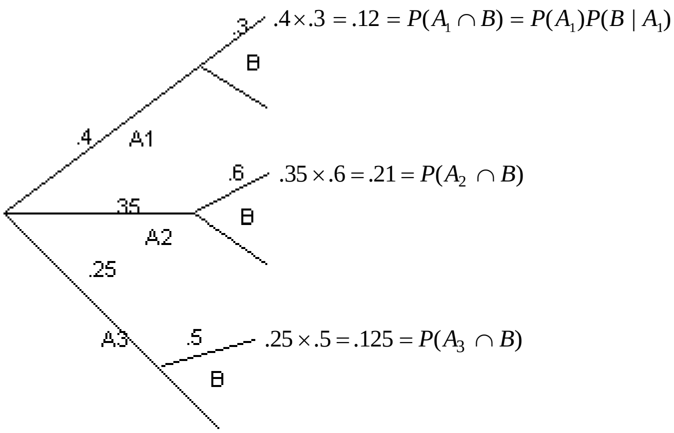
Finally, the desired probability is $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.02364}{.10} = .2364$.

56. $$P(A \mid B) + P(A' \mid B) = \frac{P(A \cap B)}{P(B)} + \frac{P(A' \cap B)}{P(B)} = \frac{P(A \cap B) + P(A' \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

57. $P(B \mid A) > P(B)$ iff $P(B \mid A) + P(B' \mid A) > P(B) + P(B' \mid A)$ iff $1 > P(B) + P(B' \mid A)$ by Exercise 56 (with the letters switched). This holds iff $1 - P(B) > P(B' \mid A)$ iff $P(B') > P(B' \mid A)$, QED.

58. $$P(A \cup B \mid C) = \frac{P[(A \cup B) \cap C]}{P(C)} = \frac{P[(A \cap C) \cup (B \cap C)]}{P(C)} = \frac{P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)}{P(C)} = P(A \mid C) + P(B \mid C) - P(A \cap B \mid C)$$

59. The required probabilities appear in the tree diagram below.

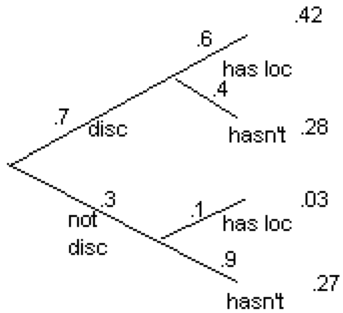


a. $P(A_2 \cap B) = .21$.

b. By the law of total probability, $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = .455$.

c. Using Bayes' theorem, $P(A_1 \mid B) = \dfrac{P(A_1 \cap B)}{P(B)} = \dfrac{.12}{.455} = .264$; $P(A_2 \mid B) = \dfrac{.21}{.455} = .462$; $P(A_3 \mid B) = 1 - .264 - .462 = .274$. Notice the three probabilities sum to 1.

60. The tree diagram below shows the probability for the four disjoint options; e.g., *P*(the flight is discovered and has a locator) = *P*(discovered)*P*(locator | discovered) = (.7)(.6) = .42.



a. $P(\text{not discovered} \mid \text{has locator}) = \dfrac{P(\text{not discovered} \cap \text{has locator})}{P(\text{has locator})} = \dfrac{.03}{.03 + .42} = .067$.

b. $P(\text{discovered} \mid \text{no locator}) = \dfrac{P(\text{discovered} \cap \text{no locator})}{P(\text{no locator})} = \dfrac{.28}{.55} = .509$.

61. The initial ("prior") probabilities of 0, 1, 2 defectives in the batch are .5, .3, .2. Now, let's determine the probabilities of 0, 1, 2 defectives in the sample based on these three cases.

- If there are 0 defectives in the batch, clearly there are 0 defectives in the sample.

P(0 def in sample | 0 def in batch) = 1.

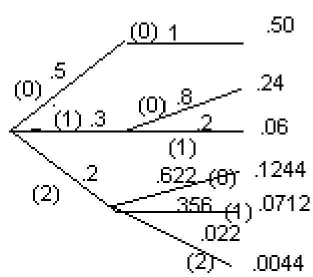
- If there is 1 defective in the batch, the chance it's discovered in a sample of 2 equals 2/10 = .2, and the probability it isn't discovered is 8/10 = .8.

P(0 def in sample | 1 def in batch) = .8, *P*(1 def in sample | 1 def in batch) = .2.

- If there are 2 defectives in the batch, the chance both are discovered in a sample of 2 equals $\frac{2}{10} \times \frac{1}{9} = .022$; the chance neither is discovered equals $\frac{8}{10} \times \frac{7}{9} = .622$; and the chance exactly 1 is discovered equals 1 – (.022 + .622) = .356.

P(0 def in sample | 2 def in batch) = .622, *P*(1 def in sample | 2 def in batch) = .356,
P(2 def in sample | 2 def in batch) = .022.

These calculations are summarized in the tree diagram below. Probabilities at the endpoints are intersectional probabilities, e.g. *P*(2 def in batch ∩ 2 def in sample) = (.2)(.022) = .0044.



a. Using the tree diagram and Bayes' rule,

P(0 def in batch | 0 def in sample) = $\dfrac{.5}{.5+.24+.1244} = .578$

P(1 def in batch | 0 def in sample) = $\dfrac{.24}{.5+.24+.1244} = .278$

P(2 def in batch | 0 def in sample) = $\dfrac{.1244}{.5+.24+.1244} = .144$

b. P(0 def in batch | 1 def in sample) = 0

P(1 def in batch | 1 def in sample) = $\dfrac{.06}{.06+.0712} = .457$

P(2 def in batch | 1 def in sample) = $\dfrac{.0712}{.06+.0712} = .543$

62. Let B = blue cab was involved, $G = B'$ = green cab was involved, and W = witness claims to have seen a blue cab. Before any witness statements, $P(B) = .15$ and $P(G)$. The witness' reliability can be coded as follows: $P(W \mid B) = .8$ (correctly identify blue), $P(W' \mid G) = .8$ (correctly identify green), and by taking complements $P(W' \mid B) = P(W \mid G) = .2$ (the two ways to mis-identify a color at night).

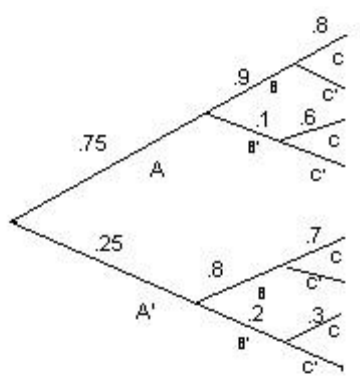
The goal is to determine $P(B \mid W)$, the chance a blue cab was involved given that's what the witness claims to have seen. Apply Bayes' Theorem:

$$P(B \mid W) = \frac{P(B)P(W \mid B)}{P(B)P(W \mid B) + P(B')P(W \mid B')} = \frac{(.15)(.8)}{(.15)(.8) + (.85)(.2)} = .4138.$$

The "posterior" probability that the cab was really blue is actually less than 50%. That's because there are so many more green cabs on the street, that it's more likely the witness mis-identified a green cab (.85 × .2) than that the witness correctly identified a blue cab (.15 × .8).

63.

a.



b. From the top path of the tree diagram, $P(A \cap B \cap C) = (.75)(.9)(.8) = .54$.

c. Event $B \cap C$ occurs twice on the diagram: $P(B \cap C) = P(A \cap B \cap C) + P(A' \cap B \cap C) = .54 + (.25)(.8)(.7) = .68$.

d. $P(C) = P(A \cap B \cap C) + P(A' \cap B \cap C) + P(A \cap B' \cap C) + P(A' \cap B' \cap C) = .54 + .045 + .14 + .015 = .74$.

e. Rewrite the conditional probability first: $P(A \mid B \cap C) = \dfrac{P(A \cap B \cap C)}{P(B \cap C)} = \dfrac{.54}{.68} = .7941$.

64. A tree diagram can help. We know that $P(\text{short}) = .6$, $P(\text{medium}) = .3$, $P(\text{long}) = .1$; also, $P(\text{Word} \mid \text{short}) = .8$, $P(\text{Word} \mid \text{medium}) = .5$, $P(\text{Word} \mid \text{long}) = .3$.

a. Use the law of total probability: $P(\text{Word}) = (.6)(.8) + (.3)(.5) + (.1)(.3) = .66$.

b. $P(\text{small} \mid \text{Word}) = \dfrac{P(\text{small} \cap \text{Word})}{P(\text{Word})} = \dfrac{(.6)(.8)}{.66} = .727$. Similarly, $P(\text{medium} \mid \text{Word}) = \dfrac{(.3)(.5)}{.66} = .227$, and $P(\text{long} \mid \text{Word}) = .045$. (These sum to .999 due to rounding error.)

65. A tree diagram can help. We know that $P(\text{day}) = .2$, $P(\text{1-night}) = .5$, $P(\text{2-night}) = .3$; also, $P(\text{purchase} \mid \text{day}) = .1$, $P(\text{purchase} \mid \text{1-night}) = .3$, and $P(\text{purchase} \mid \text{2-night}) = .2$.

Apply Bayes' rule: e.g., $P(\text{day} \mid \text{purchase}) = \dfrac{P(\text{day} \cap \text{purchase})}{P(\text{purchase})} = \dfrac{(.2)(.1)}{(.2)(.1) + (.5)(.3) + (.3)(.2)} = \dfrac{.02}{.23} = .087$.

Similarly, $P(\text{1-night} \mid \text{purchase}) = \dfrac{(.5)(.3)}{.23} = .652$ and $P(\text{2-night} \mid \text{purchase}) = .261$.

66. Let E, C, and L be the events associated with e-mail, cell phones, and laptops, respectively. We are told $P(E) = 40\%$, $P(C) = 30\%$, $P(L) = 25\%$, $P(E \cap C) = 23\%$, $P(E' \cap C' \cap L') = 51\%$, $P(E \mid L) = 88\%$, and $P(L \mid C) = 70\%$.

a. $P(C \mid E) = P(E \cap C)/P(E) = .23/.40 = .575$.

b. Use Bayes' rule: $P(C \mid L) = P(C \cap L)/P(L) = P(C)P(L \mid C)/P(L) = .30(.70)/.25 = .84$.

c. $P(C \mid E \cap L) = P(C \cap E \cap L)/P(E \cap L)$.
For the denominator, $P(E \cap L) = P(L)P(E \mid L) = (.25)(.88) = .22$.
For the numerator, use $P(E \cup C \cup L) = 1 - P(E' \cap C' \cap L') = 1 - .51 = .49$ and write
$P(E \cup C \cup L) = P(C) + P(E) + P(L) - P(E \cap C) - P(C \cap L) - P(E \cap L) + P(C \cap E \cap L)$
$\Rightarrow .49 = .30 + .40 + .25 - .23 - .30(.70) - .22 + P(C \cap E \cap L) \Rightarrow P(C \cap E \cap L) = .20$.
So, finally, $P(C \mid E \cap L) = .20/.22 = .9091$.

67. Let T denote the event that a randomly selected person is, in fact, a terrorist. Apply Bayes' theorem, using $P(T) = 1{,}000/300{,}000{,}000 = .0000033$:

$P(T \mid +) = \dfrac{P(T)P(+ \mid T)}{P(T)P(+ \mid T) + P(T')P(+ \mid T')} = \dfrac{(.0000033)(.99)}{(.0000033)(.99) + (1 - .0000033)(1 - .999)} = .003289$. That is to say, roughly 0.3% of all people "flagged" as terrorists would be actual terrorists in this scenario.

68. Let's see how we can implement the hint. If she's flying airline #1, the chance of 2 late flights is (30%)(10%) = 3%; the two flights being "unaffected" by each other means we can multiply their probabilities. Similarly, the chance of 0 late flights on airline #1 is (70%)(90%) = 63%. Since percents add to 100%, the chance of exactly 1 late flight on airline #1 is 100% – (3% + 63%) = 34%. A similar approach works for the other two airlines: the probability of exactly 1 late flight on airline #2 is 35%, and the chance of exactly 1 late flight on airline #3 is 45%.

The initial ("prior") probabilities for the three airlines are $P(A_1)$ = 50%, $P(A_2)$ = 30%, and $P(A_3)$ = 20%. Given that she had exactly 1 late flight (call that event B), the conditional ("posterior") probabilities of the three airlines can be calculated using Bayes' Rule:

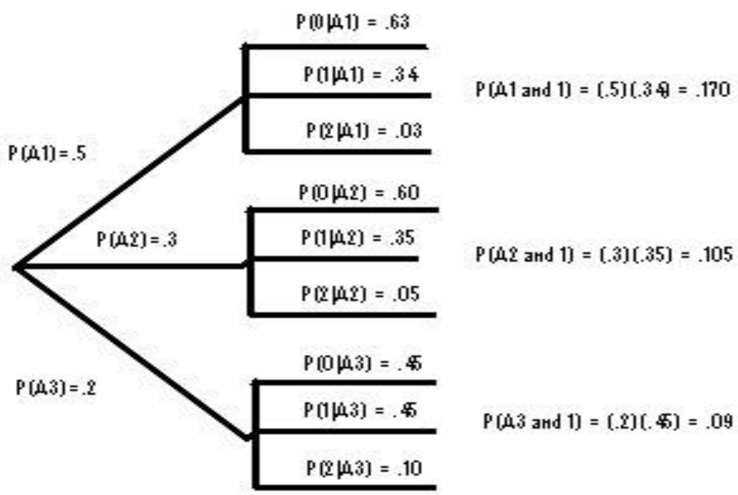
$$P(A_1 \mid B) = \frac{P(A_1)P(B \mid A_1)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + P(A_3)P(B \mid A_3)} = \frac{(.5)(.34)}{(.5)(.34) + (.3)(.35) + (.2)(.45)} = \frac{.170}{.365} = .4657;$$

$$P(A_2 \mid B) = \frac{P(A_2)P(B \mid A_2)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + P(A_3)P(B \mid A_3)} = \frac{(.3)(.35)}{.365} = .2877; \text{ and}$$

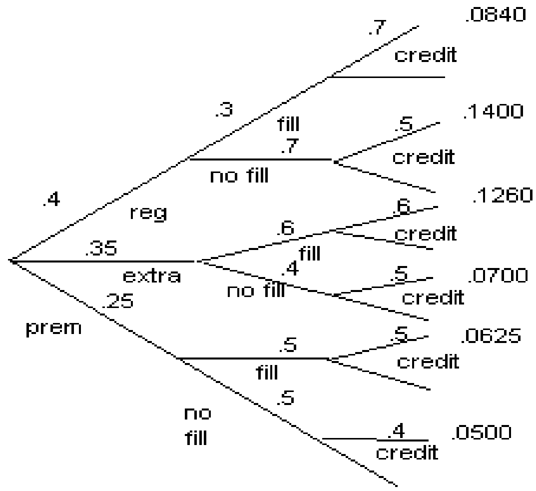
$$P(A_3 \mid B) = \frac{P(A_3)P(B \mid A_3)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + P(A_3)P(B \mid A_3)} = \frac{(.2)(.45)}{.365} = .2466.$$

Notice that, except for rounding error, these three posterior probabilities add to 1.

The tree diagram below shows these probabilities.



69. The tree diagram below summarizes the information in the exercise (plus the previous information in Exercise 59). Probabilities for the branches corresponding to paying with credit are indicated at the far right. ("extra" = "plus")



a. $P(\text{plus} \cap \text{fill} \cap \text{credit}) = (.35)(.6)(.6) = .1260$.

b. $P(\text{premium} \cap \text{no fill} \cap \text{credit}) = (.25)(.5)(.4) = .05$.

c. From the tree diagram, $P(\text{premium} \cap \text{credit}) = .0625 + .0500 = .1125$.

d. From the tree diagram, $P(\text{fill} \cap \text{credit}) = .0840 + .1260 + .0625 = .2725$.

e. $P(\text{credit}) = .0840 + .1400 + .1260 + .0700 + .0625 + .0500 = .5325$.

f. $P(\text{premium} \mid \text{credit}) = \dfrac{P(\text{premium} \cap \text{credit})}{P(\text{credit})} = \dfrac{.1125}{.5325} = .2113$.

Section 2.5

70. Using the definition, two events A and B are independent if $P(A \mid B) = P(A)$;
$P(A \mid B) = .6125$; $P(A) = .50$; $.6125 \neq .50$, so A and B are not independent.
Using the multiplication rule, the events are independent if $P(A \cap B)=P(A)P(B)$;
$P(A \cap B) = .25$; $P(A)P(B) = (.5)(.4) = .2$. $.25 \neq .2$, so A and B are not independent.

71.

a. Since the events are independent, then A' and B' are independent, too. (See the paragraph below Equation 2.7.) Thus, $P(B'|A') = P(B') = 1 - .7 = .3$.

b. Using the addition rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .4 + .7 - (.4)(.7) = .82$. Since A and B are independent, we are permitted to write $P(A \cap B) = P(A)P(B) = (.4)(.7)$.

c. $$P(AB' \mid A \cup B) = \frac{P(AB' \cap (A \cup B))}{P(A \cup B)} = \frac{P(AB')}{P(A \cup B)} = \frac{P(A)P(B')}{P(A \cup B)} = \frac{(.4)(1-.7)}{.82} = \frac{.12}{.82} = .146 .$$

72. $P(A_1 \cap A_2) = .11$ while $P(A_1)P(A_2) = .055$, so A_1 and A_2 are not independent.
$P(A_1 \cap A_3) = .05$ while $P(A_1)P(A_3) = .0616$, so A_1 and A_3 are not independent.
$P(A_2 \cap A_3) = .07$ and $P(A_2)P(A_3) = .07$, so A_2 and A_3 are independent.

73. From a Venn diagram, $P(B) = P(A' \cap B) + P(A \cap B) = P(B) \Rightarrow P(A' \cap B) = P(B) - P(A \cap B)$. If A and B are independent, then $P(A' \cap B) = P(B) - P(A)P(B) = [1 - P(A)]P(B) = P(A')P(B)$. Thus, A' and B are independent.

Alternatively, $$P(A' \mid B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{P(B) - P(A)P(B)}{P(B)} = 1 - P(A) = P(A').$$

74. Using subscripts to differentiate between the selected individuals,
$P(O_1 \cap O_2) = P(O_1)P(O_2) = (.45)(.45) = .2025$.
$P(\text{two individuals match}) = P(A_1 \cap A_2) + P(B_1 \cap B_2) + P(AB_1 \cap AB_2) + P(O_1 \cap O_2) =$
$.40^2 + .11^2 + .04^2 + .45^2 = .3762$.

75. Let event E be the event that an error was signaled incorrectly.
We want $P(\text{at least one signaled incorrectly}) = P(E_1 \cup \ldots \cup E_{10})$. To use independence, we need intersections, so apply deMorgan's law: $= P(E_1 \cup \ldots \cup E_{10}) = 1 - P(E_1' \cap \cdots \cap E_{10}')$. $P(E') = 1 - .05 = .95$,

so for 10 independent points, $P(E_1' \cap \cdots \cap E_{10}') = (.95)\ldots(.95) = (.95)^{10}$. Finally, $P(E_1 \cup E_2 \cup \ldots \cup E_{10}) =$

$1 - (.95)^{10} = .401$. Similarly, for 25 points, the desired probability is $1 - (P(E'))^{25} = 1 - (.95)^{25} = .723$.

76. Follow the same logic as in Exercise 75: If the probability of an event is p, and there are n independent "trials," the chance this event never occurs is $(1 - p)^n$, while the chance of at least one occurrence is $1 - (1 - p)^n$. With p = 1/9,000,000,000 and n = 1,000,000,000, this calculates to 1 – .9048 = .0952.

Note: For extremely small values of p, $(1 - p)^n \approx 1 - np$. So, the probability of at least one occurrence under these assumptions is roughly $1 - (1 - np) = np$. Here, that would equal 1/9.

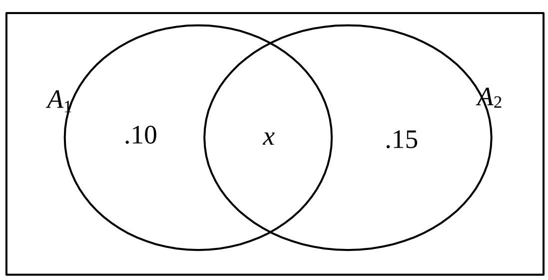
77. Let p denote the probability that a rivet is defective.

a. .15 = P(seam needs reworking) = 1 – P(seam doesn't need reworking) =
1 – P(no rivets are defective) = 1 – P(1^{st} isn't def $\cap \ldots \cap$ 25^{th} isn't def) =
$1 - (1 - p)\ldots(1 - p) = 1 - (1 - p)^{25}$.
Solve for p: $(1 - p)^{25} = .85 \Rightarrow 1 - p = (.85)^{1/25} \Rightarrow p = 1 - .99352 = .00648$.

b. The desired condition is $.10 = 1 - (1 - p)^{25}$. Again, solve for p: $(1 - p)^{25} = .90 \Rightarrow$
$p = 1 - (.90)^{1/25} = 1 - .99579 = .00421$.

78. P(at least one opens) = 1 – P(none open) = $1 - (.04)^5$ = .999999897.
P(at least one fails to open) = 1 – P(all open) = $1 - (.96)^5$ = .1846.

79. Let A_1 = older pump fails, A_2 = newer pump fails, and $x = P(A_1 \cap A_2)$. The goal is to find x. From the Venn diagram below, $P(A_1) = .10 + x$ and $P(A_2) = .05 + x$. Independence implies that $x = P(A_1 \cap A_2) = P(A_1)P(A_2)$ $= (.10 + x)(.05 + x)$. The resulting quadratic equation, $x^2 - .85x + .005 = 0$, has roots x = .0059 and x = .8441. The latter is impossible, since the probabilities in the Venn diagram would then exceed 1. Therefore, x = .0059.



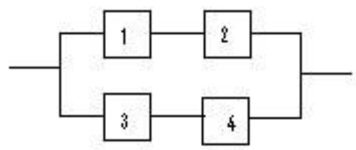
80. Let A_i denote the event that component #i works (i = 1, 2, 3, 4). Based on the design of the system, the event "the system works" is $(A_1 \cup A_2) \cup (A_3 \cap A_4)$. We'll eventually need $P(A_1 \cup A_2)$, so work that out first: $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = (.9) + (.9) - (.9)(.9) = .99$. The third term uses independence of events. Also, $P(A_3 \cap A_4)$ = (.8)(.8) = .64, again using independence.

Now use the addition rule and independence for the system:

$$\begin{aligned} P((A_1 \cup A_2) \cup (A_3 \cap A_4)) &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P((A_1 \cup A_2) \cap (A_3 \cap A_4)) \\ &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P(A_1 \cup A_2) \times P(A_3 \cap A_4) \\ &= (.99) + (.64) - (.99)(.64) = .9964 \end{aligned}$$

(You could also use deMorgan's law in a couple of places.)

81. Using the hints, let $P(A_i) = p$, and $x = p^2$. Following the solution provided in the example, P(system lifetime exceeds t_0) = $p^2 + p^2 - p^4 = 2p^2 - p^4 = 2x - x^2$. Now, set this equal to .99:
$2x - x^2 = .99 \Rightarrow x^2 - 2x + .99 = 0 \Rightarrow x = 0.9$ or $1.1 \Rightarrow p = 1.049$ or .9487. Since the value we want is a probability and cannot exceed 1, the correct answer is p = .9487.



82. A = {(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)} $\Rightarrow P(A) = \frac{6}{36} = \frac{1}{6}$; B = {(1,4)(2,4)(3,4)(4,4)(5,4)(6,4)} $\Rightarrow P(B) = \frac{1}{6}$; and C = {(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)} $\Rightarrow P(C) = \frac{1}{6}$.

$A \cap B$ = {(3,4)} $\Rightarrow P(A \cap B) = \frac{1}{36} = P(A)P(B)$; $A \cap C$ = {(3,4)} $\Rightarrow P(A \cap C) = \frac{1}{36} = P(A)P(C)$; and $B \cap C$ = {(3,4)} $\Rightarrow P(B \cap C) = \frac{1}{36} = P(B)P(C)$. Therefore, these three events are pairwise independent.

However, $A \cap B \cap C$ = {(3,4)} $\Rightarrow P(A \cap B \cap C) = \frac{1}{36}$, while $P(A)P(B)P(C) = \; = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$, so $P(A \cap B \cap C) \neq P(A)P(B)P(C)$ and these three events are not mutually independent.

83. We'll need to know P(both detect the defect) = 1 – P(at least one doesn't) = 1 – .2 = .8.

a. P(1st detects $\cap$ 2nd doesn't) = P(1st detects) – P(1st does $\cap$ 2nd does) = .9 – .8 = .1.
Similarly, P(1st doesn't $\cap$ 2nd does) = .1, so P(exactly one does)= .1 + .1= .2.

b. P(neither detects a defect) = 1 – [P(both do) + P(exactly 1 does)] = 1 – [.8+.2] = 0. That is, under this model there is a 0% probability neither inspector detects a defect. As a result, P(all 3 escape) = (0)(0)(0) = 0.

84. We'll make repeated use of the independence of the A_is and their complements.

a. $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$ = (.95)(.98)(.80) = .7448.

b. This is the complement of part **a**, so the answer is 1 – .7448 = .2552.

c. $P(A_1' \cap A_2' \cap A_3') = P(A_1')P(A_2')P(A_3')$ = (.05)(.02)(.20) = .0002.

d. $P(A_1' \cap A_2 \cap A_3) = P(A_1')P(A_2)P(A_3)$ = (.05)(.98)(.80) = .0392.

e. $P([A_1' \cap A_2 \cap A_3] \cup [A_1 \cap A_2' \cap A_3] \cup [A_1 \cap A_2 \cap A_3'])$ = (.05)(.98)(.80) + (.95)(.02)(.80) + (.95)(.98)(.20) = .07302.

f. This is just a little joke — we've all had the experience of electronics dying right after the warranty expires! ☺

85.

a. Let D_1 = detection on 1st fixation, D_2 = detection on 2nd fixation.
P(detection in at most 2 fixations) = $P(D_1) + P(D_1' \cap D_2)$; since the fixations are independent,
$P(D_1) + P(D_1' \cap D_2) = P(D_1) + P(D_1')\,P(D_2) = p + (1 - p)p = p(2 - p)$.

b. Define $D_1, D_2, \ldots, D_n$ as in **a**. Then P(at most n fixations) =
$P(D_1) + P(D_1' \cap D_2) + P(D_1' \cap D_2' \cap D_3) + \ldots + P(D_1' \cap D_2' \cap \cdots \cap D_{n-1}' \cap D_n)$ =
$p + (1 - p)p + (1 - p)^2 p + \ldots + (1 - p)^{n-1}p = p[1 + (1 - p) + (1 - p)^2 + \ldots + (1 - p)^{n-1}]$ =
$p \cdot \dfrac{1-(1-p)^n}{1-(1-p)} = 1-(1-p)^n$.
Alternatively, P(at most n fixations) = 1 – P(at least n+1 fixations are required) =
1 – P(no detection in 1st n fixations) = $1 - P(D_1' \cap D_2' \cap \cdots \cap D_n') = 1 - (1 - p)^n$.

c. P(no detection in 3 fixations) = $(1 - p)^3$.

d. P(passes inspection) = P({not flawed} ∪ {flawed and passes})
= P(not flawed) + P(flawed and passes)
= .9 + P(flawed) P(passes | flawed) = .9 + (.1)$(1 - p)^3$.

e. Borrowing from **d**, P(flawed | passed) = $\dfrac{P(\text{flawed} \cap \text{passed})}{P(\text{passed})} = \dfrac{.1(1-p)^3}{.9+.1(1-p)^3}$. For p = .5,

P(flawed | passed) = $\dfrac{.1(1-.5)^3}{.9+.1(1-.5)^3} = .0137$.

86.

a. $P(A) = \frac{2{,}000}{10{,}000}$ = .2. Using the law of total probability, $P(B) = P(A)P(B \mid A) + P(A')P(B \mid A')$ =

$(.2)\frac{1{,}999}{9{,}999} + (.8)\frac{2{,}000}{9{,}999}$ = **.2** exactly. That is, $P(B) = P(A) = .2$. Finally, use the multiplication rule: $P(A \cap B) = P(A) \times P(B \mid A) = (.2)\frac{1{,}999}{9{,}999}$ = .039984. Events A and B are *not* independent, since $P(B)$ = .2 while $P(B \mid A) = \frac{1{,}999}{9{,}999} = .19992$, and these are not equal.

b. If A and B were independent, we'd have $P(A \cap B) = P(A) \times P(B) = (.2)(.2) = .04$. This is very close to the answer .039984 from part **a**. This suggests that, for most practical purposes, we could treat events A and B in this example as if they were independent.

c. Repeating the steps in part **a**, you again get $P(A) = P(B) = .2$. However, using the multiplication rule, $P(A \cap B) = P(A) \times P(B \mid A) = \frac{2}{10} \times \frac{1}{9}$ =.0222. This is very different from the value of .04 that we'd get if A and B were independent!

The critical difference is that the population size in parts **a-b** is huge, and so the probability a second board is green *almost* equals .2 (i.e., 1,999/9,999 = .19992 ≈ .2). But in part **c**, the conditional probability of a green board shifts a lot: 2/10 = .2, but 1/9 = .1111.

87.

a. Use the information provided and the addition rule:
$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \Rightarrow P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = .55 + .65 - .80 = .40$.

b. By definition, $P(A_2 \mid A_3) = \frac{P(A_2 \cap A_3)}{P(A_3)} = \frac{.40}{.70}$ = .5714. If a person likes vehicle #3, there's a 57.14% chance s/he will also like vehicle #2.

c. No. From **b**, $P(A_2 \mid A_3) = .5714 \neq P(A_2) = .65$. Therefore, A_2 and A_3 are not independent. Alternatively, $P(A_2 \cap A_3) = .40 \neq P(A_2)P(A_3) = (.65)(.70) = .455$.

d. The goal is to find $P(A_2 \cup A_3 \mid A_1')$, i.e. $\frac{P([A_2 \cup A_3] \cap A_1')}{P(A_1')}$. The denominator is simply 1 – .55 = .45.

There are several ways to calculate the numerator; the simplest approach using the information provided is to draw a Venn diagram and observe that $P([A_2 \cup A_3] \cap A_1') = P(A_1 \cup A_2 \cup A_3) - P(A_1)$ = .88 – .55 = .33. Hence, $P(A_2 \cup A_3 \mid A_1') = \frac{.33}{.45}$ = .7333.

88. Let D = patient has disease, so $P(D)$ = .05. Let ++ denote the event that the patient gets two independent, positive tests. Given the sensitivity and specificity of the test, $P(++ \mid D)$ = (.98)(.98) = .9604, while $P(++ \mid D')$ = (1 – .99)(1 – .99) = .0001. (That is, there's a 1-in-10,000 chance of a healthy person being mis-diagnosed with the disease <u>twice</u>.) Apply Bayes' Theorem:

$$P(D \mid ++) = \frac{P(D)P(++ \mid D)}{P(D)P(++ \mid D) + P(D')P(++ \mid D')} = \frac{(.05)(.9604)}{(.05)(.9604) + (.95)(.0001)} = .9980$$

89. The question asks for P(<u>exactly</u> one tag lost | at <u>most</u> one tag lost) = $P((C_1 \cap C_2') \cup (C_1' \cap C_2) \mid (C_1 \cap C_2)')$. Since the first event is contained in (a subset of) the second event, this equals

$$\frac{P((C_1 \cap C_2') \cup (C_1' \cap C_2))}{P((C_1 \cap C_2)')} = \frac{P(C_1 \cap C_2') + P(C_1' \cap C_2)}{1 - P(C_1 \cap C_2)} = \frac{P(C_1)P(C_2') + P(C_1')P(C_2)}{1 - P(C_1)P(C_2)}$$ by independence =

$$\frac{\pi(1-\pi) + (1-\pi)\pi}{1-\pi^2} = \frac{2\pi(1-\pi)}{1-\pi^2} = \frac{2\pi}{1+\pi}.$$

Supplementary Exercises

90.

a. $\binom{10}{3}$ = 120.

b. There are 9 other senators from whom to choose the other two subcommittee members, so the answer is 1 × $\binom{9}{2}$ = 36.

c. There are 120 possible subcommittees. Among those, the number which would include <u>none</u> of the 5 most senior senators (i.e., all 3 members are chosen from the 5 most junior senators) is $\binom{5}{3}$ = 10.

Hence, the number of subcommittees with <u>at least one</u> senior senator is 120 – 10 = 110, and the chance of this randomly occurring is 110/120 = .9167.

d. The number of subcommittees that can form from the 8 "other" senators is $\binom{8}{3}$ = 56, so the probability of this event is 56/120 = .4667.

91.

a. P(line 1) = $\frac{500}{1500}$ = .333;

P(crack) = $\frac{.50(500)+.44(400)+.40(600)}{1500} = \frac{666}{1500}$ = .444.

b. This is one of the percentages provided: P(blemish | line 1) = .15.

c. P(surface defect) = $\frac{.10(500)+.08(400)+.15(600)}{1500} = \frac{172}{1500}$;

P(line 1 $\cap$ surface defect) = $\frac{.10(500)}{1500} = \frac{50}{1500}$;

so, P(line 1 | surface defect) = $\frac{50/1500}{172/1500} = \frac{50}{172}$ = .291.

92.

a. He will have one type of form left if either 4 withdrawals or 4 course substitutions remain. This means the first six were either 2 withdrawals and 4 subs or 6 withdrawals and 0 subs; the desired probability is $\frac{\binom{6}{2}\binom{4}{4}+\binom{6}{6}\binom{4}{0}}{\binom{10}{6}} = \frac{16}{210}$ =.0762.

b. He can start with the withdrawal forms or the course substitution forms, allowing two sequences: W-C-W-C or C-W-C-W. The number of ways the first sequence could arise is (6)(4)(5)(3) = 360, and the number of ways the second sequence could arise is (4)(6)(3)(5) = 360, for a total of 720 such possibilities. The total number of ways he could select four forms one at a time is $P_{4,10}$ = (10)(9)(8)(7) = 5040. So, the probability of a perfectly alternating sequence is 720/5040 = .143.

93. Apply the addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow .626 = P(A) + P(B) - .144$. Apply independence: $P(A \cap B) = P(A)P(B) = .144$.
So, $P(A) + P(B) = .770$ and $P(A)P(B) = .144$.
Let $x = P(A)$ and $y = P(B)$. Using the first equation, $y = .77 - x$, and substituting this into the second equation yields $x(.77 - x) = .144$ or $x^2 - .77x + .144 = 0$. Use the quadratic formula to solve:

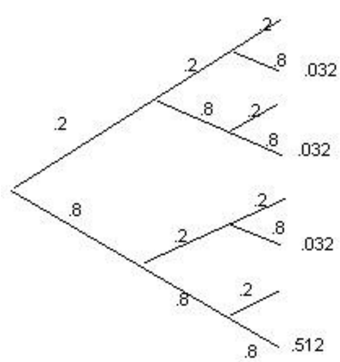
$$x = \frac{.77 \pm \sqrt{(-.77)^2 - (4)(1)(.144)}}{2(1)} = \frac{.77 \pm .13}{2}$$

= .32 or .45. Since $x = P(A)$ is assumed to be the larger probability, $x = P(A) = .45$ and $y = P(B) = .32$.

94. The probability of a bit reversal is .2, so the probability of maintaining a bit is .8.

a. Using independence, P(all three relays correctly send 1) = (.8)(.8)(.8) = .512.

b. In the accompanying tree diagram, each .2 indicates a bit reversal (and each .8 its opposite). There are several paths that maintain the original bit: no reversals or exactly two reversals (e.g., 1 → 1 → 0 → 1, which has reversals at relays 2 and 3). The total probability of these options is .512 + (.8)(.2)(.2) + (.2)(.8)(.2) + (.2)(.2)(.8) = .512 + 3(.032) = .608.



c. Using the answer from **b**, $P(1 \text{ sent} \mid 1 \text{ received}) = \frac{P(1 \text{ sent} \cap 1 \text{ received})}{P(1 \text{ received})} =$

$$\frac{P(1 \text{ sent})P(1 \text{ received} \mid 1 \text{ sent})}{P(1 \text{ sent})P(1 \text{ received} \mid 1 \text{ sent}) + P(0 \text{ sent})P(1 \text{ received} \mid 0 \text{ sent})} = \frac{(.7)(.608)}{(.7)(.608) + (.3)(.392)} = \frac{.4256}{.5432} =$$
.7835.

In the denominator, $P(1 \text{ received} \mid 0 \text{ sent}) = 1 - P(0 \text{ received} \mid 0 \text{ sent}) = 1 - .608$, since the answer from **b** also applies to a 0 being relayed as a 0.

95.

a. There are 5! = 120 possible orderings, so $P(\text{BCDEF}) = \frac{1}{120} = .0833$.

b. The number of orderings in which F is third equals 4×3×1*×2×1 = 24 (*because F must be here), so $P(\text{F is third}) = \frac{24}{120} = .2$. Or more simply, since the five friends are ordered completely at random, there is a ⅕ chance F is specifically in position three.

c. Similarly, $P(\text{F last}) = \dfrac{4\times3\times2\times1\times1}{120} = .2$.

d. P(F hasn't heard after 10 times) = P(not on #1 ∩ not on #2 ∩ … ∩ not on #10) = $\frac{4}{5}\times\cdots\times\frac{4}{5} = \left(\frac{4}{5}\right)^{10}$ = .1074.

96. Palmberg equation: $P_d(c) = \dfrac{(c/c^*)^\beta}{1+(c/c^*)^\beta}$

a. $P_d(c^*) = \dfrac{(c^*/c^*)^\beta}{1+(c^*/c^*)^\beta} = \dfrac{1^\beta}{1+1^\beta} = \dfrac{1}{1+1} = .5$.

b. The probability of detecting a crack that is twice the size of the "50-50" size c^* equals $P_d(2c^*) = \dfrac{(2c^*/c^*)^\beta}{1+(2c^*/c^*)^\beta} = \dfrac{2^\beta}{1+2^\beta}$. When $\beta = 4$, $P_d(2c^*) = \dfrac{2^4}{1+2^4} = \dfrac{16}{17} = .9412$.

c. Using the answers from **a** and **b**, P(exactly one of two detected) = P(first is, second isn't) + P(first isn't, second is) = (.5)(1 – .9412) + (1 – .5)(.9412) = .5.

d. If $c = c^*$, then $P_d(c) = .5$ irrespective of β. If $c < c^*$, then $c/c^* < 1$ and $P_d(c) \to \dfrac{0}{0+1} = 0$ as $\beta \to \infty$. Finally, if $c > c^*$ then $c/c^* > 1$ and, from calculus, $P_d(c) \to 1$ as $\beta \to \infty$.

97. When three experiments are performed, there are 3 different ways in which detection can occur on exactly 2 of the experiments: (i) #1 and #2 and not #3; (ii) #1 and not #2 and #3; and (iii) not #1 and #2 and #3. If the impurity is present, the probability of exactly 2 detections in three (independent) experiments is (.8)(.8)(.2) + (.8)(.2)(.8) + (.2)(.8)(.8) = .384. If the impurity is absent, the analogous probability is 3(.1)(.1)(.9) = .027. Thus, applying Bayes' theorem, P(impurity is present | detected in exactly 2 out of 3) $= \dfrac{P(\text{detected in exactly 2} \cap \text{present})}{P(\text{detected in exactly 2})} = \dfrac{(.384)(.4)}{(.384)(.4)+(.027)(.6)} = .905$.

98. Our goal is to find $P(A \cup B \cup C \cup D \cup E)$. We'll need all of the following probabilities:

$P(A) = P$(Allison gets her calculator back) = 1/5. This is intuitively obvious; you can also see it by writing out the 5! = 120 orderings in which the friends could get calculators (ABCDE, ABCED, …, EDCBA) and observe that 24 of the 120 have A in the first position. So, $P(A) = 24/120 = 1/5$. By the same reasoning, $P(B) = P(C) = P(D) = P(E) = 1/5$.

$P(A \cap B) = P$(Allison and Beth get their calculators back) = 1/20. This can be computed by considering all 120 orderings and noticing that six — those of the form ABxyz — have A and B in the correct positions. Or, you can use the multiplication rule: $P(A \cap B) = P(A)P(B \mid A) = (1/5)(1/4) = 1/20$. All other pairwise intersection probabilities are also 1/20.

$P(A \cap B \cap C) = P$(Allison and Beth and Carol get their calculators back) = 1/60, since this can only occur if two ways — ABCDE and ABCED — and 2/120 = 1/60. So, all three-wise intersections have probability 1/60.

$P(A \cap B \cap C \cap D) = 1/120$, since this can only occur if all 5 girls get their own calculators back. In fact, all four-wise intersections have probability 1/120, as does $P(A \cap B \cap C \cap D \cap E)$ — they're the same event.

Finally, put all the parts together, using a general inclusion-exclusion rule for unions:

$$\begin{aligned}P(A\cup B\cup C\cup D\cup E) &= P(A)+P(B)+P(C)+P(D)+P(E)\\ &-P(A\cap B)-P(A\cap C)-\cdots-P(D\cap E)\\ &+P(A\cap B\cap C)+\cdots+P(C\cap D\cap E)\\ &-P(A\cap B\cap C\cap D)-\cdots-P(B\cap C\cap D\cap E)\\ &+P(A\cap B\cap C\cap D\cap E)\end{aligned}$$

$$=5\cdot\frac{1}{5}-10\cdot\frac{1}{20}+10\cdot\frac{1}{60}-5\cdot\frac{1}{120}+\frac{1}{120}=1-\frac{1}{2}+\frac{1}{6}-\frac{1}{24}+\frac{1}{120}=\frac{76}{120}=.633$$

The final answer has the form $1-\frac{1}{2}+\frac{1}{6}-\frac{1}{24}=\frac{1}{1!}-\frac{1}{2!}+\frac{1}{3!}-\frac{1}{4!}+\frac{1}{5!}$. Generalizing to n friends, the probability at least one will get her own calculator back is $\frac{1}{1!}-\frac{1}{2!}+\frac{1}{3!}-\frac{1}{4!}+\cdots+(-1)^{n-1}\frac{1}{n!}$.

When n is large, we can relate this to the power series for e^x evaluated at $x = -1$:

$$e^x=\sum_{k=0}^{\infty}\frac{x^k}{k!}=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\Rightarrow$$

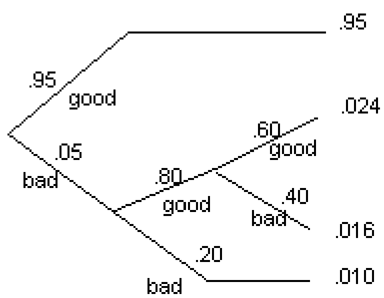
$$e^{-1}=1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots=1-\left[\frac{1}{1!}-\frac{1}{2!}+\frac{1}{3!}-\cdots\right]\Rightarrow$$

$$1-e^{-1}=\frac{1}{1!}-\frac{1}{2!}+\frac{1}{3!}-\cdots$$

So, for large n, P(at least one friend gets her own calculator back) $\approx 1 - e^{-1} = .632$. Contrary to intuition, the chance of this event does not converge to 1 (because "someone is bound to get hers back") or to 0 (because "there are just too many possible arrangements"). Rather, in a large group, there's about a 63.2%

chance someone will get her own item back (a match), and about a 36.8% chance that nobody will get her own item back (no match).

99. Refer to the tree diagram below.



a. P(pass inspection) = P(pass initially $\cup$ passes after recrimping) =
P(pass initially) + P(fails initially $\cap$ goes to recrimping $\cap$ is corrected after recrimping) =
.95 + (.05)(.80)(.60) (following path "bad-good-good" on tree diagram) = .974.

b. P(needed no recrimping | passed inspection) = $\dfrac{P(\text{passed initially})}{P(\text{passed inspection})} = \dfrac{.95}{.974} = .9754$.

100.

a. First, the probabilities of the A_i are $P(A_1) = P(JJ) = (.6)^2 = .36$; $P(A_2) = P(MM) = (.4)^2 = .16$; and $P(A_3) = P(JM \text{ or } MJ) = (.6)(.4) + (.4)(.6) = .48$.
Second, P(Jay wins | A_1) = 1, since Jay is two points ahead and, thus has won; P(Jay wins | A_2) = 0, since Maurice is two points ahead and, thus, Jay has lost; and P(Jay wins | A_3) = p, since at that moment the score has returned to deuce and the game has effectively started over. Apply the law of total probability:

$$P(\text{Jay wins}) = P(A_1)P(\text{Jay wins} \mid A_1) + P(A_2)P(\text{Jay wins} \mid A_2) + P(A_3)P(\text{Jay wins} \mid A_3)$$
$$p = (.36)(1) + (.16)(0) + (.48)(p)$$

Therefore, $p = .36 + .48p$; solving for p gives $p = \dfrac{.36}{1-.48} = .6923$.

b. Apply Bayes' rule: $P(JJ \mid \text{Jay wins}) = \dfrac{P(JJ)P(\text{Jay wins} \mid JJ)}{P(\text{Jay wins})} = \dfrac{(.36)(1)}{.6923} = .52$.

101. Let A = 1st functions, B = 2nd functions, so $P(B) = .9$, $P(A \cup B) = .96$, $P(A \cap B)=.75$. Use the addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow .96 = P(A) + .9 - .75 \Rightarrow P(A) = .81$.

Therefore, $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{.75}{.81} = .926$.

102.

a. $P(F) = 919/2026 = .4536$. $P(C) = 308/2026 = .1520$.

b. $P(F \cap C) = 110/2026 = .0543$. Since $P(F) \times P(C) = .4536 \times .1520 = .0690 \neq .0543$, we find that events F and C are not independent.

c. $P(F \mid C) = P(F \cap C)/P(C) = 110/308 = .3571$.

d. $P(C \mid F) = P(C \cap F)/P(F) = 110/919 = .1197$.

e. Divide each of the two rows, Male and Female, by its row total.

	Blue	**Brown**	**Green**	**Hazel**
Male	.3342	.3180	.1789	.1689
Female	.3906	.3156	.1197	.1741

According to the data, brown and hazel eyes have similar likelihoods for males and females. However, females are much more likely to have blue eyes than males (39% versus 33%) and, conversely, males have a much greater propensity for green eyes than do females (18% versus 12%).

103. A tree diagram can help here.

a. $P(E_1 \cap L) = P(E_1)P(L \mid E_1) = (.40)(.02) = .008$.

b. The law of total probability gives $P(L) = \sum P(E_i)P(L \mid E_i) = (.40)(.02) + (.50)(.01) + (.10)(.05) = .018$.

c. $P(E_1' \mid L') = 1 - P(E_1 \mid L') = 1 - \dfrac{P(E_1 \cap L')}{P(L')} = 1 - \dfrac{P(E_1)P(L' \mid E_1)}{1 - P(L)} = 1 - \dfrac{(.40)(.98)}{1 - .018} = .601$.

104. Let B denote the event that a component needs rework. By the law of total probability, $P(B) = \sum P(A_i)P(B \mid A_i) = (.50)(.05) + (.30)(.08) + (.20)(.10) = .069$.
Thus, $P(A_1 \mid B) = \dfrac{(.50)(.05)}{.069} = .362$, $P(A_2 \mid B) = \dfrac{(.30)(.08)}{.069} = .348$, and $P(A_3 \mid B) = .290$.

105. This is the famous "Birthday Problem" in probability.

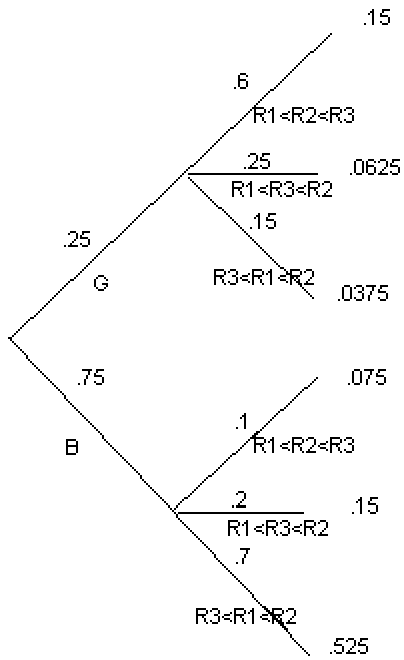
a. There are 365^{10} possible lists of birthdays, e.g. (Dec 10, Sep 27, Apr 1, …). Among those, the number with zero matching birthdays is $P_{10,365}$ (sampling ten birthdays without replacement from 365 days. So, $P(\text{all different}) = \dfrac{P_{10,365}}{365^{10}} = \dfrac{(365)(364)\cdots(356)}{(365)^{10}} = .883$. $P(\text{at least two the same}) = 1 - .883 = .117$.

b. The general formula is P(at least two the same) = 1 – $\frac{P_{k,365}}{365^k}$. By trial and error, this probability equals .476 for k = 22 and equals .507 for k = 23. Therefore, the smallest k for which k people have at least a 50-50 chance of a birthday match is 23.

c. There are 1000 possible 3-digit sequences to end a SS number (000 through 999). Using the idea from **a**, P(at least two have the same SS ending) = 1 – $\frac{P_{10,1000}}{1000^{10}}$ = 1 – .956 = .044.

Assuming birthdays and SS endings are independent, P(at least one "coincidence") = P(birthday coincidence $\cup$ SS coincidence) = .117 + .044 – (.117)(.044) = .156.

106. See the accompanying tree diagram.



a. $P(G \mid R_1 < R_2 < R_3) = \dfrac{.15}{.15 + .075} = .67$ while $P(B \mid R_1 < R_2 < R_3) = .33$, so classify the specimen as granite. Equivalently, $P(G \mid R_1 < R_2 < R_3) = .67 > ½$ so granite is more likely.

b. $P(G \mid R_1 < R_3 < R_2) = \dfrac{.0625}{.2125} = .2941 < ½$, so classify the specimen as basalt.

$P(G \mid R_3 < R_1 < R_2) = \dfrac{.0375}{.5625} = .0667 < ½$, so classify the specimen as basalt.

c. P(erroneous classification) = P(B classified as G) + P(G classified as B) =
$P(B)P$(classified as G | B) + $P(G)P$(classified as B | G) =
$(.75)P(R_1 < R_2 < R_3 \mid B) + (.25)P(R_1 < R_3 < R_2 \text{ or } R_3 < R_1 < R_2 \mid G) =$
$(.75)(.10) + (.25)(.25 + .15) = .175$.

d. For what values of p will $P(G \mid R_1 < R_2 < R_3)$, $P(G \mid R_1 < R_3 < R_2)$, and $P(G \mid R_3 < R_1 < R_2)$ all exceed ½? Replacing .25 and .75 with p and $1 - p$ in the tree diagram,

$P(G \mid R_1 < R_2 < R_3) = \dfrac{.6p}{.6p + .1(1-p)} = \dfrac{.6p}{.1 + .5p} > .5$ iff $p > \dfrac{1}{7}$;

$P(G \mid R_1 < R_3 < R_2) = \dfrac{.25p}{.25p + .2(1-p)} > .5$ iff $p > \dfrac{4}{9}$;

$P(G \mid R_3 < R_1 < R_2) = \dfrac{.15p}{.15p + .7(1-p)} > .5$ iff $p > \dfrac{14}{17}$ (most restrictive). Therefore, one would always classify a rock as granite iff $p > \dfrac{14}{17}$.

107. P(detection by the end of the nth glimpse) = 1 – P(not detected in first n glimpses) =

$1 - P(G_1' \cap G_2' \cap \cdots \cap G_n') = 1 - P(G_1')P(G_2')\cdots P(G_n')$ = 1 – (1 – p_1)(1 – p_2) … (1 – p_n) = $1 - \prod_{i=1}^{n}(1 - p_i)$.

108.

a. P(walks on 4th pitch) = P(first 4 pitches are balls) = $(.5)^4$ = .0625.

b. P(walks on 6th pitch) = P(2 of the first 5 are strikes ∩ #6 is a ball) =
P(2 of the first 5 are strikes)P(#6 is a ball) = $\binom{5}{2}(.5)^2(.5)^3(.5)$ = .15625.

c. Following the pattern from **b**, P(walks on 5th pitch) = $\binom{4}{1}(.5)^1(.5)^3(.5)$ = .125. Therefore, P(batter walks) = P(walks on 4th) + P(walks on 5th) + P(walks on 6th) =
.0625 + .125 + .15625 = .34375.

d. P(first batter scores while no one is out) = P(first four batters all walk) = $(.34375)^4$ = .014.

109.

a. P(all in correct room) = $\frac{1}{4!} = \frac{1}{24}$ = .0417.

b. The 9 outcomes which yield completely incorrect assignments are: 2143, 2341, 2413, 3142, 3412, 3421, 4123, 4321, and 4312, so P(all incorrect) = $\frac{9}{24}$ = .375.

110.

a. P(all full) = $P(A \cap B \cap C)$ = (.9)(.7)(.8) = .504.
P(at least one isn't full) = 1 – P(all full) = 1 – .504 = .496.

b. P(only NY is full) = $P(A \cap B' \cap C') = P(A)P(B')P(C')$ = (.9)(1–.7)(1–.8) = .054.
Similarly, P(only Atlanta is full) = .014 and P(only LA is full) = .024.
So, P(exactly one full) = .054 + .014 + .024 = .092.

111. Note: $s = 0$ means that the very first candidate interviewed is hired. Each entry below is the candidate hired for the given policy and outcome.

Outcome	$s = 0$	$s = 1$	$s = 2$	$s = 3$	Outcome	$s = 0$	$s = 1$	$s = 2$	$s = 3$
				4	3124				
1234	1	4	4	3	3142	3	1	4	4
1243	1	3	3	4	3214	3	1	4	2
1324	1	4	4	2	3241	3	2	1	4
1342	1	2	2	3	3412	3	2	1	1
1423	1	3	3	2	3421	3	1	1	2
1432	1	2	2	4	4123	3	2	2	1
2134	2	1	4	3	4132	4	1	3	3
2143	2	1	3	4	4213	4	1	2	2
2314	2	1	1	1	4231	4	2	1	3
2341	2	1	1	3	4312	4	2	1	1
2413	2	1	1	1	4321	4	3	1	2
2431	2	1	1			4	3	2	1

From the table, we derive the following probability distribution based on s:

s	0	1	2	3
P(hire #1)	$\frac{6}{24}$	$\frac{11}{24}$	$\frac{10}{24}$	$\frac{6}{24}$

Therefore $s = 1$ is the best policy.

112. P(at least one occurs) = $1 - P$(none occur) = $1 - (1 - p_1)(1 - p_2)(1 - p_3)(1 - p_4)$.
P(at least two occur) = $1 - P$(none or exactly one occur) =
$1 - [(1 - p_1)(1 - p_2)(1 - p_3)(1 - p_4) + p_1(1 - p_2)(1 - p_3)(1 - p_4) + (1 - p_1)\, p_2(1 - p_3)(1 - p_4) +$
$(1 - p_1)(1 - p_2)p_3(1 - p_4) + (1 - p_1)(1 - p_2)(1 - p_3)p_4]$.

113. $P(A_1) = P$(draw slip 1 or 4) = ½; $P(A_2) = P$(draw slip 2 or 4) = ½;
$P(A_3) = P$(draw slip 3 or 4) = ½; $P(A_1 \cap A_2) = P$(draw slip 4) = ¼;
$P(A_2 \cap A_3) = P$(draw slip 4) = ¼; $P(A_1 \cap A_3) = P$(draw slip 4) = ¼.
Hence $P(A_1 \cap A_2) = P(A_1)P(A_2) = ¼$; $P(A_2 \cap A_3) = P(A_2)P(A_3) = ¼$; and
$P(A_1 \cap A_3) = P(A_1)P(A_3) = ¼$. Thus, there exists pairwise independence. However,
$P(A_1 \cap A_2 \cap A_3) = P$(draw slip 4) = $¼ \neq ⅛ = P(A_1)P(A_2)P(A_3)$, so the events are not mutually independent.

114. $$P(A_1 | A_2 \cap A_3) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_2 \cap A_3)} = \frac{P(A_1)P(A_2)P(A_3)}{P(A_2)P(A_3)} = P(A_1).$$