

**Solution Manual for Signals and Systems 1st  
Edition Mahmood Nahvi 0073380709  
9780073380704**

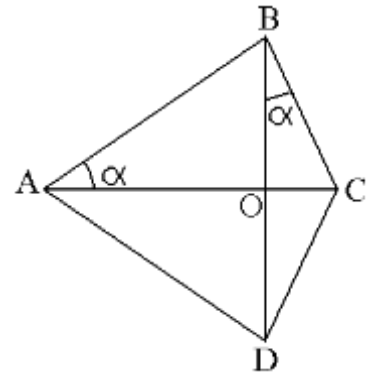
Fulllink download

Solution Manual: <https://testbankpack.com/p/solution-manual-for-signals-and-systems-1st-edition-mahmood-nahvi-0073380709-9780073380704/>

Solution Chapter Problems.  
 Mahmood Nahvi, July 2013

### Chapter Problems

**Problem 139** The four corners of a quadrilateral in the plane are labeled in clockwise direction as  $A, B, C, D$ . It is given that  $\angle ABC = \angle ADC = 90^\circ$  and  $AC \perp BD$ . The intersection of  $AC$  and  $BD$  is labeled  $O$ . Show that  $\sin^2 \alpha = \frac{OB}{OC}$  and  $\sin^2 \alpha = \frac{OD}{OA}$ .



**Solution.**

From right triangle  $ABC$  and right triangle  $ADC$  we have  $\angle BAC = \angle DAC$ . The angles  $\angle BAC$  and  $\angle OBC$  are equal. From right triangle  $BOC$  we obtain  $\sin^2 \alpha = \frac{OB}{OC}$ . From right triangle  $DOA$  we obtain  $\sin^2 \alpha = \frac{OD}{OA}$ .

The magnitude,  $f$  is the frequency (in Hz),  $\phi$  is the phase in radians. Determine the amplitude,  $A$ , and the phase,  $\phi$ , in the above form.

**Problem 140** The expression  $A \cos(2\pi f t + \phi)$  where  $A$  is the amplitude,  $f$  is the frequency,  $t$  is the time, and  $\phi$  is the phase in radians, represents a sinusoidal signal. Determine the amplitude,  $A$ , and the phase,  $\phi$ , in the following signals when represented in the above form.

- 1)  $3 \cos(3t + 45^\circ)$       2)  $5 \sin(\sqrt{2}t + 120^\circ)$       3)  $2 \cos(5t + 180^\circ)$       4)  $3 \cos(\pi t + 10^\circ)$       5)  $4 \cos(2\pi t + \pi/3)$       6)  $5 \sin(6.28t + 2\pi/3)$       7)  $2 \cos(\pi t - 0.5)$

| Solution. | signal  | $A$ | $f$                              | $\phi$      |
|-----------|---|-----|----------------------------------|-------------|
| 1)        | $3 \cos(3t + 45^\circ)$   | 3   | $\frac{3}{2\pi} = 0.4775$        | $45^\circ$  |
| 2)        | $5 \sin(\sqrt{2}t + 120^\circ) = 5 \cos(\sqrt{2}t + 150^\circ)$ | 5   | $\frac{\sqrt{2}}{2\pi} = 0.2251$ | $150^\circ$ |
| 3)        | $2 \cos(5t + 180^\circ) = 2 \cos(5t)$                           | 2   | $\frac{5}{2\pi} = 0.7958$        | $180^\circ$ |
| 4)        | $3 \cos(\pi t + 10^\circ)$                                      | 3   | $\frac{1}{2} = 0.5$              | $10^\circ$  |
| 5)        | $4 \cos(2\pi t + \pi/3) = 4 \cos(2\pi t)$                       | 4   | 1                                | $120^\circ$ |
| 6)        | $5 \sin(6.28t + 2\pi/3) = 5 \cos(6.28t - \pi/6)$                | 5   | 1                                | $330^\circ$ |
| 7)        | $2 \cos(\pi t - 0.5) = 2 \cos(\pi t)$                           | 2   | 0.5                              | $0^\circ$   |

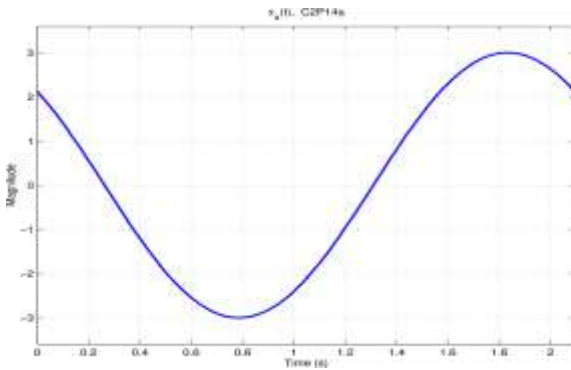
$$i) \quad \sin(6.28t - 2\pi/3) = \cos(6.28t - \pi/6) \quad \square$$

$$ii) \quad 0.2(t - 1) = \cos(0.2t - 11.46^\circ) \quad \square$$

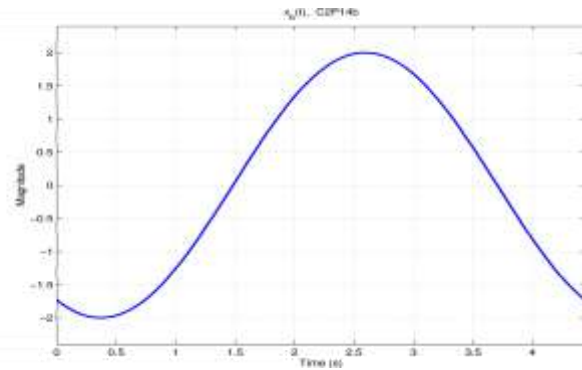
$$\frac{3.14}{\pi} = 0.9995 \quad \square$$

$$\frac{1}{10\pi} = 0.0318 \quad \square \quad 11.46^\circ$$

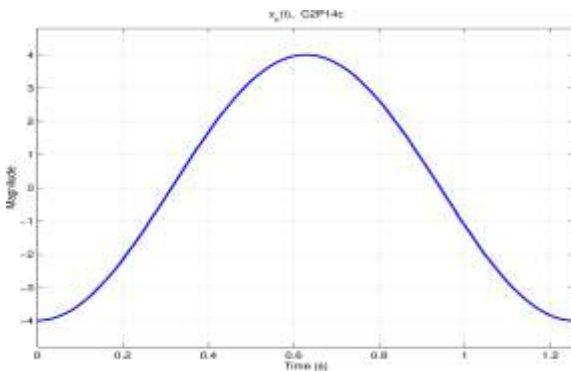
Problem 15 Sketch and label signals given in Problem 10



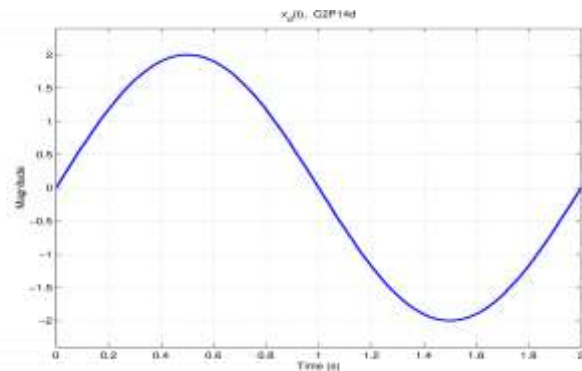
$x_a(t)$



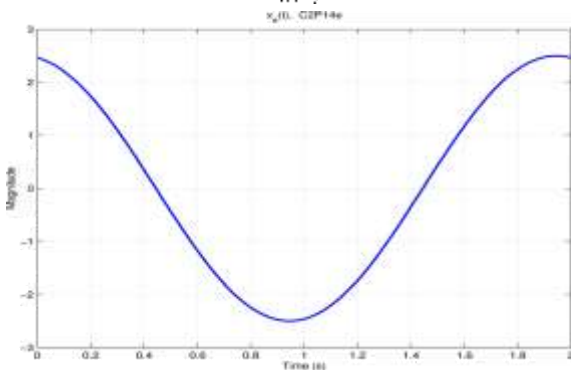
$x_b(t)$



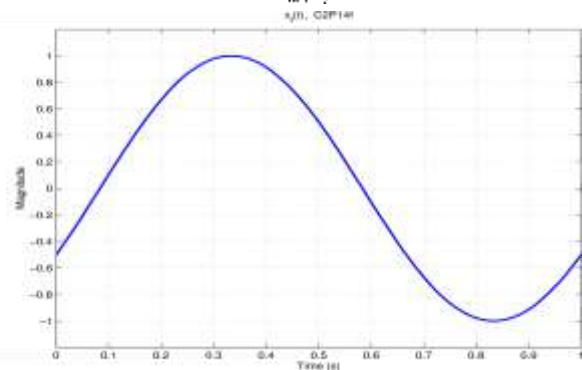
$x_c(t)$



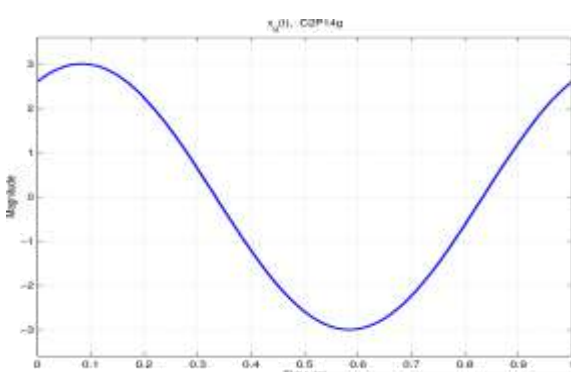
$x_d(t)$



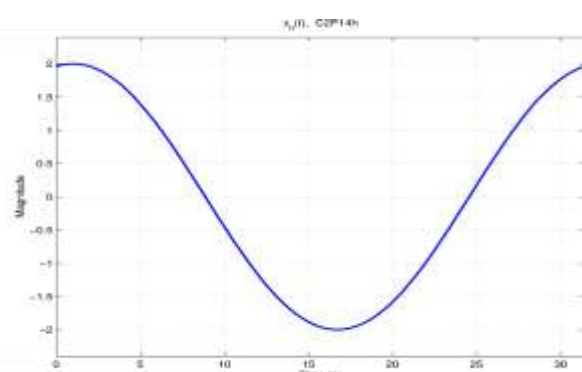
$x_e(t)$



$x_f(t)$



$x_g(t)$



$x_h(t)$

Problems 16-21. Identify the correct frequency given sinusoids.

Problem 169 The frequency of  $\cos 10^6 t$  is

- MHz
- kHz
- MHz
- kHz
- kHz
- above.

Problem 170 The frequency of  $\sin(\pi t/6)$  is

- 1/6 Hz
- 1/12 Hz
- 1/3 Hz
- 2/π Hz
- none of the above.

Problem 171 The frequency of  $\cos(3000\pi t)$  is

- kHz
- kHz
- kHz
- kHz
- kHz
- none of the above.

Problem 172 The frequency of  $\cos 10^6 t$  must nearly

- kHz
- kHz
- kHz
- kHz
- kHz
- none of the above.

Problem 173 The frequency of  $\sin(t/6)$  is

- Hz
- Hz
- Hz
- Hz
- Hz
- none of the above.

Problem 174 The frequency of  $\cos 10^3 t$  must nearly

- π/3 Hz
- 1/(12π) Hz
- 1/12 Hz
- π/3 Hz
- none of the above.

Problem 175 The frequency of  $\cos 10^6 t$  must nearly

- kHz
- kHz
- kHz
- kHz
- kHz
- none of the above.

Problems 22-31. Identify the period of the given signals.

Problem 229 The period of  $\cos 10^6 t$  is

- μs
- ns
- μs
- μs
- μs
- none of the above.

Problem 230 The period of  $\cos(2 \times 10^6 \pi t)$  is

- μs
- μs
- μs
- μs
- μs
- none of the above.

Problem 231 The period of  $\cos 20\pi t$  is

- s
- s
- s
- s
- s
- none of the above.

Problem 232 The period of  $\cos 10^6 t$  is

- μs
- μs
- μs
- μs
- μs
- none of the above.

Problem 233 The period of  $\cos 20\pi t$  is

- s
- μs
- μs
- μs
- μs
- none of the above.

Problem 234 The period of  $\cos 10^6 t$  is

- 3π/2 s
- μs
- s
- μs
- μs
- none of the above.

21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65  
66  
67  
68  
69  
70  
71  
72  
73  
74  
75  
76  
77  
78  
79  
80  
81  
82  
83  
84  
85  
86  
87  
88  
89  
90  
91  
92  
93  
94  
95  
96  
97  
98  
99  
100

Problem 239 The period is

a)  $\frac{1}{5} \text{ s}$   
b)  $\frac{1}{3} \text{ s}$   
c)  $(\frac{1}{5} + \frac{1}{3})2\pi \text{ s}$   
d)  $\frac{3\pi}{5}$   
e)  $\frac{2\pi}{5}$  above.

Problem 240 The period is

a)  $\frac{2\pi}{3} \text{ s}$   
b)  $\pi \text{ s}$   
c)  $\frac{\pi}{2}$   
d)  $\frac{\pi}{3}$   
e)  $\frac{2\pi}{5}$

The period is

a)  $\frac{1}{5} \text{ s}$   
b)  $\frac{1}{3} \text{ s}$   
c)  $(\frac{1}{5} + \frac{1}{3})2\pi \text{ s}$   
d)  $\frac{3\pi}{5}$   
e)  $\frac{2\pi}{5}$  above.

**Problem 310** The period  $2.1\pi$  s  
 is  $1.99\pi$  s  
 above.

**Problem 311** The period  $200\pi$  s  
 is  $201\pi$  s  
 above.

correct phase  $\pi$  following sinusoids when expressed in form  $\cos(\omega t + \theta)$ .

**Problems 32-37.** Identify

**Problem 32** The phase  $\cos(\pi t + 30^\circ)$   
 is  $30$  degrees  
 radians  
 radians  
 degrees  
 above.

**Problem 33** The phase  $\sin(2\pi t + 30^\circ)$   
 is  $30$  degrees  
 degrees  
 degrees  
 above.  
 The phase  $\sin(\pi t + \pi)$  must nearly  
 be  $\pi$  radians  
 radians  
 degrees  
 above.

**Problem 34** The phase  $\cos(\pi t + \pi)$   
 is  $180$  degrees  
 radians  
 radians  
 degrees  
 above.

**Problem 35** The phase  $\cos(500\pi(t + 10^{-3}))$   
 is  $180$  degrees  
 radians  
 radians  
 degrees  
 above.

**Problem 36** The phase  $\cos(\pi t + 0.5)$   
 is  $0.5$  degrees  
 radians  
 radians  
 degrees  
 above.

**Problem 37** The phase  $\cos(\pi t + \pi)$   
 is  $180$  degrees  
 radians  
 radians  
 degrees  
 above.

**Problems 38-45.** Determine the correct phase relationship between sinusoids given below.

**Problem 38**  $x_1 = \cos(t + 10^\circ)$ ,  $x_2 = \cos(t - 30^\circ)$   
 leads  $40$  degrees  
 lags  $40$  degrees  
 leads  $20$  degrees  
 lags  $20$  degrees  
 above.

**Problem 39**  $x_1 = \cos(t - 30^\circ)$ ,  
 $x_2 = \cos(t + 30^\circ)$   
 leads  $60$  degrees  
 lags  $60$  degrees  
 leads  $30$  degrees  
 lags  $30$  degrees  
 above.

**Problem 40**  $x_1 = \cos(t + 10^\circ)$ ,  $x_2 = \sin(t)$   
 leads  $10$  degrees  
 lags  $10$  degrees  
 leads  $80$  degrees  
 lags  $80$  degrees  
 above.

10.  $\cos t$  leads  $\sin t$  by  $90^\circ$ .

11.  $\sin t$  lags  $\cos t$  by  $90^\circ$ .

Problem 12.  $\sin t$  lags  $\cos t$  by  $90^\circ$ .

13.  $\cos t$  leads  $\sin t$  by  $90^\circ$ .

14.  $\sin t$  lags  $\cos t$  by  $90^\circ$ .

15.  $\cos t$  leads  $\sin t$  by  $90^\circ$ .

16.  $\sin t$  lags  $\cos t$  by  $90^\circ$ .

17.  $\cos t$  lags  $\sin t$  by  $90^\circ$ .

Problem 18.  $\sin t$  lags  $\cos t$  by  $90^\circ$ .

19.  $\cos t$  leads  $\sin t$  by  $90^\circ$ .

20.  $\sin t$  lags  $\cos t$  by  $90^\circ$ .

21.  $\cos t$  leads  $\sin t$  by  $90^\circ$ .

22.  $\sin t$  lags  $\cos t$  by  $90^\circ$ .

23.  $\cos t$  leads  $\sin t$  by  $90^\circ$ .

Problem 24.  $\sin t$  lags  $\cos t$  by  $90^\circ$ .

25.  $\cos t$  leads  $\sin t$  by  $90^\circ$ .

26.  $\sin t$  lags  $\cos t$  by  $90^\circ$ .

27.  $\cos t$  leads  $\sin t$  by  $90^\circ$ .

28.  $\sin t$  lags  $\cos t$  by  $90^\circ$ .



Problem 119  $v_1 = \cos 10^4 t$ ,  $v_2 = \cos 10^4(t - 10 \mu\text{s})$

- a) lags  $10^4$  degrees
- b) lags almost  $10^4$  phase
- c) leads  $10^4$  degrees
- d) lags  $10^4$  degrees
- e) lags  $10^4$  radian.

11

Problem 120 phasors  $\rightarrow$  convert each time function given

- a)  $3 \cos(3t + 45^\circ)$   $\rightarrow$   $2 \sin(3t - 120^\circ)$
- b)  $2 \cos(\pi t + 10^\circ) \cdot \cos(\pi t + \pi/3)$

Solution. The answers are

- a)  $3.1455 \cos(3t + 82.89^\circ)$
- b)  $2.0090 \cos(\pi t + 12.41^\circ)$

The following

Matlab code was executed to obtain the answers.

```

r1=3; p1=45*pi/180; v1=r1*exp(i*p1);
r2=2; p2=-120*pi/180-pi/2; v2=r2*exp(i*p2);
A1=abs(v1+v2);
P1=180*angle(v1+v2)/pi;
#

r1=-4; p1=0; v1=r1*exp(i*p1);
r2=2; p2=-2*pi/3; v2=r2*exp(i*p2);
A2=abs(v1+v2);
P2=180*angle(v1+v2)/pi;
#

r1=2; p1=10*pi/180; v1=r1*exp(i*p1);
r2=-1; p2=pi/3; v2=r2*exp(i*p2);
A3=abs(v1+v2);
P3=180*angle(v1+v2)/pi;
#

r1=-3; p1=-2*pi/3-pi/2; v1=r1*exp(i*p1);
r2=2; p2=-3*pi/2; v2=r2*exp(i*p2);
A4=abs(v1+v2);
P4=180*angle(v1+v2)/pi;
#

```

A1 = 3.1455, P1 = 82.89, A2 = 2.0090, P2 = 12.41, A3 = 1.6178, P3 = 68.26, A4 = 5.7287, P4 = 167.93

1455 7287 0090 6178  
8912 -167 9399 4147 3037

Problem 121  $v_1 = \cos 10^6 t$ ,  $v_2 = \cos 10^6(t - 1 \mu\text{s})$

- a) leads  $10^6$  degrees
- b) leads  $10^6$  radian
- c) lags almost  $10^6$  phase
- d) lags  $10^6$  radian
- e) none of the above.

12

below form  $\cos(2\pi f t + \phi)$

- a)  $\cos(5t)$   $\rightarrow$   $5(t - 0.5)$
- b)  $\sin(6.28t - 2\pi/3)$   $\rightarrow$   $6.28(t - 0.5)$

13

167.93°

68.26°

b) 5.7287 cos(5t

d) 1.6178 cos(6.28t

**Problem 17.10** For each time function given below determine if periodic.

periodic or aperiodic, and specify the period

determine

**a**  $\cos(5t)$

$3.1416t$

**b**  $\cos(\pi t \cdot 10^\circ) \cdot \cos(2\pi t \cdot \pi/3)$

**d**  $\cos(5t)$   $\cos(0.2(t - \pi))$

**c**  $\sin(3t) \cos(3t)$

**e**  $\sin(6.28t \cdot 2\pi/3)$

**d**  $\sin(3.141592t)$

**f**  $\sin(1.14t) \cos(3.141592t)$

**g**  $\cos(1.14t) \cos(3.141592t)$

$= k_1 T_1$ . Equivalently,

**Solution.** If  $\frac{T_1}{T_2} = \frac{k_2}{k_1}$ , where  $k_1$  and  $k_2$  are integers,

then the sum is periodic with  $T = k_1 T_1$

$$\frac{k_1}{k_2} = \frac{\omega_1}{\omega_2} \Rightarrow \frac{2\pi}{\omega_1} = \frac{2\pi}{\omega_2}$$

|          |                     | $\frac{k_1}{k_2}$                               | Periodic?  | $\frac{2\pi}{\omega_1}$ | $\frac{2\pi}{\omega_2}$              |
|----------|---------------------|---|------------|-------------------------|--------------------------------------|
| <b>a</b> | $\cos(5t)$          | $\frac{5}{\pi}$                                 | <b>no</b>  |                         |                                      |
| <b>b</b> | $3.1416$            | $\frac{6250}{3927}$                             | <b>yes</b> | $\frac{2\pi}{5}$        | $\frac{2\pi}{3.1416} = 2500\pi$      |
| <b>c</b> | $\sin(3t) \cos(3t)$ | $\frac{\pi}{2\pi}$                              | <b>yes</b> | $\frac{2\pi}{\pi}$      | $\frac{2\pi}{2\pi}$                  |
| <b>d</b> | $6.28$ , $0.2$      | $\frac{6.28}{0.2} = \frac{157}{5}$              | <b>yes</b> | $\frac{2\pi}{6.28}$     | $\frac{2\pi}{0.2} = 10\pi$           |
| <b>e</b> | $\sin(3t) \cos(3t)$ | $\frac{1}{2\pi}$                                | <b>yes</b> | $\frac{2\pi}{\pi}$      | $\frac{2\pi}{2\pi}$                  |
| <b>f</b> | $\sin(3.141592t)$   | $\frac{\pi}{3.141592}$                          | <b>no</b>  |                         |                                      |
| <b>g</b> | $1.14$ , $3.141592$ | $\frac{1.14}{3.141592} = \frac{142500}{392699}$ | <b>yes</b> | $\frac{2\pi}{1.14}$     | $\frac{2\pi}{3.141592} = 250,000\pi$ |

**Problem 119** For each time function given below determine if it is periodic or aperiodic, and specify the period if periodic.

**a)**  $\cos 2t$  or  $6.28t$

**b)**  $\cos 250t$  or  $6.2816t$

**c)**  $\cos 250t$  or  $6.28159t$

**d)**  $\cos 6.2816t$  or  $6.28159t$

**e)**  $\cos \sqrt{2}t$  or  $1.41t$

**f)**  $\cos 1.4142t$  or  $1.41t$

**ans**      **ans**       $\frac{\omega_1}{\omega_2} = \frac{k_1}{k_2}$       Periodic?      **ans**       $\frac{2\pi}{\omega_1} = \frac{2\pi}{\omega_2}$

**a)**  $2\pi$        $6.28$        $\frac{2\pi}{6.28} = \frac{k_1}{k_2}$       **ans**

**b)**  $2\pi$        $6.2816$        $\frac{2\pi}{6.2816} = \frac{k_1}{k_2}$       **ans**

**c)**  $2\pi$        $6.28159$        $\frac{2\pi}{6.28159} = \frac{k_1}{k_2}$       **ans**

**d)**  $6.2816$        $6.28159$        $\frac{6.2816}{6.28159} = \frac{628160}{628159} = \frac{k_1}{k_2}$       **ans**      **ans**       $628159 \frac{2\pi}{6.28159} = 2 \times 10^5 \pi$   
 $628160 \frac{2\pi}{6.2816} =$

**e)**  $\sqrt{2}$        $1.41$        $\frac{\sqrt{2}}{1.41} = \frac{k_1}{k_2}$       **ans**

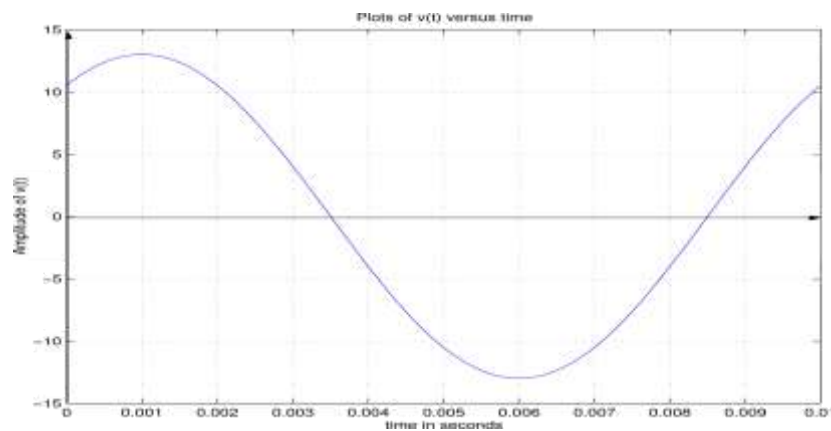
**f)**  $1.4142$        $1.41$        $\frac{1.4142}{1.41} = \frac{2357}{2350} = \frac{k_1}{k_2}$       **ans**      **ans**       $2350 \frac{2\pi}{1.41} = \frac{10^4 \pi}{3}$   
 $2357 \frac{2\pi}{1.4142}$

**Problem 100** For each time function given below determine if it is a) periodic, b) aperiodic, or c) whether more information is needed. Specify the period if periodic.

|    |       |         | $\frac{\omega_1}{\omega_2} = \frac{k_1}{k_2}$               | Periodic? | $\frac{2\pi}{\omega_1} = \frac{2\pi}{\omega_2}$          |
|----|-------|---------|---|-----------|--|
| a) | 3.14t | 3.1416t | $\frac{3.14}{3.1416} = \frac{3925}{3927} = \frac{k_1}{k_2}$ | no        | $\frac{2\pi}{3.14} = \frac{2\pi}{3.1416} = 2500\pi$      |
| b) | 3.14  | 3.1416  | $\frac{3.14}{3.1416} = \frac{3925}{3927} = \frac{k_1}{k_2}$ | yes       | $\frac{2\pi}{3.14} = \frac{2\pi}{3.1416} = 2500\pi$      |
| c) | no    | 100π    | $\frac{\pi}{100\pi} = \frac{1}{100} = \frac{k_1}{k_2}$      | yes       | $\frac{2\pi}{\pi} = \frac{2\pi}{100\pi} = \frac{2}{100}$ |
| d) | 6.28  | 1       | $\frac{6.28}{1} = \frac{157}{25} = \frac{k_1}{k_2}$         | yes       | $\frac{2\pi}{6.28} = \frac{2\pi}{1} = 100\pi$            |

**Problem 101** A sinusoidal voltage  $v(t)$  has a frequency of 100 Hz, a zero value, and a peak value of 15 V which reaches at  $t = 0$ . Write the equation for the function in time in cosine form and plot it for  $t$  from 0 to 0.01 seconds.

**Solution.**  $v(t) = 15 \cos(200\pi(t - 0.001) - \pi/5)$ .



$$v(t) = 15 \cos(200\pi t - \pi/5)$$

**Problems 102-104** For the following sinusoids determine the phase with reference to  $\cos(t)$ .

a)  $\cos(t + 30^\circ)$  and  $\cos(t - 10^\circ)$     b)  $\sin(t + 30^\circ)$  and  $\cos(t - 10^\circ)$

c)  $\cos(t)$  and  $\sin(t - 10^\circ)$     d)  $\sin(t + 30^\circ)$  and  $\sin(t - 10^\circ)$

and lead), e)  $130^\circ$ , f)  $50^\circ$

30°)

**Solution.** **A** 40°, **B** 50° (or

**Problem 592** Consider the sum

$$x(t) = \sum_{k=0}^{N-1} \cos(\omega t + \theta_k)$$

where  $\theta_k$  are values obtained above. The sinusoids having the same frequency but different phases  $x(t)$  is expressed as  $x(t) = \sum_{k=0}^{N-1} \cos(\omega t + \theta_k)$  where  $\theta_k = k\pi/4$ ,  $k = 0, 1, \dots, N-1$ . Write a Matlab program to plot  $x(t)$  for  $N = 500$  and  $t$  from 0 to 1. The following conditions are to be used:

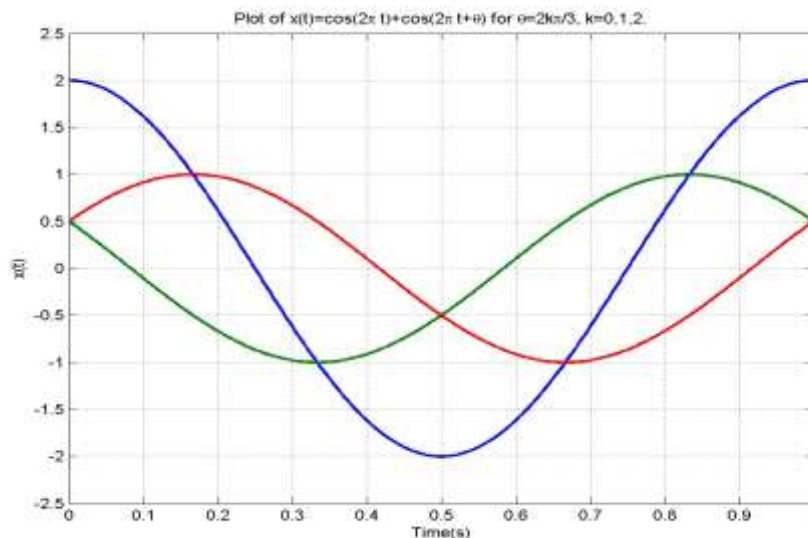
- (a)  $\omega = 2\pi$  rad/s
- (b)  $\theta_k = k\pi/4$ ,  $k = 0, 1, \dots, N-1$

**Solution.** The Matlab program in Problem 592 is modified and used.

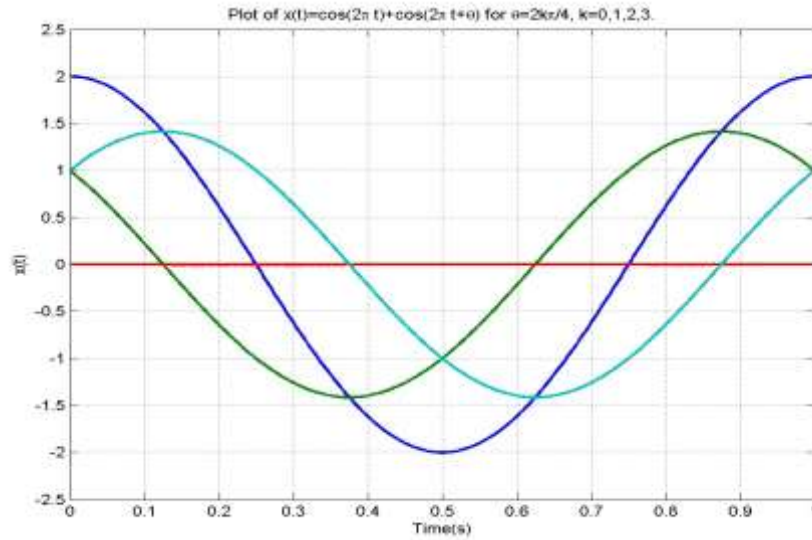
$$x(t) = \sum_{k=0}^{N-1} \cos(2\pi t + \theta_k) = \frac{2k\pi}{N}$$

Representations suggested in (a) and (b) appear analytically identical. The program shown below doesn't generate identical plots. Plots (a) and (b) are different from each other.

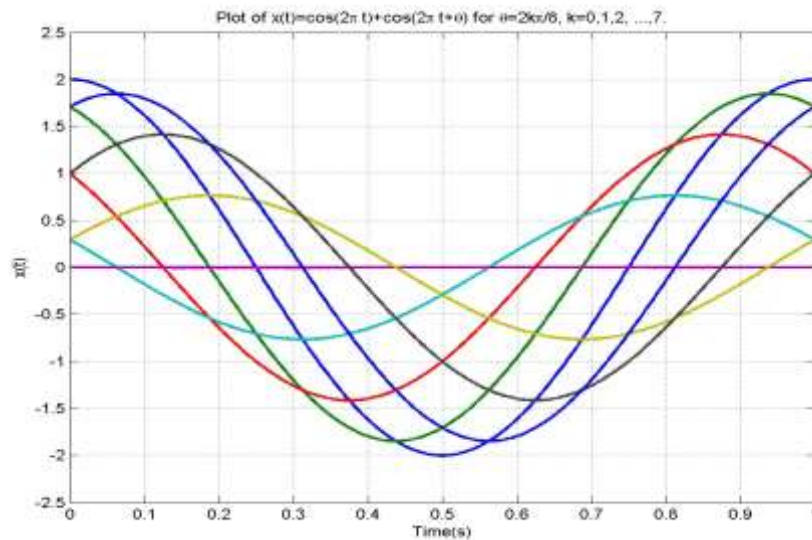
```
clear
N=500;
theta=2*pi*k/N;
t=linspace(0,1,500);
for k=0:N-1
    x(k)=cos(2*pi*t(k)+theta(k));
end
plot(t,x,'LineWidth',2);
axis([0 1 -2.5 2.5]);
```



Plots [100](#) problem [100](#) with [100](#) = [5](#)



Plots **100** problem **7%** with **N<sub>s</sub> = 4**



Plots **100** problem **7%** with **N<sub>s</sub> = 8**

**Problem 100** **3%** **1%** Our three measurements were made on a sinusoidal signal  $x(t) = A \cos(2\pi ft + \phi)$  where  $f$  is the frequency following the FFT. assumed

| $\phi$ | $A$       | $\phi = 0$ | $\phi = 30^\circ$ |
|--------|-----------|------------|-------------------|
| $x(t)$ | $-2.1213$ | $-0.4693$  | $1.5620$          |

Knowing  $f$ ,  $A$  and  $\phi$  computed? Show how  $A$  minimum  
**100** Verify above frequency assumption. **100** Find  $A$  amplitude and  $\phi$  phase which  $A$  amplitude may be  
 what is  $A$  minimum number of samples from  
 sample number  $N$  obtained.



Solution. Let  $x(t) = A \cos(2\pi f_0 t + \theta)$ . Then

$$A \cos(\alpha + \omega t) = A[\cos \alpha \cos \omega t - \sin \alpha \sin \omega t] \quad \text{where } \omega = 2\pi \times 10^6 \pi f_0$$

$$A \cos(2\alpha + \omega t) = A[\cos \alpha \cos \omega t - \sin \alpha \sin \omega t] - A \sin \alpha \sin \omega t$$

From (1) we obtain

$$\cos \alpha = \frac{x_1}{A}$$

From (2) we obtain

$$\sin \alpha = \frac{A \cos \alpha - x_2}{A \sin \omega t}$$

From (1) and (3) we obtain

$$\cos \alpha = \frac{1}{\sin \omega} \left( \cos \omega \cdot \frac{x_1}{x_0} \right)$$

From (1) and (4) we obtain

$$\cos \alpha = \frac{1}{\sin 2\alpha} \left( \cos 2\alpha \cdot \frac{x_2}{x_0} \right)$$

Equate (5) and (6) we get

$$\frac{1}{\sin \alpha} \left( \cos \alpha \cdot \frac{x_1}{x_0} \right) = \frac{1}{\sin 2\alpha} \left( \cos 2\alpha \cdot \frac{x_2}{x_0} \right)$$

The result is

$$\alpha = \cos^{-1} \left( \frac{x_0 + x_2}{2x_1} \right); \quad \theta = 10^6 \pi t$$

From (1) and (2) we get

$$A \cos(\alpha + \omega t) = A[\cos \alpha \cos \omega t - \sin \alpha \sin \omega t] - A \sin \alpha \sin \omega t$$

$$\frac{x_1}{A} = \cos \alpha$$

$$\frac{\sin \theta}{\cos \theta} = \frac{x_1}{A \sin \alpha}$$

In the present case

$$\alpha = 0.2\pi, \quad \theta = 2.1213, \quad \sin \theta = 0.4693 \text{ which results in } \theta = \frac{0.4693}{2.1213 \sin 0.2\pi} = 45^\circ$$

$$\theta = 10^6 \pi t + 45^\circ, \quad \cos(0.2\pi)$$

The answer is  $x(t) = 4 \cos(2 \times 10^6 \pi t + 45^\circ)$

The answer will be from the needed to determine the

$$\sin(\theta) = 2.1213 \cos(0.4\pi) = 1.3620 / \sin(0.4\pi) = 2.1213$$

$$\theta = \left( \frac{2.1213}{2.1213} \right) = 45^\circ, \quad \theta = 45^\circ$$

$$\text{signal } x(t) = 4 \cos(2\pi f t + \theta)$$

that produce independent equidistant signals whose measure-

Knowing the frequency, samples give enough information).

(Samples that are given in Table 2 model measurements sinusoid).

Problem 42. Each of the phase frequency,  $A \cos(2\pi f t + \theta)$

(i.e., find the amplitude, expression

Problem 43. Write the mathematical then sketch

ments **are** recorded **in**

Table 54 (Problem 54)

|          | $f$    | $x_1$  | $x_2$  |
|----------|--------|--------|--------|
|          | 1.7321 | 1.8437 | 1.9263 |
|          | 1.7321 |        | 0.7615 |
|          | 1.6209 |        | 0.1725 |
|          | 1.6209 |        | 0.1979 |
| $x_3(t)$ | 0.1732 | 0.1889 |        |
|          | 0.1732 |        |        |
| $x_4(t)$ | 1.4494 | 2.0148 | 2.5306 |
|          | 1.4494 |        |        |
|          | 1.4331 | 1.3831 | 1.3201 |
|          | 1.4331 |        |        |
| $x_6(t)$ | 4.2002 | 4.9875 |        |
|          | 4.2002 |        | 0.3516 |
|          |        |        | 5.0184 |
| $x_7(t)$ | 0.2409 |        |        |
|          | 0.2409 |        | 0.4500 |

Table 55 (Problem 55)

| signal   | period | $x_{max}$ | $x(0)$ | slope |
|----------|--------|-----------|--------|-------|
| $x_1(t)$ | 100 ns | 5         | 2.7321 | -     |
| $x_2(t)$ | 100 ns | 5         | 2.85   | .     |
| $x_3(t)$ | 100 ns | 5         | 0.3823 | .     |
| $x_4(t)$ | 100 ns | 5         | 1.0511 | ..    |
| $x_5(t)$ | 100 ns | 5         | 2.366  | ..    |
| $x_6(t)$ | 100 ns | 7.5       | 3.1823 | .     |
| $x_7(t)$ | 100 ns | 2.5       | 1.0729 | ..    |
| $x_8(t)$ | 100 ns | 5         |        | ..    |

Problem 55 max

**Solution** Problem

Using three sample values and the formulation developed compute  $x(t) = A \cos(2\pi f t + \theta)$ . The following Matlab program is a sample program.

```

fprintf('Part i\n');
t=linspace(0, 2*10^(-7), 3);
x=[ 7321 8437 9263]';
x0=1.7321; r1=1.8437; r2=1.9263;
alpha=acos((x0+x2)/(2*x1));
theta=atan((r1-r2)/(x0*(r1+r2)));
A=x0/cos(theta)

```

```

x=A*cos(2*pi*f*t+theta)

```

The results are given below.

```

Part i
x = 7321 8437 9263
alpha = 9974e+005
theta = 5238
A = 0.0003

```

```

Part ii
x = 6209 7615 1725
alpha = 9020e+005
theta = 9998

```

$\mu = 9990$

$\sigma$

Part IV

$\mu = 1732 \quad 1889 \quad 1979$

$\sigma = 0018e-005$

theta = 5241

$\mu = 2001$

Part 11  
 $x = 4494 \quad y = 0148 \quad z = 5306$   
 $\ddot{x} = 4997e-005$   
 $\theta = 2001$   
 $\rho = 0005$

Part 12  
 $x = 4330 \quad y = 3831 \quad z = 3201$   
 $\ddot{x} = 5495e+005$   
 $\theta = 3002$   
 $\rho = 5001$

Part 13  
 $x = 2092 \quad y = 9875 \quad z = 0184$   
 $\ddot{x} = 2002e+005$   
 $\theta = 6000$   
 $\rho = 1000$

Part 14  
 $x = 2499 \quad y = 3516$   
 $\ddot{x} = 5425e+005$   
 $\theta = 5009 \quad \rho = 4500$   
 $\rho = 0921$

The answers rounded decimal given

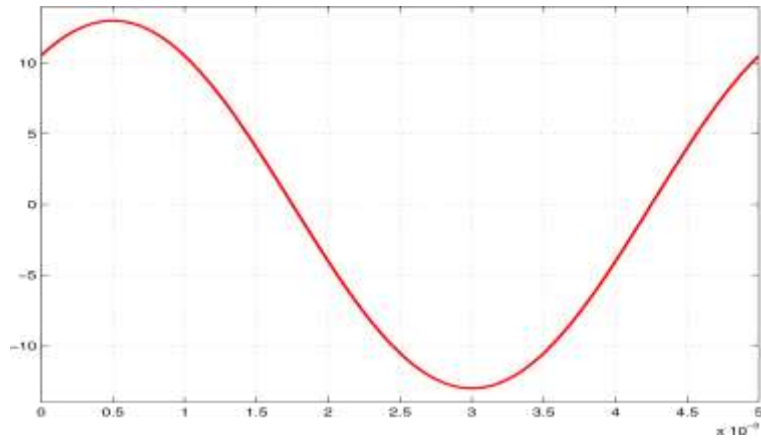
Answers Problem 11 points  
 $x = 10^5 \pi t \cdot \pi/6$   
 $y = 10^5 \pi t$   
 $z = \cos(4 \cdot 10^5 \pi t)$   
 $\ddot{x} = \cos(10^5 \pi t)$   
 $\ddot{y} = \cos(6 \cdot 10^5 \pi t \cdot \pi/6)$   
 $\ddot{z} = \cos(5 \cdot 1.2)$   
 $\rho = \cos(3.1 \cdot 10^5 \pi t \cdot 0.3)$   
 $\rho = \cos(1.24 \cdot 10^5 \pi t \cdot 0.6)$   
 $\rho = \cos(3.06 \cdot 10^5 \pi t \cdot 1.8)$

Problem 15 A sinusoidal

which reaches

Ans. below.

voltage  $v(t)$  [100] is frequency [200] Hz, is zero [100] value, [100] is peak value [10] is [10]  $t=0.5$  ms. Write [10] equation [10] is function [10] time [10] is cosine form.



$$v(t) = 100 \cos[400\pi(t - 0.0005)] = 100 \cos(400\pi t)$$

$\cdot \pi/5)$

**Problem** A low-frequency periodic signal  $s(t)$  is modulated by a value added by the sum of the first  $N$  harmonics of fundamental frequency  $f_s$  (in Hz) as given below:

$$s(t) = \sum_{n=1}^N C_n \cos(2\pi n f_s t)$$

The signal  $s(t)$  modulates a sinusoidal carrier  $\cos(2\pi f_c t)$ ,  $f_c = 100$  MHz. The modulated waveform  $x(t) = s(t) \cos(2\pi f_c t)$ ,  $f_c = 100$  kHz,  $f_s = 1$  MHz, within a band  $100$  MHz by a filter with unity amplitude gain and frequency  $f$  within the band  $100$  MHz. The phase of the filter is a sinusoidal input possible distortions. Find the output of the filter  $y(t)$  and discuss the possible distortions.

**Solution.** The input is  $\sum_{n=1}^N C_n \cos(2\pi n f_s t) \cos(2\pi f_c t) = \sum_{n=1}^N C_n x_n(t)$

$$x(t) = s(t) \cos(2\pi f_c t) = \sum_{n=1}^N C_n \cos(2\pi n f_s t) \cos(2\pi f_c t)$$

where  $x_n(t) = \cos(2\pi n f_s t) \cos(2\pi f_c t)$

Therefore,  $x_n(t) = \cos(a) \cos(b)$  phase

passing through the filter magnitude  $0.4\pi \times 10^3$  changes

| Frequency $f$                              | Phase   |
|--|---|
| $f_c$                                      | $\phi = 0.4\pi \times 10^3 \cdot f_c$   |
| $n f_s$                                    | $\phi = 0.4\pi \times 10^3 \cdot n f_s$   |
| $n f_s$                                    | $\phi = 0.4\pi \times 10^3 \cdot n f_s$   |
| Output component                           |   |
| Input component $\cos[2\pi(f_c + n f_s)t]$ | $\cos[2\pi(f_c + n f_s)t - 0.4\pi \times 10^3 \cdot n f_s]$<br>$= \cos[2\pi f_c(t - \tau_c) + 2\pi n f_s(t - \tau_s)]$<br>where $\tau_c = \frac{1}{5f_c} = 2 \times 10^{-9}$ and $\tau_s = 10^{-9}$ |
| Input component $\cos[2\pi(f_c - n f_s)t]$ | $\cos[2\pi(f_c - n f_s)t - 0.4\pi \times 10^3 \cdot n f_s]$<br>$= \cos[2\pi f_c(t - \tau_c) - 2\pi n f_s(t - \tau_s)]$  |

The input-output pair of the filter

$$x_n(t) = \cos[2\pi(f_c + n f_s)t - \phi] \cos[2\pi(f_c - n f_s)t - \phi]$$

$$y_n(t) = \cos[2\pi f_c(t - \tau_c) + 2\pi n f_s(t - \tau_s)] \cos[2\pi f_c(t - \tau_c) - 2\pi n f_s(t - \tau_s)]$$

$$\cos(2\pi f_c t) \cos(2\pi n f_c t) = \frac{1}{2} [\cos(2\pi f_c t + 2\pi n f_c t) + \cos(2\pi f_c t - 2\pi n f_c t)]$$

is

$$\cos(a) \cos(b) = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

Therefore,

$$y_n(t) = \cos(2\pi f_c(t - \tau_s)) \cos(2\pi n f_c(t - \tau_s)), \text{ where } \tau_s = 10^{-9} \text{ s}$$

$$y(t) = \sum_{n=1}^N \cos(2\pi n f_c(t - \tau_s)) \cos(2\pi f_c(t - \tau_s))$$

$$\tau_s = 10^{-9}$$

$\tau_s$



summary,

$$s(t) \cos 2\pi f_c t = s(t - \tau_s) \cos 2\pi f_c (t - \tau_s)$$

All components

signal  $s(t)$  undergo an equal delay  $\tau_s$  resulting in no distortion.

The sinusoidal modulating carrier undergoes

no delay and no other changes in  $x(t)$ .

show that  $V \cos(\omega t + \theta) = \sqrt{V_1^2 + V_2^2} \cos(\omega t + \theta)$  where

Problem 5.9 Use phasor notation

$$V = \sqrt{V_1^2 + V_2^2 + 2V_1 V_2 \cos(\theta_1 - \theta_2)}$$

$$\theta = \tan^{-1} \left\{ \frac{V_1 \sin \theta_1 + V_2 \sin \theta_2}{V_1 \cos \theta_1 + V_2 \cos \theta_2} \right\}$$

Solution.

$$V \cos(\omega t + \theta) = \text{Re} \{ V e^{j\theta} e^{j\omega t} \}$$

$$V \cos(\omega t + \theta) = \text{Re} \{ V_1 e^{j\theta_1} e^{j\omega t} \}$$

$$V \cos(\omega t + \theta) = \text{Re} \{ V_2 e^{j\theta_2} e^{j\omega t} \}$$

$$\text{Re} \{ (V_1 e^{j\theta_1} + V_2 e^{j\theta_2}) e^{j\omega t} \} = \text{Re} \{ V e^{j\theta} e^{j\omega t} \}$$

$$V e^{j\theta} = V_1 e^{j\theta_1} + V_2 e^{j\theta_2} \Rightarrow \begin{cases} V \cos \theta = V_1 \cos \theta_1 + V_2 \cos \theta_2 & \text{(Eq-1)} \\ V \sin \theta = V_1 \sin \theta_1 + V_2 \sin \theta_2 & \text{(Eq-2)} \end{cases}$$

find phase divide Eq-2 and Eq-1.

$$\theta = \tan^{-1} \frac{V_1 \sin \theta_1 + V_2 \sin \theta_2}{V_1 \cos \theta_1 + V_2 \cos \theta_2}$$

find magnitude,

square Eq-1 and Eq-2 then side-by-side.

$$V^2 = V_1^2 + V_2^2 + 2V_1 V_2 (\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2)$$

$$V^2 = V_1^2 + V_2^2 + 2V_1 V_2 \cos(\theta_1 - \theta_2)$$

Example 5.13 is constant value show that shifted

Problem 5.9 Shift periodic square-wave signal following form:

infinite series

waveform may be represented

$$|a_n| = \frac{2A}{\pi n} \quad n \text{ odd.}$$

$$w(t) = \sum_{n=1}^{\infty} |a_n| \cos \left( \frac{2\pi n t}{T} + \theta_n \right); \quad \text{where } \theta_n \text{ obtained in Example 5.13}$$

Show that shift does not affect conclusions regarding power distribution

Solution.

$$w(t) = \sum_{n=1}^{\infty} |a_n| \cos \left( \frac{2\pi n (t - \tau)}{T} + \theta_n \right)$$

$$\text{where } \theta_n = \theta_n + \frac{2\pi n \tau}{T}$$

The power spectrum description of sinusoids remains the same

$$|a_n| = \frac{2A}{\pi n} \left( \frac{1}{T} \int_0^T \cos(n\omega t) dt \right)$$

$$|a_n| = \frac{2A}{\pi n} \left( \frac{1}{T} \int_0^T \cos(n\omega t) dt \right)$$

are orthogonal to each other, their  
Time doesn't affect the power spectrum

**Problem 30** The function  $x(t) = 0$  approximated during  $T/2 \leq t \leq T/2$

$$y(t) = \frac{T}{\pi} \sum_{n=1}^N \frac{1}{n} \cos(n\omega t) \sin\left(\frac{2\pi n t}{T}\right)$$

One measure of approximation error is  $\frac{1}{T} \int_{-T/2}^{T/2} |x(t) - y(t)|^2 dt$ . Write a

amplitude is changed, is  
periodic signal.

is finite series

program to generate  $x(t)$  and  $y(t)$ ,

plot them, and compute

the error as defined above.

— ... plot error

**Solution.**

Run program

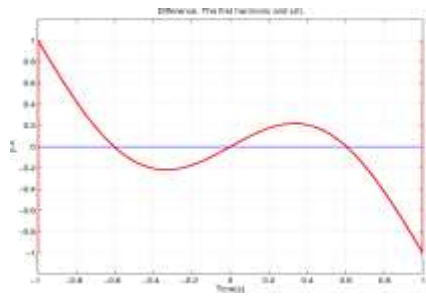
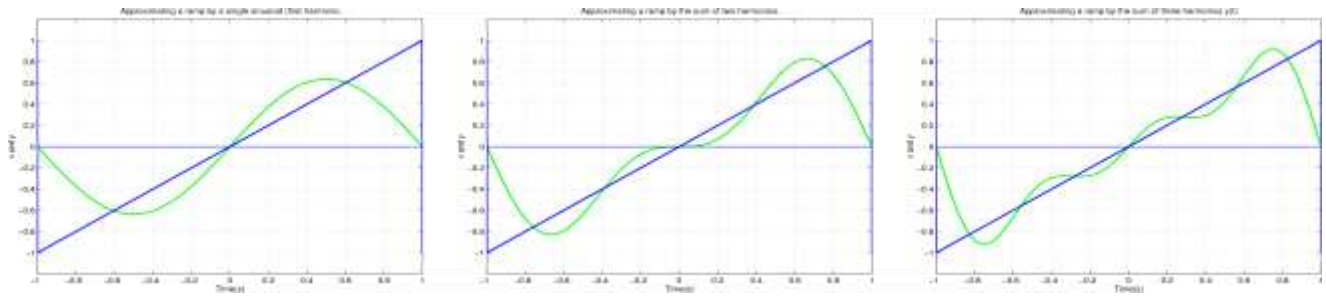
```

N=10;
t=linspace(0,1,1000*N);
%
%
for n=1:N
    for k=1:1000*N;
        p(k)=(-1)^(n-1)*sin(n*pi*t(k))/(pi*n);
    end;
end;
y=sum(p,2);

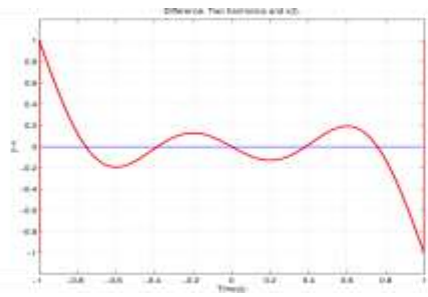
e=x-y;
e2=e.^2;
AE=sum(e2)/(1000*N)

```

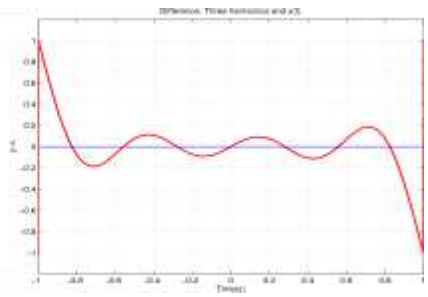
|  |        |        |        | 0.0448 | 0.0367 | 0.0311 | 0.0270 | 0.0238 | 0.0213 | 0.0193 |
|--|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|  | 0.1307 | 0.0800 | 0.0575 |        |        |        |        |        |        |        |



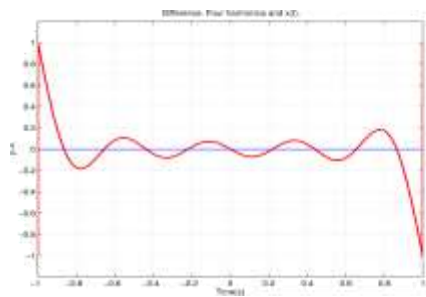
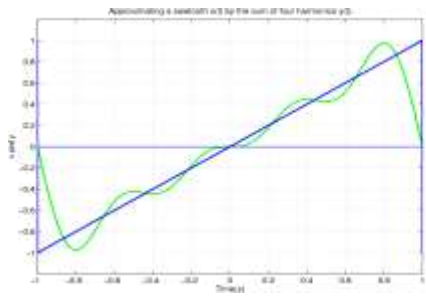
$N = 1$



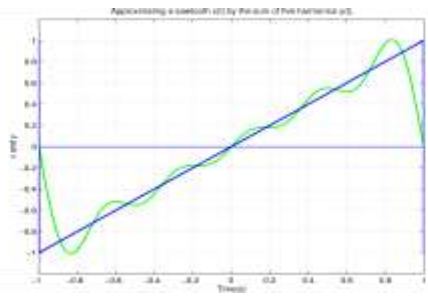
$N = 2$



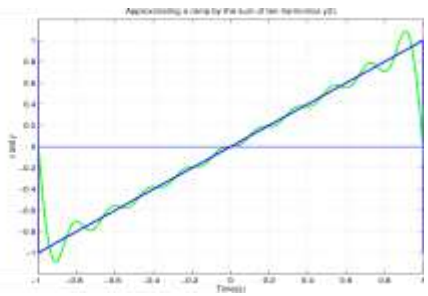
$N = 3$



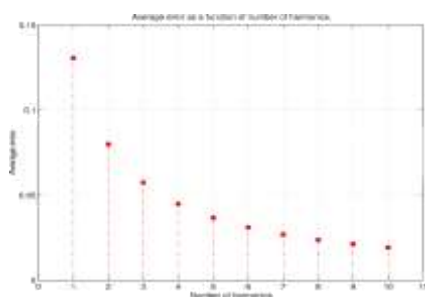
$N = 4$



$N = 5$



$N = 6$



$N = 1, 2, 3, \dots, 10$

**Problem 310** Motion of free electron in sinusoidal electric field. An electron with negative electric charge  $e = -1.602 \times 10^{-19}$  C and mass  $m_e = 9.109 \times 10^{-31}$  kg is placed in an electric field in vacuum when strength  $E = E_0 \cos(\omega t)$ . Determine the span of the motion of an electron.

☐ | experiences ☐

subjected to an electric field  $E = 10^{-6} \cos(2\pi ft)$  V/m at frequencies  $f = 10^3$  Hz,  $10^4$  kHz,  $10^6$  MHz, and  $10^9$  GHz.

**Solution.**

$$a(t) = \frac{eE(t)}{m} = \frac{eE_0}{m} \cos(2\pi ft), \quad \text{where } \frac{eE_0}{m} = 10^{-6} \frac{C}{m}$$

$$\text{where } \lambda = \frac{a_0}{(2\pi f)^2} = \frac{4454.8}{f^2}$$

$$x(t) = X_0 \cos(2\pi ft).$$

| Frequency            | 10 <sup>3</sup> Hz | 10 <sup>4</sup> kHz | 10 <sup>6</sup> MHz | 10 <sup>9</sup> GHz          |
|----------------------|--------------------|---------------------|---------------------|------------------------------|
| Span in oscillations | 1.237455 nm        | 4.455 nm            | 4.455 nm            | 4.455 × 10 <sup>-13</sup> nm |

plane. In each beam

**Problem 10.10** The following program is written to sweep from left to right and moves down an incremental value, similar to the operation of a recording element in a seismograph.

```

hold on
axis([0 10 0 50])
x=sin(2*pi*k*t)+10-2*k-0.5;
plot(x,'b'); ylabel('harmonics'); title('harmonics');
xlabel('Time');
grid;

```

Execute the program in successive steps and examine the plot to verify that it agrees with your expectation. The fourth line of the program with the `axis` command is replaced from the following:

```

x=sin(2*pi*k*t)+2*k+0.5; axis([0 10 5 50]);
x=sin(2*pi*k*t)+50-2*k-0.5; axis([0 10 5 50]);
x=sin(2*pi*k*t)+60-2*k-0.5; axis([0 10 5 50]);
x=sin(2*pi*k*t)+90-2*k-1; axis([0 10 5 50]);
x=sin(2*pi*k*t)+90-4*k-1; axis([0 10 50 50]);
y=cos(2*pi*k*t)+90-4*k-3; axis([0 10 50 50]); plot(x,y);

```

Explore alternative variables which would enhance the way appearance of the two-dimensional plot.

pattern generated by the program is shown below.

**Solution.** The two-dimensional



```
    t = linspace(0, 1, 1000);
    y = sin(2*pi*k*t);
    ylabel('harmonics');
    title('harmonics');
```

```
    x = sin(2*pi*k*t) + 10 - 2*k - 0.8;
    plot(t, x);
    xlabel('Time');
```



```

grid;
hold on
format
print -dpsc pattern_a.eps
print -dtiff pattern_a.tiff %Saves plot in encapsulated post-script
% Saves plot in tiff format
figure(2)
hold on
axis([0 10 0 10])
x=sin(2*pi*k*t)+2*k+0;
plot(x,t);
xlabel('Time (s)');
ylabel('harmonics');
title(' harmonics');
grid;
hold on
hold on
print -dpsc pattern_b.eps
%.....
figure(6)
hold on
x=sin(2*pi*k*t)+90-4*k-1;
y=cos(2*pi*k*t)+90-4*k-3;
axis([0 10 0 10])
plot(x,y);
xlabel('Time (s)');
ylabel('harmonics');
title(' harmonics');
grid;
hold on
hold on
print -dpsc pattern_f.eps

```

## Patterns.

the xy plane and obtain

## Polarization, and Lissajous

sinusoidal motion in point

### Project: Trajectories, Wave

**Purpose.** investigate trajectories from motion from patterns

**Introduction and Summary.** The motion vector drawn from origin Cartesian coordinates

trajectories.

may be found by eliminating the point  $(x, y)$  plane amplitude, phase, and frequency functions of time. The path traversed by the variable  $\theta$  from those equations. Some may be deduced from the trajectory  $\theta$  contains several classes where  $x$  and  $y$  coordinate values vary

- i)  $x(t)$  and  $y(t)$  have same frequency.
- ii)  $x(t)$  and  $y(t)$  have slightly different frequencies.
- iii) The frequencies of  $x(t)$  and  $y(t)$  are harmonics of a principle frequency.
- iv) The frequencies of  $x(t)$  and  $y(t)$  are not harmonics of a principle frequency.
- v) The effect of the sampling rate on the electric circuit.
- vi) Implementation through an

paramete-

can be stated as a time-varying  $\theta$  vector, called the trajectory, parameters of the motion (such as the  $\theta$  project you will generate and use sinusoidally with time. The project

frequency.  
frequency.

The present project may

deal with physical oscillators.

Examples from both are carried out by mathematical simulation and included.

vector is in the plane drawn

Section 10  $x(t)$  and  $y(t)$  from the origin in point have the same frequency. Consider the time-varying whose projections are  $E_x \cos(\omega t)$  and  $E_y \cos(\omega t + \phi)$

$$\begin{cases} x = E_x \cos \omega t \\ y = E_y \cos(\omega t + \phi) \end{cases}$$

are observed: linear (zero

Depending on the phase difference and amplitude ratio, three types of trajectories are observed: circular (equal amplitudes with  $\pm \pi/2$  phase); elliptical (all other values); and a straight line.

Show that  $\frac{E_y}{E_x} = \frac{a}{b}$

trajectory is a straight line. See Fig. 14-a.

coordinate system

**a) Linear Trajectory.**

Rotate the coordinate system by an angle  $\alpha$ . Show the relationship between the coordinate

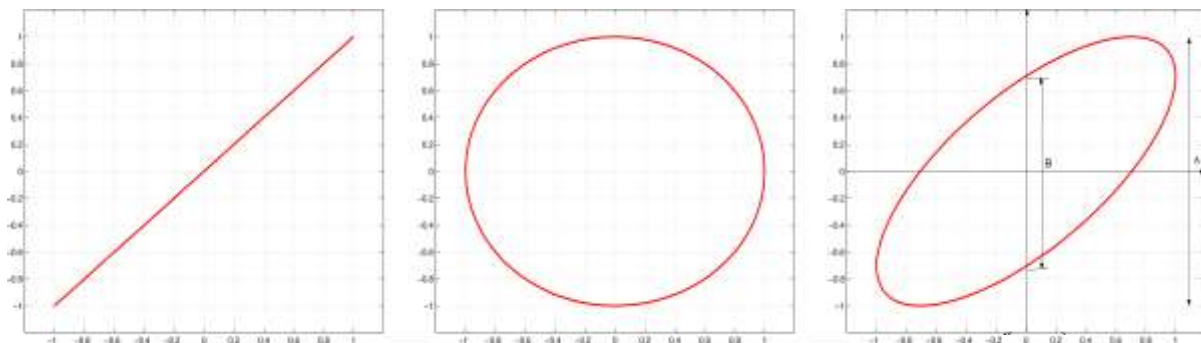
systems

value represent

$$\begin{cases} x' = x \cos \alpha + y \sin \alpha \\ y' = -x \sin \alpha + y \cos \alpha \end{cases}$$

Determine the appropriate only.

Find the equation of the trajectory in the coordinate system. is a one-dimensional vector which oscillates in time along the  $x$ -axis



(a) Linear trajectory

(b) Circular

(c) Elliptical form

$$\begin{cases} x = E_x \cos \omega t \\ y = E_y \cos(\omega t + \phi) \end{cases}$$

trajectory

$$\begin{cases} x = E_x \cos \omega t \\ y = E_y \sin \omega t \end{cases}$$

$$\begin{cases} x = E_x \cos(\omega t + \pi/4) \\ y = E_y \sin(\omega t + \pi/4) \end{cases}$$

Fig. 14 simple Lissajous patterns. Trajectories in the general case,

vector

trajectories

have

**b) Circular Trajectory.**

Show that  $\frac{E_y}{E_x} = 1$  and  $\phi = \pm \pi/2$  determine the direction of motion

$$\frac{2xy}{E_x E_y} = \sin(2\phi)$$

found from trajectory.

**c) Elliptical Trajectory.**

Show that

vector

$$\left(\frac{x}{E_x}\right)^2 + \left(\frac{y}{E_y}\right)^2 = 1$$

that angle

determine the direction of motion

trajectory.

$$\mathbf{B} = \mathbf{1}(B/A)$$

where  $\mathbf{B}$  and  $\mathbf{A}$  are shown in Fig. 14-c. Rotate the coordinate system along the major axis of the ellipse. Determine the rotation angle. Write the coordinate system.

Matlab code given below generate a linear

Simulation Computer. Run

an

trajectory.

elliptical

appropriate angle align  
equation the trajectory

trajectory.

```

w=2*pi*f; T=1/f; theta=0; % Motion parameters.
t=linspace(0,100*T,1000); % Trajectory.
x=a*cos(w*t); y=b*cos(w*t+theta) plot(x,y)

```

Change the parameters of the motion variables. Explore how each may vary. Change the shape of the trajectory. From the amplitudes and horizontal phase difference. values used in the simulation.

Parallels with Electromagnetic vector. plots the trajectories obtain between them. Compare with the electromagnetic plane direction of propagation. electric field follows:

**Wave Polarization.** The electric field vector is a time-varying vector which lies in the plane that is perpendicular to the direction of propagation. It is a sinusoidal wave with two components (each which vary sinusoidally with time) in the plane of propagation.

$$\begin{cases} x = E_x \cos \omega t \\ y = E_y \cos(\omega t + \phi) \end{cases}$$

Each component is a sinusoidal wave.

This is the same vector discussed in the beginning of this section with three possible trajectories. trajectory is associated with wave type polarization. The electromagnetic wave, therefore, is said polarized in any of the above directions. labeled as right, circular or elliptical. clockwise or counter-clockwise slightly different frequencies.

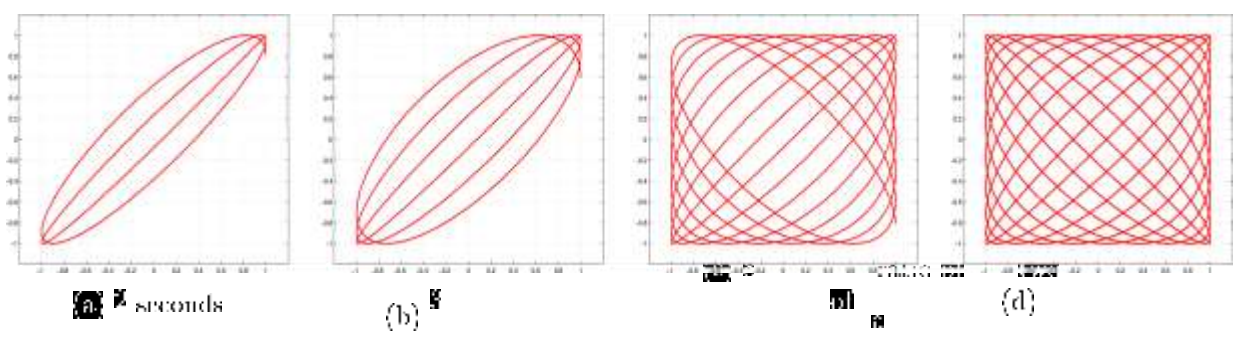
Section 11.1  $x(t)$  and  $y(t)$  have

$$\begin{cases} x(t) = a \cos \omega_1 t \\ y(t) = b \cos \omega_2 t \end{cases}$$

With  $\omega_1$  and  $\omega_2$  approximately equal (but not exactly equal), the difference in frequencies will appear as a time-varying phase difference, resulting in a trajectory which slowly moves between the above three patterns. Construct an example where

$$\begin{cases} x(t) = \cos 2\pi t \\ y(t) = b \cos 2.1\pi t \end{cases}$$

(corresponding  $\omega_1 = 2\pi$  Hz, respectively). Modify the Matlab program given in Section 11.1 plot the Lissajous patterns for  $b = 0.5, 1, 2$  seconds,  $0.5, 1, 2$  seconds,  $0.5, 1, 2$  seconds. Determine the time needed for one cycle in each plot.



Four Lissajous patterns, all with  $\omega_1 = 2\pi$  Hz. How long does it take to complete one cycle?

Explore the effect of  $\omega_2 = (k \pm 1)\omega_1$ , with  $k = 0.5, 0.1, 0.01$ . frequencies of  $x(t)$  and  $y(t)$  are harmonics of the principle frequency whose shapes are associated with the frequencies. Fig. 11.1 shows four different variations of  $y(t)$ .

**Section 11.10** The  
patterns generated  
examples shown in

11.10

11.10

complex  
Four

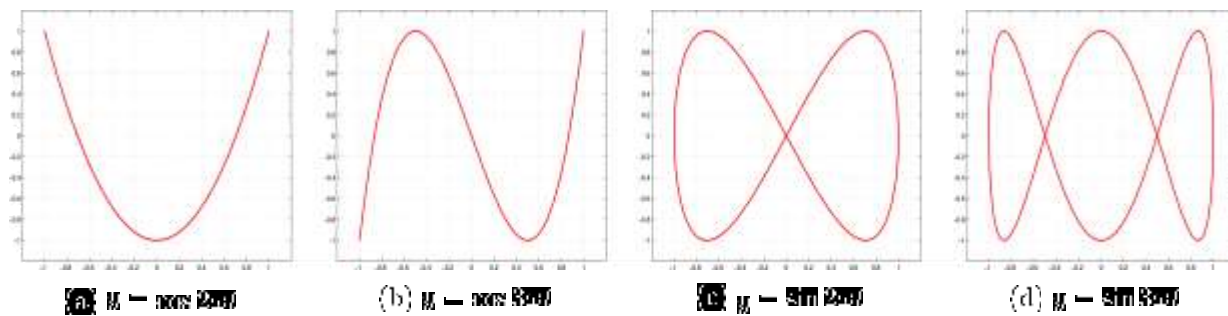


Fig. 1.10 Four Lissajous patterns with  $x = \cos(2t)$  and various  $y(t)$ .

Using Matlab, plot trajectories in each case and eliminate  $t$  between  $x = \cos(at)$  and  $y = \cos(bt)$ ,  $a/b = 5/3$ . Repeat together and verify  $y = \sin(bt)$ . Plot obtained through Matlab. Determine the number of crossings in horizontal and vertical directions. Relate the ratio  $a/b$ . Suggest a method to measure the frequency of a sinusoidal signal from Lissajous patterns.

Section 1.10 The frequencies of oscillators), with the principle of theory, the practice (e.g., simulation in Matlab) is to repeat itself. Run the code, observe the Lissajous pattern and eventually Section 1.10 Repeat the procedure.

Section 1.10 Effect of sampling rate. In plotting a trajectory, the  $x$  and  $y$  coordinates are sufficiently fast. Otherwise the plot will exhibit artifacts caused by the number of samples. Show the following examples of Lissajous patterns derived from  $x = \cos(t)$  and  $y = \sin(3t)$  with three different sampling rates. You may use the following code for this purpose.

```

%Repeat N=20
t=linspace(0,N*pi,800)
x=cos(t); y=cos(3*t); plot(x,y);
%Repeat N=20
t=linspace(0,N*pi,100)
x=cos(2*t); y=sin(3*t); plot(x,y);

```

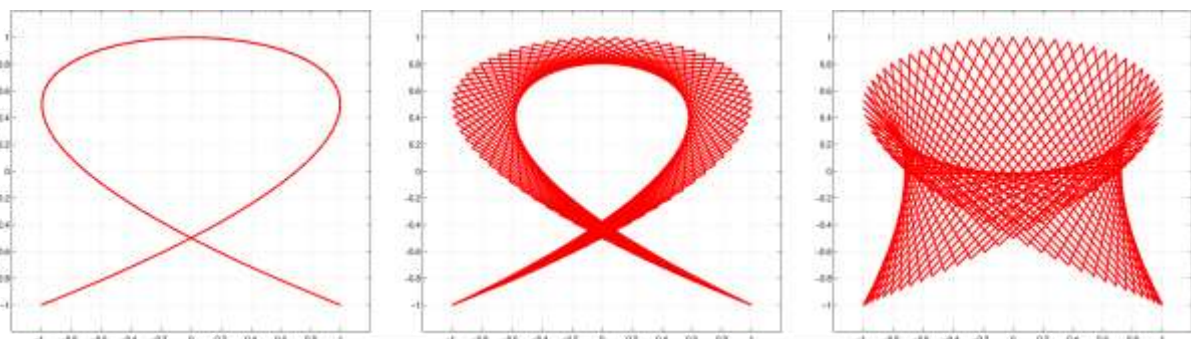


Fig. 1.11

(a)  $N = 2$

(b)  $N = 3$

10.10.10 following 10.10.10 samples

10.10.10

(T)

- i)  $N = 200, \quad \mu = 100, \quad \sigma = 20$
- ii)  $N = 500, \quad \mu = 100, \quad \sigma = 20$
- iii)  $N = 100, \quad \mu = 50, \quad \sigma = 10$
- iv)  $N = 200, \quad \mu = 2$

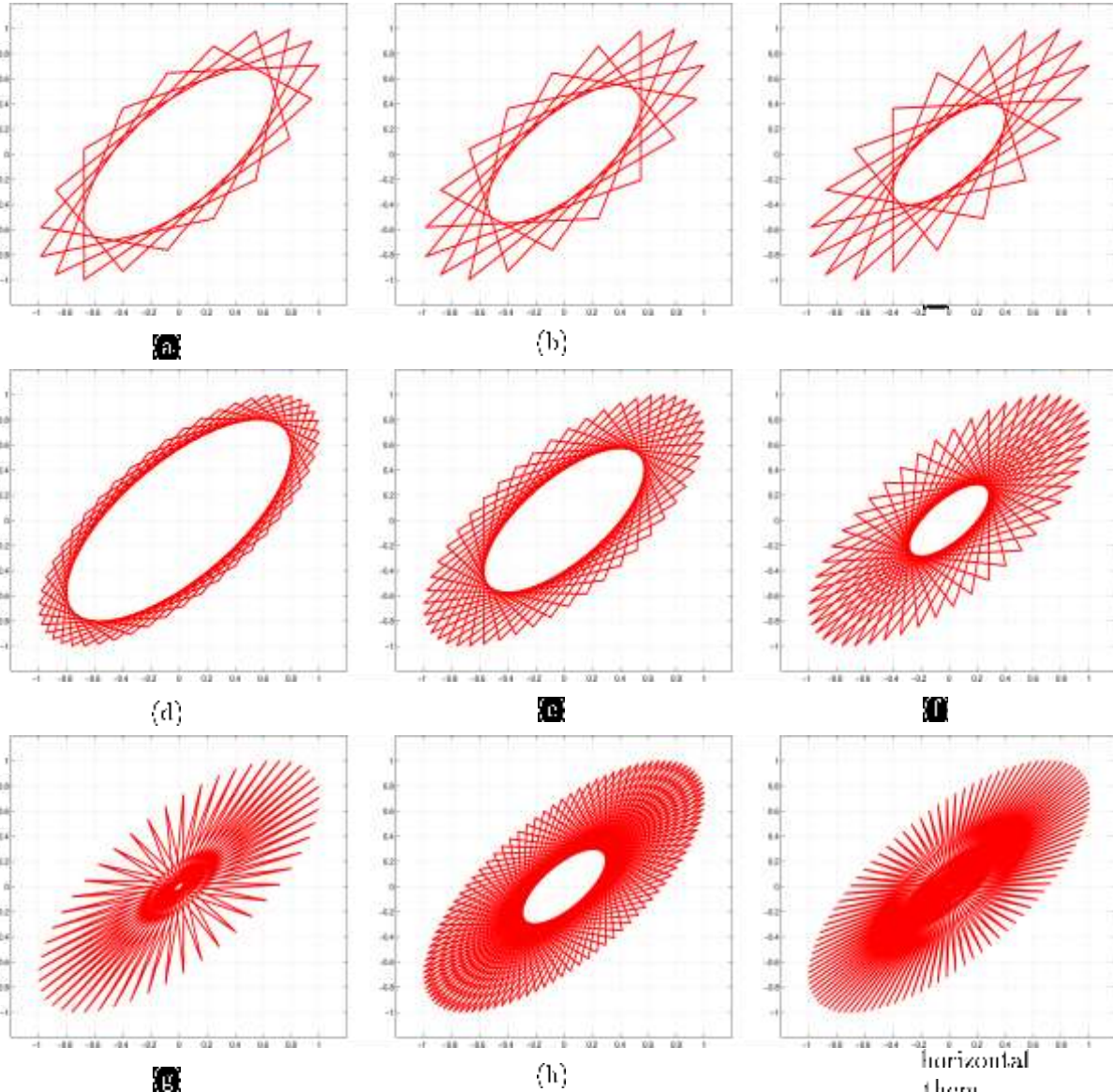
10.10.10 sampled effect

10.10.10



Mallah code given below explain trends in the plots.

```
t=linspace(0,1,100);
x=sin(2*pi*t); y=cos(2*pi*t+pi/4); plot(x,y)
```



Section 14.1 Implementation through an Electric Circuit. An oscilloscope whose deflections are controlled by two signals  $x(t)$  and  $y(t)$ , respectively, eliminates the need for a purpose, an ordinary oscilloscope can be used to display a trajectory, as shown in Fig. 16-a with a sinusoidal voltage signal. Start with the situation shown in Fig. 16-b. Connect the function generator to the oscilloscope. The voltage between the terminals of the function generator is shown in Fig. 16-c. The voltage between the terminals of the oscilloscope is shown in Fig. 16-d. The voltage between the terminals of the oscilloscope is shown in Fig. 16-e. The voltage between the terminals of the oscilloscope is shown in Fig. 16-f. The voltage between the terminals of the oscilloscope is shown in Fig. 16-g. The voltage between the terminals of the oscilloscope is shown in Fig. 16-h. The voltage between the terminals of the oscilloscope is shown in Fig. 16-i. The voltage between the terminals of the oscilloscope is shown in Fig. 16-j. The voltage between the terminals of the oscilloscope is shown in Fig. 16-k. The voltage between the terminals of the oscilloscope is shown in Fig. 16-l. The voltage between the terminals of the oscilloscope is shown in Fig. 16-m. The voltage between the terminals of the oscilloscope is shown in Fig. 16-n. The voltage between the terminals of the oscilloscope is shown in Fig. 16-o. The voltage between the terminals of the oscilloscope is shown in Fig. 16-p. The voltage between the terminals of the oscilloscope is shown in Fig. 16-q. The voltage between the terminals of the oscilloscope is shown in Fig. 16-r. The voltage between the terminals of the oscilloscope is shown in Fig. 16-s. The voltage between the terminals of the oscilloscope is shown in Fig. 16-t. The voltage between the terminals of the oscilloscope is shown in Fig. 16-u. The voltage between the terminals of the oscilloscope is shown in Fig. 16-v. The voltage between the terminals of the oscilloscope is shown in Fig. 16-w. The voltage between the terminals of the oscilloscope is shown in Fig. 16-x. The voltage between the terminals of the oscilloscope is shown in Fig. 16-y. The voltage between the terminals of the oscilloscope is shown in Fig. 16-z.

project.

16

circuit

17

using

18

19 20

21 22 23 none 24

25  $\cos(\omega t)$

26

27 28

29 30 31 above

Show that

$$29 \text{ } \frac{1}{\omega} \text{ none } 10000\omega \text{ 30 31 } \text{ } \frac{1}{\omega} \text{ } - 0.707 \text{ none } (10000\omega$$

vertical  
displays  
deflection mode.

0

32

33

relationship

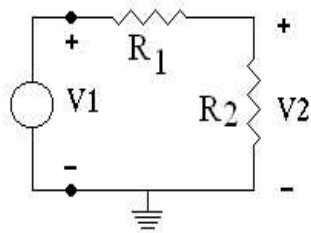
$$\frac{C^2 \omega^2}{\sqrt{1 + R^2 C^2 \omega^2}} \text{ 34 } \text{ } \cdot \tan^{-1}(RC\omega)$$

35 36

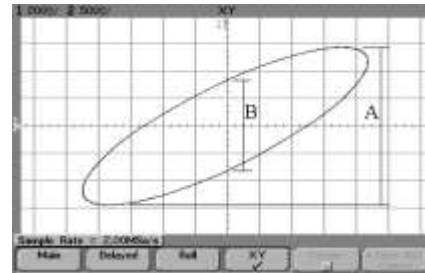
The elliptic pattern shown in Fig. 16-a should appear on the scope. Find the phase angle between the two signals from the theory. Compute the phase angle between the two signals from the equation of the trajectory on the screen.

$$\phi = \tan^{-1}(B/A)$$

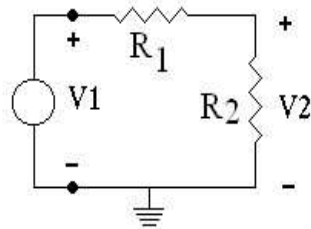
where  $A$  and  $B$  are shown in Fig. 16-b. The phase angle is expected to be  $90^\circ$ . The slope of the trajectory is changed through the horizontal and vertical gains of the signal generator. The phase angle may be changed by changing the frequency between the horizontal and vertical channels. Explore the effect of the above trajectory.



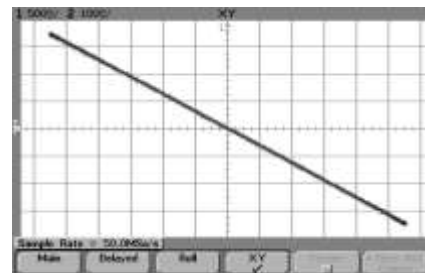
RC circuit.



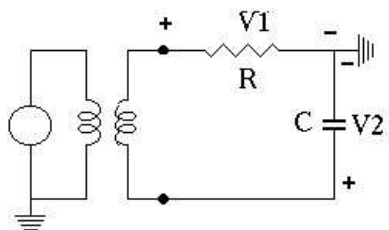
Elliptic trajectory



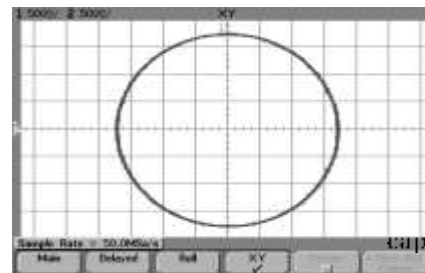
RRR circuit



Linear trajectory



RC circuit with isolation transformer



Circular trajectory

Fig. 16 Three circuits for generating coordinate signals and the resulting trajectories on the oscilloscope.

obtain a linear trajectory shown in Fig. 16-b. Relate the slope of the trajectory to the resistor values and the circuit configuration in sections III, IV, and V that directly connected to the horizontal and vertical channels.

obtain a circular trajectory former. The situations described (digital approach computer) from

Continue with real-time implementations of sinusoidal signal generators using the method obtained

**Conclusions.** Describe your overall conclusions from **177**.  
**178** draw **179** **180** which distinguishes **181** results  
**182** analog approach (using **183** function generator **184**)

project. What applications may **185** have? Where  
**186** (using **187**  
 oscilloscope)?