Solution Manual for Statics and Mechanics of Materials 4th Edition Hibbeler 0133451607 9780133451603

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Solution Manual:

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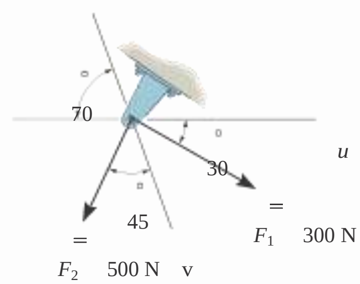
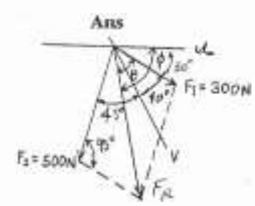
2–1. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.



SOLUTION

$$F_R = \sqrt{(300)^2 + (500)^2 - 2(300)(500)\cos 95^\circ} = 605.1 = 605 \text{ N}$$ **Ans.**

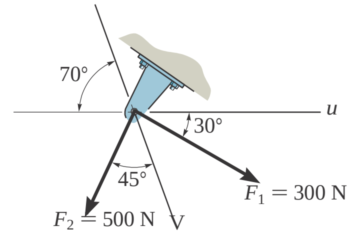


$$\frac{605.1}{\sin 95^\circ} = \frac{500}{\sin \theta}$$

$$\theta = 55.40^\circ$$

$$\phi = 55.40^\circ + 30^\circ = 85.4^\circ$$ **Ans.**

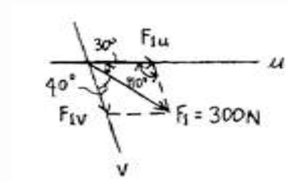
2–2. Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.



SOLUTION

$$\frac{F_{1u}}{\sin 40^\circ} = \frac{300}{\sin 110^\circ}$$

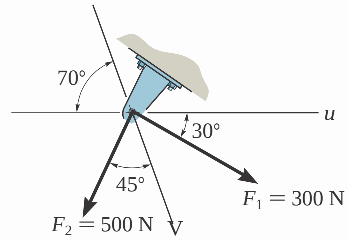
$$F_{1u} = 205 \text{ N}$$ **Ans.**



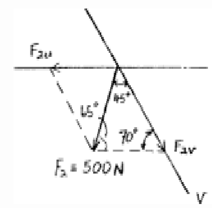
$$\frac{F_{1V}}{\sin 30^\circ} = \frac{300}{\sin 110^\circ}$$

$$F_{1V} = 160 \text{ N}$$ **Ans.**

2–3. Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.



SOLUTION



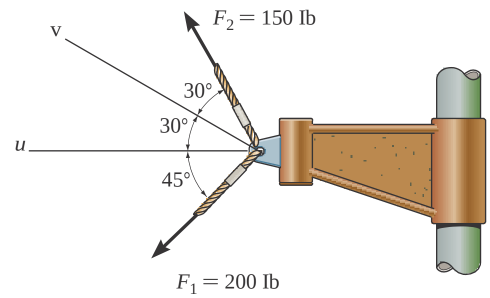
$$\frac{F_{2u}}{\sin 45°} = \frac{500}{\sin 70°}$$

$$F_{2u} = 376 \text{ N}$$ **Ans.**

$$\frac{F_{2v}}{\sin 65°} = \frac{500}{\sin 70°}$$

$$F_{2v} = 482 \text{ N}$$ **Ans.**

2--**4.** Determine the magnitude of the resultant force acting on the bracket and its direction measured counterclockwise from the positive *u* axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

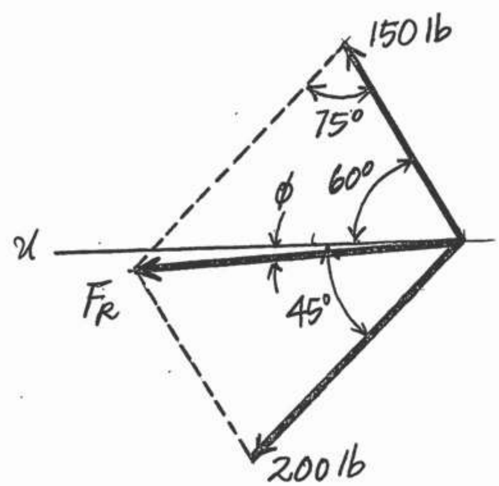
$$F_R = \sqrt{200^2 + 150^2 - 2(200)(150)\cos 75^\circ}$$
$$= 216.72 \text{ lb} = 217 \text{ lb}$$ **Ans.**

Applying the law of sines to Fig. *b* and using this result yields

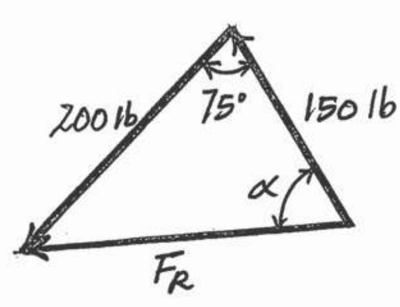
$$\frac{\sin\alpha}{200} = \frac{\sin 75^\circ}{216.72} \qquad \alpha = 63.05^\circ$$

Thus, the direction angle ϕ of $\mathbf{F}_R$, measured counterclockwise from the positive *u* axis, is

$$\phi = \alpha - 60^\circ = 63.05^\circ - 60^\circ = 3.05^\circ$$ **Ans.**

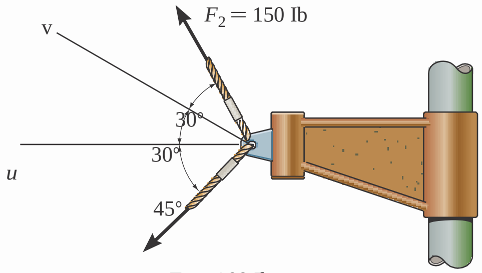


(a)



(b)

--5. Resolve $\mathbf{F}_1$ into components along the u and v axes, and determine the magnitudes of these components.



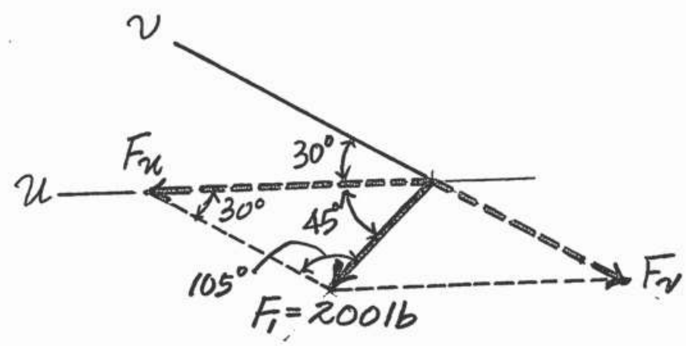
SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

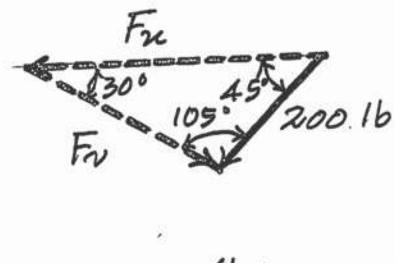
Applying the law of sines to Fig. *b*, yields

$$\frac{F_u}{\sin 105°} = \frac{200}{\sin 30°} \qquad F_u = 386 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{F_v}{\sin 45°} = \frac{200}{\sin 30°} \qquad F_v = 283 \text{ lb} \qquad \textbf{Ans.}$$

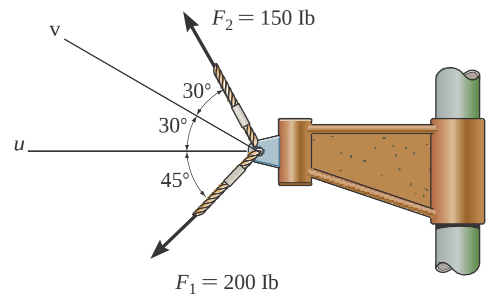


(a)



(b)

--6. Resolve $\mathbf{F}_2$ into components along the u and v axes, and determine the magnitudes of these components.



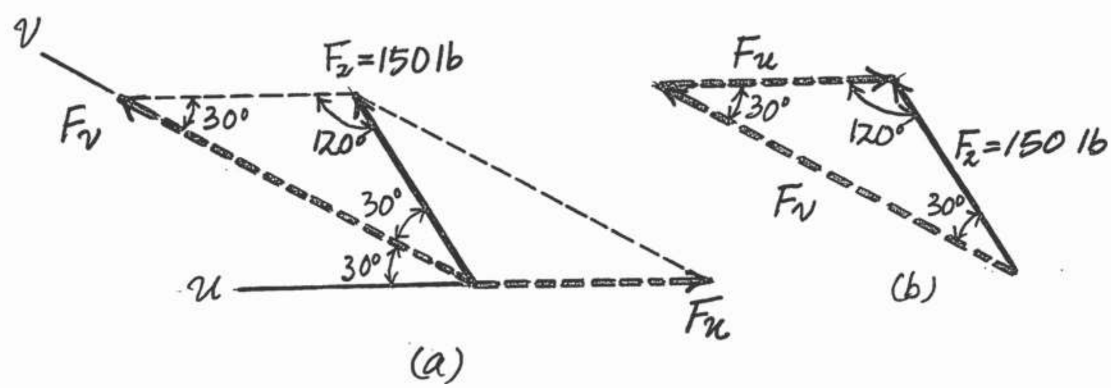
SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. a and b, respectively.

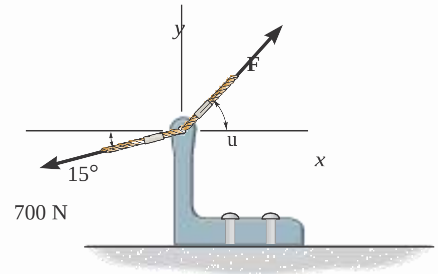
Applying the law of sines to Fig. b,

$$\frac{F_u}{\sin 30°} = \frac{150}{\sin 30°} \qquad F_u = 150 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{F_v}{\sin 120°} = \frac{150}{\sin 30°} \qquad F_v = 260 \text{ lb} \qquad \textbf{Ans.}$$



2-7. If 0 = 60° and F = 450 N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{700^2 + 450^2 - 2(700)(450)\cos 45°}$$

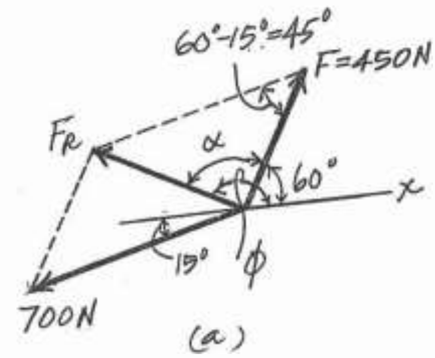
$$= 497.01\text{ N} = 497\text{ N}$$ **Ans.**

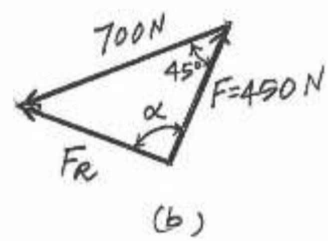
This yields

$$\frac{\sin\alpha}{700} = \frac{\sin 45°}{497.01} \qquad \alpha = 95.19°$$

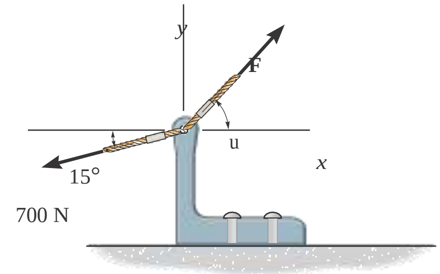
Thus, the direction of angle ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis, is

$$\phi = \alpha + 60° = 95.19° + 60° = 155°$$ **Ans.**





22-8. If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force **F** and its direction 0.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

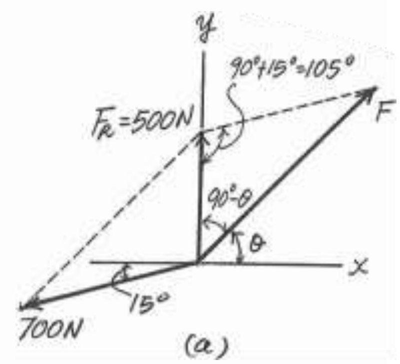
$$F = \sqrt{500^2 + 700^2 - 2(500)(700)\cos 105°}$$

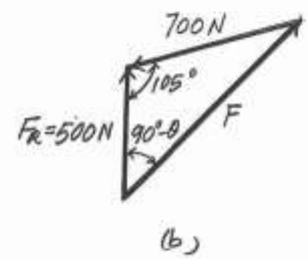
$$= 959.78\text{ N} = 960\text{ N}$$ **Ans.**

Applying the law of sines to Fig. *b*, and using this result, yields

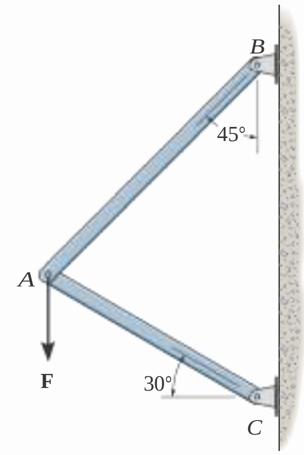
$$\frac{\sin(90° + \theta)}{700} = \frac{\sin 105°}{959.78}$$

$$\theta = 45.2°$$ **Ans.**





2-9. The vertical force **F** acts downward at A on the two-membered frame. Determine the magnitudes of the two components of **F** directed along the axes of AB and AC. Set $F = 500$ N.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

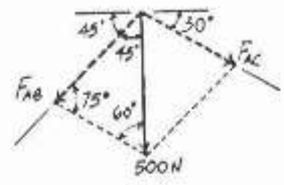
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60^\circ} = \frac{500}{\sin 75^\circ}$$

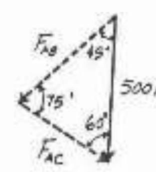
$$F_{AB} = 448 \text{ N} \qquad \textbf{Ans.}$$

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{500}{\sin 75^\circ}$$

$$F_{AC} = 366 \text{ N} \qquad \textbf{Ans.}$$

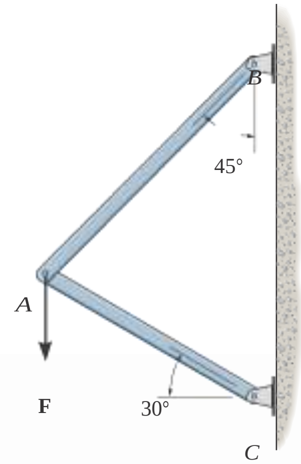


(a)



(b)

2-10. Solve Prob. 2–9 with $F = 350$ lb.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

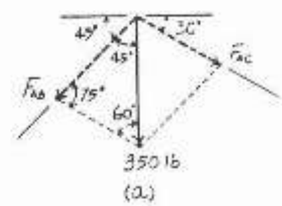
Trigonometry: Using the law of sines (Fig. *b*), we have

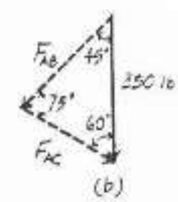
$$\frac{F_{AB}}{\sin 60^\circ} = \frac{350}{\sin 75^\circ}$$

$$F_{AB} = 314 \text{ lb} \qquad \textbf{Ans.}$$

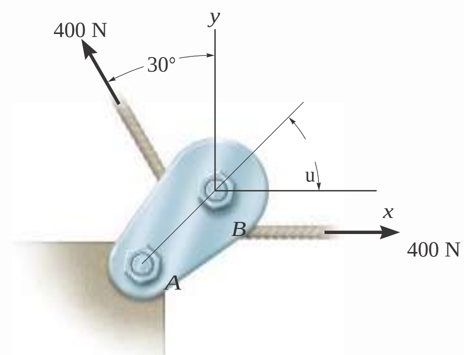
$$\frac{F_{AC}}{\sin 45^\circ} = \frac{350}{\sin 75^\circ}$$

$$F_{AC} = 256 \text{ lb} \qquad \textbf{Ans.}$$





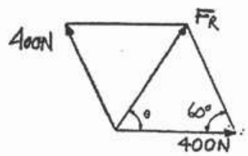
2-11. If the tension in the cable is 400 N, determine the magnitude and direction of the resultant force acting on the pulley. This angle is the same angle 0 of line *AB* on the tailboard block.

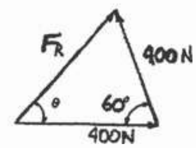


SOLUTION

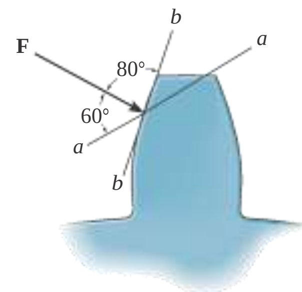
$$F_R = \sqrt{(400)^2 + (400)^2 - 2(400)(400)\cos 60^\circ} = 400\ \text{N} \quad \textbf{Ans}$$

$$\frac{\sin\theta}{400} = \frac{\sin 60^\circ}{400}; \quad \theta = 60^\circ \quad \textbf{Ans}$$





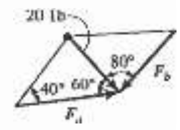
22-12. The force acting on the gear tooth is $F = 20$ lb. Resolve this force into two components acting along the lines *aa* and *bb*.



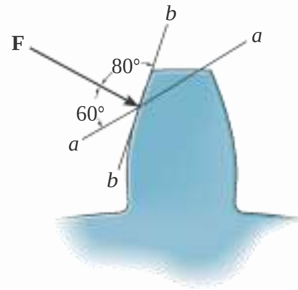
SOLUTION

$$\frac{20}{\sin 40^\circ} = \frac{F_a}{\sin 80^\circ}; \qquad F_a = 30.6 \text{ lb}$$ **Ans.**

$$\frac{20}{\sin 40^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.9 \text{ lb}$$ **Ans.**



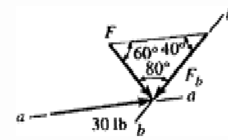
2-13. The component of force **F** acting along line *aa* is required to be 30 lb. Determine the magnitude of **F** and its component along line *bb*.



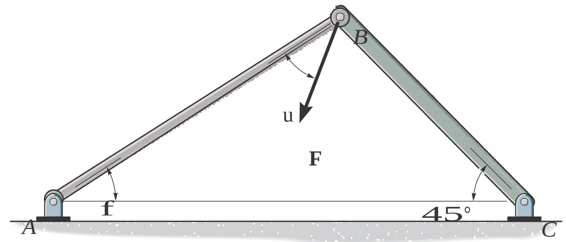
SOLUTION

$$\frac{30}{\sin 80°} = \frac{F}{\sin 40°}; \qquad F = 19.6 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{30}{\sin 80°} = \frac{F_b}{\sin 60°}; \qquad F_b = 26.4 \text{ lb} \qquad \textbf{Ans.}$$

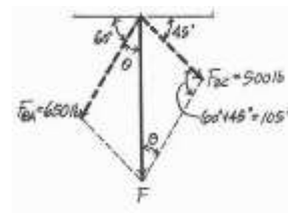


2-14. Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A, and the component acting along member BC is 500 lb, directed from B towards C. Determine the magnitude of **F** and its direction θ. Set $\phi = 60°$.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.



(a)

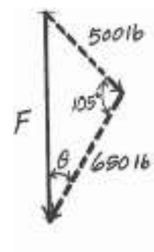
Applying the law of cosines to Fig. *b*,

$$F = \sqrt{500^2 + 650^2 - 2(500)(650)\cos 105°}$$

$$= 916.91 \text{ lb} = 917 \text{ lb} \qquad \textbf{Ans.}$$

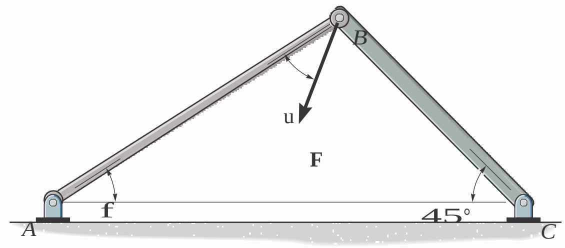
Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\theta}{500} = \frac{\sin 105°}{916.91} \qquad \theta = 31.8° \qquad \textbf{Ans.}$$



(b)

2-15. Force **F** acts on the frame such that its component acting along member *AB* is 650 lb, directed from *B* towards *A*. Determine the required angle ϕ ($0° \le \phi \le 90°$) and the component acting along member *BC*. Set $F = 850$ lb and $\theta = 30°$.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

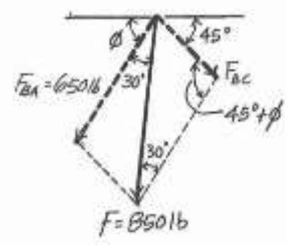
Applying the law of cosines to Fig. *b*,

$$F_{BC} = \sqrt{850^2 + 650^2 - 2(850)(650)\cos 30°}$$

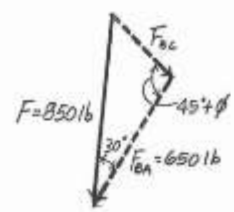
$$= 433.64 \text{ lb} = 434 \text{ lb} \qquad \textbf{Ans.}$$

Using this result and applying the sine law to Fig. *b*, yields

$$\frac{\sin(45° + \phi)}{850} = \frac{\sin 30°}{433.64} \qquad \phi = 56.5° \qquad \textbf{Ans.}$$

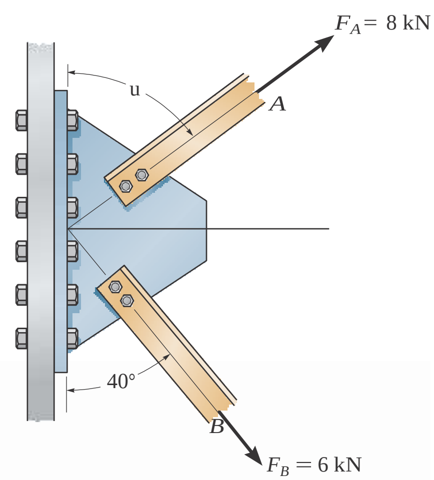


(a)



(b)

22-16. The plate is subjected to the two forces at A and B as shown. If $\theta = 60°$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of cosines (Fig. *b*), we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100°}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN}$$

The angle θ can be determined using law of sines (Fig. *b*).

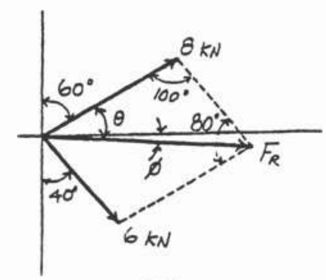
$$\frac{\sin \theta}{6} = \frac{\sin 100°}{10.80}$$

$$\sin \theta = 0.5470$$

$$\theta = 33.16°$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16° - 30° = 3.16°$$

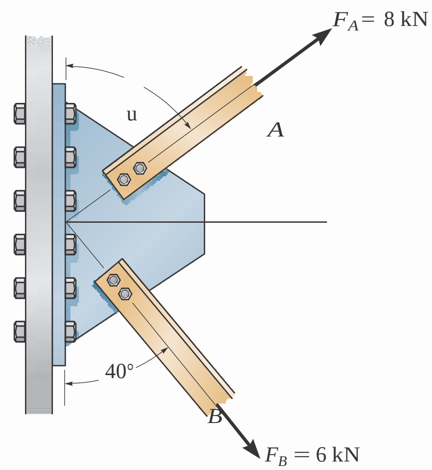


(a)

(b)

2-17. Determine the angle 0 for connecting member *A* to

horizontally to the right. Also, what is the magnitude of the resultant force?



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig .*b*), we have

$$\frac{\sin(90° - u)}{6} = \frac{\sin 50°}{8}$$

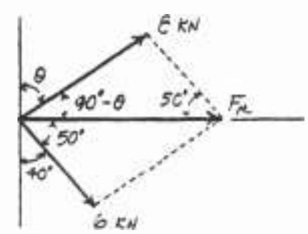
$$\sin(90° - u) = 0.5745$$

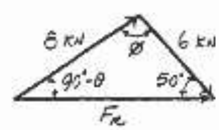
$$u = 54.93° = 54.9° \qquad \textbf{Ans.}$$

From the triangle, $f = 180° - (90° - 54.93°) - 50° = 94.93°$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93°}$$

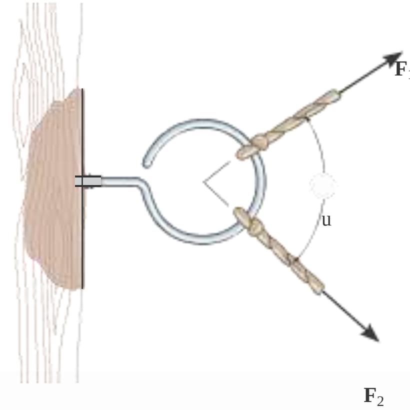
$$= 10.4 \text{ kN} \qquad \textbf{Ans.}$$





(b)

2-18. Two forces act on the screw eye. If $F_1 = 400$ N and $F_2 = 600$ N, determine the angle 0 (0° … 0 … 180°) between them, so that the resultant force has a magnitude of $F_R = 800$ N.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively. Applying law of cosines to Fig. *b*,

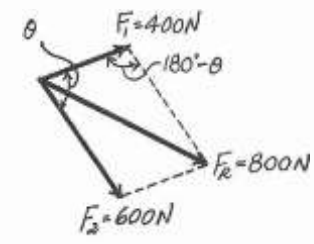
$$800 = \sqrt{400^2 + 600^2 - 2(400)(600)\cos(180° - \theta°)}$$

$$800^2 = 400^2 + 600^2 - 480000\cos(180° - \theta)$$

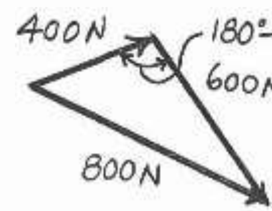
$$\cos(180° - \theta) = -0.25$$

$$180° - \theta = 104.48$$

$$\theta = 75.52° = 75.5° \qquad \textbf{Ans.}$$

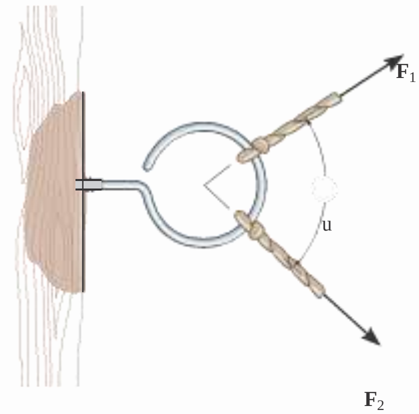


(a)



(b)

2-19. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If their lines of action are at an angle θ apart and the magnitude of each force is $F_1 = F_2 = F$, determine the magnitude of the resultant force $\mathbf{F}_R$ and the angle between $\mathbf{F}_R$ and $\mathbf{F}_1$.



SOLUTION

$$\frac{F}{\sin \phi} = \frac{F}{\sin(\theta - \phi)}$$

$$\sin(\theta - \phi) = \sin \phi$$

$$\theta - \phi = \phi$$

$$\phi = \frac{\theta}{2} \qquad \textbf{Ans.}$$

$$F_R = \sqrt{(F)^2 + (F)^2 - 2(F)(F)\cos(180° - \theta)}$$

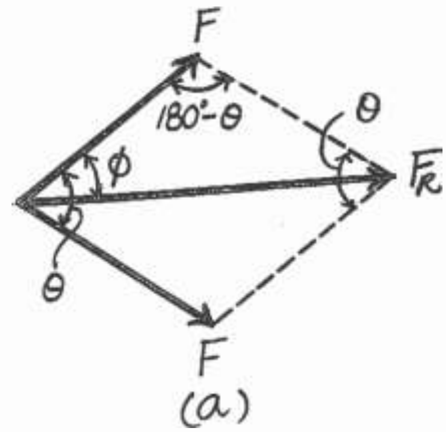
Since $\cos(180° - \theta) = -\cos\theta$

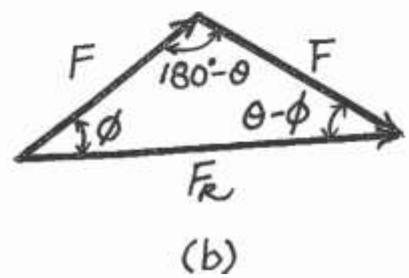
$$F_R = F\left(\sqrt{2}\right)\sqrt{1 + \cos\theta}$$

Since $\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 + \cos\theta}{2}}$

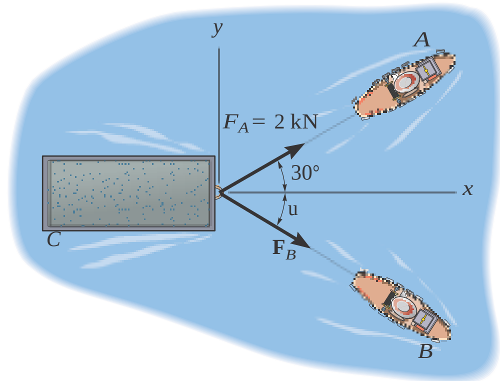
Then

$$F_R = 2F\cos\left(\frac{\theta}{2}\right) \qquad \textbf{Ans.}$$



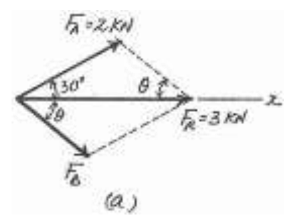


22-20. If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force $\mathbf{F}_B$ and its direction 0.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



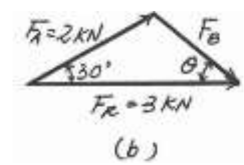
Applying the law of cosines to Fig. *b*,

$$F_B = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 30°}$$

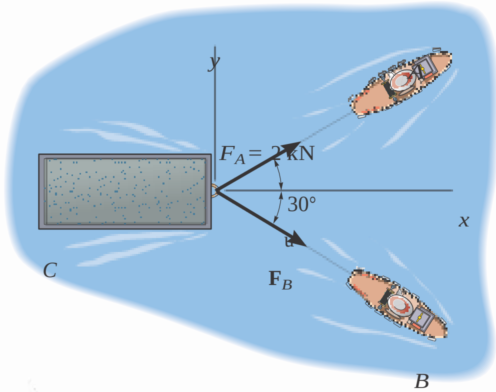
$$= 1.615\text{kN} = 1.61\text{ kN} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\theta}{2} = \frac{\sin 30°}{1.615} \qquad \theta = 38.3° \qquad \textbf{Ans.}$$

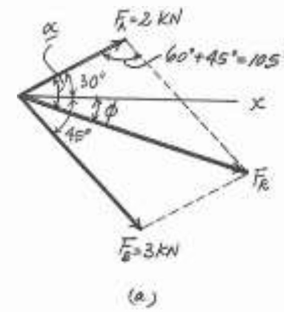


2-21. If $F_B = 3$ kN and $0 = 45°$, determine the magnitude of the resultant force of the two tugboats and its direction measured clockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



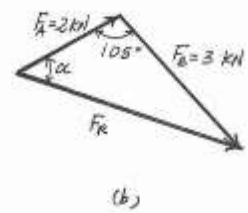
Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 105°}$$

$$= 4.013 \text{ kN} = 4.01 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b*, yields

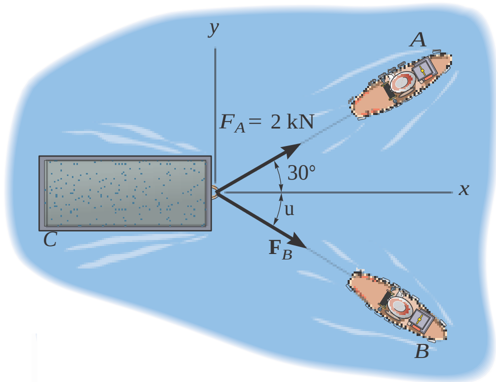
$$\frac{\sin \alpha}{3} = \frac{\sin 105°}{4.013} \qquad \alpha = 46.22°$$



Thus, the direction angle ϕ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\phi = \alpha - 30° = 46.22° - 30° = 16.2° \qquad \textbf{Ans.}$$

2-22. If the resultant force of the two tugboats is required to be directed towards the positive x axis, and $\mathbf{F}_B$ is to be a minimum, determine the magnitude of $\mathbf{F}_R$ and $\mathbf{F}_B$ and the angle 0.



SOLUTION

For $\mathbf{F}_B$ to be minimum, it has to be directed perpendicular to $\mathbf{F}_R$. Thus,

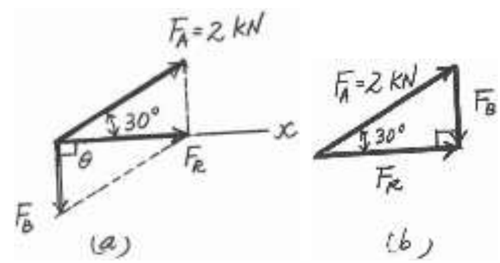
$$\theta = 90^\circ \qquad \textbf{Ans.}$$

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

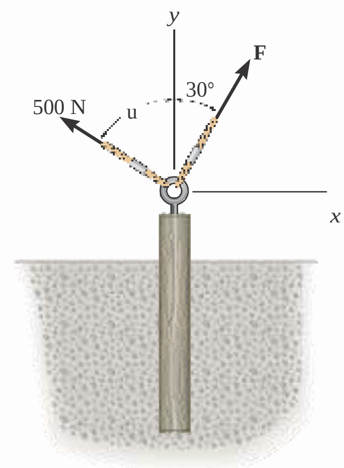
By applying simple trigonometry to Fig. *b*,

$$F_B = 2 \sin 30^\circ = 1 \text{ kN} \qquad \textbf{Ans.}$$

$$F_R = 2 \cos 30^\circ = 1.73 \text{ kN} \qquad \textbf{Ans.}$$



2-23. Two forces act on the screw eye. If $F = 600$ N, determine the magnitude of the resultant force and the angle 0 if the resultant force is directed vertically upward.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b* respectively. Applying law of sines to Fig. *b*,

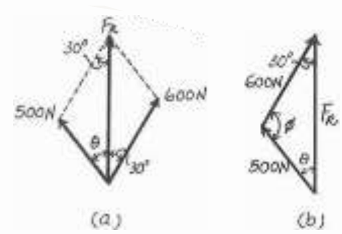
$$\frac{\sin\theta}{600} = \frac{\sin 30^\circ}{500}; \quad \sin\theta = 0.6 \quad \theta = 36.87^\circ = 36.9^\circ$$ **Ans.**

Using the result of θ,

$$\phi = 180^\circ - 30^\circ - 36.87^\circ = 113.13^\circ$$

Again, applying law of sines using the result of ϕ,

$$\frac{F_R}{\sin 113.13^\circ} = \frac{500}{\sin 30^\circ}; \quad F_R = 919.61 \text{ N} = 920 \text{ N}$$ **Ans.**



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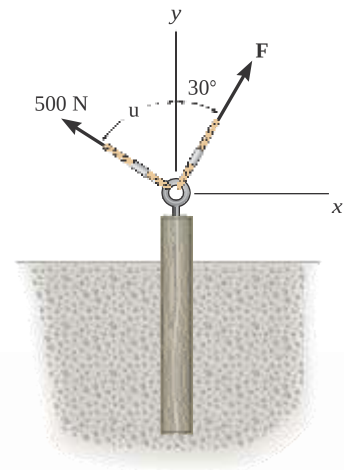
2-24. Two forces are applied at the end of a screw eye in order to remove the post. Determine the angle θ ($0° \leq \theta \leq 90°$) and the magnitude of force **F** so that the resultant force acting on the post is directed vertically upward and has a magnitude of 750 N.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig. *b*), we have

$$\frac{\sin \phi}{750} = \frac{\sin 30°}{500}$$

$$\sin \phi = 0.750$$

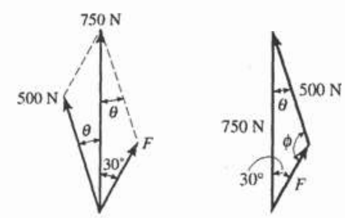
$$\phi = 131.41° \text{ (By observation, } \phi > 90°\text{)}$$

Thus,

$\theta = 180° - 30° - 131.41° = 18.59° = 18.6°$ **Ans.**

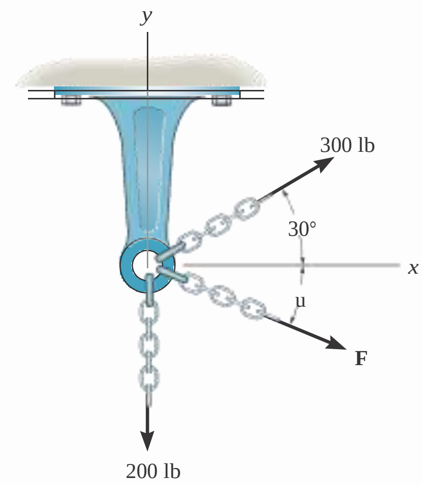
$$\frac{F}{\sin 18.59°} = \frac{500}{\sin 30°}$$

$F = 319$ N **Ans.**



(a) (b)

2-25. Three chains act on the bracket such that they create a resultant force having a magnitude of 500 lb. If two of the chains are subjected to known forces, as shown, determine the angle 0 of the third chain measured clockwise from the positive x axis, so that the magnitude of force **F** in this chain is a *minimum*. All forces lie in the x–y plane. What is the magnitude of **F**? *Hint*: First find the resultant of the two known forces. Force **F** acts in this direction.



SOLUTION

Cosine law:

$$F_{R1} = \sqrt{300^2 + 200^2 - 2(300)(200)\cos 60^\circ} = 264.6 \text{ lb}$$

Sine law:

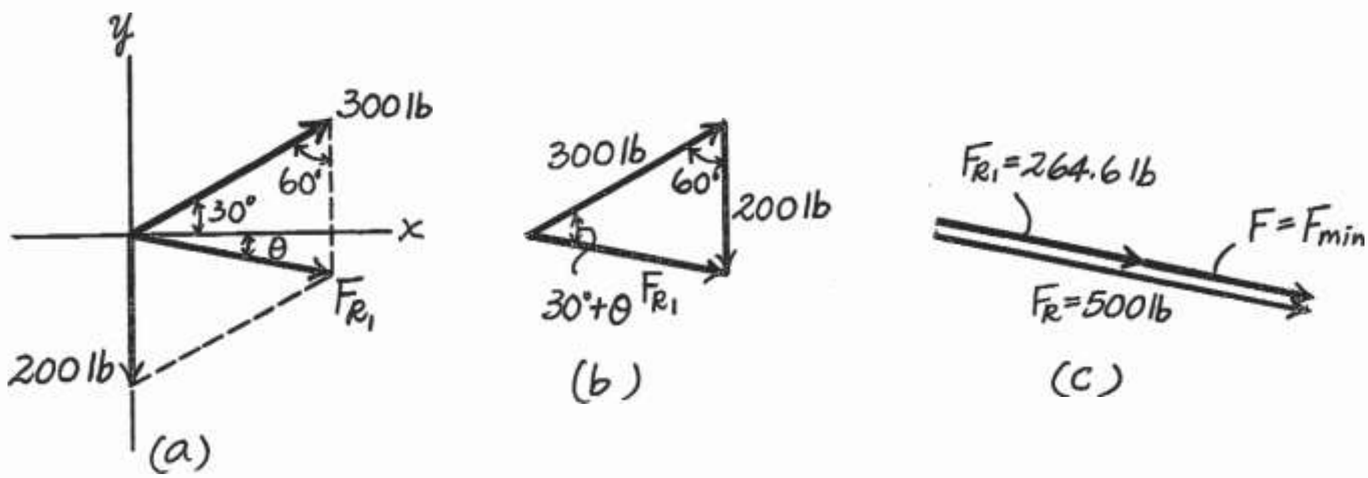
$$\frac{\sin(30^\circ + u)}{200} = \frac{\sin 60^\circ}{264.6} \qquad u = 10.9^\circ$$ **Ans.**

When **F** is directed along $\mathbf{F}_{R1}$, F will be minimum to create the resultant force.

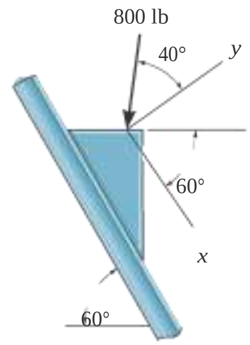
$$F_R = F_{R1} + F$$

$$500 = 264.6 + F_{min}$$

$$F_{min} = 235 \text{ lb}$$ **Ans.**



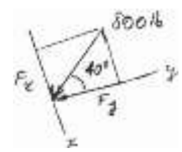
2-26. Determine the x and y components of the 800-lb force.



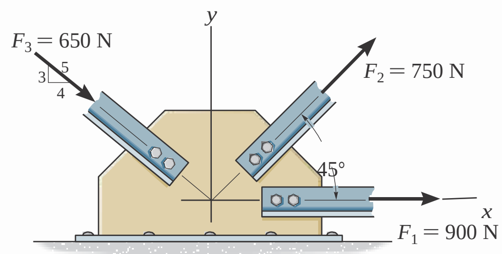
SOLUTION

$F_x = 800 \sin 40° = 514 \text{ lb}$ **Ans.**

$F_y = -800 \cos 40° = -613 \text{ lb}$ **Ans.**



2-27. Resolve each force acting on the gusset plate into its x and y components, and express each force as a Cartesian vector.



SOLUTION

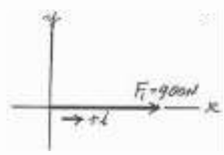
$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\}\text{ N}$ **Ans.**

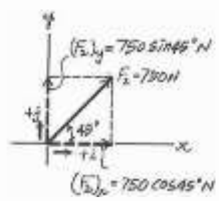
$\mathbf{F}_2 = \{750 \cos 45°(+\mathbf{i}) + 750 \sin 45°(+\mathbf{j})\}\text{ N}$

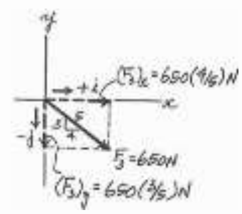
$= \{530\mathbf{i} + 530\mathbf{j}\}\text{ N}$ **Ans.**

$$\mathbf{F}_3 = \left\{650\left(\frac{4}{5}\right)(+\mathbf{i}) + 650\left(\frac{3}{5}\right)(-\mathbf{j})\right\}\text{ N}$$

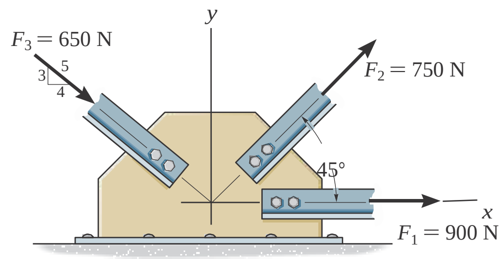
$= \{520\,\mathbf{i} - 390\mathbf{j})\}\text{ N}$ **Ans.**







22-28. Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 900 \text{ N} \qquad (F_1)_y = 0$$

$$(F_2)_x = 750 \cos 45° = 530.33 \text{ N} \qquad (F_2)_y = 750 \sin 45° = 530.33 \text{ N}$$

$$(F_3)_x = 650\left(\frac{4}{5}\right) = 520 \text{ N} \qquad (F_3)_y = 650\left(\frac{3}{5}\right) = 390 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900 + 530.33 + 520 = 1950.33 \text{ N} \rightarrow$$

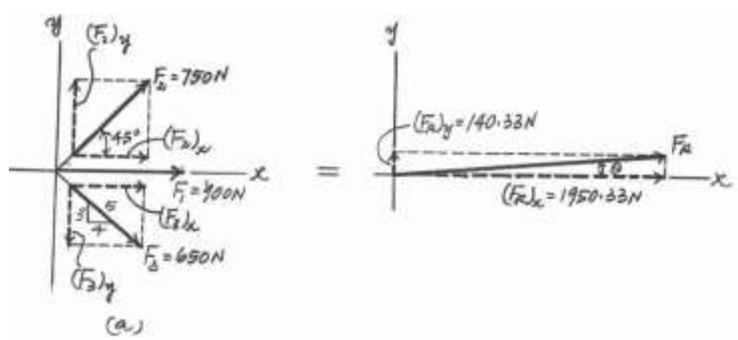
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 530.33 - 390 = 140.33 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

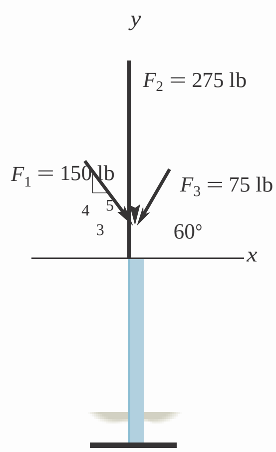
$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{1950.33^2 + 140.33^2} = 1955 \text{ N} = 1.96 \text{ kN} \quad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{140.33}{1950.33}\right) = 4.12° \qquad \textbf{Ans.}$$



2-29. Express each of the three forces acting on the column in Cartesian vector form and compute the magnitude of the resultant force.



SOLUTION

$$\mathbf{F}_1 = 150\left(\frac{3}{5}\right)\mathbf{i} - 150\left(\frac{4}{5}\right)\mathbf{j}$$

$$\mathbf{F}_1 = \{90\mathbf{i} - 120\mathbf{j}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_2 = \{-275\mathbf{j}\}\text{ lb}$$ **Ans.**

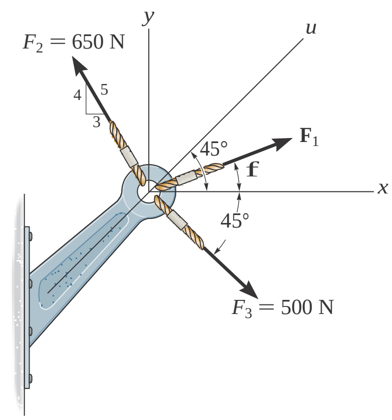
$$\mathbf{F}_3 = -75\cos 60^\circ\mathbf{i} - 75\sin 60^\circ\mathbf{j}$$

$$\mathbf{F}_3 = \{-37.5\mathbf{i} - 65.0\mathbf{j}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{52.5\mathbf{i} - 460\mathbf{j}\}\text{ lb}$$

$$F_R = \sqrt{(52.5)^2 + (-460)^2} = 463\text{ lb}$$ **Ans.**

2-30. The magnitude of the resultant force acting on the bracket is to be 400 N. Determine the magnitude of $\mathbf{F}_1$ if $f = 30°$.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = F_1 \cos 30° = 0.8660F_1 \qquad (F_1)_y = F_1 \sin 30° = 0.5F_1$$

$$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N} \qquad (F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$$

$$(F_3)_x = 500 \cos 45° = 353.55 \text{ N} \qquad (F_3)_y = 500 \sin 45° = 353.55 \text{ N}$$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 0.8660F_1 - 390 + 353.55 = 0.8660F_1 - 36.45$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 0.5F_1 + 520 - 353.55 = 0.5F_1 + 166.45$$

Since the magnitude of the resultant force is $F_R = 400$ N, we can write

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}}$$

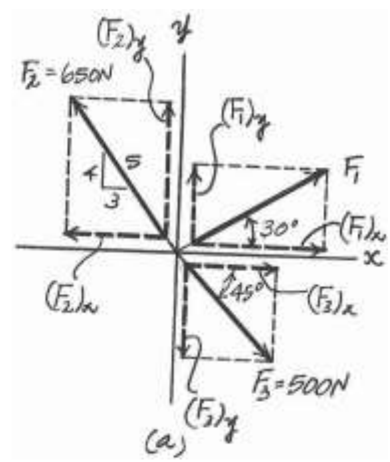
$$400 = \sqrt{(0.8660F_1 - 36.45)^2 + (0.5F_1 + 166.45)^2}$$

$$F_1^{\,2} + 103.32F_1 - 130967.17 = 0 \qquad \textbf{Ans.}$$

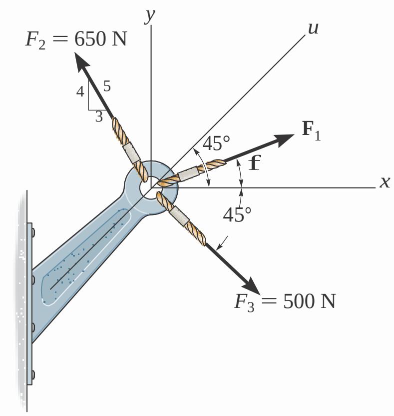
Solving,

$$F_1 = 314 \text{ N} \qquad \text{or} \qquad F_1 = -417 \text{ N} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{F}_1 = 417$ N must act in the opposite sense to that shown in the figure.



2-31. If the resultant force acting on the bracket is to be directed along the positive u axis, and the magnitude of $\mathbf{F}_1$ is required to be *minimum*, determine the magnitudes of the resultant force and $\mathbf{F}_1$.



SOLUTION

Rectangular Components: By referring to Figs. *a* and *b*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$$

$$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N} \qquad (F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$$

$$(F_3)_x = 500 \cos 45^\circ = 353.55 \text{ N} \qquad (F_3)_y = 500 \sin 45^\circ = 353.55 \text{ N}$$

$$(F_R)_x = F_R \cos 45^\circ = 0.7071 F_R \qquad (F_R)_y = F_R \sin 45^\circ = 0.7071 F_R$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad 0.7071 F_R = F_1 \cos\phi - 390 + 353.55 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad 0.7071 F_R = F_1 \sin\phi + 520 - 353.55 \qquad \textbf{(2)}$$

Eliminating F_R from Eqs. (1) and (2), yields

$$F_1 = \frac{202.89}{\cos\phi - \sin\phi} \qquad \textbf{(3)}$$

The first derivative of Eq. (3) is

$$\frac{dF_1}{d\phi} = \frac{\sin\phi + \cos\phi}{(\cos\phi - \sin\phi)^2} \qquad \textbf{(4)}$$

The second derivative of Eq. (3) is

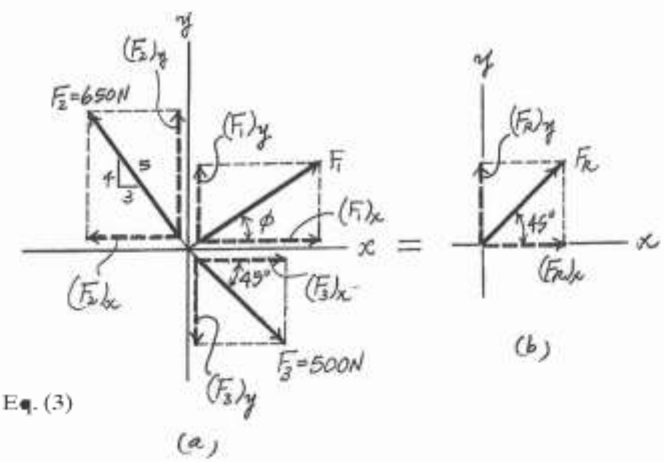
$$\frac{d^2F_1}{d\phi^2} = \frac{2(\sin\phi + \cos\phi)^2}{(\cos\phi - \sin\phi)^3} + \frac{1}{\cos\phi - \sin\phi} \qquad \textbf{(5)}$$

For $\mathbf{F}_1$ to be minimum, $\frac{dF_1}{d\phi} = 0$. Thus, from Eq. (4)

$$\sin\phi + \cos\phi = 0$$

$$\tan\phi = -1$$

$$\phi = -45^\circ$$



Substituting $\phi = -45^\circ$ into Eq. (5), yields

$$\frac{d^2F_1}{d\phi^2} = 0.7071 > 0$$

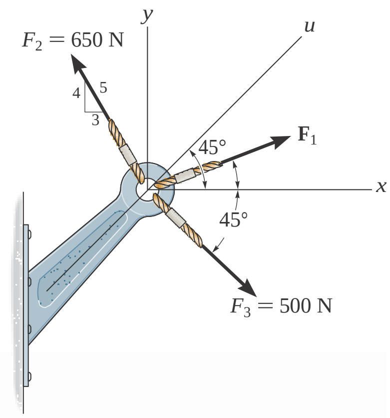
This shows that $\phi = -45^\circ$ indeed produces minimum F_1. Thus, from Eq. (3)

$$F_1 = \frac{202.89}{\cos(-45^\circ) - \sin(-45^\circ)} = 143.47 \text{ N} = 143 \text{ N} \qquad \textbf{Ans.}$$

Substituting $\phi = -45^\circ$ and $F_1 = 143.47$ N into either Eq. (1) or Eq. (2), yields

$$F_R = 91.9 \text{ N} \qquad \textbf{Ans.}$$

2-32. If the magnitude of the resultant force acting on the bracket is 600 N, directed along the positive u axis, determine the magnitude of **F** and its direction ϕ.



SOLUTION

Rectangular Components: By referring to Figs. *a* and *b*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$

$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N} \qquad (F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$

$(F_3)_x = 500 \cos 45° = 353.55 \text{ N} \qquad (F_3)_y = 500 \cos 45° = 353.55 \text{ N}$

$(F_R)_x = 600 \cos 45° = 424.26 \text{ N} \qquad (F_R)_y = 600 \sin 45° = 424.26 \text{ N}$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes, we have

$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x; \qquad 424.26 = F_1 \cos\phi - 390 + 353.55 \qquad$ **(1)**

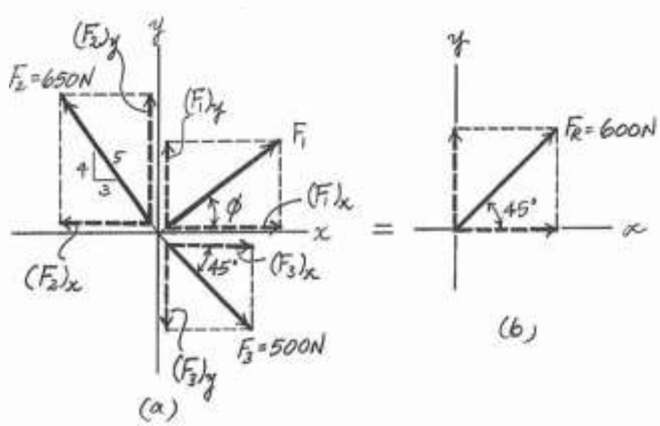
$$F_1 \cos\phi = 460.71$$

$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad 424.26 = F_1 \sin\phi + 520 - 353.55 \qquad$ **(2)**

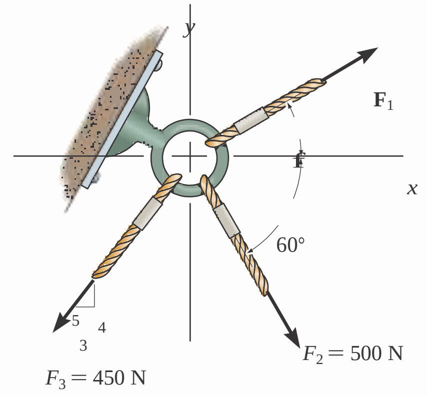
$$F_1 \sin\phi = 257.82$$

Solving Eqs. (1) and (2), yields

$\phi = 29.2° \qquad F_1 = 528 \text{ N} \qquad$ **Ans.**



2-33. If $F_1 = 600$ N and $\phi = 30°$, determine the magnitude of the resultant force acting on the eyebolt and its direction, measured clockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of each force can be written as

$$(F_1)_x = 600 \cos 30° = 519.62 \text{ N} \qquad (F_1)_y = 600 \sin 30° = 300 \text{ N}$$

$$(F_2)_x = 500 \cos 60° = 250 \text{ N} \qquad (F_2)_y = 500 \sin 60° = 433.01 \text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270 \text{ N} \qquad (F_3)_y = 450\left(\frac{4}{5}\right) = 360 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\xrightarrow{+}\ \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 519.62 + 250 - 270 = 499.62 \text{ N} \rightarrow$$

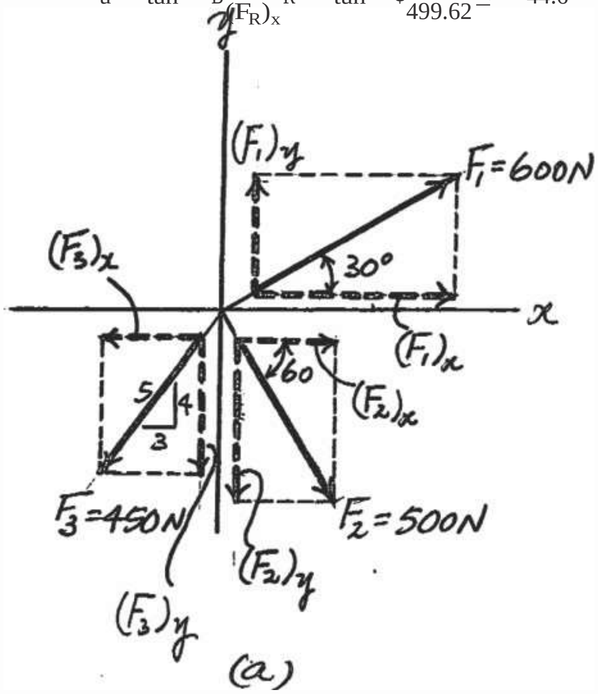
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 300 - 433.01 - 360 = -493.01 \text{ N} = 493.01 \text{ N} \downarrow$$

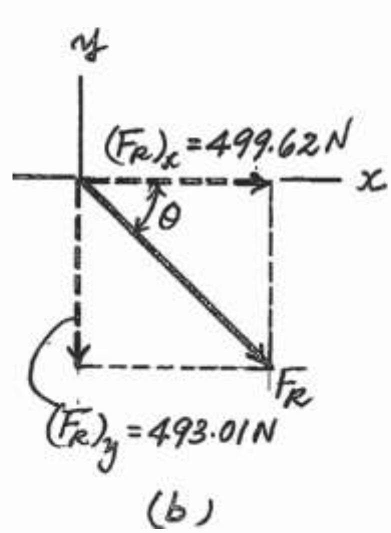
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{499.62^2 + 493.01^2} = 701.91 \text{ N} = 702 \text{ N}$$ **Ans.**

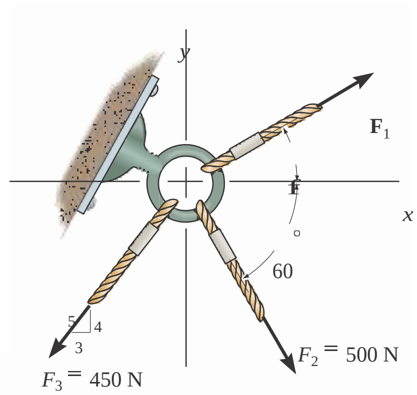
The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{493.01}{499.62}\right) = 44.6°$$ **Ans.**





2-34. If the magnitude of the resultant force acting on the eyebolt is 600 N and its direction measured clockwise from the positive x axis is $\theta = 30°$, determine the magnitude of $\mathbf{F}_1$ and the angle ϕ.



SOLUTION

Rectangular Components: By referring to Figs. *a* and *b*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = F_1 \cos \phi$ $\qquad$ $(F_1)_y = F_1 \sin \phi$

$(F_2)_x = 500 \cos 60° = 250 \text{ N}$ $\qquad$ $(F_2)_y = 500 \sin 60° = 433.01 \text{ N}$

$(F_3)_x = 450\left(\frac{3}{5}\right) = 270 \text{ N}$ $\qquad$ $(F_3)_y = 450\left(\frac{4}{5}\right) = 360 \text{ N}$

$(F_R)_x = 600 \cos 30° = 519.62 \text{ N}$ $\qquad$ $(F_R)_y = 600 \sin 30° = 300 \text{ N}$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x; \quad 519.62 = F_1 \cos \phi + 250 - 270$

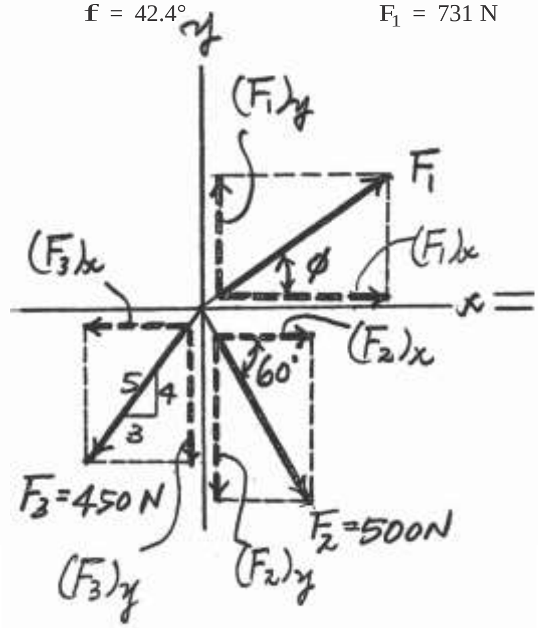
$$F_1 \cos \phi = 539.62 \qquad \textbf{(1)}$$

$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad -300 = F_1 \sin \phi - 433.01 - 360$

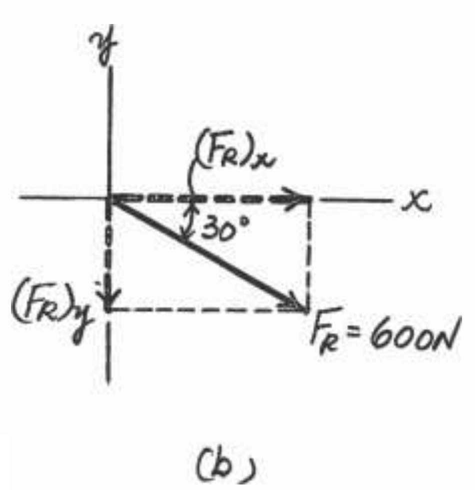
$$F_1 \sin \phi = 493.01 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$\phi = 42.4°$ $\qquad$ $F_1 = 731 \text{ N}$ $\qquad$ **Ans.**

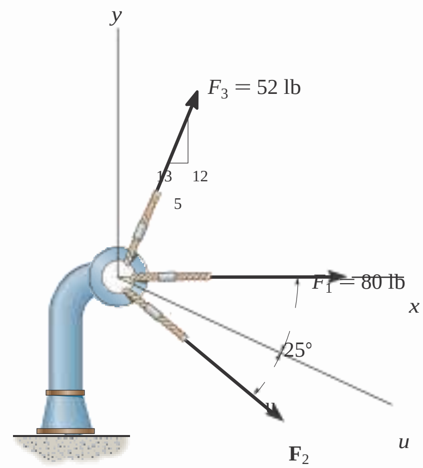


(a)



(b)

2-35. Three forces act on the bracket. Determine the magnitude and direction 0 of $\mathbf{F}_2$ so that the resultant force is directed along the positive *u* axis and has a magnitude of 50 lb.



SOLUTION

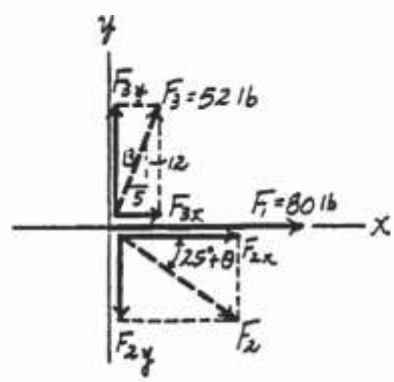
Scalar Notation: Summing the force components algebraically, we have

$$\stackrel{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad 50\cos 25° = 80 + 52\left(\frac{5}{13}\right) + F_2\cos(25° + u)$$

$$F_2\cos(25° + u) = -54.684 \qquad \textbf{(1)}$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad -50\sin 25° = 52\left(\frac{12}{13}\right) - F_2\sin(25° + u)$$

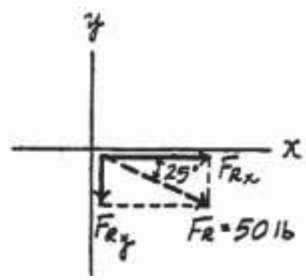
$$F_2\sin(25° + u) = 69.131 \qquad \textbf{(2)}$$



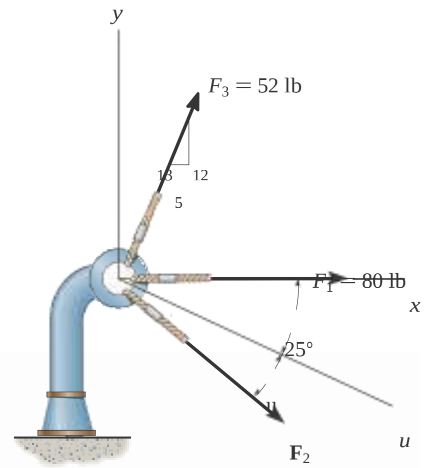
Solving Eqs. (1) and (2) yields

$25° + u = 128.35° \qquad u = 103°$ **Ans.**

$F_2 = 88.1$ lb **Ans.**



22-36. If $F_2 = 150$ lb and $\theta = 55°$, determine the magnitude and direction measured clockwise from the positive x axis, of the resultant force of the three forces acting on the bracket.



SOLUTION

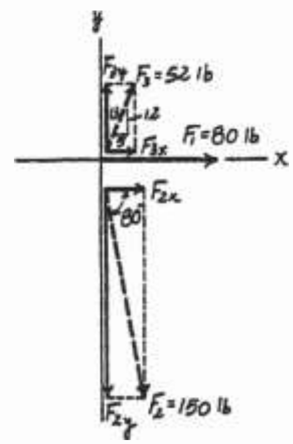
Scalar Notation: Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 80 + 52\left(\frac{5}{13}\right) + 150\cos 80°$$

$$= 126.05 \text{ lb} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 52\left(\frac{12}{13}\right) - 150\sin 80°$$

$$= -99.72 \text{ lb} = 99.72 \text{ lb} \downarrow$$

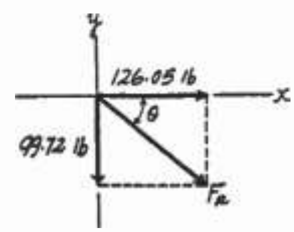


The magnitude of the resultant force $\mathbf{F}_R$ is

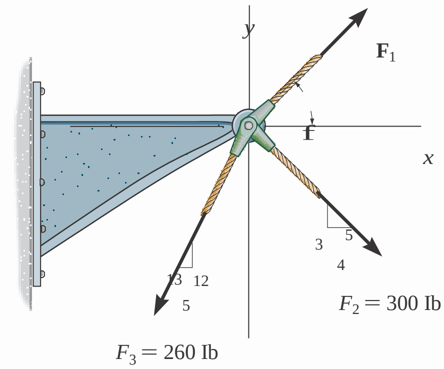
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{126.05^2 + 99.72^2} = 161 \text{ lb}$$ **Ans.**

The direction angle θ measured clockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{99.72}{126.05}\right) = 38.3°$$ **Ans.**



2-37. If $\phi = 30°$ and $F_1 = 250$ lb, determine the magnitude of the resultant force acting on the bracket and its direction measured clockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 250\cos 30° = 216.51 \text{ lb} \qquad (F_1)_y = 250\sin 30° = 125 \text{ lb}$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240 \text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180 \text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ lb}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 216.51 + 240 - 100 = 356.51 \text{ lb} \rightarrow$$

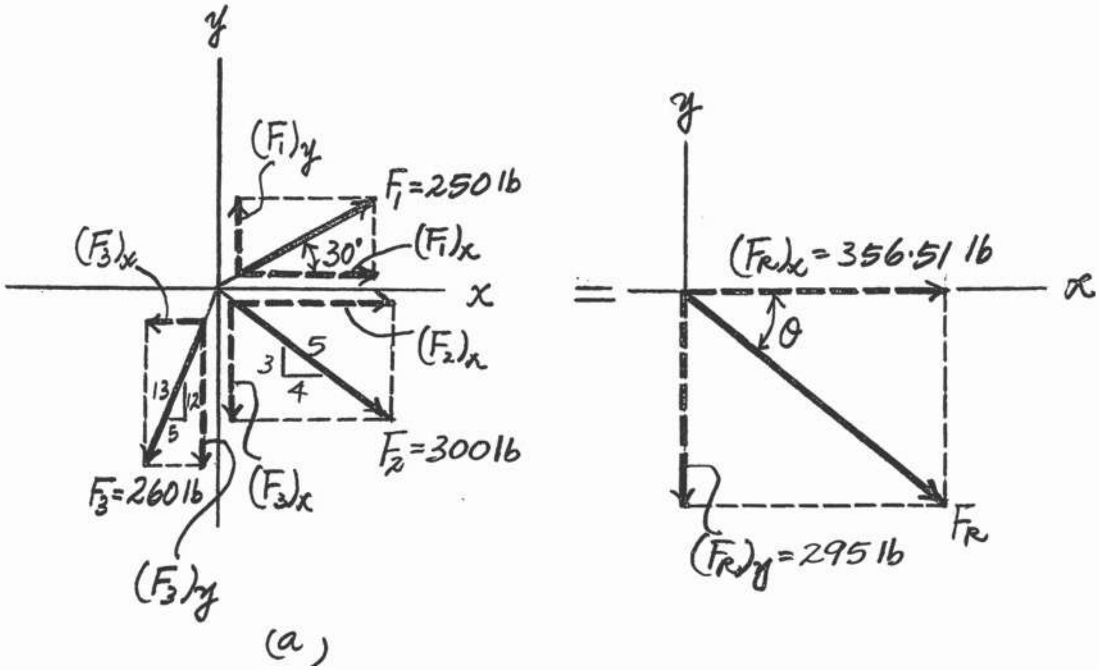
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 125 - 180 - 240 = -295 \text{ lb} = 295 \text{ lb} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

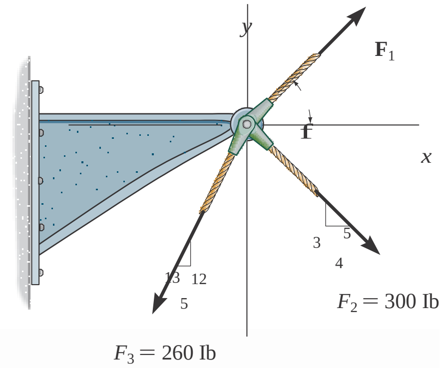
$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{356.51^2 + 295^2} = 463 \text{ lb}$$ **Ans.**

The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{295}{356.51}\right) = 39.6°$$ **Ans.**



2-38. If the magnitude of the resultant force acting on the bracket is 400 lb directed along the positive x axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240 \text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180 \text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ lb}$$

$$(F_R)_x = 400 \text{ lb} \qquad (F_R)_y = 0$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad 400 = F_1 \cos\phi + 240 - 100$$

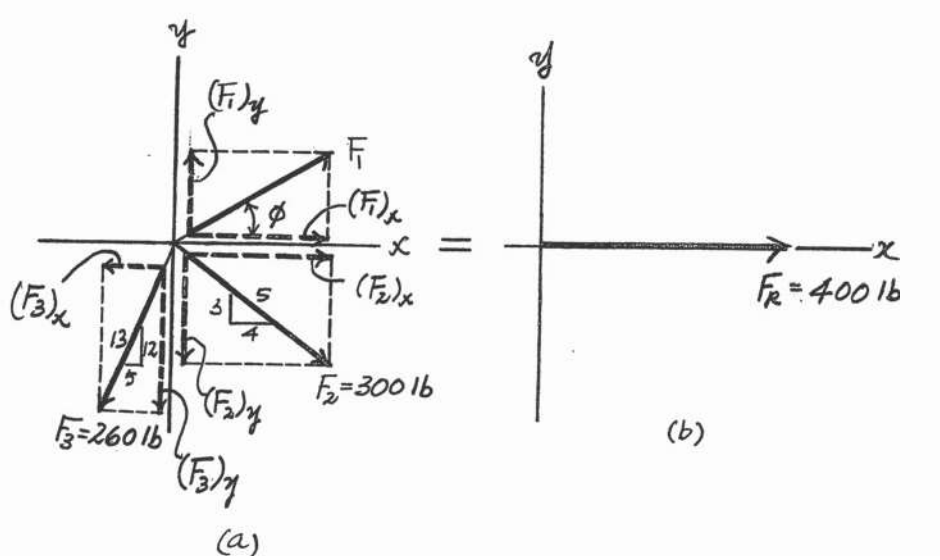
$$F_1 \cos\phi = 260 \qquad (1)$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad 0 = F_1 \sin\phi - 180 - 240$$

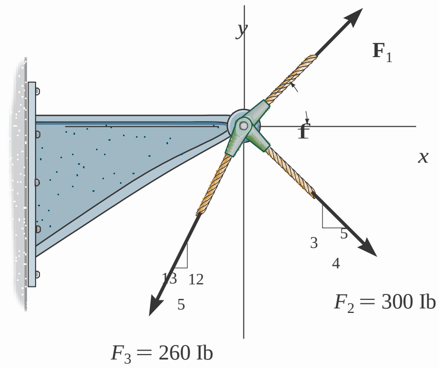
$$F_1 \sin\phi = 420 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$$\phi = 58.2^\circ \qquad F_1 = 494 \text{ lb} \qquad \textbf{Ans.}$$



2-39. If the resultant force acting on the bracket is to be directed along the positive x axis and the magnitude of $\mathbf{F}_1$ is required to be a minimum, determine the magnitudes of the resultant force and $\mathbf{F}_1$.



SOLUTION

Rectangular Components: By referring to Figs. *a* and *b*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1\cos\phi \qquad (F_1)_y = F_1\sin\phi$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240\text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180\text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ lb}$$

$$(F_R)_x = F_R \qquad (F_R)_y = 0$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad 0 = F_1\sin\phi - 180 - 240$$

$$F_1 = \frac{420}{\sin\phi} \qquad (1)$$

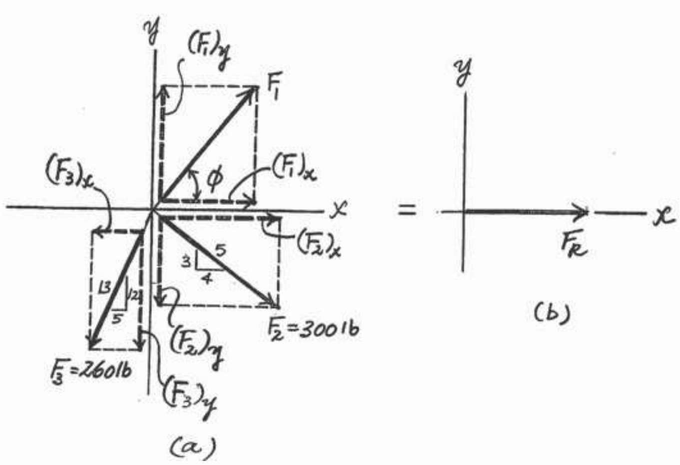
$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad F_R = F_1\cos\phi + 240 - 100 \qquad (2)$$

By inspecting Eq. (1), we realize that F_1 is minimum when $\sin\phi = 1$ or $\phi = 90°$. Thus,

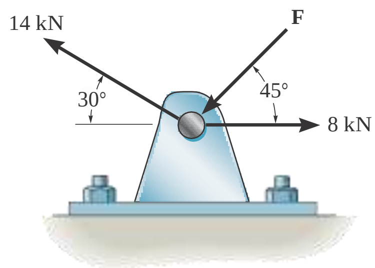
$$F_1 = 420\text{ lb}$$ **Ans.**

Substituting these results into Eq. (2), yields

$$F_R = 140\text{ lb}$$ **Ans.**



2-40. Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?



SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x;$ $\quad F_{Rx} = 8 - F\cos 45° - 14\cos 30°$

$= -4.1244 - F\cos 45°$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = -F\sin 45° + 14\sin 30°$

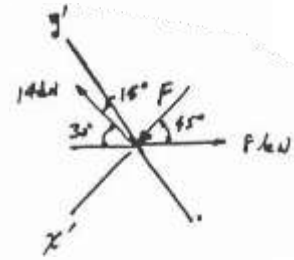
$= 7 - F\sin 45°$

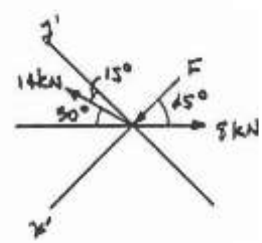
$$F_R^2 = (-4.1244 - F\cos 45°)^2 + (7 - F\sin 45°)^2 \qquad \textbf{(1)}$$

$$2F_R\frac{dF_R}{dF} = 2(-4.1244 - F\cos 45°)(-\cos 45°) + 2(7 - F\sin 45°)(-\sin 45°) = 0$$

$F = 2.03$ kN **Ans.**

From Eq. (1); $\quad F_R = 7.87$ kN **Ans.**





Also, from the figure require

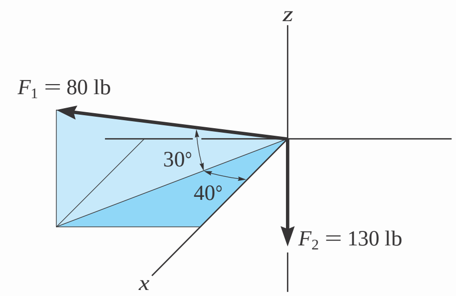
$(F_R)_{x'} = 0 = \Sigma F_{x'};$ $\quad F + 14\sin 15° - 8\cos 45° = 0$

$F = 2.03$ kN **Ans.**

$(F_R)_{y'} = \Sigma F_{y'};$ $\quad F_R = 14\cos 15° - 8\sin 45°$

$F_R = 7.87$ kN **Ans.**

2-41. Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the



SOLUTION

$\mathbf{F}_1 = \{80 \cos 30° \cos 40° \mathbf{i} - 80 \cos 30° \sin 40° \mathbf{j} + 80 \sin 30° \mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

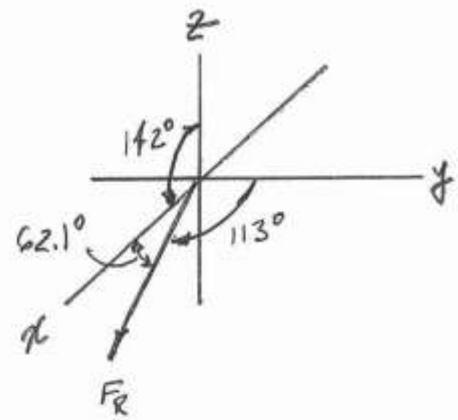
$\mathbf{F}_R = \{53.1\mathbf{i} - 44.5\mathbf{j} - 90.0\mathbf{k}\}$ lb

$F_R = \sqrt{(53.1)^2 + (-44.5)^2 + (-90.0)^2} = 114$ lb **Ans.**

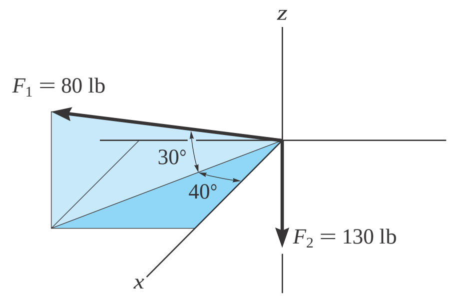
$\alpha = \cos^{-1}\left(\frac{53.1}{113.6}\right) = 62.1°$ **Ans.**

$\beta = \cos^{-1}\left(\frac{-44.5}{113.6}\right) = 113°$ **Ans.**

$\gamma = \cos^{-1}\left(\frac{-90.0}{113.6}\right) = 142°$ **Ans.**



2-42. Specify the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_2$ and express each force as a Cartesian vector.



SOLUTION

$\mathbf{F}_1 = \{80 \cos 30° \cos 40° \mathbf{i} - 80 \cos 30° \sin 40° \mathbf{j} + 80 \sin 30° \mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb **Ans.**

$\alpha_1 = \cos^{-1}\left(\frac{53.1}{80}\right) = 48.4°$ **Ans.**

$\beta_1 = \cos^{-1}\left(\frac{-44.5}{80}\right) = 124°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\frac{40}{80}\right) = 60°$ **Ans.**

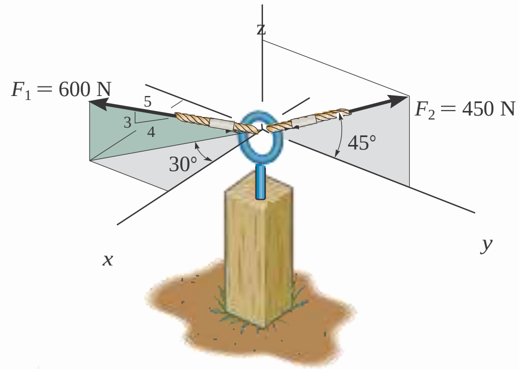
$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb **Ans.**

$\alpha_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$ **Ans.**

$\beta_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$ **Ans.**

$\gamma_2 = \cos^{-1}\left(\frac{-130}{130}\right) = 180°$ **Ans.**

2-43. Determine the coordinate direction angles of force $\mathbf{F}_1$.



SOLUTION

Rectangular Components: By referring to Figs. *a*, the *x*, *y*, and *z* components of $\mathbf{F}_1$ can be written as

$$(F_1)_x = 600\left(\frac{4}{5}\right)\cos 30°\text{ N} \qquad (F_1)_y = 600\left(\frac{4}{5}\right)\sin 30°\text{ N} \qquad (F_1)_z = 600\left(\frac{3}{5}\right)\text{ N}$$

Thus, $\mathbf{F}_1$ expressed in Cartesian vector form can be written as

$$\mathbf{F}_1 = 600\left\{\frac{4}{5}\cos 30°(+\mathbf{i}) + \frac{4}{5}\sin 30°(-\mathbf{j}) + \frac{3}{5}(+\mathbf{k})\right\}\text{ N}$$
$$= 600[0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}]\text{ N}$$

Therefore, the unit vector for $\mathbf{F}_1$ is given by

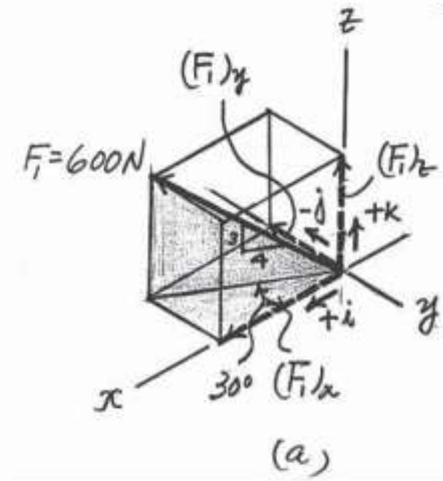
$$\mathbf{u}_{F_1} = \frac{\mathbf{F}_1}{F_1} = \frac{600(0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k})}{600} = 0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}$$

The coordinate direction angles of $\mathbf{F}_1$ are

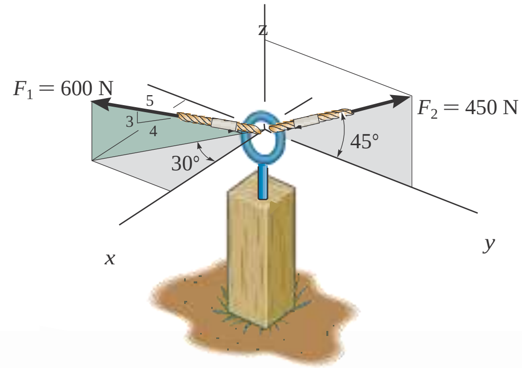
$$\alpha = \cos^{-1}(u_{F_1})_x = \cos^{-1}(0.6928) = 46.1° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}(u_{F_1})_y = \cos^{-1}(-0.4) = 114° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}(u_{F_1})_z = \cos^{-1}(0.6) = 53.1° \qquad \textbf{Ans.}$$

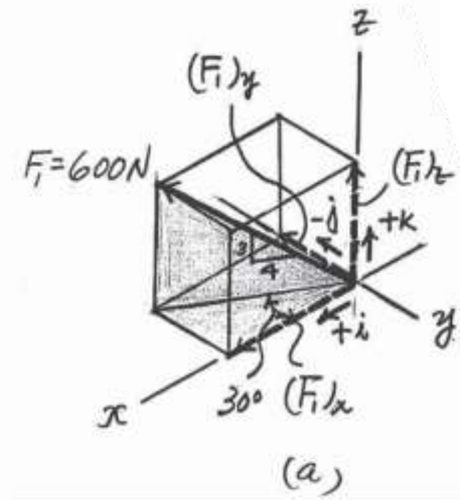


2-44. Determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, they are expressed in Cartesian vector form as



$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\sin 30°(-\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

$$= \{415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 450\cos 45°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$

$$= \{318.20\mathbf{j} + 318.20\mathbf{k}\} \text{ N}$$

Resultant Force: The resultant force acting on the eyebolt can be obtained by vectorally adding $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}) + (318.20\mathbf{j} + 318.20\mathbf{k})$$

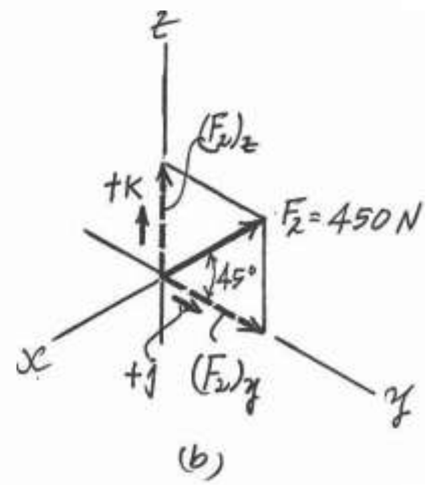
$$= \{415.69\mathbf{i} + 78.20\mathbf{j} + 678.20\mathbf{k}\} \text{ N}$$

The magnitude of $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(415.69)^2 + (78.20)^2 + (678.20)^2} = 799.29 \text{ N} = 799 \text{ N}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

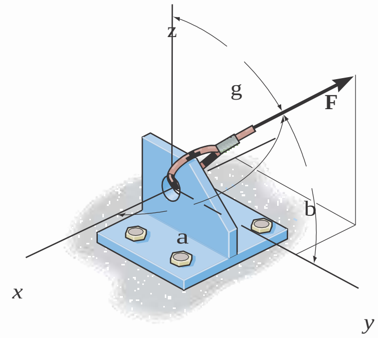


$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{415.69}{799.29}\right) = 58.7°$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{78.20}{799.29}\right) = 84.4°$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{678.20}{799.29}\right) = 32.0°$$ **Ans.**

2-45. The force **F** acts on the bracket within the octant shown. If $F = 400$ N, b = 60°, and g = 45°, determine the x, y, z components of **F**.



SOLUTION

Coordinate Direction Angles: Since β and γ are known, the third angle α can be determined from

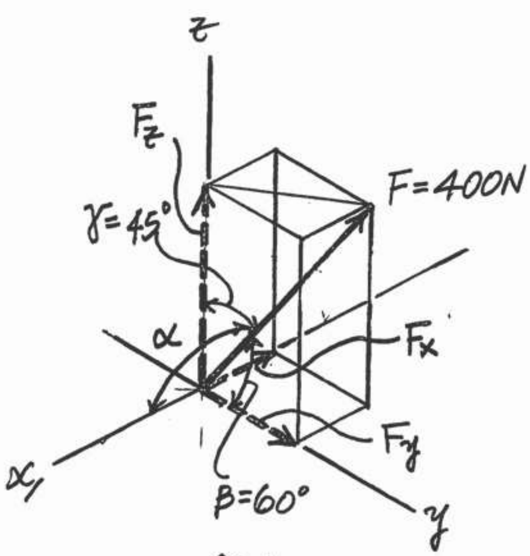
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\cos^2\alpha + \cos^2 60° + \cos^2 45° = 1$$
$$\cos\alpha = \pm 0.5$$

Since **F** is in the octant shown in Fig. a, θ_x must be greater than 90°. Thus, $\alpha = \cos^{-1}(-0.5) = 120°$.

Rectangular Components: By referring to Fig. a, the x, y, and z components of **F** can be written as

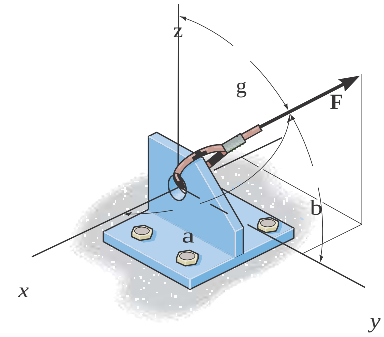
$$F_x = F\cos\alpha = 400\cos 120° = -200\text{ N} \qquad \textbf{Ans.}$$
$$F_y = F\cos\beta = 400\cos 60° = 200\text{ N} \qquad \textbf{Ans.}$$
$$F_z = F\cos\gamma = 400\cos 45° = 283\text{ N} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative x axis.



(a)

2-46. The force **F** acts on the bracket within the octant shown. If the magnitudes of the x and z components of **F** are $F_x = 300$ N and $F_z = 600$ N, respectively, and b = 60°, determine the magnitude of **F** and its y component. Also, find the coordinate direction angles a and g.



SOLUTION

Rectangular Components: The magnitude of **F** is given by

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = \sqrt{300^2 + F_y^2 + 600^2}$$

$$F^2 = F_y^2 + 450\ 000 \qquad (1)$$

The magnitude of $\mathbf{F}_y$ is given by

$$F_y = F\cos 60° = 0.5F \qquad (2)$$

Solving Eqs. (1) and (2) yields

$$F = 774.60\text{ N} = 775\text{ N} \qquad \textbf{Ans.}$$

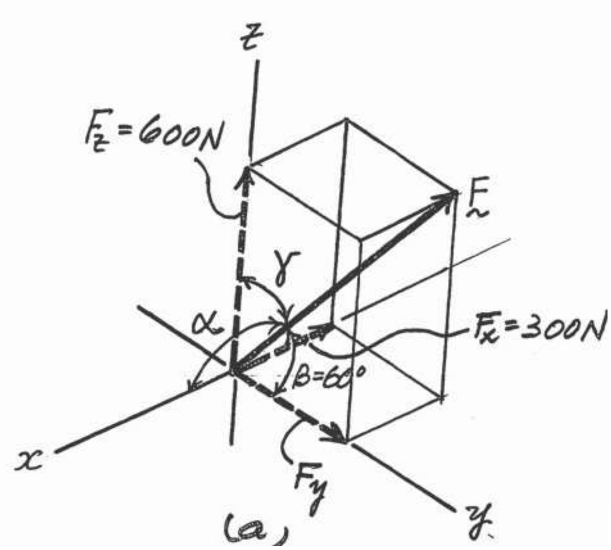
$$F_y = 387\text{ N} \qquad \textbf{Ans.}$$

Coordinate Direction Angles: Since **F** is contained in the octant so that $\mathbf{F}_x$ is directed towards the negative x axis, the coordinate direction angle θ_x is given by

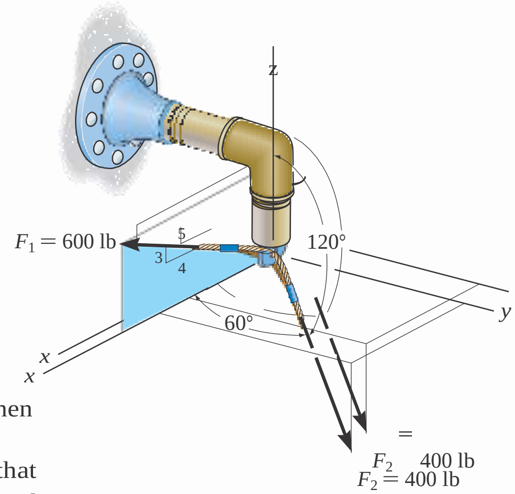
$$\alpha = \cos^{-1}\left(\frac{-F_x}{F}\right) = \cos^{-1}\left(\frac{-300}{774.60}\right) = 113° \qquad \textbf{Ans.}$$

The third coordinate direction angle is

$$\gamma = \cos^{-1}\left(\frac{-F_z}{F}\right) = \cos^{-1}\left(\frac{600}{774.60}\right) = 39.2° \qquad \textbf{Ans.}$$



2-47. Express each force acting on the pipe assembly in Cartesian vector form.



SOLUTION

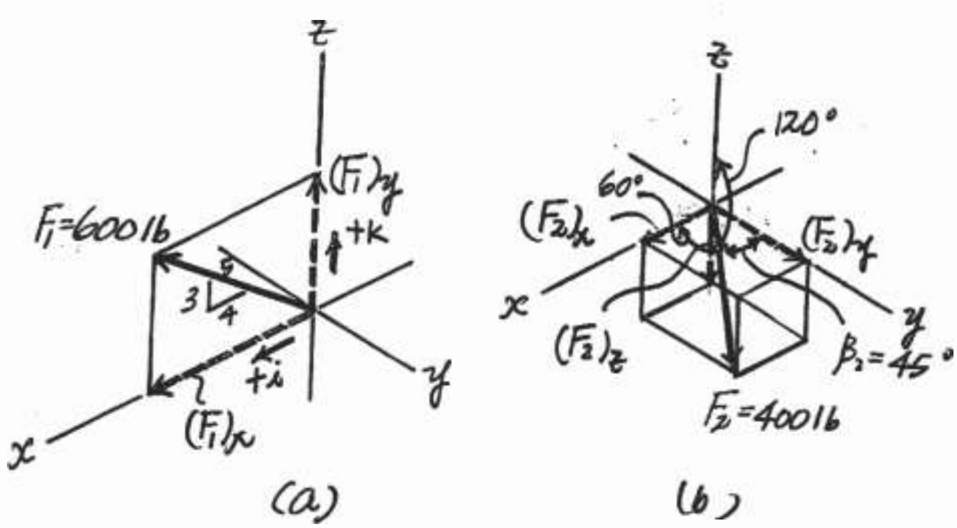
Rectangular Components: Since $\cos^2 \alpha_2 + \cos^2 \beta_2 + \cos^2 \gamma_2 = 1$, then $\cos \beta_2 = \pm\sqrt{1 - \cos^2 60° - \cos^2 120°} = \pm 0.7071$. However, it is required that $\beta_2 < 90°$, thus, $\beta_2 = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)(+\mathbf{i}) + 0\mathbf{j} + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

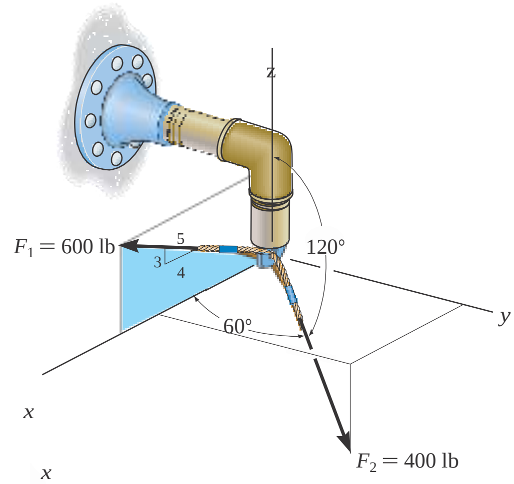
$$= [480\mathbf{i} + 360\mathbf{k}] \text{ lb}$$ **Ans.**

$$\mathbf{F}_2 = 400 \cos 60°\mathbf{i} + 400 \cos 45°\mathbf{j} + 400 \cos 120°\mathbf{k}$$

$$= [200\mathbf{i} + 283\mathbf{j} - 200\mathbf{k}] \text{ lb}$$ **Ans.**



2-48. Determine the magnitude and the direction of the resultant force acting on the pipe assembly.



SOLUTION

Force Vectors: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_2 = \pm\sqrt{1 - \cos^2 60° - \cos^2 120°} = \pm 0.7071$. However, it is required that $\beta_2 < 90°$, thus, $\beta_2 = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their *x*, *y*, and *z* components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)(+\mathbf{i}) + 0\mathbf{j} + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

$$= \{480\mathbf{i} + 360\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_2 = 400 \cos 60°\mathbf{i} + 400 \cos 45°\mathbf{j} + 400 \cos 120°\mathbf{k}$$

$$= \{200\mathbf{i} + 282.84\mathbf{j} - 200\mathbf{k}\} \text{ lb}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (480\mathbf{i} + 360\mathbf{k}) + (200\mathbf{i} + 282.84\mathbf{j} - 200\mathbf{k})$$

$$= \{680\mathbf{i} + 282.84\mathbf{j} + 160\mathbf{k}\} \text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$

$$= \sqrt{680^2 + 282.84^2 + 160^2} = 753.66 \text{ lb} = 754 \text{ lb} \qquad \textbf{Ans.}$$

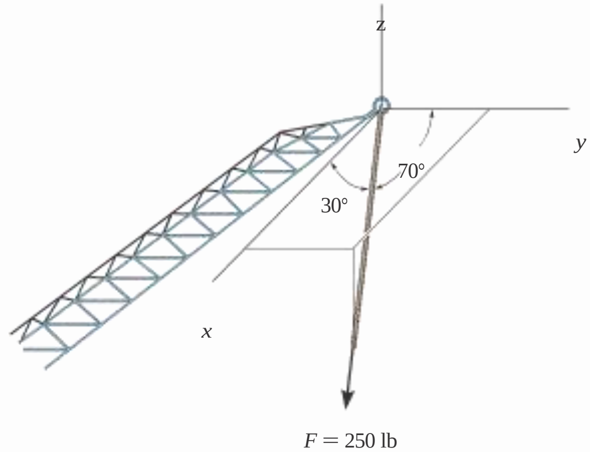
The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left(\frac{(F_R)_x}{F_R}\right) = \cos^{-1}\left(\frac{680}{753.66}\right) = 25.5° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{(F_R)_y}{F_R}\right) = \cos^{-1}\left(\frac{282.84}{753.66}\right) = 68.0° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{(F_R)_z}{F_R}\right) = \cos^{-1}\left(\frac{160}{753.66}\right) = 77.7° \qquad \textbf{Ans.}$$

2-49. Determine the magnitude and coordinate direction angles of $\mathbf{F}_1 = \{60\mathbf{i} - 50\mathbf{j} + 40\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-40\mathbf{i} - 85\mathbf{j} + 30\mathbf{k}\}$ N. Sketch each force on an *x*, *y*, *z* reference frame.



SOLUTION

$\mathbf{F}_1 = 60\,\mathbf{i} - 50\,\mathbf{j} + 40\,\mathbf{k}$

$F_1 = \sqrt{(60)^2 + (-50)^2 + (40)^2} = 87.7496 = 87.7\text{ N}$ **Ans.**

$\alpha_1 = \cos^{-1}\left(\frac{60}{87.7496}\right) = 46.9°$ **Ans.**

$\beta_1 = \cos^{-1}\left(\frac{-50}{87.7496}\right) = 125°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\frac{40}{87.7496}\right) = 62.9°$ **Ans.**

$\mathbf{F}_2 = -40\,\mathbf{i} - 85\,\mathbf{j} + 30\,\mathbf{k}$

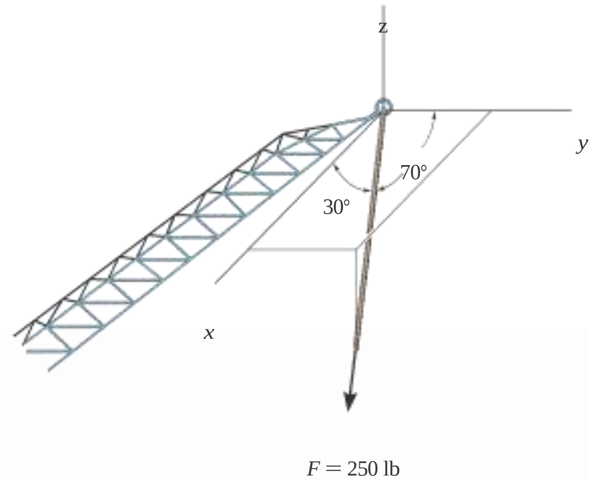
$F_2 = \sqrt{(-40)^2 + (-85)^2 + (30)^2} = 98.615 = 98.6\text{ N}$ **Ans.**

$\alpha_2 = \cos^{-1}\left(\frac{-40}{98.615}\right) = 114°$ **Ans.**

$\beta_2 = \cos^{-1}\left(\frac{-85}{98.615}\right) = 150°$ **Ans.**

$\gamma_2 = \cos^{-1}\left(\frac{30}{98.615}\right) = 72.3°$ **Ans.**

2-50. The cable at the end of the crane boom exerts a force of 250 lb on the boom as shown. Express **F** as a Cartesian vector.



SOLUTION

Cartesian Vector Notation: With a = 30° and b = 70°, the third coordinate direction angle g can be determined using Eq. 2–8.

$$\cos^2 a + \cos^2 b + \cos^2 g = 1$$

$$\cos^2 30° + \cos^2 70° + \cos^2 g = 1$$

$$\cos g = ; 0.3647$$

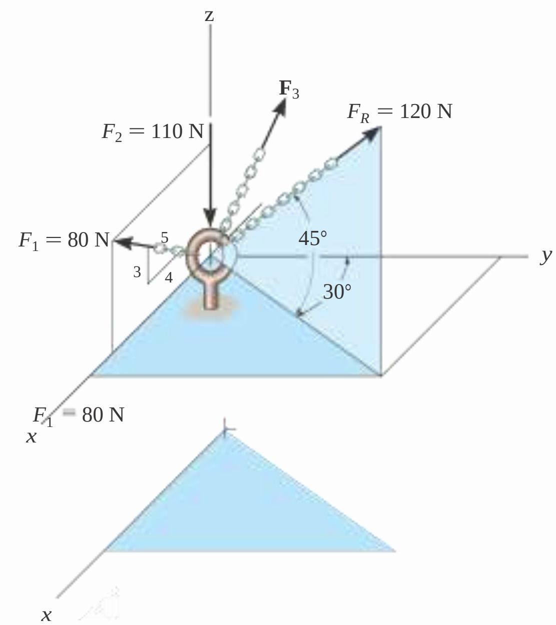
$$g = 68.61° \text{ or } 111.39°$$

By inspection, g = 111.39° since the force **F** is directed in negative octant.

$$\mathbf{F} = 2505\cos 30°\mathbf{i} + \cos 70°\mathbf{j} + \cos 111.39°6 \text{ lb}$$

$$= \{217\mathbf{i} + 85.5\mathbf{j} - 91.2\mathbf{k}\} \text{ lb}$$ **Ans.**

2-51. Three forces act on the ring. If the resultant force $\mathbf{F}_R$ has a magnitude and direction as shown, determine the magnitude and the coordinate direction angles of force $\mathbf{F}_3$.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_R = 120\{\cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k}\}\ \text{N}$$

$$= \{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_1 = 80\left\{\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right\}\ \text{N} = \{64.0\mathbf{i} + 48.0\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_2 = \{-110\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_3 = \{F_{3_x}\mathbf{i} + F_{3_y}\mathbf{j} + F_{3_z}\mathbf{k}\}\ \text{N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$\{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\} = \left\{\left(64.0 + F_{3_x}\right)\mathbf{i} + F_{3_y}\mathbf{j} + \left(48.0 - 110 + F_{3_z}\right)\mathbf{k}\right\}$$

Equating **i**, **j** and **k** components, we have

$$64.0 + F_{3_x} = 42.43 \qquad F_{3_x} = -21.57\ \text{N}$$

$$F_{3_y} = 73.48\ \text{N}$$

$$48.0 - 110 + F_{3_z} = 84.85 \qquad F_{3_z} = 146.85\ \text{N}$$

The magnitude of force $\mathbf{F}_3$ is

$$F_3 = \sqrt{F_{3_x}^2 + F_{3_y}^2 + F_{3_z}^2}$$

$$= \sqrt{(-21.57)^2 + 73.48^2 + 146.85^2}$$

$$= 165.62\ \text{N} = 166\ \text{N} \qquad \textbf{Ans.}$$

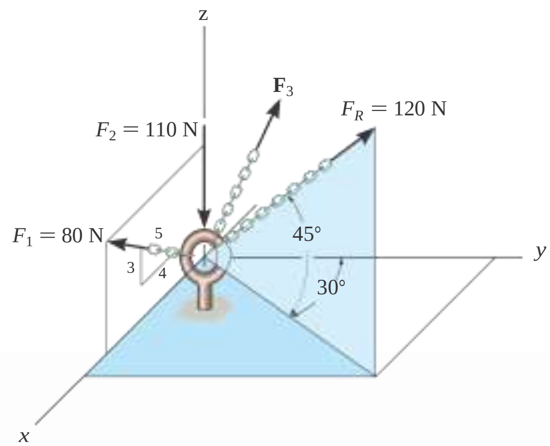
The coordinate direction angles for $\mathbf{F}_3$ are

$$\cos\alpha = \frac{F_{3_x}}{F_3} = \frac{-21.57}{165.62} \qquad \alpha = 97.5° \qquad \textbf{Ans.}$$

$$\cos\beta = \frac{F_{3_y}}{F_3} = \frac{73.48}{165.62} \qquad \beta = 63.7° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{F_{3_z}}{F_3} = \frac{146.85}{165.62} \qquad \gamma = 27.5° \qquad \textbf{Ans.}$$

***2-52.** Determine the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_R$.



SOLUTION

Unit Vector of $\mathbf{F}_1$ ***and*** $\mathbf{F}_R$**:**

$$\mathbf{u}_{F_1} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k} = 0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\mathbf{u}_R = \cos 45° \sin 30°\mathbf{i} + \cos 45° \cos 30°\mathbf{j} + \sin 45°\mathbf{k}$$

$$= 0.3536\mathbf{i} + 0.6124\mathbf{j} + 0.7071\mathbf{k}$$

Thus, the coordinate direction angles $\mathbf{F}_1$ and $\mathbf{F}_R$ are

$\cos \alpha_{F_1} = 0.8 \qquad \alpha_{F_1} = 36.9°$ **Ans.**

$\cos \beta_{F_1} = 0 \qquad \beta_{F_1} = 90.0°$ **Ans.**

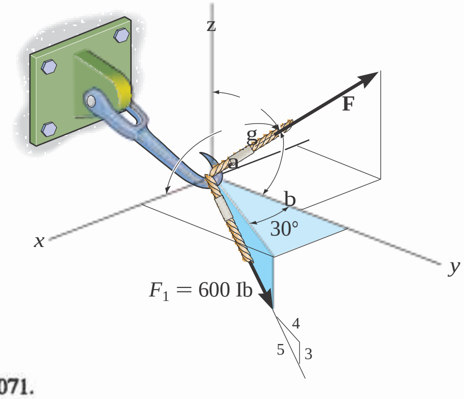
$\cos \gamma_{F_1} = 0.6 \qquad \gamma_{F_1} = 53.1°$ **Ans.**

$\cos \alpha_R = 0.3536 \qquad \alpha_R = 69.3°$ **Ans.**

$\cos \beta_R = 0.6124 \qquad \beta_R = 52.2°$ **Ans.**

$\cos \gamma_R = 0.7071 \qquad \gamma_R = 45.0°$ **Ans.**

2-53. If a = 120°, b 6 90°, g = 60°, and *F* = 400 lb, determine the magnitude and coordinate direction angles of the resultant force acting on the hook.



SOLUTION

Force Vectors: Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, then $\cos\beta = \pm\sqrt{1 - \cos^2 120° - \cos^2 60°} = \pm 0.7071$. However, it is required that $\beta < 90°$, thus, $\beta = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their *x*, *y*, and *z* components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\sin 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(-\mathbf{k})$$
$$= \{240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}\}\text{ lb}$$
$$\mathbf{F} = 400\cos 120°\mathbf{i} + 400\cos 45°\mathbf{j} + 400\cos 60°\mathbf{k}$$
$$= \{-200\mathbf{i} + 282.84\mathbf{j} + 200\mathbf{k}\}\text{ lb}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$= (240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}) + (-200\mathbf{i} + 282.84\mathbf{j} + 200\mathbf{k})$$
$$= \{40\mathbf{i} + 698.53\mathbf{j} - 160\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

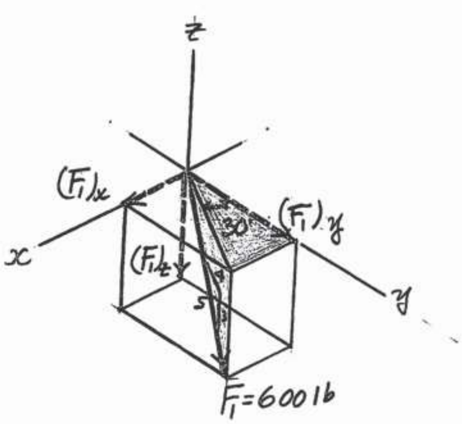
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{(40)^2 + (698.53)^2 + (-160)^2} = 717.74\text{ lb} = 718\text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

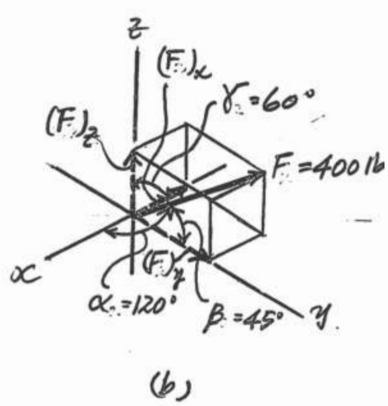
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{40}{717.74}\right) = 86.8° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{698.53}{717.74}\right) = 13.3° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-160}{717.74}\right) = 103° \qquad \textbf{Ans.}$$

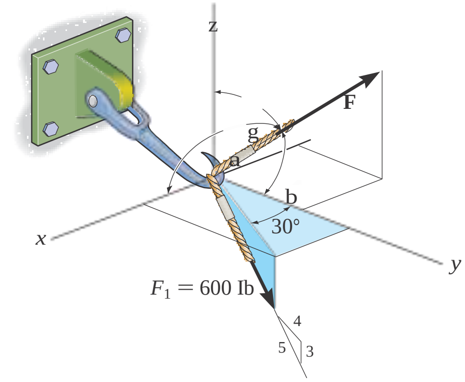


(a)



(b)

2-54. If the resultant force acting on the hook is $\mathbf{F}_R = \{-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k}\}$ lb, determine the magnitude and coordinate direction angles of **F**.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\sin 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(-\mathbf{k})$$

$$= \{240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}\}\text{ lb}$$

$$\mathbf{F} = F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$

$$-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k} = (240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}) + (F\cos\theta_x\mathbf{i} + F\cos\theta_y\mathbf{j} + F\cos\theta_z\mathbf{k})$$

$$-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k} = (240 + F\cos\alpha)\mathbf{i} + (415.69 + F\cos\beta)\mathbf{j} + (F\cos\gamma - 360)\mathbf{k}$$

Equating the **i**, **j**, and **k** components, we have

$$-200 = 240 + F\cos\theta_x$$

$$F\cos\alpha = -440 \qquad (1)$$

$$800 = 415.69 + F\cos\beta$$

$$F\cos\beta = 384.31 \qquad (2)$$

$$150 = F\cos\gamma - 360$$

$$F\cos\gamma = 510 \qquad (3)$$

Squaring and then adding Eqs. (1), (2), and (3), yields

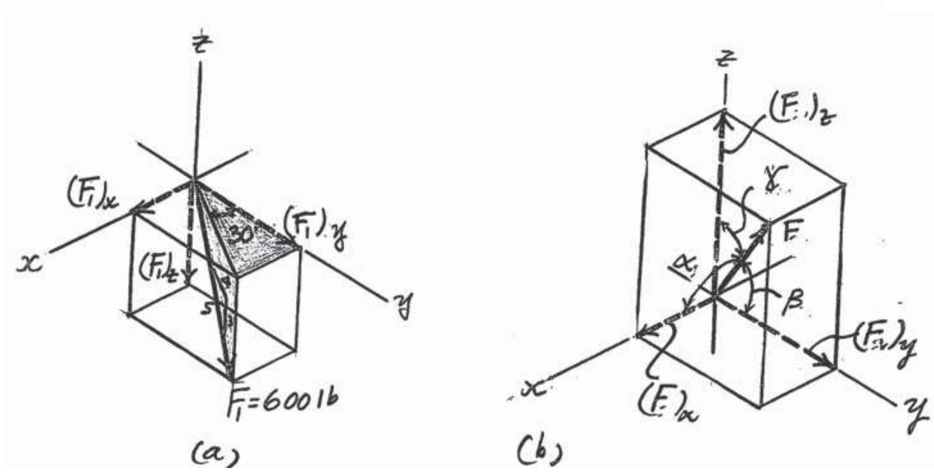
$$F^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 601392.49 \qquad (4)$$

However, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Thus, from Eq. (4)

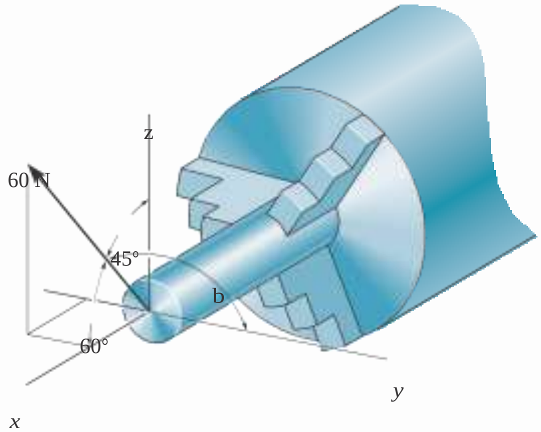
$$F = 775.49\text{ N} = 775\text{ N} \qquad \textbf{Ans.}$$

Substituting $F = 775.49$ N into Eqs. (1), (2), and (3), yields

$$\alpha = 125° \qquad \beta = 60.3° \qquad \gamma = 48.9° \qquad \textbf{Ans.}$$



2-55. The stock mounted on the lathe is subjected to a force of 60 N. Determine the coordinate direction angle b and express the force as a Cartesian vector.



SOLUTION

$$1 = \sqrt{\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma}$$

$$1 = \cos^2 60° + \cos^2 \beta + \cos^2 45°$$

$$\cos \beta = \pm 0.5$$

$$\beta = 60°, 120°$$

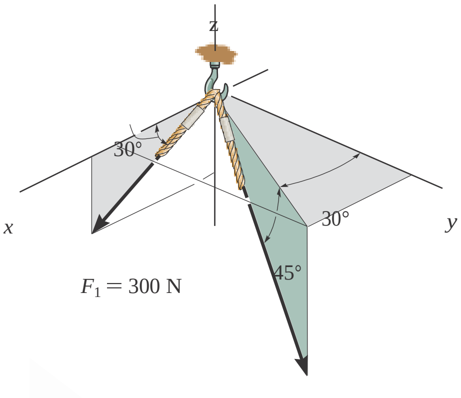
Use

$$\beta = 120°$$ **Ans.**

$$F = 60\text{ N}(\cos 60°\mathbf{i} + \cos 120°\mathbf{j} + \cos 45°\mathbf{k})$$

$$= \{30\mathbf{i} - 30\mathbf{j} + 42.4\mathbf{k}\}\text{ N}$$ **Ans.**

22-56. Express each force as a Cartesian vector.



SOLUTION

Rectangular Components: By referring to Figs. *a* and *b*, the *x*, *y*, and *z* components of $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written as

$(F_1)_x = 300 \cos 30° = 259.8$ N $\qquad (F_2)_x = 500 \cos 45° \sin 30° = 176.78$ N

$(F_1)_y = 0 \qquad (F_2)_y = 500 \cos 45° \cos 30° = 306.19$ N

$(F_1)_t = 300 \sin 30° = 150$ N $\qquad (F_2)_z = 500 \sin 45° = 353.55$ N

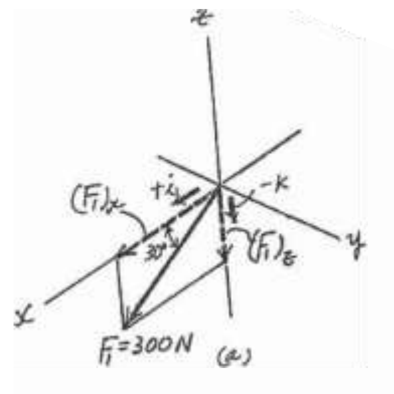
Thus, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written in Cartesian vector form as

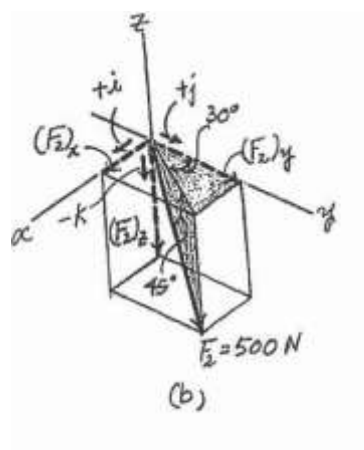
$$\mathbf{F}_1 = 259.81(+\mathbf{i}) + 0\mathbf{j} + 150(-\mathbf{k})$$

$$= \{260\mathbf{i} - 150\mathbf{k}\} \text{ N} \qquad \textbf{Ans.}$$

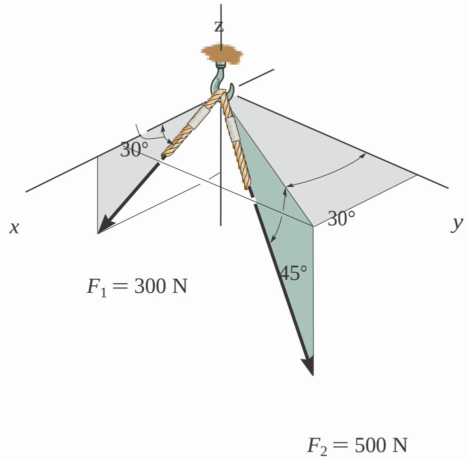
$$\mathbf{F}_2 = 176.78(+\mathbf{i}) + 306.19(+\mathbf{j}) + 353.55(-\mathbf{k})$$

$$= \{177\mathbf{i} + 306\mathbf{j} - 354\mathbf{k}\} \text{ N} \qquad \textbf{Ans.}$$





2-57. Determine the magnitude and coordinate direction angles of the resultant force acting on the hook.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expessed in Cartesian vector form as

$$\mathbf{F}_1 = 300\cos 30°(+\mathbf{i}) + 0\mathbf{j} + 300\sin 30°(-\mathbf{k})$$
$$= \{259.81\mathbf{i} - 150\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 500\cos 45°\sin 30°(+\mathbf{i}) + 500\cos 45°\cos 30°(+\mathbf{j}) + 500\sin 45°(-\mathbf{k})$$
$$= \{176.78\mathbf{i} - 306.19\mathbf{j} - 353.55\mathbf{k}\}\text{ N}$$

Resultant Force: The resultant force acting on the hook can be obtained by vectorally adding $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= (259.81\mathbf{i} - 150\mathbf{k}) + (176.78\mathbf{i} + 306.19\mathbf{j} - 353.55\mathbf{k})$$
$$= \{436.58\mathbf{i}) + 306.19\mathbf{j} - 503.55\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

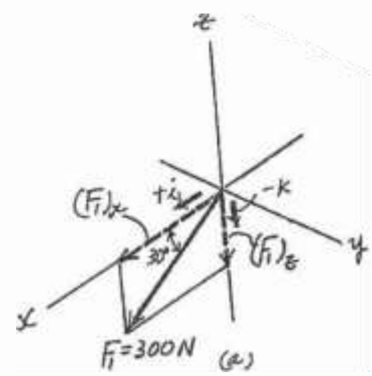
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2(F_R)_z{}^2}$$
$$= \sqrt{(436.58)^2 + (306.19)^2 + (-503.55)^2} = 733.43\text{ N} = 733\text{ N}$$ **Ans.**

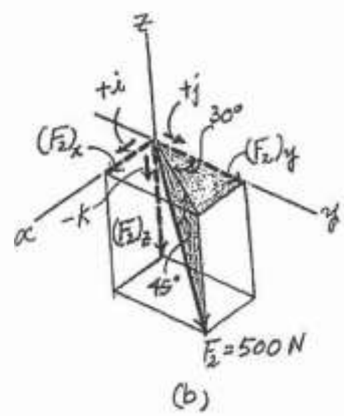
The coordinate direction angles of $\mathbf{F}_R$ are

$$\theta_x = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{436.58}{733.43}\right) = 53.5°$$ **Ans.**

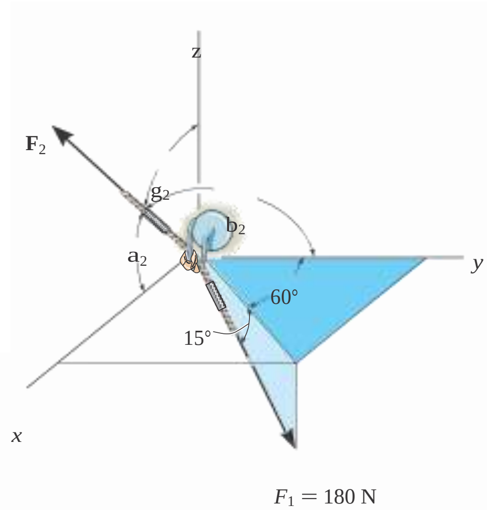
$$\theta_y = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{306.19}{733.43}\right) = 65.3°$$ **Ans.**

$$\theta_z = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-503.55}{733.43}\right) = 133°$$ **Ans.**





2-58. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$ so that the resultant of the two forces acts along the positive x axis and has a magnitude of 500 N.



SOLUTION

$$\mathbf{F}_1 = (180\cos 15°)\sin 60°\,\mathbf{i} + (180\cos 15°)\cos 60°\,\mathbf{j} - 180\sin 15°\,\mathbf{k}$$

$$= 150.57\,\mathbf{i} + 86.93\,\mathbf{j} - 46.59\,\mathbf{k}$$

$$\mathbf{F}_2 = F_2\cos\alpha_2\,\mathbf{i} + F_2\cos\beta_2\,\mathbf{j} + F_2\cos\gamma_2\,\mathbf{k}$$

$$\mathbf{F}_R = \{500\,\mathbf{i}\}\text{ N}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

i components:

$$500 = 150.57 + F_2\cos\alpha_2$$

$$F_{2x} = F_2\cos\alpha_2 = 349.43$$

j components:

$$0 = 86.93 + F_2\cos\beta_2$$

$$F_{2y} = F_2\cos\beta_2 = -86.93$$

k components:

$$0 = -46.59 + F_2\cos\gamma_2$$

$$F_{2z} = F_2\cos\gamma_2 = 46.59$$

Thus,

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2 + F_{2z}^2} = \sqrt{(349.43)^2 + (-86.93)^2 + (46.59)^2}$$

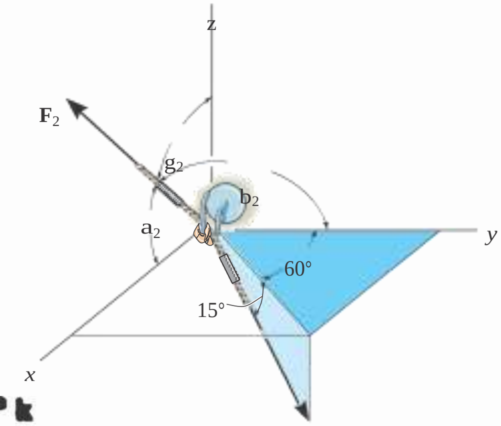
$F_2 = 363$ N **Ans**

$\alpha_2 = 15.8°$ **Ans**

$\beta_2 = 104°$ **Ans**

$\gamma_2 = 82.6°$ **Ans**

2-59. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$ so that the resultant of the two forces is zero.



SOLUTION

$$\mathbf{F}_1 = (180\cos 15°)\sin 60°\,\mathbf{i} + (180\cos 15°)\cos 60°\,\mathbf{j} - 180\sin 15°\,\mathbf{k}$$

$$= 150.57\,\mathbf{i} + 86.93\,\mathbf{j} - 46.59\,\mathbf{k}$$

$$\mathbf{F}_2 = F_2\cos\alpha_2\,\mathbf{i} + F_2\cos\beta_2\,\mathbf{j} + F_2\cos\gamma_2\,\mathbf{k}$$

$$\mathbf{F}_R = \mathbf{0}$$

i components :

$$0 = 150.57 + F_2\cos\alpha_2$$

$$F_2\cos\alpha_2 = -150.57$$

j components :

$$0 = 86.93 + F_2\cos\beta_2$$

$$F_2\cos\beta_2 = -86.93$$

k components :

$$0 = -46.59 + F_2\cos\gamma_2$$

$$F_2\cos\gamma_2 = 46.59$$

$$F_2 = \sqrt{(-150.57)^2 + (-86.93)^2 + (46.59)^2}$$

Solving.

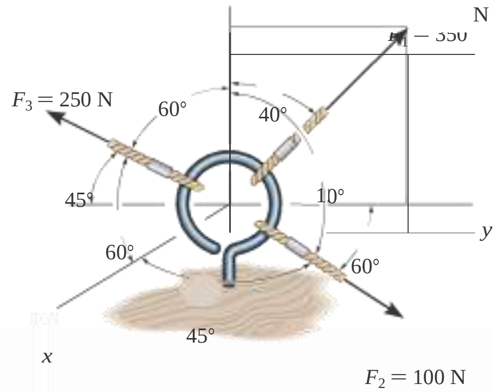
$F_2 = 180$ N **Ans**

$\alpha_2 = 147°$ **Ans**

$\beta_2 = 119°$ **Ans**

$\gamma_2 = 75.0°$ **Ans**

to the three forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Cartesian Vector Notation:

$\mathbf{F}_1 = 3505\sin 40°\mathbf{j} + \cos 40°\mathbf{k}6$ N

$= 5224.98\mathbf{j} + 268.12\mathbf{k}6$ N

$= 5225\mathbf{j} + 268\mathbf{k}6$ N **Ans.**

$\mathbf{F}_2 = 1005\cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}6$ N

$= 570.71\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}6$ N

$= 570.7\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}6$ N **Ans.**

$\mathbf{F}_3 = 2505\cos 60°\mathbf{i} + \cos 135°\mathbf{j} + \cos 60°\mathbf{k}6$ N

$= 5125.0\mathbf{i} - 176.78\mathbf{j} + 125.0\mathbf{k}6$ N

$= 5125\mathbf{i} - 177\mathbf{j} + 125\mathbf{k}6$ N **Ans.**

Resultant Force:

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$= 5170.71 + 125.02\mathbf{i} + 1224.98 + 50.0 - 176.782\mathbf{j} + 1268.12 - 50.0 + 125.02\mathbf{k}6$ N

$= 5195.71\mathbf{i} + 98.20\mathbf{j} + 343.12\mathbf{k}6$ N

The magnitude of the resultant force is

$$F_R = 2F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2$$

$$= 2195.71^2 + 98.20^2 + 343.12^2$$

$$= 407.03 \text{ N} = 407 \text{ N}$$ **Ans.**

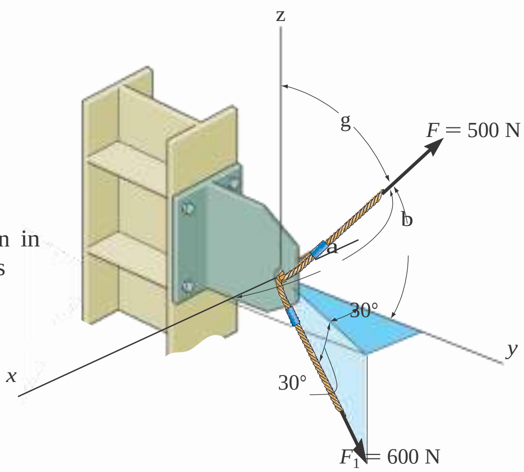
The coordinate direction angles are

$\cos a = \frac{F_{R_x}}{F_R} = \frac{195.71}{407.03}$ $a = 61.3°$ **Ans.**

$\cos b = \frac{F_{R_y}}{F_R} = \frac{98.20}{407.03}$ $b = 76.0°$ **Ans.**

$\cos g = \frac{F_{R_z}}{F_R} = \frac{343.12}{407.03}$ $g = 32.5°$ **Ans.**

2-61. If the resultant force acting on the bracket is directed along the positive y axis, determine the magnitude of the resultant force and the coordinate direction angles of **F** so that $\beta < 90°$.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}$ can be expressed in Cartesian vector form as

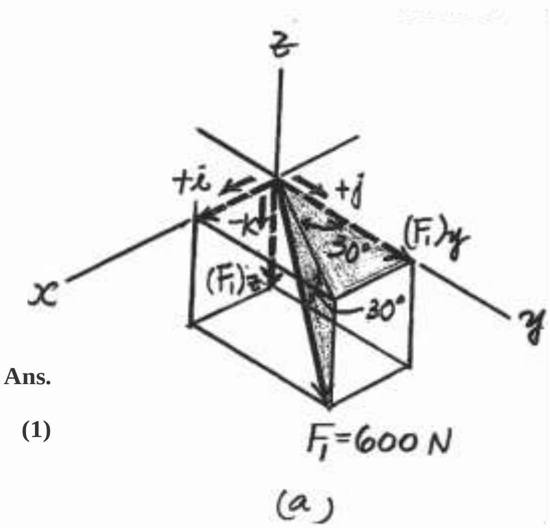
$$\mathbf{F}_1 = 600\cos 30°\sin 30°(+\mathbf{i}) + 600\cos 30°\cos 30°(+\mathbf{j}) + 600\sin 30°(-\mathbf{k})$$

$$= \{259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}\}\ \text{N}$$

$$\mathbf{F} = 500\cos\alpha\mathbf{i} + 500\cos\beta\mathbf{j} + 500\cos\gamma\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed towards the positive y axis, then

$$\mathbf{F}_R = F_R\,\mathbf{j}$$



Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$

$$F_R\,\mathbf{j} = (259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}) + (500\cos\alpha\mathbf{i} + 500\cos\beta\mathbf{j} + 500\cos\gamma\mathbf{k})$$

$$F_R\,\mathbf{j} = (259.81 + 500\cos\alpha)\mathbf{i} + (450 + 500\cos\beta)\mathbf{j} + (500\cos\gamma - 300)\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

$$0 = 259.81 + 500\cos\alpha$$

$$\alpha = 121.31° = 121° \qquad \textbf{Ans.}$$

$$F_R = 450 + 500\cos\beta \qquad \textbf{(1)}$$

$$0 = 500\cos\gamma - 300$$

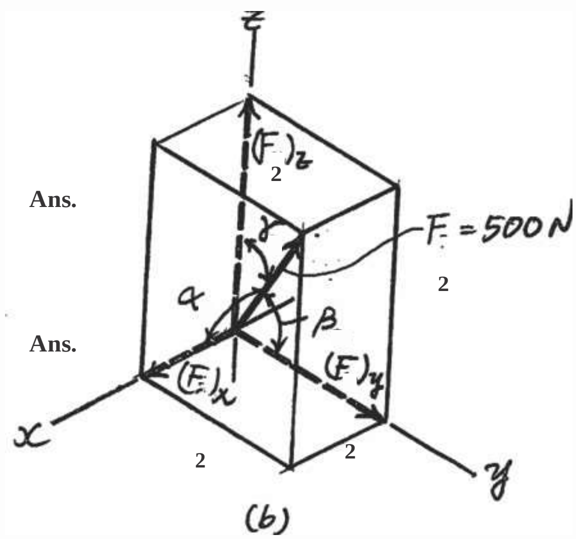
$$\gamma = 53.13° = 53.1° \qquad \textbf{Ans.}$$

However, since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, $\alpha = 121.31°$, and $\gamma = 53.13°$,

$$\cos\beta = \pm\sqrt{1 - \cos^2 121.31° - \cos^2 53.13°} = \pm 0.6083$$

If we substitute $\cos\beta = 0.6083$ into Eq. (1),

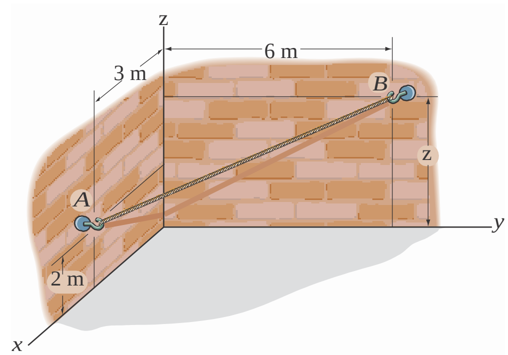
$$F_R = 450 + 500(0.6083) = 754\ \text{N} \qquad \textbf{Ans.}$$



and

$$\beta = \cos^{-1}(0.6083) = 52.5° \qquad \textbf{Ans.}$$

2-62. Determine the position vector **r** directed from point *A* to point *B* and the length of cord *AB*. Take $z = 4$ m.



SOLUTION

Position Vector: The coordinates for points *A* and *B* are $A(3, 0, 2)$ m and $B(0, 6, 4)$ m, respectively. Thus,

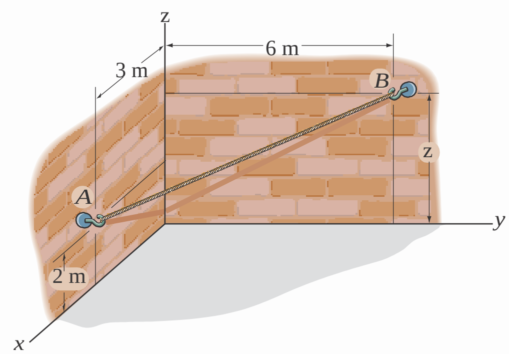
$$\mathbf{r}_{AB} = (0-3)\mathbf{i} + (6-0)\mathbf{j} + (4-2)\mathbf{k}$$

$$= \{-3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}\} \text{ m} \qquad \textbf{Ans.}$$

The length of cord *AB* is

$$r_{AB} = \sqrt{(-3)^2 + 6^2 + 2^2} = 7 \text{ m} \qquad \textbf{Ans.}$$

2-63. If the cord *AB* is 7.5 m long, determine the coordinate position $+z$ of point *B*.



SOLUTION

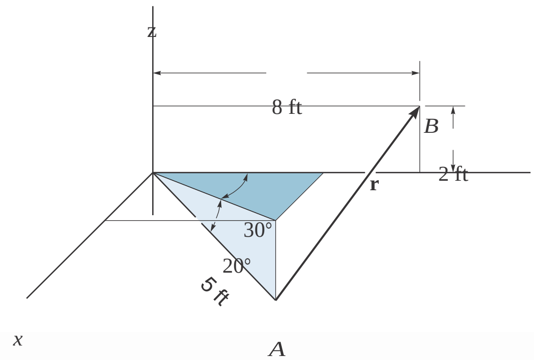
Position Vector: The coordinates for points *A* and *B* are $A(3, 0, 2)$ m and $B(0, 6, z)$ m, respectively. Thus,

$$\mathbf{r}_{AB} = (0-3)\mathbf{i} + (6-0)\mathbf{j} + (z-2)\mathbf{k}$$
$$= \{-3\mathbf{i} + 6\mathbf{j} + (z-2)\mathbf{k}\} \text{ m}$$

Since the length of cord is equal to the magnitude of $\mathbf{r}_{AB}$, then

$$r_{AB} = 7.5 = \sqrt{(-3)^2 + 6^2 + (z-2)^2}$$
$$56.25 = 45 + (z-2)^2$$
$$z - 2 = \pm 3.354$$
$$z = 5.35 \text{ m} \qquad \textbf{Ans.}$$

22-64. Express the position vector **r** in Cartesian vector form; then determine its magnitude and coordinate



SOLUTION

$$\mathbf{r} = (-5\cos 20^\circ \sin 30^\circ)\mathbf{i} + (8 - 5\cos 20^\circ \cos 30^\circ)\mathbf{j} + (2 + 5\sin 20^\circ)\mathbf{k}$$

$$\mathbf{r} = \{-2.35\mathbf{i} + 3.93\mathbf{j} + 3.71\mathbf{k}\}\ \text{ft}$$ **Ans.**

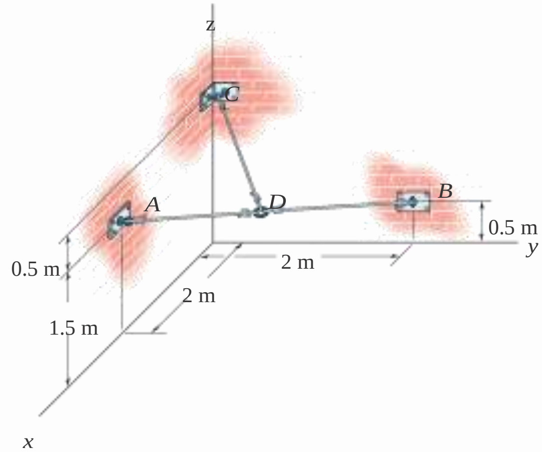
$$r = \sqrt{(-2.35)^2 + (3.93)^2 + (3.71)^2} = 5.89\ \text{ft}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{-2.35}{5.89}\right) = 113^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{3.93}{5.89}\right) = 48.2^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{3.71}{5.89}\right) = 51.0^\circ$$ **Ans.**

2-65. Determine the lengths of wires AD, BD, and CD. The ring at D is midway between A and B.



SOLUTION

$$D\left(\frac{2+0}{2}, \frac{0+2}{2}, \frac{1.5+0.5}{2}\right)\text{ m} = D(1, 1, 1)\text{ m}$$

$$\mathbf{r}_{AD} = (1-2)\mathbf{i} + (1-0)\mathbf{j} + (1-1.5)\mathbf{k}$$

$$= -1\mathbf{i} + 1\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{r}_{BD} = (1-0)\mathbf{i} + (1-2)\mathbf{j} + (1-0.5)\mathbf{k}$$

$$= 1\mathbf{i} - 1\mathbf{j} + 0.5\mathbf{k}$$

$$\mathbf{r}_{CD} = (1-0)\mathbf{i} + (1-0)\mathbf{j} + (1-2)\mathbf{k}$$

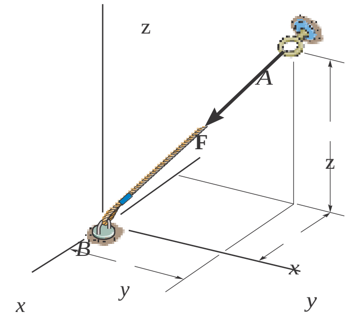
$$= 1\mathbf{i} + 1\mathbf{j} - 1\mathbf{k}$$

$$r_{AD} = \sqrt{(-1)^2 + 1^2 + (-0.5)^2} = 1.50\text{ m}$$ **Ans.**

$$r_{BD} = \sqrt{1^2 + (-1)^2 + 0.5^2} = 1.50\text{ m}$$ **Ans.**

$$r_{CD} = \sqrt{1^2 + 1^2 + (-1)^2} = 1.73\text{ m}$$ **Ans.**

2-66. If $\mathbf{F} = \{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}\}$ N and cable AB is 9 m long, determine the x, y, z coordinates of point A.



SOLUTION

Position Vector: The position vector $\mathbf{r}_{AB}$, directed from point A to point B, is given by

$$\mathbf{r}_{AB} = [0 - (-x)]\mathbf{i} + (0 - y)\mathbf{j} + (0 - z)\mathbf{k}$$

$$= x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$$

Unit Vector: Knowing the magnitude of $\mathbf{r}_{AB}$ is 9 m, the unit vector for $\mathbf{r}_{AB}$ is given by

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9}$$

The unit vector for force $\mathbf{F}$ is

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \frac{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}}{\sqrt{350^2 + (-250)^2 + (-450)^2}} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Since force $\mathbf{F}$ is also directed from point A to point B, then

$$\mathbf{u}_{AB} = \mathbf{u}_F$$

$$\frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

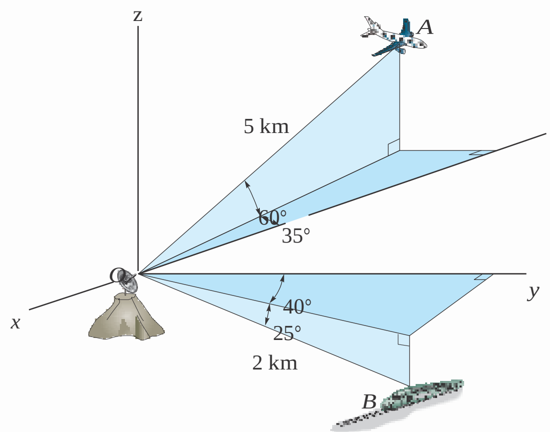
Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

$\frac{x}{9} = 0.5623$ $\qquad x = 5.06$ m **Ans.**

$\frac{-y}{9} = -0.4016$ $\qquad y = 3.61$ m **Ans.**

$\frac{-z}{9} = 0.7229$ $\qquad z = 6.51$ m **Ans.**

2-67. At a given instant, the position of a plane at A and a train at B are measured relative to a radar antenna at O. Determine the distance d between A and B at this instant. To solve the problem, formulate a position vector, directed from A to B, and then determine its magnitude.



SOLUTION

Position Vector: The coordinates of points A and B are

$$A(-5\cos 60^\circ \cos 35^\circ,\ -5\cos 60^\circ \sin 35^\circ,\ 5\sin 60^\circ)\ \text{km}$$

$$= A(-2.048,\ -1.434,\ 4.330)\ \text{km}$$

$$B(2\cos 25^\circ \sin 40^\circ,\ 2\cos 25^\circ \cos 40^\circ,\ -2\sin 25^\circ)\ \text{km}$$

$$= B(1.165,\ 1.389,\ -0.845)\ \text{km}$$

The position vector $\mathbf{r}_{AB}$ can be established from the coordinates of points A and B.

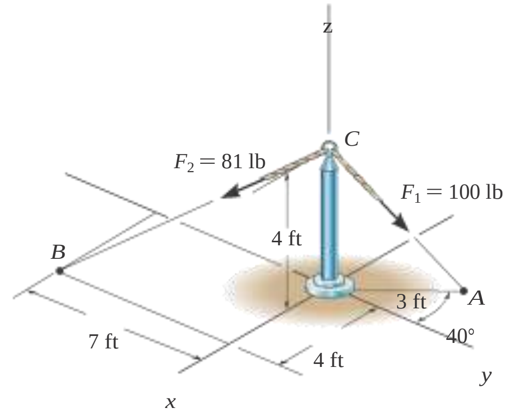
$$\mathbf{r}_{AB} = \{[1.165 - (-2.048)]\mathbf{i} + [1.389 - (-1.434)]\mathbf{j} + (-0.845 - 4.330)\mathbf{k}\}\ \text{km}$$

$$= \{3.213\mathbf{i} + 2.822\mathbf{j} - 5.175\mathbf{k}\}\ \text{km}$$

The distance between points A and B is

$$d = r_{AB} = \sqrt{3.213^2 + 2.822^2 + (-5.175)^2} = 6.71\ \text{km}$$ **Ans.**

22-68. Determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$$\mathbf{F}_1 = -100\left(\frac{3}{5}\right)\sin 40°\,\mathbf{i} + 100\left(\frac{3}{5}\right)\cos 40°\,\mathbf{j} - 100\left(\frac{4}{5}\right)\mathbf{k}$$

$$= \{-38.567\,\mathbf{i} + 45.963\,\mathbf{j} - 80\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = 81\text{ lb}\left(\frac{4}{9}\,\mathbf{i} - \frac{7}{9}\,\mathbf{j} - \frac{4}{9}\,\mathbf{k}\right)$$

$$= \{36\,\mathbf{i} - 63\,\mathbf{j} - 36\,\mathbf{k}\}\text{lb}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = \{-2.567\,\mathbf{i} - 17.04\,\mathbf{j} - 116.0\,\mathbf{k}\}\text{ lb}$$

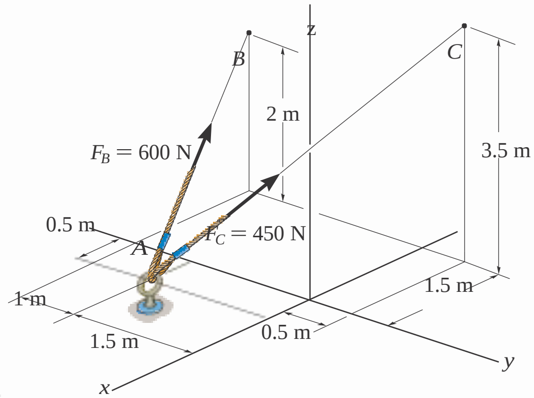
$$F_R = \sqrt{(-2.567)^2 + (-17.04)^2 + (-116.0)^2} = 117.27\text{ lb} = 117\text{ lb} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{-2.567}{117.27}\right) = 91.3° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{-17.04}{117.27}\right) = 98.4° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-116.0}{117.27}\right) = 172° \qquad \textbf{Ans}$$

2-69. Express $\mathbf{F}_B$ and $\mathbf{F}_C$ in Cartesian vector form.



SOLUTION

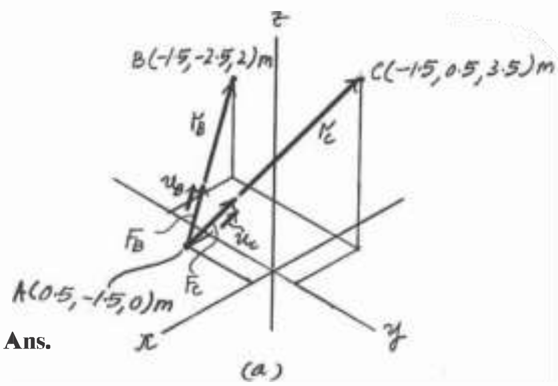
Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5 - 0.5)\mathbf{i} + [-2.5 - (-1.5)]\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [-2.5 - (-1.5)]^2 + (2 - 0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5 - 0.5)\mathbf{i} + [0.5 - (-1.5)]\mathbf{j} + (3.5 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [0.5 - (-1.5)]^2 + (3.5 - 0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$

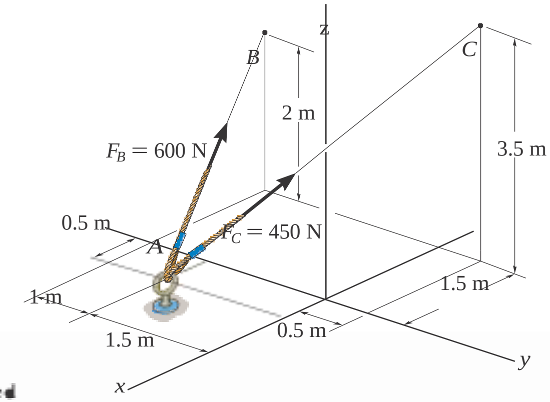


Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\text{ N}$$ **Ans.**

2-70. Determine the magnitude and coordinate direction angles of the resultant force acting at A.



SOLUTION

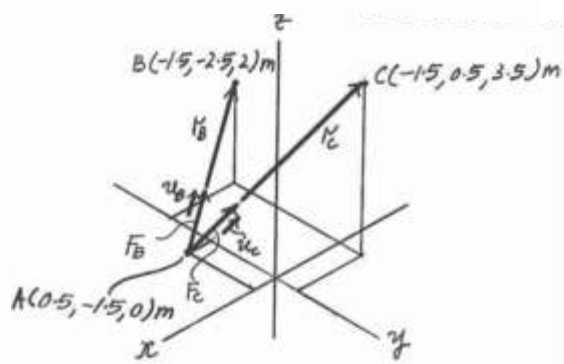
Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5 - 0.5)\mathbf{i} + [-2.5 - (-1.5)]\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [-2.5 - (-1.5)]^2 + (2 - 0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5 - 0.5)\mathbf{i} + [0.5 - (-1.5)]\mathbf{j} + (3.5 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [0.5 - (-1.5)]^2 + (3.5 - 0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$



(a)

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}) + (-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k})$$

$$= \{-600\mathbf{i} + 750\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(-600)^2 + 0^2 + 750^2} = 960.47\text{ N} = 960\text{ N}$$

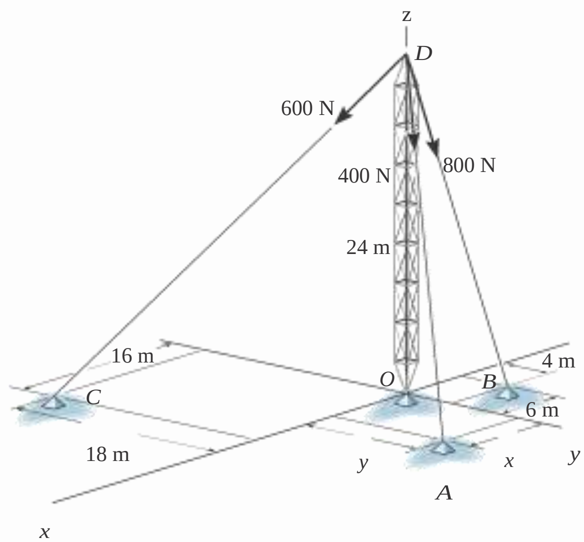
The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{-600}{960.47}\right) = 129°$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{960.47}\right) = 90°$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{760}{960.47}\right) = 38.7°$$ **Ans.**

2-71. The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Take $x = 20$ m, $y = 15$ m.



SOLUTION

$$\mathbf{F}_{DA} = 400\left(\frac{20}{34.66}\mathbf{i} + \frac{15}{34.66}\mathbf{j} - \frac{24}{34.66}\mathbf{k}\right)\text{ N}$$

$$\mathbf{F}_{DB} = 800\left(\frac{-6}{25.06}\mathbf{i} + \frac{4}{25.06}\mathbf{j} - \frac{24}{25.06}\mathbf{k}\right)\text{ N}$$

$$\mathbf{F}_{DC} = 600\left(\frac{16}{34}\mathbf{i} - \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right)\text{ N}$$

$$\mathbf{F}_R = \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC}$$

$$= \{321.66\mathbf{i} - 16.82\mathbf{j} - 1466.71\mathbf{k}\}\text{ N}$$

$$F_R = \sqrt{(321.66)^2 + (-16.82)^2 + (-1466.71)^2}$$

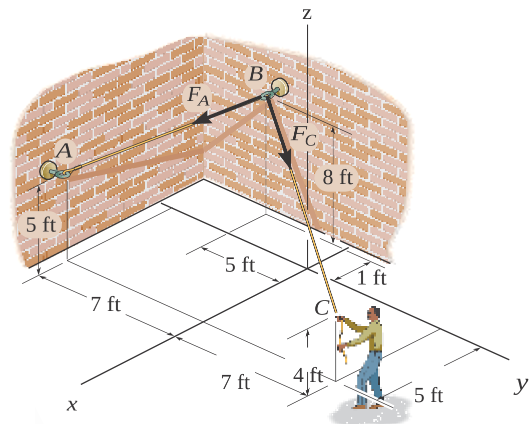
$$= 1501.66\text{ N} = 1.50\text{ kN}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{321.66}{1501.66}\right) = 77.6°$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{-16.82}{1501.66}\right) = 90.6°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-1466.71}{1501.66}\right) = 168°$$ **Ans.**

22-72. The man pulls on the rope at C with a force of 70 lb which causes the forces $\mathbf{F}_A$ and $\mathbf{F}_C$ at B to have this same magnitude. Express each of these two forces as Cartesian vectors.



SOLUTION

Unit Vectors: The coordinate points A, B, and C are shown in Fig. *a*. Thus,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{[5-(-1)]\mathbf{i} + [-7-(-5)]\mathbf{j} + (5-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7-(-5)]^2 + (5-8)^2}}$$

$$= \frac{6}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{[5-(-1)]\mathbf{i} + [-7(-5)]\mathbf{j} + (4-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7(-5)]^2 + (4-8)^2}}$$

$$= \frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

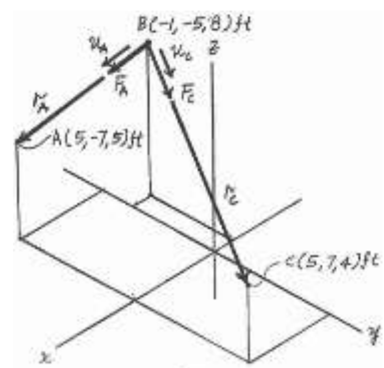
Force Vectors: Multiplying the magnitude of the force with its unit vector,

$$\mathbf{F}_A = F_A\mathbf{u}_A = 70\left(\frac{6}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}\right)$$

$$= \{60\mathbf{i} - 20\mathbf{j} + 30\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_C = F_C\mathbf{u}_C = 70\left(\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

$$= \{30\mathbf{i} + 60\mathbf{j} + 20\mathbf{k}\}\text{ lb}$$ **Ans.**



(a)

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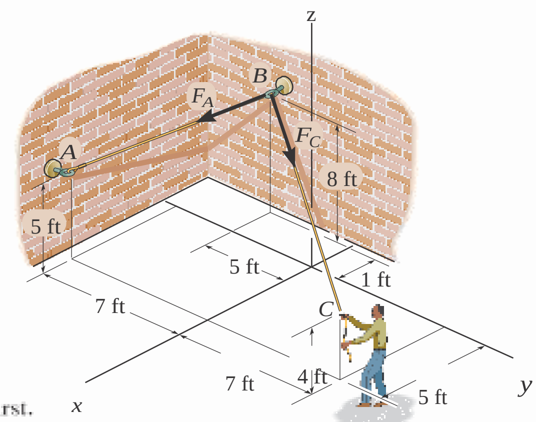
2-73. The man pulls on the rope at C with a force of 70 lb which causes the forces $\mathbf{F}_A$ and $\mathbf{F}_C$ at B to have this same magnitude. Determine the magnitude and coordinate direction angles of the resultant force acting at B.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{[5-(-1)]\mathbf{i} + [-7(-5)]\mathbf{j} + (5-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7(-5)]^2 + (5-8)^2}}$$

$$= \frac{6}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

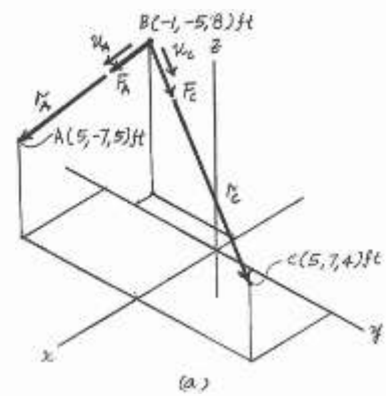
$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{[5-(-1)]\mathbf{i} + [-7(-5)]\mathbf{j} + (4-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7(-5)]^2 + (4-8)^2}}$$

$$= \frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = 70\left(\frac{6}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}\right) = \{60\mathbf{i} - 20\mathbf{j} + 30\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 70\left(\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) = \{30\mathbf{i} + 60\mathbf{j} + 20\mathbf{k}\}\text{ lb}$$



Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_C = (60\mathbf{i} - 20\mathbf{j} - 30\mathbf{k}) + (30\mathbf{i} + 60\mathbf{j} - 20\mathbf{k})$$

$$= \{90\mathbf{i} + 40\mathbf{j} - 50\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(90)^2 + (40)^2 + (-50)^2} = 110.45\text{ lb} = 110\text{ lb}$$ **Ans.**

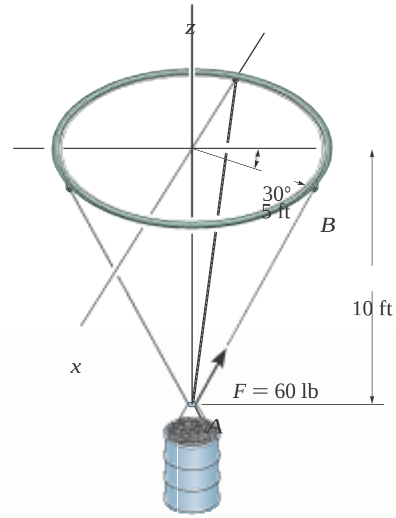
The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{90}{110.45}\right) = 35.4°$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{40}{110.45}\right) = 68.8°$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-50}{110.45}\right) = 117°$$ **Ans.**

2-74. The load at A creates a force of 60 lb in wire AB. Express this force as a Cartesian vector acting on A and directed toward B as shown.



SOLUTION

Unit Vector: First determine the position vector $\mathbf{r}_{AB}$. The coordinates of point B are

$$B\,(5\sin 30°,\ 5\cos 30°,\ 0)\text{ ft} = B\,(2.50,\ 4.330,\ 0)\text{ ft}$$

Then

$$\mathbf{r}_{AB} = \{(2.50 - 0)\mathbf{i} + (4.330 - 0)\mathbf{j} + [0 - (-10)]\mathbf{k}\}\text{ ft}$$

$$= \{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{2.50^2 + 4.330^2 + 10.0^2} = 11.180\text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}}{11.180}$$

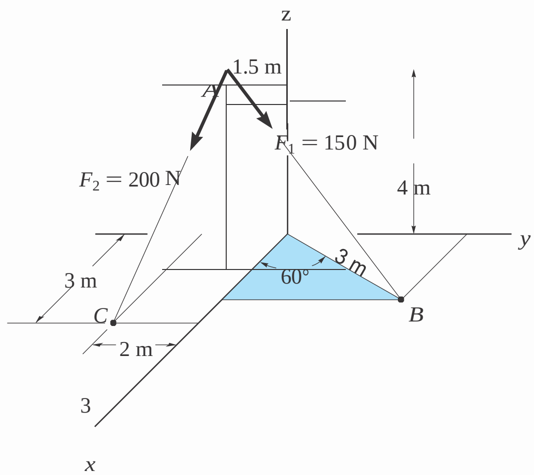
$$= 0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}$$

Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 60\,\{0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}\}\text{ lb}$$

$$= \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\}\text{ lb}$$ **Ans.**

2-75. Determine the magnitude and coordinate direction angles of the resultant force acting at point A.



SOLUTION

$$\mathbf{r}_{AC} = \{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}\}\text{ m}$$

$$|\mathbf{r}_{AC}| = \sqrt{3^2 + (-0.5)^2 + (-4)^2} = \sqrt{25.25} = 5.02494$$

$$\mathbf{F}_2 = 200\left(\frac{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}}{5.02494}\right) = (119.4044\mathbf{i} - 19.9007\mathbf{j} - 159.2059\mathbf{k})$$

$$\mathbf{r}_{AB} = (3\cos 60°\mathbf{i} + (1.5 + 3\sin 60°)\mathbf{j} - 4\mathbf{k})$$

$$\mathbf{r}_{AB} = (1.5\mathbf{i} + 4.0981\mathbf{j} + 4\mathbf{k})$$

$$|\mathbf{r}_{AB}| = \sqrt{(1.5)^2 + (4.0981)^2 + (-4)^2} = 5.9198$$

$$\mathbf{F}_1 = 150\left(\frac{1.5\mathbf{i} + 4.0981\mathbf{j} - 4\mathbf{k}}{5.9198}\right) = (38.0079\mathbf{i} + 103.8396\mathbf{j} - 101.3545\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (157.4124\mathbf{i} + 83.9389\mathbf{j} - 260.5607\mathbf{k})$$

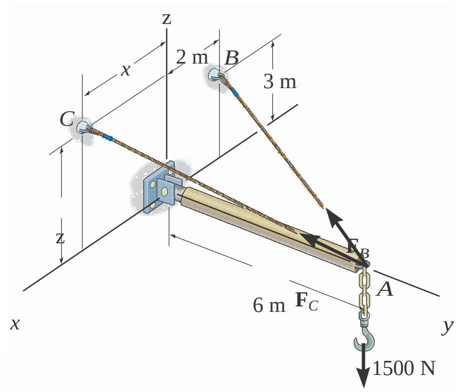
$$F_R = \sqrt{(157.4124)^2 + (83.9389)^2 + (-260.5604)^2} = 315.7786 = 316\text{ N}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{157.4124}{315.7786}\right) = 60.100° = 60.1°$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{83.9389}{315.7786}\right) = 74.585° = 74.6°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-260.5607}{315.7786}\right) = 145.60° = 146°$$ **Ans.**

2-76. Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point A towards O, determine the magnitudes of the resultant force and forces $\mathbf{F}_B$ and $\mathbf{F}_C$. Set $x = 3$ m and $z = 2$ m.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ must be determined first. From Fig. a,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i}+(0-6)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-2-0)^2+(0-6)^2+(3-0)^2}} = -\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i}+(0-6)\mathbf{j}+(2-0)\mathbf{k}}{\sqrt{(3-0)^2+(0-6)^2+(2-0)^2}} = \frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = -\frac{2}{7}F_B\mathbf{i}-\frac{6}{7}F_B\mathbf{j}+\frac{3}{7}F_B\mathbf{k}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = \frac{3}{7}F_C\mathbf{i}-\frac{6}{7}F_C\mathbf{j}+\frac{2}{7}F_C\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed along the negative y axis, and the load $\mathbf{W}$ is directed along the z axis, these two forces can be written as

$$\mathbf{F}_R = -F_R\mathbf{j} \qquad \text{and} \qquad \mathbf{W} = [-1500\mathbf{k}]\text{ N}$$

Resultant Force: The vector addition of $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{W}$ is equal to $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R\mathbf{j} = \left(-\frac{2}{7}F_B\mathbf{i}-\frac{6}{7}F_B\mathbf{j}+\frac{3}{7}F_B\mathbf{k}\right)+\left(\frac{3}{7}F_C\mathbf{i}-\frac{6}{7}F_C\mathbf{j}+\frac{2}{7}F_C\mathbf{k}\right)+(-1500\mathbf{k})$$

$$-F_R\mathbf{j} = \left(-\frac{2}{7}F_B+\frac{3}{7}F_C\right)\mathbf{i}+\left(-\frac{6}{7}F_B-\frac{6}{7}F_C\right)\mathbf{j}+\left(\frac{3}{7}F_B+\frac{2}{7}F_C-1500\right)\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

$$0 = -\frac{2}{7}F_B+\frac{3}{7}F_C \qquad (1)$$

$$-F_R = -\frac{6}{7}F_B-\frac{6}{7}F_C \qquad (2)$$

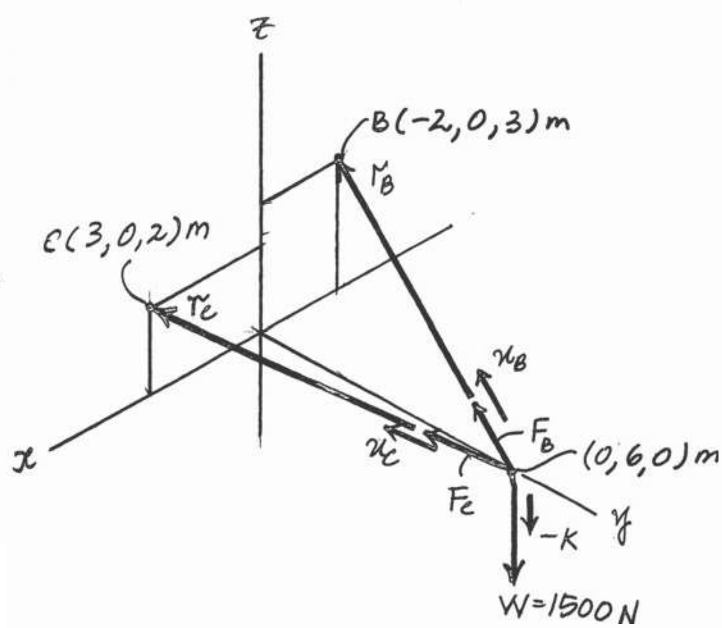
$$0 = \frac{3}{7}F_B+\frac{2}{7}F_C-1500 \qquad (3)$$

Solving Eqs. (1), (2), and (3) yields

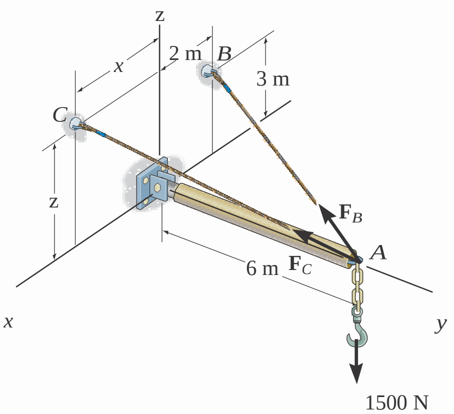
$F_C = 1615.38\text{ N} = 1.62\text{ kN}$ **Ans.**

$F_B = 2423.08\text{ N} = 2.42\text{ kN}$ **Ans.**

$F_R = 3461.53\text{ N} = 3.46\text{ kN}$ **Ans.**



2-77. Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point A towards O, determine the values of x and z for the coordinates of point C and the magnitude of the resultant force. Set $F_B = 1610$ N and $F_C = 2400$ N.



SOLUTION

Force Vectors: From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i}+(0-6)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-2-0)^2+(0-6)^2+(3-0)^2}} = -\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}$$

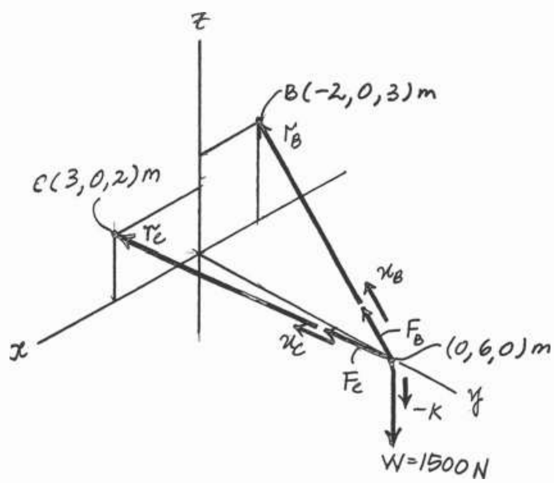
$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(x-0)\mathbf{i}+(0-6)\mathbf{j}+(z-0)\mathbf{k}}{\sqrt{(x-0)^2+(0-6)^2+(z-0)^2}} = \frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}$$

Thus,

$$\mathbf{F}_B = F_B\mathbf{u}_B = 1610\left(-\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}\right) = [-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k}]\text{N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 2400\left(\frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}\right)$$

$$= \frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}$$



Since the resultant force $\mathbf{F}_R$ is directed along the negative y axis, and the load is directed along the z axis, these two forces can be written as

$$\mathbf{F}_R = -F_R\mathbf{j} \qquad \text{and} \qquad \mathbf{W} = [-1500\mathbf{k}]\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R\mathbf{j} = (-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k}) + \left(\frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}\mathbf{k}\right)+(-1500\mathbf{k})$$

$$-F_R\mathbf{j} = \left(\frac{2400x}{\sqrt{x^2+z^2+36}}-460\right)\mathbf{i}-\left(\frac{14\,400}{\sqrt{x^2+z^2+36}}+1380\right)\mathbf{j}+\left(690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = \frac{2400x}{\sqrt{x^2+z^2+36}}-460 \qquad \frac{2400x}{\sqrt{x^2+z^2+36}} = 460 \qquad (1)$$

$$-F_R = -\left(\frac{14400}{\sqrt{x^2+z^2+36}}+1380\right) \qquad F_R = \frac{14400}{\sqrt{x^2+z^2+36}}+1380 \qquad (2)$$

$$0 = 690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500 \qquad \frac{2400z}{\sqrt{x^2+z^2+36}} = 810 \qquad (3)$$

Dividing Eq. (1) by Eq. (3), yields

$$x = 0.5679z \qquad (4)$$

Substituting Eq. (4) into Eq. (1), and solving

$$z = 2.197\text{ m} = 2.20\text{ m} \qquad \textbf{Ans.}$$

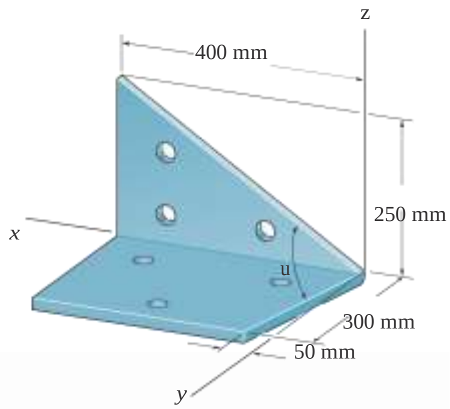
Substituting $z = 2.197$ m into Eq. (4), yields

$$x = 1.248\text{ m} = 1.25\text{ m} \qquad \textbf{Ans.}$$

Substituting $x = 1.248$ m and $z = 2.197$ m into Eq. (2), yields

$$F_R = 3591.85\text{ N} = 3.59\text{ kN} \qquad \textbf{Ans.}$$

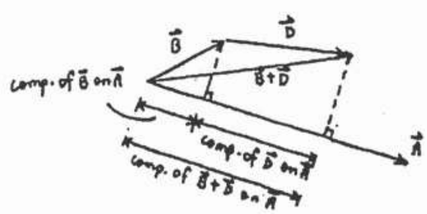
2-78. Given the three vectors **A, B,** and **D,** show that **A** · **(B + D)** = **(A** · **B)** + **(A** · **D)**.



SOLUTION

Since the component of (**B** + **D**) is equal to the sum of the components of **B** and **D**, then

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{D} \qquad \textbf{(QED)}$$



Also,

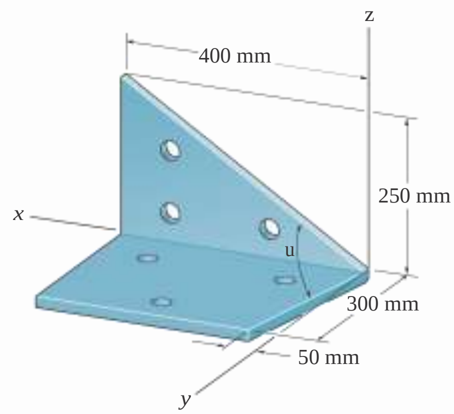
$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot[(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}]$$

$$= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z)$$

$$= (A_xB_x + A_yB_y + A_zB_z) + (A_xD_x + A_yD_y + A_zD_z)$$

$$= (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D}) \qquad \textbf{(QED)}$$

2-79. Determine the angle θ between the edges of the sheet-metal bracket.



SOLUTION

$\mathbf{r}_1 = \{400\mathbf{i} + 250\mathbf{k}\}$ mm ; $\quad r_1 = 471.70$ mm

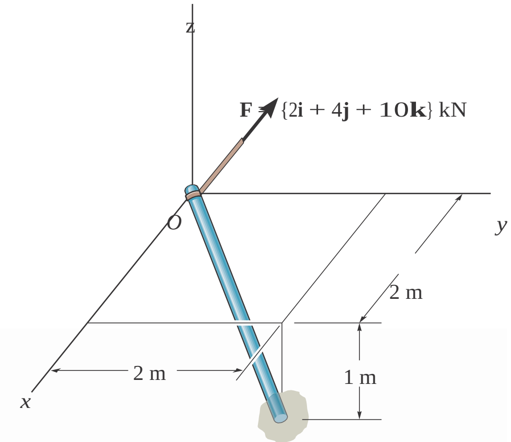
$\mathbf{r}_2 = \{50\mathbf{i} + 300\mathbf{j}\}$ mm ; $\quad r_2 = 304.14$ mm

$\mathbf{r}_1 \cdot \mathbf{r}_2 = (400)(50) + 0(300) + 250(0) = 20\,000$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{r_1 r_2}\right)$$

$$= \cos^{-1}\left(\frac{20\,000}{(471.70)(304.14)}\right) = 82.0^\circ$$ **Ans.**

***2-80.** Determine the projection of the force **F** along the pole.

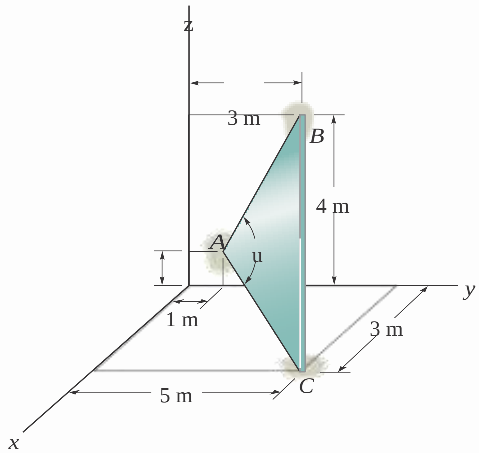


SOLUTION

$$\text{Proj } F = \mathbf{F} \cdot \mathbf{u}_a = (2\,\mathbf{i} + 4\,\mathbf{j} + 10\,\mathbf{k}) \cdot \left(\frac{2}{3}\,\mathbf{i} + \frac{2}{3}\,\mathbf{j} - \frac{1}{3}\,\mathbf{k}\right)$$

$$\text{Proj } F = 0.667 \text{ kN}$$

2-81. Determine the length of side BC of the triangular plate. Solve the problem by finding the magnitude of $\mathbf{r}_{BC}$; then check the result by first finding 0, r_{AB}, and r_{AC} and then using the cosine law.



SOLUTION

$\mathbf{r}_{BC} = \{3\,\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k}\}$ m

$r_{BC} = \sqrt{(3)^2 + (2)^2 + (-4)^2} = 5.39$ m **Ans**

Also,

$\mathbf{r}_{AC} = \{3\,\mathbf{i} + 4\,\mathbf{j} - 1\,\mathbf{k}\}$ m

$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990$ m

$\mathbf{r}_{AB} = \{2\,\mathbf{j} + 3\,\mathbf{k}\}$ m

$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056$ m

$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$

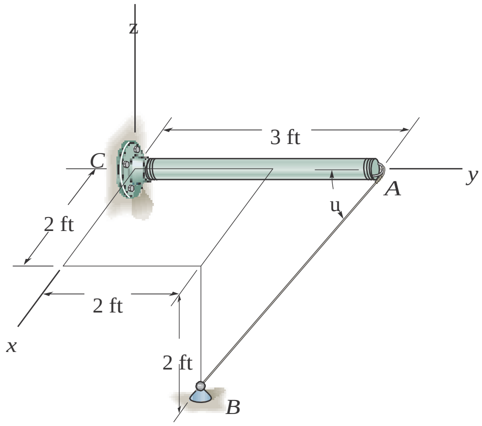
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC}\cdot\mathbf{r}_{AB}}{r_{AC}r_{AB}}\right) = \cos^{-1}\frac{5}{(5.0990)(3.6056)}$$

$\theta = 74.219°$

$$r_{BC} = \sqrt{(5.0990)^2 + (3.6056)^2 - 2(5.0990)(3.6056)\cos 74.219°}$$

$r_{BC} = 5.39$ m **Ans**

2-82. Determine the angle 0 between the y axis of the pole and the wire AB.



SOLUTION

Position Vector:

$$\mathbf{r}_{AC} = 5-3\mathbf{j}6 \text{ ft}$$

$$\mathbf{r}_{AB} = 512 - 02\mathbf{i} + 12 - 32\mathbf{j} + 1-2 - 02\mathbf{k}6 \text{ ft}$$

$$= 52\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}6 \text{ ft}$$

The magnitudes of the position vectors are

$$r_{AC} = 3.00 \text{ ft} \qquad r_{AB} = 2 2^2 + 1-1 2^2 + 1-2 2^2 = 3.00 \text{ ft}$$

The Angles Between Two Vectors U***:*** The dot product of two vectors must be determined first.

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 1-3\mathbf{j}2 \cdot 12\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}2$$

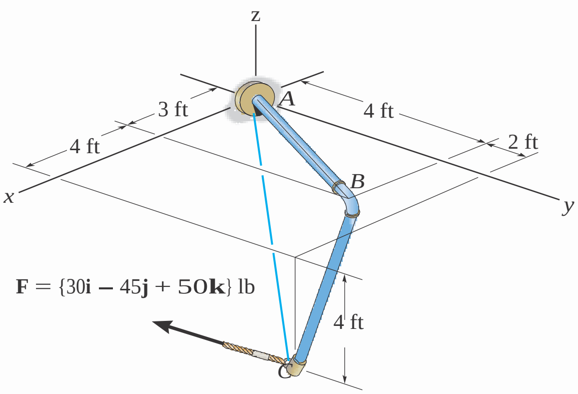
$$= 0122 + 1-321-12 + 01-22$$

$$= 3$$

Then,

$$u = \cos^{-1}\left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AB}}{r_{AO} r_{AB}}\right) = \cos^{-1}\left[\frac{3}{3.00\ 3.00}\right] = 70.5°$$ **Ans.**

2-83. Determine the magnitudes of the components of **F** acting along and perpendicular to segment *BC* of the pipe assembly.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{CB}$ must be determined first. From Fig. *a*

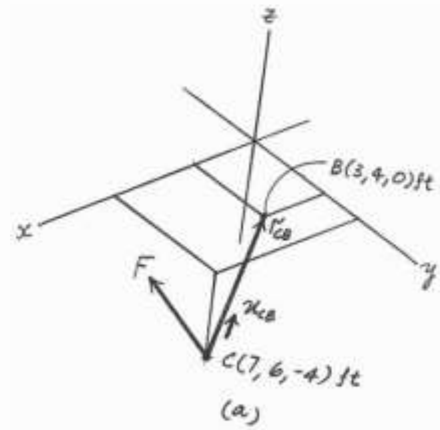
$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CB}}{r_{CB}} = \frac{(3-7)\mathbf{i} + (4-6)\mathbf{j} + [0-(-4)]\mathbf{k}}{\sqrt{(3-7)^2 + (4-6)^2 + [0-(-4)]^2}} = -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** parallel to segment *BC* of the pipe assembly is

$$(F_{BC})_{pa} = \mathbf{F}\cdot\mathbf{u}_{CB} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k})\cdot\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$$= (30)\left(-\frac{2}{3}\right) + (-45)\left(-\frac{1}{3}\right) + 50\left(\frac{2}{3}\right)$$

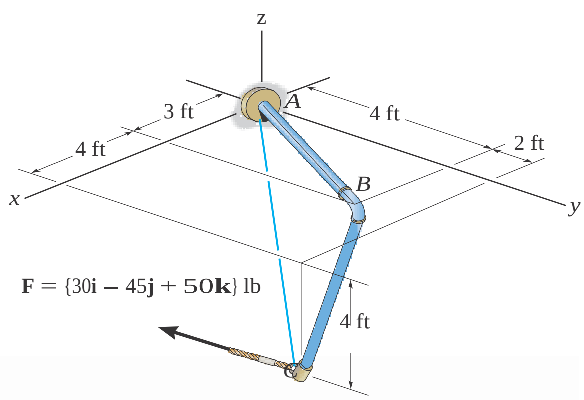
$$= 28.33 \text{ lb} = 28.3 \text{ lb}$$ **Ans.**



The magnitude of **F** is $F = \sqrt{30^2 + (-45)^2 + 50^2} = \sqrt{5425}$ lb. Thus, the magnitude of the component of **F** perpendicular to segment *BC* of the pipe assembly can be determined from

$$(F_{BC})_{pr} = \sqrt{F^2 - (F_{BC})_{pa}^2} = \sqrt{5425 - 28.33^2} = 68.0 \text{ lb}$$ **Ans.**

***2-84.** Determine the magnitude of the projected component of **F** along *AC*. Express this component as a Cartesian vector.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*

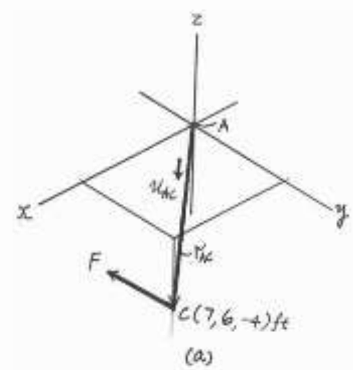
$$\mathbf{u}_{AC} = \frac{(7-0)\mathbf{i} + (6-0)\mathbf{j} + (-4-0)\mathbf{k}}{\sqrt{(7-0)^2 + (6-0)^2 + (-4-0)^2}} = 0.6965\,\mathbf{i} + 0.5970\,\mathbf{j} - 0.3980\,\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line AC is

$$F_{AC} = \mathbf{F}\cdot\mathbf{u}_{AC} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k})\cdot(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

$$= (30)(0.6965) + (-45)(0.5970) + 50(-0.3980)$$

$$= 25.87 \text{ lb}$$ **Ans.**

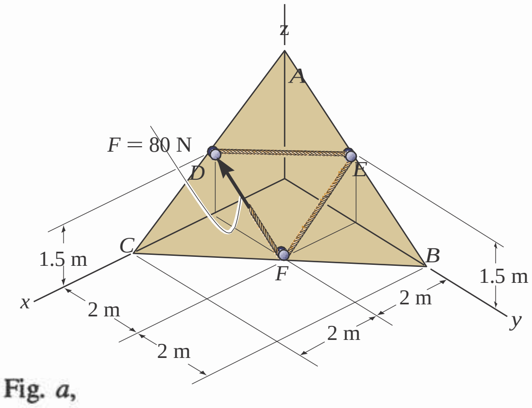


Thus, $\mathbf{F}_{AC}$ expressed in Cartesian vector form is

$$\mathbf{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = -25.87(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

$$= \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\}\text{ lb}$$ **Ans.**

2-85. Determine the projection of force $F = 80$ N along line BC. Express the result as a Cartesian vector.



SOLUTION

Unit Vectors: The unit vectors $\mathbf{u}_{FD}$ and $\mathbf{u}_{FC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{FD} = \frac{\mathbf{r}_{FD}}{r_{FD}} = \frac{(2-2)\mathbf{i} + (0-2)\mathbf{j} + (1.5-0)\mathbf{k}}{\sqrt{(2-2)^2 + (0-2)^2 + (1.5-0)^2}} = -\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

$$\mathbf{u}_{FC} = \frac{\mathbf{r}_{FC}}{r_{FC}} = \frac{(4-2)\mathbf{i} + (0-2)\mathbf{j} + (0-0)\mathbf{k}}{\sqrt{(4-2)^2 + (0-2)^2 + (0-0)^2}} = 0.7071\mathbf{i} - 0.7071\mathbf{j}$$

Thus, the force vector $\mathbf{F}$ is given by

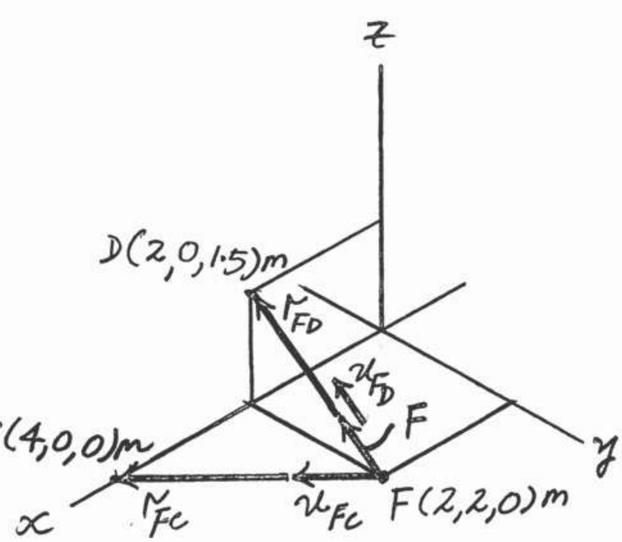
$$\mathbf{F} = F\mathbf{u}_{FD} = 80\left(-\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right) = [-64\mathbf{j} + 48\mathbf{k}]\text{ N}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}$ along line BC is

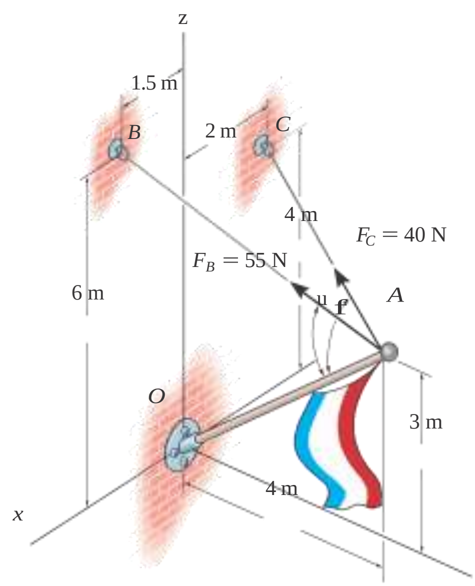
$$F_{BC} = \mathbf{F} \cdot \mathbf{u}_{FC} = (-64\mathbf{j} + 48\mathbf{k}) \cdot (0.7071\mathbf{i} - 0.7071\mathbf{j})$$
$$= (0)(0.7071) + (-64)(-0.7071) + 48(0)$$
$$= 45.25 = 45.2\text{ N} \qquad \textbf{Ans.}$$

The component of $\mathbf{F}_{BC}$ can be expressed in Cartesian vector form as

$$\mathbf{F}_{BC} = F_{BC}(\mathbf{u}_{FC}) = 45.25(0.7071\mathbf{i} - 0.7071\mathbf{j})$$
$$= \{32\mathbf{i} - 32\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$



2-86. Determine the angles θ and ϕ made between the axes *OA* of the flag pole and *AB* and *AC*, respectively, of each cable.



SOLUTION

$\mathbf{r}_{AC} = \{-2\mathbf{i} - 4\mathbf{j} + 1\mathbf{k}\}$ m; $\quad r_{AC} = 4.58$ m

$\mathbf{r}_{AB} = \{1.5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\}$ m; $\quad r_{AB} = 5.22$ m

$\mathbf{r}_{AO} = \{-4\mathbf{j} - 3\mathbf{k}\}$ m; $\quad r_{AO} = 5.00$

$\mathbf{r}_{AB} \cdot \mathbf{r}_{AO} = (1.5)(0) + (-4)(-4) + (3)(-3) = 7$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AO}}{r_{AB}\, r_{AO}}\right)$$

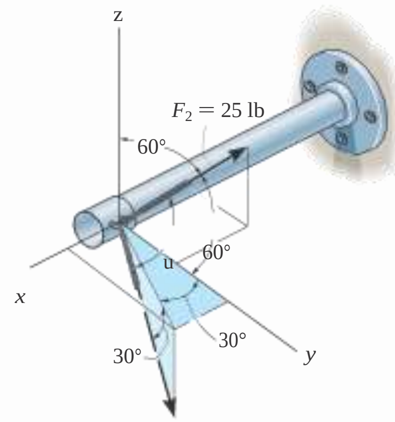
$$= \cos^{-1}\left(\frac{7}{5.22(5.00)}\right) = 74.4^\circ$$ **Ans.**

$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (-2)(0) + (-4)(-4) + (1)(-3) = 13$

$$\phi = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{13}{4.58(5.00)}\right) = 55.4^\circ$$ **Ans.**

2-87. Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



SOLUTION

Force Vector:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30° \mathbf{i} + \cos 30° \cos 30° \mathbf{j} - \sin 30° \mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{F}_1 = F_R \mathbf{u}_{F_1} = 30(0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \text{ lb}$$

$$= [12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}] \text{ lb}$$

Unit Vector: One can obtain the angle $\alpha = 135°$ for $\mathbf{F}_2$ using Eq. 2–8. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, with $\beta = 60°$ and $\gamma = 60°$. The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k} = -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

Projected Component of $\mathbf{F}_1$ Along the Line of Action of $\mathbf{F}_2$:

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

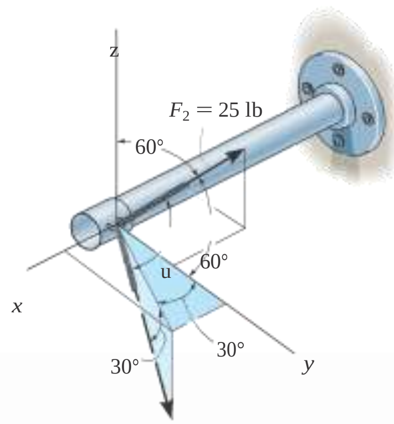
$$= (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5)$$

$$= -5.44 \text{ lb}$$

Negative sign indicates that the projected component of $(F_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

The magnitude is $(F_1)_{F_2} = 5.44$ lb **Ans.**

2-88. Determine the angle 0 between the two cables attached to the pipe.



SOLUTION

Unit Vectors:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30°\mathbf{i} + \cos 30° \cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k}$$

$$= -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

The Angles Between Two Vectors u:

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

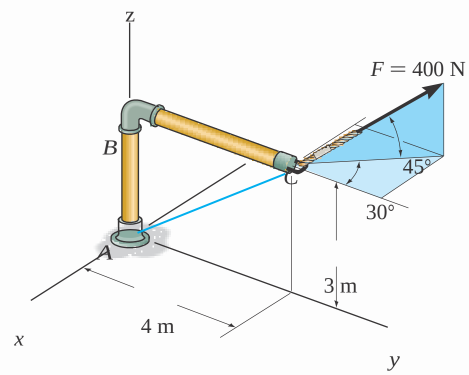
$$= 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5)$$

$$= -0.1812$$

Then,

$$u = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1812) = 100°$$ **Ans.**

2-89. Determine the projection of force $F = 400$ N acting along line AC of the pipe assembly. Express the result as a Cartesian vector.



SOLUTION

Force and Unit Vector: The force vector **F** and unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. (a)

$$\mathbf{F} = 400(-\cos 45° \sin 30°\,\mathbf{i} + \cos 45° \cos 30°\,\mathbf{j} + \sin 45°\,\mathbf{k})$$
$$= \{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\}$$

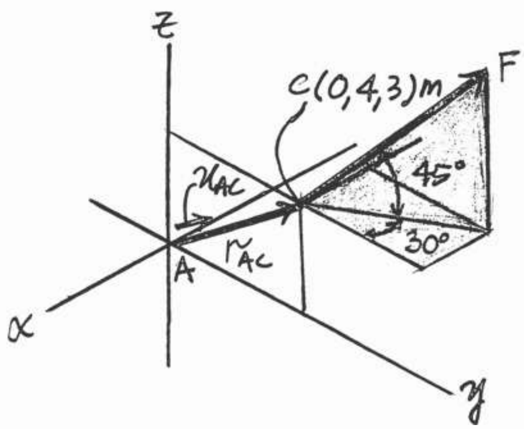
$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(0-0)\mathbf{i} + (4-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-0)^2 + (4-0)^2 + (3-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line AC is

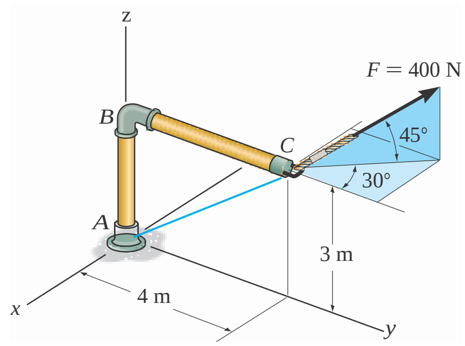
$$F_{AC} = \mathbf{F} \cdot \mathbf{u}_{AC} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}) \cdot \left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right)$$
$$= (-141.42)(0) + 244.95\left(\frac{4}{5}\right) + 282.84\left(\frac{3}{5}\right)$$
$$= 365.66 \text{ lb}$$ **Ans.**

Thus, $\mathbf{F}_{AC}$ written in Cartesian vector form is

$$\mathbf{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = 365.66\left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right) = \{293\mathbf{j} + 219\mathbf{k}\} \text{ lb}$$ **Ans.**



2-90. Determine the magnitudes of the components of force $F = 400$ N acting parallel and perpendicular to segment BC of the pipe assembly.



SOLUTION

Force Vector: The force vector **F** must be determined first. From Fig. *a*,

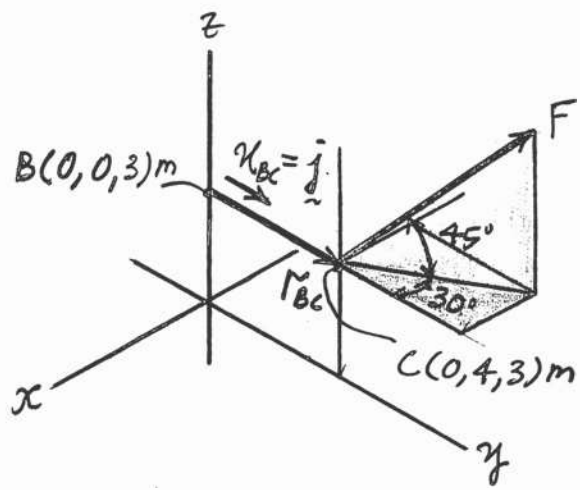
$$\mathbf{F} = 400(-\cos 45° \sin 30° \mathbf{i} + \cos 45° \cos 30° \mathbf{j} + \sin 45° \mathbf{k})$$
$$= \{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\} \text{ N}$$

Vector Dot Product: By inspecting Fig. (a) we notice that $u_{BC} = \mathbf{j}$. Thus, the magnitude of the component of **F** parallel to segment BC of the pipe assembly is

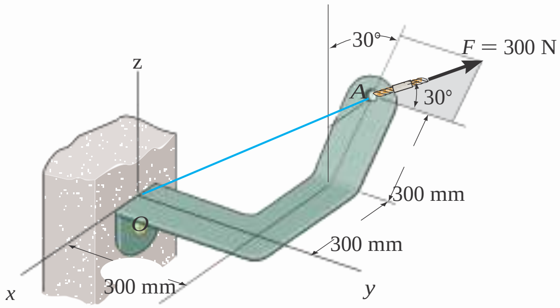
$$(F_{BC})_{paral} = \mathbf{F} \cdot \mathbf{j} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}) \cdot \mathbf{j}$$
$$= -141.42(0) + 244.95(1) + 282.84(0)$$
$$= 244.95 \text{ lb} = 245 \text{ N} \qquad \textbf{Ans.}$$

The magnitude of the component of **F** perpendicular to segment BC of the pipe assembly can be determined from

$$(F_{BC})_{per} = \sqrt{F^2 - (F_{BC})_{paral}} = \sqrt{400^2 - 244.95^2} = 316 \text{ N} \qquad \textbf{Ans.}$$



2-91. Determine the magnitudes of the projected components of the force $F = 300$ N acting along the x and y axes.



SOLUTION

Force Vector: The force vector **F** must be determined first. From Fig. *a*,

$$\mathbf{F} = -300 \sin 30° \sin 30° \mathbf{i} + 300 \cos 30° \mathbf{j} + 300 \sin 30° \cos 30° \mathbf{k}$$

$$= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}] \text{ N}$$

Vector Dot Product: The magnitudes of the projected component of **F** along the x and y axes are

$$F_x = \mathbf{F} \cdot \mathbf{i} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{i}$$

$$= -75(1) + 259.81(0) + 129.90(0)$$

$$= -75 \text{ N}$$

$$F_y = \mathbf{F} \cdot \mathbf{j} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{j}$$

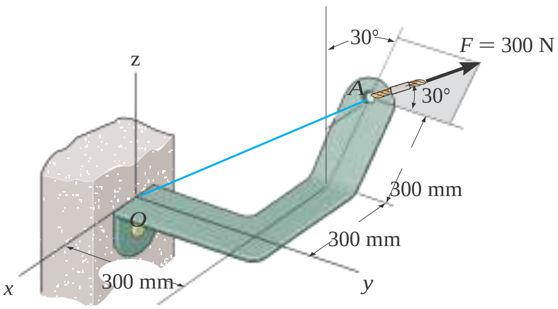
$$= -75(0) + 259.81(1) + 129.90(0)$$

$$= 260 \text{ N}$$

The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative x axis. Thus

$$F_x = 75 \text{ N}, \qquad F_y = 260 \text{ N}$$ **Ans.**

22-92. Determine the magnitude of the projected component of the force $F = 300$ N acting along line OA.



SOLUTION

Force and Unit Vector: The force vector $\mathbf{F}$ and unit vector $\mathbf{u}_{OA}$ must be determined first. From Fig. a

$$\mathbf{F} = (-300 \sin 30° \sin 30°\mathbf{i} + 300 \cos 30°\mathbf{j} + 300 \sin 30° \cos 30°\mathbf{k})$$

$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\} \text{ N}$$

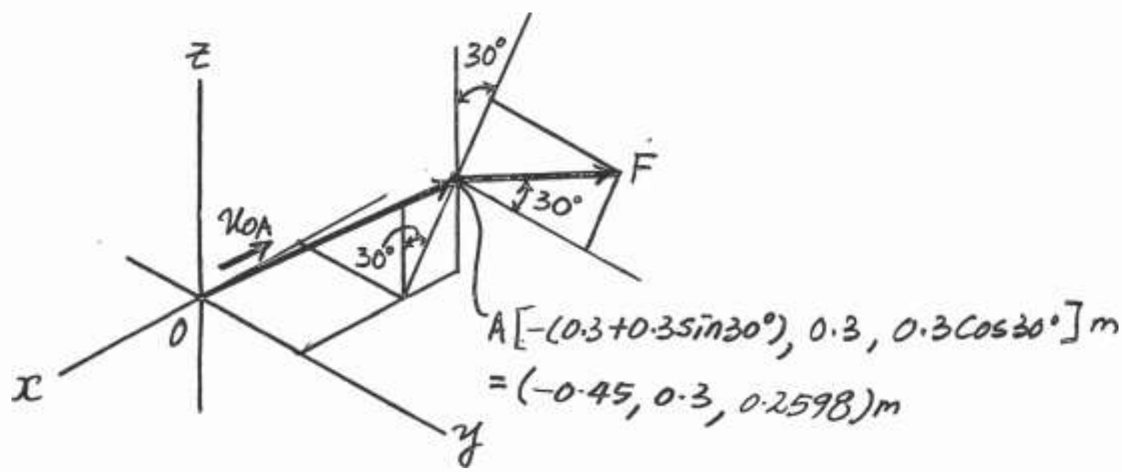
$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.2598 - 0)\mathbf{k}}{\sqrt{(-0.45 - 0)^2 + (0.3 - 0)^2 + (0.2598 - 0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}$ along line OA is

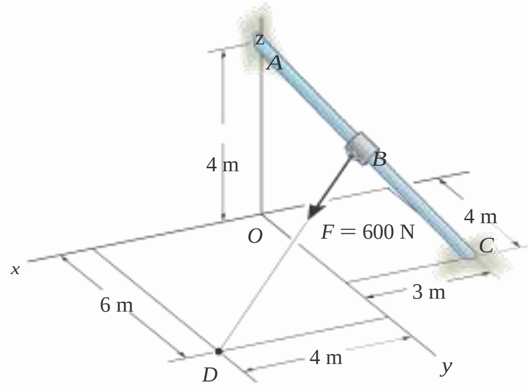
$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot (-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k})$$

$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$

$$= 242 \text{ N}$$ **Ans.**



2-93. Determine the components of **F** that act along rod *AC* and perpendicular to it. Point *B* is located at the midpoint of the rod.



SOLUTION

$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \qquad r_{AC} = \sqrt{(-3)^2 + 4^2 + (-4)^2} = \sqrt{41}\text{ m}$

$\mathbf{r}_{AB} = \dfrac{\mathbf{r}_{AC}}{2} = \dfrac{-3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}}{2} = -1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$

$\mathbf{r}_{AD} = \mathbf{r}_{AB} + \mathbf{r}_{BD}$

$\mathbf{r}_{BD} = \mathbf{r}_{AD} - \mathbf{r}_{AB}$

$= (4\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}) - (-1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$

$= \{5.5\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\}\text{ m}$

$r_{BD} = \sqrt{(5.5)^2 + (4)^2 + (-2)^2} = 7.0887\text{ m}$

$\mathbf{F} = 600\left(\dfrac{\mathbf{r}_{BD}}{r_{BD}}\right) = 465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k}$

Component of **F** along $\mathbf{r}_{AC}$ is $\mathbf{F}_{||}$

$F_{||} = \dfrac{\mathbf{F}\cdot\mathbf{r}_{AC}}{r_{AC}} = \dfrac{(465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k})\cdot(-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$

$F_{||} = 99.1408 = 99.1\text{ N}$ **Ans.**

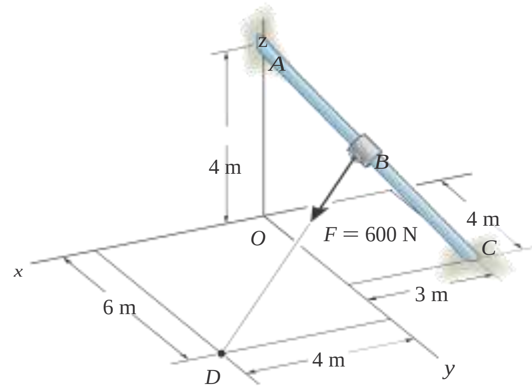
Component of *F* perpendicular to $\mathbf{r}_{AC}$ is $F_{\perp}$

$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$

$F_{\perp}^2 = 600^2 - 99.1408^2$

$F_{\perp} = 591.75 = 592\text{ N}$ **Ans.**

2-94. Determine the components of **F** that act along rod *AC* and perpendicular to it. Point *B* is located 3 m along the rod from end *C*.



SOLUTION

$\mathbf{r}_{CA} = 3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$

$r_{CA} = 6.403124$

$\mathbf{r}_{CB} = \frac{3}{6.403124}(\mathbf{r}_{CA}) = 1.40556\mathbf{i} - 1.874085\mathbf{j} + 1.874085\mathbf{k}$

$\mathbf{r}_{OB} = \mathbf{r}_{OC} + \mathbf{r}_{CB}$

$= -3\mathbf{i} + 4\mathbf{j} + \mathbf{r}_{CB}$

$= -1.59444\mathbf{i} + 2.1259\mathbf{j} + 1.874085\mathbf{k}$

$\mathbf{r}_{OD} = \mathbf{r}_{OB} + \mathbf{r}_{BD}$

$\mathbf{r}_{BD} = \mathbf{r}_{OD} - \mathbf{r}_{OB} = (4\mathbf{i} + 6\mathbf{j}) - \mathbf{r}_{OB}$

$= 5.5944\mathbf{i} + 3.8741\mathbf{j} - 1.874085\mathbf{k}$

$r_{BD} = \sqrt{(5.5944)^2 + (3.8741)^2 + (-1.874085)^2} = 7.0582$

$\mathbf{F} = 600\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = 475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k}$

$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \qquad r_{AC} = \sqrt{41}$

Component of **F** along $\mathbf{r}_{AC}$ is $\mathbf{F}_{||}$

$$F_{||} = \frac{\mathbf{F} \cdot \mathbf{r}_{AC}}{r_{AC}} = \frac{(475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k}) \cdot (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$$

$F_{||} = 82.4351 = 82.4\text{ N}$ **Ans.**

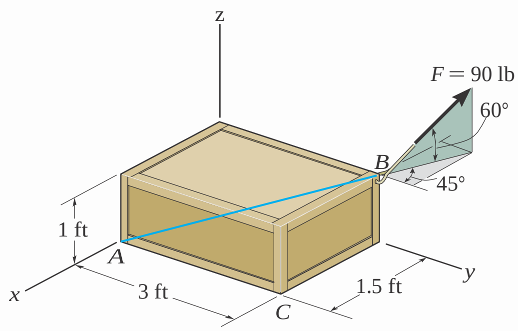
Component of **F** perpendicular to $\mathbf{r}_{AC}$ is $\mathbf{F}_{\perp}$

$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$

$F_{\perp}^2 = 600^2 - 82.4351^2$

$F_{\perp} = 594\text{ N}$ **Ans.**

2-95. Determine the magnitudes of the components of force $F = 90$ lb acting parallel and perpendicular to diagonal AB of the crate.



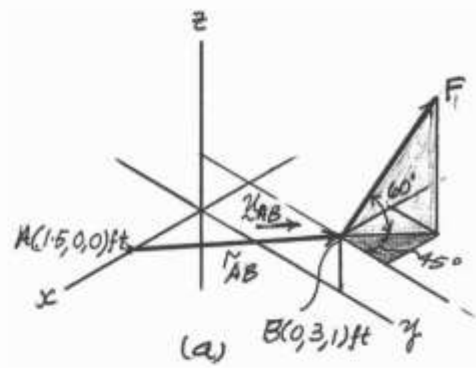
SOLUTION

Force and Unit Vector: The force vector **F** and unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*

$$\mathbf{F} = 90(-\cos 60^\circ \sin 45^\circ \mathbf{i} + \cos 60^\circ \cos 45^\circ \mathbf{j} + \sin 60^\circ \mathbf{k})$$

$$= [-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}] \text{ lb}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(0 - 1.5)\mathbf{i} + (3 - 0)\mathbf{j} + (1 - 0)\mathbf{k}}{\sqrt{(0 - 1.5)^2 + (3 - 0)^2 + (1 - 0)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$



(a)

Vector Dot Product: The magnitude of the projected component of **F** parallel to the diagonal AB is

$$[(F)_{AB}]_{\text{pa}} = \mathbf{F} \cdot \mathbf{u}_{AB} = (-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}) \cdot \left(-\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

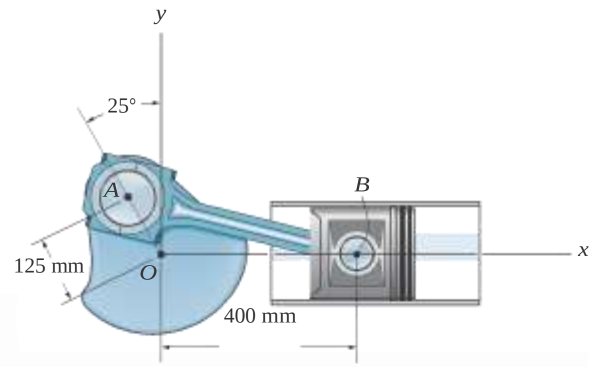
$$= (-31.82)\left(-\frac{3}{7}\right) + 31.82\left(\frac{6}{7}\right) + 77.94\left(\frac{2}{7}\right)$$

$$= 63.18 \text{ lb} = 63.2 \text{ lb}$$ **Ans.**

The magnitude of the component **F** perpendicular to the diagonal AB is

$$[(F)_{AB}]_{\text{pr}} = \sqrt{F^2 - [(F)_{AB}]_{\text{pa}}^2} = \sqrt{90^2 - 63.18^2} = 64.1 \text{ lb}$$ **Ans.**

***2-96.** Determine the length of the connecting rod AB by first formulating a Cartesian position vector from A to B and then determining its magnitude.



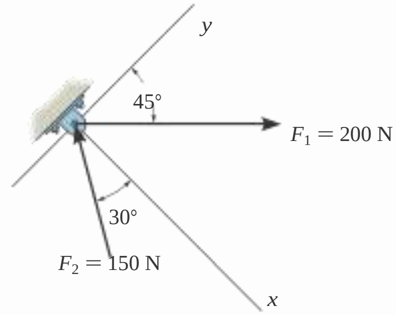
SOLUTION

$$\mathbf{r}_{AB} = [16 - (-5\sin 30°)]\mathbf{i} + (0 - 5\cos 30°)\,\mathbf{j}$$

$$= \{18.5\,\mathbf{i} - 4.330\,\mathbf{j}\}\text{ in.}$$

$$r_{AB} = \sqrt{(18.5)^2 + (4.330)^2} = 19.0\text{ in.}$$ **Ans.**

2-97. Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$.



SOLUTION

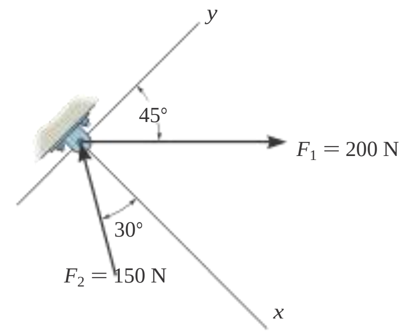
$F_{1x} = 200 \sin 45° = 141 \text{ N}$ **Ans.**

$F_{1y} = 200 \cos 45° = 141 \text{ N}$ **Ans.**

$F_{2x} = -150 \cos 30° = -130 \text{ N}$ **Ans.**

$F_{2y} = 150 \sin 30° = 75 \text{ N}$ **Ans.**

2-98. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive *x* axis.



SOLUTION

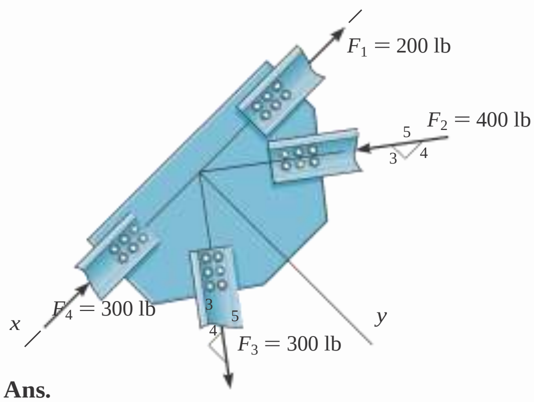
$\searrow^{+} F_{Rx} = \Sigma F_x; \quad F_{Rx} = -150 \cos 30° + 200 \sin 45° = 11.518 \text{ N}$

$\nearrow^{+} F_{Ry} = \Sigma F_y; \quad F_{Ry} = 150 \sin 30° + 200 \cos 45° = 216.421 \text{ N}$

$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217 \text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{216.421}{11.518}\right) = 87.0°$ **Ans.**

2-99. Determine the x and y components of each force acting on the *gusset plate* of the bridge truss. Show that the resultant force is zero.



SOLUTION

$F_{1x} = -200$ lb **Ans.**

$F_{1y} = 0$ **Ans.**

$F_{2x} = 400\left(\frac{4}{5}\right) = 320$ lb **Ans.**

$F_{2y} = -400\left(\frac{3}{5}\right) = -240$ lb **Ans.**

$F_{3x} = 300\left(\frac{3}{5}\right) = 180$ lb **Ans.**

$F_{3y} = 300\left(\frac{4}{5}\right) = 240$ lb **Ans.**

$F_{4x} = -300$ lb **Ans.**

$F_{4y} = 0$ **Ans.**

$F_{Rx} = F_{1x} + F_{2x} + F_{3x} + F_{4x}$

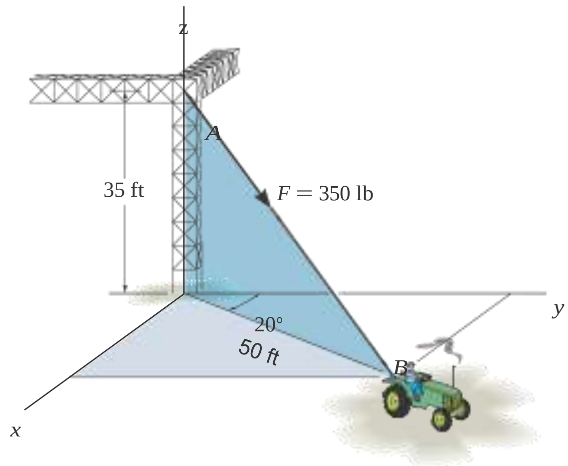
$F_{Rx} = -200 + 320 + 180 - 300 = 0$

$F_{Ry} = F_{1y} + F_{2y} + F_{3y} + F_{4y}$

$F_{Ry} = 0 - 240 + 240 + 0 = 0$

Thus, $F_R = 0$

***2-100.** The cable attached to the tractor at B exerts a force of 350 lb on the framework. Express this force as a Cartesian vector.



SOLUTION

$$\mathbf{r} = 50 \sin 20°\mathbf{i} + 50 \cos 20°\mathbf{j} - 35\mathbf{k}$$

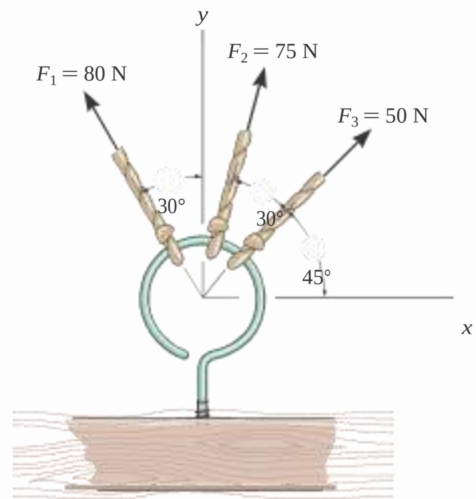
$$\mathbf{r} = \{17.10\mathbf{i} + 46.98\mathbf{j} - 35\mathbf{k}\}\ \text{ft}$$

$$r = \sqrt{(17.10)^2 + (46.98)^2 + (-35)^2} = 61.03\ \text{ft}$$

$$\mathbf{u} = \frac{\mathbf{r}}{r} = (0.280\mathbf{i} + 0.770\mathbf{j} - 0.573\mathbf{k})$$

$$\mathbf{F} = F\mathbf{u} = \{98.1\mathbf{i} + 269\mathbf{j} - 201\mathbf{k}\}\ \text{lb}$$ **Ans.**

2-101. Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_2$. Specify its direction measured counterclockwise from the positive x axis.



SOLUTION

$$F' = \sqrt{(80)^2 + (50)^2 - 2(80)(50)\cos 105^\circ} = 104.7 \text{ N}$$

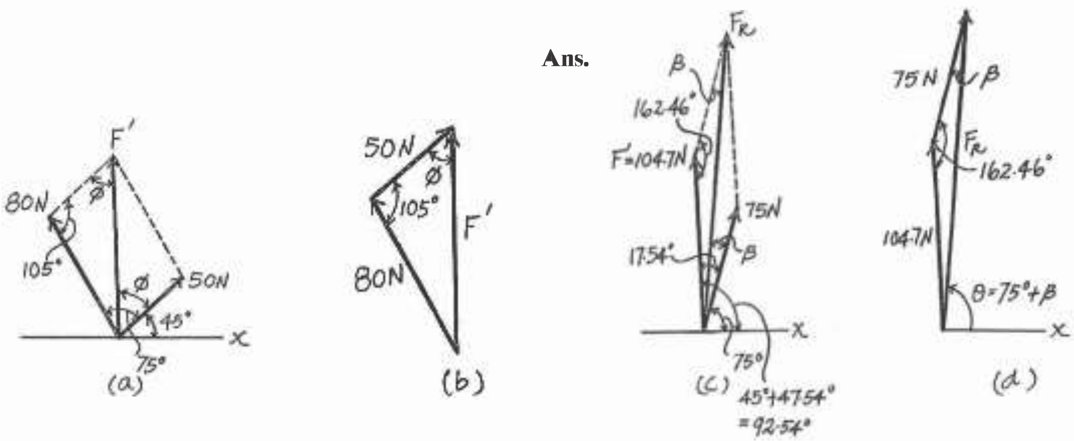
$$\frac{\sin \phi}{80} = \frac{\sin 105^\circ}{104.7}; \qquad \phi = 47.54^\circ$$

$$F_R = \sqrt{(104.7)^2 + (75)^2 - 2(104.7)(75)\cos 162.46^\circ}$$

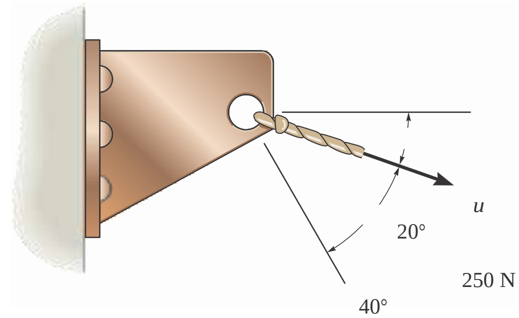
$$F_R = 177.7 = 178 \text{ N}$$ **Ans.**

$$\frac{\sin \beta}{104.7} = \frac{\sin 162.46^\circ}{177.7}; \qquad \beta = 10.23^\circ$$

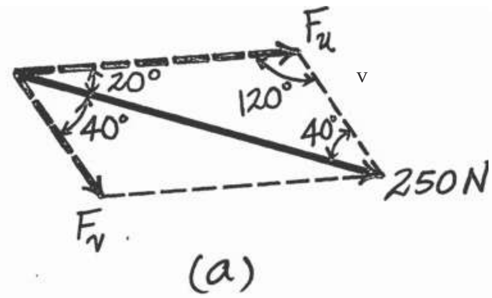
$$\theta = 75^\circ + 10.23^\circ = 85.2^\circ$$ **Ans.**



2-102. Resolve the 250-N force into components acting along the u and v axes and determine the magnitudes of these components.

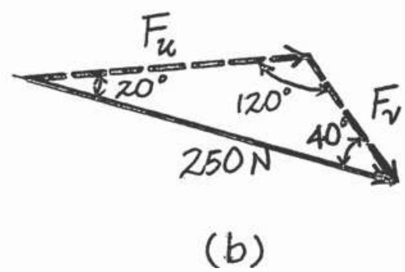


SOLUTION

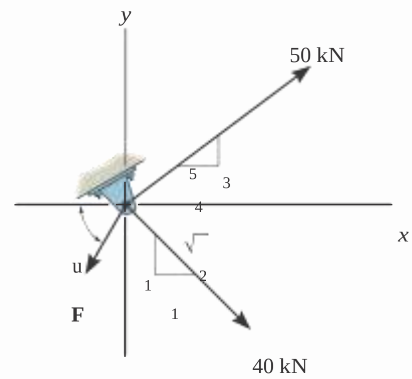


$$\frac{250}{\sin 120^\circ} = \frac{F_u}{\sin 40^\circ}; \qquad F_u = 186\text{ N} \qquad \text{Ans}$$

$$\frac{250}{\sin 120^\circ} = \frac{F_v}{\sin 20^\circ}; \qquad F_v = 98.7\text{ N} \qquad \text{Ans}$$



2-103. If $\theta = 60°$ and $F = 20$ kN, determine the magnitude of the resultant force and its direction measured clockwise from the positive x axis.



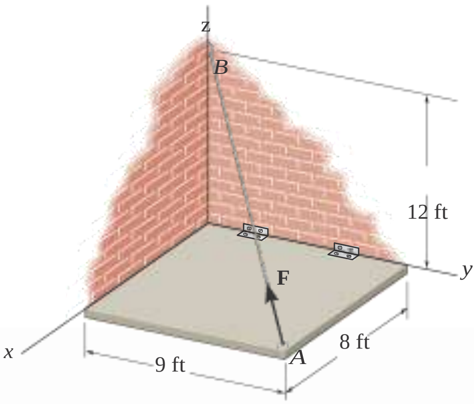
$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 50\left(\frac{4}{5}\right) + \frac{1}{\sqrt{2}}(40) - 20\cos 60° = 58.28\text{ kN}$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 50\left(\frac{3}{5}\right) - \frac{1}{\sqrt{2}}(40) - 20\sin 60° = -15.60\text{ kN}$

$F_R = \sqrt{(58.28)^2 + (-15.60)^2} = 60.3\text{ kN}$ **Ans.**

$\phi = \tan^{-1}\left[\frac{15.60}{58.28}\right] = 15.0°$ **Ans.**

***2-104.** The hinged plate is supported by the cord AB. If the force in the cord is $F = 340$ lb, express this force, directed from A toward B, as a Cartesian vector. What is the length of the cord?



SOLUTION

Unit Vector:

$$\mathbf{r}_{AB} = \{(0-8)\mathbf{i} + (0-9)\mathbf{j} + (12-0)\mathbf{k}\}\text{ ft}$$

$$= \{-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{(-8)^2 + (-9)^2 + 12^2} = 17.0\text{ ft} \qquad \textbf{Ans.}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}}{17} = -\frac{8}{17}\mathbf{i} - \frac{9}{17}\mathbf{j} + \frac{12}{17}\mathbf{k}$$

Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 340\left\{-\frac{8}{17}\mathbf{i} - \frac{9}{17}\mathbf{j} + \frac{12}{17}\mathbf{k}\right\}\text{ lb}$$

$$= \{-160\mathbf{i} - 180\mathbf{j} + 240\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$