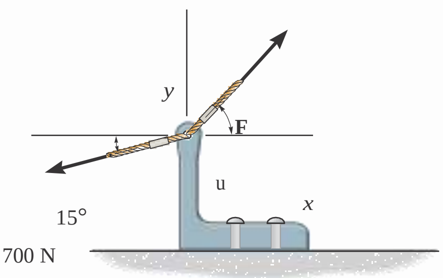
Solution Manual for Statics and Mechanics of Materials 5th Edition Hibbeler 0134382595 9780134382593

2–1.

If u = 60° and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of consines to Fig. *b*,

$$F_R = \sqrt{700^2 + 450^2 - 2(700)(450)\cos 45°}$$

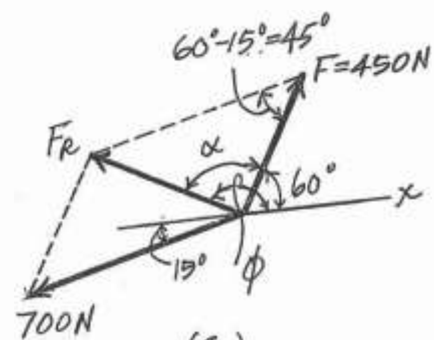
$$= 497.01 \text{ N} = 497 \text{ N} \qquad \textbf{Ans.}$$

This yields

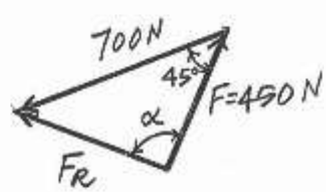
$$\frac{\sin \alpha}{700} = \frac{\sin 45°}{497.01} \qquad \alpha = 95.19°$$

Thus, the direction of angle ϕ of F_R measured counterclockwise from the positive x axis, is

$$\phi = \alpha + 60° = 95.19° + 60° = 155° \qquad \textbf{Ans.}$$



(a)

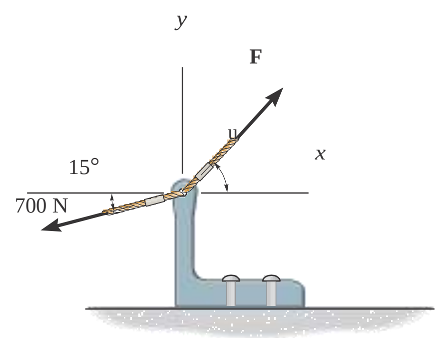


(b)

Ans:
$F_R = 497$ N
$\phi = 155°$

2–2.

If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force **F** and its direction θ.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

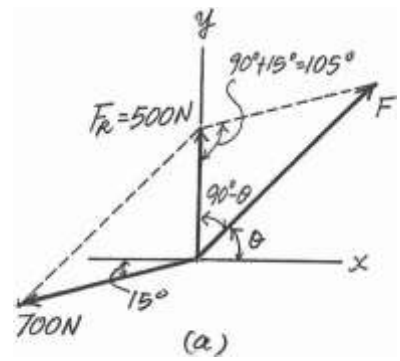
$$F = \sqrt{500^2 + 700^2 - 2(500)(700)\cos 105^\circ}$$

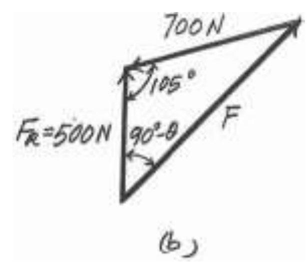
$$= 959.78 \text{ N} = 960 \text{ N} \qquad \textbf{Ans.}$$

Applying the law of sines to Fig. *b*, and using this result, yields

$$\frac{\sin(90^\circ + \theta)}{700} = \frac{\sin 105^\circ}{959.78}$$

$$\theta = 45.2^\circ \qquad \textbf{Ans.}$$

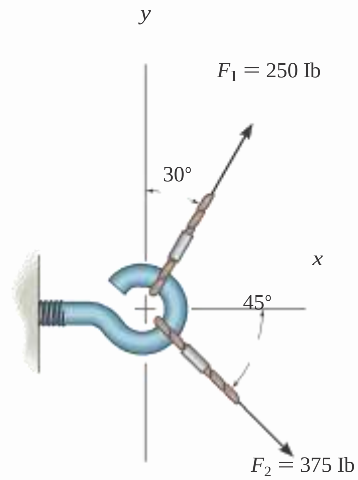




Ans:
$F = 960$ N
u = 45.2°

2–3.

Determine the magnitude of the resultant force $F_R = F_1 + F_2$ and its direction, measured counterclockwise from the positive x axis.



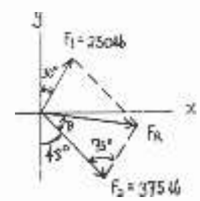
SOLUTION

$$F_R = \sqrt{(250)^2 + (375)^2 - 2(250)(375)\cos 75^\circ} = 393.2 = 393 \text{ lb}$$ **Ans.**

$$\frac{393.2}{\sin 75^\circ} = \frac{250}{\sin \theta}$$

$$\theta = 37.89^\circ$$

$$\phi = 360^\circ - 45^\circ + 37.89^\circ = 353^\circ$$ **Ans.**



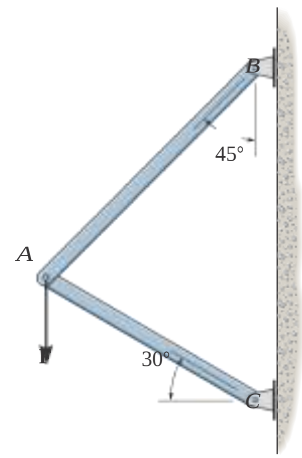
Ans:

F_R = 393 lb
ϕ = 353°

***2–4.**

The vertical force F acts downward at A on the two-membered frame. Determine the magnitudes of the two components of F directed along members AB and AC. Set $F = 500$ N.

Determine the magnitudes of the two components of F directed along the axes of AB and AC. Set F = 500 N.



SOLUTION

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Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using the law of sines (Fig. *b*), we have

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60°} = \frac{500}{\sin 75°}$$

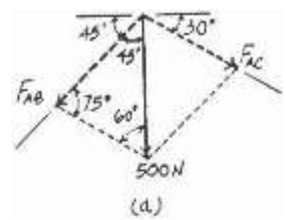
$$F_{AB} = 448 \text{ N} \qquad \textbf{Ans.}$$

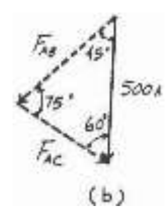
$$\frac{F_{AC}}{\sin 45°} = \frac{500}{\sin 75°}$$

$$F_{AC} = 366 \text{ N} \qquad \textbf{Ans.}$$

Ans.

Ans.

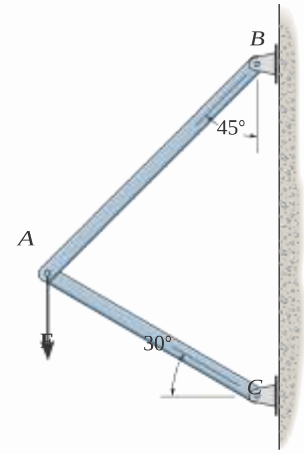




Ans:
F_{AB} = 448 N
F_{AC} = 366 N

2–5.

Solve Prob. 2–4 with $F = 350$ lb.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

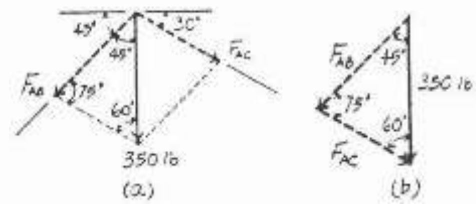
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60°} = \frac{350}{\sin 75°}$$

$$F_{AB} = 314 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{F_{AC}}{\sin 45°} = \frac{350}{\sin 75°}$$

$$F_{AC} = 256 \text{ lb} \qquad \textbf{Ans.}$$

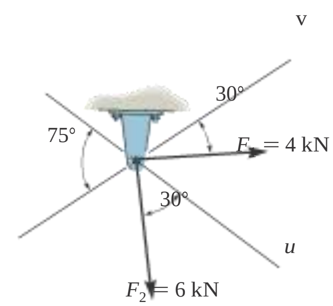


Ans:
$F_{AB} = 314$ lb

$F_{AC} = 256 \text{ lb}$

2–6.

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive *u* axis.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.
Trigonometry. Applying Law of cosines by referring to Fig. *b*,

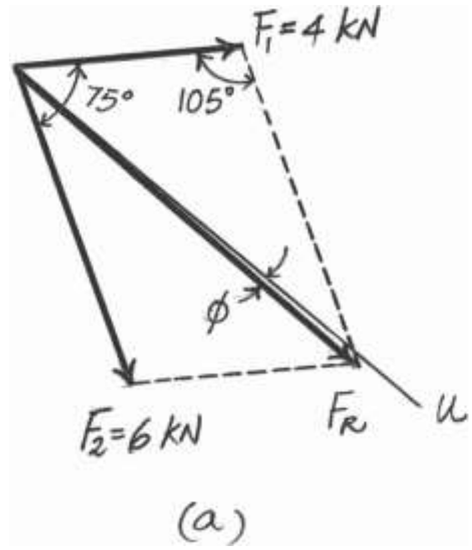
$$F_R = \sqrt{4^2 + 6^2 - 2(4)(6)\cos 105°} = 8.026\text{ kN} = 8.03\text{ kN}$$ **Ans.**

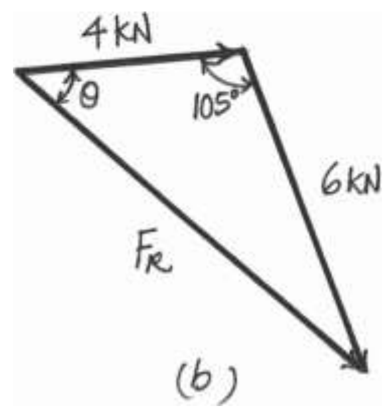
Using this result to apply Law of sines, Fig. *b*,

$$\frac{\sin u}{6} = \frac{\sin 105°}{8.026}; \qquad u = 46.22°$$

Thus, the direction f of $\mathbf{F}_R$ measured clockwise from the positive *u* axis is

$$f = 46.22° - 45° = 1.22°$$ **Ans.**



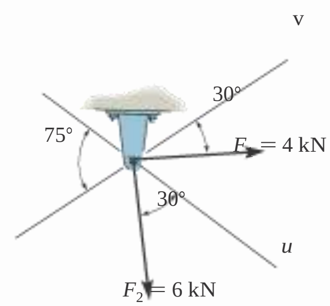


Ans:
$F_R = 8.03$ kN

$\phi = 1.22°$

2–7.

Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.

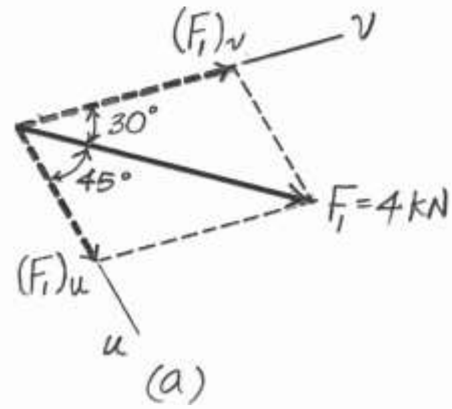


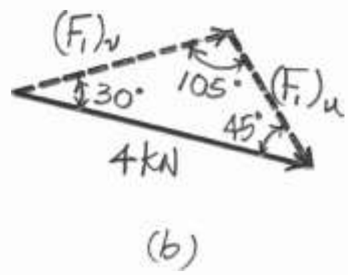
SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.
Trigonometry. Applying the sines law by referring to Fig. *b*.

$$\frac{(F_1)_v}{\sin 45^\circ} = \frac{4}{\sin 105^\circ}; \qquad (F_1)_v = 2.928 \text{ kN} = 2.93 \text{ kN} \qquad \textbf{Ans.}$$

$$\frac{(F_1)_u}{\sin 30^\circ} = \frac{4}{\sin 105^\circ}; \qquad (F_1)_u = 2.071 \text{ kN} = 2.07 \text{ kN} \qquad \textbf{Ans.}$$

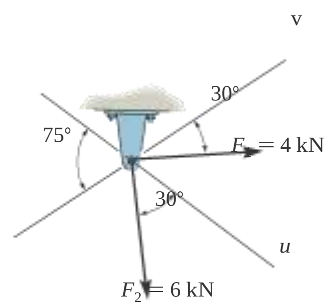




Ans:
$(F_1)_v = 2.93$ kN
$(F_1)_u = 2.07$ kN

***2–8.**

Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.

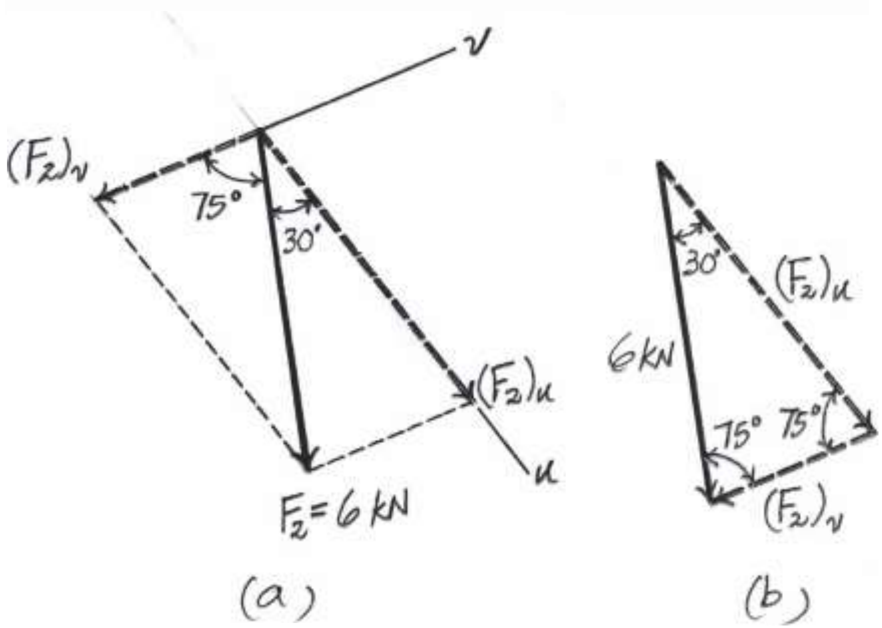


SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.
Trigonometry. Applying the sines law of referring to Fig. *b*,

$$\frac{(F_2)_u}{\sin 75^\circ} = \frac{6}{\sin 75^\circ}; \qquad (F_2)_u = 6.00\text{ kN} \qquad \textbf{Ans.}$$

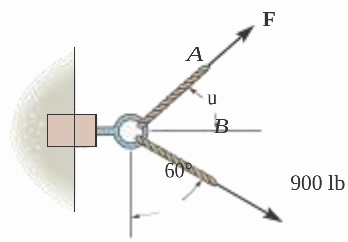
$$\frac{(F_2)_v}{\sin 30^\circ} = \frac{6}{\sin 75^\circ}; \qquad (F_2)_v = 3.106\text{ kN} = 3.11\text{ kN} \qquad \textbf{Ans.}$$



Ans:
$(F_2)_u$ = 6.00 kN
$(F_2)_v$ = 3.11 kN

2–9.

If the resultant force acting on the support is to be 1200 lb, directed horizontally to the right, determine the force **F** in rope *A* and the corresponding angle u.



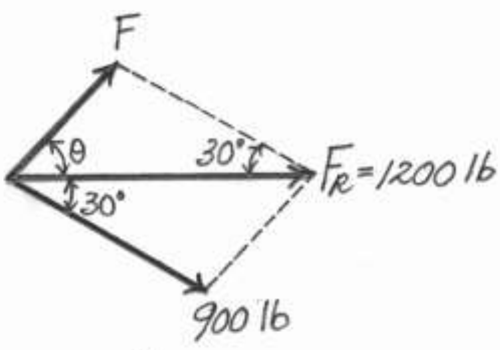
SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.
Trigonometry. Applying the law of cosines by referring to Fig. *b*,

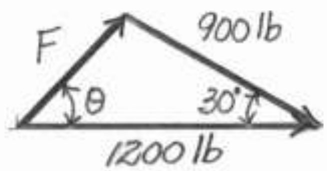
$$F = \sqrt{900^2 + 1200^2 - 2(900)(1200)\cos 30^\circ} = 615.94 \text{ lb} = 616 \text{ lb} \qquad \textbf{Ans.}$$

Using this result to apply the sines law, Fig. *b*,

$$\frac{\sin u}{900} = \frac{\sin 30^\circ}{615.94}; \qquad u = 46.94^\circ = 46.9^\circ \qquad \textbf{Ans.}$$



(a)



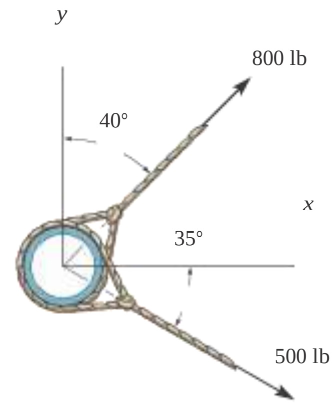
(b)

Ans:
$F = 616$ lb

$u = 46.9°$

2–10.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive *x* axis.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.

Trigonometry. Applying the law of cosines by referring to Fig. *b*,

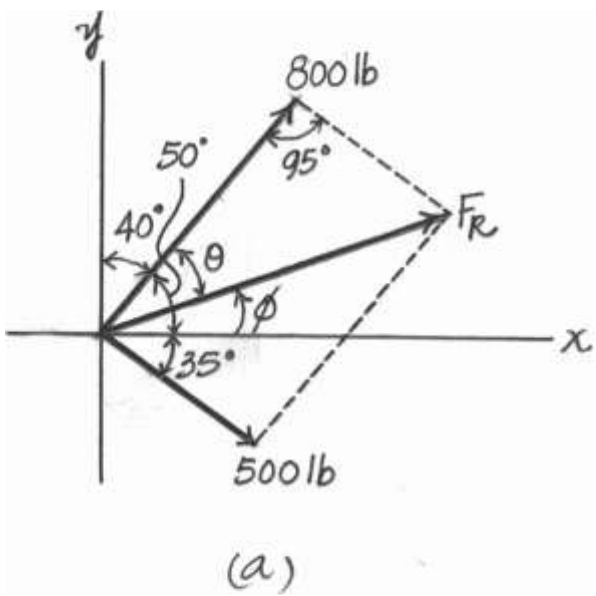
$$F_R = \sqrt{800^2 + 500^2 - 2(800)(500)\cos 95^\circ} = 979.66 \text{ lb} = 980 \text{ lb} \qquad \textbf{Ans.}$$

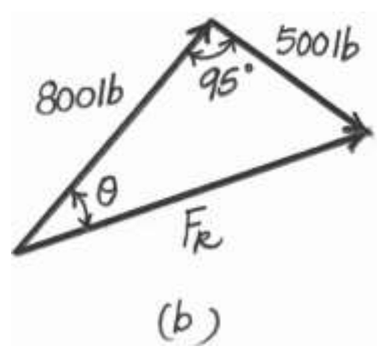
Using this result to apply the sines law, Fig. *b*,

$$\frac{\sin \theta}{500} = \frac{\sin 95^\circ}{979.66}; \qquad \theta = 30.56^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive *x* axis is

$$\phi = 50^\circ - 30.56^\circ = 19.44^\circ = 19.4^\circ \qquad \textbf{Ans.}$$



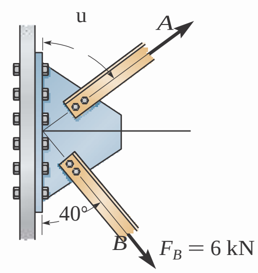


Ans:
$F_R = 980$ lb
$\phi = 19.4^\circ$

2–11.

If 0 = 60°, determine the magnitude of the resultant and its direction measured clockwise from the horizontal.

The plate is subjected to the two forces at *A* and *B* as shown. If u = 60°, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



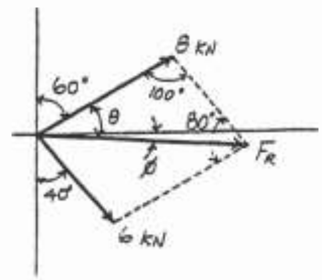
SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of cosines (Fig. *b*), we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100°}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN}$$ **Ans.**



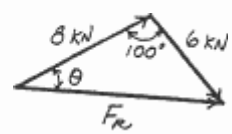
(a)

The angle u can be determined using law of sines (Fig. *b*).

$$\frac{\sin u}{6} = \frac{\sin 100°}{10.80}$$

$$\sin u = 0.5470$$

$$u = 33.16°$$



(b)

Thus, the direction f of $\mathbf{F}_R$ measured from the *x* axis is

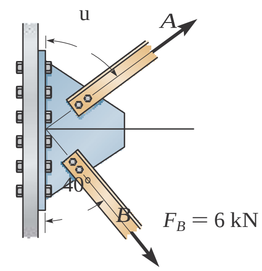
$$f = 33.16° - 30° = 3.16°$$ **Ans.**

Ans:

$F_R = 10.8$ kN
$\mathbf{f} = 3.16°$

***2–12.**

Determine the angle 0 for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



SOLUTION

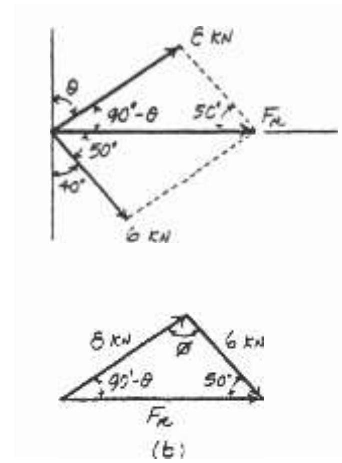
Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig .*b*), we have

$$\frac{\sin(90° - u)}{6} = \frac{\sin 50°}{8}$$

$$\sin(90° - u) = 0.5745$$

$$u = 54.93° = 54.9° \qquad \textbf{Ans.}$$



From the triangle, f $= 180° - (90° - 54.93°) - 50° = 94.93°$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

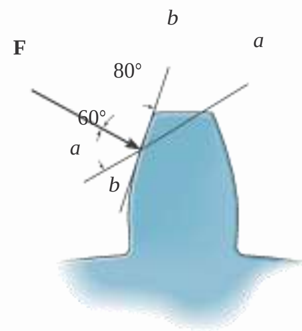
$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93°}$$

$$= 10.4 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$0 = 54.9°$
$F_R = 10.4$ kN

2–13.

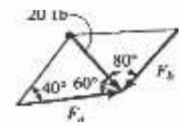
The force acting on the gear tooth is $F = 20$ lb. Resolve this force into two components acting along the lines *aa* and *bb*.



SOLUTION

$$\frac{20}{\sin 40^\circ} = \frac{F_a}{\sin 80^\circ}; \quad F_a = 30.6 \text{ lb}$$ **Ans.**

$$\frac{20}{\sin 40^\circ} = \frac{F_b}{\sin 60^\circ}; \quad F_b = 26.9 \text{ lb}$$ **Ans.**

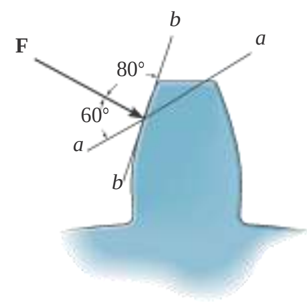


Ans:

$F_a = 30.6 \text{ lb}$
$F_b = 26.9 \text{ lb}$

2–14.

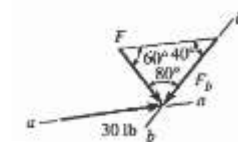
The component of force **F** acting along line *aa* is required to be 30 lb. Determine the magnitude of **F** and its component along line *bb*.



SOLUTION

$$\frac{30}{\sin 80^\circ} = \frac{F}{\sin 40^\circ}; \qquad F = 19.6 \text{ lb}$$ **Ans.**

$$\frac{30}{\sin 80^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.4 \text{ lb}$$ **Ans.**



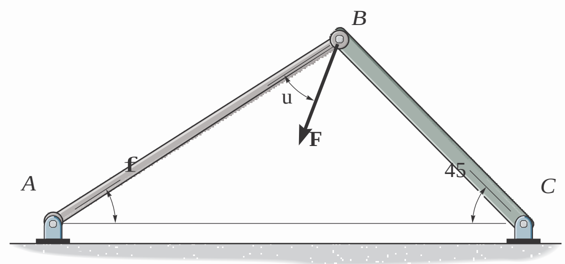
Ans:
$F = 19.6$ lb

$F_b = 26.4 \text{ lb}$

2–15.

Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A, and the component acting along member BC is 500 lb, directed from B towards C. Determine the magnitude of **F** and its direction θ. Set $\phi = 60°$.

Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A, and the component acting along member BC is 500 lb, directed from B towards C. Determine the magnitude of **F** and its direction θ. Set $\phi = 60°$.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$F = \sqrt{500^2 + 650^2 - 2(500)(650)\cos 105°}$$

$$F = 916.91 \text{ lb} = 917 \text{ lb} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b* yields

$$\frac{\sin \theta}{500} = \frac{\sin 105°}{916.91} \qquad \theta = 31.8° \qquad \textbf{Ans.}$$

SOLUTION

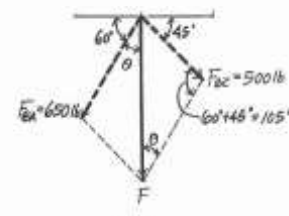
The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

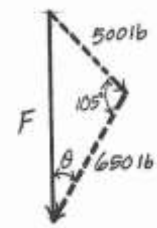
$$F = 916.91 \text{ lb} = 917 \text{ lb} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b* yields

$$\frac{\sin \theta}{500} = \frac{\sin 105°}{916.91} \qquad \theta = 31.8° \qquad \textbf{Ans.}$$



(a)



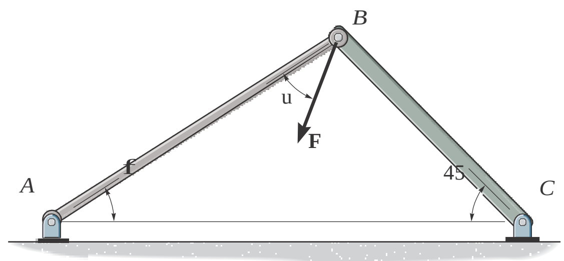
(b)

Ans:

$F = 917$ lb
$0 = 31.8°$

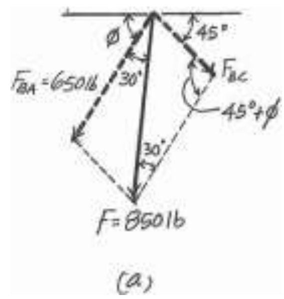
***2–16.**

Force **F** acts on the frame such that its component acting along member *AB* is 650 lb, directed from *B* towards *A*. Determine the required angle ϕ ($0° \le \phi \le 45°$) and the component acting along member *BC*. Set $F = 850$ lb and $\theta = 30°$.



SOLUTIO

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



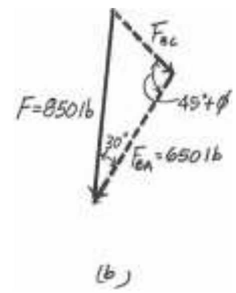
Applying the law of cosines to Fig. *b*,

$$F_{BC} = \sqrt{850^2 + 650^2 - 2(850)(650)\cos 30°}$$

$$= 433.64 \text{ lb} = 434 \text{ lb} \qquad \textbf{Ans.}$$

Using this result and applying the sine law to Fig. *b* yields

$$\frac{\sin(45° + \phi)}{850} = \frac{\sin 30°}{433.64} \qquad \phi = 33.5° \qquad \textbf{Ans.}$$

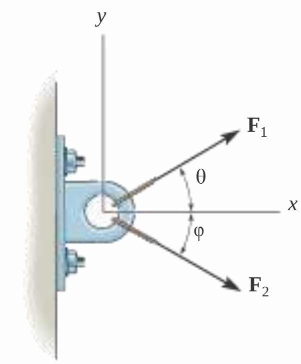


Ans:

$F_{BC} = 434$ lb
$\phi = 33.5°$

2–17.

If $F_1 = 30$ lb and $F_2 = 40$ lb, determine the angles 0 and **f** so that the resultant force is directed along the positive x axis and has a magnitude of $F_R = 60$ lb.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.
Trigonometry. Applying the law of cosine by referring to Fig. *b*,

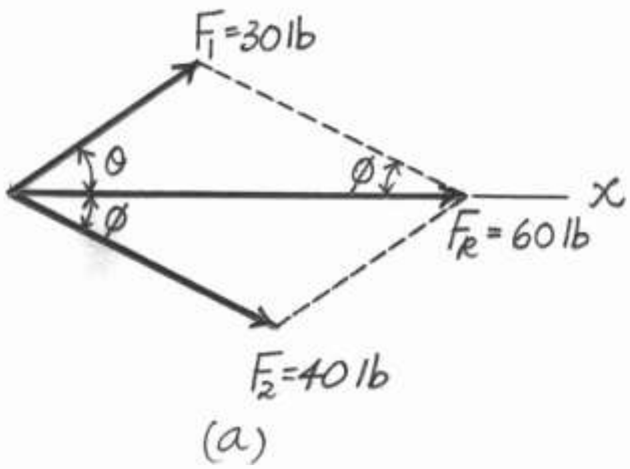
$$40^2 = 30^2 + 60^2 - 2(30)(60)\cos 0$$

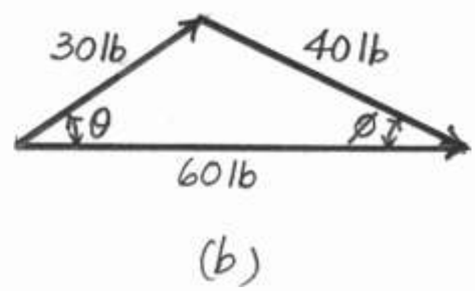
$$0 = 36.34° = 36.3°$$ **Ans.**

And

$$30^2 = 40^2 + 60^2 - 2(40)(60)\cos \mathbf{f}$$

$$\mathbf{f} = 26.38° = 26.4°$$ **Ans.**



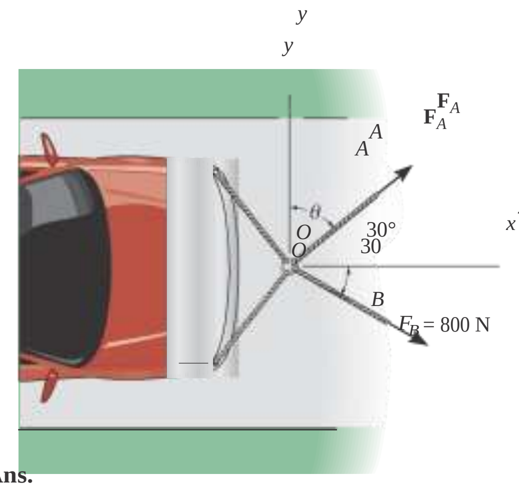


Ans:
0 = 36.3°
f = 26.4°

2–18.

Determine the magnitude and direction 6 of $\mathbf{F}_A$ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.

Determine the magnitude and direction of $\mathbf{F}_A$ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.



SOLUTION

$\xrightarrow{+} F_{R_x} = \Sigma F_x;$ $\quad F_{R_x} = F_A \sin 6 + 800 \cos 30° = 1250$

$+ \$ F_{R_y} = \Sigma F_y;$ $\quad F_{R_y} = F_A \cos 6 - 800 \sin 30° = 0$

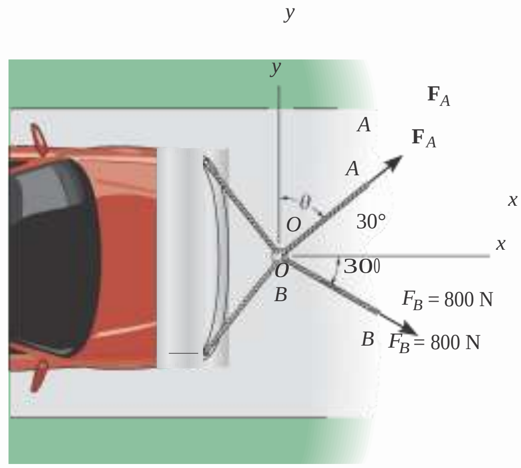
$6 = 54.3°$ **Ans.**

$F_A = 686 \text{ N}$ **Ans.**

Ans:
u = 54.3°
F_A = 686 N

2–19. Determine the magnitude and direction, measured counterclockwise from the positive x axis, of the resultant force acting on the ring at O, if $F_A = 750$ N and $\theta = 45°$.

Determine the magnitude of the resultant force acting on the ring at O if $F_A = 750$ N and $\theta = 45°$. What is its direction, measured counterclockwise from the positive x axis?



SOLUTION

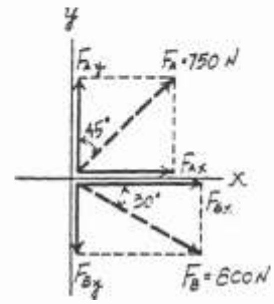
Scalar Notation: Suming the force components algebraically, we have

$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 750 \sin 45° + 800 \cos 30°$

$= 1223.15 \text{ N} \rightarrow$

$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 750 \cos 45° - 800 \sin 30°$

$= 130.33 \text{ N} \uparrow$



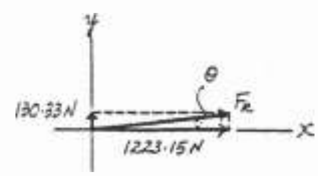
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2}$$

$$= \sqrt{1223.15^2 + 130.33^2} = 1230 \text{ N} = 1.23 \text{ kN} \qquad \textbf{Ans.}$$

The directional angle θ measured counterclockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{130.33}{1223.15}\right) = 6.08° \qquad \textbf{Ans.}$$

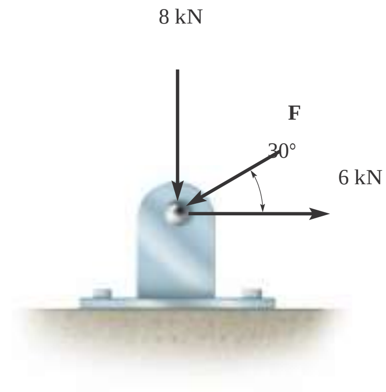


Ans:

$F_R = 1.23$ kN
0 = 6.08°

***2–20.**

Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible. What is the minimum magnitude of $\mathbf{F}_R$?



SOLUTION

Parallelogram Law. The parallelogram laws of addition for 6 kN and 8 kN and then their resultant F' and F are shown in Figs. *a* and *b*, respectively. In order for F_R to be minimum, it must act perpendicular to **F**.

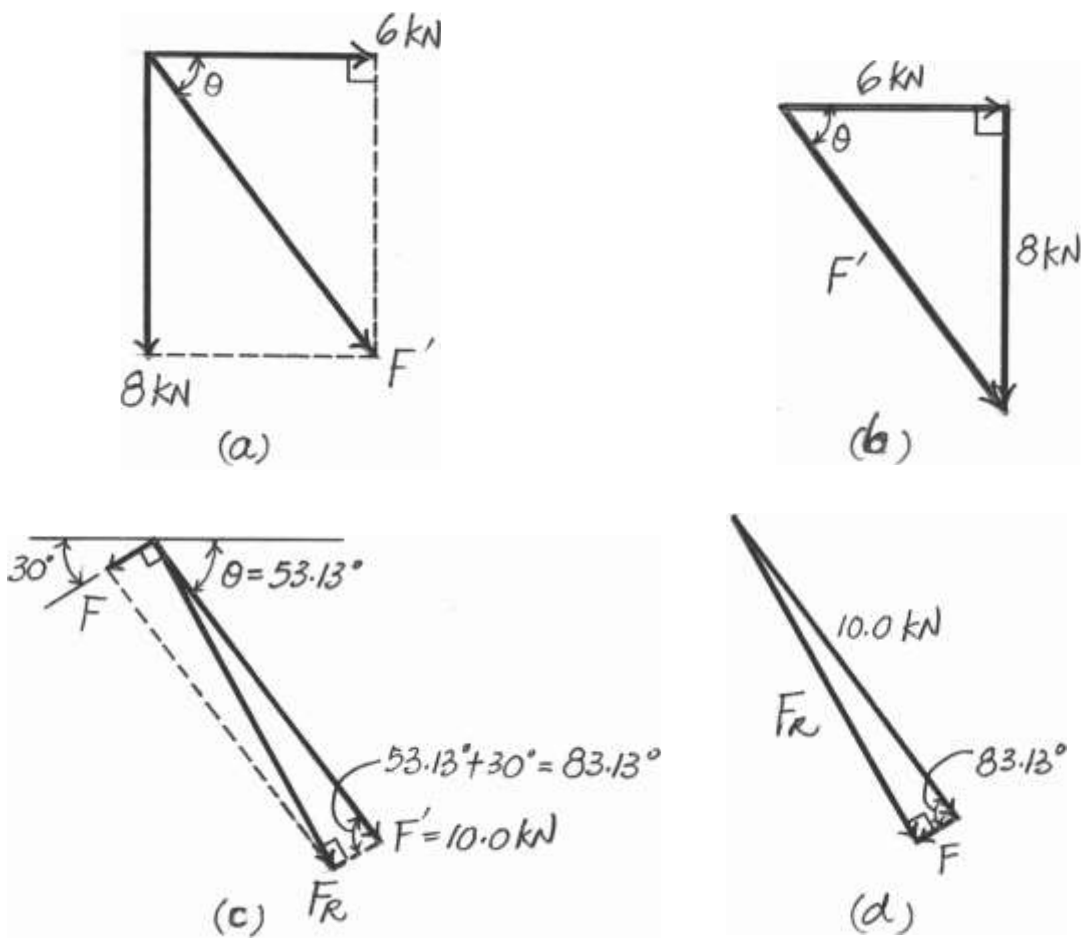
Trigonometry. Referring to Fig. *b*,

$$F' = \sqrt{6^2 + 8^2} = 10.0 \text{ kN} \qquad \theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.13°.$$

Referring to Figs. *c* and *d*,

$$F_R = 10.0 \sin 83.13° = 9.928 \text{ kN} = 9.93 \text{ kN} \qquad \textbf{Ans.}$$

$$F = 10.0 \cos 83.13° = 1.196 \text{ kN} = 1.20 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
F_R = 9.93 kN
F = 1.20 kN

2–21.

If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force $\mathbf{F}_B$ and its direction θ.

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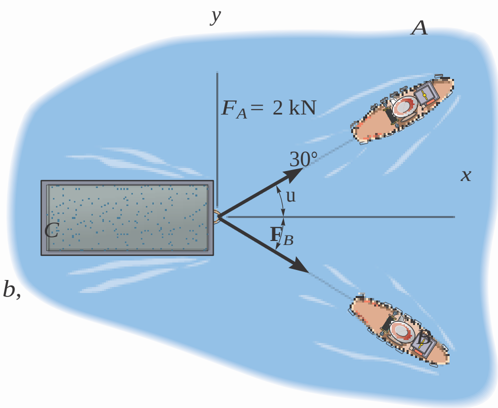


SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$F_B = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 30^\circ}$$

$$= 1.615\text{ kN} = 1.61\text{ kN}$$ **Ans.**

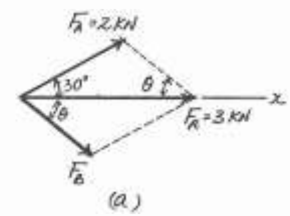
$$F_B = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 30^\circ} = 1.615\text{ kN} = 1.61\text{ kN}$$ **Ans.**

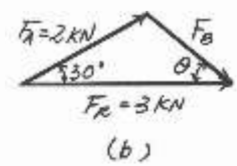
Using this result and applying the law of sines to Fig. *b* yields

$$\frac{\sin \theta}{2} = \frac{\sin 30^\circ}{1.615} \qquad \theta = 38.3^\circ$$ **Ans.**

Using this result and applying the law of sines to Fig. *b* yields

$$\frac{\sin \theta}{2} = \frac{\sin 30^\circ}{1.615} \qquad \theta = 38.3^\circ$$ **Ans.**

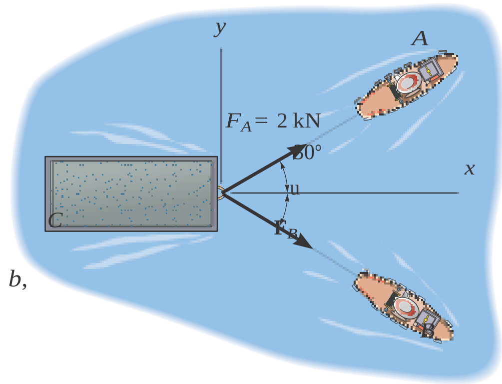




Ans:
F_B = 1.61 kN
0 = 38.3°

2–22.

If $F_B = 3$ kN and $\theta = 45°$, determine the magnitude of the resultant force of the two tugboats and its direction measured clockwise from the positive x axis.



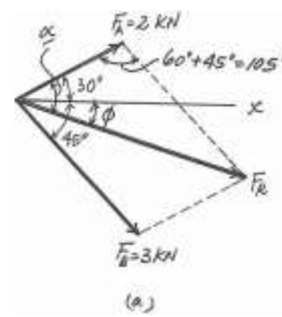
SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

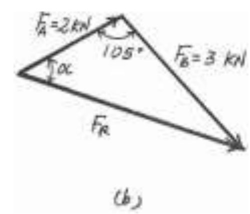
$$F_R = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 105°}$$

$$= 4.013 \text{ kN} = 4.01 \text{ kN}$$ **Ans.**



Using this result and applying the law of sines to Fig. *b* yields

$$\frac{\sin \alpha}{3} = \frac{\sin 105°}{4.013} \qquad \alpha = 46.22°$$



Thus, the direction angle ϕ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

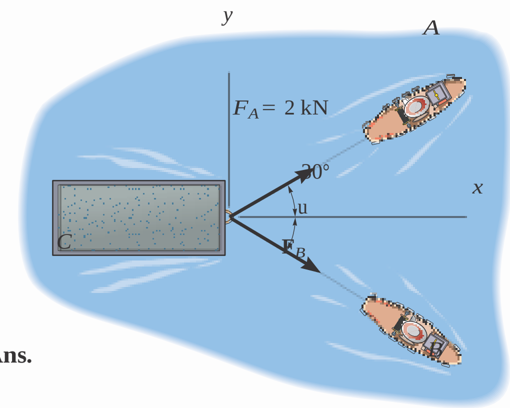
$$\phi = \alpha - 30° = 46.22° - 30° = 16.2°$$ **Ans.**

Ans:

$F_R = 4.01$ kN
$\phi = 16.2°$

2–23.

If the resultant force of the two tugboats is required to be directed towards the positive x axis, and F_B is to be a minimum, determine the magnitude of $\mathbf{F}_R$ and $\mathbf{F}_B$ and the angle u.



SOLUTION

For $\mathbf{F}_B$ to be minimum, it has to be directed perpendicular to $\mathbf{F}_R$. Thus,

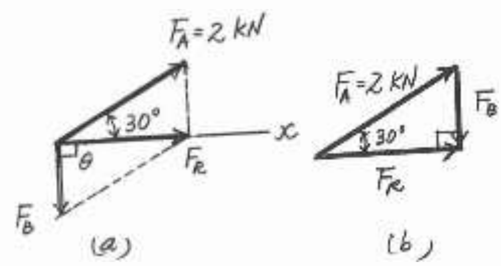
$$u = 90°$$ **Ans.**

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

By applying simple trigonometry to Fig. *b*,

$$F_B = 2 \sin 30° = 1 \text{ kN}$$ **Ans.**

$$F_R = 2 \cos 30° = 1.73 \text{ kN}$$ **Ans.**

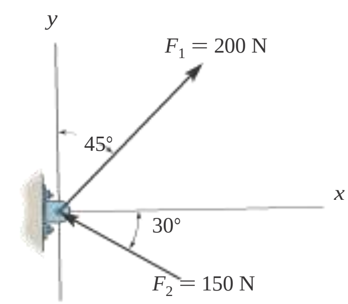


Ans:
$0 = 90°$
$F_B = 1$ kN

$F_R = 1.73 \text{ kN}$

***2–24.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200 \sin 45° - 150 \cos 30° = 11.518 \text{ N} \rightarrow$$

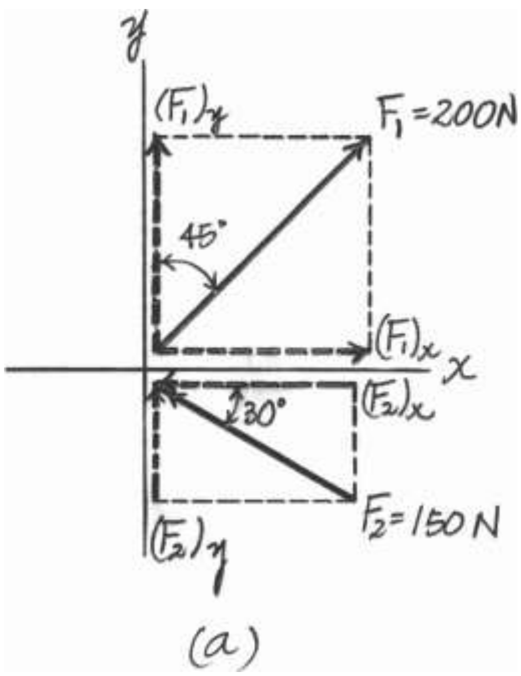
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 200 \cos 45° + 150 \sin 30° = 216.42 \text{ N} \uparrow$$

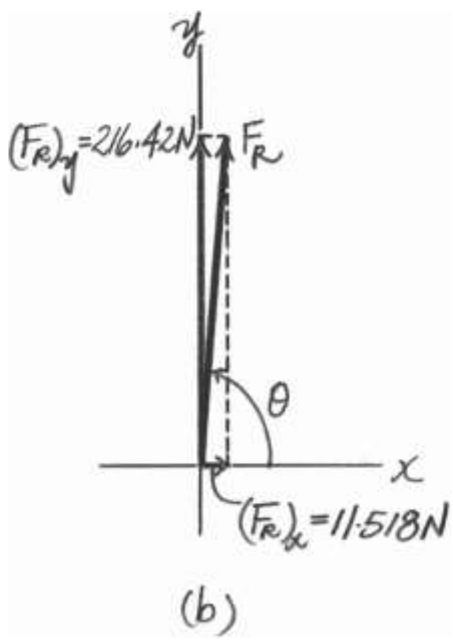
Referring to Fig. b, the magnitude of the resultant force F_R is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{11.518^2 + 216.42^2} = 216.73 \text{ N} = 217 \text{ N} \qquad \textbf{Ans.}$$

And the directional angle θ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{216.42}{11.518}\right) = 86.95° = 87.0° \qquad \textbf{Ans.}$$

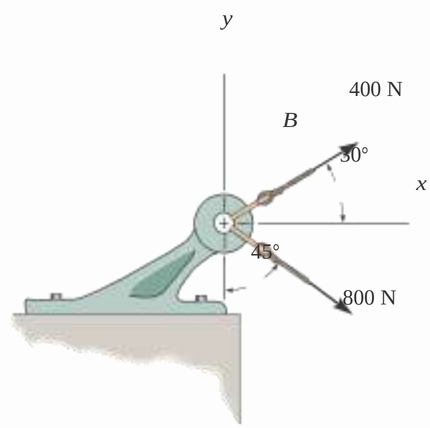




Ans:
F_R = 217 N
0 = 87.0°

2–25.

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes by referring to Fig. a,

$\xrightarrow{+} (F_R)_x = \Sigma F_x;\quad (F_R)_x = 400\cos 30° + 800\sin 45° = 912.10\text{ N} \rightarrow$

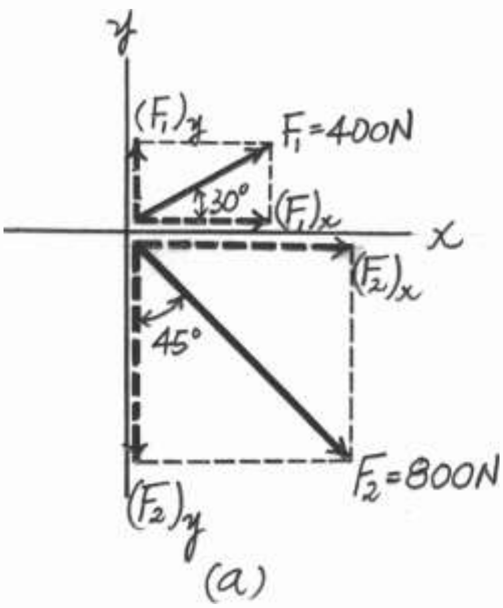
$+\uparrow (F_R)_y = \Sigma F_y;\quad (F_R)_y = 400\sin 30° - 800\cos 45° = -365.69\text{ N} = 365.69\text{ N}\downarrow$

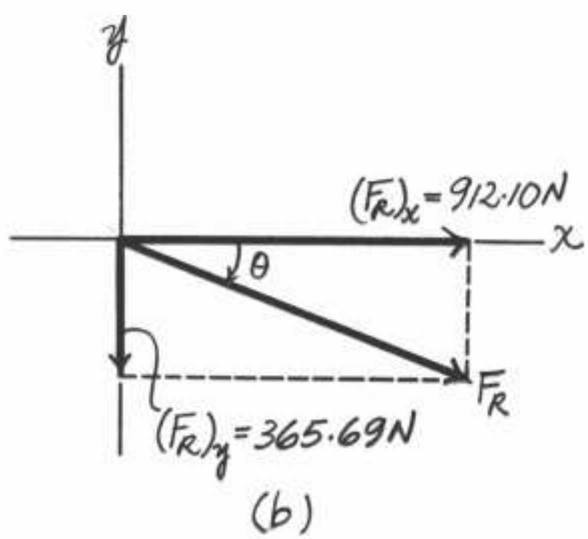
Referring to Fig. b, the magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{912.10^2 + 365.69^2} = 982.67\text{ N} = 983\text{ N} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{365.69}{912.10}\right) = 21.84° = 21.8° \qquad \textbf{Ans.}$$



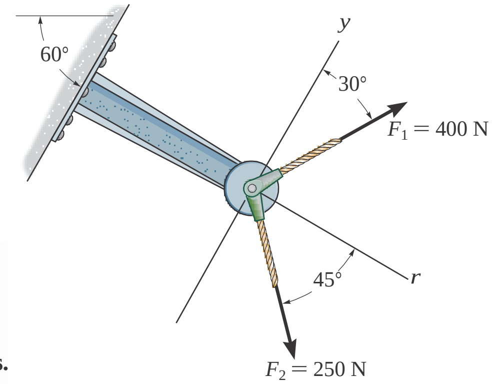


Ans:

$F_R = 983$ N
$0 = 21.8°$

2–26.

Resolve $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x and y components.



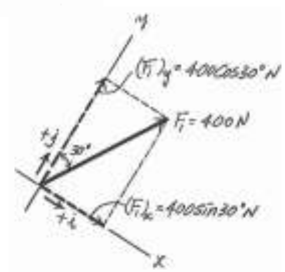
SOLUTION

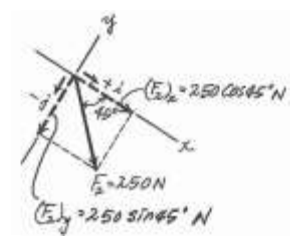
$$\mathbf{F}_1 = \{400 \sin 30°(+\mathbf{i}) + 400 \cos 30°(+\mathbf{j})\} \text{ N}$$

$$= \{200\mathbf{i} + 346\mathbf{j}\} \text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = \{250 \cos 45°(+\mathbf{i}) + 250 \sin 45°(-\mathbf{j})\} \text{ N}$$

$$= \{177\mathbf{i} - 177\mathbf{j}\} \text{ N} \qquad \textbf{Ans.}$$



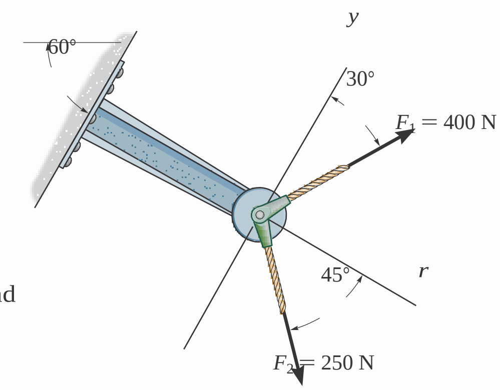


Ans:

$\mathbf{F}_1 = 5200\mathbf{i} + 346\mathbf{j}6$ N
$\mathbf{F}_2 = 5177\mathbf{i} - 177\mathbf{j}6$ N

2–27.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written as

$$(F_1)_x = 400 \sin 30° = 200 \text{ N} \qquad (F_1)_y = 400 \cos 30° = 346.41 \text{ N}$$

$$(F_2)_x = 250 \cos 45° = 176.78 \text{ N} \qquad (F_2)_y = 250 \sin 45° = 176.78 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200 + 176.78 = 376.78 \text{ N}$$

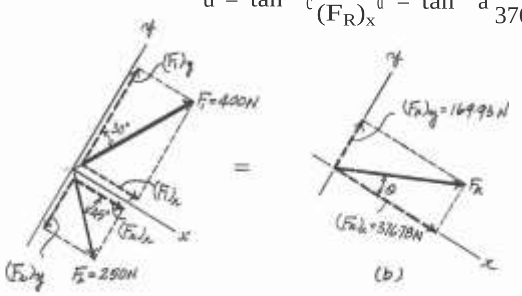
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 346.41 - 176.78 = 169.63 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{376.78^2 + 169.63^2} = 413 \text{ N} \qquad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured counterclockwise from the positive axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{169.63}{376.78}\right) = 24.2° \qquad \textbf{Ans.}$$



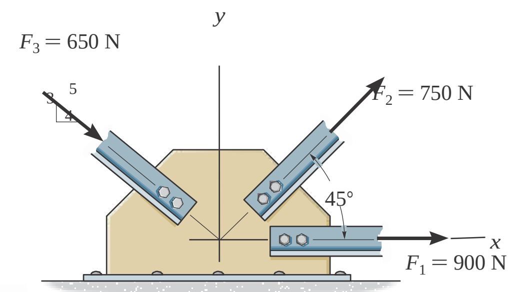
(a) (b)

Ans:
F_R = 413 N
0 = 24.2°

***2–28.**

Resolve each force acting on the gusset plate into its x and y components, and express each force as a Cartesian vector.

Resolve each force acting on the *gusset plate* into its x and y components, and express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\}\ \text{N}$$ **Ans.**

$$\mathbf{F}_2 = \{750\cos 45°(+\mathbf{i}) + 750\sin 45°(+\mathbf{j})\}\ \text{N}$$

$$= \{530\mathbf{i} + 530\mathbf{j}\}\ \text{N}$$ **Ans.**

$$\mathbf{F}_3 = \left\{650\left(\frac{4}{5}\right)(+\mathbf{i}) + 650\left(\frac{3}{5}\right)(-\mathbf{j})\right\}\ \text{N}$$

$$= \{520\mathbf{i} - 390\mathbf{j}\}\ \text{N}$$ **Ans.**

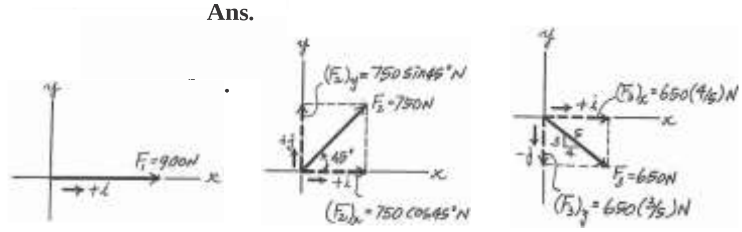
$$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\}\ \text{N}$$

$$\mathbf{F}_2 = \{750\cos 45°(+\mathbf{i}) + 750\sin 45°(+\mathbf{j})\}\ \text{N}$$

$$= \{530\mathbf{i} + 530\mathbf{j}\}\ \text{N}$$

$$\mathbf{F}_3 = \left\{650\left(\frac{4}{5}\right)(+\mathbf{i}) + 650\left(\frac{3}{5}\right)(-\mathbf{j})\right\}\ \text{N}$$

$$= \{520\mathbf{i} - 390\mathbf{j})\}\ \text{N}$$

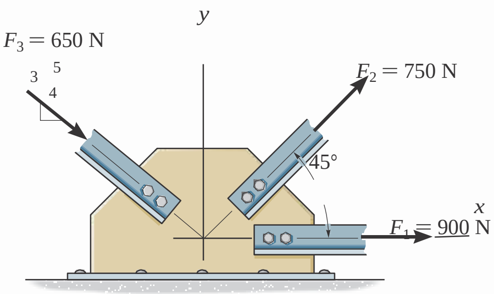


Ans:

$\mathbf{F}_1 = \{900\mathbf{i}\}$ N
$\mathbf{F}_2 = \{530\mathbf{i} + 530\mathbf{j}\}$ N
$\mathbf{F}_3 = \{520\mathbf{i} - 390\mathbf{j}\}$ N

2–29.

Determine the magnitude of the resultant force acting on the gusset plate and its direction, measured counterclockwise from the positive x axis.



Determine the magnitude of the resultant force acting on

the positive x axis.

SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 900 \text{ N} \qquad (F_1)_y = 0$$

$$(F_2)_x = 750 \cos 45° = 530.33 \text{ N} \qquad (F_2)_y = 750 \sin 45° = 530.33 \text{ N}$$

$$(F_3)_x = 650\left(\frac{4}{5}\right) = 520 \text{ N} \qquad (F_3)_y = 650\left(\frac{3}{5}\right) = 390 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900 + 530.33 + 520 = 1950.33 \text{ N} \rightarrow$$

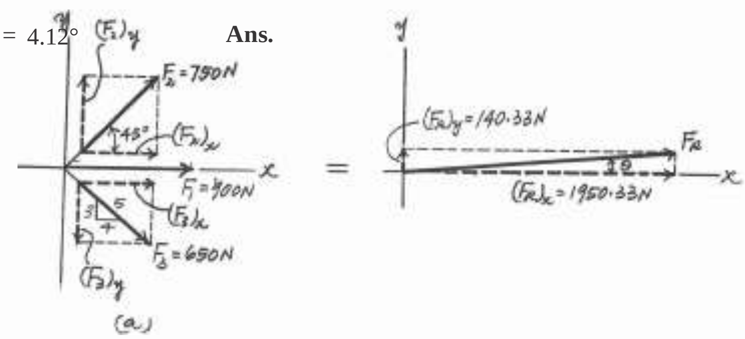
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 530.33 - 390 = 140.33 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{1950.33^2 + 140.33^2} = 1955 \text{ N} = 1.96 \text{ kN} \quad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

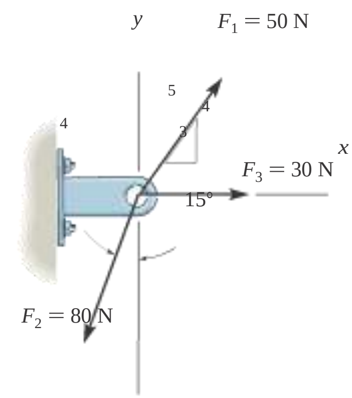
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{140.33}{1950.33}\right) = 4.12° \quad \textbf{Ans.}$$



Ans:
$F_R = 1.96$ kN
0 = 4.12°

2–30.

Express each of the three forces acting on the support in Cartesian vector form and determine the magnitude of the resultant force and its direction, measured clockwise from positive x axis.



SOLUTION

Cartesian Notation. Referring to Fig. *a*,

$$\mathbf{F}_1 = (F_1)_x\,\mathbf{i} + (F_1)_y\,\mathbf{j} = 50\left(\frac{3}{5}\right)\mathbf{i} + 50\left(\frac{4}{5}\right)\mathbf{j} = \{30\,\mathbf{i} + 40\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = -(F_2)_x\,\mathbf{i} - (F_2)_y\,\mathbf{j} = -80\sin 15°\,\mathbf{i} - 80\cos 15°\,\mathbf{j}$$
$$= \{-20.71\,\mathbf{i} - 77.27\,\mathbf{j}\}\text{ N}$$
$$= \{-20.7\,\mathbf{i} - 77.3\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$F_3 = (F_3)_x\,\mathbf{i} = \{30\,\mathbf{i}\} \qquad \textbf{Ans.}$$

Thus, the resultant force is

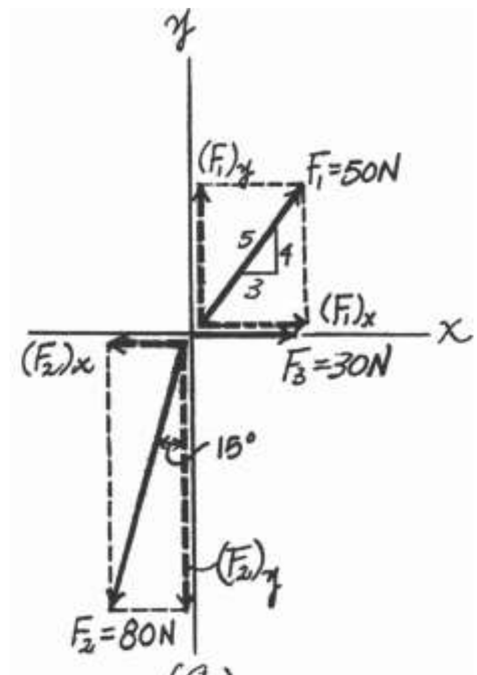
$$\mathbf{F}_R = \Sigma\mathbf{F}\,; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$= (30\mathbf{i} + 40\,\mathbf{j}) + (-20.71\mathbf{i} - 77.27\mathbf{j}) + 30\mathbf{i}$$
$$= \{39.29\,\mathbf{i} - 37.27\,\mathbf{j}\}\text{ N}$$

Referring to Fig. *b*, the magnitude of $\mathbf{F}_R$ is

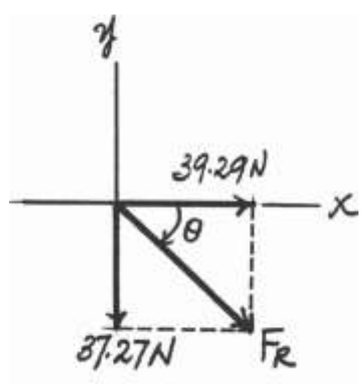
$$F_R = \sqrt{39.29^2 + 37.27^2} = 54.16\text{ N} = 54.2\text{ N} \qquad \textbf{Ans.}$$

And its directional angle 0 measured clockwise from the positive x axis is

$$0 = \tan^{-1}\left(\frac{37.27}{39.29}\right) = 43.49° = 43.5° \qquad \textbf{Ans.}$$



(a)

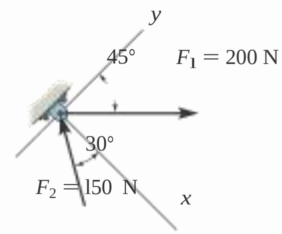


(b)

Ans:
$\mathbf{F}_1 = \{30\mathbf{i} + 40\mathbf{j}\}$ N
$\mathbf{F}_2 = \{-20.7\,\mathbf{i} - 77.3\,\mathbf{j}\}$ N
$F_3 = \{30\,\mathbf{i}\}$
$F_R = 54.2$ N
$0 = 43.5°$

2–31.

Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$.



SOLUTION

$F_{1x} = 200 \sin 45° = 141$ N **Ans.**

$F_{1y} = 200 \cos 45° = 141$ N **Ans.**

$F_{2x} = -150 \cos 30° = -130$ N **Ans.**

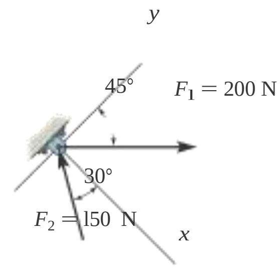
$F_{2y} = 150 \sin 30° = 75$ N **Ans.**

Ans:
$F_{1x} = 141$ N
$F_{1y} = 141$ N

$$F_{2x} = -130 \text{ N}$$
$$F_{2y} = 75 \text{ N}$$

***2–32.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x; \quad F_{Rx} = -150 \cos 30° + 200 \sin 45° = 11.518 \text{ N}$

$+\uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = 150 \sin 30° + 200 \cos 45° = 216.421 \text{ N}$

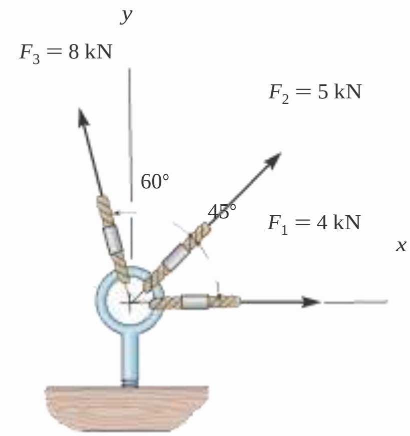
$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217 \text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{216.421}{11.518}\right) = 87.0°$ **Ans.**

Ans:
$F_R = 217$ N
$\theta = 87.0°$

2–33.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 4 + 5\cos 45° - 8\sin 15° = 5.465 \text{ kN} \rightarrow$$

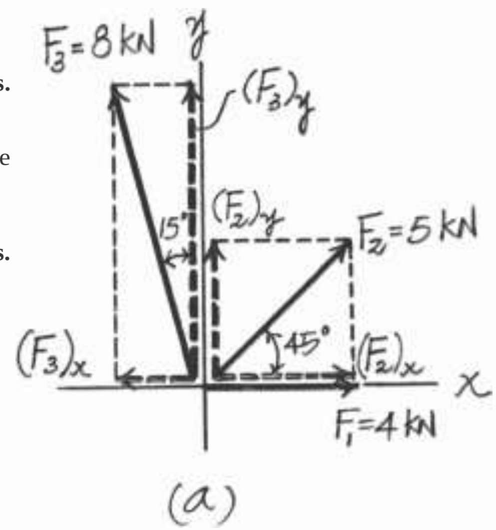
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 5\sin 45° + 8\cos 15° = 11.263 \text{ kN} \uparrow$$

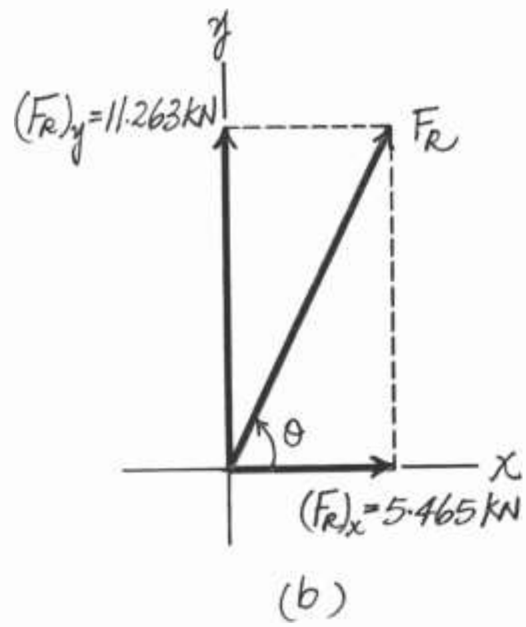
By referring to Fig. b, the magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{5.465^2 + 11.263^2} = 12.52 \text{ kN} = 12.5 \text{ kN} \qquad \textbf{Ans.}$$

And the directional angle θ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{11.263}{5.465}\right) = 64.12° = 64.1° \qquad \textbf{Ans.}$$



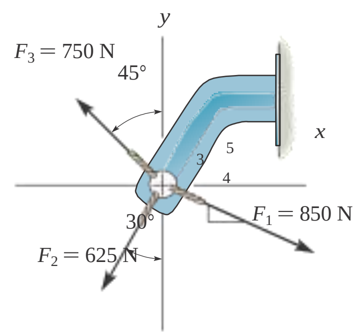


Ans:

F_R = 12.5 kN
0 = 64.1°

2–34.

Express $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ as Cartesian vectors.



SOLUTION

$$\mathbf{F}_1 = \frac{4}{5}(850)\,\mathbf{i} - \frac{3}{5}(850)\,\mathbf{j}$$

$$= \{680\,\mathbf{i} - 510\,\mathbf{j}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_2 = -625 \sin 30°\,\mathbf{i} - 625 \cos 30°\,\mathbf{j}$$

$$= \{-312\,\mathbf{i} - 541\,\mathbf{j}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_3 = -750 \sin 45°\,\mathbf{i} + 750 \cos 45°\,\mathbf{j}$$

$$= \{-530\,\mathbf{i} + 530\,\mathbf{j}\}\text{ N}$$ **Ans.**

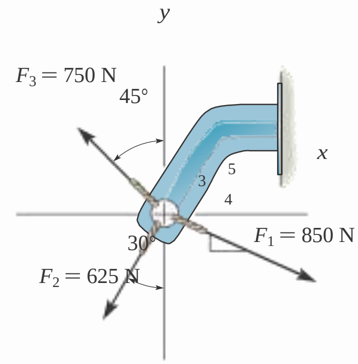
Ans:
$\mathbf{F}_1 = \{680\mathbf{i} - 510\mathbf{j}\}$ N
$\mathbf{F}_2 = \{-312\mathbf{i} - 541\mathbf{j}\}$ N

$\mathbf{F}_3 = \{-530\mathbf{i} + 530\mathbf{j}\}$ N

2–35.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

SOLUTION

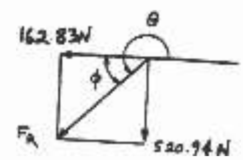
$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = \frac{4}{5}(850) - 625 \sin 30° - 750 \sin 45° = -162.83$ N

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = -\frac{3}{5}(850) - 625 \cos 30° + 750 \cos 45° = -520.94$ N

$F_R = \sqrt{(-162.83)^2 + (-520.94)^2} = 546$ N **Ans.**

$\phi = \tan^{-1}\left(\frac{520.94}{162.83}\right) = 72.64°$

$\theta = 180° + 72.64° = 253°$ **Ans.**

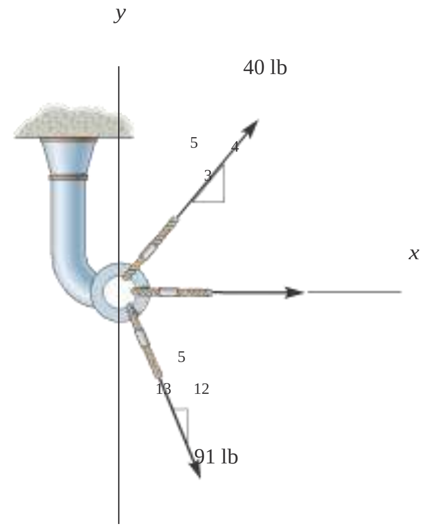


Ans:

$F_R = 546$ N
$0 = 253°$

***2–36.**

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 40\left(\frac{3}{5}\right) + 91\left(\frac{5}{13}\right) + 30 = 89 \text{ lb} \rightarrow$$

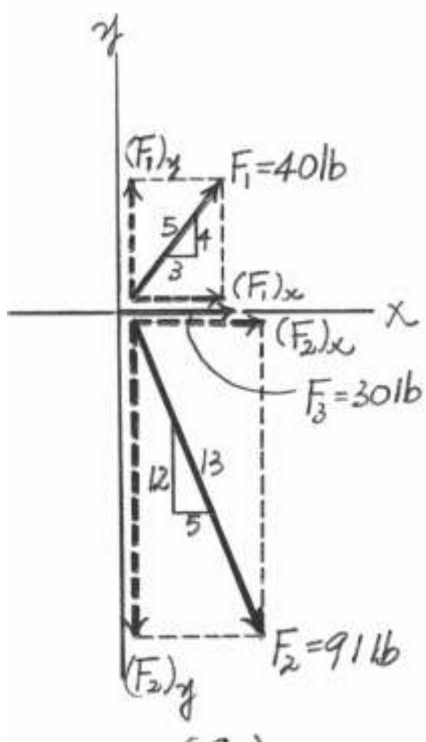
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 40\left(\frac{4}{5}\right) - 91\left(\frac{12}{13}\right) = -52 \text{ lb} = 52 \text{ lb} \downarrow$$

By referring to Fig. b, the magnitude of resultant force is

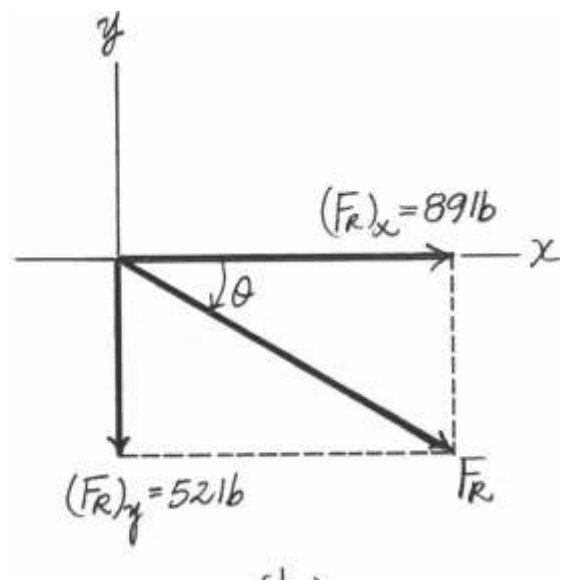
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{89^2 + 52^2} = 103.08 \text{ lb} = 103 \text{ lb} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{52}{89}\right) = 30.30° = 30.3° \qquad \textbf{Ans.}$$



(a)



(b)

Ans:

$F_R = 103$ lb
0 = 30.3°

2–37.

Determine the magnitude and direction θ of the resultant force $\mathbf{F}_R$. Express the result in terms of the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ and the angle ϕ.

$\mathbf{F}_1$ $\mathbf{F}_R$ ϕ θ $\mathbf{F}_2$

SOLUTION

$\mathbf{F}_1$ $\mathbf{F}_R$ ϕ θ $\mathbf{F}_2$

$$F_R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos(180° - \phi)$$

Since $\cos(180° - \phi) = -\cos\phi$,

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\phi}$$ **Ans.**

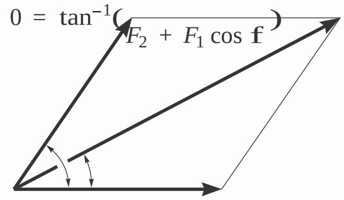
From the figure,

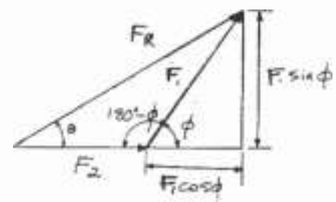
$$\tan\theta = \frac{F_1\sin\phi}{F_2 + F_1\cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{F_1\sin\phi}{F_2 + F_1\cos\phi}\right)$$ **Ans.**

Ans:
$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\phi}$
$F_1\sin\phi$

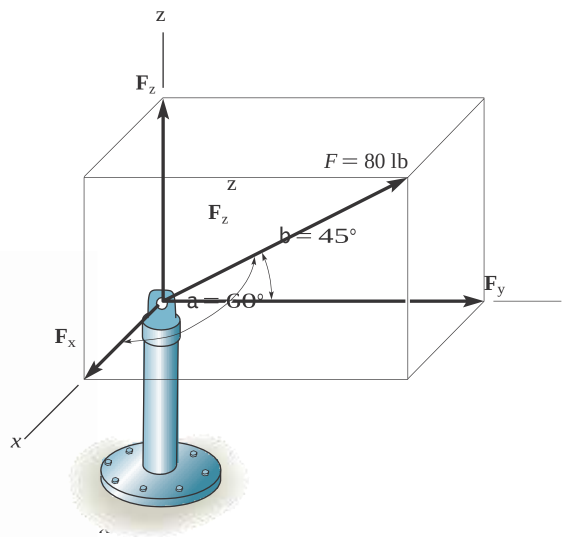
$0 = \tan^{-1}\left(F_2 + F_1 \cos \mathbf{f} \right)$





2–38.

The force **F** has a magnitude of 80 lb. Determine the magnitudes of the *x, y, z* components of **F**.



SOLUTION

$1 = \cos^2 60° + \cos^2 45° + \cos^2 g$

Solving for the positive root, $g = 60°$

$F_x = 80 \cos 60° = 40.0$ lb **Ans.**

$F_y = 80 \cos 45° = 56.6$ lb **Ans.**

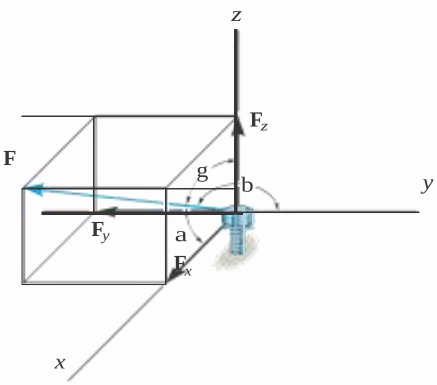
$F_z = 80 \cos 60° = 40.0$ lb **Ans.**

Ans:

$F_x = 40.0$ lb
$F_y = 56.6$ lb
$F_z = 40.0$ lb

2–39.

The bolt is subjected to the force **F**, which has components acting along the *x*, *y*, *z* axes as shown. If the magnitude of **F** is 80 N, and a = 60° and g = 45°, determine the magnitudes of its components.



SOLUTION

$$\cos b = \sqrt{1 - \cos^2 a - \cos^2 g}$$

$$= \sqrt{1 - \cos^2 60° - \cos^2 45°}$$

$$b = 120°$$

$F_x = |80 \cos 60°| = 40 \text{ N}$ **Ans.**

$F_y = |80 \cos 120°| = 40 \text{ N}$ **Ans.**

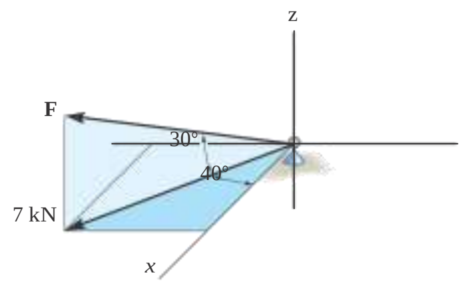
$F_z = |80 \cos 45°| = 56.6 \text{ N}$ **Ans.**

Ans:
$F_x = 40 \text{ N}$

$F_y = 40$ N
$F_z = 56.6$ N

***2–40.**

Determine the magnitude and coordinate direction angles of the force **F** acting on the support. The component of **F** in the *x–y* plane is 7 kN.



SOLUTION

Coordinate Direction Angles. The unit vector of **F** is

$$\mathbf{u}_F = \cos 30° \cos 40° \mathbf{i} - \cos 30° \sin 40° \mathbf{j} + \sin 30° \mathbf{k}$$

$$= \{0.6634\mathbf{i} - 0.5567\mathbf{j} + 0.5\mathbf{k}\}$$

Thus,

$\cos a = 0.6634; \quad a = 48.44° = 48.4°$ **Ans.**

$\cos b = -0.5567; \quad b = 123.83° = 124°$ **Ans.**

$\cos g = 0.5; \quad g = 60°$ **Ans.**

The magnitude of **F** can be determined from

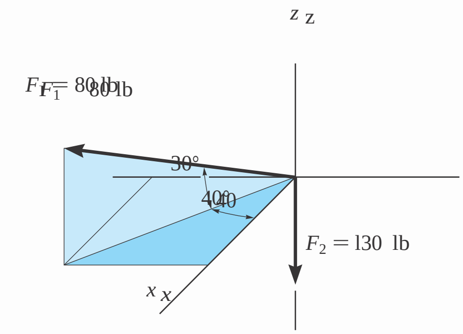
$F \cos 30° = 7; \quad F = 8.083 \text{ kN} = 8.08 \text{ kN}$ **Ans.**

Ans:
$a = 48.4°$
$b = 124°$
$g = 60°$

$F = 8.08$ kN

2–41.

Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



SOLUTION

$\mathbf{F}_1 = \{80 \cos 30° \cos 40° \mathbf{i} - 80 \cos 30° \sin 40° \mathbf{j} + 80 \sin 30° \mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

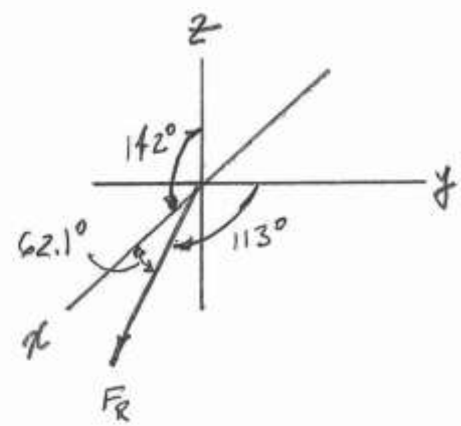
$\mathbf{F}_R = \{53.1\mathbf{i} - 44.5\mathbf{j} - 90.0\mathbf{k}\}$ lb

$F_R = \sqrt{(53.1)^2 + (-44.5)^2 + (-90.0)^2} = 114$ lb **Ans.**

$\alpha = \cos^{-1}\left(\frac{53.1}{113.6}\right) = 62.1°$ **Ans.**

$\beta = \cos^{-1}\left(\frac{-44.5}{113.6}\right) = 113°$ **Ans.**

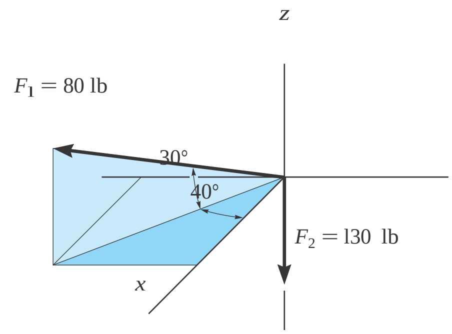
$\gamma = \cos^{-1}\left(\frac{-90.0}{113.6}\right) = 142°$ **Ans.**



Ans:
$F_R = 114$ lb
$\alpha = 62.1°$
$\beta = 113°$
$\gamma = 142°$

2–42.

Specify the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_2$ and express each force as a Cartesian vector.



SOLUTION

$\mathbf{F}_1 = \{80 \cos 30° \cos 40° \mathbf{i} - 80 \cos 30° \sin 40° \mathbf{j} + 80 \sin 30° \mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb **Ans.**

$\alpha_1 = \cos^{-1}\left(\frac{53.1}{80}\right) = 48.4°$ **Ans.**

$\beta_1 = \cos^{-1}\left(\frac{-44.5}{80}\right) = 124°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\frac{40}{80}\right) = 60°$ **Ans.**

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb **Ans.**

$\alpha_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$ **Ans.**

$\beta_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$ **Ans.**

$\gamma_2 = \cos^{-1}\left(\frac{-130}{130}\right) = 180°$ **Ans.**

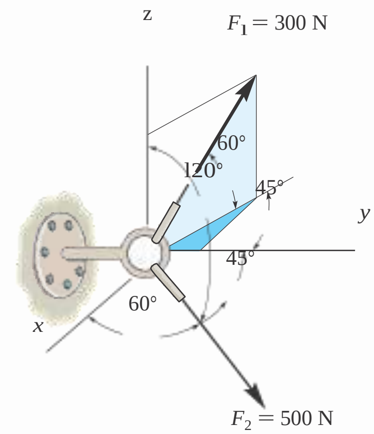
Ans:
$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb
$\alpha_1 = 48.4°$

$b_1 = 124°$
$g_1 = 60°$
$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb
$a_2 = 90°$
$b_2 = 90°$
$g_2 = 180°$

2–43.

The screw eye is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force. Find the magnitude and coordinate direction angles of the resultant force.

Express each force in Cartesian vector form and then determine the resultant force. Find the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$\mathbf{F}_1 = 300(-\cos 60° \sin 45° \mathbf{i} + \cos 60° \cos 45° \mathbf{j} + \sin 60° \mathbf{k})$

$= \{-106.07\mathbf{i} + 106.07\mathbf{j} + 259.81\mathbf{k}\}$ N

$= \{-106\mathbf{i} + 106\mathbf{j} + 260\mathbf{k}\}$ N **Ans.**

$\mathbf{F}_2 = 500(\cos 60° \mathbf{i} + \cos 45° \mathbf{j} + \cos 120° \mathbf{k})$

$= \{250.0\mathbf{i} + 353.55\mathbf{j} - 250.0\mathbf{k}\}$ N

$= \{250\mathbf{i} + 354\mathbf{j} - 250\mathbf{k}\}$ N **Ans.**

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

$= -106.07\mathbf{i} + 106.07\mathbf{j} + 259.81\mathbf{k} + 250.0\mathbf{i} + 353.55\mathbf{j} - 250.0\mathbf{k}$

$= 143.93\mathbf{i} + 459.62\mathbf{j} + 9.81\mathbf{k}$

$= \{144\mathbf{i} + 460\mathbf{j} + 9.81\mathbf{k}\}$ N **Ans.**

$F_R = \sqrt{143.93^2 + 459.62^2 + 9.81^2} = 481.73 \text{ N} = 482 \text{ N}$ **Ans.**

$\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{143.93\mathbf{i} + 459.62\mathbf{j} + 9.81\mathbf{k}}{481.73} = 0.2988\mathbf{i} + 0.9541\mathbf{j} + 0.02036\mathbf{k}$

$\cos a = 0.2988$ $\quad a = 72.6°$ **Ans.**

$\cos b = 0.9541$ $\quad b = 17.4°$ **Ans.**

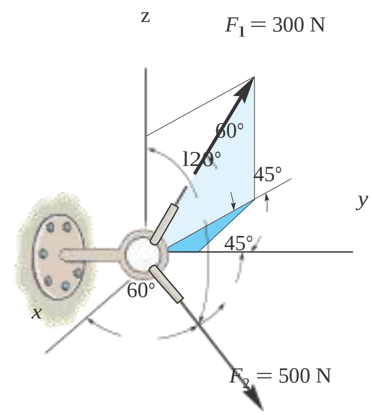
$\cos g = 0.02036$ $\quad g = 88.8°$ **Ans.**

Ans:
$\mathbf{F}_1 = \{-106\mathbf{i} + 106\mathbf{j} + 260\mathbf{k}\}$ N
$\mathbf{F}_2 = \{250\mathbf{i} + 354\mathbf{j} - 250\mathbf{k}\}$

N $\mathbf{F}_R = \{144\mathbf{i} + 460\mathbf{j} + 9.81\mathbf{k}\}$
N $F_R = 482$ N
a = 72.6°
b = 17.4°
g = 88.8°

***2–44.**

Determine the coordinate direction angles of $\mathbf{F}_1$.



SOLUTION

$$\mathbf{F}_1 = 300(-\cos 60^\circ \sin 45^\circ\,\mathbf{i} + \cos 60^\circ \cos 45^\circ\,\mathbf{j} + \sin 60^\circ\,\mathbf{k})$$

$$= \{-106.07\,\mathbf{i} + 106.07\,\mathbf{j} + 259.81\,\mathbf{k}\}\text{ N}$$

$$= \{-106\,\mathbf{i} + 106\,\mathbf{j} + 260\,\mathbf{k}\}\text{ N}$$

$$\mathbf{u}_1 = \frac{\mathbf{F}_1}{300} = -0.3536\,\mathbf{i} + 0.3536\,\mathbf{j} + 0.8660\,\mathbf{k}$$

$a_1 = \cos^{-1}(-0.3536) = 111^\circ$ **Ans.**

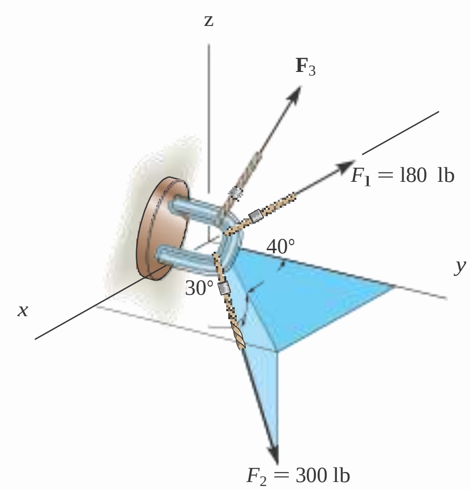
$b_1 = \cos^{-1}(0.3536) = 69.3^\circ$ **Ans.**

$g_1 = \cos^{-1}(0.8660) = 30.0^\circ$ **Ans.**

Ans:
$a_1 = 111^\circ$
$b_1 = 69.3^\circ$
$g_1 = 30.0^\circ$

2–45.

Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces acts along the positive y axis and has a magnitude of 600 lb.



SOLUTION

$F_{Rx} = \Sigma F_x; \qquad 0 = -180 + 300\cos 30° \sin 40° + F_3 \cos \alpha$

$F_{Ry} = \Sigma F_y; \qquad 600 = 300\cos 30° \cos 40° + F_3 \cos \beta$

$F_{Rz} = \Sigma F_z; \qquad 0 = -300 \sin 30° + F_3 \cos \gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Solving:

$F_3 = 428 \text{ lb}$ **Ans.**

$\alpha = 88.3°$ **Ans.**

$\beta = 20.6°$ **Ans.**

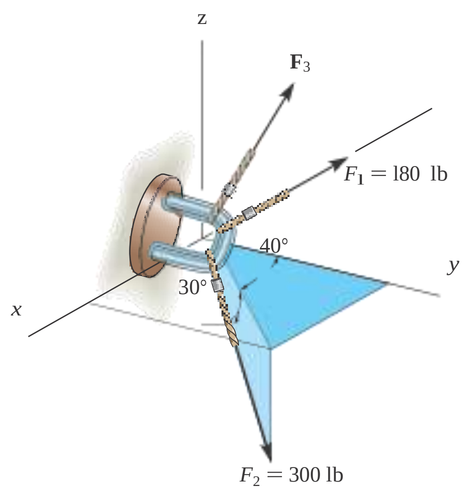
$\gamma = 69.5°$ **Ans.**

Ans:
$F_3 = 428 \text{ lb}$

a = 88.3°
b = 20.6°
g = 69.5°

2–46.

Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces is zero.



SOLUTION

F_{Rx} = ©F_x; $0 = -180 + 300 \cos 30° \sin 40° + F_3 \cos a$

F_{Ry} = ©F_y; $0 = 300 \cos 30° \cos 40° + F_3 \cos b$

F_{Rz} = ©F_z; $0 = -300 \sin 30° + F_3 \cos g$

$\cos^2 a + \cos^2 b + \cos^2 g = 1$

Solving:

$F_3 = 250$ lb **Ans.**

$a = 87.0°$ **Ans.**

$b = 143°$ **Ans.**

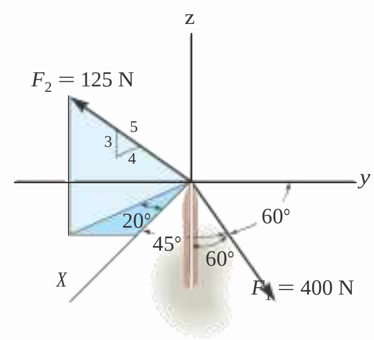
$g = 53.1°$ **Ans.**

Ans:
$F_3 = 250$ lb

a = 87.0°
b = 143°
g = 53.1°

2–47.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$\mathbf{F}_1 = 400(\cos 45°\mathbf{i} + \cos 60°\mathbf{j} - \cos 60°\mathbf{k}) = \{282.84\mathbf{i} + 200\mathbf{j} - 200\mathbf{k}\}$ N

$\mathbf{F}_2 = 125\left[\frac{4}{5}(\cos 20°)\mathbf{i} - \frac{4}{5}(\sin 20°)\mathbf{j} + \frac{3}{5}\mathbf{k}\right] = \{93.97\mathbf{i} - 34.20\mathbf{j} + 75.0\mathbf{k}\}$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= \{282.84\mathbf{i} + 200\mathbf{j} - 200\mathbf{k}\} + \{93.97\mathbf{i} - 34.20\mathbf{j} + 75.0\mathbf{k}\}$$

$$= \{376.81\mathbf{i} + 165.80\mathbf{j} - 125.00\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{376.81^2 + 165.80^2 + (-125.00)^2}$$

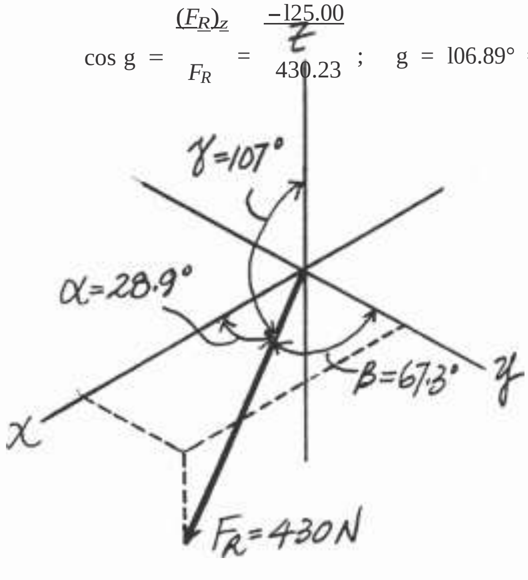
$$= 430.23\text{ N} = 430\text{ N}$$ **Ans.**

The coordinate direction angles are

$\cos a = \frac{(F_R)_x}{F_R} = \frac{376.81}{430.23}$; $a = 28.86° = 28.9°$ **Ans.**

$\cos b = \frac{(F_R)_y}{F_R} = \frac{165.80}{430.23}$; $b = 67.33° = 67.3°$ **Ans.**

$\cos g = \frac{(F_R)_z}{F_R} = \frac{-125.00}{430.23}$; $g = 106.89° = 107°$ **Ans.**

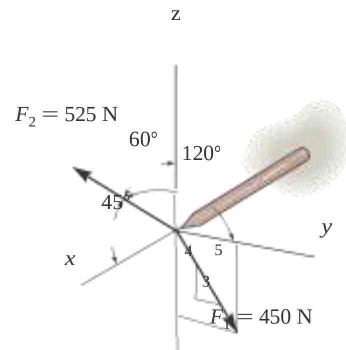


Ans:

F_R = 430 N
a = 28.9°
b = 67.3°
g = 107°

***2–48.**

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$$\mathbf{F}_1 = 450\left(\frac{3}{5}\mathbf{j} - \frac{4}{5}\mathbf{k}\right) = \{270\mathbf{j} - 360\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 525(\cos 45°\mathbf{i} + \cos 120°\mathbf{j} + \cos 60°\mathbf{k}) = \{371.23\mathbf{i} - 262.5\mathbf{j} + 262.5\mathbf{k}\}\text{ N}$$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= \{270\mathbf{j} - 360\mathbf{k}\} + \{371.23\mathbf{i} - 262.5\mathbf{j} + 262.5\mathbf{k}\}$$

$$= \{371.23\mathbf{i} + 7.50\mathbf{j} - 97.5\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{371.23^2 + 7.50^2 + (-97.5)^2}$$

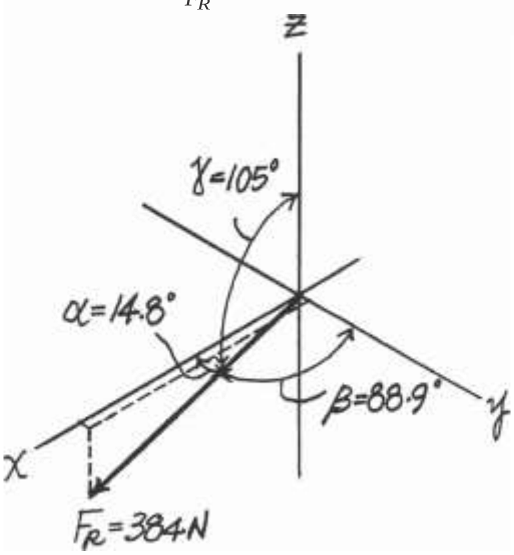
$$= 383.89\text{ N} = 384\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{371.23}{383.89}; \qquad \alpha = 14.76° = 14.8° \qquad \textbf{Ans.}$$

$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{7.50}{383.89}; \qquad \beta = 88.88° = 88.9° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-97.5}{383.89}; \qquad \gamma = 104.71° = 105° \qquad \textbf{Ans.}$$

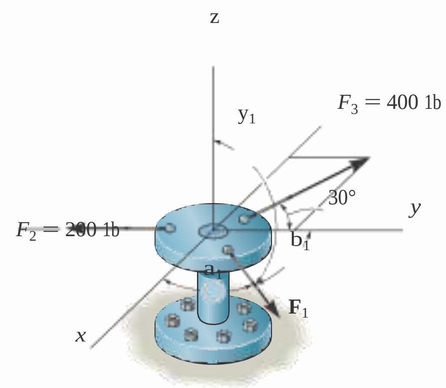


Ans:
$F_R = 384$ N
$\alpha = 14.8°$

b = 88.9°
g = l05°

2–49.

Determine the magnitude and coordinate direction angles a_1, b_1, g_1 of $\mathbf{F}_1$ so that the resultant of the three forces acting on the bracket is $\mathbf{F}_R = 5\,-350\mathbf{k}\,6$ lb.
x



SOLUTION

$\mathbf{F}_1 = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$

$\mathbf{F}_2 = -200\,\mathbf{j}$

$\mathbf{F}_3 = -400 \sin 30°\,\mathbf{i} + 400 \cos 30°\,\mathbf{j}$

$\quad = -200\,\mathbf{i} + 346.4\,\mathbf{j}$

$\mathbf{F}_R = ©\mathbf{F}$

$-350\,\mathbf{k} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k} - 200\,\mathbf{j} - 200\,\mathbf{i} + 346.4\,\mathbf{j}$

$0 = F_x - 200\,; \qquad F_x = 200 \text{ lb}$

$0 = F_y - 200 + 346.4\,; \qquad F_y = -146.4 \text{ lb}$

$F_z = -350 \text{ lb}$

$F_1 = 2(200)^2 + (-146.4)^2 + (-350)^2$

$F_1 = 425.9 \text{ lb} = 429 \text{ lb}$ **Ans.**

$a_1 = \cos^{-1} a\dfrac{200}{428.9}b = 62.2°$ **Ans.**

$b_1 = \cos^{-1} a\dfrac{-146.4}{428.9}b = 110°$ **Ans.**

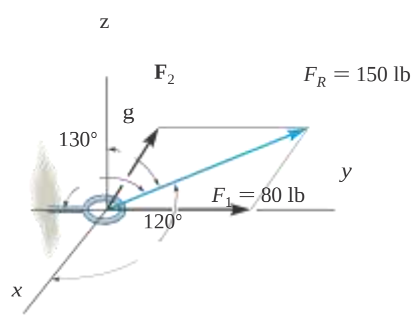
$g_1 = \cos^{-1} \left(\dfrac{-350}{428.9}\right) = 145°$ **Ans.**

Ans:

$F_1 = 429$ lb
$\alpha_1 = 62.2°$
$\beta_1 = 110°$ γ_1
$= 145°$

2–50.

If the resultant force $\mathbf{F}_R$ has a magnitude of 150 lb and the coordinate direction angles shown, determine the magnitude of $\mathbf{F}_2$ and its coordinate direction angles.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_R$, γ can be determined from

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2 120° + \cos^2 50° + \cos^2\gamma = 1$$

$$\cos\gamma = \pm 0.5804$$

Here $\gamma < 90°$, then

$$\gamma = 54.52°$$

Thus

$$\mathbf{F}_R = 150(\cos 120°\mathbf{i} + \cos 50°\mathbf{j} + \cos 54.52°\mathbf{k})$$

$$= \{-75.0\mathbf{i} + 96.42\mathbf{j} + 87.05\mathbf{k}\}\text{ lb}$$

Also

$$\mathbf{F}_1 = \{80\mathbf{j}\}\text{ lb}$$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\{-75.0\mathbf{i} + 96.42\mathbf{j} + 87.05\mathbf{k}\} = \{80\mathbf{j}\} + \mathbf{F}_2$$

$$F_2 = \{-75.0\mathbf{i} + 16.42\mathbf{j} + 87.05\mathbf{k}\}\text{ lb}$$

Thus, the magnitude of $\mathbf{F}_2$ is

$$F_2 = \sqrt{(F_2)_x^2 + (F_2)_y^2 + (F_2)_z^2} = \sqrt{(-75.0)^2 + 16.42^2 + 87.05^2}$$

$$= 116.07\text{ lb} = 116\text{ lb} \qquad \textbf{Ans.}$$

And its coordinate direction angles are

$$\cos\alpha_2 = \frac{(F_2)_x}{F_2} = \frac{-75.0}{116.07}; \qquad \alpha_2 = 130.25° = 130° \qquad \textbf{Ans.}$$

$$\cos\beta_2 = \frac{(F_2)_y}{F_2} = \frac{16.42}{116.07}; \qquad \beta_2 = 81.87° = 81.9° \qquad \textbf{Ans.}$$

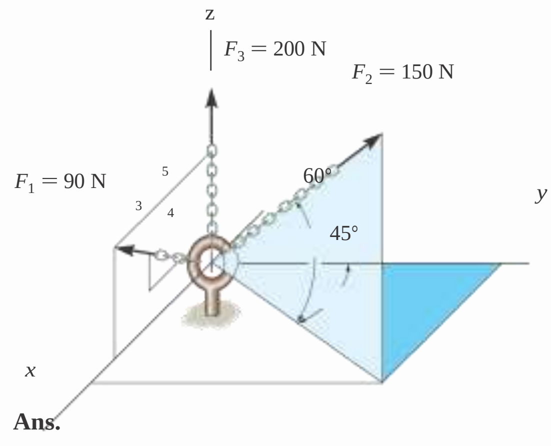
$$\cos\gamma_2 = \frac{(F_2)_z}{F_2} = \frac{87.05}{116.07}; \qquad \gamma_2 = 41.41° = 41.4° \qquad \textbf{Ans.}$$

Ans:
$F_2 = 116$ lb
$\alpha_2 = 130°$
$\beta_2 = 81.9°$

$g_2 = 41.4°$

2–51.

Express each force as a Cartesian vector.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$,

$$\mathbf{F}_1 = 90\left(\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right) = \{72.0\mathbf{i} + 54.0\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_2 = 150\,(\cos 60° \sin 45°\mathbf{i} + \cos 60° \cos 45°\mathbf{j} + \sin 60°\mathbf{k})$$

$$= \{53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}\}\text{ N}$$

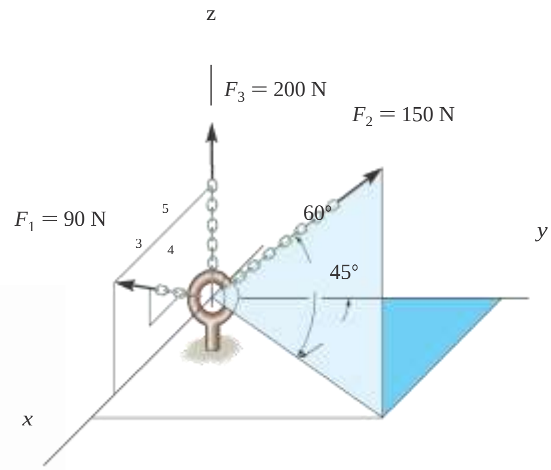
$$= \{53.0\mathbf{i} + 53.0\mathbf{j} + 130\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_3 = \{200\,\mathbf{k}\}$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{72.0\mathbf{i} + 54.0\mathbf{k}\}$ N
$\mathbf{F}_2 = \{53.0\mathbf{i} + 53.0\mathbf{j} + 130\mathbf{k}\}$ N
$\mathbf{F}_3 = \{200\,\mathbf{k}\}$

***2–52.**

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$,

$$\mathbf{F}_1 = 90\left(\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right) = \{72.0\mathbf{i} + 54.0\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 150\,(\cos 60^\circ \sin 45^\circ\mathbf{i} + \cos 60^\circ \cos 45^\circ\mathbf{j} + \sin 60^\circ\mathbf{k})$$

$$= \{53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_3 = \{200\,\mathbf{k}\}\text{ N}$$

Resultant Force.

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (72.0\mathbf{i} + 54.0\mathbf{k}) + (53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}) + (200\mathbf{k})$$

$$= \{125.03\mathbf{i} + 53.03\mathbf{j} + 383.90\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{125.03^2 + 53.03^2 + 383.90^2}$$

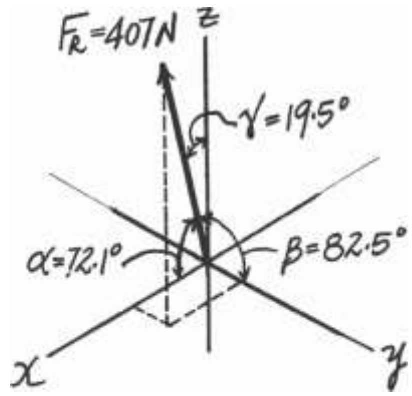
$$= 407.22\text{ N} = 407\text{ N}$$ **Ans.**

And the coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{125.03}{407.22};\qquad \alpha = 72.12^\circ = 72.1^\circ$$ **Ans.**

$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{53.03}{407.22};\qquad \beta = 82.52^\circ = 82.5^\circ$$ **Ans.**

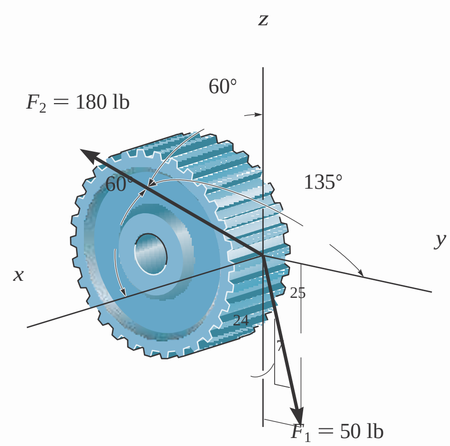
$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{383.90}{407.22};\qquad \gamma = 19.48^\circ = 19.5^\circ$$ **Ans.**



Ans:
$F_R = 407$ N
$\alpha = 72.1^\circ$
$\beta = 82.5^\circ$
$\gamma = 19.5^\circ$

2–53.

The spur gear is subjected to the two forces. Express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = \frac{7}{25}(50)\mathbf{j} - \frac{24}{25}(50)\mathbf{k} = \{14.0\mathbf{j} - 48.0\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_2 = 180\cos 60°\mathbf{i} + 180\cos 135°\mathbf{j} + 180\cos 60°\mathbf{k}$$

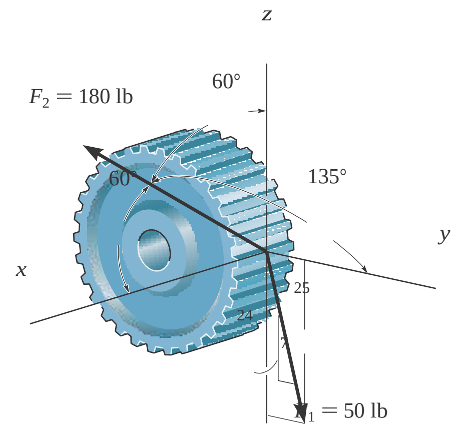
$$= \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{14.0\mathbf{j} - 48.0\mathbf{k}\}$ lb

$\mathbf{F}_2 = \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}$ lb

2–54.

The spur gear is subjected to the two forces. Determine the resultant of the two forces and express the result as a Cartesian vector.



SOLUTION

$$F_{Rx} = 180 \cos 60° = 90$$

$$F_{Ry} = \frac{7}{25}(50) + 180 \cos 135° = -113$$

$$F_{Rz} = -\frac{24}{25}(50) + 180 \cos 60° = 42$$

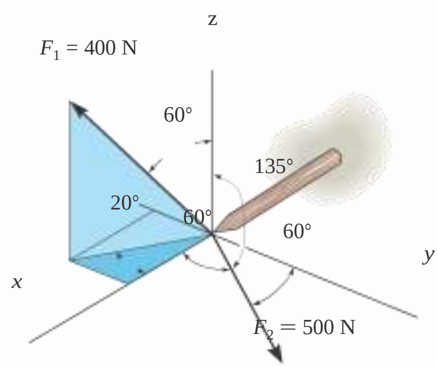
$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\} \text{ lb}$$ **Ans.**

Ans:

$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\}$ lb

2–55.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$$\mathbf{F}_1 = 400\,(\sin 60° \cos 20°\mathbf{i} - \sin 60° \sin 20°\mathbf{j} + \cos 60°\mathbf{k})$$

$$= \{325.52\mathbf{i} - 118.48\mathbf{j} + 200\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_2 = 500\,(\cos 60°\mathbf{i} + \cos 60°\mathbf{j} + \cos 135°\mathbf{k})$$

$$= \{250\mathbf{i} + 250\mathbf{j} - 353.55\mathbf{k}\}\ \text{N}$$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

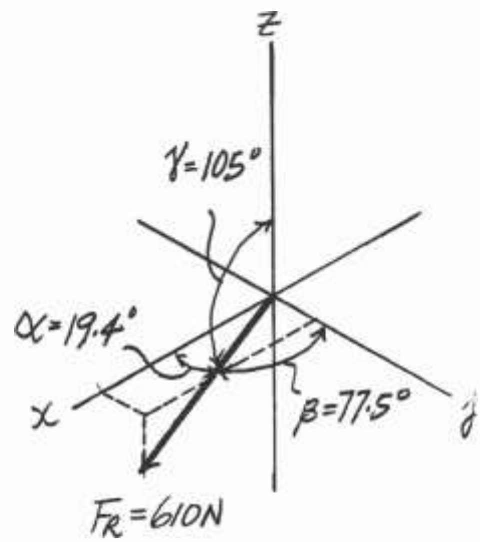
$$= (325.52\mathbf{i} - 118.48\mathbf{j} + 200\mathbf{k}) + (250\mathbf{i} + 250\mathbf{j} - 353.55\mathbf{k})$$

$$= \{575.52\mathbf{i} + 131.52\mathbf{j} - 153.55\,\mathbf{k}\}\ \text{N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{575.52^2 + 131.52^2 + (-153.55)^2}$$

$$= 610.00\ \text{N} = 610\ \text{N} \qquad \textbf{Ans.}$$



The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{575.52}{610.00} \qquad \alpha = 19.36° = 19.4° \qquad \textbf{Ans.}$$

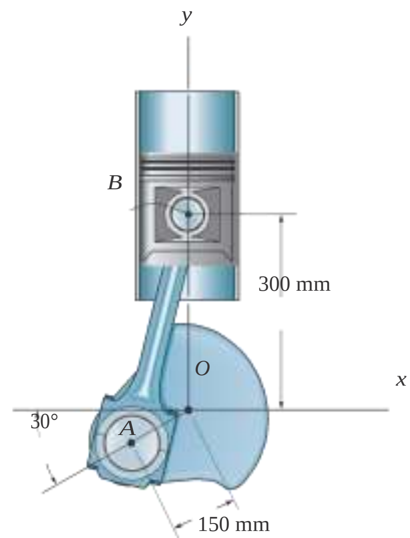
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{131.52}{610.00} \qquad \beta = 77.549° = 77.5° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-153.55}{610.00} \qquad \gamma = 104.58° = 105° \qquad \textbf{Ans.}$$

Ans:
$F_R = 610$ N
$\alpha = 19.4°$
$\beta = 77.5°$
$\gamma = 105°$

***2–56.**

Determine the length of the connecting rod AB by first formulating a position vector from A to B and then determining its magnitude.



SOLUTION

Position Vector. The coordinates of points A and B are $A(-150\cos 30°, -150\sin 30°)$ mm and $B(0, 300)$ mm respectively. Then

$$\mathbf{r}_{AB} = [0 - (-150\cos 30°)]\mathbf{i} + [300 - (-150\sin 30°)]\mathbf{j}$$

$$= \{129.90\mathbf{i} + 375\mathbf{j}\}\text{ mm}$$

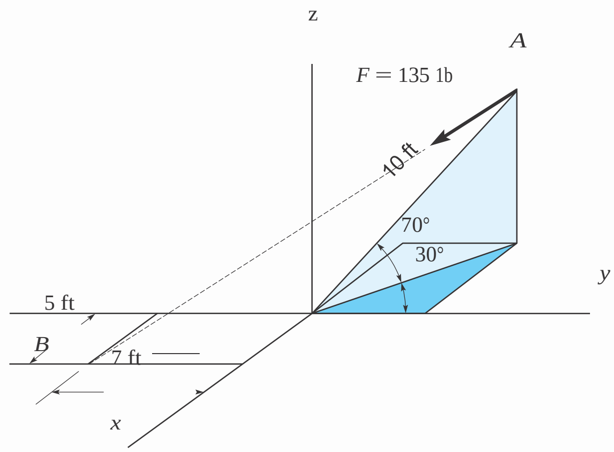
Thus, the magnitude of $\mathbf{r}_{AB}$ is

$$r_{AB} = \sqrt{129.90^2 + 375^2} = 396.86\text{ mm} = 397\text{ mm} \qquad \textbf{Ans.}$$

Ans:
r_{AB} = 397 mm

2–57.

Express force **F** as a Cartesian vector; then determine its coordinate direction angles.



SOLUTION

$$\mathbf{r}_{AB} = (5 + 10\cos 70^\circ \sin 30^\circ)\mathbf{i}$$
$$+ (-7 - 10\cos 70^\circ \cos 30^\circ)\mathbf{j} - 10\sin 70^\circ\mathbf{k}$$

$$\mathbf{r}_{AB} = \{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{(6.710)^2 + (-9.962)^2 + (-9.397)^2} = 15.25$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = (0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k})$$

$$\mathbf{F} = 135\mathbf{u}_{AB} = (59.40\mathbf{i} - 88.18\mathbf{j} - 83.18\mathbf{k})$$

$$= \{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{59.40}{135}\right) = 63.9^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{-88.18}{135}\right) = 131^\circ$$ **Ans.**

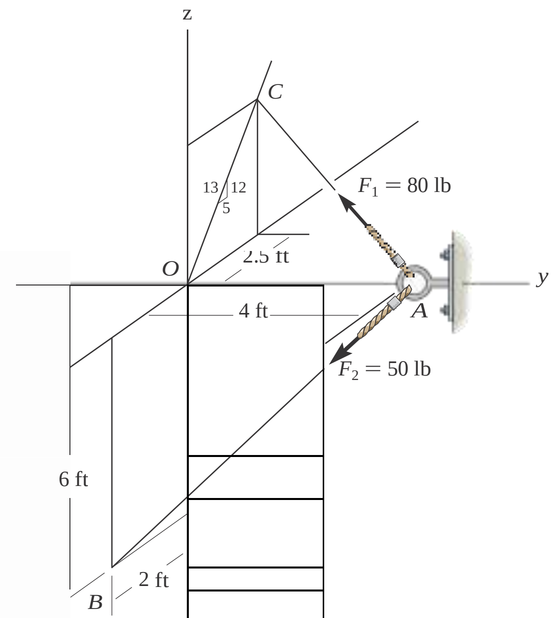
$$\gamma = \cos^{-1}\left(\frac{-83.18}{135}\right) = 128^\circ$$ **Ans.**

Ans:
$\mathbf{F} = \{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\}$ lb

a = 63.9°
b = 131°
g = 128°

2–58.

Express each force as a Cartesian vector, and then determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$$\mathbf{r}_{AC} = \left\{-2.5\,\mathbf{i} - 4\,\mathbf{j} + \frac{12}{5}(2.5)\,\mathbf{k}\right\}\text{ ft}$$

$$\mathbf{F}_1 = 80\text{ lb}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = -26.20\,\mathbf{i} - 41.93\,\mathbf{j} + 62.89\,\mathbf{k}$$

$$= \{-26.2\,\mathbf{i} - 41.9\,\mathbf{j} + 62.9\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{r}_{AB} = \{2\,\mathbf{i} - 4\,\mathbf{j} - 6\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{F}_2 = 50\text{ lb}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 13.36\,\mathbf{i} - 26.73\,\mathbf{j} - 40.09\,\mathbf{k}$$

$$= \{13.4\,\mathbf{i} - 26.7\,\mathbf{j} - 40.1\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= -12.84\,\mathbf{i} - 68.65\,\mathbf{j} + 22.80\,\mathbf{k}$$

$$= \{-12.8\,\mathbf{i} - 68.7\,\mathbf{j} + 22.8\,\mathbf{k}\}\text{ lb}$$

$$F_R = \sqrt{(-12.84)^2 + (-68.65)^2 + (22.80)^2} = 73.47 = 73.5\text{ lb}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{-12.84}{73.47}\right) = 100^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{-68.65}{73.47}\right) = 159^\circ$$ **Ans.**

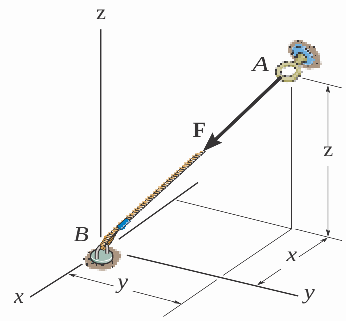
$$\gamma = \cos^{-1}\left(\frac{22.80}{73.47}\right) = 71.9^\circ$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{-26.2\,\mathbf{i} - 41.9\,\mathbf{j} + 62.9\,\mathbf{k}\}$ lb
$\mathbf{F}_2 = \{13.4\,\mathbf{i} - 26.7\,\mathbf{j} - 40.1\,\mathbf{k}\}$ lb
$F_R = 73.5$ lb

a = 100°
b = 159°
g = 71.9°

2–59.

If $\mathbf{F} = \{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}\}$ N and cable AB is 9 m long, determine the x, y, z coordinates of point A.



SOLUTION

Position Vector: The position vector $\mathbf{r}_{AB}$, directed from point A to point B, is given by

$$\mathbf{r}_{AB} = [0 - x]\mathbf{i} + (0 - y)\mathbf{j} + (0 - z)\mathbf{k}$$

$$= -x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$$

Unit Vector: Knowing the magnitude of $\mathbf{r}_{AB}$ is 9 m, the unit vector for $\mathbf{r}_{AB}$ is given by

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9}$$

The unit vector for force **F** is

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \frac{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}}{\sqrt{350^2 + (-250)^2 + (-450)^2}} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Since force **F** is also directed from point A to point B, then

$$\mathbf{u}_{AB} = \mathbf{u}_F$$

$$\frac{-x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9} = -0.5623\mathbf{i} \quad 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$\frac{x}{9} = -0.5623$ $\qquad x = -5.06$ m $\qquad$ **Ans.**

$\frac{-y}{9} = -0.4016$ $\qquad y = 3.61$ m $\qquad$ **Ans.**

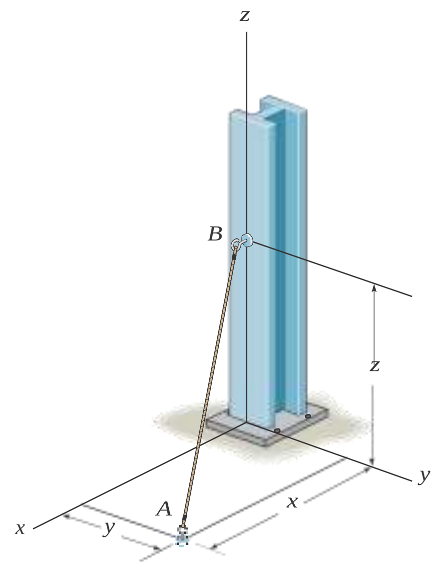
$\frac{-z}{9} = 0.7229$ $\qquad z = 6.51$ m $\qquad$ **Ans.**

Ans:
$x = -5.06$ m

$y = 3.61$ m
$z = 6.51$ m

***2–60.**

The 8-m-long cable is anchored to the ground at A. If $x = 4$ m and $y = 2$ m, determine the coordinate z to the highest point of attachment along the column.



SOLUTION

$$\mathbf{r} = \{4\mathbf{i} + 2\mathbf{j} + z\mathbf{k}\}\text{ m}$$

$$r = \sqrt{(4)^2 + (2)^2 + (z)^2} = 8$$

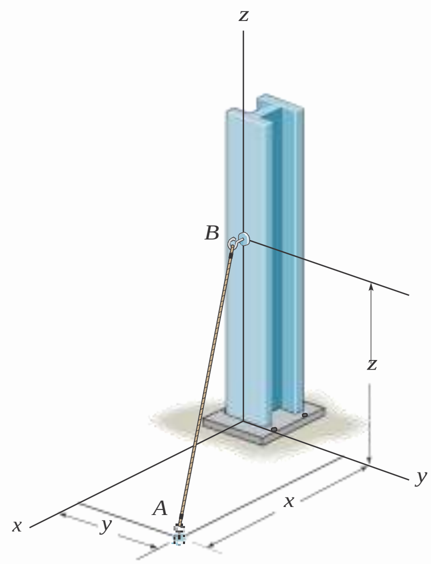
$$z = 6.63\text{ m}$$ **Ans.**

Ans:

6.63 m

2–61.

The 8-m-long cable is anchored to the ground at A. If $z = 5$ m, determine the location $+x$, $+y$ of the support at A. Choose a value such that $x = y$.



SOLUTION

$$\mathbf{r} = \{x\mathbf{i} + y\mathbf{j} + 5\mathbf{k}\}\text{ m}$$

$$r = \sqrt{(x)^2 + (y)^2 + (5)^2} = 8$$

$x = y$, thus

$$2x^2 = 8^2 - 5^2$$

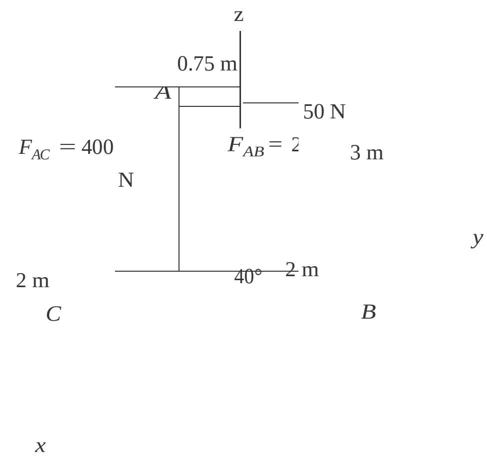
$$x = y = 4.42\text{ m}$$ **Ans.**

Ans:

$x = y = 4.42$ m

2–62.

Express each of the forces in Cartesian vector form and then determine the magnitude and coordinate direction angles of the resultant force



SOLUTION

Unit Vectors. The coordinates for points A, B and C are (0, −0.75, 3) m, B(2 cos 40°, 2 sin 40°, 0) m and C(2, −1, 0) m, respectively.

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(2\cos 40° - 0)\mathbf{i} + [2\sin 40° - (-0.75)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(2\cos 40° - 0)^2 + [2\sin 40° - (-0.75)]^2 + (0-3)^2}}$$

$$= 0.3893\mathbf{i} + 0.5172\mathbf{j} - 0.7622\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(2-0)\mathbf{i} + [-1-(-0.75)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(2-0)^2 + [-1-(-0.75)]^2 + (0-3)^2}}$$

$$= 0.5534\mathbf{i} - 0.0692\mathbf{j} - 0.8301\mathbf{k}$$

Force Vectors

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_{AB} = 250\,(0.3893\mathbf{i} + 0.5172\mathbf{j} - 0.7622\mathbf{k})$$

$$= \{97.32\mathbf{i} + 129.30\mathbf{j} - 190.56\mathbf{k}\}\text{ N}$$ **Ans.**

$$= \{97.3\mathbf{i} + 129\mathbf{j} - 191\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 400\,(0.5534\mathbf{i} - 0.06917\mathbf{j} - 0.8301\mathbf{k})$$

$$= \{221.35\mathbf{i} - 27.67\mathbf{j} - 332.02\mathbf{k}\}\text{ N}$$ **Ans.**

$$= \{221\mathbf{i} - 27.7\mathbf{j} - 332\mathbf{k}\}\text{ N}$$

Resultant Force

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$= \{97.32\mathbf{i} + 129.30\mathbf{j} - 190.56\mathbf{k}\} + \{221.35\mathbf{i} - 27.67\mathbf{j} - 332.02\mathbf{k}\}$$

$$= \{318.67\mathbf{i} + 101.63\mathbf{j} - 522.58\,\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{318.67^2 + 101.63^2 + (-522.58)^2}$$

$$= 620.46\text{ N} = 620\text{ N}$$

And its coordinate direction angles are

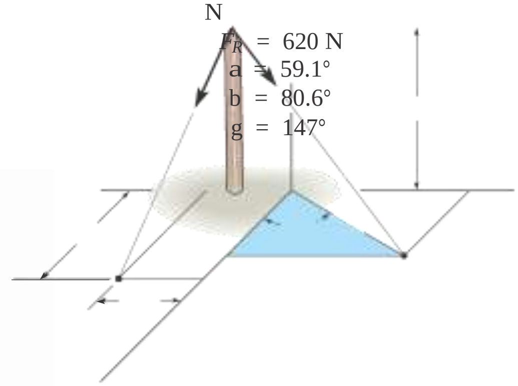
$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{318.67}{620.46}; \qquad \alpha = 59.10° = 59.1°$$ **Ans.**

$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{101.63}{620.46}; \qquad \beta = 80.57° = 80.6°$$ **Ans.**

$$\frac{(F_R)_z}{} \quad \frac{-522.58}{}$$

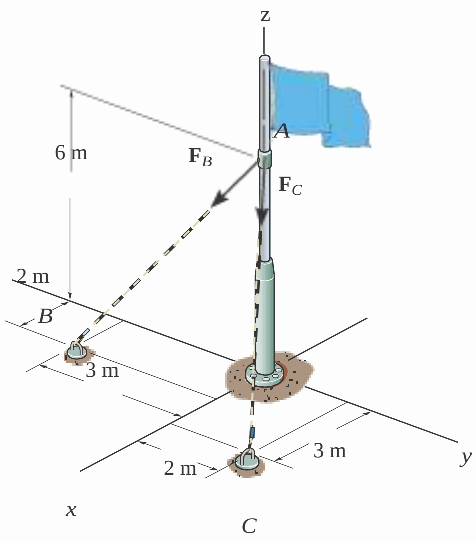
cos g = $\frac{}{F_R}$ = $\frac{}{620.46}$; g = 147.38° = 147° **Ans.**

$\mathbf{F}_{AB}$ = {97.3**i** – 129**j** – 191**k**} N
$\mathbf{F}_{AC}$ = {221**i** – 27.7**j** – 332**k**} N
F_R = 620 N
a = 59.1°
b = 80.6°
g = 147°



2–63.

If $F_B = 560$ N and $F_C = 700$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 560\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 700\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k}\}\ \text{N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}) + (300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k})$$
$$= \{460\mathbf{i} - 40\mathbf{j} - 1080\mathbf{k}\}\ \text{N}$$

The magnitude of $\mathbf{F}_R$ is

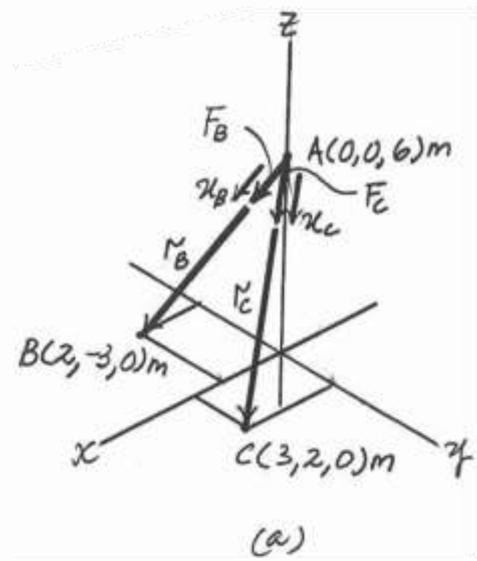
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{(460)^2 + (-40)^2 + (-1080)^2} = 1174.56\ \text{N} = 1.17\ \text{kN}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{460}{1174.56}\right) = 66.9°$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-40}{1174.56}\right) = 92.0°$$ **Ans.**

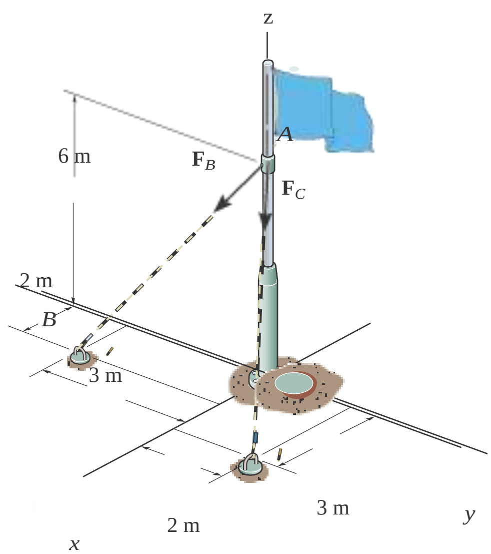
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157°$$ **Ans.**



Ans:
F_R = 1.17 kN
a = 66.9°
b = 92.0°
g = 157°

***2–64.**

If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 700\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 560\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}) + (240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k})$$
$$= \{440\mathbf{i} - 140\mathbf{j} - 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

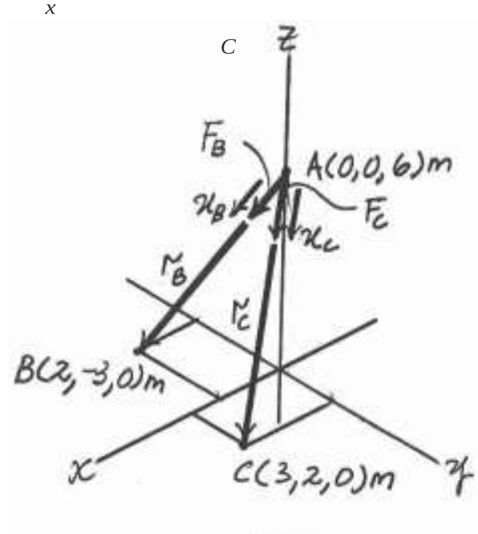
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{(440)^2 + (-140)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{440}{1174.56}\right) = 68.0°$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-140}{1174.56}\right) = 96.8°$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{1080}{1174.56}\right) = 157°$$ **Ans.**

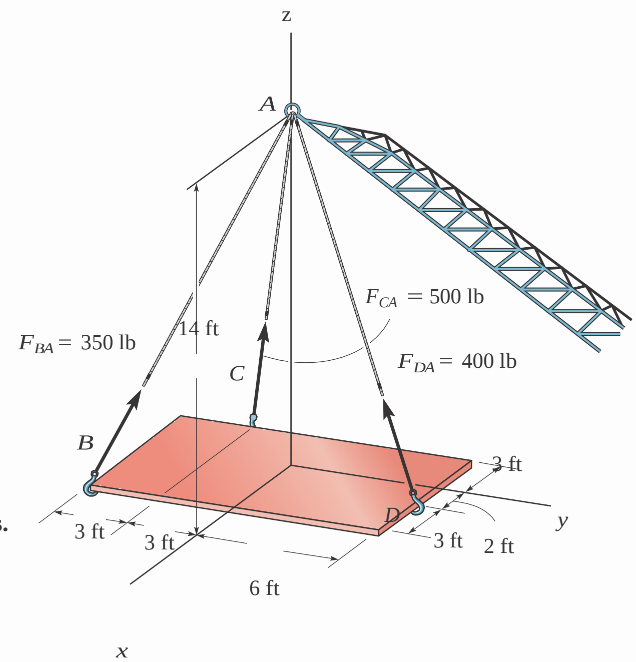


(a)

Ans:
$F_R = 1.17$ kN
a = 68.0°
b = 96.8°
g = 157°

2–65.

The plate is suspended using the three cables which exert the forces shown. Express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_{BA} = 350\left(\frac{\mathbf{r}_{BA}}{r_{BA}}\right) = 350\left(-\frac{5}{16.031}\mathbf{i} + \frac{6}{16.031}\mathbf{j} + \frac{14}{16.031}\mathbf{k}\right)$$

$$= \{-109\,\mathbf{i} + 131\,\mathbf{j} + 306\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_{CA} = 500\left(\frac{\mathbf{r}_{CA}}{r_{CA}}\right) = 500\left(\frac{3}{14.629}\mathbf{i} + \frac{3}{14.629}\mathbf{j} + \frac{14}{14.629}\mathbf{k}\right)$$

$$= \{103\,\mathbf{i} + 103\,\mathbf{j} + 479\,\mathbf{k}\}\text{ lb}$$ **Ans.**

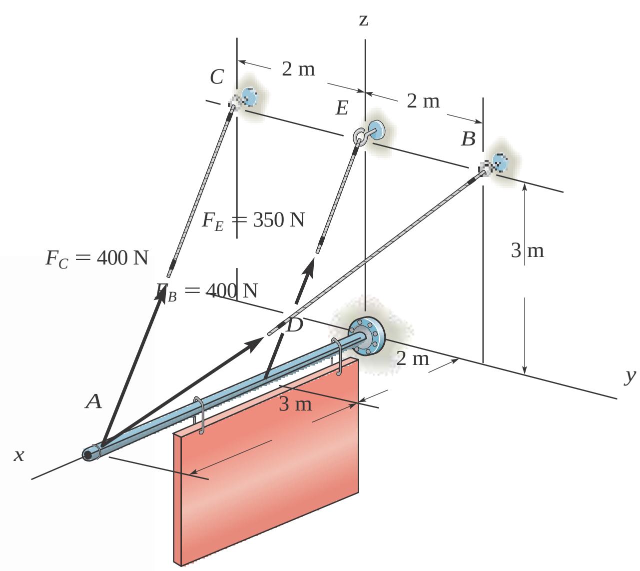
$$\mathbf{F}_{DA} = 400\left(\frac{\mathbf{r}_{DA}}{r_{DA}}\right) = 400\left(-\frac{2}{15.362}\mathbf{i} - \frac{6}{15.362}\mathbf{j} + \frac{14}{15.362}\mathbf{k}\right)$$

$$= \{-52.1\,\mathbf{i} - 156\,\mathbf{j} + 365\,\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F}_{BA} = \{-109\,\mathbf{i} + 131\,\mathbf{j} + 306\,\mathbf{k}\}$ lb
$\mathbf{F}_{CA} = \{103\,\mathbf{i} + 103\,\mathbf{j} + 479\,\mathbf{k}\}$ lb
$\mathbf{F}_{DA} = \{-52.1\,\mathbf{i} - 156\,\mathbf{j} + 365\,\mathbf{k}\}$ lb

2–66.

Represent each cable force as a Cartesian vector.



SOLUTION

$\mathbf{r}_C = (0 - 5)\mathbf{i} + (-2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\}$ m

$r_C = \sqrt{(-5)^2 + (-2)^2 + 3^2} = \sqrt{38}$ m

$\mathbf{r}_B = (0 - 5)\mathbf{i} + (2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\}$ m

$r_B = \sqrt{(-5)^2 + 2^2 + 3^2} = \sqrt{38}$ m

$\mathbf{r}_E = (0 - 2)\mathbf{i} + (0 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-2\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}\}$ m

$r_E = \sqrt{(-2)^2 + 0^2 + 3^2} = \sqrt{13}$ m

$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$

$\mathbf{F}_C = 400\left(\frac{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{38}}\right) = \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\}$ N **Ans.**

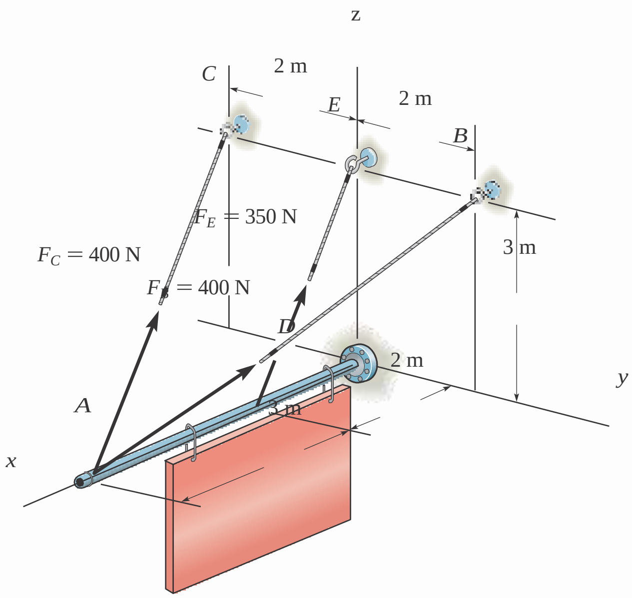
$\mathbf{F}_B = 400\left(\frac{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{\sqrt{38}}\right) = \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\}$ N **Ans.**

$\mathbf{F}_E = 350\left(\frac{-2\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}}{\sqrt{13}}\right) = \{-194\mathbf{i} + 291\mathbf{k}\}$ N **Ans.**

Ans:
$\mathbf{F}_C = \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\}$ N
$\mathbf{F}_B = \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\}$ N
$\mathbf{F}_E = \{-194\mathbf{i} + 291\mathbf{k}\}$ N

2–67.

Determine the magnitude and coordinate direction angles of the resultant force of the two forces acting at point A.



SOLUTION

$$\mathbf{r}_C = (0 - 5)\mathbf{i} + (-2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\}$$

$$r_C = \sqrt{(-5)^2 + (-2)^2 + (3)^2} = \sqrt{38}\text{ m}$$

$$\mathbf{F}_C = 400\left(\frac{\mathbf{r}_C}{r_C}\right) = 400\left(\frac{(-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})}{\sqrt{38}}\right)$$

$$\mathbf{F}_C = (-324.4428\mathbf{i} - 129.777\mathbf{j} + 194.666\mathbf{k})$$

$$\mathbf{r}_B = (0 - 5)\mathbf{i} + (2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\}$$

$$r_B = \sqrt{(-5)^2 + 2^2 + 3^2} = \sqrt{38}\text{ m}$$

$$\mathbf{F}_B = 400\left(\frac{\mathbf{r}_B}{r_B}\right) = 400\left(\frac{(-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})}{\sqrt{38}}\right)$$

$$\mathbf{F}_B = (-324.443\mathbf{i} + 129.777\mathbf{j} + 194.666\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_C + \mathbf{F}_B = (-648.89\mathbf{i} + 389.33\mathbf{k})$$

$$F_R = \sqrt{(-648.89)^2 + (389.33)^2 + 0^2} = 756.7242$$

$F_R = 757$ N **Ans.**

$\alpha = \cos^{-1}\left(\frac{-648.89}{756.7242}\right) = 149.03 = 149°$ **Ans.**

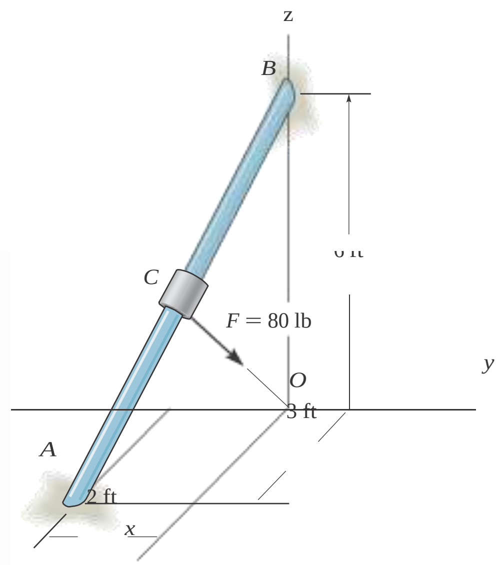
$\beta = \cos^{-1}\left(\frac{0}{756.7242}\right) = 90.0°$ **Ans.**

$\gamma = \cos^{-1}\left(\frac{389.33}{756.7242}\right) = 59.036 = 59.0°$ **Ans.**

Ans:
$F_R = 757$ N
$\alpha = 149°$
$\beta = 90.0°$
$\gamma = 59.0°$

***2–68.**

The force **F** has a magnitude of 80 lb and acts at the midpoint *C* of the rod. Express this force as a Cartesian vector.



SOLUTION

$$\mathbf{r}_{AB} = (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})$$

$$\mathbf{r}_{CB} = \frac{1}{2}\mathbf{r}_{AB} = (-1.5\mathbf{i} + 1\mathbf{j} + 3\mathbf{k})$$

$$\mathbf{r}_{CO} = \mathbf{r}_{BO} + \mathbf{r}_{CB}$$

$$= -6\mathbf{k} - 1.5\mathbf{i} + 1\mathbf{j} + 3\mathbf{k}$$

$$= -1.5\mathbf{i} + 1\mathbf{j} - 3\mathbf{k}$$

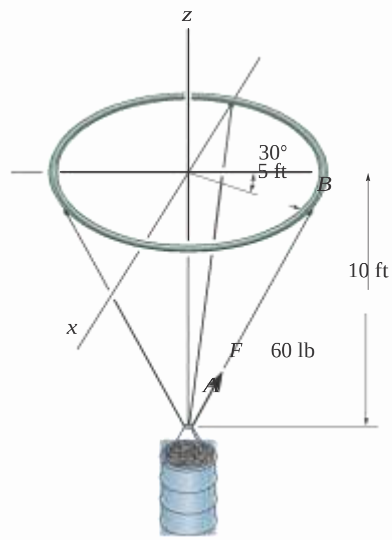
$$r_{CO} = 3.5$$

$$F = 80\left(\frac{\mathbf{r}_{CO}}{r_{CO}}\right) = \{-34.3\mathbf{i} + 22.9\mathbf{j} - 68.6\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$F = \{-34.3\mathbf{i} + 22.9\mathbf{j} - 68.6\mathbf{k}\}$ lb

2–69.

The load at *A* creates a force of 60 lb in wire *AB*. Express this force as a Cartesian vector.



SOLUTION

Unit Vector: First determine the position vector $\mathbf{r}_{AB}$. The coordinates of point B are

B (5 sin 30°, 5 cos 30°, 0) ft = B (2.50, 4.330, 0) ft

Then

$$\mathbf{r}_{AB} = 5(2.50 - 0)\mathbf{i} + (4.330 - 0)\mathbf{j} + [0 - (-10)]\mathbf{k}6 \text{ ft}$$

$$= 52.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}6 \text{ ft}$$

$$r_{AB} = 3\overline{2.50^2 + 4.330^2 + 10.0^2} = 11.180 \text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}}{11.180}$$

$$= 0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}$$

Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 60\,50.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}6 \text{ lb}$$

$$= 513.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}6 \text{ lb}$$ **Ans.**

Ans:

$$\mathbf{F} = \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\}\ \text{lb}$$

2–70.

Determine the magnitude and coordinate direction angles of the resultant force acting at point A on the post.

z

$F_{AC} = 150$ N A $F_{AB} = 200$ N

C 4 3 5 3 m 3 m y

O 2 m

4 m B

x

SOLUTION

Unit Vector. The coordinates for points A, B and C are $A(0, 0, 3)$ m, $B(2, 4, 0)$ m, and $C(-3, -4, 0)$ m, respectively.

$$\mathbf{r}_{AB} = (2 - 0)\mathbf{i} + (4 - 0)\mathbf{j} + (0 - 3)\mathbf{k} = \{2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}\} \text{ m}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}}{\sqrt{2^2 + 4^2 + (-3)^2}} = \frac{2}{\sqrt{29}}\mathbf{i} + \frac{4}{\sqrt{29}}\mathbf{j} - \frac{3}{\sqrt{29}}\mathbf{k}$$

$$\mathbf{r}_{AC} = (-3 - 0)\mathbf{i} + (-4 - 0)\mathbf{j} + (0 - 3)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}\} \text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (-3)^2}} = -\frac{3}{\sqrt{34}}\mathbf{i} - \frac{4}{\sqrt{34}}\mathbf{j} - \frac{3}{\sqrt{34}}\mathbf{k}$$

Force Vectors

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_{AB} = 200\left(\frac{2}{\sqrt{29}}\mathbf{i} + \frac{4}{\sqrt{29}}\mathbf{j} - \frac{3}{\sqrt{29}}\mathbf{k}\right)$$

$$= \{74.28\mathbf{i} + 148.56\mathbf{j} - 111.42\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 150\left(-\frac{3}{\sqrt{34}}\mathbf{i} - \frac{4}{\sqrt{34}}\mathbf{j} - \frac{3}{\sqrt{34}}\mathbf{k}\right)$$

$$= \{-77.17\mathbf{i} - 102.90\mathbf{j} - 77.17\mathbf{k}\} \text{ N}$$

Resultant Force

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$= \{74.28\mathbf{i} + 148.56\mathbf{j} - 111.42\mathbf{k}\} + \{-77.17\mathbf{i} - 102.90\mathbf{j} - 77.17\mathbf{k}\}$$

$$= \{-2.896\mathbf{i} + 45.66\mathbf{j} - 188.59\,\mathbf{k}\} \text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{(-2.896)^2 + 45.66^2 + (-188.59)^2}$$

$$= 194.06 \text{ N} = 194 \text{ N}$$ **Ans.**

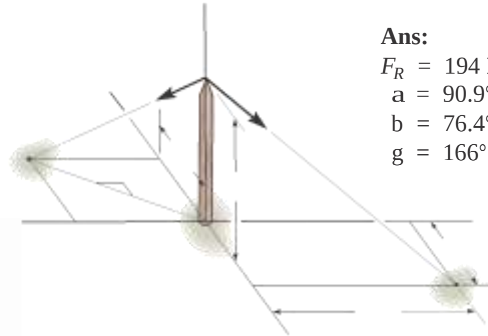
And its coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{-2.896}{194.06};\quad \alpha = 90.86° = 90.9°$$ **Ans.**

$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{45.66}{194.06};\quad \beta = 76.39° = 76.4°$$ **Ans.**

$$\frac{(F_R)_z}{} \quad \frac{-188.59}{}$$

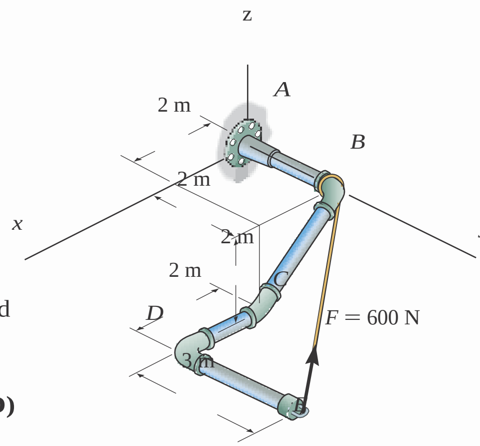
$\cos g = \frac{}{F_R} = \frac{}{194.06}$; $g = 166.36° = 166°$ **Ans.**



Ans:
$F_R = 194$ N
$a = 90.9°$
$b = 76.4°$
$g = 166°$

2–71.

Given the three vectors **A**, **B**, and **D**, show that $\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$.



SOLUTION

Since the component of $(\mathbf{B} + \mathbf{D})$ is equal to the sum of the components of **B** and **D**, then

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{D} \qquad \textbf{(QED)}$$

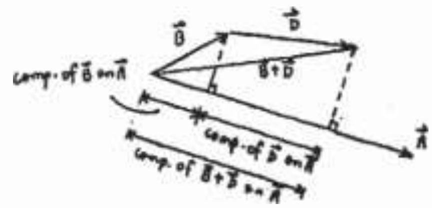
Also,

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\mathbf{k}) \cdot [(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}]$$

$$= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z)$$

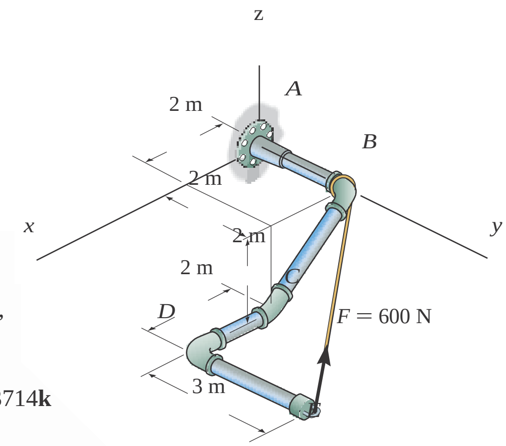
$$= (A_xB_x + A_yB_y + A_zB_z) + (A_xD_x + A_yD_y + A_zD_z)$$

$$= (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D}) \qquad \textbf{(QED)}$$



***2–72.**

Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



SOLUTION

Unit Vectors: The unit vectors $\mathbf{u}_{EB}$ and $\mathbf{u}_{ED}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{EB} = \frac{\mathbf{r}_{EB}}{r_{EB}} = \frac{(0-4)\mathbf{i} + (2-5)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-4)^2 + (2-5)^2 + [0-(-2)]^2}} = -0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}$$

$$\mathbf{u}_{ED} = -\mathbf{j}$$

Thus, the force vector **F** is given by

$$\mathbf{F} = F\mathbf{u}_{EB} = 600(-0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}) = [-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}]\ \text{N}$$

Vector Dot Product: The magnitude of the component of **F** parallel to segment DE of the pipe assembly is

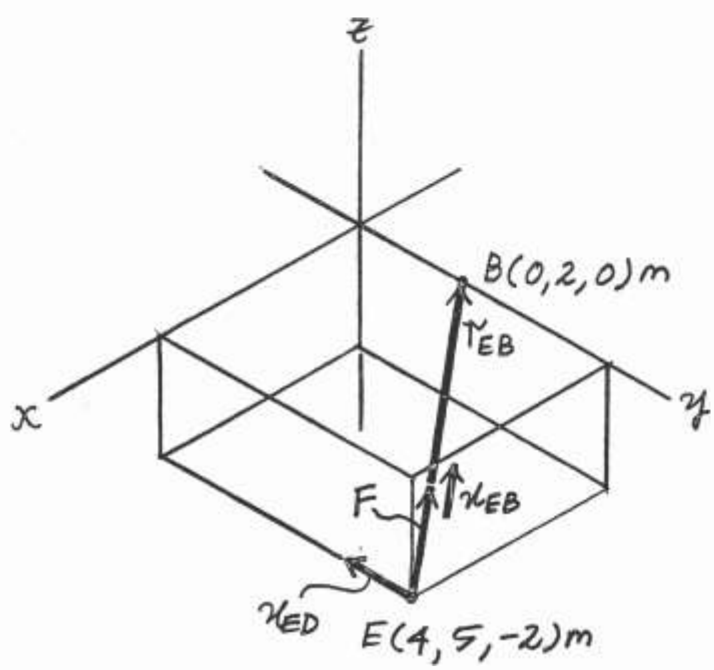
$$(F_{ED})_{paral} = \mathbf{F} \cdot \mathbf{u}_{ED} = (-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}) \cdot (-\mathbf{j})$$

$$= (-445.66)(0) + (-334.25)(-1) + (222.83)(0)$$

$$= 334.25 = 334\ \text{N} \qquad \textbf{Ans.}$$

The component of **F** perpendicular to segment DE of the pipe assembly is

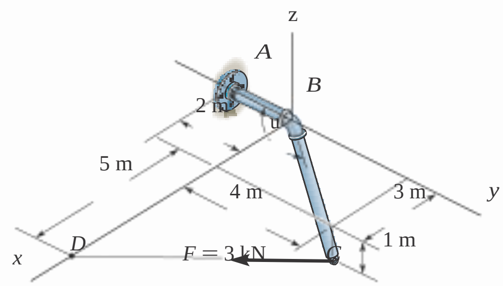
$$(F_{ED})_{per} = \sqrt{F^2 - (F_{ED})_{paral}^2} = \sqrt{600^2 - 334.25^2} = 498\ \text{N} \qquad \textbf{Ans.}$$



Ans:
$(F_{ED})_{]]} = 334$ N
$(F_{ED})_{\#} = 498$ N

2–73.

Determine the angle θ between BA and BC.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(0, -2, 0)$ m, $B(0, 0, 0)$ m and $C(3, 4, -1)$ m respectively. Thus, the unit vectors along BA and BC are

$$\mathbf{u}_{BA} = -\mathbf{j} \qquad \mathbf{u}_{BE} = \frac{(3-0)\,\mathbf{i} + (4-0)\,\mathbf{j} + (-1-0)\,\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (-1-0)^2}} = \frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{4}{\sqrt{26}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}$$

The Angle θ Between BA and BC.

$$\mathbf{u}_{BA} \cdot \mathbf{u}_{BC} = (-\mathbf{j}) \cdot \left(\frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{4}{\sqrt{26}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}\right)$$

$$= (-1)\left(\frac{4}{\sqrt{26}}\right) = -\frac{4}{\sqrt{26}}$$

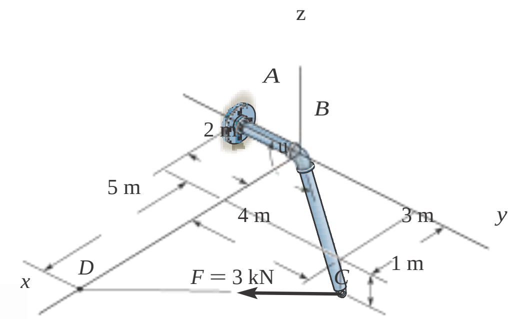
Then

$$\theta = \cos^{-1}(\mathbf{u}_{BA} \cdot \mathbf{u}_{BC}) = \cos^{-1}\left(-\frac{4}{\sqrt{26}}\right) = 141.67^\circ = 142^\circ$$ **Ans.**

Ans:
$0 = 142°$

2–74.

Determine the magnitude of the projected component of the 3 kN force acting along axis BC of the pipe.



SOLUTION

Unit Vectors. Here, the coordinates of points B, C and D are $B\,(0, 0, 0)$ m, $C(3, 4, -1)$ m and $D(8, 0, 0)$. Thus the unit vectors along BC and CD are

$$\mathbf{u}_{BC} = \frac{(3-0)\,\mathbf{i} + (4-0)\,\mathbf{j} + (-1-0)\,\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (-1-0)^2}} = \frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{4}{\sqrt{26}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}$$

$$\mathbf{u}_{CD} = \frac{(8-3)\,\mathbf{i} + (0-4)\,\mathbf{j} + [0-(-1)]\,\mathbf{k}}{\sqrt{(8-3)^2 + (0-4)^2 + [0-(-1)]^2}} = \frac{5}{\sqrt{42}}\,\mathbf{i} - \frac{4}{\sqrt{42}}\,\mathbf{j} + \frac{1}{\sqrt{42}}\,\mathbf{k}$$

Force Vector. For $\mathbf{F}$,

$$\mathbf{F} = F\mathbf{u}_{CD} = 3\left(\frac{5}{\sqrt{42}}\,\mathbf{i} - \frac{4}{\sqrt{42}}\,\mathbf{j} + \frac{1}{\sqrt{42}}\,\mathbf{k}\right)$$

$$= \left(\frac{15}{\sqrt{42}}\,\mathbf{i} - \frac{12}{\sqrt{42}}\,\mathbf{j} + \frac{3}{\sqrt{42}}\,\mathbf{k}\right)\text{ kN}$$

Projected Component of F. Along BC, it is

$$(F_{BC}) = \mathbf{F}\cdot\mathbf{u}_{BC} = \left(\frac{15}{\sqrt{42}}\,\mathbf{i} - \frac{12}{\sqrt{42}}\,\mathbf{j} + \frac{3}{\sqrt{42}}\,\mathbf{k}\right)\cdot\left(\frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{4}{\sqrt{26}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}\right)$$

$$= \left(\frac{15}{\sqrt{42}}\right)\left(\frac{3}{\sqrt{26}}\right) + \left(-\frac{12}{\sqrt{42}}\right)\left(\frac{4}{\sqrt{26}}\right) + \frac{3}{\sqrt{42}}\left(-\frac{1}{\sqrt{26}}\right)$$

$$= -\frac{6}{\sqrt{1092}} = -0.1816\text{ kN} = 0.182\text{ kN} \qquad \textbf{Ans.}$$

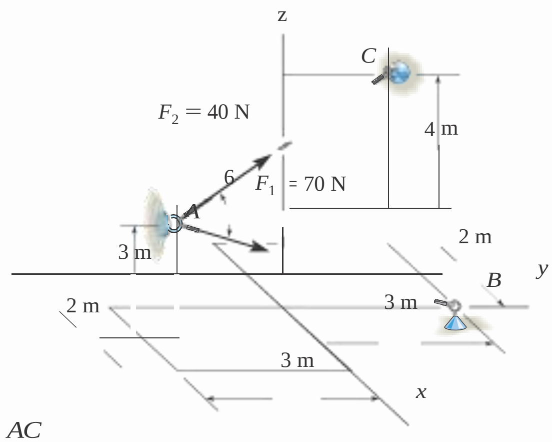
The negative signs indicate that this component points in the direction opposite to that of $\mathbf{u}_{BC}$.

Ans:

$(F_{BC}) = 0.182$ kN

2–75.

Determine the angle θ between the two cables.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(2, -3, 3)$ m, $B(0, 3, 0)$ and $C(-2, 3, 4)$ m, respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(0-2)^2 + [3-(-3)]^2 + (0-3)^2}} = -\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(-2-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (4-3)\mathbf{k}}{\sqrt{(-2-2)^2 + [3-(-3)]^2 + (4-3)^2}} = -\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}$$

The Angle θ Between AB and AC.

$$\mathbf{u}_{AB} \cdot \mathbf{u}_{AC} = \left(-\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\right) \cdot \left(-\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}\right)$$

$$= \left(-\frac{2}{7}\right)\left(-\frac{4}{\sqrt{53}}\right) + \frac{6}{7}\left(\frac{6}{\sqrt{53}}\right) + \left(-\frac{3}{7}\right)\left(\frac{1}{\sqrt{53}}\right)$$

$$= \frac{41}{7\sqrt{53}}$$

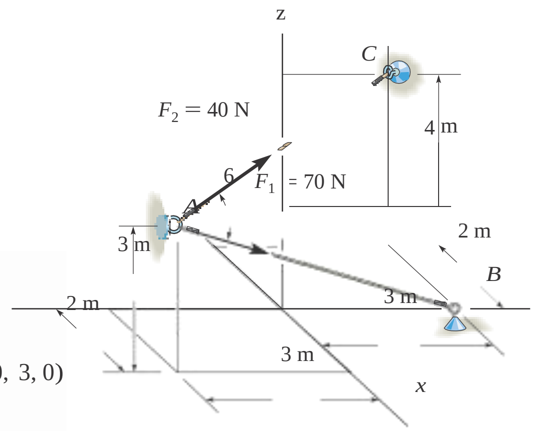
Then

$$\theta = \cos^{-1}(\mathbf{u}_{AB} \cdot \mathbf{u}_{AC}) = \cos^{-1}\left(\frac{41}{7\sqrt{53}}\right) = 36.43° = 36.4° \qquad \textbf{Ans.}$$

Ans:
$0 = 36.4°$

***2–76.**

Determine the magnitude of the projection of the force $\mathbf{F}_1$ along cable AC.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(2, -3, 3)$ m, $B(0, 3, 0)$ and $C(-2, 3, 4)$ m, respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(0-2)^2 + [3-(-3)]^2 + (0-3)^2}} = -\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(-2-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (4-3)\mathbf{k}}{\sqrt{(-2-2)^2 + [3-(-3)]^2 + (4-3)^2}} = -\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}$$

Force Vector, For $\mathbf{F}_1$,

$$\mathbf{F}_1 = F_1\mathbf{u}_{AB} = 70\left(-\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\right) = \{-20\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}\}\text{ N}$$

Projected Component of $\mathbf{F}_1$. Along AC, it is

$$(F_1)_{AC} = \mathbf{F}_1 \cdot \mathbf{u}_{AC} = (-20\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}) \cdot \left(-\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}\right)$$

$$= (-20)\left(-\frac{4}{\sqrt{53}}\right) + 60\left(\frac{6}{\sqrt{53}}\right) + (-30)\left(\frac{1}{\sqrt{53}}\right)$$

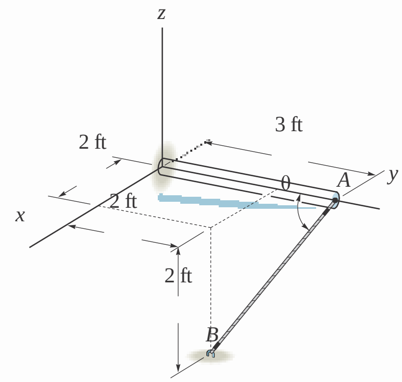
$$= 56.32\text{ N} = 56.3\text{ N}$$ **Ans.**

The positive sign indicates that this component points in the same direction as $\mathbf{u}_{AC}$.

Ans:
$(F_1)_{AC} = 56.3\text{ N}$

2–77.

Determine the angle 0 between the pole and the wire *AB*.



SOLUTION

Position Vector:

$$\mathbf{r}_{AC} = 5-3\mathbf{j}6 \text{ ft}$$

$$\mathbf{r}_{AB} = 512 - 02\mathbf{i} + 12 - 32\mathbf{j} + 1-2 - 02\mathbf{k}6 \text{ ft}$$

$$= 52\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}6 \text{ ft}$$

The magnitudes of the position vectors are

$$r_{AC} = 3.00 \text{ ft} \qquad r_{AB} = 22^2 + 1-12^2 + 1-22^2 = 3.00 \text{ ft}$$

The Angles Between Two Vectors U***:*** The dot product of two vectors must be determined first.

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 1-3\mathbf{j}2 \cdot 12\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}2$$

$$= 0122 + 1-321-12 + 01-22$$

$$= 3$$

Then,

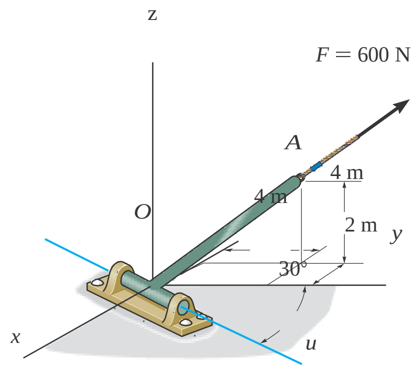
$$u = \cos^{-1}\left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AB}}{r_{AO}r_{AB}}\right) = \cos^{-1}\left[\frac{3}{3.00(3.00)}\right] = 70.5°$$ **Ans.**

Ans:
0 = 70.5°

2–78.

Determine the magnitude of the projection of force $F = 600$ N along the u axis.

the magnitude of the projection of the along the u is



SOLUTION

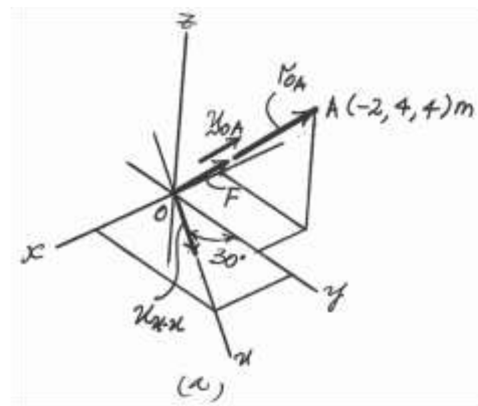
Unit Vectors: The unit vectors $\mathbf{u}_{OA}$ and $\mathbf{u}_u$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-2-0)\mathbf{i} + (4-0)\mathbf{j} + (4-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (4-0)^2 + (4-0)^2}} = -\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_u = \sin 30^\circ \mathbf{i} + \cos 30^\circ \mathbf{j}$$

Thus, the force vectors **F** is given by

$$\mathbf{F} = F\mathbf{u}_{OA} = 600\left(-\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}\}\text{ N}$$



Vector Dot Product: The magnitude of the projected component of **F** along the u axis is

$$F_u = \mathbf{F} \cdot \mathbf{u}_u = (-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}) \cdot (\sin 30^\circ \mathbf{i} + \cos 30^\circ \mathbf{j})$$

$$= (-200)(\sin 30^\circ) + 400(\cos 30^\circ) + 400(0)$$

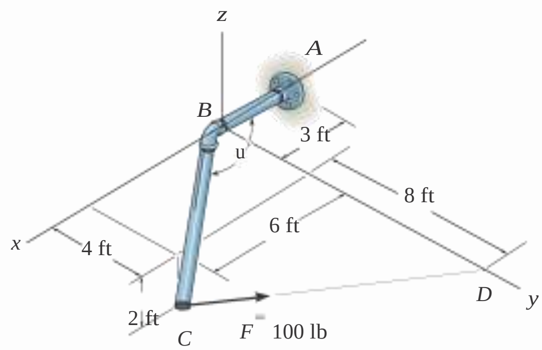
$$= 246 \text{ N}$$ **Ans.**

Ans:

$\mathbf{F}_u = 246$ N

2–79.

Determine the magnitude of the projected component of the 100-lb force acting along the axis *BC* of the pipe.



SOLUTION

$$\mathbf{r}_{BC} = \{6\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

$$\mathbf{F} = 100\frac{\{-6\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}\}}{\sqrt{(-6)^2 + 8^2 + 2^2}}$$

$$= \{-58.83\mathbf{i} + 78.45\mathbf{j} + 19.61\mathbf{k}\}\text{ lb}$$

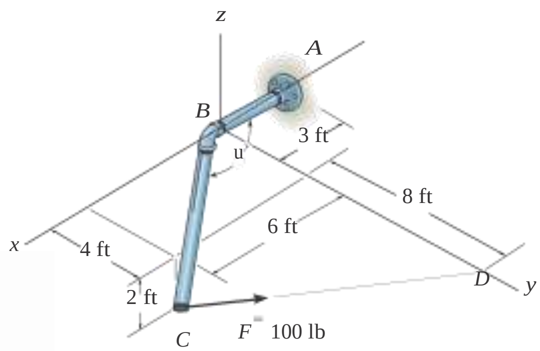
$$F_p = \mathbf{F}\cdot\mathbf{u}_{BC} = \mathbf{F}\cdot\frac{\mathbf{r}_{BC}}{|\mathbf{r}_{BC}|} = \frac{-78.45}{7.483} = -10.48$$

$$F_p = 10.5\text{ lb}$$ **Ans.**

Ans:
$F_p = 10.5$ lb

***2–80.**

Determine the angle θ between pipe segments BA and BC.



SOLUTION

$$\mathbf{r}_{BC} = \{6\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{BA} = \{-3\mathbf{i}\}\text{ ft}$$

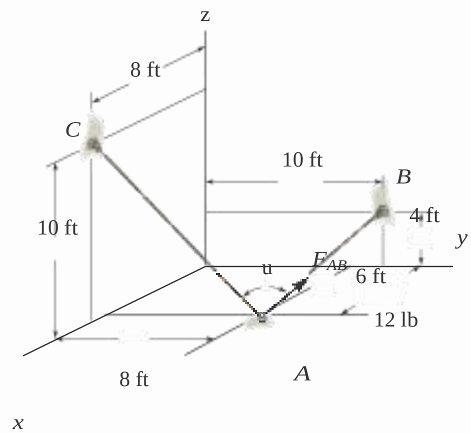
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{BC} \cdot \mathbf{r}_{BA}}{|\mathbf{r}_{BC}||\mathbf{r}_{BA}|}\right) = \cos^{-1}\left(\frac{-18}{22.45}\right)$$

$\theta = 143°$ **Ans.**

Ans:
$0 = 143°$

2–81.

Determine the angle 0 between the two cables.



SOLUTION

$$0 = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC}\, r_{AB}}\right)$$

$$= \cos^{-1}\left(\frac{(2\,\mathbf{i} - 8\,\mathbf{j} + 10\,\mathbf{k}) \cdot (-6\,\mathbf{i} + 2\,\mathbf{j} + 4\,\mathbf{k})}{\sqrt{2^2 + (-8)^2 + 10^2}\,\sqrt{(-6)^2 + 2^2 + 4^2}}\right)$$

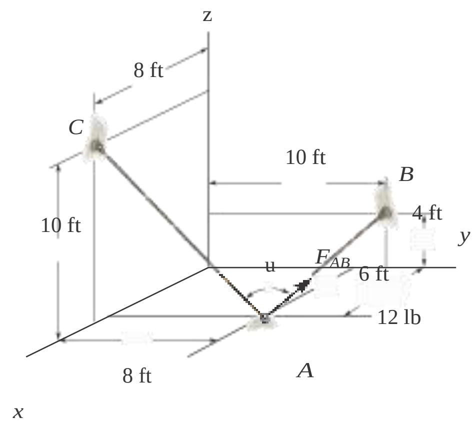
$$= \cos^{-1}\left(\frac{12}{96.99}\right)$$

$0 = 82.9°$ **Ans.**

Ans:
$0 = 82.9^\circ$

2–82.

Determine the projected component of the force acting in the direction of cable *AC*. Express the result as a Cartesian vector.



SOLUTION

$$\mathbf{r}_{AC} = \{2\,\mathbf{i} - 8\,\mathbf{j} + 10\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AB} = \{-6\,\mathbf{i} + 2\,\mathbf{j} + 4\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{F}_{AB} = 12\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 12\left(-\frac{6}{7.483}\,\mathbf{i} + \frac{2}{7.483}\,\mathbf{j} + \frac{4}{7.483}\,\mathbf{k}\right)$$

$$\mathbf{F}_{AB} = \{-9.621\,\mathbf{i} + 3.207\,\mathbf{j} + 6.414\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{u}_{AC} = \frac{2}{12.961}\,\mathbf{i} - \frac{8}{12.961}\,\mathbf{j} + \frac{10}{12.961}\,\mathbf{k}$$

$$\text{Proj } F_{AB} = \mathbf{F}_{AB} \cdot \mathbf{u}_{AC} = -9.621\left(\frac{2}{12.961}\right) + 3.207\left(-\frac{8}{12.961}\right) + 6.414\left(\frac{10}{12.961}\right)$$

$$= 1.4846$$

$$\text{Proj } \mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AC}$$

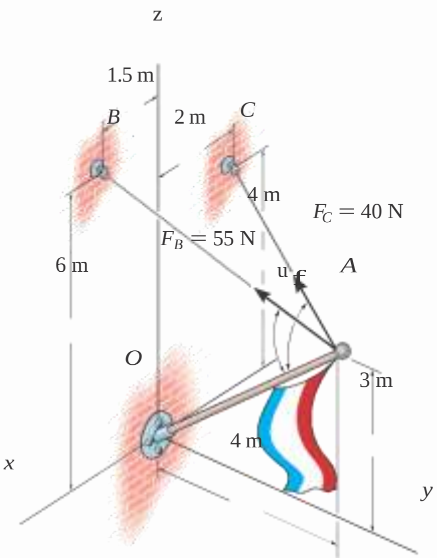
$$\text{Proj } \mathbf{F}_{AB} = (1.4846)\left(\frac{2}{12.962}\,\mathbf{i} - \frac{8}{12.962}\,\mathbf{j} + \frac{10}{12.962}\,\mathbf{k}\right)$$

$$\text{Proj } \mathbf{F}_{AB} = \{0.229\,\mathbf{i} - 0.916\,\mathbf{j} + 1.15\,\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
Proj $\mathbf{F}_{AB} = \{0.229\,\mathbf{i} - 0.916\,\mathbf{j} + 1.15\,\mathbf{k}\}$ lb

2–83.

Determine the angles θ and ϕ between the flag pole and the cables *AB* and *AC*.



SOLUTION

$\mathbf{r}_{AC} = \{-2\mathbf{i} - 4\mathbf{j} + 1\mathbf{k}\}$ m; $\quad r_{AC} = 4.58$ m

$\mathbf{r}_{AB} = \{1.5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\}$ m; $\quad r_{AB} = 5.22$ m

$\mathbf{r}_{AO} = \{-4\mathbf{j} - 3\mathbf{k}\}$ m; $\quad r_{AO} = 5.00$

$\mathbf{r}_{AB} \cdot \mathbf{r}_{AO} = (1.5)(0) + (-4)(-4) + (3)(-3) = 7$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AO}}{r_{AB}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{7}{5.22(5.00)}\right) = 74.4^\circ \qquad \textbf{Ans.}$$

$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (-2)(0) + (-4)(-4) + (1)(-3) = 13$

$$\phi = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{13}{4.58(5.00)}\right) = 55.4^\circ \qquad \textbf{Ans.}$$

Ans:
0 = 74.4°
f = 55.4°

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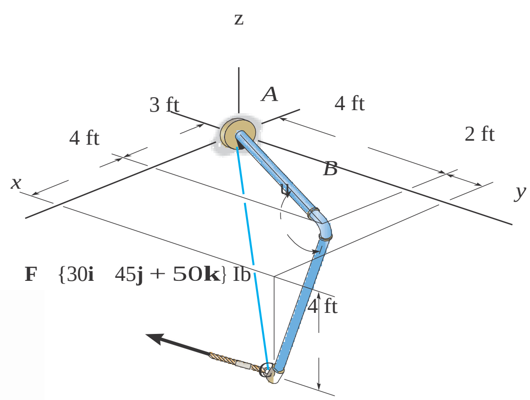
***2–84.**

Determine the magnitudes of the components of **F** acting along and perpendicular to segment *BC* of the pipe assembly.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{CB}$ must be determined first. From Fig. *a*,

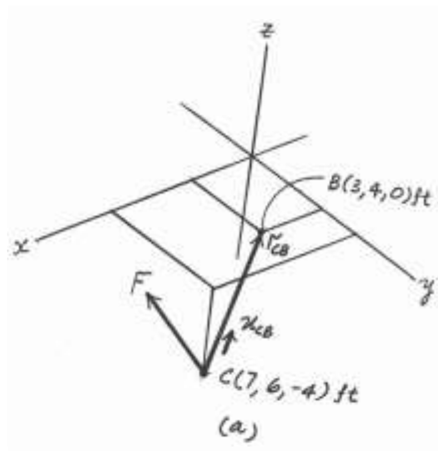
$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CB}}{r_{CB}} = \frac{(3-7)\mathbf{i} + (4-6)\mathbf{j} + [0-(-4)]\mathbf{k}}{\sqrt{(3-7)^2 + (4-6)^2 + [0-(-4)]^2}} = -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** parallel to segment *BC* of the pipe assembly is

$$(F_{BC})_{pa} = \mathbf{F} \cdot \mathbf{u}_{CB} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k}) \cdot \left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$$= (30)\left(-\frac{2}{3}\right) + (-45)\left(-\frac{1}{3}\right) + 50\left(\frac{2}{3}\right)$$

$$= 28.33 \text{ lb} = 28.3 \text{ lb}$$ **Ans.**



The magnitude of **F** is $F = \sqrt{30^2 + (-45)^2 + 50^2} = \sqrt{5425}$ lb. Thus, the magnitude of the component of **F** perpendicular to segment *BC* of the pipe assembly can be determined from

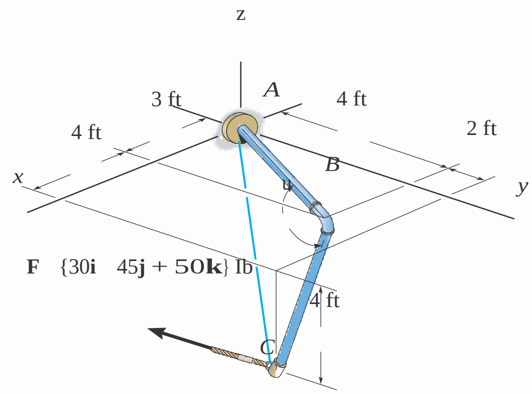
$$(F_{BC})_{per} = \sqrt{F^2 - (F_{BC})_{pa}^2} = \sqrt{5425 - 28.33^2} = 68.0 \text{ lb}$$ **Ans.**

Ans:

$$(F_{BC})_{]]} = 28.3 \text{ lb}$$
$$(F_{BC})_{\#} = 68.0 \text{ lb}$$

2–85.

Determine the magnitude of the projected component of **F** along line *AC*. Express this component as a Cartesian vector.



SOLUTION

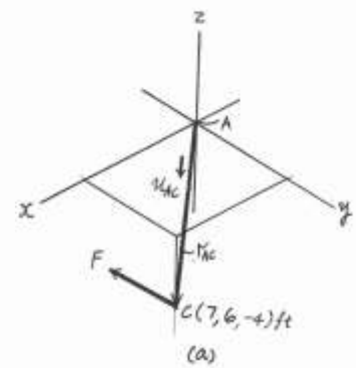
Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AC} = \frac{(7-0)\mathbf{i} + (6-0)\mathbf{j} + (-4-0)\mathbf{k}}{\sqrt{(7-0)^2 + (6-0)^2 + (-4-0)^2}} = 0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line *AC* is

$$F_{AC} = \mathbf{F} \cdot \mathbf{u}_{AC} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k}) \cdot (0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

$$= (30)(0.6965) + (-45)(0.5970) + 50(-0.3980)$$
$$= 25.87 \text{ lb}$$ **Ans.**



Thus, $\mathbf{F}_{AC}$ expressed in Cartesian vector form is

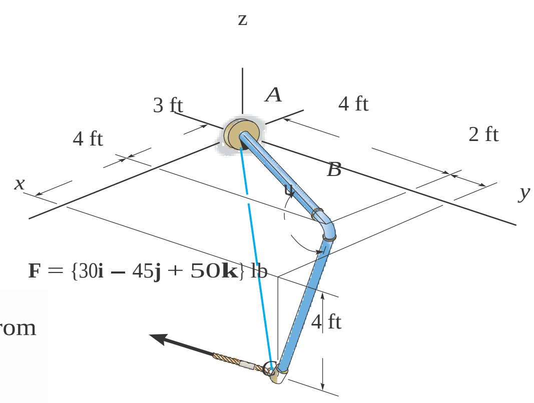
$$\mathbf{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = -25.87(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

$$= \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\} \text{ lb}$$ **Ans.**

Ans:
F_{AC} = 25.87 lb
$\mathbf{F}_{AC} = \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\}$ lb

2–86.

Determine the angle θ between the pipe segments *BA* and *BC*.



SOLUTION

Position Vectors: The position vectors $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ must be determined first. From Fig. *a*,

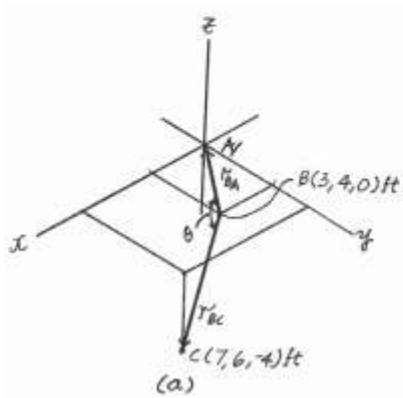
$$\mathbf{r}_{BA} = (0-3)\mathbf{i} + (0-4)\mathbf{j} + (0-0)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_{BC} = (7-3)\mathbf{i} + (6-4)\mathbf{j} + (-4-0)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}\text{ ft}$$

The magnitude of $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ are

$$r_{BA} = \sqrt{(-3)^2 + (-4)^2} = 5\text{ ft}$$

$$r_{BC} = \sqrt{4^2 + 2^2 + (-4)^2} = 6\text{ ft}$$



Vector Dot Product:

$$\mathbf{r}_{BA} \cdot \mathbf{r}_{BC} = (-3\mathbf{i} - 4\mathbf{j}) \cdot (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

$$= (-3)(4) + (-4)(2) + 0(-4)$$

$$= -20\text{ ft}^2$$

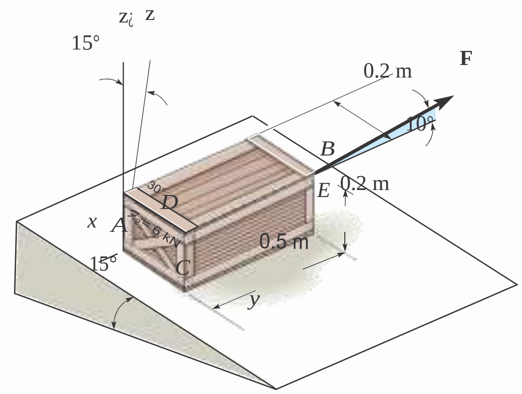
Thus,

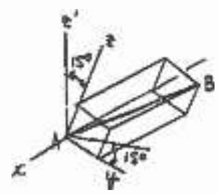
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{r_{BA}\, r_{BC}}\right) = \cos^{-1}\left[\frac{-20}{5(6)}\right] = 132^\circ$$ **Ans.**

Ans:
$0 = 132°$

2–87.

If the force $F = 100$ N lies in the plane *DBEC*, which is parallel to the x–z plane, and makes an angle of 10° with the extended line *DB* as shown, determine the angle that **F** makes with the diagonal *AB* of the crate.





SOLUTION

Use the x, y, z axes.

$$\mathbf{u}_{AB} = \left(\frac{-0.5\mathbf{i} + 0.2\mathbf{j} + 0.2\mathbf{k}}{0.57446}\right)$$

$$= -0.8704\mathbf{i} + 0.3482\mathbf{j} + 0.3482\mathbf{k}$$

$$\mathbf{F} = -100\cos 10°\mathbf{i} + 100\sin 10°\mathbf{k}$$

$$0 = \cos^{-1}\left(\frac{\mathbf{F}\cdot\mathbf{u}_{AB}}{F\,u_{AB}}\right)$$

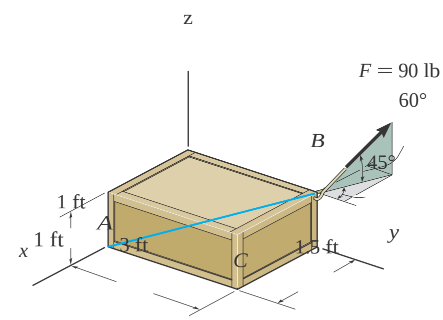
$$= \cos^{-1}\left(\frac{-100(\cos 10°)(-0.8704) + 0 + 100\sin 10°\,(0.3482)}{100(1)}\right)$$

$$= \cos^{-1}(0.9176) = 23.4°$$ **Ans.**

Ans:
$0 = 23.4°$

***2–88.**

Determine the magnitudes of the components of force $F = 90$ lb acting parallel and perpendicular to diagonal AB of the crate.



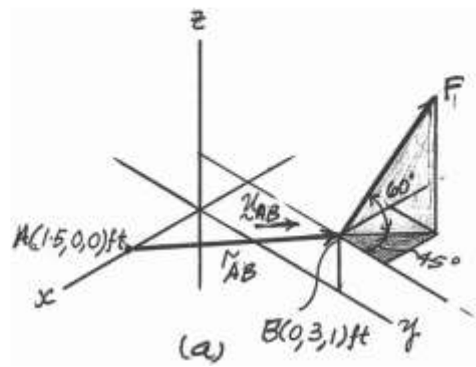
SOLUTION

Force and Unit Vector: The force vector **F** and unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*,

$$\mathbf{F} = 90(-\cos 60° \sin 45°\mathbf{i} + \cos 60° \cos 45°\mathbf{j} + \sin 60°\mathbf{k})$$

$$= \{-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}\} \text{ lb}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(0-1.5)\mathbf{i} + (3-0)\mathbf{j} + (1-0)\mathbf{k}}{\sqrt{(0-1.5)^2 + (3-0)^2 + (1-0)^2}} = -\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$



Vector Dot Product: The magnitude of the projected component of **F** parallel to the diagonal AB is

$$[(F)_{AB}]_{pa} = \mathbf{F} \cdot \mathbf{u}_{AB} = (-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}) \cdot \left(-\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

$$= (-31.82)\left(-\frac{3}{7}\right) + 31.82\left(\frac{6}{7}\right) + 77.94\left(\frac{2}{7}\right)$$

$$= 63.18 \text{ lb} = 63.2 \text{ lb} \qquad \textbf{Ans.}$$

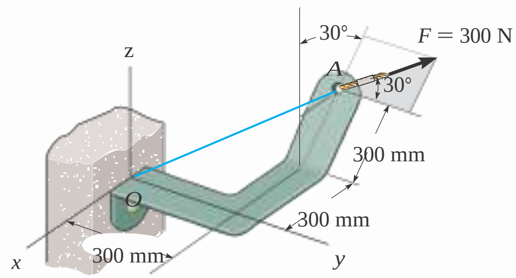
The magnitude of the component **F** perpendicular to the diagonal AB is

$$[(F)_{AB}]_{per} = \sqrt{F^2 - [(F)_{AB}]_{pa}^2} = \sqrt{90^2 - 63.18^2} = 64.1 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
3$(F)_{AB}$4]] = 63.2 lb
3$(F)_{AB}$4# = 64.1 lb

2–89.

Determine the magnitudes of the projected components of the force acting along the x and y axes.



SOLUTION

Force Vector: The force vector **F** must be determined first. From Fig. *a*,

$$\mathbf{F} = -300 \sin 30° \sin 30° \mathbf{i} + 300 \cos 30° \mathbf{j} + 300 \sin 30° \cos 30° \mathbf{k}$$

$$= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}] \text{ N}$$

Vector Dot Product: The magnitudes of the projected component of **F** along the x and y axes are

$$F_x = \mathbf{F} \cdot \mathbf{i} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot \mathbf{i}$$

$$= -75(1) + 259.81(0) + 129.90(0)$$

$$= -75 \text{ N}$$

$$F_y = \mathbf{F} \cdot \mathbf{j} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot \mathbf{j}$$

$$= -75(0) + 259.81(1) + 129.90(0)$$

$$= 260 \text{ N}$$

The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative x axis. Thus

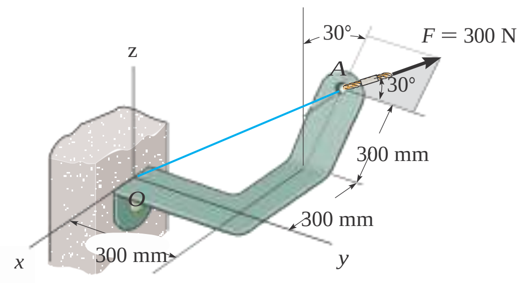
$$F_x = 75 \text{ N}, \quad F_y = 260 \text{ N}$$ **Ans.**

Ans:

$F_x = 75\text{ N}$
$F_y = 260\text{ N}$

2–90.

Determine the magnitude of the projected component of the force acting along line *OA*.



SOLUTION

Force and Unit Vector: The force vector **F** and unit vector $\mathbf{u}_{OA}$ must be determined first. From Fig. a,

$$\mathbf{F} = (-300 \sin 30° \sin 30°\mathbf{i} + 300 \cos 30°\mathbf{j} + 300 \sin 30° \cos 30°\mathbf{k})$$

$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\}\text{ N}$$

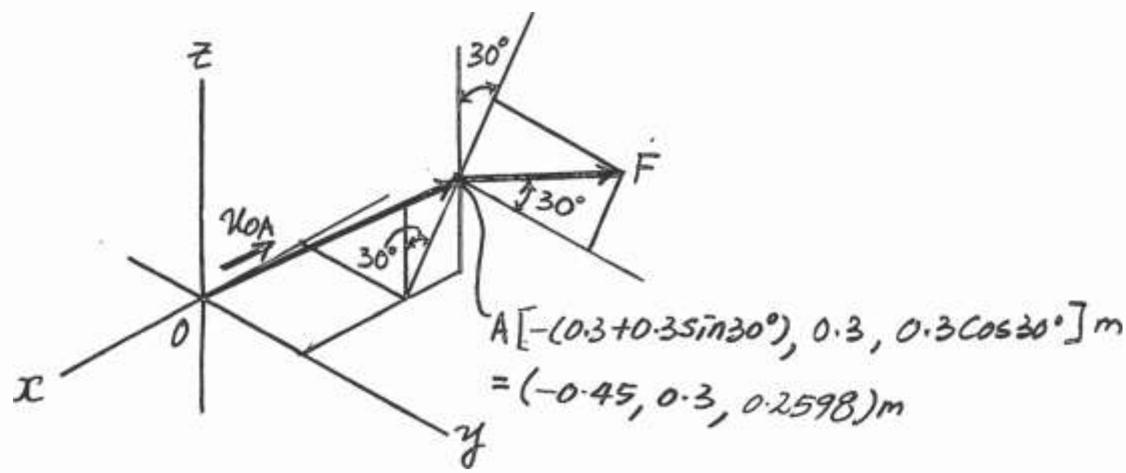
$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.2598 - 0)\mathbf{k}}{\sqrt{(-0.45 - 0)^2 + (0.3 - 0)^2 + (0.2598 - 0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line *OA* is

$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot (-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k})$$

$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$

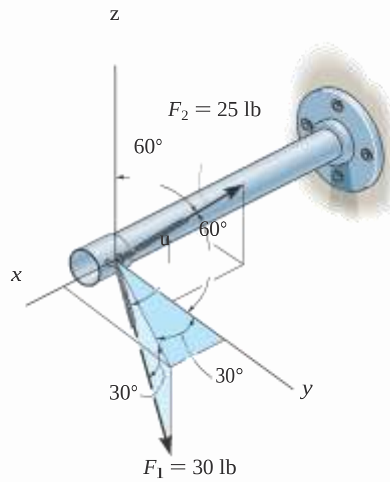
$$= 242\text{ N}$$ **Ans.**



Ans:
$F_{OA} = 242$ N

2–91.

Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



SOLUTION

Force Vector:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30°\mathbf{i} + \cos 30° \cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{F}_1 = F_R\mathbf{u}_{F_1} = 30(0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \text{ lb}$$

$$= \{12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}\} \text{ lb}$$

Unit Vector: One can obtain the angle $a = 135°$ for $\mathbf{F}_2$ using Eq. 2–8. $\cos^2 a + \cos^2 b + \cos^2 g = 1$, with $b = 60°$ and $g = 60°$. The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k} = -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

***Projected Component of* $\mathbf{F}_1$ *Along the Line of Action of* $\mathbf{F}_2$:**

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

$$= (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5)$$

$$= -5.44 \text{ lb}$$

Negative sign indicates that the projected component of $(F_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

The magnitude is $(F_1)_{F_2} = 5.44$ lb **Ans.**

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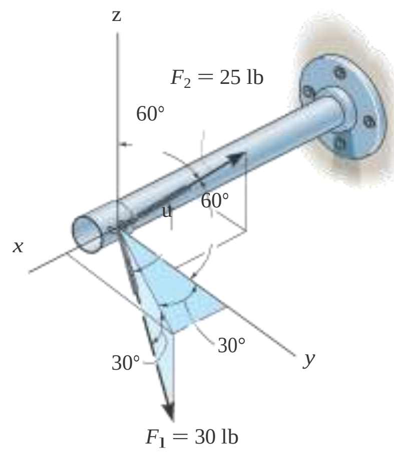
Ans:
The magnitude is $(F_1)_{F_2} = 5.44$ lb

***2–92.**

Determine the angle θ between the two forces.



SOLUTION

Unit Vectors:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30° \mathbf{i} + \cos 30° \cos 30° \mathbf{j} - \sin 30° \mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 135° \mathbf{i} + \cos 60° \mathbf{j} + \cos 60° \mathbf{k}$$

$$= -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

The Angles Between Two Vectors θ:

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

$$= 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5)$$

$$= -0.1812$$

Then,

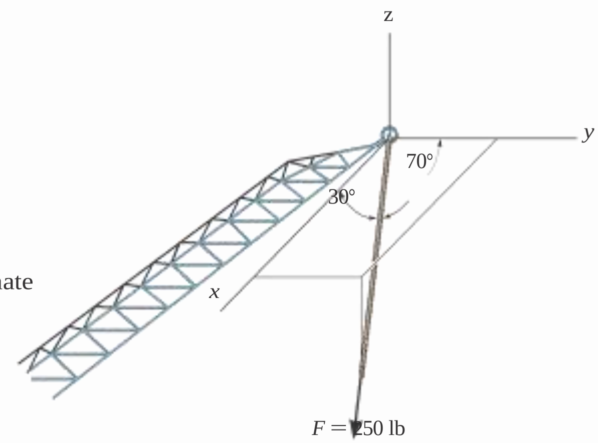
$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1812) = 100°$$ **Ans.**

Ans:

$0 = 100°$

***R2–4.**

The cable exerts a force of 250 lb on the crane boom as shown. Express this force as a Cartesian vector.



SOLUTION

Cartesian Vector Notation: With $\alpha = 30°$ and $\beta = 70°$, the third coordinate direction angle γ can be determined using Eq. 2–8.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2 30° + \cos^2 70° + \cos^2\gamma = 1$$

$$\cos\gamma = \pm 0.3647$$

$$\gamma = 68.61° \text{ or } 111.39°$$

By inspection, $\gamma = 111.39°$ since the force **F** is directed in negative octant.

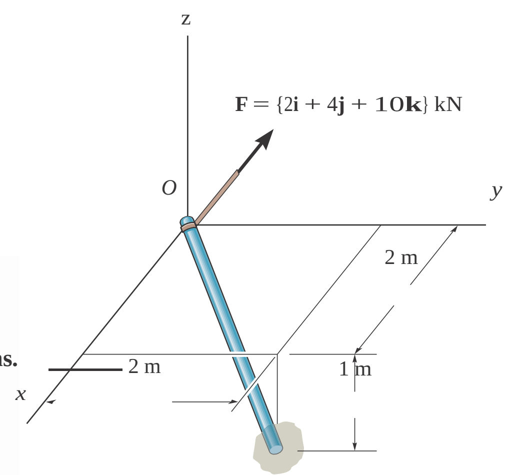
$$\mathbf{F} = 250\{\cos 30°\mathbf{i} + \cos 70°\mathbf{j} + \cos 111.39°\} \text{ lb}$$

$$= \{217\mathbf{i} + 85.5\mathbf{j} - 91.2\mathbf{k}\} \text{ lb}$$ **Ans.**

Ans:
$\mathbf{F} = \{217\mathbf{i} + 85.5\mathbf{j} - 91.2\mathbf{k}\}$ lb

***R2–8.**

Determine the projection of the force **F** along the pole.



SOLUTION

$\text{Proj } F = \mathbf{F} \cdot \mathbf{u}_a = (2\,\mathbf{i} + 4\,\mathbf{j} + 10\,\mathbf{k}) \cdot \left(\frac{2}{3}\,\mathbf{i} + \frac{2}{3}\,\mathbf{j} - \frac{1}{3}\,\mathbf{k}\right)$

$\text{Proj } F = 0.667 \text{ kN}$ **Ans.**

Ans:
$F = 0.667 \text{ kN}$

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